TECHNICAL MATHEMATICS SJDC 1



Jacquelynn Schwegel
San Joaquin Delta College

LibreTexts™

First Sandbox

This text is disseminated via the Open Education Resource (OER) LibreTexts Project (https://LibreTexts.org) and like the thousands of other texts available within this powerful platform, it is freely available for reading, printing, and "consuming."

The LibreTexts mission is to bring together students, faculty, and scholars in a collaborative effort to provide an accessible, and comprehensive platform that empowers our community to develop, curate, adapt, and adopt openly licensed resources and technologies; through these efforts we can reduce the financial burden born from traditional educational resource costs, ensuring education is more accessible for students and communities worldwide.

Most, but not all, pages in the library have licenses that may allow individuals to make changes, save, and print this book. Carefully consult the applicable license(s) before pursuing such effects. Instructors can adopt existing LibreTexts texts or Remix them to quickly build course-specific resources to meet the needs of their students. Unlike traditional textbooks, LibreTexts' web based origins allow powerful integration of advanced features and new technologies to support learning.



LibreTexts is the adaptable, user-friendly non-profit open education resource platform that educators trust for creating, customizing, and sharing accessible, interactive textbooks, adaptive homework, and ancillary materials. We collaborate with individuals and organizations to champion open education initiatives, support institutional publishing programs, drive curriculum development projects, and more.

The LibreTexts libraries are Powered by NICE CXone Expert and was supported by the Department of Education Open Textbook Pilot Project, the California Education Learning Lab, the UC Davis Office of the Provost, the UC Davis Library, the California State University Affordable Learning Solutions Program, and Merlot. This material is based upon work supported by the National Science Foundation under Grant No. 1246120, 1525057, and 1413739.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation nor the US Department of Education.

Have questions or comments? For information about adoptions or adaptions contact info@LibreTexts.org or visit our main website at https://LibreTexts.org.

This text was compiled on 01/06/2026

TABLE OF CONTENTS

Licensing

1: Review of Arithmetic

- 1.1: Place Value
- 1.2: Rounding Numbers
- 1.3: Negative Numbers
- 1.4: Decimals
- 1.5: Fractions
- 1.6: Order of Operations
- 1.7: Evaluating Algebraic Expressions and Formulas

2: Solving Equations

- 2.1: The Language of Algebra
- 2.2: Solving Equations with Addition and Subtraction Properties of Equality
- 2.3: Solving Equations with the Division and Multiplication Properties of Equality
- 2.4: Solving Equations with Variables and Constants on Both Sides
- 2.5: Solving Equations with Fractions or Decimals
- 2.6: General Strategy to Solve Linear Equations
- 2.7: Solving a Formula for a Specific Variable

3: Proportions and Variation

- 3.1: Ratios, Rates, Proportions
- 3.2: Applications of Proportionality
- 3.3: Direct and Inverse Variation

4: Percent

- 4.1: Basic Percent Problems
- 4.2: Percents in the Real World
- 4.3: Percent Increase or Decrease

5: Units of Measure and Converting Units

- 5.1: Scientific and Engineering Notation
- 5.2: Accuracy and Significant Figures
- 5.3: The US Measurement System
- 5.4: Converting Units of Measure/ Dimensional Analysis
- 5.5: The Metric System
- 5.6: Converting Between Systems
- 5.7: Conversions with Time, Rate, Area and Volume

6: Geometry

- 6.1: Polygons
- 6.2: Pythagorean Theorem for Right Triangles
- 6.3: Perimeter and Circumference

- 6.4: Area of Polygons and Circles
- 6.5: Composite Figures
- 6.6: Volume of Common Solids

7: Trigonometry

- 7.1: Angles
- 7.2: Triangles
- 7.3: Introduction to Right Triangle Trigonometry
- 7.4: Right Triangle Trigonometry in the Real World
- 7.5: Power and Impedance Triangles - An Elelctrical Example
- 7.6: Trigonometry and Single Phase AC Generation for Electricians (Flinn)
  - Licensing
  - 7.6.1: Trigonometry
    - 7.6.1.1: Angles
    - 7.6.1.2: Triangles
    - 7.6.1.3: Pythagoras
    - 7.6.1.4: Naming Right Triangle Sides
    - 7.6.1.5: Trigonometry Functions
    - 7.6.1.6: Power and Impedance Triangles
  - 7.6.2: Vectors
    - 7.6.2.1: A Vector Primer
    - 7.6.2.2: Quadrants
    - 7.6.2.3: Polar vs. Rectangular Form
    - 7.6.2.4: Vector Addition
  - 7.6.3: AC Generation
    - 7.6.3.1: Electromagnetic Induction
    - 7.6.3.2: The Alternator
    - 7.6.3.3: How a Waveform Is Generated
    - 7.6.3.4: AC Waveform Analysis
    - 7.6.3.5: Frequency and Alternators
  - Index
  - Detailed Licensing

8: Vectors

- 8.1: Angles in Standard Position
- 8.2: Trigonometric Functions of Any Angle
- 8.3: Introduction to Vectors
- 8.4: Vector Operations
- 8.5: Vector Addition in the Real World - Electrical Application

9: Sinusoidal Models

- 9.1: Periodic Behaviors in the Real World
- 9.2: Sinusoidal Graphs
- 9.3: More Sinusoidal Modeling
- 9.4: Sinusoidal Models (Exercises)
- 9.5: AC Waveform - An Electrical Application

10: Law of Sines and Cosines

- 10.1: Non-right Triangles - Law of Sines
- 10.2: Non-right Triangles - Law of Cosines

Index

Glossary

Detailed Licensing

Detailed Licensing

Licensing

*A detailed breakdown of this resource's licensing can be found in **Back Matter/Detailed Licensing**.*

CHAPTER OVERVIEW

1: Review of Arithmetic

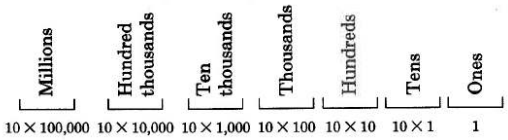
1.1: Place Value
1.2: Rounding Numbers
1.3: Negative Numbers
1.4: Decimals
1.5: Fractions
1.6: Order of Operations
1.7: Evaluating Algebraic Expressions and Formulas

1: Review of Arithmetic is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

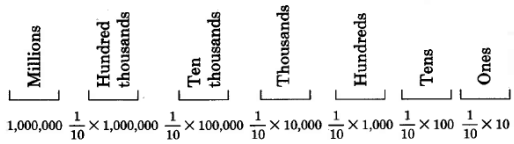
1.1: Place Value

Whole Number Place Values

Our number system is called a positional number system with base ten. Each position has a particular value. The first sevel place values for whole numbers are shown below.



This means that each position has $\frac{1}{10}$ the value of the position to its left.



Thus, a digit written to the right of the units position must have a value of $\frac{1}{10}$ of 1. Recalling that the word "of" translates to multiplication ($\cdot$), we can see that the value of the *first position* to the right of the units digit is $\frac{1}{10}$ of 1, or

$$\frac{1}{10} \cdot 1 = \frac{1}{10}$$

The value of the *second position* to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{10}$, or

$$\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^2} = \frac{1}{100}$$

The value of the third position to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{100}$, or

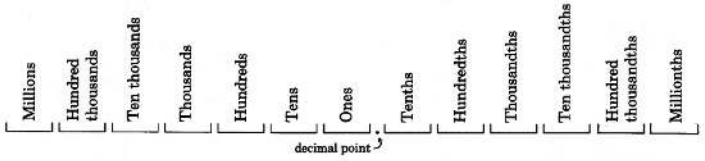
$$\frac{1}{10} \cdot \frac{1}{100} = \frac{1}{10^3} = \frac{1}{1000}$$

This pattern continues.

We can now see that if we were to write digits in positions to the right of the units positions, those positions have values that are fractions. Not only do the positions have fractional values, but the fractional values are all powers of 10 $(10, 10^2, 10^3, ...)$.

Decimal Place Values

Mathematicians denote the separation of whole number part and the fractional or less than whole-number-part of a number by writing a **decimal point**. The word *decimal* comes from the Latin prefix "deci" which means ten, and we use it because we use a base ten number system. Numbers written in this form are called **decimals**.



Notice that decimal numbers have the suffix "th."

The following numbers are examples of decimals.

42.6

The 6 is in the tenths position.

$42.6 = 42\frac{6}{10}$

9.8014

The 8 is in the tenths position.
The 0 is in the hundredths position.
The 1 is in the thousandths position.
The 4 is in the ten thousandths position.

$9.8014 = 9\frac{8014}{10,000}$

0.93

The 9 is in the tenths position.
The 3 is in the hundredths position.

$0.93 = \frac{93}{100}$

Note

Quite often a zero is inserted in front of a decimal point (in the units position) of a decimal fraction that has a value less than one. This zero helps keep us from overlooking the decimal point.

0.7

The 7 is in the tenths position.

$0.7 = \frac{7}{10}$

Note

We can insert zeros to the right of the right-most digit in a decimal fraction without changing the value of the number.

$\frac{7}{10} = 0.7 = 0.70 = \frac{70}{100} = \frac{7}{10}$

Reading Decimals

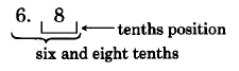
To read a decimal number,

1. Read the whole number part as usual. (If the whole number is less than 1, omit steps 1 and 2.)
2. Read the decimal point as the word "and."
3. Read the number to the right of the decimal point as if it were a whole number.
4. Say the name of the position of the last digit.

Sample Set A

Read the following numbers.

6.8

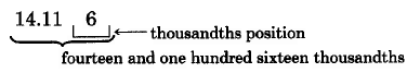


Note

Some people read this as "six point eight." This phrasing gets the message across, but technically, "six *and* eight tenths" is the correct phrasing.

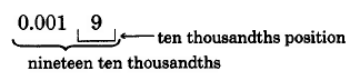
Sample Set A

14.116



Sample Set A

0.0019



Sample Set A

81

Eighty-one

In this problem, the indication is that any whole number is a decimal fraction. Whole numbers are often called *decimal numbers*.

81 = 81.0

Practice Set A

Read the following decimal fractions.

12.9

Answer

twelve and nine tenths

Practice Set A

4.86

Answer

four and eighty-six hundredths

Practice Set A

7.00002

Answer

seven and two hundred thousandths

Practice Set A

0.030405

Answer

thirty thousand four hundred five millionths

Writing Decimals

To write a decimal number,

Write the whole number part.
Write a decimal point for the word "and."
Write the decimal part of the number so that the right-most digit appears in the position indicated in the word name. If necessary, insert zeros to the right of the decimal point in order that the right-most digit appears in the correct position.

Sample Set B

Write each number.

Thirty-one and twelve hundredths.

Solution

The decimal position indicated is the hundredths position.

31.12

Sample Set B

Two and three hundred-thousandths.

Solution

The decimal position indicated is the hundred thousandths. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 3 in the correct position.

2.00003

Sample Set B

Six thousand twenty-seven and one hundred four millionths.

Solution

The decimal position indicated is the millionths position. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 4 in the correct position.

6,027.000104

Sample Set B

Seventeen hundredths.

Solution

The decimal position indicated is the hundredths position.

0.17

Practice Set B

Write each decimal fraction.

Three hundred six and forty-nine hundredths.

Answer

306.49

Practice Set B

Nine and four thousandths.

Answer

9.004

Practice Set B

Sixty-one millionths.

Answer

0.000061

Exercises

For the following three problems, give the decimal name of the position of the given number in each decimal fraction.

Exercise 1.1.1

1. 3.941
9 is in the position.
4 is in the position.
1 is in the position.

Answer

Tenths; hundredths, thousandths

Exercise 1.1.2

17.1085
1 is in the position.
0 is in the position.
8 is in the position.
5 is in the position.

Exercise 1.1.3

652.3561927
9 is in the position.
7 is in the position.

Answer

Hundred thousandths; ten millionths

For the following 7 problems, read each decimal fraction by writing it.

Exercise 1.1.4

9.2

Exercise 1.1.5

8.1

Answer

eight and one tenth

Exercise 1.1.6

10.15

Exercise 1.1.7

55.06

Answer

fifty-five and six hundredths

Exercise 1.1.8

0.78

Exercise 1.1.9

1.904

Answer

one and nine hundred four thousandths

Exercise 1.1.10

10.00011

For the following 10 problems, write each decimal fraction.

Exercise 1.1.11

Three and twenty one-hundredths.

Answer

3.20

Exercise 1.1.12

Fourteen and sixty seven-hundredths.

Exercise 1.1.13

One and eight tenths.

Answer

1.8

Exercise 1.1.14

Sixty-one and five tenths.

Exercise 1.1.15

Five hundred eleven and four thousandths.

Answer

511.004

Exercise 1.1.16

Thirty-three and twelve ten-thousandths.

Exercise 1.1.17

Nine hundred forty-seven thousandths.

Answer

0.947

Exercise 1.1.18

Two millionths.

Exercise 1.1.19

Seventy-one hundred-thousandths.

Answer

0.00071

Exercise 1.1.20

One and ten ten-millionths.

Calculator Problems

For the following 10 problems, perform each division using a calculator. Then write the resulting decimal using words.

Exercise 1.1.21

$3 \div 4$

Answer

seventy-five hundredths

Exercise 1.1.22

$1 \div 8$

Exercise 1.1.23

$4 \div 10$

Answer

four tenths

Exercise 1.1.24

$2 \div 5$

Exercise 1.1.25

$4 \div 25$

Answer

sixteen hundredths

Exercise 1.1.26

$1 \div 50$

Exercise 1.1.27

$3 \div 16$

Answer

one thousand eight hundred seventy-five ten thousandths

Exercise 1.1.28

$15 \div 8$

Exercise 1.1.29

$11 \div 20$

Answer

fifty-five hundredths

Exercise 1.1.30

$9 \div 40$

Exercises for Review

Exercise 1.1.31

Round 2,614 to the nearest ten.

Answer

2610

Exercise 1.1.32

Is 691,428,471 divisible by 3?

Exercise 1.1.33

Determine the missing numerator.

$\frac{3}{14} = \frac{?}{56}$

Answer

12

Exercise 1.1.34

Find $\frac{3}{16}$ of $\frac{32}{39}$

Exercise 1.1.35

Find the value of $\sqrt{\frac{25}{81}} + (\frac{2}{3})^2 + \frac{1}{9}$

Answer

$\frac{10}{9}$ or $1\frac{1}{9}$

This page titled 1.1: Place Value is shared under a CC BY license and was authored, remixed, and/or curated by Denny Burzynski & Wade Ellis, Jr. (OpenStax CNX) via source content that was edited to the style and standards of the LibreTexts platform.

- **6.1: Reading and Writing Decimals** by Denny Burzynski & Wade Ellis, Jr. is licensed CC BY 4.0. Original source: https://archive.org/details/cnx-org-col10615.

1.2: Rounding Numbers

Rounding Numbers

We round numbers for many reasons:

- We may not know the exact number.
- The exact number may not be important for what we are doing.
- We may need a **quick way to figure** something out.

When you are rounding numbers, use zeros to hold the places at the end of the number. Work through the following examples and exercises carefully. **Rounding is an important skill.**

Rounding to the Nearest Ten

Here is a short method to round to the nearest ten. When rounding to the nearest ten, do this:

1. Underline the tens digit. 8̲3
2. Look at the digit following in the ones place. 8̲3
3. If the digit in the ones place is less than 5, write a 0 in the ones place. Leave the tens digit as it is.
   - 4̲2, rounds to 40 (42 is nearer to 40 than to 50)
   - 1̲4, rounds to 10
   - 8̲3, rounds to 80
4. If the digit in the ones place is 5 or more, write a 0 in the ones place. Add one more ten to the tens place.
   - 3̲6, rounds to **4**0 (36 is nearer to 40 than to 30)
   - 2̲5, rounds to **3**0
   - 9̲8, rounds to **10**0 (one more ten than nine tens is ten tens)

Note

Note: If you are rounding to the nearest ten, single digits are rounded like this:

- 0, 1, 2, 3, 4 all round to **0**.
- 5, 6, 7, 8, 9 all round to **10**.

A number rounded to the nearest ten will have a zero in the ones place.

The number will end with

0, 10, 20, 30, 40, 50, 60, 70, 80, or 90.

When you round a number, use the sign that means “approximately equal” ≈.

Sample Set A

Round each number to the nearest 10. Check your work using the answer key at the end of the exercise.

a. **47** is between _____ tens and _____ tens.
47 is closest to _____ tens.
Rounded number is _____ .

b. **81** is between _____ tens and _____ tens.
81 is closest to _____ tens.
Rounded number is _____ .

c. **14** is between _____ tens and _____ tens.
14 is closest to _____ tens.
Rounded number is _____ .

d. **26** is between _____ tens and _____ tens.
26 is closest to _____ tens.
Rounded number is _____ .

e. **98** is between _____ tens and _____ tens.
98 is closest to _____ tens.
Rounded number is _____ .

f. **57** is between _____ tens and _____ tens.
57 is closest to _____ tens.
Rounded number is _____ .

g. **73** is between _____ tens and _____ tens.
73 is closest to _____ tens.
Rounded number is _____ .

Answers to Exercise 1

a. 4 tens, 5 tens
5 tens
50

b. 8 tens, 9 tens
8 tens
80

c. 1 tens, 2 tens
1 ten
10

d. 2 tens, 3 tens
3 tens
30

e. 9 tens. 10 tens
10 tens
100

f. 5 tens, 6 tens
6 tens
60

g. 7 tens, 8 tens
7 tens
70

Rounding Decimal Numbers

With a few minor changes, we can apply the same rules of rounding to decimals.

To round a decimal to a particular position:

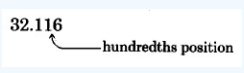
1. Mark the position of the round-off digit (with an arrow or check).
2. Note whether the digit to the immediate right of the marked digit is
   a. *less than 5*. If so, leave the round-off digit unchanged.
   b. *5 or greater*. If so, add 1 to the round-off digit.
3. If the round-off digit is
   a. to the right of the decimal point, eliminate all the digits to its right.
   b. to the left of the decimal point, replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

Sample Set B

Round each decimal to the specified position. (The numbers in parentheses indicate which step is being used.)

Round 32.116 to the nearest hundredth.

Solution

1: 

2b: The digit immediately to the right is 6, and $6 > 5$, so we add 1 to the round-off digit:
$1 + 1 = 2$

3a: The round-off digit is to the right of the decimal point, so we eliminate all digits to its right.

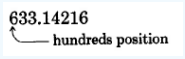
32.12

The number 32.116 rounded to the nearest hundredth is 32.12.

Sample Set C

Round 633.14216 to the nearest hundred.

Solution

1: 

2a: The digit immediately to the right is 3, and $3 < 5$ so we leave the round-off digit unchanged.

3b: The round-off digit is to the left of 0, so we replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

600

The number 633.14216 rounded to the nearest hundred is 600.

Sometimes we hear a phrase such as "round to three decimal places." This phrase means that the round-off digit is the third decimal digit (the digit in the thousandths position).

Sample Set D

67.129 rounded to the second decimal place is 67.13.

Sample Set E

67.129558 rounded to 3 decimal places is 67.130.

Now you try...

Practice Set A

Round each decimal to the specified position.

4.816 to the nearest hundredth.

Answer

4.82

Practice Set B

0.35928 to the nearest ten thousandths.

Answer

0.3593

Practice Set C

82.1 to the nearest one.

Answer

82

Practice Set D

753.98 to the nearest hundred.

Answer

800

Practice Set E

Round 43.99446 to three decimal places.

Answer

43.994

Practice Set F

Round 105.019997 to four decimal places.

Answer

105.0200

Practice Set G

Round 99.9999 to two decimal places.

Answer

100.00

Rounding Numbers in General

It is often necessary to round a number to a specified place value.

Rounding a number:

1. Locate the **rounding digit** in the place to which you are rounding.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

? Exercises 1.2.1

Round each number to the indicated place value.

1. $6,473$ (thousands)

2. $6,473$ (hundreds)

3. $6,473$ (tens)

4. 0.7049 (tenths)

5. 0.7049 (hundredths)

6. 0.7049 (thousandths)

Answer

1. $6,000$

2. $6,500$
3. $6,470$
4. 0.7
5. 0.70
6. 0.705

This page titled 1.2: Rounding Numbers is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by via source content that was edited to the style and standards of the LibreTexts platform.

- **1.5: Topic E- Rounding Numbers** is licensed CC BY 4.0. Original source: https://opentextbc.ca/alfm1.
- **6.3: Rounding Decimals** by Denny Burzynski & Wade Ellis, Jr. is licensed CC BY 4.0. Original source: https://archive.org/details/cnx-org-col10615.
- **1.3: Decimals** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

1.3: Negative Numbers

Negative numbers are a fact of life, from winter temperatures to our bank accounts. Let's practice evaluating expressions involving negative numbers.



Absolute Value

The ***absolute value*** of a number is its distance from 0. You can think of it as the size of a number without identifying it as positive or negative. Numbers with the same absolute value but different signs, such as 3 and -3, are called ***opposites***. The absolute value of -3 is 3, and the absolute value of 3 is also 3.

We use a pair of straight vertical bars to indicate absolute value; for example, $|-3| = 3$ and $|3| = 3$.

? Exercises 1.3.1

Evaluate each expression.

1. $|-5|$

2. $|5|$

Answer

1. 5
2. 5

Adding Negative Numbers

To add two negative numbers, add their absolute values (i.e., ignore the negative signs) and make the final answer negative.

? Exercises 1.3.1

Perform each addition.

3. $-8 + (-7)$

4. $-13 + (-9)$

Answer

3. -15
4. -22

To add a positive number and a negative number, we *subtract* the smaller absolute value from the larger. If the positive number has the larger absolute value, the final answer is positive. If the negative number has the larger absolute value, the final answer is negative.

Exercises 1.3.1

Perform each addition.

5. $7 + (-3)$

6. $-7 + 3$

7. $14 + (-23)$

8. $-14 + 23$

9. The temperature at noon on a chilly Monday was -7°F. By the next day at noon, the temperature had risen 25°F. What was the temperature at noon on Tuesday?

Answer

5. 4

6. -4

7. -9

8. 9

9. 18°F

If an expression consists of only additions, we can break the rules for order of operations and add the numbers in whatever order we choose.

Exercises 1.3.1

Evaluate each expression using any shortcuts that you notice.

10. $-10 + 4 + (-4) + 3 + 10$

11. $-291 + 73 + (-9) + 27$

Answer

10. 3

11. -200

Subtracting Negative Numbers

The image below shows part of a paystub in which an $ 18 payment needed to be made, but the payroll folks wanted to track the payment in the deductions category. Of course, a positive number in the deductions will subtract money away from the paycheck. Here, though, a deduction of negative 18 dollars has the effect of *adding* 18 dollars to the paycheck. Subtracting a negative amount is equivalent to adding a positive amount.

Deductions after Federal Tax	
Faculty Union Dues	$27.00
Stipend for Part Time Faculty	-$18.00
Workers Compensation Hourly Assessment	$0.13

To subtract two signed numbers, we **add** the first number to the **opposite** of the second number.

Exercises 1.3.1

Perform each subtraction.

12. $5 - 2$

13. $2 - 5$

14. $-2 - 5$

15. $-5 - 2$

16. $2 - (-5)$

17. $5 - (-2)$

18. $-2 - (-5)$

19. $-5 - (-2)$

20. One day in February, the temperature in Portland, Oregon is 43°F, and the temperature in Portland, Maine is -12°F. What is the difference in temperature?

Answer

12. 3

13. -3

14. -7

15. -7

16. 7

17. 7

18. 3

19. -3

20. 55°F difference

Multiplying Negative Numbers

Suppose you spend 3 dollars on a coffee every day. We could represent spending 3 dollars as a negative number, -3 dollars. Over the course of a 5-day work week, you would spend 15 dollars, which we could represent as -15 dollars. This shows that $-3 \cdot 5 = -15$, or $5 \cdot -3 = -15$.



If two numbers with **opposite** signs are multiplied, the product is negative.

Exercise 1.3.1

Find each product.

21. $-4 \cdot 3$

22. $5(-8)$

Answer

21. -12

22. -40

Going back to our coffee example, we saw that $5(-3) = -15$. Therefore, the *opposite* of $5(-3)$ must be positive 15. Because -5 is the opposite of 5, this implies that $-5(-3) = 15$.

If two numbers with the **same** sign are multiplied, the product is positive.

WARNING! These rules are different from the rules for addition; be careful not to mix them up.

? Exercise 1.3.1

Find each product.

23. $-2(-9)$

24. $-3(-7)$

Answer

23. 18

24. 21

Recall that an exponent represents a repeated multiplication. Let's see what happens when we raise a negative number to an exponent.

? Exercises 1.3.1

Evaluate each expression.

25. $(-2)^2$

26. $(-2)^3$

27. $(-2)^4$

28. $(-2)^5$

Answer

25. 4

26. -8

27. 16

28. -32

If a negative number is raised to an **odd** power, the result is negative.
If a negative number is raised to an **even** power, the result is positive.

Dividing Negative Numbers

Let's go back to the coffee example we saw earlier: $-3 \cdot 5 = -15$. We can rewrite this fact using division and see that $-15 \div 5 = -3$; a negative divided by a positive gives a negative result. Also, $-15 \div -3 = 5$; a negative divided by a negative gives a positive result. This means that the rules for division work exactly like the rules for multiplication.

If two numbers with **opposite** signs are divided, the quotient is negative.
If two numbers with the **same** sign are divided, the quotient is positive.

Exercises 1.3.1

Find each quotient.

29. $-42 \div 6$

30. $32 \div (-8)$

31. $-27 \div (-3)$

32. $0 \div 4$

33. $0 \div (-4)$

34. $4 \div 0$

Answer

29. -7

30. -4

31. 9

32. 0

33. 0

34. undefined

Go ahead and check those last three exercises with a calculator. Any surprises?

- 0 divided by another number is 0.
- A number divided by 0 is undefined, or not a real number.

Here's a quick explanation of why $4 \div 0$ can't be a real number. Suppose that there is a mystery number, which we'll call n, such that $4 \div 0 = n$. Then we can rewrite this division as a related multiplication, $n \cdot 0 = 4$. But because 0 times any number is 0, the left side of this equation is 0, and we get the result that $0 = 4$, which doesn't make sense. Therefore, there is no such number n, and $4 \div 0$ cannot be a real number.

This page titled 1.3: Negative Numbers is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.2: Negative Numbers** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

1.4: Decimals



Decimal notation is based on powers of 10: 0.1 is one tenth, 0.01 is one hundredth, 0.001 is one thousandth, and so on.

thousands	hundreds	tens	ones/units	.	tenths	hundredths	thousandths

? Exercises 1.4.1

Write each number.

1. ninety and twenty-three hundredths

2. seven and fifty-six thousandths

Answer

1. 90.23
2. 7.056

Adding & Subtracting Decimals

Before you add or subtract decimals, you must line up the decimal points.

? Exercises 1.4.1

Add each pair of numbers.

3. $3.75 + 12.8$

4. $71.085 + 112.93$

Answer

3. 16.55
4. 184.015

When subtracting, you may need to add zeros to the first number so you can borrow correctly.

? Exercises 1.4.1

Subtract each pair of numbers.

5. $46.57 - 38.29$

6. $82.78 - 67.024$

Answer

5. 8.28
6. 15.756

Multiplying Decimals

To multiply decimal numbers:

1. Temporarily ignore the decimal points.
2. Multiply the numbers as though they are whole numbers.
3. Add the total number of decimal digits in the two numbers you multiplied. The result will have that number of digits to the right of the decimal point.

Note: You do NOT need to line up the decimal points when you are multiplying.

? Exercises 1.4.1

Multiply each pair of numbers.

7. $13.5 \cdot 2.9$

8. $4.18 \cdot 3.7$

9. Evie worked 37.5 hours at a pay rate of $\$ 17.50$ per hour. How much did she earn in total?

Answer

7. 39.15
8. 15.466
9. $\$ 656.25$

Dividing Decimals

Let's review everyone's favorite topic, long division. The three parts of a division are named as follows: dividend ÷ divisor = quotient. When this is written with a long division symbol, the dividend is inside the symbol, the divisor is on the left, and the quotient is the answer we create on top.

$$\text{divisor}\overline{)\,\text{dividend}}\quad\text{quotient}$$

To divide by a decimal:

1. Write in long division form.
2. Move the decimal point of the divisor until it is a whole number.
3. Move the decimal point of the dividend the same number of places to the right.
4. Place the decimal point in the quotient directly above the decimal point in the dividend. Divide the numbers as though they are whole numbers.
5. If necessary, add zeros to the right of the last digit of the dividend to continue.

? Exercise 1.4.1

Divide each pair of numbers.

10. $97.4 \div 0.4$

11. $9.74 \div 0.04$

Answer

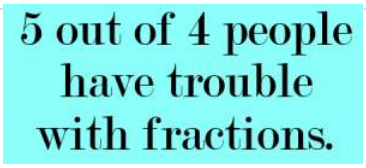
10. 243.5

11. $2,435$

This page titled 1.4: Decimals is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.3: Decimals** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

1.5: Fractions



A fraction describes equal parts of a whole: $\frac{\text{part}}{\text{whole}}$

Using official math vocabulary: $\frac{\text{numerator}}{\text{denominator}}$

? Exercises 1.5.1

The month of April had 11 rainy days and 19 days that were not rainy.

1. What fraction of the days were rainy?

2. What fraction of the days were not rainy?

Answer

1. $\frac{11}{30}$
2. $\frac{19}{30}$

Simplifying Fractions

Two fractions are equivalent if they represent the same number. (The same portion of a whole.) To build an equivalent fraction, multiply the numerator and denominator by the same number.

? Exercises 1.5.1

3. Write $\frac{4}{5}$ as an equivalent fraction with a denominator of 15.

4. Write $\frac{2}{3}$ as an equivalent fraction with a denominator of 12.

Answer

3. $\frac{12}{15}$
4. $\frac{8}{12}$

Many fractions can be ***simplified***, or reduced. Here are four special cases.

? Exercises 1.5.1

Simplify each fraction, if possible.

5. $\frac{7}{1}$

6. $\frac{7}{7}$

7. $\frac{0}{7}$

8. $\frac{7}{0}$

Answer

5. 7

6. 1

7. 0

8. undefined

A fraction is completely reduced, or in simplest form, or in lowest terms, when the numerator and denominator have no common factors other than 1. To reduce a fraction, divide the numerator and denominator by the same number.

? Exercises 1.5.1

Reduce each fraction to simplest form.

9. $\frac{9}{12}$

10. $\frac{10}{6}$

Answer

9. $\frac{3}{4}$

10. $\frac{5}{3}$

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators straight across. If possible, simplify your answer.

? Exercises 1.5.1

Multiply each pair of numbers. Be sure that each answer is in simplest form.

11. $8 \cdot \frac{1}{4}$

12. $\frac{6}{7} \cdot \frac{7}{12}$

13. $\frac{5}{8} \cdot \frac{2}{3}$

14. $\frac{6}{5} \cdot \frac{10}{12}$

Answer

11. 2

12. $\frac{1}{2}$

13. $\frac{5}{12}$

14. 1

To find a fraction **of** a number, multiply.

? Exercise 1.5.1

15. To pass his workplace training, Nathan must correctly answer at least $\frac{9}{10}$ of 50 questions. How many questions must he answer correctly to pass the training?

Answer

15. at least 45 questions

Dividing Fractions

To divide by a fraction, multiply by the reciprocal of the second number. (Flip the second fraction upside-down.)

Exercises 1.5.1

Divide. Be sure that each answer is in simplest form.

16. $12 \div \frac{3}{4}$

17. $\frac{3}{10} \div \frac{1}{2}$

18. Suppose you need to measure 2 cups of flour, but the only scoop you can find is $\frac{1}{3}$ cup. How many scoops of flour will you need?

Answer

16. 16

17. $\frac{3}{5}$

18. 6 scoops

Comparing Fractions

If two fractions have the same denominator, we can simply compare their numerators.

If two fractions have different denominators, we can rewrite them with a common denominator and then compare their numerators.

Exercise 1.5.1

19. Cookie recipe A requires $\frac{3}{4}$ cup of sugar, whereas cookie recipe B requires $\frac{2}{3}$ cup of sugar. Which recipe requires more sugar?



Answer

19. A requires $\frac{1}{12}$ cup more than B

Adding & Subtracting Fractions

To add or subtract two fractions with the same denominator, add or subtract the numerators and keep the common denominator.

Exercises 1.5.1

20. Jack ate $\frac{3}{8}$ of a pizza. Mack ate $\frac{1}{8}$ of the pizza. What fraction of the pizza did they eat together?

21. Tracy ate $\frac{5}{6}$ of a pizza. Stacy ate $\frac{1}{6}$ of the pizza. How much more of the pizza did Tracy eat?

Answer

20. $\frac{1}{2}$ of the pizza

21. $\frac{2}{3}$ more

To add or subtract two fractions with different denominators, first write them with a common denominator. Then add or subtract them.

? Exercises 1.5.1

A $\frac{3}{8}$-inch thick sheet of plywood is going to be laid onto a $\frac{1}{4}$-inch thick sheet of plywood.

22. What is the combined thickness of the two sheets?

23. What is the difference in thickness of the two sheets of plywood?

Jacqueline budgets $\frac{1}{4}$ of her monthly income for food and $\frac{1}{3}$ of her monthly income for rent.

24. What fraction of her monthly income does she budget for these two expenses combined?

25. What fraction more of her monthly income does she budget for her rent than for her food?

Answer

22. $\frac{5}{8}$ inches combined
23. $\frac{1}{8}$ inches difference
24. $\frac{7}{12}$ combined
25. $\frac{1}{12}$ more

Fractions and Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

A decimal that ends (eventually has a remainder of 0) is called a terminating decimal. Fun fact: If the denominator of a fraction has no prime factors other than 2's and 5's, the decimal will terminate. Also, the fraction can be built up to have a denominator of 10, or 100, or $1,000$…

? Exercises 1.5.1

Write each fraction as a decimal.

26. $\frac{11}{4}$

27. $\frac{7}{20}$

Answer

26. 2.75
27. 0.35

A decimal that continues a pattern of digits is called a repeating decimal. We can represent the repeating digits by using either an overbar or ellipsis (three dots)…

? Exercises 1.5.1

Write each fraction as a decimal.

28. $\frac{5}{9}$

29. $\frac{18}{11}$

Answer

28. $0.\overline{5}$ or $0.555...$

29. $1.\overline{63}$ or $1.636363...$

Mixed Numbers

A mixed number represents a **sum**. For example, $6\frac{2}{3}$ means $6+\frac{2}{3}$.

To write a mixed number as an improper fraction:

1. Multiply the whole number part by the denominator.
2. Add this result to the original numerator to get the new numerator.
3. Keep the same denominator.

? Exercises 1.5.1

Rewrite each mixed number as an improper fraction.

30. $2\frac{1}{5}$

31. $6\frac{2}{3}$

Answer

30. $\frac{11}{5}$

31. $\frac{20}{3}$

To write an improper fraction as a mixed number:

1. Divide the numerator by the denominator to get the whole number part.
2. The remainder after dividing is the new numerator.
3. Keep the same denominator.

? Exercises 1.5.1

Rewrite each improper fraction as a mixed number.

32. $\frac{23}{2}$

33. $\frac{14}{3}$

Answer

32. $11\frac{1}{2}$

33. $4\frac{2}{3}$

Adding or subtracting mixed numbers can be fairly simple or more complicated, depending on the numbers. If adding two mixed numbers would give you an improper fraction as part of your result, you'll need to carry; if subtracting two mixed numbers would give you a negative fraction as part of your result, you'll need to borrow.

Exercises 1.5.1

34. Add: $7\frac{1}{3}+2\frac{3}{4}$

35. Subtract: $7\frac{1}{3}-2\frac{3}{4}$

Answer

34. $10\frac{1}{12}$

35. $4\frac{7}{12}$

Multiplying or dividing mixed numbers is tricky. Change any mixed numbers into improper fractions before doing the calculation, then change the answer back to a mixed number if possible.

Exercises 1.5.1

36. Multiply: $3\frac{1}{2}\cdot 2\frac{1}{3}$

37. $5\frac{1}{2}$ cups of water will be divided equally into 3 jars. How much water will go into each jar?

Answer

36. $8\frac{1}{6}$

37. $1\frac{5}{6}$ cup

This page titled 1.5: Fractions is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.4: Fractions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

1.6: Order of Operations

To evaluate an expression means to simplify it and find its value.

? Exercises 1.6.1

1. Evaluate by performing the addition first: $12-2+3$

2. Evaluate by performing the subtraction first: $12-2+3$

Answer

1. 7
2. 13

When we evaluate an expression, we want to have a single correct answer. It isn't very helpful for the answer to be "maybe 7, or maybe 13". Mathematicians have decided on an order of operations, which tells us which steps should be done before other steps. Think of them as the rules of the road.

Order of Operations

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Let's finish up this module with some order of operations practice.

? Exercises 1.6.1

Evaluate each expression using the order of operations.

1. $(2-5)^2 \cdot 2+1$

2. $2-5^2 \cdot (2+1)$

3. $[7(-2)+16] \div 2$

4. $7(-2)+16 \div 2$

5. $\dfrac{1-3^4}{2(5)}$

6. $\dfrac{(1-3)^4}{2} \cdot 5$

Answer

1. 19
2. -73
3. 1
4. -6
5. -8
6. 40

Before we move on, you should be aware that there are a handful of ways to show multiplication. All of the following represent 3×4:

$$3 \cdot 4 \; 3 * 4 \; 3(4) \; (3)4 \; (3)(4)$$

In this textbook, you will most often see the dot, like $3 \cdot 4$, or parentheses directly next to a number, like $3(4)$. We tend to avoid using the 3×4 symbol because it can be mistaken for the letter x.

Exercises 1.6.1

Simplify each expression.

7. $12 \div (3 \cdot 2)$

8. $12 \div 3 \cdot 2$

9. $5(1+3)-2$

10. $5(1)+(3-2)$

Answer

7. 2

8. 8

9. 18

10. 6

An ***exponent*** indicates repeated multiplication. For example, $6^2 = 6 \cdot 6 = 36$ and $4^3 = 4 \cdot 4 \cdot 4 = 64$. The exponent tells us how many factors of the base are being multiplied together.

Exercises 1.6.1

Simplify each expression.

11. $3^2 + 4^2$

12. $(3+4)^2$

13. $(7+3)(7-5)^3$

14. $7+3(7-5)^3$

Answer

11. 25

12. 49

13. 80

14. 31

In the next set of exercises, the only differences are the parentheses, but every exercise has a different answer.

Exercises 1.6.1

Simplify each expression.

15. $39-7 \cdot 2+3$

16. $(39-7) \cdot 2+3$

17. $39-(7 \cdot 2+3)$

18. $39-7 \cdot(2+3)$

19. $(39-7) \cdot(2+3)$

Answer

15. 28

16. 67

17. 22

18. 4

19. 160

It is possible to have grouping symbols nested within grouping symbols; for example, $7+(5^2-(3(17-12 \div 4)+2 \cdot 5) \div 4)$.

To make it somewhat easier to match up the pairs of left and right parentheses, we can use square brackets instead: $7+(5^2-[3(17-12 \div 4)+2 \cdot 5] \div 4)$.

? Exercises 1.6.1

Simplify the expression.

20. $7+(5^2-[3(17-12 \div 4)+2 \cdot 5] \div 4)$

Answer

20. 19

A fraction bar is another grouping symbol; it tells us to perform all of the steps on the top and separately perform all of the steps down below. The final step is to divide the top number by the bottom number.

? Exercises 1.6.1

Simplify each expression.

21. $\frac{15-1}{6+1}$

22. $\frac{(7+2) \cdot 4}{18 \div(3+3)}$

23. $\frac{5 \cdot 4^2}{2}$

24. $\frac{(5 \cdot 4)^2}{2}$

25. $\frac{(5-1)^2}{2+6}$

26. $(5-1)^2 \div 2+6$

Answer

21. 2

22. 12

23. 40

24. 200

25. 2

26. 14

This page titled 1.6: Order of Operations is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.1: Order of Operations** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **1.2: Negative Numbers** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

1.7: Evaluating Algebraic Expressions and Formulas

One common situation where using correct order of operations is important is when evaluating an expression or formula.

In math, *evaluating* an expression or formula means replacing the variables in the formula with numbers and then performing the required calculations to find a numerical value. For example, the formula for simple interest is I = Prt. To find the simple interest, I, earned on $400 with a 5% interest rate for 6 years you need to replace P with 400, r with 0.05 and t with 6 and the do all of the multiplication like this:

$$I = Prt$$

$$I = (400)(0.05)(6)$$

$$I = \$120$$

Tips for Evaluating Algebraic Expressions and Formulas

1. Replace all occurrences of variables with open parentheses. Leave room between the parentheses to substitute the given value of the variable.
2. Substitute the given values of variables in the open parentheses prepared in the first step.
3. Evaluate the resulting expression according to the Rules Guiding Order of Operations.

Let's begin with an example.

Example 1

Evaluate the expression $x^2 - 2xy + y^2$ at $x = -3$ and $y = 2$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $x^2 - 2xy + y^2$ with open parentheses.

$$x^2 - 2xy + y^2 = (\,)^2 - 2(\,)(\,) + (\,)^2$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned} x^2 - 2xy + y^2 &\quad \text{Original expression.} \\ = (-3)^2 - 2(-3)(2) + (2)^2 &\quad \text{Substitute } -3 \text{ for } x \text{ and } 2 \text{ for } y. \\ = 9 - 2(-3)(2) + 4 &\quad \text{Evaluate exponents first.} \\ = 9 - (-6)(2) + 4 &\quad \text{Left to right, multiply } 2(-3) = -6. \\ = 9 - (-12) + 4 &\quad \text{Left to right, multiply: } (-6)(2) = -12. \\ = 9 + 12 + 4 &\quad \text{Add the opposite.} \\ = 25 &\quad \text{Add.} \end{aligned}$$

Practice Exercise

If $x = -2$ and $y = -1$, evaluate $x^3 - y^3$.

Answer

-7

Example 2

Evaluate the expression $(a - b)^2$ If $a = 3$ and $b = -5$, at $a = 3$ and $b = -5$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $(a - b)^2$ with open parentheses.

$$(a-b)^2 = (() - ())^2$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}(a-b)^2 &= ((3)-(-5))^2 && \text{Substitute 3 for } a \text{ and } -5 \text{ for } b.\\ &= (3+5)^2 && \text{Add the opposite: } (3)-(-5) = 3+5\\ &= 8^2 && \text{Simplify inside parentheses: } 3+5=8\\ &= 64 && \text{Evaluate exponent: } 8^2 = 64\end{aligned}$$

Practice Exercise

If $a = 3$ and $b = -5$, evaluate $a^2 - b^2$.

Answer

−16

Example 3

Evaluate the expression |a|−|b| at a = 5 and b = −7.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression |a|−|b| with open parentheses.

$$|a| - |b| = |(\,)| - |(\,)|$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}|a| - |b| = |(5)| = |(-7)| && \text{Substitute 5 for } a \text{ and } -7 \text{ for } b.\\ = 5 - 7 && \text{Absolute values first: } |(5)| = 5 \text{ and } |(-7)| = 7\\ = 5 + (-7) && \text{Add the opposites: } 5-7 = 5+(-7).\\ = -2 && \text{Add: } 5+(-7) = -2.\end{aligned}$$

Practice Exercise

If $a = 5$ and $b = -7$, evaluate $2|a| - 3|b|$.

Answer

−11

Example 4

Evaluate the expression $|a - b|$ at $a = 5$ and $b = -7$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $|a - b|$ with open parentheses.

$$|a-b| = |(\,) - (\,)|$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned} |a-b| &= |(5)-(-7)| && \text{Substitute 5 for } a \text{ and } -7 \text{ for } b. \\ &= |5+7| && \text{Add the opposite: } 5-(-7)=5+7. \\ &= |12| && \text{Add: } 5+7=12. \\ &= 12 && \text{Take the absolute value: } |12|=12. \end{aligned}$$

Practice Exercise

If $a = 5$ and $b = -7$, evaluate $|2a - 3b|$.

Answer

31

Example 5

Evaluate the expression

$$\frac{ad-bc}{a+b}$$

at $a = 5$, $b = -3$, $c = 2$, and $d = -4$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression with open parentheses.

$$\frac{ad-bc}{a+b} = \frac{(\,)(\,)-(\,)(\,)}{(\,)+(\,)}$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned} \frac{ad-bc}{a+b} &= \frac{(5)-(-3)(2)}{(5)+(-3)} && \text{Substitute: 5 for } a,\ -3 \text{ for } b,\ 2 \text{ for } c,\ -4 \text{ for } d. \\ &= \frac{-20-(-6)}{2} && \text{Numerator: } (5)(=4)=-20,\ (-3)(2)=-6. \\ & && \text{Denominator: } 5+(-3)=2. \\ &= \frac{-20+6}{2} && \text{Numerator: Add the opposite.} \\ &= \frac{-14}{2} && \text{Numerator: } -20+6=-14. \\ &= -7 && \text{Divide.} \end{aligned}$$

Practice Exercise

If a = −7, b = −3, c = −15, 15, and $d = -14$, evaluate:

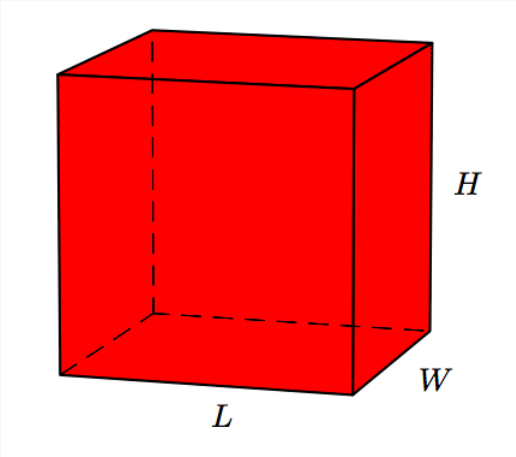
$$\frac{a^2+b^2}{c+d}$$

Answer

−2

Example 6

Pictured below is a rectangular prism.



The volume of the rectangular prism is given by the formula

$$V = LWH,$$

where L is the length, W is the width, and H is the height of the rectangular prism. Find the volume of a rectangular prism having length 12 feet, width 4 feet, and height 6 feet.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of of L, W, and H in the formula

$$V = LWH$$

with open parentheses.

$$V = (\,)(\,)(\,)$$

Next, substitute 12 ft for L, 4 ft for W, and 6 ft for H and simplify.

$$\begin{aligned} V &= (12\text{ft})(4\text{ft})(6\text{ft}) \\ &= 288\text{ft}^3 \end{aligned}$$

Hence, the volume of the rectangular prism is 288 cubic feet.

Practice Exercise

The surface area of the prism pictured in this example is given by the following formula:

$$S = 2(WH + LH + LW)$$

If L = 12, W = 4, and H = 6 feet, respectively, calculate the surface area.

Answer

288 square feet

More Practice Exercises

In Exercises 1-12, evaluate the expression at the given value of x.

1. $-3x^2 - 6x + 3$ at x = 7
2. $7x^2 - 7x + 1$ at x = −8

3. $-6x - 6$ at $x = 3$

4. $6x - 1$ at $x = -10$

5. $5x^2 + 2x + 4$ at $x = -1$

6. $4x^2 - 9x + 4$ at $x = -3$

7. $-9x - 5$ at $x = -2$

8. $-9x + 12$ at $x = 5$

9. $4x^2 + 2x + 6$ at $x = -6$

10. $-3x^2 + 7x + 4$ at $x = -7$

11. $12x + 10$ at $x = -12$

12. $-6x + 7$ at $x = 11$

In Exercises 13-28, evaluate the expression at the given values of x and y.

13. $|x|-|y|$ at $x = -5$ and $y = 4$

14. $|x|-|y|$ at $x = -1$ and $y = -2$

15. $-5x^2 + 2y^2$ at $x = 4$ and $y = 2$

16. $-5x^2 - 4y^2$ at $x = -2$ and $y = -5$

17. $|x|-|y|$ at $x = 0$ and $y = 2$

18. $|x|-|y|$ at $x = -2$ and $y = 0$

19. $|x - y|$ at $x = 4$ and $y = 5$

20. $|x - y|$ at $x = -1$ and $y = -4$

21. $5x^2 - 4xy + 3y^2$ at $x = 1$ and $y = -4$

22. $3x^2 + 5xy + 3y^2$ at $x = 2$ and $y = -1$

23. $|x - y|$ at $x = 4$ and $y = 4$

24. $|x - y|$ at $x = 3$ and $y = -5$

25. $-5x^2 - 3xy + 5y^2$ at $x = -1$ and $y = -2$

26. $3x^2 - 2xy - 5y^2$ at $x = 2$ and $y = 5$

27. $5x^2 + 4y^2$ at $x = -2$ and $y = -2$

28. $-4x^2 + 2y^2$ at $x = 4$ and $y = -5$

In Exercises 29-40, evaluate the expression at the given value of x.

29. $\frac{9+9x}{-x}$ at $x = -3$

30. $\frac{9-2x}{-x}$ at $x = -1$

31. $\frac{-8x+9}{-9+x}$ at $x = 10$

32. $\frac{2x+4}{1+x}$ at $x = 0$

33. $\frac{-4+9x}{7x}$ at $x = 2$

34. $\frac{-1-9x}{x}$ at $x = -1$

35. $\frac{-12-7x}{x}$ at $x = -1$

36. $\frac{12+11x}{3x}$ at $x = -6$

37. $\frac{6x-10}{5}$ + x at x = −6

38. $\frac{11x+11}{-4}$ + x at x = 5

39. $\frac{10x+11}{5}$ + x at x = −4

40. $\frac{6x+12}{-3}$ + x at x = 2

---

41. The formula

$$d = 16t^2$$

gives the distance (in feet) that an object falls from rest in terms of the time t that has elapsed since its release. Find the distance d (in feet) that an object falls in t = 4 seconds.

42. The formula

$$d = 16t^2$$

gives the distance (in feet) that an object falls from rest in terms of the time t that has elapsed since its release. Find the distance d (in feet) that an object falls in t = 24 seconds.

43. The formula

$$C = \frac{5(F-32)}{9}$$

gives the Celcius temperature C in terms of the Fahrenheit temperature F. Use the formula to find the Celsius temperature (◦ C) if the Fahrenheit temperature is F = 230◦ F.

44. The formula

$$C = \frac{5(F-32)}{9}$$

gives the Celcius temperature C in terms of the Fahrenheit temperature F. Use the formula to find the Celsius temperature (˚C) if the Fahrenheit temperature is F = 95 ˚F.

45. The Kelvin scale of temperature is used in chemistry and physics. Absolute zero occurs at 0˚ K, the temperature at which molecules have zero kinetic energy. Water freezes at 273˚ K and boils at K = 373˚ K. To change Kelvin temperature to Fahrenheit temperature, we use the formula

$$F = \frac{9(K-273)}{5} + 32.$$

Use the formula to change 28˚ K to Fahrenheit.

46. The Kelvin scale of temperature is used in chemistry and physics. Absolute zero occurs at 0˚ K, the temperature at which molecules have zero kinetic energy. Water freezes at 273˚ K and boils at K = 373˚ K. To change Kelvin temperature to Fahrenheit temperature, we use the formula

$$F = \frac{9(K-273)}{5} + 32.$$

Use the formula to change 248˚ K to Fahrenheit.

47. A ball is thrown vertically upward. Its velocity t seconds after its release is given by the formula

$$v = v0 - gt,$$

where v_0 is its initial velocity, g is the acceleration due to gravity, and v is the velocity of the ball at time t. The acceleration due to gravity is g = 32 feet per second per second. If the initial velocity of the ball is v_0 = 272 feet per second, find the speed of the ball after t = 6 seconds.

48. A ball is thrown vertically upward. Its velocity t seconds after its release is given by the formula

$$v = v_0 - gt,$$

where v_0 is its initial velocity, g is the acceleration due to gravity, and v is the velocity of the ball at time t. The acceleration due to gravity is g = 32 feet per second per second. If the initial velocity of the ball is v_0 = 470 feet per second, find the speed of the ball after t = 4 seconds.

---

49. **Even numbers**. Evaluate the expression 2n for the following values:

    i) n = 1

    ii) n = 2

    iii) n = 3

    iv) n = −4

    v) n = −5

    vi) Is the result always an even number? Explain.

50. **Odd numbers**. Evaluate the expression 2n + 1 for the following values:

    i) n = 1

    ii) n = 2

    iii) n = 3

    iv) n = −4

    v) n = −5

    vi) Is the result always an odd number? Explain.

Answers

1. −186

3. −24

5. 7

7. 13

9. 138

11. −134

13. 1

15. −72

17. −2

19. 1

21. 69

23. 0

25. 9

27. 36

29. −6

31. −71

33. 1

35. 5

37. 46

39. −29

41. 256 feet

43. 110 degrees

45. −409° F

47. 80 feet per second

49.

    i) 2

    ii) 4

    iii) 6

    iv) −8

    v) −10

    vi) Yes, the result will always be an even number because 2 will always be a factor of the product 2n.

This page titled 1.7: Evaluating Algebraic Expressions and Formulas is shared under a CC BY-NC-SA 3.0 license and was authored, remixed, and/or curated by David Arnold.

- **3.2: Evaluating Algebraic Expressions** by David Arnold is licensed CC BY-NC-SA 3.0.

CHAPTER OVERVIEW

2: Solving Equations

2.1: The Language of Algebra
2.2: Solving Equations with Addition and Subtraction Properties of Equality
2.3: Solving Equations with the Division and Multiplication Properties of Equality
2.4: Solving Equations with Variables and Constants on Both Sides
2.5: Solving Equations with Fractions or Decimals
2.6: General Strategy to Solve Linear Equations
2.7: Solving a Formula for a Specific Variable

2: Solving Equations is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

2.1: The Language of Algebra

Learning Objectives

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Evaluate an expression
- Identify and combine like terms
- Translate an English phrase to an algebraic expression

Use Variables and Algebraic Symbols

Suppose this year Greg is 20 years old and Alex is 23. You know that Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are variables and the 3 is a constant. The ages change ("vary") but the 3 years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by 3 years, 3 is the constant. In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age g, then we could use $g+3$ to represent Alex's age. See Table $2.1.1$.

Table $2.1.1$

Greg's age	Alex's age
12	15
20	23
35	38
g	$g+3$

The letters used to represent these changing ages are called variables. The letters most commonly used for variables are x, y, a, b, and c.

Definition: VARIABLE

A **variable** is a letter that represents a number whose value may change.

Definition: CONSTANT

A **constant** is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic **operations: addition, subtraction, multiplication,** and **division**. We'll list the symbols used to indicate these operations below (Table $2.1.2$). You'll probably recognize some of them.

Table $2.1.2$

Operation	Notation	Say:	The result is...
Addition	$a+b$	a plus b	the sum of a and b
Subtraction	$a-b$	a minus b	the difference of a and b
Multiplication	$a \cdot b, ab, (a)(b), (a)b, a(b)$	a times b	the product of a and b
Division	$a \div b, a/b, \frac{a}{b}, b\overline{)a}$	a divided by b	the quotient of a and b, a is called the dividend, and b is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words “of” and “and.”

- The *difference* of 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9-2$.
- The *product* of 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as $4 \cdot 8$.

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ (‘three times y’) or $3 \cdot x \cdot y$ (three times x times y)? To make it clear, use $\cdot$ or parentheses for multiplication.

When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

EQUALITY SYMBOL

$a=b$ is read “a is equal to b”

The symbol "$=$" is called the **equal sign**.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols "$<$" and "$>$".

INEQUALITY

$a<b$ is read “a is less than b”

a is to the left of b on the number line

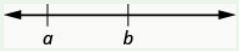


Figure 2.1.1

$a>b$ is read "a is greater than b”

a is to the right of b on the number line

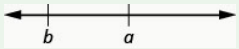


Figure 2.1.2

The expressions $a<b$ or $a>b$ can be read from left to right or right to left, though in English we usually read from left to right Table 2.1.3. In general, $a<b$ is equivalent to $b>a$. For example $7<11$ is equivalent to $11>7$. And $a>b$ is equivalent to $b<a$. For example $17>4$ is equivalent to $4<17$.

Table 2.1.3

Inequality Symbols	Words
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or not equal to b

? Exercise 2.1.1

Translate from algebra into English:

1. $17 \leq 26$
2. $8 \neq 17 - 3$
3. $12 > 27 \div 3$
4. $y + 7 < 19$

Answer

1. $17 \leq 26$, 17 is less than or equal to 26
2. $8 \neq 17 - 3$, 8 is not equal to 17 minus 3
3. $12 > 27 \div 3$, 12 is greater than 27 divided by 3
4. $y + 7 < 19$, y plus 7 is less than 19

? Exercise 2.1.2

Translate from algebra into English:

1. $14 \leq 27$
2. $19 - 2 \neq 8$
3. $12 > 4 \div 2$
4. $x - 7 < 1$

Answer

1. 14 is less than or equal to 27
2. 19 minus 2 is not equal to 8
3. 12 is greater than 4 divided by 2
4. x minus 7 is less than 1

? Exercise 2.1.3

Translate from algebra into English:

1. $19 \leq 15$
2. $7 = 12 - 5$
3. $15 \div 3 < 8$
4. $y + 3 < 6$

Answer

1. 19 is greater then or equal to 15
2. 7 is equal to 12 minus 5

3. 15 divided by 3 is less than 8
4. y plus 3 is greater than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

GROUPING SYMBOLS

Parentheses	()
Brackets	[]
Braces	{}

Here are some examples of expressions that include **grouping symbols**. We will simplify expressions like these later in this section.

$$8(14-8) \qquad 21-3[2+4(9-8)] \qquad 24\div\{13-2[1(6-5)+4]\}$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have expressions and *equations*.

EXPRESSION

An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Table **2.1.4**

Expression	Words	English Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	n minus one	the difference of n and one
$6\cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb. An **equation** is two expressions linked with an **equal sign**. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Definition: EQUATION

An **equation** is two expressions connected by an equal sign.

Here are some examples of equations.

Table **2.1.5**

Equation	English Sentence
$3+5=8$	the sum of three and five is equal to eight
$n-1=14$	n minus one equals fourteen
$6\cdot 7=42$	The product of six ans seven is equal to forty-two
$x=53$	x is equal to fifty-three
$y+9=2y-3$	y plus nine is equal to two y minus three

? Exercise 2.1.4

Determine if each is an expression or an equation:

1. $2(x+3)=10$
2. $4(y-1)+1$
3. $x\div 25$
4. $y+8=40$

Answer

1. $2(x+3)=10$. This is an equation — two expressions are connected with an equal sign.
2. $4(y-1)+1$. This is an expression — no equal sign.
3. $x\div 25$. This is an expression — no equal sign.
4. $y+8=40$. This is an equation — two expressions are connected with an equal sign.

? Exercise 2.1.5

Determine if each is an expression or an equation:

1. $3(x-7)=27$
2. $5(4y-2)-7$

Answer

1. equation
2. expression

? Exercise 2.1.6

Determine if each is an expression or an equation:

1. $y^3\div 14$
2. $4x-6=22$

Answer

1. expression
2. equation

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

EVALUATE AN EXPRESSION

To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Exercise 2.1.19

Evaluate $7x - 4$, when

1. $x = 5$
2. $x = 1$

Answer

1.

when $x = 5$	$7x - 4$
	$7(5) - 4$
Multiply.	$35 - 4$
Subtract.	31

2.

when $x = 1$	$7x - 4$
	$7(1) - 4$
Multiply.	$7 - 4$
Subtract.	3

Exercise 2.1.20

Evaluate $8x - 3$, when

1. $x = 2$
2. $x = 1$

Answer

1. 13
2. 5

Exercise 2.1.21

Evaluate $4y - 4$, when

1. $y = 3$
2. $y = 5$

Answer

1. 8
2. 16

Exercise 2.1.22

Evaluate $x = 4$, when

1. x^2
2. 3^x

Answer

1.

	x^2
Replace x with 4.	$(4)^2$
Use definition of exponent.	$4 \cdot 4$
Simplify.	16

2.

	3^x
Replace x with 4.	\(3^\)
Use definition of exponent.	$3 \cdot 3 \cdot 3 \cdot 3$
Simplify.	81

Exercise 2.1.23

Evaluate $x = 3$, when

1. x^2
2. 4^x

Answer

1. 9
2. 64

Exercise 2.1.24

Evaluate $x = 6$, when

1. x^3
2. 2^x

Answer

1. 216
2. 64

Exercise 2.1.25

Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Answer

	$2x^2+3x+8$
Substitute $x=4$.	$2x^2+3x+8$ $2(4)^2+3(4)+8$
Follow the order of operations.	$2(16)+3(4)+8$
	$32+12+8$
	52

? Exercise 2.1.26

Evaluate $3x^2+4x+1$ when $x=3$.

Answer

40

? Exercise 2.1.27

Evaluate $6x^2-4x-7$ when $x=2$.

Answer

9

Identify and Combine Like Terms

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

TERM

A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are $7, y, 5x^2, 9a$, and b^5.

The constant that multiplies the variable is called the **coefficient**.

COEFFICIENT

The **coefficient** of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x=1\cdot x$.

? Exercise 2.1.28

Identify the coefficient of each term:

1. $14y$
2. $15x^2$
3. a

Answer

1. The coefficient of $14y$ is 14
2. The coefficient of $15x^2$ is 15

3. The coefficient of a is 1 since $a = 1a$.

Exercise 2.1.29

Identify the coefficient of each term:

1. $17x$
2. $41b^2$
3. z

Answer

1. 14
2. 41
3. 1

Exercise 2.1.30

Identify the coefficient of each term:

1. $9p$
2. $13a^2$
3. y^3

Answer

1. 9
2. 13
3. 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?

$$5x \quad 7 \quad n^2 \quad 4 \quad 3x \quad 9n^2$$

The 7 and the 4 are both constant terms.

The $5x$ and the $3x$ are both terms with x.

The n^2 and the $9n^2$ are both terms with n^2.

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- x^2 and $9x^2$ are like terms.

LIKE TERMS

Terms that are either constants or have the same variables raised to the same powers are called **like terms**.

Exercise 2.1.31

Identify the like terms: $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$.

Answer

y^3 and $4y^3$ are like terms because both have y^3; the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2; the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like $9x$.

Exercise 2.1.32

Identify the like terms: $9, 2x^3, y^2, 8x^3, 15, 9y, 11y^2$.

Answer

9 and 15, y^2 and $11y^2$, $2x^3$ and $8x^3$

Exercise 2.1.33

Identify the like terms: $4x^3, 8x^2, 19, 3x^3, 24, 6x^3$.

Answer

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2 + 3x + 8$, from Example, the three terms are $2x^2$, $3x$, and 8.

Exercise 2.1.34

Identify the terms in each expression.

1. $9x^2 + 7x + 12$
2. $8x + 3y$

Answer

1. The terms of $9x^2 + 7x + 12$ are $9x^2, 7x$, and 12.
2. The terms of $8x + 3y$ are $8x$ and $3y$.

Exercise 2.1.35

Identify the terms in the expression $4x^2 + 5x + 17$.

Answer

$4x^2, 5x, 17$

Exercise 2.1.36

Identify the terms in the expression $5x + 2y$.

Answer

$5x, 2y$

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought $12x$, you would be right!

$$\begin{array}{c} 4x + 7x + x \\ x + x + x + x \quad + x + x + x + x + x + x + x \quad + x \\ 12x \end{array} \tag{2.1.1}$$

Add the coefficients and keep the same variable. It doesn't matter what x is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

Simplify: $4x + 7x + x$

Add the coefficients. $12x$

Exercise 2.1.37: How To Combine Like Terms

Simplify: $2x^2 + 3x + 7 + x^2 + 4x + 5$

Answer

Step 1. Identify the like terms.	$2x^2 + 3x + 7 + x^2 + 4x + 5$ $2x^2 + 3x + 7 + x^2 + 4x + 5$
Step 2. Rearrange the expression so the like terms are together.	$2x^2 + x^2 + 3x + 4x + 7 + 5$
Step 3. Combine like terms.	$3x^2 + 7x + 12$

Exercise 2.1.38

Simplify: $3x^2 + 7x + 9 + 7x^2 + 9x + 8$.

Answer

$10x^2 + 16x + 17$

Exercise 2.1.39

Simplify: $4y^2 + 5y + 2 + 8y^2 + 4y + 5$.

Answer

$12y^2 + 9y + 7$

COMBINE LIKE TERMS.

1. Identify like terms.
2. Rearrange the expression so like terms are together.
3. Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra, then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. Table 2.1.7 summarizes them.

Table 2.1.7

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$ (2.1.2)
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a-b$ (2.1.3)
Multiplication	a times b the product of a and b twice a	$a \cdot b, ab, a(b), (a)(b)$ (2.1.4) $2a$ (2.1.5)
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a \div b, a/b, \frac{a}{b}, b\overline{)a}$ (2.1.6)

Look closely at these phrases using the four operations:

the **sum** *of a and b*
the **difference** *of a and b*
the **product** *of a and b*
the **quotient** *of a and b*

Figure **2.1.5**

Each phrase tells us to operate on two numbers. Look for the words *of* and *and* to find the numbers.

? Exercise 2.1.40

Translate each English phrase into an algebraic expression:

1. the difference of $17x$ and 5
2. the quotient of $10x^2$ and 7.

Answer

1. The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

   the *difference of* 17*x* *and* 5

   17*x* minus 5

   17*x* – 5

2. The key word is "quotient," which tells us the operation is division.

   the *quotient of* $10x^2$ *and* 7

   divide $10x^2$ by 7

   $10x^2 \div 7$

   This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Exercise 2.1.41

Translate each English phrase into an algebraic expression:

1. the difference of $14x^2$ and 13
2. the quotient of $12x$ and 2.

Answer

1. $14x^2 - 13$
2. $12x \div 2$

Exercise 2.1.42

Translate each English phrase into an algebraic expression:

1. the sum of $17y^2$ and 19
2. the product of 7 and y.

Answer

1. $17y^2 + 19$
2. $7y$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight "more than" means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven "less than" means 7 subtracted from your present age.

Exercise 2.1.43

Translate the English phrase into an algebraic expression:

1. Seventeen more than y
2. Nine less than $9x^2$.

Answer

1. The key words are *more than*. They tell us the operation is addition. *More than* means "added to."

$$\begin{aligned} &\text{Seventeen more than } y \\ &\text{Seventeen added to } y \\ &y + 17 \end{aligned}$$

2. The key words are *less than*. They tell us to subtract. *Less than* means "subtracted from."

$$\begin{aligned} &\text{Nine less than } 9x^2 \\ &\text{Nine subtracted from } 9x^2 \\ &9x^2 - 9 \end{aligned}$$

Exercise 2.1.44

Translate the English phrase into an algebraic expression:

1. Eleven more than x
2. Fourteen less than $11a$.

Answer

1. $x + 11$

2. $11a - 14$

Exercise 2.1.45

Translate the English phrase into an algebraic expression:

1. 13 more than z
2. 18 less than $8x$.

Answer

1. $z + 13$
2. $8x - 18$

Exercise 2.1.46

Translate the English phrase into an algebraic expression:

1. five times the sum of m and n
2. the sum of five times m and n.

Answer

There are two operation words—*times* tells us to multiply and *sum* tells us to add.

1. Because we are multiplying 5 times the sum we need parentheses around the sum of m and n, $(m + n)$. This forces us to determine the sum first. (Remember the order of operations.)

$$\begin{array}{c} \text{five times the sum of } m \text{ and } n \\ 5(m + n) \end{array} \tag{2.1.7}$$

2. To take a sum, we look for the words "of" and "and" to see what is being added. Here we are taking the sum *of* five times m and \(n\.)

$$\begin{array}{c} \text{the sum of five times } m \text{ and } n \\ 5m + n \end{array} \tag{2.1.8}$$

Exercise 2.1.47

Translate the English phrase into an algebraic expression:

1. four times the sum of p and q
2. the sum of four times p and q.

Answer

1. $4(p + q)$
2. $4p + q$

Exercise 2.1.48

Translate the English phrase into an algebraic expression:

1. the difference of two times x and 8,
2. two times the difference of x and 8.

Answer

1. $2x - 8$

2. $2(x-8)$

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We'll see how to do this in the next two examples.

Exercise 2.1.49

The length of a rectangle is 6 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Answer

Write a phrase about the length of the rectangle.	6 less than the width
Substitute w for 'the width.'	6 less then w
Rewrite 'less than' as 'subtracted from.'	6 subtracted from w
Translate the phrase into algebra.	$w-6$

(2.1.9)

Exercise 2.1.50

The length of a rectangle is 7 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Answer

$w-7$

Exercise 2.1.51

The width of a rectangle is 6 less than the length. Let l represent the length of the rectangle. Write an expression for the width of the rectangle.

