MTH 65: ALGEBRA 2



Math Department
Clackamas Community College





Math 65 - Algebra 2

OER Textbook

First Edition - 2025

Welcome

Welcome to MATH 65 - Algebra 2!

This book is here to support you every step of the way as you build a strong foundation in algebra and discover how it connects to the world around you. Algebra isn't just a series of rules; it's a tool for solving real problems, spotting patterns, and making confident decisions. You'll find clear explanations, guided practice, and plenty of opportunities to check your understanding so you can learn with ease and grow your math skills with confidence.

Here's how this book was designed to help you succeed:

- **Essential Skills**: Each section opens with carefully chosen sample problems that highlight the core skills you'll need to successfully learn the new material.
- **Plain-Language Introductions**: New concepts are first presented informally in clear, everyday language and supported by visual descriptions, helping you build intuition before moving on to formal definitions and procedures.
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- **Answer Key**: All answers to practice problems are collected in the back of the book, so you can quickly confirm you're on the right track.
- **Self-Check**: At the end of each practice set, use the "self-check" questions to pinpoint any areas you'd like to revisit or get extra help with.

As you work through this book, you'll grow from reviewing essential algebra skills to mastering polynomials, exponents, radicals, and absolute value. Each step builds your confidence, sharpens your problem-solving, and prepares you for advanced math and real-world applications. Remember: learning math is a journey, and your Clackamas Community College community is here to support you every step of the way.

Let's get started on this exciting journey together!

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PARCE 2024–2025 Team Members

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CHAPTER OVERVIEW

Unit 0: Prerequesite Review

Before we dive into new algebra skills, we'll warm up with the most essential tools from previous math courses. In this unit, we'll review order of operations, working with fractions, and solving linear equations and inequalities—everything from one-step equations to systems. We'll revisit how to combine like terms, isolate variables, and interpret solutions in context. These essential skills form the algebraic toolkit we'll rely on throughout the course!

Section 0.1: Order of Operations

Section 0.1P: Practice

Section 0.2: Fractions

Section 0.2P: Practice

Section 0.3: General Strategy to Solve Linear Equations

Section 0.3P: Practice

Section 0.4: General Strategy to Solve Linear Inequalities

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Section 0.5: Solving Systems of Linear Equations

Section 0.5P: Practice

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Section 0.1: Order of Operations

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $2^3 + 3^2$
- Evaluate $3 + 4 \cdot 5$
- Evaluate $2 \cdot 3 + 2 \cdot 2$

Learning Objectives

Motivating Problem

A student simplified $8 + 2 \cdot 5$ and got 50. Another student got 18. Who's correct—and why?

Fun Fact

In the 1600s, some mathematicians used a vertical line, called a "vinculum" (similar to the one in a square root), to indicate grouping instead of parentheses. These early forms of notation eventually evolved into the clear order-of-operations conventions we use today.

The Goal

In this section, we'll review how to correctly simplify expressions using the order of operations: grouping symbols, exponents, multiplication/division (left to right), and addition/subtraction (left to right). This is the foundation we need before working with algebraic expressions.

Simplify Expressions Using the Order of Operations

To **simplify an expression** means to perform all the possible mathematical operations. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

$$4 \cdot 2 + 1$$

$$8 + 1$$

$$9$$

By not using an **equal sign** when you simplify an expression, you may avoid confusing expressions with equations.

SIMPLIFY AN EXPRESSION

To **simplify an expression**, do all operations in the expression.

You are familiar with most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings and result in different values. For example, consider the expression:

$$4 + 3 \cdot 7$$

If you simplify this expression, what do you get?

Some students incorrectly say 49 since $4 + 3$ give 7 and $7 \cdot 7$ is 49.

$$4 + 3 \cdot 7$$

$$7 \cdot 7$$

$$49$$

Others say 25 since $3 \cdot 7$ is 21 and $21 + 4$ makes 25.

$$4 + 3 \cdot 7$$

$$4 + 21$$

$$25$$

Imagine the confusion in our banking system if every problem had several different correct answers!

The same expression should give the same result, so mathematicians established some guidelines early on, which are called the **Order of Operations**.

PERFORM THE ORDER OF OPERATIONS.

1. Parentheses and Other Grouping Symbols
   - Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost grouping symbols first.
2. Exponents
   - Simplify all expressions with exponents.
3. Multiplication and Division
   - Perform all multiplication and division in order from left to right. These operations have equal priority.
4. Addition and Subtraction
   - Perform all addition and subtraction in order from left to right. These operations have equal priority.

Students often ask, "How will I remember the order?" Here is a way to help you remember: Take the first letter of each keyword and substitute the silly phrase: "Please Excuse My Dear Aunt Sally."

Parentheses	**P**lease
Exponents	**E**xcuse
Multiplication **D**ivision	**M**y **D**ear
Addition **S**ubtraction	**A**unt **S**ally

It's good that "**M**y **D**ear" goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, "**A**unt **S**ally" goes together, reminding us that **a**ddition and **s**ubtraction also have equal priority and are performed in order from left to right.

Let's try an example.

Example 1

Simplify:

1. $4 + 3 \cdot 7$
2. $(4 + 3) \cdot 7$

Solution

1.

	$4+3\cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4+3\cdot 7$
Add.	$4+21$
	25

2.

	$(4+3)\cdot 7$
Are there any **p**arentheses? Yes.	$(4+3)\cdot 7$
Simplify inside the parentheses.	$(7)7$
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

Try It 1

Simplify:

1. $8+3\cdot 9$
2. $(8+3)\cdot 9$

Answer

1. 35
2. 99

Example 2

Simplify: $18\div 6+4(5-2)$

Solution

Parentheses? Yes, subtract first.	$18 \div 6 + 4(5 - 2)$ $18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	$18 \div 6 + 4(3)$
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

? Try It 2

Simplify: $30 \div 5 + 10(3 - 2)$

Answer

16

When there are multiple grouping symbols, we simplify the innermost grouping symbols first and work outward.

✔ Example 3

Simplify: $5 + 2^3 + 3[6 - 3(4 - 2)]$.

Solution

	$5+2^3+3[6-3(4-2)]$
Are there any parentheses (or other grouping symbols)? Yes.	
Focus on the parentheses that are inside the brackets.	$5+2^3+3[6-3(4-2)]$
Subtract.	$5+2^3+3[6-3(2)]$
Continue inside the brackets and multiply.	$5+2^3+3[6-6]$
Continue inside the brackets and subtract.	$5+2^3+3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	$5+2^3+3[0]$
Simplify exponents.	$5+8+3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5+8+0$
Is there any addition or subtraction? Yes.	
Add.	$13+0$
Add.	13

? Try It 3

Simplify: $7^2-2[4(5+1)]$.

Answer

1

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Section 0.1P: Practice

Practice Makes Progress

In the following exercises, simplify each expression.

Exercise 1

5^3

Exercise 2

2^8

Exercise 3

$3 + 8 \cdot 5$

Exercise 4

$(3 + 8) \cdot 5$

Exercise 5

$2^3 - 12 \div (9 - 5)$

Exercise 6

$3 \cdot 8 + 5 \cdot 2$

Exercise 7

$2 + 8(6 + 1)$

Exercise 8

$4 \cdot 12/8$

Exercise 9

$(6 + 10) \div (2 + 2)$

Exercise 10

$20 \div 4 + 6 \cdot 5$

Exercise 11

$3^2 + 7^2$

? Exercise 12

$3(1+9\cdot 6)-4^2$

? Exercise 13

$2[1+3(10-2)]$

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions using the order of operations.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 0.2: Fractions

Be Prepared

We will rely heavily on these skills throughout this section.

- Convert $\frac{11}{4}$ to a mixed number.
- Find the least common multiple of 6 and 15.
- Determine the prime factorization of 48.

Learning Objectives

Motivating Problem

You need ¾ cup of flour and ⅔ cup of sugar for a recipe. If you want to make a half batch, how much of each ingredient should you use?

Fun Fact

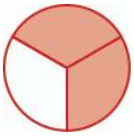
Fractions go way back—clay tablets from ancient Mesopotamia (over 4,000 years ago) show calculations with fractions in base 60! That's part of why we still use 60 minutes in an hour and 360 degrees in a circle today.

The Goal

In this section, we'll review how to add, subtract, multiply, and simplify fractions. We'll also explore how to find common denominators and reduce answers to their simplest form, enabling us to work confidently with fractions in algebra.

Simplify Fractions

A fraction is a way to represent parts of a whole. The fraction $\frac{2}{3}$ represents two of three equal parts. In the fraction $\frac{2}{3}$, the 2 is called the **numerator** and the 3 is called the **denominator**. The line is called the fraction bar.



In the circle, $\frac{2}{3}$ of the circle is shaded—2 of the 3 equal parts.

Fraction Definition

A **fraction** is written $\frac{a}{b}$, where $b \neq 0$ and

a is the **numerator** and b is the **denominator**.

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

Fractions that have the same value are **equivalent fractions**. The Equivalent Fractions

Property allows us to find equivalent fractions and also simplify fractions.

Equivalent Fractions Property

If a, b, and c are numbers where $b \neq 0, c \neq 0$,

then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

A fraction is considered simplified if there are no common factors, other than 1, in its numerator and denominator.

For example,

$\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.

$\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

We simplify, or reduce, a fraction by dividing out the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been divided out. If an expression has fractions, it is not completely simplified until the fractions are simplified.

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into prime numbers. Then divide out the common factors using the Equivalent Fractions Property.

Example 1

Simplify $\frac{-315}{770}$.

Solution

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, use a factor tree.	Rewrite 315 and 770 as the product of the primes.	$-\frac{315}{770}$ $-\frac{3\cdot3\cdot5\cdot7}{2\cdot5\cdot7\cdot11}$
Step 2. Simplify using the equivalent fractions property by dividing out common factors.	Mark the common factors 5 and 7. Divide out the common factors	$-\frac{3\cdot3\cdot\cancel{5}\cdot\cancel{7}}{2\cdot\cancel{5}\cdot\cancel{7}\cdot11}$ $-\frac{3\cdot3}{2\cdot11}$
Step 3. Multiply the remaining factors, if necessary.		$-\frac{9}{22}$

Try It 1

Simplify $-\frac{120}{192}$.

Answer

$-\frac{5}{8}$

We now summarize the steps you should follow to simplify fractions.

SIMPLIFY A FRACTION.

1. Rewrite the numerator and denominator to show the common factors.
   If needed, factor the numerator and denominator into prime numbers first.
2. Simplify using the Equivalent Fractions Property by dividing out common factors.
3. Multiply any remaining factors.

Multiply and Divide Fractions

Many people find multiplying and dividing fractions easier than adding and subtracting fractions.

To multiply fractions, we multiply the numerators and multiply the denominators.

FRACTION MULTIPLICATION

If a, b, c, and d are numbers where $b \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To multiply fractions, multiply the numerators and multiply the denominators.

When multiplying fractions, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the product as the first step. In the following example, we will multiply a negative number and a positive number, so the product will be negative.

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, for example, $3 = \frac{3}{1}$.

Example 2

Multiply: $-\frac{12}{5}(-20x)$.

Solution

The first step is to find the sign of the product. Since the signs are the same, the product is positive.

	$\frac{12}{5}(20x)$
Write $20x$ as a fraction.	$\frac{12}{5}\left(\frac{20x}{1}\right)$
Multiply.	$\frac{12 \cdot 20x}{5 \cdot 1}$
Rewrite 20 to show the common factor 5 and divide it out.	$\frac{12 \cdot 4 \cdot \cancel{5} \cdot x}{\cancel{5} \cdot 1}$
Simplify.	$48x$

Try It 2

Multiply: $\frac{11}{3}(-9a)$.

Answer

$-33a$

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, we need some vocabulary. The **reciprocal** of a fraction is found by inverting the fraction, placing the numerator in the denominator, and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. Since 4 is written in fraction form as $\frac{4}{1}$, the reciprocal of 4 is $\frac{1}{4}$.

To divide fractions, we multiply the first fraction by the reciprocal of the second.

FRACTION DIVISION

If a, b, c, and d are numbers where $b \neq 0, c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide fractions, we multiply the first fraction by the **reciprocal** of the second.

We need to say $b \neq 0$, $c \neq 0$, and $d \neq 0$, to be sure we don't divide by zero!

Example 3

Find the quotient: $-\frac{7}{18} \div (-\frac{14}{27})$.

Solution

	$-\frac{7}{18} \div (-\frac{14}{27})$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{7}{18}\left(-\frac{27}{14}\right)$
Determine the sign of the product, and then multiply.	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{7 \cdot 9 \cdot 3}{9 \cdot 2 \cdot 7 \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

Try It 3

Divide: $-\frac{7}{27} \div (-\frac{35}{36})$.

Answer

$\frac{4}{15}$

The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

Definition: COMPLEX FRACTION

A **complex fraction** is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{y}{6}}$$

To simplify a complex fraction, remember that the fraction bar means division. For example, the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

Example 4

Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Solution

$$\frac{\frac{x}{2}}{\frac{xy}{6}}$$

Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{x \cdot 3 \cdot 2}{2 \cdot x \cdot y}$
Divide common factors and simplify.	$\frac{3}{y}$

Try It 4

Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Answer

$\frac{3}{4b}$

Add and Subtract Fractions

When we multiply fractions, we multiply the numerators and denominators directly. However, to add or subtract fractions, they must have a common denominator.

FRACTION ADDITION AND SUBTRACTION

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \text{ and } \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

The **least common denominator** (LCD) of two fractions is the smallest number that can be used as a common denominator of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

LEAST COMMON DENOMINATOR

The **least common denominator** (LCD) of two fractions is the least common multiple (LCM) of their denominators.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

Example 5

Add: $\frac{7}{12} + \frac{5}{18}$.

Solution

Step 1. Do they have a common denominator? • No—rewrite each fraction with the LCD (least common denominator).	No. Find the LCD of 12, 18. We multiply the numerator and denominator of each fraction by the factor needed to get the denominator to be 36. Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $LCD = 2 \cdot 2 \cdot 3 \cdot 3$ $LCD = 36$ LCD is 36. $\frac{7}{12} + \frac{5}{18}$ $\frac{7 \cdot 3}{12 \cdot 3} + \frac{5 \cdot 2}{18 \cdot 2}$ $\frac{21}{36} + \frac{10}{36}$
Step 2. Add or subtract the fractions.	Add.	$\frac{31}{36}$
Step 3. Simplify, if possible.	Since 31 is prime, its only factors are 1 and 31. Since 31 does not go into 36, the answer is simplified.	

? Try It 5

Add: $\frac{13}{15} + \frac{17}{20}$.

Answer

$\frac{103}{60}$

ADD OR SUBTRACT FRACTIONS.

1. Do they have a common denominator?
   - Yes—go to step 2.
   - No—rewrite each fraction with the LCD (least common denominator).
     - Find the LCD.
     - Change each fraction into an equivalent fraction with the LCD as its denominator.
2. Add or subtract the fractions.
3. Simplify, if possible.

We now have all four operations for fractions. The table below summarizes fraction operations.

Fraction Multiplication	Fraction Division
$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$
Multiply the numerators and multiply the denominators	Multiply the first fraction by the reciprocal of the second.
Fraction Addition	**Fraction Subtraction**
$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$	$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$
Add the numerators and place the sum over the common denominator.	Subtract the numerators and place the difference over the common denominator.
To multiply or divide fractions, an LCD is NOT needed. To add or subtract fractions, an LCD is needed.	

When starting an exercise, always identify the operation and then recall the methods needed for that operation.

Example 6

Simplify:

a. $\frac{5x}{6}-\frac{3}{10}$

b. $\frac{5x}{6}\cdot\frac{3}{10}$.

Solution

First ask, "What is the operation?" Identifying the operation will determine whether or not we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

a.

What is the operation? Subtraction.	$\frac{5x}{6}-\frac{3}{10}$
Rewrite using the LCD.	$\frac{5x\cdot 5}{6\cdot 5}-\frac{3\cdot 3}{10\cdot 3}$
Rewrite with common denominators.	$\frac{25x}{30}-\frac{9}{30}$
Subtract the numerators.	$\frac{25x-9}{30}$
Simplify.	$\frac{25x-9}{30}$

b.

What is the operation? Multiplication.	$\frac{5x}{6}\cdot\frac{3}{10}$
To multiply fractions,multiply the numerators and multiply the denominators.	$\frac{5x\cdot 3}{6\cdot 10}$
Rewrite, showing common factors.	
Remove common factors.	$\frac{\cancel{5}x\cdot\cancel{3}}{2\cdot\cancel{3}\cdot 2\cdot\cancel{5}}$
Simplify.	$\frac{x}{4}$

Notice, we needed an LCD to add $\frac{5x}{6}-\frac{3}{10}$, but not to multiply $\frac{5x}{6}\cdot\frac{3}{10}$.

Try It 6

Simplify:

a. $\frac{3a}{4} - \frac{8}{9}$

b. $\frac{3a}{4} \cdot \frac{8}{9}$

Answer

a. $\frac{27a - 32}{36}$

b. $\frac{2a}{3}$

Use the Order of Operations to Simplify Fractions

The fraction bar in a fraction acts as a grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

SIMPLIFY AN EXPRESSION WITH A FRACTION BAR.

1. Simplify the expression in the numerator. Simplify the expression in the denominator.
2. Simplify the fraction.

Where does the negative sign go in a fraction? Usually, the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

$$\frac{-1}{3} = -\frac{1}{3} \qquad \frac{\text{negative}}{\text{positive}} = \text{negative}$$

$$\frac{1}{-3} = -\frac{1}{3} \qquad \frac{\text{positive}}{\text{negative}} = \text{negative}$$

PLACEMENT OF NEGATIVE SIGN IN A FRACTION

For any positive numbers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Example 7

Simplify: $\frac{4(-3) + 6(-2)}{-3(2) - 2}$.

Solution

The fraction bar acts like a grouping symbol. So, completely simplify the numerator and the denominator separately.

$$\frac{4(-3) + 6(-2)}{-3(2) - 2}$$

Multiply. $\frac{-12 + (-12)}{-6 - 2}$

Simplify. $\frac{-24}{-8}$

Divide. 3

Try It 7

Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Answer

4

Now we'll look at complex fractions where the numerator or denominator contains an expression that can be simplified. Therefore, we must first simplify the numerator and denominator separately, following the order of operations. Then we divide the numerator by the denominator, as the fraction bar means division.

Example 8

Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution

Step 1. Simplify the numerator.	$\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$ $\frac{\frac{1}{4}}{4+3^2}$
Step 2. Simplify the denominator.	$\frac{\frac{1}{4}}{4+9}$ $\frac{\frac{1}{4}}{13}$
Step 3. Divide the numerator by the denominator. Simplify if possible	$\frac{1}{4} \div \frac{13}{1}$ $\frac{1}{4} \cdot \frac{1}{13}$ $\frac{1}{52}$

Try It 8

Simplify: $\frac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Answer

272

SIMPLIFY COMPLEX FRACTIONS.

1. Simplify the numerator.
2. Simplify the denominator.
3. Divide the numerator by the denominator. Simplify if possible.

Example 9

Simplify: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution

It may help to put parentheses around the numerator and the denominator.

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$$

Simplify the numerator ($LCD = 6$) and simplify the denominator ($LCD = 12$). $\dfrac{\left(\frac{3}{6}+\frac{4}{6}\right)}{\left(\frac{9}{12}-\frac{2}{12}\right)}$

Simplify. $\dfrac{\left(\frac{7}{6}\right)}{\left(\frac{7}{12}\right)}$

Divide the numerator by the denominator. $\frac{7}{6} \div \frac{7}{12}$

Simplify. $\frac{7}{6} \cdot \frac{12}{7}$

Divide out common factors. $\dfrac{\cancel{7} \cdot \cancel{6} \cdot 2}{\cancel{6} \cdot \cancel{7} \cdot 1}$

Simplify. 2

Try It 9

Simplify: $\dfrac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Answer

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but can now evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the variable's value into the expression and then simplify.

Example 10

Evaluate $2x^2y$ when $x = \frac{1}{4}$ and $y = -\frac{2}{3}$.

Solution

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply; divide out the common factors. Notice we write 16 as $2\cdot 2\cdot 4$ to make it easy to remove common factors.	$-\frac{2\cdot 1\cdot 2}{2\cdot 2\cdot 4\cdot 3}$
Simplify.	$-\frac{1}{12}$

? Try It 10

Evaluate $3ab^2$ when $a=-\frac{2}{3}$ and $b=-\frac{1}{2}$.

Answer

$-\frac{1}{2}$

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Section 0.2P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$-\frac{108}{63}$

Exercise 2

$\frac{120}{252}$

Exercise 3

$\frac{14x^2}{21y}$

Exercise 4

$-\frac{210a^2}{110b^2}$

In the following exercises, perform the indicated operation and simplify.

Exercise 5

$-\frac{3}{4}\left(-\frac{4}{9}\right)$

Exercise 6

$\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$

Exercise 7

$\frac{3}{7}\cdot 21n$

Exercise 8

$\frac{5}{18}\div\left(-\frac{15}{24}\right)$

Exercise 9

$-\frac{\frac{8}{21}}{\frac{12}{35}}$

Exercise 10

$$\frac{\frac{m}{3}}{\frac{n}{2}}$$

Exercise 11

$$\frac{7}{12}+\frac{5}{8}$$

Exercise 12

$$\frac{7}{12}-\frac{9}{16}$$

Exercise 13

$$-\frac{2}{3}-\left(-\frac{3}{4}\right)$$

Exercise 14

$$\frac{x}{3}+\frac{1}{4}$$

Exercise 15

$$-\frac{5a}{3}+\left(-\frac{10}{6}\right)$$

Exercise 16

$$-\frac{5a}{3}\div\left(-\frac{10}{6}\right)$$

Exercise 17

$$\frac{5\cdot 6-3\cdot 4}{4\cdot 5-2\cdot 3}$$

Exercise 18

$$\frac{7\cdot 4-2(8-5)}{9\cdot 3-3\cdot 5}$$

Exercise 19

$$\frac{2}{\frac{1}{3}+\frac{1}{5}}$$

Exercise 20

$$\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$$

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Exercise 21

$-\frac{3}{8} \div \left(-\frac{3}{10}\right)$

Exercise 22

$-\frac{3}{8} + \frac{5}{12}$

Exercise 23

$\frac{11}{12a} \cdot \frac{9a}{16}$

Exercise 24

$\frac{1}{2} + \frac{2}{3} \cdot \frac{5}{12}$

Exercise 25

$1 - \frac{3}{5} \div \frac{1}{10}$

Exercise 26

$12\left(\frac{9}{20} - \frac{4}{15}\right)$

Exercise 27

$\dfrac{\frac{5}{8} + \frac{1}{6}}{\frac{19}{24}}$

Exercise 28

$\left(\frac{5}{9} + \frac{1}{6}\right) \div \left(\frac{2}{3} - \frac{1}{2}\right)$

Writing Exercises

Exercise 29

Why do you need a common denominator to add or subtract fractions? Explain.

Exercise 30

Explain how you find the reciprocal of a fraction.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply and divide fractions.			
add and subtract fractions.			
use the order of operations to simplify fractions.			
evaluate variable expressions with fractions.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 0.3: General Strategy to Solve Linear Equations

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $3x + 5$ when $x = 2$
- Simplify $4(4n + 1) - 15n$
- Solve $\frac{x}{4} - 2 = 3$
- Simplify $-7(\frac{1}{-7})$

Learning Objectives

Motivating Problem

Amara has $250 in her bank account and deposits $25 every week. How many weeks will it take for her to save up $625?

Fun Fact

Linear equations might seem simple, but they've played a big role in history. Ancient Babylonian tablets from nearly 4,000 years ago demonstrate linear problems involving trade, inheritance, and land measurement—solved without symbols, relying solely on clever logic and word problems.

The Goal

In this section, we'll review how to solve linear equations by combining like terms, using inverse operations, and handling equations with fractions or parentheses.

Solve Linear Equations Using a General Strategy

Solving an equation is like discovering the answer to a puzzle. The purpose of solving an equation is to find the value or values of the variable that make it a true statement. Any value of the variable that makes the equation true is called a **solution** to the equation. It is the answer to the puzzle!

Solution of an Equation

A **solution** of an equation is a value of a variable that makes a true statement when substituted into the equation.

To determine whether a number is a solution to an equation, we substitute the value for the variable in the equation. If the resulting equation is a true statement, then the number is a solution of the equation.

Determine Whether a Number is a Solution to an Equation

1. Substitute the number for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true.
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Example 1

Determine whether the values are solutions to the equation: $5y + 3 = 10y - 4$.

a. $y = \frac{3}{5}$
b. $y = \frac{7}{5}$

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

a.

	$5y+3=10y-4$
Substitute $\frac{3}{5}$ for y	$5\left(\frac{3}{5}\right)+3\stackrel{?}{=}10\left(\frac{3}{5}\right)-4$
Multiply.	$3+3\stackrel{?}{=}6-4$
Simplify.	$6\neq 2$

Since $y=\frac{3}{5}$ does not result in a true equation, $y=\frac{3}{5}$ is not a solution to the equation $5y+3=10y-4$.

b.

	$5y+3=10y-4$
Substitute $\frac{7}{5}$ for y	$5\left(\frac{7}{5}\right)+3\stackrel{?}{=}10\left(\frac{7}{5}\right)-4$
Multiply.	$7+3\stackrel{?}{=}14-4$
Simplify.	$10=10\checkmark$

Since $y=\frac{7}{5}$ results in a true equation, $y=\frac{7}{5}$ is a solution to the equation $5y+3=10y-4$.

Try It 1

Determine whether the values are solutions to the equation: $4x-2=2x+1$.

a. $x=\frac{3}{2}$
b. $x=-\frac{1}{2}$

Answer

a. yes
b. no

There are many types of equations that we will learn to solve. In this section, we will focus on a **linear equation.**

LINEAR EQUATION

A **linear equation** is an equation in one variable that can be written, where a and b are real numbers and $a\neq 0$, as:

$$ax+b=0$$

To solve a linear equation, it is beneficial to have an overall strategy that can be applied to any linear equation. In the next example, we will give the steps of a general strategy for solving any linear equation. Simplifying each side of the equation as much as possible first makes the subsequent steps easier.

Example 2

Solve: $\frac{2}{3}(3m-6)=5-m$.

Solution

	$\frac{2}{3}(3m-6)=5-m$
Distribute.	$2m-4=5-m$
Add m to both sides to get the variables only on the left.	$2m+m-4=5-m+m$
Simplify.	$3m-4=5$
Add 4 to both sides to get constants only on the right.	$3m-4+4=5+4$
Simplify.	$3m=9$
Divide both sides by three.	$\frac{3m}{3}=\frac{9}{3}$
Simplify.	$m=3$

Check:	$\frac{2}{3}(3m-6)=5-m$
Let $m=3$.	$\frac{2}{3}(3\cdot 3-6)\stackrel{?}{=}5-3$
	$\frac{2}{3}(9-6)\stackrel{?}{=}2$
	$\frac{2}{3}(3)\stackrel{?}{=}2$
	$2=2\checkmark$

? Try It 2

Solve: $\frac{1}{3}(6u+3)=7-u$.

Answer

$u=2$

These steps are summarized in the **General Strategy for Solving Linear Equations**, which is presented below.

SOLVE LINEAR EQUATIONS USING A GENERAL STRATEGY

1. Simplify each side of the equation as much as possible.
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation to make sure the result is a true statement.

We can solve equations by getting all the **variable** terms to either side of the **equal sign**. By collecting the variable terms on the side where the **coefficient** of the variable is larger, we avoid working with some negatives. This will be a good strategy when we

solve inequalities later in this chapter. It also helps us prevent errors with negatives.

Example 3

Solve: $4(x-1)-2=5(2x+3)+6$.

Solution

Distribute.		$4x-4-2=10x+15+6$
Combine like terms.		$4x-6=10x+21$
Subtract $4x$ from each side to get the variables only on the right since $10>4$.		$\begin{array}{l}4x-4x\\-6\\=10x-4x+21\end{array}$
Simplify.		$-6=6x+21$
Subtract 21 from each side to get the constants on the left.		$\begin{array}{l}-6-21\\=6x\\+21-21\end{array}$
Simplify.		$-27=6x$
Divide both sides by 6.		$\frac{-27}{6}=\frac{6x}{6}$
Simplify.		$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$	
Let $x=-\frac{9}{2}$.	$4\left(\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left(2\left(\frac{9}{2}\right)+3\right)+6$	
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$	
	$-22-2\stackrel{?}{=}5(-6)+6$	
	$-24\stackrel{?}{=}-30+6$	
	$-24=-24\checkmark$	

Try It 3

Solve: $6(p-3)-7=5(4p+3)-12$.

Answer

$p=-2$

Example 4

Solve: $10[3-8(2s-5)]=15(40-5s)$.

Solution

		$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add $160s$ to both sides to get the $160s$ to both sides to get the variables to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 from both sides to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide both sides by 85.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$, so $s=-2$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Let $s=-2$.	$10\,[3-8(2-2)-5]\stackrel{?}{=}15(40-5(-2))$	
	$10[3-8(-4-5)]\stackrel{?}{=}15(40+10)$	
	$10[3-8(-9)]\stackrel{?}{=}15(50)$	
	$10(3+72)\stackrel{?}{=}750$	
	$10(75)\stackrel{?}{=}750$	
	$750=750\checkmark$	

Try It 4

Solve: $6[4-2(7y-1)]=8(13-8y)$.

Answer

$y=-\frac{17}{5}$

Classify Equations

Whether or not an equation is true depends on the value of the variable. The equation $7x+8=-13$ is true when we replace the variable x with the value -3, but it is not true when we replace x with any other value. An equation like this is referred to as a **conditional equation**. All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values is a **conditional equation**.

Now let's consider the equation $7y+14=7(y+2)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for y.

Solve:

	$7y+14=7(y+2)$
Distribute.	$7y+14=7y+14$
Subtract $7y$ to each side to get the y's to one side.	$7y-7y+14$ $=7y-7y+14$
Simplify—the y's are eliminated.	$14=14$
	Notice $14=14$ is a true statement.

This means that the equation $7y+14=7(y+2)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is valid for all real numbers.

What happens when we solve the equation $-8z=-8z+9$?

Solve:

	$-8z=-8z+9$
Add $8z$ to both sides to leave the constant alone on the right.	$-8z+8z$ $=-8z+8z+9$
Simplify—the z's are eliminated.	$0=9$
	Notice $0=9$ is a false statement.

Solving the equation $-8z=-8z+9$ led to the false statement $0=9$. The equation $-8z=-8z+9$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a contradiction.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

The next few examples will ask us to classify an equation as conditional, an identity, or a contradiction.

Example 5

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution: $6(2n-1)+3=2n-8+5(2n+1)$.

Solution

	$6(2n-1)+3=2n-8+5(2n+1)$
Distribute.	$12n-6+3=2n-8+10n+5$
Combine like terms.	$12n-3=12n-3$
Subtract $12n$ from each side to get the n's to one side.	$12n-12n-3=12n-12n-3$
Simplify.	$-3=-3$
This is a true statement.	The equation is an identity.
	The solution is all real numbers.

Try It 5

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$.

Answer

identity; all real numbers

Example 6

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution: $8+3(a-4)=0$.

Solution

	$8+3(a-4)=0$
Distribute.	$8+3a-12=0$
Combine like terms.	$3a-4=0$
Add 4 to both sides.	$3a-4+4=0+4$
Simplify.	$3a=4$
Divide.	$\frac{3a}{3}=\frac{4}{3}$
Simplify.	$a=\frac{4}{3}$
The equation is true when $a=\frac{4}{3}$.	This is a conditional equation.
	The solution is $a=\frac{4}{3}$.

Try It 6

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution: $11(q+3)-5=19$.

Answer

conditional equation; $q=-\frac{9}{11}$

Example 7

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution: $5m+3(9+3m)=2(7m-11)$.

Solution

	$5m+3(9+3m)=2(7m-11)$
Distribute.	$5m+27+9m=14m-22$
Combine like terms.	$14m+27=14m-22$
Subtract $14m$ from both sides.	$14m+27-14m=14m-22-14m$
Simplify.	$27\neq -22$
But $27\neq -22$.	The equation is a contradiction.
	It has no solution.

Try It 7

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:$4(7d+18)=13(3d-2)-11d$.

Answer

contradiction; no solution

We summarize the methods for classifying equations in the table.

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	True for any value of the variable	All real numbers
Contradiction	False for all values of the variable	No solution

Solve Equations with Fraction or Decimal Coefficients

We could use the General Strategy to solve the next example. This method would work fine, but many students do not feel very confident when they see all those fractions. We will now present an alternative method for solving equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the **least common denominator** (LCD) of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called *clearing* the equation of fractions.

To clear an equation of decimals, we think of all the decimals in their fraction form and then find the LCD of those denominators.

Example 8

Solve: $5=\frac{1}{2}y+\frac{2}{3}y-\frac{3}{4}y$.

Solution

Find the LCD of all fractions in the equation.	$5=\frac{1}{2}y+\frac{2}{3}y-\frac{3}{4}y$
The LCD is 12.	
Multiply both sides of the equation by 12.	$12(5)=12\cdot\left(\frac{1}{2}y+\frac{2}{3}y-\frac{3}{4}y\right)$
Distribute.	$12(5)=12\cdot\frac{1}{2}y+12\cdot\frac{2}{3}y-12\cdot\frac{3}{4}y$
Simplify—notice, no more fractions.	$60=6y+8y-9y$
Combine like terms.	$60=5y$
Divide by five.	$\frac{60}{5}=\frac{5y}{5}$
Simplify.	$12=y$

Check:	$5=\frac{1}{2}y+\frac{2}{3}y-\frac{3}{4}y$
Let $y=12$.	$5\stackrel{?}{=}\frac{1}{2}(12)+\frac{2}{3}(12)-\frac{3}{4}(12)$
	$5\stackrel{?}{=}6+8-9$
	$5=5\checkmark$

Try It 8

Solve: $7=\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$.

Answer

$x=12$

Notice in the previous example that once we cleared the equation of fractions, it became similar to those we solved earlier in this chapter. We changed the problem to one we already knew how to solve. We then used the **General Strategy for Solving Linear Equations**.

SOLVE EQUATIONS WITH FRACTION OR DECIMAL COEFFICIENTS.

1. Find the least common denominator (LCD) of *all* the fractions and decimals (in fraction form) in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions and decimals.
3. Solve using the General Strategy for Solving Linear Equations.

In the next example, we'll distribute before we clear the fractions.

Example 9

Solve: $\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$.

Solution

	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2}\cdot y-\frac{1}{2}\cdot 5=\frac{1}{4}\cdot y-\frac{1}{4}\cdot 1$
Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, four.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$

An alternate way to solve this equation is to clear the fractions without distributing first. If you multiply the factors correctly, this method will be easier.

		$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Multiply by the LCD, 4.		$4\cdot\frac{1}{2}(y-5)=4\cdot\frac{1}{4}(y-1)$
Multiply four times the fractions.		$2(y-5)=1(y-1)$
Distribute.		$2y-10=y-1$
Collect the variables to the left.		$2y-y-10=y-y-1$
Simplify.		$y-10=-1$
Collect the constants to the right.		$y-10+10=-1+10$
Simplify.		$y=9$
Check:	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$	
Let $y=9$.	$\frac{1}{2}(9-5)\stackrel{?}{=}\frac{1}{4}(9-1)$	

Finish the check on your own.

? Try It 9

Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$.

Answer

$m=-1$

When you multiply both sides of an equation by the LCD of the fractions, make sure you multiply each term by the LCD—even if it does not contain a fraction.

Example 10

Solve: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$

Solution

	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Multiply both sides by the LCD, 4.	$4\left(\frac{4q+3}{2}+6\right)=4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\cdot\left(\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$
	$8q+6+24=3q+5$
	$8q+30=3q+5$
Collect the variables to the left.	$8q-3q+30=3q-3q+5$
Simplify.	$5q+30=5$
Collect the constants to the right.	$5q+30-30=5-30$
Simplify.	$5q=-25$
Divide both sides by five.	$\frac{5q}{5}=\frac{-25}{5}$
Simplify.	$q=-5$
Check:	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Let $q=-5$.	$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$
Finish the check on your own.	

Try It 10

Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$.

Answer

$r=1$

Some equations have decimals in them. This type of equation may arise when solving problems involving money or percentages. But decimals can also be expressed as fractions. For example, $0.7=\frac{7}{10}$ and $0.29=\frac{29}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the **least common denominator**.

The next example uses an equation that is typical of the ones we will see in the money applications in a later section. Notice that we will clear all decimals by multiplying by the LCD of their fraction form.

Example 11

Solve: $0.25x+0.05(x+3)=2.85$.

Solution

Look at the decimals and think of the equivalent fractions:

$$0.25=\frac{25}{100},\qquad 0.05=\frac{5}{100},\qquad 2.85=2\frac{85}{100}.$$

Notice, the LCD is 100. By multiplying by the LCD we will clear the decimals from the equation.

	$0.25x+0.05(x+3)=2.85$
Distribute first.	$0.25x+0.05x+0.15=2.85$
Combine like terms.	$0.30x+0.15=2.85$
To clear decimals, multiply by 100.	$100(0.30x+0.15)=100(2.85)$
Distribute.	$30x+15=285$
Subtract 15 from both sides.	$30x+15-15=285-15$
Simplify.	$30x=270$
Divide by 30.	$\frac{30x}{30}=\frac{270}{30}$
Simplify.	$x=9$
Check it yourself by substituting $x=9$ into the original equation.	

? Try It 11

Solve: $0.10d+0.05(d-5)=2.15$.

Answer

$d=16$

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Section 0.3P: Practice

Practice Makes Progress

In the following exercises, determine whether the given values are solutions to the equation.

Exercise 1

$6y + 10 = 12y$

a. $y = \frac{5}{3}$

b. $y = -\frac{1}{2}$

Exercise 2

$8u - 1 = 6u$

a. $u = -\frac{1}{2}$

b. $u = \frac{1}{2}$

In the following exercises, solve each linear equation.

Exercise 3

$15(y - 9) = -60$

Exercise 4

$-(w - 12) = 30$

Exercise 5

$51 + 5(4 - q) = 56$

Exercise 6

$\frac{2}{3}(9c - 3) = 22$

Exercise 7

$12 + 2(5 - 3y) = -9(y - 1) - 2$

Exercise 8

$4(p - 4) - (p + 7) = 5(p - 3)$

Exercise 9

$4[5 - 8(4c - 3)] = 12(1 - 13c) - 8$

Exercise 10

$5[2(m+4)+8(m-7)]=2[3(5+m)-(21-3m)]$

In the following exercises, solve each equation.

Exercise 11

$\frac{1}{4}x-\frac{1}{2}=-\frac{3}{4}$

Exercise 12

$\frac{1}{2}a+\frac{3}{8}=\frac{3}{4}$

Exercise 13

$\frac{1}{3}w+\frac{5}{4}=w-\frac{1}{4}$

Exercise 14

$\frac{1}{4}(p-7)=\frac{1}{3}(p+5)$

Exercise 15

$\frac{4n+8}{4}=\frac{n}{3}$

Exercise 16

$\frac{7u-1}{4}-1=\frac{4u+8}{5}$

Exercise 17

$0.4x+0.6=0.5x-1.2$

Exercise 18

$0.05n+0.10(n+8)=2.15$

Exercise 19

$0.10d+0.25(d+5)=4.05$

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction, and then state the solution.

Exercise 20

$23z+19=3(5z-9)+8z+46$

? Exercise 21

$18(5j-1)+29=47$

? Exercise 22

$7v+42=11(3v+8)-2(13v-1)$

? Exercise 23

$9(14d+9)+4d=13(10d+6)+3$

Everyday Math

? Exercise 24

Fencing Micah has 74 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, L, by solving the equation $2L+2(L-2.5)=74$.

Writing Exercises

? Exercise 25

What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

? Exercise 26

If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve linear equations using a general strategy.			
classify equations.			
solve equations with fraction or decimal coefficients.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 0.4: General Strategy to Solve Linear Inequalities

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $2x + 5$ when $x = 3$
- Solve $2x + 5 = 3x - 7$ for x
- Graph the equation $y = -x + 2$

Learning Objectives

Motivating Problem

You want to buy concert tickets that cost $45 each. You have $200 saved. How many tickets can you buy *at most*—and how would you write that as a math sentence?

Fun Fact

The symbols $<$ and $>$ were first used in print in the 16th century by the English mathematician **Thomas Harriot**. Before that, mathematicians wrote out "greater than" or "less than" in words—imagine doing that on every test!

The Goal

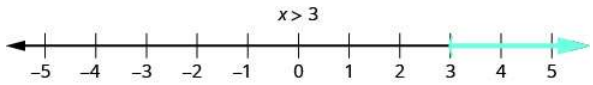
In this section, we'll review how to solve linear inequalities, graph their solutions on a number line, and interpret the meaning of these solutions. We'll also learn to reverse the inequality symbol when multiplying or dividing by a negative number.

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

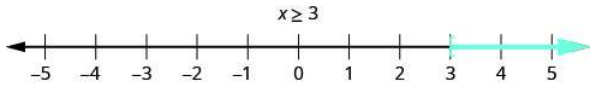
What about the solution of an inequality? What number would make the inequality $x > 3$ true? Are you thinking, 'x could be 4'? That's correct, but x could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality $x > 3$.

We illustrate the solutions to the inequality $x > 3$ on a number line by shading in all the numbers to the right of 3, indicating that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown below. Please note that the following convention is used: light blue arrows indicate the positive direction, and dark blue arrows indicate the negative direction.



The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We achieve this by placing a bracket at $x = 3$, as shown below.



The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

Example 1

Graph on the number line:

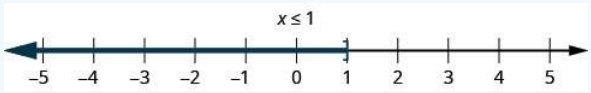
a. $x \leq 1$

b. $x < 5$

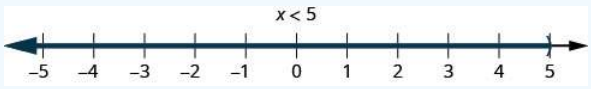
c. $x > -1$

Solution

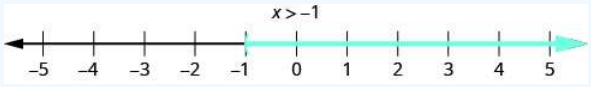
a. $x \leq 1$ This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at $x = 1$ to show that it is included.



b. $x < 5$ This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put parentheses at $x = 5$ to show that it is not included.



c. $x > -1$ This means all numbers greater than −1, but not including −1. We shade in all the numbers on the number line to the right of −1, then put parentheses at $x = -1$ to show that it is not included.



? Try It 1

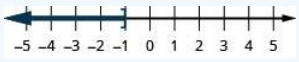
Graph on the number line:

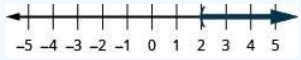
a. $x \leq -1$

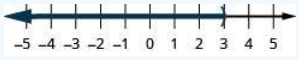
b. $x > 2$

c. $x < 3$

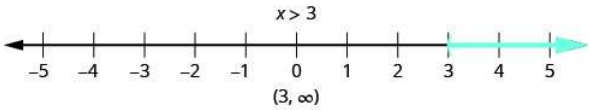
Answer

a. 

b. 

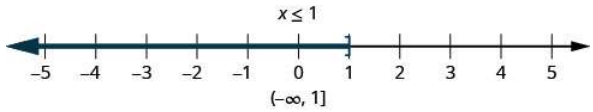
c. 

We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In **interval notation**, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as 'infinity'. It is not an actual number. The figure below shows both the number line and the interval notation.



The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as 'negative infinity'. The figure below shows both the number line and interval notation.



The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

INEQUALITIES, NUMBER LINES, AND INTERVAL NOTATION

Parentheses or brackets in the interval notation match the symbol at the endpoint of the arrow.



Example 2

Graph on the number line and write in interval notation.

a. $x \geq -3$

b. $x < 2.5$

c. $x \leq \frac{3}{5}$

Solution

a.

	$x \geq -3$
Shade to the right of −3, and put a bracket at −3.	
Write in interval notation.	$[-3, \infty)$

b.

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	
Write in interval notation.	$(-\infty, 2.5)$

c.

	$x \leq -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

Try It 2

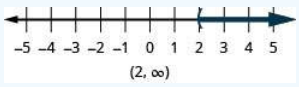
Graph on the number line and write in interval notation:

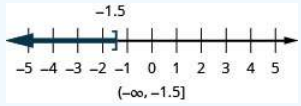
a. $x > 2$

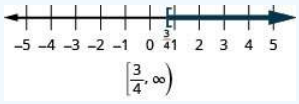
b. $x \leq -1.5$

c. $x \geq \frac{3}{4}$

Answer

a. 

b. 

c. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal. Similar properties hold true for inequalities.

PROPERTIES OF INEQUALITY

Subtraction Property of Inequality	**Addition Property of Inequality**
For any numbers $a, b,$ and c,	For any numbers $a, b,$ and c
if $a < b$	if $a < b$
then $a - c < b - c$.	then $a + c < b + c$
if $a > b$	if $a > b$
then $a - c > b - c$.	then $a + c > b + c$

We use these properties to solve inequalities, taking the same steps we used to solve equations.

Example 3

Solve the inequality $n - \frac{1}{2} \leq \frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution

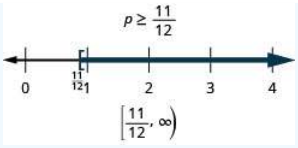
	$n - \frac{1}{2} \leq \frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	$n - \frac{1}{2} + \frac{1}{2} \leq \frac{5}{8} + \frac{1}{2}$
Simplify.	$n \leq \frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \frac{9}{8}]$

Try It 3

Solve the inequality, graphing the solution on the number line, and writing the solution in interval notation.

$p - \frac{3}{4} \geq \frac{1}{6}$

Answer



Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don't divide by 0). Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples where we multiply or divide by a positive number.

	$10 < 15$		$10 < 15$
Divide both sides by 5.	$\frac{10}{5} \,?\, \frac{15}{5}$	Multiply both sides by 5.	$10(5) \,?\, 15(5)$
Simplify.	$2 \,?\, 3$		$50 \,?\, 75$
Fill in the inequality signs.	$2 < 3$		$50 < 75$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

	$10 < 15$		$10 < 15$
Divide both sides by -5.	$\frac{10}{-5} \,?\, \frac{15}{-5}$	Multiply both sides by -5.	$10(-5) \,?\, 15(-5)$
Simplify.	$-2 \,?\, -3$		$-50 \,?\, -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

The inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the direction of the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

DIVISION AND MULTIPLICATION PROPERTIES OF INEQUALITY

For any real numbers a, b, c

$$\begin{aligned}
&\text{if } a < b \text{ and } c > 0, \text{ then } \quad \tfrac{a}{c} < \tfrac{b}{c} \text{ and } ac < bc \\
&\text{if } a > b \text{ and } c > 0, \text{ then } \quad \tfrac{a}{c} > \tfrac{b}{c} \text{ and } ac > bc \\
&\text{if } a < b \text{ and } c < 0, \text{ then } \quad \tfrac{a}{c} > \tfrac{b}{c} \text{ and } ac > bc \\
&\text{if } a > b \text{ and } c < 0, \text{ then } \quad \tfrac{a}{c} < \tfrac{b}{c} \text{ and } ac < bc
\end{aligned}$$

In short, when we **divide or multiply** an inequality by...

- ...a **positive number**, the inequality stays the **same**.
- ...a **negative number**, the inequality **reverses**.

Example 4

Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

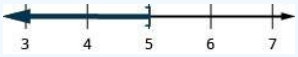
Try It 4

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$12d \leq 60$

Answer

$d \leq 5$



$(-\infty, 5]$

Example 5

Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$-10a \geq 50$
Divide both sides of the inequality by −10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

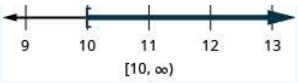
Try It 5

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$-7r \leq -70$

Answer

$r \geq 10$



SOLVING INEQUALITIES

Sometimes, when solving an inequality, the variable ends up on the right side. We can rewrite the inequality in reverse to get the variable to the left.

$$x > a \text{ has the same meaning as } a < x$$

Think about it as "If Xavier is taller than Alex, then Alex is shorter than Xavier."

Example 6

Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Solution

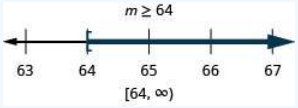
	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

Try It 6

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$$24 \leq \frac{3}{8}m$$

Answer



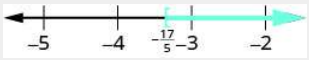
Solve Inequalities That Require Simplification

Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

Example 7

Solve the inequality $4m \leq 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Solution

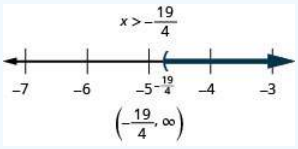
	$4m \le 9m + 17$
Subtract 9m from both sides to collect the variables on the left.	$4m - 9m \le 9m - 9m + 17$
Simplify.	$-5m \le 17$
Divide both sides of the inequality by −5, and reverse the inequality.	$\frac{-5m}{-5} \ge \frac{17}{-5}$
Simplify.	$m \ge -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

Try It 7

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$$-3x - 3 < x + 16$$

Answer



Example 8

Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

Solution

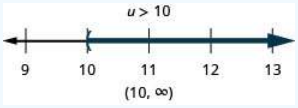
Simplify each side as much as possible.	$8p + 3(p - 12) > 7p - 28$
Distribute.	$8p + 3p - 36 > 7p - 28$
Combine like terms.	$11p - 36 > 7p - 28$
Subtract $7p$ from both sides to collect the variables on the left.	$11p - 36 - 7p > 7p - 28 - 7p$
Simplify.	$4p - 36 > -28$
Add 36 to both sides to collect the constants on the right.	$4p - 36 + 36 > -28 + 36$
Simplify.	$4p > 8$
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4} > 84$
Simplify.	$p > 2$
Graph the solution on the number line.	
Write the solution in interval notation.	$(2, \infty)$

Try It 8

Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

$$6u + 8(u - 1) > 10u + 32$$

Answer



Just as some equations are identities and some are contradictions, inequalities can also be identities or contradictions. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an **identity**. If the result is a false statement, we have a **contradiction**.

Example 9

Solve the inequality $8x - 2(5 - x) < 4(x + 9) + 6x$, graph the solution on the number line, and write the solution in interval notation.

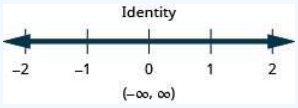
Solution

Simplify each side as much as possible.	$8x - 2(5 - x) < 4(x + 9) + 6x$
Distribute.	$8x - 10 + 2x < 4x + 36 + 6x$
Combine like terms.	$10x - 10 < 10x + 36$
Subtract 10x from both sides to collect the variables on the left.	$10x - 10 - 10x < 10x + 36 - 10x$
Simplify.	$-10 < 36$
The x's are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, \infty)$

Try It 9

Solve the inequality $4b - 3(3 - b) > 5(b - 6) + 2b$, graph the solution on the number line, and write the solution in interval notation.

Answer



Example 10

Solve the inequality $\frac{1}{3}a - \frac{1}{8}a > \frac{5}{24}a + \frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

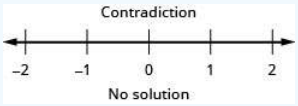
Solution

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract 5a from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Try It 10

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

Answer



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like ‘more than’ and ‘less than’. But others are not as obvious.

Think about the phrase ‘at least’ – what does it mean to be ‘at least 21 years old’? It means 21 or more. The phrase 'at least’ is the same as ‘greater than or equal to’.

The following table shows some common phrases that indicate inequalities.

