MATH 10103 TECHNICAL MATHEMATICS



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MATH 1013 Technical Mathematics

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1: NUMBER SENSE

The Number Sense Unit builds the foundation for understanding how different types of numbers are used in technical fields. In this unit, we will review whole numbers and integers, and extend our understanding to fractions, decimals, scientific notation, and radicals such as square roots and cube roots. Emphasis will be placed on interpreting, comparing, and applying these numbers in practical situations related to technical programs. Developing strong number sense will help you work confidently with measurements, calculations, and real-world problem solving throughout this course.

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1.1: WHOLE NUMBERS AND INTEGERS

Numbers are used to describe quantity, direction, and change. Whole numbers are used for counting and measuring amounts, while integers extend this idea to include values below zero. Together, they provide a foundation for working with many types of real-world data.

These ideas show up in everyday and technical situations. Whether tracking inventory, measuring distance, or recording gains and losses, understanding how numbers behave helps ensure that calculations make sense and results are interpreted correctly.

In this module, the focus is on building a strong understanding of whole numbers and integers and how they are used in context. You will represent, compare, and order values, work with different forms of whole numbers, and explore how integers behave on a number line, including opposites and absolute value.

You will also use a calculator to perform operations with integers and interpret your results in a way that reflects the situation being modeled.

Where do Math Symbols Come From

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View this YouTube Video online

This video provides a brief look at how common math symbols developed over time and how they are used to represent numbers and operations.

1.1.1: Understanding Whole Numbers

1.1.2: Prime Factorization and Exponential Form

1.1.3: Understanding Integers

1.1.4: Calculator Skills Involving Integers

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1.1.1: Understanding Whole Numbers

Use Place Value with Whole Numbers

The most basic numbers used in algebra are the numbers we use to count objects in our world: $1, 2, 3, 4$, and so on. These are called the **counting numbers**. Counting numbers are also called *natural numbers*. If we add zero to the counting numbers, we get the set of **whole numbers**.

- Counting Numbers: $1, 2, 3, \ldots$
- Whole Numbers: $0, 1, 2, 3, \ldots$

The notation "…" is called ellipsis and means "and so on," or that the pattern continues endlessly.

We can visualize counting numbers and whole numbers on a **number line** (see Figure $1.1.1.1$).

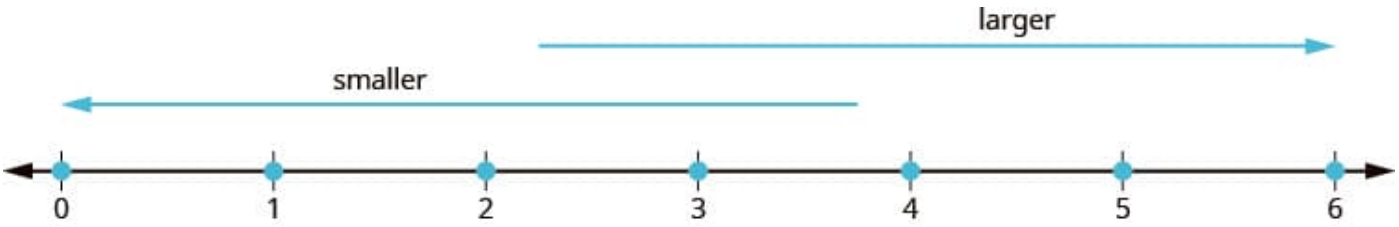


Figure $1.1.1.1$: The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left. While this number line shows only the whole numbers 0 through 6, the numbers keep going without end.

Our **number system** is called a place value system, because the value of a digit depends on its position in a number. Figure $1.1.1.2$ shows the **place values**. The place values are separated into groups of three, which are called periods. The periods are *ones, thousands, millions, billions, trillions*, and so on. In a written number, commas separate the periods.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure $1.1.1.2$: The number 5278194 is shown in the chart. The digit 5 is in the millions place. The digit 2 is in the hundred-thousands place. The digit 7 is in the ten-thousands place. The digit 8 is in the thousands place. The digit 1 is in the hundreds place. The digit 9 is in the tens place. The digit 4 is in the ones place.

Example 1

In the number 63407218, find the place value of each digit:

1. 7
2. 0
3. 1
4. 6
5. 3

Solution

Place the number in the place value chart:

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

1. The 7 is in the thousands place.
2. The 0 is in the ten thousands place.
3. The 1 is in the tens place.
4. The 6 is in the ten-millions place.
5. The 3 is in the millions place.

Exercise

For the number 27493615, find the place value of each digit:

1. 2
2. 1
3. 4
4. 7
5. 5

Answer available at the end of this section.

Exercise

For the number 519711641328, find the place value of each digit:

1. 9
2. 4
3. 2
4. 6
5. 7

Answer available at the end of this section.

Identify Place Values of Whole Numbers

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This video demonstrates two examples of identifying the place value of a specific digit within a number by analyzing its position and determining its value.

Word Form and Standard Form

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period, followed by the name of the period, without the s at the end. Start at the left, where the periods have the largest value. The ones period is not named. The commas separate the periods, so wherever there is a comma in the number, put a comma between the words (see Figure 1.1.1.3). The number 74218369 is written as seventy-four million, two hundred eighteen thousand, three hundred sixty-nine.

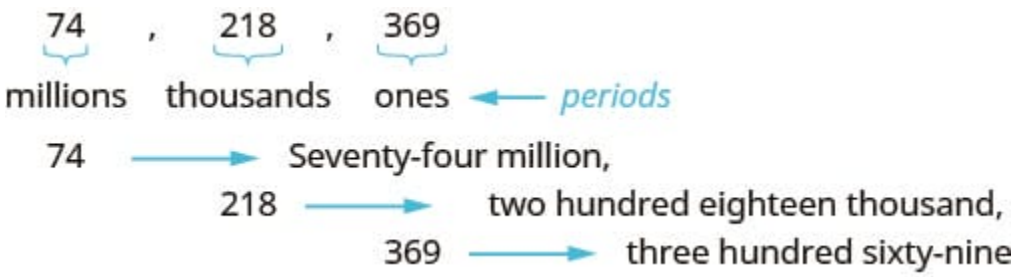


Figure 1.1.1.3

Write a Whole Number in Word Form

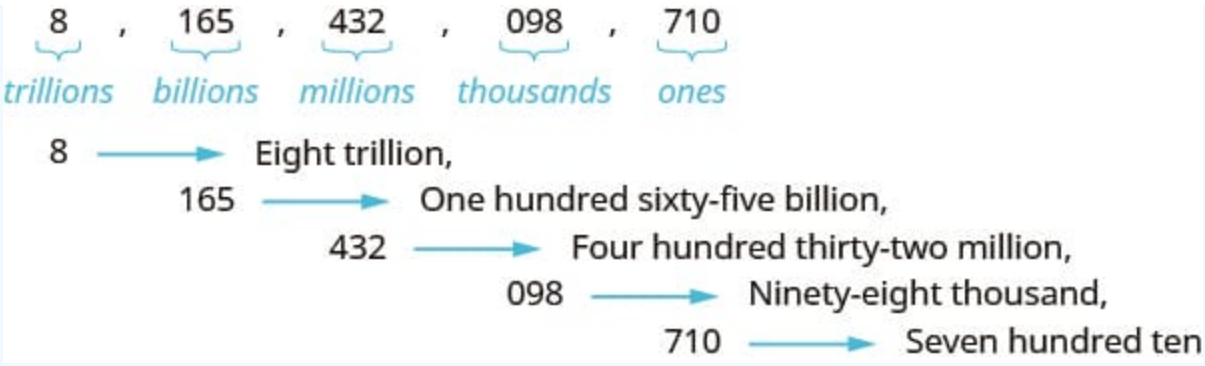
1. Start at the left and name the number in each period, followed by the period name.
2. Put commas in the number to separate the periods.
3. Do not name the ones period.

Example 2

Name the number 8165432098710 using words.

Solution

Name the number in each period, followed by the period name.



Put the commas in to separate the periods.

So, 8165432098710 is named as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Exercise

Name the number 9,258,137,904,061 using words.

Answer available at the end of this section.

Exercise

Name the number 17,864,325,619,004 using words.

Answer available at the end of this section.

Write Word Names for Large Whole Numbers

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This video walks through two examples of converting numbers from standard form into written form by carefully expressing each digit's value in words.

We are now going to reverse the process by writing the digits from the name of the number. To write the number in standard form, we first look for the clue words that indicate the periods. It is helpful to draw three blanks for the needed periods and then fill in the blanks with the numbers, separating the periods with commas.

Write a Whole Number In Standard Form

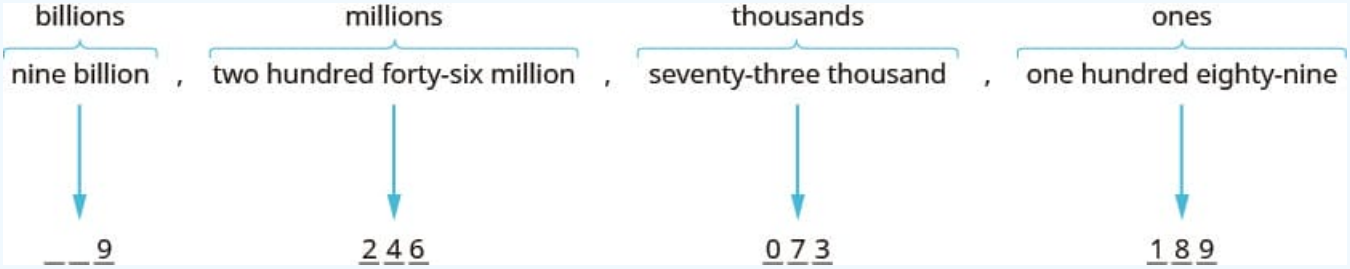
1. Identify the words that indicate periods. (Remember, the ones period is never named.)
2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
3. Name the number in each period and place the digits in the correct place value position.

Example 3

Write *nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine* in standard form.

Solution

Identify the words that indicate periods.
Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Then write the digits in each period.



The number is 9,246,073,189.

Exercise

Write the number two billion, four hundred sixty-six million, seven hundred fourteen thousand, fifty-one in standard form.

Answer available at the end of this section.

Exercise

Write the number eleven billion, nine hundred twenty-one million, eight hundred thirty thousand, one hundred six in standard form.

Answer available at the end of this section.

Number Words to Place Value

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View this YouTube Video online

In this video there are two examples of how to convert written form to standard form.

Expanded Notation

The whole numbers

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

are called *digits* and are used to construct larger whole numbers. For example, consider the whole number 222 (pronounced "two hundred twenty two"). It is made up of three twos, but the position of each two describes a different meaning or value.

2	2	2
hundreds	tens	ones

- The first two is in the "hundreds" position and represents two hundreds or 200.
- The second two is in the "tens" position and represents two tens or 20.
- The third two is in the "ones" position and represents two ones or 2.

Consider the larger number 123,456,789. The following table shows the place value of each digit.

1	2	3	4	5	6	7	8	9
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
millions			thousands			ones		

In "expanded notation," we would write

> 1 hundred million + 2 ten millions + 3 millions + 4 hundred thousands + 5 ten thousands + 6 thousands + 7 hundreds + 8 tens + 9 ones.

We read the numeral 123,456,789 as "one hundred twenty three million, four hundred fifty six thousand, seven hundred eighty nine."

Let's look at another example.

Example 4

Write the number 23,712 in expanded notation, then pronounce the result.

Solution

In expanded notation, 23,712 becomes

> 2 ten thousands + 3 thousands + 7 hundreds + 1 ten + 2 ones.

This is pronounced "twenty three thousand, seven hundred twelve."

Example 5

Write the number 203,405 in expanded notation, then pronounce the result.

Solution

In expanded notation, 203,405 becomes

2 hundred thousands + 0 ten thousands + 3 thousands + 4 hundreds + 0 tens + 5 ones.

Since 0 ten thousands is zero and 0 tens is also zero, this can also be written

2 hundred thousands + 3 thousands + 4 hundreds + 5 ones.

This is pronounced “two hundred three thousand, four hundred five.”

Exercise

Write the number 430,705 in expanded notation. Pronounce the result.

Answer available at the end of this section.

Whole Numbers: Place Value and Expanded Form (Notation)

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains place value and how to write whole numbers in expanded form.

Graphing Numbers on the Number Line

We can represent whole numbers on a **number line** by matching each number to a point. First, draw a number line, then set a tick mark at zero.



The next step is to declare a unit length.



The remainder of the whole numbers now fall easily in place on the number line.



When we are asked to graph a whole number on a number line, we shade in a solid dot at the position on the number line that corresponds to the given whole number.

Example 6

Graph the whole numbers 1, 3, and 5 on the number line.

Solution

Shade the numbers 1, 3, and 5 on the number line as solid dots.

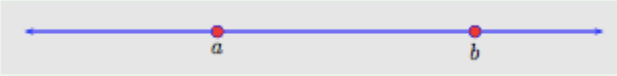


Ordering Whole Numbers

Now that we can represent whole numbers on a number line, we can compare and order them. As we move to the left, the numbers get smaller, and as we move to the right, the numbers get larger. This leads to the following idea.

Ordering the Whole Numbers

Suppose that a and b are whole numbers located on the number line so that the point representing the whole number a lies to the left of the point representing the whole number b.



Then the whole number a is "less than" the whole number b and write

$$a < b.$$

Alternatively, we can also say that the whole number b is "greater than" the whole number a and write

$$b > a.$$

Comparison Property

When comparing two whole numbers a and b, only one of three possibilities is true:

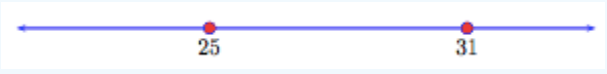
$$a < b \text{ or } a = b \text{ or } a > b.$$

Example 7

Compare the whole numbers 25 and 31.

Solution

On the number line, 25 is located to the left of 31.



Therefore, 25 is less than 31 and write $25 < 31$. Alternatively, we could also note that 31 is located to the right of 25. Therefore, 31 is greater than 25 and write $31 > 25$.

Exercise

Compare the whole numbers 18 and 12.

Answer available at the end of this section.

Comparing Whole Numbers

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View this YouTube Video online

This video explains how to compare whole numbers using a number line and the symbols <, >, and =.

Answers

Exercise

1. ten millions
2. tens
3. hundred thousands
4. millions
5. ones

Exercise

1. billions
2. ten thousands
3. tens
4. hundred thousands
5. hundred millions

Exercise

nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Exercise

seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand four

Exercise

2,466,714,051

Exercise

11,921,830,106

Exercise

4 hundred thousand + 3 thousand + 7 hundreds + 5 ones

"four hundred three thousand, seven hundred five"

Exercise

18 > 12.

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1.1.2: Prime Factorization and Exponential Form

In the statement $3 \cdot 4 = 12$, the number 12 is called the *product*, while 3 and 4 are called *factors*.

Example 1

Find all whole number factors of 18.

Solution

We need to find all whole number pairs whose product equals 18. The following pairs come to mind.

$$1 \cdot 18 = 18 \text{ and } 2 \cdot 9 = 18 \text{ and } 3 \cdot 6 = 18.$$

Hence, the factors of 18 are (in order) 1, 2, 3, 6, 9, and 18.

Exercise

Find all whole number factors of 21.

Answer available at the end of this section.

Divisibility

Because division is the inverse of multiplication, the factors of a number are also called divisors. For example, since $3 \cdot 6 = 18 =$, we can also say that 18 is divisible by 3 and by 6. This means that when working with whole numbers, the terms factor and divisor refer to the same number.

Divisible

Let a and b be whole numbers. Then a is divisible by b if and only if the remainder is zero when a is divided by b. In this case, we say that "b is a divisor of a."

When the remainder is zero, the number divides evenly, so it is both a divisor and a factor.

Factors and Divisors

If $c = a \cdot b$, then a and b are called *factors* of c. Both a and b are also called *divisors* of c.

Now that we can find the factors of a number, we can use those factors to describe different types of numbers. Some numbers have only a few factors, while others have many. This leads us to an important classification of numbers based on their factors.

Prime Numbers

We begin with the definition of a prime number.

Prime Number

A whole number (other than 1) is a prime number if its only factors (divisors) are 1 and itself. Equivalently, a number is prime if and only if it has exactly two factors (divisors).

Example 3

Which of the whole numbers 12, 13, 21, and 37 are prime numbers?

Solution

- The factors (divisors) of 12 are 1, 2, 3, 4, 6, and 12. Hence, 12 is not a prime number.
- The factors (divisors) of 13 are 1 and 13. Because its only divisors are 1 and itself, 13 is a prime number.
- The factors (divisors) of 21 are 1, 3, 7, and 21. Hence, 21 is not a prime number.
- The factors (divisors) of 37 are 1 and 37. Because its only divisors are 1 and itself, 37 is a prime number.

Exercise

Which of the whole numbers 15, 23, 51, and 59 are prime numbers?

Answer available at the end of this section.

Example 4

List all the prime numbers less than 20.

Solution

The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19.

You try it!

List all the prime numbers less than 100.

Composite Numbers

If a whole number is not a prime number, then it is called a *composite number*.

Example 5

Is the whole number 1,179 prime or composite?

Solution

Note that 1 + 1 + 7 + 9 = 18, which is divisible by both 3 and 9. Hence, 3 and 9 are both divisors of 1,179. Therefore, 1,179 is a composite number.

Exercise

Is the whole number 2,571 prime or composite?

Answer available at the end of this section.

Factor Trees

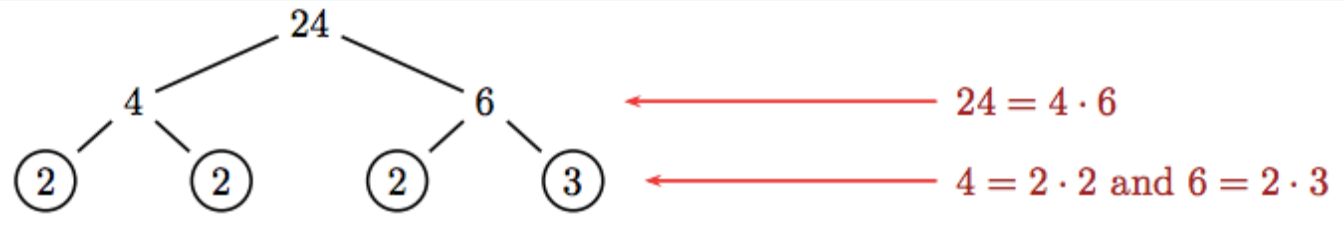
We will now learn how to express a composite number as a unique product of prime numbers. The most popular device for accomplishing this goal is the *factor tree*.

Example 6

Express 24 as a product of prime factors.

Solution

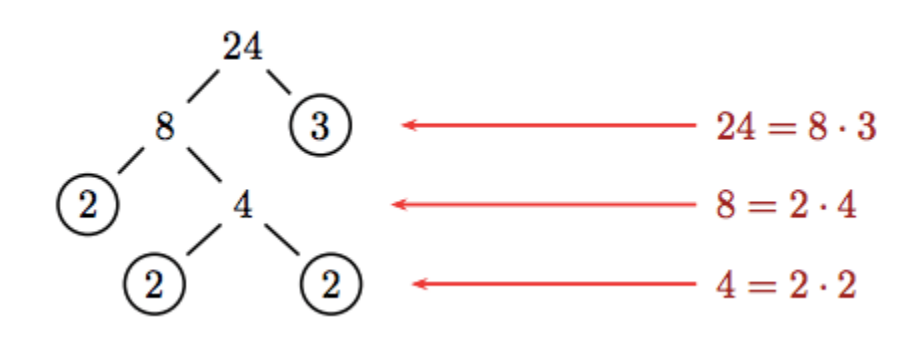
We use a factor tree to break 24 down into a product of primes.



At each level of the tree, break the current number into a product of two factors. The process is complete when all of the "circled leaves" at the bottom of the tree are prime numbers. Arranging the factors in the "circled leaves" in order,

$$24 = 2 \cdot 2 \cdot 2 \cdot 3.$$

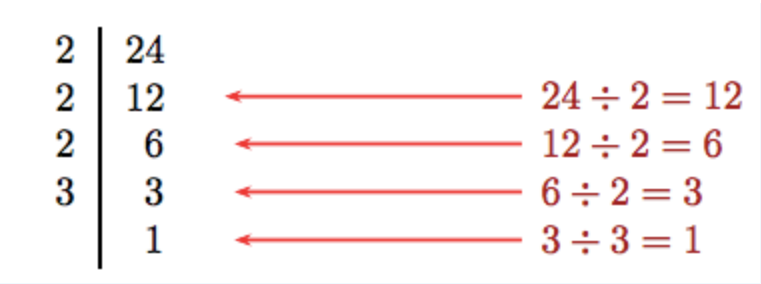
The final answer does not depend on product choices made at each level of the tree. Here is another approach.



The final answer is found by including all of the factors from the "circled leaves" at the end of each branch of the tree, which yields the same result, namely $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

Alternate Approach

Some favor repeatedly dividing by 2 until the result is no longer divisible by 2. Then try repeatedly dividing by the next prime until the result is no longer divisible by that prime. The process terminates when the last resulting quotient is equal to the number 1.



The first column reveals the prime factorization; i.e., 24 = 2 · 2 · 2 · 3.

Exercise

Express 36 as a product of prime factors.

Answer available at the end of this section.

The fact that the alternate approach in Example 6 yielded the same result is significant.

Unique Factorization Theorem

Every whole number can be **uniquely** factored as a product of primes.

This result guarantees that if the prime factors are ordered from smallest to largest, everyone will get the same result when breaking a number into a product of prime factors.

Prime Factorization

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Exponents

When a number is written as a product of repeated prime factors, we can use exponents to write the product more efficiently.

Exponents

The expression a^m is defined to mean

$$a^m = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}}$$

The number *a* is called the base of the exponential expression and the number *m* is called the exponent. The exponent *m* tells us to repeat the base *a* as a factor *m* times.

Example 7

Evaluate 2^5, 2^3, and 5^2.

Solution

- In the case of 2^5, we have

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$= 32.$$

- In the case of 3^3, we have

$$3^3 = 3 \cdot 3 \cdot 3$$

$$= 27.$$

- In the case of 5^2, we have

$$5^2 = 5 \cdot 5$$

$$= 25.$$

Exercise

Evaluate: 3^5.

Answer available at the end of this section.

Note: Calculator Tip

To evaluate expressions with exponents on a calculator, look for the ^ symbol. For example, to evaluate 3^5, enter 3^5 into your calculator.

Intro To Exponents

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Example 8

Express the solution to Example 6 in compact form using exponents.

Solution

In Example 6, we determined the prime factorization of 24.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$

Because $2 \cdot 2 \cdot 2 = 2^3$, we can write this more compactly.

$$24 = 2^3 \cdot 3$$

Exercise

Prime factor 54.

Answer available at the end of this section.

Example 9

Evaluate the expression $2^3 \cdot 3^2 \cdot 5^2$.

Solution

First raise each factor to the given exponent, then perform the multiplication in order (left to right).

$$2^3 \cdot 3^2 \cdot 5^2 = 8 \cdot 9 \cdot 25$$

$$= 72 \cdot 25$$

$$= 1800$$

Exercise

Evaluate: $3^3 \cdot 5^2$.

Answer available at the end of this section.

Answers

Exercise
1, 3, 7, and 21

Exercise
23 and 59

Exercise
Composite

Exercise
$2 \cdot 2 \cdot 3 \cdot 3$.

Exercise
243.

Exercise
$2 \cdot 3 \cdot 3 \cdot 3$

Exercise
675

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1.1.3: Understanding Integers

Do you live in a place that has very cold winters? Have you ever experienced a temperature below zero? If so, you are already familiar with negative numbers. A **negative number** is a number that is less than 0. Very cold temperatures are measured in degrees below zero and can be described by negative numbers. For example, -1° F (read as "negative one degree Fahrenheit") is 1 degree below 0. A minus sign is shown before a number to indicate that it is negative. Figure $1.1.3.1$ shows -20° F, which is 20 degrees below 0.

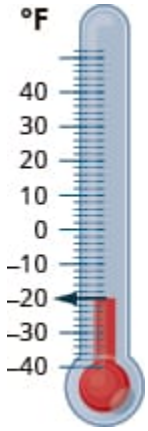


Figure $1.1.3.1$: Temperatures below zero are described by negative numbers.

Temperatures are not the only negative numbers. A bank overdraft is another example of a negative number. If a person writes a check for more than he has in his account, his balance will be negative.

Elevations can also be represented by negative numbers. The elevation at sea level is 0 feet. Elevations above sea level are positive and elevations below sea level are negative. The elevation of the Dead Sea, which borders Israel and Jordan, is about $1,302$ feet below sea level, so the elevation of the Dead Sea can be represented as $-1,302$ feet. See Figure $1.1.3.2$.

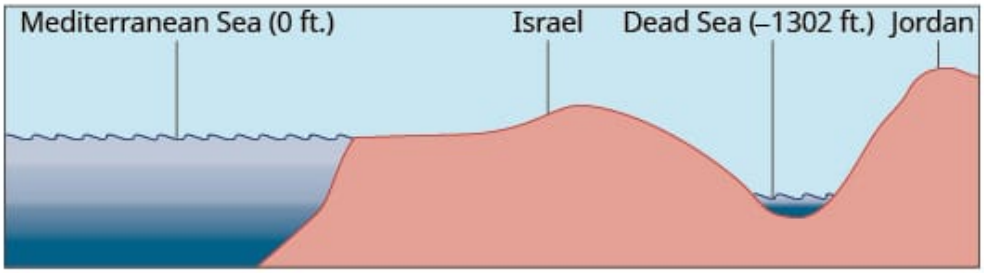


Figure $1.1.3.2$: The surface of the Mediterranean Sea has an elevation of 0 ft. The diagram shows that nearby mountains have higher (positive) elevations whereas the Dead Sea has a lower (negative) elevation.

Depths below the ocean surface are also described by negative numbers. A submarine, for example, might descend to a depth of 500 feet. Its position would then be -500 feet as labeled in Figure $1.1.3.3$.

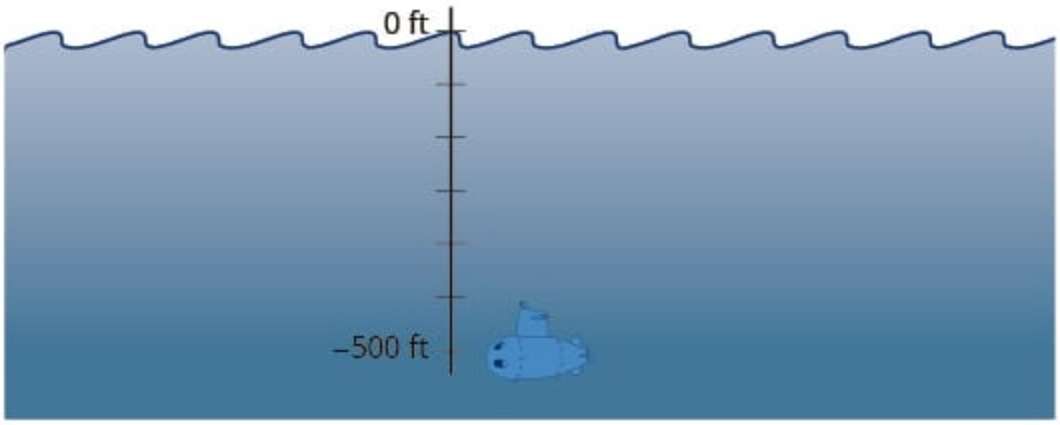


Figure 1.1.3.3: Depths below sea level are described by negative numbers. A submarine 500 ft below sea level is at −500 ft.

Both positive and negative numbers can be represented on a number line. Recall that when discussing whole numbers, the number line started at 0 and showed the counting numbers increasing to the right as shown in Figure $1.1.3.4$. The counting numbers $(1, 2, 3, ...)$ on the number line are all positive. We could write a plus sign, $+$, before a positive number such as $+2$ or $+3$, but it is customary to omit the plus sign and write only the number. If there is no sign, the number is assumed to be positive.

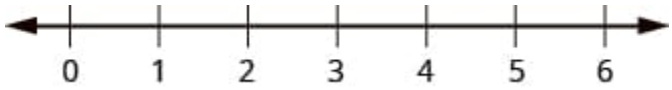


Figure 1.1.3.4

Now we need to extend the number line to include negative numbers. We mark several units to the left of zero, keeping the intervals the same width as those on the positive side. We label the marks with negative numbers, starting with -1 at the first mark to the left of $0, -2$ at the next mark, and so on. See Figure $1.1.3.5$.

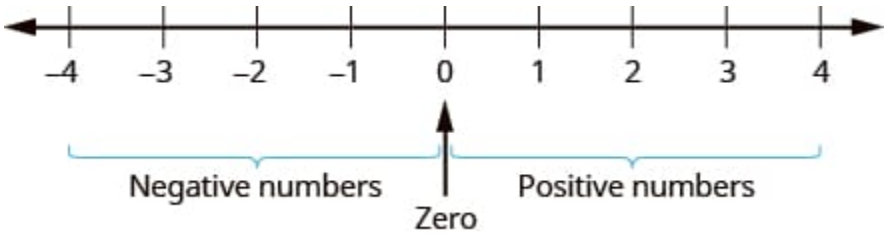


Figure 1.1.3.5: On a number line, positive numbers are to the right of zero. Negative numbers are to the left of zero. What about zero? Zero is neither positive nor negative.

The arrows at either end of the line indicate that the number line extends forever in each direction. There is no greatest positive number and there is no smallest negative number.

Example 1

Plot the numbers on a number line:

a. 3
b. -3
c. -2

Solution

Draw a number line. Mark 0 in the center and label several units to the left and right.

a. To plot 3, start at 0 and count three units to the right. Place a point as shown in Figure $1.1.3.6$.

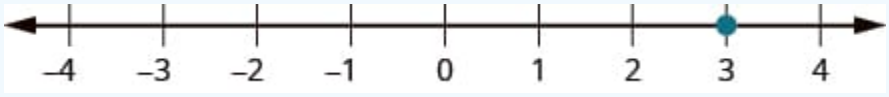


Figure $1.1.3.6$

b. To plot -3, start at 0 and count three units to the left. Place a point as shown in Figure $1.1.3.7$.

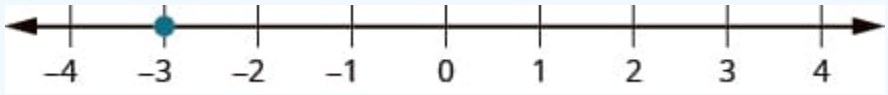


Figure $1.1.3.7$

c. To plot -2, start at 0 and count two units to the left. Place a point as shown in Figure $1.1.3.8$.

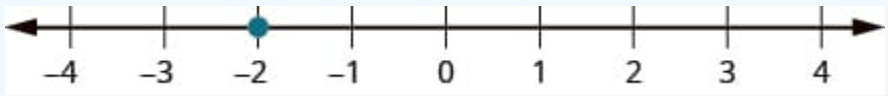


Figure $1.1.3.8$

Exercise

Plot the numbers on a number line.

$1, -1, -4$

Answer available at the end of this section.

Exercise

Plot the numbers on a number line.

$-4, 4, -1$

Answer available at the end of this section.

Now that we have seen how to graph positive and negative whole numbers on a number line, let's now formally introduce the idea of integers.

Definition: Integers

Integers are numbers that include positive and negative whole numbers, and zero.

$$\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$$

Order Positive and Negative Numbers

We can use the number line to compare and order positive and negative numbers. On the number line, values increase from left to right and decrease from right to left. See Figure 1.1.3.9.

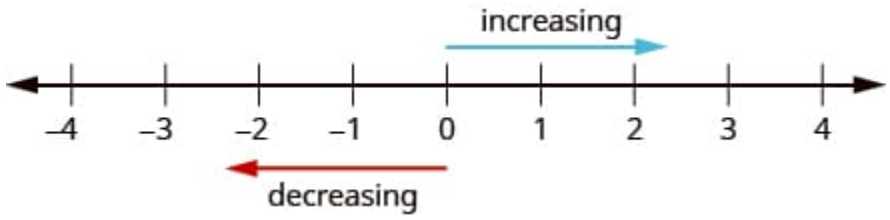


Figure 1.1.3.9

Just as we did with positive numbers, we can use inequality symbols to show the ordering of positive and negative numbers. Remember that we use the notation $a < b$ (read a is less than b) when a is to the left of b on the number line. We write $a > b$ (read a is greater than b) when a is to the right of b on the number line. This is shown for the numbers 3 and 5 in Figure 1.1.3.10.

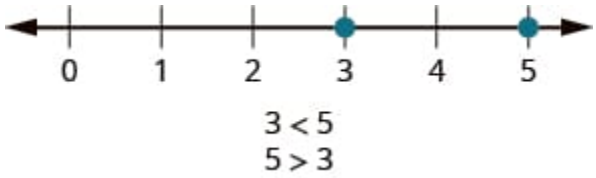
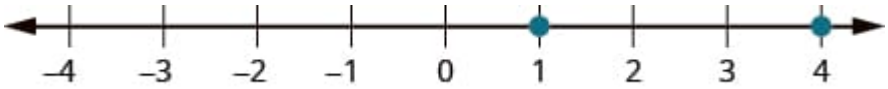


Figure 1.1.3.10: The number 3 is to the left of 5 on the number line. So 3 is less than 5, and 5 is greater than 3.

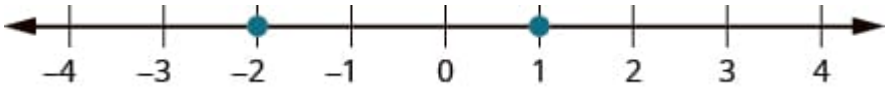
The numbers lines to follow show a few more examples.

a.



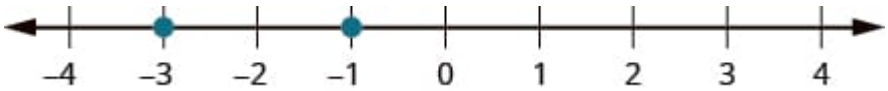
4 is to the right of 1 on the number line, so $4 > 1$. 1 is to the left of 4 on the number line, so $1 < 4$.

b.



-2 is to the left of 1 on the number line, so $-2 < 1$. 1 is to the right of -2 on the number line, so $1 > -2$.

c.



-1 is to the right of -3 on the number line, so $-1 > -3$. -3 is to the left of -1 on the number line, so $-3 < -1$.

Example 2

Order each of the following pairs of numbers using $<$ or $>$:

a. $14\_\_\_6$
b. $-1\_\_\_9$
c. $-1\_\_\_-4$
d. $2\_\_\_-20$

Solution

Begin by plotting the numbers on a number line as shown in Figure 1.1.3.11.

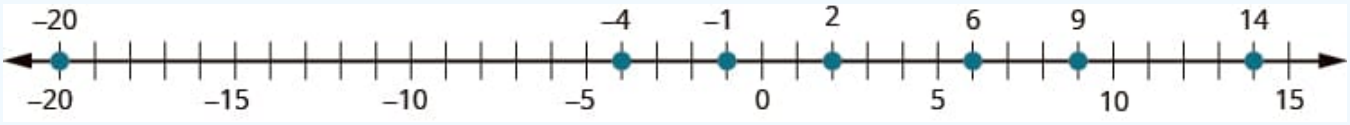


Figure 1.1.3.11

a.

Compare 14 and 6.	14___6
14 is to the right of 6 on the number line.	14 > 6

b.

Compare –1 and 9.	–1___9
–1 is to the left of 9 on the number line.	–1 < 9

c.

Compare –1 and –4.	–1___–4
–1 is to the right of –4 on the number line.	–1 > –4

d.

Compare 2 and –20.	2___–20
2 is to the right of –20 on the number line.	2 > –20

Exercise

Order each of the following pairs of numbers using $<$ or $>$.

a. $15\_\_\_7$
b. $-2\_\_\_5$
c. $-3\_\_\_-7$
d. $5\_\_\_-17$

Answer a available at the end of this section.

Exercise

Order each of the following pairs of numbers using $<$ or $>$.

a. $8\_\_\_13$
b. $3\_\_\_-4$
c. $-5\_\_\_-2$
d. $9\_\_\_-21$

Answer a available at the end of this section.

Negative Numbers

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces integers, including negative numbers, and explains how they are represented on a number line. It also demonstrates how to compare integers and determine which values are greater or less.

Find Opposites

On the number line, the negative numbers are a mirror image of the positive numbers with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they are called **opposites**. The opposite of 2 is -2, and the opposite of -2 is 2 as shown in Figure $1.1.3.12a$. Similarly, 3 and -3 are opposites as shown in Figure $1.1.3.12b$.

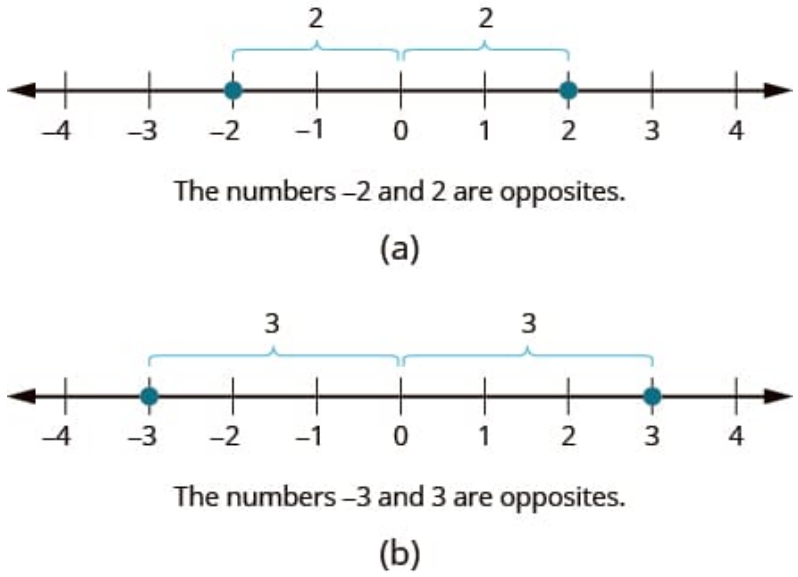


Figure $1.1.3.12$

Definition: opposite

The opposite of a number is the number that is the same distance from zero on the number line, but on the opposite side of zero.

Example 3

Find the opposite of each number:

a. 7
b. -10

Solution

a. The number -7 is the same distance from 0 as 7, but on the opposite side of 0. So -7 is the opposite of 7 as shown in Figure $1.1.3.13$.



Figure $1.1.3.13$

b. The number 10 is the same distance from 0 as -10, but on the opposite side of 0. So 10 is the opposite of -10 as shown in Figure $1.1.3.14$.

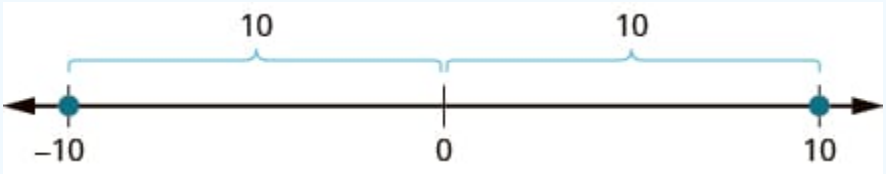


Figure $1.1.3.14$

Exercise

Find the opposite of each number:

a. 4
b. -3

Answer a available at the end of this section.

Exercise

Find the opposite of each number:

a. 8
b. -5

Answer a available at the end of this section.

Opposite Notation

Just as the same word in English can have different meanings, the same symbol in algebra can have different meanings.

The specific meaning becomes clear by looking at how it is used. You have seen the symbol "$-$", in three different ways.

10 - 4	Between two numbers, the symbol indicates the operation of subtraction. We read 10 – 4 as 10 minus 4.

-8	In front of a number, the symbol indicates a negative number. We read –8 as negative eight.
-x	In front of a variable or a number, it indicates the opposite. We read –x as the opposite of x .
–(–2)	Here we have two signs. The sign in the parentheses indicates that the number is negative 2. The sign outside the parentheses indicates the opposite. We read – (–2) as the opposite of –2.

Definition: Opposite Notation

$-a$ means the opposite of the number a.

The notation $-a$ is read the opposite of a.

Example 4

Simplify: $-(-6)$.

Solution

	–(–6)
The opposite of –6 is 6.	6

Exercise

Simplify: $-(-1)$

Answer available at the end of this section.

Exercise

Simplify: $-(-5)$

Answer available at the end of this section.

Simplifying the Opposites of Negative Integers

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to find the opposite of an integer and demonstrates the process through four examples.

Translate Word Phrases into Expressions with Integers

Now we can translate word phrases into expressions with integers. Look for words that indicate a negative sign. For example, the word *negative* in "negative twenty" indicates -20. So does the word *opposite* in "the opposite of 20."

Example 5

Translate each phrase into an expression with integers:

a. the opposite of positive fourteen
b. the opposite of -11
c. negative sixteen

Solution

a. the opposite of fourteen -14
b. the opposite of -11 $-(-11)$
c. negative sixteen -16

Exercise

Translate each phrase into an expression with integers:

a. the opposite of positive nine
b. the opposite of -15
c. negative twenty

Answer a available at the end of this section.

Exercise

Translate each phrase into an expression with integers:

a. the opposite of negative nineteen
b. the opposite of twenty-two
c. negative nine

Answer a available at the end of this section.

As we saw at the start of this section, negative numbers are needed to describe many real-world situations. We'll look at some more applications of negative numbers in the next example.

Example 6

Translate into an expression with integers:

a. The temperature is 12 degrees Fahrenheit below zero.
b. The football team had a gain of 3 yards.
c. The elevation of the Dead Sea is $1,302$ feet below sea level.
d. A checking account is overdrawn by $\$40$.

Solution

Look for key phrases in each sentence. Then look for words that indicate negative signs. Don't forget to include units of measurement described in the sentence.

a.

	The temperature is 12 degrees Fahrenheit below zero.
Below zero tells us that 12 is a negative number.	–12 °F

b.

	The football team had a gain of 3 yards.
A *gain* tells us that 3 is a positive number.	3 yards

c.

	The elevation of the Dead Sea is 1,302 feet below sea level.
Below sea level tells us that 1,302 is a negative number.	–1,302 feet

d.

	A checking account is overdrawn by $40.
Overdrawn tells us that 40 is a negative number.	–$40

Exercise

Translate into an expression with integers: The football team had a gain of 5 yards.

Answer available at the end of this section.

Exercise

Translate into an expression with integers: The scuba diver was 30 feet below the surface of the water.

Answer available at the end of this section.

Absolute Value

We saw that numbers such as 5 and -5 are opposites because they are the same distance from 0 on the number line. They are both five units from 0. The distance between 0 and any number on the number line is called the absolute value of that number. Because distance is never negative, the absolute value of any number is always nonnegative. The symbol for absolute value is two vertical lines on either side of a number. So the absolute value of 5 is written as $|5|$, and the absolute value of -5 is written as $|-5|$ as shown in Figure $1.1.3.15$.

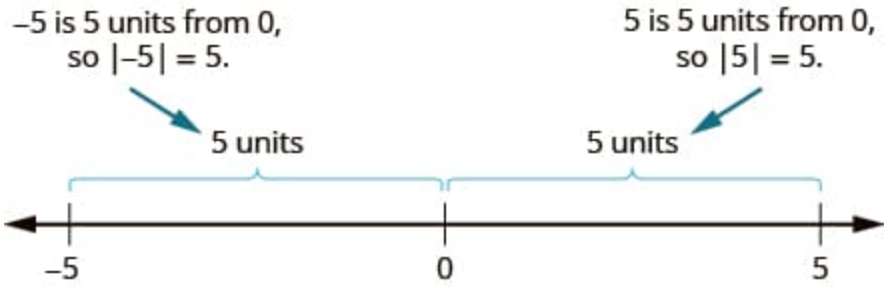


Figure 1.1.3.15

Definition: Absolute Value

The absolute value of a number is its distance from 0 on the number line.

The absolute value of a number n is written as $|n|$.

$$|n| \geq 0 \ for\ all\ numbers$$

Example 7

Simplify:

a. $|3|$
b. $|-44|$
c. $|0|$

Solution

a.

	|3|
3 is 3 units from zero.	3

b.

	|-44|
-44 is 44 units from zero.	44

c.

	|0|
0 is already at zero.	0

Exercise

Simplify:

a. $|12|$
b. $-|-28|$

Answer a available at the end of this section.

Exercise

Simplify:

a. $|9|$
b. $-|37|$

Answer a available at the end of this section.

Determine the Absolute Value of an Integer

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces absolute value and explains how to determine the distance of a number from zero on a number line.

We can also compare expressions containing absolute value. We will start by evaluating the absolute value first and then use the number line to determine which value is greater or less.

Example 8

Fill in $<$, $>$ or $=$ for each of the following:

a. $|-5|\_\_\_-|-5|$
b. $8\_\_\_-|-8|$
c. $-9\_\_\_-|-9|$
d. $-|-7|\_\_\_-7$

Solution

To compare two expressions, simplify each one first. Then compare.

a.

	|–5|___– |–5|
Simplify.	5___–5
Order.	5 > –5

b.

	8___– |–8|
Simplify.	8___–8
Order.	8 > –8

c.

	–9___– |–9|
Simplify.	–9___–9

Order.	−9 = −9

d.

	−|−7|___−7
Simplify.	−7___−7
Order.	−7 = −7

Exercise

Fill in $<$, $>$, or $=$ for each of the following:

a. $|-9|$ ___ $-|-9|$
b. 2___$-|-2|$
c. -8___$|-8|$
d. $-|-5|$___-5

Answer a available at the end of this section.

Exercise

Fill in $<$, $>$, or $=$ for each of the following:

a. 7___$-|-7|$
b. $-|-11|$___-11
c. $|-4|$___$-|-4|$
d. -1___$|-1|$

Answer a available at the end of this section.

Answers

Exercise



Exercise



Exercise

$>$
$<$
$>$
$>$

Exercise

$<$
$>$
$<$
$>$

Exercise

-4
3

Exercise

-8

5

Exercise

1

Exercise

5

Exercise

-9

15

-20

Exercise

19

-22

-9

Exercise

5 yards

Exercise

-30 feet

Exercise

12

-28

Exercise

9

-37

Exercise

$>$

$>$

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Exercise

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1.1.4: Calculator Skills Involving Integers

In this lesson, you will use a calculator to perform operations with integers. Because negative numbers can be tricky, it is important to enter expressions correctly to get accurate results.

How to Enter a Negative Number

When entering a negative number into your calculator, make sure to use the (–) button located next to ENTER and not the – above the + button.

Adding and Subtracting Integers

Before using a calculator, it is helpful to have a general idea of how to add and subtract integers by hand. Understanding how positive and negative numbers work can help you enter expressions correctly and avoid mistakes. It also allows you to check whether your answer makes sense. Below you will find four rules that will help you when adding and subtracting integers.

Rule 1: Subtracting a negative is the same as adding a positive.

Rule 2: Adding two negative integers always results in a negative integer.

Rule 3: Adding two positive integers always results in a positive integer.

Rule 4: The sign when adding integers with opposite signs is the same as the integer with the larger absolute value.

These rules are good to keep in the back of your mind, as they can serve as a quick error check when you use a calculator.

Example 1

Use your calculator to calculate 4 + (–7). Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Example 2

Use your calculator to calculate 18 – 9. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Example 3

Use your calculator to calculate 27 – (–13). Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Example 4

Use your calculator to calculate (–13) + 90. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Multiplying and Dividing Integers

Now let's look at how multiplication and division work with integers. The rules are a bit different from addition and subtraction, but once you understand how the signs affect the result, they are easy to apply. Below, you will find two rules used for multiplying and dividing integers.

Rule 1: When multiplying or dividing two integers with the same sign, the result is positive.

Rule 2: When multiplying or dividing two integers with opposite signs, the result is negative.

Just as before, these rules can serve as a quick error check when using a calculator.

Example 5

Use your calculator to calculate 4×8. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Example 6

Use your calculator to calculate $9 \times (-10)$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Example 7

Use your calculator to calculate $400/(-25)$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Example 8

Use your calculator to calculate $-750/(-3)$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Evaluating Exponential Expressions with Integers

Now let's look at how exponents work with negative values. The result can change depending on whether the negative is included inside parentheses or not. Understanding this difference will help you enter expressions correctly and interpret your answers.

Rule 1: If the negative value is inside parentheses, the entire number is being raised to the exponent.

Rule 2: If the negative value is not in parentheses, only the number is raised to the exponent, and the result is then multiplied by -1.

Just as before, these rules can serve as a quick error check when using a calculator.

Example 9

Use your calculator to calculate $(-3)^2$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Entering (–3)^2 into your calculator, the result is 9. This agrees with our expectation because

$(-3)^2 = -3 \cdot -3$

$= 9$

Example 10

Use your calculator to calculate -3^2. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Entering –3^2 into your calculator, the result is –9. This agrees with our expectation because

$-3^2 = -1 \cdot 3 \cdot 3$

$= -9$

Note: Calculator Tip

Be careful when entering expressions with exponents into your calculator. Including or omitting parentheses can change the result, so always enter the expression exactly as it is written.

Evaluating Expressions Involving Order of Operations with Integers

Rules Guiding Order of Operations

When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

When evaluating expressions, it is helpful to understand the order of operations. However, calculators follow this same order automatically when expressions are entered correctly.

Note: Calculator Tip

When using a calculator, it is important to enter the expression carefully and check that your answer is reasonable. Most calculators allow you to enter the entire expression before pressing Enter. On many scientific calculators, the open parenthesis is above the 8 key and the close parenthesis is above the 9 key.

Example 11

Use your calculator to simplify $-2 - 3(5 - 7)$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Answer available at the end of this section.

Entering $-2 - 3(5 - 7)$ into your calculator, the result is 4.

This is reasonable because the expression is evaluated by simplifying inside the parentheses first, then multiplying, and finally subtracting.

Example 12

Use your calculator to simplify $-2(2 - 4)^2 - 3(3 - 5)^3$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Entering $-2(2 - 4)^2 - 3(3 - 5)^3$ into your calculator, the result is 16.

This makes sense because the expressions inside parentheses are evaluated first, followed by exponents, then multiplication and subtraction.

Example 13

Use your calculator to simplify $-24 \div 8(-3)$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Entering $-24 \div 8(-3)$ into your calculator, the result is 9.

This is reasonable because division and multiplication are performed from left to right, so the division occurs before the multiplication.

Example 14

Use your calculator to simplify $(-2)(-3)(-2)^3$. Explain how the answer agrees with what was expected.

Answer available at the end of this section.

Entering $(-2)(-3)(-2)^3$ into your calculator, the result is -48.

This agrees with our expectation because when you simplify the expression by hand, you will start with the exponents first, then multiplication in the order that it occurs, moving left to right.

Answers

Example 1

When typing this into your calculator, you can enter $4 + (-7)$ or $4 + -7$. The parentheses around -7 are used to show that it is a single value.

After performing either calculation, we find that $4 + (-7) = -3$. Since we are adding integers with opposite signs, the sign of the answer matches the sign of the integer with the larger absolute value which $|-7|=7$.

Example 2

Using a calculator, we find that $18 - 9 = 9$. Since 18 was larger than 9, we expected the difference to be positive.

Example 3

Using a calculator, we find that 27 – (–13) = 40. Since we're subtracting a negative number, it is the same as adding a positive, so this is the same as 27 + 13 = 40.

Example 4

Using a calculator, we find that (–13) + 90 = 77. Since we are adding integers with opposite signs, the sign of the answer matches the sign of the integer with the larger absolute value, which is positive since 90 is positive.

Example 5

Entering 4 × 8 into your calculator, the result is 32. This agrees with our expectation. The numbers have the same signs, so the result is positive.

Example 6

Entering 9 × (–10) into your calculator, the result is –90. This agrees with our expectation. The numbers have opposite signs, so the result is negative.

Example 7

Entering 400/(–25) into your calculator, the result is –16. This agrees with our expectation. The numbers have opposite signs, so the result is negative.

Example 8

Entering –750/(–3) into your calculator, the result is 250. This agrees with our expectation. The numbers have the same signs, so the result is positive.

Example 9

Example 10

Example 11

Example 11

Example 12

Example 13

Example 14

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1.1.5: Whole Numbers and Integers Practice

The following set of exercises provides practice for the concepts covered in Module 1.1: Whole Numbers and Integers. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 1.1.1 - Understanding Whole Numbers

Plot the given numbers on a number line, then use < or > to compare them.

1. 15 __ 11

2. 65 __ 83

Identify the requested digit of the given number.

3. Which digit is in the hundreds place of the number 2,318,999,087?

4. Which digit is in the hundred thousands place of the number 6,125,412,255?

Write the number in standard form.

5. forty-five thousand, two hundred sixty-three

6. five hundred eighty-five thousand, nine hundred thirty-six

Write the number in expanded form. Then, write the number in words.

7. 73,475

8. 265,076,503

Section 1.1.2 - Prime Factorization and Exponential Form

Find the prime factorization of the natural number. Write the answer in compact form using exponents.

9. 320

10. 96

11. 128

Evaluate the exponential expression.

12. $2^3 \cdot 4^1$

13. $5^2 \cdot 2^3$

Section 1.1.3 - Understanding Integers

Plot the given numbers on a number line, then use < or > to compare them.

14. −6 __ 3

15. −5 __ −10

Simplify.

16. $-(8)$

17. $-(-14)$

18. $|-21|$

19. $-|-75|$

Fill in <, >, or = for each of the following pairs of numbers.

20. $4 \_\_ -|4|$

21. $-2 \_\_ |-2|$

Translate each of the following phrases into expressions with positive or negative numbers.

22. a temperature of 10 below zero

23. an elevation of 85 feet above sea level

Section 1.1.4 - Calculator Skills Involving Integers

Evaluate each expression using a calculator.

24. $-33 + (-67)$

25. $19 - (-9)$

26. $-9 \bullet 4$

27. $-120 \div (-6)$

28. -1^5

29. $(-4)^2$

30. $5^2 \cdot (-3)^3$

31. $6 + 7(-1)$

32. $18 \div 6 \cdot 1$

33. $4 - 40 \div 5 \cdot 4 + 9$

34. $(11 + 5 \cdot 2) - 10$

35. $9 - (3 - 1)^3 + 10$

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1.2: FRACTIONS

Fractions have not always looked the way they do today. Earlier systems used parts of a whole in practical ways tied to measurement, sharing, and trade. Even though the notation has changed, the idea behind fractions has stayed the same: describing quantities that are not whole.

That idea shows up more often than you might expect. Anytime something is divided, measured, or adjusted, fractions are involved. In technical settings, that might mean working with materials, interpreting measurements, or making sure quantities stay consistent when something is scaled.

In this module, the focus is on understanding what fractions represent and how to work with them in different forms. You will represent fractions using visual models and number lines, rewrite them as equivalent fractions, and compare them to determine relative size.

You will also use a calculator to perform operations with fractions and make sense of the results in context.

Story of Maths - Fractions in Egypt

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video provides a brief look at how fractions developed over time and how they were used to represent parts of a whole before modern notation.

1.2.1: Understanding Fractions

1.2.2: Equivalent Fractions

1.2.3: Compare Fractions

1.2.4: Calculator Skills Involving Fractions

1.2.5: Fractions Practice

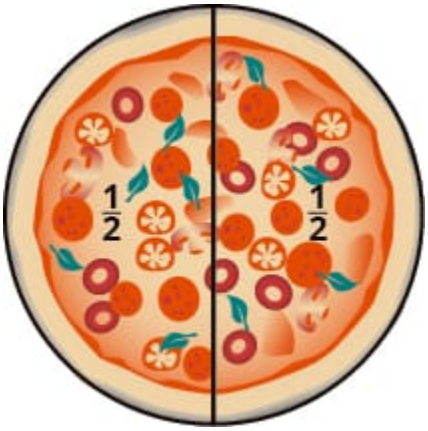
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1.2.1: Understanding Fractions

Understand the Meaning of Fractions

Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts.

In math, we write $\frac{1}{2}$ to mean one out of two parts.



On Tuesday, Andy and Bobby share a pizza with their parents, Fred and Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.



On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.



Fractions

A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In Figure 1.2.1.1, the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.

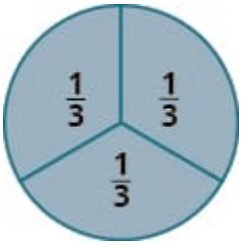


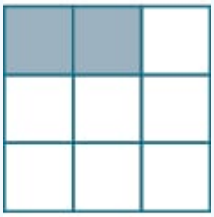
Figure 1.2.1.1

Example 1

Name the fraction of the shape that is shaded in each of the figures.



(a)



(b)

Solution

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

ⓐ

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

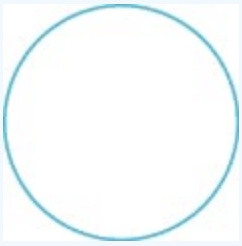
ⓑ

How many equal parts are there?	There are nine equal parts.
How many are shaded?	Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

Example 2

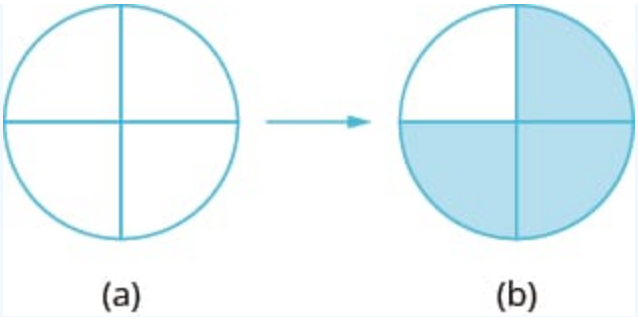
Shade $\frac{3}{4}$ of the circle.



Solution

The denominator is 4, so we divide the circle into four equal parts ⓐ.

The numerator is 3, so we shade three of the four parts ⓑ.



$\frac{3}{4}$ of the circle is shaded.

In Example 1 and Example 2, we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction tiles, as shown in Figure 1.2.1.2. Here, the whole is modeled as one long, undivided rectangular tile. Beneath it are tiles of equal length divided into different numbers of equally sized parts.

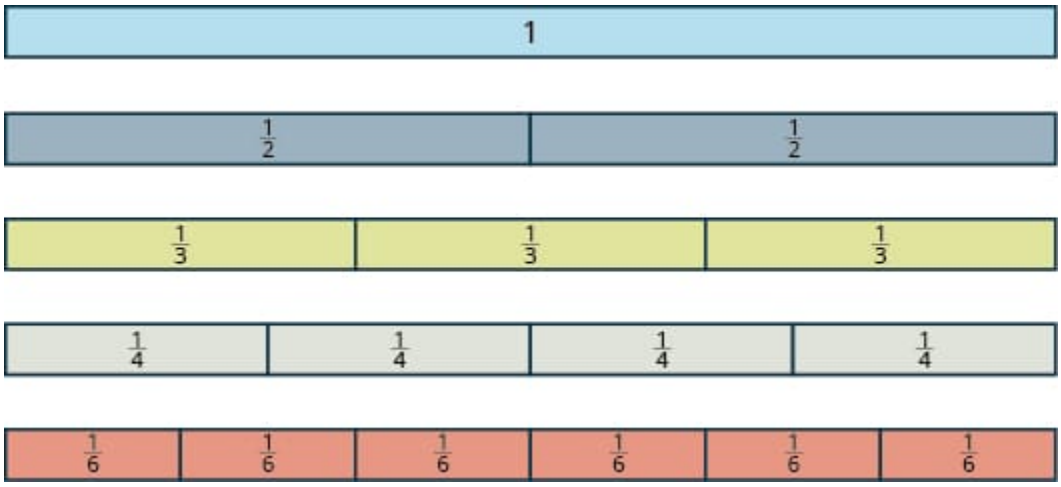


Figure 1.2.1.2

We'll be using fraction tiles to discover some basic facts about fractions. Refer to Figure 1.2.1.2 to answer the following questions:

How many $\frac{1}{2}$ tiles does it take to make one whole tile?	It takes two halves to make a whole, so two out of two is $\frac{2}{2}=1$.
How many $\frac{1}{3}$ tiles does it take to make one whole tile?	It takes three thirds, so three out of three is $\frac{3}{3}=1$.
How many $\frac{1}{4}$ tiles does it take to make one whole tile?	It takes four fourths, so four out of four is $\frac{4}{4}=1$.
How many $\frac{1}{6}$ tiles does it take to make one whole tile?	It takes six sixths, so six out of six is $\frac{6}{6}=1$.
What if the whole were divided into 24 equal parts? (We have not shown fraction tiles to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ tiles does it take to make one whole tile?	It takes 24 twenty-fourths, so $\frac{24}{24}=1$.

This leads us to the *Property of One*.

Property of One

Any number, except zero, divided by itself is one.

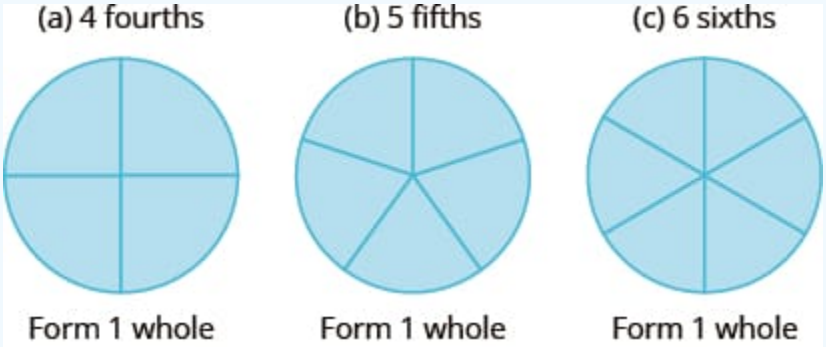
$$\frac{a}{a} = 1 \qquad (a \neq 0)$$

Example 3

Use fraction circles to make wholes using the following pieces:

1. ⓐ 4 fourths
2. ⓑ 5 fifths
3. ⓒ 6 sixths

Solution



What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

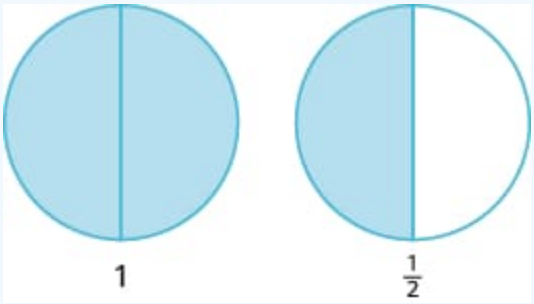
Example 4

Use fraction circles to make wholes using the following pieces:

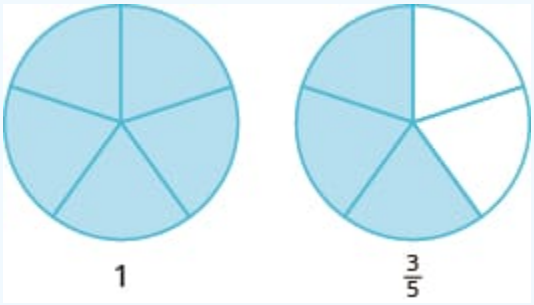
1. ⓐ 3 halves
2. ⓑ 8 fifths
3. ⓒ 7 thirds

Solution

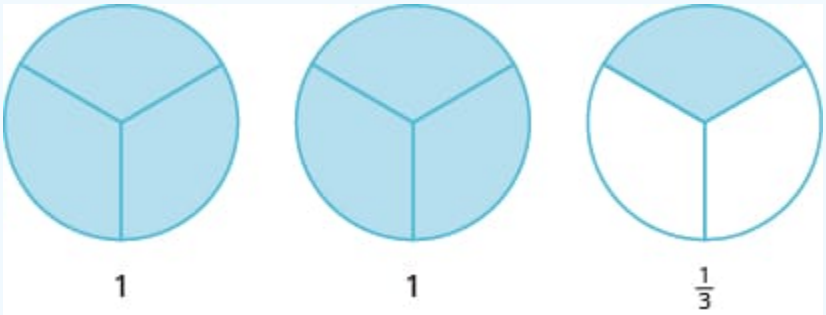
ⓐ 3 halves make 1 whole with 1 half left over.



ⓑ 8 fifths make 1 whole with 3 fifths left over.



ⓒ7 thirds make 2 wholes with 1 third left over.



Model Improper Fractions and Mixed Numbers

In Example 4 (b), you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*.

The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Mixed Numbers

A mixed number consists of a whole number a and a fraction $\frac{b}{c}$, where $c \neq 0$. It is written as:

$a\frac{b}{c}$

Fractions such as $\frac{5}{4}, \frac{3}{2}, \frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}, \frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Proper and Improper Fractions

The fraction $\frac{a}{b}$ is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

Example 5

Name the improper fraction modeled. Then write the improper fraction as a mixed number.



Answer available at the end of this section.

Solution

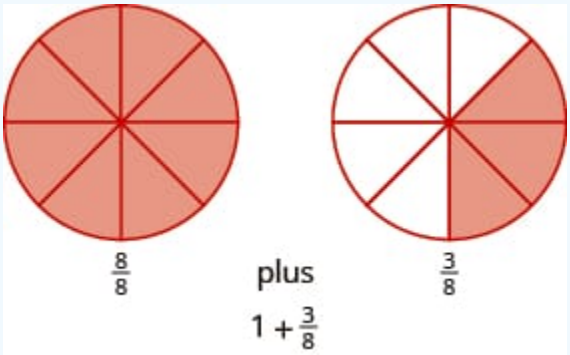
Each circle is divided into three equal pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so the total is $\frac{4}{3}$. The figure also shows one whole circle and one additional third, which is $1\frac{1}{3}$. Therefore, $\frac{4}{3} = 1\frac{1}{3}$.

Example 6

Draw a figure to model $\frac{11}{8}$

Solution

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.



So, $\frac{11}{8} = 1\frac{3}{8}$.

Example 7

Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

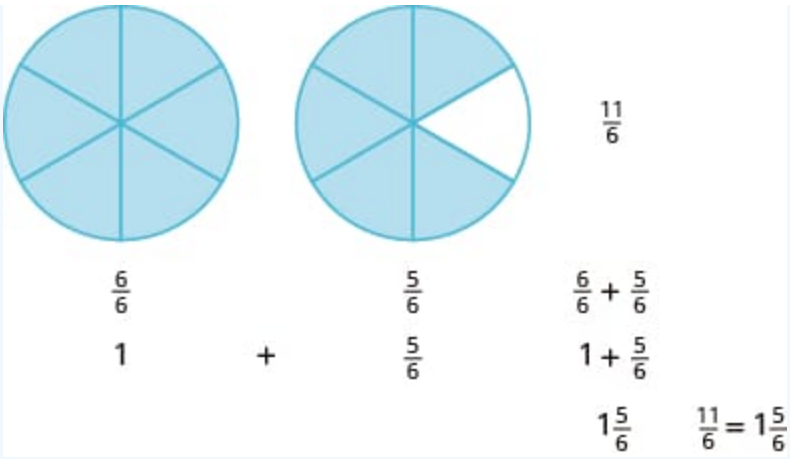
Solution

We start with $\frac{11}{6}$. We know that six sixths makes one whole.

$\frac{6}{6} = 1$

That leaves us with five more sixths, since $11 - 6 = 5$, which is $\frac{5}{6}$.

So, $\frac{11}{6} = 1\frac{5}{6}$.



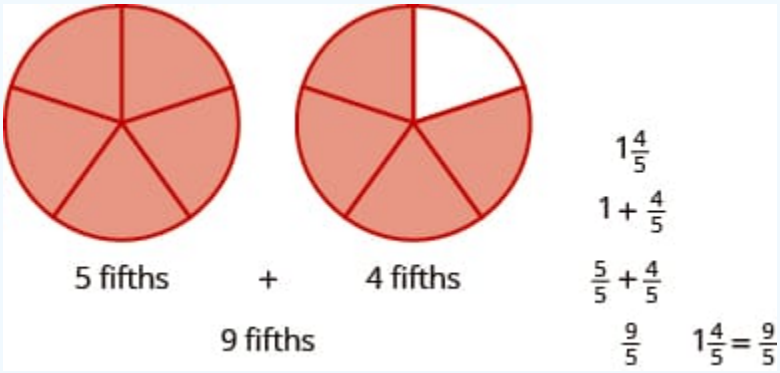
Example 8

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Solution

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths.

So, $1\frac{4}{5} = \frac{9}{5}$.



Convert between Improper Fractions and Mixed Numbers

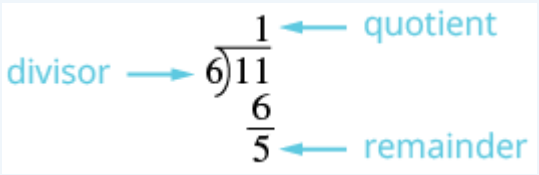
In Example 7, we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = 1\frac{5}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\overline{)11}$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example 9

Convert $\frac{11}{6}$ to a mixed number.

Solution

$\frac{11}{6}$	
Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
Identify the quotient, remainder and divisor.	
Write the mixed number as $\text{quotient}\ \frac{\text{remainder}}{\text{divisor}}$.	$1\frac{5}{6}$
So, $\frac{11}{6} = 1\frac{5}{6}$	

How To

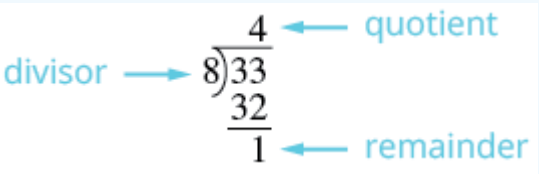
Convert an improper fraction to a mixed number.

1. Divide the denominator into the numerator.
2. Identify the quotient, remainder, and divisor.
3. Write the mixed number as $\text{quotient}\ \frac{\text{remainder}}{\text{divisor}}$.

Example 10

Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Solution

	$\frac{33}{8}$
Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $33 \div 8$.
Identify the quotient, remainder, and divisor.	
Write the mixed number as $\text{quotient}\ \frac{\text{remainder}}{\text{divisor}}$.	$4\frac{1}{8}$
So, $\frac{3}{8} = 4\frac{1}{8}$	

In Example 8, we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.

$$\frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example 11

Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Solution

	$4\frac{2}{3}$
Multiply the whole number by the denominator.	
The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$
Simplify.	$\frac{12 + \square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{12 + 2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

How To

Convert a mixed number to an improper fraction.

1. Multiply the whole number by the denominator.
2. Add the numerator to the product found in Step 1.
3. Write the final sum over the original denominator.

Example 12

Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Solution

	$10\frac{2}{7}$
Multiply the whole number by the denominator.	
The whole number is 10 and the denominator is 7.	$\frac{10 \cdot 7 + \square}{\square}$
Simplify.	$\frac{70 + \square}{\square}$
Add the numerator to the product.	

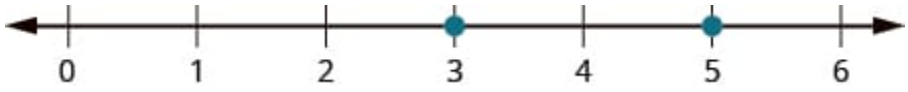
The numerator of the mixed number is 2.	$\frac{70+2}{\square}$
Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Locate Fractions and Mixed Numbers on the Number Line

Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Let us locate $\frac{1}{5}, \frac{4}{5}, 3, 3\frac{1}{3}, \frac{7}{4}, \frac{9}{2}, 5$, and $\frac{8}{3}$ on the number line.

We will start with the whole numbers 3 and 5 because they are the easiest to plot.

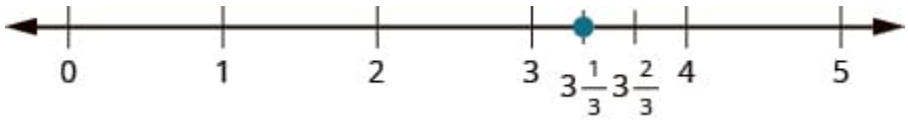


The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{4}{5}$ are located between the whole numbers 0 and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$, and $\frac{4}{5}$.

Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.



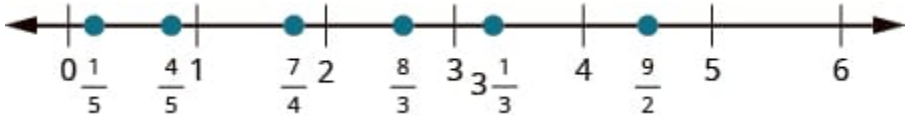
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.



Finally, look at the improper fractions $\frac{7}{4}, \frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

$$\frac{7}{4} = 1\frac{3}{4}, \quad \frac{9}{2} = 4\frac{1}{2}, \quad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.

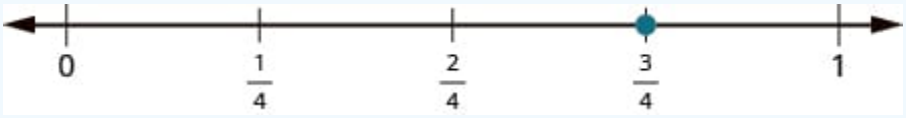


Example 13

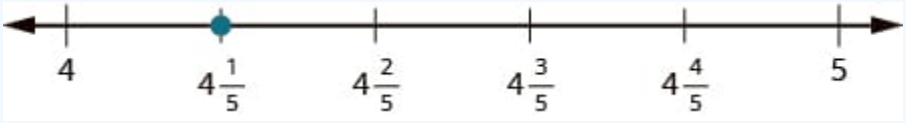
Locate and label the following on a number line:
$\frac{3}{4}, \frac{4}{3}, \frac{5}{3}, 4\frac{1}{5}, \frac{7}{2}$.

Solution

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.



Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$.

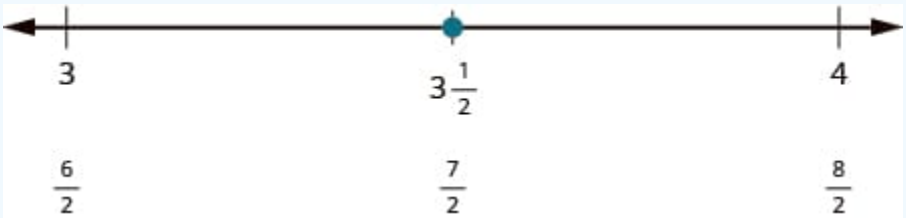
It is easier to plot them if we convert them to mixed numbers first.

$$\frac{4}{3} = 1\frac{1}{3}, \quad \frac{5}{3} = 1\frac{2}{3}$$

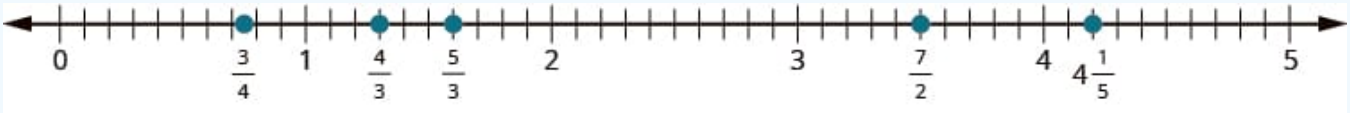
Divide the distance between 1 and 2 into thirds.



Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



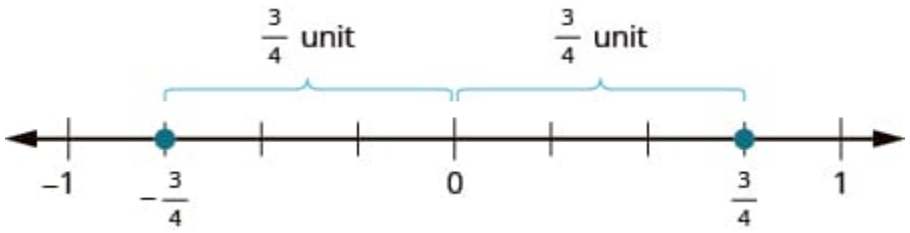
The number line shows all the numbers located on the number line.



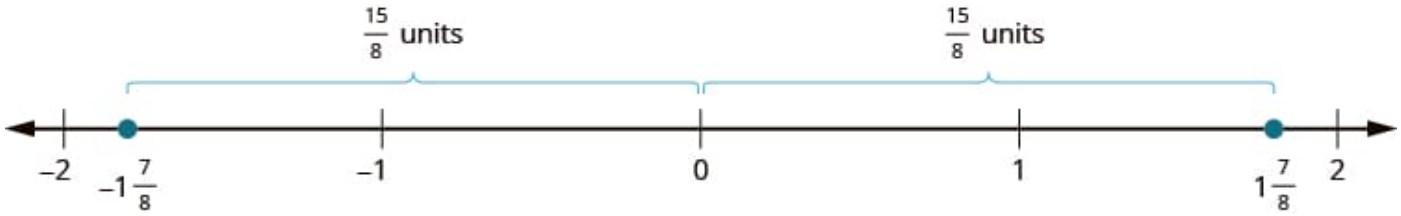
In Introduction to Integers, we defined the opposite of a number. It is the number that is the same distance from zero on the number line but on the opposite side of zero. We saw, for example, that the opposite of 7 is -7 and the opposite of -7 is 7.



Fractions have opposites, too. The opposite of $\frac{3}{4}$ is $-\frac{3}{4}$. It is the same distance from 0 on the number line, but on the opposite side of 0.



Thinking of negative fractions as the opposite of positive fractions will help us locate them on the number line. To locate $-\frac{15}{8}$ on the number line, first think of where $\frac{15}{8}$ is located. It is an improper fraction, so we first convert it to the mixed number $1\frac{7}{8}$ and see that it will be between 1 and 2 on the number line. So its opposite, $-\frac{15}{8}$, will be between -1 and -2 on the number line.



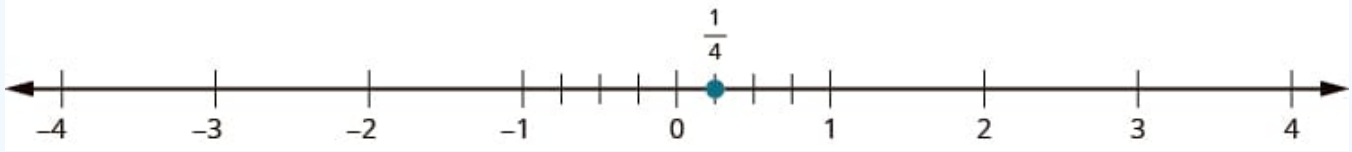
Example 14

Locate and label the following on the number line: $\frac{1}{4}, -\frac{1}{4}, 1\frac{1}{3}, -1\frac{1}{3}, \frac{5}{2}, -\frac{5}{2}$.

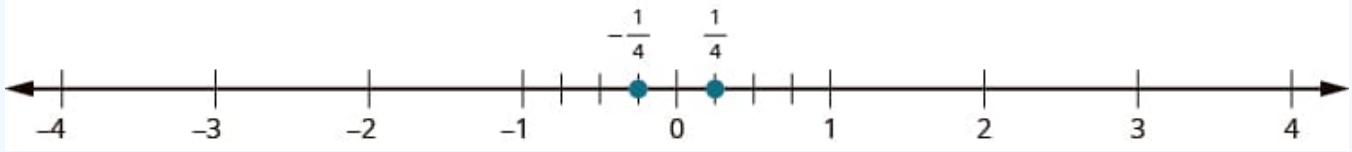
Solution

Draw a number line. Mark 0 in the middle and then mark several units to the left and right.

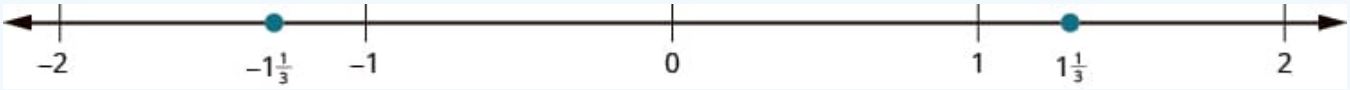
To locate $\frac{1}{4}$, divide the interval between 0 and 1 into four equal parts. Each part represents one-quarter of the distance. So plot $\frac{1}{4}$ at the first mark.



To locate $-\frac{1}{4}$, divide the interval between 0 and -1 into four equal parts. Plot $-\frac{1}{4}$ at the first mark to the left of 0.

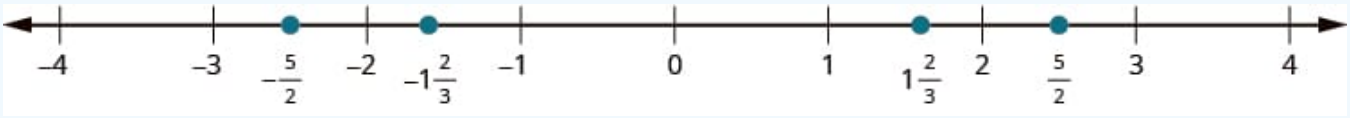


Since $1\frac{1}{3}$ is between 1 and 2, divide the interval between 1 and 2 into three equal parts. Plot $1\frac{1}{3}$ at the first mark to the right of 1. Then since $-1\frac{1}{3}$ is the opposite of $1\frac{1}{3}$ it is between -1 and -2. Divide the interval between -1 and -2 into three equal parts. Plot $-1\frac{1}{3}$ at the first mark to the left of -1.



To locate $\frac{5}{2}$ and $-\frac{5}{2}$, it may be helpful to rewrite them as the mixed numbers $2\frac{1}{2}$ and $-2\frac{1}{2}$.

Since $2\frac{1}{2}$ is between 2 and 3, divide the interval between 2 and 3 into two equal parts. Plot $\frac{5}{2}$ at the mark. Then since $-2\frac{1}{2}$ is between -2 and -3, divide the interval between -2 and -3 into two equal parts. Plot $-\frac{5}{2}$ at the mark.



Answers

Example 5

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1.2.2: Equivalent Fractions

Equivalent fractions represent the same value, even though they are written differently. One way to understand this idea is to use a visual model. Consider the image shown in Figure 1.2.2.1, where the shaded region represents 1/3 of the total area of the figure (one of three equal regions is shaded).



Figure 1.2.2.1: The shaded region is 1/3 of the whole region.

In Figure 1.2.2.2, we've shaded 2/6 of the entire region (two of six equal regions are shaded).



Figure 1.2.2.2: The shaded region is 2/6 of the whole region.

In Figure 1.2.2.3, we've shaded 4/12 of the entire region (four of twelve equal regions are shaded).

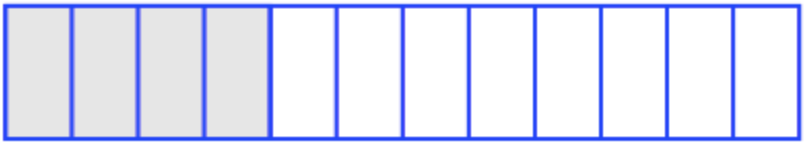


Figure 1.2.2.3: The shaded region is 4/12 of the whole region.

Let's take the diagrams from Figure 1.2.2.1, Figure 1.2.2.2, and Figure 1.2.2.3 and stack them one atop the other, as shown in Figure 1.2.2.4.

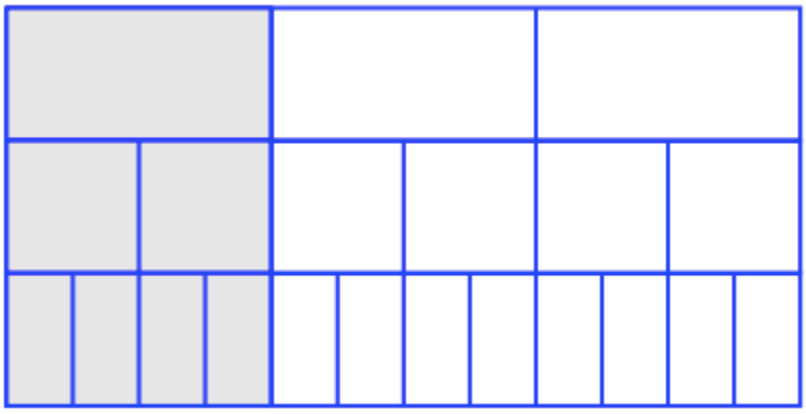


Figure 1.2.2.4: One of three equals two of six equals four of twelve.

Figure 1.2.2.4 provides solid visual evidence that the following fractions are equivalent.

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

Key Observations

1. If we start with the fraction 1/3, then multiply both numerator and denominator by 2, we get the following result.

$$\begin{aligned} \frac{1}{3} &= \frac{1 \cdot 2}{3 \cdot 2} && \text{Multiply the numerator and denominator by 2.} \\ &= \frac{2}{6} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going from Figure 1.2.2.1 to 1.2.2.2, where we double the number of available boxes (going from 3 available to 6 available) and double the number of shaded boxes (going from 1 shaded to 2 shaded).

2. If we start with the fraction 1/3, then multiply both numerator and denominator by 4, we get the following result.

$$\begin{aligned} = \frac{1}{3} = \frac{1 \cdot 4}{3 \cdot 4} && \text{Multiply numerator and denominator by 4.} \\ = \frac{4}{12} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going from Figure 1.2.2.1 to 1.2.2.3, where we multiply the number of available boxes by 4 (going from 3 available to 12 available) and multiply the number of shaded boxes by 4 (going from 1 shaded to 4 shaded).

This leads to the following results.

Creating Equivalent Fractions

If you start with a fraction, then multiply both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \cdot x}{b \cdot x}$$

Arguing in Reverse

Reversing the above argument also holds true.

1. If we start with the fraction 2/6, then divide both numerator and denominator by 2, we get the following result.

$$\begin{aligned} \frac{2}{6} &= \frac{2 \div 2}{6 \div 2} && \text{Divide numerator and denominator by 2.} \\ &= \frac{1}{3} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going backwards from Figure 1.2.2.2 to 1.2.2.1, where we divide the number of available boxes by 2 (going from 6 available to 3 available) and dividing the number of shaded boxes by 2 (going from 2 shaded to 1 shaded).

2. If we start with the fraction 4/12, then divide both numerator and denominator by 4, we get the following result.

$$\begin{aligned} \frac{4}{12} &= \frac{4 \div 4}{12 \div 4} && \text{Multiply numerator and denominator by 4.} \\ &= \frac{1}{3} && \text{Simplify numerator and denominator.} \end{aligned}$$

This is precisely the same thing that happens going backwards from Figure 1.2.2.3 to 1.2.2.1, where we divide the number of available boxes by 4 (going from 12 available to 3 available) and divide the number of shaded boxes by 4 (going from 4 shaded to 1 shaded).

The above discussion motivates the following fundamental result.

Creating Equivalent Fractions

If you start with a fraction, then divide both its numerator and denominator by the same number, the resulting fraction is equivalent (has the same numerical value) to the original fraction. In symbols,

$$\frac{a}{b} = \frac{a \div x}{b \div x}.$$

Equivalent Fractions

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces equivalent fractions and demonstrates how to find equivalent fractions by multiplying or dividing the numerator and denominator by the same factor.

The Greatest Common Divisor

To further develop our understanding of equivalent fractions, we introduce the concept of the greatest common divisor.

Divisor

If d and a are natural numbers, we say that “d divides a” if and only if when a is divided by d, the remainder is zero. In this case, we say that “d is a divisor of a.”

For example, when 36 is divided by 4, the remainder is zero. In this case, we say that “4 is a divisor of 36.” On the other hand, when 25 is divided by 4, the remainder is not zero. In this case, we say that “4 is not a divisor of 25.”

Greatest Common Divisor

Let a and b be natural numbers. The common divisors of a and b are those natural numbers that divide both a and b. The *greatest common divisor* is the largest of these common divisors.

As we have seen, a divisor can also be called a factor. For this reason, the greatest common divisor is sometimes referred to as the greatest common factor.

Example 1

Find the greatest common divisor of 18 and 24.

Solution

First list the divisors of each number, the numbers that divide each number with zero remainder.

Divisors of 18 : 1, 2, 3, 6, 9, and 18

Divisors of 24 : 1, 2, 3, 4, 6, 8, 12, and 24

The common divisors are:

Common Divisors : 1, 2, 3, and 6

The greatest common divisor is the largest of the common divisors. That is,

Greatest Common Divisor = 6.

That is, the largest number that divides both 18 and 24 is the number 6.

Exercise

Find the greatest common divisor of 12 and 18.

Answer available at the end of this section.

GCF: Greatest Common Factor

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to find the greatest common factor (GCF) by identifying common factors and selecting the largest one. The greatest common factor is another name for the greatest common divisor.

Reducing a Fraction to Lowest Terms

First, a definition.

Lowest Terms

A fraction is said to be *reduced to lowest terms* if the greatest common divisor of both numerator and denominator is 1.

Thus, for example, 2/3 is reduced to lowest terms because the greatest common divisior of 2 and 3 is 1. On the other hand, 4/6 is **not** reduced to lowest terms because the greatest common divisor of 4 and 6 is 2.

Example 2

Reduce the fraction 18/24 to lowest terms.

Solution

One technique that works well is dividing both numerator and denominator by the greatest common divisor of the numerator and denominator. In Example 1, we saw that the greatest common divisor of 18 and 24 is 6. We divide both numerator and denominator by 6 to get

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} \quad \text{Divide numerator and denominator by 6.}$$
$$= \frac{3}{4} \quad \text{Simplify numerator and dice.}$$

Note that the greatest common divisor of 3 and 4 is now 1. Thus, 3/4 is reduced to lowest terms.

There is a second way we can show division of numerator and denominator by 6. First, factor both numerator and denominator as follows:

$$\frac{18}{24} = \frac{3 \cdot 6}{4 \cdot 6} \quad \text{Factor out a 6.}$$

You can then show "division" of both numerator and denominator by 6 by "crossing out" or "canceling" a 6 in the numerator for a 6 in the denominator, like this:

$$= \frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}} \quad \text{Cancel common factor.}$$
$$= \frac{3}{4}$$

Note that we get the same equivalent fraction, reduced to lowest terms, namely 3/4.

Exercise

Reduce the fraction 12/18 to lowest terms.

Answer available at the end of this section.

Important Point

In Example 2 we saw that 6 was both a *divisor* and a *factor* of 18. The words *divisor* and *factor* are equivalent.

We used the following technique in our second solution in Example 2.

Cancellation Rule

If you express numerator and denominator as a product, then you may cancel common factors from the numerator and denominator. The result will be an equivalent fraction.

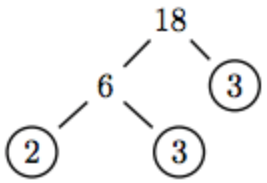
Because of the "Cancellation Rule," one of the most effective ways to reduce a fraction to lowest terms is to first find prime factorizations for both numerator and denominator, then cancel all common factors.

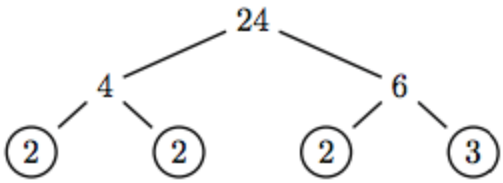
Example 3

Reduce the fraction 18/24 to lowest terms.

Solution

Use factor trees to prime factor numerator and denominator.





Once we've factored the numerator and denominator, we cancel common factors.

$$\begin{aligned}\frac{18}{24} &= \frac{2\cdot 3\cdot 3}{2\cdot 2\cdot 2\cdot 3} && \text{Prime factor numerator and denominator.}\\ &= \frac{\cancel{2}\cdot\cancel{3}\cdot 3}{\cancel{2}\cdot 2\cdot 2\cdot\cancel{3}} && \text{Cancel common factors.}\\ &= \frac{3}{2\cdot 2} && \text{Remaining factors.}\\ &= \frac{3}{4} && \text{Simplify denominator.}\end{aligned}$$

Thus, 18/24 = 3/4.

Exercise

Reduce the fraction 28/35 to lowest terms.

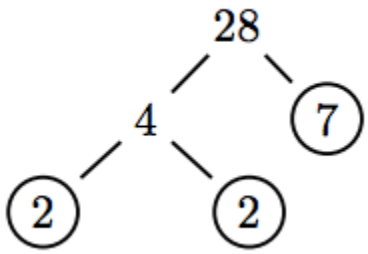
Answer available at the end of this section.

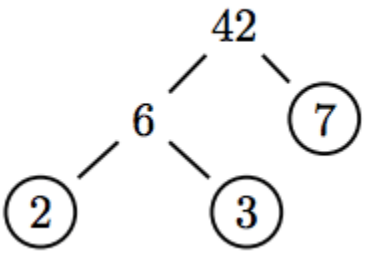
Example 4

Reduce the fraction 28/42 to lowest terms.

Solution

Use factor trees to prime factor numerator and denominator.





Now we can cancel common factors.

$$\begin{aligned}\frac{28}{42} &= \frac{2\cdot 2\cdot 7}{2\cdot 3\cdot 7} && \text{Prime factor numerator and denominator.}\\ &= \frac{\cancel{2}\cdot 2\cdot\cancel{7}}{\cancel{2}\cdot 3\cdot\cancel{7}} && \text{Cancel common factors.}\\ &= \frac{2}{3}\end{aligned}$$

Thus, 28/42 = 2/3.

Exercise

Reduce the fraction 36/60 to lowest terms.

Answer available at the end of this section.

Simplifying Fractions

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to simplify fractions using the prime factorization method and the divisibility method.

Equivalent Fractions in Higher Terms

Sometimes the need arises to find an equivalent fraction with a different, larger denominator.

Example 5

Express 3/5 as an equivalent fraction having denominator 20.

Solution

The key here is to remember that multiplying numerator and denominator by the same number produces an equivalent fraction. To get an equivalent fraction with a denominator of 20, we'll have to multiply numerator and denominator of 3/5 by 4.

$$\begin{aligned} &\frac{3}{5} && \text{Multiply numerator and denominator by 4.} \\ =\ &\frac{12}{20} && \text{Simplify numerator and denominator.} \end{aligned}$$

Therefore, 3/5 equals 12/20.

Exercise

Express 2/3 as an equivalent fraction having denominator 21.

Answer available at the end of this section.

Example 6

Express 8 as an equivalent fraction having denominator 5.

Solution

The key here is to note that

$$8 = \frac{8}{1} \quad \text{Understood denominator is 1.}$$

To get an equivalent fraction with a denominator of 5, we'll have to multiply numerator and denominator of 8/1 by 5.

$$= \frac{8 \cdot 5}{1 \cdot 5} \quad \text{Multiply numerator and denominator by 5.}$$
$$= \frac{40}{5} \quad \text{Simplify numerator and denominator.}$$

Therefore, 8 equals 40/5.

Exercise

Express 5 as an equivalent fraction having denominator 7.

Answer available at the end of this section.

Negative Fractions

We have to also deal with fractions that are negative. First, let's discuss placement of the negative sign.

- Positive divided by negative is negative, so

$$\frac{3}{-5} = -\frac{3}{5}.$$

- But it is also true that negative divided by positive is negative. Thus,

$$\frac{-3}{5} = -\frac{3}{5}.$$

These two observations imply that all three of the following fractions are equivalent (the same number):

$$\frac{3}{-5} = -\frac{3}{5} = \frac{-3}{5}.$$

Note that there are three possible placements for the negative sign: (1) the denominator, (2) the fraction bar, or (3) the numerator. Any one of these placements produces an equivalent fraction.

Fractions and Negative Signs

Let *a* and *b* be any integers. All three of the following fractions are equivalent (same number):

$$\frac{a}{-b} = -\frac{a}{b} = \frac{-a}{b}.$$

Mathematicians prefer to place the negative sign either in the numerator or on the fraction bar. The use of a negative sign in the denominator is discouraged.

Example 7

Reduce:

$$\frac{50}{-75}$$

to lowest terms.

Solution

Prime factor numerator and denominator and cancel.

$$\begin{aligned}\frac{50}{-75} &= \frac{2\cdot 5\cdot 5}{-3\cdot 5\cdot 5}\\ &= \frac{2\cdot \cancel{5}\cdot \cancel{5}}{-3\cdot \cancel{5}\cdot \cancel{5}}\\ &= \frac{2}{-3}\\ &= \frac{2}{-3}\end{aligned}$$

However, it is preferred that there be no negative signs in the denominator, so let's place the negative sign on the fraction bar (the numerator would suit as well). Thus,

$$\frac{50}{-75} = -\frac{2}{3}$$

We also have the following result.

Fractions and Negative Signs

Let a and b be any integers. Then,

$$\frac{-a}{-b} = \frac{a}{b}.$$

Example 8

Reduce:

$$\frac{-12}{-18}$$

Solution

Unlike Example 7, some like to take care of the sign of the answer first.

$$\frac{-12}{-18} = \frac{12}{18}$$

Now we can factor numerator and denominator and cancel common factors.

$$\begin{aligned}&= \frac{2\cdot 2\cdot 3}{2\cdot 3\cdot 3}\\ &= \frac{\cancel{2}\cdot 2\cancel{3}}{\cancel{2}\cdot \cancel{3}\cdot 3} \qquad = \frac{2}{3}\end{aligned}$$

Thus,

$$\frac{-12}{-18} = \frac{2}{3}.$$

Exercise

Reduce:

$$\frac{-21}{-56}$$

Answer available at the end of this section.

Answers

Exercise
6

Exercise
2/3

Exercise
4/5

Exercise
3/5

Exercise
14/21

Exercise
35/7

Exercise

$$\frac{3}{8}$$

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1.2.3: Compare Fractions

You often need to determine whether one fraction is greater than or less than another. Since a fraction represents a part of a whole, the greater fraction represents more of the whole. If two fractions have a common denominator, you can compare the numerators. If the denominators are different, you can rewrite the fractions with a common denominator and then compare the numerators.

Determining Whether Two Fractions Are Equivalent

Since equivalent fractions do not always have the same numerator and denominator, one way to determine if two fractions are equivalent is to find a common denominator and rewrite each fraction with that denominator. Once the two fractions have the same denominator, you can check to see if the numerators are equal. If they are equal, then the two fractions are equal as well.

One way to find a common denominator is to check to see if one denominator is a factor of the other denominator. If so, the greater denominator can be used as the common denominator.

Example

Are $\frac{2}{6}$ and $\frac{8}{18}$ equivalent fractions?

Solution

Does $\frac{2}{6} = \frac{8}{18}$?	To solve this problem, find a common denominator for the two fractions. This will help you compare the two fractions. Since 6 is a factor of 18, you can write both fractions with 18 as the denominator.
$\frac{2 \cdot 3}{6 \cdot 3} = \frac{6}{18}$	Start with the fraction $\frac{2}{6}$. Multiply the denominator, 6, by 3 to get a new denominator of 18. Since you multiply the denominator by 3, you must also multiply the numerator by 3.
$\frac{8}{18}$	The fraction $\frac{8}{18}$ already has a denominator of 18, so you can leave it as is.
$\frac{6}{18}$ does not equal $\frac{8}{18}$	Compare the fractions. Now that both fractions have the same denominator, 18, you can compare numerators.

$\frac{2}{6}$ and $\frac{8}{18}$ are not equivalent fractions.

When one denominator is not a factor of the other denominator, you can find a common denominator by multiplying the denominators together.

Example

Determine whether $\frac{3}{6}$ and $\frac{5}{10}$ are equivalent fractions.

Solution

$6 \cdot 10 = 60$	Use 60 as a common denominator.
$\frac{3}{6} = \frac{3 \cdot 10}{6 \cdot 10} = \frac{30}{60}$	Multiply the numerator and denominator of $\frac{3}{6}$ by 10 to get 60 in the denominator.
$\frac{5}{10} = \frac{5 \cdot 6}{10 \cdot 6} = \frac{30}{60}$	Multiply the numerator and denominator of $\frac{5}{10}$ by 6.

$\frac{30}{60} = \frac{30}{60}$	Now that the denominators are the same, compare the numerators.
Yes, $\frac{3}{6}$ and $\frac{5}{10}$ are equivalent fractions.	Since 30 is the value of the numerator for both fractions, the two fractions are equal.

Notice in the above example that you can use 30 as the least common denominator since both 6 and 10 are factors of 30. Any common denominator will work.

In some cases, you can simplify one or both of the fractions, which can result in a common denominator.

Example

Determine whether $\frac{2}{3}$ and $\frac{40}{60}$ are equivalent fractions.

Solution

$\frac{40}{60} = \frac{40 \div 10}{60 \div 10} = \frac{4}{6}$	Simplify $\frac{40}{60}$. Divide the numerator and denominator by the common factor 10.
$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$	$\frac{4}{6}$ is still not in lowest terms, so divide the numerator and the denominator again, this time by the common factor 2.
$\frac{2}{3} = \frac{2}{3}$	Compare the fractions. The numerators and denominators are the same.

Yes, $\frac{2}{3}$ and $\frac{40}{60}$ are equivalent fractions.

Note

In the example above, you could have used the common factor of 20 to simplify $\frac{40}{60}$ directly to $\frac{2}{3}$.

Determining Equivalent Fractions

To determine whether or not two fractions are equivalent:

Step 1: Rewrite one or both of the fractions so that they have common denominators.

Step 2: Compare the numerators to see if they have the same value. If so, then the fractions are equivalent.

Exercise

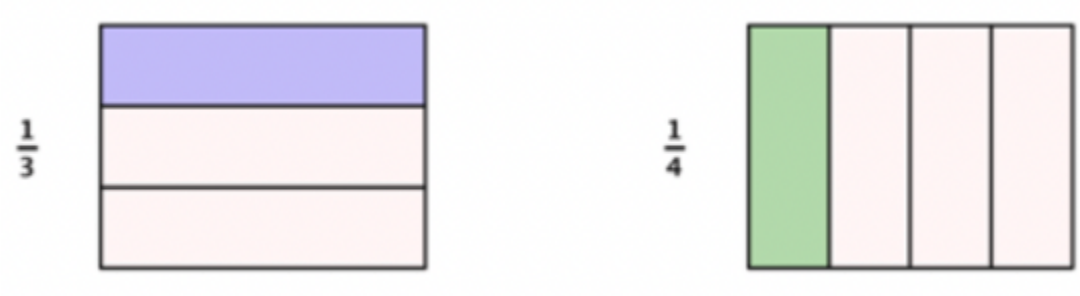
Which of the following fraction pairs are equivalent?

A. $\frac{5}{7}$ and $\frac{7}{5}$
B. $\frac{12}{30}$ and $\frac{6}{10}$
C. $\frac{4}{20}$ and $\frac{1}{5}$
D. $\frac{8}{11}$ and $\frac{8}{22}$

Answer available at the end of this section.

Comparing Fractions Using < and >

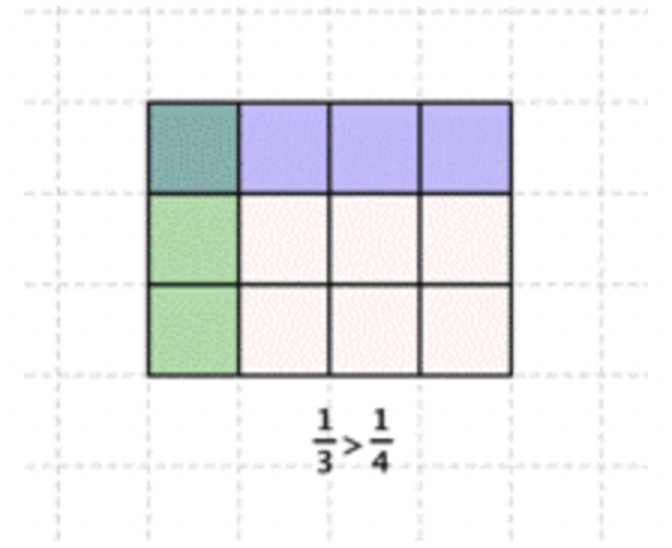
When given two or more fractions, it is often useful to know which fraction is greater than or less than the other. For example, if the discount in one store is $\frac{1}{3}$ off the original price and the discount in another store is $\frac{1}{4}$ off the original price, which store is offering a better deal? To answer this question, and others like it, you can compare fractions.



To determine which fraction is greater, you need to find a common denominator. You can then compare the fractions directly. Since 3 and 4 are both factors of 12, you will divide the whole into 12 parts, create equivalent fractions for $\frac{1}{3}$ and $\frac{1}{4}$, and then compare.



Now you see that $\frac{1}{3}$ contains 4 parts of 12, and $\frac{1}{4}$ contains 3 parts of 12. So, $\frac{1}{3}$ is greater than $\frac{1}{4}$.



As long as the denominators are the same, the fraction with the greater numerator is the greater fraction, as it contains more

parts of the whole. The fraction with the lesser numerator is the lesser fraction, as it contains fewer parts of the whole.

Recall that the symbol < means “less than”, and the symbol > means “greater than”. These symbols are inequality symbols. So, the true statement 3<8 is read as "3 is less than 8" and the statement 5>3 is read as "5 is greater than 3". One way to help you remember the distinction between the two symbols is to think that the smaller end of the symbol points to the lesser number.

As with comparing whole numbers, the inequality symbols are used to show when one fraction is “greater than” or “less than” another fraction.

Comparing Fractions

To compare two fractions:

Step 1: Compare denominators. If they are different, rewrite one or both fractions with a common denominator.

Step 2: Check the numerators. If the denominators are the same, then the fraction with the greater numerator is the greater fraction. The fraction with the lesser numerator is the lesser fraction. And, as noted above, if the numerators are equal, the fractions are equivalent.

Example

Use < or > to compare the two fractions $\frac{4}{5}$ and $\frac{14}{20}$.

Solution

Is $\frac{4}{5} > \frac{14}{20}$ or is $\frac{4}{5} < \frac{14}{20}$?	You cannot compare the fractions directly because they have different denominators. You need to find a common denominator for the two fractions.
$\frac{4}{5} = \frac{?}{20}$	Since 5 is a factor of 20, you can use 20 as the common denominator.
$\frac{4 \cdot 4}{5 \cdot 4} = \frac{16}{20}$	Multiply the numerator and denominator by 4 to create an equivalent fraction with a denominator of 20.
$\frac{16}{20} > \frac{14}{20}$	Compare the two fractions. $\frac{16}{20}$ is greater than $\frac{14}{20}$.
$\frac{4}{5} > \frac{14}{20}$	If $\frac{16}{20} > \frac{14}{20}$, then $\frac{4}{5} > \frac{14}{20}$, since $\frac{4}{5} = \frac{16}{20}$.

Exercise

Which of the following is a true statement?

A. $\frac{5}{6} < \frac{24}{30}$
B. $\frac{75}{100} > \frac{9}{12}$
C. $\frac{4}{16} > \frac{1}{3}$
D. $\frac{3}{8} < \frac{20}{40}$

Answer available at the end of this section.

Comparing Integers, Fractions, and Mixed Numbers

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to compare integers, fractions, and mixed numbers using a number line and by creating equivalent fractions with a common denominaor.

Answers

Exercise

A. Incorrect. Although the same numbers, 5 and 7, are used in each fraction, the numerators and denominators are not equal, so the fractions cannot be equivalent.
B. Incorrect. 30 is divisible by 10, and 12 is divisible by 6. However, they do not share a common multiple: $6 \cdot 2 = 12$, and $10 \cdot 3 = 30$. This means the fractions are not equivalent.
C. Correct. Take the fraction $\frac{1}{5}$ and multiply both the numerator and denominator by 4. You are left with the fraction $\frac{4}{20}$. This means that the two fractions are equivalent.
D. Incorrect. The numerators of the two fractions are the same, but the denominators are different. This means the fractions are not equivalent.

Exercise

A. Incorrect. $\frac{5}{6}$ is equivalent to $\frac{25}{30}$, and since 25>24, $\frac{25}{30} > \frac{24}{30}$, which means $\frac{5}{6} > \frac{24}{30}$. The correct answer is $\frac{3}{8} < \frac{20}{40}$.
B. Incorrect. Both fractions simplify to $\frac{3}{4}$, so one isn't greater than or less than the other one. They are equivalent. The correct answer is $\frac{3}{8} < \frac{20}{40}$.
C. Incorrect. Finding a common denominator, you can compare $\frac{12}{48}$ to $\frac{16}{48}$, and see that $\frac{12}{48} < \frac{16}{48}$, which means $\frac{4}{16} < \frac{1}{3}$. The correct answer is $\frac{3}{8} < \frac{20}{40}$.
D. Correct. Simplifying $\frac{20}{40}$, you get the equivalent fraction $\frac{1}{2}$. Since you still don't have a common denominator, write $\frac{1}{2}$ as an equivalent fraction with a denominator of 8: $\frac{1}{2} = \frac{1 \cdot 4}{2 \cdot 4} = \frac{4}{8}$. You find that $\frac{3}{8} < \frac{4}{8}$, so $\frac{3}{8} < \frac{20}{40}$ as well.

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1.2.4: Calculator Skills Involving Fractions

In technical settings, fractions often appear when working with measurements, materials, or quantities that are less than one. Calculators can perform operations with fractions quickly and accurately, but it is important to know how to enter expressions correctly. Many scientific calculators include built-in fraction features that allow you to enter and work with fractions directly, making it easier to perform calculations without relying on manual methods. In this section, you will use a calculator to add, subtract, multiply, and divide fractions and express answers as both improper fractions and mixed numbers.

Many scientific calculators, including the TI-30XS, allow you to enter fractions using a built-in fraction template.

Entering Fractions on a Scientific Calculator

Using a TI-30XS MultiView

1. Press the n/d button. This button shows a stacked fraction.
2. Enter the numerator.
3. Use the arrow key to move to the denominator.
4. Enter the denominator.

Example:

To enter $\frac{3}{4}$:

1. Press the n/d button.
2. Type 3, arrow down, then 4.

Using Other Calculators

Look for a button that shows a fraction, often stacked like $\frac{a}{b}$ or labeled n/d or ab/c.

If Your Calculator Does Not Have a Fraction Button

You can always enter fractions using division.

$\frac{3}{4} = (3 \div 4)$

Simplifying Fractions Using a Calculator

Scientific calculators can simplify fractions automatically. Instead of rewriting fractions by hand, you can enter the fraction and let the calculator display the result in simplest form.

Simplifying Fractions Using a Calculator

1. Press the n/d button.
2. Enter the numerator.
3. Use the arrow key to move to the denominator.
4. Enter the denominator.
5. Press Enter.

Example 1

Simplify $\frac{6}{8}$

Solution

1. Press n/d.
2. Enter 6, arrow down, then 8.
3. Press Enter.

$\frac{6}{8} = \frac{3}{4}$

Exercise

Simplify $\frac{10}{15}$

Answer available at the end of this section.

Entering Mixed Numbers

1. Enter the whole number.
2. Press 2nd, then press n/d.
3. Enter the numerator.
4. Use the arrow key to move to the denominator.
5. Enter the denominator.

Example 2

Simplify $2\frac{1}{3}$

Solution

1. Type 2.
2. Press 2nd, then n/d.
3. Enter 1, arrow down, then 3.
4. Press Enter.

$2\frac{1}{3} = \frac{7}{3}$

Exercise

Simplify $3\frac{2}{5}$

Answer available at the end of this section.

You may have noticed that the answer in the previous exercise was written as an improper fraction. Next, you will learn how to use your calculator to convert between improper fractions and mixed numbers.

Converting Between Mixed Numbers and Improper Fractions

To convert between forms:

1. Enter the fraction or mixed number.
2. Press Enter.
3. Press 2nd, then press x10^n.

If the fraction or mixed number is already displayed on your calculator, you can skip Step 1.

Example 3

Convert $\frac{17}{5}$ to a mixed number.

Solution

1. Enter 17 over 5 using the n/d button.
2. Press Enter.
3. Press 2nd, then x10^n.

$\frac{17}{5} = 3\frac{2}{5}$

Exercise

Convert $4\frac{1}{3}$ to an improper fraction

Answer available at the end of this section.

Convert a Decimal into a Fraction

If you enter a fraction using division instead of the n/d button, your calculator may display the result as a decimal.

To convert the result to a fraction:

1. Press 2nd.
2. Press the Table button (this is the F↔D function).

Adding, Subtracting, Multiplying and Dividing Fractions

Now that you understand how to use the fraction buttons on your calculator, you are now ready to use your calculator to perform operations with fractions.

Example 4

Perform the requested operation: $\frac{1}{2} + \frac{3}{4}$. Answer with a proper fraction or a mixed number.

Solution

1. Press n/d, enter 1, arrow down, then 2.
2. Arrow right, press +.
3. Press n/d, enter 3, arrow down, then 4.
4. Press Enter.
5. Press 2nd, then x10^n to convert to a mixed number.

$\frac{1}{2} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$

Exercise

Perform the requested operation: $2\frac{1}{3} + \frac{1}{6}$. Answer with a proper fraction or a mixed number.

Answer available at the end of this section.

Example 5

Perform the requested operation: $-\frac{1}{4} - \frac{5}{6}$. Answer with a proper fraction or a mixed number.

Solution

1. Press the negative key.
2. Press n/d, enter 1, arrow down, then 4.
3. Press –.
4. Press n/d, enter 5, arrow down, then 6.
5. Press Enter.
6. Press 2nd, then x10^n to convert to a mixed number.

$-\frac{1}{4} - \frac{5}{6} = -\frac{13}{12} = -1\frac{1}{12}$

Exercise

Perform the requested operation: $3\frac{1}{2} - \frac{5}{4}$. Answer with a proper fraction or a mixed number.

Answer available at the end of this section.

Example 6

Perform the requested operation: $2\frac{1}{4} \times \frac{2}{3}$. Answer with a proper fraction or a mixed number.

Solution

1. Enter 2, press 2nd, then n/d, then enter 1, arrow down, then 4.
2. Press ×.
3. Press n/d, enter 2, arrow down, then 3.
4. Press Enter.
5. Press 2nd, then x10^n to convert to a mixed number.

$2\frac{1}{4} \times \frac{2}{3} = \frac{3}{2} = 1\frac{1}{2}$

Exercise

Perform the requested operation: $-\frac{2}{3} \times \frac{3}{5}$. Answer with a proper fraction or a mixed number.

Answer available at the end of this section.

Example 7

Perform the requested operation: $\frac{3}{4} \div \frac{2}{5}$. Answer with a proper fraction or a mixed number.

Solution

1. Press n/d, then enter 3, arrow down, then 4.
2. Press ÷.
3. Press n/d, then enter 2, arrow down, then 5.
4. Press Enter.

$\frac{3}{4} \div \frac{2}{5} = \frac{15}{8} = 1\frac{7}{8}$

Exercise

Perform the requested operation: $2\frac{1}{2} \div \frac{3}{4}$. Answer with a proper fraction or a mixed number.

Answer available at the end of this section.

Order of Operations with Fractions

When evaluating expressions with fractions using your calculator, enter each expression exactly as shown, using parentheses when needed. The calculator will follow the correct order of operations.

Example 8

$(\frac{1}{2} + \frac{3}{4}) \div \frac{5}{6}$

Solution

$(\frac{1}{2} + \frac{3}{4}) \div \frac{5}{6} = \frac{3}{2} = 1\frac{1}{2}$

Example 9

$2\frac{1}{3} - (\frac{3}{4})^2$

Solution

$2\frac{1}{3} - (\frac{3}{4})^2 = 1\frac{37}{48}$

Answers

Exercise
$\frac{10}{15} = \frac{2}{3}$

Exercise
$3\frac{2}{5} = \frac{17}{5}$

Exercise
$4\frac{1}{3} = \frac{13}{3}$

Exercise
$2\frac{1}{3} + \frac{1}{6} = 2\frac{1}{2}$

Exercise
$3\frac{1}{2} - \frac{5}{4} = \frac{9}{4} = 2\frac{1}{4}$

Exercise
$-\frac{2}{3} \times \frac{3}{5} = -\frac{2}{5}$

Exercise
$2\frac{1}{2} \div \frac{3}{4} = \frac{10}{3} = 3\frac{1}{3}$

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1.2.5: Fractions Practice

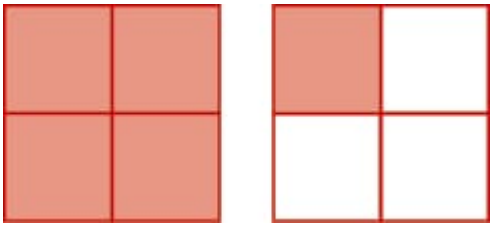
The following set of exercises provides practice for the concepts covered in Module 1.2: Fractions. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

1.2.1 - Understanding Fractions

Name the fraction of each figure that is shaded.

1. 

Name the improper fractions. Then write each improper fraction as a mixed number.

2. 

Convert the improper fraction to a mixed number.

3. $\frac{58}{15}$

Convert the mixed number to an improper fraction.