Answer

$l-6$

Exercise 2.1.52

June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Answer

Write the phrase about the number of dimes.	three less than four times the number of quarters
Substitute q for the number of quarters.	3 less than 4 times q
Translate '4 times q .'	3 less than 4q
Translate the phrase into algebra.	4q - 3

(2.1.10)

Exercise 2.1.53

Geoffrey has dimes and quarters in his pocket. The number of dimes is eight less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Answer

$4q-8$

Exercise 2.1.54

Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Answer

$7n+3$

Key Concepts

- **Notation** — **The result is…**
  - $a+b$ — the sum of a and b
  - $a-b$ — the difference of a and b
  - $a \cdot b, ab, (a)(b)(a)b, a(b)$ — the product of a and b
  - $a \div b, a/b, \frac{a}{b}, b\overline{a}$ — the quotient of a and b
- **Inequality**
  - $a < b$ is read 'a is less than b'' — a is to the left of b on the number line
  - $a > b$ is read 'a is greater than b'' — a is to the right of b on the number line
- **Inequality Symbols** — **Words**
  - $a \neq b$ — a is not equal to b
  - $a < b$ — a is less than b
  - $a \leq b$ — a is less than or equal to b
  - $a > b$ — a is greater than b
  - $a \geq b$ — a is greater than or equal to b
- **Combine Like Terms**
  1. Identify like terms.
  2. Rearrange the expression so like terms are together.
  3. Add or subtract the coefficients and keep the same variable for each group of like terms.

Glossary

coefficient

The coefficient of a term is the constant that multiplies the variable in a term.

constant

A constant is a number whose value always stays the same.

equality symbol

The symbol "=" is called the equal sign. We read $a = b$ as "a is equal to b."

equation

An equation is two expressions connected by an equal sign.

evaluate an expression

To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

expression

An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

like terms

Terms that are either constants or have the same variables raised to the same powers are called like terms.

simplify an expression

To simplify an expression, do all operations in the expression.

term

A term is a constant or the product of a constant and one or more variables.

variable

A variable is a letter that represents a number whose value may change.

This page titled 2.1: The Language of Algebra is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **1.3: Use the Language of Algebra** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

2.2: Solving Equations with Addition and Subtraction Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Verify a solution of an equation
- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same – so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Definition: Solution of an Equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

TO DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true (the left side is equal to the right side)
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Exercise 2.2.1

Determine whether $x = \frac{3}{2}$ is a solution of $4x - 2 = 2x + 1$.

Answer

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

	$4x - 2 = 2x + 1$
Substitute $\frac{3}{2}$ for x.	$4\left(\frac{3}{2}\right) - 2 \stackrel{?}{=} 2\left(\frac{3}{2}\right) + 1$
Multiply.	$6 - 2 \stackrel{?}{=} 3 + 1$
Subtract.	$4 = 4 \checkmark$

Since $x = \frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x - 2 = 2x + 1$.

Exercise 2.2.2

Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3$?

Answer

no

Exercise 2.2.3

Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

Answer

yes

Solve Equations Using the Subtraction and Addition Properties of Equality

We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in Figure 2.2.1. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?

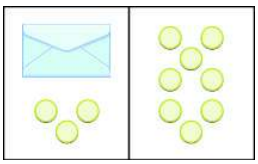


Figure 2.2.1: The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." See Figure 2.2.2 for an illustration of this process.

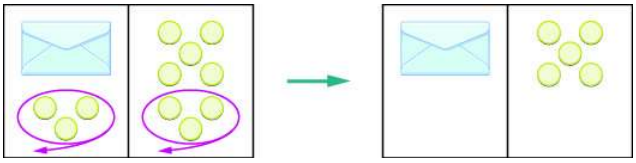


Figure 2.2.2: The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In Figure 2.2.3 each side of the workspace represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x.

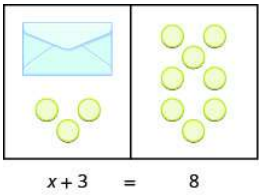


Figure 2.2.3: The illustration shows a model for the equation $x + 3 = 8$.

Let's write algebraically the steps we took to discover how many counters were in the envelope:

Table 2.2.1

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Check:

Five in the envelope plus three more does equal eight!

$$5+3=8 \tag{2.2.1}$$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

SUBTRACTION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{aligned} &\textbf{If} \quad a=b \\ &\textbf{then} \quad a-c=b-c \end{aligned} \tag{2.2.2}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

Note

Doing the Manipulative Mathematics activity "Subtraction Property of Equality" will help you develop a better understanding of how to solve equations by using the **Subtraction Property of Equality**.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Exercise 2.2.4

Solve: $y+37=-13$.

Answer

To get y by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

	$y+37=-13$
Subtract 37 from each side to 'undo' the addition.	$y+37-37=-13-37$
Simplify.	$y=-50$
Check:	$y+37=-13$
Substitute $y=-50$	$-50+37=-13$
	$-13\stackrel{?}{=}-13$ ✓

Since y=−50 makes y+37=−13 a true statement, we have the solution to this equation.

Exercise 2.2.5

Solve: $x+19=-27$.

Answer

$x=-46$

Exercise 2.2.6

Solve: $x + 16 = -34$.

Answer

$x = -50$

What happens when an equation has a number subtracted from the variable, as in the equation $x - 5 = 8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to 'undo' the subtraction we will add the number to both sides. We use the **Addition Property of Equality**.

ADDITION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{array}{ll} \textbf{If} & a = b \\ \textbf{then} & a + c = b + c \end{array} \tag{2.2.3}$$

When you add the same quantity from both sides of an equation, you still have equality.

In Exercise 2.2.4, 37 was added to the y and so we subtracted 37 to 'undo' the addition. In Exercise 2.2.7, we will need to 'undo' subtraction by using the **Addition Property of Equality**.

Exercise 2.2.7

Solve: $a - 28 = -37$.

Answer

		$a - 28 = -37$
Add 28 to each side to 'undo' the subtraction.		$a - 28 + 28 = -37 + 28$
Simplify.		$a = -9$
Check:	$a - 28 = -37$	
Substitute $a = -9$	$-9 - 28 = -37$	
	$-37 \stackrel{?}{=} -37$ ✓	
		The solution to $a - 28 = -37$ is $a = -9$.

Exercise 2.2.8

Solve: $n - 61 = -75$.

Answer

$n = -14$

Exercise 2.2.9

Solve: $p - 41 = -73$.

Answer

$p = -32$

Exercise 2.2.10

Solve: $x - \frac{5}{8} = \frac{3}{4}$

Answer

	$x - \frac{5}{8} = \frac{3}{4}$
Use the Addition Property of Equality.	$x - \frac{5}{8} + \frac{5}{8} = \frac{3}{4} + \frac{5}{8}$
Find the LCD to add the fractions on the right.	$x - \frac{5}{8} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8}$
Simplify.	$x = \frac{11}{8}$
Check:	$x - \frac{5}{8} = \frac{3}{4}$
Substitute $x = \frac{11}{8}$	$\frac{11}{8} - \frac{5}{8} \stackrel{?}{=} \frac{3}{4}$
Subtract.	$\frac{6}{8} \stackrel{?}{=} \frac{3}{4}$
Simplify.	$\frac{3}{4} = \frac{3}{4}$ ✓
	The solution to $x - \frac{5}{8} = \frac{3}{4}$ is $x = \frac{11}{8}$.

Exercise 2.2.11

Solve: $p - \frac{2}{3} = \frac{5}{6}$.

Answer

$p = \frac{9}{6} p = \frac{3}{2}$

Exercise 2.2.12

Solve: $q - \frac{1}{2} = \frac{5}{6}$.

Answer

$q = \frac{4}{3}$

The next example will be an equation with decimals.

Exercise 2.2.13

Solve: $n - 0.63 = -4.2$.

Answer

	$n - 0.63 = -4.2$
Use the Addition Property of Equality.	$n - 0.63 + 0.63 = -4.2 + 0.63$
Add.	$n = -3.57$
Check:	$n = -3.57$
Let $n = -3.57$.	$-3.57 - 0.63 \stackrel{?}{=} -4.2$
	$-4.2 = -4.2$ ✓

Exercise 2.2.14

Solve: $b - 0.47 = -2.1$.

Answer

$b = -1.63$

Exercise 2.2.15

Solve: $c - 0.93 = -4.6$.

Answer

$c = -3.67$

Solve Equations That Require Simplification

In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality.

You should always simplify as much as possible before you try to isolate the variable. Remember that to simplify an expression means to do all the operations in the expression. Simplify one side of the equation at a time. Note that **simplification** is different from the process used to solve an equation in which we apply an operation to both sides.

Exercise 2.2.16: How to Solve Equations That Require Simplification

Solve: $9x - 5 - 8x - 6 = 7$.

Answer

Step 1. Simplify the expressions on each side as much as possible.	Rearrange the terms, using the Commutative Property of Addition. Combine like terms. Notice that each side is now simplified as much as possible.	$9x - 5 - 8x - 6 = 7$ $9x - 8x - 5 - 6 = 7$ $x - 11 = 7$
Step 2. Isolate the variable.	Now isolate x. Undo subtraction by adding 11 to both sides.	$x - 11 + 11 = 7 + 11$

Step 3. Simplify the expressions on both sides of the equation.		$x = 18$
Step 4. Check the solution.		**Check:** Substitute $x = 18$. $9x - 5 - 8x - 6 = 7$ $9(18) - 5 - 8(18) - 6 \stackrel{?}{=} 7$ $162 - 5 - 144 - 6 \stackrel{?}{=} 7$ $157 - 144 - 6 \stackrel{?}{=} 7$ $13 - 6 \stackrel{?}{=} 7$ $7 = 7$ ✓ The solution to $9x - 5 - 8x - 6 = 7$ is $x = 18$.

? Exercise 2.2.17

Solve: $8y - 4 - 7y - 7 = 4$.

Answer

$y = 15$

? Exercise 2.2.18

Solve: $6z + 5 - 5z - 4 = 3$.

Answer

$z = 2$

? Exercise 2.2.19

Solve: $5(n-4)-4n=-8$.

Answer

We simplify both sides of the equation as much as possible before we try to isolate the variable.

	$5(n-4)-4n=-8$
Distribute on the left.	$5n-20-4n=-8$
Use the Commutative Property to rearrange terms.	$5n-4n-20=-8$
Combine like terms.	$n-20=-8$
Each side is as simplified as possible. Next, isolate n.	
Undo subtraction by using the Addition Property of Equality.	$n-20+20=-8+20$
Add.	$n=12$
Check. Substitute n=12. $5(n-4)-4n=-8$ $5(12-4)-4(12)\stackrel{?}{=}-8z$ $5(8)-48\stackrel{?}{=}-8$ $40-48\stackrel{?}{=}-8$ $-8=-8\checkmark$	
	The solution to $5(n-4)-4n=-8$ is $n=12$.

Exercise 2.2.20

Solve: $5(p-3)-4p=-10$.

Answer

$p=5$

Exercise 2.2.21

Solve: $4(q+2)-3q=-8$.

Answer

$q=-16$

Exercise 2.2.22

Solve: $3(2y-1)-5y=2(y+1)-2(y+3)$.

Answer

We simplify both sides of the equation before we isolate the variable.

	$3(2y-1)-5y=2(y+1)-2(y+3)$
Distribute on both sides.	$6y-3-5y=2y+2-2y-6$
Use the Commutative Property of Addition.	$6y-5y-3=2y-2y+2-6$
Combine like terms.	$y-3=-4$
Each side is as simplified as possible. Next, isolate y.	
Undo subtraction by using the Addition Property of Equality.	$y-3+3=-4+3$
Add.	$y=-1$
Check. Let y=−1. $3(2y-1)-5y=2(y+1)-2(y+3)$ $3(2(-1)-1)-5(-1)\stackrel{?}{=}2(-1+1)-2(-1+3)$ $3(-2-1)+5\stackrel{?}{=}2(0)-2(2)$ $3(-3)+5\stackrel{?}{=}-4$ $-9+5\stackrel{?}{=}-4$ $-4=-4\checkmark$	
	The solution to $3(2y-1)-5y=2(y+1)-2(y+3)3(2y-1)-5y=2(y+1)-2(y+3)$ is $y=-1$.

? Exercise 2.2.23

Solve: $4(2h-3)-7h=6(h-2)-6(h-1)$.

Answer

$h=6$

? Exercise 2.2.24

Solve: $2(5x+2)-9x=3(x-2)-3(x-4)$.

Answer

$x=2$

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating from English sentences into equations. Our first step is to look for the word (or words) that would translate to the **equals sign**. Here are some of the words that are commonly used.

Equals =

- is
- is equal to
- is the same as
- the result is
- gives
- was

- will be

The steps we use to translate a sentence into an equation are listed below.

TRANSLATE AN ENGLISH SENTENCE TO AN ALGEBRAIC EQUATION

1. Locate the “equals” word(s). Translate to an equals sign (=).
2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

Exercise 2.2.25

Translate and solve: Eleven more than x is equal to 54.

Answer

Translate.	Eleven more than x is equal to 54 $x+11 = 54$
Subtract 11 from both sides.	$x+11-11 = 54-11$
Simplify.	$x = 43$
Check: Is 54 eleven more than 43? $43+11 \stackrel{?}{=} 54$ $54 = 54\checkmark$ (2.2.4)	

Exercise 2.2.26

Translate and solve: Ten more than x is equal to 41.

Answer

$x+10=41; x=31$

Exercise 2.2.27

Translate and solve: Twelve less than x is equal to 51.

Answer

y−12=51;y=63

Exercise 2.2.28

Translate and solve: The difference of 12t and 11t is −14.

Answer

Translate.	The difference of $12t$ and $11t$ is -14 $12t - 11t = -14$
Simplify.	$t = -14$
Check: $12(-14) - 11(-14) \stackrel{?}{=} -14$ $-168 + 154 \stackrel{?}{=} -14$ (2.2.5) $-14 = -14$✓	

? Exercise 2.2.29

Translate and solve: The difference of 4x and 3x is 14.

Answer

$4x - 3x = 14; x = 14$

? Exercise 2.2.30

Translate and solve: The difference of 7a and 6a is −8.

Answer

$7a - 6a = -8; a = -8$

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real life question. To begin with that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

We will start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use q for the number of quarters if you were solving a problem about coins.

? Exercise 2.2.31: How to Solve Translate and Solve Applications

The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Answer

Step 1. Read the problem. Make sure all the words and ideas are understood.	The problem is about the weight of newspapers.	
Step 2. Identify what we are asked to find.	What are we asked to find?	"How much did the newspapers weigh the 2nd month?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Choose a variable.	Let w = weight of the newspapers the 1st month

Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.	Restate the problem. We know the weight of the newspapers the second month is 28 pounds. Translate into an equation, using the variable w.	Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds. Weight from 1st month plus 28 equals 57. $w + 28 = 57$
Step 5. Solve the equation using good algebra techniques.	Solve.	$w + 28 - 28 = 57 - 28$ $w = 29$
Step 6. Check the answer in the problem and make sure it makes sense.	Does 1st month's weight plus 2nd month's weight equal 57 pounds?	**Check:** Does 1st month's weight plus 2nd month's weight equal 57 pounds? $29 + 28 \stackrel{?}{=} 57$ $57 = 57$ ✓
Step 7. Answer the question with a complete sentence.	Write a sentence to answer "How much did the newspapers weigh the 2nd month?"	The 2nd month the newspapers weighed 29 pounds.

Exercise 2.2.32

Translate into an algebraic equation and solve:

The Pappas family has two cats, Zeus and Athena. Together, they weigh 23 pounds. Zeus weighs 16 pounds. How much does Athena weigh?

Answer

7 pounds

Exercise 2.2.33

Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Answer

42 books

SOLVE AN APPLICATION.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with the important information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Exercise 2.2.34

Randell paid $28,675 for his new car. This was $875 less than the sticker price. What was the sticker price of the car?

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	"What was the sticker price of the car?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let s = the sticker price of the car.
Step 4. Translate into an equation. Restate the problem in one sentence.	$28675 is $875 less than the sticker price
	$28675 is $875 less than s
Step 5. Solve the equation.	$28675 = s - 875$ $28675 + 875 = s - 875 + 875$ $29550 = s$
Step 6. Check the answer. Is $875 less than $29550 equal to $28675?	$29550 - 875 \stackrel{?}{=} 28675$ $28675 = 28675$✓
Step 7. Answer the question with a complete sentence.	The sticker price of the car was $29550.

Exercise 2.2.35

Translate into an algebraic equation and solve:

Eddie paid $19875 for his new car. This was $1025 less than the sticker price. What was the sticker price of the car?

Answer

$20900

Exercise 2.2.36

Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

Answer

$11.00

Key Concepts

- **To Determine Whether a Number is a Solution to an Equation**
  1. **Substitute the number in for the variable in the equation.**
  2. **Simplify the expressions on both sides of the equation.**
  3. **Determine whether the resulting statement is true.**
     - If it is true, the number is a solution.

- If it is not true, the number is not a solution.

- **Addition Property of Equality**
  - For any numbers *a*, *b*, and *c*, if a=b, then a+c=b+c.
- **Subtraction Property of Equality**
  - For any numbers *a*, *b*, and *c*, if a=b, then a−c=b−c.
- **To Translate a Sentence to an Equation**
  1. Locate the “equals” word(s). Translate to an equal sign (=).
  2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
  3. Translate the words to the right of the “equals” word(s) into an algebraic expression.
- **To Solve an Application**
  1. Read the problem. Make sure all the words and ideas are understood.
  2. Identify what we are looking for.
  3. Name what we are looking for. Choose a variable to represent that quantity.
  4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
  5. Solve the equation using good algebra techniques.
  6. Check the answer in the problem and make sure it makes sense.
  7. Answer the question with a complete sentence.

Glossary

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

This page titled 2.2: Solving Equations with Addition and Subtraction Properties of Equality is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **2.1: Solve Equations Using the Subtraction and Addition Properties of Equality** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

2.3: Solving Equations with the Division and Multiplication Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in Figure **2.3.1**.

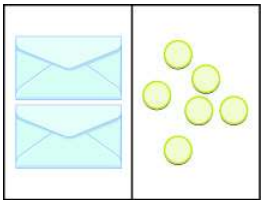


Figure **2.3.1**: The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are "hidden" in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in Figure **2.3.2**? There are two envelopes, and each contains xx counters. Together, the two envelopes must contain a total of 6 counters.

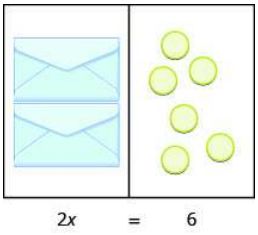


Figure **2.3.2**: The illustration shows a model of the equation $2x = 6$.

	$2x = 6$
If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2} = \frac{6}{2}$
we get:	$x = 3$

We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works! Three counters in each of two envelopes does equal six!

This example leads to the **Division Property of Equality**.

THE DIVISION PROPERTY OF EQUALITY

For any numbers a, b, and c, and $c \neq 0$,

$$\begin{array}{rcl} \text{If} & a & = & b \\ \text{then} & \frac{a}{c} & = & \frac{b}{c} \end{array} \tag{2.3.1}$$

When you divide both sides of an equation by any non-zero number, you still have equality.

Note

Doing the Manipulative Mathematics activity "**Division Property of Equality**" will help you develop a better understanding of how to solve equations by using the Division Property of Equality.

The goal in solving an equation is to 'undo' the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to 'undo' the multiplication.

Exercise 2.3.1

Solve: $5x = -27$.

Answer

To isolate x, "undo" the multiplication by 5.		$5x = -27$
Divide to 'undo' the multiplication.		$\frac{5x}{5} = -\frac{27}{5}$
Simplify.		$x = -\frac{27}{5}$
Check:	$5x = -27$	
Substitute $-\frac{27}{5}$ for x.	$5\left(-\frac{27}{5}\right) \stackrel{?}{=} -27$	
	$-27 = -27$ ✓	
		Since this is a true statement, $x = -\frac{27}{5}$ is the solution to $5x = -27$.

Exercise 2.3.2

Solve: $3y = -41$.

Answer

$y = -\frac{41}{3}$

Exercise 2.3.3

Solve: $4z = -55$.

Answer

$y = -\frac{55}{4}$

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The **Multiplication Property of Equality** will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

THE MULTIPLICATION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{array}{lll} \text{If} & a & = & b \\ \text{then} & ac & = & bc \end{array} \qquad (2.3.2)$$

If you multiply both sides of an equation by the same number, you still have equality.

Exercise 2.3.4

Solve: $\frac{y}{-7} = -14$

Answer

Here y is divided by −7. We must multiply by −7 to isolate y.

	$\frac{y}{-7} = -14$
Multiply both sides by −7.	$-7\left(\frac{y}{-7}\right) = -7(-14)$
Multiply.	$\frac{-7y}{7} = 98$
Simplify.	$y = 98$
Check: $\frac{y}{-7} = -14$	
Substitute y=98.	$\frac{98}{-7} \stackrel{?}{=} -14$
Divide.	$-14 = -14$ ✓

Exercise 2.3.5

Solve: $\frac{a}{-7} = -42$

Answer

$a = 294$

Exercise 2.3.6

Solve: $\frac{b}{-6} = -24$

Answer

$b = 144$

Exercise 2.3.7

Solve: $-n = 9$

Answer

	$-n=9$
Remember −n is equivalent to −1n.	$-1n=9$
Divide both sides by −1.	$\frac{-1n}{-1}=\frac{9}{-1}$
Divide.	$n=-9$
Notice that there are two other ways to solve −n=9. We can also solve this equation by multiplying both sides by −1 and also by taking the opposite of both sides.	
Check:	$-n=9$
Substitute n=−9.	$-(-9)\stackrel{?}{=}9$
Simplify.	$9=9$✓

? Exercise 2.3.8

Solve: $-k=8$.

Answer

$k=-8$

? Exercise 2.3.9

Solve: $-g=3$.

Answer

$g=-3$

? Exercise 2.3.10

Solve: $\frac{3}{4}x=12$

Answer

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

	$\frac{3}{4}x = 12$
Multiply by the reciprocal of $\frac{3}{4}$.	$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 12$
Reciprocals multiply to 1.	$1x = \frac{4}{3} \cdot \frac{12}{1}$
Multiply.	$x = 16$
Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.	
Check:	$\frac{3}{4}x = 12$
Substitute $x = 16$.	$\frac{3}{4} \cdot 16 \stackrel{?}{=} 12$
	$12 = 12$ ✓

Exercise 2.3.11

Solve: $\frac{2}{5}n = 14$.

Answer

$n = 35$

Exercise 2.3.12

Solve: $\frac{5}{6}y = 15$.

Answer

$y = 18$

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

Exercise 2.3.13

Solve: $\frac{8}{15} = -\frac{4}{5}x$

Answer

	$\frac{8}{15}=-\frac{4}{5}x$
Multiply by the reciprocal of $-\frac{4}{5}$.	$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right)=\left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$
Reciprocals multiply to 1.	$-\frac{\cancel{5}\cdot\cancel{4}\cdot 2}{\cancel{4}\cdot 3\cdot\cancel{5}}=1x$
Multiply.	$-\frac{2}{3}=x$
Check:	$\frac{8}{15}=-\frac{4}{5}x$
Let $x=-\frac{2}{3}$.	$\frac{8}{15}=-\frac{4}{5}\left(-\frac{2}{3}\right)$
	$\frac{8}{15}=\frac{8}{15}$ ✓

Exercise 2.3.14

Solve: $\frac{9}{25}=-\frac{4}{5}z$

Answer

$z=-\frac{9}{5}$

Exercise 2.3.15

$\frac{5}{6}=-\frac{8}{3}r$

Answer

$r=-\frac{5}{16}$

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

Exercise 2.3.16

Solve: $14-23=12y-4y-5y$.

Answer

Begin by simplify each side of the equation.

	$14-23\stackrel{?}{=}-36+12+15$
Simplify each side.	$-9=-9$ ✓
Divide both sides by 3 to isolate y.	$14-23=12y-4y-5y$
Divide.	$-9=3y$
Check:	$14-23=12y-4y-5y$
Substitute $y=-3$.	$14-23\stackrel{?}{=}12(-3)-4(-3)-5(-3)$
	$14-23\stackrel{?}{=}-36+12+15$
	$-9=-9$ ✓

Exercise 2.3.17

Solve: $18-27=15c-9c-3c$.

Answer

$c=-3$

Exercise 2.3.18

Solve: $18-22=12x-x-4x$.

Answer

$x=-\frac{4}{7}$

Exercise 2.3.19

Solve: $-4(a-3)-7=25$.

Answer

Here we will simplify each side of the equation by using the distributive property first.

	$-4(a-3)-7=25$
Distribute.	$-4a+12-7=25$
Simplify.	$-4a+5=25$
Simplify.	$-4a=20$
Divide both sides by -4 to isolate a.	$\frac{-4a}{-4}=\frac{20}{-4}$
Divide.	$a=-5$
Check:	$-4(a-3)-7=25$
Substitute $a=-5$	$-4(-5-3)-7\stackrel{?}{=}25$
	$-4(-8)-7\stackrel{?}{=}25$
	$32-7\stackrel{?}{=}25$
	$25=25$ ✓

? Exercise 2.3.20

Solve: $-4(q-2)-8=24$.

Answer

$q=-6$

? Exercise 2.3.21

Solve: $-6(r-2)-12=30$.

Answer

$r=-5$

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

PROPERTIES OF EQUALITY

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Subtraction Property of Equality	**Addition Property of Equality**
For any real numbers a, b, and c,	For any real numbers a, b, and c,
if $a=b$,	if $a=b$,
then $a-c=b-c$	then $a+c=b+c$
Divition Property of Equality	**Multiplication Property of Equality**
For any real numbers a, b, and c,	For any real numbers a, b, and c,
if $a=b$,	if $a=b$,
then $a-c=b-c$	then $a+c=b+c$

(2.3.3)

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

In the next few examples, we will translate sentences into equations and then solve the equations. You might want to review the translation table in the previous chapter.

Exercise 2.3.22

Translate and solve: The number 143 is the product of −11 and y.

Answer

Translate.	The number 143 is the product of −11 and y. $143 = -11y$
Divide by −11.	$\frac{143}{-11} = \frac{-11y}{-11}$
Simplify.	$-13 = y$
Check: $143 = -11y$ $143 \stackrel{?}{=} -11(-13)$ (2.3.4) $143 = 143\checkmark$	

Exercise 2.3.23

Translate and solve: The number 132 is the product of -12 and y.

Answer

132=−12y;y=−11

Exercise 2.3.24

Translate and solve: The number 117 is the product of -13 and z.

Answer

117=−13z;z=−9

Exercise 2.3.25

Translate and solve: n divided by 8 is −32.

Answer

Begin by translating the sentence into an equation. Translate.		n divided by 8 is –32. $\frac{n}{8} = -32$
Multiple both sides by 8.		$8 \cdot \frac{n}{8} = 8(-32)$
Simplify.		$n = -256$
Check:	Is nn divided by 8 equal to −32?	
Let $n = -256$.	Is −256 divided by 88 equal to −32?	
Translate.	$\frac{-256}{8} \stackrel{?}{=} -32$	
Simplify.	$-32 = -32$✓	

Exercise 2.3.26

Translate and solve: nn divided by 7 is equal to −21.

Answer

$\frac{n}{7} = -21; n = -147$

Exercise 2.3.27

Translate and solve: n divided by 8 is equal to −56.

Answer

$\frac{n}{8} = -56; n = -448$

Exercise 2.3.28

Translate and solve: The quotient of yy and −4 is 68.

Answer

Begin by translating the sentence into an equation.

Translate.		The quotient of y and –4 is 68. $\frac{y}{-4} = 68$
Multiply both sides by -4.		$-4\left(\frac{y}{-4}\right) = -4(68)$
Simplify.		$y = -272$
Check:	Is the quotient of y and −4 equal to 68?	
Let y=−272.	Is the quotient of −272 and −4 equal to 68?	
Translate.	$\frac{-272}{-4} \stackrel{?}{=} 68$	
Simplify.	$68 = 68$✓	

Exercise 2.3.29

Translate and solve: The quotient of q and −8 is 72.

Answer

$\frac{q}{-8} = 72; q = -576$

Exercise 2.3.30

Translate and solve: The quotient of p and −9 is 81.

Answer

$\frac{p}{-9} = 81; p = -729$

Exercise 2.3.31

Translate and solve: Three-fourths of p is 18.

Answer

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.	Three-fourths of p is 18. $\frac{3}{4}p = 18$
Multiply both sides by $\frac{4}{3}$.	$\frac{4}{3} \cdot \frac{3}{4}p = \frac{4}{3} \cdot 18$
Simplify.	$p = 24$
Check:	Is three-fourths of p equal to 18?
Let p = 24.	Is three-fourths of 24 equal to 18?
Translate.	$\frac{3}{4} \cdot 24 \stackrel{?}{=} 18$
Simplify.	$18 = 18$✓

Exercise 2.3.32

Translate and solve: Two-fifths of f is 16.

Answer

$\frac{2}{5}f = 16; f = 40$

Exercise 2.3.33

Translate and solve: Three-fourths of f is 21.

Answer

$\frac{3}{4}f = 21; f = 28$

Exercise 2.3.34

Translate and solve: The sum of three-eighths and x is one-half.

Answer

Begin by translating the sentence into an equation.

Translate.		The sum of three – eighths and x is $\frac{1}{2}$ $\frac{3}{8} + x = \frac{1}{2}$
Subtract $\frac{3}{8}$ from each side.		$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$
Simplify and rewrite fractions with common denominators.		$x = \frac{4}{8} - \frac{3}{8}$
Simplify.		$x = \frac{1}{8}$
Check:	Is the sum of three-eighths and x equal to one-half?	
Let $x = \frac{1}{8}$.	Is the sum of three-eighths and one-eighth equal to one-half?	
Translate.	$\frac{3}{8} + \frac{1}{8} \stackrel{?}{=} \frac{1}{2}$	
Simplify.	$\frac{4}{8} \stackrel{?}{=} \frac{1}{2}$	
Simplify.	$\frac{1}{2} = \frac{1}{2}$ ✓	

Exercise 2.3.35

Translate and solve: The sum of five-eighths and x is one-fourth.

Answer

$\frac{5}{8} + x = \frac{1}{4}; x = -\frac{3}{8}$

Exercise 2.3.36

Translate and solve: The sum of three-fourths and x is five-sixths.

Answer

$\frac{3}{4} + x = \frac{5}{6}; x = \frac{1}{12}$

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

Exercise 2.3.37

Denae bought 6 pounds of grapes for $10.74. What was the cost of one pound of grapes?

Answer

What are you asked to find?	The cost of 1 pound of grapes	
Assign a variable.	Let c = the cost of one pound.	
Write a sentence that gives the information to find it.	The cost of 6 pounds is $10.74	
Translate into an equation.	$6c = 10.74$	
Solve.	$\frac{6c}{c} = \frac{10.74}{6}$ $c = 1.79$	(2.3.5)
	The grapes cost $1.79 per pound.	
Check: If one pound costs $1.79, *do* 6 pounds cost $10.74?		
$6(1.79) \stackrel{?}{=} 10.74$ $10.74 = 10.74$✓		

? Exercise 2.3.38

Translate and solve:

Arianna bought a 24-pack of water bottles for $9.36. What was the cost of one water bottle?

Answer

$0.39

? Exercise 2.3.39

Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for $34.98. What is the cost for each person?

Answer

$5.83

? Exercise 2.3.40

Andreas bought a used car for $12,000. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Answer

What are you asked to find?	The original price of the car	
Assign a variable.	Let p = the original price.	
Write a sentence that gives the information to find it.	\$12000 is $\frac{3}{4}$ of the original price.	
Translate into an equation.	$12000 = \frac{3}{4}p$	(2.3.6)
	$\frac{3}{4}(12000) = \frac{4}{3} \cdot \frac{3}{4}p$	
	$16000 = p$	
Solve.		
	The original cost of the car was \$16000.	
Check: Is $\frac{3}{4}$ of \$16000 equal to \$12000		
$\frac{3}{4} \cdot 16000 \stackrel{?}{=} 12000$ $12000 = 12000$✓		

? Exercise 2.3.41

Translate and solve:

The annual property tax on the Mehta's house is \$1,800, calculated as $\frac{15}{1000}$ of the assessed value of the house. What is the assessed value of the Mehta's house?

Answer

\$120000

? Exercise 2.3.42

Translate and solve:

Stella planted 14 flats of flowers in $\frac{2}{3}$ of her garden. How many flats of flowers would she need to fill the whole garden?

Answer

21 flats

Key Concepts

- **The Division Property of Equality**—For any numbers a, b, and c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
  When you divide both sides of an equation by any non-zero number, you still have equality.
- **The Multiplication Property of Equality**—For any numbers a, b, and c, if $a = b$, then $ac = bc$.
  If you multiply both sides of an equation by the same number, you still have equality.

This page titled 2.3: Solving Equations with the Division and Multiplication Properties of Equality is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **2.2: Solve Equations using the Division and Multiplication Properties of Equality** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

2.4: Solving Equations with Variables and Constants on Both Sides

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side", and the other side of the equation to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Exercise 2.4.1

Solve: $7x + 8 = -13$.

Answer

In this equation, the variable is found only on the left side. It makes sense to call the left side the "variable" side. Therefore, the right side will be the "constant" side. We will write the labels above the equation to help us remember what goes where.

$$\overset{\text{variable}}{7x} + 8 = \overset{\text{constant}}{-13}$$

Since the left side is the "xx", or variable side, the 8 is out of place. We must "undo" adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

	$\overset{\text{variable}}{7x} + 8 = \overset{\text{constant}}{-13}$
Use the Subtraction Property of Equality.	$7x + 8 - 8 = -13 - 8$
Simplify.	$7x = -21$
Now all the variables are on the left and the constant on the right. The equation looks like those you learned to solve earlier.	
Use the Division Property of Equality.	$\frac{7x}{7} = \frac{-21}{7}$
Simplify.	$x = -3$
Check:	$7x + 8 = -13$
Let x=−3.	$7(-3) + 8 \stackrel{?}{=} -13$
	$-21 + 8 \stackrel{?}{=} -13$
	$-13 = -13$ ✓

Exercise 2.4.2

Solve: $3x + 4 = -8$.

Answer

$x = -4$

Exercise 2.4.3

Solve: $5a + 3 = -37$.

Answer

$a = -8$

Exercise 2.4.4

Solve: $8y - 9 = 31$.

Answer

Notice, the variable is only on the left side of the equation, so we will call this side the "variable" side, and the right side will be the "constant" side. Since the left side is the "variable" side, the 9 is out of place. It is subtracted from the 8y, so to "undo" subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

	variable constant $8y - 9 = 31$
Add 9 to both sides.	$8y - 9 + 9 = 31 + 9$
Simplify.	$8y = 40$
	The variables are now on one side and the constants on the other. We continue from here as we did earlier.
Divide both sides by 8.	$\frac{8y}{8} = \frac{40}{8}$
Simplify.	$y = 5$
Check:	$8y - 9 = 31$
Let y=5.	$8 \cdot 5 - 9 \stackrel{?}{=} 31$
	$40 - 9 \stackrel{?}{=} 31$
	$31 = 31$ ✓

Exercise 2.4.5

Solve: $5y - 9 = 16$.

Answer

$y = 5$

Exercise 2.4.6

Solve: $3m - 8 = 19$.

Answer

$m = 9$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Exercise 2.4.7

Solve: $9x = 8x - 6$.

Answer

Here the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

	variable constant $9x = 8x - 6$
We don't want any x's on the right, so subtract the 8x from both sides.	$9x - 8x = 8x - 8x - 6$
Simplify.	$x = -6$
We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.	
Check:	$9x = 8x - 6$
Let x=−6.	$9(-6) \stackrel{?}{=} 8(-6) - 6$
	$-54 \stackrel{?}{=} -48 - 6$
	$-54 = -54$ ✓

Exercise 2.4.8

Solve: $6n = 5n - 10$.

Answer

$n = -10$

Exercise 2.4.9

Solve: $-6c = -7c - 1$

Answer

$c = -1$

Exercise 2.4.10

Solve: $5y - 9 = 8y$

Answer

The only constant is on the left and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

	constant variable $5y-9=8y$
Subtract 5y from both sides.	$5y-5y-9=8y-5y$
Simplify.	$-9=3y$
We have the y's on the right and the constants on the left. Divide both sides by 3.	$\frac{-9}{3}=\frac{3y}{3}$
Simplify.	$-3=y$

Check:	$5y-9=8y$
Let $y=-3$.	$5(-3)-9\stackrel{?}{=}8(-3)$
	$-15-9\stackrel{?}{=}-24$
	$-24=-24$ ✓

Exercise 2.4.11

Solve: $3p-14=5p$.

Answer

$p=-7$

Exercise 2.4.12

Solve: $8m+9=5m$

Answer

$m=-3$

Exercise 2.4.13

Solve: $12x=-x+26$

Answer

The only constant is on the right, so let the left side be the "variable" side.

	variable constant $12x=-x+26$
Remove the −x from the right side by adding x to both sides.	$12x+x=-x+x+26$
Simplify.	$13x=26$
All the x's are on the left and the constants are on the right. Divide both sides by 13.	$\frac{13x}{13}=\frac{26}{13}$
Simplify.	$x=2$

? Exercise 2.4.14

Solve: $12j = -4j + 32$

Answer

$j = 2$

? Exercise 2.4.15

Solve: $8h = -4h + 12$

Answer

$h = 1$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

? Exercise 2.4.16: How to Solve Equations with Variables and Constants on Both Sides

Solve: $7x + 5 = 6x + 2$

Answer

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.	The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the "x" side. The right side will be the "constant" side.	variable constant $7x + 5 = 6x + 2$
Step 2. Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.	With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$
Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the "constant" side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$
Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of x is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16$ ✓

Exercise 2.4.17

Solve: $12x + 8 = 6x + 2$.

Answer

$x = -1$

Exercise 2.4.18

Solve: $9y + 4 = 7y + 12$.

Answer

$y = 4$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

BEGINNING STRATEGY FOR SOLVING EQUATIONS WITH VARIABLES AND CONSTANTS ON BOTH SIDES OF THE EQUATION.

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

Exercise 2.4.19

Solve: $8n - 4 = -2n + 6$.

Answer

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8 > -2$, make the left side the "variable" side.	variable constant $8n - 4 = -2n + 6$
We don't want variable terms on the right side—add 2n to both sides to leave only constants on the right.	$8n + 2n - 4 = -2n + 2n + 6$
Combine like terms.	$10n - 4 = 6$
We don't want any constants on the left side, so add 4 to both sides.	$10n - 4 + 4 = 6 + 4$
Simplify.	$10n = 10$
The variable term is on the left and the constant term is on the right. To get the coefficient of nn to be one, divide both sides by 10.	$\frac{10n}{10} = \frac{10}{10}$
Simplify.	$n = 1$
Check:	$8n - 4 = -2n + 6$
Let n=1.	$8 \cdot 1 - 4 \stackrel{?}{=} -2 \cdot 1 + 6$
	$8 - 4 \stackrel{?}{=} -2 + 6$
	$4 = 4 \checkmark$

Exercise 2.4.20

Solve: $8q - 5 = -4q + 7$

Answer

$q = 1$

Exercise 2.4.21

Solve: $7n - 3 = n + 3$

Answer

$n = 1$

Exercise 2.4.22

Solve: $7a - 3 = 13a + 7$

Answer

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since 13>7, make the right side the “variable” side and the left side the “constant” side.

	constant variable $7a-3=13a+7$
Subtract 7a from both sides to remove the variable term from the left.	$7a-7a-3=13a-7a+7$
Combine like terms.	$-3=6a+7$
Subtract 7 from both sides to remove the constant from the right.	$-3-7=6a+7-7$
Simplify.	$-10=6a$
Divide both sides by 6 to make 1 the coefficient of a.	$\frac{-10}{6}=\frac{6a}{6}$
Simplify.	$-\frac{5}{3}=a$
Check:	$7a-3=13a+7$
Let $a=-\frac{5}{3}$.	$7\left(-\frac{5}{3}\right)-3\stackrel{?}{=}13\left(-\frac{5}{3}\right)+7$
	$-\frac{35}{3}-\frac{9}{3}\stackrel{?}{=}-\frac{65}{3}+\frac{21}{3}$
	$-\frac{54}{3}=-\frac{54}{3}$ ✓

? Exercise 2.4.23

Solve: $2a-2=6a+18$

Answer

$a=-5$

? Exercise 2.4.24

Solve: $4k-1=7k+17$

Answer

$k=-6$

In the last example, we could have made the left side the “variable” side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

? Exercise 2.4.25

Solve: $\frac{4}{5}x+6=\frac{1}{4}x-2$

Answer

Since $\frac{5}{4} > \frac{1}{4}$, make the left side the "variable" side and the right side the "constant" side.

	variable constant $\frac{5}{4}x+6=\frac{1}{4}x-2$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x-\frac{1}{4}x+6=\frac{1}{4}x-\frac{1}{4}x-2$
Combine like terms.	$x+6=-2$
Subtract 6 from both sides.	$x+6-6=-2-6$
Simplify.	$x=-8$
Check: Let $x=-8$	$\frac{5}{4}x+6 = \frac{1}{4}x-2$ $\frac{5}{4}(-8)+6 \stackrel{?}{=} \frac{1}{4}(-8)-2$ $-10+6 \stackrel{?}{=} -2-2$ $-4 = -4\checkmark$

? Exercise 2.4.26

Solve: $\frac{7}{8}x-12=-\frac{1}{8}x-2$

Answer

$x=10$

? Exercise 2.4.27

Solve: $\frac{7}{6}x+11=\frac{1}{6}y+8$

Answer

$y=-3$

We will use the same strategy to find the solution for an equation with decimals.

? Exercise 2.4.28

Solve: $7.8x+4=5.4x-8$.

Answer

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

	variable side constant side $7.8x + 4 = 5.4x - 8$
Subtract 5.4x from both sides.	$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$
Combine like terms.	$2.4x + 4 = -8$
Subtract 4 from both sides.	$2.4x + 4 - 4 = -8 - 4$
Simplify.	$2.4x = -12$
Use the Division Property of Equality.	$\frac{2.4x}{2.4} = \frac{-12}{2.4}$
Simplify.	$x = -5$
Check:	$7.8x + 4 = 5.4x - 5$
Let $x = -5$	$7.8(-5) + 4 = 5.4(-5) - 8$
	$-39 + 4 \stackrel{?}{=} -27 - 8$
	$-35 = -35$ ✓

? Exercise 2.4.29

Solve: $2.8x + 12 = -1.4x - 9$

Answer

$x = -5$

? Exercise 2.4.30

Solve: $3.6y + 8 = 1.2y - 4$

Answer

$y = -5$

Key Concepts

- **Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation**
  1. Choose which side will be the "variable" side—the other side will be the "constant" side.
  2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
  3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
  5. Check the solution by substituting it into the original equation.

This page titled 2.4: Solving Equations with Variables and Constants on Both Sides is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **2.3: Solve Equations with Variables and Constants on Both Sides** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

2.5: Solving Equations with Fractions or Decimals

Learning Objectives

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$.

Table 2.5.1

	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x+\frac{1}{2}-\frac{1}{2}=\frac{1}{4}-\frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x=\frac{1}{4}-\frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x=\frac{1}{4}-\frac{2}{4}$
Subtract.	$\frac{1}{8}x=-\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1}\cdot\frac{1}{8}x=\frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x=-2$

This method worked fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called "clearing" the equation of fractions.

Let's solve a similar equation, but this time use the method that eliminates the fractions.

Exercise 2.5.1: How to Solve Equations with Fraction Coefficients

Solve: $\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$

Answer

Step 1. Find the least common denominator of *all* the fractions in the equation.	What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?	$\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$ LCD $=6$
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.	Multiply both sides of the equation by the LCD 6.	$6\left(\frac{1}{6}y-\frac{1}{3}\right)=6\left(\frac{5}{6}\right)$
	Use the Distributive Property.	$6\cdot\frac{1}{6}y-6\cdot\frac{1}{3}=6\cdot\frac{5}{6}$
	Simplify – and notice, no more fractions!	$y-2=5$

Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the "y" term, add 2. Simplify.	$y-2+2=5+2$ $y=7$

? Exercise 2.5.2

Solve: $\frac{1}{4}x+\frac{1}{2}=\frac{5}{8}$

Answer

$x=\frac{1}{2}$

? Exercise 2.5.3

Solve: $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$

Answer

$x=-2$

Notice in Exercise 2.5.1, once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

STRATEGY TO SOLVE EQUATIONS WITH FRACTION COEFFICIENTS.

1. Find the least common denominator of *all* the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

? Exercise 2.5.4

Solve: $6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$

Answer

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.		$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$
The LCD is 20.		
Multiply both sides of the equation by 20.		$20(6)=20\cdot\left(\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v\right)$
Distribute.		$20(6)=20\cdot\frac{1}{2}v+20\cdot\frac{2}{5}v-20\cdot\frac{3}{4}v$
Simplify—notice, no more fractions!		$120=10v+8v-15v$
Combine like terms.		$120=3v$
Divide by 3.		$\frac{120}{3}=\frac{3v}{3}$
Simplify.		$40=v$
Check:	$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$	
Let v=40.	$6\stackrel{?}{=}\frac{1}{2}(40)+\frac{2}{5}(40)-\frac{3}{4}(40)$	
	$6\stackrel{?}{=}20+16-30$	
	$6=6\checkmark$	

Exercise 2.5.5

Solve: $7=\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$

Answer

$x=12$

Exercise 2.5.6

Solve: $-1=\frac{1}{2}u+\frac{1}{4}u-\frac{2}{3}u$

Answer

$u=-12$

In the next example, we again have variables on both sides of the equation.

Exercise 2.5.7

Solve: $a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$

Answer

	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Find the LCD of all fractions in the equation. The LCD is 8.	
Multiply both sides by the LCD.	$8\left(a+\frac{3}{4}\right)=8\left(\frac{3}{8}a-\frac{1}{2}\right)$
Distribute.	$8\cdot a+8\cdot\frac{3}{4}=8\cdot\frac{3}{8}a-8\cdot\frac{1}{2}$
Simplify—no more fractions.	$8a+6=3a-4$
Subtract 3a3a from both sides.	$8a-3a+6=3a-3a-4$
Simplify.	$5a+6=-4$
Subtract 6 from both sides.	$5a+6-6=-4-6$
Simplify.	$5a=-10$
Divide by 5.	$\frac{5a}{5}=\frac{-10}{5}$
Simplify.	$a=-2$
Check:	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Let a=−2.	$-2+\frac{3}{4}\stackrel{?}{=}\frac{3}{8}(-2)-\frac{1}{2}$
	$-\frac{8}{4}+\frac{3}{4}\stackrel{?}{=}-\frac{16}{8}-\frac{4}{8}$
	$-\frac{5}{4}=-\frac{10}{8}$
	$-\frac{5}{4}=-\frac{5}{4}\checkmark$

? Exercise 2.5.8

Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$

Answer

$x=-1$

? Exercise 2.5.9

Solve: $c+\frac{3}{4}=\frac{1}{2}c-\frac{1}{4}$

Answer

$c=-2$

In the next example, we start by using the Distributive Property. This step clears the fractions right away.

? Exercise 2.5.10

Solve: $-5=\frac{1}{4}(8x+4)$

Answer

	$-5=\frac{1}{4}(8x+4)$
Distribute.	$-5=\frac{1}{4}\cdot 8x+\frac{1}{4}\cdot 4$
Simplify. Now there are no fractions.	$-5=2x+1$
Subtract 1 from both sides.	$-5-1=2x+1-1$
Simplify.	$-6=2x$
Divide by 2.	$\frac{-6}{2}=\frac{2x}{2}$
Simplify.	$-3=x$

Check:	$-5=\frac{1}{4}(8x+4)$
Let x=−3.	$-5\stackrel{?}{=}\frac{1}{2}(4(-3)+2)$
	$-5\stackrel{?}{=}\frac{1}{2}(-12+2)$
	$-5\stackrel{?}{=}\frac{1}{2}(-10)$
	$-5=-5$ ✓

Exercise 2.5.11

Solve: $-11=\frac{1}{2}(6p+2)$

Answer

$p=-4$

Exercise 2.5.12

Solve: $8=\frac{1}{3}(9q+6)$

Answer

$q=2$

In the next example, even after distributing, we still have fractions to clear.

Exercise 2.5.13

Solve: $\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$

Answer

	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2}\cdot y-\frac{1}{2}\cdot 5=\frac{1}{4}\cdot y-\frac{1}{4}\cdot 1$
Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, 4.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
Check:	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Let y=9.	$\frac{1}{2}(9-5)\stackrel{?}{=}\frac{1}{4}(9-1)$
Finish the check on your own.	

? Exercise 2.5.14

Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$

Answer

$n=2$

? Exercise 2.5.15

Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$

Answer

$m=-1$

? Exercise 2.5.16

Solve: $\frac{5x-3}{4}=\frac{x}{2}$

Answer

	$\frac{5x-3}{4}=\frac{x}{2}$
Multiply by the LCD, 4.	$4\left(\frac{5x-3}{4}\right)=4\left(\frac{x}{2}\right)$
Simplify.	$5x-3=2x$
Collect the variables to the right.	$5x-5x-3=2x-5x$
Simplify.	$-3=-3x$
Divide.	$\frac{-3}{-3}=\frac{-3x}{-3}$
Simplify.	$1=x$
Check:	$\frac{5x-3}{4}=\frac{x}{2}$
Let x=1.	$\frac{5(1)-3}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{2}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{1}{2}=\frac{1}{2}$✓

Exercise 2.5.17

Solve: $\frac{4y-7}{3}=\frac{y}{6}$

Answer

$y=2$

Exercise 2.5.18

Solve: $\frac{-2z-5}{4}=\frac{z}{8}$

Answer

$z=-2$

Exercise 2.5.19

Solve: $\frac{a}{6}+2=\frac{a}{4}+3$

Answer

	$\frac{a}{6}+2=\frac{a}{4}+3$
Multiply by the LCD, 12.	$12\left(\frac{a}{6}+2\right)=12\left(\frac{a}{4}+3\right)$
Distribute.	$12\cdot\frac{a}{6}+12\cdot 2=12\cdot\frac{a}{4}+12\cdot 3$
Simplify.	$2a+24=3a+36$
Collect the variables to the right.	$2a-2a+24=3a-2a+36$
Simplify.	$24=a+36$
Collect the constants to the left.	$24-36=a+36-36$
Simplify.	$a=-12$
Check:	$\frac{a}{6}+2=\frac{a}{4}+3$
Let a=−12.	$\frac{-12}{6}+2\stackrel{?}{=}\frac{-12}{4}+3$
	$-2+2\stackrel{?}{=}-3+3$
	$0=0\checkmark$

Exercise 2.5.20

Solve: $\frac{b}{10}+2=\frac{b}{4}+5$

Answer

$b=-20$

Exercise 2.5.21

Solve: $\frac{c}{6}+3=\frac{c}{3}+4$

Answer

$c=-6$

Exercise 2.5.22

Solve: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$

Answer

	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Multiply by the LCD, 4.	$4\left(\frac{4q+3}{2}+6\right)=4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\left(\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$ $8q+6+24=3q+5$ $8q+30=3q+5$
Collect the variables to the left.	$8q-3q+30=3q-3q+5$
Simplify.	$5q+30=5$
Collect the constants to the right.	$5q+30-30=5-30$
Simplify.	$5q=-25$
Divide by 5.	$\frac{5q}{5}=\frac{-25}{5}$
Simplify.	$q=-5$
Check:	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Let q=−5.	$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$
Finish the check on your own.	

? Exercise 2.5.23

Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$

Answer

$r=1$

? Exercise 2.5.24

Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$

Answer

$s=-8$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3=\frac{3}{10}$ and $0.17=\frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

? Exercise 2.5.25

Solve: $0.06x+0.02=0.25x-1.5$

Answer

Look at the decimals and think of the equivalent fractions.

$0.06 = \frac{6}{100}$ $0.02 = \frac{2}{100}$ $0.25 = \frac{25}{100}$ $1.5 = 1\frac{5}{10}$

Notice, the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both side by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now we have no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the variables to the right.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let x=8 $0.06(8) + 0.02 \stackrel{?}{=} 0.25(8) - 1.5$ $0.48 + 0.02 \stackrel{?}{=} 2.00 - 1.5$ $0.50 = 0.50$ ✓	

Exercise 2.5.26

Solve: $0.14h + 0.12 = 0.35h - 2.4$

Answer

$h = 12$

Exercise 2.5.27

Solve: $0.65k - 0.1 = 0.4k - 0.35$

Answer

$k = -1$

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

Exercise 2.5.28

Solve: $0.25x + 0.05(x + 3) = 2.85$

Answer

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting x=9 into the original equation.	

? Exercise 2.5.29

Solve: $0.25n + 0.05(n + 5) = 2.95$

Answer

$n = 9$

? Exercise 2.5.30

Solve: $0.10d + 0.05(d - 5) = 2.15$

Answer

$d = 16$

Key Concepts

- **Strategy to Solve an Equation with Fraction Coefficients**
  1. Find the least common denominator of all the fractions in the equation.
  2. Multiply both sides of the equation by that LCD. This clears the fractions.
  3. Solve using the General Strategy for Solving Linear Equations.

This page titled 2.5: Solving Equations with Fractions or Decimals is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **2.5: Solve Equations with Fractions or Decimals** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

2.6: General Strategy to Solve Linear Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Exercise 2.6.1: How to Solve Linear Equations Using the General Strategy

Solve: $-6(x+3)=24$.

Answer

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x+3)=24$ $-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x-18+18=24+18$ $-6x=42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by –6. Simplify.	$\frac{-6x}{-6}=\frac{42}{-6}$ $x=-7$
Step 5. Check the solution.	Let $x=-7$ Simplify. Multiply.	**Check:** $-6(x+3)=24$ $-6(-7+3)\stackrel{?}{=}24$ $-6(-4)\stackrel{?}{=}24$ $24=24$ ✓

Exercise 2.6.2

Solve: $5(x+3)=35$

Answer

$x=4$

Exercise 2.6.3

Solve: $6(y-4)=-18$

Answer

$y=1$

GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS.

1. **Simplify each side of the equation as much as possible.**
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. **Collect all the variable terms on one side of the equation.**
   Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
   Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term to equal to 1.**
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

Exercise 2.6.4

Solve: $-(y+9)=8$

Answer

	$-(y+9)=8$
Simplify each side of the equation as much as possible by distributing.	$-y-9=8$
The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y-9+9=8+9$
Simplify.	$-y=17$
Rewrite −y as −1y.	$-1y=17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by −1.	$\frac{-1y}{-1}=\frac{17}{-1}$
Simplify.	$y=-17$
Check:	$-(y+9)=8$
Let y=−17.	$-(-17+9)\stackrel{?}{=}8$
	$-(-8)\stackrel{?}{=}8$
	$8=8\checkmark$

Exercise 2.6.5

Solve: $-(y+8)=-2$

Answer

$y = -6$

Exercise 2.6.6

Solve: $-(z+4) = -12$

Answer

$z = 8$

Exercise 2.6.7

Solve: $5(a-3)+5 = -10$

Answer

	$5(a-3)+5=-10$
Simplify each side of the equation as much as possible.	
Distribute.	$5a-15+5=-10$
Combine like terms.	$5a-10=-10$
The only a term is on the left side, so all variable terms are on one side of the equation.	
Add 10 to both sides to get all constant terms on the other side of the equation.	$5a-10+10=-10+10$
Simplify.	$5a=0$
Make the coefficient of the variable term to equal to 11 by dividing both sides by 55.	$\frac{5a}{5}=\frac{0}{5}$
Simplify.	$a=0$
Check:	$5(a-3)+5=-10$
Let a=0.	$5(0-3)+5\stackrel{?}{=}-10$
	$5(-3)+5\stackrel{?}{=}-10$
	$-15+5\stackrel{?}{=}-10$
	$-10=-10$ ✓

Exercise 2.6.8

Solve: $2(m-4)+3 = -1$

Answer

$m = 2$

Exercise 2.6.9

Solve:$7(n-3)-8 = -15$

Answer

$n = 2$

Exercise 2.6.10

Solve: $\frac{2}{3}(6m - 3) = 8 - m$

Answer

	$\frac{2}{3}(6m-3) = 8-m$
Distribute.	$4m - 2 = 8 - m$
Add m to get the variables only to the left.	$4m + m - 2 = 8 - m + m$
Simplify.	$5m - 2 = 8$
Add 2 to get constants only on the right.	$5m - 2 + 2 = 8 + 2$
Simplify.	$5m = 10$
Divide by 5.	$\frac{5m}{5} = \frac{10}{5}$
Simplify.	$m = 2$
Check:	$\frac{2}{3}(6m-3) = 8-m$
Let m=2.	$\frac{2}{3}(6 \cdot 2 - 3) \stackrel{?}{=} 8 - 2$
	$\frac{2}{3}(12-3) \stackrel{?}{=} 6$
	$\frac{2}{3}(9) \stackrel{?}{=} 6$
	$6 = 6$ ✓

Exercise 2.6.11

Solve: $\frac{1}{3}(6u + 3) = 7 - u$

Answer

$u = 2$

Exercise 2.6.12

Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$

Answer

$x = 4$

Exercise 2.6.13

Solve: $8 - 2(3y + 5) = 0$

Answer

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by $-6-6$.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$
Check: Let $y=-13$. $8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0$ ✓	

Exercise 2.6.14

Solve: $12-3(4j+3)=-17$

Answer

$j=\frac{5}{3}$

Exercise 2.6.15

Solve: $-6-8(k-2)=-10$

Answer

$k=\frac{5}{2}$

Exercise 2.6.16

Solve: $4(x-1)-2=5(2x+3)+6$

Answer

	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract 4x to get the variables only on the right side since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 to get the constants on left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24$ ✓

Exercise 2.6.17

Solve: $6(p-3)-7=5(4p+3)-12$

Answer

$p=-2$

Exercise 2.6.18

Solve: $8(q+1)-5=3(2q-4)-1$

Answer

$q=-8$

Exercise 2.6.19

Solve: $10[3-8(2s-5)]=15(40-5s)$

Answer

		$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add 160s to get the s's to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute s=−2.	$10[3-8(2(-2)-5)]\stackrel{?}{=}15(40-5(-2))$	
	$10[3-8(-4-5)]\stackrel{?}{=}15(40+10)$	
	$10[3-8(-9)]\stackrel{?}{=}15(50)$	
	$10[3+72]\stackrel{?}{=}750$	
	$10[75]\stackrel{?}{=}750$	
	$750=750$ ✓	

? Exercise 2.6.20

Solve: $6[4-2(7y-1)]=8(13-8y)$.

Answer

$y=-\frac{17}{5}$

? Exercise 2.6.21

Solve: $12[1-5(4z-1)]=3(24+11z)$.

Answer

$z=0$

? Exercise 2.6.22

Solve: $0.36(100n+5)=0.6(30n+15)$.

Answer

	$0.36(100n + 5) = 0.6(30n + 15)$
Distribute.	$36n + 1.8 = 18n + 9$
Subtract 18n to get the variables to the left.	$36n - 18n + 1.8 = 18n - 18n + 9$
Simplify.	$18n + 1.8 = 9$
Subtract 1.8 to get the constants to the right.	$18n + 1.8 - 1.8 = 9 - 1.8$
Simplify.	$18n = 7.2$
Divide.	$\frac{18n}{18} = \frac{7.2}{18}$
Simplify.	$n = 0.4$
Check:	$0.36(100n + 5) = 0.6(30n + 15)$
Let n=0.4.	$0.36(100(0.4) + 5) \stackrel{?}{=} 0.6(30(0.4) + 15)$
	$0.36(40 + 5) \stackrel{?}{=} 0.6(12 + 15)$
	$0.36(45) \stackrel{?}{=} 0.6(27)$
	$16.2 = 16.2$ ✓

? Exercise 2.6.23

Solve: $0.55(100n + 8) = 0.6(85n + 14)$.

Answer

$n = 1$

? Exercise 2.6.24

Solve: $0.15(40m - 120) = 0.5(60m + 12)$.

Answer

$m = -1$

Classify Equations

Consider the equation we solved at the start of the last section, 7x+8=−13. The solution we found was x=−3. This means the equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3. We showed this when we checked the solution x=−3 and evaluated 7x+8=−13 for x=−3.

$$7(-3) + 8 \stackrel{?}{=} -13$$
$$-21 + 8 \stackrel{?}{=} -13$$
$$-13 = -13 \checkmark$$

If we evaluate 7x+8 for a different value of *x*, the left side will not be −13.

The equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3, but not true when we replace *x* with any other value. Whether or not the equation 7x+8=−13 is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation 2y+6=2(y+3). Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for *y*.

	$2y + 6 = 2(y + 3)$
Distribute.	$2y + 6 = 2y + 6$
Subtract 2y to get the y's to one side.	$2y - 2y + 6 = 2y - 2y + 6$
Simplify—the y's are gone!	$6 = 6$

But 6=6 is true.

This means that the equation 2y+6=2(y+3) is true for any value of *y*. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is all real numbers.

What happens when we solve the equation 5z=5z−1?

	$5z = 5z - 1$
Subtract 5z to get the constant alone on the right.	$5z - 5z = 5z - 5z - 1$
Simplify—the z's are gone!	$0 \neq -1$

But $0 \neq -1$.

Solving the equation 5z=5z−1 led to the false statement 0=−1. The equation 5z=5z−1 will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a **contradiction**.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

Exercise 2.6.25

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$

Answer

$6(2n-1)+3=2n-8+5(2n+1)$	
Distribute.	$12n-6+3=2n-8+10n+5$
Combine like terms.	$12n-3=12n-3$
Subtract 12n to get the nn's to one side.	$12n-12n-3=12n-12n-3$
Simplify.	$-3=-3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

Exercise 2.6.26

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4+9(3x-7)=-42x-13+23(3x-2)$

Answer

identity; all real numbers

Exercise 2.6.27

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$

Answer

identity; all real numbers

Exercise 2.6.28

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Answer

	$10 + 4(p - 5) = 0$
Distribute.	$10 + 4p - 20 = 0$
Combine like terms.	$4p - 10 = 0$
Add 10 to both sides.	$4p - 10 + 10 = 0 + 10$
Simplify.	$4p = 10$
Divide.	$\frac{4p}{4} = \frac{10}{4}$
Simplify.	$p = \frac{5}{2}$
The equation is true when $p = frac52$.	This is a conditional equation. The solution is $p = frac52$.

Exercise 2.6.29

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q + 3) - 5 = 19$

Answer

conditional equation; \(q = \frac{9}{11}\

Exercise 2.6.30

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6 + 14(k - 8) = 95$

Answer

conditional equation; $k = \frac{193}{14}$

Exercise 2.6.31

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m + 3(9 + 3m) = 2(7m - 11)$

Answer

	$5m + 3(9 + 3m) = 2(7m - 11)$
Distribute.	$5m + 27 + 9m = 14m - 22$
Combine like terms.	$14m + 27 = 14m - 22$
Subtract 14m from both sides.	$14m + 27 - 14m = 14m - 22 - 14m$
Simplify.	$27 \neq -22$
But $27 \neq -22$.	The equation is a contradiction. It has no solution.

Exercise 2.6.32

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$12c+5(5+3c)=3(9c-4)$

Answer

contradiction; no solution

Exercise 2.6.33

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4(7d+18)=13(3d-2)-11d$

Answer

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**
  1. Simplify each side of the equation as much as possible.
     Use the Distributive Property to remove any parentheses.
     Combine like terms.
  2. Collect all the variable terms on one side of the equation.
     Use the Addition or Subtraction Property of Equality.
  3. Collect all the constant terms on the other side of the equation.
     Use the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable term to equal to 1.
     Use the Multiplication or Division Property of Equality.
     State the solution to the equation.
  5. Check the solution.
     Substitute the solution into the original equation.

This page titled 2.6: General Strategy to Solve Linear Equations is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **2.4: Use a General Strategy to Solve Linear Equations** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

2.7: Solving a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

DISTANCE, RATE, AND TIME

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \quad \text{where} \quad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned} \tag{2.7.1}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

SOLVE AN APPLICATION (WITH A FORMULA).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Exercise 2.7.1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Exercise 2.7.2

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Answer

330 miles

Exercise 2.7.3

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Answer

7 miles

Exercise 2.7.4

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$d = rt$ $520 \stackrel{?}{=} 65 \cdot 8$ $520 = 520$✓	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

? Exercise 2.7.5

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Answer

11 hours

? Exercise 2.7.6

Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Answer

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Exercise 2.7.1 and Exercise 2.7.4, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Exercise 2.7.4, we had to find the value of t. We substituted in values of d and r and then used algebra to solve for tt. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Exercise 2.7.7

Solve the formula d=rt for t:

1. when d=520 and r=65
2. in general

Answer

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

1. when d=520 and r=65		2. in general	
Write the formula.	$d = rt$	Write the formula.	$d = rt$
Substitute.	$520 = 65t$		
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$	Divide, to isolate tt.	$\frac{d}{r} = \frac{rt}{t}$
Simplify.	$8 = t$	Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Exercise 2.7.8

Solve the formula $d = rt$ for r:

1. when d=180 and t=4
2. in general

Answer

1. $r = 45$
2. $r = \frac{d}{t}$

Exercise 2.7.9

Solve the formula $d = rt$ for r:

1. when d=780 and t=12
2. in general

Answer

1. $r = 65$
2. $r = \frac{d}{t}$

Exercise 2.7.10

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 90$ and $b = 15$
2. in general

Answer

1. when $A = 90$ and $b = 15$		2. in general	
Write the formula.	$A = \frac{1}{2}bh$	Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2}15h$	Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$	Simplify.	$2A = bh$
Solve for h.	$12 = h$	Solve for hh.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$

? Exercise 2.7.11

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 170$ and $b = 17$
2. in general

Answer

1. $h = 20$
2. $h = \frac{2A}{b}$

? Exercise 2.7.12

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 62$ and $h = 31$
2. in general

Answer

1. $b = 4$
2. $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, I, for a principal, P, invested at rate, r, for t years.

? Exercise 2.7.13

Solve the formula I=Prt to find the principal, P:

1. when I=$5,600, r=4% ,t=7years
2. in general

Answer

1. I=$5,600, r=4% ,t=7years		2. in general	
Write the formula.	$I = Prt$	Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$		
Simplify.	$5600 = P(0.28)$	Simplify.	$I = P(rt)$
Divide, to isolate P.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	Divide, to isolate P.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$	Simplify.	$\frac{I}{rt} = P$
The principal is	$20,000		$P = \frac{I}{rt}$

? Exercise 2.7.14

Solve the formula I=Prt to find the principal, P:

1. when I=$2160, r=6% ,t=3 years
2. in general

Answer

1. $12000
2. $P = \frac{I}{rt}$

? Exercise 2.7.15

Solve the formula I=Prt to find the principal, P:

1. when I=$5400, r= 12% ,t=5 years
2. in general

Answer

1. $9000
2. $P = \frac{I}{rt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually x and y. You might be given an equation that is solved for y and need to solve it for x, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

? Exercise 2.7.16

Solve the formula 3x+2y=18 for y:

1. when x=4
2. in general

Answer

1. when x=4		2. in general	
	$3x+2y=18$		$3x+2y=18$
Substitute.	$3(4)+2y=18$		
Subtract to isolate the y-term.	$12-12+2y=18-12$	Subtract to isolate the y-term.	$3x-3x+2y=18-3x$
Divide.	$\frac{2y}{2}=\frac{6}{2}$	Divide.	$\frac{2y}{2}=\frac{18}{2}-\frac{3x}{2}$
Simplify.	$y=3$	Simplify.	$y=-\frac{3x}{2}+9$

Exercise 2.7.17

Solve the formula 3x+4y=10 for *y*:

1. when $x=\frac{14}{3}$
2. in general

Answer

1. $y=-1$
2. $y=\frac{10-3x}{4}$

Exercise 2.7.18

Solve the formula 5x+2y=18 for *y*:

1. when $x=4$
2. in general

Answer

1. $y=-1$
2. $y=\frac{18-5x}{2}$

In Exercise 2.7.7 through Exercise 2.7.18 we used the numbers in part 1 as a guide to solving in general in part 2. Now we will solve a formula in general without using numbers as a guide.

Exercise 2.7.19

Solve the formula P=a+b+c for a.