>	≥	<	≤
is greater than	is greater than or equal to	is less than	is less than or equal to
is more than	is at least	is smaller than	is at most
is larger than	is no less than	has fewer than	is no more than
exceeds	is the minimum	is lower than	is the maximum

Example 11

Translate and solve. Then write the solution in interval notation.

Twelve times c is no more than 96.

Solution

Translate.	Twelve times c is no more than 96 $12c \leq 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \leq \frac{96}{12}$
Simplify.	$c \leq 8$
Write in interval notation.	$(-\infty, 8]$

Try It 11

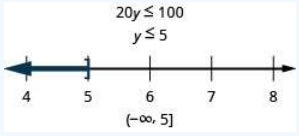
Translate and solve. Then write the solution in interval notation and graph on the number line.

a. Twenty times y is at most 100

b. Nine times z is no less than 135

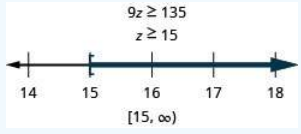
Answer

a. $20y \leq 100$
$y \leq 5$



$(-\infty, 5]$

b. $9z \geq 135$
$z \geq 15$



$[15, \infty)$

Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. In fact, inequality applications are so common that we often do not even realize we are doing algebra. For example, how many gallons of gas can be put in the car for $20? Is the rent on an apartment affordable? Is there enough time before class to go get lunch, eat it, and return to class? How much money should each family member's holiday gift cost without going over budget?

The method we will use to solve applications involving linear inequalities is very similar to the one we used when solving applications with equations. We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in a single sentence to facilitate its translation into an inequality. Then, we will solve the inequality.

Example 12

Emma has landed a new job and will need to relocate. Her monthly income will be $5,265. To qualify for an apartment rental, Emma's monthly income must be at least three times the rent amount. What is the highest rent Emma will qualify for?

Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.	the highest rent Emma will qualify for	
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let r = rent	
Step 4. Translate into an inequality.	Emma's monthly income must be at least three times the rent.	
Step 5. Solve the inequality. Remember, $a > x$ has the same meaning as $x < a$	$5265 \geq 3r$ $1755 \geq r$ $r \leq 1755$	(NaN.4.1)
Step 6. Check the answer in the problem and make sure it makes sense.	A maximum rent of \$1,755 seems reasonable for an income of \$5,265.	
Step 7. Answer the question in a complete sentence.	The maximum rent is \$1,755.	

? Try It 12

Alan is loading a pallet with boxes that each weigh 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

Answer

There can be no more than 20 boxes.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down. To check applications like this, we will round our answer to a number that is easy to compute with and ensure that this number makes the inequality true.

✓ Example 13

Dawn won a mini-grant of \$4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost \$254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what we are looking for.	
Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate. Write a sentence that	
gives the information to find it.	254.12 times the number of tablets is no more than \$4000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$
But n must be a whole number of tablets, so round to 15.	$n \leq 15$
Step 6. Check the answer in the problem and make sure it makes sense.	Rounding down the price to \$250, 15 tablets would cost \$3750, while 16 tablets would be \$4000. So a maximum of 15 tablets at \$254.12 seems reasonable.
Step 7. Answer the question in a complete sentence.	Dawn can buy a maximum of 15 tablets.

? Try It 13

Angie has \$20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs \$2.63. What is the maximum number of packs she can buy?

Answer

She can buy at most seven packs.

A common goal of most businesses is to make a profit. *Profit* is the money that remains when the expenses have been subtracted from the money earned. In the following example, we will determine the number of jobs a small businessman needs to complete each month to achieve a certain level of **profit**.

✓ Example 14

Elliot has a landscape maintenance business. His monthly expenses are \$1,100. If he charges \$60 per job, how many jobs must he do to earn a profit of at least \$4,000 a month?

Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.	the number of jobs Elliot needs	
Step 3. Name what we are looking for. Choose a variable to represent it.	Let j = the number of jobs.	
Step 4. Translate. Write a sentence that gives the information to find it.	\$60 times the number of jobs minus \$1,100 is at least \$4,000.	
Step 5. Solve the inequality.	$60j - 1100 \geq 4000$ $60j \geq 5100$ $j \geq 85$ jobs	(NaN.4.2)
Step 6. Check the answer in the problem and make sure it makes sense.	If Elliot did 90 jobs, his profit would be $60(90) - 1100 = \$4,300$, which is more than \$4,000.	
Step 7. Answer the question in a complete sentence.	Elliot must work at least 85 jobs.	

? Try It 14

Caleb has a pet sitting business. He charges \$32 per hour. His monthly expenses are \$2272. How many hours must he work to earn a profit of at least \$800 per month?

Answer

He must work at least 96 hours.

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Section 0.4P: Practice

Practice Makes Progress

In the following exercises, graph each inequality on the number line and write in interval notation.

Exercise 1

$x > 3$

Exercise 2

$x \leq -0.5$

Exercise 3

$x \geq \frac{1}{3}$

Exercise 4

$x \leq 5$

Exercise 5

$x \geq -1.5x$

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

Exercise 6

$m - 45 \leq 62$

Exercise 7

$v + 12 > 3$

Exercise 8

$b + \frac{7}{8} \geq \frac{1}{6}$

Exercise 9

$g - \frac{11}{12} < -\frac{5}{18}$

Exercise 10

$9s \geq 81$

Exercise 11

$-8v \leq 96$

Exercise 12

$40 < \frac{5}{8}k$

Exercise 13

$\frac{9}{4}g \leq 36$

Exercise 14

$9p > 14p - 18$

Exercise 15

$9y + 5(y + 3) < 4y - 35$

Exercise 16

$4x - (x - 2) \geq 7x - 26$

Exercise 17

$6n - 12(3 - n) \leq 9(n - 4) + 9n$

Exercise 18

$9u + 5(2u - 5) \geq 12(u - 1) + 7u$

Exercise 19

$\frac{5}{6}a - \frac{1}{4}a > \frac{7}{12}a + \frac{2}{3}$

Exercise 20

$12v + 3(4v - 1) \leq 19(v - 2) + 5v$

Exercise 21

$35k \geq -77$

Exercise 22

$18q - 4(10 - 3q) < 5(6q - 8)$

Exercise 23

$-\frac{21}{8}y \leq -\frac{15}{28}$

Exercise 24

$d + 29 > -61$

Exercise 25

$\frac{n}{13} \leq -6$

In the following exercises, translate and solve the given problems. Then write the solution in interval notation and graph on the number line.

Exercise 26

Six more than k exceeds 25

Exercise 27

Twelve less than x is no less than 21

Exercise 28

Negative two times s is lower than 56

Everyday Math

Exercise 29

A water taxi has a maximum load of 1,800 pounds. If the average weight of one person is 150 pounds, how many people can safely ride in the water taxi?

Exercise 30

Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Exercise 31

Joni sells kitchen aprons online for $32.50 each. How many aprons must she sell next month if she wants to earn at least $1,000?

Exercise 32

Keshad gets paid $2,400 per month plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

Exercise 33

Katherine is a personal chef. She charges $115 per four-person meal. Her monthly expenses are $3,150. How many four-person meals must she sell in order to make a profit of at least $1,900?

Exercise 34

Five student government officers want to go to the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

Exercise 35

In 2014, a high school stage collapsed in Fullerton, California, when 250 students got on stage for the finale of a musical production. Two dozen students were injured. The stage could support a maximum of 12,750 pounds. If the average weight of a student is assumed to be 140 pounds, what is the maximum number of students who could safely be on the stage?

Exercise 36

Adele and Walter found the perfect venue for their wedding reception. The cost is $9,850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

Writing Exercises

Exercise 37

Find your last month's phone bill and the hourly salary you are paid at your job. (If you do not have a job, use the hourly salary you would realistically be paid if you had a job.) Calculate the number of hours of work it would take you to earn at least enough money to pay your phone bill by writing an appropriate inequality and then solving it.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 0.5: Solving Systems of Linear Equations

Be Prepared

We will rely heavily on these skills throughout this section.

- Graph $y = 2x + 1$
- Solve $\frac{y}{2} + 3x = -2x + 3$ for y
- Identify the intersection point of $y = 2x + 3$ and $y = -3x + 1$

Learning Objectives

Motivating Problem

Two phone plans are available. Plan A charges \$10 per month plus \$0.05 per text. Plan B charges \$20 per month with unlimited texts. How many texts would be required to make the two plans cost the same?

Fun Fact

More than 3,000 years ago, ancient Chinese mathematicians used a counting board and bamboo strips to solve systems of linear equations. These early techniques, recorded in the Nine Chapters on the Mathematical Art, included methods similar to matrix elimination, centuries before it was introduced in Western mathematics.

The Goal

In this section, we'll review how to solve systems of linear equations using graphing, substitution, and elimination. We'll revisit how to determine if a system has one solution, no solution, or infinitely many, and how to apply this to real-world comparisons.

Determine Whether an Ordered Pair is a Solution of a System of Equations

We previously learned how to solve linear equations with one variable. Remember that the solution of an equation is a value of the variable that makes a true statement when substituted into the equation. Now, we will work with **systems of linear equations**, which are two or more linear equations grouped together.

Definition: SYSTEM OF LINEAR EQUATIONS

When two or more linear equations are grouped together, they form a system of linear equations.

We will focus our work here on systems of two linear equations in two unknowns. In later courses, you may solve larger systems of equations.

Below is an example of a system of two linear equations. We use a brace to show how the two equations are grouped together to form a system of equations.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

A linear equation in two variables, like $2x + y = 7$, has an infinite number of solutions. Its graph is a line. Remember, every point on the line is a solution to the equation, and every solution to the equation is a point on the line.

To solve a system of two linear equations, we want to find the values of the variables that are solutions to *both* equations. In other words, we are looking for the ordered pairs (x, y) that make *both* equations true. These are referred to as the **solutions to a system of equations**.

Definition: SolutionS OF A SYSTEM OF EQUATIONS

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

To determine if an ordered pair is a solution to a system of two equations, we substitute the values of the variables into each equation. If the ordered pair satisfies both equations, it is a solution to the system.

Let's consider the system below:

$$\begin{cases} 3x - y = 7 \\ x - 2y = 4 \end{cases}$$

Is the ordered pair $(2, -1)$ a solution?

We substitute $x = 2$ and $y = -1$ into both equations.

$$\begin{array}{ll} 3x - y = 7 & x - 2y = 4 \\ 3(2) - (-1) \stackrel{?}{=} 7 & 2 - 2(-1) \stackrel{?}{=} 4 \\ 7 = 7 \text{ true} & 4 = 4 \text{ true} \end{array}$$

The ordered pair $(2, -1)$ made both equations true. Therefore, $(2, -1)$ is a solution to this system.

Let's try another ordered pair. Is the ordered pair $(3, 2)$ a solution?

We substitute $x = 3$ and $y = 2$ into both equations.

$$\begin{array}{ll} 3x - y = 7 & x - 2y = 4 \\ 3(3) - 2 \stackrel{?}{=} 7 & 3 - 2(2) \stackrel{?}{=} 4 \\ 7 = 7 \text{ true} & -2 = 4 \text{ false} \end{array}$$

The ordered pair $(3, 2)$ made one equation true, but it made the other equation false. Since it is not a solution to **both** equations, it is not a solution to this system.

Example 1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x - y = -1 \\ 2x - y = -5 \end{cases}$

1. $(-2, -1)$
2. $(-4, -3)$

Solution

1.

$$\begin{cases} x-y=-1 \\ 2x-y=-5 \end{cases}$$

We substitute $x=-2$ and $y=-1$ into both equations

$$\begin{array}{ll} x-y=1 & 2x-y=-5 \\ -2-(-1)\stackrel{?}{=}-1 & 2(-2)-(-1)\stackrel{?}{=}-5 \\ -1=-1\checkmark & -3=-5\times \end{array}$$

$(-2,-1)$ does not make both equations true. Therefore, $(-2,-1)$ is not a solution to the system.

2.

$$\begin{cases} x-y=-1 \\ 2x-y=-5 \end{cases}$$

We substitute $x=-4$ and $y=-3$ into both equations

$$\begin{array}{ll} x-y=-1 & 2x-y=-5 \\ -4-(-3)\stackrel{?}{=}-1 & 2(-4)-(-3)\stackrel{?}{=}-5 \\ -1=-1\checkmark & -5=-5\checkmark \end{array}$$

$(-4,-3)$ makes both equations true. Therefore, $(-4,-3)$ is a solution to the system.

? Try It 1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x-3y=-8 \\ -3x-y=4 \end{cases}$

1. $(2,-2)$
2. $(-2,2)$

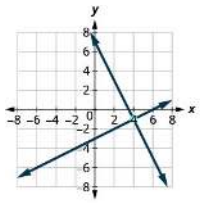
Answer

1. no
2. yes

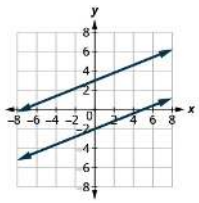
Solve a System of Linear Equations by Graphing

In this section, we will use three methods to solve a system of linear equations. The first method we'll use is graphing. The graph of a linear equation is a line. Each point on the line is a solution to the equation. For a system of two equations, we will graph two lines. Then we can see all the points that are solutions to each equation. By identifying what the lines have in common, we can find the solution to the system.

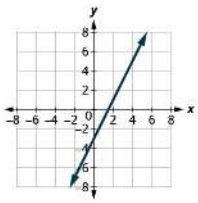
Most linear equations in one variable have a single solution, but we saw that some equations, called **contradictions**, have no solutions, while other equations, called **identities**, have all numbers as solutions. Similarly, when we solve a system of two linear equations represented by a graph of two lines in the same plane, there are three possible cases, as shown below.



The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.



The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.

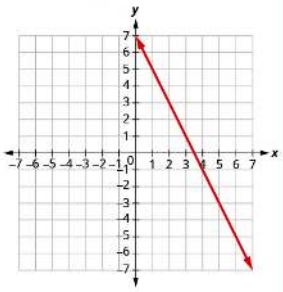
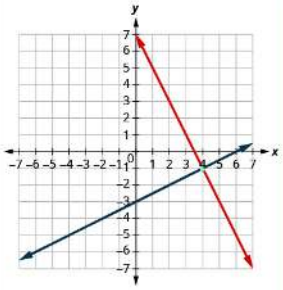


Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

✓ Example 2

Solve the system by graphing: $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Graph the first equation.	To graph the first line, write the equation in slope–intercept form. $2x + y = 7$ $y = -2x + 7$ $m = -2$ $b = 7$	$2x + y = 7$ $x - 2y = 6$ 
Step 2. Graph the second equation on the same rectangular coordinate system.	To graph the second line, use intercepts. $x - 2y = 6$ $(0, -3)$ $(6, 0)$	
Step 3. Determine whether the lines intersect, are parallel, or are the same line.	Look at the graph of the lines.	The lines intersect.
Step 4. Identify the solution to the system. If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system. If the lines are parallel, the system has no solution If the lines are the same, the system has an infinite number of solutions.	Since the lines intersect, find the point of intersection. Check the point in both equations.	The lines intersect at $(4, -1)$. $2x + y = 7$ $2(4) + (-1) \stackrel{?}{=} 7$ $8 - 1 \stackrel{?}{=} 7$ $7 = 7$ ✓ $x - 2y = 6$ $4 - 2(-1) \stackrel{?}{=} 6$ $6 = 6$ ✓ The solution is $(4, -1)$.

? Try It 2

Solve each system by graphing: $\begin{cases} x - 3y = -3 \\ x + y = 5 \end{cases}$

Answer

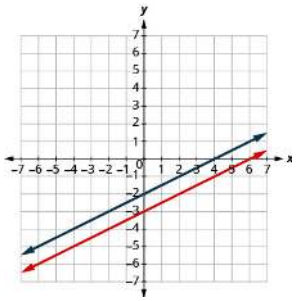
$(3, 2)$

In the following two examples, we'll examine a system of equations that has no solution and a system of equations that has an infinite number of solutions.

✓ Example 3

Solve the system by graphing: $\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$

Solution

	$y=\frac{1}{2}x-3$ $x-2y=4$
To graph the first equation, we will use its slope and y-intercept.	$y=\frac{1}{2}x-3$
	$m=\frac{1}{2}$
	$b=-3$
To graph the second equation, we will use the intercepts.	$x-2y=4$
	x | y 0 | −2 4 | 0
Graph the lines.	
Determine the point of intersection.	The lines are parallel.
	Since no point is on both lines, there is no ordered pair that makes both equations true. There is no solution to this system.

Try It 3

Solve each system by graphing: $\begin{cases} y=3x-1 \\ 6x-2y=6 \end{cases}$

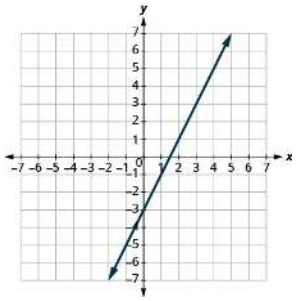
Answer

no solution

Example 4

Solve the system by graphing: $\begin{cases} y=2x-3 \\ -6x+3y=-9 \end{cases}$

Solution

	$y = 2x - 3$ $-6x + 3y = -9$
Find the slope and y-intercept of the first equation.	$y = 2x - 3$ $m = 2$ $b = -3$
Find the intercepts of the second equation.	$-6x + 3y = -9$
	x | y 0 | -3 $\frac{3}{2}$ | 0
Graph the lines.	
Determine the point of intersection.	The lines are the same!
	Since every point on the line makes both equations true, there are infinitely many ordered pairs that make both equations true.
	There are infinitely many solutions to this system.

Try It 4

Solve each system by graphing: $\begin{cases} y = \frac{1}{2}x - 4 \\ 2x - 4y = 16 \end{cases}$

Answer

infinitely many solutions

The steps for solving a system of linear equations using the graphing method are outlined below.

TO SOLVE A SYSTEM OF LINEAR EQUATIONS BY GRAPHING.

1. Graph the first equation.
2. Graph the second equation on the same rectangular coordinate system.
3. Determine whether the lines intersect, are parallel, or are the same line.
4. Identify the solution to the system.
   - If the lines intersect, identify the point of intersection. Verify that it is a solution to both equations. This is the solution to the system.
   - If the lines are parallel, the system has no solution.
   - If the lines are the same, the system has an infinite number of solutions.

Solve a System of Equations by Substitution

We will use the same system we used first for graphing.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

We will first solve one of the equations for either x or y. We can choose either equation and solve for either variable, but we'll try to make a choice that'll keep the work manageable.

Then we substitute that expression into the other equation. The result is an equation with just one variable, and we know how to solve those!

After we find the value of one variable, we will substitute that value into one of the original equations and solve for the other variable. Finally, we verify our solution to ensure it satisfies both equations.

Example 5

Solve the system by substitution. $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$

Solution

Step 1. Solve one of the equations for either variable.	We'll solve the first equation for y.	$2x+y=7$ $y=7-2x$	**Step 2.** Substitute the expression from Step 1 into the other equation.	We replace y in the second equation with the expression $7-2x$.	$x-2y=6$ $x-2(7-2x)=6$
Step 3. Solve the resulting equation.	Now we have an equation with just 1 variable. We know how to solve this!	$x-2(7-2x)=6$ $x-14+4x=6$ $5x=20$ $x=4$	**Step 4.** Substitute the solution in Step 3 into one of the original equations to find the other variable.	We'll use the first equation and replace x with 4.	$2x+y=7$ $2(4)+y=7$ $8+y=7$ $y=-1$
			Step 6. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4,-1)$ into both equations and make sure they are both true.	$2x+y=7$ $\quad x-2y=6$ $2(4)+(-1)\stackrel{?}{=}7$ $\quad 4-2(-1)\stackrel{?}{=}6$ $7=7\checkmark$ $\quad 6=6\checkmark$ Both equations are true. $(4,-1)$ is the solution to the system.
Step 5. Write the solution as an ordered pair.	The ordered pair is (x,y).	$(4,-1)$			

Try It 5

Solve the system by substitution. $\begin{cases} -2x+y=-11 \\ x+3y=9 \end{cases}$

Answer

$(6,1)$

If the equations are given in standard form, we'll need to start by solving for one of the variables. In the following example, we'll solve the first equation for y.

Example 6

Solve the system by substitution. $\begin{cases} 3x+y=5 \\ 2x+4y=-10 \end{cases}$

Solution

We need to solve one equation for one variable. Then we will substitute that expression into the other equation.

Solve for y. Substitute into the other equation.	$3x+y=5$ $y=-3x+5$ $2x+4y=-10$
Replace the y with $-3x+5$.	$2x+4(-3x+5)=-10$
Solve the resulting equation for x.	$2x-12x+20=-10$
	$-10x+20=-10$ $-10x=-30$
Substitute $x=3$ into $3x+y=5$ to find y.	$x=3$ $3x+y=5$
	$3(3)+y=5$ $9+y=5$
The ordered pair is $(3,-4)$.	$y=-4$
Check the ordered pair in both equations: $3x+y=5$ $\quad 2x+4y=-10$ $3\cdot3+(-4)\stackrel{?}{=}5$ $\quad 2\cdot3+4(-4)\stackrel{?}{=}-10$ $9-4\stackrel{?}{=}5$ $\quad 6-16\stackrel{?}{=}-10$ $5=5\checkmark$ $\quad -10=-10\checkmark$	
	The solution is $(3,-4)$.

Try It 6

Solve the system by substitution. $\begin{cases} -x+y=4 \\ 4x-y=2 \end{cases}$

Answer

$(2,6)$

Next, let's look at what happens when we use substitution to "solve" a system with no solutions.

Example 7

Solve the system by substitution. $\begin{cases} 5x-2y=-10 \\ y=\frac{5}{2}x \end{cases}$

Solution

The second equation is already solved for y, so we can substitute for y in the first equation.

Substitute x for y in the first equation.	$y=\left(\frac{5}{2}x\right)$ $5x-2y=-10$
Replace the y with $\frac{5}{2}x$.	$5x-2\left(\frac{5}{2}x\right)=-10$
Solve for x.	$5x-5x=-10$
	$0\neq-10$

Since $0=-10$ is a false statement, the graphs of the two equations would be parallel lines. The system has no solutions.

Try It 7

Solve the system by substitution. $\begin{cases} 3x+2y=9 \\ y=-\frac{3}{2}x+1 \end{cases}$

Answer

no solution

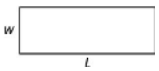
Similarly, if our work resulted in a statement which was always true, such as $5=5$, we would know that there were infinitely many solutions to our system.

In the following example, we'll use the formula for the perimeter of a rectangle, $P=2L+2W$.

Example 8

The perimeter of a rectangle is 88. The length is five more than twice the width. Find the length and width of the rectangle.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
Step 3. Name what we are looking for.	Let L be the length and W the width
Step 4. Translate into a system of equations.	The perimeter of a rectangle is 88.
	$2L+2W=P$ $2L+2W=88$
	The length is five more than twice the width.
	$L=2W+5$
The system is:	$2L+2W=88$ $L=2W+5$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for L. Substitute $2W+5$ for L in the first equation.	$L=2W+5$ $2L+2W=88$
Solve for W.	$2(2W+5)+2W=88$
	$4W+10+2W=88$
	$6W+10=88$
	$6W=78$
Substitute $W=13$ into the second equation and then solve for L.	$W=13$ $L=2W+5$
	$L=2\cdot13+5$
	$L=31$
Step 6. Check the answer in the problem.	Does a rectangle with length 31 and width 13 has a perimeter of 88? Yes.
Step 7. Answer the equation.	The length is 31 inches, and the width is 13 inches.

Try It 8

The perimeter of a rectangle is 40. The length is 4 more than the width. Find the length and width of the rectangle.

Answer

The length is 12 inches, and the width is 8 inches.

The steps for the substitution method are listed below for easy reference.

SOLVE A SYSTEM OF EQUATIONS BY SUBSTITUTION.

1. Solve one of the equations for either variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the resulting equation.
4. Substitute the solution in Step 3 into one of the original equations to find the other variable.
5. Write the solution as an ordered pair.
6. Check that the ordered pair is a solution to **both** original equations.

Notice if one of the equations in the system is given in slope–intercept form; Step 1 is already done.

Solve a System of Equations by Elimination

The **Elimination Method** is based on the Addition Property of Equality. The Addition Property of Equality says that you still have equality when you add the same quantity to both sides of an equation. We will extend the Addition Property of Equality to say that the results are equal when you add equal quantities to both sides of an equation.

For any expressions a, b, c, and d,

$$\begin{array}{ll} \textbf{if} & a=b \\ \textbf{and} & c=d \\ \textbf{then} & a+c=b+d \end{array}$$

To solve a system of equations using the elimination method, we first put both equations into standard form. Then we decide which variable will be easiest to eliminate. How do we decide? We want the coefficients of one variable to be opposites, so that we can add the equations together and eliminate that variable.

Once we get an equation with just one variable, we solve it. Then, we substitute that value into one of the original equations to solve for the remaining variable. As always, we verify our answers to ensure they are solutions to both of the original equations.

Now we'll see how to use elimination to solve the same system of equations we solved by graphing and by substitution.

Example 9

Solve the system by elimination. $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Write both equations in standard form. If any coefficients are fractions, clear them.	Both equations are in standard form, $Ax+By=C$. There are no fractions.	$2x+y=7$ $x-2y=6$
Step 2. Make the coefficients of one variable opposites. Decide which variable you will eliminate. Multiply one or both equations so that the coefficients of that variable are opposites.	We can eliminate the y's by multiplying the first equation by 2. Multiply both sides of $2x+y=7$ by 2.	$2x+y=7$ $x-2y=6$ $2(2x+y)=2(7)$ $x-2y=6$
Step 3. Add the equations resulting from Step 2 to eliminate one variable.	We add the x's, y's, and constants.	$4x+2y=14$ $x-2y=6$ $5x \quad =20$
Step 4. Solve for the remaining variable.	Solve for x.	$x=4$
Step 5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.	Substitute $x=4$ into the second equation, $x-2y=6$. Then solve for y.	$x-2y=6$ $4-2y=6$ $-2y=2$ $y=-1$
Step 6. Write the solution as an ordered pair.	Write it as (x, y).	$(4,-1)$
Step 7. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4,-1)$ into $2x+y=7$ and $x-2y=6$ Do they make both equations true? Yes!	$2x+y=7$ $\quad$ $x-2y=6$ $2(4)+(-1)\stackrel{?}{=}7$ $\quad$ $4-2(-1)\stackrel{?}{=}6$ $7=7\checkmark$ $\quad$ $6=6\checkmark$ The solution is $(4,-1)$.

Try It 9

Solve the system by elimination. $\begin{cases} 3x+y=5 \\ 2x-3y=7 \end{cases}$

Answer

$(2,-1)$

In the following example, we will be able to make the coefficients of one variable opposites by multiplying one equation by a constant.

Example 10

Solve the system by elimination. $\begin{cases} 3x-2y=-2 \\ 5x-6y=10 \end{cases}$

Solution

	$3x-2y=-2$ $5x-6y=10$
Both equations are in standard form.	
None of the coefficients are opposites.	
We can make the coefficients of y opposites by multiplying the first equation by -3.	$-3(3x-2y)=-3(-2)$ $5x-6y=10$
Simplify.	$-9x+6y=6$ $5x-6y=10$
Add the two equations to eliminate y.	$-9x+6y=6$ $5x-6y=10$ $-4x\quad\ =16$
Solve for the remaining variable, x. Substitute $x=-4$ into one of the original equations.	$x=-4$ $3x-2y=-2$
	$3(-4)-2y=-2$
Solve for y.	$-12-2y=-2$ $-2y=10$ $y=-5$
Write the solution as an ordered pair.	The ordered pair is $(-4,-5)$.
Check that the ordered pair is a solution to both original equations. $3x-2y=-2$ $3(-4)-2(-5)\stackrel{?}{=}-2$ $-12+10\stackrel{?}{=}-2$ $-2=-2\checkmark$ $5x-6y=10$ $5(-4)-6(-5)\stackrel{?}{=}10$ $-20+30\stackrel{?}{=}10$ $10=10\checkmark$	
	The solution is $(-4,-5)$.

Try It 10

Solve the system by elimination. $\begin{cases} 3x+2y=2 \\ 6x+5y=8 \end{cases}$

Answer

$(-2,4)$

When the system of equations contains fractions, we will first clear them by multiplying each equation by its least common denominator (LCD).

Example 11

Solve the system by elimination. $\begin{cases} x+\frac{1}{2}y=6 \\ \frac{3}{2}x+\frac{2}{3}y=\frac{17}{2} \end{cases}$

Solution

In this example, both equations have fractions. Our first step will be to multiply each equation by its LCD to clear the fractions.

	$x+\frac{1}{2}y=6$ $\frac{3}{2}x+\frac{2}{3}y=\frac{17}{2}$
To clear the fractions, multiply each equation by its LCD.	$2\left(x+\frac{1}{2}y\right)=2(6)$ $6\left(\frac{3}{2}x+\frac{2}{3}y\right)=6\left(\frac{17}{2}\right)$
Simplify.	$2x+y=12$ $9x+4y=51$
Now we are ready to eliminate one of the variables. Notice that both equations are in standard form.	
We can eliminate y multiplying the top equation by -4.	$-4(2x+y)=-4(12)$ $9x+4y=51$
Simplify and add.	$-8x-4y=-48$ $9x+4y=51$ $x=3$
Substitute $x=3$ into one of the original equations.	$x+\frac{1}{2}y=6$
Solve for y.	$3+\frac{1}{2}y=6$
	$\frac{1}{2}y=3$
	$y=6$
Write the solution as an ordered pair.	The ordered pair is $(3,6)$.
Check that the ordered pair is a solution to **both** original equations.	$x+\frac{1}{2}y=6$ $3+\frac{1}{2}(6)\stackrel{?}{=}6$ $3+3\stackrel{?}{=}6$ $6=6\checkmark$ $\frac{3}{2}x+\frac{2}{3}y=\frac{17}{2}$ $\frac{3}{2}(3)+\frac{2}{3}(6)\stackrel{?}{=}\frac{17}{2}$ $\frac{9}{2}+4\stackrel{?}{=}\frac{17}{2}$ $\frac{9}{2}+\frac{8}{2}\stackrel{?}{=}\frac{17}{2}$ $\frac{17}{2}=\frac{17}{2}\checkmark$
	The solution is $(3,6)$.

Try It 11

Solve the system by elimination. $\begin{cases} x+\frac{3}{5}y=-\frac{1}{5} \\ -\frac{1}{2}x-\frac{2}{3}y=\frac{5}{6} \end{cases}$

Answer

$(1,-2)$

Next, let's see what happens when we use the elimination method to "solve" a system with infinite solutions.

Example 12

Solve the system by elimination. $\begin{cases} 3x+4y=12 \\ y=3-\frac{3}{4}x \end{cases}$

Solution

$$\begin{cases} 3x+4y=12 \\ y=3-\frac{3}{4}x \end{cases}$$

Write the second equation in standard form.

$$\begin{cases} 3x+4y=12 \\ \frac{3}{4}x+y=3 \end{cases}$$

Clear the fractions by multiplying thesecond equation by 4.

$$\begin{cases} 3x+4y=12 \\ 4\left(\frac{3}{4}x+y\right)=4(3) \end{cases}$$

Simplify.

$$\begin{cases} 3x+4y=12 \\ 3x+4y=12 \end{cases}$$

To eliminate a variable, we multiply thesecond equation by -1.

$$\begin{cases} 3x+4y=12 \\ -3x-4y=-12 \end{cases}$$

$$0=0$$

Simplify and add.

This is a true statement. After we cleared the fractions in the second equation, did you notice that the two equations were the same? That means we have infinitely many solutions.

Try It 12

Solve the system by elimination. $\begin{cases} 7x-3y=-2 \\ -14x+6y=4 \end{cases}$

Answer

infinite solutions

Lastly, let's use the elimination method to solve an application problem.

Example 13

Joe stops at a burger restaurant every day on his way to work. On Monday, he had one order of medium fries and two small sodas, totaling 620 calories. On Tuesday, he had two orders of medium fries and one small soda, totaling 820 calories. How many calories are there in one order of medium fries? How many calories are in one small soda?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the number of calories in one order of medium fries and in one small soda.
Step 3. Name what we are looking for.	Let f be the number of calories in 1 order of medium fries. Let s be the number of calories in 1 small soda.
Step 4. Translate into a system of equations:	one medium fries and two small sodas had a total of 620 calories
	$f+2s=620$
	two medium fries and one small soda had a total of 820 calories.
	$2f+s=820$
Our system is:	$\begin{aligned} f+2s&=620\\ 2f+s&=820 \end{aligned}$
Step 5. Solve the system of equations. To solve the system of equations, use elimination. The equations are in standard form. To get opposite coefficients of f, multiply the top equation by -2.	$\begin{aligned} -2(f+2s)&=-2(620)\\ 2f+\ s&=820 \end{aligned}$
Simplify and add.	$\begin{aligned} -2f-4s&=-1240\\ 2f+\ s&=820\\ \hline -3s&=-420 \end{aligned}$
Solve for s.	$s=140$
Substitute $s=140$ into one of the original equations and then solve for f.	$f+2s=620$
	$f+2\cdot 140=620$
	$f+280=620$
	$f=340$
Step 6. Check the answer.	Verify that these numbers make sense in the problem, and that they are solutions to both equations. We leave this to you!
Step 7. Answer the question.	The small soda has 140 calories and the fries have 340 calories.

Try It 13

Malik stops at the grocery store to buy a bag of diapers and two cans of formula. He spends a total of $37. The following week, he stops and buys two bags of diapers and five cans of formula, totaling $87. How much does a bag of diapers cost? How much is one can of formula?

Answer

The bag of diapers costs $11, and the can of formula costs $13.

The steps for the elimination method are listed below for easy reference.

HOW TO SOLVE A SYSTEM OF EQUATIONS BY ELIMINATION.

1. Write both equations in standard form. If any coefficients are fractions, clear them.
2. Make the coefficients of one variable opposites.
   - Decide which variable you will eliminate.
   - Multiply one or both equations so that the coefficients of that variable are opposites.
3. Add the equations resulting from Step 2 to eliminate one variable.
4. Solve for the remaining variable.
5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.
6. Write the solution as an ordered pair.
7. Check that the ordered pair is a solution to **both** original equations.

Choose the Most Convenient Method to Solve a System of Linear Equations

When you have to solve a system of linear equations, you'll want to choose the method that is easiest to do and minimizes your chance of making mistakes.

Graphing	Substitution	Elimination
Use when you need a picture of the situation.	Use when one equation is already solved for one variable.	Use when the equations are in standard form.

Example 14

For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} 3x+8y=40 \\ 7x-4y=-32 \end{cases}$
2. $\begin{cases} 5x+6y=12 \\ y=\frac{2}{3}x-1 \end{cases}$

Solution

1. $\begin{cases} 3x+8y=40 \\ 7x-4y=-32 \end{cases}$

Since both equations are in standard form, using elimination will be most convenient.

2. $\begin{cases} 5x+6y=12 \\ y=\frac{2}{3}x-1 \end{cases}$

Since one equation is already solved for y, using substitution will be most convenient.

Try It 14

For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} y=2x-1 \\ 3x-4y=-6 \end{cases}$
2. $\begin{cases} 6x-2y=12 \\ 3x+7y=-13 \end{cases}$

Answer

1. Since one equation is already solved for y, using substitution will be most convenient;
2. Since both equations are in standard form, using elimination will be most convenient.

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Section 0.5P: Practice

Practice Makes Progress

In the following exercises, determine if the following points are solutions to the given system of equations.

Exercise 1

$$\begin{cases} 2x-6y=0 \\ 3x-4y=5 \end{cases}$$

a. $(3,1)$

b. $(-3,4)$

Exercise 2

$$\begin{cases} x+y=2 \\ y=\frac{3}{4}x \end{cases}$$

a. $\left(\frac{8}{7},\frac{6}{7}\right)$

b. $\left(1,\frac{3}{4}\right)$

In the following exercises, solve the following systems of equations by graphing.

Exercise 3

$$\begin{cases} -3x+y=-1 \\ 2x+y=4 \end{cases}$$

Exercise 4

$$\begin{cases} 2x+3y=6 \\ y=-2 \end{cases}$$

Exercise 5

$$\begin{cases} x-3y=-6 \\ x=-3 \end{cases}$$

Exercise 6

$$\begin{cases} -2x+4y=4 \\ y=\frac{1}{2}x \end{cases}$$

Exercise 7

$$\begin{cases} 2x+y=6 \\ -8x-4y=-24 \end{cases}$$

Solve the System by Substitution.

Exercise 8

$$\begin{cases} y=x+2 \\ y=-2x+2 \end{cases}$$

? Exercise 9

$\begin{cases} y = \frac{3}{2}x + 1 \\ y = -\frac{1}{2}x + 5 \end{cases}$

? Exercise 10

$\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$

? Exercise 11

$\begin{cases} x + y = 5 \\ 2x - y = 4 \end{cases}$

? Exercise 12

$\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$

? Exercise 13

$\begin{cases} x + y = -4 \\ -x + 2y = -2 \end{cases}$

? Exercise 14

$\begin{cases} -2x + 3y = 3 \\ x + 3y = 12 \end{cases}$

? Exercise 15

$\begin{cases} x = -3y + 4 \\ 2x + 6y = 8 \end{cases}$

? Exercise 16

$\begin{cases} x + 3y = -6 \\ 4y = -\frac{4}{3}x - 8 \end{cases}$

? Exercise 17

$\begin{cases} -3x + 2y = -2 \\ y = -x + 4 \end{cases}$

Solve the System by Elimination.

? Exercise 18

$\begin{cases} y = -2x + 1 \\ 4x + 2y = 8 \end{cases}$

? Exercise 19

$\begin{cases} 4x + 2y = 10 \\ 4x - 2y = -6 \end{cases}$

Exercise 20

$\begin{cases} y = -\frac{1}{2}x + 5 \\ x + 2y = 10 \end{cases}$

Exercise 21

$\begin{cases} y = 2x + 3 \\ 2x - y = -3 \end{cases}$

Everyday Math

Exercise 22

Molly is making strawberry-infused water. For each ounce of strawberry juice, she uses three times as many ounces of water. How many ounces of strawberry juice and how many ounces of water does she need to make 64 ounces of strawberry-infused water?

Exercise 23

Enrique is making a party mix that contains raisins and nuts. For each ounce of nuts, he uses twice the amount of raisins. How many ounces of nuts and how many ounces of raisins does he need to make 24 ounces of party mix?

Exercise 24

Leo is planning his spring flower garden. He wants to plant tulip and daffodil bulbs. He will plant 6 times as many daffodil bulbs as tulip bulbs. If he wants to plant 350 bulbs, how many tulip bulbs and how many daffodil bulbs should he plant?

Writing Exercises

Exercise 25

In a system of linear equations, the two equations have the same slope. Describe the possible solutions to the system.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an ordered pair is a solution of a system of equations.			
solve a system of linear equations by graphing.			
solve a system of equations by substitution.			
solve a system of equations by elimination.			
determine the number of solutions of a linear system.			
solve applications of systems of equations			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 1: Working With Polynomials

In this unit, we build a strong foundation in polynomial expressions and exponents. We'll learn how to add, subtract, multiply, and divide polynomials, and apply exponent rules to simplify expressions. We'll explore special patterns, such as perfect square trinomials and the difference of squares. As we practice simplifying and factoring, we'll see how algebra helps us model patterns and quantities in compact, powerful ways, preparing us for more advanced algebra and applications.

Section 1.1: Add and Subtract Polynomials

Section 1.1P: Practice

Section 1.2: Use Multiplication Properties of Exponents

Section 1.2P: Practice

Section 1.3: Multiply Polynomials

Section 1.3P: Practice

Section 1.4: Special Products

Section 1.4P: Practice

Section 1.5: Divide Monomials

Section 1.5P: Practice

Section 1.6: Divide Polynomials

Section 1.6P: Practice

Section 1.7: Integer Exponents and Scientific Notation

Section 1.7P: Practice

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Section 1.1: Add and Subtract Polynomials

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: $3(2x-5)-4x$.
- Translate into an algebraic expression: "Seven more than twice a number"
- Evaluate $\frac{y}{2}+3$ when $y=10$

Learning Objectives

Motivating Problem

You're organizing care packages for a community event. Each package include school supplies. One supplier charges you $3x+2$ dollars for a package with notebooks and pencils, and another supplier charges $5x-4$ for a package with folders and markers. Here, x, represents the number of items in each package. How can you write an expression that represents the total cost of both packages in terms of x? What does each part of your expression represent?

Fun Fact

The first use of the word *"polynomial"* originates from Greek roots: *"poly,"* meaning "many," and *"nomial,"* meaning "term." But long before the term existed, Babylonian mathematicians in 2000 BCE were already solving geometric problems that required adding and subtracting terms like today's polynomials!

The Goal

In this section, we'll learn how to recognize different types of polynomials—like monomials, binomials, and trinomials—and how to identify their degrees. We'll also practice adding and subtracting them by combining like terms and learn how to evaluate polynomial expressions for specific values.

Identify Polynomials, Monomials, Binomials, and Trinomials

You have learned that a *term* is a constant or the product of a constant and one or more variables. When it is of the form ax^m, where a is a constant and m is a whole number, it is called a *monomial*. Some examples of monomial are $8, -2x^2, 4y^3$, and $11z^7$.

Definition: Monomials

A **monomial** is a term of the form ax^m, where a is a constant and m is a positive whole number.

A monomial, or two or more monomials combined by addition or subtraction, is a polynomial. Some polynomials have special names, based on the number of terms. A monomial is a polynomial with exactly one term. A binomial has exactly two terms, and a trinomial has exactly three terms. There are no special names for polynomials with more than three terms.

Definitions: Polynomials

- **polynomial**—A monomial, or two or more monomials combined by addition or subtraction, is a *polynomial*.
- **monomial**—A polynomial with exactly one term is called a *monomial*.
- **binomial**—A polynomial with exactly two terms is called a *binomial*.
- **trinomial**—A polynomial with exactly three terms is called a *trinomial*.

Here are some examples of polynomials.

Polynomial	$b+1$	$4y^2-7y+2$	$4x^4+x^3+8x^2-9x+1$	
Monomial	14	$8y^2$	$-9x^3y^5$	-13
Binomial	$a+7$	$4b-5$	y^2-16	$3x^3-9x^2$
Trinomial	$x^2-7x+12$	$9y^2+2y-8$	$6m^4-m^3+8m$	z^4+3z^2-1

Notice that every monomial, binomial, and trinomial is also a polynomial. They are just special members of the "family" of polynomials, and so they have special names. We use the words *monomial*, *binomial*, and *trinomial* when referring to these special polynomials and call all the rest *polynomials*.

Example 1

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial.

a. $4y^2-8y-6$
b. $-5a^4b^2$
c. $2x^5-5x^3-3x+4$
d. $13-5m^3$
e. q

Solution

	Polynomial	Number of terms	Type
a.	$4y^2-8y-6$	3	Trinomial
b.	$-5a^4b^2$	1	Monomial
c.	$2x^5-5x^3-9x^2+3x+4$	5	Polynomial
d.	$13-5m^3$	2	Binomial
e.	q	1	Monomial

Try It 1

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial:

a. $5b$
b. $8y^3-7y^2-y-3$
c. $-3x^2-5x+9$
d. $81-4a^2$
e. $-5x^6$

Answer

a. monomial
b. polynomial
c. trinomial
d. binomial
e. monomial

Determine the Degree of Polynomials

The exponents of the variable determine the degree of a polynomial and the degree of its terms. A **monomial** that has no variable, just a constant, is a special case. The degree of a constant is 0, i.e., it has no variable.

Definition: Degree of a Polynomial

- The **degree of a term** is the sum of the exponents of its variables.
- The **degree of a constant** is 0.
- The **degree of a polynomial** is the highest degree of all its terms.

Let's see how this works by looking at several polynomials. We'll take it step by step, starting with monomials and then progressing to polynomials with more terms.

Monomial	14	$8y^2$	$-9x^3y^6$	$-13a$
Degree	0	2	8	1
Binomial	$a+7$	$4b^3-5b$	x^2y^2-16	$3n^3-9n^2$
Degree of each term	1 0	2 1	4 0	3 2
Degree of polynomial	1	2	4	3
Trinomial	$x^2-7x+12$	$9a^2+6ab+b^2$	$6m^4-m^3n^2+8mn^5$	z^4+3z^2-1
Degree of each term	2 1 0	2 2 2	4 5 6	4 2 0
Degree of polynomial	2	2	6	4
Polynomial	$b+1$	$4y^2-7y+2$	$4x^4+x^3+8x^2-9x+1$	
Degree of each term	1 0	2 1 0	4 3 2 1 0	
Degree of polynomial	1	2	4	

A polynomial is in **standard form** when the terms of a polynomial are written in descending order of degrees. Get in the habit of writing the term with the highest degree first.

Example 2

Find the degree of the following polynomials.

a. $10y$
b. $4x^3-7x+5$
c. -15
d. $-8b^2+9b-2$
e. $8xy^2+2y$

Solution

a. The exponent of y is 1, so the degree is 1.
b. The highest exponent among its terms is 3, so the degree is 3.
c. A nonzero constant has degree 0.
d. The highest exponent among its terms is 2, so the degree is 2.
e. The term xy^2 has total degree $1+2=3$, so the polynomial's degree is 3.

Try It 2

Find the degree of the following polynomials:

a. $-15b$
b. $10z^4+4z^2-5$
c. $12c^5d^4+9c^3d^9-7$
d. $3x^2y-4x$
e. -9

Answer

a. 1

b. 4
c. 12
d. 3
e. 0

Add and Subtract Monomials

You have learned how to simplify expressions by combining like terms. Remember, like terms must have the same variables with the same exponent. Since monomials are terms, adding and subtracting monomials is the same as combining like terms. If the monomials are like terms, we just combine them by adding or subtracting the coefficients.

Example 3

a. Add: $25y^2 + 15y^2$
b. Subtract: $16p - (-7p)$

Solution

a. The squared y terms are alike and can be added: $40y^2$
b. Subtracting a negative is the same as adding: $16p - (-7p) = 16p + 7p = 23p$

Try It 3

a. Add: $-15c^2 + 8c^2$
b. Subtract: $8m - (-5m)$

Answer

a. $-7c^2$
b. $13m$

Remember that like terms must have the same variables with the same exponents.

Example 4

Simplify each expression:

a. $c^2 + 7d^2 - 6c^2$
b. $u^2v + 5u^2 - 3v^2$

Solution

a. The squared c terms are alike and can be combined: $-5c^2 + 7d^2$
b. There are no like terms to combine.

Try It 4

Simplify each expression:

a. $8y^2 + 3z^2 - 3y^2$
b. $m^2n^2 - 8m^2 + 4n^2$

Answer

a. $5y^2 + 3z^2$
b. There are no like terms to combine.

Add and Subtract Polynomials

We can think of adding and subtracting polynomials as just adding and subtracting a series of monomials. Look for the like terms—those with the same variables and the same exponent. The Commutative Property allows us to rearrange the terms to put like terms together.

Example 5

Find the sum: $(5y^2 - 3y + 15) + (3y^2 - 4y - 11)$

Solution

Identify like terms.	$(\underline{5y^2} - \underline{3y} + \underline{15}) + (\underline{3y^2} - \underline{4y} - \underline{11})$
Rearrange to get the like terms together.	$\underline{5y^2 + 3y^2} - \underline{3y - 4y} + \underline{15 - 11}$
Combine like terms.	$8y^2 - 7y + 4$

Try It 5

Find the sum: $(7x^2 - 4x + 5) + (x^2 - 7x + 3)$

Answer

$8x^2 - 11x + 8$

Example 6

Find the difference: $(9w^2 - 7w + 5) - (2w^2 - 4)$

Solution

	$(9w^2 - 7w + 5) - (2w^2 - 4)$
Distribute and identify like terms.	$\underline{9w^2} - \underline{7w} + \underline{5} - \underline{2w^2} + \underline{4}$
Rearrange the terms.	$\underline{9w^2 - 2w^2} - \underline{7w} + \underline{5} + \underline{4}$
Combine like terms.	$7w^2 - 7w + 9$

Try It 6

Find the difference: $(9b^2 - 5b - 4) - (3b^2 - 5b - 7)$

Answer

$6b^2 + 3$

Example 7

Find the difference: $(p^2 + q^2) - (p^2 + 10pq - 2q^2)$

Solution

	$(p^2+q^2)-(p^2+10pq-2q^2)$
Distribute.	$p^2+q^2-p^2-10pq+2q^2$
Rearrange the terms, to put like terms together	$p^2-p^2-10pq+q^2+2q^2$
Combine like terms.	$-10pq+3q^2$

? Try It 7

Find the difference: $(a^2+b^2)-(a^2+5ab-6b^2)$

Answer

$7b^2-5ab$

✓ Example 8

Simplify: $(a^3-a^2b)-(ab^2+b^3)+(a^2b+ab^2)$

Solution

	$(a^3-a^2b)-(ab^2+b^3)+(a^2b+ab^2)$
Distribute.	$a^3-a^2b-ab^2-b^3+a^2b+ab^2$
Rearrange the terms, to put like terms together	$a^3-a^2b+a^2b-ab^2+ab^2-b^3$
Combine like terms.	a^3-b^3

? Try It 8

Simplify: $(x^3-x^2y)-(xy^2+y^3)+(x^2y+xy^2)$

Answer

x^3-y^3

Evaluate a Polynomial for a Given Value

We have already learned how to evaluate expressions. Since polynomials are expressions, we'll follow the same procedures to evaluate a **polynomial**. We will substitute the given value for the variable and then simplify using the order of operations.

✓ Example 9

Evaluate $5x^2-8x+4$ when

a. $x=4$
b. $x=-2$
c. $x=0$

Solution

a. $x = 4$	
	$5x^2 - 8x + 4$
Substitute 4 for x.	$5(4)^2 - 8(4) + 4$
Simplify the exponents.	$5 \cdot 16 - 8(4) + 4$
Multiply.	$80 - 32 + 4$
Simplify.	52

b. $x = -2$	
	$5x^2 - 8x + 4$
Substitute -2 for x.	$5(-2)^2 - 8(-2) + 4$
Simplify the exponents.	$5 \cdot 4 - 8(-2) + 4$
Multiply.	$20 + 16 + 4$
Simplify.	40

c. $x = 0$	
	$5x^2 - 8x + 4$
Substitute 0 for x.	$5(0)^2 - 8(0) + 4$
Simplify the exponents.	$5 \cdot 0 - 8(0) + 4$
Multiply.	$0 + 0 + 4$
Simplify.	4

Try It 9

Evaluate: $5z^2 - z - 4$ when

a. $z = -2$
b. $z = 0$
c. $z = 2$

Answer

a. 18
b. -4
c. 14

Example 10

The polynomial $-16t^2 + 250$ gives the height (in feet) of a ball t seconds after it is dropped from a 250-foot-tall building. Find the height of the ball after 2 seconds.

Solution

	$-16t^2 + 250$
Substitute $t = 2$.	$-16(2)^2 + 250$
Simplify	$-16 \cdot 4 + 250$
Simplify	$-64 + 250$
Simplify	186
	After 2 seconds the height of the ball is 186 feet.

Try It 10

The polynomial $-16t^2 + 150$ gives the height (in meters) of a ball t seconds after it is dropped from a 150-foot-tall building. Find the height of the ball after 3 seconds.

Answer

After 3 seconds, the ball's height is 6 meters.

Example 11

The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with sides x feet and sides of height y feet. Find the cost of producing a box with $x = 4$ feet and $y = 6$ feet.

Solution

	$6x^2 + 15xy$
Substitute $x = 4, y = 6$.	$6(4)^2 + 15(4)(6)$
Simplify.	$6 \cdot 16 + 15(4)(6)$
Simplify.	$96 + 360$
Simplify.	456
	The cost of producing the box is $456.