4. $12\frac{1}{4}$

Locate the numbers on a number line.

5. $\frac{5}{8}, \frac{4}{3}, 3\frac{3}{4}, 4$

Section 1.2.2 - Equivalent Fractions

Find the GCD of the given numbers.

6. 56, 96

7. 63, 21

Reduce the given fraction to lowest terms. Answer with a proper fraction or a mixed number.

8. $\frac{28}{56}$

9. $\frac{74}{62}$

10. $\frac{27}{-75}$

11. $\frac{-44}{-62}$

Find an equivalent fraction using the given denominator.

12. Express $\frac{15}{19}$ as an equivalent fraction having denominator 95.

13. Express 3 as an equivalent fraction having denominator 8.

Section 1.2.3 - Comparing Fractions

Determine whether the pair of fractions is equivalent. Write Yes or No, and briefly explain how you know.

14. $\frac{3}{5}$ and $\frac{6}{10}$

15. $\frac{4}{7}$ and $\frac{12}{20}$

Use $<$, $>$, or $=$ to compare each pair of fractions.

16. $\frac{2}{3}$ ____ $\frac{3}{4}$

17. $\frac{5}{6}$ ____ $\frac{8}{10}$

18. $-\frac{3}{4}$ ____ $-\frac{5}{6}$

Section 1.2.4 - Calculator Skills Involving Fractions

Perform the indicated operation. Answer with a simplified proper fraction or a mixed number.

19. $-\frac{22}{25}+\frac{9}{40}$

20. $6\frac{5}{12}+7\frac{7}{18}$

21. $-\frac{9}{10}-\frac{3}{4}$

22. $2\frac{5}{12}-1\frac{7}{18}$

23. $\frac{2}{9}\cdot\left(-\frac{45}{32}\right)$

24. $3\frac{1}{5}\cdot 1\frac{7}{8}$

25. $8\div 2\frac{2}{3}$

26. $\frac{\frac{5}{8}}{\frac{4}{5}}$

27. $\left(-\frac{8}{5}\right)^2$

28. $\left(-\frac{3}{4}\right)\left(\frac{1}{2}\right)-\left(\frac{3}{5}\right)\left(\frac{1}{4}\right)$

29. $\left(\frac{3}{2}\right)^{2}-\left(\frac{7}{8}\right)\left(-\frac{1}{2}\right)$

30. $\frac{3}{2}+\frac{1}{4}\left(-\frac{9}{8}\right)$

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1.3: DECIMALS

Decimals are commonly used when precision matters. Whether measuring length, tracking money, or reading values on a display, they provide a way to represent quantities that fall between whole numbers.

You will see decimals used often in technical settings. Whether working with measurements, reading instruments, or calculating costs, they offer a clear and efficient way to represent and compare values.

In this module, the focus is on understanding how decimals are structured and how they behave in calculations. You will use place value to read, write, and compare decimal numbers, and round values to a specified place when an exact value is not needed.

You will also use estimation to check whether results are reasonable and use a calculator to perform operations with decimals. The module connects decimals and fractions as different ways to represent the same value.

Decimal Song

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video reviews operations with decimals in a fun way and provides a helpful overview of adding, subtracting, multiplying, and dividing decimals.

1.3.1: Understanding Decimals

1.3.2: Rounding Whole Numbers and Decimals

1.3.3: Calculator Skills Involving Decimals

1.3.4: Fractions and Decimals

1.3.5: Decimals Practice

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1.3.1: Understanding Decimals

Recall that whole numbers are constructed by using *digits*.

The Digits

The set

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

is called the set of *digits*.

As an example, the whole number 55,555 (“fifty-five thousand five hundred fifty-five”) is constructed by using a single digit. However, the position of the digit 5 determines its value in the number 55,555. The first occurrence of the

5	5	5	5	5
ten thousands	thousands	hundreds	tens	ones
10,000	1,000	100	10	1

Table 1.3.1.1: Place value.

digit 5 happens in the ten thousands place, so its value is 5 ten thousands, or 50,000. The next occurrence of the digit 5 is in the thousands place, so its value is 5 thousands, or 5,000. Indeed, the whole number 55,555 in expanded form is

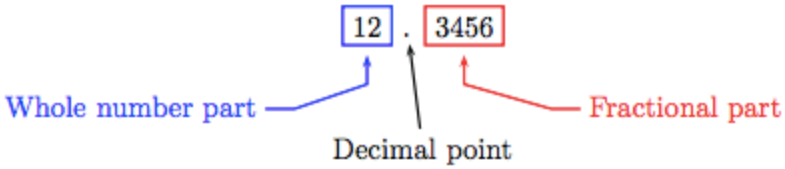
$$50000 + 5000 + 500 + 50 + 5,$$

which reflects the value of the digit 5 in each place.

Decimal Notation

In Table 1.3.1.1, each time you move one column to the left, the place value is 10 times larger than the place value of the preceding column. Vice-versa, each time you move one column to the right, the place value is 1/10 of the place value of the preceding column.

Now, consider the *decimal number* 12.3456, which consists of three parts: the whole number part, the decimal point, and the fractional part.



The whole number part of the decimal number is the part that lies strictly to the left of the decimal point, and the place value of each digit in the whole number part is given by the columns shown in Table 5.1.

The fractional part of the decimal number is the part that lies strictly to the right of the decimal point. As we saw in Table 1.3.1.1, each column has a value equal to 1/10 of the value of the column that lies to its immediate left. Thus, it should come as no surprise that:

- The first column to the right of the decimal point has place value 1/10 (tenths).
- The second column to the right of the decimal point has place value 1/100 (hundredths).
- The third column to the right of the decimal point has place value 1/1000 (thousandths).
- The fourth column to the right of the decimal point has place value 1/10000 (ten-thousandths).

These results are summarized for the decimal number 12.3456 in Table 1.3.1.2.

1	2	.	3	4	5	6
tens	ones	decimal point	tenths	hundredths	thousandths	ten-thousandths
10	1	.	1/10	1/100	1/1000	1/10000

Table 1.3.1.2: Place value.

Decimal Place Value

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains decimal place value by showing how each digit to the right of the decimal represents tenths, hundredths, thousandths, and beyond, helping you understand the value of each digit in a decimal number.

Expanded Form

Writing decimals in expanded form is similar to writing whole numbers in expanded form because each digit is expressed by its place value, but digits to the right of the decimal point are written as fractions over powers of ten (such as tenths, hundredths, and thousandths) to represent parts of a whole.

The decimal number 12.3456 is made up of 1 ten, 2 ones, 3 tenths, 4 hundredths, 5 thousandths, and 6 ten-thousandths, and can be written in *expanded form* as

$$12.3456 = 10 + 2 + \frac{3}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{6}{10000}.$$

Example 1

Write the decimal number 1,234.56 in expanded form.

Solution

$$1,234.56 = 1,000 + 200 + 30 + 4 + \frac{5}{10} + \frac{6}{100}$$

Exercise

Write the decimal number 3502.23 in expanded form.

Answer available at the end of this section.

Example 2

Write the decimal number 56.128 in expanded form.

Solution

$$56.128 = 50 + 6 + \frac{1}{10} + \frac{2}{100} + \frac{8}{1000}$$

Exercise

Write the decimal number 235.568 in expanded form.

Answer available at the end of this section.

Reading and Writing Decimal Numbers

How to Read a Decimal Number

1. Pronounce the whole number part to the left of the decimal as you would any whole number.
2. Say the word “and” for the decimal point.
3. State the fractional part to the right of the decimal as you would any whole number, followed by the place value of the digit in the rightmost column.

Example 3

Pronounce the decimal number 34.12.

Solution

The rightmost digit in the fractional part of 34.12 is in the hundredths column. Thus, 34.12 is pronounced “thirty-four and twelve hundredths.”

Exercise

Pronounce 28.73.

Answer available at the end of this section.

Important Point

In pronouncing decimal numbers, the decimal point is read as “and.” No other instance of the word “and” should appear in the pronunciation.

Example 4

Explain why “four hundred and thirty-four and two tenths” is an incorrect pronunciation of the decimal number 434.2.

Solution

The decimal point is read as “and.” No other occurrence of the word “and” is allowed in the pronunciation. The correct pronunciation should be “four hundred thirty-four and two tenths.”

Exercise

Pronounce 286.9.

Answer available at the end of this section.

Example 5

Pronounce the decimal number 5,678.123.

Solution

The rightmost digit in the fractional part of 5,678.123 is in the thousandths column. Hence, 5,678.123 is pronounced “five thousand six hundred seventy-eight and one hundred twenty-three thousandths.”

Exercise

Pronounce 7, 002.207

Answer available at the end of this section.

Example 6

Pronounce the decimal number 995.4325.

Solution

The rightmost digit in the fractional part of 995.4325 is in the ten-thousandths column. Hence, 995.4325 is pronounced "nine hundred ninety-five and four thousand three hundred twenty-five ten-thoudandths."

Exercise

Pronounce 500.1205.

Answer available at the end of this section.

Read and Write Decimals

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to read and write decimals using place value.

Decimals to Fractions

Because we now have the ability to pronounce decimal numbers, it is a simple exercise to change a decimal to a fraction. For example, 134.12 is pronounced "one hundred thirty-four and twelve hundredths," so it can easily be written as a mixed fraction.

$$134.12 = 134\frac{12}{100}$$

But this mixed fraction can be changed to an improper fraction.

$$134\tfrac{12}{100} = \tfrac{100\cdot 134+12}{100} = \tfrac{13400+12}{100} = \tfrac{13412}{100}$$

Note that the numerator is our original number without the decimal point. There are two decimal places in the original number and the denominator of the final improper fraction contains two zeros.

This discussion leads to the following result.

Changing Decimals to Improper Fractions

To change a decimal number to an improper fraction, proceed as follows:

1. Create a fraction.
2. Place the decimal number in the numerator without the decimal point.
3. Count the number of decimal places. Place an equal number of zeros in the denominator.

Example 7

Change the following decimal numbers to improper fractions: (a) 1.2345, and (b) 27.198.

Solution

In each case, place the number in the numerator without the decimal point. In the denominator, add a number of zeros equal to the number of decimal places.

a) The decimal number 1.2345 has four decimal places. Hence,

$$1.2345 = \frac{12345}{10000}$$

b) The decimal number 27.198 has three decimal places. Hence,

$$27.198 = \frac{27198}{1000}$$

Exercise

Change 17.205 to an improper fraction.

Answer available at the end of this section.

Example 8

Change each of the following decimals to fractions reduced to lowest terms: (a) 0.35, and (b) 0.125.

Solution

Place each number in the numerator without the decimal point. Place a number of zeros in the denominator equal to the number of decimal places. Reduce to lowest terms.

a) First, place 35 over 100.

$$0.35 = \frac{35}{100}$$

We can divide both numerator and denominator by the greatest common divisor.

$$\begin{aligned} &= \frac{35 \div 5}{100 \div 5} && \text{Divide numerator and denominator by 5.} \\ &= \frac{7}{20} && \text{Simplify numerator and denominator.} \end{aligned}$$

b) First, place 125 over 1000.

$$0.125 = \frac{125}{1000}$$

Prime factor and cancel common factors.

$$= \frac{5 \cdot 5 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} \quad \text{Prime factor numerator and denominator.}$$

$$= \frac{\cancel{5} \cdot \cancel{5} \cdot \cancel{5}}{2 \cdot 2 \cdot 2 \cdot \cancel{5} \cdot \cancel{5} \cdot \cancel{5}} \quad \text{Cancel common factors.}$$

$$= \frac{1}{8} \quad \text{Simplify.}$$

Exercise

Change 0.375 to a fraction, reduced to lowest terms.

Answer available at the end of this section.

Converting Base-10 Fractions

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View this YouTube Video online

This video explains how decimals represent fractions with denominators of powers of 10, showing how tenths, hundredths, and thousandths are written and interpreted in decimal form.

Comparing Decimals

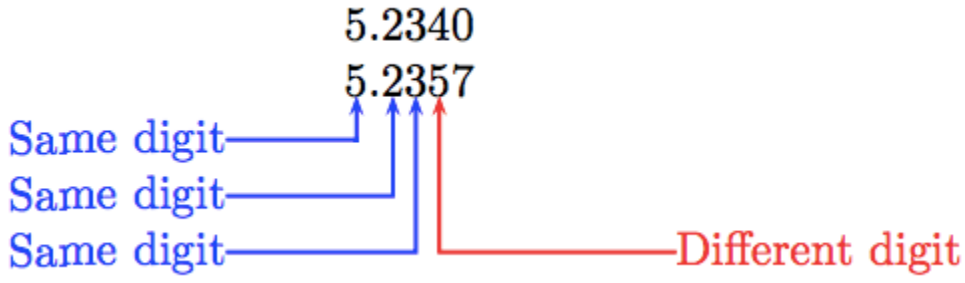
We can compare two positive decimals by comparing digits in each place as we move from left to right, place by place. For example, suppose we wish to compare the decimal numbers 5.234 and 5.2357. First, add enough trailing zeros to the decimal number with the fewer decimal places so that the numbers have the same number of decimal places. In this case, note that

$$5.234 = 5\frac{234}{1000} = 5\frac{2340}{10000} = 5.2340.$$

Important Observation

Adding trailing zeros to the end of the fractional part of a decimal number does not change its value.

Next, align the numbers as follows.



As you scan the columns, moving left to right, the first place that has different digits occurs in the thousandths place, where the digit 5 is the second number is greater than the digit 4 in the first number in the same place. Because 5 is greater than 4, the second number is larger than the first. That is:

$$5.234 < 5.2357$$

This discussion suggests the following algorithm.

Comparing Positive Decimal Numbers

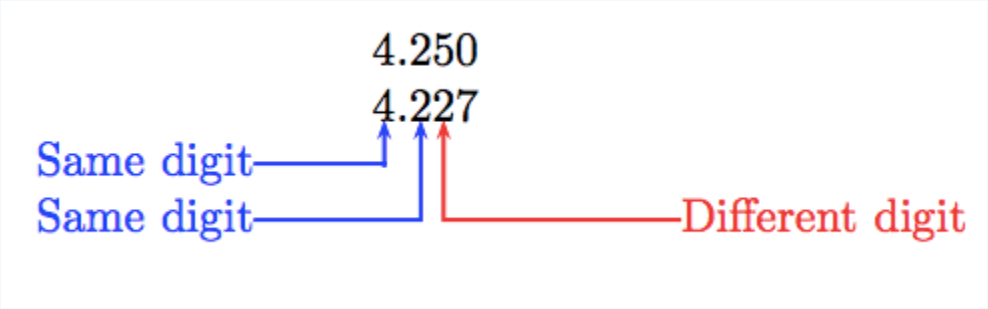
1. Add enough trailing zeros so that both numbers have the same number of decimal places.
2. Compare the digits in each place, moving left to right.
3. As soon as you find two digits in the same place that are different, the decimal number with the greatest digit in this place is the larger number.

Example 11

Compare 4.25 and 4.227.

Solution

Add a trailing zero to the first decimal number and align the numbers as follows.



The first difference is in the hundredths place, where the digit 5 in the first number is greater than the digit 2 in the same place of the second number. Hence, the first number is larger than the second; that is:

$$4.25 > 4.227$$

Exercise

Compare 8.34 and 8.348.

Answer available at the end of this section.

Comparing Decimal Numbers

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View this YouTube Video online

This video explains how to compare decimals by aligning the numbers at the decimal point and then comparing digits from left to right to determine which value is greater or less.

When comparing negative numbers, the number with the larger magnitude is the smaller number. Hence, we have to adjust our algorithm for comparing negative decimal numbers.

Comparing Negative Decimal Numbers

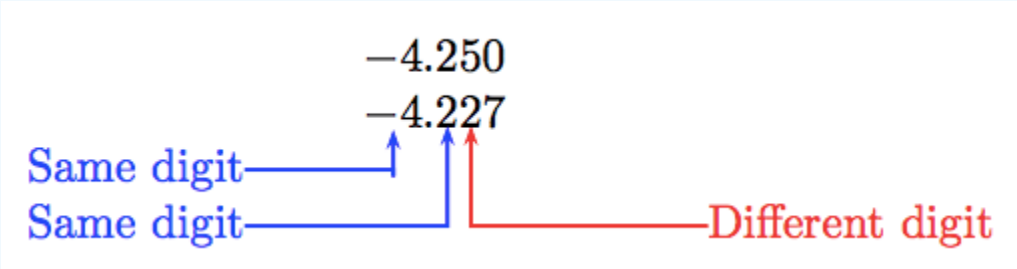
1. Add enough trailing zeros so that both numbers have the same number of decimal places.
2. Compare the digits in each place, moving left to right.
3. As soon as you find two digits in the same place that are different, the decimal number with the greatest digit in this place is the smaller number.

Example 12

Compare –4.25 and –4.227.

Solution

Add a trailing zero to the first decimal number and align the numbers as follows.



The first difference is in the hundredths place, where the digit 5 in the first number is greater than the digit 2 in the same place of the second number. Hence, the first number is **smaller** than the second; that is:

$$-4.25 < -4.227$$

Exercise

Compare –7.86 and –7.85.

Answer available at the end of this section.

Answers

Exercise

$3,000 + 500 + 2 + \frac{2}{10} + \frac{3}{100}$

Exercise

$200 + 30 + 5 + \frac{5}{10} + \frac{6}{100} + \frac{8}{1000}$

Exercise

“Twenty-eight and seventy-three hundredths”

Exercise

"Two hundred eighty six and nine tenths"

Exercise

Answer: “Seven thousand two and two hundred seven thousandths.”

Exercise

Answer: “Five hundred and one thousand two hundred five ten-thousandths.”

Exercise

$\frac{17205}{100}$

Exercise

3/8

Exercise

8.34 < 8.348

Exercise

–7.86 < –7.85

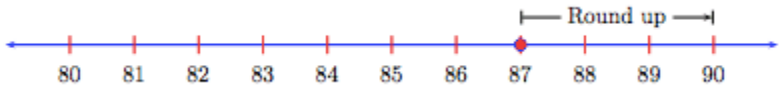
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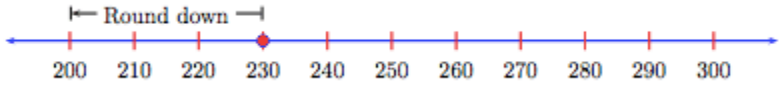
1.3.2: Rounding Whole Numbers and Decimals

Rounding Whole Numbers

When less precision is needed, we round numbers to a particular place. For example, suppose a store owner needs approximately 87 boxes of ten-penny nails, but they can only be ordered in cartons containing ten boxes.



Note that 87 is located closer to 9 tens (or 90) than it is to 8 tens (or 80). Thus, rounded to the nearest ten, 87 ≈ 90 (87 approximately equals 90). The store owner decides that 90 boxes is probably a better fit for his needs. On the other hand, the same store owner estimates that he will need 230 bags of peatmoss for his garden section.



Note that 230 is closer to 2 hundreds (or 200) than it is to 3 hundreds (or 300). The store owner worries that might have overestimated his need, so he rounds down to the nearest hundred, 230 ≈ 200 (230 approximately equals 200). There is a simple set of rules to follow when rounding.

Rules for Rounding

To round a number to a particular place, follow these steps:

1. Mark the place you wish to round to. This is called the rounding digit.

2. Check the next digit to the right of your digit marked in step 1. This is called the test digit.

    a) If the test digit is greater than or equal to 5, add 1 to the rounding digit and replace all digits to the right of the rounding digit with zeros.

    b) If the test digit is less than 5, keep the rounding digit the same and replace all digits to the right of the rounding digit with zero

Let's try these rules with an example or two.

Example 1

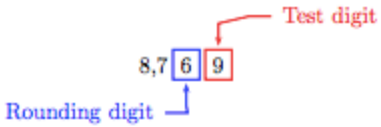
Round the number 8,769 to the nearest ten.

Solution

Mark the rounding and test digits.

Exercise

Round the number 9,443 to the nearest ten.



The test digit is greater than 5. The “Rules for Rounding” require that we add 1 to the rounding digit, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest ten,

$$8,769 \approx 8,770.$$

That is, 8,769 is *approximately* equal to 8,770.

Mathematical Notation.

The symbol

$$\approx$$

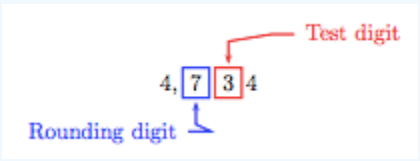
means *approximately equal*.

Example 2

Round the number 4,734 to the nearest hundred.

Solution

Mark the rounding and test digits.



The test digit is less than 5. The “Rules for Rounding” require that we keep the rounding digit the same, then make all digits to the right of the rounding digit zeros. Thus, rounded to the nearest hundred,

$$4,734 \approx 4,700.$$

Exercise 1.3.2.1

Round the number 6,656 to the nearest hundred.

Answer available at the end of this section.

Answer available at the end of this section.

Rounding to the Various Place Values

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This video demonstrates how to round the whole number 7,345,862 to various place values.

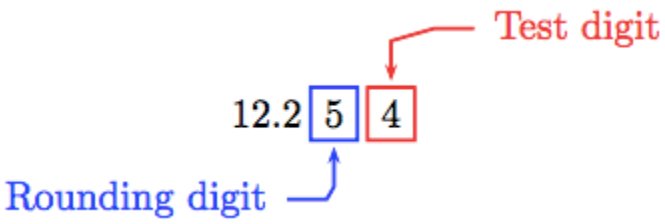
Rounding Decimals

The rules for rounding decimal numbers are almost identical to the rules for rounding whole numbers. First, a bit of terminology.

Rounding Digit and Test Digit

The digit in the place to which we wish to round is called the *rounding digit* and the digit that follows on its immediate right is called the *test* digit.

If we want to round the decimal number 12.254 to the nearest hundredth, then the rounding digit is 5 and the test digit is 4.



If we used the rules for rounding whole numbers, because the test digit 4 is less than 5, we would replace all digits to the right of the rounding digit with zeros to obtain the following approximation.

$$12.254 \approx 12.250$$

However, because

$$12.250 = 12\frac{250}{1000} = 12\frac{25}{100},$$

because trailing zeros at the end of a decimal do not change its value. Hence, we *truncate* every digit after the rounding digit and use the following approximation.

$$12.254 \approx 12.25$$

Important Observation

Deleting trailing zeros from the end of the fractional part of a decimal number does not change its value.

The above discussion motivates the following algorithm for rounding decimal numbers.

Rounding Decimal Numbers

Locate the *rounding digit* and the *test digit*.

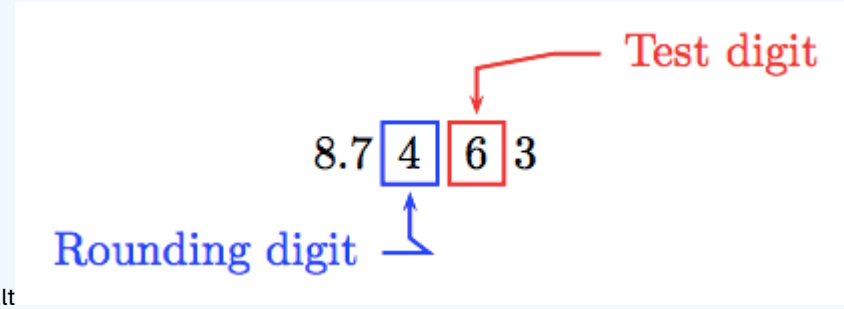
- If the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate all digits to the right of the rounding digit.
- If the test digit is less than 5, simply truncate all digits to the right of the rounding digit.

Example 3

Round 8.7463 to the nearest hundredth.

Solution

Locate the rounding digit in the hundredths place and the test digit to its immediate right.



alt

Because the test digit is greater than 5, add 1 to the rounding digit and truncate all digits to the right of the rounding digit. Hence, to the nearest hundredth:

$$8.7463 \approx 8.75$$

Exercise

Round 9.2768 to the nearest hundredth.

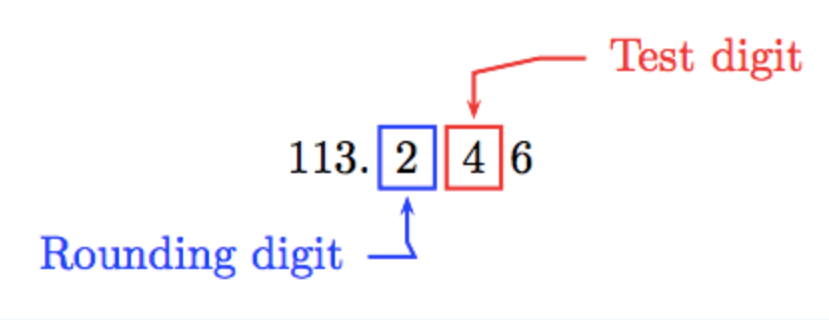
Answer available at the end of this section.

Example 4

Round 113.246 to the nearest tenth.

Solution

Locate the rounding digit in the tenths place and the test digit to its immediate right.



Because the test digit is less than 5, truncate all digits to the right of the rounding digit. Hence, to the nearest tenth:

$$113.246 \approx 113.2$$

Exercise

Round 58.748 to the nearest tenth.

Answer available at the end of this section.

Rounding

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View this YouTube Video online

This video includes an overview of rounding and includes whole number and decimal examples.

Answers

Exercise 1.3.2.1

Exercise 1.3.2.1

6,700

Exercise

9.28

Exercise

58.7

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1.3.3: Calculator Skills Involving Decimals

In this lesson, you will practice using a calculator to perform operations with decimals and utilize various strategies to determine the reasonableness of your answers.

Using Estimation to Evaluate the Reasonableness of Your Answer

Estimation is one strategy that you can use to determine if your final answer makes sense.

Definition: Estimation

Estimation is the process of determining an expected value of a computation.

Common words used in estimation are *about*, *near*, and *between*.

One strategy that we can use when estimating a result is by rounding each number.

In the rounding process, it is important to note two facts:

1. The rounding that is done in estimation does not always follow the rules of rounding discussed in the previous lesson on rounding whole numbers and decimals. Since estimation is concerned with the expected value of a computation, rounding is done using *convenience* as the guide rather than using hard-and-fast rounding rules. For example, when estimating a division problem, it may be more convenient to divide by 20 than by 30.
2. Since rounding may occur out of convenience, and different people have different ideas of what may be convenient, results of an estimation done by rounding may vary. For a particular computation, different people may get different estimated results. *Results may vary*.

Example 1

Use your calculator to calculate 5.7 + 11.9. Then estimate the sum to determine whether your answer is reasonable.

Answer available at the end of this section.

Example 2

Use your calculator to calculate 5.123 + 18.47. Then estimate the sum to determine whether your answer is reasonable.

Answer available at the end of this section.

Example 3

Use your calculator to calculate 50 – 37.42. Then estimate the difference to determine whether your answer is reasonable.

Answer available at the end of this section.

Example 4

Use your calculator to calculate 4.77 – 6.3. Then estimate the difference to determine whether your answer is reasonable.

Answer available at the end of this section.

Example 5

Use your calculator to calculate 4.5(6.107). Then estimate the product to determine whether your answer is reasonable.

Answer available at the end of this section.

Using Number Sense to Evaluate the Reasonableness of Your Answer

In the previous examples, we used estimation to evaluate the reasonableness of our answers because we were:

- adding or subtracting numbers
- multiplying numbers
- working with values that were easy to round

In these cases, we could round the numbers to simpler values and perform the calculation mentally.

But, using estimation is not always practical. Sometimes it is better to rely on number sense when we are:

- dividing with decimals
- working with multi-step expressions
- estimating would be difficult or confusing

In these cases, think about the size of the numbers and how the operation affects the result. For example:

- Dividing by a number greater than 1 makes the result smaller.
- Dividing by a number less than 1 makes the result larger.

If your answer fits what you expect, it is likely reasonable.

Example 6

Use your calculator to calculate 0.04 × 0.087. Then use estimation or number sense to determine if the answer is reasonable.

Answer available at the end of this section.

Example 7

Use your calculator to calculate 0.28 ÷ 4. Then use estimation or number sense to determine if the answer is reasonable.

Answer available at the end of this section.

Example 8

Use your calculator to calculate −23.492 ÷ (−0.04). Then use estimation or number sense to determine if the answer is reasonable.

Answer available at the end of this section.

For more complex expressions, focus on entering the expression carefully and checking that your answer makes sense based on the size of the numbers.

Exercise 9

Use your calculator to calculate $30(0.4) + (0.5)^2$.

Answer available at the end of this section.

Exercise 10

Use your calculator to calculate $1.8(22.4 + 2.6)$.

Answer available at the end of this section.

Exercise 11

Use your calculator to calculate $16 \div (0.6 + 0.4)$.

Answer available at the end of this section.

Exercise 12

Use your calculator to calculate $(3.2 + 0.8)(0.9 - 0.3)$.

Answer available at the end of this section.

Answers

Example 1

Using a calculator, we find that the sum is 17.6.

To check that this result is reasonable, we can estimate by rounding 5.7 to 6 and 11.9 to 12. Then 6 + 12 = 18, which is close to 17.6, so the answer is reasonable.

Example 2

Using a calculator, we find that the sum is 23.593.

To check that this result is reasonable, we can estimate by rounding 5.123 to 5 and 18.47 to 18. Then 5 + 18 = 23, which is close to 23.593, so the answer is reasonable

Example 3

Using a calculator, we find that the difference is 12.58.

To check that this result is reasonable, we can estimate by rounding 37.42 to 40 instead of 38, since $50 - 40$ is easier to mentally calculate than $50 - 38$. Then $50 - 40 = 10$, which is close to 12.58, so the answer is reasonable.

Example 4

Using a calculator, we find that the difference is -1.53.

To check that this result is reasonable, we can estimate by rounding 4.77 to 5 and 6.3 to 6. Then 5 – 6 = -1, which is close to -1.53, so the answer is reasonable.

Example 5

Using a calculator, we find that the product is 27.4815.

To check that this result is reasonable, we can estimate by rounding 4.5 to 5 and 6.107 to 6. Then 5(6) = 30, which is close to 27.4815, so the answer is reasonable.

Example 6

Using a calculator, we find that the product is 0.00348.

Since both numbers are less than 1, the product must be very small. Multiplying the leading digits gives 4 × 9 = 36, so the result should be around 0.003 to 0.004. Because 0.00348 is in this range, the answer is reasonable.

Example 7

Using a calculator, we find that the quotient is 0.07.

Since 0.28 is less than 1, dividing by 4 should give a smaller number. Since 28 ÷ 4 = 7, the result 0.07 makes sense.

Example 8

Using a calculator, we find that the quotient is 587.3.

To check that this result is reasonable, note that a negative divided by a negative gives a positive result. Also, dividing by a number less than 1 makes the result larger. Since 587.3 is much larger than 23.492, the answer is reasonable.

Exercise 9

Using a calculator, we find that the quotient is 12.25.

Exercise 10

Using a calculator, we find that the quotient is 45.

Exercise 11

Using a calculator, we find that the quotient is 16.

Exercise 12

Using a calculator, we find that the quotient is 2.4.

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1.3.4: Fractions and Decimals

So far, we have worked primarily with decimals and used place value, rounding, and estimation to understand them. In many situations, decimals and fractions are used interchangeably. In this section, we will convert fractions to decimals and evaluate expressions that include both forms.

When converting a fraction to a decimal, only one of two things can happen. Either the process will terminate or the decimal representation will begin to repeat a pattern of digits. In each case, the procedure for changing a fraction to a decimal is the same.

Changing a Fraction to a Decimal

To change a fraction to a decimal, divide the numerator by the denominator. *Hint: If you first reduce the fraction to lowest terms, the numbers will be smaller and the division will be a bit easier as a result.*

Terminating Decimals

Terminating Decimals

First reduce the fraction to lowest terms. If the denominator of the resulting fraction has a prime factorization consisting of strictly twos and/or fives, then the decimal representation will "terminate."

Example 1

Change 15/48 to a decimal.

Solution

First, reduce the fraction to lowest terms.

$$\begin{aligned}\frac{15}{48} &= \frac{3 \cdot 5}{3 \cdot 16} \\ &= \frac{5}{16}\end{aligned}$$

Next, note that the denominator of 5/16 has prime factorization 16 = 2·2·2·2. It consists only of twos. Hence, the decimal representation of 5/16 should terminate.

$$\begin{array}{r} 0.3125 \\ 16\overline{)5.0000} \\ \underline{4\ 8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

The zero remainder terminates the process. Hence, 5/16 = 0.3125.

Exercise

Change 10/16 to a decimal.

Answer available at the end of this section.

Example 2

Change $3\frac{7}{20}$ to a decimal.

Solution

Note that 7/20 is reduced to lowest terms and its denominator has prime factorization 20 = 2 · 2 · 5. It consists only of twos and fives. Hence, the decimal representation of 7/20 should terminate.

$$\begin{array}{r} 0.35 \\ 20\overline{)7.00} \\ \underline{6\ 0} \\ 1\ 00 \\ \underline{1\ 00} \\ 0 \end{array}$$

The zero remainder terminates the process. Hence, 7/20 = 0.35. Therefore, $3\frac{7}{20}$ = 3.35.

Exercise

Change $7\frac{11}{20}$ to a decimal.

Answer available at the end of this section.

Converting Any Fraction

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video includes several examples of how to convert a fraction to a decimal using division. In the video, there are examples that result in a terminating decimal and examples that result in a repeating decimal.

Repeating Decimals

Repeating Decimals

First reduce the fraction to lowest terms. If the prime factorization of the resulting denominator does not consist strictly of twos and fives, then the division process will never have a remainder of zero. However, repeated patterns of digits must eventually reveal themselves.

Example 3

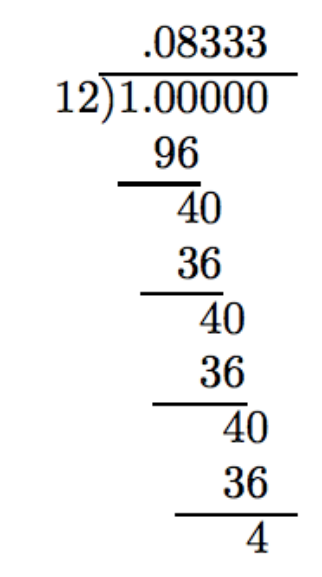
Change 1/12 to a decimal.

Solution

Note that 1/12 is reduced to lowest terms and the denominator has a prime factorization $12 = 2 \cdot 2 \cdot 3$ that does not consist strictly of twos and fives. Hence, the decimal representation of 1/12 will not "terminate." We need to carry out the division until a remainder reappears for a second time. This will indicate repetition is beginning.

$$
\begin{array}{r}
.083 \\
12\overline{)1.000} \\
\underline{96} \\
40 \\
\underline{36} \\
4
\end{array}
$$

Note the second appearance of 4 as a remainder in the division above. This is an indication that repetition is beginning. However, to be sure, let's carry the division out for a couple more places.



Note how the remainder 4 repeats over and over. In the quotient, note how the digit 3 repeats over and over. It is pretty evident that if we were to carry out the division a few more places, we would get

$$\frac{1}{12} = 0.833333\cdots$$

The ellipsis is a symbolic way of saying that the threes will repeat forever. It is the mathematical equivalent of the word "etcetera."

Exercise

Change 5/12 to a decimal.

Answer available at the end of this section.

There is an alternative notation to the ellipsis, namely

$$\frac{1}{12} = 0.08\overline{3}.$$

The bar over the 3 (called a "repeating bar") indicates that the 3 will repeat indefinitely. That is,

$$0.08\overline{3} = 0.083333....$$

Using the Repeating Bar

To use the repeating bar notation, take whatever block of digits are under the repeating bar and duplicate that block of digits infinitely to the right.

Thus, for example:

- $5.3\overline{45} = 5.3454545....$
- $0.\overline{142857} = 0.142857142857142857....$

Important Observation

Although $0.8\overline{33}$ will also produce 0.8333333 ..., as a rule we should use as few digits as possible under the repeating bar. Thus, $0.8\overline{3}$ is preferred over $0.8\overline{33}$.

Example 4

Change 23/111 to a decimal. Write your answer as a repeating decimal using a bar, then round the decimal to the nearest hundredth.

Solution

The denominator of 23/111 has prime factorization 111 = 3 ·37 and does **not** consist strictly of twos and fives. Hence, the decimal representation will not "terminate." We need to perform the division until we spot a repeated remainder.

$$\begin{array}{r} 0.207 \\ 111\overline{)23.000} \\ 22\,2 \\ \hline 800 \\ 777 \\ \hline 23 \end{array}$$

Note the return of 23 as a remainder. Thus, the digit pattern in the quotient should start anew, but let's add a few places more to our division to be sure.

$$\begin{array}{r} 0.207207 \\ 111\overline{)23.000000} \\ 22\,2 \\ \hline 800 \\ 777 \\ \hline 230 \\ 222 \\ \hline 800 \\ 777 \\ \hline 23 \end{array}$$

Aha! Again a remainder of 23. Repetition! At this point, we are confident that

$$\frac{23}{111} = 0.207207....$$

Using a "repeating bar," this result can be written

$$\frac{23}{111} = 0.\overline{207}.$$

Rounded to the nearest hundredth,

$$\frac{23}{111} \approx 0.21$$

Exercise

Change 5/33 to a decimal. Write your answer as a repeating decimal using a bar, then round the decimal to the nearest hundredth.

Answer available at the end of this section.

Expressions Containing Both Decimals and Fractions

At this point we can convert fractions to decimals, and vice-versa, we can convert decimals to fractions. Therefore, we should be able to evaluate expressions that contain a mix of fraction and decimal numbers.

Note: Calculator Tip

If you prefer to use a calculator, enter the expression exactly as written, using parentheses when needed, and confirm that your result matches the solution shown.

Example 5

Simplify: $-\frac{3}{8} - 1.25$.

Solution

Let's change 1.25 to an improper fraction.

$$\begin{aligned} 1.25 &= \frac{125}{100} && \text{Two decimal places} \Rightarrow \text{two zeroes.} \\ &= \frac{5}{4} && \text{Reduce to lowest terms.} \end{aligned}$$

In the original problem, replace 1.25 with 5/4, make equivalent fractions with a common denominator, then subtract.

$$\begin{aligned} -\frac{3}{8} - 1.25 &= -\frac{3}{8} - \frac{5}{4} && \text{Replace 1.25 with 5/4.} \\ &= -\frac{3}{8} - \frac{5 \cdot 2}{4 \cdot 2} && \text{Equivalent fractions, LCD} = 8. \\ &= -\frac{3}{8} - \frac{10}{8} && \text{Simplify the numerator and denominator.} \\ &= -\frac{3}{8} + \left(-\frac{10}{8}\right) && \text{Add the opposite.} \\ &= \frac{13}{8} && \text{Add.} \end{aligned}$$

Thus, –3/8 – 1.25 = –13/8.

Alternate Solution

Because –3/8 is reduced to lowest terms and 8 = 2 ·2 ·2 consists only of twos, the decimal representation of –3/8 will terminate.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ \underline{2\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \end{array}$$

Hence, –3/8 = –0.375. Now, replace –3/8 in the original problem with –0.375, then simplify.

$$\begin{aligned} -\frac{3}{8} - 1.25 &= -0.375 - 1.25 && \text{Replace } -3/8 \text{ with } -0.375. \\ &= -0.375 + (-1.25) && \text{Add the opposite.} \\ &= -1.625 && \text{Add.} \end{aligned}$$

Thus, –3/8 – 1.25 = –1.625.

Are They the Same?

The first method produced –13/8 as an answer; the second method produced –1.625. Are these the same results? One way to find out is to change –1.625 to an improper fraction.

$$\begin{aligned} -1.625 &= -\frac{1625}{1000} && \text{Three places } \Rightarrow \text{ three zeroes.} \\ &= -\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 13}{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5} && \text{Prime factor.} \\ &= -\frac{13}{2 \cdot 2 \cdot 2} && \text{Cancel common factors.} \\ &= -\frac{13}{8} && \text{Simplify.} \end{aligned}$$

Thus, the two answers are the same.

Exercise

Simplify: $-\frac{7}{8} - 6.5$

Answer available at the end of this section.

Example 6

Simplify: $-\frac{2}{3} + 0.35$.

Solution

Let's attack this expression by first changing 0.35 to a fraction.

$$-\frac{2}{3}+0.35=-\frac{2}{3}+\frac{35}{100} \quad \text{Change 0.35 to a fraction.}$$
$$=-\frac{2}{3}+\frac{7}{20} \quad \text{Reduce 35/100 to lowest terms.}$$

Find an LCD, make equivalent fractions, then add.

$$=-\frac{2\cdot 20}{3\cdot 20}+\frac{7\cdot 3}{20\cdot 3} \quad \text{Equivalent fractions with LCD = 60.}$$
$$=-\frac{40}{60}+\frac{21}{60} \quad \text{Simplify numerators and denominators.}$$
$$=-\frac{19}{60} \quad \text{Add.}$$

Then, $-\frac{2}{3}+0.35=-\frac{19}{60}$.

Exercise

Simplify: $-\frac{4}{9}+0.25$

Answer available at the end of this section.

In Example 6, we run into trouble if we try to change –2/3 to a decimal. The decimal representation for –2/3 is a repeating decimal (the denominator is not made up of only twos and fives). Indeed, –2/3 = $-0.\overline{6}$. To add $-0.\overline{6}$ and 0.35, we have to align the decimal points, then begin adding at the right end. But $-0.\overline{6}$ has no right end! This observation leads to the following piece of advice.

Important Observation

When presented with a problem containing both decimals and fractions, if the decimal representation of any fraction repeats, its best to first change all numbers to fractions, then simplify.

Answers

Exercise

0.625

Exercise

7.55

Exercise

0.41666...

Exercise

$0.\overline{15}$

Rounded to the nearest hundredth, $\frac{5}{33}\approx 0.15$

Exercise

$-7\frac{3}{8}$ or –7.375

Exercise

–7/36

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1.3.5: Decimals Practice

The following set of exercises provides practice for the concepts covered in Module 1.3: Decimals. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 1.3.1 - Understanding Decimals

Identify the requested digit of the given number.

1. Which digit is in the tenths column of the number 9279.0075?

2. Which digit is in the hundredths column of the number 1743.1634?

Write the given decimal number in expanded form.

4. 68.392

5. 815.64

Write the decimal in word form.

6. 974.35

7. 0.6879

Convert each decimal to a fraction. Simplify your answer and write it as a mixed number when needed.

8. 0.74

9. 1.24

10. 0.375

11. 2.125

12. 3.6

Fill in < or > for each of the following pairs of numbers.

13. 8.5934 ___ 8.554

14. 514.873553 ___ 514.87374

15. 0.000681 ___ 0.00043174

Section 1.3.2 - Rounding Whole Numbers and Decimals

16. Round 56872.21 to the nearest thousand.

17. Round 42.841 to the nearest tenth.

18. Round 53.794 to the nearest hundredth.

Section 1.3.3 - Calculator Skills Involving Decimals

Estimate each expression by rounding to convenient values. Write the new expression you used to estimate.

19. 17.45 + 12.974

20. 59.08 – 4.6

21. (9.4)(12.15)

Evaluate each expression using a calculator.

22. –19.47 – 32.58

23. 94.69 – (– 12.678)

24. (– 8.5)(1.69)

25. 12.04 ÷ 43

26. –1.15 ÷ (– 0.05)

27. $35(0.2) + (0.9)^2$

28. 1.18(46.22 + 3.71)

Section 1.3.4 - Fractions and Decimals

Convert the given fraction to a decimal. Round to the nearest hundredths, if needed.

29. $\frac{14}{5}$

30. $\frac{15}{16}$

31. $\frac{8}{9}$

32. $\frac{65}{36}$

Simplify the given expression. Answer with a simplified fraction.

33. $\frac{3}{2} - 2.73$

34. $\frac{3}{4} + 3.7$

35. $-\frac{11}{6} + 1.12$

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1.4: ROOTS AND SCIENTIFIC NOTATION

Some problems involve numbers that grow very quickly or become very small, making them difficult to work with using standard notation. Square roots, cube roots, and scientific notation provide different ways to understand and manage these values.

These ideas are especially useful in technical settings where scale matters. Whether working with measurements, interpreting data, or describing very large or very small quantities, it is important to recognize how numbers behave and how they can be represented more efficiently.

In this module, the focus is on understanding how roots relate to repeated multiplication and how they can be used to describe values. You will determine square roots of perfect squares, estimate square roots of non-perfect squares, and find cube roots of perfect cubes.

You will also learn to represent very large and very small numbers using scientific notation, making them easier to read, write, and compare.

Folding Paper Can Get You To the Moon

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View this YouTube Video online

This video explores how quantities grow when repeatedly multiplied, helping build an understanding of scale and the size of very large numbers.

1.4.1: Introduction to Square Roots

1.4.2: Introduction to Cube Roots

1.4.3: Introduction to Scientific Notation

1.4.4: Roots and Scientific Notation Practice

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1.4.1: Introduction to Square Roots

Earlier, we saw that squaring a number means multiplying the number by itself. For example, $3^2 = 9$ and $(-4)^2 = 16$.

In this section, we will use perfect squares to help us understand square roots.

Knowing common perfect squares will help you recognize square roots more quickly. Some of the most common are $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81,$ and $10^2 = 100$.

These values will be especially useful as we begin working with square roots.

Square Roots

A square root reverses the process of squaring a number. In other words, it answers the question: what number multiplied by itself gives a given number?

Square Root

If $a^2 = b$, then a is called a square root of the number b.

All positive numbers have two square roots—one positive and one negative. For example, since $3^2 = 9$, both 3 and -3 are square roots of 9.

Example 1

Find the square roots of the number 49.

Solution

To find a square root of 49, we must think of a number a such that $a^2 = 49$. Two numbers come to mind.

- $(-7)^2 = 49$. Therefore, –7 is a square root of 49.
- $7^2 = 49$. Therefore, 7 is a square root of 49.

Note that 49 has two square roots, one positive and one negative.

Exercise

Find the square roots of 256.

Answer available at the end of this section.

Example 2

Find the square roots of the number 196.

Solution

To find a square root of 196, we must think of a number a such that $a^2 = 196$. With help from the "List of Squares," two numbers come to mind.

- $(-14)^2 = 196$. Therefore, -14 is a square root of 196.
- $14^2 = 196$. Therefore, 14 is a square root of 196.

Note that 196 has two square roots, one of which is positive and the other one is negative.

Exercise

Find the square roots of 625.

Answer available at the end of this section.

Example 3

Find the square roots of the number 0.

Solution

To find a square root of 0, we must think of a number a such that $a^2 = 0$. There is only one such number, namely zero. Hence, 0 is the square root of 0.

Exercise

Find the square roots of 9.

Answer available at the end of this section.

Example 4

Find the square roots of the number -25.

Solution

To find a square root of -25, we must think of a number a such that $a^2 = -25$. This is impossible because no square of a real number (whole number, integer, fraction, or decimal) can be negative. Positive times positive is positive and negative times negative is also positive. You cannot square and get a negative answer. Therefore, -25 has no square roots.

Exercise

Find the square roots of -81.

Answer available at the end of this section.

Radical Notation

Because $(-3)^2 = 9$ and $3^2 = 9$, both -3 and 3 are square roots of 9. Special notation, called *radical notation*, is used to request these square roots.

- The radical notation $\sqrt{9}$, pronounced “the nonnegative square root of 9,” calls for the nonnegative square root of 9. Hence,

$$\sqrt{9} = 3.$$

- The radical notation $-\sqrt{9}$, pronounced “the negative square root of 9,” calls for the negative square root of 9. Hence,

$$-\sqrt{9} = -3.$$

Radical Notation

In the expression $\sqrt{9}$, the symbol $\sqrt{\ }$ is called a *radical* and the number within the radical, in this case the number 9, is called the *radicand*.

For example,

- In the expression $\sqrt{529}$, the number 529 is the radicand.
- In the expression $\sqrt{a^2 + b^2}$, the expression $a^2 + b^2$ is the radicand.

Radical Notation and Square Root

If b is a positive number, then

$\sqrt{b}$ is the positive square root of b, and

$-\sqrt{b}$ is the negative (opposite) square root of b.

Example 5

Simplify: (a) $\sqrt{121}$, (b) $-\sqrt{625}$, and (c) $\sqrt{0}$.

Solution

(a) Referring to the list of squares, we note that 11^2 = 121 and $(-11)^2$ = 121. Therefore, both 11 and –11 are square roots of 121. However, $\sqrt{121}$ calls for the nonnegative square root of 121. Thus,

$$\sqrt{121} = 11.$$

(b) Referring to the list of squares, we note that 25^2 = 625 and $(-25)^2$ = 625. Therefore, both 25 and –25 are square roots of 625. However, $-\sqrt{625}$ calls for the negative square root of 625. Thus,

$$-\sqrt{625} = -25.$$

(c) There is only one square root of zero. Therefore,

$$\sqrt{0} = 0.$$

Exercise

Simplify: a) $\sqrt{144}$ b) $-\sqrt{324}$

Answer available at the end of this section.

Example 6

Simplify: (a) $-\sqrt{25}$, and (b) $\sqrt{-25}$

Solution

(a) Because $5^2 = 25$ and $(-5)^2 = 25$, both 5 and –5 are square roots of 25. However, the notation $-\sqrt{25}$ calls for the negative square root of 25. Thus, $-\sqrt{25} = -5$.

(b) It is not possible to square a real number (whole number, integer, fraction, or decimal) and get –25. Therefore, there is no real square root of –25. That is, $\sqrt{-25}$ is not a real number.

Exercise

Simplify: a) $-\sqrt{36}$ b) $\sqrt{-36}$

Answer available at the end of this section.

Exponents and Square Roots

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces radical notation and explains how to find square roots using perfect squares and calculator methods.

Estimating Square Roots

The squares in the "List of Squares" are called *perfect squares*. Each is the square of a whole number. Not all numbers are perfect squares. For example, in the case of $\sqrt{24}$, there is no whole number whose square is equal to 24. However, this does not prevent $\sqrt{24}$ from being a perfectly good number.

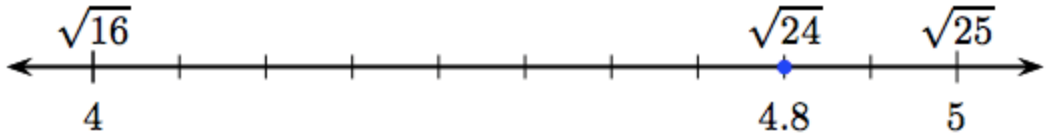
We can use the "List of Squares" to find decimal approximations when the radicand is not a perfect square.

Example 11

Estimate $\sqrt{24}$ by guessing. Use a calculator to find a more accurate result and compare this result with your guess.

Solution

From the "List of Squares," note that 24 lies betwen 16 and 25, so 24 will lie between 4 and 5, with √24 much closer to 5 than it is to 4.



Let's guess

$$\sqrt{24} \approx 4.8.$$

As a check, let's square 4.8.

$$(4.8)^2 = (4.8)(4.8) = 23.04$$

Not quite 24! Clearly, $\sqrt{24}$ must be a little bit bigger than 4.8.

Let's use a scientific calculator to get a better approximation. From our calculator, using the square root button, we find

$$\sqrt{24} \approx 4.89897948557.$$

Even though this is better than our estimate of 4.8, it is still only an approximation. Our calculator was only capable of providing 11 decimal places. However, the exact decimal representation of $\sqrt{24}$ is an infinite decimal that never terminates and never establishes a pattern of repetition.

If you were to multiply this number by itself (square the number), you would get a number that is extremely close to 24, but it would not be exactly 24.

Exercise

Use your calculator to estimate: $\sqrt{83}$. Round to the nearest tenth.

Answer available at the end of this section.

Important Observation

A calculator can only produce a finite number of decimal places. If the decimal representation of your number does not terminate within this limited number of places, then the number in your calculator window is only an approximation.

Order of Operations

With the addition of radical notation, the *Rules Guiding Order of Operations* change slightly.

Rules Guiding Order of Operations

When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents and radicals that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

The only change in the rules is in item #2, which says: “Evaluate all exponents and radicals that appear in the expression,” putting radicals on the same level as exponents.

Example 7

Simplify: $-3\sqrt{9}+12\sqrt{4}$.

Solution

According to the *Rules Guiding Order of Operations*, we must evaluate the radicals in this expression first.

$$\begin{aligned} -3\sqrt{9}+12\sqrt{4} &= -3(3)+12(2) && \text{Evaluate radicals first: } \sqrt{9}=3 \\ & && \text{and } \sqrt{4}=2. \\ &= -9+24 && \text{Multiply: } -3(3)=-9 \text{ and } 12(2)=24. \\ &= 15 && \text{Add: } -9+24=15. \end{aligned}$$

Exercise

Simplify: $2\sqrt{4}-3\sqrt{9}$

Answer available at the end of this section.

Example 8

Simplify: $-2-3\sqrt{36}$.

Solution

According to the *Rules Guiding Order of Operations,* we must evaluate the radicals in this expression first, moving left to right.

$$\begin{aligned} -2-3\sqrt{36} &= -2-36 && \text{Evaluate radicals first: } \sqrt{36}=6 \\ &= -2-18 && \text{Multiply: } 3(6)=18. \\ &= -20 && \text{Subtract: } -2-18=-2+(-18)=-20. \end{aligned}$$

Exercise

Simplify: $5-8\sqrt{169}$

Answer available at the end of this section.

Example 9

Simplify: (a) $\sqrt{9+16}$ and (b) $\sqrt{9}+\sqrt{16}$.

Solution

Apply the *Rules Guiding Order of Operations*.

a) In this case, the radical acts like grouping symbols, so we must evaluate what is inside the radical first.

$$\begin{aligned}\sqrt{9+16} &= \sqrt{25} && \text{Add: } 9+16=25.\\ &= 5 && \text{Take nonnegative square root: } \sqrt{25}=5.\end{aligned}$$

b) In this example, we must evaluate the square roots first.

$$\begin{aligned}\sqrt{9}+\sqrt{16} &= 3+4 && \text{Square root: } \sqrt{9}=3 \text{ and } \sqrt{16}=4.\\ &= 7 && \text{Add: } 3+4=7.\end{aligned}$$

Exercise

Simplify: a) $\sqrt{25+144}$ b) $\sqrt{25}+\sqrt{144}$

Answer available at the end of this section.

Sometimes you may need to use a calculator to simplify expressions with square roots. It is best to wait to round until the final answer. If you evaluate step-by-step, be sure to keep enough decimal places so your final answer is accurate.

Example 10

Simplify: $3\sqrt{7}-2\sqrt{5}$. Round to the nearest hundredth.

Solution

Sometimes you may need to use a calculator to simplify expressions with square roots. It is best to wait to round until the final answer. If evaluating step-by-step, keep several decimal places during your work.

Evaluate the radicals using a calculator:

$\sqrt{7}\approx 2.6458 \quad \text{and} \quad \sqrt{5}\approx 2.2361$

Now multiply:

$3(2.6458)\approx 7.9374 \quad \text{and} \quad 2(2.2361)\approx 4.4722$

Now subtract:

$7.9374-4.4722\approx 3.4652$

Round the final answer to the nearest hundredth:

$3\sqrt{7}-2\sqrt{5}\approx 3.47$

Exercise

Simplify: $4\sqrt{6}+3\sqrt{2}$. Round to the nearest thousandths.

Answer available at the end of this section.

Calculator Tip

Just as with order of operations, you should be able to enter the entire expression into your calculator at once and evaluate it in one step.

Fractions and Decimals

We can also find square roots of fractions and decimals.

Example 11

Simplify: (a) $\sqrt{\frac{4}{9}}$, and (b) $-\sqrt{0.49}$.

Solution

(a) Because $\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{9}$, then

$$\sqrt{\frac{4}{9}} = \frac{2}{3}.$$

(b) Because $(0.7)^2 = (0.7)(0.7) = 0.49$ and $(-0.7)^2 = (-0.7)(-0.7) = 0.49$, both 0.7 and -0.7 are square roots of 0.49. However, $-\sqrt{0.49}$ calls for the negative square root of 0.49. Hence,

$$-\sqrt{0.49} = -0.7$$

Exercise

Simplify: a) $\sqrt{\frac{25}{49}}$ b) $\sqrt{0.36}$

Answer available at the end of this section.

Example 12

Simplify: $\sqrt{0.8}$. Round to the nearest thousandths.

Solution

This is not a perfect square, so we use a calculator.

$\sqrt{0.8} \approx 0.894427$

Round to the nearest thousandth:

$\sqrt{0.8} \approx 0.894$

Exercise

Simplify: $\sqrt{0.3}$. Round to the nearest tenths.

Answer available at the end of this section.

Answers

Exercise

–16, 16

Exercise

–25, 25

Exercise

–3, 3

Exercise

There are none.

Exercise

(a) 12 (b) –18

Exercise

(a) –6 (b) Not a real number.

Exercise

9.1

Exercise

–5

Exercise

–99

Exercise

(a) 13 (b) 17

Exercise

$4\sqrt{6}+3\sqrt{2}\approx 14.041$

Exercise

(a) 5/7 (b) 0.6

Exercise

$\sqrt{0.3}\approx 0.5$

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1.4.2: Introduction to Cube Roots

Earlier, we saw that squaring a number means multiplying the number by itself. Similarly, cubing a number means multiplying the number by itself three times. For example, 2^3 = 8 and (-3)^3 = -27.

In the previous section, we used perfect squares to help us understand square roots. In this section, we will use perfect cubes to help us understand cube roots.

Knowing common perfect cubes will help you recognize cube roots more quickly. Some of the most common are 1^3 = 1, 2^3 = 8, 3^3 = 27, 4^3 = 64, 5^3 = 125, and 6^3 = 216.

These values will be especially useful as we begin working with cube roots.

Cube Roots

A cube root reverses the process of cubing a number. In other words, it answers the question: what number multiplied by itself three times gives a given number?

Cube Root

If a^3 = b, then a is called a cube root of b.

Cube roots are written using radical notation with an index of 3: ∛b.

The small 3 is called the index and tells us we are finding a cube root. If no index is written, it is understood to be 2, which represents a square root.

Unlike square roots, cube roots behave a little differently. Every real number has exactly one real cube root, including negative numbers. For example, since (-3)^3 = -27, the cube root of -27 is -3.

Introduction to cube roots

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces the idea of cube roots and provides a few examples of how to evaluate cube roots of perfect cubes.

Example 1

Simplify: ∛27

Solution

Since 3^3 = 27, ∛27 = 3.

Exercise

Evaluate: ∛64

Answer available at the end of this section.

Example 2

Simplify: ∛(-125)

Solution

Since (-5)^3 = -125, ∛(-125) = -5.

Exercise

Simplify: ∛(-8)

Answer available at the end of this section.

Estimating Cube Roots

The cubes in the list above are called perfect cubes. Each is the cube of a whole number. Not all numbers are perfect cubes. For example, there is no whole number whose cube is equal to 20. However, ∛20 is still a real number.

When the radicand is not a perfect cube, we use a calculator to approximate the value of the cube root as a decimal.

Calculator Tip

Most scientific calculators have an xth root key, written as x√. This key allows you to choose the index of the root.

To find a cube root:
1. Press 3.
2. Press the x√ key.
3. Enter the number.
4. Press =.

For example, to find ∛20:
3 x√ 20 =

Example 3

Simplify and round to the nearest tenths: ∛20

Solution

Using a calculator:
3 x√ 20 ≈ 2.7

∛20 ≈ 2.7

Exercise

Simplify and round to the nearest tenths: ∛50

Answer available at the end of this section.

Order of Operations

Cube roots follow the same rules for order of operations as square roots. When evaluating expressions, cube roots are treated the same as exponents and should be evaluated before multiplication, division, addition, and subtraction.

We will evaluate any cube roots first, then continue simplifying using the order of operations.

Example 4

Simplify: 2 + ∛8

Solution

Evaluate the cube root first:
∛8 = 2

Then add:
2 + 2 = 4

Exercise

Simplify: 5 + ∛27

Answer available at the end of this section.

Example 5

Simplify: 3∛8 - 4

Solution

Evaluate the cube root first:
∛8 = 2

Then multiply:
3(2) = 6

Then subtract:
6 - 4 = 2

Exercise

Simplify: 2∛27 + 1

Answer available at the end of this section.

Example 6

Simplify and round to the nearest tenths: 4 + ∛30

Solution

Evaluate the cube root using a calculator:
3 x√ 30 ≈ 3.1

Then add:
4 + 3.1 = 7.1

Exercise

Simplify: 6 - ∛40 (round to the nearest tenth)

Answer available at the end of this section.

Answers

Exercise
4

Exercise
-2

Exercise
3.7

Exercise
8

Exercise
7

Exercise
2.6

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1.4.3: Introduction to Scientific Notation

Sometimes we need express very large or very small numbers. For instance, how many cells are there in the human body? And how large, in inches, is a cell? The answers to questions like this can be difficult to understand when expressed in regular notation:

$$\text{Number of cells in the human body } = 37,200,000,000,000$$

$$\text{Length of a cell } = 0.0011811 \text{ inches}$$

For numbers like this, it can be helpful to express them in *scientific notation*.

Basics of Scientific Notation

Before working with scientific notation, it is helpful to understand powers of ten.

A power of ten is a number written using an exponent, such as 10^2 or 10^5. The exponent tells us how many times 10 is multiplied by itself.

$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 10 \times 10 \times 10 = 1,000$
$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$

Notice the pattern: the exponent tells us how many zeros are in the number.

$10^2 = 100, \text{ which has } 2 \text{ zeros}$
$10^5 = 100,000, \text{ which has } 5 \text{ zeros}$

This pattern makes powers of ten very useful for writing large numbers in a shorter, more manageable form.

Definition: Scientific Notation

A number is written in **scientific notation** when it is expressed as a number between 1 and 10 that is multiplied by a power of 10. As an example, we could write

$37,200,000,000,000 = 3.72 \times 10^{13}$

The exponent on the 10 tells you how many places the decimal point has been moved. In this case, the exponent was moved 13 places to the left.

Example 1

Explain why the numbers $37,200,000,000,000$ and 3.72×10^{13} are equal.

Solution

$10^{13} = 10,000,000,000,000$

This means 10^{13} is a 1 followed by 13 zeros.

Now multiply by 3.72:

$3.72 \times 10,000,000,000,000 = 37,200,000,000,000$

You can also think of this as moving the decimal point in 3.72 thirteen places to the right.

Therefore,

$37,200,000,000,000 = 3.72 \times 10^{13}$

You can also use negative exponents with powers of ten. These represent very small numbers.

$10^{-1} = 0.1$
$10^{-2} = 0.01$
$10^{-3} = 0.001$

For negative exponents, the exponent tells us how many places the decimal point moves to the left.

The following table summarizes how to write both very large and very small numbers in scientific notation.

How to write very large and very small number in scientific notation

Number Size	Exponent	Decimal Point Moves...	Example Number	Scientific Notation
Very Large	Positive	Left	$453,000,000$	4.53×10^{8}
Very Small	Negative	Right	$.000000674$	6.74×10^{-7}

Let's practice with one more example.

Example 2

Convert the number $.0000049$ to scientific notation.

Solution

We see that this is a very small number, between 0 and 1. This means that we need to use a negative exponent to move the decimal point to the right. We need to move the decimal point from its current location to the place between the 4 and the 9, so that the resulting number will be between 1 and 10. If we count places from the current location to the new location, we see that it has to move 6 places. Therefore,

$$.0000049 = 4. \times 10^{-6} \tag{1.4.3.1}$$

The process of converting from scientific notation to ordinary notation consists of simply reversing the direction that the decimal point travels, and including an appropriate number of zeroes. Let's see two examples of this.

Example 3

Convert each of the following to ordinary notation.

1. 9.362×10^{10}
2. 5.7×10^{-4}

Solution

1. There is a positive exponent here, which means we expect a very large number to result. We will move the decimal point 10 places to the *right,* since we are reversing the process of conversion. We will start putting zeroes once we run out of numbers in 9.362. Therefore,

$$9.362 \times 10^{10} = 93,620,000,000 \tag{1.4.3.2}$$

2. Here, we have a negative exponent, which means that our result will be a very small number. Since we are reversing the process, we'll move the decimal point 4 places to the *left,* adding zeroes as we go. Therefore

$$5.7 \times 10^{-4} = .00057 \tag{1.4.3.3}$$

Standard Form and Scientific Notation

Standard form	Scientific notation
$34,500,000,000$	3.45×10^{10}
0.00000000889	8.89×10^{-9}

What do “very large” and “very small” mean? The answer depends on context.

For example, the number $453,000,000$ can also be written as 4.53×10^{8}. Both forms represent the same value, but one may be more useful depending on the situation.

In everyday contexts, such as news articles, numbers are often written in standard form or described in words, such as “453 million.” In technical or scientific contexts, scientific notation is often preferred because it makes large and small numbers easier to read, write, and compare.

Comparing numbers using scientific notation

One of the benefits of scientific notation is that it allows us to compare the relative size of quantities that are very large or very small. This is especially clear when the quantities are different *orders of magnitude;* that is, when they have different exponents in scientific notation. Using scientific notation can help make this more obvious than ordinary notation.

Example 4

Which is larger? 89000000 or 9800000?

Solution

The number $89{,}000{,}000$ is larger because it has more zeros than $9{,}800{,}000$.

In scientific notation, these numbers are 8.9×10^{7} and 9.8×10^{6}. Since 10^{7} is greater than 10^{6}, the first number is larger.

How might this be useful in context? Consider the following example.

Example 5

Many chemicals are harmful if ingested, inhaled, or otherwise consumed by humans. Often, we use blood concentrations of chemicals to define the quantity of a substance that is considered harmful. Mercury and lead are two chemicals known to be harmful at very low levels. Lead can be shown to cause harm at concentrations greater than 3.5×10^{-6} grams per deciliter of blood, whereas mercury can be shown to cause harm at concentrations greater than 1×10^{-7} grams per deciliter of blood.

Use the information above to compare the effects of equal amounts of lead and mercury exposure.

Solution

Both lead and mercury are harmful at low concentrations. However, mercury is more harmful because it causes harm at a smaller value.

The harmful level for mercury is 1×10^{-7}, while for lead it is 3.5×10^{-6}. Since 10^{-7} is smaller than 10^{-6}, the mercury level is lower.

This means a smaller amount of mercury can cause harm compared to the same amount of lead.

Scientific Notation

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video provides an overview of scientific notation.

Calculator Tip

Calculators and other technology often use "E" to represent powers of 10 in scientific notation.

For example, $5.87\text{E}8$ means 5.87×10^{8}.

Be sure to recognize this format when working with large or small numbers.

Writing Numbers in Proper Scientific Notation

In scientific notation, the number in front must be at least 1 and less than 10. If it is not, we adjust the decimal and change the exponent so the value stays the same. In the following example, you will see how to modify an exponent to rewrite a number in proper scientific notation.

How to Write a Number in Proper Scientific Notation

To write a number in scientific notation, the decimal must be moved so the leading value is between 1 and 10. When the decimal moves, the exponent must be adjusted to keep the value the same.

If the decimal moves to the left, add to the exponent.
If the decimal moves to the right, subtract from the exponent.

Example 8

The given numbers are not in scientific notation (look at the decimal number and see that it is either less than 1 or greater than 10). Modify each so that your answer is in scientific notation:

a. 56.7×10^{8}

b. 88.9×10^{-7}

c. 0.55×10^{9}

d. 0.88×10^{-4}

Solution

a. $56.7 \times 10^{8} = 5.67 \times 10^{9}$ because the decimal moves left 1 place, so the exponent increases by 1.

b. $88.9 \times 10^{-7} = 8.89 \times 10^{-6}$ because the decimal moves left 1 place, so the exponent increases by 1.

c. $0.55 \times 10^{9} = 5.5 \times 10^{8}$ because the decimal moves right 1 place, so the exponent decreases by 1.

d. $0.88 \times 10^{-4} = 8.8 \times 10^{-5}$ because the decimal moves right 1 place, so the exponent decreases by 1.

Multiplication and Division using Scientific Notation

Although it is possible to use properties of exponents to multiply and divide numbers written in scientific notation, this lesson focuses on how to correctly enter these expressions into a calculator and write the result in scientific notation.

Calculator Tip

Most scientific calculators have a key labeled $\times 10^{n}$, EE, or EXP. This key is used to enter numbers in scientific notation.

For example, to enter 1.7×10^{6}:

Type: 1.7 then press the $\times 10^{n}$ key or the EE key, then type 6.

To enter a negative exponent, use the negative key for the exponent.

Using this key is often faster and more accurate than typing $\times 10$ manually.

Example 9

Perform the given operation:

a. $(2.3 \times 10^{8})(3 \times 10^{-4})$
b. $\frac{6.4 \times 10^{-9}}{3.2 \times 10^{-5}}$

Solution

a. $(2.3 \times 10^{8})(3 \times 10^{-4})$

Type: $(2.3 \times 10^{8}) \times (3 \times 10^{-4})$

The calculator may display the result in standard form, such as $69,000$.

Rewrite this in scientific notation:

$69,000 = 6.9 \times 10^{4}$

b. $\frac{6.4 \times 10^{-9}}{3.2 \times 10^{-5}}$

Type: $(6.4 \times 10^{-9}) \div (3.2 \times 10^{-5})$

The calculator will return 2×10^{-4}.

Example 10

Perform the given operation and write your answer in scientific notation:

a. $(6.2 \times 10^8)(3.0 \times 10^7)$
b. $\frac{4 \times 10^5}{8 \times 10^{-3}}$

Solution

a. $(6.2 \times 10^8)(3.0 \times 10^7) = 186,000,000,000,000,000 = 1.86 \times 10^{16}$
b. $\frac{4 \times 10^5}{8 \times 10^{-3}} = 50,000,000 = 5.0 \times 10^7$

Note: Writing 5.0×10^7 is the same as writing 5×10^7. They are interchangeable.

Example 11

Perform the given operation and write your answer in scientific notation:

$\frac{(2.1 \times 10^3)(3.2 \times 10^{-8})}{(2 \times 10^4)(3 \times 10^9)}$

Solution

$\frac{(2.1 \times 10^3)(3.2 \times 10^{-8})}{(2 \times 10^4)(3 \times 10^9)} = 0.00000000000000000112$

Rewrite in scientific notation:

$0.00000000000000000112 = 1.12 \times 10^{-18}$

Example 12

The debt of a nation is 7 trillion dollars, and there are 300 thousand inhabitants. If the debt was distributed evenly among all the inhabitants, how much would each person have to pay to pay off the debt? Answer in both scientific notation and standard form.

Solution

First write each value in scientific notation.

7 trillion dollars $= 7 \times 10^{12}$

300 thousand people $= 3 \times 10^5$

Now enter the expression into the calculator:

$\frac{7 \times 10^{12}}{3 \times 10^5} = 2,333,333.33$

Write the result in scientific notation:

$2,333,333.33 \approx 2.33 \times 10^6$

This means that each person would pay about 2.3 million dollars.

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1.4.4: Roots and Scientific Notation Practice

The following set of exercises provides practice for the concepts covered in Module 1.4: Roots and Scientific Notation. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 1.4.1 - Introduction to Square Roots

Find the square roots of each number, or state that there are none.

1. 121

2. 256

3. -16

Simplify each expression. If the result is not a real number, state this. Round to the nearest tenth when necessary.

4. $\sqrt{144}$

5. $-\sqrt{324}$

6. $\sqrt{-36}$

7. $\sqrt{\frac{25}{49}}$

8. $\sqrt{0.49}$

9. $\sqrt{30}$

10. $\sqrt{18}$

11. $-\sqrt{72}$

Simplify each expression. Round to the nearest hundredth when necessary.

12. $\sqrt{9}+\sqrt{16}$

13. $2\sqrt{25}-\sqrt{36}$

14. $3\sqrt{7}-2\sqrt{5}$

Section 1.4.2 - Introduction to Cube Roots

Simplify each expression. Round to the nearest tenth when necessary.

15. $\sqrt[3]{8}$
16. $\sqrt[3]{-125}$
17. $\sqrt[3]{20}$
18. $3+\sqrt[3]{8}$
19. $6-\sqrt[3]{27}$
20. $2+3\sqrt[3]{8}-\sqrt[3]{27}$

Section 1.4.3 - Introduction to Scientific Notation

Convert each number to standard form.

21. 4.2×10^5
22. 6.08×10^{-4}

Convert each number to scientific notation.

23. $8,200,000$
24. 0.00063

Use the information given to compare the two substances. Determine which is more harmful and explain your reasoning.

25. A machine shop monitors the concentration of metal particles in the air. Aluminum particles are considered unsafe at concentrations greater than 4.0×10^{-6} grams per cubic meter, while steel particles are considered unsafe at concentrations greater than 8.0×10^{-7} grams per cubic meter. Which particle is more harmful? Explain your reason.

26. In agriculture, pesticide levels are monitored to ensure safety. Pesticide A is considered unsafe at concentrations greater than 6.5×10^{-5} grams per liter, while Pesticide B is considered unsafe at concentrations greater than 2.0×10^{-6} grams per liter. Which pesticide is more harmful? Explain your reasoning.

Rewrite each expression in proper scientific notation.

27. 72.4×10^3
28. 0.39×10^{-4}

Use a calculator. Perform each operation and write your answer in scientific notation.

29. $(3.2 \times 10^4)(5.1 \times 10^{-7})$
30. $\frac{8.4 \times 10^8}{2.1 \times 10^2}$

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1.5: APPLICATIONS INVOLVING NUMBER SENSE

Many problems in technical settings do not start with numbers—they start with a situation. Before any calculation can happen, the situation has to be interpreted and translated into something that can be solved.

This type of thinking is used often in real-world work. Whether planning materials, estimating costs, or interpreting measurements, the challenge is not just doing the math but deciding what math is needed and how to apply it.

In this module, the focus is on building that process. You will translate real-world situations into numerical expressions and solve one-step and multi-step problems involving whole numbers.

You will also work with problems that involve integers, fractions, decimals, exponents, and roots, applying what you have learned in earlier modules to more complex situations.

Fixed Mindset Vs. Growth Mindset

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View this YouTube Video online

This video introduces the idea of growth mindset and emphasizes how problem-solving improves with practice, especially when problems feel challenging at first.

1.5.1: Translating Numerical Expessions

1.5.2: One Step Applications Involving Whole Numbers

1.5.3: Multi-Step Applications Involving Whole Numbers

1.5.4: Applications Involving Other Number Types

1.5.5: Applications Involving Number Sense Practice

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1.5.1: Translating Numerical Expessions

Translate Word Phrases to Math Notation

In this section, you will learn how to translate verbal phrases into numerical expressions. Many real-world problems are written in words, so being able to turn those words into math is an important skill. We will focus on recognizing key words and phrases that represent addition, subtraction, multiplication, and division, and then writing them using mathematical notation. Once the expression is written, you can simplify it to find a solution.

Addition

Some of the word phrases that indicate addition are listed in Table 1.5.1.1 below.

Table 1.5.1.1

Operation	Words	Example	Expression
Addition	plus	1 plus 2	1 + 2
	sum	the sum of 3 and 4	3 + 4
	increased by	5 increased by 6	5 + 6
	more than	8 more than 7	7 + 8
	total of	the total of 9 and 5	9 + 5
	added to	6 added to 4	4 + 6

Example 1

Translate and simplify: the sum of 19 and 23.

Solution

The word sum tells us to add. The words of 19 and 23 tell us the addends.

	The sum of 19 and 23
Translate.	19 + 23
Add.	42
	The sum of 19 and 23 is 42.

Exercise

Translate and simplify: the sum of 17 and 26.

Answer available at the end of this section.

Exercise

Translate and simplify: the sum of 28 and 14.

Answer available at the end of this section.

Example 2

Translate and simplify: 28 increased by 31.

Solution

The words increased by tell us to add. The numbers given are the addends.

	28 increased by 31.
Translate.	28 + 31
Add.	59
	So 28 increased by 31 is 59.

Exercise

Translate and simplify: 29 increased by 76.

Answer available at the end of this section.

Exercise

Translate and simplify: 37 increased by 69.

Answer available at the end of this section.

The Language of Addition

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Subtraction

Some of the words that indicate subtraction are listed in Table $1.5.1.2$.

Table $1.5.1.2$

Operation	Word Phrase	Example	Expression
Subtraction	minus	5 minus 1	5 - 1
	difference	the difference of 9 and 4	9 - 4
	decreased by	7 decreased by 3	7 - 3
	less than	5 less than 8	8 - 5

Operation	Word Phrase	Example	Expression
	subtracted from	1 subtracted from 6	6 - 1

Example 3

Translate and then simplify:

a. the difference of 13 and 8
b. subtract 24 from 43

Solution

a. The word *difference* tells us to subtract the two numbers. The numbers stay in the same order as in the phrase.

	the difference of 13 and 8
Translate.	13 - 8
Simplify.	5

b. The words subtract from tells us to take the second number away from the first. We must be careful to get the order correct.

	subtract 24 from 43
Translate.	43 - 24
Simplify.	19

Exercise

Translate and simplify:

a. the difference of 14 and 9
b. subtract 21 from 37

Answer a available at the end of this section.

Exercise

Translate and simplify:

a. 11 decreased by 6
b. 18 less than 67

Answer a available at the end of this section.

The Language of Subtraction

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Multiplication

Some of the words that indicate multiplication are given in Table 1.5.1.3.

Table 1.5.1.3

Operation	Word Phrase	Example	Expression
Multiplication	times	3 times 8	3 × 8, 3 • 8, (3)(8),
	product	the product of 4 and 5	(4)5, or 4(5)
	twice	twice 4	2 • 4

Example 4

Translate and simplify: the product of 12 and 27.

Solution

The word *product* tells us to multiply. The words of 12 *and* 27 tell us the two factors.

	the product of 12 and 27
Translate.	12 • 27
Multiply.	324

Exercise

Translate and simplify the product of 13 and 28.

Answer available at the end of this section.

Exercise

Translate and simplify the product of 47 and 14.

Answer available at the end of this section.

Example 5

Translate and simplify: twice two hundred eleven.

Solution

The word twice tells us to multiply by 2.

	twice two hundred eleven
Translate.	2(211)
Multiply.	422

Exercise

Translate and simplify: twice one hundred sixty-seven.

Answer available at the end of this section.

Exercise

Translate and simplify: twice two hundred fifty-eight.

Answer available at the end of this section.

The Language of Multiplication

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Division

Some of the words that indicate multiplication are given in Table $1.5.1.4$.

Table

Operation	Word Phrase	Example	Expression
Division	divided by	12 divided by 4	$12 \div 4$, $\frac{12}{4}$, $4\overline{)12}$
	quotient of	the quotient of 16 and 2	$16 \div 2$, $\frac{16}{2}$, $4\overline{)12}$
	divided into	18 divided into 6	$18 \div 6$, $\frac{18}{6}$, $6\overline{)18}$

Example 6

Translate and simplify: the quotient of 51 and 17 .

Solution

The word *quotient* tells us to divide.

	the quotient of 51 and 17
Translate.	$51 \div 17$
Divide.	3

We could just as correctly have translated *the quotient of* 51 *and* 17 using the notation

$17\overline{)51}$ or $\frac{51}{17}$.

Exercise

Translate and simplify: the quotient of 91 and 13 .

Answer available at the end of this section.

Exercise

Translate and simplify: the quotient of 52 and 13 .

Answer available at the end of this section.

The Language of Division

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Answers

Exercise

Translate: $17 + 26$; Simplify: 43

Exercise

Translate: $28 + 14$; Simplify: 42

Exercise

Translate: $29 + 76$; Simplify: 105

Exercise

Translate: $37 + 69$; Simplify: 106

Exercise

$14 - 9 = 5$

$37 - 21 = 16$

Exercise

$11 - 6 = 5$

$67 - 18 = 49$

Exercise

$13 \cdot 28 = 364$

Exercise

$47 \cdot 14 = 658$

Exercise

$2(167) = 334$

Exercise

$2(258) = 516$

Exercise

91 ÷ 13 = 7

Exercise

52 ÷ 13 = 4

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1.5.2: One Step Applications Involving Whole Numbers

In this section, you will focus on solving one-step application problems using whole numbers. These problems are based on real-world situations and require you to choose the correct operation, write an expression, and find a solution.

You will use key phrases to help determine which operation to use. Then, you will translate the situation into a mathematical expression and solve.

Guidelines for Solving Application Problems

1. Read the problem carefully to identify what is being asked.
2. Write a word phrase that represents the information needed to solve the problem.
3. Translate the phrase into a mathematical expression.
4. Simplify the expression using the appropriate operation.
5. Write a complete sentence that answers the question, including units.

Addition

Addition is used when quantities are combined to find a total. Look for key phrases such as *plus*, *sum*, *increased by*, *more than*, *total of*, *in all*, or *combined*.

Example 1

During a safety inspection, a shop must report the total volume of petroleum products stored in the building. A technician counts 19 gallons of 5W-30, 26 gallons of 10W-30, and 38 gallons of synthetic oil. What is the total number of gallons stored in the shop?

Solution

We are asked to find the total gallons of oil in the shop.

Steps to Solve this Example

Description of Step	Written Work
Write a phrase.	the sum of gallons in the shop
Translate to math notation.	19 + 26 + 38
Then we simplify by adding.	83
Write a sentence to answer the question.	There are 83 gallons of oil in the shop.

Notice that the values are measured in gallons, so the result is 83 gallons.

Exercise

A customer asks a hardware store to cut several pieces of electrical wire for a renovation project. The employee records the lengths that were cut: 52 ft, 18 ft, 24 ft, and 31 ft.

To complete the sales receipt, the employee must determine the total length of wire provided. What is the combined length of the wire pieces?

Answer available at the end of this section.

Example 2

A small landscaping company just completed a large job and is preparing the weekly payroll for the crew. The four workers earned the following amounts:

Alex – $385
Jordan – $412
Chris – $296
Taylor – $458

Before writing the checks, the manager wants to know the total amount that will be paid to employees this week. What is the total payroll?

Solution

We are asked to find the total payroll.

Description of Step	Written Work
Write a phrase.	the sum of the weekly wages
Translate to math notation.	385 + 412 + 296 + 458
Then we simplify by adding.	1551
Write a sentence to answer the question.	The total payroll is $1551.

Notice that the values are measured in dollars, so the result is $1551.

Exercise

During a home renovation consultation, a contractor prepares a rough cost estimate for a homeowner. The projected expenses include:

Cabinets – $820
Countertop – $540
Lighting – $215
Flooring – $690
Appliances – $3,420
Labor – $1,250

What is the estimated total cost of the renovation?

Answer available at the end of this section.

Example 3

During an upgrade of an older electrical system, a maintenance crew must remove several types of wire from different areas of a building. The work order lists the following lengths to be removed (in feet):

Location	Type A	Type B	Type C
L1	740	620	85
L2	92	510	470
L3	44	1,950	96
L4	1,210	190	74

(a) What is the total length of Type A wire that must be removed?
(b) What is the total length of wire that must be removed from Location L2?

Solution

(a) We are asked to find the total length of Type A wire.

Description of Step	Written Work
Write a phrase.	the total length of Type A wire
Translate to mathematical notation.	740 + 92 + 44 + 1210
Simplify.	2086
Write a complete sentence.	The total length of Type A wire is 2086 ft.

Notice that the values are measured in feet, so the result is 2086 ft.

(b) We are asked to find the total length of wire removed from Location L2.

Description of Step	Written Work
Write a phrase.	the total length of wire removed from Location L2
Translate to mathematical notation.	92 + 510 + 470
Simplify.	1072
Write a complete sentence.	The total length of wire removed from Location L2 is 1072 ft.

Notice that the values are measured in feet, so the result is 1072 ft.

Exercise

Rosie owns a small café and is reviewing her financial records after the first three months of the year. She wants to understand how the business is performing.

Her monthly income was:

January – $3,200
February – $2,450
March – $1,960

Her expenses during the same time period were:

Rent – $4,900
Supplies – $2,150
Part-time worker – $1,850
Transportation – $730

Utilities – $1,040
Advertising – $620
Miscellaneous – $210

(a) What was the total revenue for the three months?
(b) What was the total amount of expenses?

Answer available at the end of this section.

Subtraction

Subtraction is used when one quantity is taken away from another or when comparing two values. Look for key phrases such as *minus*, *difference*, *decreased by*, *less than*, *fewer than*, or *remaining*.

Example 4

A commercial painter has been hired to coat the interior wall of a warehouse storage area. The total wall surface to be painted measures 680 square feet. At the end of the first shift, progress records show that 512 square feet have already been completed. Before ordering additional supplies, the contractor needs to know the remaining surface area still requiring paint. How many square feet are left to be painted?

Solution

We are asked to find the difference between the total wall surface and the amount already painted.

Description of Step	Written Work
Write a phrase.	the difference between 680 and 512
Translate to mathematical notation.	680 - 512
Simplify.	168
Write a complete sentence.	There are 168 square feet left to be painted.

Notice that the values are measured in square feet, so the result is 168 square feet.

Exercise

A flooring contractor purchases 2,520 square feet of laminate flooring for a residential installation. After completing the job, the contractor determines that 2,356 square feet were used. How much flooring remains unused?

Answer available at the end of this section.

Example 5

An electric meter at a construction site read 1,842 kilowatt hours on June 1. On July 1, the meter reading was 11,265 kilowatt hours. Based on these readings, how many kilowatt hours of electricity were used during the month of June?

Solution

We are asked to compare the two meter readings to determine how much electricity was used during the month.

Description of Step	Written Work
Write a phrase.	the amount of electricity used during June
Translate to mathematical notation.	11,265 - 1,842
Simplify.	9,423
Write a complete sentence.	A total of 9,423 kilowatt hours were used during the month of June.

Notice that the values are measured in kilowatt hours, so the result is 9,423 kilowatt hours.

Exercise

A water meter at a job site read 3,480 gallons at the beginning of the week. At the end of the week, the meter reading was 9,125 gallons. How many gallons of water were used during the week?

Answer available at the end of this section.

Example 6

A heating and cooling business reported annual profits of $45,280 last year. This year, the company reported profits of $49,965. By how much did the company's profit increase from last year to this year?

Solution

We are asked to compare this year's profit to last year's profit to find how much it increased.

Description of Step	Written Work
Write a phrase.	the increase in profit from last year to this year
Translate to mathematical notation.	49,965 - 45,280
Simplify.	4,685
Write a complete sentence.	The company's profit increased by $4,685.

Notice that the values are measured in dollars, so the result is $4,685.

Exercise

A field technician earns an annual salary of $23,840. During the year, $4,215 is withheld for taxes. What is the technician's take-home pay for the year?

Answer available at the end of this section.

Multiplication

Multiplication is used when equal groups are combined or when a quantity is repeated. Look for key phrases such as *times*, *product*, *of*, *each*, or *per*.

Example 7

A warehouse crew is preparing a shipment of products for delivery. Boxes are stacked onto pallets before being loaded onto a truck. Each pallet holds 24 boxes. By the end of the shift, 7 pallets have been loaded. How many boxes are on the pallets in total?

Solution

We are asked to find the total number of boxes when 24 boxes are placed on each pallet.

Description of Step	Written Work
Write a phrase.	24 boxes for each of 7 pallets
Translate to mathematical notation.	24×7
Simplify.	$24 \times 7 = 168$
Write a complete sentence.	There are 168 boxes on the pallets.

Notice that the values are measured in boxes, so the result is 168 boxes.

Exercise

A greenhouse worker is preparing trays of seedlings for planting in a large outdoor garden. Each tray holds 24 seedlings. Before the end of the shift, 6 trays are filled and moved to the staging area. How many seedlings are planted in total?

Answer available at the end of this section.

Example 8

A construction crew is installing fencing around a storage yard. Each section of fence is 18 feet long and must be secured before moving to the next section. By the end of the day, the crew has installed 9 sections. What is the total length of fencing installed?

Solution

We are asked to find the total length of fencing when 18 feet are used for each of 9 sections.

Description of Step	Written Work
Write a phrase.	18 feet for each of 9 sections
Translate to mathematical notation.	18×9
Simplify.	$18 \times 9 = 162$
Write a complete sentence.	The total length of fencing installed is 162 feet.

Notice that the values are measured in feet, so the result is 162 feet.

Exercise

A manufacturing technician is monitoring a machine that produces 36 parts per hour during a production run. The machine operates continuously for 4 hours before being shut down for maintenance. How many parts are produced during this time?

Answer available at the end of this section.

Example 3

A water system technician is testing flow rates in a building. A pipe delivers water at a rate of 12 gallons per minute. If the system runs for 15 minutes during the test, how many gallons of water flow through the pipe?

Solution

We are asked to find the total amount of water when 12 gallons flow each minute for 15 minutes.

Description of Step	Written Work
Write a phrase.	12 gallons per minute for 15 minutes
Translate to mathematical notation.	12×15
Simplify.	$12 \times 15 = 180$
Write a complete sentence.	A total of 180 gallons of water flow through the pipe.

Notice that the values are measured in gallons, so the result is 180 gallons.

Exercise

A maintenance worker is filling containers with cleaning solution for use throughout a facility. The system dispenses 15 gallons per minute. If the system runs for 8 minutes, how many gallons of cleaning solution are dispensed?

Answer available at the end of this section.

Division

Division is used when a quantity is split into equal parts or when finding how many groups can be made. Look for key phrases such as *divided by*, *quotient*, *per*, *out of*, or *shared equally*.

Example 10

A parts manager is organizing a shipment of 168 bolts that just arrived at a shop. To make inventory easier to track, the bolts need to be placed into identical storage bins. The manager uses 7 bins so that each bin holds the same number of bolts. How many bolts will be placed in each bin?

Solution

We are asked to find how many bolts go in each bin when 168 bolts are divided into 7 equal groups.

Description of Step	Written Work
Write a phrase.	168 boxes divided into 7 equal stacks
Translate to mathematical notation.	$168 \div 7$
Simplify.	$168 \div 7 = 24$
Write a complete sentence.	There are 24 boxes in each stack.

Notice that the values are measured in bolts, so the result is 24 bolts per bin.

Exercise

At a clinic, a nurse prepares 144 doses of medication for patients. To organize the doses for distribution, they are placed evenly into 6 trays. How many doses will be placed in each tray?

Answer available at the end of this section.

Example 11

A welding instructor is preparing materials for a class project. The instructor has 510 inches of steel rod and needs to cut pieces that are each 85 inches long so that every student has the correct size. How many students can receive a piece?

Solution

We are asked to determine how many pieces of 85 inches can be made from 510 inches of steel rod.

Description of Step	Written Work
Write a phrase.	510 inches divided into pieces of 85 inches
Translate to mathematical notation.	$510 \div 85$
Simplify.	$510 \div 85 = 6$
Write a complete sentence.	A total of 6 students can receive a piece of steel rod.

Notice that the result represents the number of pieces (or students).

Exercise

A shipping supervisor is preparing orders to be sent out at the end of the day. There are 210 items that must be packed into boxes, and each box holds 42 items. How many boxes are needed to complete the shipment?

Answer available at the end of this section.

Example 12

A maintenance crew is cleaning a large facility and tracks how much cleaning solution is used during a shift. Over the course of 8 hours, the crew uses 120 gallons of solution. To plan for future jobs, the supervisor wants to know the average amount used each hour. What is the average usage in gallons per hour?

Solution

We are asked to find how many gallons are used per hour when 120 gallons are used over 8 hours.

Description of Step	Written Work
Write a phrase.	120 gallons over 8 hours
Translate to mathematical notation.	$120 \div 8$
Simplify.	$120 \div 8 = 15$
Write a complete sentence.	The average usage is 15 gallons per hour.

Notice that the result is a rate, so the units are gallons per hour.

Exercise

A long-haul truck driver records traveling 240 miles over a 6-hour shift. For a trip report, the driver must calculate the average number of miles driven per hour. What is the average speed?

Answer available at the end of this section.

Answers

Exercise

125 ft.

Exercise

$6,935

Exercise

(a) $7,610
(b) $11,500

Exercise

The remaining flooring is 164 square feet.

Exercise

There were 5,645 gallons of water used during the week.

Exercise

The technician's take home pay for the year was $\$19,625$.

Exercise

There are 144 seedlings planted in total.

Exercise

A total of 144 parts are produced.

Exercise

A total of 120 gallons of cleaning solution are dispensed.

Exercise

There are 24 doses in each tray.

Exercise

A total of 5 boxes are needed.

Exercise

The average speed is 40 miles per hour.

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1.5.3: Multi-Step Applications Involving Whole Numbers

In this section, you will solve application problems that require more than one step. These problems involve combining multiple operations to find a final result.

As in the previous section, you will begin by identifying what the problem is asking and writing a phrase to represent the situation. However, these problems will often require you to perform more than one operation. Pay close attention to the order in which steps should be completed and use the order of operations when simplifying expressions.

Guidelines for Solving Multi-Step Application Problems

1. Read the problem carefully to identify what is being asked.
2. Break the situation into steps and write a phrase for each part.
3. Translate each part into a mathematical expression.
4. Simplify step by step, following the order of operations.
5. Write a complete sentence that answers the question, including appropriate units.

Example 1

A contractor is installing flooring in a house with several rooms. One large room requires 320 square feet of flooring, and two smaller rooms each require 145 square feet. The contractor has already installed 410 square feet of flooring. How many square feet of flooring still need to be installed?

Solution

We are asked to find how much flooring remains after part of the job has been completed.

A helpful way to think about this problem is:

total flooring needed minus flooring already installed.

Description of Step	Written Work
Write a phrase for total flooring needed.	the sum of 320 and two groups of 145
Translate to mathematical notation.	320 + 2(145)
Simplify to find total flooring.	320 + 290 = 610
Write a phrase for flooring remaining.	the difference between 610 and 410
Translate to mathematical notation.	610 - 410
Simplify.	610 - 410 = 200
Write a complete sentence.	There are 200 square feet of flooring left to install.

Notice that we first found the total flooring needed, then subtracted the amount already installed. The result is 200 square feet.

Exercise

A network technician is preparing a computer lab for a certification testing session. One main lab area requires 48 network cables for desktop stations, and two smaller breakout rooms each require 18 cables for additional workstations. By the end of the first setup period, 60 cables have already been installed and tested. How many network cables still need to be installed before the lab is ready?

Answer available at the end of this section.

Example 2

An HVAC technician is installing air filters in a large office building. Each floor has 8 units, and each unit requires 3 filters. The technician has been assigned to service 6 floors during the day. The total number of filters used will be divided evenly between 3 supply kits for tracking purposes. How many filters will be placed in each supply kit?

Solution

We are asked to find how many filters will be placed in each supply kit after completing the installations.

A helpful way to think about this problem is:

total filters needed divided by the number of supply kits.

Description of Step	Written Work
Write a phrase for total filters needed.	8 units per floor times 3 filters per unit times 6 floors
Translate to mathematical notation.	$8 \cdot 3 \cdot 6$
Simplify to find total filters.	24
Write a phrase for filters per kit.	the total filters divided among 3 kits
Translate to mathematical notation.	$24 \div 3$
Simplify.	8
Write a complete sentence.	There are 8 filters in each supply kit.

Notice that we first found the total number of filters needed, then divided that total into equal groups. The result is 8 filters per kit.

Exercise

A hospital simulation lab is being set up for a patient care training session. Each hospital bed station is stocked with 4 trays of supplies, and each tray contains 5 syringes used for practice injections. A total of 9 bed stations are prepared for the session. After setup, all syringes are collected and divided evenly into 3 locked storage cabinets for inventory control. How many syringes are placed in each cabinet?

Answer available at the end of this section.

Example 3

An automotive shop is preparing to move vehicles into service bays for scheduled maintenance. Each bay can safely hold 8 vehicles at a time. A total of 62 vehicles are scheduled for service during the day. The shop must determine how many bays are needed so that all vehicles can be serviced.

Solution

We are asked to determine how many bays are required to hold all vehicles.

A helpful way to think about this problem is:

total vehicles divided by the number of vehicles each bay can hold, then round up to ensure all vehicles are accommodated.

Description of Step	Written Work
Write a phrase.	62 vehicles divided into groups of 8
Translate to mathematical notation.	$62 \div 8$
Simplify.	7.75
Interpret the result.	7.75 bays is not possible, so we round up to 8 bays
Write a complete sentence.	A total of 8 bays are required to service all vehicles.

Notice that the answer must be a whole number because partial bays are not possible. We round up to ensure all vehicles can be serviced.

Exercise

A machinist is finishing a production run on a CNC lathe and must organize the completed parts before they are moved to quality control. Each metal tray can safely hold 6 parts without scratching the machined surfaces. By the end of the shift, 50 parts have been produced and all of them must be placed into trays for transport. How many trays are required?

Answer available at the end of this section.

Example 4

An electrician is wiring a small commercial building. The job includes installing 12 outlets on each of 5 floors and an additional 18 outlets in common areas such as hallways and storage rooms. After installation, all outlets must be tested in equal groups over 3 work periods. How many outlets will be tested during each work period?

Solution

We are asked to determine how many outlets will be tested in each work period.

A helpful way to think about this problem is:
total outlets installed divided by the number of work periods.

Description of Step	Written Work
Write a phrase for outlets installed on floors.	12 outlets per floor for 5 floors
Translate to mathematical notation.	$12 \cdot 5$
Simplify.	60
Write a phrase for total outlets installed.	outlets on floors plus outlets in common areas
Translate to mathematical notation.	60 + 18
Simplify.	78
Write a phrase for outlets tested per period.	total outlets divided among 3 work periods
Translate to mathematical notation.	$78 \div 3$

Description of Step	Written Work
Simplify.	26
Write a complete sentence.	A total of 26 outlets will be tested during each work period.

Notice that we first found the total number of outlets installed, then divided that total into equal groups for testing.

Exercise

A diesel technician is preparing equipment for a maintenance schedule. The shop services 8 machines, and each machine requires 14 filters to be replaced. In addition, 16 extra filters are kept on hand for unexpected replacements. All filters are divided evenly among 4 service carts before the work begins. How many filters will be placed on each cart?

Answer available at the end of this section.

Answers

Exercise

$(48 + 2 \cdot 18) - 60 = 24$

There are 24 network cables left to install.

Exercise

$(4 \cdot 5 \cdot 9) \div 3 = 60$

There are 60 syringes in each cabinet.

Exercise

$50 \div 6 = 8.33 \rightarrow 9$

A total of 9 trays are required.

Exercise

$(8 \cdot 14 + 16) \div 4 = 32$

There are 32 filters on each service cart.

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1.5.4: Applications Involving Other Number Types

In this section, you will solve application problems that involve integers, fractions, decimals, and other types of numbers. These problems build on the same process used earlier, but require careful attention to how different types of numbers behave in real-world situations.

Integers

Example 1

A walk-in freezer in a restaurant kitchen is set to maintain a temperature of $-6^{\circ}\mathrm{F}$. During a busy delivery period, the temperature rises by $9^{\circ}\mathrm{F}$ while the door is repeatedly opened. After the deliveries are complete, the cooling system lowers the temperature by $7^{\circ}\mathrm{F}$. What is the final temperature inside the freezer?

Solution

We are asked to determine the final temperature after a series of changes.

A helpful way to think about this problem is:
starting temperature plus a rise, then minus a drop.

Description of Step	Written Work
Write a phrase.	start at -6, increase by 9, then decrease by 7
Translate to mathematical notation.	-6 + 9 - 7
Simplify.	3
Write a complete sentence.	The final temperature inside the freezer is $-4^{\circ}\mathrm{F}$.

Notice that the temperature first increased and then decreased, so we added and then subtracted. The final value of $-4^{\circ}F$ shows that the freezer is still below zero.

Exercise

A water tower in a small town is built on a hill that is 18 feet above ground level, while an underground pump is located 7 feet below ground level. What is the vertical distance between the pump and the top of the hill?

Answer available at the end of this section.

Exercise

An aircraft is descending toward a runway from an altitude of 31,500 feet to 13,500 feet over a period of 5 minutes. What is the average rate of change in altitude, in feet per minute?

Answer available at the end of this section.

Fractions

Example 2

A fabrication shop is cutting aluminum tubing for a custom frame. One section requires $\frac{3}{4}$ of a foot of tubing, and another section requires $\frac{5}{8}$ of a foot. What total length of tubing is required?

Solution

We are asked to find the total length of tubing needed.

A helpful way to think about this problem is:
add the two fractional lengths.

Description of Step	Written Work
Write a phrase.	the sum of $\frac{3}{4}$ and $\frac{5}{8}$
Translate to mathematical notation.	$\frac{3}{4}+\frac{5}{8}$
Use a calculator to evaluate.	$1\frac{3}{8}$
Write a complete sentence.	A total of $1\frac{3}{8}$ feet of tubing is required.

Notice that we added the lengths because both pieces are required. The result is written as a mixed number to represent the total length in a practical form.

Exercise

A roofing project requires $\frac{2}{3}$ of a bundle of shingles for each section of a roof. If 5 sections are completed, how many bundles of shingles are used in total?

Answer available at the end of this section.

Example 3

A hydraulic system in a manufacturing plant uses fluid stored in large reservoirs. Each machine cycle uses $\frac{4}{5}$ of a gallon of fluid. During a test run, the system completes 3 cycles. The reservoir initially contained 4 gallons of fluid. How much fluid remains in the reservoir after the test run?

Solution

We are asked to determine how much fluid remains after several cycles.

A helpful way to think about this problem is:
total fluid minus the amount used.

Description of Step	Written Work
Write a phrase for fluid used.	$\frac{4}{5}$ of a gallon per cycle for 3 cycles
Translate to mathematical notation.	$\frac{4}{5}$ × 3
Use a calculator to evaluate.	$2\frac{2}{5}$
Write a phrase for remaining fluid.	4 minus $2\frac{2}{5}$

Description of Step	Written Work
Translate to mathematical notation.	$4 - 2\frac{2}{5}$
Use a calculator to evaluate.	$1\frac{3}{5}$
Write a complete sentence.	$1\frac{3}{5}$ of a gallon of fluid remains in the reservoir.

Notice that we first found the total amount of fluid used, then subtracted from the starting amount to determine how much remained.

Exercise

A material storage system holds $\frac{5}{2}$ cubic feet of material. Each container can hold $\frac{1}{4}$ cubic feet. How many containers can be filled?

Answer available at the end of this section.

Decimals

Example 4

A CNC machine cuts metal rods to specific lengths. One rod measures 12.75 inches, and a second rod measures 8.6 inches. After cutting both rods, 5.25 inches of material is trimmed off as waste. What is the total length of usable material?

Solution

We are asked to determine the total usable length after waste is removed.

A helpful way to think about this problem is:
add the lengths, then subtract the waste.

Description of Step	Written Work
Write a phrase.	sum of the rod lengths minus waste
Translate to mathematical notation.	12.75 + 8.6 − 5.25
Use a calculator to evaluate.	16.10
Write a complete sentence.	The total usable material is 16.10 inches.

Notice that we first added the lengths of the rods, then subtracted the waste to determine the final usable length.

Exercise

A concrete mixer produces 2.5 cubic yards of concrete per batch. If 3.2 batches are mixed, how many cubic yards of concrete are produced?

Answer available at the end of this section.

Exercise

A water storage system holds 18.6 gallons of water. This water is distributed evenly into containers that each hold 2.4 gallons. How many full containers can be filled?

Answer available at the end of this section.

Square Roots and Cube Roots

Example 5

A square metal plate is used as a base for a machine. The area of the plate is 196 square inches. What is the length of one side of the plate?

Solution

We are asked to determine the side length of a square given its area.

A helpful way to think about this problem is:
find the square root of the area.

Description of Step	Written Work
Write a phrase.	square root of 196
Translate to mathematical notation.	$\sqrt{196}$
Use a calculator to evaluate.	14
Write a complete sentence.	The side length of the plate is 14 inches.

Notice that we used a square root because the area of a square is found by multiplying a side by itself.

Exercise

A storage tank is designed in the shape of a cube and has a volume of 125 cubic feet. What is the length of one side of the tank?

Answer available at the end of this section.

Scientific Notation

Example 6

A data storage system processes 3.2×10^6 bytes of data per second. If the system runs for 5 seconds, how many bytes of data are processed?

Solution

We are asked to determine the total amount of data processed.

A helpful way to think about this problem is:
multiply the rate by the time.

Description of Step	Written Work
Write a phrase.	data per second times number of seconds
Translate to mathematical notation.	$3.2 \times 10^6 \times 5$
Use a calculator to evaluate.	1.6×10^7
Write a complete sentence.	A total of 1.6×10^7 bytes are processed.

Notice that we multiplied the rate by time to find the total amount of data processed.

Exercise

A laboratory measures a sample with a mass of $4.5 \times 10^{-}3$ grams. If 8 identical samples are combined, what is the total mass?

Answer available at the end of this section.

Answers

Exercise

$18 - (-7) = 25$

The vertical distance between the pump and the top of the hill is 25 feet.

Exercise

$(13500 - 31500) \div 5 = -3600$

The average rate of change in altitude is –3600 feet per minute.

Exercise

$\frac{2}{3} \cdot 5 = \frac{10}{3} = 3\frac{1}{3}$

A total of $3\frac{1}{3}$ bundles of shingles are used.

Exercise

$\frac{5}{2} \div \frac{1}{4}$ = 10

Exercise

2.5 × 3.2 = 8.0

8 cubic yards of concrete are produced.

Exercise

18.6 ÷ 2.4 = 7.75

7 full containers can be filled, with some water remaining.

Exercise

$\sqrt[3]{}$125 = 5

The side length of the tank is 5 feet.

Exercise

$4.5 \times 10^{-3} \times 8 = 3.6 \times 10^{-2}$

The total mass is 3.6×10^{-2} grams.

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1.5.5: Applications Involving Number Sense Practice

The following set of exercises provides practice for the concepts covered in Module 1.5: Applications Involving Number Sense. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 1.5.1 - Translating Numerical Expressions

Translate and simplify the following expressions.

1. 9 increased by 7

2. 12 decreased by 5

3. 7 less than 20

4. 8 subtracted from 25

5. 4 times 3

6. 12 divided by 3

Section 1.5.2 - One Step Applications Involving Whole Numbers

Solve each problem. Write a complete sentence and include appropriate units.

7. A clinic is preparing exam rooms for patient appointments throughout the day. A total of 210 supply kits must be distributed evenly among 7 exam rooms so that each room is fully stocked. How many supply kits are placed in each room?

8. A delivery driver is tracking mileage for a weekly route. The truck travels 620 miles during the week. By Wednesday, 285 miles have already been completed. How many miles are driven after Wednesday?

9. A technician is running cable through a large building during a renovation. The spool originally contains 3,600 feet of cable. After completing several sections of the installation, 1,275 feet have been used. How much cable remains on the spool?

10. A shipping supervisor is organizing inventory before a delivery cutoff time. There are 240 items that must be packed into boxes, and each box holds 30 items. How many boxes are needed?

11. A farmer is harvesting crops from two different fields during the same week. One field produces 125 bushels of corn, and the other produces 198 bushels. What is the total number of bushels harvested?

12. A technician records equipment usage at the beginning and end of the month. The meter reads 1,250 hours at the start and 1,980 hours at the end. How many hours was the equipment used during the month?

13. A machine operator is overseeing a production run during a scheduled shift. The machine consistently produces 72 parts per hour. If the machine runs for 4 hours before being shut down for maintenance, how many parts are produced?

Section 1.5.3 - Multi-Step Applications Involving Whole Numbers

Solve each problem. Write a complete sentence and include units when appropriate.

14. A metal fabrication shop is installing electrical service in a new welding area. The main welding bay requires 14 outlets on each of 4 walls, and an additional 18 outlets are installed near tool stations and storage areas. By the end of the first day, 46 outlets have already been wired and tested. How many outlets still need to be installed?

15. A maintenance garage is servicing a group of diesel trucks. Each truck requires 12 quarts of oil, and 7 trucks are scheduled for service. An additional 10 quarts are stored in the shop for unexpected use. All of the oil is divided evenly among 5 service carts before work begins. How many quarts of oil will be available at each cart?

16. A commercial building renovation requires new plumbing lines in multiple sections. One section requires 240 feet of pipe, and two additional sections each require 85 feet. After installation begins, 190 feet of pipe have already been installed. How many feet of pipe still need to be installed?

17. A farm is preparing multiple fields for planting. One large field requires 320 pounds of seed, and two smaller fields each require 140 pounds. After the first day, 410 pounds of seed have already been distributed. How many pounds of seed still need to be distributed?

Section 1.5.4

Solve each problem. Write a complete sentence and include units when appropriate.

18. An HVAC system is operating at –12°F inside a freezer unit. During a defrost cycle, the temperature rises 18°F, then drops 7°F once cooling resumes. What is the final temperature?

19. A lift platform in a warehouse is positioned 10 feet above ground level. It lowers to 6 feet below ground to access storage. What is the total vertical distance traveled?

20. A drilling rig lowers a tool from 2,400 feet to 1,200 feet below ground over 6 minutes. What is the average rate of change in depth, in feet per minute?

21. An electrical installation requires two wire segments measuring $\frac{5}{8}$ ft and $\frac{3}{4}$ ft. What is the total length of wire used?

22. A concrete crew pours $\frac{3}{5}$ of a cubic yard of concrete per section. If 8 sections are completed, how much concrete is used?

23. A machinist cuts two sections of a shaft measuring 11.85 inches and 7.6 inches. The sections are joined together, but 2.75 inches must be removed due to damage at the connection point. What is the final usable length of the shaft?

24. A square steel plate used in fabrication has an area of 256 square inches. What is the length of one side?

25. A network server processes data at a rate of 5.4×10^5 bytes per second. How many bytes are processed in 4 seconds?

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2: EQUATIONS AND FORMULAS

The Equations and Formulas Unit builds on your number sense by introducing variables, expressions, equations, and formulas used in technical fields. In this unit, you will learn how to represent real-world situations using algebraic notation, simplify expressions using properties of operations, and evaluate expressions by substituting given values.

You will also develop strategies for solving equations. This includes determining whether a value is a solution, solving one-step equations, and working with more complex equations involving variables and constants on both sides. A general problem-solving approach will be emphasized to help you handle a variety of equation types with confidence.

In addition, you will work with formulas commonly used in technical settings. You will evaluate formulas, solve for unknown variables, and rewrite formulas to isolate specific quantities. These skills are essential for working with measurements, design specifications, and calculations in many technical programs.

Finally, you will translate real-world situations into algebraic equations and solve application problems. This unit focuses on connecting algebra to practical scenarios, helping you develop the skills needed to model, analyze, and solve problems you may encounter in technical careers.

2.1: Algebraic Expressions

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2.1: ALGEBRAIC EXPRESSIONS

In this module, you'll learn how to work with algebraic expressions—an important skill used across many technical fields. Whether you're estimating materials, calculating costs, or working with measurements, you'll often need to interpret and simplify expressions before you can solve a problem.

We'll start by evaluating expressions by substituting values and following the order of operations. Then you'll learn how to translate verbal phrases into algebraic expressions, which is a key step in setting up real-world problems. From there, you'll simplify expressions using the properties of multiplication and the distributive property, and finish by identifying and combining like terms to make expressions easier to work with.

To get started, watch the following video, which gives a fun example of how easily things can get confusing if expressions aren't written clearly.

MadTV Fast Food Order

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View this YouTube Video online

2.1.1: Evaluating Algebraic Expressions

2.1.2: Translating Algebraic Expressions

2.1.3: Simplifying Algebraic Expressions using Properties of Multiplication

2.1.4: Simplifying Algebraic Expressions by Combining Like Terms

2.1.5: Algebraic Expressions Practice

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2.1.1: Evaluating Algebraic Expressions

A mathematical expression is a combination of numbers, variables, and operation symbols that represents a quantity without using an equal sign. In this section, we will learn how to evaluate algebraic expressions.

A key idea in algebra is the concept of a variable.

Definition: Variable

A variable is a symbol (usually a letter) that stands for a value that may vary.

Let's add the definition of a *mathematical expression*.

Definition: Mathematical Expression

When we combine numbers and variables in a valid way, using operations such as addition, subtraction, multiplication, division, exponentiation, and other operations and functions as yet unlearned, the resulting combination of mathematical symbols is called a *mathematical expression*.

Thus,

$$2a,\ x + 5, \text{ and } y^2,$$

being formed by a combination of numbers, variables, and mathematical operators, are valid mathematical expressions. A mathematical expression must be *well-formed*. For example,

$$2 + \div 5x$$

is *not a valid expression* because there is no term following the plus sign (it is not valid to write +÷ with nothing between these operators). Similarly,

$$2 + 3(2$$

is not well-formed because parentheses are not balanced.

Introduction to Algebra: Using Variables

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This video explains what variables are and how they are used in algebraic expressions.

Evaluate Algebraic Expressions

Evaluating an expression means finding its numerical value by replacing the variable with a given number and then performing the operations in the correct order.

Definition: Evaluate an Expression

To evaluate an expression, substitute the given value for the variable and simplify the expression using the order of operations.

Example 1

Evaluate $x + 7$ when

a. $x = 3$
b. $x = 12$

Solution

a. To evaluate, substitute 3 for x in the expression, and then simplify.

Steps for Evaluating

Step in Words	Step Written Mathematically
	$x + 7$
Substitute.	$3 + 7$
Add.	10

When $x = 3$, the expression $x + 7$ has a value of 10.

b. To evaluate, substitute 12 for x in the expression, and then simplify.

Steps for Evaluating

Step in Words	Step Written Mathematically
	$x + 7$
Substitute.	$12 + 7$
Add.	19

When $x = 12$, the expression $x + 7$ has a value of 19. Notice that we got different results for parts (a) and (b) even though we started with the same expression. This is because the values used for x were different. When we evaluate an expression, the value varies depending on the value used for the variable.

Exercise

Evaluate: $y + 4$ when

a. $y = 6$
b. $y = 15$

Answer a available at the end of this section.

Exercise

Evaluate: $a - 5$ when

a. $a = 9$
b. $a = 17$

Answer a available at the end of this section.

Example 2

Evaluate $9x - 2$, when

a. $x = 5$
b. $x = 1$

Solution

Remember ab means a times b, so $9x$ means 9 times x.

a. To evaluate the expression when $x = 5$, we substitute 5 for x, and then simplify.

Steps for Evaluating

Step in Words	Step Written Mathematically
	$9x - 2$
Substitute 5 for x.	$9 \cdot 5 - 2$
Multiply.	$45 - 2$
Subtract.	43

b. To evaluate the expression when $x = 1$, we substitute 1 for x, and then simplify.

Steps for Evaluating

Step in Words	Step Written Mathematically
	$9x - 2$
Substitute 1 for x.	$9 \cdot 1 - 2$
Multiply.	$9 - 2$
Subtract.	7

Notice that in part (a) that we wrote $9 \bullet 5$ and in part (b) we wrote $9 \bullet 1$. Both the dot and the parentheses tell us to multiply.

Exercise

Evaluate: $8x - 3$, when

a. $x = 2$
b. $x = 1$

Answer a available at the end of this section.

Exercise

Evaluate: $4y - 4$, when

a. $y = 3$
b. $y = 5$

Answer a available at the end of this section.

Example 3

Evaluate x^2 when $x = 10$.

Solution

We substitute 10 for x, and then simplify the expression.

Steps for Evaluating

Step in Words	Step Written Mathematically
	x^2
Substitute 10 for x.	10^2
Use the definition of exponent.	$10 \cdot 10$
Multiply	100

When $x = 10$, the expression x^2 has a value of 100.

Exercise

Evaluate: x^2 when $x = 8$.

Answer available at the end of this section.

Exercise

Evaluate: x^3 when $x = 6$.

Answer available at the end of this section.

Example 4

Evaluate 2^x when $x = 5$.

Solution

In this expression, the variable is an exponent.

Steps for Evaluating

Step in Words	Step Written Mathematically
	2^x
Substitute 5 for x.	2^5
Use the definition of exponent.	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Step in Words	Step Written Mathematically
Multiply	32

When $x = 5$, the expression 2^x has a value of 32.

Exercise

Evaluate: 2^x when $x = 6$.

Answer available at the end of this section.

Exercise

Evaluate: 3^x when $x = 4$.

Answer available at the end of this section.

Example 5

Evaluate $3x + 4y - 6$ when $x = 10$ and $y = 2$.

Solution

This expression contains two variables, so we must make two substitutions.

