MTH 60: ALGEBRA 1



Math Department
Clackamas Community College





Math 60 - Algebra 1

OER Textbook

First Edition - 2025

Welcome

Welcome to MATH 60 - Algebra 1!

This book is here to support you every step of the way as you build a strong foundation in algebra and discover how it connects to the world around you. Algebra isn't just a series of rules; it's a tool for solving real problems, spotting patterns, and making confident decisions. You'll find clear explanations, guided practice, and plenty of opportunities to check your understanding so you can learn with ease and grow your math skills with confidence.

Here's how this book was designed to help you succeed:

- **Essential Skills**: Each section opens with carefully chosen sample problems that highlight the core skills you'll need to successfully learn the new material.
- **Plain-Language Introductions**: New concepts are first presented informally in clear, everyday language and supported by visual descriptions, helping you build intuition before moving on to formal definitions and procedures.
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- **Answer Key**: All answers to practice problems are collected in the back of the book, so you can quickly confirm you're on the right track.
- **Self-Check**: At the end of each practice set, use the "self-check" questions to pinpoint any areas you'd like to revisit or get extra help with.

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TABLE OF CONTENTS

Preface

- Title Page
- Welcome
- Acknowledgements
- Table of Contents
- Copyright and Licensing

Unit 1: Introducing Unknowns

- Section 1.1: The Language of Algebra
  - Section 1.1.P: Practice
- Section 1.2: Order of Operations
  - Section 1.2P: Practice
- Section 1.3: Exploring Expressions
  - Section 1.3P: Practice
- Section 1.4: Integers
  - Section 1.4P: Practice
- Section 1.5: Fractions
  - Section 1.5P: Practice
- Section 1.6: Dimensional Analysis
  - Section 1.6P: Practice

Unit 2: Relating Unknowns

- Section 2.1: Solving Equations - Building Blocks
  - Section 2.1P: Practice
- Section 2.2: Solving Equations - Refining the Process
  - Section 2.2P: Practice
- Section 2.3: Solving Equations - Advanced Techniques and Formulas
  - Section 2.3P: Practice
- Section 2.4: Linear Inequalities
  - Section 2.4P: Practice

Unit 3: Graphing Relationships

- Section 3.1: Cartesian Plane
  - Section 3.1P: Practice
- Section 3.2: Graph Linear Equations in Two Variables
  - Section 3.2P: Practice
- Section 3.3: Intercepts
  - Section 3.3P: Practice
- Section 3.4: Understanding the Slope of a Line

- Section 3.4P: Practice
- Section 3.5: Slope-Intercept Form of Line
  - Section 3.5P: Practice
- Section 3.6: Line Equations
  - Section 3.6P: Practice
- Section 3.7: Linear Inequalities in Two Variables
  - Section 3.7P: Practice

Unit 4: Working with Systems

- Section 4.1: Solve Systems of Equations by Graphing
  - Section 4.1P: Practice
- Section 4.2: Solve Systems of Equations by Substitution
  - Section 4.2P: Practice
- Section 4.3: Solve Systems of Equations by Elimination
  - Section 4.3P: Practice
- Section 4.4: Solve Applications with Systems of Equations
  - Section 4.4P: Practice
- Section 4.5: Graphing Systems of Linear Inequalities
  - Section 4.5P: Practice

Unit 5: Practice Problem Answers

- Section 5.1: Unit 1 Practice Answers
- Section 5.2: Unit 2 Practice Answers
- Section 5.3: Unit 3 Practice Answers
- Section 5.4: Unit 4 Practice Answers

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CHAPTER OVERVIEW

In this preface, you'll find a welcome message to set the stage for your learning, guidance on how to use the book effectively, acknowledgments of those who contributed to its creation, and a table of contents to help you navigate the material. This section will help you get oriented and ready to make the most of the resources ahead.

Title Page
Welcome
Table of Contents
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Copyright and Licensing



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TABLE OF CONTENTS

Preface

- Title Page
- Welcome
- Acknowledgements
- Table of Contents
- Copyright and Licensing

Unit 1: Introducing Unknowns

- Section 1.1: The Language of Algebra
  - Section 1.1.P: Practice
- Section 1.2: Order of Operations
  - Section 1.2P: Practice
- Section 1.3: Exploring Expressions
  - Section 1.3P: Practice
- Section 1.4: Integers
  - Section 1.4P: Practice
- Section 1.5: Fractions
  - Section 1.5P: Practice
- Section 1.6: Dimensional Analysis
  - Section 1.6P: Practice

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  - Section 2.2P: Practice
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  - Section 3.3P: Practice
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  - Section 3.5P: Practice
- Section 3.6: Line Equations
  - Section 3.6P: Practice
- Section 3.7: Linear Inequalities in Two Variables
  - Section 3.7P: Practice

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  - Section 4.1P: Practice
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  - Section 4.2P: Practice
- Section 4.3: Solve Systems of Equations by Elimination
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CHAPTER OVERVIEW

Unit 1: Introducing Unknowns

In this unit, we'll begin our journey into algebra by learning how to work with unknown values and build expressions using variables. We'll strengthen our number sense as we review operations with integers and fractions, explore the order of operations, and simplify expressions. Along the way, we'll apply our skills to real-life problems using unit conversions and basic geometry. These foundational tools will help us think algebraically and prepare us for more complex ideas ahead.

Section 1.1: The Language of Algebra

Section 1.1.P: Practice

Section 1.2: Order of Operations

Section 1.2P: Practice

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Section 1.5P: Practice

Section 1.6: Dimensional Analysis

Section 1.6P: Practice

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Section 1.1: The Language of Algebra

Be Prepared

We will rely heavily on these skills throughout this section.

- Add $43 + 69$
- Multiply $(896)201$
- Divide: $7,263 \div 9$

Learning Objectives

Motivating Problem

If Alex is always 3 years older than Greg, how can you describe Alex's age when Greg is 12? Or 35? Or any age at all? Can one rule help you find Alex's age no matter what Greg's age is?

Fun Fact

The word "algebra" comes from the Arabic word *al-jabr*, meaning "reunion of broken parts." It was the title of a 9th-century book by Persian mathematician Al-Khwarizmi, whose work laid the foundation for solving equations and whose name gave rise to the word "algorithm."

The Goal

In this section, we will begin to learn the "language" of algebra—how to use symbols, variables, and expressions to describe relationships between numbers. We'll discover how equations are just math sentences and how variables allow us to describe patterns, rules, and unknown values in a neat, powerful way.

Use Variables and Algebraic Symbols

Suppose this year Greg is 20 years old and Alex is 23. You know that Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are variables and the 3 is a constant. The ages change ("vary") but the 3 years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by 3 years, 3 is the constant. In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age g, then we could use $g + 3$ to represent Alex's age.

Greg's age	Alex's age
12	15
20	23
35	38
g	$g+3$

The letters used to represent these changing ages are called variables. The letters most commonly used for variables are x, y, a, b, and c.

Definition: VARIABLE

A **variable** is a letter that represents a number whose value may change.

Definition: CONSTANT

A **constant** is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic operations: **addition, subtraction, multiplication,** and **division**. We'll list the symbols used to indicate these operations below. You'll probably recognize some of them.

Operation	Notation	Say:	The result is...
Addition	$a+b$	a plus b	the **sum** of a and b
Subtraction	$a-b$	a minus b	the **difference** of a and b
Multiplication	$a \cdot b, ab, (a)(b), (a)b, a(b)$	a times b	the **product** of a and b
Division	$a \div b, a/b, \frac{a}{b}, b\overline{)a}$	a divided by b	the **quotient** of a and b, a is called the dividend, and b is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words "of" and "and."

- The *difference* of 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9-2$.
- The *product* of 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as $4 \cdot 8$.

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ ('three times y') or $3 \cdot x \cdot y$ (three times x times y)? To make it clear, use $\cdot$ or parentheses for multiplication.

When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

EQUALITY SYMBOL

$a = b$ is read "a is equal to b"

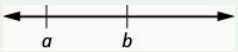
The symbol "$=$" is called the **equal sign**.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols "$<$" and "$>$".

INEQUALITY

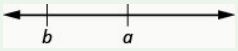
$a < b$ is read "a is less than b"

a is to the left of b on the number line



$a > b$ is read "a is greater than b"

a is to the right of b on the number line



The expressions $a < b$ or $a > b$ can be read from left to right or right to left, though in English we usually read from left to right. In general, $a < b$ is equivalent to $b > a$. For example $7 < 11$ is equivalent to $11 > 7$. And $a > b$ is equivalent to $b < a$. For example $17 > 4$ is equivalent to $4 < 17$.

Inequality Symbols	Words
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or equal to b

Example 1

Translate the following from algebra into English:

1. $17 \leq 26$
2. $8 \neq 17 - 3$
3. $12 > 27 \div 3$
4. $y + 7 < 19$

Solution

1. $17 \leq 26$, 17 is less than or equal to 26
2. $8 \neq 17 - 3$, 8 is not equal to 17 minus 3
3. $12 > 27 \div 3$, 12 is greater than 27 divided by 3
4. $y + 7 < 19$, y plus 7 is less than 19

Try It 1

Translate the following from algebra into English:

1. $19 \geq 15$
2. $7 = 12 - 5$
3. $15 \div 3 < 8$
4. $y + 3 < 6$

Answer

1. 19 is greater than or equal to 15
2. 7 is equal to 12 minus 5
3. 15 divided by 3 is less than 8
4. y plus 3 is less than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

GROUPING SYMBOLS

Parentheses	()
Brackets	[]
Braces	{}

Here are some examples of expressions that include **grouping symbols**. We will simplify expressions like these later in this section.

$$8(14 - 8) \qquad 21 - 3[2 + 4(9 - 8)] \qquad 24 \div \{13 - 2[1(6 - 5) + 4]\}$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have expressions and *equations*.

EXPRESSION

An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Expression	Words	English Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	n minus one	the difference of n and one
$6\cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb. An **equation** is two expressions linked with an **equal sign**. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Definition: EQUATION

An **equation** is two expressions connected by an equal sign.

Here are some examples of equations.

Equation	English Sentence
$3+5=8$	The sum of three and five is equal to eight
$n-1=14$	n minus one equals fourteen
$6\cdot 7=42$	The product of six and seven is equal to forty-two
$x=53$	x is equal to fifty-three
$y+9=2y-3$	y plus nine is equal to two y minus three

Example 2

Determine if each is an expression or an equation:

1. $2(x+3)=10$
2. $4(y-1)+1$
3. $x\div 25$
4. $y+8=40$

Solution

1. $2(x+3)=10$. This is an equation — two expressions are connected with an equal sign.
2. $4(y-1)+1$. This is an expression — no equal sign.
3. $x\div 25$. This is an expression — no equal sign.
4. $y+8=40$. This is an equation — two expressions are connected with an equal sign.

Try It 2

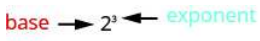
Determine if each is an expression or an equation:

1. $3(x-7)=27$
2. $5(4y-2)-7$

Answer

1. equation
2. expression

Suppose we need to multiply nine factors of 2. We could write this as $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$. This is tedious and it can be hard to keep track of all those 2s, so we use exponents. We write $2\cdot2\cdot2$ as 2^3 and $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ as 2^9. In expressions such as 2^3, the 2 is called the *base* and the 3 is called the *exponent*. The exponent tells us how many times we need to multiply the base.

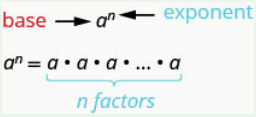


We read 2^3 as “two to the third power” or “two cubed” or "two to the power of three."

We say 2^3 is in exponential notation and $2\cdot2\cdot2$ is in expanded notation.

EXPONENTIAL NOTATION

a^n means the product of n factors of a.



The expression a^n is read a to the n^{th} power.

While we read a^n as “a to the n^{th} power,” we usually read:

- a^2 “a squared”
- a^3 “a cubed”

We’ll see later why a^2 and a^3 have special names.

The table below shows how we read some expressions with exponents.

Expression	In Words
7^2	7 to the second power or 7 squared
5^3	5 to the third power or 5 cubed
9^4	9 to the fourth power
12^5	12 to the fifth power

Example 3

Simplify: 3^4

Solution

3^4

Expand the expression	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right	$9 \cdot 3 \cdot 3$
Multiply	$27 \cdot 3$
Multiply	81

? Try It 3

Simplify:

1. 5^3
2. 1^7
3. 0^5

Answer

1. 125
2. 1
3. 0

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Section 1.1.P: Practice

Practice Makes Progress

Use Variables and Algebraic Symbols

In the following exercises, translate algebraic expressions into English.

Exercise 1

$16 - 9$

Exercise 2

$28 \div 4$

Exercise 3

$(2)(7)$

Exercise 4

$14 < 21$

Exercise 5

$36 \geq 19$

Exercise 6

$y - 1 > 6$

Exercise 7

$2 \leq 18 \div 6$

In the following exercises, determine if each is an expression or an equation.

Exercise 8

$9 \cdot 6 = 54$

Exercise 9

$5 \cdot 4 + 3$

Exercise 10

$x + 9$

Everyday Math

Exercise 11

Car insurance Justin's car insurance has a $750 deductible per incident. This means that he pays $750, and his insurance company will pay all costs beyond $750. If Justin files a claim for $2,100, how much will he pay? How much will his insurance company pay?

Writing Exercises

Exercise 12

Explain the difference between an expression and an equation.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use variables and algebraic symbols.			
translate algebraic exressions to English phrases.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.2: Order of Operations

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $5+5+5$
- Evaluate $3+4-5$
- Evaluate $2\cdot 3+2\cdot 2$

Learning Objectives

Motivating Problem

Which is correct: $5+4\times 8=72$ or $5+4\times 8=37$?

One small change in the order you do things makes a big difference in the answer!

Fun Fact

The phrase “Please Excuse My Dear Aunt Sally” is a mnemonic device used worldwide to remember the order of operations. However, did you know that this idea has its roots all the way back to 16th-century European mathematical texts? Even then, mathematicians were trying to stop people from arguing over whether to multiply or add first!

The Goal

In this section, we will learn why math has rules about which operations to do first and what happens if we don’t follow them. We’ll also practice simplifying expressions step-by-step using grouping symbols, exponents, multiplication, division, addition, and subtraction, all in the correct order.

Simplify Expressions Using the Order of Operations

To **simplify an expression** means to perform all the possible mathematical operations. For example, to simplify $4\cdot 2+1$ we’d first multiply $4\cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

$$4\cdot 2+1$$

$$8+1$$

$$9$$

By not using an **equal sign** when you simplify an expression, you may avoid confusing expressions with equations.

SIMPLIFY AN EXPRESSION

To **simplify an expression**, do all operations in the expression.

We’ve introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings and result in different values. For example, consider the expression:

$$4+3\cdot 7$$

If you simplify this expression, what do you get?

Some students incorrectly say 49 since $4+3$ give 7 and $7\cdot 7$ is 49.

$$4+3\cdot 7$$

$$7\cdot 7$$

$$49$$

Others say 25 since $3 \cdot 7$ is 21 and $21 + 4$ makes 25.

$$4 + 3 \cdot 7$$

$$4 + 21$$

$$25$$

Imagine the confusion in our banking system if every problem had several different correct answers!

The same expression should give the same result, so mathematicians established some guidelines early on, which are called the **Order of Operations**.

PERFORM THE ORDER OF OPERATIONS.

1. Parentheses and Other Grouping Symbols
   - Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost grouping symbols first.
2. Exponents
   - Simplify all expressions with exponents.
3. Multiplication and Division
   - Perform all multiplication and division in order from left to right. These operations have equal priority.
4. Addition and Subtraction
   - Perform all addition and subtraction in order from left to right. These operations have equal priority.

Students often ask, "How will I remember the order?" Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase: "Please Excuse My Dear Aunt Sally."

Parentheses	**P**lease
Exponents	**E**xcuse
Multiplication **D**ivision	**M**y **D**ear
Addition **S**ubtraction	**A**unt **S**ally

It's good that "**My Dear**" goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, "**Aunt Sally**" goes together, reminding us that **a**ddition and **s**ubtraction also have equal priority and are performed in order from left to right.

Let's try an example.

Example 1

Simplify:

1. $4 + 3 \cdot 7$
2. $(4 + 3) \cdot 7$

Solution

1.

	$4+3\cdot 7$
Are there any **p**arentheses? No.	
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply first.	$4+3\cdot 7$
Add.	$4+21$
	25

2.

	$(4+3)\cdot 7$
Are there any **p**arentheses? Yes.	$(4+3)\cdot 7$
Simplify inside the parentheses.	$(7)7$
Are there any **e**xponents? No.	
Is there any **m**ultiplication or **d**ivision? Yes.	
Multiply.	49

Try It 1

Simplify:

1. $8+3\cdot 9$
2. $(8+3)\cdot 9$

Answer

1. 35
2. 99

Example 2

Simplify: $18\div 6+4(5-2)$

Solution

Parentheses? Yes, subtract first.	$18 \div 6 + 4(5 - 2)$ $18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	$18 \div 6 + 4(3)$
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

? Try It 2

Simplify: $30 \div 5 + 10(3 - 2)$

Answer

16

When there are multiple grouping symbols, we simplify the innermost grouping symbols first and work outward.

✔ Example 3

Simplify: $5 + 2^3 + 3[6 - 3(4 - 2)]$.

Solution

	$5+2^3+3[6-3(4-2)]$
Are there any parentheses (or other grouping symbols)? Yes.	
Focus on the parentheses that are inside the brackets.	$5+2^3+3[6-3(4-2)]$
Subtract.	$5+2^3+3[6-3(2)]$
Continue inside the brackets and multiply.	$5+2^3+3[6-6]$
Continue inside the brackets and subtract.	$5+2^3+3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	$5+2^3+3[0]$
Simplify exponents.	$5+8+3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5+8+0$
Is there any addition or subtraction? Yes.	
Add.	$13+0$
Add.	13

? Try It 3

Simplify: $7^2-2[4(5+1)]$.

Answer

1

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Section 1.2P: Practice

Practice Makes Progress

Simplify Expressions Using the Order of Operations

In the following exercises, simplify each expression.

Exercise 1

5^3

Exercise 2

2^8

Exercise 3

$3+8\cdot 5$

Exercise 4

$(3+8)\cdot 5$

Exercise 5

$2^3-12\div(9-5)$

Exercise 6

$3\cdot 8+5\cdot 2$

Exercise 7

$2+8(6+1)$

Exercise 8

$4\cdot 12/8$

Exercise 9

$(6+10)\div(2+2)$

Exercise 10

$20\div 4+6\cdot 5$

Exercise 11

3^2+7^2

? Exercise 12

$3(1+9\cdot 6)-4^2$

? Exercise 13

$2[1+3(10-2)]$

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions using the order of operations.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.3: Exploring Expressions

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify $1+8\cdot 9$
- Write $2\cdot 2\cdot 2\cdot 2$ in exponential notation
- Express *the difference of 20 and 15* as a number sentence.
- Express *the product of 2 and 7, less 5* as a number sentence.

Learning Objectives

Motivating Problem

If someone gives you an expression like $4x+7x+x$, do you really need to plug in a number for x to work with it? Or can you simplify it first? How much *cleaner* can we make this expression?

Fun Fact

The idea of using letters to represent unknown quantities in expressions became widespread thanks to the French mathematician François Viète in the 1500s. Before that, equations were written entirely in words—imagine trying to solve algebra problems with *zero* symbols!

The Goal

This section helps us take apart and make sense of algebraic expressions. We will evaluate expressions by substituting values, identify like terms and coefficients, and learn how to simplify expressions by combining those like terms—making big expressions feel a lot smaller.

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when a given number replaces the variable.

EVALUATE AN EXPRESSION

To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Example 1

Evaluate $7x-4$, when

1. $x=5$
2. $x=1$

Solution

1.

when $x = 5$	$7x - 4$
	$7(5) - 4$
Multiply.	$35 - 4$
Subtract.	31

2.

when $x = 1$	$7x - 4$
	$7(1) - 4$
Multiply.	$7 - 4$
Subtract.	3

Try It 1

Evaluate $8x - 3$, when

1. $x = 2$
2. $x = 1$

Answer

1. 13
2. 5

Example 2

Evaluate $x = 4$, when

1. x^2
2. 3^x

Solution

1.

	x^2
Replace x with 4.	$(4)^2$
Use definition of exponent.	$4 \cdot 4$
Simplify.	16

2.

	3^x
Replace x with 4.	3^4
Use definition of exponent.	$3 \cdot 3 \cdot 3 \cdot 3$
Simplify.	81

Try It 2

Evaluate $x = 6$, when

1. x^3
2. 2^x

Answer

1. 216
2. 64

Example 3

Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution

	$2x^2 + 3x + 8$
Substitute $x = 4$.	$2x^2 + 3x + 8$ $2(4)^2 + 3(4) + 8$
Follow the order of operations.	$2(16) + 3(4) + 8$
	$32 + 12 + 8$
	52

Try It 3

Evaluate $6x^2 - 4x - 7$ when $x = 2$.

Answer

9

Identify and Combine Like Terms

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

TERM

A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are $7, y, 5x^2, 9a$, and $-2a^3b$.

The constant that multiplies the variable is called the **coefficient**.

COEFFICIENT

The **coefficient** of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x = 1 \cdot x$.

Example 4

Identify the coefficient of each term:

1. $14y$
2. $15x^2$
3. a

Solution

1. The coefficient of $14y$ is 14
2. The coefficient of $15x^2$ is 15
3. The coefficient of a is 1 since $a = 1a$.

Try It 4

Identify the coefficient of each term:

1. $9p$
2. $13a^2$
3. y^3

Answer

1. 9
2. 13
3. 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?

$$5x \quad 7 \quad n^2 \quad 4 \quad 3x \quad 9n^2$$

The 7 and the 4 are both constant terms.

The $5x$ and the $3x$ are both terms with x.

The n^2 and the $9n^2$ are both terms with n^2.

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- x^2 and $9x^2$ are like terms.

LIKE TERMS

Terms that are either constants or have the same variables raised to the same powers are called **like terms**.

Example 5

Identify the like terms: $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$.

Solution

y^3 and $4y^3$ are like terms because both have y^3; the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2; the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like $9x$.

Try It 5

Identify the like terms: $4x^3, 8x^2, 19, 3x^3, 24, 6x^3$.

Answer

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2 + 3x + 8$, the three terms are $2x^2$, $3x$, and 8.

Example 6

Identify the terms in each expression.

1. $9x^2 + 7x + 12$
2. $8x + 3y$

Solution

1. The terms of $9x^2 + 7x + 12$ are $9x^2, 7x$, and 12.
2. The terms of $8x + 3y$ are $8x$ and $3y$.

Try It 6

Identify the terms in the expression $5x + 2y$.

Answer

$5x, 2y$

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought $12x$, you would be right!

$$\begin{array}{c} 4x + 7x + x \\ x + x + x + x \quad + x + x + x + x + x + x + x \quad + x \\ 12x \end{array}$$

Add the coefficients and keep the same variable. It doesn't matter what x is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

Simplify: $4x + 7x + x$

Add the coefficients. $12x$

Example 7

Simplify: $2x^2 + 3x + 7 + x^2 + 4x + 5$

Solution

Step 1. Identify the like terms.	$2x^2 + 3x + 7 + x^2 + 4x + 5$ $2x^2 + 3x + 7 + x^2 + 4x + 5$
Step 2. Rearrange the expression so the like terms are together.	$2x^2 + x^2 + 3x + 4x + 7 + 5$
Step 3. Combine like terms.	$3x^2 + 7x + 12$

? Try It 7

Simplify: $4y^2 + 5y + 2 + 8y^2 + 4y + 5$.

Answer

$12y^2 + 9y + 7$

COMBINE LIKE TERMS.

1. Identify like terms.
2. Rearrange the expression so like terms are together.
3. Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra; then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that.

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a + b$
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a - b$
Multiplication	a times b the product of a and b twice a	$a \cdot b, ab, a(b), (a)(b)$ $2a$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a \div b, a/b, \frac{a}{b}$

Look closely at these phrases using the four operations:

the **sum** *of a and b*

the **difference** *of a and b*

the **product** *of a and b*

the **quotient** *of a and b*

Each phrase tells us to operate on two numbers. Look for the words *of* and *and* to find the numbers.

Example 8

Translate each English phrase into an algebraic expression:

1. the difference of $17x$ and 5
2. the quotient of $10x^2$ and 7.

Solution

1. The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

   the *difference of* $17x$ *and* 5

   $17x$ minus 5

   $17x - 5$

2. The key word is "quotient," which tells us the operation is division.

the *quotient of* $10x^2$ *and* 7

divide $10x^2$ by 7

$10x^2 \div 7$

This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Try It 8

Translate each English phrase into an algebraic expression:

1. the sum of $17y^2$ and 19
2. the product of 7 and y.

Answer

1. $17y^2 + 19$
2. $7y$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight "more than" means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven "less than" means 7 subtracted from your present age.

Example 9

Translate the English phrase into an algebraic expression:

1. Seventeen more than y
2. Nine less than $9x^2$.

Solution

1. The key words are *more than.* They tell us the operation is addition. *More than* means "added to."

   Seventeen more than y

   Seventeen added to y

   $y + 17$

2. The key words are *less than*. They tell us to subtract. *Less than* means "subtracted from."

   Nine less than $9x^2$

   Nine subtracted from $9x^2$

   $9x^2 - 9$

Try It 9

Translate the English phrase into an algebraic expression:

1. Eleven more than x
2. Fourteen less than $11a$.

Answer

1. $x + 11$
2. $11a - 14$

Example 10

Translate the English phrase into an algebraic expression:

1. five times the sum of m and n
2. the sum of five times m and n.

Solution

There are two operation words—*times* tells us to multiply and *sum* tells us to add.
1. Because we are multiplying 5 times the sum we need parentheses around the sum of m and n, $(m + n)$. This forces us to determine the sum first. (Remember the order of operations.)

$$\text{five times the sum of } m \text{ and } n$$
$$5(m + n)$$

2. To take a sum, we look for the words "of" and "and" to see what is being added. Here we are taking the sum *of* five times m and n.

$$\text{the sum of five times } m \text{ and } n$$
$$5m + n$$

Try It 10

Translate the English phrase into an algebraic expression:

1. four times the sum of p and q
2. the sum of four times p and q
3. the difference of two times x and 8
4. two times the difference of x and 8

Answer

1. $4(p + q)$
2. $4p + q$
3. $2x - 8$
4. $2(x - 8)$

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We'll see how to do this in the next two examples.

Example 11

The length of a rectangle is 6 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution

Write a phrase about the length of the rectangle.	6 less than the width
Substitute w for 'the width.'	6 less then w
Rewrite 'less than' as 'subtracted from.'	6 subtracted from w
Translate the phrase into algebra.	$w - 6$

Try It 11

The length of a rectangle is 2 times the width. Let w represent the length of the rectangle. Write an expression for the length of the rectangle.

Answer

$2w$

Example 12

June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution

Write the phrase about the number of dimes.	three less than four times the number of quarters
Substitute q for the number of quarters.	3 less than 4 times q
Translate '4 times q.'	3 less than $4q$
Translate the phrase into algebra.	$4q - 3$

Try It 12

Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Answer

$7n + 3$

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Section 1.3P: Practice

Practice Makes Progress

Evaluate an Expression

In the following exercises, evaluate the following expressions.

Exercise 1

$7x + 8$ when $x = 2$

Exercise 2

x^2 when $x = 12$

Exercise 3

x^5 when $x = 2$

Exercise 4

$x^2 + 3x - 7$ when $x = 4$

Exercise 5

$6x + 3y - 9$ when $x = 10, y = 7$

Exercise 6

$a^2 + b^2$ when $a = 3, b = 8$

Exercise 7

$2l + 2w$ when $l = 15, w = 12$

Simplify Expressions by Combining Like Terms

In the following exercises, identify the coefficient of each term.

Exercise 8

$8a$

Exercise 9

$5r^2$

In the following exercises, identify the like terms.

Exercise 10

$x^3, 8x, 14, 8y, 5, 8x^3$

Exercise 11

$9a, a^2, 16, 16b^2, 4, 9b^2$

In the following exercises, identify the terms in each expression.

Exercise 12

$15x^2 + 6x + 2$

Exercise 13

$10y^3 + y + 2$

In the following exercises, simplify the following expressions by combining like terms.

Exercise 14

$10x + 3x$

Exercise 15

$4c + 2c + c$

Exercise 16

$7u + 2 + 3u + 1$

Exercise 17

$10a + 7 + 5a - 2 + 7a - 4$

Exercise 18

$3x^2 + 12x + 11 + 14x^2 + 8x + 5$

Translate an English Phrase to an Algebraic Expression

In the following exercises, translate the phrases into algebraic expressions.

Exercise 19

the difference of 14 and 9

Exercise 20

the product of 9 and 7

Exercise 21

the quotient of 36 and 9

Exercise 22

the sum of $8x$ and $3x$

Exercise 23

the quotient of y and 3

Exercise 24

eight times the difference of y and nine

Everyday Math

Exercise 25

Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let c represent the number of classical CDs. Write an expression for the number of rock CDs.

Exercise 26

Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.

Writing Exercises

Exercise 27

Explain how you identify the like terms in the expression $8a^2 + 4a + 9 - a^2 - 1$

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
evaluate an expression.			
identify and combine like terms.			
translate English phrases to algebraic expressions.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.4: Integers

Be Prepared

We will rely heavily on these skills throughout this section.

- Plot 0, 2, and 5 on a number line
- Fill in the appropriate symbol $=$, $<$, or $>$: $2\_\_\_\_\_5$
- Simplify $3+4-5$
- Simplify $\sqrt{144}$

Learning Objectives

Motivating Problem

You check your bank account and see that you have $15. Then you accidentally use your debit card to buy a $25 pair of shoes. What does your account balance show now? How can a number be *less* than zero, and what does that mean?

Fun Fact

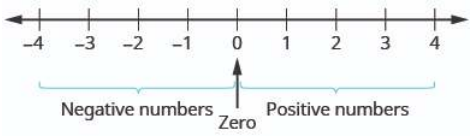
Although ancient Chinese and Indian mathematicians employed negative numbers for trade and accounting, Europeans only began accepting them during the Renaissance. Today, negative numbers are used in everything from banking to data science.

The Goal

In this section, we will discover that numbers don't stop at zero. We'll explore the world of positive and negative integers and learn how to add, subtract, multiply, and divide them. With the help of number lines and real-life examples like temperatures and money, we will understand opposites, absolute value, and how to make sense of operations with signed numbers.

Simplify Expressions with Absolute Value

A negative number is a number less than 0. The negative numbers are to the left of zero on the number line below.



You may have noticed that, on the number line, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, each one is called the **opposite** of the other. The opposite of 2 is -2, and the opposite of -2 is 2.

OPPOSITE

The **opposite** of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.



The opposite of 3 is -3.

OPPOSITE NOTATION

$-a$ means the opposite of the number a
The notation $-a$ is read as "the opposite of a."

We saw that numbers such as 3 and −3 are opposites because they are the same distance from 0 on the number line. They are both three units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

Definition: ABSOLUTE VALUE

The **absolute value** of a number is its distance from 0 on the number line.

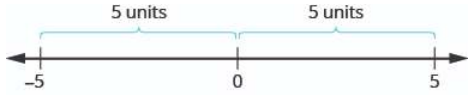
The absolute value of a number n is written as $|n|$ and $|n| \geq 0$ for all numbers.

Absolute values are always greater than or equal to zero.

For example,

$$-5 \text{ is 5 units away from 0, so } |-5| = 5.$$
$$5 \text{ is 5 units away from 0, so } |5| = 5.$$

The Figure below illustrates this idea.



The absolute value of a number is never negative because distance cannot be negative. The only number with absolute value equal to zero is the number zero itself because the distance from 0 to 0 on the number line is zero units.

In the next example, we'll order expressions with absolute values.

Example 1

Fill in $<$, $>$, or $=$ for each of the following pairs of numbers:

a. $|-5| \_\_ -|-5|$
b. $8 \_\_ -|8|$
c. $-9 \_\_ -|-9|$
d. $-(-16) \_\_ -|-16|$

Solution

a.

$$\begin{array}{lccc} & |-5| & \_ & -|-5| \\ \text{Simplify.} & 5 & \_ & -5 \\ \text{Order.} & 5 & > & -5 \\ & |-5| & > & -|-5| \end{array}$$

b.

$$\begin{array}{lccc} & 8 & \_ & -|-8| \\ \text{Simplify.} & 8 & \_ & -8 \\ \text{Order.} & 8 & > & -8 \\ & 8 & > & -|-8| \end{array}$$

c.

$$\begin{array}{lccc} & -9 & \_ & -|-9| \\ \text{Simplify.} & -9 & \_ & -9 \\ \text{Order.} & -9 & = & -9 \\ & -9 & = & -|-9| \end{array}$$

d.

$$\begin{array}{lccc} & -(-16) & \_ & -|-16| \\ \text{Simplify.} & 16 & \_ & 16 \\ \text{Order.} & 16 & = & 16 \\ & -(-16) & = & |-16| \end{array}$$

Try It 1

Fill in $<$, $>$, or $=$ for each of the following pairs of numbers:

a. $-|-10|\_-10$

b. $2\_-|-2|$

c. $-8\_|-8|$

d. $-(-9)\_|-9|$.

Answer

a. $=$

b. $>$

c. $<$

d. $=$

We now add absolute value bars to our list of grouping symbols. When we use the order of operations, first we simplify inside the absolute value bars as much as possible, then we take the absolute value of the resulting number.

GROUPING SYMBOLS

Parentheses	()	**Braces**	{}
Brackets	[]	**Absolute value**	||

In the next example, we simplify the expressions inside absolute value bars first just like we do with parentheses.

Example 2

Simplify: $24-|19-3(6-2)|$.

Solution

	$24-\|19-3(6-2)\|$
Work inside parentheses first:	
subtract 2 from 6.	$24-\|19-3(4)\|$
Multiply 3(4).	$24-\|19-12\|$
Subtract inside the absolute value bars.	$24-\|7\|$
Take the absolute value.	$24-7$
Subtract.	17

Try It 2

Simplify: $19-|11-4(3-1)|$.

Answer

16

Add and Subtract Integers

So far in our examples, we have only used the counting numbers and the whole numbers.

Counting numbers	$1, 2, 3 \ldots$
Whole numbers	$0, 1, 2, 3 \ldots .$

Our work with opposites gives us a way to define the **integers**. The whole numbers and their opposites are called the integers. The integers are the numbers ... $-3, -2, -1, 0, 1, 2, 3$ ...

Definition: INTEGERS

The whole numbers and their opposites are called the **integers**.

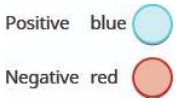
The integers are the numbers

$$\ldots -3, -2, -1, 0, 1, 2, 3 \ldots,$$

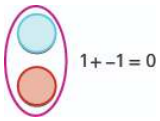
Most students are comfortable with the addition and subtraction facts for positive numbers. However, adding or subtracting with both positive and negative numbers may be more challenging.

We will use two-color counters to model the addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

We let one color (blue) represent positive. The other color (red) will represent the negatives.



If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero.



We will use the counters to show how to add:

$$5 + 3 \qquad -5 + (-3) \qquad -5 + 3 \qquad 5 + (-3)$$

The first example, $5 + 3$, adds 5 positives and 3 positives—both positives.

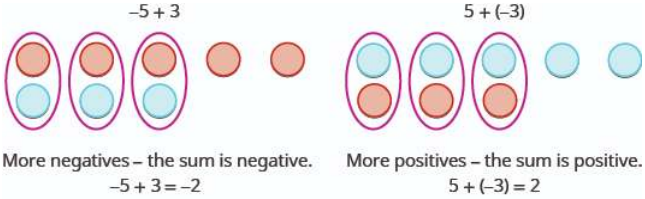
The second example, $-5 + (-3)$, adds 5 negatives and 3 negatives—both negatives.

When the signs are the same, the counters are all the same color, so we add them. In each case, we get 8—either 8 positives or 8 negatives.



So what happens when the signs are different? Let's add $-5 + 3$ and $5 + (-3)$.

When we use counters to model addition of positive and negative integers, it is easy to see whether there are more positive or more negative counters. So we know whether the sum will be positive or negative.



Example 3

Perform the following addition:

a. $-1+(-4)$

b. $-1+5$

c. $1+(-5)$

Solution

a.

	–1 + (–4)
	
1 negative plus 4 negatives is 5 negatives	–5

b.

	–1 + 5
	
There are more positives, so the sum is positive.	4

c.

	1 + (–5)
	
There are more negatives, so the sum is negative.	–4

Try It 3

Add:

a. $-2+(-4)$

b. $-2+4$

c. $2+(-4)$

Answer

a. -6

b. 2

c. -2

We will continue to use counters to model the subtraction. Perhaps when you were younger, you read "$5-3$" as "5 take away 3." When you use counters, you can think of subtraction the same way!

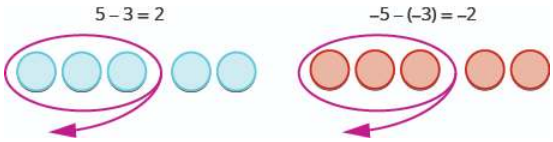
We will use the counters to show to subtract:

$$5-3 \qquad -5-(-3) \qquad -5-3 \qquad 5-(-3)$$

The first example, $5-3$, we subtract 3 positives from 5 positives and end up with 2 positives.

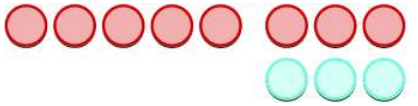
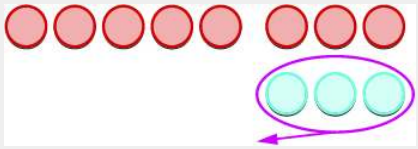
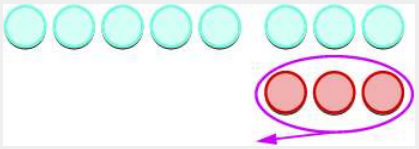
In the second example, $-5-(-3)$, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one color, and the "take away" model of subtraction was easy to apply.



What happens when we have to subtract one positive and one negative number? We'll need to use both blue and red counters as well as some neutral pairs. If we don't have the number of counters needed to take away, we add neutral pairs. Adding a neutral pair does not change the value. It is like changing quarters to nickels—the value is the same, but it looks different.

Let's look at $-5-3$ and $5-(-3)$.

	$-5-3$	$5-(-3)$
Model the first number.		
We now add the needed neutral pairs.		
We remove the number of counters modeled by the second number.		
Count what is left.		
	$-5-3=-8$	$5-(-3)=-8$
	-8	8

Example 4

Perform the following subtraction:

a. $3-1$

b. $-3-(-1)$

c. $-3-1$

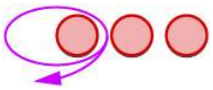
d. $3-(-1)$

Solution

a.

		3 – 1
Take 1 positive from 3 positives and get 2 positives.		2

b.

		–3 – (–1)
Take 1 positive from 3 negatives and get 2 negatives.		–2

c.

		–3 – 1
Take 1 positive from the one added neutral pair.		–4

d.

		3 – (–1)
Take 1 negative from the one added neutral pair.		4

Try It 4

Perform the following Subtraction:

a. $7-4$

b. $-7-(-4)$

c. $-7-4$

d. $7-(-4)$

Answer

a. 3

b. -3

c. -11

d. 11

Have you noticed that *subtraction of signed numbers can be done by adding the opposite*? In the last example, $-3-1$ is the same as $-3+(-1)$ and $3-(-1)$ is the same as $3+1$. You will often see this idea, the Subtraction Property, written as follows:

Definition: SUBTRACTION PROPERTY

$$a - b = a + (-b)$$

Subtracting a number is the same as adding its opposite.

Example 5

Simplify:

a. $13 - 8$ and $13 + (-8)$

b. $-17 - 9$ and $-17 + (-9)$

c. $9 - (-15)$ and $9 + 15$

d. $-7 - (-4)$ and $-7 + 4$

Solution

a.

	$13 - 8$ and	$13 + (-8)$
Subtract.	5	5

b.

	$-17 - 9$ and	$-17 + (-9)$
Subtract.	-26	-26

c.

	$9 - (-15)$ and	$9 + 15$
Subtract.	24	24

d.

	$-7 - (-4)$ and	$-7 + 4$
Subtract.	-3	-3

Try It 5

Simplify:

a. $15 - 7$ and $15 + (-7)$

b. $-14 - 8$ and $-14 + (-8)$

c. $4 - (-19)$ and $4 + 19$

d. $-4 - (-7)$ and $-4 + 7$.

Answer

a. $8, 8$

b. $-22, -22$

c. $23, 23$

d. $3, 3$

What happens when there are more than three integers? We just use the order of operations as usual.

Example 6

Simplify: $7 - (-4 - 3) - 9$.

Solution

	$7 - (-4 - 3) - 9$
Simplify inside the parentheses first.	$7 - (-7) - 9$
Subtract left to right.	$14 - 9$
Subtract.	5

? Try It 6

Simplify: $8-(-3-1)-9$.

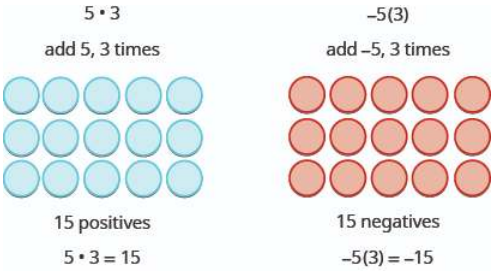
Answer

3

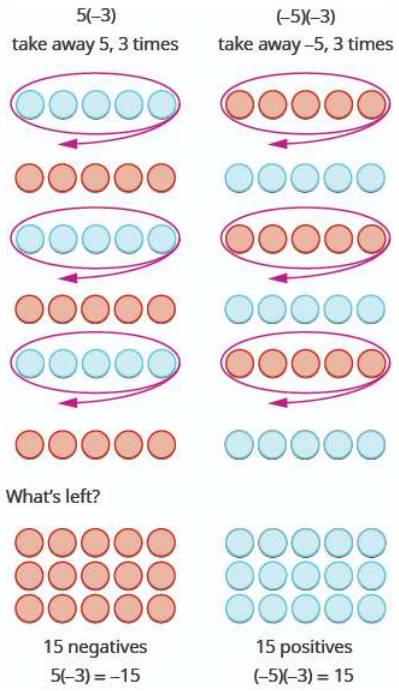
Multiply and Divide Integers

Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show multiplication of integers. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we are using the model just to help us discover the pattern.

We remember that a·b means **add** *a*, *b* **times**.



The next two examples are more interesting. What does it mean to multiply 5 by −3? It means subtract 5 from itself three times. Looking at subtraction as "taking away", it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs on the workspace.



In summary:

$$5 \cdot 3 = 15 \qquad -5(3) = -15$$
$$5(-3) = -15 \qquad (-5)(-3) = 15$$

Notice that for multiplication of two signed numbers, when the

signs are the same, the product is positive.
signs are different, the product is negative.

What about division? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $15 \cdot 3 = 15$. In words, this expression says that 15 can be divided into 3 groups of 5 each because adding five three times gives 15. If you look at some examples of multiplying integers, you might figure out the rules for dividing integers.

$$\begin{array}{llll} 5 \cdot 3 = 15 & \text{so} & 15 \div 3 = 5 & -5(3) = -15 \quad \text{so} \quad -15 \div 3 = -5 \\ (-5)(-3) = 15 & \text{so} & 15 \div (-3) = -5 & 5(-3) = -15 \quad \text{so} \quad -15 \div (-3) = 5 \end{array}$$

Division follows the same rules as multiplication with regard to signs.

MULTIPLICATION AND DIVISION OF SIGNED NUMBERS

For multiplication and division of two signed numbers:

Same signs	Result
• Two positives	Positive
• Two negatives	Positive

If the signs are the same, the result is positive.

Different signs	Result
• Positive and negative	Negative
• Negative and positive	Negative

If the signs are different, the result is negative.

Example 7

Multiply or divide:

a. $-100 \div (-4)$

b. $7 \cdot 6$

c. $4(-8)$

d. $-27 \div 3$

Solution

a. $-100 \div (-4)$
Divide, with signs that are the same the quotient is positive. 25

b. $7 \cdot 6$
Multiply, with same signs. 42

c. $4(-8)$
Multiply, with different signs. -32

d. $-27 \div 3$
Divide, with different signs, the quotient is negative. -9

Try It 7

Multiply or divide:

a. $-117 \div (-3)$

b. $3 \cdot 13$

c. $7(-4)$

d. $-42 \div 6$.

Answer

a. 39

b. 39

c. -28

d. -7

When we multiply a number by 1, the result is the same. Each time we multiply a number by −1, we get its opposite!

MULTIPLICATION BY −1

$$-1a = -a$$

Multiplying a number by -1 gives its opposite.

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember Please Excuse My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

Example 8

Simplify:

a. $(-2)^4$

b. -2^4.

Solution

Notice the difference in parts a and b. In a, the exponent applies to all of (-2), while in b, it only applies to the 2.

a.

	$(-2)^4$
Write in expanded form.	$(-2)(-2)(-2)(-2)$
Multiply.	$4(-2)(-2)$
Multiply.	$-8(-2)$
Multiply.	16

b.

	-2^4
Write in expanded form.	$-(2 \cdot 2 \cdot 2 \cdot 2)$
Multiply.	$-(4 \cdot 2 \cdot 2)$
Multiply.	$-(8 \cdot 2)$
Multiply.	-16

Try It 8

Simplify:

a. $(-3)^4$

b. -3^4.

Answer

a. 81

b. -81

The last example showed us the difference between $(-2)^4$ and -2^4. This distinction is important to prevent future errors. The following example reminds us to multiply and divide in order left to right.

Example 9

Simplify:

a. $8(-9) \div (-2)^3$

b. $-30 \div 2 + (-3)(-7)$

Solution

a.

	$8(-9) \div (-2)^3$
Exponents first.	$8(-9) \div (-8)$
Multiply.	$-72 \div (-8)$
Divide.	9

b.

	$-30 \div 2 + (-3)(-7)$
Divide first.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Try It 9

Simplify:

a. $18(-4) \div (-2)^3$

b. $-32 \div 4 + (-2)(-7)$.

Answer

a. 9

b. 6

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

Example 10

Evaluate $4x^2 - 2xy + 3y^2$ when $x = 2, y = -1$.

Solution

	$4x^2 - 2xy + 3y^2$
Substitute.	$4(2)^2 - 2(2)(-1) + 3(-1)^2$
Simplify exponents.	$4(4) - 2(2)(-1) + 3(1)$
Multiply.	$16 + 4 + 3$
Add.	23

? Try It 10

Evaluate: $3x^2 - 2xy + 6y^2$ when $x = 1, y = -2$.

Answer

31

Translate Phrases to Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

✓ Example 11

Translate and simplify: the sum of 8 and −12, increased by 3.

Solution

Translate.	$[8 + (-12)] + 3$
Simplify inside the brackets.	$-4 + 3$
Add.	-1

? Try It 11

Translate and simplify the sum of 9 and −16, increased by 4.

Answer

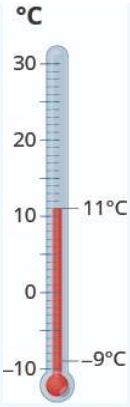
$(9 + (-16)) + 4 = -3$

Use Integers in Applications

We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

✓ Example 12

The temperature in Kendallville, Indiana one morning was 11 degrees. By mid-afternoon, the temperature had dropped to −9 degrees. What was the difference in the morning and afternoon temperatures?



Solution

Identify the problem.	**We are asked to find the difference between 11 and −9.**
Translate.	$11-(-9)$
Subtracting a negative becomes addition.	$11+9$
Add.	20

The difference in temperatures was 20 degrees.

? Try It 12

The temperature in Anchorage, Alaska one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

Answer

The difference in temperatures was 45 degrees Fahrenheit.

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Section 1.4P: Practice

Practice Makes Progress

Simplify Expressions with Absolute Value

In the following exercises, fill in $<$, $>$, or $=$ for each of the following pairs of numbers.

Exercise 1

$|-7|$ _____ $-|-7|$

Exercise 2

6 _____ $-|-6|$

Exercise 3

$|-11|$ _____ -11

Exercise 4

$-(-13)$ _____ $-|-13|$

Exercise 5

$-|2|$ _____ $-|-2|$

Exercise 6

-12 _____ $-|-12|$

Exercise 7

$|-19|$ _____ $-(-19)$

In the following exercises, simplify.

Exercise 8

$18-|2(8-3)|$

Exercise 9

$18-|12-4(4-1)+3|$

Exercise 10

$10-3|9-3(3-1)|$

Add and Subtract Integers

In the following exercises, simplify each expression.

? Exercise 11

$48 + (-16)$

? Exercise 12

$-14 + (-12) + 4$

? Exercise 13

$19 + 2(-3 + 8)$

? Exercise 14

$13 - 7$

? Exercise 15

$-13 - (-7)$

? Exercise 16

$-14 - (-27) + 9$

? Exercise 17

$27 - (-18)$

? Exercise 18

$(2 - 7) - (3 - 8)$

? Exercise 19

$-(6 - 8) - (2 - 4)$

? Exercise 20

$25 - [10 - (3 - 12)]$

Multiply and Divide Integers

In the following exercises, multiply or divide.

? Exercise 21

$-4 \cdot 8$

? Exercise 22

$13(-5)$

Exercise 23

$-24 \div 6$

Exercise 24

$-52 \div (-4)$

Exercise 25

$-28 \div 7$

Exercise 26

$-180 \div 15$

Simplify and Evaluate Expressions with Integers

In the following exercises, simplify each expression.

Exercise 27

$(-2)^6$

Exercise 28

-2^6

Exercise 29

$5(-6) + 7(-2) - 3$

Exercise 30

$-3(-5)(6)$

Exercise 31

$(8 - 11)(9 - 12)$

Exercise 32

$26 - 3(2 - 7)$

Exercise 33

$65 \div (-5) + (-28) \div (-7)$

Exercise 34

$9 - 2[3 - 8(-2)]$

Exercise 35

$8 - |2 - 4(4 - 1) + 3|$

Exercise 36

$9 - 3|2(2 - 6) - (3 - 7)|$

Exercise 37

$(-3)^2 - 24 \div (8 - 2)$

In the following exercises, evaluate each expression.

Exercise 38

$(x + y)^2$ when $x = -3$ and $y = 14$

Exercise 39

$9a - 2b - 8$ when $a = -6$ and $b = -3$

Exercise 40

$3x^2 - 4xy + 2y^2$ when $x = -2$ and $y = -3$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise 41

The sum of 3 and −15, increased by 7

Exercise 42

The difference of 10 and -18

Exercise 43

The quotient of -6 and the sum of a and b

Everyday Math

Exercise 44

Temperature: On January 15, the high temperature in Anaheim, California, was $84°$. That same day, the high temperature in Embarrass, Minnesota, was $-12°$. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

Exercise 45

Football: On the first down, the Chargers had the ball on their 25-yard line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?

Writing Exercises

Exercise 46

Explain why the sum of −8 and 2 is negative, but the sum of 8 and −2 is positive.

Exercise 47

In your own words, state the rules for multiplying and dividing integers.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with absolute value.			
add and subtract integers.			
multiply and divide integers.			
simplify and evaluate expressions with integers.			
translate English phrases to algebraic expressions.			
use integers in applications.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.5: Fractions

Be Prepared

We will rely heavily on these skills throughout this section.

- List the first 10 prime numbers.
- Find the least common multiple of 6 and 15.
- Determine the prime factorization of 48.

Learning Objectives

Motivating Problem

You need ¾ cup of flour and ⅔ cup of sugar for a recipe. If you want to make a half batch, how much of each ingredient should you use?

Fun Fact

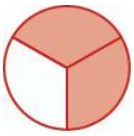
The word “fraction” comes from the Latin *fractio*, meaning “to break.” Ancient Egyptians wrote all their fractions with a numerator of 1 and used special symbols for them, employing clever techniques to represent quantities like 2/3 or 3/4 without our modern notation.

The Goal

In this section, we delve into fractions in depth, exploring what they represent, how to simplify them, and how to perform operations such as multiplication, division, addition, and subtraction. We'll learn how fractions represent parts of a whole and how to use models, patterns, and logic to work fluently with them in real-life and algebraic contexts.

Simplify Fractions

A fraction is a way to represent parts of a whole. The fraction $\frac{2}{3}$ represents two of three equal parts. In the fraction $\frac{2}{3}$, the 2 is called the **numerator** and the 3 is called the **denominator**. The line is called the fraction bar.



In the circle, $\frac{2}{3}$ of the circle is shaded—2 of the 3 equal parts.

FRACTION

A **fraction** is written $\frac{a}{b}$, where $b \neq 0$ and

a is the **numerator** and b is the **denominator**.

A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

Fractions that have the same value are **equivalent fractions**. The Equivalent Fractions

Property allows us to find equivalent fractions and also simplify fractions.

Equivalent Fractions Property

If a, b, and c are numbers where $b \neq 0, c \neq 0$,

then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

A fraction is considered simplified if there are no common factors, other than 1, in its numerator and denominator.

For example,

$\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.

$\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

We simplify, or reduce, a fraction by dividing out the common factors of the numerator and denominator. A fraction is not simplified until all common factors have been divided out. If an expression has fractions, it is not completely simplified until the fractions are simplified.

Sometimes it may not be easy to find common factors of the numerator and denominator. When this happens, a good idea is to factor the numerator and the denominator into prime numbers. Then divide out the common factors using the Equivalent Fractions Property.

Example 1

Simplify $\frac{-315}{770}$.

Solution

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, use a factor tree.	Rewrite 315 and 770 as the product of the primes.	$-\frac{315}{770}$ $-\frac{3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 5 \cdot 7 \cdot 11}$
Step 2. Simplify using the equivalent fractions property by dividing out common factors.	Mark the common factors 5 and 7. Divide out the common factors	$-\frac{3 \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}{2 \cdot \cancel{5} \cdot \cancel{7} \cdot 11}$ $-\frac{3 \cdot 3}{2 \cdot 11}$
Step 3. Multiply the remaining factors, if necessary.		$-\frac{9}{22}$

Try It 1

Simplify $-\frac{120}{192}$.

Answer

$-\frac{5}{8}$

We now summarize the steps you should follow to simplify fractions.

SIMPLIFY A FRACTION.

1. Rewrite the numerator and denominator to show the common factors.
   If needed, factor the numerator and denominator into prime numbers first.
2. Simplify using the Equivalent Fractions Property by dividing out common factors.
3. Multiply any remaining factors.

Multiply and Divide Fractions

Many people find multiplying and dividing fractions easier than adding and subtracting fractions.

To multiply fractions, we multiply the numerators and multiply the denominators.

FRACTION MULTIPLICATION

If a, b, c, and d are numbers where $b \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

To multiply fractions, multiply the numerators and multiply the denominators.

When multiplying fractions, the properties of positive and negative numbers still apply, of course. It is a good idea to determine the sign of the product as the first step. In the following example, we will multiply a negative number and a positive number, so the product will be negative.

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, for example, $3 = \frac{3}{1}$.

Example 2

Multiply: $-\frac{12}{5}(-20x)$.

Solution

The first step is to find the sign of the product. Since the signs are the same, the product is positive.

	$\frac{12}{5}(20x)$
Write $20x$ as a fraction.	$\frac{12}{5}\left(\frac{20x}{1}\right)$
Multiply.	$\frac{12 \cdot 20x}{5 \cdot 1}$
Rewrite 20 to show the common factor 5 and divide it out.	$\frac{12 \cdot 4 \cdot \cancel{5} \cdot x}{\cancel{5} \cdot 1}$
Simplify.	$48x$

Try It 2

Multiply: $\frac{11}{3}(-9a)$.

Answer

$-33a$

Now that we know how to multiply fractions, we are almost ready to divide. Before we can do that, we need some vocabulary. The **reciprocal** of a fraction is found by inverting the fraction, placing the numerator in the denominator, and the denominator in the numerator. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. Since 4 is written in fraction form as $\frac{4}{1}$, the reciprocal of 4 is $\frac{1}{4}$.

To divide fractions, we multiply the first fraction by the reciprocal of the second.

FRACTION DIVISION

If a, b, c, and d are numbers where $b \neq 0, c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide fractions, we multiply the first fraction by the **reciprocal** of the second.

We need to say $b \neq 0$, $c \neq 0$, and $d \neq 0$, to be sure we don't divide by zero!

Example 3

Find the quotient: $-\frac{7}{18} \div (-\frac{14}{27})$.