Try It 11

The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with sides x feet and sides of height y feet. Find the cost of producing a box with $x = 5$ feet and $y = 8$ feet.

Answer

The cost of producing the box is $750.

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Section 1.1P: Practice

Practice Makes Progress

In the following exercises, determine if each of the following polynomials is a monomial, binomial, trinomial, or other polynomial.

Exercise 1

$81b^5 - 24b^3 + 1$

Exercise 2

$5c^3 + 11c^2 - c - 8$

Exercise 3

$\frac{14}{15}y + \frac{1}{7}$

Exercise 4

5

Exercise 5

$4y + 17$

In the following exercises, determine the degree of each polynomial.

Exercise 6

$6a^2 + 12a + 14$

Exercise 7

$18xy^2z$

Exercise 8

$5x + 2$

Exercise 9

$y^3 - 8y^2 + 2y - 16$

Exercise 10

-24

In the following exercises, add or subtract the monomials.

Exercise 11

$7x^2 + 5x^2$

Exercise 12

$-12w + 18w$

Exercise 13

$28x - (-12x)$

Exercise 14

$-5b - 17b$

Exercise 15

$12a + 5b - 22a$

Exercise 16

$2a^2 + b^2 - 6a^2$

Exercise 17

$xy^2 - 5x - 5y^2$

Exercise 18

$a^2b - 4a - 5ab^2$

Exercise 19

Subtract $5x^6$ from $-12x^6$

In the following exercises, add or subtract the polynomials.

Exercise 20

$(5y^2 + 12y + 4) + (6y^2 - 8y + 7)$

Exercise 21

$(x^2 + 6x + 8) + (-4x^2 + 11x - 9)$

Exercise 22

$(8x^2 - 5x + 2) + (3x^2 + 3)$

Exercise 23

$(5a^2+8)+(a^2-4a-9)$

Exercise 24

$(4m^2-6m-3)-(2m^2+m-7)$

Exercise 25

$(a^2+8a+5)-(a^2-3a+2)$

Exercise 26

$(12s^2-15s)-(s-9)$

Exercise 27

Subtract $(7w^2-4w+2)$ from $(8w^2-w+6)$

Exercise 28

Find the sum of $(8a^3-8a)$ and $(a^2+6a+12)$

Exercise 29

Find the difference of

(w^2+w-42) and
$(w^2-10w+24)$.

Exercise 30

$(7m^2+mn-8n^2)+(3m^2+2mn)$

Exercise 31

$(u^2-v^2)-(u^2-4uv-3v^2)$

Exercise 32

$(x^3-x^2y)-(4xy^2-y^3)+(3x^2y-xy^2)$

In the following exercise, evaluate the polynomial for the given values.

Exercise 33

Evaluate $8y^2-3y+2$ when:

a. $y=5$
b. $y=-2$
c. $y=0$

Everyday Math

Exercise 34

A manufacturer of stereo sound speakers has found that the revenue received from selling the speakers at a cost of p dollars each is given by the polynomial $-4p^2 + 420p$. Find the revenue received when $p = 60$ dollars.

Exercise 35

The cost to rent a rug cleaner for d days is given by the polynomial $5.50d + 25$. Find the cost to rent the cleaner for 6 days.

Exercise 36

The temperature in degrees Fahrenheit is given by the polynomial $\frac{9}{5}c + 32$ where c represents the temperature in degrees Celsius. Find the temperature in degrees Fahrenheit when $c = 65°$.

Writing Exercises

Exercise 37

Using your own words, explain the difference between a polynomial with five terms and a polynomial with a degree of 5.

Exercise 38

Jonathan thinks that $\frac{1}{3}$ and $\frac{1}{x}$ are both monomials. What is wrong with his reasoning?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify polynomials, monomials, binomials, and trinomials.			
determine the degree of polynomials.			
add and subtract monomials.			
add and subtract polynomials.			
evaluate a polynomial for a given value.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.2: Use Multiplication Properties of Exponents

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: $\frac{3}{4} \cdot \frac{3}{4}$
- Simplify: $(-2)(-2)(-2)$
- Identify the base and exponent of the expression: 5^4

Learning Objectives

Motivating Problem

You're stacking trays in a cafeteria. Each level doubles the trays from the level below. After 3 levels, you have 2^3 trays. What if you had 3 more levels stacked just like that? How would you describe the total number of trays now?

Fun Fact

The use of exponents in algebra dates back to the 9th century, but **René Descartes** introduced the modern notation x^n in the 17th century. Earlier mathematicians, such as **Diophantus**, wrote powers using abbreviations or marks above letters, making algebra much more challenging to read.

The Goal

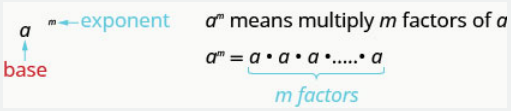
In this section, we'll explore how exponents behave when we multiply powers with the same base. We'll learn to simplify expressions using the product rule for exponents and rewrite exponential expressions more efficiently.

Simplify Expressions with Exponents

Remember that an exponent indicates repeated multiplication of the same quantity. For example, 2^4 means the product of 4 factors of 2, so 2^4 means $2 \cdot 2 \cdot 2 \cdot 2$.

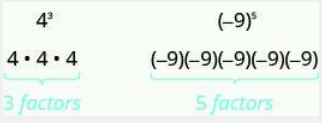
Let's review the vocabulary for expressions with exponents.

EXPONENTIAL NOTATION



This is read a to the m^{th} power.

In the expression a^m, the *exponent* m tells us how many times we use the *base* a as a factor.



Before we begin working with variable expressions containing exponents, let's simplify a few expressions involving only numbers.

Example 1

Simplify:

a. 4^3
b. 7^1
c. $\left(\frac{5}{6}\right)^2$

d. $(0.63)^2$

Solution

a.		4^3
	Multiply three factors of 4.	$4 \cdot 4 \cdot 4$
	Simplify.	64
b.		7^1
	Multiply one factor of 7.	7
c.		$\left(\frac{5}{6}\right)^2$
	Multiply two factors.	$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$
	Simplify.	$\frac{25}{36}$
d.		$(0.63)^2$
	Multiply two factors.	$(0.63)(0.63)$
	Simplify.	0.3969

Try It 1

Simplify:

a. 6^3
b. 15^1
c. $\left(\frac{3}{7}\right)^2$
d. $(0.43)^2$

Answer

a. 216
b. 15
c. $\frac{9}{49}$
d. 0.1849

Example 2

Simplify:

a. $(-5)^4$
b. -5^4

Solution

		$(-5)^4$
a.	Multiply four factors of -5	$(-5)(-5)(-5)(-5)$
	Simplify.	625
		-5^4
b.	Multiply four factors of 5.	$-(5 \cdot 5 \cdot 5 \cdot 5)$
	Simplify.	-625

Notice the similarities and differences in the two parts of the previous example! Why are the answers different? As we follow the order of operations in part 1, the parentheses indicate that we should raise (-5) to the 4^{th} power. In part 2, we raise just the 5 to the 4^{th} power and then take the opposite.

Try It 2

Simplify:

a. $(-3)^6$
b. -3^6

Answer

a. 729
b. −729

Simplify Expressions Using the Product Property for Exponents

You have seen that when you combine like terms by adding and subtracting, you need to have the same base with the same exponent. But when you multiply and divide, the exponents may be different, and sometimes the bases may be different, too.

We'll derive the properties of exponents by looking for patterns in several examples.

First, we will look at an example that leads to the Product Property.

	$x^2 \cdot x^3$
What does this mean? How many factors altogether?	$\underbrace{x \cdot x}_{2 \text{ factors}} \cdot \underbrace{x \cdot x \cdot x}_{3 \text{ factors}}$ 5 factors
So, we have	x^5
Notice that 5 is the sum of the exponents, 2 and 3.	$x^2 \cdot x^3$ is x^{2+3} or x^5

We write:

$$x^2 \cdot x^3$$
$$x^{2+3}$$
$$x^5$$

The base stayed the same, and we added the exponents. This leads to the **Product Property for Exponents**.

PRODUCT PROPERTY FOR EXPONENTS

If a is a real number, and m and n are counting numbers, then

$$a^m \cdot a^n = a^{m+n}$$

To multiply with like bases, add the exponents.

An example with numbers helps to verify this property.

$$\begin{aligned} 2^3 \cdot 2^2 &\stackrel{?}{=} 2^{2+3} \\ 4 \cdot 8 &\stackrel{?}{=} 2^5 \\ 32 &= 32\checkmark \end{aligned}$$

Example 3

Simplify: $y^5 \cdot y^6$

Solution

	$y^5 \cdot y^6$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	y^{5+6}
Simplify.	y^{11}

Try It 3

Simplify: $b^9 \cdot b^8$

Answer

b^{17}

Example 4

Simplify:

a. $a^7 \cdot a$
b. $x^{27} \cdot x^{13}$

Solution

a.

	$a^7 \cdot a$
Rewrite, $a = a^1$	$a^7 \cdot a^1$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	a^{7+1}
Simplify.	a^8

b.

	$x^{27} \cdot x^{13}$
Notice that the bases are the same, so add the exponents.	x^{27+13}
Simplify.	x^{40}

Try It 4

Simplify:

a. $p^5 \cdot p$
b. $y^{14} \cdot y^{29}$

Answer

a. p^6
b. y^{43}

We can extend the Product Property for Exponents to more than two factors.

Example 5

Simplify: $d^4 \cdot d^5 \cdot d^2$

Solution

	$d^4 \cdot d^5 \cdot d^2$
Add the exponents, since bases are the same.	d^{4+5+2}
Simplify.	d^{11}

Try It 5

Simplify: $x^6 \cdot x^4 \cdot x^8$

Answer

x^{18}

Simplify Expressions Using the Power Property for Exponents

Now let's look at an exponential expression that contains a power raised to a power. Try to identify a general property.

	$(x^2)^3$
What does this mean? How many factors altogether?	$x^2 \cdot x^2 \cdot x^2$ $x \cdot x \cdot x \cdot x \cdot x \cdot x$ 2 factors 2 factors 2 factors 6 factors
So we have	x^6
Notice that 6 is the product of the exponents, 2 and 3.	$(x^2)^3$ is $x^{2 \cdot 3}$ or x^6

We write:

$$(x^2)^3$$
$$x^{2 \cdot 3}$$
$$x^6$$

We multiplied the exponents. This leads to the **Power Property for Exponents.**

POWER PROPERTY FOR EXPONENTS

If a is a real number, and m and n are whole numbers, then

$$(a^m)^n = a^{m \cdot n}$$

To raise a power to a power, multiply the exponents.

An example with numbers helps to verify this property.

$$\begin{aligned} (3^2)^3 &\stackrel{?}{=} 3^{2 \cdot 3} \\ (9)^3 &\stackrel{?}{=} 3^6 \\ 729 &= 729 \checkmark \end{aligned}$$

Example 6

Simplify:

a. $(y^5)^9$
b. $(4^4)^7$

Solution

a.

	$(y^5)^9$
Use the power property, $(a^m)^n = a^{m \cdot n}$.	$y^{5 \cdot 9}$
Simplify.	y^{45}

b.

	$(4^4)^7$
Use the power property.	$4^{4 \cdot 7}$
Simplify.	4^{28}

Try It 6

Simplify:

a. $(b^7)^5$
b. $(5^4)^3$

Answer

a. b^{35}
b. 5^{12}

Simplify Expressions Using the Product to a Power Property

We will now look at an expression containing a product that is raised to a power. Can you find this pattern?

What does this mean?	$(2x)^3$
We group the like factors together.	$2x \cdot 2x \cdot 2x$
How many factors of 2 and of x?	$2 \cdot 2 \cdot 2 \cdot x^3$

Notice that each factor was raised to the power and $(2x)^3$ is $2^3 \cdot x^3$

We write: $(2x)^3$
$2^3 \cdot x^3$

The exponent applies to each of the factors! This leads to the **Product to Power Property for Exponents.**

PRODUCT TO POWER PROPERTY FOR EXPONENTS

If a and b are real numbers and m is a whole number, then

$$(ab)^m = a^m b^m$$

To raise a product to a power, raise each factor to that power.

An example with numbers helps to verify this property:

$$\begin{aligned}(2\cdot 3)^2 &\stackrel{?}{=} 2^2\cdot 3^2\\ 6^2 &\stackrel{?}{=} 4\cdot 9\\ 36 &= 36\checkmark\end{aligned}$$

Example 7

Simplify:

a. $(-9d)^2$
b. $(3mn)^3$.

Solution

a.

	$(-9d)^2$
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(-9)^2 d^2$
Simplify.	$81d^2$

b.

	$(3mn)^3$.
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(3)^3 m^3 n^3$
Simplify.	$27m^3n^3$

Try It 7

Simplify:

a. $(5wx)^3$
b. $(-3y)^3$

Answer

a. $125w^3x^3$
b. $-27y^3$

Simplify Expressions by Applying Several Properties

We now have three properties for multiplying expressions with exponents. Let's summarize them, and then we'll do some examples that use more than one of the properties.

PROPERTIES OF EXPONENTS

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}
Power Property	$(a^m)^n$	$=$	a^{mn}
Product to a Power	$(ab)^m$	$=$	$a^m b^m$

All exponent properties hold true for any real numbers m and n. Right now, we only use whole-number exponents.

Example 8

Simplify:

a. $(y^3)^6 (y^5)^4$
b. $(-6x^4y^5)^2$
c. $(5m)^2 (3m^3)$
d. $(3x^2y)^4 (2xy^2)^3$

Solution

a.

	$(y^3)^6 (y^5)^4$
Power Property	$y^{18} y^{20}$
Add exponents	y^{38}

b.

	$(-6x^4y^5)^2$
Product-to-a-power Property	$(-6)^2 (x^4)^2 (y^5)^2$
Simplify	$36 x^8 y^{10}$

c.

	$(5m)^2 (3m^3)$
Raise $5m$ to the 2nd power	$5^2 m^2 \cdot 3 m^3$
Multiply constants and add exponents	$75 m^5$

d.

	$(3x^2y)^4 (2xy^2)^3$
Product-to-a-power Property	$(3^4 x^8 y^4) (2^3 x^3 y^6)$
Multiply constants and add exponents	$648 x^{11} y^{10}$

Try It 8

Simplify:

a. $(a^4)^5 (a^7)^4$
b. $(-2c^4d^2)^3$
c. $(5n)^2 (3n^{10})$
d. $(c^4d^2)^5 (3cd^5)^4$

Answer

a. a^{48}
b. $-8c^{12}d^6$
c. $75n^{12}$
d. $81c^{24}d^{30}$

Multiply Monomials

Since a **monomial** is an algebraic expression, we can use the properties of exponents to multiply them.

Example 9

a. Multiply: $(3x^2) (-4x^3)$
b. Multiply: $(\frac{5}{6}x^3y) (12xy^2)$

Solution

a.

	$(3x^2) (-4x^3)$
Use the Commutative Property to rearrange the terms.	$3 \cdot (-4) \cdot x^2 \cdot x^3$
Multiply.	$-12x^5$

b.

	$(\frac{5}{6}x^3y) (12xy^2)$
Use the Commutative Property to rearrange the terms.	$\frac{5}{6} \cdot 12 \cdot x^3 \cdot x \cdot y \cdot y^2$
Multiply.	$10x^4y^3$

? Try It 9

a. Multiply: $(5y^7)(-7y^4)$

b. Multiply: $\left(\frac{2}{5}a^4b^3\right)(15ab^3)$

Answer

a. $-35y^{11}$

b. $6a^5b^6$

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Section 1.2P: Practice

Practice Makes Progress

In the following exercises, simplify each expression.

Exercise 1

$-\left(\frac{2}{3}\right)^2$

Exercise 2

$\left(-\frac{2}{3}\right)^2$

Exercise 3

-0.1^4

Exercise 4

$(-0.1)^4$

Exercise 5

$x^4 \cdot x^2$

Exercise 6

$3^{10} \cdot 3^6$

Exercise 7

$5 \cdot 5^4$

Exercise 8

$y \cdot y^3 \cdot y^5$

Exercise 9

$n^y \cdot n^2$

Exercise 10

$(b^2)^7$

Exercise 11

$(3^8)^2$

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? Exercise 12

$(x^2)^y$

? Exercise 13

$(7^a)^b$

? Exercise 14

$(-7n)^3$

? Exercise 15

$(3xyz)^4$

? Exercise 16

$(w^4)^3 \cdot (w^5)^2$

? Exercise 17

$(2xy^4)^5$

? Exercise 18

$(2r^2)^3(4r)^2$

? Exercise 19

$(3x^3)^3(x^5)^4$

? Exercise 20

$(6y^7)(-3y^4)$

? Exercise 21

$(-6c^4)(-12c)$

? Exercise 22

$\left(\frac{4}{7}rs^2\right)(14rs^3)$

? Exercise 23

$\left(\frac{2}{3}x^2y\right)\left(\frac{3}{4}xy^2\right)$

? Exercise 24

$(x^2)^4 \cdot (x^3)^2$

Exercise 25

$(a^2)^6 \cdot (a^3)^8$

Exercise 26

$(2m^6)^3$

Exercise 27

$(10x^2y)^3$

Exercise 28

$(-2a^3b^2)^4$

Exercise 29

$\left(\frac{2}{3}x^2y\right)^3$

Exercise 30

$(8a^3)^2(2a)^4$

Exercise 31

$(10p^4)^3(5p^6)^2$

Exercise 32

$\left(\frac{1}{2}x^2y^3\right)^4(4x^5y^3)^2$

Exercise 33

$(3m^2n)^2(2mn^5)^4$

Everyday Math

Exercise 34

Kate emails a flyer to ten of her friends and tells them to forward it to ten of their friends, and so on. How many people will receive the email on the sixth round?

Writing Exercise

Exercise 35

Jorge thinks $\left(\frac{1}{2}\right)^2$ is 1. What is wrong with his reasoning?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with exponents.			
simplify expressions using the Product Property for Exponents.			
simplify expressions using the Power Property for Exponents.			
simplify expressions using the Product to a Power Property.			
simplify expressions by applying several properties.			
multiply monomials.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.3: Multiply Polynomials

Be Prepared

We will rely heavily on these skills throughout this section.

- Distribute: $2(x+3)$
- Combine like terms: $x^2+9x+7x+63$
- Identify each term's degree: $2x^3+3x^2+7$

Learning Objectives

Motivating Problem

A rectangular garden has a length of $x+2$ meters and a width of $x+3$ meters. What expression represents the area of the garden? How can you multiply binomials to find the result?

Fun Fact

The ancient Indian mathematician **Bhāskara II** used methods for multiplying expressions similar to those for polynomials in the 1100s, centuries before symbolic algebra was formalized in Europe. His work in expanding binomial products helped lay the groundwork for the algebraic rules we use today!

The Goal

In this section, we'll learn how to multiply polynomials using strategies such as the distributive property and the FOIL method. We'll practice multiplying monomials, binomials, and larger expressions, and simplify the results by combining like terms.

Multiply a Polynomial by a Monomial

We have used the Distributive Property to simplify expressions like $2(x-3)$. You multiplied both terms in the parentheses, x and 3, by 2, to get $2x-6$. With this chapter's new vocabulary, you can say you were multiplying a binomial, $x-3$, by a monomial, 2.

Multiplying a **binomial** by a **monomial** is nothing new for you! Here's an example:

Example 1

Use the Distributive Property to multiply:

a. $4(x+3)$
b. $y(y-2)$
c. $7x(2x+y)$
d. $2x^3\,(x^2-8x+1)$
e. $(x+3)p$

Solution

a.

	$4(x+3)$
Distribute.	$4\cdot x+4\cdot 3$
Simplify.	$4x+12$

b.

	$y(y-2)$
Distribute.	$y \cdot y - y \cdot 2$
Simplify.	$y^2 - 2y$

c.

	$7x(2x+y)$
Distribute.	$7x \cdot 2x + 7x \cdot y$
Simplify.	$14x^2 + 7xy$

d.

	$2x^3(x^2 - 8x + 1)$
Distribute.	$2x^3 \cdot x^2 + (2x^3) \cdot (-8x) + (2x^3) \cdot 1$
Simplify.	$2x^5 - 16x^4 + 2x^3$

e.

The monomial is the second factor.	$(x+3)p$
Distribute.	$x \cdot p + 3 \cdot p$
Simplify.	$xp + 3p$

? Try It 1

Use the Distributive Property to multiply:

a. $5(x+7)$
b. $d(d-11)$
c. $2p(6p+r)$
d. $-6a^3\,(3a^2 - 2a + 6)$
e. $(a+4)2p$

Answer

a. $5x + 35$
b. $d^2 - 11d$
c. $12p^2 + 2pr$
d. $-18a^5 + 12a^4 - 36a^3$
e. $2ap + 8p$

Multiply a Binomial by a Binomial

Just like there are different ways to represent multiplication of numbers, there are several methods that can be used to multiply a **binomial** times a binomial. We will start by using the Distributive Property.

Multiply a Binomial by a Binomial Using the Distributive Property

Look at Example *Section*1.3.16, where we multiplied a binomial by a **monomial**.

Instructions	Expression
Starting Expressions	$(x+3)p$
We distributed the p to get:	$xp+3p$
What if we have $(x+7)$ instead of p?	$(x+3)(x+7)$
Distribute $(x+7)$.	$x(x+7)+3(x+7)$
Distribute again.	$x^2+7x+3x+21$
Combine like terms.	$x^2+10x+21$

Notice that before combining like terms, you had four terms. You multiplied the two terms of the first binomial by the two terms of the second binomial—four multiplications.

Example 2

Expand each expression:

a. $(y+5)(y+8)$
b. $(2y+5)(3y+4)$
c. $(4y+3)(2y-5)$
d. $(x-2)(x-y)$

Solution

a.

	$(y+5)(y+8)$
Distribute $(y+8)$.	$y(y+8)+5(y+8)$
Distribute again	$y^2+8y+5y+40$
Combine like terms.	$y^2+13y+40$

b.

	$(2y+5)(3y+4)$
Distribute $(3y+4)$.	$2y(3y+4)+5(3y+4)$
Distribute again	$6y^2+8y+15y+20$
Combine like terms.	$6y^2+23y+20$

c.

	$(4y+3)(2y-5)$
Distribute.	$4y(2y-5)+3(2y-5)$
Distribute again.	$8y^2-20y+6y-15$
Combine like terms.	$8y^2-14y-15$

d.

	$(x-2)(x-y)$
Distribute.	$x(x-y)-2(x-y)$
Distribute again.	$x^2-xy-2x+2y$
There are no like terms to combine.	

Try It 2

Expand each expression:

1. $(x+8)(x+9)$
2. $(3b+5)(4b+6)$
3. $(5y+2)(6y-3)$
4. $(a+7)(a-b)$

Answer

1. $x^2+17x+72$
2. $12b^2+38b+30$
3. $30y^2-3y-6$
4. $a^2-ab+7a-7b$

Multiply a Binomial by a Binomial Using the FOIL Method

Remember that when you multiply a binomial by a binomial, you get four terms. Sometimes you can combine like terms to get a **trinomial**, but sometimes, like in the last example, there are no like terms to combine.

Let's review part e of the last example again, paying particular attention to how we derived the four terms.

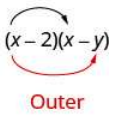
$$(x-2)(x-y)$$
$$x^2-xy-2x+2y$$

Where did the first term, x^2, come from?

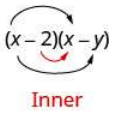
It is the product of x and x, the *first* terms in $(x-2)$ and $(x-y)$.



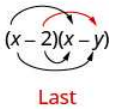
The next term, $-xy$, is the product of x and $-y$, the two *outer* terms.



The third term, $-2x$, is the product of -2 and x, the two *inner* terms.



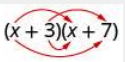
And the last term, $+2y$, came from multiplying the two *last* terms, -2 and $-y$.



We abbreviate "First, Outer, Inner, Last" as FOIL. The letters stand for '**F**irst, **O**uter, **I**nner, **L**ast'. The word FOIL is easy to remember and ensures we find all four products.

$$\begin{array}{c}(x-2)(x-y)\\ x^2-xy-2x+2y\\ F\quad O\quad I\quad L\end{array}$$

Let's look at (x+3)(x+7).

Distributive Property	FOIL
$(x+3)(x+7)$	
$x(x+7)+3(x+7)$	
$x^2+7x+3x+21$ $F\quad O\quad I\quad L$	$x^2+7x+3x+21$ $F\quad O\quad I\quad L$
$x^2+10x+21$	$x^2+10x+21$

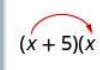
Notice how the terms in the third line fit the FOIL pattern.

Now, we will work through an example that uses the FOIL pattern to multiply two binomials.

Example 3

Multiply using the FOIL method: $(x+5)(x+9)$

Solution

Step 1. Multiply the *First* terms.	$(x+5)(x+9)$ 	$x^2+\_+\_+\_$ $F\quad O\quad I\quad L$
Step 2. Multiply the *Outer* terms.	$(x+5)(x+9)$ 	$x^2+9x+\_+\_$ $F\quad O\quad I\quad L$

Step 3. Multiply the *Inner* terms.	$(x+5)(x+9)$	$x^2+9x+5x+\_$ F O I L
Step 4. Multiply the *Last* terms.	$(x+5)(x+9)$	$x^2+9x+5x+45$ F O I L
Step 5. Combine like terms, when possible.		$x^2+14x+45$

Try It 3

Multiply using the FOIL method: $(y+6)(y+8)$

Answer

$y^2+14y+48$

We summarize the steps of the FOIL method below. The FOIL method only applies to multiplying binomials, not other polynomials!

MULTIPLY TWO BINOMIALS USING THE FOIL METHOD

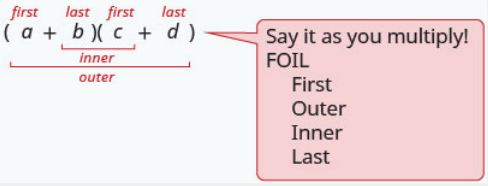
Step 1. Multiply the *First* terms.

Step 2. Multiply the *Outer* terms.

Step 3. Multiply the *Inner* terms.

Step 4. Multiply the *Last* terms.

Step 5. Combine like terms, when possible.



When you multiply by the FOIL method, drawing the lines will help your brain focus on the pattern and make it easier to apply.

Example 4

Expand each using the FOIL method:

a. $(y-7)(y+4)$

b. $(4x+3)(2x-5)$

Solution

a.

		$(y-7)(y+4)$
Multiply the *First* terms.	$(y-7)(y+4)$	$y^2+\_+\_+\_$ F O I L
Multiply the *Outer* terms.	$(y-7)(y+4)$	$y^2+4y+\_+\_$ F O I L
Multiply the *Inner* terms.	$(y-7)(y+4)$	$y^2+4y-7y+\_$ F O I L
Multiply the *Last* terms.	$(y-7)(y+4)$	$y^2+4y-7y-28$ F O I L
Combine like terms.		$y^2-3y-28$

b.

	$(4x-3)(2x-5)$ 
Multiply the *First* terms, $4x \cdot 2x$.	$8x^2 + \_ + \_ + \_$ (F O I L)
Multiply the *Outer* terms, $4x \cdot (-5)$.	$8x^2 - 20x + \_ + \_$ (F O I L)
Multiply the *Inner* terms, $3 \cdot 2x$.	$8x^2 - 2x + 6x + \_$ (F O I L)
Multiply the *Last* terms, $3 \cdot (-5)$.	$8x^2 - 20x + 6x - 15$ (F O I L)
Combine like terms.	$8x^2 - 14x - 15$

Try It 4

Expand each using the FOIL method:

1. (b−3)(b+6)
2. $(4y+5)(4y-10)$

Answer

1. $b^2+3b-18$
2. $16y^2-20y-50$

The final products in the last four examples were trinomials because we could combine the two middle terms. This is not always the case.

Example 5

Multiply: $(3x-y)(2x-5)$.

Solution

	$(3x-y)(2x-5)$
	
Multiply the *First*.	$6x^2 + \_ + \_ + \_$ (F O I L)
Multiply the *Outer*.	$6x^2 - 15x + \_ + \_$ (F O I L)
Multiply the *Inner*.	$6x^2 - 15x - 2xy + \_$ (F O I L)
Multiply the *Last*.	$6x^2 - 15x - 2xy + 5y$ (F O I L)
Combine like terms—there are none.	$6x^2-15x-2xy+5y$

Try It 5

Multiply: (10c−d)(c−6).

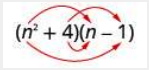
Answer

$10c^2 - 60c - cd + 6d$

Example 6

Multiply: $(n^2 + 4)(n - 1)$

Solution

	$(n^2 + 4)(n - 1)$
	
Multiply the *First*.	$n^3 + \_ + \_ + \_$ (F O I L)
Multiply the *Outer*.	$n^3 - n^2 + \_ + \_$ (F O I L)
Multiply the *Inner*.	$n^3 - n^2 + 4n + \_$ (F O I L)
Multiply the *Last*.	$n^3 - n^2 + 4n - 4$ (F O I L)
Combine like terms—there are none.	$n^3 - n^2 + 4n - 4$

Try It 6

Multiply: $(x^2 + 6)(x - 8)$

Answer

$x^3 - 8x^2 + 6x - 48$

Example 7

Multiply: $(3pq + 5)(6pq - 11)$

Solution

	$(3pq + 5)(6pq - 11)$	
Multiply the *First*.	$18p^2q^2 + \_ + \_ + \_$ (F O I L)	
Multiply the *Outer*.	$18p^2q^2 - 33pq + \_ + \_$ (F O I L)	
Multiply the *Inner*.	$18p^2q^2 - 33pq + 30pq + \_$ (F O I L)	
Multiply the *Last*.	$18p^2q^2 - 33pq + 30pq - 55$ (F O I L)	
Combine like terms—there are none.	$18p^2q^2 - 3pq - 55$	

Try It 7

Multiply: $(2ab + 5)(4ab - 4)$

Answer

$8a^2b^2 + 12ab - 20$

Multiply a Trinomial by a Binomial

We have multiplied monomials by monomials, monomials by polynomials, and binomials by binomials. Now we're ready to multiply a **trinomial** by a **binomial**. Remember, FOIL will not work in this case, but we can use either the Distributive Property or the Vertical Method. We begin by examining an example that utilizes the Distributive Property.

Example 8

Multiply using the Distributive Property: $(b+3)\left(2b^2-5b+8\right)$

Solution

	
Distribute.	
Multiply.	$2b^3-5b^2+8b+6b^2-15b+24$
Combine like terms.	$2b^3+b^2-7b+24$

Try It 8

Multiply using the Distributive Property:

1. $(y-3)\left(y^2-5y+2\right)$
2. $(x+4)\left(2x^2-3x+5\right)$

Answer

1. $y^3-8y^2+17y-6$
2. $2x^3+5x^2-7x+20$

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Section 1.3P: Practice

Practice Makes Progress

In the following exercises, multiply and simplify the expressions.

Exercise 1

$2(x-7)$

Exercise 2

$q(q+5)$

Exercise 3

$-x(x-10)$

Exercise 4

$6r(4r+s)$

Exercise 5

$-8x(x^2+2x-15)$

Exercise 6

$5q^3(q^2-2q+6)$

Exercise 7

$-4z^2(3z^2+12z-1)$

Exercise 8

$(w-6)\cdot 8$

Exercise 9

$-8(z-5)$

Exercise 10

$u(u+5)$

Exercise 11

$5p(11p-5q)$

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? Exercise 12

$3(v^2 + 10v + 25)$

? Exercise 13

$2n(4n^2 - 4n + 1)$

? Exercise 14

$(2y - 9)y$

? Exercise 15

$(x + 8)(x + 3)$

? Exercise 16

$(y - 6)(y - 2)$

? Exercise 17

$(6p + 5)(p + 1)$

? Exercise 18

$(4z - y)(z - 6)$

? Exercise 19

$(6pq - 3)(4pq - 5)$

? Exercise 20

$(y + 8)(4y^2 + y - 7)$

? Exercise 21

$5q(3q^2 - 6q + 11)$

? Exercise 22

$(4m^2 - 3m - 7)m^2$

? Exercise 23

$(5a + 7b)(5a + 7b)$

? Exercise 24

$(4y + 12z)(4y - 12z)$

Everyday Math

Exercise 25

Someone wanting to multiply 18 by 17 can use tools from this section:

a. Multiply $(20-2)(20-3)$.
b. Find $18 \cdot 17$.
c. Which way did you find easier? Why?

Writing Exercises

Exercise 26

Which method do you prefer to use when multiplying a trinomial by a binomial: the Distributive Property or the Vertical Method? Why?

Exercise 27

Multiply the following:
$(m-3)(m+3)$, $(n-10)(n+10)$, $(p-8)(p+8)$
Explain the pattern that you see in your answers.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply a polynomial by a monomial.			
multiply a binomial by a binomial.			
multiply a trinomial by a binomial.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.4: Special Products

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $(-3)^2$ and -3^2
- Expand and simplify $(x-3)(x+3)$
- Combine like terms $7x-2x^2-4x+3x^2$

Learning Objectives

Motivating Problem

A square patio has a side length of $x+5$ feet. What's the area of the patio? Can you find a quicker way to expand $(x+5)^2$ without writing it all out?

Fun Fact

The identity $a^2-b^2=(a+b)(a-b)$ was known to ancient Greek mathematicians, such as **Euclid**, who used it geometrically to calculate areas. Today, it remains a favorite trick in everything from simplifying algebra to factoring cryptographic equations.

The Goal

In this section, we'll learn how to recognize and apply special product patterns, such as the square of a binomial and the difference of squares. These shortcuts enable us to multiply certain polynomials more efficiently and help identify structure in algebraic expressions.

Square a Binomial Using the Binomial Squares Pattern

Mathematicians often seek patterns that simplify their work. A good example of this is squaring binomials. While you can always get the product by writing the **binomial** twice and using the methods of the last section, there is less work to do if you learn to use a pattern.

Let's start by looking at $(x+9)^2$.	
What does this mean?	$(x+9)^2$
It means to multiply $(x+9)$ by itself.	$(x+9)(x+9)$
Then, using FOIL, we get:	$x^2+9x+9x+81$
Combining like terms gives:	$x^2+18x+81$
Here's another one:	$(y-7)^2$
Multiply $(y-7)$ by itself.	$(y-7)(y-7)$
Using FOIL, we get:	$y^2-7y-7y+49$
And combining like terms:	$y^2-14y+49$
And one more:	$(2x+3)^2$
Multiply.	$(2x+3)(2x+3)$
Use FOIL:	$4x+6x+6x+9$
Combine like terms.	$4x^2+12x+9$

Look at these results. Do you see any patterns?

What about the number of terms? In each example, we squared a binomial and the result was a **trinomial**.

$$(a+b)^2=\_\_\_+\_\_\_+\_\_\_$$

Now look at the ***first term*** in each result. Where did it come from?

$(x+9)^2$	$(y-7)^2$	$(2x+3)^2$
$(x+9)(x+9)$	$(y-7)(y-7)$	$(2x+3)(2x+3)$
$x^2+9x+9x+81$	$y^2-7y-7y+49$	$4x^2+6x+6x+9$
$x^2+18x+81$	$y^2-14y+49$	$4x^2+12x+9$

The first term is the product of the first terms of each binomial. Since the binomials are identical, it is just the square of the first term!

$$(a+b)^2 = a^2 + \_\_\_\_ + \_\_\_\_$$

To get the ***first term*** of the product, ***square the first term***.

Where did the ***last term*** come from? Examine the examples and identify the pattern.

The last term is the product of the last terms, which is the square of the last term.

$$(a+b)^2 = \_\_\_\_ + \_\_\_\_ + b^2$$

*To get the **last term** of the product, **square the last term**.*

Finally, look at the ***middle term***. Notice it came from adding the "outer" and the "inner" terms, which are both the same! So the middle term is double the product of the two terms of the binomial.

$$(a+b)^2 = \_\_\_\_ + 2ab + \_\_\_\_$$

$$(a+b)^2 = \_\_\_\_ - 2ab + \_\_\_\_$$

*To get the **middle term** of the product, **multiply the terms and double their product**.*

Putting it all together:

Binomial Square Pattern

If a and b are real numbers, the binomial square pattern is

$$\underbrace{(a+b)^2}_{(\text{binomial})^2} = \underbrace{a^2}_{(\text{first term})^2} + \underbrace{2ab}_{2\times(\text{product of terms})} + \underbrace{a^2}_{(\text{last term})^2}$$

Applying this to two binomial types:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$

To square a binomial:

1. square the first term
2. square the last term
3. double their product

A number example helps verify the pattern.

	$(10+4)^2$
Square the first term.	$10^2 + \_\_\_\_ + \_\_\_\_$
Square the last term.	$10^2 + \_\_\_\_ + 4^2$
Double their product.	$10^2 + 2 \cdot 10 \cdot 4 + 4^2$
Simplify.	$100 + 80 + 16$
Simplify.	196

To multiply $(10+4)^2$, usually you'd follow the Order of Operations.

$$(10+4)^2$$
$$(14)^2$$
$$196$$

The pattern works!

Example 1

Expand by multiplying:

a. $(x+5)^2$
b. $(4x+6)^2$
c. $(2x-3y)^2$
d. $(4u^3+1)^2$

Solution:

a.

	$(x+5)^2$
Square the first term.	$x^2+\_\_\_+\_\_\_$
Square the last term.	$x^2+\_\_\_+5^2$
Double the product.	$x^2+2\cdot x\cdot 5+5^2$
Simplify.	$x^2+10x+25$

b.

	$(4x+6)^2$
Use the pattern.	$(4x)^2+2\cdot 4x\cdot 6+6^2$
Simplify.	$16x^2+48x+36$

c.

	$(2x-3y)^2$
Use the pattern.	$(2x)^2-2\cdot 2x\cdot 3y+(3y)^2$
Simplify.	$4x^2-12xy+9y^2$

d.

	$(4u^3+1)^2$
Use the pattern.	$(4u^3)^2+2\cdot 4u^3\cdot 1+(1)^2$
Simplify.	$16u^6+8u^3+1$

? Try It! 1

Multiply:

a. $(x+9)^2$
b. $(6z+3)^2$
c. $(2c-d)^2$
d. $(3y^3+2)^2$

Answer

a. $x^2+18x+81$
b. $36z^2+36z+9$
c. $4c^2-4cd+d^2$
d. $9y^6+12y^3+4$

Multiply Conjugates Using the Product of Conjugates Pattern

We just saw a pattern for squaring binomials that we can use to make multiplying some binomials easier. Similarly, there is a pattern for another product of binomials. However, before we proceed, we need to introduce some key vocabulary.

What do you notice about these pairs of binomials?

$$(x-9)(x+9) \qquad (y-8)(y+8) \qquad (2x-5)(2x+5)$$

Look at the first term of each **binomial** in each pair.

$$(x-9)(x+9) \qquad (y-8)(y+8) \qquad (2x-5)(2x+5)$$

Notice that the first terms are the same in each pair.

Look at the last terms of each binomial in each pair.

$$(x-9)(x+9) \qquad (y-8)(y+8) \qquad (2x-5)(2x+5)$$

Notice that the last terms are the same in each pair.

Notice how each pair has one sum and one difference.

$$\underset{\text{Difference}}{(x-9)}\underset{\text{Sum}}{(x+9)} \qquad \underset{\text{Difference}}{(y-8)}\underset{\text{Sum}}{(y+8)} \qquad \underset{\text{Difference}}{(2x-5)}\underset{\text{Sum}}{(2x+5)}$$

A pair of binomials that each have the same first term and the same last term, but one is a sum and one is a difference, has a special name. It is called a *conjugate pair* and is of the form $(a-b)$, $(a+b)$.

Definition: Conjugate Pair

A **conjugate pair** is two binomials of the form

$$(a-b),(a+b)$$

The pair of binomials each has the same first term and the same last term, but one binomial is a sum and the other is a difference.

There is a nice pattern for finding the product of conjugates. You could, of course, simply FOIL to get the product, but using the pattern makes your work easier.

Let's look for the pattern by using FOIL to multiply some conjugate pairs.

$(x-9)(x+9)$	$(y-8)(y+8)$	$(2x-5)(2x+5)$
$x^2+9x-9x-81$	$y^2+8y-8y-64$	$4x^2+10x-10x-25$
x^2-81	y^2-64	$4x^2-25$

Each **first term** is the product of the first terms of the binomials, and since they are identical, it is the square of the first term.

$$(a+b)(a-b) = a^2 - \_\_\_\_$$

To get the first term, square the first term.

The **last term** came from multiplying the last terms, the square of the last term.

$$(a+b)(a-b) = a^2 - b^2$$

To get the last term, square the last term.

What do you observe about the products?

The product of the two binomials is also a binomial! Most of the products resulting from FOIL have been trinomials.

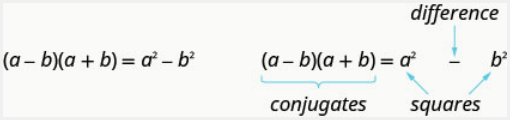
Why is there no middle term? Notice that the two middle terms you get from FOIL combine to 0 in every case, resulting from one addition and one subtraction.

The product of conjugates is always of the form $a^2 - b^2$. This is referred to as a difference of squares.

This leads to the pattern:

PRODUCT OF CONJUGATES PATTERN

If a and b are real numbers,



The product is called a **difference of squares**.

To multiply conjugates, square the first term, square the last term, and write the product as a difference of squares.

Let's test this pattern with a numerical example.

	$(10-2)(10+2)$
It is the product of conjugates, so the result will be the difference of two squares.	$\_\_\_\_ - \_\_\_\_$
Square the first term.	$10^2 - \_\_\_\_$
Square the last term.	$10^2 - 2^2$
Simplify.	$100 - 4$
Simplify.	96
What do you get using the Order of Operations?	
	$(10-2)(10+2)$
	$(8)(12)$
	96

Notice, the result is the same!

Example 2

Expand by multiplying:

a. $(x-8)(x+8)$
b. $(2x+5)(2x-5)$

Solution:

a. First, recognize this as a product of conjugates. The binomials have the same first terms, and the same last terms, and one binomial is a sum and the other is a difference.

It fits the pattern.	$(x-8)(x+8)$
Square the first term, x.	$x^2-\_\_$
Square the last term, 8.	x^2-8^2
The product is a difference of squares.	x^2-64

b. Are the binomials conjugates?

It is the product of conjugates.	$(2x+5)(2x-5)$
Square the first term, $2x$.	$(2x)^2-\_\_$
Square the last term, 5.	$(2x)^2-5^2$
Simplify. The product is a difference of squares.	$4x^2-25$

Try It! 2

Expand by multiplying:

a. $(w-3)(w+3)$
b. $(6x+5)(6x-5)$

Answer

a. w^2-9
b. $36x^2-25$

The binomials in the next example may look backwards – the variable is in the second term. But the two binomials are still conjugates, so we use the same pattern to multiply them.

Example 3

Find the product: $(3+5x)(3-5x)$

Solution:

It is the product of conjugates.	$(3+5x)(3-5x)$
Use the pattern.	$3^2-(5x)^2$
Simplify.	$9-25x^2$

Try It! 3

Multiply: $(9-2y)(9+2y)$

Answer

$81 - 4y^2$

Now we'll multiply conjugates that have two variables.

Example 4

Find the product:

a. $(5m - 9n)(5m + 9n)$
b. $(6u^2 - 11v^5)(6u^2 + 11v^5)$

Solution:

a.

This fits the pattern.	$(\underset{5m}{a} - \underset{9n}{b})(\underset{5m}{a} + \underset{9n}{b})$
Use the pattern.	$\underset{(5m)^2}{a^2} - \underset{(9n)^2}{b^2}$
Simplify.	$25m^2 - 81n^2$

b.

This fits the pattern.	$(\underset{6u^2}{a} - \underset{11v^5}{b})(\underset{6u^2}{a} + \underset{11v^5}{b})$
Use the pattern.	$\underset{(6u^2)^2}{a^2} - \underset{(11v^5)^2}{b^2}$
Simplify.	$36u^4 - 121v^{10}$

Try It! 4

Find the product:

a. $(4p - 7q)(4p + 7q)$
b. $(3x^2 - 4y^3)(3x^2 + 4y^3)$

Answer

a. $16p^2 - 49q^2$
b. $9x^4 - 16y^6$

Recognize and Use the Appropriate Special Product Pattern

We just developed special product patterns for Binomial Squares and for the Product of Conjugates. The products look similar, so it is important to recognize when it is appropriate to use each of these patterns and to notice how they differ. Look at the two patterns together and note their similarities and differences.

COMPARING THE SPECIAL PRODUCT PATTERNS

Binomial Squares	Product of Conjugates
$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)(a+b) = a^2 - b^2$
$(a-b)^2 = a^2 + 2ab + b^2$	
- Squaring a binomial	- Multiplying conjugates
- Product is a trinomial	- Product is a binomial
- Inner and outer terms with FOIL are the same	- Inner and outer terms with FOIL are opposites.
- Middle term is double the product of the terms	- There is no middle term.

Example 5

Choose the appropriate pattern and use it to find the product:

1. $(2x-3)(2x+3)$
2. $(8x-5)^2$
3. $(6m+7)^2$
4. $(5x-6)(6x+5)$

Solution:

1. $(2x-3)(2x+3)$ These are conjugates. They have the same first numbers, and the same last numbers, and one binomial is a sum and the other is a difference. It fits the Product of Conjugates pattern.

This fits the pattern.	$\underset{2x-3}{(a-b)}\underset{2x+3}{(a+b)}$
Use the pattern.	$\underset{(2x)^2-3^2}{a^2-b^2}$
Simplify.	$4x^2-9$

2. $(8x-5)^2$ We are asked to square a binomial. It fits the **binomial squares** pattern.

	$\underset{8x-5}{(a-b)^2}$
Use the pattern.	$\underset{(8x)^2-2\cdot 8x\cdot 5+5^2}{a^2-2ab+b^2}$
Simplify.	$64x^2-80x+25$

3. $(6m+7)^2$ Again, we will square a binomial so we use the **binomial squares** pattern.

	$\underset{6m+7}{(a+b)^2}$
Use the pattern.	$\underset{(6m)^2+2\cdot 6m\cdot 7+7^2}{a^2+2ab+b^2}$
Simplify.	$36m^2+84m+49$

4. $(5x-6)(6x+5)$This product does not fit the patterns, so we will use FOIL.

	$(5x-6)(6x+5)$
Use FOIL.	$30x^2+25x-36x-30$
Simplify.	$30x^2-11x-30$

Try It! 5

Choose the appropriate pattern and use it to find the product:

1. $(9b-2)(2b+9)$
2. $(9p-4)^2$
3. $(7y+1)^2$
4. $(4r-3)(4r+3)$

Answer

1. FOIL; $18b^2+77b-18$
2. Binomial Squares; $81p^2-72p+16$
3. Binomial Squares; $49y^2+14y+1$

4. Product of Conjugates; $16r^2 - 9$

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Section 1.4P: Practice

Practice Makes Progress

In the following exercises, square each binomial using the Binomial Squares Pattern.

Exercise 1

$(q+12)^2$

Exercise 2

$(y-6)^2$

Exercise 3

$(4a+10)^2$

Exercise 4

$(2y-3z)^2$

Exercise 5

$(18x-19y)^2$

Exercise 6

$(5u^2+9)^2$

Exercise 7

$(8p^3-3)^2$

In the following exercises, multiply each pair of conjugates using the Product of Conjugates Pattern.

Exercise 8

$(8j+4)(8j-4)$

Exercise 9

$(13-q)(13+q)$

Exercise 10

$(7w+10x)(7w-10x)$

Exercise 11

$(rs-\frac{2}{7})(rs+\frac{2}{7})$

? Exercise 12

$(15m^2 - 8n^4)(15m^2 + 8n^4)$

Find Each Product and simplify the expression.

? Exercise 13

$(2r + 12)^2$

? Exercise 14

$(3p + 8)(3p - 8)$

? Exercise 15

$(7a + b)(a - 7b)$

? Exercise 16

$(k - 6)^2$

Writing Exercises

? Exercise 17

Use the order of operations to show that $(3 + 5)^2$ is 64, and then use that numerical example to explain why $(a + b)^2 \neq a^2 + b^2$

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
square a binomial using the binomial squares pattern.			
multiply conjugates using the product of conjugates pattern.			
recognize and use the appropriate special product pattern.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.5: Divide Monomials

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: $\frac{8}{24}$
- Simplify: $(2m^3)^5$
- Simplify: $\frac{12x}{12y}$
- Write $x \cdot x \cdot y \cdot x \cdot y$ as an expression using exponents

Learning Objectives

Motivating Problem

Given that one megabyte is approximately 10^6 bytes and one gigabyte is approximately 10^9 bytes, determine how many megabytes are in one gigabyte.

Fun Fact

The idea of negative exponents didn't gain wide acceptance until the 18th century. **Isaac Newton** and other early mathematicians used them in calculus, but they were controversial for breaking the "rule" that powers should always multiply!

The Goal

In this section, we'll learn how to simplify quotients of monomials using the quotient rule for exponents. We'll also explore how to apply the rule when working with coefficients and how to rewrite expressions with positive exponents.

Simplify Expressions Using the Quotient Property for Exponents

Earlier, we developed the properties of exponents for multiplication. We summarize these properties below.

SUMMARY OF EXPONENT PROPERTIES FOR MULTIPLICATION

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{mn}$
Product to a Power	$(ab)^m = a^m b^m$

Now, we will examine the exponent properties for division. A quick memory refresher may be helpful before we begin. You have learned to simplify fractions by dividing out common factors from the numerator and denominator using the Equivalent Fractions Property. This property will also help you work with algebraic fractions, which are also quotients.

EQUIVALENT FRACTIONS PROPERTY

If a, b, and c are whole numbers where $b \neq 0, c \neq 0$.

$$\text{then } \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \text{ and } \frac{a \cdot c}{b \cdot c} = \frac{a}{b}$$

As before, we'll try to discover a property by looking at some examples.

Consider	$\frac{x^5}{x^2}$	**and**	$\frac{x^2}{x^3}$
What do they mean?	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$		$\frac{x \cdot x}{x \cdot x \cdot x}$
Use the Equivalent Fractions Property.	$\frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}}$		$\frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x}$
Simplify.	x^3		$\frac{1}{x}$

Notice, in each case, the bases were the same, and we subtracted exponents.

When the larger exponent was in the numerator, we were left with factors in the numerator. When the larger exponent was in the denominator, we were left with factors in the denominator—notice the numerator of 1.

We write:

$\frac{x^5}{x^2}$	$\frac{x^2}{x^3}$
x^{5-2}	$\frac{1}{x^{3-2}}$
x^3	$\frac{1}{x}$

This leads to the **Quotient Property for Exponents**.

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

1. If $m > n$ then $\frac{a^m}{a^n} = a^{m-n}$
2. If $m < n$ then $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

A couple of examples with numbers may help to verify this property.

$\frac{3^4}{3^2}$	$=$	3^{4-2}	$\frac{5^2}{5^3}$	$=$	$\frac{1}{5^{3-2}}$
$\frac{81}{9}$	$=$	3^2	$\frac{25}{125}$	$=$	$\frac{1}{5^1}$
9	$=$	$9\checkmark$	$\frac{1}{5}$	$=$	$\frac{1}{5}\checkmark$

Example 1

Simplify:

a. $\frac{x^9}{x^7}$

b. $\frac{3^{10}}{3^2}$

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

a.

Since 9 > 7, there are more factors of x in the numerator.	$\frac{x^9}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$	x^{9-7}
Simplify.	x^2

b.

Since 10 > 2, there are more factors of x in the numerator.	$\frac{3^{10}}{3^2}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$	3^{10-2}
Simplify.	3^8

Notice that when the larger exponent is in the numerator, we are left with factors in the numerator.