Steps for Evaluating

Step in Words	Step Written Mathematically
	$3x + 4y - 6$
Substitute 10 for x and 2 for y.	$3(10) + 4(2) - 6$
Multiply.	$30 + 8 - 6$
Add and subtract left to right.	32

When $x = 10$ and $y = 2$, the expression $3x + 4y - 6$ has a value of 32.

Exercise

Evaluate: $2x + 5y - 4$ when $x = 11$ and $y = 3$

Answer available at the end of this section.

Exercise

Evaluate: $5x - 2y - 9$ when $x = 7$ and $y = 8$

Answer available at the end of this section.

Example 6

Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution

We need to be careful when an expression has a variable with an exponent. In this expression, $2x^2$ means $2 \bullet x \bullet x$ and is different from the expression $(2x)^2$, which means $2x \bullet 2x$.

Steps for Evaluating

Step in Words	Step Written Mathematically
	$2x^2 + 3x + 8$
Substitute 4 for each x.	$2(4)^2 + 3(4) + 8$
Simplify 4^2.	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

Exercise

Evaluate: $3x^2 + 4x + 1$ when $x = 3$.

Answer available at the end of this section.

Exercise

Evaluate: $6x^2 - 4x - 7$ when $x = 2$.

Answer available at the end of this section.

Evaluating Algebraic Expressions

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This video demonstrates several examples of evaluating algebraic expressions for given values.

Answers

Exercise

10

19

Exercise

4

12

Exercise

13

5

Exercise

8

16

Exercise

64

Exercise

216

Exercise

64

Exercise

81

Exercise

33

Exercise

10

Exercise

40

Exercise

9

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2.1.2: Translating Algebraic Expressions

Translating Words into Mathematical Expressions

In this section we turn our attention to translating word phrases into mathematical expressions. We begin with phrases that translate into *sums*. There is a wide variety of word phrases that translate into sums. Some common examples are given in the first two columns of Table 2.1.2.1, though the list is far from complete. In like manner, a number of phrases that translate into differences are shown in the last two columns of Table 2.1.2.1.

Table 2.1.2.1: Translating words into symbols.

Phrase	Translates to:	Phrase	Translates to:
sum of x and 12	$x + 12$	difference of x and 12	$x - 12$
4 greater than b	$b + 4$	4 less than b	$b - 4$
6 more than y	$y + 6$	7 subtracted from y	$y - 7$
44 plus r	$44 + r$	44 minus r	$44 - r$
3 larger than z	$z + 3$	3 smaller than z	$z - 3$

Let's look at some examples, some of which translate into expressions involving sums, and some of which translate into expressions involving differences.

Example 1

Translate the following phrases into mathematical expressions:

a. "12 larger than x,"
b. "11 less than y," and
c. "r decreased by 9."

Solution

Here are the translations.

a. "12 larger than x" becomes $x + 12$.
b. "11 less than y" becomes $y - 11$.
c. "r decreased by 9" becomes $r - 9$.

Exercise

Translate the following phrases into mathematical expressions:

a. "13 more than x" and
b. "12 fewer than y".

Answer available at the end of this section.

Example 2

Let *W* represent the width of the rectangle. The length of a rectangle is 4 feet longer than its width. Express the length of the rectangle in terms of its width *W*.

Solution

We know that the width of the rectangle is *W*. Because the length of the rectangle is 4 feet longer that the width, we must add 4 to the width to find the length.

$$\begin{array}{cccccc} \text{Length} & \text{is} & 4 & \text{more than} & \text{the width} \\ \text{Length} & = & 4 & + & W \end{array}$$

Thus, the length of the rectangle, in terms of its width *W*, is 4 + *W*.

Exercise

The width of a rectangle is 5 inches shorter than its length *L*. Express the width of the rectangle in terms of its length *L*.

Answer available at the end of this section.

Example 3

An electrician has 15 inches of wire that needs to be cut into two pieces. Let *x* represent the length of one of the resulting pieces. Express the length of the second piece in terms of the length *x* of the first piece.

Solution

The wire has original length 15 inches. It is cut into two pieces and the first piece has length *x*. To find the length of the second piece, we must subtract the length of the first piece from the total length.

$$\begin{array}{cccccc} \text{Length of the second piece} & \text{is} & \text{Total length} & \text{minus} & \text{the length of the first piece} \\ \text{Length of the second piece} & = & 15 & - & x \end{array}$$

Thus, the length of the second piece, in terms of the length x of the first piece, is 15 – *x*.

Exercise

A tile installer has a strip of trim that is cut into two pieces, the first of which measures 12 inches. Express the total length of the trim as a function of *x*, where *x* represents the length of the second strip of trim.

Answer available at the end of this section.

There is also a wide variety of phrases that translate into products. Some examples are shown in the first two columns of Table 2.1.2.2, though again the list is far from complete. In like manner, a number of phrases translate into quotients, as shown in last two columns of Table 2.1.2.2.

Table 2.1.2.2: Translating words into symbols.

Phrase	Translates to:	Phrase	Translates to:
product of *x* and 12	12*x*	quotient of *x* and 12	*x*/12
4 times *b*	4*b*	4 divided by *b*	4/*b*

Phrase	Translates to:	Phrase	Translates to:
twice r	$2r$	the ratio of 44 to r	$44/r$

Let's look at some examples, some of which translate into expressions involving products, and some of which translate into expressions involving quotients.

Example 4

Translate the following phrases into mathematical expressions: (a) "11 times x," (b) "quotient of y and 4," and (c) "twice a."

Solution

Here are the translations. a) "11 times x" becomes $11x$. b) "quotient of y and 4" becomes $y/4$, or equivalently, $\frac{y}{4}$. c) "twice a" becomes $2a$.

Exercise

Translate into mathematical symbols: (a) "the product of 5 and x" and (b) "12 divided by y."

Answer available at the end of this section.

Example 5

A plumber has a pipe of unknown length x. He cuts it into 4 equal pieces. Find the length of each piece in terms of the unknown length x.

Solution

The total length is unknown and equal to x. The plumber divides it into 4 equal pieces. To find the length of each piece, we must divide the total length by 4.

$$\begin{array}{cccccc} \text{Length of each piece} & \text{is} & \text{Total length} & \text{divided by} & 4 \\ \text{Length of each piece} & = & x & \div & 4 \end{array}$$

Thus, the length of each piece, in terms of the unknown length x, is $x/4$, or equivalently, $\frac{x}{4}$.

Exercise

A carpenter cuts a board of unknown length L into three equal pieces. Express the length of each piece in terms of L.

Answer available at the end of this section.

Example 6

Mary invests A dollars in a savings account. She invests five times this amount in a certificate of deposit. How much does she invest in the certificate of deposit, in terms of the amount A in the savings account?

Solution

The amount in the savings account is A dollars. She invests five times this amount in a certificate of deposit.

$$\begin{array}{cccccc} \text{Amount in CD} & \text{is} & 5 & \text{times} & \text{Amount in savings} \\ \text{Amount in CD} & = & 5 & \cdot & A \end{array}$$

Thus, the amount invested in the certificate of deposit, in terms of the amount A in the savings account, is 5*A*.

Exercise

David's company has a server that can store K gigabytes of data on one drive. A backup system stores half that amount. Express the amount the backup system can store in terms of K.

Answer available at the end of this section.

Writing Expressions

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View this YouTube Video online

This video demonstrates how to translate written phrases into mathematical expressions. It walks through common keywords and phrases—such as “sum,” “difference,” “product,” and “quotient”—and shows how they correspond to mathematical operations.

Combinations

Some phrases require combinations of the mathematical operations employed in previous examples.

Example 7

A worker installs *x* feet of cable. Another section requires 3 more than twice that amount. What is the total length needed for the second section in terms of x?

Solution

The first section is *x*. The second section is 3 more than twice the first section.

$$\begin{array}{lll} \text{Second section is} & 3 \text{ more than} & \text{twice the first section} \\ \text{Second section} = & 3+ & 2x \end{array}$$

Therefore, the second section, in terms of the first section *x*, is 3 + 2*x*.

Exercise

A technician completes y service calls in one day. The next day he completes 4 less than 3 times that number of service calls. Express the number of service calls completed the next day in terms of *y*.

Answer available at the end of this section.

Example 8

A medical billing office processes *B* patient claims in one day. Another office can process 15 fewer than 3 times as many claims. What is the number of claims that the second office processes in terms of B?

Solution

The number of claims that the first office processes is *B*. The second office processes 15 fewer than 3 times as many.

$$\begin{array}{lll} \text{second office} & \text{is} & \text{3 times the first office} & \text{less} & 15 \\ \text{second office} = & & 3B - & & 15 \end{array}$$

Therefore, the second office, in terms of the first office, is 3*B* – 15.

Exercise

A daycare has C children enrolled in the infant room. The toddler room is larger and has 7 more than twice that number. Express the number of children in the toddler room in terms of C.

Answer available at the end of this section.

Writing Algebraic Expressions

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View this YouTube Video online

This video shows how to translate phrases with more than one operation into mathematical expressions, including how to group parts correctly using parentheses.

Answers

Exercise

(a) *x* + 13 and
(b) *y* – 12

Exercise

L – 5

Exercise

12 + *x*

Exercise

(a) 5*x* and (b) 12/*y*.

Exercise

L/3

Exercise

$\frac{1}{2}K$

Exercise

3*y* – 4

Exercise

2C + 7

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2.1.3: Simplifying Algebraic Expressions using Properties of Multiplication

Recall the commutative and associative properties of multiplication.

The Commutative Property of Multiplication. If *a* and *b* are any integers, then

$$a \cdot b = b \cdot a, \text{ or equivalently, } ab = ba.$$

The Associative Property of Multiplication. If a, b, and c are any integers, then

$$(a \cdot b) \cdot c = a \cdot (b \cdot c), \text{ or equivalently, } (ab)c = a(bc).$$

The commutative property allows us to change the order of multiplication without affecting the product or answer. The associative property allows us to regroup without affecting the product or answer.

Example 1

Simplify: 2(3*x*).

Solution

Use the associative property to regroup, then simplify.

$$\begin{aligned} 2(3x) &= (2 \cdot 3)x \quad \text{Regrouping with the associative property.} \\ &= 6x \quad \text{Simplify: } 2 \cdot 3 = 6. \end{aligned}$$

Exercise

Simplify: –5(7*y*)

Answer available at the end of this section.

The statement $2(3x) = 6x$ is an identity, which means it is true for every value of x. In other words, the left-hand side and the right-hand side are always equal.

The work shown in Example 1 proves this statement. However, it can still be helpful to check a few values of x to build confidence that the equation works.

If *x* = 4, then

$$\begin{aligned} 2(3x) &= 2(3(4)) \\ &= 2(12) \\ &= 24 \end{aligned} \quad \text{and} \quad \begin{aligned} 6x &= 6(4) \\ &= 24 \end{aligned}$$

If *x* = –5, then

$$\begin{aligned} 2(3x) &= 2(3(-5)) \\ &= 2(-15) \\ &= -30 \end{aligned} \quad \text{and} \quad \begin{aligned} 6x &= 6(-5) \\ &= -30 \end{aligned}$$

The above calculations show that 2(3*x*)=6*x* for both *x* = 4 and *x* = –5. Indeed, the statement 2(3*x*)=6*x* is true, regardless of what is substituted for *x*.

Example 2

Simplify: (–3*t*)(–5).

Solution

In essence, we are multiplying three numbers, –3, *t*, and –5, but the grouping symbols ask us to multiply the –3 and the t first. The associative and commutative properties allow us to change the order and regroup.

$$\begin{aligned}(-3t)(-5) &= ((-3)(-5))t \quad \text{Change the order and regroup.}\\ &= 15t \quad \text{Multiply: } (-3)(-5) = 15.\end{aligned}$$

Exercise

Simplify: (–8*a*)(5)

Answer available at the end of this section.

Example 3

Simplify: (–3*x*)(–2*y*)

Solution

In essence, we are multiplying four numbers, –3, x, –2, and y, but the grouping symbols specify a particular order. The associative and commutative properties allow us to change the order and regroup.

$$\begin{aligned}(-3x)(-2y) &= ((-3)(-2))(xy) \quad \text{Change the order and regroup.}\\ &= 6xy \quad \text{Multiply: } (-3)(-2) = 6.\end{aligned}$$

Exercise

Simplify: (–4*a*)(5*b*)

Answer available at the end of this section.

Speeding Things Up

The meaning of the expression 2 · 3 · 4 is clear. Parentheses and order of operations are really not needed, as the commutative and associative properties explain that it doesn't matter which of the three numbers you multiply together first.

- You can multiply 2 and 3 first:

$$\begin{aligned}2 \cdot 3 \cdot 4 &= (2 \cdot 3) \cdot 4\\ &= 6 \cdot 4\\ &= 24.\end{aligned}$$

- Or you can multiply 3 and 4 first:

$$\begin{aligned} 2 \cdot 3 \cdot 4 &= 2 \cdot (3 \cdot 4) \\ &= 2 \cdot 12 \\ &= 24. \end{aligned}$$

- Or you can multiply 2 and 4 first:

$$\begin{aligned} 2 \cdot 3 \cdot 4 &= (2 \cdot 4) \cdot 3 \\ &= 8 \cdot 3 \\ &= 24. \end{aligned}$$

So, it does not matter which two factors you multiply first.

Of course, this would not be the case if there were a mixture of multiplication and other operators (division, addition, subtraction). Then we would have to strictly follow the "Rules Guiding Order of Operations." But if the only operator is multiplication, the order of multiplication is irrelevant.

Thus, when we see 2(3*x*), as in Example 1, we should think "It's all multiplication and it doesn't matter which two numbers I multiply together first, so I'll multiply the 2 and the 3 and get 2(3*x*)=6*x*."

Our comments apply equally well to a product of four or more factors. It simply doesn't matter how you group the multiplication. So, in the case of (−3*x*)(−2*y*), as in Example 3, find the product of −2 and −3 and multiply the result by the product of *x* and *y*. That is, (−3*x*)(−2*y*)=6*xy*.

Example 4

Simplify: (2*a*)(3*b*)(4*c*).

Solution

The only operator is multiplication, so we can order and group as we please. So, we'll take the product of 2, 3, and 4, and multiply the result by the product of a, b, and c. That is,

$$(2a)(3b)(4c) = 24abc$$

Exercise

Simplify: (−3*x*)(−2*y*)(−4*z*)

Answer available at the end of this section.

Basic Number Properties for Algebra

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View this YouTube Video online

This video introduces key algebraic properties and shows how they can be used to simplify and rewrite expressions.

The Distributive Property

Multiplication is distributive with respect to addition.

The Distributive Property

If a, b, and c are any integers, then

$$a \cdot (b + c) = a \cdot b + a \cdot c, \text{ or equivalently, } a(b + c) = ab + ac.$$

For example, if we follow the "Rules Guiding Order of Operations" and first evaluate the expression inside the parentheses, then

$$\begin{aligned} 3(4+5) &= 3(9) && \text{Parentheses first: } 4+5=9. \\ &= 27. && \text{Multiply: } 3(9)=27. \end{aligned}$$

But if we "distribute" the 3, we get the same answer.

$$\begin{aligned} 3(4+5) &= 3(4+5) && \text{Each number in parentheses is multiplied by the number 3 outside the parentheses.} \\ &= 3(4)+3(5) \\ &= 12+15 && \text{Multiply first: } 3(4)=12,\ 3(5)=15. \\ &= 27 && \text{Add.} \end{aligned}$$

Example 5

Use the distributive property to simplify: $3(4x + 5)$.

Solution

Distribute the 3.

$$\begin{aligned} 3(4x+5) &= 3(4x)+3(5) && \text{Each number in parentheses is multiplied by the number 3 outside the parentheses.} \\ &= 12x+15 && \text{Multiply first: } 3(4x)=12x,\ 3(5)=15. \end{aligned}$$

Exercise

Use the distributive property to simplify: $2(5z+7)$.

Answer available at the end of this section.

Multiplication is also distributive with respect to subtraction.

The Distributive Property

If a, b, and c are any integers, then

$$a \cdot (b - c) = a \cdot b - a \cdot c, \text{ or equivalently, } a(b - c) = ab - ac.$$

The application of this form of the distributive property is identical to the first, the only difference being the subtraction symbol.

Example 6

Use the distributive property to simplify: $5(3x - 2)$.

Solution

Distribute the 5.

$$\begin{aligned} 5(3x-2) &= 5(3x)-5(2) && \text{Each number in parentheses is multiplied by the number 5 outside the parentheses.} \\ &= 15x-10 && \text{Multiply first: } 5(3x)=15x, 5(2)=10. \end{aligned}$$

Exercise

Use the distributive property to simplify: 7(4*a* – 5).

Answer available at the end of this section.

Example 7

Remove parentheses: (a) –9(2*t* + 7), and (b) –5(4 – 3*y*).

Solution

a) Use the distributive property.

$$\begin{aligned} -9(2t+7) &= -9(2t)+(-9)(7) && \text{Distribute multiplication by } -9. \\ &= -18t+(-63) && \text{Multiply: } -9(2t)=-18t \text{ and } -9(7)=-63. \\ &= -18t-63 && \text{Write the answer in simpler form.} \\ & && \text{Adding } -63 \text{ is the same as subtracting } 63. \end{aligned}$$

b) Use the distributive property.

$$\begin{aligned} -5(4-3y) &= -5(4)-(-5)(3y) && \text{Distribute multiplication by } -5. \\ &= -20-(-15y) && \text{Multiply: } -5(4)=-20 \text{ and } -5(3y)=-15y. \\ &= -18t-63 && \text{Write the answer in simpler form.} \\ & && \text{Subtracting } -15y \text{ is the same as adding } 15y. \end{aligned}$$

Exercise

Remove parentheses: –3(4*t* – 11).

Answer available at the end of this section.

Writing Mathematics

Example 7 stresses the importance of using as few symbols as possible to write your final answer. Hence, –18*t* – 63 is favored over –18*t* + (–63) and –20 + 15*y* is favored over –20 – (–15*y*). You should always make these final simplifications.

Moving a Bit Quicker

Once you've applied the distributive property to a number of problems, showing all the work as in Example 7, you should try to eliminate some of the steps. For example, consider again Example 7(a). It's not difficult to apply the distributive property without writing down a single step, getting:

$$-9(2t+7) = -18t-63.$$

Here's the thinking behind this technique:

1. First, multiply –9 times 2*t*, getting –18*t*.
2. Second, multiply –9 times +7, getting –63.

Note that this provides exactly the same solution found in Example 7(a).

Let try this same technique on Example 7(b).

$$-5(4-3y) = -20 + 15y$$

Here's the thinking behind this technique.

1. First, multiply –5 times 4, getting –20.
2. Second, multiply –5 times –3*y*, getting +15*y*.

Note that this provides exactly the same solution found in Example 7(b).

Introduction to the Distributive Property

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View this YouTube Video online

This video explains how to use the distributive property with variables to simplify expressions.

Extending the Distributive Property

Suppose that we add an extra term inside the parentheses.

Distributive Property

If *a*, *b*, *c*, and *d* are any integers, then

$$a(b + c + d) = ab + ac + ad.$$

Note that we "distributed" the *a* times each term inside the parentheses. Indeed, if we added still another term inside the parentheses, we would "distribute" *a* times that term as well.

Example 8

Remove parentheses: –5(2*x* – 3*y* + 8).

Solution

We will use the "quicker" technique, "distributing" –5 times each term in the parentheses mentally.

$$-5(2x - 3y + 8) = -10x + 15y - 40$$

Here is our thought process:

1. First, multiply –5 times 2*x*, getting –10*x*.

2. Second, multiply –5 times –3*y*, getting +15*y*.
3. Third, multiply –5 times +8, getting –40.

Exercise

Remove parentheses: –3(4*a* – 5*b* + 7)

Answer available at the end of this section.

Example 9

Remove parentheses: –4(–3*a* + 4*b* – 5*c* + 12).

Solution

We will use the "quicker" technique, "distributing" –4 times each term in the parentheses mentally.

$$-4(-3a+4b-5c+12)=12a-16b+20c-48$$

Here is our thought process:

1. First, multiply –4 times –3*a*, getting 12*a*.
2. Second, multiply –4 times +4*b*, getting –16*b*.
3. Third, multiply –4 times –5*c*, getting +20*c*.
4. Fourth, multiply –4 times +12, getting –48.

Exercise

Remove parentheses: –2(–2*x* + 4*y* – 5*z* – 11).

Answer available at the end of this section.

Distributing a Negative

It is helpful to recall that negating is equivalent to multiplying by –1.

Multiplying by –1

Let *a* be any integer, then

$$(-1)a=-a \text{ and } -a=(-1)a.$$

We can use this fact, combined with the distributive property to negate a sum.

Example 10

Remove parentheses: –(*a* + *b*).

Solution

Change the negative symbol into multiplying by –1, then distribute the –1.

$$-(a+b) = (-1)(a+b) \quad \text{Negating is equivalent to multiplying by } -1.$$
$$= -a - b \quad \text{Distribute the } -1.$$

We chose to use the "quicker" technique of "distributing" the –1. Here is our thinking:

1. Multiply –1 times *a*, getting –*a*.
2. Multiply –1 times +*b*, getting –*b*.

Exercise

Remove parentheses: –(4*a* – 3*c*)

Answer available at the end of this section.

The results in Example 10 and Example 11 show us how to negate a sum: Simply negate each term of the sum. Positive terms change to negative, negative terms turn to positive.

Negating a Sum

To negate a sum, simply negate each term of the sum. For example, if *a* and *b* are integers, then

–(*a* + *b*) = –*a* – *b* and – (*a* – *b*) = –*a* + *b*.

Example 12

Remove parentheses: –(5 – 7*u* + 3*t*).

Solution

Simply negate each term in the parentheses.

–(5 – 7*u* + 3*t*) = –5+7*u* – 3*t*

Exercise

Remove parentheses: –(5 – 2*x* + 4*y* – 5*z*)

Answer available at the end of this section.

Answers

Exercise
–35*y*

Exercise
–40*a*

Exercise
–20*ab*

Exercise
–24*xyz*.

Exercise
10z + 14

Exercise
28*a* – 35

Exercise

$-12t + 33$

Exercise

$-12a + 15b - 21$

Exercise

$4x - 8y + 10z + 22$

Exercise

$-4a + 3c$

Exercise

$-5+2x - 4y + 5z$

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2.1.4: Simplifying Algebraic Expressions by Combining Like Terms

We begin our discussion with the definition of a *term*.

Definition: Term

A **term** is a single number or variable, or it can be the product of a number (called its coefficient) and one or more variables (called its variable part). The terms in an algebraic expression are separated by addition symbols.

Example 1

Identify the terms in the algebraic expression

$$3x^2 + 5xy + 9y^2 + 12$$

For each term, identify its coefficient and variable part.

Solution

In tabular form, we list each term of the expression 3x^2 + 5xy + 9y^2 + 12, its coefficient, and its variable part.

Term	Coefficient	Variable Part
$3x^2$	3	x^2
$5xy$	5	xy
$9y^2$	9	y^2
12	12	None

Exercise

How many terms are in the algebraic expression $3x^2 + 2xy - 3y^2$?

Answer available at the end of this section.

Example 2

Identify the terms in the algebraic expression

$$a^3 - 3a^2b + 3ab^2 - b^3$$

For each term, identify its coefficient and variable part.

Solution

The first step is to write each difference as a sum, because the terms of an expression are defined above to be those items separated by addition symbols.

$$a^3 + (-3a^2b) + 3ab^2 + (-b^3)$$

In tabular form, we list each term of the expression $a^3 + (-3a^2b) + 3ab^2 + (-b^3)$, its coefficient, and its variable part.

Term	Coefficient	Variable Part
a^3	1	a^3
$-3a^2b$	−3	a^2b
$3ab^2$	3	ab^2
$-b^3$	−1	b^3

Exercise

How many terms are in the algebraic expression $11 - a^2 - 2ab + 3b^2a$?

Answer available at the end of this section.

Like Terms

We define what is meant by “like terms” and “unlike terms.”

Definition

Like and Unlike Terms. The variable parts of two terms determine whether the terms are like terms or unlike terms.

Like Terms. Two terms are called like terms if they have identical variable parts, which means that the terms must contain the same variables raised to the same exponential powers.

Unlike Terms. Two terms are called unlike terms if their variable parts are different.

Example 3

Classify each of the following pairs as either *like terms* or *unlike terms*: (a) $3x$ and $-7x$, (b) $2y$ and $3y^2$, (c) $-3t$ and $5u$, and (d) $-4a^3$ and $3a^3$.

Solution

Like terms must have *identical* variable parts.

a. $3x$ and $-7x$ have identical variable parts. They are “like terms.”
b. $2y$ and $3y^2$ do not have identical variable parts (the exponents differ). They are “unlike terms.”
c. $-3t$ and $5u$ do not have identical variable parts (different variables). They are “unlike terms.”
d. $-4a^3$ and $3a^3$ have identical variable parts. They are “like terms.”

Exercise

Are $-3xy$ and $11xy$ *like* or *unlike* terms?

Answer available at the end of this section.

Combining Like Terms Intro

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces the idea of combining like terms by starting with a simple example of adding identical items (like Chuck Norrises) and then connecting that idea to algebraic expressions.

Combining Like Terms

When using the distributive property, it makes no difference whether the multiplication is on the left or the right, one still distributes the multiplication times each term in the parentheses.

Distributive Property

If *a*, *b*, and *c* are integers, then

$$a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca.$$

In either case, you distribute a times each term of the sum.

"Like terms" can be combined and simplified. The tool used for combining like terms is the distributive property. For example, consider the expression $3y + 7y$, composed of two "like terms" with a common variable part. We can use the distributive property and write

$$3y + 7y = (3 + 7)y$$

Note that we are using the distributive property in reverse, "factoring out" the common variable part of each term. Checking our work, note that if we redistribute the variable part y times each term in the parentheses, we are returned to the original expression $3y + 7y$.

Example 4

Use the distributive property to combine like terms (if possible) in each of the following expressions: (a) $-5x^2 - 9x^2$, (b) $-5ab + 7ab$, (c) $4y^3 - 7y^2$, and (d) $3xy^2 - 7xy^2$.

Solution

If the terms are "like terms," you can use the distributive property to "factor out" the common variable part.

a) Factor out the common variable part x^2.

$$\begin{aligned} -5x^2 - 9x^2 &= (-5 - 9)x^2 \quad \text{Use the distributive property.} \\ &= -14x^2 \quad \text{Simplify: } -5 - 9 = -5 + (-9) = -14. \end{aligned}$$

b) Factor out the common variable part *ab*.

$$-5ab + 7ab = (-5 + 7)ab \quad \text{Use the distributive property.}$$
$$= 2ab \quad \text{Simplify: } -5 + 7 = 2.$$

c) The terms in the expression $4y^3 - 7y^2$ have different variable parts (the exponents are different). These are "unlike terms" and cannot be combined.

d) Factor out the common variable part xy^2.

$$3xy^2 - 7xy^2 = (3 - 7)xy^2 \quad \text{Use the distributive property.}$$
$$= -4xy^2 \quad \text{Simplify: } 3 - 7 = 3 + (-7) = -4.$$

Exercise

Simplify: $-8z - 11z$

Answer available at the end of this section.

Speeding Things Up a Bit

Once you've written out all the steps for combining like terms, like those shown in Example 4, you can speed things up a bit by following this rule:

Combining Like Terms

To combine like terms, simply add their coefficients and keep the common variable part.

Thus for example, when presented with the sum of two like terms, such as in $5x + 8x$, simply add the coefficients and repeat the common variable part; that is, $5x + 8x = 13x$.

Example 5

Combine like terms:

a. $-9y - 8y$,
b. $-3y^5 + 4y^5$ and
c. $-3u^2 + 2u^2$.

Solution

a) Add the coefficients and repeat the common variable part. Therefore,

$$-9y - 8y = -17y.$$

b) Add the coefficients and repeat the common variable part. Therefore,

$$-3y^5 + 4y^5 = 1y^5.$$

However, note that $1y^5 = y^5$. Following the rule that the final answer should use as few symbols as possible, a better answer is $-3y^5 + 4y^5 = y^5$.

c) Add the coefficients and repeat the common variable part. Therefore,

$$-3u^2 + 2u^2 = (-1)u^2.$$

However, note that $(-1)u^2 = -u^2$. Following the rule that the final answer should use as few symbols as possible, a better answer is $-3u^2 + 2u^2 = -u^2$.

Exercise

Combine: $-3x^2$, and $-4x^2$

Answer available at the end of this section.

Combining Like Terms Intro

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to combine like terms to simplify algebraic expressions.

Simplify

A frequently occurring instruction asks the reader to *simplify* an expression.

Simplify

The instruction simplify is a generic term that means "try to write the expression in its most compact form, using the fewest symbols possible."

One way you can accomplish this goal is by combining like terms when they are present.

Example 6

Simplify: $2x + 3y - 5x + 8y$.

Solution

Use the commutative property to reorder terms and the associative and distributive properties to regroup and combine like terms.

$$\begin{aligned} 2x + 3y - 5x + 8y &= (2x - 5x) + (3y + 8y) && \text{Reorder and regroup.} \\ &= -3x + 11y && \text{Combine like terms:} \\ & && 2x - 5x = -3x \text{ and } 3y + 8y = 11y. \end{aligned}$$

Alternate solution

Of course, you do not need to show the regrouping step. If you are more comfortable combining like terms in your head, you are free to present your work as follows:

$$2x + 3y - 5x + 8y = -3x + 11y.$$

Exercise

Simplify: $-3a + 4b - 7a - 9b$

Answer available at the end of this section.

Example 7

Simplify: $-2x - 3 - (3x + 4)$.

Solution

First, distribute the negative sign.

$$-2x - 3 - (3x + 4) = -2x - 3 - 3x - 4 \quad -(3x + 4) = -3x - 4.$$

Next, use the commutative property to reorder, then the associative property to regroup. Then combine like terms.

$$\begin{aligned} &= (-2x - 3x) + (-3 - 4) && \text{Reorder and regroup.} \\ &= -5x + (-7) && \text{Combine like terms:} \\ & && -2x - 3x = -5x. \\ &= -5x - 7 && \text{Simplify:} \\ & && -5x + (-7) = -5x - 7. \end{aligned}$$

Alternate solution

You may skip the second step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$\begin{aligned} -2x - 3 - (3x + 4) &= -2x - 3 - 3x - 4 && \text{Distribute negative sign.} \\ &= -5x - 7 && \text{Combine like terms.} \end{aligned}$$

Exercise

Simplify: $-9a - 4 - (4a - 8)$

Answer available at the end of this section.

Example 8

Simplify: $2(5 - 3x) - 4(x + 3)$.

Solution

Use the distributive property to expand, then use the commutative and associative properties to group the like terms and combine them.

$$\begin{aligned} 2(5 - 3x) - 4(x + 3) &= 10 - 6x - 4x - 12 && \text{Use the distributive property.} \\ &= (-6x - 4x) + (10 - 12) && \text{Group like terms.} \\ &= -10x - 2 && \text{Combine like terms: } -6x - 4x = -10x \text{ and } 10 - 12 = -2. \end{aligned}$$

Alternate solution

You may skip the second step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$\begin{aligned} 2(5-3x)-4(x+3) &= 10-6x-4x-12 \quad \text{Distribute.} \\ &= -10x-2 \quad \text{Combine like terms.} \end{aligned}$$

Exercise

Simplify: $-2(3a-4)-2(5-a)$

Answer available at the end of this section.

Example 9

Simplify: $-8(3x^2y-9xy)-8(-7x^2-8xy)$

Solution

We will proceed a bit quicker with this solution, using the distributive property to expand, then combining like terms mentally.

$$\begin{aligned} -8(3x^2y-9xy)-8(-7x^2y-8xy) &= -24x^2y+72xy+56x^2y+64xy \\ &= 32x^2y+136xy \end{aligned}$$

Exercise

Simplify: $(a^2-2ab)-2(3ab+a^2)$

Answer available at the end of this section.

Simplifying Algebraic Expressions

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View this YouTube Video online

This video shows how to simplify algebraic expressions by using the distributive property and combining like terms.

Answers

Exercise

3

Exercise

4

Exercise

Like terms

Exercise

$-19z$

Exercise

$-7x^2$

Exercise

$-10a - 5b$

Exercise

$-13a + 4$

Exercise

$-4a - 2$

Exercise

$-a^2 - 8ab$

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2.1.5: Algebraic Expressions Practice

The following set of exercises provides practice for the concepts covered in Module 2.1: Algebraic Expressions. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 2.1.1 - Evaluate Algebraic Expressions

Evaluate the expression for the given value.

1. $9x + 7$ when $x = 3$

2. $8x - 6$ when $x = 7$

3. $x^2 + 5x - 8$ when $x = 6$

4. $6x + 3y - 9$ when $x = 6, y = 9$

5. $(x + y)^2$ when $x = 6, y = 9$

6. $2l + 2w$ when $l = 18, w = 14$

Section 2.1.2 - Translating Algebraic Expressions

Translate the given word phrase into an algebraic expression.

7. The product of 9 and y

8. 3 less than x

9. The quotient of y and 8

10. Seven times the sum of y and four

Write an algebraic expression.

11. A computer lab has L laptops. The number of desktop computers in the computer lab is 3 more than the number of laptops. What is the number of desktop computers in terms of L?

12. At a construction site, one level has b support beams. Another level has 4 fewer beams than this. What is the number of beams on the smaller level in terms of b?

13. A mechanic has n bolts. The number of washers is 7 less than twice the number of bolts. What is the number of washers in terms of n?

14. A business orders s large storage containers. The number of small containers is 3 more than 6 times the number of large containers. What is the number of small containers in terms of s?

Section 2.1.3 - Simplifying Algebraic Expressions Using Properties of Multiplication

Simplify the expression.

15. $7(-8x)$

16. 9(6x)

17. 3(4 + 6x)

18. –2(5x + 5)

19. –(–6y + 9x – 7)

Section 2.1.4 - Simplify Expressions by Combining Like Terms

Simplify the expression.

20. –9r – 10r + 3r

21. 11 + 16s – 14 – 6s

22. $-19y^2 + 18y^3 - 5y^2 - 17y^3$

23. 10y – 6 – (–10 – 10y)

24. –6(–5n – 4) – 9(3 + 4n)

25. $-4(-4xy + 5y^3) + 6(-5xy - 9y^3)$

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2.2: SOLVING EQUATIONS

In this module, you'll learn how to solve equations using the properties of equality and a general strategy. These are skills you'll actually use in technical fields when you need to figure out an unknown value. For example, you might need to find a missing measurement in construction, calculate a value in an electrical circuit, adjust a mixture in a shop or lab, or determine time or cost in a business setting.

We'll start by checking whether a given value is a solution to an equation. Then you'll solve equations using addition, subtraction, multiplication, and division. From there, you'll work with equations that have variables and constants on one or both sides and apply a general strategy to solve more complex problems.

To help you remember the steps, watch the following video, which presents the process in a fun and memorable way.

Two-Step Equations Song

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online



2.2.1: Solve One Step Equations Using Addition and Subtraction

2.2.2: Solve One Step Equations Using Multiplication and Division

2.2.3: Solve Equations with Variables and Constants on Both Sides

2.2.4: Solve Equations Using a General Strategy

2.2.5: Solving Equations Practice

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2.2.1: Solve One Step Equations Using Addition and Subtraction

When some people hear the word *algebra*, they think of solving equations. The applications of solving equations are limitless and extend to all careers and fields. In this section, we will begin solving equations. We will start by solving basic equations, and then as we proceed through the course we will build up our skills to cover many different forms of equations.

Determine Whether a Number is a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. An algebraic equation states that two algebraic expressions are equal. To solve an equation is to determine the values of the variable that make the equation a true statement. Any number that makes the equation true is called a **solution** of the equation. It is the answer to the puzzle!

Definition: Solution of an Equation

A solution to an equation is a value of a variable that makes a true statement when substituted into the equation. The process of finding the solution to an equation is called solving the equation.

To find the solution to an equation means to find the value of the variable that makes the equation true. Can you recognize the solution of $x+2=7$? If you said 5, you're right! We say 5 is a solution to the equation $x+2=7$ because when we substitute 5 for x the resulting statement is true.

$$\begin{aligned} x+2&=7\\ 5+2&\stackrel{?}{=}7\\ 7&=7\ \checkmark \end{aligned}$$

Since $5+2=7$ is a true statement, we know that 5 is indeed a solution to the equation. The symbol $\stackrel{?}{=}$ asks whether the left side of the equation is equal to the right side. Once we know, we can change to an equal sign $(=)$ or not-equal sign $(\neq)$.

HOW TO: DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

Step 1. Substitute the number for the variable in the equation.

Step 2. Simplify the expressions on both sides of the equation.

Step 3. Determine whether the resulting equation is true.

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example 1

Determine whether $x=5$ is a solution of $6x-17=16$.

Solution

	$6x-17=16$
Substitute 5 for x.	$6\cdot 5-17\stackrel{?}{=}16$

Multiply.	$30 - 17 \stackrel{?}{=} 16$
Subtract	$13 \neq 16$

So $x = 5$ is not a solution to the equation $6x - 17 = 16$.

Exercise

Is $x = 3$ a solution of $4x - 7 = 16$?

Answer available at the end of this section.

Exercise

Is $x = 2$ a solution of $6x - 2 = 10$?

Answer available at the end of this section.

Example 2

Determine whether $y = 2$ is a solution of $6y - 4 = 5y - 2$.

Solution

Here, the variable appears on both sides of the equation. We must substitute 2 for each y.

	$6y - 4 = 5y - 2$
Substitute 2 for y.	$6(2) - 4 \stackrel{?}{=} 5(2) - 2$
Multiply.	$12 - 4 \stackrel{?}{=} 10 - 2$
Subtract	$8 = 8$ ✓

Since $y = 2$ results in a true equation, we know that 2 is a solution to the equation $6y - 4 = 5y - 2$.

Exercise

Is $y = 3$ a solution of $9y - 2 = 8y + 1$?

Answer available at the end of this section.

Exercise

Is $y = 4$ a solution of $5y - 3 = 3y + 5$?

Answer available at the end of this section.

Model the Subtraction Property of Equality

We will use a model to help you understand how the process of solving an equation is like solving a puzzle. An envelope represents

the variable – since its contents are unknown – and each counter represents one.

Suppose a desk has an imaginary line dividing it in half. We place three counters and an envelope on the left side of desk, and eight counters on the right side of the desk as in Figure 2.2.1.1. Both sides of the desk have the same number of counters, but some counters are hidden in the envelope. Can you tell how many counters are in the envelope?

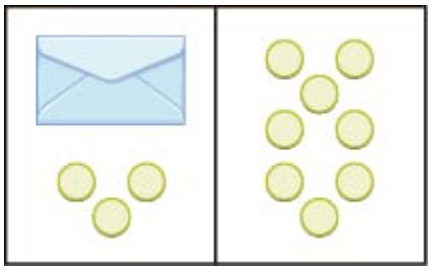


Figure 2.2.1.1

What steps are you taking in your mind to figure out how many counters are in the envelope? Perhaps you are thinking "I need to remove the 3 counters from the left side to get the envelope by itself. Those 3 counters on the left match with 3 on the right, so I can take them away from both sides. That leaves five counters on the right, so there must be 5 counters in the envelope." Figure 2.2.1.2 shows this process.

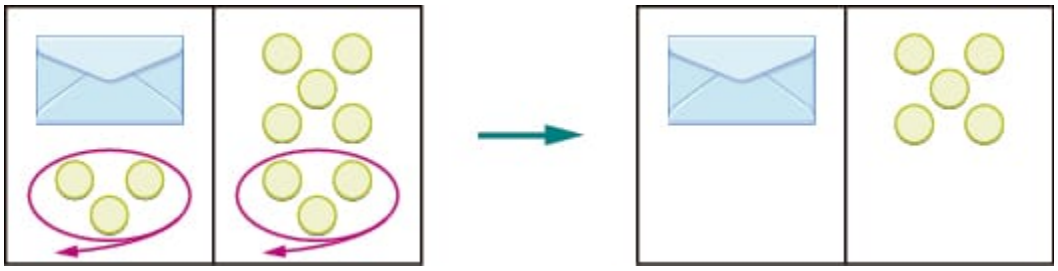


Figure 2.2.1.2

What algebraic equation is modeled by this situation? Each side of the desk represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x, so the number of counters on the left side of the desk is $x + 3$. On the right side of the desk are 8 counters. We are told that $x + 3$ is equal to 8 so our equation is $x + 3 = 8$.

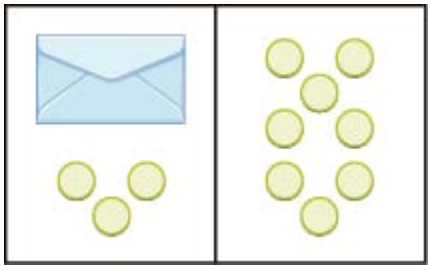


Figure 2.2.1.3

$$x + 3 = 8$$

Let's write algebraically the steps we took to discover how many counters were in the envelope.

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Now let's check our solution. We substitute 5 for x in the original equation and see if we get a true statement.

$$\begin{aligned} x+3 &= 8 \\ 5+3 &\stackrel{?}{=} 8 \\ 8 &= 8 \checkmark \end{aligned}$$

Our solution is correct. Five counters in the envelope plus three more equals eight

Example 3

Write an equation modeled by the envelopes and counters, and then solve the equation:

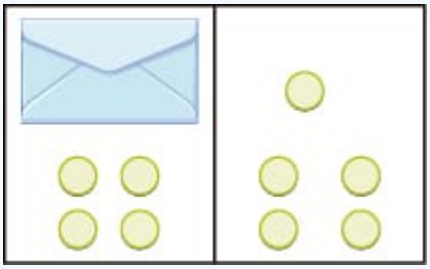


Figure 2.2.1.4

Solution

On the left, write x for the contents of the envelope, add the 4 counters, so we have x + 4.	x + 4
On the right, there are 5 counters.	5
The two sides are equal.	x + 4 = 5
Solve the equation by subtracting 4 counters from each side.	

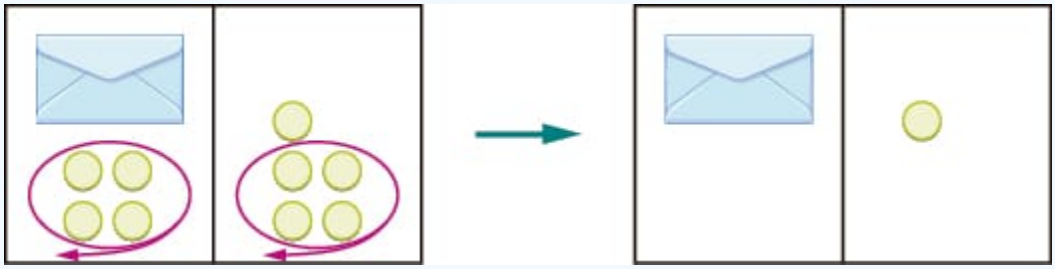


Figure 2.2.1.5

We can see that there is one counter in the envelope. This can be shown algebraically as:

$$\begin{aligned} x+4 &= 5 \\ x+4-4 &= 5-4 \\ x &= 1 \end{aligned}$$

Substitute 1 for x in the equation to check.

$$\begin{aligned} x+4 &= 5 \\ 1+4 &\stackrel{?}{=} 5 \\ 5 &= 5 \checkmark \end{aligned}$$

Since $x = 1$ makes the statement true, we know that 1 is indeed a solution.

Exercise

Write the equation modeled by the envelopes and counters, and then solve the equation:

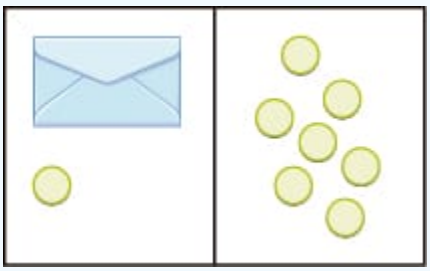


Figure 2.2.1.6

Answer available at the end of this section.

Algebra Basics

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This video explains how solving equations is like balancing a scale and demonstrates how to solve equations using the addition and subtraction properties of equality.

Solve Equations Using the Subtraction Property of Equality

Our puzzle has given us an idea of what we need to do to solve an equation. The goal is to isolate the variable by itself on one side of the equations. In the previous examples, we used the Subtraction Property of Equality, which states that when we subtract the same quantity from both sides of an equation, we still have equality.

Definition: Subtraction Property of Equality

For any numbers a, b, and c, if $a = b$ then $a - c = b - c$

Think about twin brothers Andy and Bobby. They are 17 years old. How old was Andy 3 years ago? He was 3 years less than 17, so his age was $17 - 3$, or 14. What about Bobby's age 3 years ago? Of course, he was 14 also. Their ages are equal now, and subtracting the same quantity from both of them resulted in equal ages 3 years ago.

$$a = b$$
$$a - 3 = b - 3$$

HOW TO: SOLVE AN EQUATION USING THE SUBTRACTION PROPERTY OF EQUALITY

Step 1. Use the Subtraction Property of Equality to isolate the variable.

Step 2. Simplify the expressions on both sides of the equation.

Step 3. Check the solution.

Example 4

Solve: $x + 8 = 17$.

Solution

We will use the Subtraction Property of Equality to isolate x.

	$x + 8 = 17$
Subtract 8 from both sides.	$x + 8 - 8 = 17 - 8$
Simplify.	$x = 9$
Check the solution.	$9 + 8 = 17$
	$17 = 17$ ✓

Since $x = 9$ makes $x + 8 = 17$ a true statement, we know 9 is the solution to the equation.

Exercise

Solve: $x + 6 = 19$

Answer available at the end of this section.

Exercise

Solve: $x + 9 = 14$

Answer available at the end of this section.

Example 5

Solve: $100 = y + 74$.

Solution

To solve an equation, we must always isolate the variable—it doesn't matter which side it is on. To isolate y, we will subtract 74 from both sides.

	$100 = y + 74$
Subtract 74 from both sides.	$100 - 74 = y + 74 - 74$
Simplify.	$26 = y$
Substitute 26 for y to check	$100 \stackrel{?}{=} 26 + 74$
	$100 = 100$ ✓

Since $y = 26$ makes $100 = y + 74$ a true statement, we have found the solution to this equation.

Exercise

Solve: $95 = y + 67$

Answer available at the end of this section.

Exercise

Solve: $91 = y + 45$

Answer available at the end of this section.

Solve Equations Using the Addition Property of Equality

In all the equations we have solved so far, a number was added to the variable on one side of the equation. We used subtraction to "undo" the addition in order to isolate the variable.

But suppose we have an equation with a number subtracted from the variable, such as $x - 5 = 8$. We want to isolate the variable, so to "undo" the subtraction we will add the number to both sides.

We use the Addition Property of Equality, which says we can add the same number to both sides of the equation without changing the equality. Notice how it mirrors the Subtraction Property of Equality.

Definition: Addition Property of Equality

For any numbers a, b, and c, if $a = b$ then $a + c = b + c$

Remember the 17-year-old twins, Andy and Bobby? In ten years, Andy's age will still equal Bobby's age. They will both be 27.

$$a = b$$
$$a + 10 = b + 10$$

We can add the same number to both sides and still keep the equality.

HOW TO: SOLVE AN EQUATION USING THE ADDITION PROPERTY OF EQUALITY

Step 1. Use the Addition Property of Equality to isolate the variable.

Step 2. Simplify the expressions on both sides of the equation.

Step 3. Check the solution.

Example 6

Solve: $x - 5 = 8$.

Solution

We will use the Addition Property of Equality to isolate the variable.

	$x - 5 = 8$

Add 5 to both sides.	$x-5+5=8+5$
Simplify	$x=13$
Now we can check. Let x = 13.	$13-5\stackrel{?}{=}8$
	$8=8$ ✓

Exercise

Solve: $x-9=13$

Answer available at the end of this section.

Exercise

Solve: $y-1=3$

Answer available at the end of this section.

Example 7

Solve: $27=a-16$.

Solution

We will add 16 to each side to isolate the variable.

	$27=a-16$
Add 16 to each side.	$27+16=a-16+16$
Simplify.	$43=a$
Now we can check. Let a = 43.	$27\stackrel{?}{=}43-16$
	$27=27$ ✓

The solution to $27=a-16$ is $a=43$.

Exercise

Solve: $19=a-18$

Answer available at the end of this section.

Exercise

Solve: $27=n-14$

Answer available at the end of this section.

Algebraic Equations and Their Solutions

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This video demonstrates how to solve basic equations by isolating the variable using the addition and subtraction properties of equality and the multiplication property of equality.

Answers

Exercise
no

Exercise
yes

Exercise
yes

Exercise
yes

Exercise
$x+1=7; x=6$

Exercise
$x=13$

Exercise
$x=5$

Exercise
$y=28$

Exercise
$y=46$

Exercise
$x=22$

Exercise
$y=4$

Exercise
$a=37$

Exercise
$n=41$

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2.2.2: Solve One Step Equations Using Multiplication and Division

Model the Division Property of Equality

All of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term. Now we'll see how to solve equations that involve division. We will model an equation with envelopes and counters in Figure $2.2.2.1$.

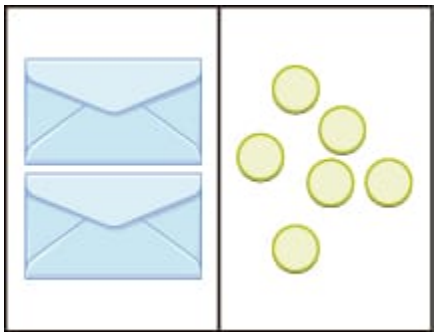


Figure $2.2.2.1$

Here, there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are "hidden" in the envelopes. So how many counters are in each envelope?

To determine the number, separate the counters on the right side into 2 groups of the same size. So 6 counters divided into 2 groups means there must be 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in Figure $2.2.2.2$? There are two envelopes, and each contains x counters. Together, the two envelopes must contain a total of 6 counters. So the equation that models the situation is $2x = 6$.

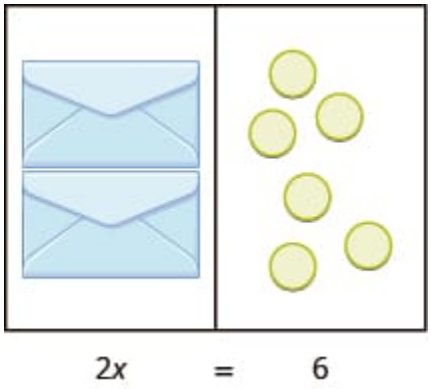


Figure $2.2.2.2$

We can divide both sides of the equation by 2 as we did with the envelopes and counters.

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

We found that each envelope contains 3 counters. Does this check? We know $2 \bullet 3 = 6$, so it works. Three counters in each of two

envelopes does equal six. Figure 2.2.2.3 shows another example.

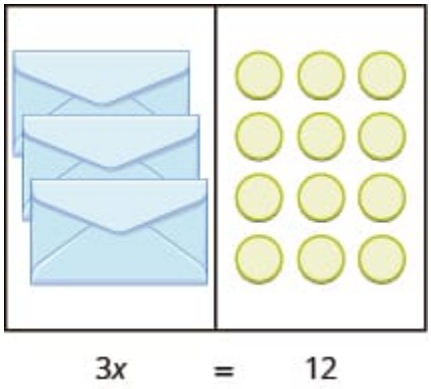


Figure 2.2.2.3

Now we have 3 identical envelopes and 12 counters. How many counters are in each envelope? We have to separate the 12 counters into 3 groups. Since $12 \div 3 = 4$, there must be 4 counters in each envelope. See Figure 2.2.2.4.

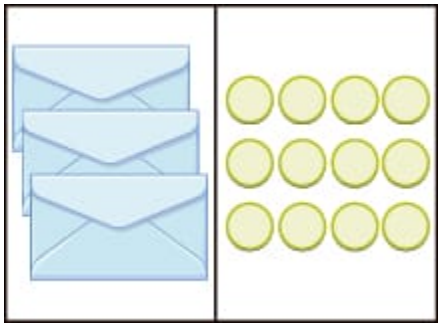


Figure 2.2.2.4

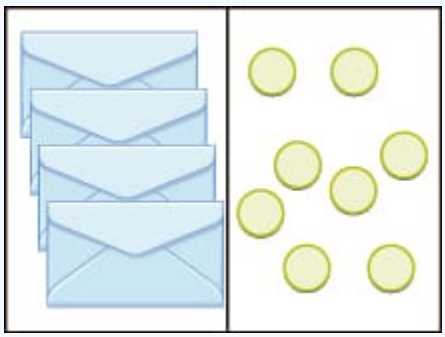
The equation that models the situation is $3x = 12$. We can divide both sides of the equation by 3.

$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

Does this check? It does because $3 \bullet 4 = 12$.

Example 1

Write an equation modeled by the envelopes and counters, and then solve it.



Solution

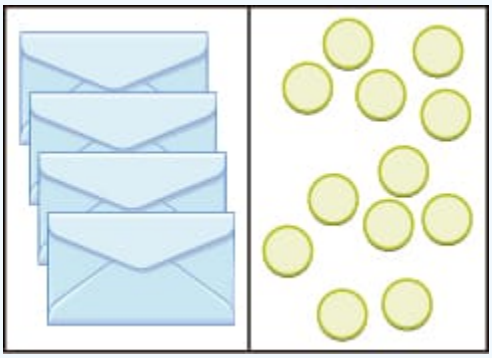
There are 4 envelopes, or 4 unknown values, on the left that match the 8 counters on the right. Let's call the unknown quantity in the envelopes x.

Write the equation.	$4x = 8$
Divide both sides by 4.	$\frac{4x}{4} = \frac{8}{4}$
Simplify.	$x = 2$

There are 2 counters in each envelope.

Exercise

Write the equation modeled by the envelopes and counters. Then solve it.



Answer available at the end of this section.

Solve Equations Using the Division Property of Equality

The previous examples lead to the Division Property of Equality. When you divide both sides of an equation by any nonzero number, you still have equality.

Definition: Division Property of Equality

For any numbers a, b, c, where $c \neq 0$,

If $a = b$ then $\frac{a}{c} = \frac{b}{c}$.

Example 2

Solve: $7x = -49$.

Solution

To isolate x, we need to undo multiplication.

Divide each side by 7.	$\frac{7x}{7} = \frac{-49}{7}$
Simplify.	$x = -7$

Check the solution.

Substitute –7 for x.	$7(-7) \stackrel{?}{=} -49$
	$-49 = -49$ ✓

Therefore, -7 is the solution to the equation.

Exercise

Solve: $8a = 56$

Answer available at the end of this section.

Exercise

Solve: $11n = 121$

Answer available at the end of this section.

Example 3

Solve: $-3y = 63$.

Solution

To isolate y, we need to undo the multiplication.

Divide each side by –3.	$\frac{-3y}{-3} = \frac{63}{-3}$
Simplify.	$y = -21$

Check the solution.

Substitute –21 for y.	$-3(-21) \stackrel{?}{=} 63$
	$63 = 63$ ✓

Since this is a true statement, $y = -21$ is the solution to the equation.

Exercise

Solve: $-8p = 96$

Answer available at the end of this section.

Exercise

Solve: $-12m = 108$

Answer available at the end of this section.

Algebra Basics

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This video demonstrates how to solve one-step equations using the multiplication and division properties of equality.

Solve Equations Using the Multiplication Property of Equality

Definition: Multiplication Property of Equality

For any numbers a, b, c, where $c \neq 0$,

If $a = b$ then $ac = bc$.

Example 4

Solve: $\frac{x}{7} = 12$

Solution

To isolate x, we need to undo the division.

Multiply each side by 7.	$\frac{x}{7} \cdot 7 = 12 \cdot 7$
Simplify.	$x = 84$

Check the solution.

Substitute 84 for x.	$\frac{84}{7} \stackrel{?}{=} 12$
	$12 = 12$ ✓

Therefore, $x = 84$ is the solution to the equation.

Exercise

Solve: $\frac{x}{2} = 19$

Answer available at the end of this section.

Example 5

Solve: $\frac{1}{2}x = 8$

Solution

To isolate x, we need to undo the multiplication.

Since x is being multiplied by the fraction $\frac{1}{2}$, you can multiply both sides by 2 to get a coeeficient of 1 for the variable.

Multiply each side by 2.	$\frac{1}{2}x = 8$ $2\left(\frac{1}{2}x\right) = 2(8)$
Simplify.	$\frac{2}{2}x = 16$ $x = 16$

Check the solution.

Substitute 16 for x.	$\frac{1}{2}(16) = 8$
	$\frac{16}{2} \stackrel{?}{=} 8$ $8 = 8\checkmark$

Therefore, $x = 16$ is the solution to the equation.

Exercise

Solve: $\left(-\frac{1}{4}\right)x = 2$

Answer available at the end of this section.

Solve One Step Equations by Multiplying by a Reciprocal

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View this YouTube Video online

This video demonstrates how to solve equations using the multiplication property of equality by multiplying both sides by a reciprocal, with four step-by-step examples.

Answers

Exercise

$4x = 12$; $x = 3$

Exercise

7

Exercise

11

Exercise

-12

Exercise

-9

Exercise

38

Exercise

$x = -8$

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2.2.3: Solve Equations with Variables and Constants on Both Sides

You may have noticed that in all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we'll see how to solve equations where the variable terms and/or constant terms are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the variable side, and the other side of the equation to be the constant side. Then, we will use the Subtraction and Addition Properties of Equality, step by step, to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that started with variables and constants on both sides into the form ax = b. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Solve an Equation with Constants on Both Sides

When an equation has constants on both sides, we begin by using the Addition or Subtraction Properties of Equality to move all constant terms to one side of the equation. This simplifies the equation and allows us to rewrite it in a form we already know how to solve. From there, we can isolate the variable using the same steps we have practiced.

Example 1

Solve: 4x + 6 = –14.

Solution

In this equation, the variable is only on the left side. It makes sense to call the left side the variable side. Therefore, the right side will be the constant side. We'll write the labels above the equation to help us remember what goes where.

Since the left side is the variable side, the 6 is out of place. We must "undo" adding 6 by subtracting 6, and to keep the equality we must subtract 6 from both sides. Use the Subtraction Property of Equality.	$4x + 6 - 6 = -14 - 6$
Simplify.	$4x = -20$
Now all the x's are on the left and the constant on the right.	
Use the Division Property of Equality.	$\frac{4x}{4} = \frac{-20}{4}$
Simplify.	$x = -5$

Check: Let x = –5.	$4x+6 = -14$ $4(-5) + 6 \stackrel{?}{=} -14$ $-20+6 \stackrel{?}{=} -14$ $-14 = -14$ ✓

Exercise

Solve: 3x + 4 = –8.

Answer available at the end of this section.

Exercise

Solve: 5a + 3 = –37.

Answer available at the end of this section.

Example 2

Solve: 2y – 7 = 15.

Solution

Notice that the variable is only on the left side of the equation, so this will be the variable side and the right side will be the constant side. Since the left side is the variable side, the 7 is out of place. It is subtracted from the 2y, so to 'undo' subtraction, add 7 to both sides.

Add 7 to both sides.	$2y-7+7=15+7$
Simplify.	$2y=22$
The variables are now on one side and the constants on the other.	
Divide both sides by 2.	$\frac{2y}{2}=\frac{22}{2}$
Simplify.	$y=11$
Check: Substitute: y = 11.	$2y-7=15$ $2\cdot 11-7 \stackrel{?}{=} 15$ $22-7 \stackrel{?}{=} 15$ $15=15$ ✓

Exercise

Solve: 5y – 9 = 16.

Answer available at the end of this section.

Exercise

Solve: 3m – 8 = 19.

Answer available at the end of this section.

Solving Two-Step Equations

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This video will demonstrate how to solve two step equations that have a constant on both sides.

Solve an Equation with Variables on Both Sides

What if there are variables on both sides of the equation? We will start like we did above—choosing a variable side and a constant side, and then use the Subtraction and Addition Properties of Equality to collect all variables on one side and all constants on the other side. Remember, what you do to the left side of the equation, you must do to the right side too.

Example 3

Solve: 5x = 4x + 7.

Solution

Here the variable, x, is on both sides, but the constants appear only on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

We don't want any variables on the right, so subtract the 4x.	$5x - 4x = 4x - 4x + 7$
Simplify.	$x = 7$
We have all the variables on one side and the constants on the other. We have solved the equation.	
Check: Substitute 7 for x.	$5x = 4x + 7$ $5(7) \stackrel{?}{=} 4(7) + 7$ $35 \stackrel{?}{=} 28 + 7$ $35 = 35$ ✓

Exercise

Solve: 6n = 5n + 10.

Answer available at the end of this section.

Exercise

Solve: −6c = −7c + 1.

Answer available at the end of this section.

Example 4

Solve: 5y − 8 = 7y.

Solution

The only constant, −8, is on the left side of the equation and variable, y, is on both sides. Let's leave the constant on the left and collect the variables to the right.

Subtract 5y from both sides.	$5y - 5y - 8 = 7y - 5y$
Simplify.	$-8 = 2y$
We have the variables on the right and the constants on the left. Divide both sides by 2.	$\frac{-8}{2} = \frac{2y}{2}$
Simplify.	$-4 = y$
Rewrite with the variable on the left.	$y = -4$
Check: Let y = −4.	$5y - 8 = 7y$ $5(-4) - 8 \stackrel{?}{=} 7(-4)$ $-20 - 8 \stackrel{?}{=} -28$ $-28 = -28$ ✓

Exercise

Solve: 3p − 14 = 5p.

Answer available at the end of this section.

Exercise

Solve: 8m + 9 = 5m.

Answer available at the end of this section.

Example 5

Solve: 7x = – x + 24.

Solution

The only constant, 24, is on the right, so let the left side be the variable side.

Remove the –x from the right side by adding x to both sides.	$7x + x = -x + x + 24$
Simplify.	$8x = 24$
All the variables are on the left and the constants are on the right. Divide both sides by 8.	$\frac{8x}{8} = \frac{24}{8}$
Simplify.	$x = 3$
Check: Substitute x = 3.	$7x = -x + 24$ $7(3) \stackrel{?}{=} -(3) + 24$ $21 \stackrel{?}{=} -3 + 24$ $21 = 21$ ✓

Exercise

Solve: 12j = –4j + 32.

Answer available at the end of this section.

Exercise

Solve: 8h = –4h + 12.

Answer available at the end of this section.

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables *and* constants on both sides of the equation. As we did before, we'll collect the variable terms to one side and the constants to the other side.

Example 6

Solve: 7x + 5 = 6x + 2.

Solution

Start by choosing which side will be the variable side and which side will be the constant side. The variable terms are 7x and 6x. Since 7 is greater than 6, make the left side the variable side and so the right side will be the constant side.

Collect the variable terms to the left side by subtracting 6x from both sides.	$7x - 6x + 5 = 6x - 6x + 2$
Simplify.	$x + 5 = 2$

Now, collect the constants to the right side by subtracting 5 from both sides.	$x + 5 - 5 = 2 - 5$
Simplify.	$x = -3$
The solution is x = –3.	
Check: Let x = –3.	$7x + 5 = 6x + 2$ $7(-3) + 5 \stackrel{?}{=} 6(-3) + 2$ $-21 + 5 \stackrel{?}{=} -18 + 2$ $-16 = -16$ ✓

Exercise

Solve: 12x + 8 = 6x + 2.

Answer available at the end of this section.

Exercise

Solve: 9y + 4 = 7y + 12.

Answer available at the end of this section.

We'll summarize the steps we took so you can easily refer to them.

HOW TO: SOLVE AN EQUATION WITH VARIABLES AND CONSTANTS ON BOTH SIDES

Step 1. Choose one side to be the variable side and then the other will be the constant side.

Step 2. Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.

Step 3. Collect the constants to the other side, using the Addition or Subtraction Property of Equality.

Step 4. Make the coefficient of the variable 1, using the Multiplication or Division Property of Equality.

Step 5. Check the solution by substituting it into the original equation.

It is a good idea to make the variable side the one in which the variable has the larger coefficient. This usually makes the arithmetic easier.

Example 7

Solve: 6n – 2 = –3n + 7.

Solution

We have 6n on the left and –3n on the right. Since 6 > – 3, make the left side the "variable" side.

We don't want variables on the right side—add 3n to both sides to leave only constants on the right.	$6n + 3n - 2$ $= -3n + 3n + 7$

Combine like terms.	$9n - 2 = 7$
We don't want any constants on the left side, so add 2 to both sides.	$9n - 2 + 2 = 7 + 2$
Simplify.	$9n = 9$
The variable term is on the left and the constant term is on the right. To get the coefficient of n to be one, divide both sides by 9.	$\frac{9n}{9} = \frac{9}{9}$
Simplify.	$n = 1$
Check: Substitute 1 for n.	$6n - 2 = -3n + 7$ $6(1) - 2 \stackrel{?}{=} -3(1) + 7$ $4 \stackrel{?}{=} -3 + 7$ $4 = 4$ ✓

Exercise

Solve: 8q – 5 = –4q + 7.

Answer available at the end of this section.

Exercise

Solve: 7n – 3 = n + 3.

Answer available at the end of this section.

Example 8

Solve: 2a – 7 = 5a + 8.

Solution

This equation has 2a on the left and 5a on the right. Since 5 > 2, make the right side the variable side and the left side the constant side.

Subtract 2a from both sides to remove the variable term from the left.	$2a - 2a - 7 = 5a - 2a + 8$
Combine like terms.	$-7 = 3a + 8$
Subtract 8 from both sides to remove the constant from the right.	$-7 - 8 = 3a + 8 - 8$
Simplify.	$-15 = 3a$
Divide both sides by 3 to make 1 the coefficient of a.	$\frac{-15}{3} = \frac{3a}{3}$
Simplify.	$-5 = a$

Check: Let a = –5.	$2a-7=5a+8$ $2(-5)-7\stackrel{?}{=}5(-5)+8$ $-10-7\stackrel{?}{=}-25+8$ $-17=-17$ ✓

Note that we could have made the left side the variable side instead of the right side, but it would have led to a negative coefficient on the variable term. While we could work with the negative, there is less chance of error when working with positives. The strategy outlined above helps avoid the negatives!

Exercise

Solve: 2a – 2 = 6a + 18.

Answer available at the end of this section.

Exercise

Solve: 4k – 1 = 7k + 17.

Answer available at the end of this section.

Example: Variables on both sides

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This video demonstrates how to solve an equation with variables and constants on both sides by working the same example two different ways.

To solve an equation with fractions, we still follow the same steps to get the solution.

Example 9

Solve: $\frac{3}{2}$x + 5 = $\frac{1}{2}$x – 3.

Solution

Since $\frac{3}{2}>\frac{1}{2}$, make the left side the variable side and the right side the constant side.

Subtract $\frac{1}{2}$x from both sides.	$\frac{3}{2}x-\frac{1}{2}x+5=\frac{1}{2}x\frac{1}{2}x-3$

Combine like terms.	$x + 5 = -3$
Subtract 5 from both sides.	$x + 5 - 5 = -3 - 5$
Simplify.	$x = -8$
Check: Let x = –8.	$\frac{3}{2}x + 5 = \frac{1}{2}x - 3$ $\frac{3}{2}(-8) + 5 \stackrel{?}{=} \frac{1}{2}(-8) - 3$ $-12 + 5 \stackrel{?}{=} -4 - 3$ $-7 = -7$ ✓

Exercise

Solve: $\frac{7}{8}$x - 12 = $-\frac{1}{8}$x – 2.

Answer available at the end of this section.

Exercise

Solve: $\frac{7}{6}$y + 11 = $\frac{1}{6}$y + 8.

Answer available at the end of this section.

We follow the same steps when the equation has decimals, too.

Solve: 3.4x + 4 = 1.6x – 5.

Solution

Since 3.4 > 1.6, make the left side the variable side and the right side the constant side.

Subtract 1.6x from both sides.	$3.4x - 1.6x + 4 = 1.6x - 1.6x - 5$
Combine like terms.	$1.8x + 4 = -5$
Subtract 4 from both sides.	$1.8x + 4 - 4 = -5 - 4$
Simplify.	$1.8x = -9$
Use the Division Property of Equality.	$\frac{1.8x}{1.8} = \frac{-9}{1.8}$
Simplify.	$x = -5$
Check: Let x = –5.	$3.4x + 4 = 1.6x - 5$ $3.4(-5) + 4 \stackrel{?}{=} 1.6(-5) - 5$ $-17 + 4 \stackrel{?}{=} -8 - 5$ $-13 = -13$ ✓

Exercise

Solve: $2.8x + 12 = -1.4x - 9$.

Answer available at the end of this section.

Exercise

Solve: $3.6y + 8 = 1.2y - 4$.

Answer available at the end of this section.

Answers

Exercise
x = -4

Exercise
a = -8

Exercise
y = 5

Exercise
m = 9

Exercise
n = 10

Exercise
c = 1

Exercise
p = -7

Exercise
m = -3

Exercise
j = 2

Exercise
h = 1

Exercise
x = -1

Exercise
y = 4

Exercise
q = 1

Exercise
n = 1

Exercise
a = -5

Exercise
k = -6

Exercise
x = 10

Exercise
y = -3

Exercise
x = -5

Exercise
y = -5

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2.2.4: Solve Equations Using a General Strategy

Solve Equations Using a General Strategy

Each of the section of this module has dealt with solving one specific form of a linear equation. It's time now to lay out an overall strategy that can be used to solve any linear equation. We call this the general strategy. Some equations won't require all the steps to solve, but many will. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

HOW TO: USE A GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS

Step 1. Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms.

Step 2. Collect all the variable terms to one side of the equation. Use the Addition or Subtraction Property of Equality.

Step 3. Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.

Step 4. Make the coefficient of the variable term to equal to 1. Use the Multiplication or Division Property of Equality. State the solution to the equation.

Step 5. Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

Example 1

Solve: 6x + 14 + 2x = 30.

Solution

Simplify each side of the equation as much as possible. Combine like terms.	$8x + 14 = 30$
Collect all variable terms on one side of the equation—all x's are already on the left side.	
Collect constant terms on the other side of the equation. Subtract 14 from each side.	$8x + 14 - 14 = 30 - 14$
Simplify.	8x = 16
Make the coefficient of the variable term equal to 1. Divide each side by 8.	$\frac{8x}{8} = \frac{30}{8}$
Simplify.	x = 2
Check: Let $x = 2$.	$6(2) + 14 + 2(2) \stackrel{?}{=} 30$ $12 + 14 + 4 \stackrel{?}{=} 30$ $30 \stackrel{?}{=} 30$ ✓

Since this is a true statement, $x = 2$ is the solution.

Exercise

Solve: 2x + 6 + 3x = 26.

Answer available at the end of this section.

Exercise

Solve: 8y – 9 + 3y = 2.

Answer available at the end of this section.

Example 2

Solve: 2 – 3x + 18 = 4x – 8.

Solution

Simplify each side of the equation as much as possible. Combine like terms.	$-3x + 20 = 4x - 8$
Collect all variable terms on one side of the equation. Subtract 4x from each side.	$-3x - 4x + 20 = 4x - 4x - 8$
Collect constant terms on the other side of the equation. Subtract 20 from each side.	$-7x + 20 - 20 = -8 - 20$
Simplify.	$-7x = -28$
Make the coefficient of the variable term equal to 1. Divide each side by –7.	$\frac{-7x}{-7} = \frac{-28}{-7}$
Simplify.	x = 4
Check: Let $x = 4$.	$2 - 3(4) + 18) \stackrel{?}{=} 4(4) - 8$ $2 - 12 + 18 \stackrel{?}{=} 16 - 8$ $8 \stackrel{?}{=} 8$ ✓

Since this is a true statement, $x = 4$ is the solution.

Exercise

Solve: 18 + 3x – 5 = 2x + 9.

Answer available at the end of this section.

Exercise

Solve: 25 – 4x – 7 = x + 3.

Answer available at the end of this section.

Example 3

Solve: 3(x + 2) = 18.

Solution

Simplify each side of the equation as much as possible. Use the Distributive Property.	$3x + 6 = 18$
Collect all variable terms on one side of the equation—all x's are already on the left side.	
Collect constant terms on the other side of the equation. Subtract 6 from each side.	$3x + 6 - 6 = 18 - 6$

Simplify.	3x = 12
Make the coefficient of the variable term equal to 1. Divide each side by 3.	$\frac{3x}{3} = \frac{12}{3}$
Simplify.	x = 4
Check: Let $x = 4$.	$3(4+2) \stackrel{?}{=} 18$ $3(6) \stackrel{?}{=} 18$ $18 \stackrel{?}{=} 18$ ✓

Since this is a true statement, $x = 4$ is the solution.

Exercise

Solve: 5(x + 3) = 35.

Answer available at the end of this section.

Exercise

Solve: 6(y – 4) = –18.

Answer available at the end of this section.

Solving an Equation with One Set of Parentheses

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View this YouTube Video online

This video demonstrates how to solve an equation with one set of parentheses by using the distributive property, followed by the addition and subtraction properties of equality.

Example 4

Solve: –(x + 5) = 7.

Solution

Simplify each side of the equation as much as possible by distributing. The only x term is on the left side, so all variable terms are on the left side of the equation.	$-x - 5 = 7$
Add 5 to both sides to get all constant terms on the right side of the equation.	$-x - 5 + 5$ $= 7 + 5$
Simplify.	$-x = 12$
Make the coefficient of the variable term equal to 1 by multiplying both sides by -1.	$-1(-x) = -1(12)$
Simplify.	$x = -12$

Check: Let x = –12.	$-(x+5)=7$ $-(-12+5)\stackrel{?}{=}7$ $-(-7)\stackrel{?}{=}7$ $7=7$ ✓

Exercise

Solve: –(y + 8) = –2.

Answer available at the end of this section.

Exercise

Solve: –(z + 4) = –12.

Answer available at the end of this section.

Example 5

Solve: 4(x – 2) + 5 = –3.

Solution

Simplify each side of the equation as much as possible. Distribute.	$4x-8+5=-3$
Combine like terms.	$4x-3=-3$
The only x is on the left side, so all variable terms are on one side of the equation.	
Add 3 to both sides to get all constant terms on the other side of the equation.	$4x-3+3=-3+3$
Simplify.	$4x=0$
Make the coefficient of the variable term equal to 1 by dividing both sides by 4.	$\frac{4x}{4}=\frac{0}{4}$
Simplify.	$x=0$
Check: Let x = 0.	$4(x-2)+5=-3$ $4(0-2)+5\stackrel{?}{=}-3$ $4(-2)+5\stackrel{?}{=}-3$ $-8+5\stackrel{?}{=}-3$ $-3=-3$ ✓

Solving a Multiple Step Linear Equation in One Variable

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View this YouTube Video online

TThis video demonstrates how to solve an equation by distributing a negative one across parentheses and then simplifying to isolate the variable.

Exercise

Solve: 2(a – 4) + 3 = –1.

Answer available at the end of this section.

Exercise

Solve: 7(n – 3) – 8 = –15.

Answer available at the end of this section.

Example 6

Solve: 8 – 2(3y + 5) = 0.

Solution

Be careful when distributing the negative.

Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$y=-\frac{1}{3}$
Divide both sides by –6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$

Check: Let y = $-\frac{1}{3}$.	$8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0$ ✓

Exercise

Solve: 12 – 3(4j + 3) = –17.

Answer available at the end of this section.

Exercise

Solve: –6 – 8(k – 2) = –10.

Answer available at the end of this section.

Solve a Linear Equation in One Variable

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View this YouTube Video online

This video demonstrates how to solve an equation by removing parentheses, combining like terms, and then using the properties of equality to isolate the variable.

Example 7

Solve: 3(x – 2) – 5 = 4(2x + 1) + 5.

Solution

Distribute.	$3x-6-5=8x+4+5$
Combine like terms.	$3x-11=8x+9$
Subtract 3x to get all the variables on the right since 8 > 3.	$3x-3x-11=8x-3x+9$
Simplify.	$-11=5x+9$
Subtract 9 to get the constants on the left.	$-11-9=5x+9-9$

Simplify.	$-20 = 5x$
Divide by 5.	$\frac{-20}{5} = \frac{5x}{5}$
Simplify.	$-4 = x$
Check: Substitute: –4 = x.	$3(x-2)-5 = 4(2x+1)+5$ $3(-4-2)-5 \stackrel{?}{=} 4[2(-4)+1]+5$ $3(-6)-5 \stackrel{?}{=} 4(-8+1)+5$ $-18-5 \stackrel{?}{=} 4(-7)+5$ $-23 \stackrel{?}{=} -28+5$ $-23 = -23$ ✓

Exercise

Solve: 6(p – 3) – 7 = 5(4p + 3) – 12.

Answer available at the end of this section.

Exercise

Solve: 8(q + 1) – 5 = 3(2q – 4) – 1.

Answer available at the end of this section.

Solve a Linear Equation One Variable - Parentheses and Variables on Both Sides

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This video demonstrates how to solve an equation with parentheses by first simplifying both sides, combining like terms, and then using the properties of equality to isolate the variable.

Example 8

Solve: $\frac{1}{2}(6x - 2) = 5 - x$.

Solution

Distribute.	$3x - 1 = 5 - x$
Add x to get all the variables on the left.	$3x - 1 + x = 5 - x + x$

Simplify.	$4x - 1 = 5$
Add 1 to get constants on the right.	$4x - 1 + 1 = 5 + 1$
Simplify.	$4x = 6$
Divide by 4.	$\frac{4x}{4} = \frac{6}{4}$
Simplify.	$x = \frac{3}{2}$
Check: Let x = $\frac{3}{2}$.	$\frac{1}{2}(6x - 2) = 5 - x$ $\frac{1}{2}\left(6 \cdot \frac{3}{2} - 2\right) \stackrel{?}{=} 5 - \frac{3}{2}$ $\frac{1}{2}(9 - 2) \stackrel{?}{=} \frac{10}{2} - \frac{3}{2}$ $\frac{1}{2}(7) \stackrel{?}{=} \frac{7}{2}$ $\frac{7}{2} = \frac{7}{2}$ ✓

Exercise

Solve: $\frac{1}{3}(6u + 3) = 7 - u$.

Answer available at the end of this section.

Exercise

Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$.

Answer available at the end of this section.

In many applications, we will have to solve equations with decimals. The same general strategy will work for these equations.

Example 9

Solve: 0.24(100x + 5) = 0.4(30x + 15).

Solution

Distribute.	$24x + 1.2 = 12x + 6$
Subtract 12x to get all the x s to the left.	$24x + 1.2 - 12x = 12x + 6 - 12x$
Simplify.	$12x + 1.2 = 6$
Subtract 1.2 to get the constants to the right.	$12x + 1.2 - 1.2 = 6 - 1.2$
Simplify.	$12x = 4.8$
Divide.	$\frac{12x}{12} = \frac{4.8}{12}$
Simplify.	$x = 0.4$

Check: Let x = 0.4.	$0.24(100x+5)=0.4(30x+15)$ $0.24[100(0.4)+5] \stackrel{?}{=} 0.4[30(0.4)+15]$ $0.24(40+5) \stackrel{?}{=} 0.4(12+15)$ $0.24(45) \stackrel{?}{=} 0.4(27)$ $10.8=10.8$ ✓

Exercise

Solve: 0.55(100n + 8) = 0.6(85n + 14).

Answer available at the end of this section.

Exercise

Solve: 0.15(40m – 120) = 0.5(60m + 12).

Answer available at the end of this section.

Answers

Exercise
x = 4
Exercise
y = 1
Exercise
x = –4
Exercise
y = 3
Exercise
x = 4
Exercise
y = 1
Exercise
y = -6
Exercise
z = 8
Exercise
a = 2
Exercise
n = 2
Exercise
$j=\frac{5}{3}$
Exercise
$k=\frac{5}{2}$
Exercise
p = -2
Exercise
q = -8
Exercise

u = 2

Exercise

x = 4

Exercise

n = 1

Exercise

n = - 1

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2.2.5: Solving Equations Practice

The following set of exercises provides practice for the concepts covered in Module 2.2: Solving Equations. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 2.2.1 - Solving One Step Equations Using Addition and Subtraction

Determine whether each given value is a solution to the equation.

1. Is y = 7 a solution of the equation y + 18 = 25?

2. Is q = 3 a solution of the equation 8q + 4 = 20

3. Is v = 11 a solution of the equation 7v – 3 = 4v + 36?

Solve each equation using the addition and subtraction properties of equality.

4. b + 5 = 13

5. x – 4 = 12

6. 9 = q – 72

Section 2.2.2 - Solving One Step Equations Using Multiplication and Division

Solving using the multiplication and division properties of equality.

7. $2x = 48$

8. $3x = 0$

9. $\frac{x}{9} = 4$

Section 2.2.3 - Solve Equations with Variables and Constants on Both Sides

Solve the equation for the variable.

10. 3a + 8 = –46

11. –12p – 3 = 15

12. 4x + 36 = 10x

13. 8y – 5 = 3y + 20

14. $\frac{5}{4}c - 3 = \frac{1}{4}c - 16$

15. 2.4w – 100 = 0.8w + 28

Section 2.2.4 - Solve an Equation Using a General Strategy

Solve the linear equation using a general strategy.

16. $3x + 5 + 2x = 20$

17. $5a - 8 + 2a = 3a + 13$

18. $4x + 7 - 2x + 5 = 3x - 8$

19. $4(y + 7) = 64$

20. $14 = -7(3n + 4)$

21. $\frac{3}{5}(10x - 5) = 27$

22. $0.5(16m + 34) = -15$

23. $-(t - 8) = 17$

24. $8(6b - 7) + 23 = 63$

25. $-9 + 6(5 - k) = 12$

26. $18 - (9r + 7) = -16$

27. $3(4n - 1) - 2 = 8n + 3$

28. $5(x - 4) - 4x = 14$

29. $18 - 2(3y + 4) = 2y - 6$

30. $5 + 6(3s - 5) = -3 - 2(8s - 1)$

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2.3: FORMULAS AND THEIR APPLICATIONS

Formulas are used throughout technical fields to describe relationships between quantities. Whether you are calculating electrical power, determining material costs, or measuring airflow in a system, formulas provide a consistent way to model real-world situations.

In this module, you will build on your work with algebraic expressions and equations to begin working with formulas. You will start by evaluating formulas by substituting known values and simplifying using the order of operations. From there, you will extend those skills to solve formulas for an unknown value. You will also learn how to rewrite formulas to isolate a specific variable, which is an important skill when working with industry-specific formulas. Finally, you will apply formulas to solve real-world problems.

To get started, you will watch a short video that explores the famous formula $E = mc^2$. While the physics behind this formula is complex, it serves as a powerful example of how formulas are used to describe relationships between quantities in the real world. Throughout this module, you will focus on using formulas in practical and meaningful ways similar to those used in technical fields.

The Physics Behind Einstein's Most Famous Equation

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View this YouTube Video online

2.3.1: Evaluate Formulas and Apply the Order of Operations

2.3.2: Evaluate Formulas and Apply Equation Solving Strategies

2.3.3: Rewriting Formulas for a Specified Variable

2.3.4: Solving Applications Involving Formulas

2.3.5: Formulas and Additional Applications Practice

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2.3.1: Evaluate Formulas and Apply the Order of Operations

In math, *evaluating* a formula means replacing the variables in the formula with numbers and then performing the required calculations to find a numerical value. For example, the formula for simple interest is I = Prt. To find the simple interest, I, earned on $400 with a 5% interest rate for 6 years you need to replace P with 400, r with 0.05 and t with 6 and the do all of the multiplication like this:

$$I = Prt$$

$$I = (400)(0.05)(6)$$

$$I = \$120$$

What are Formulas, And How Do I Use Them?

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In this video, you will be introduced to the idea of formulas and a brief overview of common formulas and how to work with them.

Tips for Evaluating Formulas

1. Replace all occurrences of variables with open parentheses. Leave room between the parentheses to substitute the given value of the variable.
2. Substitute the given values of variables in the open parentheses prepared in the first step.
3. Evaluate the resulting expression according to the Rules Guiding Order of Operations.

Example 1

Evaluate the formula $P = VI$ for $V = 120$ and $I = 5$.

Solution

Following “Tips for Evaluating Formulas,” first replace all occurrences of variables in the formula with open parentheses.

$P = (\ )(\ )$

Secondly, replace each variable with its given value, and evaluate.

$P = (120)(5)$ Substitute 120 for V and 5 for I.

$P = 600$ Multiply.

Exercise

Evaluate the given formula.

$R = pt$ for $p = 18$, $t = 6$

Answer available at the end of this section.

Answer available at the end of this section.

Exercise

Evaluate the given formula.

$V = wlh$ for $w = 20$, $l = 15$, and $h = 2$

Answer available at the end of this section.

Example 2

Evaluate the formula $A = \frac{1}{2}(b_1 + b_2)h$ for $b_1 = 8$, $b_2 = 12$, and $h = 5$.

Solution

Following "Tips for Evaluating Formulas," first replace all occurrences of variables in the formula with open parentheses.

$A = \frac{1}{2}(( ) + ( ))( )$

Secondly, replace each variable with its given value, and evaluate.

$A = \frac{1}{2}((8) + (12))(5)$ Substitute values.

$A = \frac{1}{2}(20)(5)$ Add inside parentheses.

$A = 10(5)$ Multiply by $\frac{1}{2}$.

$A = 50$ Multiply.

Exercise

Evaluate the formula.

$C = \frac{1}{3}(l + w)$ for $l = 7$, $w = 5$

Answer available at the end of this section.

Exercise

Evaluate the formula.

$D = k(a + b)c$ for $k = \frac{1}{5}$, $a = 4$, $b = 6$, $c = 3$

Answer available at the end of this section.

Example 3

Evaluate the formula $S = 2a^3 + b^2$ for $a = 2$ and $b = 5$.

Solution

Following "Tips for Evaluating Formulas," first replace all occurrences of variables in the formula with open parentheses.

$S = 2(\ )^3 + (\ )^2$

Secondly, replace each variable with its given value, and evaluate.

$S = 2(2)^3 + (5)^2$ Substitute values.

$S = 2(8) + 25$ Evaluate exponents.

$S = 16 + 25$ Multiply.

$S = 41$ Add.

Exercise

Evaluate the formula.

$S = l^2 + w^2$ for $l = 5$, $w = 12$

Answer available at the end of this section.

Exercise

Evaluate the formula.

$A = \pi r^2$ for $r = 4$, $\pi \approx 3.14$

Answer available at the end of this section.

Example 4

Evaluate the formula $T = p(1 + r)$ for $p = 80$ and $r = 0.05$.

Solution

Following "Tips for Evaluating Formulas," first replace all occurrences of variables in the formula with open parentheses.

$T = (\ )(1 + (\ ))$

Secondly, replace each variable with its given value, and evaluate.

$T = (80)(1 + 0.05)$ Substitute values.

$T = (80)(1.05)$ Add inside parentheses.

$T = 84$ Multiply.

Exercise

Evaluate the formula.

$R = q(1 + s)$ for $q = 45$, $s = 0.20$

Answer available at the end of this section.

Exercise

Evaluate the formula.

$P = a(1 + b) + 12$ for $a = 45$, $b = 0.20$

Answer available at the end of this section.

Example 5

Evaluate the formula $P = \frac{W}{t}$ for $W = 300$ and $t = 6$.

Solution

Following "Tips for Evaluating Formulas," first replace all occurrences of variables in the formula with open parentheses.

$P = \frac{(\,)}{(\,)}$

Secondly, replace each variable with its given value, and evaluate.

$P = \frac{300}{6}$ Substitute values.

$P = 50$ Divide.

Exercise

Evaluate the formula.

$R = \frac{d}{t}$ for $d = 180$, $t = 4$

Answer available at the end of this section.

Exercise

Evaluate the formula.

$M = \frac{f}{a}$ for $f = 96$, $a = 8$

Answer available at the end of this section.

Example 6

Evaluate the formula $M = \frac{wL^2}{8}$ for $w = 2$ and $L = 8$.

Solution

Following “Tips for Evaluating Formulas,” first replace all occurrences of variables in the formula with open parentheses.

$M = \frac{(\,)(\,)^2}{8}$

Secondly, replace each variable with its given value, and evaluate.

$M = \frac{(2)(8)^2}{8}$ Substitute values.

$M = \frac{2(64)}{8}$ Evaluate exponents.

$M = \frac{128}{8}$ Multiply numerator.

$M = 16$ Divide.

Exercise

Evaluate the formula.

$Q = \frac{ab^2}{10}$ for $a = 5, b = 4$

Answer available at the end of this section.

Exercise

Evaluate the formula.

$K = \frac{mv^2}{4}$ for $m = 2, v = 8$

Answer available at the end of this section.

How to Work With Formulas

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View this YouTube Video online

This video introduces four key things to know when working with formulas. It will also give you a preview for a future lesson on how to rewrite formulas for unknown variables.

Answers

Exercise

$R = 108$

Exercise

Exercise

$L = 600$

Exercise

$C = 4$

Exercise

$D = 6$

Exercise

$S = 169$

Exercise

$A = 50.24$

Exercise

$T = 54$

Exercise

$P = 66$

Exercise

$R = 45$

Exercise

$M = 12$

Exercise

$Q = 8$

Exercise

$K = 32$

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- 3.2: Evaluating Algebraic Expressions by David Arnold is licensed CC BY-NC-SA 3.0.

2.3.2: Evaluate Formulas and Apply Equation Solving Strategies

In this section, you will evaluate formulas and then solve the resulting equation to find an unknown value. This process combines substitution from the previous section with algebraic equation-solving skills from earlier in this unit.

After substituting the given values into the formula, simplify the expression. If an unknown variable remains, solve the resulting equation to find its value. As you work, including units can help ensure that your final answer is expressed in the correct units.

Tips for Evaluating Formulas and Solving

1. Substitute the given values into the formula.
2. Simplify the expression using the order of operations.
3. If needed, solve the resulting equation to isolate the unknown variable.
4. Include appropriate units in your final answer.

Example 1

Evaluate the formula $V = rt$ for $V = 210$ and $r = 7$. Find t.

Solution

$210 = (7)(t)$ Substitute the given values into the formula.

$t = \frac{210}{7}$ Divide both sides by 7.

$t = 30$ Simplify.

Exercise

Evaluate the formula $d = rt$ for $d = 180$ and $r = 20$. Find t.

Answer available at the end of this section.

Example 2

Evaluate the formula $r = \frac{P}{h}$ for $r = 16$ and $h = 5$. Find P.

Solution

$16 = \frac{P}{5}$ Substitute the given values into the formula.

$P = (16)(5)$ Multiply both sides by 5.

$P = 80$ Simplify.

Exercise

Evaluate the formula $k = \frac{F}{t}$ for $k = 9$ and $t = 4$. Find F.

Answer available at the end of this section.

Example 3

Evaluate the formula $T = 2a + b$ for $T = 42$ and $a = 10$. Find b.

Solution

$42 = 2(10) + b$ Substitute the given values into the formula.

$42 = 20 + b$ Multiply.

$b = 42 - 20$ Subtract 20 from both sides.

$b = 22$ Simplify.

Exercise

Evaluate the formula $D = pn + f$ for $D = 98$, $p = 12$, and $n = 6$. Find f.

Answer available at the end of this section.

Example 4

Evaluate the formula $L = 3(x + 4)$ for $L = 90$. Find x.

Solution

$90 = 3(x + 4)$ Substitute the given value into the formula.

$90 = 3x + 12$ Distribute 3.

$78 = 3x$ Subtract 12 from both sides.

$26 = x$ Divide both sides by 3.

$x = 26$ Simplify.

Exercise

Evaluate the formula $P = 4(y - 3)$ for $P = 52$. Find y.

Answer available at the end of this section.

Example 5

Evaluate the formula $T = 2(x + 6) + 8$ for $T = 64$. Find x.

Solution

$64 = 2(x + 6) + 8$ Substitute the given value into the formula.

$64 = 2x + 12 + 8$ Distribute 2.

$64 = 2x + 20$ Combine like terms.

$44 = 2x$ Subtract 20 from both sides.

$22 = x$ Divide both sides by 2.

$x = 22$ Simplify.

Exercise

Evaluate the formula $L = 3(y + 4) + 5$ for $L = 50$. Find y.

Answer available at the end of this section.

Example 6

Evaluate the formula $d = \frac{2x+12}{4}$ for $d = 9$. Find x.

Solution

$9 = \frac{2x+12}{4}$ Substitute the given value into the formula.

$36 = 2x + 12$ Multiply both sides by 4.

$24 = 2x$ Subtract 12 from both sides.

$12 = x$ Divide both sides by 2.

$x = 12$ Simplify.

Exercise

Evaluate the formula $m = \frac{3y+15}{5}$ for $m = 12$. Find y.

Answer available at the end of this section.

Answers

Exercise
$t = 9$

Exercise
$F = 36$

Exercise
$f = 26$

Exercise
$y = 16$

Exercise
$y = 11$

Exercise
$y = 15$

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2.3.3: Rewriting Formulas for a Specified Variable

In many scientific formulas, more than one variable is used instead of just a single variable. These formulas often use several different letters to represent different quantities.

For example, Newton's Law of Gravitation describes the force of attraction between two objects. The formula is

$$F = \frac{GMm}{r^2}$$

where m represents the mass of the smaller object, M represents the mass of the larger object, and r is the distance between the objects. The letter G is a constant.

Variable case

Pay close attention to uppercase and lowercase letters when working with formulas. Different cases represent different variables. For example, M and m represent different quantities in this formula. Always use the variables exactly as they are given and do not switch between uppercase and lowercase letters.

Earlier in this unit, we described the goal that must be met when we are asked to "solve an equation for " x.

Steps to Rewrite a Formula for a Specified Variable

Step 1. Simplify each side of the equation.
Distribute and combine like terms if needed.

Step 2. Identify the variable you are solving for.

Step 3. Move all terms with that variable to one side.

Step 4. Isolate the variable using inverse operations.

Step 5. Check that the variable is by itself and the equation is correct.

Thus, to solve an equation for x, we need to isolate the terms containing x on one side of the equation, and all remaining terms on the other side of the equation.

Example 1

Solve for x : $x + a = b$.

Solution

$$\begin{aligned} x + a &= b && \text{Identify the variable to isolate: } x. \\ x + a - a &= b - a && \text{Subtract } a \text{ from both sides.} \\ x &= b - a && \text{Simplify to isolate } x. \end{aligned}$$

Therefore, $x = b - a$.

Exercise

Solve for x : $x - c = d$.

Answer available at the end of this section.

In Example 1, notice that the formula is rewritten so that x is isolated, with all other terms on the opposite side. The result, $x = b - a$, shows x written in terms of the other variables. Now, what if we wanted to rewrite the same formula to solve for a instead?

Example 2

Solve for a : $x + a = b$.

Solution

$$\begin{aligned} x + a &= b && \text{Identify the variable to isolate: } a. \\ x + a - x &= b - x && \text{Subtract } x \text{ from both sides.} \\ a &= b - x && \text{Simplify to isolate } a. \end{aligned}$$

Therefore, $a = b - x$.

Exercise

Solve for x : $x - c = d$.

Answer available at the end of this section.

Solving for a Variable

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to rewrite the perimeter formula for a rectangle to solve for a specified variable using the subtraction and multiplication properties of equality.

Example 3

The formula $F = kx$, known as "Hooke's Law", predicts the force F required to stretch a spring x units. Solve the equation for k.

Solution

$$\begin{aligned} F &= kx && \text{Identify the variable to isolate: } k. \\ \frac{F}{x} &= \frac{kx}{x} && \text{Divide both sides by } x. \\ \frac{F}{x} &= k && \text{Simplify to isolate } k. \end{aligned}$$

Therefore, $k = \frac{F}{x}$.

Exercise

Solve for m : $E = mc^2$.

Answer available at the end of this section.

Example 4

The formula $V = RI$ is called "Ohm's Law." It helps calculate the voltage drop V across a resistor R in an electric circuit with current I. Solve the equation for R.

Solution

$$\begin{aligned} V &= RI && \text{Identify the variable to isolate: } R. \\ \frac{V}{I} &= \frac{RI}{I} && \text{Divide both sides by } I. \\ \frac{V}{I} &= R && \text{Simplify to isolate } R. \end{aligned}$$

Therefore, $R = \frac{V}{I}$.

Exercise

Solve for t : $d = st$.

Answer available at the end of this section.

Clearing Fractions

If fractions occur in a formula, clear the fractions from the formula by multiplying both sides of the formula by the common denominator.

Example 5

The formula $K = \frac{1}{2}mv^2$ is used to calculate the kinetic energy K of a particle of mass m moving with velocity v. Solve the equation for m.

Solution

$$K = \frac{1}{2}mv^2 \quad \text{Identify the variable to isolate: } m.$$
$$2K = 2\left(\frac{1}{2}mv^2\right) \quad \text{Clear the fraction by multiplying both sides by 2.}$$
$$2K = mv^2 \quad \text{Simplify.}$$
$$\frac{2K}{v^2} = \frac{mv^2}{v^2} \quad \text{Divide both sides by } v^2.$$
$$\frac{2K}{v^2} = m \quad \text{Simplify to isolate } m.$$

Therefore, $m = \frac{2K}{v^2}$.

Exercise

Solve for $g : s = \frac{1}{2}gt^2$.

Answer available at the end of this section.

Example 6

As mentioned earlier, Newton's Universal Law of Gravitation is described by the formula

$$F = \frac{GMm}{r^2}$$

Solve this equation for m.

Solution

$$F = \frac{GMm}{r^2} \quad \text{Identify the variable to isolate: } m.$$
$$r^2F = r^2\left(\frac{GMm}{r^2}\right) \quad \text{Clear the fraction by multiplying both sides by } r^2.$$
$$r^2F = GMm \quad \text{Simplify.}$$
$$\frac{r^2F}{GM} = \frac{GMm}{GM} \quad \text{Divide both sides by } GM.$$
$$\frac{r^2F}{GM} = m \quad \text{Simplify to isolate } m.$$

Therefore, $m = \frac{r^2F}{GM}$.

Exercise

Solve for $q_2 : F = \frac{kq_1q_2}{r^2}$.

Answer available at the end of this section.

Rearrange Formulas to Isolate Specific Variables

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to rewrite the area of a triangle formula by clearing a fraction using a common denominator.

Answers

Exercise

$x = c + d$

Exercise

$c = x - d$

Exercise

$m = \frac{E}{c^2}$

Exercise

$t = \frac{d}{s}$

Exercise

$g = \frac{2s}{t^2}$

Exercise

$q_2 = \frac{Fr^2}{kq_1}$

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2.3.4: Solving Applications Involving Formulas

In this section, you will apply formulas to solve problems that commonly arise in technical fields. You will use given information, substitute values into formulas, and simplify to find meaningful results. These types of calculations are used to determine quantities such as area, cost, load, and output in real-world situations.

Before working through the problems, review the following general guidelines to help you correctly evaluate formulas.

General Guidelines for Units in Formulas

Units tell you what your answer represents and help you determine whether your result is reasonable. Treat units as part of the expression—they multiply, divide, and cancel just like numbers.

- Always include units with the given values.
- Substitute numbers into the formula before simplifying.
- Multiply units when quantities are multiplied (ft × ft = ft^2).
- Cancel units that appear in both the numerator and denominator.
- Numbers without units do not change the units.
- Focus on simplifying the expression correctly.
- Attach the correct units to your final answer.

Example: Area is measured in square units (ft^2), and volume is measured in cubic units (ft^3).

Using Formulas to Calculate Values in Applications

In the first set of problems, you will evaluate formulas by substituting given values for variables and simplifying using the order of operations. After substitution, your focus should be on carefully simplifying the expression step by step. These problems emphasize accurate calculation and proper use of mathematical operations.

Example 1

When designing a support beam, a construction engineer must calculate the total load acting on the beam based on the load per foot and the length of the beam. The formula $L = wl$ is used, where L is the total load (lb), w is the load per foot (lb/ft), and l is the length of the beam (ft). If a beam carries 18 lb/ft over a span of 12 ft, find L.

Solution

$L = (18 \text{ lb/ft})(12 \text{ ft})$ Substitute the given values for w and l.

$L = 216 \text{ lb}$ Multiply and simplify the units.

Therefore, the total load is 216 lb.

Exercise

In an electrical system, the power delivered to a device depends on the voltage and current. The formula $P = VI$ is used, where P is power (watts), V is voltage (volts), and I is current (amps). If a device operates at 120 V and draws 6 A, find P.

Answer available at the end of this section.

Example 2

In HVAC system design, ducts are sometimes shaped like trapezoids. The cross-sectional area is needed to determine airflow capacity. The formula $A = \frac{1}{2}(b_1 + b_2)h$ is used, where A is area (in^2), b_1 and b_2 are the parallel sides (in.), and h is height (in.). If the sides measure 8 in. and 14 in., and the height is 6 in., find A.

Solution

$A = \frac{1}{2}(8 \text{ in} + 14 \text{ in})(6 \text{ in})$ Substitute the given values into the formula.

$A = \frac{1}{2}(22 \text{ in})(6 \text{ in})$ Add inside the parentheses first.

$A = 11 \text{ in}(6 \text{ in})$ Multiply by $\frac{1}{2}$.

$A = 66 \text{ in}^2$ Multiply and simplify the units.

Therefore, the area is 66 in².

Exercise

Grain storage bins are sometimes modeled using formulas involving base area and height to estimate capacity. The formula $V = \frac{1}{3}Bh$ is used, where V is volume (ft^3), B is base area (ft^2), and h is height (ft). If the base area is 30 ft^2 and the height is 9 ft, find V.

Answer available at the end of this section.

Example 3

To ensure a structure is square, contractors often check diagonal measurements. The relationship between the sides is given by $c^2 = a^2 + b^2$, where c is the diagonal (ft) and a and b are side lengths (ft). If the sides are 9 ft and 12 ft, find c^2.

Solution

$c^2 = (9 \text{ ft})^2 + (12 \text{ ft})^2$ Substitute the given side lengths.

$c^2 = 81 \text{ ft}^2 + 144 \text{ ft}^2$ Evaluate the exponents.

$c^2 = 225 \text{ ft}^2$ Add to find the result.

Therefore, the value of c^2 is 225 ft².

Exercise

In surveying, a distance formula is used to relate horizontal and vertical changes. The formula $d = x^2 + y^2$ is used, where d represents a squared distance value, and x and y are measured changes. If the horizontal change is 7 units and the vertical change is 24 units, find d.

Answer available at the end of this section.

Example 4

Retail businesses must calculate the total cost of an item after sales tax is added. The formula $T = p(1 + r)$ is used, where T is the total cost (dollars), p is the original price (dollars), and r is the tax rate written as a decimal. If an item costs $80 and the tax rate is 8%, find T.

Solution

$T = 80 \text{ dollars}(1 + 0.08)$ Substitute the given values into the formula.

$T = 80 \text{ dollars}(1.08)$ Add inside the parentheses first.

$T = 86.4 \text{ dollars}$ Multiply to find the total cost.

Therefore, the total cost is $86.40.

Exercise

When feed is purchased for livestock, the total cost may include both the cost per bag and a fixed delivery fee. The formula $C = pn + f$ is used, where C is the total cost (dollars), p is the price per bag (dollars), n is the number of bags, and f is the delivery fee (dollars). If feed costs $18 per bag, 5 bags are purchased, and the delivery fee is $12, find C.

Answer available at the end of this section.

Example 5

In healthcare, medication dosage is often based on a patient's weight and prescribed dosing schedule. The formula $D = \frac{wd}{k}$ is used, where D is the dosage per dose (mg), w is weight (kg), d is dosage per kilogram (mg/kg), and k is the number of doses per day. If a patient weighs 60 kg, the dosage is 3 mg/kg, and the medication is given in 2 doses per day, find D.

Solution

$D = \frac{60 \text{ kg} \cdot 3 \text{ mg/kg}}{2}$ Substitute the given values into the formula.

$D = \frac{180 \text{ mg}}{2}$ Multiply in the numerator and simplify the units.

$D = 90 \text{ mg}$ Divide to find the dosage per dose.

Therefore, the dosage is 90 mg.

Exercise

In diesel systems, engine performance can be measured using work done over time. The formula $P = \frac{W}{t}$ is used, where P is power (ft·lb/s), W is work (ft·lb), and t is time (s). If 300 ft·lb of work are completed in 5 seconds, find P.

Answer available at the end of this section.

Example 6

Structural engineers use formulas to estimate bending in beams. The formula $M = \frac{wL^2}{8}$ is used, where M is bending moment (ft·lb), w is the load per foot (lb/ft), and L is the beam length (ft). If the load is 4 lb/ft and the beam length is 10 ft, find M.

Solution

$M = \frac{4 \text{ lb/ft}(10 \text{ ft})^2}{8}$ Substitute the given values into the formula.

$M = \frac{4 \text{ lb/ft}(100 \text{ ft}^2)}{8}$ Evaluate the exponent first.

$M = \frac{400 \text{ ft·lb}}{8}$ Multiply and simplify the units.

$M = 50 \text{ ft·lb}$ Divide to find the bending moment.

Therefore, the bending moment is 50 ft·lb.

Exercise

In an IV infusion, the flow rate can be calculated using a formula that involves volume and time squared. The formula $R = \frac{v}{t^2}$ is used, where R is the flow rate (mL/hr^2), v is the volume (mL), and t is the time (hr). If 72 mL is delivered over 3 hours, find R.

Answer available at the end of this section.

Using Formulas to Determine Unknown Values in Applications

In the second set of problems, you will substitute given values into a formula and then solve the resulting equation to find an unknown quantity. After simplifying the expression, you will use algebraic steps to isolate the variable. These problems build on your equation-solving skills while continuing to reinforce correct substitution and attention to units.

Example 7

A water tank is being filled at a constant rate. The formula $V = rt$ is used, where V is volume (gallons), r is flow rate (gallons per minute), and t is time (minutes). If the tank contains 180 gallons and the flow rate is 9 gallons per minute, find t.

Solution

$180 \text{ gal} = 9 \text{ gal/min} \cdot t$ Substitute the given values into the formula.

$t = \frac{180 \text{ gal}}{9 \text{ gal/min}}$ Divide both sides by 9 gal/min.

$t = 20 \text{ min}$ Simplify and cancel units.

Therefore, the time is 20 minutes.

Exercise

A conveyor belt moves packages at a constant speed. The formula $d = rt$ is used, where d is distance (feet), r is speed (feet per minute), and t is time (minutes). If a package travels 300 feet at a speed of 20 feet per minute, find t.

Answer available at the end of this section.

Example 8

A worker's hourly wage can be found using the formula $r = \frac{P}{h}$, where r is the hourly rate (dollars per hour), P is total pay (dollars), and h is hours worked (hours). If a worker earned $144 over 9 hours, find r.

Solution

$r = \frac{P}{h}$

$r = \frac{144 \text{ dollars}}{9 \text{ hours}}$ Substitute the given values into the formula.

$r = 16 \text{ dollars per hour}$ Divide.

Therefore, the hourly wage is $16 per hour.

Exercise

In a construction project, the average cost per unit is given by the formula $c = \frac{C}{n}$, where c is cost per unit (dollars per unit), C is total cost (dollars), and n is the number of units. If the cost per unit is $12 and 15 units are produced, find C.

Answer available at the end of this section.

Example 9

In electrical work, total resistance in a simple series circuit can be found using the formula $T = R_1 + R_2$, where T is total resistance (ohms), and R_1 and R_2 are individual resistances (ohms). If the total resistance is 30 ohms and one resistor is 12 ohms, find R_2.

Solution

$30 \text{ ohms} = 12 \text{ ohms} + R_2$ Substitute the given values into the formula.

$R_2 = 30 \text{ ohms} - 12 \text{ ohms}$ Subtract 12 ohms from both sides.

$R_2 = 18 \text{ ohms}$ Simplify.

Therefore, the resistance of the second resistor is 18 ohms.

Exercise

In landscaping, the total length of fencing is given by the formula $P = 2w + 2l$, where P is total length (feet), w is width (feet), and l is length (feet). If the total fencing is 50 feet and the width is 10 feet, find l.

Answer available at the end of this section.

Example 10

In construction, the total length of material needed is given by the formula $P = 2(w + d)$, where P is total perimeter (feet), w is width (feet), and d is depth (feet). If the total perimeter is 64 feet and the width is 12 feet, find d.

Solution

$64 \text{ ft} = 2(12 \text{ ft} + d)$ Substitute the given values into the formula.

$32 \text{ ft} = 12 \text{ ft} + d$ Divide both sides by 2.

$d = 32 \text{ ft} - 12 \text{ ft}$ Subtract 12 ft from both sides.

$d = 20 \text{ ft}$ Simplify.

Therefore, the depth is 20 feet.

Exercise

In manufacturing, the total cost is given by the formula $C = 3(p + 5)$, where C is total cost (dollars) and p is the base price (dollars). If the total cost is \$90, find p.

Answer available at the end of this section.

Example 11

A software company calculates total revenue using the formula $R = p(n + 50)$, where R is total revenue (dollars), p is price per subscription (dollars), and n is the number of new customers. If the total revenue is \$2,400 and the price per subscription is \$20, find n.

Solution

$2400 \text{ dollars} = 20 \text{ dollars}(n + 50)$ Substitute the given values into the formula.

$120 = n + 50$ Divide both sides by 20.

$n = 120 - 50$ Subtract 50 from both sides.

$n = 70$ Simplify.

Therefore, the number of new customers is 70.

Exercise

A cloud storage company calculates total data usage using the formula $D = s(u + 10)$, where D is total storage used (gigabytes), s is storage per user (gigabytes), and u is the number of users. If the total storage used is 1,800 GB and each user uses 30 GB, find u.

Answer available at the end of this section.

Example 12

In a healthcare setting, the average dosage per patient is given by the formula $d = \frac{D+20}{n}$, where d is dosage per patient (mg), D is the total medication amount (mg), and n is the number of patients. If the dosage per patient is 15 mg and there are 4 patients, find D.

Solution

$15\ \text{mg} = \frac{D+20\ \text{mg}}{4}$ Substitute the given values into the formula.

$60\ \text{mg} = D + 20\ \text{mg}$ Multiply both sides by 4.

$D = 60\ \text{mg} - 20\ \text{mg}$ Subtract 20 mg from both sides.

$D = 40\ \text{mg}$ Simplify.

Therefore, the total medication amount is 40 mg.

Exercise

In a catering business, the average cost per guest is given by the formula $c = \frac{C+50}{n}$, where c is cost per guest (dollars), C is total cost (dollars), and n is the number of guests. If the cost per guest is $25 and there are 6 guests, find C.

Answer available at the end of this section.

Answers

Exercise

$P = 720 watts$

Exercise

$V = 90\ \text{ft}^3$

Exercise

$d = 625\ \text{ft}^2$

Exercise

$C = \$102$

Exercise

$P = 60\ \text{ft·lb/s}$

Exercise

$R = 8\ \text{mL/hr}^2$

Exercise

$t = 15$ minutes

Exercise

$C = 180$ dollars

Exercise

$l = 15$ feet

Exercise

$p = 25$ dollars

Exercise

$u = 50 users$

Exercise

$C = 100$ dollars

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2.3.5: Formulas and Additional Applications Practice

The following set of exercises provides practice for the concepts covered in Module 2.3: Formulas and Additional Applications. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 2.3.1 - Evaluate Formulas and Apply Order of Operations

Substitute the provided information into the formula, then apply order of operations to simplify.

1. $P = EI$ for $E = 220$, $I = 3$

2. $C = 3(m - n)$ for $m = 18$, $n = 7$

3. $W = d(e + f)$ for $d = 10$, $e = 4$, $f = 6$

4. $S = a^2 + b^2$ for $a = 7$, $b = 24$

5. $A = \pi r^2$ for $r = 3$, $\pi \approx 3.14$

6. $A = \frac{1}{2}(x + y)h$ for $x = 6$, $y = 10$, $h = 8$

7. $T = k(p + q)r$ for $k = 2$, $p = 3$, $q = 9$, $r = 4$

Section 2.3.2 - Evaluate Formulas and Apply Equation Solving Strategies

Substitute the provided information into the formula, then solve the resulting equation.

8. $D = rt$ for $r = 5$, $t = 6$. Find D

9. $d = vt$ for $v = 40$, $t = 3$. Find d

10. $V = lwh$ for $l = 8$, $w = 3$, $h = 2$. Find V

11. $F = ma$ for $m = 6$, $a = 4$. Find F

12. $A = \frac{1}{2}bh$ for $b = 12$, $h = 5$. Find A

13. $R = \frac{v}{t}$ for $v = 120$, $t = 4$. Find R

14. $V = rt$ for $V = 200$, $r = 10$. Find t

15. $N = rt$ for $N = 144$, $r = 12$. Find t

16. $P = 2l + 2w$ for $P = 64$, $l = 18$. Find w

17. $L = 3(x + 5)$ for $L = 90$. Find x

18. $T = 2(x + 4) + 6$ for $T = 50$. Find x

19. $d = \frac{D+20}{4}$ for $d = 15$. Find D

Section 2.3.3 - Rewrite Formulas for a Specified Variable

Solve the given formulas for the indicated variable.

20. $V = lwh$ for w

21. $F = ma$ for a

22. $P = 2l + 2w$ for l

23. $A = \frac{1}{2}bh$ for b

Section 2.3.4 - Solving Applications Involving Formulas

Solve each problem. Use the given formula to find the unknown value. Substitute the provided information into the formula, then determine whether to simplify using the order of operations or solve an equation. Show your work clearly and include appropriate units in your final answer.

24. A construction engineer calculates the total load on a beam using the formula $L = wl$, where L is the total load (lb), w is the load per foot (lb/ft), and l is the length of the beam (ft). If a beam carries 15 lb/ft over a span of 10 ft, find L.

25. In an electrical system, the power delivered to a device is given by the formula $P = VI$, where P is power (watts), V is voltage (volts), and I is current (amps). If a device operates at 120 V and draws 5 A, find P.

26. In HVAC system design, ducts are sometimes shaped like trapezoids. The cross-sectional area is given by $A = \frac{1}{2}(b_1 + b_2)h$, where A is area (in²), b_1 and b_2 are the parallel sides (in.), and h is the height (in.). If the sides measure 10 in. and 14 in., and the height is 6 in., find A.

27. A machinist calculates the volume of a rectangular solid using the formula $V = lwh$, where all dimensions are in inches. If $l = 8$ in., $w = 4$ in., and $h = 3$ in., find V.

28. To ensure a structure is square, contractors use the relationship $c^2 = a^2 + b^2$, where c is the diagonal (ft) and a and b are side lengths (ft). If the sides are 9 ft and 12 ft, find c^2.

29. A medical technician calculates dosage using the formula $D = \frac{wd}{k}$, where D is dosage per dose (mg), w is weight (kg), d is dosage per kilogram (mg/kg), and k is the number of doses per day. If a patient weighs 60 kg, the dosage is 2 mg/kg, and the medication is given in 3 doses per day, find D.

30. A pump moves water according to the formula $V = rt$, where V is volume (gallons), r is flow rate (gal/min), and t is time (minutes). If 180 gallons are pumped at a rate of 9 gal/min, find t.

31. A worker's hourly rate is given by the formula $r = \frac{P}{h}$, where r is the hourly rate (dollars per hour), P is total pay (dollars), and h is hours worked (hours). If a worker earned $200 over 8 hours, find r.

32. A structural engineer calculates bending moment using the formula $M = \frac{wL^2}{8}$, where M is bending moment (ft·lb), w is load per foot (lb/ft), and L is beam length (ft). If the load is 4 lb/ft and the beam length is 10 ft, find M.

33. A business calculates total output using the formula $O = 5(n - 2)$, where O is total output (units) and n is a production value. If the output is 90 units, find n.

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2.4: APPLICATIONS OF ALGEBRAIC EQUATIONS

The story is often told that Archimedes stepped into a bath, watched the water rise, and suddenly understood a problem he had been struggling with for days. The moment itself was simple, but the insight came from the time spent thinking through something that did not immediately make sense.

This module focuses on that same idea. Problems do not always begin with a clear equation. Instead, they are often described in words, and the challenge is to interpret the situation and represent it mathematically.

Algebra provides a way to do this. When a situation involves an unknown quantity—such as a missing measurement, an unknown total, or a value that must be determined—you can represent it with an equation and solve for the unknown.

In this module, you will practice translating written descriptions into algebraic equations and using those equations to solve application problems. These skills are used in many technical settings where quantities must be determined based on available information.

As you watch the following video, consider how working through uncertainty can lead to understanding, and how algebra provides a structured way to approach that process.

How Taking a Bath Led to Archimedes' Principle

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video highlights a famous story of how some of the best opportunities to learn are the moments in which we are perplexed.

