# FSW MATHEMATICAL THINKING MGF 1130



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# FSW Mathematical Thinking MGF 1130

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Detailed Licensing

# Licensing

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# CHAPTER OVERVIEW

## 1: SET AND LOGIC

It is natural for us to classify items into groups, or sets, and consider how those sets overlap with each other. We can use these sets understand relationships between groups, and to analyze survey data.


1.1: Basics of Sets and Subsets
1.2: Set Operations- Union, Intersection, and Complement
1.3: Cardinalty and Venn Diagrams with Survey Problems
1.4: Statements, Compound Statements, and Negations
1.5: Truth Tables, Conditionals, and De Morgan's Laws
1.6: Analyzing Deductive Argument
1.7: Fallacies in Reasoning


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# 1.1: Basics of Sets and Subsets



Figure 1.1.1: A spoon, fork, and knife are elements of the set of flatware. (credit: modification of work “Cupofjoy” Wikimedia CC0 1.0 Public Domain)

Georg Cantor, the father of modern set theory, was born in 1845 in Saint Petersburg, Russia, and later moved to Germany as a youth. Besides being an accomplished mathematician, he also played the violin. Cantor received his doctoral degree in Mathematics at the age of 22.

In 1870, at the age of 25 he established the uniqueness theorem for trigonometric series. His most significant work happened between 1874 and 1884, when he established the existence of transcendental numbers (also called irrational numbers) and proved that the set of real numbers are uncountably infinite—despite the objections of his former professor Leopold Kronecker.

Cantor published his final treatise on set theory in 1897 at the age of 52, and was awarded the Sylvester Medial from the Royal Society of London in 1904 for his contributions;

## Sets and Ways to Represent Them

**Definition: Set and Element**

A **set** is a collection of objects or things. The objects or things in a set are called **its elements**. $A$ set is commonly denoted by a capital letter and defined by describing the contents, or by listing the elements of the set, enclosed in braces. For example, set $A$, containing the elements 2, 3, and 4, can be written as

$$A = \{2, 3, 4\}$$

Think back to your kitchen organization. If the drawer is the set, then the forks and knives are elements in the set. Sets can be described in a number of different ways: by roster, by set-builder notation, by interval notation, by graphing on a number line, and by Venn diagrams. Sets are typically designated with capital letters.

1. **The word Description Method:** Use a sentence to define a set.
2. **The Roster Method:** Begin with a left brace, {, list each element of the set *only once*, and then end with a right brace, }.
3. **The Set-Builder Method:** A combination of the verbal and roster methods using a **variable** such as $x$.

### Roster Notation

The simplest way to represent a set with only a few members is the **roster** (or *listing*) **method**, in which the elements in a set are listed, enclosed by curly braces, and separated by commas. For example, if $B$ is the set of even numbers less than 10, then

$$B = \{2, 4, 6, 8\}$$

This method is best used when the elements of the set are few and can be listed clearly.

### Set Builder Notation

A shorthand way to write sets is through the use of set builder notation, which is a verbal description or formula that defines the set. For example, the set of all lowercase letters of the English alphabet, $A$ written in set builder notation is:

$$A = \{x \mid x \text{ is a lowercase letter of the English alphabet}\}$$

This is read as, “Set $A$ the set of all elements x such that x is a lowercase letter of the English alphabet.”

Set-builder notation is often used for sets in which the number of elements is too large or impossible to write using the roster method.

**Example 1.1.1: Name the Set**

What is the name of the given set below, and what are its elements?

$$C = \{1, 6, 2, 5\}$$

**Answer**

The name of the set is "$C$" and its elements are $1, 6, 2,$ and $5$.

**Your Turn 1.1.1: Writing a Set Using the Roster or Listing Method**

Write a set consisting of your three favorite sports and label it with a capital $S$.

**Answer**

There are multiple possible answers depending on what your three favorite sports are, but any answer must list three different sports separated by commas, such as the following:

$$S = \{\text{hockey, basketball, soccer}\}$$

**Example 1.1.2: Different way to Represent Set**

What is the name of the given set below and its form? What are its elements? Also, write the given set in roster notation as a set builder and vice versa.

1. $C$ = {x | x is a counting number less than $5$}
2. $K$ = $\{17, 19, 21, 23, 25, 27, 29, 31, 33, 35\}$
3. Write the set of letters in the word "messenger" using the roster (listing) method.

**Answer**

1. $C$ is in set builder form. The name of the set is "$C$" and its elements are $1, 2, 3,$ and $4$. In roster notation:

$$G = \{1, 2, 3, 4\}$$

2. $K$ is in set builder form. $K$= {x | x is an odd number between $17$ to $35$ inclusive }
3. The given set is in set word description form. It has no name yet, but you can give it a name, let's say it is $Y$. So

$$Y = \{\text{e, g, m, n, r, s}\}$$

**Your Turn 1.1.2: Roster and Set Builder From**

We are looking at sets written out in two different formats. Match the name of the format to each of the following sets.

$\{x \in A \mid \text{x is even}\}$ Select an answer

$\{\text{dog, cat, fish, bearkat}\}$ Select an answer

$\{$ animal | the animal makes a good pet $\}$ Select an answer

$\{4, 6, 8, 24, 78\}$ Select an answer

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## Universal Set

The **universal set** is the set that contains **all elements being considered** in a particular discussion or problem. It provides the overall "context" for all other sets.

For example, if we are studying students' majors in a class, the universal set could be **all students in that class**. All other sets (such as Biology majors or Math majors) are subsets of the universal set.

**Definition: Universal Set**

The **universal set,** usually denoted by the capital letter $U$, is a set that establishes context. It contains all the elements within the specific context that is being examined. In Venn diagrams, the universal set is shown as a **rectangle** enclosing all circles (subsets). More example of universal set.

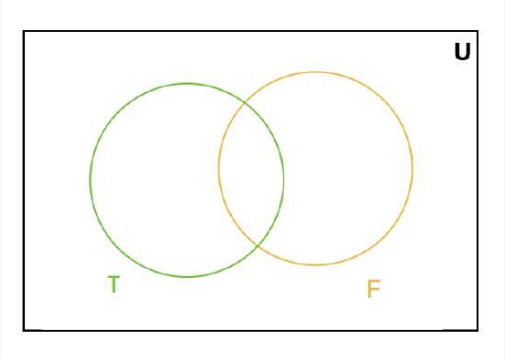
a. If we were discussing searching for books, the universal set might be all the books in the library.
b. If we were grouping your Facebook friends, the universal set would be all your Facebook friends.
c. If you were working with sets of numbers, the universal set might be all whole numbers, all integers, or all real numbers

**Example 1.1.3: Universal Set**

Consider the set $T$ = {x | x is a student at FSW who has a TikTok account} and $F$ = {y | y is a student at FSW who has a Facebook account}. Identify the universal set $U$.

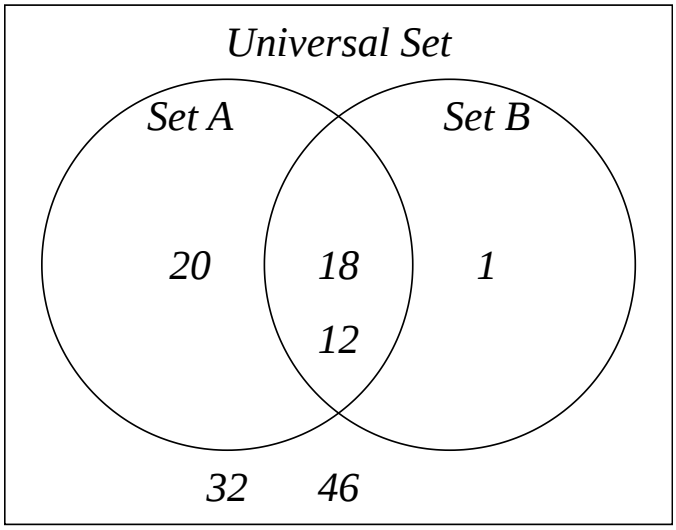
**Answer**

Example: $U$ = {s : s is a student at FSW Campus}




**Your Turn 1.1.3: Identify Universal Set**

Use the Venn diagram shown below to find the universal set $U$.




List all the elements in set $U$. Use commas to separate the numbers in the list.

Set $U = \{$ _____ $\}$

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## Well-defined sets

Let's suppose $B = \{y \mid y \text{ is a food that is tasty}\}$. The problem with this set is that, depending on personal preference, the elements of $B$ may be different for different people. I did not think onions were "tasty", so I would not include them in $B$. However, my son thinks onions are "tasty," and therefore, he would include onions in $B$. Therefore, we must ensure that we clearly specify a set so that, in every situation, the elements of the set remain the same. This is the idea behind **"well-defined".**

Let $V = \{x \mid x \text{ is a vowel in the alphabet}\}$. It should be clear that this set $V$ is not well-defined because the description of the elements does not specify which alphabet the vowels are coming from (other languages have vowels), despite the fact that the set was described in English. We can use the idea of a Universal Set to help restrict the context in which the elements are considered.

**Definition: Well-Defined**

A set is well-defined if, given any object, it is clear whether the object is or is not an element of the set.

**Example 1.1.4: Well Defined Set**

Determine if the given sets are well-defined

1. $B = \{y \mid y \text{ is a food that is tasty}\}$
2. $W = \{w \mid w \text{ is a Grammy Award-winning actress prior to } 2021\}$
3. $M = \{m \mid m \text{ is a fast car}\}$
4. $v = \{ v \mid v \text{ is a past Vice President of The United States}\}$

**Answer**

1. $B$ is not well-defined since different people may consider different foods tasty, and therefore $B$ may contain different elements for different individuals.
2. $W$ is a well-defined set because an actress was either a Grammy Award-winner or not a Grammy Award-winner prior to $2021$.
3. $M$ is not well-defined since the concept of what is "fast" for a car could vary. Is a $1968$ Volkswagen Beetle fast? What is fast?
4. $V$ is a bit tricky because, depending on the time frame, a person may or may not be included in the set. We must consider that at the time we make the determination, the list of past Vice Presidents of the United States may be finalized; therefore, the set is well-defined, even though its elements will most likely change in the future.

**Your Turn 1.1.4: Identify Well Defined Sets**

Which of the following represent a well-defined set (Choose all that apply)

- ☐ The set of Koala bears in the San Antonio zoo.
- ☐ The set of songs that you can dance to.
- ☐ The set of letters in the English alphabet.
- ☐ None of these

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**Definition: Belongs or Does not Belong**

$$\in \equiv \text{Belongs to}$$
$$\notin \equiv \text{Does not belong to}$$

For example, for set $A = \{1, 2, 3, 4\}$, $2$ is an element of the set, we'd write $2 \in A$. And $5$ is an element of the set, we'd write $5 \notin A$.

**Example 1.1.5: Belongs to or Does not Belongs to**

True or False?

1. $i \in \{a, e, i, o, u\}$
2. $y \notin \{a, e, i, o, u\}$

3. $\{i\} \in \{a, e, i, o, u\}$

**Answer**

1. True, because $i$ is in the set.
2. True, because $y$ is not in the set.
3. False, because $\{i\}$ is not in the set. Note $\{i\}$ and $i$ are different

**Your Turn 1.1.5: Belongs To or Does not Belongs To**

Let A = {1, 2, 3, 5, 7, 8}. Indicate if the statement is true or false.

$1 \in A$ ○ True ○ False

$3 \in A$ ○ True ○ False

$6 \in A$ ○ True ○ False

$8 \in A$ ○ True ○ False

$9 \in A$ ○ True ○ False

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For larger sets that have a natural ordering, an ellipsis may be used to indicate that the pattern continues. It is common practice to list the first three elements of a set to establish a pattern, write the ellipsis, and then provide the last element. Consider the set of all lowercase letters of the English alphabet, $A$. This set can be written symbolically

$$A = \{a, b, c, \ldots, x, y, z\}$$

The sets we have been discussing so far are finite sets. They all have a limited or fixed number of elements. We also use an ellipsis to indicate that the pattern continues for infinite sets, which have an unlimited number of elements.

For example, in set theory, the **set of natural numbers**, which is the set of all positive counting numbers, is represented as

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

Notice that for this set, there is no element following the ellipsis. This is because there is no largest natural number; you can always add one more to get to the next natural number. Because the set of natural numbers grows without bound, it is an infinite set.

**Set of integers**

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

The following example shows how to express a set builder using the roster (listing) method.

$$B = \{x | x \in \mathbb{N} \text{ and } 12 \leq x \leq 18\}$$
$$B = \{12, 13, 14, 15, 16, 17, 18\}$$

## Empty Set or Null Set

On January 20, 2021, Kamala Harris was sworn in as the first woman vice president of the United States of America. If we were to consider the set of all women vice presidents of the United States of America prior to January 20, 2021, this set would be known as an empty set; the number of people in this set is 0, since there were no women vice presidents before Harris. The empty set, also called the null set, is written symbolically using a pair of braces, { }, or a zero with a slash through it, $\emptyset$.

**Definition: Empty Set**

A set that contains no elements is called the **empty set** and is notated as { } or $\emptyset$.

**Checkpoint**

The set containing the number $0$, $\{0\}$, is a set with one element in it. It is not the same as the empty set $\{\ \}$, which does not have any elements in it. Symbolically: $\{0\} \neq \{\ \}$.

**Example 1.1.6: Representing the Empty Set Symbolically**

Represent each of the following sets symbolically.

1. $B = \{\ x | x$ is even natural number and $x \leq 1\}$
2. The set of birds that are also mammals.
3. $W = \{\ x \mid x$ is a woman and a president of the United States$\}$

**Answer**

1. There is no even natural number that is less than or equal to $1$, so $A = \emptyset$ or $\{\ \}$. These two different symbols for the empty set can be used interchangeably.
2. The set of birds and the set of mammals do not intersect, so the set of birds that are also mammals is empty, and we can represent it symbolically as $\emptyset$ or $\{\ \}$.
3. This set does not have any elements. So W = $\{\ \}$ or $W = \emptyset$.

**Your Turn 1.1.6:Idenfity Empty Sets**

Which of the following describe an **empty set**? (Choose all that apply).

- ☐ The set of whole numbers greater than 10 and less than 5.
- ☐ The set of real numbers whose square is negative.
- ☐ The set of animals that live in the ocean.
- ☐ None of these

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## Cardinality of a Set

Oftentimes, we are interested in the number of items (elements) in a set. This is called the cardinality of the set.

**Definition: Finite Set**

A set is a finite set if it has a finite number of elements. Any set that is not finite is an infinite set.

**Definition: Cardinality of the Set**

The number of elements in a finite set is the cardinality of that set.

The cardinality of the set $A$ is often notated as $|A|$ or $n(A)$.

**Example 1.1.7: Find the Cardinality of Sets**

1. Let $B = \{2, 4, 6, 8\}$. What is the cardinality of $B$?
2. What is the cardinality of $P$ = the set of English names for the months of the year?
3. If $W$ = {Sun, Mon, Tue, Wed, Thu, Fri, Sat}, what is $n(W)$?
4. $P$ is the set of a polygon with $2$ sides. Find $n(P)$.

**Answer**

1. $n(B) = 4$
2. $n(P) = 12$
3. $n(W) = 7$
4. $n(P) = 0$

**Your Turn 1.1.7: Determining the Cardinality of Sets**

Determine the cardinal number of the following set.

$G = \{55, 57, 59, \ldots, 71\}$

$n(G) =$ ☐

Determine the cardinal number of the following set.

$P = \{x \mid x \text{ is divisible by } 4 \text{ and } 4 \leq x < 48\}$

$n(P) =$ ☐

Find the cardinal number of the following set.

$H = \{5, 5, 5, 5, 5\}$

$n(H) =$ ☐

Find the cardinal number of the given set.

$B = \{ x \mid x \in \mathbb{N} \text{ and } 4 \leq x \leq 17 \}$

$n(B) =$ ☐

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## Equal Versus Equivalent Sets

When speaking or writing, we tend to use equal and equivalent interchangeably, but there is an important distinction between their meanings. Consider a new Ford Escape Hybrid and a new Toyota Rav4 Hybrid. Both cars are hybrid electric sport utility vehicles; in that sense, they are equivalent. They will both get you from place to place in a relatively fuel-efficient way. In this example, we are comparing the single-member set {Toyota Rav4 Hybrid} to the single-member set {Ford Escape Hybrid}. Since these two sets have the same number of elements, they are also equivalent mathematically, meaning they have the same cardinality. But they are not equal, because the two cars have different looks and features, and probably even handle differently. Each manufacturer will emphasize the features unique to their vehicle to persuade you to buy it; if the SUVs were truly equal, there would be no reason to choose one over the other.

Now consider two Honda CR-Vs that are made with exactly the same parts, on the same assembly line within a few minutes of each other—these SUVs are equal. They are identical to each other, containing the same elements without regard to order, and the only differentiator when making a purchasing decision would be varied pricing at different dealerships. The set {Honda CR-V} is equal to the set {Honda CR-V}. Symbolically, we represent equal sets as $A = B$ and equivalent sets as $A \sim B$.

Now, let us consider a Toyota dealership that has $10$ RAV4s on the lot, $8$ Prii, $7$ Highlanders, and $12$Camrys. There is a one-to-one relationship between the set of vehicles on the lot and the set consisting of the number of each type of vehicle on the lot. Therefore, these two sets are equivalent, but not equal. The set {RAV4, Prius, Highlander, Camry} is equivalent to the set $\{10, 8, 7, 12\}$ because they have the same number of elements.

**Checkpoint**

*If two sets are equal, they are also equivalent, because equal sets also have the same cardinality.*

**Example 1.1.8: Differentiating Between Equivalent and Equal Sets**

Determine if the following pairs of sets are equal, equivalent, or neither.

1. $E = \{\text{hockey, basketball, soccer, tennis, cricket}\}$ and $F = \{\text{fork, spoon, knife, meat thermometer, can opener}\}$
2. The set of vowels in the word happiness and the set of consonants in the word happiness.

**Answer**

1. Sets $E$ and $F$ have a cardinal value of $5$, but the elements in these sets are different. So, the two sets are equivalent, but they are not equal.
2. The set of vowels in the word happiness is $A = \{a, i, e\}$ and the set of consonants in the word happiness is $B = \{h, p, n, s\}$. The cardinal value of these sets is $n(A) = 3$ and $n(B) = 4$ respectively. Because the cardinality of the two sets differs, they are not equivalent. Furthermore, their elements are not identical, so they are not equal either.

**Your Turn 1.1.8: Are the Set Equilavent?**

Are the given sets equivalent?

$A = \{6, 8\}$ and $B = \{e, j\}$

- ○ no
- ○ yes

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## Subsets and Number of Subsets

Sometimes a collection might not contain all the elements of a set. For example, Chris owns three Madonna albums. While Chris's collection is a set, we can also say it is a **subset** of the larger set of all Madonna albums.

**Definition: Subsets and Proper Subsets**

A **subset** of a set $A$ is another set that contains only elements from the set $A$, but may not contain all the elements of $A$.

If $B$ is a subset of $A$, we write $B \subseteq A$

A **proper subset** is a subset that is not identical to the original set – it contains fewer elements.

If $B$ is a proper subset of $A$, we write $B \subset A$

Consider these three sets

$A =$ the set of all even numbers, $\quad B = \{2, 4, 6\}, and \quad C = \{2, 3, 4, 6\}$

Here $B \subset A$ since every element of $B$ is also an even number, so is an element of $A$.

More formally, we could say $B \subset A$ since if $x \in B$, then $x \in A$

It is also true that $B \subset C$.

$C$ is not a subset of $A$, since $C$ contains an element, $3$ , that is not contained in $A$

**Example 1.1.9: Identify the Larger Set**

The set $A = \{1, 3, 5\}$. What is a larger set this might be a subset of?

**Answer**

There are several answers: The set of all odd numbers less than $10$. The set of all odd numbers. The set of all integers. The set of all real numbers.

**Your Turn 1.1.9: Subset or Proper Subset?**

| Proper Subsets |
| --- |
| Choose the appropriate phrase to make the statement true. |
| Let $U = \{a, b, c, d, e, f, g, h, i, j\}$ |

$A = \{a, o, g, i, j\}$
$B = \{a, c, d, e, h, i\}$
$C = \{a, b\}$
$D = \{a, c, d, e, h, i\}$
$E = \{a, e, i\}$

| | | |
|---|---|---|
| $U$ | Select an answer | $C$ |
| $C$ | Select an answer | $A$ |
| $B$ | Select an answer | $U$ |
| $A$ | Select an answer | $C$ |
| $E$ | Select an answer | $D$ |
| $D$ | Select an answer | $B$ |

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$F = \{\text{fork, spoon, knife, meat thermometer, can opener}\}$. Suppose you are preparing to eat dinner, so you pull a fork and a knife from the drawer to set the table. The set $D = \{\text{fork, knife, can opener}\}$ is a subset of set $F$ because every member or element of set $D$ is also a member of set $F$. More specifically, set $D$ is a proper subset of set $F$ because there are other members of set $F$ not in set $D$. This is written as $D \subset F$. The only subset of a set that is not a proper subset of the set would be the set itself.

**Checkpoint**

*The empty set or null, $\emptyset$, is a proper subset of every set, except itself. That is $\emptyset \subset A$.*

Every set is also a subset of itself. That is $\mathbf{B \subseteq B}$

Graphically, sets are often represented as circles. In the following graphic, set $A$ is represented as a circle completely enclosed inside the circle representing set $B$, showing that set $A$ is a proper subset of set $B$. The element $x$ represents an element that is in both set $B$ and $A$.

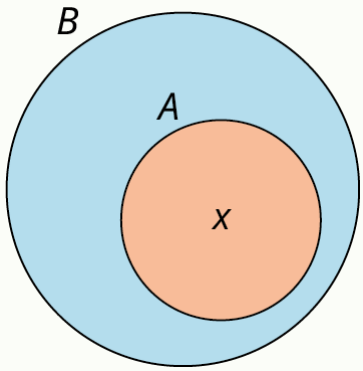



Figure 1.1.1: Subset in Diagram $A \subset B$

**Definition: Number of Subsets and Proper Subsets**

The number of subsets of a finite set $A$ is equal to 2 raised to the power of $n$, where $n$ is the number of elements in set $A$.

$$\text{Number of subsets } = 2^n$$
$$\text{Number of proper subsets} = 2^n - 1$$

**Example 1.1.10: List Subsets**

Set $L$ a set of reading materials available in a shop at the airport, $L = \{\text{newspaper, magazine, book}\}$. List all the subsets of set $L$.

**Answer**

**Step 1:** It is best to begin with the set itself, as every set is a subset of itself. In our example, the cardinality of set $L$ is $n(L) = 3$. There is only one subset of set $L$ that has the same number of elements of set $L = \{$newspaper, magazine, book $\}$.

**Step 2:** Next, list all the proper subsets of the set containing $2$ elements.There are three subsets that each contain two elements: $\{$newspaper, magazine $\}$, $\{$magazine, book $\}$, $\{$newspaper, book $\}$.

**Step 3:** Continue this process by listing all the proper subsets of the set containing one element. There are three subsets that contain one element: $\{$ newspaper $\}$; $\{$magazine$\}$; $\{$book $\}$

**Step 4:** Finally, list the subset containing $0$ elements, or the empty set: $\{\ \}$.

### Your Turn 1.1.10: List all Subsets

List the subsets of the following set. Select the correct choice.

$\{$Han, Luke, Leia$\}$

- ○ $\{\ \}$, $\{$Han$\}$, $\{$Luke$\}$, $\{$Leia$\}$
- ○ $\{\ \}$, $\{$Han, Luke, Leia$\}$
- ○ $\{$Han$\}$, $\{$Luke$\}$, $\{$Leia$\}$
- ○ $\{\ \}$, $\{$Han$\}$, $\{$Luke$\}$, $\{$Leia$\}$, $\{$Han, Luke$\}$, $\{$Han, Leia$\}$, $\{$Luke, Leia$\}$, $\{$Han, Luke, Leia$\}$
- ○ $\{$Han$\}$, $\{$Luke$\}$, $\{$Leia$\}$, $\{$Han, Luke$\}$, $\{$Han, Leia$\}$, $\{$Luke, Leia$\}$
- ○ $\{\ \}$, $\{$Han$\}$, $\{$Luke$\}$, $\{$Leia$\}$, $\{$Han, Luke, Leia$\}$
- ○ $\{$Han, Luke, Leia$\}$

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### Example 1.1.11: Find the Number of Subsets

Find the number of subsets and proper subsets of each of the following sets.

1. The set of top five scorers of all time in the NBA:
   $S = \{$ LeBron James, Kareem Abdul-Jabbar, Karl Malone, Kobe Bryant, Michael Jordan$\}$.
2. The set of the top four bestselling albums of all time:
   $A = \{$Thriller, Hotel California, The Beatles White Album, Led Zeppelin IV$\}$.

**Answer**

1. $n(S) = 5$. So, the total number of subsets is $2^5 = 32$ and the total number of proper subsets is $31$.
2. $n(A) = 4$. So, the total number of subsets is $2^4 = 16$ and the total number of proper subsets is $15$.

### Your Turn 1.1.11: Number of Subsets

Determine the number of subsets and proper subsets of the given set.

$\{$rose, orchid, daisy, lily, carnation$\}$

(a) Number of subsets = _____

(b) Number of proper subsets = _____

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**Example 1.1.12: How Many Colors?**

Suppose you have $11$ tubes of paint. How many distinct color groupings can you make with your paint? (Note: You do want some sort of color, so choosing no colors is not a valid option here. Think how this pertains to subsets and proper subsets.)

**Answer**

Here $n(H) = 11$, because there are $11$ different members in that set of paint. Therefore, the total number of subsets is $2^{11} = 2048$, which includes the empty set as well. But choosing no color is not a viable option; there are $2048 - 1 = 2047$ distinct color groupings we can make.

**Your Turn 1.1.12: Number of Hot Dogs**

You can order a hot dog with some, all, or none of these toppings: ketchup, mustard, onions, relish, cheese, tomato wedges, peppers, and a pickle. How many different hot dogs can be ordered?

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# 1.2: Set Operations- Union, Intersection, and Complement



Figure 1.2.1: A large, multigenerational family contains an intersection and a union of sets. (credit: "Family Photo Shoot Bani Syakur" by Mainur Risyada/Flickr, CC BY 2.0)

The movie *Yours, Mine, and Ours* was originally released in 1968 and starred Lucille Ball and Henry Fonda. This movie, loosely based on a true story, tells the tale of Helen, a widow with eight children, and Frank, a widower with ten children, who subsequently have an additional child together. The movie is a comedy that plays on the interpersonal and organizational struggles of feeding, bathing, and clothing twenty people in one household.

If we consider the set of Helen's children and the set of Frank's children, then the child they had together is the intersection of these two sets, and the collection of all their children combined is the union of these two sets. In this section, we will explore the operations of union and intersection as they relate to two sets.

Commonly, sets interact. For example, you and a new roommate decide to have a house party, and you both invite your circle of friends. At this party, two sets are being combined, although it may turn out that some friends were in both sets.

**Definition: Complement**

The **complement** of a set $A$ contains everything that is *not* in the set $A$, but in the universal set $U$.

The complement is notated $A'$ or $A^c$ or $\overline{A}$.

**Example 1.2.1: Find the Complement**

Consider the sets:

1. Set $U = \{1, 2, 3, ..., 10\}$ and $A = \{2, 4, 6, 8, 10\}$. Find $A^c$.
2. Given the universal set $U$ = {Billie Eilish, Donald Glover, Bruno Mars, Adele, Ed Sheeran} and set $A$ = {Donald Glover, Bruno Mars, Ed Sheeran}, find $A'$.

**Answer**

1. Complement of $A$ is the collection of all elements of $U$ that are not in $A$. So
$$A^c = \{1, 3, 5, 7, 9\}$$
2. The complement of set $A$ is the set of all elements in the universal set $U$ that are not in set $A$. So
$$A' = \{\text{Billie Eilish, Adele}\}$$

**Your Turn 1.2.1:Find the Complement**

Consider the following sets.

$U = \{1, 2, 3, \ldots, 10\}$
$A = \{4, 5, 7, 8\}$
$B = \{4, 5, 6, 7, 8\}$
$C = \{4, 5, 7, 8, 10\}$.

Select the complement $A'$ from the choices below.

- ○ {1,3,4,6,7,8}
- ○ {1,2,3,6,9,10}
- ○ {1,2,3,4,6,9}
- ○ {1,3,5,6,8,10}

{1,2,4,5,7,8}

$\emptyset$

Enter the complement of $B$ in the box below.

$B' = \{$ _____ $\}$

Enter the elements of $C'$ in the box below.

$C' = \{$ _____ $\}$

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Consider the set of digit numbers. Let this be our universal set, $U = \{1, 2, 3, ..., 10\}$. Now, let $A$ set be the subset of $U$ consisting of all the prime numbers in set $U$ and $A = \{2, 3, 5, 7\}$. The complement of set $A$ is, $A^c = \{0, 1, 4, 6, 8, 9\}$. The following Venn diagram represents this relationship graphically.

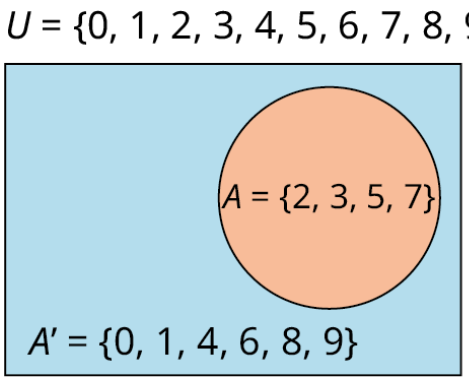



Figure 1.2.2: Complement of a Set

## The Intersection of Sets

The members that the two sets share in common are included in the intersection of the two sets. To be in the intersection of two sets, an element must be in both the first set and the second set. In this way, the intersection of two sets is a logical **AND** statement. Symbolically, $A$intersection $B$ is written as is written $A \cap B$.

**Intersection of a Sets**

The **intersection** of two sets contains only the elements that are in both sets.

The intersection is notated $A \cap B$

More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$

Let us look at Helen's and Frank's children from the movie Yours, Mine, and Ours. Helen's children consist of the set

$$H = \{\text{Colleen, Nick, Janette, Tommy, Jean, Phillip, Gerald, Theresa, Joseph}\}$$

Frank's children are included in the set

$$F = \{\text{Mike, Rusty, Greg, Rosemary, Loise, Susan, Veronica, Mary, Germaine, Joan, Joseph}\}$$

H intersection $F$ is the set of children they had together.

$$H \cap F = \{\text{Joseph}\}$$

Because Joseph is in both set $H$ and set $F$.

## Disjoint Sets

Two sets $A$ and $B$ are disjoint if they have no common element. That is $A$ and $B$ are disjoints if $A \cap B = \emptyset$

### Example 1.2.2: Find the Intersection

1. Set $A = \{1, 3, 5, 7, 9\}$ $B = \{2, 3, 5, 7\}$, Find $A \cap B$.
2. Set $P = \{h, a, p, y\}$ and $Q = \{x, z, w\}$. Find $A$ intersection $B$.

**Answer**

1. The intersection of sets $A$ and $B$ includes the elements that set $A$ and $Q$ have in common: $3$, $5$, and $7$. $A \cap B = \{3, 5, 7\}$
2. The intersection of sets $P$ and $Q$ includes the elements that set $P$ and $Q$ have in common: But there is no common element between $P$ and $Q$, so

$$P \cap Q = \emptyset$$

### Your Turn 1.2.2: Find the Intersection

Let $U$ ={1,2,3,...,10}, $A$ ={1,3,5,7}, $B$ ={1,2,3,4}, and $C$ ={3,4,6,7,9}.

Select $A^c \cap B^c$ from the choices below.

- ○ {6,8,9,10}
- ○ {1,3,6,8}
- ○ {1,4,9,10}
- ○ {1,2,6,10}
- ○ {1,4,5,9}
- ○ $\emptyset$

Hint

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Notice that if sets $A$ and $B$ are disjoint sets, then they do not share any elements in common, $A$ intersection $B$ is the empty set, as shown in the Venn diagram below.

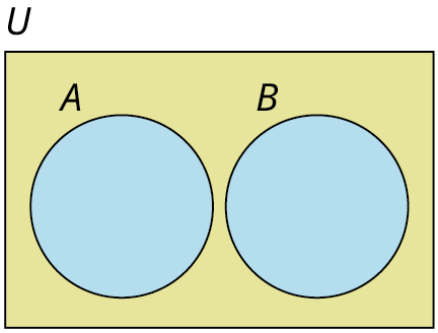



Figure 1.2.3: Disjoint Set Diagram

### Example 1.2.3: Find Intersection Between Two Disjoint Sets

$A = \{0, 2, 4, 6, 8\}$ $B = \{1, 3, 5, 7, 9\}$. Find $A \cap B$.

**Answer**

Because sets $A$ and $B$ are disjoint, they do not share any elements in common. So, the intersection of set $A$ and set $B$ is the empty set.

$A \cap B = \emptyset = \{\}$

**Your Turn 1.2.3: Intersection of Disjoint Sets**

Use the given sets below to find the new set. Enter each number separated by a comma. If there are no numbers in the resulting set, leave the answer blank. Be sure to spell the words correctly!!

$A$ ={Apple,Grape,Mango,Tomato,Watermelon} and
$B$ ={Avocado,Coconut,Orange,Pear,Squash}

$A \cap B$ ={ _____ }

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Notice that if set $A$ is a subset of set $B$, then $A$ intersection $B$ is equal to set $A$, as shown in the Venn diagram below.

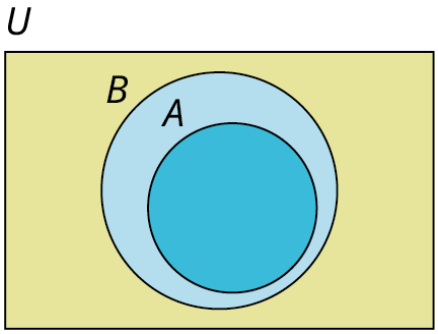



Figure 1.2.4: $A \cap B = A$.

## The Union of Sets

Like the union of two families in marriage, the union of two sets includes all the members of the first set and all the members of the second set. To be in the union of two sets, an element must be in the first set, the second set, or both. In this way, the union of two sets is a logical inclusive OR statement. Symbolically, $A$ union $B$ is written as $A \cup B$.

**Union of two Sets**

The **union** of two sets contains all the elements contained in either set (or both sets).

The union is notated $A \cup B$

More formally, $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both)

Let us consider the sets of Helen's and Frank's children from the movie Yours, Mine, and Ours again.

Helen's children is set

H = {Colleen, Nick, Janette, Tommy, Jean, Phillip, Gerald, Theresa, Joseph}

Frank's children is set

F= {Mike, Rusty, Greg, Rosemary, Loise, Susan, Veronica, Mary, Germaine, Joan, Joseph}

The union of these two sets is the collection of all nineteen of their children.

$H \cup F$ = {Colleen, Nick, Janette, Tommy, Jean, Phillip, Gerald, Theresa, Joseph, Mike, Rusty, Greg, Rosemary, Loise, Susan, Veronica, Mary, Germaine, Joan}.

Notice that Joseph is in both sets $H$ and $F$, but he is only one child, so he is listed only once in the union.

**Example 1.2.4: Find Union**

1. Set $A = \{1, 3, 5, 7, 9\}$ and set $B = \{2, 3, 5, 7\}$. Find $A \cup B$.
2. Set $A$ = {red, yellow, blue} and set $T$ = {orange, green, purple}. Find $S \cup T$.

**Answer**

1. A union B is the set formed by including all the unique elements in set $A$, set $B$, or both sets $A$ and $B$.

$$A \cup B = \{1, 3, 5, 7, 9, 2\}$$

The first five elements of the union are the five unique elements in set $A$. Even though $3$, $5$, and $7$ are also members of set $B$, these elements are only listed once. Lastly, set $B$ includes the unique element $2$, so $2$ is also included as part of the union of sets $A$ and $B$.

2. $S \cup F = \{\text{red, yellow, blue, orange, green, purple}\}$.

**Your Turn 1.2.4: Find the Intersection and Union**

Let $S$ be the universal set, where:
$S = \{1, 2, 3, \ldots, 18, 19, 20\}$
Let sets $A$ and $B$ be subsets of $S$, where:

Set $A = \{1, 2, 4, 8, 9, 11, 12, 13, 15, 17\}$

Set $B = \{1, 3, 7, 8, 9, 10, 11, 14, 15, 18, 20\}$

Find the following:

List the elements in the set $(A \cup B)$:
*Enter the elements as a list, separated by commas. If the result is the empty set, enter **DNE**.*
$(A \cup B)$ = { _____ }

List the elements in the set $(A \cap B)$:
*Enter the elements as a list, separated by commas. If the result is the empty set, enter **DNE**.*
$(A \cap B)$ = { _____ }

You may want to draw a Venn Diagram to help answer this question.

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**Facts**

For any set $A$, the following is always true.

1. $A \cap A^c = \emptyset$
2. $A \cap \emptyset = \emptyset$
3. $A \cup \emptyset = A$
4. $A \cup A^c = U$

## Applying Concepts of "AND" and "OR" to Set Operations

To become a licensed driver, you must pass some form of written test and a road test, along with several other requirements depending on your age. To keep this example simple, let us focus on the road test and the written test. If you pass the written test but fail the road test, you will not receive your license. If you fail the written test, you will not be allowed to take the road test, and you will not receive a license to drive. To receive a driver's license, you must pass the written test AND the road test. For an "AND" statement to be true, both conditions that make up the statement must be true. Similarly, the intersection of two sets $A$ and $B$ is the set of elements that are in both set $A$ and set $B$. To be a member of $A$ intersection $B$, an element must be in set $B$ and also must be in set $B$. The intersection of two sets corresponds to a logical "AND" statement.

The union of two sets is a logical inclusive "OR" statement. Say you are at a birthday party and the host offers Leah, Lenny, Maya, and you some cake or ice cream for dessert. Leah asks for cake, Lenny accepts both cake and ice cream, Maya turns down both, and you choose only ice cream. Leah, Lenny, and you are all having dessert. The “OR” statement is true if at least one of the components is true. Maya is the only one who did not have cake or ice cream; therefore, she did not have dessert and the “OR” statement is false. To be in the union of two $A$ and set $B$, an element must be in set $A$ or set $B$ or both set $A$ and set $B$.

**AND or OR**

| | | |
|---|---|---|
| **Or** | ≡ **Union** | ≡ ∪ |
| **And** | ≡ **Intersection** | ≡ ∩ |

**Example 1.2.5: Applying "AND" or "OR Operation**

Find the set consisting of elements in:

$A = \{0, 3, 6, 9, 12\}$, $B = \{0, 4, 8, 12, 16\}$, and $C = \{1, 2, 3, 5, 8, 13\}$

1. $A$ and $B$
2. $A$ or $B$
3. $A$ or $C$
4. ($B$ and $C$) or $A$

**Answer**

1. $A$ and $B$ = $A \cap B$ = $\{0, 12\}$, because only the elements $0$ and $12$ are members of both set $A$ and set $B$.
2. $A$ or $B$ = $A \cup B$ = $\{0, 3, 4, 6, 8, 9, 12, 16\}$, because the set $A$ or $B$ is the collection of all elements in $A$ or set $B$ or both.
3. $A$ or $C$ = $\{0, 1, 2, 3, 5, 6, 8, 9, 12, 13\}$, because the set $A$ or $C$ is the collection of all elements in set $A$ or set $C$ or both.
4. ($B$ and $C$) or $A$ = ($B \cap C$) ∪ $A$. Parentheses are evaluated first: ($B$ and $C$)= $B \cap C$ = {8}, because the only member that both set $B$ and set $C$ have in common is $8$. So, now we need to find $\{8\}$ or $\{0, 3, 6, 9, 12\}$, because the word translates to the union operation, the problem becomes $\{8\}$ ∪ $\{0, 3, 6, 9, 12\}$ which is equal to $\{0, 3, 6, 8, 9, 12\}$.

**Your Turn 1.2.5: And or Or**

Let $A = \{2, 4, 5, 6, 8\}$, $B = \{4, 6, 8, 10\}$, and $C = \{2, 7, 9\}$.

List the elements in each of these sets.

$A$ and $B =$ _____

$A$ or $B =$ _____

$(B$ or $C)$ and $A =$ _____

$B$ or $(C$ and $A) =$ _____

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**Example 1.2.6: Find the Elements form the Venn Diagram**

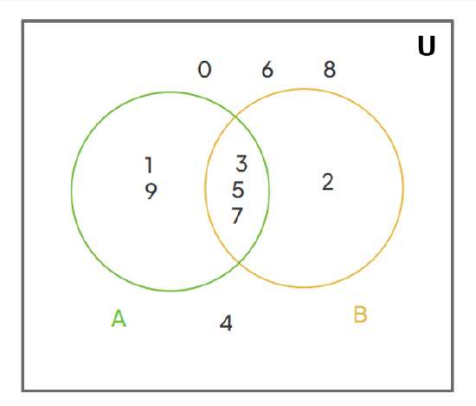



Figure 1.2.6: Venn diagram with two intersecting sets and members.

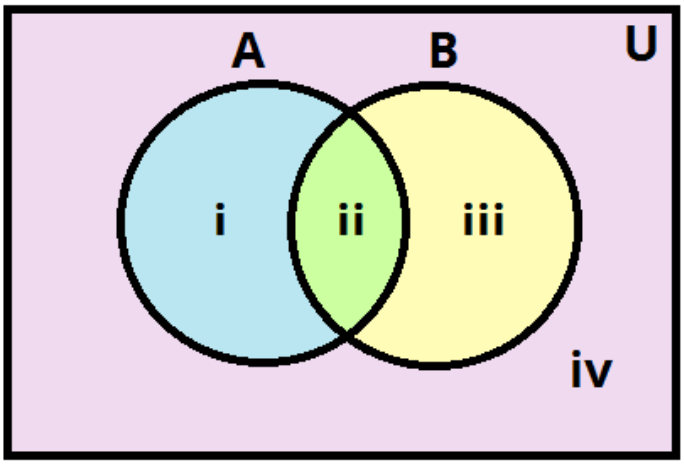
1. Find $A \cap B$.
2. Find $A \cup B$.
3. Find $A \cap B^c$.
4. Find $(A \cap B)^c$.
5. Find $(A \cup B)^c$.

**Answer**

1. $A \cap B = \{3, 5, 7\}$.
2. $A \cup B = \{1, 9, 3, 5, 7, 2\}$
3. $A \cap B^c = \{1, 9, 3, 5, 7\} \cap \{1, 9, 4, 0, 6, 8\} = \{1, 9\}$
4. $(A \cap B)^c = \{1, 9, 4, 2, 0, 6, 8\}$
5. $(A \cup B)^c = \{4, 0, 6, 8\}$

**Your Turn 1.2.6: Idenfity the Region(s)**

A two-set Venn diagram is shown below. Each of the four regions is labeled as i, ii, iii, or iv.




Determine which region(s) of the Venn diagram correspond to the set operations below. Note that $A'$ is a complement of $A$.

$(A \cup B)'$ Select an answer

$A'$ Select an answer

$B$ Select an answer

$A$ Select an answer

$A \cap B'$ Select an answer

$A \cup B$ Select an answer

$B'$ Select an answer

$A \cap B$ Select an answer

$only\ B$ Select an answer

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**Example 1.2.7: Identity the Shaded Region**

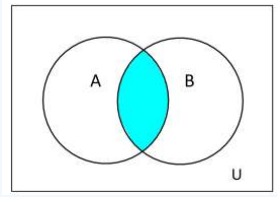


Figure 1.2.7

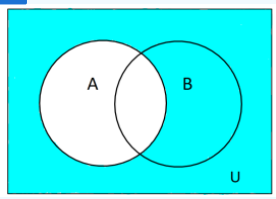


Figure 1.2.8

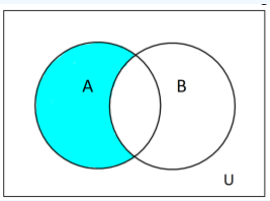


Figure 1.2.9

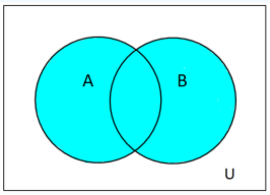


Figure 1.2.1

Write the set that represents the shaded region.

**Answer**

Figure 1.2.7 is $A \cap B$.

Figure 1.2.8 is $A^c$.

Figure 1.2.9 is only $A$.

Figure 1.2.10 is $A \cup B$.

**Your Turn 1.2.7: Place the Elements in Correct Box**

Let the universal set be:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$$

Define the following subsets of $U$:

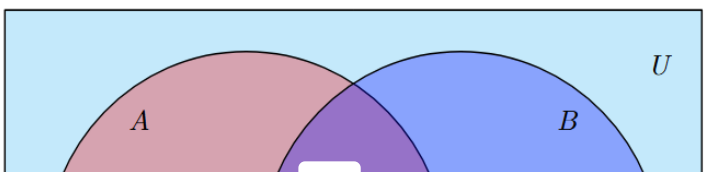
$$A = \{1, 4, 5, 6, 8, 10\}$$

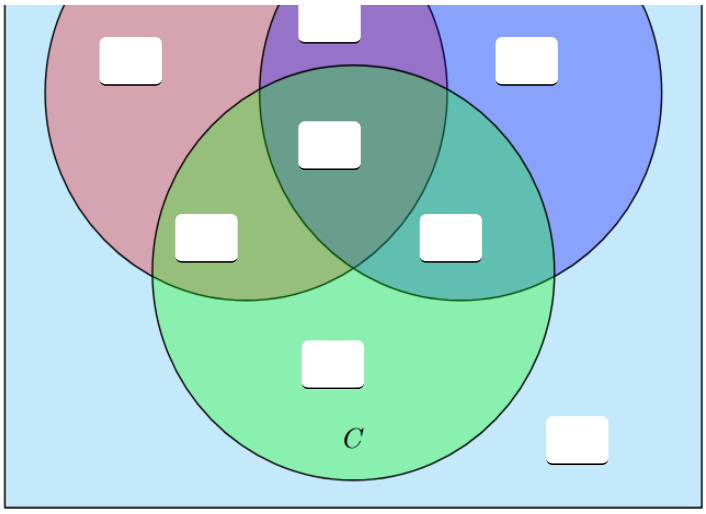
$$B = \{7, 8, 10, 11, 12, 13\}$$

$$C = \{1, 4, 5, 7, 9, 10, 13\}$$

Fill in the Venn Diagram below, placing each element from each set into the appropriate region.

*The elements in each region should be entered as a comma separated list, but the order is not important. (example: 1,3,6 and 3,6,1 would be considered identical)*






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# 1.3: Cardinalty and Venn Diagrams with Survey Problems

Oftentimes, we are interested in the number of items in a set or subset. This is called the cardinality of the set.

**Definition: Cardinality**

Recall: The number of elements in a set is the cardinality of that set.

The cardinality of the set $A$ is often notated as $|A|$ or $n(A)$.

**Example 1.3.1: Survey Problem of Two Sets**

[1] Consider the Venn diagram below. This is a Venn diagram summary of the information from a survey of students. The survey asked students if they were
taking a math class and/or a psychology class. Therefore, you see two sets; one for math and one for psychology. Describe what each region represents.

a. What is known about the **17** people listed in region I of the Venn diagram?
b. What is known about the **5** people listed in region II of the Venn diagram?
c. What is known about the **8** people listed in region III of the Venn diagram?
d. What is known about the **3** people listed in region IV of the Venn diagram?
e. How many total people were surveyed?

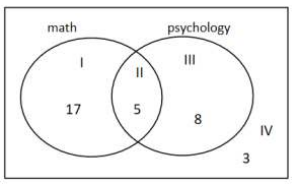


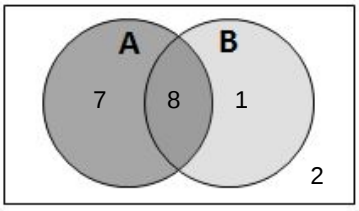

Figure 1.3.1: Math and Psychology

**Answer**

a. There are **17** students taking math but not taking psychology.
b. There are **5** students taking both math AND psychology.
c. There are **8** students taking psychology but not taking math.
d. There are **3** people that do not take math and also do not take psychology.
e. To find the number of people surveyed, find the sum of all of the students in the universal set. The sum shows $17+5+8+3=33$ students were surveyed.

**Your Turn 1.3.1: Find the Cardinality of Set from Venn Diagram**

Consider the Venn Diagram below. The numbers shown below are the cardinalities of the regions.




How many elements are in Set A? _____

How many elements are in Set B? _____

How many elements are in Set A *and* Set B? _____

How many elements are in Set A *or* Set B? _____

How many elements are *not* in Set A? _____

How many elements are *not* in Set B? _____

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**Definition: Cardinality Properties**

$$n(A \bigcup B) = n(A) + n(B) - n(A \bigcap B)$$

OR

$$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

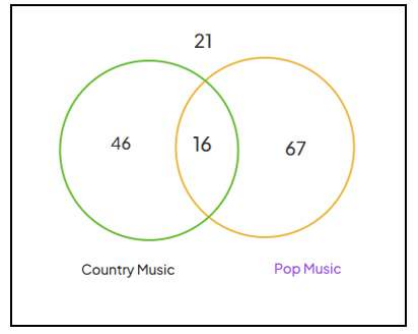
$$n(A^c) = n(U) - n(A)$$

**Example 1.3.2: Survey Problem with Two Sets**

Use the information below to complete a Venn diagram and answer the questions. A survey asked 150 people about the type of music they listened to yesterday.

**62** people listened to country music
**83** people listened to pop music
**16** people listened to country music and pop music
a. How many people listened to pop music but not country music?
b. How many people listened to country music but not pop music?
c. How many people listen to country music or pop music?
d. How many people did not listen to country music or pop music?

**Answer**




a. Pop music only $83 - 16 = 67$.

b. Country music but not pop music (Country music only) $62 - 16 = 46$.

c. Country music or pop music $46 + 16 + 67 = 129$.

d. Did not listen to country music or pop music $150 - 129 = 21$.

**Your Turn 1.3.2: Survey Problem of Two Sets**

A survey of 96 persons was conducted at TCC, and it was found that 47 persons carried a cell phone, 43 persons carried a tablet computer, and 17 carried both a cell phone and a tablet. Draw Venn diagram in paper before you answer this question.

1. How many people carried a cell phone or a tablet?

   _____

2. How many people carried neither a cell phone nor a tablet?

3. How many people carried a cell phone only?

4. How many people carried a tablet but not a cell phone?

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**Example 1.3.3: Survey Problem of Two Sets**

A survey asks: Which online services have you used in the last month?

- Twitter
- Facebook
- Have used both

The results show 40% of those surveyed have used Twitter, 70% have used Facebook, and 20% have used both. How many people have used neither Twitter nor Facebook?

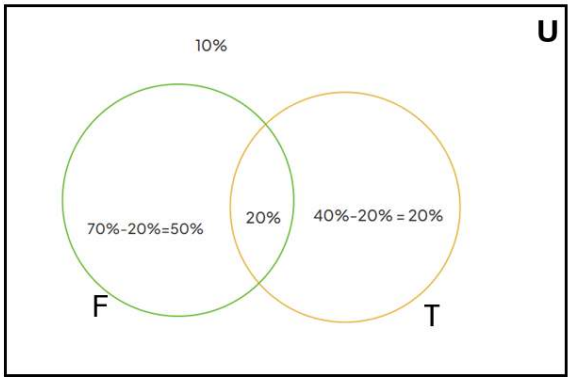
**Answer**

Let $T$ be the set of all people who have used Twitter, and $F$ be the set of all people who have used Facebook. Notice that while the cardinality of $F$ is 70% and the cardinality of $T$ is 40%, the cardinality of $F \bigcup T$ is not simply $70\% + 40\%$, since that would count those who use both services twice. To find the cardinality of $F \bigcup T$, we can add the cardinality of $F$ and the cardinality of $T$, then subtract those in the intersection that we've counted twice. In symbols,

$$n(F \bigcup T) = n(F) + n(T) - n(F \bigcap T)$$

$$n(F \bigcup T) = 70\% + 40\% - 20\% = 90\%$$

Now, to find how many people have not used either service, we're looking for the cardinality of $(F \bigcup T)^c$. Since the universal set contains 100% of people and the cardinality of $F \bigcup T = 90\%$, the cardinality of $(F \bigcup T)^c$ must be the other 10%.




**Your Turn 1.3.3: Survey Problem of Two Sets**

At a school 68 people were asked in a survey if they like statistics, English, or both. Of those people, 16 only like statistics, 17 only like English, and 15 like both.

How many people like statistics? _____

How many people like English? _____

How many do not like either statistics or English? _____

How many do not like statistics? _____

How many do not like English? _____

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**Example 1.3.5: Survey Problem of Three Sets**

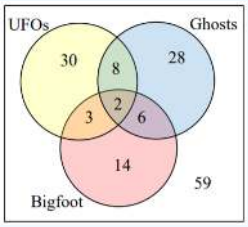
One hundred fifty people were surveyed and asked if they believed in UFOs, ghosts, and Bigfoot.

- **43** believed in UFOs
- **44** believed in ghosts
- **25** believed in Bigfoot
- **10** believed in UFOs and ghosts
- **8** believed in ghosts and Bigfoot
- **5** believed in UFOs and Bigfoot
- **2** believed in all three

How many people surveyed believed in at least one of these things?

**Answer**

Starting with the intersection of all three circles, we work our way out. Since **10** people believe in UFOs and Ghosts, and **2** believe in all three, that leaves **8** that believe in only UFOs and Ghosts. We work our way out, filling in all the regions. Once we have, we can add up all those regions, getting **91** people in the union of all three sets. This leaves **150–91 = 59** who believe in none.




**Your Turn 1.3.5: Survey Problem of Three Sets**

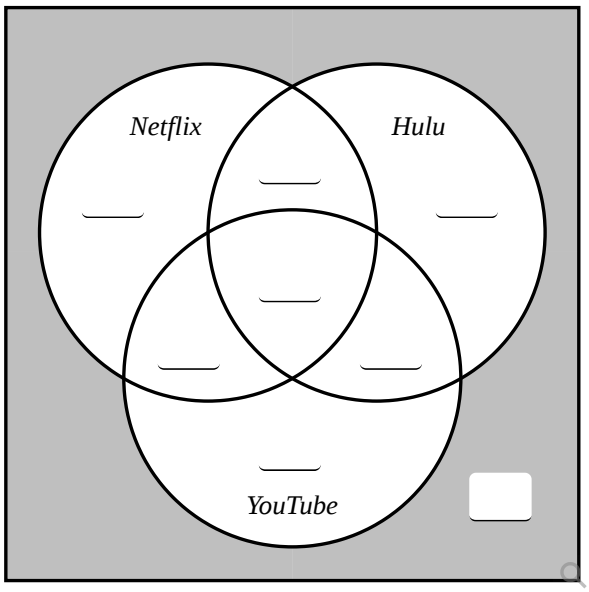
A survey asks, **"To which streaming services do you subscribe?".**

The results are as follows:

- Netflix: 24 people
- Hulu: 30 people
- YouTube: 23 people
- Both Netflix and Hulu: 12 people
- Both Hulu and YouTube: 7 people
- Both Netflix and Youtube: 10 people
- Netflix, Hulu, and YouTube: 3 people

- None of these three: 5 people

Fill in the number of people in each region of the Venn diagram below:




How many people responded that they only subscribe to a single service? ____

How many people responded that they subscribe to no more than two services? ____

How many people responded that they subscribe to two or more services? ____

How many people responded to the survey? ____

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**Example 1.3.4: Survey Problem of Three Sets**

Fifty students were surveyed, and asked if they were taking a social science (SS), humanities (HM) or a natural science (NS) course the next quarter.

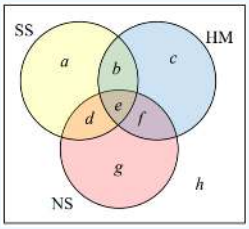
- **21** were taking a SS course
- **26** were taking a HM course
- **19** were taking a NS course
- **9** were taking SS and HM
- **7** were taking SS and NS
- **10** were taking HM and NS
- **3** were taking all three
- **7** were taking none

How many students are only taking a SS course?

**Answer**

It might help to look at a Venn diagram. From the given data, we know that there are **3** students in region $e$ and **7** students in region $h$.

Since $7$ students were taking a SS and NS course, we know that $n(d) + n(e) = 7$. Since we know there are $3$ students in region $3$, there must be $7–3 = 4$ students in region $d$.




Similarly, since there are 10 students taking HM and NS, which includes regions $e$ and $f$, there must be $10–3 = 7$ students in region $f$.

Since $9$ students were taking SS and HM, there must be $9–3 = 6$ students in region $b$.

Now, we know that $21$ students were taking a SS course. This includes students from regions $a$, $b$, $d$, and $e$. Since we know the number of students in all but region $a$, we can determine that $21–6–4–3 = 8$ students are in region $a$.

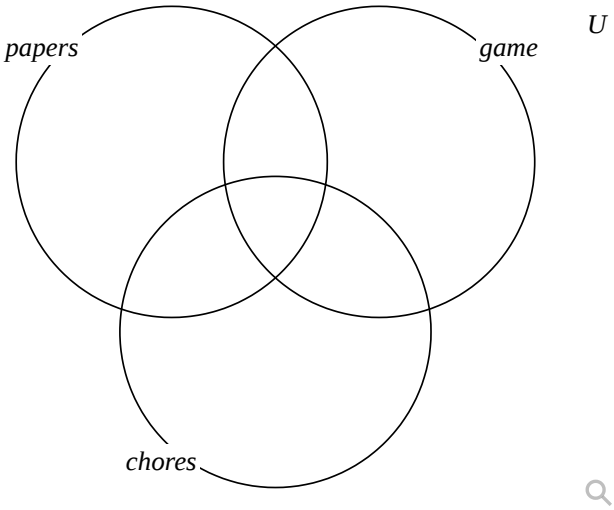
8 students are taking only a SS course.

## Your Turn 1.3.4: Survey Problem of Three Sets

Two hundred sixty-two Florida Southwestern memmber faculty members were surveyed regarding their plans for the upcoming weekend. The results were as follows:

| | |
|---|---|
| 134 | plan to grade papers |
| 115 | plan to attend a football game |
| 97 | plan to do household chores |
| 24 | plan to grade papers and attend a football game |
| 57 | plan to grade papers and do household chores |
| 28 | plan to attend a football game and do household chores |
| 7 | plan to grade papers, attend a football game, and do household chores |

Sketch a Venn diagram and answer the questions below.




1. How many planned to only grade papers?

2. _____ How many planned to grade papers or do household chores?

3. _____ How many did not plan to do any of these activities?

4. _____ How many planned to do at least two of these activities?

5. _____ How many planned to attend a football game and do household chores, but did not have any plans to grade?

6. How many planned to attend a football game or grade papers, but did not plan to do household chores?

7. How many planned to do at most one of these activities or no activities at all?

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[1] https://sites.google.com/view/mathem...eral-arts/home

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# 1.4: Statements, Compound Statements, and Negations



Figure **1.4.1**: Construction of a logical argument, like that of a house, requires you to begin with the right parts. (credit: modification of work “Barn Raising” by Robert Stinnett/Flickr, CC BY 2.0)

## Logical Statements

Have you ever built a club house, tree house, or fort with your friends? If so, you and your friends likely started by gathering some tools and supplies to work with, such as hammers, saws, screwdrivers, wood, nails, and screws. Hopefully, at least one member of your group had some knowledge of how to use the tools correctly and helped to direct the construction project. After all, if your house isn't built on a strong foundation, it will be weak and could possibly fall apart during the next big storm. This same foundation is important in logic.

In this section, we will begin with the parts that make up all logical arguments. The building block of any logical argument is a **logical statement**, or simply a statement.

**Definition: Logical Statement**

A logical statement has the form of a complete sentence, and it must make a claim that can be identified as being true or false. Every logical statement has a truth value, either true or false.

When making arguments, sometimes people make false claims. When evaluating the strength or validity of a logical argument, you must also consider the **truth values**, or the identification of true or false, of all the statements used to support the argument. While a false statement is still considered a logical statement, a strong logical argument starts with true statements.



Figure **1.4.2**: Not all roses are red. (credit: “assorted pink yellow white red roses macro” by ProFlowers/Flickr, CC BY 2.0)

An example of a logical statement with a false truth value is, “All roses are red.” It is a logical statement because it has the form of a complete sentence and makes a claim that can be determined to be either true or false. It is a false statement because not all roses are red: some roses are red, but there are also roses that are pink, yellow, and white. Requests, questions, or directives may be complete sentences, but they are not logical statements because they cannot be determined to be true or false. For example, suppose someone said to you, “Please, sit down over there.” This request does not make a claim, and it cannot be identified as true or false; therefore, it is not a logical statement.

**Exercise 1.4.1: Identifying Logical Statements**

Determine whether each of the following sentences represents a logical statement. If it is a logical statement, determine whether it is true or false.

1. Tiger Woods won the Master's golf championship at least five times.
2. Please, sit down over there.
3. All cats dislike dogs.

**Answer**

1. This is a logical statement because it is a complete sentence that makes a claim that can be identified as being true or false. As of **2021**, this statement is true: Tiger Woods won the Master's in **1997**, **2001**, **2002**, **2005** and **2019**.
2. This is not a logical statement because, although it is a complete sentence, this request does not make a claim that can be identified as being either true or false.
3. This is a logical statement because it is a complete sentence that makes a claim that can be identified as being true or false. This statement is false because some cats do like some dogs.

**Your Turn 1.4.1: Logical Statement or Not?**

Determine whether each of the following is a statement.

Close the door.

- ○ Yes, it is a statement.
- ○ No, it is not a statement.

When will I be loved?

- ○ Yes, it is a statement.
- ○ No, it is not a statement.

Honey is sweeter than sugar.

- ○ Yes, it is a statement.
- ○ No, it is not a statement.

The Sun revolves around the Earth.

- ○ Yes, it is a statement.
- ○ No, it is not a statement.

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## Representing Statements in Symbolic Form

When analyzing logical arguments that are made of multiple logical statements, symbolic form is used to reduce the amount of writing involved. Symbolic form also helps visualize the relationship between the statements in a more concise way in order to determine the strength or validity of an argument. Each logical statement is represented symbolically as a single lowercase letter, usually starting with the letter $p$, $q$, and $r$.

To begin, you will practice how to write a single logical statement in symbolic form. This skill will become more useful as you work with compound statements in later.

For example, we can represent some mammals that live in the ocean by letter $p$.

So $p$: Some mammals live in the ocean.

## Negating Statements

Consider the false statement introduced earlier, “All roses are red.” If someone said to you, “All roses are red,” you might respond with, “Some roses are not red.” You could then strengthen your argument by providing additional statements, such as, “There are also white roses, yellow roses, and pink roses, to name a few.”

The negation of a logical statement has the opposite truth value of the original statement. If the original statement is false, its negation is true, and if the original statement is true, its negation is false. Most logical statements can be negated by simply adding or removing the word *not*. For example, consider the statement, “Emma Stone has green eyes.” The negation of this statement would be, “Emma Stone does not have green eyes.” The table below gives some other examples.

**Negation**

The symbol for negation, or not, in logic is the tilde, $\sim$. So, not $p$ is represented as $\sim p$.

To negate a statement symbolically, remove or add a tilde. The negation of $\sim(\sim p)$ is $p$.

Symbolically, this equation is $\sim(\sim p) \equiv p$.

**It is not the Case, or it is not true that.**

Sometimes, the phrase "*it is not the case that*" **or** "*it is not true that*" is added before a statement to negate the statement.

Table **1.4.1**

| Logical Statement | Negation |
|---|---|
| Gordon Ramsey is a chef. $(p)$ | Gordon Ramsey is not a chef. $(\sim p)$ |
| Tony the Tiger does have spots. $(r)$ | It is not true that Tony the Tiger has spots. $(\sim r)$ |
| It is **not** raining. $(s)$ | It is raining. $(\sim s)$ |

The way you phrase your argument can impact its success. If someone presents you with a false statement, the ability to rebut that statement with its negation will provide you with the tools necessary to emphasize the correctness of your position.

## Translating Negations Between Words and Symbols

To analyze logical arguments effectively, it is essential to be able to translate between the symbolic and written forms of logical statements.

**Exercise 1.4.2: Write in Symbolic Form**

Given the statements:

$r$: Elmo is a red Muppet.

$p$: Ketchup is not a vegetable.

1. Write the symbolic form of the statement, “Elmo is not a red Muppet.”
2. Translate the statement $\sim p$ into words.

**Answer**

1. “Elmo is not a red muppet” is the negation of “Elmo is a red muppet,” which is represented as $r$. Thus $\sim r$: “Elmo is not a red muppet”.
2. Because $p$ is the symbol representing the statement, “Ketchup is not a vegetable,” $\sim p$ is equivalent to the statement, “Ketchup is a vegetable.”

**Your Turn 1.4.2: Negation of Statement**

Match the statement to the correct negation.

Statements

- [-] Sun is not cold.
- [-] Triangle has three sides.
- [-] It is not hot.
- [-] We eat ice cream.
- [-] Wonder Woman is not stronger than Captain Marvel.
- [-] There is a prime number.

Possible negations

a. Wonder Woman is stronger than Captain Marvel.
b. There is prime number and composite number.
c. Sun is very cold.
d. It is not true that sun is not cold
e. It is hot.
f. Captain Marvel is not stronger than Wonder Women.
g. Triangle has four sides.
h. There is not prime numbers.
i. We do not eat ice cream and cake.
j. It is very hot.
k. Triangle has no three sides.
l. We do not eat ice cream.

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## Quantified Statements

A quantifier is a term that expresses a numerical relationship between two sets or categories. For example, all squares are also rectangles, but only some rectangles are squares, and no squares are circles. In this example, *all*, *some*, and *none* are quantifiers. In a logical argument, the logical statements made to support the argument are called premises, and the judgment made based on the premises is called the conclusion. Logical arguments that begin with specific premises and attempt to draw more general conclusions are called inductive arguments.

**Your Turn 1.4.3: Identified Quantified Statement**

Which of the following is a quantified statement?

- ○ That tasted terrible!
- ○ The Mississippi River is the longest river in the world.
- ○ Some dogs are beagles.
- ○ Life is like a box of chocolates.
- ○ My car is blue.

Which of the following is a quantified statement?

- ○ That tasted great!
- ○ My car is purple.
- ○ Holy cow!
- ○ All dogs go to heaven.
- ○ My car is green.

Which of the following is a quantified statement?

- ○ The cow jumped over the moon.

- The cow jumped over the moon.
- If I had a hammer, I'd pound some nails.
- Your pudding is hard as a rock.
- No dogs go to heaven.
- Disney World is in Alabama.

Which of the following is a quantified statement?

- Some flowers smell nice.
- My car is green.
- When will I be loved?
- Go, Bears!
- Go, Tigers!

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## Negating Statements Containing Quantifiers

Recall that the negation of a statement will have the opposite truth value of the original statement. There are four basic forms that logical statements with quantifiers take on.

All $A$ are $B$: See Figure 1.4.1.

Some $A$ are $B$: See Figure 1.4.2.

No $A$ are $B$: See Figure 1.4.3.

Some $A$ are not $B$: See Figure 1.4.4.

We can also represent the above four quantified statements in a diagram, called an Euler Diagram, as shown below.

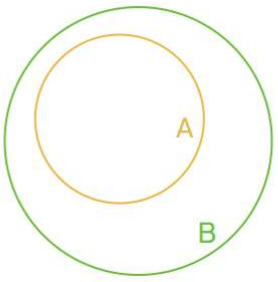



Figure **1.4.1**: All A are B

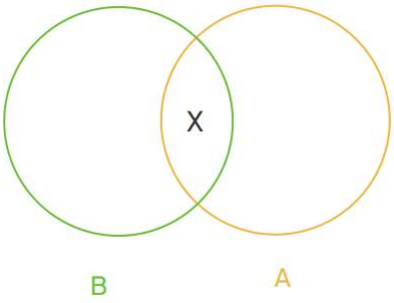



Figure **1.4.2**: Some A are B

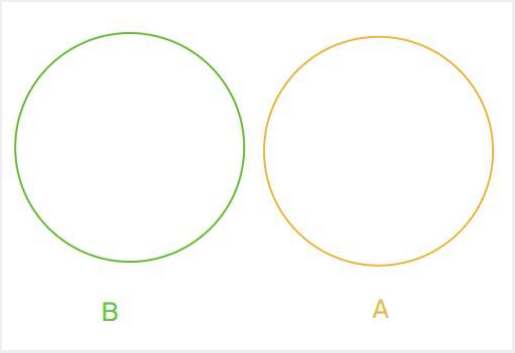



Figure **1.4.3**: No A are B

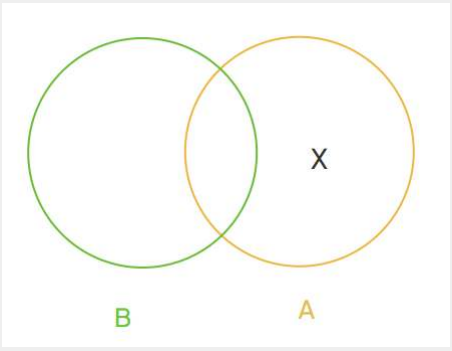



Figure **1.4.4**: Some A are not B

The negation of logical statements that use the quantifiers *all*, *some*, or *none* is a little more complicated than just adding or removing the word *not*. The following diagram shows the negation of quantified statements. The statements that are diagonally opposite each other are negations.

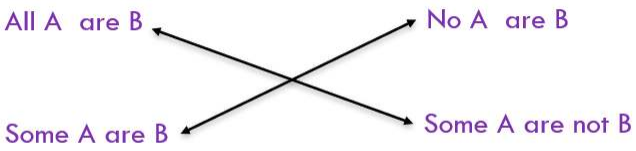
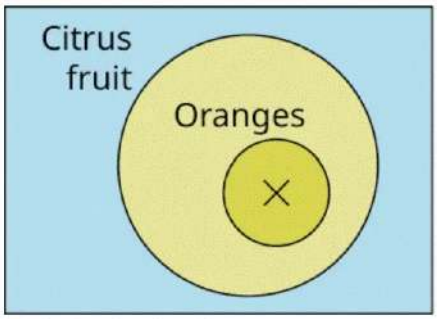
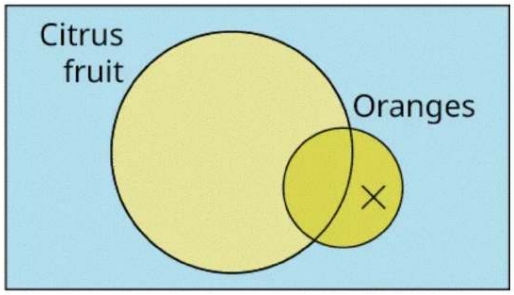
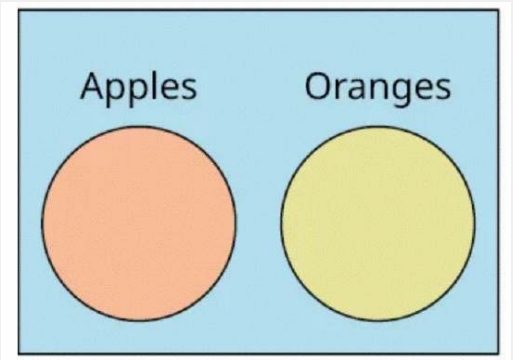
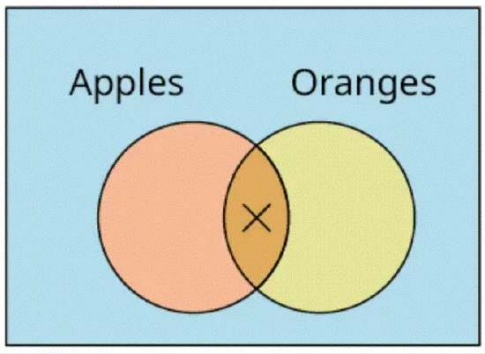



Figure **1.4.5**: Negation of Quantified Statement

Table **1.4.1** summarizes the four different forms of logical statements involving quantifiers and the forms of their associated negations, as well as the meanings of the relationships between the two categories or sets $\boldsymbol{A}$ and $\boldsymbol{B}$.

For example, consider the logical statement, “All oranges are citrus fruits.” This statement expresses as a subset relationship. The set of oranges is a subset of the set of citrus fruit. This means that there are no oranges that are outside the set of citrus fruit. The negation of this statement would have to break the subset relationship. To do this, you could say, “At least one orange is not a citrus fruit.” Or, more concisely, “Some oranges are not citrus fruit.” It is tempting to say "No, oranges are not citrus fruit," but that would be incorrect. Such a statement would go beyond breaking the subset relationship to asserting that the two sets have no elements in common. The negation of " $\boldsymbol{A}$ is a subset of $\boldsymbol{B}$ " would be to state that "$\boldsymbol{A}$ is not a subset of $\boldsymbol{B}$," as depicted by the Venn diagram in table **1.4.1**.

Table **1.4.1**

| Logical Statements with Quantifiers | Negation of Logical Statements w/Quantifiers |
| --- | --- |
| Statement: All $A$ are $B$.<br>All oranges are citrus fruits (True)<br>  <br> | Negation: Some $A$ are not $B$.<br>Some oranges are not citrus fruits. (False)<br>  <br> |
| Statement: No $A$ are $B$<br>No apples are oranges. (True)<br>  <br> | Negation: Some $A$ are $B$.<br>Some apples are oranges. (False)<br>  <br> |

**Exercise 1.4.4**: Negating Statements Containing Quantifiers *All*, *Some*, or *None*

Given the statements:

$p$: All leopards have spots.
$r$: Some apples are red.
$s$: No lemons are sweet.

Write each of the following symbolic statements in words.

1. $\sim p$
2. $\sim r$
3. $\sim s$

**Answer**

1. The statement "All leopards have spots" is $p$ and it has the form "All $A$ are $B$." According to figure 1.4.5, the negation will have the form "Some $A$ are not $B$." Thus $\sim p$ : "Some leopards do not have spots."
2. The statement "Some apples are red" has the form "Some $A$ are $B$." This indicates that the categories $A$ and $B$ overlap or intersect. According to figure 1.4.5, the negation will have the form, "No $A$ are $B$," indicating that $A$ and $B$ do not intersect. This results in the opposite truth value of the original statement, so the negation of "Some apples are red" is the statement: "No apples are red."
3. Because $s$ is the statement: "No lemons are sweet," $s$ is asserting that the set of lemons does not intersect with the set of sweet things. The negation of $s$, $\sim s$, must make the opposite claim. It must indicate that the set of lemons intersects with the set of sweet things. This means at least one lemon must be sweet. The statement, "Some lemons are sweet," is $\sim s$. The negation of the statement, "No $A$ are $B$," is the statement, "Some $A$ are $B$," as indicated in figure 1.4.5.

**Your Turn 1.4.4**: Negation of Quantified Statement

Write the negation of the following quantified statement:

**"Some trains are late."**

Select an answer trains Select an answer late.

Write the negation of the following quantified statement:

**"All mammals are land-based."**

Select an answer mammals Select an answer land-based.

Write the negation of the following quantified statement:

**"Some horses are not domesticated."**

Select an answer horses Select an answer domesticated.

Write the negation of the following quantified statement:

**"All basketball players are not tall."**

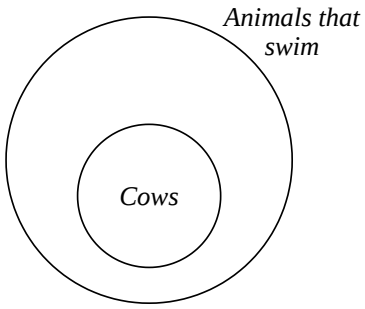
Select an answer basketball players Select an answer tall.

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The next example shows how to find negation from a diagram.

Your Turn 1.4.4A: Negation of Quantified Statement from Diagram




Select the statement that is the **negation** of the relationship depicted above.

- All cows swim.
- Some cows do not swim.
- If it is not a cow, then it can swim.

- ○ Some cows swim.
- ○ If it is not a cow, then it can not swim.
- ○ No cows swim.
- ○ If it can not swim, then it is not a cow.

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## Logical Connector

Suppose your friend is trying to get a license to drive. In most places, they will need to pass some form of written test proving their knowledge of the laws and rules for driving safely. After passing the written test, your friend must also pass a road test to demonstrate that they can perform the physical task of driving safely within the rules of the law.

Consider the statement: "My friend passed the written test, buthe did not pass the road test." This is an example of a compound statement, a statement formed by using a connective to join two independent clauses or logical statements. The statement, “My friend passed the written test,” is an independent clause because it is a complete thought or idea that can stand on its own. The second independent clause in this compound statement is, “My friend did not pass the road test.” The word "but" is the connective used to join these two statements together, forming a compound statement. So, did your friend acquire their driving license?.

This section introduces common logical connectives and their symbols, and allows you to practice translating compound statements between words and symbols. It also explores the order of operations, or dominance of connectives, when a single compound statement uses multiple connectives.

## Common Logical Connectives

Understanding the following logical connectives, along with their properties, symbols, and names, will be key to applying the topics presented in this chapter. The chapter will discuss each connective introduced here in more detail.

The joining of two logical statements with the word "and" or "but" forms a compound statement called a conjunction. In logic, for a conjunction to be true, all the independent logical statements that make it up must be true. The symbol for a conjunction is $\wedge$. Consider the compound statement, “Derek Jeter played professional baseball for the New York Yankees, and he was a shortstop.” If $p$ represents the statement, “Derrick Jeter played professional baseball for the New York Yankees,” and if $q$ represents the statement, “Derrick Jeter was a short stop,” then the conjunction will be written symbolically as $p \wedge q$.

The joining of two logical statements with the word “or” forms a compound statement called a disjunction. Consider the compound statement, "The office manager ordered cake for an employee’s birthday or they ordered ice cream.” This is a disjunction because it combines the independent clause, “The office manager ordered cake for an employee’s birthday,” with the independent clause, “The office manager ordered ice cream,” using the connective, *or*. Inclusive *or* means you can have one, or the other, or both!

### Hypothesis and Conclusion

Joining two logical statements with the word *implies*, or using the phrase “if *first statement*, then *second statement*,” is called a conditional or *implication*. The clause associated with the "if" statement is also called the hypothesis or *antecedent*, while the clause following the "then" statement or the word *implies* is called the **conclusion** or *consequent*. The conditional statement is like a one-way contract or promise.

$$\text{Hypothesis (Antecedent)} \longrightarrow \text{Conclusion (Consequent}$$

Joining two logical statements with the connective phrase “if and only if” is called a biconditional. The connective phrase "if and only if" is represented by the symbol, $p \longrightarrow q$, where $p$ is called the hypothesis or antecedent and $q$ is called the conclusion or consequent.

The table below lists the most **common connectives** used in logic, along with their symbolic forms, and the common names used to describe each connective.

Table **1.4.2**

| Connectives | Symbol | Name |
|---|---|---|
| and (but) | $\wedge$ | $p \wedge q$ is called: Conjunction |
| or | $\vee$ | $p \vee q$ is called: Disjunction |
| not | $\sim$ | $\sim p$ is called: Negation |
| if, then (implies) | $\longrightarrow$ | $p \longrightarrow q$ is called: Conditional (Implication) |
| if and only if | $\longleftrightarrow$ | $p \longleftrightarrow q$ is called: Biconditional |

**Think about Biconditional**

In everyday life, we often have a stronger meaning in mind when we use a conditional statement. Consider "If you submit your hours today, then you will be paid next Friday." What the payroll rep really means is "If you submit your hours today, then you will be paid next Friday, and if you don't submit your hours today, then you won't be paid next Friday." The conditional statement if *t*, then *p* also includes the inverse of the statement: if not *t*, then not *p*. A more compact way to express this statement is "You will be paid next Friday *if and only if* you submit your timesheet today." A statement of this form is called a **biconditional**.

# Translating Compound Statements to Symbolic Form

To translate a compound statement into symbolic form, we take the following steps.

**Steps to Write Compound Statements Into Symbolic Form**

1. First, identify if the statement is a conjunction, a disjunction, a conditional, or a biconditional.
2. Identify and label all independent logical statements with a lowercase letter, such as $p$, $q$, or $r$.
3. Identify and label any negative logical statements with a lowercase letter preceded by the negation symbol, such $\sim p$, $\sim q$, or $\sim r$.
4. Replace the connective words with the symbols that represent them, such as $\wedge$, $\vee$, $\longrightarrow$, and $\longleftrightarrow$.

Consider the previous example of your friend trying to get their driver's license. Your friend passed the written test, but they did not pass the road test. Let $p$ represent the statement, "My friend passed the written test." And, $\sim q$ represents the statement, "My friend did not pass the road test." Because the connective *but* is logically equivalent to the word *and*, the symbol for *but* is the same as the symbol for *and*; replace *but* with the symbol $\wedge$. The compound statement is symbolically written as: $p \wedge \sim q$. My friend passed the written test, but they did not pass the road test.

**Checkpoint**

*When translating English statements into symbolic form, do not assume that every connective "and" means you are dealing with a compound statement. The statement, "The stripes on the U.S. flag are red and white," is a simple statement. The word "white" doesn't express a complete thought, so it is not an independent clause and does not get its own symbol. This statement should be represented by a single letter, such as $s$: The stripes on the U.S. flag are red and white.*

**Example 1.4.5: Translating Compound Statements Into Symbolic Form**

Let us consider the following statements

$p$: It is a warm sunny day.