Try It 1

Simplify:

a. $\frac{y^{43}}{y^{37}}$

b. $\frac{10^{15}}{10^7}$

Answer

a. y^6

b. 10^8

Example 2

Simplify:

a. $\frac{b^8}{b^{12}}$

b. $\frac{7^3}{7^5}$

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

a.

Since 12 > 8, there are more factors of b in the denominator.	$\frac{b^8}{b^{12}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{1}{b^{12-8}}$
Simplify.	$\frac{1}{b^4}$

b.

Since 5 > 3, there are more factors of 3 in the denominator.	$\frac{7^3}{7^5}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{1}{7^{5-3}}$
Simplify.	$\frac{1}{7^2}$
Simplify.	$\frac{1}{49}$

Notice that when the larger exponent is in the denominator, we are left with factors in the denominator.

? Try It 2

Simplify:

a. $\frac{m^7}{m^{15}}$

b. $\frac{9^8}{9^{19}}$

Answer

a. $\frac{1}{m^8}$

b. $\frac{1}{9^{11}}$

Notice the difference in the two previous examples:

- If we start with more factors in the numerator, we will end up with factors in the numerator.
- If we start with more factors in the denominator, we will end up with factors in the denominator.

The first step in simplifying an expression using the Quotient Property for Exponents is to determine whether the exponent is larger in the numerator or the denominator.

Simplify Expressions with an Exponent of Zero

A special case of the Quotient Property is when the exponents of the numerator and denominator are equal, such as an expression like $\frac{a^m}{a^m}$. From your earlier work with fractions, you know that:

$$\frac{2}{2}=1 \quad \frac{17}{17}=1 \quad \frac{-43}{-43}=1$$

In words, a number divided by itself is 1. So, $\frac{x}{x}=1$, for any $x(x\neq 0)$, since any number divided by itself is 1.

The Quotient Property for Exponents shows us how to simplify $\frac{a^m}{a^n}$ when $m>n$ and when $n<m$ by subtracting exponents. What if $m=n$?

Consider $\frac{8}{8}$, which we know is 1.

	$\frac{8}{8}$	$=$	1
Write 8 as 2^3.	$\frac{2^3}{2^3}$	$=$	1
Subtract exponents.	2^{3-3}	$=$	1
Simplify.	2^0	$=$	1

Now we will simplify $\frac{a^m}{a^m}$ in two ways to lead us to the definition of the zero exponent. In general, for $a\neq 0$:

$\frac{a^m}{a^m}$	$\frac{a^m}{a^m}$
a^{m-m}	$\overbrace{a\cdot a\cdot\cdots\cdot a}^{m \text{ factors}}$ / $\overbrace{a\cdot a\cdot\cdots\cdot a}^{m \text{ factors}}$
a^0	1

We see $\frac{a^m}{a^m}$ simplifies to a^0 and to 1. So $a^0=1$.

ZERO EXPONENT

If a is a non-zero number, then $a^0 = 1$.

Any nonzero number raised to the power of zero is 1.

In this text, we assume that any variable raised to the power of zero is not zero.

Example 3

Simplify:

a. 9^0
b. n^0

Solution

The definition says any non-zero number raised to the power of zero is 1.

a.	9^0
Use the definition of the zero exponent.	1
b.	n^0
Use the definition of the zero exponent.	1

Try It 3

Simplify:

a. 15^0
b. m^0

Answer

a. 1
b. 1

Now that we have defined the zero exponent, we can expand all the Properties of Exponents to include whole-number exponents.

What about raising an expression to the zero power? Let's look at $(2x)^0$. We can use the product to power rule to rewrite this expression.

	$(2x)^0$
Use the product to a power rule.	2^0x^0
Use the zero exponent property.	$1 \cdot 1$
Simplify.	1

This tells us that any nonzero expression, regardless of its contents, raised to the power of zero is equal to one.

Example 4

Simplify:

a. $(5b)^0$
b. $(-4a^2b)^0$.

Solution

a.	$(5b)^0$
Use the definition of the zero exponent.	1
b.	$(-4a^2b)^0$
Use the definition of the zero exponent.	1

? Try It 4

Simplify:

a. $(11z)^0$

b. $(-11pq^3)^0$.

Answer

a. 1

b. 1

Simplify Expressions Using the Quotient to a Power Property

Now, we will examine an example that leads us to the Quotient to a Power Property.

$$\begin{array}{ll} & \left(\frac{x}{y}\right)^3 \\ \textbf{This means:} & \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y} \\ \textbf{Multiply the fractions.} & \frac{x\cdot x\cdot x}{y\cdot y\cdot y} \\ \textbf{Write with exponents.} & \frac{x^3}{y^3} \end{array}$$

Notice that the exponent applies to both the numerator and the denominator.

$$\begin{array}{ll} \textbf{We see that } \left(\frac{x}{y}\right)^3 \textbf{ is } \frac{x^3}{y^3} & \\ \textbf{We write:} & \left(\frac{x}{y}\right)^3 \\ & \frac{x^3}{y^3} \end{array}$$

This leads to the **Quotient to a Power Property for Exponents**.

QUOTIENT TO A POWER PROPERTY FOR EXPONENTS

If a and b are real numbers, $b\neq 0$, and m is a counting number, then

$$\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$$

To raise a fraction to a power, raise the numerator and denominator to that power.

An example with numbers may help you understand this property:

$$\begin{array}{c} \left(\frac{2}{3}\right)^3=\frac{2^3}{3^3} \\ \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}=\frac{8}{27} \\ \frac{8}{27}=\frac{8}{27}\checkmark \end{array}$$

✓ Example 5

Simplify:

a. $\left(\frac{3}{7}\right)^2$

b. $\left(\frac{b}{3}\right)^4$

c. $\left(\frac{k}{j}\right)^3$

Solution

a.

	$\left(\frac{3}{7}\right)^2$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{3^2}{7^2}$
Simplify.	$\frac{9}{49}$

b.

	$\left(\frac{b}{3}\right)^4$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{b^4}{3^4}$
Simplify.	$\frac{b^4}{81}$

c.

	$\left(\frac{k}{j}\right)^3$
Raise the numerator and denominator to the third power.	$\frac{k^3}{j^3}$

? Try It 5

Simplify:

a. $\left(\frac{5}{8}\right)^2$
b. $\left(\frac{p}{10}\right)^4$
c. $\left(\frac{m}{n}\right)^7$

Answer

a. $\frac{25}{64}$
b. $\frac{p^4}{10,000}$
c. $\frac{m^7}{n^7}$

Simplify Expressions by Applying Several Properties

We'll now summarize all the properties of exponents so they are all together to refer to as we simplify expressions using several properties. Notice that they are now defined for whole-number exponents.

SUMMARY OF EXPONENT PROPERTIES

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}	
Power Property	$(a^m)^n$	$=$	$a^{m \cdot n}$	
Product to a Power	$(ab)^m$	$=$	$a^m b^m$	
Quotient Property	$\frac{a^m}{a^n}$	$=$	a^{m-n},	$a \neq 0, m > n$
	$\frac{a^n}{a^n}$	$=$	1,	$a \neq 0, n > m$
Zero Exponent Definition	a^0	$=$	1,	$a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m$	$=$	$\frac{a^m}{b^m}$,	$b \neq 0$

Example 6

Simplify:

a. $\frac{(y^4)^2}{y^6}$

b. $\frac{b^{12}}{(b^2)^6}$

c. $\left(\frac{y^9}{y^4}\right)^2$

d. $\left(\frac{j^2}{k^3}\right)^4$

e. $\left(\frac{2m^2}{5n}\right)^4$

f. $\frac{(x^3)^4 (x^2)^5}{(x^6)^5}$

g. $\frac{(10p^3)^2}{(5p)^3 (2p^5)^4}$

Solution

a.

	$\frac{(y^4)^2}{y^6}$
Multiply the exponents in the numerator.	$\frac{y^8}{y^6}$
Subtract the exponents.	y^2

b.

	$\frac{b^{12}}{(b^2)^6}$
Multiply the exponents in the numerator.	$\frac{b^{12}}{b^{12}}$
Subtract the exponents.	b^0
Simplify	1

Notice that after we simplified the denominator in the first step, the numerator and the denominator were equal. So the final value is equal to 1.

c.

	$\left(\frac{y^9}{y^4}\right)^2$
Remember parentheses come before exponents.	
Notice the bases are the same, so we can simplify inside the parentheses. Subtract the exponents.	$(y^5)^2$
Multiply the exponents.	y^{10}

	Here we cannot simplify inside the parentheses first because the bases are not the same.	$\left(\frac{j^2}{k^3}\right)^4$
d.	Raise the numerator and denominator to the 4th power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$	$\frac{(j^2)^4}{(k^3)^4}$
	Use the Power Property and simplify.	$\frac{j^8}{k^{12}}$
		$\left(\frac{2m^2}{5n}\right)^4$
e.	Raise the numerator and denominator to the 4^{th} power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$	$\frac{(2m^2)^4}{(5n)^4}$
	Use the Product to a Power Property, $(ab)^m=a^mb^m$	$\frac{2^4(m^2)^4}{5^4n^4}$
	Use the Power Property and simplify.	$\frac{16m^8}{625n^4}$
f.		$\frac{(x^3)^4(x^2)^5}{(x^6)^5}$
	Use the Power Property, $(a^m)^n=a^{m\cdot n}$	$\frac{(x^{12})(x^{10})}{(x^{30})}$
	Add the exponents in the numerator.	$\frac{x^{22}}{x^{30}}$
	Use the Quotient Property, $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$	$\frac{1}{x^8}$
		$\frac{(10p^3)^2}{(5p)^3(2p^5)^4}$
	Use the Product to a Power Property, $(ab)^m=a^mb^m$	$\frac{(10)^2(p^3)^2}{(5)^3(p)^3(2)^4(p^5)^4}$
g.	Use the Power Property, $(a^m)^n=a^{m\cdot n}$	$\frac{100p^6}{125p^3\cdot 16p^{20}}$
	Add the exponents in the denominator.	$\frac{100p^6}{125\cdot 16p^{23}}$
	Use the Quotient Property, $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$	$\frac{100}{125\cdot 16p^{17}}$
	Simplify.	$\frac{1}{20p^{17}}$

Try It 6

Simplify:

a. $\frac{(m^5)^4}{m^7}$

b. $\frac{n^{12}}{(n^3)^4}$

c. $\left(\frac{r^5}{r^3}\right)^4$

d. $\left(\frac{a^3}{b^2}\right)^4$

e. $\left(\frac{7x^3}{9y}\right)^2$

f. $\frac{(a^2)^3(a^2)^4}{(a^4)^5}$

g. $\frac{(3r^3)^2(r^3)^7}{(r^3)^3}$

Answer

a. m^{13}
b. 1
c. r^8
d. $\frac{a^{12}}{b^8}$
e. $\frac{49x^6}{81y^2}$
f. $\frac{1}{a^6}$
g. $9r^{18}$

Divide Monomials

You have now been introduced to all the properties of exponents and used them to simplify expressions. Next, you'll see how to use these properties to divide monomials. Later, you'll use them to divide polynomials.

Example 7

Find the quotient:

a. $56x^7 \div 8x^3$
b. $\frac{45a^2b^3}{-5ab^5}$
c. $\frac{24a^5b^3}{48ab^4}$

Solution

a.

	$56x^7 \div 8x^3$
Rewrite as a fraction.	$\frac{56x^7}{8x^3}$
Use fraction multiplication.	$\frac{56}{8} \cdot \frac{x^7}{x^3}$
Simplify and use the Quotient Property.	$7x^4$

b. When we divide monomials with more than one variable, we write one fraction for each variable.

	$\frac{45a^2b^3}{-5ab^5}$
Use fraction multiplication.	$\frac{45}{-5} \cdot \frac{a^2}{a} \cdot \frac{b^3}{b^5}$
Simplify and use the Quotient Property.	$-9 \cdot a \cdot \frac{1}{b^2}$
Multiply.	$-\frac{9a}{b^2}$

c.

	$\frac{24a^5b^3}{48ab^4}$
Use fraction multiplication.	$\frac{24}{48} \cdot \frac{a^5}{a} \cdot \frac{b^3}{b^4}$
Simplify and use the Quotient Property.	$\frac{1}{2} \cdot a^4 \cdot \frac{1}{b}$
Multiply.	$\frac{a^4}{2b}$

Try It 7

Find the quotient:

a. $48z^8 \div 8z^2$
b. $\frac{-63c^8d^3}{7c^{12}d^2}$

c. $\dfrac{27p^4q^7}{-45p^{12}q}$

Answer

a. $6z^6$
b. $\dfrac{-9d}{c^4}$
c. $-\dfrac{3q^6}{5p^8}$

Once you become familiar with the process and have practiced it step by step several times, you may be able to simplify a fraction in one step.

Example 8

Find the quotient: $\dfrac{14x^7y^{12}}{21x^{11}y^6}$

Solution

Be very careful to simplify $\dfrac{14}{21}$ by dividing out a common factor, and to simplify the variables by subtracting their exponents.

	$\dfrac{14x^7y^{12}}{21x^{11}y^6}$
Simplify and use the Quotient Property.	$\dfrac{2y^6}{3x^4}$

Try It 8

Find the quotient: $\dfrac{30m^5n^{11}}{48m^{10}n^{14}}$

Answer

$\dfrac{5}{8m^5n^3}$

In all examples so far, there was no work to do in the numerator or denominator before simplifying the fraction. In the next example, we'll first find the product of two monomials in the numerator before we simplify the fraction. This follows the order of operations. Remember, a fraction bar is a grouping symbol.

Example 9

Find the quotient: $\dfrac{(6x^2y^3)(5x^3y^2)}{(3x^4y^5)}$

Solution

	$\dfrac{(6x^2y^3)(5x^3y^2)}{(3x^4y^5)}$
Simplify the numerator.	$\dfrac{30x^5y^5}{3x^4y^5}$
Simplify.	$10x$

Try It 9

Find the quotient: $\dfrac{(6a^4b^5)(4a^2b^5)}{12a^5b^8}$

Answer

$2ab^2$

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Section 1.5P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$\frac{t^{10}}{t^{40}}$

Exercise 2

$\frac{8^3}{8^5}$

Exercise 3

$\frac{x}{x^7}$

Exercise 4

$\frac{10}{10^3}$

Exercise 5

-15^0

Exercise 6

$-\left(15^0\right)$

Exercise 7

$7y^0(17y)^0$

Exercise 8

$\left(-93c^7d^{15}\right)^0$

Exercise 9

$\left(\frac{2}{5}\right)^2$

Exercise 10

$\left(\frac{x}{3}\right)^4$

Exercise 11

$\left(\frac{a}{b}\right)^5$

Exercise 12

$\frac{(x^4)^5}{x^{15}}$

Exercise 13

$\left(\frac{m^4}{m^7}\right)^4$

Exercise 14

$\left(\frac{a}{b^6}\right)^3$

Exercise 15

$\left(\frac{3m^5}{5n}\right)^3$

Exercise 16

$\left(\frac{5u^7}{2v^3}\right)^4$

In the following exercises, divide the monomials.

Exercise 17

$-72u^{12} \div 12u^4$

Exercise 18

$\frac{20m^8n^4}{30m^5n^9}$

Exercise 19

$\frac{45x^5y^9}{-60x^8y^6}$

Exercise 20

$\frac{(-18p^4q^7)(-6p^3q^8)}{-36p^{12}q^{10}}$

Mixed Practice

Exercise 21

$15n^{10} + 3n^{10}$

Exercise 22

$15n^{10} - 3n^{10}$

? Exercise 23

$15n^{10} \cdot 3n^{10}$

? Exercise 24

$15n^{10} \div 3n^{10}$

? Exercise 25

$\frac{56y^4z^5}{7y^3z^3} - \frac{45y^2z^2}{5y}$

Everyday Math

? Exercise 26

One gigabyte is approximately 10^9 bytes. One terabyte is approximately 10^{12} bytes. How many gigabytes are in one terabyte?

Writing Exercises

? Exercise 27

Robert thinks x^0 simplifies to 0. What would you say to convince Robert he is wrong?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions using the Quotient Property for Exponents.			
simplify expressions with zero exponents.			
simplify expressions using the Quotient to a Power Property.			
simplify expressions by applying several properties.			
divide monomials.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.6: Divide Polynomials

Be Prepared

We will rely heavily on these skills throughout this section.

- Add $\frac{3}{d}+\frac{x}{d}$
- Simplify $\frac{30xy^3}{5xy}$
- Factor $6x^2+9x$
- Combine like terms of $8a^2+12a+1+3a^2-5a+4$

Learning Objectives

Motivating Problem

You're designing a rectangular garden and want to use a polynomial $2x^2+5x+3$ to model the total area A. If you know the length of the garden is $x+1$, what expression represents the width?

Fun Fact

Did you know that as early as the 13th century, the Chinese mathematician **Qin Jiushao** was effectively using a form of synthetic division—instead of full long division—to work with higher-degree equations? In his 1247 "Mathematical Treatise in Nine Sections," he described a procedure for extracting roots of polynomials that looks almost identical to the synthetic-division technique you learn today.

The Goal

In this section, we'll learn how to divide polynomials by monomials and other polynomials using strategies like factoring and long division. We'll explore how division in algebra relates to arithmetic and unlocks a powerful tool for simplifying expressions.

Divide a Polynomial by a Monomial

In the last section, you learned how to divide a monomial by a monomial. As you continue to build up your knowledge of polynomials, the next procedure is to divide a polynomial of two or more terms by a **monomial**.

The method we'll use to divide a polynomial by a monomial is based on the properties of fraction addition. So we'll start with an example to review fraction addition.

The sum, $\frac{y}{5}+\frac{2}{5}$

simplifies to $\frac{y+2}{5}$

Now we will do this in reverse to split a single fraction into separate fractions.

We'll state the fraction addition property here just as you learned it and in reverse.

FRACTION ADDITION

If a,b, and c are numbers where $c\neq 0$, then

$$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c} \quad \text{and} \quad \frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$$

We use the form on the left to add fractions, and we use the form on the right to divide a polynomial by a monomial.

We use this form of fraction addition to divide polynomials by monomials.

DIVISION OF A POLYNOMIAL BY A MONOMIAL

To divide a polynomial by a monomial, divide each term of the polynomial by the **monomial**.

Example 1

Find the quotient: $\frac{7y^2+21}{7}$

Solution

	$\frac{7y^2+21}{7}$
Divide each term of the numerator by the denominator.	$\frac{7y^2}{7}+\frac{21}{7}$
Simplify each fraction.	y^2+3

Try It 1

Find the quotient: $\frac{8z^2+24}{4}$

Answer

$2z^2+6$

Remember that division can be represented as a fraction. When you are asked to divide a polynomial by a monomial and it is not already in fraction form, write a fraction with the polynomial in the numerator and the monomial in the denominator.

Example 2

Find the quotient: $(18x^3-36x^2)\div 6x$

Solution

	$(18x^3-36x^2)\div 6x$
Rewrite as a fraction.	$\frac{18x^3-36x^2}{6x}$
Divide each term of the numerator by the denominator.	$\frac{18x^3}{6x}-\frac{36x^2}{6x}$
Simplify.	$3x^2-6x$

Try It 2

Find the quotient: $(27b^3-33b^2)\div 3b$

Answer

$9b^2-11b$

When we divide by a negative, we must be extra careful with the signs.

Example 3

Find the quotient:

a. $\frac{12d^2-16d}{-4}$

b. $\frac{105y^5+75y^3}{5y^2}$

c. $(15x^3y-35xy^2)\div(-5xy)$

Solution

a.

	$\frac{12d^2-16d}{-4}$
Divide each term of the numerator by the denominator.	$\frac{18x^3-36x^2}{6x}$
Simplify. Remember, subtracting a negative is like adding a positive!	$-3d^2+4d$

b.

	$\frac{105y^5+75y^3}{5y^2}$
Separate the terms.	$\frac{105y^5}{5y^2}+\frac{75y^3}{5y^2}$
Simplify.	$21y^3+15y$

c.

	$(15x^3y-35xy^2)\div(-5xy)$
Rewrite as a fraction.	$\frac{15x^3y-35xy^2}{-5xy}$
Separate the terms. Be careful with the signs!	$\frac{15x^3y}{-5xy}-\frac{35xy^2}{-5xy}$
Simplify.	$-3x^2+7y$

Try It 3

Find the quotient:

a. $\frac{25y^2-15y}{-5}$

b. $\frac{216p^7-48p^5}{6p^3}$

c. $(32a^2b-16ab^2)\div(-8ab)$

Answer

a. $-5y^2+3y$

b. $36p^4-8p^2$

c. $-4a+2b$

Example 4

Find the quotient:

a. $\frac{36x^3y^2+27x^2y^2-9x^2y^3}{9x^2y}$

b. $\frac{10x^2+5x-20}{5x}$

Solution

a.

	$\frac{36x^3y^2+27x^2y^2-9x^2y^3}{9x^2y}$
Separate the terms.	$\frac{36x^3y^2}{9x^2y}+\frac{27x^2y^2}{9x^2y}-\frac{9x^2y^3}{9x^2y}$
Simplify.	$4xy+3y-y^2$

b.

	$\frac{10x^2+5x-20}{5x}$
Separate the terms.	$\frac{10x^2}{5x}+\frac{5x}{5x}-\frac{20}{5x}$
Simplify.	$2x+1-\frac{4}{x}$

? Try It 4

Find the quotient:

a. $\dfrac{35a^4b^2+14a^4b^3-42a^2b^4}{7a^2b^2}$

b. $\dfrac{10d^2-5d-2}{5d}$

Answer

a. $5a^2+2a^2b-6b^2$

b. $2d-1-\dfrac{2}{5d}$

Divide a Polynomial by a Binomial

To divide a polynomial by a **binomial**, we follow a procedure very similar to long division of numbers. So let's look carefully at the steps we take when we divide a 3-digit number, 875, by a 2-digit number, 25.

We write the long division	$25\overline{)875}$
We divide the first two digits, 87, by 25.	$\begin{array}{r} 3 \\ 25\overline{)875} \end{array}$
We multiply 3 times 25 and write the product under the 87.	$\begin{array}{r} 3 \\ 25\overline{)875} \\ \underline{75} \end{array}$
Now we subtract 75 from 87.	$\begin{array}{r} 3 \\ 25\overline{)875} \\ \underline{-75} \\ 12 \end{array}$
Then we bring down the third digit of the dividend, 5.	$\begin{array}{r} 3 \\ 25\overline{)875} \\ \underline{-75} \\ 125 \end{array}$
Repeat the process, dividing 25 into 125.	$\begin{array}{r} 35 \\ 25\overline{)875} \\ \underline{-75} \\ 125 \\ \underline{-125} \end{array}$

We check division by multiplying the quotient by the divisor.

If we did the division correctly, the product should equal the dividend.

$$35 \cdot 25$$
$$875\checkmark$$

Now we will divide a **trinomial** by a binomial. As you read through the example, notice how similar the steps are to the numerical example above.

✔ Example 5

Find the quotient: $(x^2+9x+20) \div (x+5)$

Solution

	$(x^2+9x+20)\div(x+5)$
Write it as a long division problem.	
Be sure the dividend is in standard form.	$x+5\overline{)\,x^2+9x+20}$
Divide x^2 by x. It may help to ask yourself, "What do I need to multiply x by to get x^2?"	
Put the answer, x, in the quotient over the x term.	$\begin{array}{r} x \\ x+5\overline{)\,x^2+9x+20} \end{array}$
Multiply x times $x+5$ Line up the like terms under the dividend.	$\begin{array}{r} x \\ x+5\overline{)\,x^2+9x+20} \\ x^2+5x \end{array}$
Subtract x^2+5x from x^2+9x.	
You may find it easier to change the signs and then add. Then bring down the last term, 20.	$\begin{array}{r} x \\ x+5\overline{)\,x^2+9x+20} \\ -x^2+(-5x) \\ \hline 4x+20 \end{array}$
Divide $4x$ by x. It may help to ask yourself, "What do I need to multiply x by to get $4x$?"	
Put the answer, 4, in the quotient over the constant term.	$\begin{array}{r} x+4 \\ x+5\overline{)\,x^2+9x+20} \\ -x^2+(-5x) \\ \hline 4x+20 \end{array}$
Multiply 4 times $x+5$.	$\begin{array}{r} x+4 \\ x+5\overline{)\,x^2+9x+20} \\ -x^2+(-5x) \\ \hline 4x+20 \\ 4x+20 \end{array}$
Subtract $4x+20$ from $4x+20$.	$\begin{array}{r} x+4 \\ x+5\overline{)\,x^2+9x+20} \\ -x^2+(-5x) \\ \hline 4x+20 \\ -4x+(-20) \\ \hline 0 \end{array}$
Check:	
Multiply the quotient by the divisor.	
$(x+4)(x+5)$	
You should get the dividend.	
$x^2+9x+20$ ✓	

? Try It 5

Find the quotient: $(m^2+9m+20)\div(m+4)$

Answer

$m+5$

When the divisor has a subtraction sign, we must be extra careful when we multiply the partial quotient and then subtract. It may be safer to show that we change the signs and then add.

Example 6

Find the quotient: $(2x^2 - 5x - 3) \div (x - 3)$

Solution

	$(2x^2 - 5x - 3) \div (x - 3)$
Write it as a long division problem.	
Be sure the dividend is in standard form.	$x - 3\overline{)2x^2 - 5x - 3}$
Divide $2x^2$ by x. Put the answer, $2x$, in the quotient over the x term.	$\begin{array}{r} 2x \\ x - 3\overline{)2x^2 - 5x - 3} \end{array}$
Multiply $2x$ times $x - 3$. Line up the like terms under the dividend.	$\begin{array}{r} 2x \\ x - 3\overline{)2x^2 - 5x - 3} \\ 2x^2 - 6x \end{array}$
Subtract $2x^2 - 6x$ from $2x^2 - 5x$. Change the signs and then add. Then bring down the last term.	$\begin{array}{r} 2x \\ x - 3\overline{)2x^2 - 5x - 3} \\ -2x^2 + 6x \\ \hline x - 3 \end{array}$
Divide x by x. Put the answer, 1, in the quotient over the constant term.	$\begin{array}{r} 2x + 1 \\ x - 3\overline{)2x^2 - 5x - 3} \\ -2x^2 - (-6x) \\ \hline x - 3 \end{array}$
Multiply 1 times $x - 3$.	$\begin{array}{r} 2x + 1 \\ x - 3\overline{)2x^2 - 5x - 3} \\ -2x^2 + 6x \\ \hline x - 3 \\ x - 3 \end{array}$
Subtract $x - 3$ from $x - 3$ by changing the signs and adding.	$\begin{array}{r} 2x + 1 \\ x - 3\overline{)2x^2 - 5x - 3} \\ -2x^2 + 6x \\ \hline x - 3 \\ -x + 3 \\ \hline 0 \end{array}$
To check, multiply $(x - 3)(2x + 1)$.	
The result should be $2x^2 - 5x - 3$.	

Try It 6

Find the quotient: $(2x^2 - 3x - 20) \div (x - 4)$

Answer

$2x + 5$

When we divided 875 by 25, we had no remainder. However, division of numbers sometimes leaves a remainder. The same is true when we divide polynomials. In the following example, we'll have a division that leaves a remainder. We write the remainder as a fraction with the divisor as the denominator.

Example 7

Find the quotient: $(x^3 - x^2 + x + 4) \div (x + 1)$

Solution

	$(x^3 - x^2 + x + 4) \div (x + 1)$
Write it as a long division problem.	
Be sure the dividend is in standard form.	$x+1 \overline{)\, x^3 - x^2 + x + 4}$
Divide x^3 by x. Put the answer, x^2, in the quotient over the x^2 term. Multiply x^2 times $x + 1$. Line up the like terms under the dividend.	$\begin{array}{r} x^2 \\ x+1 \overline{)\, x^3 - x^2 + x + 4} \\ x^3 + x^2 \end{array}$
Subtract $x^3 + x^2$ from $x^3 - x^2$ by changing the signs and adding. Then bring down the next term.	$\begin{array}{r} x^2 \\ x+1 \overline{)\, x^3 - x^2 + x + 4} \\ -x^3 + (-x^2) \\ \hline -2x^2 + x \end{array}$
Divide $-2x^2$ by x. Put the answer, $-2x$, in the quotient over the x term. Multiply $-2x$ times $x + 1$. Line up the like terms under the dividend.	$\begin{array}{r} x^2 - 2x \\ x+1 \overline{)\, x^3 - x^2 + x + 4} \\ -x^3 + (-x^2) \\ \hline -2x^2 + x \\ -2x^2 - 2x \end{array}$
Subtract $-2x^2 - 2x$ from $-2x^2 + x$ by changing the signs and adding. Then bring down the last term.	$\begin{array}{r} x^2 - 2x \\ x+1 \overline{)\, x^3 - x^2 + x + 4} \\ -x^3 + (-x^2) \\ \hline -2x^2 + x \\ +2x^2 + 2x \\ \hline 3x + 4 \end{array}$
Divide $3x$ by x. Put the answer, 3, in the quotient over the constant term. Multiply 3 times $x + 1$. Line up the like terms under the dividend.	$\begin{array}{r} x^2 - 2x + 3 \\ x+1 \overline{)\, x^3 - x^2 + x + 4} \\ -x^3 + (-x^2) \\ \hline -2x^2 + x \\ +2x^2 + 2x \\ \hline 3x + 4 \\ 3x + 3 \end{array}$
Subtract $3x + 3$ from $3x + 4$ by changing the signs and adding. Write the remainder as a fraction with the divisor as the denominator.	$\begin{array}{r} x^2 - 2x + 3 + \dfrac{1}{x+1} \\ x+1 \overline{)\, x^3 - x^2 + x + 4} \\ -x^3 + (-x^2) \\ \hline -2x^2 + x \\ +2x^2 + 2x \\ \hline 3x + 4 \\ -3x + (-3) \\ \hline 1 \end{array}$
To check, multiply $(x+1)\left(x^2 - 2x + 3 + \dfrac{1}{x+1}\right)$ The result should be $x^3 - x^2 + x + 4$	

Try It 7

Find the quotient: $(x^3 + 5x^2 + 8x + 6) \div (x + 2)$

Answer

$$x^2 + 3x + 2 + \frac{2}{x+2}$$

Look back at the dividends in our previous examples. The terms were written in descending order of degrees, and there were no missing degrees. The dividend in the following example will be $x^4 - x^2 + 5x - 2$. It is missing an x^3 term. We will add in $0x^3$ as a placeholder.

Example 8

Find the quotient: $(x^4 - x^2 + 5x - 2) \div (x + 2)$

Solution

Notice that there is no x^3 term in the dividend. We will add $0x^3$ as a placeholder.

	$(x^4 - x^2 + 5x - 2) \div (x + 2)$
Write it as a long division problem. Be sure the dividend is in standard form with placeholders for missing terms.	$x + 2\overline{)\,x^4 - 0x^3 - x^2 + 5x - 2}$
Divide x^4 by x. Put the answer, x^3, in the quotient over the x^3 term. Multiply x^3 times $x + 2$. Line up the like terms. Subtract and then bring down the next term.	x^3 $x + 2\overline{)\,x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ It may be helpful to change the signs and add.
Divide $-2x^3$ by x. Put the answer, $-2x^2$, in the quotient over the x^2 term. Multiply $-2x^2$ times $x + 1$. Line up the like terms. Subtract and bring down the next term.	$x^3 - 2x^2$ $x + 2\overline{)\,x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ $-(-2x^3 - 4x^2)$ $3x^2 + 5x$ It may be helpful to change the signs and add.
Divide $3x^2$ by x. Put the answer, $3x$, in the quotient over the x term. Multiply $3x$ times $x + 1$. Line up the like terms. Subtract and bring down the next term.	$x^3 - 2x^2 + 3x$ $x + 2\overline{)\,x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ $-(-2x^3 - 4x^2)$ $3x^2 + 5x$ $-(3x^2 + 6x)$ $-x - 2$ It may be helpful to change the signs and add.
Divide $-x$ by x. Put the answer, -1, in the quotient over the constant term. Multiply -1 times $x + 1$. Line up the like terms. Change the signs, add.	$x^3 - 2x^2 + 3x - 1$ $x + 2\overline{)\,x^4 + 0x^3 - x^2 + 5x - 2}$ $-(x^4 + 2x^3)$ $-2x^3 - x^2$ $-(-2x^3 - 4x^2)$ $3x^2 + 5x$ $-(3x^2 + 6x)$ $-x - 2$ $-(-x - 2)$ 0 It may be helpful to change the signs and add.
To check, multiply $(x + 2)(x^3 - 2x^2 + 3x - 1)$	
The result should be $x^4 - x^2 + 5x - 2$	

Try It 8

Find the quotient: $(x^3 + 3x + 14) \div (x + 2)$

Answer

$x^2 - 2x + 7$

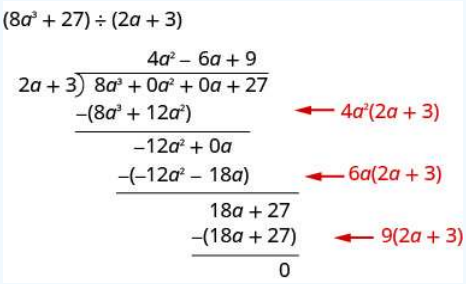
In the following example, we will divide by $2a - 3$. As we divide, we must consider both the constants and the variables.

Example 9

Find the quotient: $(8a^3 + 27) \div (2a + 3)$

Solution

This time, we will show the division all in one step. We need to add two placeholders in order to divide.



To check, multiply $(2a + 3)(4a^2 - 6a + 9)$

The result should be $8a^3 + 27$

Try It 9

Find the quotient: $(x^3 - 64) \div (x - 4)$

Answer

$x^2 + 4x + 16$

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Section 1.6P: Practice

Practice Makes Progress

In the following exercises, divide each polynomial by the monomial.

Exercise 1

$\frac{30b+75}{5}$

Exercise 2

$(55w^2-10w)\div 5w$

Exercise 3

$\frac{72r^5s^2+132r^4s^3-96r^3s^5}{12r^2s^2}$

Exercise 4

$\frac{63a^3-108a^2+99a}{9a^2}$

In the following exercises, divide each polynomial by the binomial.

Exercise 5

$(x^2-14x+49)\div(x-7)$

Exercise 6

$(4x^2-17x-15)\div(x-5)$

Exercise 7

$(x^2+2x-30)\div(x-5)$

Exercise 8

$(2n^3-10n+28)\div(n+3)$

Exercise 9

$(m^3+1000)\div(m+10)$

Exercise 10

$(x^3-216)\div(x-6)$

Everyday Math

? Exercise 11

At a company meeting, every employee shakes hands with every other employee. The number of handshakes is given by the expression $\frac{n^2-n}{2}$ where n represents the number of employees. How many handshakes will there be if there are 10 employees at the meeting?

Writing Exercises

? Exercise 12

Divide $\frac{10x^2+x-12}{2x}$ and explain with words how you get each term of the quotient.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
divide a polynomial by a monomial.			
divide a polynomial by a binomial.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.7: Integer Exponents and Scientific Notation

Be Prepared

We will rely heavily on these skills throughout this section.

- Determine the place value of the 6 in the number 648951
- Write these numberes using words: a. $7,200,000$ and b. 0.0006
- Which is greater: one billion or one million squared? Explain how you know.

Learning Objectives

Motivating Problem

The average distance from Earth to the Sun is about 93,000,000 miles. How would you write that number in a way that's easier to read, compare, and calculate with—especially if you're an astronaut?

Not-so-Fun Fact

In July 1962, NASA launched Mariner 1—the first American probe bound for Venus. Less than five minutes into flight, the rocket veered wildly off course and was destroyed. What should have been a tiny correction factor—written in scientific notation form—was dropped, causing the guidance computer to treat normal trajectory wiggles as large errors and send the rocket wildly off-target.

In cutting-edge rocketry, a single wrong exponent in scientific notation can cascade into a total mission failure.

The Goal

In this section, we'll explore the rules for working with integer exponents, including zero and negative exponents. We'll also learn how to convert between standard form and scientific notation to make sense of very large or very small numbers.

Use the Definition of a Negative Exponent

We saw that the Quotient Property for Exponents, introduced earlier in this chapter, has two forms, depending on whether the exponent is larger in the numerator or the denominator.

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and$n$ are whole numbers, then

1. If $m > n$ then $\frac{a^m}{a^n} = a^{m-n}$
2. If $n > m$ then $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

What if we just subtract exponents regardless of which is larger?

Let's consider $\frac{x^2}{x^5}$.

We subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{x^2}{x^5}$$
$$x^{2-5}$$
$$x^{-3}$$

We can also simplify $\frac{x^2}{x^5}$ by dividing out common factors:

$$\frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$$

$$\frac{1}{x^3}$$

This implies that $x^{-3} = \frac{1}{x^3}$, which leads us to the definition of a **negative exponent**.

Definition: NEGATIVE EXPONENT

If n is an integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$

The **negative exponent** tells us we can rewrite the expression by taking the reciprocal of the base and then changing the sign of the exponent.

Any expression that has negative exponents is not considered to be in its simplest form. We will use the definition of a **negative exponent** and other properties of exponents to write the expression with only positive exponents.

For example, if after simplifying an expression we end up with the expression x^{-3}, we will take one more step and write $\frac{1}{x^3}$. The answer is considered to be in its simplest form when it has only positive exponents.

Example 1

Simplify:

a. 4^{-2}
b. 10^{-3}

Solution

a.

	4^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$,	$\frac{1}{4^2}$
Simplify.	$\frac{1}{16}$

b.

	10^{-3}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$,	$\frac{1}{10^3}$
Simplify.	$\frac{1}{1000}$

Try It 1

Simplify:

a. 2^{-3}
b. 10^{-7}

Answer

a. $\frac{1}{8}$
b. $\frac{1}{10^7}$

In the previous example, we raised an integer to a negative exponent. What happens when we raise a fraction to a negative exponent? We'll start by looking at what happens to a fraction whose numerator is one and whose denominator is an integer raised to a negative exponent.

	$\frac{1}{a^{-n}}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$	$\frac{1}{\frac{1}{a^n}}$
Simplify the complex fraction.	$1\cdot\frac{a^n}{1}$
Multiply.	a^n

This leads to the **Property of Negative Exponents**.

PROPERTY OF NEGATIVE EXPONENTS

If n is an integer and $a\neq 0$, then $\frac{1}{a^{-n}}=a^n$.

Example 2

Simplify:

a. $\frac{1}{y^{-4}}$

b. $\frac{1}{3^{-2}}$

Solution

a.

	$\frac{1}{y^{-4}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}}=a^n$.	y^4

b.

	$\frac{1}{3^{-2}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}}=a^n$.	3^2
Simplify.	9

Try It 2

Simplify:

1. $\frac{1}{p^{-8}}$
2. $\frac{1}{4^{-3}}$

Answer

1. p^8
2. 64

Suppose now we have a fraction raised to a negative exponent. Let's use our definition of negative exponents to lead us to a new property.

	$\left(\frac{3}{4}\right)^{-2}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^n}$	$\frac{1}{\left(\frac{3}{4}\right)^2}$
Simplify the denominator.	$\frac{1}{\frac{9}{16}}$
Simplify the complex fraction.	$\frac{16}{9}$
But we know that $\frac{16}{9}$ is $\left(\frac{4}{3}\right)^2$	
This tells us that:	$\left(\frac{3}{4}\right)^{-2}=\left(\frac{4}{3}\right)^2$

To get from the original fraction raised to a negative exponent to the final result, we took the reciprocal of the base—the fraction—and changed the sign of the exponent.

This leads us to the **Quotient to a Negative Power Property**.

QUOTIENT TO A NEGATIVE EXPONENT PROPERTY

If a and b are real numbers, $a\neq 0, b\neq 0$, and n is an integer, then $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n$

Example 3

Simplify:

a. $\left(\frac{5}{7}\right)^{-2}$

b. $\left(-\frac{2x}{y}\right)^{-3}$

Solution

a.

	$\left(\frac{5}{7}\right)^{-2}$
Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n$. Take the reciprocal of the fraction and change the sign of the exponent. .	$\left(\frac{7}{5}\right)^2$
Simplify.	$\frac{49}{25}$

b.

	$\left(-\frac{2x}{y}\right)^{-3}$
Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n$. Take the reciprocal of the fraction and change the sign of the exponent.	$\left(-\frac{y}{2x}\right)^3$
Simplify.	$-\frac{y^3}{8x^3}$

Try It 3

Simplify:

a. $\left(\frac{2}{3}\right)^{-4}$

b. $\left(-\frac{6m}{n}\right)^{-2}$

Answer

a. $\frac{81}{16}$

b. $\frac{n^2}{36m^2}$

When simplifying an expression with exponents, it is essential to accurately identify the base.

Example 4

Simplify:

a. $(-3)^{-2}$

b. -3^{-2}

c. $\left(-\frac{1}{3}\right)^{-2}$

d. $-\left(\frac{1}{3}\right)^{-2}$

Solution

a. Here, the exponent applies to the base −3.

	$(-3)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{(-3)^2}$
Simplify.	$\frac{1}{9}$

b. The expression -3^{-2} means "find the opposite of 3^{-2}". Here the exponent applies to the base 3.

	-3^{-2}
Rewrite as a product with -1	$-1 \cdot 3^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \frac{1}{3^2}$
Simplify.	$-\frac{1}{9}$

c. Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.

	$\left(-\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\left(-\frac{3}{1}\right)^{2}$
Simplify.	9

d. The expression $-\left(\frac{1}{3}\right)^{-2}$ means "find the opposite of $\left(\frac{1}{3}\right)^{-2}$". Here the exponent applies to the base $\left(\frac{1}{3}\right)$.

	$-\left(\frac{1}{3}\right)^{-2}$
Rewrite as a product with -1	$-1 \cdot \left(\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \left(\frac{3}{1}\right)^{2}$
Simplify.	-9

Try It 4

Simplify:

1. $(-5)^{-2}$
2. -5^{-2}
3. $\left(-\frac{1}{5}\right)^{-2}$

4. $-\left(\frac{1}{5}\right)^{-2}$

Answer

1. $\frac{1}{25}$
2. $-\frac{1}{25}$
3. 25
4. -25

We must be careful to follow the Order of Operations. In the next example, parts (a) and (b) look similar, but the results are different.

Example 5

Simplify:

a. $4 \cdot 2^{-1}$

b. $(4 \cdot 2)^{-1}$

Solution

a.

Do exponents before multiplication.	$4 \cdot 2^{-1}$
Use $a^{-n} = \frac{1}{a^n}$	$4 \cdot \frac{1}{2^1}$
Simplify.	2

b.

	$(4 \cdot 2)^{-1}$
Simplify inside the parentheses first.	$(8)^{-1}$
Use $a^{-n} = \frac{1}{a^n}$	$\frac{1}{8^1}$
Simplify.	$\frac{1}{8}$

Try It 5

Simplify:

a. $6 \cdot 3^{-1}$

b. $(6 \cdot 3)^{-1}$

Answer

a. 2

b. $\frac{1}{18}$

When a variable is raised to a negative exponent, we apply the definition in the same way we did with numbers. We will assume all variables are non-zero.

Example 6

Simplify:

a. x^{-6}

b. $(u^4)^{-3}$

Solution

a.

Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$ $\quad \frac{1}{x^6}$

b. $(u^4)^{-3}$

Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$ $\quad \frac{1}{(u^4)^3}$

Simplify. $\quad \frac{1}{u^{12}}$

Try It 6

Simplify:

a. y^{-7}
b. $(z^3)^{-5}$

Answer

a. $\frac{1}{y^7}$
b. $\frac{1}{z^{15}}$

When there is a product and an exponent, we have to be careful to apply the exponent to the correct quantity. According to the Order of Operations, we simplify expressions in parentheses before applying exponents. We'll see how this works in the next example.

Example 7

Simplify:

a. $5y^{-1}$
b. $(5y)^{-1}$
c. $(-5y)^{-1}$

Solution

a. Notice the exponent applies to just the base y.

$5y^{-1}$

Take the reciprocal of the base y, and change the sign of the exponent. $\quad 5 \cdot \frac{1}{y^1}$

Simplify. $\quad \frac{5}{y}$

b. Here, the parentheses make the exponent apply to the base $5y$

$(5y)^{-1}$

Take the reciprocal of the base $5y$, and change the sign of the exponent. $\quad \frac{1}{(5y)^1}$

Simplify. $\quad \frac{1}{5y}$

c. The base here is $-5y$

	$(-5y)^{-1}$
Take the reciprocal of the base $-5y$ and change the sign of the exponent.	$\frac{1}{(-5y)^1}$
Simplify.	$\frac{1}{-5y}$
Use $\frac{a}{-b}=-\frac{a}{b}$	$-\frac{1}{5y}$

? Try It 7

Simplify:

a. $8p^{-1}$
b. $(8p)^{-1}$
c. $(-8p)^{-1}$

Answer

a. $\frac{8}{p}$
b. $\frac{1}{8p}$
c. $-\frac{1}{8p}$

With negative exponents, the Quotient Rule needs only one form $\frac{a^m}{a^n}=a^{m-n}$, for $a\neq 0$. When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative.

Simplify Expressions with Integer Exponents

All of the exponent properties we developed earlier in the chapter with whole-number exponents apply to integer exponents, too. We restate them here for reference.

SUMMARY OF EXPONENT PROPERTIES

If a and b are real numbers, and m and n are integers, then

Product Property	$a^m\cdot a^n$	$=$	a^{m+n}
Power Property	$(a^m)^n$	$=$	$a^{m\cdot n}$
Product to a Power	$(ab)^m$	$=$	$a^m b^m$
Quotient Property	$\frac{a^m}{a^n}$	$=$	$a^{m-n},\quad a\neq 0$
Zero Exponent Property	a^0	$=$	$1,\quad a\neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m$	$=$	$\frac{a^m}{b^m},\quad b\neq 0$
Properties of Negative Exponents	a^{-n}	$=$	$\frac{1}{a^n}$ and $\frac{1}{a^{-n}}=a^n$
Quotient to a Negative Exponents	$\left(\frac{a}{b}\right)^{-n}$	$=$	$\left(\frac{b}{a}\right)^n$

✓ Example 8

Simplify:

a. $x^{-4}\cdot x^6$
b. $y^{-6}\cdot y^4$
c. $z^{-5}\cdot z^{-3}$

Solution

a.

	$x^{-4} \cdot x^{6}$
Use the Product Property, $a^m \cdot a^n = a^{m+n}$	x^{-4+6}
Simplify.	x^{2}

b.

	$y^{-6} \cdot y^{4}$
Notice the same bases, so add the exponents.	y^{-6+4}
Simplify.	y^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$	$\frac{1}{y^2}$

c.

	$z^{-5} \cdot z^{-3}$
Add the exponents, since the bases are the same.	z^{-5-3}
Simplify.	z^{-8}
Use the definition of a negative exponent to take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{z^8}$

Try It 8

Simplify:

a. $a^{-1} \cdot a^{6}$
b. $b^{-8} \cdot b^{4}$
c. $c^{-8} \cdot c^{-7}$

Answer

a. a^5
b. $\frac{1}{b^4}$
c. $\frac{1}{c^{15}}$

In the next example, we'll start by using the Commutative Property to group the same variables together. This makes it easier to identify the like bases before using the Product Property.

Example 9

Simplify:

a. $(m^4n^{-3})(m^{-5}n^{-2})$
b. $(2x^{-6}y^{8})(-5x^{5}y^{-3})$

Solution

a.

	$(m^4n^{-3})(m^{-5}n^{-2})$
Use the Commutative Property to get like bases together.	$m^4m^{-5} \cdot n^{-2}n^{-3}$
Add the exponents for each base.	$m^{-1} \cdot n^{-5}$
Take reciprocals and change the signs of the exponents.	$\frac{1}{m^1} \cdot \frac{1}{n^5}$
Simplify.	$\frac{1}{mn^5}$

b.

	$(2x^{-6}y^{8})\,(-5x^{5}y^{-3})$
Rewrite with the like bases together.	$2(-5)\cdot(x^{-6}x^{5})\cdot(y^{8}y^{-3})$
Multiply the coefficients and add the exponents of each variable.	$-10\cdot x^{-1}\cdot y^{5}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$	$-10\cdot\frac{1}{x^{1}}\cdot y^{5}$
Simplify.	$\frac{-10y^{5}}{x}$

Try It 9

Simplify:

a. $(p^{6}q^{-2})\,(p^{-9}q^{-1})$
b. $(3u^{-5}v^{7})\,(-4u^{4}v^{-2})$

Answer

a. $\frac{1}{p^{3}q^{3}}$
b. $-\frac{12v^{5}}{u}$

In the next example, we'll use the Power Property and the Product to a Power Property.

Example 10

Simplify:

a. $(6k^{3})^{-2}$
b. $(5x^{-3})^{2}$

Solution

a.

	$(6k^{3})^{-2}$
Use the Product to a Power Property, $(ab)^{m}=a^{n}b^{m}$	$(6)^{-2}\,(k^{3})^{-2}$
Use the Power Property, $(a^{m})^{n}=a^{m\cdot n}$	$6^{-2}k^{-6}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$	$\frac{1}{6^{2}}\cdot\frac{1}{k^{6}}$
Simplify.	$\frac{1}{36k^{6}}$

b.

	$(5x^{-3})^{2}$
Use the Product to a Power Property, $(ab)^{m}=a^{n}b^{m}$	$5^{2}\,(x^{-3})^{2}$
Simplify 5^{2} and multiply the exponents of x using the Power Property, $(a^{m})^{n}=a^{m\cdot n}$.	$25\cdot x^{-6}$
Rewrite x^{-6} by using the Definition of a Negative Exponent, $a^{-n}=\frac{1}{a^{n}}$	$25\cdot\frac{1}{x^{6}}$
Simplify.	$\frac{25}{x^{6}}$

Try It 10

Simplify:

a. $(-4x^{4})^{-2}$
b. $(8a^{-4})^{2}$

Answer

a. $\frac{1}{16x^8}$

b. $\frac{64}{a^8}$

To simplify a fraction, we use the Quotient Property and subtract the exponents.

Example 11

Simplify: $\frac{r^5}{r^{-4}}$

Solution

	$\frac{r^5}{r^{-4}}$
Use the Quotient Property, $\frac{a^n}{a^n} = a^{m-n}$	$r^{5-(-4)}$
Simplify.	r^9

Try It 11

Simplify: $\frac{x^8}{x^{-3}}$

Answer

x^{11}

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on. Consider the numbers 4,000 and 0.004. We know that 4,000 means $4 \times 1,000$ and 0.004 means $4 \times \frac{1}{1,000}$.

If we write 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

$4,000$	0.004
$4 \times 1,000$	$4 \times \frac{1}{1,000}$
4×10^3	$4 \times \frac{1}{10^3}$
	4×10^{-3}

When a number is written as a product of two numbers, where the first factor is a number greater than or equal to one but less than 10, and the second factor is a power of 10 written in exponential form, it is said to be in **Scientific Notation**.

SCIENTIFIC NOTATION

A number is expressed in **scientific notation** when it is of the form

$$a \times 10^n \text{ where } 1 \leq a < 10 \text{ and } n \text{ is an integer}$$

It is customary in scientific notation to use as the $\times$ multiplication sign, even though we avoid using this sign elsewhere in algebra.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.

$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
$4000. = 4 \times 10^3$	$0.004 = 4 \times 10^{-3}$
Moved the decimal point 3 places to the left.	Moved the decimal point 3 places to the right.

In both cases, the decimal was moved 3 places to get the first factor between 1 and 10.

The power of 10 is positive when the number is larger than 1 : $4,000 = 4 \times 10^3$

The power of 10 is negative when the number is between 0 and 1 : $0.004 = 4 \times 10^{-3}$

Example 12

Write in scientific notation: 37000.