Solution

	$-\frac{7}{18} \div (-\frac{14}{27})$
To divide, multiply the first fraction by the reciprocal of the second.	$-\frac{7}{18}\left(-\frac{27}{14}\right)$
Determine the sign of the product, and then multiply.	$\frac{7 \cdot 27}{18 \cdot 14}$
Rewrite showing common factors.	$\frac{7 \cdot 9 \cdot 3}{9 \cdot 2 \cdot 7 \cdot 2}$
Remove common factors.	$\frac{3}{2 \cdot 2}$
Simplify.	$\frac{3}{4}$

Try It 3

Divide: $-\frac{7}{27} \div (-\frac{35}{36})$.

Answer

$\frac{4}{15}$

The numerators or denominators of some fractions contain fractions themselves. A fraction in which the numerator or the denominator is a fraction is called a **complex fraction**.

Definition: COMPLEX FRACTION

A **complex fraction** is a fraction in which the numerator or the denominator contains a fraction.

Some examples of complex fractions are:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}$$

To simplify a complex fraction, remember that the fraction bar means division. For example, the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ means $\frac{3}{4} \div \frac{5}{8}$.

Example 4

Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Solution

$$\frac{\frac{x}{2}}{\frac{xy}{6}}$$

Rewrite as division.	$\frac{x}{2} \div \frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2} \cdot \frac{6}{xy}$
Multiply.	$\frac{x \cdot 6}{2 \cdot xy}$
Look for common factors.	$\frac{x \cdot 3 \cdot 2}{2 \cdot x \cdot y}$
Divide common factors and simplify.	$\frac{3}{y}$

? Try It 4

Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Answer

$\frac{3}{4b}$

Add and Subtract Fractions

When we multiply fractions, we multiply the numerators and denominators directly. However, to add or subtract fractions, they must have a common denominator.

FRACTION ADDITION AND SUBTRACTION

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \text{ and } \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

To add or subtract fractions, add or subtract the numerators and place the result over the common denominator.

The **least common denominator** (LCD) of two fractions is the smallest number that can be used as a common denominator of the fractions. The LCD of the two fractions is the least common multiple (LCM) of their denominators.

LEAST COMMON DENOMINATOR

The **least common denominator** (LCD) of two fractions is the least common multiple (LCM) of their denominators.

After we find the least common denominator of two fractions, we convert the fractions to equivalent fractions with the LCD. Putting these steps together allows us to add and subtract fractions because their denominators will be the same!

✓ Example 5

Add: $\frac{7}{12} + \frac{5}{18}$.

Solution

Step 1. Do they have a common denominator? • No—rewrite each fraction with the LCD (least common denominator).	No. Find the LCD of 12, 18. We multiply the numerator and denominator of each fraction by the factor needed to get the denominator to be 36. Do not simplify the equivalent fractions! If you do, you'll get back to the original fractions and lose the common denominator!	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $LCD = 2 \cdot 2 \cdot 3 \cdot 3$ $LCD = 36$ LCD is 36. $\frac{7}{12} + \frac{5}{18}$ $\frac{7 \cdot 3}{12 \cdot 3} + \frac{5 \cdot 2}{18 \cdot 2}$ $\frac{21}{36} + \frac{10}{36}$
Step 2. Add or subtract the fractions.	Add.	$\frac{31}{36}$
Step 3. Simplify, if possible.	Since 31 is prime, its only factors are 1 and 31. Since 31 does not go into 36, the answer is simplified.	

? Try It 5

Add: $\frac{13}{15} + \frac{17}{20}$.

Answer

$\frac{103}{60}$

ADD OR SUBTRACT FRACTIONS.

1. Do they have a common denominator?
   - Yes—go to step 2.
   - No—rewrite each fraction with the LCD (least common denominator).
     - Find the LCD.
     - Change each fraction into an equivalent fraction with the LCD as its denominator.
2. Add or subtract the fractions.
3. Simplify, if possible.

We now have all four operations for fractions. The table below summarizes fraction operations.

Fraction Multiplication	Fraction Division
$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
Multiply the numerators and multiply the denominators	Multiply the first fraction by the reciprocal of the second.
Fraction Addition	**Fraction Subtraction**
$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$
Add the numerators and place the sum over the common denominator.	Subtract the numerators and place the difference over the common denominator.
To multiply or divide fractions, an LCD is NOT needed. To add or subtract fractions, an LCD is needed.	

When starting an exercise, always identify the operation and then recall the methods needed for that operation.

Example 6

Simplify:

a. $\frac{5x}{6} - \frac{3}{10}$

b. $\frac{5x}{6} \cdot \frac{3}{10}$.

Solution

First ask, "What is the operation?" Identifying the operation will determine whether or not we need a common denominator. Remember, we need a common denominator to add or subtract, but not to multiply or divide.

a.

Erroneous nesting of equation structures

b.

What is the operation? Multiplication.	$\frac{5x}{6} \cdot \frac{3}{10}$
To multiply fractions,multiply the numerators and multiply the denominators.	$\frac{5x \cdot 3}{6 \cdot 10}$
Rewrite, showing common factors.	
Remove common factors.	$\frac{\cancel{5}x \cdot \cancel{3}}{2 \cdot \cancel{3} \cdot 2 \cdot \cancel{5}}$
Simplify.	$\frac{x}{4}$

Notice, we needed an LCD to add $\frac{5x}{6} - \frac{3}{10}$, but not to multiply $\frac{5x}{6} \cdot \frac{3}{10}$.

Try It 6

Simplify:

a. $\frac{3a}{4} - \frac{8}{9}$

b. $\frac{3a}{4} \cdot \frac{8}{9}$

Answer

a. $\frac{27a - 32}{36}$

b. $\frac{2a}{3}$

Use the Order of Operations to Simplify Fractions

The fraction bar in a fraction acts as a grouping symbol. The order of operations then tells us to simplify the numerator and then the denominator. Then we divide.

SIMPLIFY AN EXPRESSION WITH A FRACTION BAR.

1. Simplify the expression in the numerator. Simplify the expression in the denominator.
2. Simplify the fraction.

Where does the negative sign go in a fraction? Usually, the negative sign is in front of the fraction, but you will sometimes see a fraction with a negative numerator, or sometimes with a negative denominator. Remember that fractions represent division. When the numerator and denominator have different signs, the quotient is negative.

$$\frac{-1}{3} = -\frac{1}{3} \qquad \frac{\text{negative}}{\text{positive}} = \text{negative}$$

$$\frac{1}{-3} = -\frac{1}{3} \qquad \frac{\text{positive}}{\text{negative}} = \text{negative}$$

PLACEMENT OF NEGATIVE SIGN IN A FRACTION

For any positive numbers a and b,

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Example 7

Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$.

Solution

The fraction bar acts like a grouping symbol. So, completely simplify the numerator and the denominator separately.

$$\frac{4(-3)+6(-2)}{-3(2)-2}$$

Multiply. $\frac{-12+(-12)}{-6-2}$

Simplify. $\frac{-24}{-8}$

Divide. 3

Try It 7

Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Answer

4

Now we'll look at complex fractions where the numerator or denominator contains an expression that can be simplified. Therefore, we must first simplify the numerator and denominator separately, following the order of operations. Then we divide the numerator by the denominator, as the fraction bar means division.

Example 8

Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution

Step 1. Simplify the numerator.	$\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$ $\frac{\frac{1}{4}}{4+3^2}$
Step 2. Simplify the denominator.	$\frac{\frac{1}{4}}{4+9}$ $\frac{\frac{1}{4}}{13}$
Step 3. Divide the numerator by the denominator. Simplify if possible	$\frac{1}{4} \div \frac{13}{1}$ $\frac{1}{4} \cdot \frac{1}{13}$ $\frac{1}{52}$

Try It 8

Simplify: $\frac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Answer

272

SIMPLIFY COMPLEX FRACTIONS.

1. Simplify the numerator.
2. Simplify the denominator.
3. Divide the numerator by the denominator. Simplify if possible.

Example 9

Simplify: $\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution

It may help to put parentheses around the numerator and the denominator.

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$$

Simplify the numerator ($LCD = 6$) and simplify the denominator ($LCD = 12$). $\frac{\left(\frac{3}{6}+\frac{4}{6}\right)}{\left(\frac{9}{12}-\frac{2}{12}\right)}$

Simplify. $\frac{\left(\frac{7}{6}\right)}{\left(\frac{7}{12}\right)}$

Divide the numerator by the denominator. $\frac{7}{6} \div \frac{7}{12}$

Simplify. $\frac{7}{6} \cdot \frac{12}{7}$

Divide out common factors. $\frac{\cancel{7} \cdot \cancel{6} \cdot 2}{\cancel{6} \cdot \cancel{7} \cdot 1}$

Simplify. 2

? Try It 9

Simplify: $\frac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Answer

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but can now evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the variable's value into the expression and then simplify.

✓ Example 10

Evaluate $2x^2y$ when $x = \frac{1}{4}$ and $y = -\frac{2}{3}$.

Solution

	$2x^2y$
Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply; divide out the common factors. Notice we write 16 as $2 \cdot 2 \cdot 4$ to make it easy to remove common factors.	$-\frac{2 \cdot 1 \cdot 2}{2 \cdot 2 \cdot 4 \cdot 3}$
Simplify.	$-\frac{1}{12}$

Try It 10

Evaluate $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$.

Answer

$-\frac{1}{2}$

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Section 1.5P: Practice

Practice Makes Progress

Simplify Fractions

In the following exercises, simplify.

Exercise 1

$-\frac{108}{63}$

Exercise 2

$\frac{14x^2}{21y}$

Exercise 3

$-\frac{210a^2}{110b^2}$

Multiply and Divide Fractions

In the following exercises, perform the indicated operation.

Exercise 4

$-\frac{3}{4}\left(-\frac{4}{9}\right)$

Exercise 5

$\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$

Exercise 6

$\frac{3}{7}\cdot 21n$

Exercise 7

$\frac{3}{4}\div\frac{x}{11}$

Exercise 8

$\frac{8u}{15}\div\frac{12v}{25}$

Exercise 9

$\frac{3}{4}\div(-12)$

In the following exercises, simplify.

Exercise 10

$$-\frac{\left(\frac{8}{21}\right)}{\left(\frac{12}{35}\right)}$$

Exercise 11

$$-\frac{\left(\frac{4}{5}\right)}{2}$$

Exercise 12

$$\frac{\left(\frac{m}{3}\right)}{\left(\frac{n}{2}\right)}$$

Add and Subtract Fractions

In the following exercises, add or subtract.

Exercise 13

$$\frac{7}{12}-\frac{9}{16}$$

Exercise 14

$$-\frac{39}{56}-\frac{22}{35}$$

Exercise 15

$$-\frac{2}{3}-\left(-\frac{3}{4}\right)$$

Exercise 16

$$\frac{x}{3}+\frac{1}{4}$$

Exercise 17

$$-\frac{4x}{9}-\frac{5}{6}$$

Exercise 18

$$-\frac{4k}{9}\cdot\frac{5}{6}$$

Exercise 19

$$-\frac{5a}{3}+\left(-\frac{10}{6}\right)$$

? Exercise 20

$-\frac{5a}{3} \div \left(-\frac{10}{6}\right)$

Use the Order of Operations to Simplify Fractions

In the following exercises, simplify.

? Exercise 21

$\frac{5 \cdot 6 - 3 \cdot 4}{4 \cdot 5 - 2 \cdot 3}$

? Exercise 22

$\frac{5^2 - 3^2}{3 - 5}$

? Exercise 23

$\frac{9(8-2) - 3(15-7)}{6(7-1) - 3(17-9)}$

? Exercise 24

$\frac{2^3 + 4^2}{\left(\frac{2}{3}\right)^2}$

? Exercise 25

$\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

? Exercise 26

$\frac{2}{\frac{1}{3} + \frac{1}{5}}$

? Exercise 27

$\frac{\frac{7}{8} - \frac{2}{3}}{\frac{1}{2} + \frac{3}{8}}$

Mixed Practice

In the following exercises, simplify.

? Exercise 28

$-\frac{3}{8} \div \left(-\frac{3}{10}\right)$

Exercise 29

$-\frac{7}{15}-\frac{y}{4}$

Exercise 30

$\frac{11}{12a}\cdot\frac{9a}{16}$

Exercise 31

$\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$

Exercise 32

$1-\frac{3}{5}\div\frac{1}{10}$

Exercise 33

$\frac{3}{8}-\frac{1}{6}+\frac{3}{4}$

Exercise 34

$\frac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$

Exercise 35

$\left(\frac{5}{9}+\frac{1}{6}\right)\div\left(\frac{2}{3}-\frac{1}{2}\right)$

Evaluate Variable Expressions with Fractions

In the following exercises, evaluate.

Exercise 36

$2x^2y^3$ when $x=-\frac{2}{3}$ and $y=-\frac{1}{2}$

Exercise 37

$\frac{a+b}{a-b}$ when $a=-3$ and $b=8$

Writing Exercises

Exercise 38

Why do you need a common denominator to add or subtract fractions? Explain.

Exercise 39

Explain how you find the reciprocal of a fraction.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply and divide fractions.			
add and subtract fractions.			
use the order of operations to simplify fractions.			
evaluate variable expressions with fractions.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 1.6: Dimensional Analysis

Be Prepared

We will rely heavily on these skills throughout this section.

- Multiply $4.29(1000)$
- Simplify $\frac{30}{54}$
- Multiply $\frac{7}{15} \cdot \frac{25}{28}$

Learning Objectives

Motivating Problem

A recipe from France calls for 500 milliliters of milk, but your measuring cup is in ounces. How can you figure out how much milk to use without guessing—or making the cake too soggy?

Fun Fact

The metric system was first introduced in France during the French Revolution as a way to unify the country's many measurement systems. Today, it's used by every country in the world *except* three: the U.S., Myanmar, and Liberia!

The Goal

In this section, we learn how to convert between different units of measurement using multiplication, logic, and a clear visual structure. We'll explore both the U.S. and metric systems and practice solving real-world problems, such as converting miles to kilometers, minutes to seconds, or gallons to ounces. Finally, we'll see how dimensional analysis can help us in science, travel, and even baking!

Make Unit Conversions in the U.S. System

There are two systems of measurement commonly used around the world. Most countries use the metric system. The U.S. uses a different system of measurement, usually called the **U.S. system**. We will look at the U.S. system first.

The U.S. measurement system uses units of inch, foot, yard, and mile to measure length and pound and ton to measure weight. For capacity, the units used are cup, pint, quart, and gallons. Both the U.S. system and the metric system measure time in seconds, minutes, and hours.

The table below shows the equivalencies of measurements. It also shows, in parentheses, the common abbreviations for each measurement.

U.S. System of Measurement

Length	1 foot (ft.)	=	12 inches (in.)
	1 yard (yd.)	=	3 feet (ft.)
	1 mile (mi.)	=	5280 feet (ft.)
Volume	3 teaspoons (t)	=	1 tablespoon (T)
	16 tablespoons (T)	=	1 cup (C)
	1 cup (C)	=	8 fluid ounces (fl.oz.)
	1 pint (pt.)	=	2 cups (C)
	1 quart (qt.)	=	2 pints (pt.)
	1 gallon (gal)	=	4 quarts (qt.)
Weight	1 pound (lb.)	=	16 ounces (oz.)
	1 ton	=	2000 pounds (lb.)
Time	1 minute (min)	=	60 seconds (sec)
	1 hour (hr)	=	60 minutes (min)
	1 day	=	24 hours (hr)
	1 week (wk)	=	7 days
	1 year (yr)	=	365 days

In many real-life applications, we need to convert between units of measurement, such as feet and yards, minutes and seconds, quarts and gallons, etc. We will use the identity property of multiplication to make these conversions. We'll restate the identity property of multiplication here for easy reference.

IDENTITY PROPERTY OF MULTIPLICATION

Because for any real number a it is true that $a \cdot 1 = a$ and that $1 \cdot a = a$, we call **1** the **multiplicative identity**.

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to change inches to feet. We know that 1 foot is equal to 12 inches, so we will write 1 as the fraction $\frac{1 \text{ foot}}{12 \text{ inches}}$. When we multiply by this fraction we do not change the value, but just change the units.

But $\frac{12 \text{ inches}}{1 \text{ foot}}$ also equals 1. How do we decide whether to multiply by $\frac{1 \text{ foot}}{12 \text{ inches}}$ or $\frac{12 \text{ inches}}{1 \text{ foot}}$? We choose the fraction that will make the units we want to convert *from* divide out. Treat the unit words like **factors** and "divide out" common units like we do common factors. If we want to convert 66 inches to feet, which multiplication will eliminate the inches?

$$66 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \quad \text{or} \quad 66 \text{ inches} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$$

The first form works since $66 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$.

The inches divide out and leave only feet. The second form has no units that will divide out and so will not help us.

Example 1

Mary Anne is 66 inches tall. Convert her height into feet.

Solution

Step 1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.	Multiply 66 inches by 1, writing 1 as a fraction relating inches and feet. We need inches in the denominator so that the inches will divide out!	$66 \text{ inches} \cdot 1$ $66 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$
Step 2. Multiply.	Think of 66 inches as $\frac{66 \text{ inches}}{1}$.	$\frac{66 \text{ inches} \cdot 1 \text{ foot}}{12 \text{ inches}}$
Step 3. Simplify the fraction.	Notice: inches divide out.	$\frac{66 \text{ inches} \cdot 1 \text{ foot}}{12 \text{ inches}}$ $\frac{66 \text{ feet}}{12}$
Step 4. Simplify.	Divide 66 by 12.	5.5 feet

Try It 1

Rene bought a hose that is 18 yards long. Convert the length to feet.

Answer

54 feet

Example 2

Ndula, an elephant at the San Diego Safari Park, weighs almost 3.2 tons. Convert her weight to pounds.

Solution

We will convert 3.2 tons into pounds. We will use the identity property of multiplication, writing 1 as the fraction $\frac{2000 \text{ pounds}}{1 \text{ ton}}$.

	3.2 tons
Multiply the measurement to be converted, by 1.	$3.2 \text{ tons} \cdot 1$
Write 1 as a fraction relating tons and pounds.	$3.2 \text{ tons} \cdot \frac{2000 \text{ pounds}}{1 \text{ ton}}$
Simplify.	$\frac{3.2 \text{ tons} \cdot 2{,}000 \text{ pounds}}{1 \text{ ton}}$
Multiply.	6400 pounds
	Ndula weighs almost 6400 pounds.

Try It 2

Arnold's SUV weighs about 4.3 tons. Convert the weight to pounds.

Answer

8600 pounds

Sometimes, to convert from one unit to another, we may need to use several other units in between, so we must multiply several fractions.

Example 3

Juliet is going with her family to their summer home. She will be away from her boyfriend for 9 weeks. Convert the time to minutes.

Solution

To convert weeks into minutes, we will convert weeks into days, days into hours, and then hours into minutes. To do this, we will multiply by the conversion factors of 1.

	9 weeks
Write 1 as $\frac{7 \text{ days}}{1 \text{ week}}$, and $\frac{60 \text{ minutes}}{1 \text{ hour}}$.	$\frac{9 \text{ wk}}{1} \cdot \frac{7 \text{ days}}{1 \text{ wk}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$
Divide out the common units.	9 weeks Write 1 as $\frac{7 \text{ days}}{1 \text{ week}}, \frac{24 \text{ hours}}{1 \text{ day}}$, and $\frac{60 \text{ minutes}}{1 \text{ hour}}$. $\frac{9 \text{ wk}}{1} \cdot \frac{7 \text{ days}}{1 \text{ wk}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$ Divide out the common units. $\frac{9 \cancel{\text{wk}}}{1} \cdot \frac{7 \cancel{\text{days}}}{1 \cancel{\text{wk}}} \cdot \frac{24 \cancel{\text{hr}}}{1 \cancel{\text{day}}} \cdot \frac{60 \text{ min}}{1 \cancel{\text{hr}}}$ Multiply. $\frac{9 \cdot 7 \cdot 24 \cdot 60 \text{ min}}{1 \cdot 1 \cdot 1 \cdot 1}$ Multiply. 90,720 min
Multiply.	$\frac{9 \cdot 7 \cdot 24 \cdot 60 \text{ min}}{1 \cdot 1 \cdot 1 \cdot 1}$
Multiply.	90,720 min

Juliet and her boyfriend will be apart for 90,720 minutes (although it may seem like an eternity!).

Try It 3

The astronauts of Expedition 28 on the International Space Station spend 15 weeks in space. Convert the time to minutes.

Answer

151,200 minutes

Example 4

How many ounces are in 1 gallon?

Solution

We will convert gallons to ounces by multiplying by several conversion factors.

	1 gallon
Multiply the measurement to be converted by 1.	$\frac{1 \text{ gallon}}{1} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} \cdot \frac{8 \text{ ounces}}{1 \text{ cup}}$
Use conversion factors to get to the right unit. Simplify.	1 gallon Multiply the measurement to be converted by 1. $\frac{1 \text{ gallon}}{1} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} \cdot \frac{8 \text{ ounces}}{1 \text{ cup}}$ Use conversion factors to get to the right unit. $\frac{1 \text{ gallon}}{1} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} \cdot \frac{8 \text{ ounces}}{1 \text{ cup}}$ Simplify. Multiply. $\frac{1 \cdot 4 \cdot 2 \cdot 2 \cdot 8 \text{ ounces}}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}$ Simplify. 128 ounces There are 128 ounces in a gallon.
Multiply.	$\frac{1 \cdot 4 \cdot 2 \cdot 2 \cdot 8 \text{ ounces}}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}$
Simplify.	128 ounces

Try It 4

How many teaspoons are in 1 cup?

Answer

48 teaspoons

Use Mixed Units of Measurement in the U.S. System

We often use mixed units of measurement in everyday situations. Suppose Joe is 5 feet 10 inches tall, stays at work for 7 hours and 45 minutes, and then eats a 1-pound 2-ounce steak for dinner—all these measurements have mixed units.

Performing arithmetic operations on measurements with mixed units of measure requires care. Be sure to add or subtract like units!

Example 5

Seymour bought three steaks for a barbecue. Their weights were 14 ounces, 1 pound 2 ounces, and 1 pound 6 ounces. How many total pounds of steak did he buy?

Solution

We will add the weights of the steaks to find the total weight of the steaks.

Add the ounces. Then add the pounds.	14 ounces 1 pound 2 ounces + 1 pound 6 ounces 2 pounds 22 ounces
Convert 22 ounces to pounds and ounces.	2 pounds + 1 pound, 6 ounces 3 pounds, 6 ounces
Add the pounds.	Seymour bought 3 pounds 6 ounces of steak.

Try It 5

Stan cut two pieces of crown molding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the molding?

Answer

21 ft. 6 in.

Example 6

Anthony bought four planks of wood that were each 6 feet 4 inches long. What is the total length of the wood he purchased?

Solution

We will multiply the length of one plank to find the total length.

Multiply the inches and then the feet.	6 feet 4 inches × 4 24 feet 16 inches
Convert the 16 inches to feet. Add the feet.	24 feet + 1 foot 4 inches 25 feet 4 inches
	Anthony bought 25 feet and 4 inches of wood.

Try It 6

Joellen wants to double a solution of 5 gallons 3 quarts. How many gallons of solution will she have in all?

Answer

11 gallons 2 qt.

Make Unit Conversions in the Metric System

In the **metric system**, units are related by powers of 10. The root words of their names reflect this relation. For example, the basic unit for measuring length is a meter. One kilometer is 1,000 meters; the prefix *kilo* means *thousand*. One centimeter is $\frac{1}{100}$ of a meter, just like one cent is $\frac{1}{100}$ of one dollar.

The table below shows the equivalencies of measurements in the metric system. The common abbreviations for each measurement are given in parentheses.

Metric System of Measurement		
Length	Mass	Capacity
1 kilometer (km) = 1,000 m	1 kilogram (kg) = 1,000 g	1 kiloliter (kL) = 1,000 L
1 hectometer (hm) = 100 m	1 hectogram (hg) = 100 g	1 hectoliter (hL) = 100 L
1 dekameter (dam) = 10 m	1 dekagram (dag) = 10 g	1 dekaliter (daL) = 10 L
1 meter (m) = 1 m	1 gram (g) = 1 g	1 liter (L) = 1 L
1 decimeter (dm) = 0.1 m	1 decigram (dg) = 0.1 g	1 deciliter (dL) = 0.1 L
1 centimeter (cm) = 0.01 m	1 centigram (cg) = 0.01 g	1 centiliter (cL) = 0.01 L
1 millimeter (mm) = 0.001 m	1 milligram (mg) = 0.001 g	1 milliliter (mL) = 0.001 L
1 meter = 100 centimeters	1 gram = 100 centigrams	1 liter = 100 centiliters
1 meter = 1,000 millimeters	1 gram = 1,000 milligrams	1 liter = 1,000 milliliters

To convert in the metric system, we will use the same technique we used in the US system. Using the identity property of multiplication, we will multiply by a conversion factor of one to get to the correct units.

Have you ever run a 5K or 10K race? The length of those races is measured in kilometers. The metric system is commonly used in the United States when discussing race length.

Example 7

Nick ran a 10K race. How many meters did he run?

Solution

We will convert kilometers to meters using the identity property of multiplication.

	10 kilometers
Multiply the measurement to be converted by 1.	$10 \text{ kilometers} \cdot 1$
Write 1 as a fraction relating kilometers and meters.	$10 \text{ kilometers} \cdot \frac{1{,}000 \text{ meters}}{1 \text{ kilometers}}$
Simplify.	$\frac{10 \cancel{\text{kilometers}} \cdot 1{,}000 \text{ m}}{1 \cancel{\text{kilometers}}}$
Multiply.	10,000 meters
	Nick ran 10,000 meters.

Try It 7

Herman bought a rug 2.5 meters in length. How many centimeters is the length?

Answer

250 centimeters

Example 8

Eleanor's newborn baby weighed 3,200 grams. How many kilograms did the baby weigh?

Solution

We will convert grams into kilograms.

	$3{,}200 \text{ grams}$
Multiply the measurement to be converted by 1.	$3{,}200 \text{ grams} \cdot 1$
Write 1 as a function relating kilograms and grams.	$3{,}200 \text{ grams} \cdot \frac{1 \text{ kg}}{1{,}000 \text{ grams}}$
Simplify.	$3{,}200 \text{ grams} \cdot \frac{1 \text{ kg}}{1{,}000 \text{ grams}}$
Multiply.	$\frac{3{,}200 \text{ kilograms}}{1{,}000}$
Divide.	3.2 kilograms The baby weighed 3.2 kilograms.

Try It 8

Anderson received a package that was marked 4,500 grams. How many kilograms did this package weigh?

Answer

4.5 kilograms

As you become familiar with the metric system, you may see a pattern. Since the system is based on multiples of ten, the calculations involve multiplying by multiples of ten. We have learned how to simplify these calculations by just moving the decimal.

To multiply by 10, 100, or 1,000, we move the decimal to the right one, two, or three places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left one, two, or three places, respectively.

We can apply this pattern when we make measurement conversions in the metric system. In the previous example, we changed 3,200 grams to kilograms by multiplying by $\frac{1}{1000}$ (or 0.001). This is the same as moving the decimal three places to the left.

$$3{,}200 \cdot \frac{1}{1{,}000} \qquad 3{,}200.$$
$$3.2 \qquad 3.2$$

Example 9

Convert

1. 350 L to kiloliters
2. 4.1 L to milliliters.

Solution

1. We will convert liters to kiloliters using the fact that 1 kiloliter = 1,000 liters and 1 kiloliter = 1,000 liters.

	350 L
Multiply by 1, writing 1 as a fraction relating liters to kiloliters.	$350\ \text{L}\cdot\frac{1\ \text{kL}}{1000\ \text{L}}$
Simplify.	$350\ \text{/L}\cdot\frac{1\ \text{kL}}{1000\ \text{/L}}$
Multiply by 1, writing 1 as a fraction relating liters to kiloliters. Simplify. Move the decimal 3 units to the left. (350.)	350 L $350\ \text{/L}\cdot\frac{1\ \text{kL}}{1{,}000\ \text{/L}}$ 0.35 kL

2. We will convert liters to milliliters using the fact that 1 liter = 1,000 milliliters and 1 liter = 1,000 milliliters.

	4.1 L
Multiply by 1, writing 1 as a fraction relating liters to milliliters.	$4.1\ \text{L}\cdot\frac{1{,}000\ \text{mL}}{1\ \text{L}}$
Simplify.	$4.1\ \text{/L}\cdot\frac{1{,}000\ \text{mL}}{1\ \text{/L}}$
Move the decimal 3 units to the right.	4,100 mL
	4,100 mL

Try It 9

Convert:

1. 350 hL to liters
2. 4.1 L to centiliters

Answer

1. 35,000 liters
2. 410 centiliters

Use Mixed Units of Measurement in the Metric System

Performing arithmetic operations on measurements with mixed units of measure in the metric system requires the same care we used in the US system. But it may be easier because of the relation of the units to the powers of 10. Make sure to add or subtract like units.

Example 10

Ryland is 1.6 meters tall. His younger brother is 85 centimeters tall. How much taller is Ryland than his younger brother?

Solution

We can convert both measurements to either centimeters or meters. Since meters is the larger unit, we will subtract the lengths in meters. We convert 85 centimeters to meters by moving the decimal 2 places to the left.

Write the 85 centimeters as meters.	1.60m
	-0.85m
	0.75m

Ryland is 0.75 m taller than his brother.

Try It 10

The fence around Hank's yard is 2 meters high. Hank is 96 centimeters tall. How much shorter is Hank than the fence? Write the answer in meters.

Answer

1.04 meters

Example 11

Dena's lentil soup recipe calls for 150 milliliters of olive oil. If she wants to triple the recipe, how many liters of olive oil will she need?

Solution

We will find the amount of olive oil in milliliters then convert to liters.

	Triple 150 mL
Translate to algebra.	$3 \cdot 150\ \text{mL}$
Multiply.	$450\ \text{mL}$
Convert to liters.	$450 \cdot \frac{0.001\ \text{L}}{1\ \text{ml}}$
Simplify.	$0.45\ \text{L}$
	Dena needs 0.45 liters of olive oil.

Try It 11

To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Answer

2.4 kilograms

Convert Between the U.S. and the Metric Systems of Measurement

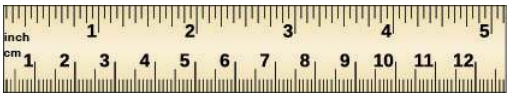
Many measurements in the United States are made in metric units. Our soda may come in 2-liter bottles, our calcium may come in 500-mg capsules, and we may run a 5K race. To work easily in both systems, we need to be able to convert between the two systems.

The table below shows some of the most common conversions.

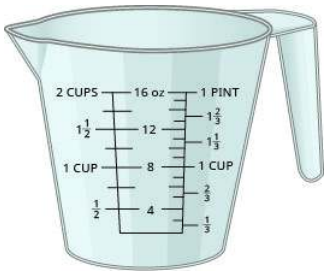
Conversion Factors Between U.S. and Metric Systems

Length	Mass	Capacity
1 in. = 2.54 cm	1 lb. = 0.45 kg	1 qt. = 0.95 L
1 ft. = 0.305 m	1 oz. = 28 g	1 fl. oz. = 30 ml
1 yd. = 0.914 m	1 kg = 2.2 lb	1 L = 1.06 lb
1 mi. = 1.61 km		
1 m = 3.28 ft		

The figure below shows how inches and centimeters are related on a ruler.



The figure below shows the ounce and milliliter markings on a measuring cup.



The figure below shows how pounds and kilograms are marked on a bathroom scale.



We convert between the systems just as we do within them—by multiplying by unit conversion factors.

Example 12

Lee's water bottle holds 500 mL of water. How many ounces are in the bottle? Round to the nearest tenth of an ounce.

Solution

	500 mL
Multiplying by a unit conversion factor relating mL and ounces	$500 \text{ milliliters} \cdot \frac{1 \text{ ounce}}{30 \text{ milliliters}}$
Simplify.	$\frac{50 \text{ ounce}}{30}$
Divide.	16.7 ounces
	The water bottle has 16.7 ounces

Try It 12

How many liters are in 4 quarts of milk?

Answer

3.8 liters

Example 13

Soleil was on a road trip and saw a sign that said the next rest stop was in 100 kilometers. How many miles until the next rest stop?

Solution

	100 kilometers
Multiplying by a unit conversion factor relating km and mi.	$100 \text{ kilometers} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}}$
Simplify.	$\frac{100 \text{ miles}}{1.61}$
Divide.	62 miles
	Soleil will travel 62 miles.

Try It 13

The height of Mount Kilimanjaro is 5,895 meters. Convert the height to feet.

Answer

19,335.6 feet

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Section 1.6P: Practice

Practice Makes Progress

Make Unit Conversions in the U.S. System

In the following exercises, convert the units.

Exercise 1

A park bench is 6 feet long. Convert the length to inches.

Exercise 2

A football field is 160 feet wide. Convert the width to yards.

Exercise 3

On a baseball diamond, the distance from home plate to first base is 30 yards. Convert the distance to feet.

Exercise 4

A killer whale weighs 4.6 tons. Convert the weight to pounds.

Exercise 5

An empty bus weighs 35,000 pounds. Convert the weight to tons.

Exercise 6

Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.

Exercise 7

JJ's cat, Posy, weighs 14 pounds. Convert her weight to ounces.

Exercise 8

Crista will serve 20 cups of juice at her son's party. Convert the volume to gallons.

Exercise 9

Jon is 6 feet 4 inches tall. Convert his height to inches.

Exercise 10

The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days.

Exercise 11

Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.

Use Mixed Units of Measurement in the U.S. System

In the following exercises, solve.

Exercise 12

Eli caught three fish. The weights of the fish were 2 pounds 4 ounces, 1 pound 11 ounces, and 4 pounds 14 ounces. What was the total weight of the three fish?

Exercise 13

One day Anya kept track of the number of minutes she spent driving. She recorded 45, 10, 8, 65, 20, and 35. How many hours did Anya spend driving?

Exercise 14

Renee attached a 6-foot 6-inch extension cord to her computer's 3-foot 8-inch power cord. What was the total length of the cords?

Exercise 15

Leilani wants to make 8 placemats. For each placemat, she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

Exercise 16

Ghalib ran 5 kilometers. Convert the length to meters.

Exercise 17

Estrella is 1.55 meters tall. Convert her height to centimeters.

Exercise 18

Mount Whitney is 3,072 meters tall. Convert the height to kilometers.

Exercise 19

June's multivitamin contains 1,500 milligrams of calcium. Convert this to grams.

Exercise 20

One stick of butter contains 91.6 grams of fat. Convert this to milligrams.

Exercise 21

A bottle of wine contains 750 milliliters. Convert this to liters.

Use Mixed Units of Measurement in the Metric System

In the following exercises, solve.

? Exercise 22

Matthias is 1.8 meters tall. His son is 89 centimeters tall. How much taller is Matthias than his son?

? Exercise 23

A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, of the weights of a duck and a dove?

? Exercise 24

Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?

? Exercise 25

Jonas drinks 200 milliliters of water 8 times a day. How many liters of water does Jonas drink in a day?

Convert Between the U.S. and the Metric Systems of Measurement

In the following exercises, make the unit conversions. Round to the nearest tenth.

? Exercise 26

Bill is 75 inches tall. Convert his height to centimeters.

? Exercise 27

Marcus passed a football 24 yards. Convert the pass length to meters

? Exercise 28

A 5K run is 5 kilometers long. Convert this length to miles.

? Exercise 29

Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.

? Exercise 30

Ozzie put 14 gallons of gas in his truck. Convert the volume to liters.

Everyday Math

? Exercise 31

Nutrition Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?

Writing Exercises

? Exercise 32

Would you prefer to run 10 kilometers or 10 miles? Justify your response.

Exercise 33

Suzanne measured the length of her bed in inches and found the equivalent measurement in feet is 6.2 feet. Suzanne thinks this is the same as 6 ft 2 in, but she is unsure. Help Suzanne figure out how to write 6.2 feet as a mixed measurement in terms of feet and inches.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
define U.S. units of measurement and convert from one unit to another.			
use U.S units of measurement.			
define metric units of measurement and convert from one unit to another.			
use metric units of measurement.			
convert between the U.S. and the metric system of measurement.			
convert between Fahrenheit and Celsius temperatures.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 2: Relating Unknowns

Now that we've learned how to write and simplify expressions, we'll shift our focus to solving equations and inequalities. We'll explore techniques for isolating variables, including working with fractions, decimals, and multi-step problems. We'll also learn how to rearrange formulas and apply our algebra skills to everyday situations like budgeting and travel. By building these connections, we'll deepen our understanding of how quantities relate to one another.

Section 2.1: Solving Equations - Building Blocks

Section 2.1P: Practice

Section 2.2: Solving Equations - Refining the Process

Section 2.2P: Practice

Section 2.3: Solving Equations - Advanced Techniques and Formulas

Section 2.3P: Practice

Section 2.4: Linear Inequalities

Section 2.4P: Practice

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Section 2.1: Solving Equations - Building Blocks

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $3x + 5$ when $x = 2$
- Simplify $4(4n + 1) - 15n$
- True of False? $2x = 10$ when $x = 6$
- Simplify $-7(\frac{1}{-7})$

Learning Objectives

Motivating Problem

Imagine you're splitting a bill at a restaurant. You paid $18 more than your friend, and together, you spent $54. How can you determine the amount each person paid?

This kind of everyday problem leads naturally into the world of solving equations. With a little math, we can find those unknowns—and maybe settle the bill fairly!

Fun Fact

The equals sign (=) was invented in 1557 by Welsh mathematician **Robert Recorde**, who got tired of writing "is equal to" over and over again. He chose two parallel lines because "no two things can be more equal." Efficient *and* poetic!

The Goal

This section helps students learn how to isolate variables and solve equations step-by-step using the rules of algebra. From one-step problems to those that involve combining like terms or clearing fractions, students will build confidence in solving for unknowns and checking their solutions.

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose of solving an equation is to find the value or values of the variable that make each side of the equation the same, so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Definition: Solution of an Equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

How to determine whether a number is a *solution* to an equation...

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true (the left side is equal to the right side)
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Example 1

Determine whether $x = \frac{3}{2}$ is a solution of $4x - 2 = 2x + 1$.

Solution

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

	$4x-2=2x+1$
Substitute $\frac{3}{2}$ for x	$4\left(\frac{3}{2}\right)-2\stackrel{?}{=}2\left(\frac{3}{2}\right)+1$
Multiply.	$6-2\stackrel{?}{=}3+1$
Subtract.	$4=4\checkmark$

Since $x=\frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x-2=2x+1$.

Try It 1

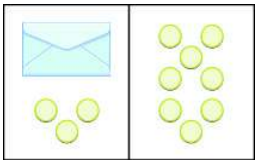
Is $y=\frac{4}{3}$ a solution of $9y+2=6y+3$?

Answer

no

Solve Equations Using the Subtraction and Addition Properties of Equality

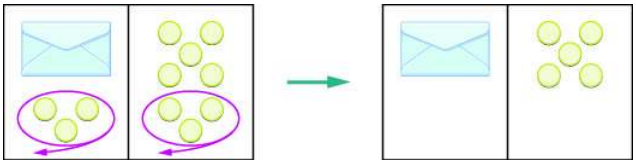
We will use a model to clarify the process of solving an equation. An envelope represents the variable, since its contents are unknown, and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in the figure below. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?



The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

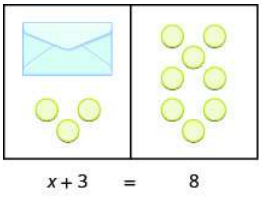
What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." Below is an illustration of this process.



The illustration shows a model for solving an equation with one variable. On both sides of the workspace, remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? The following figure illustrates that each side of the workspace represents an expression, and the center line serves as the equal sign. We will call the contents of the envelope x.



The illustration shows a model for the equation $x+3=8$.

Let's write algebraically the steps we took to discover how many counters were in the envelope:

	$x+3=8$
First, we took away three from each side.	$x+3-3=8-3$
Then we were left with five.	$x=5$

Check:

Five in the envelope plus three more equals eight!

$$5+3=8$$

Our model has given us an idea of how to solve a specific type of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

SUBTRACTION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{aligned} &\textbf{If} && a=b \\ &\textbf{then} && a-c=b-c \end{aligned}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. We verify our solutions by substituting the value into the equation to ensure we have a true statement.

Example 2

Solve: $y+37=-13$.

Solution

To get y by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

		$y+37=-13$
Subtract 37 from each side to 'undo' the addition.		$y+37=-13$ $-37\quad -37$
Simplify.		$y=-50$
Check:	$y+37=-13$	
Substitute $y=-50$	$-50+37\stackrel{?}{=}-13$	
	$-13=-13\checkmark$	

Since $y=-50$ makes $y+37=-13$ a true statement, we have the solution to this equation.

Try It 2

Solve: $x+19=-27$.

Answer

$x=-46$

What happens when an equation has a number subtracted from the variable, as in the equation $x-5=8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to 'undo' the subtraction, we will add the number to both sides. We use the **Addition Property of Equality**.

ADDITION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{aligned} &\textbf{If} && a=b \\ &\textbf{then} && a+c=b+c \end{aligned}$$

When you add the same quantity from both sides of an equation, you still have equality.

In the Example below, we will need to 'undo' subtraction by using the **Addition Property of Equality**.

Example 3

Solve: $a-28=-37$.

Solution

		$a-28=-37$
Add 28 to each side to 'undo' the subtraction.		$a-28+28=-37+28$
Simplify.		$a=-9$
Check:	$a-28=-37$	
Substitute $a=-9$	$-9-28\stackrel{?}{=}-37$	
	$-37=-37\checkmark$	
		The solution to $a-28=-37$ is $a=-9$.

Try It 3

Solve: $p-41=-73$.

Answer

$p=-32$

Example 4

Solve: $x-\frac{5}{8}=\frac{3}{4}$

Solution

		$x-\frac{5}{8}=\frac{3}{4}$
Use the Addition Property of Equality.		$x-\frac{5}{8}+\frac{5}{8}=\frac{3}{4}+\frac{5}{8}$
Find the LCD to add the fractions on the right.		$x=\frac{6}{8}+\frac{5}{8}$
Simplify.		$x=\frac{11}{8}$
Check:	$x-\frac{5}{8}=\frac{3}{4}$	
Substitute $x=\frac{11}{8}$	$\frac{11}{8}-\frac{5}{8}\stackrel{?}{=}\frac{3}{4}$	
Subtract.	$\frac{6}{8}\stackrel{?}{=}\frac{3}{4}$	
Simplify.	$\frac{3}{4}=\frac{3}{4}\checkmark$	
		The solution to $x-\frac{5}{8}=\frac{3}{4}$ is $x=\frac{11}{8}$.

Try It 4

Solve: $p-\frac{2}{3}=\frac{5}{6}$.

Answer

$p=\frac{9}{6}$ or $p=\frac{3}{2}$

The next example will be an equation with decimals.

Example 5

Solve: $n-0.63=-4.2$.

Solution

		$n-0.63=-4.2$
Use the Addition Property of Equality.		$n-0.63+0.63=-4.2+1.63$
Add.		$n=-3.57$
Check:	$n=-3.57$	
Let $n=-3.57$.	$-3.57-0.63\stackrel{?}{=}-4.2$	
	$-4.2=-4.2\checkmark$	

Try It 5

Solve: $c-0.93=-4.6$.

Answer

$c=-3.67$

Solve Equations That Require Simplification

In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality.

You should always simplify as much as possible before you try to isolate the variable. Remember that to simplify an expression means to do all the operations in the expression. Simplify one side of the equation at a time. Note that **simplification** is different from the process used to solve an equation, in which we apply an operation to both sides.

Example 6

Solve: $9x - 5 - 8x - 6 = 7$.

Solution

Step 1. Simplify the expressions on each side as much as possible.	Rearrange the terms, using the Commutative Property of Addition. Combine like terms. Notice that each side is now simplified as much as possible.	$9x - 5 - 8x - 6 = 7$ $9x - 8x - 5 - 6 = 7$ $x - 11 = 7$
Step 2. Isolate the variable.	Now isolate x. Undo subtraction by adding 11 to both sides.	$x - 11 + 11 = 7 + 11$
Step 3. Simplify the expressions on both sides of the equation.		$x = 18$
Step 4. Check the solution.		**Check:** Substitute $x = 18$. $9x - 5 - 8x - 6 = 7$ $9(18) - 5 - 8(18) - 6 \stackrel{?}{=} 7$ $162 - 5 - 144 - 6 \stackrel{?}{=} 7$ $157 - 144 - 6 \stackrel{?}{=} 7$ $13 - 6 \stackrel{?}{=} 7$ $7 = 7$ ✓ The solution to $9x - 5 - 8x - 6 = 7$ is $x = 18$.

Try It 6

Solve: $8y - 4 - 7y - 7 = 4$.

Answer

$y = 15$

Example 7

Solve: $5(n - 4) - 4n = -8$.

Solution

We simplify both sides of the equation as much as possible before we try to isolate the variable.

	$5(n-4)-4n=-8$
Distribute on the left.	$5n-20-4n=-8$
Use the Commutative Property to rearrange terms.	$5n-4n-20=-8$
Combine like terms.	$n-20=-8$
Each side is as simplified as possible. Next, isolate n.	
Undo subtraction by using the Addition Property of Equality.	$n-20+20=-8+20$
Add.	$n=12$
Check. Substitute $n=12$. $5(n-4)-4n=-8$ $5(12-4)-4(12)\stackrel{?}{=}-8$ $5(8)-48\stackrel{?}{=}-8$ $40-48\stackrel{?}{=}-8$ $-8=-8\checkmark$	
	The solution to $5(n-4)-4n=-8$ is $n=12$.

? Try It 7

Solve: $5(p-3)-4p=-10$.

Answer

$p=5$

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating from English sentences into equations. Our first step is to look for the word (or words) that would translate to the **equals sign**. Here are some of the words that are commonly used.

Equals =

- is
- is equal to
- is the same as
- the result is
- gives
- was
- will be

The steps we use to translate a sentence into an equation are listed below.

TRANSLATE AN ENGLISH SENTENCE TO AN ALGEBRAIC EQUATION

1. Locate the “equals” word(s). Translate to an equals sign (=).
2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

Example 8

Translate and solve: Eleven more than x is equal to 54.

Solution

Translate.	Eleven more than x is equal to 54 $x+11 = 54$
Subtract 11 from both sides.	$x+11-11 = 54-11$
Simplify.	$x = 43$
Check: Is 54 eleven more than 43? $43+11 = 54$✓	

Try It 8

Translate and solve: Twelve less than x is equal to 51.

Answer

$y-12=51; \quad y=63$

Example 9

Translate and solve: The difference of $12t$ and $11t$ is -14.

Solution

Translate.	The difference of $12t$ and $11t$ is -14 $12t-11t = -14$
Simplify.	$t = -14$
Check: $12(-14)-11(-14) \stackrel{?}{=} -14$ $-168+154 \stackrel{?}{=} -14$ $-14 = -14$✓	

Try It 9

Translate and solve: The difference of $7a$ and $6a$ is -8.

Answer

$7a-6a=-8; \quad a=-8$

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real life question. To begin with, that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

We will start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use q for the number of quarters if you were solving a problem about coins.

Example 10

The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	The problem is about the weight of newspapers.	
Step 2. Identify what we are asked to find.	What are we asked to find?	"How much did the newspapers weigh the 2nd month?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Choose a variable.	Let w = weight of the newspapers the 1st month
Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.	Restate the problem. We know the weight of the newspapers the second month is 28 pounds. Translate into an equation, using the variable w.	Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds. Weight from 1st month plus 28 equals 57. $w + 28 = 57$
Step 5. Solve the equation using good algebra techniques.	Solve.	$w + 28 - 28 = 57 - 28$ $w = 29$
Step 6. Check the answer in the problem and make sure it makes sense.	Does 1st month's weight plus 2nd month's weight equal 57 pounds?	**Check:** Does 1st month's weight plus 2nd month's weight equal 57 pounds? $29 + 28 \stackrel{?}{=} 57$ $57 = 57$ ✓

The final step in all application problems is to write the answer in a complete sentence, using the appropriate units: "The first month, the newspapers weighed 29 pounds."

Try It 10

Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Answer

42 books

SOLVE AN APPLICATION.

1. **Read** the problem. Ensure that all words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in a single sentence that includes the most essential information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example 11

Randell paid $28,675 for his new car. This was $875 less than the sticker price. What was the sticker price of the car?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	"What was the sticker price of the car?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let s = the sticker price of the car.
Step 4. Translate into an equation. Restate the problem in one sentence.	$28675 is $875 less than the sticker price
	$28675 is $875 less than s
	$28675 = s - 875$
Step 5. Solve the equation.	$28675 + 875 = s - 875 + 875$
	$29550 = s$
Step 6. Check the answer. Is $875 less than $29550 equal to $28675?	
$29550 - 875 \stackrel{?}{=} 28675$	
$28675 = 28675$✓	
Step 7. Answer the question with a complete sentence.	The sticker price of the car was $29550.

Try It 11

Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

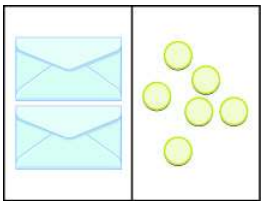
Answer

$11.00

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant, and so will require division to isolate the variable.

Let's look at our puzzle again, this time with the envelopes and counters.

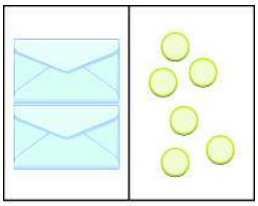


The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration, two identical envelopes contain the same number of counters. Remember, the left side of the workspace must equal the right side; however, the counters on the left side are *hidden* in the envelopes. So, how many counters are in each envelope?

How do we determine the number? We need to separate the counters on the right side into two groups of equal size to correspond with the two envelopes on the left side. The 6 counters are divided into two equal groups, giving 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in the figure below? There are two envelopes, each containing x counters. Together, the two envelopes must contain a total of 6 counters.



The illustration shows a model of the equation $2x = 6$.

	$2x = 6$
If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2} = \frac{6}{2}$
we get:	$x = 3$

We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works! Three counters in each of two envelopes equals six!

This example leads to the **Division Property of Equality**.

THE DIVISION PROPERTY OF EQUALITY

For any numbers a, b, and c, and $c \neq 0$,

$$\begin{aligned} &\text{If} && a &&= b \\ &\text{then} && \tfrac{a}{c} &&= \tfrac{b}{c} \end{aligned}$$

When you divide both sides of an equation by any non-zero number, you still have equality.

Note

Doing the Manipulative Mathematics activity "**Division Property of Equality**" will help you develop a better understanding of how to solve equations by using the Division Property of Equality.

The goal in solving an equation is to 'undo' the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to 'undo' the multiplication.

Example 12

Solve: $5x = -27$.

Solution

To isolate x, "undo" the multiplication by 5.		$5x = -27$
Divide to 'undo' the multiplication.		$\frac{5x}{5} = -\frac{27}{5}$
Simplify.		$x = -\frac{27}{5}$
Check:	$5x = -27$	
Substitute $-\frac{27}{5}$ for x.	$5\left(-\frac{27}{5}\right) \stackrel{?}{=} -27$	
	$-27 = -27$ ✓	
		Since this is a true statement, $x = -\frac{27}{5}$ is the solution to $5x = -27$.

Try It 12

Solve: $3y = -41$.

Answer

$y = -\frac{41}{3}$

Consider the equation $\frac{x}{4} = 3$. We want to know what number, when divided by 4, gives 3. So to "undo" the division, we will need to multiply by 4. The **Multiplication Property of Equality** will allow us to do this. This property states that if we start with two equal quantities and multiply both by the same number, the results will be equal.

THE MULTIPLICATION PROPERTY OF EQUALITY

For any numbers *a*, *b*, and *c*,

$$\begin{array}{lrcl} \textbf{If} & a & = & b \\ \textbf{then} & ac & = & bc \end{array}$$

If you multiply both sides of an equation by the same number, you still have equality.

Example 13

Solve: $\frac{y}{-7} = -14$

Solution

Here y is divided by −7. We must multiply by −7 to isolate y.

	$\frac{y}{-7}=-14$
Multiply both sides by −7.	$-7\left(\frac{y}{-7}\right)=-7(-14)$
Multiply.	$\frac{-7y}{-7}=98$
Simplify.	$y=98$
Check: $\frac{y}{-7}=-14$	
Substitute y=98.	$\frac{98}{-7}\stackrel{?}{=}-14$
Divide.	$-14=-14$ ✓

Try It 13

Solve: $\frac{a}{-7}=-42$

Answer

$a=294$

Example 14

Solve: $\frac{3}{4}x=12$

Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

	$\frac{3}{4}x=12$
Multiply by the reciprocal of $\frac{3}{4}$.	$\frac{4}{3}\cdot\frac{3}{4}x=\frac{4}{3}\cdot 12$
Reciprocals multiply to 1.	$1x=\frac{4}{3}\cdot\frac{12}{1}$
Multiply.	$x=16$
Notice that we could have divided both sides of the equation $\frac{3}{4}x=12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.	
Check:	$\frac{3}{4}x=12$
Substitute $x=16$.	$\frac{3}{4}\cdot 16\stackrel{?}{=}12$
	$12=12$ ✓

Try It 14

Solve: $\frac{2}{5}n=14$.

Answer

$n=35$

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

Example 15

Solve: $\frac{8}{15} = -\frac{4}{5}x$

Solution

	$\frac{8}{15} = -\frac{4}{5}x$
Multiply by the reciprocal of $-\frac{4}{5}$.	$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right) = \left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$
Reciprocals multiply to 1.	$\frac{\cancel{5}\cdot\cancel{4}\cdot 2}{\cancel{4}\cdot 3\cdot\cancel{5}} = 1x$
Multiply.	$-\frac{2}{3} = x$
Check:	$\frac{8}{15} = -\frac{4}{5}x$
Let $x = -\frac{2}{3}$.	$\frac{8}{15} = -\frac{4}{5}\left(-\frac{2}{3}\right)$
	$\frac{8}{15} = \frac{8}{15}$ ✓

Try It 15

Solve: $\frac{9}{25} = -\frac{4}{5}z$

Answer

$z = -\frac{9}{20}$

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations, the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

Example 16

Solve: $14 - 23 = 12y - 4y - 5y$.

Solution

Begin by simplifying each side of the equation.

	$14-23=12y-4y-5y$
Simplify each side.	$-9=3y$
Divide both sides by 3 to isolate y.	$-3=y$
Check:	$14-23=12y-4y-5y$
Substitute $y=-3$.	$14-23\stackrel{?}{=}12(-3)-4(-3)-5(-3)$
Multiply.	$14-23\stackrel{?}{=}-36+12+15$
Simplify each side.	$-9=-9$ ✓

Try It 16

Solve: $18-27=15c-9c-3c$.

Answer

$c=-3$

We have now covered all four properties of equality: subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

PROPERTIES OF EQUALITY

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Subtraction Property of Equality
For any real numbers $a, b,$ and c,
if $a=b$,
then $a-c=b-c$

Addition Property of Equality
For any real numbers $a, b,$ and c,
if $a=b$,
then $a+c=b+c$

Division Property of Equality
For any real numbers $a, b,$ and c,
if $a=b$,
then $\frac{a}{c}=\frac{b}{c}$

Multiplication Property of Equality
For any real numbers $a, b,$ and c,
if $a=b$,
then $a\cdot c=b\cdot c$

(Section 2.1.1)

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

In the following examples, we will translate sentences into equations and then solve them. You might want to review the translation table in the previous chapter.

Example 17

Translate and solve: The number 143 is the product of −11 and y.

Solution

Translate.	The number 143 is the product of –11 and y. $143 = -11y$
Divide by −11.	$\frac{143}{-11} = \frac{-11y}{-11}$
Simplify.	$-13 = y$
Check: $143 = -11y$ $143 \stackrel{?}{=} -11(-13)$ $143 = 143\checkmark$	

Try It 17

Translate and solve: The number 132 is the product of -12 and y.

Answer

$132 = -12y;\ y = -11$

Example 18

Translate and solve: Three-fourths of p is 18.

Solution

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.	Three-fourths of p is 18. $\frac{3}{4}p = 18$
Multiply both sides by $\frac{4}{3}$.	$\frac{4}{3} \cdot \frac{3}{4}p = \frac{4}{3} \cdot 18$
Simplify.	$p = 24$
Check:	Is three-fourths of p equal to 18?
Let p = 24.	Is three-fourths of 24 equal to 18?
Translate.	$\frac{3}{4} \cdot 24 \stackrel{?}{=} 18$
Simplify.	$18 = 18\checkmark$

Try It 18

Translate and solve: Two-fifths of f is 16.