Answer

We will isolate aa on one side of the equation.	$P=a+b+c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P-b-c=a+b+c-b-c$
Simplify.	$P-b-c=a$ $a=P-b-c$

Exercise 2.7.20

Solve the formula P=a+b+c for *b*.

Answer

b=P−a−c

Exercise 2.7.21

Solve the formula P=a+b+c for *c*.

Answer

c=P−a−b

Exercise 2.7.22

Solve the formula 6x+5y=13 for *y*.

Answer

	$6x + 5y = 13$
Subtract 6x from both sides to isolate the term with y.	$6x - 6x + 5y = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13 - 6x}{5}$
Simplify.	$y = \frac{13 - 6x}{5}$

The fraction is simplified. We cannot divide 13−6x by 5.

Exercise 2.7.23

Solve the formula 4x+7y=9 for *y*.

Answer

$y = \frac{9-4x}{7}$

Exercise 2.7.24

Solve the formula 5x+8y=1 for *y*.

Answer

$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.

5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: d=rt where d = distance, r = rate, t = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

This page titled 2.7: Solving a Formula for a Specific Variable is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **2.6: Solve a Formula for a Specific Variable** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

CHAPTER OVERVIEW

3: Proportions and Variation

3.1: Ratios, Rates, Proportions

3.2: Applications of Proportionality

3.3: Direct and Inverse Variation

3: Proportions and Variation is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

3.1: Ratios, Rates, Proportions

Ratios & Rates

A ratio is the quotient of two numbers or the quotient of two quantities with the same units.

When writing a ratio as a fraction, the first quantity is the numerator and the second quantity is the denominator.

Exercise 3.1.1

1. Find the ratio of 45 minutes to 2 hours. Simplify the fraction, if possible.



Answer

1. $\frac{3}{8}$

A rate is the quotient of two quantities with different units. You must include the units.

When writing a rate as a fraction, the first quantity is the numerator and the second quantity is the denominator. Simplify the fraction, if possible. Include the units in the fraction.

Exercise 3.1.1

2. A car travels 105 miles in 2 hours. Write the rate as a fraction.

Answer

2. $\frac{105 \text{ mi}}{2 \text{ hr}}$

A unit rate has a denominator of 1. If necessary, divide the numerator by the denominator and express the rate as a mixed number or decimal.

Exercise 3.1.1

3. A car travels 105 miles in 2 hours. Write as a unit rate.

Answer

3. $\frac{52.5 \text{ mi}}{1 \text{ hr}}$ or 52.5 miles per hour

A unit price is a rate with the price in the numerator and a denominator equal to 1. The unit price tells the cost of one unit or one item. You can also simply divide the cost by the size or number of items.

Exercises 3.1.1

4. An 18-ounce box of cereal costs $ 3.59. Find the unit price.

5. A 12-ounce box of cereal costs $ 2.99. Find the unit price.

6. Which box has a lower unit price?



Answer

4. $ 0.199/oz, or around 20 cents per ounce
5. $ 0.249/oz, or around 25 cents per ounce
6. the 18-ounce box has the lower unit price

Proportions

A proportion says that two ratios (or rates) are equal.

Exercises 3.1.1

Determine whether each proportion is true or false by simplifying each fraction.

7. $\frac{6}{8} = \frac{21}{28}$

8. $\frac{10}{15} = \frac{16}{20}$

Answer

7. $\frac{3}{4} = \frac{3}{4}$; true
8. $\frac{2}{3} \neq \frac{4}{5}$; false

A common method of determining whether a proportion is true or false is called cross-multiplying or finding the ***cross products***. We multiply diagonally across the equal sign. In a true proportion, the cross products are equal.

$$\frac{a}{b} = \frac{c}{d} \rightarrow a \cdot d = b \cdot c$$

Exercises 3.1.1

Determine whether each proportion is true or false by cross-multiplying.

9. $\frac{6}{8} = \frac{21}{28}$

10. $\frac{10}{15} = \frac{16}{20}$
11. $\frac{14}{4} = \frac{15}{5}$
12. $\frac{0.8}{4} = \frac{5}{25}$

Answer

9. $168 = 168$; true
10. $200 \neq 240$; false
11. $70 \neq 60$; false
12. $20 = 20$; true

As we saw in a previous module, we can use a variable to stand for a missing number. If a proportion has a missing number, we can use cross multiplication to solve for the missing number. This is as close to algebra as we get in this textbook.

To solve a proportion for a variable:

1. Set the cross products equal to form an equation of the form $a \cdot d = b \cdot c$.
2. Isolate the variable by rewriting the multiplication equation as a division equation.
3. Check the solution by substituting the answer into the original proportion and finding the cross products.

You may discover slightly different methods that you prefer.[1] If you think "Hey, can't I do this a different way?", you may be correct.

Exercises 3.1.1

Solve for the variable.

13. $\frac{8}{10} = \frac{x}{15}$
14. $\frac{3}{2} = \frac{7.5}{n}$
15. $\frac{3}{k} = \frac{18}{24}$
16. $\frac{w}{6} = \frac{15}{9}$
17. $\frac{5}{4} = \frac{13}{x}$
18. $\frac{3.2}{7.2} = \frac{m}{4.5}$ (calculator recommended)

Answer

13. $x = 12$
14. $n = 5$
15. $k = 4$
16. $w = 10$
17. $x = 10.4$
18. $m = 2.0$

Now, let's try some with more than one variable in parts of the proportion.

Example 3.1.3

$\frac{n}{n+14}=\frac{5}{7}$.

		$\frac{n}{n+14}=\frac{5}{7}$
Multiply both sides by the LCD.		$7(n+14)\left(\frac{n}{n+14}\right)=7(n+14)\left(\frac{5}{7}\right)$
Remove common factors on each side.		$7n=5(n+14)$
Simplify.		$7n=5n+70$
Solve for n.		$2n=70$
		$n=35$
Check.		
	$\frac{n}{n+14}=\frac{5}{7}$	
Substitute $n=35$.	$\frac{35}{35+14}\stackrel{?}{=}\frac{5}{7}$	
Simplify.	$\frac{35}{49}\stackrel{?}{=}\frac{5}{7}$	
Show common factors.	$\frac{5\cdot 7}{7\cdot 7}\stackrel{?}{=}\frac{5}{7}$	
Simplify.	$\frac{5}{7}=\frac{5}{7}$ ✓	

TRy it 3.1.5

$\frac{y}{y+55}=\frac{3}{8}$.

Answer

33

TRy it 3.1.6

$\frac{z}{z-84}=-\frac{1}{5}$.

Answer

14

Example 3.1.4

$\frac{p+12}{9}=\frac{p-12}{6}$.

		$\frac{p+12}{9}=\frac{p-12}{6}$
Multiply both sides by the LCD, 18.		$18\left(\frac{p+12}{9}\right)=18\left(\frac{p-12}{6}\right)$
Simplify.		$2(p+12)=3(p-12)$
Distribute.		$2p+24=3p-36$
Solve fpr p.		$60=p$
Check.		
	$\frac{p+12}{9}=\frac{p-12}{6}$	
Substitute $p=60$.	$\frac{60+12}{9}\stackrel{?}{=}\frac{60-12}{6}$	
Simplify.	$\frac{72}{9}\stackrel{?}{=}\frac{48}{6}$	
Divide.	$8=8$ ✓	

Try it 3.1.7

$\frac{v+30}{8}=\frac{v+66}{12}$.

Answer

42

Try it 3.1.8

$\frac{2x+15}{9}=\frac{7x+3}{15}$.

Answer

6

1. The steps in the box are designed to avoid mentioning the algebraic step of dividing both sides of the equation by a number. If you are comfortable with basic algebra, then you would phrase step 2 differently. ↵

This page titled 3.1: Ratios, Rates, Proportions is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.10: Ratios, Rates, Proportions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **8.7: Solve Proportion and Similar Figure Applications** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

3.2: Applications of Proportionality

In the previous section, we studied proportions, and used them to solve problems involving ratios. In this section, we continue our study of proportions, and investigate two different types of proportionality.

In this section, you will learn to:

Recognize direct proportionality relationships, and use them to answer questions involving direct proportionality

- Compute and interpret the constant of proportionality in context
- Recognize inverse proportionality relationships, and use them to answer questions involving inverse proportionality

There are two main types of proportionality. We will learn about the more common one first.

3.2.1: Direct Proportionality

Definition: Directly Proportional

Two quantities are **directly proportional** if, as one quantity increases, the other quantity also increases at the same rate.

Direct proportionality describes all of the proportion problems we've seen before. Here is another example that shows how direct proportionality works, and introduces the next important notion.

Example 3.2.1

At an hourly wage job, you work for 5 hours, and get paid $\$83.75$. How much money will you earn if you work 7 hours? (Assume that you are not making overtime pay, or any other sort of special pay rate.)

Solution

This is a situation described by direct proportionality. Since this is an hourly wage job, and we're told that there is no overtime pay or other special pay rate, we can safely assume that we make the same amount per hour in this job. In other words, the rate of pay will be the same no matter how many hours we work. That means that if the number of hours worked increases, the amount you're paid increases at the same rate, no matter the number of hours worked.

This is a situation described by direct proportionality. Since this is an hourly wage job, and we're told that there is no overtime pay or other special pay rate, we can safely assume that we make the same amount per hour in this job. In other words, the rate of pay will be the same no matter how many hours we work. That means that if the number of hours worked increases, the amount you're paid increases at the same rate, no matter the number of hours worked.

We can solve this using a proportion, using the same techniques as in the last section. We will set up the following proportion equation:

$$\frac{\$83.75}{5 \text{ hours}} = \frac{\$x}{7 \text{ hours}} \tag{3.2.1}$$

Notice that we have picked a variable, x, to denote the answer we are trying to find -- the number of dollars earned for working 7 hours. Also notice how we've labeled our units, and have made sure that the corresponding quantities are together. If you do this every time -- label your units, and make sure corresponding quantities stay together -- you can solve any direct proportion problem.

Now for the process that will actually help us solve for x. First, rewrite the equation without labels:

$$\frac{83.75}{5} = \frac{x}{7} \tag{3.2.2}$$

Next, apply Cross Multiplication

$$5x = 83.75 \times 7 \tag{3.2.3}$$

Next, simplify the right side (using a calculator):

$$5x = 586.25 \tag{3.2.4}$$

Finally, apply Division undoes Multiplication to find x:

$$x = \frac{586.25}{5} = 117.25 \tag{3.2.5}$$

That means that if you work 7 hours, you will make $\$117.25$. Think for a moment to see if that's reasonable: it's more, but not too much more, than you made working for 5 hours. So, it seems like a sensible answer.

The approach above works just fine to find the desired answer. However, what if you wanted to know how much you'd make working for 3 hours? Or 4 hours? Or 10 hours? You could just reproduce the work above each time, you wanted. But you may also find the following approach quicker:

Definition: Constant of Proportionality

In a situation involving directly proportional quantities, the **constant of proportionality** is the common ratio that describes the comparison of any two corresponding quantities. In other words, it is the constant rate of change between the two quantities.

Let's see how to find a constant of proportionality, and how to interpret it.

Example 3.2.1

You're in the same situation as the previous example: you work for 5 hours, and earn $\$83.75$. What is the constant of proportionality in this example, and what does it mean in context?

Solution

The constant of proportionality is the common ratio that describes the comparison of any two corresponding quantities. In this situation, we actually have two sets of corresponding quantities, one of which we found in the previous example. You know that you'll make $\$83.75$ working for 5 hours, and we calculated above that you'll make $\$117.25$ if you work for 7 hours. Let's look at these two ratios:

$$\frac{\$83.75}{5 \text{ hours}} \quad \text{and} \quad \frac{\$117.25}{7 \text{ hours}} \tag{3.2.6}$$

Both of these ratios are fractions, and we can simply divide the top by the bottom to reduce them to a single number. Using a calculator, we can see that

$$\frac{\$83.75}{5 \text{ hours}} = \$83.75 \div 5 \text{ hours} = \$16.75 \text{ per hour} \tag{3.2.7}$$

and

$$\frac{\$117.25}{7 \text{ hours}} = \$117.25 \div 7 \text{ hours} = \$16.75 \text{ per hour} \tag{3.2.8}$$

These are the same answer! Do you see why? We originally found our value for x in the previous example by setting the two ratios equal. So, they must give the same answer when divided.

This shared rate -- $\$16.75$ per hour -- is the *constant of proportionality* in this situation. It is the shared value of all ratios described by this problem, where the top of the ratio is money earned, and the bottom is hours worked.

What does this constant of proportionality mean in this situation? In this case, the constant of proportionality is your hourly pay rate. *In other words, it's how much you make per hour.*

A few more comments on the example above: now that you know this rate, it's quite simple to find how much money you'll make if you work $3, 4,$ or 10 hours. You just multiply your hourly rate by the number of hours worked. For example, if you worked 4 hours, you could calculate:

$$\underset{\text{hours}}{4} \times \$16.75 \text{ per hour} = \$67.00 \tag{3.2.9}$$

This means you would make $67.00 working for 4 hours. Notice that if we reverse the process -- in other words, if we try to extract the constant of proportionality knowing that we make $67.00 in 4 hours, we get:

$$\frac{\$67.00}{4 \text{ hours}} = \$67.00 \div 4 \text{ hours} = \$16.75 \text{ per hour} \tag{3.2.10}$$

It's the same rate we found before. This is why it's called a *constant* of proportionality -- it stays the same, even as the corresponding quantities change.

Constants of proportionality will change in meaning depending on the context of the problem. For example, you might ask: If 5 people eat a total of 10 slices of pizza, how many slices would 7 people eat? In this case, the constant of proportionality could be found by:

$$\frac{10 \text{ slices}}{5 \text{ people}} = 2 \text{ slices per person} \tag{3.2.11}$$

In this case, a correct interpretation would be: "The constant of proportionality is 2 slices per person, which means that each person eats 2 slices of pizza." When asked for an interpretation, you should write a sentence similar to the previous -- your goal is the explain the meaning of the constant of proportionality in context of the situation. You will need to read carefully and use critical thinking to deduce a meaningful interpretation. As with many questions in this class, there are multiple good answers to these types of questions!

Problems that involve rates, ratios, scale models, etc. can be solved with proportions. When solving a real-world problem using a proportion, be consistent with the units.

3.2.2: Inverse Proportionality

Let's start this section with a question to illustrate the main concept. Before reading ahead, try to answer this question on your own:

Example 3.2.2

Suppose it takes 6 sanitation workers, all working simultaneously, 4 hours to pick up the trash and recycling in a given neighborhood. How many sanitation workers would it take to pick up the trash and recycling in the same neighborhood in 3 hours? (Assume that all sanitation workers work at the same rate, and can work independently.)

Did you try to answer the question yourself? What did you come up with? If you're like many students who carefully read the previous sections, you might have written this down:

$$\frac{6 \text{ sanitation workers}}{4 \text{ hours}} = \frac{x \text{ sanitation workers}}{3 \text{ hours}} \tag{3.2.12}$$

Then you'd apply Cross Multiplication to get:

$$4x = 18 \tag{3.2.13}$$

and then you'd use Division which undoes Multiplication to get

$$x = \frac{18}{4} = 4.5 \tag{3.2.14}$$

Now, the numerical answer 4.5 is a bit nonsensical, because it's talking about a number of people. So you could round up to 5, and say "It would take 5 sanitation workers to pick up the trash and recycling in 3 hours."

But wait a second: this does not make sense! Think about it: if it takes 6 workers 4 hours to accomplish this task, shouldn't it take 5 workers *more* time than 4 hours? After all, there is the same amount of work to be done, but fewer people to do it! So the answer "5 workers" cannot possibly be correct. We expect a number of workers that is *larger* than 6 to get the task done in a shorter amount of time.

We can learn two things from the previous discussion:

- It is important to evaluate whether or not an answer to a question makes sense in context by asking: What sort of answer would I expect to get? Does my answer seem reasonable?
- Not all problems can be solved using direct proportionality!

The good news is that this type of problem can be solved in a relatively simple way. We define the main concept in this section to see how these problems work.

Definition: Inversely Proportional

Two quantities are **inversely proportional** if, as one quantity increases, the other quantity decreases at the same rate.

Note how similar this definition is to the previous definition of direct proportionality. The only difference here is that one quantity increases while the other decreases:

Direct Proportionality	Inverse Proportionality
As one quantity *increases*	As one quantity *increases*
the other quantity *increases*	the other quantity *decreases*

In order to determine what type of problem you're working on, you'll need to think critically about the quantities involved, and use clues from your experience and the context of the problem to determine how the quantities are related. Things like the previous problem -- when a group of people are working together to accomplish a specific task -- are one of the primary examples of inverse proportionality. Let's see the same example again, and this time find the correct answer.

Example 3.2.2

Suppose it takes 6 sanitation workers, all working simultaneously, 4 hours to pick up the trash and recycling in a given neighborhood. How many sanitation workers would it take to pick up the trash and recycling in the same neighborhood in 3 hours? (Assume that all sanitation workers work at the same rate, and can work independently.)

Solution

The way to approach this is to find the number of *worker hours* needed to accomplish the task of picking up the trash and recycling in this neighborhood. A worker hour is defined to be an hour of work done by a worker, and that number will remain constant no matter the number of workers used.

To find the number of worker hours needed for this particular neighborhood, we simply multiply the known number of workers by the known number of hours:

$$6 \underset{\text{workers}}{} \times 4 \underset{\text{hours}}{} = 24 \underset{\text{worker hours}}{} \tag{3.2.15}$$

That means that it will require 24 *worker hours* to pick up the trash and recycling in this neighborhood.

To find the number of workers needed to pick up the trash and recycling in 3 hours, we divide the number of worker hours by the number of hours to find the number of workers:

$$\frac{24 \text{ worker hours}}{3 \text{ hours}} = 24 \underset{\text{worker hours}}{} \div 3 \underset{\text{hours}}{} = 8 \underset{\text{workers}}{} \tag{3.2.16}$$

This means it will take 8 workers 3 hours to pick up the trash and recycling. This makes sense -- it's larger than 6, which was the number of workers needed to accomplish the task in 4 hours.

All inverse proportionality problems work this way -- multiply the two known corresponding quantities, and then divide to find the answer. As always, label your units, and check to see if your answers make sense!

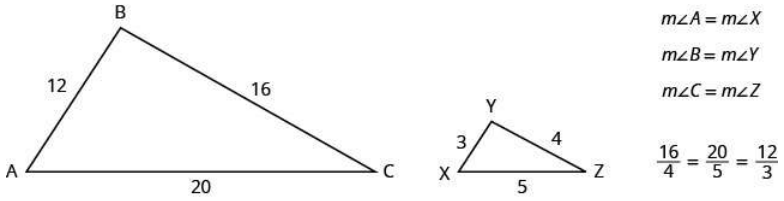
3.2.3: Solve Similar Figure Applications

When you shrink or enlarge a photo on a phone or tablet, figure out a distance on a map, or use a pattern to build a bookcase or sew a dress, you are working with **similar figures**. If two figures have exactly the same shape, but different sizes, they are said to be *similar.* One is a scale model of the other. All their corresponding angles have the same measures and their corresponding sides are in the same ratio.

Definition: SIMILAR FIGURES

Two figures are similar if the measures of their corresponding angles are equal and their corresponding sides are in the same ratio.

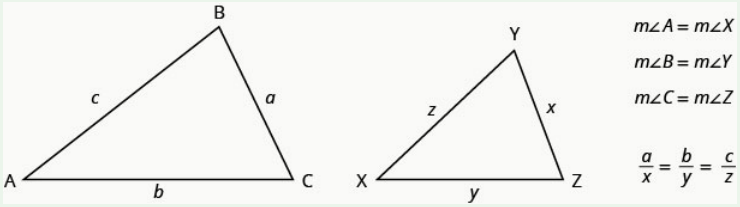
For example, the two triangles in Figure are similar. Each side of ΔABC is 4 times the length of the corresponding side of ΔXYZ.



This is summed up in the Property of Similar Triangles.

Definition: PROPERTY OF SIMILAR TRIANGLES

If ΔABC is similar to ΔXYZ



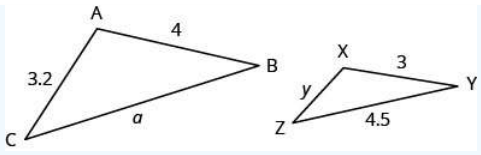
To solve applications with similar figures we will follow the Problem-Solving Strategy for Geometry Applications we used earlier.

Definition: SOLVE GEOMETRY APPLICATIONS.

1. **Read** the problem and make all the words and ideas are understood. Draw the figure and label it with the given information.
2. **Identify** what we are looking for.
3. **Name** what we are looking for by choosing a variable to represent it.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve the equation** using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

? Example 3.2.8

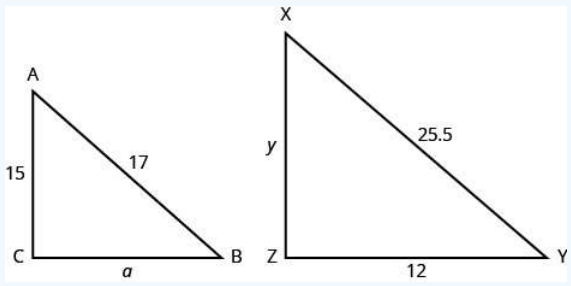
ΔABC is similar to ΔXYZ



Step 1. Read the problem. Draw the figure and label it with the given information.	Figure is given.
Step 2. Identify what we are looking for.	the length of the sides of similar triangles.
Step 3. Name the variables.	Let a= length of the third side of ΔABC. y= length of the third side of ΔXYZ.
Step 4. Translate.	Since the triangles are similar, the corresponding sides are proportional.
We need to write an equation that compares the side we are looking for to a known ratio. Since the side $AB = 4$ corresponds to the side $XY = 3$ we know $\frac{AB}{XY} = \frac{4}{3}$. So we write equations with $\frac{AB}{XY}$ to find the sides we are looking for. Be careful to match up corresponding sides correctly.	$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$. To find a: To find y: sides of large triangle → $\frac{AB}{XY} = \frac{BC}{YZ}$ $\frac{AB}{XY} = \frac{AC}{XZ}$ sides of small triangle →
Substitute.	$\frac{4}{3} = \frac{a}{4.5}$ $\frac{4}{3} = \frac{3.2}{y}$
Step 5. Solve the equation.	$3a = 4(4.5)$ Figure_08_07_030f_img_new.jpg" />
	$a = 6$ $y = 2.4$
Step 6. Check.	
Step 7. Answer the question.	The third side of ΔABC is 6 and the third side of ΔXYZ is 2.4.

TRy it 3.2.15

ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure.



Find the length of side a

Answer

8

TRy it 3.2.16

ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure.

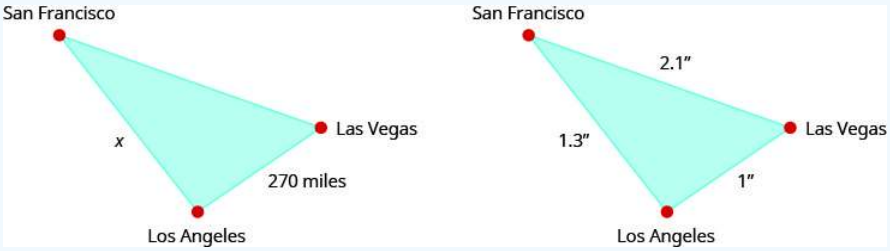
Answer

22.5

The next example shows how similar triangles are used with maps.

Example 3.2.9

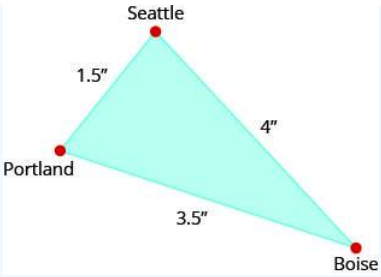
On a map, San Francisco, Las Vegas, and Los Angeles form a triangle whose sides are shown in the figure below. If the actual distance from Los Angeles to Las Vegas is 270 miles find the distance from Los Angeles to San Francisco.



Read the problem. Draw the figures and label with the given information.	The figures are shown above.
Identify what we are looking for.	The actual distance from Los Angeles to San Francisco.
Name the variables.	Let x= distance from Los Angeles to San Francisco.
Translate into an equation. Since the triangles are similar, the corresponding sides are proportional. We'll make the numerators "miles" and the denominators "inches."	$\frac{x \text{ miles}}{1.3 \text{ inches}} = \frac{270 \text{ miles}}{1 \text{ inch}}$
	$1.3\left(\frac{x}{1.3}\right) = 1.3\left(\frac{270}{1}\right)$
Solve the equation.	$x = 351$
Check.	
On the map, the distance from Los Angeles to San Francisco is more than the distance from Los Angeles to Las Vegas. Since 351 is more than 270 the answer makes sense.	
Check $x = 351$ in the original proportion. Use a calculator. $\frac{x \text{ miles}}{1.3 \text{ inches}} = \frac{270 \text{ miles}}{1 \text{ inch}}$ $\frac{351 \text{ miles}}{1.3 \text{ inches}} \stackrel{?}{=} \frac{270 \text{ miles}}{1 \text{ inch}}$ $\frac{270 \text{ miles}}{1 \text{ inch}} = \frac{270 \text{ miles}}{1 \text{ inch}}$ ✓	
Answer the question.	The distance from Los Angeles to San Francisco is 351 miles.

TRY IT 3.2.17

On the map, Seattle, Portland, and Boise form a triangle whose sides are shown in the figure below. If the actual distance from Seattle to Boise is 400 miles, find the distance from Seattle to Portland.



Answer

150 miles

Try it 3.2.18

Using the map above, find the distance from Portland to Boise.

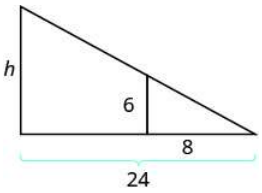
Answer

350 miles

We can use **similar figures** to find heights that we cannot directly measure.

Example 3.2.10

Tyler is 6 feet tall. Late one afternoon, his shadow was 8 feet long. At the same time, the shadow of a tree was 24 feet long. Find the height of the tree.

Read the problem and draw a figure.	
We are looking for h, the height of the tree.	
We will use similar triangles to write an equation.	
The small triangle is similar to the large triangle.	$\frac{h}{24}=\frac{6}{8}$
Solve the proportion.	$24\left(\frac{6}{8}\right)=24\left(\frac{h}{24}\right)$
Simplify.	$18=h$
Check.	
Tyler's height is less than his shadow's length so it makes sense that the tree's height is less than the length of its shadow.	
Check $h=18$ in the original proportion. $\frac{6}{8}=\frac{h}{24}$ $\frac{6}{8}\stackrel{?}{=}\frac{18}{24}$ $\frac{3}{4}=\frac{3}{4}$ ✓	

? Try it 3.2.19

A telephone pole casts a shadow that is 50 feet long. Nearby, an 8 foot tall traffic sign casts a shadow that is 10 feet long. How tall is the telephone pole?

Answer

40 feet

? Try it 3.2.20

A pine tree casts a shadow of 80 feet next to a 30-foot tall building which casts a 40 feet shadow. How tall is the pine tree?

Answer

60 feet

3.2.4: Exercises

Make sure that when you are asked to *interpret* something, you write a complete sentence describing the meaning of your numerical answer in the context of the problem.

1. Your car uses 10 gallons of gas to go 300 miles.
   - How many gallons of gas will you need to go 400 miles?
   - Is this situation described by direct or inverse proportionality, and why? Give a one-sentence answer.
   - What is the constant of proportionality in this situation, and how would you interpret it?
2. It takes 2 math professors a total of 6 hours to grade the exams for a large math class. Assume all professor grade at the same rate.
   - How many professors would it take to grade the same exams in 4 hours?
   - Is this situation described by direct or inverse proportionality, and why? Give a one-sentence answer.
3. A family drinks 2 gallons of milk every 9 days. How many gallons of milk will they use in 2 weeks? Be careful with units here! (Round to one decimal place.)
4. At a rate of 30 miles per hour, a certain trip takes 2 hours. How long would the same trip take at 40 miles per hour? (Round to one decimal place or give a fractional answer.)
5. Think of a real-world example of direct proportionality that is different than ones we've covered in this section. Give a 2-3 sentence description of the quantities involved, and why you think they are directly proportional. If you use a source, please cite it by providing a URL.
6. Think of a real-world example of inverse proportionality that is different than ones we've covered in this section. Give a 2-3 sentence description of the quantities involved, and why you think they are inversely proportional. If you use a source, please cite it by providing a URL.
7. Tonisha drove her car 320 miles and used 12.5 gallons of gas. At this rate, how far could she drive using 10 gallons of gas?
8. Marcus worked 14 hours and earned $\$ 210$. At the same rate of pay, how long would he have to work to earn $\$ 300$?
9. A picture of your author appearing on *Jeopardy!* that is 375 pixels high and 475 pixels wide needs to be reduced in size so that it is 150 pixels high. If the height and width are kept proportional, what is the width of the picture after it has been reduced?





10. At a fast-food restaurant, a 22-ounce chocolate shake has 850 calories. How many calories are in their 12-ounce chocolate shake? Round your answer to nearest whole number.
11. Yurianna is going to Europe and wants to change $800 dollars into Euros. At the current exchange rate, $1 US is equal to 0.738 Euro. How many Euros will she have for her trip? **Answer**

7. **256** miles
8. **20** hours
9. **190** pixels wide
10. 464 calories
11. 590.4 Euros

3.2: Applications of Proportionality is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

- **1.10: Ratios, Rates, Proportions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **8.7: Solve Proportion and Similar Figure Applications** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

3.3: Direct and Inverse Variation

Learning Objectives

By the end of this section, you will be able to:

- Solve direct variation problems
- Solve inverse variation problems

Note

Before you get started, take this readiness quiz.

If you miss a problem, go back to the section listed and review the material.

1. Find the multiplicative inverse of −8
   If you missed this problem, review Exercise 1.10.13.
2. Solve for n: 45=20n
   If you missed this problem, review Exercise 2.2.1.
3. Evaluate $5x^2$ when x=10
   If you missed this problem, review Exercise 1.3.25.

When two quantities are related by a proportion, we say they are *proportional* to each other. Another way to express this relation is to talk about the *variation* of the two quantities. We will discuss direct variation and inverse variation in this section.

Solve Direct Variation Problems

Lindsay gets paid $15 per hour at her job. If we let *s* be her salary and *h* be the number of hours she has worked, we could model this situation with the equation

s=15h

Lindsay's salary is the product of a constant, 15, and the number of hours she works. We say that Lindsay's salary *varies directly* with the number of hours she works. Two variables vary directly if one is the product of a constant and the other.

Definition: DIRECT VARIATION

For any two variables *x* and *y*, *y* varies directly with *x* if

y=kx, where $n \neq 0$

In applications using direct variation, generally we will know values of one pair of the variables and will be asked to find the equation that relates *x* and *y*. Then we can use that equation to find values of *y* for other values of *x*.

How to Solve Direct Variation Problems

Example 3.3.1

If y varies directly with x and y=20 when x=8, find the equation that relates *x* and *y*.

Answer

Step 1. Write the formula for direct variation.	The direct variation formula is $y = kx$.	$y = kx$	**Step 2.** Substitute the given values for the variables.	We are given $y = 20$, $x = 8$.	$20 = k \cdot 8$
Step 3. Solve for the constant of variation.	Divide both sides of the equation by 8, then simplify.	$\frac{20}{8} = k$ $k = 2.5$	**Step 4.** Write the equation that relates *x* and *y*.	Rewrite the general equation with the value we found for *k*.	$y = 2.5x$

Example 3.3.2

If y varies directly as x and y=3, when x=10, find the equation that relates *x* and *y*.

Answer

$y = \frac{3}{10}x$

Example 3.3.3

If y varies directly as x and y=12 when x=4, find the equation that relates *x* and *y*.

Answer

y=3x

We'll list the steps below.

Definition: SOLVE DIRECT VARIATION PROBLEMS

1. Write the formula for direct variation.
2. Substitute the given values for the variables.
3. Solve for the constant of variation.
4. Write the equation that relates x and y.

Now we'll solve a few applications of direct variation.

Example 3.3.4

When Raoul runs on the treadmill at the gym, the number of calories, c, he burns varies directly with the number of minutes, m, he uses the treadmill. He burned 315 calories when he used the treadmill for 18 minutes.

1. Write the equation that relates c and m.
2. How many calories would he burn if he ran on the treadmill for 25 minutes?

Answer

1.

	The number of calories, c varies directly with the number of minutes, m, on the treadmill, and c=315 when m=18
Write the formula for direct variation.	$y = kx$
We will use c in place of y and mm in place of x.	$c = km$
Substitute the given values for the variables.	$315 = k \cdot 18$
Solve for the constant of variation.	$\frac{315}{18} = \frac{k \cdot 18}{18}$
	$17.5 = k$
Write the equation that relates c and m.	$c = km$
Substitute in the constant of variation.	$c = 17.5m$

2.

	Find c when m=25.
Write the equation that relates c and m.	$c = 17.5m$
Substitute the given value for m.	$c = 17.5(25)$
Simplify.	$c = 437.5$
	Raoul would burn 437.5 calories if he used the treadmill for 25 minutes.

Example 3.3.5

The number of calories, c, burned varies directly with the amount of time, t, spent exercising. Arnold burned 312 calories in 65 minutes exercising.

1. Write the equation that relates c and t.
2. How many calories would he burn if he exercises for 90 minutes?

Answer

1. c=4.8t
2. 432 calories

Example 3.3.6

The distance a moving body travels, d, varies directly with time, t, it moves. A train travels 100 miles in 2 hours

1. Write the equation that relates d and t.
2. How many miles would it travel in 5 hours?

Answer

1. d=50t
2. 250 miles

In the previous example, the variables c and m were named in the problem. Usually that is not the case. We will have to name the variables in the next example as part of the solution, just like we do in most applied problems.

Example 3.3.7

The number of gallons of gas Eunice's car uses varies directly with the number of miles she drives. Last week she drove 469.8 miles and used 14.5 gallons of gas.

1. Write the equation that relates the number of gallons of gas used to the number of miles driven.
2. How many gallons of gas would Eunice's car use if she drove 1000 miles?

Answer

1.

	The number of gallons of gas varies directly with the number of miles driven.
First we will name the variables.	Let g= number of gallons of gas. m= number of miles driven.
Write the formula for direct variation.	$y = kx$
We will use g in place of y and m in place of x.	$g = km$
Substitute the given values for the variables	$g = 14.5$ when $m = 469.8$
	$14.5 = k(469.8)$
Solve for the constant of variation.	$\frac{14.5}{469.8} = \frac{k(469.8)}{469.8}$
We will round to the nearest thousandth.	$0.031 = k$
Write the equation that relates g and m.	$g = km$
Substitute in the constant of variation.	$g = 0.031m$

2.

	Find g when m=1000.
Write the equation that relates g and m.	g=0.031m
Substitute the given value for m.	g=0.031(1000)
Simplify.	g=31
	Eunice's car would use 31 gallons of gas if she drove it 1,000 miles.

Notice that in this example, the units on the constant of variation are gallons/mile. In everyday life, we usually talk about miles/gallon.

Example 3.3.8

The distance that Brad travels varies directly with the time spent traveling. Brad travelled 660 miles in 12 hours,

1. Write the equation that relates the number of miles travelled to the time.
2. How many miles could Brad travel in 4 hours?

Answer

1. m=55h
2. 220 miles

Example 3.3.9

The weight of a liquid varies directly as its volume. A liquid that weighs 24 pounds has a volume of 4 gallons.

1. Write the equation that relates the weight to the volume.
2. If a liquid has volume 13 gallons, what is its weight?

Answer

1. w=6v
2. 78 pounds

In some situations, one variable varies directly with the square of the other variable. When that happens, the equation of direct variation is $y = kx^2$.

Example 3.3.10

The maximum load a beam will support varies directly with the square of the diagonal of the beam's cross-section. A beam with diagonal 4" will support a maximum load of 75 pounds.

1. Write the equation that relates the maximum load to the cross-section.
2. What is the maximum load that can be supported by a beam with diagonal 8"?

Answer

1.

	The maximum load varies directly with the square of the diagonal of the cross-section.
Name the variables.	Let L= maximum load. c= the diagonal of the cross-section
Write the formula for direct variation, where y varies directly with the square of x.	$y = kx^2$
We will use L in place of y and c in place of x.	$L = kc^2$
Substitute the given values for the variables.	$L = 75$ when $c = 4$
	$75 = k \cdot 4^2$
Solve for the constant of variation.	$\frac{75}{16} = \frac{k \cdot 16}{16}$
	$4.6875 = k$
Write the equation that relates L and c.	$L = kc^2$
Substitute in the constant of variation.	$L = 4.6875c^2$

2.

	Find L when c=8.
Write the equation that relates L and c.	$\boldsymbol{L = 4.6875c^2}$
Substitute the given value for c.	$\boldsymbol{L = 4.6875(8)^2}$
Simplify.	L=300
	A beam with diagonal 8” could supporta maximum load of 300 pounds.

Example 3.3.11

The distance an object falls is directly proportional to the square of the time it falls. A ball falls 144 feet in 3 seconds.

1. Write the equation that relates the distance to the time.
2. How far will an object fall in 4 seconds?

Answer

1. $\boldsymbol{d = 16t^2}$
2. 256 feet

Example 3.3.12

The area of a circle varies directly as the square of the radius. A circular pizza with a radius of 6 inches has an area of 113.04 square inches.

1. Write the equation that relates the area to the radius.
2. What is the area of a pizza with a radius of 9 inches?

Answer

1. $\boldsymbol{A = 3.14r^2}$
2. 254.34 square inches

Solve Inverse Variation Problems

Many applications involve two variable that *vary inversely*. As one variable increases, the other decreases. The equation that relates them is $\boldsymbol{y = \frac{k}{x}}$.

Definition: INVERSE VARIATION

For any two variables x and y, y varies inversely with x if

$y=\frac{k}{x}$,where $\boldsymbol{k \neq 0}$

The word ‘inverse’ in inverse variation refers to the multiplicative inverse. The multiplicative inverse of x is $\frac{1}{x}$.

We solve inverse variation problems in the same way we solved direct variation problems. Only the general form of the equation has changed. We will copy the procedure box here and just change ‘direct’ to ‘inverse’.

Definition: SOLVE INVERSE VARIATION PROBLEMS

1. Write the formula for inverse variation.
2. Substitute the given values for the variables.
3. Solve for the constant of variation.
4. Write the equation that relates x and y.

Example 3.3.13

If y varies inversely with x and y=20 when x=8 x and y.

Answer

Write the formula for inverse variation.	$y = \frac{k}{x}$
Substitute the given values for the variables.	$y = 20$ when $x = 8$
	$20 = \frac{k}{8}$
Solve for the constant of variation.	$8(20) = 8\left(\frac{k}{8}\right)$
	$160 = k$
Write the equation that relates x and y.	$y = \frac{k}{x}$
Substitute in the constant of variation.	$y = \frac{160}{x}$

Example 3.3.14

If p varies inversely with q and p=30 when q=12 find the equation that relates p and q.

Answer

$p = \frac{360}{q}$

Example 3.3.15

If y varies inversely with x and y=8 when x=2 find the equation that relates x and y.

Answer

$y = \frac{16}{x}$

Example 3.3.16

The fuel consumption (mpg) of a car varies inversely with its weight. A car that weighs 3100 pounds gets 26 mpg on the highway.

1. Write the equation of variation.
2. What would be the fuel consumption of a car that weighs 4030 pounds?

Answer

1.

	The fuel consumption varies inversely with the weight.
First we will name the variables.	Let f= fuel consumption. w= weight.
Write the formula for inverse variation.	$y = \frac{k}{x}$
We will use f in place of y and w in place of x.	$f = \frac{k}{w}$
Substitute the given values for the variables.	$f = 26$ when $w = 3100$
	$26 = \frac{k}{3100}$
Solve for the constant of variation.	$3100(26) = 3100\left(\frac{k}{3100}\right)$
	$80{,}600 = k$
Write the equation that relates f and w.	$f = \frac{k}{w}$
Substitute in the constant of variation.	$f = \frac{80{,}600}{w}$

2.

	Find f when w=4030.
Write the equation that relates f and w.	$f = \frac{80{,}600}{w}$
Substitute the given value for w.	$f = \frac{80{,}600}{4030}$
Simplify.	f=20
	A car that weighs 4030 pounds would have fuel consumption of 20 mpg.

Example 3.3.17

A car's value varies inversely with its age. Elena bought a two-year-old car for $20,000.

1. Write the equation of variation.
2. What will be the value of Elena's car when it is 5 years old?

Answer

1. $v = \frac{40,000}{a}$
2. $8,000

Example 3.3.18

The time required to empty a pool varies inversely as the rate of pumping. It took Lucy 2.5 hours to empty her pool using a pump that was rated at 400 gpm (gallons per minute).

1. Write the equation of variation.
2. How long will it take her to empty the pool using a pump rated at 500 gpm?

Answer

1. $t = \frac{1000}{r}$
2. 2 hours

Example 3.3.19

The frequency of a guitar string varies inversely with its length. A 26" long string has a frequency of 440 vibrations per second.

1. Write the equation of variation.
2. How many vibrations per second will there be if the string's length is reduced to 20" by putting a finger on a fret?

Answer

1.

	The frequency varies inversely with the length.
Name the variables.	Let f= frequency. L=length.
Write the formula for inverse variation.	$y = \frac{k}{x}$
We will use f in place of y and L in place of x	$f = \frac{k}{L}$
Substitute the given values for the variables.	$f = 440$ when $L = 26$
	$440 = \frac{k}{26}$
Solve for the constant of variation.	$26(440) = 26\left(\frac{k}{26}\right)$
	$11,440 = k$
Write the equation that relates f and L.	$f = \frac{k}{L}$
Substitute in the constant of variation.	$f = \frac{11,440}{L}$

2.

	Find f when L=20.
Write the equation that relates f and L.	$f = \frac{11,440}{L}$
Substitute the given value for L.	$f = \frac{11,440}{20}$
Simplify.	f=572
	A 20" guitar string has frequency 572 vibrations per second.

Example 3.3.20

The number of hours it takes for ice to melt varies inversely with the air temperature. Suppose a block of ice melts in 2 hours when the temperature is 65 degrees.

1. Write the equation of variation.
2. How many hours would it take for the same block of ice to melt if the temperature was 78 degrees?

Answer

1. $h = \frac{130}{t}$
2. $1\frac{2}{3}$ hours

? Example 3.3.21

The force needed to break a board varies inversely with its length. Richard uses 24 pounds of pressure to break a 2-foot long board.

1. Write the equation of variation.
2. How many pounds of pressure is needed to break a 5-foot long board?

Answer

1. $F = \frac{48}{l}$
2. 9.6 pounds

This page titled 3.3: Direct and Inverse Variation is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **8.9: Use Direct and Inverse Variation** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/elementary-algebra-2e.

CHAPTER OVERVIEW

4: Percent

4.1: Basic Percent Problems

4.2: Percents in the Real World

4.3: Percent Increase or Decrease

This page titled 4: Percent is shared under a CC BY-NC-SA 3.0 license and was authored, remixed, and/or curated by David Arnold.

4.1: Basic Percent Problems

When one hears the word “percent,” other words come immediately to mind, words such as “century,” “cents,” or “centimeters.” A century equals 100 years. There are one hundred cents in a dollar and there are 100 centimeters in a meter. Thus, it should come as no surprise that percent means “parts per hundred.”

In the world we live in we are constantly bombarded with phrases that contain the word “percent.” The sales tax in California is 8.25%. An employee is asking his boss for a 5% raise. A union has seen a 6.25% increase in union dues. The population of a town is increasing at a rate of 2.25% per year.

It is important to remember that when performing calculations with percents, the number must be written in its decimal form. To convert a number from percent form to decimal form, drop the % and move the decimal point LEFT TWO PLACES. The percent values from the examples above are shown below converted to decimal form

8.25% = 0.0825

5% = 0.05

6.25% = 0.0625

2.25% = 0.0225

This course focuses on solving problems that involve percents. There are three basic types of percent problems:

1. *Find a given percent of a given number.* For example, find 25% of 640.
2. *Find a percent given two numbers.* For example, 15 is what percent of 50?
3. *Find a number that is a given percent of another number.* For example, 10% of what number is 12?

Let’s begin with the first of these types.

Find a Given Percent of a Given Number

Let’s begin with our first example.

Example 1

What number is 25% of 640?

Solution

Let x represent the unknown number. Translate the words into an equation.

What number	is	25	of	640
x	$=$	25%	$\cdot$	640

Now, solve the equation for x.

\cdot is only supported in math mode

Thus, 25% of 640 is 160.

Alternate Solution

We could also change 25% to a fraction.

$$\begin{aligned} x &= 25\% \cdot 640 && \text{Original equation.} \\ x &= \frac{1}{4} \cdot 640 && \text{Change 25} \\ x &= \frac{640}{4} && \text{Multiply numerators and denominators.} \\ x &= 160 && \text{Divide: } 640/4 = 160. \end{aligned}$$

Same answer.

? Exercise

What number is 36% of 120?

Answer

43.2

✓ Example 2

What is number $8\frac{1}{3}\%$ of 120?

Solution

Let x represent the unknown number. Translate the words into an equation.

What number	is	8 (1/3)	of	120
x	$=$	$8\frac{1}{3}\%$	$\cdot$	120

Now, solve the equation for x. Because

$$8\frac{1}{3}\% = 8.3\% = 0.08\overline{3},$$

working with decimals requires that we work with a repeating decimal. To do so, we would have to truncate the decimal representation of the percent at some place and satisfy ourselves with an approximate answer. Instead, let's change the percent to a fraction and seek an exact answer.

$$\begin{aligned} 8\frac{1}{3}\% &= \frac{8\frac{1}{3}}{100} && \text{Percent: Parts per hundred.} \\ &= \frac{\frac{25}{3}}{100} && \text{Mixed to improper fraction.} \\ &= \frac{25}{3} \cdot \frac{1}{100} && \text{Invert and multiply.} \\ &= \frac{25}{300} && \text{Multiply numerators and denominators.} \\ &= \frac{1}{12} && \text{Reduce: Divide numerator and denominator by 25.} \end{aligned}$$

Now we can solve our equation for x.

$$\begin{aligned} &= 8\frac{1}{3}\% \cdot 120 && \text{Original equation.} \\ x &= \frac{1}{12} \cdot 120 && 8\tfrac{1}{3}\% = 1/12. \\ x &= \frac{120}{12} && \text{Multiply numerators and denominators.} \\ x &= 10 && \text{Divide: } 120/12 = 10. \end{aligned}$$

Thus, $8\frac{1}{3}\%$ of 120 is 10.

? Exercise

What number is $4\frac{1}{6}\%$ of 1,200?

Answer

50

Example 3

What number is $105\frac{1}{4}\%$ of 18.2?

Solution

Let x represent the unknown number. Translate the words into an equation.

What number	is	105 (1/4)	of	18.2
x	$=$	$105\frac{1}{4}\%$	$\cdot$	18.2

In this case, the fraction terminates as 1/4=0.25, so

$$105\frac{1}{4}\% = 105.25 \tag{4.1.1}$$

Now we can solve our equation for x.

$$x = 105\frac{1}{4}\% \cdot 18.2 \quad \text{Original equation.}$$
$$x = 1.0525 \cdot 18.2 \quad 5\frac{1}{4}\% = 1.0525.$$
$$x = 19.1555 \quad \text{Multiply.}$$

Thus, $105\frac{1}{4}\%$ of 18.2 is 19.1555.

Exercise

What number is $105\frac{3}{4}\%$ of 222?

Answer

234.765

Find a Percent Given Two Numbers

Now we'll address our second item on the list at the beginning of the section.

Example 4

15 is what percent of 50?

Solution

Let x represent the unknown percent. Translate the words into an equation.

15	is	what percent	of	50
15	$=$	x	$\cdot$	50

The commutative property of multiplication allows us to change the order of multiplication on the right-hand side of this equation.

$$15 = 50x.$$

Now we can solve our equation for x.

$$15 = 50x \quad \text{Original equation.}$$
$$\frac{15}{50} = \frac{50x}{50} \quad \text{Divide both sides by 50.}$$
$$\frac{15}{50} = x \quad \text{Simplify right-hand side.}$$
$$x = 0.30 \quad \text{Divide: } 15/50 = 0.30.$$

But we must express our answer as a percent. To do this, move the decimal two places to the right and append a percent symbol.

$$0.30 = 0\,30.\% = 30\%$$

Thus, 15 is 30% of 50.

Alternative Conversion

At the third step of the equation solution, we had

$$x = \frac{15}{50}.$$

We can convert this to an equivalent fraction with a denominator of 100.

$$x = \frac{15 \cdot 2}{50 \cdot 2} = \frac{30}{100}$$

Thus, 15/50 = 30/100 = 30%.

Exercise

14 is what percent of 25?

Answer

56%

Example 5

10 is what percent of 80?

Solution

Let x represent the unknown percent. Translate the words into an equation.

10	is	what percent	of	80
10	=	x	·	80

The commutative property of multiplication allows us to write the right-hand side as

$$10 = 80x.$$

Now we can solve our equation for x.

$$\begin{aligned} 10 &= 80x && \text{Original equation.} \\ \frac{10}{80} &= \frac{80x}{80} && \text{Divide both sides by 80.} \\ \frac{1}{8} &= x && \text{Reduce: } 10/80 = 1/8. \\ 0.125 &= x && \text{Divide: } 1/8 = 0.125. \end{aligned}$$

But we must express our answer as a percent. To do this, move the decimal two places to the right and append a percent symbol.

$$0.125 = 0\,12.5\% = 12.5\%$$

Thus, 10 is 12.5% of 80.

Alternative Conversion

At the third step of the equation solution, we had

$$x = \frac{1}{8}.$$

We can convert this to an equivalent fraction with a denominator of 100 by setting up the proportion

$$\frac{1}{8} = \frac{n}{100}$$

Cross multiply and solve for n.

$$\begin{aligned} 8n &= 100 && \text{Cross multiply.} \\ \frac{8n}{8} &= \frac{100}{8} && \text{Divide both sides by 8.} \\ n &= \frac{25}{8} && \text{Reduce: Divide numerator and denominator by 4.} \\ n &= 12\frac{1}{2} && \text{Change 25/2 to mixed fraction.} \end{aligned}$$

Hence,

$$\frac{1}{8} = \frac{12\frac{1}{2}}{100} = 12\frac{1}{2}\%.$$

Same answer.

? Exercise

10 is what percent of 200?

Answer

5%

Find a Number that is a Given Percent of Another Number

Let's address the third item on the list at the beginning of the section.

✔ Example 6

10% of what number is 12?

Solution

Let x represent the unknown number. Translate the words into an equation.

10	of	what number	is	12
10%	·	x	=	12

Change 10% to a fraction: 10% = 10/100 = 1/10.

$$\frac{1}{10}x = 12$$

Now we can solve our equation for x.

$$\begin{aligned} 10\left(\frac{1}{10}x\right) &= 10(12) && \text{Multiply both sides by 10.} \\ x &= 120 && \text{Simplify.} \end{aligned}$$

Thus, 10% of 120 is 12.

Alternative Solution

We can also change 10% to a decimal: 10% = 0.10. Then our equation becomes

$$0.10x = 12$$

Now we can divide both sides of the equation by 0.10.

$$\begin{aligned} \frac{0.10x}{0.10} &= \frac{12}{0.10} && \text{Divide both sides by 0.10.} \\ x &= 120 && \text{Divide: } 12/0.10 = 120. \end{aligned}$$

Same answer.

Exercise

20% of what number is 45?

Answer

225

Example 7

$11\frac{1}{9}\%$ of what number is 20?

Solution

Let x represent the unknown number. Translate the words into an equation.

11 (1/9)	of	what number	is	20
$11\frac{1}{9}\%$	$\cdot$	x	$=$	20

Change $11\frac{1}{9}\%$ to a fraction.

$$\begin{aligned} 11\frac{1}{9}\% &&& \text{Percent: Parts per hundred.} \\ = \frac{\frac{100}{9}}{100} &&& \text{Mixed to improper: } 11\frac{1}{9} = 100/9. \\ = \frac{100}{9} \cdot \frac{1}{100} &&& \text{Invert and multiply.} \\ = \frac{\cancel{100}}{9} \cdot \frac{1}{\cancel{100}} &&& \text{Cancel.} \\ = \frac{1}{9} &&& \text{Simplify.} \end{aligned}$$

Replace $11\frac{1}{9}\%$ with 1/9 in the equation and solve for x.

$$\begin{aligned} \frac{1}{9}x &= 20 && 11\frac{1}{9}\% = 1/9/ \\ 9\left(\frac{1}{9}x\right) &= 9(20) && \text{Multiply both sides by 9.} \\ x &= 180 \end{aligned}$$

Thus, $11\frac{1}{9}\%$ of 180 is 20.

Exercise

$12\frac{2}{3}\%$ of what number is 760?

Answer

6,000

Exercises

1. What number is 22.4% of 125?
2. What number is 159.2% of 125?
3. 60% of what number is 90?
4. 25% of what number is 40?
5. 200% of what number is 132?
6. 200% of what number is 208?
7. 162.5% of what number is 195?
8. 187.5% of what number is 90?
9. 126.4% of what number is 158?
10. 132.5% of what number is 159?
11. 27 is what percent of 45?
12. 9 is what percent of 50?
13. 37.5% of what number is 57?
14. 162.5% of what number is 286?
15. What number is 85% of 100?
16. What number is 10% of 70?
17. What number is 200% of 15?
18. What number is 50% of 84?
19. 50% of what number is 58?
20. 132% of what number is 198?
21. 5.6 is what percent of 40?
22. 7.7 is what percent of 35?
23. What number is 18.4% of 125?
24. What number is 11.2% of 125?
25. 30.8 is what percent of 40?
26. 6.3 is what percent of 15?
27. 7.2 is what percent of 16?
28. 55.8 is what percent of 60?
29. What number is 89.6% of 125?
30. What number is 86.4% of 125?
31. 60 is what percent of 80?
32. 16 is what percent of 8?
33. What number is 200% of 11?
34. What number is 150% of 66?
35. 27 is what percent of 18?
36. 9 is what percent of 15?

37. $133\frac{1}{3}\%$ of what number is 80?

38. $121\frac{2}{3}\%$ of what number is 73?

39. What number is $54\frac{1}{3}\%$ of 6?

40. What number is $82\frac{2}{5}\%$ of 5?

41. What number is $62\frac{1}{2}\%$ of 32?

42. What number is $118\frac{3}{4}\%$ of 32?

43. $77\frac{1}{7}\%$ of what number is 27?

44. $82\frac{2}{3}\%$ of what number is 62?

45. What number is $142\frac{6}{7}\%$ of 77?

46. What number is $116\frac{2}{3}\%$ of 84?

47. $143\frac{1}{2}\%$ of what number is 5.74?

48. $77\frac{1}{2}\%$ of what number is 6.2?

49. $141\frac{2}{3}\%$ of what number is 68?

50. $108\frac{1}{3}\%$ of what number is 78?

51. What number is $66\frac{2}{3}\%$ of 96?

52. What number is $79\frac{1}{6}\%$ of 48?

53. $59\frac{1}{2}\%$ of what number is 2.38?

54. $140\frac{1}{5}\%$ of what number is 35.05?

55. $78\frac{1}{2}\%$ of what number is 7.85?

56. $73\frac{1}{2}\%$ of what number is 4.41?

57. What number is $56\frac{2}{3}\%$ of 51?

58. What number is $64\frac{1}{2}\%$ of 4?

59. What number is $87\frac{1}{2}\%$ of 70?

60. What number is $146\frac{1}{4}\%$ of 4?

---

61. It was reported that 80% of the retail price of milk was for packaging and distribution. The remaining 20% was paid to the dairy farmer. If a gallon of milk cost $3.80, how much of the retail price did the farmer receive?

62. At $1.689 per gallon of gas the cost is distributed as follows:

Crude oil supplies	**$0.95**
Oil Companies	**$0.23**
State and City taxes	**$0.23**
Federal tax	**$0.19**
Service Station	**$0.10**

Data is from Money, March 2009 p. 22, based on U. S. averages in December 2008. Answer the following questions rounded to the nearest whole percent.

a) What % of the cost is paid for crude oil supplies?

b) What % of the cost is paid to the service station?

Answers

1. 28
3. 150
5. 66
7. 120
9. 125
11. 60
13. 152
15. 85
17. 30
19. 116
21. 14
23. 23
25. 77
27. 45
29. 112
31. 75
33. 22
35. 150
37. 60
39. 3.26
41. 20
43. 35
45. 110
47. 4
49. 48
51. 64
53. 4
55. 10
57. 28.9
59. 61.25
61. $0.76

This page titled 4.1: Basic Percent Problems is shared under a CC BY-NC-SA 3.0 license and was authored, remixed, and/or curated by David Arnold.

- **7.3: Solving Basic Percent Problems** by David Arnold is licensed CC BY-NC-SA 3.0.

4.2: Percents in the Real World

In this section we will look at an assortment of practical problems involving percent.

Example 1

Myrna notes that 20% of her class is absent. If the class has 45 students, how many students are absent?

Solution

Let n represent the number of students that are absent. Then we can translate the problem statement into words and symbols.

Number absent	is	20	of	total number of students in the class
n	$=$	20%	$\cdot$	45

Because 20% = 0.20,

$$n = 0.20 \cdot 45 \quad 20$$
$$n = 9 \quad \text{Multiply: } 0.20 \cdot 45 = 9.$$

Therefore, 9 students are absent.

Exercise

Aaron notes that 15% of his class is absent. If the class has 80 students, how many students are absent?

Answer

12

Example 2

Misty answered 90% of the questions on her mathematics examination correctly. If Misty had 27 correct answers, how many questions were on the exam?

Solution

Let N represent the number of questions on the examination.

Number of correct answers	is	90	of	total number of questions
27	$=$	90%	$\cdot$	N

Because 90% = 0.90, this last equation can be written as

$$27 = 0.90N.$$

Solve for N.

$$\frac{27}{0.90} = \frac{0.90N}{0.90} \quad \text{Divide both sides by 0.90.}$$
$$30 = N \quad \text{Divide: } 27/0.90 = 30.$$

Hence, there were 30 questions on the examination.

Exercise

Erin asnwered 85% of the questions on her english examination correctly. If she had 34 correct answers, how many questions were on her exam?

Answer

40

Example 3

Misty answered 30 of 40 possible questions on her sociology examination correctly. What percent of the total number of questions did Misty mark correctly?

Solution

Let p represent the percent of the total number of questions marked correctly. Then we can translate the problem statement into words and symbols.

Number of correct answers	is	what percent	of	total number of questions
30	=	p	·	40

Because multiplication is commutative, we can write the last equation in the form

$$30 = 40p.$$

Solve for p.

$$\begin{aligned} \frac{30}{40} &= \frac{40p}{40} && \text{Divide both sides by 40.} \\ \frac{3}{4} &= p && \text{Reduce: } 30/40 = 3/4. \end{aligned}$$

We need to change p = 3/4 to a percent. There are two ways to do this:

- We can divide 3 by 4 to get

$$\begin{aligned} p &= \frac{3}{4} \\ &= 0.75 && \text{Divide: } 3/4{=}0.75. \\ &= 75\% && \text{Move decimal point 2 places right.} \end{aligned}$$

- We can create an equivalent fraction with a denominator of 100; i.e.,

$$\begin{aligned} p &= \frac{3}{4} \\ &= \frac{3 \cdot 25}{4 \cdot 25} && \text{Multiply numerator and denominator by 25.} \\ &= \frac{75}{100} && \text{Simplify numerator and denominator.} \\ &= 75\%. && \text{Percent means parts per hundred.} \end{aligned}$$

Either way, Misty got 75% of the questions on her sociology examination correct.

Exercise

Alphonso answered 19 of 25 questions on his biology test correctly. What percent of the questions did he mark correctly?

Answer

76%

Example 4

35 millilitres of a 60 millilitre solution is hydrochloric acid. What percent of the solution is hydrochloric acid?

Solution

Let p represent the percent of the percent of the solution that is hydrochloric acid. Then we can translate the problem statement into words and symbols.

Amount of hydrochloric acid	is	what percent	of	the tota amount of solution
35	=	p	·	60

Because multiplication is commutative, we can write the right-hand side of the last equation as follows.

$$35 = 60p$$

Now we can solve for p.

$$\frac{35}{60} = \frac{60p}{60} \quad \text{Divide both sides by 60.}$$
$$\frac{7}{12} = p \quad \text{Reduce: Divide numerator and denominator by 5.}$$

Now we must change p to a percent. We can do this exactly by creating an equivalent fraction with a denominator of 100.

$$\frac{7}{12} = \frac{n}{100}$$

Solve for n.

$$12n = 700 \quad \text{Cross multiply.}$$
$$\frac{12n}{12} = \frac{700}{12} \quad \text{Divide both sides by 12.}$$
$$n = \frac{175}{3} \quad \text{Reduce: Divide numerator and denominator by 4.}$$
$$n = 58\frac{1}{3} \quad \text{Change improper to mixed fraction.}$$

Hence,

$$p = \frac{7}{12} = \frac{58\frac{1}{3}}{100} = 58\frac{1}{3}\%.$$

Thus, $58\frac{1}{3}\%$ of the solution is hydrochloric acid.

Approximate Solution

If all that is needed is an approximate answer, say correct to the nearest tenth of a percent, then we would take a different approach starting with the line from above that has

$$\frac{35}{60} = p.$$

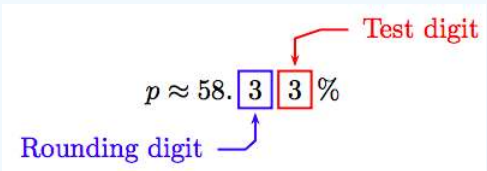
We would divide 35 by 60 to get

$$p \approx 0.5833.$$

Move the decimal two places to the right and append a percent symbol.

$$p \approx 0.5833 \approx 0\,58.33\% \approx 58.33\%.$$

Round to the nearest tenth of a percent.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 58.3\%.$$

Note that p ≈ 58.3% is approximate, but $p = 58\frac{1}{3}\%$ is exact.

? Exercise

25 millilitres of a 40 millilitre solution is sulfuric acid. What percent of the solution is sulfuric acid?

Answer

62.5%

Exercises

1. 31 millilitres of a 250 millilitre solution is sulphuric acid. What percent of the solution is sulphuric acid? Round your answer to the nearest tenth of a percent.

2. 34 millilitres of a 211 millilitre solution is phosphoric acid. What percent of the solution is phosphoric acid? Round your answer to the nearest tenth of a percent.

3. A family has completed 186 miles of a planned 346 mile trip. Find the percentage of the planned trip already traveled. Round your answer to the nearest percent.

4. A family has completed 153 miles of a planned 431 mile trip. Find the percentage of the planned trip already traveled. Round your answer to the nearest percent.

5. Erin takes roll in her fifth grade class and finds that 19 out of 34 total students on her roster are present. Find the percentage of the class that is present, correct to the nearest percent.

6. Barbara takes roll in her fifth grade class and finds that 15 out of 38 total students on her roster are present. Find the percentage of the class that is present, correct to the nearest percent.

7. Raven answered 135 of 150 possible questions on the meteorology examination correctly. What percent of the total number of questions did Raven mark correctly?

8. Liz answered 30 of 50 possible questions on the algebra examination correctly. What percent of the total number of questions did Liz mark correctly?

9. A family has traveled 114 miles of a planned trip. This is 37% of the total distance they must travel on the trip. Find, correct to the nearest mile, the total distance they will travel on their trip.

10. A family has traveled 102 miles of a planned trip. This is 23% of the total distance they must travel on the trip. Find, correct to the nearest mile, the total distance they will travel on their trip.

11. Trudy takes roll in her class at the university and finds that 65 students are present. If this is 50% of the total class enrollment, how many students are in the class?

12. Sandra takes roll in her class at the university and finds that 104 students are present. If this is 80% of the total class enrollment, how many students are in the class?

13. Bill earns a commission on all sales he makes. He sells a bed for $591 and earns a commission of $43. Find the percent commission, rounded to the nearest tenth of a percent.

14. Ira earns a commission on all sales he makes. He sells a sofa for $408 and earns a commission of $39. Find the percent commission, rounded to the nearest tenth of a percent.

15. Tami answered 70% of the questions on the physics examination correctly. If Tami had 98 correct answers, how many questions were on the exam?

16. Trinity answered 90% of the questions on the chemistry examination correctly. If Trinity had 99 correct answers, how many questions were on the exam?

17. A state charges 8% sales tax on all sales. If the sales tax on a computer is $20, find the sales price of the computer, correct to the nearest dollar.

18. A state charges 6.5% sales tax on all sales. If the sales tax on a bed is $33, find the sales price of the bed, correct to the nearest dollar.

19. Kenon earns 6% commission all his sales. If the sale of a computer earns him a $37 commission, find the sales price of the computer, correct to the nearest dollar.

20. Donald earns 4.5% commission all his sales. If the sale of a dryer earns him a $24 commission, find the sales price of the dryer, correct to the nearest dollar.

21. A 23% nitric acid solution contains 59 millilitres of nitric acid. How many total millilitres of solution are present? Round your answer to the nearest millilitre.

22. A 27% sulphuric acid solution contains 67 millilitres of sulphuric acid. How many total millilitres of solution are present? Round your answer to the nearest millilitre.

23. In a state, a television sold for $428 is assessed a sales tax of $45. Find the sales tax rate, rounded to the nearest tenth of a percent.

24. In a state, a refrigerator sold for $503 is assessed a sales tax of $44. Find the sales tax rate, rounded to the nearest tenth of a percent.

---

25. **Mars gravity.** The force of gravity on Mars is only 38% of the force of gravity on earth. If you weigh 150 pounds on earth, how much will you weigh on Mars?

26. **Wiretaps.** In 2008, there were a total of 1,891 applications to federal and state judges to authorize the interception of wire, oral, or electronic communications. If 94% of all wiretap applications were for a portable device such as a cell phone or pager, how many applications were made to tap mobile devices? Round-off to the nearest application. *Associated Press Times-Standard 4/28/09*

27. **Seniors.** 13% of Humboldt County's population is age 65 and older, about 2% more than the state's average. If the population of Humboldt County is approximately 130,000, how many people in Humboldt County are age 65 and older? *Times-Standard 6/10/2009*

28. **Antibiotics.** "The U.S. used about 35 million pounds of antibiotics last year. 70 percent of the drugs went to pigs, chickens, and cows." How many million pounds of antibiotics went to the pigs, chickens, and cows? Associated Press-Times-Standard 12/29/09 Pressure rises to stop antibiotics in agriculture.

29. **Grow faster.** "Approximately 28 million pounds of antibiotics were fed to farm animals in the US during 2008. Thirteen percent of that was fed to healthy animals to make them grow faster." How many pounds of antibiotics were fed to healthy animals? *Associated Press-Times-Standard 12/29/09 Pressure rises to stop antibiotics in agriculture.*

30. **CO2 emissions.** The accord agreed to by the US at the Copenhagen climate talks had greenhouse gas emissions held to 3.5% of 1990 levels. If 1990 levels were 5022 MMT (millions of metric tons), how many millions of metric tons might greenhouse emissions be held to? Round the result to the nearest MMT. *Associated Press-Times-Standard 12/19/09 Elements of new Copenhagen accord.*

31. **Water supply.** A new water desalination plant, the largest in the Western hemisphere, could come online by 2012 in Carlsbad, California, providing 50 million gallons of drinking water per day, or 10% of the supply for San Diego County. What is the total amount of drinking water supplied to San Diego County daily? *Associated Press-Times-Standard*

32. **Earthquake damage.** After the recent earthquake in Chile, an estimated 33 million gallons of Chilean wine, or 13% of annual production, was lost. Estimate the total annual production of Chilean wine rounded to the nearest millions of gallons. *Associated Press-Times Standard 03/24/10 Hemorrhaging cabernet: Earthquake hits winemakers in Chile.*

33. **Snowpack.** At a meadow near Echo Summit in the northern Sierra Nevada, water officials measured the snow at 65.7 inches. The water content was 25.9 inches, which is 92% of the average for this time of year. Determine the average water content for this time of year rounded to the nearest tenth of an inch. *Associated Press-Times Standard 04/02/10 California's Sierra snowpack slightly above normal.*

34. **Storefronts.** According to the Times-Standard, as of April 2008 the Bayshore Mall had 55 occupied storefronts and 17 vacant storefronts. What percent of total storefronts are vacant? Round your answer to the nearest whole number. *Times-Standard 4/19/09*

35. **Recovered.** In Humboldt County, California, 427 of the 499 vehicles stolen between August 2008 and August 2009 were recovered. What percent of the stolen vehicles were recovered? Round your result to the nearest tenth of a percent. *Times-Standard CHP offers tips on avoiding vehicle theft.*

36. **Freshman admissions.** Stanford University sent acceptance letters to 2, 300 of 32, 022 freshman applicants. What percent of freshman applicants got acceptance letters, rounded to the nearest percent? *Associated Press-Times-Standard 03/30/10 Stanford U. reports record-low admission rate.*

37. **Reduce.** Each year, Americans throw out an average of about 1, 600 pounds of waste per person. Arcata, CA resident Michael Winkler only uses one trash bag every year – totaling at most 40 pounds. Find the percent of average annual waste per person Mr. Winkler throws out to a tenth of a percent. *Times-Standard Allison White 12/26/09 Waste not...*

38. **Population decrease.** The table below shows the population of Detroit, Michigan. *Associated Press-Times-Standard 03/09/10 Detroit wants to save itself by shrinking.*

Year	Population
1950	1,849,568
1990	1,027,974
2005	890,963

What is the population of Detroit in 2005 as a percent of the population in 1950? Round your result to the nearest percent.