$q$: The family will go to the beach.

Identify the type of the following statements and write the symbolic form of each of the following compound statements.

1. If it is a warm sunny day, then the family will go to the beach.
2. The family will go to the beach, and it is a warm sunny day.
3. The family will not go to the beach if and only if it is not a warm sunny day.
4. The family does not go to the beach, or it is a warm sunny day.

**Answer**

1. Because the connective is "*if..., then...*", this statement is conditional. The compound statement is written symbolically as: $p \rightarrow q$.
2. Because the connective "*and*", this statement is a conjunction. The compound statement is written symbolically as: $q \wedge p$.
3. Because the connective is "*if and only if*", this statement is biconditional. The compound statement is written symbolically as: $\sim q \leftrightarrow \sim p$.
4. Because the connective "*or* " is a disjunction. The compound statement is written symbolically as: $\sim q \vee p$.

**Your Turn 1.4.5: Compound Statements in Symbolic Forms**

For the simple statements

*p:* It is a frog.
*q:* It has feathers.

Write each compound statement in symbolic form.

It does not have feathers if it is a frog. ? ? ?

It is a frog or it has feathers. ? ? ?

If it does not have feathers, then it is a frog. ? ? ?

It does not have feathers or it is a frog. ? ? ?

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## Translating Compound Statements in Symbolic Form with Parentheses into Words

When parentheses are written in a logical argument, they group a compound statement together just like when calculating numerical or algebraic expressions. Any statement in parentheses should be treated as a single component of the expression. If multiple parentheses are present, work with the innermost parentheses first.

Consider your friend's struggles to obtain their driver's license.

Let $p$ represent the statement, "My friend passed the written test," let $q$ represent the statement, "My friend passed the road test," and let $r$ represent the statement, "My friend received a driver's license."

The statement $(p \wedge q) \rightarrow r$ can be translated into words as follows: the statement $p \wedge q$ is grouped together to form the hypothesis of the conditional statement, and $r$ is the conclusion. The conditional statement has the form "if $p \wedge q$, then $r$." Therefore, the written form of this statement is: "If my friend passed the written test and they passed the road test, then my friend received a driver's license."

Sometimes a compound statement within parentheses may need to be negated as a group. To accomplish this, add the phrase, "it is not the case that" before the translation of the phrase in parentheses. For example, using $p$, $q$, and $r$ of your friend obtaining a license, let's translate the statement $\sim (p \wedge q) \rightarrow \sim r$ into words.

In this case, the hypothesis of the conditional statement is $\sim (p \wedge q)$ and the conclusion is $\sim r$. To negate the hypothesis, add the phrase "it is not the case" before translating what is in parentheses. The translation of the hypothesis is the sentence, "It is not the case that my friend passed the written test and they passed the road test," and the translation of the conclusion is, "My friend did not receive a driver's license." So, a translation of the complete conditional statement, $\sim (p \wedge q) \rightarrow \sim r$ is: "If it is not the case that my friend passed the written test and the road test, then my friend did not receive a driver's license."

**Steps to Write Symbolic Statements Into Words**

1. First, identify if the statement is a conjunction, a disjunction, a conditional, or a biconditional.
2. Identify each simple statement (such as $p$, $q$, and $r$) and determine what each represents in words.
3. Identify and translate the logical symbols used in the expression.
4. Follow the parentheses carefully, translating the innermost expressions first.
5. Combine the translated parts in the correct order to form a complete sentence.
6. Read the final sentence to make sure it accurately reflects the original symbolic logic.

NOTE: **The symbolic statements in parentheses appear on the same side of the comma in English translation.**

**Your Turn 1.4.6: Write Comound Statement Into Symbolic Form**

Consider the following simple statements.

$r$: The car costs $\$80K$.

$s$: The car goes $100$ mph.

$t$: The car is yellow.

Represent the following compound statement in symbolic form.

The car does not cost $\$80K$ if and only if the car is yellow and does not go $100$ mph.

**Note: Click in the answer box to see available logic symbols. Use symbol $\Rightarrow$ for conditional $\rightarrow$.**

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## Exercise 1.4.7: Translate Symbolic Form to Word

Suppose

$p$: My child finished their homework.

$q$: My child cleaned her room.

$r$: My child played video games.

$s$: My child streamed a movie.

Translate each of the following symbolic statements into words.

1. $\sim (p \wedge q)$
2. $p \rightarrow (r \vee s)$
3. $(p \rightarrow r) \vee s$
4. $\sim (r \vee s) \leftrightarrow \sim (p \wedge q)$

**Answer**

1. This statement is a conjunction. Replace $\sim$ with "It is not the case," and $\wedge$ with "and." Translation is: "It is not the case that my child finished their homework and cleaned their room."
2. This statement is a conditional. The hypothesis of the conditional statement is, "My child finished their homework." The conclusion of the conditional statement is, "My child played video games or streamed a movie." Translation of the entire statement is: "If my child finished their homework, then they played video games or streamed a movie."
3. This statement is a disjunction. Translation of the entire statement is: "If my child finished their homework, then they played video games, or streamed a movie."
4. This statement is a biconditional. The hypothesis of the biconditional statement is $\sim (r \vee s)$ and is written in words as: "It is not the case that my child played video games or streamed a movie." The conclusion of the biconditional statement is $\sim (p \wedge q)$, which translates to: "It is not the case that my child finished their homework and cleaned their room." Because the biconditional, $\leftrightarrow$ translates to if and only if. Translation of the statement is: "It is not the case that my child played video games or streamed a movie if and only if it is not the case that my child finished their homework and cleaned their room."

## Your Turn 1.4.7: Symbolic Form to Word

Let p, q, and r represent the following statements.

p: The ice cream truck is here.
q: The pool is open.
r: It is summertime.

Enter the statement below that matches the following symbolic statement: $p \leftrightarrow (q \wedge r)$

*Select from the choices below. If any of the options should not be used, select "(blank)" in that space.*

Select an answer | Select an answer | Select an answer | Select an answer
? | Select an answer

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## Necessary and Sufficient Condition

One sentence is a **sufficient condition** for another sentence if the truth of the first would guarantee the truth of the second. The truth of the first is *enough* – all you need, *sufficient* – to ensure the truth of the second. Having your head cut off is a sufficient condition for being dead. There are many other ways to die, but decapitation is enough to ensure it.

And one sentence is a **necessary condition** for a second if the truth of the second sentence is required—is needed, is *necessary*—for the truth of the first sentence. For example, paying your tuition fees is a necessary condition for graduating. If you are to graduate, you must pay your fees.

In the previous example, paying your fees is necessary, but not sufficient, for graduating. It is necessary because no one graduates without paying their fees, but it is not sufficient because there are other things you must also do (like pass the required number of credit hours).

In conditional statement $p \longrightarrow q$

1. The antecedent, $p$, is a sufficient condition for the consequent, $q$.
2. The consequent, $q$, is a necessary condition for the antecedent, $p$.

In order to remember, you think this way

$$\boxed{\text{Sufficient} \longrightarrow \text{Necessary}}$$

### Example 1.4.8: Conditional Statement in Necessary and Sufficient Form

If you pass the final exam, then you will pass the course.

**Answer**

Note: $\text{passing final} \longrightarrow \text{passing the course}$

Sufficient: passing the final

Necessary: passing the course

Passing the final is *sufficient to* pass the course.

Passing this course is necessary to pass the final.

### Your Turn 1.4.8: Necessary and Sufficient

Which of the following are equivalent to the following statement:

Staying up late but not drinking coffee is sufficient for not staying awake in class.

- ○ **If** I stay up late **and** don**'t** drink coffee, **then** I will **not** stay awake in class.
- ○ **If** I ca**n't** stay awake in class, **then** I stayed up late **but** did**n't** drink coffee.
- ○ I ca**n't** stay awake in class **if and only if** I stayed up late **but** did**n't** drink coffee.
- ○ I can stay awak in class **if and only if** I do**n't** stay up late **or** I have coffee.

Having a ticket is necessary for boarding the airplane.

- ○ **If** you board the airplane, **then** you have a ticket.
- ○ **If** you have a ticket, **then** you can board the airplane.
- ○ You can board the airplane **if and only if** you have a ticket.

You can board the airplane **if and only if** you have a ticket.

You ca**n't** board the airplane **if and only if** you do**n't** have a ticket.

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# 1.5: Truth Tables, Conditionals, and De Morgan's Laws

Before we focus on truth tables, we're going to recall the symbols that are commonly used for *and*, *or*, and *not*.

### Symbols

The symbol $\wedge$ is used for **and**: $p$ and  is notated $p \wedge q$: $p \wedge q$ is called conjunction.

The symbol $\vee$ is used for **or**: $p$ or $q$ is notated $p \vee q$: $p \vee q$ is call disjunction.

The symbol $\sim$ is used for **not**: not $p$ is notated $\sim p$.

### Boolean Statement

A **Boolean statement** is a statement that can be evaluated as **either true or false,** and *only* true or false. It's the basis of logic, mathematics, and computer programming.

Compound Boolean statements can get tricky to think about. We can create a **truth table** to keep track of what truth values for the simple statements make the complex statement true and false.

### Truth Table

A table showing what the resulting truth value of a compound statement is for all the possible truth values of the simple statements.

While working on truth value and truth table, we write **T** for "True" and **F** for "False".

Suppose you're picking out a new couch, and your significant other says, "Get a sectional *or* something with a chaise".

This is a complex statement made of two simpler conditions: "is a sectional", and "has a chaise". For simplicity, let's use $p$ to designate "is a sectional", and $q$ to designate "has a chaise".

A truth table for this situation would look like this:

| $p$ | $q$ | $p$ or $q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

In the table, T is used for true, and F for false. In the first row, if $p$ is true and $q$ is also true, then the complex statement "$p$ or $q$" is true. This sectional also features a chaise, which meets our desire. **(Remember that *or* in logic is not exclusive; if the couch has both features, it meets the condition.)**

In the previous example about the couch, the truth table was really just summarizing what we already know about how the *or* statement works. The truth tables for the basic *and*, *or*, and *not* statements are shown below.

### Basic Truth Tables for Conjunction, Disjunction, and Negation

**Conjunction**

| $p$ | $q$ | $p \wedge q$ | |
|---|---|---|---|
| T | T | T | T $\wedge$ T : T |
| T | F | F | T $\wedge$ F : F |
| F | T | F | F $\wedge$ T : F |
| F | F | F | F $\wedge$ F : F |

So the conjunction $p \wedge q$ is only **true** when both $p$ and $q$ are true. And the conjunction $p \wedge q$ is **false** whenever at least one of the statements is false.

**Disjunction**

| $p$ | $q$ | $p \vee q$ | |
|---|---|---|---|
| T | T | T | T $\vee$ T : T |
| T | F | T | T $\vee$ F : T |
| F | T | T | F $\vee$ T : T |
| F | F | F | F $\vee$ F : F |

So the disjunction $p \vee q$ is **true** whenever at least one of the statements is true. And the disjunction $p \vee q$ is only **false** if both statements are false.

**Negation**

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

Truth tables really become useful when we analyze more complex Boolean statements.

**Example 1.5.1: Truth Table for Disjunction**

Create a truth table for the statement $p \vee \sim q$

**Answer**

When we create the truth table, we need to list all the possible truth value combinations for $p$ and $q$. Notice how the first column contains **2** Ts followed by **2 Fs**, and the second column alternates **T, F, T**, F. This pattern ensures that all **4** combinations are considered.

After creating columns with those initial values, we create a third column for the expression $\sim B$.

Next, we can find the truth values of $p \vee \sim q$, using the first and third columns.

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ |
|---|---|---|---|
| T | T | F | T |
| T | F | T | T |
| F | T | F | F |
| F | F | T | T |

The truth table shows that $p \vee \sim q$ is true in three cases and false in one case.

**Your Turn 1.5.1A: Complete Truth Table**

1. Truth values for "and" statements (conjunctions):

   a. The conjunction $(p \wedge q)$ is **only true** whenever Select an answer .

   b. The conjunction $(p \wedge q)$ is **false** whenever Select an answer .

2. Truth values for "or" statements (disjunctions):

   a. The disjunction $(p \vee q)$ is **true** whenever Select an answer .

   b. The disjunction $(p \vee q)$ is **only false** whenever Select an answer .

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**Your Turn 1.5.1B: Complete Truth Table**

**Complete the truth tables below:**

Truth table for $(\sim p \wedge q)$

| $p$ | $q$ | $\sim p$ | $\sim p \wedge q$ |
|---|---|---|---|
| T | T | ? | ? |

Truth table for $(\sim p \vee q)$

| $p$ | $q$ | $\sim p$ | $\sim p \vee q$ |
|---|---|---|---|
| T | T | ? | ? |

| | | | |
|---|---|---|---|
| T | F | ? | ? |
| F | T | ? | ? |
| F | F | ? | ? |

| | | | |
|---|---|---|---|
| T | F | ? | ? |
| F | T | ? | ? |
| F | F | ? | ? |

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**Example 1.5.2**: Truth Table with Three Statements

Create a truth table for the statement $p \wedge \sim (q \vee r)$.

**Answer**

It helps to work from the inside out when creating a truth table and to create columns in the table for intermediate operations. We start by listing all the possible truth value combinations for $p, q$, and $r$. Notice how the first column contains 4 Ts followed by **4Fs**, the second column contains **2Ts, 2Fs**, then repeats, and the last column alternates **T, F, T, F ...** This pattern ensures that all 8 combinations are considered. After creating columns with those initial values, we create a fourth column for the innermost expression, $q \vee r$. The fifth column would be for $\sim (q \vee r)$ and the last sixth column for $p \wedge \sim (q \vee r)$.

| $p$ | $q$ | $r$ | $q \vee r$ | $\sim (q \vee r)$ | $p \wedge \sim (q \vee r)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | T | F | T | F | F |
| T | F | T | T | F | F |
| T | F | F | F | T | T |
| F | T | T | T | F | F |
| F | T | F | T | F | F |
| F | F | T | T | F | F |
| F | F | F | F | T | F |

**MyOpenMath 1.5.2**: Complete Truth Table

**Complete the truth table below for the given logical expression:**

Truth table for $(p \wedge \sim q) \vee \sim r$

| $p$ | $q$ | $r$ | $\sim q$ | $p \wedge \sim q$ | $\sim r$ | $(p \wedge \sim q) \vee \sim r$ |
|---|---|---|---|---|---|---|
| T | T | T | ? | ? | ? | ? |
| T | T | F | ? | ? | ? | ? |
| T | F | T | ? | ? | ? | ? |
| T | F | F | ? | ? | ? | ? |
| F | T | T | ? | ? | ? | ? |
| F | T | F | ? | ? | ? | ? |
| F | F | T | ? | ? | ? | ? |
| F | F | F | ? | ? | ? | ? |

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## Conditional Statement and Truth Value

We discussed conditional statements earlier, in which we take an action based on the value of the condition. We are now going to examine another version of a conditional, sometimes referred to as an implication, which states that the second part must logically follow from the first. As we recall

A conditional is a logical compound statement in which a statement $p$, called the antecedent, implies a statement $q$, called the consequent.

A conditional is written as $p \to q$ and is translated as "if $p$, then $q$".

**Implication**

If P is true, then Q must also be true.

The conditional statement “If it is raining, then there are clouds in the sky” is an implication. It makes sense because if the antecedent “it is raining” is true, then the consequent “there are clouds in the sky” must also be true.

Notice that the statement tells us nothing of what to expect if it is not raining; there might be clouds in the sky, or there might not. If the antecedent is false, then the consequent becomes irrelevant.

In traditional logic, a conditional is considered true as long as there are no cases in which the antecedent is true and the consequent is false.

**Truth Table for the Conditional**

| $p$ | $q$ | $p \to q$ | |
|---|---|---|---|
| T | T | T | T → T : T |
| T | F | F | T → F : F |
| F | T | T | F → T : T |
| F | F | T | F → F : T |

So the conditional statement $p \to q$ is **false** only if the hypothesis is true and the conclusion is false. Otherwise, it is always **true**.

Again, if the antecedent $p$ is false, we cannot prove that the statement is a lie, so the result of the third and fourth rows is true.

**Your Turn 1.5.3: Complete Truth Table**

In a conditional statement, when the wording "If..., then..." is used, the Select an answer comes after "If...", and the Select an answer comes after "then...".

A conditional statement is only *false* when the hypothesis is Select an answer and the conclusion is Select an answer otherwise the conditional statement is *true*.

Fill out the following truth table using the rule you described above.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | ? |
| T | F | ? |
| F | T | ? |
| F | F | ? |

When the hypothesis is to the right of the conclusion within the truth table, we have to be careful. The rule says that a conditional statement is only *false* when the Select an answer is true and the Select an answer is false. It doesn't matter which one is on the left and which one is on the right.

Carefully fill out the following truth table using the rule you described above and making careful note of the

Carefully fill out the following truth table using the rule you described above and making careful note of the fact that the hypothesis is $q$ and the conclusion is $p$.

| $p$ | $q$ | $q \to p$ |
|---|---|---|
| T | T | ? |
| T | F | ? |
| F | T | ? |
| F | F | ? |

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**Example 1.5.4**

Suppose you order a team jersey online on Tuesday and want to receive it by Friday so you can wear it to Saturday's game. The website says that if you pay for expedited shipping, you will receive the jersey by Friday. In what situation is the website telling a lie?

There are four possible outcomes:

1) You pay for expedited shipping and receive the jersey by Friday

2) You pay for expedited shipping and don't receive the jersey by Friday

3) You don't pay for expedited shipping and receive the jersey by Friday

4) You don't pay for expedited shipping and don't receive the jersey by Friday

**Answer**

Only one of these outcomes proves that the website was lying: The second outcome in which you pay for expedited shipping but don't receive the jersey by Friday.

The first outcome is exactly what was promised, so there's no problem with that.

The third outcome is not a lie because the website never said what would happen if you didn't pay for expedited shipping; maybe the jersey would arrive by Friday, whether you paid for expedited shipping or not.

The fourth outcome is not a lie because, again, the website didn't make any promises about when the jersey would arrive if you didn't pay for expedited shipping.

It may seem strange that the third outcome in the previous example, in which the first part is false but the second part is true, is not a lie. Remember, though, that if the antecedent is false, we cannot make any judgment about the consequent. The website never said that paying for expedited shipping was the *only* way to receive the jersey by Friday.

**Your Turn 1.5.4: Analyze Truth Value**

Suppose the following statement is true: A friend tells you

**"If you upload that picture to Facebook, you'll lose your job."**

Under what conditions can you say that your friend was wrong?

- ○ You don't upload the picture and don't lose your job
- ○ You upload the picture and don't lose your job.
- ○ You upload the picture and lose your job
- ○ You don't upload the picture and lose your job

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**Example 1.5.5: Truth Table for Three Statements**

Construct a truth table for the statement $(p \wedge \sim q) \rightarrow r$

**Answer**

We start by constructing a truth table with 8 rows to cover all possible scenarios.

| $p$ | $q$ | $r$ | $\sim q$ | $p \wedge \sim q$ | $r$ | $(p \wedge \sim q) \rightarrow r$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T |
| T | T | F | F | F | F | T |
| T | F | T | T | T | T | T |
| T | F | F | T | T | F | F |
| F | T | T | F | F | T | T |
| F | T | F | F | F | F | T |
| F | F | T | T | F | T | T |
| F | F | F | T | F | F | T |

When $p$ is true, $q$ is false, and $r$ is false- -the fourth row of the table-- then the antecedent $p \wedge \sim q$ will be true but the consequent false, resulting in an invalid conditional; every other case gives a valid conditional.

If you want a real-life situation that could be modeled by $(p \wedge \sim q) \rightarrow r$, consider this: let $p =$ we order meatballs, $q =$ we order pasta, and $r =$ Rob is happy. The statement $(p \wedge \sim q) \rightarrow r$ is "if we order meatballs and don't order pasta, then Rob is happy". If $p$ is true (we order meatballs), $q$ is false (we don't order pasta), and $r$ is false (Rob is not happy), then the statement is false, because we satisfied the antecedent but Rob did not satisfy the consequent.

**Your Turn 1.5.5: Construct Truth Table**

Complete the truth table for the following compound statement.

$$(q \rightarrow p) \vee \sim r$$

| p | q | r | $\sim r$ | $q \rightarrow p$ | $(q \rightarrow p) \vee \sim r$ |
|---|---|---|---|---|---|
| T | T | T | ? | ? | ? |
| T | T | F | ? | ? | ? |
| T | F | T | ? | ? | ? |
| T | F | F | ? | ? | ? |
| F | T | T | ? | ? | ? |
| F | T | F | ? | ? | ? |
| F | F | T | ? | ? | ? |
| F | F | F | ? | ? | ? |

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## Converse, Inverse, and Contrapositive

For any conditional, there are three related statements: the converse, the inverse, and the contrapositive.

| Statements | Symbolic forms | |
|---|---|---|
| conditionals | **If $p$, then $q$**<br>$p \longrightarrow q$ | $p \longrightarrow q$ is equivalent to $\sim q \longrightarrow \sim p$<br>AND<br>$q \longrightarrow p$ is equivalent to $\sim p \longrightarrow \sim q$<br>In other words, the original statement and the contrapositive must agree with each other; they must both be true, or they must both be false. Similarly, the converse and the inverse must agree with each other; they must both be true, or they must both be false. |
| Converse | **If $q$, then $p$**<br>$q \longrightarrow p$ | |
| Inverse | **If not $p$, then not $q$**<br>$\sim p \longrightarrow \sim q$ | |
| Contrapositive | **If not $q$, then not $p$**<br>$\sim q \longrightarrow \sim p$ | |

Looking at truth tables, we can see that the original conditional and the contrapositive are logically equivalent, and that the converse and inverse are logically equivalent.

| | | Conditional | Converse | Inverse | Contrapositive |
|---|---|---|---|---|---|
| $p$ | $q$ | $p \to q$ | $q \to p$ | $\sim p \to \sim q$ | $\sim q \to \sim p$ |
| T | T | T | T | T | T |
| T | F | F | T | T | F |
| F | T | T | F | F | T |
| F | T | T | T | T | T |

Equivalent

Figure 1.5.1: Truth Table for Converse, Inverse, and Contrapositive

**Example 1.5.6: Equivalent Statements**

Consider again the conditional "If it is raining, then there are clouds in the sky." It seems reasonable to assume that this is true.

**Answer**

The converse would be "If there are clouds in the sky, then it is raining." This is not always true.

The inverse would be "If it is not raining, then there are no clouds in the sky." Likewise, this is not always true.

The contrapositive would be "If there are no clouds in the sky, then it is not raining." This statement is true and is equivalent to the original condition.

**Your Turn 1.5.6: Converse, Inverse, and Contrapositive**

Consider the conditional statement...

"*If Hopi does not eat all the pizza today, then he has leftovers tomorrow.*"

(a) Write the **converse** of the original statement:

If Select an answer , then Select an answer .

(b) Write the **inverse** of the original statement:

If Select an answer ,then Select an answer .

(c) Write the **contrapositive** of the original statement:

If Select an answer , then Select an answer .

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Be aware that symbolic logic cannot represent the English language perfectly. For example, we may need to change the verb tense to show that one thing occurred before another.

**Example 1.5.7: Find Equivalent Statement**

Suppose this statement is true: “If I eat this giant cookie, then I will feel sick.” Identify which of the following statements are the converse, inverse, and contrapositive. Which of the following statements must also be true?

a. If I feel sick, then I ate that giant cookie.
b. If I don’t eat this giant cookie, then I won’t feel sick.
c. If I don’t feel sick, then I didn’t eat that giant cookie.

**Answer**

First statement (a) is the converse, which is not necessarily true. I could feel sick for some other reason, such as drinking sour milk.

Second statement (b) is the inverse, which is not necessarily true. Again, I could feel sick for some other reason; avoiding the cookie doesn’t guarantee that I won’t feel sick.

Third statement (c) is the contrapositive, which is true, but we have to think somewhat backward to explain it. If I ate the cookie, I would feel sick, but since I don’t feel sick, I must not have eaten the cookie.

Notice again that the original statement and the contrapositive have the same truth value (both are true), and the converse and the inverse have the same truth value (both are false).

**Your Turn 1.5.7: Find Equivalent Statement**

If the following statement is true, which of the following statements must also be true?

**“"If you microwave salmon in the staff kitchen, then I will be mad at you."”**

- ○ If I am mad at you, then you microwaved salmon in the staff kitchen.
- ○ If I am not mad at you, then you didn't microwave salmon in the staff kitchen
- ○ If you don't microwave salmon in the staff kitchen, then I won't be mad at you.

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Consider the statement “If you park here, then you will get a ticket.” What set of conditions would prove this statement false?

a. You don’t park here and you get a ticket.
b. You don’t park here and you don’t get a ticket.
c. You park here and you don’t get a ticket.

The first two statements are irrelevant because we don’t know what will happen if you park somewhere else. The third statement, however, contradicts the conditional statement “If you park here, then you will get a ticket” because you parked here but didn’t get a ticket. This example demonstrates a general rule; the negation of a conditional can be written as a conjunction: “It is not the case that if you park here, then you will get a ticket” is equivalent to “You park here *and* you do not get a ticket.”

**The Negation of a Conditional Statement**

The negation of a conditional statement is logically equivalent to a conjunction of the antecedent and the negation of the consequent.

$\sim (p \rightarrow q)$ is equivalent to $p \wedge \sim q$

**Example 1.5.8: Negation of Conditional**

Which of the following statements is equivalent to the negation of "If you don't grease the pan, then the food will stick to it"?

a. I didn't grease the pan and the food didn't stick to it.
b. I didn't grease the pan and the food stuck to it.
c. I greased the pan and the food didn't stick to it.

**Answer**

This is correct; it is the conjunction of the antecedent and the negation of the consequent. To disprove that not greasing the pan will cause the food to stick, I have to not grease the pan and have the food not stick. This is essentially the original statement with no negation; the "if…then" has been replaced by "and". This essentially agrees with the original statement and cannot disprove it.

**Your Turn 1.5.8: Negation of Conditional**

Use the equivalency

$$\sim (p \rightarrow q) \equiv p \wedge \sim q$$

to select the statement that is logically equivalent to the ***negation of the conditional statement.***....

"*If it is warm, then Anushka goes swimming.*"

Select an answer | Select an answer | Select an answer .

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## Biconditional Statement and Truth Value

In everyday life, we often have a stronger meaning in mind when we use a conditional statement. Consider "If you submit your hours today, then you will be paid next Friday." What the payroll rep really means is "If you submit your hours today, then you will be paid next Friday, and if you don't submit your hours today, then you won't be paid next Friday." The conditional statement if *t*, then *p* also includes the inverse of the statement: if not *t*, then not *p*. A more compact way to express this statement is "You will be paid next Friday *if and only if* you submit your timesheet today." A statement of this form is called a **biconditional**.

**Biconditional Statement**

A biconditional is a logical conditional statement in which the antecedent and consequent are interchangeable.

A biconditional is written as $p \leftrightarrow q$ and is translated as "$p$ if and only if $q$".

Because a biconditional statement $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$, we may think of it as a conditional statement combined with its converse: if $p$, then $q$ and if $q$, then $p$. The double-headed arrow shows that the conditional statement goes from left to right and from right to left. A biconditional is considered true as long as the antecedent and the consequent have the same truth value; that is, they are either both true or both false.

**Truth Table for the Biconditional**

| $p$ | $q$ | $p \leftrightarrow q$ | |
|---|---|---|---|
| T | T | T | T ↔ T : T |
| T | F | F | T ↔ F : F |
| F | T | F | F ↔ T : F |
| F | F | T | F ↔ F : T |

So the biconditional statement $p \leftrightarrow q$ is **true** only if both statements have the same truth value.

Notice that the fourth row, where both components are false, is true; if you don't submit your timesheet and you don't get paid, the person from payroll told you the truth.

**Example 1.5.9: Analyzing Biconditional Statement**

Suppose this statement is true: “The garbage truck comes down my street if and only if it is Thursday morning.” Which of the following statements could be true?

1. It is Thursday and the garbage truck did not come down my street this morning.
2. It is Monday, and the garbage truck is coming down my street.
3. It is Wednesday at 11:59PM, and the garbage truck did not come down my street today.

**Answer**

1. This cannot be true. This is like the second row of the truth table; it is true that I just experienced Thursday morning, but it is false that the garbage truck came.
2. This cannot be true. This is like the third row of the truth table; it is false that it is Thursday, but it is true that the garbage truck came.
3. This could be true. This is like the fourth row of the truth table; it is false that it is Thursday, but it is also false that the garbage truck came, so everything worked out like it should.

**Your Trun 1.5.9: Is Biconditional Statement True or Not**

Suppose the following statement is true: **"I wear my running shoes if and only if I am exercising."**

Determine whether the following statements must be true.

- ○ I am wearing my running shoes and I am not exercising.
- ○ I am exercising and I am not wearing my running shoes.
- ○ I am not exercising and I am not wearing my running shoes

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A contemporary of Boole’s, Augustus De Morgan, formalized two rules of logic that had previously been known informally. They allow us to rewrite the negation of a conjunction as a disjunction, and vice-versa.

For example, suppose you want to schedule a meeting with two colleagues at **4 : 30 PM** on Friday, and you need both of them to be available at that time. What situation would make it impossible to have the meeting? It is NOT the case that colleague $a$ is available AND colleague $b$ is available: $\sim (a \wedge b)$. This situation is equivalent to either colleague $a$ NOT being available OR colleague $b$ NOT being available: $\sim a \vee \sim b$

**De Morgan's Laws**

The negation of a conjunction is equivalent to the disjunction of the negation of the statements making up the conjunction. To negate an “and” statement, negate each part and change the “and” to “or”.

$$\sim (p \wedge q) \text{ is equivalent to } \sim p \vee \sim q$$

The negation of a disjunction is equivalent to the conjunction of the negation of the statements making up the disjunction. To negate an “or” statement, negate each part and change the “or” to “and”.

$$\sim (p \vee q) \text{ is equivalent to } \sim p \wedge \sim q$$

**Note**

Do you know what the following are equivalent to each other?

1. $\sim (\sim p \wedge \sim q) \equiv p \vee q$
2. $\sim (p \vee \sim q) \equiv \sim p \wedge q$
3. $\sim [p \rightarrow (q \wedge r)] \equiv p \wedge \sim (q \wedge r) \equiv p \wedge (\sim q \vee \sim r)$

**Example 1.5.10: Using De Morgan Law**

Write the negation of the following statement

"If Amy changes her major, then she will have to take summer school and won't graduate next year."

**Answer**

Since this conditional statement's hypothesis has a conjunction, we use the negation of the conditional statement using

$\sim (p \rightarrow q)$ is equivalent to $p \wedge \sim q$, and then we use De Morgan Law in the second part of the previous negation (SEE NOTE 3 ABOVE).

Lets say: $p$ = change her major, $q$ = take summer school, and $r$ = won't graduate

Note 3 above says: $\sim [p \rightarrow (q \wedge r)] \equiv p \wedge (\sim q \vee \sim r)$. So negation would be

Amy changes his major, **and** she doesn't have to take summer school **or** she will graduate next year.

**Your Trun 1.5.10: Use De Morgan Law**

Select the **negation** for each of the following statements. The sun is shining or it is raining.

- ○ The sun is shining and it is not raining.
- ○ The sun is not shining and it is raining.
- ○ The sun is not shining or it is raining.
- ○ The sun is shining or it is not raining.
- ○ The sun is not shining and it is not raining.
- ○ None of these.

Bob works for the state and he is not getting a raise.

- ○ Bob does not work for the state and he is getting a raise.
- ○ Bob works for the state or he is not getting a raise.
- ○ Bob works for the state and he is getting a raise.
- ○ Bob works for the state or he is getting a raise.
- ○ Bob does not work for the state or he is getting a raise.
- ○ None of these.

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**Example 1.5.11: Find Truth Value**

Define the following statements:

$p$ = "The Earth orbits the Sun."

$q$ = "The chemical symbol for water is $H_2O$."

$r$ = "A triangle has four sides."

Evaluate the truth value of the following compound statement:

$$(p \vee \sim r) \leftrightarrow (\sim q \rightarrow r)$$

**Answer**

$p$ is True, $q$ is True, and $r$ is False.

$$\begin{aligned} (T \vee \sim F) &\leftrightarrow (\sim T \rightarrow F) \\ (T \vee T) &\leftrightarrow (F \rightarrow F) \\ T &\leftrightarrow T \\ T & \end{aligned} \tag{1.5.1}$$

**Your Turn 1.5.11: Determining the Truth Value**

Define the following statements:

- **p** = "The boiling point of water at sea level is 100°C."
- **q** = "Bananas are yellow when ripe."
- **r** = "The chemical symbol for gold is $H_2O$."

Evaluate the truth value of the following compound statement:

$$(\sim p \rightarrow q) \wedge \sim r$$

Select the correct answer:

- ○ False
- ○ True

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**Example 1.5.12: Find the Truth Value of Compound Statement**

Consider the following compound statements. Use appropriate entries from truth tables to determine whether each statement is **true or false.**

1. If Mount Everest is the tallest mountain, then cats are mammals.
2. If cows eat grass, then apples are blue.
3. Tokyo is the largest city in Japan, and cats are amphibious.
4. The Mona Lisa was painted by Leonardo da Vinci, or all odd numbers are divisible by 2.

**Answer**

1. $T \rightarrow T : T$, so it is True.
2. $T \rightarrow F : F$, so it is False
3. $T \wedge F : F$, so it is False.
4. $T \vee F : T$, so it is True.

**Your Trun 1.5.12: Determining the Truth Value**

Consider the following compound statements. Use appropriate entries from truth tables to determine whether each statement is **true or false.**

a) If the Earth orbits the Sun, then 2 + 2 equals 4.

True ○ False ○

b) If pigs have tails, then humans can fly.

True ○ False ○

c) Water freezes at 0 degrees Celsius if and only if ice is solid.

True False

d) Fish live in treehouses if and only if the Eiffel Tower is in Paris.

True False

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# 1.6: Analyzing Deductive Argument

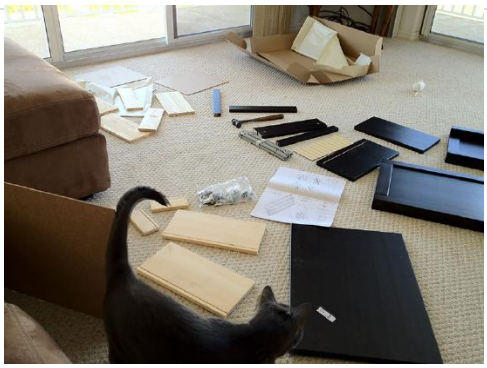


Figure **1.6.1**: When assembling furniture, instructions with images are easier to follow, just like how set relationships are easier to understand when depicted graphically. (credit: "Time to assemble more Ikea furniture!" by Rod Herrea/Flickr, CC BY 2.0)

Have you ever ordered a new dresser or bookcase that required assembly? When your package arrives, you excitedly open it and spread out the pieces. Then you check the assembly guide and verify that you have all the parts required to assemble your new dresser. Now, the work begins. Luckily for you, the assembly guide includes step-by-step instructions with images that show you how to put together your product. If you are really lucky, the manufacturer may even provide a URL or QR code connecting you to an online video that demonstrates the complete assembly process. We can likely all agree that assembly instructions are much easier to follow when they include images or videos, rather than just written directions. The same goes for the relationships between sets.

## Creating Venn Diagrams (Euler Diagram)

The main purpose of a Venn diagram is to help you visualize the relationship between sets. As such, it is necessary to be able to draw Venn diagrams from a written or symbolic description of the relationship between sets.

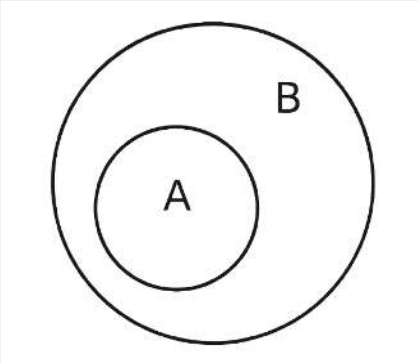



Figure **1.6.1**: All A is B Or If it A, then it is B.

We can interpret the above Euler Diagram in three different ways

- If it is $A$, then it is $B$.
- All $A$ are $B$.
- If it is not $B$, then it can not be $A$

Note:

- In a logical sense, "Some" Means "at least one" (and potentially all). In the colloquial sense, "Some" Usually implies "some, but not all."

  In logic if In logic, if you say "Some of the stars are hot," you are technically correct even though all of them are hot.

**Example 1.6.1**: All A are B

Draw a Venn diagram to represent the relationship between each of the sets.

1. All rectangles are parallelograms.
2. If she is a woman, then she is a person.

**Answer**

1. The set of rectangles is a subset of the set of parallelograms.

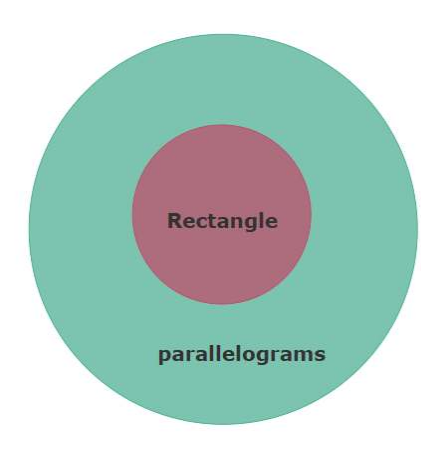



Figure **1.6.2**: All rectangles are parallelogram

Additionally, we can say, "If it is a rectangle, it is also a parallelogram." OR "If it is not a parallelogram, it can not be a rectangle".

2. The set of all women is a subset of the set of people.

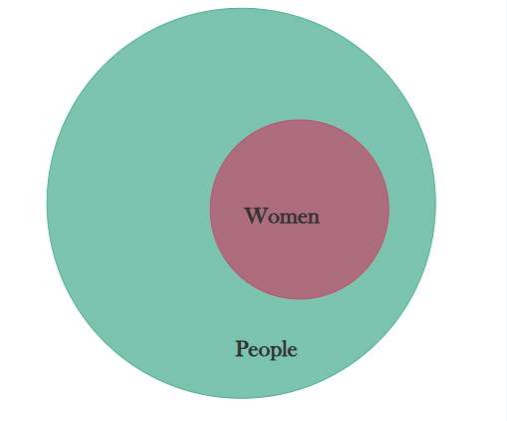
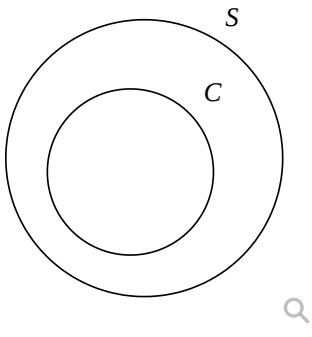


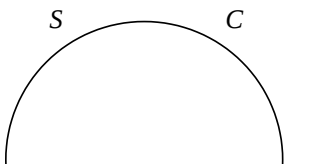

Figure **1.6.3**: All women are people

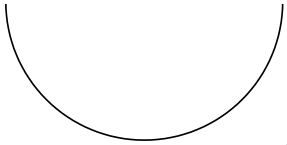
**Your Turn 1.6.1**: All A are B

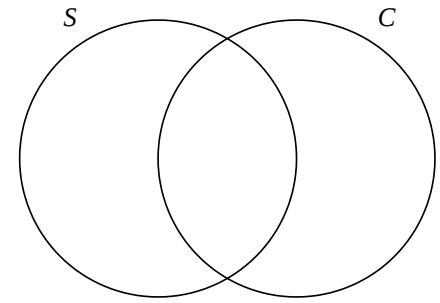
Which diagrams below *could* we have if the statement below is true?

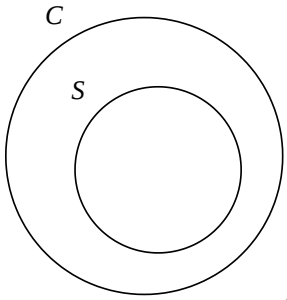
No sociology majors ever cut class.
S -- sociology majors
C -- people who cut class

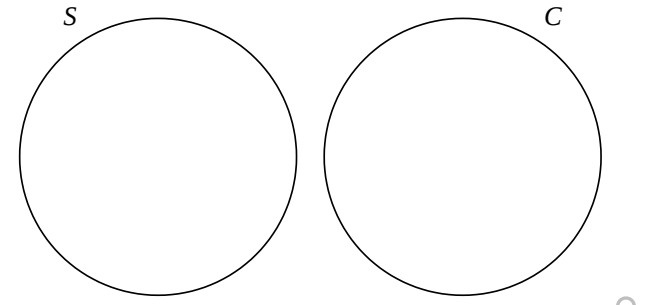







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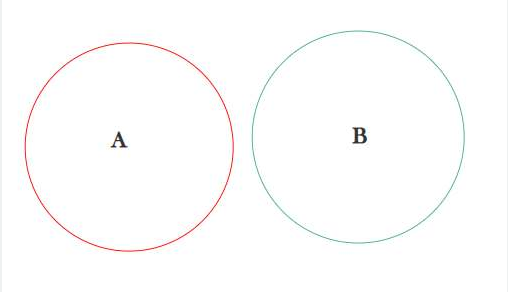



Figure **1.6.4**: No A is B

We can interpret the above Euler Diagram in the following different ways

- No $\boldsymbol{A}$ is $\boldsymbol{B}$.
- If it is $\boldsymbol{A}$, then it can not be $\boldsymbol{B}$.

**Example 1.6.2: Some A are B**

All bicycles and all cars have wheels, but no bicycle is a car. Draw a Venn diagram to represent this relationship.

**Answer**

These two sets are disjoint and must be drawn as two circles that do not touch or overlap.

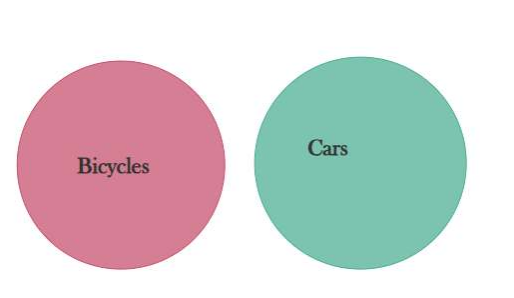
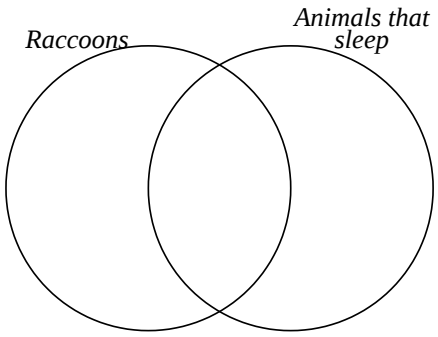



Figure 1.6.5: No bicycles are Cars

Also, we can say "If it is a bicycle, it can not be a car".

**Your Turn 1.6.2: Some A are B**




Select the statement that best describes the relationship depicted above.

- ○ If it is a raccoon, then it can not sleep.
- ○ All raccoons sleep.
- ○ No raccoons sleep.
- ○ If it is a raccoon, then it can sleep.
- ○ If it can not sleep, then it is not a raccoon.
- ○ Some raccoons sleep.

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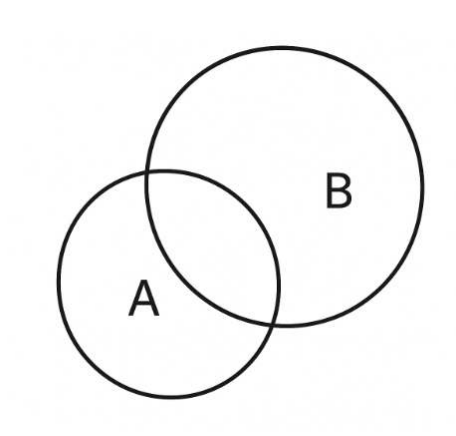



Figure **1.6.6**: Soms A are B and vice versa

We can interpret the above Euler Diagram in the following different ways

- Some $A$ are $B$.
- Some $A$ are not $B$

**Example 1.6.3**: Some A are not B

Draw a Venn diagram to represent this relationship.

1. Some oranges are not citrus fruits.
2. Some men are doctors.

**Answer**

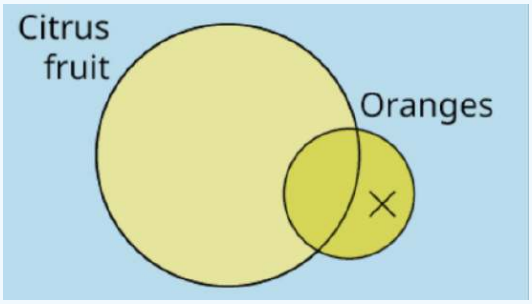



Figure **1.6.7**: Some oranges are not citrus fruit

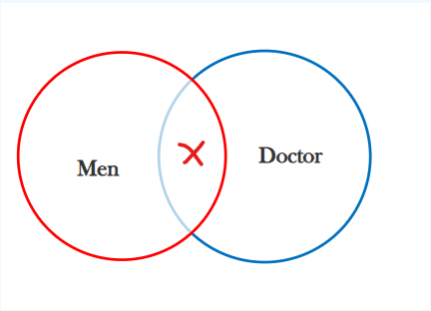



Figure **1.6.8**: Some men are doctor

## Arguments

**Definition: Arguments**

In logic and philosophy, an **argument** is not a disagreement or a shouting match. Instead, it is a structured set of statements where some statements (the **premises**) are intended to provide support or evidence for another statement (the **conclusion**).

A logical argument is a claim that a set of premises supports a conclusion. There are two general types of arguments: inductive and deductive arguments.

| Inductive Argument | Deductive Argument |
|---|---|
| An **inductive** argument uses a collection of specific examples as its premises and uses them to propose a general conclusion.<br>**Inductive:** Starts with **specific observations** → moves to a **general rule** | A **deductive** argument uses a collection of general examples as its premises and uses them to propose a specific conclusion.<br>**Deductive:** Starts with a **general example** →, moves to a **specific case**. |
| **Premise:** I forgot my purse last week.<br>**Premise:** I forgot my purse today.<br>**Conclusion:** I always forget my purse. | **Premise:** All humans are mortal.<br>**Premise:** Socrates is a human.<br>**Conclusion:** Therefore, Socrates is mortal. |
| **Premise:** The first three movies by this director were excellent.<br>**Premise:** The fourth movie was also excellent.<br>**Conclusion:** Therefore, all movies by this director are likely excellent. | **Premise:** If it is raining, then the ground is wet.<br>**Premise:** the ground is wet.<br>**Conclusion:** Therefore, It is raining. |

### Evaluating Inductive Arguments

An inductive argument is never able to prove the conclusion true, but it can provide either weak or strong evidence to suggest that it may be true.

Many scientific theories, such as the Big Bang theory, can never be proven. Instead, they are inductive arguments supported by a wide variety of evidence. Typically in science, an idea is considered a hypothesis until it has been thoroughly tested, at which point it is considered a theory. Common scientific theories, such as Newton's theory of gravity, have withstood years of testing and evidence, although they sometimes require adjustments based on new evidence, as seen with Einstein's proposal of the theory of general relativity.

### Your Turn 1.6.4: Inductive or Deductive?

Which reasoning process (inductive or deductive) is shown in each of the following?

Argument 1: A anthropology course policy states that if you turn in at least 70% of the homework, your lowest exam grade will be dropped. I turned in 80% of the homework in that anthropology course, so my lowest exam grade will be dropped.

Argument 2: All triangles have three sides. All squares have four sides. Therefore, there can't be a shape that is both a square and a triangle.

- ○ Argument 1: Inductive; Argument 2: Inductive
- ○ Argument 1: Deductive; Argument 2: Inductive
- ○ Argument 1: Inductive; Argument 2: Deductive
- ○ Argument 1: Deductive; Argument 2: Deductive

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A deductive argument is either clearly valid or not, which makes it easier to evaluate.

### Evaluating Deductive Arguments

A deductive argument is considered **valid** if, assuming that all the premises are true, the conclusion follows logically from those premises. In other words, when the premises are all true, the conclusion must be true. A valid argument cannot have true premises and a false conclusion.
A valid argument is called **sound** if all of the premises are true.

An argument that is not valid is referred to as a **fallacy**.

### Analyzing Deductive Argument with Euler Diagram

To analyze an argument with an Euler diagram:

1. Draw an Euler diagram based on the first premises of the argument
2. Draw an Euler diagram for the second premise on top of one of the first premises.
3. The argument is valid if and only if every possible diagram justifies the conclusion of the argument.
4. The argument is invalid if and only if (even one) possible diagram contradicts the conclusion.

### Example 1.6.5: Valid

Consider the deductive argument "All cats are mammals and a tiger is a cat, so a tiger is a mammal." Is this argument valid?

**Answer**

**Premise:** **All cats are mammals.**
**Premise:** **A tiger is a cat.**

---

**Conclusion:** **A tiger is a mammal.**

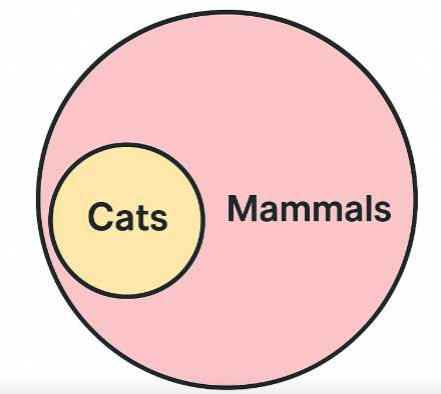



Figure 1.6.9: VALID

Both premises are true. To see that the premises must logically lead to the conclusion, we can use a Venn diagram. From the first premise, we draw the set of cats as a subset of the set of mammals. From the second premise, we are told that a tiger is contained within the set of cats. From that, we can see in the Venn diagram that the tiger must also be inside the set of mammals, so the conclusion is valid.

### Example 1.6.6: Valid

Determine if the conclusion follows logically from the premises.

**Premise:** **All children are energetic.**
**Premise:** **Some actors are children.**

---

**Conclusion:** **Some actors are energetic.**

**Answer**

For the first premise: Children's circle is inside an energetic people circle. For the second premise, the children's circle intersects with the actor circle. The final diagram clearly says that some actors are energetic (CONCLUSION). So the argument is valid.

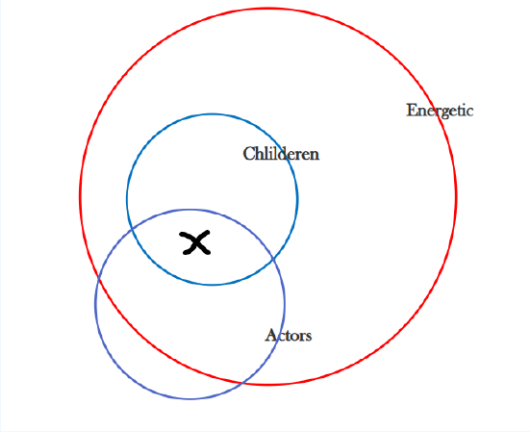



Figure **1.6.10**: VALID

**Example 1.6.7**: Invalid

Determine the validity of this argument:

**Premise:** **All firefighters know CPR.**
**Premise:** **Jill knows CPR.**

---

**Conclusion:** **Jill is a firefighter.**

**Answer**

From the first premise, we know that firefighters all lie inside the set of those who know CPR. (Firefighters are a subset of people who know CPR.) From the second premise, we know that Jill is a member of that larger set, but we do not have enough information to know whether she also is a member of the smaller subset that is firefighters.

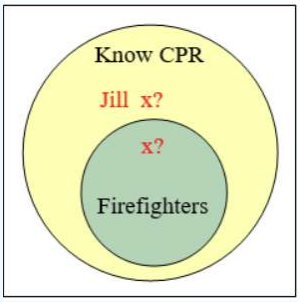



Figure **1.6.11**: Invalid

Since the conclusion does not necessarily follow from the premises, this is an invalid argument. It's possible that Jill is a firefighter, but the structure of the argument doesn't allow us to conclude that she definitely is.

It is important to note that whether or not Jill is actually a firefighter is not important in evaluating the validity of the argument; we are concerned with whether the premises are enough to prove the conclusion.

**Example 1.6.8**: Invalid

Determine the validity of this argument:

**Premise:** **No cows are purple.**
**Premise:** **Fido is not a cow.**

---

**Conclusion:** **Fido is purple.**

**Answer**

Invalid.

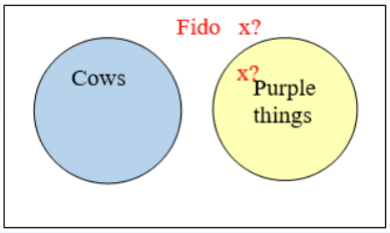



Figure **1.6.12**: INVALID

Since no cows are purple, we know there is no overlap between the set of cows and the set of purple things. We know Fido is not in the cow set, but that is not enough to conclude that Fido is in the purple things set.

**Example 1.6.9**: Invalid

**Premise:** **If you live in Seattle, you live in Washington.**
**Premise:** **Marcus does not live in Seattle.**

---

**Conclusion:** **Marcus does not live in Washington.**

**Answer**

From the first premise, we know that the set of people who live in Seattle is inside the set of those who live in Washington. From the second premise, we know that Marcus does not live in the Seattle set, but we have insufficient information to know whether Marcus lives in Washington or not. This is an invalid argument.

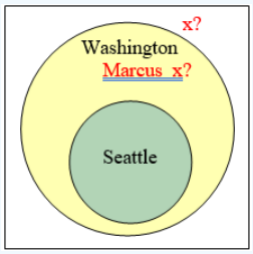



Figure **1.6.13**: INVALID

**Your Turn 1.6.10**: Valid or Invalid

*Determine if the conclusion follows logically from the premises.*

All children are energetic.
Some actors are children.
Therefore, some actors are energetic.

- ○ Valid argument
- ○ Invalid argument

All nerds wear glasses.
Grandpa wears glasses.
Therefore, Grandpa is a nerd.

- ○ Valid argument
- ○ Invalid argument

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## Standard Forms of Valid Arguments

Rather than making a truth table for every argument, we may be able to recognize certain common forms of arguments that are valid (or invalid). If we can determine that an argument fits one of the common forms, we can immediately state whether it is valid or invalid.

### Direct Reasoning (*Modus Ponens*)

The general form is:

**Premise:** $p \to q$
**Premise:** $p$
**Conclusion:** $q$

The Latin name, *modus ponens*, translates to “mode that affirms”.

**Example 1.6.11: Valid**

Recall this argument from an earlier example:

**Premise:** **If you bought bread, then you went to the store.**
**Premise:** **You bought bread.**
**Conclusion:** **You went to the store.**

**Answer**

In symbolic form:

**Premise:** $b \to s$
**Premise:** $b$
**Conclusion:** $s$

This argument has the structure described by the law of detachment. (The second premise and the conclusion are simply the two parts of the first premise detached from each other.) Instead of making a truth table, we can say that this argument is valid by stating that it satisfies the law of detachment.

You can also see the validity of the above argument by drawing an Euler Diagram.

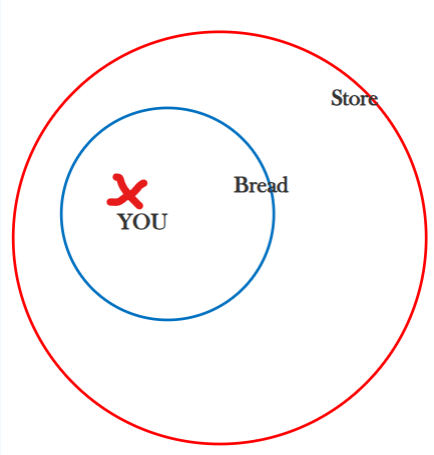



Figure **1.6.14**: VALID

### Contrapositive Reasoning (*Modus Tollens*)

The general form is:

**Premise:** $p \to q$
**Premise:** $\sim q$
**Conclusion:** $\sim p$

The Latin name, *modus tollens*, translates to “mode that denies”.

Notice that the second premise and the conclusion look like the contrapositive of the first premise, $\sim q \rightarrow \sim p$, but they have been detached. You can think of the law of contraposition as a combination of the law of detachment and the fact that the contrapositive is logically equivalent to the original statement.

**Example 1.6.12: Valid**

Determine the validity of this argument:

**Premise:** If I drop my phone into the swimming pool, my phone will be ruined.
**Premise:** My phone isn't ruined.

**Conclusion:** I didn't drop my phone into the swimming pool.

**Answer**

If we let $d =$ I drop the phone in the pool and $r =$ the phone is ruined, then we can represent the argument this way:

**Premise:** $d \rightarrow r$
**Premise:** $\sim r$

**Conclusion:** $\sim d$

The form of this argument matches what we need to invoke the contrapositive reasoning, so it is a valid argument.

You can also see the validity of the above argument by drawing an Euler Diagram.

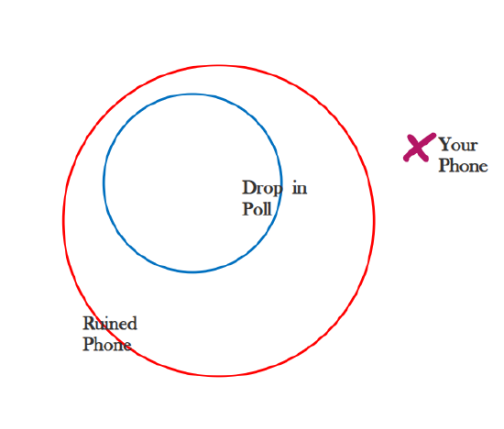



Figure 1.6.15: VALID

**Note**

Keep in mind that, when you are determining the validity of an argument, you must assume that the premises are true. If you don't agree with one of the premises, you need to keep your personal opinion out of it. Your job is to pretend that the premises are true and then determine whether they force you to accept the conclusion. You may attack the premises in a court of law or a political discussion, of course, but here we are focusing on the structure of the arguments, not the truth of what they actually say.

**Disjunctive Reasoning**

In a disjunctive reasoning, the premises consist of an *or* statement and the negation of one of the options. The conclusion is the other option. The general form is:

**Premise:** $p \vee q$
**Premise:** $\sim p$

**Conclusion:** $q$

Note: The order of the two parts of the disjunction isn't important. In other words, we could have the premises $p \vee q$ and $\sim q$, and the conclusion $p$.

**Example 1.6.13: Valid**

Determine the validity of this argument:

**Premise:** I can either drive or take the train.
**Premise:** I refuse to drive.

**Conclusion:** I will take the train.

**Answer**

If we let $p$ = I drive and $q$ = take the train, then the symbolic representation of the argument is:

\(

$$\begin{array}{ll} \textbf{Premise:} & p \vee q \\ \textbf{Premise:} & \sim p \\ \hline \textbf{Conclusion:} & q \end{array} \qquad (1.6.1)$$

This argument is valid because it has the form of a disjunctive reasoning. I have two choices, and one of them is not going to happen, so the other one must take its place.

### The Transitive Property (Hypothetical Syllogism)

The transitive property has as its premises a series of conditionals, where the consequent of one is the antecedent of the next. The conclusion is a conditional with the same antecedent as the first premise and the same consequent as the final premise. The general form is:

$$\begin{array}{ll} \textbf{Premise:} & p \rightarrow q \\ \textbf{Premise:} & q \rightarrow r \\ \hline \textbf{Conclusion:} & p \rightarrow r \\ & \sim r \rightarrow \sim p \end{array}$$

The earlier example about buying a shirt at the mall is an example illustrating the transitive property. It describes a chain reaction: if the first thing happens, then the second thing happens, and if the second thing happens, then the third thing happens. Therefore, if we want to ignore the second thing, we can say that if the first thing happens, then we know the third thing will happen. We don't have to mention the part about buying jeans; we can simply say that the first event leads to the final event. We could even have more than two premises; as long as they form a chain reaction, the transitive property will give us a valid argument.

### Example 1.6.14: Valid

Determine the validity of this argument:

$$\begin{array}{ll} \textbf{Premise:} & \textbf{If a soccer player commits a reckless foul, she will receive a yellow card.} \\ \textbf{Premise:} & \textbf{If Hayley receives a yellow card, she will be suspended for the next match.} \\ \hline \textbf{Conclusion:} & \textbf{If Hayley commits a reckless foul, she will be suspended for the next match.} \end{array}$$

**Answer**

If we let $r =$ committing a reckless foul, $y =$ receiving a yellow card, and $s =$ being suspended, then our argument looks like this:

$$\begin{array}{ll} \textbf{Premise:} & r \rightarrow y \\ \textbf{Premise:} & y \rightarrow s \\ \hline \textbf{Conclusion:} & r \rightarrow s \end{array}$$

This argument has the exact structure required to use the transitive property, so it is a valid argument.}\)

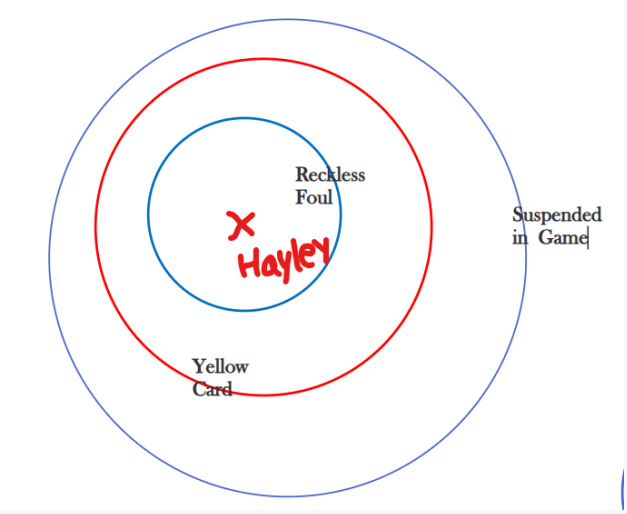



Figure 1.6.16: VALID

**Example 1.6.15: Valid**

Determine the validity of this argument:

**Premise:** If I work hard, I'll get a raise.
**Premise:** If I get a raise, I'll buy a boat.

---

**Conclusion:** If I don't buy a boat, I must not have worked hard.

**Answer**

If we let $p$ work hard, $q$ get a raise, and $r$= buying a boat, then we can represent our argument symbolically:

**Premise:** $p \rightarrow q$
**Premise:** $q \rightarrow r$

---

**Conclusion:** $\sim r \rightarrow \sim p$

Using the transitive property with the two premises, we can conclude that $p \rightarrow r$; if I work hard, then I will buy a boat. When we learned about the contrapositive, we saw that the conditional statement $p \rightarrow r$ is equivalent to $\sim r \rightarrow \sim p$. Therefore, the conclusion is indeed a logical syllogism derived from the premises.

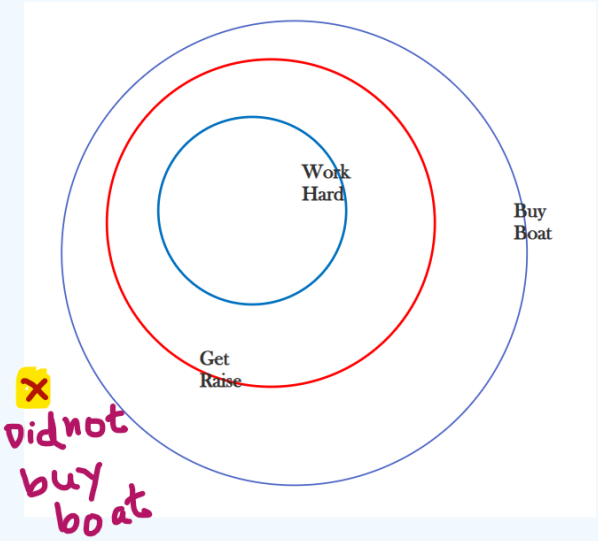



Figure 1.6.17: VALID

**Your Turn 1.6.15: Draw a Valid Conclusion**

**Supply a conclusion to make each argument valid.**

1. Bill will go to the movies or Sam will play golf.
   Sam does not play golf.
   Select an answer
2. If you pass the exam, then you will pass the course.
   If you pass the course, then you can sell your book.
   Select an answer
3. If it snows, then the ballgame will be rescheduled.
   The ballgame is not rescheduled.
   Select an answer
4. All sailors are big spenders.
   No big spenders are miserly.
   Select an answer

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**Let us summarize the four standard forms of valid argument presented above.**

| Direct Reasoning (*Modus Ponens*) | Contrapositive Reasoning(*Modus Tollens*) | Disjunctive Reasoning | The Transitive Property (Hypothetical Syllogism) |
|---|---|---|---|
| Premise: $p \to q$<br>Premise: $p$<br>Conclusion: $q$ | Premise: $p \to q$<br>Premise: $\sim q$<br>Conclusion: $\sim p$ | Premise: $p \vee q$<br>Premise: $\sim p$<br>Conclusion: $q$ | Premise: $p \to q$<br>Premise: $q \to r$<br>Conclusion: $p \to r$<br>$\sim r \to \sim p$ |

We have just looked at four forms of valid arguments; there are also four common forms that represent *invalid* arguments, which are also called *fallacies*.

## Standard Form of Invalid Arguments

### The Fallacy of the Converse

The fallacy of the converse arises when a conditional and its consequent are given as premises, and the antecedent is the conclusion. The general form is:

Premise: $p \to q$
Premise: $q$
Conclusion: $p$

Notice that the second premise and the conclusion look like the converse of the first premise, $q \to p$, but they have been detached. The fallacy of the converse incorrectly tries to assert that the converse of a statement is equivalent to that statement.

### Example 1.6.16: Invalid

Determine the validity of this argument:

**Premise:** **If you drink coffee after noon, then you have a hard time falling asleep that night.**
**Premise:** **You had a hard time falling asleep last night.**
**Conclusion:** **You drank coffee after noon yesterday.**

**Answer**

If we let $c =$ You drink coffee after noon and $h =$ You have a hard time falling asleep, then our argument looks like this:

Premise: $c \to h$
Premise: $h$
Conclusion: $c$

This argument employs converse reasoning, rendering it an invalid argument. There could be plenty of other reasons why you couldn't fall asleep: you could be worried about money, or your neighbors might have been setting off fireworks.

You can also see the validity of the above argument by drawing an Euler Diagram.

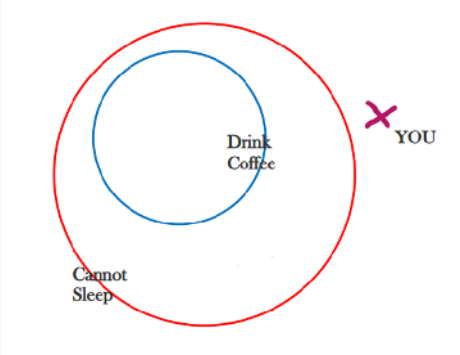



Figure 1.6.18: INVALID

## The Fallacy of the Inverse

The fallacy of the inverse occurs when a conditional and the negation of its antecedent are given as premises, and the negation of the consequent is the conclusion. The general form is:

**Premise:** $p \to q$
**Premise:** $\sim p$
**Conclusion:** $\sim q$

Again, notice that the second premise and the conclusion look like the inverse of the first premise, $\sim p \to \sim q$, but they have been detached. The fallacy of the inverse incorrectly asserts that the inverse of a statement is equivalent to that statement.

### Example 1.6.17: Invalid

Determine the validity of this argument:

**Premise:** If you listen to the Grateful Dead, then you are a hippie.
**Premise:** Sky doesn't listen to the Grateful Dead.
**Conclusion:** Sky is not a hippie.

**Answer**

If we let $g =$ listen to the Grateful Dead and $h =$ is a hippie, then this is the argument:

**Premise:** $g \to h$
**Premise:** $\sim g$
**Conclusion:** $\sim h$

This argument is invalid because it uses inverse reasoning. The first premise does not imply that all hippies listen to the Grateful Dead; there could be some hippies who listen to Phish instead.

You can also see the validity of the above argument by drawing an Euler Diagram.

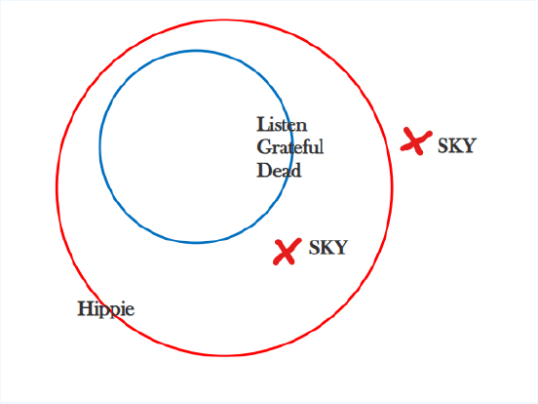



Figure 1.6.19: INVALID

## Misuse of Transitive Property

**Premise:** $p \to q$
**Premise:** $q \to r$
**Conclusion:** $r \to p$

This argument is not valid.

### Example 1.6.18: Invalid

Determine the validity of this argument:

**Premise:** If the old lady swallows a fly, she will swallow a spider.
**Premise:** If the old lady swallows a spider, she will swallow a bird.
**Conclusion:** If she will swallow a bird, she will swallow a fly.

**Answer**

If we let $p =$ the old lady swallows a fly, $q =$ she will swallow a spider, and $r =$ she will swallow a bird, then our argument looks like this:

**Premise:** $p \to q$
**Premise:** $q \to r$

---

**Conclusion:** $r \to p$

This argument has the exact structure of misuse of the transitive property, so it is an Invalid argument.

You can also see the validity of the above argument by drawing an Euler Diagram.

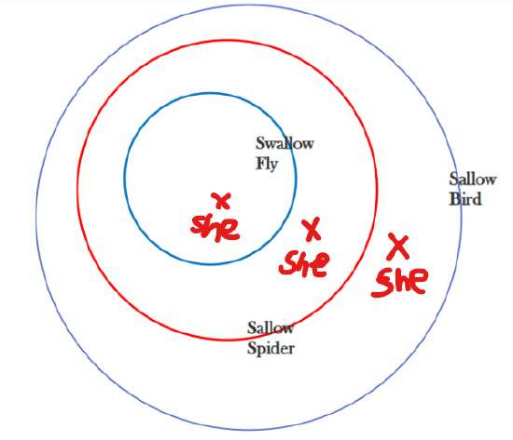



Figure **1.6.20**: INVALID

## Misuse of Disjunctive Reasoning

In a disjunctive syllogism, the premises consist of an *or* statement and the negation of one of the options. The conclusion is the other option. The general form is:

**Premise:** $p \vee q$
**Premise:** $p$

---

**Conclusion:** $\sim q$

The order of the two parts of the disjunction isn't important. In other words, we could have the premises $p \vee q$ and $q$, and the conclusion $\sim p$

### Example **1.6.19**: Invalid

Determine the validity of this argument:

**Premise:** **I can either drive or take the train.**
**Premise:** **I drive.**
**Conclusion:** **I did not take the train.**

**Answer**

If we let $p = I$ drive and $q = I$ take the train, then the symbolic representation of the argument is:

**Premise:** $p \vee q$
**Premise:** $p$

---

**Conclusion:** $\sim q$

This argument is invalid because it has the form of a misuse of disjunctive syllogism.

**Let us summarize the four standard forms of invalid argument presented above.**

| The Fallacy of the Converse | The Fallacy of the Inverse | Misuse of Disjunctive Reasoning | Misuse of Transitive Property |
|---|---|---|---|
| **Premise:** $p \to q$<br>**Premise:** $q$<br>**Conclusion:** $p$ | **Premise:** $p \to q$<br>**Premise:** $\sim p$<br>**Conclusion:** $\sim q$ | **Premise:** $p \vee q$<br>**Premise:** $p$<br>**Conclusion:** $\sim q$ | **Premise:** $p \to q$<br>**Premise:** $q \to r$<br>**Conclusion:** $r \to p$<br>$\sim p$<br>$\to \sim r$ |

**Example 1.6.20: Invalid**

Is this argument valid?

**Premise:** If I go to the party, I'll be really tired tomorrow.
**Premise:** If I go to the party, I'll get to see friends.
**Conclusion:** If I don't see friends, I won't be tired tomorrow.

**Answer**

Let $p =$ go to party, $t =$ be tired, and $f =$ see friends.

**Premise:** $p \rightarrow t$
**Premise:** $p \rightarrow f$

**Conclusion:** $\sim f \rightarrow \sim t$

We could try to rewrite the second premise using the contrapositive to state $\sim f \rightarrow \sim p$, but that does not allow us to form a syllogism. If I don't see friends, then I didn't go to the party, but that is not sufficient to claim I won't be tired tomorrow. Maybe I stayed up all night watching movies.

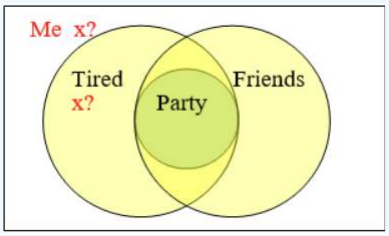



Figure 1.6.23: INVALID

A Venn diagram can help if we set it up correctly. The "party" circle must be completely contained within the intersection of the other circles. We know that I am somewhere outside the "friends" circle, but we cannot determine whether I am in the "tired" circle. All we really know for sure is that I didn't go to the party.

**Your Turn 1.6.20: Valid or Invalid**

**If I vote, then my candidate will win.**
**My Candidate won**

**Therefore, I voted**

Identify the common form of the argument or fallacy.

- ○ Direct Reasoning
- ○ Contrapositive Reasoning
- ○ Transitive Reasoning
- ○ Disjunctive Reasoning
- ○ Fallacy of the Converse
- ○ Fallacy of the Inverse
- ○ Misuse of Transitive Reasoning
- ○ Misuse of Disjunctive Reasoning

Determine whether the argument is valid or invalid.

- ○ Valid
- ○ Invalid

---

**If I am social, then I like parties.**
**I do not like the party,**

**Therefore, I am not social**

Identify the common form of the argument or fallacy.

- ○ Direct Reasoning
- ○ Contrapositive Reasoning
- ○ Fallacy of the Converse
- ○ Fallacy of the Inverse

- ○ Transitive Reasoning
- ○ Misuse of Transitive Reasoning
- ○ Disjunctive Reasoning
- ○ Misuse of Disjunctive Reasoning

Determine whether the argument is valid or invalid.

- ○ Valid
- ○ Invalid

---

**If Eulid Kepchoge runs the 2018 Berlin Marathon, then he will set a new marathon world record.**
**Eliud Kipchoge ran the 2018 Berlin marathon.**

**Therefore, he set a new world record in the marathon.**

Identify the common form of the argument or fallacy.

- ○ Direct Reasoning
- ○ Fallacy of the Converse
- ○ Contrapositive Reasoning
- ○ Fallacy of the Inverse
- ○ Transitive Reasoning
- ○ Misuse of Transitive Reasoning
- ○ Disjunctive Reasoning
- ○ Misuse of Disjunctive Reasoning

Determine whether the argument is valid or invalid.

- ○ Valid
- ○ Invalid

---

**If Aisha enjoys music class, then she will take guitar lessons.**
**Aisha did not enjoy music class.**

**Therefore, she did not take guitar lessons.**

Identify the common form of the argument or fallacy.

- ○ Direct Reasoning
- ○ Fallacy of the Converse
- ○ Contrapositive Reasoning
- ○ Fallacy of the Inverse
- ○ Transitive Reasoning
- ○ Misuse of Transitive Reasoning
- ○ Disjunctive Reasoning
- ○ Misuse of Disjunctive Reasoning

Determine whether the argument is valid or invalid.

- ○ Valid
- ○ Invalid

---

**Batman will defeat the Joker or he did not see the Bat singal.**
**Batman defeated Joker.**

**Therefore, Batman did see the Bat singnal.**

Identify the common form of the argument or fallacy.

- ○ Direct Reasoning
- ○ Fallacy of the Converse

- ○ Direct Reasoning
- ○ Fallacy of the Converse
- ○ Contrapositive Reasoning
- ○ Fallacy of the Inverse
- ○ Transitive Reasoning
- ○ Misuse of Transitive Reasoning
- ○ Disjunctive Reasoning
- ○ Misuse of Disjunctive Reasoning

Determine whether the argument is valid or invalid.

○ Valid ○ Invalid

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## 1.7: Fallacies in Reasoning

In the previous discussion, we saw that logical arguments can be invalid when the premises are not true, when the premises are not sufficient to guarantee the conclusion, or when there are invalid chains in logic. There are a number of other ways in which arguments can be invalid, a sampling of which are given here.

### Ad hominem (Personal Attack)

An ad hominem argument attacks the person making the argument, ignoring the argument itself.

Disagreeing by insulting the person — not by proving their argument wrong

**Example 1.7.1**

"Jane says that whales aren't fish, but she's only in the second grade, so she can't be right."

**Answer**

Here, the argument is attacking Jane, not the validity of her claim, so this is an ad hominem argument..

**Example 1.7.2**

"Jane says that whales aren't fish, but everyone knows that they're really mammals. She's so stupid."

**Answer**

This certainly isn't very nice, but it is not ad hominem since a valid counterargument is made along with the personal insult..

### Appeal to Ignorance

This type of argument assumes something is true because it hasn't been proven false.

Assuming something is true or false **because there's no proof otherwise**.

**Example 1.7.3**

"Nobody has proven that photo isn't of Bigfoot, so it must be Bigfoot." "I know there are aliens in space because science has not yet eliminated the possibility."

### Appeal to Authority

These arguments attempt to use the authority of a person to prove a claim. While often authority can provide strength to an argument, problems can occur when the person's opinion is not shared by other experts, or when the authority is irrelevant to the claim. An **appeal to authority** means trusting someone's opinion *as proof* instead of looking at actual evidence or logic.

**Example 1.7.4**

"A diet high in bacon can be healthy; Doctor Atkins said so."

**Answer**

Here, an appeal to the authority of a doctor is used for the argument. This generally would provide strength to the argument, except that the opinion that eating a diet high in saturated fat runs counter to general medical opinion. More supporting evidence would be needed to justify this claim.

**Example 1.7.5**

"Jennifer Hudson lost weight with Weight Watchers, so their program must work."

**Answer**

Here, there is an appeal to the authority of a celebrity. While her experience does provide evidence, it provides no more than any other person's experience would..

### Appeal to Consequence

An appeal to consequence concludes that a premise is true or false based on whether the consequences are desirable or not.

**Example 1.7.6**

"Humans will travel faster than light: faster-than-light travel would be beneficial for space travel."

**Your Turn 1.7.7: Categorize Fallacy**

Categorize the following logical fallacy.

I haven't seen anyone get killed by the police; therefore, it is not an issue.

Select an answer

Categorize the following logical fallacy.

John has really worked hard on his term project, and he will really be depressed if he does not make an A. For these reasons, you just have to give him an A on his project.

Select an answer

Categorize the following logical fallacy.

Old man Brown claims that he saw a flying saucer in his farm, but he never got beyond the fourth grade in school and can hardly read or write. He is completely ignorant of what scientists have written on the subject, so his report cannot possibly be true.

Select an answer

Categorize the following logical fallacy.

You should buy this brand of cookie. Six out of seven weight loss experts recommend it.

Select an answer

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**False Dilemma (Limited Choice)**

A false dilemma argument falsely frames an argument as an "either-or" choice, without allowing for additional options.

A **false dilemma** happens when a situation is presented as **"either this or that"**, ignoring other real possibilities.

**Example 1.7.8**

"Either those lights in the sky were an airplane or aliens. There are no airplanes scheduled for tonight, so it must be aliens."

**Answer**

This argument ignores the possibility that the lights could be something other than an airplane or aliens..

**Circular Reasoning**

Circular reasoning is an argument that relies on the conclusion being true for the premise to be true.

**The argument goes in a circle** — it assumes what it's trying to prove.

**Example 1.7.9**

"I shouldn't have gotten a C in that class; I'm an A student!"

**Answer**

In this argument, the student is claiming that because he is an A student, he shouldn't have gotten a C. But because he got a C, he is not an A student.

**Post Hoc (False Cause)**

A post hoc argument claims that because two things happened sequentially, then the first must have caused the second.

**Example 1.7.10**

"Today I wore a red shirt, and my football team won! I need to wear a red shirt every time they play to make sure they keep winning."

**Straw man**

A straw man argument involves misrepresenting the argument in a less favorable way to make it easier to attack.

Arguing against a **distorted or weaker version** of someone's real argument.

**Example 1.7.11**

"Senator Jones has proposed reducing military funding by 10%. Apparently, he wants to leave us defenseless against attacks by terrorists."

**Answer**

Here, the arguer has represented a 10% funding cut as equivalent to leaving us defenseless, making it easier to attack Senator Jones' position.

**Red Herring**

A **red herring** is a type of logical fallacy or distraction that misleads people from the main issue or argument. It's when someone brings up an irrelevant topic to divert attention from the real point

**Example 1.7.12**

Student: "Why did I get a bad grade on my test?"

Teacher: "You should be thankful you even have a school to attend."

**Answer**

The teacher's response distracts from the main issue (the student's test grade) by bringing up something unrelated (having a school).

**Your Turn 1.7.13: Categorize the Fallacy**

Categorize the following logical fallacy.

Reporter: "Why has the government not addressed rising food prices?" Politician: "Our nation has one of the best education systems in the world."

Select an answer

Categorize the following logical fallacy.

I can see that you are in the market for a new car. We're having a sale on Pintos this month. Do you want to pay cash or do you want the installment plan?