Answer

$\frac{2}{5}f = 16; f = 40$

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

Example 19

Denae bought 6 pounds of grapes for $10.74. What was the cost of one pound of grapes?

Solution

What are you asked to find?	The cost of 1 pound of grapes
Assign a variable.	Let c = the cost of one pound.
Write a sentence that gives the information to find it.	The cost of 6 pounds is $10.74
Translate into an equation.	$6c = 10.74$
Solve.	$\frac{6c}{6} = \frac{10.74}{6}$ $c = 1.79$
	The grapes cost $1.79 per pound.
Check: If one pound costs $1.79, *do* 6 pounds cost $10.74?	
	$6(1.79) \stackrel{?}{=} 10.74$ $10.74 = 10.74$✓

Try It 19

Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for $34.98. What is the cost for each person?

Answer

$5.83

Example 20

Andreas bought a used car for $12,000. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Solution

What are you asked to find?	The original price of the car
Assign a variable.	Let p = the original price.
Write a sentence that gives the information to find it.	\$12000 is $\frac{3}{4}$ of the original price.
Translate into an equation.	$12000 = \frac{3}{4}p$
	$\frac{4}{3}(12000) = \frac{4}{3} \cdot \frac{3}{4}p$
	$16000 = p$
Solve.	
	The original cost of the car was \$16000.
Check: Is $\frac{3}{4}$ of \$16000 equal to \$12000	
$\frac{3}{4} \cdot 16000 \stackrel{?}{=} 12000$	
$12000 = 12000$✓	

? Try It 20

Translate and solve:

The annual property tax on the Mehta's house is \$1,800, calculated as $\frac{15}{1000}$ of the assessed value of the house. What is the assessed value of the Mehta's house?

Answer

\$120000

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Section 2.1P: Practice

Practice Makes Progress

Verify a Solution of an Equation

In the following exercises, determine whether the given value is a solution to the equation.

Exercise 1

Is $y = \frac{5}{3}$ a solution of $6y + 10 = 12y$?

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction and Addition Properties of Equality.

Exercise 2

$x + 24 = 35$

Exercise 3

$b + \frac{1}{4} = \frac{3}{4}$

Exercise 4

$x - \frac{1}{3} = 2$

Exercise 5

$q + \frac{3}{4} = \frac{1}{2}$

Solve Equations that Require Simplification

In the following exercises, solve each equation.

Exercise 6

$c + 31 - 10 = 46$

Exercise 7

$5(y - 6) - 4y = -6$

Exercise 8

$3(5n - 1) - 14n + 9 = 10(n - 4) - 6n - 4(n + 1)$

Exercise 9

$-\left(\frac{1}{4}a - \frac{3}{4}\right) + \frac{5}{4}a = -2$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve it.

? Exercise 10

Nine more than x is equal to 52.

? Exercise 11

Ten less than m is -14.

? Exercise 12

The sum of y and -30 is 40.

? Exercise 13

The difference of $9x$ and $8x$ is 107.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

? Exercise 14

Eva's daughter is 15 years younger than her son. Eva's son is 22 years old. How old is her daughter?

? Exercise 15

Ron's paycheck this week was $17.43 less than his paycheck last week. His paycheck this week was $103.76. How much was Ron's paycheck last week?

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the Division and Multiplication Properties of Equality and check the solution.

? Exercise 16

$-5c = 55$

? Exercise 17

$0.25z = 3.25$

? Exercise 18

$\frac{x}{4} = 35$

? Exercise 19

$-20 = \frac{q}{-5}$

? Exercise 20

$-y = 6$

Exercise 21

$\frac{2}{3}y = 48$

Exercise 22

$-\frac{2}{5} = \frac{1}{10}a$

Exercise 23

$-\frac{5}{18} = -\frac{10}{9}u$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

Exercise 24

$\frac{7}{8}n - \frac{3}{4}n = 9 + 2$

Exercise 25

$-10(q - 4) - 57 = 93$

Translate to an Equation and Solve

In the following exercises, translate each phrase into an equation and then solve.

Exercise 26

133 is the product of -19 and n

Exercise 27

r divided by 12 is equal to -48

Exercise 28

The quotient of b and -6 is 18

Exercise 29

Three-tenths of x is 15

Exercise 30

The sum of two-fifths and f is one-half.

Exercise 31

The difference of q and one-eighth is three-fourths.

Everyday Math

Exercise 32

Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. How many balloons did she use in each bunch?

Exercise 33

Serena paid $12.96 for a pack of 12 pairs of sport socks. What was the price of pair of sport socks?

Exercise 34

John's SUV gets 18 miles per gallon (mpg). This is half as many mpg as his wife's hybrid car. How many miles per gallon does the hybrid car get?

Exercise 35

Miguel wants to drill a hole for a $\frac{5}{8}$ inch screw. The hole should be $\frac{1}{12}$ inch smaller than the screw. Let d equal the size of the hole he should drill. Solve the equation $d = \frac{5}{8} - \frac{1}{12}$ to see what size the hole should be.

Writing Exercises

Exercise 36

Emiliano thinks $x = 40$ is the solution to the equation $\frac{1}{2}x = 80$. Explain why he is wrong.

Exercise 37

Is -8 a solution to the equation $3x = 16 - 5x$? How do you know?

Self Check

Use these checklists to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
verify a solution of an equation.			
solve equations using the subtraction and addition properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

I can...	Confidently	With some help	No-I don't get it!
solve equations using the Division and Multiplication Properties of equality.			
solve equations that require simplification.			
translate to an equation and solve.			
translate and solve applications.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.2: Solving Equations - Refining the Process

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify $5-2(n+1)$
- Multiply $\frac{3}{2}(12x+20)$
- Simplify $4y-9+2y-3$
- Solve $\frac{1}{2}x+3=7$

Learning Objectives

Motivating Problem

Micah has 44 feet of fencing to make a dog run in his yard. They want the length to be 2.5 feet more than the width. Find the length of the dog run.

Fun Fact

Did you know? The ancient Babylonians were solving equations over 4,000 years ago—using clay tablets and a number system based on 60 instead of 10. That's why we still measure angles and time in 60 seconds today.

The Goal

This section focuses on equations that involve variables and constants on both sides, and introduces strategies to solve more complex equations. We'll continue building fluency with solving linear equations and learn how to clear messy-looking equations into something manageable.

Solve Equations with Variables on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation, with the constants on the other side. This does not happen all the time, so we will now learn to solve equations in which the variable terms, constant terms, or both appear on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side" and the other side to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

The following example will be the first to have variables and constants on both sides of the equation. Solving this equation may require several steps, so we need a clear and organized strategy.

Example 1

Solve: $9x=8x-6$.

Solution

Here, the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

	$9x = 8x - 6$
We don't want any x's on the right, so subtract the 8x from both sides.	$9x - 8x = 8x - 8x - 6$
Simplify.	$x = -6$
We successfully separated the variables on one side and the constants on the other, obtaining the solution.	
Check:	$9x = 8x - 6$
Let x=−6.	$9(-6) \stackrel{?}{=} 8(-6) - 6$
	$-54 \stackrel{?}{=} -48 - 6$
	$-54 = -54$ ✓

Try It 1

Solve: $6n = 5n - 10$.

Answer

$n = -10$

Example 2

Solve: $5y - 9 = 8y$

Solution

The only constant is on the left, and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

	constant variable $5y - 9 = 8y$
Subtract 5y from both sides.	$5y - 5y - 9 = 8y - 5y$
Simplify.	$-9 = 3y$
We have the y's on the right and the constants on the left. Divide both sides by 3.	$\frac{-9}{3} = \frac{3y}{3}$
Simplify.	$-3 = y$
Check:	$5y - 9 = 8y$
Let $y = -3$.	$5(-3) - 9 \stackrel{?}{=} 8(-3)$
	$-15 - 9 \stackrel{?}{=} -24$
	$-24 = -24$ ✓

Try It 2

Solve: $8m + 9 = 5m$

Answer

$m = -3$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. Solving this equation may require several steps, so we need a clear and organized strategy.

Example 3

Solve: $7x + 5 = 6x + 2$

Solution

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.	The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the "x" side. The right side will be the "constant" side.	variable constant $7x + 5 = 6x + 2$
Step 2. Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.	With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$
Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the "constant" side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$
Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of x is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16$ ✓

Try It 3

Solve: $12x + 8 = 6x + 2$.

Answer

$x = -1$

We'll list the steps below so you can easily refer to them. However, we will refer to this as the 'Beginning Strategy' because we will add some steps later in this chapter.

BEGINNING STRATEGY FOR SOLVING EQUATIONS WITH VARIABLES AND CONSTANTS ON BOTH SIDES OF THE EQUATION.

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants on the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

Example 4

Solve: $8n-4=-2n+6$.

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8>-2$, make the left side the "variable" side.	variable constant $8n-4=-2n+6$
We don't want variable terms on the right side—add 2n to both sides to leave only constants on the right.	$8n+2n-4=-2n+2n+6$
Combine like terms.	$10n-4=6$
We don't want any constants on the left side, so add 4 to both sides.	$10n-4+4=6+4$
Simplify.	$10n=10$
The variable term is on the left, and the constant term is on the right. To get the coefficient of nn to be one, divide both sides by 10.	$\frac{10n}{10}=\frac{10}{10}$
Simplify.	$n=1$
Check:	$8n-4=-2n+6$
Let n=1.	$8\cdot 1-4\stackrel{?}{=}-2\cdot 1+6$
	$8-4\stackrel{?}{=}-2+6$
	$4=4\checkmark$

Try It 4

Solve: $8q-5=-4q+7$

Answer

$q=1$

Example 5

Solve: $7a-3=13a+7$

Solution

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since 13>7, make the right side the “variable” side and the left side the “constant” side.

	constant variable $7a-3=13a+7$
Subtract 7a from both sides to remove the variable term from the left.	$7a-7a-3=13a-7a+7$
Combine like terms.	$-3=6a+7$
Subtract 7 from both sides to remove the constant from the right.	$-3-7=6a+7-7$
Simplify.	$-10=6a$
Divide both sides by 6 to make 1 the coefficient of a.	$\frac{-10}{6}=\frac{6a}{6}$
Simplify.	$-\frac{5}{3}=a$
Check:	$7a-3=13a+7$
Let $a=-\frac{5}{3}$.	$7\left(-\frac{5}{3}\right)-3\stackrel{?}{=}13\left(-\frac{5}{3}\right)+7$
	$-\frac{35}{3}-\frac{9}{3}\stackrel{?}{=}-\frac{65}{3}+\frac{21}{3}$
	$-\frac{54}{3}=-\frac{54}{3}$ ✓

Try It 5

Solve: $2a-2=6a+18$

Answer

$a=-5$

In the last example, we could have made the left side the “variable” side, but it would have led to a negative coefficient on the variable term. (Try it!) While we can work with the negative, there is a lower chance of errors when working with the positive. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, follow the steps of our strategy to obtain the solution.

Example 6

Solve: $\frac{5}{4}x+6=\frac{1}{4}x-2$

Solution

Since $\frac{5}{4} > \frac{1}{4}$, make the left side the "variable" side and the right side the "constant" side.

	variable constant $\frac{5}{4}x+6=\frac{1}{4}x-2$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x-\frac{1}{4}x+6=\frac{1}{4}x-\frac{1}{4}x-2$
Combine like terms.	$x+6=-2$
Subtract 6 from both sides.	$x+6-6=-2-6$
Simplify.	$x=-8$
Check: Let $x=-8$	$\frac{5}{4}x+6 = \frac{1}{4}x-2$ $\frac{5}{4}(-8)+6 \stackrel{?}{=} \frac{1}{4}(-8)-2$ $-10+6 \stackrel{?}{=} -2-2$ $-4 = -4$✓

Try It 6

Solve: $\frac{7}{8}x-12=-\frac{1}{8}x-2$

Answer

$x=10$

We will use the same strategy to find the solution for an equation with decimals.

Example 7

Solve: $7.8x+4=5.4x-8$.

Solution

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

	variable side constant side $7.8x+4=5.4x-8$
Subtract 5.4x from both sides.	$7.8x-5.4x+4=5.4x-5.4x-8$
Combine like terms.	$2.4x+4=-8$
Subtract 4 from both sides.	$2.4x+4-4=-8-4$
Simplify.	$2.4x=-12$
Use the Division Property of Equality.	$\frac{2.4x}{2.4}=\frac{-12}{2.4}$
Simplify.	$x=-5$
Check:	$7.8x+4=5.4x-8$
Let $x=-5$	$7.8(-5)+4 \stackrel{?}{=} 5.5(-5)-8$
Multiply.	$-39+4 \stackrel{?}{=} -27-8$
Simplify.	$-35=-35$✓

Try It 7

Solve: $2.8x + 12 = -1.4x - 9$

Answer

$x = -5$

Solve Equations Using the General Strategy

Until now, we have dealt with solving one specific form of a linear equation. It is now time to outline a comprehensive strategy that can be applied to solve any linear equation. Some equations we solve will not require all these steps, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Example 8

Solve: $-6(x + 3) = 24$.

Solution

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x + 3) = 24$ $-6x - 18 = 24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x - 18 + 18 = 24 + 18$ $-6x = 42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by -6. Simplify.	$\frac{-6x}{-6} = \frac{42}{-6}$ $x = -7$
Step 5. Check the solution.	Let $x = -7$ Simplify. Multiply.	**Check:** $-6(x + 3) = 24$ $-6(-7 + 3) \stackrel{?}{=} 24$ $-6(-4) \stackrel{?}{=} 24$ $24 = 24$ ✓

Try It 8

Solve: $6(y - 4) = -18$

Answer

$y = 1$

GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS.

1. **Simplify each side of the equation as much as possible.**
   Use the Distributive Property to remove any parentheses.
   Combine like terms.

2. **Collect all the variable terms on one side of the equation.**
   Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
   Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term equal to 1.**
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

Example 9

Solve: $5(a-3)+5=-10$

Solution

	$5(a-3)+5=-10$
Simplify each side of the equation as much as possible.	
Distribute.	$5a-15+5=-10$
Combine like terms.	$5a-10=-10$
There is only a term on the left side, so all variable terms are on one side of the equation.	
Add 10 to both sides to isolate all constant terms on one side of the equation.	$5a-10+10=-10+10$
Simplify.	$5a=0$
Make the coefficient of the variable term to equal to 11 by dividing both sides by 55.	$\frac{5a}{5}=\frac{0}{5}$
Simplify.	$a=0$
Check:	$5(a-3)+5=-10$
Let a=0.	$5(0-3)+5\stackrel{?}{=}-10$
	$5(-3)+5\stackrel{?}{=}-10$
	$-15+5\stackrel{?}{=}-10$
	$-10=-10$ ✓

Try It 9

Solve: $2(m-4)+3=-1$

Answer

$m=2$

Example 10

Solve: $\frac{2}{3}(6m-3)=8-m$

Solution

	$\frac{2}{3}(6m-3)=8-m$
Distribute.	$4m-2=8-m$
Add m to get the variables only to the left.	$4m+m-2=8-m+m$
Simplify.	$5m-2=8$
Add 2 to get constants only on the right.	$5m-2+2=8+2$
Simplify.	$5m=10$
Divide by 5.	$\frac{5m}{5}=\frac{10}{5}$
Simplify.	$m=2$
Check:	$\frac{2}{3}(6m-3)=8-m$
Let m=2.	$\frac{2}{3}(6\cdot 2-3)\stackrel{?}{=}8-2$
	$\frac{2}{3}(12-3)\stackrel{?}{=}6$
	$\frac{2}{3}(9)\stackrel{?}{=}6$
	$6=6\checkmark$

Try It 10

Solve: $\frac{2}{3}(9x-12)=8+2x$

Answer

$x=4$

Example 11

Solve: $4(x-1)-2=5(2x+3)+6$

Solution

	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract 4x to get the variables only on the right side since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 to get the constants on the left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24$ ✓

Try It 11

Solve: $6(p-3)-7=5(4p+3)-12$

Answer

$p=-2$

Example 12

Solve: $0.36(100n+5)=0.6(30n+15)$.

Solution

	$0.36(100n + 5) = 0.6(30n + 15)$
Distribute.	$36n + 1.8 = 18n + 9$
Subtract $18n$ to get the variables to the left.	$36n - 18n + 1.8 = 18n - 18n + 9$
Simplify.	$18n + 1.8 = 9$
Subtract 1.8 to get the constants to the right.	$18n + 1.8 - 1.8 = 9 - 1.8$
Simplify.	$18n = 7.2$
Divide.	$\frac{18n}{18} = \frac{7.2}{18}$
Simplify.	$n = 0.4$
Check:	$0.36(100n + 5) = 0.6(30n + 15)$
Let n=0.4.	$0.36(100(0.4) + 5) \stackrel{?}{=} 0.6(30(0.4) + 15)$
	$0.36(40 + 5) \stackrel{?}{=} 0.6(12 + 15)$
	$0.36(45) \stackrel{?}{=} 0.6(27)$
	$16.2 = 16.2$ ✓

Try It 12

Solve: $0.55(100n + 8) = 0.6(85n + 14)$.

Answer

$n = 1$

Classify Equations

Consider the equation we solved at the start of the last section, $7x + 8 = -13$. The solution we found was $x = -3$. This means the equation $7x + 8 = -13$ is true when we replace the variable, x, with the value −3. We showed this when we checked the solution $x = -3$ and evaluated $7x + 8 = -13$ for $x = -3$.

$$7(-3) + 8 \stackrel{?}{=} -13$$
$$-21 + 8 \stackrel{?}{=} -13$$
$$-13 = -13 \checkmark$$

If we evaluate $7x + 8$ for a different value of x, the left side will not be −13.

The equation $7x + 8 = -13$ is true when we replace the variable x with the value −3, but it is not true when we replace x with any other value. Whether or not the equation $7x + 8 = -13$ is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values is a **conditional equation**.

Now let's consider the equation $2y + 6 = 2(y + 3)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for y.

	$2y + 6 = 2(y + 3)$
Distribute.	$2y + 6 = 2y + 6$
Subtract 2y to get the y's to one side.	$2y - 2y + 6 = 2y - 2y + 6$
Simplify—the y's are gone!	$6 = 6$

But $6 = 6$ is true.

This means that the equation $2y + 6 = 2(y + 3)$ is true for any value of y. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable, like this, is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**. The solution of an identity is all real numbers.

What happens when we solve the equation $5z = 5z - 1$?

	$5z = 5z - 1$
Subtract 5z to get the constant alone on the right.	$5z - 5z = 5z - 5z - 1$
Simplify—the z's are gone!	$0 \neq -1$

But $0 \neq -1$.

Solving the equation $5z = 5z - 1$ led to the false statement $0 = -1$. The equation $5z = 5z - 1$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a **contradiction**.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**. A contradiction has no solution.

Example 13

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$

Solution

$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$	
Distribute.	$12n - 6 + 3 = 2n - 8 + 10n + 5$
Combine like terms.	$12n - 3 = 12n - 3$
Subtract 12n to get the nn's to one side.	$12n - 12n - 3 = 12n - 12n - 3$
Simplify.	$-3 = -3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

Try It 13

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution:

$4+9(3x-7)=-42x-13+23(3x-2)$

Answer

identity; all real numbers

Example 14

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Solution

	$10+4(p-5)=0$
Distribute.	$10+4p-20=0$
Combine like terms.	$4p-10=0$
Add 10 to both sides.	$4p-10+10=0+10$
Simplify.	$4p=10$
Divide.	$\frac{4p}{4}=\frac{10}{4}$
Simplify.	$p=\frac{5}{2}$
The equation is true when $p=frac52$.	This is a conditional equation. The solution is $p=frac52$.

Try It 14

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution: $11(q+3)-5=19$

Answer

conditional equation; $q=-\frac{9}{11}$

Example 15

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m+3(9+3m)=2(7m-11)$

Solution

	$5m + 3(9 + 3m) = 2(7m - 11)$
Distribute.	$5m + 27 + 9m = 14m - 22$
Combine like terms.	$14m + 27 = 14m - 22$
Subtract 14m from both sides.	$14m + 27 - 14m = 14m - 22 - 14m$
Simplify.	$27 \neq -22$
But $27 \neq -22$.	The equation is a contradiction. It has no solution.

Try It 15

Classify the equation as a conditional equation, an identity, or a contradiction, and then state the solution:

$12c + 5(5 + 3c) = 3(9c - 4)$

Answer

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

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Section 2.2P: Practice

Practice Makes Progress

Solve Equations with Variables on Both Sides

In the following exercises, solve the equations with variables on both sides.

Exercise 1

$21k = 20k - 11$

Exercise 2

$b = -4b - 15$

Exercise 3

$4x + \frac{3}{4} = 3x$

Solve Equations with Variables and Constants on Both Sides

In the following exercises, solve the equations with variables and constants on both sides.

Exercise 4

$6x - 17 = 5x + 2$

Exercise 5

$12q - 5 = 9q - 20$

Exercise 6

$7x - 17 = -8x + 13$

Exercise 7

$\frac{7}{4}m - 7 = \frac{3}{4}m - 13$

Exercise 8

$11 - \frac{1}{5}a = \frac{4}{5}a + 4$

Exercise 9

$\frac{3}{5}p + 2 = \frac{4}{5}p - 1$

Exercise 10

$2.7w - 80 = 1.2w + 10$

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

Exercise 11

$-16(3n+4)=32$

Exercise 12

$-(t-19)=28$

Exercise 13

$21+2(m-4)=25$

Exercise 14

$8(6t-5)-35=-27$

Exercise 15

$\frac{3}{5}(10x-5)=27$

Exercise 16

$15-(3r+8)=28$

Exercise 17

$18-2(y-3)=32$

Exercise 18

$-2(a-6)=4(a-3)$

Exercise 19

$9(2m-3)-8=4m+7$

Exercise 20

$5(x-4)-4x=14$

Exercise 21

$7(2n-5)=8(4n-1)-9$

Exercise 22

$-(7m+4)-(2m-5)=14-(5m-3)$

Exercise 23

$3[-14+2(15k-6)]=8(3-5k)-24$

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? Exercise 24

$4(2.5v - 0.6) = 7.6$

? Exercise 25

$0.5(16m + 34) = -15$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction, and then state the solution.

? Exercise 26

$9(a - 4) + 3(2a + 5) = 7(3a - 4) - 6a + 7$

? Exercise 27

$30(2n - 1) = 5(10n + 8)$

? Exercise 28

$5(p + 4) + 8(2p - 1) = 9(3p - 5) - 6(p - 2)$

? Exercise 29

$60(2x - 1) = 15(8x + 5)$

? Exercise 30

$11(8c + 5) - 8c = 2(40c + 25) + 5$

Everyday Math

? Exercise 31

Jovani has 150 feet of fencing to make a rectangular garden in his backyard. He wants the length to be 15 feet more than the width. Find the width, w, by solving the equation $150 = 2w + 30 + 2w$.

? Exercise 32

Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n + 0.10(2n - 1) = 1.90$.

Writing Exercises

? Exercise 33

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Exercise 34

Solve the equation $\frac{1}{4}(8x + 20) = 3x - 4$, explaining all the steps of your solution as in the examples in this section.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

I can...	Confidently	With some help	No-I don't get it!
solve equations using the general strategy for solving linear equations.			
classify equations.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.3: Solving Equations - Advanced Techniques and Formulas

Be Prepared

We will rely heavily on these skills throughout this section.

- Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$
- Solve $6x + 24 = -2x + 96$
- Identify the reciprocal of $\frac{2}{3}$

Learning Objectives

Motivating Problem

You're planning a road trip and you know the distance and your speed, but you want to figure out how long it will take. You've heard the formula is $d = rt$, but the thing you *don't* know is time.

So, how do you rewrite the formula to solve for the variable you actually care about?

Fun Fact

The formula $E = mc^2$ became famous thanks to Einstein, but did you know solving it for m or c is just algebra? Rearranging formulas like this is precisely what scientists and engineers do every day to make predictions, design technology, and understand the universe—just with cooler variables!

The Goal

This section introduces strategies for solving more complex equations that involve fractions, decimals, or multiple steps. We'll then continue building fluency with solving linear equations and learn how to clear messy-looking formulas into something manageable and useful.

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x+\frac{1}{2}-\frac{1}{2}=\frac{1}{4}-\frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x=\frac{1}{4}-\frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x=\frac{1}{4}-\frac{2}{4}$
Subtract.	$\frac{1}{8}x=-\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1}\cdot\frac{1}{8}x=\frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x=-2$

This method worked fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called "clearing" the equation of fractions.

Let's solve a similar equation, but this time use the method that eliminates the fractions.

Example 1

Solve: $\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$

Solution

Step 1. Find the least common denominator of *all* the fractions in the equation.	What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?	$\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$ LCD $=6$
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.	Multiply both sides of the equation by the LCD 6. Use the Distributive Property. Simplify – and notice, no more fractions!	$6\left(\frac{1}{6}y-\frac{1}{3}\right)=6\left(\frac{5}{6}\right)$ $6\cdot\frac{1}{6}y-6\cdot\frac{1}{3}=6\cdot\frac{5}{6}$ $y-2=5$
Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the "y" term, add 2. Simplify.	$y-2+2=5+2$ $y=7$

Try It 1

Solve: $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$

Answer

$x=-2$

Notice that in the previous exercise, once we cleared the equation of fractions, the equation resembled those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

STRATEGY TO SOLVE EQUATIONS WITH FRACTION COEFFICIENTS.

1. Find the least common denominator of *all* the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

Example 2

Solve: $6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$

Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.	$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$
The LCD is 20.	
Multiply both sides of the equation by 20.	$20(6)=20\cdot\left(\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v\right)$
Distribute.	$20(6)=20\cdot\frac{1}{2}v+20\cdot\frac{2}{5}v-20\cdot\frac{3}{4}v$
Simplify—notice, no more fractions!	$120=10v+8v-15v$
Combine like terms.	$120=3v$
Divide by 3.	$\frac{120}{3}=\frac{3v}{3}$
Simplify.	$40=v$
Check:	$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$
Let v=40.	$6\stackrel{?}{=}\frac{1}{2}(40)+\frac{2}{5}(40)-\frac{3}{4}(40)$
	$6\stackrel{?}{=}20+16-30$
	$6=6\checkmark$

Try It 2

Solve: $-1=\frac{1}{2}u+\frac{1}{4}u-\frac{2}{3}u$

Answer

$u=-12$

In the next example, we again have variables on both sides of the equation.

Example 3

Solve: $a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$

Solution

	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Find the LCD of all fractions in the equation. The LCD is 8.	
Multiply both sides by the LCD.	$8\left(a+\frac{3}{4}\right)=8\left(\frac{3}{8}a-\frac{1}{2}\right)$
Distribute.	$8\cdot a+8\cdot\frac{3}{4}=8\cdot\frac{3}{8}a-8\cdot\frac{1}{2}$
Simplify—no more fractions.	$8a+6=3a-4$
Subtract 3a3a from both sides.	$8a-3a+6=3a-3a-4$
Simplify.	$5a+6=-4$
Subtract 6 from both sides.	$5a+6-6=-4-6$
Simplify.	$5a=-10$
Divide by 5.	$\frac{5a}{5}=\frac{-10}{5}$
Simplify.	$a=-2$
Check:	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Let a=−2.	$-2+\frac{3}{4}\stackrel{?}{=}\frac{3}{8}(-2)-\frac{1}{2}$
	$-\frac{8}{4}+\frac{3}{4}\stackrel{?}{=}-\frac{16}{8}-\frac{4}{8}$
	$-\frac{5}{4}=-\frac{10}{8}$
	$-\frac{5}{4}=-\frac{5}{4}\checkmark$

? Try It 3

Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$

Answer

$x=-1$

In the next example, we start by using the Distributive Property. This step clears the fractions right away.

✓ Example 4

Solve: $-5=\frac{1}{4}(8x+4)$

Solution

	$-5=\frac{1}{4}(8x+4)$
Distribute.	$-5=\frac{1}{4}\cdot 8x+\frac{1}{4}\cdot 4$
Simplify. Now there are no fractions.	$-5=2x+1$
Subtract 1 from both sides.	$-5-1=2x+1-1$
Simplify.	$-6=2x$
Divide by 2.	$\frac{-6}{2}=\frac{2x}{2}$
Simplify.	$-3=x$
Check:	$-5=\frac{1}{4}(8x+4)$
Let x=−3.	$-5\stackrel{?}{=}\frac{1}{2}(4(-3)+2)$
	$-5\stackrel{?}{=}\frac{1}{2}(-12+2)$
	$-5\stackrel{?}{=}\frac{1}{2}(-10)$
	$-5=-5$ ✓

Try It 4

Solve: $-11=\frac{1}{2}(6p+2)$

Answer

$p=-4$

Example 5

Solve: $\frac{5x-3}{4}=\frac{x}{2}$

Solution

	$\frac{5x-3}{4}=\frac{x}{2}$
Multiply by the LCD, 4.	$4\left(\frac{5x-3}{4}\right)=4\left(\frac{x}{2}\right)$
Simplify.	$5x-3=2x$
Collect the variables to the right.	$5x-5x-3=2x-5x$
Simplify.	$-3=-3x$
Divide.	$\frac{-3}{-3}=\frac{-3x}{-3}$
Simplify.	$1=x$
Check:	$\frac{5x-3}{4}=\frac{x}{2}$
Let x=1.	$\frac{5(1)-3}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{2}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{1}{2}=\frac{1}{2}$ ✓

Try It 5

Solve: $\frac{4y-7}{3}=\frac{y}{6}$

Answer

$y=2$

Example 6

Solve: $\frac{a}{6}+2=\frac{a}{4}+3$

Solution

	$\frac{a}{6}+2=\frac{a}{4}+3$
Multiply by the LCD, 12.	$12\left(\frac{a}{6}+2\right)=12\left(\frac{a}{4}+3\right)$
Distribute.	$12\cdot\frac{a}{6}+12\cdot 2=12\cdot\frac{a}{4}+12\cdot 3$
Simplify.	$2a+24=3a+36$
Collect the variables to the right.	$2a-2a+24=3a-2a+36$
Simplify.	$24=a+36$
Collect the constants to the left.	$24-36=a+36-36$
Simplify.	$a=-12$
Check:	$\frac{a}{6}+2=\frac{a}{4}+3$
Let a=−12.	$\frac{-12}{6}+2\stackrel{?}{=}\frac{-12}{4}+3$
	$-2+2\stackrel{?}{=}-3+3$
	$0=0\checkmark$

Try It 6

Solve: $\frac{b}{10}+2=\frac{b}{4}+5$

Answer

$b=-20$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3=\frac{3}{10}$ and $0.17=\frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

Example 7

Solve: $0.06x+0.02=0.25x-1.5$

Solution

Look at the decimals and think of the equivalent fractions.

$0.06=\frac{6}{100}\quad 0.02=\frac{2}{100}\quad 0.25=\frac{25}{100}\quad 1.5=1\frac{5}{10}$

Notice, the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both side by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now we have no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the variables to the right.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let x=8 $0.06(8) + 0.02 \stackrel{?}{=} 0.25(8) - 1.5$ $0.48 + 0.02 \stackrel{?}{=} 2.00 - 1.5$ $0.50 = 0.50$ ✓	

? Try It 7

Solve: $0.65k - 0.1 = 0.4k - 0.35$

Answer

$k = -1$

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

✓ Example 8

Solve: $0.25x + 0.05(x + 3) = 2.85$

Solution

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting x=9 into the original equation.	

? Try It 8

Solve: $0.10d + 0.05(d - 5) = 2.15$

Answer

$d = 16$

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

DISTANCE, RATE, AND TIME

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \quad \text{where} \quad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

SOLVE AN APPLICATION (WITH A FORMULA).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Example 9

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Try It 9

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Answer

7 miles

Example 10

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$d = rt$ $520 \stackrel{?}{=} 65 \cdot 8$ $520 = 520$✓	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

? Try It 10

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Answer

11 hours

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine, they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is vital to be familiar with formulas and be able to manipulate them easily.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

✓ Example 11

Solve the formula $d = rt$ for t:

1. when $d = 520$ and $r = 65$
2. in general

Solution

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

1. when $d = 520$ and $r = 65$		2. in general	
Write the formula.	$d = rt$	Write the formula.	$d = rt$
Substitute.	$520 = 65t$		
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$	Divide, to isolate tt.	$\frac{d}{r} = \frac{rt}{t}$
Simplify.	$8 = t$	Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Try It 11

Solve the formula $d = rt$ for r:

1. when $d = 780$ and $t = 12$
2. in general

Answer

1. $r = 65$
2. $r = \frac{d}{t}$

Example 12

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 90$ and $b = 15$
2. in general

Solution

1. when $A = 90$ and $b = 15$		2. in general	
Write the formula.	$A = \frac{1}{2}bh$	Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2}15h$	Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$	Simplify.	$2A = bh$
Solve for h.	$12 = h$	Solve for hh.	$\frac{2A}{b} = h$

We can now find the height of a triangle if we know the area and the base by using the formula $h = \frac{2A}{b}$

Try It 12

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 62$ and $h = 31$
2. in general

Answer

1. $b = 4$
2. $b = \frac{2A}{h}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually x and y. You might be given an equation that is solved for y and need to solve it for x, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

Example 13

Solve the formula $3x + 2y = 18$ for y:

1. when $x = 4$
2. in general

Solution

1. when $x = 4$		2. in general	
	$3x + 2y = 18$		$3x + 2y = 18$
Substitute.	$3(4) + 2y = 18$		
Subtract to isolate the y-term.	$12 - 12 + 2y = 18 - 12$	Subtract to isolate the y-term.	$3x - 3x + 2y = 18 - 3x$
Divide.	$\frac{2y}{2} = \frac{6}{2}$	Divide.	$\frac{2y}{2} = \frac{18}{2} - \frac{3x}{2}$
Simplify.	$y = 3$	Simplify.	$y = -\frac{3x}{2} + 9$

Try It 13

Solve the formula $5x + 2y = 18$ for y:

1. when $x = 4$
2. in general

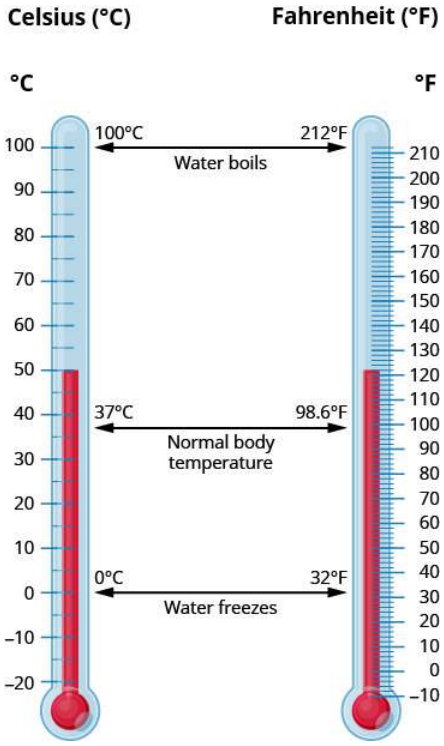
Answer

1. $y = -1$
2. $y = \frac{18-5x}{2}$

Convert between Fahrenheit and Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for 22°C, what does that mean?

The U.S. and metric systems use different scales to measure temperature. The U.S. system uses degrees Fahrenheit, written °F. The metric system uses degrees Celsius, written °C. The figure below shows the relationship between the two systems.



TEMPERATURE CONVERSION

To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula

$$C = \frac{5}{9}(F - 32)$$

To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula

$$F = \frac{9}{5}C + 32$$

Example 14

Convert 50° Fahrenheit into degrees Celsius.

Solution

We will substitute 50°F into the formula to find C.

	$C = \frac{5}{9}(F - 32)$
Substitute 50 for F.	$C = \frac{5}{9}(50 - 32)$
Simplify in parentheses.	$C = \frac{5}{9}(18)$
Multiply.	$C = 10$
	So we found that 50°F is equivalent to 10°C.

Try It 14

Convert the Fahrenheit temperature to degrees Celsius: 59° Fahrenheit.

Answer

15°C

Example 15

While visiting Paris, Woody noticed that the temperature was 20 °C. Convert the temperature into degrees Fahrenheit.

Solution

We will substitute 20°C into the formula to find F.

	$F=\frac{9}{5}C+32$
Substitute 20 for C.	$F=\frac{9}{5}(20)+32$
Multiply.	$F=36+32$
Add.	$F=68$
	So we found that 20°C is equivalent to 68°F.

Try It 15

Convert the Celsius temperature to degrees Fahrenheit: The temperature in Helsinki, Finland, was 15 °C.

Answer

59°F

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Section 2.3P: Practice

Practice Makes Progress

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

Exercise 1

$\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

Exercise 2

$\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

Exercise 3

$\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$

Exercise 4

$\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

Exercise 5

$\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

Exercise 6

$\frac{1}{5}(q+3) = \frac{1}{2}(q-3)$

Exercise 7

$\frac{1}{3}(x+5) = \frac{5}{6}$

Exercise 8

$\frac{3p+6}{3} = \frac{p}{2}$

Exercise 9

$\frac{d}{6} + 3 = \frac{d}{8} + 2$

Exercise 10

$\frac{3v-6}{2} + 5 = \frac{11v-4}{5}$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

Exercise 11

$2.1k + 3 = 7.2$

Exercise 12

$0.48x + 1.56 = 0.58x - 0.64$

Exercise 13

$0.05n + 0.10(n + 7) = 3.55$

Exercise 14

$0.05(q - 8) + 0.25q = 4.10$

Using Formulas as Equations

In the following exercises, use the $d = rt$ formula to solve.

Exercise 15

Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

Exercise 16

Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

Exercise 17

Megan is taking the bus from New York City to Montreal. The distance is 380 miles, and the bus travels at a steady rate of 76 miles per hour. How long will the bus ride be?

Exercise 18

Kareem wants to ride his bike from St. Louis to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

Exercise 19

Alejandra is driving to Cincinnati, 450 miles away. If she wants to arrive in 6 hours, at what rate must she drive?

Exercise 20

Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Exercise 21

Solve for t when $d = 240$ and $r = 60$.

In the following exercise, use the formula $A = \frac{1}{2}bh$

Exercise 22

Solve for h in general.

Exercise 23

Solve for h when $A = 176$ and $b = 22$

Solving Equations of Multiple Variables

In the following exercises, solve.

Exercise 24

Solve $180 = a + b + c$ for c

Exercise 25

Solve the formula $-5x + y = -1$ for y

Exercise 26

Solve the formula $3x + 2y = 11$ for y

Exercise 27

Solve the formula $x - y = -3$ for y

Exercise 28

Solve the formula $P = 2L + 2W$ for W

Exercise 29

Solve the formula $C = \pi d$ for π

Exercise 30

Solve the formula $V = LWH$ for H

Convert between Fahrenheit and Celsius Temperatures

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

Exercise 31

86° Fahrenheit

Exercise 32

0° Fahrenheit

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

? Exercise 33

−10° Celsius

? Exercise 34

22° Celsius

Everyday Math

? Exercise 35

Stamps: Paula bought $22.82 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 8 less than the number of 49-cent stamps. Solve the equation $0.49s + 0.21(s - 8) = 22.82$ for s, to find the number of 49-cent stamps Paula bought.

? Exercise 36

Converting temperature: Yon was visiting the United States, and he saw that the temperature in Seattle one day was $50°$. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the Celsius temperature.

Writing Exercises

? Exercise 37

If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

? Exercise 38

In the equation $0.35x + 2.1 = 3.85$ what is the LCD? How do you know?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

I can...	Confidently	With some help	No-I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 2.4: Linear Inequalities

Be Prepared

We will rely heavily on these skills throughout this section.

- Evaluate $2x + 5$ when $x = 3$
- Solve $2x + 5 = 3x - 7$ for x
- Identify which is larger, -7 or 9

Learning Objectives

Motivating Problem

Cesar is planning a 4-day trip to visit his friend at a college in another state. It will cost him $198 for airfare, $56 for local transportation, and $45 per day for food. He has $189 in savings and can earn $35 for each lawn he mows. How many lawns must he mow to have more than enough money to pay for the trip?

Fun Fact

Ancient Chinese mathematicians used *counting rods* to solve inequalities more than 2,000 years ago—long before the Western world formalized the idea. They didn't have the < or > symbols yet, but they understood the logic just as well!

The Goal

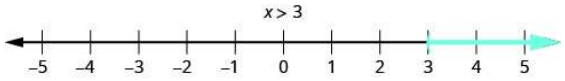
In this section, we will learn how to solve, graph, and interpret linear inequalities. We'll use strategies similar to solving equations, but we must be mindful of the direction of the inequality, especially when multiplying or dividing by a negative number. This skill helps model real-life limits and thresholds in contexts like budgeting, science, and planning.

Graph Inequalities on the Number Line

Do you remember what it means for a number to be a solution to an equation? A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

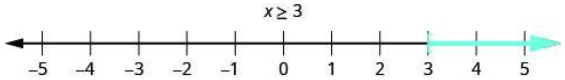
What about the solution of an inequality? What number would make the inequality $x > 3$ true? Are you thinking, 'x could be 4'? That's correct, but x could be 5 too, or 20, or even 3.001. Any number greater than 3 is a solution to the inequality $x > 3$.

We illustrate the solutions to the inequality $x > 3$ on a number line by shading in all the numbers to the right of 3, indicating that all numbers greater than 3 are solutions. Because the number 3 itself is not a solution, we put an open parenthesis at 3. The graph of $x > 3$ is shown below. Please note that the following convention is used: light blue arrows indicate the positive direction, and dark blue arrows indicate the negative direction.



The inequality $x > 3$ is graphed on this number line.

The graph of the inequality $x \geq 3$ is very much like the graph of $x > 3$, but now we need to show that 3 is a solution, too. We achieve this by placing a bracket at $x = 3$, as shown below.



The inequality $x \geq 3$ is graphed on this number line.

Notice that the open parentheses symbol, (, shows that the endpoint of the inequality is not included. The open bracket symbol, [, shows that the endpoint is included.

Example 1

Graph on the number line:

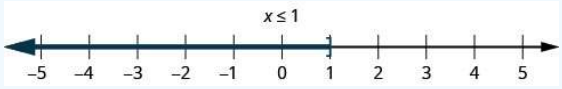
a. $x \leq 1$

b. $x < 5$

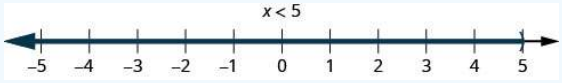
c. $x > -1$

Solution

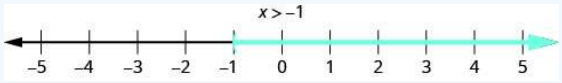
a. $x \leq 1$ This means all numbers less than or equal to 1. We shade in all the numbers on the number line to the left of 1 and put a bracket at $x = 1$ to show that it is included.



b. $x < 5$ This means all numbers less than 5, but not including 5. We shade in all the numbers on the number line to the left of 5 and put parentheses at $x = 5$ to show that it is not included.



c. $x > -1$ This means all numbers greater than −1, but not including −1. We shade in all the numbers on the number line to the right of −1, then put parentheses at $x = -1$ to show that it is not included.



? Try It 1

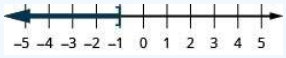
Graph on the number line:

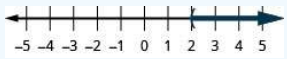
a. $x \leq -1$

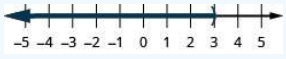
b. $x > 2$

c. $x < 3$

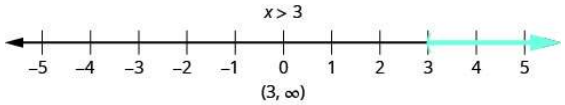
Answer

a. 

b. 

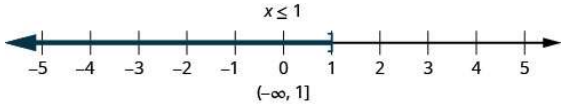
c. 

We can also represent inequalities using *interval notation.* As we saw above, the inequality $x > 3$ means all numbers greater than 3. There is no upper end to the solution to this inequality. In **interval notation**, we express $x > 3$ as $(3, \infty)$. The symbol ∞ is read as 'infinity'. It is not an actual number. The figure below shows both the number line and the interval notation.



The inequality $x > 3$ is graphed on this number line and written in interval notation.

The inequality $x \leq 1$ means all numbers less than or equal to 1. There is no lower end to those numbers. We write $x \leq 1$ in interval notation as $(-\infty, 1]$. The symbol $-\infty$ is read as 'negative infinity'. The figure below shows both the number line and interval notation.



The inequality $x \leq 1$ is graphed on this number line and written in interval notation.

INEQUALITIES, NUMBER LINES, AND INTERVAL NOTATION

Parentheses or brackets in the interval notation match the symbol at the endpoint of the arrow.



Example 2

Graph on the number line and write in interval notation.

a. $x \geq -3$

b. $x < 2.5$

c. $x \leq \frac{3}{5}$

Solution

a.

	$x \geq -3$
Shade to the right of −3, and put a bracket at −3.	
Write in interval notation.	[−3, ∞)

b.

	$x < 2.5$
Shade to the left of 2.5, and put a parenthesis at 2.5.	
Write in interval notation.	(−∞, 2.5)

c.

	$x \leq -\frac{3}{5}$
Shade to the left of $-\frac{3}{5}$, and put a bracket at $-\frac{3}{5}$.	
Write in interval notation.	$\left(-\infty, -\frac{3}{5}\right]$

Try It 2

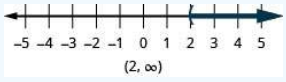
Graph on the number line and write in interval notation:

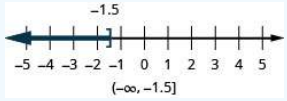
a. $x > 2$

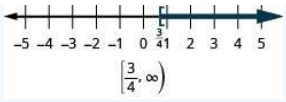
b. $x \leq -1.5$

c. $x \geq \frac{3}{4}$

Answer

a. 

b. 

c. 

Solve Inequalities using the Subtraction and Addition Properties of Inequality

The Subtraction and Addition Properties of Equality state that if two quantities are equal, when we add or subtract the same amount from both quantities, the results will be equal.

PROPERTIES OF EQUALITY

Subtraction Property of Equality	**Addition Property of Equality**
For any numbers $a, b,$ and c,	For any numbers $a, b,$ and c
if $a = b$,	if $a = b$
then $a - c = b - c$.	then $a + c = b + c$

Similar properties hold true for inequalities.

For example, we know that −4 is less than 2.	$-4 < 2$
If we subtract 5 from both quantities, is the left side still less than the right side?	$-4 - 5 \,?\, 2 - 5$
We get −9 on the left and −3 on the right.	$-9 \,?\, -3$
And we know −9 is less than −3.	$-9 < -3$
	The inequality sign stayed the same.

Similarly, we can also show that the inequality remains the same for addition.

This leads us to the Subtraction and Addition Properties of Inequality.

PROPERTIES OF INEQUALITY

Subtraction Property of Inequality	**Addition Property of Inequality**
For any numbers $a, b,$ and c,	For any numbers $a, b,$ and c
if $a < b$	if $a < b$
then $a - c < b - c$.	then $a + c < b + c$
if $a > b$	if $a > b$
then $a - c > b - c$.	then $a + c > b + c$

We use these properties to solve inequalities, taking the same steps we used to solve equations. Solving the inequality $x + 5 > 9$, the steps would look like this:

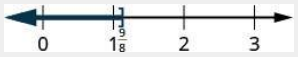
	$x + 5$	$>$	9
Subtract 5 from both sides to isolate x.	$x + 5 - 5$	$>$	$9 - 5$
	x	$>$	4

Any number greater than 4 is a solution to this inequality.

Example 3

Solve the inequality $n - \frac{1}{2} \leq \frac{5}{8}$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$n-\frac{1}{2}\leq\frac{5}{8}$
Add $\frac{1}{2}$ to both sides of the inequality.	$n-\frac{1}{2}+\frac{1}{2}\leq\frac{5}{8}+\frac{1}{2}$
Simplify.	$n\leq\frac{9}{8}$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty,\frac{9}{8}]$

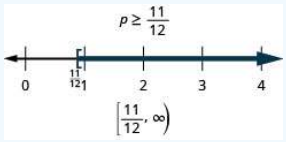
Try It 3

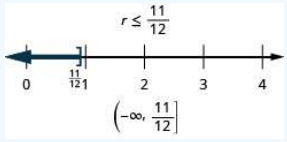
Solve each inequality, graphing the solution on the number line, and writing the solution in interval notation.

a. $p-\frac{3}{4}\geq\frac{1}{6}$

b. $r-\frac{1}{3}\leq\frac{7}{12}$

Answer

a. 

b. 

Solve Inequalities using the Division and Multiplication Properties of Inequality

The Division and Multiplication Properties of Equality state that if two quantities are equal, when we divide or multiply both quantities by the same amount, the results will also be equal (provided we don't divide by 0).

PROPERTIES OF EQUALITY

Division Property of Equality	**Mutiplication Property of Equality**
For any numbers a, b, c, and c ≠ 0	**For any numbers a, b, c**
if $a=b$	**if** $a=b$
then $\frac{a}{c}=\frac{b}{c}$	**then** $ac=bc$

Are there similar properties for inequalities? What happens to an inequality when we divide or multiply both sides by a constant?

Consider some numerical examples where we multiply or divide by a positive number.

	$10<15$		$10<15$
Divide both sides by 5.	$\frac{10}{5}\,?\,\frac{15}{5}$	Multiply both sides by 5.	$10(5)\,?\,15(5)$
Simplify.	$2\,?\,3$		$50\,?\,75$
Fill in the inequality signs.	$2<3$		$50<75$

The inequality signs stayed the same.

Does the inequality stay the same when we divide or multiply by a negative number?

	$10 < 15$		$10 < 15$
Divide both sides by -5.	$\frac{10}{-5} \; ? \; \frac{15}{-5}$	Multiply both sides by -5.	$10(-5) \; ? \; 15(-5)$
Simplify.	$-2 \; ? \; -3$		$-50 \; ? \; -75$
Fill in the inequality signs.	$-2 > -3$		$-50 > -75$

The inequality signs reversed their direction.

When we divide or multiply an inequality by a positive number, the inequality sign stays the same. When we divide or multiply an inequality by a negative number, the direction of the inequality sign reverses.

Here are the Division and Multiplication Properties of Inequality for easy reference.

DIVISION AND MULTIPLICATION PROPERTIES OF INEQUALITY

For any real numbers a, b, c

$$\text{if } a < b \text{ and } c > 0, \text{ then } \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc$$
$$\text{if } a > b \text{ and } c > 0, \text{ then } \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc$$
$$\text{if } a < b \text{ and } c < 0, \text{ then } \frac{a}{c} > \frac{b}{c} \text{ and } ac > bc$$
$$\text{if } a > b \text{ and } c < 0, \text{ then } \frac{a}{c} < \frac{b}{c} \text{ and } ac < bc$$

When we **divide or multiply** an inequality by a:

- If the number is **positive**, the inequality stays the **same**.
- If the number is **negative**, the inequality **reverses**.

Example 4

Solve the inequality $7y < 42$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$7y < 42$
Divide both sides of the inequality by 7. Since $7 > 0$, the inequality stays the same.	$\frac{7y}{7} < \frac{42}{7}$
Simplify.	$y < 6$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, 6)$

Try It 4

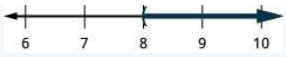
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

a. $9c > 72$

b. $12d \leq 60$

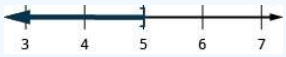
Answer

a. $c > 8$



$(8, \infty)$

b. $d \leq 5$



$(-\infty, 5]$

Example 5

Solve the inequality $-10a \geq 50$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$-10a \geq 50$
Divide both sides of the inequality by −10. Since $-10 < 0$, the inequality reverses.	$\frac{-10a}{-10} \leq \frac{50}{-10}$
Simplify.	$a \leq -5$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -5]$

Try It 5

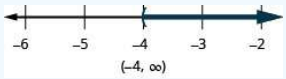
Solve each inequality, graph the solution on the number line, and write the solution in interval notation.

a. $-8q < 32$

b. $-7r \leq -70$

Answer

a. $q > -4$



b. $r \geq 10$



SOLVING INEQUALITIES

Sometimes, when solving an inequality, the variable ends up on the right side. We can rewrite the inequality in reverse to get the variable to the left.

$$x > a \text{ has the same meaning as } a < x$$

Think about it as "If Xavier is taller than Alex, then Alex is shorter than Xavier."

Example 6

Solve the inequality $-20 < \frac{4}{5}u$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$-20 < \frac{4}{5}u$
Multiply both sides of the inequality by $\frac{5}{4}$. Since $\frac{5}{4} > 0$, the inequality stays the same.	$\frac{5}{4}(-20) < \frac{5}{4}\left(\frac{4}{5}u\right)$
Simplify.	$-25 < u$
Rewrite the variable on the left.	$u > -25$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-25, \infty)$

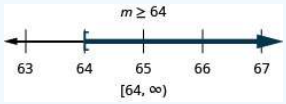
Try It 6

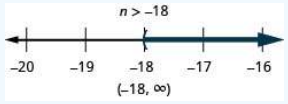
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

a. $24 \leq \frac{3}{8}m$

b. $-24 < \frac{4}{3}n$

Answer

a. 

b. 

Example 7

Solve the inequality $\frac{t}{-2} \geq 8$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$\frac{t}{-2} \geq 8$
Multiply both sides of the inequality by −2. Since $-2 < 0$, the inequality reverses.	$-2\left(\frac{t}{-2}\right) \leq -2(8)$
Simplify.	$t \leq -16$
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty, -16]$

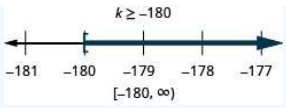
Try It 7

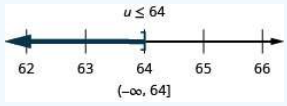
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

a. $\frac{k}{-12} \leq 15$

b. $\frac{u}{-4} \geq -16$

Answer

a. 

b. 

Solve Inequalities That Require Simplification

Most inequalities will take more than one step to solve. We follow the same steps we used in the general strategy for solving linear equations, but be sure to pay close attention during multiplication or division.

Example 8

Solve the inequality $4m \leq 9m + 17$, graph the solution on the number line, and write the solution in interval notation.

Solution

	$4m \leq 9m + 17$
Subtract 9m from both sides to collect the variables on the left.	$4m - 9m \leq 9m - 9m + 17$
Simplify.	$-5m \leq 17$
Divide both sides of the inequality by −5, and reverse the inequality.	$\frac{-5m}{-5} \geq \frac{17}{-5}$
Simplify.	$m \geq -\frac{17}{5}$
Graph the solution on the number line.	
Write the solution in interval notation.	$\left[-\frac{17}{5}, \infty\right)$

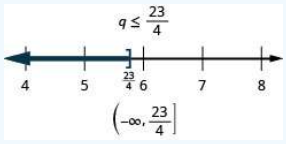
Try It 8

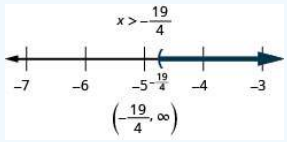
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

a. $3q \geq 7q - 23$

b. $6x < 10x + 19$

Answer

a. 

b. 

Example 9

Solve the inequality $8p + 3(p - 12) > 7p - 28$, graph the solution on the number line, and write the solution in interval notation.

Solution

Simplify each side as much as possible.	$8p+3(p-12)>7p-28$
Distribute.	$8p+3p-36>7p-28$
Combine like terms.	$11p-36>7p-28$
Subtract $7p$ from both sides to collect the variables on the left.	$11p-36-7p>7p-28-7p$
Simplify.	$4p-36>-28$
Add 36 to both sides to collect the constants on the right.	$4p-36+36>-28+36$
Simplify.	$4p>8$
Divide both sides of the inequality by 4; the inequality stays the same.	$\frac{4p}{4}>84$
Simplify.	$p>2$
Graph the solution on the number line.	
Write the solution in interval notation.	$(2,\infty)$

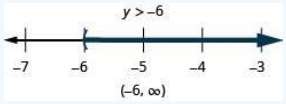
Try It 9

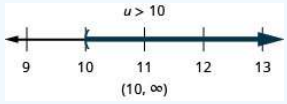
Solve the inequality, graph the solution on the number line, and write the solution in interval notation.

a. $9y+2(y+6)>5y-24$

b. $6u+8(u-1)>10u+32$

Answer

a. 

b. 