Answers

1. 12.4

3. 54

5. 56

7. 90

9. 308 mi

11. 130 students

13. 7.3

15. 140

17. $250

19. $617

21. 257 ml

23. 10.5

25. 57 pounds

27. 16,900

29. 3.84 million pounds

31. 500 million gallons

33. The average water content is 28.2 inches.

35. 85.6% of the stolen vehicles were recovered.

37. Mr. Winkler throws out 2.5% of the average American's waste.

This page titled 4.2: Percents in the Real World is shared under a CC BY-NC-SA 3.0 license and was authored, remixed, and/or curated by David Arnold.

- **7.4: General Applicants of Percent** by David Arnold is licensed CC BY-NC-SA 3.0.

4.3: Percent Increase or Decrease

A person's salary can increase by a percentage. A town's population can decrease by a percentage. A clothing firm can discount its apparel. These are the types of applications we will investigate in this section.

Percent Increase

Example 1

Add text here.

Solution

Let x represent the salesperson's salary increase. Then we can translate the problem into words and symbols.

Salary increase	is	5	of	original salary
x	$=$	5%	$\cdot$	4000

Solve for x.

\cdot is only supported in math mode

Therefore, the salary increase is \$200. To compute the new salary N, we must add this increase to the original salary.

New salary	is	original salary	plus	increase
N	$=$	4000	$+$	200

Thus, the new salary is N = \$4, 200 per month.

Alternative Solution

If the salesperson is to receive a 5% increase in his salary, then his new salary will be 105% of his original salary. Let N represent his new monthly salary. Then,

New salary	is	105	of	original salary
N	$=$	105%	$\cdot$	4000

Solve for N.

$$N = 1.05 \cdot 4000 \quad 105\% = 1.05.$$
$$N = 4200 \quad \text{Multiply: } 1.05 \cdot 4000 = 4200.$$

Same answer.

Exercise

A computer technician is granted a 4% salary increase. If the salesperson's current salary is \$2,800 per month, what will be his new salary?

Answer

\$2,912

Example 2

A salesperson making a salary of \$4,500 per month has his salary increased to \$5,000 per month. What is the percent increase?

Solution

To find the increase in salary, first subtract the original salary from the new salary.

$$\begin{aligned}\text{Salary increase} &= \text{new salary} - \text{original salary}\\ &= 5000 - 4500\\ &= 500\end{aligned}$$

Hence, the salesperson sees an increase in salary of $500.

Next, let p represent the salesperson's percent salary increase. Then we can translate the problem into words and symbols.

Salary increase	is	what percent	of	original salary
500	=	p	·	4500

The commutative property of multiplication allows us to change the order of multiplication on the right-hand side of this last equation.

$$500 = 4500p$$

Solve for p.

$$\begin{aligned}\frac{500}{4500} &= \frac{4500p}{4500} && \text{Divide both sides by 4500.}\\ \frac{1}{9} &= p && \text{Reduce by dividing numerator and denominator of 500/4500 by 500.}\end{aligned}$$

We need to change p = 1/9 to a percent. We can find an exact answer by creating an equivalent fraction with a denominator of 100.

$$\begin{aligned}\frac{1}{9} &= \frac{n}{100} && \text{Make equivalent fraction.}\\ 9n &= 100 && \text{Cross multiply.}\\ \frac{9n}{9} &= \frac{100}{9} && \text{Divide both sides by 9.}\\ n &= 11\frac{1}{9} && \text{Convert 100/9 to mixed fraction.}\end{aligned}$$

Hence, the percent increase is

$$p = \frac{1}{9} = \frac{11\frac{1}{9}}{100} = 11\frac{1}{9}\%.$$

Alternative Solution

An alternative approach is to ask what percent of the original salary equals the new salary. In this approach, let p represent the percent of the original salary that equals the new salary.

New salary	is	what percent	of	original salary
5000	=	p	·	4500

Solve for p.

$$\begin{aligned}5000 &= 4500p && \text{Change the order of multiplication.}\\ \frac{5000}{4500} &= \frac{4500p}{4500} && \text{Divide both sides by 4500.}\\ \frac{10}{9} &= p && \text{Reduce: Divide numerator and denominator of 5000/4500 by 500.}\end{aligned}$$

We need to change 10/9 to a percent. Again, create an equivalent fraction with a denominator of 100.

$$\begin{aligned}\frac{10}{9} &= \frac{n}{100} && \text{Make equivalent fraction.}\\ 9n &= 1000 && \text{Cross multiply.}\\ \frac{9n}{9} &= \frac{1000}{9} && \text{Divide both sides by 9.}\\ n &= 111\frac{1}{9} && \text{Convert 1000/9 to a mixed fraction.}\end{aligned}$$

Thus,

$$p = \frac{10}{9} = \frac{111\frac{1}{9}}{100} = 111\frac{1}{9}\%.$$

Hence, the new salary is $111\frac{1}{9}\%$ of the original salary. To find the percent increase, subtract 100% from $111\frac{1}{9}\%$.

$$111\frac{1}{9}\% - 100\% = 11\frac{1}{9}\%$$

This represents an $11\frac{1}{9}\%$ increase in salary, which is the same answer garnered by the first solution technique.

? Exercise

A statistician making a salary of $3,200 per month has his salary increased to $3,368 per month. What is the percent increase?

Answer

$5\frac{1}{4}\%$

Percent Decrease

✓ Example 3

Due to a mill closure, the population of Silvertown decreases by 8.5%. If the original population was 10,200 hardy souls, what is the new population?

Solution

Let x represent the population decrease. Then we can translate the problem into words and symbols.

Population decrease	is	8.5	of	original population
x	$=$	8.5%	$\cdot$	10200

Solve for x.

$$x = 0.085 \cdot 10200 \quad 8.5\% = 0.085.$$
$$x = 867 \quad \text{Multiply: } 0.085 \cdot 10200 = 867.$$

Therefore, the population decrease is 867. To compute the new population P, we must subtract this decrease from the original population.

New population	is	original population	minus	population decrease
P	$=$	10200	$-$	867

Thus, the new population is P = 9,333 hardy souls.

Alternative Solution

Subtract 8.5% from 100% to obtain

$$100\% - 8.5\% = 91.5\%.$$

Thus, if 8.5% of the population leaves town, then 91.5% of the population stays. Thus, the new population P is calculated from the original as follows:

New population	is	91.5	of	original population
P	$=$	91.5%	$\cdot$	10200

Solve for P.

$$P = 0.915 \cdot 10200 \quad 91.5\% = 0.915.$$
$$P = 9333 \quad \text{Multiply: } 0.915 \cdot 10200 = 9333.$$

Same answer.

Exercise

Several retail stores close and the population of Athens decreases by 7.2% as a result. If the original population was 12,500, what is the new population?

Answer

11,600

Example 4

Millertown falls on hard times and its population decreases from 11,256 to 10,923 in the space of one year. What is the percent decrease, rounded to the nearest hundredth of a percent?

Solution

To find the decrease in population, first subtract the current population from the original population.

$$\begin{aligned}\text{Population decrease} &= \text{original population} - \text{current population}\\ &= 11256 - 10923\\ &= 333\end{aligned}$$

Hence, the population has decreased by 333 people.

Next, let p represent the percent population decrease. Then we can translate the problem into words and symbols.

Population decrease	is	what percent	of	original population
333	=	p	·	11256

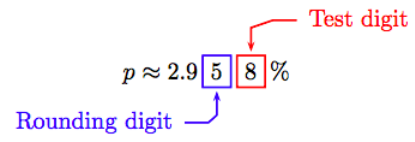
Solve for p.

\approx is only supported in math mode

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 0.02958 \approx 0\,02.958\% \approx 2.958\%.$$

We are asked to round to the nearest hundredth of a percent.



Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate. That is,

$$p \approx 2.96\%.$$

Thus, the population of Millertown decreases approximately 2.96%.

Alternative Solution

An alternative approach is to ask what percent of the original population equals the new population.

New population	is	what percent	of	original population
10923	=	p	·	11256

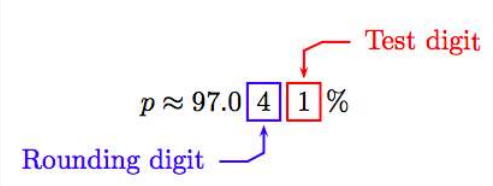
Solve for p.

$$\begin{aligned} 10923 &= 11256p && \text{Change the order of multiplication.} \\ \frac{10923}{11256} &= \frac{11256p}{11256} && \text{Divide both sides by 11256.} \\ 0.97041 &\approx p && \text{Divide: } 10923/11256 \approx 0.97041. \end{aligned}$$

To change p to a percent, move the decimal two places to the right and append a percent symbol.

$$p \approx 0.97041 \approx 0\,97.041\% \approx 97.041\%.$$

We are asked to round to the nearest hundredth of a percent.



Because the test digit is less than 5, leave the rounding digit alone and truncate. That is,

$$p \approx 97.04\%.$$

Thus, 97.04% of the Millertown population remains. To find the percent decrease (the percent who left), subtract 97.04% from 100%.

$$100\% - 97.04\% = 2.96\%$$

Hence, the population of Millertown decreases by 2.96%. Same answer.

? Exercise

A textile mill closure results in the population of the adjacent town decreasing from 8,956 to 7,800. What is the percent decrease in the population, rounded to the nearest tenth of a percent?

Answer

12.9%

Discount

Another important application of percent is the discounting of goods.

✔ Example 5

A pair of skis is marked at $310. However, a sign in the shop indicates that skis are being discounted at 15%. What will be the new selling price of the skis?

Solution

Let D represent the discount (in dollars) given for our pair of skis. Then, in words and symbols:

Discount	is	15	of	original marked price
D	$=$	15%	$\cdot$	310

Solve for D.

$$\begin{aligned} D &= 0.15 \cdot 310 && 15\% = 0.15. \\ D &= 46.50 && \text{Multiply: } 0.15 \cdot 310 = 46.50. \end{aligned}$$

Hence, the discount is $46.50. To find the new selling price, subtract this discount from the original selling price.

$$\begin{aligned}\text{New selling price} &= \text{original selling price} - \text{discount}\\ &= 310 - 46.50\\ &= 263.50\end{aligned}$$

Hence, the new selling price is $263.50.

Alternate Solution

Subtract 15% from 100% to obtain

$$100\% - 15\% = 85\%.$$

That is, if an item is discounted 15%, then its new selling price S is 85% of its original marked price.

New selling price	is	85	of	original marked price
S	$=$	85%	$\cdot$	310

Solve for S.

$$\begin{aligned}S &= 0.85 \cdot 310 && 85\% = 0.85.\\ S &= 263.50 && \text{Multiply: } 0.85 \cdot 310 = 263.50.\end{aligned}$$

Thus, the new selling price is $263.50. Same answer.

Exercise

A pair of hiking boots is marked at $200. During a sale, the boots are discounted by 8%. What is the new price of the boots?

Answer

$184

Example 6

A pair of ski boots marked at $210 is sold for $180. Find the percent discount, correct to the nearest tenth of a percent.

Solution

We can find the discount (in dollars) by subtracting the sale price from the original marked price.

$$\begin{aligned}\text{Discount} &= \text{original marked price} - \text{sale price}\\ &= 210 - 180\\ &= 30\end{aligned}$$

Hence, the boots are discounted $30.

Let p represent the percent discount. Then, in words and symbols:

Discount	is	percent discount	of	original marked price
30	$=$	p	$\cdot$	210

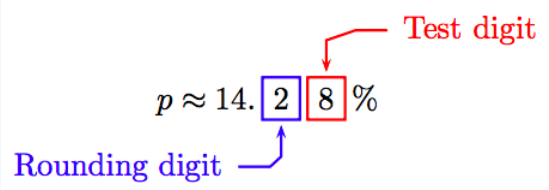
Solve for p.

$$\begin{aligned}30 &= 210p && \text{Change order of multiplication.}\\ \frac{30}{210} &= \frac{210p}{210} && \text{Divide both sides by 210.}\\ \frac{1}{7} &= p && \text{Reduce: Divide numerator and denominator of 30/210 by 30.}\\ p &\approx && \text{Divide: } 1/7 \approx 0.1428.\end{aligned}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 0.1428 \approx 0\,14.28\% \approx 14.28\%.$$

Round to the nearest tenth of a percent.



Because the test digit is greater than or equal to 5, add 1 to the rounding digit and truncate. Thus, correct to the nearest tenth of a percent, the percent discount is

$$p \approx 14.3\%.$$

Alternate Solution

An alternative approach is to ask what percent p of the original marked price equals the selling price.

New selling price	is	what percent	of	original marked price
180	=	p	·	210

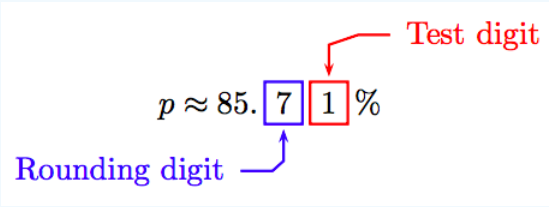
Solve for p.

$$\begin{aligned} 180 &= 210p && \text{Change the order of multiplication.} \\ \frac{180}{210} &= \frac{210p}{210} && \text{Divide both sides by 210.} \\ \frac{6}{7} &= p && \text{Reduce: Divide numerator and denominator of 180/210 by 30.} \\ p &\approx && \text{Divide: } 6/7 \approx 0.8571. \end{aligned}$$

To change p to a percent, move the decimal point two places to the right and append a percent symbol.

$$p \approx 0.8571 \approx 0\,85.71\% \approx 85.71\%.$$

Round to the nearest tenth of a percent.



Because the test digit is less than 5, do nothing to the rounding digit and truncate. Thus, correct to the nearest tenth of a percent,

$$p \approx 85.7\%.$$

Thus, the new selling price is 85.7% of the original marked price. Subtract 85.7% from 100%.

$$100\% - 85.7\% = 14.3\%.$$

That is, if the new selling price is 85.7% of the original price, then the percent discount is 14.3%. This is the same answer found with the first method.

Exercise

A computer marked at $2,000 is sold at a discount for $1,850. Find the percent discount, correct to the nearest tenth of a percent.

Answer

7.5%

Exercises

1. A television set is marked at $447. However, a sign in the shop indicates that the television set is being discounted at 20.5%. What will be the new selling price of the television set? Round your answer to the nearest penny.

2. A stereo set is marked at $380. However, a sign in the shop indicates that the stereo set is being discounted at 7.5%. What will be the new selling price of the stereo set? Round your answer to the nearest penny.

3. Due to a ball bearing plant closure, Anselm falls on hard times and its population decreases from 10,794 to 8,925 in the space of one year. What is the percent decrease, rounded to the nearest hundredth of a percent?

4. Due to a logging mill closure, Carlytown falls on hard times and its population decreases from 12,113 to 10,833 in the space of one year. What is the percent decrease, rounded to the nearest hundredth of a percent?

5. A car rack is marked at $500. However, a sign in the shop indicates that the car rack is being discounted at 3.5%. What will be the new selling price of the car rack? Round your answer to the nearest penny.

6. A car rack is marked at $405. However, a sign in the shop indicates that the car rack is being discounted at 17.5%. What will be the new selling price of the car rack? Round your answer to the nearest penny.

7. Due to a textile mill closure, the population of Silvertown decreases by 4.1%. If the original population was 14,678 hardy souls, what is the new population, correct to the nearest person?

8. Due to a department store closure, the population of Petroria decreases by 5.3%. If the original population was 14,034 hardy souls, what is the new population, correct to the nearest person?

9. A bartender is granted a 4.6% salary increase. If the bartender's current salary is $2,500 per month, find the bartender's new monthly salary, rounded to the nearest dollar.

10. A bartender is granted a 5.5% salary increase. If the bartender's current salary is $2,900 per month, find the bartender's new monthly salary, rounded to the nearest dollar.

11. A car rack marked at $358 is sold for $292. Find the percent discount, correct to the nearest tenth of a percent.

12. A bicycle marked at $328 is sold for $264. Find the percent discount, correct to the nearest tenth of a percent.

13. Due to a auto manufacturing plant closure, Carlytown falls on hard times and its population decreases from 14,393 to 12,623 in the space of one year. What is the percent decrease, rounded to the nearest hundredth of a percent?

14. Due to a ball bearing plant closure, Mayville falls on hard times and its population decreases from 8,494 to 6,609 in the space of one year. What is the percent decrease, rounded to the nearest hundredth of a percent?

15. Due to a auto manufacturing plant closure, the population of Silvertown decreases by 2.4%. If the original population was 8,780 hardy souls, what is the new population, correct to the nearest person?

16. Due to a textile mill closure, the population of Ghosttown decreases by 6.1%. If the original population was 14,320 hardy souls, what is the new population, correct to the nearest person?

17. A clerk making a salary of $2,600 per month has her salary increased to $2,950 per month. Find the percent increase correct to the nearest tenth of a percent.

18. A clerk making a salary of $3,600 per month has her salary increased to $4,100 per month. Find the percent increase correct to the nearest tenth of a percent.

19. A bartender making a salary of $4,200 per month has her salary increased to $4,300 per month. Find the percent increase correct to the nearest tenth of a percent.

20. A bartender making a salary of $3,200 per month has her salary increased to $3,550 per month. Find the percent increase correct to the nearest tenth of a percent.

21. A gardener is granted a 5.1% salary increase. If the gardener's current salary is $3,200 per month, find the gardener's new monthly salary, rounded to the nearest dollar.

22. A secretary is granted a 2.8% salary increase. If the secretary's current salary is $3,600 per month, find the secretary's new monthly salary, rounded to the nearest dollar.

23. A television set marked at $437 is sold for $347. Find the percent discount, correct to the nearest tenth of a percent.

24. A camera marked at $390 is sold for $328. Find the percent discount, correct to the nearest tenth of a percent.

---

25. Suppose that the price of an 8-ounce can of tomato sauce increased from $0.20 to $0.28.

    a) What was the amount of increase?

    b) What was the percent increase?

26. The following table summarizes summertime gasoline prices in San Francisco, CA. The price is the number of dollars required to purchase one gallon of unleaded gasoline. *Data from gasbuddy.com.*

Year	Price per gallon
2003	1.80
2004	2.28
2005	2.57
2006	3.20
2007	3.28
2008	4.61
2009	3.01

What is the percent increase or decrease from 2003 to 2005? Round your answer to the nearest whole percent.

27. Refer to the table of gas prices in Exercise 26. What is the percent increase or decrease from 2005 to 2007? Round your answer to the nearest whole percent.

28. Refer to the table of gas prices in Exercise 26. What is the percent increase or decrease from 2007 to 2009? Round your answer to the nearest whole percent.

29. **Rate hike.** A family paying $858 monthly for health coverage is faced with a 39% hike in rates. What will be their new monthly cost after the increase? *Associated Press-Times-Standard 02/09/10 HHS secretary asks Anthem Blue Cross to justify rate hike.*

30. **Parking fine.** San Francisco's Metropolitan Transportation Agency was expected to consider raising fines for the use of fake, stolen, or borrowed disabled parking tags from $100 to $825. What is the percent increase for this fine? *Associated Press-Times-Standard 01/06/10 Fines for fake disabled parking tags may go up in San Francisco.*

31. **Industrial move.** Regulations in California have caused factory farmers to move out of state. Idaho's industrial dairy went from 180, 000 cows in 1990 to 530, 000 cows in 2009. What is the percent increase for Idaho industrial dairy cows, rounded to the nearest percent? *Associated Press-Times-Standard 02/09/10 Idaho, others prepare for California egg farm exodus.*

32. **Whooping crane.** Whooping crane populations made a remarkable comeback from just 15 birds in 1941 to about 400 birds worldwide in 2010. What is the percent increase for the whooping crane population over the past seventy years, rounded to the nearest percent? *Associated Press-Times-Standard 03/17/10 Plucky whooping crane gives wildlife experts hope.*

33. **Underwater.** The loss of Arctic sea ice will allow for an underwater fiber optic cable that will cut the time it takes to send a message from London to Tokyo from a current 140 milliseconds down to 88 milliseconds. *Associated Press-Times-Standard 01/22/10 Global warming opens up Northwest Passage for underwater fiber optic cable.*

    a) What is the estimated percent decrease in communication time?

    b) At a cost of $1.2 billion, what is the cost per millisecond of saving (rounded to the nearest dollar)?

34. **Chinook salmon.** During the 2009 season in the Sacramento river basin, a record low 39,500 chinook salmon were recorded, way down from the more than 750,000 counted in 2002. What is the percent decrease in the salmon count from 2002 to 2009, rounded to the nearest percent? *Associated Press-Times-Standard 02/25/10 Feds predict better year for California salmon.*

35. **Daylight hours.** In middle latitudes, summer days can have as many as 14 hours of daylight, while winter days can have a few as 10 hours of daylight. What percent more daylight is there in summer than in winter?

36. **Cyber-experts.** Defense Secretary Robert Gates said the Pentagon will increase the number of cyber-experts it can train each year from 80 to 250 by 2011. What percent increase is this? Round your answer to the nearest percent. *Associated Press Times-Standard 4/19/09*

37. **Home prices.** Real estate data for Humboldt County, California, is given below. *Associated Press-Times-Standard 01/17/10 How is our local real estate market compared to other regions?*

Year	Number of homes sold	Average home price
2000	1,358	$152,257
2005	1,432	$344,500
2009	833	$285,000

a) What percent change in average home price occurred between 2000 and 2009?

b) What percent change in homes sold occurred between 2000 and 2009?

Answers

1. $355.36

3. 17.32%

5. $482.50

7. 14,076 people

9. $2,615

11. 18.4%

13. 12.30%

15. 8,569

17. 13.5%

19. 2.4%

21. $3,363

23. 20.6%

25.

a) $0.08

b) 40%

27. 28% increase

29. $1,193

31. 194% increase

33.

a) 37%

b) About $23,076,923 per millisecond

35. 40% more daylight

37.

a) 87% increase in average home price

b) 39% decrease in home sales

This page titled 4.3: Percent Increase or Decrease is shared under a CC BY-NC-SA 3.0 license and was authored, remixed, and/or curated by David Arnold.

- **7.5: Percent Increase or Decrease** by David Arnold is licensed CC BY-NC-SA 3.0.

CHAPTER OVERVIEW

5: Units of Measure and Converting Units

5.1: Scientific and Engineering Notation
5.2: Accuracy and Significant Figures
5.3: The US Measurement System
5.4: Converting Units of Measure/ Dimensional Analysis
5.5: The Metric System
5.6: Converting Between Systems
5.7: Conversions with Time, Rate, Area and Volume

5: Units of Measure and Converting Units is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

1.5: Scientific and Engineering Notation



Powers of Ten

Decimal notation is based on powers of 10: 0.1 is $\frac{1}{10^1}$, 0.01 is $\frac{1}{10^2}$, 0.001 is $\frac{1}{10^3}$, and so on.

We represent these powers with negative exponents: $\frac{1}{10^1} = 10^{-1}$, $\frac{1}{10^2} = 10^{-2}$, $\frac{1}{10^3} = 10^{-3}$, etc.

Negative exponents: $\frac{1}{10^n} = 10^{-n}$

Note: This is true for any base, not just 10*, but we will focus only on* 10 *in this course.*

With our base 10 number system, any power of 10 can be written as a 1 in a certain decimal place.

10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
$10,000$	$1,000$	100	10	1	0.1	0.01	0.001	0.0001

If you haven't watched the video "Powers of Ten" from 1977 on YouTube, take ten minutes right now and check it out. Your mind will never be the same again.

Scientific Notation

Let's consider how we could rewrite some different numbers using these powers of 10.

Let's take $50,000$ as an example. $50,000$ is equal to $5 \times 10,000$ or 5×10^4.[1]

Looking in the other direction, a decimal such as 0.0007 is equal to 7×0.0001 or 7×10^{-4}.

The idea behind scientific notation is that we can represent very large or very small numbers in a more compact format: a number between 1 and 10, multiplied by a power of 10.

A number is written in scientific notation if it is written in the form $a \times 10^n$, where n is an integer and a is any real number such that $1 \leq a < 10$.

Note: An ***integer*** *is a number with no fraction or decimal part:* $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

? Exercise 1.5.1

1. The mass of the Earth is approximately $5,970,000,000,000,000,000,000,000$ kilograms. The mass of Mars is approximately $639,000,000,000,000,000,000,000$ kilograms. Can you determine which mass is larger?

Answer

Clearly, it is difficult to keep track of all those zeros. Let's rewrite those huge numbers using scientific notation.

Earth's mass is larger because it's a 25-digit number and Mars' mass is a 24-digit number, but it might take a lot of work counting the zeros to be sure.

? Exercise 1.5.2

2. The mass of the Earth is approximately 5.97×10^{24} kilograms. The mass of Mars is approximately 6.39×10^{23} kilograms. Can you determine which mass is larger?

Answer

Earth's mass is about ten times larger, because the power of 10 is 1 higher than that of Mars.

It is much easier to compare the powers of 10 and determine that the mass of the Earth is larger because it has a larger power of 10. You may be familiar with the term ***order of*** magnitude; this simply refers to the difference in the powers of 10 of the two numbers. Earth's mass is one order of magnitude larger because 24 is 1 more than 23.

We can apply scientific notation to small decimals as well.

? Exercise 1.5.3

3. The radius of a hydrogen atom is approximately 0.000000000053 meters. The radius of a chlorine atom is approximately 0.00000000018 meters. Can you determine which radius is larger?

Answer

A chlorine atom's radius is larger because it has 9 zeros before the significant digits begin, but a hydrogen atom's radius has 10 zeros before the significant digits begin. As above, counting the zeros is a pain in the neck.

Again, keeping track of all those zeros is a chore. Let's rewrite those decimal numbers using scientific notation.

? Exercise 1.5.4

4. The radius of a hydrogen atom is approximately 5.3×10^{-11} meters. The radius of a chlorine atom is approximately 1.8×10^{-10} meters. Can you determine which radius is larger?

Answer

The chlorine atom has a larger radius because its power of 10 is 1 higher than that of the hydrogen atom. (Remember that -10 is larger than -11 because -10 is farther to the right on a number line.)

The radius of the chlorine atom is larger because it has a larger power of 10; the digits 1 and 8 for chlorine begin in the tenth decimal place, but the digits 5 and 3 for hydrogen begin in the eleventh decimal place.

Scientific notation is very helpful for really large numbers, like the mass of a planet, or really small numbers, like the radius of an atom. It allows us to do calculations or compare numbers without going cross-eyed counting all those zeros.

? Exercises 1.5.5

Write each of the following numbers in scientific notation.

5. $1,234$

6. $10,200,000$

7. 0.00087

8. 0.0732

Convert the following numbers from scientific notation to standard decimal notation.

9. 3.5×10^4

10. 9.012×10^7

11. 8.25×10^{-3}

12. 1.4×10^{-5}

Answer

5. 1.234×10^3
6. 1.02×10^7
7. 8.7×10^{-4}
8. 7.32×10^{-2}
9. $35,000$
10. $90,120,000$
11. 0.00825
12. 0.000014

You may be familiar with a shortcut for multiplying numbers with zeros on the end; for example, to multiply $300 \times 4,000$, we can multiply the significant digits $3 \times 4 = 12$ and count up the total number of zeros, which is five, and write five zeros on the back end of the 12: $1,200,000$. This shortcut can be applied to numbers in scientific notation.

To multiply powers of 10, add the exponents: $10^m \cdot 10^n = 10^{m+n}$

? Exercises 1.5.6

Multiply each of the following and write the answer in scientific notation.

13. $(2 \times 10^3)(4 \times 10^4)$

14. $(5 \times 10^4)(7 \times 10^8)$

15. $(3 \times 10^{-2})(2 \times 10^{-3})$

16. $(8 \times 10^{-5})(6 \times 10^9)$

Answer

13. 8×10^7
14. 3.5×10^{13}
15. 6×10^{-5}
16. 4.8×10^5

When the numbers get messy, it's probably a good idea to use a calculator. If you are dividing numbers in scientific notation with a calculator, you may need to use parentheses carefully.

? Exercises 1.5.7

The mass of a proton is 1.67×10^{-27} kg. The mass of an electron is 9.11×10^{-31} kg.

17. Divide these numbers using a calculator to determine approximately how many times greater the mass of a proton is than the mass of an electron.

18. What is the approximate mass of one million protons? (Note: one million is 10^6.)

19. What is the approximate mass of one billion protons? (Note: one billion is 10^9.)

Answer

17. the proton's mass is roughly $1,830$ or 1.83×10^3 times larger

18. 1.67×10^{-21} kg

19. 1.67×10^{-18} kg

Engineering Notation

Closely related to scientific notation is engineering notation, which uses only multiples of $1,000$. This is the way large numbers are often reported in the news; if roughly $37,000$ people live in Oregon City, we say "thirty-seven thousand" and we might see it written as "37 thousand"; it would be unusual to think of it as $3.7 \times 10,000$ and report the number as "three point seven ten thousands".

One thousand = 10^3, one million = 10^6, one billion = 10^9, one trillion = 10^{12}, and so on.

In engineering notation, the power of 10 is always a multiple of 3, and the other part of the number must be between 1 and $1,000$.

A number is written in engineering notation if it is written in the form $a \times 10^n$, where n is a multiple of 3 and a is any real number such that $1 \leq a < 1,000$.

Note: Prefixes for large numbers such as kilo, mega, giga, and tera are essentially engineering notation, as are prefixes for small numbers such as micro, nano, and pico. We'll see these next.

? Exercises 1.5.8

Write each number in engineering notation, then in scientific notation.

20. The U.S. population is around 330.2 million people.[2]

21. The world population is around 7.68 billion people.[3]

22. The U.S. national debt is around 26.6 trillion dollars.[4]

Answer

20. 330.2×10^6; 3.302×10^8

21. 7.68×10^9; 7.68×10^9 \

22. (26.6 \times 10^{12}\); 2.66×10^{13}

Measurement Prefixes: Larger

Now let's turn our attention to converting units based on their prefixes. We'll start with large units of measure.

tera- (T)	giga- (G)	mega- (M)	kilo- (k)	**[base unit]**
trillion	billion	million	thousand	one
$1,000,000,000,000$	$1,000,000,000$	$1,000,000$	$1,000$	1
10^{12}	10^9	10^6	10^3	10^0

Notice that the powers of these units are multiples of 3, just as with the engineering notation we saw in a previous module. Each prefix is $1,000$ times the next smaller prefix, so moving one place in the chart means moving the decimal point three places. Also notice that capitalization is important; megagram (which is also called a metric ton) is Mg with a capital M, but milligram is mg with a lowercase m.

Using computer memory as an example:

1 kilobyte = $1,000$ bytes

1 megabyte = 1,000 kilobytes = 1,000,000 bytes

1 gigabyte = 1,000 megabytes = 1,000,000 kilobytes, etc.

1 terabyte = 1,000 gigabytes = 1,000,000 megabytes, etc.

Note: There can be inconsistencies with different people's understanding of these prefixes with regards to computer memory, which is counted in powers of 2, not 10. Computer engineers originally defined 1 kilobyte as 1,024 bytes because $2^{10} = 1,024$, which is very close to 1,000. However, we will consider these prefixes to be powers of 1,000, not 1,024. There is an explanation at https://physics.nist.gov/cuu/Units/binary.html.

? Exercises 1.5.1



23. A $5\frac{1}{4}$ inch floppy disk from the 1980s could store about 100 kB; a $3\frac{1}{2}$ inch floppy disk from the 1990s could store about 1.44 MB. By what factor was the storage capacity increased?

24. How many times greater is the storage capacity of a 2 TB hard drive than a 500 GB hard drive?

25. In an article describing small nuclear reactors that are designed to be retrofitted into coal plants, Dr. Jose Reyes of Oregon State University says "One module will produce 60 megawatts of electricity. That's enough for about 50 thousand homes."[5] How much electricity per home is this?

26. In the same article, Dr. Reyes says "a 60 megawatt module could produce about 60 million gallons of clean water per day using existing technologies in reverse osmosis." What is the rate of watts per gallon?

27. The destructive power of nuclear weapons is measured in kilotons (the equivalent of 1,000 tons of TNT) or megatons (the equivalent of 1,000,000 tons of TNT). The first nuclear device ever tested, the US's *Trinity*, was measured at roughly 20 kilotons on July 16, 1945. The largest thermonuclear weapon ever detonated, at 50 megatons, was the USSR's *Tsar Bomba*, on October 31, 1961.[6] (Video of *Tsar Bomba* was declassified almost 60 years later, in August 2020.) How many times more powerful was *Tsar Bomba* than *Trinity*?

Answer

23. the capacity increased by a factor of 14.4

24. 4 times greater

25. 1,200 megawatts per home

26. 1 watt per gallon

27. 2,500 times more powerful

Measurement Prefixes: Smaller

Now we'll go in the other direction and look at small units of measure.

[base unit]	milli- (m)	micro- (μ or mc)	nano- (n)	pico (p)
one	thousandth	millionth	billionth	trillionth
1	0.001	0.000001	0.000000001	0.000000000001
10^0	10^{-3}	10^{-6}	10^{-9}	10^{-12}

The symbol for micro- is the Greek letter μ (pronounced “myoo”). Because this character can be difficult to replicate, you may see the letter “u” standing in for “μ” in web-based or plaintext technical articles… or you may see the prefix “mc” instead.

Again, the powers are multiples of 3; each prefix gets smaller by a factor of $\frac{1}{1000}$. The negative exponents can sometime be complicated to work with, and it may help to think about things in reverse.

1 meter = 10^3 millimeters = 10^6 micrometers = 10^9 nanometers = 10^{12} picometers

1 second = 10^3 milliseconds = 10^6 microseconds = 10^9 nanoseconds = 10^{12} picoseconds

…and so on.

See https://physics.nist.gov/cuu/Units/prefixes.html for a list of more prefixes.

? Exercises 1.5.1

28. An article about network latency compares the following latency times: “So a 10 Mbps link adds 0.4 milliseconds to the RTT, a 100 Mbps link 0.04 ms and a 1 Gbps link just 4 microseconds.”[7] Rewrite these times so that they are all in terms of milliseconds, then rewrite them in terms of microseconds.

29. The wavelength of red light is around 700 nm. Infrared radiation has a wavelength of approximately 10 μm.[8] Find the ratio of these wavelengths.

30. Nuclear radiation is measured in units called Sieverts, but because this unit is too large to be practical when discussing people’s exposure to radiation, milliSieverts and microSieverts are more commonly used. In 1986, workers cleaning up the Chernobyl disaster were exposed to an estimated dose of 250 mSv.[9] A typical chest x-ray exposes a person to 100 μSv.[10] How many chest x-rays’ worth of radiation were the workers exposed to?

Answer

28. 0.4 ms, 0.04 ms, 0.004 ms; 400 μs, 40 μs, 4 μs the ratio of the wavelengths of red and infrared is 7 to 100;
29. the ratio of the wavelengths of infrared and red is around 14 to 1
30. this is equivalent to 2,500 chest x-rays

1. For some reason, although we generally try to avoid using the "x" shaped multiplication symbol, it is frequently used with scientific notation. ↵
2. August 27, 2020 estimate from *https://www.census.gov/popclock/*↵
3. August 27, 2020 estimate from *https://www.census.gov/popclock/*↵
4. August 27, 2020 data from *fiscaldata.treasury.gov/datasets/debt-to-the-penny/debt-to-the-penny*↵
5. https://www.kgw.com/article/news/local/oregon-company-get-approval-to-build-nuclear-power-plants/283-7b26b8cd-12d5-4116-928a-065731f7a0f6 ↵
6. https://en.Wikipedia.org/wiki/Nuclear_weapon_yield ↵
7. https://www.noction.com/blog/network-latency-effect-on-application-performance ↵
8. http://labman.phys.utk.edu/phys222core/modules/m6/The%20EM%20spectrum.html ↵
9. https://en.Wikipedia.org/wiki/Chernobyl_disaster ↵
10. https://www.cancer.org/treatment/understanding-your-diagnosis/tests/understanding-radiation-risk-from-imaging-tests.html ↵

This page titled 1.5: Scientific and Engineering Notation is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.11: Scientific Notation** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **1.16: Other Conversions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

5.2: Accuracy and Significant Figures



Photo by Tudor Barker on flickr.

Every measured quantity contains some error. A standard sheet of paper is 8.5 inches wide and 11 inches high, but it's possible that the actual measurements could be closer to 8.4999 and 11.0001 inches. Even if we measure something very carefully, with very sensitive instruments, we should assume that there could be some small measurement error.[1]

Exact Values and Approximations

A number is an ***exact value*** if it is the result of counting or a definition.

A number is an ***approximation*** if it is the result of a measurement or of rounding.

? Exercises

Identify each number as an exact value or an approximation.

1. An inch is $\frac{1}{12}$ of a foot.
2. This board is 78 inches long.
3. There are 14 students in class.
4. A car's tachometer reads $3,000$ rpm.
5. A right angle measures $90°$.
6. The angle of elevation of a ramp is $4°$.

Accuracy and Significant Figures



African-American women were vital to NASA's success in the 1960s, as shown in the movie *Hidden Figures*. Photo by NASA/Kim Shiflett on flickr.

Because measurements are inexact, we need to consider how accurate they are. This requires us to think about ***significant figures*** —often abbreviated "sig figs" in conversation—which are the digits in the measurement that we trust to be correct. The ***accuracy*** of a number is equal to the number of significant figures. (By the way, the terms "significant digits" and "significant figures" are used interchangeably.) The following rules aren't particularly difficult to understand but they can take time to absorb and internalize, so we'll include lots of examples and exercises.

Significant Figures

1. All nonzero digits are significant.
   Ex: $12,345$ has five sig figs, and 123.45 has five sig figs.

2. All zeros between other nonzero digits are significant.
   Ex: $10,045$ has five sig figs, and 100.45 has five sig figs.
3. Any zeros to the right of a decimal number are significant.
   Ex: 123 has three sig figs, but 123.00 has five sig figs.
4. Zeros on the left of a decimal number are NOT significant.
   Ex: 0.123 has three sig figs, and 0.00123 has three sig figs.
5. Zeros on the right of a whole number are NOT significant unless they are marked with an overbar.
   Ex: $12,300$ has three sig figs, but $12,30\overline{0}$ has five sig figs.

Another way to think about #4 and #5 above is that zeros that are merely showing the place value—where the decimal point belongs—are NOT significant.

Exercises

Determine the accuracy (i.e., the number of significant figures) of each number.

7. $63,400$
8. $63,040$
9. $63,004$
10. 0.085
11. 0.0805
12. 0.08050



Mt. Everest, Lohtse, and Nupse in the early morning. Photo by Ralf Kayser on flickr.

In 1856, the first official measurement of the height of Mount Everest—called *Sagarmatha* in Nepal and *Chomolungma* in Tibet—was announced. The height was determined to be exactly $29,000$ feet, but there was concern that people would think this was only a rough estimate rounded to the nearest thousand feet. Therefore, the height was announced as $29,002$ feet, so that everyone seeing that number would believe that the measurement was correct to the nearest foot.[2] Yes, to demonstrate the correctness of the measurement, an incorrect measurement was announced.

Instead of fudging a number like $29,000$ to show that it is correct to the nearest foot, we can write it with an an ***overbar*** to indicate that the zeros are significant. Putting $29,00\overline{0}$ in a newspaper headline in 1856 would probably have confused people, but you can handle it because you're in a math class. Writing $29,00\overline{0}$ is our way of saying "Really, to the nearest foot, it's exactly $29,000$ feet!"

Exercises

Determine the accuracy (i.e., the number of significant figures) of each number.

13. $29,000$
14. $29,\overline{0}00$
15. $29,0\overline{0}0$
16. $29,00\overline{0}$

Two things to remember: we don't put an overbar over a nonzero digit, and we don't need an overbar for any zeros on the right of a decimal number because those are already understood to be significant.

Accuracy-Based Rounding

It is often necessary to round a number. We often round to a certain place value, such as the nearest hundredth, but there is another way to round. ***Accuracy-based rounding*** considers the number of significant figures rather than the place value.

Accuracy-based rounding:

1. Locate the **rounding digit** to which you are rounding by counting from the left until you have the correct number of significant figures.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

? Exercises

Round each number so that it has the indicated number of significant figures.

17. $51,837$ (three sig figs)
18. $51,837$ (four sig figs)
19. 4.2782 (two sig figs)
20. 4.2782 (three sig figs)

When the rounding digit of a whole number is a 9 that gets rounded up to a 0, we must write an overbar above that 0.

Similarly, when the rounding digit of a decimal number is a 9 that gets rounded up to a 0, we must include the 0 in that decimal place.

? Exercises

Round each number so that it has the indicated number of significant figures. Be sure to include trailing zeros or an overbar if necessary.

21. $13,997$ (two sig figs)
22. $13,997$ (three sig figs)
23. 2.596 (two sig figs)
24. 2.596 (three sig figs)

The height of Mount Everest has changed over the years due to plate tectonics and earthquakes. In December 2020, it was jointly announced by Nepal and China that the summit of Mount Everest has an elevation of $29,031.69$ ft.[3]

25. Round $29,031.69$ ft to two sig figs.
26. Round $29,031.69$ ft to three sig figs.
27. Round $29,031.69$ ft to four sig figs.
28. Round $29,031.69$ ft to five sig figs.
29. Round $29,031.69$ ft to six sig figs.

Accuracy when Multiplying and Dividing

Suppose you needed to square the number $3\frac{1}{3}$. You could rewrite $3\frac{1}{3}$ as the improper fraction $\frac{10}{3}$ and then figure out that $(\frac{10}{3})^2 = \frac{100}{9}$, which equals the repeating decimal $11.111\ldots$

Because most people prefer decimals to fractions, we might decide to round $3\frac{1}{3}$ to 3.33 and find that $3.33^2 = 11.0889$. The answer 11.0889 looks very accurate, but it is a false accuracy because there is round-off error involved. Only when we round to three sig figs do we get an accurate result: 11.0889 rounded to three sig figs is 11.1, which is accurate because $11.111\ldots$ rounded to three sig

figs is also **11.1**. It turns out that because **3.33** has only three significant figures, our answer must be rounded to three significant figures.

When ***multiplying or dividing*** approximate numbers, the answer must be rounded to the same number of significant figures as the ***least*** accurate of the original numbers.

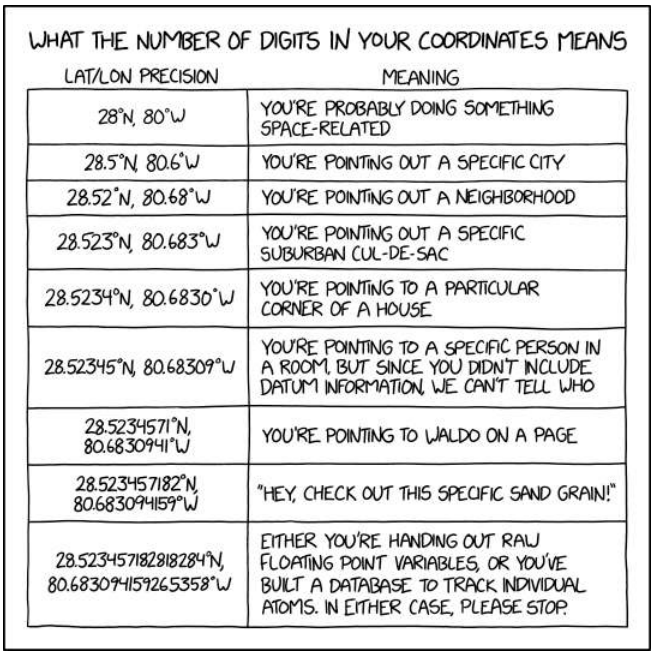
Don't round off the original numbers; do the necessary calculations first, then round the answer as your last step.

? Exercises

Use a calculator to multiply or divide as indicated. Then round to the appropriate level of accuracy.

30. $8.75 \cdot 12.25$
31. $355.12 \cdot 1.8$
32. $77.3 \div 5.375$
33. $53.2 \div 4.5$
34. Suppose you are filling a 5-gallon can of gasoline. The gasoline costs $\$4.579$ per gallon, and you estimate that you will buy 5.0 gallons. How much should you expect to spend?

Bonus material: Here is a comic strip from xkcd.com showing that including a lot of decimal digits can give a false sense of accuracy.



From the web comic xkcd.

1. Here is an example of a speed camera with gigantic measurement error: https://youtube.com/shorts/cCYy29RuhV8↵
2. Source: https://en.Wikipedia.org/wiki/Mount_Everest#cite_ref-tas1982_33-0↵
3. Source: https://www.washingtonpost.com/world/asia_pacific/mount-everest-height-nepal-china/2020/12/08/a7b3ad1e-389a-11eb-aad9-8959227280c4_story.html↵

5.2: Accuracy and Significant Figures is shared under a CC BY-NC-SA 4.0 license and was authored, remixed, and/or curated by LibreTexts.

5.3: The US Measurement System

Measurement

Measurement is a number that describes the size or amount of something. You can measure many things like length, area, capacity, weight, temperature and time. In the United States, two main systems of measurement are used: **U.S. customary measurement system** and the **metric system**. This topic addresses the measurement of length using the U.S. customary measurement system.

There are two important skills relating to units of measure. The first is a general knowledge of how much a unit of measure represents. For example, most US school children can visualize 12 inches as a length of a typical classroom ruler. If you have used hand weights or bought a bag of flour, you have a general sense of what 5 pounds feels like. And most of us can visualize a one gallon jug of milk.

The second important skill is the ability to change the units being used to describe a quantity. This may be called doing unit conversions. For example, you may know your height in both inches and feet. Juan is 6 feet tall which means Juan is 72 inches tall. 6 feet and 72 inches describe the same quantity, Juan's height, just using two different units of measure.

In order to convert from one unit of measure to another you will need some unit equivalencies or unit ratios or conversion ratios. In the previous example with Juan's height, the unit ratio used was 1 foot = 12 inches. There are A LOT of both exact and approximate unit ratios for measurements. It's okay if you don't have the conversion ratios memorized; just be sure to have them available. If you discover other conversion ratios that aren't provided here, go ahead and write them down!

U.S. System: Measurements of Length

Length is the distance from one end of an object to the other end, or from one object to another. For example, the length of a letter-sized piece of paper is 11 inches. The system for measuring length in the United States is based on the four customary units of length: **inch**, **foot**, **yard**, and **mile**.

1 foot = 12 inches

1 yard = 3 feet

1 mile = $5,280$ feet

? Exercises 5.3.1

1. How many inches are in 4.5 feet?

2. How many feet make up 18 yards?

3. 1 yard is equal to how many inches?

4. 1 mile is equivalent to how many yards?

5. How many feet is 176 inches?

6. 45 feet is what length in yards?

7. Convert $10,560$ feet into miles.

8. How many yards are the same as $1,080$ inches?

Answer

1. 54 in
2. 54 ft
3. 36 in
4. $1,760$ yd
5. $14\frac{2}{3}$ ft or 14 ft 8 in

6. 15 yd
7. 2 mi
8. 30 yd

Notice that Exercises 3 & 4 give us two more conversion ratios that we could add to our list.

U.S. System: Measurements of Weight or Mass

In everyday usage, **weight and mass** are often used interchangeably to mean how heavy an object is. If you are studying physics or engineering, you will learn that there are differences between weight and mass but we do not need focus on those differences in this course. In the US Customary System, we measure weight in **ounces** and **pounds.**

1 pound = 16 ounces

1 ton = 2,000 pounds

? Exercises 5.3.1

9. How many ounces are in 2.5 pounds?

10. How many pounds are equivalent to 1.2 tons?

11. Convert 300 ounces to pounds.

12. 1 ton is equivalent to what number of ounces?

Answer

9. 40 oz
10. 2,400 lb
11. 18.75 lb
12. 32,000 oz

U.S. System: Measurements of Volume or Capacity

Volume and capacity are also often used interchangeably to measure of how much space an object takes up or how much space is inside an object. In the US Customary System, we measure capacity using many different units such as **fluid ounces, cups** and **gallons.** Volume or capacity may also be measure in cubic units like cubic inches or cubic feet. More on that later when we study geometry.

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

There are plenty of other conversions that could be provided, such as the number of fluid ounces in a gallon, but let's keep the list relatively short.

? Exercises 5.3.1

13. How many fluid ounces are in 6 cups?

14. How many pints are in 3.5 quarts?

15. 1 gallon is equal to how many pints?

16. How many cups equal 1.25 quarts?

17. Convert 20 cups into gallons.

18. How many fluid ounces are in one half gallon?

Answer

13. 48 fl oz
14. 7 pt
15. 8 pt
16. 5 c
17. 1.25 gal
18. 64 fl oz

U.S. System: Using Mixed Units of Measurement

Measurements are frequently given with mixed units, such as a person's height being given as 5 ft 7 in instead of 67 in, or a newborn baby's weight being given as 8 lb 3 oz instead of 131 oz. This can sometimes make the calculations more complicated, but if you can convert between improper fractions and mixed numbers, you can handle this.

? Exercises 5.3.1

19. A bag of apples weighs 55 ounces. What is its weight in pounds and ounces?

20. A carton of orange juice contains 59 fluid ounces. Determine its volume in cups and fluid ounces.

21. A hallway is 182 inches long. Give its length in feet and inches.

22. The maximum loaded weight of a Ford F-150 pickup truck is $8,500$ lb. Convert this weight into tons and pounds.

Answer

19. 3 lb 7 oz
20. 7 c 3 fl oz
21. 15 ft 2 in
22. 4 t 500 lb

We'll finish up this module by adding and subtracting with mixed units. Again, it may help to think of them as mixed numbers, with a whole number part and a fractional part.

? Exercises 5.3.1



Comet weighs 8 lb 7 oz and Fred weighs 11 lb 9 oz.

23. Comet and Fred are being put into a cat carrier together. What is their combined weight?

24. How much heavier is Fred than Comet?

Two tables are 5 ft 3 in long and 3 ft 10 in long.

25. If the two tables are placed end to end, what is their combined length?

26. What is the difference in length between the two tables?

Answer

23. **20** lb or **20** lb **0** oz combined

24. **3** lb **2** oz heavier

25. **9** ft **1** in combined

26. **1** ft **5** in longer

Throughout this section, most of the unit conversions were done using a single multiplication or division operation. In the coming sections, as the conversions become more complex, we will learn an important conversion technique called *dimensional analysis*.

This page titled 5.3: The US Measurement System is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **6.1.1: Length** by The NROC Project is licensed CC BY-NC-SA 4.0. Original source: https://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html.
- **1.13: The US Measurement System** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

5.4: Converting Units of Measure/ Dimensional Analysis

Dimensional Analysis

Remember you will need to use unit ratios or conversion factors to convert a measurement, such as feet, to another type of measurement, such as inches.

When using *dimensional analysis* to convert from one unit of measure to another, you will set up a series of unit ratio multiplications. Each quantity in the multiplication must include its units. As we know, when multiplying fractions, if we have the same factor in the numerator of one fraction as in the denominator of another, those factors divide out or cancel. Dimensional analysis applies this same logic to units. If you have the same units in the numerator of one fraction as in the denominator of another, those units will cancel. Your unit conversion is complete when the only units remaining are the units to which you are converting.

Several examples of dimensional analysis to convert units of measure are shown below.

Example

How many inches are in $3\frac{1}{2}$ feet?

Solution

$3\frac{1}{2}\text{ feet} = ?\text{ inches}$	Begin by reasoning about your answer. Since a foot is longer than an inch, this means the answer would be greater than $3\frac{1}{2}$.
$3\frac{1}{2}\text{ feet} \cdot \frac{12\text{ inches}}{1\text{ foot}} = ?\text{ inches}$	Find the conversion factor that compares inches and feet, with "inches" in the numerator, and multiply.
$\frac{7\text{ feet}}{2} \cdot \frac{12\text{ inches}}{1\text{ foot}} = ?\text{ inches}$	Rewrite the mixed number as an improper fraction before multiplying.
$\frac{7\text{ feet}}{2} \cdot \frac{12\text{ inches}}{1\text{ foot}} = ?\text{ inches}$ $\frac{7}{2} \cdot \frac{12\text{ inches}}{1} = ?\text{ inches}$	You can cancel similar units when they appear in the numerator *and* the denominator. So here, cancel the similar units "feet" and "foot." This eliminates this unit from the problem.
$\frac{7 \cdot 12\text{ inches}}{2 \cdot 1} = ?\text{ inches}$	Rewrite as multiplication of numerators and denominators.
$\frac{84\text{ inches}}{2} = ?\text{ inches}$	Multiply.
$\frac{84\text{ inches}}{2} = 42\text{ inches}$	Divide.

There are 42 inches in $3\frac{1}{2}$ feet.

Notice how you can cancel the units out of the problem, just as if they were numbers. You can only cancel if the unit being cancelled is in both the numerator and denominator of the fractions you are multiplying.

In the problem above, you cancelled *feet* and *foot* leaving you with *inches*, which is what you were trying to find.

$$\frac{7\text{ feet}}{2} \cdot \frac{12\text{ inches}}{1\text{ foot}} = ?\text{ inches}$$

What if you had used the wrong conversion factor?

$$\frac{7\text{ feet}}{2} \cdot \frac{1\text{ foot}}{12\text{ inches}} =$$

You could not cancel the feet because the unit is not the same in *both* the numerator and the denominator. So if you complete the computation, you would still have both feet and inches in the answer and no conversion would take place.

Here is another example of a length conversion using dimensional analysis.

Example

How many yards is 7 feet?

Solution

$7 \text{ feet } =? \text{ yards}$	Start by reasoning about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. So your answer will be less than 7.
$7 \text{ feet } \cdot \frac{1 \text{ yard}}{3 \text{ feet}} =? \text{ yards}$	Find the conversion factor that compares feet and yards, with yards in the numerator.
$\frac{7 \text{ feet}}{1} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} =? \text{ yards}$	Rewrite the whole number as a fraction in order to multiply.
$\frac{7 \cancel{\text{ feet}}}{1} \cdot \frac{1 \text{ yard}}{3 \cancel{\text{ feet}}} =? \text{ yards}$	Cancel the similar units “feet” and “feet” leaving only yards.
$\frac{7}{1} \cdot \frac{1 \text{ yard}}{3} =? \text{ yards}$ $\frac{7 \cdot 1 \text{ yard}}{1 \cdot 3} =? \text{ yards}$	Multiply.
$\frac{7 \text{ yards}}{3} = 2\frac{1}{3} \text{ yards}$	Divide, and write as a mixed number.

7 feet equals $2\frac{1}{3}$ yards.

Note that if the units do not cancel to give you the answer you are trying to find, you may not have used the correct conversion factor.

Exercise

How many feet are in $2\frac{1}{2}$ miles?

A. 10,560 feet
B. 30 feet
C. 2,112 feet
D. 13,200 feet

Answer

A. Incorrect. There are 5,280 feet in a mile, so multiply $2\frac{1}{2}$, not 2, by 5,280. The correct answer is 13,200 feet.
B. Incorrect. A mile is much longer than 30 feet. There are 5,280 feet in a mile, so multiply 5,280 by $2\frac{1}{2}$, not $2\frac{1}{2}$ by 12, to find the number of feet in $2\frac{1}{2}$ miles. The correct answer is 13,200 feet
C. Incorrect. Multiply, don’t divide, 5,280 by $2\frac{1}{2}$. The correct answer is 13,200 feet.
D. Correct. There are 5,280 feet in a mile, so multiply $2\frac{1}{2}$ by 5,280 to get 13,200 feet.

Applying Unit Conversions

Example

An interior decorator needs border trim for a home she is wallpapering. She needs 15 feet of border trim for the living room, 30 feet of border trim for the bedroom, and 26 feet of border trim for the dining room. How many yards of border trim does she need?

Solution

$15 \text{ feet } + 30 \text{ feet } + 26 \text{ feet } = 71 \text{ feet}$	You need to find the total length of border trim that is needed for all three rooms in the house. Since the measurements for each room are given in feet, you can add the numbers.
$71 \text{ feet } = \text{ ? yards}$	How many yards is 71 feet?eason about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. Expect your answer to be less than 71.
$\frac{71 \text{ feet}}{1} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} = \text{? yards}$	Use the conversion factor $\frac{1 \text{ yard}}{3 \text{ feet}}$.
$\frac{71 \cancel{\text{ feet}}}{1} \cdot \frac{1 \text{ yard}}{3 \cancel{\text{ feet}}} = \text{? yards}$	Since "feet" is in the numerator and denominator, you can cancel this unit.
$\frac{71}{1} \cdot \frac{1 \text{ yard}}{3} = \text{? yards}$ $\frac{71 \cdot 1 \text{ yard}}{1 \cdot 3} = \text{? yards}$ $\frac{71 \text{ yards}}{3} = \text{? yards}$	Multiply.
$\frac{71 \text{ yards}}{3} = 23\frac{2}{3} \text{ yards}$	Divide, and write as a mixed number.

The interior decorator needs $23\frac{2}{3}$ yards of border trim.

The next example uses dimensional analysis to solve a problem that requires converting from miles to feet.

Example

Two runners were comparing how much they had trained earlier that day. Jo said, "According to my pedometer, I ran 8.3 miles." Alex said, "That's a little more than what I ran. I ran 8.1 miles." How many more feet did Jo run than Alex?

Solution

$8.3 \text{ miles } - 8.1 \text{ miles } = 0.2$ $0.2 \text{ mile } = \frac{2}{10} \text{ mile}$	You need to find the difference between the distance Jo ran and the distance Alex ran. Since both distances are given in the same unit, you can subtract and keep the unit the same.
$\frac{2}{10} \text{ mile } = \text{? feet}$	Since the problem asks for the difference in *feet*, you must convert from miles to feet. How many feet is 0.2 miles? Reason about the size of your answer. Since a mile is longer than a foot, the distance when expressed as feet will be a number greater than 0.2.
$\frac{2 \text{ miles}}{10} \cdot \frac{5{,}280 \text{ feet}}{1 \text{ mile}} = \text{? feet}$	Use the conversion factor $\frac{5{,}280 \text{ feet}}{1 \text{ mile}}$.
$\frac{2 \cancel{\text{ miles}}}{10} \cdot \frac{5{,}280 \text{ feet}}{1 \cancel{\text{ mile}}} = \text{? feet}$	Since "mile" is in the numerator and denominator, you can cancel this unit.
$\frac{2}{10} \cdot \frac{5{,}280 \text{ feet}}{1} = \text{? feet}$ $\frac{2 \cdot 5{,}280 \text{ feet}}{10 \cdot 1} = \text{? feet}$ $\frac{10{,}560 \text{ feet}}{10} = \text{? feet}$	Multiply. Divide.
$\frac{10{,}560 \text{ feet}}{10} = 1{,}056 \text{ feet}$	

Jo ran 1,056 feet farther than Alex.

Here's an example in which converting the units allows us to compare pricing.

Example

You are walking through a hardware store and notice two sales on tubing.

3 yards of Tubing A costs $5.49.

Tubing B sells for $1.88 for 2 feet.

Either tubing is acceptable for your project. Which tubing is less expensive?

Solution

Tubing A	
$3 \text{ yards} = \$5.49$	Find the unit price for each tubing. This will make it easier to compare.
$\frac{\$5.49 \div 3}{3 \text{ yards} \div 3} = \frac{\$1.83}{1 \text{ yard}}$	Find the cost per yard of Tubing A by dividing the cost of 3 yards of the tubing by 3.
Tubing B	
$2 \text{ feet} = \$1.88$	Tubing B is sold by the foot. Find the cost per foot by dividing $1.88 by 2 feet.
$\frac{\$1.88 \div 2}{2 \text{ feet} \div 2} = \frac{\$0.94}{1 \text{ foot}}$ $\frac{\$0.94}{1 \text{ foot}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = \frac{\$?}{? \text{ yard}}$	To compare the prices, you need to have the same unit of measure.
$\frac{\$0.94}{1 \text{ foot}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = \frac{\$2.82}{1 \text{ yard}}$ $2.82 per yard	Use the conversion factor $\frac{3 \text{ feet}}{1 \text{ yard}}$, cancel and multiply.
Tubing A: $1.83 per yard Tubing B: $2.82 per yard	Compare prices for 1 yard of each tubing.

Tubing A is less expensive than Tubing B.

In the problem above, you could also have found the price per foot for each kind of tubing and compared the unit prices of each per foot.

Exercise

A fence company is measuring a rectangular area in order to install a fence around its perimeter. If the length of the rectangular area is 130 yards and the width is 75 feet, what is the total length of the distance to be fenced?

A. 410 yards
B. 930 feet
C. 710 feet
D. 465 feet

Answer

A. Incorrect. The distance around the rectangle is two times the length plus two times the width, but you cannot perform this computation unless the units are the same. Convert yards to feet and then compute. The correct answer is 930 feet.
B. Correct. 130 yards is equivalent to 390 feet. To find the perimeter, add length + length + width + width: $390 \text{ feet} + 390 \text{ feet} + 75 \text{ feet} + 75 \text{ feet} = 930 \text{ feet}$
C. Incorrect. Convert 130 yards to feet by multiplying by 3. Then, double to get the distance of the two long sides of the rectangle in feet. Distance for the width is already given in feet, so multiply by 2 to get the length of both short sides of the rectangle. The correct answer is 930 feet.
D. Incorrect. Fencing is needed for 4 sides, not just two sides. The correct answer is 930 feet.

This page titled 5.4: Converting Units of Measure/ Dimensional Analysis is shared under a CC BY-NC-SA 4.0 license and was authored, remixed, and/or curated by The NROC Project via source content that was edited to the style and standards of the LibreTexts platform.

- **6.1.1: Length** by The NROC Project is licensed CC BY-NC-SA 4.0. Original source: https://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html.

5.5: The Metric System

Introduction

In the United States, both the **U.S. customary measurement system** and the **metric system** are used, especially in medical, scientific, and technical fields. In most other countries, the metric system is the primary system of measurement. If you travel to other countries, you will see that road signs list distances in kilometers and milk is sold in liters. People in many countries use words like “kilometer,” “liter,” and “milligram” to measure the length, volume, and weight of different objects. These measurement units are part of the metric system.

Unlike the U.S. customary system of measurement, the metric system is based on 10s. For example, a liter is 10 times larger than a deciliter, and a centigram is 10 times larger than a milligram. This idea of “10” is not present in the U.S. customary system: there are 12 inches in a foot, 3 feet in a yard, and 5,280 feet in a mile!

So, what if you have to find out how many milligrams are in a decigram? Or, what if you want to convert meters to kilometers? Understanding how the metric system works is a good start.

What is Metric?

The metric system uses units such as **meter**, **liter**, and **gram** to measure length, liquid volume, and mass, just as the U.S. customary system uses feet, quarts, and ounces to measure these.

In addition to the difference in the basic units, the metric system is based on 10s, and different measures for length include kilometer, meter, decimeter, centimeter, and millimeter. Notice that the word “meter” is part of all of these units.

The metric system also applies the idea that units within the system get larger or smaller by a power of 10. This means that a meter is 100 times larger than a centimeter, and a kilogram is 1,000 times heavier than a gram. You will explore this idea a bit later. For now, notice how this idea of “getting bigger or smaller by 10” is very different than the relationship between units in the U.S. customary system, where 3 feet equals 1 yard and 16 ounces equals 1 pound.

Length, Mass, and Volume

The table below shows the basic units of the metric system. Note that the names of all metric units follow from these three basic units.

Length	Mass	Volume
	basic units	
meter	gram	liter

In the metric system, the basic unit of length is the meter. A meter is slightly larger than a yardstick, or just over three feet.

The basic metric unit of mass is the gram. A regular-sized paperclip has a mass of about 1 gram.

Among scientists, one gram is defined as the mass of water that would fill a 1-centimeter cube. You may notice that the word “mass” is used here instead of “weight.” In the sciences and technical fields, a distinction is made between weight and mass. Weight is a measure of the pull of gravity on an object. For this reason, an object’s weight would be different if it was weighed on Earth or on the moon because of the difference in the gravitational forces. However, the object’s mass would remain the same in both places because mass measures the amount of substance in an object. As long as you are planning on only measuring objects on Earth, you can use mass/weight fairly interchangeably, but it is worth noting that there is a difference!

Finally, the basic metric unit of volume is the liter. A liter is slightly larger than a quart.

		
The handle of a shovel is about 1 meter.	A paperclip weighs about 1 gram.	A medium-sized container of milk is about 1 liter.

Though it is rarely necessary to convert between the customary and metric systems, sometimes it helps to have a mental image of how large or small some units are. The table below shows the relationship between some common units in both systems.

	Common Measurements in Customary and Metric Systems
Length	1 centimeter is a little less than half an inch.
	1.6 kilometers is about 1 mile.
	1 meter is about 3 inches longer than 1 yard.
Mass	1 kilogram is a little more than 2 pounds.
	28 grams is about the same as 1 ounce.
Volume	1 liter is a little more than 1 quart.
	4 liters is a little more than 1 gallon.

Prefixes in the Metric System

The metric system is a base 10 system. This means that each successive unit is 10 times larger than the previous one.

The names of metric units are formed by adding a prefix to the basic unit of measurement. To tell how large or small a unit is, you look at the **prefix**. To tell whether the unit is measuring length, mass, or volume, you look at the base.

Prefixes in the Metric System

kilo-	1,000 times **larger** than base unit
hecto-	100 times **larger** than base unit
deka-	10 times **larger** than base unit
meter, gram, liter	base units
deci-	10 times **smaller** than base unit
centi-	100 times **smaller** than base unit
milli-	1,000 times **smaller** than base unit

? Exercises 5.5.1

From each of the four choices, choose the most reasonable measure.

1. The length of a car:

5 kilometers, 5 meters, 5 centimeters, 5 millimeters

2. The height of a notebook:

28 kilometers, 28 meters, 28 centimeters, 28 millimeters

3. The distance to the next town:

3.8 kilometers, 3.8 meters, 3.8 centimeters, 3.8 millimeters

4. An adult woman's height:

1.6 kilometers, 1.6 meters, 1.6 centimeters, 1.6 millimeters

5. An adult woman's height:

160 kilometers, 160 meters, 160 centimeters, 160 millimeters

6. The thickness of a pane of glass:

3 kilometers, 3 meters, 3 centimeters, 3 millimeters

7. The mass of an apple:

100 kilograms, 100 grams, 100 milligrams

8. The mass of an adult man:

80 kilograms, 80 grams, 80 milligrams

9. The amount of active ingredient in a pain relief pill:

500 kilograms, 500 grams, 500 milligrams

10. The base vehicle weight of a GMC Sierra 1500:

$2,000$ kilograms, $2,000$ grams, $2,000$ milligrams

11. The capacity of a car's gas tank:

50 liters, 50 milliliters

12. A dosage of liquid cough medicine:

30 liters, 30 milliliters

Answer

1. 5 m
2. 28 cm
3. 3.8 km
4. 1.6 m
5. 160 cm
6. 3 mm
7. 100 g
8. 80 kg
9. 500 mg
10. $2,000$ kg
11. 50 L
12. 30 mL

Converting Units Up and Down the Metric Scale

The metric system was designed to be based on the natural world, and different units are related to each other by powers of 10 instead of weird numbers like 3, 12, 16, and $5,280$… This makes converting between metric units incredibly simple.

kilo- (k)	hecta- (h)	deka- (da)	**[base unit]**	deci- (d)	centi- (c)	milli- (m)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Notice that because deka- and deci- both start with d, the abbreviation for deka- is da.