Select an answer

Categorize the following logical fallacy.

It's illegal to drive faster than the speed limit and breaking the law makes you a criminal. Drivers who exceed the speed limit are criminals.

Select an answer

Categorize the following logical fallacy.

The senator voted no for the school funding bill so they must not care about our children's future.

Select an answer

Categorize the following logical fallacy.

The oven was working fine until you started using it, so you must have broken it.

Select an answer

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**Correlation implies causation**

Similar to post hoc, but without the requirement of sequence, this fallacy assumes that just because two things are related one must have caused the other. Often, there is a third variable not considered.

**Example 1.7.14**

"Months with high ice cream sales also have a high rate of deaths by drowning. Therefore, ice cream must be causing people to drown."

**Answer**

This argument implies a causal relation, when really both are more likely dependent on the weather; that ice cream and drowning are both more likely during warm summer months.

**Your Turn 1.7.15: Match Fallacy with Its Definition**

Match each logical fallacy with its definition

- [-] is based on the assumption that tradition must be better than a newer alternative.
- [-] attempts to discredit an argument or position by drawing attention to certain traits or qualities of the person who is making the argument or who holds the position
- [-] takes the form misinterpreting an argument or position in a way that makes it weaker, false, or ridiculous
- [-] concludes that a premise is true or false based on whether or not the outcome is desirable
- [-] argues that a proposition must be true because it has never been proven false or there is no evidence that contradicts it.

a. Post Hoc
b. Circular reasoning
c. False Dilemma
d. Appeal to Ignorance
e. Appeal to Consequence
f. Ad Hominem
g. Straw Man
h. Appeal to Tradition
i. Appeal to Authority

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# CHAPTER OVERVIEW

## 2: NUMBER THEORY

In previous math courses, you've no doubt run into the infamous "word problems." Unfortunately, these problems rarely resemble the type of problems we actually encounter in everyday life. In math books, you usually are told exactly which formula or procedure to use and are given exactly the information you need to answer the question. In real life, problem-solving requires identifying an appropriate formula or procedure, and determining what information you need (and won't need) to answer the question.

In this chapter, we will review several basic but powerful algebraic ideas: percents, rates, and proportions. We will then focus on the problem-solving process, and explore applying these ideas to solve problems where we don't have perfect information.


2.1: Ratios, Rates, and Proportions

2.2: Percent, Discounts, and Taxes

2.3: Primes, Composites, and Divisibility


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# 2.1: Ratios, Rates, and Proportions

## Ratios and Rates

**Ratios** are used to compare amounts or quantities or to describe a relationship between two amounts or quantities. For example, a ratio might be used to describe the cost of a month's rent as compared to the income earned in one month. You may also use a ratio to compare the number of elephants to the total number of animals in a zoo, or the amount of calories per serving in two different brands of ice cream.

Ratios compare quantities using division. This means that you can set up a ratio between two quantities as a division expression between those same two quantities.

**Definition: Ratio**

A **ratio** is a comparison of two quantities that shows how much of one quantity there is compared to another. It tells the relationship between the quantities using division.

A ratio that compares two different quantities with the same units of measurement.

Here is an example. If you have a platter containing $10$ sugar cookies and $20$ chocolate chip cookies, you can compare the cookies using a ratio.

The ratio of sugar cookies to chocolate chip cookies is:

$$\frac{\text{sugar cookies}}{\text{chocolate chip cookies}} = \frac{10}{20}$$

The ratio of chocolate chip cookies to sugar cookies is:

$$\frac{\text{chocolate chip cookies}}{\text{sugar cookies}} = \frac{20}{10}$$

You can write the ratio using words, a fraction, and also using a colon as shown below.

Ratio of sugar cookies to chocolate chip cookies

$10$ to $20$ or
$\frac{10}{20}$ or
$10:20$
$1:2$ (Simplied form)

Some people think about this ratio as: "For every $10$ sugar cookies I have, I have $20$ chocolate chip cookies."

**How to Write a Ratio?**

A ratio can be written in three different ways:

- with the word "to": $3$ to $4$
- as a fraction: $\frac{3}{4}$
- with a colon: $3:4$

A ratio is simplified if it is equivalent to a fraction that has been simplified.

Below are two additional examples that illustrate how to compare quantities using a ratio and how to express the ratio in its simplified form.

**Example 2.1.1: Find Ratio**

A basketball player takes $50$ jump shots during a practice. She makes $28$ of them. What is the ratio of shots made to shots taken? Simplify the ratio.

**Answer**

$$\begin{aligned} \frac{\text{shots made}}{\text{shots taken}} &= \frac{28}{50} && \text{Unsimplified form} \\ &= \frac{14}{25} && \text{Divide by 2 to write simplified form.} \end{aligned}$$

The ratio of shots made to shots taken is $\frac{14}{25}$, $14:25$, or $14$ to $25$.

Often, one quantity in the ratio is greater than the second quantity. You do not have to write the ratio so that the lesser quantity comes first; the important thing is to maintain a consistent relationship.

Your Turn 2.1.1:Writing Ratios

| Representing Ratios in Multiple Ways | | |
|---|---|---|
| *Represent the following scenario as ratios in the indicated ways. Then determine if the comparison is part to part or part* | | |
| Jon was reviewing his pretest from math class. He answered 40 questions correctly for every 5 questions he answe | | |
| ***Form*** | ***Ratio of Questions Answered Correctly to Total Questions*** | ***Ratio of Questions Answered I Total Questions*** |
| ***Fraction*** | **Unsimplified:** ☐ **Simplified:** ☐ | **Unsimplified:** ☐ **Simplified:** ☐ |
| ***a to b Language*** | **Unsimplified:** ☐ **to** ☐ **Simplified:** ☐ **to** ☐ | **Unsimplified:** ☐ **to** ☐ **Simplified:** ☐ **to** ☐ |
| ***Are the ratios you found Part-Whole comparison or Part-Part comparison?*** Select an answer | | |

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Note that ratios can compare a *part to a part* or a *part to a whole*.

## Rates

**Definition: Rate**

A **rate** is a ratio in which the units of the two quantities being compared are different.

**Rates** are a special type of ratio used to describe a relationship between different units of measure, such as speed, wages, or prices. A car can be described as traveling 60 miles per hour; a landscaper might earn $35 per lawn mowed; gas may be sold at $3 per gallon.

A rate is a ratio that compares two different quantities with different units of measurement. A rate is a comparison that provides information such as dollars per hour, feet per second, miles per hour, and dollars per quart, for example. The word "per" usually indicates you are dealing with a rate. Rates can be written using words, using a colon, or as a fraction. It is essential to understand which quantities are being compared.

For example, an employer wants to rent 6 buses to transport a group of 300 people on a company outing. The rate to describe the relationship can be written using words, using a colon, or as a fraction, and you must include the units.

$$\begin{aligned} &\text{six buses per 300 people} \\ &6 \text{ buses} : 300 \text{ people} \\ &\frac{6 \text{ buses}}{300 \text{ people}} \\ &\frac{1 \text{ buses}}{50 \text{ people}} \end{aligned}$$

Divide by 6 to reduce fraction

This fraction means that the rate of buses to people is 6 to 300 or, simplified, 1 bus for every 50 people.

**Example 2.1.2: Find Rate**

Write the rate as a simplified fraction: 8 phone lines for 36 employees.

**Answer**

$$\frac{\text{phone lines}}{\text{employees}} = \frac{8 \text{ phone lines}}{36 \text{ employees}} \quad \text{Indentify the relationship}$$
$$= \frac{2 \text{ phone lines}}{9 \text{ employees}} \quad \text{Divide by 4 to reduce fraction}$$

The rate of phone lines for employees can be expressed as $\frac{2 \text{ phone lines}}{9 \text{ employees}}$.

**Your Turn 2.1.2: Find Rate**

Simplify the ratio of cans of soda compared to people

$$\frac{145 \text{ cans of soda}}{20 \text{ people}}$$

$$\frac{\_\_\_\_ \text{ cans of soda}}{\_\_\_\_ \text{ people}}$$

Now find and simplify the ratio of people compared to cans of soda

$$\frac{20 \text{ people}}{145 \text{ cans of soda}}$$

$$\frac{\_\_\_\_ \text{ people}}{\_\_\_\_ \text{ cans of soda}}$$

>

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## Finding Unit Rates

**Definition: Unit Rate**

A **unit rate** is a rate expressed so that the denominator of the corresponding ratio is one. (Recall that the "denominator" is the bottom number of the ratio when it is expressed in fraction form.)

Think about the "miles per gallon" example above. If we hear, "this car gets **30** miles per gallon," we know that for every 30 miles driven, the car uses 1 gallon of gas. The number 1 is implicit in the way we express most rates. Even if we are given a rate that is not a unit rate, we can convert it to a unit rate using proportions, as in the example below.




Figure **2.1.1**: Copy and Paste Caption here. (Copyright; author via source)

A **unit rate** compares a quantity to one unit of measure. You often see the speed at which an object is traveling in terms of its unit rate.

For example, if you wanted to describe the speed of a boy riding his bike—and you had the measurement of the distance he traveled in miles in **2** hours—you would most likely express the speed by describing the distance traveled in *one* hour. This is a unit rate; it gives the distance traveled per one hour. The denominator of a unit rate will always be one.

Consider the example of a car that travels **300** miles in **5** hours. To find the unit rate, you find the number of miles traveled in ***one*** hour.

$$\begin{aligned} \frac{\text{miles}}{\text{hours}} &= \frac{300 \text{ miles}}{5 \text{ hours}} && \text{Indentify the relationship} \\ &= \frac{60 \text{ miles}}{1 \text{ hour}} && \text{Divide by 5 to reduce fraction} \end{aligned}$$

A common way to write this unit rate is **60** miles per hour.

**Note**

The word "per." Sometimes, a slanted line / is used to mean "per." Per means division. For example, **60** miles per hour can be written as **60** miles/hour or $\frac{60 \text{ miles}}{1 \text{ hr}}$.

**Example 2.1.3: Find Unit Rate**

1. A crowded subway train has **375** passengers distributed evenly among **5** cars. What is the unit rate of passengers per subway car?
2. Vancouver Island has a population of **734, 860**, and a land mass of **32, 134** square kilometres. What is the rate of number of people per square kilometre? (This is called population density.) Round your answer to the nearest whole number.
3. Amy bought **10** lb of oranges for $**4.99**. What is the rate in cents per pound? Find the rate.

**Answer**

1. We need to find the passengers per subway car. Subway cars must be in the denominator.

$$\begin{aligned} \frac{\text{passengers}}{\text{subway cars}} &= \frac{375 \text{ passengers}}{5 \text{ subway cars}} \\ &= \frac{75 \text{ passengers}}{1 \text{ subway cars}} && \text{Divide by 5 to reduce fraction} \end{aligned}$$

The unit rate of the subway car is **75** riders per subway car.

2. We need to find the people per square kilometer. So the area must be in the denominator.

$$\begin{aligned} \frac{\text{population}}{\text{area}} &= \frac{734,860 \text{ population}}{32,134 \text{ area}} \\ &= \frac{23 \text{ people}}{1 \text{ area}} && \text{Divide} \end{aligned}$$

So population density is **23** people/$\text{km}^2$.

3. We need to price per pound. Therefore, the weight (in pounds) must be in the denominator.

$$\begin{aligned} \frac{4.99 \text{ dollars}}{10 \text{ lb}} &= \frac{0.499 \text{ dollar}}{1 \text{ lb}} \\ &= \$0.449 \text{ per pound} \\ &= 44.9 \text{ cent per pound} \end{aligned}$$

**Your Turn 2.1.3: Finding Unit Rates**

A car travels **330 miles** in **4.75 hours**. For this trip, the car uses **13 gallons** of gasoline. The total cost of gasoline is **76 dollars. Answer the following question**. Round your answer to **two decimal places**.

1. How many miles does the car travel in one gallon?
   _____ Select an answer
2. How many miles does the car travel in one hour?
   _____ Select an answer
3. How much cost of the gas per hour? $ _____
   Select an answer

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A **unit price** is a unit rate that expresses the price of something. The unit price always describes the price of *one* unit, so that you can easily compare prices.

You may have noticed that grocery shelves are marked with the unit price (as well as the total price) of each product. This unit price makes it easy for shoppers to compare the prices of competing brands and different package sizes.

Consider the two containers of blueberries shown below. It might be difficult to decide which is the better buy just by looking at the prices; the container on the left is cheaper, but you also get fewer blueberries. A better indicator of value is the price *per single ounce* of blueberries for each container.




Figure 2.1.1: Unit Price

Look at the unit prices—the container on the right is actually a better deal, since the price per ounce is *lower* than the unit price of the container on the left. You pay more money for the larger container of blueberries, but you also get more blueberries than you would with the smaller container. Put simply, the container on the right is a better value than the container on the left.

So, how do you find the unit price?

Imagine a shopper wanted to use unit prices to compare a **3**-pack of tissue for **$4.98** to a single box of tissue priced at **$1.60**. Which is the better deal?

Find the unit price of the **3**-pack: $\frac{\$4.98}{3 \text{ boxes}}$

Since the price given is for **3** boxes, divide both the numerator and the denominator by **3** to get the price of **1** box, the unit price. The unit price is **$1.66** per box.

The unit price of the **3**-pack is **$1.66** per box; compare this to the price of a single box at **$1.60**. Surprisingly, the **3**-pack has a higher unit price! Purchasing the single box is the better value.

Like rates, unit prices are often described with the word “per.” Sometimes, a slanted line / is used to mean “per.” The price of the tissue might be written **$1.60**/box, which is read "**$1.60** per box."

The following example shows how to use the unit price to compare two products and determine which has the lower price.

**Example 2.1.4: Compare Two Brand**

Sami is trying to decide between two brands of crackers. Which brand has the lower unit price?

Brand A: $1.12 for 8 ounces

Brand B: $1.56 for 12 ounces

**Answer**

Brand A: Find unit Price (price per ounce)

$$\begin{aligned}\frac{\text{dollars}}{\text{ounces}} &= \frac{1.12 \text{ dollars}}{8 \text{ ounces}} \\ &= 0.14 \text{ dollar per ounce} \\ &= 14 \text{ cent per ounce}\end{aligned}$$

Brand B: Find unit Price (price per ounce)

$$\begin{aligned}\frac{\text{dollars}}{\text{ounces}} &= \frac{1.56 \text{ dollars}}{12 \text{ ounces}} \\ &= 0.13 \text{ dollar per ounce} \\ &= 13 \text{ cent per ounce}\end{aligned}$$

The unit price of Brand A crackers is 14 cents/ounce, and the unit price of Brand B is 13 cents/ounce. Brand B has a lower unit price and represents the better value.

**Your Turn 2.1.4: Rate Comparision**

| Using Unit Rates for Comparison |
|---|
| Write each of the following as a UNIT RATE.<br>Round your answer to the nearest hundredth. |
| A small box of cereal is $7.60 for 5.8 ounces.<br><br>A large box is $19.35 for 15 ounces. |

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| Small Box Unit Rate<br>Dollars per Ounce | Large Box Unit Rate<br>Dollars per Ounce |
|---|---|
| [ ] Dollars per Ounce | [ ] Dollars per Ounce |

The [Select an answer] is a better buy because it costs [Select an answer] dollars per ounce.

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## Summary

Ratios and rates are used to compare quantities and express relationships between quantities measured in the same units of measure and in different units of measure. They both can be written as a fraction, using a colon, or using the words “to” or “per”. Since rates compare two quantities measured in different units of measurement, such as dollars per hour or sick days per year, they must include their units. A unit rate or unit price is a rate that describes the rate or price for one unit of measure.

## Proportion

**Definition: Proportion**

A true **proportion** is an equation that states that two **ratios** are equal or equivalent.

If you know one ratio in a proportion, you can use that information to find values in the other equivalent ratio. Using proportions can help you solve problems such as increasing a recipe to feed a larger crowd of people, creating a design with certain consistent features, or enlarging or reducing an image to scale.

For example, imagine you want to enlarge a $5$-inch by $8$-inch photograph to fit a wood frame that you purchased. If you want the shorter edge of the enlarged photo to measure $10$ inches, how long does the photo have to be for the image to scale correctly? You can set up a proportion to determine the length of the enlarged photo.

### Determining Whether a Proportion is True or False

A proportion is usually written as two equivalent fractions. For example:

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{36 \text{ inches}}{3 \text{ feet}}$$

Notice that the equation has a ratio on each side of the equal sign. Each ratio compares the same units, inches and feet, and the ratios are equivalent because the units are consistent, and $\frac{12}{1}$ is equivalent to $\frac{36}{3}$.

Proportions might also compare two ratios with the same units. For example, Juanita has two different-sized containers of lemonade mix. She wants to compare them. She could set up a proportion to compare the number of ounces in each container to the number of servings of lemonade that can be made from each container.

$$\frac{40 \text{ ounces}}{84 \text{ ounces}} = \frac{10 \text{ servings}}{21 \text{ servings}}$$

Since the units for each ratio are the same, you can express the proportion without the units:

$$\frac{40}{84} = \frac{10}{21}$$

When using this type of proportion, it is important that the numerators represent the same situation. In the example above, they represent 40 ounces for 10 servings. The denominators should also represent the same situation, which is ounces for servings.

Juanita could also have set up the proportion to compare the ratios of the container sizes to the number of servings of each container.

$$\frac{40 \text{ ounces}}{10 \text{ servings}} = \frac{84 \text{ ounces}}{21 \text{ servings}}$$

Here are some proportions in different forms:

| Proportion | Fraction Form | Read like this… |
|---|---|---|
| $1 : 2 = 2 : 4$ | $\frac{1}{2} = \frac{2}{4}$ | $1$ is to $2$ or $2$ is to $4$ |
| $18 : 9 = 10 : 5$ | $\frac{18}{9} = \frac{10}{5}$ | $18$ is to $9$ as $10$ is to $5$ |

Problems often give incomplete information; that is, one of the terms is missing. To solve such problems, you first find the comparison or ratio that is given. It may be:

- A quantity of one thing that is mixed with a larger quantity of something else
- A scale of measurement given on a map, such as 1 cm on the map, represents 100 km distance on land
- Cost for a certain number of items
- Time to travel a certain distance

The problem will then give one term of the second ratio in the proportion. For example, if you have been told that 3 heads of lettuce cost $1.49, you may be asked to find the cost of 7 heads of lettuce.

The missing term is the second cost. The proportion will be:

$$\frac{\text{number of heads of lettuce}}{\text{cost}} = \frac{\text{number of heads of lettuce}}{\text{cost}}$$

$$\frac{3}{\$1.49} = \frac{7}{?}$$

$$3 : \$1.49 = 7 : ?$$

The most important thing to remember is to keep the order of comparison the same in the first and second ratios in a proportion. If the first ratio compares time to distance, then the second ratio in the proportion must compare time to distance.

$$\frac{\text{time}}{\text{distance}} = \frac{\text{time}}{\text{distance}}$$

Or it could be:

$$\frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{\text{time}}$$

Once you have decided on the order of comparison, it is a simple matter to write the proportion using the numbers given in the problem. Use a letter to stand for the missing term.

How would you find a missing term?

## Finding an Unknown Quantity in a Proportion

If you know that the relationship between quantities is proportional, you can use proportions to find missing quantities. Below is an example.

**Definition: Steps to Solve Proportion**

**Step 1:** Write the first ratio using the information given.
**Step 2:** Write the proportion, using a letter in place of the missing term. Be sure the **order of comparison is the same** in both the first and second ratios in your proportion.
**Step 3:** Write the proportion in the fraction form. (Try to **simplify** the ratio **before** you do all the calculations).
**Step 4:** Cross-multiply and set the cross-products equal to each other.
**Step 5:** Divide both sides of the equation by the number with the unknown term.
**Step 6:** Check by putting your answer back into the original proportion and cross-multiplying.

**Example 2.1.5: Solve Proportion**

1. Solve proportion: $\frac{8}{10} = \frac{x}{80}$
2. Solve proportion: $\frac{n}{15} = \frac{25}{20}$

**Answer**

1. Given proportion

$$\frac{8}{10} = \frac{x}{80}$$

Cross multiply:

$$\begin{aligned} 8 \times 80 &= 10 \times x \\ 640 &= 10x \\ \frac{640}{10} &= \frac{10x}{10} \quad \text{Divide both side by 10} \\ \frac{\cancel{10}\, x}{\cancel{10}} &= \frac{640}{10} \\ x &= 64 \end{aligned}$$

2. Cross multiply

$$
\begin{aligned}
20 \times n &= 15 \times 25 \\
20n &= 375 \\
\frac{\cancel{20}n}{\cancel{20}} &= \frac{375}{20} \qquad \text{Divide both side by 20} \\
n &= 18.75
\end{aligned}
$$

The numbers in a ratio often are common fractions, decimals or mixed numbers. Follow exactly the same steps that you have been using to solve whole number proportions. The calculations will use your skills with fractions.

**Your Turn 2.1.5: Solve Proportions**

| Solving Proportions | |
|---|---|
| **Solve each of the proportions below. Be sure to write your answer as an equation in simplified form. For example, $y = 20$ , NOT just $20$ .** | |
| Proportion | Solution |
| $\frac{9}{36} = \frac{r}{60}$ | |
| $\frac{18}{108} = \frac{4}{b}$ | |
| $\frac{8}{t} = \frac{28}{52.5}$ | |
| $\frac{m}{10.5} = \frac{12}{63}$ | |

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**Example 2.1.6: Applying Proportion to Solve Problem**

A recipe says to use $2$ teaspoons of baking powder for every $4$ cups of flour. If a recipe uses $10$ cups of flour, how much baking powder is needed?

**Answer**

The missing term is the teaspoons of baking powder for $10$ cups of flour. Call this term $x$.

$$
\frac{\text{baking powder}}{\text{flour}}\ (\text{recipe}) = \frac{\text{baking powder}}{\text{flour}}\ (\text{You make})
$$

$$
\frac{2}{4} = \frac{x}{10}
$$

Cross multiply:

$$
\begin{aligned}
4 \times x &= 2 \times 10 \\
4x &= 20 \\
\frac{4x}{4} &= \frac{20}{4} \qquad \text{Divide both side by 4} \\
\frac{\cancel{4}x}{\cancel{4}} &= \frac{20}{4} \\
x &= 5
\end{aligned}
$$

So $5$ teaspoons of baking powder are needed for $10$ cups of flour.

**Your Turn 2.1.6: Applying Proportion**

| | |
|---|---|
| The recipe calls for $1\frac{1}{4}$ cups of sugar to make 20 cookies. How much sugar should be used if you want to make 6 cookies? Enter your answer as a fraction or whole number. | Need Calculator? Show Calculator |

To make 6 cookies, you will need to use [ ] cups of sugar.

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Now, back to the original example. Imagine you want to enlarge a 5-inch-by-8-inch photograph to a length of 10 inches while maintaining the same width-to-length proportion. You can set up a proportion to determine the width of the enlarged photo.

$$\frac{\text{width}}{\text{length}} \text{ (First photo)} = \frac{\text{width}}{\text{length}} \text{ (Second photo)}$$

$$\frac{5}{8} = \frac{10}{n}$$

$$5 \cdot n = 8 \cdot 10$$

$$\frac{5n}{5} = \frac{80}{5}$$

$$n = 16$$

The length of the enlarged photograph is 16 inches.




Figure 2.1.2

## Solving Application Problems Using Proportions

Setting up and solving a proportion is a helpful strategy for solving a variety of proportional reasoning problems. In these problems, it is always essential to determine the unknown value and then identify a proportional relationship that can be used to solve for the unknown value. Below are some examples.

**Example 2.1.7: Application of Proportion**

Among a species of tropical birds, 30 out of every 50 birds are female. In other words, 30 birds are female and 20 birds are male. If a certain bird sanctuary has a population of 1,150 of these birds, how many of them would you expect to be female?

**Answer**

Let $x =$ the number of female birds in the sanctuary.

Cross multiply:

$$\frac{30 \text{ female birds}}{50 \text{ total birds}} = \frac{x \text{ female birds in sanctuary}}{1,150 \text{ total birds in sanctuary}}$$

$$\frac{30}{50} = \frac{x}{1,150}$$

$$50 \times x = 30 \times 1,150$$

$$50x = 34,500$$

$$\frac{50x}{50} = \frac{34,500}{50} \quad \text{Divide both side by 50}$$

$$\frac{50x}{50} = \frac{34,500}{50}$$

$$x = 690$$

You would expect **690** birds in the sanctuary to be female.

**Your Turn 2.1.7: Application of Proportion**

| Using Ratios to Solve Application Problems | |
|---|---|
| **In this problem, you will encounter ratios and use them to solve application problems using proportion. A Unit Analysis is provided to help you determine the correct ratio to use when solving application problems.** | |
| An animal shelter cares for cats and dogs. They shelter **5 cats** for every **3 dog**. Use this information to find the following ratios. Write your answers in fraction form. | |
| Ratio of cats to either type of animal | |
| | |
| Ratio of dogs to either type of animal | |
| Ratio of either type of animal to cats | |
| Ratio of either type of animal to dogs | |
| Suppose the shelter has ***a total*** of **32** animals. How many ***cats*** do they house? | |
| Suppose the shelter has ***a total of*** **24** ***animal***s. How many ***dogs*** do they house? | |

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**Example 2.1.8: Use Proportion**

A map scale indicates that $\frac{1}{2}$ inch on the map corresponds with **3** real miles. How many miles apart are two cities that are $2\frac{1}{4}$ inches apart on the map?

**Answer**

We can set up a proportion by setting equal two $\frac{\textbf{map inches}}{\textbf{real miles}}$ rates, and introducing a variable, $x$, to represent the unknown quantity – the mile distance between the cities.

$$\frac{\text{inches}}{\text{miles}} = \frac{\text{inches}}{\text{miles}}$$
$$\frac{\frac{1}{2}}{3} = \frac{\frac{9}{4}}{x}$$
$$\frac{1}{2} \times x = 3 \times \frac{9}{4}$$
$$\frac{1}{2}x = \frac{27}{4}$$
$$\frac{\frac{1}{2}x}{\frac{1}{2}} = \frac{\frac{27}{4}}{\frac{1}{2}}$$
$$x = 13.5$$

So $x = 13.5 = 13\frac{1}{2}$ miles

**Your Turn 2.1.8: Use Proportions**

| | |
|---|---|
| Ben goes to the grocery store at a rate of **3** times **3** week. How many times would he be expected to go to the grocery store in **18** weeks? | |

Suppose $x$ is the number of times Ben goes to the store in $18$ weeks.

Translate to a proportion: $\frac{\_\_\_\_ \text{ times}}{\_\_\_\_ \text{ weeks}} = \frac{x}{\_\_\_\_ \text{ weeks}}$

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Ben goes $\_\_\_\_$ times in $18$ weeks.

Round to the nearest week.

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**Example 2.1.9: Use Proportion**

Your car can drive **300** miles on a tank of **15** gallons. How far can it drive on **40** gallons? How many gallons are needed to drive **50** miles?

**Answer**

We could certainly answer this question using a proportion: $\frac{300 \text{ miles}}{15 \text{ gallons}} = \frac{x \text{ miles}}{40 \text{ gallons}}$

Cross multiply:

$$\begin{aligned} 300 \times 40 &= 15 \times x \\ 15x &= 12,000 \\ \frac{15x}{15} &= \frac{12,000}{15} \qquad \text{Divide both side by 4} \\ \frac{\cancel{15}\, x}{\cancel{15}} &= \frac{12,000}{15} \\ x &= 800 \end{aligned}$$

So **800** miles can be driven by **40** gallons.

Notice if instead we were asked "how many gallons are needed to drive **50** miles?"

Set proportion as we did before

$$\frac{300 \text{ miles}}{15 \text{ gallons}} = \frac{50 \text{ miles}}{x \text{ gallons}}$$

Cross multiply:

$$\begin{aligned} 300 \times x &= 15 \times 50 \\ 300x &= 750 \\ \frac{300x}{300} &= \frac{750}{300} \qquad \text{Divide both side by 300} \\ \frac{\cancel{300}\, x}{\cancel{300}} &= \frac{750}{300} \\ x &= 2.5 \end{aligned}$$

So **2.5** gallons are needed to drive **50** miles.

## Summary

A proportion is an equation comparing two ratios. If the ratios are equivalent, the proportion is true. If not, the proportion is false. Finding a cross product is another method for determining whether a proportion is true or false. Cross-multiplying is also helpful for finding an unknown quantity in a proportional relationship. Setting up and solving proportions is a valuable skill that can be applied to a wide range of problems.

If you wanted to power the city of Seattle using wind power, how many windmills would you need to install? Questions like these can be answered using rates and proportions.

## Notes:

4) https://www.cia.gov/library/publications/the-world-factbook/rankorder/2042rank.html

5) http://data.worldbank.org/indicator/SP.POP.TOTL

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# 2.2: Percent, Discounts, and Taxes

## Introduction

Three common formats for numbers are fractions, decimals, and percents. Percentages are often used to communicate a *relative* amount. You have probably seen them used for discounts, where the percentage of discount can apply to different prices. Percentages are also used when discussing taxes and interest rates on savings and loans.

## The Meaning of Percent

A **percent** is a ratio of a number to $100$. *Per cent* means "per $100$," or "how many out of $100$." You use the symbol % after a number to indicate a percent.

Notice that $57$ of the $100$ squares in the grid below have been shaded green. This represents $57$ percent $57$ per $100$).

$$57\% = 57 \text{ percent } = 57 \text{ parts out of } 100 = \frac{57}{100}$$

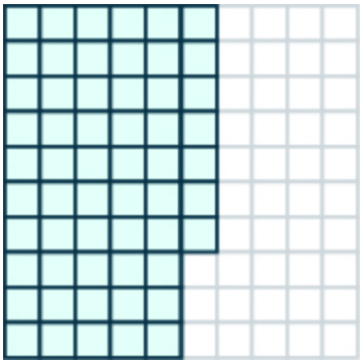


Figure **2.2.1**: Copy and Paste Caption here. (Copyright; author via source)

How many of the squares in the grid above are unshaded? Since $100$ are shaded and there are a total of $100$ squares, $43$ are unshaded. The unshaded portion of the whole grid is $43$ parts out of $100$ , or $43$ % of the grid. Notice that the shaded and unshaded portions together make $100$ % of the grid ( $100$ out of $100$ squares).

## Convert Between Percent, Decimal, and Fractional Values

**Formula: Percent to Decimal**

The fraction form of $n\%$ is found by dividing $\frac{n}{100}$ and calculating the decimal value of the fraction.

Or you move the decimal to the left (two places) and remove the percent sign.

**Do You Know?**

A percent need not be an integer and does not have to be less than $100$.

**Example 2.2.1: Rewriting a Percent as a Fraction or Decimal Form**

Rewrite the following as fractions and decimal form:

1. $8\%$
2. $38.7\%$
3. $\frac{1}{4}\%$
4. $213\%$

**Answer**

1. Using the definition an $n = 8$. Decimal form = $0.08$ and fractional form
$$\frac{8}{100} = \frac{2}{25}$$
2. Using the definition and $n = 38.7$. Decimal form = $0.387$ and fractional form
$$\frac{387}{100}$$
3. Using the definition and $n = \frac{1}{4} = 0.25$. Decimal form = $0.0025$ and fractional form

$$\frac{0.25}{100} = \frac{0.25 \times 100}{100 \times 100} = \frac{25}{10,000} = \frac{1}{400}$$

4. Using the definition and $n = 213$. Decimal form = $2.13$ and fractional form

$$\frac{213}{100}$$

When any calculation with a percent is to be performed, the form of the percent must be changed to its fractional form or its **decimal form**. We can convert a percent to a decimal by dividing the percent by $100$ and representing the result as a decimal.

**FORMULA**

To convert the number $x$ from decimal form (or fraction form) to percent, multiply $x$ by $100$ and place a percent sign, $\%$, after the number.

Or to convert from decimal form to percent form, you can simply move the decimal two places to the right without performing the multiplication.

**Example 2.2.2: Converting the Decimal Form or Fraction to Percent**

Convert each of the following to a percent:

1. $0.34$
2. $4.15$
3. $0.0391$
4. $\frac{3}{8}$

**Answer**

1. Using the formula and $x = 0.34$, we calculate, $0.34 \times 100$, which gives us $34\%$.
2. Using the formula and $x = 4.15$, we calculate, $4.15 \times 100$, which gives us $415\%$.
3. Using the formula and $x = 0.0391$, we calculate, $0.0391 \times 100$, which gives us $3.91\%$.
4. Using the formula and $x = \frac{3}{8}$, we calculate, $(\frac{3}{8} \times 100\%)$, which gives us $37.5\%$.

**Your Turn 2.2.2: Percent to Decimal and Vice Versa**

Fill in the blanks by converting between percents, decimals, and fractions. If needed, round decimals to four decimal places (to the nearest ten thousandth) and percents to two decimal places (to the nearest hundredth).

| Percent | Decimal | Reduced Fraction |
|---|---|---|
| _____ % | _____ | $\frac{17}{25}$ |
| _____ % | 0.52 | |
| 0.73 % | _____ | |
| _____ % | _____ | $\frac{8}{15}$ |
| $5\frac{7}{10}$ % | _____ | |

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## Summary

Percentages are a common way to represent fractional amounts, just as decimals and fractions are. Any number that can be written as a decimal, fraction, or percent can also be written using the other two representations.

In the 2004 vice-presidential debates, Edwards claimed that US forces have suffered "$90\%$ of the coalition casualties" in Iraq. Cheney disputed this, saying that in fact Iraqi security forces and coalition allies "have taken almost 50 percent" of the casualties (1). Who is correct? How can we make sense of these numbers?

## Percent Total, and Part

There are three components in the percent problem part, total (whole), and percent.

**Part**: The specific amount or quantity being compared to the whole (often associated with the word "is" in a word problem).

**Total (Whole)**: The entire amount or the base quantity to which the part is being compared (often associated with the word "of" in a word problem).

**Percent**: A ratio that expresses how many parts are in every 100 parts of the whole, represented by the symbol "%".

For example, let's say you are shopping during a sale. A jacket has a red tag showing a sale price of $56, that is a $80 − $56 = $24 discount, which the sign says is 30% of the original price of $80.
Here Percent is 30%, part is $56, and whole is $80

The formula and its variations are as follows:

### Definition: Percent, Total, and Part

If we have a **part** that is some percent of a **whole**, then

$$\begin{aligned} \text{percent} &= \frac{\text{part}}{\text{whole}} \\ \text{part} &= \text{percent} \cdot \text{whole} \\ \text{whole} &= \frac{\text{part}}{\text{percent}} \end{aligned}$$

We can find any missing component if the other two are known by using the above formula.

To perform the calculations, we convert the percent to a decimal. Recall, to rewrite the percent as a decimal, we move the decimal over two places to the left, or, equivalently, divide the percent number by 100.

### Example 2.2.3: Find Percent

243 people out of 400 state that they like dogs. What percent is this?

**Answer**

Here, 400 is the whole, and the part is 243. We are looking for a percent.

$$\begin{aligned} \text{Percent} &= \frac{\text{part}}{\text{whole}} \\ &= \frac{243}{400} = 0.6075 = 60.75\% \end{aligned}$$

So 60.75% says they like dogs. Notice that the percent can be found from the equivalent decimal by moving the decimal point two places to the right.

### Your Turn 2.2.3: Find Percent

Identify the percent, part, and whole in this problem. For the unknown value use $X$. And find the value of $X$.

You spend $\$240$ on groceries out of a $\$2050$ monthly budget. What percent of the budget is this? Round your answer to two decimal places.

Percent = [ ] %

Part = $[ ]

Whole = $[ ]

$X$ = [ ] %

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**Example 2.2.4: Find Part**

The sales tax in a town is **9.4%**. How much tax will you pay on a **$140** purchase?

**Answer**

Here, **$140** is the whole, and we want to find **9.4%** of **$140**. We are looking for a part. We start by writing the percent as a decimal by moving the decimal point two places to the left (which is equivalent to dividing by **100**). So **9.4%** is **0.094** as a decimal.

$$\begin{aligned} \text{part} &= \text{percent} \cdot \text{whole} \\ &= 0.094 \cdot 140 \\ &= 13.16 \end{aligned}$$

Tax is **$13.16**.

**Your Turn 2.2.4: Find Part**

Identify the percent, whole, and part in this problem. For the unknown value, use $X$. And find the value of $X$.

In the news, you hear "tuition is expected to increase by $22\%$ next year." If tuition this year is $\$1780$ per quarter, how much will the tution increase in next year? Round your answer to two decimal places.

Percent = [ ] %

Part = $[ ]

Whole = $[ ]

$X$ = $[ ]

How much will be tution next year?

$[ ]

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**Example 2.2.5: Find Whole**

A student scored **72** points, which is **60%** of the total possible points on a test. What is the total number of points on the test?

**Answer**

Her part is **72**. A percent is **60% = 0.60**; we are looking for the whole.

$$\begin{aligned} \text{whole} &= \frac{\text{percent}}{\text{whole}} \\ &= \frac{72}{0.60} \\ &= 120 \end{aligned}$$

The total number of points in the test is **120**.

## Your Turn 2.2.5: Find Whole

Identify the percent, part, and whole in this problem. For the unknown value, use $X$. And find the value of $X$.

If $62$ students represent $10\%$ of a class, what is the total number of students? Round your **answer to the nearest whole number.**

Percent = [ ] %

Whole = [ ]

Part = [ ]

$X$ = [ ]

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## Solving Percentage Problem

If the final value (or amount) is $p\%$ more than the initial

$$\text{Final} = (100+p)\% \times \text{Initial}$$
$$\text{Initial} = \frac{\text{final}}{(100+p)\%}$$

If the final value (or amount) is $p\%$ less than the initial

$$\text{Final} = (100-p)\% \times \text{Initial}$$
$$\text{Initial} = \frac{\text{final}}{(100-p)\%}$$

**Note:** When you solve a percent problem, you always change percent to decimal either by dividing by $100$ or by moving the decimal two places to the left.

## Example 2.2.6: Find Initial Value

The average price of a gallon of gas in a city increased by $12.2\%$ over the past month. If the average price was $\$3.57$ at the end of the month, what was the average gas price at the beginning of the month?

**Answer**

We are looking for the initial value before the price increase. Final value is $3.57\%$ more than the initial.

$$\begin{aligned}\text{Initial} &= \frac{\text{final}}{(100+p)\%} \\ &= \frac{3.57}{(100+12.2)\%} \\ &= \frac{3.57}{112.2\%} \\ &= \frac{3.57}{1.122} \\ &= 3.1818\end{aligned}$$

He weighed $\$3.18$ at the beginning of the month.

**Your Turn 2.2.6: Find Original Price**

You purchased a new car for a total cost of $\$23,975.00$ , which includes the $7\%$ sales tax. What was the original price of the car?

The original price was $\$$ _____

Round your answer to two decimal places.

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**Your Example 2.2.7: Find Final Value**

Marisol's family went to dinner. Together they purchased food and drinks totaling $\$108.4$. They need to pay $1.45\%$ sales tax and $27\%$ of the tip. And what is the total price of their breakfast, including tax and tip?

**Answer**

We are looking for the final amount with sales tax and tips. Final value is $(1.45 + 27)\% = 28.45\%$ more than the initial.

$$\begin{aligned} \text{Final} &= (100 + p)\% \times \text{Initial} \\ &= (100 + 28.45)\% \times 108.4 \\ &= (128.45)\% \times 108.4 \\ &= 1.2845 \times 108.4 \\ &= 139.24 \end{aligned}$$

So their total bill will be $\$139.24$ with tax and tips.

**Your Turn 2.2.7: Find Cost Before Tax**

The final cost of your new shoes is $\$98.44$ . The local sales tax rate is $6.8\%$.

The cost before tax is $\$$ _____

Round your answer to two decimal places.

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**Example 2.2.8**: Find the Initial Amount

After a diet, James lost $6\%$ of his weight. He now weighs $205$ pounds. How much did he weigh before his diet (to the nearest pound)?

**Answer**

We are looking for the initial value before weight loss. Final value is $6\%$ less than the initial.

$$\begin{aligned}\text{Initial} &= \frac{\text{final}}{(100-p)\%} \\ &= \frac{205}{(100-6)\%} \\ &= \frac{205}{94\%} \\ &= \frac{205}{0.94} \\ &= 218.0851\end{aligned}$$

He weighed $218$ lb before his diet.

**Definition: Absolute Change and Relative Change**

Given two quantities,

$$\text{Absolute Change} = |\text{Final Value} - \text{Initial Value}|$$

Here $|\ |$ (ABSOLUTE VALUE) means you change the negative to the positive value of the difference.

$$\text{Relative change} = \frac{\text{Absolute Change}}{\text{Initial Value}} \times 100\%$$

Absolute change has the same units as the original quantity.

Relative change gives a percent change. The starting **(Initial)** quantity is the **base** of the percent change.

**Using Percentage as Comparison:**
If we compare A and B, and A is more **than B**, then the base is B. Note that the base value follows the word "than".

**Example 2.2.9**: Find Absolute and Relative Change

The value of a car dropped from $\$7,400$ to $\$6,800$ over the last year. What percent decrease is this?

**Answer**

To compute the percent change, we first need to find the dollar value change: $\$6800 - \$7400 = -\$600$. Often, we take the absolute value of this amount, which is called the **absolute change**: $|-600| = 600$.

Since we are computing the percent decrease relative to the initial value, we compute this percent out of $\$7,400$:

$$\begin{aligned}\text{Relative Change} &= \frac{\text{Absolute Change}}{\text{Initial}} \times 100\% \\ &= \frac{600}{7400} \times 100\% \\ &= 8.1\%\end{aligned}$$

**Your Turn 2.2.9**: Absolute Chnage and Relative Change

The price of a latte dropped from $\$3.77$ to $\$3.49$ . Find the absolute change and relative change of the price of a latte.

Absolute change: $\$$ _____

Round to two decimal places

Relative change: _____ %

Round your answer to the tenth of a percent.

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The base of a percent is very important. For example, while Nixon was president, it was argued that marijuana was a "gateway" drug, claiming that 80% of marijuana smokers went on to use harder drugs like cocaine. The problem is, this isn't true. The true claim is that 80% of harder drug users first smoked marijuana. The difference is one of base: 80% of marijuana smokers use hard drugs, vs. 80% of hard drug users having smoked marijuana. These numbers are not equivalent. As it turns out, only one in 2,400 marijuana users actually go on to use harder drugs (2).

**Example 2.2.10: Compare Two Sizes**

There are about 75 QFC supermarkets in the U.S. Albertsons has about 215 stores. Compare the size of the two companies.

**Answer**

When we make comparisons, we must ask first whether it is an absolute or a relative comparison. The absolute difference is $215–75 = 140$. From this, we could say "Albertsons has 140 more stores than QFC." Or "QFC has 140 fewer stores than Albertsons." However, if you wrote this in an article or paper, that number does not mean much. The relative difference may be more meaningful. There are two different relative changes we could calculate, depending on which store we use as the base:

Using QFC as the base

$$\begin{aligned}\text{Relative Change} &= \frac{\text{Absolute Change}}{\text{Initial}} \times 100\% \\ &= \frac{140}{75} \times 100\% \\ &= 186.7\%\end{aligned}$$

This tells us Albertsons is 186.7% larger than QFC.

Using Albertsons as the base

$$\begin{aligned}\text{Relative Change} &= \frac{\text{Absolute Change}}{\text{Initial}} \times 100\% \\ &= \frac{140}{215} \times 100\% \\ &= 65.1\%\end{aligned}$$

This tells us QFC is 65.1% smaller than Albertsons.

In the above examples, notice that both show percent *differences*. We could also calculate the size of Albertsons relative to QFC:

$\frac{215}{75} = 2.867$, which tells us Albertsons is 2.867 times the size of QFC.

Likewise, we could calculate the size of QFC relative to Albertsons:

$\frac{75}{215} = 0.349$, which tells us that QFC is 34.9% of the size of Albertsons.

**Your Turn 2.2.10: Comparing Percentage**

The state of Indiana has $64,200$ teachers, and the state of Ohio has $121,300$ teachers.

**Compare Ohio to Indiana using absolute and relative change (Here, Indiania is the base.)**

**Absolute Change:** Ohio has _____ more teachers than Indiana.

**Relative Change:** Ohio has _____ % more teachers than Indiana.

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**Compare Indiana to Ohio using absolute and relative change**

**Compare Indiana to Ohio using absolute and relative change (Here, Ohio is the base.)**

**Absolute Change:** Indiana has [ ] fewer teachers than Ohio.

**Relative Change:** Indiana has [ ] % fewer teachers than Ohio.

(Round the percent to the hundredth place.)

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We'll wrap up our review of percents with a couple of cautions. First, when discussing a change in quantities that are already measured in percentages, we must be careful in how we describe the change. When you discuss **the percent change of a percentage**, you are essentially treating the original percentage as a number and calculating the percent change in the usual manner.

**Definition: Percentage Point**

A **percentage point** is a **unit used to describe the difference between two percentages**.

**Example 2.2.11: Percentage vs Percentage Point**

A politician's support increases from $40\%$ of voters to $50\%$ of voters. Describe the change.

**Answer**

We could describe this using an absolute change: $|40\% - 50\%| = 10\%$. Notice that since the original quantities were percents, this change also has the units of percent. In this case, it is best to describe this as an increase of **10 percentage points**.

In contrast, we could compute the percent change: $\frac{10\%}{40\%} \times 100\% = 25\%$ increase. This is the relative change, and we'd say the politician's support has increased by $25\%$.

**Your Turn \( \PageIndex{11}\): Percentage Point**

After missing one credit card payment, Slavano's interest rate went from $13.99\%$ APR to $30.99\%$ APR on his credit card.

**Find and interpret the absolute change**

The credit card rate [Select an answer] [Select an answer] [Select an answer] after the missed payment

**Find and interpret the relative change**

The credit card rate [Select an answer] [Select an answer] [Select an answer] after the missed payment

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Notes:

1) http://www.factcheck.org/cheney_edwards_mangle_facts.html

2) http://tvtropes.org/pmwiki/pmwiki.php/Main/LiesDamnedLiesAndStatistics

3) http://www.whitehouse.gov/sites/default/files/omb/budget/fy2013/assets/hist07z1.xls

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## 2.3: Primes, Composites, and Divisibility

Encryption, which is needed for the secure exchange of information (i.e., online banking or shopping) is based on prime numbers. Encryption uses a composite number that is the product of two very large prime numbers. To break the encryption, the two primes that were used to form the composite number need to be determined. If the two prime numbers used are sufficiently large, even the fastest computer cannot determine those two prime numbers in a reasonable amount of time. It would take a computer 300 trillion years to crack the current encryption standard.



Figure **2.3.1**: Computers are protected using encryption based on prime numbers. (credit: "Data Security" by Blogtrepreneur/Flickr, CC BY 2.0)

### Applying Divisibility Rules

Before we begin our investigation of divisibility, we need to know some facts about important sets of numbers:

- The counting numbers are also called the natural numbers. This set of numbers, $\{1, 2, 3, 4, ...\}$, is denoted with the symbol $\mathbb{N}$.
- Another important set of numbers is the integers. The integers are the natural numbers, along with 0, and the negatives of the natural numbers. This set is often written as $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$. We denote the integers with the symbol $\mathbb{Z}$.
- Notice that $\mathbb{N}$ is a proper subset of $\mathbb{Z}$, or, $\mathbb{N} \subset \mathbb{Z}$. All the ideas of this section apply to the natural numbers, while only some apply to all the integers.

**Divisibility and Divisor (Factors)**

Divisibility is when the integer $a$ is divided by $b$, and there is no remainder. We called $b$ divisor of $a$. Or $b$ is a factor of $a$.

There are many occasions when separating items into equal groups comes into play to ensure an equal distribution of whole items. For example, Francis, a preschool art teacher, has $15$ students in one class. Francis has $225$ sheets of construction paper and wants to give each student an equal number of sheets. To know if he will use all the construction paper, Francis is really asking if $225$ can be evenly divided into $15$ groups.

| Divisor | Rule |
|---|---|
| 2 | Last digit is even |
| 3 | Add the digits of the number together. If that sum is divisible by 3 , then so is the original number |
| 4 | Look at only the last two digits. If this number is divisible by 4 , so is the original number |
| 5 | Look at only the last digit. If it is a 5 or a 0 , then the original number is divisible by 5 |
| 6 | If the number passes the rule for divisibility by 2 and for 3 , then the number is divisible by 6 |
| 7 | Take the last digit of a number, double it, and subtract that result from the rest of the number. If the new number is divisible by 7, the original number is divisible by 7. |
| 8 | Look at the last three digits. If this number is divisible by 8, then the original number is. |
| 9 | Add the digits of the number together. If that number is divisible by 9 , then so is the original number |
| 10 | Look at only the last digit. If it is a 0, then the original number is divisible by 10. |
| 12 | If the number passes the rule for 3 and 4 , the number is divisible by 12 |

**Example 2.3.1: Using Divisibility Rules**

1. Using divisibility rules, determine if $245$ is divisible by $5$.
2. Using divisibility rules, determine if $25,983$ is divisible by $9$.
3. Using divisibility rules, determine if $1,946$ is divisible by $7$.

**Answer**

1. Since the last digit is a $5$, the number $2455$ is divisible by $5$ because the rule states if the last digit of the number is a $5$ or a $0$ , then the original number is divisible by $5$.
2. The divisibility rule for $9$ is that when the digits of the number are added, the sum is divisible by $9$. So, we calculate the sum of the digits.$2+5+9+8+3=27$. Since $27$ is divisible by $9$ , so is the original number $25,983$ .
3. Last digit is $6$. Double of $6$ is $12$. Now $194-12=182$. Repeat the same process. Double of $2$ is $4$. Now $18-4=14$. $14$ is a multiple of $7$. Therefore, $1946$ is divisible by $7$.

**Your Turn 2.3.1: Applying Divisibility Rule**

**Write Y for yes and N for no.**

| | |
|---|---|
| Is 5735 divisible by 2? ___ | Is 5735 divisible by 6? ___ |
| Is 5735 divisible by 3? ___ | Is 5735 divisible by 9? ___ |
| Is 5735 divisible by 4? ___ | Is 5735 divisible by 10? ___ |
| Is 5735 divisible by 5? ___ | |

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## Prime and Composite Numbers

Sometimes, a natural number has only two unique divisors, **1** and itself. For instance, **7** and **19** are prime. In other words, there is no way to divide a prime number into groups with an equal number of things, unless there is only one group, or those groups have one item per group. Other natural numbers have more than two unique divisors, such as **4**, or **26**. These numbers are called composite. The number **1** is special; it is neither prime nor composite.

To determine if a number is prime or composite, you have to determine if the number has any divisors other than **1** and itself. The divisibility rules are useful here and can quickly show you if a number has a divisor on that list.

However, if none of those divide the number, you still have to check all other possible prime divisors. What are the prime numbers that are possibly divisors of the number you are checking? You need only check the prime numbers up to the square root of the number in question. For instance, if you want to know if **2,117** is prime, you need to determine if any primes up to the square root of **2,117** (which is **46.0** when rounded to one decimal place) divide **2,117**. If any of those primes are divisors of the number in question, then the number is composite. If none of those primes work, then the number is itself prime.

We can check divisibility with whatever tool we wish. Divisibility rules are quick for some prime divisors (**2** and **5** come to mind) but aren't quick for other values (like 11). In place of divisibility rules, we could just use a calculator. If the prime number divides the number evenly (that is, there is no decimal or fractional part), then the number is divisible by that prime. Table **2.3.1** is a quick list of the prime numbers up to **6**. There are **15** prime numbers less than **50** .

Table **2.3.1** Prime Numbers Less than **50**

| | | | | |
|---|---|---|---|---|
| **2** | **3** | **5** | **7** | **9** |
| **11** | **13** | **17** | **19** | **23** |
| **29** | **31** | **37** | **41** | **43** and **47** |

### Example 2.3.2: Determining if a Number Is Prime or Composite

Determine if **423** is prime or composite.

**Answer**

The square root of **423** is **20.57** (rounded to two decimal places). So, we need to check if **423** is divisible by any prime up to **20**.

**Step 1:** Check **2**. We can tell **423** is not divisible by **2**, as the last digit isn't even.

**Step 2:** Check **5**. It is not divisible by **5** (the last digit isn't **0** or **5**).

**Step 3:** Check **3**. To check if **423** is divisible by **3**, we use the divisibility rule for **3**. When we take the sum of the digits of **423**, the result is **9**. Since **9** is divisible by **3**, so is **423**.

Since **423** is divisible by **3**, then **423** is a composite number. Writing **423** as the product of **3** and another natural number, **141** Or

\(423=3\times 141\).

### Your Turn 2.3.2: Identify Prime and Composite

Sort the numbers into the appropriate category.

| Composites | Primes |
|---|---|
| | |

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**Who Knew?: ILLEGAL PRIMES**

Large primes are a hot commodity. Using two very large primes (some have more than 22 million digits!) is necessary for secure encryption. Anyone who has a new prime that is large enough can use that prime to create a new encryption. Of course, whoever discovers a large prime could sell it to a security company. These primes are so useful for encryption that it is necessary to protect that intellectual property. In fact, at least one prime number was declared illegal.

## Finding the Prime Factorization of Composite Numbers

Before we can start with prime factorization, let's remind ourselves what it means to factor a number. We factor a number by identifying two (or more) numbers that, when multiplied, result in the original number. For instance, 3 and 24, when multiplied, give 72. So, 72 can be factored $3 \times 24$. Notice that we could have factored the 72 differently, say

$$72 = 6 \times 12$$
$$72 = 18 \times 4$$
$$72 = 6 \times 4 \times 3$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

Finding the prime factorization of a composite number means writing the number as the product of all of its prime factors. For example,

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

. Notice that all the numbers being multiplied on the right-hand side are prime numbers. Sometimes, prime numbers repeat themselves in the factorization. When prime factors do repeat, we may write the prime factorization using exponents, as in $80 = 2^4 \times 5$.

Two forms of prime factorization

Expanded from

$$80 = 2 \times 2 \times 2 \times 2 \times 5$$

Exponential form:

$$80 = 2^4 \times 5$$

One has to wonder if finding the prime factorization could result in different factorizations. The **Fundamental Theorem of Arithmetic** tells us that there is only one prime factorization for a given natural number.

## Fundamental Theorem of Arithmetic

Every natural number, other than 1, can be expressed in exactly one way, apart from the arrangement, as a product of primes.

The process of finding the prime factorization of a number is iterative, which means we do a step, then repeat it until we cannot do the step any longer. The step we use is to identify one prime factor of the number, then write the number as the prime factor times another factor. We repeat this step on the other, newly found, factor. This step is repeated until no more primes can be factored from the remaining factor.

### Factor Trees

A useful tool for helping with prime factorization is a **factor tree**. To create a factor tree for the natural number $n$ (where $n$ not 1), perform the following steps:

**Step 1:** If $n$ is prime, you're done. If $n$ is composite, continue to the next step.

**Step 2:** Identify two divisors of $n$, all them $a$ and $b$.

**Step 3:** Write the number $n$ down, and draw two branches extending down (or to the right) of the number $n$.

**Step 4:** Label the end of one branch as seen in Figure 2.3.3.

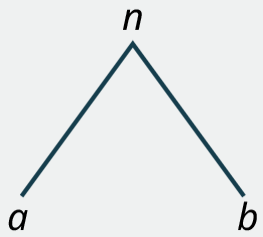



Figure 2.3.3

**Step 5:** If $a$ and $b$ are prime, the tree is complete. When a number at this step is a prime number, we refer to it as a leaf of the tree diagram.

**Step 6:** If either$a$ is $b$ composite, repeat Steps $2$ through $4$ for $a$ and $b$

**Step 7:** The process stops when the leaves are all prime.

**Step 8:** The prime factorization is then the product of all the leaves.

**Example 2.3.3: Finding Prime Factorization**

Find the prime factorization of $66$.

**Answer**

Since $66$ is even, $2$ is a factor.

**Step 1:** Factor out the $2$. The factorization is $66 = 2 \times 33$.

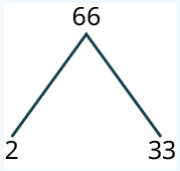



Figure 2.3.4

Since Figure $2$ is a factor, that branch is done, and $2$ is a leaf.

**Step 2:** The $33$ , though, is divisible by $3$, and is the product of $3$ and $11$ . We attach that to the factor tree.

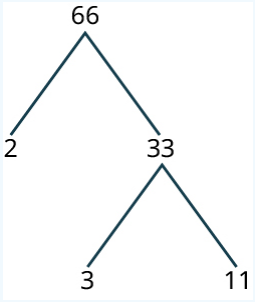



Figure 2.3.5

Since the $2$, $3$, and $11$ are all prime, the factor tree is done.

The prime factorization of $66$ is the product of the leaves, $66 = 2 \times 3 \times 11$. The factorization is complete.

**Your Turn 2.3.3: Find Prime Factorization**

Prime Factorization

*Find the prime factorizations of the given numbers. Give your answer in Expanded Form and Exponential Form.*

*The example in the first row will guide you on how to enter your answers.*

***Notes:***
*1. Make sure to place your factors in increasing order. (i.e.* $2 \cdot 3$ *not* $3 \cdot 2$*)*
*2. If there aren't repeated factors, your answers for both forms will be identical.*
*3. Use the "^" button (shift 6) for exponents. So 3^2 is* $3^2$

*For multiplication, use the "*" button (shift 8) on the keyboard.*

| Number | Prime Factorization Expanded Form | Prime Factorization Exponential Form |
|---|---|---|
| Example: $44$ | $2 \cdot 2 \cdot 11$ | $2^2 \cdot 11$ |
| $64$ | | |
| $76$ | | |
| $54$ | | |
| $70$ | | |

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**Example 2.3.4: Finding the Prime Factorization**

Find the prime factorization of $135$.

**Answer**

The number $135$ is divisible by $3$, and so $3$ is a factor of $135$.

**Step 1:** Factor out the $3$. The factorization is $135 = 3 \times 45$ shown in Figure 2.3.6

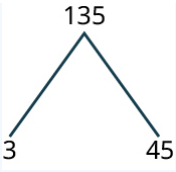



Figure 2.3.6

45 is also divisible by 3.

**Step 2:** Factor out a 3 from 45. The other factor is 15. The factor tree is shown in Figure 2.3.7.

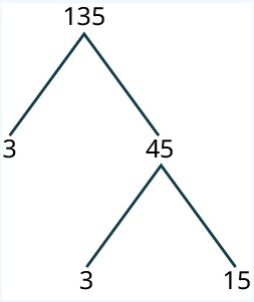



Figure 2.3.7

15 is also divisible by 3.

**Step 3:** The factors of 15 are 3 and 5. The factor tree is shown in Figure 2.3.8.

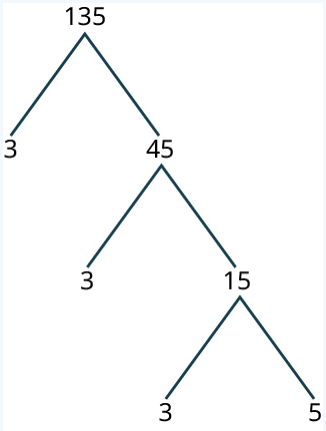



Figure 2.3.8

All the leaves are prime, so the process is complete. The prime factorization of 135 is $3^5 \times 5$.

Your Turn 2.3.4: Find Prime Factorization

For multiplication, use the "*" button (shift 8) on the keyboard.

Find the prime factorization of 360. Use exponential form, as in $12 = 2^2 \cdot 3$.

$360 =$

For multiplication, use the "*" button (shift 8) on the keyboard.

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## Finding the Greatest Common Divisor

Two numbers often have more than one divisor in common (all pairs of natural numbers have the common divisor 1). When listing the common divisors, it's often the case that the largest is of interest. This divisor is called the greatest common divisor and is denoted **GCD**. It is also sometimes referred to as the greatest common factor (GCF).

For instance, $6$ is the greatest common divisor of $12$ and $18$. We can see this by listing all the divisors of each number and, by inspection, selecting the largest value that shows up in each list.

| | |
|---|---|
| The divisors of $12$ | $1, 2, 3, 4, 6, 12$ |
| The divisors of $18$ | $1, 2, 3, 6, 9, 18$ |

It is easy to see that $6$ is the greatest value that appears in both lists.

**Example 2.3.5: Finding the Greatest Common Factor Using Lists**

Find the greatest common divisor of $400$ and $250$ by listing all divisors of each number.

**Answer**

We create a list of all the divisors of $400$ and of $250$, and choose the largest one.

The divisors of $400$ are

$1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 80, 100, 200$.

The divisors of $250$ are

$1, 2, 5, 10, 25, 50, 125, 250$.

The largest value that appears on both lists is $50$, so the greatest common divisor of $400$ and $250$ is $50$.

**Your Turn 2.3.5: Finding GCD Using Divisor**

List the *positive* divisors of $a$ and $b$ in **increasing order,** separated by commas (for example, Divisors of $6 = 1, 2, 3, 6$ ). Then find $\gcd(a, b)$.

Let $a$ = 56 and $b$ = 72. Then

Divisors of $a$ = _____

Divisors of $b$ = _____

$\gcd(a, b)$ = _____

**Do not put spaces between the numbers in your answer.**

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## Finding the Greatest Common Divisor Using Prime Factorization

Listing all the divisors of the numbers in the set will always work, but for some relatively small numbers, the set of all divisors can become pretty big, and finding them all can be a chore. Another approach to finding the greatest common divisor is to use the prime factorization of the numbers. To do so, use the following steps:

**Step 1:** Find the prime factorization of the numbers.

**Step 2:** Identify the prime factors that appear in every number's prime factorization. These are called the common prime factors.

**Step 3:** Identify the smallest exponent of each prime factor identified in Step 2 in the prime factorizations.

**Step 4:** Multiply the prime factors identified in Step 2 raised to the powers identified in Step 3. The result is the greatest common divisor.

### Example 2.3.6: Finding the Greatest Common Divisor Using Prime Factorization

Find the greatest common divisor of 24 and 90 by using their prime factorizations.

**Answer**

**Step 1:** Find the prime factorizations of the numbers.

The prime factorization of 24 is $2^3 \times 3$
The prime factorization of 90 is $2 \times 3^2 \times 5$

**Step 2:** Identify the prime factors that appear in every number's prime factorization.

There is only one common prime factor, 2 and 3.

**Step 3:** Identify the smallest exponent of each prime from those identified in Step 2 in the prime factorizations.

The exponent of 2 in the prime factorization of 24 is 3. The exponent of 2 in the prime factorization of 90 is 1. So, the smallest exponent of 2 is 1. The exponent of 3 in the prime factorization of 24 is 1. The exponent of 3 in the prime factorization of 90 is 2. So, the smallest exponent of 3 is 1.

**Step 4:** Multiply the prime factors identified in Step 2 raised to the powers identified in Step 3.

This gives $2^1 \times 3^1 = 6$Math input error

## Applications of the Greatest Common Divisor

The greatest common divisor has uses that are related to other mathematics (reducing fractions) but also in everyday applications. We'll look at two such applications, which have very similar underlying structures. In each case, something must divide the groups or measurements equally.

### Example 2.3.7: Party Bags

Sarah is planning a party. She has 36 candies and 48 toys to place in party favor bags. If she needs each party favor bag to be identical, what is the greatest number of party favor bags she can make? How many candies will be in each bag? How many toys will be in each bag?

**Answer**

Since we want the greatest number of party favor bags, we need the GCD of 38 and 48.

**Step 1:** Find the prime factorizations of the numbers.

The prime factorization of 36 is $2^2 \times 3^2$

The prime factorization of 48 is $2^4 \times 3$

**Step 2:** Identify the prime factors that appear in every number's prime factorization.

The common prime factors are 2 and 3.

**Step 3:** Identify the smallest exponent of each prime identified in Step 2 in the prime factorizations.

The smallest exponent for 2 in both prime factorization is 2 .

The smallest exponent for 3 in both prime factorization is 1 .

**Step 4:** Multiply the prime factors identified in Step 2 raised to the powers identified in Step 3.

This gives $2^2 \times 3^1 = 12$. The GCD of 48 and 36 is 12 , so the greatest number of party favor bags is 12.

Number of candies in each bag is $\frac{36}{12} = 3$.

The number of toys in each bag is $\frac{48}{12} = 4$.

### Your Turn 2.3.7: Application of GCD

Alfredo is preparing platters of quiches and empanadas for a party; he has 76 quiches and 52 empanadas

If he would like each plate to be identical, with no items left over, what is the greatest number of platters Alfredo can create?

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## Finding the Least Common Multiple

The flip side of a divisor of a number is a multiple of that number. For example, $5$ is a divisor of $45$ and so $45$ is a multiple of $5$. More generally, if the number $a$ divides the number $b$, then $b$ is a multiple of $a$.

This drives the idea of the least common multiple of a set of numbers. A common multiple of a set of numbers is a multiple of each of those numbers. For instance, $45$ is a common multiple of $9$ and $5$, because $45$ is a multiple of $9$ ($9$ divides $45$) and $45$ is also a multiple of $5$ ($5$ divides $45$). The least common multiple (LCM) of a set of numbers is the smallest positive common multiple of that set of numbers.

There are (at least) three ways to find the LCM of a set of numbers, and we will explore two of them. One way is to create a list of multiples of each number in the set, and then identify the smallest multiple that appears in those lists.

**Example 2.3.8: Finding the Least Common Multiple Using Lists**

Find the LCM of $24$ and $90$ by listing multiples and choosing the smallest common multiple.

**Answer**

Create a list of the multiples of each number.

**Step 1:** The first $15$ multiples for $24$ :

$24, 48, 72, 96, 120, 144, 168, 192, 216, 240, 264, 288, 312, 336, 360$.

**Step 2:** The first $15$ multiples for $90$:

$90, 180, 270, 360, 450, 540, 630, 720, 810, 900, 990, 1080, 1170, 1260, 1350$.

There is one multiple common to these lists: $360$. So,$360$ is the LCM of $24$ and $90$.

**Your Turn 2.3.8: Find LCM Using Mulitples**

List the first 12 multiples of 6 and 5.
*Write answers in a list. Usa a comma between numbers.*

6: _____

5: _____

The least common multiple of 6 and 5 is ____.

Please click on Sieve VM and use a sieve to find the multiples.
*Click and hold a color, then drag and drop it onto a number.*
*Click* **Toss** *repeatedly to see successive multiples.*
*Click* **Toss All** *to see all multiples of a number.*

***NOTE:*** *You may resize the sieve by holding down the* Ctrl *key and pressing the* - *key several times.*

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The second method we can use is to find the prime factorizations of the numbers in the set and build the LCM based on their prime divisors. The LCM can be built from the prime factorizations of the numbers in the set, in a similar way to finding the greatest common divisor. Here are the steps for using the prime factorization method to find the LCM.

**Step 1:** Find the prime factorization of each number.

**Step 2:** Identify each prime that appears in any prime factorization.

**Step 3:** Identify the largest exponent of each prime identified in Step **2** in the prime factorizations.

**Step 4:** Multiply the prime factors identified in Step **2** raised to the powers identified in Step **3**.

**Example 2.3.9: Finding the Least Common Multiple Using Prime Factorization**

Use the prime factorizations of **24** and **90** to identify their LCM.

**Answer**

**Step 1:** Find the prime factorization of each number.

$$24 = 2^3 \times 3$$

$$90 = 2 \times 3^2 \times 5$$

**Step 2:** Identify each prime that appears in any prime factorization.

The prime numbers present in the prime factorizations are **2**, **3**, and **5**.

**Step 3:** Identify the largest exponent of each prime identified in Step **2** in the prime factorizations.

| Prime | 2 | 3 | 5 |
|---|---|---|---|
| Largest exponent | 3 | 2 | 1 |

**Step 4:** Compute the LCM by multiplying the prime factors identified in Step **2** raised to the powers identified in Step **3**.

The LCM for **24** and **90**

$$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$$

**Your Turn 2.3.9: Find GCD and LCM**

**Using Prime Factorization to find the Greatest Common Factor and Least Common Multiple**

**Need Calculator?** **Show Calculator**

Write the prime factorizations of the given numbers in expanded form and then find their Greatest Common Factor(GCF). Use the GCF to find the LCM.

***Recall, the***

**LCM = product of the two numbers ÷ GCF**

The example in the first row will guide you on how to enter your answers.

**Notes:**
If the two numbers do not have any common prime factors, **their GCF is 1.**
Make sure to put the factors in increasing order (i.e. $2 \cdot 3$ not $3 \cdot 2$)

For multiplication, use the "*" button (shift 8) on the keyboard.

| Numbers | Prime Factorization of First Number | Prime Factorization of Second Number | Greatest Common Factor | Least Common Multiple |
|---|---|---|---|---|
| Example: 12 and 42 | $2 \cdot 2 \cdot 3$ | $2 \cdot 3 \cdot 7$ | Find $2 \cdot 3$ Enter the result: 6 | Find $12 \cdot 42 \div 6$ Enter the result: 84 |
| 6 and 18 | | | | |
| 33 and 27 | | | | |
| 32 and 16 | | | | |

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## Applications of the Least Common Multiple

Some applications of LCM involve events that repeat at fixed intervals, such as visits to a location. Other applications involve ensuring equal magnitudes when using parts of different sizes. In each case, we may be looking to determine when two processes "line up."

**Example 2.3.10: Application of LCM**

Alejandra is buying chocolate chip cookies and snickerdoodles for a class reunion. Each package of chocolate chip cookies contains 12 chocolate chip cookies. Each package of snickerdoodles contains 16 snickerdoodles. What is the least number of each kind of cookie she must buy to have an equal number of each? How many packages of chocolate chip cookies and snickerdoodles does she need to buy?

**Answer**

This is an example where the least number of each kind of cookie she must buy to have an equal number of each.

If we list the days that each student will volunteer, it becomes clear that we could solve this problem using the LCM of 12and 16.

Multiple of 12 are $12, 36, 48, 60, 72, 48 \cdots$

Multiple of 16 are $16, 32, 48, 64, \cdots 48$.

The smallest number appearing on both lists is 48. So she must buy 48 of each kind of cookies. She needs to buy $\frac{48}{12} = 4$ packages of chocolate chip cookies and $\frac{36}{12} = 3$ packages of snickerdoodles.

**Your Turn 2.3.10: Application of LCM**

Elijah and Van both did their laundry today. Elijah does laundry every 8 days and Van does laundry every 12 days.

How many days will it be until Elijah and Van both do laundry on the same day again?

_____ days

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## Your Turn 2.3.11: Application of GCD and LCM

**Chee is buying chocolate chip cookies and snickerdoodles for a class reunion. Each package of chocolate chip cookies contains 25 chocolate chip cookies. Each package of snickerdoodles contains 30 snickerdoodles.**

**What is the least number of each kind of cookie she must buy to have an equal number of each?**

She must buy _____ of each kind of cookie.

She needs to buy _____ packages of chocolate chip cookies and _____ packages of snickerdoodles.

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**Eva is planning a party. She has 56 candies and 49 toys to place in party favor bags.**

**Using all the candies and toys (no leftovers), what is the greatest number of identical party favor bags she can make?**

She can make _____ party favor bags.

There will be _____ candies and _____ toys in each bag.

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# CHAPTER OVERVIEW

## 3: MEASUREMENT


3.1: Dimensional Analysis and the U.S. Customary System

3.2: The Metric System

3.3: Conversion Between the Metric and USCS

3.4: Temperature, Time, and Medicine Doses


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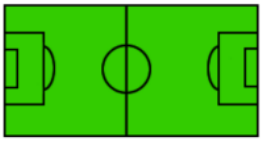
# 3.1: Dimensional Analysis and the U.S. Customary System

**Measurement** is a number that describes the size or amount of something. You can measure many things like length, area, capacity, weight, temperature and time. In the United States, two main systems of measurement are used: the **metric system** and the **U.S. customary measurement system**. This section addresses the measurement of length, area, volume, and weight using the U.S. customary measurement system.

Suppose you want to purchase tubing for a project, and you see two signs in a hardware store: $1.88 for 2 feet of tubing and $5.49 for 3 yards of tubing. If both types of tubing will work equally well for your project, which is the better price? You need to know about two **units of measurement**, yards and feet, in order to determine the answer.

## Length

**Length** is the distance from one end of an object to the other end, or from one object to another. For example, the length of a letter-sized piece of paper is 11 inches. The system for measuring length in the United States is based on the four customary units of length: **inch, foot, yard,** and **mile**. Below are examples to show measurement in each of these units.

| Unit | Description | Image |
|---|---|---|
| Inch/Inches | Some people donate their hair to be made into wigs for cancer patients who have lost hair as a result of treatment. One company requires hair donations to be at least 8 inches long. |  |
| Inches | Frame size of a bike: the distance from the center of the crank to the top of the seat tube. Frame size is usually measured in inches. This frame is 16 inches. |  |
| Foot/Feet | Rugs are typically sold in standard lengths. One typical size is a rug that is 8 feet wide and 11 feet long. This is often described as an 8 by 11 rug. |  |
| Yard/Yards | Soccer fields vary some in their size. An official field can be any length between 100 and 130 yards. |  |
| Mile/Miles | A marathon is 26.2 miles long. One marathon route is shown in the map to the right. |  |

You can use any of these four U.S. customary measurement units to describe the length of something, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the length of a rug in feet rather than miles, and to describe a marathon in miles rather than inches.

You may need to convert between units of measurement. For example, you might want to express your height using feet and inches (5 feet 4 inches) or using only inches (64 inches). You need to know the unit equivalents in order to make these conversions between units.

The table below shows equivalents and conversion factors for the four customary units of measurement of length.

| Unit Equivalents | Conversion Factors (longer to shorter units of measurement) | Conversion Factors (shorter to longer units of measurement) |
|---|---|---|
| 1 foot = 12 inches | $\frac{12 \text{ inches}}{1 \text{ foot}}$ | $\frac{1 \text{ foot}}{12 \text{ inches}}$ |
| 1yard = 3 feet | $\frac{3 \text{ feet}}{1 \text{ yard}}$ | $\frac{1 \text{ yard}}{3 \text{ feet}}$ |
| 1 mile = 5,280 feet | $\frac{5280 \text{ feet}}{1 \text{ mile}}$ | $\frac{1 \text{ mile}}{5280 \text{ feet}}$ |

Note that each of these conversion factors is a ratio of equal values, so each conversion factor equals 1. Multiplying a measurement by a conversion factor does not change the size of the measurement at all, since it is the same as multiplying by 1; it just changes the units that you are using to measure.

## Converting Between Units of Length

You can use the conversion factors to convert a measurement, such as feet, to another type of measurement, such as inches.

Note that there are many more inches for a measurement than there are feet for the same measurement, as feet is a longer unit of measurement. You could use the conversion factor $\frac{12 \text{ inches}}{1 \text{ foot}}$.

If a length is measured in feet, and you'd like to convert the length to yards, you can think, "I am converting from a shorter unit to a longer one, so the length in yards will be less than the length in feet." You could use the conversion factor $\frac{1 \text{ yard}}{3 \text{ feet}}$.

If a distance is measured in miles, and you want to know how many feet it is, you can think, "I am converting from a longer unit of measurement to a shorter one, so the number of feet would be greater than the number of miles." You could use the conversion factor $\frac{5280 \text{ feet}}{1 \text{ mile}}$.

**Important to Remember During Factor Label Method (Dimensional Analysis)**

When performing unit conversion, first identify the unit you want in your answer and the unit you wish to eliminate. The desired unit should appear in the numerator (top) of the fraction, while the unit you want to eliminate should be placed in the denominator (bottom) of the fraction.

**Factor Label Method**

The **factor label method (also called dimensional analysis)** to convert one unit of measure to another using the conversion factors. In the factor label method, you multiply by unit fractions (conversion factor) to convert a measurement from one unit to another.

$$\cdots \text{Unit 1} \times \frac{\cdots \text{Unit 2}}{\cdots \text{Unit 1}} = \cdots \text{Unit 2}$$

$$\cdots \text{Unit 1} \times \frac{\cdots \text{Unit 2}}{\cdots \text{Unit 1}} \times \frac{\cdots \text{Unit 3}}{\cdots \text{Unit 2}} = \cdots \text{Unit 3}$$

The example below shows how the factor label method can be used to convert $3\frac{1}{2}$ feet into an equivalent number of inches.

**Example 3.1.1: Feet to Inches**

How many inches are in $3.5$ feet?

**Answer**

Begin by reasoning about your answer. Since a foot is longer than an inch, this means the answer would be greater than $3.5$.

$$3.5 \text{ feet} = \_\_\_\_\_ \text{ inches}$$

Find the conversion factor that compares inches and feet, with "inches" in the numerator, and multiply

$$\frac{12 \text{ inches}}{1 \text{ foot}}$$

You can cancel similar units when they appear in the numerator *and* the denominator. So here, reduce the similar units "feet" and "foot." This eliminates this unit from the problem.

$$\begin{aligned} 3.5 \text{ ft} &= 3.5 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} && \text{Apply conversion factor.} \\ &= 3.5 \cancel{\text{ft}} \times \frac{12 \text{ in}}{1 \cancel{\text{ft}}} && \text{Cancel common units.} \\ &= \frac{3.5 \times 12}{1} \text{ in} && \text{Multiply fractions.} \\ &= 42 \text{ in} \end{aligned}$$

There are $42$ inches in $3.5$ feet.

Notice that by using the factor label method, you can reduce the units out of the problem, just as if they were numbers. You can only reduce if the unit being reduced is in both the numerator and denominator of the fractions you are multiplying.

In the problem above, you converted feet to inches, yielding the desired result.

$$3.5 \text{ feet} = \frac{7 \text{ feet}}{2} = \frac{7 \text{ feet}}{2} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$$

**Note: How do you know if you chose the wrong conversion factor?**

What if you had used the wrong conversion factor?

$$\frac{7 \text{ feet}}{2} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$$

You could not reduce the feet because the units are not the same in both the numerator and the denominator. So if you complete the computation, you would still have both feet and inches in the answer, and no conversion would take place.

Here is another example of a length conversion using the factor label method.

**Example 3.1.2: Feet to Yards**

How many yards is $7$ feet?

**Answer**

Start by reasoning about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. So your answer will be less than $7$.

$$7 \text{ feet} = \_\_\_\_\_ \text{ yards}$$

Find the conversion factor that compares feet and yards, with yards in the numerator.

$$\frac{1 \text{ yard}}{3 \text{ feet}}$$

$$\begin{aligned} 7 \text{ ft} &= 7 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} && \text{Apply conversion factor.} \\ &= 7 \cancel{\text{ft}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} && \text{Cancel common units.} \\ &= \frac{7 \times 1}{3} \text{yd} && \text{Multiply fractions.} \\ &= \frac{7}{3} \text{ yd} \end{aligned}$$

7 feet equals $2\frac{1}{3}$ yards.

**Your Turn 3.1.2: Inches to Feet**

| Using the Factor Label Method |
| --- |
| 3 feet = 1 yard<br>Use the Factor-Label Method to convert 94 feet to yards.<br>Round your answer to the nearest tenth as needed. |
| $\frac{94 \text{ feet}}{1} \times \frac{\_\_\_\_}{\_\_\_\_} = \_\_\_\_$ |

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**Note**

Note that if the units do not reduce to give you the answer you are trying to find, you may not have used the correct conversion factor.

## Applying Unit Conversions

There are times when you will need to perform computations on measurements that are given in different units. For example, consider the tubing problem given earlier. You must decide which of the two options is a better price, and you have to compare prices given in different unit measurements.

In order to compare, you need to convert the measurements into one single, common unit of measurement. To be sure you have made the computation accurately, think about whether the unit you are converting to is smaller or larger than the number you have. Its relative size will tell you whether the number you are trying to find is greater or lesser than the given number.

**Example 3.1.3: Feet to Yards**

An interior decorator needs border trim for a home she is wallpapering. She needs **15** feet of border trim for the living room, **30** feet of border trim for the bedroom, and **26** feet of border trim for the dining room. How many yards of border trim does she need?

**Answer**

You need to find the total length of border trim that is needed for all three rooms in the house. Since the measurements for each room are given in feet, you can add the numbers.

$$15 \text{ feet} + 30 \text{ feet} + 26 \text{ feet} = 71 \text{ feet}$$

How many yards is **71** feet? Reason about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. Expect your answer to be less than **71**.

$$71 \text{ feet} = \_\_\_\_\_ \text{ yards}$$

Use the conversion factor $\frac{1 \text{ yard}}{3 \text{ feet}}$.

Since "feet" is in the numerator and denominator, you can reduce this unit.

$$\frac{71 \text{ feet}}{1} \cdot \frac{1 \text{ yard}}{3 \text{ feet}}$$

$$\frac{71 \text{ yards}}{3} = 23\frac{2}{3} \text{ yards}$$

**Answer**: The interior decorator needs $23\frac{2}{3}$ yards of border trim.

**Your Turn 3.1.3: Add Different Units**

An electrician has the following lengths of three cables. What is the total length of the cable? Round to the nearest hundredth.

240 inches, 190 inches, and 25 feet.

1 feet = 12 inches, 1 yard = 3 feet

**How much cable does he have in feet?**

_____ **feet.**

**How much cable does he have in yards?**

_____ **yards.**

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The next example uses the factor label method to solve a problem that requires converting from miles to feet.