Just as some equations are identities and some are contradictions, inequalities can also be identities or contradictions. We recognize these forms when we are left with only constants as we solve the inequality. If the result is a true statement, we have an **identity**. If the result is a false statement, we have a **contradiction**.

Example 10

Solve the inequality $8x-2(5-x)<4(x+9)+6x$, graph the solution on the number line, and write the solution in interval notation.

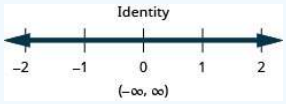
Solution

Simplify each side as much as possible.	$8x-2(5-x)<4(x+9)+6x$
Distribute.	$8x-10+2x<4x+36+6x$
Combine like terms.	$10x-10<10x+36$
Subtract 10x from both sides to collect the variables on the left.	$10x-10-10x<10x+36-10x$
Simplify.	$-10<36$
The x's are gone, and we have a true statement.	The inequality is an identity. The solution is all real numbers.
Graph the solution on the number line.	
Write the solution in interval notation.	$(-\infty,\infty)$

Try It 10

Solve the inequality $4b-3(3-b)>5(b-6)+2b$, graph the solution on the number line, and write the solution in interval notation.

Answer



Example 11

Solve the inequality $\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$, graph the solution on the number line, and write the solution in interval notation.

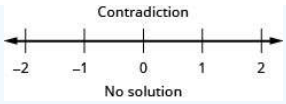
Solution

	$\frac{1}{3}a-\frac{1}{8}a>\frac{5}{24}a+\frac{3}{4}$
Multiply both sides by the LCD, 24, to clear the fractions.	$24\left(\frac{1}{3}a-\frac{1}{8}a\right)>24\left(\frac{5}{24}a+\frac{3}{4}\right)$
Simplify.	$8a-3a>5a+18$
Combine like terms.	$5a>5a+18$
Subtract 5a from both sides to collect the variables on the left.	$5a-5a>5a-5a+18$
Simplify.	$0>18$
The statement is false!	The inequality is a contradiction.
	There is no solution.
Graph the solution on the number line.	
Write the solution in interval notation.	There is no solution.

Try It 11

Solve the inequality $\frac{1}{4}x-\frac{1}{12}x>\frac{1}{6}x+\frac{7}{8}$, graph the solution on the number line, and write the solution in interval notation.

Answer



Translate to an Inequality and Solve

To translate English sentences into inequalities, we need to recognize the phrases that indicate the inequality. Some words are easy, like ‘more than’ and ‘less than’. But others are not as obvious.

Think about the phrase ‘at least’ – what does it mean to be ‘at least 21 years old’? It means 21 or more. The phrase 'at least’ is the same as ‘greater than or equal to’.

The following table shows some common phrases that indicate inequalities.

>	≥	<	≤
" class="lt-math-186559">is greater than	is greater than or equal to	is less than	is less than or equal to
" class="lt-math-186559">is more than	is at least	is smaller than	is at most
" class="lt-math-186559">is larger than	is no less than	has fewer than	is no more than
" class="lt-math-186559">exceeds	is the minimum	is lower than	is the maximum

Example 12

Translate and solve. Then write the solution in interval notation.

Twelve times c is no more than 96.

Solution

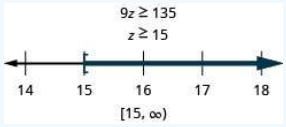
Translate.	Twelve times c is no more than 96 $12c \le 96$
Solve—divide both sides by 12.	$\frac{12c}{12} \le \frac{96}{12}$
Simplify.	$c \le 8$
Write in interval notation.	$(-\infty, 8]$

Try It 12

Translate and solve. Then write the solution in interval notation and graph on the number line.

Nine times z is not less than 135.

Answer



Example 13

Translate and solve. Then write the solution in interval notation and graph on the number line.

Thirty less than x is at least 45.

Solution

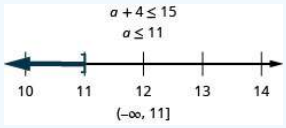
Translate.	Thirty less than x is at least 45. $x - 30 \geq 45$
Solve—add 30 to both sides.	$x - 30 + 30 \geq 45 + 30$
Simplify.	$x \geq 75$
Write in interval notation.	$[75, \infty)$
Graph on the number line.	

Try It 13

Translate and solve. Then write the solution in interval notation and graph on the number line.

Four more than a is at most 15.

Answer



Solve Applications with Linear Inequalities

Many real-life situations require us to solve inequalities. In fact, inequality applications are so common that we often do not even realize we are doing algebra. For example, how many gallons of gas can be put in the car for $20? Is the rent on an apartment affordable? Is there enough time before class to go get lunch, eat it, and return to class? How much money should each family member's holiday gift cost without going over budget?

The method we will use to solve applications involving linear inequalities is very similar to the one we used when solving applications with equations. We will read the problem and make sure all the words are understood. Next, we will identify what we are looking for and assign a variable to represent it. We will restate the problem in a single sentence to facilitate its translation into an inequality. Then, we will solve the inequality.

Example 14

Emma has landed a new job and will need to relocate. Her monthly income will be $5,265. To qualify for an apartment rental, Emma's monthly income must be at least three times the rent amount. What is the highest rent Emma will qualify for?

Solution

Step 1. Read the problem.		
Step 2. Identify what we are looking for.	the highest rent Emma will qualify for	
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let r = rent	
Step 4. Translate into an inequality.	Emma's monthly income must be at least three times the rent.	
Step 5. Solve the inequality. Remember, $a > x$ has the same meaning as $x < a$	$5265 \geq 3r$ $1755 \geq r$ $r \leq 1755$	(Section 2.4.1)
Step 6. Check the answer in the problem and make sure it makes sense.	A maximum rent of \$1,755 seems reasonable for an income of \$5,265.	
Step 7. Answer the question in a complete sentence.	The maximum rent is \$1,755.	

? Try It 14

Alan is loading a pallet with boxes that each weigh 45 pounds. The pallet can safely support no more than 900 pounds. How many boxes can he safely load onto the pallet?

Answer

There can be no more than 20 boxes.

Sometimes an application requires the solution to be a whole number, but the algebraic solution to the inequality is not a whole number. In that case, we must round the algebraic solution to a whole number. The context of the application will determine whether we round up or down. To check applications like this, we will round our answer to a number that is easy to compute with and ensure that this number makes the inequality true.

✓ Example 15

Dawn won a mini-grant of \$4,000 to buy tablet computers for her classroom. The tablets she would like to buy cost \$254.12 each, including tax and delivery. What is the maximum number of tablets Dawn can buy?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the maximum number of tablets Dawn can buy
Step 3. Name what we are looking for.	
Choose a variable to represent that quantity.	Let n = the number of tablets.
Step 4. Translate. Write a sentence that	
gives the information to find it.	254.12 times the number of tablets is no more than $4000.
Translate into an inequality.	$254.12n \leq 4000$
Step 5. Solve the inequality.	$n \leq 15.74$
But n must be a whole number of tablets, so round to 15.	$n \leq 15$
Step 6. Check the answer in the problem and make sure it makes sense.	Rounding down the price to $250, 15 tablets would cost $3750, while 16 tablets would be $4000. So a maximum of 15 tablets at $254.12 seems reasonable.
Step 7. Answer the question in a complete sentence.	Dawn can buy a maximum of 15 tablets.

Try It 15

Angie has $20 to spend on juice boxes for her son's preschool picnic. Each pack of juice boxes costs $2.63. What is the maximum number of packs she can buy?

Answer

She can buy at most seven packs.

Example 16

Sergio and Lizeth have a tight vacation budget. They plan to rent a car from a company that charges $75 a week plus $0.25 a mile. How many miles can they travel and still keep within their $200 budget?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of miles Sergio and Lizeth can travel
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let m = the number of miles.
Step 4. Translate. Write a sentence that gives the information to find it.	$\$75 + 0.25m$ is less than or equal to $\$200$.
Step 5. Solve the inequality.	$0.25m \leq 125$ $m \leq 500$ miles
Step 6. Check the answer in the problem and make sure it makes sense.	Yes, $75 + 0.25(500) = 200$.
Step 7. Answer the question in a complete sentence.	Sergio and Lizeth can travel 500 miles and still stay on budget.

(Section 2.4.2)

? Try It 16

Taleisha's phone plan costs her $28.80 a month plus $0.20 per text message. How many text messages can they use and keep their monthly phone bill no more than $50?

Answer

They can use no more than 106 text messages a month.

A common goal of most businesses is to make a profit. *Profit* is the money that remains when the expenses have been subtracted from the money earned. In the following example, we will determine the number of jobs a small businessman needs to complete each month to achieve a certain level of **profit**.

✓ Example 17

Elliot has a landscape maintenance business. His monthly expenses are $1,100. If he charges $60 per job, how many jobs must he do to earn a profit of at least $4,000 a month?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number of jobs Elliot needs
Step 3. Name what we are looking for. Choose a variable to represent it.	Let j = the number of jobs.
Step 4. Translate. Write a sentence that gives the information to find it.	$60 times the number of jobs minus $1,100 is at least $4,000.
Step 5. Solve the inequality.	$60j - 1100 \geq 4000$ $60j \geq 5100$ $j \geq 85$ jobs
Step 6. Check the answer in the problem and make sure it makes sense.	If Elliot did 90 jobs, his profit would be $60(90) - 1100 = \$4,300$, which is more than $4,000.
Step 7. Answer the question in a complete sentence.	Elliot must work at least 85 jobs.

(Section 2.4.3)

Try It 17

Caleb has a pet sitting business. He charges $32 per hour. His monthly expenses are $2272. How many hours must he work to earn a profit of at least $800 per month?

Answer

He must work at least 96 hours.

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Section 2.4P: Practice

Practice Makes Progress

Graph Inequalities on the Number Line

In the following exercises, graph each inequality on the number line

Exercise 1

$x > 1$

Exercise 2

$x < -2$

Exercise 3

$x \geq -3$

In the following exercises, graph each inequality on the number line and write in interval notation.

Exercise 4

$x > 3$

Exercise 5

$x \leq -0.5$

Exercise 6

$x \geq \frac{1}{3}$

Solve Inequalities using the Subtraction and Addition Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

Exercise 7

$m - 45 \leq 62$

Exercise 8

$g - \frac{11}{12} < -\frac{5}{18}$

Solve Inequalities using the Division and Multiplication Properties of Inequality

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

Exercise 9

$6y < 48$

Exercise 10

$-8v \leq 96$

Exercise 11

$40 < \frac{5}{8}k$

Exercise 12

$\frac{b}{-10} \geq 30$

Exercise 13

$7s < -28$

Exercise 14

$\frac{3}{5}x \leq -45$

Solve Inequalities That Require Simplification

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

Exercise 15

$5u \leq 8u - 21$

Exercise 16

$9y + 5(y + 3) < 4y - 35$

Exercise 17

$6n - 12(3 - n) \leq 9(n - 4) + 9n$

Exercise 18

$\frac{5}{6}a - \frac{1}{4}a > \frac{7}{12}a + \frac{2}{3}$

Exercise 19

$12v + 3(4v - 1) \leq 19(v - 2) + 5v$

Mixed practice

In the following exercises, solve each inequality, graph the solution on the number line, and write the solution in interval notation.

Exercise 20

$35k \geq -77$

Exercise 21

$-\frac{21}{8}y \leq -\frac{15}{28}$

Exercise 22

$\frac{n}{13} \leq -6$

Translate to an Inequality and Solve

In the following exercises, translate and solve the given problems. Then write the solution in interval notation and graph on the number line.

Exercise 23

Six more than k exceeds 25

Exercise 24

Negative two times s is lower than 56

Exercise 25

Fifteen less than a is at least -7

Everyday Math

Exercise 26

Order Size Arleen got a $20 gift card for the coffee shop. Her favorite iced drink costs $3.79. What is the maximum number of drinks she can buy with the gift card?

Exercise 27

Paychecks Keshad earns $2,400 per month, plus 6% of his sales. His brother earns $3,300 per month. For what amount of total sales will Keshad's monthly pay be higher than his brother's monthly pay?

Exercise 28

Water Bill Jake's water bill is $24.80 per month, plus $2.20 per 100 cubic feet (ccf) of water. What is the maximum number of ccf Jake can use if he wants his bill to be no more than $60?

Exercise 29

Savings Moshde runs a hairstyling business from her home. She charges $45 for a haircut and style. Her monthly expenses are $960. She wants to be able to put at least $1,200 per month into her savings account to open her own salon. How many "cut & styles" must she do to save at least $1,200 per month?

Exercise 30

Student Trip Five student government officers wish to attend the state convention. It will cost them $110 for registration, $375 for transportation and food, and $42 per person for the hotel. There is $450 budgeted for the convention in the student

government savings account. They can earn the rest of the money they need by having a car wash. If they charge $5 per car, how many cars must they wash in order to have enough money to pay for the trip?

Exercise 31

Fighter pilots The maximum height, *h*, of a fighter pilot is 77 inches. Write this as an inequality.

Exercise 32

Wedding Budget Adele and Walter found the perfect venue for their wedding reception. The cost is $9,850 for up to 100 guests, plus $38 for each additional guest. How many guests can attend if Adele and Walter want the total cost to be no more than $12,500?

Writing Exercises

Exercise 33

Give an example from your life using the phrase 'at most'.

Exercise 34

Explain why it is necessary to reverse the inequality when solving $\frac{n}{-3} < 12$.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph inequalities on the number line.			
solve inequalities using the Subtraction and Addition Properties of Inequality.			
solve inequalities using the Division and Multiplication Properties of Inequality.			
solve inequalities that require simplification.			
translate to an inequality and solve.			

I can...	Confidently	With some help	No-I don't get it!
solve applications with linear inequalities.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 3: Graphing Relationships

In this unit, we'll bring algebra to life by visualizing relationships on the coordinate plane. We'll graph equations and inequalities, explore slope as a rate of change, and write equations to model patterns we see in real-world data. We'll also interpret graphs to make predictions and decisions. Through these activities, we'll connect algebraic thinking with geometry and strengthen our ability to communicate mathematical ideas visually.

Section 3.1: Cartesian Plane

Section 3.1P: Practice

Section 3.2: Graph Linear Equations in Two Variables

Section 3.2P: Practice

Section 3.3: Intercepts

Section 3.3P: Practice

Section 3.4: Understanding the Slope of a Line

Section 3.4P: Practice

Section 3.5: Slope-Intercept Form of Line

Section 3.5P: Practice

Section 3.6: Line Equations

Section 3.6P: Practice

Section 3.7: Linear Inequalities in Two Variables

Section 3.7P: Practice

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Section 3.1: Cartesian Plane

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify: $-3+7$
- Identify the opposite of -5
- Solve: $x-4=9$
- Evaluate $2x+1$ when $x=-2$

Learning Objectives

Motivating Problem

You're using a map app to meet a friend. They tell you they're 3 blocks east and 2 blocks north of the coffee shop. If the shop is at $0,0$, how can you describe exactly where they are—and in what quadrant of the coordinate plane their location falls?

Fun Fact

The Cartesian Plane is named after **René Descartes**, a French philosopher and mathematician. Legend says he developed the idea while watching a fly crawl across his ceiling and wondering how to describe its location with numbers!

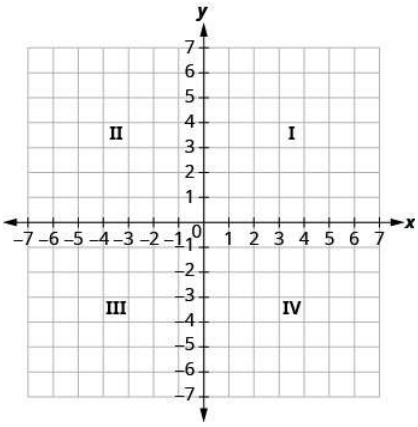
The Goal

In this section, we will learn how to locate and label points on a coordinate plane using ordered pairs. We'll also practice identifying quadrants, understanding axes, and interpreting visual information, which is foundational for graphing equations.

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane, the coordinate plane, or the Cartesian Plane.

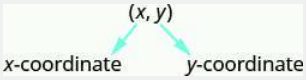
The horizontal number line is called the *x-axis*, and the vertical number line is called the *y-axis*. Together, the *x*-axis and the *y*-axis form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise.



In the **rectangular coordinate system**, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

ORDERED PAIR

An **ordered pair**, (x, y), gives the coordinates of a point in a rectangular coordinate system.



The first number is the *x*-coordinate.

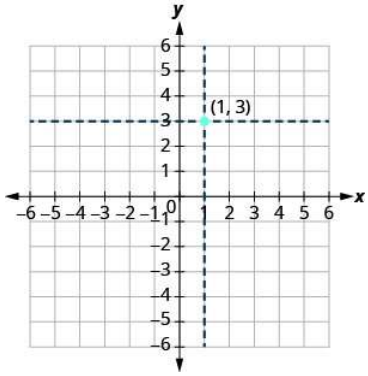
The second number is the *y*-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

THE ORIGIN

The point $(0, 0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet — that is the point with coordinates $(1, 3)$.



Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

Example 1

Plot each point in the rectangular coordinate system and identify the **quadrant** in which the point is located:

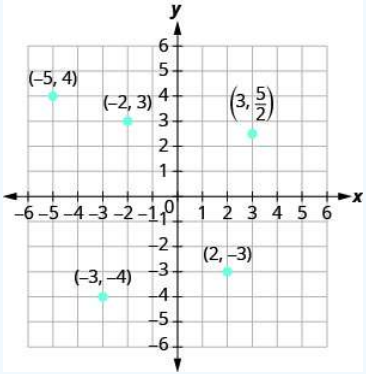
a. (−5,4)
b. (−3,−4)
c. (2,−3)
d. (−2,3)
e. $(3, \frac{5}{2})$

Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

a. Since x=−5, the point is to the left of the *y*-axis. Also, since y=4, the point is above the *x*-axis. The point (−5,4) is in Quadrant II.
b. Since x=−3, the point is to the left of the *y*-axis. Also, since y=−4, the point is below the *x*-axis. The point (−3,−4) is in Quadrant III.
c. Since x=2, the point is to the right of the *y*-axis. Since y=−3, the point is below the *x*-axis. The point (2,−3) is in Quadrant IV.
d. Since x=−2, the point is to the left of the *y*-axis. Since y=3, the point is above the *x*-axis. The point (−2,3) is in Quadrant II.

e. Since x=3, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.



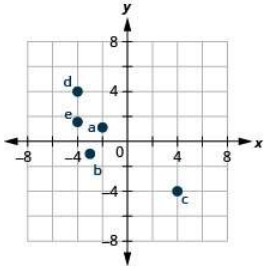
Try It 1

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

a. (−2,1)
b. (−3,−1)
c. (4,−4)
d. (−4,4)
e. $(-4, \frac{3}{2})$

Answer

The point a (-2,1) is in Quadrant II.
The point b (-3,-1) is in Quadrant III.
The point c (4,-4) is in Quadrant IV.
The point d (-4,4) is in Quadrant II.
The point e $(-4, \frac{3}{2})$ is in Quadrant II.



How do the signs affect the location of the points? As you graphed the points in the previous example, you may have noticed some patterns.

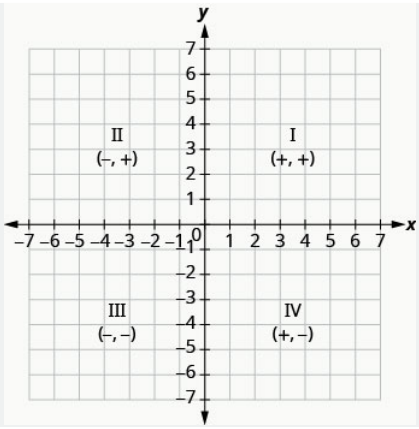
For the point in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point (−2,5) is located? In which quadrant is (2,−5) located?

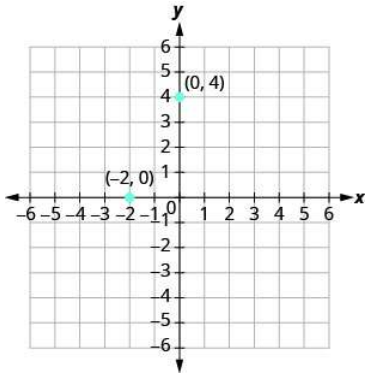
QUADRANTS

We can summarize the sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$



What if one coordinate is zero, as shown in the figure below? Where is the point (0,4) located? Where is the point (−2,0) located?



The point (0,4) is on the *y*-axis and the point (−2,0) is on the *x*-axis.

POINTS ON THE AXES

Points with a ***y*-coordinate** equal to 0 are on the *x*-axis, and have coordinates (a,0).

Points with an ***x*-coordinate** equal to 0 are on the *y*-axis, and have coordinates (0,b).

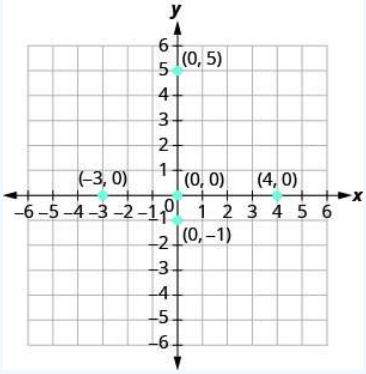
Example 2

Plot each point:

1. (0,5)
2. (4,0)
3. (−3,0)
4. (0,0)
5. (0,−1)

Solution

1. Since x=0, the point whose coordinates are (0,5) is on the *y*-axis.
2. Since y=0, the point whose coordinates are (4,0) is on the *x*-axis.
3. Since y=0, the point whose coordinates are (−3,0) is on the *x*-axis.
4. Since x=0 and y=0, the point whose coordinates are (0,0) is the origin.
5. Since x=0, the point whose coordinates are (0,−1) is on the *y*-axis.

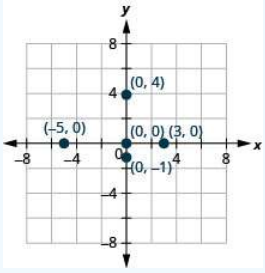


Try It 2

Plot each point:

1. (−5,0)
2. (3,0)
3. (0,0)
4. (0,−1)
5. (0,4).

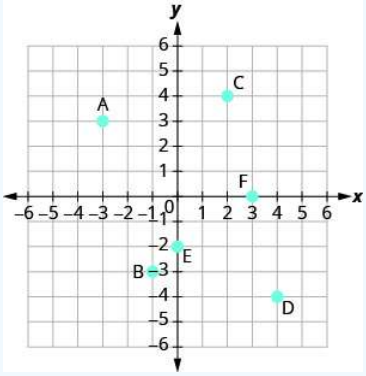
Answer



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the **ordered pair,** use the correct order, (x,y).

Example 3

Name the ordered pair of each point shown in the rectangular coordinate system.



Solution

Point A: Point A is above −3 on the *x*-axis, so the *x*-coordinate is −3. It is to the left of 3 on the *y*-axis, so the *y*-coordinate is 3. The coordinates of the point are (−3, 3).

Point B: Point B is below −1 on the *x*-axis, so the *x*-coordinate is −1. It is to the left of −3 on the *y*-axis, so the *y*-coordinate is −3. The coordinates of the point are (−1, −3).

Point C: Point C is above 2 on the *x*-axis, so the *x*-coordinate is 2. It is to the right of 4 on the *y*-axis, so the *y*-coordinate is 4. The coordinates of the point are (2, 4).

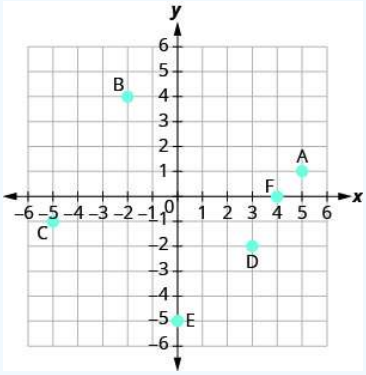
Point D: Point D is below 4 on the *x*-axis, so the *x*-coordinate is 4. It is to the right of −4 on the *y*-axis, so the *y*-coordinate is −4. The coordinates of the point are (4, −4).

Point E: Point E is on the *y*-axis at y = −2. The coordinates of point E are (0, −2).

Point F: Point F is on the *x*-axis at x = 3. The coordinates of point F are (3, 0).

? Try It 3

Name the ordered pair of each point shown in the rectangular coordinate system.



Answer

A: (5,1) B: (−2,4) C: (−5,−1) D: (3,−2) E: (0,−5) F: (4,0)

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like x=4. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x+5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$ (Section 3.1.1)

But equations can have more than one variable. Equations with two variables may be of the form Ax+By=C. Equations of this form are called **linear equations in two variables**.

LINEAR EQUATION

An equation of the form Ax+By=C, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$Ax + By = C$$
$$x + 4y = 8$$
$$A = 1,\ B = 4,\ C = 8$$

The equation $y = -3x + 5$ is also a **linear equation**. But it does not appear to be in the form Ax+By=C. We can use the Addition Property of Equality and rewrite it in Ax+By=C form.

	y	$=$	$-3x + 5$
Add to both sides.	$y + 3x$	$=$	$-3x + 5 + 3x$
Simplify.	$y + 3x$	$=$	5
Use the Commutative Property to put it in $Ax + By = C$ form.	$3x + y$	$=$	5

By rewriting $y = -3x + 5$ as $3x + y = 5$, we can easily see that it is a linear equation in two variables because it is of the form Ax+By=C. When an equation is in the form Ax+By=C, we say it is in *standard form*.

STANDARD FORM OF LINEAR EQUATION

A linear equation is in standard form when it is written Ax+By=C.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x,y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution OF A LINEAR EQUATION IN TWO VARIABLES

An **ordered pair** (x,y) is a **solution** of the linear equation Ax+By=C, if the equation is a true statement when the x- and y-values of the ordered pair are substituted into the equation.

Example 4

Determine which ordered pairs are solutions to the equation $x + 4y = 8$.

(a) (0,2)

(b) (2,−4)

(c) (−4,3)

Solution

Substitute the *x*- and *y*-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \stackrel{?}{=} 8$	$2 + 4(-4) \stackrel{?}{=} 8$	$-4 + 4 \cdot 3 \stackrel{?}{=} 8$
$0 + 8 \stackrel{?}{=} 8$	$2 + (-16) \stackrel{?}{=} 8$	$-4 + 12 \stackrel{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Try It 4

Which of the following ordered pairs are solutions to the equation $4x - y = 8$?

1. (0,8)
2. (2,0)
3. (1,−4)

Answer

$(2, 0)$ and $(1, -4)$ are solutions to the equation $4x - y = 8$.

Example 5

Which of the following ordered pairs are solutions to the equation $y = 5x - 1$?

(a) (0,−1)

(b) (1,4)

(c) (−2,−7)

Solution

Substitute the *x*- and *y*-values from each **ordered pair** into the equation and determine if it results in a true statement.

(a)	(b)	(c)
(0, –1)	(1, 4)	(–2, –7)
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
(0, –1) is a solution.	(1, 4) is a solution.	(–2, –7) is not a solution.

Try It 5

Which of the following ordered pairs are solutions to the equation $y = 4x - 3$?

1. (0,3)
2. (1,1)
3. (−1,−1)

Answer

$(1, 1)$ is a solution to the equation $y = 4x - 3$.

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*- and *y*-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation $y = 5x - 1$ that we found in Example 5. We can summarize this information in the table of solutions below.

$y = 5x - 1$		
x	y	(x, y)
0	−1	(0,−1)
1	4	(1,4)

To find a third solution, we'll select another value for x. Let's let $x = 2$, and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

The ordered pair (2,9) is a solution to $y = 5x - 1$. We will add it to our table.

$y = 5x - 1$		
x	y	(x, y)
0	−1	(0,−1)
1	4	(1,4)
2	9	(2,9)

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Example 6

Complete the table to find three solutions to the equation $y = 4x - 2$.

$y = 4x - 2$		
x	y	(x, y)
0		
−1		
2		

Solution

Substitute $x = 0$, $x = -1$, and $x = 2$ into $y = 4x - 2$.

$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = 0 - 2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in the following table.

$y = 4x - 2$		
x	y	(x, y)
0	−2	(0,−2)
−1	−6	(−1,−6)
2	6	(2,6)

Try It 6

Complete the table to find three solutions to the equation $y = 3x - 1$.

$y = 3x - 1$		
x	y	(x, y)
0		
−1		
2		

Answer

$y = 3x - 1$		
x	y	(x, y)
0	-1	(0, -1)
−1	-4	(-1, -4)
2	5	(2, 5)

Example 7

Complete the following table to find three solutions to the equation $5x - 4y = 20$.

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution

Substitute the given value into the equation $5x - 4y = 20$ and solve for the other variable. Then, fill in the values in the table.

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in the table below.

$5x-4y=20$		
x	y	(x,y)
0	−5	(0,−5)
4	0	(4,0)
8	5	(8,5)

Try It 7

Complete the table to find three solutions to the equation $2x-5y=20$.

$2x-5y=20$		
x	y	(x,y)
	-4	
	0	
-5		

Answer

$2x-5y=20$		
x	y	(x,y)
0	−4	(0,−4)
10	0	(10,0)
−5	−6	(−5,−6)

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable, choosing a number that's easy to work with is a good idea.

When the equation is in *y*-form, with *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Example 8

Find three solutions to the equation $y=-3x+2$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in y-form, it will be easier to substitute in values of x. Let's pick $x = 0$, $x = 1$, and $x = -1$.

	$x = 0$	$x = 1$	$x = -1$
	$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$
Substitute the value into the equation.	$y = -3 \cdot 0 + 2$	$y = -3 \cdot 1 + 2$	$y = -3(-1) + 2$
Simplify.	$y = 0 + 2$	$y = -3 + 2$	$y = 3 + 2$
Simplify.	$y = 2$	$y = -1$	$y = 5$
Write the ordered pair.	(0, 2)	(1, -1)	(-1, 5)
Check.			
$y=-3x+2$	$y=-3x+2$	$y=-3x+2$	
$2 \stackrel{?}{=} -3 \cdot 0 + 2$	$-1 \stackrel{?}{=} -3 \cdot 1 + 2$	$5 \stackrel{?}{=} -3(-1) + 2$	
$2 \stackrel{?}{=} 0 + 2$	$-1 \stackrel{?}{=} -3 + 2$	$5 \stackrel{?}{=} -3 + 2$	
$2 = 2$✓	$-1 = -1$✓	$5 = 5$✓	

So, (0,2), (1,−1) and (−1,5) are all solutions to $y = -3x + 2$. These solutions are summarized in the table below.

$y = -3x + 2$		
x	y	(x, y)
0	2	(0,2)
1	−1	(1,−1)
−1	5	(−1,5)

? Try It 8

Find three solutions to the equation $y = -2x + 3$.

Answer

Answers will vary. Answers might include (0,3), (1,1) and (−1,5).

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when $x = 0$, find a second solution when y=0, and then find a third solution.

✓ Example 9

Find three solutions to the equation $3x + 2y = 6$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first $x = 0$, then $y = 0$, and then find a third point.

	$x = 0$	$y = 0$	$x = 1$

Substitute the value into the equation.	$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.	$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.	$2y=6$	$3x=6$	$2y=3$
	$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.	(0, 3)	(2, 0)	$(1,\frac{3}{2})$
Check.			
$3x+2y=6$	$3x+2y=6$	$3x+2y=6$	
$3\cdot0+2\cdot3\stackrel{?}{=}6$	$3\cdot2+2\cdot0\stackrel{?}{=}6$	$3\cdot1+2\cdot\frac{3}{2}\stackrel{?}{=}6$	
$0+6\stackrel{?}{=}6$	$6+0\stackrel{?}{=}6$	$3+3\stackrel{?}{=}6$	
$6=6$✓	$6=6$✓	$6=6$✓	

So (0,3), (2,0), and $(1,\frac{3}{2})$ are all solutions to the equation $3x+2y=6$. These solutions are summarized in the table below.

$3x+2y=6$		
x	y	(x,y)
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1,\frac{3}{2})$

? Try It 9

Find three solutions to the equation $2x+3y=6$.

Answer

Answers will vary. Answers might include (0,2), (3,0), and (6, −2).

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Section 3.1P: Practice

Practice Makes Progress

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

Exercise 1

$(-4,2)$

Exercise 2

$(-1,-2)$

Exercise 3

$(3,-5)$

Exercise 4

$(-3,5)$

Exercise 5

$(\frac{5}{3},2)$

Exercise 6

$(-2,0)$

Exercise 7

$(-3,0)$

Exercise 8

$(0,0)$

Exercise 9

$(0,4)$

Exercise 10

$(0,2)$

In the following exercises, identify the quadrant in which the point is located.

Exercise 11

(3,−1)

Exercise 12

(−3,1)

Exercise 13

(−2,2)

Exercise 14

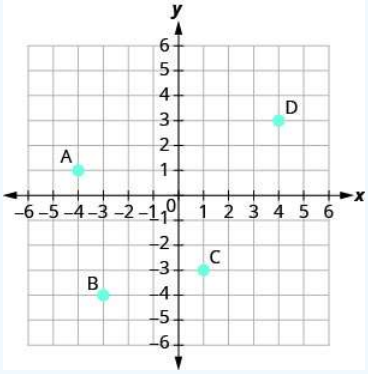
(−4,−3)

Exercise 15

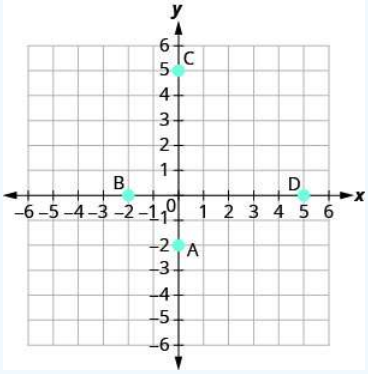
$(1, \frac{14}{5})$

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise 16



Exercise 17



Verify Solutions to an Equation in Two Variables

In the following exercises, determine if the ordered pair is a solution to $2x + y = 6$.

? Exercise 18

(1,4)

? Exercise 19

(3,0)

? Exercise 20

(2,3)

Determine if the ordered pair is a solution to $4-2y=8$.

? Exercise 21

(3,2)

? Exercise 22

(1,4)

? Exercise 23

(0,−4)

Determine if the ordered pair is a solution to $y=4x+3$.

? Exercise 24

(4,3)

? Exercise 25

(−1,−1)

? Exercise 26

$(\frac{1}{2},5)$

Finding and Representing Solutions to a Linear Equation

Use the given x value to find the corresponding y value and then state the point.

? Exercise 27

$y=2x-4;\ x=-1$

? Exercise 28

$y=-x+5;\ x=3$

Exercise 29

$y = \frac{1}{3}x + 1;\ x = 6$

Use the given y value to find the corresponding x value and state the point.

Exercise 30

$y = -\frac{3}{2}x - 2;\ y = 1$

Exercise 31

$x + 3y = 6;\ y = 0$

Exercise 32

$2x - 5y = 10;\ y = \frac{1}{2}$

Everyday Math

Exercise 33

Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age (in months) and weight (in pounds) are listed as ordered pairs below.

- (0, 7)
- (2, 11)
- (4, 15)
- (6, 16)
- (8, 19)
- (10, 20)
- (12, 21)

Plot the points on a coordinate plane and explain why, in this context, only Quadrant I is needed.

Writing Exercises

Exercise 34

Explain in words how you plot the point (4,−2) in a rectangular coordinate system.

Exercise 35

Is the point (−3,0) on the *x*-axis or *y*-axis? How do you know?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
plot points in a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to a linear equation.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.2: Graph Linear Equations in Two Variables

Be Prepared

We will rely heavily on these skills throughout this section.

- Identify which number is the x coordinate in the point $(4, -2)$
- Plot the point $(0, 3)$
- Evaluate $y = 2x - 1$ when $x = 2$
- Solve $y - 3 = x + 5$ for y

Learning Objectives

Motivating Problem

You earn $15 per hour at a part-time job. You want to know how much money you'll make after working different numbers of hours. That's a perfect setup for a linear equation—and a graph makes it easy to see the pattern!

Fun Fact

The very first computer-generated images—what we now call computer graphics—were made by plotting linear equations on a screen in the 1950s. Straight lines were the building blocks of early animation, from rocket trajectories to the first video games!

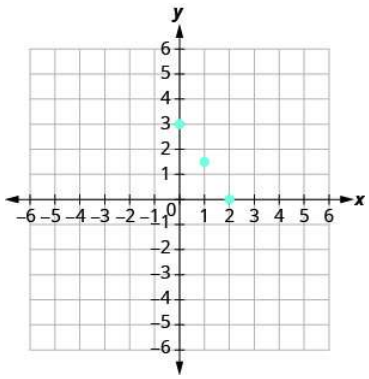
The Goal

This section introduces how to graph linear equations by finding solutions, plotting points, and connecting them to form a line. We will connect equations to visual patterns and begin recognizing how algebra and geometry work together.

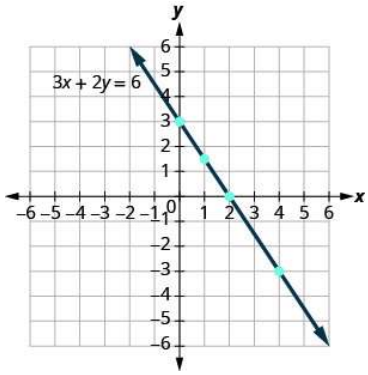
Recognize the Relationship Between the Solutions of an Equation and Its Graph

In the previous section, we found several solutions to the equation $3x + 2y = 6$. They are listed in the following table. So, the ordered pairs $(0, 3)$, $(2, 0)$, and $(1, \frac{3}{2})$ are some solutions to the equation $3x + 2y = 6$. We can plot these solutions in the rectangular coordinate system as shown below.

$3x + 2y = 6$		
x	y	(x, y)
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

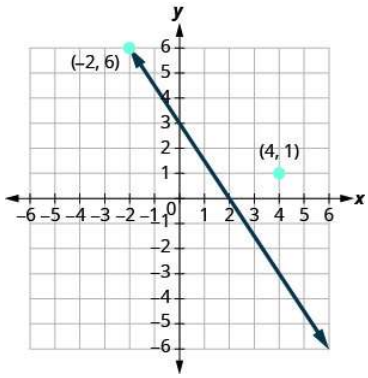


Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation $3x + 2y = 6$. See the Figure below. Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.



Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are $(-2, 6)$ is on the line shown in the following Figure. If you substitute $x = -2$ and $y = 6$ into the equation, you find that it is a solution to the equation $3x + 2y = 6$.



Test $(-2, 6)$

$$3x + 2y = 6$$
$$3(-2) + 2(6) = 6$$
$$-6 + 12 = 6$$
$$6 = 6 \checkmark$$

So the point $(-2, 6)$ is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are $(-2, 6)$" is often shortened to "the point $(-2, 6)$.")

Notice that the point $(4, 1)$ is not on the line. What does that tell us about $(4, 1)$ being a solution to the equation $3x + 2y = 6$?

What about $(4, 1)$?

$$3x + 2y = 6$$
$$3 \cdot 4 + 2 \cdot 1 = 6$$
$$12 + 2 \stackrel{?}{=} 6$$
$$14 \neq 6$$

We see that $(4, 1)$ is not a solution to the equation $3x + 2y = 6$. In general, points not on the line are not solutions to the equation that the line represents.

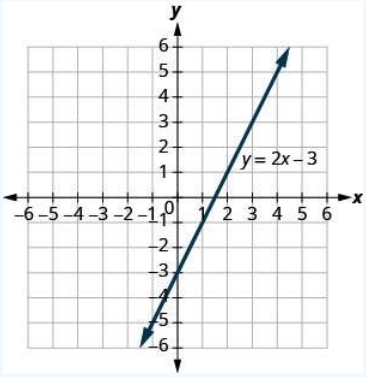
GRAPH OF A LINEAR EQUATION

The **graph of a linear equation** Ax+By=C is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Example 1

The graph of $y = 2x - 3$ is shown.



For each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

A $(0, -3)$ B $(3, 3)$ C $(2, -3)$ D $(-1, -5)$

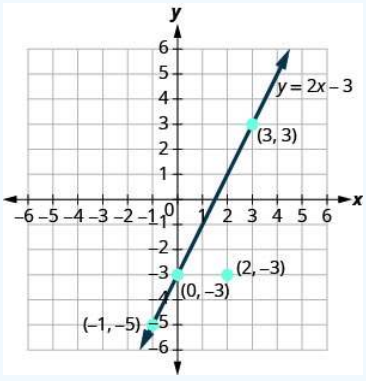
Solution

Substitute the *x*- and *y*-values into the equation to check if the ordered pair is a solution to the equation.

1.

A: (0, –3)	B: (3, 3)	C: (2, –3)	D: (–1, –5)
$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$
$-3 \stackrel{?}{=} 2(0) - 3$	$3 \stackrel{?}{=} 2(3) - 3$	$-3 \stackrel{?}{=} 2(2) - 3$	$-5 \stackrel{?}{=} 2(-1) - 3$
$-3 = -3$ ✓	$3 = 3$ ✓	$-3 \neq 1$	$-5 = -5$ ✓
(0, –3) is a solution.	(3, 3) is a solution.	(2, –3) is not a solution.	(–1, –5) is a solution.

2. Plot the points A $(0, -3)$, B $(3, 3)$, C $(2, -3)$, and D $(-1, -5)$ on the graph of $y = 2x - 3$.



The points that are solutions to $y = 2x - 3$ are on the line, but the point that is not a solution is not on the line.

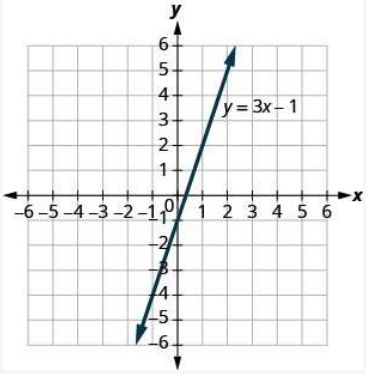
The points $(0, -3)$, $(3, 3)$, and $(-1, -5)$ are on the line $y = 2x - 3$, and the point $(2, -3)$ is not on the line.

Try It 1

Use the graph of $y = 3x - 1$ to decide whether each ordered pair is:

- a solution to the equation.
- on the line.

1. (0,-1)
2. (2,5)
3. (-2,2)



Answer

1. yes, yes
2. yes, yes
3. no, no

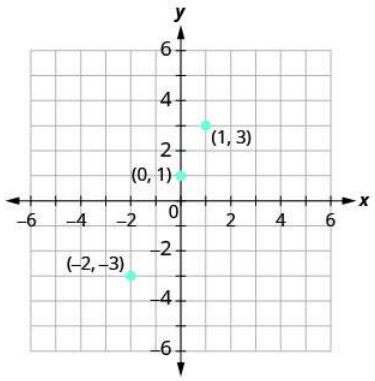
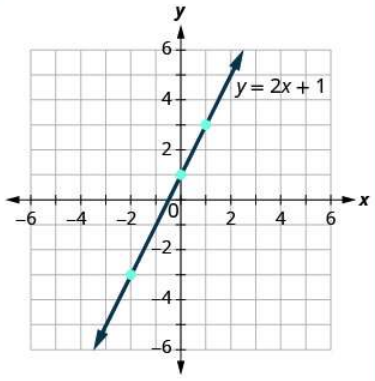
Graph a Linear Equation by Plotting Points

Several methods can be used to graph a linear equation. The method we used to graph $3x + 2y = 6$ is called plotting points, or the Point–Plotting Method.

Example 2

Graph the equation $y = 2x + 1$ by plotting points.

Solution

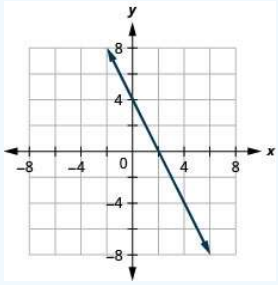
Step 1. Find three points whose coordinates are solutions to the equation.	You can choose any values for x or y. In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.	$y = 2x + 1$ $x = 0$ $y = 2x + 1$ $y = 2 \cdot 0 + 1$ $y = 0 + 1$ $y = 1$ $x = 1$ $y = 2x + 1$ $y = 2 \cdot 1 + 1$ $y = 2 + 1$ $y = 3$ $x = -2$ $y = 2x + 1$ $y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$
Organize the solutions in a table.	Put the three solutions in a table.	$y = 2x + 1$ x | y | (x, y) 0 | 1 | (0, 1) 1 | 3 | (1, 3) –2 | –3 | (–2, –3)
Step 2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!	Plot: (0, 1), (1, 3), (–2, –3). Do the points line up? Yes, the points line up.	
Step 3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.	This line is the graph of $y = 2x + 1$.	

Be sure to use a straight edge, such as a ruler or the edge of an ID card, to draw a clear and accurate line through three points.

Try It 2

Graph the equation by plotting points: $y = -2x + 4$.

Answer



The steps to take when graphing a linear equation by plotting points are summarized below.

GRAPH A LINEAR EQUATION BY PLOTTING POINTS

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Use a straight edge to draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line, but it will not represent the solutions to the equation—it will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong, and you need to check your work. Look at the difference between part (a) and part (b) below.



(a) (b)

Let’s do another example. This time, we’ll show the last two steps all on one grid.

Example 3

Graph the equation $y = -3x$.

Solution

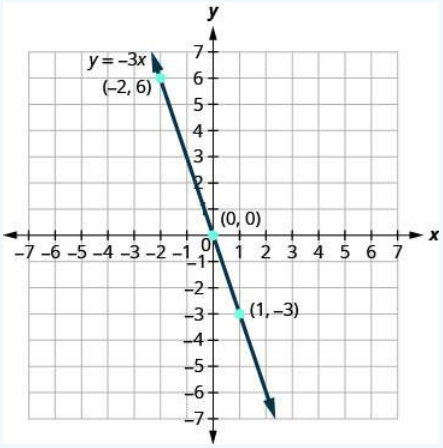
Find three points that are solutions to the equation. Here, again, it’s easier to choose values for x. Do you see why?

$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3x$	$y = -3x$
$y = -3 \cdot 0$	$y = -3 \cdot 1$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$

We list the points in this Table.

$y=-3x$		
x	y	(x,y)
0	0	(0,0)
1	-3	(1,-3)
-2	6	(-2,6)

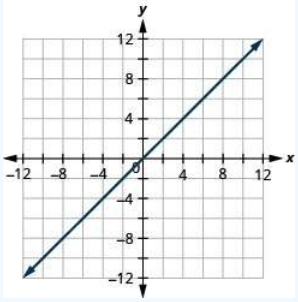
Plot the points, check that they line up, and draw the line.



Try It 3

Graph the equation $y=x$.

Answer



When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for x. But the math is easier if we make 'good' choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

Example 4

Graph the equation $y=\frac{1}{2}x+3$.

Solution

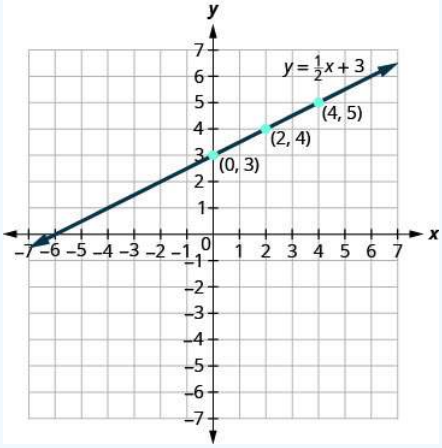
Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?

$x=0$	$x=2$	$x=4$
$y=\frac{1}{2}x+3$	$y=\frac{1}{2}x+3$	$y=\frac{1}{2}x+3$
$y=\frac{1}{2}(0)+3$	$y=\frac{1}{2}(2)+3$	$y=\frac{1}{2}(4)+3$
$y=0+3$	$y=1+3$	$y=2+3$
$y=3$	$y=4$	$y=5$

The points are shown in this table.

$y=\frac{1}{2}x+3$		
x	y	(x,y)
0	3	(0,3)
2	4	(2,4)
4	5	(4,5)

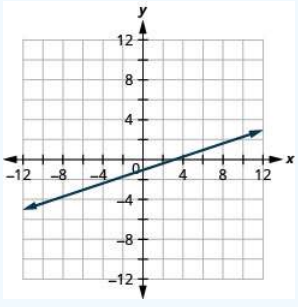
Plot the points, check that they line up, and draw the line.



Try It 4

Graph the equation $y=\frac{1}{3}x-1$.

Answer



So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation $2x+y=3$. If $y=0$, what is the value of x?

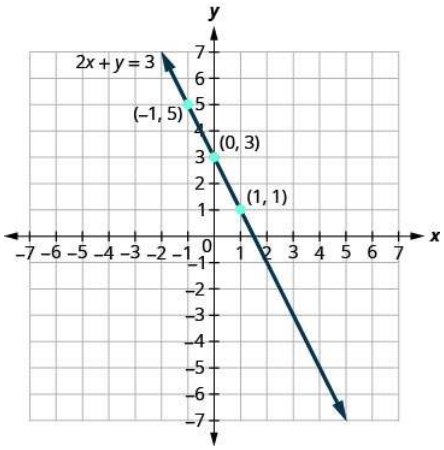
$$y = 0$$
$$2x + y = 3$$
$$2x + 0 = 3$$
$$2x = 3$$
$$x = \frac{3}{2}$$
$$\left(\frac{3}{2}, 0\right)$$

This point has a fraction for the *x*-coordinate; while we could graph this point, it is hard to precisely graph fractions. Remember in the example $y = \frac{1}{2}x + 3$, we carefully chose values for x so as not to graph fractions at all. If we solve the equation $2x + y = 3$ for y, it will be easier to find three solutions to the equation.

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned} \qquad \text{(Section 3.2.1)}$$

The solutions for $x = 0$, $x = 1$, and $x = -1$ are shown in the following table and the graph below.

$2x + y = 3$		
x	y	(x, y)
0	3	(0,3)
1	1	(1,1)
-1	5	(-1,5)



Can you locate the point $(\frac{3}{2}, 0)$ which we found by letting $y = 0$, on the line?

Example 5

Graph the equation $3x + y = -1$.

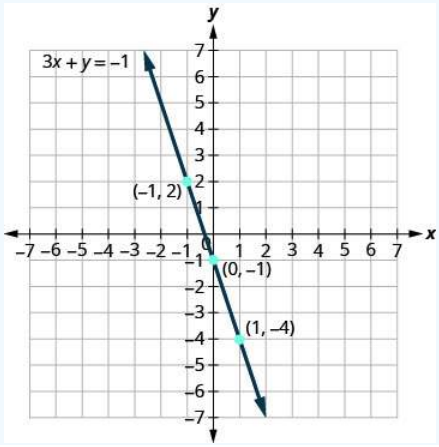
Solution

Find three points that are solutions to the equation. $3x + y = -1$

First solve the equation for y. $y = -3x - 1$

The solutions for $x = 0$, $x = 1$, and $x = -1$ are shown in the following table and the graph below.

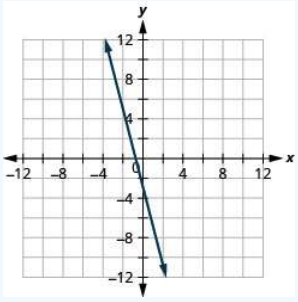
$3x + y = -1$		
x	y	(x, y)
0	-1	(0,-1)
1	-4	(1,-4)
-1	2	(-1,2)



Try It 5

Graph the equation $4x + y = -3$.

Answer



If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the *x*- and *y*-axis are the same, the graphs match!

The equation in Example 5 was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form, it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting $x = 0$ and solving for y. We can plot a second point by letting $y = 0$ and then solving for x. Then we will plot a third point by using some other value for x or y.

Example 6

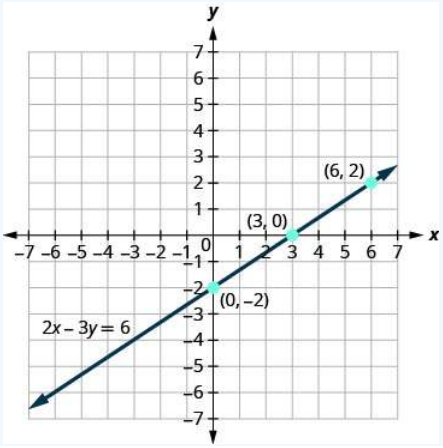
Graph the equation $2x - 3y = 6$.

Solution

Find three points that are solutions to the equation.	$2x-3y$	$=$	6
First let $x=0$.	$2(0)-3y$	$=$	6
Solve for y.	$-3y$	$=$	6
	y	$=$	-2
Now let $y=0$.	$2x-3(0)$	$=$	6
Solve for x.	$2x$	$=$	6
	x	$=$	3
We need a third point. Remember, we can choose any value for x or y. We'll let x = 6.	$2(6)-3y$	$=$	6
	$12-3y$	$=$	6
Solve for y.	$-3y$	$=$	-6
	y	$=$	2

We list the ordered pairs in this table, plot the points, check that they line up, and draw the line below them.

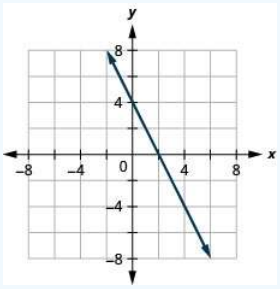
$2x-3y=6$		
x	y	(x,y)
0	-2	(0,-2)
3	0	(3,0)
6	2	(6,2)



Try It 6

Graph the equation $4x+2y=8$.

Answer



Graph Vertical and Horizontal Lines

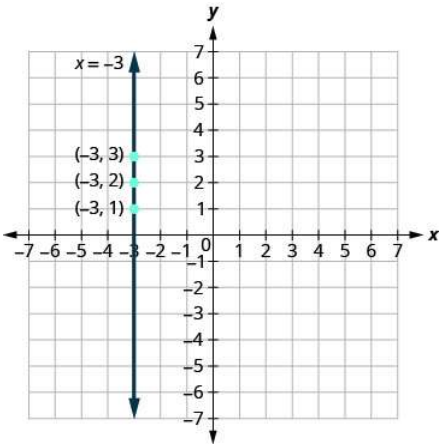
Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let's consider the equation $x = -3$. This equation has only one variable, x. The equation says that x is *always* equal to -3, so its value does not depend on y. No matter what y is, the value of x is always -3.

So to make a table of values, write -3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we'll use 1, 2, and 3 for the *y*-coordinates.

$x = -3$		
x	y	(x, y)
-3	1	(-3,1)
-3	2	(-3,2)
-3	3	(-3,3)

Plot the points from the Table above and connect them with a straight line. Notice that we have graphed a *vertical line*.



VERTICAL LINE

A **vertical line** is the graph of an equation of the form x=a.

The line passes through the *x*-axis at $(a, 0)$.

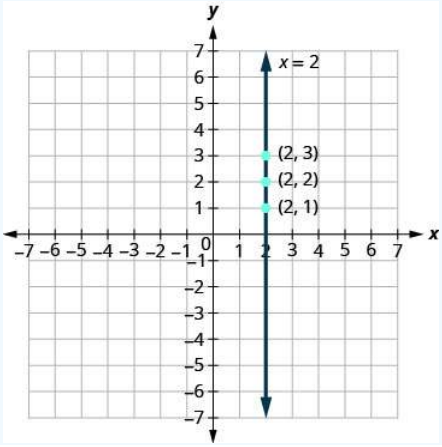
Example 7

Graph the equation $x = 2$.

Solution

The equation has only one variable, x, and x always equals 2. We create a Table where x is always 2 and then put in any values for y. The graph is a vertical line passing through the *x*-axis at 2.

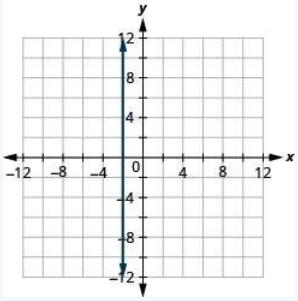
$x = 2$		
x	y	(x, y)
2	1	(2,1)
2	2	(2,2)
2	3	(2,3)



Try It 7

Graph the equation $x = -2$.