Converting between metric units of measure requires knowledge of the metric prefixes and an understanding of the decimal system. That's about it.

For instance, you can figure out how many centigrams are in one dekagram by using the table above. One dekagram is larger than one centigram, so you expect that one dekagram will equal many centigrams.

In the table, each unit is 10 times larger than the one to its immediate right. This means that $1 \text{ dekagram} = 10 \text{ grams}$; $10 \text{ grams} = 100 \text{ decigrams}$; and $100 \text{ decigrams} = 1,000 \text{ centigrams}$. So, $1 \text{ dekagram} = 1,000 \text{ centigrams}$.

Exercises 5.5.2

How many milligrams are in one decigram?

Solution

kg | hg | dag | g | dg | cg | mg ^ ^	Identify locations of milligrams and decigrams. Decigrams (dg) are larger than milligrams (mg), so you expect there to be many mg in one dg.
×10 ×10 kg | hg | dag | g | dg | cg | mg ^ → → ^	Dg is 10 times larger than a cg, and a cg is 10 times larger than a mg.
$1\text{dg} \cdot 10 \cdot 10 = 100\text{mg}$	Since you are going from a larger unit to a smaller unit, multiply. Multiply: $1 \cdot 10 \cdot 10$, to find the number of milligrams in one decigram.

There are 100 milligrams (mg) in 1 decigram (dg).

Exercises 5.5.3

Convert 1 centimeter to kilometers.

Solution

km hm dam m dm cm mm ^ ^	Identify locations of kilometers and centimeters. Kilometers (km) are larger than centimeters (cm), so you expect there to be less than one km in a cm.
÷10 ÷10 ÷10 ÷10 ÷10 km hm dam m dm cm mm ^ ← ← ← ← ^	Cm is 10 times smaller than a dm; a dm is 10 times smaller than a m, etc. Since you are going from a smaller unit to a larger unit, divide.
1 centimeter $\div 10 \div 10 \div 10 \div 10 \div 10 = 0.00001$ kilometers	Divide: $1 \div 10 \div 10 \div 10 \div 10 \div 10$ to find the number of kilometers in one centimeter.

1 centimeter (cm) = 0.00001 kilometers (km).

Once you begin to understand the metric system, you can use a shortcut to convert among different metric units. The size of metric units increases tenfold as you go up the metric scale. The decimal system works the same way: a tenth is 10 times larger than a hundredth; a hundredth is 10 times larger than a thousandth, etc. By applying what you know about decimals to the metric system, converting among units is as simple as moving decimal points.

Here is the first problem from above: How many milligrams are in one decigram? You can recreate the order of the metric units as shown below:

kg hg dag g dg cg mg
1 2

This question asks you to start with 1 decigram and convert that to milligrams. As shown above, milligrams is two places to the right of decigrams. You can just move the decimal point two places to the right to convert decigrams to milligrams:

1 dg = 100. mg
1 2

The same method works when you are converting from a smaller to a larger unit, as in the problem: Convert 1 centimeter to kilometers.

km hm dam m dm cm mm
5 4 3 2 1

Note that instead of moving to the right, you are now moving to the left, so the decimal point must do the same:

1 cm = 0.00001 km
1 2 3 4 5

? Exercises 5.5.4

1. Convert 3.7 meters to centimeters.
2. Convert 3.7 meters to millimeters.
3. Convert 2.45 kilometers to meters.
4. Convert 2.45 kilometers to centimeters.
5. Convert 342 millimeters to meters.
6. Convert 342 millimeters to centimeters.

7. Convert 528 meters to kilometers.

8. Convert 45 centimeters to meters.

Answer

1. 370 cm
2. $3,700$ mm
3. $2,450$ m
4. $245,000$ cm
5. 0.342 m
6. 34.2 cm
7. 0.528 km
8. 0.45 m

? Exercises 5.5.5

9. Convert 0.813 grams to centigrams.

10. Convert 0.813 grams to milligrams.

11. Convert 1.25 kilograms to grams.

12. Convert 1.25 kilograms to milligrams.

13. Convert 960 milligrams to grams.

14. Convert 960 milligrams to centigrams.

15. Convert $1,350$ grams to dekagrams.

16. Convert 7.5 centigrams to grams.

Answer

9. 81.3 cg
10. 813 g
11. $1,250$ g
12. $1,250,000$ mg
13. 0.96 g
14. 96 cg
15. 135 dag
16. 0.075 g

? Exercises 5.5.6

17. Convert 2.8 liters to milliliters.

18. Convert 2.8 liters to deciliters.

19. Convert 15 dekaliters to liters.

20. Convert 0.75 deciliters to milliliters.

21. Convert 600 milliliters to centiliters.

22. Convert 600 milliliters to liters.

23. Convert 4.5 deciliters to liters.

24. Convert 550 centiliters to liters.

25. Flying on IcelandAir, you happen to notice that one mini bottle of booze is labeled 50 **mL**, but another mini bottle is labeled 5 **cL**. How do the two bottles compare in size?

26. How many 500-milliliter bottles of Coke[1] are equivalent to one 2-liter bottle?

27. The engine displacement of a Yamaha Majesty scooter is 125 cc (cubic centimeters), and the engine displacement of a Chevrolet Spark automobile is 1.4 L (liters). What is the approximate ratio of these engine displacements?

Answer

17. $2,800$ mL
18. 28 dL
19. 150 L
20. 75 mL
21. 60 cL
22. 0.6 L
23. 0.45 L
24. 5.5 L
25. they are equal in size
26. 4 bottles
27. about 11 to 1

This page titled 5.5: The Metric System is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **6.2.1: The Metric System** by The NROC Project is licensed CC BY-NC-SA 4.0. Original source: https://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html.
- **1.14: The Metric System** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

5.6: Converting Between Systems

Converting between the U.S. system and metric system is important in today's global economy; like it or not, the metric system is infiltrating our lives.

To convert between systems we will return dimensional analysis.

The numbers in these conversion ratios are usually difficult to work with, so we will use a calculator whenever necessary and pay attention to rounding. If you discover other conversion ratios that aren't provided here, write them down!

Converting Measurements of Length

You can use the conversion ratios in this table…

- 1 in = 2.54 cm
- 1 ft ≈ 0.305 m
- 1 yd ≈ 0.914 m
- 1 mi ≈ 1.609 km

…or the equivalent conversion ratios in this table.

- 1 cm ≈ 0.394 in
- 1 m ≈ 3.281 ft
- 1 m ≈ 1.094 yd
- 1 km ≈ 0.621 mi

Okay, full disclosure: these conversion ratios aren't exactly equivalent. For example, if we reverse the conversion ratio 1 in = 2.54 cm[1], we discover that 1 cm is actually $\frac{1}{2.54} \approx 0.3937$ inches. All of these conversion have been rounded to three decimal places, which means that we may get a slightly different result depending on which version of the conversion ratio we use.

? Exercises 5.6.1

1. How many centimeters is 6 inches?

2. Convert 200 yards into meters.

3. 12 feet is equal to how many centimeters?

4. How many feet is 50 meters?

5. 15 meters is what length in yards?

6. Convert 75 centimeters into inches.

7. Is 21 kilometers equivalent to 13 miles? If not, what is the percent error?



8. Is 66 kilometers equivalent to 41 miles? If not, what is the percent error?



Answer

1. 15.2 cm
2. 183 m
3. 366 cm
4. 164 ft
5. 16.4 yd
6. 29.5 in
7. not exactly; the error is around 0.3%.
8. they are essentially the same; the error is around 0.015%.

Converting Measurements of Weight or Mass

You can use the conversion ratios in this table…

- 1 oz ≈ 28.350 g
- 1 lb ≈ 0.454 kg

…or the (almost) equivalent conversion ratios in this table.

- 1 g ≈ 0.0353 oz
- 1 kg ≈ 2.205 lb

Continue to practice dimensional analysis as you work through the example exercises.

? Exercises 5.6.1

9. Convert 4 ounces into grams.

10. How many kilograms are equivalent to 120 pounds?

11. Convert 50 grams to ounces.

12. Convert 5 kilograms to pounds.

13. How many grams is a half pound of ground beef?

14. In around 2010, the National Collector's Mint (not affiliated with the U.S. Mint) ran a TV commercial selling an imitation $\$ 50$ gold coin modeled after the U.S. "buffalo" nickel. The commercial made the following claims. *This replica coin is coated in* 31 *milligrams of pure gold! And the price of gold keeps going up; gold is worth about* $\$ 1,000$ *per ounce! But you can order these fake coins for only* $\$ 19.95$ *apiece!* What is the approximate dollar value of the gold in one of these coins?

15. In 2020, the National Collector's Mint is still selling the imitation $ **50** gold "buffalo" nickel for $ **19.95**. This version of the coin is coated in **14** milligrams of pure gold.[2] As of September 2, 2020, the price of gold is $ **1,940** per ounce.[3] What is the approximate dollar value of the gold in one of these coins?

Answer

9. **113** g
10. **54.5** kg
11. **1.76** oz
12. **11.0** lb
13. **227** g
14. about $ **1.09**
15. about $ **0.96**

Converting Measurements of Volume or Capacity

You can use the conversion ratios in this table…

- **1** fl oz ≈ **29.6** mL
- **1** qt ≈ **0.946** L
- **1** gal ≈ **3.79** L

…or the (almost) equivalent conversion ratios in this table. This table includes one extra entry.

- **1** mL ≈ **0.0338** fl oz
- **1** L ≈ **1.06** qt
- **1** L ≈ **0.264** gal
- **1** L ≈ **33.8** fl oz

? Exercises 5.6.1

16. A toilet uses **3.5** gallons of water per flush. How many liters is this?

17. How many milliliters of drink are in a **12**-ounce can?

18. How many fluid ounces are in a **50** milliliter mini-sized bottle of alcohol?

19. What part of a gallon is in a **2**-liter bottle of soda pop?

Answer

16. **13.3** L
17. **355** mL
18. **1.69** fl oz
19. **0.53** gal

Converting Measurements: Extensions

Let's finish up with some rates that require conversions.

Exercises 5.6.1

Maxine is driving across Canada. Her car has a 14.2-gallon gas tank and gets an average of 26 miles per gallon.

20. Approximately how many kilometers—actually, the Canadian spelling is kilometres. Approximately how many kilometres can she travel on a full tank of gas?

21. Of course, Canada measures gas in liters. Actually, litres. Convert Maxine's mileage rate, 26 miles per gallon, to kilometres per litre.

Answer

20. a bit less than 600 km

21. 11 km/L

---

1. An inch is defined to be exactly 2.54 cm, if you were curious. ↵
2. https://ncmint.com/2020-buffalo-tribute-proof/ ↵
3. https://markets.businessinsider.com/commodities/gold-price ↵

This page titled 5.6: Converting Between Systems is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.15: Converting Between Systems** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

5.7: Conversions with Time, Rate, Area and Volume

Converting Measurements of Time and Rate

You probably know all of the necessary conversions for time. When we get to units of time larger than weeks, however, we encounter problems because not all months have the same number of days, a year is not exactly 52 weeks, and the time it takes for the Earth to orbit the Sun is not exactly 365 days. Therefore, it doesn't make sense to expect an exact answer to a question like "how many minutes are in one month?" We will have to use our best judgment in situations such as these.

1 min = 60 sec

1 hr = 60 min

1 day (dy) = 24 hr

1 week (wk) = 7 dy

1 year (yr) = 365 dy

Exercises 5.7.1

1. How many minutes is one standard 365-day year?[1]

2. Have you been alive for 1 billion seconds? Is this even possible?

Answer

1. $525,600$ min
2. this is roughly 31.7 years, which is indeed possible

Exercises 5.7.1

Usain Bolt holds the world record time for the 100-meter dash, 9.58 seconds.

3. What was his average speed in kilometers per hour?

4. What was his average speed in miles per hour?

Answer

3. 37.6 km/hr
4. 23.3 mi/hr

The more information we know, the more things we can figure out.

Exercises 5.7.1



An F-15 fighter jet can reach a sustained top speed of roughly Mach 2.3; this is 2.3 times the speed of sound, which is 770 miles per hour.[2]

5. What is the jet's top speed in miles per hour?

6. At this speed, how many miles would the jet travel in one minute?

The jet's range at this speed before it runs out of fuel is around 600 miles.

7. If the jet flies 600 miles at top speed, for how many minutes will it fly?

The jet's maximum fuel capacity is $3,475$ gallons of fuel.

8. If the jet flies 600 miles and burns $3,475$ gallons of fuel, find the jet's fuel efficiency, in miles per gallon.

9. Rewrite the jet's fuel efficiency, in gallons per mile.

10. How many gallons of fuel does the jet consume in one minute?

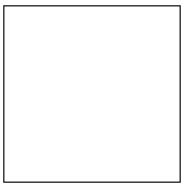
Answer

5. $1,770$ mi/hr
6. 29.5 mi in 1 min
7. 20.3 min
8. 0.17 mi/gal
9. 5.8 gal/mi
10. 171 gal in 1 min

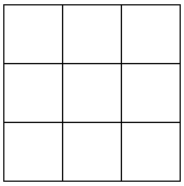
Converting Measurements of Area

In the US Customary System

Consider a square yard; the area of a square with sides 1 yard long.



1 yard = 3 feet, so we can divide the square into three sections vertically and three sections horizontally to convert both dimensions of the square from yards to feet. This forms a 3 by 3 grid, which shows us visually that 1 square yard equals 9 square feet, not 3 square feet!



The linear conversion ratio of 1 to 3 means that that the conversion ratio for the areas is 1 to 3^2, or 1 to 9.

Here's another way to think about it without a diagram: $1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^2 = (3 \text{ ft})^2$. To remove the parentheses, we must square the number *and* square the units: $(3 \text{ ft})^2 = 3^2 \text{ ft}^2 = 9 \text{ ft}^2$.

More generally, we need to **square** the linear conversion factors when converting units of area. If the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2.

Exercises 5.7.2

1. An acre is defined as the area of a 660 foot by 66 foot rectangle. (That's a furlong by a chain, if you were curious.) How many square feet are in 1 acre?

2. How many square yards are in 1 acre?

3. How many square inches equals 1 square foot?

Answer

1. $43,560\ \text{ft}^2$
2. $4,840\text{yd}^2$
3. $144\ \text{in}^2$

It should be no surprise that this module will be full of conversion ratios. As always, if you discover other conversion ratios that aren't provided here, it would be a good idea to write them down so you can use them as needed.

- $1\ \text{ft}^2 = 144\ \text{in}^2$
- $1\ \text{yd}^2 = 9\ \text{ft}^2$
- $1\ \text{acre (ac)} = 43,560\ \text{ft}^2$
- $1\ \text{ac} = 4,840\ \text{yd}^2$
- $1\ \text{mi}^2 = 27,878,400\ \text{ft}^2$
- $1\ \text{mi}^2 = 3,097,600\ \text{yd}^2$
- $1\ \text{mi}^2 = 640\ \text{ac}$

An acre is defined as a unit of area; it would be wrong to say "acres squared" or put an exponent of 2 on the units.

Exercises 5.7.2

4. A hallway is 9 yards long and 2 yards wide. How many square feet of linoleum are needed to cover the hallway?

5. A proposed site for an elementary school is 600 feet by 600 feet. Find its area, in acres.

Answer

4. $162\ \text{ft}^2$
5. $8.3\ \text{ac}$

In the Metric System

- $1\ \text{cm}^2 = 100\ \text{mm}^2$
- $1\ \text{m}^2 = 1,000,000\ \text{mm}^2$
- $1\ \text{m}^2 = 10,000\ \text{cm}^2$
- $1\ \text{hectare (ha)} = 10,000\ \text{m}^2$
- $1\ \text{km}^2 = 1,000,000\ \text{m}^2$
- $1\ \text{km}^2 = 100\ \text{ha}$

A hectare is defined as a square with sides 100 meters long. Dividing a square kilometer into ten rows and ten columns will make a 10 by 10 grid of 100 hectares. As with acres, it would be wrong to say "hectares squared" or put an exponent of 2 on the units.

Exercises 5.7.2

6. A hallway is 9 meters long and 2 meters wide. How many square centimeters of linoleum are needed to cover the hallway?

7. A proposed site for an elementary school is 200 meters by 200 meters. Find its area, in hectares.

Answer

6. $180,000\ \text{cm}^2$

7. $4\ \text{ha}$

Between Systems

Converting between the U.S. and metric systems will involve messy decimal values. For example, because $1\ \text{in} = 2.54\ \text{cm}$, we can square both numbers and find that $(1\ \text{in})^2 = (2.54\ \text{cm})^2 = 6.4516\ \text{cm}^2$. The conversions are rounded to three or four significant digits in the table below.

- $1\ \text{in}^2 \approx 6.45\ \text{cm}^2 \leftrightarrow 1\ \text{cm}^2 \approx 0.155\ \text{in}^2$
- $1\ \text{in}^2 \approx 6.45\ \text{cm}^2 \leftrightarrow 1\ \text{cm}^2 \approx 0.155\ \text{in}^2$
- $1\ \text{yd}^2 \approx 0.836\ \text{m}^2 \leftrightarrow 1\ \text{m}^2 \approx 1.196\ \text{yd}^2$
- $1\ \text{mi}^2 \approx 2.59\ \text{km}^2 \leftrightarrow 1\ \text{km}^2 \approx 0.386\ \text{mi}^2$
- $1\ \text{ac} \approx 0.405\ \text{ha} \leftrightarrow 1\ \text{ha} \approx 2.47\ \text{ac}$

? Exercises 5.7.2

8. The area of Portland is $145\ \text{mi}^2$. Convert this area to square kilometers.

9. How many hectares is a $5,000$ acre ranch?

10. A sheet of paper measures 8.5 inches by 11 inches. What is the area in square centimeters?

11. A soccer field is 100 meters long and 70 meters wide. What is its area in square feet?

Answer

8. $376\ \text{km}^2$

9. $2,000\ \text{ha}$ (to one sig fig) or $2,\overline{0}00\ \text{ha}$ (to two sig figs)

10. $6\overline{0}0\ \text{cm}^2$ (to two sig figs)

11. $75,300\ \text{ft}^2$



Converting Measurements of Volume

In the US Customary System

Just as we saw with area, converting between units of volume requires us to be careful because cubic units behave differently than linear units.

Quantities of mulch, dirt, or gravel are often measured by the cubic yard. How many cubic feet are in one cubic yard?

1 yard = 3 feet, so we can divide the length into three sections, the width into three sections, and the height into three sections to convert all three dimensions of the cube from yards to feet. This forms a 3 by 3 by 3 cube, which shows us that 1 cubic yard equals 27 cubic feet. The linear conversion ratio of 1 to 3 means that that the conversion ratio for the volumes is 1 to 3^3, or 1 to 27.

Here's another way to think about it: $1\ \text{yd} = 3\ \text{ft}$, so $(1\ \text{yd})^3 = (3\ \text{ft})^3$. To remove the parentheses, we must cube the number *and* cube the units: $(3\ \text{ft})^3 = 3^3\ \text{ft}^3 = 27\ \text{ft}^3$.

More generally, we need to **cube** the linear conversion factors when converting units of volume. If the linear units have a ratio of 1 to n, the cubic units will have a ratio of 1 to n^3.

$$1 \text{ ft}^3 = 1,728 \text{ in}^3$$

$$1 \text{ yd}^3 = 27 \text{ ft}^3$$

$$1 \text{ yd}^3 = 46,656 \text{ in}^3$$

? Exercises 5.7.3

1. Determine the number of cubic inches in 1 cubic foot.

2. Determine the number of cubic inches in 1 cubic yard.

3. Determine the number of cubic millimeters in 1 cubic centimeter.

4. Determine the number of cubic centimeters in 1 cubic meter.

Answer

1. $1,728 \text{ in}^3$
2. $46,656 \text{ in}^3$
3. $1,000 \text{ mm}^3$
4. $1,000,000 \text{ cm}^3$

? Exercises 5.7.3

5. True story: A friend at the National Guard base gave us three long wooden crates to use as raised planting beds. (The crates probably carried some kind of weapons or ammunition, but our friend wouldn't say.) Henry, who was taking geometry in high school, was asked to measure the crates and figure out how much soil we needed. The inside dimensions of each crate were 112 inches long, 14 inches wide, and 14 inches deep. We wanted to fill them most of the way full with soil, leaving about 4 inches empty at the top. How many cubic yards of soil did we need to order from the supplier?

6. True story, continued: I decided to check our answer and did a rough estimate by rounding each dimension to the nearest foot, then figuring out the volume from there. Did this give the same result?

Answer

5. the result is very close to 1 cubic yard: $(112 \text{ in} \cdot 14 \text{ in} \cdot 10 \text{ in}) \cdot 3 \text{ crates} = 47,040 \text{ in}^3 \approx 1.01 \text{ yd}^3$
6. this estimate is also 1 cubic yard: $(9 \text{ ft} \cdot 1 \text{ ft} \cdot 1 \text{ ft}) \cdot 3 \text{ crates} = 27 \text{ ft}^3 = 1 \text{ yd}^3$

We can convert between units of volume and liquid capacity. As you might expect, the numbers are messy in the U.S. system.

$$1 \text{ fl oz} \approx 1.805 \text{ in}^3 \leftrightarrow 1 \text{ in}^3 \approx 0.554 \text{ fl oz}$$

$$1 \text{ ft}^3 \approx 7.48 \text{ gal} \leftrightarrow 1 \text{ gal} \approx 0.1337 \text{ ft}^3$$

? Exercises 5.7.3



7. A wading pool has a diameter of roughly 5 feet and a depth of 6 inches. How many gallons of water are required to fill it about 80% of the way full?

8. A standard U.S. soda pop can has a diameter of $2\frac{1}{2}$ inches and a height of $4\frac{3}{4}$ inches. Verify that the can is able to hold 12 fluid ounces of liquid.

Answer

7. around 60 gallons

8. yes, the can is able to hold 12 fluid ounces; the can's volume is roughly 23.3 cubic inches ≈ 12.9 fluid ounces.

In the Metric System

$$1 \text{ cm}^3 = 1 \text{ cc} = 1 \text{ mL}$$

$$1 \text{ cm}^3 = 1,000 \text{ mm}^3$$

$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$

$$1 \text{ L} = 1,000 \text{ cm}^3$$

$$1 \text{ m}^3 = 1,000 \text{ L}$$

It's no surprise that the metric conversion ratios are all powers of 10.

? Exercise 5.7.3

9. A can of Perrier mineral water has a diameter of 5.6 cm and a height of 14.7 cm. Verify that the can is able to hold 330 milliliters of liquid.

Answer

yes, the can is able to hold 330 milliliters; the can's volume is roughly 362 cubic centimeters, which is equivalent to 362 milliliters.

Between Systems

Converting between the U.S. and metric systems will of course involve messy decimal values. For example, because 1 in $= 2.54$ cm, we can cube both numbers and find that $1 \text{ in}^3 = (2.54 \text{ cm})^3 \approx 16.387 \text{ cm}^3$. The conversions are rounded to three or four significant figures in the table below.

$$1 \text{ in}^3 \approx 16.39 \text{ cm}^3 \leftrightarrow 1 \text{ cm}^3 \approx 0.0612 \text{ in}^3$$
$$1 \text{ ft}^3 \approx 0.0284 \text{ m}^3 \leftrightarrow 1 \text{ m}^3 \approx 35.29 \text{ ft}^3$$
$$1 \text{ yd}^3 \approx 0.7646 \text{ m}^3 \leftrightarrow 1 \text{ m}^3 \approx 1.308 \text{ yd}^3$$

? Exercises 5.7.3



10. A "two yard" dumpster has a volume of 2 cubic yards. Convert this to cubic meters.

11. Convert 240 in^3 to cm^3.

12. Convert 500 cm^3 to in^3.

13. Convert $1,000\ \text{ft}^3$ to m^3.

14. Convert $45\ \text{m}^3$ to yd^3.

Answer

10. $1.53\ \text{m}^3$
11. $3,930\ \text{cm}^3$
12. $30.5\ \text{in}^3$
13. $28.3\ \text{m}^3$
14. $59\ \text{yd}^3$

This page titled 5.7: Conversions with Time, Rate, Area and Volume is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.16: Other Conversions** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **1.21: Converting Units of Area** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **1.25: Converting Units of Volume** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

CHAPTER OVERVIEW

6: Geometry

6.1: Polygons
6.2: Pythagorean Theorem for Right Triangles
6.3: Perimeter and Circumference
6.4: Area of Polygons and Circles
6.5: Composite Figures
6.6: Volume of Common Solids

6: Geometry is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

6.1: Polygons

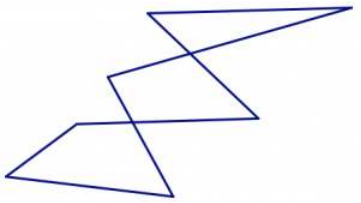
Definition

A **polygon** is:

1. a plane figure
2. that is bounded by a finite number of straight line segments
3. in which each segment meets exactly two others, one at each of its endpoints.

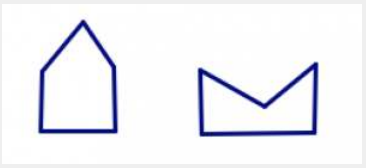
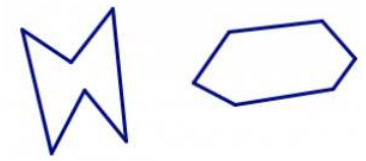
A few comments about polygons:

- The line segments that make up a polygon are called its **edges** and the points where they meet are called its **vertices** (singular: **vertex**).
- Because of properties (2) and (3) in the definition, the boundaries of polygons are not self-intersecting.



Not a polygon.

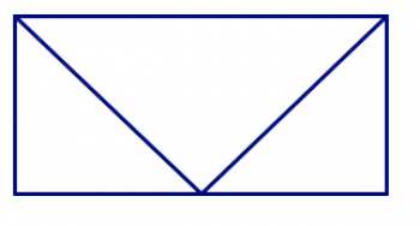
- Polygons are named based on the number of sides they have.

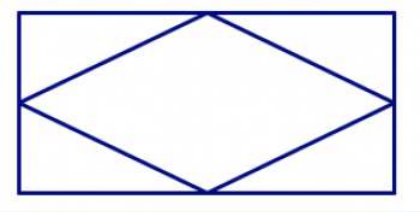
name	# of sides	examples
triangle	3	
quadrilateral	4	
pentagon	5	
hexagon	6	
heptagon	7	
octagon	8	
nonagon	9	
decagon	10	

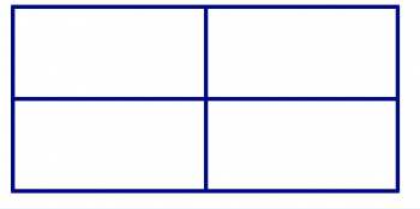
- In general, we call a polygon with *n* sides an *n-gon*.

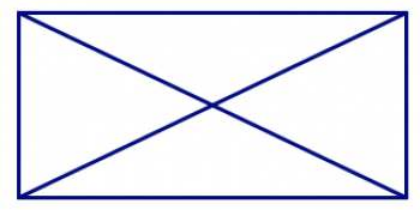
Problem 6

In the pictures below, there are polygons hidden in the design. In each design, find all of the triangles, quadrilaterals, pentagons, and hexagons. How can you be sure you've found them all and haven't counted any twice?









Angle Sum

You know that the sum of the interior angles in any triangle is 180°. Can you say anything about the angles in other polygons?

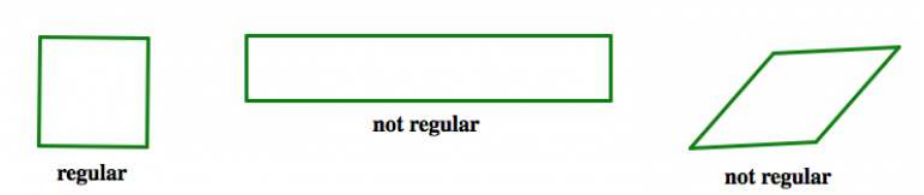
You probably know that rectangles have four 90° angles. So *if if all quadrilaterals have the same interior angle sum*, it must be 360° (since 4 × 90° = 360°).

In general, the sum of the angles inside a polygon with n sides is (n - 2) × 180°. For example, the interior angles in a triangle, which has n = 3 sides, is (3 - 2) × 180° = 180°

Definition

A **regular polygon** has all sides the same length and all angles the same measure.

For example, squares are regular quadrilaterals — all four sides are the same length, and all four angles measure 90°. But a non-square rectangle is *not regular*. Even though all of the angles are 90°, the sides are not all the same length. Similarly, a non-square rhombus is *not regular*. Even though the sides of a rhombus are all the same length, the angles can be different.



Problem 8

Since a square is a regular quadrilateral, you know that every angle in a regular quadrilateral measure 90°. What about angles in other regular polygons?

1. What is the measure of each angle in a regular triangle? Explain how you know you are right.
2. What is the measure of each angle in a regular pentagon? Explain how you know you are right.
3. What is the measure of each angle in a regular hexagon? Explain how you know you are right.
4. What is the measure of each angle in a regular *n-gon*? Explain how you know you are right.

This page titled 6.1: Polygons is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Michelle Manes via source content that was edited to the style and standards of the LibreTexts platform.

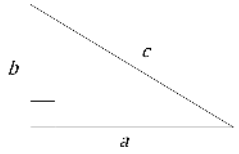
- **7.4: Polygons** by Michelle Manes is licensed CC BY-SA 4.0. Original source: pressbooks.oer.hawaii.edu/mathforelementaryteachers.

6.2: Pythagorean Theorem for Right Triangles

Pythagoras and the Pythagorean Theorem

Pythagoras was a Greek philosopher who lived around 500 BC. He is credited as being a philosopher and mathematician. Much of what we know of Pythagoras is from writings that were copied down hundreds of years after his death, so the validity of what we do know is questionable. He is credited with Pythagoras' theorem when actually it has been proven that Babylonians and Indians were using variations of it for hundreds of years before he even came along.

The **Pythagorean theorem**, also known as **Pythagoras' theorem**, is a relation in Euclidean geometry among the three sides of a right triangle. 'It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.



Theorem 6.2.1: The Pythagorean Theorem

In a right triangle with legs a and b and hypotenuse c,

$$a^2 + b^2 = c^2$$

When we look at the formula, there is one important thing to remember: C is always the longest side. A and B can be swapped around, but when using this formula, C is always the longest side (which is also the side opposite the 90-degree angle).

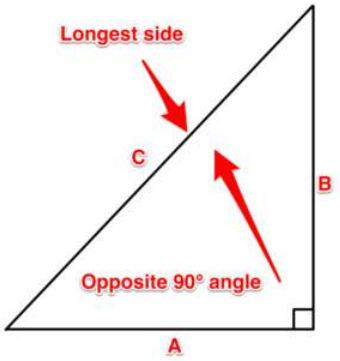


Figure 15. Longest side triangle

Basically, the Pythagoras' theorem says that you can figure out any side of a right triangle as long as you have the other two sides. And, if you know the lengths of all three sides of a triangle, you can use the Pythagorean theorem to verify whether the triangle is a right triangle or not. The ancient Egyptians used this method for surveying when they needed to redraw boundaries after the yearly flooding of the Nile washed away their previous markings.[1]

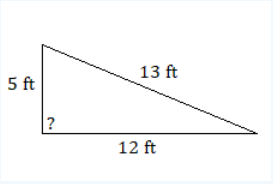


Video! *This video walks through how to apply Pythagoras' theorem on a right triangle.*

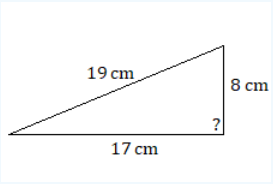
Exercises 6.2.1

Use the Pythagorean theorem to determine whether either of the following triangles is a right triangle.

8.



9.



Answer

8. right triangle, because $5^2 + 12^2 = 13^2$

9. not a right triangle, because $8^2 + 17^2 \neq 19^2$

Before we continue, we need to briefly discuss square roots. Calculating a square root is the opposite of squaring a number. For example, $\sqrt{49} = 7$ because $7^2 = 49$. If the number under the square root symbol is not a perfect square like 49, then the square root will be an irrational decimal that we will round off as necessary.

Exercises 6.2.1

Use a calculator to find the value of each square root. Round to the hundredths place.

10. $\sqrt{50}$

11. $\sqrt{296}$

12. $\sqrt{943}$

Answer

10. 7.07

11. 17.20

12. 30.71

We most often use the Pythagorean theorem to calculate the length of a missing side of a right triangle. Here are three different versions of the Pythagorean theorem arranged to find a missing side, so you don't have to use algebra with $a^2 + b^2 = c^2$.

The Pythagorean Theorem, three other versions

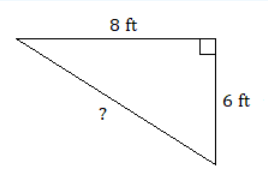
$c = \sqrt{a^2 + b^2}$

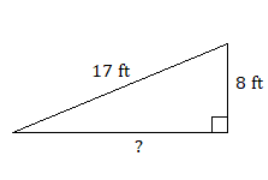
$b = \sqrt{c^2 - a^2}$

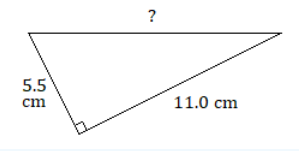
$a = \sqrt{c^2 - b^2}$

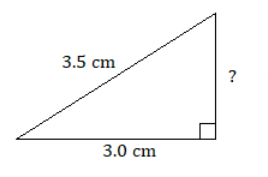
Exercises 6.2.1

Find the length of the missing side for each of these right triangles. Round to the nearest tenth, if necessary.

13. 

14. 

15. 

16. 

Answer

13. 10 ft
14. 15 ft
15. 12.3 cm
16. 1.8 cm

1. The surveyors were called "rope-stretchers" because they used a loop of rope 12 units long with 12 equally-spaced knots. Three rope-stretchers each held a knot, forming a triangle with lengths 3, 4, and 5 units. When the rope was stretched tight, they knew that the angle between the 3-unit and 4-unit sides was a right angle because $3^2 + 4^2 = 5^2$. From *Discovering Geometry: an Inductive Approach* by Michael Serra, Key Curriculum Press, 1997. ↵

This page titled 6.2: Pythagorean Theorem for Right Triangles is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.18: Triangles** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

6.3: Perimeter and Circumference

You may use a calculator throughout this module if needed.

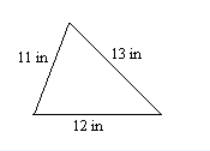
Perimeter

A polygon is a closed geometric figure with straight sides. Common polygons include triangles, squares, rectangles, parallelograms, trapezoids, pentagons, hexagons, octagons… The perimeter of a polygon is the distance around the outside. In general, to find the perimeter of a polygon, you can add up the lengths of all of its sides.

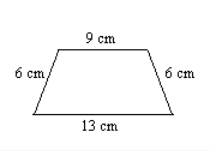
Also, if you haven't already, now is the time to get in the habit of including units in your answers.

Exercises 6.3.1

1. Find the perimeter of the triangle.



2. Find the perimeter of the trapezoid.



Answer

1. **36** in
2. **34** cm

If we know that some of the sides of a polygon are equal, we can use a formula as an alternative to adding up all of the lengths individually. The first formula shown below uses the variable s for the side of a square. The rectangle formulas use l for length and w for width, or b for base and h for height; these terms are interchangeable.

Perimeter Formulas

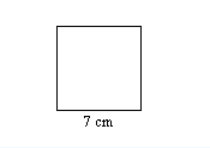
Square: $P = 4s$

Rectangle: $P = 2l + 2w$ or $P = 2b + 2h$

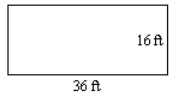
Rectangle: $P = 2(l + w)$ or $P = 2(b + h)$

Exercises 6.3.1

3. Find the perimeter of the square.



4. Find the perimeter of the rectangle.



5. A storage area, which is a rectangle that is 45 feet long and 20 feet wide, needs to be fenced around all four sides. How many feet of fencing is required? (To keep it simple, ignore any gates or other complications.)

6. Giancarlo is putting crown molding around the edge of the ceiling of his living room. If the room is a 12-foot by 16-foot rectangle, how much crown molding does he need?

Answer

3. 28 cm
4. 104 ft
5. 130 ft
6. 56 ft

The sides of a regular polygon are all equal in length. Therefore, multiplying the length of a side by the number of sides will give us the perimeter.

Perimeter Formula

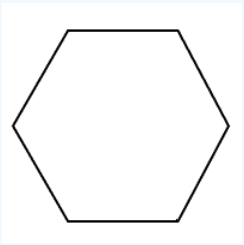
Regular Polygon with n sides of length s:

$$P = n \cdot s$$

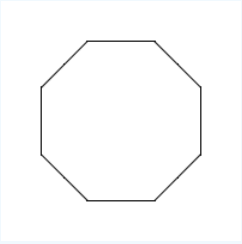
Exercises 6.3.1

Find the perimeter of each regular polygon.

7. Each side of the hexagon is 4 inches long.



8. Each side of the octagon is 2.5 centimeters long.



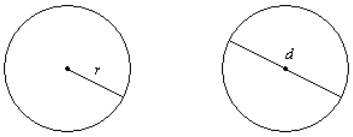
Answer

7. 24 in
8. 20 cm

Circumference

Instead of calling it the perimeter, the distance around the outside of circle is called the ***circumference***. Let's review some circle vocabulary before moving on.

Every point on a circle is the same distance from its center. This distance from the center to the edge of the circle is called the ***radius***. The distance from one edge to another, through the center of the circle, is called the ***diameter***. As you can see, the diameter is twice the length of the radius.



Throughout history, different civilizations have discovered that the circumference of a circle is slightly more than 3 times the length of its diameter. By the year 2000 BCE, the Babylonians were using the value $3\frac{1}{8} = 3.125$ and the Egyptians were using the value $3\frac{13}{81} \approx 3.1605$.[1] The value $3\frac{1}{7} \approx 3.1429$ is an even better approximation for the ratio of the circumference to the diameter. However, the actual value cannot be written as an exact fraction. It is the irrational number π, pronounced "pie", which is approximately 3.14159.

Circumference Formulas

$$C = \pi d$$

$$C = 2\pi r$$

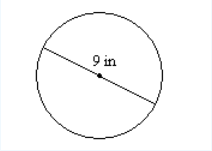
Any scientific calculator will have a π key; using this will give you the most accurate result, although you should be sure to round your answer appropriately. (See this module if you need a refresher on rounding with multiplying or dividing.) Many people use 3.14 as an approximation for π, but this can lead to round-off error; if you must use an approximation, 3.1416 is better than 3.14.

Sometimes we bend the rules in this textbook and ask you to round to a certain place value instead of rounding to a certain number of significant digits.

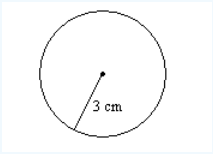
? Exercises 6.3.1

Calculate the circumference of each circle. Round to the nearest tenth.

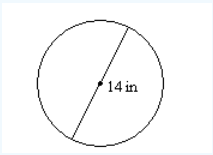
9.

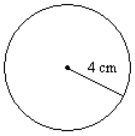


10.



11.



12. 

Answer

9. 28.3 in
10. 18.8 cm
11. 44.0 in
12. 25.1 cm

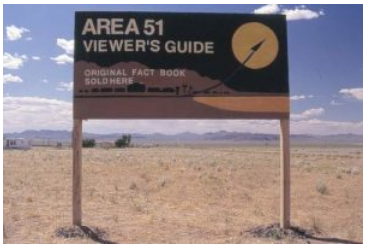
1. This information comes from Chapter 1 of the book *A History of Pi* by Petr Beckmann. It is a surprisingly interesting read. ↵

This page titled 6.3: Perimeter and Circumference is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.8: Perimeter and Circumference** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

6.4: Area of Polygons and Circles

You may use a calculator for most of this module as needed.

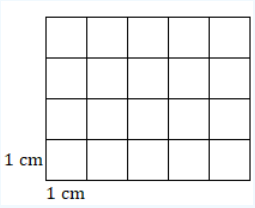


We have seen that the perimeter of a polygon is the distance around the outside. Perimeter is a length, which is one-dimensional, and so it is measured in linear units (feet, centimeters, miles, etc.). The area of a polygon is the amount of two-dimensional space inside the polygon, and it is measured in square units: square feet, square centimeters, square miles, etc.

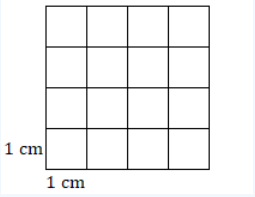
You can always think of area as the number of squares required to completely fill in the shape.

Exercises 6.4.1

1. Find the area of this rectangle.



2. Find the area of this square.



Answer

1. $20\ \text{cm}^2$
2. $16\ \text{cm}^2$

Rectangles and Squares

There are of course formulas for finding the areas of rectangles and squares; we don't have to count little squares.

Area of a Rectangle

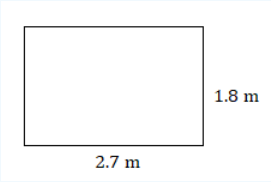
$A = lw$[1] or $A = bh$

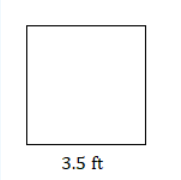
Area of a Square

$$A = s^2$$

Exercises 6.4.1

Find the area of each figure.

3. 

4. 

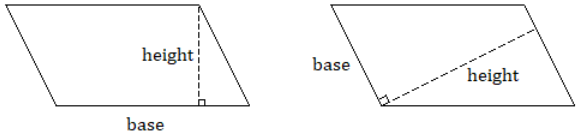
Answer

3. $4.86\ \text{m}^2$

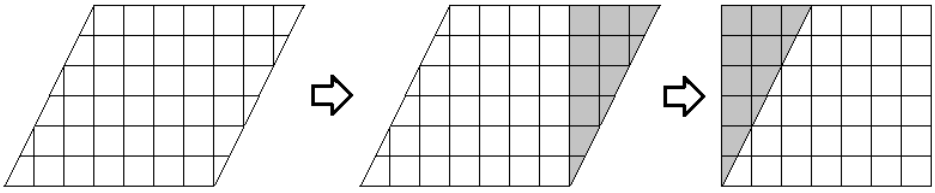
4. $12.25\ \text{ft}^2$

Parallelograms

Another common polygon is the ***parallelogram***, which looks like a tilted rectangle. As the name implies, the pairs of opposite sides are parallel and have the same length. Notice that, if we label one side as a base of the parallelogram, we have a perpendicular height which is *not* the length of the other sides.



The following set of diagrams shows that we can cut off part of a parallelogram and rearrange the pieces into a rectangle with the same base and height as the original parallelogram. A parallelogram with a base of **7** units and a vertical height of **6** units is transformed into a **7** by **6** rectangle, with an area of **42** square units.



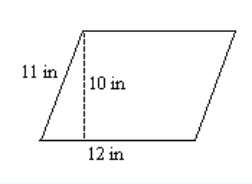
Therefore, the formula for the area of a parallelogram is identical to the formula for the area of a rectangle, provided that we are careful to use the base and the height, which must be perpendicular.

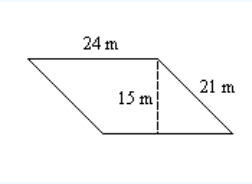
Area of a Parallelogram

$A = bh$

Exercises 6.4.1

Find the area of each parallelogram.

5. 

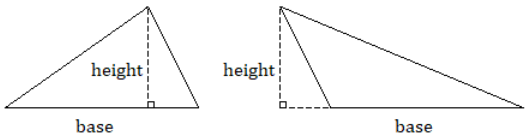
6. 

Answer

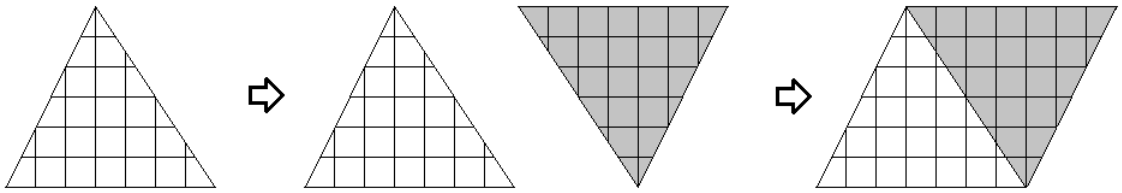
5. $120\ \text{in}^2$
6. $360\ \text{m}^2$

Triangles

When we need to find the area of a triangle, we need to identify a base and a height that is perpendicular to that base. If the triangle is obtuse, you may have to imagine the height outside of the triangle and extend the base line to meet it.



As shown below, any triangle can be doubled to form a parallelogram. Therefore, the area of a triangle is one half the area of a parallelogram with the same base and height.



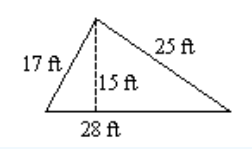
Area of a Triangle

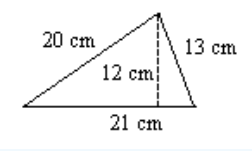
$A = \frac{1}{2}bh$ or $A = bh \div 2$

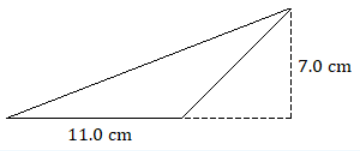
As with a parallelogram, remember that the height must be perpendicular to the base.

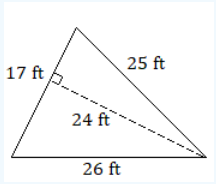
Exercises 6.4.1

Find the area of each triangle.

7. 

8. 

9. 

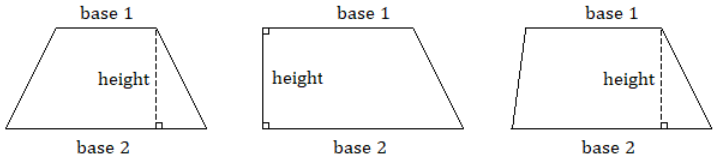
10. 

Answer

7. $210\ \text{ft}^2$
8. $126\ \text{cm}^2$
9. $38.5\ \text{cm}^2$
10. $204\ \text{ft}^2$

Trapezoids

A somewhat less common quadrilateral is the ***trapezoid***, which has exactly one pair of parallel sides, which we call the bases. The first example shown below is called an isosceles trapezoid because, like an isosceles triangle, its two nonparallel sides have equal lengths.



There are a number of ways to show where the area formula comes from, but the explanations are better in video because they can be animated.[2][3][4]

Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

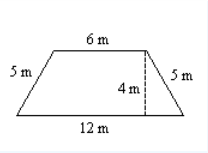
or

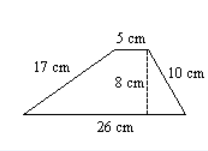
$$A = (b_1 + b_2)h \div 2$$

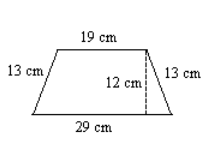
Don't be intimidated by the subscripts on b_1 and b_2; it's just a way to name two different measurements using the same letter for the variable. (Many people call the bases a and b instead; feel free to write it whichever way you prefer.) Whatever you call them, you just add the two bases, multiply by the height, and take half of that.

? Exercises 6.4.1

Find the area of each trapezoid.

11. 

12. 

13. 

Answer

11. $36\ \text{m}^2$
12. $124\ \text{cm}^2$
13. $288\ \text{cm}^2$

Circles

The area of a circle is π times the square of the radius: $A = \pi r^2$. The units are still square units, even though a circle is round. (Think of the squares on a round waffle.) Because we can't fit a whole number of squares—or an exact fraction of squares—inside the circle, the area of a circle will be an approximation.



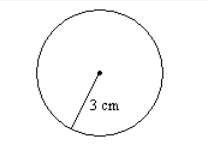
Area of a Circle

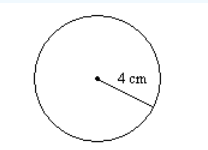
\[A=\pi{r^2}\]

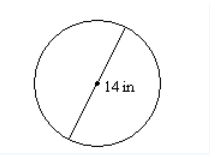
Remember that $\pi \approx 3.1416$.

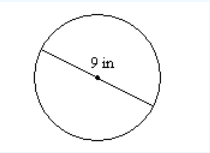
Exercises 6.4.1

Find the area of each circle. Round to the nearest tenth or to three significant figures, whichever seems appropriate.

14. 

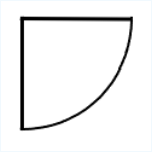
15. 

16. 

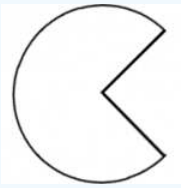
17. 

Each figure is a fraction of a circle. Calculate each area.

18. The radius of the quarter circle is **5** meters.



19. A quarter circle has been removed from a circle with a diameter of **7** feet.



Answer

14. **28.3 cm^2**
15. **50.3 cm^2**
16. **153.9 in^2**
17. **63.6 in^2**
18. **19.6 m^2**
19. **28.9 ft^2**

1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1". ↵
2. https://youtu.be/yTnYRpcZA9c ↵
3. https://youtu.be/WZtO3oERges ↵
4. https://youtu.be/uLHc6Br2veg ↵

This page titled 6.4: Area of Polygons and Circles is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.19: Area of Polygons and Circles** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

6.5: Composite Figures



This sunburst pattern is composed of **276** geometric tiles.

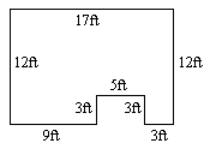
You may use a calculator throughout this module as needed.

Many objects have odd shapes made up of simpler shapes. A composite figure is a geometric figure which is formed by—or *composed* of—two or more basic geometric figures. We will look at a handful of fairly simple examples, but this concept can of course be extended to much more complicated figures.

To find the area of a composite figure, it is generally a good idea to divide it into simpler shapes and either add or subtract their areas as necessary.

? Exercises 6.5.1

A floor plan of a room is shown. The room is a **12**-foot by **17**-foot rectangle, with a **3**-foot by **5**-foot rectangle cut out of the south side.



1. Determine the amount of molding required to go around the perimeter of the room.

2. Determine the amount of flooring required to cover the entire area.

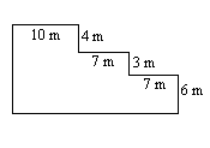
Answer

1. **64 ft**
2. **189 ft^2**

You may need to figure out some unmarked dimensions.

? Exercises 6.5.1

A plan for an irregular parking lot is shown.



3. Calculate the perimeter.

4. Calculate the area.

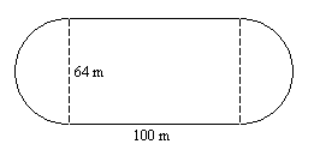
Answer

3. **74 m**
4. **235 m^2**

Sometimes, composite figures involve parts of circles coupled with polygons.

Exercises 6.5.1

A high school is building a track at its athletic fields. The track, which is formed by two straight sides and two semicircles as shown in the plans below, is supposed to have a total length of 400 meters.



5. Determine the distance around the track. Will the track be the right length?

6. After the new track is built, landscapers need to lay sod on the field inside the track. What is the area of the field inside the track?

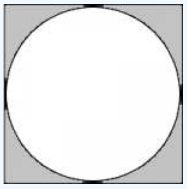
Answer

5. Based on the stated measurements, the distance around the track will be 401 meters, which appears to be 1 meter too long. In real life, precision would be very important here, and you might ask for the measurements to be given to the nearest tenth of a meter.

6. around $9,620\ \text{m}^2$

If we need to determine a fraction or percent out of the whole, we may be able to solve the problem without knowing any actual measurements.

Exercise 6.5.1



7. Circular disks are being cut from squares of sheet metal, with the remainder around the corners being discarded. Assuming that the circles are made as large as possible, what percent of the sheet metal will be discarded?

Answer

around 21.5% (Hint: Make up an easy number for the side of the square, like 2 or 10.)

This page titled 6.5: Composite Figures is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.20: Composite Figures** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

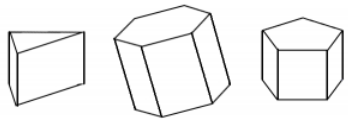
6.6: Volume of Common Solids

Volume

Volume is the amount of space inside the solid. Volume is three-dimensional, measured in cubic units. You can imagine the volume as the number of cubes required to completely fill up the solid.

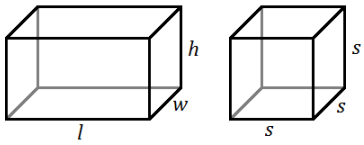
Prism

A solid with two equal sized polygons as its bases and rectangular lateral faces is called a right-angle prism. Some examples are shown below. We will refer to them simply as ***prisms*** in this textbook. (We will not be working with oblique prisms, which have parallelograms for the lateral faces.)



If you know the area of one of the bases, multiplying it by the height gives you the volume of the prism. In the formula below, we are using a capital B to represent the *area* of the base.

Rectangular solids, typically called boxes and cubes, are common types of prisms.



For a rectangular solid with length l, width w, and height h:

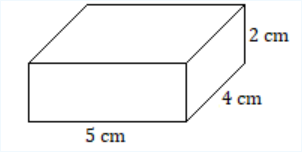
$$V = lwh$$

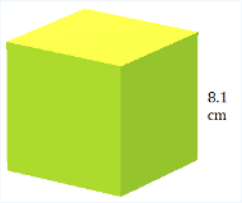
For a cube with side length s:

$$V = s^3$$

? Exercises 6.6.1

Find the volume of each solid.

1. 

2. 

Answer

1. $40\ \text{cm}^3$
2. $531\ \text{cm}^3$

Generalized Volume of a Prism

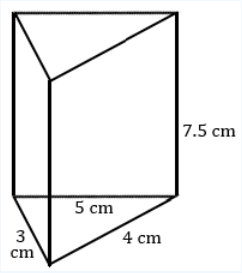
For a prism with base area B and height h:

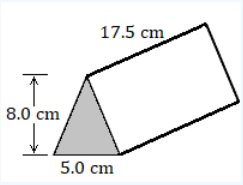
$$V = Bh$$

If the prism is lying on its side, the "height" will look like a length. No matter how the prism is oriented, the height is the dimension that is perpendicular to the planes of the two parallel bases.

Exercises 6.6.1

Find the volume of each prism.

3. 

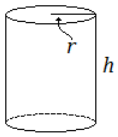
4. 

Answer

3. $45\ \text{cm}^3$

4. $350\ \text{cm}^3$

A cylinder can be thought of as a prism with bases that are circles, rather than polygons. Just as with a prism, the volume is the area of the base multiplied by the height.



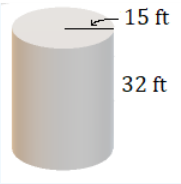
Volume of a Cylinder

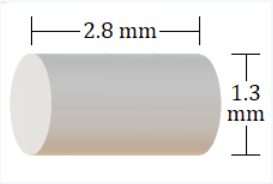
For a cylinder with radius r and height h:

$$V = \pi r^2 h$$

Exercises 6.6.1

Find the volume of each cylinder.

6. 

7. 

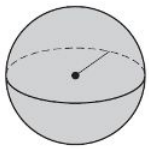
Answer

6. $22,600\ \text{ft}^3$

7. $3.7\ \text{mm}^3$

Volume of a Sphere

A sphere is a ball. Typically spheres are described either by their diameter or their radius which are like the diameter and radius of a 2-dimensional circle.



For a sphere with radius r:

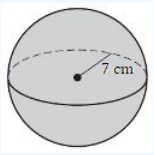
$$V = \frac{4}{3}\pi r^3$$

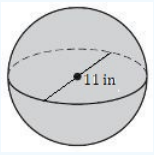
or

$$V = 4\pi r^3 \div 3$$

? Exercises 6.6.1

Find the volume of each sphere.

8. 

9. 

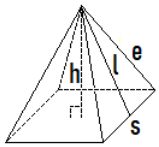
Answer

8. $1,440\ \text{cm}^3$

9. $700\ \text{in}^3$

Pyramids

A *pyramid* is a geometric solid with a polygon base and triangular faces with a common vertex (called the *apex* of the pyramid). Pyramids are named according to the shape of their bases. The most common pyramids have a square or another regular polygon for a base, making all of the faces identical isosceles triangles. The height, h, is the distance from the apex straight down to the center of the base. Two other measures used with pyramids are the edge length e, the sides of the triangular faces, and the slant height l, the height of the triangular faces.



Volume of a Pyramid

In general, the volume of a pyramid with base of area B and height h is

$$V = \frac{1}{3}Bh$$

or

$$V = Bh \div 3$$

If the base is a square with side length s, the volume is

$$V = \frac{1}{3}s^2h$$

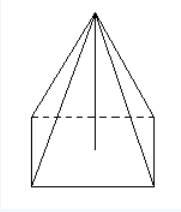
or

$$V = s^2h \div 3$$

Interestingly, the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.

? Exercises 6.6.1

1. A pyramid has a square base with sides **16** centimeters long, and a height of **15** centimeters. Find the volume of the pyramid.



2. The Great Pyramid at Giza in Egypt has a height of 137 meters and a square base with sides 230 meters long.[1] Find the volume of the pyramid.



Answer

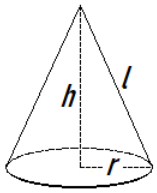
1. $1,280\ \text{cm}^3$

2. 2.4 million m^3

The lateral surface area (LSA) of a pyramid is found by adding the area of each triangular face.

Cones

A cone is like a pyramid with a circular base.



You may be able to determine the height h of a cone (the altitude from the apex, perpendicular to the base), or the slant height l (which is the length from the apex to the edge of the circular base). Note that the height, radius, and slant height form a right triangle with the slant height as the hypotenuse. We can use the Pythagorean theorem to determine the following equivalences.

The slant height$l$, height h, and radius r of a cone are related as follows:

$$l = \sqrt{r^2 + h^2}$$

$$h = \sqrt{l^2 - r^2}$$

$$r = \sqrt{l^2 - h^2}$$

Just as the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height, the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.

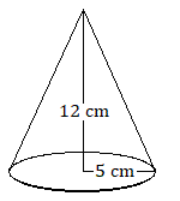
Volume of a Cone

The volume of a cone with a base radius r and height h is

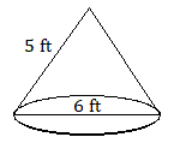
$$V = \frac{1}{3}\pi r^2 h \)or\ (V = \pi r^2 h \div 3$$

? Exercises 6.6.1

5. The base of a cone has a radius of 5 centimeters, and the vertical height of the cone is 12 centimeters. Find the volume of the cone.



6. The base of a cone has a diameter of 6 feet, and the slant height of the cone is 5 feet. Find the volume of the cone.



Answer

5. 314 cm^3

6. $37.7\ \text{ft}^3$

Composite Solids

Of course, not every three-dimensional object is a prism, cylinder, or sphere. A composite solid is made up of two or more simpler solids. As with two-dimensional composite figures, breaking the figure into recognizable solids is a good first step.

Now that we have looked at the five major solids—prism, cylinder, sphere, pyramid, cone—you should be able to handle composite solids made from these shapes. Just remember to take them in pieces.

? Exercise 6.6.1

A **250**-gallon propane tank is roughly in the shape of a cylinder with a hemisphere on each end. The length of the cylindrical part is **6** feet long, and the cross-sectional diameter of the tank is **2.5** feet.



9. Calculate the volume of the tank in cubic feet.

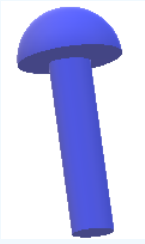
10. Verify that the tank can hold **250** gallons of liquid propane.

Answer

9. $37.6\ \text{ft}^3$ (the cylinder's volume $\approx 29.45\ \text{ft}^3$ and the two hemispheres' combined volume $\approx 8.18\ \text{ft}^3$)

10. $37.6\ \text{ft}^3 \approx 281.5\ \text{gal}$, which is more than $250\ \text{gal}$.

? Exercise 6.6.1



10. A rivet is formed by topping a cylinder with a hemisphere. The width of the cylindrical part (the rivet pin) is **1.6** cm and the length is **7** cm. The width of the hemisphere-shaped top (the rivet head) is **3.2** cm. Find the rivet's volume.

Answer

$31.2\ \text{cm}^3$ (the cylinder's volume $\approx 14.07\ \text{cm}^3$ and the hemisphere's volume $\approx 17.16\ \text{cm}^3$.)

This page titled 6.6: Volume of Common Solids is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.24: Volume of Common Solids** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.
- **1.26: Pyramids and Cones** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

CHAPTER OVERVIEW

7: Trigonometry

7.1: Angles

7.2: Triangles

7.3: Introduction to Right Triangle Trigonometry

7.4: Right Triangle Trigonometry in the Real World

7.5: Power and Impedance Triangles - An Elelctrical Example

7: Trigonometry is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

7.1: Angles

What's the deal with angles anyway?

Before we even get into trigonometry, we need to discuss angles and some vocabulary related to angles.

Degree. One-three-hundred-and-sixtieth of the circumference of a circle. It is also the unit by which we measure angles.

Figure 1. Degrees

Angle. This is the space between two intersecting lines.

Figure 2. Angle

Complementary angles. These are two angles whose sum equals 90 degrees.

Figure 3. Complementary angle

Supplementary angles. These are two angles whose sum equals 180 degrees.

Figure 4. Supplementary angle

Acute angle. An angle that is less than 90 degrees.

Figure 5. Acute angle

Obtuse angle. An angle that is greater than 90 degrees.

Figure 6. Obtuse angle

Similar angles. It is possible for triangles to each have different sized sides but share the same sized angles. These are called similar angles.

Figure 7. Similar angles

Right angle. This is an angle that is 90 degrees.

Figure 8. Right angle



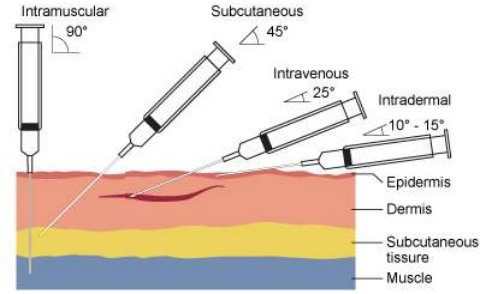
an angle grinder

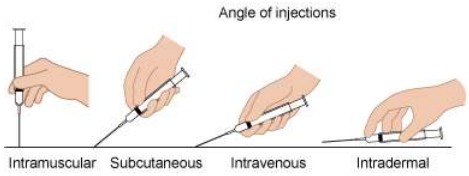
Angle measurement is important in construction, surveying, physical therapy, and many other fields. We can visualize an angle as the figure formed when two line segments share a common endpoint. We can also think about an angle as a measure of rotation. A full rotation or a full circle is $360°$, so a half rotation or U-turn is $180°$, and a quarter turn is $90°$.

We often classify angles by their size.

- **Acute Angle:** between $0°$ and $90°$
- **Right Angle:** exactly $90°$
- **Obtuse Angle:** between $90°$ and $180°$
- **Straight Angle:** exactly $180°$
- **Reflexive Angle:** between $180°$ and $360°$

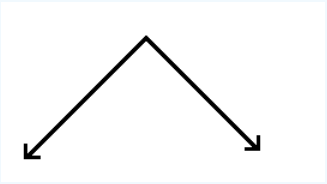
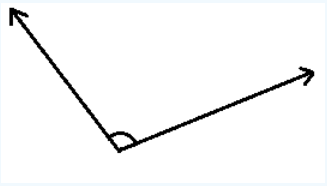
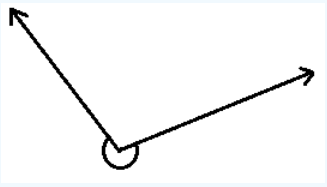
Lines that form a $90°$ angle are called ***perpendicular***. As shown below, the needle should be perpendicular to the body surface for an intramuscular injection.

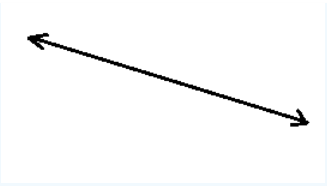


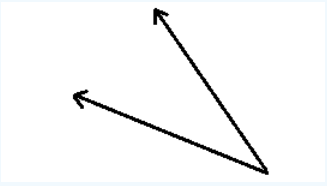


Exercises 7.1.1

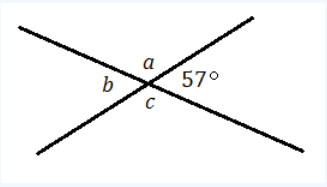
Identify each angle shown below as acute, right, obtuse, straight, or reflexive.

1. 
2. 
3. 

4. 

5. 

Find the measure of each unknown angle.

6. 

Answer

1. right angle
2. obtuse angle
3. reflexive angle
4. straight angle
5. acute angle
6. $a = 123°$; $b = 57°$; $c = 123°$

Angles in Triangles

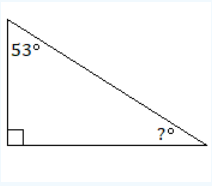
If you need to find the measures of the angles in a triangle, there are a few rules that can help.

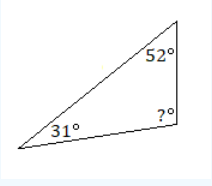
The sum of the angles of every triangle is $180°$.

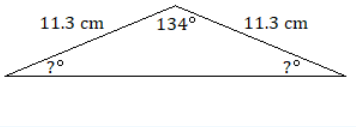
If any sides of a triangle have equal lengths, then the angles opposite those sides will have equal measures.

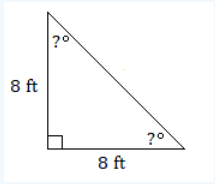
? Exercises 7.1.1

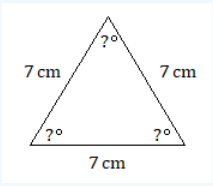
Find the measures of the unknown angles in each triangle.

7. 

8. 

9. 

10. 

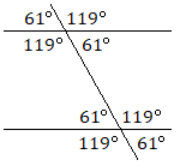
11. 

Answer

7. **37°**
8. **97°**
9. **23°** each
10. **45°** each
11. **60°** each

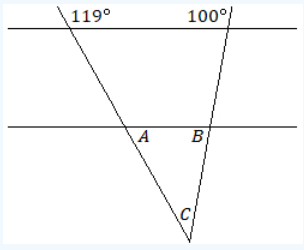
Angles and Parallel Lines

Two lines that point in the exact same direction and will never cross are called parallel lines. If two parallel lines are crossed by a third line, sets of equally-sized angles will be formed, as shown in the following diagram. All four acute angles will be equal in measure, all four obtuse angles will be equal in measure, and any acute angle and obtuse angle will have a combined measure of **180°**.



Example 7.1.1

Find the measures of angles ***A***, ***B***, and ***C***.



Solution

A. 61°

B. 80°

C. 39°

Degrees, Minutes, Seconds

It is possible to have angle measures that are not a whole number of degrees. It is common to use decimals in these situations, but the older method—called the ***degrees-minutes-seconds*** or DMS system—divides a degree using fractions out of 60: a minute is $\frac{1}{60}$ of a degree, and a second is $\frac{1}{60}$ of a minute, which means a second is $\frac{1}{3,600}$ of a degree. (Fortunately, these conversions work exactly like time; think of 1 degree as 1 hour.) For example, $2.5° = 2°30'$.



We will look at the procedure for converting between systems, but there are online calculators such as the one at https://www.fcc.gov/media/radio/dms-decimal which will do the conversions for you.

If you have latitude and longitude in DMS, like N $18°54'40''$ W $155°40'51''$, and need to convert it to decimal degrees, the process is fairly simple with a calculator.

Converting from DMS to Decimal Degrees

Enter $\text{degrees} + \text{minutes} \div 60 + \text{seconds} \div 3600$ in your calculator. Round the result to the fourth decimal place, if necessary.[1]

? Exercises 7.1.1

Convert each angle measurement from degrees-minutes-seconds into decimal form. Round to the nearest ten-thousandth, if necessary.

13. $67°48'54''$

14. $19°37'25''$

15. $34°14'12''$

Answer

13. $67.815°$
14. $19.6236°$
15. $34.2367°$

Going from decimal degrees to DMS is a more complicated process.

Converting from Decimal Degrees to DMS

1. The whole-number part of the angle measurement gives the number of degrees.
2. Multiply the decimal part by 60. The whole number part of this result is the number of minutes.
3. Multiply the decimal part *of the minutes* by 60. This gives the number of seconds (including any decimal part of seconds).

For example, let's convert $15.374°$.

1. The *degrees* part of our answer will be 15.
2. The decimal part times 60 is $0.374 \cdot 60 = 22.44$ minutes. The *minutes* part of our answer will be 22.
3. The decimal part times 60 is $0.44 \cdot 60 = 26.4$ seconds. The *seconds* part of our answer will be 26.4.