**Example 3.1.4: Yards to Miles**

Two runners were comparing how much they had trained earlier that day. Jo said, "According to my pedometer, I ran 2 miles." Alex said, "I ran 4000 yards." Who ran more distance?

**Answer**

We know 1 mi = 5280 ft and 1 yd = 3 ft. Let's convert how much Jo runs in yards to compare to Alex.

$$\begin{aligned} 2\text{ mi} &= 2\text{ mi} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ yd}}{3\text{ ft}} && \text{Apply conversion factors.} \\ &= 2\ \cancel{\text{mi}} \times \frac{5280\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}} \times \frac{1\text{ yd}}{3\ \cancel{\text{ft}}} && \text{Cancel common units.} \\ &= \frac{2 \times 5280 \times 1}{1 \times 3}\text{ yd} && \text{Multiply fractions.} \\ &= 3520\text{ yd} \end{aligned}$$

So Jo runs 3520 yards, which is less than Alex ran, which is 4,000 yards

**Your Turn 3.1.4: Yard to Feet and Mile**

Convert 1795 yards to feet and miles. Round your answer to two decimal places.

1795 yards = _____ feet

1795 yards = _____ miles

**Use: 3 feet = 1 yard, 1 mile = 5280 feet, or 1 mile = 1760 yards**

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Now let's revisit the question from earlier.

**Example 3.1.5: Comparing Two Distances**

Jamir is training for a race and is running laps around a field. If the distance around the field is **475** yards, how many complete laps would he need to run at least **6** miles?

**Answer**

We know **1** mi = **5280** ft and **1** yd = **3** ft. Let's convert how **6** miles to feet.

$$\begin{aligned} 6\text{ mi} &= 6\text{ mi} \times \frac{5,280\text{ ft}}{1\text{ mi}} && \text{Apply conversion factors.} \\ &= 6\ \cancel{\text{mi}} \times \frac{5,280\text{ mi}}{1\ \cancel{\text{mi}}} && \text{Cancel common units.} \\ &= \frac{6 \times 5,280}{1}\text{ ft} && \text{Multiply fractions.} \\ &= 31,680\text{ ft} \end{aligned}$$

So Jamir wants to run at least **31,680** feet. Let's convert how **475** yards to feet.

We know **1** yd = **3** ft. Let's convert how much Jo runs in yards to compare to Alex.

$$\begin{aligned} 475\text{ yd} &= 475\text{ yard} \times \frac{3\text{ ft}}{1\text{ yd}} && \text{Apply conversion factors.} \\ &= 475\ \cancel{\text{yd}} \times \frac{3\text{ ft}}{1\ \cancel{\text{yd}}} && \text{Cancel common units.} \\ &= \frac{475 \times 3}{1}\text{ ft} && \text{Multiply fractions.} \\ &= 1,425\text{ ft} \end{aligned}$$

So

$$\text{number of lap} = \frac{31,680\text{ ft}}{1,425\text{ ft}} = 22.23$$

So **23** laps must be completed to run at least **6** miles.

**Your Turn 3.1.5: Find Number of Laps**

Jamir is training for a race and is running laps around a field. If the distance around the field is **200** yards, how many complete laps would he need to run at least **7** miles?

_____ laps

**1 yard = 3 feet and 1 mile = 5280 feet**

**Hint**: Convert **7** miles to feet and **200** yards to feet.

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## Summary

The four basic units of measurement that are used in the U.S. customary measurement system are: inch, foot, yard, and mile. Typically, people use yards, miles, and sometimes feet to describe long distances. Measurement in inches is common for shorter objects or lengths.

You need to convert from one unit of measure to another if you are solving problems that include measurements involving more than one type of measurement. Each of the units can be converted to one of the other units using the table of equivalents, the conversion factors, and/or the factor label method shown in this topic.

### Weight

#### Introduction

When you mention how heavy or light an object is, you are referring to its weight. In the U.S. customary system of measurement, weight is measured in ounces, pounds, and tons. Like other units of measurement, you can convert between these units and you sometimes need to do this to solve problems.

In **2010**, the post office charges **$0.44** to mail something that weighs an ounce or less. The post office charges **$0.17** for each additional ounce, or fraction of an ounce, of weight. How much will it cost to mail a package that weighs two pounds three ounces? To answer this question, you need to understand the relationship between ounces and pounds.

## Units of Weight

You often use the word **weight** to describe how heavy or light an object or person is. Weight is measured in the U.S. customary system using three units: ounces, pounds, and tons. An **ounce** is the smallest unit for measuring weight, a **pound** is a larger unit, and a **ton** is the largest unit.

| | |
|---|---|
| Whales are some of the largest animals in the world. Some species can reach weights of up to **200** tons- that's equal to **400,000** pounds |  |
| Meat is a product that is typically sold by the pound. One pound of ground beef makes about four hamburger patties. |  |
| Ounces are used to measure lighter objects. A stack of **11** pennies is equal to about one ounce. |  |

You can use any of the customary measurement units to describe the weight of something, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the weight of a human being in pounds rather than tons. It makes more sense to describe the weight of a car in tons rather than ounces.

**1** pound (lb) = **16** ounces (oz)

**1** ton (t) = **2,000** pounds (lb)

## Converting Between Units of Weight

Four ounces is a typical serving size of meat. Since meat is sold by the pound, you might want to convert the weight of a package of meat from pounds to ounces in order to determine how many servings are contained in a package of meat.

The weight capacity of a truck is often provided in tons. You might need to convert pounds into tons if you are trying to determine whether a truck can safely transport a big shipment of heavy materials.

The table below shows the unit conversions and conversion factors that are used to make conversions between customary units of weight.

| Unit Equivalents | Conversion Factors (heavier to lighter units of measurement) | Conversion Factors (lighter to heavier units of measurement) |
|---|---|---|
| **1** pound = **16** ounces | $\frac{16 \text{ ounces}}{1 \text{ pound}}$ | $\frac{1 \text{ pound}}{16 \text{ ounces}}$ |
| **1** ton = **2,000** pounds | $\frac{2000 \text{ pounds}}{1 \text{ ton}}$ | $\frac{1 \text{ ton}}{2000 \text{ pounds}}$ |

You can use the *factor label method* to convert one customary unit of weight to another customary unit of weight. This method uses conversion factors, which allow you to "reduce" units to end up with your desired unit of measurement.

Each of these conversion factors is a ratio of equal values, so each conversion factor equals **2**. Multiplying a measurement by a conversion factor does not change the size of the measurement at all, since it is the same as multiplying by **1**. It just changes the units that you are using to measure it in.

Two examples illustrating the factor label method are shown below.

### Example 3.1.6: Tons to Pounds

How many tons is **6,500** pounds?

**Answer**

Begin by reasoning about your answer. Since a ton is heavier than a pound, expect your answer to be a number less than 6,500.

$$6500 \text{ pounds} = \_\_\_\_\_ \text{tons}$$

Multiply by the conversion factor that relates tons to pounds: $\frac{1 \text{ ton}}{2000 \text{ pounds}}$

$$\begin{aligned} 6500 \text{ lb} &= 6500 \text{ lb} \times \frac{1 \text{ tons}}{2000 \text{ lb}} && \text{Apply conversion factor.} \\ &= 6500 \cancel{\text{lb}} \times \frac{1 \text{ tons}}{2000 \cancel{\text{lb}}} && \text{Cancel common units.} \\ &= \frac{6500 \times 1}{2000} \text{ tons} && \text{Multiply fractions.} \\ &= 3.25 \text{ tons} \end{aligned}$$

There are **6,500** pounds in $3\frac{1}{4}$ tons.

### Your Turn 3.1.6: Pounds to Tons

| Pounds to Tons | |
|---|---|
| A municipal trash facility allows a person to throw away a maximum of 40 pounds of trash per week. Last week, 155 people threw away the maximum allowable trash. How many tons of trash did this equal? **Round the answer to two decimal places and use** | |

many tons of trash did this equal? **Round the answer to two decimal places and use the conversion factor** $1$ **ton** $= 2,000$ **pounds**

**This is** _____ **tons of trash.**

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## Applying Unit Conversions

There are times when you need to perform calculations on measurements that are given in different units. To solve these problems, you need to convert one of the measurements to the same unit of measurement as the other measurement.

Think about whether the unit you are converting to is smaller or larger than the unit you are converting from. This will help you be sure that you are making the right computation. You can use the factor label method to make the conversion from one unit to another.

Here is an example of a problem that requires converting between units.

**Example 3.1.7: How Many People Can Ride in Elevator?**

A standard elevator in a mid-rise building can hold a maximum weight of about **2.25** tons. Assuming an average adult weight of **234** pounds, what is the maximum number of adults who can safely ride the elevator?

**Answer**

Let's find out how much pound weight of the elevator would weigh

We know **1** ton = **2,000** lb.

$$\begin{aligned} 2.5 \text{ ton} &= 2.5 \text{ ton} \times \frac{2,000 \text{ lb}}{1 \text{ ton}} && \text{Apply conversion factors.} \\ &= 2.25 \cancel{\text{ton}} \times \frac{2,000 \text{ lb}}{1 \cancel{\text{ton}}} && \text{Cancel common units.} \\ &= \frac{2.25 \times 2,000}{1} \text{ yd} && \text{Multiply fractions.} \\ &= 4,500 \text{ lb} \end{aligned}$$

So

$$\text{number of people} = \frac{4,500 \text{ lb}}{234 \text{ lb}} = 19.23$$

The maximum number of adults who can safely ride the elevator is **19** people.

Let's revisit the post office problem that was posed earlier. We can use unit conversion to solve this problem.

**Your Turn 3.1.7: Find Maxium Number of People**

A standard elevator in a midrise building can hold a maximum weight of about $3\frac{1}{3}$ tons. Assuming an average adult weight of $229$ pounds, what is the maximum number of adults who can safely ride the elevator?

_____ people

**Use** $1$ **ton** $= 2000$ **lb**

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## Summary

In the U.S. customary system of measurement, weight is measured in three units: ounces, pounds, and tons. A pound is equivalent to **16** ounces, and a ton is equivalent to **2,000** pounds. While an object's weight can be described using any of these units, it is typical to describe very heavy objects using tons and very light objects using an ounce. Pounds are used to describe the weight of many objects and people. Often, in order to compare the weights of two objects or people or to solve problems involving weight, you must convert from one unit of measurement to another. Using conversion factors with the factor label method is an effective strategy for converting units and solving problems.

## Volume Capacity

### Introduction

**Capacity** is the amount of liquid (or other pourable substance) that an object can hold when it's full. When a liquid, such as milk, is being described in gallons or quarts, this is a measure of capacity.

Understanding units of capacity can help you solve problems like this: Sven and Johanna were hosting a potluck dinner. They did not ask their guests to tell them what they would be bringing, and three people ended up bringing soup. Erin brought $1$ quart, Richard brought $3$ pints, and LeVar brought $9$ cups. How many cups of soup did they have altogether?

## Units of Capacity

There are five main units for measuring capacity in the U.S. customary measurement system. The smallest unit of measurement is a **fluid ounce**. "Ounce" is also used as a measure of weight, so it is important to use the word "fluid" with ounce when you are talking about capacity. Sometimes the prefix "fluid" is not used when it is clear from the context that the measurement is capacity, not weight.

The other units of capacity in the customary system are the **cup**, **pint**, **quart**, and **gallon**. The table below describes each unit of capacity and provides an example to illustrate the size of the unit of measurement.

| | |
|---|---|
| **Fluid Ounce**<br>A unit of capacity equal to $\frac{1}{8}$ of a cup.<br>One fluid ounce of water at $62$°F weighs about one ounce. The amount of liquid medicine is often measured in fluid ounces. |  |
| **Cup**<br>A unit equal to $8$ fluid ounces. The capacity of a standard measuring cup is one cup. |  |
| **Pint**<br>A unit equal to $16$ fluid ounces, or $2$ cups. The capacity of a carton of ice cream is often measured in pints. |  |
| **Quart**<br>A unit equal to $32$ fluid ounces, or $4$ cups. You often see quarts of milk being sold in the supermarket. |  |
| **Gallon**<br>A unit equal to $4$ quarts, or $128$ fluid ounces. When you fill up your car with gasoline, the price of gas is often listed in dollars per gallon. |  |

You can use any of these five measurement units to describe the capacity of an object, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the capacity of a swimming pool in gallons and the capacity of an expensive perfume in fluid ounces.

Sometimes you will need to convert between units of measurement. For example, you might want to express $5$ gallons of lemonade in cups if you are trying to determine how many $8$-fluid-ounce servings the amount of lemonade would yield.

The table below shows some of the most common equivalents and conversion factors for the five customary units of measurement of capacity.

| Unit Equivalents | Conversion Factors (heavier to lighter units of measurement) | Conversion Factors (lighter to heavier units of measurement) |
|---|---|---|
| $1$ cup = $8$ fluid ounces | $\frac{1 \text{ cup}}{8 \text{ fluid ounces}}$ | $\frac{8 \text{ fluid ounces}}{1 \text{ cup}}$ |
| $1$ pint = $2$ cups | $\frac{1 \text{ pint}}{2 \text{ fluid cups}}$ | $\frac{2 \text{ fluid cups}}{1 \text{ pint}}$ |
| $1$ quart = $2$ pints | $\frac{1 \text{ quart}}{2 \text{ pints}}$ | $\frac{2 \text{ pints}}{1 \text{ quart}}$ |
| $1$ quart = $4$ cups | $\frac{1 \text{ quart}}{4 \text{ cups}}$ | $\frac{4 \text{ cups}}{1 \text{ quart}}$ |
| $1$ gallon = $4$ quarts | $\frac{1 \text{ gallon}}{4 \text{ quarts}}$ | $\frac{4 \text{ quarts}}{1 \text{ gallon}}$ |
| $1$ gallon = $16$ cups | $\frac{1 \text{ gallon}}{16 \text{ cups}}$ | $\frac{16 \text{ cups}}{1 \text{ gallon}}$ |

## Converting Between Units of Capacity

As with converting units of length and weight, you can use the factor label method to convert from one unit of capacity to another. An example of this method is shown below.

### Example 3.1.8: Gallons to Pints

How many pints is $2\frac{3}{4}$ gallons?

**Answer**

Begin by reasoning about your answer. Since a gallon is larger than a pint, expect the answer in pints to be a number greater than $2\frac{3}{4}$.

$$2\frac{3}{4} \text{ gallons} = \frac{11}{4} \text{ gallons} = \_\_\_\_\_ \text{ pints}$$

The table above does not contain a conversion factor for gallons and pints, so you cannot convert it in one step. However, you can use quarts as an intermediate unit, as shown here. Set up the equation so that two sets of labels reduce—gallons and quarts.

$$\frac{11 \text{ gallons}}{4} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}}$$

Multiply and simplify.

$$\frac{11 \cdot 4 \cdot 2 \text{ pints}}{4 \cdot 1 \cdot 1}$$
$$\frac{88 \text{ pints}}{4} = 22 \text{ pints}$$

Hence, $2\frac{3}{4}$ gallons is $22$ pints.

**Your Turn 3.1.8: Volume Conversion**

Convert $29$ gallons into cups. In other words, how many cups are in $29$ gallons? [ ] cups

You buy a $29$ -gallon container of ice cream from the local ice cream parlor to serve at a party. If $\frac{1}{2}$ cup $= 1$ serving, how many people can you serve ice cream to at your party? (Assume each person eats only $1$ one serving of ice cream.)

[ ] servings.

Common Volume Conversions

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**Example 3.1.9: Gallons to fluid Ounces**

How many gallons are $32$ fluid ounces?

**Answer**

Begin by reasoning about your answer. Since gallons is a larger unit than fluid ounces, expect the answer to be less than $32$ .

$$32 \text{ fluid ounces} = \_\_\_\_\_ \text{ gallons}$$

Please note that your answer should be expressed in gallons. Ensure that the units you want to remove are on opposite sides of the fraction so that they cancel each other.

$$\begin{aligned} 1 \text{ cup} &= 8 \text{ fl oz} \\ 1 \text{ pt} &= 2 \text{ cups} \\ 1 \text{ qt} &= 2 \text{ pt} \\ 1 \text{ gal} &= 4 \text{ qt} \end{aligned}$$

$$\frac{32 \text{ fl oz}}{1} \times \frac{1 \text{ cup}}{8 \text{ fl oz}} \times \frac{1 \text{ pt}}{2 \text{ cup}} \times \frac{1 \text{ qt}}{2 \text{ pt}} \times \frac{1 \text{ gal}}{4 \text{ qt}} = \frac{32 \text{ gal}}{128} = 0.25 \text{ gal}$$

$32$ fluid ounces equal to $0.25$ gallons.

**Your Turn 3.1.9: Add Different Volume Units**

Steven and Johanna were hosting a potluck dinner. They did not ask their guests to tell them what they would be bringing, and three people ended up bringing soup. Erin brought $3$ quart, Richard brought $4.75$ pints, and LeVar brought $10.6$ cups. How much soup did they have in total?

$3$ quart = [ ] cup

$4.75$ pints = [ ] cups

Total soup is = [ ] cups

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**Example 3.1.10: Quarts to Gallons**

Natasha is making lemonade to bring to the beach. She has two containers. One holds one gallon and the other holds **2** quarts. If she fills both containers, how many cups of lemonade will she have?

**Answer**

This problem requires you to find the sum of the capacity of each container and then convert that sum to cups.

$$1 \text{ gallon} + 2 \text{ quarts} = \_\_\_\_\_ \text{ cups}$$

First, find the sum in quarts. 1 gallon is equal to 4 quarts.

$$4 \text{ quarts} + 2 \text{ quarts} = 6 \text{ quarts}$$

Since the problem asks for the capacity in cups, convert 6 quarts to cups.

$$\frac{6 \text{ quarts}}{1} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}}$$

Reduce units that appear in both the numerator and denominator.

$$\frac{6}{1} \cdot \frac{2}{1} \cdot \frac{2 \text{ cups}}{1}$$

Multiply.

$$6 \cdot 2 \cdot 2 = 24 \text{ cups}$$

Hence, Natasha will have **24** cups of lemonade.

Another way to work the problem above would be to first change **1** gallon to **16** cups and change **2** quarts to **8** cups. Then add: **16 + 8** = **24** cups.

**Your Turn 3.1.10: Volume Conversion**

You are in charge of drinks for a community barbecue. You need to supply at least 120 cups of beverage to provide enough for the projected number of people who will attend. So far, you have received the following donations:

- Enough mix to make 4 gallons of lemonade
- 6 bottles of fruit juice that each contain 64 fl. oz.

How many cups of beverage do you have?

Will you have enough for the barbecue?

- no
- yes

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## Summary

There are five basic units for measuring capacity in the U.S. customary measurement system. These are the fluid ounce, cup, pint, quart, and gallon. These measurement units are related to one another, and capacity can be described using any of the units. Typically, people use gallons to describe larger quantities and fluid ounces, cups, pints, or quarts to describe smaller quantities. Often, in order to compare or to solve problems involving the amount of liquid in a container, you need to convert from one unit of measurement to another.

## Measuring Areas in USCS System

Area measures the **amount of surface** a shape covers. In the U.S. Customary System, area is measured using **square units**.

| Area Unit (USCS) | Based On | Where It's Used |
|---|---|---|
| Square Inch ($in^2$) | 1 inch × 1 inch | Small surfaces (screens, tiles, paper pieces) |
| Square Foot ($ft^2$) | 1 foot × 1 foot | Rooms, flooring, houses |
| Square Yard ($yd^2$) | 1 yard × 1 yard | Carpet, land plots, larger surfaces |
| One Acre | 43,560 $ft^2$ | Land, real estate, farming |
| Square Mile ($mi^2$) | 1 mile × 1 mile | Cities, counties, and large land areas |

| Units | Relationship | Conversion Factor |
|---|---|---|
| $ft^2$ to $in^2$ | $1\text{ ft} \times 1\text{ ft} = 12\text{ in} \times 12\text{ in}$<br>$1\text{ ft}^2 = 144\text{ in}^2$ | $1\text{ ft}^2 = 144\text{ in}^2$ |
| $yd^2$ to $ft^2$ | $1\text{ yd} \times 1\text{ yd} = 3\text{ ft} \times 3\text{ ft}$<br>$1\text{ yd}^2 = 9\text{ ft}^2$ | $1\text{ yd}^2 = 9\text{ ft}^2$ |
| acre to $ft^2$ | $1\text{ acre} = 43,560\text{ ft}^2$ | $1\text{ acre} = 43,650\text{ ft}^2$ |

**Example 3.1.11**: Area Conversion

How many square yards of carpet are needed to cover the floor of a rectangular room that measures 24 feet by 21 feet?

**Answer**

First, convert the measures from feet to yards.

$$24\text{ ft} = 24\text{ ft} \times \tfrac{1\text{ yd}}{3\text{ ft}} = 8\text{ yd}$$
$$21\text{ ft} = 21\text{ ft} \times \tfrac{1\text{ yd}}{3\text{ ft}} = 7\text{ yd}$$

Use the area formula to determine the area of the room in square yards.

$$8\text{ yd} \times 7\text{ yd} = 56\text{ yd}^2$$

So the area of the room is $56\text{ yd}^2$.

Another way to solve this problem is given below

$$\text{Area} = 24\text{ ft} \times 21\text{ ft} = 504\text{ ft}^2$$

Find the conversion factor that compares square feet and square yards, with square feet in the numerator.

$$\frac{1\text{ yd}^2}{9\text{ ft}^2}$$

$$504\text{ ft}^2 = 504\text{ ft}^2 \times \frac{1\text{ yd}^2}{9\text{ ft}^2} \quad \text{Apply conversion factor.}$$
$$= 504\ \cancel{\text{ft}^2} \times \frac{1\text{ yd}^2}{9\ \cancel{\text{ft}^2}} \quad \text{Cancel common units.}$$
$$= \frac{504 \times 1}{9}\text{yd}^2 \quad \text{Multiply fractions.}$$
$$= 56\text{ yd}^2$$

**Your Turn 3.1.11**: Area Conversion

Convert the following English measurements for area. Round to the nearest whole number.

a) 7 acre = ____ square feet

b) 256 square miles = ____ acres

b) 4000 acre = ____ square miles

Common USCS Area Conversions

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## Your Turn 3.1.12: Mix Conversion

| Multi-Step US Conversions |
| --- |
| Convert the US measurements as indicated. Round your results to two decimal places as needed. |
| 56 pints = ____ gallons |
| 3.6 miles = ____ yards |
| 4 square yards = ____ square inches |

Common Conversions

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## Your Turn 3.1.13: Select the Best Estimate

**Select the best estimate for the measure of the following.**

Which unit is most appropriate for measuring the width of a football field?

- ○ inch
- ○ mile
- ○ foot
- ○ yard

Which unit is best for measuring the amount of milk in a jug?

- ○ fluid ounce
- ○ cup
- ○ teaspoon
- ○ gallon

Which unit is most appropriate for measuring the weight of a bag of apples?

- ○ cup
- ○ pound
- ○ ounce
- ○ ton

Which unit is most appropriate for measuring the height of a building?

- ○ inch
- ○ foot
- ○ mile
- ○ yard

Which unit is most appropriate for measuring the area of National Park?

- ○ square yard

square yard

square mile

square inch

square feet

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## 3.2: The Metric System

Even if you don't travel outside of the United States, many specialty grocery stores utilize the metric system. For example, if you want to make authentic tamales you might visit the nearest Hispanic grocery store. While shopping, you discover that there are two brands of mass for the same price, but one bag is marked $1,200$ g and the other $1$ kg. Which one is the better deal? Understanding the metric system allows you to understand that $1,200$ grams (g) is equivalent to $1.2$ kilogram (kg), so the $1,200$ g bag is the better deal.



Figure **3.2.1**: A scale that measures weight in both metric and customary units. (credit: "Weighing the homemade cheddar" by Ruth Hartnup/Flickr, CC BY 2.0)

### Units of Measurement in the Metric System

Units of measurement provide common standards so that, regardless of where or when an object or substance is measured, the results are consistent. When measuring distance, the units of measure might be feet, meters, or miles. Weight might be expressed in terms of pounds or grams. Volume or capacity might be measured in gallons, or liters. Understanding how metric units of measure relate to each other is essential to understanding the metric system:

- The metric unit for distance is the meter (m). A person's height might be written as $1.8$ meters ($1.8$ m). A meter is slightly longer than a yard ($3$ feet), while a centimeter is slightly less than half an inch.
- The metric unit for area is the square meter ($m^2$). The area of a professional soccer field is $7,140$ square meters ($7,140\ m^2$).
- The metric base unit for volume is the cubic meter ($m^3$). However, the liter (L), which is a metric unit of capacity, is used to describe the volume of liquids. Soda is often sold in $2$-liter ($2$ L) bottles.
- The gram (g) is a metric unit of mass, but is commonly used to express weight. The weight of a paper clip is approximately $1$ gram ($1$ g).
- The metric unit for temperature is degrees Celsius (°C). The temperature on a warm summer day might be $24$ °C.

While the U.S. Customary System of Measurement uses ounces and pounds to distinguish between weight units of different sizes, in the metric system, a base unit is combined with a prefix, such as kilo in kilogram, to identify the relationship between smaller or larger units.

| USCS | Metric | Comparison |
|---|---|---|
| Yard | Meter | A meter is a little more than a yard.<br>$1$ yard is about $0.9$ meters |
| Pound | Kilogram | A kilogram is heavier than a Pound<br>$1$ kilogram is about $2.2$ pounds |
| Quarts | Liter | Liter is a little more than a quart<br>$1$ quart is about $0.95$ liters |
| Centimeter | Inches | An inch is longer than a centimeter<br>$1$ inch is about $2.54$ centimeter |
| Miles | Kilometer | A mile is longer than a kilometer<br>$1$ mile is about $1.609$ kilometer |

**Checkpoint**

*When using abbreviations to represent metric measures, always separate the quantity and the units with a space, with no spaces between the letters or symbols in the units. For example, 7 millimeters is written as 7 mm, not 7mm.*

Let's be honest. Most of us use computers or smartphones to perform many of the calculations and conversions we were taught in math class. However, there is value in understanding the metric system, as it is ubiquitous, and most importantly, knowing how the different metric units relate to each other allows you to

compare prices, find the right tool in a workshop, or acclimate when in another country. No matter the circumstance, you cannot avoid the metric system.

**Your Turn 3.2.1**: Matching Appropriate Measurement with Units

**Measurment - Matching measurement with units.**

Match each measurement with the best appropirate unit.

| | |
|---|---|
| - Meter | a. Length of typical text book. |
| - Kilometer | b. Amount of water in a swimming pool. |
| - Centimeter | c. Weight of one serving of peanuts. |
| - Kilogram | d. Distance from Atlanta to Fort Myers. |
| - Grams | e. Length of an electrical wire. |
| - Liter | f. Height of a bicycle. |

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While there are other base units in the metric system, our discussions in this chapter will be limited to units used to express length, area, volume, weight, and temperature.

## Metric Prefixes

Unlike the U.S. Customary System of Measurement, in which $12$ inches is equal to $1$ foot and $3$ feet are equal to $1$ yard, the metric system is structured so that the units within the system get larger or smaller by a power of $10$. For example, a centimeter is $10^2$ or $100$ times smaller than a meter, while the kilometer is $10^3$, or $1,000$ times larger than a meter.

The metric system combines base units and unit prefixes reasonably according to the size of a measured object or substance. The most used prefixes are shown in Table $3.2.1$. An easy way to remember the order of the prefixes, from largest to smallest, is the mnemonic ***King Henry Died From Drinking Chocolate Milk.***

Table $3.2.1$ Metric Prefixes

| Prefix | kilo– | hecto– | deca– | base unit | deci– | centi– | milli– |
|---|---|---|---|---|---|---|---|
| **Abbreviation** | k | ha | da | meter (m), gram (g), liter (L) | d | c | m |
| **Magnitude** | $10^3$ | $10^2$ | $10^1$ | $10^0 = 1$ | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ |

## Converting Metric Units of Measure

Imagine you order a textbook online and the shipping detail indicates the weight of the book is $1$ kg. By attaching the letter “k” to the base unit of gram (g), the unit used to express the measure is $10^3$ or $1,000$ times greater than a gram. One kilogram is equivalent to $1,000$ grams.

The tip of a highlighter measures approximately $1$ cm. The letter “c” attached to the base unit of meter (m) means the unit used to express the of $10^{-2}$ or $\frac{1}{100}$ a meter. One meter is equivalent to $100$ centimeters.

A **conversion factor** is used to convert between smaller metric units and larger metric units, and vice versa. It is a number that, when used with multiplication or division, converts units from one metric system to another, both of which have the same base unit. In the metric system, these conversion factors are directly related to the powers of 10. The most commonly used conversion factors are shown in figure $3.2.2$.

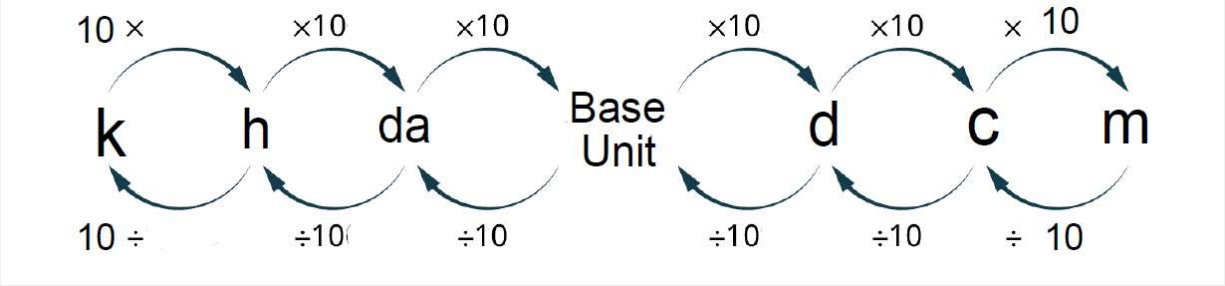



Figure **3.2.2**: Base unit could be meter (m), gram (g), and liter(L)

In the metric system, weight is expressed in terms of grams or kilograms, with a kilogram being equal to **1,000**grams. A paper clip weighs about **1** gram. A liter of water weighs about **1** kilogram. In fact, in the same way that 1 liter is equal in volume to **1** cubic decimeter, the kilogram was originally defined as the mass of 1 liter of water. In some cases, particularly in scientific or medical settings where small amounts of materials are used, the milligram is used to express weight. At the other end of the scale is the metric ton (mt), which is equivalent to **1,000** kilograms. The average car weighs about **2** metric tons.

Any discussion about metric weight must also include a conversation about mass. Scientifically, mass is the amount of matter in an object, whereas weight is the force exerted on an object by gravity. The amount of mass of an object remains constant no matter where the object is. Identical objects located on Earth and on the moon will have the same mass, but the weight of the objects will differ because the moon has a weaker gravitational force than Earth. So, objects with the same mass will weigh less on the moon than on Earth.

Since there is no easy way to measure mass, and since gravity is just about the same no matter where on Earth you go, people in countries that use the metric system often use the words mass and weight interchangeably. While scientifically the kilogram is only a unit of mass, in everyday life it is often used as a unit of weight as well.

To have an essential understanding of metric weight, you must be able to identify reasonable values for weight. When testing for reasonableness, you should assess both the unit and the unit value. Only by examining both can you determine whether the given weight is reasonable for the situation.

**Definition: Changing Units within Metric System**

In figure **3.2.2**, the units on the left are bigger than the units on the right.

1. If you go **from large → small**, multiply by **10** for each step to the right. (move decimal **right**).
2. If you go **from small → large**, divide by **10** for each step to the left. (move decimal **left**).

**Your Turn 3.2.2: Metric Conversion**

Fill in the table of metric equivalents based on the given value. **75** hm

| km | hm | dam | m | dm | cm | mm |
|---|---|---|---|---|---|---|
| | | | | | | |

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**Example 3.2.3: Converting Metric Distances Using Multiplication**

1. The firehouse is **13.45** km from the library. How many meters is it from the firehouse to the library?
2. How many liters is **3,565** milliliters?
3. Convert **125** centigram to milligrams.
4. Convert **24,300,000** milligrams to kilograms.
5. Change **0.025** kilograms to centigram

**Answer**

1. When converting from a larger unit to a smaller unit, use multiplication, see in the figure **3.2.2**. We need to multiply by **10** three times. It is the same as saying multiply by **1000** or move the decimal **3** places to the right.

$$13.45 \times 1,000 = 13,450$$

So, the firehouse is **13,450** meters away from the firehouse.

2. When converting from a smaller unit to a larger unit, use division, see in the figure **3.2.2**. We need to divide by **10** three times. It is the same as saying divide by **1,000** or move the decimal **3** places to the left.

$$3,565 \div 1,000 = 3.565$$

So, **3,565** milliliters is **3.565** liters.

3. When converting from a larger unit to a smaller unit, use multiplication to calculate the result. See in the figure **3.2.2**. We need to multiply by **10**. It is the same as saying move the decimal **1** places to the right..

$$125 \times 10 = 1,250$$

So, **125** centigrams are **1,250** milligrams.

4. Use division to convert from a smaller metric weight unit to a larger metric weight unit. To convert from milligrams to kilograms, divide the value of the weight in milligrams by **1,000** to first convert from milligrams to grams and divide by **1,000** again to convert from grams to kilograms.

$$\frac{24,300,000}{1,000,000} = 24.3$$

So, **24,300,000** milligrams are equivalent to **24.3** kilograms.

5. Use multiplication to convert from a larger metric weight unit to a smaller metric weight unit. To convert from kilograms to centigrams, multiply the value of the weight by **10**, five times .

$$0.025 \times 10 \times 10 \times 10 \times 10 \times 10 = 2,500$$

So the **0.025** kilogram is equivalent to **2,500** centigrams.

**Your Turn 3.2.3: Metric Conversion**

Convert the following measurements. **Do not round the results.**

a. Austin ran $1783$ meters. How many kilometers did he run?

He ran _____ km.

b. Zach poured $0.43$ liters of juice for a smoothie. How many centiliters is this?

He poured _____ centiliters.

c. A turtle crawled $0.07491$ kilometers. How many millimeters is this?

The turtle crawled _____ mm.

d. Alexis weighed $0.63$ grams of a substance for a chemistry experiment. How many milligrams did she weigh?

She weighed _____ mg of the substance.

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## Example 3.2.4: Identifying Reasonable Units for Weight

1. Which is the more reasonable value for the weight of a newborn baby: 3.5 kg or 3.5 g?
2. Which of the following represents a reasonable value for the weight of three lemons? 250 g, 2,500g, or 250 kg?
3. What is a reasonable value for the length of a person's thumb: 5 meters, 5 centimeters, or 5 millimeters?
4. What is a reasonable value for the width of an electric chord: 4 meters, 4 centimeters, or 4 millimeters?
5. The blue whale is the largest living mammal on Earth. Which of the following is a reasonable value for the weight of a blue whale: 149 g, 149 kg, or 149 mt? Explain your answer.

**Answer**

1. Using our reference weights, a baby weighs more than 3.5 paperclips, so 3.5 kilograms is a more reasonable value for the weight of a newborn baby.
2. Because a kilogram is about 2.2 pounds, we can eliminate 250 kg as it is way too heavy. 2,500 grams is equivalent to 2.5 kilograms, or about five pounds, which is again, too heavy. So, a reasonable value for the weight of three lemons would be 250 grams.
3. Given that a meter is slightly longer than a yard, 5 meters is not a reasonable value for the length of a person's thumb. Since a millimeter is 10 times smaller than a centimeter, which is approximately half an inch, 5 millimeters is not a reasonable estimate for the length of a person's thumb. The correct answer is 5 centimeters.
4. 4 millimeters.
5. A reasonable value for the weight of a blue whale is 149 metric tons. Both 149 g and 149 kg are much too small a value for the largest living mammal on Earth.

## Your Turn 3.2.4: Select the Best Estimate

**Select the best estimate for the measure of the following.**

What is a reasonable estimate for the length of a hallway?

- ○ 2.5 milimeter
- ○ 2.5 centimeter
- ○ 2.5 meter
- ○ 2.5 kilometer

What is a reasonable value for the length of a page of a book:?

- ○ 2.50 millimeter
- ○ 25 millimeter
- ○ 2500 millimeter
- ○ 250 millimeter

Which of the following is a reasonable value for the weight of a butterfly?

- ○ 30 gram
- ○ 30 centigram
- ○ 30 ton
- ○ 30 miligram

Which represents a reasonable value for the weight of a penny?

- ○ 2.5 gram
- ○ 2.5 decagram

2.5 decagram

2.5 kilogram

2.5 miligram

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**Who Knew?: How Do You Measure the Weight of a Whale?**

It is impossible to weigh a living whale. Fredrik Christiansen from the Aarhus Institute of Advanced Studies in Denmark developed an innovative way to measure the weight of whales. Using images taken from a drone and computer modeling, the weight of a whale can be estimated with great accuracy

**Who Knew?: The United States and the Metric System**

Did you know that the metric system pervades daily life in the United States already? While Americans still may purchase gallons of milk and measure house sizes in square feet, there are many instances of the metric system. Photographers buy 35 mm film and use 50 mm lenses. When you have a headache, you might take 600 mg of ibuprofen. And if you are eating a low-carb diet you probably restrict your carb intake to fewer than 20 g of carbs daily. Did you know even the dollar is metric? In the video, Neil DeGrasse Tyson and comedian co-host Chuck Nice provide an amusing perspective on the metric system.

The International System of Units (SI) is the current international standard metric system and is the most widely used system around the world. In most English-speaking countries SI units such as meter, liter, and metric ton are spelled metre, litre, and tonne.

**WORK IT OUT: Get to Know the Metric System**

Just how much is the metric system a part of your life now? Probably more than you think. For the next **24** hours, take notice as you move through your daily activities. When you are shopping, are the package sizes provided in metric units? Change the weather app on your phone to display the temperature in degrees Celsius. Are you able to tell what kind of day it will be now? While the United States is not officially using the metric system, you will still find the metric system all around you

## Measuring Areas

Area is the size of a surface. It could be a piece of land, a rug, a wall, or any other two-dimensional surface with attributes that can be measured in the metric unit for distance, meters. Determining the area of a surface is important to many everyday activities. For example, when purchasing paint, you'll need to know how many square units of surface area need to be painted to determine how much paint to buy.



Figure **3.2.3**: A painter uses an extension roller to paint a wall. (credit: "Paint Rollers are effective" by WILLPOWER STUDIOS/Flickr, CC BY 2.0)

Square units indicate that two measures in the same units have been multiplied together. For example, to find the area of a rectangle, multiply the length units and the width units to determine the area in square units.

$$1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^2$$

Note that to accurately calculate area, each of the measures being multiplied must be of the same units. For example, to find an area in square centimeters, both length measures (length and width) must be in centimeters.

**FORMULA: To Find the Area of Rectangle**

The formula used to determine the area depends on the shape of that surface. Here, we will limit our discussions to the area of rectangular-shaped objects like the one in figure **3.2.3**. Given this limitation, the basic formula for area is:

$$\text{Area} = \text{lenght} \times \text{width}$$

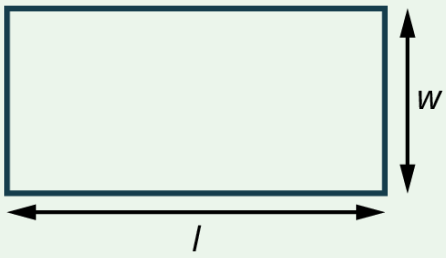



Figure **3.2.4**: Rectangle with Length ($l$) and Width ($w$) labeled

## Reasonable Values for Area

Because area is determined by multiplying two lengths, the magnitude of the difference between different square units is exponential. In other words, while a meter is **100** times greater in length than a centimeter, a square meter ( $\mathbf{m^2}$ ) is $100 \times 100$, or **10,000** times greater in area than a square centimeter ($\mathbf{cm^2}$). The relationships between benchmark metric area units are shown in the following table.

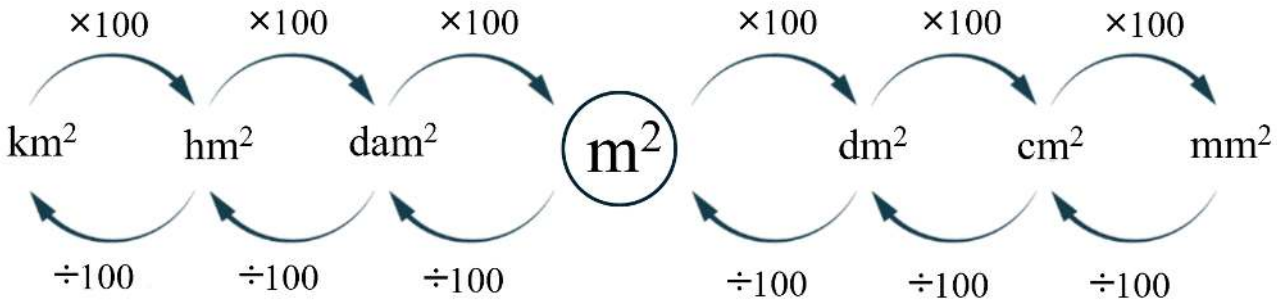



Figure **3.2.5**: Area Conversion Chart

| Units | Relationship | Conversion Factor |
|---|---|---|
| $\mathbf{km^2}$ to $\mathbf{m^2}$ | $1\text{ km} \times 1\text{ km} = 1{,}000\text{ m} \times 1{,}000\text{ m}$<br>$1\text{ km}^2 = 1{,}000{,}000\text{ m}^2$ | $1\text{ km}^2 = 1{,}000{,}000\text{ m}^2$ |
| $\mathbf{m^2}$ to $\mathbf{cm^2}$ | $1\text{ m} \times 1\text{ m} = 100\text{ cm} \times 100\text{ cm}$<br>$1\text{ m}^2 = 10{,}000\text{ cm}^2$ | $1\text{ m}^2 = 10{,}000\text{ cm}^2$ |
| $\mathbf{cm^2}$ to $\mathbf{mm^2}$ | $1\text{ cm} \times 1\text{ cm} = 10\text{ mm} \times 10\text{ mm}$<br>$1\text{ cm}^2 = 100\text{ mm}^2$ | $1\text{ cm}^2 = 100\text{ mm}^2$ |

A fundamental understanding of metric area is essential for identifying reasonable values for area. When testing for reasonableness, you should assess both the unit and the unit value. Only by examining both can you determine whether the given area is reasonable for the situation.

**Example 3.2.5: Determining Reasonable Units for Area**

1. Which unit of measure is most reasonable to describe the area of a sheet of paper: $\mathbf{km^2}$, $\mathbf{cm^2}$, or $\mathbf{mm^2}$ ?
2. You want to paint the walls of your bedroom. Which represents a reasonable value for the area of the walls: **100** $\text{cm}^2$, **100** $\text{m}^2$, or **100** $\text{km}^2$

**Answer**

1. In the U.S. Customary System of Measurement, the length and width of paper are usually measured in inches. In the metric system, centimeters are used for measures usually expressed in inches. Thus, the most reasonable unit of measure to describe the area of a sheet of paper is square centimeters. Square kilometers is too large a unit, and square millimeters is too small a unit.
2. An area of **100** $\text{cm}^2$ is equivalent to a surface of **10** cm × **10** cm, which is much too small for the walls of a bedroom. An area of **100** $\text{km}^2$ is equivalent to a surface of **10** km × **10** km, which is much too large for the walls of a bedroom. So, a reasonable value for the area of the walls is **100** $\text{m}^2$.

## Your Turn 3.2.5: Reasonable Unit for Area

**Select the best estimate for the measure of the following.**

Which is the most reasonable area for a parking lot?

- ○ 800 square meter
- ○ 8 square centimeter
- ○ 8 square kilometer
- ○ 80 square centimeter

Which is the most reasonable area for a sticky note?

- ○ 25 square centimeter
- ○ 25 square meter
- ○ 0.25 square kilometer
- ○ 2500 square meter

Which is the most reasonable area for a city?

- ○ 400 square meter
- ○ 4000 squaremeter
- ○ 4 square centimeter
- ○ 400 square kilometer

Which is the most reasonable area for a postage stamp?

- ○ 5 square meter
- ○ 50 square centimeter
- ○ 5 square milimeter
- ○ 500 square milimeter

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### Converting Units of Measure for Area

Just like converting units of measure for distance, you can convert units of measure for area. However, the conversion factor, or the number used to multiply or divide to convert from one area unit to another, is not the same as the conversion factor for metric distance units. Recall that the conversion factor for area is exponentially relative to the conversion factor for distance. The most frequently used conversion factors are shown in Figure 9.7.

## Example 3.2.6: Converting Units of Measure for Area

1. A plot of land has an area of $237,500,000$ $m^2$. What is the area in square kilometers?
2. A plot of land has an area of $0.004046$ $km^2$. What is the area of the land in square meters?

**Answer**

1. Use division to convert from a smaller metric area unit to a larger metric area unit. To convert from $m^2$ to $km^2$, divide the value of the area by $1,000,000$ (divide the value of the area of by $100$ three times). See figure $3.2.5$ or move the decimal six units to the left.

$$\frac{237,500,000}{1,000,000} = 237.5$$

So the area of the plot is $237.5$ $km^2$

2. Use multiplication to convert from a larger metric area unit to a smaller metric area unit. To convert from $km^2$ to $m^2$, multiply the value of the area by $1,000,000$ or move the decimal six decimal places to the right.

$$0.004046 \times 1,000,000 = 4,046$$

So the area of the plot is $4,406$ $m^2$

## Your Turn 3.2.6: Converting Units of Measure of Area

Convert $70.75$ square decameters to square kilometers.

Select an answer

Need Calculator? Show Calculator

Measurement and Conversion Table

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## Your Turn 3.2.7: Determining Area by Converting Units of Measure for Length First

1. A computer chip measures $10$ mm by $15$ mm. How many square centimeters is the computer chip?
2. A piece of fabric measures $110$ cm by $116$ cm. What is the area of the fabric in square meters?

**Answer**

1. First, convert the measures of the computer chip into centimeters

$$10 \text{ mm} = 1 \text{ cm}$$
$$15 \text{ mm} = 1.5 \text{ cm}$$

Use the area formula to determine the area of the chip.

$$1 \times 1.5 = 1.5$$

The computer chip has an area of $1.5$ $cm^2$.

2. First, convert the measures of the fabric into meters from centimeters.

Use the area formula to determine the area of the fabric

$$1.10 \text{ m} \times 1.16 \text{ m} = 1.276 \text{ m}^2$$

So the area of the fabric is $1.276$ $m^2$.

When calculating area, ensure that both distance measurements are expressed in the same units of distance. Sometimes you must convert one measurement before using the area formula.

**Your Turn 3.2.7: Square Centimeter and Square Meter**

A rectangular sign measures $340$ cm by $253$ cm. What is the area of the sign in square meters?

Round your answer to the nearest hundredth.

square meters

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When calculating area, you may need to use multiple steps, such as converting units and subtracting irrelevant areas.

**Who Knew?: The Origin of the Metric System**

The metric system is the official measurement system for every country in the world except the United States, Liberia, and Myanmar. But did you know it originated in France during the French Revolution in the late 18th century? At the time there were over $250,000$ different units of weights and measures in use, often determined by local customs and economies. For example, land was often measured in days, referring to the amount of land a person could work in a day.

## Measuring Volume

Volume is a measure of the space contained within or occupied by three-dimensional objects. It could be a box, a pool, a storage unit, or any other three-dimensional object with attributes that can be measured in the metric unit for distance–meters. For example, when purchasing an SUV, you may want to compare how many cubic units of cargo the SUV can hold.



Figure 3.2.6: Packing cartons sit on a loading dock ready to be filled. (credit: “boxing day” by Erich Ferdinand/Flickr, CC BY 2.0)

Cubic units indicate that three measures in the same units have been multiplied together. For example, to find the volume of a rectangular prism, you would multiply the length units by the width units and the height units to determine the volume in square units:

$$1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$$

Note that to accurately calculate volume, each of the measures being multiplied must be of the same units. For example, to find a volume in cubic centimeters, each of the measures must be in centimeters.

**FORMULA**

The formula used to determine volume depends on the shape of the three-dimensional object. Here we will limit our discussions to the area to rectangular prisms like the one in Figure 3.2.7. Given this limitation, the basic formula for volume is:

$$V = l \times w \times h$$

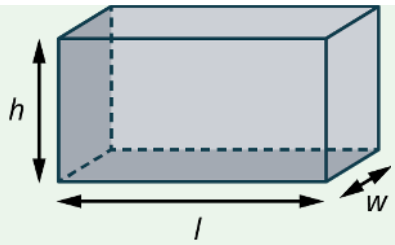



Figure **3.2.7**: Rectangular Prism with Height $(h)$, length$(l)$, and width$(w)$ Labeled.

## Reasonable Values for Volume

Because volume is determined by multiplying three lengths, the magnitude of the difference between different cubic units is exponential. In other words, while a meter is 100 times greater in length than a centimeter, a cubic meter, $m^3$, is $100 \times 100 \times 100$, or $1{,}000{,}000$ times greater in area than a cubic centimeter, $cm^3$. This relationship between benchmark metric volume units is shown in the following table.

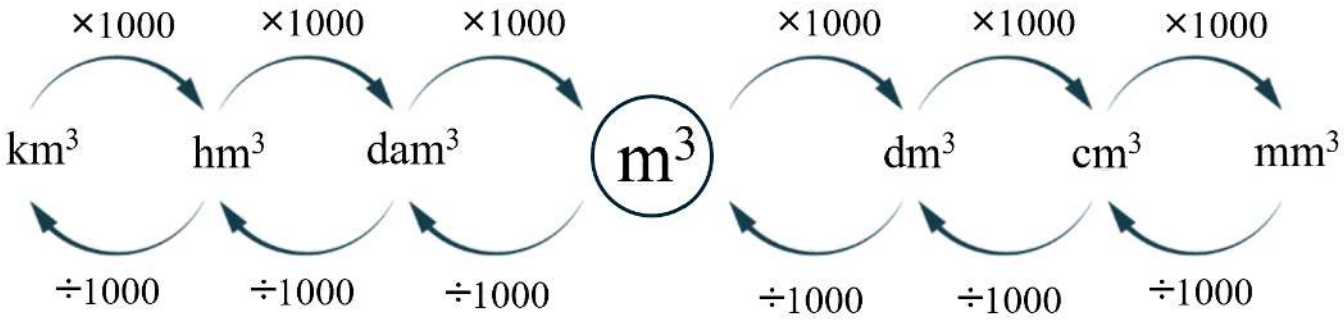



Figure **3.2.8**: Volume Conversion Chart

| Units | Relationship | Conversion Factor |
|---|---|---|
| $km^3$ to $m^3$ | $1\text{ km} \times 1\text{ km} \times 1\text{ km} = 1{,}000\text{ m} \times 1{,}000\text{ m} \times 1{,}000\text{ m}$<br>$1\text{ km}^3 = 1{,}000{,}000{,}000\text{ m}^3$ | 1 $km^3$ = 1,000,000,000 $m^3$ |
| $m^3$ to $dm^3$ | $1\text{ m} \times 1\text{ m} \times 1\text{ m} = 10\text{ dm} \times 10\text{ dm} \times 10\text{ dm}$<br>$1\text{ m}^3 = 1{,}000\text{ dm}^3$ | 1 $m^3$ = 1,000 $dm^3$ |
| $cm^3$ to $mm^3$ | $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 10\text{ mm} \times 10\text{ mm} \times 10\text{ mm}$<br>$1\text{ cm}^3 = 1{,}000\text{ mm}^3$ | 1 $cm^3$ = 1,000 $mm^3$ |

To have an essential understanding of metric volume, you must be able to identify reasonable values for volume. When testing for reasonableness you should assess both the unit and the unit value. Only by examining both can you determine whether the given volume is reasonable for the situation.

### Example 3.2.8: Determining Reasonable Values for Volume

1. A grandparent wants to send cookies to their grandchild away at college. Which represents a reasonable value for the volume of a box to ship the cookies: $3{,}375$ $km^3$, $3{,}375$ $m^3$, or $3{,}375$ $cm^3$?
2. A food manufacturer is prototyping new packaging for one of its most popular products. Which represents a reasonable value for the volume of the box:

   $2$ $dm^3$, $2$ $cm^3$, or $2$ $mm^3$ ?

**Answer**

1. A volume of $3{,}375\ \text{km}^3$ is equivalent to a rectangular prism with dimensions of $15\text{ km} \times 15\text{ km} \times 15\text{ km}$, which is far too large for a shipping box. An area of $3{,}375\ \text{m}^3$ is equivalent to a surface of $15\text{ m} \times 15\text{ m} \times 15\text{ m}$, which is also too large. A reasonable value for the volume of the box is $3{,}375\ \text{cm}^3$.
2. A decimeter is equal to $10$ centimeters. A box with a volume of $2\ \text{dm}^3$might have the dimensions $1\text{ dm} \times 1\text{ dm} \times 1\text{ dm}$, or $10\text{ cm} \times 10\text{ cm} \times 10\text{ cm}$ , which is reasonable. A box with a volume of $2\ \text{cm}^3$ or $2\ \text{mm}^3$ would be too small.

Your Turn **3.2.8**: Select the Best Estimate Measure

**Select the best estimate for the measure of the following.**

Which is the most reasonable volume of gasoline in a car tank?

- ○ 6 kiloliter
- ○ 60 mililiter
- ○ 60 liter
- ○ 60 militer

Which is the most reasonable volume for liquid medicine in a spoon?

- ○ 50 liter
- ○ 5 kiloLiter
- ○ 5 mililiter
- ○ 500 mililiter

Which represents a reasonable value for the volume of a storage area?

- ○ 8 cubic kilometer
- ○ 8 cubic meter
- ○ 8 cubic milimeter
- ○ 8 cubic centimeter

Which represents a reasonable value for the volume of a fish tank?

- ○ 40,000 cubic meter
- ○ 40,000 cubic milimeter
- ○ 40,000 cubic decameter
- ○ 40,000 cubic centimeter

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## Converting Like Units of Measure for Volume

Just like converting units of measure for distance, you can convert units of measure for volume. However, the conversion factor, the number used to multiply or divide to convert from one volume unit to another, is different from the conversion factor for metric distance units. Recall that the conversion factor for volume is exponentially relative to the conversion factor for distance. The most frequently used conversion factors are illustrated in Figure **3.2.3**.

### Example 3.2.9: Converting Units of Measure for Volume

1. A pencil case has a volume of $1,700$ $cm^3$. What is the volume in cubic millimeters?
2. A shipping container has a volume of $33.2$ $m^3$. What is the volume in cubic centimeters?
3. A holding tank at the local aquarium has a volume of $22,712,000,000$ $dm^3$. What is the volume in cubic meters?

**Answer**

1. Use multiplication to convert from a larger metric volume unit to a smaller metric volume unit. To convert from $cm^3$ to $mm^3$, multiply the value of the volume by $1,000$.

$$1,700 \times 1,000 = 1,700,000$$

The pencil case has a volume of $1,700,000$ $mm^3$.

2. Use multiplication to convert a larger metric volume unit to a smaller metric volume unit. To convert from $m^3$to $cm^3$, first multiply the value of the volme by $1,000$ to convert from $mm^3$ to $dm^3$ and then multiply again by $1,000$ to convert form $dm^3$ to $m^3$.

$$33.2 \times 1,000 \times 1,000 = 33,200,000$$

The shipping container has a volume of $32,200,000$ $cm^3$.

3. Figure 3.2.3 indicates that when converting from a smaller metric volume unit to a larger metric volume unit you divide using the given conversion factor. To convert from $cm^3$ to $m^3$, divide the value of the volume by $1,000$ to first convert from $cm^3$ to $dm^3$, then divide again to convert from $dm^3$ to $m^3$. Or divide by $1,000,000$.

$$\frac{22,712,000,000}{1,000,000} = 22,712$$

The holding tank has a volume of $22,712$ $m^3$.

### Your Turn 3.2.9: Converting Units of Measure for Volume

Convert 16.291 $km^3$ to $dam^3$

16.291 $km^3$ = _____ [ ]

**Enter the numerical answer in the first box and the correct unit abbreviation in the second box. Do not round your answer.**

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## Understanding Other Metric Units of Volume

When was the last time you purchased a bottle of soda? Was the volume of the bottle expressed in cubic centimeters or liters? The liter (L) is a metric unit of capacity often used to express the volume of liquids. A liter is equal in volume to $1$ cubic decimeter. A milliliter is equal in volume to $1$ cubic centimeter. So, when a doctor orders $10$cc (cubic centimeters) of saline to be administered to a patient, they are referring to $10$ $mL$ of saline.

The most frequently used factors for converting from cubic meters to liters are listed in the following table.

| Equivalency: Water Volume in Cubic Units and Water Volume in Liters |
|---|
| $1$ $cm^3$ $=$ $1mL$ OR $1,000$ $cm^3$ $=$ $1L$ |
| $1$ $dm^3$ $=$ $1L$ |
| $1$ $m^3$ $=$ $1kL$ |

### Example 3.2.10: Converting Different Units of Measure for Volume

A holding tank at the local aquarium has a volume of $22,712,000,000$ $cm^3$? What is the capacity of the holding tank in liters?

**Answer**

Use division to convert from cubic centimeters to liters. To determine the equivalent volume in liters, convert from $cm^3$ to $L$ by dividing the value of the volume in $cm^3$ by $1,000$.

$$
\begin{aligned}
22,712,000,000\ \text{cm}^3 &= 22,712,000,000\ \text{cm}^3 \times \frac{1\ \text{L}}{1,000\ \text{cm}^3} \\
&= 22,712,000,000\ \cancel{\text{cm}^3} \times \frac{1\ \text{L}}{1,000\ \cancel{\text{cm}^3}} \qquad \text{Cancel common units.} \\
&= 22,712,000\ \text{L}
\end{aligned}
$$

The holding tank holds $22,712,000\ \text{L}$ of water.

**Your Turn 3.2.10: Converting Different Units of Measure for Volume Using Multiplication**

An airplane used $150\ \text{m}^3$ of fuel to fly from New York to Hawaii. How many liters of fuel did the airplane use?

**Answer**

Because $1$ liter is equivalent to $1$ cubic decimeter, use multiplication to convert from $\text{m}^3$ to $\text{dm}^3$. Multiply the value of the volume by $1,000$ to convert from $\text{m}^3$ to $\text{dm}^3$. Because $\(1\)\ \text{dm}^3 = 1\ \text{L}$, the resulting value is equivalent to the number of liters used.

The airplane used $150,000$ liters of fuel.

---

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# 3.3: Conversion Between the Metric and USCS

## Estimate equivalence between the Metric Units and US Customary Units

Before converting metric units to US customary units and vice versa, you need to familiarize the approximate metric equivalents of common customary units. The table below summarizes commonly used metrics and US customary units, along with their approximate equivalences. The notations enclosed are the abbreviations of each unit.

| Metric Units | US Customary Units |
|---|---|
| **Lengths** | |
| **1** meter (m) | **3.38** feet (ft. ) |
| **1** kilometer (km) | **0.62** mile (m) |
| **1**centimeter (cm) | **0.39** inch (in) |
| **0.9144** meter (m) | **1** yard (yd.) |
| **1.852** kilometers (km) | **1** nautical mile (NM) |
| **Weight** | |
| **1** kilogram (kg) | **2.2** pounds (lb) |
| **28.3** grams (g) | **1** ounce (oz.) |
| **1** milligram (mg) | **0.002** teaspoon (tsp.) |
| **Fluid Capacity** | |
| **29.57** milliliters (mL) | **1** fluid ounce (fl. Oz.) |
| **1** liter (L) | **4.23** cups (c) |
| **3.79** liters (L) | **1**gallon (gal.) |
| **473.176** milliliters (mL) | **1** pint (pt.) |
| **0.95** liters (L) | **1** quart (qt.) |
| **158.987** liters (L) | **1** barrel (bbl.) |

| Metric Units | US Customary Units |
|---|---|
| **Area** | |
| **6.45** square centimeters ($cm^2$) | **1** square inch ($in^2$) |
| **1** square meter ($m^2$) | **1.196** square yard ($yd^2$) |
| **4046.873** square meters ($m^2$) | **1** acre (ac.) |
| **Volume and Capacity** | |
| **16.39** milliliter (mL) | **1** cubic inch ($in^3$) |
| **28.32** liters (L) | **1** cubic foot ($ft^3$) |
| **764.55** liters (L) | **1** cubic yard ($yd^3$) |
| **1233.482** cubic meters ($m^3$) | **1** cubic foot ($ft^3$) |

Based on the above table, answer the following question.

**Your Turn 3.3.1: Comparing Units**

**Select the correct answer. Note that both units being compared have the same numerical value. For example, in the first question, we are comparing 5 yards and 5 meters**

Which is the greater distance?

- ○ a meter

- a yard

---

Which is the greater distance?

- a meter
- a foot

---

Which is the greater distance?

- a inch
- a centimeter

---

Which is the greater distance?

- a mile
- a kilometer

---

Which is the greater weight?

- a kilogram
- a pound

---

Which is the greater weight?

- a gram
- an ounce

---

Which is the greater volume?

- a gallon
- a liter

---

Which is the greater volume?

- a liter
- a quart

---

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## Convert Metric Units to US Customary Units and Vice Versa

You have familiarized yourself with the commonly used equivalence between metric and US customary units. At this point, you will learn to convert metric units to US customary units and vice versa.

To convert metric units to US customary units, use their estimated equivalence. We use the factor label method, which was introduced in the last two sections. In this method, we write the equivalence as a ratio and multiply it by the given measurement (in metric) to convert. Ensure that the units of measure being converted cancel out. For instance, to convert liters to gallons, be sure to cancel out liters and retain gallons as the final unit.

### Example 3.3.2: Miles to Kilometer

A car's speedometer shows that a family has currently traveled **154** miles on the route to their vacation destination. Convert this distance to kilometers.

**Answer**

We know **1** mi = **1.6093** km. So the conversion factor we use here is

Find the conversion factor that compares miles and kilometers, with "miles" in the numerator, and multiply by it. The conversion factor is

$$\frac{1.6093 \text{ km}}{1 \text{ mi}}$$

Now multiply by conversion factor

$$\begin{aligned} 154 \text{ mi} &= 154 \text{ mi} \times \frac{1.6093 \text{ km}}{1 \text{ mi}} && \text{Apply conversion factor.} \\ &= 154 \cancel{\text{mi}} \times \frac{1.6093 \text{ km}}{1 \cancel{\text{mi}}} && \text{Cancel common units.} \\ &= \frac{154 \times 1.6093}{1} \text{ km} && \text{Multiply fractions.} \\ &= 247.8 \text{ km} \end{aligned}$$

Hence, **154** miles is approximately **247.8** kilometers.

**Alternate Solution**

Note that this would work equally well if we chose **1** km ≈ **0.6214** mi.

$$\begin{aligned} 154 \text{ mi} &= 154 \text{ mi} \times \frac{1 \text{ km}}{0.6214 \text{ mi}} && \text{Apply conversion factor.} \\ &= 154 \cancel{\text{mi}} \times \frac{1 \text{ km}}{0.6214 \cancel{\text{mi}}} && \text{Cancel common units.} \\ &= \frac{154 \times 1}{0.6214} \text{ km} && \text{Multiply fractions.} \\ &= 247.8 \text{ km} \end{aligned}$$

### Your Turn 3.3.2: Convert Miles to Kilometer

Convert 28 miles to kilometers. Show all conversion factors needed. Round final answer to 2 decimal places if needed.

$$\frac{28 \text{ miles}}{1} \cdot \frac{\_\_\_\_ \text{ Select an answer}}{\_\_\_\_ \text{ Select an answer}} = \_\_\_\_ \text{ Select an answer}$$

**Must use 1 mile 1.61 ≈ km or 1 km ≈ 0.62 mi**

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### Example 3.3.3: Square Miles to Square Kilometer

Convert **35** square miles into square kilometers. When appropriate, round the answer off to the nearest hundredth. Use **1** mile = **1.609** kilometers.

**Answer**

Let's find the conversion factor

$$\begin{aligned} 1 \text{ mi} &= 1.609 \text{ km} \\ 1 \text{ mi} \times 1 \text{ mi} &= 1.609 \text{ km} \times 1.609 \text{ km} \\ 1 \text{ mi}^2 &= 1.609^2 \text{ km}^2 \end{aligned}$$

Use the conversion factor that compares square miles and square kilometers, with "square miles" in the denominator, and multiply by it.

$$\begin{aligned} 35 \text{ mi}^2 &= 35 \text{ mi}^2 \times \frac{1.609^2 \text{ km}^2}{1 \text{ mi}^2} && \text{Apply conversion factor.} \\ &= 35 \text{ mi}^2 \times \frac{1.609 \times 1.609 \text{ km}^2}{1 \text{ mi}^2} && \text{Cancel common units.} \\ &= \frac{35 \times 1.609 \times 1.609}{1} \text{ km}^2 && \text{Multiply fractions.} \\ &= 90.61 \text{ km}^2 \end{aligned}$$

## Your Turn 3.3.3: Change Square Kilometers to Square Miles

A certain city covers an area of 400 square kilometer. How many square miles is this? Use the conversion factor given below. Round final answer to 2 decimal places if needed.

$$\frac{400 \text{ square kilometer}}{1} \cdot \frac{\_\_\_\_ \text{ Select an answer}}{\_\_\_\_ \text{ Select an answer}} = \_\_\_\_ \text{ Select an answer}$$

**Must use 1 square mile $\approx 1.609^2$ square kilometer.**

Note: Use ^ button from keyboard to type exponent.

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## Example 3.3.4: Cubic Inches to Cubic Centimeter

A party balloon holds $650 \text{ in}^3$ of helium. What is the volume of the balloon in cubic centimeters $\text{cm}^3$?

**Answer**

$$\begin{aligned} 1 \text{ in} &= 2.54 \text{ cm} \\ 1 \text{ in} \times 1 \text{ in} \times 1 \text{ in} &= 2.54 \text{ cm} \times 2.54 \text{ cm} \times 2.54 \text{ cm} \\ 1 \text{ in}^3 &= 2.54^3 \text{ cm}^3 \end{aligned}$$

Use the conversion factor that compares cubic inches and cubic centimeters, with "cubic inches" in the denominator, and multiply

$$\begin{aligned} 650 \text{ in}^3 &= 650 \text{ in}^3 \times \frac{2.54^3 \text{ cm}^3}{1 \text{ in}^3} && \text{Apply conversion factor.} \\ &= 650 \text{ in}^3 \times \frac{2.54 \times 2.54 \times 2.54 \text{ cm}^3}{1 \text{ in}^3} && \text{Cancel common units.} \\ &= \frac{650 \times 2.54 \times 2.54 \times 2.54}{1} \text{ cm}^3 && \text{Multiply fractions.} \\ &= 10,651.59 \text{ cm}^3 \end{aligned}$$

## Your Turn 3.3.4: Cubic Centimeter to Cubic Inches

A party balloon holds 560 cubic centimeters of helium. What is the volume of the balloon in cubic inches? **Round to your**

**answer to 2 decimal places.**

Use 1 inch = 2.54 centimeter

[ ] cubic inches.

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## Example 3.3.5: Cubic Meter to Cubic Feet

Christine plans to put up a short-course swimming pool at her house. A short-course swimming pool usually measures 22 meters in length, 18 meters in width, and 4 meters in depth. How much water (in cubic feet) can that swimming pool hold when it is filled? Round to the nearest whole number.

**Answer**

Use 1 foot = 0.3048 meter to convert each foot to meters.

$$\begin{aligned} 22\text{ m} &= 22\text{ m} \times \tfrac{1\text{ ft}}{0.3048\text{ m}} = 72.1785\text{ ft} \\ 18\text{ m} &= 18\text{ m} \times \tfrac{1\text{ ft}}{0.3048\text{ m}} = 59.0551\text{ ft} \\ 4\text{ m} &= 4\text{ m} \times \tfrac{1\text{ ft}}{0.3048\text{ m}} = 13.1233\text{ ft} \end{aligned}$$

Use the volume formula to determine the volume of the pool water.

$$72.1785\text{ ft} \times 59.0551\text{ ft} \times 13.1233\text{ ft} = 55,938.18\text{ ft}^3$$

So the swimming pool can hold about $55,938\text{ ft}^3$ water.

## Your Turn 3.3.5: Cubic Feet to Cubic Meter

**Conversions Between US and Metric Systems**

Convert between the US and Metric measurements as indicated. **Round your results to the nearest whole number.**

Convert 50000 $ft^3$ to $m^3$.

[ ] $m^3$

**(Use 1 $m^3$ = 35.31 $ft^3$)**

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### Example 3.3.6: Kilogram to Ounces

Change 2.3 kilograms to ounces.

**Answer**

One kilogram weighs 2.2 pounds, and there are 16 ounces in a pound.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &\approx 80.96\text{ oz} \end{aligned}$$

Hence, 2.3 kilograms weighs 80.96 ounces.

**Alternate Solution**

Another approach utilizes the fact that 1 kilogram equals 1000 grams and 1 ounce equals approximately 28.35 grams.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{1000\text{ g}}{1\text{ kg}} \cdot \frac{1\text{ oz}}{28.35\text{ g}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \cdot \frac{1\text{ oz}}{28.35\ \cancel{\text{g}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 1000 \cdot 1}{1 \cdot 28.35}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

**Roundoff Error**

Why the discrepancy in answers? This difference in approximations is due to something called *round-off error*. Indeed, in the first calculation, we used a conversion factor that is rounded to the nearest tenth; i.e., 1 kg ≈ 2.2 lb. If we use a more accurate conversion factor to change kilograms to pounds, namely 1 kg ≈ 2.2046 lb, we get the following result.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2046\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2046\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2046 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions. } \approx 81.13\text{ oz} \end{aligned}$$

Note that this result is in better agreement with the second result above.

### Example 3.3.7: Dekaliters to Gallons

Change 2.5 dekalitres to gallons.

**Answer**

Recall that 1 daL = 10 L and 1 L = 0.264 gal.

$$\begin{aligned} 2.5\text{ daL} &\approx 2.5\ \cancel{\text{daL}} \times \frac{10\ \cancel{\text{L}}}{1\ \cancel{\text{daL}}} \times \frac{0.264\text{ gal}}{1\ \cancel{\text{L}}} && \text{Apply conversion factors.} \\ &\approx \frac{25 \times 10 \times 0.264}{1}\text{ gal} && \text{Multiply fractions.} \\ &\approx 6.6\text{ gal} \end{aligned}$$

Hence, 2.5 dekalitres is approximately equal to 6.6 gallons.

### Your Turn 3.3.7: Hectares to Acres

Convert 97 hectares to acres.

Round your answer to two decimal places.

_____ acres

**Must Use: 1 acre ≈ 0.4 hectare (ha)**

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## Converting the Rates

Modern speedometers often show a car's speed in both miles per hour and kilometers per hour.




Figure **3.3.1**: Speedometer

**Example 3.3.8**: Miles Per Hours to Meters Per Minutes

Daniel drove **40** miles in two hours. What distance (in meters) did he cover in one minute?

**Answer**

Please note that your answer should be expressed in meters per minute. This means that the meter unit must be in the numerator and the minute unit must be in the denominator in the final answer. Ensure that the units that you want to remove are on opposite sides of the fraction so they cancel each other.

$$\begin{aligned} 40 \text{ mi} &= 2 \text{ hr} \\ 0.62 \text{ mi} &= 1 \text{ km} \\ 1 \text{ km} &= 1000 \text{ m} \\ 1 \text{ hr} &= 60 \text{ min} \end{aligned}$$

Convert **40** miles in two hours to kilometers per hour.

$$\frac{40 \cancel{\text{ mi}}}{2 \cancel{\text{ hr}}} \times \frac{1 \cancel{\text{ km}}}{0.62 \cancel{\text{ mi}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{ km}}} \times \frac{1 \cancel{\text{ hr}}}{60 \text{ min}} = \frac{40,000 \text{ m}}{74.4 \text{ min}} = 537.63 \text{ m/min}$$

Daniel has covered **537.63** meters in one minute.

**Your Turn 3.3.8**: Miles Per Hour to Meter Per Second

Convert 53 miles per hour to meters per second.

meters per second

Round answers to the nearest hundredth, if necessary.

(Use 1 mile = 5280 feet, 1 foot = 0.3048 meter, 1 hr = 3600 seconds)

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### Example 3.3.9: Cubic Yards to Liters

How many liters of water are needed to fill an Olympic swimming pool with a total volume of **3,300** cubic yards? Assume that the water level is as deep as the pool. Use the US customary to metric units' equivalence: **1** cubic yard ($yd^3$) = **746.56** liters (L).

**Answer**

$$\begin{aligned} 3,300\ \text{yd}^3 &= 3,300\ \text{yd}^3 \times \frac{746.56\ \text{L}}{1\ \text{yd}^3} && \text{Apply conversion factors.} \\ &= 3,300\ \cancel{\text{yd}^3} \times \frac{746.56\ \text{L}}{1\ \cancel{\text{yd}^3}} && \text{Cancel common units.} \\ &= \frac{3,300 \times 746.56}{1}\ \text{gal} && \text{Multiply fractions.} \\ &= 2,463,648\ \text{L} \end{aligned}$$

At most **2,463,648** liters of water are needed to fill the swimming pool.

### Your Turn 3.3.9: Liters to Cubic Yards

Convert 833,427 liters to cubic yards. Show all conversion factors needed. Round final answer to 2 decimal places if needed.

$$\frac{833{,}427 \text{ liters}}{1} \cdot \frac{\_\_\_\_ \text{ Select an answer}}{\_\_\_\_ \text{ Select an answer}} = \_\_\_\_ \text{ Select an answer}$$

**Must use 1 cubic yard ≈ 746.56 Liters**

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### Example 3.3.10: Cost for Driving Car

Robert's new car requires at most **57** liters of fuel. If the current average price for regular gas is $**2.864** per gallon, how much will it cost him to fill his car's tank? Use the US customary to metric units' equivalence: **1** gallon = **3.79** liters

**Answer**

$$\begin{aligned} 57\ \text{L} &= 57\ \text{L} \times \frac{1\ \text{gal}}{3.79\ \text{L}} && \text{Apply conversion factors.} \\ &= 57\ \cancel{\text{L}} \times \frac{1\ \text{gal}}{3.79\ \cancel{\text{L}}} && \text{Cancel common units.} \\ &= \frac{57 \times 1}{3.79}\ \text{gal} && \text{Multiply fractions.} \\ &= 15.0396\ \text{gal} \end{aligned}$$

Since **1** gallon = **2.864** dollars, we can use $\frac{2.864\ \text{dollar}}{1\ \text{gal}}$ conversion factor to change gallon to dollars.

$$15.0396\ \cancel{\text{gal}} \times \frac{2.864\ \text{dollar}}{1\ \cancel{\text{gal}}} = \frac{15.0396 \times 2.864}{1}\ \text{dollar} = 43.07\ \text{dollar}$$

It will cost Robert approximately $**43.07** to fill his car's tank.

Your Turn **3.3.10**: Cost of Driving Car

Alex's new car requires $37$ liters of fuel to fill the tank. How many gallons does he need to put in his car? If the current average price for regular gas is $ $3.248$ per gallon, how much will it cost him to fill his car's tank?

Use $1$ gallon = $3.79$ liters

Alex need to use [ ] gallos of gas to fill the tank.(Round to 2 decimal places)

Total cost of the gas is $[ ]

(Round to the nearest cent)

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# 3.4: Temperature, Time, and Medicine Doses

## Measuring Temperature




Figure 3.4.1: A thermometer that measures temperature in both customary and metric units. (credit: "Thermometer" by Jeff Djevdet/Flickr, CC BY 2.0)

When you touch something and it feels warm or cold, what is that really telling you about that substance? Temperature is a measure of how fast atoms and molecules are moving in a substance, whether that be the air, a stove top, or an ice cube. The faster those atoms and molecules move, the higher the temperature.

In the metric system, temperature is measured using the Celsius (°C) scale. Because temperature is a condition of the physical properties of a substance, the Celsius scale was created with 100 degrees separating the point at which water freezes, 0 °C, and the point at which water boils, 100 °C. Scientifically, these are the points at which water molecules change from one state of matter to another—from solid (ice) to liquid (water) to gas (water vapor).

**Checkpoint**

*When reading temperatures, it's important to look beyond the degree symbol to determine which temperature scale the units express. For example, 13 °C reads "13 degrees Celsius," indicating that the temperature is expressed using the Celsius scale, while 13 °F reads "13 degrees Fahrenheit," indicating that the temperature is expressed using the Fahrenheit scale.*

**Who Knew?: How Many Temperature Scales Are There?**

Did you know that in addition to Fahrenheit and Celsius, there is a third temperature scale widely used throughout the world? The Kelvin scale starts at absolute zero, the lowest possible temperature at which there is no heat energy present at all. It is primarily used by scientists to measure very high or very low temperatures when water is not involved.

## Converting Between Fahrenheit and Celsius Temperatures

Understanding how to convert between Fahrenheit and Celsius temperatures is an essential skill in understanding metric temperatures. You likely know that below $32$°F means freezing temperatures and perhaps that the same holds true for $0°C$. While it may be difficult to recall that water boils at $212°F$, knowing that it boils at $100°C$ is a fairly easy thing to remember.

But what about all the temperatures in between? What is the temperature in degrees Celsius on a scorching summer day? What about a cool autumn afternoon? If a recipe instructs you to preheat the oven to $350°F$ , what Celsius temperature do you set the oven at?

Figure 3.4.2 lists common temperatures on both scales. Since we don't use Celsius temperatures daily, it's challenging to recall them. Fortunately, we don't have to. Instead, we can convert temperatures from Fahrenheit to Celsius and from Celsius to Fahrenheit using a simple algebraic expression.

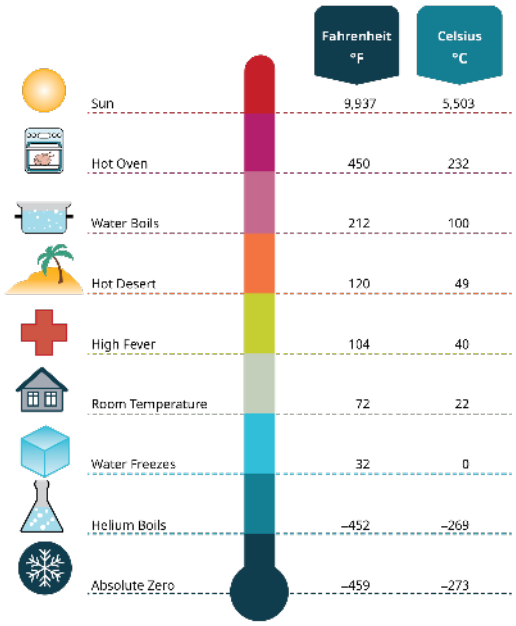



Figure **3.4.2**: Common Temperatures

The following example uses the chart above to estimate the temperature in various situations.

**Your Turn 3.4.1: Estimate Best Temperature**

**Select the best estimate temperature.**

Approximate comfortable room temperature

- ○ 70 degree Celsius
- ○ 50 degree Celsius
- ○ 20 degree Celsius
- ○ 30 degree Celsius

Very hot day

- ○ 15 degree Celsius
- ○ 35 degree Celsius
- ○ 80 degree Celsius
- ○ 65 degree Celsius

Thermostat settiing in a home

- ○ 40 degree Celsius
- ○ 80 degree Celsius
- ○ 60 degree Celsius
- ○ 20 degree Celsius

Oven Temperature for cooking a rost.

- ○ 100 degree Celsius
- ○ 350 degree Celsius
- ○ 80 degree Celsius
- ○ 175 degree Celsius

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**Definition: Temperature Conversion Formula (EXACT CONVERSION)**

| | |
|---|---|
| **Temperature** | Celsius to Fahrenheit: $$F = \frac{9}{5}C + 32$$ Celsius to Kelvin: $$K = C + 273.15$$ Fahrenheit to Celsius: $$C = \frac{5}{9}(F - 32)$$ Kelvin to Celsius: $$C = K - 273.15$$ |

**Example 3.4.2: F to C**

Convert $73°F$ to Celsius.

**Answer**

To convert from Fahrenheit to Celsius, we need the equation $C = \frac{5}{9}(F - 32)$. Substitute in the given value for Fahrenheit to find the answer. Round to the nearest degree.

$$C = \tfrac{5}{9}(F - 32)$$
$$C = \tfrac{5}{9}(73 - 32)$$
$$C = \tfrac{5}{9}(41)$$
$$C = 23.3°C$$

**Example 3.4.3: C to F**

Convert $28°C$ to Fahrenheit.

**Answer**

Use the formula to determine the temperature in degrees Fahrenheit.

$$F = \tfrac{9}{5}C + 32$$
$$F = \tfrac{9}{5}(28) + 32 = 82.4$$

A temperature of $82.4°F$ is equal to $28°C$.

**Example 3.4.4: K to F**

Convert $3763$ K to Fahrenheit.

**Answer**

First, we need to convert to Celsius, then we can convert to Fahrenheit. Round to two decimal places.

$C = K - 273.15 = 3763 - 273.15 = 3489.85°C$

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(3489.85) + 32$$
$$F = 6281.73 + 32$$
$$F = 6313.73°C$$

Thus, $3763$ K $= 6313.73$ $F$

**Your Turn 3.4.4: Temperature Conversion**

(a) Blood frozen and stored at (minus)$-144$ $^{o}$C can last for up to $5$ years. Express this temperature in Fahrenheit.