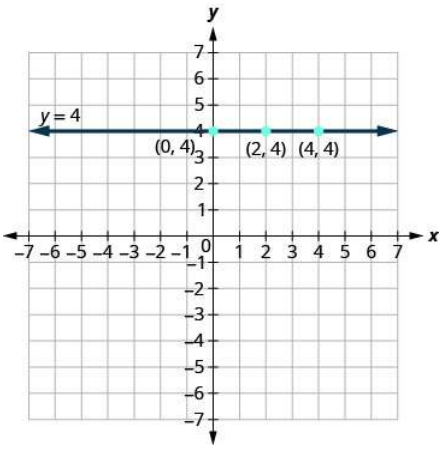
Answer



What if the equation has y but no x? Let's graph the equation $y = 4$. This time the y- value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y's in the following Table and then choose any values for x. We'll use 0, 2, and 4 for the x-coordinates.

$y = 4$		
x	y	(x, y)
0	4	(0,4)
2	4	(2,4)
4	4	(4,4)

The graph is a horizontal line passing through the y-axis at 4.



HORIZONTAL LINE

A **horizontal line** is the graph of an equation of the form $y = b$.

The line passes through the *y*-axis at $(0, b)$.

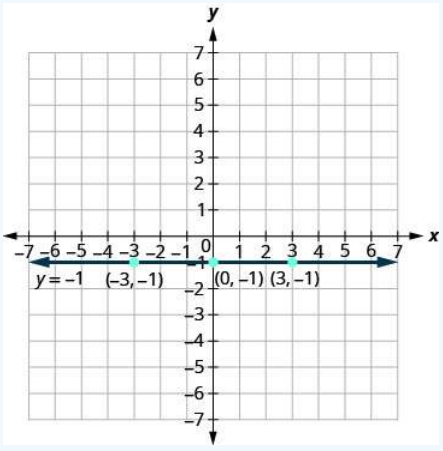
Example 8

Graph the equation $y = -1$.

Solution

The equation $y = -1$ has only one variable, y, and y always equals -1. We create a table where y is always -1 and then add any value for x. The graph is a horizontal line passing through the *y*-axis at -1.

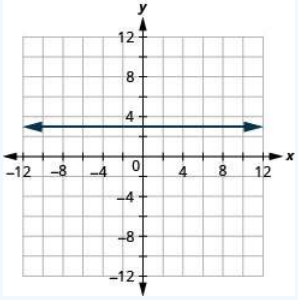
$y = -1$		
x	y	(x, y)
0	-1	(0,-1)
3	-1	(3,-1)
-3	-1	(-3,-1)



Try It 8

Graph the equation $y = 3$.

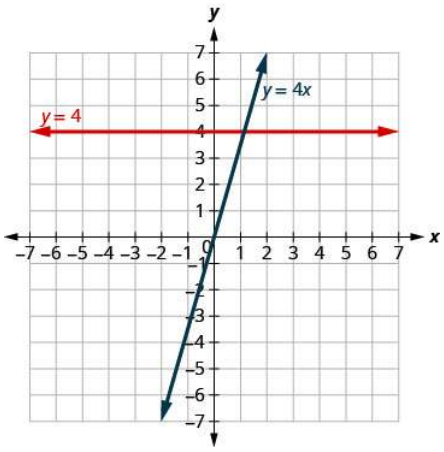
Answer



The equations for vertical and horizontal lines look very similar to equations like $y = 4x$. What is the difference between the equations $y = 4x$ and $y = 4$?

The equation $y = 4x$ has both x and y. The value of y depends on the value of x. The *y*-coordinate changes according to the value of x. The equation $y = 4$ has only one variable. The value of y is constant. The *y*-coordinate is always 4. It does not depend on the value of x. See the tables and graph below.

$y = 4x$				$y = 4$		
x	y	(x, y)		x	y	(x, y)
0	0	(0,0)		0	4	(0,4)
1	4	(1,4)		1	4	(1,4)
2	8	(2,8)		2	4	(2,4)



Notice that in the graph above, the equation $y = 4x$ gives a slanted line, while $y = 4$ gives a horizontal line.

Example 9

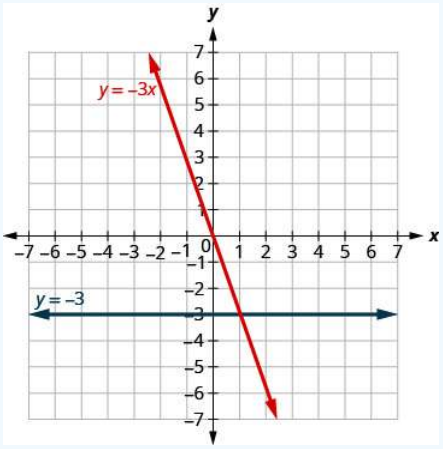
Graph $y = -3x$ and $y = -3$ in the same rectangular coordinate system.

Solution

Notice that the first equation has the variable x in the tables, while the second does not. The two graphs are shown below the tables.

$y = -3x$		
x	y	(x, y)
0	0	(0,0)
1	-3	(1,-3)
2	-6	(2,-6)

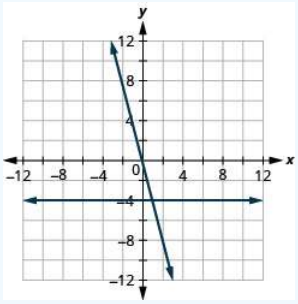
$y = -3$		
x	y	(x, y)
0	-3	(0,-3)
1	-3	(1,-3)
2	-3	(2,-3)



Try It 9

Graph $y = -4x$ and $y = -4$ in the same rectangular coordinate system.

Answer



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Section 3.2P: Practice

Practice Makes Progress

Recognize the Relationship Between the Solutions of an Equation and its Graph

In the following exercises, for each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

Exercise 1

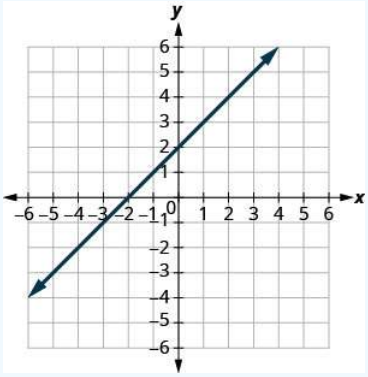
$y = x + 2$

(a) (0, 2)

(b) (1, 2)

(c) (-1, 1)

(d) (-3, -1)



Exercise 2

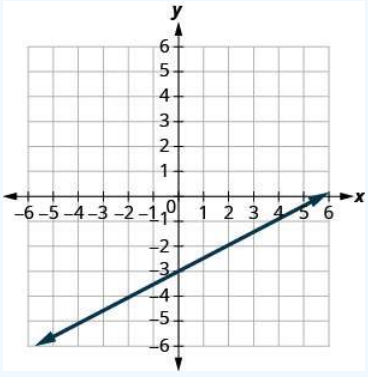
$y = \frac{1}{2}x - 3$

(a) (0, -3)

(b) (2, -2)

(c) (-2, -4)

(d) (4, 1)



Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

Exercise 3

$y = 3x - 1$

Exercise 4

$y = -2x + 2$

Exercise 5

$y = 2x$

Exercise 6

$y = -4x$

Exercise 7

$y = \frac{1}{2}x + 2$

Exercise 8

$y = -\frac{2}{5}x + 1$

Exercise 9

$x + y = 6$

Exercise 10

$x + y = -3$

Exercise 11

$x - y = 2$

Exercise 12

$x - y = -1$

Exercise 13

$3x + y = 7$

Exercise 14

$\frac{1}{3}x + y = 2$

? Exercise 15

$2x + 3y = 12$

? Exercise 16

$x - 6y = 3$

Graph Vertical and Horizontal Lines

In the following exercises, graph each equation.

? Exercise 17

$x = 4$

? Exercise 18

$y = -5$

? Exercise 19

$x = \frac{7}{3}$

? Exercise 20

$y = -\frac{15}{4}$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

? Exercise 21

$y = 2x$ and $y = 2$

? Exercise 22

$y = -\frac{1}{2}x$ and $y = -\frac{1}{2}$

Mixed Practice

In the following exercises, graph each equation.

? Exercise 23

$y = -\frac{1}{2}x + 3$

? Exercise 24

$y = -x$

? Exercise 25

$4x + y = 2$

? Exercise 26

$2x + 6y = 12$

? Exercise 27

$x = 3$

Everyday Math

? Exercise 28

Motor home cost. The Robinsons rented a motor home for one week to go on vacation. It cost them \$594 plus \$0.32 per mile to rent the motor home, so the linear equation $y = 594 + 0.32x$ gives the cost, y, for driving x miles. Calculate the rental cost for driving 400, 800, and 1200 miles, and then graph the line.

Writing Exercises

? Exercise 29

Explain how you would choose three x-values to make a table to graph the line $y = \frac{1}{5}x - 2$

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the solutions of an equation and its graph.			
graph a linear equation by plotting points.			
graph vertical and horizontal lines.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to ensure a strong foundation before you move on. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.3: Intercepts

Be Prepared

We will rely heavily on these skills throughout this section.

- Identify the y-coordinate in the point $(3, 5)$
- Solve $y = 2x - 1$ for x when $y = 0$
- Solve $y - 2x = 3$ for y when $x = 0$

Learning Objectives

Motivating Problem

A heat wave is coming so you set up a lemonade stand outside the Wacheno Welcome Center. You've spent $10 on supplies so far, and you're charging $0.50 per cup of lemonade. How many cups of lemonade do you need to sell before you break even (earn back the money you have put into the lemonade stand)?

Fun Fact

French mathematician Pierre de Fermat used x- and y-intercepts to study curves long before graphing calculators existed. His ideas helped pave the way for analytic geometry and modern calculus—and today, intercepts still play a key role in forecasting profits, analyzing data, and engineering designs.

The Goal

In this section, we won't guess where lines cross the axes—we'll learn how to find those key points directly from equations or graphs. We'll use x- and y-intercepts as shortcuts for graphing lines and as tools to better understand how linear relationships behave.

Identify the x- and y-Intercepts on a Graph

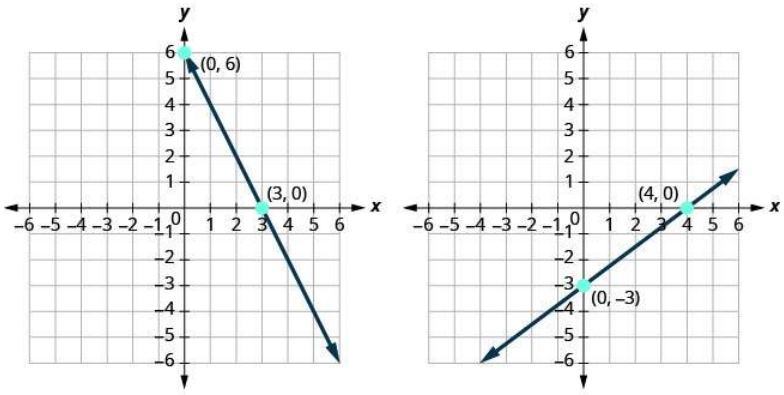
Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the x-axis and the y-axis. These points are called the *intercepts* of the line.

INTERCEPTS OF A LINE

The points where a line crosses the x-axis and the y-axis are called the **intercepts of a line**.

Let's look at the graphs of the lines provided in the figure.



(a) $2x + y = 6$ (b) $3x - 4y = 12$

(c) $x - y = 5$ (d) $y = -2x$

First, notice where each of these lines crosses the x-axis.

figure	The line crosses the x-axis at:	Ordered pair of this point
figure (a)	3	(3,0)
figure (b)	4	(4,0)
figure (c)	5	(5,0)
figure (d)	0	(0,0)

Do you see a pattern?

For each row, the y-coordinate of the point where the line crosses the x-axis is zero. The point where the line crosses the x-axis has the form $(a, 0)$ and is called the **x-intercept of a line**. The x-intercept occurs when y is zero. Let's look at the points where these lines cross the y-axis.

figure	The line crosses the x-axis at:	Ordered pair of this point
figure (a)	6	(0,6)
figure (b)	−3	(0,−3)
figure (c)	−5	(0,5)
figure (d)	0	(0,0)

What is the pattern here?

In each row, the x-coordinate of the point where the line crosses the y-axis is zero. This point has the form $(0, b)$ and is called the **y-intercept of the line**. The y-intercept occurs when x is zero.

x- INTERCEPT AND y-INTERCEPT OF A LINE

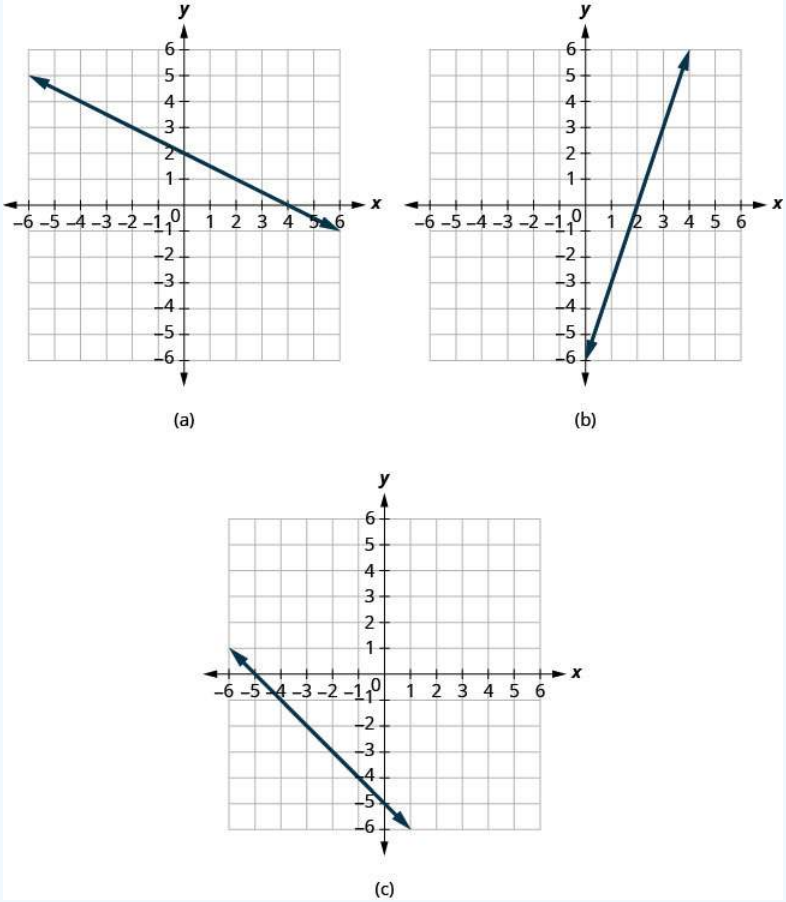
The x-intercept is the point $(a, 0)$ where the line crosses the x-axis.

The y-intercept is the point $(0, b)$ where the line crosses the y-axis.

	x	y
• The x-intercept occurs when y is zero.	a	0
• The y-intercept occurs when x is zero.	0	b

Example 1

Find the x- and y-intercepts on each graph.



Solution

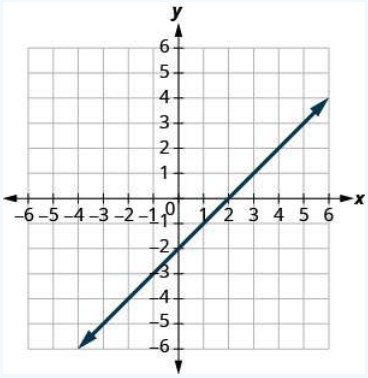
(a) The graph crosses the x-axis at the point $(4, 0)$. The x-intercept is $(4, 0)$.
The graph crosses the y-axis at the point $(0, 2)$. The y-intercept is $(0, 2)$.

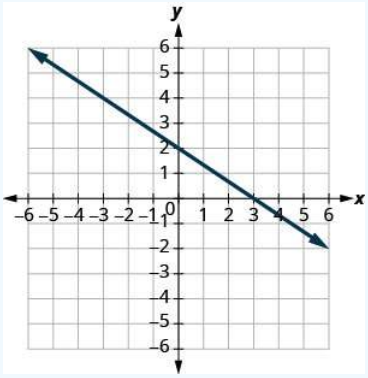
(b) The graph crosses the x-axis at the point $(2, 0)$. The x-intercept is $(2, 0)$
The graph crosses the y-axis at the point $(0, -6)$. The y- intercept is $(0, -6)$.

(c) The graph crosses the x-axis at the point $(-5, 0)$. The x-intercept is $(-5, 0)$.
The graph crosses the y-axis at the point $(0, -5)$. The y-intercept is $(0, -5)$.

Try It 1

Find the x- and y-intercepts on the graphs.

a. 

b. 

Answer

a. x-intercept: $(2, 0)$; y-intercept: $(0, -2)$

b. x-intercept: $(3, 0)$, y-intercept: $(0, 2)$

Find the x- and y-Intercepts from an Equation of a Line

Recognizing that the x-intercept occurs when y is zero and that the y-intercept occurs when x is zero gives us a method to find the intercepts of a line from its equation. To find the **x-intercept**, let $y = 0$ and solve for x. To find the **y-intercept**, let $x = 0$ and solve for y.

x- AND y-INTERCEPTS FROM THE EQUATION OF A LINE

Use the equation of the line. To find:

- the x-intercept of the line, let $y = 0$ and solve for x.
- the y-intercept of the line, let $x = 0$ and solve for y.

Example 2

Find the intercepts of $2x + y = 6$.

Solution

We will let $y = 0$ to find the x-intercept, and let $x = 0$ to find the y-intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$		
x	y	
	0	x-intercept
0		y-intercept

To find the x-intercept, let $y=0$.

	$2x+y=6$
Let $y=0$.	$2x+0=6$
Simplify.	$2x=6$
	$x=3$
The x-intercept is	(3, 0)
To find the y-intercept, let x = 0.	
	$2x+y=6$
Let x = 0.	$2\cdot 0+y=6$
Simplify.	$0+y=6$
	$y=6$
The y-intercept is	(0, 6)

The intercepts are the points $(3,0)$ and $(0,6)$.

Try It 2

a. Find the intercepts of $3x+y=12$.

b. Find the intercepts of $x+4y=8$.

Answer

a. x-intercept: $(4,0)$, y-intercept: $(0,12)$

b. x-intercept: $(8,0)$, y-intercept: $(0,2)$

Example 3

Find the intercepts of $4x-3y=12$.

Solution

To find the x-intercept, let $y = 0$.	
	$4x - 3y = 12$
Let $y = 0$.	$4x - 3 \cdot 0 = 12$
Simplify.	$4x - 0 = 12$
	$4x = 12$
	$x = 3$
The x-intercept is	(3, 0)
To find the y-intercept, let $x = 0$.	
	$4x - 3y = 12$
Let $x = 0$.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The y-intercept is	(0, −4)

The intercepts are the points $(3, 0)$ and $(0, -4)$.

Try It 3

a. Find the intercepts of $3x - 4y = 12$.

b. Find the intercepts of $2x - 4y = 8$.

Answer

a. x-intercept: $(4, 0)$, y-intercept: $(0, -3)$

b. x-intercept: $(4, 0)$, y-intercept: $(0, -2)$

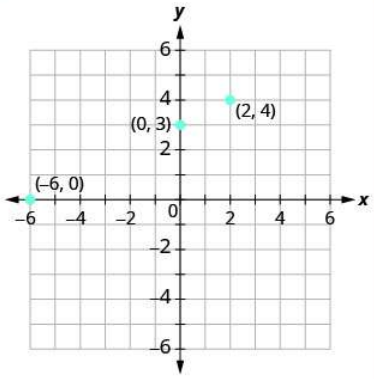
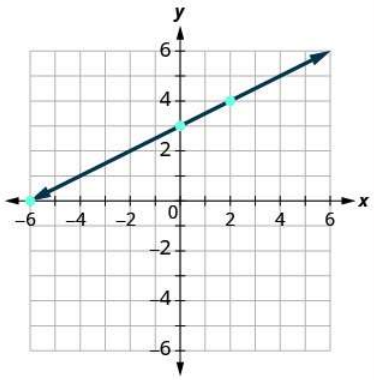
Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the x- and y-intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

Example 4

Graph $-x + 2y = 6$ using the intercepts.

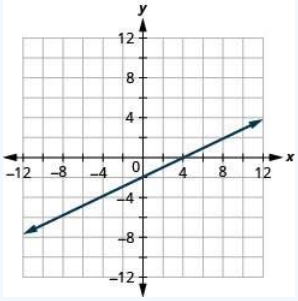
Solution

Step 1. Find the x- and y-intercepts of the line. Let $y = 0$ and solve for x. Let $x = 0$ and solve for y.	Find the x-intercept.	Let $y = 0$ $-x + 2y = 6$ $-x + 2(0) = 6$ $-x = 6$ $x = -6$ The x-intercept is $(-6, 0)$.
	Find the y-intercept.	Let $x = 0$ $-x + 2y = 6$ $-0 + 2y = 6$ $2y = 6$ $y = 3$ The y-intercept is $(0, 3)$.
Step 2. Find another solution to the equation.	We'll use $x = 2$.	Let $x = 2$ $-x + 2y = 6$ $-2 + 2y = 6$ $2y = 8$ $y = 4$ A third point is $(2, 4)$.
Step 3. Plot the three points. Check that the points line up.	x | y | (x, y) -6 | 0 | $(-6, 0)$ 0 | 3 | $(0, 3)$ 2 | 4 | $(2, 4)$	
Step 4. Draw the line.	See the graph.	

? Try It 4

Graph $x - 2y = 4$ using the intercepts.

Answer



The steps to graph a linear equation using the intercepts are summarized below.

GRAPH A LINEAR EQUATION USING THE INTERCEPTS.

1. Find the x- and y-intercepts of the line.
   - Let $y = 0$ and solve for x.
   - Let $x = 0$ and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Use a straight edge to draw a line through the three points.

Example 5

Graph $4x - 3y = 12$ using the intercepts.

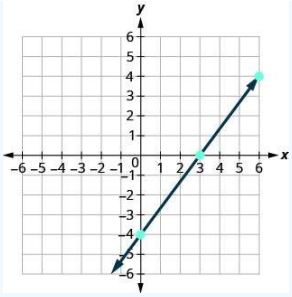
Solution

Find the intercepts and a third point.

x-intercept, let $y = 0$	y-intercept, let $x = 0$	third point, let $y = 4$
$4x - 3y = 12$	$4x - 3y = 12$	$4x - 3y = 12$
$4x - 3(0) = 12$	$4(0) - 3y = 12$	$4x - 3(4) = 12$
$4x = 12$	$-3y = 12$	$4x - 12 = 12$
$x = 3$	$y = -4$	$4x = 24$
		$x = 6$

We list the points in this table and show the graph below.

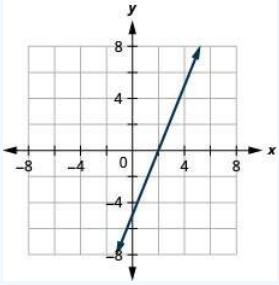
$4x - 3y = 12$		
x	y	(x, y)
3	0	(3,0)
0	−4	(0,−4)
6	4	(6,4)



Try It 5

Graph $5x - 2y = 10$ using the intercepts.

Answer



Example 6

Graph $y = 5x$ using the intercepts.

Solution

x-intercept	y-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
(0, 0)	(0, 0)

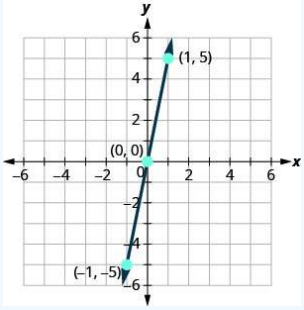
This line has only one intercept. It is the point $(0, 0)$.

To ensure accuracy we need to plot three points. Since the x- and y-intercepts are the same point, we need *two* more points to graph the line.

Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

$y = 5x$		
x	y	(x, y)
0	0	(0,0)
1	5	(1,5)
−1	−5	(−1,−5)

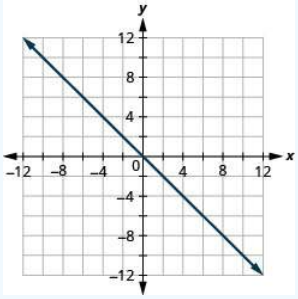
Plot the three points, check that they line up, and draw the line.



Try It 6

Graph $y = -x$ using the intercepts.

Answer



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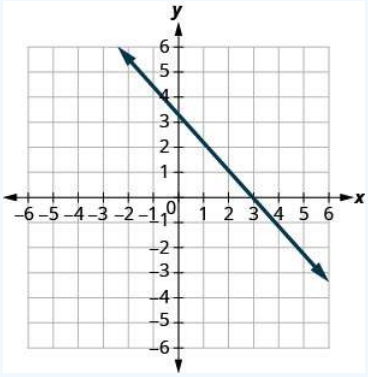
Section 3.3P: Practice

Practice Makes Progress

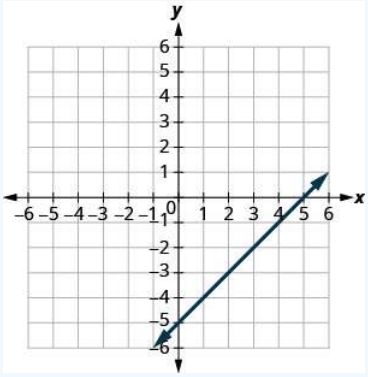
Identify the x- and y-Intercepts on a Graph

In the following exercises, find the x- and y-intercepts on each graph.

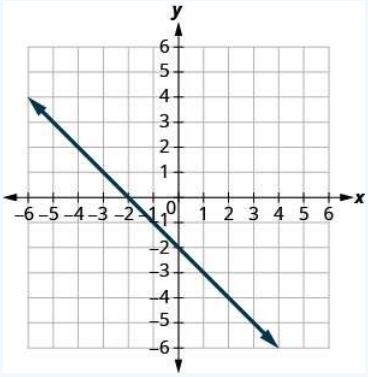
Exercise 1



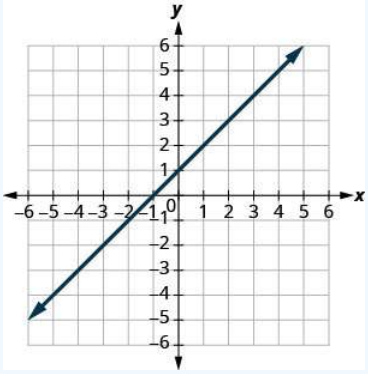
Exercise 2



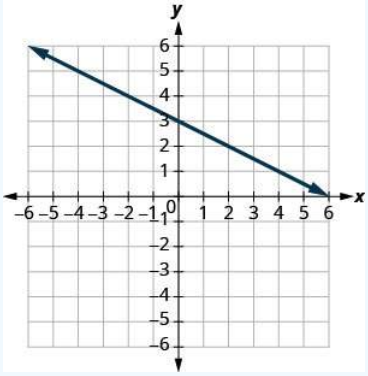
Exercise 3



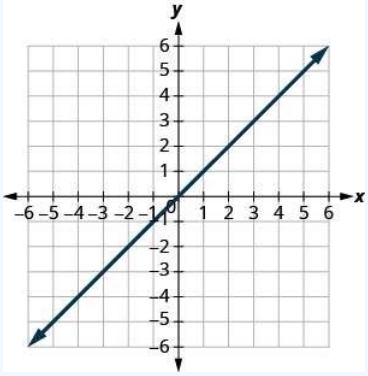
Exercise 4



Exercise 5



Exercise 6



Find the x- and y-Intercepts from an Equation of a Line

In the following exercises, find the intercepts for each equation.

Exercise 7

$x + y = 4$

https://math.libretexts.org/@go/page/188504

? Exercise 8

$x + y = -2$

? Exercise 9

$x - y = 5$

? Exercise 10

$x - y = -3$

? Exercise 11

$3x + y = 6$

? Exercise 12

$x - 3y = 12$

? Exercise 13

$2x + 5y = 10$

? Exercise 14

$y = \frac{1}{3}x + 1$

? Exercise 15

$y = \frac{1}{5}x + 2$

? Exercise 16

$y = 3x$

? Exercise 17

$y = -4x$

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

? Exercise 18

$-x + 5y = 10$

? Exercise 19

$x + 2y = 4$

Exercise 20

$x + y = -3$

Exercise 21

$x - y = 1$

Exercise 22

$4x + y = 4$

Exercise 23

$3x - 2y = 6$

Exercise 24

$3x - y = -6$

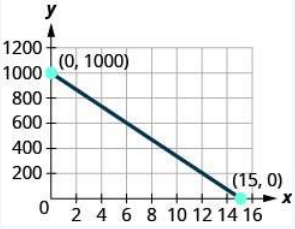
Exercise 25

$y = x$

Everyday Math

Exercise 26

Road trip. Damien is driving from Chicago to Denver, a distance of 1000 miles. The graph below shows his distance (in miles) remaining over time (in hours).



Find the x- and y-intercepts and explain what they mean for Damien.

Writing Exercises

Exercise 27

How do you find the x-intercept of the graph of $3x - 2y = 6$?

Exercise 28

Do you prefer to use the method of plotting points or using intercepts to graph $y = \frac{2}{3}x - 2$? Why?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify the *x* and *y* intercepts on a graph.			
find the *x* and *y* intercepts from an equation of a line.			
graph a line using the intercepts.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.4: Understanding the Slope of a Line

Be Prepared

We will rely heavily on these skills throughout this section.

- Add: $\frac{2}{3}+\frac{1}{6}$
- Simplify: $\frac{1-4}{8-2}$
- Find the x-intercept of the equation $2x+3y=6$

Learning Objectives

Motivating Problem

You're hiking a trail that gains 600 feet of elevation over a 1.5-mile stretch. How steep is the trail? If another trail climbs the same amount over just 1 mile, which one is harder to climb—and how can you compare them using math?

Fun Fact

The ancient Egyptians used a form of slope called the *seked* to construct pyramids, describing how far a pyramid rose vertically for every cubit it extended horizontally. Their knowledge of slope was practical—and precise enough to build wonders that still stand today!

The Goal

In this section, we'll stop thinking of lines as just visual slants and learn how to describe their tilt precisely using numbers. We'll explore the meaning of slope, learn how to calculate it from graphs and coordinates, and connect it to everyday ideas like steepness and rate of change.

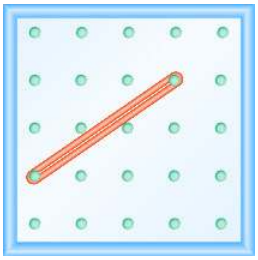
When you graph linear equations, you may notice that some lines tilt up as they go from left to right, and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down, or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, the grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

Use Geoboards to Model Slope

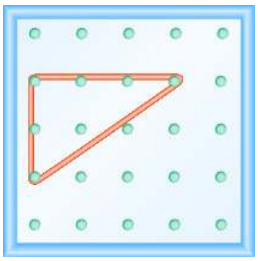
A **geoboard** is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

As shown below, we'll start by stretching a rubber band between two pegs.



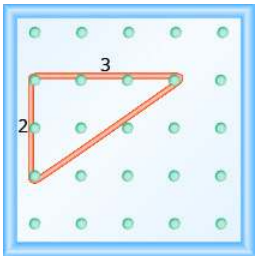
Doesn't it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in the following Figure.



We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise,** and the horizontal distance is called the **run**.



The rise on this geoboard is 2, as the rubber band goes up two units.

What is the run?

The rubber band goes across 3 units. The run is 3.

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

SLOPE OF A LINE

The **slope of a line** of a line is $m = \frac{\text{rise}}{\text{run}}$.

The **rise** measures the vertical change, and the **run** measures the horizontal change between two points on the line.

What is the slope of the line on our geoboard?

$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{2}{3}$$

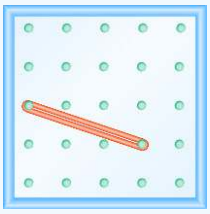
(Section 3.4.1)

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up, it is positive, and if it goes down, it is negative. The run will go from left to right and be positive.

Example 1

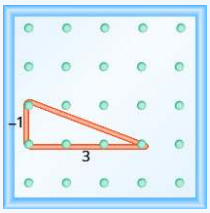
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg. The rise will be negative since the right peg is lower than the left peg.



The rise is -1. $m = \frac{-1}{\text{run}}$

The run is 3. $m = \frac{-1}{3}$

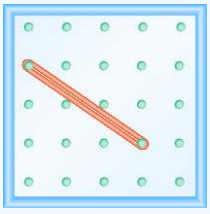
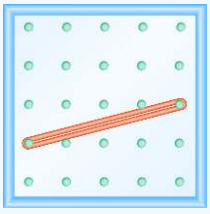
$m = -\frac{1}{3}$

The slope is $-\frac{1}{3}$

This means that the line drops 1 unit for every 3 units of run.

Try It 1

What is the slope of the line on the geoboard?

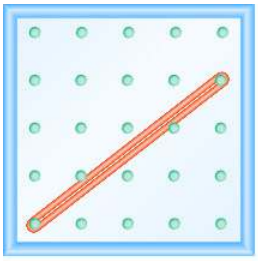
a.  b. 

Answer

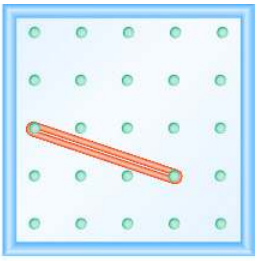
a. $m = -\frac{2}{3}$

b. $m = \frac{1}{4}$

Do you notice any difference in the two lines shown below?



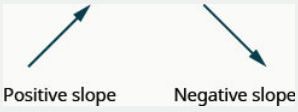
$m = \frac{3}{4}$ (a)



$m = -\frac{1}{3}$ (b)

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line in the left figure is going up; it has a **positive slope**. The line in the right figure is going down; it has a **negative slope**.

POSITIVE AND NEGATIVE SLOPES



Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

Now, we'll examine some graphs on the xy-coordinate plane and determine their slopes. The method will be very similar to that we used to model our geoboards.

To find the slope, we must count out the rise and the run. But where do we start?

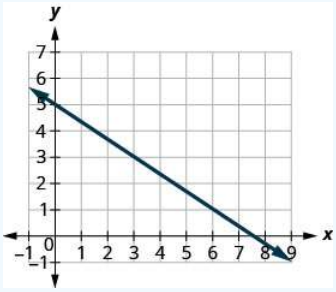
We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

FIND THE SLOPE OF A LINE FROM ITS GRAPH

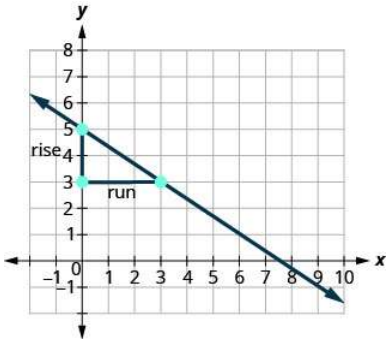
1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$.

Example 2a

Find the slope of the line shown.

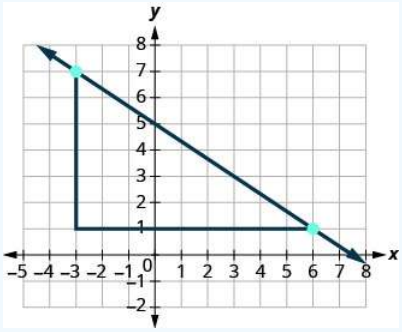


Solution

Locate two points on the graph whose coordinates are integers.	$(0,5)$ and $(3,3)$
Which point is on the left?	$(0,5)$
Starting at (0,5), sketch a right triangle to (3,3).	
Count the rise—it is negative.	The rise is -2.
Count the run.	The run is 3.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y decreases by 2 units as x increases by 3 units.

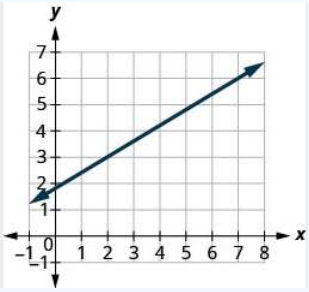
What if we used the points $(-3, 7)$ and $(6, 1)$ to find the slope of the line?



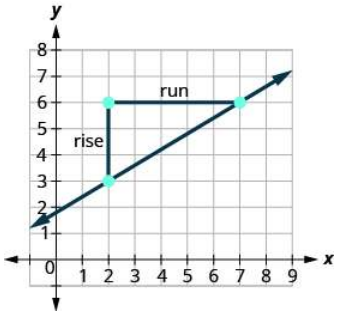
The rise would be -6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

Example 2b

Find the slope of the line shown.



Solution

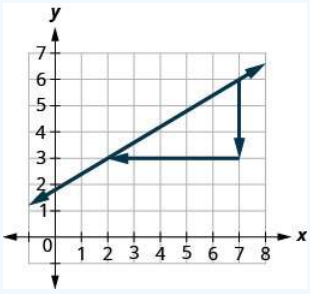
Locate two points on the graph whose coordinates are integers.	$(2, 3)$ and $(7, 6)$
Which point is on the left?	$(2, 3)$
Starting at (2,3), sketch a right triangle to (7,6).	
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

This means that y increases by 3 units as x increases by 5 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way, the run was always positive, and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let's use the points $(2,3)$ and $(7,6)$ again, but now we'll start at $(7,6)$.

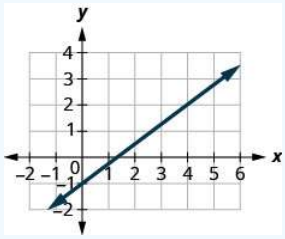


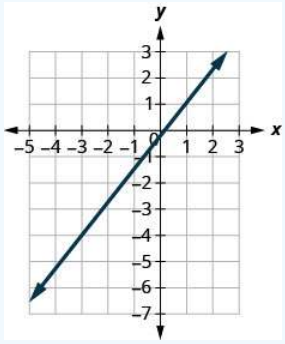
Count the rise.	**The rise is -3.**
Count the run. It goes from right to left, so it is negative.	**The run is-5.**
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5}$
	The slope of the line is $\frac{3}{5}$

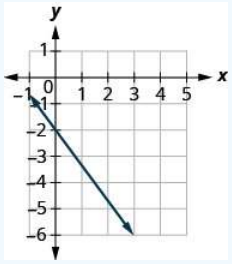
It does not matter where you start—the slope of the line is always the same.

Try It 2

Find the slope of the line shown.

a. 

b. 

c. 

Answer

a. $m = \frac{3}{4}$

b. $m = \frac{5}{4}$

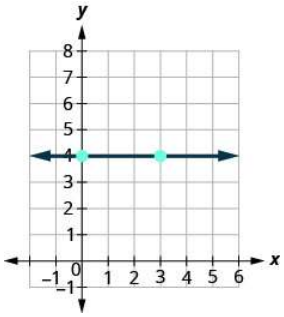
c. $m = -\frac{4}{3}$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $y = b$	**Vertical line** $x = a$
y **-coordinates are the same.**	x **-coordinates are the same.**

So, how do we find the slope of the horizontal line y=4? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's consider the points $(0,4)$ and $(3,4)$ on the graph of $y = 4$.



What is the rise?	The rise is 0.
What is the run?	The run is 3.
	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{0}{3}$
What is the slope?	$m = 0$
	The slope of the horizontal line $y = 4$ is 0.

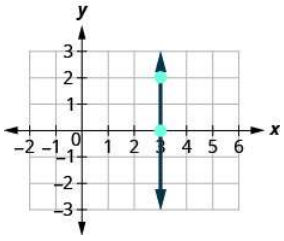
All horizontal lines have slope 0. When the y-coordinates are the same, the rise is 0.

SLOPE OF A HORIZONTAL LINE

The slope of a horizontal line, $y = b$, is 0.

Your room's floor is horizontal, and its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Let's switch to the vertical line $x = 3$ and consider the points $(3, 0)$ and $3, 2)$.



What is the rise?	The rise is 2.
What is the run?	The run is 0.
	$m = \frac{\text{rise}}{\text{run}}$
What is the slope?	$m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line $x = 3$ is undefined.

The slope of any vertical line is undefined. When the x-coordinates of a line are all the same, the run is 0.

SLOPE OF A VERTICAL LINE

The slope of a vertical line, $x = a$, is undefined.

Example 3

Find the slope of each line:

a. $x = 8$

b. $y = -5$

Solution

a. This is a vertical line, so the slope is undefined.

b. This is a horizontal line, so the slope is $m = 0$.

Try It 3

Find the slope of the line:

a. $y = 7$

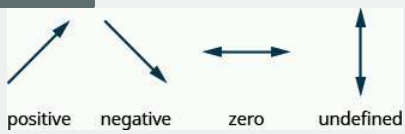
b. $x = -4$

Answer

a. $m = 0$

b. The slope is undefined

QUICK GUIDE TO THE SLOPES OF LINES



Remember, we 'read' a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Sometimes we'll need to find the slope of a line between two points when we don't have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we'll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x, y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x, y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

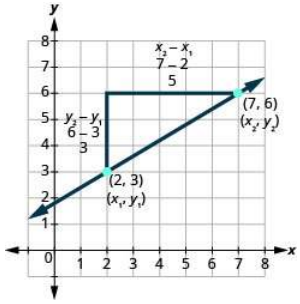
(x_1, y_1) read 'x sub $1, y$ sub 1'
(x_2, y_2) read 'x sub $2, y$ sub 2'

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third-grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points $(2, 3)$ and $(7, 6)$.



Since we have two points, we will use subscript notation, $\begin{pmatrix} x_1, y_1 \\ 2, 3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 7, 6 \end{pmatrix}$.

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3.

$$3 = 6 - 3$$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2.

$$5 = 7 - 2$$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$

But 6 is y_2, the *y*-coordinate of the second point and 3 is y_1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y2-y1}{7-2}$

Also, 7 is x_2, the *x*-coordinate of the second point and 2 is x_1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y2-y1}{x2-x1}$

We've shown that $m = \frac{y2-y1}{x2-x1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

SLOPE FORMULA

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This is the **slope formula**.

The slope is:

$$\begin{gathered} y \text{ of the second point minus } y \text{ of the first point} \\ \text{over} \\ x \text{ of the second point minus } x \text{ of the first point.} \end{gathered}$$

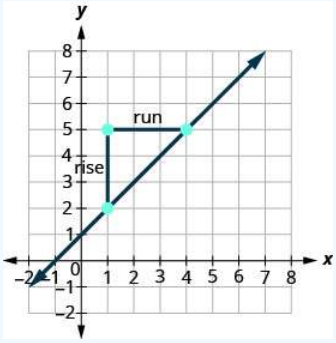
Example 4a

Use the **slope formula** to find the slope of the line between the points $(1, 2)$ and $(4, 5)$.

Solution

Misplaced '#'

Let's confirm this by counting out the slope on a graph using $m = \frac{\text{rise}}{\text{run}}$.



It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

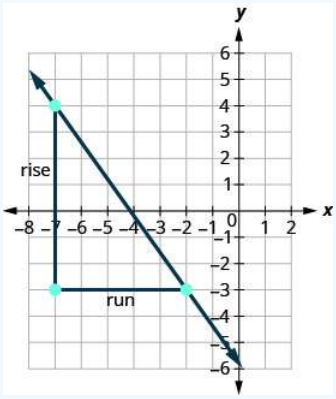
Example 4b

Use the slope formula to find the slope of the line through the points $(-2,-3)$ and $(-7,4)$.

Solution

Misplaced '#'

Let's verify this slope on the graph shown.



$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{-7}{5}$$
$$m = -\frac{7}{5}$$

(Section 3.4.2)

Try It 4

Use the slope formula to find the slope of the line through the points:

a. $(-3,4)$ and $(2,-1)$

b. $(-2,6)$ and $(-3,-4)$

Answer

a. $m = -1$

b. $m = 10$

Graph a Line Given a Point and the Slope

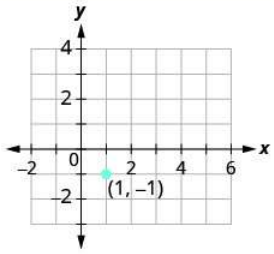
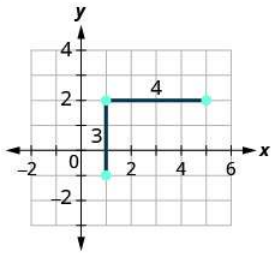
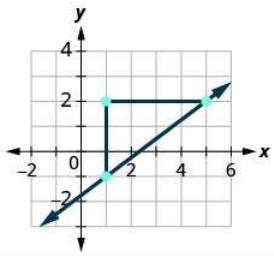
In this chapter, we have graphed lines by plotting points, using intercepts, and recognizing horizontal and vertical lines.

Another method for graphing lines is the **point–slope method**. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the slope definition to draw the graph of the line.

Example 5

Graph the line passing through the point $(1,-1)$ whose slope is $m = \frac{3}{4}$.

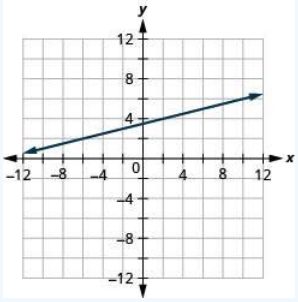
Solution

Step 1. Plot the given point.	Plot (1, –1).	
Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ rise = 3 run = 4
Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	
Step 4. Connect the points with a line.	Connect the two points with a line.	

Try It 5

Graph the line passing through the point $(-2, 3)$ with the slope $m = \frac{1}{4}$.

Answer



GRAPH A LINE GIVEN A POINT AND THE SLOPE.

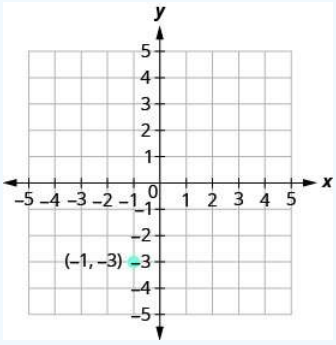
1. Plot the given point.
2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Use a straight edge to connect the points with a line.

Example 6

Graph the line passing through the point $(-1, -3)$ whose slope is $m = 4$.

Solution

Plot the given point.

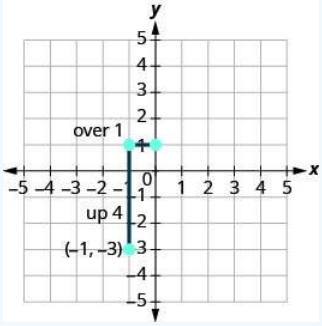


Identify the rise and the run. $m = 4$

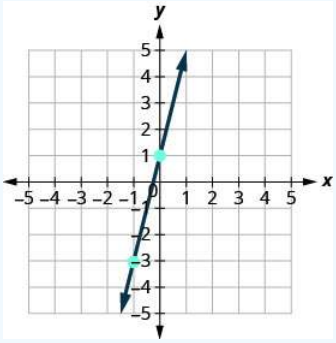
Write 4 as a fraction. $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$

$\text{rise} = 4 \quad \text{run} = 1$

Count the rise and run and mark the second point.



Connect the two points with a line.



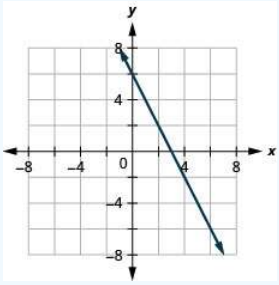
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You can check your work by finding a third point. Since the slope is $m = 4$, it can be written as $m = \frac{-4}{-1}$. Go back to $(-1, -3)$ and count out the rise, -4, and the run, -1.

Try It 6

Graph the line with the point $(4, -2)$ and slope $m = -2$.

Answer

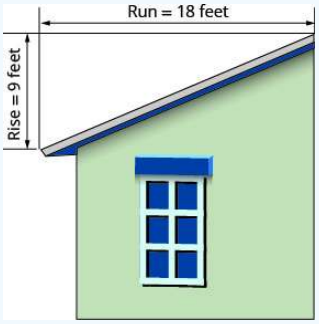


Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

Example 7

The 'pitch' of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{rise}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$

The slope of the roof is $\frac{1}{2}$.

The roof rises 1 foot for every 2 feet of horizontal run.

Try It 7

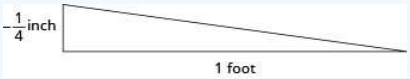
Consider two roofs; Roof A has a rise of 14 feet and a run of 24 feet, while Roof B has a rise of 15 feet and a run of 36 feet. Which roof has a steeper pitch?

Answer

Roof A has a pitch of $\frac{7}{12}$ while Roof B has a pitch of $\frac{5}{12}$. Roof A is steeper since $\frac{7}{12} > \frac{5}{12}$.

Example 8

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{-\frac{1}{4}\text{inch}}{1\text{ foot}}$
Change units.	$m = \frac{-\frac{1}{4}\text{ inch}}{12\text{ inches}}$
Simplify.	$m = -\frac{1}{48}$
	The slope of the pipe is $-\frac{1}{48}$

The pipe drops 1 inch for every 48 inches of horizontal run.

Try It 8

a. Find the slope of a pipe that goes down $\frac{1}{3}$ inch per foot.

b. Find the slope of a ramp that goes up $\frac{3}{4}$ inch per yard.

Answer

a. The pipe has a slope of $-\frac{1}{36}$

b. The ramp has a slope of $\frac{1}{48}$

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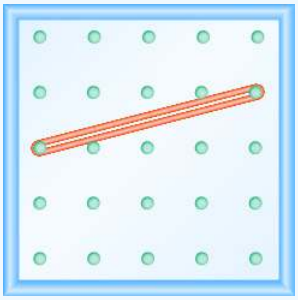
Section 3.4P: Practice

Practice Makes Progress

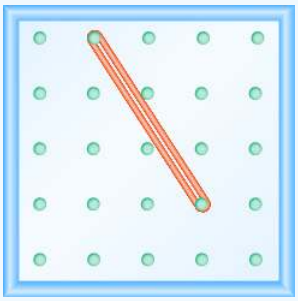
Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

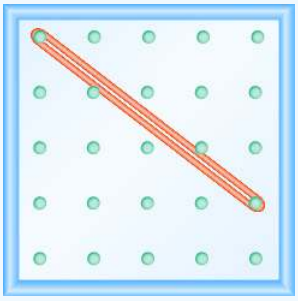
Exercise 1



Exercise 2



Exercise 3



In the following exercises, model each slope. Draw a picture to show your results.

Exercise 4

$\frac{2}{3}$

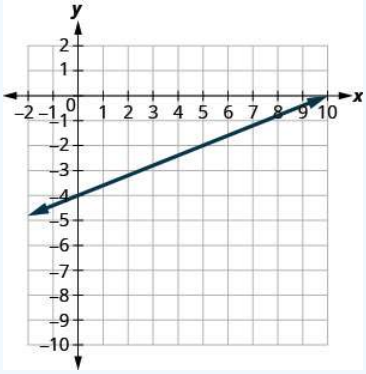
Exercise 5

$-\frac{2}{3}$

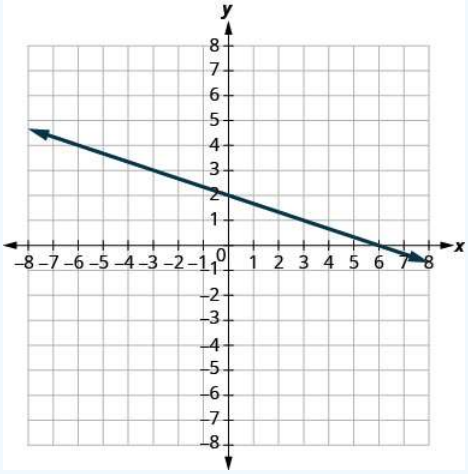
Use $m = \frac{rise}{run}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

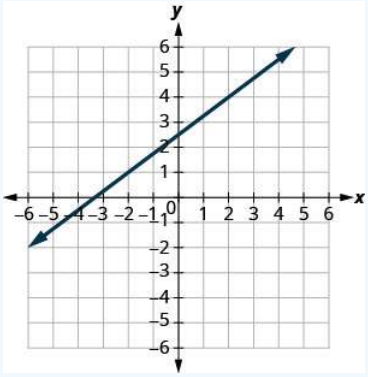
Exercise 6



Exercise 7

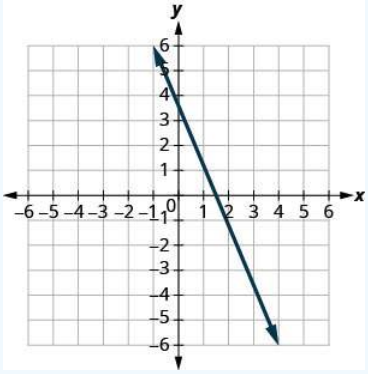


Exercise 8

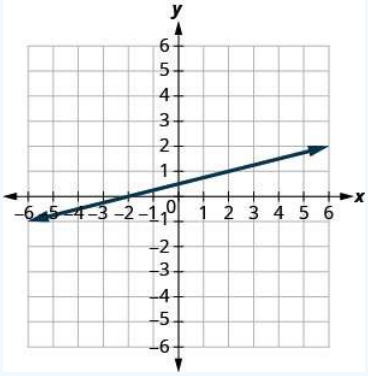


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Exercise 9



Exercise 10



Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise 11

$y = 3$

Exercise 12

$x = 4$

Use the Slope Formula to Find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise 13

$(1, 4), (3, 9)$

Exercise 14

$(-3, 3), (4, -5)$

Exercise 15

$(-1,-2),(2,5)$

Exercise 16

$(4,-5),(1,-2)$

Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

Exercise 17

$(1,-2)$; $m=\frac{3}{4}$

Exercise 18

$(-3,4)$; $m=-\frac{3}{2}$

Exercise 19

y-intercept $=3$; $m=-\frac{2}{5}$

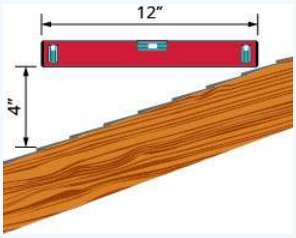
Exercise 20

$(-3,3)$; $m=2$

Everyday Math

Exercise 21

Slope of a roof. Builders refer to roof slope as “x-in-12” pitch. What is the slope in this diagram?



Exercise 22

Road grade. A local road has a grade of 6%. What is the slope as a fraction? What rise and run represents this?

Exercise 23

Wheelchair ramp. Rules require a 1-inch rise for each 12 inches of run. How long must the ramp be for a 24-inch rise

Writing Exercises

Exercise 24

What does the sign of the slope tell you about a line?

Exercise 25

Why is the slope of a vertical line “undefined”?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it
use geoboards to model slope.			
use $m = \frac{rise}{run}$ to find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.5: Slope-Intercept Form of Line

Be Prepared

We will rely heavily on these skills throughout this section.

Identify the y-intercept of $y = x + 3$

Evaluate $y = -2x + 5$ when $x = -1$

Evaluate $3x + 4$ when $x = 2$

Learning Objectives

Motivating Problem

You're using a ride-share app that charges a $2 base fee plus $1.50 per mile. What equation can you use to predict your fare—and how can the graph of this equation help you compare trips?

Fun Fact

Before GPS was common, airline pilots used linear equations—often in slope-intercept form—to chart flight paths and adjust for wind drift. By plotting lines quickly and understanding their direction, they could navigate long distances using just a map, compass, and some math.

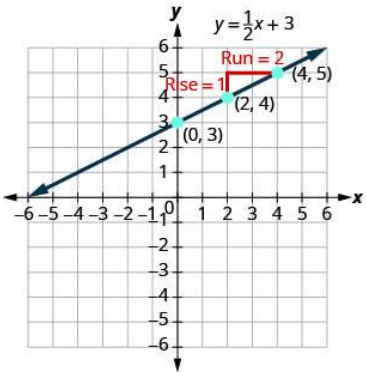
The Goal

In this section, we won't just look at equations—we'll learn how to read their structure to graph a line instantly. We'll use the slope and y-intercept to sketch lines quickly, see patterns, and connect equations to real-world meaning.

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method to graph lines.

In a previous section, we graphed the line of the equation $y = \frac{1}{2}x + 3$ by plotting points. Let's find the slope of this line.