So $15.374° = 15°22'26.4''$.

? Exercises 7.1.1

Convert each angle measurement from decimal into degrees-minutes-seconds form.

16. $26.785°$

17. $58.216°$

18. $41.13°$

Answer

16. $26°47'6''$
17. $58°12'57.6''$
18. $41°7'48''$

---

1. We round to four decimal places because 1 second of angle is $\frac{1}{3,600}$ of a degree. This is a smaller fraction than $\frac{1}{1,000}$ so our precision is slightly better than the thousandths place. ↵

This page titled 7.1: Angles is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.17: Angles** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

7.2: Triangles



Triangle Basics

- All triangles have three sides. (File this fact under the “thank you Captain Obvious” category.)
- All triangles contain 180 degrees.

Figure 9. Triangle

Isosceles triangle. This triangle has two sides that are equal, and two angles that are equal.

Figure 10. Isosceles triangle

Equilateral triangle. All three sides of this triangle are equal, and all three of its angles are equal too.

Figure 11. Equilateral triangle

Similar triangles. These triangles each have different sized sides, but they share the same sized angles.

Figure 12. Similar triangles

Right triangles. These triangles have one right angle (equal to 90 degrees). This means that the other two angles are complementary, that is, they must add to 90 degrees.

Figure 13. Complementary angles.

Scalene triangle. These triangles have three sides all of different lengths.

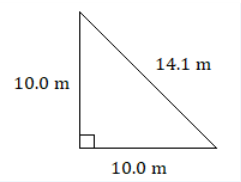
Obtuse triangle. These triangles have one obtuse angle, so one angle whose measure is greater than 90 degrees.

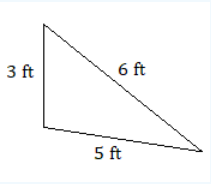
Acute triangle. These triangles have three acute angels, so all three angles measure less than 90 degrees.

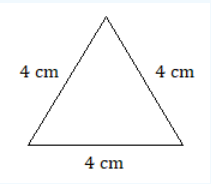
It is possible to describe a triangles based on two key characteristics, its side lengths (equilateral, isosceles or scalene) and its angle measures (right, obtuse or acute).

? Exercises 7.2.1

Classify each triangle by angle and side. For example, “acute scalene”.)

1. 

2. 

3. 

Answer

1. right isosceles triangle
2. obtuse scalene triangle
3. acute equilateral triangle (yes, an equilateral triangle will always be acute)

This page titled 7.2: Triangles is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by Morgan Chase (OpenOregon) via source content that was edited to the style and standards of the LibreTexts platform.

- **1.18: Triangles** by Morgan Chase is licensed CC BY-NC-SA 4.0. Original source: https://openoregon.pressbooks.pub/techmath.

7.3: Introduction to Right Triangle Trigonometry

Trigonometry

Trigonometry is the study of the relationship that exists between the sides and the angles of a triangle. The study of trigonometry begins by studying the interrelationships between the angles and the side lengths in every right triangle.

We have already learned how to determine the side lengths of a triangle using the Pythagorean theorem. Next up is using those sides to determine the angles. Lucky for us we know that in a right triangle we must have one 90-degree angle. And, if we know either of the other two angles, we can easily find third angle measure because the sum of all angles in a triangle is 180 degrees. Unknown angles are often labeled using letters from the Greek alphabet. In the examples below the Greek letter theta, θ.

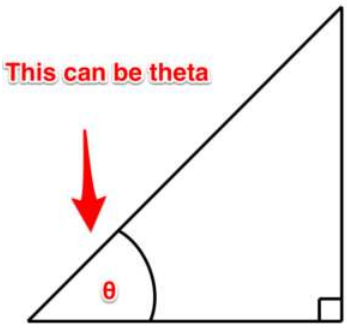


Figure 16. Theta 1

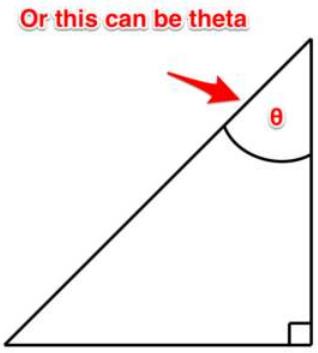


Figure 17. Theta 2

Now once you have figured out which angle is your theta, we identify the sides of the triangle base on their position in relation to theta.

Adjacent. This is the side that sits *adjacent* to theta. This side is one of the lines that touches or forms the angle.

Opposite. This is the side that sits *opposite* to theta. This side is across the triangle from the angle.

Hypotenuse. This is the side that sits *opposite* the 90-degree angle. The hypotenuse is always the longest side of the triangle.

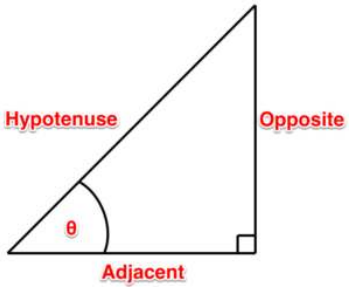


Figure 18. Hypotenuse

And if we switch which angle is labeled theta, then the names of the sides change as in Figure 19.

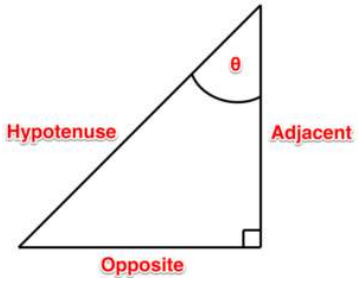


Figure 19. Adjacent and Opposite sides switch places

Defining Trigonometric Functions

When determining the designate angle we can use different ratios of sides.

- We can use a ratio of the opposite to the hypotenuse.
- We can use a ratio of the adjacent to the hypotenuse.
- We can use a ration of the opposite to the adjacent.

The trigonometric functions sine, cosine and tangent relate the measure of angle to the ratios of side lengths in right triangle containing that angle.

- sin θ = opposite/hypotenuse
- cos θ = adjacent/hypotenuse
- tan θ = opposite/adjacent

One way of remembering the ratios are these mnemonics:

- SOH – Sine is opposite/ hypotenuse
- CAH – Cosine is adjacent/hypotenuse
- TOA – Tangent is opposite/ adjacent

By the way,

- sin is short for sine
- cos is short for cosine
- tan is short for tangent

This page titled 7.3: Introduction to Right Triangle Trigonometry is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

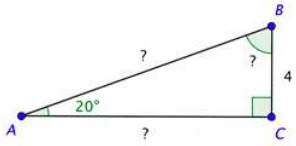
7.4: Right Triangle Trigonometry in the Real World

Learning Objectives

- Find the missing lengths and angles of a right triangle.
- Solve applied problems using right-triangle trigonometry.

Introduction

Suppose you have to build a ramp and don't know how long it needs to be. You know certain angle measurements and side lengths, but you need to find the missing pieces of information.



There are six trigonometric functions, or ratios, that you can use to compute what you don't know. You will now learn how to use these six functions to solve right-triangle application problems.

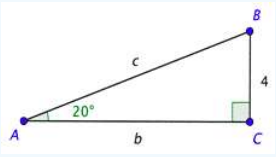
Solving Right Triangles

Determining all of the side lengths and angle measures of a right triangle is known as *solving* a right triangle. Let's look at how to do this when you're given one side length and one acute angle measure. Once you learn how to solve a right triangle, you'll be able to solve many real world applications, such as the ramp problem at the beginning of this lesson, and the only tools you'll need are the definitions of the trigonometric functions, the Pythagorean Theorem, and a calculator.

Example 7.4.2

Problem: You need to build a ramp with the following dimensions. Solve the right triangle shown below. Use the approximations $\sin 20° \approx 0.342$ and $\tan 20° \approx 0.364$, and give the lengths to the nearest tenth.

Answer



Remember that the acute angles in a right triangle are complementary, which means their sum is $90°$. Since $m\angle A = 20°$, it follows that $m\angle B = 90° - 20° = 70°$.

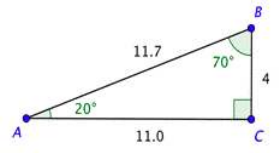
You can use the definition of sine to find c. Substitute the measure of the angle on the left side of the equation and use the triangle to set up the ratio on the right. Solving the equation and rounding to the nearest tenth gives you $c \approx 11.7$.

$$\begin{aligned} \sin 20° &= \tfrac{\text{opposite}}{\text{hypotenuse}} \\ 0.342 &\approx \tfrac{4}{c} \\ c(0.342) &\approx 4 \\ c &\approx \tfrac{4}{0.342} \approx 11.695 \end{aligned}$$

In a similar way, you can use the definition of tangent and the measure of the angle to find b. Solving the equation and rounding to the nearest tenth gives you $b \approx 11.0$.

$$\tan 20^\circ = \frac{\text{opposite}}{\text{adjacent}}$$
$$0.364 \approx \frac{4}{b}$$
$$0.364b \approx 4$$
$$b \approx \frac{4}{0.364} \approx 10.989$$

The ramp needs to be 11.7 feet long.

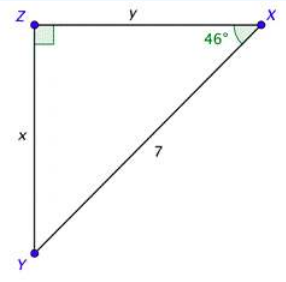


In the problem above, you were given the values of the trigonometric functions. In the next problem, you'll need to use the trigonometric function keys on your calculator to find those values.

Example 7.4.3

Problem: Solve the right triangle shown below. Give the lengths to the nearest tenth.

Answer



The acute angles are complementary, which means their sum is 90°. Since Since $m\angle X = 46^\circ$, it follows that $m\angle Y = 90^\circ - 46^\circ = 44^\circ$.

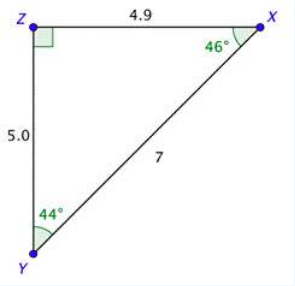
You can use the definition of sine to find x. Use your calculator to find the value of $\sin 46^\circ$ and the triangle to set up the ratio on the right. Solving the equation and rounding to the nearest tenth gives you $x \approx 5.0$.

$$\sin 46^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$0.72 \approx \frac{x}{7}$$
$$7(0.72) \approx x$$
$$5.04 \approx x$$

To find y, you can either use another trigonometric function (such as cosine) or you can use the Pythagorean Theorem. Solving the equation and rounding to the nearest tenth gives you $y \approx 4.9$.

$$5.0^2 + y^2 \approx 7^2$$
$$25 + y^2 \approx 49$$
$$y^2 \approx 24$$
$$y \approx \sqrt{24}$$
$$y \approx 4.899$$

We now know all three sides and all three angles. Their values are shown in the drawing.

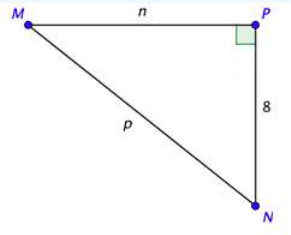


Sometimes you may be given enough information about a right triangle to solve the triangle, but that information may not include the measures of the acute angles. In this situation, you will need to use the inverse trigonometric function keys on your calculator to solve the triangle.

Example 7.4.4

Problem: Solve the right triangle shown below, given that $\tan N = \frac{5}{4}$. Find the exact side lengths and approximate the angles to the nearest degree.

Answer



You are not given an angle measure, but you can use the definition of tangent to find the value of n.

Use the ratio you are given on the left side and the information from the triangle on the right side. Cross-multiply and solve for n.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$
$$\frac{5}{4} = \frac{n}{8}$$
$$40 = 4n$$
$$n = 10$$

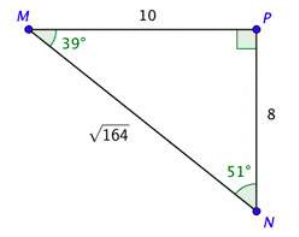
Use the Pythagorean Theorem to find the value of p.

$$10^2 + 8^2 = p^2$$
$$100 + 64 = p^2$$
$$164 = p^2$$
$$\sqrt{164} = p$$

We can use the triangle to find a value of the tangent and the inverse tangent key on your calculator to find the angle that yields that value. Rounding to the nearest degree, $\angle M$ is approximately $39°$, $M \approx 39°$. Subtract $39°$, from $90°$ to get $N \approx 51°$.

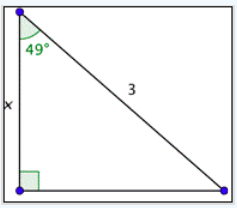
$$\tan M = \frac{8}{10} = 0.8$$
$$M = \tan^{-1} 0.8$$
$$M \approx 38.7°$$

We now know all three sides and all three angles. Their values are shown in the drawing.



? Try It 7.4.1

What is the value of x to the nearest hundredth?



A. 4.57
B. 1.97
C. 0.90
D. 0.22

Answer

A. 4.57. Incorrect. You probably set up the ratio incorrectly, equating $\cos 49°$ and $\frac{3}{x}$. A correct way to set up the equation is $\cos 49° = \frac{x}{3}$. The correct answer is 1.97.

B. 1.97. Correct. One way to set up a correct equation is to use the definition of cosine. This will give you $\cos 49° = \frac{x}{3}$. The solution to this equation is: $x = 3 \cdot \cos 49° \approx 3 \cdot (0.656) = 1.968$. This rounds to 1.97.

C. 0.90. Incorrect. You probably set up the correct equation, $\cos 49° = \frac{x}{3}$, and solved it correctly. However, your calculator was not set to degrees. The correct answer is 1.97.

D. 0.22. Incorrect. You may have correctly set up your equation as $\cos 49° = \frac{x}{3}$, but then incorrectly solved it as $x = \frac{\cos 49°}{3}$. The correct answer is 1.97.

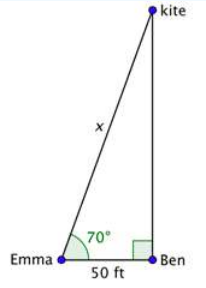
Using Trigonometry in Real-World Problems

There are situations in the real world, such as building a ramp for a loading dock, in which you have a right triangle with certain information about the sides and angles, and you wish to find unknown measures of sides or angles. This is where understanding trigonometry can help you.

✔ Example ✔ 7.4.5

Problem: Ben and Emma are out flying a kite. Emma can see that the kite string she is holding is making a $70°$ angle with the ground. The kite is directly above Ben, who is standing 50 feet away. To the nearest foot, how many feet of string has Emma let out?

Answer



We want to find the length of string let out. It is the hypotenuse of the right triangle shown.

Since the 50-foot distance measures the adjacent side to the $70°$ angle, you can use the cosine function to find x.

$$\cos 70° = \frac{50}{x}$$

Solve the equation for x. Use a calculator to find a numerical value. The answer rounds to 146.

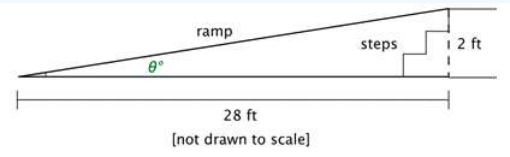
$$\begin{aligned} x \cdot \cos 70° &= 50 \\ x &= \frac{50}{\cos 70°} \\ x &= \frac{50}{0.342...} \\ x &= 146.19... \end{aligned}$$

Emma has let out approximately 146 feet of string.

In the example above, you were given one side and an acute angle. In the next one, you're given two sides and asked to find an angle. Finding an angle will usually involve using an inverse trigonometric function. The Greek letter theta, θ, is commonly used to represent an unknown angle. In this example, θ represents the angle of elevation.

✔ Example ✔ 7.4.6

Problem: A wheelchair ramp is placed over a set of stairs so that one end is 2 feet off the ground. The other end is at a point that is a horizontal distance of 28 feet away, as shown in the diagram. What is the angle of elevation to the nearest tenth of a degree?



Answer

The angle of elevation is labeled $\theta°$ in the diagram. The lengths given are the sides opposite and adjacent to this angle, so you can use the tangent function to find θ.

You want to find the measure of an angle that gives you a certain tangent value. This means that you need to find the inverse tangent. Remember that you have to use the keys 2ND and TAN on your calculator. Look at the hundredths place to round to the nearest tenth.

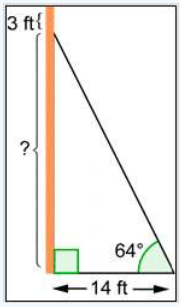
$$\begin{aligned} \tan\theta° &= \frac{2}{28} \\ \tan\theta° &= \frac{1}{14} \\ \theta° &= \tan^{-1}\left(\frac{1}{14}\right) \\ \theta &= 4.0856... \end{aligned}$$

The angle of elevation is approximately $4.1°$.

Sometimes the right triangle can be part of a bigger picture.

Try It 7.4.2

A guy wire is attached to a telephone pole 3 feet below the top of the pole, as shown below. The guy wire is anchored 14 feet from the telephone pole and makes a $64°$ angle with the ground. How high up the pole is the guy wire attached? Round your answer to the nearest tenth of a foot.



A. $14 \cdot \sin 64° \approx 12.6$ feet
B. $14 \cdot \tan 64° \approx 28.7$ feet
C. $14 \cdot \tan 64° + 3 \approx 31.7$ feet
D. $\frac{14}{\cos 64°} \approx 31.9$ feet

Answer

A. $14 \cdot \sin 64° \approx 12.6$ feet . Incorrect. You may have been confused as to which ratio corresponds to which trigonometric function. You need to solve the equation $\tan 64° = \frac{x}{14}$, where x represents the vertical distance from the base of the telephone pole up to where the guy wire is attached. The correct answer is $14 \cdot \tan 64° \approx 28.7$ feet .

B. $14 \cdot \tan 64° \approx 28.7$ feet . Correct. Let x represent the vertical distance from the base of the telephone pole up to where the guy wire is attached. Then $\tan 64° = \frac{x}{14}$. Solving this equation for x gives you $x = 14 \cdot \tan 64° \approx 28.7$ feet .

C. $14 \cdot \tan 64° + 3 \approx 31.7$ feet . Incorrect. It looks like you set up and solved the correct equation to find the unknown length. However, you misread the problem. When you added the 3 you found the height of the entire pole. The correct answer is $14 \cdot \tan 64° \approx 28.7$ feet .

D. $\frac{14}{\cos 64°} \approx 31.9$ feet . Incorrect. It looks like you set up and solved an equation to find the length of the wire (the hypotenuse of the triangle). You need to solve the equation $\tan 64° = \frac{x}{14}$, where x represents the vertical distance from the base of the telephone pole up to where the guy wire is attached. The correct answer is $14 \cdot \tan 64° \approx 28.7$ feet .

Summary

There are many ways to find the missing side lengths or angle measures in a right triangle. Solving a right triangle can be accomplished by using the definitions of the trigonometric functions and the Pythagorean Theorem. This process is called solving a right triangle. Being able to solve a right triangle is useful in solving a variety of real-world problems such as the construction of a wheelchair ramp.

This page titled 7.4: Right Triangle Trigonometry in the Real World is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by The NROC Project via source content that was edited to the style and standards of the LibreTexts platform.

- **19.1.2: Right Triangle Trigonometry** by The NROC Project is licensed CC BY-NC-SA 4.0. Original source: https://content.nroc.org/DevelopmentalMath.HTML5/Common/toc/toc_en.html.

7.5: Power and Impedance Triangles - An Elelctrical Example

Impedance triangles

When dealing with DC circuits the only thing that opposes current is the **resistance** in the circuit.

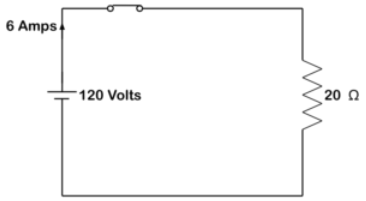


Figure 20. DC resistive circuit

AC adds a component that opposes current as well. This is called **reactance** and it runs 90 degrees to the circuit resistance. This means it is not possible to add them together arithmetically; it has to be done using the Pythagorean theorem. When you add these two together, you get a total opposition to current flow called **impedance**.

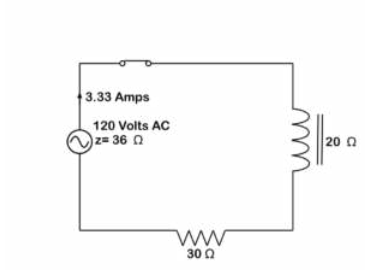


Figure 21. DC inductive circuit

The triangle that is created when adding the resistance to the reactance is known as an **impedance triangle**.

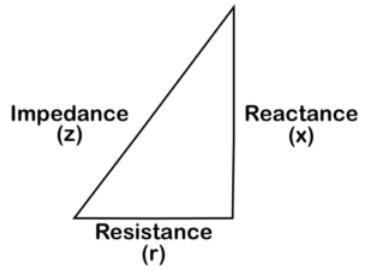


Figure 22. Impedance triangle

In an impedance triangle, the resistance (r) is always on the bottom of the triangle, the reactance (x) always goes on the side and the hypotenuse is always the impedance (z).

Power triangles

When dealing with a purely resistive circuit, the power being dissipated is in the form of heat or light and is measured in watts and is known as **true or active power**. It is a product of I^2R.

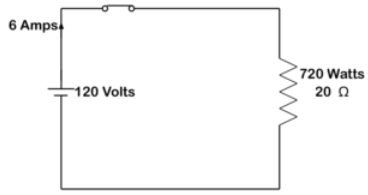


Figure 23. Resistive power circuit

In an AC circuit with inductance, watts are still present. There is also a reactive power present as current passes across the reactance. This power is called **reactive power** and is also called **wattless or quadrature power**. Its unit is the Vars.

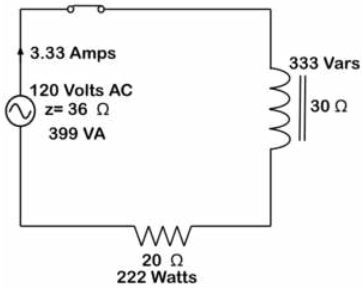


Figure 24. Inductive power circuit

Much like the impedance triangle, we can not just add the two powers together to get overall power. They must be added using the Pythagoras' theorem. Their sum is equal to the **apparent power** (VA).

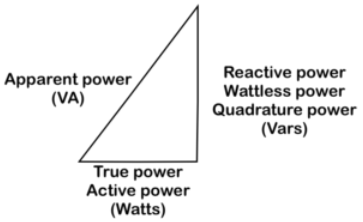


Figure 25. Power triangle

When calculating for reactive power, we are still able to use the power formulas. We just have to use them with reactance instead of resistance.

- I^2X = Vars
- E^2 (inductor voltage) /X = Vars
- I x E (inductor voltage) = Vars

Remember

When building an impedance or power triangle, the resistive component always goes on the bottom of the triangle and the reactive component always goes on the side.

This page titled 7.5: Power and Impedance Triangles - An Elelctrical Example is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

CHAPTER OVERVIEW

8: Vectors

8.1: Angles in Standard Position

8.2: Trigonometric Functions of Any Angle

8.3: Introduction to Vectors

8.4: Vector Operations

8.5: Vector Addition in the Real World - Electrical Application

---

This page titled 8: Vectors is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

8.1: Angles in Standard Position

Angle Vocabulary

To study vectors, we need a more general definition of an angle. We say that an **angle** is formed by rotating a ray $\overrightarrow{OA}$ about the endpoint O (called the **vertex**), so that the ray is in a new position, denoted by the ray $\overrightarrow{OB}$. The ray $\overrightarrow{OA}$ is called the **initial side** of the angle, and $\overrightarrow{OB}$ is the **terminal side** of the angle (see Figure 1.4.1(a)).

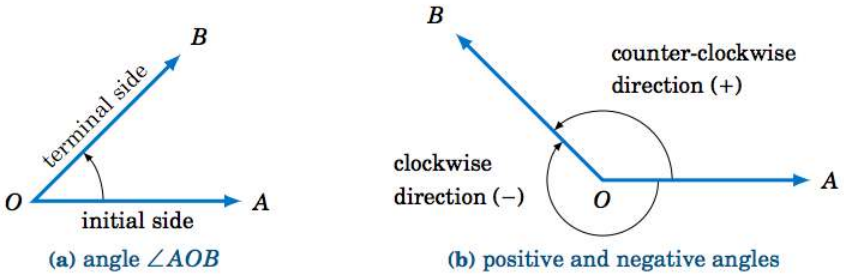


Figure 1.4.1 Definition of a general angle

We denote the angle formed by this rotation as $\angle AOB$, or simply $\angle O$, or even just O. If the rotation is counter-clockwise then we say that the angle is **positive**, and the angle is **negative** if the rotation is clockwise (see Figure 1.4.1(b)).

One full counter-clockwise rotation of $\overrightarrow{OA}$ back onto itself (called a **revolution**), so that the terminal side coincides with the initial side, is an angle of $360°$; in the clockwise direction this would be $-360°$. Not rotating $\overrightarrow{OA}$ constitutes an angle of $0°$. More than one full rotation creates an angle greater than $360°$. For example, notice that $30°$ and $390°$ have the same terminal side in Figure 1.4.2, since $30 + 360 = 390$.

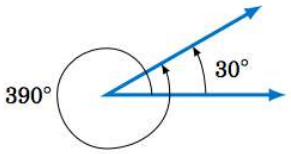


Figure 1.4.2 Angle greater than $360°$

We say that an angle θ is in **standard position** if its initial side is the positive x-axis and its vertex is the origin $(0, 0)$.

Quadrants and Signs

Recall that the **xy-coordinate plane** consists of points denoted by pairs (x, y) of real numbers. The first number, x, is the point's **x coordinate**, and the second number, y, is its **y coordinate**. The x and y coordinates are measured by their positions along the **x-axis** and **y-axis**, respectively, which determine the point's position in the plane. This divides the xy-coordinate plane into four **quadrants** (denoted by QI, QII, QIII, QIV), based on the signs of x and y (see Figure 1.4.3(a)-(b)).

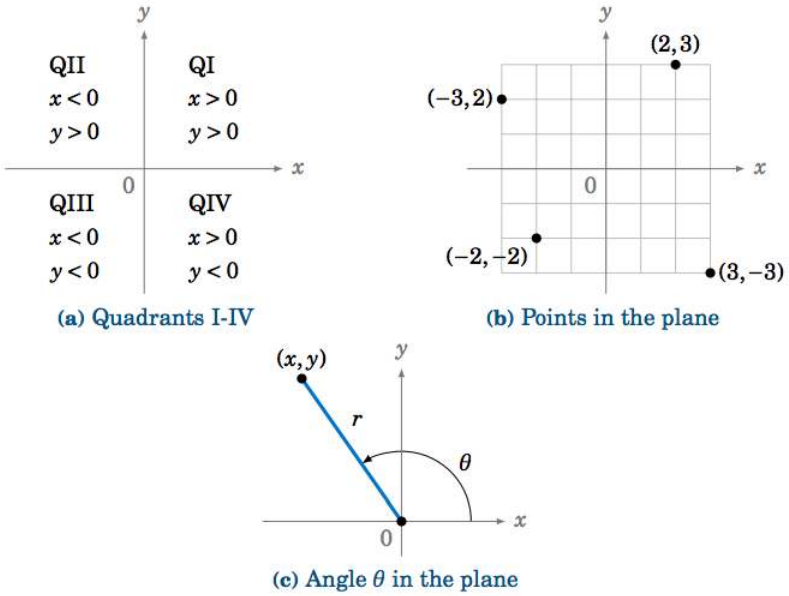


Figure 1.4.3 xy-coordinate plane

Contributors

- Michael Corral (Schoolcraft College). The content of this page is distributed under the terms of the GNU Free Documentation License, Version 1.2.

This page titled 8.1: Angles in Standard Position is shared under a CC BY-SA 1.3 license and was authored, remixed, and/or curated by Michael Corral via source content that was edited to the style and standards of the LibreTexts platform.

- **1.4: Trigonometric Functions of Any Angle** by Michael Corral is licensed GNU FDL. Original source: http://www.mecmath.net/trig/index.html.

8.2: Trigonometric Functions of Any Angle

Expanding Our Understanding of Trigonometry

Recall that we say an angle θ is in **standard position** if its initial side is the positive x-axis and its vertex is the origin $(0,0)$. Pick *any* point (x,y) on the terminal side of θ a distance $r>0$ from the origin (see Figure 1.4.3(c)). (Note that $r=\sqrt{x^2+y^2}$. Why?) We then define the trigonometric functions of θ as follows:

$$\sin\theta=\frac{y}{r} \qquad \cos\theta=\frac{x}{r} \qquad \tan\theta=\frac{y}{x} \tag{8.2.1}$$

As in the acute case, by the use of similar triangles these definitions are well-defined (i.e. they do not depend on which point (x,y) we choose on the terminal side of θ). Also, notice that $|\sin\theta|\le 1$ and $|\cos\theta|\le 1$, since $|y|\le r$ and $|x|\le r$ in the above definitions.

Notice that in the case of an acute angle these definitions are equivalent to our earlier definitions in terms of right triangles: draw a right triangle with angle θ such that $x=$ **adjacent side**, $y=$ **opposite side**, and $r=$ **hypotenuse**. For example, this would give us $\sin\theta=\frac{y}{r}=\frac{\text{opposite}}{\text{hypotenuse}}$ and $\cos\theta=\frac{x}{r}=\frac{\text{adjacent}}{\text{hypotenuse}}$, just as before (see Figure 1.4.4(a)).

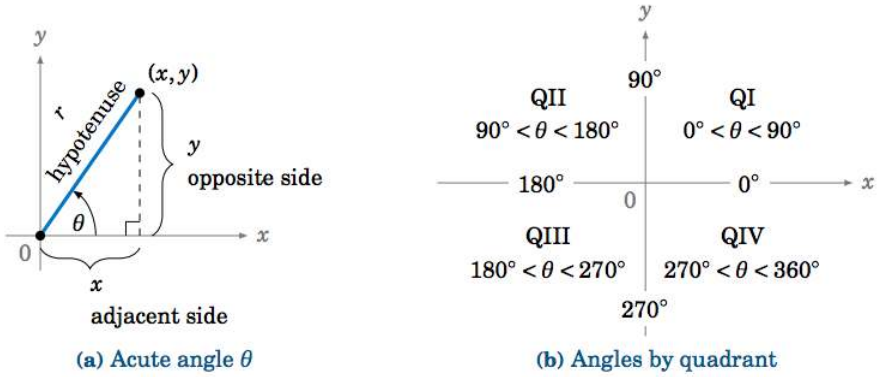


Figure 1.4.4

Since $360°$ represents one full revolution, the trigonometric function values repeat every $360°$. For example, $\sin 360°=\sin 0°$, $\cos 390°=\cos 30°$, $\tan 540°=\tan 180°$, $\sin(-45°)=\sin 315°$, etc. In general, if two angles differ by an integer multiple of $360°$ then each trigonometric function will have equal values at both angles. Angles such as these, which have the same initial and terminal sides, are called **coterminal**.

Earlier, we saw how the values of trigonometric functions of an acute angle θ were found using ratios of the side lengths of a right triangle. Now we learn that every angle larger than $90°$ is coterminal with an acute angle. That acute angle has a special name: if θ is a nonacute angle then we say that the **reference angle** for θ is the acute angle formed by the terminal side of θ and either the positive or negative x-axis. For example, $60°$ is the reference angle for the nonacute angle $\theta=120°$ and $45°$ is the reference angle for $\theta=225°$. The trigonometric function values for the nonacute angle will be the same magnitude as the trigonometric function values for the corresponding acute reference angle but the signs may be different depending on the quadrant where the terminal side of the angle is located.

Example 1.24

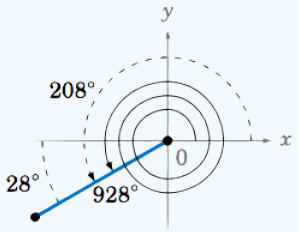


Figure 1.4.7

a. Which angle between $0°$ and $360°$ has the same terminal side (and hence the same trigonometric function values) as $928°$?

a. Which angle between $0°$ and $360°$ has the same terminal side (and hence the same trigonometric function values) as θ?
b. What is the reference angle for θ?

Solution

(a) Since $928° = 2 \times 360° + 208°$, then θ has the same terminal side as $208°$, as in Figure 1.4.7.

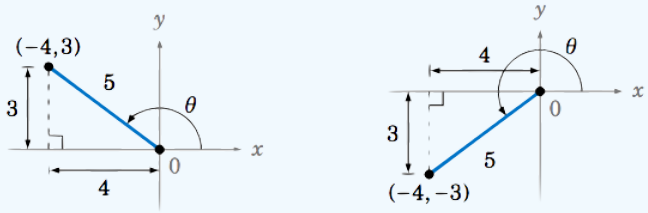
(b) $928°$ and $208°$ have the same terminal side in QIII, so the reference angle for $\theta = 928°$ is $208° - 180° = 28°$.

Example 1.25

Suppose that $\cos\ \theta = -\frac{4}{5}$. Find the exact values of $\sin\ \theta$ and $\tan\ \theta$. Also find θ in each quadrant.

Solution

Draw a right triangle and interpret $\cos\ \theta$ as the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ of two of its sides. Since $\cos\ \theta = -\frac{4}{5}$, we can use 4 as the length of the adjacent side and 5 as the length of the hypotenuse. By the Pythagorean Theorem, the length of the opposite side must then be 3. Since $\cos\ \theta$ is negative, we know from Figure 1.4.5 that θ must be in either QII or QIII. Thus, we have *two* possibilities, as shown in Figure 1.4.8 below:



(a) θ in QII (b) θ in QIII

Figure 1.4.8 $\cos\theta = -\frac{4}{5}$

When θ is in QII, we see from Figure 1.4.8(a) that the point $(-4, 3)$ is on the terminal side of θ, and so we have $x = -4$, $y = 3$, and $r = 5$. Thus, $\sin\ \theta = \frac{y}{r} = \frac{3}{5}$ and $\tan\ \theta = \frac{y}{x} = \frac{3}{-4}$.

When θ is in QIII, we see from Figure 1.4.8(b) that the point $(-4, -3)$ is on the terminal side of θ, and so we have $x = -4$, $y = -3$, and $r = 5$. Thus, $\sin\ \theta = \frac{y}{r} = \frac{-3}{5}$ and $\tan\ \theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$.

Thus, either $\boxed{\sin\ \theta = \frac{3}{5} \text{ and } \tan\ \theta = -\frac{3}{4}}$ or $\boxed{\sin\ \theta = -\frac{3}{5} \text{ and } \tan\ \theta = \frac{3}{4}}$.

If $\cos\ \theta = -\frac{4}{5}$ were solved we would get θ = 143.14, which lies in quadrant II. The reference angle is 36.87. To determine the angle in quadrant III, θ = 180 + 36.87 = 216.87.

For an angle θ in standard position and a point (x, y) on its terminal side:

a. $\sin\ \theta$ has the same sign as y
b. $\cos\ \theta$ has the same sign as x
c. $\tan\ \theta$ is positive when x and y have the same sign
d. $\tan\ \theta$ is negative when x and y have opposite signs

8.2: Trigonometric Functions of Any Angle is shared under a not declared license and was authored, remixed, and/or curated by LibreTexts.

- **1.4: Trigonometric Functions of Any Angle** by Michael Corral is licensed GNU FDL. Original source: http://www.mecmath.net/trig/index.html.

8.3: Introduction to Vectors

Learning Objectives

- View vectors geometrically.
- Find magnitude and direction.

Vector Basics

A **vector** is a quantity that possesses magnitude and direction. A vector is a specific quantity drawn as a line segment with an arrowhead at one end. It has an **initial point**, where it begins, and a **terminal point**, where it ends. A vector is defined by its magnitude, or the length of the line, and its direction, indicated by an arrowhead at the terminal point. Thus, a vector is a directed line segment. There are various symbols that distinguish vectors from other quantities:

- Lower case, boldfaced type, with or without an arrow on top such as $\mathbf{u}$, $\mathbf{w}$, $\vec{\mathbf{v}}$, $\vec{\mathbf{u}}$, $\vec{\mathbf{w}}$.
- Given initial point P and terminal point Q, a vector can be represented as $\overrightarrow{PQ}$. The arrowhead on top is what indicates that it is not just a line, but a directed line segment.
- Given an initial point of $(0,0)$ and terminal point (a,b), a vector may be represented as $\langle a,b\rangle$.
- Given an initial point of (a,b) and terminal point (c,d), a vector may be represented as $\langle c-a,d-b\rangle$.

A vector specifies only magnitude and direction, not an initial point. Unless we are given other information, we typically assume the initial point of a vector is the origin, (0,0). A vector whose initial point is the origin may be referred to as a **position vector.**

In Figure 8.3.2, we see the original vector $\overrightarrow{CD}$ and the position vector $\overrightarrow{AB}$.

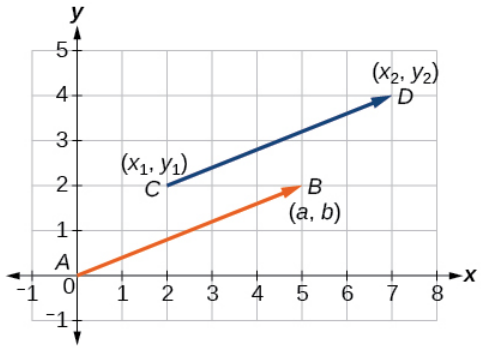


***Figure* 8.3.2**

PROPERTIES OF VECTORS

A vector is a directed line segment with an initial point and a terminal point. Vectors are identified by magnitude, or the length of the line, and direction, represented by the arrowhead pointing toward the terminal point. The position vector has an initial point at $(0,0)$ and is identified by its terminal point $\langle a,b\rangle$.

Example 8.3.1*A*: Find the Position Vector

Consider the vector whose initial point is $P(2,3)$ and terminal point is $Q(6,4)$. Find the position vector.

Solution

The position vector is found by subtracting one x-coordinate from the other x-coordinate, and one y-coordinate from the other y-coordinate. Thus

$$\begin{aligned} v &= \langle 6-2, 4-3\rangle \\ &= \langle 4,1\rangle \end{aligned}$$

The position vector begins at $(0,0)$ and terminates at $(4,1)$. The graphs of both vectors are shown in Figure 8.3.3.

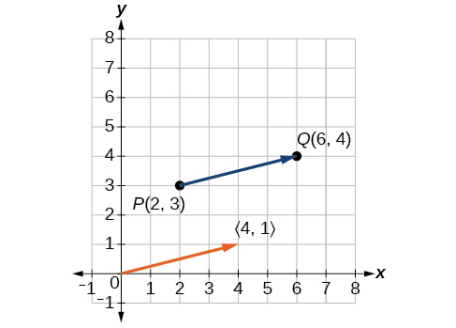


***Figure* 8.3.3**

We see that the position vector is $\langle 4,1\rangle$.

Example 8.3.1*B*: Drawing a Vector with the Given Criteria and Its Equivalent Position Vector

Find the position vector given that vector v has an initial point at $(-3,2)$ and a terminal point at $(4,5)$, then graph both vectors in the same plane.

Solution

The position vector is found using the following calculation:

$$\begin{aligned} v &= \langle 4-(-3), 5-2\rangle \\ &= \langle 7,3\rangle \end{aligned}$$

Thus, the position vector begins at $(0,0)$ and terminates at $(7,3)$. See Figure 8.3.4.

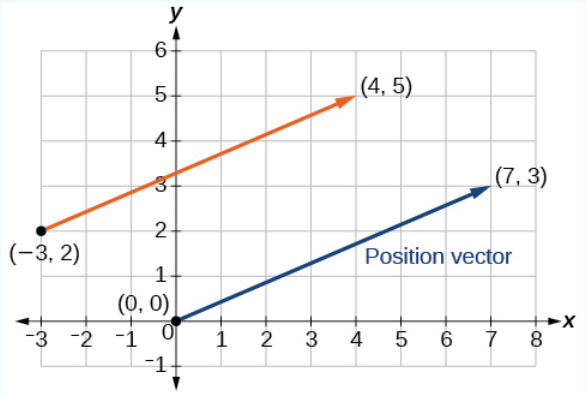


***Figure* 8.3.4**

Exercise 8.3.1

Draw a vector $\vec{v}$ that connects from the origin to the point $(3,5)$.

Answer

https://math.libretexts.org/@go/page/194244

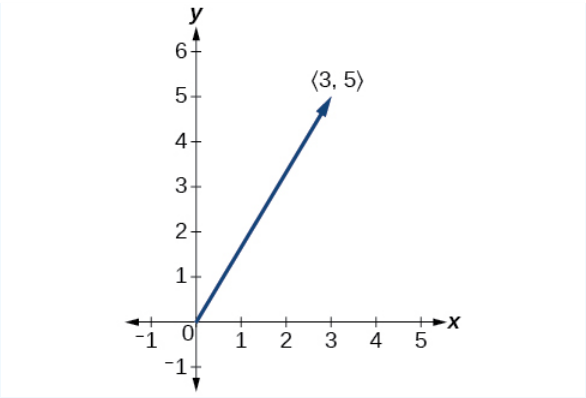


***Figure* 8.3.5**

Finding Magnitude and Direction

The **magnitude of a vector** is its length or distance which we can find by visualizing the vector as the hypotenuse of a right triangle and using the Pythagorean Theorem. When the magnitude of a vector may be represented with absolute value bars around the vector as in |<a,b>| or |v|.

The **direction of a vector** is described using an angle and can be found using the inverse of our tangent function on a scientific calculator. Traditionally the direction of a vector should be the smallest positive angle formed by the positive x-axis and the position vector.

MAGNITUDE AND DIRECTION OF A VECTOR

Given a position vector $\vec{v} = \langle a, b \rangle$,the magnitude is found by $|v| = \sqrt{a^2 + b^2}$.The direction is equal to the angle formed with the x -axis, or with the y-axis, depending on the application. For a position vector, the direction is found by $\tan\theta = \left(\frac{b}{a}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$, as illustrated in Figure 8.3.6.

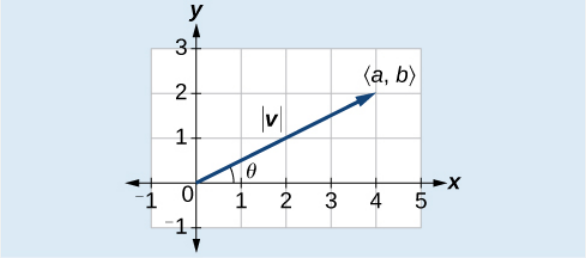


***Figure* 8.3.6**

Two vectors $\vec{v}$ and $\vec{u}$ are considered equal if they have the same magnitude and the same direction. Additionally, if both vectors have the same position vector, they are equal.

Example 8.3.2A: Finding the Magnitude and Direction of a Vector

Find the magnitude and direction of the vector with initial point $P(-8, 1)$ and terminal point $Q(-2, -5)$.Draw the vector.

Solution

First, find the position vector.

$$u = \langle -2, -(-8), -5 - 1 \rangle$$
$$= \langle 6, -6 \rangle$$

We use the Pythagorean Theorem to find the magnitude.

$$|u| = \sqrt{(6)^2 + (-6)^2}$$
$$= \sqrt{72}$$
$$= \sqrt{62}$$

The direction is given as

$$\tan\theta = \frac{-6}{6} = -1 \rightarrow \theta = \tan^{-1}(-1)$$
$$= -45°$$

However, the angle terminates in the fourth quadrant, so we add $360°$ to obtain a positive angle. Thus, $-45°+360°= 315°$. See Figure 8.3.7.

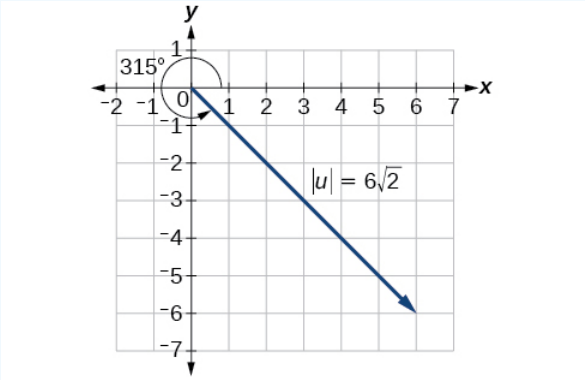


***Figure* 8.3.7**

Example 8.3.2*B*: Showing That Two Vectors Are Equal

Show that vector $\vec{v}$ with initial point at $(5, -3)$ and terminal point at $(-1, 2)$ is equal to vector $\vec{u}$ with initial point at $(-1, -3)$ and terminal point at $(-7, 2)$. Draw the position vector on the same grid as $\vec{v}$ and $\vec{u}$. Next, find the magnitude and direction of each vector.

Solution

As shown in Figure 8.3.8, draw the vector $\vec{v}$ starting at initial $(5, -3)$ and terminal point $(-1, 2)$. Draw the vector $\vec{u}$ with initial point $(-1, -3)$ and terminal point $(-7, 2)$. Find the standard position for each.

Next, find and sketch the position vector for $\vec{v}$ and $\vec{u}$. We have

$$v = \langle -1 - 5, 2 - (-3) \rangle$$
$$= \langle -6, 5 \rangle u$$
$$= \langle -7 - (-1), 2 - (-3) \rangle$$
$$= \langle -6, 5 \rangle$$

Since the position vectors are the same, $\vec{v}$ and $\vec{u}$ are the same.

An alternative way to check for vector equality is to show that the magnitude and direction are the same for both vectors. To show that the magnitudes are equal, use the Pythagorean Theorem.

$$
\begin{aligned}
|v| &= \sqrt{(-1-5)^2 + (2-(-3))^2} \\
&= \sqrt{(-6)^2 + (5)^2} \\
&= \sqrt{36+25} \\
&= \sqrt{61} \\
|u| &= \sqrt{(-7-(-1))^2 + (2-(-3))^2} \\
&= \sqrt{(-6)^2 + (5)^2} \\
&= \sqrt{36+25} \\
&= \sqrt{61}
\end{aligned}
$$

As the magnitudes are equal, we now need to verify the direction. Using the tangent function with the position vector gives

$$
\begin{aligned}
\tan\theta = -\frac{5}{6} \Rightarrow \theta &= \tan^{-1}\left(-\frac{5}{6}\right) \\
&= -39.8°
\end{aligned}
$$

However, we can see that the position vector terminates in the second quadrant, so we add $180°$. Thus, the direction is $-39.8° + 180° = 140.2°$.

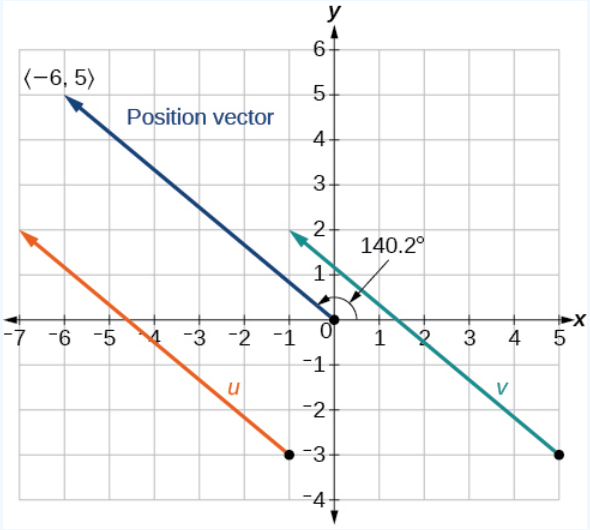


Figure 8.3.8

This page titled 8.3: Introduction to Vectors is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

8.4: Vector Operations

Learning Objectives

- Perform vector addition and scalar multiplication.
- Find the component form of a vector.
- Find the unit vector in the direction of v.
- Perform operations with vectors in terms of i and j.
- Find the dot product of two vectors.

Performing Vector Addition and Scalar Multiplication

Now that we understand the properties of vectors, we can perform operations involving them. While it is convenient to think of the vector $u = \langle x, y \rangle$ as an arrow or directed line segment from the origin to the point (x, y), vectors can be situated anywhere in the plane. The sum of two vectors $\vec{u}$ and $\vec{v}$, or vector addition, produces a third vector $\overrightarrow{u+v}$, the resultant vector.

To find $\overrightarrow{u+v}$, we first draw the vector $\vec{u}$, and from the terminal end of $\vec{u}$, we drawn the vector $\vec{v}$. In other words, we have the initial point of $\vec{v}$ meet the terminal end of $\vec{u}$. This position corresponds to the notion that we move along the first vector and then, from its terminal point, we move along the second vector. The sum $\overrightarrow{u+v}$ is the resultant vector because it results from addition or subtraction of two vectors. The resultant vector travels directly from the beginning of $\vec{u}$ to the end of $\vec{v}$ in a straight path, as shown in Figure 8.4.9.

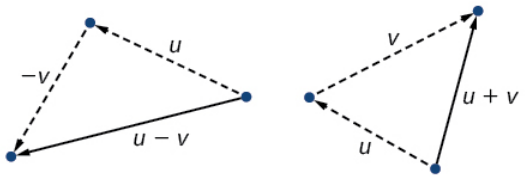


***Figure* 8.4.9**

Vector subtraction is similar to vector addition. To find $\overrightarrow{u-v}$, view it as $\overrightarrow{u+(-v)}$. Adding $\overrightarrow{-v}$ is reversing direction of $\vec{v}$ and adding it to the end of $\vec{u}$. The new vector begins at the start of $\vec{u}$ and stops at the end point of $\overrightarrow{-v}$. See Figure 8.4.10 for a visual that compares vector addition and vector subtraction using parallelograms.

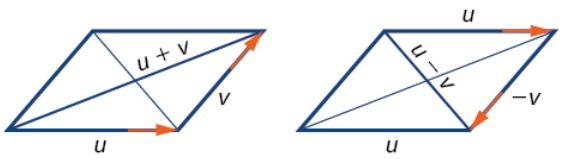


***Figure* 8.4.10**

Example 8.4.3: Adding and Subtracting Vectors

Given $u = \langle 3, -2 \rangle$ and $v = \langle -1, 4 \rangle$, find two new vectors $\overrightarrow{u+v}$, and $\overrightarrow{u-v}$.

Solution

To find the sum of two vectors, we add the components. Thus,

$$\begin{aligned} u + v &= \langle 3, -2 \rangle + \langle -1, 4 \rangle \\ &= \langle 3 + (-1), -2 + 4 \rangle \\ &= \langle 2, 2 \rangle \end{aligned}$$

See Figure 8.4.11*a*.

To find the difference of two vectors, add the negative components of $\vec{v}$ to $\vec{u}$. Thus,

$$\begin{aligned} u+(-v) &= \langle 3,-2\rangle + \langle 1,-4\rangle \\ &= \langle 3+1,-2+(-4)\rangle \\ &= \langle 4,-6\rangle \end{aligned}$$

See Figure 8.4.11*b*.

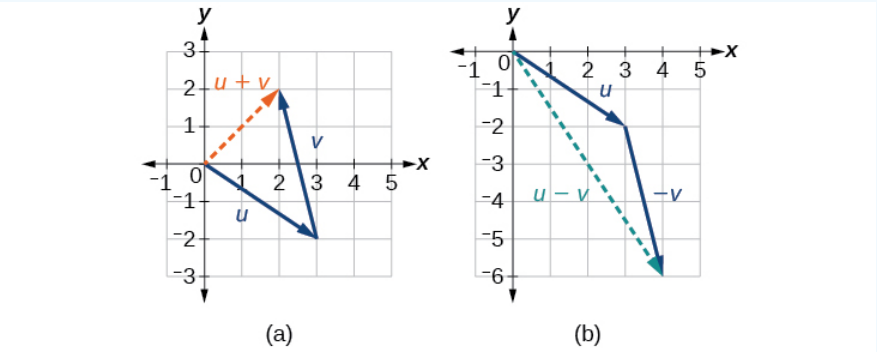


***Figure* 8.4.11:** *(a) Sum of two vectors (b) Difference of two vectors*

Multiplying By a Scalar

While adding and subtracting vectors gives us a new vector with a different magnitude and direction, the process of multiplying a vector by a **scalar**, a constant, changes only the magnitude of the vector or the length of the line. Scalar multiplication has no effect on the direction unless the scalar is negative, in which case the direction of the resulting vector is opposite the direction of the original vector.

SCALAR MULTIPLICATION

Scalar multiplication involves the product of a vector and a scalar. Each component of the vector is multiplied by the scalar. Thus, to multiply $v = \langle a, b\rangle$ by k, we have

$$kv = \langle ka, kb\rangle$$

Only the magnitude changes, unless k is negative, and then the vector reverses direction.

Example 8.4.4: Performing Scalar Multiplication

Given vector $\vec{v} = \langle 3, 1\rangle$, find $3\vec{v}$, $\frac{1}{2}$, and $-\vec{v}$.

Solution

See Figure 8.4.12 for a geometric interpretation. If $\vec{v} = \langle 3, 1\rangle$, then

$$\begin{aligned} 3v &= \langle 3\cdot 3, 3\cdot 1\rangle \\ &= \langle 9, 3\rangle \\ \frac{1}{2}v &= \langle \frac{1}{2}\cdot 3, \frac{1}{2}\cdot 1\rangle \\ &= \langle \frac{3}{2}, \frac{1}{2}\rangle \\ -v &= \langle -3, -1\rangle \end{aligned}$$

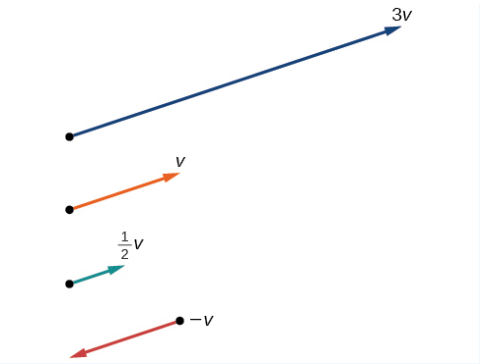


***Figure* 8.4.12**

Analysis

Notice that the vector $3\vec{v}$ is three times the length of $\vec{v}$, $\frac{1}{2}\vec{v}$ is half the length of $\vec{v}$, and $\overrightarrow{-v}$ is the same length of $\vec{v}$, but in the opposite direction.

? Exercise 8.4.2

Find the scalar multiple $3u$ given $\vec{u} = \langle 5, 4 \rangle$.

Answer

$3u = \langle 15, 12 \rangle$

✓ Example 8.4.5

Find a linear equation to solve for the following unknown quantities: One number exceeds another number by 17 and their sum is 31. Find the two numbers.

Solution

First, we must multiply each vector by the scalar.

$$\begin{aligned} 3u &= 3\langle 3, -2 \rangle \\ &= \langle 9, -6 \rangle \\ 2v &= 2\langle -1, 4 \rangle \\ &= \langle -2, 8 \rangle \end{aligned}$$

Then, add the two together.

$$\begin{aligned} w &= 3u + 2v \\ &= \langle 9, -6 \rangle + \langle -2, 8 \rangle \\ &= \langle 9 - 2, -6 + 8 \rangle \\ &= \langle 7, 2 \rangle \end{aligned}$$

So, $w = \langle 7, 2 \rangle$.

Finding Component Form

In some applications involving vectors, it is helpful for us to be able to break a vector down into its components. Vectors are comprised of two components: the horizontal component is the x direction, and the vertical component is the y direction. For example, we can see in the graph in Figure **8.4.13** that the position vector $\langle 2, 3 \rangle$ comes from adding the vectors v_1 and v_2. We have v_2 with initial point $(0, 0)$ and terminal point $(2, 0)$.

$$v_1 = \langle 2-0, 0-0 \rangle$$
$$= \langle 2, 0 \rangle$$

We also have v_2 with initial point $(0, 0)$ and terminal point $(0, 3)$.

$$v_2 = \langle 0-0, 3-0 \rangle$$
$$= \langle 0, 3 \rangle$$

Therefore, the position vector is

$$v = \langle 2+0, 3+0 \rangle$$
$$= \langle 2, 3 \rangle$$

Using the Pythagorean Theorem, the magnitude of v_1 is 2, and the magnitude of v_2 is 3. To find the magnitude of v, use the formula with the position vector.

$$|v| = \sqrt{|v_1|^2 + |v_2|^2}$$
$$= \sqrt{2^2 + 3^2}$$
$$= \sqrt{13}$$

The magnitude of v is $\sqrt{13}$. To find the direction, we use the tangent function $\tan\theta = \frac{y}{x}$.

$$\tan\theta = \frac{v_2}{v_1}$$
$$\tan\theta = \frac{3}{2}$$
$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$$

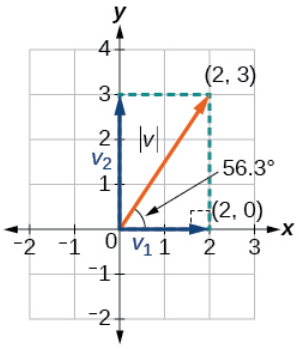


***Figure* 8.4.13**

Thus, the magnitude of $\vec{v}$ is $\sqrt{13}$ and the direction is $56.3°$ off the horizontal.

Example 8.4.6: Finding the Components of the Vector

Find the components of the vector $\vec{v}$ with initial point $(3, 2)$ and terminal point $(7, 4)$.

Solution

First find the standard position.

$$v = \langle 7-3, 4-2 \rangle$$
$$= \langle 4, 2 \rangle$$

See the illustration in Figure 8.4.14.

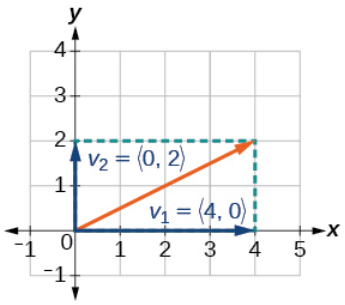


***Figure* 8.4.14**

The horizontal component is $\vec{v_1} = \langle 4, 0 \rangle$ and the vertical component is $\vec{v_2} = \langle 0, 2 \rangle$.

Finding the Unit Vector in the Direction of v

In addition to finding a vector's components, it is also useful in solving problems to find a vector in the same direction as the given vector, but of magnitude 1. We call a vector with a magnitude of 1 a unit vector. We can then preserve the direction of the original vector while simplifying calculations.

Unit vectors are defined in terms of components. The horizontal unit vector is written as $\vec{i} = \langle 1, 0 \rangle$ and is directed along the positive horizontal axis. The vertical unit vector is written as $\vec{j} = \langle 0, 1 \rangle$ and is directed along the positive vertical axis. See Figure 8.4.15.

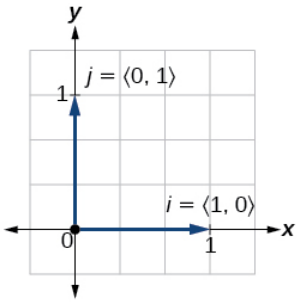


***Figure* 8.4.15**

THE UNIT VECTORS

If $\vec{v}$ is a nonzero vector, then $\frac{v}{|v|}$ is a unit vector in the direction of v. Any vector divided by its magnitude is a unit vector. Notice that magnitude is always a scalar, and dividing by a scalar is the same as multiplying by the reciprocal of the scalar.

Example 8.4.7: Finding the Unit Vector in the Direction of v

Find a unit vector in the same direction as $v = \langle -5, 12 \rangle$.

Solution

First, we will find the magnitude.

$$\begin{aligned} |v| &= \sqrt{(-5)^2 + (12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Then we divide each component by $|v|$, which gives a unit vector in the same direction as $\vec{v}$:

$$\frac{v}{|v|} = -\frac{5}{13}i + \frac{12}{13}j$$

or, in component form

$$\frac{v}{|v|} = \left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle$$

See Figure 8.4.16.

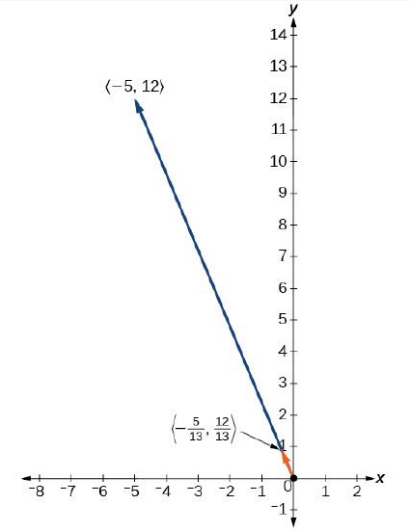


***Figure* 8.4.16**

Verify that the magnitude of the unit vector equals 1. The magnitude of $-\frac{5}{13}i + \frac{12}{13}j$ is given as

$$\begin{aligned}\sqrt{\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} &= \sqrt{\frac{25}{169} + \frac{144}{169}} \\ &= \sqrt{\frac{169}{169}} \\ &= 1\end{aligned}$$

The vector $u = \frac{5}{13}i + \frac{12}{13}j$ is the unit vector in the same direction as $v = \langle -5, 12 \rangle$.

Performing Operations with Vectors in Terms of i and j

So far, we have investigated the basics of vectors: magnitude and direction, vector addition and subtraction, scalar multiplication, the components of vectors, and the representation of vectors geometrically. Now that we are familiar with the general strategies used in working with vectors, we will represent vectors in rectangular coordinates in terms of i and j.

VECTORS IN THE RECTANGULAR PLANE

Given a vector $\vec{v}$ with initial point $P = (x_1, y_1)$ and terminal point $Q = (x_2, y_2)$, $\vec{v}$ is written as

$$v = (x_2 - x_1)i + (y_1 - y_2)j \tag{8.4.1}$$

The position vector from $(0, 0)$ to (a, b), where $(x_2 - x_1) = a$ and $(y_2 - y_1) = b$, is written as $\vec{v} = \vec{a}i + \vec{b}j$. This vector sum is called a linear combination of the vectors $\vec{i}$ and $\vec{j}$.

The magnitude of $\vec{v} = \vec{a}i + \vec{b}j$ is given as $|v| = \sqrt{a^2 + b^2}$. See Figure 8.4.17.

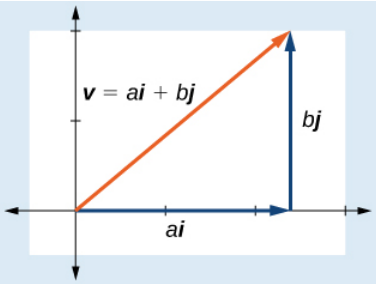


Figure 8.4.17

Example 8.4.8A: Writing a Vector in Terms of i and j

Given a vector $\vec{v}$ with initial point $P = (2, -6)$ and terminal point $Q = (-6, 6)$, write the vector in terms of $\vec{i}$ and $\vec{j}$.

Solution

Begin by writing the general form of the vector. Then replace the coordinates with the given values.

$$\begin{aligned} v &= (x_2 - x_1)i + (y_2 - y_1)j \\ &= (-6 - 2)i + (6 - (-6))j \\ &= -8i + 12j \end{aligned}$$

Example 8.4.8B: Writing a Vector in Terms of i and j Using Initial and Terminal Points

Given initial point $P_1 = (-1, 3)$ and terminal point $P_2 = (2, 7)$, write the vector $\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$.

Solution

Begin by writing the general form of the vector. Then replace the coordinates with the given values.

$$\begin{aligned} v &= (x_2 - x_1)i + (y_2 - y_1)j \\ v &= (2 - (-1))i + (7 - 3)j \\ &= 3i + 4j \end{aligned}$$

Exercise 8.4.3

Write the vector $\vec{u}$ with initial point $P = (-1, 6)$ and terminal point $Q = (7, -5)$ in terms of $\vec{i}$ and $\vec{j}$.

Answer

$u = 8i - 11j$

Performing Operations on Vectors in Terms of i and j

When vectors are written in terms of i and j, we can carry out addition, subtraction, and scalar multiplication by performing operations on corresponding components.

ADDING AND SUBTRACTING VECTORS IN RECTANGULAR COORDINATES

Given $v = ai + bj$ and $u = ci + dj$, then

$$\begin{aligned} v + u &= (a + c)i + (b + d)j \\ v - u &= (a - c)i + (b - d)j \end{aligned}$$

Example 8.4.9: Finding the Sum of the Vectors

Find the sum of $v_1 = 2i - 3j$ and $v_2 = 4i + 5j$.

Solution

$$\begin{aligned} v_1 + v_2 &= (2+4)i + (-3+5)j \\ &= 6i + 2j \end{aligned}$$

Calculating the Component Form of a Vector: Direction

We have seen how to draw vectors according to their initial and terminal points and how to find the position vector. We have also examined notation for vectors drawn specifically in the Cartesian coordinate plane using i and j. For any of these vectors, we can calculate the magnitude. Now, we want to combine the key points, and look further at the ideas of magnitude and direction.

Calculating direction follows the same straightforward process we used for polar coordinates. We find the direction of the vector by finding the angle to the horizontal. We do this by using the basic trigonometric identities, but with $|v|$ replacing r.

VECTOR COMPONENTS IN TERMS OF MAGNITUDE AND DIRECTION

Given a position vector $v = \langle x, y \rangle$ and a direction angle θ,

$$\cos\theta = \frac{x}{|v|} \text{ and } \sin\theta = y|v|$$
$$x = |v|\cos\theta$$
$$y = |v|\sin\theta$$

Thus, $v = xi + yj = |v|\cos\theta i + |v|\sin\theta j$, and magnitude is expressed as $|v| = \sqrt{x^2 + y^2}$.

Example 8.4.10: Writing a Vector in Terms of Magnitude and Direction

Write a vector with length 7 at an angle of $135°$ to the positive x-axis in terms of magnitude and direction.

Solution

Using the conversion formulas $x = |v|\cos\theta i$ and $y = |v|\sin\theta j$, we find that

$$\begin{aligned} x &= 7\cos(135°)i \\ &= -\frac{7\sqrt{2}}{2} \\ y &= 7\sin(135°)j \\ &= \frac{7\sqrt{2}}{2} \end{aligned}$$

This vector can be written as $v = 7\cos(135°)i + 7\sin(135°)j$ or simplified as

$$v = -\frac{7\sqrt{2}}{2}i + \frac{7\sqrt{2}}{2}j$$

Exercise 8.4.4

A vector travels from the origin to the point $(3, 5)$. Write the vector in terms of magnitude and direction.

Answer

$v = \sqrt{34}\cos(59°)i + \sqrt{34}\sin(59°)j$

Magnitude = 34

$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$

Finding the Dot Product of Two Vectors

As we discussed earlier in the section, scalar multiplication involves multiplying a vector by a scalar, and the result is a vector. As we have seen, multiplying a vector by a number is called scalar multiplication. If we multiply a vector by a vector, there are two possibilities: the *dot product* and the *cross product*. We will only examine the dot product here; you may encounter the cross product in more advanced mathematics courses.

The dot product of two vectors involves multiplying two vectors together, and the result is a scalar.

DOT PRODUCT

The dot product of two vectors $v = \langle a, b \rangle$ and $u = \langle c, d \rangle$ is the sum of the product of the horizontal components and the product of the vertical components.

$$v \cdot u = ac + bd \tag{8.4.2}$$

To find the angle between the two vectors, use the formula below.

$$\cos\theta = \frac{v}{|v|} \cdot \frac{u}{|u|} \tag{8.4.3}$$

Example 8.4.11A: Finding the Dot Product of Two Vectors

Find the dot product of $v = \langle 5, 12 \rangle$ and $u = \langle -3, 4 \rangle$.

Solution

Using the formula, we have

$$\begin{aligned} v \cdot u &= \langle 5, 12 \rangle \cdot \langle -3, 4 \rangle \\ &= 5 \cdot (-3) + 12 \cdot 4 \\ &= -15 + 48 \\ &= 33 \end{aligned}$$

Example 8.4.11B: Finding the Dot Product of Two Vectors and the Angle between Them

Find the dot product of $v_1 = 5i + 2j$ and $v_2 = 3i + 7j$. Then, find the angle between the two vectors.

Solution

Finding the dot product, we multiply corresponding components.

$$\begin{aligned} v_1 \cdot v_2 &= \langle 5, 2 \rangle \cdot \langle 3, 7 \rangle \\ &= 5 \cdot 3 + 2 \cdot 7 \\ &= 15 + 14 \\ &= 29 \end{aligned}$$

To find the angle between them, we use the formula $\cos\theta = \frac{v}{|v|} \cdot \frac{u}{|u|}$.

$$\begin{aligned} \frac{v}{|v|} \cdot \frac{u}{|u|} &= \left\langle \frac{5}{\sqrt{29}} + \frac{2}{\sqrt{29}} \right\rangle \cdot \left\langle \frac{3}{\sqrt{58}} + \frac{7}{\sqrt{58}} \right\rangle \\ &= \frac{5}{\sqrt{29}} \cdot \frac{3}{\sqrt{58}} + \frac{2}{\sqrt{29}} \cdot \frac{7}{\sqrt{58}} \\ &= \frac{15}{\sqrt{1682}} + \frac{14}{\sqrt{1682}} \\ &= \frac{29}{\sqrt{1682}} \\ &= 0.707107 \end{aligned}$$

$$\cos^{-1}(0.707107) = 45°$$

See Figure 8.4.18.