2.4.1: Translating Algebraic Equations

2.4.2: Solving Applications Involving Equations

2.4.3: Algebraic Applications Practice

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2.4.1: Translating Algebraic Equations

Earlier in this unit, we translated word phrases into expressions. In this section, we will translate sentences into equations. The first step is to look for the word (or words) that translate(s) to the equal sign. Table 2.4.1.1 reminds us of some of the words that translate to the equal sign.

Table 2.4.1.1

Equals (=)						
is	is equal to	is the same as	the result is	gives	was	will be

HOW TO: TRANSLATE A WORD SENTENCE TO AN ALGEBRAIC EQUATION

Step 1. Locate the "equals" word(s). Translate to an equal sign.

Step 2. Translate the words to the left of the "equals" word(s) into an algebraic expression.

Step 3. Translate the words to the right of the "equals" word(s) into an algebraic expression.

Now we are ready to try an example.

Example 1

Translate and solve: five more than x is equal to 26.

Solution

Translate.	$\underbrace{\text{Five more than } x}_{x+5} \quad \underbrace{\text{is equal to}}_{=} \quad \underbrace{26}_{26}$
Subtract 5 from both sides.	$x + 5 - 5 = 26 - 5$
Simplify.	x = 21
Check: Is 26 five more than 21?	$21 + 5 \stackrel{?}{=} 26$ $26 = 26$ ✓

The solution checks.

Exercise

Translate and solve: Eleven more than x is equal to 41.

Answer available at the end of this section.

Exercise

Translate and solve: Twelve less than y is equal to 51.

Answer available at the end of this section.

Example 2

Translate and solve: The difference of 5p and 4p is 23.

Solution

Translate.	$\underbrace{\text{The difference of } 5p \text{ and } 4p}_{5p-4p} \quad \underbrace{\text{is}}_{=} \quad \underbrace{23}_{23}$
Simplify.	p = 23
Check.	$5p-4p=23$ $5(23)-4(23) \stackrel{?}{=} 23$ $115-92 \stackrel{?}{=} 23$ $23=23$ ✓

The solution checks.

Exercise

Translate and solve: The difference of 4x and 3x is 14.

Answer available at the end of this section.

Exercise

Translate and solve: The difference of 7a and 6a is –8.

Answer available at the end of this section.

Example 3

Translate and solve: The number 143 is the product of –11 and *y*.

Solution

Translate.	$\underbrace{\text{The number } 143}_{143} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{the product of } -11 \text{ and } y.}_{-11y}$
Divide by –11.	$\frac{143}{-11}=\frac{-11y}{-11}$

Simplify.	$-13 = y$
	Check: $143 = -11y$ $143 \stackrel{?}{=} -11(-13)$ $143 = 143\checkmark$

Exercise

Translate and solve: The number 132 is the product of *–12* and *y*.

Answer available at the end of this section.

Exercise

Translate and solve: The number 117 is the product of *–13* and *z*.

Answer available at the end of this section.

Example 4

Translate and solve: *n* divided by 8 is –32.

Solution

Begin by translating the sentence into an equation. Translate.	n divided by 8 is -32. $\frac{n}{8} = -32$
Multiple both sides by 8.	$8 \cdot \frac{n}{8} = 8(-32)$
Simplify.	$n = -256$
Check:	Is -256 divided by 8 equal to -32? $\frac{-256}{8} \stackrel{?}{=} -32$ $-32 = -32\checkmark$

Exercise

Translate and solve: n divided by 7 is equal to –21.

Answer available at the end of this section.

Exercise

Translate and solve: n divided by 8 is equal to –56.

Answer available at the end of this section.

Example 5

Translate and solve: The quotient of y and –4 is 68.

Solution

Begin by translating the sentence into an equation.

Translate.	The quotient of y and -4 is 68. $\frac{y}{-4} = 68$
Multiply both sides by -4.	$-4\left(\frac{y}{-4}\right) = -4(68)$
Simplify.	$y = -272$
Check:	Is the quotient of -272 and -4 equal to 68? $\frac{-272}{-4} \stackrel{?}{=} 68$ $68 = 68$✓

Exercise

Translate and solve: The quotient of q and –8 is 72.

Answer available at the end of this section.

Exercise

Translate and solve: The quotient of p and –9 is 81.

Answer available at the end of this section.

Example 6

Translate and solve: Three-fourths of p is 18.

Solution

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.	Three-fourths of p is 18. $\frac{3}{4}p = 18$
Multiply both sides by $\frac{4}{3}$.	$\frac{4}{3} \cdot \frac{3}{4}p = \frac{4}{3} \cdot 18$
Simplify.	$p = 24$
Check:	Is three-fourths of 24 equal to 18? $\frac{3}{4} \cdot 24 \stackrel{?}{=} 18$ $18 = 18$✓

Exercise

Translate and solve: Two-fifths of f is 16.

Answer available at the end of this section.

Exercise

Translate and solve: Three-fourths of f is 21.

Answer available at the end of this section.

Example 7

Translate and solve: The sum of three-eighths and x is one-half.

Solution

Begin by translating the sentence into an equation.

Translate.	The sum of three – eighths and x is $\frac{1}{2}$ $\frac{3}{8} + x = \frac{1}{2}$
Subtract $\frac{3}{8}$ from each side.	$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$
Simplify and rewrite fractions with common denominators.	$x = \frac{4}{8} - \frac{3}{8}$
Simplify.	$x = \frac{1}{8}$

Check:	Is the sum of $\frac{3}{8}$ and $\frac{1}{8}$ equal to $\frac{1}{2}$? $\frac{3}{8}+\frac{1}{8}\stackrel{?}{=}\frac{1}{2}$ $\frac{4}{8}\stackrel{?}{=}\frac{1}{2}$ $\frac{1}{2}=\frac{1}{2}$✓

Exercise

Translate and solve: The sum of five-eighths and *x* is one-fourth.

Answer available at the end of this section.

Exercise

Translate and solve: The sum of three-fourths and *x* is five-sixths.

Answer available at the end of this section.

Answers

Exercise

x + 11 = 41; x = 30

Exercise

y - 12 = 51; y = 63

Exercise

4x - 3x = 14; x = 14

Exercise

7a - 6a = -8; a = -8

Exercise

132=−12y; y=−11

Exercise

117=−13z; z=−9

Exercise

$\frac{n}{7}=-21; n=-147$

Exercise

$\frac{n}{8}=-56; n=-448$

Exercise

$\frac{-q}{-8}=72; q=-576$

Exercise

$\frac{p}{-9}=81; p=-729$

Exercise

$\frac{2}{5}f=16; f=40$

Exercise

$\frac{3}{4}f=21; f=28$

Exercise

$\frac{5}{8}+x=\frac{1}{4}; x=-\frac{3}{8}$

Exercise

$\frac{3}{4}+x=\frac{5}{6}; x=\frac{1}{12}$

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2.4.2: Solving Applications Involving Equations

In the previous section, you translated word sentences into algebraic equations. In this section, you will apply those skills to solve application problems. Rather than being given an equation, you will define a variable, write an equation that represents the situation, and then solve it. These problems are based on technical and workplace contexts such as measurements, materials, equipment, production, and rates.

Steps for Solving an Application Problem

1. Read the problem carefully and identify the quantity to be found.
2. Define a variable to represent the unknown quantity.
3. Write an algebraic equation that models the situation.
4. Solve the equation.
5. State the solution in a complete sentence, including appropriate units.
6. Verify that the solution is reasonable in the context of the problem.

Example 1

A metal rod used in a fabrication project is cut into two pieces. One piece is 2.5 times as long as the other. The total length of the rod is 84 inches. Find the length of each piece.

Solution

Let x represent the length of the shorter piece in inches.

Then the length of the longer piece is $2.5x$.

Write an equation for the total length.

$x + 2.5x = 84$

Combine like terms.

$3.5x = 84$

Divide both sides by 3.5.

$3.5x \div 3.5 = 84 \div 3.5$

$x = 24$

Find the length of the longer piece.

$2.5(24) = 60$

Answer: The shorter piece is 24 inches long and the longer piece is 60 inches long.

Check: $24 + 60 = 84$

Exercise

A steel bar used in a machining process is cut into two pieces. One piece is 1.8 times as long as the other. The total length of the bar is 70 inches. Find the length of each piece.

Answer available at the end of this section.

Example 2

A hydraulic hose assembly is made from three sections. The second section is 1.5 times the length of the first section, and the third section is twice the length of the first section. The total length of the hose is 63 inches. Find the length of each section.

Solution

Let x represent the length of the first section in inches.

Then the second section is $1.5x$, and the third section is $2x$.

Write an equation for the total length.

$x + 1.5x + 2x = 63$

Combine like terms.

$4.5x = 63$

Divide both sides by 4.5.

$4.5x \div 4.5 = 63 \div 4.5$

$x = 14$

Find the lengths of the other sections.

$1.5(14) = 21$

$2(14) = 28$

Answer: The first section is 14 inches long, the second section is 21 inches long, and the third section is 28 inches long.

Check: $14 + 21 + 28 = 63$

Exercise

A set of three conveyor belt sections is installed in a manufacturing system. The second section is 1.2 times the length of the first section, and the third section is 2.3 times the length of the first section. The total length of the conveyor system is 74 feet. Find the length of each section.

Answer available at the end of this section.

Example 3

An HVAC technician charges a service fee of $\$85$ for the first hour and $\$65$ for each additional hour. A customer's total bill for a repair was $\$475$. How many hours did the technician work?

Solution

Let x represent the total number of hours worked.

The first hour costs $\$85$, and the remaining $x-1$ hours cost $\$65$ each.

Write an equation for the total cost.

$85+65(x-1)=475$

Distribute.

$85+65x-65=475$

Combine like terms.

$65x+20=475$

Subtract 20 from both sides.

$65x+20-20=475-20$

$65x=455$

Divide both sides by 65.

$65x \div 65=455 \div 65$

$x=7$

Answer: The technician worked 7 hours.

Check: $85+65(7-1)=85+65(6)=85+390=475$

Exercise

A mobile equipment technician charges a service fee of $\$95$ for the first hour and $\$72$ for each additional hour. A customer's total bill for a repair was $\$527$. How many hours did the technician work?

Answer available at the end of this section.

Example 4

A diesel technician earns $\$24.50$ per hour for regular time and $\$36.75$ per hour for overtime. In one week, the technician worked 40 regular hours and earned a total of $\$1{,}347.50$. How many hours of overtime did the technician work?

Solution

Let x represent the number of overtime hours.

The technician earns $\$24.50$ for each of the 40 regular hours and $\$36.75$ for each overtime hour.

Write an equation for the total earnings.

$24.50(40) + 36.75x = 1347.50$

Multiply.

$980 + 36.75x = 1347.50$

Subtract 980 from both sides.

$980 + 36.75x - 980 = 1347.50 - 980$

$36.75x = 367.50$

Divide both sides by 36.75.

$36.75x \div 36.75 = 367.50 \div 36.75$

$x = 10$

Answer: The technician worked 10 hours of overtime.

Check: $24.50(40) + 36.75(10) = 980 + 367.50 = 1347.50$

Exercise

An industrial maintenance technician earns $\$22.80$ per hour for regular time and $\$34.20$ per hour for overtime. In one week, the technician worked 40 regular hours and earned a total of $\$1{,}231.20$. How many hours of overtime did the technician work?

Answer available at the end of this section.

Example 5

In a manufacturing process, $\frac{1}{6}$ of the parts produced are rejected during quality inspection. A production run resulted in $1{,}050$ acceptable parts. How many total parts were produced?

Solution

Let x represent the total number of parts produced.

Since $\frac{1}{6}$ of the parts are rejected, $\frac{5}{6}$ of the parts are acceptable.

Write an equation for the number of acceptable parts.

$\frac{5}{6}x = 1050$

Multiply both sides by $\frac{6}{5}$.

$\frac{5}{6}x \cdot \frac{6}{5} = 1050 \cdot \frac{6}{5}$

$x = 1260$

Answer: A total of 1260 parts were produced.

Check: $\frac{5}{6}(1260) = 1050$

Exercise

In a machining process, $\frac{1}{8}$ of the parts produced are rejected during inspection. A production run resulted in $1{,}400$ acceptable parts. How many total parts were produced?

Answer available at the end of this section.

Example 6

A tractor is used to apply fertilizer to a field. The tractor applies fertilizer at a rate of 3.2 acres per hour. The operator worked long enough to cover 86.4 acres. How many hours did the tractor operate?

Solution

Let x represent the number of hours the tractor operated.

The tractor covers 3.2 acres per hour, so the total area covered is $3.2x$.

Write an equation for the total area.

$3.2x = 86.4$

Divide both sides by 3.2.

$3.2x \div 3.2 = 86.4 \div 3.2$

$x = 27$

Answer: The tractor operated for 27 hours.

Check: $3.2(27) = 86.4$

Exercise

A data technician is transferring files to a server at a rate of 3.6 gigabytes per minute. A total of 97.2 gigabytes of data were transferred. How many minutes did the transfer take?

Answer available at the end of this section.

Example 7

A contractor ordered a shipment of tiles to be used for flooring. The tiles were divided evenly into 8 boxes. After removing 15 tiles from one box, 45 tiles remained in that box. How many tiles were in the original shipment?

Solution

Let x represent the total number of tiles in the shipment.

Since the tiles were divided evenly into 8 boxes, each box contained $\frac{x}{8}$ tiles.

After removing 15 tiles, 45 remained.

Write an equation.

$\frac{x}{8} - 15 = 45$

Add 15 to both sides.

$\frac{x}{8} - 15 + 15 = 45 + 15$

$\frac{x}{8} = 60$

Multiply both sides by 8.

$\frac{x}{8} \cdot 8 = 60 \cdot 8$

$x = 480$

Answer: There were 480 tiles in the original shipment.

Check: $\frac{480}{8} = 60$ and $60 - 15 = 45$

Exercise

A warehouse receives a shipment of parts that are divided evenly into 6 containers. After removing 12 parts from one container, 48 parts remain in that container. How many parts were in the original shipment?

Answer available at the end of this section.

Example 8

A medical supply technician stocked a cabinet with supplies at the beginning of the shift. During the day, 38 items were used. At the end of the shift, 124 items remained. How many items were in the cabinet at the beginning of the shift?

Solution

Let x represent the number of items in the cabinet at the beginning of the shift.

During the day, 38 items were used, leaving 124 items.

Write an equation.

$x - 38 = 124$

Add 38 to both sides.

$x - 38 + 38 = 124 + 38$

$x = 162$

Answer: There were 162 items in the cabinet at the beginning of the shift.

Check: $162 - 38 = 124$

Exercise

A tool room supervisor counted drill bits at the end of the day. During the shift, 27 drill bits were issued to workers, and 91 drill bits remained in storage. How many drill bits were in storage at the beginning of the shift?

Answer available at the end of this section.

Example 9

A landscaping crew is installing edging along a walkway. They install 12 sections of edging, each with the same length. After removing 3 feet from each section for adjustments, the total length of edging installed is 216 feet. What was the original length of each section?

Solution

Let x represent the original length of each section in feet.

After removing 3 feet from each section, each section has length $x - 3$.

There are 12 sections, so the total length is $12(x - 3)$.

Write an equation.

$12(x - 3) = 216$

Distribute.

$12x - 36 = 216$

Add 36 to both sides.

$12x - 36 + 36 = 216 + 36$

$12x = 252$

Divide both sides by 12.

$12x \div 12 = 252 \div 12$

$x = 21$

Answer: The original length of each section was 21 feet.

Check: $12(21 - 3) = 12(18) = 216$

Exercise

A farmer is installing irrigation lines across a field. He installs 10 sections of pipe, each with the same length. After cutting 2 feet from each section for adjustments, the total length of pipe installed is 180 feet. What was the original length of each section?

Answer available at the end of this section.

Common Mistakes

- **Not defining the variable clearly**
  Always state what the variable represents before writing the equation. This helps ensure the equation matches the situation.

- **Forgetting what x represents**
  If x is the total, do not treat it as a part. If x is a part, be careful when writing expressions for totals.

• **Missing $(x-1)$ in "first plus additional" situations**
If the first unit is handled differently, the remaining amount is often $x-1$, not x.

• **Incorrectly combining terms**
Expressions like $x+2.5x$ should be combined to $3.5x$. Do not multiply unless appropriate.

• **Not using parentheses when needed**
Expressions like $12(x-3)$ must include parentheses before distributing.

• **Stopping too early**
After solving the equation, make sure to answer the question. Some problems require finding an additional value.

• **Forgetting units in the final answer**
Always include units such as inches, hours, or parts in the final answer.

• **Not checking the solution**
Substitute your answer back into the equation or context to verify it is correct.

Answers

Exercise

Equation: $x+1.8x=70$

Answer: The shorter piece is 25 inches long and the longer piece is 45 inches long.

Exercise

Equation: $x+1.2x+2.3x=74$

Answer: The first section is 16.8 feet long, the second section is 20.16 feet long, and the third section is 38.64 feet long.

Exercise

Equation: $95+72(x-1)=527$

Answer: The technician worked 7 hours.

Exercise

Equation: $22.80(40)+34.20x=1231.20$

Answer: The technician worked 9 hours of overtime.

Exercise

Equation: $\frac{7}{8}x=1400$

Answer: A total of 1600 parts were produced.

Exercise

Equation: $3.6x=97.2$

Answer: The transfer took 27 minutes.

Exercise

Equation: $\frac{x}{6}-12=48$

Answer: There were 360 parts in the original shipment.

Exercise

Equation: $x-27=91$

Answer: There were 118 drill bits in storage at the beginning of the shift.

Exercise

Equation: $10(x-2)=180$

Answer: The original length of each section was 20 feet.

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2.4.3: Algebraic Applications Practice

The following set of exercises provides practice for the concepts covered in Module 2.4: Algebraic Applications. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 2.4.1 - Translating Algebraic Equations

Translate to an equation and then solve.

1. The quotient of c and -19 is 38.

2. Five more than x is equal to 21.

3. The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.

4. u divided by 7 is equal to -49.

5. The sum of $-4n$ and $5n$ is -32.

6. Four-thirds of w is 36.

7. The difference of $9x$ and $8x$ is 17.

8. -184 is the product of 23 and p.

9. The sum of nine-tenths and g is two-thirds.

10. h divided by -13 is equal to -65.

Section 2.4.2 - Solving Applications Involving Equations

Solve each problem. Write a complete sentence and include units in your answer.

11. A steel rod is cut into two pieces. One piece is 2.2 times as long as the other. The total length is 96 inches. Find the length of each piece.

12. A set of three pipes is installed in a system. The second pipe is 1.4 times the length of the first pipe, and the third pipe is 2.1 times the length of the first pipe. The total length is 84 feet. Find the length of each pipe.

13. A storage tank contained some amount of fluid at the beginning of the day. During the day, 72 gallons were used, and 188 gallons remained. How many gallons were in the tank at the beginning of the day?

14. A contractor installs 14 sections of railing, each with the same length. After cutting 2 feet from each section for adjustments, the total installed length is 224 feet. What was the original length of each section?

15. A pump moves water at a rate of 4.8 gallons per minute. A total of 168 gallons were moved. How many minutes did the pump run?

16. In a manufacturing process, $\frac{1}{5}$ of the parts produced are rejected. A production run resulted in $1{,}280$ acceptable parts. How many total parts were produced?

17. A service technician charges a fee of $\$90$ for the first hour and $\$55$ for each additional hour. A customer's total bill was $\$420$. How many hours did the technician work?

18. A conveyor system moves materials at a rate of 18 feet per minute. How many minutes will it take to move 900 feet of

material?

19. A shipment of materials is divided evenly into 9 containers. After removing 6 items from one container, 42 items remain in that container. How many items were in the original shipment?

20. A mechanic earns $\$26.50$ per hour for regular time and $\$39.75$ per hour for overtime. In one week, the mechanic worked 40 regular hours and earned a total of $\$1{,}457.50$. How many hours of overtime did the mechanic work?

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3: PROPORTIONAL REASONING AND APPLICATIONS

The Proportional Reasoning and Applications Unit focuses on understanding relationships between quantities and applying those relationships in practical situations. In this unit, you will work with ratios and rates, simplify and interpret them, and use them to compare quantities in meaningful ways. You will also determine whether relationships are proportional and solve problems involving proportions.

You will extend this understanding to direct and inverse proportionality, learning how different quantities change in relation to one another. These concepts are essential for analyzing situations where values increase or decrease together or vary based on changing conditions.

In addition, you will work with percents and their connections to fractions and decimals. You will convert between forms and use both proportions and equations to solve percent problems. This foundation will support more advanced applications involving percent relationships.

The unit also emphasizes real-world applications of proportional reasoning. You will solve problems involving percent increase and decrease, relative error and tolerance, and common financial situations such as sales tax, discounts, wages, and budgeting. These skills are widely used in technical fields and everyday decision-making.

3.1: Ratios and Proportional Reasoning

3.1.1: Introduction to Ratios and Rates

3.1.2: Introduction to Proportions

3.1.3: Applications of Proportionality

3.1.4: Rates and Proportional Reasoning Practice

3.2: Percents

3.2.1: Percents, Decimals and Fractions

3.2.2: Solving Basic Percent Problems

3.2.3: Percents Practice

3.3: Applications of Percent

3.3.1: Applications of Basic Percent Problems

3.3.2: Percent Increase and Decrease

3.3.3: Applications of Percent Practice

3.4: Financial Applications

3.4.1: Sales Tax and Markup

3.4.2: Discounts and Markdowns

3.4.3: Wages and Overtime

3.4.4: Personal Budgeting

3.4.5: Financial Applications Practice

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3.1: RATIOS AND PROPORTIONAL REASONING

What seems like a minor annoyance — a faucet that will not quite stop dripping — turns out to be a useful math problem. How fast is the water dripping? How much will be wasted in an hour, a day, or a month? Answering these questions requires the same kind of thinking you will use throughout this module: comparing quantities, finding rates, and understanding how relationships scale over time.

This type of thinking shows up often in technical work. A welder adjusting a gas-to-wire feed ratio, an automotive technician interpreting a gear ratio, a medical professional calculating a dose based on weight, or a technician adjusting measurements for a larger project are all working with ratios and proportions.

In this module, you will write and interpret ratios in different formats, simplify them, and determine unit rates in real-world contexts. You will also decide whether two ratios form a proportion and solve applied problems involving proportional relationships.

You will finish the module by working with direct and inverse proportional relationships, learning to distinguish between them and apply them in context.

How Fast Does Water Drip From a Faucet

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explores how a quantity can build over time, providing a useful way to think about rates and how values are related in real-world situations.

3.1.1: Introduction to Ratios and Rates

3.1.2: Introduction to Proportions

3.1.3: Applications of Proportionality

3.1.4: Rates and Proportional Reasoning Practice

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3.1.1: Introduction to Ratios and Rates

We use *ratios* to compare two numeric quantities or quantities with the same units.

Ratio

A *ratio* is the quotient of two numerical quantities or two quantities with the same physical units.

For example, ancient Greek geometers believed that the most pleasing rectangle to the eye had length and width such that the ratio of length to width was a specific number, called the *Golden Ratio*, approximately equal to 1.6180339887.... Architects to this day use this ratio in their designs.

There are a number of equivalent ways of expressing ratios, three of which we will use in this text: fraction notation, “to” notation, and “colon” notation.

- 3/4 is a ratio, read as “the ratio of 3 to 4.”
- 3 to 4 is a ratio, read as “the ratio of 3 to 4.”
- 3:4 is a ratio, read as “the ratio of 3 to 4.”

Example 1

Express each of the following ratios as a fraction reduced to lowest terms: (a) 36 to 24, and (b) 0.12 : 0.18.

Solution

(a) To express the ratio “36 to 24” as a fraction, place 36 over 24 and reduce.

$\frac{36}{24} = \frac{3}{2}$

Thus, the ratio 36 to 24 equals 3/2.

(b) To express the ratio “0.12:0.18” as a fraction, place 0.12 over 0.18 and reduce.

$\frac{0.12}{0.18} = \frac{2}{3}$

Thus, the ratio 0.12:0.18 equals 2/3.

Exercise

Express 0.12 : 0.3 as a fraction reduced to lowest terms.

Answer available at the end of this section.

Example 2

A rectangular metal plate used in fabrication has a length of $3\frac{1}{4}$ feet and a width of $2\frac{1}{2}$ feet. Express the ratio of length to width as a fraction reduced to lowest terms.

Solution.

Write the ratio of length to width and simplify.

$\frac{3\frac{1}{4}\text{ ft}}{2\frac{1}{2}\text{ ft}} = \frac{13}{10}$

Hence, the ratio of length to width is $\frac{13}{10}$.

Exercise

A solution is made by mixing $3\frac{1}{4}$ gallons of water with $2\frac{1}{2}$ gallons of concentrate. Express the ratio of water to concentrate as a fraction reduced to lowest terms.

Answer available at the end of this section.

Ratios and Rates

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View this YouTube Video online

This video introduces how fractions and ratios can be used to compare quantities and explains how a rate is a type of ratio that compares a quantity to a period of time.

Rates

We now introduce the concept of *rate*, a special type of ratio.

Rate

A *rate* is a quotient of two measurements with different units.

The physical interpretation of a rate in terms of its units is an important skill.

Example 3

A water storage tank is filled with 224 gallons of water in 12 minutes. Express the ratio of water filled to time as a fraction reduced to lowest terms. Write a short sentence explaining the meaning of your result. Include units in your description.

Solution

Place water filled over time and reduce.

$\frac{224\text{ gal}}{12\text{ min}} = \frac{56\text{ gal}}{3\text{ min}}$

Thus, the rate is 56 gallons to 3 minutes. This means 56 gallons of water are added every 3 minutes.

Exercise

A medical clinic administers 180 vaccine doses over a 12-hour shift. Express the ratio of doses administered to time as a fraction reduced to lowest terms. Write a short sentence explaining the meaning of your result. Include units in your description.

Answer available at the end of this section.

Unit Rates

When making comparisons, it is helpful to have a rate in a form where the denominator is 1. Such rates are given a special name.

Unit Rate

A *unit rate* is a rate whose denominator is 1.

Example 4

A service truck travels between job sites, driving a total of 120 miles in 4 hours of travel time. Find the average rate of speed.

Solution

Place distance over time and simplify.

$$\frac{120 \text{ miles}}{4 \text{ hours}} = \frac{30 \text{ miles}}{1 \text{ hour}} = 30 \text{ miles per hour} \tag{3.1.1.1}$$

Hence, the average rate of speed is 30 miles per hour.

Exercise

A construction crew is installing fencing along a property line. The crew installs 72 feet of fencing in 3 hours. Find the average rate of installation.

Answer available at the end of this section.

Example 5

An employee records 8.5 hours of work and earns $95 for the shift. Find the average hourly pay rate.

Solution

Place earnings over hours worked and simplify.

$$\frac{95 \text{ dollars}}{8.5 \text{ hours}} \approx 11.18 \text{ dollars per hour} \tag{3.1.1.2}$$

Hence, his hourly pay rate is approximately $11.18 per hour.

Exercise

A retail store tracks the number of items sold during a shift. Over a 3.5-hour period, the store sells 45 items. Find the average sales rate.

Answer available at the end of this section.

Example 6

A fleet manager is comparing the fuel efficiency of two service vehicles.

Vehicle A travels 422 miles using 15 gallons of fuel.
Vehicle B travels 354 miles using 13 gallons of fuel.

Which vehicle has the better fuel efficiency?

Solution

Find the unit rate (miles per gallon) for each vehicle.

$\frac{422 \text{ miles}}{15 \text{ gal}} \approx 28.13 \text{ miles per gallon}$

$\frac{354 \text{ miles}}{13 \text{ gal}} \approx 27.23 \text{ miles per gallon}$

Compare the unit rates.

Since $28.13 > 27.23$, Vehicle A has the better fuel efficiency.

Hence, Vehicle A gets better gas mileage.

Exercise

Two machines are being compared for efficiency.

Machine A produces 360 parts using 12 hours of operation.
Machine B produces 325 parts using 10 hours of operation.

Which machine has the higher production rate?

Answer

Answer available at the end of this section.

Rates and Unit Rates

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View this YouTube Video online

This video introduces rates and unit rates and demonstrates how to compare values using unit rates to determine the best buy through several examples.

Answers

Exercise

2/5

Exercise

$\frac{13}{10}$

Exercise

$$\frac{15 \text{ doses}}{1 \text{ hr}} \tag{3.1.1.3}$$

This means the clinic administers 15 doses per hour.

Exercise

24 feet per hour

Exercise

12.86 items per hour

Exercise

Machine B

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3.1.2: Introduction to Proportions

In the previous lesson, we introduced the concepts of ratio and rate. In this section, we set these ratios equal to each other to form a proportion.

Proportions

A proportion is a statement that shows two ratios or rates are equal.

For example, each of the equations

$$\frac{1}{3}=\frac{2}{6},\ \frac{15\text{ miles}}{2\text{ hours}}=\frac{30\text{ miles}}{4\text{ hours}},\ \text{and}\ \frac{a}{b}=\frac{c}{d}$$

compare two ratios or rates and is a proportion.

The proportion

$$\frac{1}{3}=\frac{2}{6}$$

is read “one is to three as two is to six.” The four numbers that make up this proportion are called the *terms* of the proportion and are ordered in a natural manner.

$$\begin{aligned}\text{First term}\longrightarrow\ &\frac{1}{3}=\frac{2}{6}\longleftarrow\text{Third term}\\ \text{Second term}\longrightarrow\ &\frac{1}{3}=\frac{2}{6}\longleftarrow\text{Fourth term}\end{aligned}$$

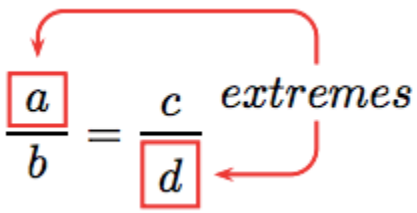
Extremes and Means

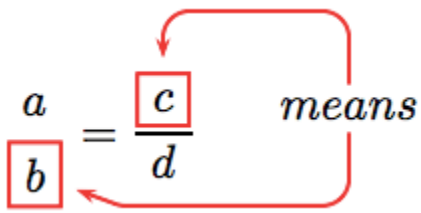
The first and fourth terms are called the *extremes* of the proportion. The second and third terms are called the *means* of the proportion.

In the proportion

$$\frac{a}{b}=\frac{c}{d},$$

the terms *a* and *d* are the extremes; the terms *b* and *c* are the means.





If we multiply both sides of the proportion by the common denominator,

$$bd\left(\frac{a}{b}\right)=bd\left(\frac{c}{d}\right)$$

then cancel,

$$\cancel{b}\,d\left(\frac{a}{\cancel{b}}\right) = b\cancel{d}\left(\frac{c}{\cancel{d}}\right)$$

we get the following result.

$$ad = bc$$

This leads to the following observation.

Product of Extremes and Means

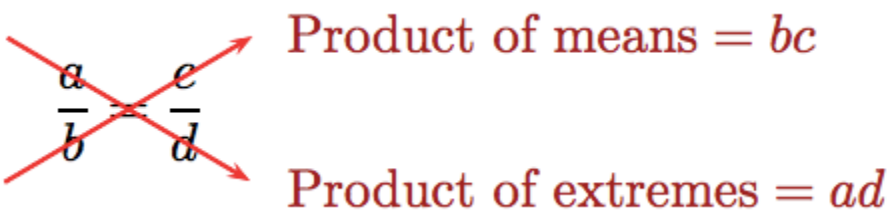
In the proportion

$$\frac{a}{b} = \frac{c}{d}$$

the product of the means equals the product of the extremes. That is,

$$ad = bc.$$

We can get an equivalent result using a technique called *cross multiplication*.

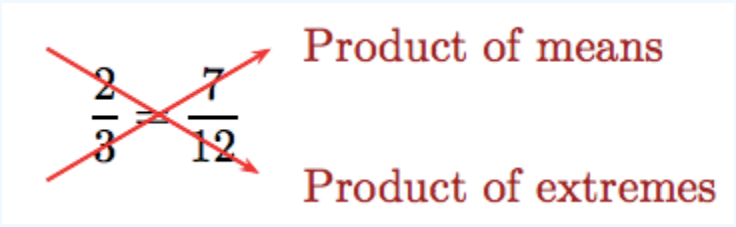


Example 1

Which of the following is a valid proportion: (a) $\frac{2}{3} = \frac{7}{12}$, or (b) $\frac{4}{9} = \frac{12}{27}$.

Solution

(a) Cross multiply



to get

24 = 21.

Hence, the product of the extremes does not equal the product of the means, so 2/3=7/12 is **not** a valid proportion.

(b) Cross multiply



to get

$$108 = 108.$$

Hence, the product of the extremes equals the product of the means, so 4/9 = 12/27 **is** a valid proportion.

Exercise

Is the following a valid proportion? $\frac{4}{3} = \frac{16}{11}$

Answer available at the end of this section.

Proportions

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View this YouTube Video online

This video introduces proportions as equations that show two ratios are equal and demonstrates how to solve proportions using cross multiplication through several examples.

Solving Proportions

We already have all the tools needed to solve proportions. Let's begin with the first example.

Example 2

Solve the proportion for *x*: $\frac{3}{4} = \frac{x}{12}$.

Solution

Cross multiply, then solve the resulting equation.

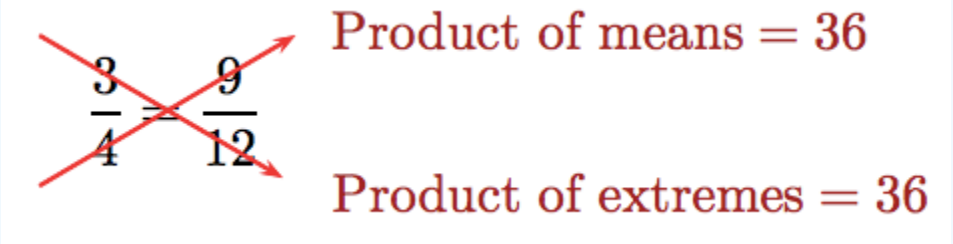
$$\begin{aligned} \frac{3}{4} &= \frac{x}{12} && \text{Original proportion.} \\ 4 \cdot x &= 3 \cdot 12 && \text{Products of means and extremes are equal.} \\ 4x &= 36 && \text{Simplify.} \\ \frac{4x}{4} &= \frac{36}{4} && \text{Divide both sides by 4.} \\ x &= 9 && \text{Simplify.} \end{aligned}$$

Check

Substitute 9 for x into the original proportion and check.

$$\begin{aligned} \frac{3}{4} &= \frac{x}{12} && \text{Original proportion.} \\ \frac{3}{4} &= \frac{9}{12} && \text{Substitute 9 for } x. \end{aligned}$$

Cross multiply.



Thus, the solution 9 checks.

Exercise

Solve the proportion for *n*: $\frac{2}{3} = \frac{n}{9}$

Answer available at the end of this section.

Example 3

Solve the proportion for *n*: $\frac{3}{2} = \frac{24}{n}$.

Solution

Cross multiply, then solve the resulting equation.

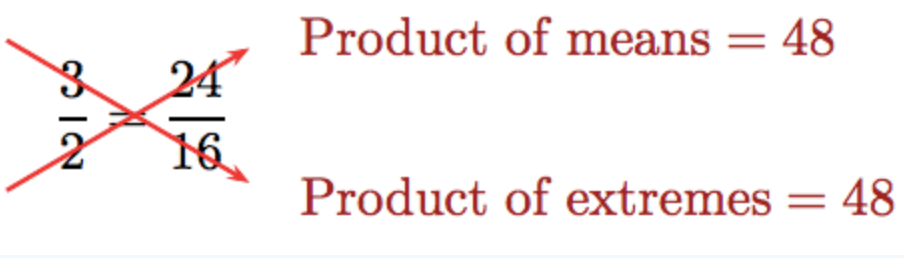
$$\begin{aligned} \frac{3}{2} &= \frac{24}{n} && \text{Original proportion.} \\ 3 \cdot n &= 2 \cdot 24 && \text{Products of means and extremes are equal.} \\ 3n &= 48 && \text{Simplify.} \\ \frac{3n}{3} &= \frac{48}{3} && \text{Divide both sides by 3.} \\ n &= 16 && \text{Simplify.} \end{aligned}$$

Check

Substitute 16 for *n* into the original proportion and check.

$$\begin{aligned} \frac{3}{2} &= \frac{24}{n} && \text{Original proportion.} \\ \frac{3}{2} &= \frac{24}{16} && \text{Substitute 16 for } n. \end{aligned}$$

Cross multiply.



Thus, the solution 16 checks.

Exercise

Solve the proportion for *m*: $\frac{9}{6} = \frac{m}{4}$

Answer available at the end of this section.

Example 4

Solve the proportion for *x*: $\frac{2x+1}{15} = \frac{1}{3}$.

Solution

Cross multiply, then solve the resulting equation.

$$\begin{aligned} \frac{2x+1}{15} &= \frac{1}{3} && \text{Original proportion.} \\ 3(2x+1) &= 15(1) && \text{Products f means and extremes are equal.} \\ 6x+3 &= 15 && \text{On the left, distribute.} \\ & && \text{On the right, multiply.} \\ 6x+3-3 &= 15-3 && \text{Subtract 3 from both sides.} \\ 6x &= 12 && \text{Simplify.} \\ \frac{6x}{6} &= \frac{12}{6} && \text{Divide both sides by 6.} \\ x &= 2 && \text{Simplify both sides.} \end{aligned}$$

Check

We'll leave it to our readers to check this solution.

Exercise

Solve the proportion for *y*: $\frac{6+2y}{18} = \frac{8}{9}$

Answer available at the end of this section.

Applications

A number of practical applications involve solving a proportion.

Example 5

A contractor purchases 5 concrete blocks for $11.50. How many concrete blocks can be purchased for $34.50, assuming a constant price per block?

Solution

Set up a proportion and solve.

$\frac{5}{11.50} = \frac{x}{34.50}$ Original proportion.

$11.50x = 5 \cdot 34.50$ Cross multiply.

$11.50x = 172.5$ Multiply.

$\frac{11.50x}{11.50} = \frac{172.5}{11.50}$ Divide both sides by 11.50.

$x = 15$ Simplify.

Hence, 15 concrete blocks can be purchased.

Exercise

A supplier sells 8 bolts for $6.40. How many bolts can be purchased for $20.00, assuming a constant price per bolt?

Answer available at the end of this section.

Checking Units is Extremely Important

When setting up a proportion, check to make sure that both numerators have the same units and both denominators have the same units.

For example, in Example 5, both numerators have blocks as units and both denominators have dollars as units.

$$\frac{5 \text{ blocks}}{11.50 \text{ dollars}} = \frac{15 \text{ blocks}}{x \text{ dollars}}$$

This proportion is set up correctly because both numerators have the same units and both denominators have the same units.

On the other hand, if we had set up the proportion incorrectly as follows,

$$\frac{5 \text{ blocks}}{11.50 \text{ dollars}} = \frac{x \text{ dollars}}{15 \text{ blocks}}$$

a quick check of the units reveals the error; that is, the numerators have different units and the denominators have different units. Checking units helps us avoid errors.

Example 6

On a site plan, 1.2 centimeters represents 2 miles. What is the actual distance if a measured route is 10.6 centimeters? Round your answer to the nearest tenth of a mile.

Solution

$\frac{1.2}{2} = \frac{10.6}{x}$ Original proportion.

$1.2x = 2 \cdot 10.6$ Cross multiply.

$x \approx 17.7$ Simplify.

Hence, the distance is approximately 17.7 miles.

Exercise

On a blueprint, 1 inch represents 8 feet. A wall measures 6 inches on the drawing. What is the actual length of the wall?

Answer available at the end of this section.

Example 7

A chemical solution is mixed using 3 gallons of water for every 2 gallons of concentrate. If 9 gallons of water are used, how much concentrate is needed?

Solution

$\frac{3}{2} = \frac{9}{x}$ Original proportion.

$3x = 2 \cdot 9$ Cross multiply.

$x = 6$ Simplify.

Hence, 6 gallons of concentrate are needed.

Exercise

A fuel mixture requires 5 gallons of gasoline for every 2 gallons of oil. If 8 gallons of oil are used, how many gallons of gasoline are needed?

Answer available at the end of this section.

Answers

Exercise
No

Exercise
6

Exercise
6

Exercise
5

Exercise
25 bolts

Exercise
48 feet

Exercise
20 gallons of gasoline

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3.1.3: Applications of Proportionality

In the previous section, we studied proportions, and used them to solve problems involving ratios. In this section, we continue our study of proportions, and investigate two different types of proportionality.

There are two main types of proportionality. We will learn about the more common one first.

Direct Proportionality

Definition: Directly Proportional

Two quantities are **directly proportional** if, as one quantity increases, the other quantity also increases at the same rate.

Direct proportionality describes all of the proportion problems we've seen before. Here is another example that shows how direct proportionality works, and introduces the next important notion.

Example 1

At an hourly wage job, you work for 5 hours, and get paid $\$83.75$. How much money will you earn if you work 7 hours? (Assume that you are not making overtime pay, or any other sort of special pay rate.)

Solution

This situation represents direct proportionality because the amount earned increases at a constant rate with the number of hours worked.

Set up a proportion and solve.

$\frac{83.75}{5} = \frac{x}{7}$ Original proportion.

$5x = 83.75 \cdot 7$ Cross multiply.

$5x = 586.25$ Multiply.

$\frac{5x}{5} = \frac{586.25}{5}$ Divide both sides by 5.

$x = 117.25$ Simplify.

Hence, the earnings for 7 hours are $\$117.25$.

The approach above works well to find the desired answer. However, what if you wanted to know how much you would earn working 333 hours? Or 444 hours? Or 101010 hours? You could repeat the same process each time, but there is a quicker method.

Definition: Constant of Proportionality

In a situation involving directly proportional quantities, the **constant of proportionality** is the common ratio that describes the comparison of any two corresponding quantities. In other words, it is the constant rate of change between the two quantities.

Let's see how to find a constant of proportionality, and how to interpret it.

Example 2

You're in the same situation as the previous example: you work for 5 hours, and earn $\$83.75$. What is the constant of proportionality in this example, and what does it mean in context?

Solution

The constant of proportionality is the common ratio between corresponding quantities.

$\frac{83.75 \text{ dollars}}{5 \text{ hours}} = 16.75 \text{ dollars per hour}$ Divide.

$\frac{117.25 \text{ dollars}}{7 \text{ hours}} = 16.75 \text{ dollars per hour}$ Divide.

Both ratios give the same value.

The constant of proportionality is 16.75 dollars per hour.

This means the earnings increase at a constant rate of $\$16.75$ per hour.

Now that you know the rate, you can quickly find earnings for any number of hours by multiplying the hourly rate by the number of hours worked.

Example 3

Using the constant of proportionality from the previous example, determine how much you will earn if you work 4 hours.

Solution

Multiply the hourly rate by the number of hours worked.

$4 \text{ hours} \times 16.75 \text{ dollars per hour} = 67.00 \text{ dollars}$ Multiply.

Hence, the earnings for 4 hours are $\$67.00$.

The relationship described above can be written using the direct proportionality formula.

Direct Proportionality Formula

$y = kx$ where k represents the constant of proportionality.

Multiply to find y.

Example 4

A contractor is installing flooring in a series of identical rooms. Based on previous work, the contractor can install 120 square feet of flooring per hour.

If the contractor works for 6 hours, how much flooring will be installed? What if the contractor works for 8 hours?

Solution

Since y is directly proportional to x, use the formula

$y = kx$ Direct proportion formula.

Substitute $k = 120$.

$y = 120x$ Substitute for k.

For $x = 6$:

$y = 120(6) = 720$ Multiply.

For $x = 8$:

$y = 120(8) = 960$ Multiply.

Hence, 720 square feet will be installed in 6 hours and 960 square feet in 8 hours.

Inverse Proportionality

To introduce this idea, consider the following question.

Example 5

Suppose it takes 6 sanitation workers, all working at the same rate, 4 hours to complete a job. How many workers are needed to complete the same job in 3 hours?

Solution

A common mistake is to treat this as a direct proportion.

$\frac{6}{4} = \frac{x}{3}$ Incorrect setup.

$4x = 18$

$x = 4.5$

Now, the numerical answer 4.5 does not make sense, since it represents a number of workers. You might round to 5, but this still does not make sense in context.

If it takes 6 workers 4 hours to complete the job, using fewer workers should require more time, not less. Since the job must be completed in 3 hours, more workers are needed, not fewer. Therefore, the answer cannot be correct.

From this discussion, we can conclude:

- It is important to check whether an answer makes sense in context. Ask yourself: What kind of answer should I expect? Does my answer seem reasonable?
- Not all problems can be solved using direct proportionality.

This type of problem can be solved using a different type of relationship, which we will define next.

Definition: Inversely Proportional

Two quantities are **inversely proportional** if, as one quantity increases, the other quantity decreases at the same rate.

Note how similar this definition is to the previous definition of direct proportionality. The only difference here is that one quantity increases while the other decreases:

Direct Proportionality	Inverse Proportionality
As one quantity *increases*	As one quantity *increases*
the other quantity *increases*	the other quantity *decreases*

In order to determine what type of problem you're working on, you'll need to think critically about the quantities involved, and use clues from your experience and the context of the problem to determine how the quantities are related. Things like the previous problem -- when a group of people are working together to accomplish a specific task -- are one of the primary examples of inverse proportionality. Let's see the same example again, and this time find the correct answer.

Example 6

Suppose it takes 6 sanitation workers, all working simultaneously, 4 hours to pick up the trash and recycling in a given neighborhood. How many sanitation workers would it take to pick up the trash and recycling in the same neighborhood in 3 hours? (Assume that all sanitation workers work at the same rate, and can work independently.)

Solution

The total amount of work stays the same.

$6 \times 4 = 24$ worker-hours Total work.

$\frac{24}{3} = 8$ Divide by time.

Hence, 8 workers are needed.

This type of relationship can be written using the inverse proportionality formula.

Inverse Proportionality Formula

$y = \frac{k}{x}$ where k represents the constant of proportionality.

Divide to find y.

Example 7

A construction crew is pouring a concrete slab that must be completed in one continuous process. Based on the size of the slab, the job requires a total of 24 worker-hours to finish.

If 4 workers are assigned to the job, how long will it take to complete the pour? What if 8 workers are assigned instead?

Solution

Since y is inversely proportional to x, use the formula

$y = \frac{k}{x}$ Inverse proportion formula.

Substitute $k = 24$.

$y = \frac{24}{x}$ Substitute for k.

For $x = 4$:

$y = \frac{24}{4} = 6$ Divide.

For $x = 8$:

$y = \frac{24}{8} = 3$ Divide.

Hence, the job will take 6 hours with 4 workers and 3 hours with 8 workers.

Direct and Inverse Variation

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View this YouTube Video online

This video introduces direct and inverse variation (proportional relationships) and demonstrates how to identify each type and solve related problems using appropriate methods.

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3.1.4: Rates and Proportional Reasoning Practice

Section 3.1.1 - Introduction to Ratios and Rates

Express the given ratio as a fraction reduced to lowest terms.

1. 132 : 84

2. 12 : 28

3. 77 : 121

4. 0.74 : 0.2

5. 0.93 : 0.39

6. $3\frac{2}{3} : 2\frac{4}{9}$

7. $2\frac{4}{9} : 1\frac{1}{3}$

Find the requested rate or unit rate. Round your answer to the nearest hundredth when necessary.

8. A construction worker earns $105 for working 8.5 hours. What is the hourly pay rate, rounded to the nearest cent?

9. A technician installs wiring across a job site, covering 120 feet in 5 hours. Find the average installation rate.

10. Two irrigation pumps are being evaluated for efficiency.

    Pump A moves 202.9 gallons of water using 13.9 units of energy.
    Pump B moves 221.6 gallons of water using 11.8 units of energy.

    Which pump is more efficient?

11. Two delivery routes are being compared for fuel efficiency.

    Route A requires 10.7 gallons of fuel to travel 297.6 miles.
    Route B requires 12.6 gallons of fuel to travel 298.1 miles.

    Which route is more fuel efficient?

Section 3.1.2 - Introduction to Proportions

Determine if the proportion is a true proportion.

12. $\frac{6}{7} = \frac{18}{21}$

13. $\frac{8}{9} = \frac{3}{8}$

Solve the given proportion.

14. $\frac{16}{5} = \frac{x}{20}$

15. $\frac{4}{21} = \frac{12}{x}$

16. $\frac{3}{22} = \frac{21}{x}$

17. $\frac{7x+3}{8} = \frac{5}{4}$

Solve each problem using a proportion. Round your answer to the nearest hundredth when necessary.

18. A hardware supplier sells 13 fasteners for $1.97. What is the cost of 7 fasteners?

19. A construction crew purchases 3 bolts for $1.23. How many bolts can be purchased for $6.15?

20. A technician installs 90 feet of cable in 3 hours. How long will it take to install 240 feet at the same rate?

21. A wind energy project uses 25 turbines to supply electricity to 1,000 homes. Estimate how many homes can be powered by 40 turbines.

22. A forestry plan allows for 267 million board-feet of timber to build approximately 27,000 two-bedroom homes.

    i) Estimate how much timber is needed to build 10,000 homes.

    ii) Estimate how much timber is needed to build 15,000 homes.

Section 3.1.3 - Applications of Proportionality

Solve each problem.

23. y varies directly with x. If $y = 18$ when $x = 3$, find y when $x = 7$.

24. y varies directly with x. If $y = 32$ when $x = 4$, find x when $y = 56$.

25. y varies inversely with x. If $y = 6$ when $x = 4$, find y when $x = 12$.

26. y varies inversely with x. If $y = 8$ when $x = 6$, find x when $y = 4$.

Determine whether the relationship is direct or inverse, then solve.

27. A technician installs 90 feet of cable in 3 hours. How many feet of cable will be installed in 8 hours at the same rate?

28. A job requires 36 worker-hours to complete. How long will it take 6 workers to complete the job?

29. A crew of 8 workers can complete a task in 9 hours. How long will it take 12 workers to complete the same task?

30. A tank can be filled in 6 hours using 3 pumps. How long will it take to fill the tank using 9 pumps?

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3.2: PERCENTS

You see percents every day—20% tip at a restaurant, a 15% off sale, or a 95% on a test. But what does that number actually mean, and how does it connect to the fractions and decimals you already know? That connection is the foundation of this module.

Percents show up in a lot of real situations. You might use them to figure out a discount, track a commission, estimate a change, or make sense of a rate. Being comfortable with percents—and knowing how to move between percents, decimals, and fractions—is a practical skill you'll use often.

Before you get started, watch the video below—it's a little silly, but it just might get percents stuck in your head in the best possible way.

In this module, you'll learn to convert between percents, decimals, and fractions in any direction. Then you'll use that understanding to solve percent problems using both proportions and equations, so you have more than one way to approach them.

Percentages (Math Song)

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video is a silly song about percentages.

3.2.1: Percents, Decimals and Fractions

3.2.2: Solving Basic Percent Problems

3.2.3: Percents Practice

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3.2.1: Percents, Decimals and Fractions

In the square shown in Figure 3.2.1.1, a large square has been partitioned into ten rows of ten little squares in each row and we've shaded 20 of 100 possible little squares, or 20% of the total number of little squares.

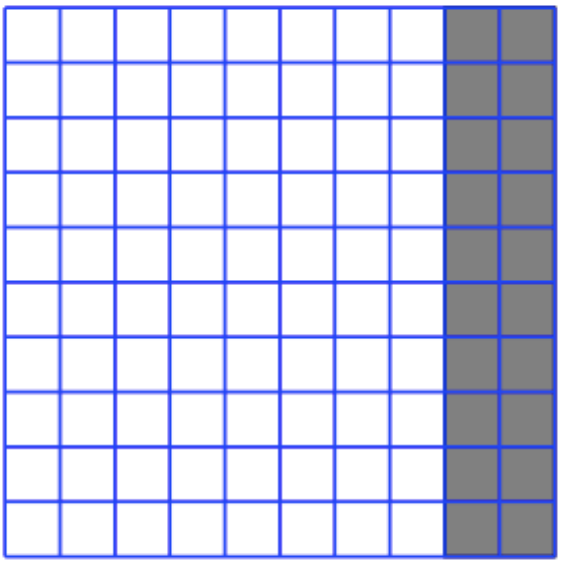


Figure 3.2.1.1: Shading 20 of 100 little squares, or 20% of the total number of little squares.

The Meaning of Percent

Percent means "parts per hundred."

In Figure 3.2.1.1, 80 out of a possible 100 squares are left unshaded. Thus, 80% of the little squares are unshaded. If instead we shaded 35 out of the 100 squares, then 35% of the little squares would be shaded. If we shaded all of the little squares, then 100% of the little squares would be shaded (100 out of 100).

So, when you hear the word "percent," think "parts per hundred."

What are Percentages?

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces percents as "per 100" and shows how to convert percents to fractions by writing them over 100 and simplifying.

Changing a Percent to a Fraction

Based on the discussion above, it is fairly straightforward to change a percent to a fraction.

Percent to Fraction

To change a percent to a fraction, drop the percent sign and put the number over 100.

Example 1

Change 24% to a fraction.

Solution

Drop the percent symbol and put 24 over 100.

$$\begin{aligned} 24\% &= \frac{24}{100} && \text{Percent: Parts per hundred.} \\ &= \frac{6}{25} && \text{Reduce.} \end{aligned}$$

Hence, 24% = 6/25.

Exercise

Change 36% to a fraction reduced to lowest terms.

Answer available at the end of this section.

Example 2

Change $14\frac{2}{7}\%$ to a fraction.

Solution

Drop the percent symbol and put $14\frac{2}{7}$ over 100.

$$\begin{aligned} 14\frac{2}{7}\% &= \frac{14\frac{2}{7}}{100} && \text{Percent: Parts per hundred.} \\ &= \frac{\frac{100}{7}}{100} && \text{Mixed to improper fraction.} \\ &= \frac{100}{7} \cdot \frac{1}{100} && \text{Invert and multiply.} \\ &= \frac{\cancel{100}}{7} \cdot \frac{1}{\cancel{100}} && \text{Cancel.} \\ &= \frac{1}{7} \end{aligned}$$

Hence, $14\frac{2}{7}\% = 1/7$.

Exercise

Change $11\frac{1}{9}\%$ to a fraction reduced to lowest terms.

Answer available at the end of this section.

Example 3

Change 28.4% to a fraction.

Solution

Drop the percent symbol and put 28.4 over 100.

$$\begin{aligned} 28.4\% &= \frac{28.4}{100} && \text{Percent: Parts per hundred.} \\ &= \frac{28.4 \cdot 10}{100 \cdot 10} && \text{Multiply numerator and denominator by 10.} \\ &= \frac{284}{1000} && \text{Multiplying by 10 moves decimal point one place right.} \\ &= \frac{71 \cdot 4}{250 \cdot 4} && \text{Factor.} \\ &= \frac{71}{250} && \text{Cancel common factor.} \end{aligned}$$

Exercise

Change 87.5% to a fraction reduced to lowest terms.

Answer available at the end of this section.

Changing a Percent to a Decimal

To change a percent to a decimal, we need only remember that percent means “parts per hundred.”

Changing a Percent to a Decimal

To change a percent to a decimal, drop the percent sign and put the number over 100. Then write the result as a decimal.

You may also see this done by moving the decimal point two places to the left, but this is just a shortcut for dividing by 100.

Example 4

Change 23.25% to a decimal.

Solution

Drop the percent symbol and put 23.25 over 100.

$$\begin{aligned} 23.25\% &= \frac{23.25}{100} && \text{Percent: Parts per hundred.} \\ &= 0.2325 && \text{Dividing by 100 moves decimal point 2 places left.} \end{aligned}$$

Therefore, 23.25% = 0.2325.

Exercise

Change 2.4% to a decimal.

Answer available at the end of this section.

Example 5

Change $5\frac{1}{2}\%$ to a decimal.

Solution

First convert the mixed number to a decimal.

$5\frac{1}{2}\% = 5.5\%$ Convert $\frac{1}{2} = 0.5$.

$= \frac{5.5}{100}$ Percent means per 100.

$= 0.055$

Thus, $5\frac{1}{2}\% = 0.055$.

Exercise

Change $6\frac{3}{4}\%$ to a decimal.

Answer available at the end of this section.

Changing a Decimal to a Percent

Changing a decimal to a percent is the opposite of changing a percent to a decimal. In that process, we drop the percent symbol and divide by 100. The following rule reverses this process.

Changing a Decimal to a Percent

To change a decimal to a percent, multiply by 100 and add a percent symbol.

You may also see this done by moving the decimal point two places to the right, but this is just a shortcut for multiplying by 100.

Example 6

Change 0.0725 to a percent.

Solution

Multiply by 100 and add a percent symbol.

$0.0725 \times 100 = 7.25$ Multiply by 100.

$7.25 = 7.25\%$

Thus, $0.0725 = 7.25\%$.

Exercise

Change to 0.0375 to a percent.

Answer available at the end of this section.

Example 7

Change 1.025 to a percent.

Solution

Multiply by 100 and add a percent symbol.

$1.025 \times 100 = 102.5 \quad \text{Multiply by } 100.$

$102.5 = 102.5\%$

Exercise

Change 0.525 to a percent.

Answer available at the end of this section.

Changing a Fraction to a Percent

One way to proceed is to first change the fraction to a decimal, then change the resulting decimal to a percent.

Fractions to Percents

To change a fraction to a percent, follow these steps:

1. Use your calculator to convert the fraction to a decimal or divide the numerator by the denominator.
2. Multiply the result by 100 and add a percent symbol.

Example 8

Change $\frac{5}{8}$ to a percent.

Solution

Change $\frac{5}{8}$ to a decimal, then change the decimal to a percent.

To change $\frac{5}{8}$ to a decimal, divide 5 by 8. Since the denominator is a product of twos, the decimal should terminate.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

To change 0.625 to a percent, multiply by 100 and add a percent symbol.

$0.625 \times 100 = 62.5$

$62.5 = 62.5\%$

Exercise

Change $\frac{5}{16}$ to a percent.

Answer available at the end of this section.

Exercise

Change $\frac{4}{9}$ to a percent.

Answer available at the end of this section.

Sometimes we will be content with an approximation.

Example 10

Change $\frac{4}{13}$ to a percent. Round your answer to the nearest tenth of a percent.

Solution

To change $\frac{4}{13}$ to a decimal, divide 4 by 13.

$\frac{4}{13} \approx 0.3077$

Since we are rounding to the nearest tenth of a percent, it is helpful to carry the decimal to four places before converting.

To change the decimal to a percent, multiply by 100 and add a percent symbol.

$0.3077 \times 100 = 30.77$

$30.77 = 30.77\%$

Now round to the nearest tenth of a percent.

$30.77\% \approx 30.8\%$

Thus, $\frac{4}{13} \approx 30.8\%$.

Exercise

Change $\frac{4}{17}$ to a percent. Round your answer to the nearest tenth of a percent.

Answer available at the end of this section.

Relating Fractions, Decimals, and Percents

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View this YouTube Video online

This video demonstrates three examples of how to convert decimals to percents and fractions.

Answers

Exercise
9/25

Exercise
1/9

Exercise
7/8

Exercise
0.024

Exercise
0.0675

Exercise
3.75%

Exercise
52.5%

Exercise
31.35%

Exercise
$44\frac{4}{9}\%$

Exercise
23.5%

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3.2.2: Solving Basic Percent Problems

In this section, we will solve percent problems using a percent proportion as the primary method and also demonstrate how a percent equation can be used. Success with either approach depends on using keywords to translate the problem into a proportion or an equation.

We begin by solving these problems using a percent proportion.

Write Percent Problems As Proportions

The proportion method for solving percent problems involves a percent proportion. A **percent proportion** is an equation where a percent is equal to an equivalent ratio.

For example, 60% = $\frac{60}{100}$ and we can simplify $\frac{60}{100} = \frac{3}{5}$. Since the equation $\frac{60}{100} = \frac{3}{5}$ shows a percent equal to an equivalent ratio, we call it a **percent proportion**.

Definition: Percent Proportion

The amount is to the base as the percent is to 100.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

Now that we understand the percent proportion, we need a way to identify the part and the whole in percent problems.

Use Keywords to Write a Percent Proportions

Many times, keywords can help identify the part and the whole in a percent problem.

The value near “of” is usually the whole.
The value near “is” is usually the part.

If either value is written as “what number,” let xxx represent the unknown value.

Example 1

Translate to a proportion. What number is 75% of 90?

Solution

Use keywords to identify the part and the whole.

The value near “of” is the whole: 90
The value near “is” is the part: n

The percent is 75%.

Set up the percent proportion.

$\frac{n}{90} = \frac{75}{100}$

Exercise

Translate to a proportion: What number is 60% of 105?

Answer available at the end of this section.

Exercise

Translate to a proportion: What number is 40% of 85?

Answer available at the end of this section.

Example 2

Translate to a proportion. 19 is 25% of what number?

Solution

Use keywords to identify the part and the whole.

The value near “is” is the part: 19
The value near “of” is the whole: x

The percent is 25%.

Set up the percent proportion.

$$\frac{19}{x}=\frac{25}{100}$$

Exercise

Translate to a proportion: 36 is 25% of what number?

Answer available at the end of this section.

Exercise

Translate to a proportion: 27 is 36% of what number?

Answer available at the end of this section.

Example 3

Translate to a proportion. What percent of 27 is 9?

Solution

Use keywords to identify the part and the whole.

The value near “of” is the whole: 27
The value near “is” is the part: 9

The percent is unknown: $x\%$

Set up the percent proportion.

$$\frac{9}{27} = \frac{x}{100}$$

Exercise

Translate to a proportion: What percent of 52 is 39?

Answer available at the end of this section.

Exercise

Translate to a proportion: What percent of 92 is 23?

Answer available at the end of this section.

Translate and Solve Percent Proportions

Now that we have written percent equations as proportions, we are ready to solve the proportions.

Example 4

Translate and solve using proportions: What number is 45% of 80?

Solution

Use keywords to identify the part and the whole.

The value near “of” is the whole: 80
The value near “is” is the part: n

The percent is 45%.

Set up the percent proportion.

$$\frac{n}{80} = \frac{45}{100}$$

Solve the proportion.

$100n = 80 \cdot 45$

$100n = 3600$

$n = 36$

Exercise

Translate and solve using proportions: What number is 65% of 40?

Answer available at the end of this section.

Exercise

Translate and solve using proportions: What number is 85% of 40?

Answer available at the end of this section.

Solve a Percent Problem Using a Percent Proportion (Example 1)

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to use a percent proportion to solve a percent problem with an unknown part.

In the next example, the percent is more than 100, which is more than one whole. So the unknown number will be more than the base.

Example 5

Translate and solve using proportions: 125% of 25 is what number?

Solution

Use keywords to identify the part and the whole.

The value near “of” is the whole: 80
The value near “is” is the part: n

The percent is 45%.

Set up the percent proportion.

$$\frac{n}{80} = \frac{45}{100}$$

Solve the proportion.

$$100n = 80 \cdot 45$$

$$100n = 3600$$

$$n = 36$$

Exercise

Translate and solve using proportions: 125% of 64 is what number?

Answer available at the end of this section.

Exercise

Translate and solve using proportions: 175% of 84 is what number?

Answer available at the end of this section.

Percents with decimals and money are also used in proportions.

Example 6

Translate and solve: 6.5% of what number is $1.56?

Solution

Use keywords to identify the part and the whole.

The value near “is” is the part: 1.56
The value near “of” is the whole: x

The percent is 6.5%.

Set up the percent proportion.

$$\frac{1.56}{x} = \frac{6.5}{100}$$

Solve the proportion.

$$100(1.56) = 6.5x$$

$$156 = 6.5x$$

$$x = 24$$

Exercise

Translate and solve using proportions: 8.5% of what number is $3.23?

Answer available at the end of this section.

Exercise

Translate and solve using proportions: 7.25% of what number is $4.64?

Answer available at the end of this section.

Solve a Percent Problem Using a Percent Proportion (Example 2)

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to use a percent proportion to solve a percent problem with an unknown whole.

Example 7

Translate and solve using proportions: What percent of 72 is 9?

Solution

Use keywords to identify the part and the whole.

The value near “of” is the whole: 72
The value near “is” is the part: 9

The percent is unknown: $x\%$

Set up the percent proportion.

$$\frac{9}{72} = \frac{x}{100}$$

Solve the proportion.

$$72x = 900$$

$$x = 12.5$$

Check.

9 is a little more than $\frac{1}{8}$ of 72, and 12.5% is a little more than $\frac{1}{8}$ of 100, so the answer is reasonable.

Thus, 9 is 12.5% of 72.

Exercise

Translate and solve using proportions: What percent of 72 is 27?

Answer available at the end of this section.

Exercise

Translate and solve using proportions: What percent of 92 is 23?

Answer available at the end of this section.

Solve Percent Problems Using Percent Equations

In addition to using proportions, percent problems can be solved using equations. This method uses keywords to translate the problem directly into an equation, which can then be solved using algebra.

Determind What Percent One Number is of Another Using a Percent Proportion (Example 3)

Video text/ introduction.

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to use a percent proportion to solve a percent problem with an unknown percent.

Use Keyword to Write Percent Equations

Use keywords to translate a percent problem into an equation.

“is” means equals
“of” means multiply
“what” represents the unknown x

Write the percent as a decimal before solving the equation.

The following examples revisit earlier problems using the equation method.

Example 8

Translate and solve using an equation: What number is 45% of 80?

Solution

Use keywords to write an equation.

“is” means equals
“of” means multiply
“what number” represents x

Rewrite the percent as a decimal.

$45\% = 0.45$

Translate the problem into an equation.

What number is 45% of 80?

$\text{what number} \rightarrow x \qquad \text{is} \rightarrow = \qquad 45\% \text{ of } 80 \rightarrow 0.45 \cdot 80$

$x = 0.45 \cdot 80$

Solve by multiplying.

$x = 36$

Example 9

Translate and solve using an equation: What percent of 72 is 9?

Solution

Use keywords to write an equation.

“is” means equals
“of” means multiply
“what percent” represents x

Since the percent is unknown, there is no need to convert to a decimal. Instead, convert the final answer to a percent.

Translate the problem into an equation.

What percent of 72 is 9?

$$\text{what percent} \to x \qquad \text{of } 72 \to \cdot 72 \qquad \text{is } 9 \to = 9$$

$$x \cdot 72 = 9$$

Solve by dividing.

$$x = \frac{9}{72}$$

$$x = 0.125$$

Convert to a percent.

$$0.125 = 12.5\%$$

Percent Questions Using a Percent Equation

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to solve percent problems using a percent equation through four examples.

Answers

Exercise

$\frac{n}{105} = \frac{60}{100}$

Exercise

$\frac{n}{85} = \frac{40}{100}$

Exercise

$\frac{36}{n} = \frac{25}{100}$

Exercise

$\frac{27}{n} = \frac{36}{100}$

Exercise

$\frac{n}{100} = \frac{39}{52}$

Exercise

$\frac{n}{100} = \frac{23}{92}$

Exercise

26

Exercise

34

Exercise

80

Exercise

147

Exercise

38

Exercise

64

Exercise

37.5%

Exercise

25%

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3.2.3: Percents Practice

The following set of exercises provides practice for the concepts covered in Module 3.2: Percents. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 3.2.1 - Fractions, Decimals and Percents

Convert the given percent to a fraction, and simplify the result.

1. 15.2%
2. 5%
3. 177%
4. 42.5%
5. $4\frac{9}{10}\%$

Convert the given percent to a decimal.

6. 4%
7. 5.4%
8. 0.3344%
9. 125%

Convert the given decimal to a percent.

10. 0.29
11. 3.372
12. 0.064
13. 0.0075

Convert the given fraction to a percent. Round the answer to the nearest tenth of a percent, if necessary.

14. $\frac{4}{5}$
15. $\frac{7}{20}$
16. $\frac{7}{8}$
17. $\frac{5}{6}$
18. $\frac{3}{2}$
19. $\frac{10}{7}$

Section 3.2.2 - Solving Percent Problems

Translate and solve using a percent proportion or a percent equation.

20. What number is 10% of 70?

21. 9 is what percent of 50?

22. 25% of what number is 40?

23. 7.7 is what percent of 35?

24. What number is 159.2% of 125?

25. 200% of what number is 208?

26. 132% of what number is 198?

27. What number is 150% of 66?

28. 187.5% of what number is 90?

29. $77\frac{1}{2}\%$ of what number is 6.2?

30. $121\frac{2}{3}\%$ of what number is 73?

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3.3: APPLICATIONS OF PERCENT

A few years ago, a dozen eggs cost about a dollar fifty. Today, that same carton might run you four dollars or more. That kind of change has a name—inflation—and it's measured using percents. Whether prices are going up or down, wages are changing, or items are marked up or discounted, percent is the language we use to describe that change.

You'll see this kind of math in a lot of real situations. A small business owner tracking profits, a buyer comparing material costs, or a worker deciding if a raise actually keeps up with expenses—all of them are thinking in terms of percent change.

Before you get started, watch the video below for a look at inflation in action. It gives you a real-world example of the kind of change you'll be working with in this module.

In this module, you'll solve real-world problems involving percent relationships and connect the math to situations you might actually encounter. You'll also learn how to calculate percent increase and percent decrease so you can describe and interpret change clearly—whether values are going up, going down, or changing more than once.

How the Consumer-Price Index Measures Inflation

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces inflation and shows how prices change over time using percent increase.

3.3.1: Applications of Basic Percent Problems

3.3.2: Percent Increase and Decrease

3.3.3: Applications of Percent Practice

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3.3.1: Applications of Basic Percent Problems

In this section, you will continue working with percent problems using percent proportions. However, unlike the previous section, these problems involve real-world applications related to technical fields. The first and most important step is carefully reading the problem to identify the part, whole, and percent. This will not always be as straightforward as it was in the previous section, so careful interpretation is essential.

Steps for Solving Basic Percent Problems

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.
2. Set up a percent proportion: $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$
3. Solve the proportion.
4. Write your answer as a complete sentence with appropriate units.

Finding the Part

In the following examples, the percent and whole are given, and you will find the part.

Example 1

A hydraulic system requires fluid that is 22% additive. If a tank contains 3,450 milliliters of fluid, how many milliliters are additive?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Percent} = 22\%$

$\text{Whole} = 3450 \text{ mL}$

$\text{Part} = x$

Notice that the total amount of fluid in the tank is given, so this represents the whole.

2. Set up a percent proportion.

$\frac{x}{3450} = \frac{22}{100}$

3. Solve the proportion.

$100x = 3450 \cdot 22$

$100x = 75900$

$x = 759$

4. Write your answer as a complete sentence with appropriate units.

There are 759 milliliters of additive in the tank.

Exercise

A pallet of concrete blocks contains 480 blocks. If 15% of the blocks are damaged during delivery, how many blocks are damaged?

Answer available at the end of this section.

Exercise

A fabrication shop is running at increased capacity to meet demand. During one shift, the shop produced 135% of its standard output of 240 parts. How many parts were produced during the shift?

Answer available at the end of this section.

Example 2

A diesel technician is servicing a cooling system after replacing a radiator hose on a piece of heavy equipment. The service manual states that the cooling system should be filled with a mixture that is 40% antifreeze. After draining and flushing the system, the technician confirms that the system holds 6.8 liters total. How many liters of antifreeze are needed to prepare the correct mixture?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Percent} = 40\%$

$\text{Whole} = 6.8 \text{ L}$

$\text{Part} = x$

The total capacity of the cooling system is 6.8 liters, so that is the whole.

2. Set up a percent proportion.

$\frac{x}{6.8} = \frac{40}{100}$

3. Solve the proportion.

$100x = 6.8 \cdot 40$

$100x = 272$

$x = 2.72$

4. Write your answer as a complete sentence with appropriate units.

The technician needs 2.72 liters of antifreeze to prepare the correct mixture.

Exercise

A production supervisor at a plastics manufacturing plant is reviewing quality control data from a recent shift. During the shift, 2,400 parts were produced on an injection molding machine. The quality report shows that 6.5% of the parts did not meet specifications and were rejected.

How many parts were rejected during the shift?

Answer available at the end of this section.

Finding the Whole

In the following examples, the part and percent are given, and you will find the whole.

Example 3

A technician determines that 36 meters of wire have been installed, which represents 12% of the total wiring needed for a project. How many meters of wire are needed in total?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Part} = 36 \text{ m}$

$\text{Percent} = 12\%$

$\text{Whole} = x$

The amount of wire installed is part of the total, so it represents the part.

2. Set up a percent proportion.

$\frac{36}{x} = \frac{12}{100}$

3. Solve the proportion.

$12x = 36 \cdot 100$

$12x = 3600$

$x = 300$

4. Write your answer as a complete sentence with appropriate units.

There are 300 meters of wire needed in total.

Exercise

A clinic reports that 58 patients missed their appointments in one week. This represents 4% of all scheduled appointments. How many appointments were scheduled that week?

Answer available at the end of this section.

Exercise

A training coordinator reports that 45 employees have completed a required certification, which represents 15% of all employees who need the training. How many employees need the training in total?

Answer

Answer available at the end of this section.

Example 4

A fabrication shop reports that it produced 324 parts during a shift. This represents 135% of its standard production level. What is the standard number of parts produced in a shift?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Part} = 324$

$\text{Percent} = 135\%$

$\text{Whole} = x$

Notice that the number of parts produced is greater than the standard amount, so the percent is greater than 100%.

2. Set up a percent proportion.

$\frac{324}{x} = \frac{135}{100}$

3. Solve the proportion.

$135x = 324 \cdot 100$

$135x = 32400$

$x = 240$

4. Write your answer as a complete sentence with appropriate units.

The standard number of parts produced in a shift is 240 parts.

Exercise

A machine is rated to produce 520 units during a standard shift. On a busy day, the machine produced 702 units. What percent of the standard production was achieved?

Answer available at the end of this section.

Example 5

A construction project manager is reviewing progress on a large commercial building. By the end of the week, the crew has completed 864 square feet of roofing. According to the project schedule, this amount represents 18% of the total roofing work required for the building. What is the total amount of roofing, in square feet, required for the project?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Part} = 864 \text{ ft}^2$

$\text{Percent} = 18\%$

$\text{Whole} = x$

The completed roofing is part of the total roof area, so it represents the part.

2. Set up a percent proportion.

$\frac{864}{x} = \frac{18}{100}$

3. Solve the proportion.

$18x = 864 \cdot 100$

$18x = 86400$

$x = 4800$

4. Write your answer as a complete sentence with appropriate units.

The total roof area is 4800 square feet.

Exercise

A service manager is reviewing repair data for a fleet of vehicles. During the past month, 54 vehicles required brake service, which represents 9% of the total number of vehicles in the fleet. The manager needs to know the total fleet size to schedule future maintenance. How many vehicles are in the fleet?

Answer available at the end of this section.

Find the Percent

In the following examples, the part and whole are given, and you will find the percent.

Example 6

A machining process produces 250 metal components during a shift. Out of these, 20 components required additional finishing before they could be approved for use. What percent of the components required additional finishing?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Part} = 20$

$\text{Whole} = 250$

$\text{Percent} = x$

The components requiring additional finishing are part of the total production.

2. Set up a percent proportion.

$\frac{20}{250} = \frac{x}{100}$

3. Solve the proportion.

$250x = 20 \cdot 100$

$250x = 2000$

$x = 8$

4. Write your answer as a complete sentence with appropriate units.

The percent of components requiring additional finishing is 8%.

Exercise

A farmer harvested 210 bushels of corn from a field that is expected to produce 600 bushels. What percent of the expected yield has been harvested?

Answer available at the end of this section.

Exercise

A delivery company typically completes 75 deliveries per day. On a busy day, the company completed 96 deliveries. What percent of the typical daily deliveries were completed?

Answer available at the end of this section.

Example 7

An electrician is installing lighting fixtures in a commercial building. The project requires a total of 320 fixtures to be installed. By the end of the day, 68 fixtures have been installed and tested. The electrician needs to report progress as a percentage of completion. What percent of the lighting fixtures have been installed?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Part} = 68$

$\text{Whole} = 320$

$\text{Percent} = x$

The number of fixtures installed is part of the total required.

2. Set up a percent proportion.

$\frac{68}{320} = \frac{x}{100}$

3. Solve the proportion.

$320x = 68 \cdot 100$

$320x = 6800$

$x = 21.25$

4. Write your answer as a complete sentence with appropriate units.

The percent of lighting fixtures installed is 21.25%.

Exercise

A plumber tests a newly installed water system to check for leaks. Out of 480 total pipe connections, 84 connections passed the pressure test on the first inspection. What percent of the connections passed the test?

Answer available at the end of this section.

Example 8

A technician is preparing a coolant mixture for a vehicle after replacing a radiator. The service manual requires a mixture that includes water and coolant, and the technician needs to verify the composition before filling the system. The technician measures and mixes 14 milliliters of concentrated coolant with 36 milliliters of water in a graduated cylinder. What percent of the resulting mixture is water?

Solution

1. Carefully read the problem and identify the part, whole, and percent in the problem. Let x represent the unknown part, whole, or percent.

$\text{Water} = 36 \text{ mL}$

$\text{Coolant} = 14 \text{ mL}$

$\text{Total mixture} = 36 + 14 = 50 \text{ mL}$

$\text{Part} = 36$

$\text{Whole} = 50$

$\text{Percent} = x$

The total mixture is the whole, and the amount of water is the part.

2. Set up a percent proportion.

$\frac{36}{50} = \frac{x}{100}$

3. Solve the proportion.

$50x = 36 \cdot 100$

$50x = 3600$

$x = 72$

4. Write your answer as a complete sentence with appropriate units.

72% of the resulting mixture is water.

Answers

Exercise

There are 72 damaged blocks.

Exercise

The shop produced 324 parts during the shift.

Exercise

There were 156 rejected parts during the shift.

Exercise

There were 1450 appointments scheduled that week.

Exercise

There are 300 employees who need the training in total.

Exercise

The machine produced 135% of the standard production.

Exercise

There are 600 vehicles in the fleet.

Exercise

35% of the expected yield has been harvested.

Exercise

The company completed 128% of the typical daily deliveries.

Exercise

17.5% of the pipe has been installed.

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3.3.2: Percent Increase and Decrease

In this section, you will calculate percent increase and percent decrease. These are used to describe how a value changes compared to where it started.

The most important idea is that percent change is always based on the original amount, not the new amount. Because of this, percent increase and percent decrease are not the same in reverse. If a value increases by a certain percent and then decreases by that same percent, it will not return to the original value.

In the following examples, we will use proportions to calculate percent change and apply these ideas to real-world situations.

Calculating Percent Change

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces percent increase and percent decrease and shows how each is calculated using the original amount.

Percent Increase

In the following examples, we will look at percent increase. In technical fields, this might be used to describe increases in production, changes in measurements like pressure or flow rate, or the number of units completed during a shift.

Percent Increase Formula

To find the percent increase of two values, we use the following formula.

$$\frac{\text{new} - \text{original}}{\text{original}} = \frac{\text{percent increase}}{100}$$

Example 1

An HVAC system moved 160 cubic feet of air per minute during one test and 200 cubic feet of air per minute during a later test. What is the percent increase?

Solution

1. Identify the original and new amounts. Let x represent the percent increase.

$\text{Original} = 160$

$\text{New} = 200$

2. Use the percent increase formula.

$\frac{200-160}{160} = \frac{x}{100}$

$\frac{40}{160} = \frac{x}{100}$

3. Solve the proportion.

$160x = 40 \cdot 100$

$160x = 4000$

$x = 25$

4. Write your answer.

The airflow increased by 25%.

Exercise

An automated sorting system in a distribution center separates packages by destination. During one shift, the system sorted 85 packages, and during the next shift, it sorted 110 packages. What is the percent increase?

Answer available at the end of this section.

Example 2

A greenhouse is tracking plant growth during a controlled growing cycle. At the start of the week, a sample plant measured 18 centimeters in height. After adjustments to light exposure and nutrient levels, the plant measured 24 centimeters at the end of the week. What is the percent increase in the height of the plant? Round to the nearest tenth of a percent.

Solution

1. Identify the original and new amounts. Let x represent the percent increase.

$\text{Original} = 18 \text{ cm}$

$\text{New} = 24 \text{ cm}$

2. Use the percent increase formula.

$\frac{24-18}{18} = \frac{x}{100}$

$\frac{6}{18} = \frac{x}{100}$

3. Solve the proportion.

$18x = 6 \cdot 100$

$18x = 600$

$x \approx 33.3$

4. Write your answer.

The height of the plant increased by approximately 33.3%.

Exercise

A production line completed 27 parts during one shift. After adjustments were made to improve efficiency, the line completed 35 parts during the next shift. What is the percent increase? Round to the nearest tenth of a percent.

Answer available at the end of this section.

Example 3

A crew installs 240 feet of piping during a shift. The amount installed increases by 15%. How many additional feet are installed?

Solution

1. Identify the original amount and the percent increase. Let x represent the amount of increase.

$\text{Original} = 240$

$\text{Percent increase} = 15\%$

2. Use the percent increase formula.

$\frac{x}{240} = \frac{15}{100}$

3. Solve the proportion.

$100x = 240 \cdot 15$

$100x = 3600$

$x = 36$

4. Write your answer.

36 additional feet are installed.

Exercise

A facility uses 180 kilowatt-hours of energy during one period. Usage increases by 12%. How many additional kilowatt-hours are used?

Answer available at the end of this section.

Finding the New Value After a Percent Increase

To find the new value after a percent increase, we use the following formula:

$$\frac{\text{original} - \text{new}}{\text{original}} = \frac{100 + \text{percent increase}}{100}$$

Example 4

A farm harvests 160 bushels of produce in one day. The harvest increases by 25%. How many bushels are harvested after the increase?

Solution

1. Identify the original amount and the percent change. Let x represent the new amount.

$\text{Original} = 160$

$\text{Percent increase} = 25\%$

2. Use the percent increase formula.

$\frac{x}{160} = \frac{100+25}{100}$

$\frac{x}{160} = \frac{125}{100}$

3. Solve the proportion.

$100x = 160 \cdot 125$

$100x = 20000$

$x = 200$

4. Write your answer.

The farm harvested 200 bushels after the increase.

Exercise

A web server handles 220 user requests per minute. After a 15% increase in traffic, how many requests are handled?

Answer available at the end of this section.

Example 5

A machine produces 400 units during a shift. Due to increased demand, production increases by 25%. The following day, production increases again by 25%. How many units are produced after the second increase?

Solution

1. Identify the original amount and the percent increase. Let x represent the new amount after each step.

$\text{Original} = 400$

$\text{Percent increase} = 25\%$

2. First increase.

$\frac{x}{400} = \frac{100+25}{100}$

$\frac{x}{400} = \frac{125}{100}$

$100x = 400 \cdot 125$

$100x = 50000$

$x = 500$

After the first increase, the machine produces 500 units.

3. Second increase.

$\frac{x}{500} = \frac{125}{100}$

$100x = 500 \cdot 125$

$100x = 62500$

$x = 625$

4. Write your answer.

The machine produces 625 units after the second increase.

Note: A 50% increase from 400 units would result in 600 units. The final value of 625 units is larger because each 25% increase is applied to a new, larger amount. Percent increases are not additive.

Exercise

A data processing system handles 320 tasks per minute. Due to increased demand, the number of tasks processed increases by 15% and then increases again by 15%. How many tasks per minute are processed after the second increase?

Answer available at the end of this section.

Percent Decrease

In the following examples, we will look at percent decrease. In technical fields, this might be used to describe a decrease in cycle time on a machine, a reduction in material waste, or fewer service calls after equipment adjustments.

Percent Decrease Formula

To find the percent decrease of two values, we use the following formula:

$$\frac{\text{original} - \text{new}}{\text{original}} = \frac{\text{percent decrease}}{100}$$

Note: When finding the change, subtract the smaller value from the larger value to keep the change positive.

Example 6

A storage tank held 180 gallons of water. After use, the tank now holds 135 gallons. What is the percent decrease?

Solution

1. Identify the original and new amounts. Let x represent the percent decrease.

$\text{Original} = 180$

$\text{New} = 135$

2. Use the percent decrease formula.

$\frac{180-135}{180} = \frac{x}{100}$

$\frac{45}{180} = \frac{x}{100}$

3. Solve the proportion.

$180x = 45 \cdot 100$

$180x = 4500$

$x = 25$

4. Write your answer.

The amount of water decreased by 25%.

Exercise

An air compressor's pressure decreased from 160 psi to 124 psi. What is the percent decrease?

Answer available at the end of this section.

Example 7

A warehouse tracked the number of damaged items after updating its packaging process. Before the change, 48 items were damaged during shipping. After the update, only 31 items were damaged. What is the percent decrease in the number of damaged items? Round to the nearest tenth of a percent.

Solution

1. Identify the original and new amounts. Let x represent the percent decrease.

$\text{Original} = 48$

$\text{New} = 31$

2. Use the percent decrease formula.

$\frac{48-31}{48} = \frac{x}{100}$

$\frac{17}{48} = \frac{x}{100}$

3. Solve the proportion.

$48x = 17 \cdot 100$

$48x = 1700$

$x \approx 35.4$

4. Write your answer.

The number of damaged items decreased by approximately 35.4%.

Exercise

A lab tracked the number of samples processed per hour. The rate decreased from 95 samples per hour to 68 samples per hour. What is the percent decrease? Round to the nearest tenth of a percent.

Answer available at the end of this section.

Example 8

A welding shop completes 300 welds during a shift. The number of welds completed decreases by 18%. How many fewer welds are completed?

Solution

1. Identify the original amount and the percent decrease. Let x represent the amount of decrease.

$\text{Original} = 300$

$\text{Percent decrease} = 18\%$

2. Use the percent decrease formula.

$\frac{x}{300} = \frac{18}{100}$

3. Solve the proportion.

$100x = 300 \cdot 18$

$100x = 5400$

$x = 54$

4. Write your answer.

The numbre of welds decreases by 54.

Exercise

A machinist produces 250 parts during a shift. The number of parts produced decreases by 16%. How many fewer parts are produced?

Answer available at the end of this section.

Finding the New Value After a Percent Decrease

To find the new value after a percent decrease, we use the following formula:

$$\frac{\text{original} - \text{new}}{\text{original}} = \frac{100 \text{ - percent decrease}}{100}$$

Example 9

An automotive shop completes 200 vehicle inspections during a day. The number of inspections decreases by 18%. How many inspections are completed after the decrease?

Solution

1. Identify the original amount and the percent change. Let x represent the new amount.

$\text{Original} = 200$

$\text{Percent decrease} = 18\%$

2. Use the percent decrease formula.

$\frac{x}{200} = \frac{100-18}{100}$

$\frac{x}{200} = \frac{82}{100}$

3. Solve the proportion.

$100x = 200 \cdot 82$

$100x = 16400$

$x = 164$

4. Write your answer.

The shop completes 164 inspections after the decrease.

Exercise

A lumber yard has 350 boards in inventory. After discarding defective boards, the inventory decreases by 12%. How many boards remain?

Answer available at the end of this section.

Example 10

A roofing crew installs 500 shingles during one shift. Due to maintenance delays, the number of shingles installed decreases by 20%. The following day, it decreases again by 20%. How many shingles are installed after the second decrease?

Solution

1. Identify the original amount and the percent decrease. Let x represent the new amount after each step.

$\text{Original} = 500$

$\text{Percent decrease} = 20\%$

2. First decrease.

$\frac{x}{500} = \frac{100-20}{100}$

$\frac{x}{500} = \frac{80}{100}$

$100x = 500 \cdot 80$

$100x = 40000$

$x = 400$

After the first decrease, 400 shingles are installed.

3. Second decrease.

$\frac{x}{400} = \frac{80}{100}$

$100x = 400 \cdot 80$

$100x = 32000$

$x = 320$

4. Write your answer.

320 shingles are installed after the second decrease.

Note: A 40% decrease from 500 shingles would result in 300 shingles. The final value of 320 shingles is larger because each 20% decrease is applied to a new, smaller amount. Percent decreases are not additive.

Exercise

A wastewater treatment facility processes 600 gallons of water per minute. Due to two separate adjustments, the flow rate decreases by 15% and then decreases again by 10%. How many gallons per minute are processed after the second decrease?

Answer available at the end of this section.

Applying Percent Increase and Decrease Together

In the following examples, you will calculate both percent increase and percent decrease for the same situation. Notice that the original amount is different in each calculation, which leads to different percent values.

Example 11

An office tracks the number of documents processed during a workday. The number increases from 200 documents to 260 documents and then decreases back to 200 documents. What is the percent increase and the percent decrease? Round to the nearest tenth of a percent.

Solution

Increase:

$\frac{260-200}{200} = \frac{x}{100}$

$\frac{60}{200} = \frac{x}{100}$

$200x = 60 \cdot 100$

$200x = 6000$

$x = 30$

Percent increase is 30%.

Decrease:

$\frac{260-200}{260} = \frac{x}{100}$

$\frac{60}{260} = \frac{x}{100}$

$260x = 60 \cdot 100$

$260x = 6000$

$x \approx 23.1$

Percent decrease is approximately 23.1%.

These values are not the same because percent change is always based on the original amount.

Exercise

A sheet metal shop tracks the number of panels formed per hour. The number increases from 150 panels per hour to 210 panels per hour and then decreases back to 150 panels per hour. Find the percent increase and the percent decrease. Round to the nearest tenth of a percent.

Answer available at the end of this section.

Answers

Exercise
The number of packages increased by approximately 29.4%.

Exercise
The output increased by approximately 29.6%.

Exercise
21.6 kilowatt-hours are used.

Exercise
The server handles 253 requests per minute after the increase.

Exercise
The system produces 423.2 tasks after the second increase.

Exercise
The pressure decreased by 22.5%.

Exercise
The rate decreased by approximately 28.4%.

Exercise
The number of parts produced decreases by 40.

Exercise
308 boards remain.

Exercise
459 gallons per minute are processed after the second decrease.

Exercise
The increase is 40%. The decrease is approximately 28.6%.

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3.3.3: Applications of Percent Practice

The following set of exercises provides practice for the concepts covered in Module 3.3: Applications of Percent. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 3.3.1 - Applications of Basic Percent Problems

Solve each problem. Write a complete sentence and include units when appropriate.

1. During a busy shipping day, a warehouse prepares 600 boxes for delivery. If 25% are loaded onto trucks in the morning, how many boxes are shipped?

2. A maintenance team is working through a checklist of tasks. By midday, they have completed 48 tasks, which is 20% of the total. How many tasks are on the full checklist?

3. At an automotive shop, 90 vehicles are inspected during a safety check. Of these, 18 need repairs before being approved. What percent of the vehicles need repairs?

4. A land survey shows that 12 acres are currently being used for equipment storage, which is 30% of the property. What is the total size of the property?

5. In a school computer lab, 20 out of 80 computers are flagged for software updates before the next semester. What percent of the computers need updates?

6. During a routine system inspection, a technician completes 36 checks, which represents 12% of the full inspection process. How many checks are required in total?

7. A manufacturing machine is pushed beyond its usual capacity and produces 324 parts in one shift, which is 135% of its standard output. What is the machine's standard output?

8. A technician prepares a cooling solution by mixing 14 mL of coolant with 36 mL of water. What percent of the mixture is water?

9. On a machining line, 1280 components are produced in one shift. Quality control reports that 7.5% require additional polishing before moving forward. How many components require polishing?

10. During a facility audit, 42 machines are found to be offline, representing 3.5% of all machines in the plant. How many machines are in the facility?

11. A storage tank used in daily operations currently holds 37.5 gallons of liquid, which is 62.5% of its full capacity. What is the tank's total capacity?

12. A flooring crew is working on a large installation project. So far, they have installed 2150 square feet, which represents 43% of the total job. What is the total square footage?

13. An electronics technician tests 384 connections on a control panel. Of these, 29 fail inspection and must be repaired. What percent of the connections failed? Round to the nearest tenth of a percent.

Section 3.3.2 - Percent Increase and Decrease

Solve each problem. Round to the nearest tenth of a percent when necessary.

14. In one hour, 140 packages are processed at a distribution center. The next hour, output rises to 175 packages. What is the percent increase?

15. A clinic records 120 patient visits in a day. The following day, visits decrease by 15%. How many visits occur after the decrease?

16. Under normal conditions, a web server handles 300 requests per minute. During peak traffic, this increases by 20%. How many additional requests are handled each minute?

17. A lab processes 90 samples per hour, but after a system issue, output drops to 72 samples per hour. What is the percent decrease?

18. A delivery service typically completes 250 deliveries per day. On a busier day, deliveries increase by 12%. How many deliveries are completed?

19. Enrollment in a training program starts at 200 participants and later decreases by 18%. How many fewer participants are enrolled?

20. A data center processes 400 tasks per minute. During a surge, the workload increases by 10% and then increases again by another 10%. How many tasks per minute are processed after the second increase?

21. During one hour, a small store sells 180 items. The next hour, sales drop to 135 items. What is the percent decrease?

22. A recycling center processes 500 pounds of material in a day. Due to slower intake, the amount decreases by 8%. How many pounds are processed after the decrease?

23. A print shop produces 220 flyers in one hour. After upgrading equipment, production increases by 25%. How many flyers are produced after the increase?

24. After improvements are made, a maintenance team reduces system errors from 75 to 54. What is the percent decrease?

25. Output at a fabrication shop increases from 150 units to 210 units to meet demand, then returns to 150 units the next day. Find the percent increase and the percent decrease.

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3.4: FINANCIAL APPLICATIONS

Most people learn financial math the hard way — a total at checkout that is higher than expected, a paycheck that does not go as far as planned, or a “good deal” that turns out to be less of a deal once you look at the numbers. Understanding how these calculations work is not just useful; it helps you make better decisions.

These ideas show up in everyday situations. A worker calculating commission, someone setting prices with markup, or a shopper comparing sale prices are all using the same core skills — whether they think of it as math or not.

Before you get started, watch the video below for a broad introduction to how money works in the real world. It covers a range of topics — taxes, interest, credit scores, inflation, and more. Not all of these will be used directly in this module, but the overview helps show where this math fits and why it matters.

In this module, you will work with sales tax and markup, then move into discounts and sale prices so you can evaluate costs more accurately. You will also solve problems involving wages — including hourly pay, overtime, and commission — and finish by building and evaluating a simple personal budget based on income and expenses.

Explaining Basic Financial Concepts YOU Should Understand

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video covers a range of topics related to financial math including taxes, interest, credit scores, inflation, and more.

3.4.1: Sales Tax and Markup

3.4.2: Discounts and Markdowns

3.4.3: Wages and Overtime

3.4.4: Personal Budgeting

3.4.5: Financial Applications Practice

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3.4.1: Sales Tax and Markup

In many real-world situations, a value increases by a certain percent from its original amount. You have already learned how to calculate percent increase and how to find a new value after an increase. In this section, you will apply those same ideas to two common financial situations, sales tax and markup, where the prices or costs are increased by a percent.

Using Percent Increase in Financial Situations

A percent increase is applied to an original amount.

Use these relationships when working with the amount of increase, the original amount, and the rate:

$$\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{rate}}{100}$$

$$\text{original amount} + \text{amount of increase} = \text{new amount}$$

Use this relationship when working with the new amount, the original amount, and the rate:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 + \text{rate}}{100}$$

Solve Sales Tax Applications

Sales tax is a common example of a percent increase. Instead of finding how much a value increased, we are adding a percent of the purchase price (original amount) to find the total cost (new amount).

When you buy something, sales tax is calculated as a percent of the purchase price. This gives the sales tax (amount of increase), which is then added to the purchase price to find the total cost.

You can use a proportion, like you did in the previous lesson, to find the sales tax (amount of increase) or the total cost (new amount).

Connection: You can also find the sales tax by multiplying the purchase price by the decimal form of the tax rate. Both methods give the same result.

Example 1

A repair shop purchased a piece of equipment for $\$392$ in Washington, where the sales tax rate was 6.5% of the purchase price. What was (a) the sales tax and (b) the total cost of the equipment?

Solution

(a) Let x represent the amount of increase (sales tax).

Identify the parts:
- original amount (purchase price) $= 392$
- rate $= 6.5\%$

Use the relationship:

$$\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.1.1}$$

Set up a proportion.

$$\frac{x}{392} = \frac{6.5}{100} \tag{3.4.1.2}$$

Cross multiply.

$$100x = 392(6.5) \tag{3.4.1.3}$$

$$100x = 2548 \tag{3.4.1.4}$$

Divide both sides by 100.

$$x = 25.48 \tag{3.4.1.5}$$

The sales tax is $\$25.48$.

(b) Find the new amount (total cost).

$$392 + 25.48 = 417.48 \tag{3.4.1.6}$$

The total cost is $\$417.48$.

Exercise

Find (a) the sales tax and (b) the total cost: A company purchased insulated work jackets for employees for $\$250$ in St. Louis, where the sales tax rate was 8.2% of the purchase price.

Answer a available at the end of this section.

Answer b available at the end of this section.

Sales Tax Given Percent

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to calculate the sales tax when given te original price and the sales tax rate.

Example 2

Evelyn bought a new smartphone for $499 plus tax. She was surprised when she got the receipt and saw that the tax was $42.42. What was the sales tax rate for this purchase?

Solution

Let r represent the rate.

Identify the parts:
- amount of increase (sales tax) $= 42.42$
- original amount (purchase price) $= 499$

Use the relationship:

$$\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.1.7}$$

Set up a proportion.

$$\frac{42.42}{499} = \frac{r}{100} \tag{3.4.1.8}$$

Cross multiply.

$$499r = 42.42(100) \tag{3.4.1.9}$$

$$499r = 4242 \tag{3.4.1.10}$$

Divide both sides by 499.

$$r = 8.5 \tag{3.4.1.11}$$

The sales tax rate was 8.5%.

Exercise

Diego bought a new car for $26,525. He was surprised that the dealer then added $2,387.25. What was the sales tax rate for this purchase?

Answer available at the end of this section.

Example 3

A customer paid $\$216.00$ total for a tool after sales tax was added. If the sales tax rate was 8%, what was the original purchase price of the tool?

Solution

Let x represent the original amount (purchase price).

Identify the parts:
- new amount (total cost) $= 216$
- rate $= 8\%$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 + \text{rate}}{100} \tag{3.4.1.12}$$

Set up a proportion.

$$\frac{216}{x} = \frac{100 + 8}{100} \tag{3.4.1.13}$$

$$\frac{216}{x} = \frac{108}{100} \tag{3.4.1.14}$$

Cross multiply.

$$216(100) = 108x \tag{3.4.1.15}$$

$$21600 = 108x \tag{3.4.1.16}$$

Divide both sides by 108.

$$x = 200 \tag{3.4.1.17}$$

The purchase price was $\$200$.

Exercise

A maintenance department paid $\$322.50$ for supplies after sales tax was added. If the sales tax rate was 7.5%, what was the original purchase price?

Answer available at the end of this section.

Example 4

A manufacturing company paid $\$535.00$ total for a piece of equipment after sales tax was added. If the original purchase price was $\$500$, what was the sales tax rate?

Solution

Let r represent the rate.

Identify the parts:
- original amount (purchase price) $= 500$
- new amount (total cost) $= 535$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 + \text{rate}}{100} \tag{3.4.1.18}$$

Set up a proportion.

$$\frac{535}{500} = \frac{100 + r}{100} \tag{3.4.1.19}$$

Cross multiply.

$$535(100) = 500(100 + r) \tag{3.4.1.20}$$

$$53500 = 50000 + 500r \tag{3.4.1.21}$$

Subtract 50000 from both sides.

$$3500 = 500r \tag{3.4.1.22}$$

Divide both sides by 500.

$$r = 7 \tag{3.4.1.23}$$

The sales tax rate was 7%.

Exercise

A landscaping company paid $\$787.50$ total for equipment after sales tax was added. If the original purchase price was $\$750$, what was the sales tax rate?

Answer available at the end of this section.

Solve Mark-up Applications

Markup is a common example of a percent increase used in business and technical settings. Instead of adding tax to a purchase price, a business adds a percent of its cost (original amount) to find the selling price (new amount).

The amount a business pays for an item is called the cost (sometimes called the wholesale price). The markup is the amount of increase, and it is usually calculated as a percent of the cost, called the markup rate. The markup is then added to the cost to find the selling price (sometimes called the list price).

You can use a proportion, like you did in the previous lesson, to find the markup (amount of increase) or the selling price (new amount).

Connection: You can also find the markup by multiplying the cost by the decimal form of the markup rate. Both methods give the same result.

The selling price (list price) should always be greater than the cost (wholesale price).

Example 5

A small equipment supplier bought a tool for $\$250$. The supplier marked the price up 40%. Find the
(a) amount of markup and
(b) the selling price of the tool.

Solution

(a) Let x represent the amount of increase (markup).

Identify the parts:
- original amount (cost) $= 250$
- rate $= 40\%$

Use the relationship:

$$\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.1.24}$$

Set up a proportion.

$$\frac{x}{250} = \frac{40}{100} \tag{3.4.1.25}$$

Cross multiply.

$$100x = 250(40) \tag{3.4.1.26}$$

$$100x = 10000 \tag{3.4.1.27}$$

Divide both sides by 100.

$$x = 100 \tag{3.4.1.28}$$

The markup is $\$100$.

(b) Find the new amount (selling price).

$$250 + 100 = 350 \tag{3.4.1.29}$$

The selling price is $\$350$.

Exercise

An auto resale business bought a car for $\$8{,}500$. The business marked the price up 35%. Find the
(a) amount of markup and
(b) the selling price of the car.

Answer a available at the end of this section.

Answer b available at the end of this section.

Example 6

A small engine repair business bought a mower for $\$640$. The business marked the price up $\$160$. What was the markup rate?

Solution

Let r represent the rate.

Identify the parts:
- amount of increase (markup) $= 160$
- original amount (cost) $= 640$

Use the relationship:

$$\frac{\text{amount of increase}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.1.30}$$

Set up a proportion.

$$\frac{160}{640} = \frac{r}{100} \tag{3.4.1.31}$$

Cross multiply.

$$640r = 160(100) \tag{3.4.1.32}$$

$$640r = 16000 \tag{3.4.1.33}$$

Divide both sides by 640.

$$r = 25 \tag{3.4.1.34}$$

The markup rate was 25%.

Exercise

A printing company bought materials for $\$750$. The company marked the price up $\$225$. What was the markup rate?

Answer available at the end of this section.

Example 7

A parts supplier sold a tool for $\$1{,}080$ after marking up the price 20%. What was the original cost of the tool?

Solution

Let x represent the original amount (cost).

Identify the parts:
- new amount (selling price) $= 1080$
- rate $= 20\%$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 + \text{rate}}{100} \tag{3.4.1.35}$$

Set up a proportion.

$$\frac{1080}{x} = \frac{100 + 20}{100} \tag{3.4.1.36}$$

$$\frac{1080}{x} = \frac{120}{100} \tag{3.4.1.37}$$

Cross multiply.

$$1080(100) = 120x \tag{3.4.1.38}$$

$$108000 = 120x \tag{3.4.1.39}$$

Divide both sides by 120.

$$x = 900 \tag{3.4.1.40}$$

The original cost was $\$900$.

Exercise

A hardware store sold equipment for $\$1{,}380$ after marking up the price 15%. What was the original cost of the equipment?

Answer available at the end of this section.

Example 8

A manufacturing company bought a piece of equipment for $\$1{,}200$ and sold it for $\$1{,}380$. What was the markup rate?

Solution

Let r represent the rate.

Identify the parts:
- original amount (cost) $= 1200$
- new amount (selling price) $= 1380$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 + \text{rate}}{100} \tag{3.4.1.41}$$

Set up a proportion.

$$\frac{1380}{1200} = \frac{100 + r}{100} \tag{3.4.1.42}$$

Cross multiply.

$$1380(100) = 1200(100 + r) \tag{3.4.1.43}$$

$$138000 = 120000 + 1200r \tag{3.4.1.44}$$

Subtract 120000 from both sides.

$$18000 = 1200r \tag{3.4.1.45}$$

Divide both sides by 1200.

$$r = 15 \tag{3.4.1.46}$$

The markup rate was 15%.

Exercise

A farm equipment dealer bought a machine for $\$2{,}000$ and sold it for $\$2{,}500$. What was the markup rate?

Answer available at the end of this section.

Answers

Exercise
$20.50

Exercise
$270.50

Exercise
9%

Exercise
$300

Exercise
5%

Exercise
$2,975

Exercise
$11,475

Exercise
30%

Exercise
$1,200

Exercise
25%

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3.4.2: Discounts and Markdowns

In many real-world situations, a value decreases by a certain percent from its original amount. You have already learned how to calculate percent decrease and how to find a new value after a decrease. In this section, you will apply those same ideas to two common financial situations, sale prices and wholesale pricing, where prices are reduced by a percent.

Using Percent Decrease in Financial Situations

A percent decrease is applied to an original amount.

Use these relationships when working with the amount of decrease, the original amount, and the rate:

$$\frac{\text{amount of decrease}}{\text{original amount}} = \frac{\text{rate}}{100}$$

$$\text{original amount} - \text{amount of decrease} = \text{new amount}$$

Use this relationship when working with the new amount, the original amount, and the rate:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 - \text{rate}}{100}$$

Solve Sale Price Applications

A sale price is a common example of a percent decrease. Instead of adding a percent to an original amount, we subtract a percent of the original price to find the sale price.

When an item is on sale, the discount is calculated as a percent of the original price. This gives the discount (amount of decrease), which is then subtracted from the original price to find the sale price (new amount).

You can use a proportion, like you did in the previous lesson, to find the discount (amount of decrease) or the sale price (new amount).

Connection: You can also find the discount by multiplying the original price by the decimal form of the discount rate. Both methods give the same result.

Example 1

A hardware store is offering a 25% discount on a tool that originally cost $\$240$. Find the
(a) amount of decrease (discount) and
(b) the sale price of the tool.

Solution

(a) Let x represent the amount of decrease (discount).

Identify the parts:
- original amount $= 240$
- rate $= 25\%$

Use the relationship:

$$\frac{\text{amount of decrease}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.2.1}$$

Set up a proportion.

$$\frac{x}{240} = \frac{25}{100} \tag{3.4.2.2}$$

Cross multiply.

$$100x = 240(25) \tag{3.4.2.3}$$

$$100x = 6000 \tag{3.4.2.4}$$

Divide both sides by 100.

$$x = 60 \tag{3.4.2.5}$$

The discount is $\$60$.

(b) Find the new amount (sale price).

$$240 - 60 = 180 \tag{3.4.2.6}$$

The sale price is $\$180$.

Exercise

An equipment supplier is offering a 30% discount on a piece of equipment that originally cost $\$350$. Find the
(a) amount of decrease (discount) and
(b) the sale price of the equipment.

Answer a available at the end of this section.

Example 2

An auto parts store is offering a discount of $\$54$ on a part that originally cost $\$450$. What is the discount rate?

Solution

Let r represent the rate.

Identify the parts:
- amount of decrease (discount) $= 54$
- original amount $= 450$

Use the relationship:

$$\frac{\text{amount of decrease}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.2.7}$$

Set up a proportion.

$$\frac{54}{450} = \frac{r}{100} \tag{3.4.2.8}$$

Cross multiply.

$$450r = 54(100) \tag{3.4.2.9}$$

$$450r = 5400 \tag{3.4.2.10}$$

Divide both sides by 450.

$$r = 12 \tag{3.4.2.11}$$

The discount rate is 12%.

Exercise

A welding supply store is offering a discount of $\$45$ on equipment that originally cost $\$300$. What is the discount rate?

Answer available at the end of this section.

Example 3

A printing company sold materials for $\$272$ after offering a 15% discount. What was the original price of the materials?

Solution

Let x represent the original amount.

Identify the parts:
- new amount (sale price) $= 272$
- rate $= 15\%$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 - \text{rate}}{100} \tag{3.4.2.12}$$

Set up a proportion.

$$\frac{272}{x} = \frac{100 - 15}{100} \tag{3.4.2.13}$$

$$\frac{272}{x} = \frac{85}{100} \tag{3.4.2.14}$$

Cross multiply.

$$272(100) = 85x \tag{3.4.2.15}$$

$$27200 = 85x \tag{3.4.2.16}$$

Divide both sides by 85.

$$x = 320 \tag{3.4.2.17}$$

The original price was $320.

Exercise

A farm supply store sold equipment for $328 after offering an 18% discount. What was the original price of the equipment?

Answer available at the end of this section.

Find the Original Price Given the Discounted Price

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video uses a percent proportion to calculate the original price when given the sale price and the discount rate.

Example 4

A machine shop reduced the price of a tool from $500 to $425. What was the discount rate?

Solution

Let r represent the rate.

Identify the parts:
- original amount $= 500$
- new amount (sale price) $= 425$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 - \text{rate}}{100} \tag{3.4.2.18}$$

Set up a proportion.

$$\frac{425}{500} = \frac{100 - r}{100} \tag{3.4.2.19}$$

Cross multiply.

$$425(100) = 500(100 - r) \tag{3.4.2.20}$$

$$42500 = 50000 - 500r \tag{3.4.2.21}$$

Subtract 50000 from both sides.

$$-7500 = -500r \tag{3.4.2.22}$$

Divide both sides by -500.

$$r = 15 \tag{3.4.2.23}$$

The discount rate is 15%.

Exercise

A catering business reduced the price of supplies from $\$500$ to $\$460$. What was the discount rate?

Answer available at the end of this section.

Solve Wholesale Applications

Wholesale pricing is another example of a percent decrease used in business and technical settings. Instead of selling an item at the regular price, a business may reduce the price by a percent to offer a wholesale price.

The regular price is the original amount. The discount is the amount of decrease, and it is usually calculated as a percent of the regular price, called the discount rate. The discount is then subtracted from the regular price to find the wholesale price (new amount).

You can use a proportion, like you did in the previous lesson, to find the discount (amount of decrease) or the wholesale price (new amount).

Connection: You can also find the discount by multiplying the regular price by the decimal form of the discount rate. Both methods give the same result.

The wholesale price should always be less than the regular price.

Example 5

A machine shop receives a 20% wholesale discount on materials with a regular price of $\$750$. Find the
(a) amount of decrease (discount) and
(b) the wholesale price.

Solution

(a) Let x represent the amount of decrease (discount).

Identify the parts:
- original amount (regular price) $= 750$
- rate $= 20\%$

Use the relationship:

$$\frac{\text{amount of decrease}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.2.24}$$

Set up a proportion.

$$\frac{x}{750} = \frac{20}{100} \tag{3.4.2.25}$$

Cross multiply.

$$100x = 750(20) \tag{3.4.2.26}$$

$$100x = 15000 \tag{3.4.2.27}$$

Divide both sides by 100.

$$x = 150 \tag{3.4.2.28}$$

The discount is $\$150$.

(b) Find the new amount (wholesale price).

$$750 - 150 = 600 \tag{3.4.2.29}$$

The wholesale price is $\$600$.

Exercise

A packaging company offers a 16% discount on supplies with a regular price of $\$800$. Find the
(a) amount of decrease (discount) and
(b) the wholesale price.

Answer available at the end of this section.

Example 6

A distributor offers a discount of $\$180$ on materials with a regular price of $\$900$. What is the discount rate?

Solution

Let r represent the rate.

Identify the parts:
- amount of decrease (discount) $= 180$
- original amount (regular price) $= 900$

Use the relationship:

$$\frac{\text{amount of decrease}}{\text{original amount}} = \frac{\text{rate}}{100} \tag{3.4.2.30}$$

Set up a proportion.

$$\frac{180}{900} = \frac{r}{100} \tag{3.4.2.31}$$

Cross multiply.

$$900r = 180(100) \tag{3.4.2.32}$$

$$900r = 18000 \tag{3.4.2.33}$$

Divide both sides by 900.

$$r = 20 \tag{3.4.2.34}$$

The discount rate is 20%.

Exercise

A janitorial supply company offers a discount of $\$96$ on cleaning supplies with a regular price of $\$600$. What is the discount rate?

Answer available at the end of this section.

Example 7

A landscaping company purchased supplies for $\$680$ after receiving a 15% wholesale discount. What was the regular price of the supplies?

Solution

Let x represent the original amount (regular price).

Identify the parts:
- new amount (wholesale price) $= 680$
- rate $= 15\%$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 - \text{rate}}{100} \tag{3.4.2.35}$$

Set up a proportion.

$$\frac{680}{x} = \frac{100 - 15}{100} \tag{3.4.2.36}$$

$$\frac{680}{x} = \frac{85}{100} \tag{3.4.2.37}$$

Cross multiply.

$$680(100) = 85x \tag{3.4.2.38}$$

$$68000 = 85x \tag{3.4.2.39}$$

Divide both sides by 85.

$$x = 800 \tag{3.4.2.40}$$

The regular price was $\$800$.

Exercise

A restaurant purchased supplies for $\$540$ after receiving a 10% wholesale discount. What was the regular price of the supplies?

Answer available at the end of this section.

Example 8

A retail business paid $\$720$ for materials that had a regular price of $\$900$. What was the wholesale discount rate?

Solution

Let r represent the rate.

Identify the parts:
- original amount (regular price) $= 900$
- new amount (wholesale price) $= 720$

Use the relationship:

$$\frac{\text{new amount}}{\text{original amount}} = \frac{100 - \text{rate}}{100} \tag{3.4.2.41}$$

Set up a proportion.

$$\frac{720}{900} = \frac{100 - r}{100} \tag{3.4.2.42}$$

Cross multiply.

$$720(100) = 900(100 - r) \tag{3.4.2.43}$$

$$72000 = 90000 - 900r \tag{3.4.2.44}$$

Subtract 90000 from both sides.

$$-18000 = -900r \tag{3.4.2.45}$$

Divide both sides by -900.

$$r = 20 \tag{3.4.2.46}$$

The wholesale discount rate was 20%.

Exercise

A small business paid $\$1{,}350$ for materials that had a regular price of $\$1{,}800$. What was the wholesale discount rate?

Answer available at the end of this section.

Answers

Exercise

$\$105$

$\$245$

Exercise

15%

Exercise

$\$400$

Exercise

8%

Exercise

$\$128, \quad \$672$

Exercise

16%

Exercise

$\$600$

Exercise

25%

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3.4.3: Wages and Overtime

Getting paid isn't always as simple as an hourly rate times the number of hours worked. Depending on the job, earnings can include overtime pay, commissions, and different types of deductions. Understanding how these pieces fit together helps you make sense of your paycheck and plan your finances.

In this section, you will calculate earnings based on hourly wages, account for taxes and deductions, and explore how overtime and commission affect total pay. These skills are important when comparing job offers, budgeting income, and making financial decisions after graduation.

Wages

Many jobs pay workers based on an hourly wage. To determine total earnings, you multiply the hourly rate by the number of hours worked. This can be extended to find weekly, monthly, or yearly income.

However, the amount you earn is not always the amount you take home. Deductions such as taxes, Social Security, and other withholdings are taken out of your gross pay (total earnings) to determine your net pay (take-home pay).

Understanding Pay

Gross pay is the total amount earned before deductions.

Net pay (or take-home pay) is the amount you actually receive after deductions are taken out.

Deductions may include taxes, insurance, and other required payments.

Example 1

Avery works as a warehouse employee earning $18.50 per hour. She works 40 hours per week for 52 weeks per year. Find Avery's total yearly pay.

Solution

Step 1: Find weekly pay

$\$18.50 \times 40 = \$740$

Step 2: Find yearly pay

$\$740 \times 52 = \$38,480$

Avery earns $\$38,480$ per year.

Exercise

Luis works as an HVAC technician earning $22.40 per hour. He works 38 hours per week for 50 weeks per year. Find Luis's total yearly pay.

Answer available at the end of this section.

How to Calculate Gross Weekly, Yearly and Monthly Salary

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to calculate weekly, yearly and monthly salaries based on an hourly pay.

Example 2

Maya works as a dental assistant earning $20.25 per hour. She works 40 hours per week for 52 weeks per year. Each year, $4,200 is taken out of her pay for taxes and insurance.

a. Find Maya's total yearly pay.
b. Find how much she takes home after deductions.

Solution

a. Total yearly pay

Step 1: Find weekly pay

$\$20.25 \times 40 = \$810$

Step 2: Find yearly pay

$\$810 \times 52 = \$42,120$

Maya's total yearly pay is $\$42,120$.

b. Take-home pay

Step 1: Subtract deductions

$\$42,120 - \$4,200 = \$37,920$

Maya takes home $\$37,920$ per year.