Solution

Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.	Remember, there is a decimal at the end of 37,000. Move the decimal after the 3. 3.700 is between 1 and 10.	37,000.
Step 2. Count the number of decimal places, n, that the decimal point was moved.	The decimal point was moved 4 places to the left.	37000.
Step 3. Write the number as a product with a power of 10. If the original number is: Greater than 1, the power of 10 will be 10^n. Between 0 and 1, the power of 10 will be 10^{-n}.	37,000 is greater than 1 so the power of 10 will have exponent 4.	3.7×10^4
Step 4. Check.	Check to see if your answer makes sense.	10^4 is 10,000 and 10,000 times 3.7 will be 37,000. $37,000 = 3.7 \times 10^4$

Try It 12

Write in scientific notation: 48300.

Answer

4.83×10^4

HOW TO: Convert from decimal notation to scientific notation

1. Step 1. Move the decimal point so that the first factor is greater than or equal to 1 but less than 10.
2. Step 2. Count the number of decimal places, n, that the decimal point was moved.
3. Step 3. Write the number as a product with a power of 10.
   If the original number is:
   - greater than 1, the power of 10 will be 10^n.
   - between 0 and 1, the power of 10 will be 10^{-n}.
4. Step 4. Check.

Example 13

Write in scientific notation: 0.0052.

Solution

The original number, 0.0052, is between 0 and 1 so we will have a negative power of 10.

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	0.0052
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}
Check.	
5.2×10^{-3} $5.2 \times \frac{1}{10^3}$ $5.2 \times \frac{1}{1000}$ 5.2×0.001	
0.0052	$0.0052 = 5.2 \times 10^{-3}$

Try It 13

Write in scientific notation: 0.0129

Answer

1.29×10^{-2}

Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.

9.12×10^4	9.12×10^{-4}
$9.12 \times 10,000$	9.12×0.0001
$91,200$	0.000912

If we examine the location of the decimal point, we can see an easy method for converting a number from scientific notation to decimal form.

$$9.12 \times 10^4 = 91,200 \quad 9.12 \times 10^{-4} = 0.000912$$

$9.12 \times 10^4 = 91,200$	$9.12 \times 10^{-4} = 0.000912$
$9.12 \times 10^4 = 91,200$	$9.12 \times 10^{-4} = 0.000912$
Move the decimal point 4 places to the right.	Move the decimal point 4 places to the left.

In both cases, the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

Example 14

Convert to decimal form: 6.2×10^3

Solution

Step 1. Determine the exponent, n, on the factor 10.	The exponent is 3.	6.2×10^3
Step 2. Move the decimal n places, adding zeros if needed. If the exponent is positive, move the decimal point n places to the right. If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	The exponent is positive, so move the decimal point 3 places to the right. We need to add 2 zeros as placeholders.	6.200. 6,200
Step 3. Check to see if your answer makes sense.		10^3 is 1000 and 1000 times 6.2 will be 6,200. $6.2 \times 10^3 = 6{,}200$

Try It 14

Convert to decimal form: 9.25×10^4

Answer

$92,500$

The steps are summarized below.

HOW TO

Convert scientific notation to decimal form.

To convert scientific notation to decimal form:

1. Step 1. Determine the exponent, n, on the factor 10.
2. Step 2. Move the decimal n places, adding zeros if needed.
   - If the exponent is positive, move the decimal point n places to the right.
   - If the exponent is negative, move the decimal point $|n|$ places to the left.
3. Step 3. Check.

Multiply and Divide Using Scientific Notation

Astronomers use very large numbers to describe distances in the universe and ages of stars and planets. Chemists use very small numbers to describe the size of an atom or the charge on an electron. When scientists perform calculations with very large or very small numbers, they use scientific notation. Scientific notation provides a way to perform calculations without writing a large number of zeros. We will see how the Properties of Exponents are used to multiply and divide numbers in scientific notation.

Example 15

a. Multiply. Write answers in decimal form: $(4 \times 10^5)(2 \times 10^{-7})$

b. Divide. Write answers in decimal form: $\dfrac{9 \times 10^3}{3 \times 10^{-2}}$

Solution

a.

	$(4 \times 10^5)(2 \times 10^{-7})$
Use the Commutative Property to rearrange the factors.	$4 \cdot 2 \cdot 10^5 \cdot 10^{-7}$
Multiply.	8×10^{-2}
Change to decimal form by moving the decimal two places left.	0.08

b.

	$\frac{9 \times 10^3}{3 \times 10^{-2}}$
Separate the factors, rewriting as the product of two fractions.	$\frac{9}{3} \times \frac{10^3}{10^{-2}}$
Divide.	3×10^5
Change to decimal form by moving the decimal five places right.	300000

Try It 15

a. Multiply $(3 \times 10^6)(2 \times 10^{-8})$. Write answers in decimal form.

b. Divide $\frac{8 \times 10^2}{4 \times 10^{-2}}$. Write answers in decimal form.

Answer

a. 0.06

b. $20,000$

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Section 1.7P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$\frac{1}{t^{-9}}$

Exercise 2

$\frac{1}{10^{-4}}$

Exercise 3

$\left(\frac{7}{2}\right)^{-3}$

Exercise 4

$\left(-\frac{3}{xy^2}\right)^{-3}$

Exercise 5

$y^5 \cdot y^{-5}$

Exercise 6

$y \cdot y^5$

Exercise 7

$y \cdot y^{-5}$

Exercise 8

$x^4 \cdot x^{-2} \cdot x^{-3}$

Exercise 9

$(m^3n^{-3})(m^{-5}n^{-1})$

Exercise 10

$(pq^{-4})(p^{-6}q^{-3})$

Exercise 11

$(-5m^4n^6)(8m^{-5}n^{-3})$

Exercise 12

$(2p^{-5})^2$

Exercise 13

$\frac{n^5}{n^{-2}}$

In the following exercises, write your answer in scientific notation.

Exercise 14

1,290,000

Exercise 15

0.041

Exercise 16

0.00000103

In the following exercises, write your answer in decimal form.

Exercise 17

8.3×10^2

Exercise 18

1.93×10^{-5}

Exercise 19

$(2 \times 10^2)(1 \times 10^{-4})$

Exercise 20

$\frac{5 \times 10^{-2}}{1 \times 10^{-10}}$

Everyday Math

Exercise 21

The probability of winning the 2010 Megamillions lottery was about 0.0000000057. Write the number in scientific notation.

Exercise 22

At the start of 2012, the US federal budget had a deficit of more than $\$1.5 \times 10^{13}$. Convert this number to decimal form.

? Exercise 23

In 1942, the U.S. Mint produced 154,500,000 nickels. Write 154,500,000 in scientific notation.

Writing Exercises

? Exercise 24

When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of a negative exponent.			
simplify expressions with integer exponents.			
convert from decimal notation to scientific notation.			
convert scientific notation to decimal form.			
multiply and divide using scientific notation.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 2: Roots and Radicals

Now it’s time to explore roots and radicals—expressions that help us reverse powers. We’ll start with square roots and then go deeper into simplifying, estimating, and operating with radicals. You’ll learn how to add, subtract, multiply, and divide radical expressions, solve square root equations, and convert between radicals and rational exponents. These skills are essential in science, geometry, and real-world modeling, providing new ways to express and simplify complex relationships.

Section 2.1: Simplify and Use Square Roots

Section 2.1P: Practice

Section 2.2: Simplify Square Roots

Section 2.2P: Practice

Section 2.3: Add and Subtract Square Roots

Section 2.3P: Practice

Section 2.4: Multiply Square Roots

Section 2.4P: Practice

Section 2.5: Divide Square Roots

Section 2.5P: Practice

Section 2.6: Rational Exponents

Section 2.6P: Practice

Section 2.7: Solve Equations with Square Roots

Section 2.7P: Practice

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Section 2.1: Simplify and Use Square Roots

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: 9^2, $(-9)^2$, and -9^2
- Round 3.846 to the nearest hundredth
- Identify all perfect squares between 1 and 100

Learning Objectives

Motivating Problem

A square sandbox has an area of 49 square feet. How long is one side?

What if the area were 50 square feet—how could you estimate the side length then?

Fun Fact

The square root symbol $\sqrt{}$ was first used by German mathematician **Christoph Rudolff** in 1525. Before that, mathematicians described roots using words or abbreviations, making even simple expressions much longer to write!

The Goal

In this section, we'll learn what square roots represent and how to simplify them when they're perfect squares. We'll also learn to estimate non-perfect square roots and apply them to real-world problems, such as finding the side lengths of squares.

Simplify Expressions with Roots

In Foundations, we briefly looked at square roots. Remember that when a real number n is multiplied by itself, we write n^2 and read it 'n^2 squared'. This number is called the **square** of n, and n is called the **square root**. For example,

13^2 is read "13 squared"

169 is called the square of 13, since $13^2 = 169$

13 is a square root of 169

Definition: Square and Square Root of a Number

Square

If $n^2 = m$, then m is the **square** of n.

Square Root

If $n^2 = m$, then n is a **square root** of m.

Notice $(-13)^2 = 169$ also, so -13 is also a square root of 169. Therefore, both 13 and -13 are square roots of 169.

So, every positive number has two square roots—one positive and one negative. What if we only wanted the positive square root of a positive number? We use a *radical sign*, and write, $\sqrt{m}$, which denotes the positive square root of m. The positive square root is also called the **principal square root**.

We also use the radical sign for the square root of zero. Because $0^2 = 0, \sqrt{0} = 0$. Notice that zero has only one square root.

Definition: Square Root Notation

$\sqrt{m}$ is read "the square root of m."

If $n^2 = m$, then $n = \sqrt{m}$, for $n \geq 0$.

radical sign $\longrightarrow \sqrt{m} \longleftarrow$ radicand

We know that every positive number has two square roots, and the radical sign indicates the positive one. We write $\sqrt{169} = 13$. To find the negative square root of a number, we place a negative sign in front of the radical sign. For example, $-\sqrt{169} = -13$.

Example 1

Simplify:

a. $\sqrt{144}$
b. $-\sqrt{289}$

Solution:

a. Since $12^2 = 144$ we have $\sqrt{144} = \sqrt{12^2} = 12$.

b. Since $17^2 = 289$ we have $\sqrt{289} = \sqrt{17^2} = 17$.

Try It 1

Simplify:

a. $-\sqrt{64}$
b. $\sqrt{225}$

Answer

a. -8
b. 15

Can we simplify $\sqrt{-49}$? Is there a number whose square is -49?

$$(\_\_\_)^2 = -49$$

Any positive number squared is positive. Any negative number squared is positive. There is no real number equal to $\sqrt{-49}$. The square root of a negative number is not a real number.

Example 2

Simplify:

a. $\sqrt{-196}$
b. $-\sqrt{64}$

Solution:

a. Since there is no real number whose square is -196, $\sqrt{-196}$ is not a real number.

b. Since $8^2 = 64$ we have $-\sqrt{64} = -\sqrt{8^2} = -8$.

Try It 2

Simplify:

a. $-\sqrt{49}$
b. $\sqrt{-121}$

Answer

a. -7

b. not a real number

So far, we have only talked about squares and square roots. Let's now extend our work to include higher powers and higher roots.

Let's review some vocabulary first.

We write:	We say:
n^2	n squared
n^3	n cubed
n^4	n to the fourth power
n^5	n to the fifth power

The terms 'squared' and 'cubed' come from the formulas for the area of a square and the volume of a cube.

It will be helpful to have a table of the powers of the integers from -5 to 5.

Number	Square	Cube	Fourth power	Fifth power
n	n^2	n^3	n^4	n^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
x	x^2	x^3	x^4	x^5
x^2	x^4	x^6	x^8	x^{10}

Number	Square	Cube	Fourth power	Fifth power
n	n^2	n^3	n^4	n^5
–1	1	–1	1	–1
–2	4	–8	16	–32
–3	9	–27	81	–243
–4	16	–64	256	–1024
–5	25	–125	625	–3125

Notice the signs in the table. All powers of positive numbers are positive, of course. But when we have a negative number, the *even* powers are positive and the *odd* powers are negative. We'll copy the row with the powers of -2 to help you see this.

n	n^2	n^3	n^4	n^5
–2	4	–8	16	–32

Even power
Positive result

Odd power
Negative result

We will now extend the definition of the square root to higher roots.

Definition: Nth Root of a Number

If $b^n = a$, then b is an n^{th} root of a.

The principal n^{th} root of a is written $\sqrt[n]{a}$.

The n is called the **index** of the radical.

Just like we use the word 'cubed' for b^3, we use the term 'cube root' for $\sqrt[3]{a}$.

Refer to the earlier table to find higher roots.

$$4^3 = 64 \qquad \sqrt[3]{64} = 4$$
$$3^4 = 81 \qquad \sqrt[4]{81} = 3$$
$$(-2)^5 = -32 \qquad \sqrt[5]{-32} = -2$$

Could we have an even root of a negative number? We know that the square root of a negative number is not a real number. The same is true for any even root. *Even* roots of negative numbers are not real numbers. *Odd* roots of negative numbers are real numbers.

Properties of $\sqrt[n]{a}$

When n is an even number and

- $a \geq 0$, then $\sqrt[n]{a}$ is a real number.
- $a < 0$, then $\sqrt[n]{a}$ is not a real number.

When n is an odd number, $\sqrt[n]{a}$ is a real number for all values of a.

We will apply these properties in the next two examples.

Example 3

Simplify:

a. $\sqrt[3]{64}$
b. $\sqrt[4]{81}$
c. $\sqrt[5]{32}$

Solution:

a. Since $4^3 = 64$ we have $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$.

b. Since $3^4 = 81$ we have $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$.

c. Since $2^5 = 32$ we have $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$.

Try It 3

Simplify:

a. $\sqrt[3]{1000}$
b. $\sqrt[4]{16}$
c. $\sqrt[5]{243}$

Answer

a. 10
b. 2
c. 3

In this example, be alert to both negative signs and even and odd powers.

Example 4

Simplify:

a. $\sqrt[3]{-125}$
b. $\sqrt[4]{16}$
c. $\sqrt[5]{-243}$

Solution:

a. Since $(-5)^3 = -125$ we have $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$.

b. $\sqrt[4]{-16}$ has no real solution because no real number raised to the 4th power equals a negative number.

c. Since $(-3)^5 = -243$ we have $\sqrt[5]{-243} = \sqrt[5]{(-3)^5} = -3$.

Try It 4

Simplify:

a. $\sqrt[3]{-27}$
b. $\sqrt[4]{-256}$
c. $-\sqrt[5]{-32}$

Answer

a. -3
b. not real

c. 2

Estimate and Approximate Roots

When we see a number with a radical sign, we often don't think about its numerical value. While we probably know that the $\sqrt{4} = 2$, what is the value of $\sqrt{21}$ or $\sqrt[3]{50}$? In some situations, a quick estimate is meaningful, and in others, it is convenient to have a decimal approximation.

To get a numerical estimate of a square root, we look for **perfect square** numbers closest to the radicand. To find an estimate of $\sqrt{11}$, we see 11 is between perfect square numbers 9 and 16, *closer* to 9. Its square root then will be between 3 and 4, but closer to 3.

Number	Square Root
4	2
9	3
16	4
25	5

11 → $\sqrt{11}$

$9 < 11 < 16$

$3 < \sqrt{11} < 4$

Number	Cube Root
8	2
27	3
64	4
125	5

91 → $\sqrt[3]{91}$

$64 < 91 < 125$

$4 < \sqrt[3]{91} < 5$

Similarly, to estimate $\sqrt[3]{91}$, we see 91 is between perfect cube numbers 64 and 125. The cube root then will be between 4 and 5.

Example 5

Estimate each root between two consecutive whole numbers:

a. $\sqrt{105}$
b. $\sqrt[3]{43}$

Solution:

a. Think of the perfect square numbers closest to 105. Make a small table of these perfect squares and their square roots.

	$\sqrt{105}$
	105 → $\sqrt{105}$
	Number / Square Root: 81 / 9; 100 / 10; 121 / 11; 144 / 12
Locate 105 between two consecutive perfect squares.	$100 < 105 < 121$
$\sqrt{105}$ is between their square roots.	$10 < \sqrt{105} < 11$

b. Similarly we locate 43 between two perfect cube numbers.

	$\sqrt[3]{43}$
	43 → $\sqrt[3]{43}$
	Number / Cube Root: 8 / 2; 27 / 3; 64 / 4; 125 / 5
Locate 43 between two consecutive perfect cubes.	$27 < 43 < 64$
$\sqrt[3]{43}$ is between their cube roots.	$3 < \sqrt[3]{43} < 4$

Try It 5

Estimate each root between two consecutive whole numbers:

a. $\sqrt{84}$
b. $\sqrt[3]{152}$

Answer

a. $9 < \sqrt{84} < 10$
b. $5 < \sqrt[3]{152} < 6$

There are mathematical methods to approximate square roots, but nowadays most people use a calculator to find square roots. To find a square root, you will use the $\sqrt{x}$ key on your calculator. To find a cube root, or any root with a higher index, you will use the $\sqrt[n]{x}$ key.

When you use these keys, you get an approximate value. It is an approximation, accurate to the number of digits shown on your calculator's display. The symbol for an approximation is $\approx$ and it is read 'approximately'.

Suppose your calculator has a 10 digit display. You would see that

$$\sqrt{5} \approx 2.236067978 \text{ rounded to two decimal places is } \sqrt{5} \approx 2.24$$

$$\sqrt[4]{93} \approx 3.105422799 \text{ rounded to two decimal places is } \sqrt[4]{93} \approx 3.11$$

How do we know these values are approximations and not the exact values? Look at what happens when we square them:

$$(2.236067978)^2 = 5.000000002 \qquad (3.105422799)^4 = 92.999999991$$
$$(2.24)^2 = 5.0176 \qquad (3.11)^4 = 93.54951841$$

The results are close to 5 and 93, but not exact, so the used values are approximations, not exact answers.

Example 6

Round to two decimal places:

a. $\sqrt{17}$
b. $\sqrt[3]{49}$
c. $\sqrt[4]{51}$

Solution:

a. $\sqrt{17} \approx 4.12$

b. $\sqrt[3]{49} \approx 3.66$

c. $\sqrt[4]{51} \approx 2.67$

Try It 6

Round to two decimal places:

1. $\sqrt{11}$
2. $\sqrt[3]{71}$
3. $\sqrt[4]{127}$

Answer

a. ≈ 3.32
b. ≈ 4.14
c. ≈ 3.36

Simplify Variable Expressions with Square Roots

What if we have to find the square root of an expression with a variable? Consider $\sqrt{9x^2}$. Can you think of an expression whose square is $9x^2$?

$$(?)^2 = 9x^2$$
$$(3x)^2 = 9x^2 \quad \text{so} \sqrt{9x^2} = 3x$$

When we use the radical sign to take the square root of a variable expression, we should specify that x≥0x≥0 to make sure we get the *principal square root*.

However, in this chapter, we will assume that each variable in a square-root expression represents a non-negative number, and so we will not write $x \geq 0$ next to every radical.

What about square roots of higher powers of variables? Think about the Power Property of Exponents we used earlier.

$$(a^m)^n = a^{m \cdot n}$$

If we square a^m, the exponent will become 2m.

$$(a^m)^2 = a^{2m}$$

How does this help us take square roots? Let's look at a few:

$$\sqrt{25u^8} = 5u^4 \quad \text{Because} (5u^4)^2 = 25u^8$$
$$\sqrt{16r^{20}} = 4r^{10} \quad \text{Because} (4r^{10})^2 = 16r^{20}$$
$$\sqrt{196q^{36}} = 14q^{18} \quad \text{Because} (14r^{18})^2 = 196q^{36}$$

Example 7

Simplify:

a. $\sqrt{x^6}$
b. $\sqrt{y^{16}}$

Solution

a. Since $(x^3)^2 = x^6$ we have $\sqrt{x^6} = \sqrt{(x^3)^2} = x^3$.
b. Since $(y^8)^2 = y^{16}$ we have $\sqrt{y^{16}} = \sqrt{(y^8)^2} = y^8$.

Try It 7

Simplify:

a. $\sqrt{m^4}$
b. $\sqrt{b^{10}}$.

Answer

a. m^2
b. b^5

Example 8

Simplify: $\sqrt{16n^2}$

Solution

Since $(4n)^2 = 16n^2$ we have $\sqrt{16n^2} = \sqrt{(4n)^2} = 4n$.

Try It 8

Simplify: $\sqrt{169y^2}$.

Answer

$13y$

Example 9

Simplify: $-\sqrt{81c^2}$.

Solution

Since $(9c)^2 = 81c^2$ we have $\sqrt{81c^2} = \sqrt{(9c)^2} = 9c$.

Try It 9

Simplify: $-\sqrt{100p^2}$.

Answer

$-10p$

Example 10

Simplify: $\sqrt{36x^2y^2}$.

Solution

Since $36x^2y^2 = (6xy)^2$ we have $\sqrt{36x^2y^2} = \sqrt{(6xy)^2} = 6xy$

Try It 10

Simplify: $\sqrt{225m^2n^2}$.

Answer

15mn

Example 11

Simplify: $\sqrt{121a^6b^8}$

Solution

Since $121a^6b^8 = (11a^3b^4)^2$ we have $\sqrt{121a^6b^8} = \sqrt{(11a^3b^4)^2} = 11a^3b^4$

Try It 11

Simplify: $\sqrt{169x^{10}y^{14}}$

Answer

$13x^5y^7$

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Section 2.1P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$\sqrt{36}$

Exercise 2

$\sqrt{64}$

Exercise 3

$\sqrt{100}$

Exercise 4

$-\sqrt{4}$

Exercise 5

$-\sqrt{1}$

Exercise 6

$\sqrt{-121}$

Exercise 7

$\sqrt{9+16}$

Exercise 8

$\sqrt{9}+\sqrt{16}$

In the following exercises, estimate each square root between two consecutive whole numbers.

Exercise 9

$\sqrt{70}$

Exercise 10

$\sqrt{200}$

In the following exercises, approximate each square root and round to two decimal places.

Exercise 11

$\sqrt{19}$

Exercise 12

$\sqrt{53}$

In the following exercises, simplify. Assume all variables are non-negative.

Exercise 13

$\sqrt{y^2}$

Exercise 14

$\sqrt{a^{14}}$

Exercise 15

$\sqrt{49x^2}$

Exercise 16

$\sqrt{121m^{20}}$

Exercise 17

$-\sqrt{81x^{18}}$

Exercise 18

$\sqrt{9c^8d^{12}}$

Everyday Math

Exercise 19

Denise wants to have a square accent of designer tiles in her new shower. She can afford to buy 625 square centimeters of the designer tiles. How long can a side of the accent be?

Writing Exercises

Exercise 20

Why is there no real number equal to $\sqrt{-64}$?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			
simplify variable expressions with square roots.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.2: Simplify Square Roots

Be Prepared

We will rely heavily on these skills throughout this section.

- Find the prime factorization of 36
- Identify: Is 49 a perfect square? What about 48?
- Simplify $-\sqrt{25}$
- Multiply: $3 \cdot 2 \cdot 5$

Learning Objectives

Motivating Problem

Melissa accidentally drops a pair of sunglasses from the top of a roller coaster, 64 feet above the ground. We will need to simplify $\sqrt{\frac{64}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground.

Fun Fact

The ancient Babylonians developed a method for estimating square roots more than 4,000 years ago. Clay tablets show they could estimate roots like $\sqrt{2}$ accurately to several decimal places—without calculators!

The Goal

In this section, we'll learn how to simplify square roots that are not perfect squares by factoring out perfect square factors. We'll break down radicals using prime factorization and simplify expressions to a cleaner, more usable form.

In the last section, we estimated the square root of a number between two consecutive whole numbers. We can say that $\sqrt{50}$ is between 7 and 8. This is fairly easy to do when the numbers are small enough to use our table of square root values.

But what if we want to estimate $\sqrt{500}$? If we simplify the square root first, we'll be able to estimate it easily. There are other reasons, too, to simplify square roots as you'll see later in this chapter.

A square root is considered *simplified* if its radicand contains no perfect square factors.

Definition: SIMPLIFIED SQUARE ROOT

$\sqrt{a}$ is considered simplified if a has no perfect square factors.

So $\sqrt{31}$ is simplified. But $\sqrt{32}$ is not simplified, because 16 is a perfect square factor of 32.

Use the Product Property to Simplify Square Roots

The properties we will use to simplify expressions with square roots are similar to the properties of exponents. We know that $(ab)^m = a^m b^m$. The corresponding property of square roots says that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Definition: PRODUCT PROPERTY OF SQUARE ROOTS

If a, b are non-negative real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

We use the Product Property of Square Roots to remove all perfect square factors from a radical.

Example 1

Simplify: $\sqrt{50}$.

Solution

Step 1. Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect square factor.	25 is the largest perfect square factor of 50. $50 = 25 \cdot 2$ Always write the perfect square factor first.	$\sqrt{50}$ $\sqrt{25 \cdot 2}$
Step 2. Use the product rule to rewrite the radical as the product of two radicals.		$\sqrt{25} \cdot \sqrt{2}$
Step 3. Simplify the square root of the perfect square.		$5\sqrt{2}$

Try It 1

Simplify: $\sqrt{48}$.

Answer

$4\sqrt{3}$

Notice in the previous example that the simplified form of $\sqrt{50}$ is $5\sqrt{2}$, which is the product of an integer and a square root. We always write the integer in front of the square root.

Definition: SIMPLIFY A SQUARE ROOT USING THE PRODUCT PROPERTY.

1. Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect-square factor.
2. Use the product rule to rewrite the radical as the product of two radicals.
3. Simplify the square root of the perfect square.

Example 2

Simplify: $\sqrt{500}$.

Solution

	$\sqrt{500}$
Rewrite the radicand as a product using the largest perfect square factor	$\sqrt{100 \cdot 5}$
Rewrite the radical as the product of two radicals	$\sqrt{100} \cdot \sqrt{5}$
Simplify	$10\sqrt{5}$

Try It 2

Simplify:$\sqrt{432}$.

Answer

$12\sqrt{3}$

We could use the simplified form $10\sqrt{5}$ to estimate $\sqrt{500}$. We know $\sqrt{5}$ is between 2 and 3, and $\sqrt{500}$ is $10\sqrt{5}$. So $\sqrt{500}$ is between 20 and 30.

The next example is similar to the previous ones, but with variables.

Example 3

Simplify: $\sqrt{x^3}$.

Solution

	$\sqrt{x^3}$
Rewrite the radicand as a product using the largest perfect square factor	$\sqrt{x^2 \cdot x}$
Rewrite the radical as the product of two radicals	$\sqrt{x^2} \cdot \sqrt{x}$
Simplify	$x\sqrt{x}$

Try It 3

Simplify:$\sqrt{b^5}$.

Answer

$b^2\sqrt{b}$

We follow the same procedure when there is a coefficient in the radical, too.

Example 4

Simplify: $\sqrt{25y^5}$.

Solution

	$\sqrt{25y^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{25y^4 \cdot y}$
Rewrite the radical as the product of two radicals.	$\sqrt{25y^4} \cdot \sqrt{y}$
Simplify.	$5y^2\sqrt{y}$

Try It 4

Simplify: $\sqrt{16x^7}$.

Answer

$4x^3\sqrt{x}$

In the next example, both the constant and the variable have perfect square factors.

Example 5

Simplify: $\sqrt{72n^7}$.

Solution

	$\sqrt{72n^7}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{36n^6 \cdot 2n}$
Rewrite the radical as the product of two radicals.	$\sqrt{36n^6} \cdot \sqrt{2n}$
Simplify.	$6n^3\sqrt{2n}$

Try It 5

Simplify: $\sqrt{32y^5}$.

Answer

$4y^2\sqrt{2y}$

Example 6

Simplify: $\sqrt{63u^3v^5}$.

Solution

	$\sqrt{63u^3v^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{9u^2v^4 \cdot 7uv}$
Rewrite the radical as the product of two radicals.	$\sqrt{9u^2v^4} \cdot \sqrt{7uv}$
Simplify.	$3uv^2\sqrt{7uv}$

Try It 6

Simplify: $\sqrt{98a^7b^5}$.

Answer

$7a^3b^2\sqrt{2ab}$

We have seen how to use the Order of Operations to simplify some expressions with radicals. To simplify $\sqrt{25}+\sqrt{144}$, we must simplify each square root separately first, then add to get the sum of 17.

The expression $\sqrt{17}+\sqrt{7}$ cannot be simplified—to begin, we'd need to simplify each square root, but neither 17 nor 7 contains a perfect square factor.

In the next example, we have the sum of an integer and a square root. We simplify the square root, but cannot add the resulting expression to the integer.

Example 7

Simplify: $3+\sqrt{32}$.

Solution

	$3+\sqrt{32}$
Rewrite the radicand as a product using the largest perfect square factor.	$3+\sqrt{16 \cdot 2}$
Rewrite the radical as the product of two radicals.	$3+\sqrt{16} \cdot \sqrt{2}$
Simplify.	$3+4\sqrt{2}$

The terms are not like, and so we cannot add them. Trying to add an integer and a radical is like trying to add an integer and a variable—they are not like terms!

Try It 7

Simplify: $5+\sqrt{75}$.

Answer

$5+5\sqrt{3}$

The next example includes a fraction with a radical in the numerator. Remember that in order to simplify a fraction you need a common factor in the numerator and denominator.

Example 8

Simplify: $\frac{4-\sqrt{48}}{2}$.

Solution

	$\frac{4-\sqrt{48}}{2}$
Rewrite the radicand as a product using thelargest perfect square factor.	$\frac{4-\sqrt{16 \cdot 3}}{2}$
Rewrite the radical as the product of two radicals.	$\frac{4-\sqrt{16} \cdot \sqrt{3}}{2}$
Simplify.	$\frac{4-4\sqrt{3}}{2}$
Factor the common factor from thenumerator.	$\frac{4(1-\sqrt{3})}{2}$
Remove the common factor, 2, from thenumerator and denominator.	$2(1-\sqrt{3})$

Try It 8

Simplify: $\frac{10-\sqrt{75}}{5}$.

Answer

$2-\sqrt{3}$

Use the Quotient Property to Simplify Square Roots

Whenever you have to simplify a square root, the first step you should take is to determine whether the radicand is a perfect square. A *perfect square fraction* is a fraction in which both the numerator and the denominator are perfect squares.

Example 9

Simplify: $\sqrt{\frac{9}{64}}$.

Solution

	$\sqrt{\frac{9}{64}}$
Since $(\frac{3}{8})^2$	$\frac{3}{8}$

Try It 9

Simplify: $\sqrt{\frac{25}{16}}$.

Answer

$\frac{5}{4}$

If the numerator and denominator have any common factors, remove them. You may find a perfect square fraction!

Example 10

Simplify: $\sqrt{\frac{45}{80}}$.

Solution

	$\sqrt{\frac{45}{80}}$
Simplify inside the radical first. Rewrite the expression to show the common factors of the numerator and denominator.	$\sqrt{\frac{5 \cdot 9}{5 \cdot 16}}$
Simplify the fraction by removing common factors.	$\sqrt{\frac{9}{16}}$
Simplify. $(\frac{3}{4})^2 = \frac{9}{16}$	$\frac{3}{4}$

? Try It 10

Simplify: $\sqrt{\frac{98}{162}}$.

Answer

$\frac{7}{9}$

In the last example, our first step was to simplify the fraction under the radical by removing common factors. In the next example, we will use the Quotient Property to simplify expressions under radicals. We divide the like bases by subtracting their exponents, $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$.

✔ Example 11

Simplify: $\sqrt{\frac{m^6}{m^4}}$.

Solution

	$\sqrt{\frac{m^6}{m^4}}$
Simplify the fraction inside the radical first	$\sqrt{m^2}$
Divide the like bases by subtracting the exponents.	
Simplify.	m

? Try It 11

Simplify: $\sqrt{\frac{x^{14}}{x^{10}}}$.

Answer

x^2

✔ Example 12

Simplify: $\sqrt{\frac{48p^7}{3p^3}}$.

Solution

	$\sqrt{\frac{48p^7}{3p^3}}$
Simplify the fraction inside the radical first.	$\sqrt{16p^4}$
Simplify.	$4p^2$

Try It 12

Simplify: $\sqrt{\frac{75x^5}{3x}}$.

Answer

$5x^2$

Remember the Quotient to a Power Property? It stated that we can raise a fraction to a power by raising the numerator and denominator to the power separately.

$(\frac{a}{b})^m = \frac{a^m}{b^m}, b \neq 0$

We can use a similar property to simplify the square root of a fraction. After removing all common factors from the numerator and denominator, if the fraction is not a perfect square, we simplify the numerator and denominator separately.

Definition: QUOTIENT PROPERTY OF SQUARE ROOTS

If a, b are non-negative real numbers and $b \neq 0$, then

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example 13

Simplify: $\sqrt{\frac{21}{64}}$.

Solution

	$\sqrt{\frac{21}{64}}$
We cannot simplify the fraction inside the radical.	$\frac{\sqrt{21}}{\sqrt{64}}$
Rewrite using the quotient property.	
Simplify the square root of 64. The numerator cannot be simplified.	$\frac{\sqrt{21}}{8}$

Try It 13

Simplify: $\sqrt{\frac{19}{49}}$.

Answer

$\frac{\sqrt{19}}{7}$

How to Use the Quotient Property to Simplify a Square Root.

Example 14

Simplify: $\sqrt{\frac{27m^3}{196}}$.

Solution

Step 1. Simplify the fraction in the radicand, if possible.	$\frac{27m^3}{196}$ cannot be simplified.	$\sqrt{\frac{27m^3}{196}}$
Step 2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.	We rewrite $\sqrt{\frac{27m^3}{196}}$ as the quotient of $\sqrt{27m^3}$ and $\sqrt{196}$.	$\frac{\sqrt{27m^3}}{\sqrt{196}}$

Step 3. Simplify the radicals in the numerator and the denominator.	$9m^2$ and 196 are perfect squares.	$\frac{\sqrt{9m^2}\cdot\sqrt{3m}}{\sqrt{196}}$ $\frac{3m\sqrt{3m}}{14}$

? Try It 14

Simplify: $\sqrt{\frac{48x^5}{100}}$

Answer

$\frac{2x^2\sqrt{3x}}{5}$

Definition: SIMPLIFY A SQUARE ROOT USING THE QUOTIENT PROPERTY.

1. Simplify the fraction in the radicand, if possible.
2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.
3. Simplify the radicals in the numerator and the denominator.

✓ Example 15

Simplify: $\sqrt{\frac{45x^5}{y^4}}$.

Solution

	$\sqrt{\frac{45x^5}{y^4}}$
We cannot simplify the fraction inside the radical. Rewrite using the quotient property.	$\frac{\sqrt{45x^5}}{\sqrt{y^4}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{9x^4}\sqrt{5x}}{y^2}$
Simplify.	$\frac{3x^2\sqrt{5x}}{y^2}$

? Try It 15

Simplify: $\sqrt{\frac{80m^3}{n^6}}$

Answer

$\frac{4m\sqrt{5m}}{n^3}$

Be sure to simplify the fraction in the radicand first, if possible.

✓ Example 16

Simplify: $\sqrt{\frac{81d^9}{25d^4}}$.

Solution

	$\sqrt{\frac{81d^9}{25d^4}}$
Simplify the fraction in the radicand.	$\sqrt{\frac{81d^5}{25}}$
Rewrite using the quotient property.	$\frac{\sqrt{81d^5}}{\sqrt{25}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{81d^4}\sqrt{d}}{5}$
Simplify.	$\frac{9d^2\sqrt{d}}{5}$

Try It 16

Simplify: $\sqrt{\frac{64x^7}{9x^3}}$.

Answer

$\frac{8x^2}{3}$

Example 17

Simplify: $\sqrt{\frac{18p^5q^7}{32pq^2}}$.

Solution

	$\sqrt{\frac{18p^5q^7}{32pq^2}}$
Simplify the fraction in the radicand.	$\sqrt{\frac{9p^4q^5}{16}}$
Rewrite using the quotient property.	$\frac{\sqrt{9p^4q^5}}{\sqrt{16}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{9p^4q^4}\sqrt{q}}{4}$
Simplify.	$\frac{3p^2q^2\sqrt{q}}{4}$

Try It 17

Simplify: $\sqrt{\frac{50x^5y^3}{72x^4y}}$.

Answer

$\frac{5y\sqrt{x}}{6}$

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Section 2.2P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$\sqrt{125}$

Exercise 2

$\sqrt{200}$

Exercise 3

$\sqrt{450}$

Exercise 4

$\sqrt{800}$

Exercise 5

$\sqrt{675}$

Exercise 6

$\sqrt{x^7}$

Exercise 7

$\sqrt{p^3}$

Exercise 8

$\sqrt{m^{13}}$

Exercise 9

$\sqrt{49n^{17}}$

Exercise 10

$\sqrt{81r^{15}}$

Exercise 11

$\sqrt{98m^5}$

Exercise 12

$\sqrt{300p^9q^{11}}$

Exercise 13

$\sqrt{242m^{13}n^{21}}$

Exercise 14

$5+\sqrt{12}$

Exercise 15

$1+\sqrt{45}$

Exercise 16

$\frac{10-\sqrt{24}}{2}$

Exercise 17

$\frac{3+\sqrt{90}}{3}$

Exercise 18

$\sqrt{\frac{45}{125}}$

Exercise 19

$\sqrt{\frac{x^{10}}{x^6}}$

Exercise 20

$\sqrt{\frac{y^4}{y^8}}$

Exercise 21

$\sqrt{\frac{200x^7}{2x^3}}$

Exercise 22

$\sqrt{\frac{96x^7}{121}}$

Exercise 23

$\sqrt{\frac{75r^9}{s^8}}$

Exercise 24

$\sqrt{\frac{28p^7}{q^2}}$

Exercise 25

$\sqrt{\frac{121p^5}{81p^2}}$

Exercise 26

$\sqrt{\frac{27p^2q^{10}}{8p^5q^3}}$

Everyday Math

Exercise 27

a. Elliott decides to construct a square garden that will take up 288 square feet of his yard. Simplify $\sqrt{288}$ to determine the length and the width of his garden. Round to the nearest tenth of a foot.

b. Suppose Elliott decides to reduce the size of his square garden so that he can create a 5-foot-wide walking path on the north and east sides of the garden. Simplify $\sqrt{288}-5$ to determine the length and width of the new garden. Round to the nearest tenth of a foot.

Writing Exercises

Exercise 28

Explain why $\sqrt{x^4}=x^2$. Then explain why $\sqrt{x^{16}}=x^8$.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the Product Property to simplify square roots.			
use the Quotient Property to simplify square roots.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.3: Add and Subtract Square Roots

Be Prepared

We will rely heavily on these skills throughout this section.

- Combine like terms: $3x - 5y - 2x + 3y$
- Determine all possible factorizations of 36
- List the perfect squares less than 100

Learning Objectives

Motivating Problem

You're landscaping a garden with square tiles. One path is made of tiles with an area of 50 square meters, and another with an area of 18 square meters. Can you estimate and combine the total length of the paths?

Fun Fact

In the 9th century, Persian mathematician **Al-Khwarizmi** used early versions of square root expressions in his algebraic work. The word "algebra" comes from the title of his book, *Al-Kitab al-Mukhtasar fi Hisab al-Jabr wal-Muqabala* which roughly translates as "The Compendious Book on Calculation by Completion and Balancing".

The Goal

In this section, we'll learn how to simplify, add, and subtract square roots by identifying like terms. We'll also practice combining simplified radicals and recognizing when terms can or can't be combined.

We know that we must follow the order of operations to simplify expressions with square roots. The radical is a grouping symbol, so we work inside the radical first. We simplify $\sqrt{2+7}$ in this way:

	$\sqrt{2+7}$
Add inside the radical.	$\sqrt{9}$
Simplify.	3

So if we have to add $\sqrt{2}+\sqrt{7}$, we must not combine them into one radical.

$\sqrt{2}+\sqrt{7} \neq \sqrt{2+7}$

Trying to add square roots with different radicands is similar to adding unlike terms.

But, just like we can	$x+x$	**we can add**	$\sqrt{3}+\sqrt{3}$
	$x+x=2x$		$\sqrt{3}+\sqrt{3}=2\sqrt{3}$

Adding square roots with the same radicand is just like adding like terms. We call square roots with the same radicand like square roots to remind us they work the same as like terms.

Definition: LIKE SQUARE ROOTS

Square roots with the same radicand are called **like square roots**.

We add and subtract like square roots in the same way we add and subtract like terms. We know that \(3x+8x\) is \(11x\). Similarly we add $3\sqrt{x}+8\sqrt{x}$ and the result is $11\sqrt{x}$.

Add and Subtract Like Square Roots

Consider combining like terms with variables as you work through the following few examples. When you have like radicands, you just add or subtract the coefficients. When the radicands are not like, you cannot combine the terms.

Example 1

Simplify: $2\sqrt{2}-7\sqrt{2}$.

Solution

	$2\sqrt{2}-7\sqrt{2}$
Since the radicals are like, we subtract the coefficients.	$-5\sqrt{2}$

Try It 1

Simplify: $5\sqrt{3}-9\sqrt{3}$.

Answer

$-4\sqrt{3}$

Example 2

Simplify: $3\sqrt{y}+4\sqrt{y}$.

Solution

	$3\sqrt{y}+4\sqrt{y}$
Since the radicals are like, we add the coefficients.	$7\sqrt{y}$

Try It 2

Simplify: $5\sqrt{u}+3\sqrt{u}$.

Answer

$8\sqrt{u}$

Example 3

Simplify: $4\sqrt{x}-2\sqrt{y}$

Solution

	$4\sqrt{x}-2\sqrt{y}$
Since the radicals are not like, we cannot subtract them. We leave the expression as is.	$4\sqrt{x}-2\sqrt{y}$

Try It 3

Simplify: $7\sqrt{p}-6\sqrt{q}$.

Answer

$7\sqrt{p}-6\sqrt{q}$

Example 4

Simplify: $5\sqrt{13}+4\sqrt{13}+2\sqrt{13}$.

Solution

	$5\sqrt{13}+4\sqrt{13}+2\sqrt{13}$
Since the radicals are like, we add the coefficients.	$11\sqrt{13}$

Try It 4

Simplify: $6\sqrt{10}+2\sqrt{10}+3\sqrt{10}$.

Answer

$11\sqrt{10}$

Example 5

Simplify: $2\sqrt{6}-6\sqrt{6}+3\sqrt{3}$.

Solution

	$2\sqrt{6}-6\sqrt{6}+3\sqrt{3}$
Since the first two radicals are like, we subtract their coefficients.	$-4\sqrt{6}+3\sqrt{3}$

Try It 5

Simplify: $3\sqrt{7}-8\sqrt{7}+2\sqrt{5}$.

Answer

$-5\sqrt{7}+2\sqrt{5}$

Example 6

Simplify: $2\sqrt{5n}-6\sqrt{5n}+4\sqrt{5n}$.

Solution

	$2\sqrt{5n}-6\sqrt{5n}+4\sqrt{5n}$
Since the radicals are like, we combine them.	$-0\sqrt{5n}$
Simplify.	0

Try It 6

Simplify: $4\sqrt{3y}-7\sqrt{3y}+2\sqrt{3y}$.

Answer

$-\sqrt{3y}$

When radicals contain more than one variable, as long as all the variables and their exponents are identical, the radicals are like.

Example 7

Simplify: $\sqrt{3xy}+5\sqrt{3xy}-4\sqrt{3xy}$.

Solution

	$\sqrt{3xy}+5\sqrt{3xy}-4\sqrt{3xy}$
Since the radicals are like, we combine them.	$2\sqrt{3xy}$

? Try It 7

Simplify: $3\sqrt{7mn}+\sqrt{7mn}-4\sqrt{7mn}$.

Answer

0

Add and Subtract Square Roots that Need Simplification

Remember that we always simplify square roots by removing the largest perfect-square factor. Sometimes, when we have to add or subtract square roots that do not appear to have **like radicals**, we find like radicals after simplifying the square roots.

✓ Example 8

Simplify: $\sqrt{20}+3\sqrt{5}$.

Solution

	$\sqrt{20}+3\sqrt{5}$
Simplify the radicals, when possible.	$\sqrt{4}\cdot\sqrt{5}+3\sqrt{5}$
	$2\sqrt{5}+3\sqrt{5}$
Combine the like radicals.	$5\sqrt{5}$

? Try it 8

Simplify: $\sqrt{18}+6\sqrt{2}$.

Answer

$9\sqrt{2}$

✓ Example 9

Simplify: $\sqrt{48}-\sqrt{75}$

Solution

	$\sqrt{48}-\sqrt{75}$
Simplify the radicals.	$\sqrt{16}\cdot\sqrt{3}-\sqrt{25}\cdot\sqrt{3}$
	$4\sqrt{3}-5\sqrt{3}$
Combine the like radicals.	$-\sqrt{3}$

? Try It 9

Simplify: $\sqrt{20}-\sqrt{45}$.

Answer

$-\sqrt{5}$

Just like we use the Associative Property of Multiplication to simplify 5(3x) and get \(15x\), we can simplify $5(3\sqrt{x})$ and get $15\sqrt{x}$. We will use the Associative Property in the next example.

Example 10

Simplify: $5\sqrt{18}-2\sqrt{8}$.

Solution

	$5\sqrt{18}-2\sqrt{8}$
Simplify the radicals.	$5\cdot\sqrt{9}\cdot\sqrt{2}-2\cdot\sqrt{4}\cdot\sqrt{2}$
	$5\cdot 3\cdot\sqrt{2}-2\cdot 2\cdot\sqrt{2}$
	$15\sqrt{2}-4\sqrt{2}$
Combine the like radicals.	$11\sqrt{2}$

Try It 10

Simplify: $3\sqrt{20}-7\sqrt{45}$.

Answer

$-15\sqrt{5}$

Example 11

Simplify: $\frac{3}{4}\sqrt{192}-\frac{5}{6}\sqrt{108}$.

Solution

	$\frac{3}{4}\sqrt{192}-\frac{5}{6}\sqrt{108}$
Simplify the radicals.	$\frac{3}{4}\sqrt{64}\cdot\sqrt{3}-\frac{5}{6}\sqrt{36}\cdot\sqrt{3}$
	$\frac{3}{4}\cdot 8\cdot\sqrt{3}-\frac{5}{6}\cdot 6\cdot\sqrt{3}$
	$6\sqrt{3}-5\sqrt{3}$
Combine the like radicals.	$\sqrt{3}$

Try It 11

Simplify: $\frac{3}{5}\sqrt{200}-\frac{3}{4}\sqrt{128}$.

Answer

0

Example 12

Simplify: $\frac{2}{3}\sqrt{48}-\frac{3}{4}\sqrt{12}$.

Solution

	$\frac{2}{3}\sqrt{48}-\frac{3}{4}\sqrt{12}$
Simplify the radicals.	$\frac{2}{3}\sqrt{16}\cdot\sqrt{3}-\frac{3}{4}\sqrt{4}\cdot\sqrt{3}$
	$\frac{2}{3}\cdot 4\cdot\sqrt{3}-\frac{3}{4}\cdot 2\cdot\sqrt{3}$
	$\frac{8}{3}\sqrt{3}-\frac{3}{2}\sqrt{3}$
Find a common denominator to subtract the coefficients of the like radicals.	$\frac{16}{6}\sqrt{3}-\frac{9}{6}\sqrt{3}$
Simplify.	$\frac{7}{6}\sqrt{3}$

Try It 12

Simplify: $\frac{2}{5}\sqrt{32}-\frac{1}{3}\sqrt{8}$

Answer

$\frac{14}{15}\sqrt{2}$

In the next example, we will remove constant and variable factors from the square roots.

Example 13

Simplify: $\sqrt{18n^5}-\sqrt{32n^5}$

Solution

	$\sqrt{18n^5}-\sqrt{32n^5}$
Simplify the radicals.	$\sqrt{9n^4}\cdot\sqrt{2n}-\sqrt{16n^4}\cdot\sqrt{2n}$
	$3n^2\sqrt{2n}-4n^2\sqrt{2n}$
Combine the like radicals.	$-n^2\sqrt{2n}$

Try It 13

Simplify: $\sqrt{27p^3}-\sqrt{48p^3}$

Answer

$-p\sqrt{3p}$

Example 14

Simplify: $2\sqrt{8x^2}-5x\sqrt{32}+5\sqrt{18x^2}$.

Solution

	$2\sqrt{8x^2}-5x\sqrt{32}+5\sqrt{18x^2}$
Simplify the radicals.	$2\sqrt{4x^2}\cdot\sqrt{2}-5x\sqrt{16}\cdot\sqrt{2}+5\sqrt{9x^2}\cdot\sqrt{2}$
	$2\cdot 2x\cdot\sqrt{2}-5x\cdot 4\cdot\sqrt{2}+5\cdot 3x\cdot\sqrt{2}$
	$4x\sqrt{2}-20x\sqrt{2}+15x\sqrt{2}$
Combine the like radicals.	$-x\sqrt{2}$

? Try It 14

Simplify: $3\sqrt{12x^2} - 2x\sqrt{48} + 4\sqrt{27x^2}$

Answer

$10x\sqrt{3}$

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Section 2.3P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$3\sqrt{5}+6\sqrt{5}$

Exercise 2

$9\sqrt{7}-10\sqrt{7}$

Exercise 3

$\sqrt{a}-4\sqrt{a}$

Exercise 4

$8\sqrt{a}-2\sqrt{b}$

Exercise 5

$8\sqrt{7}+2\sqrt{7}+3\sqrt{7}$

Exercise 6

$5\sqrt{3ab}+\sqrt{3ab}-2\sqrt{3ab}$

Exercise 7

$\sqrt{50}+4\sqrt{2}$

Exercise 8

$\sqrt{80}-3\sqrt{5}$

Exercise 9

$\sqrt{27}-\sqrt{75}$

Exercise 10

$\sqrt{48}+\sqrt{27}$

Exercise 11

$2\sqrt{50}-3\sqrt{72}$

Exercise 12

$2\sqrt{12}+3\sqrt{48}$

Exercise 13

$\frac{1}{6}\sqrt{27}-\frac{3}{8}\sqrt{48}$

Exercise 14

$\frac{1}{4}\sqrt{98}-\frac{1}{3}\sqrt{128}$

Exercise 15

$\sqrt{72a^5}-\sqrt{50a^5}$

Exercise 16

$2\sqrt{50r^8}+4\sqrt{54r^8}$

Exercise 17

$3\sqrt{20x^2}-4\sqrt{45x^2}+5x\sqrt{80}$

Exercise 18

$\sqrt{175k^4}-\sqrt{63k^4}$

Exercise 19

$9\sqrt{2}-8\sqrt{2}$

Exercise 20

$8\sqrt{13}-4\sqrt{13}-3\sqrt{13}$

Exercise 21

$\sqrt{80a^5}-\sqrt{45a^5}$

Exercise 22

$21\sqrt{19}-2\sqrt{19}$

Exercise 23

$\frac{5}{6}\sqrt{27}+\frac{5}{8}\sqrt{48}$

Exercise 24

$\sqrt{75}-\sqrt{108}$

? Exercise 25

$4\sqrt{24x^2} - \sqrt{54x^2} + 3x\sqrt{6}$

Everyday Math

? Exercise 26

A decorator decides to use square tiles as an accent strip in the design of a new shower, but she wants to rotate the tiles to look like diamonds. She will use 9 large tiles, each measuring 8 inches on a side, and 8 small tiles, each measuring 2 inches on a side. Determine the width of the accent strip by simplifying the expression $9(8\sqrt{2}) + 8(2\sqrt{2})$. (Round to the nearest tenth of an inch.)