The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

$$m = \frac{\text{rise}}{\text{run}} \qquad \text{(Section 3.5.1)}$$

$$m = \frac{1}{2}$$

What is the y-intercept of the line? The y-intercept is where the line crosses the y-axis, so the y-intercept is $(0, 3)$. The equation of this line is:

$y = \frac{1}{2}x + 3$

Notice, the line has:

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

When a linear equation is solved for y, the coefficient of the x-term is the slope, and the constant term is the y-coordinate of the y-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$m = \frac{1}{2}$; y-intercept is $(0, 3)$

$y = \frac{1}{2}x + 3$

$y = mx + b$

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE

The **slope–intercept form** of an equation of a line with slope m and y-intercept, $(0, b)$ is,

$$y = mx + b$$

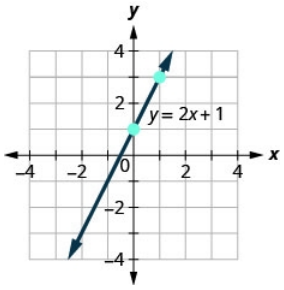
Example 1

Use the graph to find the slope and y-intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Solution

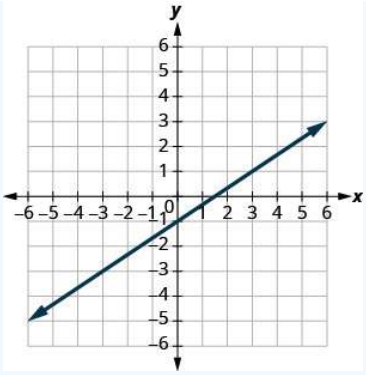
To find the slope of the line, we need to choose two points on the line. We'll use the points $(0, 1)$ and $(1, 3)$.

	
Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the y-intercept of the line.	The y-intercept is the point $(0, 1)$.
We found slope $m = 2$ and y-intercept $(0, 1)$.	$y = 2x + 1$ $y = mx + b$

The slope is the same as the coefficient of x, and the y-coordinate of the y-intercept is the same as the constant term.

Try It 1

Use the graph to find the slope and y-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.



Answer

slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

Identify the Slope and y-Intercept From an Equation of a Line

In a previous section, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the y-intercept as the point, and then count out the slope from there. Let's practice finding the values of the slope and y-intercept from the equation of a line.

Example 2

Identify the slope and y-intercept of the line with equation $y = -3x + 5$.

Solution

We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the y-intercept.	y-intercept is $(0, 5)$

Try It 2

Identify the slope and y-intercept of the line

a. $y = \frac{2}{5}x - 1$

b. $y = -\frac{4}{3}x + 1$

Answer

a. slope $m = \frac{2}{5}$; y-intercept $(0,-1)$

b. slope $m = -\frac{4}{3}$; y-intercept $(0,1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

Example 3

Identify the slope and y-intercept of the line with equation $x + 2y = 6$.

Solution

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form, we must first solve the equation for y.

Solve for y.	$x+2y=6$
Subtract x from each side.	$2y=-x+6$
Divide both sides by 2.	$\frac{2y}{2}=\frac{-x+6}{2}$
Simplify.	$\frac{2y}{2}=\frac{-x}{2}+\frac{6}{2}$
(Remember: $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$)	
Simplify.	$y=-\frac{1}{2}x+3$
Write the slope–intercept form of the equation of the line.	$y=mx+b$
Write the equation of the line.	$y=-\frac{1}{2}x+3$
Identify the slope.	$m=-\frac{1}{2}$
Identify the y-intercept.	y-intercept is $(0, 3)$

? Try It 3

Identify the slope and y-intercept of the line

a. $x+4y=8$

b. $3x+2y=12$

Answer

a. slope $m=-\frac{1}{4}$; y-intercept $(0,2)$

b. slope $m=-\frac{3}{2}$; y-intercept $(0,6)$

Graph a Line Using Its Slope and Intercept

Now that we know how to find the slope and y-intercept of a line from its equation, we can graph the line by plotting the y-intercept and then using the slope to find another point.

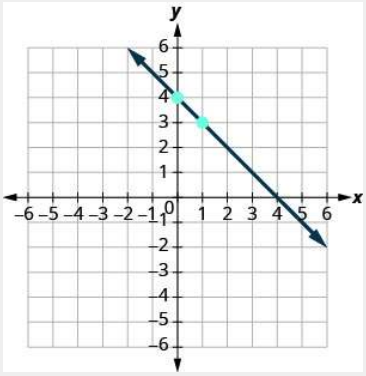
GRAPH A LINE USING ITS SLOPE AND y-INTERCEPT.

1. Find the slope-intercept form of the equation of the line.
2. Identify the slope and y-intercept.
3. Plot the y-intercept.
4. Use the slope formula $\frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
5. Starting at the y-intercept, count out the rise and run to mark the second point.
6. Use a straight edge to draw a line connecting the points.

✓ Example 4

Graph the line of the equation $y=-x+4$ using its slope and y-intercept.

Solution

	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and y-intercept.	$m = -1$
	y-intercept is $(0, 4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise -1, run 1
Draw the line.	
To verify your work, you can find another point on the line and ensure it is a solution to the equation. In the graph, we see the line goes through $(4, 0)$.	

check.

$y = -x + 4$

$0 \stackrel{?}{=} -4 + 4$

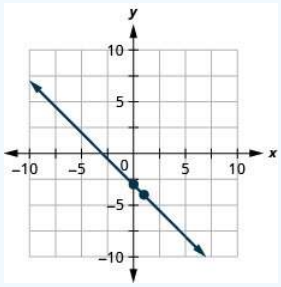
$0 = 0$✓

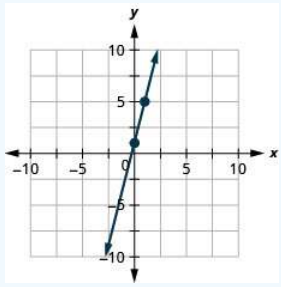
Try It 4

a. Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.

b. Graph the line of the equation $y = 4x + 1$ using its slope and y-intercept.

Answer

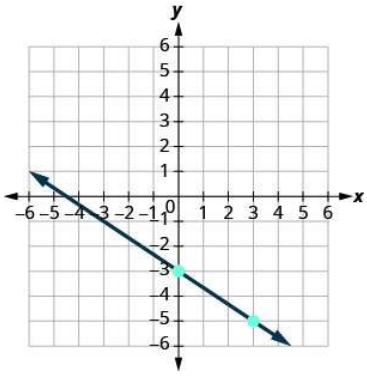
a. 

b. 

Example 5

Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and y-intercept.

Solution

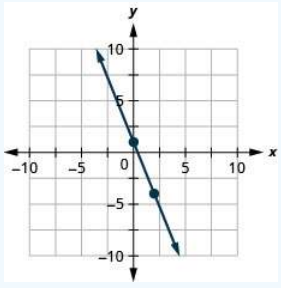
	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and y-intercept.	$m = -\frac{2}{3}$; y-intercept is $(0, -3)$
Plot the y-intercept.	See the graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	
Draw the line.	

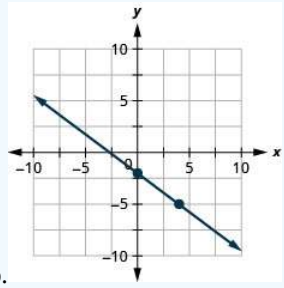
Try It 5

a. Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and y-intercept.

b. Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and y-intercept.

Answer

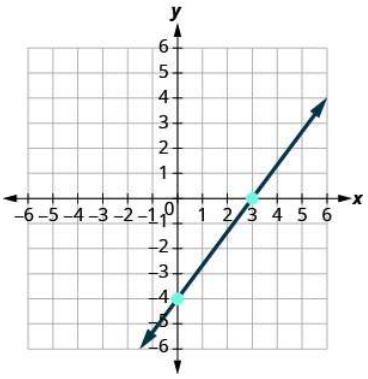
a. 

b. 

Example 6

Graph the line of the equation $4x - 3y = 12$ using its slope and y-intercept.

Solution

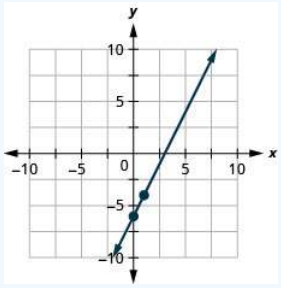
	$4x-3y=12$
Find the slope–intercept form of the equation.	$-3y=-4x+12$
	$-\frac{3y}{3}=\frac{-4x+12}{-3}$
The equation is now in slope–intercept form.	$y=\frac{4}{3}x-4$
Identify the slope and y-intercept.	$m=\frac{4}{3}$
	y-ntercept is $(0,-4)$
Plot the y-intercept.	See the graph below.
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

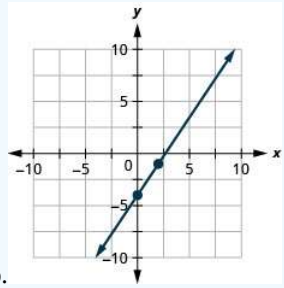
Try It 6

a. Graph the line of the equation $2x-y=6$ using its slope and y-intercept.

b. Graph the line of the equation $3x-2y=8$ using its slope and y-intercept.

Answer

a. 

b. 

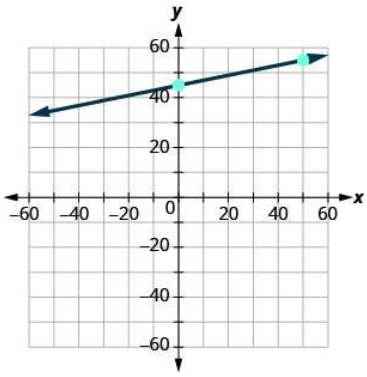
We've used a grid with x and y both ranging from approximately -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications involving real-world data, we need to extend the axes to accommodate larger positive or smaller negative numbers.

Example 7

Graph the line of the equation $y=0.2x+45$ using its slope and y-intercept.

Solution

We'll use a grid with the axes going from about -80 to 80.

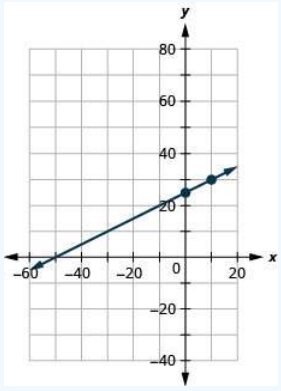
	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and y-intercept.	$m = 0.2$
	The y-intercept is $(0, 45)$
Plot the y-intercept.	See the graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

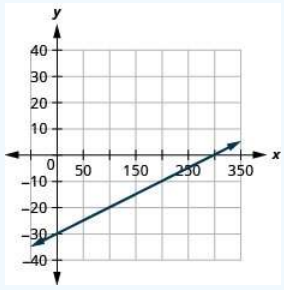
Try It 7

a. Graph the line of the equation $y = 0.5x + 25$ using its slope and y-intercept.

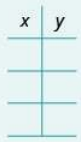
b. Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Answer

a. 

b. 

Now that we have graphed lines by using the slope and y-intercept, let's summarize all the methods we have used to graph lines.

Methods to Graph Lines			
Point Plotting	**Slope–Intercept**	**Intercepts**	**Recognize Vertical and Horizontal Lines**
	$y = mx + b$		
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods for graphing lines, how do we know which method to use for a given equation?

While we can plot points, use the slope–intercept form, or find the intercepts for any equation, recognizing the most convenient way to graph a certain type of equation makes our work easier. Generally, plotting points is not the most efficient method for graphing a line. Let's look for some patterns to help determine the most convenient method to graph a line.

Here are the six equations we graphed in this chapter, along with the method used to graph each one.

Misplaced '#'

Equations #1 and #2 each have just one variable. Remember, in equations of this form, the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the x-intercept and $x = 0$ to find the y-intercept, and then found a third point by choosing another value for x or y.

Equations #5 and #6 are written in slope–intercept form. After identifying the slope and y-intercept from the equation, we used them to graph the line.

This leads to the following strategy.

STRATEGY FOR CHOOSING THE MOST CONVENIENT METHOD TO GRAPH A LINE

Consider the form of the equation.

- If it has only one variable, it is represented by a vertical or horizontal line.
  - $x = a$ is a vertical line passing through the x-axis at a.
  - $y = b$ is a horizontal line passing through the y-axis at b.
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
  - Identify the slope and y-intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
  - Find the x- and y-intercepts, a third point, and then graph.

Example 8

Determine the most convenient method to graph each line.

1. $y = -6$
2. $5x - 3y = 15$
3. $x = 7$
4. $y = \frac{2}{5}x - 1$.

Solution

1. This equation has only one variable, y. Its graph is a horizontal line crossing the y-axis at -6.
2. This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.
3. There is only one variable, x. The graph is a vertical line crossing the x-axis at 7.
4. Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and y-intercept.

? Try It 8

Determine the most convenient method to graph each line:

1. $3x + 2y = 12$
2. $y = 4$
3. $y = \frac{1}{5}x - 4$
4. $x = -7$

Answer

1. intercepts
2. horizontal line
3. slope–intercept
4. vertical line

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will examine a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually, when a linear equation models a real-world situation, different letters are used for the variables, instead of x and y. The variable names remind us of what quantities are being measured.

✓ Example 9

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

1. Find the Fahrenheit temperature for a Celsius temperature of 0.
2. Find the Fahrenheit temperature corresponding to a Celsius temperature of 20.
3. Interpret the slope and F-intercept of the equation.
4. Graph the equation.

Solution

1. | | |
   |---|---|
   | Find the Fahrenheit temperature for a Celsius temperature of 0. | $F = \frac{9}{5}C + 32$ |
   | Find F when $C = 0$. | $F = \frac{9}{5}(0) + 32$ |
   | Simplify. | $F = 32$ |

2. | | |
   |---|---|
   | Find the Fahrenheit temperature for a Celsius temperature of 20. | $F = \frac{9}{5}C + 32$ |
   | Find F when $C = 20$. | $F = \frac{9}{5}(20) + 32$ |
   | Simplify. | $F = 36 + 32$ |
   | Simplify. | $F = 68$ |

3. Interpret the slope and F-intercept of the equation.

Even though this equation uses F and C, it is still in slope–intercept form.

$y = mx + b$

$F = mC + b$

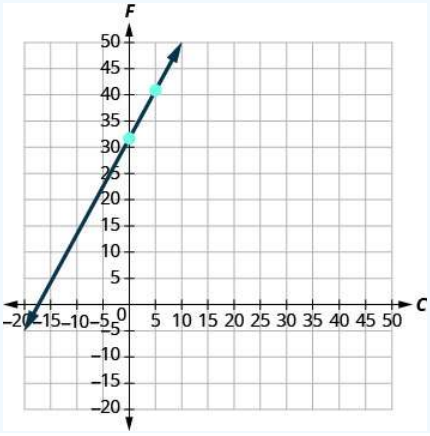
$F = \frac{9}{5}C + 32$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees.

The F-intercept means that when the temperature is $0°$ on the Celsius scale, it is $32°$ on the Fahrenheit scale.

4. Graph the equation.

We'll need to use a larger scale than our usual. Start at the F-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point.



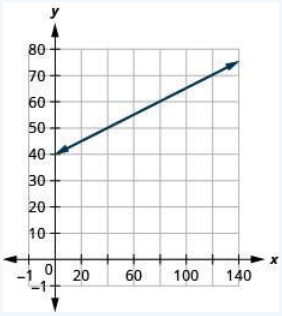
Try It 9

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, T, based on the number of cricket chirps, n, in one minute.

1. Estimate the temperature when there are no chirps.
2. Estimate the temperature when the number of chirps in one minute is 100.
3. Interpret the slope and T-intercept of the equation.
4. Graph the equation.

Answer

1. 40 degrees
2. 65 degrees
3. The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (F) increases 1 degree when the number of chirps, n, increases by 4. The T-intercept means that when the number of chirps is 0, the temperature is $40°$.
4.



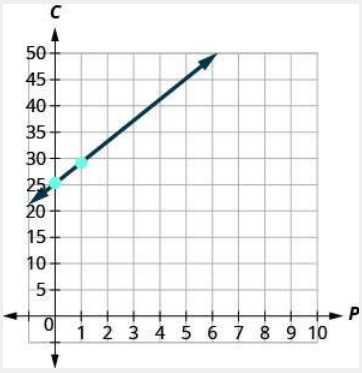
The cost of running certain types of businesses has two components: a fixed cost and a variable cost. The fixed cost remains constant regardless of the number of units produced. Some examples include the costs of rent, insurance, equipment, advertising, and other recurring expenses. The variable cost depends on the number of units produced. These include the material and labor needed to produce each item.

Example 10

Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C, in dollars and the number of pizzas, p, that she sells.

1. Find Stella's cost for a week when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Solution

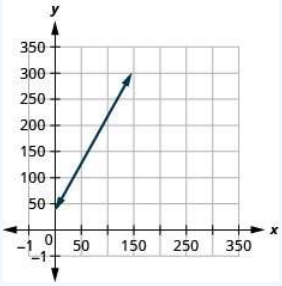
1. Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is $\$25$ when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are $\$85$ when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by $\$4$ for each pizza Stella sells. The C-intercept means that even when Stella sells no pizzas, her costs for the week are $\$25$.
4. Graph the equation. We'll need to use a larger scale than our usual. Start at the C-intercept $(0, 25)$ then count out the rise of 4 and the run of 1 to get a second point.	

Try It 10

Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, C, in dollars and the number of wedding invitations, n, that she writes.

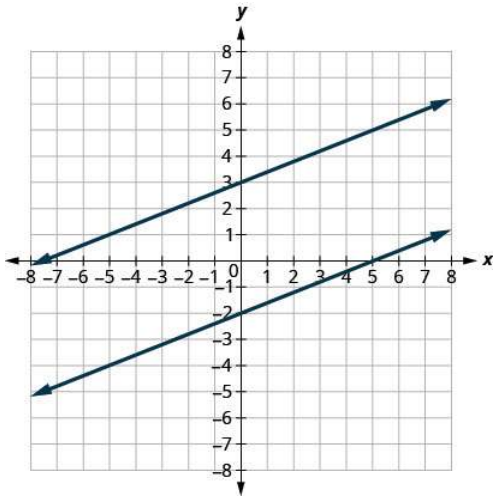
1. Find Loreen's cost for a week when she writes no invitations.
2. Find the cost for a week when she writes 75 invitations.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $\$35$
2. $\$170$
3. The slope, 1.8, means that the weekly cost, C, increases by $\$1.80$ when the number of invitations, n, increases by 1. The C-intercept means that when the number of invitations is 0, the weekly cost is $\$35$.
4. 

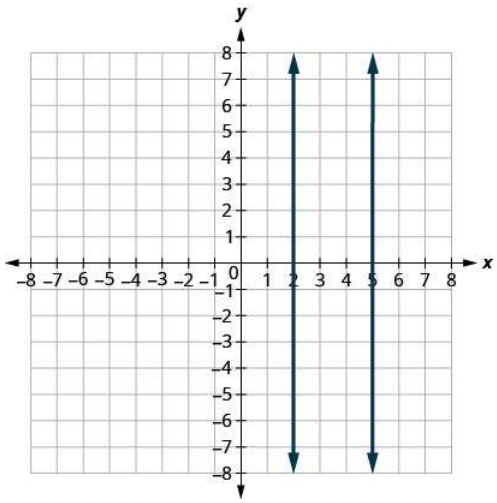
Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.



Both lines have the same slope, $m = \frac{2}{5}$, and different y-intercepts.

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different x-intercepts are parallel.



Vertical lines with different x-intercepts are parallel.

PARALLEL LINES

Parallel lines are lines in the same plane that do not intersect.

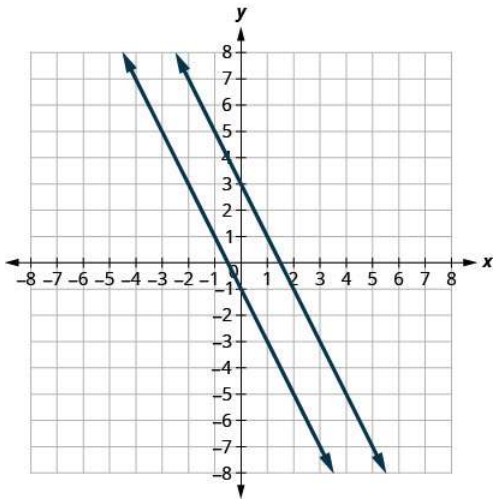
- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope–intercept form: $y = -2x + 3$. We solve the second equation for y:

$$\begin{aligned} 2x + y &= -1 \\ y &= -2x - 1 \end{aligned}$$

(Section 3.5.2)

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the y-intercept of each line?

$$\begin{array}{llll} y & = mx + b & y = mx + b \\ y & = -2x + 3 & y = -2x - 1 \\ m & = -2 & m = -2 \\ b & b = 3 & b = -1 \end{array}$$

The slopes of the lines are the same, and the y-intercept of each line, $(0,3)$ and $(0,1)$, is different. So we know these lines are parallel.

Since parallel lines have the same slope and different y-intercepts, we can now just look at the slope–intercept form of the equations of lines and decide if the lines are parallel.

Example 11

Use slopes and y-intercepts to determine if the lines $3x-2y=6$ and $y=\frac{3}{2}x+1$ are parallel.

Solution

Solve the first equation for y.

$$\begin{aligned} 3x-2y &= 6 \\ -2y &= -3x+6 \\ \frac{-2y}{-2} &= \frac{-3x+6}{-2} \\ y &= \frac{3}{2}x-3 \end{aligned}$$

The equation is now in slope–intercept form.

The equation of the second line is already in slope–intercept form.

Identify the slope and y-intercept of both lines.

$y=\frac{3}{2}x+1$	$y=\frac{3}{2}x-3$
$y=mx+b$	$y=mx+b$
$m=\frac{3}{2}$	$m=\frac{3}{2}$
y-intercept is $(0,1)$	y-intercept is $(0,-3)$

The lines have the same slope and different y-intercepts; therefore, they are parallel. You may want to graph the lines to confirm whether they are parallel.

Try It 11

a. Use slopes and y-intercepts to determine if the lines $2x-5y=5$ and $y=-\frac{2}{5}x-4$ are parallel.

b. Use slopes and y-intercepts to determine if the lines $4x-3y=6$ and $y=\frac{4}{3}x-1$ are parallel.

Answer

a. Not parallel

b. parallel

Example 12

a. Use slopes and y-intercepts to determine if the lines $y=-4$ and $y=3$ are parallel.

b. Use slopes and y-intercepts to determine if the lines $x=-2$ and $x=-5$ are parallel.

Solution

a.

Write each equation in slope-intercept form.

Since there is no x term we write $0x$.

Identify the slope and y-intercept of both lines.

		and	
	$y=-4$		$y=3$
	$y=0x-4$		$y=0x+3$
	$y=mx+b$		$y=mx+b$
	$m=0$		$m=0$
	y-intercept is $(0,-4)$		y-intercept is $(0,3)$

The lines have the same slope and different y-intercepts; therefore, they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y-axis at $y=-4$ and at $y=3$, we

know the y-intercepts are $(0,-4)$ and $(0,3)$. The lines have the same slope and different y-intercepts; therefore, they are parallel.

b.

$$x=-2 \text{ and } x=-5$$

Since there is no y, the equations cannot be put in slope–intercept form. But we recognize them as equations of vertical lines. Their x-intercepts are -2 and -5. Since their x-intercepts are different, the vertical lines are parallel.

Try It 12

a. Use slopes and y-intercepts to determine if the lines $x=1$ and $x=-5$ are parallel.

b. Use slopes and y-intercepts to determine if the lines $y=1$ and $y=-5$ are parallel.

Answer

a. parallel

b. parallel

Example 13

Use slopes and y-intercepts to determine if the lines $y=2x-3$ and $-6x+3y=-9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution

The first equation is already in slope-intercept form. $y=2x-3$

Solve the second equation for y

$$\begin{aligned} -6x+3y &= -9 \\ 3y &= 6x-9 \\ \tfrac{3y}{3} &= \tfrac{6x-9}{3} \\ y &= 2x-3 \end{aligned}$$

The second equation is now also in slope–intercept form.

Identify the slope and y-intercept of both lines.

$y=2x-3$	$y=2x-3$
$y=mx+b$	$y=mx+b$
$m=2$	$m=2$
The y-intercept is $(0,-3)$	The y-intercept is $(0,-3)$

The lines have the same slope, but they also have the same y-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Try It 13

a. Use slopes and y-intercepts to determine if the lines $y=\frac{1}{2}x+1$ and $x+2y=2$ are parallel.

b. Use slopes and y-intercepts to determine if the lines $y=\frac{3}{4}x-3$ and $3x-4y=12$ are parallel.

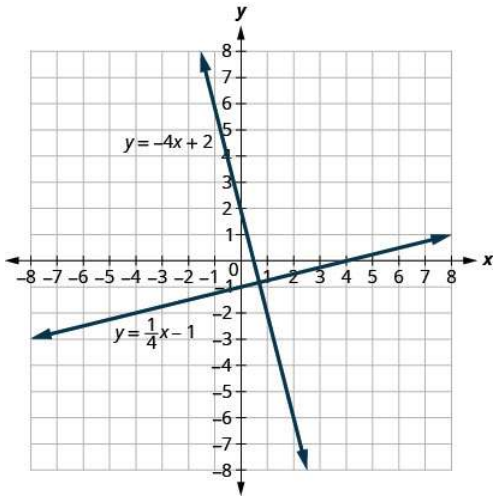
Answer

a. not parallel; intersecting lines

b. not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y = \frac{1}{4}x - 1$ and $y = -4x + 2$.



These lines lie in the same plane and intersect at right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y = \frac{1}{4}x - 1$ rises, so its slope is positive. The line $y = -4x + 2$ drops from left to right, indicating a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1 = \frac{1}{4}$, and the slope of the second line, $m_2 = -4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

$$\begin{array}{c} m_1 \cdot m_2 \\ \frac{1}{4}(-4) \\ -1 \end{array}$$

This is always true for **perpendicular lines** and leads us to this definition.

PERPENDICULAR LINES

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

$$m_1 \cdot m_2 = -1 \text{ and } m_1 = \frac{-1}{m_2}$$

Vertical lines and horizontal lines are always perpendicular to each other.

We examined the slope–intercept form of linear equations and determined whether the lines were parallel. We can do the same for perpendicular lines.

We find the slope–intercept form of the equation and then determine if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same y-intercepts.

Example 14

Use slopes to determine if the lines $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution

The first equation is already in slope–intercept form: $y = -5x - 4$

Solve the second equation for y.

$$\begin{aligned} x-5y &= 5 \\ -5y &= -x+5 \\ \tfrac{-5y}{-5} &= \tfrac{-x+5}{-5} \\ y &= \tfrac{1}{5}x-1 \end{aligned}$$

The second equation is now also in slope-intercept form.

Identify the slope of each line.

$$\begin{aligned} y &= -5x-4 & \qquad y &= \tfrac{1}{5}x-1 \\ y &= mx+b & \qquad y &= mx+b \\ m_1 &= -5 & \qquad m_2 &= \tfrac{1}{5} \end{aligned}$$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

$$\begin{aligned} & m_1 \cdot m_2 \\ & -5\left(\tfrac{1}{5}\right) \\ & -1\checkmark \end{aligned}$$

Try It 14

a. Use slopes to determine if the lines $y=-3x+2$ and $x+3y=4$ are perpendicular.

b. Use slopes to determine if the lines $y=2x-5$ and $x+2y=-6$ are perpendicular.

Answer

a. not perpendicular

b. perpendicular

Example 15

Use slopes to determine if the lines, $7x+2y=3$ and $2x+7y=5$ are perpendicular.

Solution

Solve the equations for y.

$$\begin{aligned} 7x+2y &= 3 & \qquad 2x+7y &= 5 \\ 2y &= -7x+3 & \qquad 7y &= -2x+5 \\ \tfrac{2y}{2} &= \tfrac{-7x+3}{2} & \qquad \tfrac{7y}{7} &= \tfrac{-2x+5}{7} \\ y &= -\tfrac{7}{2}x+\tfrac{3}{2} & \qquad y &= \tfrac{-2}{7}x+\tfrac{5}{7} \end{aligned}$$

Identify the slope of each line.

$$\begin{aligned} y &= mx+b & \qquad y &= mx+b \\ m_1 &= -\tfrac{7}{2} & \qquad m_2 &= -\tfrac{2}{7} \end{aligned}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Try It 15

a. Use slopes to determine if the lines $5x+4y=1$ and $4x-5y=3$ are perpendicular.

b. Use slopes to determine if the lines $2x-9y=3$ and $9x-2y=1$ are perpendicular.

Answer

a. perpendicular

b. not perpendicular

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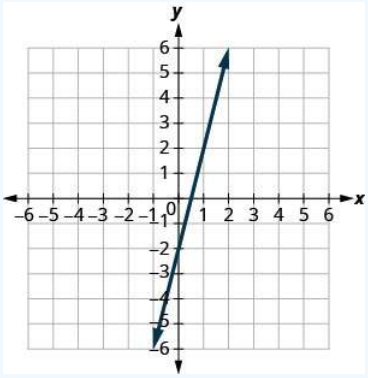
Section 3.5P: Practice

Practice Makes Progress

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

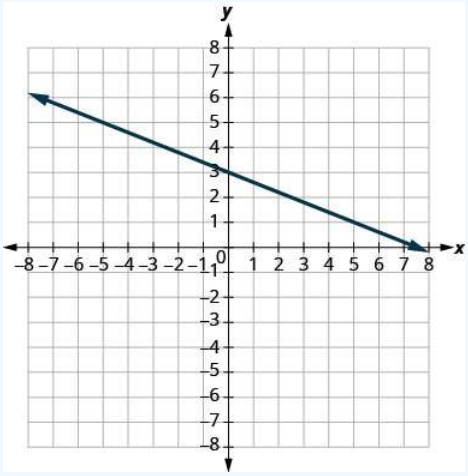
In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise 1



$y = 4x - 2$

Exercise 2



$y = -\frac{2}{5}x + 3$

Identify the Slope and y-Intercept From an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise 3

$y = 4x - 10$

Exercise 4

$8x + 3y = 12$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

Exercise 5

$y = x + 4$

Exercise 6

$y = -x + 3$

Exercise 7

$y = -\frac{2}{5}x - 3$

Exercise 8

$4x - 3y = 6$

Choose the Most Convenient Method to Graph a Line

In the following exercises, determine the most convenient method to graph each line.

Exercise 9

$y = 4$

Exercise 10

$y = -5x + 2$

Exercise 11

$y = \frac{4}{5}x - 3$

Exercise 12

$y = -1$

Exercise 13

$2x - 5y = -10$

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise 14

$6x - 3y = 9; \quad 2x - y = 3$

Exercise 15

$8x + 6y = 6; \quad 12x + 9y = 12$

Exercise 16

$x = 7; \quad x = -8$

Exercise 17

$y = 5; \quad y = 1$

Exercise 18

$4x + 4y = 8; \quad x + y = 2$

Exercise 19

$4x - 8y = 16; \quad x - 2y = 4$

Exercise 20

$9x - 5y = 4; \quad 5x + 9y = -1$

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise 21

$x - 4y = 8; \quad 4x + y = 2$

Exercise 22

$3x - 4y = 8; \quad 4x - 3y = 6$

Exercise 23

$2x + 4y = 3; \quad 6x + 3y = 2$

Exercise 24

$2x - 6y = 4; \quad 12x + 4y = 9$

Exercise 25

$8x - 2y = 7; \quad 3x + 12y = 9$

Everyday Math

Exercise 26

The equation $P = 28 + 2.54w$ models the relation between the amount of Randy's monthly water bill payment, P, in dollars, and the number of units of water, w, used.

a. Find the payment for a month when Randy used 0 units of water.
b. Find the payment for a month when Randy used 15 units of water.
c. Interpret the slope and P-intercept of the equation.
d. Graph the equation.

Exercise 27

Patel's weekly salary includes base pay plus commission on his sales. The equation $S = 750 + 0.09c$ models the relation between his weekly salary, S, in dollars, and the amount of his sales, c, in dollars.

a. Find Patel's salary for a week when his sales were 0.
b. Find Patel's salary for a week when his sales were $18,540$.
c. Interpret the slope and S-intercept of the equation.
d. Graph the equation.

Exercise 28

Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, C, of the banquet and the number of guests, g.

a. Find the cost if the number of guests is 50.
b. Find the cost if the number of guests is 100.
c. Interpret the slope and C-intercept of the equation.
d. Graph the equation.

Exercise 29

The equation $n = 4T - 160$ is used to estimate the number of cricket chirps, n, in one minute based on the temperature in degrees Fahrenheit, T.

a. Explain what the slope of the equation means.
b. Explain what the n-intercept of the equation means. Is this a realistic situation?

Writing Exercises

Exercise 30

Why are all horizontal lines parallel?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the graph and the slope-intercept form of an equation of a line.			
identify the slope and *y*-intercept from an equation of a line.			
graph a line using its slope and intercept.			
choose the most convenient method to graph a line.			
graph and interpret applications of slope-intercept.			
use slopes to identify parallel lines.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.6: Line Equations

Be Prepared

We will rely heavily on these skills throughout this section.

- Determine the slope of the line $y = \frac{1}{2}x - 1$
- Determine whether the point $(4, 1)$ lies on the line $y = -x + 5$
- Find the y-intercept of the equation $4x - 2y = 10$

Learning Objectives

Motivating Problem

You're designing a wheelchair ramp that must rise 2 feet for every 12 feet of horizontal run. The ramp starts at ground level in front of the entrance. If someone asks you to describe the slope of the ramp and write an equation to model its incline, what form should you use—and how do you decide?

Fun Fact

The idea of using different forms to represent lines dates back to early analytic geometry. French mathematician François Viète laid the groundwork in the 1500s, and later, Descartes formalized it—giving us the flexible language of line equations we still use today.

The Goal

In this section, we won't just plug into formulas—we'll learn how to build equations of lines based on the information we have. Whether we're given a slope and a point, two points, or a relationship like "parallel to" or "perpendicular to," we'll decide which method fits the situation and write the equation that models it.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section, we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and y-Intercept

If the equation is written in slope-intercept form, $y = mx + b$, we can easily determine the slope and y-intercept of a line. Now, we will do the reverse—we will start with the slope and y-intercept and use them to find the equation of the line.

Example 1

Find an equation of a line with slope -7 and y-intercept $(0, -1)$.

Solution

Since we are given the slope and y-intercept of the line, we can substitute the necessary values into the slope–intercept form, $y = mx + b$.

Name the slope.	$m = -7$
Name the y-intercept.	y-intercept $(0, -1)$
Substitute the values into y=mx+b.	$y = mx + b$
	$y = -7x + (-1)$
	$y = -7x + -1$

Try It 1

a. Find an equation of a line with slope $\frac{2}{5}$ and y-intercept $(0, 4)$.

b. Find an equation of a line with slope -1 and y-intercept $(0, -3)$.

Answer

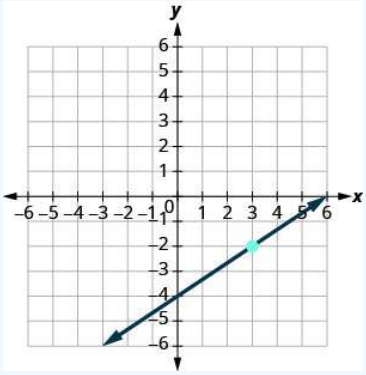
a. $y = \frac{2}{5}x + 4$

b. $y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

Example 2

Find the equation of the line shown.



Solution

We need to find the slope and y-intercept of the line from the graph so that we can substitute the necessary values into the slope–intercept form, $y = mx + b$.

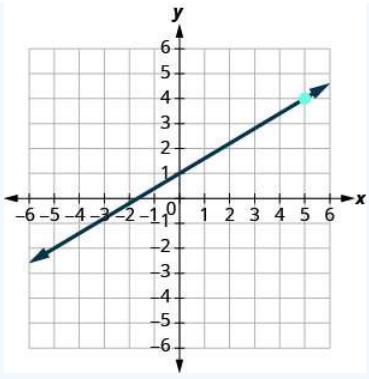
To find the slope, we choose two points on the graph.

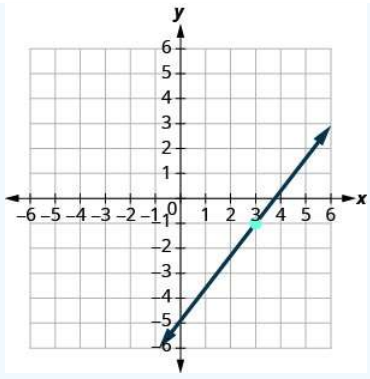
The y-intercept is $(0, -4)$ and the graph passes through $(3, -2)$.

Find the slope by counting the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{3}$
Find the y-intercept.	y-intercept $(0, -4)$
Substitute the values into $y = mx + b$.	$y = mx + b$
	$y = \frac{2}{3}x - 4$

Try It 2

Find the equation of the line shown in the graph.

a. 

b. 

Answer

a. $y = \frac{3}{5}x + 1$

b. $y = \frac{4}{3}x - 5$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope–intercept form of the equation works well when you are given the slope and y-intercept or when you read them off a graph. But what happens when you have another point instead of the y-intercept?

We will use the slope formula to derive another form of the line's equation. Suppose we have a line that has slope m and that contains some specific point (x_1, y_1) and some other point, which we will just call (x, y). We can write the slope of this line and then change it to a different form.

	m	$= \frac{y-y_1}{x-x_1}$
Multiply both sides of the equation by $x - x_1$.	$m(x-x_1)$	$= \left(\frac{y-y_1}{x-x_1}\right)(x-x_1)$
Simplify.	$m(x-x_1)$	$= y - y_1$
Rewrite the equation with the y terms on the left.	$y - y_1$	$= m(x-x_1)$

This format is known as the point–slope form of a line's equation.

POINT–SLOPE FORM OF AN EQUATION OF A LINE

The **point–slope form** of an equation of a line with slope m and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

We can use the point–slope form of an equation to find the equation of a line when given the slope and one point. We will then rewrite the equation in slope–intercept form. Most applications of linear equations use the slope–intercept form.

FIND AN EQUATION OF A LINE GIVEN THE SLOPE AND A POINT.

1. Identify the slope.
2. Identify the point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Example 3

Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point $(6, -4)$. Write the equation in slope–intercept form.

Solution

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = -\frac{1}{3}$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ 6, & -4 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-4) = -\frac{1}{3}(x - 6)$
Simplify.	$y + 4 = -\frac{1}{3}x + 2$
Write in slope–intercept form.	$y = -\frac{1}{3}x - 2$

Try It 3

Find an equation of a line with slope $m = -\frac{2}{5}$ and containing the point $(10, -5)$.

Answer

$y = -\frac{2}{5}x - 1$

Example 4

Find an equation of a horizontal line that contains the point $(-1, 2)$. Write the equation in slope–intercept form.

Solution

Every horizontal line has slope 0. We can substitute the slope and points into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = 0$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ -1, & 2 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - 2 = 0(x - (-1))$
Simplify.	$y - 2 = 0(x + 1)$
	$y - 2 = 0$
	$y = 2$
Write in slope–intercept form.	It is in y-form, but could be written as $y = 0x + 2$.

Try It 4

a. Find an equation of a vertical line containing the point $(-3, 8)$.

b. Find an equation of a horizontal line containing the point $(-1, 4)$.

Answer

a. $x = -3$

b. $y = 4$

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example, we'll see how to find the equation of a line when just two points are given.

We have two options so far for finding the equation of a line: the slope–intercept form or the point–slope form. Since we will know two points, we will use the point–slope form.

FIND AN EQUATION OF A LINE GIVEN TWO POINTS.

1. Find the slope using the given points.
2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Example 5

Find an equation of a line that contains the points $(-3, -1)$ and $(2, -2)$. Write the equation in slope–intercept form.

Solution

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

Find the slope of the line through $(-3, -1)$ and $(2, -2)$.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
	$m = \frac{-2 - (-1)}{2 - (-3)}$
	$m = \frac{-1}{5}$
	$m = -\frac{1}{5}$
Choose either point.	(x_1, y_1) $(2, -2)$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-2) = -\frac{1}{5}(x - 2)$
	$y + 2 = -\frac{1}{5}x + \frac{2}{5}$
Write in slope–intercept form.	$y = -\frac{1}{5}x - \frac{8}{5}$

Try It 5

Find an equation of a line containing the points $(-2, -4)$ and $(1, -3)$.

Answer

$y = \frac{1}{3}x - \frac{10}{3}$

Example 6

Find an equation of a line that contains the points $(-2, 4)$ and $(-2, -3)$. Write the equation in slope–intercept form.

Solution

Again, the first step will be to find the slope.

Find the slope of the line through $(-2,4)$ and $(-2,-3)$

$$\begin{aligned} m &= \frac{y_2-x_1}{x_2-x_1} \\ m &= \frac{-3-4}{-2-(-2)} \\ m &= \frac{-7}{0} \end{aligned}$$

The slope is undefined.

This tells us it is a vertical line. Both of our points have an x-coordinate of -2. So our equation of the line is $x = -2$. Since there is no y, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

? Try It 6

Find an equation of a line containing the points $(5,1)$ and $(5,-4)$.

Answer

$x = 5$

We have seen that we can use either the slope–intercept form or the point–slope form to find the equation of a line. Which form we use will depend on the information we are given. This is summarized below.

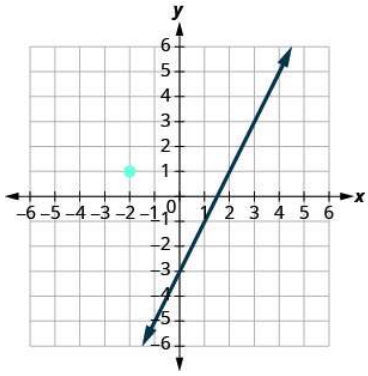
To Write an Equation of a Line		
If given:	Use:	Form:
Slope and y-intercept	slope–intercept	$y = mx + b$
Slope and a point	point–slope	$y - y_1 = m(x - x_1)$
Two points	point–slope	$y - y_1 = m(x - x_1)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope, just what we need to use the point–slope equation.

First, let's look at this graphically.

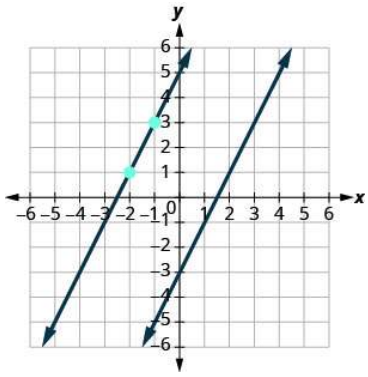
The graph shows the graph of $y = 2x - 3$. We want to graph a line parallel to this line and passing through the point $(-2,1)$.



We know that parallel lines have the same slope. So, the second line will have the same slope as $y = 2x - 3$. That slope is $m_{\parallel} = 2$. We'll use the notation $m_{\parallel}$ to represent the slope of a line parallel to a line with slope m. (Notice that the subscript $\parallel$ looks like two

parallel lines.)

The second line will pass through $(-2, 1)$ and have $m = 2$. To graph the line, we start at $(-2, 1)$ and count out the rise and run. With $m = 2$ (or $m = \frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.



Do the lines appear parallel? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically.

We can use either the slope–intercept form or the point–slope form to find the equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

Example 7

Find an equation of a line parallel to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_1 = 2$
Step 3. Identify the point.	The given point is, (–2, 1).	$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Does this equation make sense? What is the y-intercept of the line? What is the slope?

Try It 7

Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Answer

$y = \frac{1}{2}x + 1$

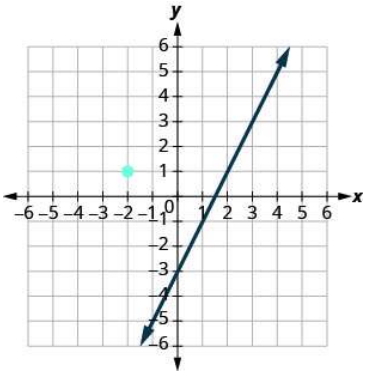
FIND AN EQUATION OF A LINE PARALLEL TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the parallel line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line that passes through a specific point and is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals of each other. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of $y = 2x - 3$. Now, we want to graph a line perpendicular to this line and passing through $(-2, 1)$.

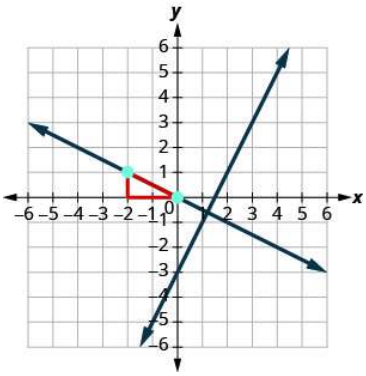


We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_\perp$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $_\perp$ looks like the right angles made by two perpendicular lines.)

$$\begin{array}{ll} y = 2x - 3 & \text{perpendicular line} \\ m = 2 & m_\perp = -\frac{1}{2} \end{array}$$

We now know the perpendicular line will pass through $(-2, 1)$ with $m_\perp = -\frac{1}{2}$.

To graph the line, we will start at $(-2, 1)$ and count out the rise -1 and the run 2. Then we draw the line.



Do the lines appear perpendicular? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find the equation of a line. In this example, we know one point, and can find the slope, so we will use the point–slope form.

Example 8

Find an equation of a line perpendicular to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_{\perp} = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, (–2, 1)	$\begin{pmatrix} x_1 & y_1 \\ -2, & 1 \end{pmatrix}$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = -\frac{1}{2}(x - (-2))$ $y - 1 = -\frac{1}{2}(x + 2)$ $y - 1 = -\frac{1}{2}x - 1$
Step 5. Write the equation in slope–intercept form.		$y = -\frac{1}{2}x$

Try It 8

Find an equation of a line perpendicular to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

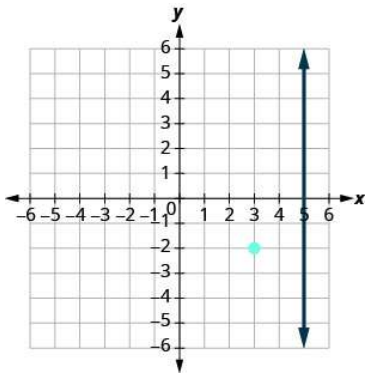
Answer

b. $y = -2x + 16$

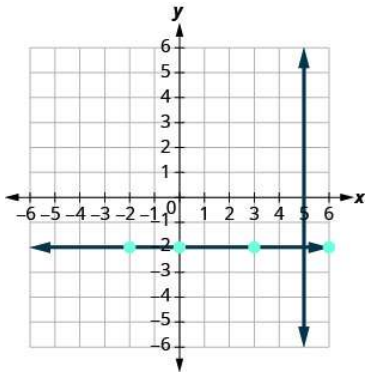
FIND AN EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the perpendicular line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Suppose we want to find a line that is perpendicular to $x = 5$ and contains the point $(3, -2)$. The graph shows us the line $x = 5$ and the point $(3, -2)$.



We know every line perpendicular to a vertical line is horizontal, so we will sketch the horizontal line through $(3, -2)$.



Do the lines appear perpendicular?

If we examine a few points on this horizontal line, we notice that they all have y-coordinates of -2. Therefore, the equation of the line perpendicular to the vertical line $x = 5$ is $y = -2$.

Example 9

Find an equation of a line that is perpendicular to $y = -4$ that contains the point $(-4, 2)$. Write the equation in slope–intercept form.

Solution

The line $y = -4$ is horizontal. Any line perpendicular to it must be vertical, in the form $x = a$. Since the perpendicular line is vertical and passes through $(-4, 2)$, every point on it has an x-coordinate of -4. The equation of the perpendicular line is $x = -4$. You may want to sketch the lines. Do they appear perpendicular?

Try It 9

Find an equation of a line that is perpendicular to the line $x = 1$ that contains the point $(-5, 2)$. Write the equation in slope–intercept form.

Answer

$y = 2$

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Section 3.6P: Practice

Practice Makes Progress

Find an Equation of the Line

Find an Equation of the Line Given the Slope and y-Intercept

In the following exercises, find the equation of a line with the given slope and y-intercept. Write the equation in slope–intercept form.

Exercise 1

slope 8 and y-intercept $(0, -6)$

Exercise 2

slope -3 and y-intercept $(0, -1)$

Exercise 3

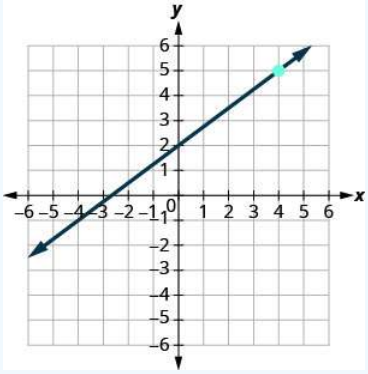
slope $-\frac{2}{3}$ and y-intercept $(0, -3)$

Exercise 4

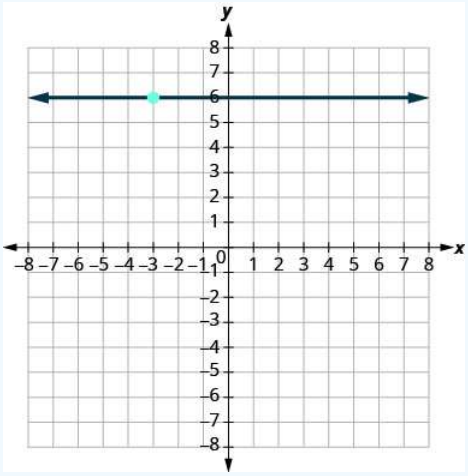
slope -4 and y-intercept $(0, 0)$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

Exercise 5



Exercise 6



Find an Equation of the Line Given the Slope and a Point

Exercise 7

$m = \frac{5}{6}$, point $(6, 7)$

Exercise 8

$m = -\frac{1}{3}$, point $(-9, -8)$

Exercise 9

Horizontal line containing $(-1, -7)$

Exercise 10

$m = -4$, point $(-2, -3)$

Find an Equation of the Line Given Two Points

Exercise 11

$(3, 1)$ and $(2, 5)$

Exercise 12

$(-5, -3)$ and $(4, -6)$

Exercise 13

$(3, -2)$ and $(-4, 4)$

Exercise 14

$(4, 2)$ and $(4, -3)$

? Exercise 15

$(6, 2)$ and $(-3, 2)$

? Exercise 16

$(0, 0)$ and $(1, 4)$

? Exercise 17

$(3, 5)$ and $(-7, 5)$

Find an Equation of a Line Parallel to a Given Line

? Exercise 18

line $y = -3x - 1$, point $(2, -3)$

? Exercise 19

line $2x + 3y = 6$, point $(0, 5)$

? Exercise 20

line $x - 6 = 0$, point $(4, -3)$

? Exercise 21

line $y + 7 = 0$, point $(1, -1)$

Find an Equation of a Line Perpendicular to a Given Line

? Exercise 22

line $y = \frac{2}{3}x - 4$, point $(2, -4)$

? Exercise 23

line $4x + 5y = -3$, point $(0, 0)$

? Exercise 24

line $y - 6 = 0$, point $(-5, -3)$

? Exercise 25

line y-axis, point $(2, 1)$

Mixed Practice

? Exercise 26

Containing the points $(2, 7)$ and $(3, 8)$

? Exercise 27

$m = \frac{5}{6}$, containing point $(6, 7)$

? Exercise 28

Parallel to the line $2x + 3y = 6$, containing point $(0, 5)$

? Exercise 29

Perpendicular to the line y-axis, point $(-6, 2)$

? Exercise 30

Containing the points $(-2, 0)$ and $(-3, -2)$

? Exercise 31

Parallel to the line $x = -4$, containing point $(-3, -5)$

? Exercise 32

Perpendicular to the line $4x + 3y = 1$, containing point $(0, 0)$

Everyday Math

? Exercise 33

Fuel consumption. The city mpg, x, and highway mpg, y, of two cars are $(29, 40)$ and $(19, 28)$. Find a linear equation that models the relationship between city and highway mpg.

Writing Exercises

? Exercise 34

Explain in your own words why the slopes of two perpendicular lines must have opposite signs.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the equation of the line given the slope and *y*-intercept.			
find an equation of the line given the slope and a point.			
find an equation of the line given two points.			
find an equation of a line parallel to a given line.			
find an equation of a line perpendicular to a given line.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 3.7: Linear Inequalities in Two Variables

Be Prepared

We will rely heavily on these skills throughout this section.

- Identify if $(3,4)$ is a solution to $y = 2x + 1$
- Plot $y = -x + 5$ and determine if $(0,3)$ is above or below the line
- Solve and graph the solutions for the inequality: $2x + 3 > 7 - x$

Learning Objectives

Motivating Problem

You're setting ticket prices for a concert: General Admission (GA) tickets cost $20, and VIP tickets cost $30. Your goal is to earn more than $10,000. What combinations of ticket sales will meet your goal?

Fun Fact

The concept of using regions to illustrate solutions dates back to ancient Babylonian mathematicians, who used tablets to track acceptable ranges for items such as grain prices or crop yields. They didn't use graphs, but they were doing inequality logic thousands of years ago!

The Goal

This section helps us learn how to graph linear inequalities in two variables. We'll practice plotting boundary lines, deciding whether to use dashed or solid lines, testing points, and shading solution regions—skills that allow them to visualize not just a single solution, but whole ranges of possibilities.

Verify Solutions to an Inequality in Two Variables

We have learned how to solve inequalities in one variable. Now, we will look at inequalities in two variables. Inequalities in two variables have many applications. If you ran a business, for example, you would want your revenue to exceed your costs, so that your business would generate a profit.

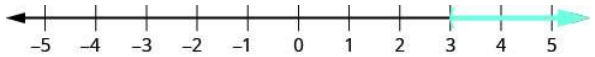
LINEAR INEQUALITY

A **linear inequality** is an inequality that can be written in one of the following forms:

$$Ax + By > C \quad Ax + By \geq C \quad Ax + By < C \quad Ax + By \leq C$$

where A and B are not both zero.

Do you remember that an inequality with one variable had many solutions? The solution to the inequality $x > 3$ is any number greater than 3. We illustrate this on the number line by shading in the area to the right of 3 and placing an open parenthesis at 3.



Similarly, inequalities in two variables have many solutions. Any ordered pair (x, y) that makes the inequality true when we substitute in the values is a solution of the inequality.

Solution OF A LINEAR INEQUALITY

An ordered pair (x, y) is a **solution of a linear inequality** if the inequality is true when we substitute the values of x and y.

Example 1

Determine whether each ordered pair is a solution to the inequality $y > x + 4$:

1. $(0, 0)$
2. $(1, 6)$
3. $(2, 6)$
4. $(-5, -15)$
5. $(-8, 12)$

Solution

1.

$(0, 0)$	$y > x + 4$
Substitute 0 for x and 0 for y.	$0 \stackrel{?}{>} 0 + 4$
Simplify.	$0 \not> 4$ So, $(0, 0)$ is not a solution to $y > x + 4$.

2.

$(1, 6)$	$y > x + 4$
Substitute 1 for x and 6 for y.	$6 \stackrel{?}{>} 1 + 4$
Simplify.	$6 > 5$ So, $(1, 6)$ is a solution to $y > x + 4$.

3.

$(2, 6)$	$y > x + 4$
Substitute 2 for x and 6 for y.	$6 \stackrel{?}{>} 2 + 4$
Simplify.	$6 \not> 6$ So, $(2, 6)$ is not a solution to $y > x + 4$.

4.

$(-5, -15)$	$y > x + 4$
Substitute -5 for x and -15 for y.	$-15 \stackrel{?}{>} -5 + 4$
Simplify.	$-15 \not> -1$ So, $(-5, -15)$ is not a solution to $y > x + 4$.

5.

$(-8, 12)$	$y > x + 4$
Substitute -8 for x and 12 for y.	$12 \stackrel{?}{>} -8 + 4$
Simplify.	$12 > -4$ So, $(-8, 12)$ is a solution to $y > x + 4$.

? Try It 1

Determine whether each ordered pair is a solution to the inequality $y < x + 1$:

1. $(0, 0)$
2. $(8, 6)$
3. $(-2, -1)$
4. $(3, 4)$
5. $(-1, -4)$

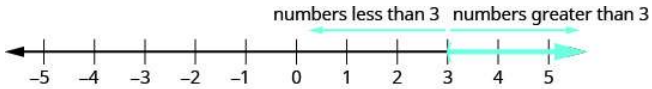
Answer

1. yes
2. yes
3. no
4. no
5. yes

Recognize the Relation Between the Solutions of an Inequality and Its Graph

Now, we will look at how the solutions of an inequality relate to its graph.

Let's think about the number line representing all solutions to $x > 3$. The point $x = 3$ separated that number line into two parts. On one side of 3 are all the numbers less than 3. On the other side of 3, all the numbers are greater than 3.



The solution to $x > 3$ is the shaded part of the number line to the right of $x = 3$.

Similarly, the line $y = x + 4$ separates the plane into two regions. On one side of the line are points with $y < x + 4$. On the other side of the line are the points with $y > x + 4$. We refer to the line $y = x + 4$ as a boundary line.