Exercise

Derrick works as a welder earning $21.80 per hour. He works 42 hours per week for 50 weeks per year. Each year, $5,600 is taken out of his pay for taxes and insurance.

a. Find Derrick's total yearly pay.
b. Find how much he takes home after deductions.

Answer available at the end of this section.

Example 3

Ethan works as a machine operator earning $19.80 per hour. He works 40 hours per week for 52 weeks per year and is paid every two weeks.

Each paycheck has the following deductions:
- $185 for taxes
- $72 for insurance

a. Find Ethan's gross pay per paycheck.
b. Find the total amount deducted from each paycheck.
c. Find Ethan's take-home pay per paycheck.

Solution

a. Gross pay per paycheck

Step 1: Find weekly pay

$\$19.80 \times 40 = \$792$

Step 2: Find pay for two weeks

$\$792 \times 2 = \$1,584$

Gross pay per paycheck is $\$1,584$.

b. Total deductions

$\$185 + \$72 = \$257$

Total deductions are $\$257$.

c. Take-home pay

$\$1,584 - \$257 = \$1,327$

Ethan takes home $\$1,327$ per paycheck.

Exercise

Marcus works as a forklift operator earning $20.60 per hour. He works 40 hours per week and is paid every two weeks.

Each paycheck has the following deductions:
- $210 for taxes
- $85 for insurance

a. Find Marcus's gross pay per paycheck.
b. Find the total amount deducted from each paycheck.
c. Find Marcus's take-home pay per paycheck.

Answer available at the end of this section.

Example 4

Sofia works as an office assistant earning $18.90 per hour. She works 40 hours per week for 52 weeks per year.

Each year, 16% of her total pay is deducted for taxes.

a. Find Sofia's total yearly pay.
b. Find the amount taken out for taxes.
c. Find how much she takes home after taxes.

Solution

a. Total yearly pay

$\$18.90 \times 40 = \$756$

$\$756 \times 52 = \$39,312$

Sofia's total yearly pay is $\$39,312$.

b. Amount taken out for taxes

Set up a proportion

$$\frac{x}{39,312} = \frac{16}{100}$$

Cross multiply

$100x = 39,312 \times 16$

$100x = 628,992$

$x = 6,289.92$

Taxes are $\$6,289.92$.

c. Take-home pay

$\$39,312 - \$6,289.92 = \$33,022.08$

Sofia takes home $\$33,022.08$.

Exercise

Caleb works as an aviation technician earning $23.40 per hour. He works 40 hours per week for 50 weeks per year.

Each year, 15% of his total pay is deducted for taxes.

a. Find Caleb's total yearly pay.
b. Find the amount taken out for taxes.
c. Find how much he takes home after taxes.

Answer available at the end of this section.

Earning Overtime

In many jobs, workers earn additional pay when they work more than their regular hours. A standard work week is typically 40 hours. Any hours worked beyond 40 are considered overtime.

Overtime is often paid at a higher rate, commonly called *time-and-a-half*. This means the worker earns 1.5 times their regular

hourly rate for each overtime hour.

Example 5

Noah works as a construction worker earning $24.00 per hour. During one week, he works 46 hours.

Overtime is paid at time-and-a-half for any hours over 40.

Find Noah's total pay for the week.

Solution

Step 1: Find regular pay

Regular hours: 40

$\$24.00 \times 40 = \$960$

Step 2: Find overtime hours

$46 - 40 = 6$ hours

Step 3: Find overtime rate

$1.5 \times \$24.00 = \$36.00$

Step 4: Find overtime pay

$\$36.00 \times 6 = \$216$

Step 5: Find total pay

$\$960 + \$216 = \$1,176$

Noah earns $\$1,176$ for the week.

Exercise

Jalen works as an HVAC technician earning $22.50 per hour. During one week, he works 48 hours.

Overtime is paid at time-and-a-half for any hours over 40.

Find Jalen's total pay for the week.

Answer available at the end of this section.

Commission

Salespeople often earn a commission, which is a percent of their total sales. Their income may be based only on commission, or it may include a base hourly wage or salary plus commission.

To find the commission, we use a percent proportion. The commission is the part, the total sales is the whole, and the rate of commission is the percent.

The rate of commission is the percent earned on the total amount sold.

Commission

$$\frac{\text{commission}}{\text{total sales}} = \frac{\text{rate}}{100}$$

The commission is the part and the total sales is the whole.

Solve the proportion to find the commission.

Alternatively, you can find the commission on a sale by multiplying the rate of commission, as a decimal, by the total sales.

Example 6

Jamal works as a parts salesperson at an automotive shop and earns a 6% commission on total sales. During one week, he sells $8,450 worth of parts.

Solution

Find the amount of commission Jamal earned.

Step 1: Set up a proportion

$$\frac{x}{8,450} = \frac{6}{100}$$

Step 2: Cross multiply

$100x = 8,450 \times 6$

$100x = 50,700$

Step 3: Solve

$x = 507$

Jamal earns $507 in commission.

Exercise

Fernando receives 18% commission when he makes a computer sale. How much commission will he receive for selling a computer for $2,190?

Answer available at the end of this section.

Commission Amount Given Percent of Sales

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to calculate the commission earned based on a sales amount and a commission rate.

Example 7

Rikki works in appliance sales. She earned $87 in commission when she sold a $1,450 stove.

Solution

Find the rate of commission.

Step 1: Set up a proportion

$$\frac{87}{1,450} = \frac{x}{100}$$

Step 2: Cross multiply

$$1,450x = 87 \times 100$$

$$1,450x = 8,700$$

Step 3: Solve

$$x = \frac{8,700}{1,450} = 6$$

The rate of commission is 6%.

Exercise

Alex works in equipment sales for a farm and tractor dealership. He earned $1,140 in commission after selling a piece of equipment for $28,500. Find the rate of commission.

Answer available at the end of this section.

Example 8

Lena works in HVAC equipment sales. She earns a base salary of $520 per week plus a 4% commission on her total sales. During one week, Lena sells $9,800 worth of equipment. Find Lena's total earnings for the week.

Solution

Step 1: Find commission using a proportion

$$\frac{x}{9,800} = \frac{4}{100}$$

Cross multiply

$$100x = 9,800 \times 4$$

$$100x = 39,200$$

Solve

$$x = 392$$

Commission is $392.

Step 2: Add base pay

$520 + $392 = $912

Lena earns $912 for the week.

Exercise

Darius works in construction supply sales. He earns a base salary of $480 per week plus a 5% commission on his total sales. During one week, he sells $8,600 worth of materials. Find Darius's total earnings for the week.

Answer available at the end of this section.

Example 9

Mia is deciding between two jobs at an auto parts store.

Job A pays $21.50 per hour for a 40-hour work week.
Job B pays $450 per week plus a 6% commission on sales.

During a typical week, Mia expects to sell $6,000 worth of parts.

Which job pays more for that week?

Solution

Step 1: Find weekly pay for Job A

$21.50 \times 40 = \$860$

Step 2: Find commission for Job B

$$\frac{x}{6,000} = \frac{6}{100}$$

$100x = 6,000 \times 6$

$100x = 36,000$

$x = 360$

Commission is $360.

Step 3: Find total pay for Job B

$450 + $360 = $810

Step 4: Compare

$860 > $810

Job A pays more for that week.

Exercise

Ethan is comparing two jobs at an industrial supply company.

Job A pays $20.75 per hour for a 40-hour work week.
Job B pays $400 per week plus a 7% commission on sales.

During a typical week, Ethan expects to sell $6,800 worth of equipment.

Which job pays more for that week?

Answer available at the end of this section.

Answers

Exercise

$42,560

Exercise

a. $45,780 b. $40,180

Exercise

a. $1,648 b. $295 c. $1,353

Exercise

a. $46,800 b. $7,020 c. $39,780

Exercise

$1,215

Exercise

$394.20

Exercise

4%

Exercise

$910

Exercise

Job A: $830 Job B: $876 Job B pays more.

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3.4.4: Personal Budgeting

A budget is a plan for how money and resources are used. Businesses, colleges, and individuals all use them—but do you?

Creating a realistic budget helps you understand your financial priorities and limits. It ensures your income covers necessities, allows for wants like dining out or travel, and supports saving for future goals. Without a plan, it's easy to fall into debt or struggle to keep up with expenses.

While budgeting can feel overwhelming at first, the hardest part is getting started. Over time, you can adjust both the amounts and the categories to fit your needs.

Creating a Budget

Think of a budget as a tool to help you reach long-term financial goals. It estimates your income and expenses so you can track progress and make better financial decisions.

Many tools—like Goodbudget, Mint, or Google Sheets—can help, but the process starts before choosing an app.

Steps to Create a Monthly Budget:

1. **Track income and expenses**
   Review the past 6–12 months to understand your spending habits.
2. **Set your income baseline**
   List all after-tax income sources. If income varies, use an average.

Checkpoint

For income that is not steady, such as gig work or freelance work, use the previous 6 to 12 months of income to find an average income from that gig or freelance work. Use this average in the budgeting process.

3. **Determine your expenses**
   Use bills and statements to identify fixed (rent, insurance) and variable (groceries, gas) expenses. Convert yearly or quarterly costs into monthly amounts.

Checkpoint

Some expenses are yearly, such as insurance or property taxes. Other expanses may be quarterly (four times per year) or semiannual (twice per year). To budget for such bills by month, divide the bill total by the number of months the bill covers.

4. **Categorize expenses**
   Group expenses into categories that make sense for you. More detail gives better insight.
5. **Compare income and expenses**
   If expenses are higher, make adjustments. If there's extra income, decide how to use it.
6. **Plan for unexpected costs**
   Build a cushion for things like repairs or emergencies.
7. **Adjust as needed**
   Update your budget regularly to reflect changes in income or expenses.

7 Steps on How to Create a Budget

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video discusses the seven steps to creating a budget.

In this section, we will focus on income and expenses. One of the easiest ways to manage a budget is to create a table, with one column containing income sources, another with income values, a third with expense categories, and a last containing expenses. An example is shown in the table below.

Table with Budget

Income Source	Amount	Expense	Amount
Full-time job	$3,565	Rent	$975
Uber	$185	Car Payment	$355
TOTAL	$3,750	Student Loan	$418
		Electric	$76
		Food	$400
		Gasoline	$250
		Car Insurance	$165
		Clothing	$100
		Entertainment	$100
		TOTAL	$2,839

Who Knew?: Gross Pay and Take Home Pay

If you've ever had a paycheck, you know that taxes are taken out of your pay before you get your check. This amount of money varies from state to state, and sometimes even city to city. For a person making $50,000 per year gross salary in Salt Lake City, Utah, take home pay is about 75.6% of gross salary. In Detroit, Michigan, take home pay is about 74.5% of gross salary. Lakeland, Florida, take home pay is about 80.5% of gross salary. These also change based on how much a person earns! Before choosing a place to live, it makes sense to determine how much deductions from pay will impact your income.

Example 1

Heather has graduated college and currently works as a nurse for a rural medical group. Her net monthly income from that job is $3,765.40. She also works part-time on the weekends, earning another $672.00 per month. Her monthly expenses are rent at $1,050, car payments at $489, student loan payments at $728, car insurance at $139, utilities at $130, clothing at $150, entertainment (going out with friends, Netflix, Amazon Prime, movies) at $300, credit card debt at $200, food at $360, and gasoline at $275. Create her budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

Solution

Step 1: To begin, we create the table with appropriate headings.

Income Source	Amount	Expense	Amount

Step 2: Her income categories are her nursing job, with $3,765.40 per month, and her part-time job, with $672.00 per month. Entering these into the table, we have the following.

Income Source	Amount	Expense	Amount
Nursing	$3,765.40		
Part-time	$672.00		

Step 3: Her monthly expenses are listed above. Entering the categories and the amount for each of those expenses, the table is now

Income Source	Amount	Expense	Amount
Nursing	$3,765.40	Rent	$1,050
Part-time	$672.00	Car Payment	$489
		Student Loan	$728
		Car Insurance	$139
		Utilities	$130
		Clothing	$150
		Entertainment	$300
		Credit Card	$200
		Food	$400
		Gasoline	$250

Step 4: Totaling the income and expenses, we see that her total income is $4,437.40 per month, and her total expenses are $3,836 per month. Comparing these, we see that Heather has $601.40 in excess income per month. This provides a cushion in her budget.

Exercise

Mateo works as a union electrician in a suburban area. His monthly take-home pay is $3,375, and he earns an additional average of $300 per month from side jobs.

His monthly expenses are:

- Mortgage: $986.78
- Truck payment: $589.00
- Truck insurance: $312
- Utilities: $167
- Clothing: $150

- Entertainment: $400
- Credit card payment: $325
- Food: $470
- Gasoline: $375

a. Find Mateo's total monthly income.
b. Find Mateo's total monthly expenses.
c. Compare his income and expenses to determine whether he has excess income or a shortage each month.
d. State how much he has left over or how much he falls short each month.

Answer available at the end of this section.

Example 2

Carol is working in a dental lab, creating dentures and bridges. Monthly her take home pay is $2,816 (based on $22 per hour minus payroll taxes). She also receives $320 per month in child support for her one daughter. Her monthly expenses are rent at $700, car payments at $229, student loan payments at $250, car insurance at $119, health insurance at $225, utilities at $80, clothing at $75, entertainment at $200, food at $275, and gasoline at $275. Create Carol's budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

Solution

Step 1: To begin, we create the table with appropriate headings.

Income Source	Amount	Expense	Amount

Step 2: Her income categories are from work, $2,816, and child support, $320, per month. Entering these into the table, we have the following.

Income Source	Amount	Expense	Amount
Job	$2,816.00		
Child support	$320.00		

Step 3: Her monthly expenses are listed above. Entering the categories and the amount for each of those expenses, the table is now

Income Source	Amount	Expense	Amount
Job	$2,816.00	Rent	$700
Child support	$320.00	Car Payment	$229
		Student Loan	$250
		Car Insurance	$119
		Utilities	$80
		Health insurance	$225

Income Source	Amount	Expense	Amount
		Clothing	$75
		Entertainment	$200
		Food	$275
		Gasoline	$275

Step 4: Totaling the income and expenses, we see that her total income is $3,136.00 per month, and her total expenses are $2,428.00 per month. Comparing these, we see that Carol has $708.00 in excess income per month. This is the cushion in her budget.

Exercise

Maddie works as a mechanical engineer and earns $6,093.75 per month after payroll taxes.

Her monthly expenses are:
- Mortgage: $1,452.89
- Car payment: $627.38
- Car insurance: $179.00
- Health insurance: $265.00
- Utilities: $320
- Clothing: $150
- Entertainment: $400
- Credit card payment: $450
- Food: $370
- Gasoline: $175

a. Find Maddy's total monthly income.

b. Find Maddy's total monthly expenses.

c. Compare her income and expenses to determine whether she has excess income or a shortage.

d. State how much she has left over or how much she falls short each month.

Answer available at the end of this section.

Using the budget process, we can make decisions on adding expenses to the budget. To do so, check the cushion of the budget to see if there is room in the budget for the new expense.

Example 3

In example 2, we determined that Carol had excess income of $708.00. She looks up the cost of before-school care for her daughter. She finds that, monthly, the cost would be $252.00 per month. Is this an affordable program for Carol? Add this expense to her budget table.

Answer available at the end of this section.

Exercise

Recall Heather's budget from Example 1. She decides she wants to buy her own home, which would increase her expenses. Instead of $1,050.00 in rent, she would pay $1,240.00 for her mortgage. Her utilities costs would increase to $295.00 per month. Add these to Heather's budget to determine if the changes are affordable.

Answer available at the end of this section.

The 50-30-20 Budget Philosophy

It can be difficult to decide how to divide your income across expenses. A common approach is the **50–30–20 rule**, which gives a simple guideline for budgeting:

- **50% for needs** – essential expenses like rent, utilities, groceries, insurance, transportation, childcare, and minimum debt payments.
- **30% for wants** – nonessential items such as dining out, subscriptions, travel, hobbies, or a gym membership.
- **20% for savings and debt** – saving for emergencies, retirement, investments, or paying down additional debt.

Distinguishing between needs and wants is important, since the line can be blurry. Needs are required for basic living, while wants improve quality of life but are optional. Both matter—ignoring wants entirely can make a budget hard to maintain.

The 20% category helps build financial stability. It can fund an emergency account (often 3 months of income), support long-term savings, and cover unexpected expenses without disrupting your budget.

Keep in mind that these categories are guidelines, not strict rules. Your actual budget may include additional or different expenses depending on your situation.

Necessary Expenses and Expenses that are Wants

What counts as a **need** versus a **want** can vary from person to person. For example, internet service may be a need for someone who works from home or has children in school, but a want for someone who can rely on public access or a phone hotspot.

Cars are another example. For many people, a car is a need, but alternatives like biking, public transportation, or walking may make it a want instead. Even when something is a need, the **level of spending** matters. A car may be necessary, but choosing a more expensive model than your budget allows turns part of that cost into a want.

The same idea applies to housing. If your budget allows $900 for rent but you choose a $1,250 apartment, the extra $350 is a want.

The goal is to carefully evaluate your expenses and decide what is truly necessary. If your expenses exceed your income, adjustments may be needed—such as reducing costs, choosing more affordable options, or cutting back on nonessential spending. These guidelines can help keep your finances manageable, especially as you plan for life after graduation.

Example 4

In example 3, after Carol added before school care for her daughter to the budget, her budget was as shown below. Evaluate Carol's budget using the 50-30-20 budget philosophy.

Income:
• Job: $2,816.00
• Child support: $320.00

Expenses:
• Rent: $700
• Car payment: $229

- Student loan: $250
- Car insurance: $119
- Utilities: $80
- Health insurance: $225
- Clothing: $75
- Entertainment: $200
- Food: $275
- Gasoline: $275
- Before-school care: $252

Solution

Carol's total income is $3,136.00. Applying the 50-30-20 budget philosophy to this income requires the calculation of each of those percentages.

For the necessities, Carol should budget 50% of her income, or $0.5 \times \$3{,}136.00 = \$1{,}568.00$.

For her wants, she should budget 30% of her income, or $0.3 \times \$3{,}136.00 = \$940.80$.

For savings and extra debt service, she should budget 20% of her income, or $0.2 \times \$3{,}136.00 = \$627.20$.

In her budget, her necessities include all expenses except for entertainment. These expenses total $2,480, which exceeds the suggested budget amount of $1568.00. To follow the guidelines, Carol would have to cut back on these necessities.

For her wants, she spends $200.00 on entertainment, which is well below the suggested budget amount of $940.80. If she modifies how much she spends on needs, she may be able to increase the spending on her wants.

Her excess income is $456.00, which is below what she should be saving and using to pay down extra debt. If she does adjust how much she spends on needs, she could increase the amount for savings.

Exercise

Recall Heather's budget from Example 1, before she thought of moving. That budget is below. Evaluate Heather's budget using the 50-30-20 budget philosophy.

Income Source	Amount	Expense	Amount
Nursing	$3,765.40	Rent	$1,050
Part-time	$672.00	Car Payment	$489
		Student Loan	$728
		Car Insurance	$139
		Utilities	$130
		Clothing	$150
		Entertainment	$300
		Credit Card	$200
		Food	$400
		Gasoline	$250

Answer available at the end of this section.

Example 5

Carmen is about to graduate and has been offered a job at a bank as a data scientist. She estimates her monthly take home pay to be $5,662.50. Apply the 50-30-20 philosophy to that monthly income. How should Carmen use this information?

Solution

Carmen's monthly income: $5,662.50

50% (needs): $2,831.25
30% (wants): $1,698.75
20% (savings and debt): $1,132.50

She should plan her budget so that essential expenses stay within $2,831.25, discretionary spending stays within $1,698.75, and $1,132.50 is set aside for savings and debt. This helps guide her choices about housing, transportation, and spending priorities.

Exercise

Elijah has finished an apprenticeship and is about to start his first job as an HVAC (heating, ventilation, and air conditioning) tech. He estimates that his net monthly income will be $3,263.44. Apply the 50-30-20 budget philosophy to his income to set guidelines for Elijah's budget. How should Elijah use this information?

Answer available at the end of this section.

Example 6

Steve is thinking of moving out of his family's home. He works full-time earning $18 per hour, which is approximately $29,180 per year after taxes (based on 40 hours per week for 52 weeks). He pays $218.00 per month on student loans and $162.00 per month for his car.

a. Find Steve's monthly income.
b. Apply the 50-30-20 budget philosophy to determine how much he should spend on needs, wants, and savings.
c. After paying his car payment and student loan, how much does he have left for other necessities?
d. How much is available for wants and for savings?
e. Based on this information, discuss whether moving out is affordable for Steve.

Solution

Steve's monthly income: $2,431.67

50% (needs): $1,215.83
30% (wants): $729.50
20% (savings): $486.33

Car payment and student loan total: $380.00

Amount available for other necessities: $835.83

Amount for wants: $729.50
Amount for savings: $486.33

Conclusion: After paying his car and student loan, Steve has $835.83 left for other necessities such as rent, utilities, food, insurance, and gasoline. This is likely not enough to cover basic living expenses. Even if he uses money from savings, his budget is still very tight. It does not appear that moving out is affordable without increasing income or reducing expenses.

Exercise

Fran is considering a new job that will require her to move to an area with a higher cost of living. Her new job will provide a net annual income of $43,700. She currently pays $295.00 per month for her car, $264.00 per month for student loans, and $200.00 per month toward credit card debt.

a. Find Fran's monthly income.
b. Apply the 50-30-20 budget philosophy to determine how much she should spend on needs, wants, and savings.
c. After paying her car payment, student loan, and credit card debt, how much does she have left for other necessities?
d. How much is available for wants and for savings?
e. Based on this information, discuss whether changing jobs and moving is affordable for Fran.

Answer available at the end of this section.

Budgeting and the 50:30:20 Rule

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains the 50-30–20 Budget Philosophy.

Answers

Exercise

a. $3,675
b. $3,774.78
c. Expenses are greater than income.
d. Mateo falls short by $99.78 each month.

Exercise

a. $6,093.75
b. $4,389.27
c. Income is greater than expenses.
d. Maddy has $1,704.48 in excess income each month.

Example 3

She can afford this, as the cost for the before school program is $252.00 and she had extra income of $708.00. Adding this to her budget, her budget table is now

Income Source	Amount	Expense	Amount
Job	$2,816.00	Rent	$700
Child support	$320.00	Car Payment	$229
		Student Loan	$250
		Car Insurance	$119
		Utilities	$80
		Health insurance	$225
		Clothing	$75
		Entertainment	$200
		Food	$275

Income Source	Amount	Expense	Amount
		Gasoline	$275
		Before-school care	$252

Now, she has $456.00 in excess income per month.

Exercise

The changes are affordable, but they leave a much smaller cushion in her budget.

Exercise

Total income: $4,437.40

Needs (rent, car payment, student loan, car insurance, utilities, food, gasoline): $3,186
50% guideline: $2,218.70

Wants (clothing, entertainment): $450
30% guideline: $1,331.22

Savings/Debt (credit card): $200
20% guideline: $887.48

Conclusion: Heather is spending too much on needs and not enough on savings. Her budget does not follow the 50-30-20 guideline.

Exercise

Elijah's monthly income: $3,263.44

50% (needs): $1,631.72
30% (wants): $979.03
20% (savings and debt): $652.69

He should plan his budget so that essential expenses stay within $1,631.72, discretionary spending stays within $979.03, and $652.69 is set aside for savings and debt. This helps guide his choices about housing, transportation, and spending priorities.

Exercise

Fran's monthly income: $3,641.67

50% (needs): $1,820.84
30% (wants): $1,092.50
20% (savings): $728.33

Car, student loan, and credit card total: $759.00

Amount available for other necessities: $1,061.84

Amount for wants: $1,092.50
Amount for savings: $728.33

Conclusion: After paying her existing debts, Fran has $1,061.84 left for other necessities such as housing, utilities, food, and transportation. This may be tight depending on the higher cost of living. The move may be affordable, but it will likely require careful budgeting and possibly reducing wants or savings.

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3.4.5: Financial Applications Practice

The following set of exercises provides practice for the concepts covered in Module 3.4: Financial Applications. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 3.4.1 - Sales Tax and Markup

Solve each problem. Write a complete sentence and include units when appropriate. Round to the nearest cent when necessary.

1. Jordan stops by a supply store to pick up a new power tool priced at 480 dollars. The sales tax rate is 8%. Find (a) the sales tax and (b) the total cost.

2. After checking out at a store, a customer sees a total of 129.60 dollars on the receipt, which includes 8% sales tax. What was the original price of the purchase?

3. Maria purchases a piece of equipment for 725 dollars and pays 58 dollars in sales tax. What is the tax rate?

4. A supplier purchases equipment for 300 dollars and marks it up 25% before selling it. Find (a) the markup and (b) the selling price.

5. A tool is listed for sale at 1380 dollars after a 15% markup has been applied. What was the original cost?

6. A small business buys equipment for 950 dollars, marks it up 20%, and then applies 8% sales tax to the selling price. What is the final price paid by the customer?

Section 3.4.2 - Discount and Markdown

Solve each problem. Write a complete sentence and include units when appropriate. Round to the nearest cent when necessary.

7. At a local auto parts store, Marcus notices a promotion on a brake component that normally costs 350 dollars. The store advertises a 20% discount. Find (a) the discount and (b) the sale price.

8. While ordering equipment, a technician notices that a 96 dollar discount has been applied to an item that originally cost 600 dollars. What is the discount rate?

9. After a discount is applied, Elena's company pays 765 dollars for a set of materials. The discount rate was 15%. What was the original price?

10. A piece of equipment that originally cost 900 dollars is marked down and sold for 765 dollars. What is the discount rate?

11. A small business owner orders materials listed at 500 dollars and receives an 18% wholesale discount from the supplier. Find (a) the discount and (b) the wholesale price.

Section 3.4.3 - Wages and Overtime

Solve each problem. Write a complete sentence and include units when appropriate. Round to the nearest cent when necessary.

12. Jordan works full-time as a technician earning 19.40 dollars per hour and works 40 hours each week for 52 weeks. Find Jordan's total yearly pay.

13. Luis is a machinist who earns 20.80 dollars per hour and works 40 hours per week for 52 weeks. Each year, 5200 dollars is deducted for taxes and insurance. Find Luis's take-home pay.

14. Derrick works 45 hours in one week at a rate of 23.50 dollars per hour. Overtime is paid at time-and-a-half for hours over

40. Find Derrick's total weekly pay.

15. Maria is an HVAC technician who works 52 hours in one week and earns 26.00 dollars per hour. Overtime is paid at time-and-a-half after 40 hours. Find Maria's weekly pay.

16. Kayla works as a sales associate and earns a 5% commission on all sales. In one week, her sales total 7800 dollars. Find Kayla's commission.

17. Ethan earns a base pay of 500 dollars per week plus a 6% commission on sales. During one week, his sales total 6500 dollars. Find Ethan's total weekly earnings.

18. Noah is deciding between two job offers. Job A pays 22.00 dollars per hour for a 40-hour work week. Job B pays 420 dollars per week plus a 5% commission on 7200 dollars in sales. Which job pays more for that week?

Section 3.4.4 - Personal Budgeting

Solve each problem. Write a complete sentence and include units when appropriate. Round to the nearest cent when necessary.

19. Tasha earns $3250 per month and has monthly expenses of $2980. Determine whether she has excess income or a shortage, and find the amount.

20. Brandon is a technician who earns $4180 per month and spends $4365 each month. Determine whether he has excess income or a shortage, and find the amount.

21. Luis is a machinist who earns $3720 per month. His monthly expenses include rent $1050, car payment $410, insurance $180, utilities $160, food $420, gasoline $210, entertainment $300, and credit card payment $250. Find his total expenses and determine whether he has excess income or a shortage.

22. Erica earns $3950 per month and currently spends $3540. Her rent increases by $180 and her utilities increase by $75. Determine whether her new budget is affordable.

23. Monica is a technician who earns $4200 per month. Find the amounts she should budget for needs 50%, wants 30%, and savings 20%.

24. Caleb earns $3400 per month. His current budget includes needs of $2050, wants of $820, and savings of $200. Determine whether his budget follows the 50-30-20 guideline.

25. Jasmine earns $4600 per month. Her budget includes needs of $2900, wants of $1200, and savings of $300. Determine whether her budget follows the 50-30-20 guideline.

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4: MEASUREMENT AND UNIT CONVERSIONS

Measurements show up everywhere — in construction, manufacturing, healthcare, transportation, and everyday problem solving. But working with measurements is not just about getting an answer. It is about understanding how reliable that answer is and making sure the units make sense for the situation.

This unit is divided into three main parts. First, you will learn how to work with measured numbers. This includes significant digits, accuracy, precision, and error. These ideas help you decide how much of a number you can trust and how to report results correctly. You will practice identifying meaningful digits, applying rules for operations, and working with concepts like relative error, tolerance, and greatest possible error (GPE).

Next, you will focus on unit conversions within a single system. You will work with both the U.S. Customary system and the Metric system, converting measurements of length, weight (mass), time, area, and volume. You will also apply these conversions in practical situations so that the math connects to real-world tasks.

Finally, you will combine everything by working between systems. Many real situations require switching between U.S. Customary and Metric units. You will practice converting between systems, working with compound units like speed and flow rates, and solving application problems that require multiple steps and careful attention to units.

The goal of this unit is to help you work confidently with measurements — not just performing calculations, but understanding what the numbers represent and how to use them appropriately in real situations.

4.1: Measurements and Significant Digits

4.1.1: Accuracy and Significant Digits

4.1.2: Precision

4.1.3: Error and Tolerance

4.1.4: Measurement and Significant Digits Practice

4.2: U.S. Customary Measurements

4.2.1: Length, Weight and Time in the U.S. Customary System

4.2.2: Area and Volume in U.S. Customary Units

4.2.3: Applications Involving U.S. Customary Measurements

4.2.4: U.S. Customary Measurements Practice

4.3: Metric Measurements

4.3.1: Length and Mass in the Metric System

4.3.2: Area and Volume in the Metric System

4.3.3: Applications Involving the Metric System

4.3.4: Metric Measurements Practice

4.4: Metric and U.S. Customary Conversions

4.4.1: Length and Mass Between Systems

4.4.2: Area and Volume Between Systems

4.4.3: Temperature Conversions

4.4.4: Applications Involving Metric and U.S Customary Conversion

4.4.5: Metric and U.S. Customary Conversion Practice

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4.1: MEASUREMENTS AND SIGNIFICANT DIGITS

What does it mean for a measurement to be “close enough”? In this module, you’ll explore three connected ideas that help describe how reliable a number really is. Before getting started, watch the hook video featuring the legend of William Tell — who must shoot an apple off his son’s head — shown in two different scenarios. The comparison gives a clear, intuitive sense of the difference between accuracy and precision before we define those terms formally.

The module begins with significant digits and accuracy, where you’ll learn how to identify which digits in a number are meaningful and how that affects addition and subtraction. Next, you’ll study precision — a related but different idea — and apply it when multiplying and dividing measurements. Finally, you’ll examine error, including how to calculate relative error, work within tolerances, and determine the greatest possible error (GPE) for a given measurement.

By the end of the module, you should have a practical understanding of how to work with measured values — not just how to calculate with them, but how to interpret what those numbers actually mean.

What's the Difference Between Accuracy and Precision?

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This video uses the story of William Tell to give a clear, intuitive sense of the difference between accuracy and precision.

4.1.1: Accuracy and Significant Digits

4.1.2: Precision

4.1.3: Error and Tolerance

4.1.4: Measurement and Significant Digits Practice

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4.1.1: Accuracy and Significant Digits

In earlier sections, you have already looked at the rules for rounding numbers when needed. Now it is time to take a closer look at *why* rounding matters.

In real applications, most numbers come from measurements, and every measurement has some level of uncertainty. Whether you are measuring a length, reading a gauge, or working with a digital display, the value you record depends on the precision of the measuring tool. For example, a ruler marked to the nearest tenth of an inch cannot guarantee anything more precise than that, and a meter that reads 12.612.612.6 may represent a value that is slightly higher or lower.

Because of this, not every digit in a number carries the same meaning. Some digits are known with confidence, while others are estimates. Understanding which digits are meaningful—and how to round appropriately—helps you communicate results clearly and avoid implying more precision than your measurements actually support.

This is where accuracy and significant digits come in.

Exact Values and Approximations

Exact Values vs. Approximations

In practice, numbers fall into two categories.

A number is exact if it comes from counting or from a definition.

A number is an approximation if it comes from a measurement or from rounding.

Example 1

Identify each number as exact or an approximation.

a. A metal bracket measures 12.4 centimeters.
b. A box contains 24 screws.
c. A pressure gauge reads 85 psi.

Solution

a. 12.4 cm $\rightarrow$ approximation Measured value; limited by precision of the tool.
b. 24 screws $\rightarrow$ exact Counted value; no uncertainty.
c. 85 psi $\rightarrow$ approximation Instrument reading; may not be exact.

Exercise

Identify each number as exact or an approximation.

a. There are 32 students enrolled in a course.
b. One hour contains 60 minutes.
c. A thermometer reads 68°F.
d. A pipe is 2.5 meters long.

Answer available at the end of this section.

Precision, Accuracy, Measurement and Significant Figures

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video provides an overview of the topics covered in sections 4.1.1 and 4.1.2 and relates them to using a ruler.

Accuracy and Significant Digits

Because measurements are not exact, we need a way to describe how reliable they are. This is where accuracy and significant digits come in.

In everyday language, something is accurate if it is “correct” or very close to the true value. In this lesson, we are not focusing on how close a measurement is to the true value. Instead, we are focusing on what the digits in a number represent.

Significant digits are the digits in a number that actually tell us something about a measurement. They include all the digits we know for sure, along with one digit that is estimated.

Understanding significant digits helps make sure that reported values do not suggest more information than the measurement actually supports.

The rules for identifying significant digits are not difficult, but they do take some practice. The examples and exercises that follow will help you get comfortable using them.

Significant Digits

1. All nonzero digits are significant.
Example: $12{,}345$ has five significant digits, and 123.45 has five significant digits.

2. Zeros between nonzero digits are significant.
Example: $10{,}045$ has five significant digits, and 100.45 has five significant digits.

3. Zeros to the right of a decimal point are significant.
Example: 123 has three significant digits, but 123.00 has five significant digits.

4. Zeros to the left of the first nonzero digit are not significant.
Example: 0.123 has three significant digits, and 0.00123 has three significant digits.

5. Zeros at the end of a whole number are not significant unless they are marked with an overbar.
Example: $12{,}300$ has three significant digits, but $12{,}30\overline{0}$ has five significant digits.

Zeros that are only used to show place value are not significant.

Example 2

Determine the number of significant digits in each number.

a. $72{,}500$
b. $72{,}050$
c. $72{,}005$

Solution

a. $72{,}500 \rightarrow 3$ significant digits Trailing zeros in a whole number are not significant.
b. $72{,}050 \rightarrow 4$ significant digits The zero between nonzero digits is significant; the final zero is not.
c. $72{,}005 \rightarrow 5$ significant digits Zeros between nonzero digits are significant.

Example 3

Determine the number of significant digits in each number.

a. 0.094
b. 0.0904
c. 0.09040

Solution

a. $0.094 \rightarrow 2$ significant digits Leading zeros are not significant.

b. $0.0904 \rightarrow 3$ significant digits The zero between nonzero digits is significant.

c. $0.09040 \rightarrow 4$ significant digits Trailing zeros to the right of a decimal are significant.

Exercise

Determine the number of significant digits in each number.

a. $58{,}300$
b. $58{,}030$
c. 0.072
d. 0.0702
e. 0.07020

Answer available at the end of this section.

In 1856, the first official measurement of the height of Mount Everest—called Sagarmatha in Nepal and Chomolungma in Tibet—was announced. The height was determined to be $29{,}000$ feet, but there was concern that people would think this was a rough estimate rounded to the nearest thousand feet.

To avoid this confusion, the height was reported as $29{,}002$ feet so that it appeared to be measured to the nearest foot. In other words, a slightly incorrect number was used to make the measurement seem more precise.

Rather than changing the number, we can use an overbar to show that trailing zeros are significant. For example, writing $29{,}00\overline{0}$ indicates that the value is known to the nearest foot and that all five digits are significant.

This notation allows us to communicate the meaning of the measurement without changing its value.

Example 4

Determine the number of significant digits in each number.

a. $41{,}000$
b. $41{,}\overline{0}00$
c. $41{,}0\overline{0}0$
d. $41{,}00\overline{0}$

Solution

a. 2
b. 5
c. 4
d. 3

Two things to remember: we don't put an overbar over a nonzero digit, and we don't need an overbar for any zeros on the right of a decimal number because those are already understood to be significant.

Measurement and Significant Figures (Digits)

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces significant digits and how to use them when working with measurements.

Accuracy-Based Rounding

As we saw in Unit 1, it is often necessary to round a number. We usually round to a specific place value, such as the nearest hundredth, but there is another approach. Rounding based on significant digits focuses on the number of significant digits rather than the place value.

Accuracy-Based Rounding

1. Locate the rounding digit by counting from the left until you reach the desired number of significant digits.
2. Look at the test digit directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and remove all digits to its right. If the test digit is less than 5, keep the rounding digit the same and remove all digits to its right.

Example 5

Round each number to the indicated number of significant digits.

a. $58{,}947$ (three significant digits)
b. $58{,}947$ (four significant digits)

Solution

a. $58{,}947 \rightarrow 58{,}900$ Round to 3 significant digits.
b. $58{,}947 \rightarrow 58{,}950$ Round to 4 significant digits.

Example 6

Round each number to the indicated number of significant digits.

a. 7.2683 (two significant digits)
b. 7.2683 (three significant digits)

Solution

a. $7.2683 \rightarrow 7.3$ Round to 2 significant digits.
b. $7.2683 \rightarrow 7.27$ Round to 3 significant digits.

Exercise

Round each number to the indicated number of significant digits. Include trailing zeros or an overbar if necessary.

a. $63{,}998$ (two significant digits)
b. $63{,}998$ (three significant digits)
c. 4.995 (two significant digits)
d. 4.995 (three significant digits)

Answer available at the end of this section.

In these examples, we rounded by counting significant digits from left to right and using the next digit to decide whether to round up or down.

When rounding whole numbers, be careful with trailing zeros. If a zero is created by rounding and is meant to be significant, it must be shown with an overbar.

For decimal numbers, trailing zeros are included to show that they are significant.

Example 7

A survey measurement records the elevation of a site as $24{,}786.39$ feet.

Round the measurement to the indicated number of significant digits.

a. Two significant digits
b. Three significant digits
c. Four significant digits

Solution

a. $24{,}786.39 \rightarrow 25{,}000$ Round to 2 significant digits.
b. $24{,}786.39 \rightarrow 24{,}800$ Round to 3 significant digits.
c. $24{,}786.39 \rightarrow 24{,}790$ Round to 4 significant digits.

Exercise

A measurement shows a distance of $16{,}493.72$ meters.

Round the measurement to the indicated number of significant digits.

a. Two significant digits
b. Three significant digits
c. Four significant digits

Answer available at the end of this section.

How to Round Numbers to Significant Numbers (Figures)

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to round to a certain number of significant digits.

Accuracy when Multiplying and Dividing

Suppose you need to square the number $3\frac{1}{3}$. You can rewrite $3\frac{1}{3}$ as the improper fraction $\frac{10}{3}$, then compute $\left(\frac{10}{3}\right)^2 = \frac{100}{9}$, which is the repeating decimal $11.111\ldots$

Because decimals are often easier to work with, you might instead round $3\frac{1}{3}$ to 3.33 and calculate $3.33^2 = 11.0889$. This result looks very precise, but that precision is misleading because rounding was introduced at the start.

To report the result appropriately, we round based on significant digits. Since 3.33 has three significant digits, the final answer should also have three significant digits. Rounding 11.0889 to three significant digits gives 11.1, which matches $11.111\ldots$ rounded to three significant digits.

When multiplying or dividing approximate numbers, the result should be rounded to the same number of significant digits as the factor with the fewest significant digits.

Do not round intermediate values. Perform the calculation first, then round the final answer.

Example 8

Multiply and round the result to the correct number of significant digits.

a. 2.46×3.1

Solution

$$2.46 \times 3.1 = 7.626$$
$$\rightarrow 7.6 \quad \text{Round to 2 significant digits (least in original numbers).}$$

Example 9

Divide and round the result to the correct number of significant digits.

a. $18.72 \div 4.5$

Solution

$18.72 \div 4.5 = 4.16$
$\rightarrow 4.2$ Round to 2 significant digits (least in original numbers).

Exercise

Multiply or divide. Round each result to the correct number of significant digits.

a. 4.72×2.3
b. $9.84 \div 3.2$
c. 5.6×1.27
d. $14.5 \div 2.8$

Answer available at the end of this section.

Example 10

A machine produces parts at a rate of 3.6 parts per minute. It runs for 12.4 minutes. How many parts are produced?

Solution

$3.6 \times 12.4 = 44.64$
$\rightarrow 45$ Round to 2 significant digits.

Approximately 45 parts are produced.

Example 11

A 15.8-meter cable is cut into pieces that are each 2.4 meters long. How many pieces can be made?

Solution

$15.8 \div 2.4 = 6.5833\ldots$
$\rightarrow 6.6$ Round to 2 significant digits.

Approximately 6.6 pieces can be made.

Exercise

Multiply or divide. Round each result to the correct number of significant digits.

a. A drill operates at 1.8 centimeters per second for 42.5 seconds. How far does it drill?

b. A container holds 72.4 liters and is filled at a rate of 8.6 liters per minute. How many minutes does it take to fill?

Answer available at the end of this section.

Answers

Exercise

a. Exact
b. Exact
c. Approximation
d. Approximation

Exercise

a. 3
b. 4
c. 2
d. 3
e. 4

Exercise

a. $64{,}000$
b. $64{,}0\overline{0}$
c. 5.0
d. 5.00

Exercise

a. $16{,}000$
b. $16{,}500$
c. $16{,}490$

Exercise

a. 11
b. 3.1
c. 7.1
d. 5.2

Exercise

a. 77 centimeters
b. 8.4 minutes

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4.1.2: Precision

In many situations, measurements are not recorded exactly—they are rounded to a convenient level of detail. For example, the length of a pipe might be recorded as 12.4 meters, or the weight of a shipment might be listed as 850 pounds. These values give useful information, but they are not exact.

The level of detail used when recording a measurement is called precision. A measurement reported to the nearest tenth is more precise than one reported to the nearest whole number.

Whenever a measurement is rounded, there is a limit to how far the reported value could be from the actual value. This limit is called the greatest possible error.

Understanding precision and greatest possible error helps you interpret measurements correctly and recognize how much variation is possible in a reported value.

Precision

Precision

The precision of a number is the place value of the rightmost significant figure. For example, $82,000$ is precise to the thousands place, $81,999$ is precise to the ones place, and something like $81,999.2$ would be precise to the tenths place.

Example 1

Identify the precision (the place value of the rightmost significant digit) of each number.

a. $41{,}000$
b. $41,000\overline{}$
c. $41{,}050$

Solution

a. $41{,}000 \rightarrow$ thousands place Trailing zeros are not significant.

b. $41{,}000\overline{} \rightarrow$ ones place Overbar shows the zero is significant.

c. $41{,}050 \rightarrow$ tens place Last significant digit is in the tens place.

Example 2

Identify the precision (the place value of the rightmost significant digit) of each number.

a. 0.042
b. 0.0405
c. 0.04050

Solution

a. $0.042 \rightarrow$ thousandths place Leading zeros are not significant.

b. $0.0405 \rightarrow$ ten-thousandths place Last significant digit is the 5.

c. $0.04050 \rightarrow$ hundred-thousandths place Trailing zero is significant.

Exercise

Identify the precision (the place value of the rightmost significant digit) of each number.

a. $63{,}000$
b. $63{,}\overline{000}$
c. $63{,}020$
d. 0.058
e. 0.0506
f. 0.05060

Answer available at the end of this section.

Finding the Precision of a Measurement

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video walks through three examples of finding precision of a given measurement.

Precision when Adding and Subtracting

In Unit 1, we rounded numbers to a specific place value, such as the nearest tenth. In this lesson, we will use that same idea, but now the focus is on how precision affects calculations.

When adding or subtracting measurements, the result should be rounded to the least precise place value among the original numbers.

Suppose the attendance at a large event is estimated at $25{,}000$ people. If 3 people leave, the new attendance is not reported as $24{,}997$. The original estimate was only given to the nearest thousand, so reporting a more precise number would be misleading.

However, if the estimate changes by about $1{,}000$ people, the new attendance could be reported as $24{,}000$. This keeps the same level of precision as the original estimate.

When adding or subtracting numbers with different levels of precision, the result must be rounded to the least precise place value among the original numbers.

Do not round the original numbers. Perform the calculation first, then round the final answer.

Example 3

Add and round the result to the correct level of precision.

$12.4 \text{ cm} + 3.78 \text{ cm}$

Solution

$12.4 + 3.78 = 16.18$
$\rightarrow 16.2 \text{ cm}$ Round to the nearest tenth.

Example 4

Subtract and round the result to the correct level of precision.

$45.62 \text{ m} - 8.3 \text{ m}$

Solution

$45.62 - 8.3 = 37.32$
$\rightarrow 37.3 \text{ m}$ Round to the nearest tenth.

Exercise

Add or subtract. Round each result to the correct level of precision.

a. $7.25 \text{ L} + 2.8 \text{ L}$
b. $18.6 \text{ kg} - 4.32 \text{ kg}$
c. $3.482 \text{ m} + 1.7 \text{ m}$
d. $52.1 \text{ cm} - 6.78 \text{ cm}$

Answer available at the end of this section.

Example 5

A technician measures two sections of wire. One section is 2.35 meters long, and the other is 1.8 meters long. What is the total length?

Solution

$2.35 + 1.8 = 4.15$
$\rightarrow 4.2 \text{ m}$ Round to the nearest tenth.

The total length is 4.2 meters.

Example 6

A container holds 15.7 liters of fluid. After use, 6.28 liters remain. How much fluid was used?

Solution

$15.7 - 6.28 = 9.42$
$\rightarrow 9.4 \text{ L}$ Round to the nearest tenth.

Approximately 9.4 liters of fluid were used.

Exercise

Solve each problem. Round your answer to the correct level of precision.

a. A board measures 4.75 feet and another measures 2.6 feet. What is the total length?

b. A tank contains 28.4 gallons of fuel. After use, 9.73 gallons remain. How much fuel was used?

c. A pipe section is 6.382 meters long. A 2.4-meter section is removed. What is the remaining length?

d. Two cables measure 1.25 meters and 3.7 meters. What is their total length?

Answer available at the end of this section.

Before moving on, consider what happens when multiplying or dividing with exact numbers.

If one of the numbers is exact, it does not limit the precision of the result. For example, suppose an object weighs 4.37 ounces and you want the total weight of 3 identical objects. Multiplying 4.37×3 is the same as adding $4.37 + 4.37 + 4.37 = 13.11$ ounces. The result is still precise to the hundredths place.

Exact numbers, such as counted values, are treated as having unlimited precision. They do not affect how the final answer is rounded.

Answers

Exercise

a. thousands place
b. tens place
c. tens place
d. thousandths place
e. ten-thousandths place
f. hundred-thousandths place

Exercise

a. $10.1\ \text{L}$
b. $14.3\ \text{kg}$
c. $5.2\ \text{m}$
d. $45.3\ \text{cm}$

Exercise

a. 7.4 ft
b. 18.7 gal
c. 4.0 m
d. 5.0 m

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4.1.3: Error and Tolerance

In real-world situations, measurements and calculations are rarely exact. Whether you are measuring a length, weighing a part, or using a formula, there is usually some difference between the value you obtain and the true value.

To understand and describe this difference, we use several types of error. These include absolute error, relative error, tolerance, and greatest possible error. Each of these helps us measure how far a result is from what is expected and how much variation is acceptable.

Understanding error allows you to interpret results more accurately and make better decisions in technical settings.

Relative Error

Every measurement includes some amount of error, no matter how carefully it is taken. It is often helpful to think about the size of the error compared to the size of what is being measured. For example, a difference of 50 cents is important when pricing a low-cost item but not very important when pricing something much more expensive. In the same way, an error of one-eighth of an inch could matter when fitting a part into a tight space, but it would not matter when measuring a much larger object.

The expected outcome is the value we would have in a perfect situation. If a window screen is supposed to be exactly 25 inches wide, this value is the expected outcome. We treat it as though it has infinitely many significant digits.

To find the absolute error, subtract the measured value and the expected outcome. Because the expected outcome is treated as exact, the result should be reported using the same precision as the measurement.

Relative error is found by dividing the absolute error by the expected outcome. The result is often written as a percent. Some sources call this percent error, but the calculation is the same. This process is the same idea used when finding percent increase or percent decrease.

To Find Relative Error Written as a Percent

1. Subtract the measured value and the expected outcome to find the absolute error.
2. Divide the absolute error by the expected outcome to find the relative error.
3. Convert the result to a percent.

Example 1

Find the absolute error and relative error (written as a percent).

A board is supposed to be 24 inches long, but it is measured as 23.6 inches.

Solution

$$\text{Absolute error} = |23.6 - 24| = 0.4 \text{ in} \quad \text{Difference between measured and expected values.}$$

$$\begin{aligned} \text{Relative error} &= \frac{0.4}{24} = 0.0167 \\ &\to 1.67\% \quad \text{Convert to a percent.} \end{aligned}$$

Example 2

Find the absolute error and relative error (written as a percent).

A tank is expected to hold 50 liters, but it actually holds 52.3 liters.

Solution

$$\text{Absolute error} = |52.3 - 50| = 2.3 \text{ L}$$

$$\text{Relative error} = \frac{2.3}{50} = 0.046$$
$$\rightarrow 4.6\%$$

Exercise

Find the absolute error and relative error (written as a percent) for each situation.

a. A cable is expected to be 10 meters long but measures 9.7 meters.

b. A container should hold 32 gallons but actually holds 30.8 gallons.

c. A part is designed to be 5.0 cm wide but measures 5.3 cm.

d. A shipment is expected to weigh 120 kg but weighs 118.5 kg.

Answer available at the end of this section.

Precision, Accuracy, and Error

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This videos explains the various types of error and how they relate.

Tolerance

Tolerance is the maximum amount that a measurement is allowed to differ from the expected outcome.

For example, the U.S. Mint requires coins to have consistent size and weight so they function properly in machines. A dime has a weight of 2.268 grams with a tolerance of ± 0.091 grams.

This means the smallest acceptable weight is

$$2.268 - 0.091 = 2.177 \text{ grams}$$

and the largest acceptable weight is

$$2.268 + 0.091 = 2.359 \text{ grams}$$

Any dime with a weight outside this range would be considered unacceptable.

Tolerance Notation

A measurement written in the form

$\text{value} \pm \text{tolerance}$

means the actual value may be higher or lower than the stated value by the amount of the tolerance.

For example,

$2.268 \pm 0.091 \text{ g}$

means the measurement can vary by 0.091 grams above or below 2.268 grams.

Example 3

Find the minimum and maximum acceptable values.

A bolt is designed to be 5.00 ± 0.02 cm long.

Solution

$\text{Minimum} = 5.00 - 0.02 = 4.98 \text{ cm}$

$\text{Maximum} = 5.00 + 0.02 = 5.02 \text{ cm}$

The acceptable values range from 4.98 cm to 5.02 cm.

Example 4

Find the minimum and maximum acceptable values.

A tank is designed to hold 120 liters with a tolerance of ± 5 liters.

Solution

$\text{Minimum} = 120 - 5 = 115 \text{ L}$

$\text{Maximum} = 120 + 5 = 125 \text{ L}$

The acceptable values range from 115 L to 125 L.

Exercise

Find the minimum and maximum acceptable values for each measurement.

a. A bolt is 5.00 ± 0.02 cm long.

b. A tank holds 120 ± 5 liters.

c. A cable is 8.5 m long with a tolerance of ± 0.1 m.

d. A board is 12.0 inches long with a tolerance of ± 0.3 inches.

e. A pipe diameter is 3.25 ± 0.05 cm.

f. A container holds 45 gallons with a tolerance of ± 2 gallons.

Answer available at the end of this section.

Example 5

Find the absolute error and relative error (written as a percent).

A measurement is 8.50 ± 0.10 m.

Solution

$$\text{Absolute error} = 0.10 \text{ m} \quad \text{The tolerance gives the absolute error.}$$

$$\begin{aligned} \text{Relative error} &= \frac{0.10}{8.50} = 0.0118 \\ &\rightarrow 1.18\% \quad \text{Write as a percent.} \end{aligned}$$

Exercise

Find the absolute error and relative error (written as a percent) for each measurement.

a. 6.40 ± 0.08 m

b. 250 ± 5 kg

Answer available at the end of this section.

Greatest Possible Measurement Error (GPE)

Tolerance and greatest possible error are closely related ideas.

When a measurement is given with a tolerance, the tolerance represents the largest amount the value can differ from the expected outcome. In this case, the tolerance is the greatest possible error.

When a measurement is rounded instead of given with a tolerance, the greatest possible error comes from the rounding. The actual value may be slightly higher or lower than the reported value.

For example, if a scale reports a weight of 23 pounds to the nearest pound, the actual weight could be up to 0.5 pounds higher or lower than the reported value. This can be written as 23 ± 0.5.

As the level of precision increases, the greatest possible error decreases. More precise measurements have smaller possible errors.

Greatest Possible Error (GPE)

If a tolerance is not given, the greatest possible error can be calculated using the precision of the measurement.

The greatest possible error is found by dividing the precision by 2.

$$\text{GPE} = \frac{\text{precision}}{2}$$

The precision is the place value to which the measurement is rounded. For example, rounding to the nearest tenth means the precision is 0.1.

Example 6

Find the greatest possible error.

A measurement is 12.0 ± 0.3 cm.

Solution

$\text{GPE} = 0.3 \text{ cm}$ The tolerance gives the greatest possible error.

Exercise

Find the greatest possible error for each measurement.

a. 8.5 ± 0.1 m

b. 250 ± 5 kg

Answer available at the end of this section.

Example 7

Find the greatest possible error for each measurement.

a. 45 cm
b. 12.3 m

Solution

a. $\text{Precision} = 1 \text{ cm}$ Rightmost significant digit is in the ones place.

$$\text{GPE} = \frac{1}{2} = 0.5 \text{ cm}$$
$$\rightarrow 45 \pm 0.5 \text{ cm}$$

b. $\text{Precision} = 0.1 \text{ m}$ Rightmost significant digit is in the tenths place.

$$\text{GPE} = \frac{0.1}{2} = 0.05 \text{ m}$$
$$\rightarrow 12.3 \pm 0.05 \text{ m}$$

Example 8

Find the greatest possible error for each measurement.

a. 78.6 kg
b. 320 L

Solution

a. Precision $= 0.1$ kg Rightmost significant digit is in the tenths place.

$$\text{GPE} = \frac{0.1}{2} = 0.05 \text{ kg}$$
$$\rightarrow 78.6 \pm 0.05 \text{ kg}$$

b. Precision $= 10$ L Rightmost significant digit is in the tens place.

$$\text{GPE} = \frac{10}{2} = 5 \text{ L}$$
$$\rightarrow 320 \pm 5 \text{ L}$$

Exercise

Find the greatest possible error for each measurement.

a. 62 ft
b. 9.4 cm
c. 150 kg
d. 3.27 m

Answer available at the end of this section.

Greatest Possible Error

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to find the GPE of three measurement values.

Answers

Exercise

a. Absolute error: 0.3 m, Relative error: 3%
b. Absolute error: 1.2 gal, Relative error: 3.75%
c. Absolute error: 0.3 cm, Relative error: 6%
d. Absolute error: 1.5 kg, Relative error: 1.25%

Exercise

a. Minimum: 4.98 cm, Maximum: 5.02 cm
b. Minimum: 115 L, Maximum: 125 L
c. Minimum: 8.4 m, Maximum: 8.6 m
d. Minimum: 11.7 in, Maximum: 12.3 in
e. Minimum: 3.20 cm, Maximum: 3.30 cm
f. Minimum: 43 gal, Maximum: 47 gal

Exercise

a. Absolute error: 0.08 m, Relative error: 1.25%
b. Absolute error: 5 kg, Relative error: 2%

Exercise

a. 0.1 m
b. 5 kg

Exercise

a. 62 ± 0.5 ft
b. 9.4 ± 0.05 cm
c. 150 ± 5 kg

d. 3.27 ± 0.005 m

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4.1.4: Measurement and Significant Digits Practice

The following set of exercises provides practice for the concepts covered in Module 4.1: Measurement and Significant Digits. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 4.1.1 - Accuracy and Significant Digits

Identify each number as exact or an approximation.

1. 36 students

2. 7.5 meters

Determine the number of significant digits in each number.

3. $4{,}200$

4. 0.0305

Round each number to the indicated number of significant digits.

5. $58{,}739$ (three significant digits)

6. 2.864 (two significant digits)

Section 4.1.2 - Precision

Identify the precision (place value of the rightmost significant digit).

7. $72{,}000$

8. 6.40

Sections 4.1.1 & 4.1.2 Mixed Practice

Perform each operation and round to the correct accuracy or precision as needed.

9. $12.4\ \text{m} + 3.78\ \text{m}$

10. 2.46×3.1

11. $45.2\ \text{kg} - 7.83\ \text{kg}$

12. $18.72 \div 4.5$

13. A board measures 4.75 ft and another measures 2.3 ft. Find the total length.

14. A machine produces 3.6 parts per minute for 12.4 minutes. How many parts are produced?

Section 4.1.3 - Error and Tolerance

Find the absolute error and relative error written as a percent.

15. A board should be 24 in but measures 23.5 in.

16. A tank should hold 50 L but holds 52.1 L.

Find the minimum and maximum acceptable values.

17. 6.50 ± 0.05 m

18. 120 ± 5 L

Find the absolute error and relative error written as a percent.

19. 8.50 ± 0.10 m

Find the greatest possible error.

20. 12.0 ± 0.3 cm

21. 150 ± 5 kg

22. 18 ft

23. 7.2 m

24. 320 cm

25. 0.84 m

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4.2: U.S. CUSTOMARY MEASUREMENTS

How did a system of measurement end up with 12 inches in a foot, 3 feet in a yard, 5,280 feet in a mile, and 16 ounces in a pound? Before beginning this module, watch the video, which tells the story of two kingdoms — the Metric Kingdom and the Customary Kingdom. Life in the Customary Kingdom is harder, messier, and far more reliant on fractions. The frustration eventually reaches a breaking point. Keep that story in mind as you work through the conversions ahead; the irregularity you'll encounter is not a coincidence.

This module covers three areas of the U.S. Customary system. You will begin with units of length, weight, and time, learning to convert between them accurately within the system. From there, you will extend those skills to units of area and volume, where the conversions become less intuitive and require careful reasoning. The module closes with practical applications that draw on all of these skills together, asking you to solve real-world problems using U.S. Customary measurements.

By the end, you will be able to move fluently between units in the U.S. Customary system — even when the relationships between them are anything but simple.

Metric vs. Customary Measurement

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View this YouTube Video online

This video tells the story of Metric Kingdom and the Customary Kingdom as a way to illustrate the differences between the two units.

4.2.1: Length, Weight and Time in the U.S. Customary System

4.2.2: Area and Volume in U.S. Customary Units

4.2.3: Applications Involving U.S. Customary Measurements

4.2.4: U.S. Customary Measurements Practice

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4.2.1: Length, Weight and Time in the U.S. Customary System

In everyday work, measurements show up constantly — distances on a job site, weights of materials, and time spent completing a task. In the United States, these measurements are often given in U.S. customary units such as feet, pounds, and hours. Being able to convert between these units allows you to compare quantities, follow instructions accurately, and avoid costly mistakes. In this section, you will review common U.S. customary units for length, weight, and time, practice converting between them, and apply these ideas to compound units such as speed, where measurements are combined to describe real-world situations.

Units of Length

The most common units of length are the inch, foot, yard, and mile. Our focus will be on the technique used to convert from one unit of length to another.

American Units of Length

Facts relating common units of length.

- 1 foot (ft) = 12 inches (in)
- 1 yard (yd) = 3 feet (ft)
- 1 mile (mi) = 5280 feet (ft)

Take for example, the fact that there are 3 feet in 1 yard, which can be stated as an equation, using the common abbreviations for feet (ft) and yards (yd).

$$3 \text{ ft} = 1 \text{ yd}$$

If we divide both sides of the equation by 3 ft,

$$\frac{3 \text{ ft}}{3 \text{ ft}} = \frac{1 \text{ yd}}{3 \text{ ft}},$$

or equivalently,

$$1 = \frac{1 \text{ yd}}{3 \text{ ft}}.$$

The key observation is the fact that the ratio 1 yd/3 ft equals the number 1. Consequently, multiplying by the “conversion factor” 1 yd/3 ft is equivalent to multiplying by 1. This can be used to change a measurement in feet to yards.

Example 1

Change 36 feet to yards.

Solution

Multiply by the conversion factor 1 yd/3 ft.

$$
\begin{aligned}
36\text{ ft} &= 36\text{ ft} \cdot 1 && \text{Multiplicative Identity Property.}\\
&= 36\text{ ft} \cdot \frac{1\text{ yd}}{3\text{ ft}} && \text{Replace 1 with 1 yd/3 ft.}\\
&= 36\,\cancel{\text{ft}} \cdot \frac{1\text{ yd}}{3\,\cancel{\text{ft}}} && \text{Cancel common unit.}\\
&= \frac{36 \cdot 1}{3}\text{ yd} && \text{Multiply fractions.}\\
&= \frac{36}{3}\text{ yd} && \text{Simplify.}\\
&= 12\text{ yd} && \text{Divide.}
\end{aligned}
$$

Hence, 36 feet equals 12 yards.

Exercise

Change 81 feet to yards.

Answer available at the end of this section.

Example 2

Change 18 yards to feet.

Solution

Multiply by the conversion factor 3 ft/1 yd.

$$
\begin{aligned}
18\text{ yd} &= 18\text{ yd} \cdot 1 && \text{Multiplicative Identity Property.}\\
&= 18\text{ yd} \cdot \frac{3\text{ ft}}{1\text{ yd}} && \text{Replace 1 with } \frac{3\text{ ft}}{1\text{ yd}}.\\
&= 18\,\cancel{\text{yd}} \cdot \frac{3\text{ ft}}{1\,\cancel{\text{yd}}} && \text{Cancel common unit.}\\
&= \frac{18 \cdot 3}{1}\text{ ft} && \text{Multiply fractions.}\\
&= 54\text{ ft} && \text{Simplify.}
\end{aligned}
$$

Hence, 18 yards equals 54 feet.

Exercise

Change 15 yards to feet.

Answer available at the end of this section.

How to Convert Units

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This video includes a "formula" for how to set up conversion problems.

Note

Notice that in Example 1, we converted feet to yards, so we used the conversion factor $\frac{1\text{ yd}}{3\text{ ft}}$ to cancel feet and leave yards.

In the next example, we reversed the process by choosing a conversion factor that canceled yards and left feet.

Example 3

Change 24 inches to feet.

Solution

Multiply by the conversion factor 1 ft/12 in.

$$\begin{aligned} 24\text{ in} &= 24\text{ in} \cdot 1 && \text{Multiplicative Identity Property.} \\ &= 24\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} && \text{Replace 1 with 1 ft/12 in.} \\ &= 24\,\cancel{\text{in}} \cdot \frac{1\text{ in}}{12\,\cancel{\text{in}}} && \text{Cancel common unit.} \\ &= \frac{24 \cdot 1}{12}\text{ ft} && \text{Multiply fractions.} \\ &= 2\text{ ft} && \text{Simplify.} \end{aligned}$$

Hence, 24 inches equals 2 feet.

Exercise

Change 48 inches to feet.

Answer available at the end of this section.

We provide a summary of conversion factors for units of length in Table 4.2.1.1.

Table 4.2.1.1: Conversion factors for units of length.

Convert	Conversion Factor	Convert	Conversion Factor
feet to inches	$\frac{12\text{ in}}{1\text{ ft}}$	inches to feet	$\frac{1\text{ ft}}{12\text{ in}}$
yards to feet	$\frac{3\text{ ft}}{1\text{ yd}}$	feet to yards	$\frac{1\text{ yd}}{3\text{ ft}}$
miles to feet	$\frac{5280\text{ ft}}{1\text{ mi}}$	feet to miles	$\frac{1\text{ mi}}{5280\text{ ft}}$

Some conversions require more than one application of a conversion factor.

Example 4

Change 4 yards to inches.

Solution

We multiply by a chain of conversion factors, the first to change yards to feet, the second to change feet to inches.

$$
\begin{aligned}
4\text{ yd} &= 4\text{ yd} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{12\text{ in}}{1\text{ ft}} && \text{Multiply by conversion factors.}\\
&= 4\ \cancel{\text{yd}} \cdot \frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}} \cdot \frac{12\text{ in}}{1\ \cancel{\text{ft}}} && \text{Cancel common units.}\\
&= \frac{4 \cdot 3 \cdot 12}{1 \cdot 1}\text{ in} && \text{Multiply fractions.}\\
&= 144\text{ in} && \text{Simplify.}
\end{aligned}
$$

Hence, 4 yards equals 144 inches.

Exercise

Change 8 yards to inches.

Answer available at the end of this section.

Example 5

Change 2 miles to yards.

Solution

We multiply by a chain of conversion factors, the first to change miles to feet, the second to change feet to yards.

$$
\begin{aligned}
2\text{ mi} &= 2\text{ mi} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{1\text{ yd}}{3\text{ ft}} && \text{Multiply by conversion factors.}\\
&= 2\ \cancel{\text{mi}} \cdot \frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{mi}}} \cdot \frac{1\text{ yd}}{3\ \cancel{\text{ft}}} && \text{Cancel common units.}\\
&= \frac{2 \cdot 5280 \cdot 1}{1 \cdot 3}\text{ yd} && \text{Multiply fractions.}\\
&= 3520\text{ yd} && \text{Simplify.}
\end{aligned}
$$

Hence, 2 miles equals 3,520 yards.

Exercise

Change 5 miles to yards.

Answer available at the end of this section.

Common Customary Length Conversions

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View this YouTube Video online

This video shows examples of converting between U.S. Customary length units.

Units of Weight

The most common units of weight are the ounce, pound, and ton. Our focus will remain on how to convert from one unit to another.

American Units of Weight

Facts relating common units of weight.

- 1 pound (lb) = 16 ounces (oz)
- 1 ton = 2000 pounds (lb)

The above facts lead to the conversion factors in Table 4.2.1.2.

Table 4.2.1.2: Conversion factors for units of weight.

Convert	Conversion Factor	Convert	Conversion Factor
pounds to ounces	$\frac{16\text{ oz}}{1\text{ lb}}$	ounces to pounds	$\frac{1\text{ lb}}{16\text{ oz}}$
tons to pounds	$\frac{2000\text{ lb}}{1\text{ ton}}$	pounds to tons	$\frac{1\text{ ton}}{2000\text{ lb}}$

Example 6

Change $2\frac{1}{2}$ pounds to ounces.

Solution

Multiply by the appropriate conversion factor.

$$\begin{aligned} 2\frac{1}{2}\text{ lb} &= 2\frac{1}{2}\text{ lb}\cdot\frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factor.}\\ &= 2\frac{1}{2}\cancel{\text{lb}}\cdot\frac{16\text{ oz}}{1\cancel{\text{ lb}}} && \text{Cancel common units.}\\ &= \left(2\frac{1}{2}\cdot 16\right)\text{ oz} && \text{Multiply fractions.}\\ &= \left(\frac{5}{2}\cdot 16\right)\text{ oz} && \text{Mixed to improper fraction.}\\ &= \frac{80}{2}\text{ oz} && \text{Multiply.}\\ &= 40\text{ oz} && \text{Divide.} \end{aligned}$$

Hence, $2\frac{1}{2}$ pounds equals 40 ounces.

Exercise

Change $6\frac{1}{4}$ pounds to ounces.

Answer available at the end of this section.

Example 7

Change 3.2 tons to ounces.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$\begin{aligned} 3.2 \text{ ton } &= 3.2 \text{ ton } \cdot \frac{2000 \text{ lb}}{1 \text{ ton}} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} && \text{Multiply by conversion factors.} \\ &= 3.2 \cancel{\text{ ton}} \cdot \frac{2000 \cancel{\text{ lb}}}{1 \cancel{\text{ ton}}} \cdot \frac{16 \text{ oz}}{1 \cancel{\text{ lb}}} && \text{Cancel common units.} \\ &= \frac{3.2 \cdot 2000 \cdot 16}{1 \cdot 1} \text{ oz} && \text{Multiply fractions.} \\ &= 102,400 \text{ oz} && \text{Simplify.} \end{aligned}$$

Hence, 3.2 tons equals 102,400 ounces.

Exercise

Change 4.1 tons to ounces.

Answer available at the end of this section.

Common Customary Weight Conversions

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View this YouTube Video online

This video shows examples of converting between U.S. Customary weight units.

Units of Time

The most common units of time are seconds, minutes, hours, days, and years.

American Units of Time

Facts relating common units of time.

- 1 minute (min) = 60 seconds (s)
- 1 hour (hr) = 60 minutes (min)
- 1 day (day) = 24 hours (hr)
- 1 year (yr) = 365 days (day)

These facts lead to the conversion factors in Table 6.4.

Table 4.2.1.4: Conversion factors for units of time.

Convert	Conversion Factor	Convert	Conversion Factor
minutes to seconds	$\frac{60 \text{ s}}{1 \text{ min}}$	seconds to minutes	$\frac{1 \text{ min}}{60 \text{ s}}$
hours to minutes	$\frac{60 \text{ min}}{1 \text{ hr}}$	minutes to hours	$\frac{1 \text{ hr}}{60 \text{ min}}$
days to hours	$\frac{24 \text{ hr}}{1 \text{ day}}$	hours to days	$\frac{1 \text{ day}}{24 \text{ hr}}$

Convert	Conversion Factor	Convert	Conversion Factor
years to days	$\frac{365\text{ day}}{1\text{ yr}}$	days to years	$\frac{1\text{ yr}}{365\text{ day}}$

Example 9

How many seconds in a year?

Solution

A chain of conversion factors is needed.

$$1\text{ yr} = 1\text{ yr} \cdot \frac{365\text{ day}}{1\text{ yr}} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{60\text{ s}}{1\text{ min}} \quad \text{Conversion factors.}$$

$$= 1\ \cancel{\text{yr}} \cdot \frac{365\ \cancel{\text{day}}}{1\ \cancel{\text{yr}}} \cdot \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \cdot \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \cdot \frac{60\text{ s}}{1\ \cancel{\text{min}}} \quad \text{Cancel common units.}$$

$$= \frac{1 \cdot 365 \cdot 24 \cdot 60 \cdot 60}{1 \cdot 1 \cdot 1 \cdot 1}\text{ s} \quad \text{Multiply fractions.}$$

$$= 31,536,000\text{ s} \quad \text{Simplify.}$$

Thus, 1 year equals 31,536,000 seconds.

Exercise

How many seconds in a day?

Answer available at the end of this section.

Convert Hours and Days to Seconds Using Unit Fractions

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This video shows examples of converting between time units.

Converting Units of Speed

Example 10

A professional pitcher can throw a baseball at 95 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Solution

There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

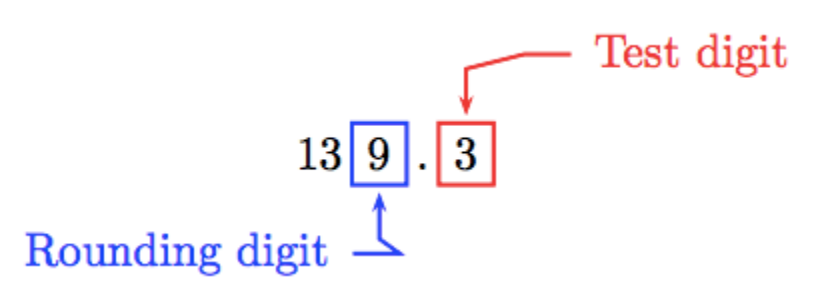
$$95\frac{\text{mi}}{\text{h}} \approx 95\frac{\text{mi}}{\text{h}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} \cdot \frac{1\ \text{h}}{60\ \text{min}} \cdot \frac{1\ \text{min}}{60\ \text{s}} \quad \text{Conversion factors.}$$

$$\approx 95\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\ \text{ft}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ \text{s}} \quad \text{Cancel common units.}$$

$$\approx \frac{95 \cdot 5280 \cdot 1 \cdot 1}{1 \cdot 60 \cdot 60}\frac{\text{ft}}{\text{s}} \quad \text{Multiply fractions.}$$

$$\approx 139.3\frac{\text{ft}}{\text{s}} \quad \text{Multiply and divide.}$$

To round to the nearest foot per second, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, to the nearest foot per second, the speed is approximately 139 feet per second.

Whew! Since the batter stands at home plate, which is about 60 feet from where the pitch is delivered, the batter has less than 1/2 a second to react to the pitch!

Exercise

A women's softball pitcher can throw her fastball at 60 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Answer available at the end of this section.

Convert Miles Per Hour to Feet Per Second

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This video demonstrates how to convert miles per hour to feet per second.

Answers

Exercise
27 yards

Exercise
45 feet

Exercise
4 feet

Exercise
288 inches

Exercise
8,800 yards

Exercise
100 ounces

Exercise
128,000 ounces

Exercise
86,400 seconds

Exercise
88 feet per second

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4.2.2: Area and Volume in U.S. Customary Units

Units of Area

Area measures the amount of surface covered. It is expressed in square units, which are based on multiplying a length by a width.

The most common units of area in U.S. customary measurement are square inches, square feet, square yards, and acres. We will focus on converting from one unit to another.

Area of Conversion Facts

Facts relating common units of area.

- $1\ \text{ft}^2 = 144\ \text{in}^2$
- $1\ \text{yd}^2 = 9\ \text{ft}^2$
- $1\ \text{yd}^2 = 1296\ \text{in}^2$
- $1\ \text{mi}^2 = 640\ \text{acres}$

Remember that area units are squared, so the conversion factor must also be squared.

These facts lead to the conversion factors listed in Table 4.2.2.1.

Table 4.2.2.1: Conversion factors for units of area.

Convert	Conversion Factor	Convert	Conversion Factor
square feet to square inches	$\frac{1\ \text{ft}^2}{144\ \text{in}^2}$	square inches to square feet	$\frac{144\ \text{in}^2}{1\ \text{ft}^2}$
square yards to square feet	$\frac{1\ \text{yd}^2}{9\ \text{ft}^2}$	square feet to square yards	$\frac{9\ \text{ft}^2}{1\ \text{yd}^2}$
square yards to square inches	$\frac{1\ \text{yd}^2}{1296\ \text{in}^2}$	square inches to square yards	$\frac{1296\ \text{in}^2}{1\ \text{yd}^2}$
square miles to acres	$\frac{1\ \text{mi}^2}{640\ \text{acres}}$	acres to square miles	$\frac{640\ \text{acres}}{1\ \text{mi}^2}$

Example 1

Change $7\ \text{yd}^2$ to square feet.

Solution

$$7\ \text{yd}^2 = 7\ \text{yd}^2 \cdot \frac{9\ \text{ft}^2}{1\ \text{yd}^2} \qquad \text{Choose a conversion factor that cancels square yards.}$$

$$= 7\ \cancel{\text{yd}^2} \cdot \frac{9\ \text{ft}^2}{1\ \cancel{\text{yd}^2}} \qquad \text{Cancel common units.}$$

$$= 63\ \text{ft}^2 \qquad \text{Simplify.}$$

Hence, $7\ \text{yd}^2$ equals $63\ \text{ft}^2$.

Example 2

Change $2\ \text{yd}^2$ to square inches.

Solution

$$\begin{aligned} 2\ \text{yd}^2 &= 2\ \text{yd}^2 \cdot \frac{9\ \text{ft}^2}{1\ \text{yd}^2} && \text{Convert to square feet.} \\ &= 18\ \text{ft}^2 \cdot \frac{144\ \text{in}^2}{1\ \text{ft}^2} && \text{Convert to square inches.} \\ &= 2592\ \text{in}^2 && \text{Simplify.} \end{aligned}$$

Exercise

Change $5\ \text{ft}^2$ to square inches.

Answer

Answer available at the end of this section.

Unit Conversions of Area and Volume

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This video demonstrates how to convert U.S. Customary Units with squared units.

Units of Volume and Liquid Capacity

Volume measures space, while liquid capacity describes how much a container can hold. Both represent the same idea but use different units depending on the situation.

Because of this, liquid capacity is typically measured using its own set of units rather than cubic units.

The most common units of liquid capacity are fluid ounces, cups, pints, quarts, and gallons. We will focus on converting from one unit to another.

Liquid Capacity Conversion Facts

Facts relating common units of liquid capacity.

- 1 cup (c) = 8 fluid ounces (fl oz)
- 1 pint (pt) = 2 cups (c)
- 1 quart (qt) = 2 pints (pt)
- 1 gallon (gal) = 4 quarts (qt)

These facts lead to the conversion factors listed in Table 4.2.2.2.

Table 4.2.2.2: Conversion factors for units of liquid capacity.

Convert	Conversion Factor	Convert	Conversion Factor
cups to ounces	$\frac{8\ \text{fl oz}}{1\ \text{c}}$	ounces to cups	$\frac{1\ \text{c}}{8\ \text{fl oz}}$

Convert	Conversion Factor	Convert	Conversion Factor
pints to cups	$\frac{2\text{ c}}{1\text{ pt}}$	cups to pints	$\frac{1\text{ pt}}{2\text{ c}}$
quarts to pints	$\frac{2\text{ pt}}{1\text{ qt}}$	pints to quarts	$\frac{1\text{ qt}}{2\text{ pt}}$
gallons to quarts	$\frac{4\text{ qt}}{1\text{ gal}}$	quarts to gallons	$\frac{1\text{ gal}}{4\text{ qt}}$

Example 3

Change 5.6 gallons to pints.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$\begin{aligned} 5.6\text{ gal} &= 5.6\text{ gal} \cdot \frac{4\text{ qt}}{1\text{ gal}} \cdot \frac{2\text{ pt}}{1\text{ qt}} && \text{Multiply by conversion factors.} \\ &= 5.6\ \cancel{\text{gal}} \cdot \frac{4\ \cancel{\text{qt}}}{1\ \cancel{\text{gal}}} \cdot \frac{2\text{ pt}}{1\ \cancel{\text{qt}}} && \text{Cancel common units.} \\ &= \frac{5.6 \cdot 4 \cdot 2}{1 \cdot 1}\text{ pt} && \text{Multiply fractions.} \\ &= 44.8\text{ pt} && \text{Simplify.} \end{aligned}$$

Hence, 5.6 gallons equals 44.8 pints.

Exercise

Change 6 quarts to cups.

Answer available at the end of this section.

Fluid Ounces to Gallons

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This video demonstrates how to convert from ounces to gallons.

While liquid capacity uses units like quarts and gallons, volume is often measured using cubic units. Since these two types of units are connected, we can convert between liquid capacity units and cubic units.

Cubic Units of Volume Conversion Facts

Facts relating common units of liquid capacity.

- $1\text{ ft}^3 = 1728\text{ in}^3$
- $1\text{ yd}^3 = 27\text{ ft}^3$
- $1\text{ yd}^3 = 46{,}656\text{ in}^3$

- $1\text{ gal} = 231\text{ in}^3$
- $1\text{ ft}^3 \approx 7.48\text{ gal}$
- $1\text{ fl oz} \approx 1.805\text{ in}^3$
- $1\text{ pt} = 28.875\text{ in}^3$

These facts lead to the conversion factors listed in Table 4.2.2.3.

Table 4.2.2.3: Conversion factors for cubic units of volume.

Convert	Conversion Factor	Convert	Conversion Factor
cubic feet to cubic inches	$\frac{1728\text{ in}^3}{1\text{ ft}^3}$	cubic inches to cubic feet	$\frac{1\text{ ft}^3}{1728\text{ in}^3}$
cubic yards to cubic feet	$\frac{27\text{ ft}^3}{1\text{ yd}^3}$	cubic feet to cubic yards	$\frac{1\text{ yd}^3}{27\text{ ft}^3}$
cubic yards to cubic inches	$\frac{46656\text{ in}^3}{1\text{ yd}^3}$	cubic inches to cubic yards	$\frac{1\text{ yd}^3}{46656\text{ in}^3}$
gallon to cubic inches	$\frac{231\text{ in}^3}{1\text{ gal}}$	cubic inches to gallon	$\frac{1\text{ gal}}{231\text{ in}^3}$
cubic feet to gallons	$\frac{7.48\text{ gal}}{1\text{ ft}^3}$	gallons to cubic feet	$\frac{1\text{ ft}^3}{7.48\text{ gal}}$
fluid ounces to cubic inches	$\frac{1.805\text{ in}^3}{1\text{ fl oz}}$	cubic inches to fluid ounces	$\frac{1\text{ fl oz}}{1.805\text{ in}^3}$
pint to cubic inches	$\frac{28.875\text{ in}^3}{1\text{ pt}}$	cubic inches to pint	$\frac{1\text{ pt}}{28.875\text{ in}^3}$

Example 4

Change 4 ft^3 to cubic inches.

Solution

$$\begin{aligned} 4\text{ ft}^3 &= 4\text{ ft}^3 \cdot \frac{1728\text{ in}^3}{1\text{ ft}^3} && \text{Choose a conversion factor that cancels cubic feet.} \\ &= 4\,\cancel{\text{ft}^3} \cdot \frac{1728\text{ in}^3}{1\,\cancel{\text{ft}^3}} && \text{Cancel common units.} \\ &= 6912\text{ in}^3 && \text{Simplify.} \end{aligned}$$

Hence, 4 ft^3 equals 6912 in^3.

Example 5

Change 2 yd^3 to cubic inches.

Solution

$$\begin{aligned} 2\text{ yd}^3 &= 2\text{ yd}^3 \cdot \frac{27\text{ ft}^3}{1\text{ yd}^3} && \text{Convert to cubic feet.} \\ &= 54\text{ ft}^3 \cdot \frac{1728\text{ in}^3}{1\text{ ft}^3} && \text{Convert to cubic inches.} \\ &= 93{,}312\text{ in}^3 && \text{Simplify.} \end{aligned}$$

Exercise

Change 5 ft^3 to gallons.

Answer

Answer available at the end of this section.

Unit Conversions of Area and Volume

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This video demonstrates how to convert square units and cubic units within the U.S. Customary system.

Answers

Exercise

720 in^2

Exercise

24 cups

Exercise

37.4 gal

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4.2.3: Applications Involving U.S. Customary Measurements

In technical work, measurements rarely stay in one unit. A length might be given in feet but needed in inches. A fluid amount might be measured in gallons but used in quarts. Rates like speed or flow add another layer by combining units.

The key skill is not memorizing conversions — it is setting them up so the units cancel correctly. Once the setup is right, the calculation becomes straightforward.

In this section, you will apply unit conversions in a variety of real-world situations, including length, weight, area, volume, liquid capacity, and compound units.

Applications Involving Length

Example 1

A section of conduit runs 7.5 feet along a wall. What is the length of the conduit in inches? Round your answer to the nearest whole inch.

Solution

$$7.5 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 90 \text{ in} \quad \text{Convert feet to inches.}$$

The conduit is 90 inches long.

Example 2

A technician installs 4 sections of cable, each measuring 2.25 yards. What is the total length of cable in feet?

Solution

Solution

$$4 \cdot 2.25 \text{ yd} = 9 \text{ yd} \quad \text{Find the total length in yards.}$$
$$9 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} = 27 \text{ ft} \quad \text{Convert yards to feet.}$$

The total length of cable is 27 feet.

Exercise

A fiber optic line will be installed along a 2.4-mile long. What is the total distance in feet?

Answer available at the end of this section.

Applications Involving Weight

Example 3

A shipment of metal parts weighs 2.75 tons. What is the total weight in pounds?

Solution

$$2.75 \text{ tons} \cdot \frac{2000 \text{ lb}}{1 \text{ ton}} = 5500 \text{ lb} \quad \text{Convert tons to pounds.}$$

The shipment weighs 5500 pounds.

Example 4

A mechanic installs 6 identical parts, each weighing 18.5 ounces. What is the total weight in pounds?

Solution

$$\begin{aligned} 6 \cdot 18.5 \text{ oz} &= 111 \text{ oz} && \text{Find the total weight in ounces.} \\ 111 \text{ oz} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} &= 6.9375 \text{ lb} && \text{Convert ounces to pounds.} \\ &\approx 6.9 \text{ lb} && \text{Round to the nearest tenth.} \end{aligned}$$

The total weight of the parts is 6.9 pounds.

Exercise

A truck is carrying 3.5 tons of gravel. The gravel will be divided evenly into 7 smaller loads. What is the weight of each load in pounds?

Answer available at the end of this section.

Applications Involving Time

Example 4

A machine runs continuously for 2.5 hours during a production cycle. The machine's output is tracked in seconds. How long does the machine run in seconds?

Solution

$$\begin{aligned} 2.5 \text{ hr} \cdot \frac{60 \text{ min}}{1 \text{ hr}} &= 150 \text{ min} && \text{Convert hours to minutes.} \\ 150 \text{ min} \cdot \frac{60 \text{ s}}{1 \text{ min}} &= 9000 \text{ s} && \text{Convert minutes to seconds.} \end{aligned}$$

The machine runs for 9000 seconds.

Example 5

A quality control system runs a 45-second test cycle. It completes 120 cycles. How long does the testing take in minutes?

Solution

$$\begin{aligned} 120 \cdot 45 \text{ s} &= 5400 \text{ s} && \text{Find the total time in seconds.} \\ 5400 \text{ s} \cdot \frac{1 \text{ min}}{60 \text{ s}} &= 90 \text{ min} && \text{Convert seconds to minutes.} \end{aligned}$$

The testing takes 90 minutes.

Exercise

A recording session lasts 375 minutes. How long is the session in hours? Round your answer to the nearest tenth of an hour.

Answer available at the end of this section.

Applications Involving Area

Example 5

A panel has an area of 6 square feet. What is the area in square inches?

Solution

$$6\ \text{ft}^2 \cdot \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^2 = 6 \cdot 144\ \text{in}^2 \quad \text{Convert square feet to square inches.}$$
$$= 864\ \text{in}^2 \quad \text{Multiply.}$$

The area is 864 square inches.

Example 6

An office space includes two sections measuring 180 square feet and 135 square feet. Carpet is sold by the square yard. How many square yards of carpet are needed?

Solution

$$180\ \text{ft}^2 + 135\ \text{ft}^2 = 315\ \text{ft}^2 \quad \text{Find the total area in square feet.}$$
$$315\ \text{ft}^2 \cdot \left(\frac{1\ \text{yd}}{3\ \text{ft}}\right)^2 = 315 \cdot \frac{1}{9}\ \text{yd}^2 \quad \text{Convert to square yards.}$$
$$= 35\ \text{yd}^2 \quad \text{Simplify.}$$

35 square yards of carpet are needed.

Exercise

A property includes 1.2 acres of land. A landscaping plan requires measurements in square feet. What is the area of the property in square feet?

Answer available at the end of this section.

Applications Involving Liquid Capacity

Example 7

A cooling system holds 6 quarts of fluid. How many pints does it hold?

Solution

$6 \text{ qt} \cdot \frac{2 \text{ pt}}{1 \text{ qt}} = 12 \text{ pt}$ Convert quarts to pints.

The system holds 12 pints.

Example 8

A storage tank contains 5 gallons of cleaning solution. After use, 3.5 quarts remain. How many gallons of solution were used? Round your answer to the nearest tenth of a gallon.

Solution

$$\begin{aligned} 5 \text{ gal} \cdot \frac{4 \text{ qt}}{1 \text{ gal}} &= 20 \text{ qt} && \text{Convert gallons to quarts.} \\ 20 \text{ qt} - 3.5 \text{ qt} &= 16.5 \text{ qt} && \text{Find the amount used.} \\ 16.5 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} &= 4.125 \text{ gal} && \text{Convert quarts to gallons.} \\ &\approx 4.1 \text{ gal} && \text{Round to the nearest tenth.} \end{aligned}$$

4.1 gallons of solution were used.

Exercise

A tank contains 20 quarts of liquid. The liquid will be poured evenly into containers that each hold 2.5 gallons. How many containers can be filled?

Answer available at the end of this section.

Applications Involving Volume

Example 7

A storage box has a volume of 2 cubic feet. What is the volume in cubic inches?

Solution

$$\begin{aligned} 2 \text{ ft}^3 \cdot \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)^3 &= 2 \cdot 1728 \text{ in}^3 && \text{Convert cubic feet to cubic inches.} \\ &= 3456 \text{ in}^3 && \text{Multiply.} \end{aligned}$$

The volume is 3456 cubic inches.

Example 8

A load of material measures 3 cubic yards. After use, 18 cubic feet remain. How much material was used in cubic yards?

Solution

$$3\ \text{yd}^3 \cdot \left(\frac{3\ \text{ft}}{1\ \text{yd}}\right)^3 = 81\ \text{ft}^3 \quad \text{Convert cubic yards to cubic feet.}$$
$$81\ \text{ft}^3 - 18\ \text{ft}^3 = 63\ \text{ft}^3 \quad \text{Find the amount used.}$$
$$63\ \text{ft}^3 \cdot \left(\frac{1\ \text{yd}}{3\ \text{ft}}\right)^3 = 63 \cdot \frac{1}{27}\ \text{yd}^3 \quad \text{Convert back to cubic yards.}$$
$$= 2.333\ldots\ \text{yd}^3 \quad \text{Simplify.}$$
$$\approx 2.3\ \text{yd}^3 \quad \text{Round to the nearest tenth.}$$

2.3 cubic yards of material were used.

Exercise

A bin contains 864 cubic inches of material. The material will be divided evenly into containers that each hold 0.25 cubic feet. How many containers can be filled?

Answer available at the end of this section.

Applications Involving Compound Units

Example 9

A vehicle travels at a speed of 65 miles per hour. What is this speed in feet per second? Round your answer to the nearest whole number.

Solution

$$65\frac{\text{mi}}{\text{hr}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} \cdot \frac{1\ \text{hr}}{3600\ \text{s}} = \frac{343{,}200}{3600}\frac{\text{ft}}{\text{s}} \quad \text{Convert miles to feet and hours to seconds.}$$
$$\approx 95\frac{\text{ft}}{\text{s}} \quad \text{Round to the nearest whole number.}$$

The speed is approximately 95 feet per second.

Example 10

A pump fills a tank at a rate of 18 gallons per minute. How many quarts per hour does the pump deliver?

Solution

$$18\frac{\text{gal}}{\text{min}} \cdot \frac{4\ \text{qt}}{1\ \text{gal}} \cdot \frac{60\ \text{min}}{1\ \text{hr}} = 4320\frac{\text{qt}}{\text{hr}} \quad \text{Convert gallons to quarts and minutes to hours.}$$

The pump delivers 4320 quarts per hour.

Exercise

A processing system moves material at a rate of 480 pounds per hour. How many tons per day does the system move? Round your answer to the nearest tenth of a ton.

Answer available at the end of this section.

Answers

Exercise

$12{,}672$ feet

Exercise

1000 pounds

Exercise

6.3 hours

Exercise

$52{,}272$ square feet

Exercise

2 containers

Exercise

2 containers

Exercise

5.8 tons per day

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4.2.4: U.S. Customary Measurements Practice

The following set of exercises provides practice for the concepts covered in Module 4.2: U.S. Customary Measurements. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 4.2.1 - Length, Weight and Time Unit Conversions in the U.S. Customary System

Convert the following measurements. Round answers as indicated.

1. 244 inches to yards (nearest tenth)
2. $198{,}550$ inches to miles (nearest tenth)
3. $19{,}257$ feet to miles (nearest tenth)
4. Convert $3\frac{1}{16}$ pounds to ounces. Round to the nearest tenth.
5. $9{,}499$ pounds to tons (nearest tenth)
6. $4{,}672{,}133$ seconds to days (nearest tenth)
7. 99 miles per hour to feet per second (nearest foot per second)

Section 4.2.2 - Area and Volume Units in the U.S. Customary System

Convert the following measurements. Round all answers to the nearest tenth, as needed.

8. $6\ \text{ft}^2$ to square inches
9. $3\ \text{yd}^2$ to square inches
10. $720\ \text{in}^2$ to square feet
11. $2\ \text{mi}^2$ to acres
12. 7 quarts to cups
13. $3\ \text{yd}^3$ to cubic feet
14. $5\ \text{ft}^3$ to gallons

Section 4.2.3 - Applications Involving U.S. Customary Units

Solve each problem. Use appropriate unit conversions and show all work. If your answer is a decimal, round as indicated and include units in your final answer.

15. A pipeline extends 2.6 miles. A section measuring 850 feet is removed. What is the remaining length in feet?
16. A system processes 9 parts, each weighing 12.5 ounces. What is the total weight in pounds? Round to the nearest tenth.
17. A tank holds 6 gallons of liquid. After use, 5.5 quarts remain. How many gallons were used? Round to the nearest tenth.

18. A workspace includes sections measuring 150 square feet and 210 square feet. What is the total area in square yards?

19. A storage unit holds 2.5 cubic feet. If 864 cubic inches are removed, how much volume remains in cubic feet? Round to the nearest tenth.

20. A pump delivers 15 gallons per minute. How many quarts per hour does it deliver?

21. A shipment weighs 1.8 tons. Each crate holds 450 pounds. How many crates are needed?

22. Two panels measure 3 square feet and 5 square feet. What is the total area in square inches?

23. A load contains 1.75 cubic yards of material. The material is divided into containers that each hold 9 cubic feet. How many containers can be filled?

24. A vehicle travels at 62 miles per hour for 30 seconds. How many feet does it travel? Round to the nearest whole number.

25. A processing system moves 520 pounds per hour. How many tons per day does it move? Round to the nearest tenth.

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4.3: METRIC MEASUREMENTS

What do the French Revolution and a $125 million NASA spacecraft have in common? Before beginning this module, watch the video below, which traces the metric system from its origins in the chaos of 18th-century France — where dozens of competing local measurements made commerce and governance a nightmare — all the way to 1999, when a unit conversion error between metric and U.S. customary measurements caused the Mars Climate Orbiter to burn up in the Martian atmosphere. The metric system was designed to make measurement simple, universal, and exact. The consequences of ignoring it can be catastrophic.

This module covers three areas of the metric system. You will begin with units of length, weight, and temperature, learning to convert between them using the system's base-ten structure. From there, you will extend those skills to units of area and volume, where the base-ten relationships make conversions far more predictable than their U.S. Customary counterparts. The module closes with practical applications that draw on all of these skills, asking you to solve real-world problems using metric measurements.

By the end, you will be able to move confidently between metric units — and you will understand why most of the world never looked back.

Why the Metric System Matters

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View this YouTube Video online

This video traces the metric system from its origins in the chaos of 18th century France all the way to 1999.

4.3.1: Length and Mass in the Metric System

4.3.2: Area and Volume in the Metric System

4.3.3: Applications Involving the Metric System

4.3.4: Metric Measurements Practice

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4.3.1: Length and Mass in the Metric System

The metric system of units is the standard system of units preferred by scientists. It is based on the base ten number system and its decimal format is more friendly to users of this system. There is a common set of prefixes adopted by the metric system to indicate a power of ten to apply to the base unit.

Metric System Preferences

This is a list of standard prefixes for the metric system and their meanings.

$$\begin{array}{ll} \text{deka} = 10 & \text{deci} = 1/10 \\ \text{hecto} = 100 & \text{centi} = 1/100 \\ \text{kilo} = 1000 & \text{milli} = 1/1000 \end{array}$$

Thus, for example, a dekameter is 10 meters, a hectoliter is 100 liters, and a kilogram is 1000 grams. Similarly, a decimeter is 1/10 of a meter, a centiliter is 1/100 of a liter, and a milligram is 1/1000 of a gram.

Intro to the Metric System

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View this YouTube Video online

This video gives an overview of the Metric system and the common prefixes used.

Converting Units of Length

The standard measure of length in the metric system is the meter. We can apply the standard prefixes to get the following result.

Metric Units of Length

These units of length are used in the metric system.

Unit Length	Unit Abbreviation
1 kilometer = 1000 meters	km
1 hectometer = 100 meters	hm
1 dekameter = 10 meters	dam
1 meter	m
1 decimeter = $\frac{1}{10}$ meter	dm
1 centimeter = $\frac{1}{100}$ meter	cm
1 millimeter = $\frac{1}{1000}$ meter	mm

We can use these facts to build conversion factors as we did in Section 6.3. For example, because

$$1 \text{ km} = 1000 \text{ m},$$

we can divide both sides by 1000 m to produce the conversion factor

$$1 = \frac{1 \text{ km}}{1000 \text{ m}}.$$

This conversion factor can help change meters into kilometers.

Before using this conversion factor in an example, we repeat here the rules for multiplying and dividing by powers of ten. We will be making heavy use of these rules in this section.

Multiplying and Dividing by Powers of Ten

- Multiplying a decimal number by 10^n will move the decimal point n places to the right. For example, $3.2567 \cdot 10^2 = 3.2567 \cdot 100 = 325.67$.
- Dividing a decimal number by 10^n will move the decimal point n places to the left. For example, $3.2567/10^2 = 3.2567/100 = 0.032567$.

And now the example.

Example 1

Change 2,326 meters to kilometers.

Solution

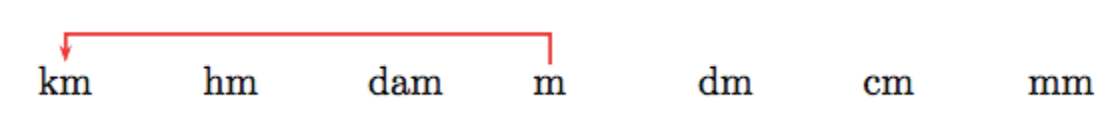
Multiply by the conversion factor 1 km/1000m.

$$\begin{aligned} 2326 \text{ m} &= 2326 \text{ m} \cdot \frac{1 \text{ km}}{1000 \text{ m}} && \text{Apply conversion factor.} \\ &= 2326 \cancel{\text{m}} \cdot \frac{1 \text{ km}}{1000 \cancel{\text{m}}} && \text{Cancel common units.} \\ &= \frac{2326 \cdot 1}{1000} \text{ km} && \text{Multiply fractions.} \\ &= 2.326 \text{ km} && \text{Simplify.} \end{aligned}$$

In the last step, note that dividing by 1000 moves the decimal point three places to the left. Thus, 2326 meters is equal to 2.326 kilometers.

Alternate Solution

A second solution depends upon the fact that multiplying or dividing by a power of ten will move the decimal point right or left a number of places equal to the number of zeros present in the multiplier or divisor. Thus, as we saw above, dividing by 1000 moved the decimal point 3 places to the left. Suppose that we arrange the metric units of length in order, from largest to smallest, as shown below.



Note that we must move 3 places left to move from the meters (m) to the kilometers (km). In like manner, if we write 2,326 meters as 2,326.0meters, then we can convert to kilometers by moving the decimal 3 places to the left.

$$2,326.0\,\text{m} = 2.236\,0\,\text{km} = 2.326\,\text{km}$$

Exercise

Change 1,156 meters to kilometers

Answer available at the end of this section.

Example 2

Change 537 centimeters to meters.

Solution

We know that

$$1\ \text{cm} = \frac{1}{100}\ \text{m},$$

or multiplying both sides of this result by 100,

$$100\ \text{cm} = 1\ \text{m}.$$

Dividing both sides of this last result by 100 cm, we obtain the conversion factor 1 m/100 cm.

$$\begin{aligned} 537\ \text{cm} &= 537\ \text{cm} \cdot \frac{1\ \text{m}}{100\ \text{cm}} && \text{Apply conversion factor.} \\ &= 537\ \cancel{\text{cm}} \cdot \frac{1\ \text{m}}{100\ \cancel{\text{cm}}} && \text{Cancel common units.} \\ &= \frac{537 \cdot 1}{100}\ \text{m} && \text{Multiply fractions.} \\ &= 5.37\ \text{m} \end{aligned}$$

In the last step, note that dividing by 100 moves the decimal point two places to the left. Alternately, we can set up our ordered list of units.

km	hm	dam	m	dm	cm	mm

Note that we must move 2 places left to move from the centimeters (cm) to the meters (m). In like manner, if we write 537 centimeters as 537.0 centimeters, then we can convert to meters by moving the decimal 2 places to the left.

$$537.0\,\text{cm} = 5.37\,0\,\text{m} = 5.37\,\text{m}$$

Sometimes more than one conversion factor is needed.

Exercise

Change 276 centimeters to meters.

Answer available at the end of this section.

Example 3

Change 10.2 dekameters to centimeters.

Solution

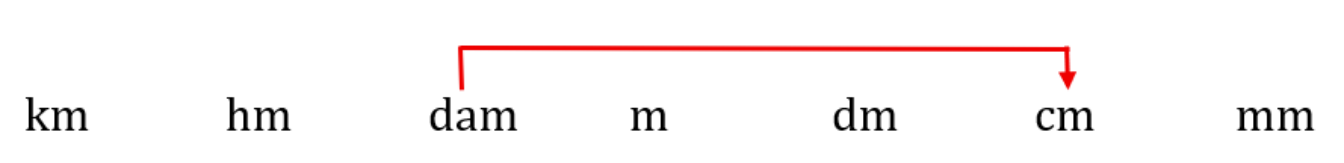
We have two facts:

- 1 dam=10 m, which yields the conversion factor 10 m/1 dam.
- 1 cm=(1/100) m or 100 cm=1 m, which yields the conversion factor 100 cm/1 m.

$$\begin{aligned} 10.2 \text{ dam} &= 10.2 \text{ dam} \cdot \frac{10 \text{ m}}{1 \text{ dam}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} && \text{Apply conversion factor.} \\ &= 10.2 \cancel{\text{dam}} \cdot \frac{10 \cancel{\text{m}}}{1 \cancel{\text{dam}}} \cdot \frac{100 \text{ cm}}{1 \cancel{\text{m}}} && \text{Cancel common units.} \\ &= \frac{10.2 \cdot 10 \cdot 100}{1} \text{ cm} && \text{Multiply fractions.} \\ &= 10,200 \text{ cm} \end{aligned}$$

In the last step, note that multiplying by 10, then by 100, moves the decimal point three places to the right.

Alternately, we can set up our ordered list of units.



Note that we must move 3 places right to move from the dekameters (dam) to the centimeters (cm). In like manner, we can convert 10.2 dekameters to centimeters by moving the decimal 3 places to the right.

$$10.2 \text{ dam} = 10\,200 \text{ cm} = 10,200 \text{ cm}$$

Exercise

Change 13.5 dekameters to centimeters

Answer available at the end of this section.

Common Metric Length Conversions

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to convert common units of length within the Metric system using Unit Fractions.

Converting Units of Mass

The fundamental unit of mass in the metric system is called a gram. Originally, it was defined to be equal to one cubic centimeter of water measured at the temperature of melting ice. Now it is defined in terms of the kilogram, the standard unit of mass in the metric system.

The mass of an object is not the same as its weight; mass describes how much matter an object contains, while weight depends on gravity. Because the metric system is based on mass, we use grams and kilograms to describe how heavy an object is.

The same metric system prefixes apply.

Metric Units of Mass

These units of mass are used in the metric system.

Units of Mass	**Unit Abbreviation**
1 kilogram = 1000 grams	kg
1 hectogram = 100 grams	hg
1 dekagram = 10 grams	dag
1 gram	g
1 decigram = $\frac{1}{10}$ gram	dg
1 centigram = $\frac{1}{100}$ gram	cg
1 milligram = $\frac{1}{1000}$ gram	mg

Example 4

Convert 0.025 dekagrams to milligrams.

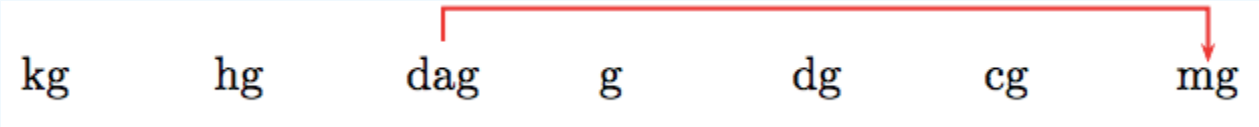
Solution

We'll use two conversion factors:

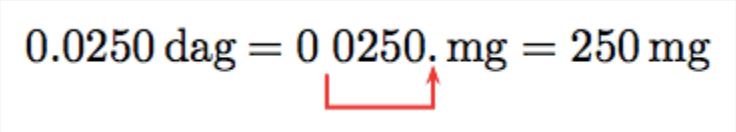
- 1 dag=10 g, which yields the conversion factor 10 g/1 dag.
- 1 mg=(1/1000) g, which yields the conversion factor 1000 mg/1 g.

$$\begin{aligned} 0.025 \text{ dag} &= 0.025 \text{ dag} \cdot \frac{10 \text{ g}}{1 \text{ dag}} \cdot \frac{1000 \text{ mg}}{1 \text{ g}} && \text{Apply conversion factor.} \\ &= 0.025 \text{ } \cancel{\text{dag}} \cdot \frac{10 \text{ } \cancel{\text{g}}}{1 \text{ } \cancel{\text{dag}}} \cdot \frac{1000 \text{ mg}}{1 \text{ } \cancel{\text{g}}} && \text{Cancel common units.} \\ &= \frac{0.025 \cdot 10 \cdot 1000}{1} \text{ mg} && \text{Multiply fractions.} \\ &= 250 \text{ mg} \end{aligned}$$

Alternately, we can set up our ordered list of units.



Note that we must move 4 places right to move from the dekagrams (dag) to the milligrams (mg). In like manner, we can convert 0.025 dekagrams to milligrams by moving the decimal 4 places to the right.



Exercise

Convert 0.05 dekagrams to milligrams.

Answer available at the end of this section.

Common Metic Weight Conversions

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to convert common units of mass/weight within the Metric system using Unit Fractions.

Converting Units of Speed

Example 5

A high-speed train travels at 180 kilometers per hour. How fast is this in meters per second? Round your answer to the nearest meter per second.

Solution

There are 1000 meters in a kilometer and 3600 seconds in an hour.

$$\begin{aligned} 180\frac{\text{km}}{\text{h}} &= 180\frac{\text{km}}{\text{h}} \cdot \frac{1000\ \text{m}}{1\ \text{km}} \cdot \frac{1\ \text{h}}{3600\ \text{s}} && \text{Apply conversion factors.} \\ &= \frac{180 \cdot 1000}{3600}\frac{\text{m}}{\text{s}} && \text{Simplify.} \\ &= 50\frac{\text{m}}{\text{s}} \end{aligned}$$

To round to the nearest meter per second, note that the result is already a whole number. Thus, the speed is 50 meters per second.

Exercise

A delivery driver travels at 72 kilometers per hour. How fast is this in meters per second? Round your answer to the nearest meter per second.

Answer available at the end of this section.

Answers

Exercise

1.156 kilometers

Exercise

2.76 meters

Exercise

13,500 centimeters

Exercise

500 milligrams

Exercise

$20\ \text{m/s}$

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- 6.5: Unit Conversion- Metric System by David Arnold is licensed CC BY-NC-SA 3.0.

4.3.2: Area and Volume in the Metric System

Units of Area

The fundamental unit of area in the metric system is the square meter. A square meter represents the area of a square with each side measuring one meter.

For larger areas, the metric system also uses the hectare. One hectare is equal to $10{,}000\ \text{m}^2$.

The same metric system prefixes apply.

Metric Units of Area

These units of area are used in the metric system.

Unit of Area	Unit Abbreviation
1 square kilometer = $1{,}000{,}000$ square meters	km^2
$1\ \text{hectometer}^2 = 10{,}000\ \text{meter}^2 = 1\ \text{hactare}$	ha
1 square dekameter = 100 square meters \hspace{1.75in} dam^2	dam^2
$1\ \text{meters}^2$	m^2
$1\ \text{decimeter}^2 = \frac{1}{100}\text{meters}^2$	dm^2
$1\ \text{centimeters}^2 = \frac{1}{10{,}000}\ \text{meters}^2$	cm^2
$1\ \text{millameters}^2 = \frac{1}{1{,}000{,}000}\ \text{meters}^2$	mm^2

The unit hectare is equal to one square hectometer $(1\ \text{ha} = 1\ \text{hm}^2)$, but the name hectare is used instead of square hectometer in practice.

Example 1

Convert $11{,}725$ square centimeters to square meters.

Solution

We'll use one conversion factor:

$1\ \text{m}^2 = 10{,}000\ \text{cm}^2$, which yields the conversion factor $\frac{1\ \text{m}^2}{10{,}000\ \text{cm}^2}$.

$$
\begin{aligned}
11{,}725\ \text{cm}^2 &= 11{,}725\ \text{cm}^2 \cdot \frac{1\ \text{m}^2}{10{,}000\ \text{cm}^2} && \text{Apply conversion factor.}\\
&= 11{,}725\ \cancel{\text{cm}^2} \cdot \frac{1\ \text{m}^2}{10{,}000\ \cancel{\text{cm}^2}} && \text{Cancel common units.}\\
&= \frac{11{,}725}{10{,}000}\ \text{m}^2 && \text{Divide.}\\
&= 1.1725\ \text{m}^2 && \text{Simplify.}
\end{aligned}
$$

Alternately, we can set up our ordered list of units.

$$\text{km}^2 \quad \text{hm}^2 \quad \text{dam}^2 \quad \text{m}^2 \quad \text{dm}^2 \quad \text{cm}^2 \quad \text{mm}^2$$

Note that we must move 4 places left to move from square centimeters (cm^2) to square meters (m^2). Since $1\ \text{m} = 100\ \text{cm}$, squaring this gives $1\ \text{m}^2 = 10{,}000\ \text{cm}^2$. Because we are dividing by $10{,}000$, the decimal moves 4 places to the left.

$$11275\ \text{cm}^2 = 11275.0\ \text{cm}^2 = 1.1275\ \text{m}^2$$

Exercise

Convert 2.75 square meters to square millimeters.

Answer available at the end of this section.

Units of Volume

The fundamental unit of volume in the metric system is called a liter. Currently, 1 liter is defined as 1 cubic decimeter (imagine a cube with each edge 1/10 of a meter).

The same metric system prefixes apply.

Metric Units of Volume

These units of volume are used in the metric system.

Unit of Volume	Unit Abbreviation
1 kiloliter = 1000 liters	kL
1 hectoliter = 100 liters	hL
1 dekaliter = 10 liters	daL
1 liter	L
1 deciliter = $\frac{1}{10}$ liter	dL
1 centiliter = $\frac{1}{100}$ liter	cL
1 milliliter = $\frac{1}{1000}$ liter	mL

Example 5

Convert 11,725 milliliters to dekaliters.

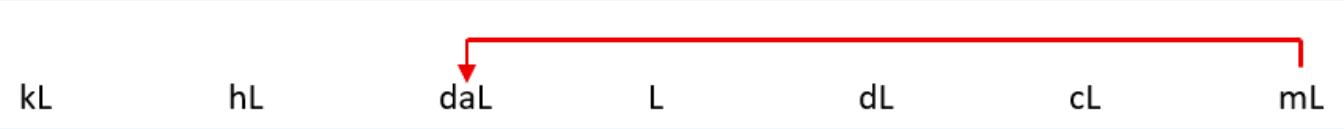
Solution

We'll use two conversion factors:

- 1 daL=10 L, which yields the conversion factor 1 daL/10 L.
- 1 mL=(1/1000) L, which yields the conversion factor 1 L/1000 mL.

$$\begin{aligned} 11,725\text{ mL} &= 11,725\text{ mL} \cdot \frac{1\text{ L}}{1000\text{ mL}} \cdot \frac{1\text{ daL}}{10\text{ L}} && \text{Apply conversion factors.} \\ &= 11,725\ \cancel{\text{mL}} \cdot \frac{1\ \cancel{\text{L}}}{1000\ \cancel{\text{mL}}} \cdot \frac{1\text{ daL}}{10\ \cancel{\text{L}}} && \text{Cancel common units.} \\ &= \frac{11,725 \cdot 1 \cdot 1}{1000 \cdot 10}\text{ daL} && \text{Multiply fractions.} \\ &= 1.1725\text{ daL} \end{aligned}$$

Alternately, we can set up our ordered list of units.



Note that we must move 4 places left to move from the milliliters (mL) to the dekaliters (daL). In like manner, we can convert 11,725 milliliters to dekaliters by moving the decimal 4 places to the left.

$$11,725.0\text{ mL} = 1.1725\ 0\text{ daL} = 1.1725\text{ daL}$$

Exercise

Convert 5,763 milliliters to dekaliters.

Answer available at the end of this section.

Volume can also be measured using cubic units. In the metric system, 1 cubic centimeter $= 1$ milliliter.

Example 4.3.2.1

Convert 0.34 L to cubic centimeters.

Solution

We'll use two facts:

$1\text{ L} = 1000\text{ mL}$, which yields the conversion factor $\frac{1000\text{ mL}}{1\text{ L}}$.

$1\text{ cm}^3 = 1\text{ mL}$, which allows us to replace milliliters with cubic centimeters.

$$\begin{aligned} 0.34\text{ L} &= 0.34\text{ L} \cdot \frac{1000\text{ mL}}{1\text{ L}} && \text{Convert liters to milliliters.} \\ &= 340\text{ mL} && \text{Simplify.} \\ &= 340\text{ cm}^3 && \text{Use } 1\text{ cm}^3 = 1\text{ mL}. \end{aligned}$$

Exercise

Convert $875\ \text{cm}^3$ to milliliters.

Answer available at the end of this section.

Common Metric Volume Conversions

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to convert common units of volume within the Metric system using Unit Fractions.

Answers

Exercise

$2{,}750{,}000\ \text{mm}^2$

Exercise

0.5763 dekaliters

Exercise

$875\ \text{mL}$

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- 6.5: Unit Conversion- Metric System by David Arnold is licensed CC BY-NC-SA 3.0.

4.3.3: Applications Involving the Metric System

In many technical settings, measurements are recorded and used in the metric system. Unlike U.S. Customary units, metric units are based on powers of ten, which makes conversions more consistent across different types of measurements.

A length might be given in meters but needed in millimeters. A mass might be measured in grams but reported in kilograms. Area and volume introduce squared and cubic units, and these follow the same pattern of scaling by powers of ten.

The key skill is still setting up conversions so that units cancel correctly. Once the setup is correct, the calculation becomes straightforward.

In this section, you will apply metric unit conversions in a variety of situations involving length, mass, area, and volume.

Applications Involving Length

Example 1

A steel rod measures 0.85 meters. What is the length in centimeters?

Solution

$$0.85 \text{ m} \cdot \frac{100 \text{ cm}}{1 \text{ m}} = 85 \text{ cm} \quad \text{Convert meters to centimeters.}$$

The rod measures 85 centimeters.

Example 2

A cable is 2.4 meters long. After cutting off 350 millimeters, what is the remaining length in meters?

Solution

$$350 \text{ mm} \cdot \frac{1 \text{ m}}{1000 \text{ mm}} = 0.35 \text{ m} \quad \text{Convert millimeters to meters.}$$
$$2.4 - 0.35 = 2.05 \text{ m} \quad \text{Find the remaining length.}$$

The remaining length is 2.05 meters.

Exercise

A pipeline is 1.8 kilometers long. What is the length in meters?

Answer available at the end of this section.

Applications Involving Mass

Example 3

A sample has a mass of 0.65 kilograms. What is the mass in grams?

Solution

$$0.65\ \text{kg} \cdot \frac{1000\ \text{g}}{1\ \text{kg}} = 650\ \text{g} \quad \text{Convert kilograms to grams.}$$

The mass is 650 grams.

Example 4

A container holds 2400 grams of material. The material is divided evenly into 6 portions. What is the mass of each portion in kilograms?

Solution

$$2400\ \text{g} \cdot \frac{1\ \text{kg}}{1000\ \text{g}} = 2.4\ \text{kg} \quad \text{Convert grams to kilograms.}$$
$$2.4 \div 6 = 0.4\ \text{kg} \quad \text{Find the mass of each portion.}$$

Each portion has a mass of 0.4 kilograms.