Writing Exercises

? Exercise 27

Explain the difference between like radicals and unlike radicals. Ensure your answer is sensible for radicals that contain both numbers and variables.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract like square roots.			
add and subtract square roots that need simplification.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.4: Multiply Square Roots

Be Prepared

We will rely heavily on these skills throughout this section.

- Multiply $3 \cdot 2 \cdot x \cdot 4 \cdot x$
- Factor 72 into the product of a perfect square and one other number
- Multiply $\sqrt{4} \cdot \sqrt{9}$

Learning Objectives

Motivating Problem

You're helping to design a rectangular garden. The length is $\sqrt{18}$ feet, and the width is $\sqrt{2}$ feet. To order the right amount of soil, you need to know the garden's exact area. What is the area of the garden?

Fun Fact

The √ symbol is thought to come from the Latin word *radix*, meaning "root." This is why we also refer to square roots as radicals. The modern radical symbol appeared in European math texts around the 1500s.

The Goal

In this section, we'll explore how to multiply square roots using the product rule for radicals. We'll also combine like terms when possible and simplify radical expressions by recognizing perfect square factors.

Multiply Square Roots

We have used the Product Property of Square Roots to simplify square roots by removing the perfect square factors. The Product Property of Square Roots says

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

We can use the Product Property of Square Roots 'in reverse' to multiply square roots.

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

Remember, we assume all variables are greater than or equal to zero.

We will rewrite the Product Property of Square Roots so we see both ways together.

Definition: PRODUCT PROPERTY OF SQUARE ROOTS

If a, b are nonnegative real numbers, then

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$.

So we can multiply $\sqrt{3} \cdot \sqrt{5}$ in this way:

$$\sqrt{3} \cdot \sqrt{5}$$
$$\sqrt{3 \cdot 5}$$
$$\sqrt{15}$$

Sometimes the product gives us a perfect square:

$$\sqrt{2} \cdot \sqrt{8}$$
$$\sqrt{2 \cdot 8}$$
$$\sqrt{16}$$
$$4$$

Even when the product is not a perfect square, we must look for perfect-square factors and simplify the radical whenever possible.

Multiplying radicals with coefficients is much like multiplying variables with coefficients. To multiply $4x \cdot 3y$, we multiply the coefficients together and then the variables. The result is $12xy$. Keep this in mind as you do these examples.

Example 1

Simplify:

a. $\sqrt{2} \cdot \sqrt{6}$
b. $(4\sqrt{3})(2\sqrt{12})$

Solution

a.	$\sqrt{2} \cdot \sqrt{6}$
Multiply using the Product Property.	$\sqrt{12}$
Simplify the radical.	$\sqrt{4} \cdot \sqrt{3}$
Simplify.	$2\sqrt{3}$
b.	$(4\sqrt{3})(2\sqrt{12})$
Multiply using the Product Property.	$8\sqrt{36}$
Simplify the radical.	$8 \cdot 6$
Simplify.	48

Try It 1

Simplify:

a. $\sqrt{5} \cdot \sqrt{10}$
b. $(6\sqrt{3})(5\sqrt{6})$

Answer

a. $5\sqrt{2}$
b. $90\sqrt{2}$

Example 2

Simplify: $(6\sqrt{2})(3\sqrt{10})$

Solution

	$(6\sqrt{2})(3\sqrt{10})$
Multiply using the Product Property.	$18\sqrt{20}$
Simplify the radical.	$18\sqrt{4} \cdot \sqrt{5}$
Simplify.	$18 \cdot 2 \cdot \sqrt{5}$
	$36\sqrt{5}$

Try It 2

Simplify: $(3\sqrt{2})(2\sqrt{30})$

Answer

$12\sqrt{15}$

When we have to multiply square roots, we first find the product and then remove any perfect square factors.

Example 3

Simplify:

a. $(\sqrt{8x^3})(\sqrt{3x})$
b. $(\sqrt{20y^2})(\sqrt{5y^3})$

Solution

a.	$(\sqrt{8x^3})(\sqrt{3x})$
Multiply using the Product Property.	$\sqrt{24x^4}$
Simplify the radical.	$\sqrt{4x^4}\cdot\sqrt{6}$
Simplify.	$2x^2\sqrt{6}$
b.	$(\sqrt{20y^2})(\sqrt{5y^3})$
Multiply using the Product Property.	$\sqrt{100y^5}$
Simplify the radical.	$10y^2\sqrt{y}$

Try It 3

Simplify:

a. $(\sqrt{6x^5})(\sqrt{2x})$
b. $(\sqrt{12y^2})(\sqrt{3y^5})$

Answer

a. $2x^3\sqrt{3}$
b. $6y^2\sqrt{y}$

Example 4

Simplify: $(10\sqrt{6p^3})(3\sqrt{18p})$

Solution

	$(10\sqrt{6p^3})(3\sqrt{18p})$
Multiply.	$30\sqrt{108p^4}$
Simplify the radical.	$30\sqrt{36p^4}\cdot\sqrt{3}$
	$30\cdot 6p^2\cdot\sqrt{3}$
	$180p^2\sqrt{3}$

Try It 4

Simplify: $(\sqrt{62x^2})(8\sqrt{45x^4})$

Answer

$24x^3\sqrt{310}$

Example 5

Simplify:

a. $(\sqrt{2})^2$
b. $(-\sqrt{11})^2$.

Solution

a.	$(\sqrt{2})^2$
Rewrite as a product.	$(\sqrt{2})(\sqrt{2})$
Multiply.	$\sqrt{4}$
Simplify.	2
b.	$(-\sqrt{11})^2$
Rewrite as a product.	$(-\sqrt{11})(-\sqrt{11})$
Multiply.	$\sqrt{121}$
Simplify.	11

Try It 5

Simplify:

a. $(\sqrt{12})^2$
b. $(-\sqrt{15})^2$.

Answer

a. 12
b. 15

The results of the previous example lead us to this property.

Definition: SQUARING A SQUARE ROOT

If a is a nonnegative real number, then

$(\sqrt{a})^2 = a$

By realizing that squaring and taking a square root are 'opposite' operations, we can simplify $(\sqrt{2})^2$ and get 2 right away. When we multiply the two **like square roots** in part (a) of the next example, it is the same as squaring.

Example 6

Simplify:

a. $(2\sqrt{3})(8\sqrt{3})$
b. $(3\sqrt{6})^2$.

Solution

a.	$(2\sqrt{3})(8\sqrt{3})$
Multiply. Remember, $(\sqrt{3^2})$	16·3
Simplify.	48
b.	$(3\sqrt{6})^2$
Multiply.	9·6
Simplify.	54

Try It 6

Simplify:

a. $(6\sqrt{11})(5\sqrt{11})$
b. $(5\sqrt{8})^2$.

Answer

a. 330
b. 200

Use Polynomial Multiplication to Multiply Square Roots

In the following few examples, we will use the Distributive Property to multiply expressions with square roots.

We will first distribute and then simplify the square roots when possible.

Example 7

Simplify:

a. $3(5-\sqrt{2})$
b. $\sqrt{2}(4-\sqrt{10})$.

Solution

a.	$3(5-\sqrt{2})$
Distribute.	$15-3\sqrt{2})$
b.	$\sqrt{2}(4-\sqrt{10})$
Distribute.	$4\sqrt{2}-\sqrt{20}$
Simplify.	$4\sqrt{2}-2\sqrt{5}$

Try It 7

Simplify:

a. $6(2+\sqrt{6})$
b. $\sqrt{7}(1+\sqrt{14})$.

Answer

a. $12+\sqrt{6}$
b. $\sqrt{7}+7\sqrt{2}$

Example 8

Simplify:

a. $\sqrt{5}(7+2\sqrt{5})$
b. $\sqrt{6}(\sqrt{2}+\sqrt{18})$.

Solution

a.	$\sqrt{5}(7+2\sqrt{5})$
Distribute.	$7\sqrt{5}+2\cdot 5$
Simplify.	$7\sqrt{5}+10$
	$10+7\sqrt{5}$
b.	$\sqrt{6}(\sqrt{2}+\sqrt{18})$
Multiply.	$\sqrt{12}+\sqrt{108}$
Simplify.	$(\sqrt{4}\cdot\sqrt{3}+\sqrt{36}\cdot\sqrt{3})$
	$2\sqrt{3}+6\sqrt{3}$
Combine like radicals.	$8\sqrt{3}$

Try It 8

Simplify:

a. $\sqrt{6}(1+3\sqrt{6})$
b. $\sqrt{12}(\sqrt{3}+\sqrt{24})$

Answer

a. $18+\sqrt{6}$
b. $6+12\sqrt{2}$

When we worked with polynomials, we multiplied binomials by binomials. Remember, this gave us four products before we combined any like terms. To ensure we obtain all four products, we organized our work using the FOIL method.

Example 9

Simplify: $(2+\sqrt{3})(4-\sqrt{3})$

Solution

	$(2+\sqrt{3})(4-\sqrt{3})$
Multiply.	$8-2\sqrt{3}+4\sqrt{3}-3$
Combine like terms.	$5+2\sqrt{3}$

Try it 9

Simplify: $(1+\sqrt{6})(3-\sqrt{6})$.

Answer

$-3+2\sqrt{6}$

Example 10

Simplify: $(3-2\sqrt{7})(4-2\sqrt{7})$.

Solution

	$(3-2\sqrt{7})(4-2\sqrt{7})$
Multiply.	$12-6\sqrt{7}-8\sqrt{7}+4\cdot 7$
Simplify.	$12-6\sqrt{7}-8\sqrt{7}+28$
Combine like terms.	$40-14\sqrt{7}$

Try It 10

Simplify: $(6-3\sqrt{7})(3+4\sqrt{7})$.

Answer

$-66+15\sqrt{7}$

Example 11

Simplify: $(3\sqrt{2}-\sqrt{5})(\sqrt{2}+4\sqrt{5})$.

Solution

	$3\sqrt{2}-\sqrt{5})(\sqrt{2}+4\sqrt{5})$
Multiply.	$3\cdot 2+12\sqrt{10}-\sqrt{10}-4\cdot 5$
Simplify.	$6+12\sqrt{10}-\sqrt{10}-20$
Combine like terms.	$-14+11\sqrt{10}$

Try It 11

Simplify: $(\sqrt{6}-3\sqrt{8})(2\sqrt{6}+\sqrt{8})$

Answer

$-12-20\sqrt{3}$

Example 12

Simplify: $(4-2\sqrt{x})(1+3\sqrt{x})$.

Solution

	$(4-2\sqrt{x})(1+3\sqrt{x})$.
Multiply.	$4+12\sqrt{x}-2\sqrt{x}-6x$
Combine like terms.	$4+10\sqrt{x}-6x$

Try It 12

Simplify: $(10+3\sqrt{n})(1-5\sqrt{n})$

Answer

$10-47\sqrt{n}-15n$

Note that some special products made our work easier when we multiplied binomials earlier. This is true when we multiply square roots, too. The special product formulas we used are shown below.

Definition: SPECIAL PRODUCT FORMULAS

Binomial Squares	Product of Conjugates
$(a+b)^2=a^2+2ab+b^2$	$(a-b)(a+b)=a^2-b^2$
$(a-b)^2=a^2-2ab+b^2$	

We will use the special product formulas in the next few examples. We will start with the Binomial Squares formula.

Example 13

Simplify:

a. $(2+\sqrt{3})^2$
b. $(4-2\sqrt{5})^2$.

Solution

Be sure to include the $2ab$ term when squaring a binomial.

a.

	$(a+b)^2$ $(2+\sqrt{3})^2$
Multiply using the binomial square pattern.	$a^2+2ab+b^2$ $2^2+2\cdot 2\cdot\sqrt{3}+(\sqrt{3})^2$
Simplify.	$4+4\sqrt{3}+3$
Combine like terms.	$7+4\sqrt{3}$

b.

	$(a-b)^2$ $(4-2\sqrt{5})^2$
Multiply using the binomial square pattern.	$a^2-2ab+b^2$ $4^2-2\cdot 4\cdot 2\sqrt{5}+(2\sqrt{5})^2$
Simplify.	$16-16\sqrt{5}+4\cdot 5$ $16-16\sqrt{5}+20$
Combine like terms.	$36-16\sqrt{5}$

Try It 13

Simplify:

a. $(10+\sqrt{2})^2$
b. $(9-2\sqrt{10})^2$.

Answer

a. $102+20\sqrt{2}$
b. $121-36\sqrt{10}$

Example 14

Simplify: $(1+3\sqrt{x})^2$.

Solution

	$(a+b)^2$ $(1+3\sqrt{x})^2$
Multiply using the binomial square pattern.	$a^2+2ab+b^2$ $1^2+2\cdot 1\cdot 3\sqrt{x}+(3\sqrt{x})^2$
Simplify.	$1+6\sqrt{x}+9x$

Try It 14

Simplify: $(2+5\sqrt{m})^2$.

Answer

$4+20\sqrt{m}+25m$

In the next two examples, we will find the product of conjugates.

Example 15

Simplify: $(4-\sqrt{2})(4+\sqrt{2})$.

Solution

	$(4-\sqrt{2})(4+\sqrt{2})$
Multiply using the binomial square pattern.	$4^2-(\sqrt{2})^2$
Simplify.	$16-2$ 14

Try It 15

Simplify: $(2-\sqrt{3})(2+\sqrt{3})$

Answer

1

Example 16

Simplify: $(5-2\sqrt{3})(5+2\sqrt{3})$

Solution

	$(5-2\sqrt{3})(5+2\sqrt{3})$
Multiply using the binomial square pattern.	$5^2-(2\sqrt{3})^2$
Simplify.	$25-4\cdot 3$ 13

Try It 16

Simplify: $(4+5\sqrt{7})(4-5\sqrt{7})$.

Answer

-159

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Section 2.4P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$\sqrt{2} \cdot \sqrt{8}$

Exercise 2

$(3\sqrt{3})(2\sqrt{18})$

Exercise 3

$\sqrt{7} \cdot \sqrt{14}$

Exercise 4

$(4\sqrt{8})(5\sqrt{8})$

Exercise 5

$(5\sqrt{2})(3\sqrt{6})$

Exercise 6

$(-2\sqrt{3})(3\sqrt{18})$

Exercise 7

$(5\sqrt{6})(-\sqrt{12})$

Exercise 8

$(-2\sqrt{7})(-2\sqrt{14})$

Exercise 9

$(2\sqrt{5b^3})(4\sqrt{15b})$

Exercise 10

$(6\sqrt{3d^3})(4\sqrt{12d^5})$

Exercise 11

$(\sqrt{11}+\sqrt{5})(\sqrt{11}+6\sqrt{5})$

Exercise 12

$(4\sqrt{6}+7\sqrt{13})(8\sqrt{6}-3\sqrt{13})$

Exercise 13

$(9-\sqrt{w})(2+\sqrt{w})$

Exercise 14

$(6+5\sqrt{n})(11+3\sqrt{n})$

Exercise 15

$(1+8\sqrt{2})(1-8\sqrt{2})$

Exercise 16

$(9-4\sqrt{3})(9+4\sqrt{3})$

Exercise 17

$(4\sqrt{6})(-\sqrt{18})$

Exercise 18

$(-5\sqrt{7})(6\sqrt{21})$

Exercise 19

$(\sqrt{35y^3})(\sqrt{7y^3})$

Exercise 20

$(\sqrt{29})^2$

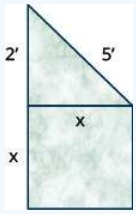
Exercise 21

$(-4+\sqrt{17})(-3+\sqrt{17})$

Everyday Math

Exercise 22

a. Use the Pythagorean Theorem to find the length of a side of the square base. Round to the nearest tenth of a foot.



b. Find the exact area of the face of the square base.

Writing Exercises

Exercise 23

a. Explain why $(-\sqrt{n})^2$ is always positive, for $n \geq 0$.
b. Explain why $-(\sqrt{n})^2$ is always negative, for $n \geq 0$.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply square roots.			
use polynomial multiplication to multiply square roots.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.5: Divide Square Roots

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify $\frac{24}{3}$
- Simplify $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{8}}$
- Simplify $\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

Learning Objectives

Motivating Problem

You're measuring a diagonal brace that's $12\frac{1}{\sqrt{2}} \cdot 21$ feet long—but your carpenter's calculator doesn't accept square roots in the denominator. How can you write this length in a more usable form?

Fun Fact

Rationalizing the denominator wasn't always required—early mathematicians often left radicals in the denominator. The modern preference for writing radicals in the numerator is mainly for consistency and convenience in computation!

The Goal

In this section, we'll learn how to divide square roots and simplify radical quotients. We'll also learn how to rationalize denominators by removing square roots using multiplication, so expressions are written in standard form.

Divide Square Roots

We know that we simplify fractions by removing factors that are common to both the numerator and the denominator. When we have a fraction with a square root in the numerator, we first simplify the square root. Then we can look for common factors.

Common Factors	No common factors
$\frac{3\sqrt{2}}{3 \cdot 5}$	$\frac{2\sqrt{3}}{3 \cdot 5}$

Example 1

Simplify: $\frac{\sqrt{54}}{6}$

Solution

	$\frac{\sqrt{54}}{6}$
Simplify the radical.	$\frac{\sqrt{9} \cdot \sqrt{6}}{6}$
Simplify.	$\frac{3\sqrt{6}}{6}$
Remove the common factors.	$\frac{3\sqrt{6}}{3 \cdot 2}$
Simplify.	$\frac{\sqrt{6}}{2}$

Try It 1

Simplify: $\frac{\sqrt{75}}{15}$.

Answer

$\frac{\sqrt{3}}{3}$

Example 2

Simplify: $\frac{6-\sqrt{24}}{12}$.

Solution

	$\frac{6-\sqrt{24}}{12}$
Simplify the radical.	$\frac{6-\sqrt{4}\cdot\sqrt{6}}{12}$
Simplify.	$\frac{6-2\sqrt{6}}{12}$
Factor the common factor from the numerator.	$\frac{2(3-\sqrt{6})}{12}$
Remove the common factors.	$\frac{2(3-\sqrt{6})}{2\cdot 6}$
Simplify.	$\frac{3-\sqrt{6}}{6}$

Try It 2

Simplify: $\frac{8-\sqrt{40}}{10}$.

Answer

$\frac{4-\sqrt{10}}{5}$

We have used the Quotient Property of Square Roots to simplify square roots of fractions. The Quotient Property of Square Roots says

$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}, b\neq 0.$

Sometimes we will need to use the Quotient Property of Square Roots 'in reverse' to simplify a fraction with square roots.

$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}, b\neq 0.$

We will rewrite the Quotient Property of Square Roots so we see both ways together. Remember: we assume all variables are greater than or equal to zero so that their square roots are real numbers.

Definition: QUOTIENT PROPERTY OF SQUARE ROOTS

If a, b are non-negative real numbers and $b\neq 0$, then

$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ and $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

We will use the Quotient Property of Square Roots 'in reverse' when the fraction we start with is the quotient of two square roots, and neither radicand is a perfect square. When we write the fraction in a single square root, we may find common factors in the numerator and denominator.

Example 3

Simplify: $\frac{\sqrt{27}}{\sqrt{75}}$

Solution

	$\frac{\sqrt{27}}{\sqrt{75}}$
Neither radicand is a perfect square, so rewrite using the quotient property of square root.	$\sqrt{\frac{27}{75}}$
Remove common factors in the numerator and denominator.	$\sqrt{\frac{9}{25}}$
Simplify.	$\frac{3}{5}$

Try It 3

Simplify: $\frac{\sqrt{48}}{\sqrt{108}}$

Answer

$\frac{2}{3}$

We will use the Quotient Property for Exponents, $\frac{a^m}{a^n} = a^{m-n}$, when we have variables with exponents in the radicands.

Example 4

Simplify: $\frac{\sqrt{6y^5}}{\sqrt{2y}}$

Solution

	$\frac{\sqrt{6y^5}}{\sqrt{2y}}$
Neither radicand is a perfect square, so rewrite using the quotient property of square root.	$\sqrt{\frac{6y^5}{2y}}$
Remove common factors in the numerator and denominator.	$\sqrt{3y^4}$
Simplify.	$y^2\sqrt{3}$

Try It 4

Simplify: $\frac{\sqrt{12r^3}}{\sqrt{6r}}$.

Answer

$r\sqrt{2}$

Example 5

Simplify: $\frac{\sqrt{72x^3}}{\sqrt{162x}}$

Solution

	$\frac{\sqrt{72x^3}}{\sqrt{162x}}$
Rewrite using the quotient property of square roots.	$\sqrt{\frac{72x^3}{162x}}$
Remove common factors.	$\sqrt{\frac{18\cdot 4\cdot x^2\cdot x}{18\cdot 9\cdot x}}$
Simplify.	$\sqrt{\frac{4x^2}{9}}$
Simplify the radical.	$\frac{2x}{3}$

Try It 5

Simplify: $\frac{\sqrt{75q^5}}{\sqrt{108q}}$.

Answer

$\frac{5q^2}{6}$

Example 6

Simplify: $\frac{\sqrt{147ab^8}}{\sqrt{3a^3b^4}}$.

Solution

	$\frac{\sqrt{147ab^8}}{\sqrt{3a^3b^4}}$
Rewrite using the quotient property of square roots.	$\sqrt{\frac{147ab^8}{3a^3b^4}}$
Remove common factors.	$\sqrt{\frac{49b^4}{a^2}}$
Simplify the radical.	$\frac{7b^2}{a}$

Try It 6

Simplify: $\frac{\sqrt{162x^{10}y^2}}{\sqrt{2x^6y^6}}$.

Answer

$\frac{9x^2}{y^2}$

Rationalize a One-Term Denominator

Before the calculator became a tool of everyday life, tables of square roots were used to find approximate values of square roots. The table below shows a portion of a table of squares and square roots. Square roots are approximated to five decimal places in this table.

n	n^2	$\sqrt{n}$
200	40,000	14.14214
201	40,401	14.17745
202	40,804	14.21267
203	41,209	14.24781
204	41,616	14.28286
205	42,025	14.31782
206	42,436	14.35270
207	42,849	14.38749
208	43,264	14.42221
209	43,681	14.45683
210	44,100	14.49138

A table of square roots was used to find approximate values of square roots before the invention of calculators.

If someone needed to approximate a fraction with a square root in the denominator, it meant doing long division with a five-decimal-place divisor. This was a very cumbersome process.

For this reason, a process called rationalizing the denominator was developed. A fraction with a radical in the denominator is converted to an equivalent fraction whose denominator is an integer. This process is still used today and is also helpful in other areas of mathematics.

Definition: RATIONALIZING THE DENOMINATOR

The process of converting a fraction with a radical in the denominator to an equivalent fraction whose denominator is an integer is called **rationalizing the denominator**.

The square roots of numbers that are not perfect squares are irrational numbers. When we **rationalize the denominator**, we write an equivalent fraction with a rational number in the denominator.

Let's look at a numerical example.

Suppose we need an approximate value for the fraction.	$\frac{1}{\sqrt{2}}$
A five decimal place approximation to $\sqrt{2}$ is 1.41421	$\frac{1}{1.41421}$
Without a calculator, would you want to do this division?	$1.41421\overline{)1.0}$

But we can find a fraction equivalent to $\frac{1}{\sqrt{2}}$ by multiplying the numerator and denominator by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}}$$
$$\frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$
$$\frac{\sqrt{2}}{2}$$

Now, if we need an approximate value, we divide $2\overline{)1.41421}$. This is much easier.

Even though calculators are available nearly everywhere, a fraction with a radical in the denominator still must be rationalized. It is not considered simplified if the denominator contains a square root.

Similarly, a square root is not considered simplified if the radicand contains a fraction.

Definition: SIMPLIFIED SQUARE ROOTS

A square root is considered simplified if there are

- no perfect-square factors in the radicand
- no fractions in the radicand
- no square roots in the denominator of a fraction

To rationalize a denominator, we use the property that $(\sqrt{a})^2 = a$. If we square an irrational square root, we get a rational number.

We will use this property to rationalize the denominator in the next example.

Example 7

Simplify: $\frac{4}{\sqrt{3}}$.

Solution

To remove the square root from the denominator, we multiply it by itself. To keep the fractions equivalent, we multiply both the numerator and denominator by the same factor.

	$\frac{4}{\sqrt{3}}$
Multiply both the numerator and denominator by $\sqrt{3}$	$\frac{4 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$
Simplify.	$\frac{4\sqrt{3}}{3}$

Try It 7

Simplify: $\frac{6}{\sqrt{5}}$.

Answer

$\frac{6\sqrt{5}}{5}$

Example 8

Simplify: $-\frac{8}{3\sqrt{6}}$

Solution

To remove the square root from the denominator, we multiply it by itself. To keep the fractions equivalent, we multiply both the numerator and denominator by $\sqrt{6}$.

	$-\frac{8}{3\sqrt{6}}$
Multiply both the numerator and the denominator by $\sqrt{6}$.	$-\frac{8 \cdot \sqrt{6}}{3\sqrt{6} \cdot \sqrt{6}}$
Simplify.	$-\frac{8\sqrt{6}}{3 \cdot 6}$
Remove common factors.	$-\frac{4 \cdot \cancel{2}\sqrt{6}}{3 \cdot \cancel{2} \cdot 3}$
Simplify.	$-\frac{4\sqrt{6}}{9}$

Try It 8

Simplify: $-\frac{9}{4\sqrt{3}}$.

Answer

$-\frac{3\sqrt{3}}{4}$

Always simplify the radical in the denominator first, before you rationalize it. This way the numbers stay smaller and easier to work with.

Example 9

Simplify: $\sqrt{\frac{5}{12}}$.

Solution

	$\sqrt{\frac{5}{12}}$
The fraction is not a perfect square, so rewrite using the Quotient Property.	$\frac{\sqrt{5}}{\sqrt{12}}$
Simplify the denominator.	$\frac{\sqrt{5}}{2\sqrt{3}}$
Rationalize the denominator.	$\frac{\sqrt{5}\cdot\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}}$
Simplify.	$\frac{\sqrt{15}}{2\cdot 3}$
Simplify.	$\frac{\sqrt{15}}{6}$

? Try It 9

Simplify: $\sqrt{\frac{7}{18}}$.

Answer

$\frac{\sqrt{14}}{6}$

✓ Example 10

Simplify: $\sqrt{\frac{11}{28}}$

Solution

	$\sqrt{\frac{11}{28}}$
Rewrite using the Quotient Property.	$\frac{\sqrt{11}}{\sqrt{28}}$
Simplify the denominator.	$\frac{\sqrt{11}}{2\sqrt{7}}$
Rationalize the denominator.	$\frac{\sqrt{11}\cdot\sqrt{7}}{2\sqrt{7}\cdot\sqrt{7}}$
Simplify.	$\frac{\sqrt{77}}{2\cdot 7}$
Simplify.	$\frac{\sqrt{77}}{14}$

? Try It 10

Simplify: $\sqrt{\frac{3}{27}}$.

Answer

$\frac{1}{3}$

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Section 2.5P: Practice

Practice Makes Progress

In the following exercises, simplify.

Exercise 1

$\frac{\sqrt{27}}{6}$

Exercise 2

$\frac{2-\sqrt{32}}{8}$

Exercise 3

$\frac{\sqrt{80}}{\sqrt{125}}$

Exercise 4

$\frac{\sqrt{75r^5}}{108r}$

Exercise 5

$\frac{\sqrt{98}}{14}$

Exercise 6

$\frac{5+\sqrt{125}}{15}$

Exercise 7

$\frac{\sqrt{26y^7}}{2y}$

In the following exercises, simplify and rationalize the denominator.

Exercise 8

$\frac{3}{\sqrt{13}}$

Exercise 9

$\frac{10}{3\sqrt{10}}$

Exercise 10

$\sqrt{\frac{3}{20}}$

Exercise 11

$\sqrt{\frac{19}{175}}$

Exercise 12

$\frac{3}{3+\sqrt{11}}$

Exercise 13

$\frac{8}{1-\sqrt{5}}$

Exercise 14

$\frac{\sqrt{3}}{\sqrt{m}-\sqrt{5}}$

Exercise 15

$\frac{\sqrt{150x^2y^6}}{\sqrt{6x^4y^2}}$

Exercise 16

$\frac{15}{\sqrt{5}}$

Exercise 17

$\sqrt{\frac{8}{54}}$

Exercise 18

$\frac{\sqrt{2}}{\sqrt{x}-\sqrt{3}}$

Everyday Math

Exercise 19

A supply kit is dropped from an airplane flying at an altitude of 250 feet. Simplify $\sqrt{\frac{250}{16}}$ to determine how many seconds it takes for the supply kit to reach the ground.

Writing Exercises

Exercise 20

a. Simplify $\sqrt{\frac{27}{3}}$ and explain all your steps.
b. Simplify $\sqrt{\frac{27}{5}}$ and explain all your steps.
c. Why are the two methods of simplifying square roots different?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
divide square roots.			
rationalize a one-term denominator.			
rationalize a two-term denominator.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.6: Rational Exponents

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify $\sqrt{8} \cdot \sqrt{200}$
- Determine the value of $2^3 \cdot 5^3$
- Use the Power Property: Simplify $(x^4)^2$

Learning Objectives

Motivating Problem

You're analyzing how fast a population grows over time. You see a formula like $P = 100 \cdot (2)^{\frac{3}{2}t}$. What could that exponent mean? Why would someone use a fraction instead of a whole number?

Fun Fact

The use of fractional exponents gained popularity in the 17th century, thanks to mathematicians like **Descartes** and **Newton**, who sought ways to describe square and cube roots without using radical symbols.

The Goal

In this section, we'll learn how to rewrite and evaluate expressions with **rational (fractional) exponents**, and how they relate to **roots**. We'll also simplify expressions using the exponent rules we've already learned.

Simplify Expressions with $a^{\frac{1}{n}}$

Rational exponents are another way of writing expressions with radicals. When we use **rational exponents**, we can apply the properties of exponents to simplify expressions.

The Power Property for Exponents says that $(a^m)^n = a^{m \cdot n}$ when m and n are whole numbers. Let's assume we are now not limited to whole numbers.

Suppose we want to find a number p such that $(8^p)^3 = 8$. We will use the Power Property of Exponents to find the value of p.

	$(8^p)^3 = 8$
Multiply the exponents on the left.	$8^{3p} = 8$
Write the exponent 1 on the right.	$8^{3p} = 8^1$
The exponents must be equal.	$3p = 1$
Solve for p.	$p = \frac{1}{3}$

But we know also $(\sqrt[3]{8})^3 = 8$. Then it must be that $8^{\frac{1}{3}} = \sqrt[3]{8}$

This same logic can be used for any positive integer exponent n to show that $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Definition: RATIONAL EXPONENT $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ is a real number and $n \geq 2$, $a^{\frac{1}{n}} = \sqrt[n]{a}$.

There will be times when working with expressions will be easier if you use **rational exponents** and times when it will be easier if you use radicals. In the first few examples, you'll practice converting expressions between these two notations.

Example 1

Write as a radical expression:

a. $x^{\frac{1}{2}}$

b. $y^{\frac{1}{3}}$
c. $z^{\frac{1}{4}}$

Solution

a.	$x^{\frac{1}{2}}$
The denominator of the exponent is 2, so the index of the radical is 2. We do not show the index when it is 2.	$\sqrt{x}$
b.	$y^{\frac{1}{3}}$
The denominator of the exponent is 3, so the index is 3.	$\sqrt[3]{y}$
c.	$z^{\frac{1}{4}}$
The denominator of the exponent is 4, sothe index is 4.	$\sqrt[4]{z}$

Try It 1

Write as a radical expression:

a. $t^{\frac{1}{2}}$
b. $m^{\frac{1}{3}}$
c. $r^{\frac{1}{4}}$

Answer

a. $\sqrt{t}$
b. $\sqrt[3]{m}$
c. $\sqrt[4]{r}$

Example 2

Write with a rational exponent:

a. $\sqrt{x}$
b. $\sqrt[3]{y}$
c. $\sqrt[4]{z}$

Solution

We want to write each radical in the form $a^{\frac{1}{n}}$.

a.	$\sqrt{x}$
No index is shown, so it is 2. The denominator of the exponent will be 2.	$x^{\frac{1}{2}}$
b.	$\sqrt[3]{y}$
The index is 3, so the denominator of the exponent is 3.	$y^{\frac{1}{3}}$
c.	$\sqrt[4]{z}$
The index is 4, so the denominator of the exponent is 4.	$z^{\frac{1}{4}}$

Try It 2

Write with a rational exponent:

a. $\sqrt{v}$
b. $\sqrt[3]{p}$
c. $\sqrt[4]{p}$

Answer

a. $v^{\frac{1}{2}}$
b. $p^{\frac{1}{3}}$
c. $p^{\frac{1}{4}}$

Example 3

Write with a rational exponent:

a. $\sqrt{5y}$
b. $\sqrt[3]{4x}$
c. $3\sqrt[4]{5z}$

Solution

a.	$\sqrt{5y}$
No index is shown, so it is 2. The denominator of the exponent will be 2.	$(5y)^{\frac{1}{2}}$
b.	$\sqrt[3]{4x}$
The index is 3, so the denominator of the exponent is 3.	$(4x)^{\frac{1}{3}}$
c.	$3\sqrt[4]{5z}$
The index is 4, so the denominator of the exponent is 4.	$3(5z)^{\frac{1}{4}}$

Try It 3

Write with a rational exponent:

a. $\sqrt{10m}$
b. $\sqrt[5]{3n}$
c. $3\sqrt[4]{6y}$

Answer

a. $(10m)^{\frac{1}{2}}$
b. $(3n)^{\frac{1}{5}}$
c. \3(6y)^{\frac{1}{4}}\)

In the next example, you may find it easier to simplify the expressions if you rewrite them as radicals first.

Example 4

Simplify:

a. $25^{\frac{1}{2}}$
b. $64^{\frac{1}{3}}$

c. $256^{\frac{1}{4}}$

Solution

a.	$25^{\frac{1}{2}}$
Rewrite as a square root.	$\sqrt{25}$
Simplify.	5
b.	$64^{\frac{1}{3}}$
Rewrite as a cube root.	$\sqrt[3]{64}$
Recognize 64 is a perfect cube.	$\sqrt[3]{4^3}$
Simplify.	4
c.	$256^{\frac{1}{4}}$
Rewrite as a fourth root.	$\sqrt[4]{256}$
Recognize 256 is a perfect fourth power.	$\sqrt[4]{4^4}$
Simplify.	4

? Try It 4

Simplify:

a. $36^{\frac{1}{2}}$
b. $8^{\frac{1}{3}}$
c. $16^{\frac{1}{4}}$

Answer

a. 6
b. 2
c. 2

Be careful of the placement of the negative signs in the next example. We will need to use the property $a^{-n} = \frac{1}{a^n}$ in one case.

✓ Example 5

Simplify:

a. $(-64)^{\frac{1}{3}}$
b. $-64^{\frac{1}{3}}$
c. $(64)^{-\frac{1}{3}}$

Solution

a.	$(-64)^{\frac{1}{3}}$
Rewrite as a cube root.	$\sqrt[3]{-64}$
Rewrite 64 as a perfect cube.	$\sqrt[3]{(-4)^3}$
Simplify.	-4
b.	$-64^{\frac{1}{3}}$
The exponent applies only to the 64.	$-(64^{\frac{1}{3}})$
Rewrite as a cube root.	$-\sqrt[3]{64}$
Rewrite 64 as 4^3.	$-\sqrt[3]{4^3}$
Simplify.	-4
c.	$(64)^{-\frac{1}{3}}$
Rewrite as a fraction with a positive exponent, using the property, $a^{-n}=\frac{1}{a^n}$. Write as a cube root.	$\frac{1}{\sqrt[3]{64}}$
Rewrite 64 as 4^3.	$\frac{1}{\sqrt[3]{4^3}}$
Simplify.	$\frac{1}{4}$

? Try It 5

Simplify:

a. $(-125)^{\frac{1}{3}}$
b. $-125^{\frac{1}{3}}$
c. $(125)^{-\frac{1}{3}}$.

Answer

a. -5
b. -5
c. $\frac{1}{5}$

✔ Example 6

Simplify:

a. $(-16)^{\frac{1}{4}}$
b. $-16^{\frac{1}{4}}$
c. $(16)^{-\frac{1}{4}}$

Solution

a.	$(-16)^{\frac{1}{4}}$
Rewrite as a fourth root.	$\sqrt[4]{-16}$
There is no real number whose fourth power is −16.	
b.	$-16^{\frac{1}{4}}$
The exponent applies only to the 16.	$-(16^{\frac{1}{4}})$
Rewrite as a fourth root.	$-\sqrt[4]{16}$
Rewrite 16 as 2^4	$-\sqrt[4]{2^4}$
Simplify.	−2
c.	$(16)^{-\frac{1}{4}}$
Rewrite as a fraction with a positive exponent, using the property, $a^{-n}=\frac{1}{a^n}$.	$\frac{1}{(16)^{\frac{1}{4}}}$
Rewrite as a fourth root.	$\frac{1}{\sqrt[4]{16}}$
Rewrite 16 as 2^4.	$\frac{1}{\sqrt[4]{2^4}}$
Simplify.	$\frac{1}{2}$

Try It 6

Simplify:

a. $(-64)^{\frac{1}{2}}$
b. $-64^{\frac{1}{2}}$
c. $(64)^{-\frac{1}{2}}$

Answer

a. Not a real number.
b. −8
c. $\frac{1}{8}$

Simplify Expressions with $a^{\frac{m}{n}}$

Let's work with the Power Property for Exponents some more.

Suppose we raise $a^{\frac{1}{n}}$ to the power m.

	$(a^{\frac{1}{n}})^m$
Multiply the exponents.	$a^{\frac{1}{n}\cdot m}$
Simplify.	$a^{\frac{m}{n}}$
So $a^{\frac{m}{n}}=(\sqrt[n]{a})^m$ also.	

Now suppose we take a^m to the $\frac{1}{n}$ power.

	$(a^m)^{\frac{1}{n}}$
Multiply the exponents.	$a^{m\cdot\frac{1}{n}}$
Simplify.	$a^{\frac{m}{n}}$
So $a^{\frac{m}{n}}=\sqrt[n]{a^m}$ also.	

Which form do we use to simplify an expression? We usually take the root first—that way we keep the numbers in the radicand smaller.

Definition: RATIONAL EXPONENT $a^{\frac{m}{n}}$

For any positive integers m and n,

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Example 7

Write with a rational exponent:

a. $\sqrt{y^3}$
b. $\sqrt[3]{x^2}$
c. $\sqrt[4]{z^3}$

Solution

We want to use $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ to write each radical in the form $a^{\frac{m}{n}}$.

a.
The numerator of the exponent is the exponent of y, 3. $\sqrt{y^3}$
The denominator of the exponent is the index of the radical, 2. $y^{\frac{3}{2}}$

b.
The numerator of the exponent is the exponent of x, 2. $\sqrt{x^2}$
The denominator of the exponent is the index of the radical, 3. $x^{\frac{2}{3}}$

c.
The numerator of the exponent is the exponent of z, 3. $\sqrt{z^3}$
The denominator of the exponent is the index of the radical, 4. $z^{\frac{3}{4}}$

Try It 7

Write with a rational exponent:

a. $\sqrt{x^5}$
b. $\sqrt[4]{z^3}$
c. $\sqrt[5]{y^2}$

Answer

a. $x^{\frac{5}{2}}$
b. $z^{\frac{3}{4}}$
c. $y^{\frac{2}{5}}$

Example 8

Simplify:

a. $9^{\frac{3}{2}}$
b. $125^{\frac{2}{3}}$
c. $81^{\frac{3}{4}}$

Solution

We will rewrite each expression as a radical first using the property, $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$. This form lets us take the root first and so we keep the numbers in the radicand smaller than if we used the other form.

1.	$9^{\frac{3}{2}}$
The power of the radical is the numerator of the exponent, 3. Since the denominator of the exponent is 2, this is a square root.	$(\sqrt{9})^3$
Simplify.	3^3
	27
2.	$125^{\frac{2}{3}}$
The power of the radical is the numerator of the exponent, 2. Since the denominator of the exponent is 3, this is a cube root.	$(\sqrt[3]{125})^2$
Simplify.	5^2
	25
3.	$81^{\frac{3}{4}}$
The power of the radical is the numerator of the exponent, 3. Since the denominator of the exponent is 4, this is a fourth root.	$(\sqrt[4]{81})^3$
Simplify.	3^3
	27

Try It 8

Simplify:

a. $4^{\frac{3}{2}}$
b. $27^{\frac{2}{3}}$
c. $625^{\frac{3}{4}}$

Answer

a. 8
b. 9
c. 125

Remember that $b^{-p} = \frac{1}{b^p}$. The negative sign in the exponent does not change the sign of the expression.

Example 9

Simplify:

a. $16^{-\frac{3}{2}}$
b. $32^{-\frac{2}{5}}$
c. $4^{-\frac{5}{2}}$

Solution

We will rewrite each expression first using $b^{-p} = \frac{1}{b^p}$ and then change to radical form.

a.	$16^{-\frac{3}{2}}$
Rewrite using $b^{-p} = \frac{1}{b^p}$.	$\frac{1}{16^{\frac{3}{2}}}$
Change to radical form. The power of the radical is the numerator of the exponent, 3. The index is the exponent's denominator, 2.	$\frac{1}{(\sqrt{16})^3}$
Simplify.	$\frac{1}{4^3}$
	$\frac{1}{64}$
b.	$32^{-\frac{2}{5}}$
Rewrite using $b^{-p} = \frac{1}{b^p}$.	$\frac{1}{32^{\frac{2}{5}}}$
Change to radical form.	$\frac{1}{(\sqrt[5]{32})^2}$
Rewrite the radicand as a power.	$\frac{1}{(\sqrt[5]{2^5})^2}$
Simplify.	$\frac{1}{2^2}$
	$\frac{1}{4}$
c.	$4^{-\frac{5}{2}}$
Rewrite using $b^{-p} = \frac{1}{b^p}$.	$\frac{1}{4^{\frac{5}{2}}}$
Change to radical form.	$\frac{1}{(\sqrt{4})^5}$
Simplify.	$\frac{1}{2^5}$
	$\frac{1}{32}$

Try It 9

Simplify:

a. $4^{-\frac{3}{2}}$
b. $27^{-\frac{2}{3}}$
c. $625^{-\frac{3}{4}}$

Answer

a. $\frac{1}{8}$
b. $\frac{1}{9}$
c. $\frac{1}{125}$

Example 10

Simplify:

a. $-25^{\frac{3}{2}}$
b. $-25^{-\frac{3}{2}}$
c. $(-25)^{\frac{3}{2}}$

Solution

a.	$-25^{\frac{3}{2}}$
Rewrite in radical form.	$-(\sqrt{25})^3$
Simplify the radical	-5^3
Simplify.	-125
b.	$-25^{-\frac{3}{2}}$
Rewrite using $b^{-p}=\frac{1}{b^p}$.	$-(\frac{1}{25^{\frac{3}{2}}})$
Rewrite in radical form.	$-(\frac{1}{(\sqrt{25})^3})$
Simplify the radical.	$-(\frac{1}{5^3})$
Simplify.	$-\frac{1}{125}$
c.	$(-25)^{\frac{3}{2}}$.
Rewrite in radical form.	$(\sqrt{-25})^3$
There is no real number whose square root is -25.	Not a real number.

? Try It 10

Simplify:

a. $-81^{\frac{3}{2}}$
b. $-81^{-\frac{3}{2}}$
c. $(-81)^{-\frac{3}{2}}$

Answer

a. -729
b. $-\frac{1}{729}$
c. not a real number

Use the Laws of Exponents to Simplify Expressions with Rational Exponents

The same laws of exponents that we already used apply to rational exponents, too. We will list the Exponent Properties here to have them for reference as we simplify expressions.

SUMMARY OF EXPONENT PROPERTIES

If a,b are real numbers and m,n are rational numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0, m > n$
	$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, a \neq 0, n > m$
Zero Exponent Definition	$a^0 = 1, a \neq 0$
Quotient to a Power Property	$(\frac{a}{b})^m = \frac{a^m}{b^m}, b \neq 0$

When we multiply the same base, we add the exponents.

Example 11

Simplify:

a. $2^{\frac{1}{2}} \cdot 2^{\frac{5}{2}}$
b. $x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$
c. $z^{\frac{3}{4}} \cdot z^{\frac{5}{4}}$

Solution

a.	$2^{\frac{1}{2}} \cdot 2^{\frac{5}{2}}$
The bases are the same, so we add the exponents.	$2^{\frac{1}{2}+\frac{5}{2}}$
Add the fractions.	$2^{\frac{6}{2}}$
Simplify the exponent.	2^3
Simplify.	8
b.	$x^{\frac{2}{3}} \cdot x^{\frac{4}{3}}$
The bases are the same, so we add the exponents.	$x^{\frac{2}{3}+\frac{4}{3}}$
Add the fractions.	$x^{\frac{6}{3}}$
Simplify.	x^2
c.	$z^{\frac{3}{4}} \cdot z^{\frac{5}{4}}$
The bases are the same, so we add the exponents.	$z^{\frac{3}{4}+\frac{5}{4}}$
Add the fractions.	$z^{\frac{8}{4}}$
Simplify.	z^2

Try It 11

Simplify:

a. $3^{\frac{2}{3}} \cdot 3^{\frac{4}{3}}$
b. $y^{\frac{1}{3}} \cdot y^{\frac{8}{3}}$
c. $m^{\frac{1}{4}} \cdot m^{\frac{3}{4}}$

Answer

a. 9
b. y^3
c. m

We will use the Power Property in the next example.

Example 12

Simplify:

a. $(x^4)^{\frac{1}{2}}$
b. $(y^6)^{\frac{1}{3}}$
c. $(z^9)^{\frac{2}{3}}$

Solution

a.	$(x^4)^{\frac{1}{2}}$
To raise a power to a power, we multiply the exponents.	$x^{4\cdot\frac{1}{2}}$
Simplify.	x^2
b.	$(y^6)^{\frac{1}{3}}$
To raise a power to a power, we multiply the exponents.	$y^{6\cdot\frac{1}{3}}$
Simplify.	y^2
c.	$(z^9)^{\frac{2}{3}}$
To raise a power to a power, we multiply the exponents.	$z^{9\cdot\frac{2}{3}}$
Simplify.	z^6

? Try It 12

Simplify:

a. $(r^6)^{\frac{5}{3}}$
b. $(s^{12})^{\frac{3}{4}}$
c. $(m^9)^{\frac{2}{9}}$

Answer

a. r^{10}
b. s^9
c. m^2

The Quotient Property tells us that when we divide with the same base, we subtract the exponents.

✓ Example 13

Simplify:

a. $\frac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$
b. $\frac{y^{\frac{3}{4}}}{y^{\frac{1}{4}}}$
c. $\frac{z^{\frac{2}{3}}}{z^{\frac{5}{3}}}$

Solution

a.	$\frac{x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$
To divide with the same base, we subtract the exponents.	$x^{\frac{4}{3}-\frac{1}{3}}$
Simplify.	x
b.	$\frac{y^{\frac{3}{4}}}{y^{\frac{1}{4}}}$
To divide with the same base, we subtract the exponents.	$y^{\frac{3}{4}-\frac{1}{4}}$
Simplify.	$y^{\frac{1}{2}}$
c.	$\frac{z^{\frac{2}{3}}}{z^{\frac{5}{3}}}$
To divide with the same base, we subtract the exponents.	$z^{\frac{2}{3}-\frac{5}{3}}$
Rewrite without a negative exponent.	$\frac{1}{z}$

Try It 13

Simplify:

a. $\frac{u^{\frac{5}{4}}}{u^{\frac{1}{4}}}$
b. $\frac{v^{\frac{5}{6}}}{v^{\frac{2}{6}}}$
c. $\frac{x^{\frac{2}{3}}}{x^{\frac{5}{3}}}$

Answer

a. u
b. $v^{\frac{1}{6}}$
c. $\frac{1}{x}$

Sometimes we need to use more than one property. In the next two examples, we will use both the Product to a Power Property and then the Power Property.

Example 14

Simplify:

a. $(27u^{\frac{1}{2}})^{\frac{2}{3}}$
b. $(8v^{\frac{1}{4}})^{\frac{2}{3}}$

Solution

a.	$(27u^{\frac{1}{2}})^{\frac{2}{3}}$
First we use the Product to a Power Property.	$(27)^{\frac{2}{3}}(u^{\frac{1}{2}})^{\frac{2}{3}}$
Rewrite 27 as a power of 3.	$(3^3)^{\frac{2}{3}}(u^{\frac{1}{2}})^{\frac{2}{3}}$
To raise a power to a power, we multiply the exponents.	$(3^2)(u^{\frac{1}{3}})$
Simplify.	$9u^{\frac{1}{3}}$
b.	$(8v^{\frac{1}{4}})^{\frac{2}{3}}$.
First we use the Product to a Power Property.	$(8)^{\frac{2}{3}}(v^{\frac{1}{4}})^{\frac{2}{3}}$
Rewrite 8 as a power of 2.	$(2^3)^{\frac{2}{3}}(v^{\frac{1}{4}})^{\frac{2}{3}}$
To raise a power to a power, we multiply the exponents.	$(2^2)(v^{\frac{1}{6}})$
Simplify.	$4v^{\frac{1}{6}}$

Try It 14

Simplify:

a. $(16m^{\frac{1}{3}})^{\frac{3}{2}}$
b. $(81n^{\frac{2}{5}})^{\frac{3}{2}}$

Answer

a. $64m^{\frac{1}{2}}$
b. $729n^{\frac{3}{5}}$

We will use both the Product and Quotient Properties in the next example.

Example 15

Simplify:

a. $\frac{x^{\frac{3}{4}}\cdot x^{-\frac{1}{4}}}{x^{-\frac{6}{4}}}$
b. $\frac{y^{\frac{4}{3}}\cdot y}{y^{-\frac{2}{3}}}$

Solution

a.	$\frac{x^{\frac{3}{4}}\cdot x^{-\frac{1}{4}}}{x^{-\frac{6}{4}}}$
Use the Product Property in the numerator, add the exponents.	$\frac{x^{\frac{2}{4}}}{x^{-\frac{6}{4}}}$
Use the Quotient Property, subtract the exponents.	$x^{\frac{8}{4}}$
Simplify.	x^2
b.	$\frac{y^{\frac{4}{3}}\cdot y}{y^{-\frac{2}{3}}}$
Use the Product Property in the numerator, add the exponents.	$\frac{y^{\frac{7}{3}}}{y^{-\frac{2}{3}}}$
Use the Quotient Property, subtract the exponents.	$y^{\frac{9}{3}}$
Simplify.	y^3

Try It 15

Simplify:

a. $\frac{m^{\frac{2}{3}} \cdot m^{-\frac{1}{3}}}{m^{-\frac{5}{3}}}$

b. $\frac{n^{\frac{1}{6}} \cdot n}{n^{-\frac{11}{6}}}$.

Answer

a. m^2

b. n^3

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Section 2.6P: Practice

Practice Makes Progress

In the following exercises, write as a radical expression.

Exercise 1

$r^{\frac{1}{2}}$

Exercise 2

$s^{\frac{1}{3}}$

Exercise 3

$t^{\frac{1}{4}}$

In the following exercises, write with a rational exponent.

Exercise 4

$\sqrt[4]{r^7}$

Exercise 5

$\sqrt[5]{s^3}$

Exercise 6

$\sqrt[3]{t^7}$

In the following exercises, simplify.