BOUNDARY LINE

The line with equation $Ax + By = C$ is the **boundary line** that separates the region where $Ax + By > C$ from the region where $Ax + By < C$.

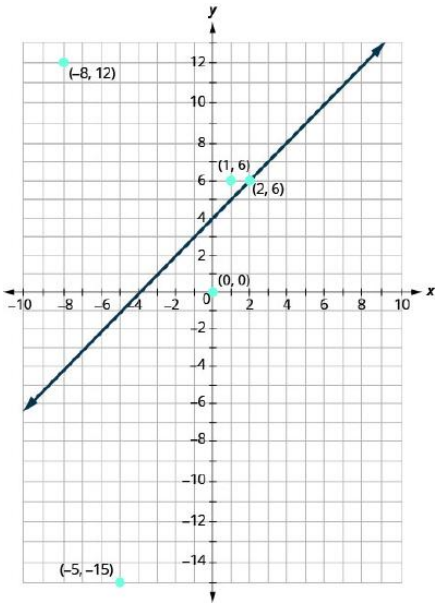
For an inequality in one variable, the endpoint is shown with a parenthesis or a bracket, depending on whether or not a is included in the solution:



Similarly, for an inequality in two variables, the boundary line is shown with a solid or dashed line to indicate whether or not it the line is included in the solution.

$Ax + By < C$	$Ax + By \leq C$
$Ax + By > C$	$Ax + By \geq C$
The boundary line is not included in the solution.	The boundary line is included in the solution.
The boundary line is dashed.	**The boundary line is solid.**

Now, let's take a look at what we found in our first example exploring $y > x + 4$. We'll start by graphing the line $y = x + 4$, and then we'll plot the five points we tested.



We previously found that some of the points we tested were solutions to the inequality $y > x + 4$ and some were not.

Which of the points we plotted are solutions to the inequality $y > x + 4$? The points $(1, 6)$ and $(-8, 12)$ are solutions to the inequality $y > x + 4$. Notice that they are both on the same side of the boundary line $y = x + 4$.

The two points $(0, 0)$ and $(-5, -15)$ are on the other side of the boundary line $y = x + 4$, and they are not solutions to the inequality $y > x + 4$. For those two points, $y < x + 4$.

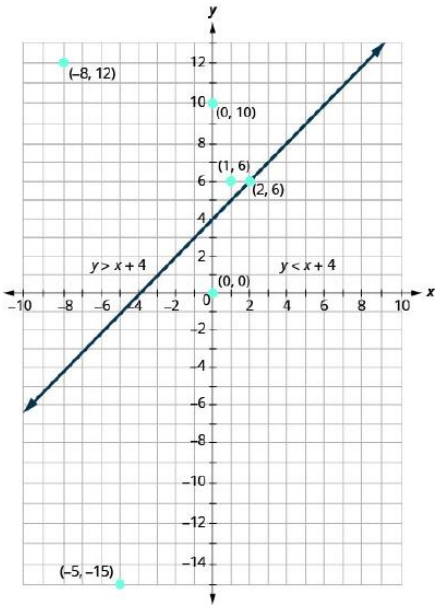
What about the point $(2, 6)$? Because $6 = 2 + 4$, the point is a solution to the equation $y = x + 4$. So the point $(2, 6)$ is on the boundary line.

Let's take another point on the left side of the boundary line and test whether or not it is a solution to the inequality $y > x + 4$. The point $(0, 10)$ clearly looks to be to the left of the boundary line, doesn't it? Is it a solution to the inequality?

$$y > x + 4$$
$$10 \stackrel{?}{>} 0 + 4$$
$$10 > 4 \qquad \text{So, } (0, 10) \text{ is a solution to } y > x + 4.$$

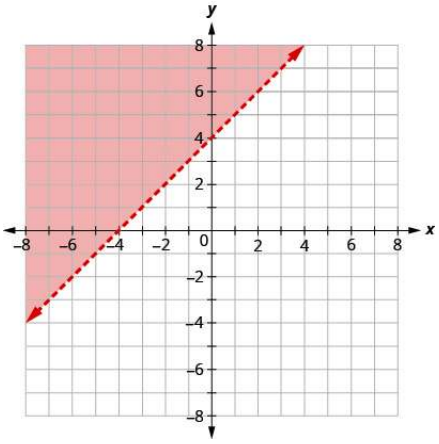
Any point you choose on the left side of the boundary line is a solution to the inequality $y > x + 4$. All points on the left are solutions.

Similarly, all points on the right side of the boundary line, the side with $(0, 0)$ and $(-5, -15)$, are not solutions to $y > x + 4$.



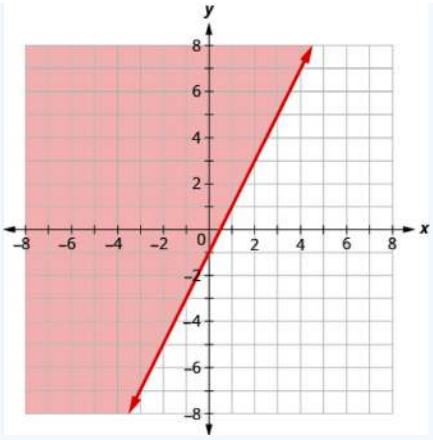
The graph of the inequality $y > x + 4$ is shown in the Figure below. The line $y = x + 4$ divides the plane into two regions. The shaded area shows the solutions to the inequality $y > x + 4$.

The points on the boundary line, those where $y = x + 4$, are *not solutions* to the inequality $y > x + 4$, so the line itself is not part of the solution. We show that by making the line dashed, not solid.



Example 2

The boundary line shown is $y = 2x - 1$. Write the inequality shown by the graph.



Solution

The line $y = 2x - 1$ is the boundary line. On one side of the line are the points with $y > 2x - 1$ and on the other side of the line are the points with $y < 2x - 1$.

Let's test the point $(0, 0)$ and see which inequality describes its side of the boundary line.

At $(0, 0)$, which inequality is true:

$y > 2x - 1$	or	$y < 2x - 1$?
$y > 2x - 1$		$y < 2x - 1$
$0 > 2 \cdot 0 - 1$		$0 < 2 \cdot 0 - 1$
$0 > -1$ True		$0 < -1$ False

Since $y > 2x - 1$ is true, the side of the line with $(0, 0)$, is the solution. The shaded region shows the solution of the inequality $y > 2x - 1$.

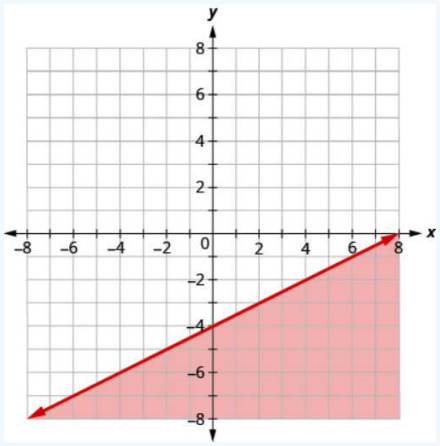
Since the boundary line is graphed with a solid line, the inequality includes the equal sign.

The graph shows the inequality $y \geq 2x - 1$.

We could use any point as a test point, provided it is not on the line. Why did we choose $(0, 0)$? Because it's the easiest to evaluate. You may want to pick a point on the other side of the boundary line and check that $y < 2x - 1$.

? Try It 2

Write the inequality shown by the graph with the boundary line $y = \frac{1}{2}x - 4$.

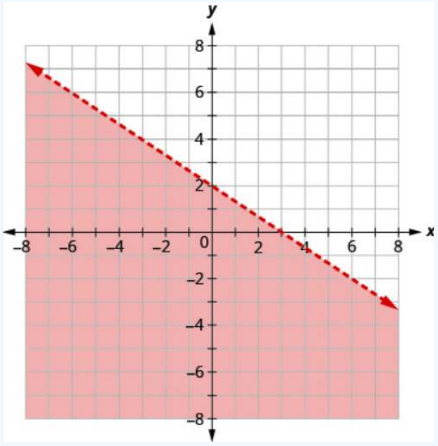


Answer

$y \leq \frac{1}{2}x - 4$

Example 3

The boundary line shown is $2x + 3y = 6$. Write the inequality shown by the graph.



Solution

The line $2x + 3y = 6$ is the boundary line. On one side of the line are the points with $2x + 3y > 6$ and on the other side of the line are the points with $2x + 3y < 6$.

Let's test the point $(0, 0)$ and see which inequality describes its side of the boundary line.

At $(0, 0)$, which inequality is true:

$2x + 3y > 6$		or	$2x + 3y < 6$?	
$2x + 3y > 6$			$2x + 3y < 6$	
$2(0) + 3(0) > 6$			$2(0) + 3(0) < 6$	
$0 > 6$	False		$0 < 6$	True

So the side with $(0, 0)$ is the side where $2x + 3y < 6$.

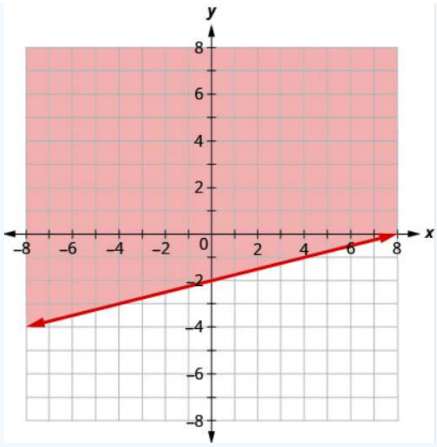
(You may want to pick a point on the other side of the boundary line and check that $2x + 3y > 6$.)

Since the boundary line is graphed as a dashed line, the inequality does not include an equal sign.

The graph illustrates the solution to the inequality $2x + 3y < 6$.

Try It 3

Write the inequality shown by the shaded region in the graph with the boundary line $x - 4y = 8$.



Answer

$x - 4y \leq 8$

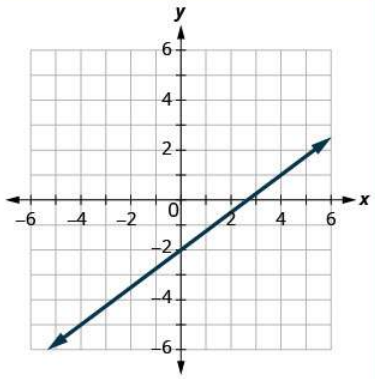
Graph Linear Inequalities

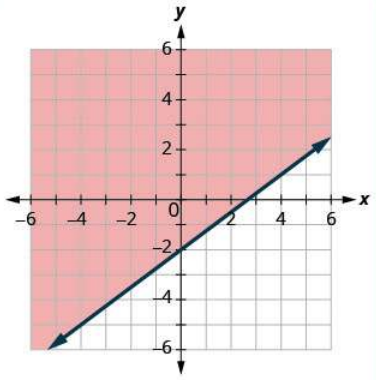
Now, we're ready to put all this together to graph linear inequalities.

Example 4

Graph the **linear inequality** $y \geq \frac{3}{4}x - 2$.

Solution

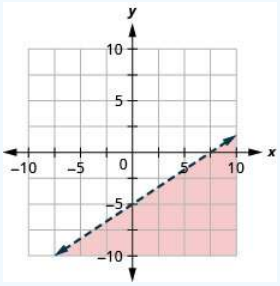
Step 1. Identify and graph the boundary line. • If the inequality is $\leq$ or $\geq$, the boundary line is solid. • If the inequality is $<$ or $>$, the boundary line is dashed.	Replace the inequality sign with an equal sign to find the boundary line. Graph the boundary line $y = \frac{3}{4}x - 2$. The inequality sign is $\geq$, so we draw a solid line.	
Step 2. Test a point that is not on the boundary line. Is it a solution of the inequality?	We'll test (0, 0). Is it a solution of the inequality?	At (0, 0), is $y \geq \frac{3}{4}x - 2$? $0 \stackrel{?}{\geq} \frac{3}{4}(0) - 2$ $0 \geq -2$ So, (0, 0) is a solution.

Step 3. Shade in one side of the boundary line. • If the test point is a solution, shade in the side that includes the point. • If the test point is not a solution, shade in the opposite side.	The test point (0, 0), is a solution to $y \geq \frac{3}{4}x - 2$. So we shade in that side.	

Try It 4

Graph the linear inequality $y < \frac{2}{3}x - 5$.

Answer



The steps for graphing a linear inequality are summarized here.

GRAPH A LINEAR INEQUALITY.

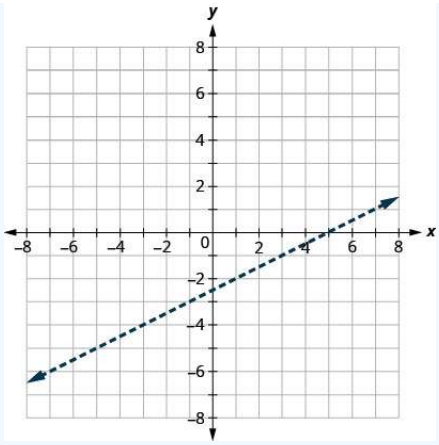
1. Identify and graph the boundary line.
   - If the inequality is $\leq$ or $\geq$, the boundary line is solid.
   - If the inequality is $<$ or $>$, the boundary line is dashed.
2. Test a point that is not on the boundary line. Is it a solution of the inequality?
3. Shade in one side of the boundary line.
   - If the test point is a solution, shade in the side that includes the point.
   - If the test point is not a solution, shade in the opposite side.

Example 5

Graph the linear inequality $x - 2y < 5$.

Solution

First, we graph the boundary line $x - 2y = 5$. The inequality is $<$, so we draw a dashed line.



Then we test a point. We'll use $(0,0)$ again because it is easy to evaluate and it is not on the boundary line.

Is $(0,0)$ a solution of $x-2y<5$?

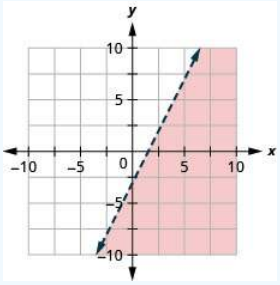
$$\begin{aligned} 0-2(0) &\stackrel{?}{<} 5 \\ 0-0 &\stackrel{?}{<} 5 \\ 0 &< 5 \end{aligned}$$

The point $(0,0)$ is a solution of $x-2y<5$, so we shade in that side of the boundary line.

Try It 5

Graph the linear inequality $2x-y>3$.

Answer



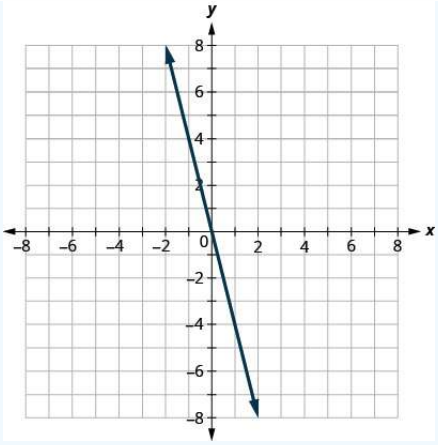
What if the boundary line goes through the origin? Then we won't be able to use $(0,0)$ as a test point. No problem—we'll just choose some other point that is not on the boundary line.

Example 6

Graph the linear inequality $y \leq -4x$.

Solution

First, we graph the boundary line $y=-4x$. It is in slope–intercept form, with $m=-4$ and $b=0$. The inequality is $\leq$, so we draw a solid line.



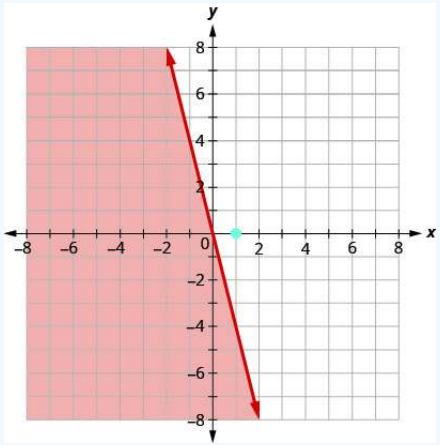
Now, we need a test point. We can see that the point $(1, 0)$ is not on the boundary line.

Is $(1, 0)$ a solution of $y \leq -4x$?

$0 \stackrel{?}{\leq} -4(1)$

$0 \not\leq -4$

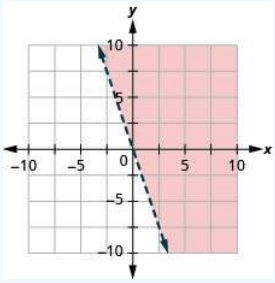
The point $(1, 0)$ is not a solution to $y \leq -4x$, so we shade in the opposite side of the boundary line.



Try It 6

Graph the linear inequality $y > -3x$.

Answer



Some linear inequalities have only one variable. They may have an x but no y, or a y but no x. In these cases, the boundary line will be either a vertical or a horizontal line. Do you remember?

$x = a$ vertical line
$y = b$ horizontal line

Example 7

Graph the linear inequality $y > 3$.

Solution

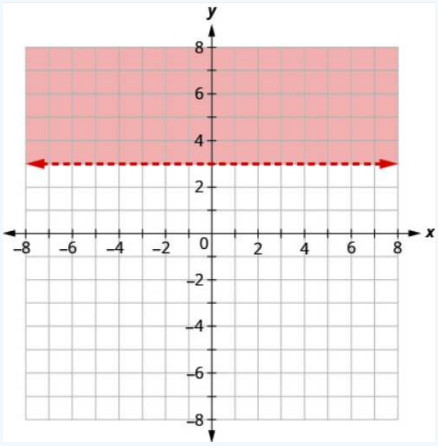
First, we graph the boundary line $y = 3$. It is a horizontal line. The inequality is $>$, so we draw a dashed line.

We test the point $(0, 0)$.

$$y > 3$$
$$0 \not> 3$$

$(0, 0)$ is not a solution to $y > 3$.

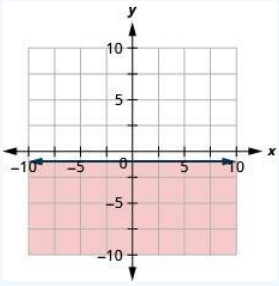
So we shade the side that does not include $(0, 0)$.



Try It 7

Graph the linear inequality $y \leq -1$.

Answer



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Section 3.7P: Practice

Practice Makes Progress

Verify Solutions to an Inequality in Two Variables

In the following exercises, determine whether each ordered pair is a solution to the given inequality.

Exercise 1

Determine whether each ordered pair is a solution to the inequality $y > x - 3$:

a. $(0, 0)$
b. $(2, 1)$
c. $(-1, -5)$
d. $(-6, -3)$
e. $(1, 0)$

Exercise 2

Determine whether each ordered pair is a solution to the inequality $y < x + 5$:

a. $(-3, 0)$
b. $(1, 6)$
c. $(-6, -2)$
d. $(0, 1)$
e. $(5, -4)$

Exercise 3

Determine whether each ordered pair is a solution to the inequality $x + y > 2$:

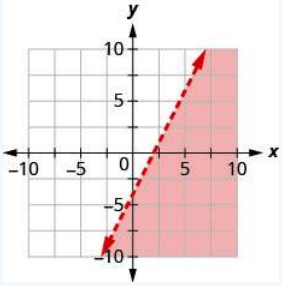
a. $(1, 1)$
b. $(4, -3)$
c. $(0, 0)$
d. $(-8, 12)$
e. $(3, 0)$

Recognize the Relation Between the Solutions of an Inequality and its Graph

In the following exercises, write the inequality shown by the shaded region.

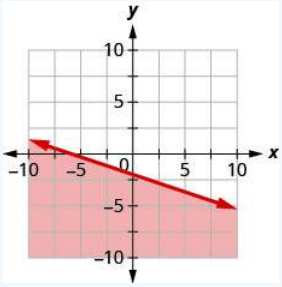
Exercise 4

Write the inequality shown by the graph with the boundary line $y = 2x - 4$.



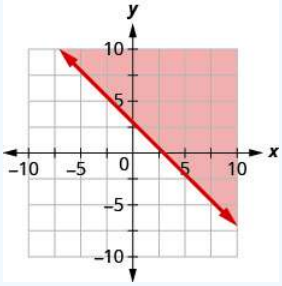
Exercise 5

Write the inequality shown by the graph with the boundary line $y = -\frac{1}{3}x - 2$.



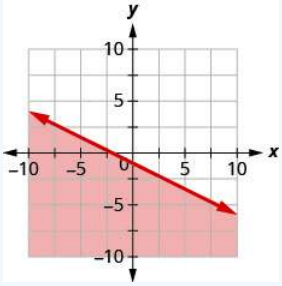
Exercise 6

Write the inequality shown by the shaded region in the graph with the boundary line $x + y = 3$.



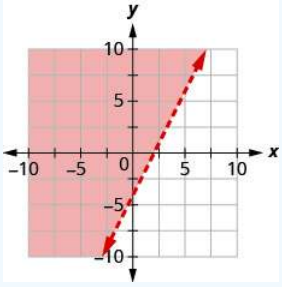
Exercise 7

Write the inequality shown by the shaded region in the graph with the boundary line $x + 2y = -2$.



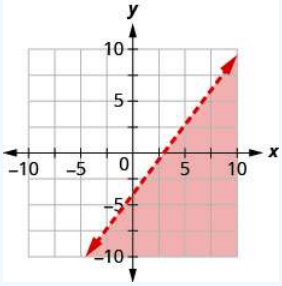
Exercise 8

Write the inequality shown by the shaded region in the graph with the boundary line $2x - y = 4$.



Exercise 9

Write the inequality shown by the shaded region in the graph with the boundary line $4x - 3y = 12$.



Graph Linear Inequalities

In the following exercises, graph each linear inequality.

Exercise 10

Graph the linear inequality $y < \frac{3}{5}x + 2$

Exercise 11

Graph the linear inequality $y \geq -\frac{1}{3}x - 2$

Exercise 12

Graph the linear inequality $x - y \geq -2$

Exercise 13

Graph the linear inequality $x + 5y < -5$

Exercise 14

Graph the linear inequality $4x + 2y \geq -8$

Exercise 15

Graph the linear inequality $y > x$

Exercise 16

Graph the linear inequality $y \leq -3x$

Exercise 17

Graph the linear inequality $y < -1$

Exercise 18

Graph the linear inequality $y \geq 2$

Exercise 19

Graph the linear inequality $x > -2$

Exercise 20

Graph the linear inequality $x \leq 4$

Exercise 21

Graph the linear inequality $x - y < -3$

Exercise 22

Graph the linear inequality $y \leq \frac{5}{4}x$

Exercise 23

Graph the linear inequality $y < -3x - 4$

Exercise 24

Graph the linear inequality $x \geq 0$

Everyday Math

Exercise 25

Shopping. Tula has $20 to spend at the used book sale. Hardcover books cost $2 each and paperback books cost $0.50 each. If x is the number of hardcover books Tula can buy and y is the number of paperback books she can buy, the inequality $2x + \frac{1}{2}y \leq 20$ models the situation.

a. Graph the inequality.
b. List three solutions to the inequality $2x + \frac{1}{2}y \leq 20$ where both x and y are whole numbers.

Writing Exercises

Exercise 26

Explain why in some graphs of linear inequalities the boundary line is solid but in other graphs it is dashed.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
verify solutions to an inequality in two variables.			
recognize the relation between the solutions of an inequality and its graph.			
graph linear inequalities.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 4: Working with Systems

This unit brings together everything we've learned to tackle systems of equations and inequalities. We'll use graphing, substitution, and elimination to solve problems involving two or more unknowns, and we'll apply these strategies to real-world contexts like pricing, mixtures, and constraints. We'll also analyze solution sets and interpret what they mean in practical situations. These skills allow us to model more complex scenarios and make informed decisions using algebra.

Section 4.1: Solve Systems of Equations by Graphing

Section 4.1P: Practice

Section 4.2: Solve Systems of Equations by Substitution

Section 4.2P: Practice

Section 4.3: Solve Systems of Equations by Elimination

Section 4.3P: Practice

Section 4.4: Solve Applications with Systems of Equations

Section 4.4P: Practice

Section 4.5: Graphing Systems of Linear Inequalities

Section 4.5P: Practice

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Section 4.1: Solve Systems of Equations by Graphing

Be Prepared

We will rely heavily on these skills throughout this section.

- Graph $y = 2x + 1$
- Graph $y \leq -x + 3$
- Identify the intersection point of $y = 2x + 3$ and $y = -3x + 1$

Learning Objectives

Motivating Problem

Two friends are tracking savings. One starts with \$50 and saves \$5 a week, the other starts with \$20 and saves \$10 a week. When will they have the same amount?

When two situations change over time, their graphs can cross—and that crossing point often tells you exactly what you need to know.

Fun Fact

In 1888, Lewis Carroll (author of *Alice in Wonderland*) gave a public lecture on solving simultaneous equations—and made it a *logic puzzle* for his audience. Turns out he was just as into systems as he was into wonderlands!

The Goal

In this section, we will learn how to solve systems of linear equations by graphing both equations and identifying the point where they intersect. This method helps visualize solutions and lays the groundwork for more abstract techniques later.

Determine Whether an Ordered Pair is a Solution of a System of Equations

We previously learned how to solve linear equations with one variable. Remember that the solution of an equation is a value of the variable that makes a true statement when substituted into the equation. Now, we will work with **systems of linear equations**, which are two or more linear equations grouped together.

Definition: SYSTEM OF LINEAR EQUATIONS

When two or more linear equations are grouped together, they form a system of linear equations.

We will focus our work here on systems of two linear equations in two unknowns. In later courses, you may solve larger systems of equations.

Below is an example of a system of two linear equations. We use a brace to show how the two equations are grouped together to form a system of equations.

$$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$$

A linear equation in two variables, like \(2*x* + *y* = 7\), has an infinite number of solutions. Its graph is a line. Remember, every point on the line is a solution to the equation, and every solution to the equation is a point on the line.

To solve a system of two linear equations, we want to find the values of the variables that are solutions to *both* equations. In other words, we are looking for the ordered pairs (x, y) that make *both* equations true. These are referred to as the **solutions to a system of equations**.

Definition: SolutionS OF A SYSTEM OF EQUATIONS

Solutions of a system of equations are the values of the variables that make all the equations true. A solution of a system of two linear equations is represented by an ordered pair (x, y).

To determine if an ordered pair is a solution to a system of two equations, we substitute the values of the variables into each equation. If the ordered pair satisfies both equations, it is a solution to the system.

Let's consider the system below:

$$\begin{cases} 3x - y = 7 \\ x - 2y = 4 \end{cases}$$

Is the ordered pair $(2, -1)$ a solution?

We substitute $x = 2$ and $y = -1$ into both equations.

$$\begin{aligned} 3x - y &= 7 & \qquad x - 2y &= 4 \\ 3(2) - (-1) &\stackrel{?}{=} 7 & \qquad 2 - 2(-1) &\stackrel{?}{=} 4 \\ 7 &= 7 \text{ true} & \qquad 4 &= 4 \text{ true} \end{aligned}$$

The ordered pair $(2, -1)$ made both equations true. Therefore, $(2, -1)$ is a solution to this system.

Let's try another ordered pair. Is the ordered pair $(3, 2)$ a solution?

We substitute $x = 3$ and $y = 2$ into both equations.

$$\begin{aligned} 3x - y &= 7 & \qquad x - 2y &= 4 \\ 3(3) - 2 &\stackrel{?}{=} 7 & \qquad 2 - 2(2) &\stackrel{?}{=} 4 \\ 7 &= 7 \text{ true} & \qquad -2 &= 4 \text{ false} \end{aligned}$$

The ordered pair $(3, 2)$ made one equation true, but it made the other equation false. Since it is not a solution to **both** equations, it is not a solution to this system.

Example 1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x - y = -1 \\ 2x - y = -5 \end{cases}$

1. $(-2, -1)$
2. $(-4, -3)$

Solution

1.

$$\begin{cases} x - y = -1 \\ 2x - y = -5 \end{cases}$$

We substitute $x = -2$ and $y = -1$ into both equations

$$\begin{aligned} x - y &= 1 & \qquad 2x - y &= -5 \\ -2 - (-1) &= -1 & \qquad 2(-2) - (-1) &\stackrel{?}{=} -5 \\ -1 &= -1 \checkmark & \qquad -3 &= -5 \times \end{aligned}$$

$(-2, -1)$ does not make both equations true. Therefore, $(-2, -1)$ is not a solution to the system.

2.

We substitute $x = -4$ and $y = -3$ into both equations.

$$\begin{array}{ll} x - y = -1 & 2x - y = -5 \\ -4 - (-3) \stackrel{?}{=} -1 & 2(-4) - (-3) \stackrel{?}{=} -5 \\ -1 = -1 \checkmark & -5 = -5 \checkmark \end{array}$$

$(-4, -3)$ makes both equations true. Therefore, $(-4, -3)$ is a solution to the system.

Try It 1

Determine whether the ordered pair is a solution to the system: $\begin{cases} x - 3y = -8 \\ -3x - y = 4 \end{cases}$

1. $(2, -2)$
2. $(-2, 2)$

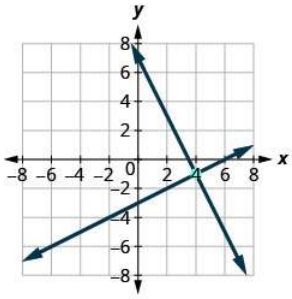
Answer

1. no
2. yes

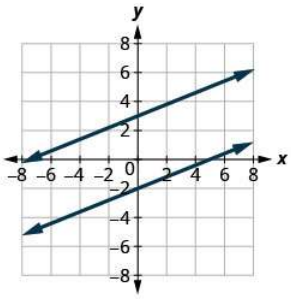
Solve a System of Linear Equations by Graphing

In this section, we will use three methods to solve a system of linear equations. The first method we'll use is graphing. The graph of a linear equation is a line. Each point on the line is a solution to the equation. For a system of two equations, we will graph two lines. Then we can see all the points that are solutions to each equation. By identifying what the lines have in common, we can find the solution to the system.

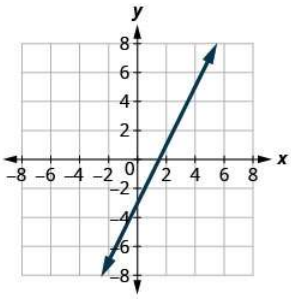
Most linear equations in one variable have a single solution, but we saw that some equations, called **contradictions**, have no solutions, while other equations, called **identities**, have all numbers as solutions. Similarly, when we solve a system of two linear equations represented by a graph of two lines in the same plane, there are three possible cases, as shown below.



The lines intersect.
Intersecting lines have one point in common.
There is one solution to this system.



The lines are parallel.
Parallel lines have no points in common.
There is no solution to this system.



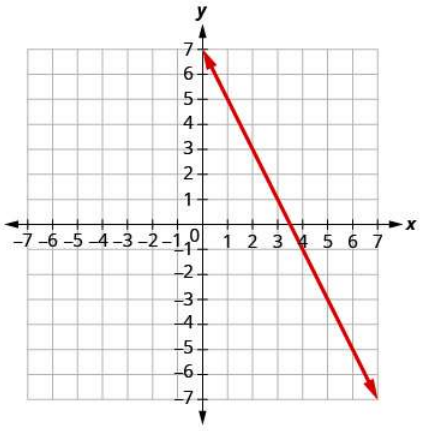
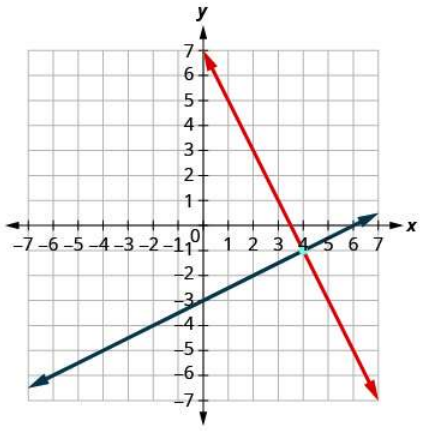
Both equations give the same line.
Because we have just one line, there are infinitely many solutions.

For the first example of solving a system of linear equations in this section and the following two sections, we will solve the same system of two linear equations. But we'll use a different method in each section. After seeing the third method, you'll decide which method is the most convenient way to solve this system.

Example 2

Solve the system by graphing: $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$

Solution

Step 1. Graph the first equation.	To graph the first line, write the equation in slope–intercept form. $2x + y = 7$ $y = -2x + 7$ $m = -2$ $b = 7$	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ 
Step 2. Graph the second equation on the same rectangular coordinate system.	To graph the second line, use intercepts. $x - 2y = 6$ $(0, -3)$ $(6, 0)$	
Step 3. Determine whether the lines intersect, are parallel, or are the same line.	Look at the graph of the lines.	The lines intersect.
Step 4. Identify the solution to the system. If the lines intersect, identify the point of intersection. Check to make sure it is a solution to both equations. This is the solution to the system. If the lines are parallel, the system has no solution If the lines are the same, the system has an infinite number of solutions.	Since the lines intersect, find the point of intersection. Check the point in both equations.	The lines intersect at $(4, -1)$. $2x + y = 7$ $2(4) + (-1) \stackrel{?}{=} 7$ $8 - 1 \stackrel{?}{=} 7$ $7 = 7$ ✓ $x - 2y = 6$ $4 - 2(-1) \stackrel{?}{=} 6$ $6 = 6$ ✓ The solution is $(4, -1)$.

? Try It 2

Solve each system by graphing: $\begin{cases} x - 3y = -3 \\ x + y = 5 \end{cases}$

Answer

$(3, 2)$

The steps for solving a system of linear equations using the graphing method are outlined below.

TO SOLVE A SYSTEM OF LINEAR EQUATIONS BY GRAPHING.

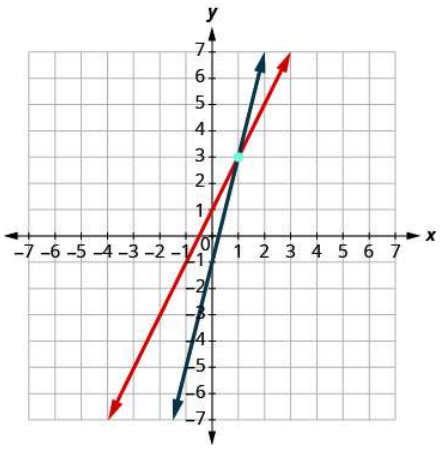
1. Graph the first equation.
2. Graph the second equation on the same rectangular coordinate system.
3. Determine whether the lines intersect, are parallel, or are the same line.
4. Identify the solution to the system.
   - If the lines intersect, identify the point of intersection. Verify that it is a solution to both equations. This is the solution to the system.
   - If the lines are parallel, the system has no solution.
   - If the lines are the same, the system has an infinite number of solutions.

Example 3

Solve the system by graphing: $\begin{cases} y = 2x + 1 \\ y = 4x - 1 \end{cases}$

Solution

Both of the equations in this system are in slope-intercept form, so we will use their slopes and y-intercepts to graph them.
$\begin{cases} y = 2x + 1 \\ y = 4x - 1 \end{cases}$

Find the slope and y-intercept of the first equation.	$y = 2x + 1$ $m = 2$ $b = 1$
Find the slope and y-intercept of the first equation.	$y = 4x - 1$ $m = 4$ $b = -1$
Graph the two lines.	
Determine the point of intersection.	The lines intersect at (1, 3).
	
Check the solution in both equations.	$y = 2x + 1$ $\quad$ $y = 4x - 1$ $3 \stackrel{?}{=} 2 \cdot 1 + 1$ $\quad$ $3 \stackrel{?}{=} 4 \cdot 1 - 1$ $3 = 3\checkmark$ $\quad$ $3 = 3\checkmark$
	The solution is (1, 3).

Try It 3

Solve the system by graphing: $\begin{cases} y = 3x + 3 \\ y = -x + 7 \end{cases}$

Answer

$(1, 6)$

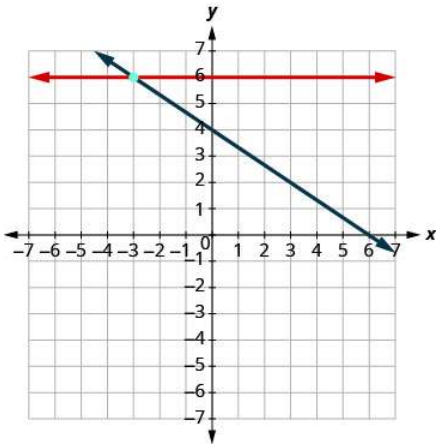
Usually, when equations are given in standard form, the most convenient way to graph them is by using the intercepts. We'll do this in the next Example.

Do you remember how to graph a linear equation with just one variable? It will be either a vertical or a horizontal line.

Example 4

Solve the system by graphing: $\begin{cases} y = 6 \\ 2x + 3y = 12 \end{cases}$

Solution

	$y = 6$ $2x + 3y = 12$
We know the first equation represents a horizontal line whose y-intercept is 6.	$y = 6$
The second equation is most conveniently graphed using intercepts.	$2x + 3y = 12$
To find the intercepts, let $x = 0$ and then $y = 0$.	x | y 0 | 4 6 | 0
Graph the lines.	
Determine the point of intersection.	The lines intersect at $(-3, 6)$.
Check the solution to both equations.	$y = 6$: $6 \stackrel{?}{=} 6$, $6 = 6$✓ $2x + 3y = 12$: $2(-3) + 3(6) \stackrel{?}{=} 12$, $-6 + 18 \stackrel{?}{=} 12$, $12 = 12$✓
	The solution is $(-3, 6)$.

Try It 4

Solve each system by graphing: $\begin{cases} x = 4 \\ 3x - 2y = 24 \end{cases}$

Answer

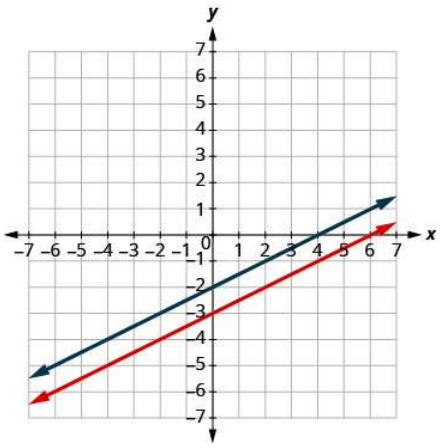
$(4, -6)$

In all the systems of linear equations so far, the lines intersected, and the solution was one point. In the following two examples, we'll examine a system of equations that has no solution and a system of equations that has an infinite number of solutions.

Example 5

Solve the system by graphing: $\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$

Solution

	$y = \frac{1}{2}x - 3$ $x - 2y = 4$
To graph the first equation, we will use its slope and y-intercept.	$y = \frac{1}{2}x - 3$
	$m = \frac{1}{2}$
	$b = -3$
To graph the second equation, we will use the intercepts.	$x - 2y = 4$
	$\begin{array}{\|c\|c\|} \hline x & y \\ \hline 0 & -2 \\ \hline 4 & 0 \\ \hline \end{array}$
Graph the lines.	
Determine the point of intersection.	The lines are parallel.
	Since no point is on both lines, there is no ordered pair that makes both equations true. There is no solution to this system.

Try It 5

Solve the system by graphing: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 6 \end{cases}$

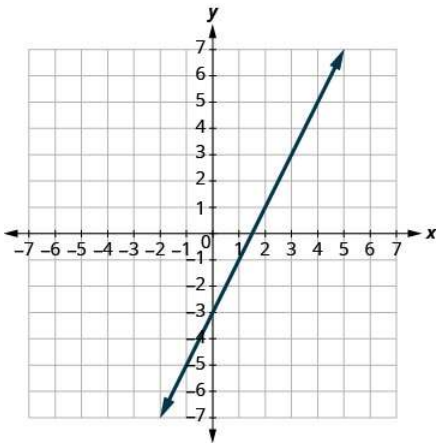
Answer

no solution

Example 6

Solve the system by graphing: $\begin{cases} y = 2x - 3 \\ -6x + 3y = -9 \end{cases}$

Solution

	$y = 2x - 3$ $-6x + 3y = -9$
Find the slope and y-intercept of the first equation.	$y = 2x - 3$ $m = 2$ $b = -3$
Find the intercepts of the second equation.	$-6x + 3y = -9$
	x | y 0 | -3 $\frac{3}{2}$ | 0
Graph the lines.	
Determine the point of intersection.	The lines are the same!
	Since every point on the line makes both equations true, there are infinitely many ordered pairs that make both equations true.
	There are infinitely many solutions to this system.

Try It 6

Solve each system by graphing: $\begin{cases} y = \frac{1}{2}x - 4 \\ 2x - 4y = 16 \end{cases}$

Answer

infinitely many solutions

If you write the second equation in the previous example in slope-intercept form, you may recognize that the equations have the same slope and y-intercept.

When we graphed the second line in the last example, we drew it right over the first line. We say the two lines are coincident. Coincident lines have the same slope and the same y-intercept.

COINCIDENT LINES

Coincident lines have the same slope and the same y-intercept.

Determine the Number of Solutions of a Linear System

There will be times when we want to know how many solutions exist to a system of linear equations, but we may not actually need to find the solutions. It will be helpful to determine this without graphing.

We have seen that two lines in the same plane must either intersect or be parallel to each other. The systems of equations in the previous examples all had two intersecting lines, and each system had one solution.

A system with parallel lines has no solution, and a system with **coincident lines** has infinitely many solutions.

We'll organize these results in the Figure below:

Graph	Number of solutions
2 intersecting lines	1
Parallel lines	None
Same line	Infinitely many

Parallel lines have the same slope but different y-intercepts. So, if we write both equations in a system of linear equations in slope–intercept form, we can see how many solutions there will be without graphing! Look at the system we solved in a previous example.

$$\begin{cases} y = \frac{1}{2}x - 3 \\ x - 2y = 4 \end{cases}$$

The first line is in slope–intercept form. $y = \frac{1}{2}x - 3$

$m = \frac{1}{2}, b = -3$

If we solve the second equation for y, we get

$$x - 2y = 4$$
$$x - 2y = -x + 4$$
$$y = \frac{1}{2}x - 2$$
$$m = \frac{1}{2}, b = -2$$

The two lines have the same slope but different y-intercepts. They are parallel lines, and thus, they have no points in common. We call a system of equations like this **an inconsistent system**, it has no solutions. We call a system of equations that has at least one solution **a consistent system**.

The figure below shows how to determine the number of solutions of a linear system by examining its slopes and intercepts.

Number of Solutions of a Linear System of Equations			
Slopes	**Intercepts**	**Type of Lines**	**Number of Solutions**
Different		Intersecting	1 point
Same	Different	Parallel	No solution
Same	Same	Coincident	Infinitely many solutions

CONSISTENT AND INCONSISTENT SYSTEMS

A **consistent system** of equations is a system of equations with at least one solution.

An **inconsistent system** of equations is a system of equations with no solution.

We also categorize the equations in a system of equations by labeling them as *independent* or *dependent*.

If two equations are **independent**, they each have their own set of solutions. When graphed, **independent equations** will appear as Intersecting or parallel lines.

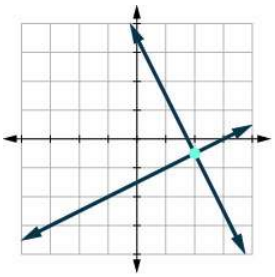
If two equations are **dependent**, all the solutions of one equation are also solutions of the other equation. When we graph two **dependent equations**, we get coincident lines.

INDEPENDENT AND DEPENDENT EQUATIONS

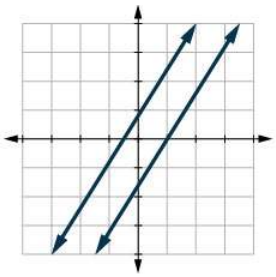
Two equations are **independent** if they have different solutions.

Two equations are **dependent** if all the solutions of one equation are also solutions of the other equation.

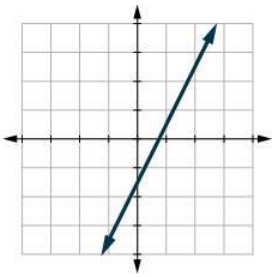
Let's summarize this by examining the graphs of the three types of systems.



Intersecting



Parallel



Coincident

Lines	Intersecting	Parallel	Coincident
Number of solutions	1 point	No solution	Infinitely many
Consistent/inconsistent	Consistent	Inconsistent	Consistent
Dependent/independent	Independent	Independent	Dependent

Example 7

Without graphing, determine the number of solutions and then classify the system of equations: $\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$

Solution

We will compare the slopes and intercepts of the two lines.

$$\begin{cases} y = 3x - 1 \\ 6x - 2y = 12 \end{cases}$$

The first equation is already in slope-intercept form.

$$y = 3x - 1$$

Write the second equation inslope–intercept form.

$$\begin{aligned} 6x - 2y &= 12 \\ -2y &= -6x - 12 \\ \tfrac{-2y}{-2} &= \tfrac{-6x+12}{-2} \\ y &= 3x - 6 \end{aligned}$$

Find the slope and intercept of each line.

$$\begin{aligned} y &= 3x - 1 & y &= 3x - 6 \\ m &= 3 & m &= 3 \\ b &= -1 & b &= -6 \end{aligned}$$

Since the slopes are the same and y-intercepts are different, the lines are parallel.

A system of equations whose graphs are parallel lines has no solution and is inconsistent and independent.

Try It 7

Without graphing, determine the number of solutions and then classify the system of equations.

$$\begin{cases} 4x - 5y = 20 \\ y = \frac{4}{5}x - 4 \end{cases}$$

Answer

infinitely many solutions, consistent, dependent

Solve Applications of Systems of Equations by Graphing

We will use the same problem-solving strategy we used previously to set up and solve applications of systems of linear equations. We'll modify the strategy slightly here to make it appropriate for systems of equations.

USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Ensure that all words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Verify** the answer to the problem and ensure it is logical and sensible.
7. **Answer** the question with a complete sentence.

Step 5 is where we will use the method introduced in this section. We will graph the equations and find the solution.

Example 10

Sondra is making 10 quarts of punch from fruit juice and club soda. The number of quarts of fruit juice is 4 times the number of quarts of club soda. How many quarts of fruit juice and club soda does Sondra need?

Solution

Step 1. Read the problem.

Step 2. Identify what we are looking for.

We are looking for the number of quarts of fruit juice and the number of quarts of club soda that Sondra will need.

Step 3. Name what we are looking for. Choose variables to represent those quantities.

Let f be the number of quarts of fruit juice and c be the number of quarts of club soda.

Step 4. Translate into a system of equations.

$$\underbrace{\text{The number of quarts of fruit juice}}_{f} \text{ and } \underbrace{\text{the number of quarts of club soda}}_{c} \text{ is } 10 \quad f + c = 10$$

$$\underbrace{\text{The number of quarts of fruit juice}}_{f} \text{ is } \underbrace{\text{four times the number of quarts of club soda}}_{4c} \quad f = 4c$$

We now have the system. $\begin{cases} f + c = 10 \\ f = 4c \end{cases}$

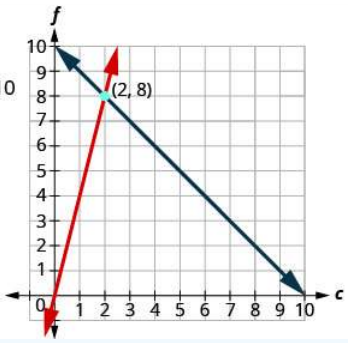
Step 5. Solve the system of equations using good algebra techniques.

$f = 4c$ $\quad f + c = 10$

$m = 4$ $\quad f = -c + 10$

$b = 0$ $\quad m = -1$

$b = 10$



The point of intersection $(2, 8)$ is the solution. This means Sondra needs 2 quarts of club soda and 8 quarts of fruit juice.

Step 6. Verify the answer to the problem and ensure it is logical and sensible.

Does this make sense in the context of the problem?

Yes, the number of quarts of fruit juice, 8 is 4 times the number of quarts of club soda, 2.

Yes, 10 quarts of punch is 8 quarts of fruit juice plus 2 quarts of club soda.

Step 7. Answer the question with a complete sentence.

Sondra needs 8 quarts of fruit juice and 2 quarts of soda.

? Try It 10

Manny is making 12 quarts of orange juice from concentrate and water. The number of quarts of water is 3 times the number of quarts of concentrate. How many quarts of concentrate and how many quarts of water does Manny need?

Answer

Manny needs 3 quarts of juice concentrate and 9 quarts of water.

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Section 4.1P: Practice

Practice Makes Progress

Determine Whether an Ordered Pair is a Solution of a System of Equations

In the following exercises, determine if the following points are solutions to the given system of equations.

Exercise 1

$\begin{cases} 2x-6y=0 \\ 3x-4y=5 \end{cases}$

(3,1)

Exercise 2

$\begin{cases} 7x-4y=-1 \\ -3x-2y=1 \end{cases}$

$\left(-\frac{3}{13},-\frac{2}{13}\right)$

Exercise 3

$\begin{cases} 2x+y=5 \\ x+y=1 \end{cases}$

(2,0)

Exercise 4

$\begin{cases} x+y=2 \\ y=\frac{3}{4}x \end{cases}$

$\left(1,\frac{3}{4}\right)$

Exercise 5

$\begin{cases} x+5y=10 \\ y=\frac{3}{5}x+1 \end{cases}$

$\left(\frac{5}{4},\frac{7}{4}\right)$

Solve a System of Linear Equations by Graphing

In the following exercises, solve the following systems of equations by graphing.

Exercise 6

$\begin{cases} 3x+y=-3 \\ 2x+3y=5 \end{cases}$

Exercise 7

$\begin{cases} -3x+y=-1 \\ 2x+y=4 \end{cases}$

Exercise 8

$$\begin{cases} y = x + 2 \\ y = -2x + 2 \end{cases}$$

Exercise 9

$$\begin{cases} y = \frac{3}{2}x + 1 \\ y = -\frac{1}{2}x + 5 \end{cases}$$

Exercise 10

$$\begin{cases} -x + y = -3 \\ 4x + 4y = 4 \end{cases}$$

Exercise 11

$$\begin{cases} -3x + y = -1 \\ 2x + y = 4 \end{cases}$$

Exercise 12

$$\begin{cases} x + y = 5 \\ 2x - y = 4 \end{cases}$$

Exercise 13

$$\begin{cases} x + y = 2 \\ x - y = 0 \end{cases}$$

Exercise 14

$$\begin{cases} x + y = -5 \\ x - y = 3 \end{cases}$$

Exercise 15

$$\begin{cases} 2x + y = 6 \\ -8x - 4y = -24 \end{cases}$$

Exercise 16

$$\begin{cases} x + y = -4 \\ -x + 2y = -2 \end{cases}$$

Exercise 17

$$\begin{cases} 4x - 3y = 8 \\ 8x - 6y = 14 \end{cases}$$

Exercise 18

$$\begin{cases} -2x + 3y = 3 \\ x + 3y = 12 \end{cases}$$

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Exercise 19

$$\begin{cases} 2x+3y=6 \\ y=-2 \end{cases}$$

Exercise 20

$$\begin{cases} x-3y=-3 \\ y=2 \end{cases}$$

Exercise 21

$$\begin{cases} -2x+4y=4 \\ y=\frac{1}{2}x \end{cases}$$

Exercise 22

$$\begin{cases} 2x-y=-1 \\ x=1 \end{cases}$$

Exercise 23

$$\begin{cases} x-3y=-6 \\ x=-3 \end{cases}$$

Exercise 24

$$\begin{cases} x=-3y+4 \\ 2x+6y=8 \end{cases}$$

Exercise 25

$$\begin{cases} x+3y=-6 \\ 4y=-\frac{4}{3}x-8 \end{cases}$$

Exercise 26

$$\begin{cases} -3x+2y=-2 \\ y=-x+4 \end{cases}$$

Determine the Number of Solutions of a Linear System

Without graphing the following systems of equations, determine the number of solutions and then classify the system of equations.

Exercise 27

$$\begin{cases} y=2x+3 \\ 2x-y=-3 \end{cases}$$

Exercise 28

$$\begin{cases} y=\frac{2}{3}x+1 \\ 2x-3y=7 \end{cases}$$

? Exercise 29

$$\begin{cases} 4x + 2y = 10 \\ 4x - 2y = -6 \end{cases}$$

? Exercise 30

$$\begin{cases} y = -\frac{1}{2}x + 5 \\ x + 2y = 10 \end{cases}$$

Everyday Math

? Exercise 31

Molly is making strawberry-infused water. For each ounce of strawberry juice, she uses three times as many ounces of water. How many ounces of strawberry juice and how many ounces of water does she need to make 64 ounces of strawberry-infused water?

? Exercise 32

Enrique is making a party mix that contains raisins and nuts. For each ounce of nuts, he uses twice the amount of raisins. How many ounces of nuts and how many ounces of raisins does he need to make 24 ounces of party mix?

? Exercise 33

Leo is planning his spring flower garden. He wants to plant tulip and daffodil bulbs. He will plant 6 times as many daffodil bulbs as tulip bulbs. If he wants to plant 350 bulbs, how many tulip bulbs and how many daffodil bulbs should he plant?

Writing Exercises

? Exercise 34

In a system of linear equations, the two equations have the same slope. Describe the possible solutions to the system.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an ordered pair is a solution of a system of equations.			
solve a system of linear equations by graphing.			
determine the number of solutions of a linear system.			
solve applications of systems of equations by graphing.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before

proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 4.2: Solve Systems of Equations by Substitution

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify $-5(3-x)+2x$
- Solve $3x-9y+2(x-1)=9-x+y$ for y
- Determine if $(2,7)$ is a solution to $y>2x+3$

Learning Objectives

Motivating Problem

You're choosing between two cell phone plans: one has a flat monthly rate, the other charges per minute. At what point do they cost the same?

Substitution helps us plug one situation into another—and find where they line up.

Fun Fact

In 1850 BCE, ancient Egyptian scribes were already solving systems of equations by hand! In the **Rhind Mathematical Papyrus**, one problem involved finding how many loaves of bread two workers earned at different rates. Their method? A kind of substitution, trial and error with clever logic.

The Goal

In this section, we will solve systems by substituting one equation into the other, reducing the system to a single-variable equation. This algebra-based method emphasizes flexibility and prepares us for more complex algebraic modeling later.

Solving systems of linear equations by graphing is a helpful method for visualizing the types of solutions that may result. However, there are many cases where solving a system by graphing is inconvenient or imprecise. If the graphs extend beyond the small grid with x and y both between -10 and 10, graphing the lines may be cumbersome. And if the solutions to the system are not integers, it can be hard to read their values precisely from a graph.

In this section, we will solve systems of linear equations by the substitution method.

Solve a System of Equations by Substitution

We will use the same system we used first for graphing.

$$\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$$

We will first solve one of the equations for either x or y. We can choose either equation and solve for either variable, but we'll try to make a choice that'll keep the work manageable.

Then we substitute that expression into the other equation. The result is an equation with just one variable, and we know how to solve those!

After we find the value of one variable, we will substitute that value into one of the original equations and solve for the other variable. Finally, we verify our solution to ensure it satisfies both equations.

Example 1

Solve the system by substitution. $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Solve one of the equations for either variable.	We'll solve the first equation for y.	$2x + y = 7$ $y = 7 - 2x$
Step 2. Substitute the expression from Step 1 into the other equation.	We replace y in the second equation with the expression $7 - 2x$.	$x - 2y = 6$ $x - 2(7 - 2x) = 6$
Step 3. Solve the resulting equation.	Now we have an equation with just 1 variable. We know how to solve this!	$x - 2(7 - 2x) = 6$ $x - 14 + 4x = 6$ $5x = 20$ $x = 4$
Step 4. Substitute the solution in Step 3 into one of the original equations to find the other variable.	We'll use the first equation and replace x with 4.	$2x + y = 7$ $2(4) + y = 7$ $8 + y = 7$ $y = -1$
Step 5. Write the solution as an ordered pair.	The ordered pair is (x, y).	$(4, -1)$
Step 6. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4, -1)$ into both equations and make sure they are both true.	$2x + y = 7$ $\quad$ $x - 2y = 6$ $2(4) + (-1) \stackrel{?}{=} 7$ $\quad$ $4 - 2(-1) \stackrel{?}{=} 6$ $7 = 7$ ✓ $\quad$ $6 = 6$ ✓ Both equations are true. $(4, -1)$ is the solution to the system.

Try It 1

Solve the system by substitution. $\begin{cases} -2x + y = -11 \\ x + 3y = 9 \end{cases}$

Answer

(6,1)

SOLVE A SYSTEM OF EQUATIONS BY SUBSTITUTION.