Exercise

A package has a mass of 3.8 kilograms. After removing 450 grams, what is the remaining mass in kilograms?

Answer available at the end of this section.

Applications Involving Area

Example 5

A metal plate has an area of 0.35 square meters. What is the area in square millimeters?

Solution

$$0.35\ \text{m}^2 \cdot \left(\frac{1000\ \text{mm}}{1\ \text{m}}\right)^2 = 0.35 \cdot 1{,}000{,}000\ \text{mm}^2 \quad \text{Convert square meters to square millimeters.}$$
$$= 350{,}000\ \text{mm}^2 \quad \text{Multiply.}$$

The area is $350{,}000$ square millimeters.

Example 6

One piece of fabric has an area of 1.8 square meters and another has an area of 9500 square centimeters...

Solution

$$9500\ \text{cm}^2 \cdot \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 0.95\ \text{m}^2 \quad \text{Convert to square meters.}$$
$$1.8 + 0.95 = 2.75\ \text{m}^2 \quad \text{Add the areas.}$$

The total area is 2.75 square meters.

Exercise

A section of flooring measures 0.6 square meters. After covering 1800 square centimeters, how much area remains in square meters?

Answer available at the end of this section.

Applications Involving Volume

Example 7

A beaker holds 2.4 liters of liquid. What is the volume in milliliters?

Solution

$2.4\ \text{L} \cdot \dfrac{1000\ \text{mL}}{1\ \text{L}} = 2400\ \text{mL}$ Convert liters to milliliters.

The volume is 2400 milliliters.

Example 8

A bottle of cleaning solution contains 750 cubic centimeters of liquid. The liquid is divided evenly into 3 equal portions. What is the volume in liters of each portion?

Solution

$750\ \text{cm}^3 \cdot \frac{1\ \text{mL}}{1\ \text{cm}^3} = 750\ \text{mL}$ Use $1\ \text{cm}^3 = 1\ \text{mL}$.

$750 \div 3 = 250\ \text{mL}$ Divide the volume.

$250\ \text{mL} \cdot \frac{1\ \text{L}}{1000\ \text{mL}} = 0.25\ \text{L}$ Convert to liters.

Exercise

One storage tank holds 0.004 cubic meters of liquid and another tank holds 3.5 liters. How much more liquid does the larger tank hold? Express your answer in liters.

Answer available at the end of this section.

Applications Involving Compound Units

Example 9

A conveyor belt moves at a speed of 1.8 meters per second. What is this speed in meters per minute?

Solution

$1.8\frac{\text{m}}{\text{s}} \cdot \frac{60\ \text{s}}{1\ \text{min}} = 108\frac{\text{m}}{\text{min}}$ Convert seconds to minutes.

The conveyor belt moves at 108 meters per minute.

Example 10

A pump delivers 18 liters per minute. How many milliliters per second does the pump deliver?

Solution

$18\frac{\text{L}}{\text{min}} \cdot \frac{1000\ \text{mL}}{1\ \text{L}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 300\frac{\text{mL}}{\text{s}}$ Convert liters to milliliters and minutes to seconds.

The pump delivers 300 milliliters per second.

Exercise

A processing line moves material at a rate of 240 kilograms per hour. How many grams per minute does the line move?

Answer available at the end of this section.

Answers

Exercise

1800 meters

Exercise

3.35 kilograms

Exercise

0.42 square meters

Exercise

0.5 liters

Exercise

4000 grams per minute

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4.3.4: Metric Measurements Practice

The following set of exercises provides practice for the concepts covered in Module 4.3: Metric Measurements. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 3.4.1 - Length and Mass in the Metric System

Convert the following measurements.

1. $8{,}528$ millimeters to meters
2. 12 meters to centimeters
3. 4.9 kilometers to meters
4. $1{,}929$ meters to kilometers
5. 223 milligrams to centigrams
6. 70 kilograms to grams
7. $6{,}693$ grams to kilograms
8. 72 kilometers per hour to meters per second, rounded to the nearest meter per second
9. 20 meters per second to kilometers per hour, rounded to the nearest kilometer per hour

Section 4.3.2 - Area and Volume in the Metric System

Convert the following measurements.

10. $8{,}400$ square centimeters to square meters
11. $2{,}750{,}000$ square millimeters to square meters
12. 0.45 liters to cubic centimeters
13. 3.2 cubic meters to liters
14. 6.5 square meters to square millimeters
15. 1.8 liters to milliliters

Section 4.3.3 - Applications Involving Metric Conversions

Solve each problem. Use appropriate unit conversions and show all work. If your answer is a decimal, round as indicated and include units in your final answer.

16. A cable is 3.6 meters long. After removing 450 millimeters, what is the remaining length in meters?
17. A batch has a mass of 2.75 kilograms. After removing 320 grams, what is the remaining mass in kilograms?
18. Two pieces of fabric measure 1.4 square meters and 8500 square centimeters. What is the total area in square meters?

19. A section of flooring measures 0.8 square meters. After covering 2500 square centimeters, how much area remains in square meters?

20. A bottle of cleaning solution contains 900 cubic centimeters of liquid. The liquid is divided evenly into 3 equal portions. What is the volume in liters of each portion?

21. A tank holds 0.006 cubic meters of liquid. How many liters does it hold?

22. A conveyor belt moves at 2.4 meters per second for 45 seconds. How many meters does it move?

23. A pump delivers 15 liters per minute for 8 minutes. How many liters are delivered?

24. A flow rate is 350 milliliters per second. How many liters per minute is this? Round to the nearest tenth.

25. A cyclist travels at 18 kilometers per hour for 30 minutes. How far does the cyclist travel in kilometers?

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4.4: METRIC AND U.S. CUSTOMARY CONVERSIONS

What if the United States had gone metric from the very beginning? Before starting this module, watch the video — a Saturday Night Live sketch imagining George Washington's dream of choosing his own system of weights and measures. It's played for laughs, but the underlying tension is real: the United States stands almost alone in the world in its continued use of the U.S. Customary system, and the gap between that system and the metric system has practical consequences every time the two worlds meet.

This module is about bridging that gap. You will begin with conversions between the two systems for units of length and weight, where a reliable set of conversion factors will carry most of the work. From there, you will tackle compound conversions involving area and volume — cases where a single unit conversion must be applied more than once, and where careful reasoning matters more than memorization. Temperature conversions between Fahrenheit and Celsius require a different approach entirely, involving a formula rather than a simple ratio, and those are covered next. The module closes with practical applications that draw on all of these skills together, presenting real-world problems that require you to move fluidly between both systems.

By the end, you will be equipped to handle the conversions that come up in science, medicine, engineering, and everyday life — wherever the customary and metric worlds collide.

Washington's Dream - SNL Skit

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View this YouTube Video online

This video is a SNL skit imagining George Washington's dream of choosing his own system of weights and measures.

4.4.1: Length and Mass Between Systems

4.4.2: Area and Volume Between Systems

4.4.3: Temperature Conversions

4.4.4: Applications Involving Metric and U.S Customary Conversion

4.4.5: Metric and U.S. Customary Conversion Practice

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4.4.1: Length and Mass Between Systems

We often need to convert from the U.S. Customary system of units to the Metric system of units or vice-versa (imagine traveling to a European country using the metric system). This section will focus on lenght, weight/mass, and speed.

Converting Units of Length

One meter is slightly longer than one yard. Indeed,

$$1\,\text{m} \approx 1.0936\,\text{yd}.$$

If we divide both sides of this equation by 1.0936, then

$$\frac{1\text{ m}}{1.0936} \approx \frac{1.0936\text{ yd}}{1.0936}$$
$$0.9144\text{ m} \approx 1\text{ yd}$$

Further conversions can be made. For example, to change meters to feet, we make the following conversions.

$$\begin{aligned} 1\text{ m} &\approx 1\text{ m} \cdot 1\frac{\text{yd}}{0.9144\text{ m}} \cdot \frac{3\text{ ft}}{1\text{ yd}} && \text{Apply conversion factors.} \\ &\approx 1\,\cancel{\text{m}} \cdot \frac{1\ \cancel{\text{yd}}}{0.9144\ \cancel{\text{m}}} \cdot \frac{3\text{ ft}}{1\ \cancel{\text{yd}}} && \text{Cancel common units.} \\ &\approx \frac{1 \cdot 1 \cdot 3}{0.9144 \cdot 1}\text{ ft} && \text{Multiply fractions.} \\ &\approx 3.2808\text{ ft} \end{aligned}$$

Table 4.4.1.1 shows some of the most common conversions between American units of length and metric units of length.

Table 4.4.1.1: Length conversions: American — Metric.

American to Metric	Metric to American
1 in ≈ 2.54 cm	1 cm ≈ 0.3937 in
1 ft ≈ 0.3048 m	1 m ≈ 3.2808 ft
1 yd ≈ 0.9144 m	1 m ≈ 1.0936 yd
1 mi ≈ 1.6093 km	1 km ≈ 0.6214 mi

Example 1

A car's speedometer shows that a family has currently traveled 154 miles in route to their vacation destination. Convert this distance to kilometers.

Solution

Choose 1 mi = 1.6093 km from Table 4.4.1.1.

$$\begin{aligned} 154 \text{ mi} &\approx 154 \text{ mi} \cdot \frac{1.6093 \text{ km}}{1 \text{ mi}} && \text{Apply conversion factor.} \\ &\approx 154 \cancel{\text{mi}} \cdot \frac{1.6093 \text{ km}}{1 \cancel{\text{mi}}} && \text{Cancel common units.} \\ &\approx \frac{154 \cdot 1.6093}{1} \text{ km} && \text{Multiply fractions.} \\ &\approx 247.8 \text{ km} \end{aligned}$$

Hence, 154 miles is approximately 247.8 kilometers.

Alternate Solution

Note that this would work equally well if we chose 1 km ≈ 0.6214 mi from Table 6.5.

$$\begin{aligned} 154 \text{ mi} &\approx 154 \text{ mi} \cdot \frac{1 \text{ km}}{0.6214 \text{ mi}} && \text{Apply conversion factor.} \\ &\approx 154 \cancel{\text{mi}} \cdot \frac{1 \text{ km}}{0.6214 \cancel{\text{mi}}} && \text{Cancel common units.} \\ &\approx \frac{154 \cdot 1}{0.6214} \text{ km} && \text{Multiply fractions.} \\ &\approx 247.8 \text{ km} \end{aligned}$$

Exercise

Change 227 miles to kilometers. Round to the nearest tenth of a kilometer.

Answer available at the end of this section.

Convert Lengths Between the Standard and Metric Systems

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View this YouTube Video online

This video shows examples of converting lengths between U.S. Customary and the Metric systems.

Converting Units of Weight and Mass

It is known that

$$1\,\text{kg} \approx 2.2\,\text{lb}.$$

Dividing both sides of this equation by 2.2,

$$\begin{aligned} \frac{1 \text{ kg}}{2.2} &\approx \frac{2.2 \text{ lb}}{2.2} \\ 0.454 \text{ kg} &\approx 1\text{lb} \end{aligned}$$

A summary of the more common conversion factors regarding mass and weight are given in Table 4.4.1.2.

Table 4.4.1.2: Mass—Weight conversions: American — Metric.

American to Metric	Metric to American
1 oz ≈ 28.35 g	1 g ≈ 0.035 oz
1 lb ≈ 0.454 kg	1 kg ≈ 2.2 lb

Example 2

Change 2.3 kilograms to ounces.

Solution

One kilogram weighs 2.2 pounds and there are 16 ounces in a pound.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &\approx 80.96\text{ oz} \end{aligned}$$

Hence, 2.3 kilograms weighs 80.96 ounces.

Alternate Solution

Another approach uses the facts that 1 kilogram equals 1000 grams and 1 ounce equals 28.35 grams.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{1000\text{ g}}{1\text{ kg}} \cdot \frac{1\text{ oz}}{28.35\text{ g}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \cdot \frac{1\text{ oz}}{28.35\ \cancel{\text{g}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 1000 \cdot 1}{1 \cdot 28.35}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

Roundoff Error

Why the discrepancy in answers? This difference in approximations is due to something called *round-off error*. Indeed, in the first calculation, we used a conversion factor that is rounded to the nearest tenth; i.e., 1 kg ≈ 2.2 lb. If we use a more accurate conversion factor to change kilograms to pounds, namely 1 kg ≈ 2.2046 lb, we get the following result.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2046\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2046\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2046 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

Note that this result is in better agreement with the second result above.

Exercise

Change 5.7 kilograms to ounces. Round to the nearest ounce.

Answer available at the end of this section.

Convert Weight Between the Standard and Metric Systems

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View this YouTube Video online

This video shows examples of converting mass/weight between U.S. Customary and the Metric systems.

Converting Units of Speed

Modern speedometers often show a car's speed in both miles per hour and kilometers per hour.

Example 4

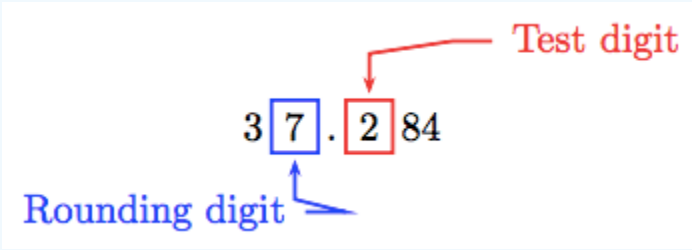
A car's speedometer is showing it speeding along at 60 kilometers per hour. How fast is it traveling in miles per hour? Round your answer to the nearest mile per hour.

Solution

From Table 6.5, 1 km ≈ 0.6214 mi.

$$\begin{aligned} 60\frac{\text{km}}{\text{h}} &\approx 60\frac{\text{km}}{\text{h}} \cdot \frac{0.6214\text{ mi}}{1\text{ km}} && \text{Apply conversion factor.} \\ &\approx 60\frac{\cancel{\text{km}}}{\text{h}} \cdot \frac{0.6214\text{ mi}}{1\ \cancel{\text{km}}} && \text{Cancel common units.} \\ &\approx \frac{60 \cdot 0.6214}{1}\frac{\text{mi}}{\text{h}} && \text{Multiply fractions.} \\ &\approx 37.284\frac{\text{mi}}{\text{h}} \end{aligned}$$

To round to the nearest mile per hour, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit as it is and drop all digits to the right. Thus, to the nearest mile per hour, the speed is approximately 37 miles per hour.

Exercise

A car's speedometer registers 45 kilometers per hour. Change this speed to miles per hour. Round your answer to the nearest mile per hour.

Answer available at the end of this section.

Answers

Exercise

365.3 kilometers

Exercise

201 ounces

Exercise

28 miles per hour

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- 6.6: American Units to Metric Units and Vice-Versa by David Arnold is licensed CC BY-NC-SA 3.0.

4.4.2: Area and Volume Between Systems

In this section, we will focus on converting units of area and volume from the U.S. Customary system of units to the Metric system of units or vice-versa.

Converting Area Units

Area conversions are based on squaring length conversions. Since area involves two dimensions, each unit conversion factor must also be squared.

For example, we know that

$$1 \text{ in} \approx 2.54 \text{ cm}$$

Squaring both sides gives

$$(1 \text{ in})^2 \approx (2.54 \text{ cm})^2$$
$$1 \text{ in}^2 \approx 6.452 \text{ cm}^2$$

Similarly, since

$$1 \text{ ft} \approx 0.3048 \text{ m}$$

we square both sides to obtain

$$(1 \text{ ft})^2 \approx (0.3048 \text{ m})^2$$
$$1 \text{ ft}^2 \approx 0.0929 \text{ m}^2$$

These relationships show that area conversion factors are not linear — they are the result of squaring length conversions.

Again, these conversion factors for area have been well established. The more common conversion factors for area are shown in Table 4.4.2.1.

Table 4.4.2.1: Area conversions: American — Metric.

American to Metric	Metric to American
$1 \text{ in}^2 \approx 6.452 \text{ cm}^2$	$1 \text{ cm}^2 \approx 0.1550 \text{ in}^2$
$1 \text{ ft}^2 \approx 0.0929 \text{ m}^2$	$1 \text{ m}^2 \approx 10.764 \text{ ft}^2$
$1 \text{ yd}^2 \approx 0.836 \text{ m}^2$	$1 \text{ m}^2 \approx 1.196 \text{ yd}^2$
$1 \text{ acre} \approx 0.405 \text{ ha}$	$1 \text{ ha} \approx 2.47 \text{ acres}$

Example 1

Convert 18 ft^2 to square meters.

Solution

Recall that $1 \text{ ft}^2 \approx 0.0929 \text{ m}^2$.

$$\begin{aligned} 18\ \text{ft}^2 &\approx 18\ \text{ft}^2 \cdot \frac{0.0929\ \text{m}^2}{1\ \text{ft}^2} && \text{Apply conversion factors.} \\ &\approx 18\ \cancel{\text{ft}^2} \cdot \frac{0.0929\ \text{m}^2}{1\ \cancel{\text{ft}^2}} && \text{Cancel common units.} \\ &\approx \frac{18 \cdot 0.0929}{1}\ \text{m}^2 && \text{Multiply fractions.} \\ &\approx 1.6722\ \text{m}^2 \end{aligned}$$

Hence, $18\ \text{ft}^2$ is approximately equal to $1.67\ \text{m}^2$.

Exercise

Convert $45\ \text{yd}^2$ to square meters. Round area conversions to the nearest tenth.

Answer available at the end of this section.

Example 2

Convert $2.5\ \text{ft}^2$ to square centimeters.

Solution

Recall that $1\ \text{ft}^2 \approx 0.0929\ \text{m}^2$ and $1\ \text{m}^2 = 10{,}000\ \text{cm}^2$.

$$\begin{aligned} 2.5\ \text{ft}^2 &\approx 2.5\ \text{ft}^2 \cdot \frac{0.0929\ \text{m}^2}{1\ \text{ft}^2} \cdot \frac{10{,}000\ \text{cm}^2}{1\ \text{m}^2} && \text{Apply conversion factors.} \\ &\approx 2.5\ \cancel{\text{ft}^2} \cdot \frac{0.0929\ \cancel{\text{m}^2}}{1\ \cancel{\text{ft}^2}} \cdot \frac{10{,}000\ \text{cm}^2}{1\ \cancel{\text{m}^2}} && \text{Cancel common units.} \\ &\approx \frac{2.5 \cdot 0.0929 \cdot 10{,}000}{1 \cdot 1}\ \text{cm}^2 && \text{Multiply fractions.} \\ &\approx 2322.5\ \text{cm}^2 \end{aligned}$$

Hence, $2.5\ \text{ft}^2$ is equal to $2322.5\ \text{cm}^2$.

Exercise

Change $3.8\ \text{yd}^2$ to square centimeters. Round your answer to the nearest whole number.

Answer available at the end of this section.

Converting Units of Volume

It is known that

$$1\ \text{qt} \approx 0.946\ \text{L}.$$

Dividing both sides of this equation by 0.946,

$$1\ \text{L} \approx 1.06\ \text{qt}.$$

Again, these conversion factors for volume have been well worked out by scientists. The more common conversion factors for volume are shown in Table 4.4.2.2.

American to Metric	Metric to American
$1\ \text{tsp} = 5\ \text{mL}$	$1\ \text{mL} = 0.2\ \text{tsp}$
$1\ \text{tbsp} = 15\ \text{mL}$	$1\ \text{mL} \approx 0.067\ \text{tbsp}$
1 fl oz ≈ 0.030 L	1 L ≈ 33.8 fl oz
1 pt ≈ 0.473 L	1 L ≈ 2.1 pt
1 qt ≈ 0.946 L	1 L ≈ 1.06 qt
1 gal ≈ 3.785 L	1 L ≈ 0.264 gal
$1\ \text{in}^3 \approx 16.387\ \text{cm}^3$	$1\ \text{cm}^3 \approx 0.0610\ \text{in}^3$
$1\ \text{yd}^3 \approx 0.765\ \text{m}^3$	$1\ \text{m}^3 \approx 1.31\ \text{yd}^3$

Table 4.4.2.2: Volume conversions: American — Metric.

Example 3

Convert $3.2\ \text{yd}^3$ to cubic meters.

Solution

$$\begin{aligned} 3.2\ \text{yd}^3 &\approx 3.2\ \text{yd}^3 \cdot \frac{0.765\ \text{m}^3}{1\ \text{yd}^3} && \text{Apply conversion factors.} \\ &\approx 3.2\ \cancel{\text{yd}^3} \cdot \frac{0.765\ \text{m}^3}{1\ \cancel{\text{yd}^3}} && \text{Cancel common units.} \\ &\approx \frac{3.2 \cdot 0.765}{1}\ \text{m}^3 && \text{Multiply fractions.} \\ &\approx 2.448\ \text{m}^3 \end{aligned}$$

Hence, $3.2\ \text{yd}^3$ is equal to $2.448\ \text{m}^3$.

Exercise

Change $275\ \text{in}^3$ to cubic meters. Round your answer to the nearest thousandth of a cubic meter.

Answer available at the end of this section.

Example 4

Change 2.5 dekaliters to gallons.

Solution

Recall that 1 daL = 10 L and 1 L = 0.264 gal.

$$\begin{aligned} 2.5\ \text{daL} &\approx 2.5\ \text{daL} \cdot \frac{10\ \text{L}}{1\ \text{daL}} \cdot \frac{0.264\ \text{gal}}{1\ \text{L}} && \text{Apply conversion factors.} \\ &\approx 2.5\ \cancel{\text{daL}} \cdot \frac{10\ \cancel{\text{L}}}{1\ \cancel{\text{daL}}} \cdot \frac{0.264\ \text{gal}}{1\ \cancel{\text{L}}} && \text{Cancel common units.} \\ &\approx \frac{2.5 \cdot 10 \cdot 0.264}{1 \cdot 1}\ \text{gal} && \text{Multiply fractions.} \\ &\approx 6.6\ \text{gal} \end{aligned}$$

Hence, 2.5 dekalitres is approximately equal to 6.6 gallons.

Exercise

Change $18.5\ \text{fl oz}$ to milliliters. Round your answer to the nearest whole number.

Answer available at the end of this section.

Convert Volume Between the Standard and Metric Systems

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video shows examples of converting volume between U.S. Customary and the Metric systems.

Example 5

Change $2.8\ \text{g/cm}^3$ to pounds per cubic foot. Round your answer to the nearest tenth.

Solution

Recall that $1\ \text{lb} \approx 454\ \text{g}$ and $1\ \text{ft}^3 \approx 28{,}317\ \text{cm}^3$.

$$\begin{aligned} 2.8\frac{\text{g}}{\text{cm}^3} &\approx 2.8\frac{\text{g}}{\text{cm}^3} \cdot \frac{1\ \text{lb}}{454\ \text{g}} \cdot \frac{28{,}317\ \text{cm}^3}{1\ \text{ft}^3} && \text{Apply conversion factors.} \\ &\approx 2.8\frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \cdot \frac{1\ \text{lb}}{454\ \cancel{\text{g}}} \cdot \frac{28{,}317\ \cancel{\text{cm}^3}}{1\ \text{ft}^3} && \text{Cancel common units.} \\ &\approx \frac{2.8 \cdot 1 \cdot 28{,}317}{454 \cdot 1}\frac{\text{lb}}{\text{ft}^3} && \text{Multiply fractions.} \\ &\approx 174.8\frac{\text{lb}}{\text{ft}^3} \end{aligned}$$

Hence, $2.8\ \text{g/cm}^3$ is approximately equal to $174.8\ \text{lb/ft}^3$.

Exercise

Change $185\ \text{lb/ft}^3$ to grams per cubic centimeter. Round your answer to the nearest hundredth.

Answer available at the end of this section.

Answers

Exercise

$37.6\ \text{m}^2$

Exercise

$3176\ \text{cm}^2$

Exercise

$0.005\ \mathrm{m}^3$

Exercise

555 mL

Exercise

$2.96\ \mathrm{g/cm}^3$

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4.4.3: Temperature Conversions

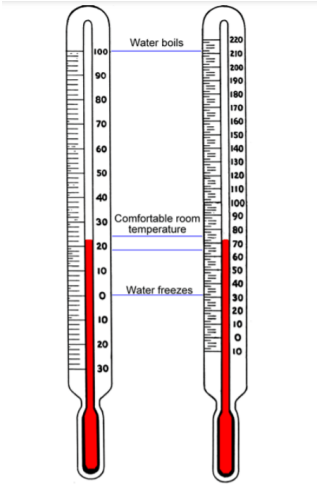
Turn on the television any morning and you will see meteorologists talking about the day's weather forecast. In addition to telling you what the weather conditions will be like (sunny, cloudy, rainy, muggy), they also tell you the day's forecast for high and low temperatures. A hot summer day may reach 100° in Philadelphia, while a cool spring day may have a low of 40° in Seattle.

If you have been to other countries, though, you may notice that meteorologists measure heat and cold differently outside of the United States. For example, a TV weatherman in San Diego may forecast a high of 89°, but a similar forecaster in Tijuana, Mexico—which is only 20 miles south— may look at the same weather pattern and say that the day's high temperature is going to be 32°. What's going on here?

The difference is that the two countries use different temperature scales. In the United States, temperatures are usually measured using the **Fahrenheit** scale, while most countries that use the metric system use the **Celsius** scale to record temperatures. Learning about the different scales— including how to convert between them—will help you figure out what the weather is going to be like, no matter which country you find yourself in.

Measuring Temperature on Two Scales

Fahrenheit and Celsius are two different scales for measuring temperature.

A thermometer measuring a temperature of 22° Celsius is shown here.		A thermometer measuring a temperature of 72° Fahrenheit is shown here.
On the Celsius scale, water freezes at 0° and boils at 100°.		On the Fahrenheit scale, water freezes at 32° and boils at 212°.
If the United States were to adopt the Celsius scale, forecast temperatures would rarely go below -30° or above 45°. (A temperature of -18° may be forecast for a cold winter day in Michigan, while a temperature of 43° may be predicted for a hot summer day in Arizona.)		In the United States, forecast temperatures measured in Fahrenheit rarely go below - 20° or above 120°. (A temperature of 0° may be forecast for a cold winter day in Michigan, while a temperature of 110° may be predicted for a hot summer day in Arizona.)
Most office buildings maintain an indoor temperature between 18°C and 24°C to keep employees comfortable.		Most office buildings maintain an indoor temperature between 65°F and 75°F to keep employees comfortable.

Converting Between the Scales

By looking at the two thermometers shown, you can make some general comparisons between the scales. For example, many people tend to be comfortable in outdoor temperatures between 50°F and 80°F (or between 10°C and 25°C). If a meteorologist predicts an average temperature of 0°C (or 32°F), then it is a safe bet that you will need a winter jacket.

Sometimes, it is necessary to convert a Celsius measurement to its exact Fahrenheit measurement or vice versa. For example, what if you want to know the temperature of your child in Fahrenheit, and the only thermometer you have measures temperature in Celsius measurement? Converting temperature between the systems is a straightforward process as long as you use the formulas provided below.

Temperature

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View this YouTube Video online

This video explains the Fahrenheit and Celsius temperature scales and shows how to use the formulas to convert between them with step-by-step examples.

Temperature Conversion Formulas

Temperature Conversion Formulas

To convert a Fahrenheit measurement to a Celsius measurement, use this formula.

$$C = \frac{5}{9}(F - 32)$$

To convert a Celsius measurement to a Fahrenheit measurement, use this formula.

$$F = \frac{9}{5}C + 32$$

How were these formulas developed? They came from comparing the two scales. Since the freezing point is 0° in the Celsius scale and 32° on the Fahrenheit scale, we subtract 32 when converting from Fahrenheit to Celsius, and add 32 when converting from Celsius to Fahrenheit.

There is a reason for the fractions $\frac{5}{9}$ and $\frac{9}{5}$, also. There are 100 degrees between the freezing (0°) and boiling points (100°) of water on the Celsius scale and 180 degrees between the similar points (32° and 212°) on the Fahrenheit scale. Writing these two scales as a ratio, $\frac{F°}{C°}$, gives $\frac{180°}{100°} = \frac{180° \div 20}{100° \div 20} = \frac{9}{5}$. If you flip the ratio to be $\frac{C°}{F°}$, you get $\frac{100°}{180°} = \frac{100° \div 20}{180° \div 20} = \frac{5}{9}$. Notice how these fractions are used in the conversion formulas.

The example below illustrates the conversion of Celsius temperature to Fahrenheit temperature, using the boiling point of water, which is 100° C.

Example 1

The boiling point of water is 100°C. What temperature does water boil at in the Fahrenheit scale?

Solution

A Celsius temperature is given. To convert it to the Fahrenheit scale, use the formula below.

$$F = \frac{9}{5}C + 32$$

Substitute 100 for C and multiply.

$$F = \frac{9}{5}(100) + 32$$

$$F = \frac{900}{5} + 32$$

Simplify $\frac{900}{5}$ by dividing numerator and denominator by 5.

$$F = \frac{900 \div 5}{5 \div 5} + 32$$

$$F = \frac{180}{1} + 32$$

Add 180 + 32.

$$F = 212$$

Answer: The boiling point of water is 212°F.

Example 2

Water freezes at 32°F. On the Celsius scale, what temperature is this?

Solution

A Fahrenheit temperature is given. To convert it to the Celsius scale, use the formula below.

$$C = \frac{5}{9}(F - 32)$$

Substitute 32 for F and subtract.

$$C = \frac{5}{9}(32 - 32)$$

Any number multiplied by 0 is 0

$$C = \frac{5}{9}(0)$$

$$C = 0$$

Answer: The freezing point of water is 0°C.

The two previous problems used the conversion formulas to verify some temperature conversions that were discussed earlier—the boiling and freezing points of water. The next example shows how these formulas can be used to solve a real-world problem using different temperature scales.

Example 3

Two scientists are doing an experiment designed to identify the boiling point of an unknown liquid. One scientist gets a result of 120°C; the other gets a result of 250°F. Which temperature is higher and by how much?

Solution

One temperature is given in °C, and the other is given in °F. To find the difference between them, we need to measure them on the same scale.

What is the difference between 120°C and 250°F?

Use the conversion formula to convert 120°C to °F. (You could convert 250°F to °C instead; this is explained in the text after this example.)

$$F = \frac{9}{5}C + 32$$

Substitute 120 for C.

$$F = \frac{9}{5}(120) + 32$$

Multiply.

$$F = \frac{1080}{5} + 32$$

Simplify $\frac{1080}{5}$ by dividing numerator and denominator by 5.

$$F = \frac{1080 \div 5}{5 \div 5} + 32$$

Add 216 + 32.

$$F = \frac{216}{1} + 32$$

You have found that 120°C = 248°F.

$$F = 248$$

To find the difference between 248°F and 250°F, subtract.

$$250\ °F – 248\ °F = 2\ °F$$

Answer: 250°F is the higher temperature by 2°F

You could have converted 250°F to °C instead, and then found the difference in the two measurements. (Had you done it this way, you would have found that 250°F = 121.1°C, and that 121.1°C is 1.1°C higher than 120°C.) Whichever way you choose, it is important to compare the temperature measurements within the same scale, and to apply the conversion formulas accurately.

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4.4.4: Applications Involving Metric and U.S Customary Conversion

In many real-world situations, measurements are not all given in the same system. A job site might use feet and inches, while equipment specifications are listed in meters. A product label may list volume in liters, while a container is measured in gallons. To work accurately, you need to recognize the type of measurement, choose the correct conversion, and decide how to combine values when they are given in different units.

In this section, you will solve problems that require converting between the Metric and U.S. Customary systems. Some problems involve a single conversion, while others require multiple steps or operations such as addition, subtraction, multiplication, or division. As you work, focus on identifying the units, selecting appropriate conversion factors, and keeping track of units throughout the solution.

Applications Involving Length

Example 1

A metal rod measures 2.4 meters. What is the length of the rod in inches? Round to the nearest whole inch.

Solution

$$\begin{aligned} 2.4 \text{ m} \cdot \frac{39.3701 \text{ in}}{1 \text{ m}} &= 94.48824 \text{ in} \quad \text{Convert meters to inches.} \\ &\approx 94 \text{ in} \quad \text{Round to the nearest whole inch.} \end{aligned}$$

The rod is approximately 94 inches long.

Example 2

A landscaping border uses 18 sections of edging, each 2.5 feet long. What is the total length of edging in meters? Round to the nearest hundredth of a meter.

Solution

$$\begin{aligned} 18 \cdot 2.5 \text{ ft} &= 45 \text{ ft} \quad \text{Find the total length in feet.} \\ 45 \text{ ft} \cdot \frac{0.3048 \text{ m}}{1 \text{ ft}} &= 13.716 \text{ m} \quad \text{Convert feet to meters.} \\ &\approx 13.72 \text{ m} \quad \text{Round to the nearest hundredth.} \end{aligned}$$

The total length of edging is approximately 13.72 meters.

Exercise

A trail marker lists a distance of 2.3 kilometers. What is the distance in miles? Round to the nearest tenth of a mile.

Answer available at the end of this section.

Applications Involving Weight

Example 3

A package has a mass of 850 grams. What is the mass in ounces? Round to the nearest tenth of an ounce.

Solution

$$850\text{ g}\cdot\frac{0.035274\text{ oz}}{1\text{ g}}=29.9829\text{ oz}\quad\text{Convert grams to ounces.}$$
$$\approx 30.0\text{ oz}\quad\text{Round to the nearest tenth.}$$

The package has a mass of approximately 30.0 ounces.

Example 4

A shipment weighs 18 pounds. After removing 1.5 kilograms, what is the remaining weight in pounds? Round to the nearest tenth of a pound.

Solution

$$1.5\text{ kg}\cdot\frac{2.20462\text{ lb}}{1\text{ kg}}=3.30693\text{ lb}\quad\text{Convert kilograms to pounds.}$$
$$18-3.30693=14.69307\text{ lb}\quad\text{Find the remaining weight.}$$
$$\approx 14.7\text{ lb}\quad\text{Round to the nearest tenth.}$$

The remaining weight is approximately 14.7 pounds.

Exercise

A load has a mass of 12 pounds. What is the mass in grams? Round to the nearest gram.

Answer available at the end of this section.

Applications Involving Area

Example 5

A field covers 2.4 acres. What is the area in hectares? Round to the nearest hundredth of a hectare.

Solution

$$2.4\text{ acres}\cdot\frac{0.404686\text{ ha}}{1\text{ acre}}=0.9712464\text{ ha}\quad\text{Convert acres to hectares.}$$
$$\approx 0.97\text{ ha}\quad\text{Round to the nearest hundredth.}$$

The area is approximately 0.97 hectares.

Example 6

A sign has an area of 1.5 square meters. A portion measuring 4 square feet is removed. What is the remaining area in square meters? Round to the nearest hundredth of a square meter.

Solution

$$4\ \text{ft}^2 \cdot \frac{0.092903\ \text{m}^2}{1\ \text{ft}^2} = 0.371612\ \text{m}^2 \quad \text{Convert square feet to square meters.}$$
$$1.5 - 0.371612 = 1.128388\ \text{m}^2 \quad \text{Find the remaining area.}$$
$$\approx 1.13\ \text{m}^2 \quad \text{Round to the nearest hundredth.}$$

The remaining area is approximately 1.13 square meters.

Exercise

A sheet of plastic has an area of 150 square inches. What is the area in square meters? Round to the nearest tenth of a square meter.

Answer available at the end of this section.

Applications Involving Volume

Example 7

A storage bin has a volume of 5.2 cubic feet. What is the volume in cubic meters? Round to the nearest hundredth of a cubic meter.

Solution

$$5.2\ \text{ft}^3 \cdot \frac{0.0283168\ \text{m}^3}{1\ \text{ft}^3} = 0.14724736\ \text{m}^3 \quad \text{Convert cubic feet to cubic meters.}$$
$$\approx 0.15\ \text{m}^3 \quad \text{Round to the nearest hundredth.}$$

The volume is approximately 0.15 cubic meters.

Example 8

A tank holds 2 gallons of liquid. After removing 1.5 liters, what is the remaining volume in gallons? Round to the nearest hundredth of a gallon.

Solution

$$1.5\ \text{L} \cdot \frac{0.264172\ \text{gal}}{1\ \text{L}} = 0.396258\ \text{gal} \quad \text{Convert liters to gallons.}$$
$$2 - 0.396258 = 1.603742\ \text{gal} \quad \text{Find the remaining volume.}$$
$$\approx 1.60\ \text{gal} \quad \text{Round to the nearest hundredth.}$$

The remaining volume is approximately 1.60 gallons.

Exercise

A container holds 120 cubic inches of liquid. What is the volume in liters? Round to the nearest hundredth of a liter.

Answer available at the end of this section.

Applications Involving Compound Units

Example 9

A vehicle travels at a speed of 60 miles per hour. What is this speed in meters per second? Round to the nearest whole number.

Solution

$$60\frac{\text{mi}}{\text{hr}} \cdot \frac{1609.34\text{ m}}{1\text{ mi}} \cdot \frac{1\text{ hr}}{3600\text{ s}} = 26.8223\frac{\text{m}}{\text{s}} \quad \text{Convert miles to meters and hours to seconds.}$$
$$\approx 27\frac{\text{m}}{\text{s}} \quad \text{Round to the nearest whole number.}$$

The speed is approximately 27 meters per second.

Example 10

A pump delivers 12 gallons per minute for 5 minutes. How many liters of liquid are delivered? Round to the nearest whole liter.

Solution

$$12\frac{\text{gal}}{\text{min}} \cdot 5\text{ min} = 60\text{ gal} \quad \text{Find the total volume in gallons.}$$
$$60\text{ gal} \cdot \frac{3.78541\text{ L}}{1\text{ gal}} = 227.1246\text{ L} \quad \text{Convert gallons to liters.}$$
$$\approx 227\text{ L} \quad \text{Round to the nearest whole number.}$$

The pump delivers approximately 227 liters.

Exercise

A conveyor belt moves material at a rate of 2.5 meters per second for 3 minutes. How many feet does the material travel? Round to the nearest whole foot.

Answer available at the end of this section.

Answers

Exercise
1.4 miles

Exercise
5443 grams

Exercise
0.1 square meters

Exercise
1.97 liters

Exercise
1476 feet

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4.4.5: Metric and U.S. Customary Conversion Practice

The following set of exercises provides practice for the concepts covered in Module 4.4: Metric and U.S. Customary Conversions. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 4.4.1 - Length and Mass Between Systems

Convert each of the given values. Round your answer as indicated.

1. 42 inches to centimeters, rounded to the nearest tenth of a centimeter.
2. 22 centimeters to inches, rounded to the nearest tenth of an inch.
3. 39 miles to kilometers, rounded to the nearest tenth of a kilometer.
4. 1522 centimeters to yards, rounded to the nearest tenth of a yard.
5. 4.2 kilometers to miles, rounded to the nearest tenth of a mile.
6. 19.4 kilograms to pounds, rounded to the nearest tenth of a pound.
7. 33 ounces to grams, rounded to the nearest tenth of a gram.
8. 56 miles per hour to kilometers per hour, rounded to the nearest kilometer per hour.

Section 4.4.2 - Area and Volume Between Systems

Convert each of the given values. Round your answer as indicated.

9. $18.5\ \text{ft}^2$ to square meters, rounded to the nearest tenth.
10. $3.6\ \text{m}^2$ to square feet, rounded to the nearest tenth.
11. $4.2\ \text{ft}^3$ to cubic meters, rounded to the nearest thousandth.
12. $185\ \text{in}^3$ to cubic centimeters, rounded to the nearest whole number.
13. 55.9 liters to gallons, rounded to the nearest tenth of a gallon.

Section 4.4.3 - Temperature Conversions

Use $F = \frac{9}{5}C + 32$ or $C = \frac{5}{9}(F - 32)$.

14. 22°C to degrees Fahrenheit.
15. 68°F to degrees Celsius.

Section 4.4.4 - Applications Involving Metric and the U.S. Customary Systems

Solve each problem. Use appropriate unit conversions. Round your answer as indicated and include units.

16. A cable measures 4.2 meters. What is the length in feet? Round to the nearest tenth.

17. A piece of plastic covers 120 square inches. What is the area in square centimeters? Round to the nearest whole number.

18. A container holds 2.8 liters of liquid. What is the volume in gallons? Round to the nearest hundredth.

19. A vehicle travels at 55 miles per hour. What is the speed in meters per second? Round to the nearest whole number.

20. A board is 6 feet long. After cutting off 0.5 meters, what is the remaining length in feet? Round to the nearest tenth.

21. A load has a mass of 3.6 kilograms. After removing 1.2 pounds, what is the remaining mass in kilograms? Round to the nearest hundredth.

22. A tank holds 1.5 gallons of liquid. What is the volume in liters? Round to the nearest hundredth.

23. A pump delivers 10 liters per minute for 6 minutes. How many gallons are delivered? Round to the nearest hundredth.

24. A conveyor belt moves at 75 feet per minute. What is the speed in meters per second? Round to the nearest hundredth.

25. A cyclist travels at 20 kilometers per hour for 45 minutes. How many miles does the cyclist travel? Round to the nearest tenth.

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5: GEOMETRY

Geometry is the study of shapes, measurements, and the relationships between figures. In technical fields, geometry is used any time you measure a space, cut material, build a frame, read a blueprint, or calculate how much material is needed for a job.

In this unit, you will begin by working with angles, polygons, and triangles. You will learn how to identify and classify different figures, use angle relationships to find missing measures, and use triangle similarity and congruence to solve problems.

You will also work with two-dimensional figures by finding perimeter and area. These skills are useful for tasks such as measuring borders, estimating flooring or paint, and planning material needs.

Finally, you will extend these ideas to three-dimensional figures. You will find lateral surface area, total surface area, and volume of prisms and cylinders, and you will apply those skills to realistic situations involving containers, materials, and space.

5.1: Angles and Geometric Relationships

5.1.1: Angles and Angle Relationships

5.1.2: Polygons and Angle Measurements

5.1.3: Triangles and Similarity

5.1.4: Angles and Relationships Practice

5.2: Perimeter and Area

5.2.1: Perimeter and Circumference

5.2.2: Area

5.2.3: Applications Involving Perimeter and Area

5.2.4: Perimeter and Area Practice

5.3: Volume and Surface Area

5.3.1: Volume and Surface Area of Prisms

5.3.2: Volume and Surface Area of Cylinders and Spheres

5.3.3: Applications of Volume and Surface Area

5.3.4: Volume and Surface Area Practice

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5.1: ANGLES AND GEOMETRIC RELATIONSHIPS

Geometry shows up any time you measure, build, or work with shapes—whether you are reading a blueprint, cutting materials, or laying out a space. In this module, you will start with one of the most basic ideas in geometry: angles. You will learn how to identify, measure, and classify angles, and use angle relationships to find missing measures.

Next, you will work with polygons, which are closed figures made up of straight sides. You will identify different types of polygons and learn how to find the measures of their interior and exterior angles. These skills will help you understand how shapes are structured and how their parts relate to each other.

You will also focus on triangles, one of the most important shapes in geometry. You will classify triangles and explore similarity and congruence. These ideas allow you to determine when triangles have the same shape or the same size, and they are useful for finding missing measurements.

Before you begin, watch the video on parallel lines and transversals. The angle relationships you see there will be used throughout this module.

Parallel Lines and Transversals

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video introduces the idea of parallel lines and transversals, a topic that will be used throughout the module.

5.1.1: Angles and Angle Relationships

5.1.2: Polygons and Angle Measurements

5.1.3: Triangles and Similarity

5.1.4: Angles and Relationships Practice

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5.1.1: Angles and Angle Relationships

We begin our study of angles with a description of how angles are formed and how they are classified. An angle is the joining of two rays, which sweep out as the sides of the angle, with a common endpoint. The common endpoint is called the vertex. We will often need to refer to more than one vertex, so you will want to know the plural of vertex, which is vertices.

In Figure $5.1.1.1$, let the ray $\overrightarrow{AB}$ stay put. Rotate the second ray $\overrightarrow{AC}$ in a counterclockwise direction to the size of the angle you want. The angle is formed by the amount of rotation of the second ray. When the ray $\overrightarrow{AC}$ continues to rotate in a counterclockwise direction back to its original position coinciding with ray $\overrightarrow{AB}$, the ray will have swept out 360°. We call the rays the “sides” of the angle.

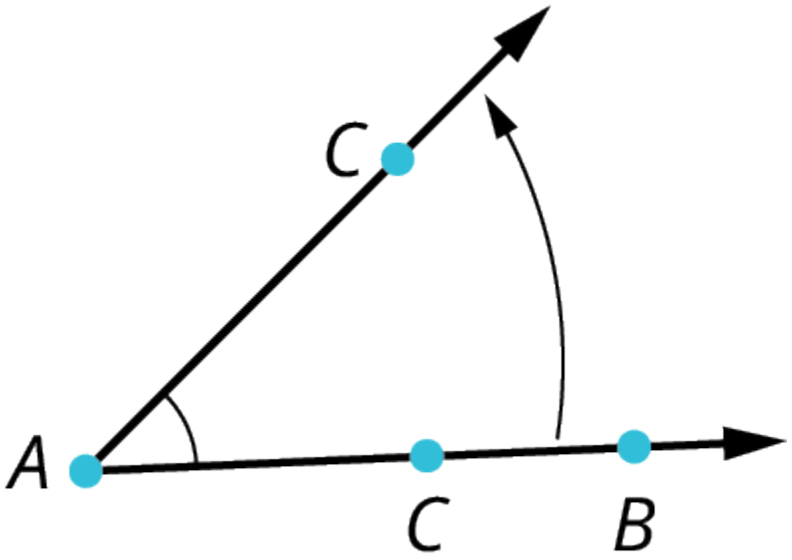


Figure $5.1.1.1$: Vertex and Sides of an Angle

Classifying Angles

Several angles are referred to so often that they have been given special names. A straight angle measures 180°; a right angle measures 90°; an acute angle is any angle whose measure is less than 90°; and an obtuse angle is any angle whose measure is between 90° and 180°. See Figure $5.1.1.2$.

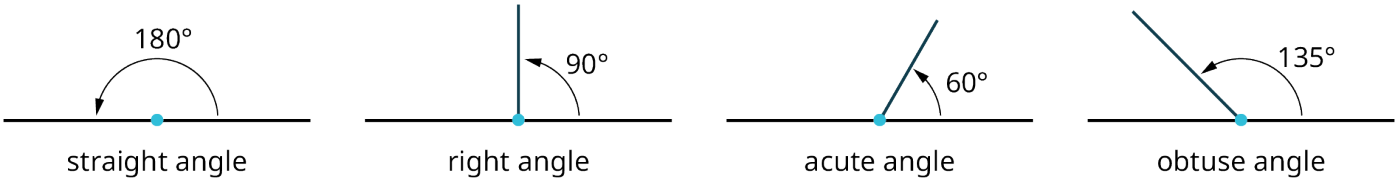


Figure $5.1.1.2$: Classifying and Naming Angles

An easy way to measure angles is with a protractor (Figure $5.1.1.3$). A protractor is a very handy little tool, usually made of transparent plastic, like the one shown here.

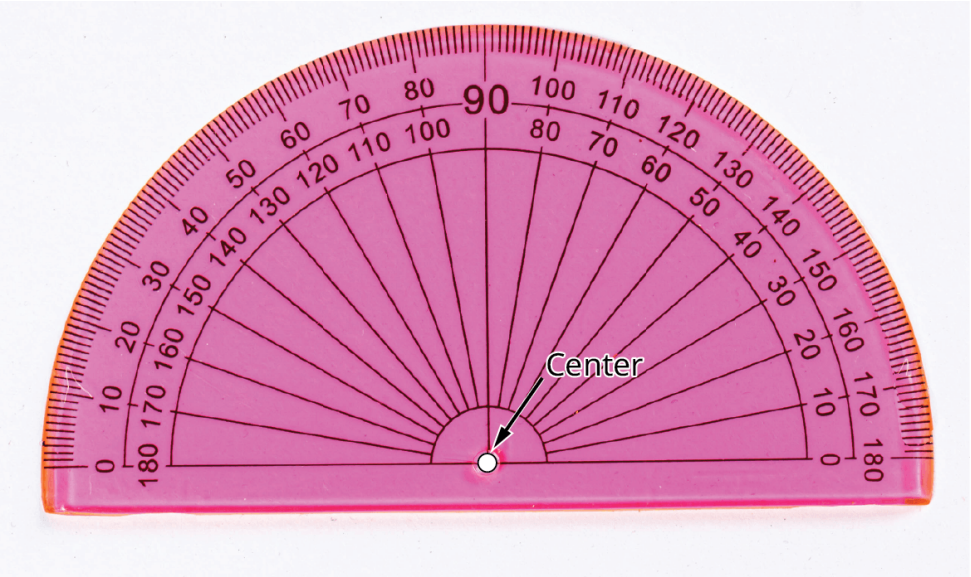


Figure 5.1.1.3: Protractor (credit: modification of work “School drawing tools” by Marco Verch/Flickr, CC BY 2.0)

With a protractor, you line up the straight bottom with the horizontal straight line of the angle. Be sure to have the center hole lined up with the vertex of the angle. Then, look for the mark on the protractor where the second ray lines up. As you can see from the image, the degrees are marked off. Where the second ray lines up is the measurement of the angle.

Checkpoint

Make sure you correctly match the center mark of the protractor with the vertex of the angle to be measured. Otherwise, you will not get the correct measurement. Also, keep the protractor in a vertical position.

Notation

Naming angles can be done in couple of ways. We can name the angle by three points, one point on each of the sides and the vertex point in the middle, or we can name it by the vertex point alone. Also, we can use the symbols $\angle$ or $\measuredangle$ before the points. When we are referring to the measure of the angle, we use the symbol $m\measuredangle$. See Figure 5.1.1.4.

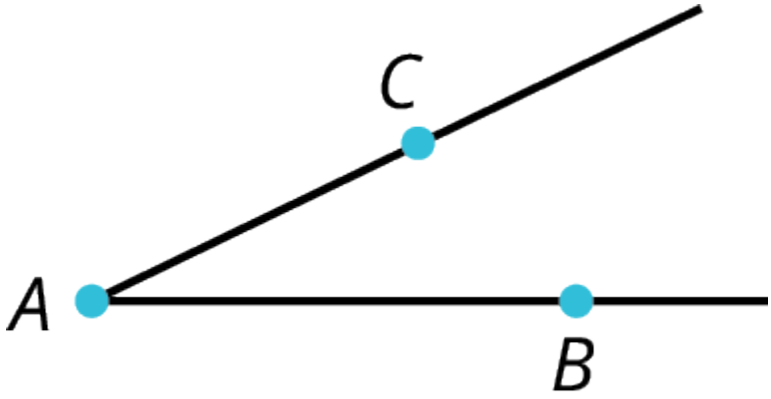
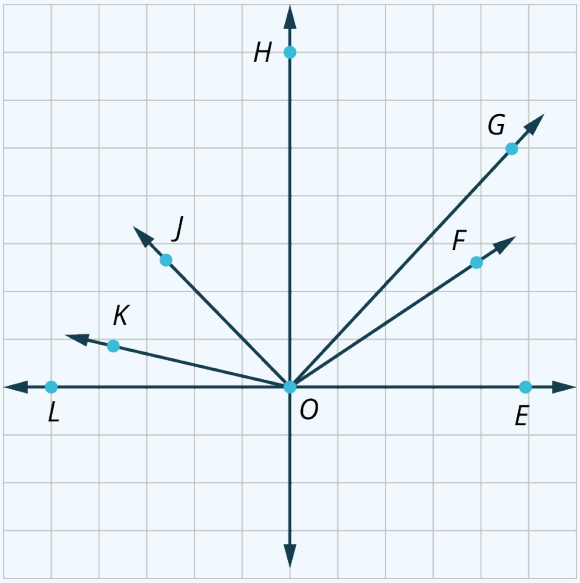


Figure 5.1.1.4: Naming an Angle

We can name this angle $\measuredangle BAC$, or $\measuredangle CAB$, or $\measuredangle A$.

Example 1

Determine which angles are acute, right, obtuse, or straight on the graph. You may want to use a protractor for this one.



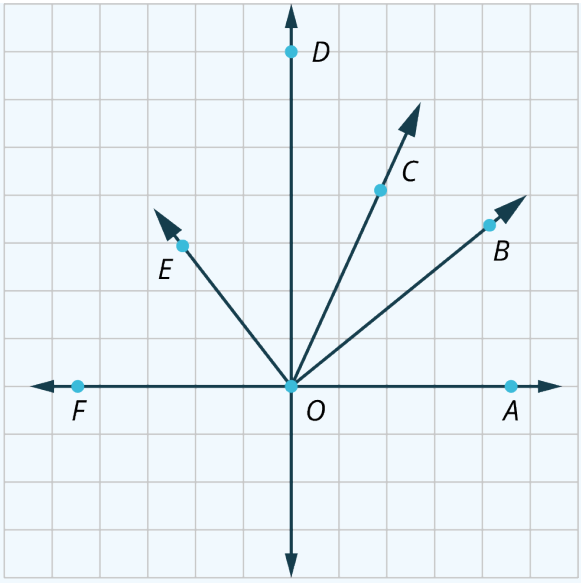
Solution

Acute angles measure less than 90°.	**Obtuse angles measure between** 90° **and** 180°.	**Right angles measure** 90°.	**Straight angles measure** 180°.
∠EOF	∠EOJ	∠EOH	∠EOL
∠EOG	∠EOK	∠HOL	
∠FOG	∠FOJ	∠GOJ	
∠GOH	∠FOK		
∠FOH	∠FOL		
∠HOJ	∠GOK		
∠HOK	∠GOL		
∠JOK			
∠KOL			
∠JOL			

Most angles can be classified visually or by description. However, if you are unsure, use a protractor.

Exercise

Determine which angles are acute, obtuse, right, and straight in the graph.



Answer available at the end of this section.

Types of Angles and Angle Relationships

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video brings together everything you need to know about angles, from basic types to how angles relate to each other.

Adjacent Angles

Two angles with the same starting point or vertex and one common side are called adjacent angles. In Figure $5.1.1.8$, angle $\angle DBC$ is adjacent to $\angle CBA$. Notice that the way we designate an angle is with a point on each of its two sides and the vertex in the middle. When adjacent angles are next to each other like this, their measures add together to equal the measure of the larger angle they form. Therefore, $m\angle DBC + m\angle CBA = m\angle DBA$.

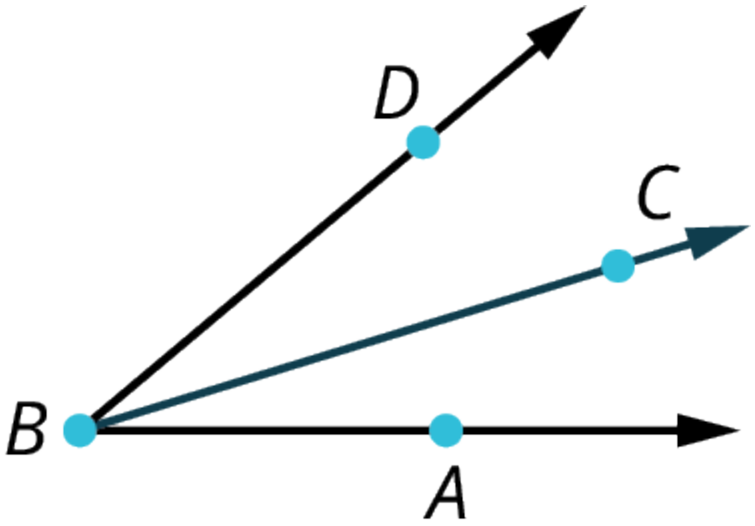
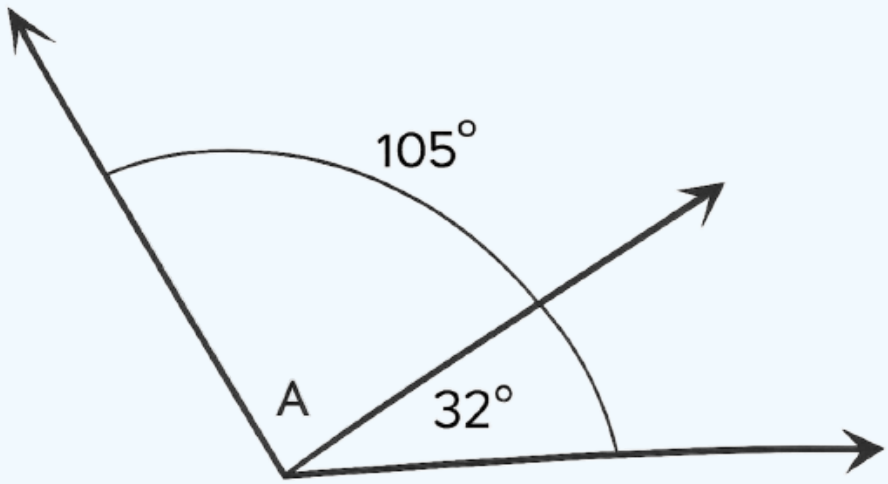


Figure 5.1.1.5: Adjacent Angles

Example 2

In the figure provided, find the m ∠ A.



Solution

$A + 32^\circ = 105^\circ$

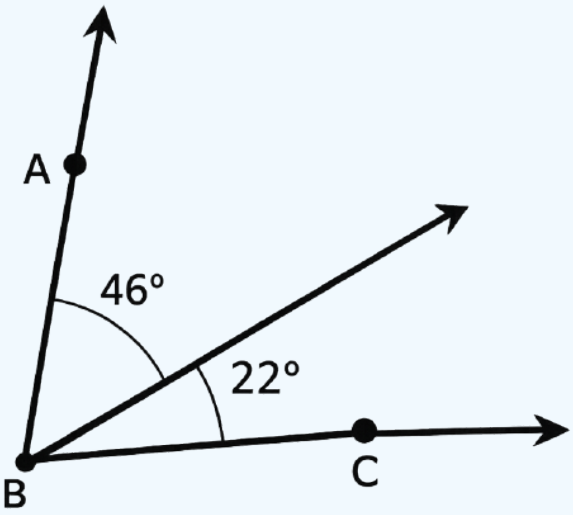
$A = 105^\circ - 32^\circ$

$A = 73^\circ$

The measure of angle A is 73°.

Exercise

In the figure provided, find the m ∠ ABC.



Answer available at the end of this section.

Determine the Measure of Missing Angles from a Figure

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video shows how to find a missing angle when two adjacent angles make up a larger angle.

Supplementary Angles

Two angles are supplementary if the sum of their measures equals 180°. In Figure $5.1.1.6$, the $m\angle ABE = 145^\circ$. The $m\angle FBE = 35^\circ$. Angles ABE and FBE are supplementary angles.

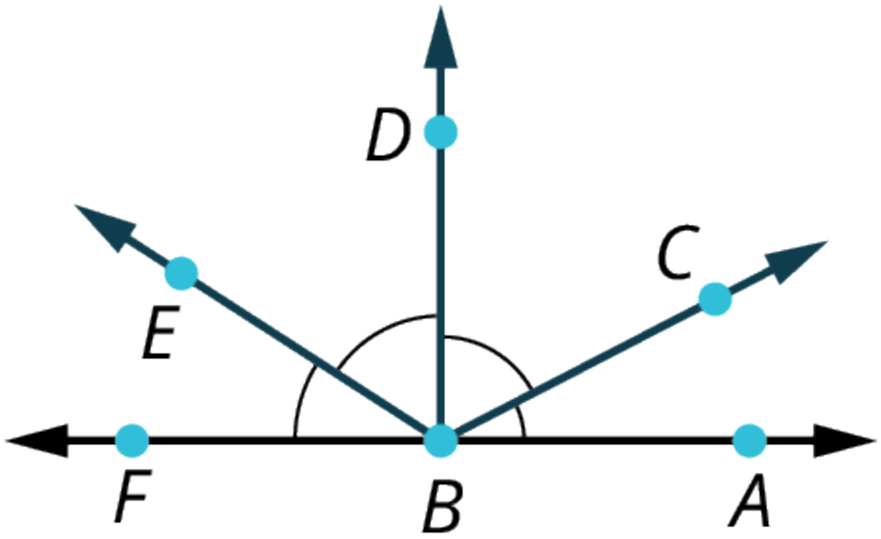
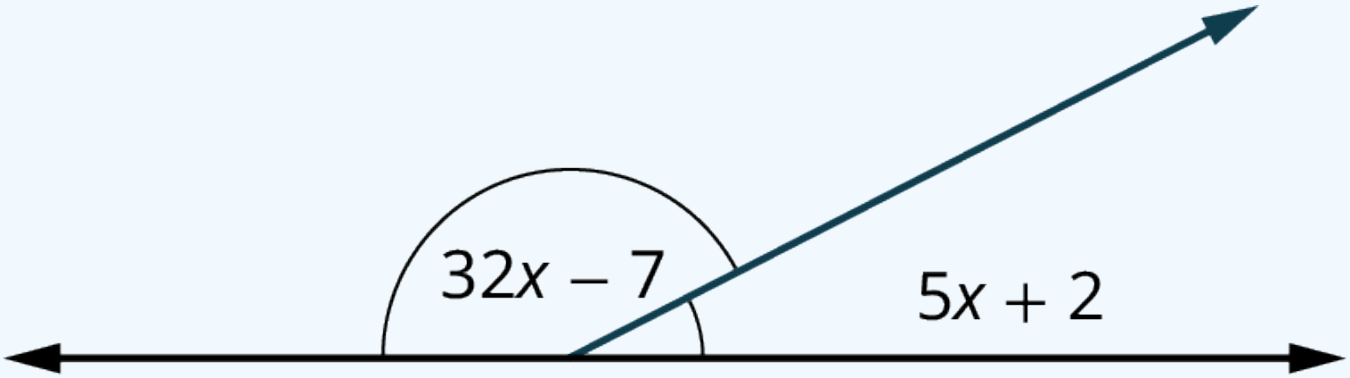


Figure 5.1.1.6: Supplementary Angles

Example 2

Solve for the angle measurements in the figure shown.



Solution

Step 1: These are supplementary angles. We can see this because the two angles are part of a horizontal line, and a horizontal line represents 180°. Therefore, the sum of the two angles equals 180°.

Step 2:

$$\begin{aligned}(32x-7)+(5x+2)&=180\\37x-5&=180\\37x&=185\\x&=5\end{aligned}$$

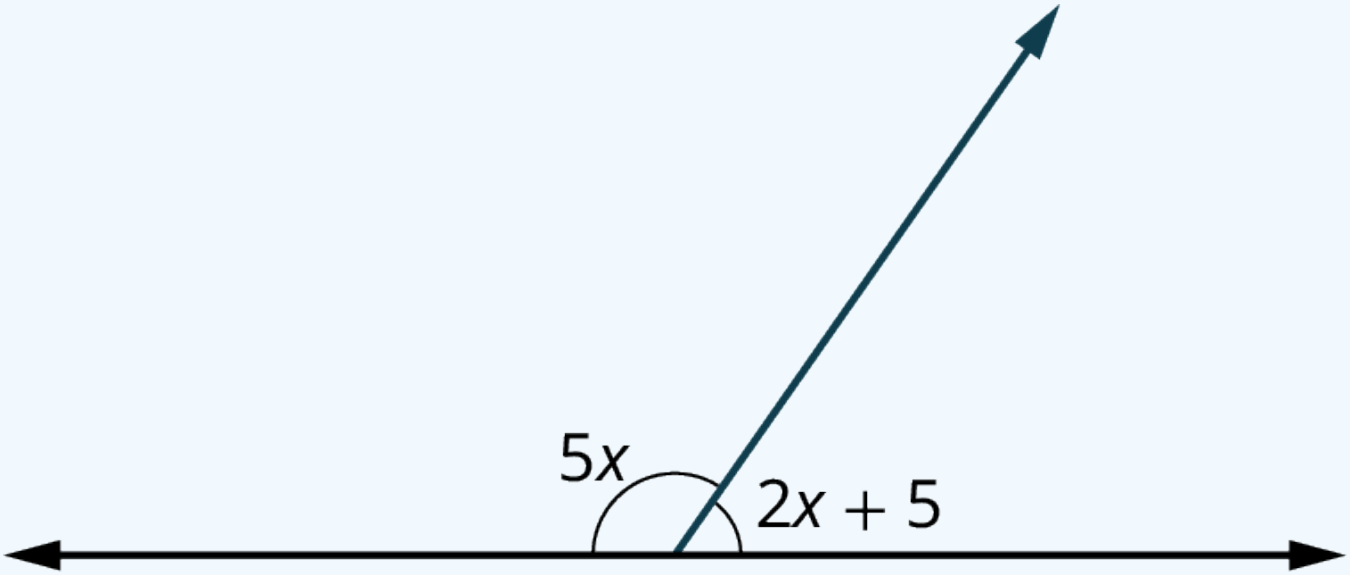
Step 3: Find the measure of each angle:

$$\begin{aligned}32x-7&=32(5)-7\\&=153^\circ\\5x+2&=5(5)+2\\&=27^\circ\end{aligned}$$

Step 4: We check: $153^\circ+27^\circ=180^\circ$.

Exercise

Solve for the angle measurements in the figure shown.



Answer available at the end of this section.

Complementary Angles

Two angles are complementary if the sum of their measures equals 90°. In Figure we have $m\measuredangle ABC = 30^\circ$, and $m\measuredangle ABD = 90^\circ$. What is the $m\measuredangle CBD$? These are complementary angles. Therefore, because $90^\circ - 30^\circ = 60^\circ$, the $\measuredangle CBD = 60^\circ$.

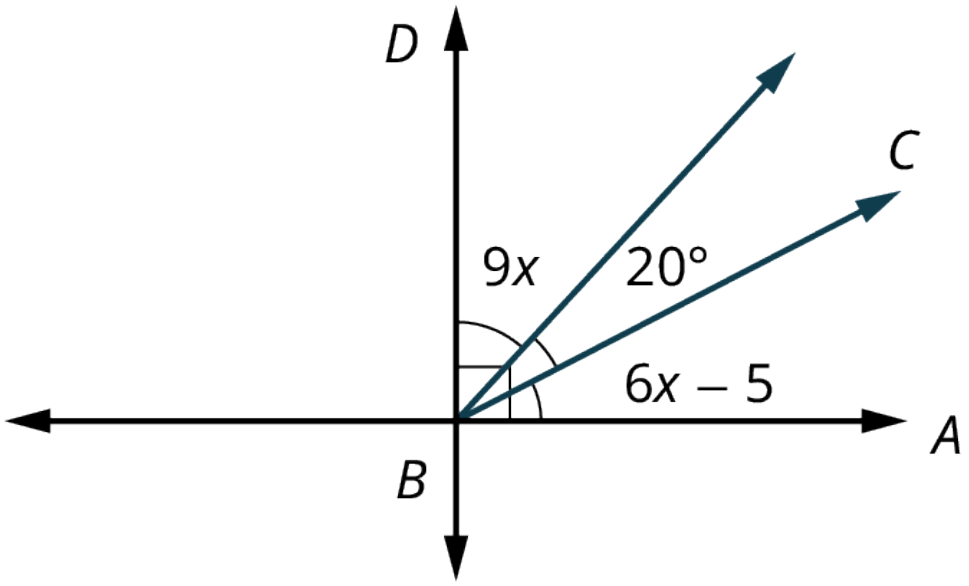
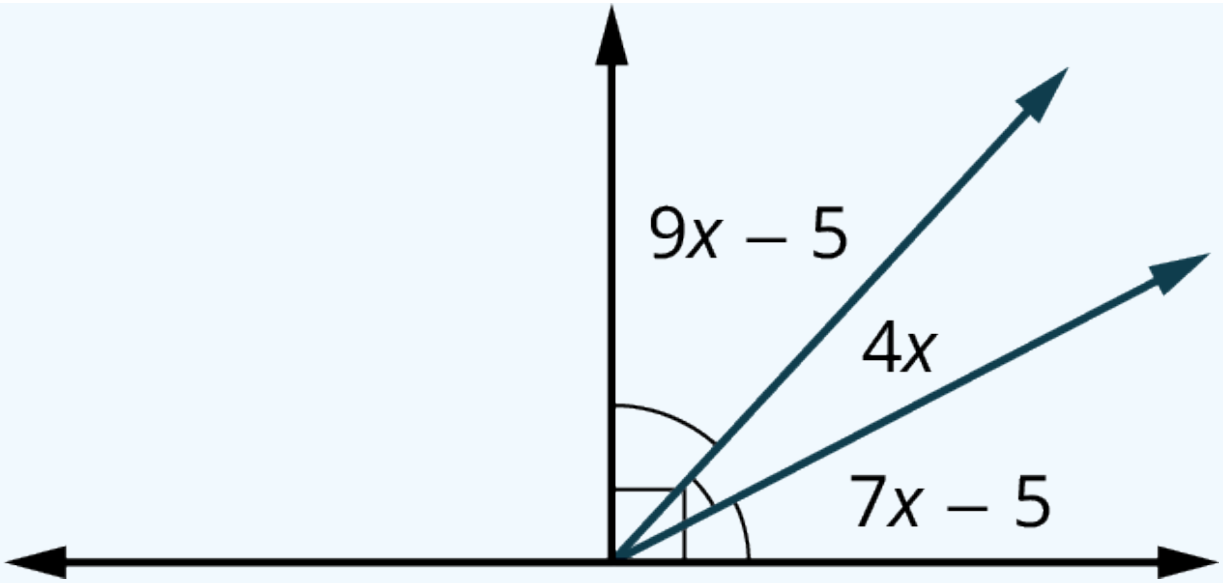


Figure 5.1.1.7: Complementary Angles

Example 3

Solve for the angle measurements in given figure.



Solution

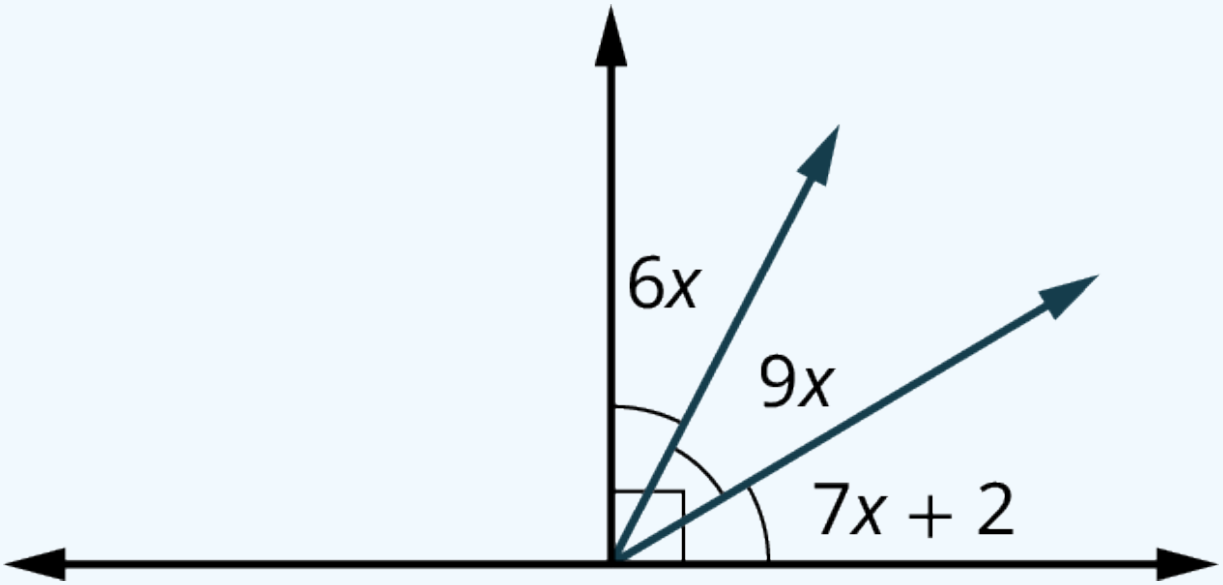
We have that

$$\begin{aligned}(9x-5)+4x+(7x-5)&=90\\20x&=100\\x&=5\end{aligned}$$

Then, $m\measuredangle(9x-5)=40^\circ, m\measuredangle(4x)=20^\circ$, and $m\measuredangle(7x-5)=30^\circ$.

Exercise

Find the measure of each angle in the given figure.



Answer available at the end of this section.

Angle Basics

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video provides additional examples of complementary and supplementary angles.

Vertical Angles

When two lines intersect, the opposite angles are called vertical angles, and vertical angles have equal measure. For example, Figure $5.1.1.15$ shows two straight lines intersecting each other. One set of opposite angles shows angle markers; those angles have the same measure. The other two opposite angles have the same measure as well.

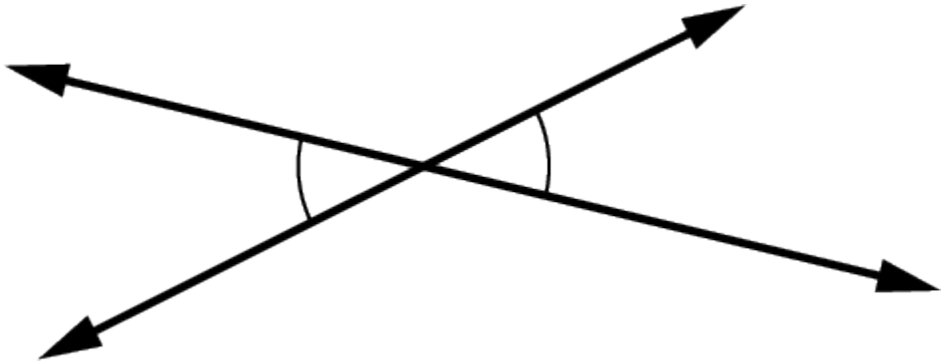
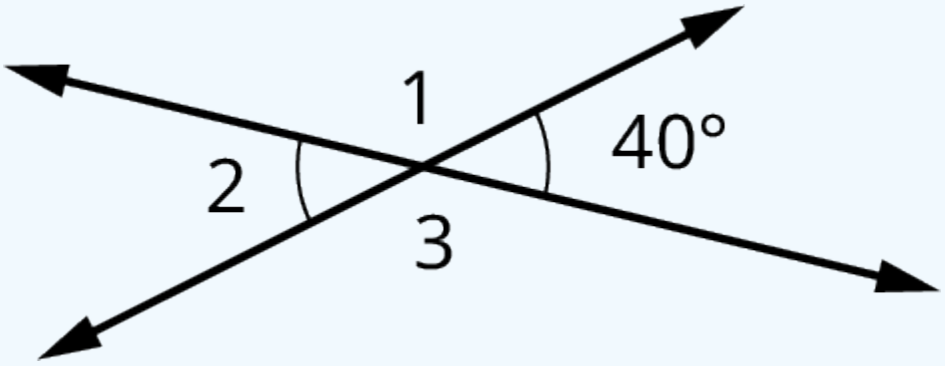


Figure $5.1.1.8$: Vertical Angles

Example 4

In given figure, one angle measures 40°. Find the measures of the remaining angles.



Solution

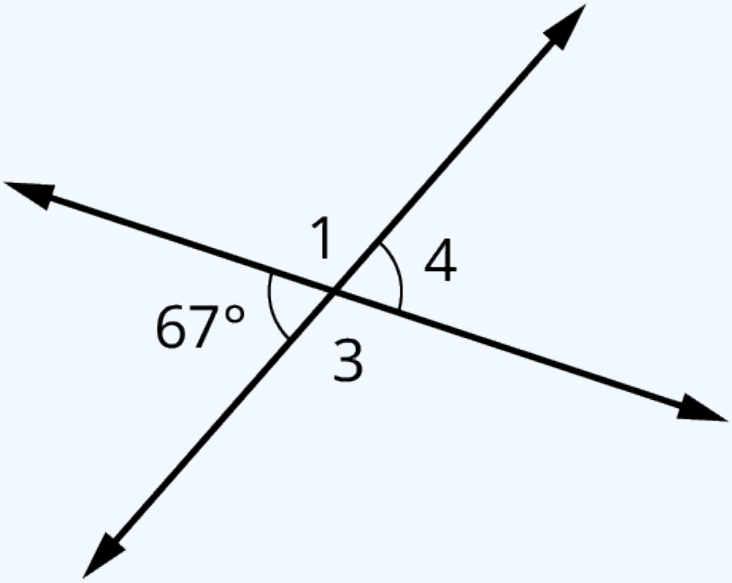
The 40-degree angle and $\angle 2$ are vertical angles. Therefore, $m\measuredangle 2 = 40^\circ$.

Notice that $\measuredangle 2$ and $\measuredangle 1$ are supplementary angles, meaning that the sum of $m\measuredangle 2$ and $m\measuredangle 1$ equals 180°. Therefore, $m\measuredangle 1 = 180^\circ - 40^\circ = 140^\circ$.

Since $\measuredangle 1$ and $\measuredangle 3$ are vertical angles, then $m\measuredangle 3$ equals 140°.

Exercise

Given the two intersecting lines in the figure shown and $m\measuredangle 2 = 67^\circ$, find the measure of the remaining angles.



Answer available at the end of this section.

Transversals

When two parallel lines are crossed by a straight line or **transversal**, eight angles are formed, including alternate interior angles, alternate exterior angles, corresponding angles, vertical angles, and supplementary angles. See Figure 5.1.1.18. Angles 1, 2, 7, and 8 are called exterior angles, and angles 3, 4, 5, and 6 are called interior angles.

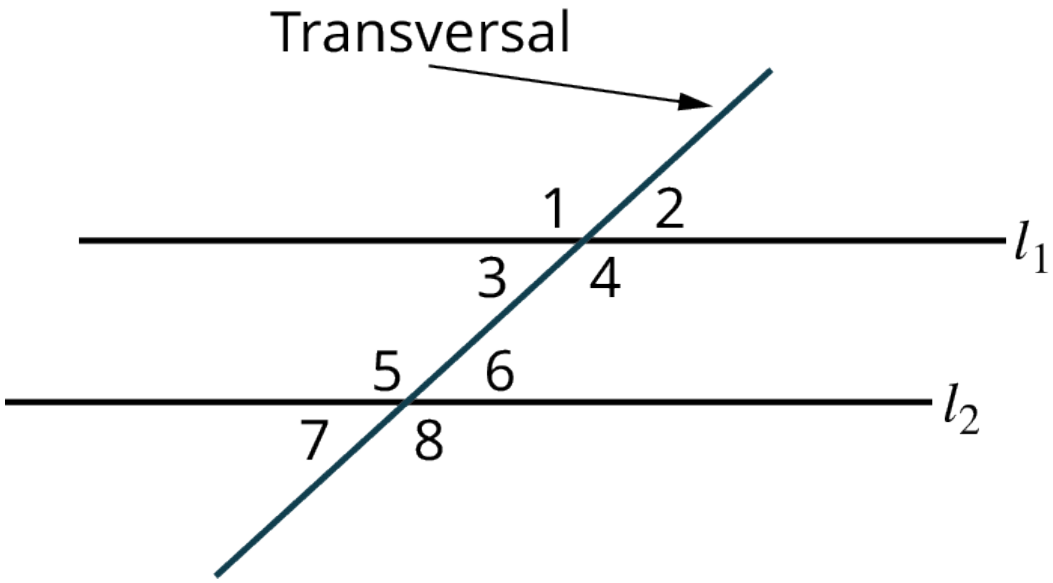


Figure 5.1.1.9: Transversal

Alternate Interior Angles

Alternate interior angles are the interior angles on opposite sides of the transversal. These two angles have the same measure. For example, $\measuredangle 3$ and $\measuredangle 6$ are alternate interior angles and have equal measure; $\measuredangle 4$ and $\measuredangle 5$ are alternate interior angles and have equal measure as well. See Figure 5.1.1.19.

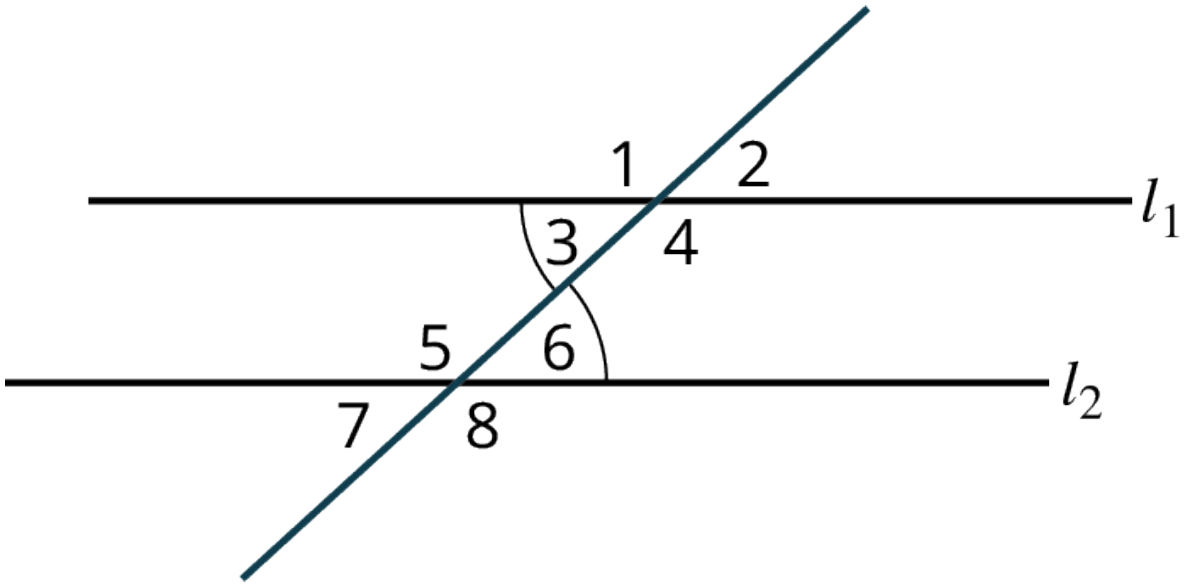


Figure 5.1.1.10: Alternate Interior Angles

Alternate Exterior Angles

Alternate exterior angles are exterior angles on opposite sides of the transversal and have the same measure. For example, in Figure 5.1.1.20, $\measuredangle 2$ and $\measuredangle 7$ are alternate exterior angles and have equal measures; $\measuredangle 1$ and $\measuredangle 8$ are alternate exterior angles and have equal measures as well.

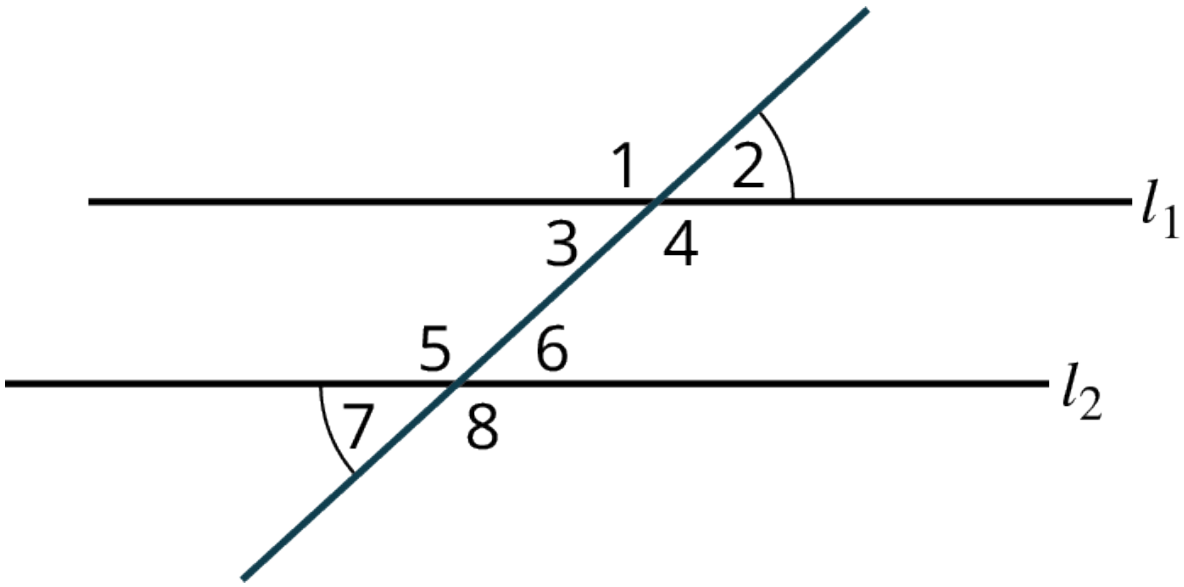


Figure 5.1.1.11: Alternate Exterior Angles

Corresponding Angles

Corresponding angles refer to one exterior angle and one interior angle on the same side as the transversal, which have equal measures. In Figure 5.1.1.21, $\measuredangle 1$ and $\measuredangle 5$ are corresponding angles and have equal measures; $\measuredangle 3$ and $\measuredangle 7$ are corresponding angles and have equal measures; $\measuredangle 2$ and $\measuredangle 6$ are corresponding angles and have equal measures; $\measuredangle 4$ and $\measuredangle 8$ are corresponding angles and have equal measures as well.

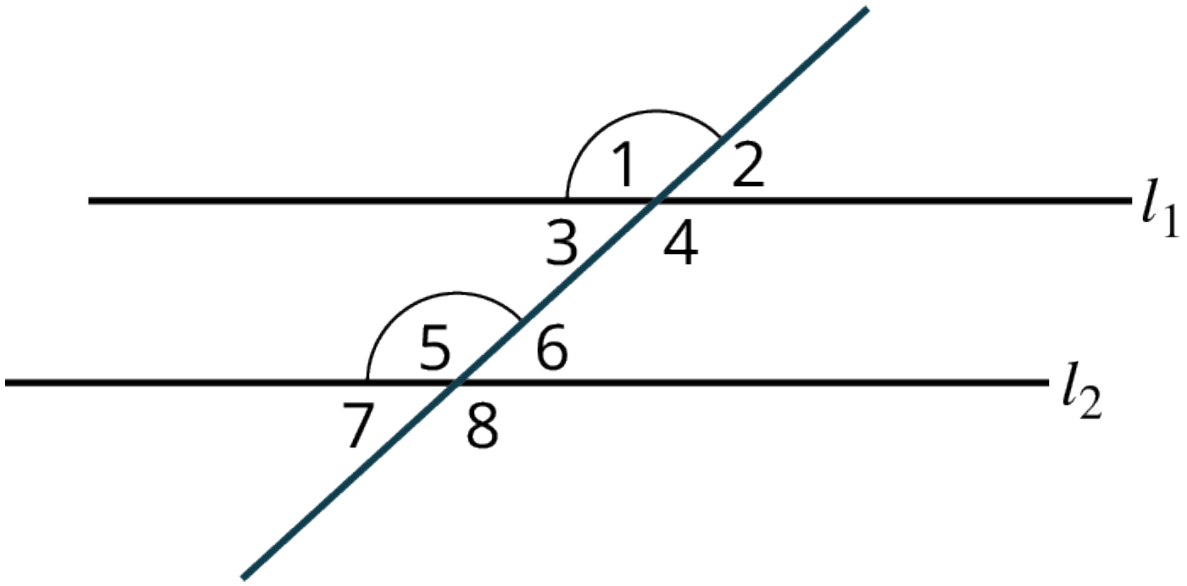
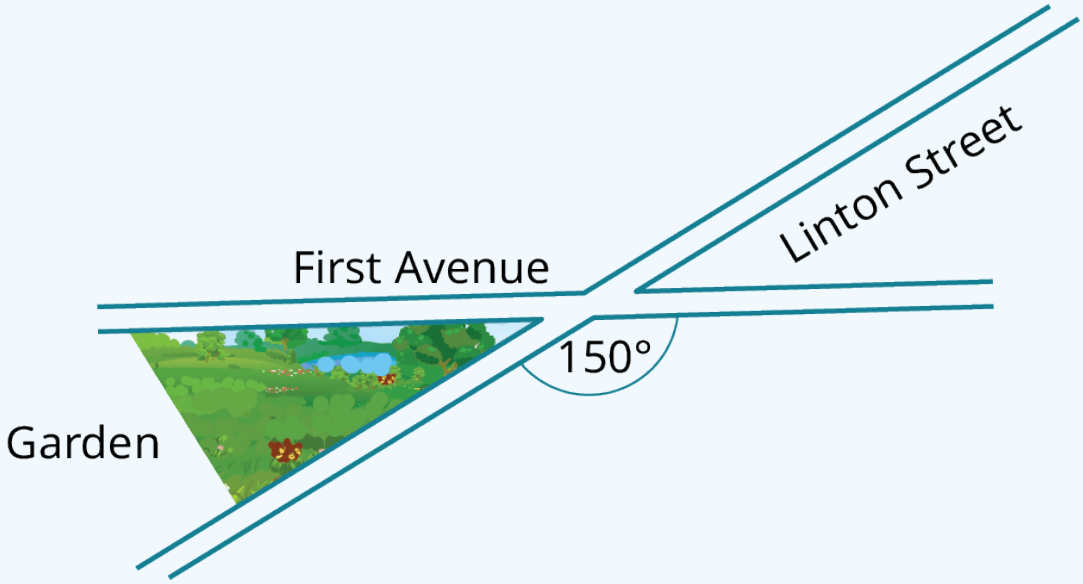


Figure 5.1.1.12: Corresponding Angles

Example 5

You live on the corner of First Avenue and Linton Street. You want to plant a garden in the far corner of your property and fence off the area. However, the corner of your property does not form the traditional right angle. You learned from the city that the streets cross at an angle equal to 150°. What is the measure of the angle that will border your garden?



Solution

As the angle between Linton Street and First Avenue is 150°, the supplementary angle is 30°. Therefore, the garden will form a 30° angle at the corner of your property.

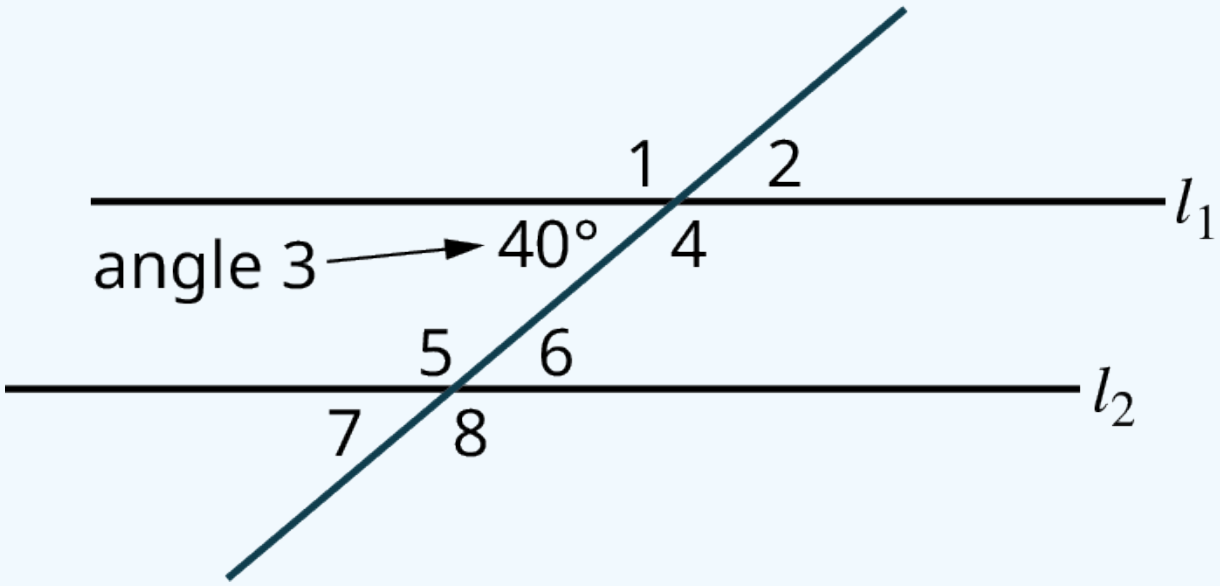
Exercise

Suppose you have a similar property to the one in example 5, but the angle that corresponds to the garden corner is 50°. What is the measure between the two cross streets?

Answer available at the end of this section.

Example 6

In the provided figure given that angle 3 measures 40°, find the measures of the remaining angles and give a reason for your solution.



Solution

$m\measuredangle 2 = m\measuredangle 3 = 40^\circ$ by vertical angles.

$\measuredangle 3 = m\measuredangle 7$ by corresponding angles.

$m\measuredangle 7 = m\measuredangle 6 = 40^\circ$ by vertical angles.

$m\measuredangle 1 = 180 - 40 = 140^\circ$ by supplementary angles.

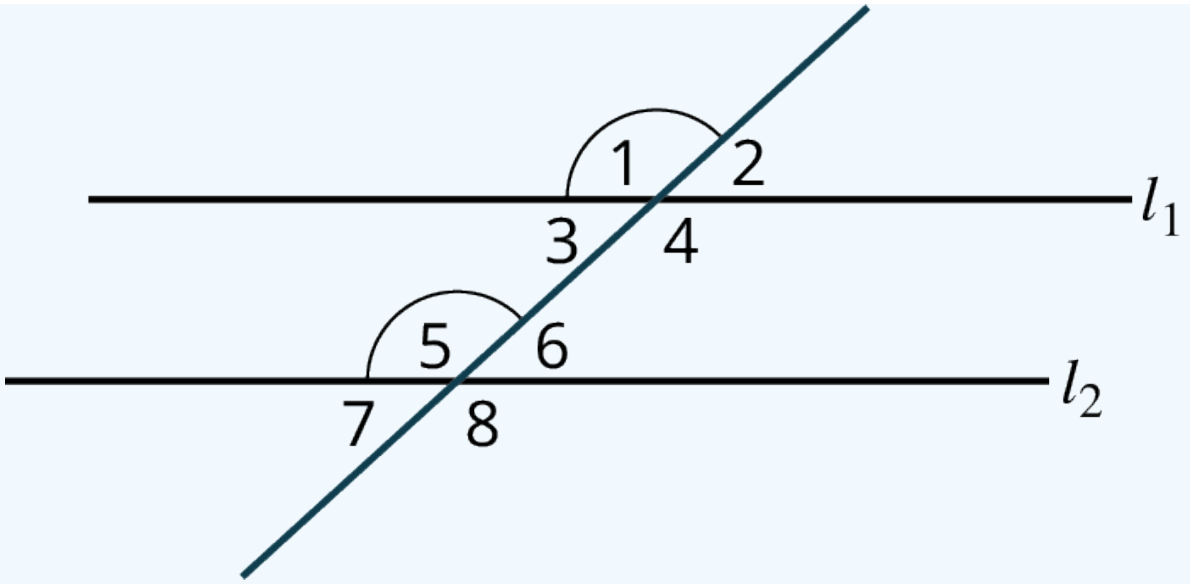
$m\measuredangle 4 = m\measuredangle 1 = 140^\circ$ by vertical angles.

$m\measuredangle 8 = m\measuredangle 1 = 140^\circ$ by alternate exterior angles.

$m\measuredangle 5 = m\measuredangle 8 = 140^\circ$ by vertical angles.

Exercise

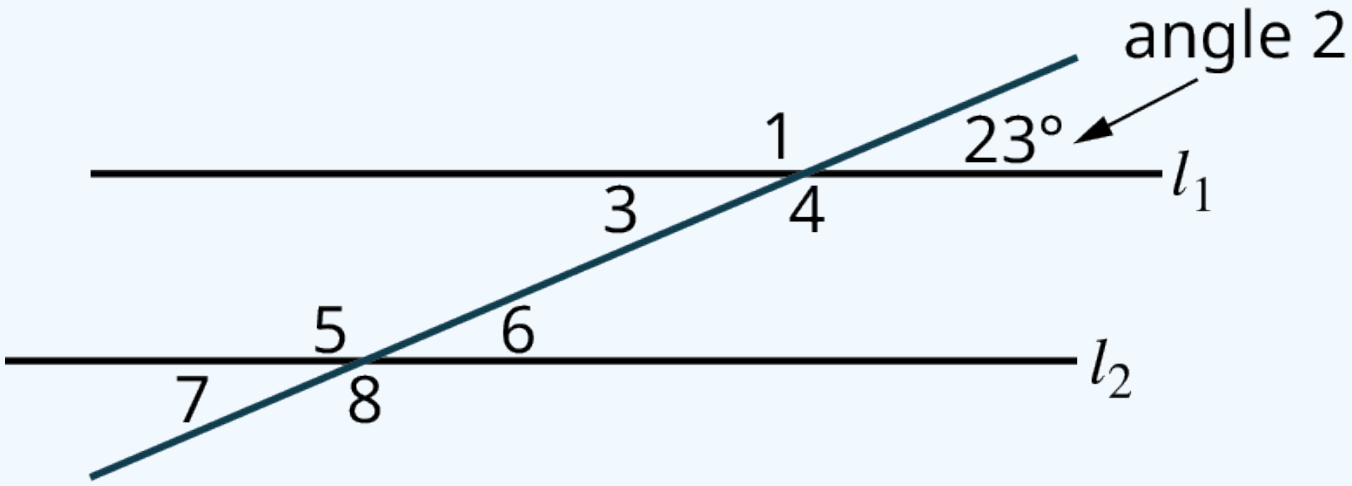
In the given figure if $m\measuredangle 1 = 120^\circ$, find the $m\measuredangle 5$, $m\measuredangle 4$, and $m\measuredangle 8$.



Answer available at the end of this section.

Example 7

In the provided figure given that angle 2 measures 23°, find the measure of the remaining angles and state the reason for your solution.



Solution

$m\measuredangle 2 = m\measuredangle 3 = 23^\circ$ by vertical angles, because $\measuredangle 2$ and $\measuredangle 3$ are the opposite angles formed by two intersecting lines.

$m\measuredangle 1 = 157^\circ$ by supplementary angles to $m\measuredangle 2$ or $m\measuredangle 3$. We see that $\measuredangle 1$ and $\measuredangle 2$ form a straight angle as does $\measuredangle 1$ and $\measuredangle 3$. A straight angle measures 180°, so $180^\circ - 23^\circ = 157^\circ$.

$m\measuredangle 4 = m\measuredangle 1 = 157^\circ$ by vertical angles, because $\measuredangle 4$ and $\measuredangle 1$ are the two opposite angles formed by two intersecting lines.

$m\measuredangle 5 = m\measuredangle 1 = 157^\circ$ by corresponding angles because they are the same angle formed by the transversal crossing two parallel lines, one exterior and one interior.

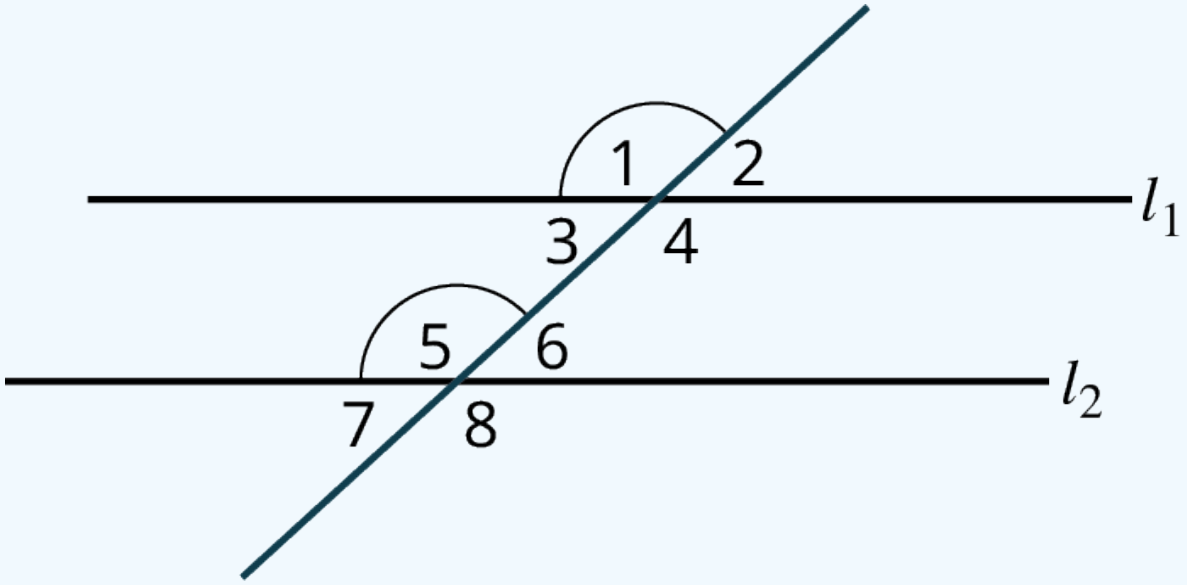
$m\measuredangle 8 = m\measuredangle 5 = 157^\circ$ by vertical angles because $\measuredangle 8$ and $\measuredangle 5$ are the two opposite angles formed by two intersecting lines.

$m\measuredangle 7 = m\measuredangle 2 = 23^\circ$ by alternate exterior angles because, like vertical angles, these angles are the opposite angles formed by the transversal intersecting two parallel lines.

$m\measuredangle 6 = m\measuredangle 7 = 23^\circ$ by vertical angles because these are the opposite angles formed by two intersecting lines.

Exercise

In the provided figure given that the $m\measuredangle 2 = 48^\circ$, find $m\measuredangle 1$, and $m\measuredangle 5$.



Answer available at the end of this section.

Angles Formed Between Transversals and Parallel Lines

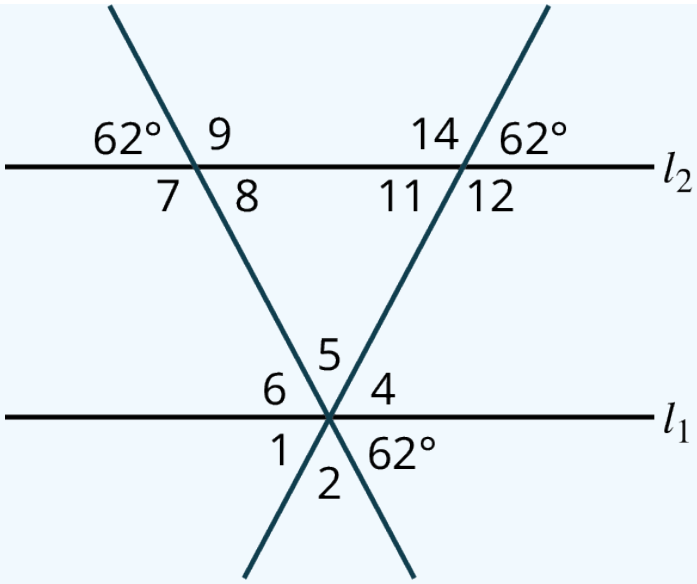
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View this YouTube Video online

This video explains how parallel lines cut by a transversal form different types of angles and how to identify them.

Example 8

Find the measures of the angles 1, 2, 4, 11, 12, and 14 in given figure and the reason for your answer given that l_1 and l_2 are parallel.



Solution

$m\measuredangle 12 = 118^\circ$, supplementary angles

$m\measuredangle 14 = 118^\circ$, vertical angles

$m\measuredangle 11 = 62^\circ$, vertical angles

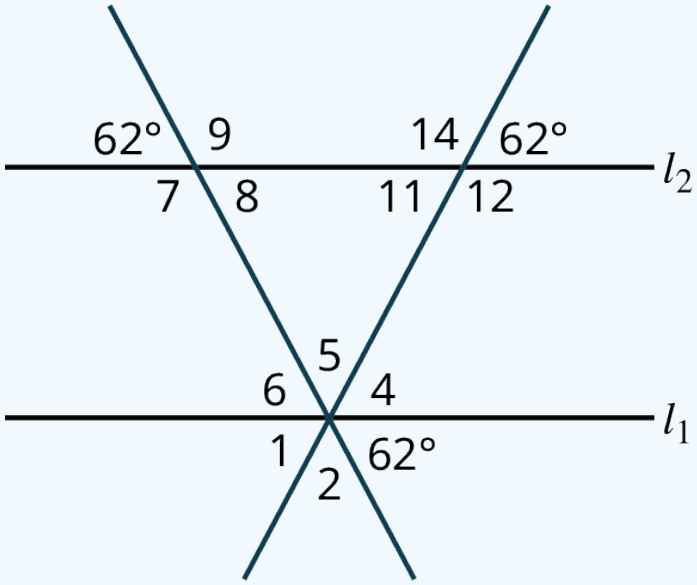
$m\measuredangle 4 = 62^\circ$, corresponding angles

$m\measuredangle 1 = 62^\circ$, vertical angles

$m\measuredangle 2 = 56^\circ$, supplementary angles

Exercise

Using the figure given in example 8, find the measures of angles 5, 6, 7, 8, and 9.



Answer available at the end of this section.

Answer available at the end of this section.

Answers

Exercise

Acute angles measure less than 90°.	Obtuse angles measure between 90° and 180°.	Right angles measure 90°.	Straight angles measure 180°.
∠AOB ∠AOC ∠BOC ∠BOD ∠COD ∠COE ∠DOE ∠EOF	∠AOE ∠BOF ∠COF	∠AOD ∠BOE ∠DOF	∠AOF

Exercise

88°

Exercise

$125^\circ, 55^\circ$

Exercise

$24^\circ, 36^\circ, 30^\circ$

Exercise

$m\angle 1 = 113^\circ$, $m\angle 3 = 113^\circ$, and $m\angle 4 = 67^\circ$.

Exercise

130°

Exercise

$m\measuredangle 5 = 120^\circ$, $m\measuredangle 4 = 60^\circ$, and $\mathrm{m}\measuredangle 8 = 120^\circ$.

Exercise

$m\angle 1 = 132^\circ$, $m\angle 5 = m\angle 1 = 132^\circ$, and $m\angle 8 = m\angle 5 = 157^\circ$.

Exercise

Exercise

$m\measuredangle 5 = 56^\circ$, $m\measuredangle 6 = 62^\circ$, $m\measuredangle 7 = 118^\circ$, $m\measuredangle 8 = 62^\circ$, and $\mathrm{m}\measuredangle 9 = 118^\circ$.

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5.1.2: Polygons and Angle Measurements

Polygonal shapes are everywhere, from road signs and furniture to buildings and everyday objects. These shapes are often chosen because their structure and angles make them useful for specific purposes.

Polygons are also used in design, where patterns are created using repeated shapes such as triangles, quadrilaterals, and hexagons. In these cases, the arrangement of sides and angles plays an important role in how the pattern looks and fits together.

In this section, we will focus on polygons and their angle measures. You will learn how to identify different types of polygons and calculate the measures of their interior and exterior angles.

Identifying Polygons

A polygon is a closed, two-dimensional shape classified by the number of straight-line sides. See Figure $5.1.2.1$ for some examples. We show only up to eight-sided polygons, but there are many, many more.

Number of Sides	Name	Shape
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	

Figure $5.1.2.1$: Types of Polygons

If all the sides of a polygon have equal lengths and all the angles are equal, it is called a regular polygon. Any closed shape made of line segments is a polygon.

In Figure $5.1.2.2$, the first two shapes are pentagons because they each have five sides and five vertices. The third shape is a hexagon because it has six sides and six vertices.

Polygons can also be classified as convex or concave. The first two shapes are convex because none of their interior angles are greater than 180°. The third shape is concave because it has an interior angle greater than 180°, causing the shape to “cave in.”

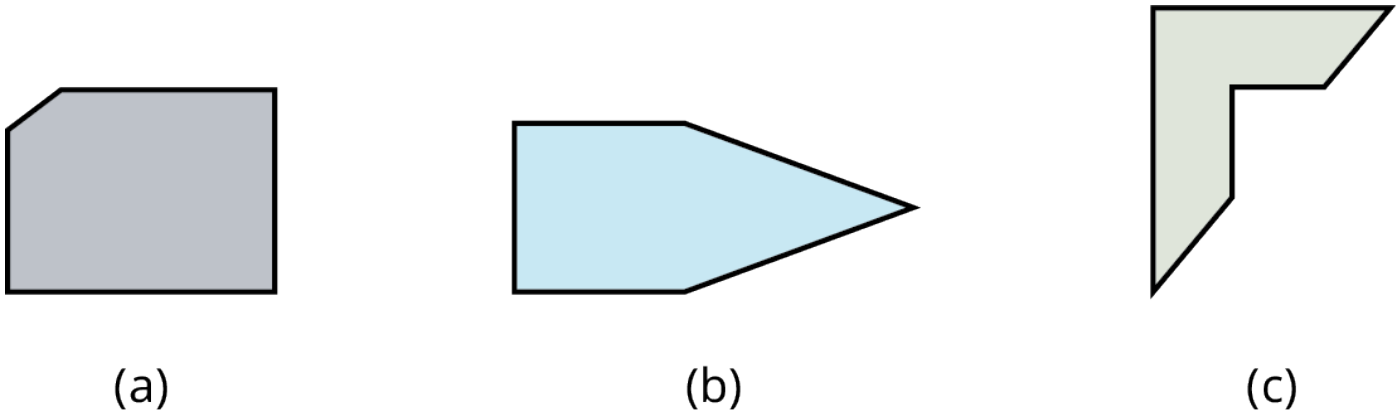


Figure 5.1.2.2 : Polygons

Polygons

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View this YouTube Video online

This video introduces polygons and explains how to identify them. It also describes how polygons are classified based on the number of sides and vertices.

While there are variations of all polygons, quadrilaterals contain an additional set of figures classified by angles and whether there are one or more pairs of parallel sides. See Figure 5.1.2.3.

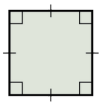
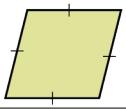
Description	Shape
A **trapezoid** has one pair of parallel sides.	
A **parallelogram** has two sets of parallel sides and no right angles.	
A **rectangle** is a parallelogram with four right angles and two sets of parallel sides.	
A **square** is a rectangle with four equal sides.	
A **rhombus** is a parallelogram with all sides equal, two sets of parallel sides.	

Figure 5.1.2.3 : Types of Quadrilaterals

Quadrilaterals

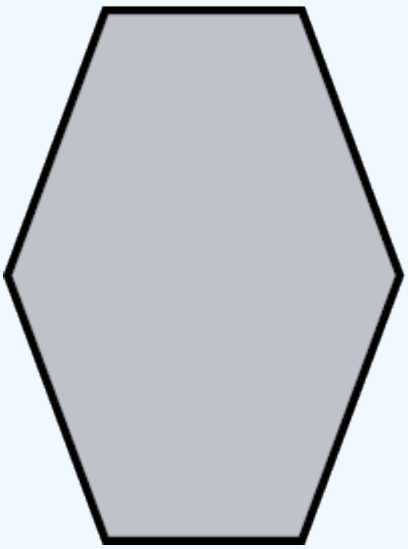
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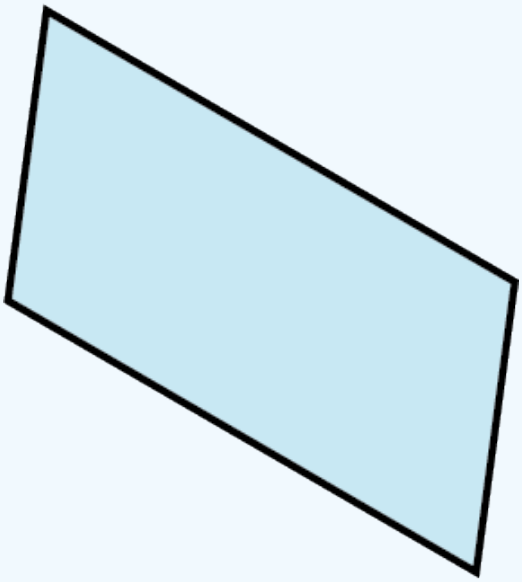
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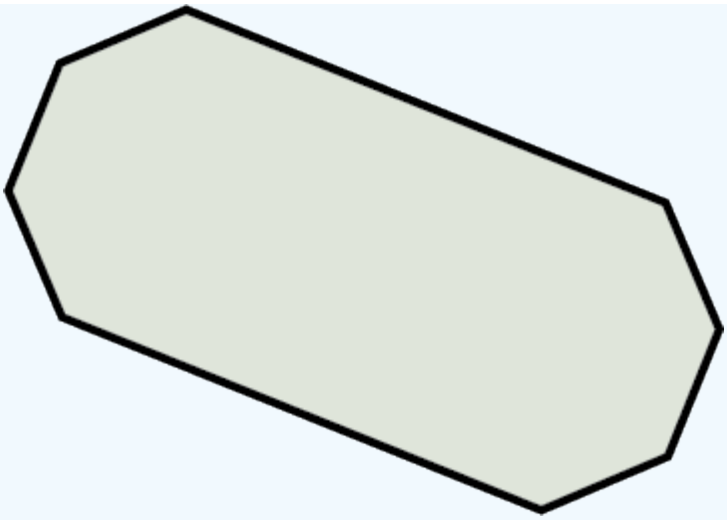
This video introduces quadrilaterals and explains how to identify and classify them based on their sides, angles, and properties.

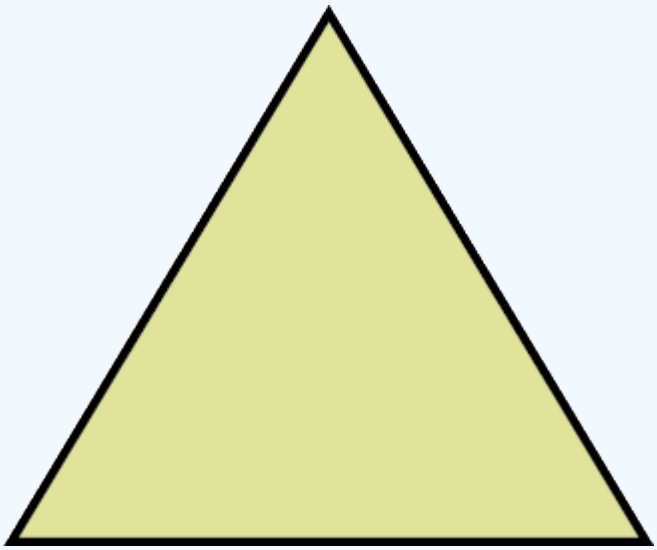
Example 1

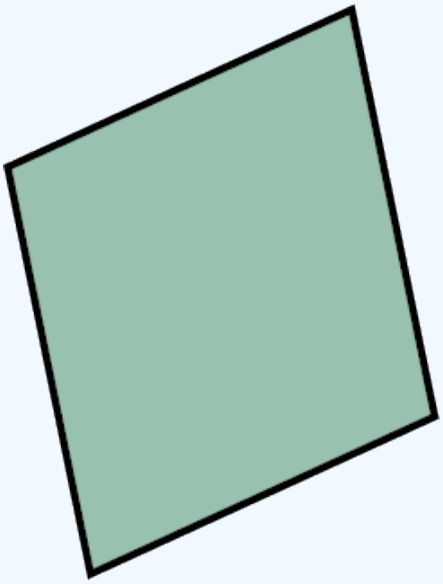
Identify each polygon.

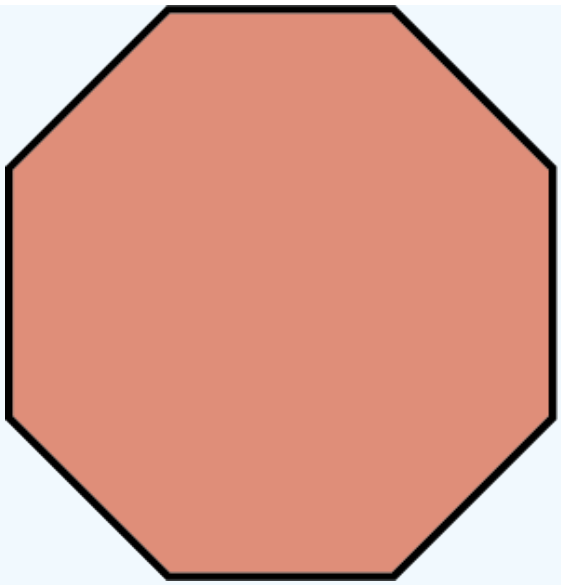
1. 

2. 

3. 

4. 

5. 

6. 