Exercise 7

$625^{\frac{1}{4}}$

Exercise 8

$243^{\frac{1}{5}}$

Exercise 9

$32^{\frac{1}{5}}$

Exercise 10

$(-49)^{\frac{1}{2}}$

Exercise 11

$-49^{\frac{1}{2}}$

Exercise 12

$(49)^{-\frac{1}{2}}$

Exercise 13

$(-1)^{\frac{1}{4}}$

Exercise 14

$(1)^{-\frac{1}{4}}$

Exercise 15

$-1^{\frac{1}{4}}$

Exercise 16

$1000^{\frac{2}{3}}$

Exercise 17

$25^{\frac{3}{2}}$

Exercise 18

$32^{\frac{3}{5}}$

Exercise 19

$100^{\frac{3}{2}}$

Exercise 20

$49^{-\frac{5}{2}}$

Exercise 21

$(-100)^{\frac{3}{2}}$

Exercise 22

$-64^{\frac{3}{2}}$

Exercise 23

$-64^{-\frac{3}{2}}$

Exercise 24

$(-64)^{\frac{3}{2}}$

Exercise 25

$6^{\frac{4}{3}} \cdot 6^{\frac{5}{3}}$

Exercise 26

$n^{\frac{2}{10}} \cdot n^{\frac{8}{10}}$

Exercise 27

$q^{\frac{2}{5}} \cdot q^{\frac{13}{5}}$

Exercise 28

$\frac{s^{\frac{11}{6}}}{s^{\frac{6}{6}}}$

Exercise 29

$\frac{z^{\frac{7}{3}}}{z^{\frac{1}{3}}}$

Exercise 30

$\frac{w^{\frac{2}{7}}}{w^{\frac{9}{7}}}$

Exercise 31

$(81r^{\frac{4}{5}})^{\frac{1}{4}}$

Exercise 32

$(64s^{\frac{3}{7}})^{\frac{1}{6}}$

Exercise 33

$(r^8 s^4)^{\frac{1}{4}}$

Exercise 34

$(u^{15}v^{20})^{\frac{1}{5}}$

Exercise 35

$\frac{m^{\frac{7}{4}} \cdot m^{-\frac{5}{4}}}{m^{-\frac{2}{4}}}$

? Exercise 36

$\frac{n^{\frac{3}{7}} \cdot n}{n^{-\frac{4}{7}}}$

? Exercise 37

$n^{\frac{2}{6}} \cdot n^{\frac{4}{6}}$

? Exercise 38

$(b^{10})^{\frac{3}{5}}$

? Exercise 39

$(64s^{\frac{3}{5}})^{\frac{1}{6}}$

Everyday Math

? Exercise 40

Trevon wants to make a square patio in his yard. He has enough concrete to pave an area of 242 square feet. Simplify $242^{\frac{1}{2}}$ to find the length of each side of his patio, round to the nearest tenth of a foot.

? Exercise 41

An airplane dropped a flare from a height of 1024 feet above a lake. Simplify $\frac{1024^{\frac{1}{2}}}{16^{\frac{1}{2}}}$ to find how many seconds it took for the flare to reach the water.

Writing Exercises

? Exercise 42

Explain why the expression $(-16)^{\frac{3}{2}}$ cannot be evaluated.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
convert from radical to rational exponent notation.			
convert from rational exponent to radical notation.			
simplify expression with rational exponents			
use the laws of exponents to simplify expressions with rational exponents			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.7: Solve Equations with Square Roots

Be Prepared

We will rely heavily on these skills throughout this section.

- Solve $5(x+1)-4=3(2x-7)$
- Simplify $\sqrt{9}$ and 9^2
- Solve $5x+7=-12$

Learning Objectives

Motivating Problem

You're programming a robot to stop moving after it travels 64 inches. The robot's distance is tracked with a sensor that reports the square root of the distance traveled.

If the display shows $10=8\sqrt{d}$, how far has it gone? What if the display shows $15=10\sqrt{d}$?

Fun Fact

The concept of "extraneous solutions" was already known in the 9th century—mathematicians like **Al-Khwarizmi** recognized that squaring both sides of an equation could introduce invalid answers!

The Goal

In this section, we'll learn how to solve linear equations that involve square roots by isolating the radical, squaring both sides, and checking for extraneous solutions. We'll keep our focus on equations that stay linear after solving.

Solve Radical Equations

In this section, we will solve equations that have the variable in the radicand of a square root. Equations of this type are called radical equations.

Definition: RADICAL EQUATION

An equation in which the variable is in the radicand of a square root is called a **radical equation**.

As usual, in solving these equations, what we do to one side of an equation we must do to the other side as well. Since squaring a quantity and taking a square root are 'opposite' operations, we will square both sides in order to remove the radical sign and solve for the variable inside.

But remember that when we write $\sqrt{a}$ we mean the principal square root. So $\sqrt{a}\geq 0$ always. When we solve radical equations by squaring both sides, we may get an algebraic solution that would make $\sqrt{a}$ negative. This algebraic solution would not be a solution to the original **radical equation**; it is an *extraneous solution*. We encountered extraneous solutions when solving rational equations as well.

Example 1

For the equation $\sqrt{x+2}=x$:

a. Is x=2 a solution?

b. Is x=−1 a solution?

Solution

a. Is x=2 a solution?

	$\sqrt{x+2}=x$
Let $x = 2$.	$\sqrt{2+2} \stackrel{?}{=} 2$
Simplify.	$\sqrt{4} \stackrel{?}{=} 2$
	$2 = 2$ ✓
	2 is a solution.

b. Is x=−1 a solution?

	$\sqrt{x+2}=x$
Let $x = -1$.	$\sqrt{-1+2} \stackrel{?}{=} -1$
Simplify.	$\sqrt{1} \stackrel{?}{=} -1$
	$1 \neq -1$
	−1 is not a solution.
	−1 is an extraneous solution to the equation.

? Try It 1

For the equation $\sqrt{x+6}=x$:

a. Is x=−2 a solution?
b. Is x=3 a solution?

Answer

a. no
b. yes

Now we will see how to solve a radical equation. Our strategy is based on the relation between taking a square root and squaring.

For $a \geq 0$, $(\sqrt{a})^2 = a$

How to Solve Radical Equations

✔ Example 2

Solve: $\sqrt{2x-1}=7$

Solution

Step 1. Isolate the radical on one side of the equation.	$\sqrt{2x-1}$ is already isolated on the left side.	$\sqrt{2x-1}=7$
Step 2. Square both sides of the equation.	Remember, $(\sqrt{a})^2 = a$	$(\sqrt{2x-1})^2 = (7)^2$

Step 3. Solve the new equation.		$2x-1=49$ $2x=50$ $x=25$
Step 4. Check the answer.		Check: $\sqrt{2x-1}=7$ $\sqrt{2(25)-1}\stackrel{?}{=}7$ $\sqrt{50-1}\stackrel{?}{=}7$ $\sqrt{49}\stackrel{?}{=}7$ $7=7\checkmark$ The solution is $x=25$.

Try It 2

Solve: $\sqrt{3x-5}=5$.

Answer

10

Definition: SOLVE A RADICAL EQUATION.

a. Isolate the radical on one side of the equation.
b. Square both sides of the equation.
c. Solve the new equation.
d. Check the answer.

Example 3

Solve: $\sqrt{5n-4}-9=0$.

Solution

	$\sqrt{5n-4}-9=0$
To isolate the radical, add 9 to both sides.	$\sqrt{5n-4}-9+9=0+9$
Simplify.	$\sqrt{5n-4}=9$
Square both sides of the equation.	$(\sqrt{5n-4})^2=(9)^2$
Solve the new equation.	$5n-4=81$
	$5n=85$
	$n=17$
Check the answer. $\sqrt{5n-4}-9=0$ $\sqrt{5(17)-4}-9\stackrel{?}{=}0$ $\sqrt{85-4}-9\stackrel{?}{=}0$ $\sqrt{81}-9\stackrel{?}{=}0$ $9-9\stackrel{?}{=}0$ $0=0\checkmark$	
	The solution is $n = 17$.

? Try It 3

Solve: $\sqrt{3m+2}-5=0$.

Answer

$\frac{23}{3}$

✓ Example 4

Solve: $\sqrt{3y+5}+2=5$.

Solution

	$\sqrt{3y+5}+2=5$
To isolate the radical, subtract 2 from both sides.	$\sqrt{3y+5}+2-2=5-2$
Simplify.	$\sqrt{3y+5}=3$
Square both sides of the equation.	$(\sqrt{3y+5})^2=(3)^2$
Solve the new equation.	$3y+5=9$
	$3y=4$
	$y=\frac{4}{3}$
Check the answer. $\sqrt{3y+5}+2=5$ $\sqrt{3\left(\frac{4}{3}\right)+5}+2\stackrel{?}{=}5$ $\sqrt{4+5}+2\stackrel{?}{=}5$ $\sqrt{9}+2\stackrel{?}{=}5$ $3+2\stackrel{?}{=}5$ $5=5\checkmark$	
	The solution is $y=\frac{4}{3}$

Try It 4

Solve: $\sqrt{5q+1}+4=6$.

Answer

$\frac{3}{5}$

When we use a radical sign, we mean the principal or positive root. If an equation has a square root equal to a negative number, that equation will have no solution.

Example 5

Solve: $\sqrt{9k-2}+1=0$.

Solution

	$\sqrt{9k-2}+1=0$
To isolate the radical, subtract 1 from both sides.	$\sqrt{9k-2}+1-1=0-1$
Simplify.	$\sqrt{9k-2}=-1$
Since the square root is equal to a negative number, the equation has no solution.	

Try It 5

Solve: $\sqrt{2r-3}+5=0$

Answer

no solution

When there is a coefficient in front of the radical, we must square it, too.

Example 6

Solve: $3\sqrt{3x-5}-8=4$.

Solution

	$3\sqrt{3x-5}-8=4$
Isolate the radical.	$3\sqrt{3x-5}=12$
Square both sides of the equation.	$(3\sqrt{3x-5})^2=(12)^2$
Simplify, then solve the new equation.	9(3x−5)=144
Distribute.	27x−45=144
Solve the equation.	27x=189
	x=7
Check the answer. $x=7$ $3\sqrt{3x-5}-8=4$ $3\sqrt{3(7)-5}-8\stackrel{?}{=}4$ $3\sqrt{21-5}-8\stackrel{?}{=}4$ $3\sqrt{16}-8\stackrel{?}{=}4$ $3(4)-8\stackrel{?}{=}4$ $4=4\checkmark$	The solution is x=7.

Try It 6

Solve: $2\sqrt{9a+3}+14=15$.

Answer

$-\frac{11}{36}$

Example 7

Solve: $\sqrt{4z-3}=\sqrt{3z+2}$.

Solution

	$\sqrt{4z-3}=\sqrt{3z+2}$
The radical terms are isolated	$\sqrt{4z-3}=\sqrt{3z+2}$
Square both sides of the equation.	$(\sqrt{4z-3})^2=(\sqrt{3z+2})^2$
Simplify, then solve the new equation	4z−3=3z+2
	z−3=2
	z=5
Check the answer. We leave it to you to show that 5 checks!	The solution is z=5.

? Try It 7

Solve: $\sqrt{2x-5}=\sqrt{5x+3}$.

Answer

$x=\frac{-8}{3}$

Use Square Roots in Applications

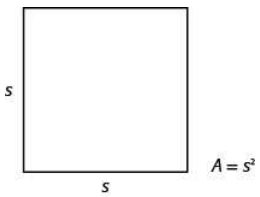
As you progress through your college courses, you'll encounter formulas that include square roots in many disciplines. We have already used formulas to solve geometry applications.

We will use our Problem-Solving Strategy for Geometry Applications, with slight modifications, to develop a plan for solving applications involving formulas from any discipline.

Definition: SOLVE APPLICATIONS WITH FORMULAS.

a. **Read** the problem carefully and ensure that all the words and ideas are understood. When appropriate, draw a figure and label it with the given information.
b. **Identify** what we are looking for.
c. **Name** what we are looking for by choosing a variable to represent it.
d. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
e. **Solve the equation** using good algebra techniques.
f. **Check** the answer in the problem and make sure it makes sense.
g. **Answer** the question with a complete sentence.

We used the formula A=L·W to find the area of a rectangle with length *L* and width *W*. A square is a rectangle in which the length and width are equal. If we let *s* be the length of a side of a square, the area of the square is \(s^2\).

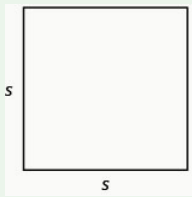


The formula $A=s^2$ gives us the area of a square if we know the length of a side. What if we want to find the length of a side for a given area? Then we need to solve the equation for *s*.

	$A = s^2$
Take the square root of both sides.	$\sqrt{A} = \sqrt{s^2}$
Simplify.	$s = \sqrt{A}$

We can use the formula $s = \sqrt{A}$ to find the length of a side of a square for a given area.

Definition: AREA OF A SQUARE



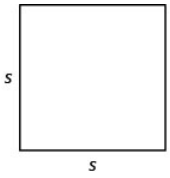
Area, A	$A = s^2$
Length of a side, s	$s = \sqrt{A}$

We will illustrate this concept in the next example.

Example 8

Mike and Lychelle want to make a square patio. They have enough concrete to pave an area of 200 square feet. Use the formula $s = \sqrt{A}$ to find the length of each side of the patio. Round your answer to the nearest tenth of a foot.

Solution

Step 1. Read the problem. Draw a figure and label it with the given information.	
	A = 200 square feet
Step 2. Identify what you are looking for.	The length of a side of the square patio.
Step 3. Name what you are looking for by choosing a variable to represent it.	Let s = the length of a side.
Step 4. Translate into an equation by writing the appropriate formula or model for the situation. Substitute the given information.	$s = \sqrt{A}$, and $A = 200$ $s = \sqrt{200}$
Step 5. Solve the equation using good algebra techniques. Round to one decimal place.	$s = 14.14213...$ $s \approx 14.1$
Step 6. Check the answer in the problem and make sure it makes sense.	
$14.1^2 \stackrel{?}{\approx} 200$ $14.1^2 \approx 198.81$ ✓	
This is close enough because we rounded the square root. Is a patio with a side of 14.1 feet reasonable? Yes.	
Step 7. Answer the question with a complete sentence.	Each side of the patio should be 14.1 feet.

Try It 8

Katie wants to plant a square lawn in her front yard. She has enough sod to cover an area of 370 square feet. Use the formula $s = \sqrt{A}$ to find the length of each side of her lawn. Round your answer to the nearest tenth of a foot.

Answer

19.2 feet

Another application of square roots involves gravity.

Definition: FALLING OBJECTS

On Earth, if an object is dropped from a height of hh feet, the time in seconds it will take to reach the ground is found by using the formula,

$t = \frac{\sqrt{h}}{4}$

For example, if an object is dropped from a height of 64 feet, we can find the time it takes to reach the ground by substituting h=64 into the formula.

	$t = \frac{\sqrt{h}}{4}$
	$t = \frac{\sqrt{64}}{4}$
Take the square root of 64.	$t = \frac{8}{4}$
Simplify the fraction.	$t = 2$

It would take 2 seconds for an object dropped from a height of 64 feet to reach the ground.

Example 9

Christy dropped her sunglasses from a bridge 400 feet above a river. Use the formula $t = \frac{\sqrt{h}}{4}$ to find how many seconds it took for the sunglasses to reach the river.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	The time it takes for the sunglasses to reach the river.
Step 3. Name what you are looking for by choosing a variable to represent it.	Let t = time.
Step 4. Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.	$t = \frac{\sqrt{h}}{4}$, and $h = 400$ $t = \frac{\sqrt{400}}{4}$
Step 5. Solve the equation using good algebra techniques.	$t = \frac{20}{4}$ $t = 5$
Step 6. Check the answer in the problem and make sure it makes sense.	
$5 \stackrel{?}{=} \frac{\sqrt{400}}{4}$ $5 \stackrel{?}{=} \frac{20}{4}$ 5=5✓	
Does 5 seconds seem reasonable? Yes.	
Step 7. Answer the question with a complete sentence.	It will take 5 seconds for the sunglasses to hit the water.

Try It 9

A helicopter dropped a rescue package from a height of 1,296 feet. Use the formula $t = \frac{\sqrt{h}}{4}$ to find how many seconds it took for the package to reach the ground.

Answer

9 seconds

Police officers investigating car accidents measure the length of the skid marks on the pavement. Then they use square roots to determine the speed, in miles per hour, a car was going before applying the brakes.

Definition: SKID MARKS AND SPEED OF A CAR

If the length of the skid marks is d feet, then the speed, s, of the car before the brakes were applied can be found by using the formula,

$s = \sqrt{24d}$

Example 10

After a car accident, the skid marks for one car measured 190 feet. Use the formula $s = \sqrt{24d}$ to find the speed of the car before the brakes were applied. Round your answer to the nearest tenth.

Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what we are looking for.	The speed of a car.
Step 3. **Name** what we are looking for.	Let s = the speed.
Step 4. **Translate** into an equation by writing the appropriate formula.	$s = \sqrt{24d}$, and $d = 190$
Substitute the given information.	$s = \sqrt{24(190)}$
Step 5. **Solve the equation**.	$s = \sqrt{4560}$
	$s = 67.52777...$
Round to 1 decimal place.	$s \approx 67.5$
Step 6. **Check** the answer in the problem. $67.5 \approx ?\sqrt{24(190)}$ $67.5 \approx ?\sqrt{4560}$ $67.5 \approx ?67.5277...$	
Is 67.5 mph a reasonable speed?	Yes.
Step 7. **Answer** the question with a complete sentence.	The car's speed was approximately 67.5 miles per hour.

? Try It 10

An accident investigator measured the skid marks of the car. The length of the skid marks was 76 feet. Use the formula $s = \sqrt{24d}$ to find the speed of the car before the brakes were applied. Round your answer to the nearest tenth.

Answer

42.7 mph

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Section 2.7P: Practice

Practice Makes Progress

In the following exercises, check whether the given values are solutions.

Exercise 1

For the equation $\sqrt{x+12}=x$: Is $x=4$ a solution?

Exercise 2

For the equation $\sqrt{x+12}=x$: Is $x=-3$ a solution?

In the following exercises, solve.

Exercise 3

$\sqrt{5y+1}=4$

Exercise 4

$\sqrt{2m-3}-5=0$

Exercise 5

$\sqrt{6v-2}-10=0$

Exercise 6

$\sqrt{5q+3}-4=0$

Exercise 7

$\sqrt{5v-2}+5=0$

Exercise 8

$\sqrt{v-10}+10=v$

Exercise 9

$\sqrt{y+4}-y+2=0$

Exercise 10

$2\sqrt{5x+1}-8=0$

Exercise 11

$\sqrt{4v+3}=\sqrt{v-6}$

Exercise 12

$\sqrt{r}+6=\sqrt{r+8}$

Exercise 13

$\sqrt{s-3}+2=\sqrt{s+4}$

Exercise 14

$\sqrt{8u+1}+9=0$

Exercise 15

$\sqrt{9p+9}=\sqrt{10p-6}$

Everyday Math

Round approximations to one decimal place.

Exercise 16

Vince wants to make a square patio in his yard. He has enough concrete to pave an area of 130 square feet. Use the formula $s=\sqrt{A}$ to find the length of each side of his patio. Round your answer to the nearest tenth of a foot.

Exercise 17

An airplane dropped a flare from a height of 1024 feet above a lake. Use the formula $t=\frac{\sqrt{h}}{4}$ to find how many seconds it took for the flare to reach the water.

Exercise 18

A construction worker dropped a hammer while building the Grand Canyon skywalk, 4000 feet above the Colorado River. Use the formula $t=\frac{\sqrt{h}}{4}$ to find how many seconds it took for the hammer to reach the river.

Exercise 19

The skid marks for a car involved in an accident measured 216 feet. Use the formula $s=\sqrt{24d}$ to find the speed of the car before the brakes were applied. Round your answer to the nearest tenth.

Exercise 20

An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 117 feet. Use the formula $s=\sqrt{24d}$ to find the speed of the vehicle before the brakes were applied. Round your answer to the nearest tenth.

Writing Exercises

Exercise 21

a. Solve the equation $\sqrt{r+4}-r+2=0$.
b. Explain why one of the “solutions” that was found was not actually a solution to the equation.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine if given values makes equations involving rational exponents true			
solve equations involving rational exponents			
solve applications involving rational exponents			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 3: Measuring Magnitude

In this final unit, we'll explore absolute value as a way to measure distance—how far a number is from zero, no matter the direction. We'll learn how to solve equations and inequalities involving absolute value, both symbolically and graphically. With the help of tools like Desmos, we'll see how absolute value defines ranges, tolerances, and real-world limits. Whether it's GPS coordinates or error margins in science, absolute value helps us define what "close enough" means.

Section 3.1: Absolute Value

Section 3.1P: Practice

Section 3.2: Absolute Value Equations

Section 3.2P: Practice

Section 3.3: Absolute Value Inequalities

Section 3.3P: Practice

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Section 3.1: Absolute Value

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: a. $|-7|$ and b. $|-3+2|-1$
- Plot the points $(-3,3)$ and $(2,2)$ on graph paper
- Simplify: $-(2-3x)-(x-4)$

Learning Objectives

Motivating Problem

You're standing at a subway station at position 0. A friend is at -5, and another friend is at +7. Who's farther from you? Does direction matter if you're only interested in distance?

Fun Fact

Karl Weierstrass first introduced the absolute value symbol $|x|$ in the 1800s as a means to formalize the concept of distance from zero, an idea that is widely used in science, finance, and navigation.

The Goal

In this section, we'll learn what absolute value means and how to represent it on a number line and a coordinate plane. We'll explore symmetry and how absolute value reflects distance, regardless of direction.

Absolute Value

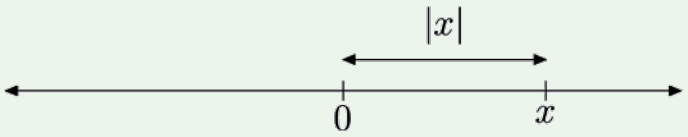
In a sense, the absolute value of a number is a measure of its magnitude, its distance from zero, without its sign. Thus,

$$|7|=7 \qquad \text{and} \qquad |-7|=7$$

Here is the formal definition of the absolute value of a real number.

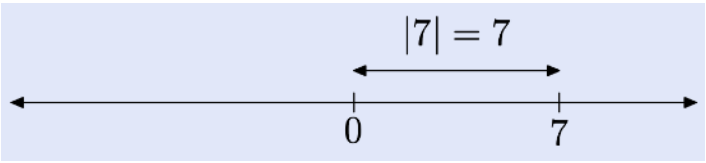
Definition: Absolute Value

To find the absolute value of any real number, first locate the number on the real line.

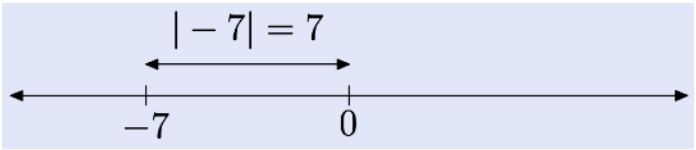


The absolute value of the number is defined as its distance from the origin.

For example, to find the absolute value of 7, locate 7 on the real line and then find its distance from the origin.



To find the absolute value of -7, locate -7 on the real line and then find its distance from the origin.



Some like to say that taking the absolute value "produces a number that is always positive." However, this ignores an important exception, that is,

$$|0| = 0$$

When we take the absolute value of a number, the result is always nonnegative; that is, the result is either positive or zero. In symbols,

$$|x| \geq 0 \text{ for all real numbers } x$$

This makes sense when we think of the $|x|$ as the distance from x to zero, since distance is always nonnegative.

However, the discussion above is not of sufficient depth to handle more sophisticated problems involving absolute value.

A Piecewise Definition of Absolute Value

Because absolute value is intimately connected with distance, mathematicians and scientists find it an invaluable tool for measurement and error analysis. However, we will need a formulaic definition of the absolute value if we want to use this tool effectively. We need to develop a piecewise definition of the absolute value function that defines the absolute value for any arbitrary real number x.

We begin with a few observations. Remember, the absolute value of a number is always nonnegative (positive or zero).

1. If a number is negative, negating that number will make it positive. $|-5| = -(-5) = 5$, and similarly, $|-12| = -(-12) = 12$. Thus, if $x < 0$ (if x is negative), then $|x| = -(x)$.
2. If $x = 0$, then $|x| = 0$.
3. If a number is positive, taking the absolute value of that number does not change anything.

$$|5| = 5, \text{ and similarly, } |12| = 12$$

Thus, if $x > 0$ (if x is positive), then $|x| = x$.

We can summarize these three cases with a piecewise definition.

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ x, & \text{if } x > 0 \end{cases}$$

It is the first line in our piecewise definition that usually leaves students scratching their heads. They might say, "I thought absolute value makes a number positive (or zero), yet you have $|x| = -x$; that is, you have the absolute value of x equal to a negative x." Try as they might, this seems contradictory. Does it seem so to you?

However, there is no contradiction. If $x < 0$, that is, if x is a negative number, then $-x$ is a positive number. For example, if $x = -8$, then $-x = 8$, and even though we say "negative x," in this case, $-x$ is a positive number.

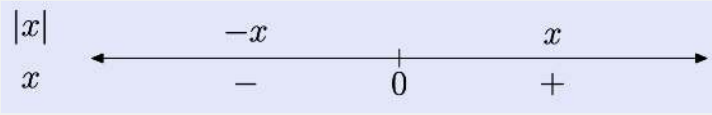
If this still has you running confused, consider the simple fact that x and $-x$ must have "opposite signs." If one is positive, the other is negative, and vice versa. Consequently,

- if x is positive, then $-x$ is negative, but
- if x is negative, then $-x$ is positive.

Let's summarize what we've learned thus far.

Summarizing the Definition on a Number Line

We like to use a number line to help summarize the definition of the absolute value of x.



Some remarks are in order for this summary on the number line.

- We first draw the real line, then mark the "critical value" for the expression inside the absolute value bars on the number line. The number zero is a critical value for the expression x, because x changes sign as you move from one side of zero to the other.
- To the left of zero, x is a negative number. We indicate this with the minus sign below the number line. To the right of zero, x is a positive number, indicated with a plus sign below the number line.
- Above the number line, we simplify the expression $|x|$. To the left of zero, x is a negative number (look below the line), so $|x| = -x$. Note how the result $-x$ is placed above the line to the left of zero. Similarly, to the right of zero, x is a positive number (look below the line), so $|x| = x$. Note how the result x is placed above the line to the right of zero.

In the piecewise definition of $|x|$, we have three distinct pieces, one for each of the cases discussed above. However, because $|0| = 0$, we can include this case with the piece $|x| = x$, if we adjust the condition to include zero.

Definition

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Note that this piecewise definition agrees with our discussion to date.

1. If x is a negative number (i.e., if $x < 0$), then the absolute value must change x to a positive number by negating. That is, $|x| = -x$.
2. If x is positive or zero (i.e., if $x \geq 0$), then there's nothing to do except remove the absolute value bars. That is, $|x| = x$.

Constructing Piecewise Definitions

Let's see if we can determine piecewise definitions for other expressions involving absolute value.

Example 1

Determine a piecewise definition for $|x - 2|$.

Solution

First, find the "critical value" of the expression, the point at which the expression within the absolute value bars will change sign. This occurs when $x - 2 = 0$.

$$\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}$$

Draw a real line and mark this critical value of x on the line. Place the expression $x - 2$ below the line at its left end.



Next, determine the sign of $x - 2$ for values of x on each side of 2. This is easily done by "testing" a point on each side of 2 in the expression $x - 2$.

Take $x = 1$, which lies to the left of the critical value 2 on our number line. Substitute this value of x in the expression $x - 2$, obtaining

$$x-2=1-2=-1$$

which is negative. Indeed, regardless of which x-value you pick to the left of 2, when inserted into the expression $x-2$, you will get a negative result (you should check this for other values of x to the left of 2). We indicate that the expression $x-2$ is negative for values of x to the left of 2 by placing a minus (-) sign below the number line to the left of 2.

Next, pick $x=3$, which lies to the right of the critical value 2 on the number line. Substitute this value of x into the expression $x-2$, obtaining

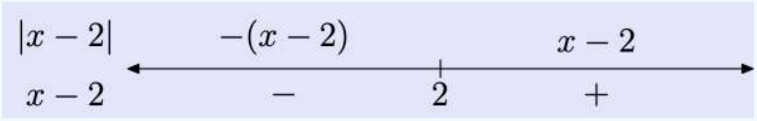
$$x-2=3-2=1$$

which is positive. Indeed, regardless of which x-value you pick to the right of 2, when inserted into the expression $x-2$, you will get a positive result (you should check this for other values of x to the right of 2). We indicate that the expression $x-2$ is positive for values of x to the right of 2 by placing a plus (+) sign below the number line to the right of 2.



The next step is to remove the absolute value bars from the expression $|x-2|$, depending on the sign of $x-2$.

To the left of 2, the expression $x-2$ is negative (note the minus sign (-) below the number line), so $|x-2|=-(x-2)$. That is, we have to negate $x-2$ to make it positive. This is indicated by placing $-(x-2)$ above the line to the left of 2.



To the right of 2, the expression $x-2$ is positive (note the plus sign (+) below the line), so $|x-2|=x-2$. That is, we simply remove the absolute value bars because the quantity inside is already positive. This is indicated by placing $x-2$ above the line to the right of 2 (see the number line above).

We can use this last number line summary to construct a piecewise definition of the expression $|x-2|$.

$$|x-2|=\begin{cases}-(x-2), & \text{if } x<2,\\ x-2, & \text{if } x\geq 2\end{cases}=\begin{cases}-x+2, & \text{if } x<2\\ x-2, & \text{if } x\geq 2\end{cases}$$

Our number line and piecewise definition agree: $|x-2|=-(x-2)$ to the left of 2 and $|x-2|=x-2$ to the right of 2. Further, note how we've included the critical value of 2 "on the right" in our piecewise definition.

Let's summarize the method we followed to construct the piecewise function above.

Constructing a Piecewise Definition for Absolute Value

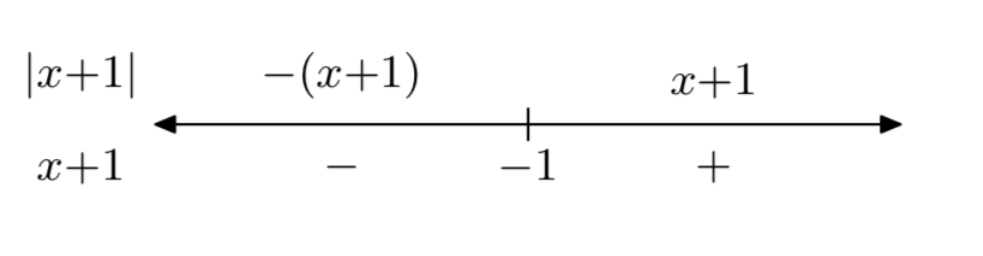
When presented with the absolute value of an algebraic expression, perform the following steps to remove the absolute value bars and construct an equivalent piecewise definition.

1. Take the expression that is inside the absolute value bars, and set that expression equal to zero. Then solve for x. This value of x is called a "critical value." (Note: The expression inside the absolute value bars could have more than one critical value. We will not encounter such problems in this text.)
2. Place your critical value on a number line.
3. Place the expression inside the absolute value bars below the number line at the left end.
4. Test the sign of the expression inside the absolute value bars by inserting a value of x from each side of the critical value and marking the result with a plus (+) or minus (-) sign below the number line.
5. Place the original expression, the one including the absolute value bars, above the number line at the left end.
6. Use the sign of the expression inside the absolute value bars (indicated by the plus and minus signs below the number line) to remove the absolute value bars, placing the results above the number line on each side of the critical value.
7. Construct a piecewise definition that organizes the results on the number line.

? Try It 1

Determine a piecewise definition for $y = |x+1|$

Answer



$$f(x) = \begin{cases} -x-1, & \text{if } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$$

Let's apply this technique to another example.

✔ Example 2

Determine a piecewise definition for $|3-2x|$.

Solution

Step 1: First set the expression inside the absolute value bars equal to zero and solve for x.

$$\begin{aligned} 3-2x &= 0 \\ x &= 3/2 \end{aligned}$$

Note that $3-2x=0$ at $x=\frac{3}{2}$. This is the "critical value" for this expression.

Steps 2 and 3: Draw a number line and mark this critical value on the line. The next step requires that we place the expression inside the absolute value bars, namely $3-2x$, underneath the line at its left end.



Step 4: Next, determine the sign of $3-2x$ for values of x on each side of $\frac{3}{2}$. This is easily done by "testing" a point on each side of $\frac{3}{2}$ in the expression $3-2x$.

- Take $x=1$, which lies to the left of $\frac{3}{2}$. Substitute this value of x into the expression $3-2x$, obtaining

  $$3-2x = 3-2(1) = 1$$

  which is positive. Indicate this result by placing a plus sign (+) below the number line to the left of $\frac{3}{2}$.
- Next, pick $x=2$, which lies to the right of $\frac{3}{2}$. Substitute this value of x into the expression $3-2x$, obtaining

  $$3-2x = 3-2(2) = -1$$

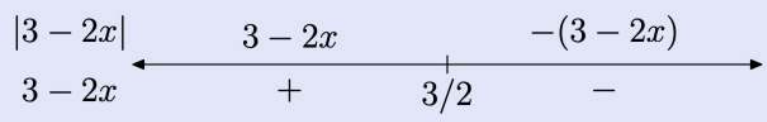
  which is negative. Indicate this result by placing a negative sign (-) below the line to the right of $\frac{3}{2}$ (see the number line above).



Steps 5 and 6: Place the original expression, namely $|3-2x|$, above the number line at the left end. The next step is to remove the absolute value bars from the expression $|3-2x|$.

- To the left of $\frac{3}{2}$, the expression $3-2x$ is positive (note the plus sign (+) below the number line), so $|3-2x| = 3-2x$. Indicate this result by placing the expression $3-2x$ above the number line to the left of $\frac{3}{2}$.

- To the right of $\frac{3}{2}$, the expression $3-2x$ is negative (note the minus sign (-) below the numberline), so $|3-2x| = -(3-2x)$. That is, we have to negate $3-2x$ to make it positive. This is indicated by placing the expression $-(3-2x)$ above the line to the right of $\frac{3}{2}$.



Step 7: We can use this last number line summary to write a piecewise definition for the expression $|3-2x|$.

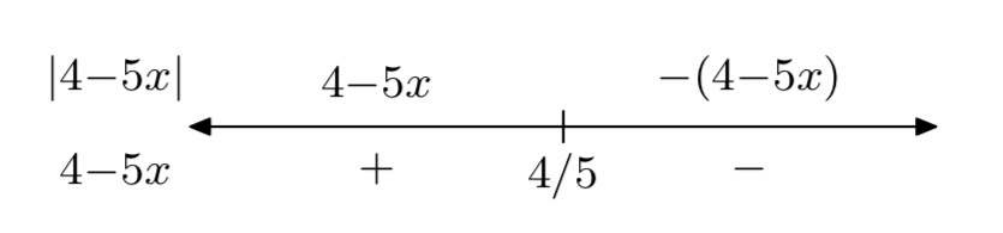
$$|3-2x| = \begin{cases} 3-2x, & \text{if } x < 3/2. \\ -(3-2x), & \text{if } x \geq 3/2 \end{cases} = \begin{cases} 3-2x, & \text{if } x < 3/2 \\ -3+2x, & \text{if } x \geq 3/2 \end{cases}$$

Again, note how we've included the critical value of $\frac{3}{2}$ "on the right."

? Try It 2

Determine the piecewise definition for $y = |4-5x|$.

Answer



$$g(x) = \begin{cases} 4-5x, & \text{if } x < \frac{4}{5} \\ -4+5x, & \text{if } x \geq \frac{4}{5} \end{cases}$$

Drawing the Graph of an Absolute Value Equation

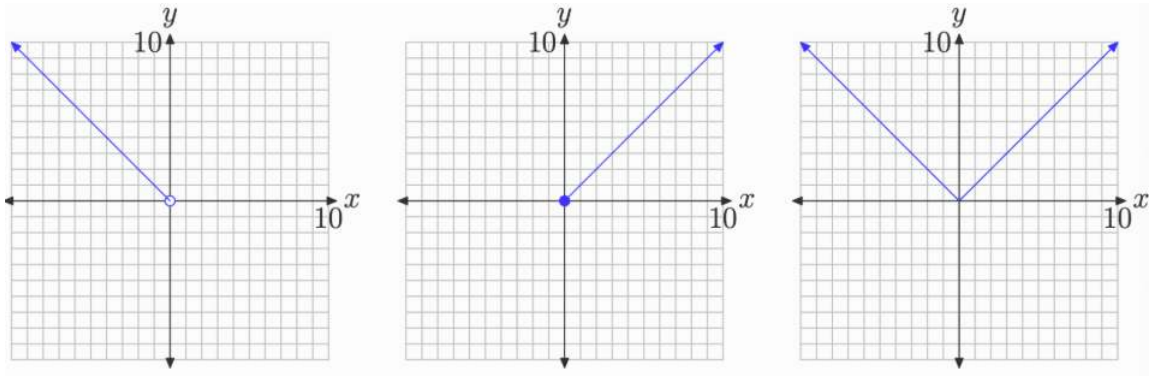
Now that we know how to construct a piecewise definition for an expression containing absolute value bars, we can use what we know about graphing lines.

Consider again the basic definition of the absolute value of x.

$$y = |x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Some basic observations are:

- If $x < 0$, then $y = -x$. This ray starts at the origin and extends to the left with slope -1. Its graph is pictured on the left in the figure below.
- If $x \geq 0$, then $y = x$. This ray starts at the origin and extends to the right with slope 1. Its graph is pictured in the center of the figure below.
- We combine these graphs to produce the graph of $y = |x|$, pictured on the right in the figure below.



You should commit the graph of $y = |x|$ to memory. Things to note:

- The graph of $y = |x|$ is "V-shaped."
- The vertex of the graph is at the point $(0, 0)$.
- The left-hand branch has the equation $y = -x$ and a slope of -1.
- The right-hand branch has the equation $y = x$ and a slope of 1.

Now that we know how to draw the graph of $y = |x|$, we can sketch a number of simple graphs involving absolute value.

Example 3

Sketch a graph of $y = |x + 1|$

Solution

We constructed the following piecewise definition of $y = |x + 1|$:

$$y = |x+1| = \begin{cases} -(x+1), & \text{if } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$$

We now sketch each piece of this equation based on its definition.

If $x < -1$, then $y = -(x + 1)$. This defines a ray starting at $x = -1$ and extending to the left. To find the endpoint of the ray, evaluate $y = -(x + 1)$ at $x = -1$:

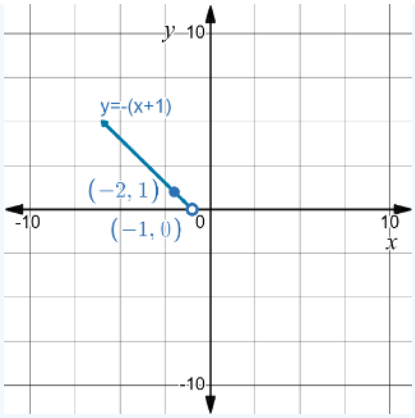
$$y = -(-1 + 1) = -0 = 0$$

Thus, the endpoint of the ray is located at $(-1, 0)$. Since x must be strictly less than -1, this endpoint is shown as an open circle on the graph.

Next, choose a value of x to the left of -1, such as $x = -2$, to find another point:

$$y = -(-2 + 1) = -(-1) = 1$$

Thus, a second point on the ray is $(-2, 1)$. Using these points, we construct a graph containing the two evaluated points and a ray connecting them.



If $x \geq -1$, then $y = x + 1$. This defines a ray starting at $x = -1$ and extending to the right. Evaluate $y = x + 1$ at $x = -1$:

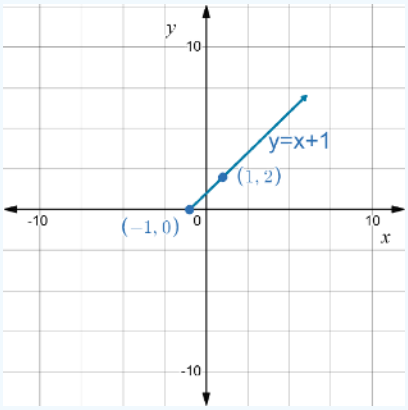
$$y = -1 + 1 = 0$$

So the endpoint of the ray is at $(-1, 0)$. Since x may equal -1 in this case, the endpoint is shown as a closed circle on the graph.

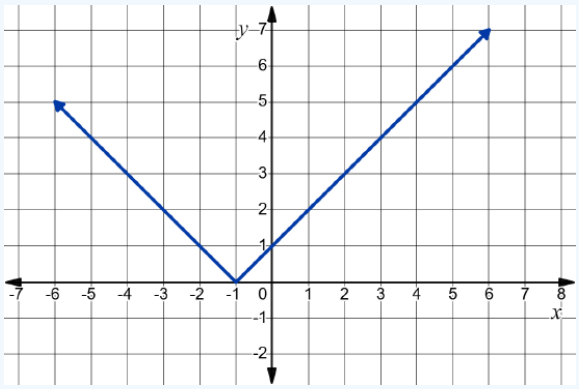
Now pick a value of x to the right of -1, such as $x = 1$:

$$y = 1 + 1 = 2$$

Thus, another point on the ray is $(1, 2)$. Using these points, we construct a graph containing the two evaluated points and a ray connecting them.



To sketch the graph of $y = |x + 1|$, we combine the two rays we just graphed.

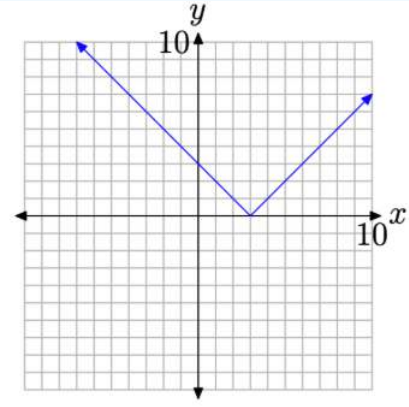


Note the “V-shape” of the graph. We will refer to the point at the tip of the “V” as the **vertex** of the absolute value function.

? Try It 3

Sketch the graph of $y = |x - 3|$.

Answer



✔ Example 4

Sketch the graph of $y = |3 - 2x|$.

Solution

We constructed the following piecewise definition of $y = |3 - 2x|$ in an earlier example.

$$y = |3 - 2x| = \begin{cases} 3 - 2x, & \text{if } x < 3/2 \\ -3 + 2x, & \text{if } x \geq 3/2 \end{cases}$$

We now sketch each piece of this equation based on its definition.

If $x < \frac{3}{2}$, then $y = 3 - 2x$. This defines a ray, starting at $x = \frac{3}{2}$ and extending to the left. We first find the endpoints of the ray by evaluating $y = 3 - 2x$ at $x = \frac{3}{2}$

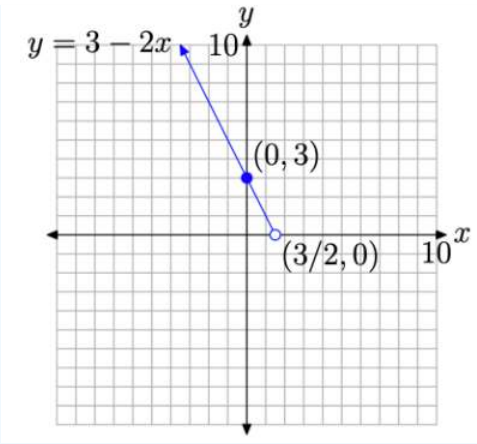
$$y = 3 - 2(3/2) = 3 - 3 = 0$$

Thus, the endpoint of the ray is located at $(\frac{3}{2}, 0)$. Since x must be strictly less than $\frac{3}{2}$, this endpoint is shown as an open circle on the graph.

Next, pick a value of x that lies to the left of $\frac{3}{2}$, such as $x = 0$, to find another point on the ray.

$$y = 3 - 2(0) = 3 - 0 = 3$$

Thus, a second point on the ray is $(0, 3)$. Using these points, we construct a graph containing the two evaluated points and a ray connecting them.



If $x \geq \frac{3}{2}$, then $y = -3 + 2x$. This is a ray, starting at $x = \frac{3}{2}$ and extending to the right. We find the endpoint of the ray by evaluating $y = -3 + 2x$ at $x = \frac{3}{2}$,

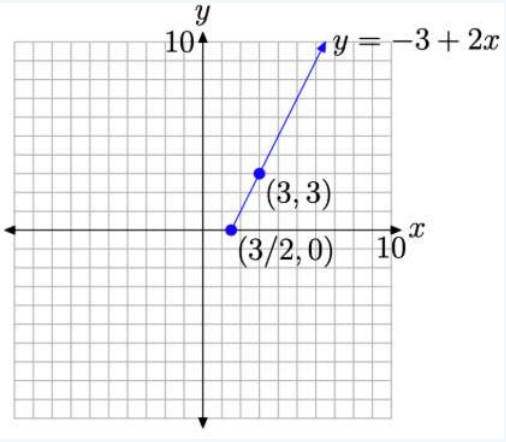
$$y = -3 + 2(3/2) = -3 + 3 = 0$$

Thus, the endpoint of the ray is located at $(\frac{3}{2}, 0)$. Since x may equal $\frac{3}{2}$ in this case, the endpoint is shown as a closed circle on the graph.

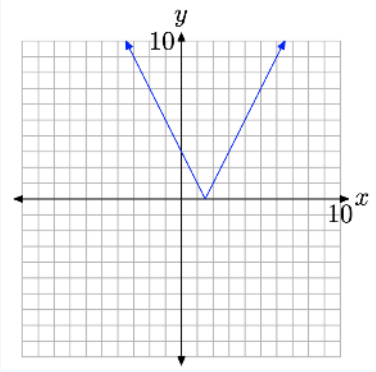
Next, to find another point on the ray, pick a value of x that lies to the right of $\frac{3}{2}$ such as $x = 3$,

$$y = -3 + 2(3) = -3 + 6 = 3$$

Thus, a second point on the ray is $(3, 3)$. Using these points, we construct a graph containing the two evaluated points and a ray connecting them.



To sketch the graph of $y = |3 - 2x|$, we need only combine the two pieces from the two rays we previously graphed.



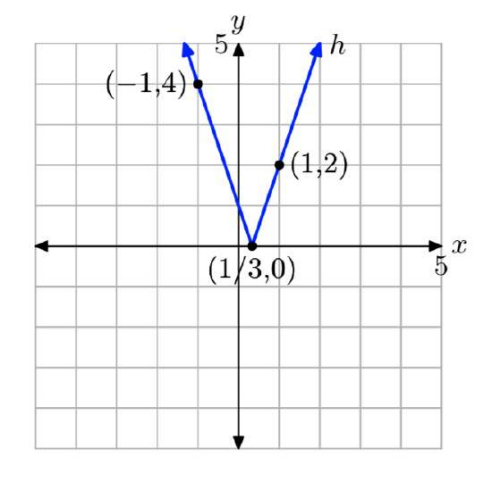
In this graph, the equation of the left-hand branch of the "V" is $y = 3 - 2x$. An alternative approach to drawing this branch is to note that its graph is contained within the graph of the whole line $y = 3 - 2x$, which has a slope of -2 and a y-intercept at $(0, 3)$. Thus, one could draw the entire line using the slope and y-intercept, then erase the part of the line that lies to the right of $x = \frac{3}{2}$. A similar strategy would work for the right-hand branch of $y = |3 - 2x|$.

? Try It 4

Sketch a graph for $y = |1 - 3x|$.

Answer

$$y = \begin{cases} 1 - 3x, & \text{if } x < \frac{1}{3} \\ -1 + 3x, & \text{if } x \geq \frac{1}{3} \end{cases}$$



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Section 3.1P: Practice

Practice Makes Progress

For the following exercises...

a. Mark the “critical value” on a number line.
b. Mark the sign of the expression inside the absolute value bars below the number line.
c. Above the number line, remove the absolute value bars according to the sign of the expression you marked below the number line.
d. Create a piecewise definition for the given absolute value equation.

Exercise 1

$y = |x + 1|$

Exercise 2

$y = |4 - 5x|$

Exercise 3

$y = |-x - 5|$

Exercise 4

$y = x + |x|$

For the following exercises...

a. Create a piecewise definition for the given expression
b. Use your piecewise definition to sketch the graph of the given absolute value equation on its own coordinate system.

Exercise 5

$y = |x - 1|$

Exercise 6

$y = |2x - 1|$

Exercise 7

$y = |1 - 3x|$

Exercise 8

$y = x - |x|$

Writing Exercises

? Exercise 9

Use a graphing calculator to draw the graphs of $y = |x|$, $y = |x + 2|$, $y = |x + 4|$, and $y = |x + 6|$ on the same viewing window. In your own words, explain what you learned in this exercise.

? Exercise 10

Explain how writing a piecewise definition helps you to graph an absolute value relation.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine the "critical value" of an absolute value expression			
create a piecewise definition for a given absolute value expression			
graph an absolute value expression			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.2: Absolute Value Equations

Be Prepared

We will rely heavily on these skills throughout this section.

- Solve $x + 3 = 7$ for x
- Solve $3y - 2x - 1 = 5 - 2y + x$ for y
- Graph the lines $y = x + 3$ and $y = 7$ and determine the point of their intersection

Learning Objectives

Motivating Problem

You're using GPS to find out how far you are from a coffee shop. Whether you're two blocks east or two blocks west, the distance is still 2. If the total walking distance is six blocks, how far could you be from the shop?

Fun Fact

Absolute value equations are used in real life to define tolerances—for example, a machine part must be within ± 0.01 inches of a target measurement. It's all about how far off from an error of zero you're allowed to be!

The Goal

In this section, we'll learn to solve absolute value equations both algebraically and graphically. We'll explore what it means to say "distance equals a number" and use Desmos to visualize where two expressions intersect.

Solve Absolute Value Equations

As we prepare to solve absolute value equations, we review our definition of **absolute value**.

ABSOLUTE VALUE

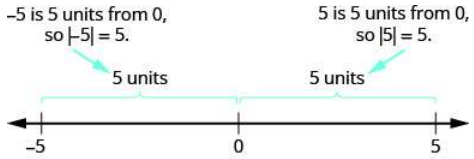
The absolute value of a number is its distance from zero on the number line.

The absolute value of a number n is written as $|n|$ and $|n| \geq 0$ for all numbers.

Absolute values are always greater than or equal to zero.

We learned that both a number and its opposite are the same distance from zero on the number line. Since they have the same distance from zero, they have the same absolute value. For example:

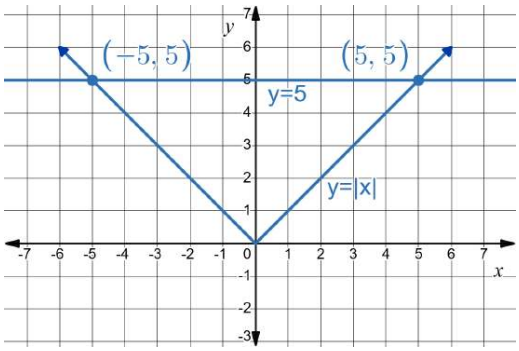
- -5 is 5 units away from 0, so $|-5| = 5$.
- 5 is 5 units away from 0, so $|5| = 5$.



For the equation |x|=5,|x|=5, we are looking for all numbers that make this a true statement. We are looking for the numbers whose distance from zero is 5. We just saw that both 5 and -5-5 are five units from zero on the number line. They are the solutions to the equation.

$$\begin{array}{ll} \text{If} & |x| = 5 \\ \text{then} & x = -5 \text{ or } x = 5 \end{array}$$

Let's visualize this graphically. Consider the graphs of $y = |x|$ and $y = 5$. The equation $|x| = 5$ has a solution corresponding to each point of intersection on the graph. We can see from the graph that the intersection points occur when $x = -5$ and $x = 5$.



The solution can be simplified to a single statement by writing $x = \pm 5$. This is read, "x is equal to positive or negative 5".

We can generalize this to the following property for absolute value equations.