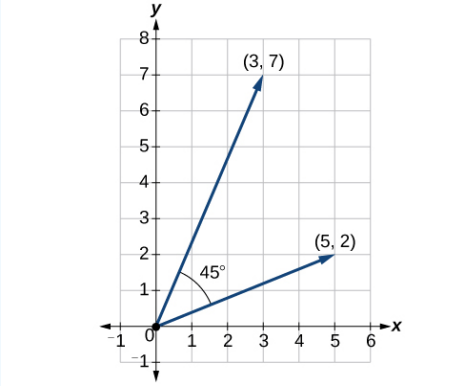


***Figure* 8.4.18**

Example 8.4.11C: Finding the Angle between Two Vectors

Find the angle between $u = \langle -3, 4 \rangle$ and $v = \langle 5, 12 \rangle$.

Solution

Using the formula, we have

$$\begin{aligned} \theta &= \cos^{-1}\left(\frac{u}{|u|} \cdot \frac{v}{|v|}\right) \\ \left(\frac{u}{|u|} \cdot \frac{v}{|v|}\right) &= \frac{-3i + 4j}{5} \cdot \frac{5i + 12j}{13} \\ &= \left(-\frac{3}{5} \cdot \frac{5}{13}\right) + \left(\frac{4}{5} \cdot \frac{12}{13}\right) \\ &= -\frac{15}{65} + \frac{48}{65} \\ &= \frac{33}{65} \\ \theta &= \cos^{-1}\left(\frac{33}{65}\right) \\ &= 59.5^\circ \end{aligned}$$

See Figure 8.4.19.

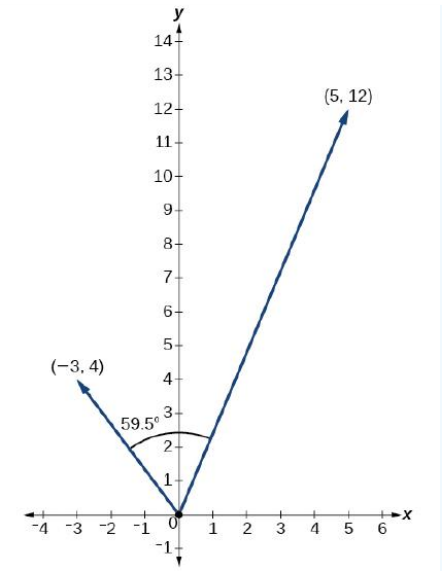


***Figure* 8.4.19**

Media: Access these online resources for additional instruction and practice with vectors.

- Introduction to Vectors
- Vector Operations
- The Unit Vector

Key Concepts

- The position vector has its initial point at the origin. See Example **8.4.1**.
- If the position vector is the same for two vectors, they are equal. See Example **8.4.2**.
- Vectors are defined by their magnitude and direction. See Example **8.4.3**.
- If two vectors have the same magnitude and direction, they are equal. See Example **8.4.4**.
- Vector addition and subtraction result in a new vector found by adding or subtracting corresponding elements. See Example **8.4.5**.
- Scalar multiplication is multiplying a vector by a constant. Only the magnitude changes; the direction stays the same. See Example **8.4.6** and Example **8.4.7**.
- Vectors are comprised of two components: the horizontal component along the positive x-axis, and the vertical component along the positive y-axis. See Example **8.4.8**.
- The unit vector in the same direction of any nonzero vector is found by dividing the vector by its magnitude.
- The magnitude of a vector in the rectangular coordinate system is $|v| = \sqrt{a^2 + b^2}$. See Example **8.4.9.**
- In the rectangular coordinate system, unit vectors may be represented in terms of ii and jj where i represents the horizontal component and j represents the vertical component. Then, $v = ai + bj$ is a scalar multiple of v by real numbers a and b. See Example **8.4.10** and Example **8.4.11**.
- Adding and subtracting vectors in terms of i and j consists of adding or subtracting corresponding coefficients of i and corresponding coefficients of j. See Example **8.4.12**.
- A vector $v = ai + bj$ is written in terms of magnitude and direction as $v = |v| \cos \theta i + |v| \sin \theta j$. See Example **8.4.13**.
- The dot product of two vectors is the product of the i terms plus the product of the j terms. See Example **8.4.14**.
- We can use the dot product to find the angle between two vectors. Example **8.4.15** and Example **8.4.16**.
- Dot products are useful for many types of physics applications. See Example **8.4.17**.

Contributors and Attributions

- Jay Abramson (Arizona State University) with contributing authors. Textbook content produced by OpenStax College is licensed under a Creative Commons Attribution License 4.0 license. Download for free

at https://openstax.org/details/books/precalculus.

This page titled 8.4: Vector Operations is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

8.5: Vector Addition in the Real World - Electrical Application

Electrical Notation for Vectors

We have seen that vectors can be expressed using several different notations. v = <0,1> and v = 1j are two notations we have seen. In an electrical context you may see another notation where the magnitude is given first in volts and the direction is given second in degrees. For example 1V@(\0^\circ\).

An Application of Vector Addition

AC values are constantly changing magnitude and direction. We will talk about this more in-depth in the AC generation portion of the course. Eventually, we will be required to add these values together. The sum of the vectors is called the **resultant**. This is all well and good when vectors are heading in the same direction…

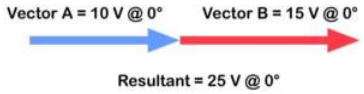


Figure 26. Vectors in the same direction

… because you can just add them together.

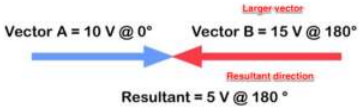


Figure 27. Vectors in the opposite direction

It isn't even bad if they are heading in the opposite direction. You can just subtract them. The only thing when *adding* them in opposite directions is that you have to pay attention to which vector has the greatest value. This will become the new direction of the sum of the vectors.

The problem arises when they are heading in completely different directions.

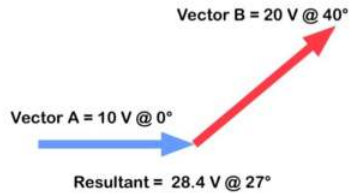


Figure 28. Vectors moving in different direction

When adding vectors, we have to find some common ground. This is why we focus on the X and Y coordinates. Each vector can be broken down into X and Y coordinates. This allows us to find some common ground as the X coordinates are heading in the same direction and the Y coordinates are heading in the same direction. Let's look at the example in Figure 39.

In this example, we have one vector that is 38 V @ 20 degrees and another that is 100 V @ 135 degrees.

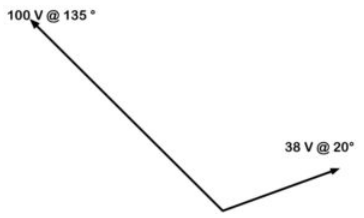


Figure 39. Adding vectors

The first step is to draw in the X and Y axes. (See Figure 40.) This will help provide us with a reference when determining our X and Y coordinates.

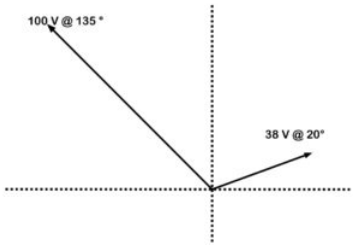


Figure 40. Adding vectors draw quadrant

Next we will work out the X and the Y for each vector, in Figure 41.

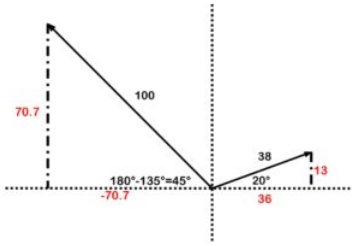


Figure 41. Adding vectors determining X and Y

Next add an X-Y chart and enter the coordinates (Figure 42).

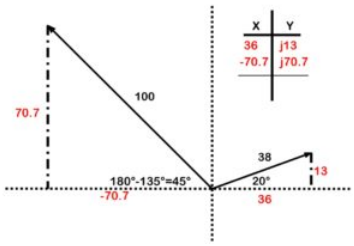


Figure 42. Adding vectors X and Y chart

Add up your X coordinates and your Y coordinates and you have your answer in rectangular form (Figure 43).

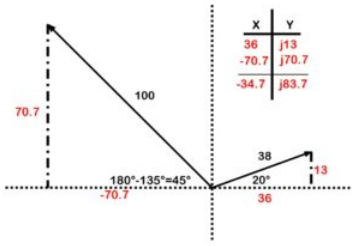


Figure 43. Adding vectors for rectangular form

Take your rectangular form and chart it on its own (Figure 44).

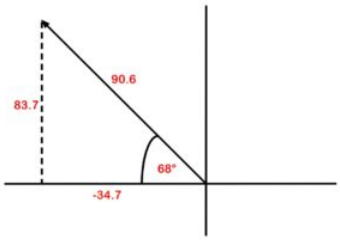


Figure 44. Adding vectors for polar form

Take the resultant and the angle, and convert it to polar form: 90.6 @ (180°−68°)90.6 @ 112°.

There you have it. If you have more vectors, you just keep adding other rows to your X-Y chart.



Video! This video walks through how to add vectors heading in completely different directions.

This page titled 8.5: Vector Addition in the Real World - Electrical Application is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

CHAPTER OVERVIEW

9: Sinusoidal Models

9.1: Periodic Behaviors in the Real World

9.2: Sinusoidal Graphs

9.3: More Sinusoidal Modeling

9.4: Sinusoidal Models (Exercises)

9.5: AC Waveform - An Electrical Application

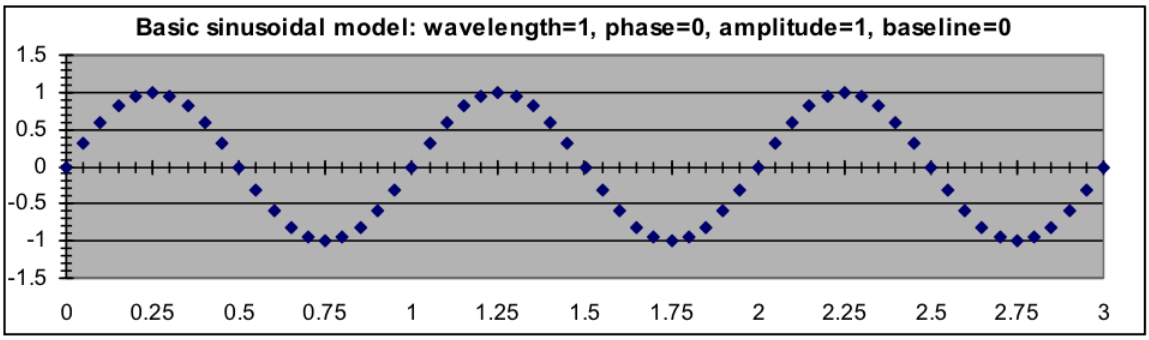
9: Sinusoidal Models is shared under a not declared license and was authored, remixed, and/or curated by LibreTexts.

9.1: Periodic Behaviors in the Real World

Sinusoidal Models

Repeating behavior is very often encountered in the world, with examples such as ocean waves, heartbeats, vibrations of a spring, rotation of wheels, the phases of the moon, and the pattern of the seasons. Because none of the modeling functions we have discussed so far have repeating features, we need something more to be able to model repetitive phenomena.

For most repetitive situations, models based on the same sine function used in trigonometry will serve our needs. This may seem surprising—what do triangles and waves have to do with each other? One reason that mathematics is so powerful is that the same patterns show up in a great variety of different situations, which means that mathematical tools that were developed for one purpose often turn out to be useful in other contexts.

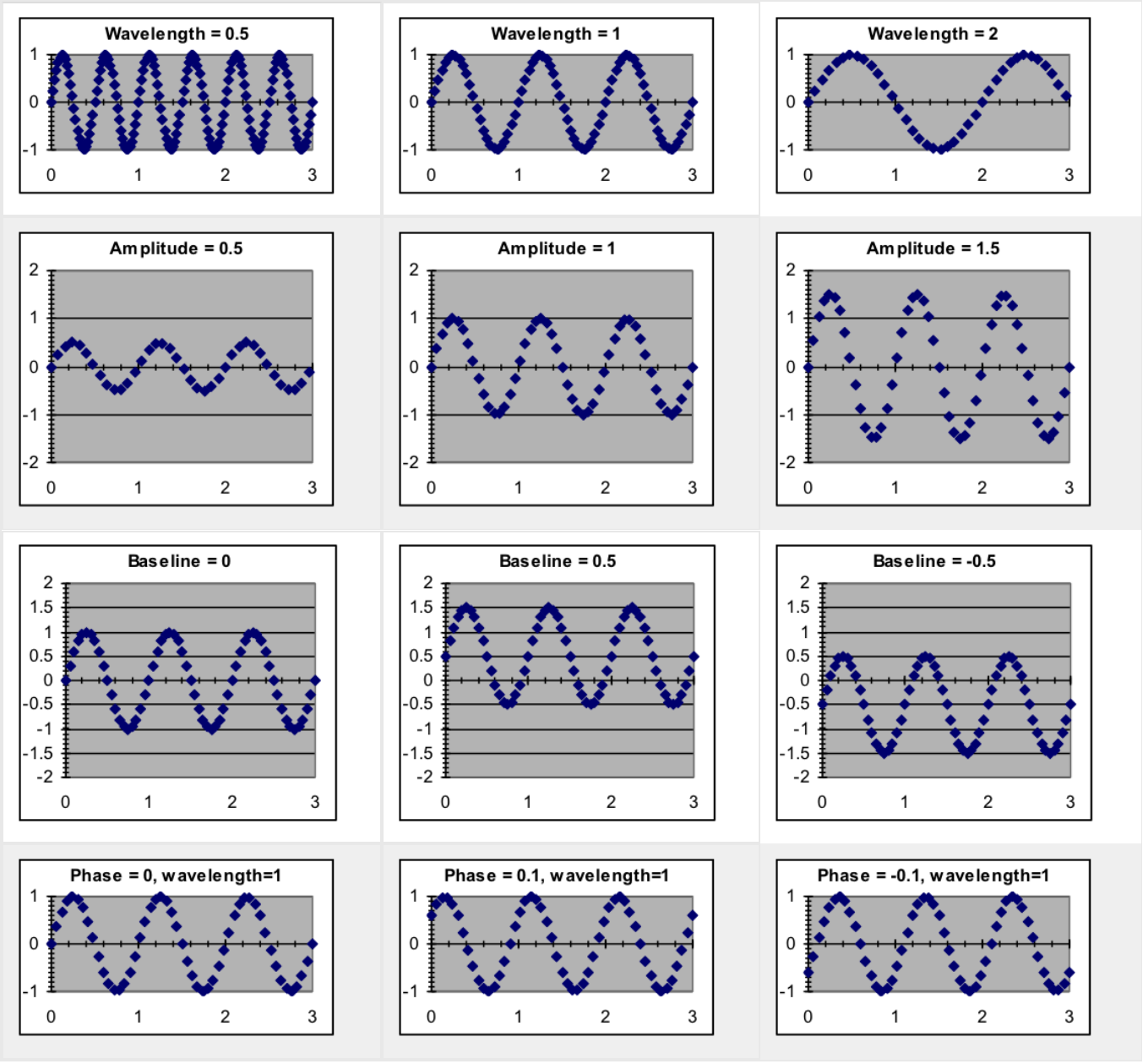


For modeling, we will want to be able to modify the shape of the standard sine function in several ways. First, we want to control how often the pattern repeats—the **wavelength**—so that we are not limited to the wavelength of the sine function (2π, or about 6.2832). Secondly, we need to be able to shift the pattern to the right or left so that the peaks and valleys of the model match the peaks and valleys of the data—this shift is called the **phase** of the model. Thirdly, we need to be able to match the amount of up-and-down variation in the model—the **amplitude**—to the amount of variation in the data. When the average value is not zero, we will also need to add a constant **baseline** to the model.

Summary of sinusoidal parameters

- **Wavelength**—How often does the pattern repeat? Wavelength is in the same units as the *x* data, and the ratio of *x* to the model wavelength is multiplied by 2π because that is the natural wavelength of the SIN function. When the *x* values are based on time, the wavelength is sometimes referred to as the *period* or as a *cycle* of the sinusoid.
- **Phase** (or **phase shift**)—Where in the pattern is the $x = 0$ point? The basic pattern (which has a phase shift of zero) can be shifted to the left or the right. Note that because the pattern repeats, a shift to the left of ¼ wavelength would result in the same pattern as a shift to the right of ¾ wavelength.
- **Amplitude**—This is simply a scale factor that determines how far the maximum and minimum *y* are from the average. While it is mathematically possible to use a negative amplitude (the pattern turns upside down), this is not done since the same effect can be obtained by a half-wavelength phase shift.

- **Baseline**—The average around which the pattern varies. This can be modeled by addition to the result of the SIN function after multiplying by the amplitude.



CC licensed content, Shared previously

- Mathematics for Modeling. **Authored by**: Mary Parker and Hunter Ellinger. **License**: *CC BY: Attribution*

9.1: Periodic Behaviors in the Real World is shared under a not declared license and was authored, remixed, and/or curated by LibreTexts.

9.2: Sinusoidal Graphs

There is A LOT of information is this section relating to both the sine curve and the cosine curve.

Focus on sine. We will not use cosine models in this class.

Periodic Functions

The London Eye (London Eye photo by authors, 2010, CC-BY) is a huge Ferris wheel 135 meters (394 feet) tall in London, England, which completes one rotation every 30 minutes. When we look at the behavior of this Ferris wheel it is clear that it completes 1 cycle, or 1 revolution, and then repeats this revolution over and over again.



This is an example of a periodic function, because the Ferris wheel repeats its revolution or one cycle every 30 minutes, and so we say it has a period of 30 minutes.

In this section, we will work to sketch a graph of a rider's height above the ground over time and express this height as a function of time.

Definition: PERIODIC FUNCTIONS

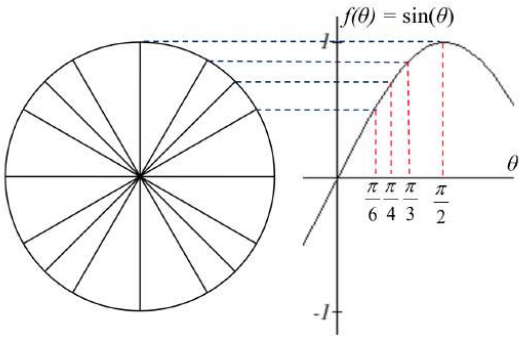
A **periodic function** is a function for which a specific horizontal shift, P, results in the original function: $f(x+P)=f(x)$ for all values of x. When this occurs we call the smallest such horizontal shift with $P>0$ the **period** of the function.

You might immediately guess that there is a connection here to finding points on a circle, since the height above ground would correspond to the y value of a point on the circle. We can determine the y value by using the sine function. To get a better sense of this function's behavior, we can create a table of values we know, and use them to sketch a graph of the sine and cosine functions.

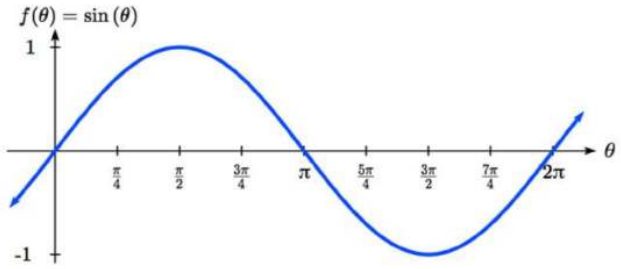
Listing some of the values for sine and cosine on a unit circle,

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

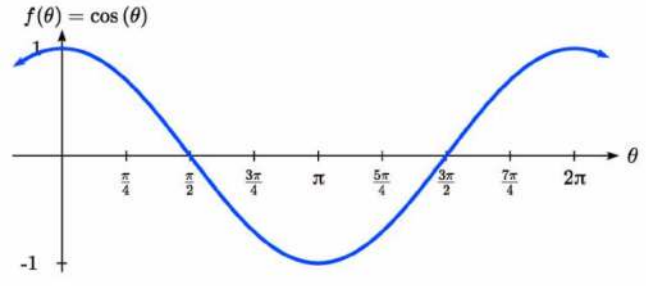
Here you can see how for each angle, we use the y value of the point on the circle to determine the output value of the sine function.



Plotting more points gives the full shape of the sine and cosine functions.



Notice how the sine values are positive between 0 and π, which correspond to the values of sine in quadrants 1 and 2 on the unit circle, and the sine values are negative between π and 2π, corresponding to quadrants 3 and 4.



Like the sine function we can track the value of the cosine function through the 4 quadrants of the unit circle as we place it on a graph.

Both of these functions are defined for all real numbers, since we can evaluate the sine and cosine of any angle. By thinking of sine and cosine as coordinates of points on a unit circle, it becomes clear that the range of both functions must be the interval [-1, 1].

DOMAIN AND RANGE OF SINE AND COSINE

The domain of sine and cosine is all real numbers, $(-\infty, \infty)$.

The range of sine and cosine is the interval [-1, 1].

Both these graphs are called **sinusoidal** graphs.

In both graphs, the shape of the graph begins repeating after 2π. Indeed, since any coterminal angles will have the same sine and cosine values, we could conclude that

$$\sin(\theta + 2\pi) = \sin(\theta)$$

and

$$\cos(\theta + 2\pi) = \cos(\theta)$$

In other words, if you were to shift either graph horizontally by 2π, the resulting shape would be identical to the original function. Sinusoidal functions are a specific type of periodic function.

PERIOD OF SINE AND COSINE

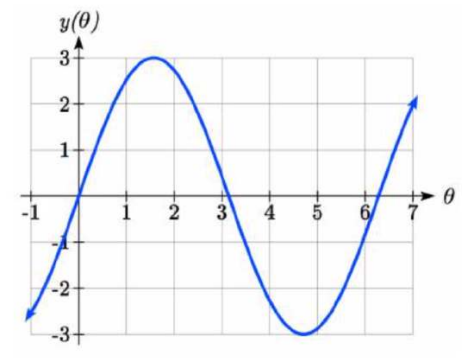
The periods of the sine and cosine functions are both 2π.

Transforming Sine and Cosine

Example 9.2.2

A point rotates around a circle of radius 3. Sketch a graph of the y coordinate of the point.

Solution



Recall that for a point on a circle of radius r, the y coordinate of the point is $y = r\sin(\theta)$, so in this case, we get the equation

$$y(\theta) = 3\sin(\theta)$$

The constant 3 causes a vertical stretch of the y values of the function by a factor of 3.

Notice that the period of the function does not change.

Since the outputs of the graph will now oscillate between -3 and 3, we say that the **amplitude** of the sine wave is 3.

Exercise 9.2.1

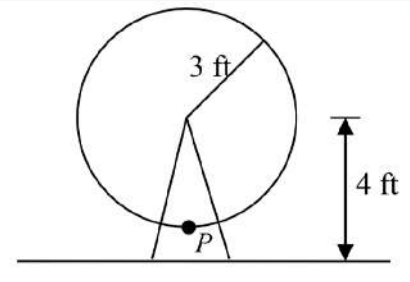
What is the amplitude of the function $f(\theta) = 7\cos(\theta)$? Sketch a graph of this function.

Answer

7

Example 9.2.3

A circle with radius 3 feet is mounted with its center 4 feet off the ground. The point closest to the ground is labeled P. Sketch a graph of the height above ground of the point P as the circle is rotated, then find a function that gives the height in terms of the angle of rotation.



Solution

Sketching the height, we note that it will start 1 foot above the ground, then increase up to 7 feet above the ground, and continue to oscillate 3 feet above and below the center value of 4 feet.

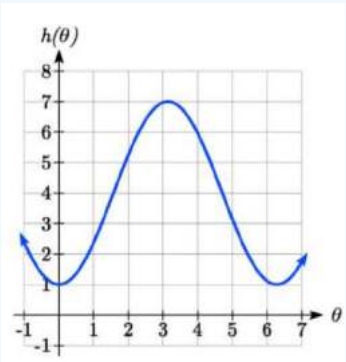
Although we could use a transformation of either the sine or cosine function, we start by looking for characteristics that would make one function easier to use than the other.

We decide to use a cosine function because it starts at the highest or lowest value, while a sine function starts at the middle value. A standard cosine starts at the highest value, and this graph starts at the lowest value, so we need to incorporate a vertical reflection.

Second, we see that the graph oscillates 3 above and below the center, while a basic cosine has an amplitude of one, so this graph has been vertically stretched by 3, as in the last example.

Finally, to move the center of the circle up to a height of 4, the graph has been vertically shifted up by 4. Putting these transformations together,

$$h(\theta) = -3\cos(\theta) + 4$$



Definition: MIDLINE

The center value of a sinusoidal function, the value that the function oscillates above and below, is called the **midline** of the function, corresponding to a vertical shift.

The function $f(\theta) = \cos(\theta) + k$ has midline at $y = k$.

Exercise 9.2.2

What is the midline of the function $f(\theta) = 3\cos(\theta) - 4$? Sketch a graph of the function.

Answer

-4

To answer the Ferris wheel problem at the beginning of the section, we need to be able to express our sine and cosine functions at inputs of time. To do so, we will utilize composition. Since the sine function takes an input of an angle, we will look for a function that takes time as an input and outputs an angle. If we can find a suitable $\theta(t)$ function, then we can compose this with our $f(\theta) = \cos(\theta)$ function to obtain a sinusoidal function of time: $f(t) = \cos(\theta(t))$.

Example 9.2.4

A point completes 1 revolution every 2 minutes around a circle of radius 5. Find the x coordinate of the point as a function of time, if it starts at (5, 0).

Solution

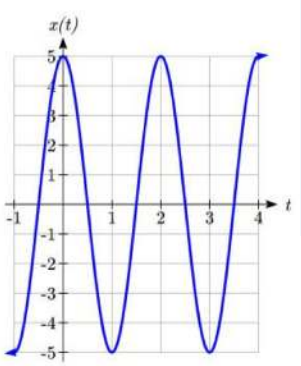
Normally, we would express the x coordinate of a point on a unit circle using $x = r\cos(\theta)$, here we write the function $x(\theta) = 5\cos(\theta)$.

The rotation rate of 1 revolution every 2 minutes is an angular velocity. We can use this rate to find a formula for the angle as a function of time. The point begins at an angle of 0. Since the point rotates 1 revolution = 2π radians every 2 minutes, it rotates π radians every minute. After t minutes, it will have rotated:

$$\theta(t) = \pi t \text{ radians}$$

Composing this with the cosine function, we obtain a function of time.

$$x(t) = 5\cos(\theta(t)) = 5\cos(\pi t)$$



Notice that this composition has the effect of a horizontal compression, changing the period of the function.

To see how the period relates to the stretch or compression coefficient B in the equation $f(t) = \sin(Bt)$, note that the period will be the time it takes to complete one full revolution of a circle.

If a point takes P minutes to complete 1 revolution, then the angular velocity is $\frac{2\pi \text{ radians}}{P \text{ minutes}}$. Then $\theta(t) = \frac{2\pi}{P}t$. Composing with a sine function,

$$f(t) = \sin(\theta(t)) = \sin\left(\frac{2\pi}{P}t\right)$$

From this, we can determine the relationship between the coefficient B and the period:

$$B = \frac{2\pi}{P}$$

Notice that the stretch or compression coefficient B is a ratio of the "normal period of a sinusoidal function" to the "new period." If we know the stretch or compression coefficient B, we can solve for the "new period":

$$P = \frac{2\pi}{B}$$

Summarizing our transformations so far:

TRANSFORMATIONS OF SINE AND COSINE

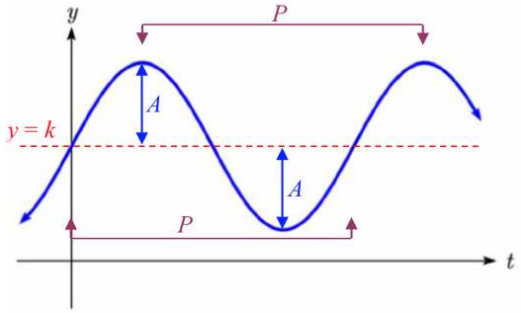
Given an equation in the form

$$f(t) = A\sin(Bt) + k \text{ or } f(t) = A\cos(Bt) + k \tag{9.2.1}$$

A is the vertical stretch, and is the **amplitude** of the function.

B is the horizontal stretch/compression, and is related to the **period,** P, by $P = \frac{2\pi}{B}$.

k is the vertical shift and determines the **midline** of the function.



Example 9.2.5

What is the period of the function $f(t) = \sin\left(\frac{\pi}{6}t\right)$?

Solution

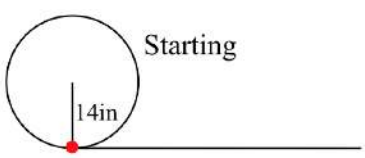
Using the relationship above, the stretch/compression factor is $B = \frac{\pi}{6}$, so the period will be

$$P = \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{6}} = 2\pi \cdot \frac{6}{\pi} = 12$$

While it is common to compose sine or cosine with functions involving time, the composition can be done so that the input represents any reasonable quantity.

Example 9.2.6

A bicycle wheel with radius 14 inches has the bottom-most point on the wheel marked in red. The wheel then begins rolling down the street. Write a formula for the height above ground of the red point after the bicycle has travelled x inches.

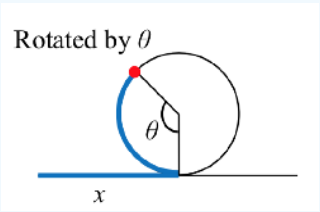


Solution

The height of the point begins at the lowest value, 0, increases to the highest value of 28 inches, and continues to oscillate above and below a center height of 14 inches. In terms of the angle of rotation, θ:

$$h(\theta) = -14\cos(\theta) + 14$$

In this case, x is representing a linear distance the wheel has travelled, corresponding to an arclength along the circle. Since arclength and angle can be related by $s = r\theta$, in this case we can write $x = 14\theta$, which allows us to express the angle in terms of x:



$$\theta(x) = \frac{x}{14}$$

Composing this with our cosine-based function from above,

$$h(x) = h(\theta(x)) = -14\cos\left(\frac{x}{14}\right) + 14 = -14\cos\left(\frac{1}{14}x\right) + 14$$

The period of this function would be $P = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{14}} = 2\pi \cdot 14 = 28\pi$, the circumference of the circle. This makes sense – the wheel completes one full revolution after the bicycle has travelled a distance equivalent to the circumference of the wheel.

Example 9.2.7

Determine the midline, amplitude, and period of the function $f(t) = 3\sin(2t) + 1$.

Solution

The amplitude is 3

The period is $P = \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

The midline is at $y = 1$

Amplitude, midline, and period, when combined with vertical flips, allow us to write equations for a variety of sinusoidal situations.

Exercise 9.2.3

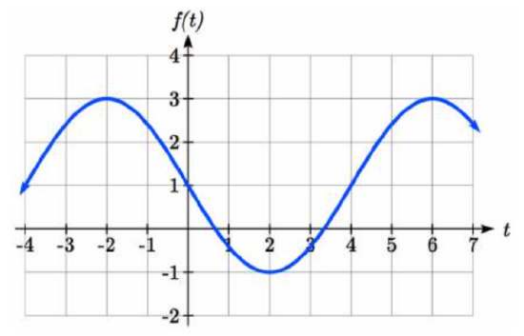
If a sinusoidal function starts on the midline at point (0,3), has an amplitude of 2, and a period of 4, write a formula for the function.

Answer

$$f(x) = 2\sin\left(\frac{\pi}{2}x\right) + 3$$

Example 9.2.8

Find a formula for the sinusoidal function graphed here.



Solution

The graph oscillates from a low of -1 to a high of 3, putting the midline at $y = 1$, halfway between.

The amplitude will be 2, the distance from the midline to the highest value (or lowest value) of the graph.

The period of the graph is 8. We can measure this from the first peak at $x = -2$ to the second at $x = 6$. Since the period is 8, the stretch/compression factor we will use will be

$$B = \frac{2\pi}{P} = \frac{2\pi}{8} = \frac{\pi}{4}$$

At $x = 0$, the graph is at the midline value, which tells us the graph can most easily be represented as a sine function. Since the graph then decreases, this must be a vertical reflection of the sine function. Putting this all together,

$$f(t) = -2\sin\left(\frac{\pi}{4}t\right) + 1$$

With these transformations, we are ready to answer the Ferris wheel problem from the beginning of the section.

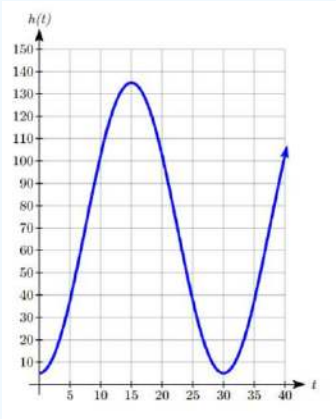
Example 9.2.9

The London Eye is a huge Ferris wheel in London, England, which completes one rotation every 30 minutes. The diameter of the wheel is 120 meters, but the passenger capsules sit outside the wheel. Suppose the diameter at the capsules is 130 meters, and riders board from a platform 5 meters above the ground. Express a rider's height above ground as a function of time in minutes.

Solution

It can often help to sketch a graph of the situation before trying to find the equation.

With a diameter of 130 meters, the wheel has a radius of 65 meters. The height will oscillate with amplitude of 65 meters above and below the center.



Passengers board 5 meters above ground level, so the center of the wheel must be located 65 + 5 = 70 meters above ground level. The midline of the oscillation will be at 70 meters.

The wheel takes 30 minutes to complete 1 revolution, so the height will oscillate with period of 30 minutes.

Lastly, since the rider boards at the lowest point, the height will start at the smallest value and increase, following the shape of a flipped cosine curve.

Putting these together:

Amplitude: 65

Midline: 70

Period: 30, so

$$B = \frac{2\pi}{30} = \frac{\pi}{15}$$

Shape: negative cosine

An equation for the rider's height would be

$$h(t) = -65\cos\left(\frac{\pi}{15}t\right) + 70$$

? Exercise 9.2.4

The Ferris wheel at the Puyallup Fair (Photo by photogirl7.1, www.flickr.com/photos/kitkaph...86205/sizes/z/, CC-BY) has a diameter of about 70 feet and takes 3 minutes to complete a full rotation. Passengers board from a platform 10 feet above the ground. Write an equation for a rider's height above ground over time.



Answer

$$h(t) = -35\cos\left(\frac{2\pi}{3}t\right) + 45$$

While these transformations are sufficient to represent many situations, occasionally we encounter a sinusoidal function that does not have a vertical intercept at the lowest point, highest point, or midline. In these cases, we need to use horizontal shifts. Since we are combining horizontal shifts with horizontal stretches, we need to be careful. Recall that when the inside of the function is factored, it reveals the horizontal shift.

HORIZONTAL SHIFTS OF SINE AND COSINE

Given an equation in the form

$$f(t) = A\sin\left(B(t-h)\right) + k \text{ or } f(t) = A\cos\left(B(t-h)\right) + k \tag{9.2.2}$$

h is the horizontal shift of the function

✓ Example 9.2.10

Sketch a graph of $f(t) = 3\sin\left(\frac{\pi}{4}t - \frac{\pi}{4}\right)$.

Solution

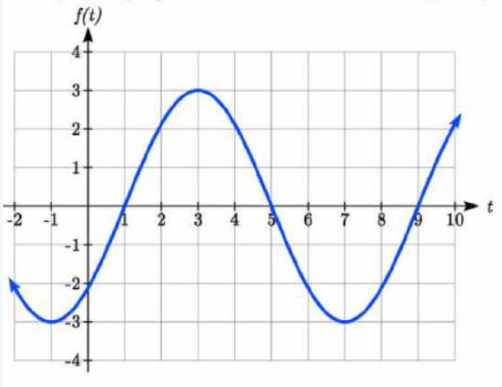
To reveal the horizontal shift, we first need to factor inside the function:

$$f(t) = 3\sin\left(\frac{\pi}{4}(t-1)\right)$$

This graph will have the shape of a sine function, starting at the midline and increasing, with an amplitude of 3. The period of the graph will be

$$P = \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi \cdot \frac{4}{\pi} = 8$$

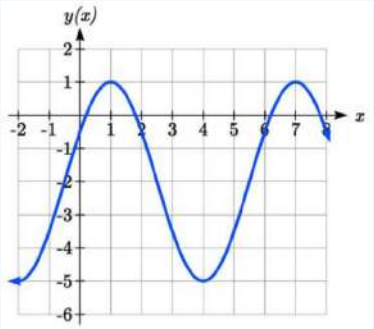
Finally, the graph will be shifted to the right by 1.



In some physics and mathematics books, you will hear the horizontal shift referred to as **phase shift**. In other physics and mathematics books, they would say the phase shift of the equation above is $\frac{\pi}{4}$, the value in the unfactored form. Because of this ambiguity, we will not use the term phase shift any further, and will only talk about the horizontal shift.

Example 9.2.11

Find a formula for the function graphed here.



Solution

With highest value at 1 and lowest value at -5, the midline will be halfway between at -2.

The distance from the midline to the highest or lowest value gives an amplitude of 3.

The period of the graph is 6, which can be measured from the peak at $x = 1$ to the next peak at $x = 7$, or from the distance between the lowest points. This gives

$$B = \frac{2\pi}{P} = \frac{2\pi}{6} = \frac{\pi}{3}$$

For the shape and shift, we have more than one option. We could either write this as:

A cosine shifted 1 to the right

A negative cosine shifted 2 to the left

A sine shifted 1/2 to the left

A negative sine shifted 2.5 to the right

While any of these would be fine, the cosine shifts are easier to work with than the sine shifts in this case, because they involve integer values. Writing these:

$$y(x) = 3\cos\left(\frac{\pi}{3}(x-1)\right) - 2$$

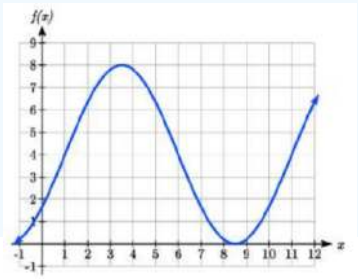
or

$$y(x) = -3\cos\left(\frac{\pi}{3}(x+2)\right) - 2$$

Again, these functions are equivalent, so both yield the same graph.

? Exercise 9.2.5

Write a formula for the function graphed here.



Answer

Two possibilities:

$$f(x) = 4\cos\left(\frac{\pi}{5}(x-3.5)\right) + 4 \text{ or } f(x) = 4\sin\left(\frac{\pi}{5}(x-1)\right) + 4$$

Important Topics of This Section

- Periodic functions
- Sine and cosine function from the unit circle
- Domain and range of sine and cosine functions
- Sinusoidal functions
- Negative angle identity
- Simplifying expressions
- Transformations
- Amplitude
- Midline
- Period
- Horizontal shifts

This page titled 9.2: Sinusoidal Graphs is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by David Lippman & Melonie Rasmussen (The OpenTextBookStore) via source content that was edited to the style and standards of the LibreTexts platform.

- **6.1: Sinusoidal Graphs** by David Lippman & Melonie Rasmussen is licensed CC BY-SA 4.0. Original source: http://www.opentextbookstore.com/details.php?id=30.

9.3: More Sinusoidal Modeling

Suppose we charted the average daily temperatures in New York City over the course of one year. We would expect to find the lowest temperatures in January and February and highest in July and August. This familiar cycle repeats year after year, and if we were to extend the graph over multiple years, it would resemble a periodic function.



Figure 9.3.1: The hands on a clock are periodic: they repeat positions every twelve hours. (credit: "zoutedrop"/Flickr)

Many other natural phenomena are also periodic. For example, the phases of the moon have a period of approximately 28 days, and birds know to fly south at about the same time each year. So how can we model an equation to reflect periodic behavior? First, we must collect and record data. We then find a function that resembles an observed pattern. Finally, we make the necessary alterations to the function to get a model that is dependable. In this section, we will take a deeper look at specific types of periodic behavior and model equations to fit data.

Determining the Amplitude and Period of a Sinusoidal Function

Any motion that repeats itself in a fixed time period is considered **periodic motion** and can be modeled by a **sinusoidal function**. The amplitude of a sinusoidal function is the distance from the midline to the maximum value, or from the midline to the minimum value. The midline is the average value. Sinusoidal functions oscillate above and below the midline, are periodic, and repeat values in set cycles. Recall from Graphs of the Sine and Cosine Functions that the **period** of the sine function and the cosine function is 2π. In other words, for any value of x,

$$\sin(x \pm 2\pi k) = \sin x \text{ and } \cos(x \pm 2\pi k) = \cos x \tag{9.3.1}$$

where k is an integer.

STANDARD FORM OF SINUSOIDAL EQUATIONS

The general forms of a sinusoidal equation are given as

$$y = A\sin(Bt - C) + D \tag{9.3.2}$$

or

$$y = A\cos(Bt - C) + D \tag{9.3.3}$$

where $\text{amplitude} = |A|$, B is related to period such that the $\text{period} = \frac{2\pi}{B}$, C is the phase shift such that $\frac{C}{B}$ denotes the horizontal shift, and D represents the vertical shift from the graph's parent graph.

Note that the models are sometimes written as

$$y = a\sin(\omega t \pm C) + D \tag{9.3.4}$$

or

$$y = a\cos(\omega t \pm C) + D, \tag{9.3.5}$$

with a period that is given as $\frac{2\pi}{\omega}$.

The difference between the sine and the cosine graphs is that the sine graph begins with the average value of the function and the cosine graph begins with the maximum or minimum value of the function.

Example 9.3.1: Showing How the Properties of a Trigonometric Function Can Transform a Graph

Show the transformation of the graph of $y = \sin x$ into the graph of $y = 2\sin(4x - \frac{\pi}{2}) + 2$.

Solution

Consider the series of graphs in Figure 9.3.1 and the way each change to the equation changes the image.

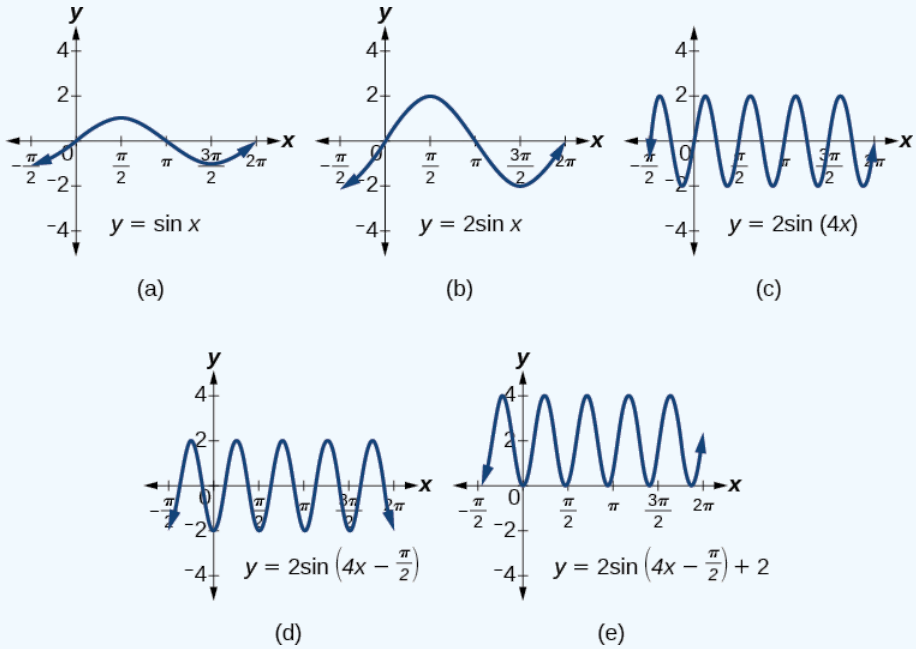


Figure 9.3.2:

a. The basic graph of $y = \sin x$
b. Changing the amplitude from 1 to 2 generates the graph of $y = 2\sin x$.
c. The period of the sine function changes with the value of B, such that $\text{period} = \frac{2\pi}{B}$. Here we have $B = 4$, which translates to a period of $\frac{\pi}{2}$. The graph completes one full cycle in $\frac{\pi}{2}$ units.
d. The graph displays a horizontal shift equal to $\frac{C}{B}$, or $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$.
e. Finally, the graph is shifted vertically by the value of D. In this case, the graph is shifted up by 2 units.

Example 9.3.2: Finding the Amplitude and Period of a Function

Find the amplitude and period of the following functions and graph one cycle.

a. $y = 2\sin(\frac{1}{4}x)$
b. $y = -3\sin(2x + \frac{\pi}{2})$
c. $y = \cos x + 3$

Solution

We will solve these problems according to the models.

a. $y = 2\sin(\frac{1}{4}x)$ involves sine, so we use the form

$$y = A\sin(Bt + C) + D$$

We know that $|A|$ is the amplitude, so the amplitude is 2. Period is $\frac{2\pi}{B}$ so the period is

$$\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{4}}$$
$$= 8\pi$$

See the graph in Figure 9.3.3.

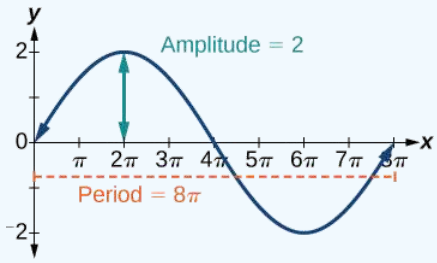


Figure 9.3.3:

b. $y = -3\sin(2x + \frac{\pi}{2})$ involves sine, so we use the form

$$y = A\sin(Bt - C) + D$$

Amplitude is $|A|$, so the amplitude is $|-3| = 3$. Since A is negative, the graph is reflected over the x-axis. Period is $\frac{2\pi}{B}$,so the period is

$$\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$$

The graph is shifted to the left by $\frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$ units. See Figure 9.3.4.

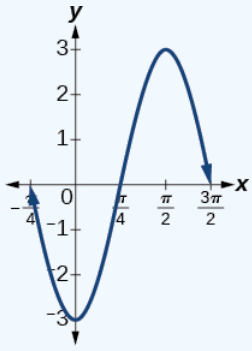


Figure 9.3.4:

c. $y = \cos x + 3$ involves cosine, so we use the form

d.
$$y = A\cos(Bt \pm C) + D$$

Amplitude is $|A|$, so the amplitude is 1 and the period is 2π (Figure 9.3.5. This is the standard cosine function shifted up three units.

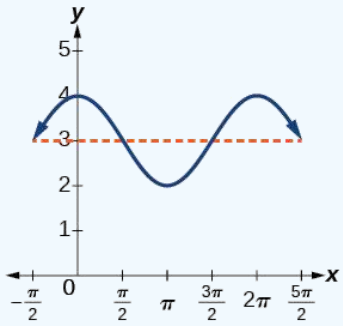


Figure 9.3.5

Exercise 9.3.1:

What are the amplitude and period of the function $y = 3\cos(3\pi x)$?

Answer

The amplitude is 3, and the period is $\frac{2}{3}$.

Finding Equations and Graphing Sinusoidal Functions

One method of graphing sinusoidal functions is to find five key points. These points will correspond to intervals of equal length representing $\frac{1}{4}$ of the period. The key points will indicate the location of maximum and minimum values. If there is no vertical shift, they will also indicate x-intercepts. For example, suppose we want to graph the function $y = \cos\theta$. We know that the period is 2π, so we find the interval between key points as follows.

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

Starting with $\theta = 0$, we calculate the first y-value, add the length of the interval $\frac{\pi}{2}$ to 0, and calculate the second y-value. We then add $\frac{\pi}{2}$ repeatedly until the five key points are determined. The last value should equal the first value, as the calculations cover one full period. Making a table similar to Table 9.3.1, we can see these key points clearly on the graph shown in Figure 9.3.6.

Table 9.3.1

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos\theta$	1	0	$-1-1$	0	1

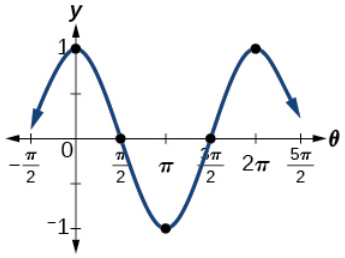


Figure 9.3.6

Example 9.3.3: Graphing Sinusoidal Functions Using Key Points

Graph the function $y = -4\cos(\pi x)$ using amplitude, period, and key points.

Solution

The amplitude is $|-4| = 4$. The period is $\frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$. (Recall that we sometimes refer to B as ω.) One cycle of the graph can be drawn over the interval $[0, 2]$. To find the key points, we divide the period by 4. Make a table similar to Figure 9.3.1:, starting with $x = 0$ and then adding $\frac{1}{2}$ successively to x and calculate y. See the graph in Figure 9.3.7.

Table 9.3.1

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = -4\cos(\pi x)$	-4	0	4	0	-4

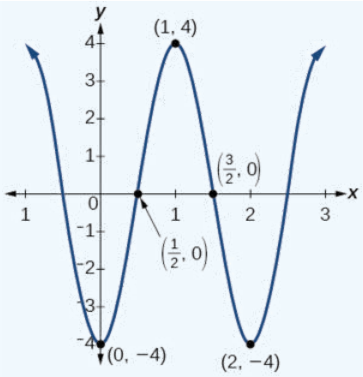


Figure 9.3.7

? Exercise 9.3.2:

Graph the function $y = 3\sin(3x)$ using the amplitude, period, and five key points.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$3\sin(3x)$	0	3	0	-3	0

Answer

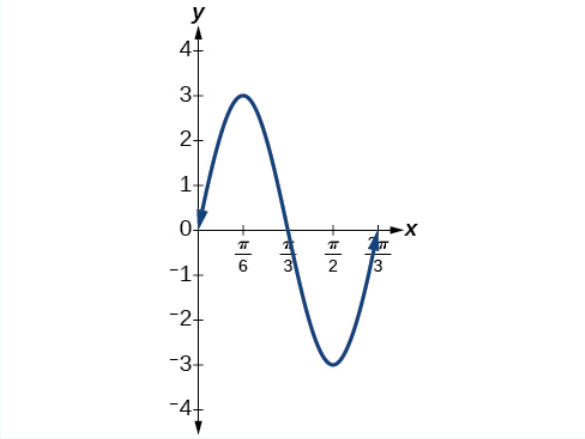


Figure 9.3.8

Modeling Periodic Behavior

We will now apply these ideas to problems involving periodic behavior.

✓ Example 9.3.4: Modeling an Equation and Sketching a Sinusoidal Graph to Fit Criteria

The average monthly temperatures for a small town in Oregon are given in Table 9.3.1. Find a sinusoidal function of the form $y = A\sin(Bt - C) + D$ that fits the data (round to the nearest tenth) and sketch the graph.

Table 9.3.2

Month	Temperature, ^oF
January	42.5
February	44.5
March	48.5
April	52.5
May	58
June	63
July	68.5
August	69
September	64.5
October	55.5
November	46.5
December	43.5

Solution

Recall that amplitude is found using the formula

$$A = \frac{\text{largest value} - \text{smallest value}}{2} \tag{9.3.6}$$

Thus, the amplitude is

$$\begin{aligned} |A| &= \frac{69 - 42.5}{2} \\ &= 13.25 \end{aligned}$$

The data covers a period of 12 months, so $\frac{2\pi}{B} = 12$ which gives $B = \frac{2\pi}{12} = \frac{\pi}{6}$.

The vertical shift is found using the following equation.

$$D = \frac{\text{highest value} + \text{lowest value}}{2} \tag{9.3.7}$$

Thus, the vertical shift is

$$D = \frac{69 + 42.5}{2} \qquad = 55.8$$

So far, we have the equation $y = 13.3\sin(\frac{\pi}{6}x - C) + 55.8$.

To find the horizontal shift, we input the x and y values for the first month and solve for C.

$$\begin{aligned} 42.5 &= 13.3\sin(\frac{\pi}{6}(1) - C) + 55.8 \\ -13.3 &= 13.3\sin(\frac{\pi}{6} - C) \\ -1 &= \sin(\frac{\pi}{6} - C) \qquad \sin\theta = -1 \rightarrow \theta = -\frac{\pi}{2} \\ \frac{\pi}{6} - C &= -\frac{\pi}{2} \\ \frac{\pi}{6} + \frac{\pi}{2} &= C \\ &= \frac{2\pi}{3} \end{aligned}$$

We have the equation $y = 13.3\sin(\frac{\pi}{6}x - \frac{2\pi}{3}) + 55.8$. See the graph in Figure 9.3.9.

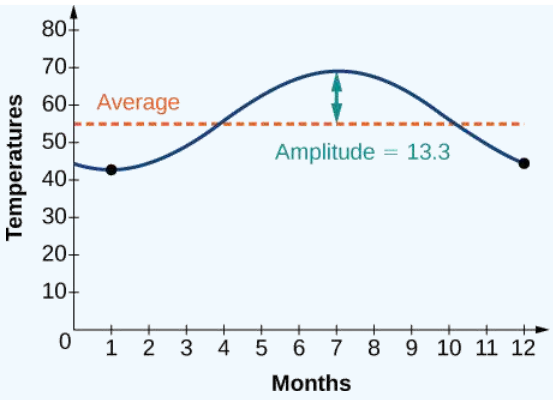


Figure 9.3.9

Example 9.3.5: Describing Periodic Motion

The hour hand of the large clock on the wall in Union Station measures 24 inches in length. At noon, the tip of the hour hand is 30 inches from the ceiling. At 3 PM, the tip is 54 inches from the ceiling, and at 6 PM, 78 inches. At 9 PM, it is again 54 inches from the ceiling, and at midnight, the tip of the hour hand returns to its original position 30 inches from the ceiling. Let y equal the distance from the tip of the hour hand to the ceiling x hours after noon. Find the equation that models the motion of the clock and sketch the graph.

Solution

Begin by making a table of values as shown in Table 9.3.3.

Table 9.3.3

x	y	Points to plot
Noon	30 in	$(0, 30)$
3 PM	54 in	$(3, 54)$
6 PM	78 in	$(6, 78)$
9 PM	54 in	$(9, 54)$
Midnight	30 in	$(12, 30)$

To model an equation, we first need to find the amplitude.

$$\begin{aligned} |A| &= \left|\frac{78-30}{2}\right| \\ &= 24 \end{aligned}$$

The clock's cycle repeats every 12 hours. Thus,

$$\begin{aligned} B &= \frac{2\pi}{12} \\ &= \frac{\pi}{6} \end{aligned}$$

The vertical shift is

$$\begin{aligned} D &= \frac{78+30}{2} \\ &= 54 \end{aligned}$$

There is no horizontal shift, so $C = 0$. Since the function begins with the minimum value of y when $x = 0$ (as opposed to the maximum value), we will use the cosine function with the negative value for A. In the form $y = A\cos(Bx \pm C) + D$, the equation is

$$y = -24\cos(\frac{\pi}{6}x) + 54 \tag{9.3.8}$$

See Figure 9.3.10.

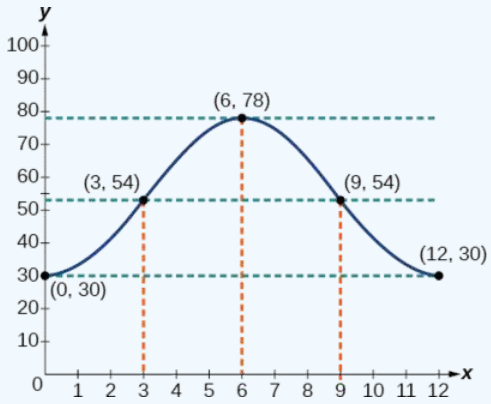


Figure 9.3.10

Example 9.3.6: Determining a Model for Tides

The height of the tide in a small beach town is measured along a seawall. Water levels oscillate between 7 feet at low tide and 15 feet at high tide. On a particular day, low tide occurred at 6 AM and high tide occurred at noon. Approximately every 12 hours, the cycle repeats. Find an equation to model the water levels.

Solution

As the water level varies from 7 ft to 15 ft, we can calculate the amplitude as

$$|A| = \left|\frac{(15-7)}{2}\right| = 4$$

The cycle repeats every 12 hours; therefore, B is

$$\frac{2\pi}{12} = \frac{\pi}{6}$$

There is a vertical translation of $\frac{(15+8)}{2} = 11.5$. Since the value of the function is at a maximum at $t = 0$, we will use the cosine function, with the positive value for A.

$$y = 4\cos(\frac{\pi}{6})t + 11$$

See Figure 9.3.11.

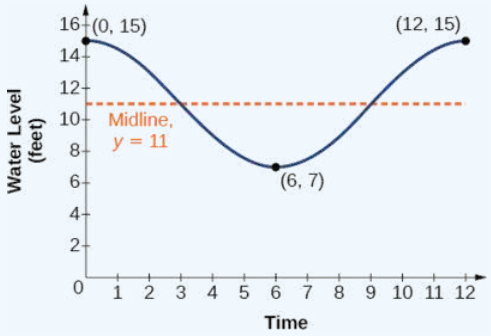


Figure 9.3.11

Exercise 9.3.3:

The daily temperature in the month of March in a certain city varies from a low of $24°F$ to a high of $40°F$. Find a sinusoidal function to model daily temperature and sketch the graph. Approximate the time when the temperature reaches the freezing point $32°F$. Let $t = 0$ correspond to noon.

Answer

$$y = 8\sin\left(\frac{\pi}{12}t\right) + 32$$

The temperature reaches freezing at noon and at midnight.

Exercise 9.3.4: Interpreting the Periodic Behavior Equation

The average person's blood pressure is modeled by the function $f(t) = 20\sin(160\pi t) + 100$, where $f(t)$ represents the blood pressure at time t, measured in minutes. Interpret the function in terms of period and frequency. Sketch the graph and find the blood pressure reading.

Analysis

Blood pressure of $\frac{120}{80}$ is considered to be normal. The top number is the maximum or systolic reading, which measures the pressure in the arteries when the heart contracts. The bottom number is the minimum or diastolic reading, which measures the pressure in the arteries as the heart relaxes between beats, refilling with blood. Thus, normal blood pressure can be modeled by a periodic function with a maximum of 120 and a minimum of 80.

Example 9.3.7: Interpreting the Periodic Behavior Equation

The average person's blood pressure is modeled by the function $f(t) = 20\sin(160\pi t) + 100$, where $f(t)$ represents the blood pressure at time t, measured in minutes. Interpret the function in terms of period and frequency. Sketch the graph and find the blood pressure reading.

Solution

The period is given by

$$\begin{aligned}\frac{2\pi}{\omega} &= \frac{2\pi}{160\pi} \\ &= \frac{1}{80}\end{aligned}$$

In a blood pressure function, frequency represents the number of heart beats per minute. Frequency is the reciprocal of period and is given by

$$\begin{aligned}\frac{\omega}{2\pi} &= \frac{160\pi}{2\pi} \\ &= 80\end{aligned}$$

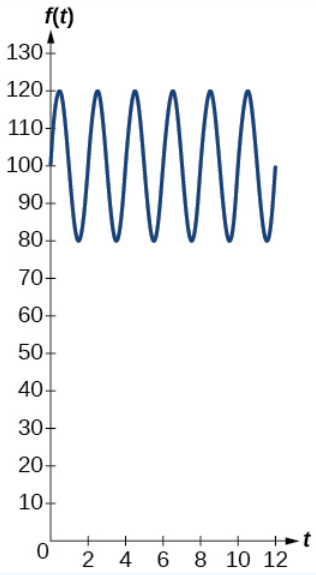


Figure 9.3.12: The blood pressure reading on the graph is $\frac{120}{80}$ ($\frac{\text{maximum}}{\text{minimum}}$).

Analysis

Blood pressure of $\frac{120}{80}$ is considered to be normal. The top number is the maximum or systolic reading, which measures the pressure in the arteries when the heart contracts. The bottom number is the minimum or diastolic reading, which measures the pressure in the arteries as the heart relaxes between beats, refilling with blood. Thus, normal blood pressure can be modeled by a periodic function with a maximum of 120 and a minimum of 80.

Simple Harmonic Motion

Harmonic motion is a form of periodic motion, but there are factors to consider that differentiate the two types. While general **periodic motion** applications cycle through their periods with no outside interference, **harmonic motion** requires a restoring force. Examples of harmonic motion include springs, gravitational force, and magnetic force.

A type of motion described as **simple harmonic motion** involves a restoring force but assumes that the motion will continue forever. Imagine a weighted object hanging on a spring, When that object is not disturbed, we say that the object is at rest, or in equilibrium. If the object is pulled down and then released, the force of the spring pulls the object back toward equilibrium and harmonic motion begins. The restoring force is directly proportional to the displacement of the object from its equilibrium point. When $t = 0, d = 0$.

SIMPLE HARMONIC MOTION

We see that **simple harmonic motion** equations are given in terms of displacement:

$$d = a\cos(\omega t) \text{ or } d = a\sin(\omega t) \tag{9.3.9}$$

where $|a|$ is the amplitude, $\frac{2\pi}{\omega}$ is the period, and $\frac{\omega}{2\pi}$ is the frequency, or the number of cycles per unit of time.

Exercise 9.3.5: Finding the Displacement, Period, and Frequency, and Graphing a Function

For each the given functions:

a. $y = 5\sin(3t)$
b. $y = 6\cos(\pi t)$
c. $y = 5\cos(\frac{\pi}{2})t$

address the following questions:

1. Find the maximum displacement of an object.

2. Find the period or the time required for one vibration.
3. Find the frequency.
4. Sketch the graph.

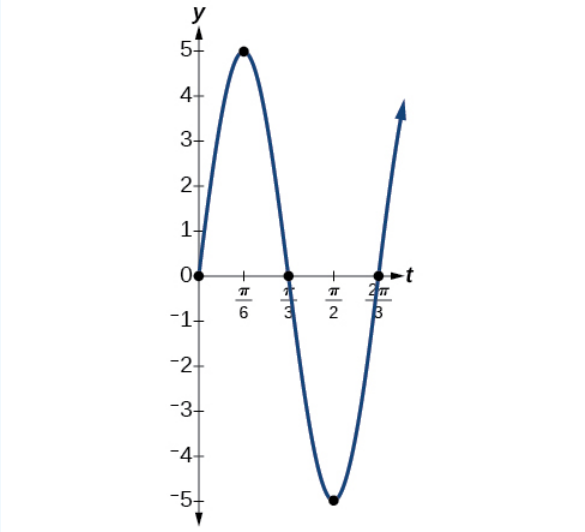


Figure 9.3.13

Answer a

1. The maximum displacement is equal to the amplitude, $|a|$, which is 5.
2. The period is $\frac{2\pi}{\omega} = \frac{2\pi}{3}$.
3. The frequency is given as $\frac{\omega}{2\pi} = \frac{3}{2\pi}$.
4. See Figure 9.3.13. The graph indicates the five key points.

Answer b

1. The maximum displacement is 6.
2. The period is $\frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$.
3. The frequency is $\frac{\omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$.
4. See Figure 9.3.14.

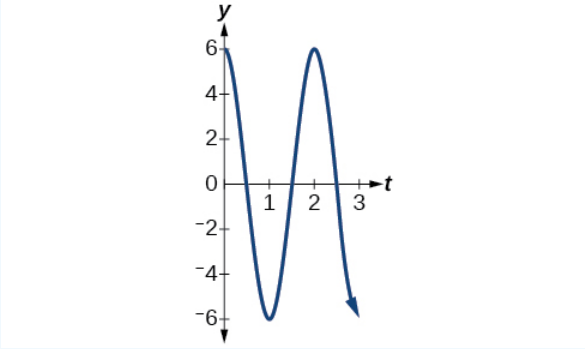


Figure 9.3.14

Answer c

1. The maximum displacement is 5.
2. The period is $\frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}} = 4$.
3. The frequency is $\frac{1}{4}$.
4. See Figure 9.3.15.

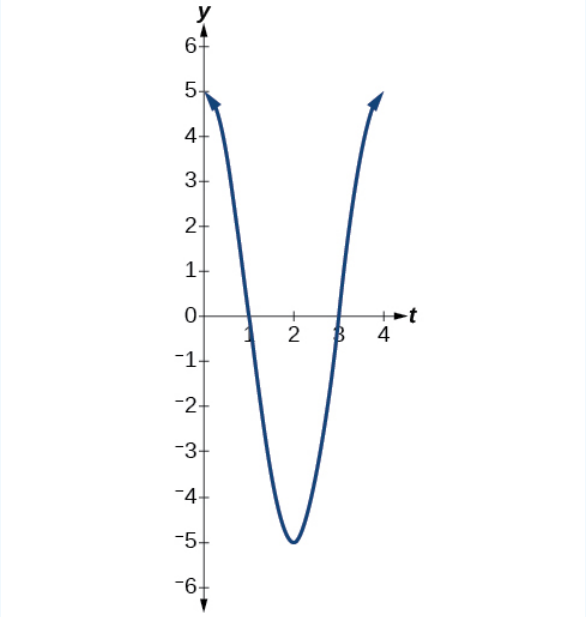


Figure 9.3.15

Key Equations

Standard form of sinusoidal equation	$y = A\sin(Bt - C) + D$ or $y = A\cos(Bt - C) + D$
Simple harmonic motion	$d = a\cos(\omega t)$ or $d = a\sin(\omega t)$
Damped harmonic motion	$f(t) = ae^{-ct}\sin(\omega t)$ or $f(t) = ae^{-ct}\cos(\omega t)$

Key Concepts

- Sinusoidal functions are represented by the sine and cosine graphs. In standard form, we can find the amplitude, period, and horizontal and vertical shifts. See Example and Example.
- Use key points to graph a sinusoidal function. The five key points include the minimum and maximum values and the midline values. See Example.
- Periodic functions can model events that reoccur in set cycles, like the phases of the moon, the hands on a clock, and the seasons in a year. See Example, Example, Example and Example.
- Harmonic motion functions are modeled from given data. Similar to periodic motion applications, harmonic motion requires a restoring force. Examples include gravitational force and spring motion activated by weight. See Example.
- Damped harmonic motion is a form of periodic behavior affected by a damping factor. Energy dissipating factors, like friction, cause the displacement of the object to shrink. See Example, Example, Example, Example, and Example.
- Bounding curves delineate the graph of harmonic motion with variable maximum and minimum values. See Example.

This page titled 9.3: More Sinusoidal Modeling is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

- **7.6: Modeling with Trigonometric Equations** by OpenStax is licensed CC BY 4.0. Original source: https://openstax.org/details/books/precalculus.

9.4: Sinusoidal Models (Exercises)

Section 6.1 Exercises

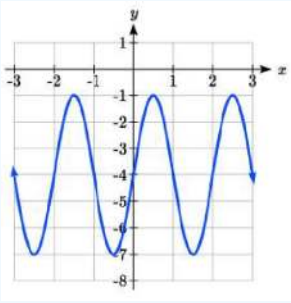
1. Sketch a graph of $f(x) = -3\sin(x)$.

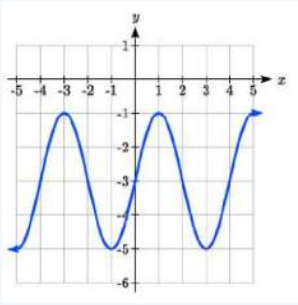
2. Sketch a graph of $f(x) = 4\sin(x)$.

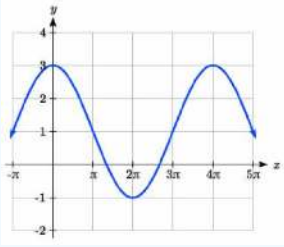
3. Sketch a graph of $f(x) = 2\cos(x)$.

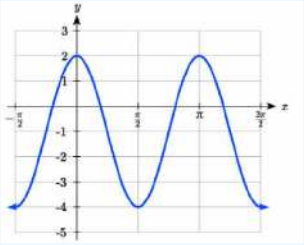
4. Sketch a graph of $f(x) = -4\cos(x)$.

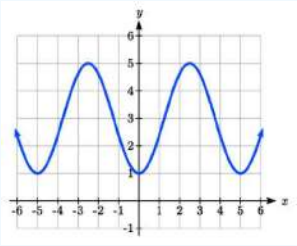
For the graphs below, determine the amplitude, midline, and period, then find a formula for the function.