_____ Select an answer

(Round to the nearest tenth)

---

(b) What is $66.1$ $^{o}$F in Celsius ($^{o}$C)?

_____ $^{o}$C

(Round to the nearest tenth)

---

(c) For a temperature of $87$ Kelvin, find the equivalent temperature in Degrees Celsius and Degrees Fahrenheit.

$87$ Kelvin = _____ $^{o}$C

(Round to the nearest tenth)

$87$ Kelvin = _____ $^{o}$F

(Round to the nearest tenth)

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## Reasonable Values for Temperature

While knowing the exact temperature is important in most cases, sometimes an approximation will do. When trying to assess the reasonableness of values for temperature, there is a quicker way to convert temperatures for an approximation using mental math. These simpler formulas are listed in Table **3.4.3**.

**FORMULA: Estimate Temperature**

The formulas **used to estimate** temperatures from Fahrenheit to Celsius or from Celsius to Fahrenheit are outlined in the following table.

Table **3.4.3**: Estimate Temperature Conversion

| Fahrenheit to Celsius | Celsius to Fahrenheit |
|---|---|
| Subtract **30** from Fahrenheit and and divide by **2** $C = \frac{F-30}{2}$ | Multiply Celsius by **2** and add **30** $F = 2C + 30$ |

### Example 3.4.5: Using Benchmark Temperatures to Determine Reasonable Values for Temperature

1. Which is the more reasonable value for the temperature of a freezer? **5°C** or **−5°C**?
2. The average body temperature is generally accepted as **98.6°F** What is a reasonable value for the average body temperature in degrees Celsius? **98.6°C** or **64.3°C** or **34.3°C**?

**Answer**

1. We know that water freezes at **0°C**. So, the more reasonable value for the temperature of a freezer is **−5°C** which is below **0°C**. At a temperature of **5°C**, it is above freezing.
2. To estimate the average body temperature in degrees Celsius, subtract **30** from the temperature in degrees Fahrenheit, and divide the result by **2**.

$$\frac{98.6-30}{2} = \frac{68.6}{2} = 34.3$$

A reasonable value for average body temperature is **34.3°C**.

### Your Turn 3.4.5: Estimate the Temperature

Which of the following is the closest value to the temperature in degree Fahrenheit for 9 degrees Celsius?

- ○ 264 Fahrenheit
- ○ 49 Fahrenheit
- ○ -13 Fahrenheit
- ○ 282 Fahrenheit

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### Example 3.4.6: Using Conversion to Determine Reasonable Values for Temperaturesk

Which is a reasonable temperature for storing chocolate: **28°C** or **18°C** or **2°C**?

**Answer**

Use the formula in Table **3.4.2** to determine the temperature in degrees Fahrenheit.

$$F = \frac{9}{5}C + 32$$
$$F = \frac{9}{5}(28) + 32 = 82.4$$
$$F = \frac{9}{5}(18) + 32 = 64.4$$
$$F = \frac{9}{5}(2) + 32 = 35.6$$

A temperature of **82.4°F** would be too hot, causing the chocolate to melt. A temperature of **35.6°F** is very close to freezing, which would affect the look and feel of the chocolate. So, a reasonable temperature for storing chocolate is **18°C**, or **64.4°F**.

### Your Turn 3.4.6: Which City is Hotter?

Determine which location has the hotter average temperature and by how many degrees Fahrenheit.

Berbera, Somalia: $84.6$ degrees Fahrenheit; Nema, Mauritania: $30.7$ degrees Celsius

What is the name of the city with the hottest average temperature?

- ○ Berbera
- ○ Nema

How many degrees Fahrenheit hotter is that city? Round to the nearest tenth of a degree.

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**Definition: Time**

**Time** is used to describe the amount of existence between events.

Some of the common units (along with their abbreviations) for temperature and time are listed in the following table.

| Unit Conversion Table for Time | |
|---|---|
| Time | **1** minute (min) = **60** seconds (sec)<br>**1** hour ( hr) = **60** minutes<br>**1** day (da) = **24** hours<br>**1** week (wk) = **7** days<br>**1** year (yr)= **52** weeks |

**Example 3.4.7: Hours to Week**

Convert $2,016$ hr to weeks.

**Answer**

Looking in the unit conversion table under *time*, we see that $1 \text{ wk} = 7 \text{ da}$ and that $1 \text{ da} = 24 \text{ hr}$. To convert from hours to weeks, we must first convert from hours to days and then from days to weeks. We need two unit fractions.

The unit fraction needed for converting from hours to days is $\frac{1 \text{ da}}{24 \text{ hr}}$. The unit fraction needed for converting from days to weeks is $\frac{1 \text{ wk}}{7 \text{ da}}$.

$$\begin{aligned} 2,016 \text{ hr} &= \frac{2,016 \text{ hr}}{1} \cdot \frac{1 \text{ da}}{24 \text{ hr}} \cdot \frac{1 \text{ wk}}{7 \text{ da}} && \text{Divide out common units.} \\ &= \frac{2,016 \cancel{\text{ hr}}}{1} \cdot \frac{1 \cancel{\text{ da}}}{24 \cancel{\text{ hr}}} \cdot \frac{1 \text{ wk}}{7 \cancel{\text{ da}}} \\ &= \frac{2,016 \cdot 1 \text{ wk}}{24 \cdot 7} && \text{Simplify} \\ &= 12 \text{ wk} \end{aligned}$$

Thus, $2,016 \text{ hr} = 12 \text{ wk}$.

**Your Turn 3.4.7: Time Conversion**

**Using Tables for Conversions**

Use that 60 minutes = 1 hour and 60 seconds = 1 minute to complete the table below.

Then answer the corresponding questions.

| Hours | 0 hr | 1 hr | 2 hr | 3 hr | 4 hr |
|---|---|---|---|---|---|
| Minutes | _____ min | _____ min | _____ min | _____ min | _____ min |

| | | | | | |
|---|---|---|---|---|---|
| **Seconds** | _____ **sec** | _____ **sec** | _____ **sec** | _____ **sec** | _____ **sec** |

How many seconds are in 120 minutes? _____ **sec**

How many hours are in 240 minutes? _____ **hr**

How many seconds are in 3 hours? _____ **sec**

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## To Your Health: Dosages

A medicine can be more harmful than helpful if it is not taken in the proper dosage. A dosage (or dose) is the specific amount of a medicine that is known to be therapeutic for an ailment in a patient of a certain size. Dosages of the active ingredient in medications are usually described by units of mass, typically grams or milligrams, and generally are equated with a number of capsules or teaspoonfuls to be swallowed or injected. The amount of the active ingredient in a medicine is carefully controlled so that the proper number of pills or spoonfuls contains the proper dose.

Most drugs must be taken in just the right amount. If too little is taken, the desired effects will not occur (or will not occur fast enough for comfort); if too much is taken, there may be potential side effects that are worse than the original ailment. Some drugs are available in multiple dosages. For example, tablets of the medication levothyroxine sodium, a synthetic thyroid hormone for those suffering from decreased thyroid gland function, are available in 11 different doses, ranging from 25 micrograms (µg) to 300 µg. It is a doctor's responsibility to prescribe the correct dosage for a patient, and it is a pharmacist's responsibility to provide the patient with the correct medicine at the dosage prescribed. Thus, proper quantities—which are expressed using numbers and their associated units—are crucial for keeping us healthy.

## Effects are Dose-Dependent

Chemicals are the most common things for which doses are measured, but there are others, such as radiation exposure. For humans, most doses of micronutrients and medications are measured in milligrams (mg), but some are measured in micrograms because of their potency. Nonmedicinal poisons span the measurement scale; some poisons are so dangerous that a single microgram of it could be deadly, whereas other substances take much more. For example, even water is toxic when consumed in large enough quantities.

Dosage (the size of each dose) determines the strength and duration of the health benefits of nutrients, and also of the therapeutic effects of medical treatments. Dosage also determines the severity of adverse effects of treatments and toxins.

Duration of exposure, that is, the period of time over which the dose was received (all at once or gradually) also determines its effects (the body may build tolerance to gradual exposure to a drug, while a large immediate dose could be deadly).

The route by which a dose is exposed to, may affect the outcome, because some medications have different effects depending on whether they are inhaled, ingested, taken transdermally, injected, or inserted.

The dosage, route, concentration, and division over time may all be critical considerations in the administration of drugs or in responding to exposure to a toxin. In nutrition, the route is usually a given, as nutrients are generally consumed, while dosage and the frequency of nutrient ingestion are very important variables in preventing disease and promoting overall health.

## Calculation of Dose

Calculating drug dosages for humans based on the doses used in animal studies can be based on weight (e.g., mg/kg) or surface area (e.g., $mg/m^2$) based on weight. Drug dosage calculation is required if the physician's order is different from what is available.

### ✔ Example 3.4.8: Find the Number of Tablets

1. A physician ordered **100** mg of Demerol. Demerol is available as **50** mg per tablet. How many tablets should the nurse administer?
2. The doctor's order is **1.2** g of Folic Acid. Folic Acid is available as **800** mg per tablet. How many tablets should be taken?

**Answer**

1. We start with the given, **100** mg. We want to change the unit from *mg* to *tablets*. There are **50** mg in **1** tablet (Remember that *per* tablet means *one* tablet. From this relationship, we can construct the following conversion factor.

$$50 \text{ mg} = 1 \text{ tablet}$$

We use the conversion factor that will cancel out the original unit, *mg*, and introduce the unit we are converting to, which is *tablet*.

$$100 \cancel{\text{mg}} \times \frac{1 \text{ tablet}}{50 \cancel{\text{mg}}} = 2 \text{ tablets}$$

Hence, the nurse should administer **2** tablets.

2. We start with the given amount, **1.2** g, and we want to conver*t it* to the number of *tablets*. First, we convert **1.2** g to mg and then convert mg to tablets. We need a conversion factor for each step. Here are the conversion factors that we can use.

$$800 \text{ mg} = 1 \text{ tablet}$$
$$1,000 \text{ mg} = 1 \text{ g}$$

$$1.2 \cancel{\text{g}} \times \frac{1,000 \cancel{\text{mg}}}{1 \cancel{\text{g}}} \times \frac{1 \text{ tablet}}{800 \cancel{\text{mg}}} = 1.5 \text{ tablets}$$

Hence, exact **1.5** tablets should be taken. If we round to the nearest whole number, **2** tablets should be taken.

**Your Turn 3.4.8: Find the Number of Tablets**

Order: codeine 240 mg
Drug available: 40 mg per tablet.
Find the number of tablets.

[ ] tablets.

(Round to the nearest whole tablet)

---

If one tablet contains 350 milligrams of a medication, how many tablets would you need to take if you had been prescribed 3.5 gram of the medication? Be sure to pick the correct label for your answer.

Round your answer to one decimal place.

[ ]

(Round to the nearest whole tablet)

- ○ g
- ○ capsules
- ○ tablets
- ○ mg

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Drug Dosage Calculation based on Body Weight. Many drugs (especially in children) are dosed according to body weight (mg/kg). These calculations are carried in 3-step conversions. The first step is to convert the body weight from *pounds (lbs)* to *kg*. The second step is to convert *kg* to *mg* (the total *mg dose* calculated based on body weight). And, finally, the *mg dose* is converted to the number of *tablets*.

**Example 3.4.9: Find the Number of Tablets**

Demerol is ordered **1.5** mg/kg for a patient that is **220** lbs. Demerol is available as **50** mg per tablet. How many tablets should the nurse administer?

**Answer**

We start with the given, **220** lbs. We want to change the unit from lbs to kg, and then, from kg to total dose (mg) and then the mg dose to tablets. The first conversion factor will cancel out the original unit, lbs, and introduce the unit we are converting to, which is *kg*. The second conversion factor will cancel out *kg*, and introduce the unit of the *dose* (usually mg) and then mg to tablet. The following are the conversion factors that we can use.

$$\begin{aligned} 1\text{ kg} &= 2.2\text{ lb} \\ 1.5\text{ mg} &= 1\text{ kg} \\ 1\text{ tablet} &= 50\text{ mg} \end{aligned}$$

$$220\,\text{lbs} \times \frac{1\,\text{kg}}{2.2\,\text{lbs}} \times \frac{1.5\,\text{mg}}{1\,\text{kg}} \times \frac{1\text{ tablet}}{50\,\text{mg}} = 3\text{ tablets}$$

Hence, the nurse should administer **3** tablets.

**Your Turn 3.4.9: Find the Number of Tablets**

A doctor prescribes amoxicillin 85 mg/kg to a child weighing 77.5 lbs. Amoxicillin is available as 500 mg tablets.

How many tablets should the nurse administer?

_____ tablets.

(Round to the nearest whole tablet)

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**Example 3.4.10: Find the Number of Tablets in a Week**

The antihistamine Benadryl is often prescribed for allergies. A typical dose for a **95**-a pound person is **30** mg every **5** hours. Following this dosage, how many **10.5** mg chewable tablets would be taken in a week?

**Answer**

Note that we are looking for tablets per week, so the "tablet" unit must be in the numerator and the "week" unit must be in the denominator. The following are the conversion factors that we can use.

$$\begin{aligned} 30\text{ mg} &= 5\text{ hr} \\ 24\text{ hr} &= 1\text{ days} \\ 1\text{ week} &= 7\text{ days} \\ 1\text{ tablet} &= 10.5\text{ mg} \end{aligned}$$

$$\frac{30\,\text{mg}}{5\,\text{hrs}} \times \frac{24\,\text{hr}}{1\,\text{day}} \times \frac{7\,\text{days}}{1\text{ week}} \times \frac{1\text{ tablet}}{10.5\,\text{mg}} = 96\,\frac{\text{tablet}}{\text{week}}$$

Hence, the nurse administers **96** tablets in a week.

**Your Turn 3.4.10: Find the Number of Tablets in a Week**

The antihistamine Benadryl is often prescribed for allergies. A typical dose for a 90 -pound person is 20 mg every 7 hours.

Following this dosage, how many 11.5 mg chewable tablets would be taken in a week?

_____ tablets per week (Round to the nearest tablet)

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**Example 3.4.11: Doses Calculation**

Suppose a dose of $9,000$ units/kg of penicillin is prescribed every $8$ hours for treatment of a bacterial infection. For penicillin, $400,000$ units = $250$ mg.

(a) Express the dose in mg per kg of body weight.

(b) How many milligrams of penicillin would a $22$-kg child take in one day?

**Answer**

(a) Note that we are looking for mg per kg, so the "mg" unit must be in the numerator and the "kg" unit must be in the denominator. The following are the conversion factors that we can use.

$$9,000 \text{ units} = 1 \text{ kg}$$
$$400,000 \text{ units} = 250 \text{ mg}$$

$$\frac{9,000 \cancel{\text{units}}}{1 \cancel{\text{kg}}} \times \frac{250 \cancel{\text{mg}}}{1 \cancel{\text{units}}} = 5.63 \frac{\text{mg}}{\text{kg}}$$

Hence, the dose will be $5.63$ milligram per kilogram.

(b) Note that we are looking for mg per day, so the "mg" unit must be in the numerator and the "day" unit must be in the denominator. The following are the conversion factors that we can use.

$$9,000 \text{ units} = 1 \text{ kg}$$
$$400,000 \text{ units} = 250 \text{ mg}$$
$$1 \text{ day} = 24 \text{ hr}$$

$$\frac{9,000 \cancel{\text{units}}}{1 \text{ kg} \times 8 \cancel{\text{hr}}} \times \frac{250 \text{ mg}}{400,000 \cancel{\text{units}}} \times \frac{22 \cancel{\text{kg}}}{1} \times \frac{24 \cancel{\text{hr}}}{1 \text{ day}} = 371.25 \frac{\text{mg}}{\text{day}}$$

Hence, the dose will be $371.25$ milligrams per day.

**Your Turn 3.4.11: Doses Calculation**

Suppose a dose of $9000$ units/kg of penicillin is prescribed every $6$ hours for treatment of a bacterial infection. For penicillin, $400,000$ units = $250$ mg.

(a) Express the dose in mg per kg of body weight.

_____ mg/kg of body weight (Round to two decimal places.)

(b) How many milligrams of penicillin would a 28-kg child take in one day?

_____ mg per day.

(Round to two decimal places.)

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## References

1. The Use of Body Surface Area as a Criterion of Drug Dosage in Cancer Chemotherapy(opens in new window) [cancerres.aacrjournals.org] "The Use of Body Surface Area as a Criterion of Drug Dosage in Cancer Chemotherapy" D Pinkel. Cancer Research 1958

## Contributors and Attributions

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# CHAPTER OVERVIEW

## 4: GEOMETRY


4.1: Basic Geometric Concepts and Figures

4.2: Perimeter, Circumference, and Area

4.3: Volume and Surface Area

4.4: Similar Triangles and Pythagorean Theorem


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# 4.1: Basic Geometric Concepts and Figures

## Introduction

You use geometric terms in everyday language, often without thinking about it. For example, any time you say “walk along this line” or “watch out, this road quickly angles to the left” you are using geometric terms to make sense of the environment around you. You use these terms flexibly, and people generally know what you are talking about.

In the world of mathematics, each of these geometric terms has a specific definition. It is important to know these definitions—as well as how different figures are constructed—to become familiar with the language of geometry. Let’s start with a basic geometric figure: the plane.

## Figures on a Plane

A **plane** is a flat surface that continues forever (or, in mathematical terms, infinitely) in every direction. It has two dimensions: length and width.

You can visualize a plane by placing a piece of paper on a table. Now imagine that the piece of paper stays perfectly flat and extends as far as you can see in two directions, left-to-right and front-to-back. This gigantic piece of paper gives you a sense of what a geometric plane is like: it continues infinitely in two directions. (Unlike the piece of paper example, though, a geometric plane has no height.)

A plane can contain a number of geometric figures. The most basic geometric idea is a **point**, which has no dimensions. A point is simply a location on the plane. It is represented by a dot. Three points that don’t lie in a straight line will determine a plane.

The image below shows four points, labeled $A$, $B$, $C$, $D$.

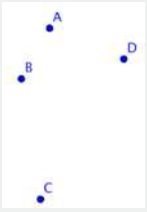



Figure **4.1.1**: A set of points

Two points on a plane determine a line. A line is a one-dimensional figure that is made up of an infinite number of individual points placed side by side. In geometry, all lines are assumed to be straight; if they bend they are called a curve. A line continues infinitely in two directions.

Below is line *AB* or, in geometric notation, $\overleftrightarrow{AB}$. The arrows indicate that the line keeps going forever in the two directions. This line could also be called line *BA*. While the order of the points does not matter for a line, it is customary to name the two points in alphabetical order.

The image below shows the points A and B and the line $\overleftrightarrow{AB}$.

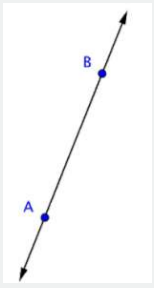



Figure **4.1.2**: Line *AB*

There are two more figures to consider. The section between any two points on a line is called a line segment. A line segment can be very long, very short, or somewhere in between. The difference between a line and a **line segment** is that the line segment has two endpoints and a line goes on forever. A line segment is denoted by its two endpoints, as in $\overline{CD}$.

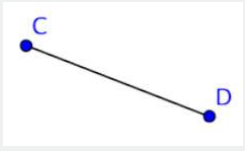



Figure **4.1.3**: Line segment *CD*

A **ray** has one endpoint and goes on forever in one direction. Mathematicians name a ray with notation like $\overrightarrow{EF}$, where point *E* is the endpoint and *F* is a point on the ray. When naming a ray, we always say the endpoint first. Note that $\overrightarrow{FE}$ would have the endpoint at *F*, and continue through *E*, which is a different ray than $\overrightarrow{EF}$, which would have an endpoint at E, and continue through F.

The term “ray” may be familiar because it is a common word in English. “Ray” is often used when talking about light. While a ray of light resembles the geometric term “ray,” it does not go on forever, and it has some width. A geometric ray has no width; only length.

Below is an image of ray *EF* or $\overrightarrow{EF}$. Notice that the endpoint is *E*.




Figure 4.1.4: Ray *EF*

## Example 4.1.1

Identify each line and line segment in the picture below.

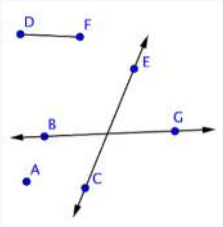



Figure 4.1.5: Line and Line Segment

**Answer**

Two points define a line, and a line is denoted with arrows. There are two lines in this picture: $\overleftrightarrow{CE}$ and $\overleftrightarrow{BG}$.

A line segment is a section between two points. $\overline{DF}$ is a line segment. But there are also two more line segments on the lines themselves: $\overline{CE}$ and $\overline{BG}$.

Lines: $\overleftrightarrow{CE}$, $\overleftrightarrow{BG}$

Line segments: $\overline{DF}$, $\overline{CE}$, $\overline{BG}$.

## Your Turn 4.1.1

Identify each point and ray in the picture below.

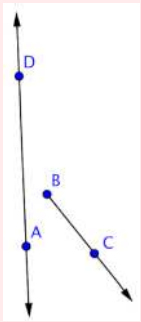



Figure 4.1.6: Point and Ray

**Answer**

There are four points: *A*, *B*, *C*, and *D*. There are also three rays, though only one may be obvious.

Ray $\overrightarrow{BC}$ begins at point *B* and goes through *C*. Two more rays exist on line $\overleftrightarrow{AD}$: they are $\overrightarrow{DA}$ and $\overrightarrow{AD}$.

Points: *A*, *B*, *C*, *D*

Rays: $\overrightarrow{BC}$, $\overrightarrow{AD}$, $\overrightarrow{DA}$

## Angles

Lines, line segments, points, and rays are the building blocks of other figures. For example, two rays with a common endpoint make up an **angle**. The common endpoint of the angle is called the **vertex**.

The angle ABC is shown below. This angle can also be called ∠ABC, ∠CBA or simply ∠B. When you are naming angles, be careful to include the vertex (here, point B) as the middle letter.

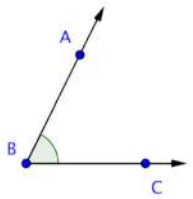



Figure 4.1.7: Angle

The image below shows a few angles on a plane. Notice that the label of each angle is written "point-vertex-point," and the geometric notation is in the form ∠ABC.

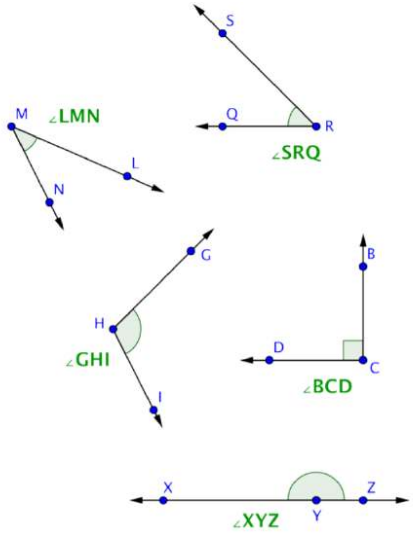



Figure 4.1.8: Naming Angles

Sometimes angles are very narrow; sometimes they are very wide. When people talk about the "size" of an angle, they are referring to the arc between the two rays. The length of the rays has nothing to do with the size of the angle itself. Drawings of angles will often include an arc (as shown above) to help the reader identify the correct 'side' of the angle.

Think about an analog clock face. The minute and hour hands are both fixed at a point in the middle of the clock. As time passes, the hands rotate around the fixed point, making larger and smaller angles as they go. The length of the hands does not impact the angle that is made by the hands.

An angle is measured in degrees, represented by the symbol °. A circle is defined as having $360°$. (In skateboarding and basketball, "doing a 360" refers to jumping and doing one complete body rotation.

A **right angle** is any degree that measures exactly $90°$. This represents exactly one-quarter of the way around a circle. Rectangles contain exactly four right angles. A corner mark is often used to denote a right angle, as shown in right angle DCB below.

Angles that are between $0°$ and $90°$ (smaller than right angles) are called **acute angles**. Angles that are between $90°$ and $180°$ (larger than right angles and less than $180°$) are called **obtuse angles**. An angle that measures exactly $180°$ is called a **straight angle** because it forms a straight line.

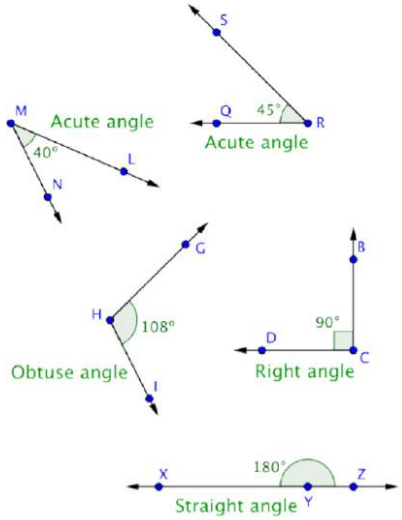



Figure 4.1.9: Different Type of Angles

**Example 4.1.2**

Label each angle below as acute, right, or obtuse.

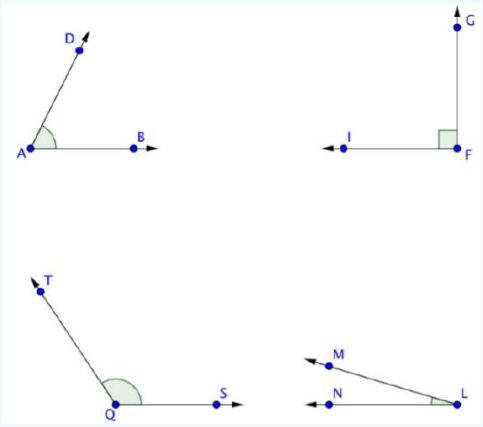



Figure 4.1.10: Identify the Angles

**Answer**

You can start by identifying any right angles.

$\angle GFI$ is a right angle, as indicated by the corner mark at vertex F.

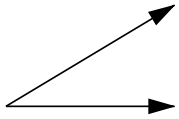
Acute angles will be smaller than $\angle GFI$ (or less than $90°$). This means that $\angle DAB$ and $\angle MLN$ are acute.

$\angle TQS$ is larger than $\angle GFI$, so it is an obtuse angle.

$\angle DAB$ and $\angle MLN$ are acute angles. $\angle GFI$ is a right angle. $\angle TQS$ is an obtuse angle.

**Your Turn 4.1.2: Identify the Angle**

Classify the angle shown as Acute, Obtuse, or Right



- Acute
- Obtuse
- Right

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## Properties of Angles

### Introduction

Imagine two separate and distinct lines on a plane. There are two possibilities for these lines: they will either intersect at one point, or they will never intersect. When two lines intersect, four angles are formed. Understanding how these angles relate to each other can help you figure out how to measure them, even if you only have information about the size of one angle.

### Parallel and Perpendicular

Parallel lines are two or more lines that never intersect. Likewise, parallel line segments are two line segments that never intersect even if the line segments were turned into lines that continued forever. Examples of parallel line segments are all around you, in the two sides of this page and in the shelves of a bookcase. When you see lines or structures that seem to run in the same direction, never cross one another, and are always the same distance apart, there's a good chance that they are parallel.

Perpendicular lines are two lines that intersect at a **90°** (right) angle. And perpendicular line segments also intersect at a **90°** (right) angle. You can see examples of perpendicular lines everywhere as well—on graph paper, in the crossing pattern of roads at an intersection, to the colored lines of a plaid shirt. In our daily lives, you may be happy to call two lines perpendicular if they merely seem to be at right angles to one another. When studying geometry, however, you need to make sure that two lines intersect at a 90º angle before declaring them to be perpendicular.

The image below shows some parallel and perpendicular lines. The geometric symbol for parallel is ||, so you can show that $AB \parallel CD$. Parallel lines are also often indicated by the marking >> on each line (or just a single > on each line). Perpendicular lines are indicated by the symbol ⊥, so you can write $\overleftrightarrow{WX} \perp \overleftrightarrow{YZ}$.

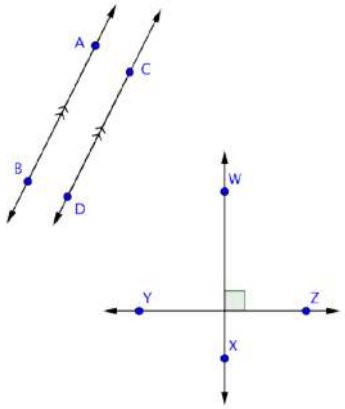



Figure **4.1.11**: Parallel and Perpendicular Lines

If two lines are parallel, then any line that is perpendicular to one line will also be perpendicular to the other line. Similarly, if two lines are both perpendicular to the same line, then those two lines are parallel to each other. Let's take a look at one example and identify some of these types of lines.

**Example 4.1.3**

Identify a set of parallel lines and a set of perpendicular lines in the image below.

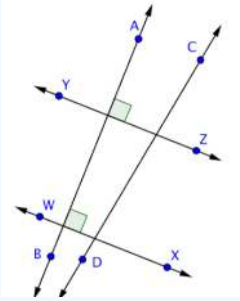


Figure 4.1.12: Intersecting Lines

**Answer**

Parallel lines never meet, and perpendicular lines intersect at a right angle. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ do not intersect in this image, but if you imagine extending both lines, they will intersect soon. So, they are neither parallel nor perpendicular.

$\overleftrightarrow{AB}$ is perpendicular to both $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$, as indicated by the right-angle marks at the intersection of those lines.

Since $\overleftrightarrow{AB}$ is perpendicular to both lines, then $\overleftrightarrow{WX}$ and $\overleftrightarrow{YZ}$ are parallel.

$\overleftrightarrow{WX} \parallel \overleftrightarrow{YZ}$

$\overleftrightarrow{AB} \perp \overleftrightarrow{WX}$, $\overleftrightarrow{AB} \perp \overleftrightarrow{YZ}$

**Your Turn 4.1.3: Matching with Definition**

Match the word with its definition.

- [-] The length of the shortest path connecting two points in a plane.
- [-] A section of a line that consists of two endpoints and all the points between them.
- [-] Two lines that intersect to form a right angle.
- [-] Two lines in the same plane that do not intersect.
- [-] A location in space that is represented by a dot and has no dimension.
- [-] A set of points consisting of two different rays that have the same endpoint.
- [-] Has one dimension and extends in opposite directions without end. It is determined by two points in a plane.

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## Supplementary and Complementary

In the example above, m∠BAC and m∠DAC add up to $180°$. Two angles whose measures add up to $180°$ are called **supplementary angles**. There's also a term for two angles whose measurements add up to $90°$, they are called **complementary angles**.

One way to remember the difference between the two terms is that "corner" and "complementary" each begin with *c* (a $90°$ angle looks like a corner), while straight and "supplementary" each begin with *s* (a straight angle measures $180°$).

If you can identify supplementary or complementary angles within a problem, finding missing angle measurements is often simply a matter of adding or subtracting.

## Example 4.1.4

Two angles are supplementary. If one of the angles measures **48°**, what is the measurement of the other angle?

**Answer**

Two supplementary angles make up a straight angle, so the measurements of the two angles will be **180°**.

$$m\angle A + m\angle B = 180°$$

You know the measurement of one angle. To find the measurement of the second angle, subtract **48°** from **180°**.

$$48° + m\angle B = 180°$$

$$m\angle B = 180° - 48°$$

$$m\angle B = 132°$$

The measurement of the other angle is **132°**.

## Your Turn 4.1.4: Complementary or Supplementary?

$m\angle T = 32°$ and $m\angle R = 164°$. Angles $T$ and $R$ are

- ○ complementary.
- ○ supplementary.
- ○ neither complementary nor supplementary.

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## Example 4.1.5

Find the measurement of ∠AXZ.

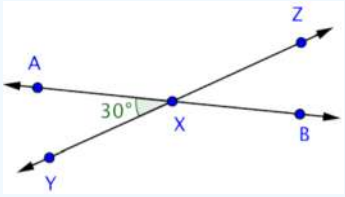



Figure **4.1.13**

**Answer**

This image shows two intersecting lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{YZ}$. They intersect at point *X*, forming four angles. Angles ∠AXY and ∠AXZ are supplementary because together they make up the straight angle <XYZ.

$$m\angle AXY + m\angle AXZ = m\angle YXZ$$

$$30° + m\angle AXZ = 180°$$

$$m\angle AXZ = 150°$$

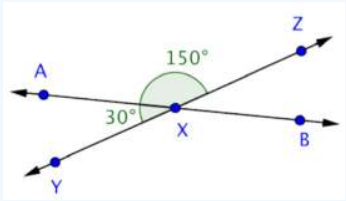
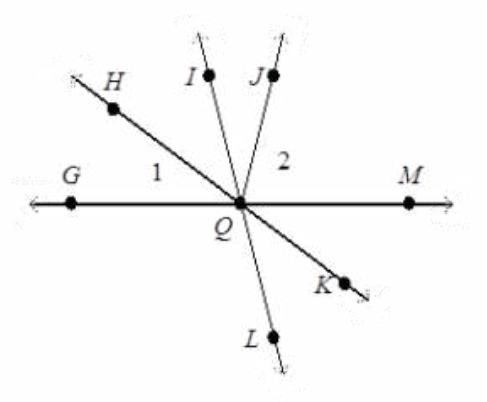



Figure **4.1.14**

m∠AXZ = **150°**

**Your Turn 4.1.5: Complementary or Supplementary?**

Pick an angle supplementary to $\angle HQJ$.




- ○ $\angle 1$
- ○ $\angle JQK$
- ○ $\angle GQJ$
- ○ $\angle HQJ$

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## Vertical Angles

When two lines intersect, the opposite angles are called vertical angles, and vertical angles have equal measure. For example, Figure 4.1.15 shows two straight lines intersecting each other. One set of opposite angles shows angle markers; those angles have the same measure. The other two opposite angles have the same measure as well.

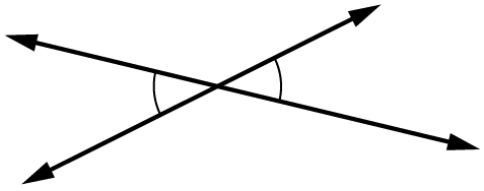


Figure 4.1.15: Vertical Angles

**Example 4.1.6: Calculating Vertical Angles**

In Figure 4.1.16, one angle measures $40°$. Find the measures of the remaining angles.

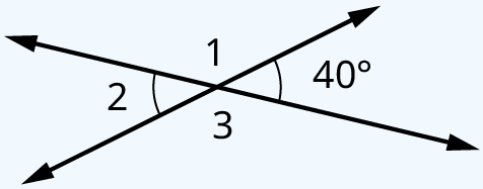



Figure 4.1.16: Vertical Angles

**Answer**

The $40°$ angle and $\angle 2$ vertical angles. Therefore, $m\angle 2 = 40°$

Notice that $\angle 2$ and $\angle 1$ are supplementary angles, meaning that the sum of $m\angle 2$ and $m\angle 2$ equals $180°$.

Therefore, $m\angle 1 = 180° - 40° = 140°$

Since $\angle 3$ and $\angle 1$ are vertical angles, then $m\angle 2 = 140°$

### Finding Angle Measurements

In the example below, notice how you can use the same technique as shown above (using straight angles) to find the measurement of a missing angle.

**Example 4.1.7: Evaluating Space**

You live on the corner of First Avenue and Linton Street. You want to plant a garden in the far corner of your property (Figure 4.1.17) and fence off the area. However, the corner of your property does not form the traditional right angle. You learned from the city that the streets intersect at an angle of $150°$. What is the measure of the angle that will border your garden?

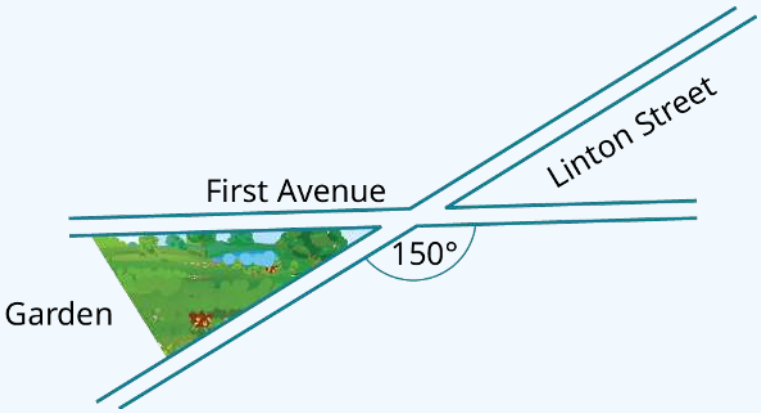



Figure 4.1.17: Intersection of Two Streets

**Answer**

As the angle between Linton Street and First Avenue is$150°$, the supplementary angle is $30°$.Therefore, the garden will from a $30°$ angle at the corner of your property.

**Example 4.1.8: Solving for Angle Measurements and Supplementary Angles**

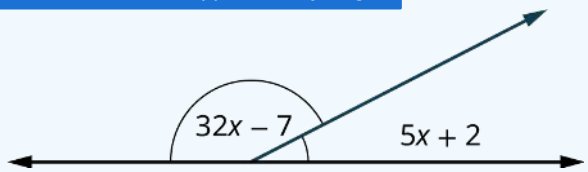



Figure 4.1.18: Supplement Angles

**Answer**

**Step 1**: These are supplementary angles. We can see this because the two angles are part of a horizontal line, and a horizontal line represents $180°$. Therefore, the sum of the two angles equals $180°$.

**Step 2**:

$$\begin{aligned}(32x-7)+(5x+2) &= 180\\ 37x-5 &= 180\\ 37x &= 185\\ x &= 5\end{aligned}$$

**Step 3:** Find the measure of each angle:

$$\begin{aligned}32x-7 &= 32(5)-7\\ &= 153°\\ 5x+2 &= 5(5)+2\\ &= 27°\end{aligned}$$

**Step 4**: We check: $153° + 27° = 180°$.

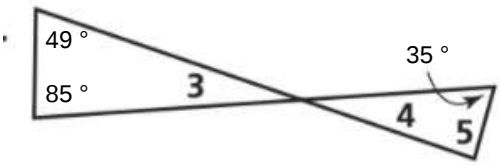
**Your Turn 4.1.8: Find Missing Angles**

Find the...

$m\angle 3$ =

$m\angle 4$ = _____

$m\angle 5$ = _____



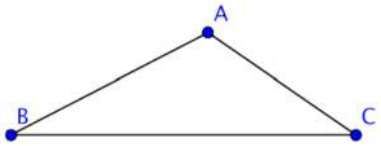

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## Triangles

Geometric shapes, also known as figures, are a crucial part of the study of geometry. The triangle is one of the basic shapes in geometry. It is the simplest shape within a classification of shapes called polygons. All triangles have three sides and three angles, but they come in many different shapes and sizes. Within the group of all triangles, the characteristics of a triangle's sides and angles are used to classify it even further. Triangles have some important characteristics, and understanding these characteristics allows you to apply the ideas in real-world problems.

There is an established convention for naming triangles. The labels of the vertices of the triangle, which are generally capital letters, are used to name a triangle.



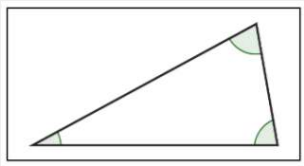
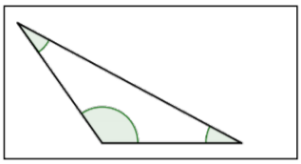
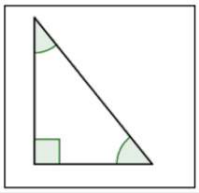

You can call this triangle *ABC* or ΔABC since *A*, *B*, and *C* are the vertices of the triangle. When naming the triangle, you can begin with any vertex. Then keep the letters in order as you go around the polygon. The triangle above could be named in a variety of ways: ΔABC, or ΔCBA. The sides of the triangle are line segments *AB*, *AC*, and *CB*.

Just as triangles can be classified as acute, obtuse, or right based on their angles, they can also be classified by the length of their sides. Sides of equal length are called **congruent** sides. While we designate a segment joining points *A* and *B* by the notation $\overline{AB}$, we designate the length of a segment joining points *A* and *B* by the notation *AB* without a segment bar over it. The length *AB* is a number, and the segment $\overline{AB}$ is the collection of points that make up the segment.

### Classifying and Naming Triangles

A polygon is a closed plane figure with three or more straight sides. Polygons each have a special name based on the number of sides they have. For example, the polygon with three sides is called a triangle because "tri" is a prefix that means "three." Its name also indicates that this polygon has three angles. The prefix "poly" means many.

The table below shows and describes three classifications of triangles. Notice how the types of angles in the triangle are used to classify the triangle.

| Name of Triangle | Picture of a Triangle | Description |
|---|---|---|
| **Acute Triangle** |  | A triangle with **3** acute angles (**3** angles measuring between $0°$ and $90°$ ). |
| **Obtuse Triangle** |  | A triangle with **1** obtuse angle (**1** angle measuring between $90°$ and $180°$). |
| **Right Triangle** |  | A triangle containing one right angle (**1** angle that measures $90°$). Note that the right angle is indicated by a corner mark and does not require labeling as $90°$. |

The sum of the measures of the three interior angles of a triangle is always $180°$. This fact can be applied to find the measure of the third angle of a triangle, if you are given the other two. Consider the examples below.

**Example 4.1.9**

A triangle has two angles that measure $35°$ and $75°$. Find the measure of the third angle.

**Answer**

The sum of the three interior angles of a triangle is $180°$.

$$35° + 75° + x = 180°$$

Find the value of $x$.

$$110° + x = 180°$$

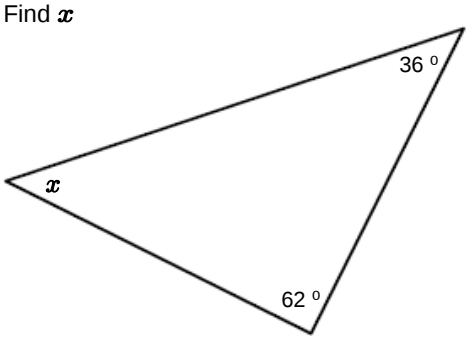
$$x = 180° - 110°$$

$$x = 70°$$

The third angle of the triangle measures $70°$.

**Your Turn 4.1.9: Find Missing Angle of Triangle**

Find $x$



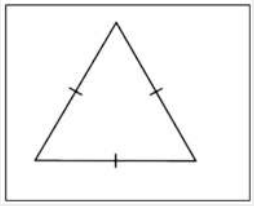
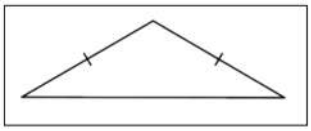
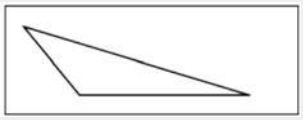

*** diagram is not drawn to scale!

$x =$ ____ °

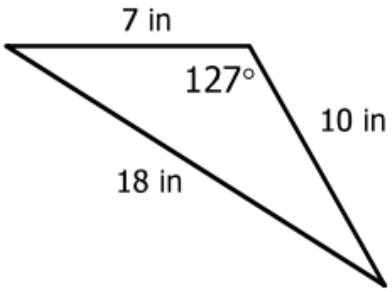
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Mathematicians show congruency by putting a hash mark symbol through the middle of the sides of equal length. If the hash mark is the same on one or more sides, then those sides are congruent. If the sides have different hash marks, they are *not* congruent. The table below shows the classification of triangles by their side lengths.

| Name of Triangle | Picture of a Triangle | Description |
|---|---|---|
| **Equilateral Triangle** |  | A triangle whose three sides have the same length. These sides of equal length are called congruent sides. |
| **Isosceles Triangle** |  | A triangle with exactly two congruent sides. |
| **Scalene Triangle** |  | A triangle in which all three sides are of different lengths. |

**Your Turn 4.1.10: Classify Triangle**




Classify the triangle based on its sides:

- ○ Equilateral
- ○ Isosceles
- ○ Scalene

Classify the triangle based on its angles:

- ○ Right
- ○ Equiangular
- ○ Obtuse
- ○ Acute

○ Acute

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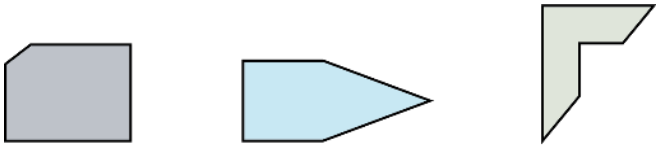
## Polygons

A polygon is a closed, two-dimensional shape classified by the number of straight-line sides. See Figure 10.82 for some examples. We show only up to eight-sided polygons, but there are many, many more.

| Number of Sides | Name | Shape |
|---|---|---|
| 3 | Triangle |  |
| 4 | Quadrilateral |  |
| 5 | Pentagon |  |
| 6 | Hexagon |  |
| 7 | Heptagon |  |
| 8 | Octagon |  |

Figure 4.1.19: Types of Polygons

If all the sides of a polygon have equal lengths and all the angles are equal, they are called **regular polygons**. However, any shape with sides that are line segments can be classified as a polygon. For example, the first two shapes, shown in Figure 4.1.20, are both pentagons because they each have five sides and five vertices. The third shape Figure 4.1.20 is a hexagon because it has six sides and six vertices. We should note here that the hexagon in Figure 4.1.20 is a concave hexagon, as opposed to the first two shapes, which are convex pentagons. Technically, what makes a polygon concave is having an interior angle that measures greater than $180°$. They are hollowed out, or cave in, so to speak. Convex refers to the opposite effect, where the shape is rounded out or pushed out.



(a) (b) (c)

Figure 4.1.20: Not Regular Polygons

The formula is used to find the sum of all interior angles of a polygon.

**Definition: Sum of Interior Angles**

Sum of Interior Angles Formula for a polygon with $n$ sides

$$\text{Sum} = (n-2) \times 180°$$

If the polygon is regular (all sides and angles equal):

$$\text{Each interior angle} = \frac{\text{Sum}}{n}$$

For triangle $n = 3$

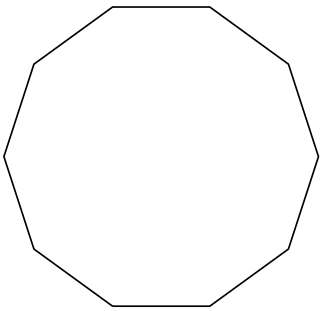
$$\text{sum} = (3-2) \times 180^\circ = 1 \times 180^\circ = 180^\circ$$

For quadrilateral $n = 4$

$$\text{sum} = (4-2) \times 180^\circ = 2 \times 180^\circ = 360^\circ$$

**Your Turn 4.1.11: Find the Angel of Regular Polygon**

The polygon is regular. Find the following.



Sum of the interior angles =

Measure of an interior angle =

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## Quadrilaterals

**Quadrilaterals** are a special type of polygon. As with triangles and other polygons, quadrilaterals have special properties and can be classified by characteristics of their angles and sides. Understanding the properties of different quadrilaterals can help you in solving problems that involve this type of polygon. While there are variations of all polygons, quadrilaterals contain an additional set of figures classified by angles and whether there is one or more pairs of parallel sides. See Figure **4.1.21**.

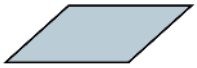
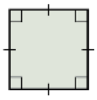
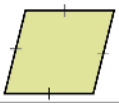
| Description | Shape |
|---|---|
| A **trapezoid** has one pair of parallel sides. |  |
| A **parallelogram** has two sets of parallel sides and no right angles. |  |
| A **rectangle** is a parallelogram with four right angles and two sets of parallel sides. |  |
| A **square** is a rectangle with four equal sides. |  |
| A **rhombus** is a parallelogram with all sides equal, two sets of parallel sides. |  |

Figure **4.1.21**: Types of Quadrilaterals

**Example 4.1.12: Determining Multiple Polygons**

What polygons make up Figure **4.1.22**?

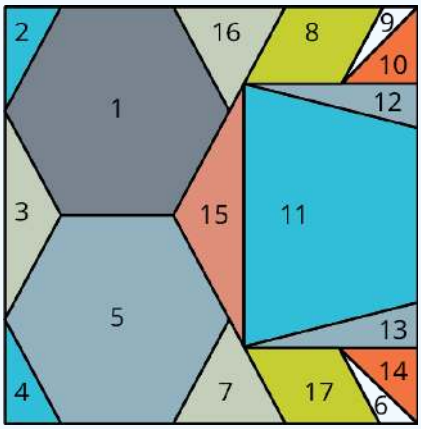



Figure **4.1.22**: Naming Polygons

**Answer**

Shapes **1** and **5** are hexagons; shapes **2, 3, 4, 6, 7, 9, 10, 12, 13, 14, 15,** and **16** are triangles; shapes **8** and **17** are parallelograms; and shape **11** is a trapezoid.

## Interior Angles of a Quadrilateral

The sum of the interior angles of any quadrilateral is **360°**. Consider the two examples below.

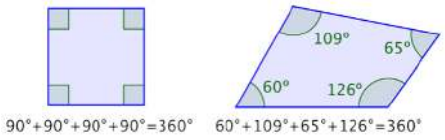



Figure **4.1.23**: Sum of Interior Angles of Quadrilateral

You could draw many quadrilaterals, such as these, and carefully measure the four angles. You would find that for every quadrilateral, the sum of the interior angles will always be **360°**.

**Sum of the Interior Angles of a Quadrilateral**

Since the sum of the interior angles of any triangle is **180°** and there are two triangles in a quadrilateral, the sum of the angles for each quadrilateral is **360°**.

You can use the properties of parallelograms to solve problems. Consider the example that follows.

**Example 4.1.13: Find Missing Angle of a Quadrilateral**

Determine the measures of three unknown angles.

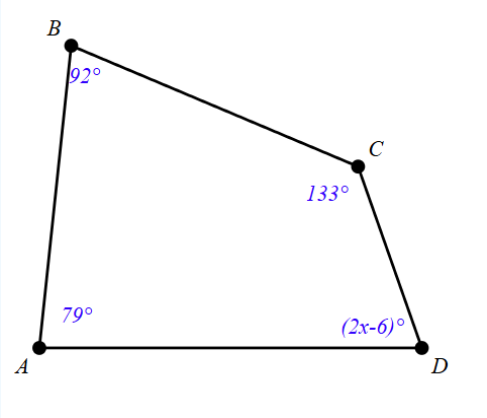



Figure 4.1.24: Missing Angle of Quadrilateral

**Answer**

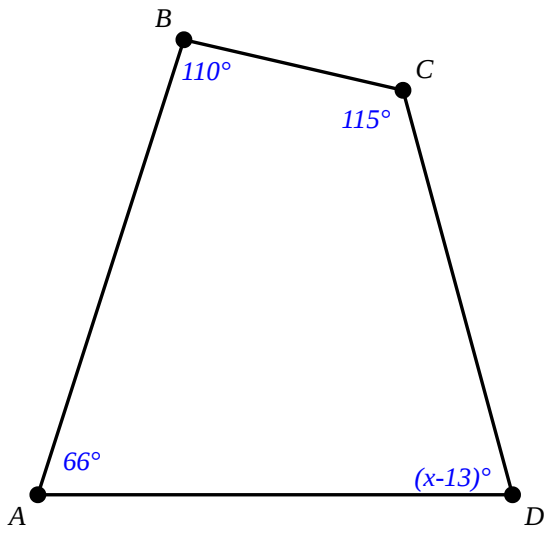
The total of the sum of the interior angles is 360°

$$\begin{aligned} 92+79+133+(2x-6) &= 360 \\ 292+2x &= 360 \\ 2x &= 360-298 \\ 2x &= 62 \\ x &= \frac{62}{2} \\ x &= 31 \end{aligned}$$

So the unknown angles are

$$2x-6 \quad = 2\times 31-6 \quad = 56$$

**Your Turn 4.1.13: Find Missing Angle of Quadrilateral**




Find $x$. Then find the $m\angle D$.

$x =$ ☐

$m\angle D =$ ☐ °

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The diagram below illustrates the relationship between the different types of quadrilaterals.

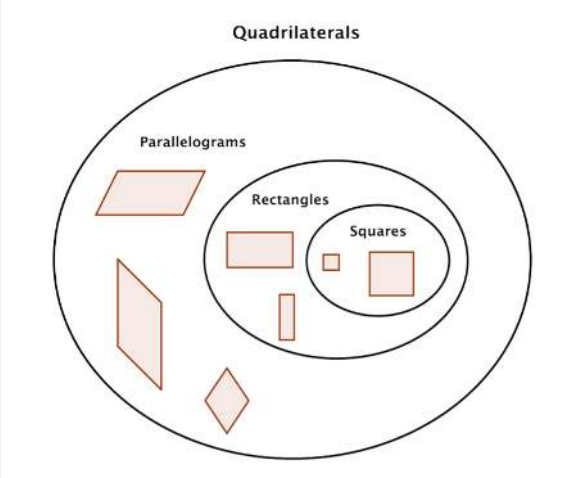



Figure 4.1.25: Relationship Between Quadrilaterals

**Your Turn 4.1.14: Identify Quadrilateral**

A trapezoid is_____ a square.

- ○ Always
- ○ Sometimes
- ○ Never

A rhombus is_____ a parallelogram.

- ○ Always
- ○ Sometimes
- ○ Never

A squre is _____a rhombus.

- ○ Always
- ○ Sometimes
- ○ Never

A rectangles are_____ square.

- ○ Always
- ○ Sometimes

Never

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# 4.2: Perimeter, Circumference, and Area

## Perimeter and Area

### Introduction

**Perimeter** and **area** are two important and fundamental mathematical topics. They help you to quantify physical space and also provide a foundation for more advanced mathematics found in algebra, trigonometry, and calculus. Perimeter is a measurement of the distance around a shape and area gives us an idea of how much surface the shape covers.

Knowledge of area and perimeter is applied practically by people on a daily basis, such as architects, engineers, and graphic designers, and is math that is very much needed by people in general. Understanding how much space you have and learning how to fit shapes together exactly will help you when you paint a room, buy a home, remodel a kitchen, or build a deck.

**Your Turn 4.2.1: Area or Perimeter?**



The red distance around the outside of the rectangle is called the Select an answer



The amount of space the blue inside of the rectangle covers is called the Select an answer

For each of the following, identify it as needing a perimeter or area

a) The number of feet Marcel travels when he walks around the block. Select an answer

b) The amount of paint needed to cover a wall of a living room. Select an answer

c) The number of tiles needed to cover the floor of a bathroom. Select an answer

d) The amount of fence needed to enclose a back yard. Select an answer

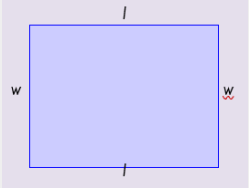
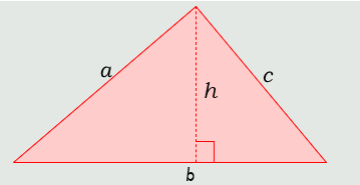
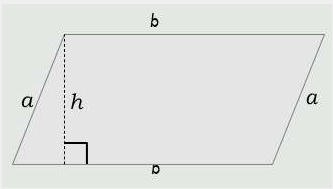
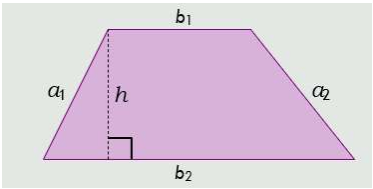
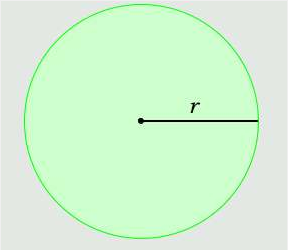
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### Perimeter

The perimeter of a two-dimensional shape is the distance around the shape. You can think of wrapping a string around a triangle. The length of this string would be the perimeter of the triangle. Or, walking around the outside of a park, you walk the distance of the park's perimeter. Some people find it useful to think "peRIMeter" because the edge of an object is its rim and pe**RIM**eter has the word "rim" in it.

If the shape is a **polygon**, then you can add up all the lengths of the sides to find the perimeter. Be sure to measure all lengths in the same units. You measure perimeter in linear units, which is a one-dimensional measurement. Examples of units of measure for length are inches, centimeters, or feet.

| Name | Figure | Area ($A$) and Perimeter ($P$) |
|---|---|---|
| Rectangle/Square |   | $A = l \cdot w$<br>$P = 2(l + w)$<br>$l$ is a length and $w$ is width. |
| Triangle |   | $A = \frac{1}{2} \cdot b \cdot h$<br>$P = a + b + c$<br>$b$ is a base and $h$ is a height. |
| Parallelogram |   | $A = b \cdot h$<br>$P = 2(b + a)$ |
| Trapezoid |   | $A = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$<br>$P = b_1 + b_2 + a_1 + a_2$<br>$b_1$ and $b_2$ are length of parallel bases. |
| Circle |   | $A = \pi r^2$<br>$C = 2\pi r$<br>$r$ is a radius and $C$ is Cirumference (Perimeter) |

Remember that all cubes are rectangular prisms, so the formula for finding the volume of a cube is the area of the base of the cube times the height.

### Example 4.2.2: Find the Perimeter

Find the perimeter of the given figure. All measurements indicated are inches.

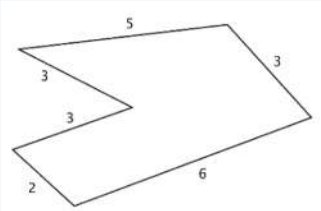



Figure **4.2.1**: Polygon Perimeter

**Answer**

Since all the sides are measured in inches, just add the lengths of all six sides to get the perimeter.

$$P = 5 + 3 + 6 + 2 + 3 + 3$$

Remember to include units.

$P = 22$ inches. This means that a tightly wrapped string running the entire distance around the polygon would measure 22 inches long.

**Example 4.2.3:Sides are Given as Algebraic Expression**

Find the perimeter of a triangle with sides. Write your answer in terms of $x$.

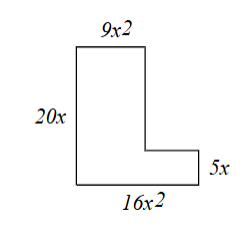


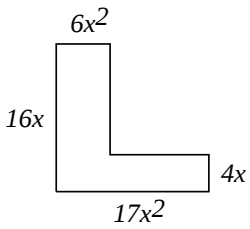

Figure 4.2.2: Polygon Sides in Terms of $x$

**Answer**

$$\text{Perimeter} = 16x^2 + 5x + 9x^2 + 20x + (16x^2 - 9x^2) + (20x - 5x) = 16x^2 + 5x + 9x^2 + 20x + 7x^2 + 14x = 32x^2 + 40x$$

**Your Turn 4.2.3: Find Perimeter**

Write the polynomial that represents the perimeter of the figure pictured below.




Perimeter =

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Sometimes, you need to use what you know about a polygon in order to find the perimeter. Let's look at the rectangle in the next example.

**Example 4.2.4: Perimeter of Rectangle**

A rectangle has a length of 8 centimeters and a width of 3 centimeters. Find the perimeter.

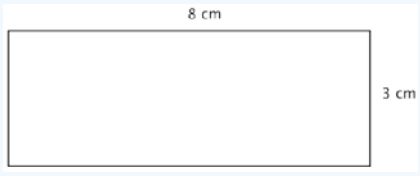



Figure 4.2.3: Rectangle with Known Sides

**Answer**

Since this is a rectangle, the opposite sides have the same lengths, 3 cm. and 8 cm. Add up the lengths of all four sides to find the perimeter.

$$P = 3 + 3 + 8 + 8 = 22 \text{ cm}$$

Notice that the perimeter of a rectangle always has two pairs of equal-length sides.

The formula for the perimeter of a rectangle is often written as $P = 2l + 2w$, where $l$ is the length of the rectangle and $w$ is the width of the rectangle.

So you could have also written

$$P = 2(3) + 2(8) = 6 + 16 = 22 \text{ cm}$$

.

**Example 4.2.5: Find Missing Length**

Find the length of the rectangle pictured above, if the perimeter is 96.

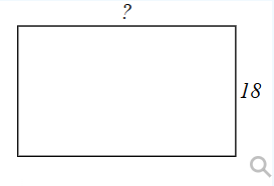



Figure 4.2.4: Rectangle with Unknown Side

**Answer**

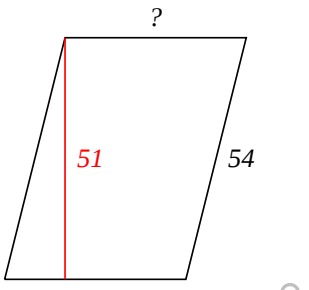
Two opposite sides of a rectangle have equal length. So the two opposite sides have a total length of $18 + 18 = 36$. The remaining two sides must total 60. So unknown side must be $\frac{60}{2} = 30$.

Another way

$$\begin{aligned} 96 &= 2l + 2 \times 18 \\ 96 &= 2l + 36 \\ 96 - 36 &= 2l \\ 60 &= 2l \\ l &= 30 \end{aligned}$$

**Your Turn 4.2.5: Find Missing Side if Perimeter if Given**

Find the width of the parallelogram shown below, if the perimeter is 196.




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## Triangles

Triangles are the strongest geometric shape. Roof trusses, bridges, and towers use triangular frames to distribute weight evenly. Ladders, bicycle frames, tents, and cranes use triangular supports. Foldable stands and racks often use triangular shapes for stability. The following example shows how to find the area and perimeter of a triangle.

### Example 4.2.6: Area and Perimeter of a Triangle

Find the perimeter and area of the figure below.

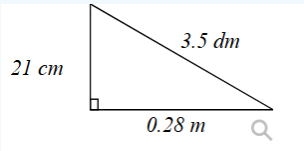



Figure 4.2.5: Right Triangle

**Answer**

First change centimeter (cm) and decimeter (dm) to meter (m).

$$\begin{aligned} 21 \text{ cm} &= 0.21 \text{ m} \\ 3.5 \text{ dm} &= 0.35 \text{ m} \end{aligned}$$

The base measures $0.28$ m and the height measures $0.21$ m . Using the formula, we can calculate the area:

$$A = \frac{1}{2}(0.28)(0.21) = 0.0294 \text{ m}^2$$

The perimeter is

$$\P = 0.21 + 0.28 + 0.35 = 0.84 \text{ m}$$

### Example 4.2.7: Finding Area of a Triangle

Find the area of this triangle that has a base of $4$ cm and a height of $6$ cm Figure 4.2.6.

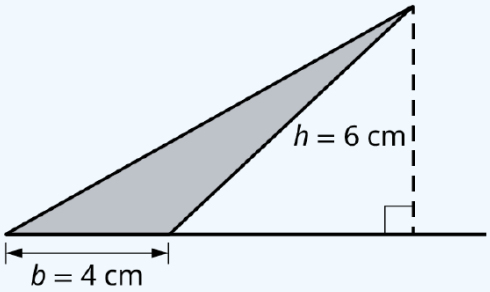



Figure ✔ 4.2.6: Obtuse Triangle

**Answer**

The base measures $4$ cm and the height measures $6$ cm. Using the formula, we can calculate the area:

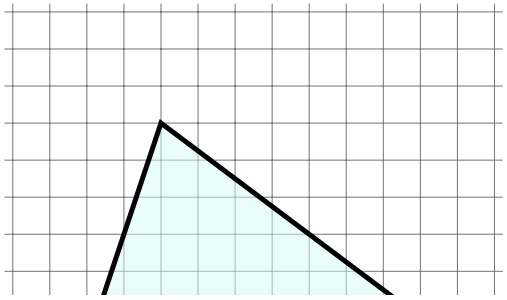
$$A = \frac{1}{2}(4)(6) = 12 \text{ cm}^2$$

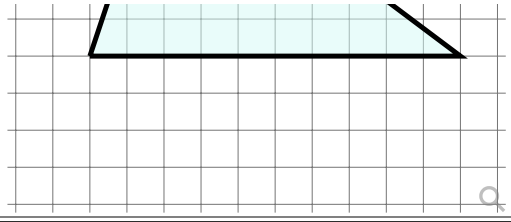
### Your Turn 4.2.7: Find Area of Triangle

**Finding the Area of a Triangle**

Use the diagram of the triangle in **Figure A** to answer the questions.
Each small square has a side length of 1 mm and an area of $1\text{mm}^2$.
**Note: Use a side of the triangle that has a whole number length as the base.**

**Figure A**





What it the length of the base of the triangle? _____ mm

What it the height of the triangle? _____ mm

What it the area of the triangle? _____ $mm^2$

***Round your result to two decimal places as needed.***

Question Help: Video Written Example

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## Quadrilaterals

A quadrilateral is a four-sided polygon with four vertices. Some quadrilaterals have either one or two sets of parallel sides. The set of quadrilaterals include the square, the rectangle, the parallelogram, the trapezoid, and the rhombus. The most common quadrilaterals are the square and the rectangle.

### Square and Rectangle

Many everyday applications require the use of the perimeter and area formulas. Suppose you are remodeling your home and you want to replace all the flooring. You need to know how to calculate the area of the floor to purchase the correct amount of tile, hardwood, or carpet. If you want to paint the rooms, you need to calculate the area of the walls and the ceiling to know how much paint to buy, and the list goes on. Can you think of other situations where you might need to calculate area?

**Example 4.2.8: Finding Area of a Rectangle**

You have a garden with an area of $196$ square feet. How wide can the garden be if the length is $28$ feet?

**Answer**

The area of a rectangular region is $A = lw$. Letting the width equal $w$ :

$$\begin{aligned} A &= lw \\ 196 &= 28w \\ \frac{196}{28} &= w \\ 7 &= w \end{aligned}$$

**Your Turn 4.2.8: Find the Number of Tiles**

Jerome is planning on tiling his bathroom floor with square tiles measuring 8 inches on each side. The bathroom measures 11 feet by 14 feet.

(a) What is the area of the floor, in square feet?

_____ square feet

(b) Convert this area to square inches.

_____ square inches

(c) How many 8-inch square tiles will Jerome need to cover the floor?

(c) How many 8-inch square tiles will Jerome need to cover the floor?

**Round UP to the nearest whole number.**

_____ tiles

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## Parallelogram

The area of a parallelogram can be found using the formula for the area of a triangle. If we cut a diagonal across the parallelogram from one vertex to the opposite vertex, we have two triangles. If we multiply the area of a triangle by **2** , we have the area of a parallelogram:

**Example 4.2.9: Finding the Area of a Parallelogram**

Use the diagram of the parallelogram in the following figure to answer the questions.

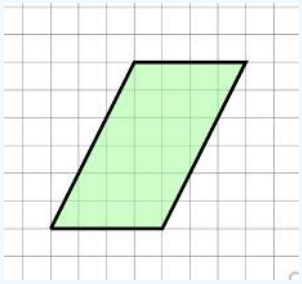


Figure **4.2.7**: Parallelogram; One Small square of Side Length **1** ft.

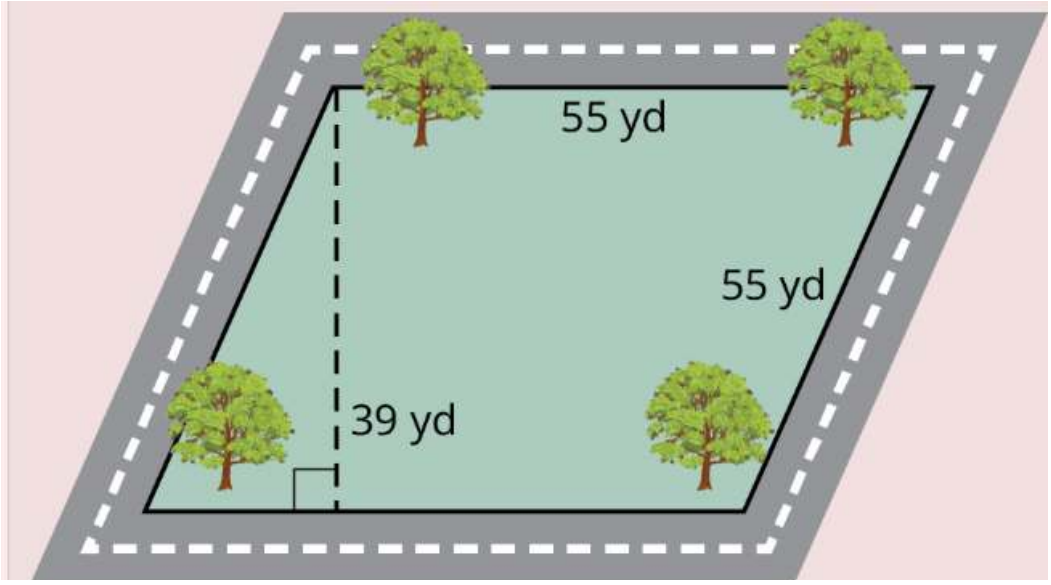
**Answer**

Note that the base is **4** feet and the height is **6** feet. Using the formula $A = b \times h$, we have

$$A = 6 \text{ ft} \times 4 \text{ ft} = 24 \text{ ft}^2$$

**Your Turn 4.2.9: Find the Cost to Put Sods**

The boundaries of a city park form a parallelogram in the follwoing figure.



The park occupies one city block, bounded by two sets of parallel streets. Each street measures 55 yd long. The perpendicular distance between streets is 39 yd.

(a) How much sod, sold by the square foot, should the city purchase to cover the entire park?

_____ square feet

(b) If the sod is sold for $2 per square foot and installation is $2 per square foot, how much will it cost if the city wants to cover the entire park?

$ _____ (Round to the nearest dollar)

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Trapezoid

Another quadrilateral is the trapezoid. A trapezoid has one set of parallel sides or bases. An example is provided below. Notice that the height of a trapezoid will always be perpendicular to the bases (just like when you find the height of a parallelogram).

**Example 4.2.10: Finding the Area of a Trapezoid**

>

The given figure is a regular trapezoid. Find the exact perimeter and then find the area.

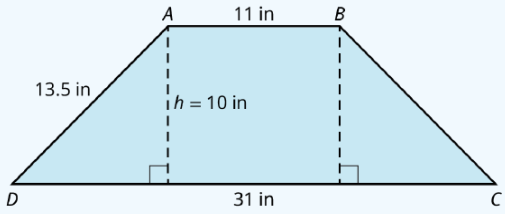



Figure 4.2.8: Regular Trapezoid (Non-Parallel Sides have Equal Length)

**Answer**

The perimeter is the measure of the boundary of a shape, so we simply add up the lengths of its sides. We have

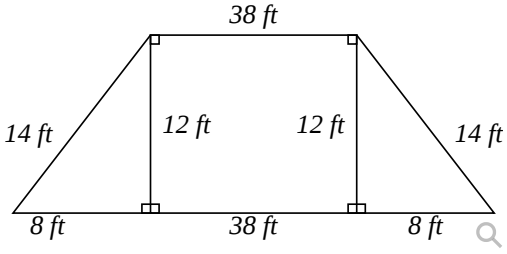
$$\text{Perimeter} = 31 + 13.5 + 11 + 13.5 = 69 \text{ in}$$

Then, the area of the trapezoid using the formula is

$$A = \frac{1}{2}(11 + 31) \cdot 10 = 210 \text{ in}^2$$

**Your Turn 4.2.10: Area and Perimeter of Trapezoid**

Find the perimeter and area of the following composite figure:



Perimeter = [ ] feet (If needed, enter answer to 1 decimal place.)

Area = [ ] square feet (If needed, enter answer to 1 decimal place.)

**NOTE: Figures are NOT to scale.**

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## Circles

### Introduction

Circles are a common shape. You see them all over—wheels on a car, Frisbees passing through the air, compact discs delivering data. These are all circles.

A circle is a two-dimensional figure, just like polygons and quadrilaterals. However, circles are measured differently from these other shapes—you even have to use some different terms to describe them. Let's take a look at this interesting shape.

### Properties of Circles

A circle represents a set of points, all of which are the same distance away from a fixed, middle point. This fixed point is called the center. The distance from the center of the circle to any point on the circle is called the **radius**.

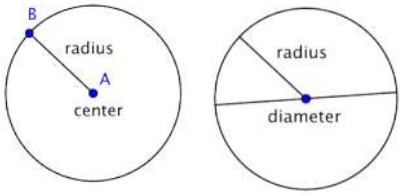



Figure **4.2.9**: Circles

When two radii (the plural of radius) are put together to form a line segment across the circle, you have a **diameter**. The diameter of a circle passes through the center of the circle and has its endpoints on the circle itself.

The diameter of any circle is two times the length of that circle's radius. It can be represented by the expression $2r$, or "two times the radius." So, if you know a circle's radius, you can multiply it by $2$ to find the diameter; this also means that if you know a circle's diameter, you can divide by $2$ to find the radius.

### Circumference

The distance around a circle is called the circumference. (Recall, the distance around a polygon is the perimeter.)

One interesting property about circles is that the ratio of a circle's circumference and its diameter is the same for all circles. No matter the size of the circle, the ratio of the circumference to the diameter will be the same.

Some actual measurements of different items are provided below. The measurements are accurate to the nearest millimeter or quarter-inch (depending on the unit of measurement used). Look at the ratio of the circumference to the diameter for each one—although the items are different, the ratio for each is approximately the same.

| Item | Circumference ($C$) (rounded to nearest hundredth) | Diameter ($d$) | Ratio $\frac{C}{d}$ |
|---|---|---|---|
| Cup | $253$ mm | $79$ mm | $\frac{253}{79} = 3.2025...$ |
| Quarter | $84$ mm | $79$ mm | $\frac{84}{27} = 3.1111...$ |
| Bowl | $37.25$ in | $11.5$ in | $\frac{37.25}{11.75} = 3.1702...$ |

The circumference and the diameter are approximate measurements since there is no precise way to measure these dimensions exactly. If you were able to measure them more precisely, however, you would find that the ratio $\frac{C}{d}$ would move towards $3.14$ for each of the items given. The mathematical name for the ratio $\frac{C}{d}$ is **pi** and is represented by the Greek letter $\pi$.

$\pi$ is a non-terminating, non-repeating decimal, so it is impossible to write it out completely? The first 10 digits of $\pi$ are $3.141592653$; it is often rounded to $3.14$ or estimated as the fraction $\frac{22}{7}$. Note that both $3.14$ and $\frac{22}{7}$ are *approximations* of $\pi$, and are used in calculations where it is not important to be precise.

### Exact and Approximate Answers

When we calculate the area and circumference of a circle, there are two ways to write the answers. If we write the answer in terms of $\pi$, the answer is called an exact answer. If we substitute $\pi = 3.14$ and simplify the given decimal places, the answer is called an approximate answer.

### Example 4.2.11: Circumference of a Circle

Find the circumference of a circle with a radius of $2.5$ yards. Write the answer in terms of pi (exact answer) and use $\pi = 3.14$ to find an approximate answer and round your answer to two decimal places.

**Answer**

To calculate the circumference of a circle given a radius of $2.5$ yards, use the formula $C = 2\pi r$.

$$C = 2\pi r$$

$$C = 2\pi \cdot 2.5$$

$$C = 5\pi$$

So the exact answer is $C = 5\pi \text{ yd}$. Use $3.14$ as an approximation for $\pi$.

$$C \approx 3.14 \cdot 5$$

$$C \approx 15.7 \text{ yd}$$

### Example 4.2.12: Tire Circumference

A $22$-inch wheel rim has a diameter of $22$ inches. When a tire is mounted and fully inflated, the wheel will expand another $2$ inches from the rim.

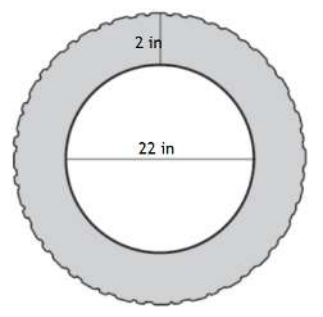



Figure 4.2.10: Circumference of Tire

What is the circumference of the inflated tire?

**Answer**

For an inflated tire, the radius is $r = 11 + 2 = 13 \text{ in}$.

To calculate the circumference of an inflated tire with a radius of $12$ in, use the formula $C = 2\pi r$.

$$C = 2\pi r$$

$$C = 2\pi \cdot 13$$

$$C = 26\pi$$

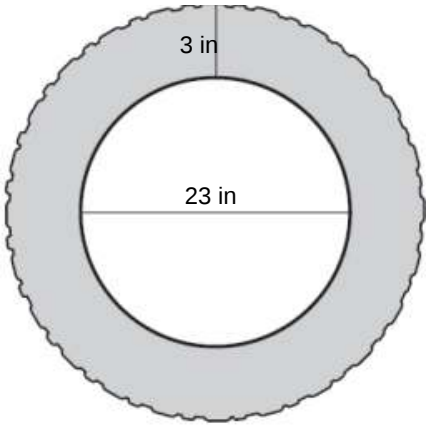
So the exact answer is $C = 26\pi \text{ in}$. Use $3.14$ as an approximation for $\pi$.

$$C \approx 26 \cdot 3.14$$

$$C \approx 81.64 \text{ in}$$

### Your Turn 4.2.12: Circumference of Tire

A $23$ -inch wheel rim has a diameter of $23$ inches. When a tire is mounted and fully inflated, the wheel will expand another $3$ inches from the rim.




What is the circumference of the inflated tire? **Round your answer to two decimal places.**

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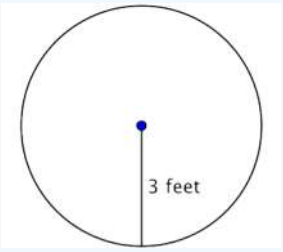
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## Area of Circle

Pi, $\pi$, is an important number in geometry. You have already used it to calculate the circumference of a circle. You use $\pi$ when you are figuring out the area of a circle, too.

### Example 4.2.13: Area of a Circle

Find the area of the circle.




**Answer**

To find the area of this circle, use the formula $A = \pi r^2$. Remember to write the answer in terms of square units, since you are finding the area.

$$A = \pi r^2$$

$$A = \pi \cdot 3^2$$

$$A = \pi \cdot 9$$

$$A = 3.14 \cdot 9$$

$$A = 28.26$$

Answer The area is $9\pi$ or approximately $28.26 \ \text{feet}^2$.

### Example 4.2.14: Area of a Pizza

The diameter of a pizza is **14** inches. If the pizza is cut into **10** slices, what is the area of one slice of pizza? If a whole pizza costs $**14.50**, what is the price of one square inch? *Round your answer to the nearest hundredth.*

**Answer**

To find the area of this circle, use the formula $A = \pi r^2$. Remember to write the answer in terms of square units, since you are finding the area.

$$A = \pi r^2$$

$$A = \pi \cdot 7^2$$

$$A = \pi \cdot 49$$

$$A = 3.14 \cdot 49$$

$$A = 153.86 \text{ in}^2$$

The area of one slice of pizza is approximately $\frac{153.86}{10} = 15.39 \text{ in}^2$.

Price for $1 \text{ in}^2$ slice of pizza is $\frac{\$14.50}{15.39} = \$0.94$.

**Your Turn 4.2.14: Which Pizza Place has Best Deal?**

At most pizza places, the menu will show the diameter of their different pizza sizes, along with the price of each size. Most pizza places offer "deals" that may or may not save you money. After doing some research at his 3 favorite pizza places, Gerardo need your help to figure out ... WHICH PIZZA PLACE HAS THE BEST DEAL?

| | *Pi* *by Pythagoras | Big Caesars Pizza | Ray's Famous Pizza |
|---|---|---|---|
| **DIAMETER** | 10 in | 13 in | 11 in |
| **PRICE** | $12.36 | $12.83 | $12.18 |
| **AREA OF PIZZA** *(Round to nearest tenth)* | _____ sq. inches | _____ sq. inches | _____ sq. inches |
| **COST PER SQ. INCH** | $ _____ | $ _____ | $ _____ |

Select an answer has the best deal because their pizza cost the least per square inch.

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## Working More with Perimeter and Area (Composite Figures)

Often, you need to find the area or perimeter of a shape that is not a standard polygon. Artists and architects, for instance, often work with complex shapes. However, even complex shapes can be thought of as composed of smaller, less complicated shapes, such as rectangles, trapezoids, and triangles.

To find the perimeter of non-standard shapes, you still find the distance around the shape by adding together the length of each side.

Finding the area of non-standard shapes is a bit different. You need to create regions within the shape for which you can find the area and add these areas together. Have a look at how this is done below.

You can also use what you know about perimeter and area to help solve problems about situations like buying fencing or paint, or determining how big a rug is needed in the living room. Here's a fencing example.

**Example 4.2.15: Area of Composite Figure**

Find the area of the shaded region. (Object is not drawn to scale.)

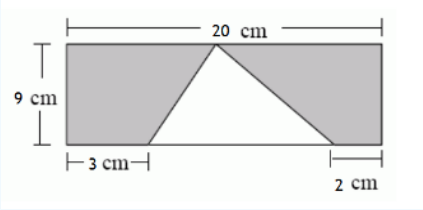



Figure 4.2.11: Composite Figure

**Answer**

This shape is a combination of two simpler shapes: a rectangle and a trapezoid. Find the area of each.

Find the area of the rectangle.

$$A = l \cdot w$$

$$A = 9 \cdot 20$$

$$A = 180 \text{ cm}^2$$

Find the area of the white triangle.

\(A = \dfrac{b\dot h}{2})

$$A = \frac{(20-5) \times 9}{2}$$

$$A = \frac{135}{2}$$

$$A = 67.5 \text{ ft}^2$$

The shaded area is

$$180 \text{ cm}^2 - 67.5 \text{ cm}^2 = 112.5 \text{ cm}^2$$

## Example 4.2.16: Area of Composite Figure

Find the area of the shaded region.