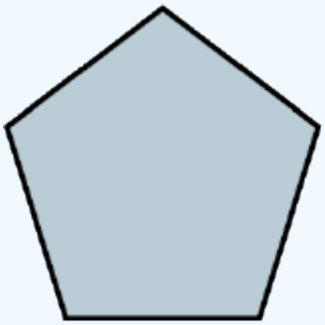
Solution

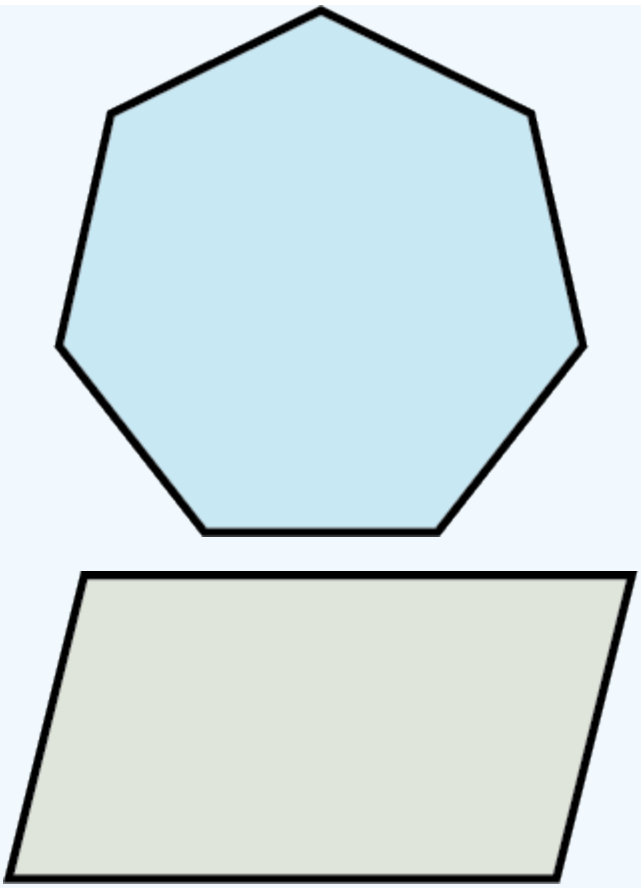
1. This shape has six sides. Therefore, it is a hexagon.
2. This shape has four sides, so it is a quadrilateral. It has two pairs of parallel sides making it a parallelogram.
3. This shape has eight sides making it an octagon.
4. This is an equilateral triangle, as all three sides are equal.
5. This is a rhombus; all four sides are equal.
6. This is a regular octagon, eight sides of equal length and equal angles.

Exercise

Identify the shape.



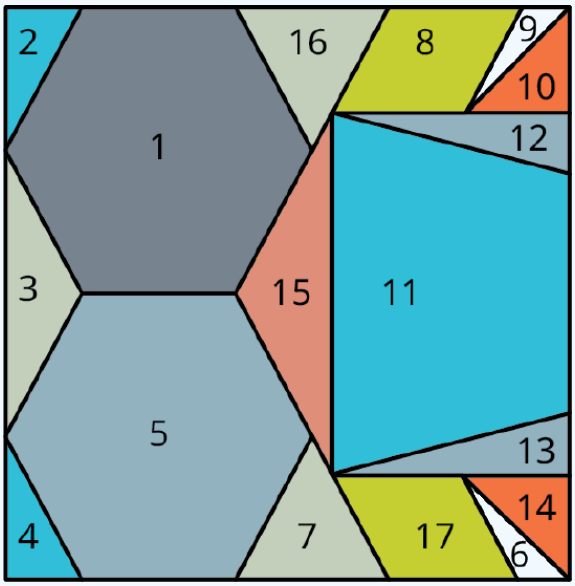




Answer available at the end of this section.

Example 2

What polygons make up the figure shown?

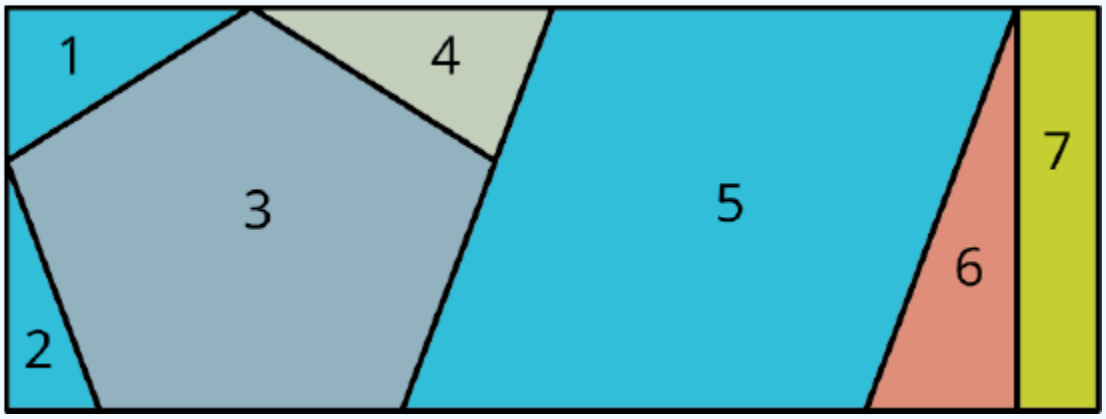


Solution

Shapes 1 and 5 are hexagons; shapes 2, 3, 4, 6, 7, 9, 10, 12, 13, 14, 15, and 16 are triangles; shapes 8 and 17 are parallelograms; and shape 11 is a trapezoid.

Exercise

What polygons make up the figure shown?



Answer available at the end of this section.

Sum of Interior and Exterior Angles

In the previous section, we worked with angles and angle relationships. Now that we can identify and classify polygons, we can apply those ideas to analyze their angle measures. We begin with triangles, where the interior angles follow a simple pattern that will help us extend these ideas to polygons with more sides.

The sum of the interior angles of any triangle is 180°.

Example 3

A triangle has two angles that measure 35° and 75°. Find the measure of the third angle.

Solution

$$\begin{aligned} x + 35^\circ + 75^\circ &= 180^\circ \\ x + 110^\circ &= 180^\circ \\ x &= 70^\circ \end{aligned}$$

The measure of the third angle is 70°.

Exercise

A triangle has two angles that measure 48° and 67°. Find the measure of the third angle.

Answer available at the end of this section.

We can extend this idea to polygons with more sides by dividing them into triangles and using the sum of 180° for each triangle. This leads to a formula for finding the sum of the interior angles of any polygon.

Formula for the Sum of Interior Angles of a Polygon

The sum of the interior angles of a polygon with n sides is given by

$$S = (n-2)180^\circ.$$

For example, if we want to find the sum of the interior angles in a parallelogram, we have

$$\begin{aligned} S &= (4-2)180^\circ \\ &= 2(180) = 360^\circ \end{aligned}$$

Similarly, to find the sum of the interior angles inside a regular heptagon, we have

$$\begin{aligned} S &= (7-2)180^\circ \\ &= 5(180) \\ &= 900^\circ. \end{aligned}$$

To find the measure of each interior angle of a regular polygon with n sides, we have the following formula.

Formula to Find the Measure of Each Interior Angle of a Polygon

The measure of each interior angle of a regular polygon with n sides is given by

$$a = \frac{(n-2)180^\circ}{n}$$

For example, find the measure of an interior angle of a regular heptagon, as shown in Figure Figure $5.1.2.4$. We have

$$a = \frac{(7-2)180^\circ}{7} = 128.57^\circ$$

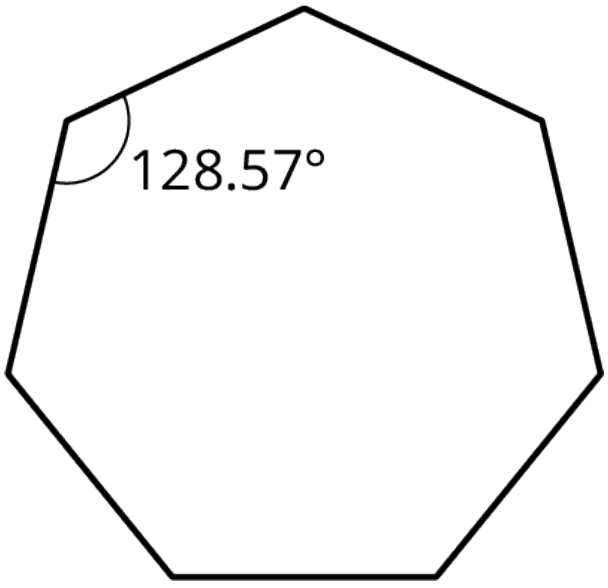


Figure $5.1.2.4$: Interior Angles

Example 3

Find the measure of an interior angle in a regular octagon using the formula, and then find the sum of all the interior angles using the sum formula.

Solution

An octagon has eight sides, so $n = 8$.

Step 1: Using the formula $a = \frac{(n-2)180^\circ}{8}$:

$$\begin{aligned} a &= \frac{(8-2)180^\circ}{8} \\ &= \frac{(6)180^\circ}{8} \\ &= 135^\circ. \end{aligned}$$

So, the measure of each interior angle in a regular octagon is 135°.

Step 2: The sum of the angles inside an octagon, so using the formula:

$$\begin{aligned} S &= (n-2)180^\circ \\ &= (8-2)180^\circ \\ &= 6(180) \\ &= 1,080^\circ. \end{aligned}$$

Step 3: We can test this, as we already know the measure of each angle is 135°. Thus, $8\,(135^\circ) = 1,080^\circ$.

Exercise

Find the measure of each interior angle of a regular pentagon and then find the sum of the interior angles.

Answer available at the end of this section.

Example 4

An interior designer is installing identical regular hexagonal tiles on a floor so that three tiles meet at each vertex.

Determine whether the tiles will fit together without gaps or overlaps by using angle measures.

Solution

First, find the measure of each interior angle of a regular hexagon.

$$\begin{aligned} a &= \frac{(n-2)180^\circ}{n} \\ a &= \frac{(6-2)180^\circ}{6} \\ a &= \frac{4(180^\circ)}{6} \\ a &= 120^\circ \end{aligned}$$

Since three tiles meet at each point, add three interior angles.

$$120^\circ + 120^\circ + 120^\circ = 360^\circ$$

The hexagonal tiles will fit together without gaps or overlaps because the angles add to 360°.

Exercise

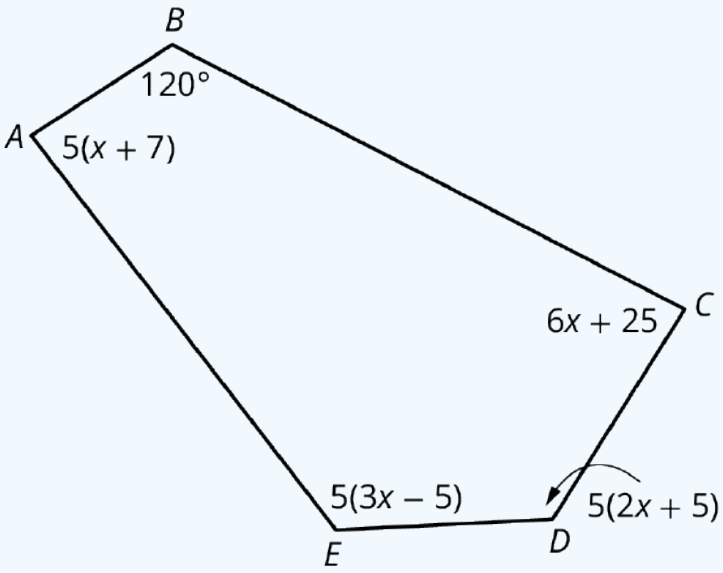
An interior designer is installing identical regular pentagonal tiles on a floor so that three tiles meet at each point where their corners touch.

Determine whether the tiles will fit together without gaps or overlaps by using angle measures.

Answer available at the end of this section.

Example 5

Use algebra to calculate the measure of each interior angle of the given five-sided polygon.



Solution

Step 1: Let us find out what the total of the sum of the interior angles should be. Use the formula for the sum of the angles in a polygon with n sides: $S = (n-2)180^\circ$. So, $S = (5-2)180^\circ = 540^\circ$.

Step 2: We add up all the angles and solve for x :

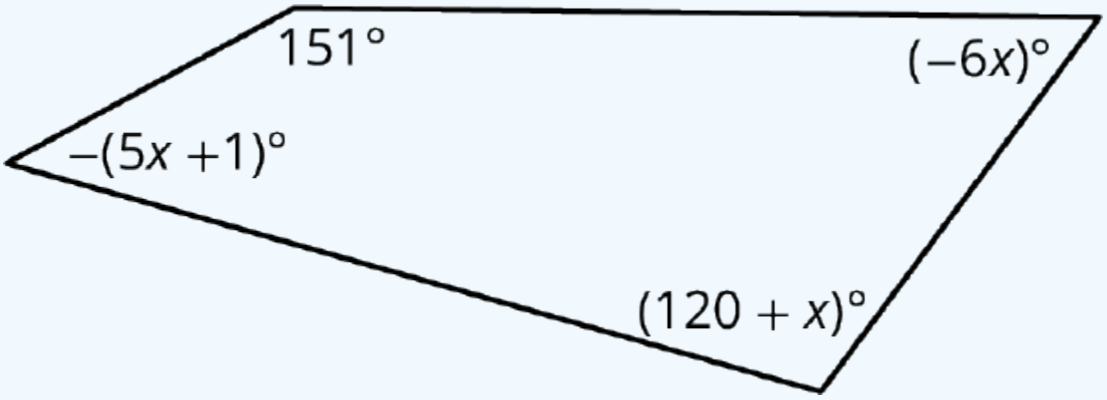
$$\begin{aligned} 5(x+7) + 120 + (6x+25) + 5(2x+5) + 5(3x-5) &= 540 \\ 5x + 6x + 10x + 15x + 180 &= 540 \\ 36x &= 360 \\ x &= 10 \end{aligned}$$

Step 3: We can then find the measure of each interior angle:

$$\begin{aligned} m\measuredangle A &= 5(10+7) = 85^\circ \\ m\measuredangle B &= 120^\circ \\ m\measuredangle C &= 6(10) + 25 = 85^\circ \\ m\measuredangle D &= 5\,(2{*}10 + 5) = 125^\circ \\ m\measuredangle E &= 5\,(3{*}10 - 5) = 125^\circ \end{aligned}$$

Exercise

Find the sum of the measures of the interior angles and then find the measure of each interior angle in the figure shown.



Answer available at the end of this section.

Angles in Polygons

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

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This video explains how to find the sum of the interior angles of a polygon by dividing it into triangles and applying the 180° angle sum.

An exterior angle of a regular polygon is an angle formed by extending a side length beyond the closed figure. The measure of an exterior angle of a regular polygon with n sides is found using the following formula:

Formula to Find the Measure of Exterior Angles of a Polygon

To find the measure of an exterior angle of a regular polygon with n sides, use the formula

$$b = \frac{360^\circ}{n}$$

In Figure Figure $5.1.2.5$, we have a regular hexagon $ABCDEF$. By extending the lines of each side, an angle is formed on the exterior of the hexagon at each vertex. The measure of each exterior angle is found using the formula, $b = \frac{360^\circ}{6} = 60^\circ$.

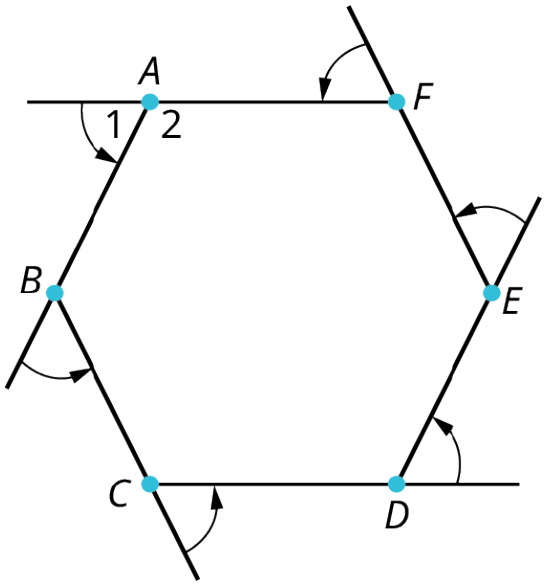
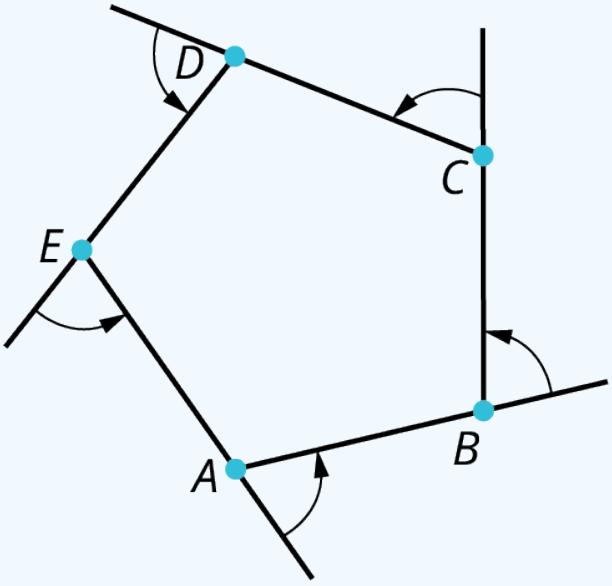


Figure 5.1.2.5: Exterior Angles

Now, an important point is that the sum of the exterior angles of a regular polygon with n sides equals 360°. This implies that when we multiply the measure of one exterior angle by the number of sides of the regular polygon, we should get 360°. For the example in Figure 5.1.2.5, we multiply the measure of each exterior angle, 60°, by the number of sides, six. Thus, the sum of the exterior angles is $6\,(60^\circ) = 360^\circ$.

Example 6

Find the measure of each exterior angle of the polygon shown, then find the sum of the exterior angles.

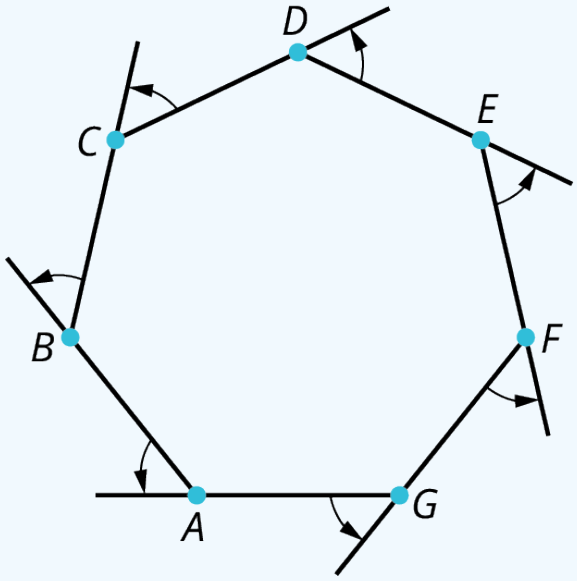


Solution

Each individual angle measures $\frac{360}{5} = 72^\circ$. Then, the sum of the exterior angles is $5\,(72^\circ) = 360^\circ$.

Exercise

Find the measure of each exterior angle of the polygon shown. Round to the nearest tenth.



Answer available at the end of this section.

Answers

Exercise

Rectangle, Pentagon, Octagon, Parallelogram

Exercise

Shapes 1, 2, 4, and 6 are triangles; shape 3 is a pentagon; shape 5 is a parallelogram; and shape 7 is a rectangle.

Exercise

65°

Exercise

Each interior angle: 108°

Sum of interior angles: 540°

Exercise

No. Each interior angle of a regular pentagon is 108°, and $108^\circ + 108^\circ + 108^\circ = 324^\circ$. Since the angles do not add to 360°, the tiles will leave gaps.

Exercise

$x = -9$

Angle measures: $44^\circ, 151^\circ, 54^\circ, 111^\circ$

Exercise

$\frac{360^\circ}{7} \approx 51.4^\circ$

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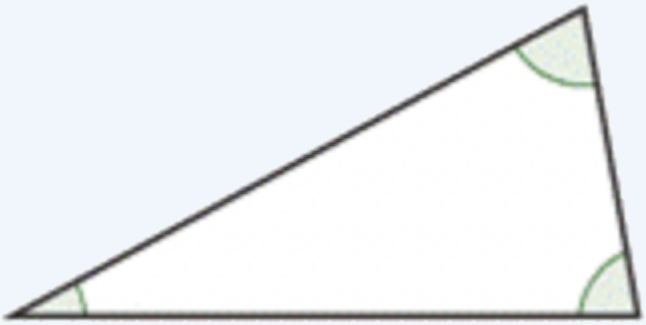
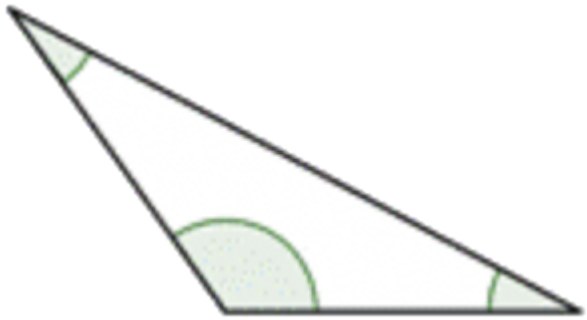
- 10.4: Polygons, Perimeter, and Circumference by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/contemporary-mathematics.

5.1.3: Triangles and Similarity

Now that we have worked with triangles and their angle measures, we can explore how triangles are classified and compared. On this page, we will classify triangles, determine whether triangles are similar or congruent, and find missing measurements in similar triangles.

Classifying and Naming Triangles

Triangles can be classified by their angle measures, as shown in the table below.

Name of Triangle	Picture of Triangle	Description
Acute Triangle		A triangle with 3 acute angles (3 angles measuring between 0° and 90°).
Obtuse Triangle		A triangle with 1 obtuse angle (1 angle measuring between 90° and 180°)
Right Triangle		A triangle containing one right angle (1 angle that measures 90°). Note that the right angle is shown with a corner mark and does not need to be labeled 90°.

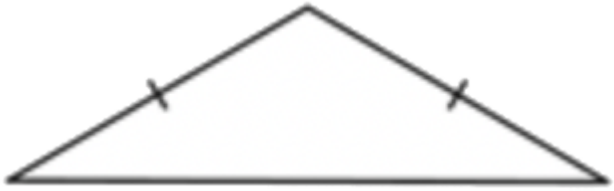
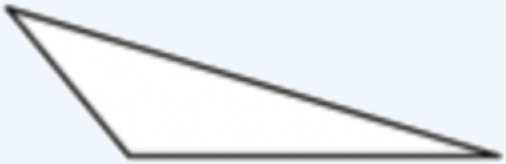
There is a standard way to name triangles. The vertices of a triangle are labeled with capital letters, and these letters are used to name the triangle.



Figure 5.1.3.1: Triangle ABC

In the diagram above, the triangle has vertices A, B, and C. This triangle can be named $\triangle ABC$ or $\triangle CBA$. When naming a triangle, you may start at any vertex, but the letters should follow the order as you move around the triangle.

Triangles can also be classified by the lengths of their sides. Sides that have the same length are called congruent sides. In diagrams, congruent sides are shown using matching marks on the sides.

Name of Triangle	Picture of Triangle	Description
Equilateral Triangle		A triangle whose three sides have the same length. These sides of equal length are called congruent sides.
Isosceles Triangle		A triangle with exactly two congruent sides.
Scalene Triangle		A triangle in which all three sides are a different length.

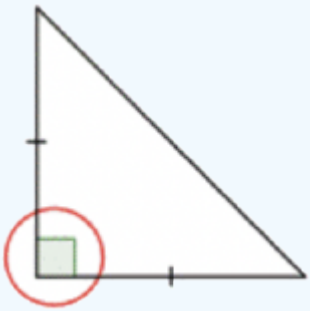
To describe a triangle even more specifically, you can use information about both its sides and its angles. Consider this example.

Example 1

Classify the triangle below.



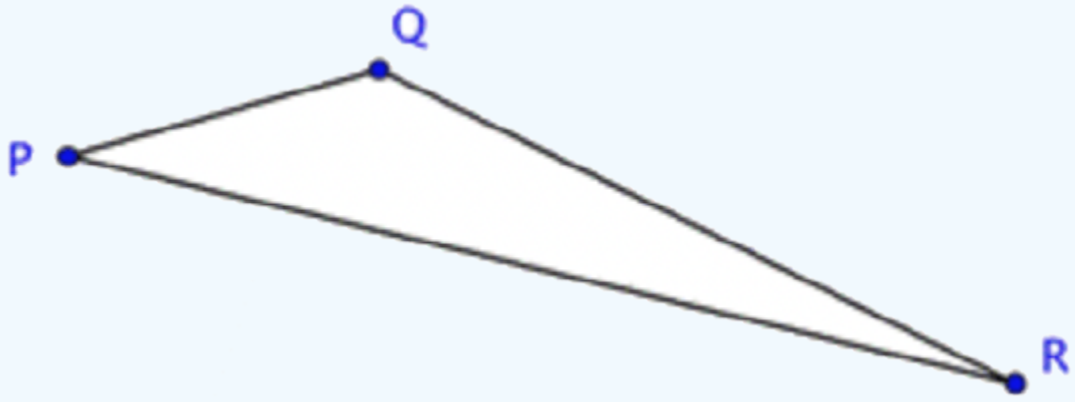
Solution

	Notice what kind of angles the triangle has. Since one angle is a right angle, this is a right triangle.
	Notice the lengths of the sides. Are there congruence marks or other labels? The congruence marks tell us there are two sides of equal length. So, this is an isosceles triangle.

This is an isosceles right triangle.

Exercise

Determine whether each classification is correct or incorrect. Explain your reasoning.



A. acute scalene
B. right isosceles
C. obtuse scalene
D. obtuse isosceles

Answer available at the end of this section.

Triangles

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View this YouTube Video online

This video reviews triangles and explains how they are classified by their side lengths and angle measures.

Identifying Congruent and Similar Triangles

Two triangles are congruent if they are exactly the same size and shape. In congruent triangles, corresponding sides have equal lengths and corresponding angles have equal measures.

Consider the two triangles shown.

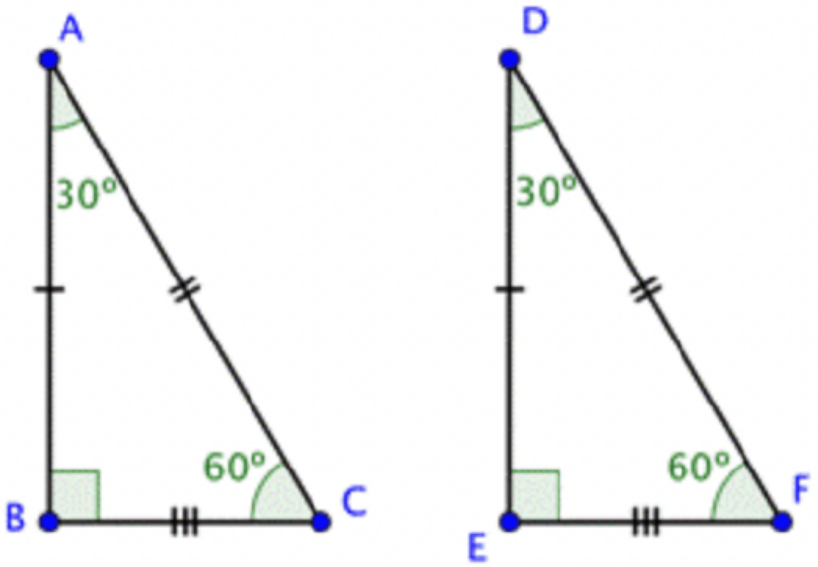


Figure 5.1.3.2: Triangle ABC and Triangle DEF

Since $\angle B$ and $\angle E$ are right angles, both triangles are right triangles. We can name these triangles $\triangle ABC$ and $\triangle DEF$.

It is important to match corresponding parts in the same order when comparing triangles.

Corresponding angles and sides match as follows:

$$\angle B \leftrightarrow \angle E$$
$$\angle A \leftrightarrow \angle D$$
$$\angle C \leftrightarrow \angle F$$
$$\overline{AB} \leftrightarrow \overline{DE}$$
$$\overline{AC} \leftrightarrow \overline{DF}$$
$$\overline{BC} \leftrightarrow \overline{EF}$$

Since all corresponding sides and angles are equal, $\triangle ABC$ and $\triangle DEF$ are congruent.

Let's look at another pair of triangles. Below are $\triangle ABC$ and $\triangle RST$.

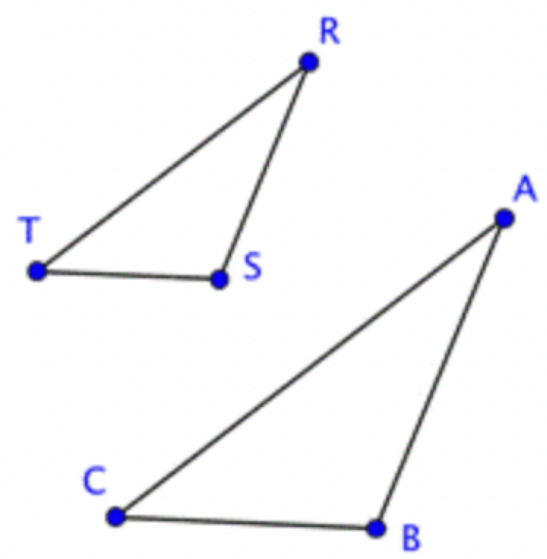


Figure 5.1.3.3: Triangle RST and Triangle ABC

These triangles are not congruent because $\triangle RST$ is smaller than $\triangle ABC$. However, they appear to have the same shape, so they may be similar. To determine if they are similar, we compare their corresponding angles. We say that triangles are similar when their corresponding angles have the same measure.

Congruent angles are marked with matching arc marks, just as congruent sides are marked with matching hash marks.

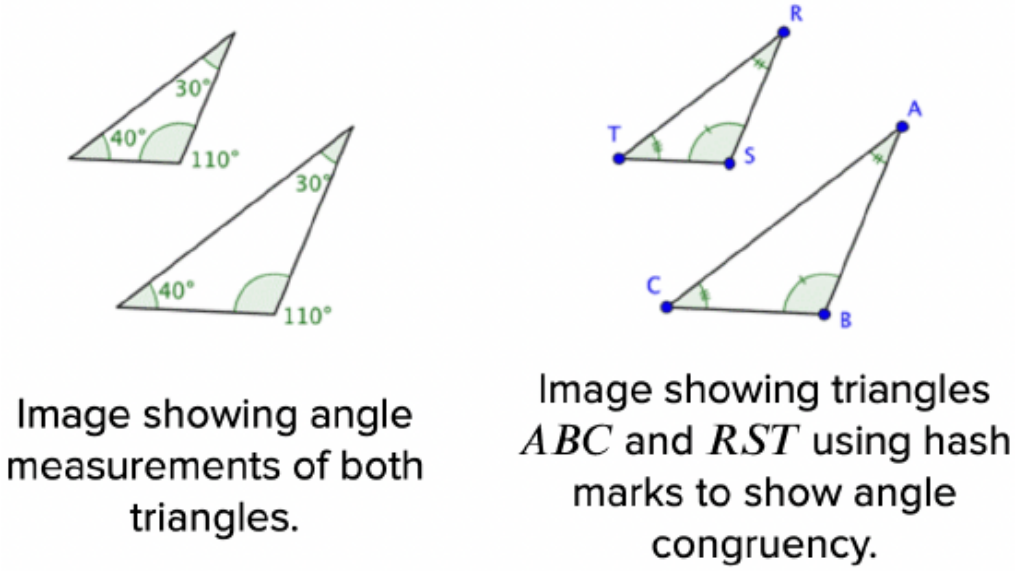
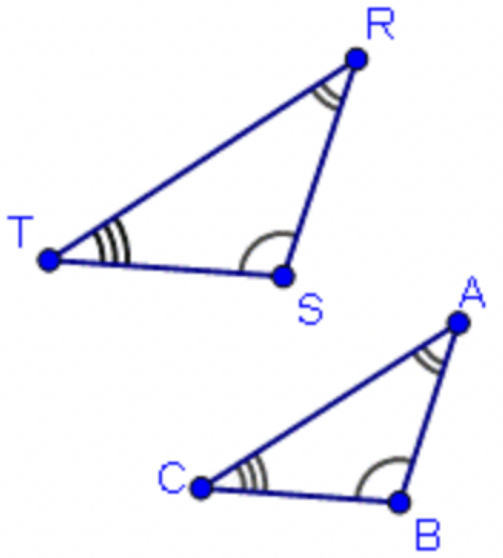


Figure 5.1.3.4

From the diagrams with the marked angles, we can see that the corresponding angles are congruent, so the two triangles are similar.

We can also show congruent angles by using multiple bands within the angle, rather than multiple hash marks on one band. Below is an image using multiple bands within the angle.



Figure

Congruent and Similar Triangles

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View this YouTube Video online

This video explains congruent and similar triangles. It shows how to identify each type and how to compare triangles using their corresponding sides and angles.

Corresponding Sides of Similar Triangles

There is another method for determining similarity of triangles that involves comparing the ratios of the lengths of the corresponding sides.

If the ratios of the pairs of corresponding sides are equal, the triangles are similar.

Consider the two triangles below.

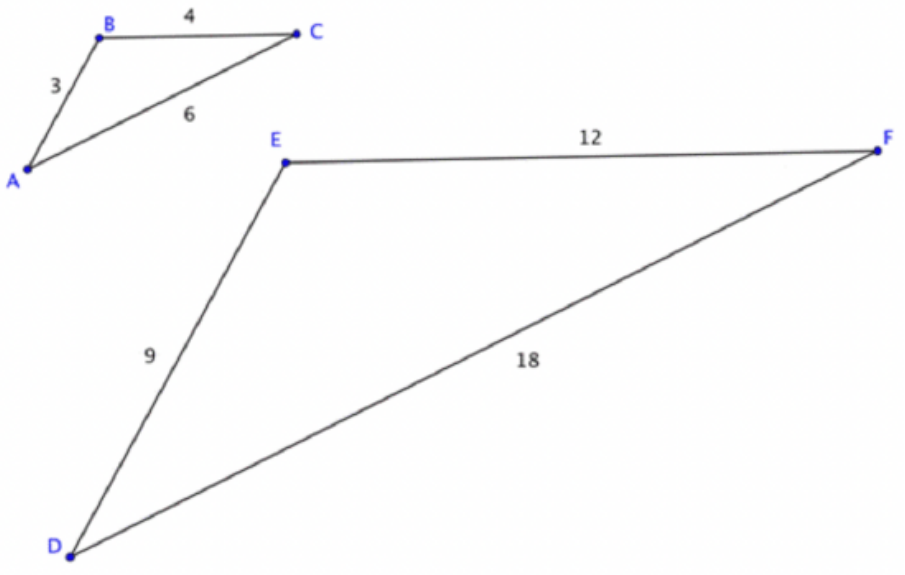


Figure 5.1.3.6: Triangle ABC and Triangle DEF

$\triangle ABC$ is *not* congruent to $\triangle DEF$ because the side lengths of $\triangle DEF$ are longer than those of $\triangle ABC$. So, are these triangles similar? If they are, the corresponding sides should be proportional.

Since these triangles are oriented in the same way, you can pair the left, right, and bottom sides: $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, $\overline{AC}$ and $\overline{DE}$. (You might have called these the two shortest sides, the two longest sides, and the two leftover sides and arrived at the same ratios). Now we will look at the ratios of their lengths.

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Substituting the side length values into the proportion, you see that it is true:

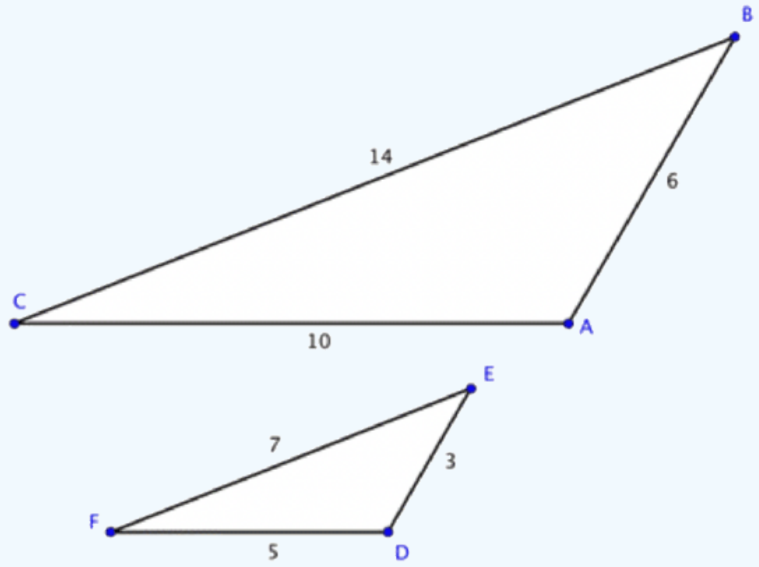
$\frac{3}{9} = \frac{4}{12} = \frac{6}{18}$

If the corresponding sides are proportional, then the triangles are similar. Triangles ABC and DEF are similar, but not congruent.

Let's use this idea of proportional corresponding sides to determine whether two more triangles are similar.

Example 2

Determine if the triangles below are similar by seeing if their corresponding sides are proportional.



Solution

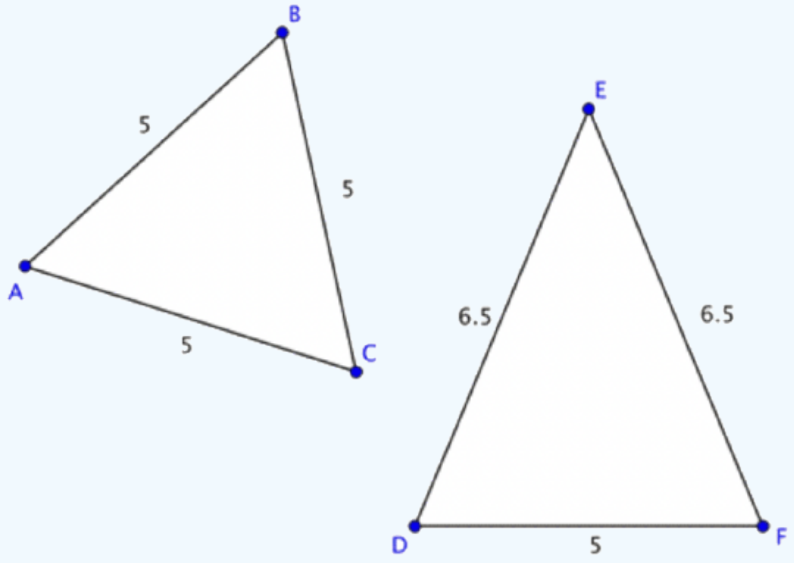
$\overline{CA} \leftrightarrow \overline{FD}$ $\overline{AB} \leftrightarrow \overline{DE}$ $\overline{BC} \leftrightarrow \overline{EF}$	First determine the corresponding sides, which are opposite corresponding angles.
$\frac{CA}{FD} = \frac{AB}{DE}$ $= \frac{BC}{EF}$	Write the corresponding side lengths as ratios.
$\frac{10}{5} = \frac{6}{3} = \frac{14}{7}$ $2 = 2 = 2$	Substitute the side lengths into the ratios, and determine if the ratios of the corresponding sides are equivalent. They are, so the triangles are similar.

$\triangle ABC$ and $\triangle DEF$ are similar.

The mathematical symbol ~ means “is similar to.” So, you can write $\triangle ABC$ is similar to $\triangle DEF$ as $\triangle ABC \sim \triangle DEF$.

Exercise

Determine whether the two triangles are similar, congruent, or neither.



A. $\triangle ABC$ and $\triangle DEF$ are congruent.
B. $\triangle ABC$ and $\triangle DEF$ are similar.
C. $\triangle ABC$ and $\triangle DEF$ are similar and congruent.
D. $\triangle ABC$ and $\triangle DEF$ are neither similar nor congruent.

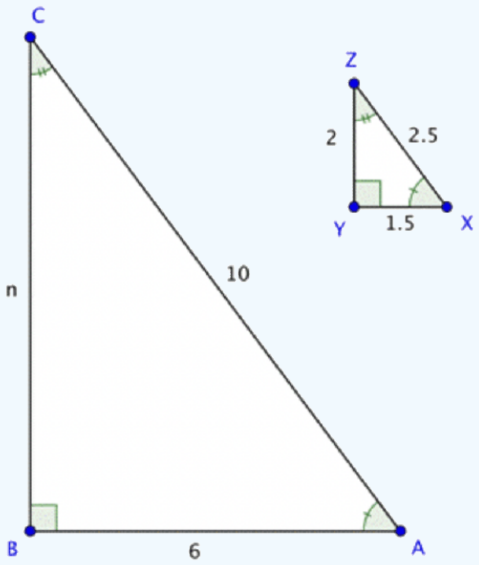
Answer available at the end of this section.

Finding Missing Measurements in Similar Triangles

You can find the missing measurements in a triangle if you know some measurements of a similar triangle. Let's look at an example.

Example 3

$\triangle ABC$ and $\triangle XYZ$ are similar triangles. What is the length of side BC?



Solution

$\frac{BC}{YZ} = \frac{AB}{XY}$	In similar triangles, the ratios of corresponding sides are proportional. Set up a proportion of two ratios, one that includes the missing side.
$\frac{n}{2} = \frac{6}{1.5}$	Substitute in the known side lengths for the side names in the ratio. Let the unknown side length be n.
$2 \cdot 6 = 1.5 \cdot n$ $12 = 1.5n$ $8 = n$	Solve for n using cross multiplication.

The missing length of side BC is 8 units.

This process is fairly straightforward, but be careful that your ratios represent corresponding sides, recalling that corresponding sides are opposite corresponding angles.

Solving Application Problems Involving Similar Triangles

Triangles, similarity, and congruence can be used to solve real-world problems. For example, triangles can help find unknown distances or heights.

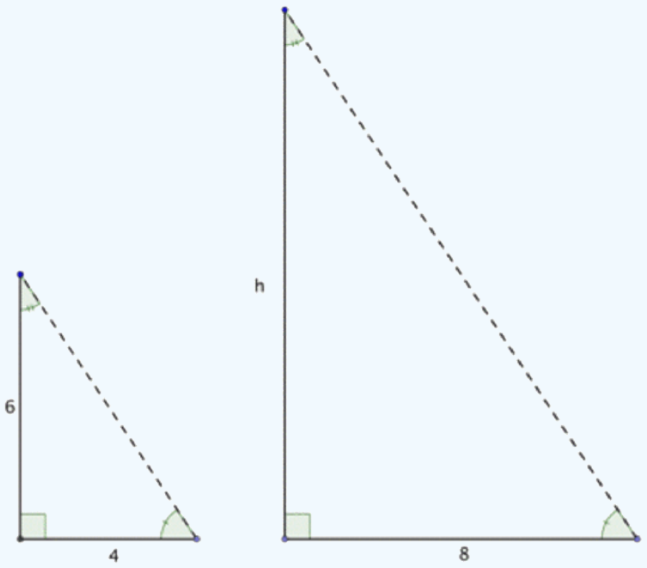
Consider two trees and their shadows. One tree is 6 feet tall, and the height of the other is unknown. By measuring the lengths of their shadows, we can use triangle similarity to find the unknown height.

The trees and their shadows form similar triangles. The height of each tree corresponds to one side, the shadow corresponds to another, and the line from the top of the tree to the tip of the shadow forms the third side.

Using these similar triangles, we can set up a proportion to find the unknown height.

Example 4

When the sun is at a certain angle in the sky, a 6--foot tree will cast a 4-foot shadow. How tall is a tree that casts an 8-foot shadow?



Solution

The angle measurements are the same, so the triangles are similar triangles. Since they are similar triangles, you can use proportions to find the size of the missing side.

$\frac{\text{Tree 1}}{\text{Tree 2}} = \frac{\text{Shadow 1}}{\text{Shadow 2}}$	Set up a proportion comparing the heights of the trees and the lengths of their shadows.
$\frac{6}{h} = \frac{4}{8}$	Substitute in the known lengths. Call the missing tree height h.
$6 \cdot 8 = 4h$ $48 = 4h$ $12 = h$	Solve for h using cross-multiplication.

The tree is 12 feet tall.

Answers

Exercise

A. Incorrect. The triangle is obtuse, not acute.
B. Incorrect. The triangle does not have a right angle.
C. Correct. The triangle is obtuse scalene.
D. Incorrect. The triangle does not have equal sides.

Exercise

A. Incorrect. The triangles are not the same size.
B. Incorrect. The side lengths are not proportional.
C. Incorrect. The triangles are not congruent.
D. Correct. The triangles are neither similar nor congruent.

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5.1.4: Angles and Relationships Practice

The following set of exercises provides practice for the concepts covered in Module 5.1: Angles and Relationships. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

Section 5.1.1 - Angles and Angle Relationships

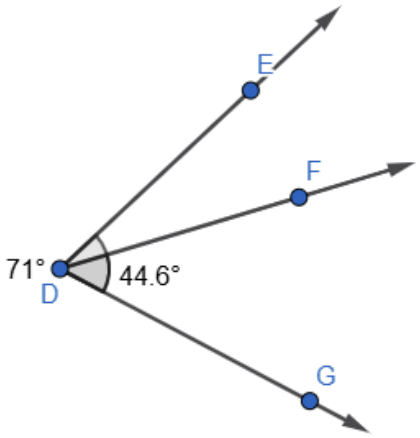
Classify the angles in the following exercises as acute, obtuse, right, or straight.

1. $m\angle = 32^\circ$

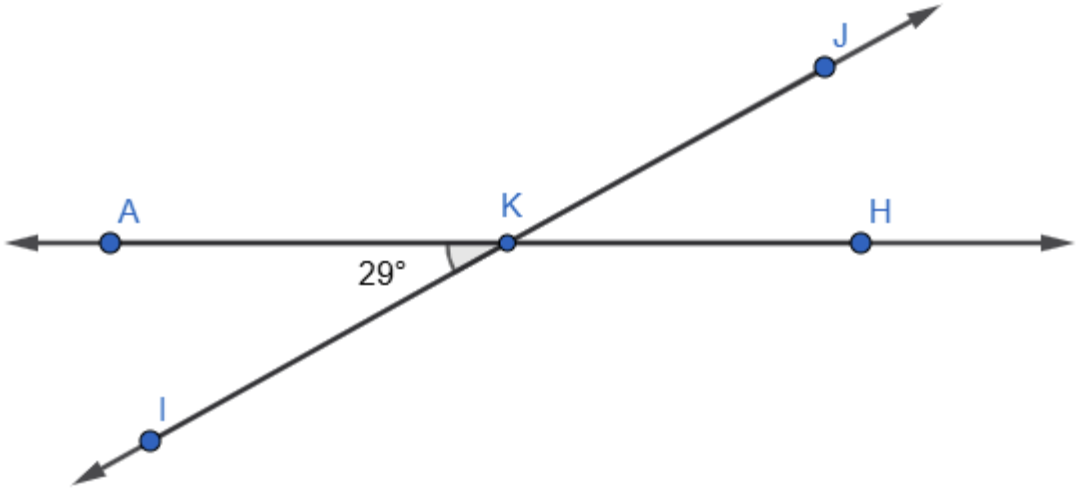
2. $m\angle = 97^\circ$

Use the given figure to solve for the angle measurements.

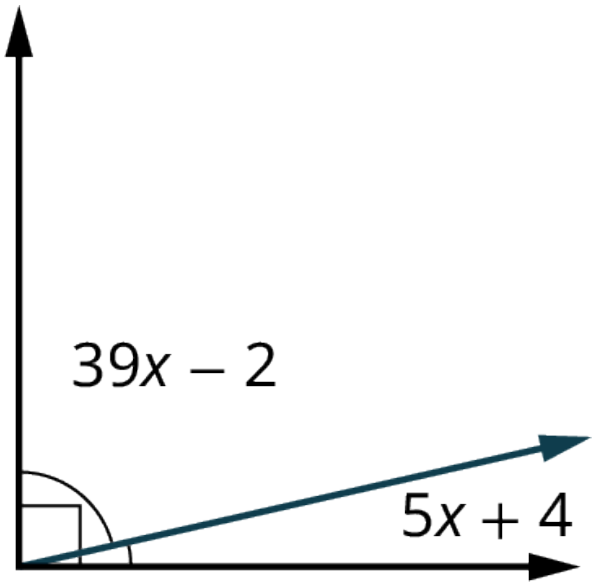
3. The measure of $\angle EDG$ is 71°. Find the measure of $\angle EDF$.



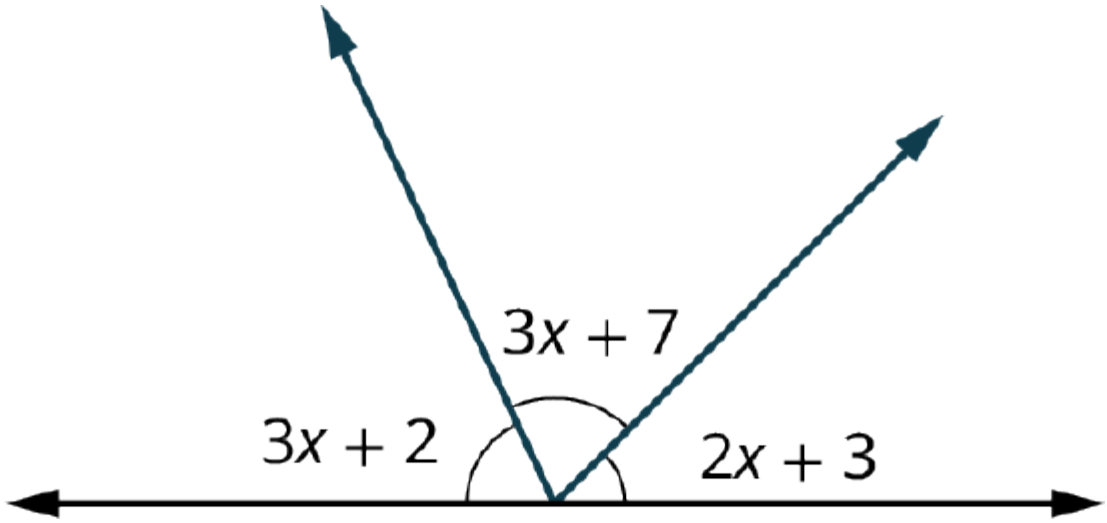
4. Find the measures of $\angle AKH$ and $\angle JKH$.



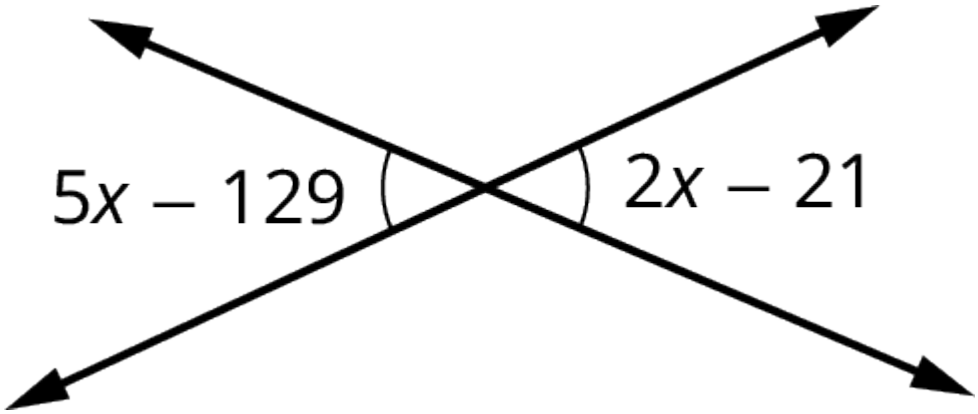
5. Find x.



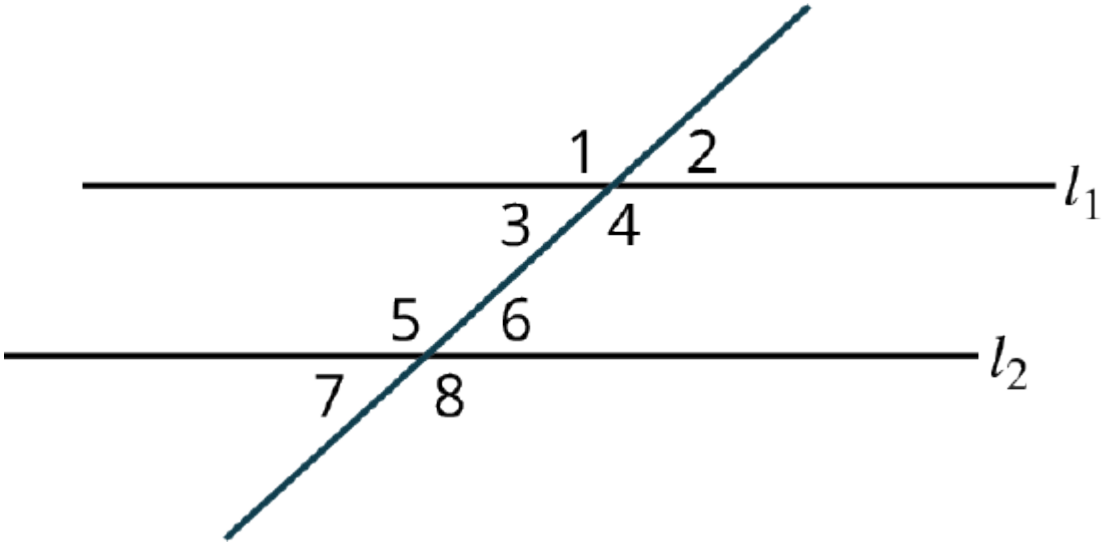
6. Find x.



7. Find x.



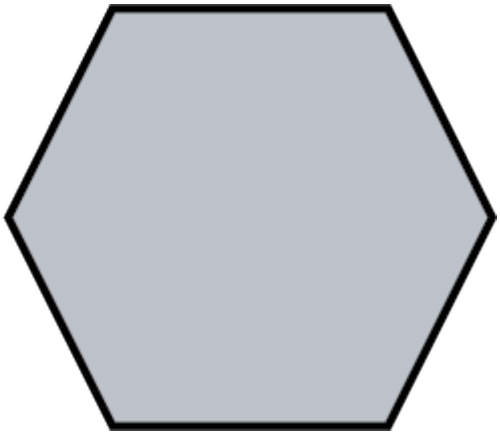
8. In the figure below, let angle 2 measure 35°. Find the measure of all the other angles and state the reason for your solution.



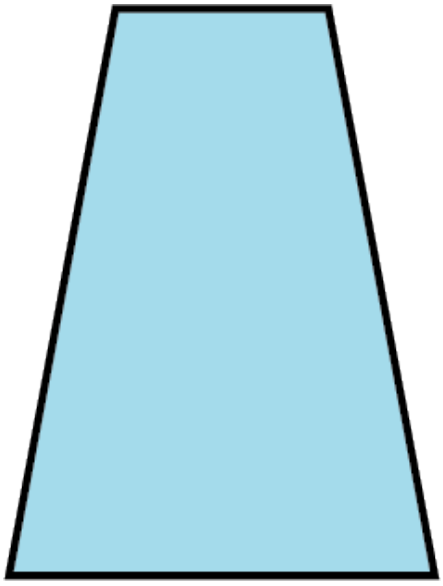
Section 5.1.2 - Polygons and Angle Measurements

In the following exercises, identify the polygons.

9.



10.



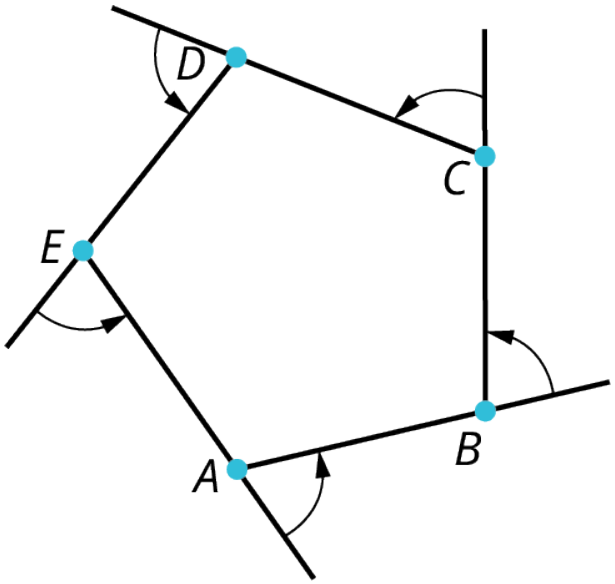
Find the measure of the third angle in each triangle.

11. A triangle has two angles that measure 35° and 65°. Find the measure of the third angle.

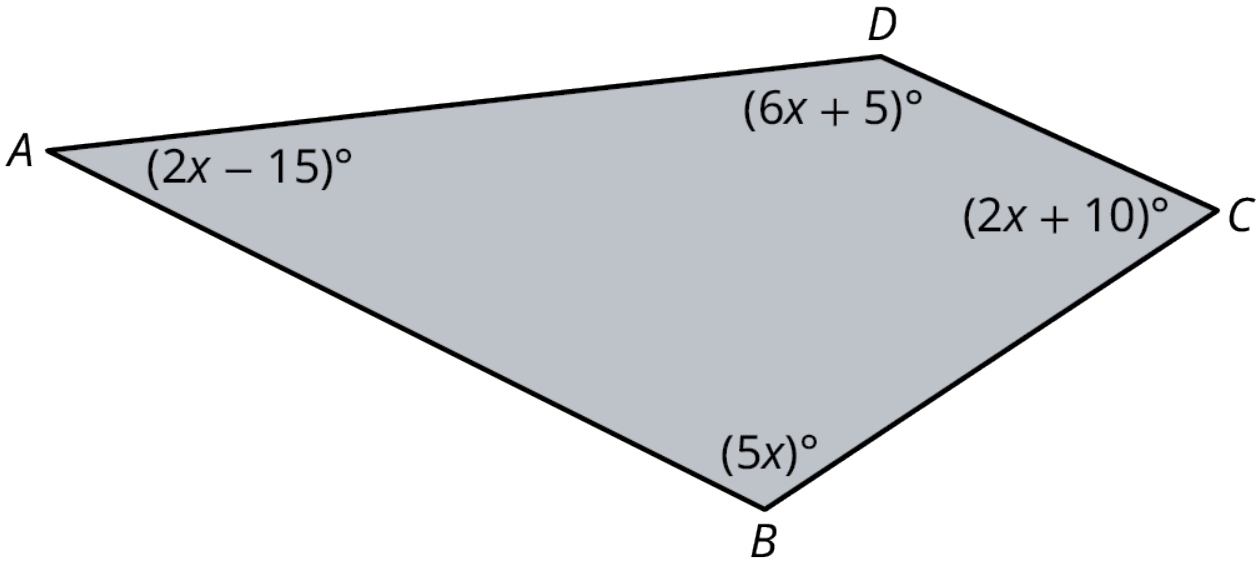
12. A triangle has two angles that measure 58° and 47°. Find the measure of the third angle.

Find the requested measurement.

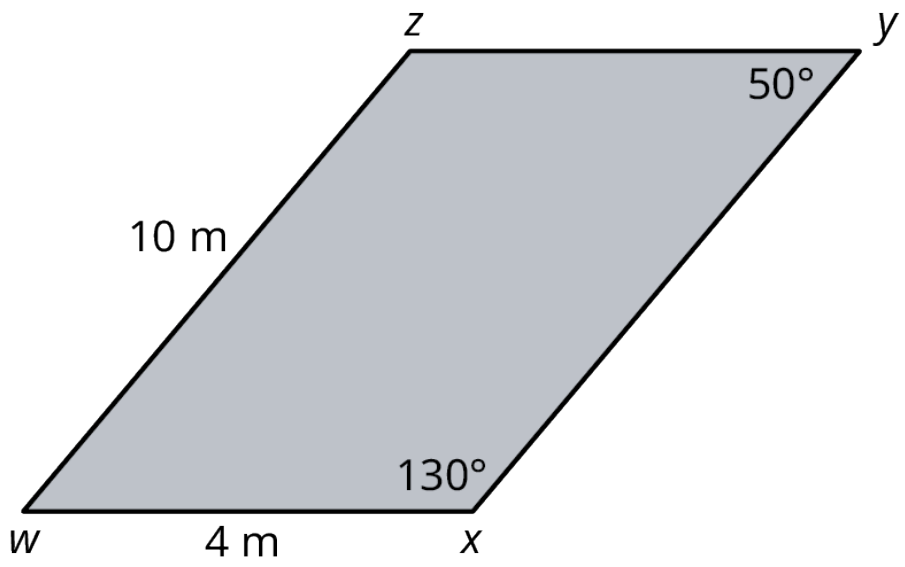
13. What is the measure of each interior angle and exterior angle of a regular pentagon?



14. Use algebra to find the measure of each angle of the quadrilateral shown.



15. Find the missing sides and angles of the parallelogram shown.



Section 5.1.3 - Triangles and Similarity

Find the classification of each triangle by angle or by side.

16. A triangle has angle measures 45°, 45°, and 90°.

17. A triangle has side lengths 7, 7, and 10.

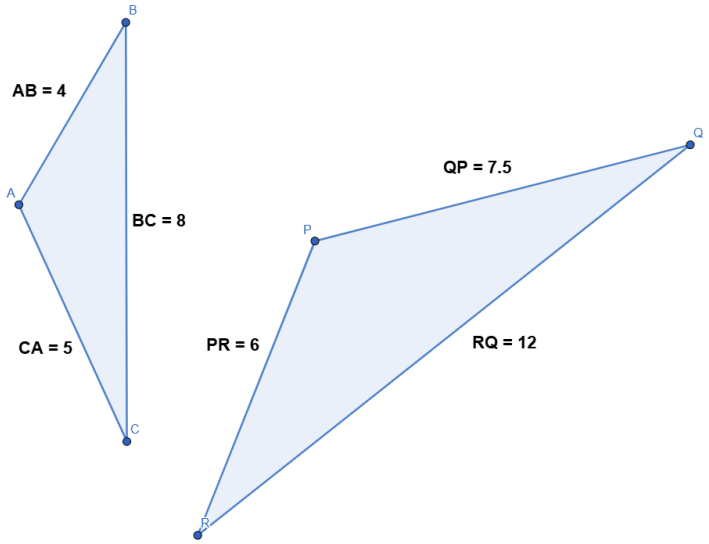
18. A triangle has angle measures 30°, 60°, and 90°.

19. A triangle has side lengths 5, 6, and 7.

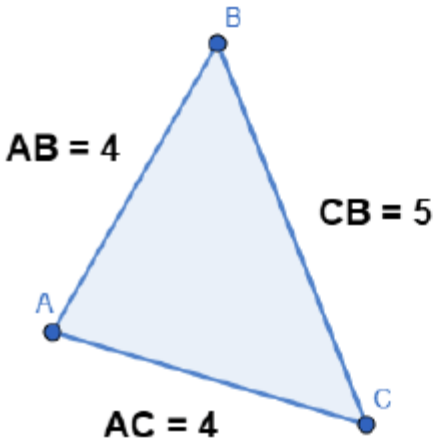
Determine whether the triangles are congruent, similar, or neither by comparing corresponding side lengths.

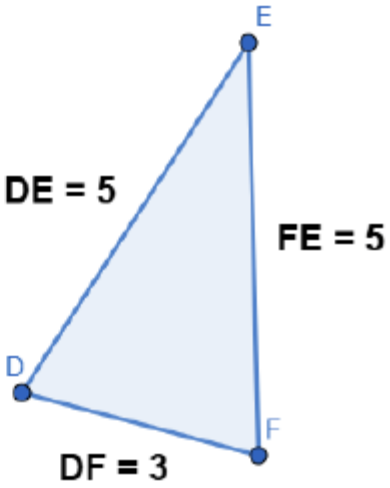
20.

Parallelogram WXYZ. Angle x measures 130 degrees and Angle Y measures 50 degrees. Side WZ is 10 m and WX is 4 m.



21.





Find the missing side length. Assume the triangles are similar.

22. Two similar triangles have corresponding sides 6 and 9. A side in the smaller triangle is 8. Find the corresponding side in the larger triangle.

23. Two similar triangles have corresponding sides of 5 and 10. A side in the larger triangle is 14. Find the corresponding side in the smaller triangle.

24. A scale drawing of a triangular metal plate is made using a scale factor of $\frac{1}{4}$. One side of the actual plate measures 12 ft. What is the corresponding side length in the scale drawing?

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5.2: PERIMETER AND AREA

Measurement is one of the most practical branches of geometry. Whether you are buying flooring for a room, fencing a backyard, or designing a garden, the ability to measure and calculate the space around and inside a shape is an essential real-world skill. In this unit, you will learn how to find the perimeter of geometric figures — the total distance around the outside — and the area — the amount of space contained within.

We will work with a variety of two-dimensional figures, building a toolkit of formulas that apply to shapes you encounter in everyday life. From rectangles and triangles to circles and composite figures, each shape has its own set of properties that guide how we measure it. Along the way, you will notice that the number pi appears whenever circles are involved, connecting the diameter of a circle to its circumference and area in a relationship that has fascinated mathematicians for centuries.

Once you have those foundational skills in place, you will apply them to real-world problems involving two-dimensional figures. These applications will ask you to read carefully, identify the relevant shape and measurements, and choose the right formula to find a solution.

Before diving in, enjoy the clip from Person of Interest below. In it, you will hear a memorable explanation of pi and the remarkable fact that its decimal expansion goes on forever — containing, within it, every number you could ever imagine. It is a fitting introduction to a unit where pi will play an important role.

Explanation of Pi

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This clip from Person of Interest includes a memorable explanation of pi.

5.2.1: Perimeter and Circumference

5.2.2: Area

5.2.3: Applications Involving Perimeter and Area

5.2.4: Perimeter and Area Practice

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5.2.1: Perimeter and Circumference

Perimeter refers to the outside measurements of some area or region given in linear units. For example, to find out how much fencing you would need to enclose your backyard, you will need the perimeter. The general definition of perimeter is the sum of the lengths of the sides of an enclosed region. For some geometric shapes, we have formulas. For other shapes, it is a matter of just adding up the side lengths.

Perimeter of a Rectangle

Perimeter of a Rectangle

The formula for the perimeter P of a rectangle is $P = 2L + 2W$, twice the length L plus twice the width W.

Alternatively, the perimeter can be found by adding together the length of all four sides.

Example 1

Find the perimeter of the given figure.



Solution

$P = 2(6) + 2(4) = 20$ m

Exercise

Find the perimeter of the given figure.



Answer available at the end of this section.

Perimeter of a Regular Polygon

Perimeter of a Regular Polygon

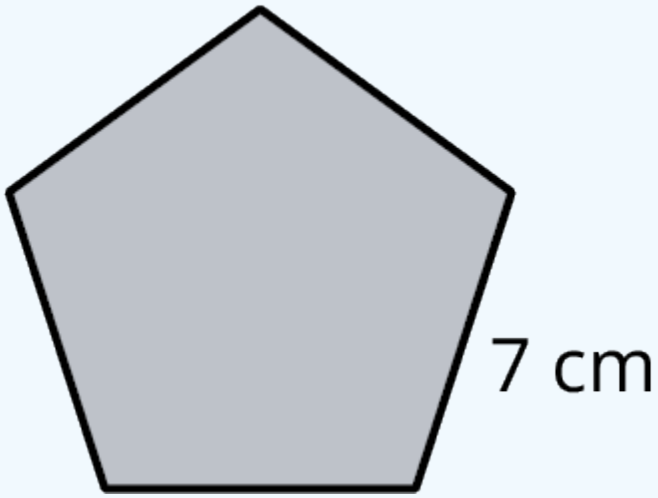
The perimeter of a regular polygon with n sides is given as

$P = n \cdot s.$

For example, the perimeter of an equilateral triangle, a triangle with three equal sides, and a side length of 7 cm is $P = 3(7) = 21 \text{ cm}.$

Example 2

Find the perimeter of a regular pentagon with a side length of 7 cm.



Solution

A regular pentagon has five equal sides. Therefore, the perimeter is equal to $P = 5(7) = 35$ cm.

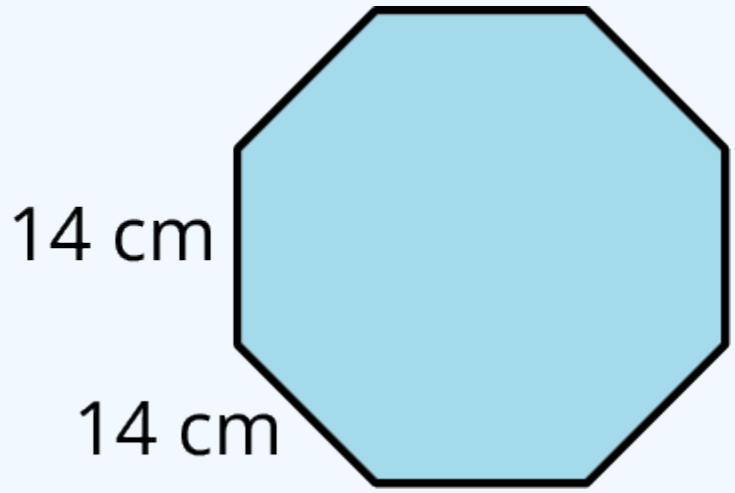
Exercise

Find the perimeter of a square table that measures 30 inches across from one side to its opposite side.

Answer available at the end of this section.

Example 3

Find the perimeter of a regular octagon with a side length of 14 cm.

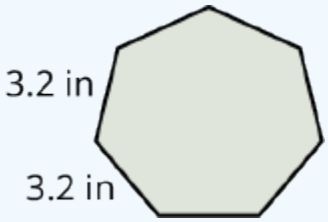


Solution

A regular octagon has eight sides of equal length. Therefore, the perimeter of a regular octagon with a side length of 14 cm is $P = 8(14) = 112$ cm.

Exercise

Find the perimeter of a regular heptagon with a side length of 3.2 in as shown in the figure.



Answer available at the end of this section.

Perimeter of a Parallelogram

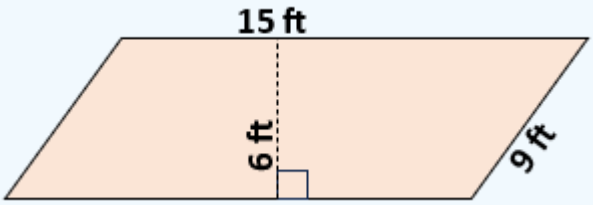
Perimeter of a Parallelogram

The perimeter of a parallelogram with base, b, and side, a, is given as

$P = 2b + 2a.$

Example 4

Find the perimeter of the given figure.



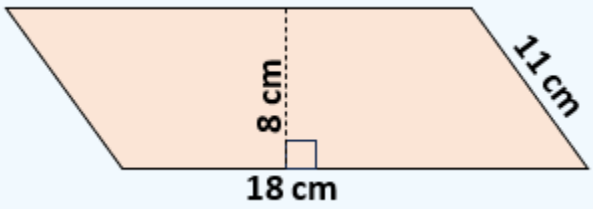
Octagon wiht side length 14 cm

Solution

$P = 2(15) + 2(9) = 48 \text{ ft}$

Exercise

Find the perimeter of the given figure.



Answer available at the end of this section.

Perimeter of a Trapezoid

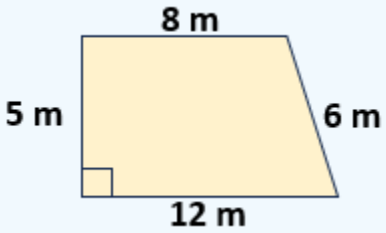
Perimeter of a Trapezoid

The perimeter of a trapezoid with bases b_1 and b_2, and sides s_1 and s_2, is given as

$P = b_1 + b_2 + s_1 + s_2$

Example 5

Find the perimeter of the given figure.

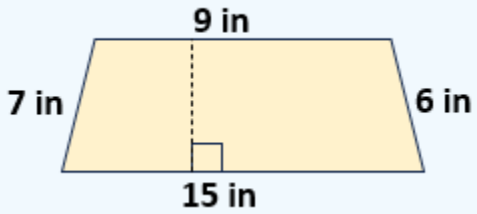


Solution

$P = 12 + 8 + 5 + 6 = 31 \text{ m}$

Exercise

Find the perimeter of the given figure.



Answer available at the end of this section.

Perimeter

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to find the perimeter of polygons by adding the lengths of all sides, including regular and irregular shapes.

Circles and Circumference

The perimeter of a circle is called the **circumference**.

Circumference of a Circle Formula

The circumference of a circle is found using the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

The radius is $1/2$ of the diameter of a circle. The symbol $\pi = 3.141592654\ldots$ is the ratio of the circumference to the diameter. Because this ratio is constant, our formula is accurate for any size circle. See Figure $5.2.1.1$.

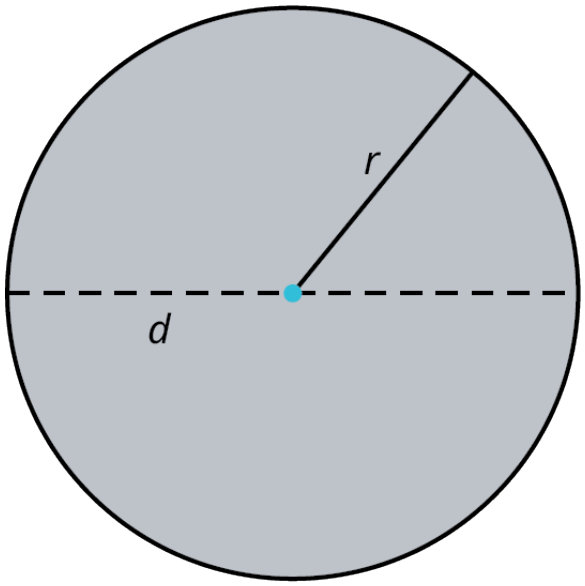


Figure $5.2.1.1$: Circle Diameter and Radius

Let the radius be equal to 3.5 inches. Then, the circumference is

$$\begin{aligned} C &= 2\pi(3.5) \\ &= 21.99\text{in}. \end{aligned}$$

Example 6

Find the circumference of a circle with diameter 10 cm.

Solution

If the diameter is 10 cm, the circumference is $C = 10\pi = 31.42\ \text{cm}$.

Exercise

Find the circumference of a circle with a radius of 2.25 cm.

Answer available at the end of this section.

Example 7

Find the radius of a circle with a circumference of 12 in.

Solution

If the circumference is 12 in, then the radius is

$$12 = 2\pi r$$
$$\frac{12}{2\pi} = r = 1.91\text{in}.$$

Exercise

Find the radius of a circle with a circumference of $15.71\ \text{cm}$.

Answer available at the end of this section.

Circumference of a Circle

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to find the circumference of a circle using the appropriate formula and shows how the radius or diameter is used in the calculation.

Perimeter of Irregular Polygons and Composite Figures

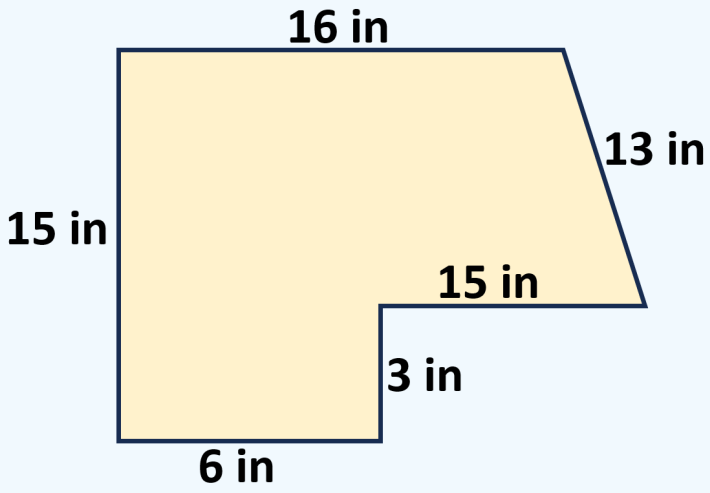
A composite figure is a shape made by combining two or more simple shapes, such as polygons or circles, into a single figure.

Perimeter of an Irregular Polygons and Composite Figures

To find the perimeter of an irregular polygon or composite figure, add the lengths of all the outer edges and curved parts. Only include the boundary around the outside of the figure.

Example 8

Find the perimeter of the given figure.



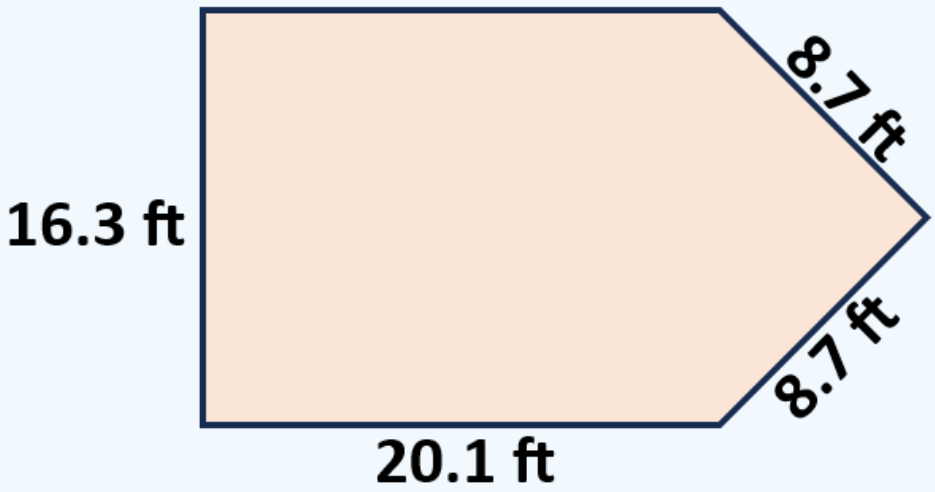
Solution

$P = 16 + 13 + 15 + 3 + 6 + 15$
$= 68$

The perimeter is 68 inches.

Exercise

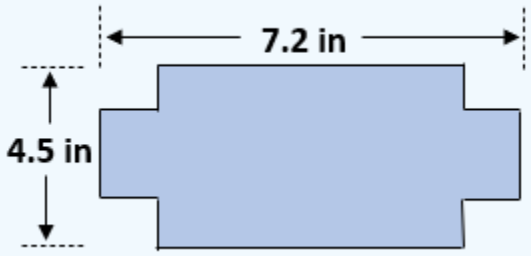
Find the perimeter of the given figure.



Answer available at the end of this section.

Example 9

Find the perimeter of the given figure.

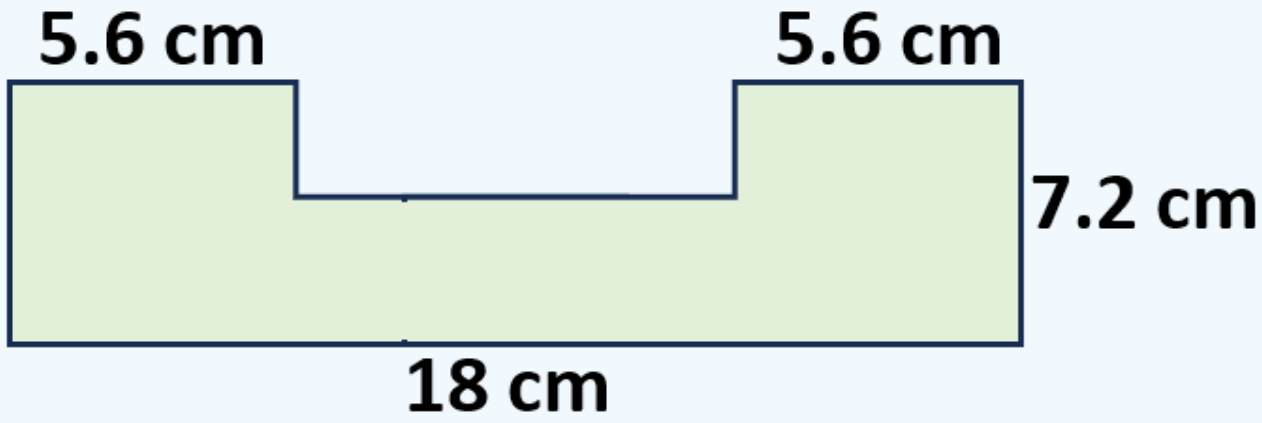


Solution

$$
\begin{aligned}
P &= 2(7.2 \text{ in}) + 2(4.5 \text{ in}) \\
&= 14.4 \text{ in} + 9 \text{ in} \\
&= 23.4 \text{ in}
\end{aligned}
$$

Exercise

Find the perimeter of the given figure.



Answer available at the end of this section.

Determine the Perimeter of a an L-shaped Polygon

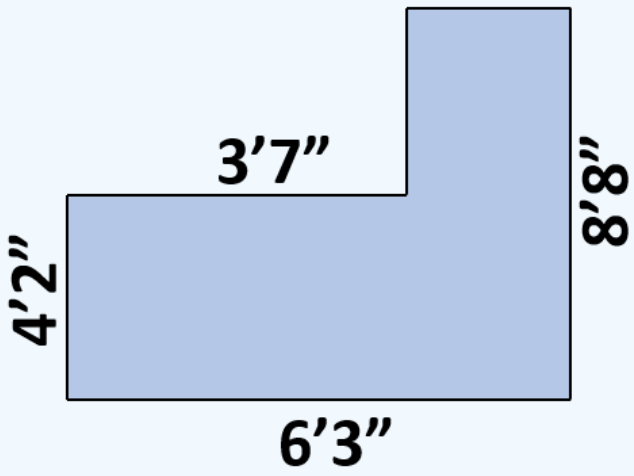
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This video explains how to find the perimeter of an L-shaped polygon by determining any missing side lengths and adding all sides.

Example 10

Find the perimeter of the given figure.



Solution

To find the perimeter, we first need to determine the missing side lengths.

The total height of the figure is $8'8''$. One vertical side is $4'2''$, so the remaining vertical segment is:

$$8'8'' - 4'2'' = 4'6''$$

The total width of the figure is $6'3''$. One horizontal segment is $3'7''$, so the remaining horizontal segment is:

$$6'3'' - 3'7'' = 2'8''$$

Now add all the sides:

$$P = 6'3'' + 8'8'' + 2'8'' + 4'6'' + 3'7'' + 4'2''$$

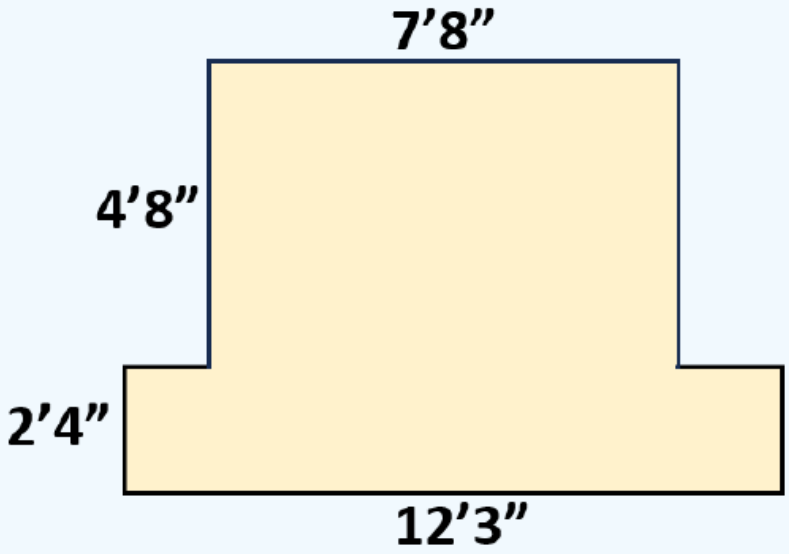
$$P = 29'34''$$

$$P = 31'10''$$

The perimeter is $31'10''$.

Exercise

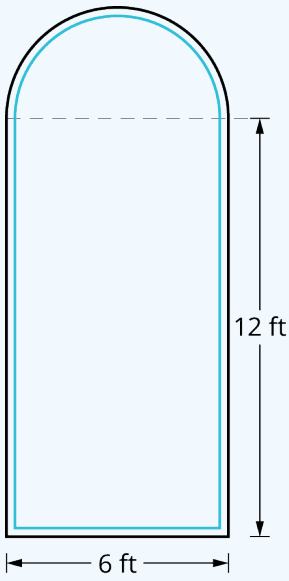
Find the perimeter of the given figure.



Answer available at the end of this section.

Example 11

You decide to make a trim for the window shown below. How many feet of trim do you need to buy?



Solution

The trim will cover the 6 ft along the bottom, the two 12 ft sides, and the semicircle on top. The arc length of a semicircle is one-half the circumference of a full circle. The diameter of the semicircle is 6 ft.

$C = \frac{1}{2}\pi d$

$C = \frac{1}{2}\pi(6) = 3\pi \approx 9.4 \text{ ft}$

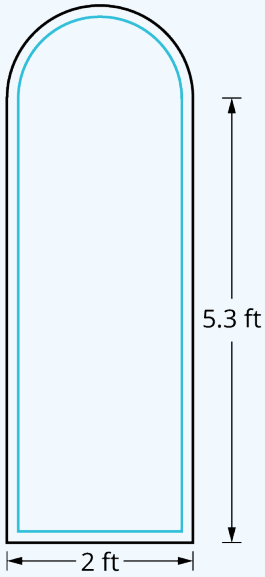
Now add all parts of the perimeter.

$6 + 12 + 12 + 9.4 = 39.4$ ft

You need to buy 39.4 ft of trim.

Exercise

To make a trim for the window in the figure shown, you need the perimeter. How many feet of trim do you need to buy?



Answer available at the end of this section.

Determine Perimeter Involving a Rectangle and Circle

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains how to find the perimeter of a composite figure made from a rectangle and a circle by identifying and adding the appropriate side lengths and curved sections.

Answers

Exercise

P = 32 feet

Exercise

120 inches

Exercise

22.4 in.

Exercise

P = 58 cm

Exercise

$P = 37$ in

Exercise

$C = 4.5\pi = 14.14 \text{ cm}$

Exercise

$r = 2.5\text{cm}$

Exercise

$P = 121.13\text{m}$

Exercise

$P = 50.4\text{cm}$

Exercise

$P = 26'8''$

Exercise

13.9 ft

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5.2.2: Area

Area measures the amount of space inside a region. It is expressed in square units, such as in^2 or cm^2. For example, when buying tile for a floor, you need to know the area because tile is sold by the square foot. Carpeting is often sold by the square yard.

Area is different from perimeter. Perimeter measures the distance around a shape and is expressed in linear units, such as feet or inches. These linear measurements describe the boundary of a shape, while area describes the space inside that boundary. Although the units may vary depending on the situation, you must know the linear dimensions in order to calculate the area.

Many common geometric shapes, such as rectangles, triangles, and circles have formulas for finding area. In this section, we will focus on shapes with known formulas. Area is written using square units, such as in^2 or cm^2. Linear measurements do not use exponents and can be thought of as having an exponent of 1.

Area of Triangles

The formula for the area of a triangle is given as follows.

Area of a Triangle

The area of a triangle is given by $A = \frac{1}{2}bh$, where b represents the base and h represents the height.

For example, consider the triangle in Figure $5.2.2.1$.

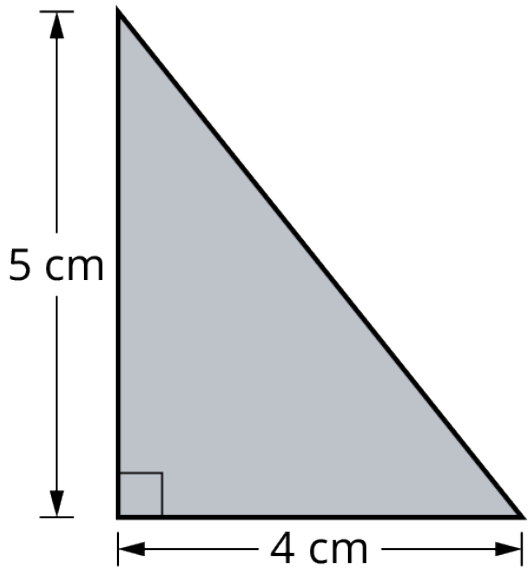


Figure $5.2.2.1$: Triangle 1

The base measures 4 cm and the height measures 5 cm . Using the formula, we can calculate the area:

$$A = \frac{1}{2}(4)(5) = \frac{1}{2}(20) = 10 \text{ cm}^2$$

In Figure $5.2.2.2$:, the triangle has a base equal to 7 cm and a height equal to 3.5 cm . Notice that we can only find the height by dropping a perpendicular to the base. The area is then

$$A = \frac{1}{2}(7)(3.5) = 12.25 \text{ cm}^2$$

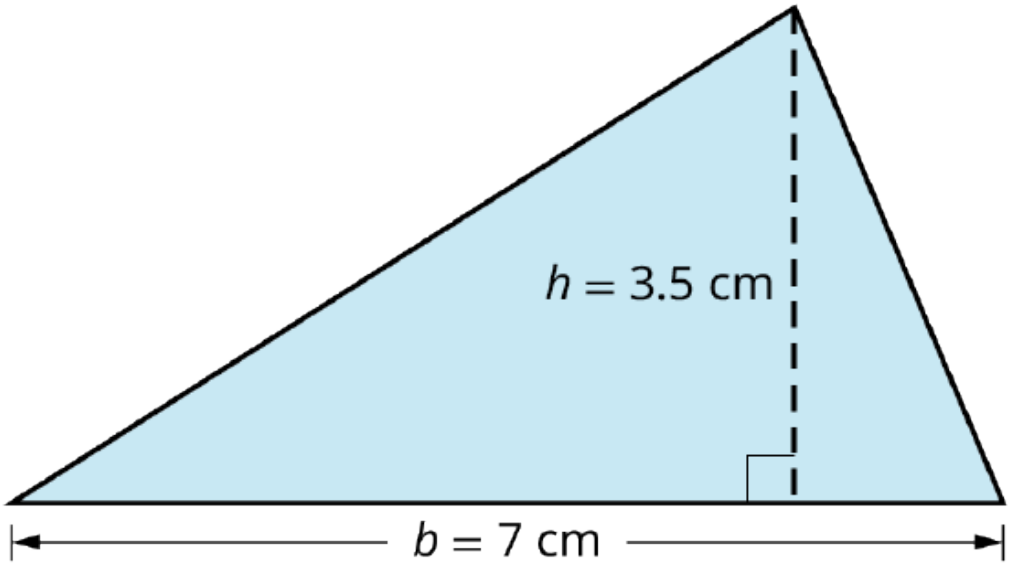
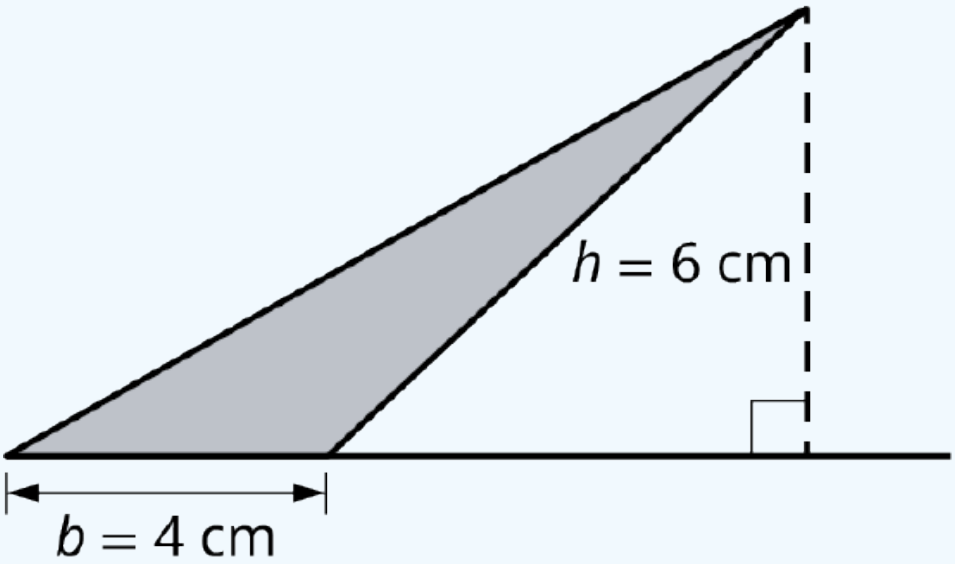


Figure 5.2.2.2: Triangle 2

Example 1

Find the area of this triangle.

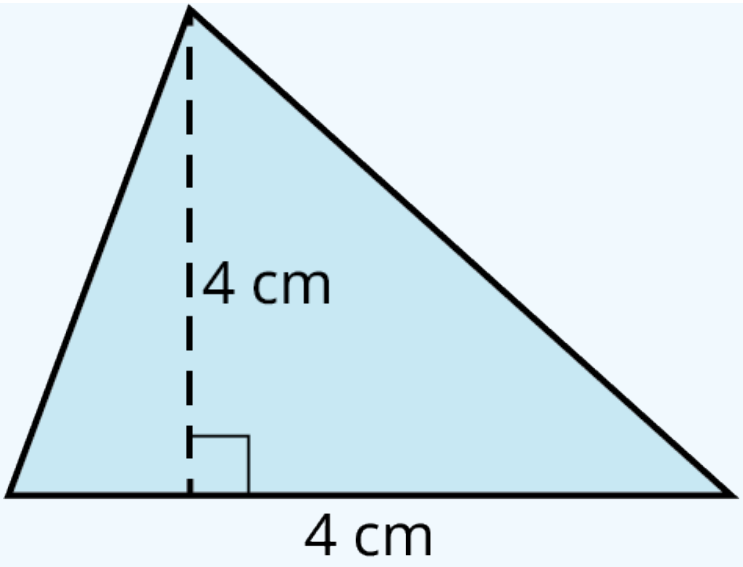


Solution

Using the formula, we have $A = \frac{1}{2}(4 \text{ cm})(6 \text{ cm}) = \frac{24}{2} \text{ cm}^2 = 12 \text{ cm}^2$.

Exercise

Find the area of the triangle.



Answer available at the end of this section.

Find the Area of a Triangle

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video works through two examples of finding the area of a triangle.

Area of Quadrilaterals

A quadrilateral is a four-sided polygon with four vertices. Some quadrilaterals have either one or two sets of parallel sides. The set of quadrilaterals include the square, the rectangle, the parallelogram, the trapezoid, and the rhombus. The most common quadrilaterals are the square and the rectangle.

Square

In Figure $5.2.2.3$, a $12\text{ in} \times 12\text{ in}$ grid is represented with twelve $1\text{ in} \times 1\text{ in}$ squares across each row and twelve $1\text{ in} \times 1\text{ in}$ squares down each column. If you count the little squares, the total is 144 squares. Of course, you do not have to count squares to find area—we have a formula. Thus, the formula for the area of a square, where s is the length of a side, is $A = s \cdot s$. The area of the square in Figure $5.2.2.3$ is $A = 12\text{ in} \times 12\text{ in} = 144\text{ in}^2$.

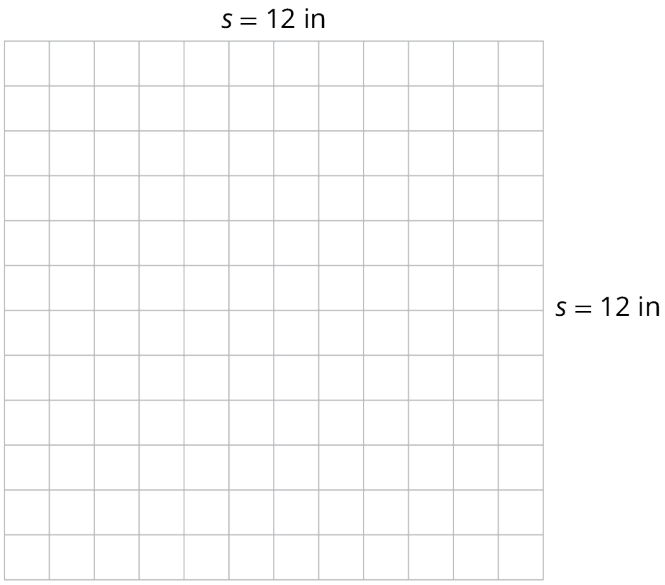


Figure 5.2.2.3: Area of a Square

Area of a Square

The formula for the area of a square is $A = s \cdot s$ or $A = s^2$.

Rectangle

Similarly, the area of a rectangle is found by multiplying length times width. The rectangle in Figure 5.2.2.4 has a width of 5 in and a length of 12 in. The area is $A = 5(12) = 60$ in^2.

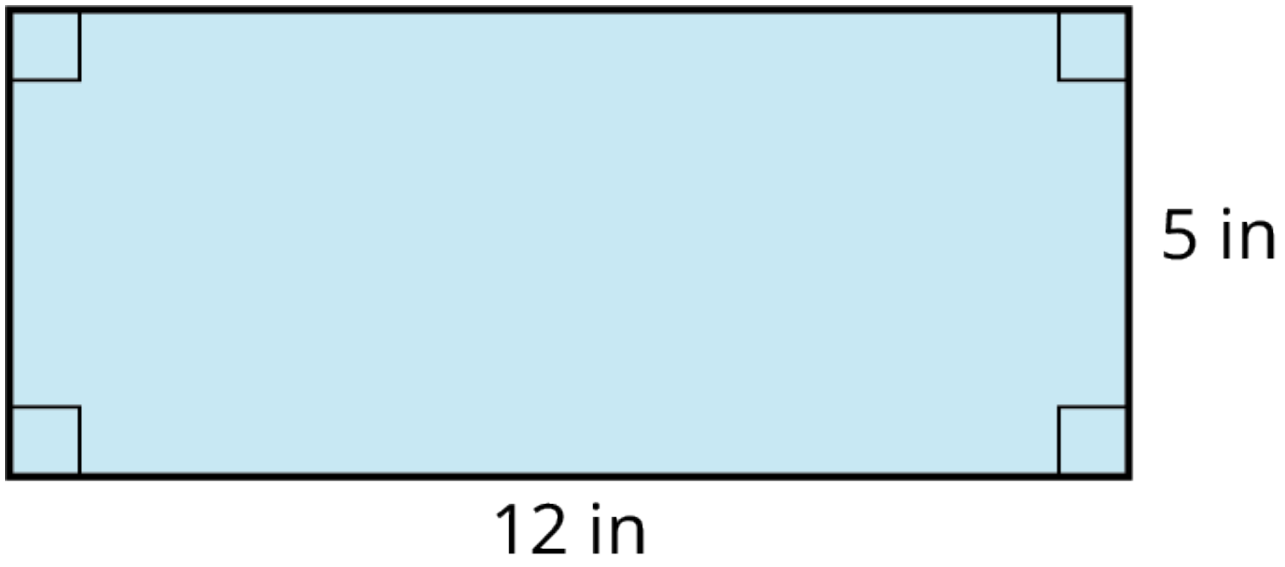


Figure 5.2.2.4: Area of a Rectangle

Area of a Rectangle

The area of a rectangle is given by $A = lw$.

Many everyday applications require the use of the perimeter and area formulas. Suppose you are remodeling your home and you want to replace all the flooring. You need to know how to calculate the area of the floor to purchase the correct amount of tile, or hardwood, or carpet. If you want to paint the rooms, you need to calculate the area of the walls and the ceiling to know how much paint to buy, and the list goes on. Can you think of other situations where you might need to calculate area?

Example 2

You have a garden with an area of 196 square feet. How wide can the garden be if the length is 28 feet?

Solution

The area of a rectangular region is $A = lw$. Letting the width equal w :

$$196 = 28w$$
$$\frac{196}{28} = w = 7\text{ft}$$

Exercise

Find the area of a rectangular region with a length of 18 feet and a width 1/3 of the length.

Answer available at the end of this section.

Example 3

Jennifer is planning to install vinyl floor tile in her rectangular-shaped basement, which measures 29 ft by 16 ft. How many boxes of floor tile should she buy and how much will it cost if one box costs $50 and covers 20 ft ?

Solution

The area of the basement floor is $A = 29(16) = 464\text{ft}^2$. We will divide this area by 20 ft . Thus, $\frac{464}{20} = 23.2$. Therefore, Jennifer will have to buy 24 boxes of tile at a cost $\$1,200$.

Exercise

You remodel your kitchen and decide to change out the tile floor. The floor measures 30 ft by 15 ft. One box of tile costs $45 and covers 10 ft^2. How many boxes of tile should you buy, and what will it cost?

Answer available at the end of this section.

Parallelogram

The area of a parallelogram can be found using the formula for the area of a triangle. Notice in Figure $5.2.2.8$, if we cut a diagonal across the parallelogram from one vertex to the opposite vertex, we have two triangles. If we multiply the area of one triangle by 2, we have the area of the parallelogram:

$$A = 2\left(\frac{1}{2}bh\right)$$
$$A = bh$$

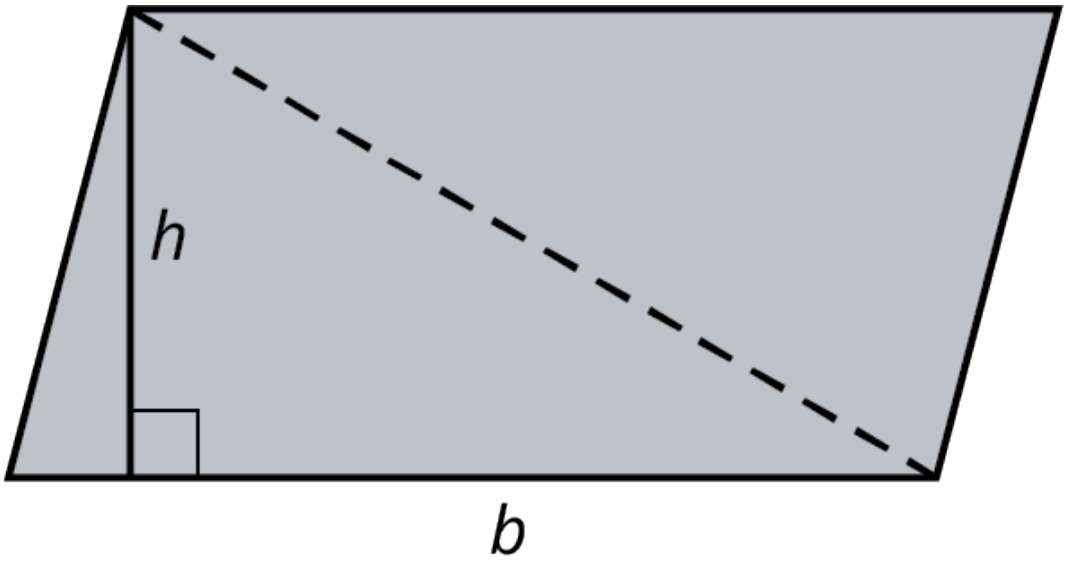


Figure 5.2.2.5: Area of a Parallelogram

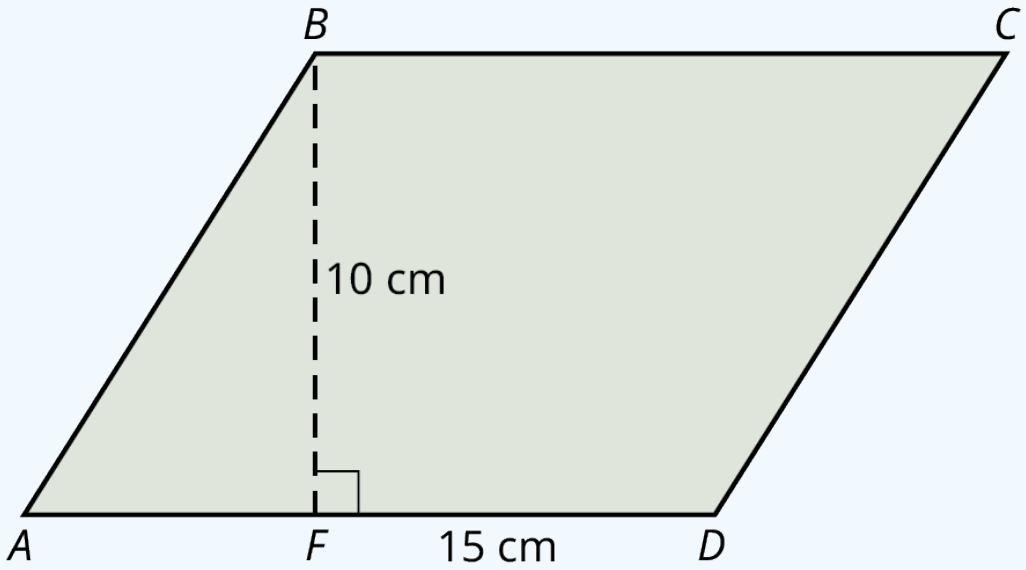
Area of a Parallelogram

The area of a parallelogram is given by $A = bh$.

For example, if we have a parallelogram with the base be equal to 10 inches and the height equal to 5 inches, the area will be $A = (10)(5) = 50 \text{ in}^2$.

Example 4

In the parallelogram, if $\overline{FB} = 10$, $\overline{AD} = 15$, $\overline{FB} = 10$, $\overline{AD} = 15$, find the exact area of the parallelogram.

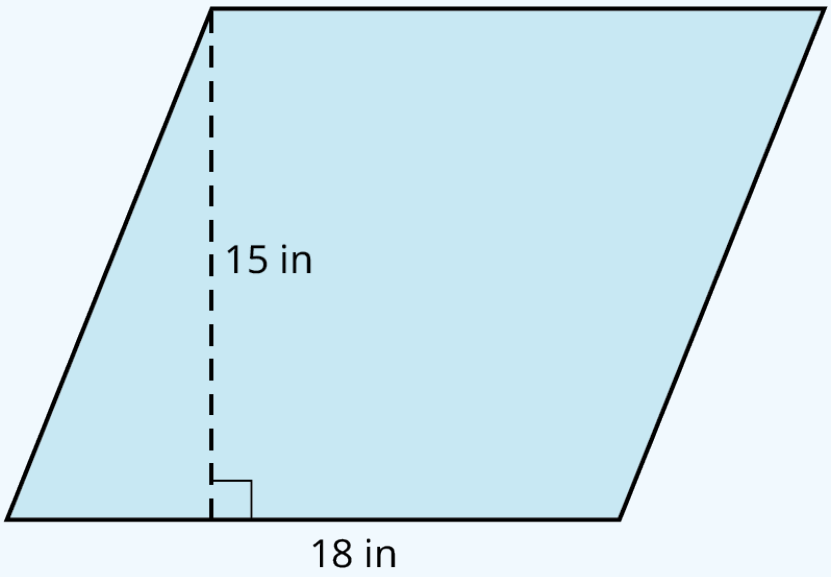


Solution

Using the formula of $A = bh$, we have $A = 10(15) = 150 \text{ cm}^2$.

Exercise

Find the area of the parallelogram.



Answer available at the end of this section.

Area of a Parallelogram

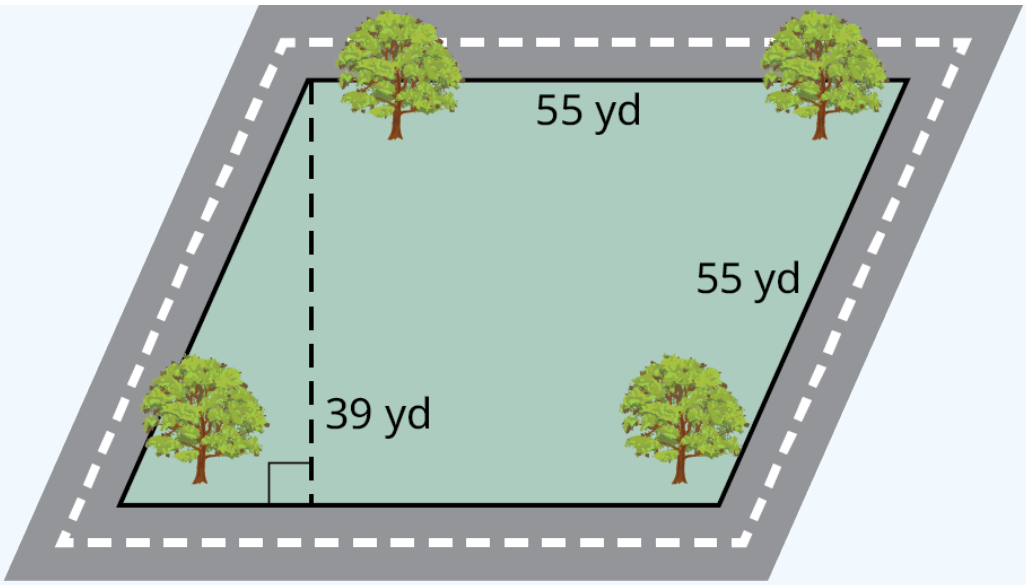
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View this YouTube Video online

This video demonstrates how to find the area of a parallelogram using the formula $A = bh$.

Example 5

The boundaries of a city park form a parallelogram. The park takes up one city block, which is contained by two sets of parallel streets. Each street measures 55 yd long. The perpendicular distance between streets is 39 yd. How much sod, sold by the square foot, should the city purchase to cover the entire park and how much will it cost? The sod is sold for $0.50 per square foot, installation is $1.50 per square foot, and the cost of the equipment for the day is $100.



Solution

Step 1: As sod is sold by the square foot, the first thing we have to do is translate the measurements of the park from yards to feet. There are 3 ft to a yard, so 55 yd is equal to 165 ft, and 39 yd is equal to 117 ft.

Step 2: The park has the shape of a parallelogram, and the formula for the area is $A = bh$:

$A = 165(117) = 19,305\text{ft}^2$

Step 3: The city needs to purchase $19,305\text{ft}^2$ of sod. The cost will be $\$0.50$ per square foot for the sod and $\$1.50$ per square foot for installation, plus $\$100$ for equipment:

$$\begin{aligned} 19,305(\$0.50) + 19,305(\$1.50) &= \$9,652.5 + \$28,957.5 + \$100.00 \\ &= \$38,710.00 \end{aligned}$$

Exercise

Suppose your city has a park just like the one in Example 5. The length of each street is 49 yd, and the perpendicular distance between streets is 31 yd. How much sod, sold by the square foot at $0.45 per square foot plus $1.00 per square foot for installation, and a flat $50 fee for equipment, should be purchased and what will the cost be?

Answer available at the end of this section.

Trapezoid

Another quadrilateral is the trapezoid. A trapezoid has one set of parallel sides or bases. The formula for the area of a trapezoid with parallel bases a and b and height h is given here.

Area of a Trapezoid

The area of a trapezoid is given by $A = \frac{1}{2}h(a + b)$.

For example, the area of the trapezoid in Figure $5.2.2.6$ that has base a equal to 8 cm, base b equal to 6 cm, and height equal to

6 cm is $A = \frac{1}{2}(6)(6+8) = 42 \text{ cm}^2$.

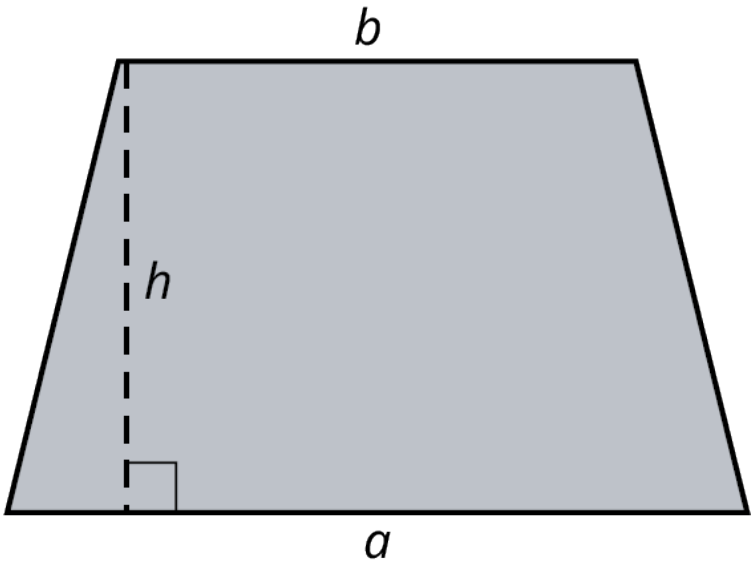
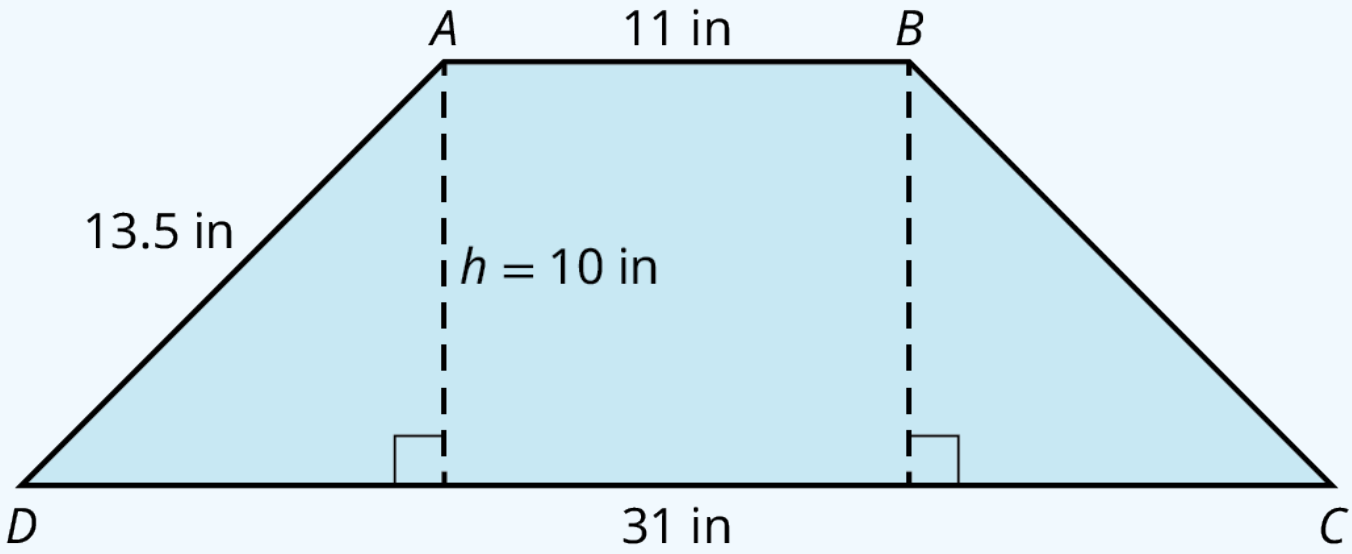


Figure 5.2.2.6: Area of a Trapezoid

Example 6

Find the area of the given figure.

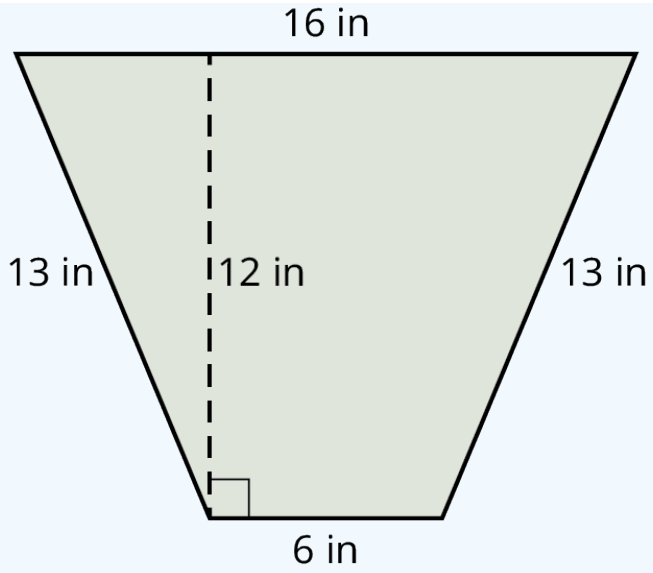


Solution

$A = \frac{1}{2}(10)(11+31) = 210 \text{ in}^2$.

Exercise

Find the area of the figure shown.



Answer available at the end of this section.

Find the Area of a Trapezoid

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to find the area of a trapezoid using the formula $A = \frac{1}{2}(b_1 + b_2)h$.

Area of Regular Polygons

To find the area of a regular polygon, we need to learn about a few more elements. First, the apothem a of a regular polygon is a line segment that starts at the center and is perpendicular to a side. The radius r of a regular polygon is also a line segment that starts at the center but extends to a vertex. See Figure $5.2.2.7$.