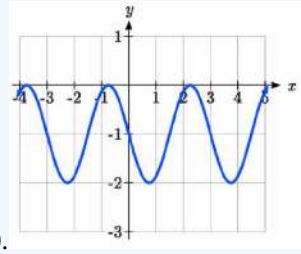
5. 

6. 

7. 

8. 

9. 

10. 

For each of the following equations, find the amplitude, period, horizontal shift, and midline.

11. $y = 3\sin(8(x+4)) + 5$

12. $y = 4\sin\left(\frac{\pi}{2}(x-3)\right) + 7$

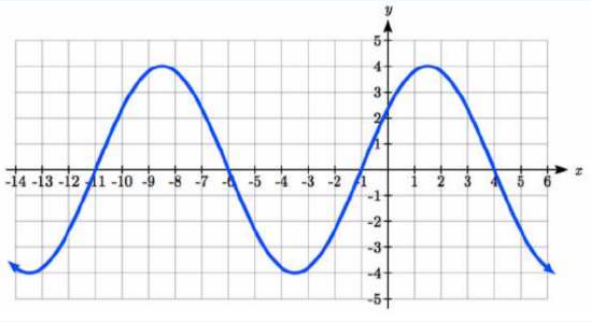
13. $y = 2\sin(3x - 21) + 4$

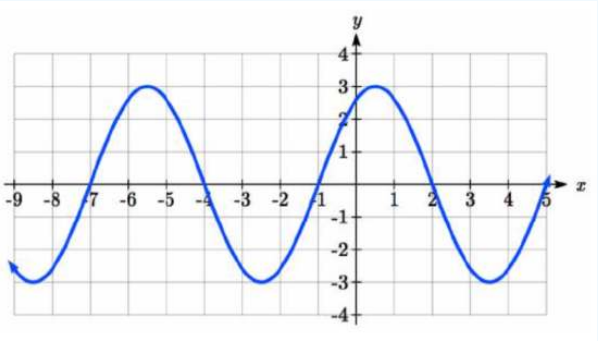
14. $y = 5\sin(5x + 20) - 2$

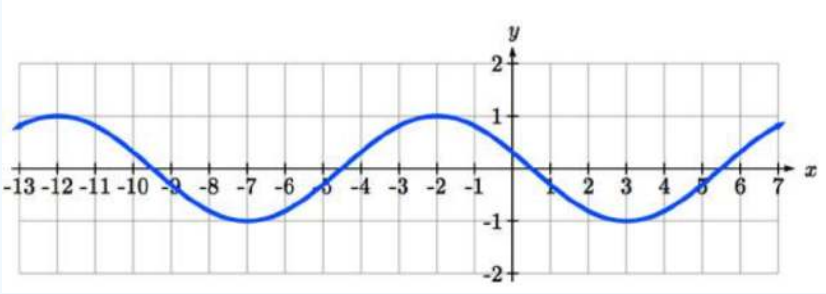
15. $y = \sin\left(\frac{\pi}{6}x + \pi\right) - 3$

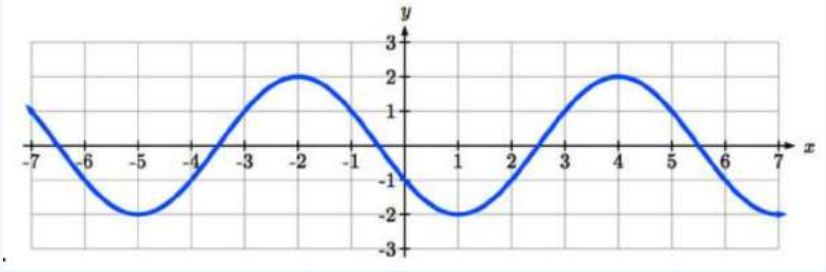
16. $y = 8\sin\left(\frac{7\pi}{6}x + \frac{7\pi}{2}\right) + 6$

Find a formula for each of the functions graphed below.

17. 

18. 

19. 

20. 

21. Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature is 50 degrees at midnight and the high and low temperature during the day are 57 and 43 degrees, respectively. Assuming t is the number of hours since midnight, find a function for the temperature, D, in terms of t.

22. Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature is 68 degrees at midnight and the high and low temperature during the day are 80 and 56 degrees, respectively. Assuming t is the number of hours since midnight, find a function for the temperature, D, in terms of t.

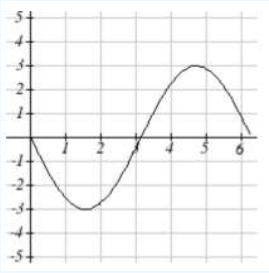
23. A Ferris wheel is 25 meters in diameter and boarded from a platform that is 1 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function $h(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn.

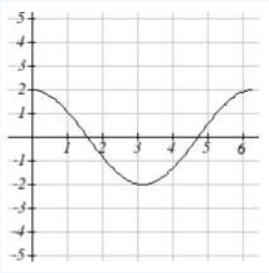
a. Find the amplitude, midline, and period of $h(t)$.
b. Find a formula for the height function $h(t)$.
c. How high are you off the ground after 5 minutes?

24. A Ferris wheel is 35 meters in diameter and boarded from a platform that is 3 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 8 minutes. The function $h(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn.

a. Find the amplitude, midline, and period of $h(t)$.
b. Find a formula for the height function $h(t)$.
c. How high are you off the ground after 4 minutes?

Answer

1. 

3. 

5. Amp: 3. Period = 2. Midline: $y = -4$. $f(t) = 3\sin(\pi t) - 4$

7. Amp: 2. Period = 4π. Midline: $y = 1$. $f(t) = 2\cos(\frac{1}{2}t) + 1$

9. Amp: 2. Period = 5. Midline: $y = 3$. $f(t) = -2\cos(\frac{2\pi}{5}t) + 3$

11. Amp: 3, Period = $\frac{\pi}{4}$, Shift: 4 left, Midline: $y = 5$

13. Amp: 2, Period = $\frac{2\pi}{3}$, Shift: 7 left, Midline: $y = 4$

15. Amp: 1, Period = 12, Shift: 6 left, Midline: $y = -3$

17. $f(x) = 4\sin(\frac{\pi}{5}(x+1))$

19. $f(x) = \cos(\frac{\pi}{5}(x+2))$

21. $D(t) = 50 - 7\sin(\frac{\pi}{12}t)$

23. a. Amp: 12.5. Midline: $y = 13.5$. Period: 10
b. $h(t) = -12.5\cos(\frac{\pi}{5}t) + 13.5$
c. $h(t) = 26$ meters

This page titled 9.4: Sinusoidal Models (Exercises) is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by David Lippman & Melonie Rasmussen (The OpenTextBookStore) via source content that was edited to the style and standards of the LibreTexts platform.

- **6.1E: Sinusoidal Graphs (Exercises)** by David Lippman & Melonie Rasmussen is licensed CC BY-SA 4.0. Original source: http://www.opentextbookstore.com/details.php?id=30.

9.5: AC Waveform - An Electrical Application

Sinusoidal Model for Current

The sine wave is extremely important when it comes to modeling electrical generation.

Any time you pass a conductor through a magnetic field, you induce a voltage. If we take that conductor and turn it into a loop and spin it continually through that magnetic field, we have created an alternator.

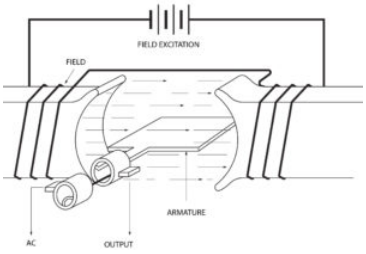


Figure 50. Alternator picture

This means that a voltage will constantly be induced. However, this is not a flat line voltage like direct current. It creates an oscillating voltage that rises and falls. As the conductor spins through the field, there will be times that it does not cut any lines of flux. There will be times when it cuts some of the lines of flux, and there will be times that it is cutting the maximum amount of flux lines that it can. This means that at some point during the sine wave generation there will be no voltage generated. Then there will be some voltage generated, and then there will be a maximum voltage generated. This creates this thing of beauty. A sine wave.

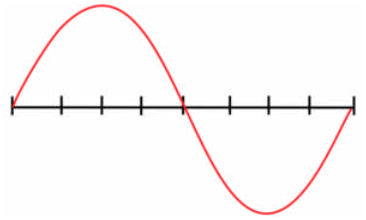
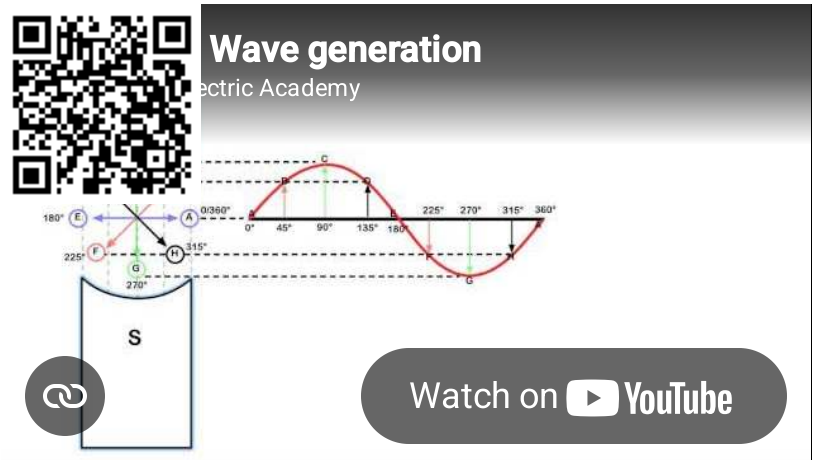


Figure 51. Sine wave

This video explains how a sine wave is generated in an alternator.



This page titled 9.5: AC Waveform - An Electrical Application is shared under a CC BY 4.0 license and was authored, remixed, and/or curated by Chad Flinn (BCcampus) via source content that was edited to the style and standards of the LibreTexts platform.

CHAPTER OVERVIEW

10: Law of Sines and Cosines

10.1: Non-right Triangles - Law of Sines
10.2: Non-right Triangles - Law of Cosines

10: Law of Sines and Cosines is shared under a CC BY-SA license and was authored, remixed, and/or curated by LibreTexts.

10.1: Non-right Triangles - Law of Sines

Learning Objectives

- Use the Law of Sines to solve oblique triangles.
- Solve applied problems using the Law of Sines.

Suppose two radar stations located 20 miles apart each detect an aircraft between them. The angle of elevation measured by the first station is 35 degrees, whereas the **angle of elevation** measured by the second station is 15 degrees. How can we determine the altitude of the aircraft? We see in Figure 10.1.1 that the triangle formed by the aircraft and the two stations is not a right triangle, so we cannot use what we know about right triangles. In this section, we will find out how to solve problems involving **non-right triangles**.

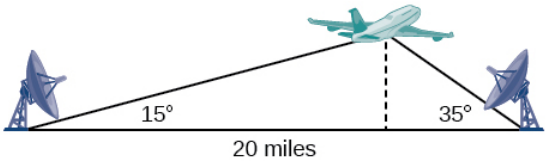


Figure 10.1.1

Using the Law of Sines to Solve Oblique Triangles

In any triangle, we can draw an **altitude**, a perpendicular line from one vertex to the opposite side, forming two right triangles. It would be preferable, however, to have methods that we can apply directly to non-right triangles without first having to create right triangles.

Any triangle that is not a right triangle is an **oblique triangle**. Solving an oblique triangle means finding the measurements of all three angles and all three sides. To do so, we need to start with at least three of these values, including at least one of the sides. We will investigate three possible oblique triangle problem situations:

ASA (angle-side-angle) We know the measurements of two angles and the included side. See Figure 10.1.2.

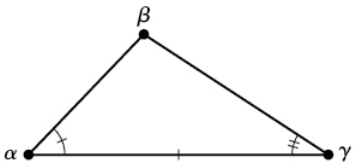


Figure 10.1.2

AAS (angle-angle-side) We know the measurements of two angles and a side that is not between the known angles. See Figure 10.1.3.

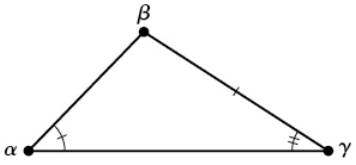


Figure 10.1.3

SSA (side-side-angle) We know the measurements of two sides and an angle that is not between the known sides. See Figure 10.1.4.

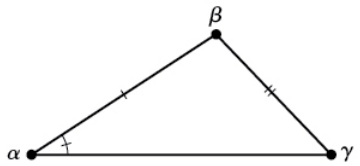


***Figure* 10.1.4**

Knowing how to approach each of these situations enables us to solve oblique triangles without having to drop a perpendicular to form two right triangles. Instead, we can use the fact that the ratio of the measurement of one of the angles to the length of its opposite side will be equal to the other two ratios of angle measure to opposite side. Let's see how this statement is derived by considering the triangle shown in Figure **10.1.5**.

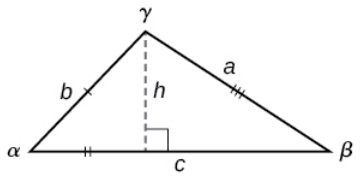


***Figure* 10.1.5**

Using the right triangle relationships, we know that $\sin\alpha = \frac{h}{b}$ and $\sin\beta = \frac{h}{a}$. Solving both equations for h gives two different expressions for h.

$$h = b\sin\alpha \text{ and } h = a\sin\beta$$

We then set the expressions equal to each other.

$$\begin{aligned} b\sin\alpha &= a\sin\beta \\ \left(\frac{1}{ab}\right)(b\sin\alpha) &= (a\sin\beta)\left(\frac{1}{ab}\right) \qquad \text{Multiply both sides by } \frac{1}{ab} \\ \frac{\sin\alpha}{a} &= \frac{\sin\beta}{b} \end{aligned}$$

Similarly, we can compare the other ratios.

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c} \text{ and } \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Collectively, these relationships are called the **Law of Sines**.

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Note the standard way of labeling triangles: angle α (alpha) is opposite side a; angle β (beta) is opposite side b; and angle γ (gamma) is opposite side c. See Figure **10.1.6**.

While calculating angles and sides, be sure to carry the exact values through to the final answer. Generally, final answers are rounded to the nearest tenth, unless otherwise specified.

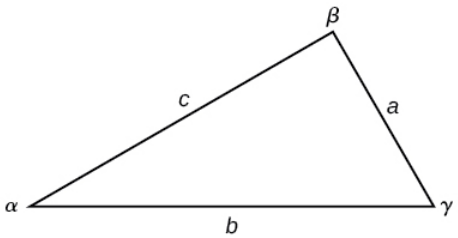


***Figure* 10.1.6**

Note: LAW OF SINES

Given a triangle with angles and opposite sides labeled as in Figure **10.1.6**, the ratio of the measurement of an angle to the length of its opposite side will be equal to the other two ratios of angle measure to opposite side. All proportions will be equal. The **Law of Sines** is based on proportions and is presented symbolically two ways.

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \tag{10.1.1}$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \tag{10.1.2}$$

To solve an oblique triangle, use any pair of applicable ratios.

Example 10.1.1: Solving for Two Unknown Sides and Angle of an AAS Triangle

Solve the triangle shown in Figure **10.1.7** to the nearest tenth.

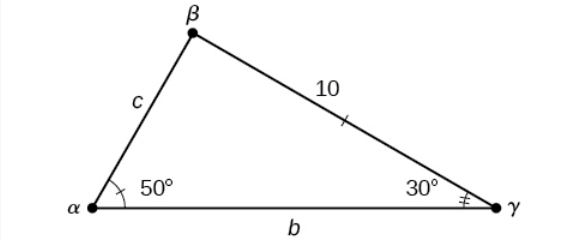


Figure 10.1.7

Solution

The three angles must add up to 180 degrees. From this, we can determine that

$$\begin{aligned}\beta &= 180^\circ - 50^\circ - 30^\circ \\ &= 100^\circ\end{aligned}$$

To find an unknown side, we need to know the corresponding angle and a known ratio. We know that angle $\alpha = 50°$and its corresponding side $a = 10$. We can use the following proportion from the Law of Sines to find the length of c.

$$\begin{aligned}\frac{\sin(50^\circ)}{10} &= \frac{\sin(30^\circ)}{c} \\ c\frac{\sin(50^\circ)}{10} &= \sin(30^\circ) \quad \text{Multiply both sides by } c \\ c &= \sin(30^\circ)\frac{10}{\sin(50^\circ)} \quad \text{Multiply by the reciprocal to isolate } c \\ c &\approx 6.5\end{aligned}$$

Similarly, to solve for b, we set up another proportion.

$$\begin{aligned}\frac{\sin(50^\circ)}{10} &= \frac{\sin(100^\circ)}{b} \\ b\sin(50^\circ) &= 10\sin(100^\circ) \quad \text{Multiply both sides by } b \\ b &= \frac{10\sin(100^\circ)}{\sin(50^\circ)} \quad \text{Multiply by the reciprocal to isolate } b \\ b &\approx 12.9\end{aligned}$$

Therefore, the complete set of angles and sides is

$$\begin{aligned}\alpha &= 50^\circ \quad a = 10 \\ \beta &= 100^\circ \quad b \approx 12.9 \\ \gamma &= 30^\circ \quad c \approx 6.5\end{aligned}$$

Exercise 10.1.1

Solve the triangle shown in Figure 10.1.8 to the nearest tenth.

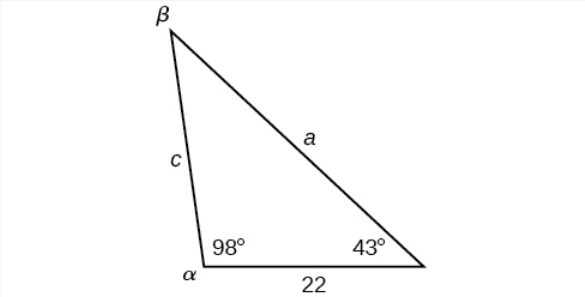


Figure 10.1.8

Answer

$\alpha = 98^\circ \quad a = 34.6$
$\beta = 39^\circ \quad b = 22$
$\gamma = 43^\circ \quad c = 23.8$

Using The Law of Sines to Solve SSA Triangles

We can use the Law of Sines to solve any oblique triangle, but some solutions may not be straightforward. In some cases, more than one triangle may satisfy the given criteria, which we describe as an ambiguous case. Triangles classified as SSA, those in which we know the lengths of two sides and the measurement of the angle opposite one of the given sides, may result in one or two solutions, or even no solution.

Note: POSSIBLE OUTCOMES FOR SSA TRIANGLES

Oblique triangles in the category SSA may have four different outcomes. Figure 10.1.9 illustrates the solutions with the known sides a and b and known angle α.

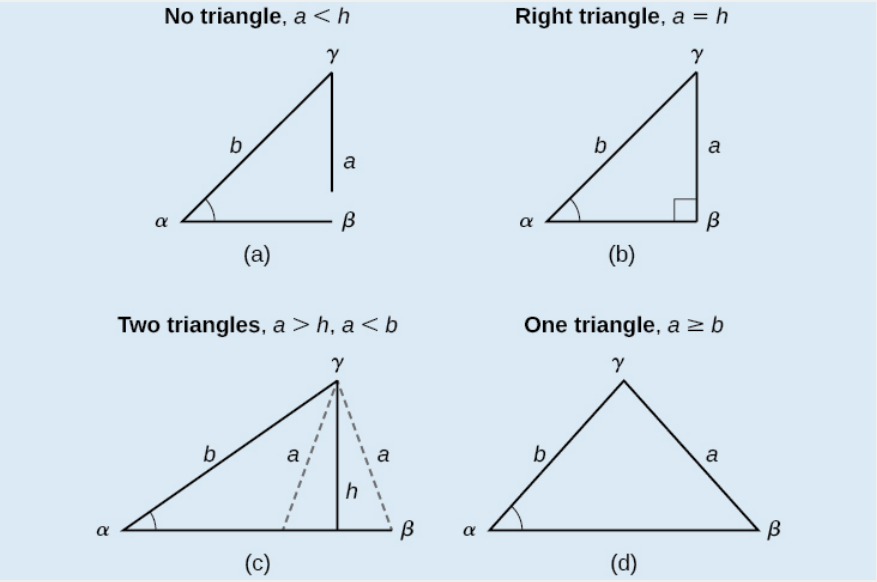


Figure 10.1.9

https://math.libretexts.org/@go/page/194249

Example 10.1.2: Solving an Oblique SSA Triangle

Solve the triangle in Figure 10.1.10 for the missing side and find the missing angle measures to the nearest tenth.

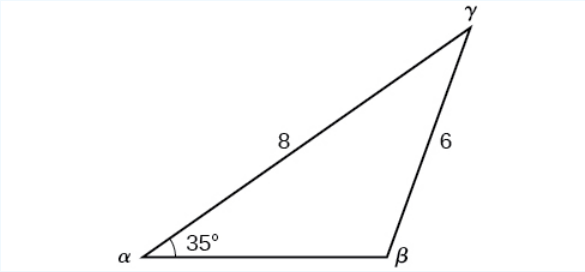


Figure 10.1.10

Solution

Use the Law of Sines to find angle β and angle γ, and then side c. Solving for β, we have the proportion

$$\begin{aligned}\frac{\sin\alpha}{a} &= \frac{\sin\beta}{b}\\ \frac{\sin(35°)}{6} &= \frac{\sin\beta}{8}\\ \frac{8\sin(35°)}{6} &= \sin\beta\\ 0.7648 &\approx \sin\beta\\ \sin^{-1}(0.7648) &\approx 49.9°\\ \beta &\approx 49.9°\end{aligned}$$

However, in the diagram, angle β appears to be an obtuse angle and may be greater than $90°$. How did we get an acute angle, and how do we find the measurement of β? Let's investigate further. Dropping a perpendicular from γ and viewing the triangle from a right angle perspective, we have Figure 10.1.11. It appears that there may be a second triangle that will fit the given criteria.

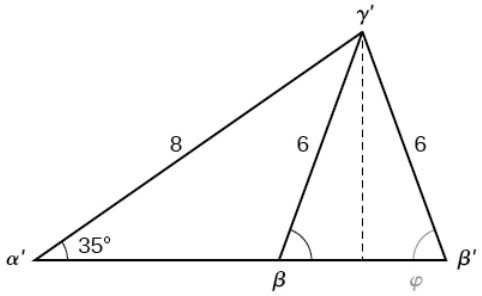


Figure 10.1.11

The angle supplementary to β is approximately equal to $49.9°$, which means that $\beta = 180° - 49.9° = 130.1°$. (Remember that the sine function is positive in both the first and second quadrants.) Solving for γ, we have

$$\begin{aligned}\gamma &= 180° - 35° - 130.1°\\ &\approx 14.9°\end{aligned}$$

We can then use these measurements to solve the other triangle. Since β' is supplementary to β, we have

$$\begin{aligned}\gamma' &= 180° - 35° - 49.5°\\ &\approx 95.1°\end{aligned}$$

Now we need to find c and c'.

We have

$$\frac{c}{\sin(14.9°)} = \frac{6}{\sin(35°)}$$
$$c = \frac{6\sin(14.9°)}{\sin(35°)}$$
$$\approx 2.7$$

Finally,

$$\frac{c'}{\sin(95.1°)} = \frac{6}{\sin(35°)}$$
$$c' = \frac{6\sin(95.1°)}{\sin(35°)}$$
$$\approx 10.4$$

To summarize, there are two triangles with an angle of $35°$, an adjacent side of 8, and an opposite side of 6, as shown in Figure 10.1.12.

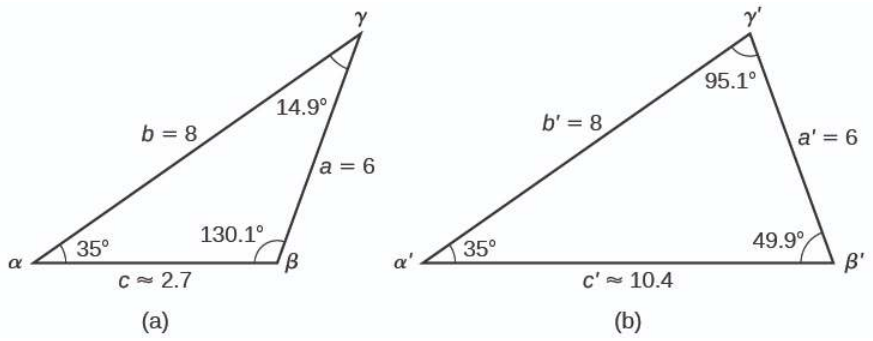


***Figure* 10.1.12**

However, we were looking for the values for the triangle with an obtuse angle β. We can see them in the first triangle (a) in Figure 10.1.12.

? Exercise 10.1.2

Given $\alpha = 80°$, $a = 120$, and $b = 121$, find the missing side and angles. If there is more than one possible solution, show both.

Answer

Solution 1

$\alpha = 80°$ $a = 120$
$\beta \approx 83.2°$ $b = 121$
$\gamma \approx 16.8°$ $c \approx 35.2$

Solution 2

$\alpha' = 80°$ $a' = 120$
$\beta' \approx 96.8°$ $b' = 121$
$\gamma' \approx 3.2°$ $c' \approx 6.8$

✓ Example 10.1.3: Solving for the Unknown Sides and Angles of a SSA Triangle

In the triangle shown in Figure 10.1.13, solve for the unknown side and angles. Round your answers to the nearest tenth.

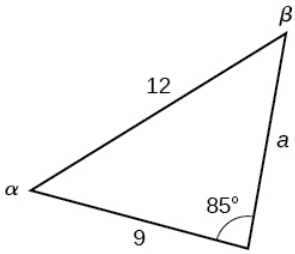


Figure 10.1.13

Solution

In choosing the pair of ratios from the Law of Sines to use, look at the information given. In this case, we know the angle, $\gamma = 85°$, and its corresponding side $c = 12$, and we know side $b = 9$. We will use this proportion to solve for β.

$$\frac{\sin(85°)}{12} = \frac{\sin\beta}{9} \qquad \text{Isolate the unknown.}$$
$$\frac{9\sin(85°)}{12} = \sin\beta$$

To find β, apply the inverse sine function. The inverse sine will produce a single result, but keep in mind that there may be two values for β. It is important to verify the result, as there may be two viable solutions, only one solution (the usual case), or no solutions.

$$\beta = \sin^{-1}\left(\frac{9\sin(85°)}{12}\right)$$
$$\beta \approx \sin^{-1}(0.7471)$$
$$\beta \approx 48.3°$$

In this case, if we subtract β from $180°$, we find that there may be a second possible solution. Thus, $\beta = 180° - 48.3° \approx 131.7°$. To check the solution, subtract both angles, $131.7°$ and $85°$, from $180°$. This gives

$$\alpha = 180° - 85° - 131.7°$$
$$\approx -36.7°$$

which is impossible, and so $\beta \approx 48.3°$.

To find the remaining missing values, we calculate $\alpha = 180° - 85° - 48.3° \approx 46.7°$. Now, only side a is needed. Use the Law of Sines to solve for a by one of the proportions.

$$\frac{\sin(85°)}{12} = \frac{\sin(46.7°)}{a}$$
$$a\frac{\sin(85°)}{12} = \sin(46.7°)$$
$$a = \frac{12\sin(46.7°)}{\sin(85°)}$$
$$\approx 8.8$$

The complete set of solutions for the given triangle is

$$\alpha \approx 46.7° \quad a \approx 8.8$$
$$\beta \approx 48.3° \quad b = 9$$
$$\gamma = 85° \quad c = 12$$

? Exercise 10.1.3

Given $\alpha = 80°$, $a = 100$, $b = 10$, find the missing side and angles. If there is more than one possible solution, show both. Round your answers to the nearest tenth.

Answer

$\beta \approx 5.7°, \gamma \approx 94.3°, c \approx 101.3$

Example 10.1.4: Finding the Triangles That Meet the Given Criteria

Find all possible triangles if one side has length 4 opposite an angle of $50°$, and a second side has length 10.

Solution

Using the given information, we can solve for the angle opposite the side of length 10. See Figure $10.1.14$.

$$\frac{\sin\alpha}{10} = \frac{\sin(50°)}{4}$$
$$\sin\alpha = \frac{10\sin(50°)}{4}$$
$$\sin\alpha \approx 1.915$$

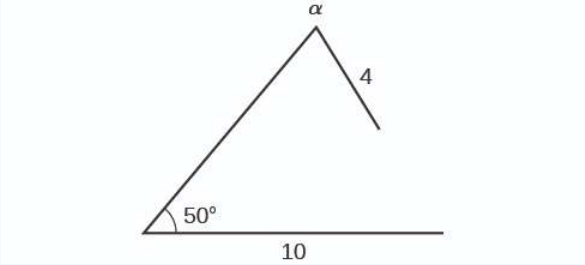


Figure 10.1.14

We can stop here without finding the value of α. Because the range of the sine function is $[-1, 1]$, it is impossible for the sine value to be 1.915. In fact, inputting $\sin^{-1}(1.915)$ in a graphing calculator generates an ERROR DOMAIN. Therefore, no triangles can be drawn with the provided dimensions.

Exercise 10.1.4

Determine the number of triangles possible given $a = 31, b = 26, \beta = 48°$.

Answer

two

Solving Applied Problems Using the Law of Sines

The more we study trigonometric applications, the more we discover that the applications are countless. Some are flat, diagram-type situations, but many applications in calculus, engineering, and physics involve three dimensions and motion.

Example 10.1.6: Finding an Altitude

Find the altitude of the aircraft in the problem introduced at the beginning of this section, shown in Figure $10.1.16$. Round the altitude to the nearest tenth of a mile.

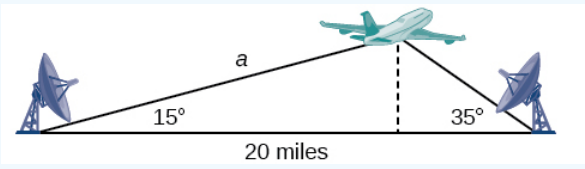


Figure 10.1.16

Solution

To find the elevation of the aircraft, we first find the distance from one station to the aircraft, such as the side a, and then use right triangle relationships to find the height of the aircraft, h.

Because the angles in the triangle add up to 180 degrees, the unknown angle must be $180°-15°-35°=130°$. This angle is opposite the side of length 20, allowing us to set up a Law of Sines relationship.

$$\begin{aligned}\frac{\sin(130°)}{20} &= \frac{\sin(35°)}{a}\\ a\sin(130°) &= 20\sin(35°)\\ a &= \frac{20\sin(35°)}{\sin(130°)}\\ a &\approx 14.98\end{aligned}$$

The distance from one station to the aircraft is about 14.98 miles.

Now that we know a, we can use right triangle relationships to solve for h.

$$\begin{aligned}\sin(15°) &= \frac{opposite}{hypotenuse}\\ \sin(15°) &= \frac{h}{a}\\ \sin(15°) &= \frac{h}{14.98}\\ h &= 14.98\sin(15°)\\ h &\approx 3.88\end{aligned}$$

The aircraft is at an altitude of approximately 3.9 miles.

Exercise 10.1.6

The diagram shown in Figure $10.1.17$ represents the height of a blimp flying over a football stadium. Find the height of the blimp if the angle of elevation at the southern end zone, point A, is $70°$, the angle of elevation from the northern end zone, point B, is $62°$, and the distance between the viewing points of the two end zones is 145 yards.

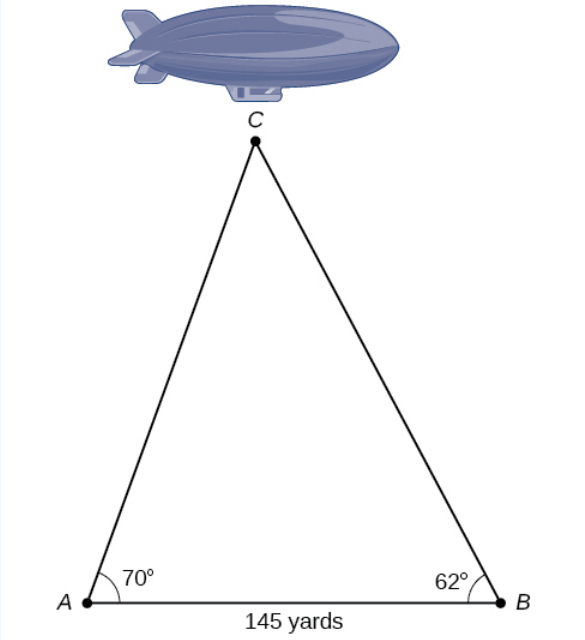


***Figure* 10.1.17**

Answer

161.9 yd.

Media

Access these online resources for additional instruction and practice with trigonometric applications.

- Law of Sines: The Basics
- Law of Sines: The Ambiguous Case

Key Equations

Law of Sines	$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$ $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
Area for oblique triangles	$Area = \frac{1}{2}bc\sin\alpha$ $= \frac{1}{2}ac\sin\beta$ $= \frac{1}{2}ab\sin\gamma$

Key Concepts

- The Law of Sines can be used to solve oblique triangles, which are non-right triangles.
- According to the Law of Sines, the ratio of the measurement of one of the angles to the length of its opposite side equals the other two ratios of angle measure to opposite side.
- There are three possible cases: ASA, AAS, SSA. Depending on the information given, we can choose the appropriate equation to find the requested solution. See Example **10.1.1**.
- The ambiguous case arises when an oblique triangle can have different outcomes.
- There are three possible cases that arise from SSA arrangement—a single solution, two possible solutions, and no solution. See Example **10.1.2** and Example **10.1.3**.
- The Law of Sines can be used to solve triangles with given criteria. See Example **10.1.4**.
- The general area formula for triangles translates to oblique triangles by first finding the appropriate height value. See Example **10.1.5**.
- There are many trigonometric applications. They can often be solved by first drawing a diagram of the given information and then using the appropriate equation. See Example **10.1.6**.

Contributors and Attributions

- Jay Abramson (Arizona State University) with contributing authors. Textbook content produced by OpenStax College is licensed under a Creative Commons Attribution License 4.0 license. Download for free at https://openstax.org/details/books/precalculus.

This page titled 10.1: Non-right Triangles - Law of Sines is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

10.2: Non-right Triangles - Law of Cosines

Learning Objectives

In this section, you will:

- Use the Law of Cosines to solve oblique triangles.
- Solve applied problems using the Law of Cosines.

Suppose a boat leaves port, travels 10 miles, turns 20 degrees, and travels another 8 miles as shown in Figure 10.2.1 How far from port is the boat?

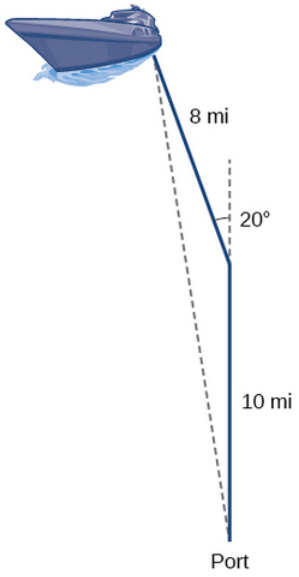


***Figure* 10.2.1**

Unfortunately, while the Law of Sines enables us to address many non-right triangle cases, it does not help us with triangles where the known angle is between two known sides, a SAS (side-angle-side) triangle, or when all three sides are known, but no angles are known, a SSS (side-side-side) triangle. In this section, we will investigate another tool for solving oblique triangles described by these last two cases.

Using the Law of Cosines to Solve Oblique Triangles

The tool we need to solve the problem of the boat's distance from the port is the *Law of Cosines*, which defines the relationship among angle measurements and side lengths in oblique triangles. Three formulas make up the Law of Cosines. At first glance, the formulas may appear complicated because they include many variables. However, once the pattern is understood, the Law of Cosines is easier to work with than most formulas at this mathematical level.

Understanding how the Law of Cosines is derived will be helpful in using the formulas. The derivation begins with the Generalized Pythagorean Theorem, which is an extension of the Pythagorean Theorem to non-right triangles. Here is how it works: An arbitrary non-right triangle ABC is placed in the coordinate plane with vertex A at the origin, side c drawn along the x-axis, and vertex C located at some point (x, y) in the plane, as illustrated in Figure 10.2.2. Generally, triangles exist anywhere in the plane, but for this explanation we will place the triangle as noted.

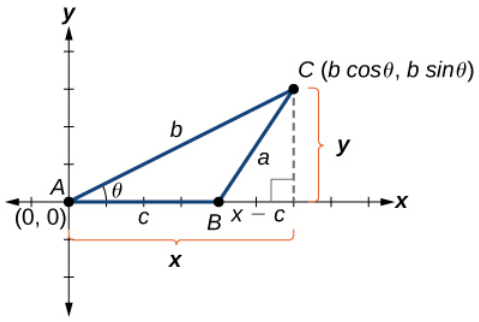


***Figure* 10.2.2**

We can drop a perpendicular from C to the x-axis (this is the altitude or height). Recalling the basic trigonometric identities, we know that

$$\cos\theta = \frac{x(adjacent)}{b(hypotenuse)} \text{ and } \sin\theta = \frac{y(opposite)}{b(hypotenuse)}$$

In terms of θ, $x = b\cos\theta$ and $y = b\sin\theta$. The (x, y) point located at C has coordinates $(b\cos\theta, b\sin\theta)$. Using the side $(x - c)$ as one leg of a right triangle and y as the second leg, we can find the length of hypotenuse a using the Pythagorean Theorem. Thus,

$$\begin{aligned}
a^2 &= (x-c)^2 + y^2 \\
&= (b\cos\theta - c)^2 + (b\sin\theta)^2 && \text{Substitute } (b\cos\theta) \text{ for } x \text{ and } (b\sin\theta) \text{ for } y \\
&= (b^2\cos^2\theta - 2bc\cos\theta + c^2) + b^2\sin^2\theta && \text{Expand the perfect square.} \\
&= b^2\cos^2\theta + b^2\sin^2\theta + c^2 - 2bc\cos\theta && \text{Group terms noting that } \cos^2\theta + \sin^2\theta = 1 \\
&= b^2(\cos^2\theta + \sin^2\theta) + c^2 - 2bc\cos\theta && \text{Factor out } b^2 \\
a^2 &= b^2 + c^2 - 2bc\cos\theta
\end{aligned}$$

The formula derived is one of the three equations of the Law of Cosines. The other equations are found in a similar fashion.

Keep in mind that it is always helpful to sketch the triangle when solving for angles or sides. In a real-world scenario, try to draw a diagram of the situation. As more information emerges, the diagram may have to be altered. Make those alterations to the diagram and, in the end, the problem will be easier to solve.

The LAW OF COSINES

The Law of Cosines states that the square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of the other two sides and the cosine of the included angle.

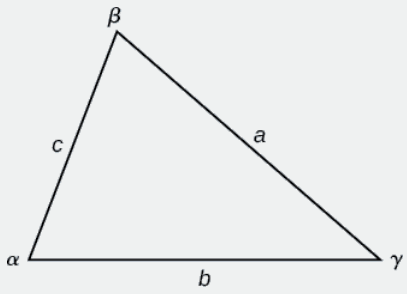


***Figure* 10.2.3**

For triangles labeled as in Figure **10.2.3**, with angles α, β and γ, and opposite corresponding sides a, b, and c, respectively, the Law of Cosines is given as three equations.

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \tag{10.2.1}$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta \tag{10.2.2}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \tag{10.2.3}$$

To solve for a missing side measurement, the corresponding opposite angle measure is needed.

When solving for an angle, the corresponding opposite side measure is needed. We can use another version of the Law of Cosines to solve for an angle.

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \tag{10.2.4}$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} \tag{10.2.5}$$

$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab} \tag{10.2.6}$$

How to: Given two sides and the angle between them (SAS), find the measures of the remaining side and angles of a triangle

1. Sketch the triangle. Identify the measures of the known sides and angles. Use variables to represent the measures of the unknown sides and angles.
2. Apply the Law of Cosines to find the length of the unknown side or angle.
3. Apply the Law of Sines or Cosines to find the measure of a second angle.
4. Compute the measure of the remaining angle.

Example 10.2.1: Finding the Unknown Side and Angles of a SAS Triangle

Find the unknown side and angles of the triangle in Figure 10.2.4.

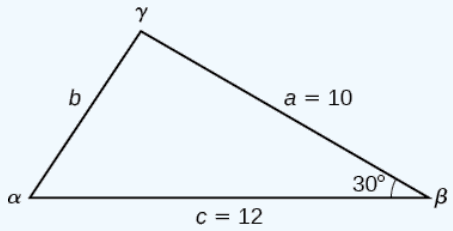


***Figure* 10.2.4**

Solution

First, make note of what is given: two sides and the angle between them. This arrangement is classified as SAS and supplies the data needed to apply the Law of Cosines.

Each one of the three laws of cosines begins with the square of an unknown side opposite a known angle. For this example, the first side to solve for is side b, as we know the measurement of the opposite angle β.

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$b^2 = 10^2 + 12^2 - 2(10)(12)\cos(30°) \quad \text{Substitute the measurements for the known quantities.}$$

$$b^2 = 100 + 144 - 240\left(\frac{\sqrt{3}}{2}\right) \quad \text{Evaluate the cosine and begin to simplify.}$$

$$b^2 = 244 - 120\sqrt{3}$$

$$b = \sqrt{244 - 120\sqrt{3}} \quad \text{Use the square root property.}$$

$$b \approx 6.013$$

Because we are solving for a length, we use only the positive square root. Now that we know the length b, we can use the Law of Sines to fill in the remaining angles of the triangle. Solving for angle α, we have

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin\alpha}{10} = \frac{\sin(30°)}{6.013}$$

$$\sin\alpha = \frac{10\sin(30°)}{6.013} \qquad \text{Multiply both sides of the equation by 10.}$$

$$\alpha = \sin^{-1}\left(\frac{10\sin(30°)}{6.013}\right) \qquad \text{Find the inverse sine of } \frac{10\sin(30°)}{6.013}.$$

$$\alpha \approx 56.3°$$

The other possibility for α would be $\alpha = 180° - 56.3° \approx 123.7°$. In the original diagram, α is adjacent to the longest side, so α is an acute angle and, therefore, $123.7°$ does not make sense. Notice that if we choose to apply the Law of Cosines, we arrive at a unique answer. We do not have to consider the other possibilities, as cosine is unique for angles between $0°$ and $180°$. Proceeding with $\alpha \approx 56.3°$, we can then find the third angle of the triangle.

$$\begin{aligned}\gamma &= 180° - 30° - 56.3° \\ &\approx 93.7°\end{aligned}$$

The complete set of angles and sides is

$$\alpha \approx 56.3° \; a = 10$$

$$\beta = 30° \; b \approx 6.013$$

$$\gamma \approx 93.7° \; c = 12$$

Exercise 10.2.1

Find the missing side and angles of the given triangle: $\alpha = 30°$, $b = 12$, $c = 24$.

Answer

$a \approx 14.9$, $\beta \approx 23.8°$, $\gamma \approx 126.2°$.

Example 10.2.2: Solving for an Angle of a SSS Triangle

Find the angle α for the given triangle if side $a = 20$, side $b = 25$, and side $c = 18$.

Solution

For this example, we have no angles. We can solve for any angle using the Law of Cosines. To solve for angle α, we have

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$20^2 = 25^2 + 18^2 - 2(25)(18)\cos\alpha \qquad \text{Substitute the appropriate measurements.}$$

$$400 = 625 + 324 - 900\cos\alpha \qquad \text{Simplify in each step.}$$

$$400 = 949 - 900\cos\alpha$$

$$-549 = -900\cos\alpha \qquad \text{Isolate } \cos\alpha.$$

$$0.61 \approx \cos\alpha \qquad \text{Find the inverse cosine.}$$

$$\alpha \approx 52.4°$$

See Figure 10.2.5.

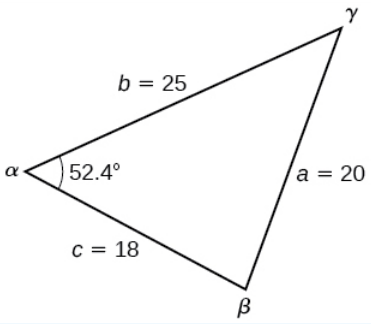


***Figure* 10.2.5**

Analysis

Because the inverse cosine can return any angle between 0 and 180 degrees, there will not be any ambiguous cases using this method.

? Exercise 10.2.2

Given $a = 5$, $b = 7$, and $c = 10$, find the missing angles.

Answer

$\alpha \approx 27.7°$, $\beta \approx 40.5°$, $\gamma \approx 111.8°$

Solving Applied Problems Using the Law of Cosines

Just as the Law of Sines provided the appropriate equations to solve a number of applications, the Law of Cosines is applicable to situations in which the given data fits the cosine models. We may see these in the fields of navigation, surveying, astronomy, and geometry, just to name a few.

✓ Example 10.2.3A: Using the Law of Cosines to Solve a Communication Problem

On many cell phones with GPS, an approximate location can be given before the GPS signal is received. This is accomplished through a process called triangulation, which works by using the distances from two known points. Suppose there are two cell phone towers within range of a cell phone. The two towers are located 6000 feet apart along a straight highway, running east to west, and the cell phone is north of the highway. Based on the signal delay, it can be determined that the signal is 5050 feet from the first tower and 2420 feet from the second tower. Determine the position of the cell phone north and east of the first tower, and determine how far it is from the highway.

Solution

For simplicity, we start by drawing a diagram similar to Figure $10.2.6$ and labeling our given information.

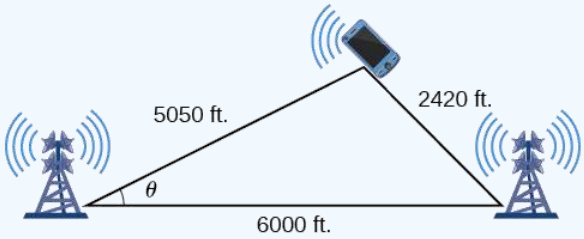


***Figure* 10.2.6**

Using the Law of Cosines, we can solve for the angle θ. Remember that the Law of Cosines uses the square of one side to find the cosine of the opposite angle. For this example, let $a = 2420$, $b = 5050$, and $c = 6000$. Thus, θ corresponds to the opposite side $a = 2420$.

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$
$$(2420)^2 = (5050)^2 + (6000)^2 - 2(5050)(6000)\cos\theta$$
$$\cos\theta \approx 0.9183$$
$$\theta \approx \cos^{-1}(0.9183)$$
$$\theta \approx 23.3°$$

To answer the questions about the phone's position north and east of the tower, and the distance to the highway, drop a perpendicular from the position of the cell phone, as in Figure 10.2.7. This forms two right triangles, although we only need the right triangle that includes the first tower for this problem.

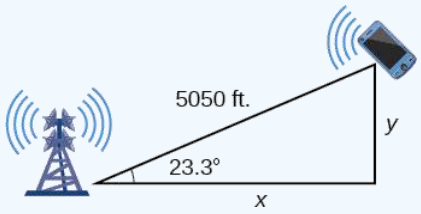


***Figure* 10.2.7**

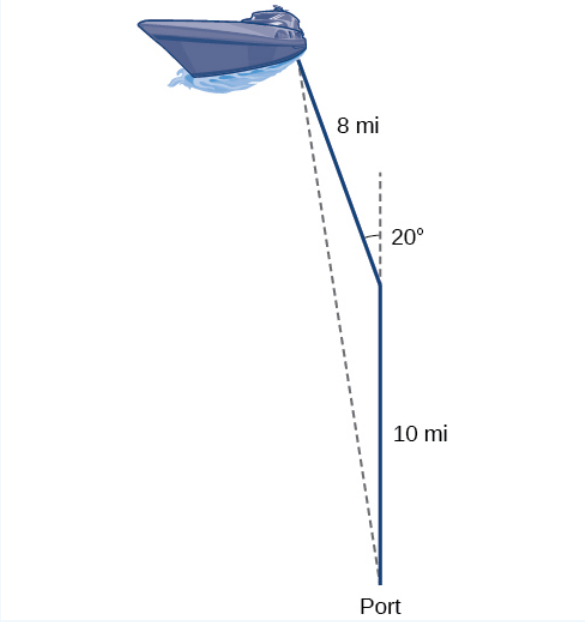
Using the angle $\theta = 23.3°$ and the basic trigonometric identities, we can find the solutions. Thus

$$\cos(23.3°) = \frac{x}{5050}$$
$$x = 5050\cos(23.3°)$$
$$x \approx 4638.15\ feet$$
$$\sin(23.3°) = \frac{y}{5050}$$
$$y = 5050\sin(23.3°)$$
$$y \approx 1997.5\ feet$$

The cell phone is approximately 4638 feet east and 1998 feet north of the first tower, and 1998 feet from the highway.

Example 10.2.3*B*: Calculating Distance Traveled Using a SAS Triangle

Returning to our problem at the beginning of this section, suppose a boat leaves port, travels 10 miles, turns 20 degrees, and travels another 8 miles. How far from port is the boat? The diagram is repeated here in Figure 10.2.8.



https://math.libretexts.org/@go/page/194250

Figure 10.2.8

Solution

The boat turned 20 degrees, so the obtuse angle of the non-right triangle is the supplemental angle, $180°-20°=160°$. With this, we can utilize the Law of Cosines to find the missing side of the obtuse triangle—the distance of the boat to the port.

$$x^2 = 8^2 + 10^2 - 2(8)(10)\cos(160°)$$
$$x^2 = 314.35$$
$$x = \sqrt{314.35}$$
$$x \approx 17.7\, miles$$

The boat is about 17.7 miles from port.

Media

Access these online resources for additional instruction and practice with the Law of Cosines.

- Law of Cosines
- Law of Cosines: Applications
- Law of Cosines: Applications 2

Key Equations

Law of Cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = a^2 + c^2 - 2ac\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Key Concepts

- The Law of Cosines defines the relationship among angle measurements and lengths of sides in oblique triangles.
- The Generalized Pythagorean Theorem is the Law of Cosines for two cases of oblique triangles: SAS and SSS. Dropping an imaginary perpendicular splits the oblique triangle into two right triangles or forms one right triangle, which allows sides to be related and measurements to be calculated. See Example 10.2.1 and Example 10.2.2.
- The Law of Cosines is useful for many types of applied problems. The first step in solving such problems is generally to draw a sketch of the problem presented. If the information given fits one of the three models (the three equations), then apply the Law of Cosines to find a solution. See Example 10.2.3 and Example 10.2.4.

Contributors and Attributions

- Jay Abramson (Arizona State University) with contributing authors. Textbook content produced by OpenStax College is licensed under a Creative Commons Attribution License 4.0 license. Download for free at https://openstax.org/details/books/precalculus.

This page titled 10.2: Non-right Triangles - Law of Cosines is shared under a CC BY-SA 4.0 license and was authored, remixed, and/or curated by OpenStax via source content that was edited to the style and standards of the LibreTexts platform.

Index

A

Absolute Value
- 1.3: Negative Numbers

alternator
- 7.6.3.2: The Alternator

angle
- 7.1: Angles

area
- 6.4: Area of Polygons and Circles

Area of oblique triangles
- 10.1: Non-right Triangles - Law of Sines

arithmetic
- 1.1: Place Value

C

circumference
- 6.3: Perimeter and Circumference

D

damped harmonic motion
- 9.3: More Sinusoidal Modeling

decimals
- 1.1: Place Value
- 1.4: Decimals

E

equilateral triangle
- 7.2: Triangles

exercises
- 1.1: Place Value

F

fraction
- 1.5: Fractions

fractions
- 1.1: Place Value

H

Heron's Formula
- 10.2: Non-right Triangles - Law of Cosines

Hypotenuse
- 7.3: Introduction to Right Triangle Trigonometry
- 7.6.1.4: Naming Right Triangle Sides

I

impedance
- 7.5: Power and Impedance Triangles - An Elelctrical Example
- 7.6.1.6: Power and Impedance Triangles

impedance triangle
- 7.5: Power and Impedance Triangles - An Elelctrical Example
- 7.6.1.6: Power and Impedance Triangles

induction
- 7.6.3.1: Electromagnetic Induction

isosceles triangle
- 7.2: Triangles

L

law of cosines
- 10.2: Non-right Triangles - Law of Cosines

Law of Sines
- 10.1: Non-right Triangles - Law of Sines

learning
- 1.1: Place Value

liter
- 5.5: The Metric System

M

metric system
- 5.5: The Metric System

mixed numbers
- 1.5: Fractions

multiplying fractions
- 1.5: Fractions

N

negative numbers
- 1.3: Negative Numbers

number
- 1.1: Place Value

O

objectives
- 1.1: Place Value

obtuse triangle
- 7.2: Triangles

Order of Operations
- 1.6: Order of Operations

P

parallel lines
- 7.1: Angles

parallelogram
- 6.4: Area of Polygons and Circles

perimeter
- 6.3: Perimeter and Circumference
- 6.4: Area of Polygons and Circles

perpendicular lines
- 7.1: Angles

polarity
- 7.6.2.2: Quadrants

power
- 7.5: Power and Impedance Triangles - An Elelctrical Example
- 7.6.1.6: Power and Impedance Triangles

Practice
- 1.1: Place Value

prisms
- 6.6: Volume of Common Solids

proportion
- 3.1: Ratios, Rates, Proportions

Pythagorean Theorem
- 7.2: Triangles

Q

Quadrant
- 7.6.2.2: Quadrants

R

rate
- 3.1: Ratios, Rates, Proportions

ratio
- 3.1: Ratios, Rates, Proportions

right triangle
- 7.2: Triangles

right triangles
- 7.2: Triangles

S

scalar
- 8.4: Vector Operations

similar triangles
- 7.2: Triangles

simple harmonic motion
- 9.3: More Sinusoidal Modeling

simplifying fractions
- 1.5: Fractions

standard form of sinusoidal equation
- 9.3: More Sinusoidal Modeling

system
- 1.1: Place Value

T

triangles
- 7.2: Triangles

V

vector
- 7.6.2.1: A Vector Primer
- 8.3: Introduction to Vectors

vector addition
- 7.6.2.4: Vector Addition
- 8.5: Vector Addition in the Real World - Electrical Application

vector magnitude
- 8.4: Vector Operations

vectors
- 8.4: Vector Operations

volume
- 6.6: Volume of Common Solids

W

writing
- 1.1: Place Value

Glossary

Sample Word 1 | Sample Definition 1

Detailed Licensing

Overview

Title: Technical Mathematics SJDC 1

Webpages: 95

Applicable Restrictions: Noncommercial

All licenses found:

- CC BY-SA 4.0: 41.1% (39 pages)
- CC BY 4.0: 30.5% (29 pages)
- Undeclared: 18.9% (18 pages)
- CC BY-NC-SA 3.0: 5.3% (5 pages)
- CC BY-NC-SA 4.0: 3.2% (3 pages)
- CC BY-SA 1.3: 1.1% (1 page)

By Page

- Technical Mathematics SJDC 1 - *CC BY-NC-SA 4.0*
  - Front Matter - *Undeclared*
    - TitlePage - *Undeclared*
    - InfoPage - *Undeclared*
    - Table of Contents - *Undeclared*
    - Licensing - *Undeclared*
  - 1: Review of Arithmetic - *CC BY-SA 4.0*
    - 1.1: Place Value - *CC BY 4.0*
    - 1.2: Rounding Numbers - *CC BY 4.0*
    - 1.3: Negative Numbers - *CC BY-SA 4.0*
    - 1.4: Decimals - *CC BY-SA 4.0*
    - 1.5: Fractions - *CC BY-SA 4.0*
    - 1.6: Order of Operations - *CC BY-SA 4.0*
    - 1.7: Evaluating Algebraic Expressions and Formulas - *CC BY-NC-SA 3.0*
  - 2: Solving Equations - *CC BY-SA 4.0*
    - 2.1: The Language of Algebra - *CC BY-SA 4.0*
    - 2.2: Solving Equations with Addition and Subtraction Properties of Equality - *CC BY-SA 4.0*
    - 2.3: Solving Equations with the Division and Multiplication Properties of Equality - *CC BY-SA 4.0*
    - 2.4: Solving Equations with Variables and Constants on Both Sides - *CC BY-SA 4.0*
    - 2.5: Solving Equations with Fractions or Decimals - *CC BY-SA 4.0*
    - 2.6: General Strategy to Solve Linear Equations - *CC BY-SA 4.0*
    - 2.7: Solving a Formula for a Specific Variable - *CC BY-SA 4.0*
  - 3: Proportions and Variation - *CC BY-SA 4.0*
    - 3.1: Ratios, Rates, Proportions - *CC BY-SA 4.0*
    - 3.2: Applications of Proportionality - *CC BY-SA 4.0*
    - 3.3: Direct and Inverse Variation - *CC BY-SA 4.0*
  - 4: Percent - *CC BY-NC-SA 3.0*
    - 4.1: Basic Percent Problems - *CC BY-NC-SA 3.0*
    - 4.2: Percents in the Real World - *CC BY-NC-SA 3.0*
    - 4.3: Percent Increase or Decrease - *CC BY-NC-SA 3.0*
  - 5: Units of Measure and Converting Units - *CC BY-SA 4.0*
    - 5.1: Scientific and Engineering Notation - *CC BY-SA 4.0*
    - 5.2: Accuracy and Significant Figures - *CC BY-NC-SA 4.0*
    - 5.3: The US Measurement System - *CC BY-SA 4.0*
    - 5.4: Converting Units of Measure/ Dimensional Analysis - *CC BY-NC-SA 4.0*
    - 5.5: The Metric System - *CC BY-SA 4.0*
    - 5.6: Converting Between Systems - *CC BY-SA 4.0*
    - 5.7: Conversions with Time, Rate, Area and Volume - *CC BY-SA 4.0*
  - 6: Geometry - *CC BY-SA 4.0*
    - 6.1: Polygons - *CC BY-SA 4.0*
    - 6.2: Pythagorean Theorem for Right Triangles - *CC BY 4.0*
    - 6.3: Perimeter and Circumference - *CC BY-SA 4.0*
    - 6.4: Area of Polygons and Circles - *CC BY-SA 4.0*
    - 6.5: Composite Figures - *CC BY-SA 4.0*
    - 6.6: Volume of Common Solids - *CC BY-SA 4.0*
  - 7: Trigonometry - *CC BY-SA 4.0*
    - 7.1: Angles - *CC BY-SA 4.0*
    - 7.2: Triangles - *CC BY-SA 4.0*

- 7.3: Introduction to Right Triangle Trigonometry - *CC BY 4.0*
- 7.4: Right Triangle Trigonometry in the Real World - *CC BY-SA 4.0*
- 7.5: Power and Impedance Triangles - An Elelctrical Example - *CC BY 4.0*
- 7.6: Trigonometry and Single Phase AC Generation for Electricians (Flinn) - *CC BY 4.0*
  - Front Matter - *Undeclared*
    - Licensing - *Undeclared*
  - 7.6.1: Trigonometry - *CC BY 4.0*
    - 7.6.1.1: Angles - *CC BY 4.0*
    - 7.6.1.2: Triangles - *CC BY 4.0*
    - 7.6.1.3: Pythagoras - *CC BY 4.0*
    - 7.6.1.4: Naming Right Triangle Sides - *CC BY 4.0*
    - 7.6.1.5: Trigonometry Functions - *CC BY 4.0*
    - 7.6.1.6: Power and Impedance Triangles - *CC BY 4.0*
  - 7.6.2: Vectors - *CC BY 4.0*
    - 7.6.2.1: A Vector Primer - *CC BY 4.0*
    - 7.6.2.2: Quadrants - *CC BY 4.0*
    - 7.6.2.3: Polar vs. Rectangular Form - *CC BY 4.0*
    - 7.6.2.4: Vector Addition - *CC BY 4.0*
  - 7.6.3: AC Generation - *CC BY 4.0*
    - 7.6.3.1: Electromagnetic Induction - *CC BY 4.0*
    - 7.6.3.2: The Alternator - *CC BY 4.0*
    - 7.6.3.3: How a Waveform Is Generated - *CC BY 4.0*
    - 7.6.3.4: AC Waveform Analysis - *CC BY 4.0*
    - 7.6.3.5: Frequency and Alternators - *CC BY 4.0*
  - Back Matter - *Undeclared*
    - Index - *Undeclared*
    - Detailed Licensing - *Undeclared*
- 8: Vectors - *CC BY 4.0*
  - 8.1: Angles in Standard Position - *CC BY-SA 1.3*
  - 8.2: Trigonometric Functions of Any Angle - *Undeclared*
  - 8.3: Introduction to Vectors - *CC BY 4.0*
  - 8.4: Vector Operations - *CC BY-SA 4.0*
  - 8.5: Vector Addition in the Real World - Electrical Application - *CC BY 4.0*
- 9: Sinusoidal Models - *Undeclared*
  - 9.1: Periodic Behaviors in the Real World - *Undeclared*
  - 9.2: Sinusoidal Graphs - *CC BY-SA 4.0*
  - 9.3: More Sinusoidal Modeling - *CC BY 4.0*
  - 9.4: Sinusoidal Models (Exercises) - *CC BY-SA 4.0*
  - 9.5: AC Waveform - An Electrical Application - *CC BY 4.0*
- 10: Law of Sines and Cosines - *CC BY-SA 4.0*
  - 10.1: Non-right Triangles - Law of Sines - *CC BY-SA 4.0*
  - 10.2: Non-right Triangles - Law of Cosines - *CC BY-SA 4.0*
- Back Matter - *Undeclared*
  - Index - *Undeclared*
  - Glossary - *Undeclared*
  - Detailed Licensing - *Undeclared*
  - Detailed Licensing - *Undeclared*

Detailed Licensing

Overview

Title: Technical Mathematics SJDC 1

Webpages: 95

Applicable Restrictions: Noncommercial

All licenses found:

- CC BY-SA 4.0: 41.1% (39 pages)
- CC BY 4.0: 30.5% (29 pages)
- Undeclared: 18.9% (18 pages)
- CC BY-NC-SA 3.0: 5.3% (5 pages)
- CC BY-NC-SA 4.0: 3.2% (3 pages)
- CC BY-SA 1.3: 1.1% (1 page)

By Page

- Technical Mathematics SJDC 1 - *CC BY-NC-SA 4.0*
  - Front Matter - *Undeclared*
    - TitlePage - *Undeclared*
    - InfoPage - *Undeclared*
    - Table of Contents - *Undeclared*
    - Licensing - *Undeclared*
  - 1: Review of Arithmetic - *CC BY-SA 4.0*
    - 1.1: Place Value - *CC BY 4.0*
    - 1.2: Rounding Numbers - *CC BY 4.0*
    - 1.3: Negative Numbers - *CC BY-SA 4.0*
    - 1.4: Decimals - *CC BY-SA 4.0*
    - 1.5: Fractions - *CC BY-SA 4.0*
    - 1.6: Order of Operations - *CC BY-SA 4.0*
    - 1.7: Evaluating Algebraic Expressions and Formulas - *CC BY-NC-SA 3.0*
  - 2: Solving Equations - *CC BY-SA 4.0*
    - 2.1: The Language of Algebra - *CC BY-SA 4.0*
    - 2.2: Solving Equations with Addition and Subtraction Properties of Equality - *CC BY-SA 4.0*
    - 2.3: Solving Equations with the Division and Multiplication Properties of Equality - *CC BY-SA 4.0*
    - 2.4: Solving Equations with Variables and Constants on Both Sides - *CC BY-SA 4.0*
    - 2.5: Solving Equations with Fractions or Decimals - *CC BY-SA 4.0*
    - 2.6: General Strategy to Solve Linear Equations - *CC BY-SA 4.0*
    - 2.7: Solving a Formula for a Specific Variable - *CC BY-SA 4.0*
  - 3: Proportions and Variation - *CC BY-SA 4.0*
    - 3.1: Ratios, Rates, Proportions - *CC BY-SA 4.0*
    - 3.2: Applications of Proportionality - *CC BY-SA 4.0*
    - 3.3: Direct and Inverse Variation - *CC BY-SA 4.0*
  - 4: Percent - *CC BY-NC-SA 3.0*
    - 4.1: Basic Percent Problems - *CC BY-NC-SA 3.0*
    - 4.2: Percents in the Real World - *CC BY-NC-SA 3.0*
    - 4.3: Percent Increase or Decrease - *CC BY-NC-SA 3.0*
  - 5: Units of Measure and Converting Units - *CC BY-SA 4.0*
    - 5.1: Scientific and Engineering Notation - *CC BY-SA 4.0*
    - 5.2: Accuracy and Significant Figures - *CC BY-NC-SA 4.0*
    - 5.3: The US Measurement System - *CC BY-SA 4.0*
    - 5.4: Converting Units of Measure/ Dimensional Analysis - *CC BY-NC-SA 4.0*
    - 5.5: The Metric System - *CC BY-SA 4.0*
    - 5.6: Converting Between Systems - *CC BY-SA 4.0*
    - 5.7: Conversions with Time, Rate, Area and Volume - *CC BY-SA 4.0*
  - 6: Geometry - *CC BY-SA 4.0*
    - 6.1: Polygons - *CC BY-SA 4.0*
    - 6.2: Pythagorean Theorem for Right Triangles - *CC BY 4.0*
    - 6.3: Perimeter and Circumference - *CC BY-SA 4.0*
    - 6.4: Area of Polygons and Circles - *CC BY-SA 4.0*
    - 6.5: Composite Figures - *CC BY-SA 4.0*
    - 6.6: Volume of Common Solids - *CC BY-SA 4.0*
  - 7: Trigonometry - *CC BY-SA 4.0*
    - 7.1: Angles - *CC BY-SA 4.0*
    - 7.2: Triangles - *CC BY-SA 4.0*

- 7.3: Introduction to Right Triangle Trigonometry - *CC BY 4.0*
- 7.4: Right Triangle Trigonometry in the Real World - *CC BY-SA 4.0*
- 7.5: Power and Impedance Triangles - An Elelctrical Example - *CC BY 4.0*
- 7.6: Trigonometry and Single Phase AC Generation for Electricians (Flinn) - *CC BY 4.0*
  - Front Matter - *Undeclared*
    - Licensing - *Undeclared*
  - 7.6.1: Trigonometry - *CC BY 4.0*
    - 7.6.1.1: Angles - *CC BY 4.0*
    - 7.6.1.2: Triangles - *CC BY 4.0*
    - 7.6.1.3: Pythagoras - *CC BY 4.0*
    - 7.6.1.4: Naming Right Triangle Sides - *CC BY 4.0*
    - 7.6.1.5: Trigonometry Functions - *CC BY 4.0*
    - 7.6.1.6: Power and Impedance Triangles - *CC BY 4.0*
  - 7.6.2: Vectors - *CC BY 4.0*
    - 7.6.2.1: A Vector Primer - *CC BY 4.0*
    - 7.6.2.2: Quadrants - *CC BY 4.0*
    - 7.6.2.3: Polar vs. Rectangular Form - *CC BY 4.0*
    - 7.6.2.4: Vector Addition - *CC BY 4.0*
  - 7.6.3: AC Generation - *CC BY 4.0*
    - 7.6.3.1: Electromagnetic Induction - *CC BY 4.0*
    - 7.6.3.2: The Alternator - *CC BY 4.0*
    - 7.6.3.3: How a Waveform Is Generated - *CC BY 4.0*
    - 7.6.3.4: AC Waveform Analysis - *CC BY 4.0*
    - 7.6.3.5: Frequency and Alternators - *CC BY 4.0*
  - Back Matter - *Undeclared*
    - Index - *Undeclared*
    - Detailed Licensing - *Undeclared*
- 8: Vectors - *CC BY 4.0*
  - 8.1: Angles in Standard Position - *CC BY-SA 1.3*
  - 8.2: Trigonometric Functions of Any Angle - *Undeclared*
  - 8.3: Introduction to Vectors - *CC BY 4.0*
  - 8.4: Vector Operations - *CC BY-SA 4.0*
  - 8.5: Vector Addition in the Real World - Electrical Application - *CC BY 4.0*
- 9: Sinusoidal Models - *Undeclared*
  - 9.1: Periodic Behaviors in the Real World - *Undeclared*
  - 9.2: Sinusoidal Graphs - *CC BY-SA 4.0*
  - 9.3: More Sinusoidal Modeling - *CC BY 4.0*
  - 9.4: Sinusoidal Models (Exercises) - *CC BY-SA 4.0*
  - 9.5: AC Waveform - An Electrical Application - *CC BY 4.0*
- 10: Law of Sines and Cosines - *CC BY-SA 4.0*
  - 10.1: Non-right Triangles - Law of Sines - *CC BY-SA 4.0*
  - 10.2: Non-right Triangles - Law of Cosines - *CC BY-SA 4.0*
- Back Matter - *Undeclared*
  - Index - *Undeclared*
  - Glossary - *Undeclared*
  - Detailed Licensing - *Undeclared*
  - Detailed Licensing - *Undeclared*