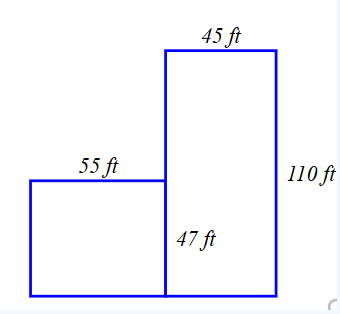



Figure 4.2.12:Composite Figure

**Answer**

Area of a rectangle, which is the length times the width.

$$A = l \times w$$

Substitute 55 ft for the length and 47 ft for the width.

$$A = 55 \times 47 = 2,585 \text{ ft}^2.$$

Area of a right rectangle, which is again the length times the width.

Substitute 110 ft for the length and 45 ft for the width.

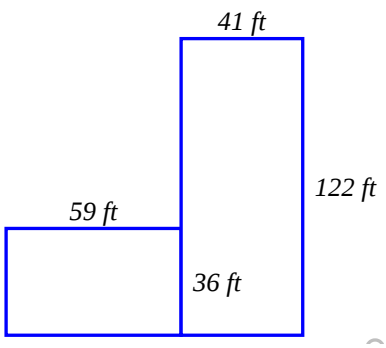
$$A = 110 \times 45 = 4,950 \text{ ft}^2$$

So the area of the figure is.

$$\text{Total area} = 2,585 \text{ ft}^2 + 4,950 \text{ ft}^2 = 7,535 \text{ ft}^2$$

## Your Turn 4.2.16: Find the Area and Perimeter

Here is a figure made of two rectangles.



(a) What is the **area** of the figure? _____ square feet

(b) What is the **perimeter** of the figure? _____ feet

(c) How much greater is the area of the figure than its perimeter? _____

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You know how to calculate the circumference and area of a circle, you can use this knowledge to find the perimeter and area of composite figures. The trick to figuring out these types of problems is to identify shapes (and parts of shapes) within the composite figure, calculate their individual dimensions, and then add them together.

**Example 4.2.17: Area and Perimeter of Composite Figure**

You decide to make a trim for the window in the figure. How many feet of trim do you need to buy? What is the area of the window?

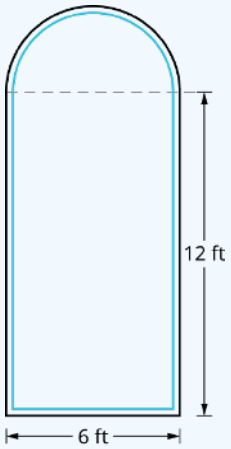



Figure **4.2.18**: Wind with Semicircle Trim in the Top

**Answer**

The trim will cover the **6** feet along the bottom and the two **12**-ft sides plus the half circle on top. The circumference of a semicircle is ½ the circumference of a circle. The diameter of the semicircle is **6** ft. So its radius is **3** ft. Then, the circumference of the semicircle would be

$$C = \frac{1}{2} \times 2\pi r$$

$$C = \frac{1}{2}\pi \times 2\pi \cdot 3$$

$$C = 9.4$$

So the circumference of the top semicircle is

$$C \approx 9.4 \text{ ft}$$

Therefore, the total perimeter of the window is $6 + 12 + 12 + 9.4 = 39.4$ ft.

The area of a rectangle is calculated by multiplying the length by the width.

$$A = l \times w$$

Substitute **6** in for the length and **12** in for the width.

$$A = 6 \cdot 12 = 72 \text{ ft}^2.$$

Area of semicircles of $r = 3$ is

\[\begin{aligned}
A &= \frac{1}{2}\pi r^2\

&= \frac{1}{2}\pi\times 3^2\

&=\frac{1}{2}\times 3.14\times 9\

&=14.13\text{~ft}^2\
\end{aligned} \nonumber \]

$$\text{Total area of window} = 72 + 14.13\pi = 86.13\pi \text{ ft}^2$$

### Example 4.2.18: Area and Circumference of Composite Figure

Find the perimeter and area of the composite figure, which is made up of two semicircles, one square, and one rectangle. Write exact answer (in terms of pi) and round your answer to two decimal places with $pi = 3.14$.

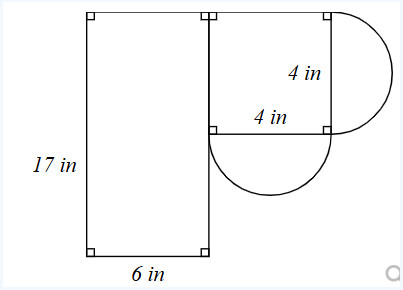



Figure 4.2.19: Composite Figure

**Answer**

Identify smaller shapes within the composite figure. This figure contains a rectangle, a square, and two semicircles.

Area of a rectangle, which is the length times the width.

$$A = l \cdot w$$

Substitute $17$ in for the length and $6$ in for the width.

$$A = 17 \cdot 6 = 102 \text{ in}^2$$

Area of a square, which is the length times the width.

$$A = l \cdot w$$

Substitute $4$ in for the length and $4$ in for the width.

$$A = 4 \cdot 4 = 16 \text{ in}^2.$$

Note that two semicircles combine to make one circle with a diameter of $4$, hence a radius of $r = 2$.

$$A = \pi r^2$$

$$A = \pi 2^2$$

$$A = 4\pi$$

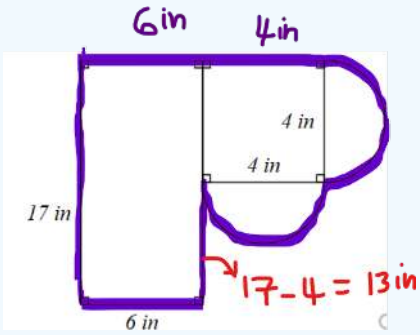
$$\text{Total area of figure } = 102 + 16 + 4\pi = 118\pi \text{ in}^2$$

For approximate answer put $\pi = 3.14$

$$\text{Total area of figure } = 118 \times 3.14 = 370.52 \text{ in}^2$$

For the perimeter, let's find the circumference of two circles. Note that two semicircles combine to make one circle with a diameter of $4$, hence a radius of $r = 2$.

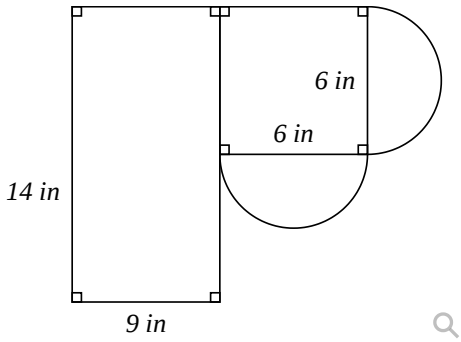
$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 2 \\ &= 4\pi \text{ inches} \end{aligned}$$



Therefore, the total perimeter of the window is $13+6+17+6+4+4\pi=(46+4\pi)$ ft. Substitute $pi=3.14$ to find an approximate answer.

## Your Turn 4.2.18: Area and Perimeter of Composite Figure

Find the perimeter and area of the following composite figure. Give the exact answers and the approximations using the fact that $\pi \approx 3.14$. Write pi to write pi symbol in the exact answer box.




| | Exact Answer | Approximate Answer |
|---|---|---|
| Perimeter | $in$ | $in$ |
| Area | $in^2$ | $in^2$ |

**NOTE: Figures are NOT to scale.**

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## Example 4.2.19: Area and Perimeter of Composite Figure

Find the area and perimeter of the polygon.

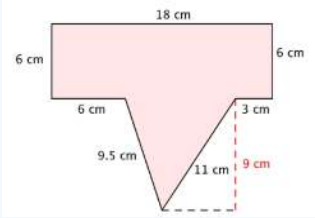



Figure 4.2.20: Composite Figure

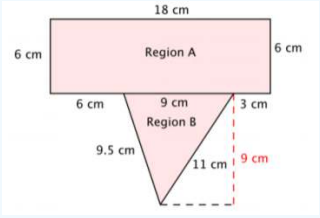
**Answer**

To find the perimeter, add together the lengths of the sides. Start at the top and work clockwise around the shape.

$$P=18+6+3+11+9.5+6+6$$

$$P=59.5 \text{ cm}$$

To find the area, divide the polygon into two separate, simpler regions. The area of the entire polygon will equal the sum of the areas of the two regions.



$$\text{Area of Polygon } = (\text{Area of A}) + (\text{Area of B})$$

Region A is a rectangle. To find the area, multiply the length $18$ by the width $6$.

$$\begin{aligned}\text{Area of Region: } A &= l \times w \\ &= 18 \times 6 \\ &= 108\end{aligned}$$

The area of Region A is $108\ \text{cm}^2$.

Region B is a triangle. To find the area, use the formula $\frac{1}{2}bh$, where the base is $9$ and the height is $9$.

$$\begin{aligned}\text{Area of Region } : B &= \frac{1}{2}b \times h \\ &= \frac{1}{2} \times 9 \times 9 \\ &= \frac{1}{2} \times 81 \\ &= 40.5\end{aligned}$$

The area of Region B is $40.5\ \text{cm}^2$.

Add the regions together.

$$108\ \text{cm}^2 + 40.5\ \text{cm}^2 = 148.5\ \text{cm}^2.$$

Perimeter = $59.5\ \text{cm}$

Area = $148.5\ \text{cm}^2$

---

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# 4.3: Volume and Surface Area

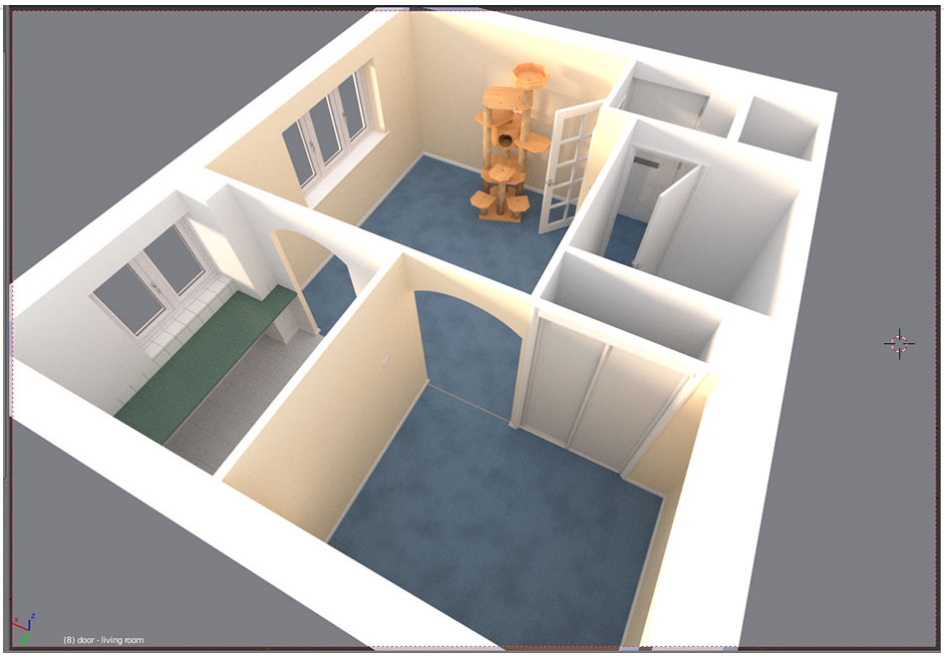


Figure **4.3.1**: Volume is illustrated in this 3-dimensional view of an interior space. This gives a buyer a more realistic interpretation of space. (credit: "beam render 10 with sun and cat tree" by monkeywing/Flickr, CC BY 2.0)

Volume and surface area are two measurements that are part of our daily lives. We use volume every day, even though we do not focus on it. When you purchase groceries, volume is the key to pricing. Judging how much paint to buy or how many square feet of siding to purchase is based on surface area. The list goes on. An example is a three-dimensional rendering of a floor plan. These types of drawings make building layouts far easier to understand for the client. It allows the viewer a realistic idea of the product at completion; you can see the natural space, the volume of the rooms. This section gives you practical information you will use consistently. You may not remember every formula, but you will remember the concepts, and you will know where to go should you want to calculate volume or surface area in the future.

We will concentrate on a few particular types of three-dimensional objects: right prisms and right cylinders. The adjective “right” refers to objects such that the sides form a right angle with the base. We will look at right rectangular prisms, right triangular prisms, right hexagonal prisms, right octagonal prisms, and right cylinders. Although the principles learned here apply to all right prisms.

Living in a two-dimensional world would be pretty boring. Thankfully, all of the physical objects that you see and use every day—computers, phones, cars, shoes—exist in three dimensions. They all have length, width, and height. (Even very thin objects like a piece of paper are three-dimensional. The thickness of a piece of paper may be a fraction of a millimeter, but it does exist.)

In the world of geometry, it is common to see three-dimensional figures. In mathematics, a flat side of a three-dimensional figure is called a face. Polyhedrons are shapes that have four or more faces, each one being a polygon. These include cubes, prisms, and pyramids. Sometimes you may even see single figures that are composites of two of these figures. Let’s take a look at some common polyhedra.

## Three-Dimensional Objects

In geometry, three-dimensional objects are called geometric solids. Surface area refers to the flat surfaces that surround the solid and is measured in square units. Volume refers to the space inside the solid and is measured in cubic units. Imagine that you have a square flat surface with width and length. Adding the third dimension adds depth or height, depending on your viewpoint, and now you have a box. One way to visualize this concept is in three-dimensional Cartesian space. The $x$-axis and the $y$-axis are, as you would expect, two dimensions and suitable for plotting two-dimensional graphs and shapes. Adding the $z$-axis, which shoots through the origin perpendicular to the $xy$-plane, and we have a third dimension.

Here is another view taking the two-dimensional square to a third dimension. See Figure **4.3.2**.

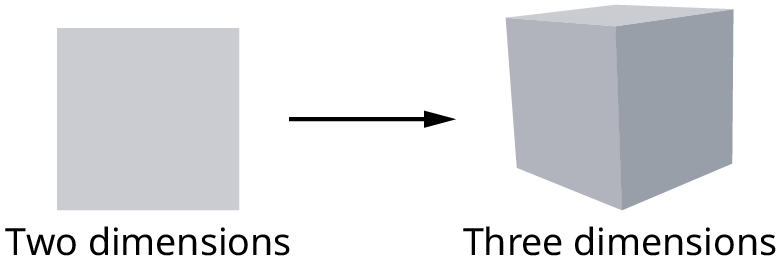
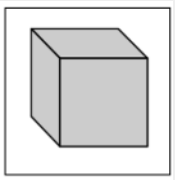
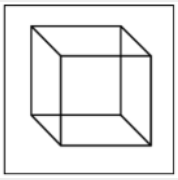
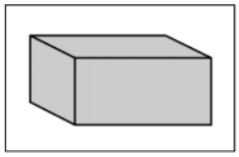
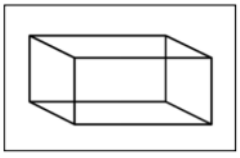
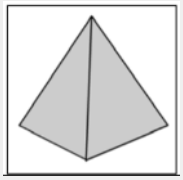
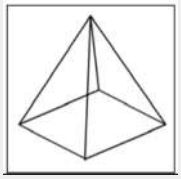



Figure **4.3.2**: Going from Two Dimensions to Three Dimensions

To study objects in three dimensions, we need to consider the formulas for surface area and volume. For example, suppose you have a box with a hinged lid that you want to use for keeping photos, and you want to cover the box with decorative paper. You would need to find the surface area to calculate the amount of paper needed. Suppose you need to know how much water will fill your swimming pool. In that case, you would need to calculate the volume of the pool. These are just a couple of examples of why these concepts should be understood, and familiarity with the formulas will allow you to make use of these ideas as related to right prisms and right cylinders.

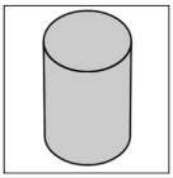
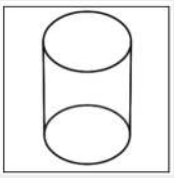
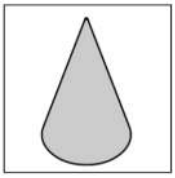
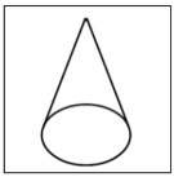
## Identifying Solids

The first set of solids contains rectangular bases. Take a look at the table below, which displays each figure in both solid and transparent forms.

| Name | Definition | Solid Form | Transparent Form |
|---|---|---|---|
| Cube | A six-sided polyhedron that has congruent squares as faces. |  |  |
| Rectangular prism | A polyhedron that has three pairs of congruent, rectangular, parallel faces. |  |  |
| Pyramid | A polyhedron with a polygonal base and a collection of triangular faces that meet at a point. |  |  |

Notice the different names that are used for these figures. A **cube** is different than a square, although they are sometimes confused with each other—a cube has three dimensions, while a square only has two. Likewise, you would describe a shoebox as a **rectangular prism** (not simply a rectangle), and the ancient **pyramids** of Egypt as…well, as pyramids (not triangles).

In this next set of solids, each figure has a circular base.

| Name | Definition | Solid Form | Transparent Form |
|---|---|---|---|
| Cylinder | A solid figure with a pair of circular, parallel bases and a round, smooth face between them. |  |  |
| Cone | A solid figure with a single circular base and a round, smooth face that diminishes to a single point. |  |  |

Take a moment to compare a pyramid and a **cone**. Notice that a pyramid has a rectangular base and flat, triangular faces; a cone has a circular base and a smooth, rounded body.

Finally, let's examine a shape that is unique: The **sphere**.

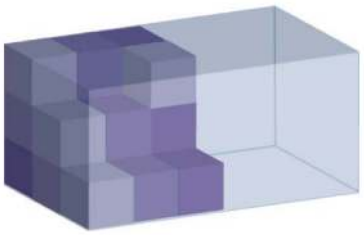
| Name | Definition | Solid Form | Transparent Form |
|---|---|---|---|
| Sphere | A solid, round figure where every point on the surface is the same distance from the center. |  |  |

There are many spherical objects all around you—soccer balls, tennis balls, and baseballs being three common items. While they may not be perfectly spherical, they are generally referred to as spheres.

## Volume

Recall that perimeter measures one dimension (length), and area measures two dimensions (length and width). To measure the amount of space a three-dimensional figure takes up, you use another measurement called **volume**.

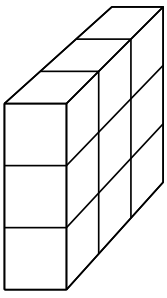
To visualize what "volume" measures, look back at the transparent image of the rectangular prism mentioned earlier (or just think of an empty shoebox). Imagine stacking identical cubes inside that box so that there are no gaps between any of the cubes. Imagine filling up the entire box in this manner. If you counted the number of cubes that fit inside that rectangular prism, you would have its volume.



Volume is measured in cubic units.

The shoebox illustrated above may be measured in cubic inches (usually represented as $\text{in}^3$ or $\text{inches}^3$), while the Great Pyramid of Egypt would be more appropriately measured in cubic meters ($\text{m}^3$ or $\text{meters}^3$).

**Your Turn 4.3.1: Find the Volume**



The total volume is _____ unit cubes.

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To find the volume of a geometric solid, you could create a transparent version of the solid, create a bunch of $1 \times 1 \times 1$ cubes, and then stack them carefully inside. However, that would take a long time! A much easier way to find the volume is to become familiar with some geometric formulas and to use those instead.

Let's go through the geometric solids once more and list the volume formula for each.

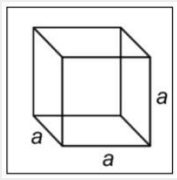
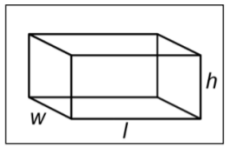
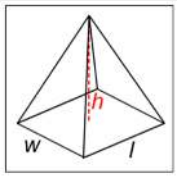
As you look through the list below, you may notice that some of the volume formulas look similar to their area formulas. To find the volume of a rectangular prism, you find the area of the base and then multiply that by the height.

## Surface Area

**Surface area** is the **total area of all the outer surfaces** of a three-dimensional object. It tells you how much material would be needed to **cover** the shape completely. It's measured in **square units** (e.g., $cm^2$, $m^2$, $in^2$).

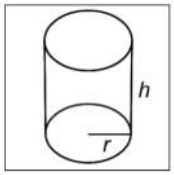
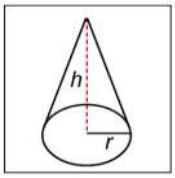
### Lateral Surface and Total Surface

There are two different kinds of surface area that are important: the ***lateral surface area*** (LSA) and the ***total surface area*** (TSA). To visualize the difference between LSA and TSA, consider a can of soup. The lateral surface area would be used to measure the size of the paper label around the can. The total surface area would be used to determine the amount of sheet metal required to manufacture the can. In other words, the total surface area includes the top and bottom, whereas the lateral surface area does not.

| Name | Transparent Form | Volume and Surface Area Formula |
|---|---|---|
| Cube |  | $V = a \cdot a \cdot a = a^3$<br>$a$ = the length of one side<br>$SA = 6a^2$ |
| Rectangular prism |  | $V = l \cdot w \cdot h$<br>$l$ = length<br>$w$ = width<br>$h$ = height<br>$SA = lw + lh + wh$ |
| Pyramid |  | $V = \frac{l \cdot w \cdot h}{3} = \frac{B \cdot h}{3}$<br>$l$ = length<br>$w$ = width<br>$h$ = height<br>$B = l \times w$ = area of the base |

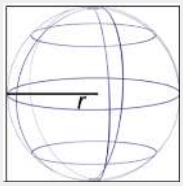
Remember that all cubes are rectangular prisms, so the formula for finding the volume of a cube is the area of the base of the cube times the height.

Now let's look at solids that have a circular base.

| Name | Transparent Form | Volume and Surface Area Formula |
|---|---|---|
| Cylinder |   | $V = \pi \cdot r^2 \cdot h$<br>$r$ = radius<br>$h$ = height<br>$SA = 2\pi rh$: Lateral surface area of a cylinder (just the curved outside)<br>$SA = 2\pi rh + 2\pi r^2$: Total surface area including top and bottom circular parts |
| Cone |   | $V = \frac{\pi \cdot r^2 \cdot h}{3}$<br>$r$ = radius<br>$h$ = height<br>$SA = \pi r\sqrt{r^2 + h^2}$: Lateral surface area (just the curved outside) |

Compare the formula for the volume of a cone ($V = \frac{\pi \cdot r^2 \cdot h}{3}$) with the formula for the volume of a pyramid ($V = \frac{l \cdot w \cdot h}{3}$). The numerator of the cone formula is the volume formula for a cylinder, and the numerator of the pyramid formula is the volume formula for a rectangular prism. Then divide each by $3$ to find the volume of the cone and the pyramid. Looking for patterns and similarities in the formulas can help you remember which formula refers to a given solid.

Finally, the formula for a sphere is provided below. Notice that the radius is cubed, not squared, and that the quantity $\pi r^3$ is multiplied by $\frac{4}{3}$.

| Name | Wireframe Form | Volume Formula |
|---|---|---|
| Sphere |   | $V = \frac{4}{3}\pi r^3$<br>$r$ = radius<br>$V = 4\pi r^2$<br>Note: For the hemisphere, you have to divide by $2$ in the above formula. |

You know how to identify the solids, and you also know the volume formulas for these solids. To calculate the actual volume of a given shape, all you need to do is substitute the solid's dimensions into the formula and calculate.

**Answer with $\pi$ and without $\pi$**

**Note:** If you keep $\pi$ in your answer, it is called an **exact answer**. If you replace $\pi$ with $3.14$, the answer is an **approximate answer**.

**Your Turn 4.3.2: Identify the Solid**

**Figure 1**

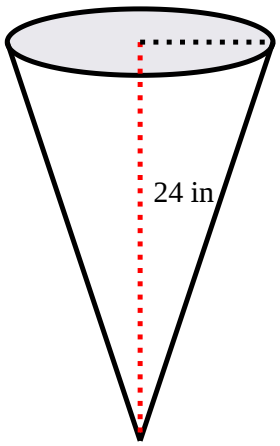



**Figure 2**

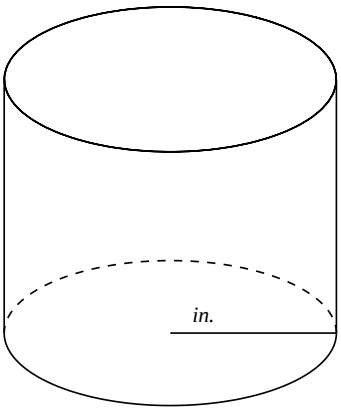


**Figure 3**

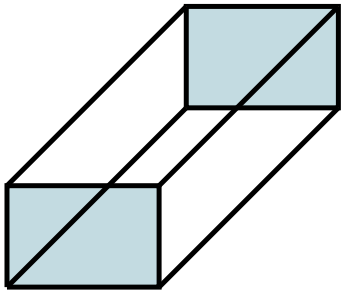


**Figure 4**

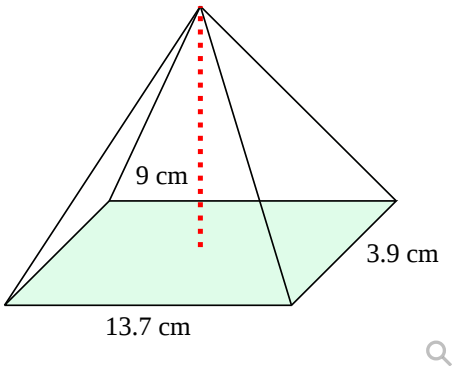



**FIgure 5**

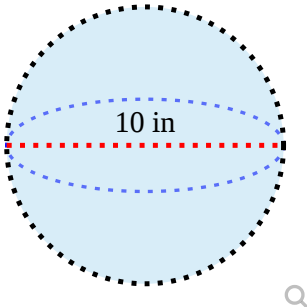
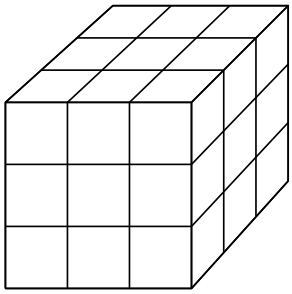



**Figure 6**



**Match the Name of the 3-dimensional object to their figure to which it applies.**

| Name of Solids | Figure |
|---|---|
| - Pyramid | a. Figure 1 |
| - Cylinder | b. Figure 2 |
| - Rectangular Box | c. Figure 3 |
| - Sphere | d. Figure 6 |
| - Cone | e. Figure 5 |
| - Cube | f. Figure 4 |

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## Example 4.3.3: Calculating Surface Area and Volume of a Rectangular Prism

Find the surface area and volume of the rectangular prism that has a width of 10 cm, a length of 5 cm, and a height of 3 cm (Figure 4.3.6).

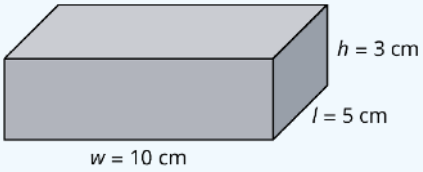



Figure 4.3.3: Rectangular Solid

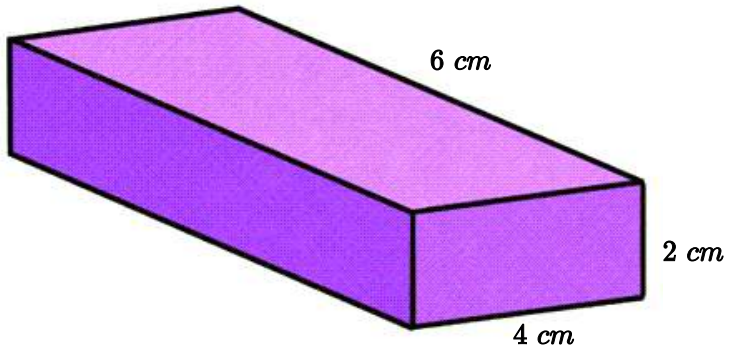
**Answer**

The surface area is

$$SA = 2(10)(5) + 2(5)(3) + 2(10)(3) = 190 \text{ cm}^2$$

## Your Turn 4.3.3: Surface Area and Volume of a Box

Find the surface area (SA) and Volume (V) of the rectangular solid with the given dimensions.




$SA =$ _____ Select an answer

$V =$ _____ Select an answer

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## Example 4.3.4: Volume of Pyramid

Find the volume of the shape shown below.

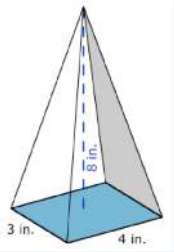



Figure **4.3.4**: Pyramid with Rectangular Base

**Answer**

Identify the shape. It has a rectangular base and rises to a point, so it is a pyramid.

Identify the proper formula to use. $l$ = length, $w$ = width, and $h$ = height

$$V = \frac{l \cdot w \cdot h}{3}$$

Use the image to identify the dimensions. $4$ = length $3$ = width $8$ = height. Then substitute $l = 4$, $w = 3$, and $h = 8$ into the formula.

$$V = \frac{4 \cdot 3 \cdot 8}{3} = \frac{96}{3} = 32$$

The volume of the pyramid is $32\ \text{in}^3$

**Your Turn 4.3.4**: Volume of Pyramid

**Volume of a Pyramid**

Use the figures below to answer the questions.

The shaded portion of the 3D figure corresponds to the 2D figure shown in Figure B.

**Round your final results to two decimal places as needed.**

**Figure A: 3-Dimensional Figure**

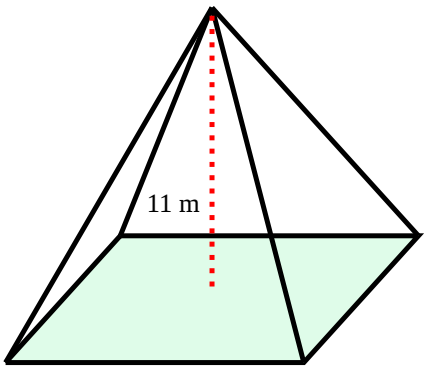
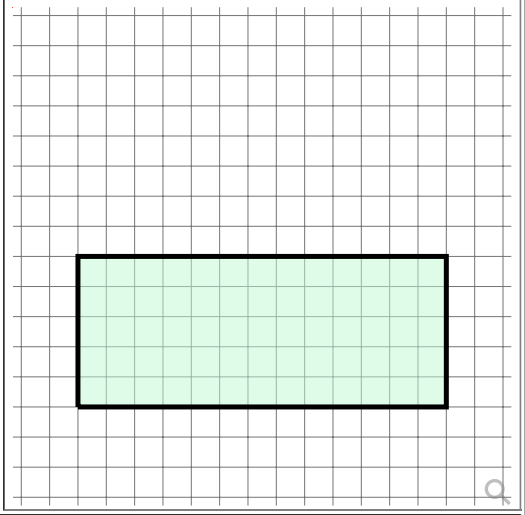



**Figure B: 2-Dimensional View of Base**



What is the area of the base of the pyramid?

Select an answer

What is the height of the pyramid?

Select an answer

What is the volume of the pyramid?

Select an answer

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## Example 4.3.5: Box and Ball

Callie is sending her nephew a soccer ball for his birthday. She packs the ball in a cubic box with side length 24 in. The diameter of the soccer ball is 22 in. She needs to determine how much space needs to be filled with packing material. What is the volume of the box and the ball? What is the remaining volume of the box that needs to be filled with packing material? Use 3.14 for $\pi$, and round the answer to the nearest hundredth.



Figure 4.3.5: Box and Ball

**Answer**

Volume of the ball

$$\begin{aligned}\text{Volume of a ball} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \cdot 22^3 \\ &= 5575.45\pi \text{ in}^3\end{aligned}$$

Volume of the box

$$\begin{aligned}\text{Volume of a box} &= a^3 \\ &= 24^3 \\ &= 13,824\pi \text{ in}^3\end{aligned}$$

$$\text{Volume of the remaining for packaging material} = 13,824 - 5,575.45 = 8,251.55 \text{ in}^3$$

## Your Turn 4.3.5: Cylinder and Ball



Consider the canister of tennis balls. Each ball has a 2.6 inches diameter. Round to two decimal places as needed.

What is the volume of the empty space in the canister? Use $\pi = 3.14$.

cubic inches



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**Example 4.3.6: Volume of Cylinder**

Brianna bought a can of soup. She measured its dimensions and recorded them as shown in the image above. How much soup can the can hold? One cubic inch is approximately equal to $0.554$ fluid ounces. How many fluid ounces of soup does the can hold?

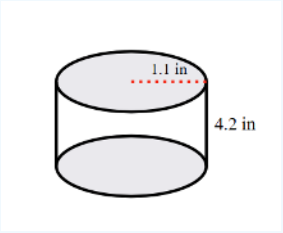



Figure 4.3.6: Cylinder

**Answer**

Calculate the volume, using $\pi = 3.14$ as an approximation for $\pi$.

$$\begin{aligned} V &= \pi \times r^2 \times h \\ &= 3.14 \times 1.1^2 \times 4.2 \\ &= 15.96 \text{ in}^3 \end{aligned}$$

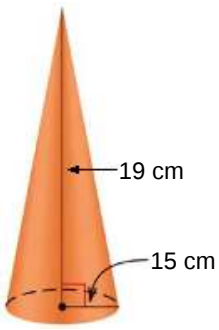
Now convert $15.96 \text{ in}^3$ to fluid ounce using $0.554 \text{ fluid ounces} = 1 \text{ in}^3$

$$\begin{aligned} 15.96 \text{ in}^3 &= \frac{15.96 \cancel{\text{ in}^3}}{1} \times \frac{0.554 \text{ fluid ounces}}{1 \cancel{\text{ in}^3}} \\ &= \frac{(15.96 \times 0.554) \text{ fluid ounces}}{1} \\ &= 8.84 \text{ fluid ounces} \end{aligned}$$

Now you try to find the volume for another solid, called a cone.

**Your Turn 4.3.6: Find the Volume of Cone**

**The figure is not drawn to scale.**




What is the volume of the cone?

$cm^3$

Round your answer to at least 2 decimal places.

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**Example 4.3.7: Can of Dog Food**

Your co-worker has designed a can of dog food that has a height of **10.1** cm and a **diameter** of **8.6** cm. Your job is to determine the area of the paper label that will cover the lateral area of the can. Because the ends of the paper label will overlap, assume that you will need the amount of paper to be **10%** more than the lateral surface area of the can. What is the area of one paper label? Round to the nearest square centimeter.



Figure 4.3.7: Can of Dog Food

**Answer**

$$\begin{aligned}\text{Lateral surface area of the can} &= 2\pi rh \\ &= 2 \cdot 3.14 \cdot 4.3 \cdot 10.1 \\ &= 272.74 \text{ cm}^2\end{aligned}$$

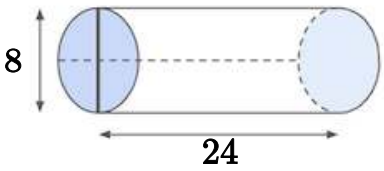
Since we need **10%** more than the lateral surface area. We need $0.10 \cdot 272.74 = 27.2$ more than $272.74$.

The area of the paper needed is

$$272.74 + 27.27 = 300.01 = 300 \text{ cm}^2$$

**Your Turn 4.3.7: Volume and Surface Area of Cylinder**

Find the surface area and volume of the cylinder.




Round answer to 2 decimal places if needed. Use $\pi = 3.14$

**Area of the 2 Bases together** =

**Lateral Surface Area** =

**Total Surface Area** =

**Volume** =

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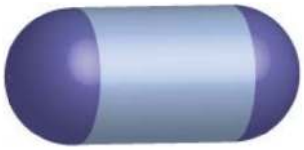
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## Composite Solids

Composite geometric solids are made from two or more geometric solids. You can find the volume of these solids as well, as long as you are able to figure out the individual solids that make up the composite shape.

Look at the image of a capsule below. Each end is a half-sphere. You can find the volume of the solid by taking it apart. What solids can you break this shape into?



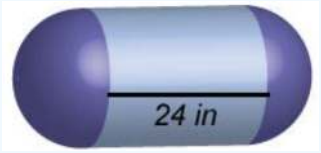
You can break it into a cylinder and two half-spheres.



Two half-spheres form a whole one, so if you know the volume formulas for a cylinder and a sphere, you can find the volume of this capsule.

**Example 4.3.8: Composite Solid**

If the radius of the spherical ends is **6** inches, find the volume of the solid below. Use **3.14** for $\pi$. Round your final answer to the nearest whole number.




**Answer**

Identify the composite solids. This capsule can be thought of as a cylinder with a half-sphere on each end.




Identify the proper formulas to use.

$$\text{Volume of a cylinder: } \pi \cdot r^2 \cdot h$$

$$\text{Volume of a sphere: } \frac{4}{3}\pi r^3$$

The height of a cylinder refers to the section between the two circular bases. This dimension is given as **24** inches, so $h = 24$. The radius of the sphere is **6** inches. You can use $r = 6$ in both formulas. Substitute the dimensions into the formulas.

For Cylinder

$$\begin{aligned}\text{Volume of a cylinder} &= \pi \cdot 6^2 \cdot 24 \\ &= \pi \cdot 36 \cdot 24 \\ &= 864\pi\end{aligned}$$

For Sphere (Two Hemisphere)

$$\begin{aligned}\text{Volume of a sphere} &= \frac{4}{3}\pi \cdot 6^3 \\ &= \frac{4}{3}\pi \cdot 216 \\ &= 288\pi\end{aligned}$$

The total exact volume is

$$\text{Volume of capsule} = 864\pi + 288\pi = 1152\pi$$

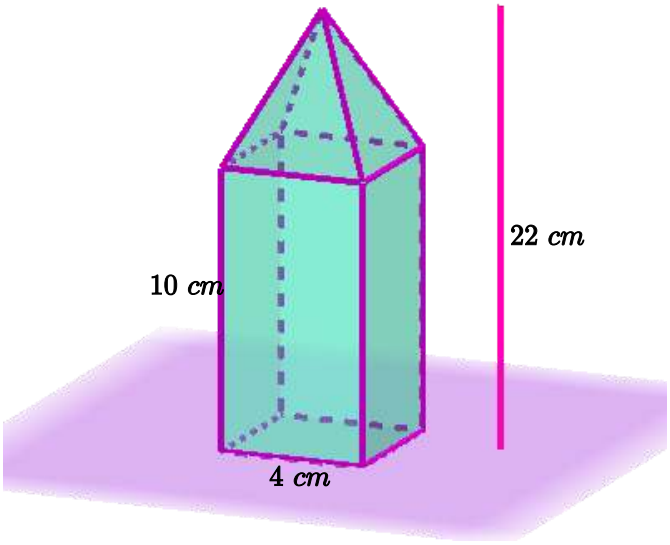
The total approximate volume is

$$\text{Volume of capsule} = 1152 \times 3.14 \approx 3617.28$$

**Your Turn 4.3.8: Find the Volume of Composite Solid**

Part 1 of 3

Find the volume of the composite figure below. Note that this figure has a square base.




Calculate the volume of a rectangular solid (VR).

$VR =$ _____ $cm^3$

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**Example 4.3.8: Composite Solid**

This figure is comprised of a hemisphere, cylinder, and cone. The lengths shown are each the vertical parts of each figure. Find the total surface area and the volume of the figure. Write your answer in term of $\pi$. Note that dimensions are given in inches.

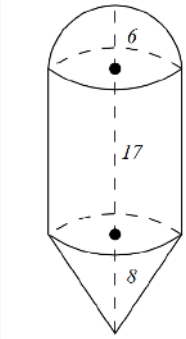



Figure 4.3.1: Copy and Paste Caption here. (Copyright; author via source)

**Answer**

$$\begin{aligned}\text{Volume of a cylinder} &= \pi r^2 h \\ &= \pi \cdot 6^2 \cdot 17 \\ &= 612\pi \text{ in}^3\end{aligned}$$

$$\begin{aligned}\text{Volume of a hemisphere} &= \frac{4}{3}\pi r^3 \div 2 \\ &= \frac{4}{3}\pi \cdot 6^3 \div 2 \\ &= 144\pi \text{ in}^3\end{aligned}$$

$$\begin{aligned}\text{Volume of a Cone} &= \frac{1}{3}\pi r^2 \cdot h \\ &= \frac{1}{3}\pi 6^2 \cdot 8 \\ &= 96\pi \text{ in}^3\end{aligned}$$

So **Volume of Figure:** $144\pi + 612\pi + 96\pi = 852\pi$

$$\begin{aligned}\text{Surface area of cylinder} &= 2\pi r h \\ &= 2\pi \cdot 6 \cdot 17 \\ &= 204\pi \text{ in}^2\end{aligned}$$

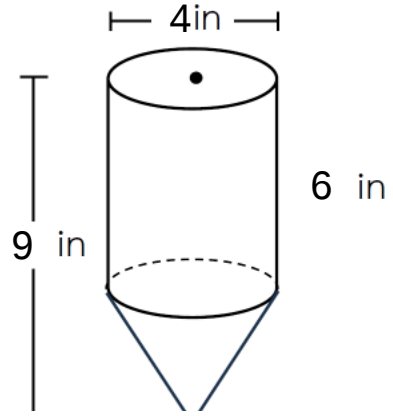
$$\begin{aligned}\text{Surface area of a hemisphere} &= 4\pi r^2 \div 2 \\ &= 4\pi \cdot 6^2 \div 2 \\ &= 72\pi \text{ in}^2\end{aligned}$$

$$\begin{aligned}\text{Surface area of a Cone} &= \pi r \sqrt{r^2 + h^2} \\ &= \pi \cdot 6 \cdot \sqrt{6^2 + 8^2} \\ &= \pi \cdot 6 \cdot \sqrt{100} \\ &= 60\pi \text{ in}^2\end{aligned}$$

So **Total Surface area of the Figure:** $204\pi + 72\pi + 60\pi = 336\pi$

**Your Turn 4.3.9: Composite Solid**

Find the volume of the figure.



Volume = _____ $in^3$

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# 4.4: Similar Triangles and Pythagorean Theorem



Figure 4.4.1: In the lower left corner of the fresco *The School of Athens* by Raphael, the figure in white writing in the book represents Pythagoras. Alongside him, to the right, the figure with the long, light-brown hair is said to depict Archimedes. (credit: modification of work "School of Athens" by Raphael ( 1483–1520), Vatican Museums/Wikimedia, Public Domain)

This is another excerpt from Raphael's *The School of Athens.* The man writing in the book represents Pythagoras, the namesake of one of the most widely used formulas in geometry, engineering, architecture, and many other fields, the Pythagorean Theorem. However, there is evidence that the theorem was known as early as 1900–1100 BC by the Babylonians. The Pythagorean Theorem is a formula used for finding the lengths of the sides of right triangles.

Born in Greece, Pythagoras lived from 569–500 BC. He initiated a cult-like group called the Pythagoreans, which was a secret society composed of mathematicians, philosophers, and musicians. Pythagoras believed that everything in the world could be explained through numbers. Besides the Pythagorean Theorem, Pythagoras and his followers are credited with the discovery of irrational numbers, the musical scale, the relationship between music and mathematics, and many other concepts that left an immeasurable influence on future mathematicians and scientists.

The focus of this section is on right triangles. We will look at how the Pythagorean Theorem is used to find the unknown sides of a right triangle, and we will also study the *special* triangles, those with set ratios between the lengths of sides. By ratios we mean the relationship of one side to another side. When you think about ratios, you should think about fractions. A fraction is a ratio, the ratio of the numerator to the denominator. Finally, we will preview trigonometry. We will learn about the basic trigonometric functions, sine, cosine and tangent, and how they are used to find not only unknown sides but unknown angles, as well, with little information.

## Pythagorean Theorem

The Pythagorean Theorem is used to find unknown sides of right triangles. The theorem states that the sum of the squares of the two legs of a right triangle equals the square of the hypotenuse (the longest side of the right triangle).

**Formula: Pythagorean Theorm**

The Pythagorean Theorem states

$$a^2 + b^2 = c^2$$

where $a$ and $b$ are two sides (legs) of a right triangle and $c$ is the hypotenuse, as shown in Figure 4.4.2. Note that $c$ is longer than legs $a$ and $b$.

So we do have a formula for one of the sides if the other two are known. The following three formulas can be obtained from $a^2 + b^2 = c^2$.

$$c = \sqrt{a^2 + b^2}$$
$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$

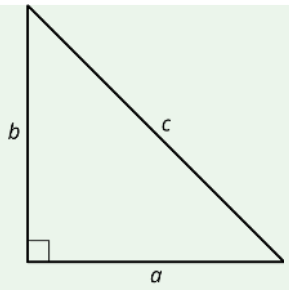



Figure **4.4.2**: Pythagorean Theorem for Right Triangle

## Example 4.4.1: Using the Pythagorean Theorem

Find the length of the missing side of the triangle in Figure **4.4.3**.

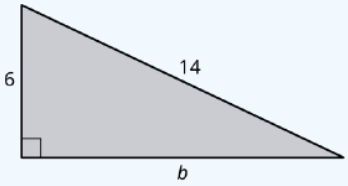



Figure **4.4.3**

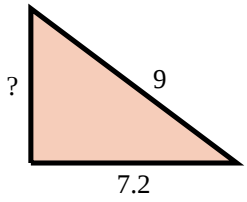
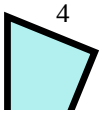
**Answer**

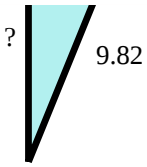
So we do have a formula for one of the sides if the other two are known

$$\begin{aligned} b &= \sqrt{c^2 - a^2} \\ &= \sqrt{14^2 - 6^2} \\ &= \sqrt{196 - 36} \\ &= \sqrt{160} \\ &= \sqrt{16 \times 10} \\ &= 4\sqrt{10} \quad \text{Exact answer} \\ &= 12.65 \quad \text{Approximate answer} \end{aligned}$$

.

## Your Turn 4.4.1: Find Missing Sides

| **Finding a Missing Side Using the Pythagorean Theorem** | |
|---|---|
| Use the diagrams to find the missing sides of the right triangles. Round your answer to one decimal place. | |
| **Figure A**<br>  <br> | **missing side** = _____ |
| **Figure D**<br>  <br> | |




missing side = _____

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## Example 4.4.2: Calculating Actual Distance

You live on the corner of First Street and Maple Avenue, and work at Star Enterprises on Tenth Street and Elm Drive (Figure ="lt-icon-d4.4.4 ). You want to calculate how far you walk to work every day and how it compares to the actual distance. Each block measures 200 ft by 200 ft.

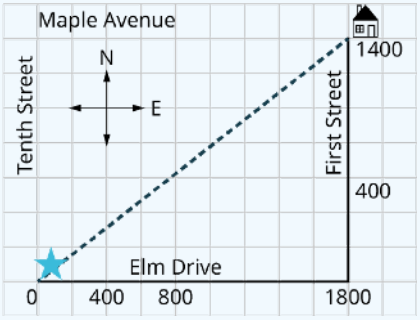



Figure 4.4.4

**Answer**

You travel 7 blocks south and 9 blocks west. If each block measures 200 ft by 200 ft, so everyday walk will be

$$9(200) + 7(200) = 1,800 \text{ ft} + 1,400 \text{ ft} = 3,200 \text{ ft}$$

The actual distance can be found using the Pythagorean theorem

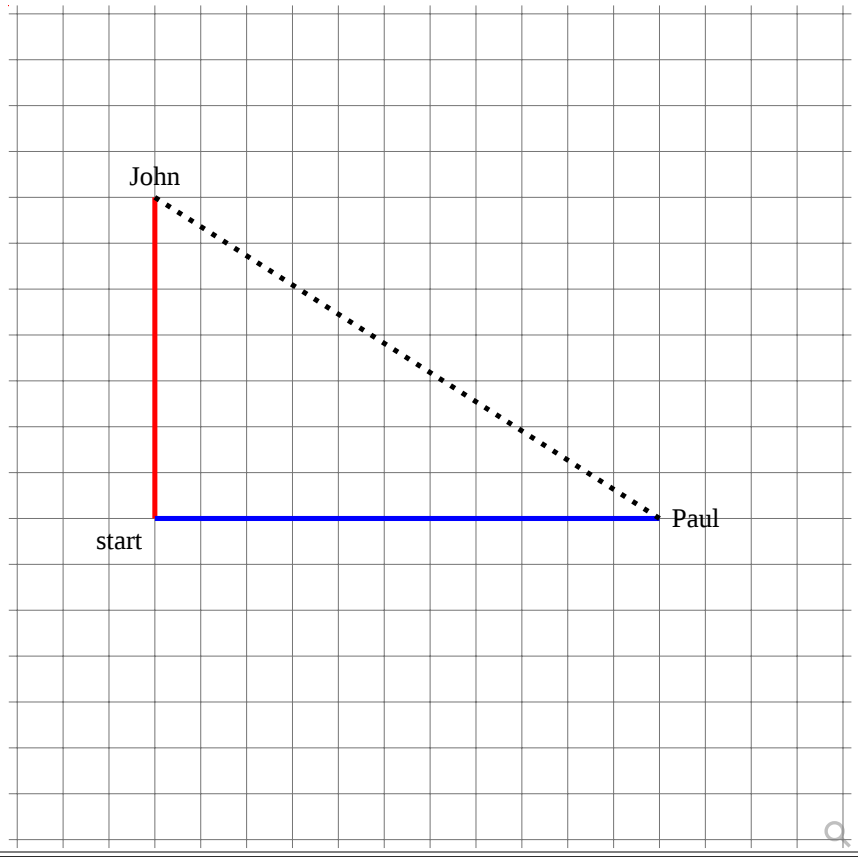
$$\begin{aligned} d &= \sqrt{(1,800)^2 + (1,400)^2} \\ &= \sqrt{3,240,000 + 1,960,000} \\ &= \sqrt{5,200,000} \\ &= 2280.4 \text{ ft} \end{aligned}$$

## Your Turn 4.4.2: Find Hypotenous

| Solving Applications Using the Pythagorean Theorem |
| --- |
| John and Paul are going to survey a lot for building a split-level home. They start off in the same spot and walk in perpendicular directions to set their stakes.<br><br>**Figure A** shows their paths and the locations where they each set their stake. |

( ***Note:*** The small squares in the grid have side lengths of one yard and areas of 1 square yard.)

**Figure A**




The red vertical line represents the path John walked.

How far did John walk before he drove in his stake? _____ **yards**

The blue horizontal line represents the path Paul walked.

How far did Paul walk before he drove in his stake? _____ **yards**

What is the distance between the spot where John planted his stake and the spot where Paul planted his stake? _____ **yards**

***Round your result to two decimal places as needed.***

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**Example 4.4.3: Find TV Size**

TV screens are measured on the diagonal. If we have a TV-cabinet that is **76** inches long and **64** inches high, how large a TV could we put in the space (leave **2**-inches on all sides for the edging of the TV)? Round your answer to the nearest hundredth.

**Answer**

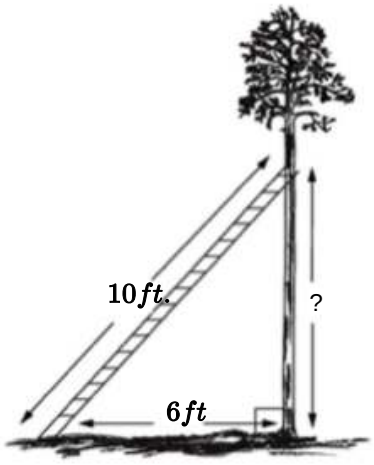
If we leave a space of **2** inches on all sides, the length of the TV must be $76-4=72$ inches and the width must be $64-4=60$ inches. So to find the size of the TV, we need to find the diagonal of the TV, which is the hypotenuse of a right triangle with legs **72** inches and **60** inches.

$$\begin{aligned} c &= \sqrt{a^2+b^2} \\ &= \sqrt{72^2+60^2} \\ &= \sqrt{5184+3600} \\ &= \sqrt{8784} \\ &= 93.72 \end{aligned}$$

So **93.72** inch TV can be placed in the cabinet.

**Your Turn 4.4.3: Find the Length of a Ladder**

A $10$ -foot ladder is leaning on a tree. The bottom of the ladder on the ground at a distance of $6$ feet from the base of the tree. The base of the tree and the ground form a right angle as shown.




[ *not drawn to scale* ]

What is the distance, in feet, between the ground and the top of the ladder?
Round your answer to the nearest hundredth.

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**People in Mathematics: Pythagoras and the Pythagoreans**

The Pythagorean Theorem is so widely used that most people assume that Pythagoras (570–490 BC) discovered it. The philosopher and mathematician uncovered evidence of the right triangle concepts in the teachings of the Babylonians dating around 1900 BC. However, it was Pythagoras who found countless applications of the theorem leading to advances in geometry, architecture, astronomy, and engineering.

Among his accolades, Pythagoras founded a school for the study of mathematics and music. Students were called the Pythagoreans, and the school's teachings could be classified as a religious indoctrination just as much as an academic experience. Pythagoras believed that spirituality and science coexist, that the intellectual mind is superior to the senses, and that intuition should be honored over observation.

Pythagoras was convinced that the universe could be defined by numbers, and that the natural world was based on mathematics. His primary belief was *All is Number*. He even attributed certain qualities to certain numbers, such as the number 8 represented justice and the number 7 represented wisdom. There was a quasi-mythology that surrounded Pythagoras. His followers thought that he was more of a spiritual being, a sort of mystic that was all-knowing and could travel through time and space. Some believed that Pythagoras had mystical powers, although these beliefs were never substantiated.

Pythagoras and his followers contributed more ideas to the field of mathematics, music, and astronomy besides the Pythagorean Theorem. The Pythagoreans are credited with the discovery of irrational numbers and of proving that the morning star was the planet Venus and not a star at all. They are also credited with the discovery of the musical scale and that different strings made different sounds based on their length. Some other concepts attributed to the Pythagoreans include the properties relating to triangles other than the right triangle, one of which is that the sum of the interior angles of a triangle equals $180°$. These geometric principles, proposed by the Pythagoreans, were proven 200 years later by Euclid.

The Pythagorean Theorem is perhaps one of the most useful formulas you will learn in mathematics because there are so many applications of it in real world settings. Architects and engineers frequently use this formula when designing ramps, bridges, and buildings.

## Similar Triangles

Two triangles are said to be similar if they have equal sets of angles. In Figure 4.4.5, $\triangle ABC$ is similar to $\triangle DEF$. The angles that are equal are called **corresponding angles**. In figure 4.4.5,

$$\angle A \text{ corresponds to } \angle D$$
$$\angle B \text{ corresponds to } \angle E$$
$$\angle C \text{ corresponds to } \angle F$$

The sides joining corresponding vertices are called **corresponding sides**. In figure \(\PageIndex{5}\)

$$AB \text{ corresponds to } DE$$
$$BC \text{ corresponds to } EF$$
$$AC \text{ corresponds to } DF$$

**Symbol for Similar**

The symbol for similar is $\sim$. The similarity statement $\triangle ABC \sim \triangle DEF$ will always be written so that corresponding vertices appear in the same order.

For the triangles in Figure 4.4.5, we could also write

$$\triangle BAC \sim \triangle BDF$$
$$\triangle ACB \sim \triangle DFE$$

But never

$$\triangle ABC \sim \triangle EDF$$
$$\triangle ACB \sim \triangle DEF$$

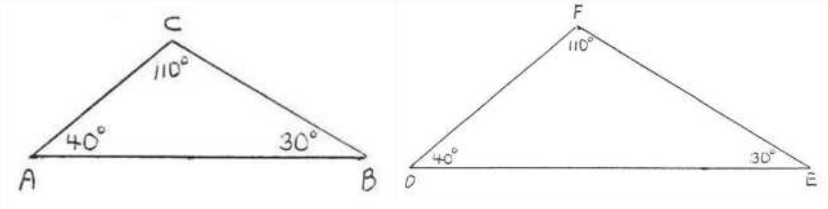



Figure 4.4.5: $\triangle ABC$ is similar to $\triangle DEF$.

We can determine which sides correspond based on the similarity statement. For example, if $\triangle ABC \sim \triangle DEF$, then side $AB$ corresponds to side $DE$ because both are the first two letters of each triangle. $BC$ corresponds to $EF$ because both are the last two letters of a triangle, and $AC$ corresponds to $DF$ because both consist of the first and last letters.

## AAA Rule for Similarity

If two triangles have all three corresponding angles equal, then the triangles are similar.

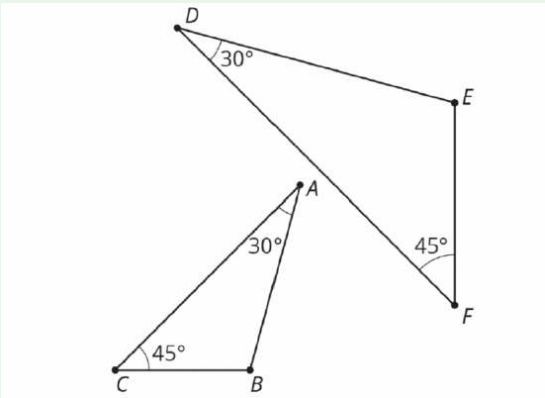



Figure 4.4.6: AAA Similarty

$\triangle ABC \sim \triangle DEF$ if

$$\angle A = \angle D \text{ and } \angle B = \angle D = 60°$$

We see that the above triangles are similar because

$$\angle B = 180° - (45° + 30°) = 180° - 75° = 105° = \angle E$$

## SSS Rule for Similarity

Two triangles are similar if either their corresponding sides are proportional.

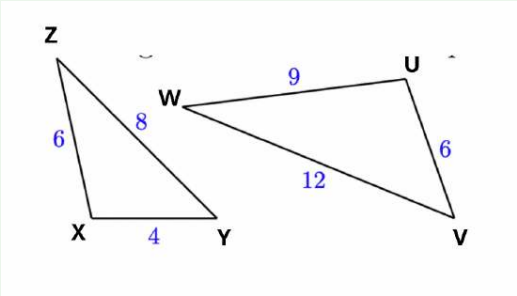



Figure 4.4.7: AAA Similarity

$\triangle UVW \sim \triangle XYZ$ if

$$\frac{\overline{UV}}{\overline{XY}} = \frac{\overline{VW}}{\overline{YZ}} = \frac{\overline{WU}}{\overline{ZX}}$$

We can see that $\triangle UVW \sim \triangle XYZ$. Substitute the side lengths into the ratios, and determine if the ratios of the corresponding sides are equivalent. They are, so the triangles are similar.

$$\frac{6}{4} = \frac{12}{8} = \frac{9}{6}$$

$$1.5 = 1.5 = 1.5$$

**SAS Rule for Similarity**

Two triangles are similar if the ratio of two pairs of corresponding sides is equal and the angles between those sides (the included angles) are congruent (equal).

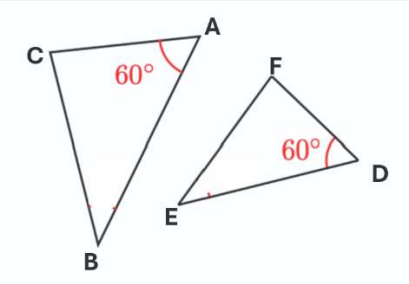



Figure 4.4.8: SAS Similarity

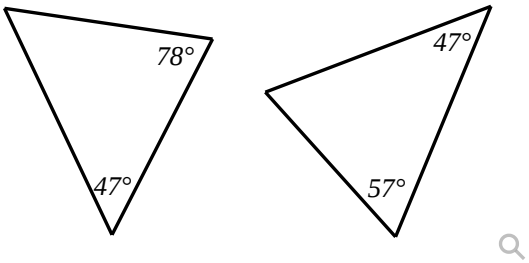
$\triangle ABC \sim \triangle DEF$ if

$$\frac{AC}{DF} = \frac{AB}{DE} \text{ and } \angle A = \angle D = 60°$$

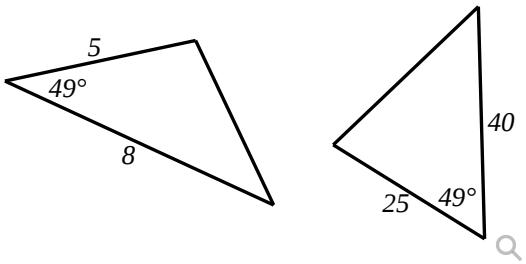
**Your Turn 4.4.4: Determine if Two Triangles are Similar**

For each pair of triangles, determine whether they are similar and by which theorem:

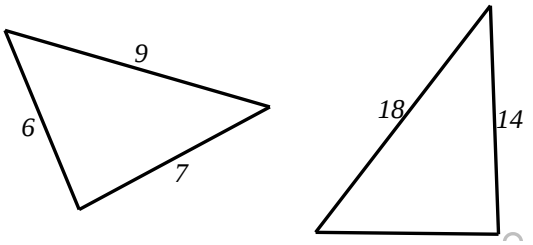
These triangles are...




- ☐ Similar by AAA
- ☐ Not similar by AAA failure
- ☐ Similar by SAS
- ☐ Not similar by SAS failure
- ☐ Similar by SSS
- ☐ Not similar by SSS failure




- ☐ Similar by AAA
- ☐ Not similar by AAA failure
- ☐ Similar by SAS
- ☐ Not similar by SAS failure
- ☐ Similar by SSS
- ☐ Not similar by SSS failure




- ☐ Similar by AAA
- ☐ Not similar by AAA failure
- ☐ Similar by SAS
- ☐ Not similar by SAS failure
- ☐ Similar by SSS
- ☐ Not similar by SSS failure

12

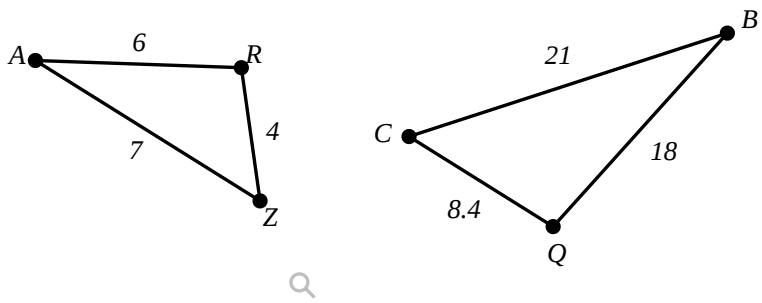
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**Your Turn 4.4.5: Determine if the Two Triangles are Similar**

Part 1 of 3

Given the two triangles below, answer the following questions. *Note the triangles are not to scale*




If any number in the diagram isn't fully visible, click the magnifying glass in the bottom right corner of that diagram for clarification.

What similar triangle criterion will be the most useful to prove these triangles are similar?

- ○ SAS
- ○ SSS
- ○ AAA

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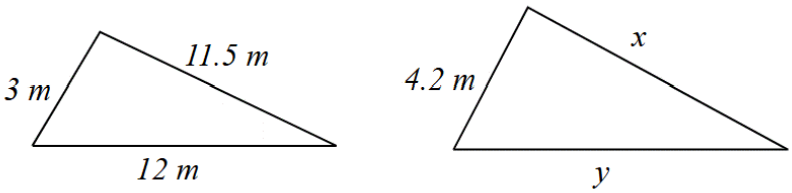
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## Finding Missing Measurements in Similar Triangles

You can find the missing measurements in a triangle if you know some measurements of a similar triangle. Let's look at an example.

**Example 4.4.6: Find Unknown Side**

Find the unknown lengths for the similar triangles below.




**Answer**

In similar triangles, the ratios of corresponding sides are proportional. Set up a proportion of two ratios, one that includes the missing side.

$$\frac{3}{4.2} = \frac{12}{y} = \frac{11.5}{x}$$

Choose the first and second ratios to solve for $y$

$$\frac{3}{4.2} = \frac{12}{y}$$

Solve for $y$using cross multiplication.

$$\begin{aligned} 3 \times y &= 4.2 \times 12 \\ 3y &= 50.4 \\ y &= \frac{50.4}{3} \\ y &= 16.8 \end{aligned}$$

Choose the first and third ratios to solve for $x$

$$\frac{3}{4.2} = \frac{11.5}{x}$$

Solve for $x$using cross multiplication.

$$\begin{aligned} 3 \cdot x &= 11.5 \cdot 4.2 \\ 3x &= 48.3 \\ x &= \frac{48.3}{3} \\ x &= 16.1 \end{aligned}$$

This process is fairly straightforward—but be careful that your ratios represent corresponding sides, recalling that corresponding sides are opposite corresponding angles.

**Your Turn 4.4.6: Find Unknown Side**

The triangles are similar. Find the value for $x$, and include the correct units.

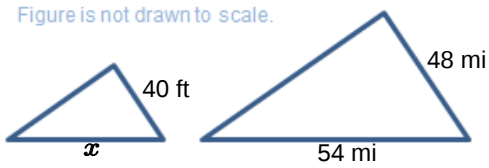



Image Description

$x =$ _____

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## Solving Application Problems Involving Similar Triangles

Applying knowledge of triangles, similarity, and congruence can be very useful for solving problems in real life. Just as you can solve for missing lengths of a triangle drawn on a page, you can use triangles to find unknown distances between locations or objects.

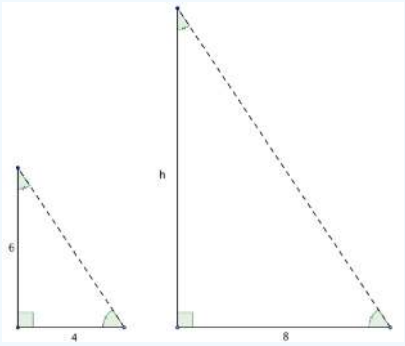
Let's consider the example of two trees and their shadows. Suppose the sun is shining down on two trees, one that is 6 feet tall and the other whose height is unknown. By measuring the length of each shadow on the ground, you can use triangle similarity to find the unknown height of the second tree.

First, let's figure out where the triangles are in this situation. The trees themselves create one pair of corresponding sides. The shadows cast on the ground are another pair of corresponding sides. The third side of these imaginary similar triangles runs from the top of each tree to the tip of its shadow on the ground. This is the hypotenuse of the triangle.

If you know that the trees and their shadows form similar triangles, you can set up a proportion to find the height of the tree.

### Example 4.4.7: Find Height of Tree

When the sun is at a certain angle in the sky, a **6**-foot tree will cast a **4**-foot shadow. How tall is a tree that casts an **8**-foot shadow?




**Answer**

The angle measurements are the same, so the triangles are similar triangles. Since they are similar triangles, you can use proportions to find the size of the missing side. Set up a proportion comparing the heights of the trees and the lengths of their shadows.

$$\frac{\text{Height of Tree 1}}{\text{Height of Tree 2}} = \frac{\text{Length of Shadow of Tree 1}}{\text{Length of Shadow of Tree 2}}$$

Substitute in the known lengths. Call the missing tree height $h$.

$$\frac{6}{h} = \frac{4}{8}$$

Solve for $h$ using cross-multiplication.

$$6 \cdot 8 = 4h$$

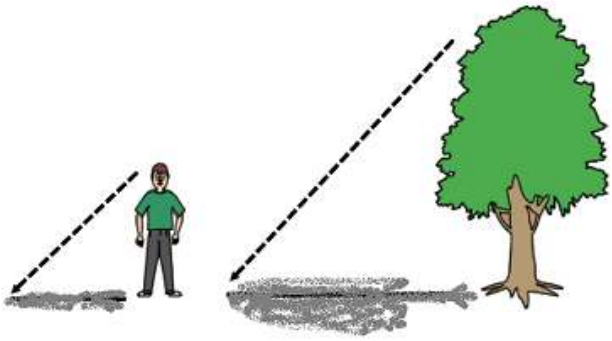
$$48 = 4h$$

$$12 = h$$

The tree is **12** feet tall.

## Your Turn 4.4.7: Find the Height of a Tree

Ranger Ben is trying to find the height of a tree in the forest. At a certain time of day he notices that his shadow is $14.5$ ft long while the tree's shadow is $21$ ft long. If Ben is $6.25$ ft tall, use these measurements to find the height of the tree.



NOTE: figure is not necessarily drawn to scale!

Height of tree: [ ] [?]

If necessary, round to 2 decimal places!

---

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## Example 4.4.8: Find the Length of Shadow

If a **30**-meter flagpole casts a shadow that is **50** meters long, how long is the shadow cast by a tree that is **36** meters high? Ans 60 meters

**Answer**

$$\frac{\text{flagpole height}}{\text{tree height}} = \frac{\text{flagpole shadow}}{\text{tree shadow}}$$

Substitute in the known lengths. Call the missing tree shadow, $l$.

$$\frac{30}{36} = \frac{50}{l}$$

Solve for $h$ using cross-multiplication.

$$50 \cdot 36 = 30l$$

$$180 = 30l$$

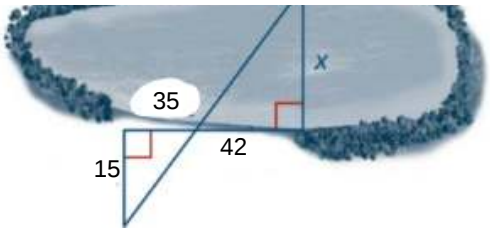
$$60 = l$$

So **36** meters high tree cast **60** meter shadow.

## Your Turn 4.4.8: Find the Legth of a Lake

Find the distance $x$ across the lake. The figure is not drawn to scale. Distances are given in meters. Type a whole number answer.






$x =$ _____ meter

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# CHAPTER OVERVIEW

## 5: BASIC OF STATISTICS

Like most people, you probably feel that it is important to "take control of your life," but what does this mean? Partly, it means being able to properly evaluate data and claims that bombard you every day. If we cannot distinguish good reasoning from faulty reasoning, then we are vulnerable to manipulation and decisions that are not in our best interest. Statistics provides tools that we need in order to react intelligently to information we hear or read. In this sense, statistics is one of the most important things that we can study.

To be more specific, here are some claims that we have heard on several occasions. (We are not saying that each one of these claims is true!)

- 4 out of 5 dentists recommend Dentyne.
- Almost 85% of lung cancers in men and 45% in women are tobacco-related.
- Condoms are effective 94% of the time.
- Native Americans are significantly more likely to be hit crossing the streets than are people of other ethnicities.
- People tend to be more persuasive when they look others directly in the eye and speak loudly and quickly.
- Women make 75 cents to every dollar a man makes when they work the same job.
- A surprising new study shows that eating egg whites can increase one’s life span.
- People predict that it is very unlikely there will ever be another baseball player with a batting average over 400.
- There is an 80% chance that in a room full of 30 people that at least two people will share the same birthday.
- 79.48% of all statistics are made up on the spot.

All of these claims are statistical in character. We suspect that some of them sound familiar; if not, we bet that you have heard other claims like them. Notice how diverse the examples are; they come from psychology, health, law, sports, business, etc. Indeed, data and datainterpretation show up in discourse from virtually every facet of contemporary life.

Statistics are often presented in an effort to add credibility to an argument or advice. We can see this by paying attention to television advertisements. Many of the numbers thrown about in this way do not represent careful statistical analysis. They can be misleading, and push consumers into decisions that might find cause to regret. For these reasons, learning about statistics is a long step towards taking “control of your life.” (It is not, of course, the only step needed for this purpose.) These next two chapters will help you learn statistical essentials. It will make you into an intelligent consumer of statistical claims.

You can take the first step right away. To be an intelligent consumer of statistics, your first reflex must be to question the statistics that you encounter. The British Prime Minister Benjamin Disraeli famously said, "There are three kinds of lies -- lies, damned lies, and statistics." This quote reminds us why it is so important to understand statistics. So, let us invite you to reform your statistical habits from now on. No longer will you blindly accept numbers or findings. Instead, you will begin to think about the numbers, their sources, and most importantly, the procedures used to generate them.

We have put the emphasis on defending ourselves against fraudulent claims wrapped up as statistics. Just as important as detecting the deceptive use of statistics is the appreciation of the proper use of statistics. You must also learn to recognize statistical evidence that supports a stated conclusion. When a research team is testing a new treatment for a disease, statistics allows them to conclude based on a relatively small trial that there is good evidence their drug is effective. Statistics allowed prosecutors in the 1950’s and 60’s to demonstrate racial bias existed in jury panels. Statistics are all around you, sometimes used well, sometimes not. We must learn how to distinguish the two cases.


5.1: Samples and Polulations

5.2: Data Type and Study Type

5.3: Frequency Tables and Histograms

5.4: Understanding Statistical Graphs and Tables

5.5: Measures of Center and Spread

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## 5.1: Samples and Polulations

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- People tend to be more persuasive when they look others directly in the eye and speak loudly and quickly.
- Women make 75 cents to every dollar a man makes when they work the same job.
- A surprising new study suggests that consuming egg whites may increase one's lifespan.
- People predict that it is very unlikely there will ever be another baseball player with a batting average over 400.
- There is a 70% chance that in a room full of 30 people, at least two people will share the same birthday.
- 79.48% of all statistics are made up on the spot.

Before going further, however, two distinct meanings of the word “statistics” should be clarified. First, “statistics” could mean the academic discipline, a field of study, involving the collection, presentation, interpretation, and use of data. When used in this sense, the word is considered singular (though it looks plural). Second, in a narrower sense, a number collected in a survey or study is referred to as a “statistic.” For instance, the average height of a sample of 100 people would be a (one) statistic. This is a singular word (of course, two or more of these numbers would be referred to as “statistics”). Be aware of the difference when you encounter this word.

All of these claims are statistical in character. We suspect that some of them sound familiar; if not, we bet that you have heard other claims like them. Notice how diverse the examples are; they come from psychology, health, law, sports, business, etc. Indeed, data and data-interpretation show up in discourse from virtually every facet of contemporary life.

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Before we begin gathering and analyzing data, we need to characterize the **population** we are studying. If we want to study the amount of money spent on textbooks by a typical first-year college student, our population might be all first-year students at your college. Or it might be the following:

- All first-year community college students in the state of Washington.
- All first-year students at public colleges and universities in the state of Washington.
- All first-year students at all colleges and universities in the state of Washington.
- All first-year students at all colleges and universities in the entire United States.
- And so on.

**Definition: Individual**

In statistics, an individual is a single object or member of a population or sample that is being studied.

If a study records the heights of 30 students, **each student is an individual**, and **height** is a variable measured on those individuals.

**Definition: Population**

In statistics, a population is the entire group of individuals or items that we want to study or draw conclusions about.

The population includes all individuals of interest, not just some of them.

Sometimes, the intended population is referred to as the **target population.** If we design our study poorly, the collected data may not accurately represent the intended population.

Why is it important to specify the population? We might get different answers to our question as we vary the population we are studying. First-year students at the University of Washington might take slightly more diverse courses than those at your college, and some of these courses may require less popular textbooks that cost more; or, on the other hand, the University Bookstore might have a larger pool of used textbooks, reducing the cost of these books to the students. Whichever the case (and it is likely that some combination of these and other factors is in play), the data we gather from your college will probably not be the same as that from the University of Washington. Particularly when conveying our results to others, we want to be clear about the population we are describing with our data.

### Example 5.1.1: Describe Population

A newspaper website contains a poll asking people their opinion on a recent news article. What is the population?

**Answer**

While the target (intended) population may have been all people, the real population of the survey is readers of the website

If we were able to gather data on every member of our population, say the average (we will define "average" more carefully in a subsequent section) amount of money spent on textbooks by each first-year student at your college during the 2009-2010 academic year, the resulting number would be called a **parameter**.

#### Definition: Parameter

A **parameter** is a value (average, percentage, etc.) calculated using all the data from a population.

We seldom see parameters; however, since surveying an entire population is usually very time-consuming and expensive, unless the population is very small or we already have the data collected.

#### Definition: Census and Survey

In a **census**, we gather information from every member of a population, and a **census** is a survey of an entire population.
In a census, we gather information from every member of a population. In a **survey**, we gather information from a sample representing the population.

You are probably familiar with two common censuses: the official government Census that attempts to count the population of the U.S. every ten years, and voting, which asks the opinion of all eligible voters in a district. The first of these demonstrates one additional problem with a census: the difficulty in finding and getting participation from everyone in a large population, which can bias or skew the results.

There are occasional instances when a census is appropriate, typically when the population is relatively small. For example, if the manager of Starbucks wanted to know the average number of hours her employees worked last week, she should be able to pull up payroll records or ask each employee directly.

Since surveying an entire population is often impractical, we usually select a sample to study.

#### Definition: Sample

A **sample** is a smaller subset of the entire population, ideally one that is fairly representative of the whole population.

We discuss sampling methods in greater detail in a later section. For now, let us assume that samples are chosen in an appropriate manner. If we survey a sample, say 100 first-year students at your college, and find the average amount of money spent by these students on textbooks, the resulting number is called a statistic.

### Your Turn 5.1.1: Idenfity the Components of Statistical Study

In the following examples of statistical studies, identify the population of interest and provide the sample size.

1. An employee at the local ice cream parlor asks seven customers if they like chocolate ice cream.

   - Population: Select an answer
   - Sample size: _____

2. A sample of nine thousand randomly selected adults were asked whether they drink at least 48 oz of water each day and only 28% said 'yes'.

   - Population: Select an answer
   - Sample size:

3. In a poll of four hundred randomly selected college students, 58% answered "yes" when asked "Do you have a television in your dorm room?"

- Population: Select an answer
- Sample size:

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**Definition: Statistic**

A **statistic** is a value (average, percentage, etc.) calculated using the data from a sample.

**Note:** Compare the statistic with the parameter.

**Example 5.1.2: Parameter or Statistic**

A researcher wanted to know how citizens of Tacoma felt about a voter initiative. To study this, she goes to the Tacoma Mall and randomly selects 500 shoppers and asks them their opinion. 60% indicates they are supportive of the initiative. What is the sample and population? Is the 60%value a parameter or a statistic?

**Answer**

The sample is the 500 shoppers questioned. The population is less clear. While the intended population of this survey was Tacoma citizens, the effective population was mall shoppers. There is no reason to assume that the 500 shoppers questioned would be representative of all Tacoma citizens.

The 60% value was based on the sample, so it is a statistic.

**Your Turn 5.1.2: Identify the Components of Statistical Study**

In 2010, the Pew Research Center questioned 756 adults in the U.S. to estimate the proportion of the population favoring marijuana use for medical purposes. It was found that 80% are in favor of using marijuana for medical purposes. State the individual, variable, population, sample, parameter and statistic.

- Sample
- Population
- Variable
- Parameter
- Statistic
- Individual/Unit

a. All adults in the U.S.
b. The 80 % in favor of using marijuana in the U.S.
c. one adult in the U.S.
d. The 80 % in favor of using marijuana in the study.
e. Whether or not an adult favors marijuana use for medical purposes
f. The 756 adults in the U.S. surveyed

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## Types of Sampling Methods

As stated before, if you want to know something about a population, it is often impossible or impractical to examine the whole population. It might be too expensive in terms of time or money. It might be impractical – you can't test all batteries for their length of lifetime because there wouldn't be any batteries left to sell. You need to look at a sample. Hopefully, the sample behaves the same as the population.

When you choose a sample, you want it to be as similar to the population as possible. If you want to test a new painkiller for adults, you would want the sample to include people who are fat, skinny, old, young, healthy, not healthy, male, female, etc.

There are many ways to collect a sample. None are perfect, and you are not guaranteed to collect a representative sample. That is, unfortunately, the limitations of sampling. However, several techniques can result in samples that give you a semi-accurate picture of the population. Just remember to be aware that the sample may not be representative. As an example, you can take a random sample of a group of people that are equally males and females, yet by chance, everyone you choose is female. If this happens, it may be a good idea to collect a new sample if you have the time and money.

There are many sampling techniques, though only five will be presented here.

1. Simple Random Sample
2. Stratified Sample
3. Systematic Sample
4. Cluster Sample
5. Convenience Sample

## Simple Random Sample

**Definition: SRS**

A **simple random sample (SRS)** of size $n$ is a sample that is selected from a population in a way that ensures that every different possible sample of size $n$ has the same chance of being selected. Also, every individual associated with the population has the same chance of being selected

1. Put all names in a hat and draw a certain number of names out.
2. Assign each individual a number and use a random number table or a calculator, or a computer to randomly select the individuals that will be measured.

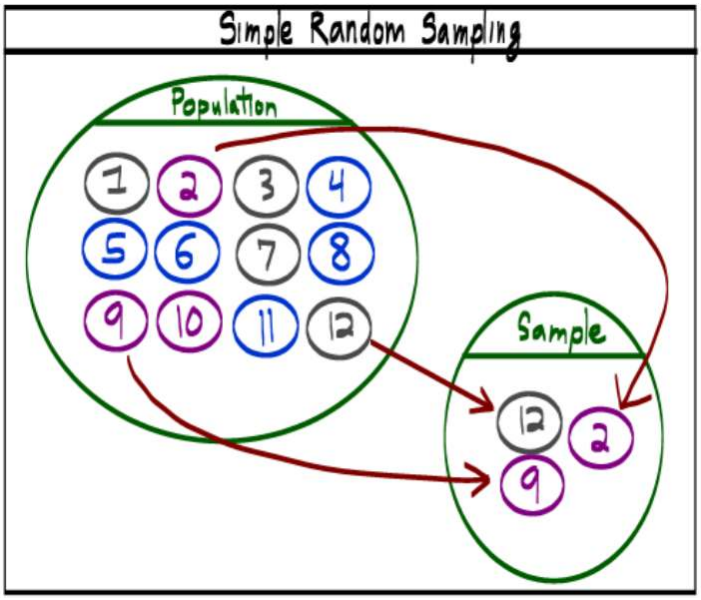



Figure 5.1.1: "Simple Random Sample Diagram" by Toros Berberyan(opens in new window) is licensed under CC BY-SA 4.0(opens in new window)

In the simple random sampling, there is an original population of 12 balls. We conduct a random sample by randomly selecting three balls from the population to be part of the sample. In this case, we have picked the ones that have the numbers 2, 9, and 12.