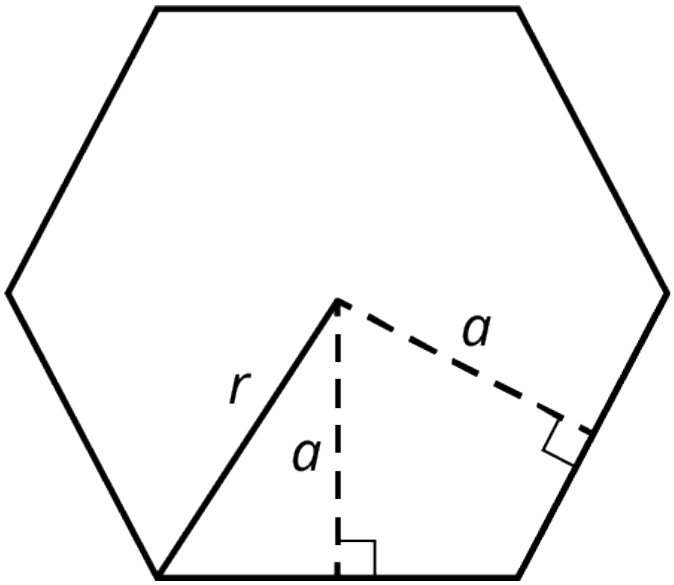


Figure 5.2.2.7: Apothem and Radius of a Polygon

Area of a Regular Polygon

The area of a regular polygon is given by $A = \frac{1}{2}ap$, where a is the apothem and p is the perimeter.

For example, consider the regular hexagon shown in Figure 5.2.2.8 with a side length of 4 cm, and the apothem measures $a = 2\sqrt{3}$.

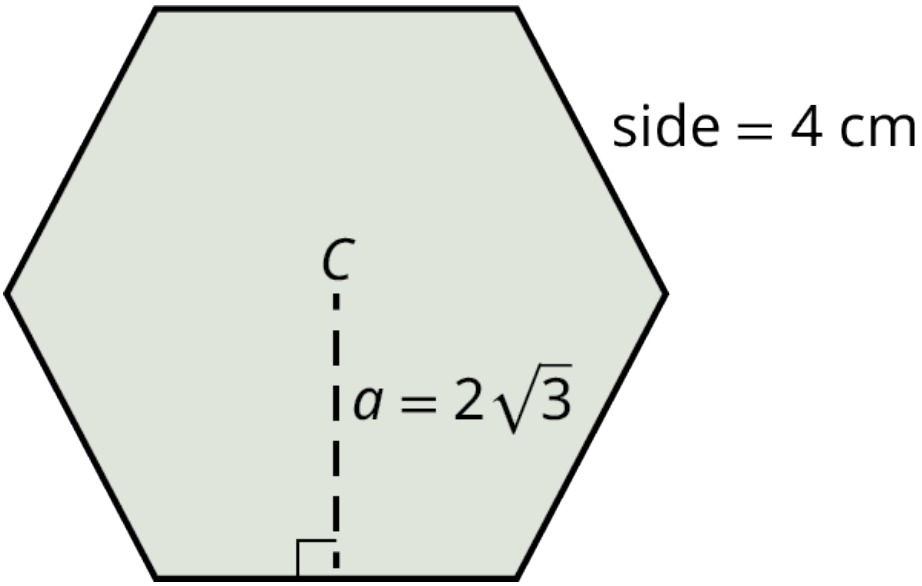


Figure 5.2.2.8: Area of a Hexagon

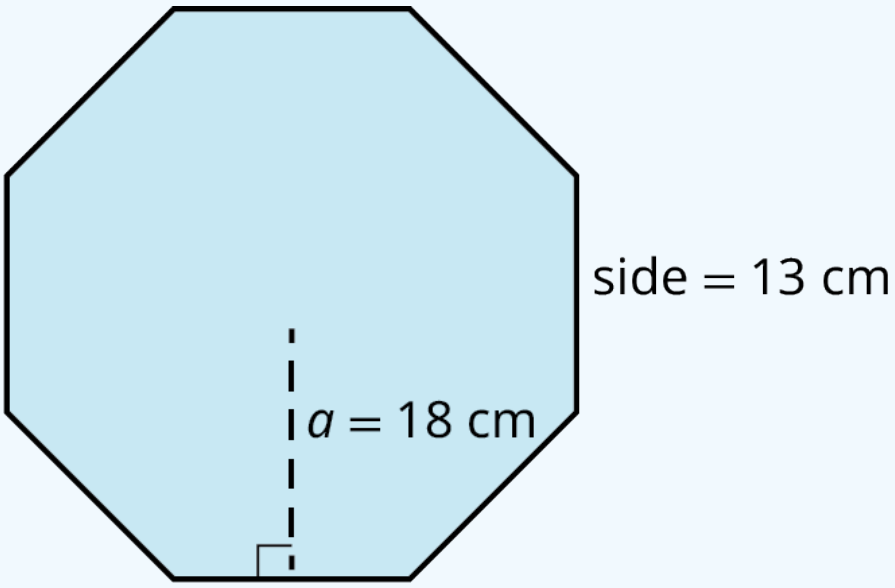
We have the perimeter, $p = 6(4) = 24$ cm. We have the apothem as $a = 2\sqrt{3}$ cm. Then, the area is:

$$\begin{aligned} A &= \frac{1}{2}(2\sqrt{3})(24) \\ &= 24\sqrt{3} \\ &\approx 41.57 \end{aligned}$$

$$A \approx 41.57 \text{ cm}^2$$

Example 9

Find the area of a regular octagon.

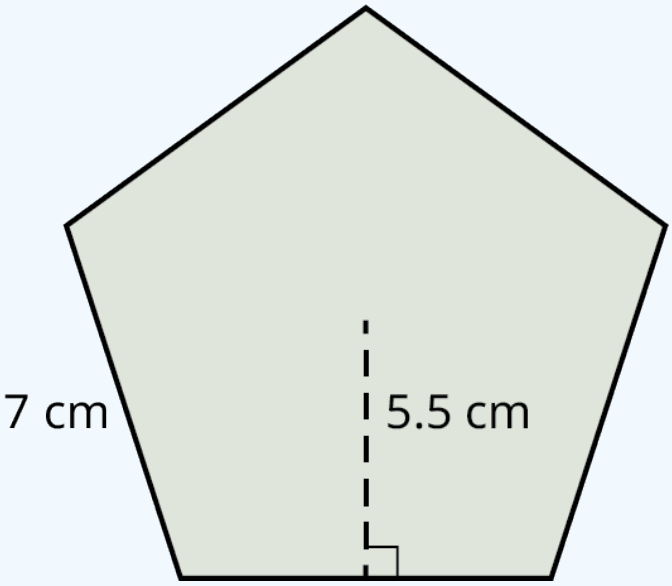


Solution

$A = \frac{1}{2}(18)(104) = 936 \text{ cm}^2$.

Exercise

Find the area of the regular pentagon.



Answer available at the end of this section.

Area of Circles

Just as the circumference of a circle includes the number π, so does the formula for the area of a circle. Recall that π is a non-terminating, non-repeating decimal number: $\pi = 3.14159\ldots$. It represents the ratio of the circumference to the diameter, so it is a critical number in the calculation of circumference and area.

Area of a Circle

The area of a circle is given by $A = \pi r^2$, where r is the radius.

For example, to find the of the circle with radius equal to 3 cm, as shown in Figure 5.2.2.9, is found using the formula $A = \pi r^2$.

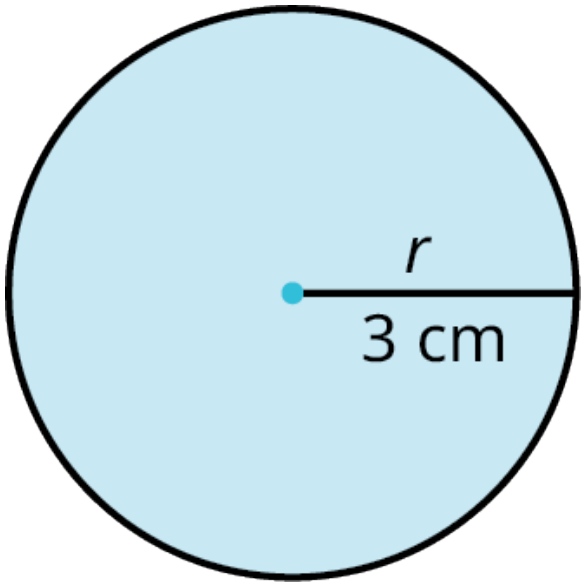


Figure 5.2.2.9: Circle with Radius 3

We have

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(3\text{ cm})^2 \\ &= 9\pi\text{ cm}^2 \\ &\approx 28.27\text{ cm}^2 \end{aligned}$$

Example 10

Find the area of a circle with diameter of 16 cm.

Solution

The formula for the area of a circle is given in terms of the radius, so we cut the diameter in half. Then, the area is $A = \pi(8)^2 = 201.1\text{ cm}^2$.

Exercise

Find the area of the circle with a radius of 3 cm.

Answer available at the end of this section.

Determine the Area of a Circle

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to find the area of a circle using the formula $A = \pi r^2$.

Example 11

You decide to order a pizza to share with your friend for dinner. The price for an 8-inch diameter pizza is \$7.99. The price for 16-inch diameter pizza is \$13.99. Which one do you think is the better value?

Solution

The area of the 8 -inch diameter pizza is $A = \pi(4)^2 = 50.3\text{in}^2$. The area of the 16-inch diameter pizza is $A = \pi(8)^2 = 201.1\text{in}^2$. Next, we divide the cost of each pizza by its area in square inches. Thus, $\frac{13.99}{201.1} = \$0.07$ per square inch and $\frac{7.99}{50.3} = \$0.16$ per square inch. So clearly, the 16 -inch pizza is the better value.

Exercise

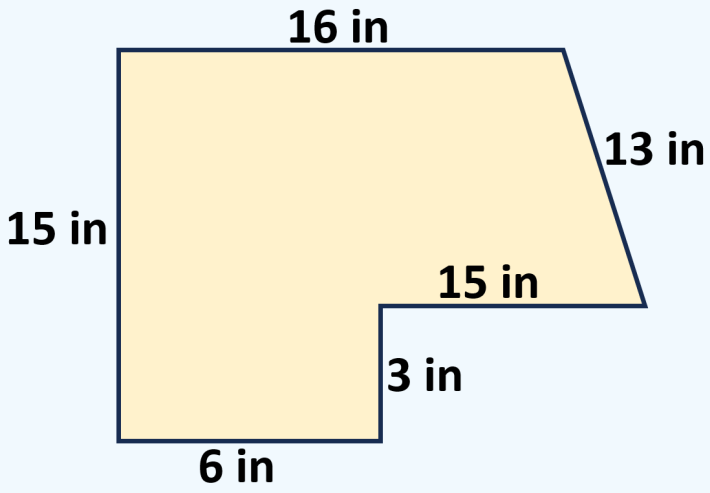
You can buy a 9-inch diameter pizza for \$10.99, or a 15-inch diameter pizza for \$14.99. Which pizza is the better value?

Answer available at the end of this section.

Area of Composite Figures

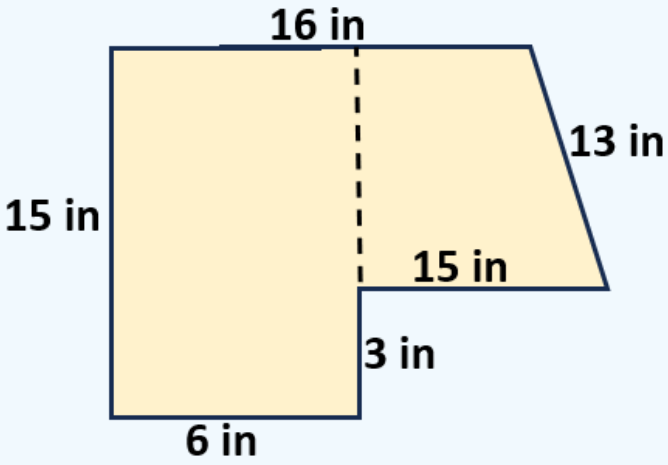
Example 12

Find the area of the given composite figure.



Solution

To find the area, split the figure into a rectangle and a trapezoid.



First find the area of the rectangle.

$A = lw$

$A = 6(15)$

$A = 90 \text{ in}^2$

Now find the height of the trapezoid.

$15 - 3 = 12 \text{ in}$

Now find the area of the trapezoid.

$A = \frac{1}{2}(b_1 + b_2)h$

$A = \frac{1}{2}(16 + 15)(12)$

$A = \frac{1}{2}(31)(12)$

$A = 186 \text{ in}^2$

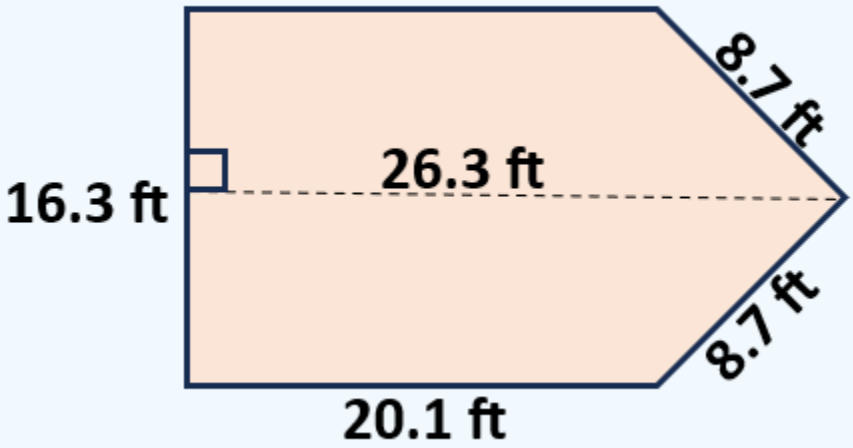
Now add the two areas.

$90 + 186 = 276 \text{ in}^2$

The area of the figure is 276 in^2.

Exercise

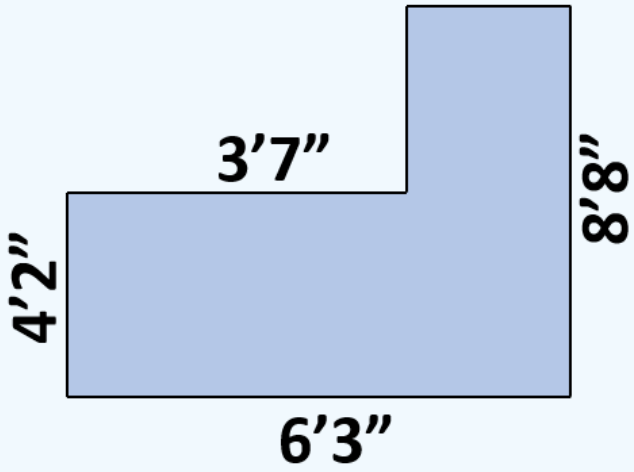
Find the area of the given composite figure.



Answer available at the end of this section.

Example 13

Find the area of the composite figure in square inches.



Solution

To find the area, split the figure into two rectangles.

First, find the missing dimensions.

Top section height:

$8'8'' - 4'2'' = 4'6''$

Right section width:

$6'3'' - 3'7'' = 2'8''$

Convert all measurements to inches.

$6'3'' = 75 \text{ in}$

$4'2'' = 50 \text{ in}$

$2'8'' = 32 \text{ in}$

$4'6'' = 54 \text{ in}$

Now find the area of each rectangle.

Bottom rectangle:

$A = 75 \times 50 = 3750 \text{ in}^2$

Top rectangle:

$A = 32 \times 54 = 1728 \text{ in}^2$

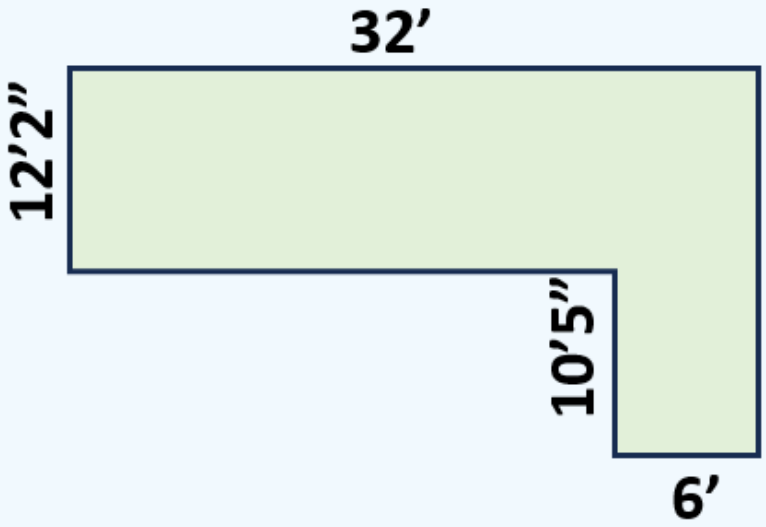
Now add the areas.

$3750 + 1728 = 5478 \text{ in}^2$

The area of the figure is 5478 in^2.

Exercise

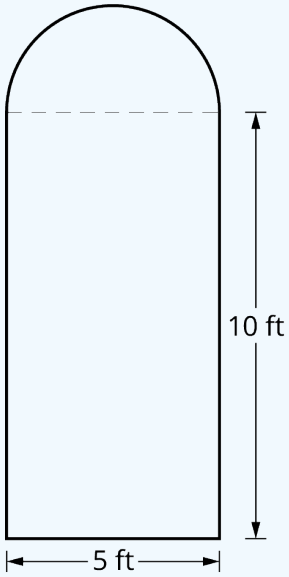
Find the area of the given composite figure in square inches.



Answer available at the end of this section.

Example 12

You want to purchase a tinted film, sold by the square foot for the window below. The bottom part of the window is a rectangle, and the top part is a semicircle. Find the area and calculate the amount of film to purchase.



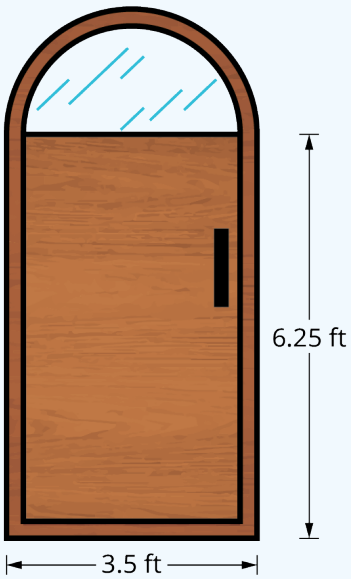
Solution

First, the rectangular portion has $A = 5(10) = 50\text{ft}^2$. For the top part, we have a semicircle with a diameter of 5 ft , so the radius is 2.5 ft . We want one half of the area of a circle with radius 2.5 ft , so the area of the top semicircle part is $A = \frac{1}{2}\pi(2.5)^2 = 9.8\text{ft}^2$. Add the area of the rectangle to the area of the semicircle. Then, the total area to be covered with the window film is

$A = 50 + 9.8 = 59.8\text{ft}^2$

Exercise

You decide to install a new front door with a semicircle top as shown in the figure. How much area will the new door occupy?



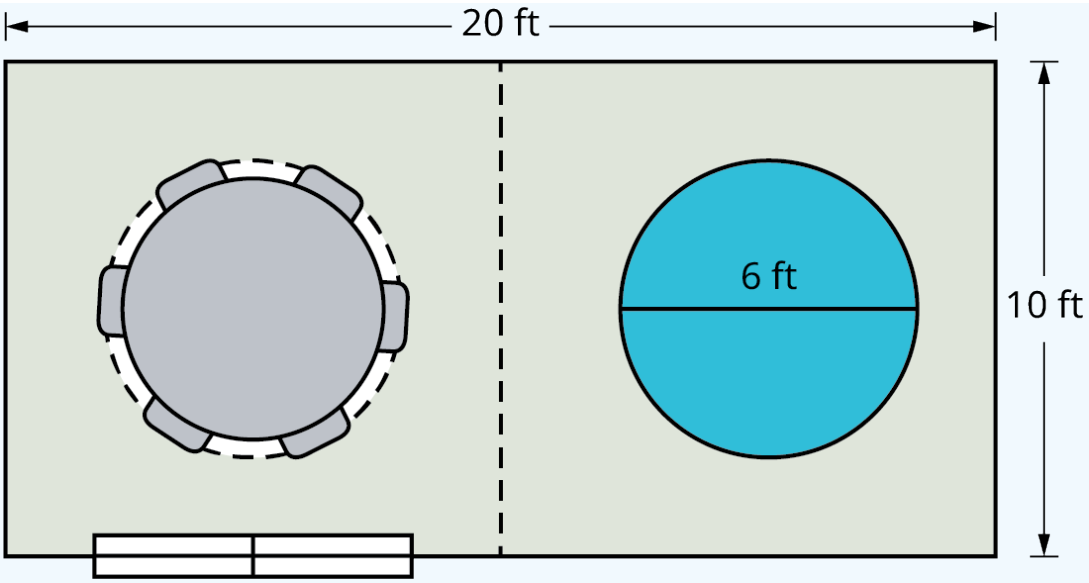
Answer available at the end of this section.

Area within Area

Suppose you want to install a round hot tub on your backyard patio. How would you calculate the space needed for the hot tub? Or, let's say that you want to purchase a new dining room table, but you are not sure if you have enough space for it. These are common issues people face every day. So, let's take a look at how we solve these problems.

Example 13

The patio in your backyard measures 20 ft by 10 ft. On one-half of the patio, you have a 4-foot diameter table with six chairs taking up an area of approximately 36 sq feet. On the other half of the patio, you want to install a hot tub measuring 6 ft in diameter. How much room will the table with six chairs and the hot tub take up? How much area is left over?

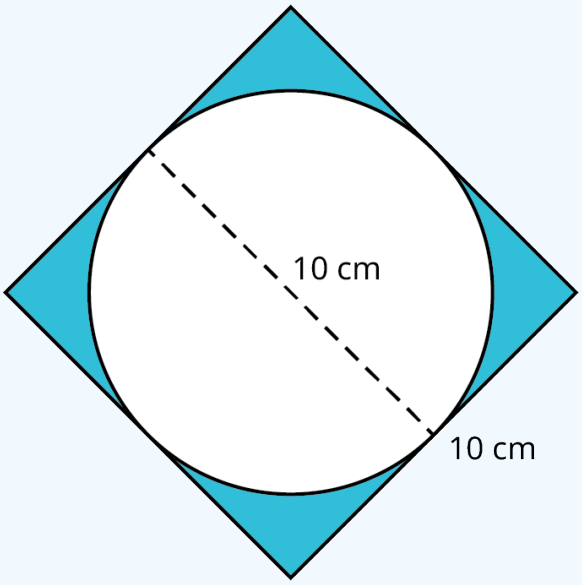


Solution

The hot tub has a radius of 3 ft . That area is then $A = \pi(3)^2 = 9\pi = 28.27\text{ft}^2$. The total square feet taken up with the table and chairs and the hot tub is $36 + 28.27 = 64.27\text{ft}^2$. The area left over is equal to the total area of the patio, 200ft^2 minus the area for the table and chairs and the hot tub. Thus, the area left over is $200 - 64.27 = 135.7\text{ft}^2$.

Exercise

Find the area of the shaded region in the given figure.



Answer available at the end of this section.

Example 14

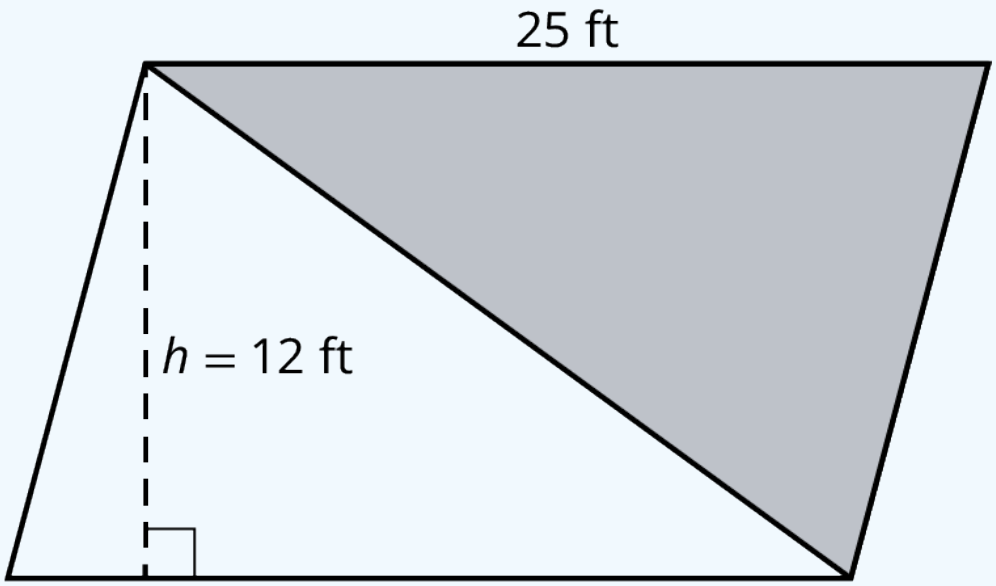
A sod farmer wants to fertilize a rectangular plot of land that is 150 ft by 240 ft. A bag of fertilizer covers $5{,}000\ \text{ft}^2$ and costs $\$200$. How much will it cost to fertilize the entire plot of land?

Solution

The plot of land is $36,000\text{ft}^2$. It will take 7.2 bags of fertilizer to cover the land area. Therefore, the farmer will have to purchase 8 bags of fertilizer at $\$200$ a bag, which comes to $1,600.

Exercise

You want to install sod on one-half of your parallelogram-shaped backyard as shown. The patio covers the other half. Sod costs $\$50$ a bag and covers 25ft^2. How much will it cost to buy the sod?



Answer available at the end of this section.

Answers

Exercise

$A = 8\ \text{cm}^2$

Exercise

$A = 108\ \text{ft}^2$

Exercise

45 boxes, $2025

Exercise

$A = 270\ \text{in}^2$

Exercise

$\text{Area} = 1519\ \text{yd}^2 = 13{,}671\ \text{ft}^2,\ \text{Cost} = \$19{,}189.95$

Exercise

$A = 108\ \text{in}^2$

Exercise

$A = 96.25\ \text{cm}^2$

Exercise

$A = 28.3 \text{ cm}^2$

Exercise

15-inch pizza

Exercise

$A = 378.095 \text{ ft}^2$

Exercise

$A = 44088 \text{ in}^2$

Exercise

$A = 24.5 \text{ ft}^2$

Exercise

$A = 400 - 25\pi \approx 321.5 \text{ cm}^2$

Exercise

$300

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5.2.3: Applications Involving Perimeter and Area

In many technical and workplace settings, measurements are used to plan materials, estimate costs, and solve real problems. Perimeter is used when working with distances around objects, such as fencing, trim, or safety barriers. Area is used when covering surfaces, such as flooring, concrete, or metal panels. In this section, you will apply perimeter and area formulas to practical situations, including composite shapes and figures with sections removed. These problems require you to interpret diagrams, identify needed measurements, and decide which formulas to use.

Applications Involving Perimeter

Example 1

An event coordinator is installing temporary fencing around a rectangular equipment storage area that measures 78 ft long and 42 ft wide. Fencing costs $\$12.50$ per linear foot. How much will it cost to install the fencing?

Solution

$P = 2l + 2w$

$P = 2(78) + 2(42)$

$P = 156 + 84 = 240 \text{ ft}$

$240 \times 12.50 = 3000$

The cost to install the fencing is $3000.

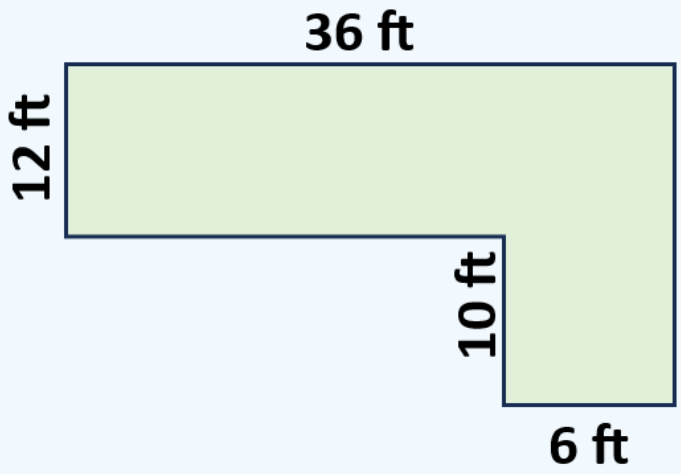
Exercise

A construction crew is installing safety fencing around a rectangular storage area that measures 64 ft long and 38 ft wide. Fencing costs $\$9.75$ per linear foot. How much will it cost to install the fencing?

Answer available at the end of this section.

Example 2

A metal fabricator is installing a guard rail around the platform shown below. How many feet of guard rail are needed to go around the entire platform?



Solution

First, determine the missing side lengths.

Right side: $12 + 10 = 22$ ft

Inner horizontal: $36 - 6 = 30$ ft

Now add all outer sides to find the perimeter.

$P = 36 + 22 + 6 + 10 + 30 + 12$

$P = 116$ ft

The fabricator needs 116 ft of guard rail.

Exercise

A stage technician is installing LED strip lighting around an L-shaped performance platform. The platform has a top length of 42 ft, a left vertical side of 14 ft, an inner vertical drop of 9 ft, and a bottom right horizontal section of 8 ft. How many feet of lighting are needed to go around the entire platform?

Answer available at the end of this section.

Example 3

A landscaping company is installing metal edging around a circular equipment pad with a radius of 18 ft. The edging is sold by the linear foot. How many feet of edging are needed? Use $\pi \approx 3.14$.

Solution

$C = 2\pi r$

$C = 2(3.14)(18)$

$C = 113.04$ ft

The landscaping company needs approximately 113.04 ft of edging.

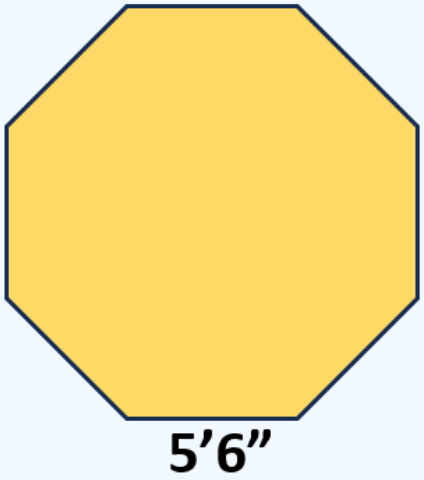
Exercise

A park maintenance worker is installing a circular border around a concrete pad with a radius of 14 ft. The border material is sold by the linear foot. How many feet of border are needed? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Example 4

A contractor is installing trim around a regular octagonal skylight. Each side of the skylight measures $5'6''$. How many feet of trim are needed to go around the skylight?



Solution

First convert the side length to feet.

$5'6'' = 5.5$ ft

Now find the perimeter.

$P = ns$

$P = 8(5.5)$

$P = 44$ ft

The contractor needs 44 ft of trim.

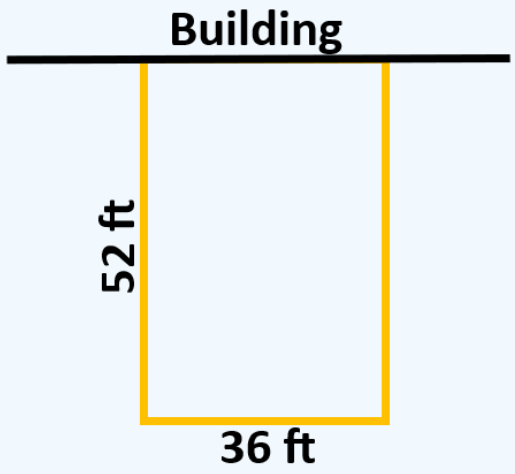
Exercise

A metalworker is installing trim around a regular hexagonal platform. Each side of the platform measures 7.25 ft. How many feet of trim are needed to go around the platform?

Answer available at the end of this section.

Example 5

A warehouse supervisor is marking off a loading zone along the wall of a building using safety tape. The loading zone is rectangular, measuring 52 ft long and 36 ft wide, but one 36 ft side runs along the building and does not need tape. Safety tape costs $\$1.85$ per foot. How much will it cost to outline the loading zone?



Solution

First find the total perimeter.

$P = 2l + 2w$

$P = 2(52) + 2(36)$

$P = 104 + 72 = 176 \text{ ft}$

Since one 36 ft side does not need tape, subtract that length.

$176 - 36 = 140 \text{ ft}$

Now find the cost.

$140 \times 1.85 = 259$

The cost to outline the loading zone is $259.

Exercise

A school facilities manager is installing protective padding along the walls of a rectangular gym that measures 84 ft by 60 ft. One 12 ft section along the wall is left open for an equipment storage area. How many feet of padding are needed?

Answer available at the end of this section.

Applications Involving Area

Example 6

An HVAC technician is installing a triangular duct cover. The base of the cover is 18 ft and the height is 12 ft. How many square feet of material are needed to cover the panel?

Solution

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(18)(12)$

$A = 108 \text{ ft}^2$

The crew needs 108 square feet of material.

Exercise

A fabrication shop is cutting a triangular metal plate with a base of 22 ft and a height of 15 ft. How many square feet of metal are needed?

Answer available at the end of this section.

Example 7

A contractor is preparing a sloped concrete pad shaped like a parallelogram. The base of the pad measures 34 ft and the perpendicular height is 12 ft. How many square feet of concrete are needed?

Solution

$A = bh$

$A = 34 \times 12$

$A = 408 \text{ ft}^2$

The contractor needs 408 square feet of concrete.

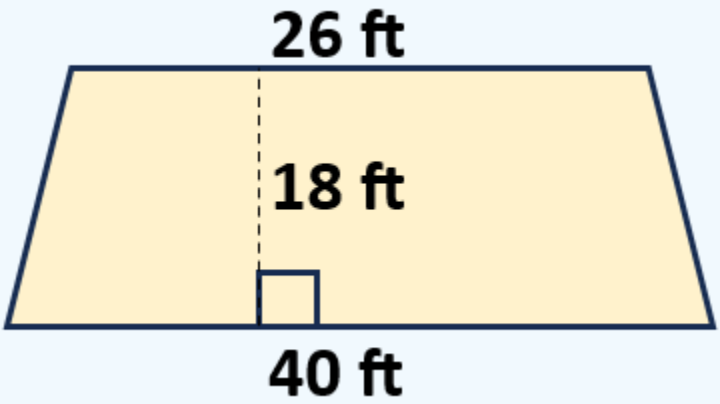
Exercise

A solar technician is installing a panel array section shaped like a parallelogram. The base measures 28 ft and the perpendicular height is 16 ft. How many square feet of panel area is needed?

Answer available at the end of this section.

Example 8

A civil engineering technician is preparing a trapezoidal test surface. The two parallel sides measure 26 ft and 40 ft, and the perpendicular height is 18 ft. How many square feet of material are needed?



Solution

$A = \frac{1}{2}(b_1 + b_2)h$

$A = \frac{1}{2}(26 + 40)(18)$

$A = \frac{1}{2}(66)(18)$

$A = 33 \times 18$

$A = 594 \text{ ft}^2$

The contractor needs 594 square feet of concrete.

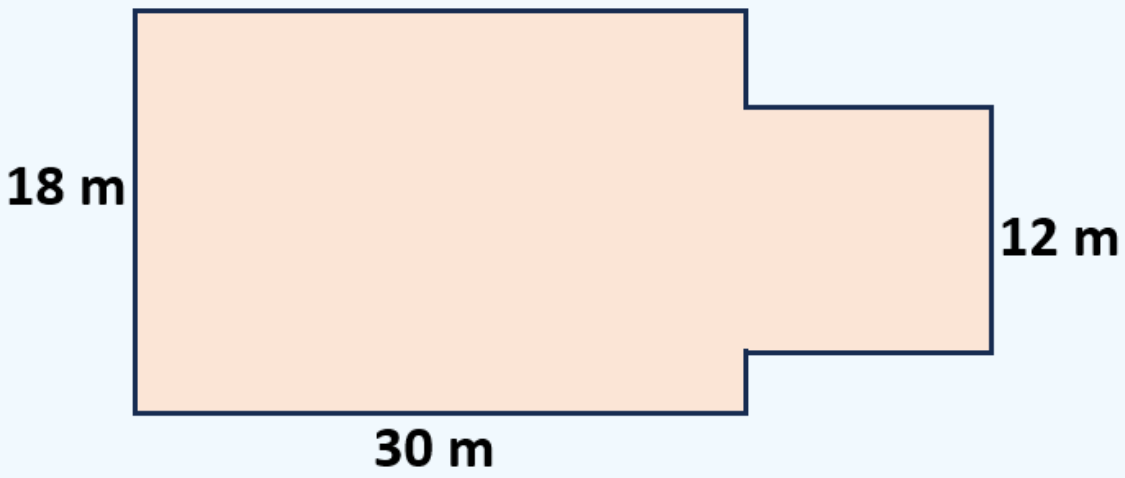
Exercise

A shipbuilder is cutting a trapezoidal section of metal for a hull panel. The two parallel sides measure 32 ft and 48 ft, and the perpendicular height is 15 ft. How many square feet of metal are needed?

Answer available at the end of this section.

Example 9

A workshop floor is being resurfaced in two connected sections. One section is a rectangle measuring 30 m by 18 m. Attached to it is a square section with side length 12 m. How many square meters of flooring material are needed in total?



Solution

First find the area of each section.

Rectangle:

$A = lw$

$A = 30 \times 18 = 540 \text{ m}^2$

Square:

$A = s^2$

$A = 12^2 = 144 \text{ m}^2$

Now add the areas.

$540 + 144 = 684 \text{ m}^2$

The workshop floor requires 684 square meters of material.

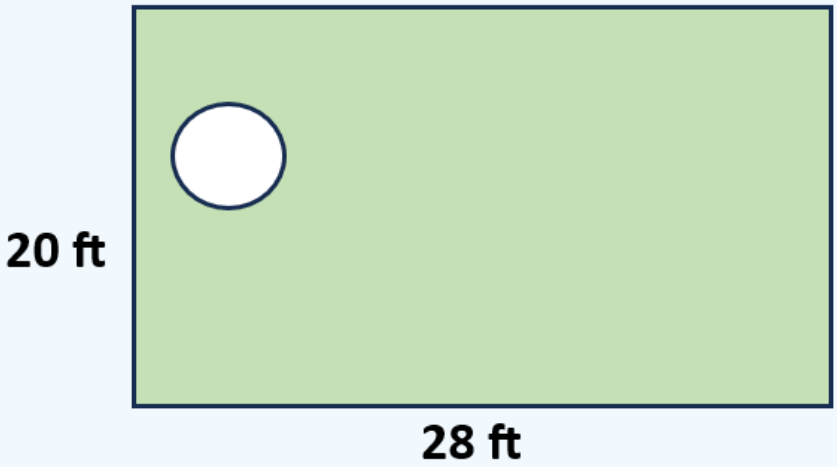
Exercise

A warehouse floor is being resurfaced in two connected sections. One section is a rectangle measuring 24 m by 16 m. Attached to one side is a triangular section with a base of 10 m and a height of 12 m. How many square meters of flooring material are needed in total?

Answer available at the end of this section.

Example 10

A facilities manager is installing flooring in a rectangular room that measures 28 ft by 20 ft. A circular area with a radius of 4 ft is cut out for a column and will not be covered. How many square feet of flooring are needed? Use $\pi \approx 3.14$.



Solution

First find the area of the entire room.

$A = lw$

$A = 28 \times 20 = 560 \text{ ft}^2$

Now find the area of the circular section.

$A = \pi r^2$

$A = 3.14 \times 4^2$

$A = 3.14 \times 16 = 50.24 \text{ ft}^2$

Subtract to find the area to be covered.

$560 - 50.24 = 509.76 \text{ ft}^2$

The contractor needs approximately 509.76 square feet of flooring.

Exercise

A flooring installer is covering a trapezoid-shaped lobby. The parallel sides of the lobby measure 34 ft and 50 ft, and the perpendicular height is 18 ft. A rectangular section measuring 10 ft by 6 ft is cut out for a built-in desk and will not be covered. How many square feet of flooring are needed?

Answer available at the end of this section.

Answers

Exercise

$\$1989.00$

Exercise

138 ft

Exercise

87.92 ft

Exercise

43.5 ft

Exercise

276 ft

Exercise

165 ft^2

Exercise

448 ft^2

Exercise

600 ft^2

Exercise

444 m^2

Exercise

696 ft^2

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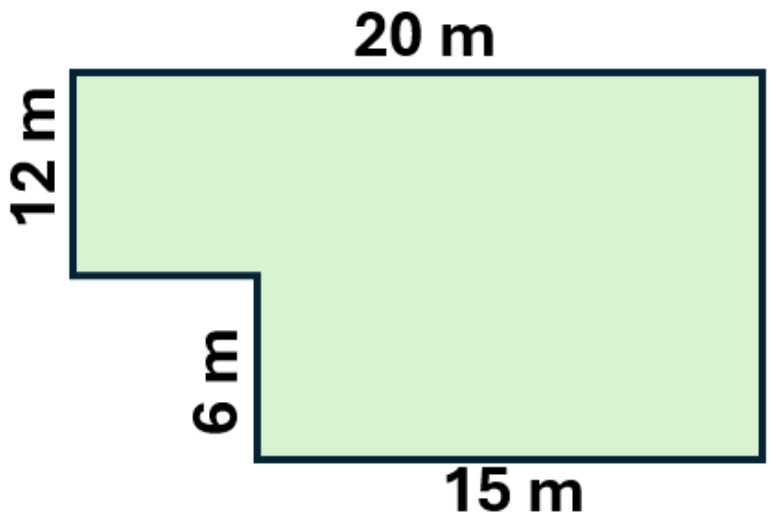
5.2.4: Perimeter and Area Practice

The following set of exercises provides practice for the concepts covered in Module 5.2: Perimeter and Area. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

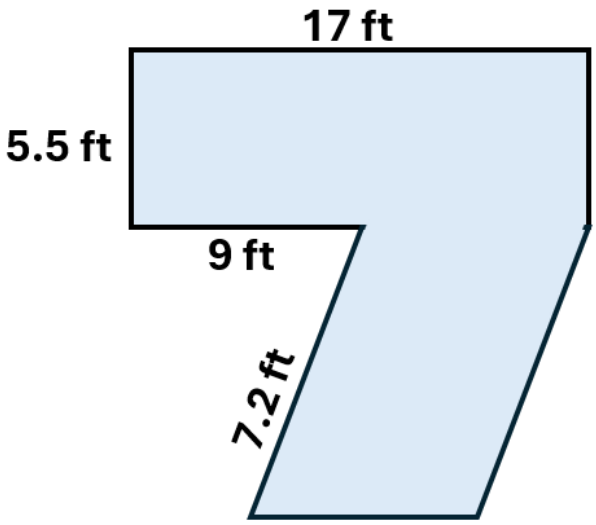
Section 5.2.1 - Perimeter and Circumference

Answer the following questions.

1. Find the perimeter of a square with side length 14 cm.
2. Find the perimeter of a regular octagon with side length 5.5 cm.
3. Find the perimeter of a parallelogram with base 16 ft and side length 9 ft.
4. Find the perimeter of a trapezoid with side lengths 12 ft, 18 ft, 7 ft, and 9 ft.
5. Find the circumference of a circle with diameter 10 in. Use $\pi \approx 3.14$.
6. Find the circumference of a circle with radius 6.5 ft. Use $\pi \approx 3.14$.
7. Find the perimeter of the given figure.



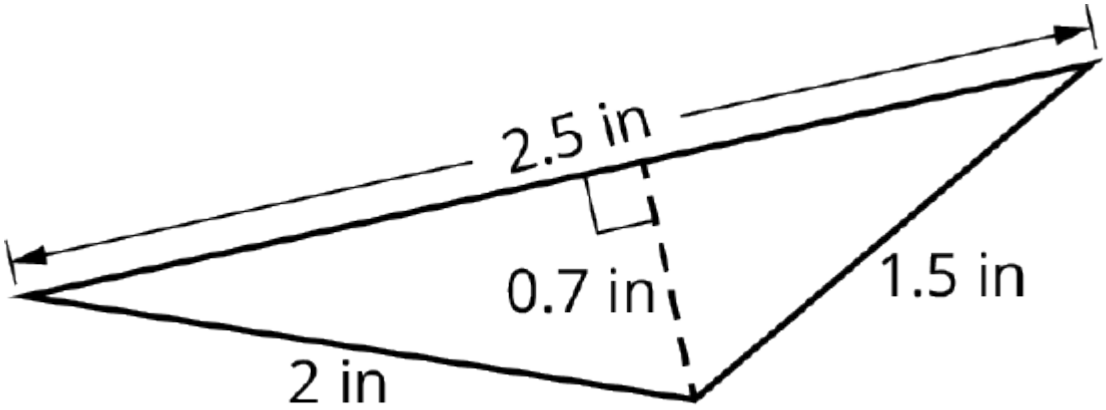
8. Find the perimeter of the given figure.



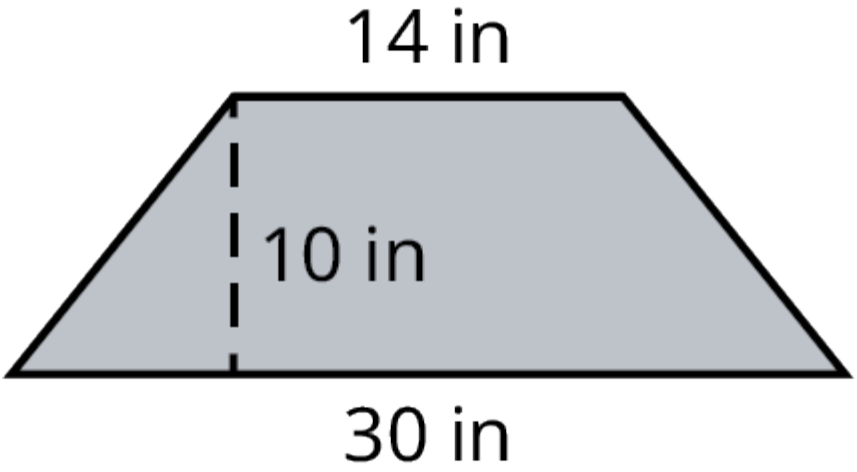
Section 5.2.2 - Area

Answer the following questions.

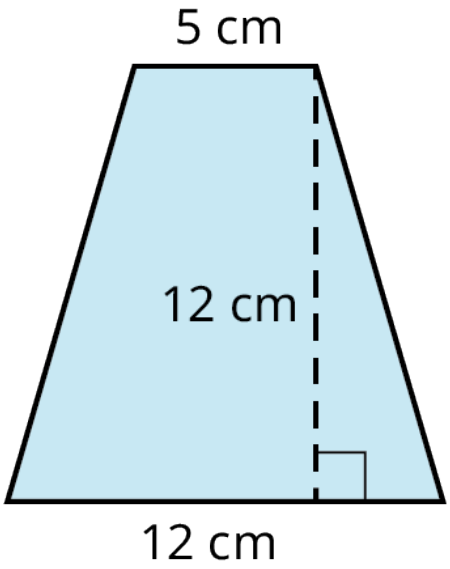
9. Find the area of a triangle with base equal to 2.5 in, height equal to 0.7 in as shown.



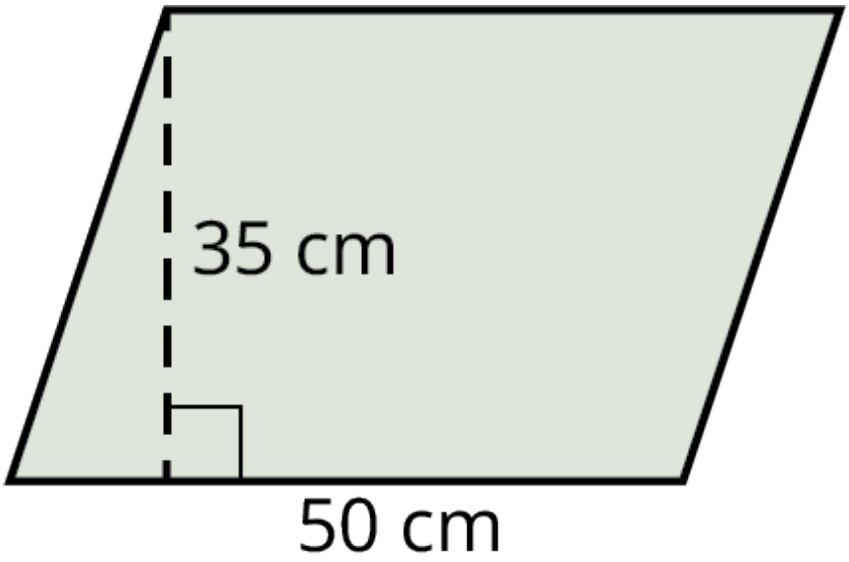
10. Find the area of a trapezoid with $b_1 = 30\ in,\ \ b_2 = 14\ in$, and $h = 10\ in$ as shown.



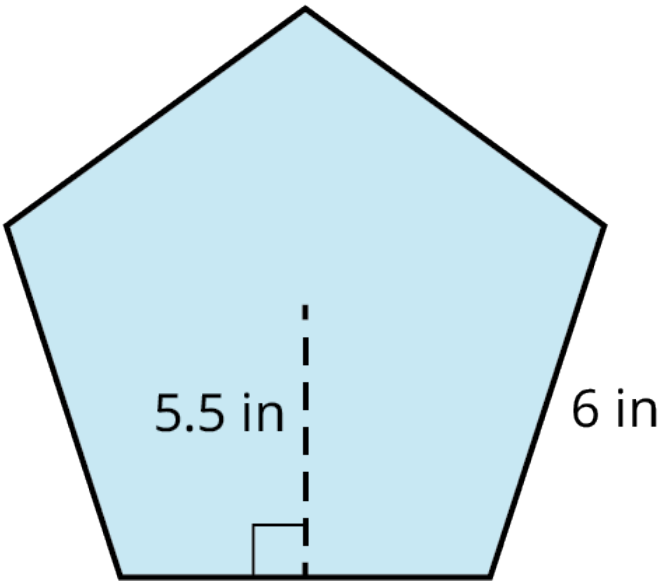
11.Find the area of the trapezoid shown.



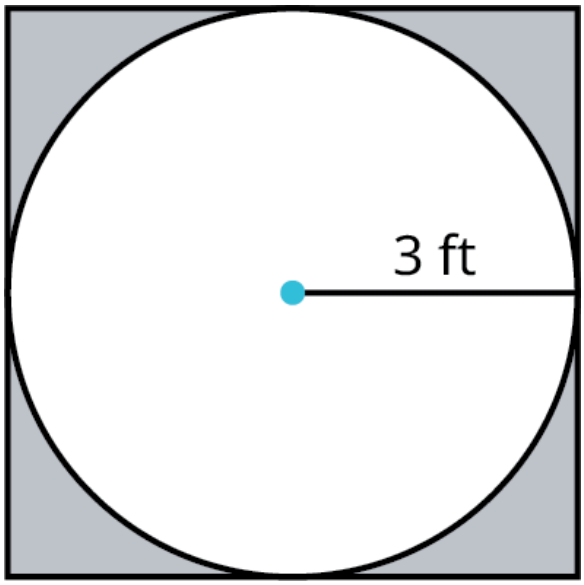
12. Find the area of the parallelogram with base equal to 50 cm and height equal to 35 cm as shown.



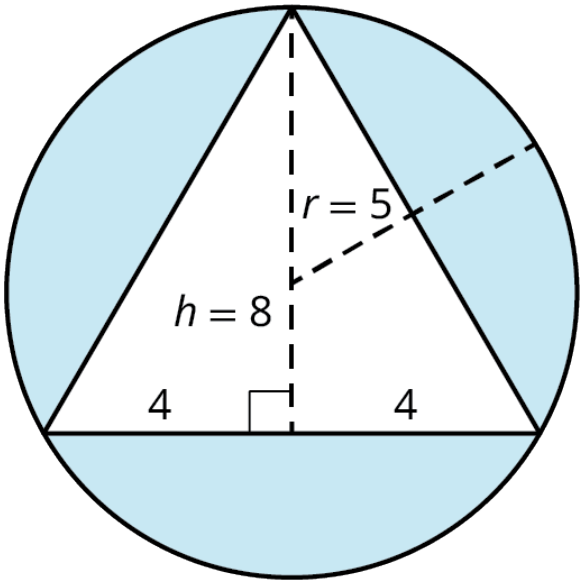
13. Find the area of a regular pentagon with the apothem equal to 5.5 in and the side length equal to 6 in as shown.



14. Find the area of the shaded region in the given figure.



15. Find the area of the shaded region in the given figure. All measurements in centimeters.



Section 5.2.3 - Applications of Perimeter and Area

Answer the following questions.

16. A technician is installing a protective cover over a rectangular control panel that measures 14 ft by 9 ft. The material is sold by the square foot. How much material is needed?

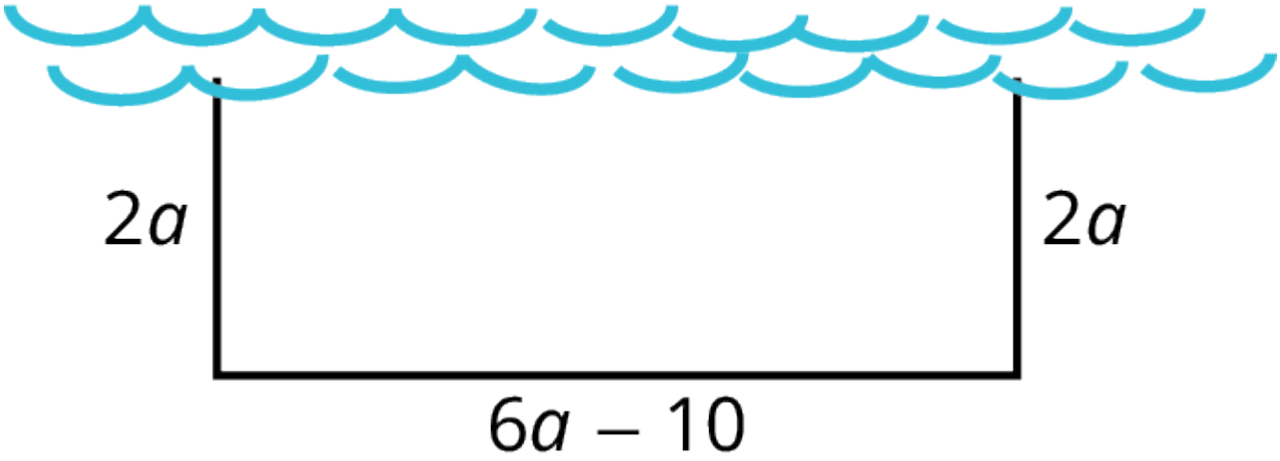
17. A triangular support brace is attached to a frame. The base is 16 ft and the height is 11 ft. The material is sold by the square foot. How much material is needed to cover the brace?

18. A circular fountain has a radius of 10 ft. How many feet of trim are needed to go around the edge? Use $\pi \approx 3.14$.

19. A technician is testing a vehicle and drives it 500 ft. The car tire has a radius of 14 in. Approximately how many revolutions does the tire make?

20. A storage area measures 40 ft by 22 ft, but one side is along a wall and does not need fencing. Fencing costs $\$8.50$ per foot. How much will it cost to fence the area?

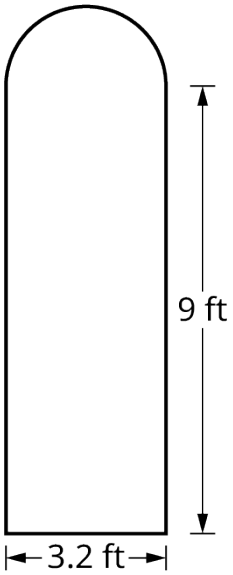
21. You have 140 ft of fencing to enclose a rectangular region that borders a river. You do not have to fence in the side that borders the river. The width is equal to $2a$, and the length is equal to six times the width less 10. Find the dimensions of the region to be fenced.



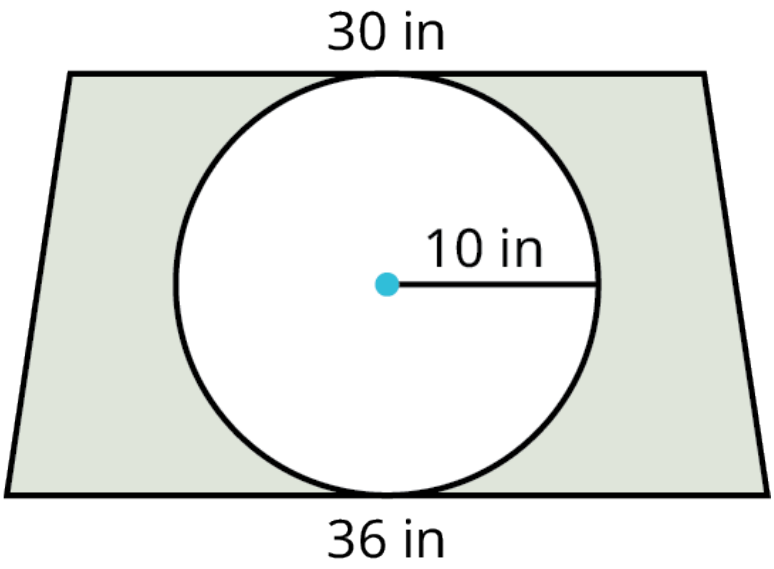
22. An L-shaped walkway is shown. Some side lengths are not labeled but can be determined from the diagram. How many feet of border are needed?

23. A rectangular floor measures 30 ft by 18 ft. A square section with side length 6 ft will not be covered. How many square feet of flooring are needed?

24. Find the area and perimeter of the window shown.



25. You are installing a countertop in the shape of a trapezoid with a round sink as shown in the figure. After the sink is installed, how much area is left on the countertop?



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5.3: VOLUME AND SURFACE AREA

The world we live in is three-dimensional, and geometry gives us the tools to measure and understand the space that objects occupy. In this unit, we move beyond flat figures and begin working with three-dimensional shapes, focusing on two of the most common: prisms and cylinders. You will learn how to calculate lateral surface area, total surface area, and volume for each of these figures, building skills that are directly applicable to fields like manufacturing, construction, packaging, and engineering.

Surface area tells us how much material covers the outside of a shape, while volume tells us how much space exists inside it. These are different questions with different formulas, and knowing when to use each one is an important part of solving real-world problems. By the end of this unit, you will be able to look at a three-dimensional figure, identify its dimensions, and determine exactly how to measure it.

You will also apply these skills to practical situations involving three-dimensional objects, practicing the kind of careful reading and problem-solving that real applications require.

Before we begin, watch the video below about bees and their remarkable use of hexagons in building their hives. Bees instinctively construct structures that maximize storage space while minimizing the material needed to build them — a perfect real-world example of the relationship between surface area and volume that sits at the heart of this unit.

Why do Honeybees Love Hexagons

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

Additional text/introduction.

5.3.1: Volume and Surface Area of Prisms

5.3.2: Volume and Surface Area of Cylinders and Spheres

5.3.3: Applications of Volume and Surface Area

5.3.4: Volume and Surface Area Practice

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5.3.1: Volume and Surface Area of Prisms

Volume and surface area show up in everyday situations—how much something can hold and how much material is needed to cover it. For example, volume tells you how much a container can hold, while surface area helps estimate materials like paint, flooring, or siding that will cover the container.

Three-Dimensional Objects

In geometry, three-dimensional objects are called geometric solids. To describe these solids, we use measurements like surface area and volume. Surface area is the total area of all the outer faces and is measured in square units, while volume is the amount of space inside the solid and is measured in cubic units.

A two-dimensional shape has length and width. When a third dimension—height—is added, the shape becomes a three-dimensional solid, such as a box. This same idea can be represented using a coordinate system. The x-axis and y-axis describe two dimensions, and adding the z-axis creates a third dimension, forming three-dimensional space.

See Figure 5.3.1.1.

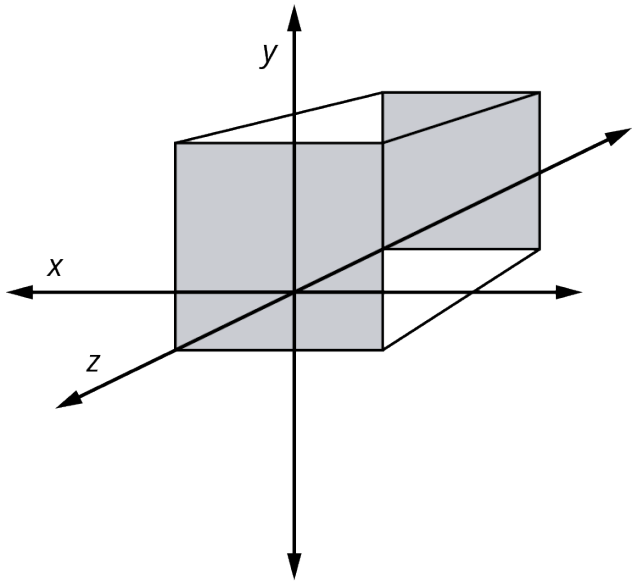


Figure 5.3.1.1: Three-Dimensional Space

Another way to visualize this is by starting with a flat square and extending it upward to form a solid. See Figure 5.3.1.2.

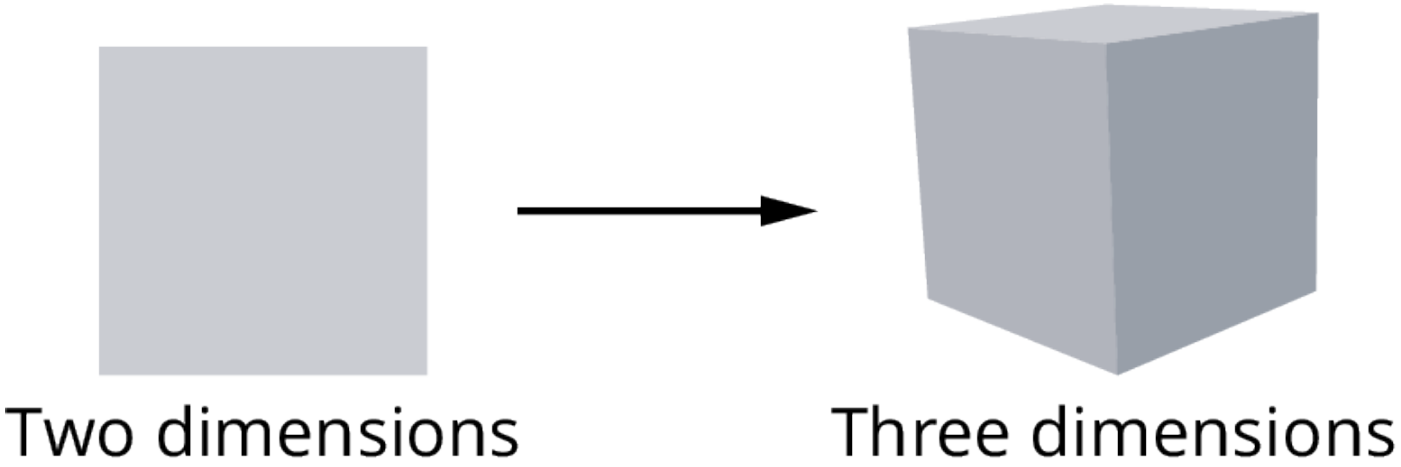


Figure 5.3.1.2: Going from Two Dimensions to Three Dimensions

To study objects in three dimensions, we use formulas for surface area and volume. For example, if you want to cover a box with decorative paper, you need to find the surface area to determine how much paper is required. If you want to know how much water a swimming pool can hold, you need to calculate its volume. In this lesson, we will focus on finding the surface area and volume of right prisms.

Volume

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video explains the idea of 3D solids and highlights the difference between volume and surface area.

Right Prisms

A right prism is a particular type of three-dimensional object. It has a polygon-shaped base and a congruent, regular polygon-shaped top, which are connected by the height of its lateral sides, as shown in Figure 5.3.1.3. The lateral sides form a right angle with the base and the top. There are rectangular prisms, hexagonal prisms, octagonal prisms, triangular prisms, and so on.

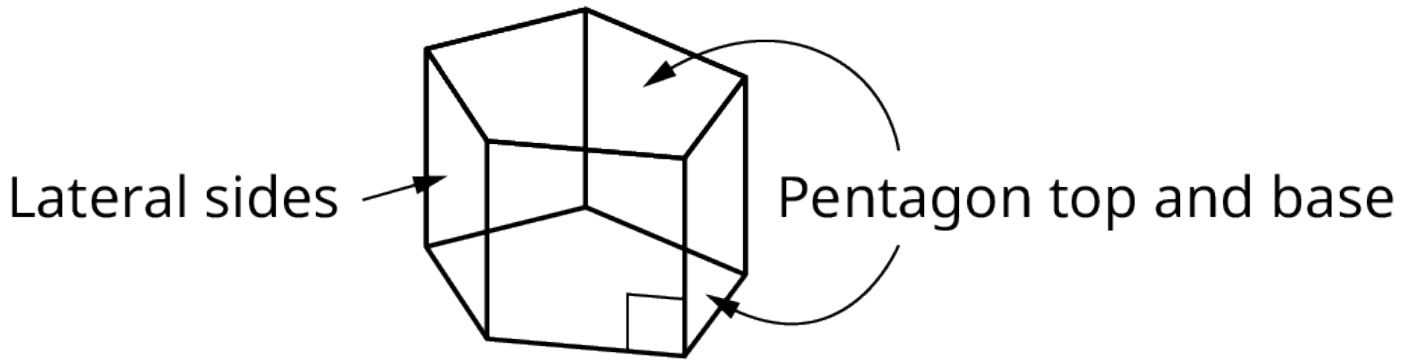


Figure 5.3.1.3: Pentagonal Prism

When working with right prisms, there are three main measurements we can calculate: lateral surface area, total surface area, and volume.

Lateral surface area and total surface area both describe the outside of the prism, but they measure different parts. Lateral surface area includes only the sides of the prism and does not include the top or bottom (the bases). Total surface area includes every outer surface, so it is the lateral surface area plus the area of the two bases.

Volume is different from both of these. Instead of measuring the outside, volume measures the space inside the prism. It tells how much the prism can hold.

In summary, lateral surface area measures just the sides, total surface area measures the entire outside, and volume measures the space inside the solid.

Formulas for Right Prisms

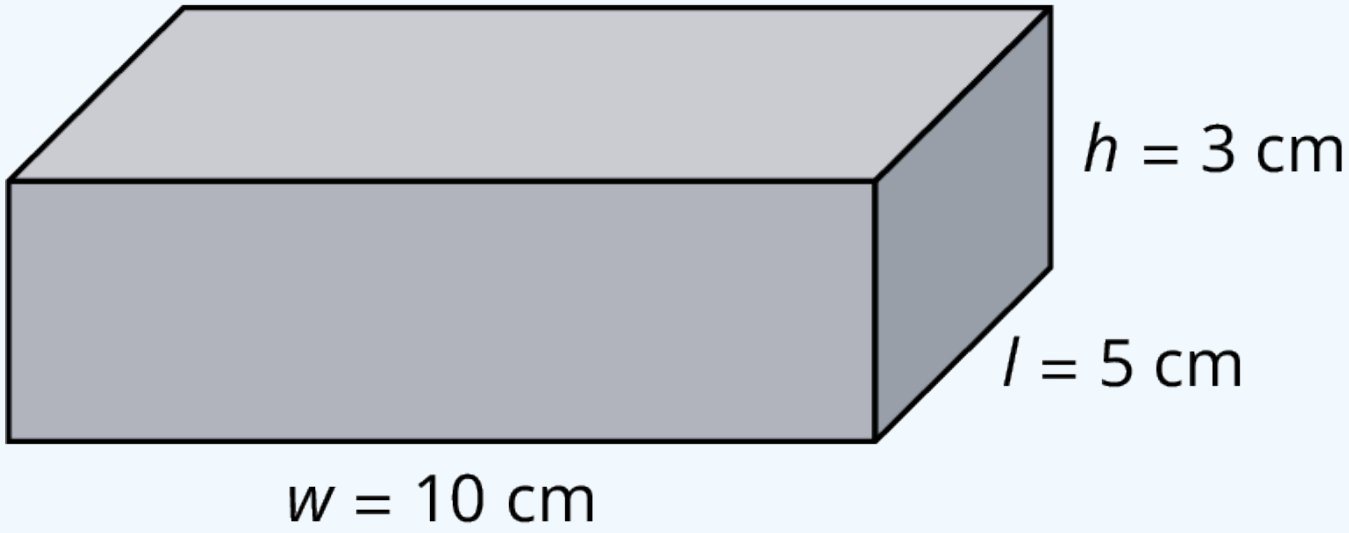
Lateral Surface Area: $L = ph$, where p is the perimeter of the base and h is the height.

Total Surface Area: $T = L + 2A$, where L is equal to the lateral surface area and A is equal to the area of the base.

Volume: $V = A \cdot h$, where A is the area of the base and h is the height.

Example 1

Find the lateral surface area, the total surface area and the volume of the given prism.



Solution

First, identify the dimensions of the rectangular prism.

$l = 5 \text{ cm}, \quad w = 10 \text{ cm}, \quad h = 3 \text{ cm}$

Step 1: Find the perimeter of the base.

$p = 2l + 2w = 2(5) + 2(10) = 10 + 20 = 30 \text{ cm}$

Step 2: Find the lateral surface area.

$L = ph = (30)(3) = 90 \text{ cm}^2$

Step 3: Find the area of the base.

$A = lw = (5)(10) = 50 \text{ cm}^2$

Step 4: Find the total surface area.

$T = L + 2A = 90 + 2(50) = 90 + 100 = 190 \text{ cm}^2$

Step 5: Find the volume.

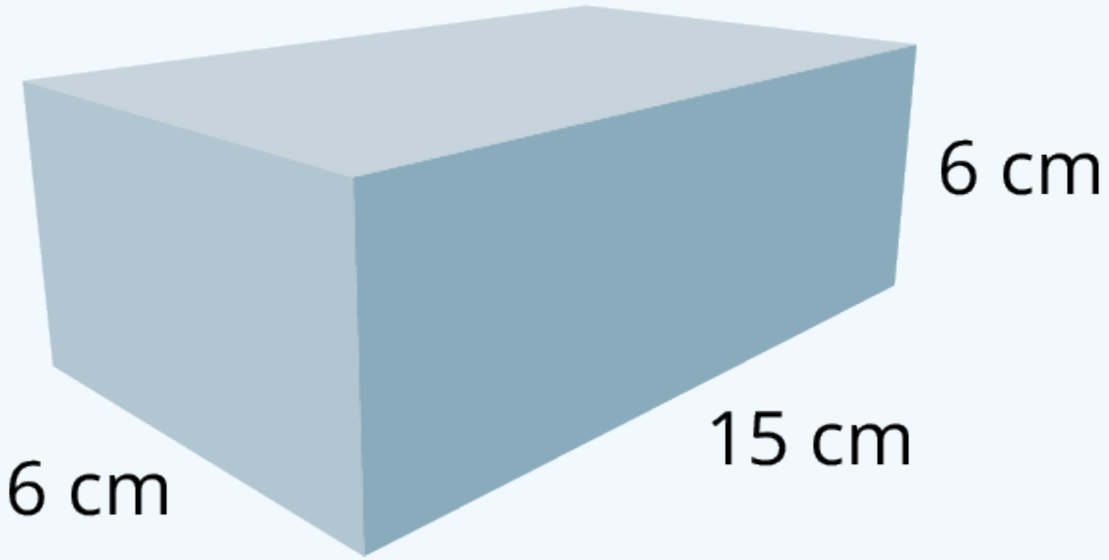
$V = A \cdot h = (50)(3) = 150 \text{ cm}^3$

Final Answers:

$L = 90 \text{ cm}^2, \quad T = 190 \text{ cm}^2, \quad V = 150 \text{ cm}^3$

Exercise

Find the lateral surface area, total surface area and volume of the given prism.



Answer available at the end of this section.

In Figure 5.3.1.4, we have three views of a right hexagonal prism. The regular hexagon is the base and top, and the lateral faces are the rectangular regions perpendicular to the base. We call it a right prism because the angle formed by the lateral sides to the base is 90°.

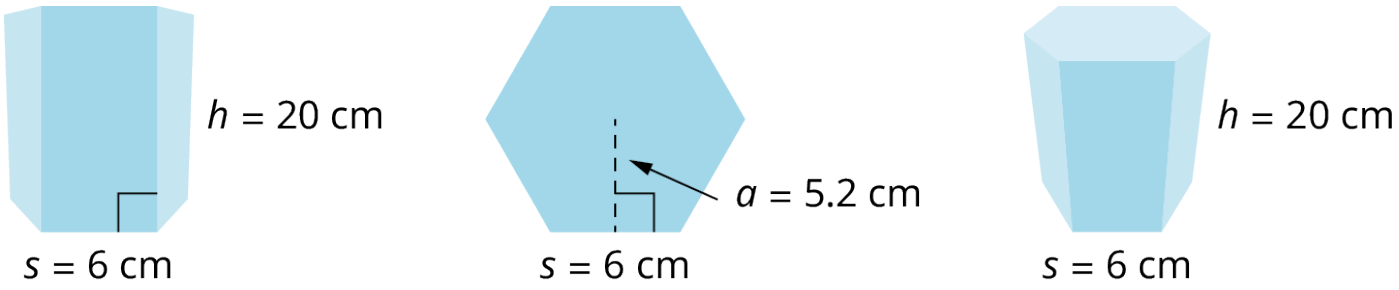


Figure 5.3.1.4: Right Hexagonal Prism

The first image is a view of the figure straight on with no rotation in any direction. The middle figure is the base or the top. The last figure shows you the solid in three dimensions.

Example 2

Find the lateral surface area, total surface area, and the volume of the hexagonal prism given in Figure 5.3.1.4.

Solution

The base is a regular hexagon with side length $s = 6$ cm and height $h = 20$ cm.

Step 1: Find the perimeter of the base.

$p = 6s = 6(6) = 36$ cm

Step 2: Find the lateral surface area.

$L = ph = (36)(20) = 720$ cm^2

Step 3: Find the area of the base.

$A = \frac{3\sqrt{3}}{2}s^2 = \frac{3\sqrt{3}}{2}(6^2) = \frac{3\sqrt{3}}{2}(36) = 54\sqrt{3}$ cm^2

Step 4: Find the total surface area.

$T = L + 2A = 720 + 2(54\sqrt{3}) = 720 + 108\sqrt{3}$ cm^2

Step 5: Find the volume.

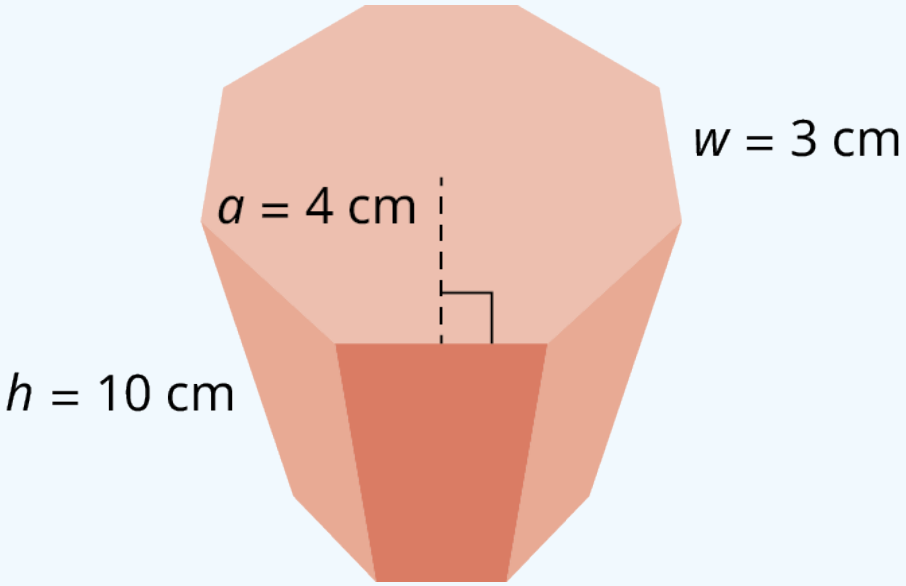
$V = A \cdot h = (54\sqrt{3})(20) = 1080\sqrt{3}$ cm^3

Final Answers:

$L = 720$ cm^2, $T \approx 907.1$ cm^2, $V \approx 1870.6$ cm^3

Exercise

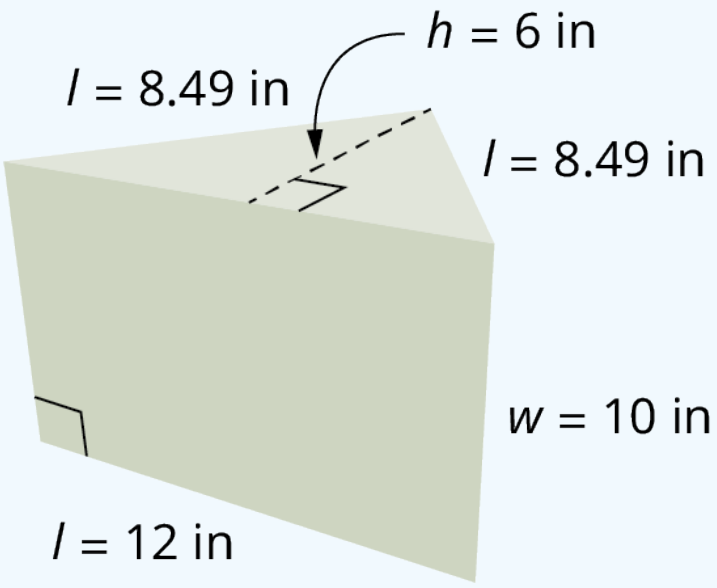
Find the lateral surface area, total surface area, and the volume of the given prism.



Answer available at the end of this section.

Example 3

Find the lateral surface area, total surface area, and volume of the given prism.



Solution

The base is an isosceles triangle. The height of the prism is 10 in.

Step 1: Find the perimeter of the base.

$p = 8.49 + 8.49 + 12 = 28.98$ in

Step 2: Find the lateral surface area.

$L = ph = (28.98)(10) = 289.8$ in^2

Step 3: Find the area of the base.

$A = \frac{1}{2}bh = \frac{1}{2}(12)(6) = 36$ in^2

Step 4: Find the total surface area.

$T = L + 2A = 289.8 + 2(36) = 289.8 + 72 = 361.8$ in^2

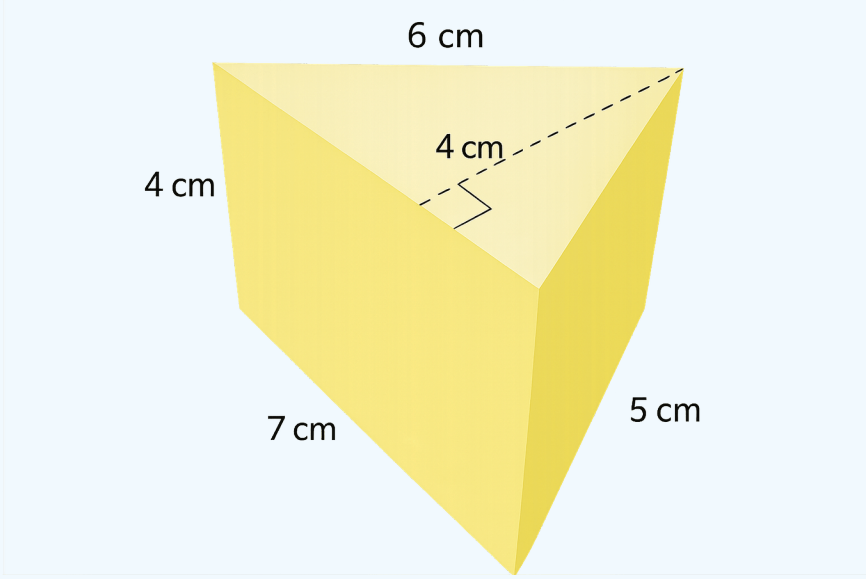
Step 5: Find the volume.

$V = A \cdot h = (36)(10) = 360$ in^3

Final Answers: $L = 289.8$ in^2, $T = 361.8$ in^2, $V = 360$ in^3

Exercise

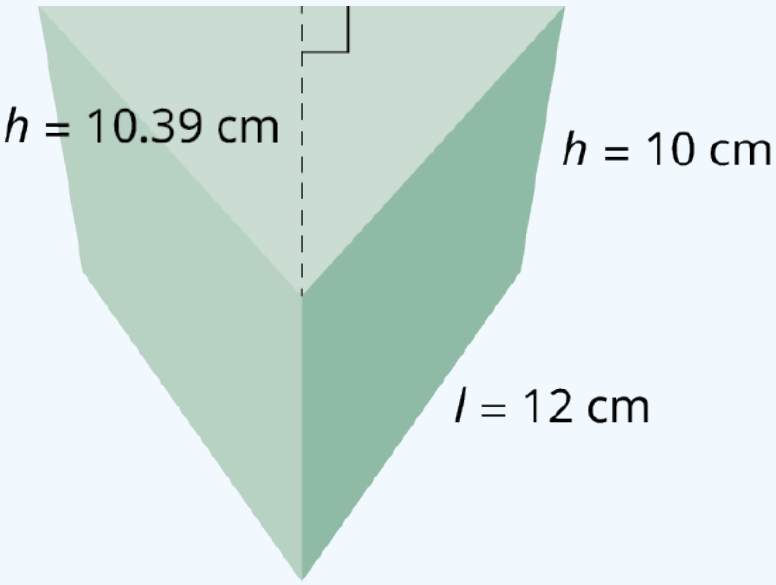
Find the lateral surface area, total surface area, and volume of the given prism.



Answer available at the end of this section.

Example 4

Find the lateral surface area, total surface area, and the volume of the given prism with an equilateral triangle as the base.



Solution

The base is an equilateral triangle with side length 12 cm. The height of the prism is 10 cm.

Step 1: Find the perimeter of the base.

$p = 3s = 3(12) = 36$ cm

Step 2: Find the lateral surface area.

$L = ph = (36)(10) = 360 \text{ cm}^2$

Step 3: Find the area of the base.

$A = \frac{1}{2}bh = \frac{1}{2}(12)(10.39) = 62.34 \text{ cm}^2$

Step 4: Find the total surface area.

$T = L + 2A = 360 + 2(62.34) = 360 + 124.68 = 484.68 \text{ cm}^2$

Step 5: Find the volume.

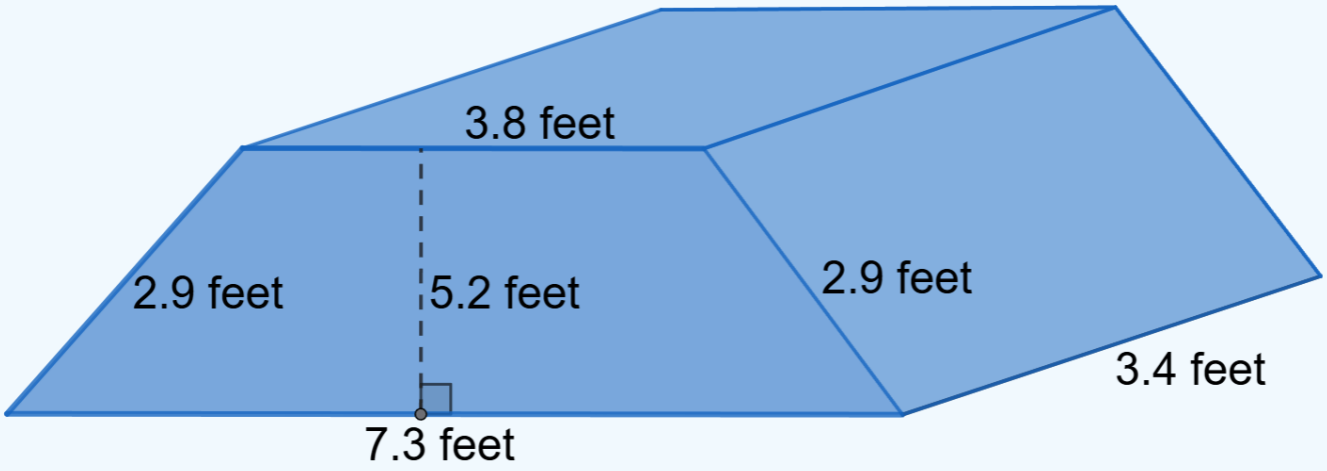
$V = A \cdot h = (62.34)(10) = 623.4 \text{ cm}^3$

Final Answer:

$L = 360 \text{ cm}^2, \quad T = 484.68 \text{ cm}^2, \quad V = 623.4 \text{ cm}^3$

Example 5

Find the lateral surface area, total surface area, and the volume of the right trapezoidal prism.



Solution

The base is a trapezoid with bases 7.3 ft and 3.8 ft, height 5.2 ft, and side lengths 2.9 ft and 2.9 ft. The height of the prism is 3.4 ft.

Step 1: Find the area of the base.

$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(7.3 + 3.8)(5.2) = \frac{1}{2}(11.1)(5.2) = 28.86 \text{ ft}^2$

Step 2: Find the perimeter of the base.

$p = 7.3 + 3.8 + 2.9 + 2.9 = 16.9 \text{ ft}$

Step 3: Find the lateral surface area.

$L = ph = (16.9)(3.4) = 57.46 \text{ ft}^2$

Step 4: Find the total surface area.

$T = L + 2A = 57.46 + 2(28.86) = 57.46 + 57.72 = 115.18 \text{ ft}^2$

Step 5: Find the volume.

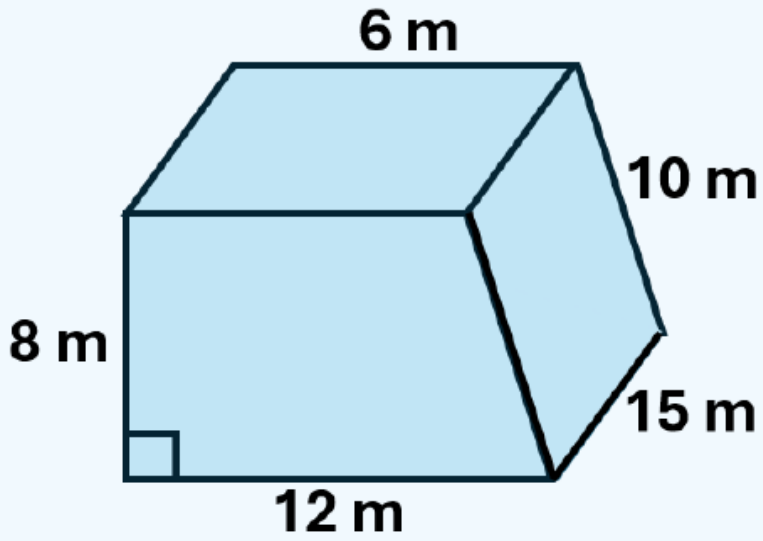
$V = A \cdot h = (28.86)(3.4) = 98.12 \text{ ft}^3$

Final Answer:

$L = 57.46 \text{ ft}^2, \quad T = 115.18 \text{ ft}^2, \quad V = 98.12 \text{ ft}^3$

Exercise

Find the lateral surface area, total surface area, and the volume of the right trapezoidal prism.



Answer available at the end of this section.

Answers

Exercise

$L = 252 \text{ cm}^2, \quad T = 324 \text{ cm}^2, \quad V = 540 \text{ cm}^3$

Exercise

$L = 240 \text{ cm}^2, \quad T = 336 \text{ cm}^2, \quad V = 960 \text{ cm}^3$

Exercise

$L = 72 \text{ cm}^2, \quad T = 100 \text{ cm}^2, \quad V = 56 \text{ cm}^3$

Exercise

$L = 540 \text{ m}^2$
$T = 684 \text{ m}^2$
$V = 1080 \text{ m}^3$

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5.3.2: Volume and Surface Area of Cylinders and Spheres

So far, we have focused on right prisms, where the bases are polygons and the sides are flat faces. Many of the same ideas carry over to other three-dimensional objects, including cylinders and spheres. While the shapes differ, the way we think about surface area and volume remains very similar. Surface area still measures how much space covers the outside of an object, and volume measures how much space is inside.

Cylinders

A cylinder is similar to a prism. Instead of polygonal bases, a right cylinder has two parallel congruent circles. The lateral surface of a right prism meets the base at a 90° angle, and the side of a cylinder also meets the base at a 90° angle.

Right cylinders are common in everyday life, such as soup cans, drink cans, pipes, and air hoses.

In Figure $5.3.2.1$, imagine cutting the cylinder along its side and unrolling it. The curved surface becomes a rectangle. The total surface area includes the two circular bases and the rectangular lateral surface. The length of the rectangle is the circumference of the base, which leads to the surface area formula for a cylinder.

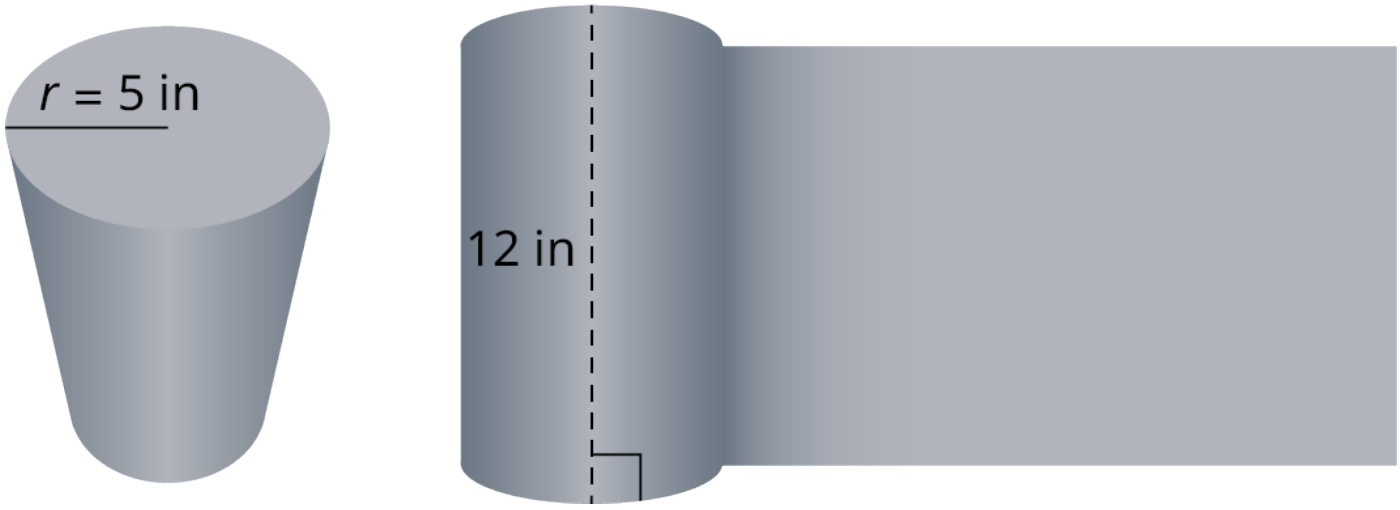


Figure $5.3.2.1$: Right Cylinder

Formulas for Right Cylinders

Lateral Surface Area: $L = Ch$, or $L = 2\pi rh = \pi dh$,

Total Surface Area: $T = 2\pi r(r + h)$, and

Volume: $V = \pi r^2 h$, or $V = \frac{1}{4}\pi d^2 h$

where C is the circumference of the base, r is the radius of the base, d is the diameter of the base, and h is the altitude of the cylinder.

Example 1

Given the cylinder in Figure $5.3.2.1$, which has a radius of 5 inches and a height of 12 inches, find the surface area and the volume.

Solution

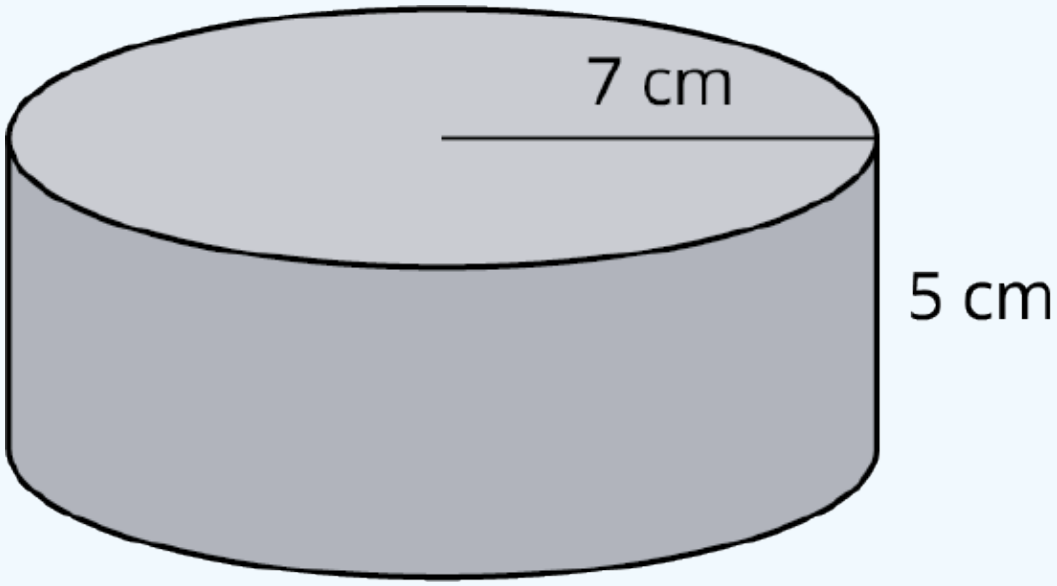
$L = 120\pi \text{ in}^2 \approx 377.0 \text{ in}^2$

$T = 170\pi \text{ in}^2 \approx 534.1 \text{ in}^2$

$V = 300\pi \text{ in}^3 \approx 942.5 \text{ in}^3$

Exercise

Find the surface area and volume of the given cylinder.



Answer available at the end of this section.

Cylinder Volume and Surface Area

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to find the volume and surface area of a cylinder.

Spheres

A sphere is a three-dimensional object in which every point on the surface is the same distance from the center. Unlike prisms and cylinders, a sphere has no edges, faces, or flat surfaces. Because of this, we cannot “unwrap” a sphere the way we can with a cylinder. Instead, we use formulas to find its surface area and volume. Spheres are common in everyday life, such as balls, globes, and bubbles.

Formulas for Spheres

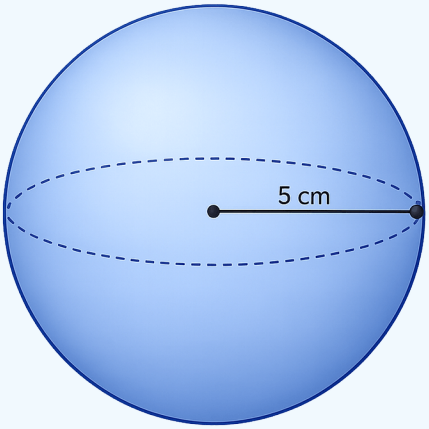
Surface Area: $A = 4\pi r^2$, or $A = \pi d^2$ and

Volume: $V = \frac{4}{3}\pi r^3$, or $V = \frac{1}{6}\pi d^3$

where r is the radius and d is the diameter of the sphere.

Example 2

Find the surface area and volume of the given figure.



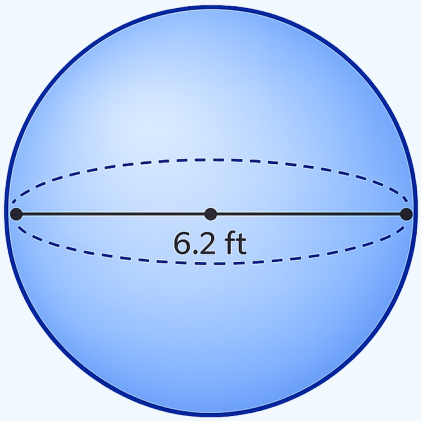
Solution

$A = 4\pi(5)^2 = 100\pi = 314.2 \text{ cm}^2$

$V = \frac{4}{3}\pi(5)^3 = 523.6 \text{ cm}^3$

Exercise

Find the surface area and volume of the given figure.



Answer available at the end of this section.

Find the Surface Area of a Sphere

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to find the surface area of a sphere.

Volume of a Sphere

This interactive element is not available in this format. Visit the link or scan the QR code to access it online.

View this YouTube Video online

This video demonstrates how to find the volume of a sphere.

Example 3

Find the surface area and volume of a hemisphere that has a radius of 12 ft.

Solution

Since a hemisphere is half of a sphere, so we find the surface area and volume of the sphere and divide by 2.

$A_{\text{sphere}} = 4\pi(12)^2 = 576\pi$

$A = \frac{576\pi}{2} = 288\pi \approx 904.8 \text{ ft}^2$

$V_{\text{sphere}} = \frac{4}{3}\pi(12)^3 = 2304\pi$

$V = \frac{2304\pi}{2} = 1152\pi \approx 3619.1 \text{ ft}^3$

Exercise

Find the surface area and volume of a hemisphere that has a diameter of 7.2 in.

Answer available at the end of this section.

Answers

Exercise

$L = 219.9 \text{ cm}^2$
$T = 527.8 \text{ cm}^2$
$V = 769.7 \text{ cm}^3$

Exercise

$A = 85.0 \text{ ft}^2$
$V = 73.6 \text{ ft}^3$

Exercise

$A = 81.4 \text{ in}^2$
$V = 97.7 \text{ in}^3$

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5.3.3: Applications of Volume and Surface Area

In many technical and workplace settings, three-dimensional measurements are used to determine how much material is needed, how much a container can hold, or how much surface needs to be covered. Volume is used when filling or storing materials, such as concrete, water, or fuel. Surface area is used when covering or coating objects, such as painting tanks or lining containers. In this section, you will apply formulas for volume, lateral surface area, and total surface area of right prisms, cylinders, and spheres to solve real-world problems.

Applications Involving Prisms

Example 1

A construction crew is pouring a rectangular concrete slab for a small equipment pad. The slab measures 12 ft long, 8 ft wide, and 0.5 ft thick. How many cubic feet of concrete are needed?

Solution

$V = lwh$

$V = 12 \times 8 \times 0.5$

$V = 48 \text{ ft}^3$

The crew needs 48 cubic feet of concrete.

Exercise

A landscaping crew is building a raised garden bed and needs to fill it with soil. The bed measures 15 ft long, 10 ft wide, and 0.4 ft deep. How many cubic feet of soil are needed to fill the bed?

Answer available at the end of this section.

Example 2

A water treatment facility is installing a rectangular settling tank. The tank measures 18 ft long, 12 ft wide, and 0.5 ft deep. The volume of the tank is measured in cubic yards. How many cubic yards of water can the tank hold?

$\text{Note: } 1 \text{ yd}^3 = 27 \text{ ft}^3$

Solution

First find the volume in cubic feet.

$V = lwh$

$V = 18 \times 12 \times 0.5$

$V = 108 \text{ ft}^3$

Now convert to cubic yards.

$\frac{108}{27} = 4$

The tank can hold $4\ \text{yd}^3$ of water.

Exercise

A farm supply company is filling a rectangular storage bin with feed. The bin measures 27 ft long, 9 ft wide, and 0.5 ft deep. Feed is measured in cubic yards. How many cubic yards of feed are needed to fill the bin?

Note: $1\ \text{yd}^3 = 27\ \text{ft}^3$

Answer available at the end of this section.

Example 3

A contractor is installing metal siding on the sides of a rectangular storage shed. The shed is 20 ft long, 12 ft wide, and 10 ft tall. The roof and floor will not be covered. How many square feet of siding are needed?

Solution

The lateral surface area includes only the four side walls.

$L = 2lh + 2wh$

$L = 2(20)(10) + 2(12)(10)$

$L = 400 + 240$

$L = 640\ \text{ft}^2$

The contractor needs 640 square feet of siding.

Exercise

A water treatment tank has a rectangular shape with dimensions 18 ft long, 10 ft wide, and 8 ft tall. The inside walls need to be coated, but the bottom and top are not included. How many square feet of coating are needed?

Answer available at the end of this section.

Example 4

A manufacturer is packaging a rectangular crate and needs to wrap the entire outside with protective material. The crate measures 16 ft long, 10 ft wide, and 6 ft tall. How many square feet of material are needed to cover the entire crate?

Solution

The total surface area includes all six faces.

$SA = 2lw + 2lh + 2wh$

$SA = 2(16 \times 10) + 2(16 \times 6) + 2(10 \times 6)$

$SA = 2(160) + 2(96) + 2(60)$

$SA = 320 + 192 + 120$

$SA = 632 \text{ ft}^2$

The manufacturer needs 632 square feet of material.

Exercise

A storage container measures 14 ft long, 9 ft wide, and 5 ft tall. The entire outside of the container will be painted. How many square feet of paint coverage is needed?

Answer available at the end of this section.

Example 5

A masonry crew is building a rectangular retaining wall that measures 24 ft long, 1.5 ft thick, and 6 ft tall. The front face and the two end faces will be covered with brick, but the back face, top, and bottom will not be covered. How many square feet of brick are needed?

Solution

Front face:

$A = 24 \times 6 = 144 \text{ ft}^2$

Two end faces:

$A = 2(1.5 \times 6)$

$A = 2(9) = 18 \text{ ft}^2$

Now add the areas that will be covered.

$144 + 18 = 162 \text{ ft}^2$

The crew needs 162 square feet of brick.

Exercise

A maintenance crew is lining part of a rectangular drainage channel that measures 30 ft long, 2 ft wide, and 4 ft deep. The bottom and the two long side walls will be lined, but the top and two end faces will not be lined. How many square feet of liner are needed?

Answer available at the end of this section.

Applications Involving Cylinders

Example 6

A technician is filling a cylindrical coolant tank. The tank has a radius of 2 ft and a height of 5 ft. How many cubic feet of coolant can the tank hold? Use $\pi \approx 3.14$.

Solution

$V = \pi r^2 h$

$V = 3.14 \times 2^2 \times 5$

$V = 3.14 \times 4 \times 5$

$V = 62.8 \text{ ft}^3$

The tank can hold 62.8 cubic feet of coolant.

Exercise

A water treatment facility uses a cylindrical storage tank with a radius of 3 ft and a height of 4 ft. How many cubic feet of water can the tank hold? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Example 7

A generator uses a cylindrical fuel tank with a radius of 2.5 ft and a height of 6 ft. Fuel is measured in gallons. How many gallons of fuel can the tank hold? Use $\pi \approx 3.14$.

Note: $1 \text{ ft}^3 = 7.48 \text{ gallons}$

Solution

First find the volume in cubic feet.

$V = \pi r^2 h$

$V = 3.14 \times 2.5^2 \times 6$

$V = 3.14 \times 6.25 \times 6$

$V = 117.75 \text{ ft}^3$

Now convert to gallons.

$117.75 \times 7.48 = 880.77$

The tank can hold approximately 880.77 gallons of fuel.

Exercise

A farm uses a cylindrical tank to store liquid fertilizer. The tank has a radius of 3 ft and a height of 5 ft. How many gallons of fertilizer can the tank hold? Use $\pi \approx 3.14$.

Note: $1 \text{ ft}^3 = 7.48 \text{ gallons}$

Answer available at the end of this section.

Example 8

A storage tank has a cylindrical shape with a radius of 4 ft and a height of 10 ft. The side of the tank will be painted, but the top and bottom will not be painted. How many square feet of surface need to be painted? Use $\pi \approx 3.14$.

Solution

The lateral surface area of a cylinder is:

$L = 2\pi rh$

$L = 2(3.14)(4)(10)$

$L = 251.2 \text{ ft}^2$

The tank requires 251.2 square feet of paint coverage.

Exercise

A cylindrical water pipe has a radius of 1.5 ft and a length of 12 ft. The outside of the pipe will be coated for protection. How many square feet of surface need to be coated? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Example 9

A cylindrical storage tank holds 314 cubic feet of liquid. The radius of the tank is 5 ft. How tall is the tank? Use $\pi \approx 3.14$.

Solution

$V = \pi r^2 h$

Substitute the known values.

$314 = 3.14 \times 5^2 \times h$

$314 = 3.14 \times 25 \times h$

$314 = 78.5h$

Now solve for h.

$\frac{314}{78.5} = h$

$h = 4 \text{ ft}$

The tank is 4 ft tall.

Exercise

A cylindrical fuel tank holds 565.2 cubic feet of fuel. The tank is 6 ft tall. What is the radius of the tank? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Applications Involving Spheres

Example 10

A water treatment facility uses a spherical storage tank with a radius of 3 ft. How many cubic feet of water can the tank hold? Use $\pi \approx 3.14$.

Solution

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}(3.14)(3^3)$

$V = \frac{4}{3}(3.14)(27)$

$V = \frac{4}{3}(84.78)$

$V = 113.04 \text{ ft}^3$

The tank can hold 113.04 cubic feet of water.

Exercise

A spherical fuel float inside a tank has a radius of 4 ft. How many cubic feet of material would be needed to make the float if it were solid? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Example 11

A metal sphere is being coated with a protective layer. The sphere has a radius of 5 ft. How many square feet of material are needed to cover the entire surface? Use $\pi \approx 3.14$.

Solution

$SA = 4\pi r^2$

$SA = 4(3.14)(5^2)$

$SA = 4(3.14)(25)$

$SA = 4(78.5)$

$SA = 314 \text{ ft}^2$

The sphere requires 314 square feet of coating.

Exercise

A spherical buoy is painted for visibility. The buoy has a radius of 6 ft. How many square feet of paint coverage are needed? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Example 12

A spherical tank stores water with a radius of 2 ft. How many liters of water can the tank hold? Use $\pi \approx 3.14$.

Note: $1 \text{ ft}^3 = 28.32 \text{ liters}$

Solution

First find the volume in cubic feet.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}(3.14)(2^3)$

$V = \frac{4}{3}(3.14)(8)$

$V = \frac{4}{3}(25.12)$

$V = 33.49 \text{ ft}^3$

Now convert to liters.

$33.49 \times 28.32 = 948.44$

The tank can hold approximately 948.44 liters of water.

Exercise

A hollow metal sphere is being filled with oil. The sphere has a radius of 3 ft. How many liters of oil can it hold? Use $\pi \approx 3.14$.

Note: $1 \text{ ft}^3 = 28.32 \text{ liters}$

Answer available at the end of this section.

Example 13

A maintenance crew is painting the outside of a spherical water tank. The tank has a diameter of 9 ft. One gallon of paint covers 275 ft^2. How many gallons of paint are needed to cover the tank? Use $\pi \approx 3.14$.

Solution

First find the radius.

$r = \frac{9}{2} = 4.5 \text{ ft}$

Now find the surface area.

$SA = 4\pi r^2$

$SA = 4(3.14)(4.5^2)$

$SA = 4(3.14)(20.25)$

$SA = 254.34 \text{ ft}^2$

Now find the number of gallons of paint needed.

$\frac{254.34}{275} \approx 0.92$

Since paint must be purchased by the gallon, round up.

The crew needs 1 gallon of paint.

Exercise

A facilities worker is coating the outside of a spherical storage vessel. The vessel has a diameter of 11 ft. One gallon of coating covers $300\ \text{ft}^2$. How many gallons of coating are needed? Use $\pi \approx 3.14$.

Answer available at the end of this section.

Answers

Exercise
$60\ \text{ft}^3$

Exercise
$4.5\ \text{yd}^3$

Exercise
$448\ \text{ft}^2$

Exercise
$412\ \text{ft}^2$

Exercise
$300\ \text{ft}^2$

Exercise
$113.04\ \text{ft}^3$

Exercise
1057.43 gallons

Exercise
$113.04\ \text{ft}^2$

Exercise
5.5 ft

Exercise
$267.95\ \text{ft}^3$

Exercise
$452.16\ \text{ft}^2$

Exercise
3203.74 liters

Exercise
2 gallons

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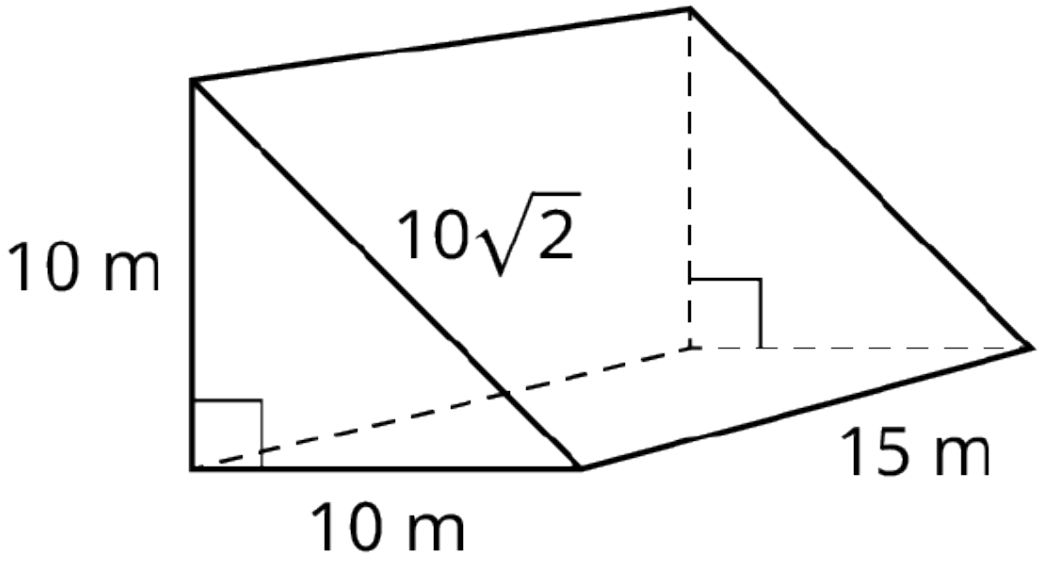
5.3.4: Volume and Surface Area Practice

The following set of exercises provides practice for the concepts covered in Module 5.3: Volume and Surface Area. Complete these problems to reinforce your understanding and build fluency with the skills introduced in this module.

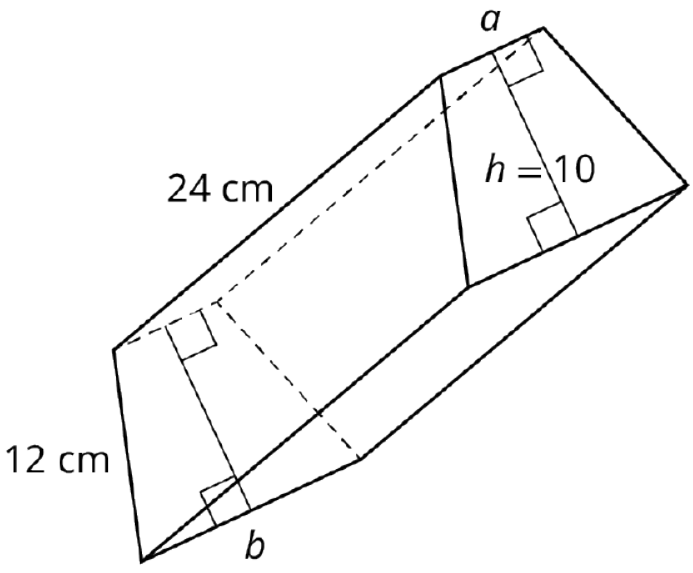
Section 5.3.1 - Volume and Surface Area of Prisms

For each prism, find the volume, lateral surface area, and total surface area.

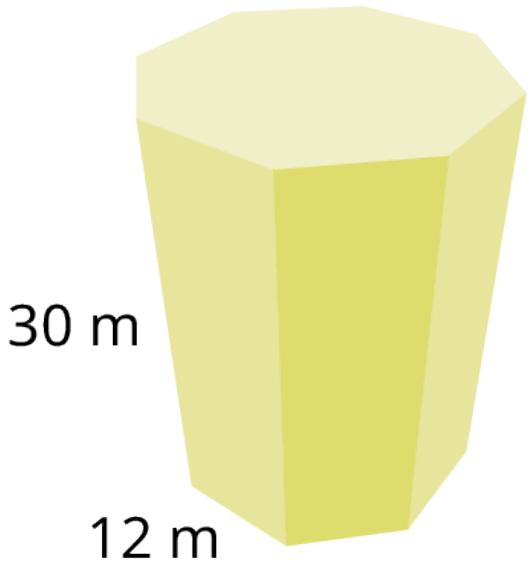
1. The right triangular prism shown below.

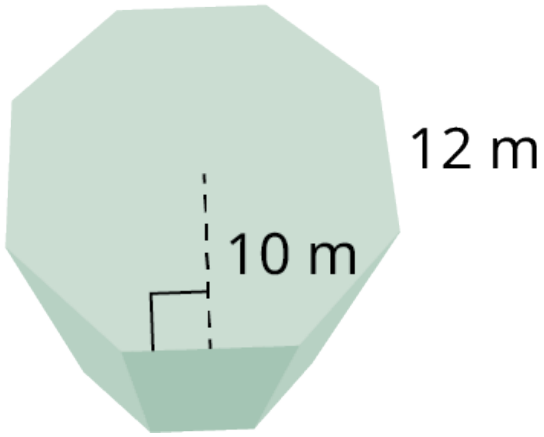


2. The trapezoidal prism shown below.



3. The octagonal prism shown below.





Section 5.3.2 - Volume and Surface Area of Cylinders and Spheres

For each of cylinder, find the volume, lateral surface area, and total surface area.

4. A cylinder has radius 12.2 in and height 30 in.

5. A cylinder has diameter 11.4 m and height 4.4 m.

For each sphere, find the volume and surface area.

6. A sphere has radius 6 cm.

7. A sphere has diameter 14 ft.

Section 5.3.3 - Applications Involving Volume and Surface Area

Find the requested value in each problem.

9. A rectangular concrete slab measures 14 ft long, 10 ft wide, and 0.5 ft thick. How many cubic feet of concrete are needed?

10. A storage bin measures 18 ft long, 12 ft wide, and 0.5 ft deep. Soil is measured in cubic yards. How many cubic yards of soil are needed? Note: $1\ \text{yd}^3 = 27\ \text{ft}^3$

11. A rectangular room measures 20 ft by 12 ft by 9 ft. The four walls will be painted. How many square feet of wall area need to be painted?

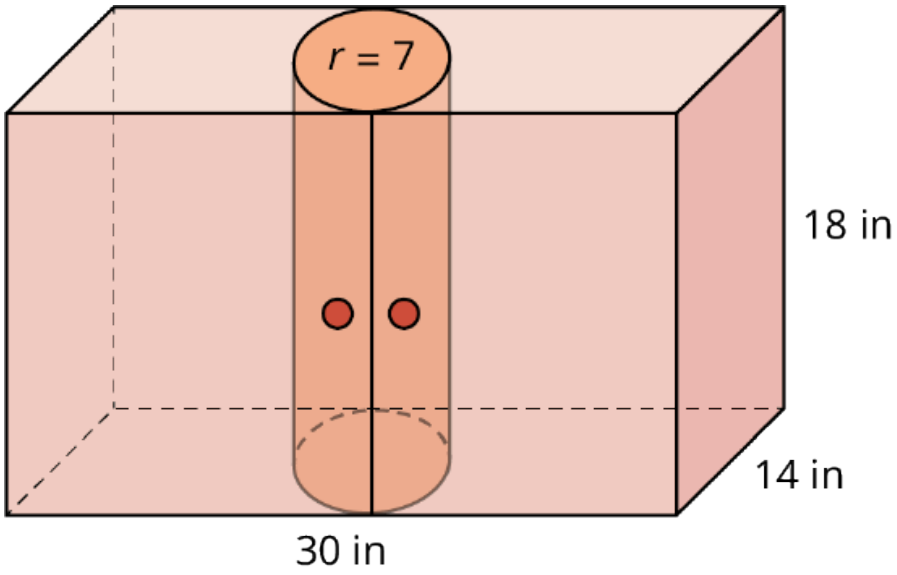
12. A cylindrical tank has a radius of 3 ft and a height of 8 ft. How many cubic feet of liquid can it hold?

13. A cylindrical fuel tank has a radius of 2.5 ft and a height of 6 ft. How many gallons of fuel can it hold? Note: $1\ \text{ft}^3 = 7.48\ \text{gallons}$

14. A cylindrical tank has a volume of 314 cubic feet and a radius of 5 ft. How tall is the tank?

15. A spherical tank has a diameter of 9 ft. One gallon of paint covers $275\ \text{ft}^2$. How many gallons of paint are needed?

16. You have remodeled your kitchen and the exhaust pipe above the stove must pass through an overhead cabinet as shown in the figure. Find the volume of the remaining space in the cabinet.



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