ABSOLUTE VALUE EQUATIONS

For any algebraic expression, u, and any positive real number, a,

$$\begin{array}{ll} \textbf{if} & |u| = a \\ \textbf{then} & u = -a \text{ or } u = a \end{array}$$

Remember that an absolute value cannot be a negative number.

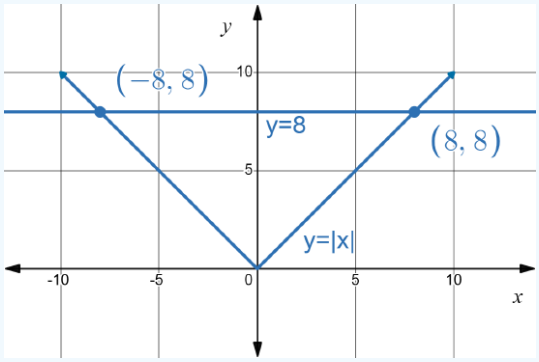
Example 1

Solve each equation graphically:

a. $|x| = 8$
b. $|x| = -6$
c. $|x| = 0$

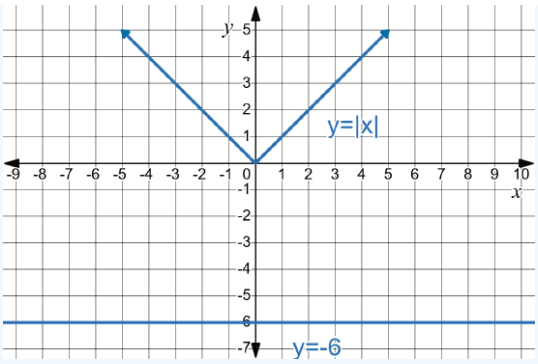
Solution

a. The graph of $y = |x|$ intersects the graph of $y = 8$ twice.



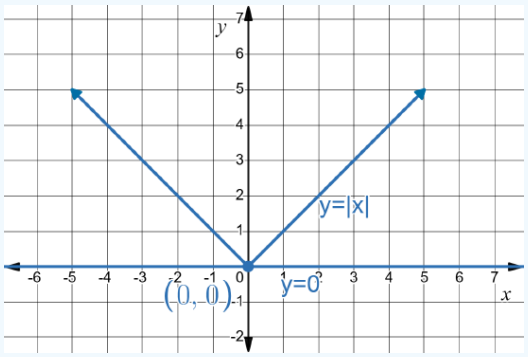
$$\begin{array}{ll} & |x| = 8 \\ \textbf{Write the equivalent equations.} & x = -8 \text{ or } x = 8 \\ & x = \pm 8 \end{array}$$

b. The graph of $y = |x|$ never intersects the graph of $y = -6$ since an absolute value is always positive. Therefore, there are no solutions to this equation.



$|x| = -6$
No solution

c. The graph of $y = |x|$ intersects the graph of $y = 0$ (the x-axis) exactly once.



	$\|x\| = 0$
Write the equivalent equations.	$x = -0$ **or** $x = 0$
Since $-0 = 0$,	$x = 0$

Both equations tell us that $x = 0$, and so there is only one solution.

? Try It 1

Solve each equation graphically:

a. $|x| = 2$
b. $|x| = -4$
c. $|x| = 0$

Answers

a. ± 2
b. no solution
c. 0

To solve an **absolute value equation**, we first isolate the absolute value expression using the same procedures we used to solve linear equations. Once we isolate the absolute value expression, we rewrite it as the two equivalent equations.

How to Solve Absolute Value Equations

✓ Example 2

Solve: $|x + 2| = 3$.

Solution

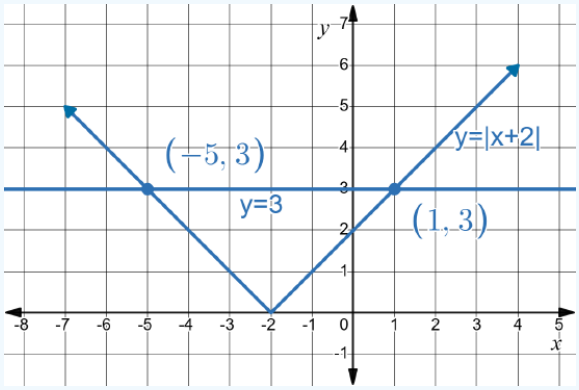
We begin by solving the equation algebraically. In this case, the argument "inside" of the absolute value is $x+2$ and must be equal to 3 or -3.

$$|x+2| = 3$$
$$\pm 3$$

Therefore, to solve this absolute value equation, set $x+2$ equal to ± 3 and solve each linear equation as usual.

$$|x+2| = 3$$
$$x+2 = -3 \quad \text{or} \quad x+2 = 3$$
$$x = -5 \qquad\qquad x = 1$$

To visualize these solutions, graph the relations on the same set of coordinate axes and find where they intersect.



From the graph, we can see the points of intersection occur when $x = -5$ and $x = 1$.

Solution Check...

Check $x = -5$	Check $x = 1$
$\|x+2\| \stackrel{?}{=} 3$	$\|x+2\| \stackrel{?}{=} 3$
$\|-5+2\| \stackrel{?}{=} 3$	$\|1+2\| \stackrel{?}{=} 3$
$\|-3\| \stackrel{?}{=} 3$	$\|3\| \stackrel{?}{=} 3$
$3 = 3\checkmark$	$3 = 3\checkmark$

Try It 2

Solve: $|2x+3| = 4$.

Answer

The solutions are $x = -\frac{7}{2}$ and $x = \frac{1}{2}$

The steps for solving an absolute value equation are summarized here.

SOLVE ABSOLUTE VALUE EQUATIONS.

1. Isolate the absolute value expression.
2. Write the equivalent equations.
3. Solve each equation.
4. Check each solution.

Example 3

Solve $2|x-7|+5=9$.

Solution

$$\begin{aligned} 2|x-7|+5&=9\\ 2|x-7|&=4\\ |x-7|&=2\\ x-7=-2 \quad &\text{or} \quad x-7=2\\ x&=5 \qquad\qquad x=9 \end{aligned}$$

Check $x=5$	Check $x=9$
$2\|x-7\|+5\stackrel{?}{=}9$ $2\|5-7\|+5\stackrel{?}{=}9$ $2\|-2\|+5\stackrel{?}{=}9$ $2(2)+5\stackrel{?}{=}9$ $4+5=9\checkmark$	$2\|x-7\|+5\stackrel{?}{=}9$ $2\|9-7\|+5\stackrel{?}{=}9$ $2\|2\|+5\stackrel{?}{=}9$ $2(2)+5\stackrel{?}{=}9$ $4+5=9\checkmark$

Try It 3

Solve: $3|x-4|-4=8$.

Answer

$x=8,\ x=0$

Example 4

Solve: $2|5x-1|-3=9$.

Solution

$$\begin{aligned} 2|5x-1|-3&=9\\ 2|5x-1|&=12\\ |5x-1|&=6\\ 5x-1=-6 \quad &\text{or} \quad 5x-1=6\\ 5x&=-5 \qquad\qquad 5x=7\\ x&=-1 \qquad\qquad x=\frac{7}{5} \end{aligned}$$

Check $x=-1$	Check $x=\frac{7}{5}$
$2\|5x-1\|-3\stackrel{?}{=}9$ $2\|5(-1)-1\|-3\stackrel{?}{=}9$ $2\|-5-1\|-3\stackrel{?}{=}9$ $2\|-6\|-3\stackrel{?}{=}9$ $2(6)-3=9$ $12-3=9\checkmark$	\($2\|5x-1\|-3\stackrel{?}{=}9$ $2\left\|5\left(\frac{7}{5}\right)-1\right\|-3\stackrel{?}{=}9$ $2\|7-1\|-3\stackrel{?}{=}9$ $2(6)-3=9$ $12-3=9\checkmark$ \) (Section 3.2.1)

Remember, an absolute value is always positive!

Try It 4

Solve: $|\frac{3}{4}x-5|+9=4$.

Answer

No solution

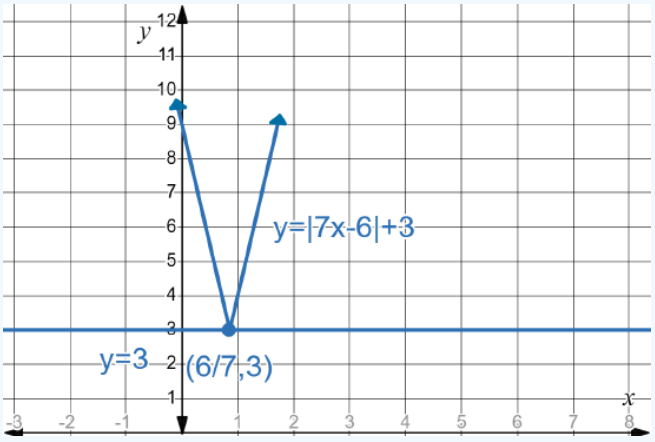
Example 5

Solve: $|7x-6|+3=3$.

Solution

$$|7x-6|+3=3$$
$$|7x-6|=0$$
$$7x-6=0$$
$$7x=6$$
$$x=\frac{6}{7}$$

Geometrically, we can see that the single intersection corresponds to the only solution.



Try It 5

Solve $9+|7x+1|=9$

Answer

$x=-\frac{1}{7}$

Some of our absolute value equations could be of the form $|u|=|v|$ where u and v are algebraic expressions. For example, $|x-3|=|2x+1|$.

How would we solve them? If two algebraic expressions are equal in absolute value, then they are either equal to each other or negatives of each other. The property for absolute value equations says that for any algebraic expression, u, and a positive real number, a, if $|u|=a$, then $u=-a$ or $u=a$.

This tells us that

if	$\|u\|=\|v\|$		
then	$\|u\|=v$	**or**	$\|u\|=-v$
and so	$u=v$ **or** $u=-v$	**or**	$u=-v$ **or** $u=-(-v)$

This leads us to the following property for equations with two absolute values.

EQUATIONS WITH TWO ABSOLUTE VALUES

For any algebraic expressions, u and v,

$$\begin{array}{ll} \textbf{if} & |u| = |v| \\ \textbf{then} & u = -v \text{ or } u = v \end{array}$$

When we take the opposite of a quantity, we must be careful with the signs and add parentheses where needed.

Example 6

Solve: $|5x - 1| = |2x + 3|$.

Solution

$$|5x - 1| = |2x + 3|$$

Write the equivalent equations.	$5x - 1 = -(2x + 3)$	or	$5x - 1 = 2x + 3$
	$5x - 1 = -2x - 3$	or	$3x - 1 = 3$
Solve each equation.	$7x - 1 = -3$		$3x = 4$
	$7x = -2$		$x = \frac{4}{3}$
	$x = -\frac{2}{7}$	or	$x = \frac{4}{3}$

Check.
We leave the check to you.

Try It 6

Solve: $|7x - 3| = |3x + 7|$.

Answer

$x = -\frac{2}{5},\ x = \frac{5}{2}$

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Section 3.2P: Practice

Practice Makes Progress

Use Desmos to graphically solve.

Exercise 1

$|x| = 3$

Exercise 2

$|y| = -1$

Exercise 3

$|z| = 0$

Exercise 4

$|2x - 3| - 4 = 1$

Exercise 5

$|4x - 1| - 3 = 0$

Exercise 6

$|3x - 4| + 5 = 7$

Exercise 7

$|4x + 7| + 2 = 5$

Exercise 8

$4|x - 1| + 2 = 10$

Exercise 9

$3|x - 4| + 2 = 11$

Exercise 10

$3|4x - 5| - 4 = 11$

Exercise 11

$3|x + 2| - 5 = 4$

? Exercise 12

$-2|x-3|+8=-4$

Solve each equation using algebraic methods.

? Exercise 13

$-3|x-4|+4=-5$

? Exercise 14

$|35x-2|+5=2$

? Exercise 15

$|14x+3|+3=1$

? Exercise 16

$|4x+3|=|2x+1|$

? Exercise 17

$|6-x|=|3-2x|$

Writing Exercise

? Exercise 18

Explain how a graph can be used to solve an absolute value equation.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve absolute value equations graphically.			
solve absolute value equations algebraically.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.3: Absolute Value Inequalities

Be Prepared

We will rely heavily on these skills throughout this section.

- Solve $-x+4<7$ for x
- Graph the inequality $y<x-5$
- Determine if $x=\frac{3}{2}$ makes $|x-2|<2$ true.

Learning Objectives

Motivating Problem

You're baking cookies that need to be in the oven at exactly 350°F. The oven manual says it fluctuates by up to 15°F during baking. What temperature range should you expect, and how can you express that as a mathematical sentence?

Fun Fact

Astronomers use absolute value inequalities to describe how far off a measurement can be when locating distant stars or planets. Even tiny errors—like being off by 0.001 light-years—can lead to big navigational mistakes!

The Goal

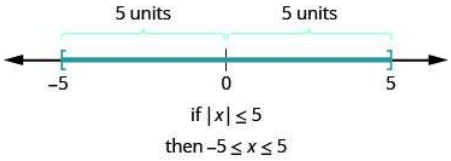
In this section, we'll learn to solve **absolute value inequalities** by translating them into statements about distance. We'll explore symbolic strategies **and use Desmos** to graph and visually analyze solution sets.

Solve Absolute Value Inequalities with "Less Than"

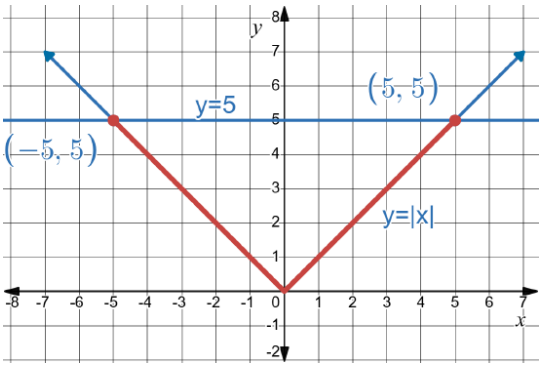
Let's look now at what happens when we have an **absolute value inequality**. Everything we've learned about solving inequalities still holds, but we must consider how the absolute value impacts our work. Again, we will look at our definition of absolute value. The absolute value of a number is its distance from zero on the number line. For the equation $|x|=5$, we saw that both 5 and -5 are five units from zero on the number line. They are the solutions to the equation.

$$|x|=5$$
$$x=-5 \quad \text{or} \quad x=5$$

What about the inequality $|x|\leq 5$? Where are the numbers whose distance is less than or equal to 5? We know -5 and 5 are both five units from zero. All the numbers between -5 and 5 are less than five units from zero.

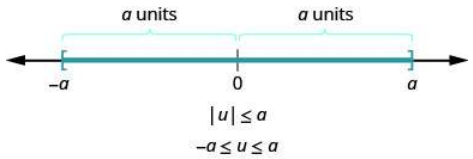


Let's visualize this graphically. Consider the graphs of $y=|x|$ and $y=5$. The solutions consist of all x-values where the graph of $y=|x|$ is less than, or equal to, the graph of $y=5$. Because we are comparing y values, "less than" is below vertically.



In this case, we see that the red portion of $|x|$ satisfies the inequality $|x| \leq 5$. Therefore, the solution is x in $[-5, 5]$.

In a more general way, we can see that if $|u| \leq a$, then $-a \leq u \leq a$.



This result is summarized here.

ABSOLUTE VALUE INEQUALITIES WITH < OR ≤

For any algebraic expression, u, and any positive real number, a,

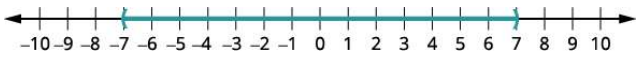
$$\begin{aligned} &\text{if} \quad |u| < a, \quad \text{then} \quad -a < u < a \\ &\text{if} \quad |u| \leq a, \quad \text{then} \quad -a \leq u \leq a \end{aligned}$$

After solving an inequality, it is often helpful to check some points to see if the solution makes sense. The graph of the solution divides the number line into three sections. Choose a value in each section and substitute it in the original inequality to see if it makes the inequality true or not. While this is not a complete check, it often helps verify the solution.

Example 1

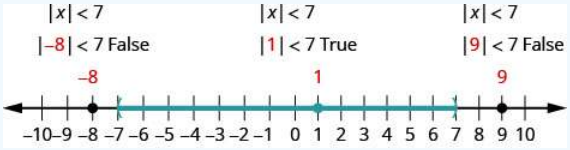
Solve $|x| < 7$. Graph the solution on a number line and write the solution in interval notation.

Solution

	$\lvert x\rvert < 7$
Write the equivalent inequality.	$-7 < x < 7$
Graph the solution.	
Write the solution using interval notation.	$(-7, 7)$

Check:

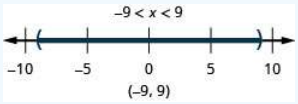
To verify, check a value in each section of the number line showing the solution. Choose numbers such as -8, 1, and 9.



Try It 1

Graph the solution on a number line and write the solution in interval notation: $|x| < 9$.

Answer



In this section, we will solve absolute value inequalities in two ways: with algebra and with graphs.

In the algebra method, we remove the absolute value bars by using the definition of absolute value. This often gives us two separate inequalities to solve. We then solve them using the same steps we use for regular linear inequalities.

In the graph method, we draw both sides of the inequality on the same coordinate plane. Then we look for the parts of the x-axis where the absolute value graph satisfies the inequality.

Using both methods helps us check our answers and understand how absolute value inequalities work.

Example 2

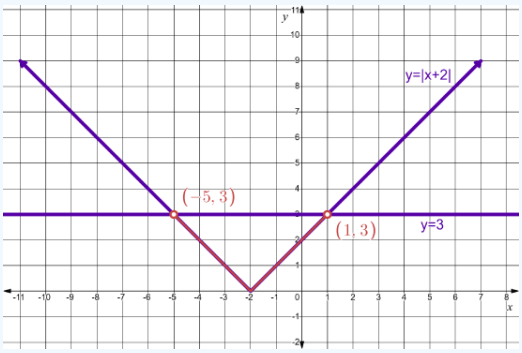
Solve using algebraic and graphical methods: $|x + 2| < 3$.

Solution

To solve algebraically, we bound the argument $x + 2$ by -3 and 3 and solve.

$$\begin{gathered} |x + 2| < 3 \\ -3 < x + 2 < 3 \\ -3-2<x + 2-2<3-2 \\ -5 < x < 1 \end{gathered}$$

To solve the system graphically, we graph both $y = |x + 2|$ and $y = 3$ to determine on what interval the absolute value graph is less than (or below) the linear graph.



Using both approaches, we determine the solution to be all x in the interval $(-5, 1)$.

Try It 2

Solve using algebraic and graphical methods: $|x - 3| < 5$

Answer

All x in the interval $(-2, 8)$.

Example 3

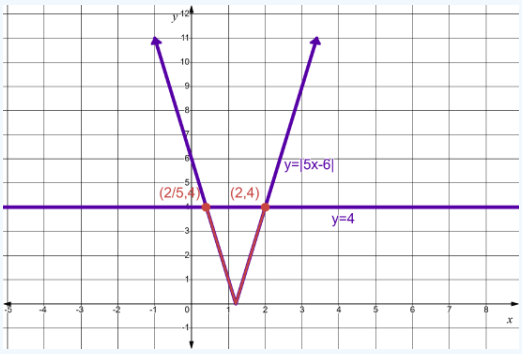
Solve using algebraic and graphical methods: $|5x-6| \leq 4$.

Solution

To solve algebraically, we bound the argument $5x-6$ by -4 and 4 and solve.

$$\begin{gathered} |5x-6| \leq 4 \\ -4 \leq 5x-6 \leq 4 \\ -4+6 \leq 5x-6+6 \leq 4+6 \\ 2 \leq 5x \leq 10 \\ \frac{2}{5} \leq x \leq 2 \end{gathered}$$

To solve the system graphically, we graph both $y=|5x-6|$ and $y=4$ to determine on what interval the absolute value graph is less than or equal to the linear graph.



Using both approaches, we determine the solution to be all x in the interval $\left[\frac{2}{5}, 2\right]$.

Try It 3

Solve using algebraic and graphical methods: $|2x-1| \leq 5$

Answer

All x in the interval $[-2, 3]$.

SOLVE ABSOLUTE VALUE INEQUALITIES WITH $<$ OR $\leq$

1. Isolate the absolute value expression.
2. Write the equivalent compound inequality.

$$\begin{aligned} |u| < a \quad &\text{is equivalent to} \quad -a < u < a \\ |u| \leq a \quad &\text{is equivalent to} \quad -a \leq u \leq a \end{aligned}$$

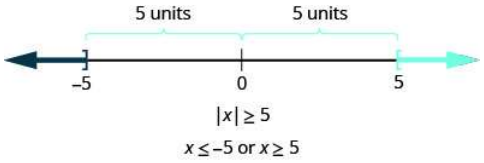
3. Solve the compound inequality.
4. Graph the solution
5. Write the solution using interval notation.

Solve Absolute Value Inequalities with "Greater Than"

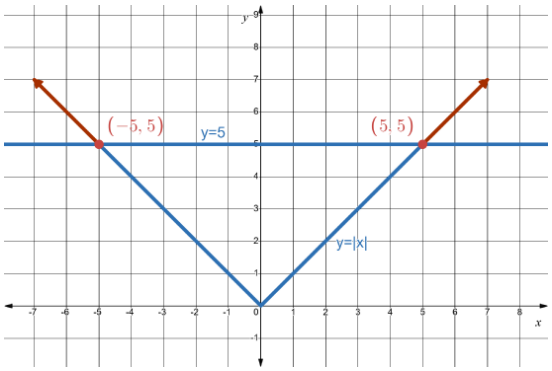
What happens for absolute value inequalities that have "greater than"? Again, we will look at our definition of absolute value. The absolute value of a number is its distance from zero on the number line.

Now we want to look at the inequality $|x| \geq 5$. Where are the numbers whose distance from zero is greater than or equal to five?

Again, both -5 and 5 are five units from zero and so are included in the solution. Numbers whose distance from zero is greater than five units would be less than -5 or greater than 5 on the number line.

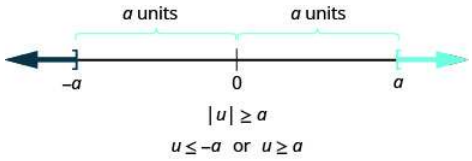


Let's visualize this graphically. Consider the graphs of $y = |x|$ and $y = 5$. The solutions consist of all x-values where the graph of $y = |x|$ is greater than, or equal to, the graph of $y = 5$. Because we are comparing y values, "greater than" is above vertically.



In this case, we see that the red portions of $|x|$ satisfy the inequality $|x| \geq 5$, so the solution has two parts. The left portion corresponds to x less than or equal to -5: $(-\infty, -5]$. The right portion corresponds to x greater than or equal to 5: $[5, \infty)$. We can use the union symbol, $\cup$, to represent "or" and the solution becomes $(-\infty, -5] \cup [5, \infty)$.

In a more general way, we can see that if $|u| \geq a$, then $u \leq -a$ or $u \geq a$.



This result is summarized here.

ABSOLUTE VALUE INEQUALITIES WITH > OR ≥

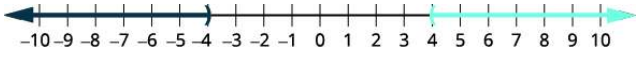
For any algebraic expression, u, and any positive real number, a,

if	$\|u\| > a$,	then $u < -a$ or $u > a$
if	$\|u\| \geq a$,	then $u \leq -a$ or $u \geq a$

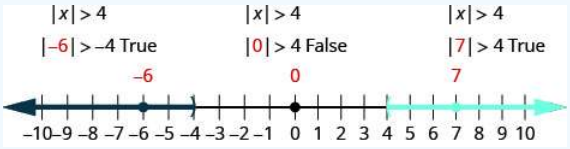
Example 4

Solve $|x| > 4$. Graph the solution on a number line and write the solution in interval notation.

Solution

	$\|x\| > 4$
Write the equivalent inequality.	$x < -4$ or $x > 4$
Graph the solution.	
Write the solution using interval notation.	$(-\infty, -4) \cup (4, \infty)$
Check:	

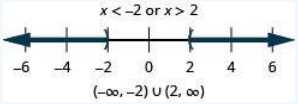
To verify, check a value in each section of the number line showing the solution. Choose numbers such as −6, 0, and 7.



Try It 4

Solve $|x| > 2$. Graph the solution and write the solution in interval notation.

Answer



Example 5

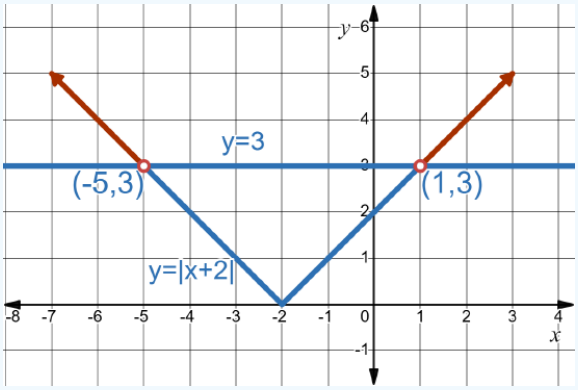
Solve using algebraic and graphical methods: $|x + 2| > 3$.

Solution

To solve algebraically, we consider the two cases where $x + 2$ is greater than 3 or less than -3.

$$|x + 2| > 3$$
$$x + 2 < -3 \quad \text{or} \quad x + 2 > 3$$
$$x < -5 \qquad\qquad x > 1$$

To solve the system graphically, we graph both $y = |x + 2|$ and $y = 3$ to determine for which values of x the absolute value graph is greater than the constant graph.



Using both approaches, we determine the solution to be all x in the interval $(-\infty, -5) \cup (1, \infty)$.

Try It 5

Solve using algebraic and graphical methods: $|4x - 3| \geq 5$.

Answer

All x in the interval $\left(-\infty, -\frac{1}{2}\right] \cup [2, \infty)$.

SOLVE ABSOLUTE VALUE INEQUALITIES WITH > OR ≥.

1. Isolate the absolute value expression.
2. Write the equivalent compound inequality.

$$|u| > a \quad \text{is equivalent to} \quad u < -a \quad \text{or} \quad u > a$$
$$|u| \geq a \quad \text{is equivalent to} \quad u \leq -a \quad \text{or} \quad u \geq a$$

3. Solve the compound inequality.
4. Graph the solution
5. Write the solution using interval notation.

Solve Applications with Absolute Value

Absolute value inequalities are often used in the manufacturing process. An item must be made with near-perfect specifications. Usually, there is a certain *tolerance* for the difference from the specifications that is allowed. If the difference from the specifications exceeds the tolerance, the item is rejected.

$$|\text{actual-ideal}| \leq \text{tolerance}$$

Example 6

The ideal diameter of a rod needed for a machine is 60 mm. The actual diameter can vary from the ideal diameter by 0.075 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Solution

	Let $x =$ the actual measurement
Use an absolute value inequality to express this situation.	$\|\text{actual-ideal}\| \leq \text{tolerance}$
	$\|x - 60\| \leq 0.075$
Rewrite as a compound inequality.	$-0.075 \leq x - 60 \leq 0.075$
Solve the inequality.	$59.925 \leq x \leq 60.075$
Answer the question.	The diameter of the rod can be between 59.925 and 60.075 mm.

Try It 6

The ideal diameter of a rod needed for a machine is 75 mm. The actual diameter can vary from the ideal diameter by 0.05 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Answer

The diameter of the rod can be between 74.95 and 75.05 mm.

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Section 3.3P: Practice

Practice Makes Progress

Solve each inequality algebraically.

Exercise 1

$|x| < 5$

Exercise 2

$|x| < 1$

Exercise 3

$|x| \leq 3$

Exercise 4

$|3x - 3| \leq 6$

Exercise 5

$|2x - 5| \leq 3$

Exercise 6

$|x| > 6$

Exercise 7

$|x| \geq 2$

Exercise 8

$|x| \geq 5$

Exercise 9

$|x - 5| > -2$

Exercise 10

$|3x - 2| > 4$

Exercise 11

$|2x + 3| + 5 < 4$

? Exercise 12

$|3x-7|+3<1$

? Exercise 13

$|6x-5|<7$

? Exercise 14

$|5x+1|\leq -2$

? Exercise 15

$|2x-1|>5$

? Exercise 16

$|x-7|\geq 1$

? Exercise 17

$3|x|+4\geq 1$

? Exercise 18

$5|x|+6\geq 1$

Everyday Math

? Exercise 19

An organic juice bottler ideally produces 215,000 bottles per day. But this total can vary by as much as 7,500 bottles. What is the maximum and minimum expected production at the bottling company?

? Exercise 20

At Lilly's Bakery, the ideal weight of a loaf of bread is 24 ounces. By law, the actual weight can vary from the ideal by 1.5 ounces. What range of weight will be acceptable to the inspector without causing the bakery to be fined?

Writing Exercises

? Exercise 21

In your own words, explain how a graph can be used to solve the absolute value inequality, $|3x-2|\geq 4$.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve absolute value inequalities with "less than".			
solve absolute value inequalities with "greater than".			
solve applications with absolute value.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 4: Practice Problem Answers

This unit provides the answers to all the practice problems from the textbook. It's here to help you confirm whether your solutions are correct and to support your learning by giving you immediate feedback. But remember: knowing the final answer is only part of doing math. You're still responsible for showing clear, step-by-step work that demonstrates your thinking and understanding.

With great power comes great responsibility—so use these answers wisely. They are meant to *check* your work, not *replace* it. If you find a mistake, go back, try again, and figure out where things went off track. That process is how real learning happens.

Section 4.1: Unit 0 Practice Answers

Section 4.2: Unit 1 Practice Answers

Section 4.3: Unit 2 Practice Answers

Section 4.4: Unit 3 Practice Answers

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Section 4.1: Unit 0 Practice Answers

Unit 0 Answers

Section 0.1P: Order of Operations

1. 125
2. 256
3. 43
4. 55
5. 5
6. 34
7. 58
8. 6
9. 4
10. 35
11. 58
12. 149
13. 50

Section 0.2P: Fractions

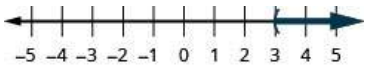
1. $-\frac{12}{7}$
2. $\frac{10}{21}$
3. $\frac{2x^2}{3y}$
4. $-\frac{21a^2}{11b^2}$
5. $\frac{1}{3}$
6. $-\frac{21}{50}$
7. $9n$
8. $-\frac{4}{9}$
9. $-\frac{10}{9}$
10. $\frac{2m}{3n}$
11. $\frac{29}{24}$
12. $\frac{1}{48}$
13. $\frac{1}{12}$
14. $\frac{4x+3}{12}$
15. $-\frac{5(a+1)}{3}$
16. a
17. $\frac{9}{7}$
18. $\frac{11}{6}$
19. $\frac{15}{4}$
20. $\frac{5}{21}$
21. $\frac{5}{4}$
22. $\frac{1}{24}$
23. $\frac{33}{64}$

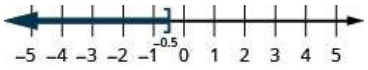
24. $\frac{7}{9}$
25. -5
26. $\frac{11}{5}$
27. 1
28. $\frac{13}{3}$
29. Answers will vary
30. Answers will vary

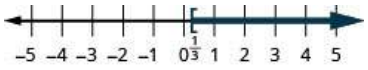
Section 0.3P: General Strategy to Solve Linear Equations

1. a. yes
   b. no
2. a. no
   b. yes
3. $y = 5$
4. $w = -18$
5. $q = 3$
6. $c = 4$
7. $y = -5$
8. $p = -4$
9. $c = -4$
10. $m = 6$
11. $x = -1$
12. $a = \frac{3}{4}$
13. $w = \frac{9}{4}$
14. $p = -41$
15. $n = -3$
16. $u = 3$
17. $x = 18$
18. $n = 9$
19. $d = 8$
20. identity; all real numbers
21. conditional equation; $j = \frac{2}{5}$
22. contradiction; no solution
23. identity; all real numbers
24. The length is 19.75 feet
25. Answers will vary
26. Answers will vary

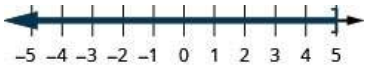
Section 0.4P: General Strategy to Solve Linear Inequalities

1. 

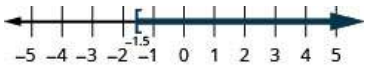
$(3, \infty)$
2. 

$(-\infty, -0.5]$
3. 

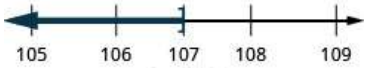
$\left[\frac{1}{3}, \infty\right)$

4. 

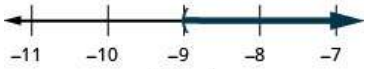
$(-\infty, 5]$

5. 

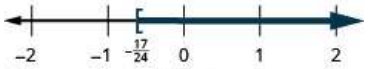
$[-1.5, \infty)$

6. $m \leq 107$


$(-\infty, 107]$

7. $v > -9$


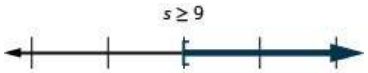
$(-9, \infty)$

8. $b \geq -\frac{17}{24}$


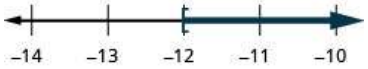
$\left[-\frac{17}{24}, \infty\right)$

9. $g < \frac{23}{36}$

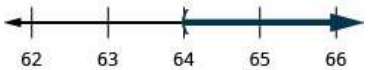

$\left(-\infty, \frac{23}{36}\right)$

10. $s \geq 9$


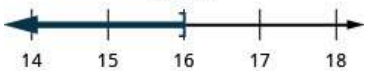
$[9, \infty)$

11. $v \geq -12$


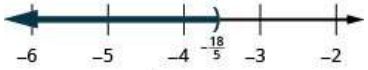
$[-12, \infty)$

12. $k > 64$


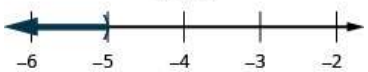
$(64, \infty)$

13. $g \leq 16$


$(-\infty, 16]$

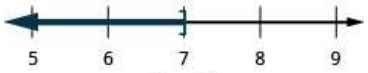
14. $p < \frac{18}{5}$


$\left(-\infty, \frac{18}{5}\right)$

15. $y < -5$


$(-\infty, -5)$

16. $x \le 7$



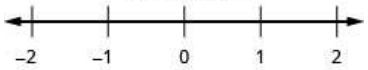
$(-\infty, 7]$

17. identity



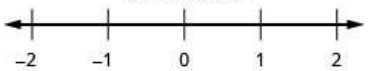
$(-\infty, \infty)$

18. Contradiction



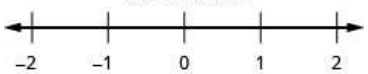
No solution

19. Contradiction



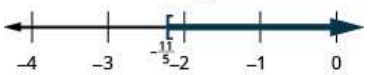
No solution

20. Contradiction



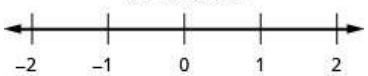
No solution

21. $k \ge -\frac{11}{5}$



$\left[-\frac{11}{5}, \infty\right)$

22. Contradiction



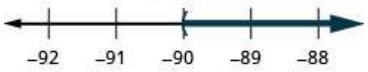
No solution

23. $y \ge \frac{10}{49}$



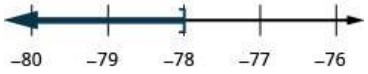
$\left[\frac{10}{49}, \infty\right)$

24. $d > -90$



$(-90, \infty)$

25. $n \le -78$

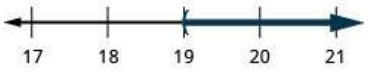


$(-\infty, -78]$

26. $k + 6 > 25$

$k > 19$

$(19, \infty)$



27. $x - 12 \geq 21$

$x \geq 33$

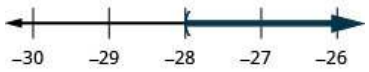
$[33, \infty)$



28. $-2s < 56$

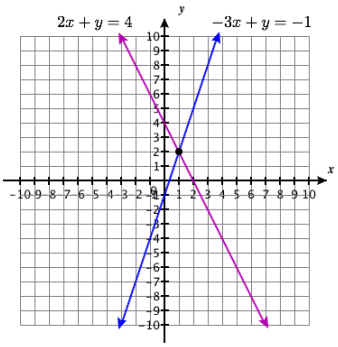
$s > -28$

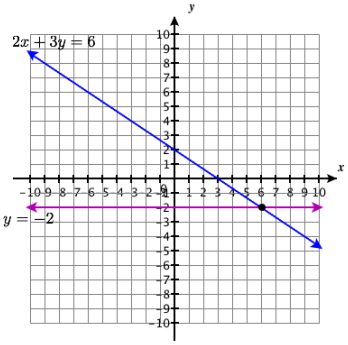
$(-28, \infty)$



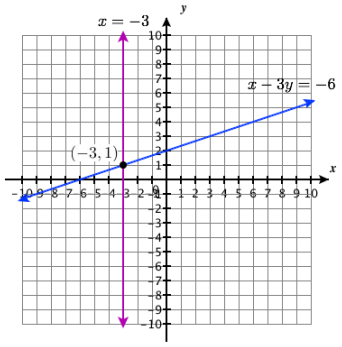
29. up to 12 people
30. up to 5 drinks
31. 31 aprons or more
32. more than $15,000 in sales
33. 44 meals or more
34. 49 cars or more
35. up to 91 students
36. 169 or fewer guests
37. Answers will vary.

Section 0.5P: Solving Systems of Linear Equations

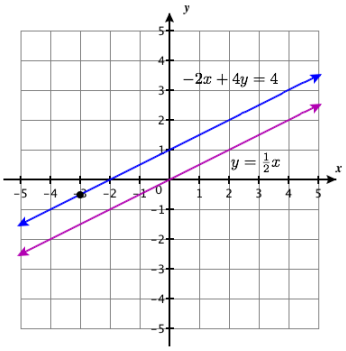
1. a. yes
   b. no
2. a. yes
   b. no
3. 

Solution Point: $(1, 2)$
4. 

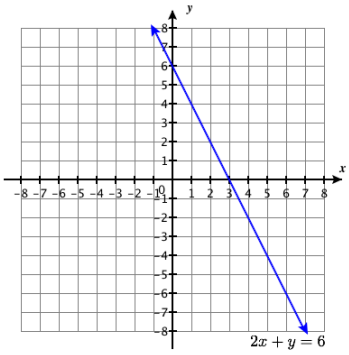
Solution Point: $(6, -2)$

5. 

Solution Point: $(-3, 1)$

6. 

no solution

7. 

infinitely many solutions with solution set: $\{(x, y) | 2x + y = 6\}$

8. $(0, 2)$
9. $(2, 4)$
10. $(1, 2)$
11. $(3, 2)$
12. $(1, 1)$
13. $(-2, -2)$
14. $(3, 3)$
15. infinitely many solutions: $\{(x, y) \mid y = -\frac{1}{3}x + \frac{4}{3}\}$
16. infinitely many solutions: $\{(x, y) \mid y = -\frac{1}{3}x - 2\}$
17. $(2, 2)$
18. no solution
19. $(\frac{1}{2}, 4)$
20. infinitely many solutions: $\{(x, y) \mid y = -\frac{1}{2}x + 5\}$
21. infinitely many solutions: $\{(x, y) \mid y = 2x + 3\}$
22. Molly needs 16 ounces of strawberry juice and 48 ounces of water.
23. Enrique needs 8 ounces of nuts and 16 ounces of raisins.
24. Leo should plant 50 tulip bulbs and 300 daffodil bulbs.
25. Answers will vary

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Section 4.2: Unit 1 Practice Answers

Unit 1 Answers

Section 1.1P: Add and Subtract Polynomials

1. trinomial
2. polynomial
3. binomial
4. monomial
5. binomial
6. 2
7. 4
8. 1
9. 3
10. 0
11. $12x^2$
12. $6w$
13. $40x$
14. $-22b$
15. $-10a + 5b$
16. $-4a^2 + b^2$
17. $xy^2 - 5x - 5y^2$
18. $a^2b - 4a - 5ab^2$
19. $-17x^6$
20. $11y^2 + 4y + 11$
21. $-3x^2 + 17x - 1$
22. $11x^2 - 5x + 5$
23. $6a^2 - 4a - 1$
24. $2m^2 - 7m + 4$
25. $11a + 3$
26. $12s^2 - 16s + 9$
27. $w^2 + 3w + 4$
28. $8a^3 + a^2 - 2a + 12$
29. $11w - 66$
30. $10m^2 + 3mn - 8n^2$
31. $4uv + 2v^2$
32. $x^3 + 2x^2y - 5xy^2 + y^3$
33. a. 187
    b. 46
    c. 2
34. $\$10,800$
35. $\$58$
36. $149°\text{F}$
37. Answers will vary.
38. Answers will vary.

Section 1.2P: Use Multiplication Properties of Exponents

1. $-\frac{4}{9}$
2. $\frac{4}{9}$
3. -0.0001

4. 0.0001
5. x^6
6. 3^{16}
7. 5^5
8. y^9
9. n^{y+2}
10. b^{14}
11. 3^{16}
12. x^{2y}
13. 7^{ab}
14. $-343n^3$
15. $81x^4y^4z^4$
16. w^{22}
17. $32x^5y^{20}$
18. $128r^8$
19. $27x^{29}$
20. $-18y^{11}$
21. $72c^5$
22. $8r^2s^5$
23. $\frac{1}{2}x^3y^3$
24. x^{14}
25. a^{36}
26. $8m^{18}$
27. $1000x^6y^3$
28. $16a^{12}b^8$
29. $\frac{8}{27}x^6y^3$
30. $1024a^{10}$
31. $25000p^{24}$
32. $x^{18}y^{18}$
33. $144m^8n^{22}$
34. 1,000,000
35. Answers will vary.

Section 1.3P: Multiply Polynomials

1. $2x-14$
2. q^2+5q
3. $-x^2+10x$
4. $24r^2+6rs$
5. $-8x^3-16x^2+120x$
6. $5q^5-10q^4+30q^3$
7. $-12z^4-48z^3+4z^2$
8. $8w-48$
9. $-8z+40$
10. u^2+5u
11. $55p^2-25pq$
12. $3v^2+30v+75$
13. $8n^3-8n^2+2n$
14. $2y^2-9y$
15. $x^2+11x+24$
16. $y^2-8y+12$

17. $6p^2 + 11p + 5$
18. $4z^2 - 24z - zy + 6y$
19. $24p^2q^2 - 42pq + 15$
20. $4y^3 + 33y^2 + y - 56$
21. $15q^3 - 30q^2 + 55q$
22. $4m^4 - 3m^3 - 7m^2$
23. $25a^2 + 70ab + 49b^2$
24. $16y^2 - 144z^2$
25. a. 306
    b. 306
    c. Answers will vary.
26. Answers will vary.
27. Answers may vary.

Section 1.4P: Special Products

1. $q^2 + 24q + 144$
2. $y^2 - 12y + 36$
3. $16a^2 + 80a + 100$
4. $4y^2 - 12yz + 9z^2$
5. $164x^2 - 136xy + 181y^2$
6. $25u^4 + 90u^2 + 81$
7. $64p^6 - 48p^3 + 9$
8. $64j^2 - 16$
9. $169 - q^2$
10. $49w^2 - 100x^2$
11. $r^2s^2 - \frac{4}{49}$
12. $225m^4 - 64n^8$
13. $4r^2 + 48r + 144$
14. $9p^2 - 64$
15. $7a^2 - 48ab - 7b^2$
16. $k^2 - 12k + 36$
17. Answers will vary.

Section 1.5P: Divide Monomials

1. $\frac{1}{t^{30}}$
2. $\frac{1}{64}$
3. $\frac{1}{x^6}$
4. $\frac{1}{100}$
5. -1
6. -1
7. 7
8. 1
9. $\frac{4}{25}$
10. $\frac{x^4}{81}$
11. $\frac{a^5}{b^5}$
12. x^5
13. $\frac{1}{m^{12}}$

14. $\frac{a^3}{b^{18}}$
15. $\frac{27m^{15}}{125n^3}$
16. $\frac{625u^{28}}{16v^{12}}$
17. $-6u^8$
18. $\frac{2m^3}{3n^5}$
19. $\frac{-3y^3}{4x^3}$
20. $\frac{-3q^5}{p^5}$
21. $18n^{10}$
22. $12n^{10}$
23. $45n^{20}$
24. 5
25. $-yz^2$
26. 10^3
27. Answers will vary.

Section 1.6P: Divide Polynomials

1. $6b+15$
2. $11w-2$
3. $6r^3+11r^2s-8rs^3$
4. $7a-12+\frac{11}{a}$
5. $x-7$
6. $4x+3$
7. $x+7+\frac{5}{x-5}$
8. $2n^2-6n+8+\frac{4}{n+3}$
9. $m^2-10m+100$
10. $x^2+6x+36$
11. 45
12. Answers will vary.

Section 1.7P: Integer Exponents and Scientific Notation

1. t^9
2. 10000
3. $\frac{8}{343}$
4. $-\frac{x^3y^6}{27}$
5. 1
6. y^6
7. $\frac{1}{y^4}$
8. $\frac{1}{x}$
9. $\frac{1}{m^2n^4}$
10. $\frac{1}{p^5q^7}$
11. $-\frac{40n^3}{m}$
12. $\frac{4}{p^{10}}$
13. n^7
14. 1.29×10^6

15. 4.1×10^{-2}
16. 1.03×10^{-6}
17. 830
18. 0.0000193
19. 0.02
20. 500,000,000
21. 5.7×10^{-9}
22. 15,000,000,000,000
23. 1.545×10^{8}
24. Answers will vary

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Section 4.3: Unit 2 Practice Answers

Unit 2 Answers

Section 2.1P: Simplify and Use Square Roots

1. 6
2. 8
3. 10
4. −2
5. −1
6. not a real number
7. 5
8. 7
9. $8 < \sqrt{70} < 9$
10. $14 < \sqrt{200} < 15$
11. 4.36
12. 7.28
13. y
14. a^7
15. $7x$
16. $11m^{10}$
17. $-9x^9$
18. $3c^4d^6$
19. 25 centimeters
20. Answers will vary.

Section 2.2P: Simplify Square Roots

1. $5\sqrt{5}$
2. $10\sqrt{2}$
3. $15\sqrt{2}$
4. $20\sqrt{2}$
5. $15\sqrt{3}$
6. $x^3\sqrt{x}$
7. $p\sqrt{p}$
8. $m^6\sqrt{m}$
9. $7n^8\sqrt{n}$
10. $9r^7\sqrt{r}$
11. $7m^2\sqrt{2m}$
12. $10p^4q^5\sqrt{3pq}$
13. $11m^6n^{10}\sqrt{2mn}$
14. $5 + 2\sqrt{3}$
15. $1 + 3\sqrt{5}$
16. $5 - \sqrt{6}$
17. $1 + \sqrt{10}$
18. $\frac{3}{5}$
19. x^2
20. $\frac{1}{y^2}$
21. $10x^2$
22. $\frac{4x^3\sqrt{6x}}{11}$
23. $\frac{5r^4\sqrt{3r}}{s^4}$

24. $\frac{4p^3\sqrt{7p}}{q}$
25. $\frac{11p\sqrt{p}}{9}$
26. $\frac{1}{2pq\sqrt{p}}$
27. 17.0 feet
28. 15.0 feet
29. Answers will vary.

Section 2.3P: Add and Subtract Square Roots

1. $9\sqrt{5}$
2. $-\sqrt{7}$
3. $-3\sqrt{a}$
4. $8\sqrt{a}-2\sqrt{b}$
5. $13\sqrt{7}$
6. $4\sqrt{3ab}$
7. $9\sqrt{2}$
8. $\sqrt{5}$
9. $-2\sqrt{3}$
10. $7\sqrt{3}$
11. $-8\sqrt{2}$
12. $16\sqrt{3}$
13. $-\sqrt{3}$
14. $-\frac{3}{4}\sqrt{2}$
15. $a^2\sqrt{2a}$
16. $10r^4\sqrt{2}+12r^4\sqrt{6}$
17. $14x\sqrt{5}$
18. $2k^2\sqrt{7}$
19. $\sqrt{2}$
20. $\sqrt{13}$
21. $a^2\sqrt{5a}$
22. $19\sqrt{19}$
23. $5\sqrt{3}$
24. $-\sqrt{3}$
25. $8x\sqrt{6}$
26. 124.5 inches
27. Answers will vary.

Section 2.4P: Multiply Square Roots

1. 4
2. $18\sqrt{6}$
3. $7\sqrt{2}$
4. 160
5. $30\sqrt{3}$
6. $-18\sqrt{6}$
7. $-30\sqrt{2}$
8. $28\sqrt{2}$
9. $40b^2\sqrt{3}$
10. $144d^4$
11. $41+7\sqrt{55}$
12. $-81+44\sqrt{78}$
13. $18+7\sqrt{w}-w$

14. $66+73\sqrt{n}+15n$
15. −127
16. 33
17. $-24\sqrt{3}$
18. $-210\sqrt{3}$
19. $7y^3\sqrt{5}$
20. 29
21. $29-7\sqrt{17}$
22. 4.6 feet
23. 21 sq. feet
24. Answers will vary.
25. Answers will vary.

Section 2.5P: Divide Square Roots

1. $\frac{\sqrt{3}}{2}$
2. $\frac{1-2\sqrt{2}}{4}$
3. $\frac{4}{5}$
4. $\frac{5r}{6}$
5. $\frac{\sqrt{2}}{2}$
6. $\frac{1+\sqrt{5}}{3}$
7. $y^3\sqrt{13}$
8. $\frac{3\sqrt{13}}{13}$
9. $\frac{\sqrt{10}}{3}$
10. $\frac{\sqrt{15}}{10}$
11. $\frac{\sqrt{133}}{35}$
12. $\frac{3(3-\sqrt{11})}{-2}$
13. $-2(1+\sqrt{5})$
14. $\frac{\sqrt{3}(\sqrt{m}+\sqrt{5})}{m-5}$
15. $\frac{5y^2}{x}$
16. $3\sqrt{5}$
17. $\frac{2\sqrt{3}}{9}$
18. $\frac{\sqrt{2}(\sqrt{x}+\sqrt{3})}{x-3}$
19. $\frac{5\sqrt{10}}{4}$ seconds
20. Answers will vary.

Section 2.6P: Rational Exponents

1. $\sqrt{r}$
2. $\sqrt[3]{s}$
3. $\sqrt[4]{t}$
4. $r^{\frac{7}{4}}$
5. $s^{\frac{3}{5}}$
6. $t^{\frac{7}{3}}$
7. 5
8. 3
9. 2
10. not a real number
11. −7
12. $\frac{1}{7}$
13. not a real number
14. 1

15. −1
16. 100
17. 125
18. 8
19. 1000
20. $\frac{1}{16,807}$
21. not a real number
22. −512
23. $-\frac{1}{512}$
24. not a real number
25. 216
26. n
27. q^3
28. s
29. z^2
30. $\frac{1}{w}$
31. $3r^{\frac{1}{5}}$
32. $2s^{\frac{1}{14}}$
33. r^2s
34. u^3v^4
35. m
36. n^2
37. n
38. b^6
39. $2s^{\frac{1}{10}}$
40. 15.6 feet
41. 8 seconds
42. Answers will vary.

Section 2.7P: Solve Equations with Square Roots

1. yes
2. no
3. 3
4. 14
5. 17
6. $\frac{13}{5}$
7. no solution
8. 10, 11
9. 5
10. 3
11. not a real number
12. no solution
13. $\frac{57}{16}$
14. no solution
15. 15
16. 11.4 feet
17. 8 seconds
18. 15.8 seconds
19. 72 feet
20. 53.0 feet

21. Answers will vary.

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