## Stratified Sampling

**Definition: Stratified Sampling**

**Stratified sampling** involves dividing the population into groups, called strata, and then selecting a simple random sample from each stratum.

1. If you want to look at musical preference, you could divide the individuals into age groups and then conduct simple random samples within each group.
2. To calculate the average price of textbooks, you could divide the individuals into groups by major and then conduct simple random samples within each group.

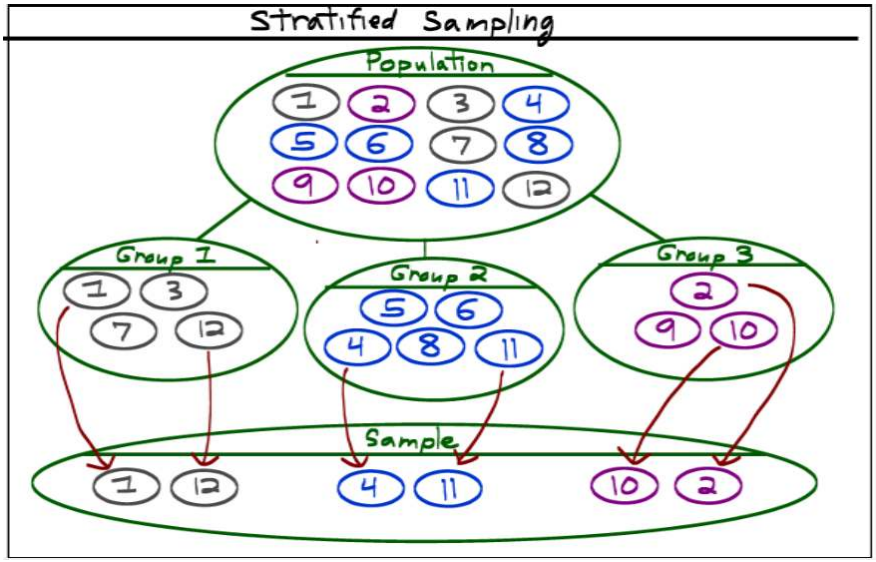



Figure 5.1.2: "Stratified Sampling Diagram" by Toros Berberyan(opens in new window) is licensed under CC BY-SA 4.0(opens in new window)

The illustration above is an example of a stratified sample. There is an original population of 12 balls. They are grouped based on a common characteristic, in this case, color. We have 4 dark grey balls in the first group, 5 blue balls in the second group, and 3 purple balls in the third group. From **all groups**, **some members** are randomly selected to be part of the sample. From the first group, the balls that are numbered 1 and 12 are selected to be part of the sample. From the second group, the balls that are numbered 4 and 11 are selected to be part of the sample. From the third group, the balls that are numbered 2 and 10 are selected to be part of the sample. Based on stratified sampling, we ended up with a sample of 6 balls numbered 1 , 2, 4 , 10 , 11 , and 12.

**Definition: Systematic Sampling**

**Systematic sampling** is where you randomly choose a starting place and then select every $k$th individual to measure.

1. You select every 5 $^{th}$ item on an assembly line
2. You select every 10 $^{th}$ name on the list
3. You select every 3 $^{rd}$ customer that comes into the store.

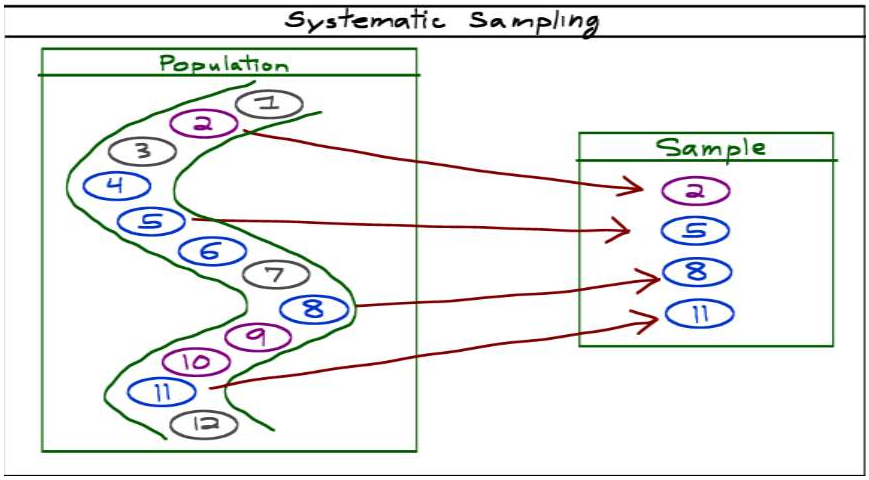



Figure 5.1.3: "Systematic Sampling Diagram" by Toros Berberyan(opens in new window) is licensed under CC BY-SA 4.0(opens in new window)

The diagram above represents a systematic sample. We have a population of 12 balls inside a curvy road. The first one is picked randomly. In this example, it will be the second one from above. Then, the rest of them are selected below the first one in a certain pattern. The diagram shows that every third one is selected to be part of the sample. Based on systematic sampling, the selected individuals in the sample are those numbered 2, 5, 8, and 11 .

**Definition : Cluster Sampling**

**Cluster sampling** involves dividing the population into groups, known as clusters. Randomly pick some clusters, then poll all individuals in those clusters.

1. A large city wants to poll all businesses in the city. They divide the city into sections (clusters), maybe a square block for each section, and use a random number generator to pick some of the clusters. Then, they poll all businesses in each chosen cluster.

2. You want to measure whether a tree in the forest is infected with bark beetles. Instead of having to walk all over the forest, you divide the forest into sectors and then randomly pick the sectors that you will travel to. Then record whether a tree is infected or not for every tree in that sector.

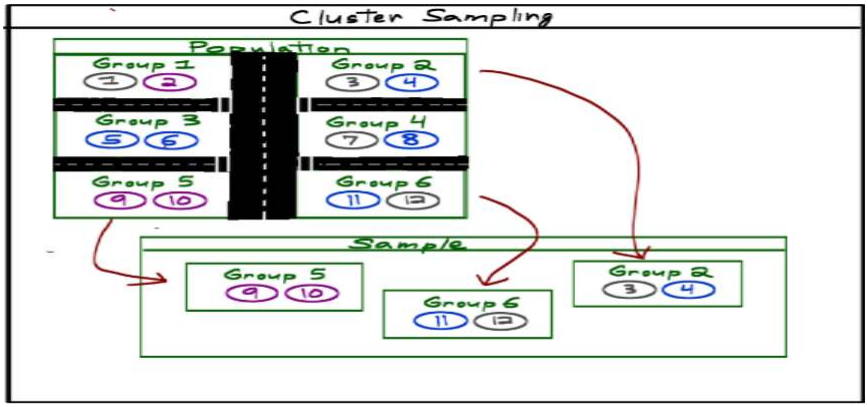



Figure **5.1.4**: "Cluster Sampling Diagram" by Toros Berberyan(opens in new window) is licensed under CC BY-SA 4.0(opens in new window)

The illustration above is an example of a **cluster sample**. There is an original population of **12** balls. They are grouped into clusters (blocks). Each group has two balls. Then, **some groups** are selected, and **all members** from those groups are chosen to be part of the sample. In this example, the group that only had balls numbered **3** and **4** is selected to be part of the sample. Then, a second group with only balls numbered **9** and **10** is selected to be part of the sample. Finally, a third group with only balls numbered **11** and **12** is selected to also be part of the sample. Based on cluster sampling, we ended up with three groups and all its members being part of the sample. The sample included balls with the numbers **3, 4, 9, 10, 11,** and **12**.

**Note**

Many people confuse stratified sampling and cluster sampling. In **stratified sampling**, you use **all the groups** and **some members** from each group. Cluster sampling is the opposite approach. **Cluster sampling** involves **selecting** a subset of **groups** and **all the members** within each group.

The four sampling techniques that were presented all have advantages and disadvantages. There is another sampling technique that is sometimes utilized because either the researcher is unfamiliar with it or it is easier to implement. This sampling technique is known as a convenience sample.

**Definition: Convenience Sampling**

**A convenience sample** is one where the researcher selects individuals to be included who are readily accessible to the researcher for collection.

If you want to know the opinion of people about the criminal justice system, you stand on a street corner near the county courthouse, and question the first **10** people who walk by. The people who walk by the county courthouse are most likely involved in some way with the criminal justice system, and their opinions would not necessarily represent the views of all individuals.

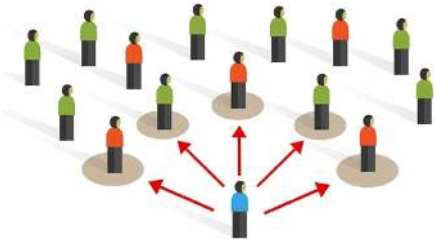


Figure **5.1.5**: Convenience Sampling. (CS)

**Note**

A **convenience sample** will not yield a representative sample and **should be avoided**.

**Example 5.1.3: Identify the Type of Sampling**

Banner Health is a multi-state nonprofit chain of hospitals. Management wants to assess the incidence of complications after surgery. They wish to use a sample of surgery patients. Several sampling techniques are described below. Categorize each technique as a simple random sample, stratified sample, systematic sample, cluster sample, or convenience sampling.

1. Obtain a list of patients who had surgery at all Banner Health facilities. Divide the patients according to the type of surgery. Draw simple random samples from each group.

2. Obtain a list of patients who had surgery at all Banner Health facilities. Number these patients, and then use a random number table to obtain the sample.
3. Randomly select some Banner Health facilities from each of the seven states, and then include all the patients on the surgery lists of the states.
4. At the beginning of the year, instruct each Banner Health facility to record any complications from every $100^{th}$ surgery.
5. Instruct each Banner Health facility to record any complications from **20** surgeries this week and send in the results.

**Answer**

1. This is a stratified sample since the patients were separated into different strata, and then random samples were taken from each stratum. The problem with this is that some types of surgeries may have a higher chance of complications than others. Of course, the stratified sample would show you this.
2. This is a random sample since each patient has the same chance of being chosen. The problem with this one is that it will take a while to collect the data.
3. This is a cluster sample since all patients are questioned in each of the selected hospitals. The problem with this is that you could have, by chance, selected hospitals that have no complications.
4. This is a systematic sample since they selected every $100^{th}$ surgery. The problem with this is that if every $90^{th}$ surgery has complications, you wouldn't see this come up in the data.
5. This is a convenience sample since they left it up to the facility on how to do it. The problem with convenience samples is that the person collecting the data will probably collect data from surgeries that had no complications.

**Your Turn 5.1.3: Identify the Type of Sampling Method**

Match the name of the sampling method descriptions given.

Situations

- [-] selecting a classroom number at random then surveying all the students in that room
- [-] number every name on a list and use a random number generator to select the first 50 numbers
- [-] dividing the population by Gender, and choosing 30 people of each gender
- [-] surveying every 3rd driver coming through a tollbooth
- [-] asking people on the street

Sampling Method

a. Simple Random
b. Convenience
c. Stratified
d. Systematic
e. Cluster

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## Guidelines for Planning a Statistical Study

1. Identify the individuals that you are interested in studying. Realize that you can only make conclusions for these individuals. As an example, if you use a fertilizer on a certain genus of plant, you cannot say how the fertilizer will work on any other types of plants. However, if you diversify too much, then you may not be able to tell if there really is an improvement since you have too many factors to consider.
2. Specify the variable. You want to make sure the variable is something that you can measure, and make sure that you control for all other factors too. For example, if you are trying to determine if a fertilizer works by measuring the height of the plants on a particular day, you need to make sure you can control how much fertilizer you put on the plants (which is what we call a treatment), and make sure that all the plants receive the same amount of sunlight, water, and temperature.

3. Specify the population. This is important in order for you to know for whom and what conclusions you can make.
4. Specify the method for taking measurements or making observations.
5. Determine if you are taking a census or sample. If taking a sample, decide on the sampling method.
6. Collect the data.
7. Use appropriate descriptive statistics methods and make decisions using appropriate inferential statistics methods.
8. Note any concerns you might have about your data collection methods and list any recommendations for future.

## Attribution:

3. Levine JD, Gordon NC, Smith R, Fields HL. (1981) Analgesic responses to morphine and placebo in individuals with postoperative pain. Pain. 10:379-89.

https://stats.libretexts.org/Bookshe...ion_Principles

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https://stats.libretexts.org/Courses...mpling_Methods

[CS] https://www.simplypsychology.org/con...-sampling.html

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# 5.2: Data Type and Study Type

## Types of Data

Once we have gathered data, we might wish to classify it. Roughly speaking, data can be classified into two categories: qualitative data (also known as categorical data) and quantitative data.

**Definition: Qualitative and Quantitative Data**

Categorical (qualitative) data are pieces of information that allow us to classify the objects under investigation into various categories.

Quantitative data are responses that are numerical in nature and with which we can perform meaningful arithmetic calculations.

**Example 5.2.1: Identify the Data Type**

1. We might conduct a survey to determine the favorite movie that each person in a math class has seen in a movie theater.
2. A survey could ask the number of movies they have seen in a movie theater in the past **12** months (**0, 1, 2, 3, 4, ...**)
3. Suppose we gather respondents' ZIP codes in a survey to track their geographical location.

**Answer**

1. When we conduct such a survey, the responses would look like: *Finding Nemo*, *The Avengers*, or *Dunkirk*. We might count the number of people who give each answer, but the answers themselves do not have any numerical values: we cannot perform computations with an answer like "*Finding Nemo*." This would be categorical data.
2. This would be quantitative data. Other examples of quantitative data would be the running time of the movie you saw most recently (**104** minutes, **137** minutes, **104** minutes, ...) or the amount of money you paid for a movie ticket the last time you went to a movie theater (**$5.50, $7.75, $9, ...**). ZIP codes are numbers, but we can't do any meaningful mathematical calculations with them (it doesn't make sense to say that **98036** is "twice" **49018** — that's like saying that Lynnwood, WA is "twice" Battle Creek, MI, which doesn't make sense at all), so ZIP codes are really categorical data.

**Your Turn 5.2.1: Classify the Data**

Classify each variable as categorical or quantitative.

(a) Disease presence (healthy, infected, etc.) is Select an answer

(b) Plant height (in inches) is Select an answer

(c) Plant health rating (poor, good, excellent) is Select an answer

(d) Flower color is Select an answer

(e) Days to germination is Select an answer

(f) Growth rate (cm/week) is Select an answer

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## Bias in Statistical Study

Consider the scenario. Suppose we are hired by a politician to determine the amount of support he has among the electorate, should he decide to run for another term. What population should we study? Every person in the district? Not every person is eligible to vote, and regardless of how strongly someone likes or dislikes the candidate, they don't have much to do with his being re-elected if they are not able to vote.

What about eligible voters in the district? That might be better, but if someone is eligible to vote but does not register by the deadline, they won't have any say in the election either. What about registered voters? Many people are registered but choose not to vote. What about "likely voters?"

This is the criterion used in much political polling; however, it can be challenging to define a "likely voter." Is it someone who voted in the last election? In the last general election? In the last presidential election? Should we consider someone who just turned 18 a "likely voter?" They weren't eligible to vote in the past, so how do we judge the likelihood that they will vote in the next election?

In November 1998, former professional wrestler Jesse "The Body" Ventura was elected governor of Minnesota. Up until right before the election, most polls showed he had little chance of winning. There were several contributing factors to the polls not reflecting the actual intent of the electorate:

- Ventura was running on a third-party ticket, and most polling methods are better suited to a two-candidate race.
- Many respondents to polls may have been embarrassed to tell pollsters that they were planning to vote for a professional wrestler.
- The mere fact that the polls showed Ventura had little chance of winning might have prompted some people to vote for him in protest, sending a message to the major-party candidates.

But one of the major contributing factors was that Ventura recruited a substantial amount of support from young people, particularly college students, who had never voted before and who registered specifically to vote in the gubernatorial election. The polls did not deem these young people likely voters (since, in most cases, young people have a lower rate of voter registration and a turnout rate for elections), and so the polling samples were subject to **sampling bias**: they omitted a portion of the electorate that was weighted in favor of the winning candidate.

Identifying the population can be a challenging task, but once it is identified, how do we choose an appropriate sample? Remember, although we would prefer to survey all members of the population, this is usually impractical unless the population is very small, so we choose a sample. There are many ways to sample a population, but one key goal we must keep in mind is that we want the sample to be **representative of the population.**

There are a number of ways that a study can be ruined before you even start collecting data. The first one we have already explored – **sampling** or **selection bias**, which is when the sample is not representative of the population. One example of this is **voluntary response bias**, which occurs when data is collected only from individuals who volunteer to participate. This is not the only potential source of bias.

### Sources of Bias

- **Sampling bias** – when the sample is not representative of the population
- **Voluntary response bias** – the sampling bias that often occurs when the respondents in the sample volunteered to participate
- **Self-interest study** – bias that can occur when the researchers have an interest in the outcome
- **Response bias** – when the respondent gives inaccurate responses for any reason
- **Perceived lack of anonymity** – when the respondent fears giving an honest answer might negatively affect them
- **Loaded questions** – when the question wording influences the responses
- **Non-response bias** – when people refusing to participate in the study can influence the validity of the outcome

### Example 5.2.1: Identify the Bias

Identify a potential source of bias for the study.

1. A recent study found that chewing gum may raise math grades in teenagers [1]. This study was conducted by the Wrigley Science Institute, a branch of the Wrigley chewing gum company. Identify a potential source of bias for the study.
2. A survey asks people, "When was the last time you visited your doctor?"
3. A survey asks participants a question about their interactions with individuals from other racial backgrounds.
4. An employer puts out a survey asking their employees if they have a drug abuse problem and need treatment help.
5. A survey asks, "Do you support funding research of alternative energy sources to reduce our reliance on high-polluting fossil fuels?"
6. A telephone poll asks the question "Do you often have time to relax and read a book?", and 50% of the people called refused to answer the survey.

**Answer**

1. This is an example of a **self-interest study, one in which the researchers have a vested interest in the study's outcome**. While this does not necessarily indicate that the study was biased, it certainly suggests that we should subject it to extra scrutiny.
2. This survey may be subject to **response bias**, as many individuals may not recall the exact date of their last doctor's visit and provide inaccurate responses.
3. Here, a **perceived lack of anonymity** could influence the outcome. The respondent might not want to be perceived as racist, even if they are, and give an untruthful answer.
4. Here, both **response bias and perceived lack of anonymity** may be sources of bias. Answering truthfully might have consequences; responses may not be accurate if employees do not feel their answers are anonymous or fear retribution from their employer.
5. **Loaded questions** can be intentionally created by pollsters with an agenda or accidentally occur due to poor question wording. Additionally, a concern is the **question order**, as the sequence of questions alters the results. A psychology researcher provides an example [2].
6. It is unlikely that the results will be representative of the entire population. This is an example of **non-response bias**, which is introduced when people refuse to participate in a study or drop out of an experiment. When people refuse to participate, we can no longer be so certain that our sample is representative of the population.

## Your Turn 5.2.2: Identify the Bias

**In each situation, identify a potential source of bias.**

A survey asks how many sexual partners a person has had in the last year.

- ○ perceived lack of anonymity
- ○ voluntary response bias
- ○ sampling bias
- ○ response bias

A radio station asks readers to phone in their choice in a daily poll.

- ○ perceived lack of anonymity
- ○ response bias
- ○ voluntary response bias
- ○ sampling bias

A substitute teacher wants to know how students in the class did on their last test. The teacher asks the 10 students sitting in the front row to state their latest test score.

- ○ voluntary response bias
- ○ sampling bias
- ○ response bias
- ○ perceived lack of anonymity

High school students are asked if they have consumed alcohol in the last two weeks.

- ○ perceived lack of anonymity
- ○ voluntary response bias
- ○ sampling bias
- ○ self interest study

The Beef Council releases a study stating that consuming red meat poses little cardiovascular risk.

- ○ sampling bias
- ○ voluntary response bias
- ○ perceived lack of anonymity
- ○ self interest study

A poll asks "Do you support a new transportation tax, or would you prefer to see our public transportation system fall apart?"

system fall apart?

- ○ voluntary response bias
- ○ self interest study
- ○ perceived lack of anonymity
- ○ loaded question

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## Type of Studies

**Observational studies** – studies in which conclusions are drawn from observations of a sample or the population. In some cases, these observations might be unsolicited, such as studying the percentage of cars that turn right at a red light even when there is a "no turn on red" sign. In other cases, the observations are solicited, like in a survey or a poll.

In contrast, experiments are commonly used when exploring how subjects react to an external influence. In an experiment, a **treatment** is applied to the subjects, and the results are measured and recorded.

**Definition: Observational and Experimental**

An **observational study** is a type of statistical study in which researchers observe and measure variables without changing or influencing the subjects.

An **experimental study** involves manipulating variables and assigning treatments to determine cause and effect.

**Example 5.2.3: Observational Study**

Specific research questions: *Do the majority of college students listen to music while they study? Do the majority of college students believe that listening to music improves their learning?*

**Answer**

To investigate these questions, the statistics students conduct a survey in their other classes. They ask these two questions:

Do you listen to music while you study? Do you think listening to music improves your concentration and memory?

This observational study aims to address two questions about a population of college students. Each question contains a claim about the population of college students. We can use the data from this study to determine if these claims are accurate. But data from this study cannot provide evidence of a cause-and-effect relationship between listening to music while studying and improvements in learning

**Example 5.2.4: Experimental Study**

Specific research question: *Does listening to music improve students' ability to quickly identify information?*

**Answer**

To investigate this question, the instructor uses word-search puzzles. She divides the class into two groups. Students on one side of the room do a word puzzle for 3 minutes while listening to music on an iPod. Students on the other side of the room do a word puzzle for 3 minutes without music. The instructor calculates the average number of words found by each group.

This study is an experiment. The instructor manipulates music to investigate the impact on puzzle completion. Data from this study can provide evidence of a cause-and-effect relationship between listening to music while studying and improvements in learning. But the improvement in learning is more narrowly defined as the ability to quickly identify information, such as words in a puzzle

**Your Turn 5.2.4: Observational or Experimental?**

Decide if each of the following describe an observational study or an experiment.

Select an answer ▾ The haircolor of shoppers at the mall was recorded.

Select an answer A drug is given to a group of people, and their reactions observed.

Select an answer A group of students are told to listen to music while taking a test and their results are compared to a group not listening to music.

Select an answer The temperature on randomly selected days throughout the year were measured.]

Select an answer The gender of children born in January were tallied.

Select an answer The growth rate of bacteria is compared before and after adding alcohol.

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When conducting experiments, it is essential to isolate the treatment being tested.

Suppose a middle school (junior high) finds that its students are not scoring well on the state's standardized math test. They decide to run an experiment to see if an alternate curriculum would improve scores. To run the test, they hire a math specialist to come in and teach a class using the new curriculum. To their delight, they see an improvement in test scores.

The difficulty with this scenario is that it is not clear whether the curriculum is responsible for the improvement or whether the improvement is due to a math specialist teaching the class. This is called **confounding** – when it is not clear which factor or factors caused the observed effect. Confounding is the downfall of many experiments, although it is sometimes hidden.

**Definition: Confounding**

**Confounding** occurs when there are two potential variables that could have caused the outcome, and it is not possible to determine which actually caused the result.

A drug company study about a weight loss pill might report that people lost an average of 8 pounds while using their new drug. However, in the fine print, you find a statement saying that participants were encouraged to also diet and exercise. In this case, it is unclear whether the weight loss is due to the pill, diet, exercise, or a combination of these factors. In this case, confounding has occurred.

**Your Turn 5.2.5: Identify Confounding Variables**

**Identify the Confounding Variable:** Identify a primary confounding variable if a confounding variable exists in the scenarios.

a) Researchers wanted to investigate why a city had an increase in crime. They observed that as the incidence of crime increased the number of schools in the city also increased. Is there a confounding variable in this scenario?

- ○ Yes, a confounding variable could be population density in the area.
- ○ Yes, a confounding variable could be the number of crimes in the area.
- ○ No, there are no confounding variables in this situation.
- ○ Yes, a confounding variable could be the number of schools in the area.
- ○ Yes, a confounding variable could be the temperature.

b) To determine whether increasing the amount of vegetables eaten affects body weight researchers

randomly placed individuals into two groups, one where the individuals did not change anything about their lifestyle and the other group was required to eat a kale salad prepared by a certain recipe as part of their lunch every day for a month. It was determined that the individuals who ate the salad lost more weight than the those who did not change their lifestyle. Is there a confounding variable in this scenario?

- ○ Yes, a confounding variable could be the amount of vegetables eaten by the observational units.
- ○ Yes, a confounding variable could be the amount of high calorie foods eaten by the observational units.
- ○ Yes, a confounding variable could be the weight of the observational units.
- ○ Yes, a confounding variable could be the time of the year the study was conducted.
- ○ No, there are no confounding variables in this situation.

---

c) Orthopedic Surgeons investigated the question, "Does running more than 25 miles in a week affect a patient's damage to the knee or knee cartilage?" The surgeons followed individuals who self-placed into two groups, those who ran more than 25 miles a week and those who did not run more than 25 miles a week. After a year, the surgeons did not find any convincing evidence of a difference of knee or cartilage damage in the two groups. Is there a confounding variable in this scenario?

- ○ No, there are no confounding variables in this situation.
- ○ Yes, a confounding variable could be the weight of the individuals.
- ○ Yes, a confounding variable could be the knee damage or cartilage damage in the knee of the individual.
- ○ Yes, a confounding variable could be what type of residence the individuals live in.
- ○ Yes, a confounding variable could be the time the individuals spent running per week

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Several measures can be implemented to reduce the likelihood of confounding. The primary measure is to use a **control group**.

### Control Group and Treatment Group

The treatment group is the group that receives the treatment or factor being tested.

Example: If a study is testing a new medicine, the people who receive the medicine are in the treatment group.

The control group is the group that does not receive the treatment. It is used as a baseline for comparison.

Example: In the same study, people who receive a placebo (or no medicine) are in the control group. The treatment group is the group that receives the treatment or factor being tested.

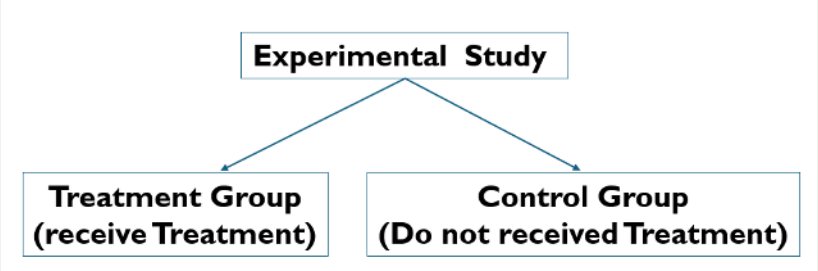



Figure **5.2.1**: Type of Experimental Study

Ideally, the groups are as similar as possible, isolating the treatment as the only potential source of difference between the groups. For this reason, the method of dividing groups is important. Some researchers attempt to ensure that the groups have similar characteristics (the same number of females, the same number of people over 50, etc.), but it is nearly impossible to control for every characteristic. Due to this, random assignment is widely used.

Sometimes not giving the control group anything does not completely control for confounding variables. For example, suppose a medicine study is testing a new headache pill by giving the treatment group the pill and the control group nothing. If the treatment group showed improvement, we would not know whether it was due to the medicine in the pill, or a response to having taken any pill. This is called a **placebo effect**.

In some cases, it is more appropriate to compare to a conventional treatment than a placebo. For example, in a cancer research study, it would not be ethical to deny any treatment to the control group or to give a placebo treatment. In this case, the currently accepted medicine would be given to the control group, sometimes referred to as a **comparison group**.

If participants know they are receiving a placebo, it defeats the purpose of using it. So there is blinding in the experimental study.

### Single Blind or Double Blind Study

**A single-blind study** is one in which the participant is unaware whether they are receiving the treatment or a placebo. A **double-blind study** is one in which both the participants and the experimenter are unaware of who is in the treatment group and who is in the control group.

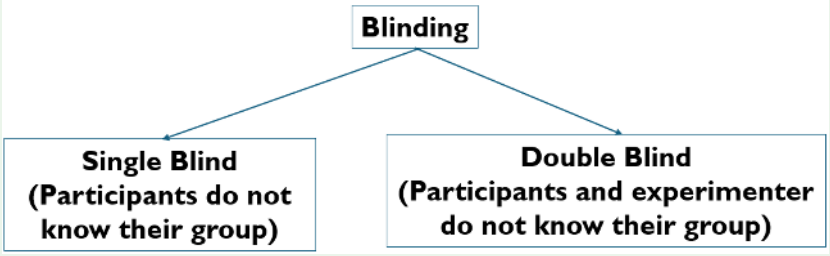



Figure 5.2.2: Type of Blinding

### Definition: Placebo

A **placebo** is a dummy treatment given to control for the placebo effect. An experiment that gives the control group a placebo is called a **placebo-controlled experiment**.

In a study for a new medicine that is dispensed in a pill form, a sugar pill could be used as a placebo.

In a study on the effect of alcohol on memory, a non-alcoholic beer might be given to the control group as a placebo.

In a study of a frozen meal diet plan, the treatment group would receive the diet food, and the control group could be given standard frozen meals stripped of their original packaging.

### Definition: Placebo Effect

The **placebo effect** occurs when the effectiveness of a treatment is influenced by the patient's perception of how effective they believe the treatment will be, resulting in a positive outcome even if the treatment is ineffective.

1. To determine if a two-day prep course would help high school students improve their scores on the SAT test, a group of students was randomly divided into two subgroups.

   The first group, the treatment group, received a two-day preparatory course. The second group, the control group, was not given the prep course. Afterwards, both groups were given the SAT.
2. In a study about anti-depression medication, you would not want the psychological evaluator to know whether the patient is in the treatment or control group, either, as it might influence their evaluation; therefore, the experiment should be conducted as a double-blind study.
3. A study found that when doing painful dental tooth extractions, patients told they were receiving a strong painkiller, while actually receiving a saltwater injection, found as much pain relief as patients receiving a dose of morphine.[1]
4. If a researcher is testing whether a new fabric can withstand fire, she simply needs to torch multiple samples of the fabric – there is no need for a control group.

### Example 5.2.6: Blind or Double-Blind?

To test a new lie detector, two groups of subjects are given the new test. One group is asked to answer all the questions truthfully, and the second group is asked to lie on one set of questions. The person administering the lie detector test does not know what group each subject is in.

Does this experiment have a control group? Is it blind, double-blind, or neither?

**Answer**

The truth-telling group could be considered the control group, but in reality, both groups are treatment groups here, as it is essential for the lie detector to accurately identify lies and not misclassify truth-telling as lying. This study is double-blinded, as the person administering the test is

unaware of which group each subject belongs to.

## Your Turn 5.2.6: Identify the Component of Experimental Study

Determine if the following experiemnt describes a double-blind experiment, a single-blind experiment, or an experiment with no blinding.

### Part A

**Problem Text**

In a fitness study, participants are enrolled in a program to evaluate a new high-intensity exercise regimen. Both the individuals performing the exercises and the trainers supervising them are informed about the specific routines being tested.

**Input Your Answer:**

Select an answer

### Part B

**Problem Text**

During a pharmaceutical trial, neither the laboratory technicians analyzing blood samples nor the patients know which samples correspond to those who received the drug.

**Input Your Answer:**

Select an answer

### Part C

**Problem Text**

A hearing test study provides participants with headphones, where they don't know if the sound adjustments are experimental, but the audio engineers managing the settings do.

**Input Your Answer:**

Select an answer

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Notes:

3. Levine JD, Gordon NC, Smith R, Fields HL. (1981) Analgesic responses to morphine and placebo in individuals with postoperative pain. Pain. 10:379-89.

1. References (10)
2. www.whitehouse.gov/sites/defa...s/hist07z1.xls
3. [1] Levine JD, Gordon NC, Smith R, Fields HL. (1981) Analgesic responses to morphine and placebo in individuals with postoperative pain. Pain. 10:379-89.
4. [1] Reuters. news.yahoo.com/s/nm/20090423/...k_gum_learning. Retrieved 4/27/09

   [2] Swartz, Norbert. www.umich.edu/~newsinfo/MT/01...01/mt6f01.html. Retrieved 3/31/2009

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# 5.3: Frequency Tables and Histograms

## Displaying Qualitative Data

Categorical, or qualitative, data are pieces of information that allow us to classify the objects under investigation into various categories. We usually begin working with categorical data by summarizing the data into a **frequency table**.

**Definition: Frequency Table**

A **frequency table** is a table that shows how often each value or category appears in a data set. It organizes data by listing the values (or groups of values) and the number of times each one occurs, making the data easier to read, analyze, and interpret.

If the data we're visualizing is categorical, then we want a quick way to represent graphically the relative numbers of units that fall in each category. When we created the frequency distributions in the last section, all we did was count the number of units in each category and record that number (this was the *frequency* of that category). Frequencies are nice when we're organizing and summarizing data; they're easy to compute, and they're always whole numbers. But they can be difficult to understand for an outsider who's being introduced to your data.

**Definition: Cumulative Frequency**

**Cumulative frequency** is the **running total** of frequencies as you move through a data set from smallest to largest.

Cumulative Frequency is the sum of all frequencies up to that point.

Let's consider a quick example. Suppose you surveyed some people and asked for their favorite color. You communicated your results using a frequency distribution. Jerry is interested in data on favorite colors, so he reads your frequency distribution. The first row indicates that twelve people listed green as their favorite color. However, Jerry has no way of knowing if that's a lot of people without knowing how many people took your survey in total. Twelve is a pretty significant number if only twenty-five people took the survey, but it's next to nothing if you recorded a thousand responses. For that reason, we will often summarize categorical data not with frequencies, but with proportions. The proportion of data that falls into a particular category is computed by dividing the frequency for that category by the total number of units in the data.

**Definition: Proportion and Relative Frequency**

$$\text{Relative Frequency} = \frac{\text{Category frequency}}{\text{Total number of data}}$$

Proportions (Relative Frequency) can be expressed as fractions, decimals, or percentages. For example, relative frequency could be $0.35$ or $35\%$.

**Example 5.3.1: Finding Proportion**

Recall the example in which a teacher recorded the responses on the first question of a multiple-choice quiz, with five possible responses (A, B, C, D, and E). The raw data was as follows.

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | A | C | A | B | B | A | E | A | C | A | A | A | C |
| E | A | B | A | A | C | A | B | E | E | A | A | C | C |

We computed a frequency distribution, relative frequency, and cumulative frequency.

**Answer**

To compute a proportion, we need the frequency (which is provided in the table above) and the total number of units represented in our data. We can find that by adding up the frequencies from all the categories: $0+14+4+4+6=28$.

To find the proportions, we divide the frequency by the total. For the first category ("A"), the proportion is $\frac{14}{28}=0.50=50\%$. We can compute the other proportions similarly, filling in the rest of the table:

| Category of Data | Frequency | Proportion (Relative Frequency) | Cumulative Frequency |
|---|---|---|---|
| D | $0$ | $\frac{0}{28}=0=0\%$ | $0$ |
| A | $14$ | $\frac{14}{28}=0.50=50\%$ | $0+14=14$ |
| B | $4$ | $\frac{4}{28}=0.143=14.3\%$ | $14+4=18$ |
| E | $4$ | $\frac{4}{28}=0.143=14.3\%$ | $18+4=22$ |
| C | $6$ | $\frac{6}{28}=0.214=21.4\%$ | $22+6=28$ |

Check your work: If you add up your proportions, you should get 1 or 100%. And frequency should be added to the total number of data in the data set.

**Your Turn 5.3.1: Find Missing Value in the Chart**

In a student survey, fifty part-time students were asked how many courses they were taking this term. The (incomplete) results are shown below:

Please round your answers to 4 decimal places.

| # of Courses | Frequency | Relative Frequency | Cumulative Frequency |
|---|---|---|---|
| 1 | _____ | _____ | 11 |
| 2 | 19 | 0.38 | _____ |
| 3 | 20 | 0.4 | 50 |

What percent of students take exactly one courses? _____ %

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Sometimes we need an even more intuitive way of displaying data. This is where charts and graphs come in. There are many, many ways of displaying data graphically, but we will concentrate on one very useful type of graph called a bar graph. In this section, we will work with bar graphs that display categorical data; the next section will be devoted to graphs that display quantitative data.

**Definition: Bar Graph**

A **bar graph** is a graph that displays a bar for each category, with the length of each bar indicating the frequency of that category.

Note: A bar graph is only used to display **categorical data**.

To construct a bar graph, we need to draw a vertical axis and a horizontal axis. The vertical direction will have a scale and measure the frequency of each category; the horizontal axis has no scale in this instance, but only the names of each category. The construction of a bar chart is most easily described by use of an example.

An insurance company determines vehicle insurance premiums based on known risk factors. If a person is considered a higher risk, their premiums will be higher. One potential factor is the color of your car. The insurance company believes that people with some color cars are more likely to get in accidents. To research this, they examine police reports for recent total-loss collisions. The data is summarized in the frequency table below.

| Color | Frequency | Relative Frequency |
|---|---|---|
| Blue | 25 | $\frac{25}{216} = 0.1157 = 11.57\%$ |
| Green | 52 | $\frac{52}{216} = 0.2407 = 24.07\%$ |
| Red | 41 | $\frac{41}{216} = 0.1898 = 18.98\%$ |
| White | 36 | $\frac{36}{216} = 0.1667 = 16.67\%$ |
| Black | 39 | $\frac{39}{216} = 0.1806 = 18.06\%$ |
| Grey | 23 | $\frac{23}{216} = 0.1065 = 10.65\%$ |
| Total | 216 | $1 = 100\%$ |

Using our car data from above, note the highest frequency is 52, so our vertical axis needs to go from 0 to 52, but we might as well use 0 to 55, so that we can put a hash mark every 5 units:

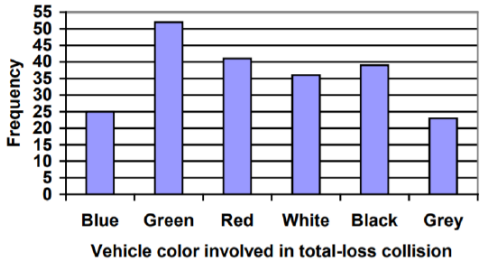



Figure 5.3.1: Bar Graph

Notice that the height of each bar is determined by the frequency of the corresponding color. The horizontal gridlines are a nice touch, but not necessary. In practice, you will find it useful to draw bar graphs using graph paper, where the gridlines are already in place, or by using technology. Instead of gridlines, we might also list the frequencies at the top of each bar, like this:

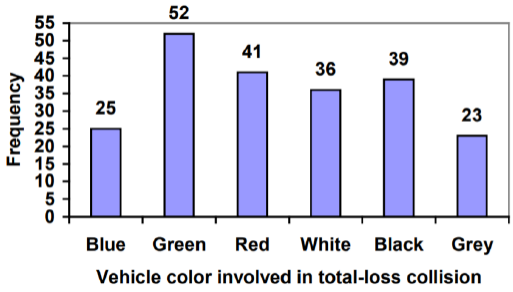



Figure 5.3.2: Bar Graph with Frequency Labelled

In this case, our chart might benefit from being reordered from largest to smallest frequency values. This arrangement can make it easier to compare similar values in the chart, even without gridlines. When we arrange categories in decreasing frequency order, it is called a **Pareto chart**.

**Definition: Pareto Chart**

A **Pareto chart** is a bar graph ordered from highest to lowest frequency.

Transforming our bar graph from earlier into a Pareto chart, we get:

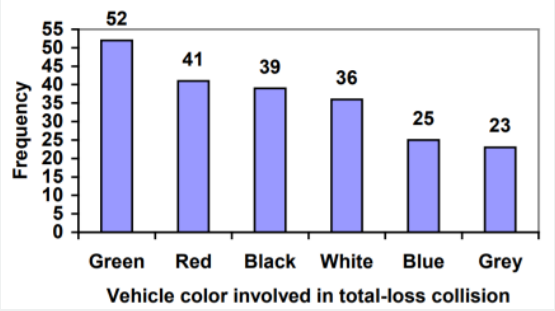



Figure 5.3.3: Pareto Chart

**Your Turn 5.3.2: Complete Frequency and Relative Frequency Table**

The class ranks of a very small high school are shown in the table below.

| | | | | |
|---|---|---|---|---|
| Jr | Jr | Sr | Fr | So |
| Sr | So | Jr | So | Fr |
| Sr | Fr | So | Fr | Jr |
| Fr | So | Fr | Fr | So |

Complete the frequency table. Round the relative frequencies to 2 decimal places as needed.

| Class rank | Frequency | Relative frequency |
|---|---|---|
| Freshmen | | |

| | | |
|---|---|---|
| Sophomores | | |
| Juniors | | |
| Seniors | | |

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Another way to represent qualitative data is with a **pie chart**. A pie chart is useful because it shows how a whole is divided into parts. It allows people to quickly compare proportions or percentages and easily see which categories make up the largest or smallest parts of the data. Pie charts are especially helpful when there are only a few categories and the goal is to show how each category contributes to the total (100%).

**Definition: Pie Chart**

A **pie chart** is a circle with wedges cut out of varying sizes marked out like slices of pie or pizza. The relative sizes of the wedges correspond to the relative frequencies of the categories.

**Example 5.3.3: Pi Chart**

For our vehicle color data in the above table, draw a pie chart.

**Answer**

We use relative frequency to draw a pie chart.

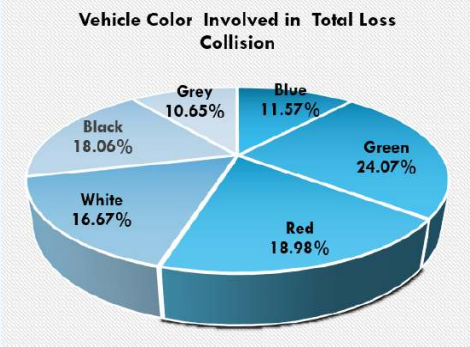



Figure 5.3.4: Pi Chart

Pie charts look nice but are harder to draw by hand than bar charts since to draw them accurately, we would need to compute the angle each wedge cuts out of the circle, then measure the angle with a protractor. Computers are much better suited to drawing pie charts. Common software programs like Microsoft Word or Excel, OpenOffice.org Write or Calc, or Google Docs are able to create bar graphs, pie charts, and other graph types. There are also numerous online tools that can create graphs [2].

**Your Turn 5.3.3: Create Bar Graph and Pie Chart**

Six students were randomly selected from each of the four classes/strata (freshmen, sophomore, junior, and senior) at Bedford High School to provide their homecoming theme preferences (Disney, Marvel, SuperHereos, and The Roaring 20's). The homecoming theme preferences of the 24 selected students are shown in the table below.

| |
|---|
| Disney |
| Disney |
| Disney |
| Marvel |
| Marvel |
| Marvel |

| |
|---|
| SuperH |
| Disney |
| SuperH |
| Roar20s |
| Roar20s |
| SuperH |
| Roar20s |
| SuperH |
| Disney |
| Disney |
| Roar20s |
| Roar20s |
| Disney |
| SuperH |
| Roar20s |
| Roar20s |
| Marvel |
| Disney |

---

a) Since data were collected for [ ? ] [ Select an answer ] variable(s), the correct graph to make is a [ Select an answer ] .

b) Complete the frequency/relative frequency table. Round to nearest hundredth.

| Theme | Frequency | Relative Frequency |
|---|---|---|
| Disney | _____ | _____ |
| Marvel | _____ | _____ |
| SuperH | _____ | _____ |
| Roar20s | _____ | _____ |

c) What is the population?

- ○ all high school students
- ○ all Bedford high school students
- ○ 24 randomly selected Bedford high school students
- ○ 24 randomly selected Bedford high school students in proportion to the class rank
- ○ a randomly selected high school student
- ○ a randomly selected Bedford high school student
- ○ Theme
- ○ Marvel

d) What is the sample?

- ○ all high school students
- ○ all Bedford high school students
- ○ 24 randomly selected Bedford high school students
- ○ 24 randomly selected Bedford high school students in proportion to the class rank
- ○ a randomly selected high school student
- ○ a randomly selected Bedford high school student

- Theme
- Marvel

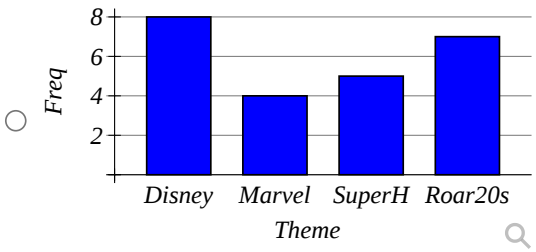
e) What is the individual?

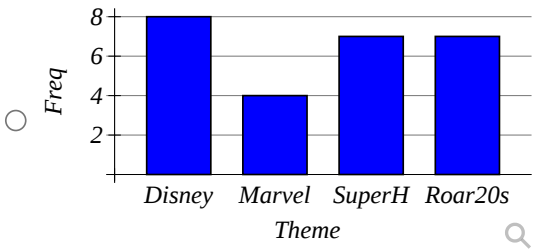
- all high school students
- all Bedford high school students
- 24 randomly selected Bedford high school students
- 24 randomly selected Bedford high school students in proportion to the class rank
- a randomly selected high school student
- a randomly selected Bedford high school student
- Theme
- Marvel

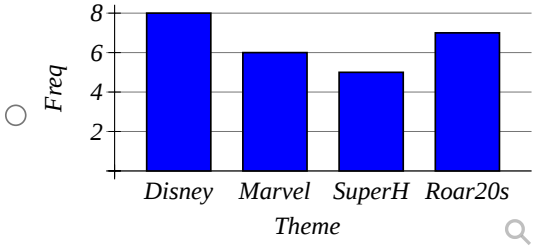
f) What type of sampling was done?

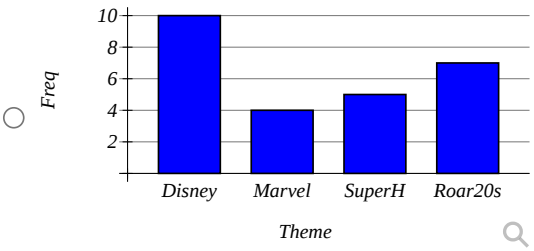
- systematic sampling
- stratified sampling
- random sampling
- cluster sampling
- convenience sampling

g) Which of the following is the correct frequency bar chart for the given data?

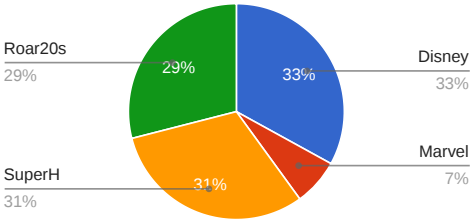


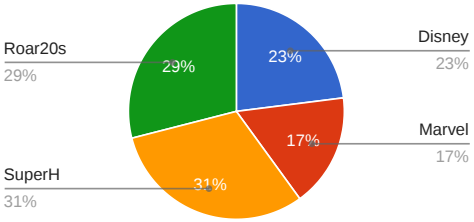



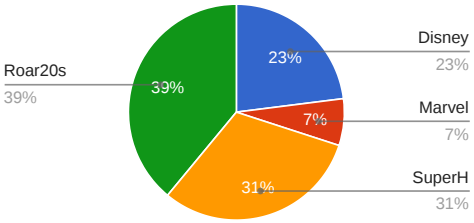



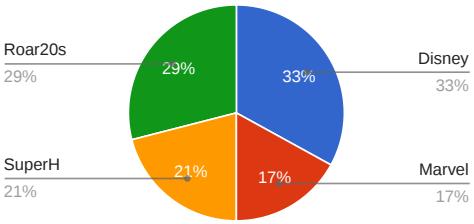



h) Which of the following is the correct pie chart for the given data?




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Don’t get fancy with graphs! People sometimes add features to graphs that don’t help to convey their information. For example, 3-dimensional bar charts like the one shown below are usually not as effective as their two-dimensional counterparts.

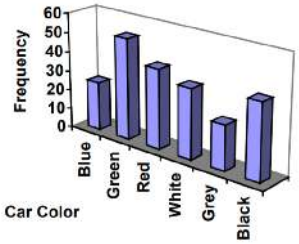



Figure 5.3.5: Somehow Misleading 3DBar Graph

Here is another way that fanciness can lead to trouble. Instead of plain bars, it is tempting to substitute meaningful images. This type of graph is called a **pictogram**.

**Definition: Pictogram**

A **pictograph** (also known as a *pictogram*) is a type of chart that uses **images, icons, or symbols** to represent data.

A labor union might produce the graph to the right to show the difference between the average manager salary and the average worker salary.




Figure 5.3.6: Pictograph Manager Salaires Vs Worker Salaries

Looking at the picture, it would be reasonable to guess that the manager's salary is 4 times as large as the worker's salary – the area of the bag looks about 4 times as large. However, the manager's salaries are in fact only twice as large as worker salaries, which was reflected in the picture by making the manager's bag twice as tall.

The following is another example of a pictograph.

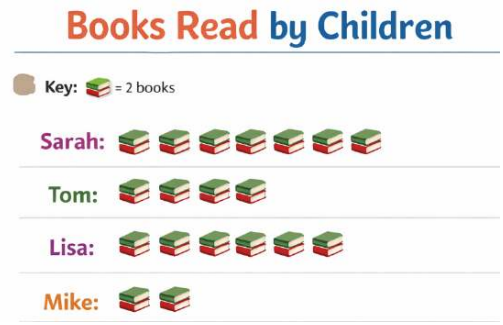



Figure 5.3.7: Pictograph: Books Read by Chlidren

## Displaying Quantitative Data

Quantitative, or numerical, data can also be summarized into frequency tables and graphs. A representation of quantitative data is called a **histogram**. The horizontal axis is a number

Some words we use to describe distributions are *uniform* (data are equally distributed across the range), *symmetric* (data are bunched up in the middle, then taper off in the same way above and below the middle), *left-skewed* (data are bunched up at the high end or larger values, and taper off toward the low end or smaller values), and *right-skewed* (data are bunched up at the low end, and taper off toward the high end). See the figures below.

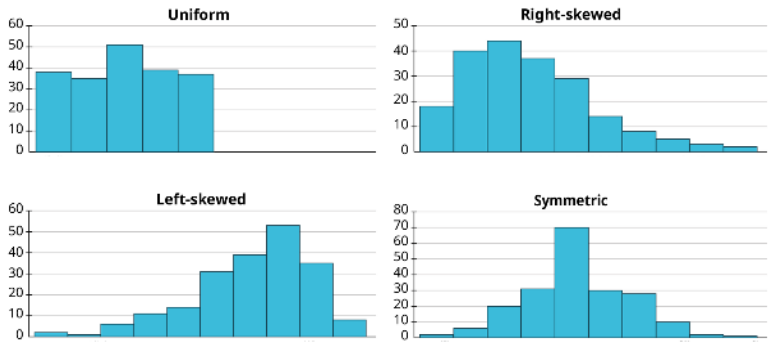



Figure 5.3.8: Distribution of Histogram

**Example 5.3.4: Construct Frequency and Relative Frequency Table**

A teacher records scores on a 20-point quiz for the 30 students in his class. The scores are

19 20 18 18 17 17 19 17 20 18 20 16 20 15 17 12 18 19 18 19 17 20 18 16 15 18 20 5 0 0

These scores could be summarized into a frequency table by grouping like values:

| Score | Frequency | Relative Frequency |
|---|---|---|
| 0 | 2 | $\frac{2}{30} = 0.067 = 6.7\%$ |
| 5 | 1 | $\frac{1}{30} = 0.033 = 3.3\%$ |
| 12 | 1 | $\frac{1}{30} = 0.033 = 3.3\%$ |
| 15 | 2 | $\frac{2}{30} = 0.067 = 6.7\%$ |
| 16 | 2 | $\frac{2}{30} = 0.067 = 6.7\%$ |
| 17 | 4 | $\frac{4}{30} = 0.133 = 13.33\%$ |
| 18 | 8 | $\frac{8}{30} = 0.2667 = 26.67\%$ |
| 19 | 4 | $\frac{4}{30} = 0.1333 = 13.33\%$ |
| 20 | 6 | $\frac{6}{30} = 0.2000 = 20.00\%$ |
| Total | 30 | $1 = 100\%$ |

Using this table, it would be possible to create a standard bar chart from this summary, as we did for categorical data:

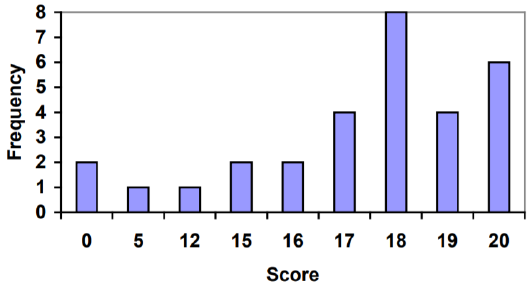



Figure 5.3.9: Histogram

**Bar Graph Vs Histogram**

**The Bar Graph (For Qualitative Data)**

In a bar graph, each bar represents a separate category. For example, if you survey people on their favorite fruit, "Apple" and "Banana" are independent categories. There is no "in-between" value between an apple and a banana, which is why we leave spaces between the bars.

**The Histogram (For Quantitative Data)**

A histogram groups numerical data into **bins** (class intervals). For example, if you are measuring the heights of people in a room, you might have a bin for "150 cm to 160 cm" and another for "160 cm to 170 cm." The bars touch because 160.10 cm follows immediately after 160 cm—the data flows continuously.

Here's a clear comparison between **bar graphs** and **histograms**:

| Feature | Bar Graph | Histogram |
|---|---|---|
| **Data Type** | Categorical (qualitative) data | Numerical (quantitative) data |
| **Bars** | Separated by gaps | Bars touch each other (continuous data) |

**Your Turn 5.3.4: Construct Frequency Distribution**

The table below shows scores on a Math test

| | | | |
|---|---|---|---|
| 80 | 50 | 90 | 100 |
| 70 | 80 | 60 | 90 |
| 50 | 30 | 50 | 90 |
| 100 | 100 | 80 | 100 |
| 60 | 60 | 100 | 100 |
| 100 | 60 | 70 | 60 |

Complete the frequency table for the Math test scores.

| Score | Frequency |
|---|---|
| 30 | |
| 40 | |
| 50 | |
| 60 | |
| 70 | |
| 80 | |
| 90 | |
| 100 | |

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If we have a large number of widely varying data values, creating a frequency table that lists every possible value as a category would lead to an exceptionally long frequency table and probably would not reveal any patterns. For this reason, it is common with quantitative data to group data into **class intervals**. A **class interval** is the range of values used to group data in a frequency distribution.

**Definition: Class Intervals**

Class intervals are groupings of the data. In general, we define class intervals so that

- Each interval is equal in size. For example, if the first class contains values from $120-129$, the second class should include values from $130-139$.
- Each interval has a **lower limit** and an **upper limit**, e.g., for interval $120-129$, $120$ is the lower limit and $129$ is the upper limit.
- The **class width** is the difference between two consecutive lower limits.
- The **class width** is the same for every interval in the frequency table.
- We typically have between $5$ and $20$ classes, depending on the amount of data we're working with.

Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of $263 - 121 = 142$. We could create 7 intervals with a width of around 20, 14 intervals with a width of around 10 , or somewhere in between. Often, we have to experiment with a few possibilities to find something that accurately represents the data. Let us try using an interval width of 15. We could start at 121 , or at 120 since it is a nice round number.

| Interval | Frequency |
|---|---|
| $120 - 134$ | 4 |
| $135 - 149$ | 14 |
| $150 - 164$ | 16 |
| $165 - 189$ | 28 |
| $180 - 194$ | 12 |
| $195 - 209$ | 8 |
| $210 - 224$ | 7 |
| $225 - 239$ | 6 |
| $240 - 254$ | 2 |
| $255 - 269$ | 3 |

Notice, the class width is 15 since $150 - 135 = 15$, $165 - 150 = 15$, and so on.

A histogram of this data would look like:

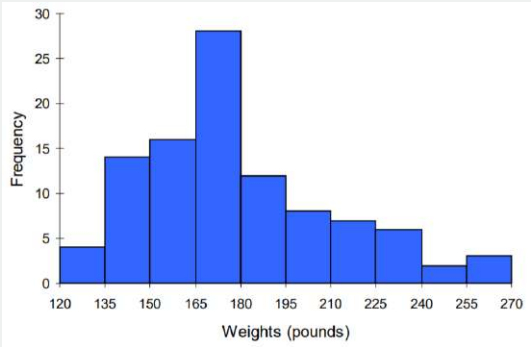
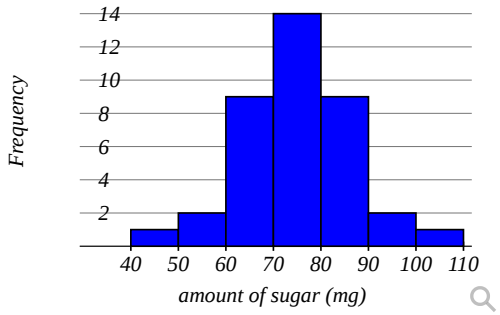



Figure 5.3.10: Histogram

**Your Turn 5.3.5: Meaning of Bin**

Data was collected for a sample of organic snacks. The amount of sugar (in mg) in each snack is summarized in the histogram below.




Which statement below best describes the meaning of the fifth bin (or class)?

- ○ There were between 80 and 90 snacks that contained 9 milligrams of sugar.
- ○ There were 9 snacks that contained 80 milligrams of sugar.
- ○ There were 9 snacks that contained 90 milligrams of sugar.

There were 90 snacks that contained 9 milligrams of sugar.

There were 80 snacks that contained 9 milligrams of sugar.

There were 9 snacks that contained between 80 and 90 milligrams of sugar.

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In many software packages, you can create a graph similar to a histogram by putting the class intervals as the labels on a bar chart.

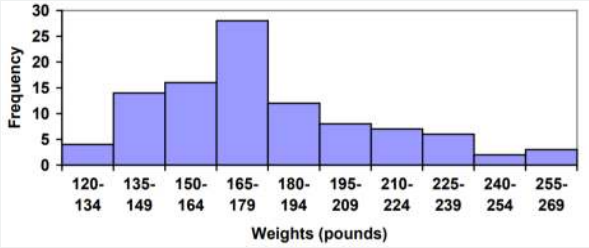



Figure 5.3.11: Histogram

Other graph types, such as pie charts, are possible for quantitative data. The usefulness of different graph types will vary depending on the number of intervals and the type of data being represented. For example, a pie chart of our weight data is difficult to read due to the large number of intervals we used.

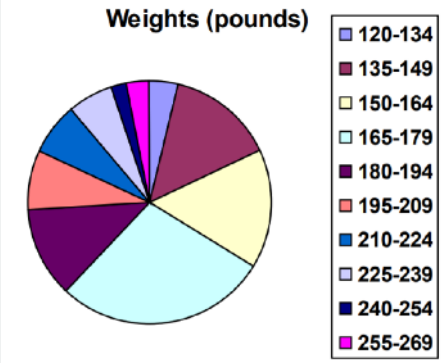



Figure 5.3.11: Pie Chart

## Your Turn 5.3.6: Complete Frequency and Relative Frequency Table

The following is the average daily temperature for Frederick, Maryland for the month of June:

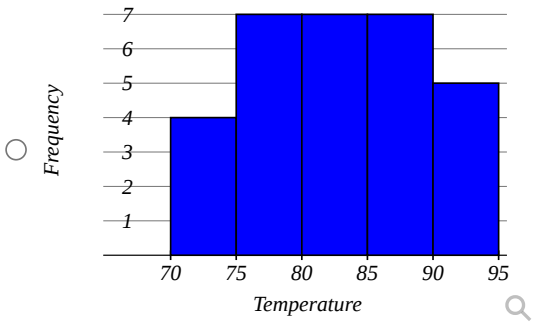
| | | | | | |
|---|---|---|---|---|---|
| 74 | 87 | 81 | 84 | 79 | 89 |
| 79 | 76 | 70 | 72 | 80 | 81 |
| 82 | 91 | 85 | 92 | 92 | 75 |
| 72 | 85 | 88 | 86 | 81 | 78 |
| 80 | 77 | 86 | 94 | 91 | 76 |

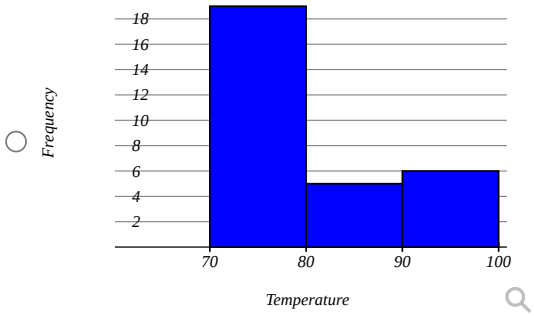
(a) Complete the frequency distribution for the data. **Express the relative frequencies in decimal form, accurate to at least 3 decimal places.**

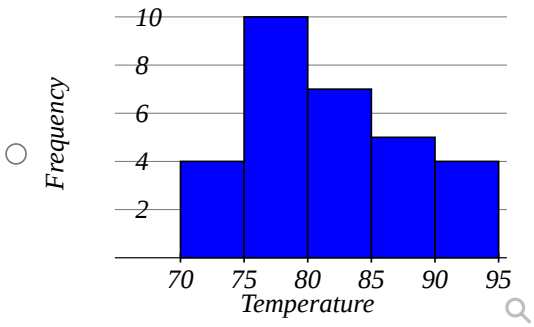
| Age | Frequency | Relative Frequency |
|---|---|---|
| 70-74 | _____ | _____ |
| 75-79 | | |

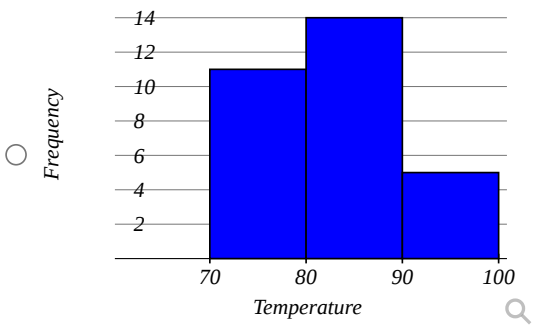
| | | |
|---|---|---|
| 80-84 | | |
| 85-89 | | |
| 90-94 | | |

(b) Which of the following is the correct histogram for this data?



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**Definition: Frequency Polygon**

An alternative representation is a frequency polygon. A **frequency polygon** begins like a histogram, but instead of drawing a bar, a point is placed at the midpoint of each interval, with a height equal to the frequency.

Typically, the points are connected with straight lines to emphasize the distribution of the data. A **frequency polygon** is useful when we want to **compare two or more** groups on one chart.

This graph makes it easier to see that reaction times were generally shorter for the larger target and that the reaction times for the smaller target were more spread out.

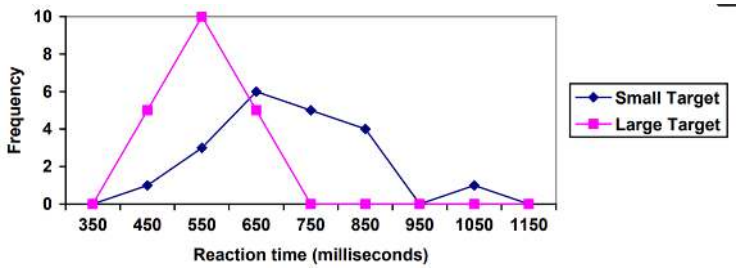
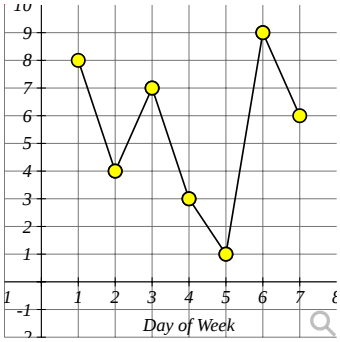



Figure **5.3.12**: Frequency Polygon

**Your Turn 5.3.7: Read the Frequency Polygon**

The line graph below shows the number of times Maria drove her car each day of the week.




How many times did Maria drive her car on day 4?

P.S. What type of graph is this?

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## Numerical Summaries of Data

It is often desirable to use a few numbers to summarize a distribution. One important aspect of a distribution is where its center is located. Measures of central tendency are discussed first. A second aspect of a distribution is how spread out it is. In other words, how much the data in the distribution vary from one

another. The second section describes measures of variability

## Notes:

1. Gallup Poll. March 5-8, 2009. http://www.pollingreport.com/enviro.htm
2. For example: http://nces.ed.gov/nceskids/createAgraph/ or http://docs.google.com
3. CNN/Opinion Research Corporation Poll. Dec 19-21, 2008, from http://www.pollingreport.com/civil.htm

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# 5.4: Understanding Statistical Graphs and Tables




Figure **5.4.1**: Data visualizations can help people quickly understand important features of a dataset. (credit: "Group of diverse people having a business meeting" by Rawpixel Ltd/Flickr, CC BY 2.0)

## Visualizing Categorical Data

A bar chart is a visualization of categorical data that consists of a series of rectangles arranged side-by-side (but not touching). Each rectangle corresponds to one of the categories. All of the rectangles have the same width. The height of each rectangle corresponds to either the number of units in the corresponding category or the proportion of the total units that fall into the category.

### Example 5.4.1: Construct Bar Graph

The students in a statistics class were asked to provide their majors. A bar graph is drawn to visualize this data. Answer the following questions.

1. How many people participated in the survey?
2. How many more people plan to major in Biology than in Education?
3. Is the data on the horizontal axis categorical or quantitative?
4. Is this graph a bar chart, a histogram, or a Pareto chart?
5. What is the proportion of students who plan to major in Education?
6. What percentage of students are unsure about their major?

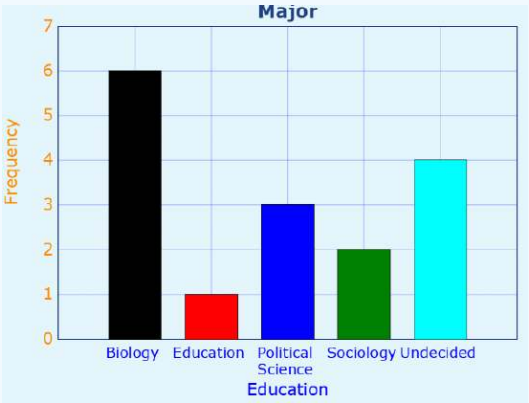


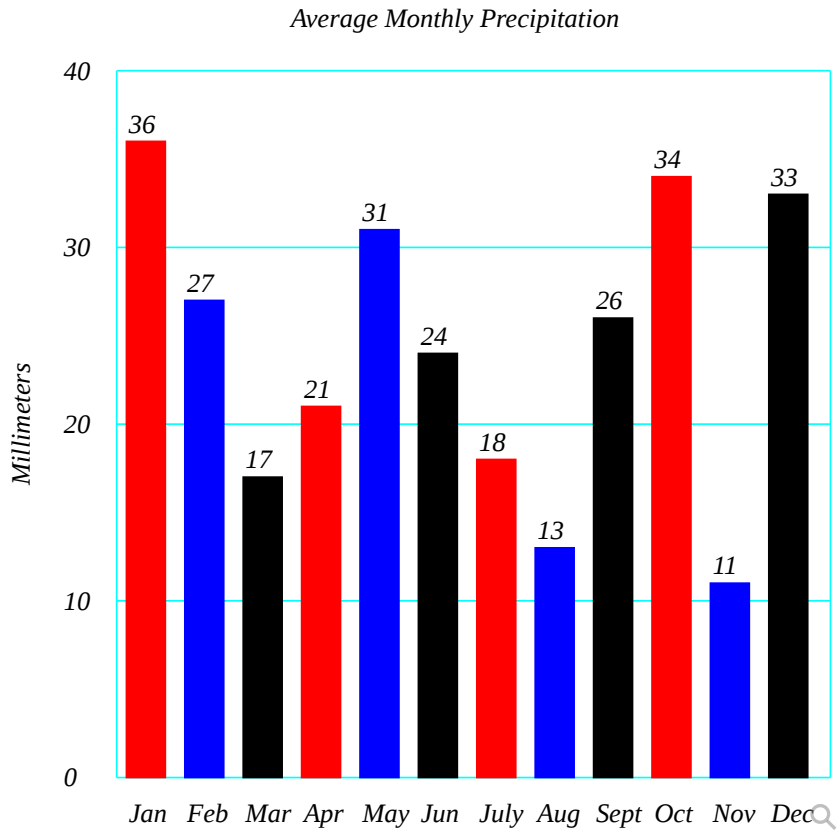

Figure **5.4.1**: Copy and Paste Caption here. (Copyright; author via source)

**Answer**

1. $6+1+3+2+4=16$
2. $6-1=5$ more students majoring in Biology than in Education.
3. Categorical (Qualitative)

4. Not Histogram. Not Pareto chart.
5. $\frac{1}{16} = 0.0625$
6. $\frac{4}{16} = 0.25 = 25\%$

**Your Turn 5.4.1: Average Monthly Precipitation**




Using the graph to answer the following questions:

a) Which month of the year gets the greatest amount of precipitation? Select an answer

b) Which month of the year gets the least amount of precipitation? Select an answer

c) Is the data on the horizontal axis of this chart categorical or quantitative? Select an answer

d) Is this a bar chart or a histogram? Select an answer Explain your thinking.

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If the data we're visualizing is categorical, then we want a quick way to represent graphically the relative numbers of units that fall in each category. When we created the frequency distributions in the last section, all we did was count the number of units in each category and record that number (this was the *frequency* of that category). Frequencies are nice when we're organizing and summarizing data; they're easy to compute, and they're always whole numbers. But they can be difficult to understand for an outsider who's being introduced to your data.

Let's get some practice reading bar graphs.

**Example 5.4.2: Reading Bar Graph**

The bar graph shown gives data on **2020** model year cars available in the United States. Analyze the graph to answer the following questions.

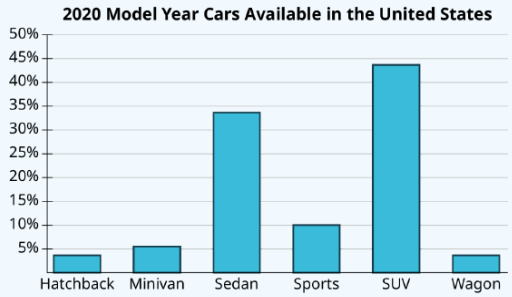


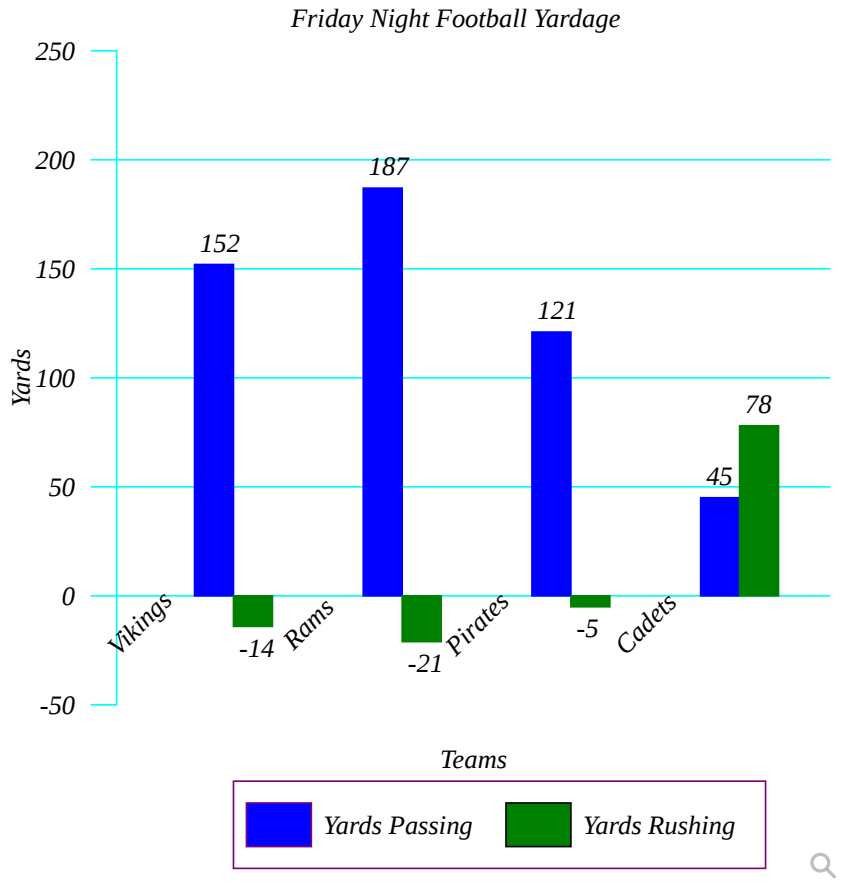

Figure 5.4.3 (data source: consumerreports.org/cars)

1. What proportion of available cars were sports cars?
2. What proportion of available cars were sedans?
3. Which categories of cars each made up less than **5%** of the models available?

**Answer**

1. The bar for sports cars goes up to **10%**, so the proportion of models that are considered sports cars is 10%.
2. The bar corresponding to sedan goes up past **30%** but not quite to **35%**. It looks like the proportion we want is between **33%** and **34%**.
3. We're looking for the bars that don't make it all the way to the **5%** line. Those categories are hatchback and wagon.

**Your Turn 5.4.2: Understanding Bar Graph**




Using the bar graph from above to answer the following questions:

a) Which team gained the greatest amount of yards by rushing? Select an answer

a) Which team gained the greatest amount of yards by rushing? Select an answer

How many yards was it? _____

b) Which team gained the least amount of yards by rushing? Select an answer

How many yards was it? _____

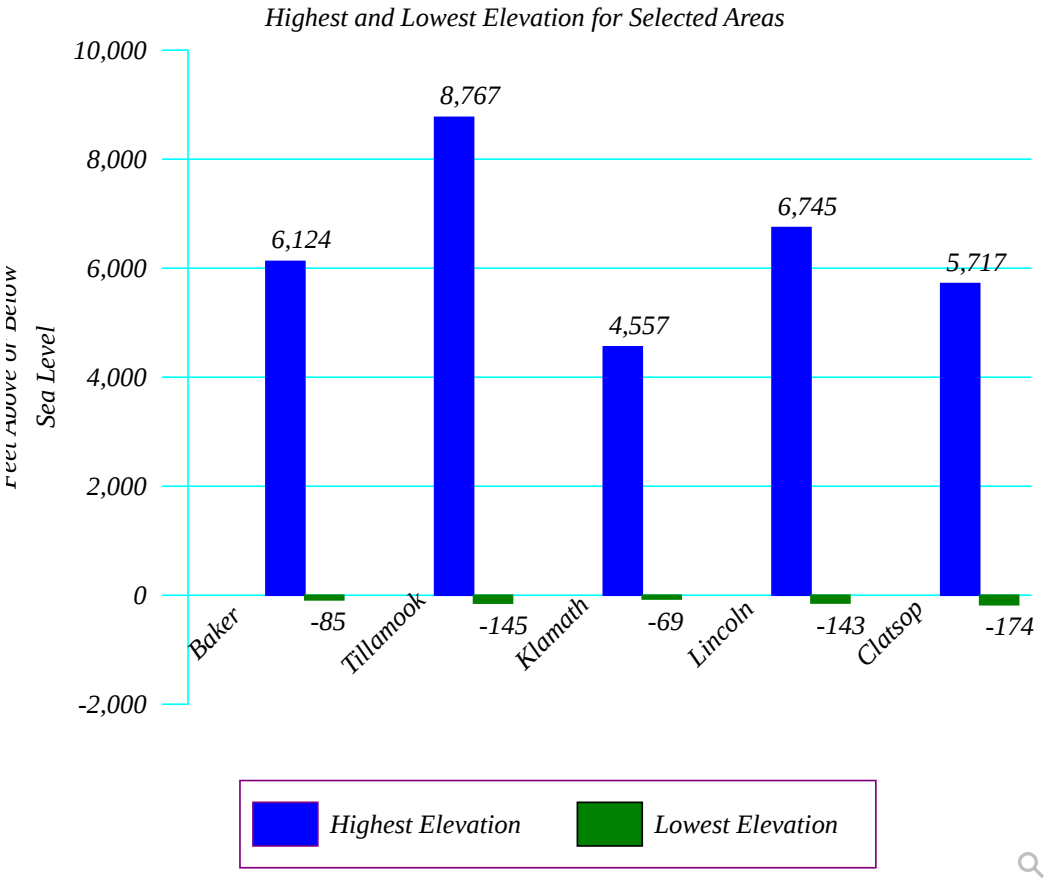
c) Which team gained the least amount of yards by passing? Select an answer

d) Which team gained the greatest amount of yards by passing? Select an answer

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## Your Turn 5.4.3: Reading Bar Graphs




Using the bar graph from above to answer the following questions:

a) What is the difference between the highest elevation in Tillamook and the lowest elevation in Baker ?

_____ feet

b) What is the difference in elevation between the lowest place in Klamath and the lowest place in Baker ?

_____ feet

c) What is the difference in elevation between the highest place in Clatsop and the highest place in Klamath ?

feet

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## Pie Charts

A pie chart consists of a circle divided into wedges, with each wedge corresponding to a category. The proportion of the area of the entire circle that each wedge represents corresponds to the proportion of the data in that category. Pie charts are difficult to create without technology because they require careful measurements of angles and precise circles, both of which are tasks better suited for computers.

### Example 5.4.4: Construct Bar Graph

Use the data that follows to generate a pie chart.

Table 5.4.1 (data source: www.consumerreports.org/cars)

| Type | Percent | Type | Percent |
|---|---|---|---|
| SUV | 43.6% | Minivan | 5.5% |
| Sedan | 33.6% | Hatchback | 3.6% |
| Sports | 10.0% | Wagon | 3.6% |

**Answer**

First, enter the chart above into a new sheet in Google Sheets. Next, click and drag to select the full table (including the header row). Click on the "Insert" menu, then select "Chart." The result may be a pie chart by default; if it isn't, you can change it to a pie chart using the "Chart type" drop-down menu in the Chart Editor.

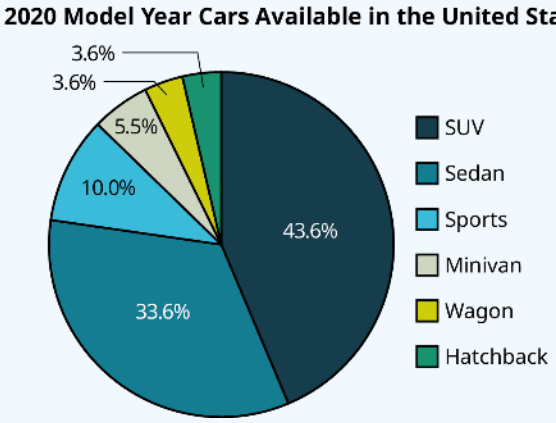
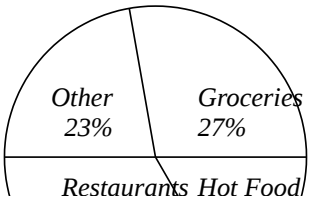


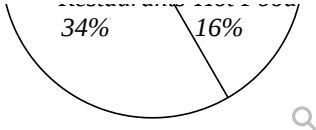

Figure 5.4.4 (data source: consumerreports.org/cars)

You can choose to use a legend to identify the categories, as well as label the slices with the relevant percentages.

### Your Turn 5.4.4: Understanding Pie Chart

Shelby categorized her food spending for this month into four categories: Groceries, Other, Hot Food, and Restaurants. The percents she spent in each category are pictured here.






If Shelby spent a total of $1190 this month, how much did she spend on Groceries?

- $309
- $27
- $321

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**People in Mathematics: Florence Nightingale**

Florence Nightingale (1820–1910) is best remembered today for her contributions in the medical field; after witnessing the horrors of field hospitals that tended to the wounded during the Crimean War, she championed reforms that encouraged sanitary conditions in hospitals. For those efforts, she is today considered the founder of modern nursing.



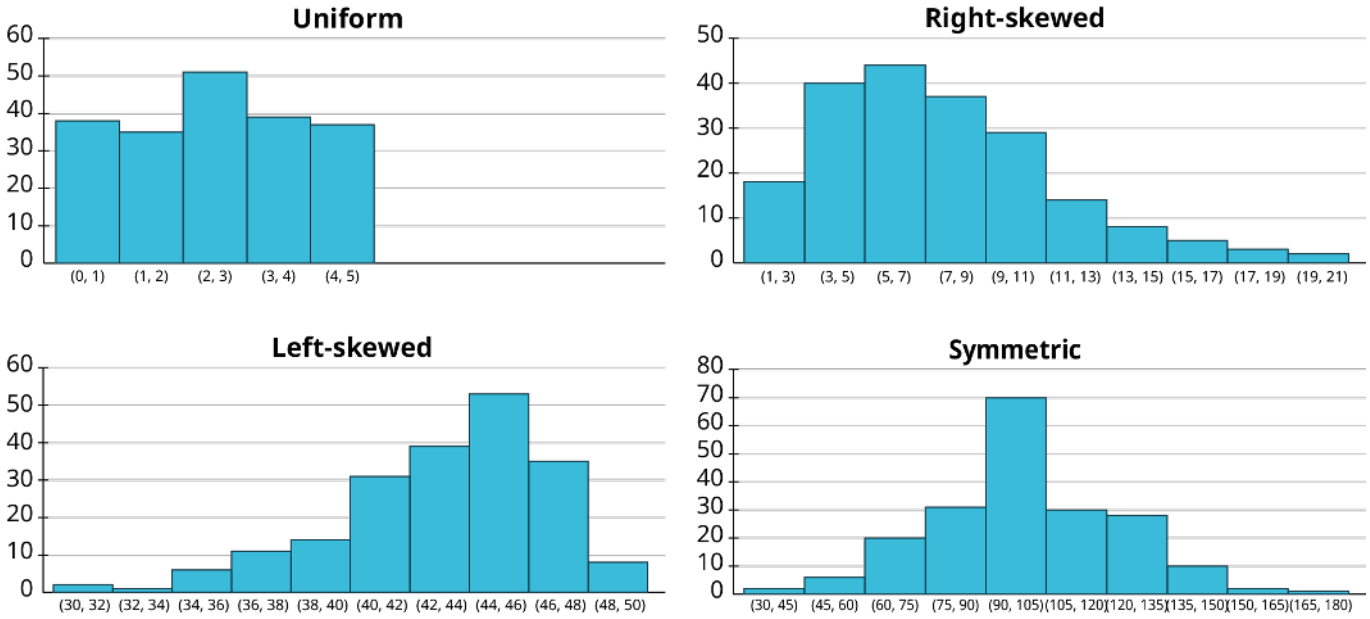
Figure 5.4.5: Florence Nightingale's significant contribution to the field of statistical graphics cannot be understated. (credit: "Florence Nightingale" by Library of Congress Prints and Photographs Division/http://hdl.loc.gov/loc.pnp/pp.print, public domain)

Nightingale is also remembered for her contributions in statistics, especially in the ways we visualize data. She developed a version of the pie chart that is today known as a *polar area diagram*, which she used to visualize the causes of death among the soldiers in the war, highlighting the number of preventable deaths the British Army suffered in that conflict.

In 1859, the Royal Statistical Society honored her for her contributions to the discipline by electing her to join the organization. She was the first woman to be so honored. She was later named an honorary member of the American Statistical Association. Nightingale's status as a revered pioneer in both nursing and statistics is a complex one, because some of her writings and opinions demonstrate a colonialist mindset and disregard for those who lost their lives and lands at the hands of the British. Her core statistical writings indicated that she felt superior to the Indigenous people she was treating. Members of both fields continue to debate her near-iconic role.

## Understating Quantitative Data

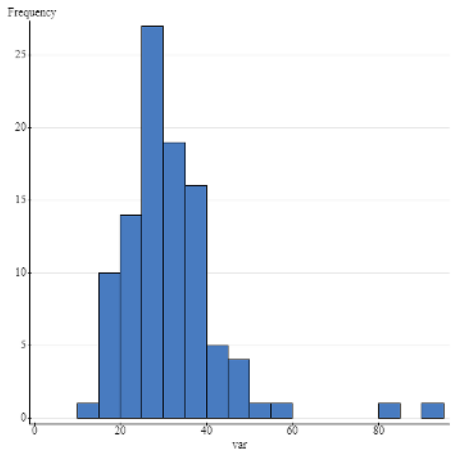
As we see in the previous section, there are four distributions that describe quantitative data. They are: *uniform* (data are equally distributed across the range), *symmetric* (data are bunched up in the middle, then taper off in the same way above and below the middle), *left-skewed* (data are bunched up at the high end or larger values, and taper off toward the low end or smaller values), and *right-skewed* (data are bunched up at the low end, and taper off toward the high end). See below figures.



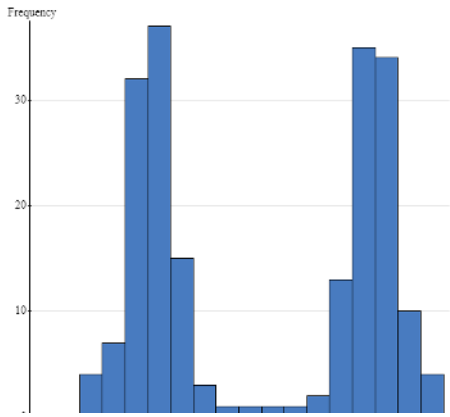

**5.4.6**: Distribution of Histogram

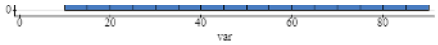
Looking back at the stem-and-leaf plot in the previous example, we can see that the data are bunched up at the low end and taper off toward the high end; that set of data is right-skewed. Knowing the distribution of a set of data gives us useful information about the property that the data are measuring.

**Your Turn 5.4.5**: Identity Distribution

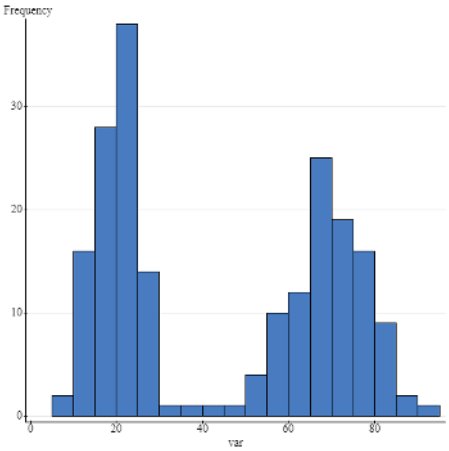



Identify the shape of the distribution of the data ABOVE: Select an answer





Identify the shape of the distribution of the data ABOVE: Select an answer




Identify the shape of the distribution of the data ABOVE: Select an answer

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**Example 5.4.6: Find Information from Histogram**

The data in "AvgSAT" contains the average SAT score for students attending every institution of higher learning in the US for which data is available. Create a histogram in Google Sheets of the average SAT scores. Use bins of width **50**. Are the data uniformly distributed, symmetric, left-skewed, or right-skewed?

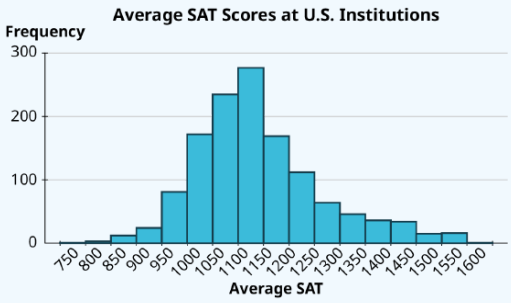


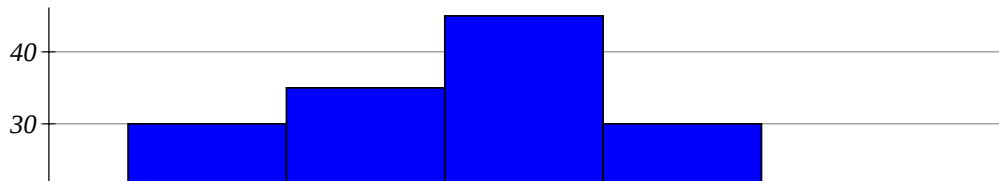

Figure 5.4.7: Fairly Symmetric Graph

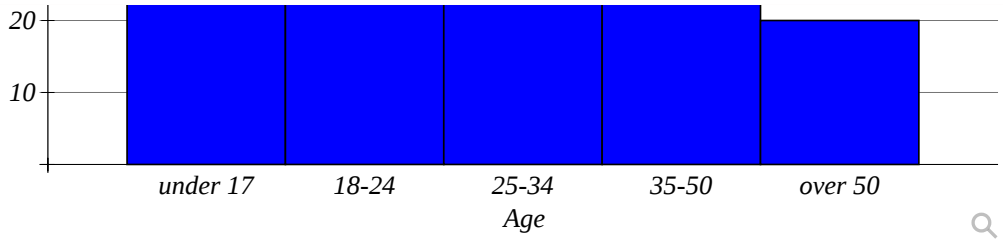
**Answer**

The data are fairly symmetric, but slightly right-skewed.

**Your Turn 5.4.7: Understanding Histogram**

A store surveyed their customers to find out their ages. The bar graph below shows the number of customers in each age group.






What percent of customers surveyed were under 17? Round your answer to 1 decimal place.

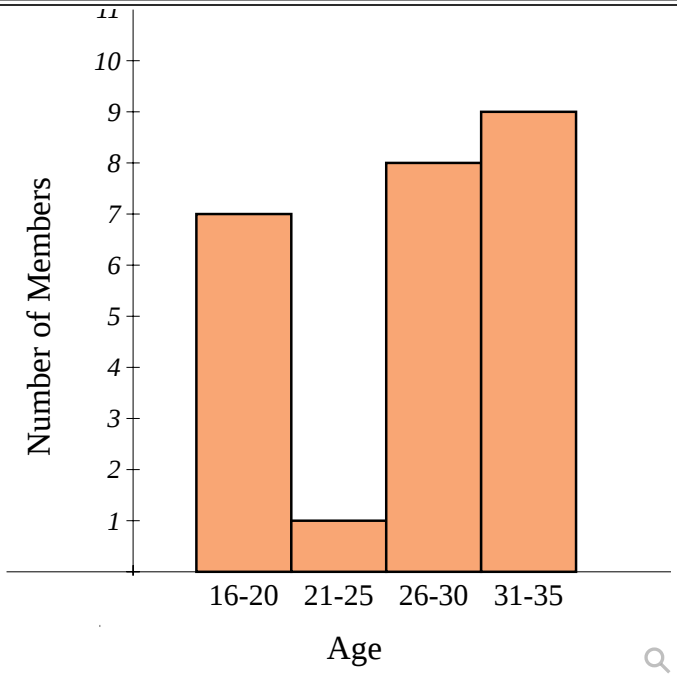
_____ %

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## Your Turn 5.4.8: Understanding Histogram

**Interpreting Histograms**




A frisbee-golf club recorded the ages of its members and used the results to construct the histogram shown above.

Find the total number of members in the club.

_____ members

Find the number of members 30 years of age or younger.

_____ members

Find the percent of total members that are 30 years of age or younger.
***Round the percent to two decimal places as needed.***

percent

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## Misleading Graphs

Graphical representations of data can be manipulated in ways that intentionally mislead the reader. There are two primary ways this can be done: by adjusting the scales on the axes and by manipulating or misrepresenting the areas of the bars. Let's look at some examples of these.

**Example 5.4.9: Misleading Graph**

The table below shows the teams and their payrolls in the English Premier League, the top soccer organization in the United Kingdom.

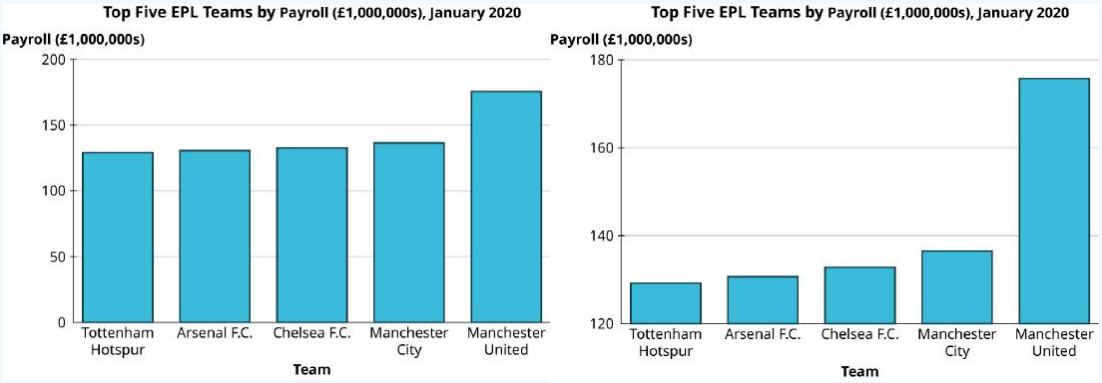



Figure 5.4.8: Misleading Bar Graph

How might someone present this data in a misleading way?

**Answer**

You should notice that despite using the same data, these two graphs look strikingly different. In the second graph, the gap between Manchester United and the other four teams looks significantly larger than in the first graph. The scale on the vertical axis has been manipulated here. The first graph's axis starts at zero, while the lowest value on the second graph's axis is 120. This trick has a strong impact on the viewer's perception of the data.

To further emphasize the difference this creates in our perception, let's examine the data again, but this time using graphics instead of colored areas on our bar graph.

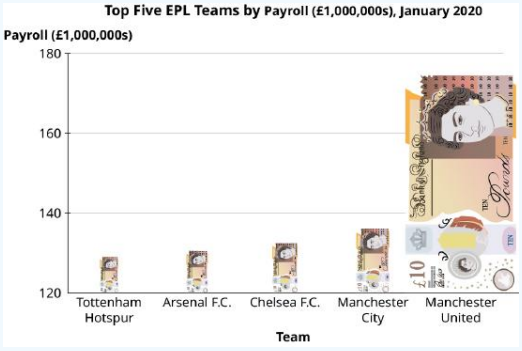



Figure ✔ 5.4.9: (data source: www.spotrac.com)

This graph uses an image of a £10 banknote in place of the bars. Using an image that evokes the context of the data in place of a standard, "boring" bar is a common tool that people use when creating *infographics*. However, this is generally not a good practice because it distorts the data. Notice that our "bars" (the banknotes) are just as tall here as they were in the previous figure. But, to maintain the right proportions, the *widths* had to be adjusted as well, which changes the area (height × width) of each bar. A key point is that when looking at rectangles, the human eye tends to process areas more easily than heights.

Checkpoint

*Beware of infographics! Areas overemphasize a difference that should be measured with a height!*

Now, let's look at all 20 teams. This histogram indicates that the data are right-skewed, with the highest number of teams having a payroll between £40 million and £80 million:

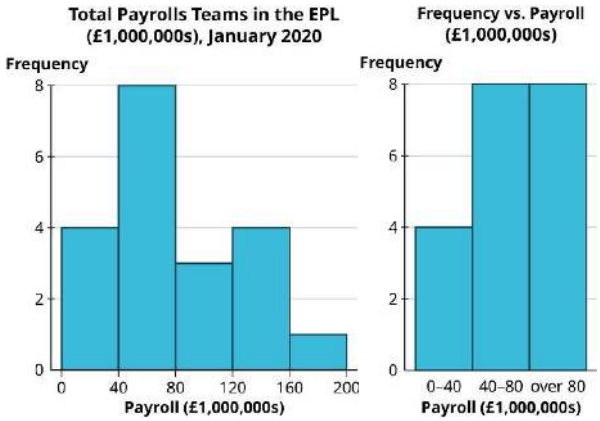



Figure 5.4.10: Misleading Bar Graphs

Even though this chart uses the same data, the skew seems to be reversed. Why? Well, even though this graph *looks* like a histogram, it isn't. Look closely at the labels on the horizontal axis; they don't correspond to spots on the axis, but instead provide a range, meaning this is a bar graph based on a binned frequency distribution.

When we review these ranges, we can see that the last range is misleading as it consists of all data "over 80." If the bins all had the same width, that last bin would run from 80 to 120. However, we can see from the histogram that the maximum value for this data is between 160 and 200. If the last bin in this bar graph were labeled honestly, it would read "80–200," which would drive home the fact that the width of that bar is misleading.

Your Turn 5.4.9: Misleading Graph

**What is misleading with the following graph?**




- ○ Based the numbers, the shorter bar should be half the larger.
- ○ Without labels on the y-axis the data seems really far apart when they are actually close together.
- ○ The labels are wrong.
- ○ The bars should be going horizontally.
- ○ The use of color is misleading.

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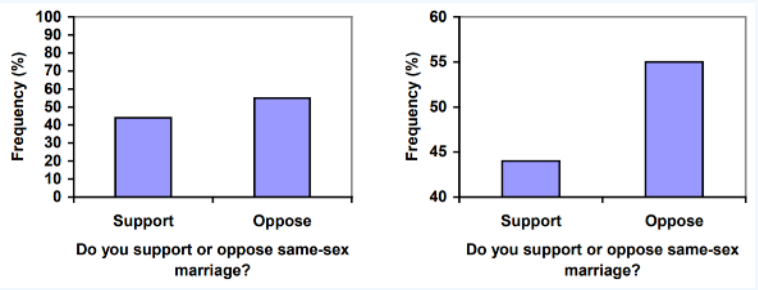
**Checkpoint**

*Always check the horizontal axis on histograms! The widths of all the bars should be equal.*

Minard's chart is remarkable in that it shows not just how the size of Napoleon's army shrank drastically over time, but also the location on the map, the direction the army was traveling at the time, and the temperature during the retreat.

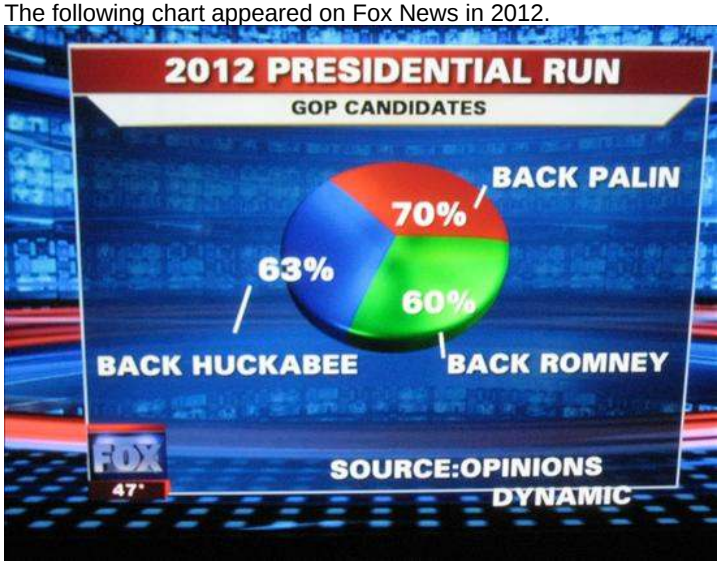
**Example 5.4.10: Misleading Bar Graph**

Compare the two graphs below showing support for same-sex marriage rights from a poll taken in December 2008 [3]. The difference in the vertical scale on the first graph suggests a different story than the true differences in percentages; the second graph makes it appear as though twice as many people oppose marriage rights as support them.




**Your Turn 5.4.10: Misleading Pie Chart**

The following chart appeared on Fox News in 2012.




a. Write 2-4 sentences to describe at least one issue you see in the way the information is represented and how that could lead this graph to be misleading.

Write 1-2 sentences to describe how you would create a graph to more accurately portray the information.

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## 5.5: Measures of Center and Spread

Let's begin by trying to find the most "typical" value of a data set.

Note that we just used the word "typical" although in many cases you might think of using the word "average." We need to be cautious with the word "average" as it has different meanings to different people in various contexts. One of the most common uses of the word "average" is what mathematicians and statisticians call the **arithmetic mean**, or just plain old **mean** for short. "Arithmetic mean" sounds rather fancy, but it is likely you have calculated a mean many times without realizing it; the mean is what most people think of when they use the word "average".

**Definition: Mean**

The **mean** of a set of data is the sum of the data values divided by the number of values.

**Example 5.5.1: Find Mean**

Marci's exam scores for her last math class were: $79, 86, 82, 94$. The mean of these values would be:

$$\frac{79+86+82+94}{4} = 85.25$$

Typically, we round the mean to one more decimal place than the original data. In this case, we would round $85.25$ to $85.3$. Thus, we can say Marci's average score on her math exams was $85.25$ or about $85.3$.

Let's look at the following two examples in the box, and how the mean changes with data value(s)

The $15$ families in a particular neighborhood are asked about their annual household income, rounded to the nearest $\$1,000$.

$$25\ 15\ 35\ 15\ 30\ 15\ 20\ 20\ 50\ 50\ 45\ 50\ 40\ 50\ 50$$

Calculating the mean annual household income

$$\frac{25+15+35+15+30+15+20+20+50+50+45+50+40+50+50}{15} = 34$$

The mean household income of our sample is $34$ thousand dollars ($\$34,00$).

Building on the last example, suppose a new family moves into the neighborhood, with a household income of $\$2$ million ($\$2,000,000$). Adding this to our sample, our mean (after rounding to the nearest dollar) is now:

$$\frac{25+15+35+15+30+15+20+20+50+50+45+50+40+50+50+2,000}{16} = 157$$

The mean household income of our sample is $157$ thousand dollars ($\$157,00$).

Imagine the data values on a see-saw or balance scale. The mean is the value that balances the data, as shown in the picture below.



If we graph our household data, the $\$2$ million data value is so far out to the right that the mean has to adjust up to keep things in balance. The $\$2$ million data value is called an outlier.



For this reason, when working with data that has **outlier** values far outside the primary grouping, it is common to use a different measure of center, the **median**.

**Outliers**

An outlier is a data value that is very different from the rest of the data and is far enough from the center.

**Definition: Median**

The **median** of a set of data is the value in the middle when the data is in order

To find the **median**, follow these simple steps:

Step $1$: Arrange the data. Put all the data values in order from smallest to largest.

Step $2$: Count the number of values.

Let $n$ be the total number of data values.

Step $3$: Find the middle.

- **If $n$ is odd:**
  The median is the middle number of the ordered data set (there is only one middle value). It is the data value in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position or ordered data set.
- **If $n$ is even:**
  The median is the average of the two middle numbers. These are the values in the $\frac{n}{2}^{\text{th}}$ and $\left(\frac{n}{2}+1\right)^{\text{th}}$ positions of the ordered data set.

*We can interpret the median as "half of the data is less than the median, and the other half is more than the median." Of course, we can rewrite this in the context of the problem.*

Look at the following ordered $31$ data.

$$37\ \ 33\ \ 33\ \ 32\ \ 29\ \ 28\ \ 28\ \ 23\ \ 22\ \ 22\ \ 22\ \ 21\ \ 21\ \ 21\ \ 20$$

$$20\ \ 19\ \ 19\ \ 18\ \ 18\ \ 18\ \ 18\ \ 16\ \ 15\ \ 14\ \ 14\ \ 14\ \ 12\ \ 12\ \ 9\ \ 6$$

Since $n = 31$ data values, an odd number, there is only one middle number, which is $\left(\frac{31+1}{2} = 16\right)^{\text{th}}$ data value, which is $20$.

So there are $15$ values below $20$ and $15$ above it.

**Example 5.5.2: Find the Median**

Find the median of these quiz scores:

$$5 \;\; 10 \;\; 8 \;\; 6 \;\; 4 \;\; 8 \;\; 2 \;\; 5 \;\; 7 \;\; 7$$

**Answer**

We start by listing the data in order:

$$2 \;\; 4 \;\; 5 \;\; 5 \;\; 6 \;\; 7 \;\; 7 \;\; 8 \;\; 8 \;\; 10$$

Since there are $10$ data values, an even number, there are two middle numbers. So, we find the mean of the two middle numbers. The position of those two middle numbers are $\left(\frac{10}{2} = 5\right)^{\text{th}}$ and $\left(\frac{10}{2} + 1 = 6\right)^{\text{th}}$. Which are $6$ and $7$.

$$\frac{(6+7)}{2} = 6.5.$$

The median quiz score was $6.5$.

*We can say that half of the quiz scores were lower than* $6.5$*, and the other half were higher than* $6.5$.

**Your Turn 5.5.2: Mean or Median?**

According to Bloomberg News, in 2011 the average annual salary for a player in the National Hockey League (NHL) can be described by the numbers $780,000 and $1,900,000. Which one of these numbers is the mean and which is the median? Explain your reasoning.



- ○ The mean is $780,000 and the median is $1,900,000 since the mean is always smaller than the median.
- ○ The mean is $1,900,000 and the median is $780,000 since there are a small number of hockey players with exceptionally large salaries which increase the mean but have little effect on the median.
- ○ The mean is $780,000 and the median is $1,900,000 since there are a small number of hockey players with exceptionally small salaries which decrease the mean but have little effect on the median.
- ○ The mean is $1,900,000 and the median is $780,000 since the mean is always larger than the median.

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Let us return now to our original household income data. To find the median, let's order the data

$$15 \;\; 15 \;\; 15 \;\; 20 \;\; 20 \;\; 25 \;\; 30 \;\; 35 \;\; 40 \;\; 45 \;\; 50 \;\; 50 \;\; 50 \;\; 50 \;\; 50$$

Since there are $15$ data values, an odd number, the median will be the middle number, the $\frac{15+1}{2} = 8^{\text{th}}$ data value. So $8^{\text{th}}$ data value is $35$.

The median income in this neighborhood sample is $35 thousand. *Thus, half of the households' earned income is less than* $35,000 *and the other half earned more than* $35,000.

If we add the new neighbor with a $2 million household income, then there will be $16$ data values.

$$15 \;\; 15 \;\; 15 \;\; 20 \;\; 20 \;\; 25 \;\; 30 \;\; 35 \;\; 40 \;\; 45 \;\; 50 \;\; 50 \;\; 50 \;\; 50 \;\; 50 \;\; 20{,}00$$

The position of those two middle numbers are $\left(\frac{16}{2} = 8\right)^{\text{th}}$ and $\left(\frac{16}{2} + 1 = 9\right)^{\text{th}}$. Which are $35$ and $40$.

$$\frac{(35+40)}{2} = 37.5$$

As we discovered in the last example, the median is $35 thousand. Notice that the new neighbor did not affect the median in this case. **The median is less affected by outliers than the mean.**

Let's think about the previous example. When we added the $16^{\text{st}}$ family's income, the mean was $157,000 from $34,00. That's a big difference in the average household income. We observe that the mean is influenced by the values of the data; that is, the mean can increase or decrease depending on the values of the data. However, when calculating the median including the $16^{\text{st}}$ family's income, the median wasn't influenced at all.

In fact, the median is generally considered a better statistic for household income, as there is a wide range of income among families. Thus, the values of the data influence the mean, but not the median.

In addition to the mean and the median, there is one other common measurement of the "typical" value of a data set: the **mode**.

**Definition: Mode**

The **mode** is the observed value of the data set that occurs most frequently.

The mode is most commonly used for categorical data, for which the median and mean cannot be computed. Additionally, the mode is the only central tendency used for both categorical and quantitative data. The mean and median are only used with quantitative data.

**Example 5.5.3: Finding Mode**

1. The 15 families in a particular neighborhood are asked about their annual household income, rounded to the nearest $1,000. Find the mode.

$$25\ 15\ 35\ 15\ 30\ 15\ 20\ 20\ 50\ 50\ 45\ 50\ 40\ 50\ 50$$

2. Five real estate exam scores are given below. Find the mode.

$$430, 430, 480, 480, 495$$

**Answer**

1. Mode is $50,000$.
2. The data set is bimodal because the scores $430$ and $480$ each occur twice.

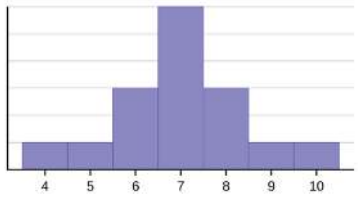
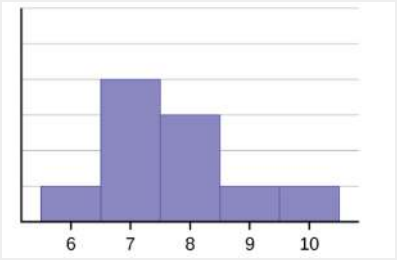
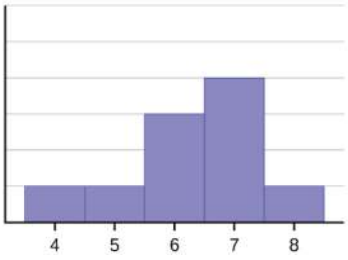
A data set can have more than one mode if several categories have the same frequency, or no modes if every category occurs only once.

## Which Is Better: Mode, Median, or Mean?

If the mode, median, and mean all purport to measure the same thing (centrality), why do we need all three? The answer is complicated, as each measure has its own strengths and weaknesses. The mode is simple to compute, but there may be more than one. Further, if no data value appears more than once, the mode is entirely unhelpful. As for the mean and median, the main difference between these two measures is how each is affected by extreme values.

Consider this example: the mean and median of $1, 2, 3, 4, 5$ are both $3$. But what if the dataset is instead $1, 2, 3, 4, 10$? The median is still $3$, but the mean is now $4$. What this example shows is that the mean is sensitive to extreme values, while the median isn't. This knowledge can help us decide which of the two is more relevant for a given dataset. If it is important that the really high or really low values are reflected in the measure of centrality, then the mean is the better option. If very high or low values are not important, however, then we should stick with the median.

The decision between mean and median only really matters if the data are skewed. If the data are symmetric, then the mean and median are going to be approximately equal, and the distinction between them is irrelevant. If the data are skewed, the mean is pulled in the direction of the skew (i.e., if the data are right-skewed, then the mean will be greater than the median; if the data are left-skewed, the opposite relation holds). The relationship between mean, mode, and median is given in the table below

| | | |
|---|---|---|
| Symmetric Data<br>$4, 5, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 9, 10$ |  | The mean is $7$, the median is $7$, and the mode is $7$.<br>$\text{mean} = \text{mode} = \text{median}$ |
| Skewed Right Data<br>$6, 7, 7, 7, 7, 8, 8, 8, 9, 10$ |  | The mean is $7.7$, the median is $7.5$, and the mode is $7$.<br>$\text{mean} > \text{median} > \text{mode}$ |
| Skew Left Data<br>$6, 7, 7, 7, 7, 8, 8, 8, 9, 10$ |  | The mean is $6.3$, the median is $6.5$, and the mode is $7$.<br>$\text{mean} < \text{median} < \text{mode}$ |

**Example 5.5.4: Which is Better Measure?**

Suppose that in a small town of $50$ people, one person earns $5,000,000$ per year and the other $49$ each earn $30,000$. Which is the better measure of the "center": the mean or the median?

**Answer**

There are $49$ people who earn $30,000$. So we add $30,000$, $49$ times. And one person who earns $5,000,000$.

$$\text{Mean} = \frac{5,000,000 + 49(30,000)}{50} = 129,400$$

$$\text{Median} = 30,000$$

The median is a better measure of the "center" than the mean because $49$ of the values are $30,000$ and one is $5,000,000$. The $5,000,000$ is an outlier. The $30,000$ gives us a better sense of the middle of the data.

Your Turn **5.5.4**: Comparing Mean and Median

A random sample of Print-O-Matic printing company's employee salaries (in dollars) are recorded in the table below.

| Salary |
|---|
| 98837 |
| 65270 |
| 57380 |
| 64553 |
| 73877 |
| 74235 |
| 66346 |
| 64553 |
| 75311 |
| 272500 |

For the data shown above, find the following. Round the answer in the first blank to 2 decimal places. Put the correct units in the second blank.

- Find the mean: _____
- Find the median: _____
- Find the mode: _____

Remove the high CEO salary of 272500 dollars from the data.Recompute the following. Round the answer in the first blank to 2 decimal places. Put the correct units in the second blank.

- Find the mean: _____
- Find the median: _____
- Find the mode: _____

Which of the following statistics was most affected by removing the high CEO salary from the data?

- ○ mode
- ○ median
- ○ mean

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## Measure of Dispression

Consider these three sets of quiz scores:

Section A:

**5 5 5 5 5 5 5 5 5 5**

Section B:

**0 0 0 0 0 10 10 10 10 10**

Section C:

**4 4 4 5 5 5 5 6 6 6**

All three of these sets of data have a mean of **5** and a median of **5**, yet the sets of scores are clearly quite different. In section A, everyone had the same score; in section B, half the class got no points and the other half got a perfect score, assuming this was a **10**-point quiz. Section C was not as consistent as Section A, but not as widely varied as Section B.

In addition to the mean and median, which are measures of the “typical” or “middle” value, we also need a measure of how “spread out” or varied each data set is.

There are several ways to measure this “spread” of the data. The first is the simplest and is called the **range**.

**Definition: Range**

The range is the difference between the maximum value and the minimum value of the data set

Using the quiz scores from above,

For section A, the range is **0** since both maximum and minimum are **5** and $5–5 = 0$

For section B, the range is **10** since $10–0 = 10$

For section C, the range is **2** since $6–4 = 2$

In the last example, the range appears to reveal how spread out the data is. However, suppose we add a fourth section, Section D, with scores

$$0 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 10$$

This section also has a mean and median of **5**. The range is **10**, yet this data set is quite different from Section B. To better illuminate the differences, we'll have to turn to more sophisticated measures of variation.

**Definition: Standard Deviation**

The standard deviation is a measure of variation that indicates the distance each data value deviates from, or differs from, the mean. A few important characteristics:

- Standard deviation is always positive. The standard deviation will be zero if all the data values are equal, and it will increase as the data values spread out.
- Standard deviation has the same units as the original data.
- Standard deviation, like the mean, can be highly influenced by outliers.

Let's use the data from Section D of the quiz score.

$$0 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 5 \quad 10$$

We would like to get an idea of the "average" deviation from the mean, but if we find the average of the values in the second column, the negative and positive values cancel each other out (this will always happen), so to prevent this, we square every value in the second column:

| Data Value | Deviation: Data Value - Mean | $(\text{Deviation})^2$ |
|---|---|---|
| 0 | $0-5=-5$ | $(-5)^2=25$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 5 | $5-5=0$ | $(0)^2=0$ |
| 10 | $10-5=5$ | $(5)^2=25$ |

We then add the squared deviations up to get $25+0+0+0+0+0+0+0+0+25=50$. Ordinarily, we would then divide by the number of scores, $n$, (in this case, 10) to find the mean of the deviations. But we only do this if the data set represents a population; if the data set represents a sample (as it almost always does), we instead divide by $n-1$ (in this case, $10-1=9$) [4].

So, in our example, we would have $\frac{50}{10}=5$ if section D represents a population and $\frac{50}{9}=$ about **5.56** if section D represents a sample. These values (**5** and **5.56**) are called, respectively, the **population variance** and the **sample variance** for section D.

Variance can be a useful statistical concept, but note that the units of variance in this instance would be points-squared since we squared all of the deviations. What are points-squared? Good question. We would rather deal with the units we started with (points in this case), so to convert back we take the square root and get:

$$\text{Population Standard Deviation} = \sqrt{\frac{50}{10}} = \sqrt{5} \approx 2.2$$

$$\text{Sample Standard Deviation} = \sqrt{\frac{50}{9}} \approx 2.4$$

If we are unsure whether the data set represents a sample or a population, we typically assume it is a sample and round answers to one more decimal place than the original data, as we have done above.

**To Compute Standard Deviation:**

1. Find the deviation of each data point from the mean. In other words, subtract the mean from the data value.
2. Square each deviation.
3. Add the squared deviations.
4. Divide by $n$, the number of data values, if the data represents a whole population; **divide by $n$–1 if the data is from a sample.**
5. Compute the square root of the result. (This result is the **standard deviation**.)

**Example 5.5.5: Finding Standard Deviat**

Computing the standard deviation for Section B above, we first calculate that the mean is **5**. Using a table can help keep track of your computations for the standard deviation:

| Data Value | Deviation: Data Value - Mean | $(\text{Deviation})^2$ |
|---|---|---|
| 0 | $0-5=-5$ | $(-5)^2=25$ |
| 0 | $0-5=-5$ | $(-5)^2=25$ |
| 0 | $0-5=-5$ | $(-5)^2=25$ |
| 0 | $0-5=-5$ | $(-5)^2=25$ |
| 0 | $0-5=-5$ | $(-5)^2=25$ |
| 10 | $10-5=5$ | $(5)^2=25$ |
| 10 | $10-5=5$ | $(5)^2=25$ |
| 10 | $10-5=5$ | $(5)^2=25$ |
| 10 | $10-5=5$ | $(5)^2=25$ |
| 10 | $10-5=5$ | $(5)^2=25$ |

Assuming this data represents a population, we will add the squared deviations, divide by $10-1=9$, the number of data values, and compute the square root:

$$\sqrt{\frac{25+25+25+25+25+25+25+25+25+25}{10-1}}=\sqrt{\frac{250}{9}}=5.27$$

Notice that the standard deviation of this data set is much larger than that of section D, since the data in this set is more spread out.

**For comparison, the standard deviations of all four sections are**

| | |
|---|---|
| Section A: 5 5 5 5 5 5 5 5 5 5 | Standard deviation: 0 |
| Section B: 0 0 0 0 0 10 10 10 10 10 | Standard deviation: 5 |
| Section C: 4 4 4 5 5 5 6 6 6 | Standard deviation: 0.8 |
| Section D: 0 5 5 5 5 5 5 5 5 10 | Standard deviation: 2.2 |

**Your Turn 5.5.5: Finding Standard Deviation**

(a) The table below provides some data for a sub-group of European countries. Fill in the blanks for the table and calculate the mean and standard deviation.

**Round all final answers to 2 decimal places**

European Mean = _____

European Standard Deviation = _____

| European Country | $Data$ | $(Data-Mean)$ | $(Data-Mean)^2$ |
|---|---|---|---|
| Germany | 22 | _____ | _____ |
| United Kingdom | 7 | _____ | _____ |
| France | 13 | _____ | _____ |
| Netherlands | 70 | _____ | _____ |
| **Totals** | _____ | | _____ |

(b) The table below provides some data for a sub-group of Asian countries. Fill in the blanks for the table and calculate the mean and standard deviation.

**Round all final answers to 2 decimal places**

Asian Mean = _____

Asian Standard Deviation = _____

| Asian Country | $Data$ | $(Data-Mean)$ | $(Data-Mean)^2$ |
|---|---|---|---|
| China | 49 | _____ | _____ |
| Japan | 66 | _____ | _____ |

| | | | |
|---|---|---|---|
| South Korea | 61 | | |
| India | 64 | | |
| **Totals** | | | |

(c) Which subgroup of trade partners has more variability?

- ○ European
- ○ Asian
- ○ Neither

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Where standard deviation is a measure of variation based on the mean, **quartiles** are based on the median.

**Definition: Quartiles**

Quartiles are values that divide a dataset into four equal parts, with each part containing **25%** of the data points

The first quartile $Q_1$ is the value so that **25%** of the data values are below or equal to it. The third quartile $Q_3$ is the value so that **75%** of the data values are less than or equal to it.

You may have guessed that the second quartile is the same as the median, $Q_1$. The median is the value such that **50%** of the data values are below or equal to it.

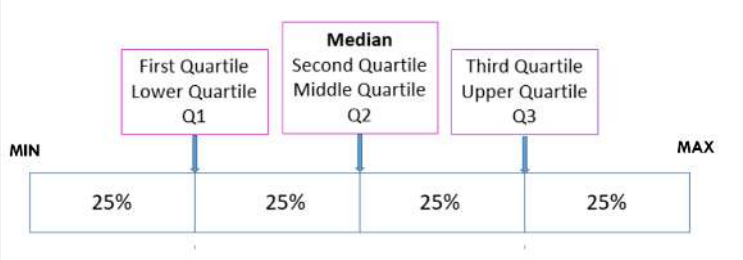



Figure **5.5.1**: Quartiles (QT)

Quartiles divide the data into quarters, as we see above Figure **5.5.1**.

- **25%** of the data is between the minimum and $Q_1$.
- **25%** of the data is between $Q_1$ and the median ($Q_2$).
- **50%** of the data is between $Q_1$ and $Q_3$.
- **75%** of the data is above $Q_3$.

First Quartile ≡ Lower Quartile ≡ $Q_1$.

Second Quartile ≡ Median, $Q_2$ ≡ Middle Quartile.

Third Quartile ≡ Upper Quartile ≡ $Q_3$.

While quartiles are not a single-number summary of variation like standard deviation, they are used in conjunction with the median, minimum, and maximum values to form a **5-number summary** of the data.

**Definition: Five-Number Summary**

The five-number summary takes this form

Minimum, $Q_1$, Median, $Q_3$, Maximum

How to find the quartiles in the data set?

**How to Find $Q_1$ and $Q_3$**

1. Begin by ordering the data from smallest to largest.
2. Find the median of the data values, which is the middle quartile.
3. Find the median of the data values that are smaller than the median. Those data values are referred to as the lower half. Which is the lower quartile, $Q_1$.
4. Find the median of the data values that are larger than the median. Those data values are referred to as the upper half. Which is the upper quartile, $Q_3$.

Let's find the quartiles for the ordered data given below

**45 47 52 52 53 55 56 58 62 80**

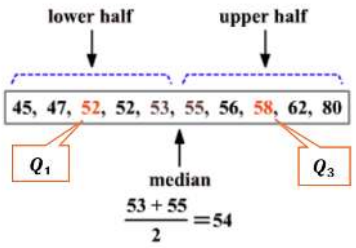



Figure **5.5.2**: Finding Quartiles [FQ]

Let's look at some examples. We can also calculate the five-number summary using calculators or scientific software such as Excel, Minitab, or R. However, in this course, we only get our feet wet with statistics, so we can calculate these values quickly by hand.

**Example 5.5.6**: Finding Quartiles

Suppose we have measured **9** females and their heights (in inches), sorted from smallest to largest, are given below. Find the Quartiles.

$$59 \quad 60 \quad 62 \quad 64 \quad 66 \quad 67 \quad 69 \quad 70 \quad 72$$

**Answer**

Median is **66** inches. We can say that **50%** or $\frac{1}{2}$ of females are shorter than or equal to **66** inches, and the other **50%** or $\frac{1}{2}$ are taller than **65** inches.

Data values that are smaller than the medians are

$$59 \quad 60 \quad 62 \quad 64$$

$$\text{Lower Quartile} = \frac{(60+62)}{2} = 61$$

We can say that **25%** or $\frac{1}{4}$ of females are shorter than or equal to **61** inches, and the other **75%** or $\frac{3}{4}$ are taller than **61** inches.

Data values that are larger than the medians are

$$67 \quad 69 \quad 70 \quad 72$$

$$\text{Upper Quartile} = \frac{(69+70)}{2} = 69.5$$

We can say that **75%** or $\frac{3}{4}$ of females are shorter than or equal to **69.5** inches, and the other **25%** or $\frac{1}{4}$ are taller than **69.5** inches.

**Your Turn 5.5.6**: Find Quartiles

Find the following measures of dispersion for the data below:

| | | | | |
|---|---|---|---|---|
| 5 | 9 | 6 | 12 | 8 |
| 10 | 16 | 15 | 12 | 20 |
| 19 | 18 | 3 | 16 | 14 |
| 13 | 10 | 16 | 9 | |

**Sorted Data**

_____

**Minimum**

_____

**Quartile 1**

_____

**Median**

_____

**Quartile 3**

_____

**Maximum**

_____

**Range**

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The 5-number summary combines the first and third quartiles with the minimum, median, and maximum values.

In the example with a sample of 9 females, the median is 66, the minimum is 59, and the maximum is 72. Hence, the 5-number summary is:

59 61 66 69.5 72

Note that the five-number summary divides the data into four intervals, each of which will contain about 25% of the data. For visualizing data, there is a graphical representation of a five-number summary called a box plot, or box and whisker graph.

For visualizing data, there is a graphical representation of a five-number summary called a **box plot**, or box and whisker graph.

**Definition: Box Plot**

A **box plot** is a graphical representation of a five-number summary.

To create a box plot, a number line is first drawn with equidistant tick marks. A box is drawn from the first quartile to the third quartile, and a line is drawn through the box at the median. "Whiskers" are extended out to the minimum and maximum values.

**Example 5.5.7: Five Number Summary**

What is the box plot of the data in ✔ example 5.5.6?

**Answer**

The box plot below is based on the 5-number summary from the sample of 9 female heights:

59 61 66 69.5 72

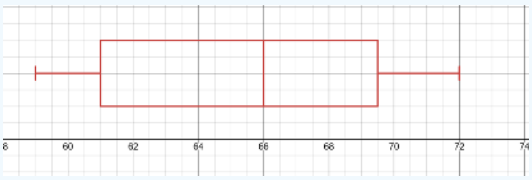



Figure 5.5.3: Box Plot

**Your Turn 5.5.7: Five Number Summary**

Find the 5 number summary for the data shown

| | |
|---|---|
| 2 | 8 |
| 11 | 17 |
| 39 | 41 |
| 52 | 53 |
| 56 | 62 |
| 74 | 75 |
| 79 | 84 |
| 88 | |

5 number summary:

_____, _____, _____, _____, _____

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Box plots are particularly useful for comparing data from two populations or samples. In fact, when comparing two samples, it is always preferable to use box plots.

**Example 5.5.8: Reading Box Plot**

The box plot of service times for two fast-food restaurants is shown below.

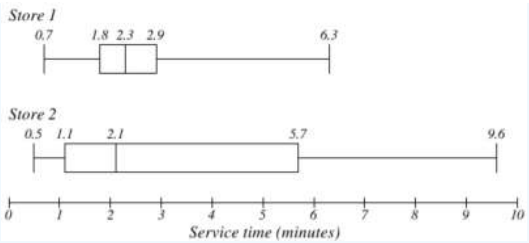



Figure **5.5.4**: Side by Side Two Box Plot

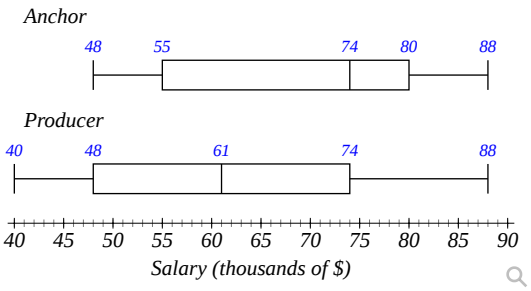
1. Which fast-food restaurant has a smaller median service time but a more spread-out distribution of service times?
2. In Store **1**, what percentage of customers were served within **2.9** minutes?
3. In Store **1**, **75%**of customers were served in less than what amount of time?
4. In Store **2**, what percentage of customers waited more than **5.7** minutes?
5. If Alex waited **4 minutes**, and this wait time is longer than that of **90%** of customers, was he in Store **1** or **Store 2**?

**Answer**

1. While Store **2** had a slightly shorter median service time (**2.1** minutes vs. **2.** minutes), Store **2** is less consistent, with a wider spread of the data.
2. In Store **1**, **75%** of customers were served within **2.9** minutes.
3. In Store **2**, **75%**of customers were served within less than **5.7** minutes.
4. **25%**
5. If Alex waited **4** minutes, and this wait time is longer than that of **90%** of customers, he is in Store **1**.

**Your Turn 5.5.8**: Reading Box Plot

The boxplots below show salaries for tv news producers and tv news anchors.




a. Hayden makes the minimum salary for a tv news anchor. Stephanie makes the minimum salary for a tv news producer.

Who makes more money? Select an answer

b. Jasmine makes $79,000, which is the 58th percentile of salaries in her occupation. Is Jasmine a tv news producer or a tv news anchor? Select an answer

c. TRUE or FALSE: The dataset for tv news producer salaries has more values (e.g., a larger *n*) than the dataset for tv news anchor salaries.

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## Weighted Average

Have you considered how your grade point average (GPA) is calculated? Your business program requires the successful completion of many courses. Your grades in each course combine to determine your GPA; however, not every course necessarily has the same level of importance as measured by your course credits.

Perhaps your math course takes one hour daily while your communications course is only delivered in one-hour sessions three times per week. Consequently, the college assigns the math course five credit hours and the communications course three credit hours. If you want an average, these different credit hours mean that the two courses do not share the same level of importance, and therefore a simple average cannot be calculated.

In a weighted average, not all pieces of data share the same level of importance or they do not occur with the same frequency. The data cannot represent a percent change or a series of numbers intended to be multiplied with each other. To calculate a weighted average, you require two components:

1. The data itself—you need the value for each piece of data.
2. The weight of the data—you need to know how important each piece of data is to the average. This is either an assigned value or a reflection of the number of times each piece of data occurs (the frequency).

To calculate a weighted average, add the products of the weights and data for the entire data set, and then divide this total by the total of the weights.

## Example 5.5.9: Find GPA

A mark transcript received by a student at a local college. The chart shows how the grade translates into a grade point.

Calculate the student's grade point average (GPA). Round your final answer to two decimals.

| Course | Letter Grade | Credit hours (Weight) |
|---|---|---|
| ENC 1100 | B | 4 |
| MAC 2302 | A | 5 |
| PSY 1203 | B+ | 3 |
| PHY 2017 | C | 4 |
| HUM 1048 | A+ | 3 |
| MGF 1130 | D | 4 |

| Grade | Grade Point Value |
|---|---|
| A+ | 4.5 |
| A | 4.0 |
| B+ | 3.5 |
| B | 3.0 |
| C+ | 2.5 |
| C | 2.0 |
| D | 1.0 |
| F | 0.0 |

**Answer**

The courses do not carry equal weights as they have different credit hours. Therefore, to calculate the GPA, you must find a weighted average.

| Course | Grade | Grade Point (Data Value) | Credit hours (Weight) | Value x Weight |
|---|---|---|---|---|
| ENC 1100 | B | 3.0 | 4 | $3 \times 4 = 12$ |
| MAC 2302 | A | 4.0 | 5 | $4 \times 5 = 20$ |
| PSY 1203 | B+ | 3.5 | 3 | $3.5 \times 3 = 10.5$ |
| PHY 2017 | C | 2.0 | 4 | $2 \times 4 = 8$ |
| HUM 1048 | A+ | 4.5 | 3 | $4.5 \times 3 = 13.5$ |
| MGF 1130 | D | 1.0 | 4 | $1 \times 4 = 4$ |
| TOTAL | | | 23 | 68 |

$$\text{GPA} = \frac{68}{23} = 2.96$$

## Your Turn 5.5.9: Find GPA

| FALL QUARTER 2024 | | |
|---|---|---|
| Course | Letter Grade | Credits |
| CHEM 140 | B+ | 3 |
| CHEM 141 | A | 2 |
| ENGL 101 | C- | 5 |
| MATH&151 | D | 5 |

| Grade | Grade point values |
|---|---|
| A | 4.0 |
| A- | 3.7 |
| B+ | 3.3 |
| B | 3.0 |
| B- | 2.7 |
| C+ | 2.3 |
| C | 2.0 |
| C- | 1.7 |
| D+ | 1.3 |
| D | 1.0 |
| F | 0.0 |

The above data comes from a Álvaro's transcript. Using the transcript and the conversion chart calculate the GPA for Álvaro for FALL QUARTER 2024 to two decimal places.

The GPA for Álvaro for FALL QUARTER 2024 is _____

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**Example 5.5.7: Weighted Average**

In your Mathematical Thinking class, your final grade is based on six categories. The categories are tests, labs, homework assignments, and a final exam. The final average for the course is the weighted average of scores earned in these categories with the following weights.

| Assignments | Tests | Project | Homework | Final Exam | Quiz | Discussion |
|---|---|---|---|---|---|---|
| Weights | 50% | 5% | 15% | 20% | 5% | 5% |

Suppose you earned the following grades on average in each of the categories: **85%** on tests, **0%** on the project,**90%** on homework assignments, **50%** on the final exam, **80%** on the quiz, and **100%** on the discussion. Determine your weighted average in the course. Record the average below as a percentage accurate to two decimal places.

**Answer**

Multiply every group grade average by its associated weight.

| Course | Weight | Average Score (Data Value) | Value x Weight |
|---|---|---|---|
| Tests | 0.50 | 85% | $0.50 \times 85\% = 42.5\%$ |
| Project | 0.05 | 0% | $0.05 \times 0\% = 0\%$ |
| Homework | 0.15 | 90% | $0.15 \times 90\% = 13.5\%$ |
| Final Exam | 0.20 | 50% | $0.20 \times 50\% = 10\%$ |
| Quiz | 0.05 | 80% | $0.05 \times 80\% = 4\%$ |
| Discussion | 0.05 | 100% | $0.05 \times 100\% = 5\%$ |
| Total (Weighted Average) | 1 | | 75% |

$$\text{Weight Average in the Course} = \frac{42.5\% + 0\% + 13.5\% + 10\% + 4\% + 5\%}{1} = 75\%$$

**Your Turn 5.5.10: Find Weighted Average**

In your precalculus class, your final grade is based on four categories. The categories are tests, labs, homework assignments, and a final exam. The final average for the course is the weighted average of scores earned in these categories with the following weights.

| Assignments | Tests | Labs | Homework | Final Exam |
|---|---|---|---|---|
| Weights | 38% | 15% | 16% | 31% |

Suppose you earned the following grades on each of the categories; 83% on tests, 82% on labs, 98% on homework assignments, and 48% on the final exam. Determine your weighted average in the course. Record the average below as a percentage accurate to two decimal places.

Course Average: _____ %

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Notes:

4. The reason we do this is highly technical, but we can see how it might be useful by considering the case of a small sample from a population that contains an outlier, which would increase the average deviation: the outlier very likely won't be included in the sample, so the mean deviation of the sample would underestimate the mean deviation of the population; thus, we divide by a slightly smaller number to get a slightly bigger average deviation.

5. van Vliet, P.K. and Gupta, J.M. (1973) Sodium bicarbonate in idiopathic respiratory distress syndrome. *Arch. Disease in Childhood*, **48**, 249–255. As quoted on http://openlearn.open.ac.uk/mod/oucontent/view.php?id=398296§ion=1.1.3

[QT] https://www.onlinemathlearning.com/quartile.html

[FQ] https://math.fandom.com/wiki/Interquartile_range

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# 5.6: Normal Distribution and Percentiles

When you take an exam, what is often as important as your actual score on the exam is the way your score compares to other students' performance. If you made a $70$ but the average score (whether the mean, median, or mode) was $85$, you did relatively poorly. If you made a $70$ but the average score was only $55$ then you did relatively well. In general, the significance of one observed value in a data set strongly depends on how that value compares to the other observed values in a data set. Therefore, we wish to attach a number to each observed value that measures its relative position.

As we know, quartiles in the previous section. Now we will introduce the percentile.

## Percentiles

Anyone who has taken a national standardized test is familiar with the idea of being given both a score on the exam and a "percentile ranking" of that score. You may be told that your score was $625$ and that it is the $85^{th}$ percentile. The first number tells how you actually did on the exam; the second says that $85\%$ of the scores on the exam were less than or equal to your score, $625$.

A **percentile** is a measure of ranking. It represents a location measurement of a data value to the rest of the values. Many standardized tests give the results as a percentile. Doctors also use percentiles to track a child's growth.

**Definition: percentile of data**

The **kth percentile** is the data value that has $k\%$ of the data at or below that value.

The P$^{th}$ percentile cuts the data set in two so that approximately $P\%$ of the data lie below or equal to it and $(100-P)\%$ of the data lie above it. In particular, the three percentiles that cut the data into fourths are called the quartiles of a data set. The quartiles are the three numbers $Q_1$, $Q_2$, $Q_3$ that divide the data set approximately into fourths.a

Given an observed value $x$ in a data set, $x$ is the $P^{th}$ percentile of the data if $P\%$ of the data are less than or equal to $x$. The number $P$ is the percentile rank of $x$.

**Example 5.6.1: Interpreting Percentile**

a. What does a score of the $90^{th}$ percentile mean? If a sample consists of $800$ test scores, how many of them would be *at or below* the $90^{th}$ percentile?
b. How many scores are above the $90^{th}$ percentile?

**Answer**

1. This means that $90\%$ of the scores were at or below this score. (A person did better than $90\%$ of the test takers.)

$$0.90 \times 800 = 720$$

So $720$ score would be below or at $90^{th}$ percentile.
2. $800 - 720 = 80$ of the scores were above $90^{th}$ percentile.

**Example 5.6.2: Percentile Vs Score**

If the test was out of $100$ points and you scored at the $80^{th}$ percentile, what was your score on the test?

**Answer**

You don't know! All you know is that you scored the same as or better than $80\%$ of the people who took the test. If all the scores were low, you could have still failed the test. On the other hand, if many of the scores were high, you could have gotten.

**Your Turn 5.6.2: Interpret Percentile**

Complete the following statements.

a. In general, _____ % of the values in a data set lie *at or below* the $76^{th}$ percentile.

b. _____ % of the values in a data set lie *above* the $68^{th}$ percentile.

c. If a sample consists of 500 test scores, _____ of them would be *at or below* the $82^{th}$ percentile .

d. If a sample consists of 500 test scores, _____ of them would be *above* the $18^{th}$ percentile.

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## $z$-Scores and Normal Distribution

You have probably seen a "bell-shaped curve" before. It turns out that many data sets, particularly when the sample size is sufficiently large, are distributed symmetrically, with most data values around the center (mean) and few outliers, resulting in a bell-shaped curve. This is called a "normal distribution." For instance, the weights of all the people in a certain country are likely distributed normally, and so are their heights. Normal distributions can describe all sorts of data involving people's cholesterol levels, blood pressure, duration of pregnancy, IQ (intelligence quotient) scores, credit scores, and many more. For instance, the pregnancy (gestation) period for humans is known to have an average of 266 days (38 weeks) with a standard deviation of 16 days. The average IQ score is defined as $100$ with a standard deviation of $15$ . Your SAT and other standardized test scores are defined in similar ways.

But what does it mean, and how does this information help us?

When data values are distributed normally, given their mean and standard deviation, we can then standardize the normal distribution and calculate various percentages related to scores/values. For example, given that the IQ scores have a mean of $100$ and a standard deviation of $15$ , one can conclude that only about$2.3\%$ of the population has an IQ of $130$ or higher.

Here is how this works. First, let us define the "Standard Normal Distribution."

**Definition: Standard Normal Distribution**

The standard normal distribution is a normal distribution (bell-shaped curve) with a mean of 0 and a standard deviation of 1.

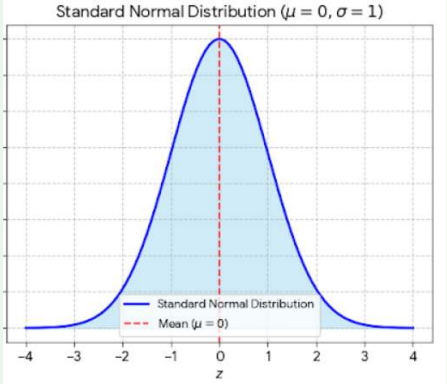



Figure $5.6.1$: Standard Normal Distribution with Mean $0$ and Standard Deviation $1$.

It turns out that there is a simple way to "convert" any normal distribution to the standard one. It involves what is known as the "$z$-score."

Another way to locate a particular observation $x$ in a data set is to compute its distance from the mean in units of standard deviation. The $z$-score indicates how many standard deviations an individual observation $x$ is from the center of the data set, its mean. It is used on distributions that have been *standardized*, which allows us to better understand its properties. If $z$ is negative, then $x$ is below average. If $z$ is $0$ then $x$ is equal to the average. If $z$ is positive, then $x$ is above the average.

**What is z-score?**

A **z-score** tells you how many standard deviations a value is away from the mean (average)

- Positive Z-score: The value is above average.
- Negative Z-score: The value is below average.
- Z-score of $0$: The value is exactly the average.

The following formula is used to calculate the z-score (z) of a data value (x) when the mean, standard deviation, and data value (x) are provided

**FORMULA**

$$z-\text{score of data} = \frac{\text{data value}(x) - \text{mean (M)}}{\text{standard deviation (SD)}}$$

Sometime $z$ score of data value $x$ is denoted by $z_x$.

**Another notation:**

Mean: $\mu$ or M

Standard Deviation: $\sigma$ or SD

**Example 5.6.3**: Finding z-Score

On a nationwide math test, the mean was **65** and the standard deviation was **10**. If Robert scored **81**, what was his $z$–score?

**Answer**

$$\begin{aligned} z &= \frac{x - \text{M}}{\text{SD}} \\ &= \frac{81 - 65}{10} \\ &= \frac{16}{10} \\ &= 1.6 \end{aligned}$$

This means Robert's score was **1.6** standard deviations above the mean.

**Your Turn 5.6.3**: Find z-score

A set of data items is normally distributed with a mean of $102$ and a standard deviation of $26$ . Convert each of the following data items to a z-score. Round your answer to the nearest hundredth.

(a) $108$

z score = _____

(b) $28$

z score = _____

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The following formula is used to calculate the data value (x) when the mean, standard deviation, and z-score are provided.

**FORMULA**

$$\text{data} = \text{mean} + z - \text{data value} \times \text{standard deviation}$$

**Example 5.6.4**: Find the Score or Data Value

A math test has mean **70** and standard deviation **8**. A student has a z-score of **1.5** on this test. What is the student's actual score?

**Answer**

Here, we need to find the student's score (data value), denoted as $x$.

$$\begin{aligned} \text{data}(x) &= \text{M} + z - \text{score} \times \text{SD} \\ &= 70 + 1.5 \times 8 \\ &= 82 \end{aligned}$$

This means Robert's score was **82** corresponding to z-score **1.5**.

**Your Turn 5.6.4**: Find the Score or Data Value

A set of data items is normally distributed with a mean of $17$ and a standard deviation of $4.5$. Find the data value in the distribution that corresponds to each of the following z-scores. Round your answers to the nearest tenth.

(a) z = $-1.25$

(b) z = $1.72$

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**Example 5.6.5: Compare Using z-Score**

A server at a restaurant tracks their weekly tips earned and hours worked. Over several weeks, the average weekly tips earned is $500 with a standard deviation of $100. This week, the server earned $650 in tips. Meanwhile, the average number of hours worked per week is 30 hours with a standard deviation of 5 hours, and this week, the server worked 400 hours.

Using z-scores, determine whether the server's weekly tips or weekly hours worked deviate more from the average, indicating where their performance was stronger relative to past trends.

**Answer**

Using the z-score formula:

z-score for weekly tips:

$$z = \frac{650 - 500}{100} = \frac{150}{100} = 1.5$$

z-score for weekly hours worked:

$$z = \frac{40 - 30}{5} = \frac{10}{5} = 2.0$$

The z-score for hours worked 92 is higher than the z-score for tips earned 1.5. This means that, relative to their average, the server worked significantly more hours than usual, compared to the amount of tips they earned. Therefore, their performance in terms of hours worked deviated more from the average than their tips.

**Your Turn 5.6.5: Who is Taller?**

The heights of adult men in America are normally distributed, with a mean of $69.8$ inches and a standard deviation of $2.63$ inches. The heights of adult women in America are also normally distributed, but with a mean of $64.1$ inches and a standard deviation of $2.55$ inches.

a) If a man is $6$ feet $3$ inches tall, what is his z-score (to two decimal places)?

z score of a man = _____

b) If a woman is $5$ feet $11$ inches tall, what is her z-score of woman (to two decimal places)?

z score of woman = _____

c) Who is relatively taller?

- ○ The 6 foot 3 inch American man

○ The 5 foot 11 inch American woman

(Hint: Make sure to change height in inches)

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**Z-Score Percentile Table**

A percentile table (or z-table) is a reference chart that converts a z-score into a **percentile rank**. A percentile tells you the percentage of people or data points that scored below you. If you want to find the percentile for a given z score. You can use the following table.

Let's say you have a data value, and if you calculate a Z-score for your data value, it is **0.20**. You then look it up in the table and find the corresponding percentile value,**57.93**.

Meaning: This indicates that **57.93%** of all observations are less than or equal to your value. In other words, your score is in the **58**th percentile. In other words, only **100** observations are more than your score.

z-Scores and Percentiles

| z-Score | Percentiles | z-Score | Percentiles | z-Score | Percentiles | z-Score | Percentiles |
|---|---|---|---|---|---|---|---|
| -4.0 | 0.003 | -1.0 | 15.87 | 0.0 | 50.00 | 1.1 | 86.43 |
| -3.5 | 0.02 | -0.95 | 17.11 | 0.05 | 51.99 | 1.2 | 88.49 |
| -3.0 | 0.13 | -0.90 | 18.41 | 0.10 | 53.98 | 1.3 | 90.32 |
| -2.9 | 0.19 | -0.85 | 19.77 | 0.15 | 55.96 | 1.4 | 91.92 |
| -2.8 | 0.26 | -0.80 | 21.19 | 0.20 | 57.93 | 1.5 | 93.32 |
| -2.7 | 0.35 | -0.75 | 22.66 | 0.25 | 59.87 | 1.6 | 94.52 |
| -2.6 | 0.47 | -0.70 | 24.20 | 0.30 | 61.79 | 1.7 | 95.54 |
| -2.5 | 0.62 | -0.65 | 25.78 | 0.35 | 63.68 | 1.8 | 96.41 |
| -2.4 | 0.82 | -0.60 | 27.43 | 0.40 | 65.54 | 1.9 | 97.13 |
| -2.3 | 1.07 | -0.55 | 29.12 | 0.45 | 67.36 | 2.0 | 97.72 |
| -2.2 | 1.39 | -0.50 | 30.85 | 0.50 | 69.15 | 2.1 | 98.21 |
| -2.1 | 1.79 | -0.45 | 32.64 | 0.55 | 70.88 | 2.2 | 98.61 |
| -2.0 | 2.28 | -0.40 | 34.46 | 0.60 | 72.57 | 2.3 | 98.93 |
| -1.9 | 2.87 | -0.35 | 36.32 | 0.65 | 74.22 | 2.4 | 99.18 |
| -1.8 | 3.59 | -0.30 | 38.21 | 0.70 | 75.80 | 2.5 | 99.38 |
| -1.7 | 4.46 | -0.25 | 40.13 | 0.75 | 77.34 | 2.6 | 99.53 |
| -1.6 | 5.48 | -0.20 | 42.07 | 0.80 | 78.81 | 2.7 | 99.65 |
| -1.5 | 6.68 | -0.15 | 44.04 | 0.85 | 80.23 | 2.8 | 99.74 |
| -1.4 | 8.08 | -0.10 | 46.02 | 0.90 | 81.59 | 2.9 | 99.81 |
| -1.3 | 9.68 | -0.05 | 48.01 | 0.95 | 82.89 | 3.0 | 99.87 |
| -1.2 | 11.51 | 0.0 | 50.00 | 1.0 | 84.13 | 3.5 | 99.98 |
| -1.1 | 13.57 | | | | | 4.0 | 99.997 |

**Example 5.6.6: Find the Percentage**

Find the percentage of data items in a normal distribution that lie either above, below, or between.

1. above **1.20** and below **1.20**.
2. below **−1.80** and above **−1.80**.
3. between **0.20** and **1.40**.

**Answer**

According to the above table

1. **88.49%** below and **(100% − 88.49%) = 11.51%** above.
2. **3.59%** below and **(100% − 3.59%) = 96.41%** above.

3. For $z = 0.20$; $57.93\%$ are below and for $z = 1.4$; $91.92\%$ are below. So $91.92\% - 57.93\% = 33.99\%$ in between.

**Example 5.6.7: Find the Percentile**

Suppose that the cholesterol levels are normally distributed with a mean of 160 (mg/dl) with a standard deviation of 40 (mg/dl).

a. What is the $z$-score corresponding to the cholesterol level of $200$ (mg/dl)? How about $260$ (mg/dl)?
b. What percent of people have a cholesterol level between $160$ and $200$? How about $260$ or higher?

**Answer**

a. Note that data value is $200$ mg/dl. So its z-score is

$$z\text{-score} = \frac{200 - 160}{40} = 1$$

Similarly, z- score of $260$ is

$$z\text{-score} = \frac{260 - 160}{40} = 2.5$$

b. z- score of $160$ is

$$z\text{-score} = \frac{160 - 160}{40} = 0$$

The percent (percentile ) between $160$ and $200$ would correspond to the z-score between $0$ and $1$.

For $z = 0.0$; $50\%$ are below and for $z = 1$; $84.13\%$ are below. So $84.13\% - 50\% = 34.13\%$ in between.

In other words, about $34\%$ of all people have a cholesterol level between $160$ and $200$.

However, $160$ mg/dl corresponds to $z = 2.5$. The table shows $99.38\%$ below $z = 2.5$. Since $100\% - 99.38\% = 0.62\%$. Only $0.62\%$, i.e., less than $1\%$ of the population, has a cholesterol level $160$ or higher. This person with a cholesterol level of $260$ should go see the doctor immediately.

**Your Turn 5.6.7: Find the Percentage**

**z-Scores and Percentiles**

| z-Score | Percentiles | z-Score | Percentiles | z-Score | Percentiles | z-Score | Percentiles |
|---|---|---|---|---|---|---|---|
| -4.0 | 0.003 | -1.0 | 15.87 | 0.0 | 50.00 | 1.1 | 86.43 |
| -3.5 | 0.02 | -0.95 | 17.11 | 0.05 | 51.99 | 1.2 | 88.49 |
| -3.0 | 0.13 | -0.90 | 18.41 | 0.10 | 53.98 | 1.3 | 90.32 |
| -2.9 | 0.19 | -0.85 | 19.77 | 0.15 | 55.96 | 1.4 | 91.92 |
| -2.8 | 0.26 | -0.80 | 21.19 | 0.20 | 57.93 | 1.5 | 93.32 |
| -2.7 | 0.35 | -0.75 | 22.66 | 0.25 | 59.87 | 1.6 | 94.52 |
| -2.6 | 0.47 | -0.70 | 24.20 | 0.30 | 61.79 | 1.7 | 95.54 |
| -2.5 | 0.62 | -0.65 | 25.78 | 0.35 | 63.68 | 1.8 | 96.41 |
| -2.4 | 0.82 | -0.60 | 27.43 | 0.40 | 65.54 | 1.9 | 97.13 |
| -2.3 | 1.07 | -0.55 | 29.12 | 0.45 | 67.36 | 2.0 | 97.72 |
| -2.2 | 1.39 | -0.50 | 30.85 | 0.50 | 69.15 | 2.1 | 98.21 |
| -2.1 | 1.79 | -0.45 | 32.64 | 0.55 | 70.88 | 2.2 | 98.61 |
| -2.0 | 2.28 | -0.40 | 34.46 | 0.60 | 72.57 | 2.3 | 98.93 |
| -1.9 | 2.87 | -0.35 | 36.32 | 0.65 | 74.22 | 2.4 | 99.18 |
| -1.8 | 3.59 | -0.30 | 38.21 | 0.70 | 75.80 | 2.5 | 99.38 |
| -1.7 | 4.46 | -0.25 | 40.13 | 0.75 | 77.34 | 2.6 | 99.53 |
| -1.6 | 5.48 | -0.20 | 42.07 | 0.80 | 78.81 | 2.7 | 99.65 |
| -1.5 | 6.68 | -0.15 | 44.04 | 0.85 | 80.23 | 2.8 | 99.74 |
| -1.4 | 8.08 | -0.10 | 46.02 | 0.90 | 81.59 | 2.9 | 99.81 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -1.3 | 9.68 | -0.05 | 48.01 | 0.95 | 82.89 | 3.0 | 99.87 |
| -1.2 | 11.51 | 0.0 | 50.00 | 1.0 | 84.13 | 3.5 | 99.98 |
| -1.1 | 13.57 | | | | | 4.0 | 99.997 |

Scores on a test are approximately normally distributed with a mean of 74 and a standard deviation of 8.3. The teacher wants to give A's to students who earned a scored of 91 above on the exam.

a) Find the z-score associated with a score of 91. Round to two decimal places.

[ ] ←Round to 1 decimal places.

b) About what percentage of scores were greater than 91? Show your work and round your answer to the nearest whole percent.←Round to 2 decimal places.

[ ] %

c) If 52 people took the test, how many would you expect to have earned an A?

[ ] ←Round to the nearest whole number.

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You probably have a good intuitive grasp of what the average of a data set says about that data set. In the following section, we begin to learn what the standard deviation reveals about the nature of the data set.

## The Empirical Rule

We start by examining a specific set of data. Table **5.6.1** shows the heights in inches of **100** randomly selected adult men. A relative frequency histogram for the data is shown in Figure **5.6.1**. The mean and standard deviation of the data, rounded to two decimal places, are $M = 69.92$ and $SD = 1.70$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 68.7 | 72.3 | 71.3 | 72.5 | 70.6 | 68.2 | 70.1 | 68.4 | 68.6 | 70.6 |
| 73.7 | 70.5 | 71.0 | 70.9 | 69.3 | 69.4 | 69.7 | 69.1 | 71.5 | 68.6 |
| 70.9 | 70.0 | 70.4 | 68.9 | 69.4 | 69.4 | 69.2 | 70.7 | 70.5 | 69.9 |
| 69.8 | 69.8 | 68.6 | 69.5 | 71.6 | 66.2 | 72.4 | 70.7 | 67.7 | 69.1 |
| 68.8 | 69.3 | 68.9 | 74.8 | 68.0 | 71.2 | 68.3 | 70.2 | 71.9 | 70.4 |
| 71.9 | 72.2 | 70.0 | 68.7 | 67.9 | 71.1 | 69.0 | 70.8 | 67.3 | 71.8 |
| 70.3 | 68.8 | 67.2 | 73.0 | 70.4 | 67.8 | 70.0 | 69.5 | 70.1 | 72.0 |
| 72.2 | 67.6 | 67.0 | 70.3 | 71.2 | 65.6 | 68.1 | 70.8 | 71.4 | 70.2 |
| 70.1 | 67.5 | 71.3 | 71.5 | 71.0 | 69.1 | 69.5 | 71.1 | 66.8 | 71.8 |
| 69.6 | 72.7 | 72.8 | 69.6 | 65.9 | 68.0 | 69.7 | 68.7 | 69.8 | 69.7 |

Table **5.6.1**: Height of Men

If we go through the data and count the number of observations that are within one standard deviation of the mean, that is, that are between $69.92 - 1.70 = 68.22$ and $69.92 + 1.70 = 71.62$ inches, there are **69** of them.

If we count the number of observations that are within two standard deviations of the mean, that is, that are between $69.92 - 2(1.70) = 66.52$ and $69.92 + 2(1.70) = 73.32$ inches, there are **95** of them.

All of the measurements are within three standard deviations of the mean, that is, between $69.92 - 3(1.70) = 64.822$ and $69.92 + 3(1.70) = 75.02$ inches.

These tallies are not coincidences but rather align with the following result, which has been found to be widely applicable.

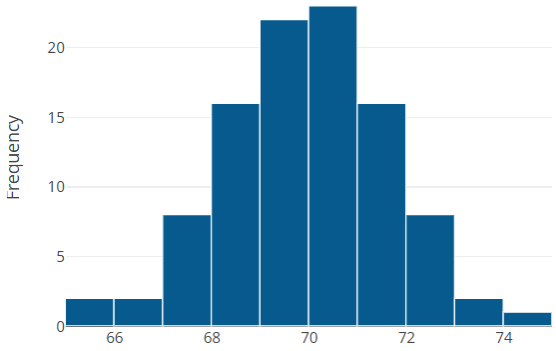



*Figure* **5.6.2**: *Normal (Bell Shaped) Distribution*

**Empirical Rule**

The Empirical Rule (also called the **68–95–99.7** rule describes how data are distributed in a normal (bell-shaped) distribution

1. approximately **68%** of the data lie between one standard deviation mean, i.e., within $(M - 1 \times SD, M + 1 \times SD)$.
2. approximately **95%** of the data lie between one standard deviation mean, i.e., within $(M - 2 \times SD, M + 2 \times SD)$.
3. approximately **99.7%** of the data lie between one standard deviation mean, i.e., within $(M - 3 \times SD, M + 3 \times SD)$.

Where M is the mean, and SD is the standard deviation.

Two key points in regard to the Empirical Rule are that the data distribution must be approximately bell-shaped and that the percentages are only approximately true. The Empirical Rule does not apply to data sets with severely asymmetric distributions, and the actual percentage of observations in any of the intervals specified by the rule could be either greater or less than those given in the rule.

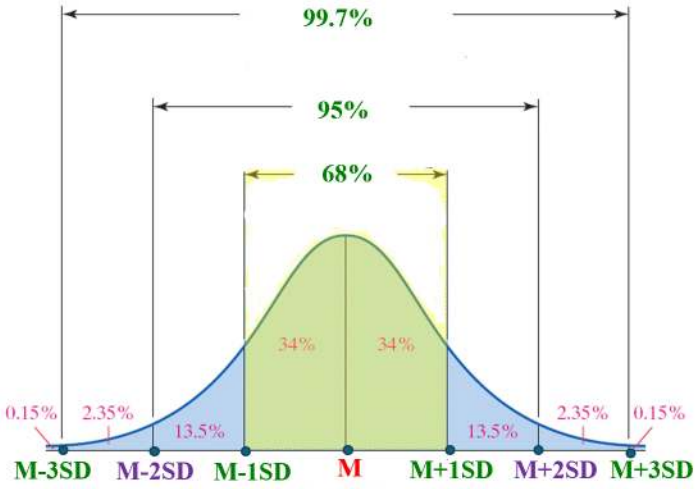



Figure 5.6.3: M as a Mean and SD as a Standard Deviation

We see this with the example of the heights of the men: the Empirical Rule suggested 68 observations between 68.22 and 71.62 inches, but we counted 69.

**Example 5.6.8: Using Empirical Rule/span>**

Scores on IQ tests have a bell-shaped distribution with a mean of 100 and a standard deviation of 10. Discuss what the Empirical Rule is after sketching a normal distribution curve. What is the percentile of IQ scores of 110 and 90?

***Answer***

A sketch of the IQ distribution is given in Figure 5.6.3. The Empirical Rule states that

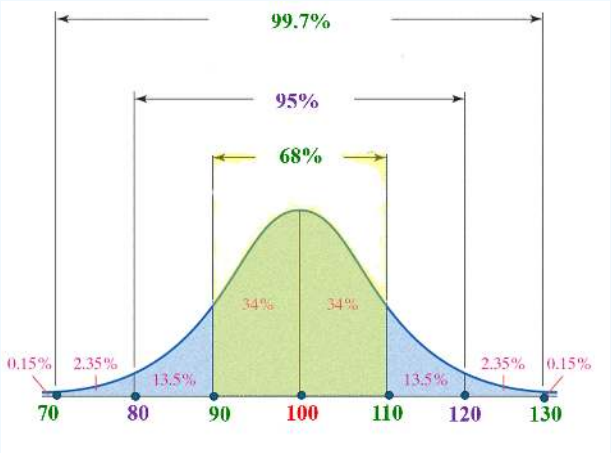



Figure 5.6.4: Distribution of IQ Scores

1. approximately 68% of the IQ scores in the population lie between 90 and 110
2. approximately 95% of the IQ scores in the population lie between 80 and 120
3. approximately 99.7% of the IQ scores in the population lie between 70 and 130.

Since 84% data are less than or equal to 110, we conclude that the IQ score 110 is the $84^{th}$ percentile. Similarly, Since 16% data are less than or equal to 90, we conclude that the IQ score 90 is the $16^{th}$ percentile.

**Your Turn 5.6.8: Use Empirical Rule**

**Use the Empirical Rule to find the following:**

A set of 1100 exam scores is normally distributed with a mean 74 and standard deviation 8.

1. How many students scored higher than 74 ?

2. How many students scored between $66$ and $82$ ? _____
3. How many students scored between $58$ and$90$ ? _____
4. How many students scored between $74$ and $82$ ? _____
5. How many students scored lower than $66$ ? _____
6. How many students scored lower than $82$ ? _____

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**Example 5.6.9: Using Empirical Rule**

1. About what percentage of all such men are between $68.2$ and $71$ inches tall?
2. What interval centered on the mean should contain about $95\%$ of all such men?
3. What percentage of all such men are less than $66.8$ inches in tall?

***Answer***

1. A sketch of the distribution of heights is given in Figure $5.6.3$.Since the interval from $68.2$ to $71.0$ has endpoints $\text{Mean} - 1 \times \text{SD}$ and $\text{Mean} + 1 \times \text{SD}$, by the Empirical Rule about $68\%$ of all $18$-year-old males should have heights in this range
2. By the Empirical Rule the shortest such interval has endpoints

$$\text{Mean} - 2 \times \text{SD} = 69.6 - 2(1.4) = 66.8$$

$$\text{Mean} + 2 \times \text{SD} = 69.6 + 2(1.4) = 72.4$$

The interval in question is the interval from $66.8$ inches to $72.4$ inches.

3. $0.15\% + 2.35\% = 2.5\%$

1.

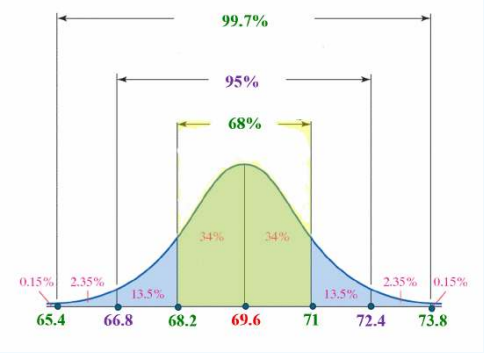
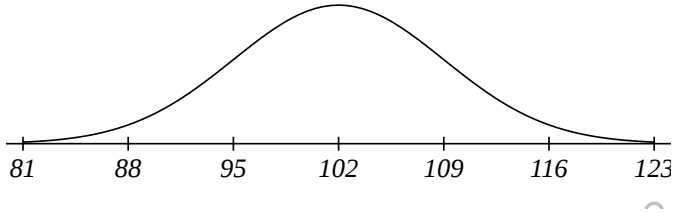



Figure 5.6.5: Distribution of Heights

**Your Turn 5.6.9: Use Empirical Rule**



Based on the graph of this normal distribution,

a. The mean is _____.

b. The standard deviation is _____.

c. 68% of the data values are between _____ and _____.

d. 95% of the data values are between _____ and _____.

e. 99.7% of the data values are between _____ and _____.

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