1. Solve one of the equations for either variable.
2. Substitute the expression from Step 1 into the other equation.
3. Solve the resulting equation.
4. Substitute the solution in Step 3 into one of the original equations to find the other variable.
5. Write the solution as an ordered pair.
6. Check that the ordered pair is a solution to **both** original equations.

Notice if one of the equations in the system is given in slope–intercept form; Step 1 is already done.

Example 2

Solve the system by substitution. $\begin{cases} x+y=-1 \\ y=x+5 \end{cases}$

Solution

The second equation is already solved for y. We will substitute the expression for y in the first equation.

	$x+y=-1$ $y=x+5$
The second equation is already solved for y. We will substitute into the first equation.	
Replace the y with $x+5$.	$y=x+5$ $x+y=-1$
Solve the resulting equation for x.	$x+x+5=-1$
	$2x+5=-1$
	$2x=-6$
Substitute $x=-3$ into $y=x+5$ to find y.	$x=-3$ $y=x+5$
	$y=-3+5$
The ordered pair is $(-3,2)$.	$y=2$
Check the ordered pair in both equations: $x+y=-1 \quad y=x+5$ $-3+2 \stackrel{?}{=} -1 \quad 2 \stackrel{?}{=} -3+5$ $-1=-1\checkmark \quad 2=2\checkmark$	
	The solution is $(-3,2)$.

Try It 2

Solve the system by substitution. $\begin{cases} 2x-y=1 \\ y=-3x-6 \end{cases}$

Answer

$(-1,-3)$

If the equations are given in standard form, we'll need to start by solving for one of the variables. In the following example, we'll solve the first equation for y.

Example 3

Solve the system by substitution. $\begin{cases} 3x+y=5 \\ 2x+4y=-10 \end{cases}$

Solution

We need to solve one equation for one variable. Then we will substitute that expression into the other equation.

Solve for \(y\).	$3x + y = 5$
Substitute into the other equation.	$y = -3x + 5$ $2x + 4y = -10$
Replace the y with $-3x + 5$.	$2x + 4(-3x + 5) = -10$
Solve the resulting equation for x.	$2x - 12x + 20 = -10$
	$-10x + 20 = -10$ $-10x = -30$
Substitute $x = 3$ into $3x + y = 5$ to find y.	$x = 3$ $3x + y = 5$
	$3(3) + y = 5$ $9 + y = 5$
The ordered pair is $(3, -4)$.	$y = -4$
Check the ordered pair in both equations:	
$3x + y = 5$ $3 \cdot 3 + (-4) \stackrel{?}{=} 5$ $9 - 4 \stackrel{?}{=} 5$ $5 = 5\checkmark$	$2x + 4y = -10$ $2 \cdot 3 + 4(-4) \stackrel{?}{=} -10$ $6 - 16 \stackrel{?}{=} -10$ $-10 = -10\checkmark$
	The solution is $(3, -4)$.

Try It 3

Solve the system by substitution. $\begin{cases} -x + y = 4 \\ 4x - y = 2 \end{cases}$

Answer

$(2, 6)$

In the previous example, it was easiest to solve for y in the first equation because it had a coefficient of 1. In the following example, it will be easier to solve for x.

Example 4

Solve the system by substitution. $\begin{cases} x - 2y = -2 \\ 3x + 2y = 34 \end{cases}$

Solution

We will solve the first equation for x and then substitute the expression into the second equation.

	$x-2y=-2$
Solve for x. Substitute into the other equation.	$x=2y-2$ $3x+2y=34$
Replace the x with $2x-2$.	$3(2y-2)+2y=34$
Solve the resulting equation for y.	$6y-6+2y=34$
Substitute $y=5$ into $x-2y=-2$ to find x.	$8y-6=34$ $8y=40$ $y=5$ $x-2y=-2$ $x-2\cdot 5=-2$ $x-10=-2$ $x=8$
The ordered pair is $(8,5)$.	
Check the ordered pair in both equations: $x-2y=-2$: $8-2\cdot 5\stackrel{?}{=}-2$, $8-10\stackrel{?}{=}-2$, $-2=-2\checkmark$ $3x+2y=34$: $3\cdot 8+2\cdot 5\stackrel{?}{=}34$, $24+10\stackrel{?}{=}34$, $34=34\checkmark$	
	The solution is $(8,5)$.

Try It 4

Solve the system by substitution. $\begin{cases} x-5y=13 \\ 4x-3y=1 \end{cases}$

Answer

$(-2,-3)$

When both equations are already solved for the same variable, it is easy to substitute!

Example 5

Solve the system by substitution. $\begin{cases} y=-2x+5 \\ y=\frac{1}{2}x \end{cases}$

Solution

Since both equations are solved for y, we can substitute one into the other.

Substitute $\frac{1}{2}x$ for y in the first equation.	$y=\frac{1}{2}x$ $y=-2x+5$
Replace the y with $\frac{1}{2}x$	$\frac{1}{2}x=-2x+5$
Solve the resulting equation. Start by clearing the fraction.	$2\left(\frac{1}{2}x\right)=2(-2x+5)$
Solve for x.	$x=-4x+10$
	$5x=10$
Substitute $x=2$ into $y=\frac{1}{2}x$ to find y.	$x=2$ $y=\frac{1}{2}x$ $y=\frac{1}{2}\cdot 2$ $y=1$
The ordered pair is $(2,1)$.	
Check the ordered pair in both equations: $y=\frac{1}{2}x \quad y=-2x+5$ $1\stackrel{?}{=}\frac{1}{2}\cdot 2 \quad 1\stackrel{?}{=}-2\cdot 2+5$ $1=1\checkmark \quad 1=-4+5$ $1=1\checkmark$	
	The solution is $(2,1)$.

Try It 5

Solve the system by substitution. $\begin{cases} y=-x+10 \\ y=\frac{1}{4}x \end{cases}$

Answer

$(8,2)$

Be very careful with the signs in the following example.

Example 6

Solve the system by substitution. $\begin{cases} 4x+2y=4 \\ 6x-y=8 \end{cases}$

Solution

We need to solve one equation for one variable. We will solve the first equation for y.

	$4x+2y=4$
Solve the first equation for y y.	$2y=-4x+4$
Substitute $-2x+2$ for y in the second equation.	$y=-2x+2$ $6x-y=8$
Replace the y with $-2x+2$.	$6x-(-2x+2)=8$
Solve the equation for x.	$6x+2x-2=8$
	$8x-2=8$ $8x=10$
Substitute $x=\frac{5}{4}$ into $4x+2y=4$ to find y.	$x=\frac{5}{4}$ $4x+2y=4$ $4\left(\frac{5}{4}\right)+2y=4$ $5+2y=4$ $2y=-1$ $y=-\frac{1}{2}$
The ordered pair is $(\frac{5}{4},-\frac{1}{2})$.	
Check the ordered pair in both equations.	
	The solution is $\left(\frac{5}{4},-\frac{1}{2}\right)$.

Check the ordered pair in both equations.

$$\begin{aligned} 4x+2y &= 4 \\ 4(\tfrac{5}{4})+2(-\tfrac{1}{2}) &\stackrel{?}{=} 4 \\ 5-1 &\stackrel{?}{=} 4 \\ 4 &= 4\checkmark \end{aligned} \qquad \begin{aligned} 6x-y &= 8 \\ 6(\tfrac{5}{4})-(-\tfrac{1}{2}) &\stackrel{?}{=} 8 \\ \tfrac{15}{4}-(-\tfrac{1}{2}) &\stackrel{?}{=} 8 \\ \tfrac{16}{2} &\stackrel{?}{=} 8 \\ 8 &= 8\checkmark \end{aligned}$$

Try It 6

Solve the system by substitution. $\begin{cases} 4x-y=0 \\ 2x-3y=5 \end{cases}$

Answer

$(-\frac{1}{2},-2)$

Solving one equation for x or y in the following example will take a little more work.

Example 7

Solve the system by substitution. $\begin{cases} 4x-3y=6 \\ 15y-20x=-30 \end{cases}$

Solution

We need to solve one equation for one variable. We will solve the first equation for x.

	$4x - 3y = 6$
Solve the first equation for x.	$4x = 3y + 6$
Substitute $\frac{3}{4}y + \frac{3}{2}$ for x in the second equation.	$x = \frac{3}{4}y + \frac{3}{2}$ $15y - 20x = -30$
Replace the x with $\frac{3}{4}y + \frac{3}{2}$	$15y - 20\left(\frac{3}{4}y + \frac{3}{2}\right) = -30$
Solve for y.	$15y - 15y - 30 = -30$
	$0 - 30 = -30$
	$0 = 0$

Since 0 = 0 is a true statement, the system is consistent. The equations are dependent. The graphs of these two equations would give the same line. The system has infinitely many solutions.

Try It 7

Solve the system by substitution. $\begin{cases} 2x - 3y = 12 \\ -12y + 8x = 48 \end{cases}$

Answer

infinitely many solutions

Example 8

Solve the system by substitution. $\begin{cases} 5x - 2y = -10 \\ y = \frac{5}{2}x \end{cases}$

Solution

The second equation is already solved for y, so we can substitute for y in the first equation.

Substitute x for y in the first equation.	$y = \frac{5}{2}x$ $5x - 2y = -10$
Replace the y with $\frac{5}{2}x$.	$5x - 2\left(\frac{5}{2}x\right) = -10$
Solve for x.	$5x - 5x = -10$
	$0 \neq -10$

Since $0 = -10$ is a false statement, the equations are inconsistent. The graphs of the two equations would be parallel lines. The system has no solutions.

Try It 8

Solve the system by substitution. $\begin{cases} 3x + 2y = 9 \\ y = -\frac{3}{2}x + 1 \end{cases}$

Answer

no solution

Solve Applications of Systems of Equations by Substitution

We'll copy here the problem-solving strategy we used in the previous section for solving systems of equations. Now that we know how to solve systems by substitution, that's what we'll do in Step 5.

HOW TO USE A PROBLEM SOLVING STRATEGY FOR SYSTEMS OF LINEAR EQUATIONS.

1. **Read** the problem. Ensure that all words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Verify** the answer to the problem and ensure it is logical.
7. **Answer** the question with a complete sentence.

Some people find setting up word problems with two variables easier than setting them up with just one variable. Choosing the variable names is easier when all you need to do is write down two letters. Think about this in the following example—how would you have done it with just one variable?

Example 9

The sum of two numbers is zero. One number is nine less than the other. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name what we are looking for.	Let n be the first number Let m be the second number
Step 4. Translate into a system of equations.	The sum of two numbers is zero.
	$n+m=0$
	One number is nine less than the other.
	$n=m-9$
The system is:	$\begin{aligned} n+m&=0\\ n&=m-9 \end{aligned}$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for n.	
Substitute $m-9$ for n in the first equation.	$n=m-9$ $n+m=0$
Solve for m.	$m-9+m=0$
	$2m-9=0$
	$2m=9$
Substitute $m=\frac{9}{2}$ into the second equation and then solve for n.	$m=\frac{9}{2}$ $n=m-9$
	$m=\frac{9}{2}-9$
	$m=\frac{9}{2}-\frac{18}{2}$
	$n=-\frac{9}{2}$
Step 6. Check the answer in the problem.	Do these numbers make sense in the problem? We will leave this to you!
Step 7. Answer the question.	The numbers are $\frac{9}{2}$ and $-\frac{9}{2}$.

? Try It 9

The sum of two numbers is 10. One number is 4 less than the other. Find the numbers.

Answer

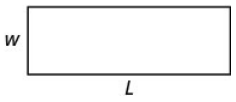
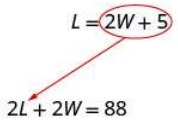
The numbers are 3 and 7.

In the following example, we'll use the formula for the perimeter of a rectangle, $P=2L+2W$.

Example 10

The perimeter of a rectangle is 88. The length is five more than twice the width. Find the length and width of the rectangle.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
Step 3. Name what we are looking for.	Let L be the length and W the width
Step 4. Translate into a system of equations.	The perimeter of a rectangle is 88.
	$2L + 2W = P$ $2L + 2W = 88$
	The length is five more than twice the width.
	$L = 2W + 5$
The system is:	$2L + 2W = 88$ $L = 2W + 5$
Step 5. Solve the system of equations. We will use substitution since the second equation is solved for L. Substitute $2W + 5$ for L in the first equation.	
Solve for W.	$2(2W + 5) + 2W = 88$
	$4W + 10 + 2W = 88$
	$6W + 10 = 88$
	$6W = 78$
Substitute $W = 13$ into the second equation and then solve for L.	$W = 13$ $L = 2W + 5$
	$L = 2 \cdot 13 + 5$
	$L = 31$
Step 6. Check the answer in the problem.	Does a rectangle with length 31 and width 13 has a perimeter of 88? Yes.
Step 7. Answer the equation.	The length is 31 inches, and the width is 13 inches.

Try It 10

The perimeter of a rectangle is 40. The length is 4 more than the width. Find the length and width of the rectangle.

Answer

The length is 12 inches, and the width is 8 inches.

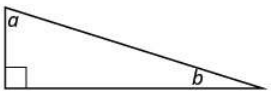
For the following example, we need to remember that the sum of the measures of the angles of a triangle is 180 degrees and that a right triangle has one 90-degree angle.

Example 11

The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle. Find the measures of both angles.

Solution

We will draw and label a figure.

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the measures of the angles.
Step 3. Name what we are looking for.	Let a be the measure of the 1st angle and b the measure of the 2nd angle
Step 4. Translate into a system of equations.	The measure of one of the small angles of a right triangle is ten more than three times the measure of the other small angle.
	$a = 3b + 10$
	The sum of the measures of the angles of a triangle is 180.
	$a + b + 90 = 180$
The system is:	$a = 3b + 10$ $a + b + 90 = 180$
Step 5. Solve the system of equations. We will use substitution since the first equation is solved for a.	$a = 3b + 10$ $a + b + 90 = 180$
Substitute $3b + 10$ for a in the second equation.	$(3b + 10) + b + 90 = 180$
Solve for b.	$4b + 100 = 180$
	$4b = 80$
	$b = 20$ $a = 3b + 10$
Substitute $b = 20$ into the first equation and then solve for a.	$a = 3 \cdot 20 + 10$ $a = 70$
Step 6. Check the answer in the problem.	We will leave this to you!
Step 7. Answer the question.	The measures of the small angles are 20 and 70.

Try It 11

The measure of one of the small angles of a right triangle is 18 degrees less than twice the measure of the other small angle. Find the measure of both angles.

Answer

The measure of the angles are 36 degrees and 54 degrees.

Example 12

Heather has been offered two options for her salary as a trainer at the gym. Option A would pay her $25,000 plus $15 for each training session. Option B would pay her $10,000 + $40 for each training session. How many training sessions would be required to make the salary options equal?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the number of training sessions that would make the pay equal.
Step 3. Name what we are looking for.	Let s be Heather's salary and n the number of training sessions
Step 4. Translate into a system of equations.	Option A would pay her $25,000 plus $15 for each training session.
	$s = 25{,}000 + 15n$
	Option B would pay her $10,000 + $40 for each training session
	$s = 10{,}000 + 40n$
The system is:	$s = 25{,}000 + 15n$ $s = 10{,}000 + 40n$
Step 5. Solve the system of equations. We will use substitution.	$s = 25{,}000 + 15n$ $s = 10{,}000 + 40n$
Substitute \(25,000+15n\) for s in the second equation.	$25{,}000 + 15n = 10{,}000 + 40n$
Solve for n.	$25{,}000 = 10{,}000 + 25n$
	$15{,}000 = 25n$
	$600 = n$
Step 6. Check the answer.	Are 600 training sessions a year reasonable? Are the two options equal when n = 600?
Step 7. Answer the question.	The salary options would be equal for 600 training sessions.

Try It 12

Two insurance companies have offered Geraldine positions. The first company pays a salary of $12,000 plus a commission of $100 for each policy sold. The second pays a salary of $20,000 plus a commission of $50 for each policy sold. How many policies would need to be sold to make the total pay the same?

Answer

There would need to be 160 policies sold to make the same pay.

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Section 4.2P: Practice

Practice Makes Progress

Solve a System of Equations by Substitution

In the following exercises, solve the systems of equations by substitution.

Exercise 1

$$\begin{cases} 15x + 2y = 6 \\ -5x + 2y = -4 \end{cases}$$

Exercise 2

$$\begin{cases} 4x + 2y = 8 \\ 8x - y = 1 \end{cases}$$

Exercise 3

$$\begin{cases} 2x + 5y = 1 \\ y = \frac{1}{3}x - 2 \end{cases}$$

Exercise 4

$$\begin{cases} 2x + y = 5 \\ x - 2y = -15 \end{cases}$$

Exercise 5

$$\begin{cases} 3x - 2y = 6 \\ y = \frac{2}{3}x + 2 \end{cases}$$

Exercise 6

$$\begin{cases} 3x + y = 1 \\ -4x + y = 15 \end{cases}$$

Exercise 7

$$\begin{cases} y = -4x \\ 4x + y = 1 \end{cases}$$

Exercise 8

$$\begin{cases} 2x + y = 10 \\ -x + y = -5 \end{cases}$$

Exercise 9

$$\begin{cases} y = -2x - 1 \\ y = -\frac{1}{3}x + 4 \end{cases}$$

Exercise 10

$$\begin{cases} 2x + 16y = 8 \\ -x - 8y = -4 \end{cases}$$

Exercise 11

$$\begin{cases} x - 2y = -5 \\ 2x - 3y = -4 \end{cases}$$

Exercise 12

$$\begin{cases} y = 2x - 8 \\ y = \frac{3}{5}x + 6 \end{cases}$$

Exercise 13

$$\begin{cases} 5x - 2y = -6 \\ y = 3x + 3 \end{cases}$$

Exercise 14

$$\begin{cases} y = 3x \\ 6x - 2y = 0 \end{cases}$$

Exercise 15

$$\begin{cases} y = \frac{7}{8}x + 4 \\ -7x + 8y = 6 \end{cases}$$

Exercise 16

$$\begin{cases} x + 3y = 1 \\ 3x + 5y = -5 \end{cases}$$

Exercise 17

$$\begin{cases} 2x + y = -4 \\ 3x - 2y = -6 \end{cases}$$

Exercise 18

$$\begin{cases} 2x + 3y = 3 \\ y = -x + 3 \end{cases}$$

In the following exercises, translate to a system of equations and solve.

Exercise 19

The sum of two numbers is 15. One number is 3 less than the other. Find the numbers.

Exercise 20

The sum of two numbers is −26. One number is 12 less than the other. Find the numbers.

? Exercise 21

The perimeter of a rectangle is 60. The length is 10 more than the width. Find the length and width.

? Exercise 22

The perimeter of a rectangle is 84. The length is 10 more than three times the width. Find the length and width.

? Exercise 23

The measure of one of the small angles of a right triangle is 26 degrees more than 3 times the measure of the other small angle. Find the measure of both angles.

? Exercise 24

The measure of one of the small angles of a right triangle is 45 degrees less than twice the measure of the other small angle. Find the measure of both angles.

Everyday Math

? Exercise 25

Two cable companies have offered Jackie positions. The first company pays a salary of $14,000 plus a commission of $100 for each cable package sold. The second pays a salary of $20,000 plus a commission of $25 for each cable package sold. How many cable packages would need to be sold to make the total pay the same?

? Exercise 26

Mitchell currently sells stoves for Company A at a salary of $12,000 plus a $150 commission for each stove he sells. Company B offers him a position with a salary of $24,000 plus a $50 commission for each stove he sells. How many stoves would Mitchell need to sell for the options to be equal?

Writing Exercises

? Exercise 27

Solve the system of equations $\begin{cases} 3x+y=12 \\ x=y-8 \end{cases}$ by substitution and explain all your steps in words.

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a system of equations by substitution.			
solve applications of systems of equations by substitution.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 4.3: Solve Systems of Equations by Elimination

Be Prepared

We will rely heavily on these skills throughout this section.

- Simplify $\frac{1}{3}x+\frac{5}{8}=\frac{31}{24}$
- Solve $2x+3y=7$ if $x=y-1$

Learning Objectives

Motivating Problem

You're buying snacks for a study group. You get 2 granola bars and 1 smoothie for $7. Later, you buy 1 granola bar and 2 smoothies for $9. How much does each item cost?

Fun Fact

Elimination is the basis of a method called Gaussian elimination, developed by **Carl Friedrich Gauss**, one of history's greatest mathematicians, when he was just a teenager! This method is used extensively in advanced math courses and is nearly identical to the process we are learning in this section, only with more variables.

The Goal

This section introduces the elimination method, where we add or subtract equations to eliminate a variable, thereby determining the value of both variables. It's a powerful technique, especially when equations are already aligned.

We have solved systems of linear equations using both graphing and substitution methods. Graphing works well when the variable coefficients are small and the solution has integer values. Substitution works well when we can easily solve one equation for one of the variables and not have too many fractions in the resulting expression.

The third method of solving systems of linear equations is called the Elimination Method. When we solved a system by substitution, we started with two equations and two variables and reduced it to one equation with one variable. This is what we'll do with the elimination method, too, but we'll have a different way to get there.

Solve a System of Equations by Elimination

The **Elimination Method** is based on the Addition Property of Equality. The Addition Property of Equality says that you still have equality when you add the same quantity to both sides of an equation. We will extend the Addition Property of Equality to say that the results are equal when you add equal quantities to both sides of an equation.

For any expressions a, b, c, and d,

$$\begin{array}{ll} \textbf{if} & a=b \\ \textbf{and} & c=d \\ \textbf{then} & a+c=b+d \end{array}$$

To solve a system of equations using the elimination method, we begin by putting both equations into standard form. Then we decide which variable will be easiest to eliminate. How do we decide? We want the coefficients of one variable to be opposites, so that we can add the equations together and eliminate that variable.

Notice how that works when we add these two equations together:

$$\begin{array}{r} 3x+y=5 \\ \underline{2x-y=0} \\ 5x \quad\quad =5 \end{array}$$

The y's add to zero, and we have one equation with one variable.

Let's try another one:

$$\begin{cases} x+4y=2 \\ 2x+5y=-2 \end{cases}$$

This time, we don't see a variable that can be immediately eliminated by adding the equations.

But if we multiply the first equation by -2, we will make the coefficients of x opposites. We must multiply every term on both sides of the equation by -2.

$$\begin{cases} -2(x+4y)=-2(2) \\ 2x+5y=-2 \end{cases}$$

$$\begin{cases} -2x-8y=-4 \\ 2x+5y=-2 \end{cases} \nonumber \qquad \text{(Section 4.3.1)}$$

Now we see that the coefficients of the x terms are opposites, so x will be eliminated when we add these two equations.

Add the equations yourself—the result should be $-3y=-6$. And that looks easy to solve, doesn't it? Here is what it would look like.

$$\begin{array}{rcl} -2x-8y & = & -4 \\ +2x+5y & = & -2 \\ \hline -3y & = & -6 \end{array}$$

We'll do one more:

$$\begin{cases} 4x-3y=10 \\ 3x+5y=-7 \end{cases}$$

It doesn't appear that we can get the coefficients of one variable to be opposites by multiplying one of the equations by a constant, unless we use fractions. So instead, we'll have to multiply both equations by a constant.

We can make the coefficients of x be opposites if we multiply the first equation by 3 and the second by -4, so we get $12x$ and $-12x$.

$3(4x-3y)=3(10)$

$-4(3x+5y)=-4(-7)$

This gives us these two new equations:

$$\begin{cases} 12x-9y=30 \\ -12x-20y=28 \end{cases}$$

When we add these equations,

$$\begin{cases} 12x-9y=30 \\ \underline{-12x-20y=28} \\ -29y=58 \end{cases}$$

the x's are eliminated and we just have $-29y=58$.

Once we get an equation with just one variable, we solve it. Then, we substitute that value into one of the original equations to solve for the remaining variable. As always, we verify our answers to ensure they are solutions to both of the original equations.

Now we'll see how to use elimination to solve the same system of equations we solved by graphing and by substitution.

Example 1

Solve the system by elimination. $\begin{cases} 2x+y=7 \\ x-2y=6 \end{cases}$

Solution

Step 1. Write both equations in standard form. If any coefficients are fractions, clear them.	Both equations are in standard form, $Ax + By = C$. There are no fractions.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$
Step 2. Make the coefficients of one variable opposites. Decide which variable you will eliminate. Multiply one or both equations so that the coefficients of that variable are opposites.	We can eliminate the y's by multiplying the first equation by 2. Multiply both sides of $2x + y = 7$ by 2.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ $\begin{cases} 2(2x + y) = 2(7) \\ x - 2y = 6 \end{cases}$
Step 3. Add the equations resulting from Step 2 to eliminate one variable.	We add the x's, y's, and constants.	$\begin{cases} 4x + 2y = 14 \\ x - 2y = 6 \end{cases}$ $5x \quad = 20$
Step 4. Solve for the remaining variable.	Solve for x.	$x = 4$
Step 5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.	Substitute $x = 4$ into the second equation, $x - 2y = 6$. Then solve for y.	$x - 2y = 6$ $4 - 2y = 6$ $-2y = 2$ $y = -1$
Step 6. Write the solution as an ordered pair.	Write it as (x, y).	$(4, -1)$
Step 7. Check that the ordered pair is a solution to **both** original equations.	Substitute $(4, -1)$ into $2x + y = 7$ and $x - 2y = 6$ Do they make both equations true? Yes!	$2x + y = 7$ $\quad$ $x - 2y = 6$ $2(4) + (-1) \stackrel{?}{=} 7$ $\quad$ $4 - 2(-1) \stackrel{?}{=} 6$ $7 = 7$ ✓ $\quad$ $6 = 6$ ✓ The solution is $(4, -1)$.

? Try It 1

Solve the system by elimination. $\begin{cases} 3x + y = 5 \\ 2x - 3y = 7 \end{cases}$

Answer

$(2, -1)$

The steps are listed below for easy reference.

HOW TO SOLVE A SYSTEM OF EQUATIONS BY ELIMINATION.

1. Write both equations in standard form. If any coefficients are fractions, clear them.
2. Make the coefficients of one variable opposites.
   - Decide which variable you will eliminate.
   - Multiply one or both equations so that the coefficients of that variable are opposites.
3. Add the equations resulting from Step 2 to eliminate one variable.
4. Solve for the remaining variable.

5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.
6. Write the solution as an ordered pair.
7. Check that the ordered pair is a solution to **both** original equations.

First, we'll give an example of eliminating one variable immediately.

Example 2

Solve the system by elimination. $\begin{cases} x+y=10 \\ x-y=12 \end{cases}$

Solution

	$\begin{array}{l} x+y=10 \\ x-y=12 \end{array}$
Both equations are in standard form.	
The coefficients of y are already opposites.	
Add the two equations to eliminate y. The resulting equation has only one variable, x.	$\begin{array}{l} x+y=10 \\ x-y=12 \\ \hline 2x \quad =22 \end{array}$
Solve for x, the remaining variable. Substitute x = 11 into one of the original equations.	$x=11$ $x+y=10$
	$11+y=10$
Solve for the other variable, y.	$y=-1$
Write the solution as an ordered pair.	The ordered pair is (11, -1).
Check that the ordered pair is a solution to **both** original equations. $\begin{array}{rcl} x+y & = & 10 \\ 11+(-1) & \stackrel{?}{=} & 10 \\ 10 & = & 10\checkmark \end{array}$ $\quad$ $\begin{array}{rcl} x-y & = & 12 \\ 11-(-1) & \stackrel{?}{=} & 12 \\ 12 & = & 12\checkmark \end{array}$	
	The solution is (11, -1).

Try It 2

Solve the system by elimination. $\begin{cases} x+y=3 \\ -2x-y=-1 \end{cases}$

Answer

$(-2, 5)$

In the next example, we will be able to make the coefficients of one variable opposites by multiplying one equation by a constant.

Example 3

Solve the system by elimination. $\begin{cases} 3x-2y=-2 \\ 5x-6y=10 \end{cases}$

Solution

	$\begin{aligned} 3x-2y&=-2\\ 5x-6y&=10 \end{aligned}$
Both equations are in standard form.	
None of the coefficients are opposites.	
We can make the coefficients of y opposites by multiplying the first equation by -3.	$\begin{aligned} -3(3x-2y)&=-3(-2)\\ 5x-6y&=10 \end{aligned}$
Simplify.	$\begin{aligned} -9x+6y&=6\\ 5x-6y&=10 \end{aligned}$
Add the two equations to eliminate y.	$\begin{aligned} -9x+6y&=6\\ 5x-6y&=10\\ \hline -4x\quad&=16 \end{aligned}$
Solve for the remaining variable, x. Substitute $x=-4$ into one of the original equations.	$\begin{aligned} x&=-4\\ 3x-2y&=-2 \end{aligned}$
	$3(-4)-2y=-2$
Solve for y.	$\begin{aligned} -12-2y&=-2\\ -2y&=10\\ y&=-5 \end{aligned}$
Write the solution as an ordered pair.	The ordered pair is $(-4,-5)$.
Check that the ordered pair is a solution to both original equations.	$\begin{aligned} 3x-2y&=-2 & \qquad 5x-6y&=10\\ 3(-4)-2(-5)&\stackrel{?}{=}-2 & 5(-4)-6(-5)&\stackrel{?}{=}10\\ -12+10&\stackrel{?}{=}-2 & -20+30&\stackrel{?}{=}10\\ -2&=-2\checkmark & 10&=10\checkmark \end{aligned}$
	The solution is $(-4,-5)$.

Try It 3

Solve the system by elimination. $\begin{cases} 3x+2y=2\\ 6x+5y=8 \end{cases}$

Answer

$(-2,4)$

Now, we'll do an example where we need to multiply both equations by constants to make the coefficients of one variable equal to the opposite.

Example 4

Solve the system by elimination. $\begin{cases} 4x-3y=9\\ 7x+2y=-6 \end{cases}$

Solution

In this example, we cannot multiply just one equation by any constant to get opposite coefficients. So we will strategically multiply both equations by a constant to get the opposites.

	$\begin{aligned} 4x-3y&=9\\ 7x+2y&=-6 \end{aligned}$
Both equations are in standard form. To get opposite coefficients of y, we will multiply the first equation by 2 and the second equation by 3.	$\begin{aligned} 2(4x-3y)&=2(9)\\ 3(7x+2y)&=3(-6) \end{aligned}$
Simplify.	$\begin{aligned} 8x-6y&=18\\ 21x+6y&=-18 \end{aligned}$
Add the two equations to eliminate y.	$\begin{aligned} 8x-6y&=18\\ 21x+6y&=-18\\ \hline 39x&=0 \end{aligned}$
Solve for x. Substitute $x=0$ into one of the original equations.	$x=0$ $7x+2y=-6$
	$7\cdot 0+2y=-6$
Solve for y.	$2y=-6$
	$y=-3$
Write the solution as an ordered pair.	The ordered pair is $(0,-3)$.
Check that the ordered pair is a solution to **both** original equations. $\begin{aligned} 4x-3y&=9\\ 4(0)-3(-3)&\stackrel{?}{=}9\\ 9&=9\checkmark \end{aligned}$ $\quad$ $\begin{aligned} 7x+2y&=-6\\ 7(0)+2(-3)&\stackrel{?}{=}-6\\ -6&=-6\checkmark \end{aligned}$	
	The solution is $(0,-3)$.

Try It 4

Solve the system by elimination. $\begin{cases} 7x+8y=4\\ 3x-5y=27 \end{cases}$

Answer

$(4,-3)$

When the system of equations contains fractions, we will first clear them by multiplying each equation by its least common denominator (LCD).

Example 5

Solve the system by elimination. $\begin{cases} x+\frac{1}{2}y=6\\ \frac{3}{2}x+\frac{2}{3}y=\frac{17}{2} \end{cases}$

Solution

In this example, both equations have fractions. Our first step will be to multiply each equation by its LCD to clear the fractions.

	$x+\frac{1}{2}y=6$ $\frac{3}{2}x+\frac{2}{3}y=\frac{17}{2}$
To clear the fractions, multiply each equation by its LCD.	$2\left(x+\frac{1}{2}y\right)=2(6)$ $6\left(\frac{3}{2}x+\frac{2}{3}y\right)=6\left(\frac{17}{2}\right)$
Simplify.	$2x+y=12$ $9x+4y=51$
Now we are ready to eliminate one of the variables. Notice that both equations are in standard form.	
We can eliminate y multiplying the top equation by -4.	$-4(2x+y)=-4(12)$ $9x+4y=51$
Simplify and add. Substitute $x=3$ into one of the original equations.	$-8x-4y=-48$ $9x+4y=51$ $x=3$ $x+\frac{1}{2}y=6$
Solve for y.	$3+\frac{1}{2}y=6$
	$\frac{1}{2}y=3$
	$y=6$
Write the solution as an ordered pair.	The ordered pair is $(3,6)$.
Check that the ordered pair is a solution to **both** original equations. $x+\frac{1}{2}y=6$ $3+\frac{1}{2}(6)\stackrel{?}{=}6$ $3+3\stackrel{?}{=}6$ $6=6\checkmark$ $\frac{3}{2}x+\frac{2}{3}y=\frac{17}{2}$ $\frac{3}{2}(3)+\frac{2}{3}(6)\stackrel{?}{=}\frac{17}{2}$ $\frac{9}{2}+4\stackrel{?}{=}\frac{17}{2}$ $\frac{9}{2}+\frac{8}{2}\stackrel{?}{=}\frac{17}{2}$ $\frac{17}{2}=\frac{17}{2}\checkmark$	
	The solution is $(3,6)$.

Try It 5

Solve the system by elimination. $\begin{cases} x+\frac{3}{5}y=-\frac{1}{5} \\ -\frac{1}{2}x-\frac{2}{3}y=\frac{5}{6} \end{cases}$

Answer

$(1,-2)$

In the previous section, we saw that not all systems of linear equations have a single ordered pair as a solution. There were infinitely many solutions when the two equations were the same line. We called that a consistent system. When the two equations described parallel lines, there was no solution. We called that an inconsistent system.

Example 6

Solve the system by elimination. $\begin{cases} 3x+4y=12 \\ y=3-\frac{3}{4}x \end{cases}$

Solution

$$\begin{cases} 3x+4y=12 \\ y=3-\frac{3}{4}x \end{cases}$$

Write the second equation in standard form. $\begin{cases} 3x+4y=12 \\ \frac{3}{4}x+y=3 \end{cases}$

Clear the fractions by multiplying thesecond equation by 4. $\begin{cases} 3x+4y=12 \\ 4\left(\frac{3}{4}x+y\right)=4(3) \end{cases}$

Simplify. $\begin{cases} 3x+4y=12 \\ 3x+4y=12 \end{cases}$

To eliminate a variable, we multiply thesecond equation by -1. $\begin{cases} 3x+4y=12 \\ \underline{-3x-4y=-12} \end{cases}$

$$0=0$$

Simplify and add.

This is a true statement. The equations are consistent but dependent. Their graphs would be the same line. The system has infinitely many solutions.

After we cleared the fractions in the second equation, did you notice that the two equations were the same? That means we have coincident lines.

Try It 6

Solve the system by elimination. $\begin{cases} 5x-3y=15 \\ y=-5+\frac{5}{3}x \end{cases}$

Answer

infinitely many solutions

Example 7

Solve the system by elimination. $\begin{cases} -6x+15y=10 \\ 2x-5y=-5 \end{cases}$

Solution

The equations are in standard form. $\begin{cases} -6x+15y=10 \\ 2x-5y=-5 \end{cases}$

Multiply the second equation by 3 to eliminate a variable. $\begin{cases} -6x+15y=10 \\ 3(2x-5y)=3(-5) \end{cases}$

Simplify and add. $\begin{cases} -6x+15y=10 \\ \underline{6x-15y=-15} \end{cases}$

$$0\neq 5$$

This statement is false. The equations are inconsistent, and so their graphs would be parallel lines.

The system does not have a solution.

Try It 7

Solve the system by elimination. $\begin{cases} 7x-3y=-2 \\ -14x+6y=8 \end{cases}$

Answer

no solution

Solve Applications of Systems of Equations by Elimination

Some application problems can be translated directly into standard-form equations, so we will use the elimination method to solve them. As before, we use our Problem-Solving Strategy to help us stay focused and organized.

Example 8

The sum of two numbers is 39. Their difference is 9. Find the numbers.

Solution

Step 1. Read the problem	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name what we are looking for.	Let n = the first number. m = the second number
Step 4. Translate into a system of equations.	
	The sum of two numbers is 39. $n+m=39$ Their difference is 9. $n-m=9$
The system is:	$\begin{cases} n+m=39 \\ n-m=9 \end{cases}$
Step 5. Solve the system of equations. To solve the system of equations, use elimination. The equations are in standard form and the coefficients of m are opposites. Add.	$\begin{cases} n+m=39 \\ \underline{n-m=9} \\ 2n \quad =48 \end{cases}$
Solve for n.	$n=24$
Substitute n=24 into one of the original equations and solve form.	$n+m=39$ $24+m=39$ $m=15$
Step 6. Check the answer.	Since 24+15=39 and 24-15=9, the answers check.
Step 7. Answer the question.	The numbers are 24 and 15.

Try It 8

The sum of two numbers is -15. Their difference is -35. Find the numbers.

Answer

The numbers are -25 and 10.

Example 9

Joe stops at a burger restaurant every day on his way to work. On Monday, he had one order of medium fries and two small sodas, totaling 620 calories. On Tuesday, he had two orders of medium fries and one small soda, totaling 820 calories. How many calories are there in one order of medium fries? How many calories are in one small soda?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the number of calories in one order of medium fries and in one small soda.
Step 3. Name what we are looking for.	Let f be the number of calories in 1 order of medium fries. Let x be the number of calories in 1 small soda.
Step 4. Translate into a system of equations:	one medium fries and two small sodas had a total of 620 calories
	$f+2s=620$
	two medium fries and one small soda had a total of 820 calories.
	$2f+s=820$
Our system is:	$\begin{aligned} f+2s&=620\\ 2f+s&=820 \end{aligned}$
Step 5. Solve the system of equations. To solve the system of equations, use elimination. The equations are in standard form. To get opposite coefficients of f, multiply the top equation by -2.	$\begin{aligned} -2(f+2s)&=-2(620)\\ 2f+\ s&=820 \end{aligned}$
Simplify and add.	$\begin{aligned} -2f-4s&=-1240\\ 2f+\ s&=820\\ \hline -3s&=-420 \end{aligned}$
Solve for s.	$s=140$
Substitute $x=140$ into one of the original equations and then solve for f.	$f+2s=620$
	$f+2\cdot 140=620$
	$f+280=620$
	$f=340$
Step 6. Check the answer.	Verify that these numbers make sense in the problem, and that they are solutions to both equations. We leave this to you!
Step 7. Answer the question.	The small soda has 140 calories and the fries have 340 calories.

? Try It 9

Malik stops at the grocery store to buy a bag of diapers and 2 cans of formula. He spends a total of $37. The next week, he stops and buys 2 bags of diapers and 5 cans of formula for a total of $87. How much does a bag of diapers cost? How much is one can of formula?

Answer

The bag of diapers costs $11, and the can of formula costs $13.

Choose the Most Convenient Method to Solve a System of Linear Equations

When you have to solve a system of linear equations in a later math class, you will usually not be told which method to use. You will need to make that decision yourself. So you'll want to choose the method that is easiest to do and minimizes your chance of making mistakes.

Graphing	Substitution	Elimination
Use when you need a picture of the situation.	Use when one equation is already solved for one variable.	Use when the equations are in standard form.

✓ Example 10

For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} 3x+8y=40 \\ 7x-4y=-32 \end{cases}$
2. $\begin{cases} 5x+6y=12 \\ y=\frac{2}{3}x-1 \end{cases}$

Solution

1. $\begin{cases} 3x+8y=40 \\ 7x-4y=-32 \end{cases}$

Since both equations are in standard form, using elimination will be most convenient.

2. $\begin{cases} 5x+6y=12 \\ y=\frac{2}{3}x-1 \end{cases}$

Since one equation is already solved for y, using substitution will be most convenient.

? Try It 10

For each system of linear equations, decide whether it would be more convenient to solve it by substitution or elimination. Explain your answer.

1. $\begin{cases} y=2x-1 \\ 3x-4y=-6 \end{cases}$
2. $\begin{cases} 6x-2y=12 \\ 3x+7y=-13 \end{cases}$

Answer

1. Since one equation is already solved for y, using substitution will be most convenient;
2. Since both equations are in standard form, using elimination will be most convenient.

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Section 4.3P: Practice

Practice Makes Progress

Solve a System of Equations by Elimination

In the following exercises, solve the systems of equations by elimination.

Exercise 1

$$\begin{cases} -3x + y = -9 \\ x - 2y = -12 \end{cases}$$

Exercise 2

$$\begin{cases} 3x - y = -7 \\ 4x + 2y = -6 \end{cases}$$

Exercise 3

$$\begin{cases} 3x + 2y = 6 \\ -6x - 4y = -12 \end{cases}$$

Exercise 4

$$\begin{cases} x + y = -8 \\ x - y = -6 \end{cases}$$

Exercise 5

$$\begin{cases} -7x + 6y = -10 \\ x - 6y = 22 \end{cases}$$

Exercise 6

$$\begin{cases} 5x - 3y = 15 \\ y = \frac{5}{3}x - 2 \end{cases}$$

Exercise 7

$$\begin{cases} 5x + 2y = 1 \\ -5x - 4y = -7 \end{cases}$$

Exercise 8

$$\begin{cases} 3x - 4y = -11 \\ x - 2y = -5 \end{cases}$$

Exercise 9

$$\begin{cases} 6x - 5y = -75 \\ -x - 2y = -13 \end{cases}$$

Exercise 10

$$\begin{cases} 2x - 5y = 7 \\ 3x - y = 17 \end{cases}$$

Exercise 11

$$\begin{cases} 7x + y = -4 \\ 13x + 3y = 4 \end{cases}$$

Exercise 12

$$\begin{cases} 3x - 5y = -9 \\ 5x + 2y = 16 \end{cases}$$

Exercise 13

$$\begin{cases} -11x + 12y = 60 \\ -22x + 24y = 90 \end{cases}$$

Exercise 14

$$\begin{cases} 4x + 7y = 14 \\ -2x + 3y = 32 \end{cases}$$

Exercise 15

$$\begin{cases} 3x + 8y = -3 \\ 2x + 5y = -3 \end{cases}$$

Exercise 16

$$\begin{cases} -3x - y = 8 \\ 6x + 2y = -16 \end{cases}$$

Exercise 17

$$\begin{cases} 3x + 8y = 67 \\ 5x + 3y = 60 \end{cases}$$

Exercise 18

$$\begin{cases} \frac{1}{3}x - y = -3 \\ x + \frac{5}{2}y = 2 \end{cases}$$

Exercise 19

$$\begin{cases} x + \frac{1}{3}y = -1 \\ \frac{1}{2}x - \frac{1}{3}y = -2 \end{cases}$$

Exercise 20

$$\begin{cases} 2x + y = 3 \\ 6x + 3y = 9 \end{cases}$$

In the following exercises, translate to a system of equations and solve.

Exercise 21

The sum of two numbers is 65. Their difference is 25. Find the numbers.

Exercise 22

The sum of two numbers is −27. Their difference is −59. Find the numbers.

Choose the Most Convenient Method to Solve a System of Linear Equations

In the following exercises, decide whether it would be more convenient to solve the system of equations by substitution or elimination.

Exercise 23

$$\begin{cases} 8x - 15y = -32 \\ 6x + 3y = -5 \end{cases}$$

Exercise 24

$$\begin{cases} x = 4y - 3 \\ 4x - 2y = -6 \end{cases}$$

Exercise 25

$$\begin{cases} y = 4x + 9 \\ 5x - 2y = -21 \end{cases}$$

Exercise 26

$$\begin{cases} 9x - 4y = 24 \\ 3x + 5y = -14 \end{cases}$$

Everyday Math

Exercise 27

Andrea is buying some new shirts and sweaters. She is able to buy 3 shirts and 2 sweaters for $114 or she is able to buy 2 shirts and 4 sweaters for $164. How much does a shirt cost? How much does a sweater cost?

Exercise 28

The total amount of sodium in 2 hot dogs and 3 cups of cottage cheese is 4720 mg. The total amount of sodium in 5 hot dogs and 2 cups of cottage cheese is 6300 mg. How much sodium is in a hot dog? How much sodium is in a cup of cottage cheese?

Writing Exercises

Exercise 29

Solve the system

$$\begin{cases} x + y = 10 \\ 5x + 8y = 56 \end{cases}$$

a. by substitution

b. by graphing

c. Which method do you prefer? Why?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve a system of equations by elimination.			
solve applications of systems of equations by elimination.			
choose the most convenient method to solve a system of linear equations.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 4.4: Solve Applications with Systems of Equations

Be Prepared

We will rely heavily on these skills throughout this section.

- Two times a number and nine is thirty-one. What is the number?
- How far does someone who rides a bike $3\frac{1}{2}$ hours at a rate of 10 miles per hour go?
- Simplify $4.025(1,562) + 254.1$

Learning Objectives

Motivating Problem

You're creating a custom trail mix with peanuts and dried fruit. One is $4 per pound, and the other is $6 per pound. You want 2 pounds of mix, which costs $5 per pound, total. How much of each do you need?

When two ingredients, speeds, or prices are blended together, systems of equations let you find the exact combo that works.

Fun Fact

Apothecaries in medieval Europe used systems of equations to blend herbs and tinctures at precise ratios for potions—chemistry and medicine have always had a mathematical component.

The Goal

In this section, we will apply systems of equations to solve real-world problems involving mixtures, motion, pricing, and comparisons. This is where algebra becomes a tool for decision-making!

In a previous section, we solved several applications with systems of linear equations. In this section, we'll examine specific types of applications that relate two quantities. We'll translate the words into linear equations, determine which method is most convenient to use, and then solve them.

Seven-Step Strategy For Solving Systems of Linear Equations:

1. **Read** the problem. Ensure that all words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose variables to represent those quantities.
4. **Translate** into a system of equations.
5. **Solve** the system of equations using good algebra techniques.
6. **Verify** the answer to the problem and ensure it is logical.
7. **Answer** the question with a complete sentence.

Translate to a System of Equations

Let's look at an example where we stop after finding the system of equations.

Example 1

Translate to a system of equations:

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution

We are looking for the amount that the husband and wife each earn.	Let $h =$ the amount the husband earns.
Translate.	$w =$ the amount the wife earns
	A married couple together earns \$110,000.
	$w + h = 110000$
	The wife earns \$16,000 less than twice what husband earns.
	$w = 2h - 16,000$
The system of equations is:	$\begin{cases} w + h = 110,000 \\ w = 2h - 16,000 \end{cases}$

? Try It 1

Translate to a system of equations:

A senior employee makes \$5 less than twice what a new employee makes per hour. Together they make \$43 per hour. How much does each employee make per hour?

Answer

$$\begin{cases} s = 2n - 5 \\ s + n = 43 \end{cases}$$

Solve Direct Translation Applications

In the previous example, we set up but did not solve the systems of equations. Now, we'll translate a situation into a system of equations *and then solve it*.

✓ Example 2

Translate to a system of equations and then solve:

Devon is 26 years older than his son Cooper. The sum of their ages is 50. Find their ages.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the ages of Devon and Cooper.
Step 3. Name what we are looking for.	Let d be Devon's age and c Cooper's age.
Step 4. Translate into a system of equations.	Devon is 26 years older than Cooper: $$d = c + 26$$
	The sum of their ages is 50: $$d + c = 50$$
	Then, the system is: $$\begin{cases} d = c + 26, \\ d + c = 50 \end{cases}$$
Step 5. Solve the system of equations using substitution.	Substitute $c + 26$ into the second equation for d: $$\begin{aligned} d + c &= 50 \\ (c + 26) + c &= 50 \\ 2c + 26 &= 50 \end{aligned}$$
	Solve for c: $$\begin{aligned} 2c + 26 &= 50 \\ 2c &= 24 \\ c &= 12 \end{aligned}$$
	Substitute $c = 12$ into the first equation and solve for d: $$\begin{aligned} d &= c + 26 \\ d &= (12) + 26 \\ d &= 38 \end{aligned}$$
Step 6. Check the answer in the problem.	Is Devon's age 26 more than Cooper's? Yes, 38 is 26 more than 12. Is the sum of their ages 50? Yes, 38 plus 12 is 50.
Step 7. Answer the question.	Devon is 38, and Cooper is 12.

Try It 2

Translate to a system of equations and then solve:

Ali is 12 years older than his youngest sister, Jameela. The sum of their ages is 40. Find their ages.

Answer

Ali is 26 and Jameela is 14.

Example 3

Translate to a system of equations and then solve:

When Jenna spent 10 minutes on the elliptical trainer and then completed a 20-minute circuit training session, her fitness app indicated that she had burned 278 calories. When she spent 20 minutes on the elliptical trainer and 30 minutes circuit training,

she burned 473 calories. How many calories does she burn each minute on the elliptical trainer? How many calories does she burn for each minute of circuit training?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for the number of calories burned each minute on the elliptical trainer and each minute of circuit training.
Step 3. Name what we are looking for.	Let e be the number of calories burned per minute on the elliptical trainer and c the number of calories burned per minute while circuit training.
Step 4. Translate into a system of equations.	10 minutes on the elliptical and circuit training for 20 minutes, burned 278 calories
	$10e + 20c = 278$
	20 minutes on the elliptical and 30 minutes of circuit training burned 473 calories
	$20e + 30c = 473$
The system is:	$\begin{aligned} 10e + 20c &= 278 \\ 20e + 30c &= 473 \end{aligned}$
Step 5. Solve the system of equations.	
Multiply the first equation by −2 to get opposite coefficients of e.	$\begin{aligned} -2(10e + 20c) &= -2(278) \\ 20e + 30c &= 473 \end{aligned}$
Simplify and add the equations. Solve for c.	$\begin{aligned} -20e - 40c &= -556 \\ 20e + 30c &= 473 \\ \hline -10c &= -83 \\ c &= 8.3 \end{aligned}$
Substitute $c = 8.3$ into one of the original equations to solve for e.	$10e + 20c = 278$
	$10e + 20(8.3) = 278$
	$10e + 166 = 278$
	$10e = 112$
	$e = 11.2$
Step 6. Check the answer in the problem.	Check the math on your own.
$\begin{aligned} 10(11.2) + 20(8.3) &\stackrel{?}{=} 278 \\ 20(11.2) + 30(8.3) &\stackrel{?}{=} 473 \end{aligned}$	
Step 7. Answer the question.	Jenna burns 8.3 calories per minute circuit training and 11.2 calories per minute while on the elliptical trainer.

Try It 3

Translate to a system of equations and then solve:

Erin spent 30 minutes on the rowing machine and 20 minutes lifting weights at the gym, burning a total of 430 calories. During her next visit to the gym, she spent 50 minutes on the rowing machine and 10 minutes lifting weights, burning a total of 600 calories. How many calories did she burn for each minute on the rowing machine? How many calories did she burn for each minute of weight lifting?

Answer

Erin burned 11 calories for each minute on the rowing machine and 5 calories for each minute of weight lifting.

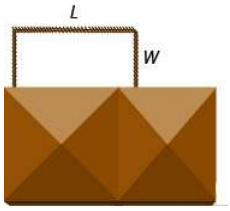
Many real-world problems involve relationships between quantities, like length and width, that can be described using variables and equations. By translating these situations into algebraic expressions, we can build and solve equations—or even systems of equations—to find unknown values that make sense in context.

Example 4

Translate to a system of equations and then solve:

Randall has 125 feet of fencing to enclose the rectangular part of his backyard adjacent to his house. He will only need to fence around three sides, because the fourth side will be the wall of the house. He wants the length of the fenced yard (parallel to the house wall) to be 5 feet more than four times the width. Find the length and the width.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for the length and width.
	
Step 3. Name what we are looking for.	Let L be the length of the fenced yard and W the width of the fenced yard.
Step 4. Translate into a system of equations.	One length and two widths equal 125.
	$L + 2W = 125$
	The length will be 5 feet more than four times the width.
	$L = 4W + 5$
The system is: **Step 5. Solve** the system of equations by substitution.	$L + 2W = 125$ $L = 4W + 5$ $L + 2W = 125$
Substitute $L = 4W + 5$ into the first equation, then solve for W.	$4W + 5 + 2W = 125$
	$6W + 5 = 125$
	$6W = 120$
Substitute 20 for W in the second equation, then solve for L.	$W = 20$ $L = 4W + 5$
	$L = 4 \cdot 20 + 5$
	$L = 80 + 5$
	$L = 85$
Step 6. Check the answer in the problem. $20 + 85 + 20 = 125$ ✓ $85 = 4 \cdot 20 + 5$ ✓	
Step 7. Answer the equation.	The length is 85 feet and the width is 20 feet.

Try It 4

Translate to a system of equations and then solve:

Alexis wants to build a rectangular dog run in her yard adjacent to her neighbor's fence. She will use 136 feet of fencing to completely enclose the rectangular dog run. The length of the dog run along the neighbor's fence will be 16 feet less than twice the width. Find the length and width of the dog run.

Answer

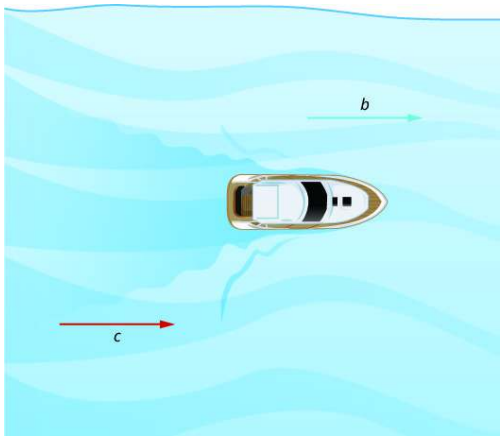
The length is 60 feet and the width is 38 feet.

Many real-world applications of uniform motion arise because of the effects of currents—of water or air—on a vehicle's actual speed. Cross-country airplane flights in the United States generally take longer to travel west than to travel east due to the prevailing wind currents.

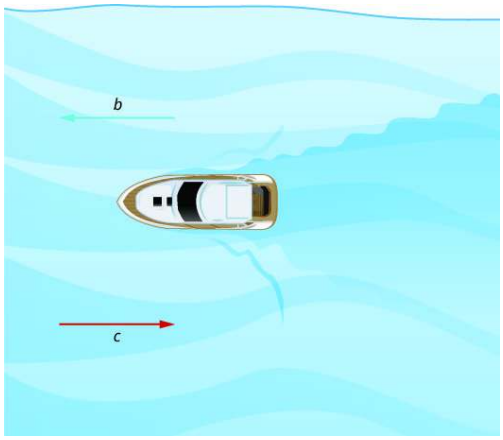
Let’s consider a boat traveling on a river. Depending on the direction the boat is traveling, the water current either slows it down or speeds it up.

The figures below show how a river current affects a boat's actual speed. We’ll call the speed of the boat in still water b and the speed of the river current c.

In the first figure, the boat is going downstream, in the same direction as the river current. The current helps push the boat, so its actual speed is faster than its speed in still water. The actual speed at which the boat is moving is $b+c$.



In the second figure, the boat travels upstream against the river's current. The current is going against the boat, so its actual speed is slower than its speed in still water. The actual speed of the boat is $b-c$.



We’ll put some numbers to this situation in the next example.

Example 5

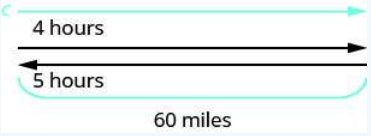
Translate to a system of equations and then solve:

A river cruise ship sailed 60 miles downstream for 4 hours and then took 5 hours sailing upstream to return to the dock. Find the speed of the ship in still water and the speed of the river current.

Solution

Read the problem.

This is a uniform motion problem, and a picture will help us visualize the situation.



Identify what we are looking for.	We are looking for the speed of the ship in still water and the speed of the current.
Name what we are looking for.	Let s be the rate of the ship in still water and c the rate of the current.
A chart will help us organize the information. The ship goes downstream and then upstream. Going downstream, the current helps the ship; therefore, the ship's actual rate is $s+c$. Going upstream, the current slows the ship; therefore, the actual rate is $s-c$.	

	Rate	• Time =	Distance
downstream	$s+c$	4	60
upstream	$s-c$	5	60

Downstream it takes 4 hours. Upstream it takes 5 hours. Each way, the distance is 60 miles.	
Translate into a system of equations. Since rate times time is distance, we can write the system of equations.	$\begin{cases} 4(s+c)=60 \\ 5(s-c)=60 \end{cases}$
Solve the system of equations. Distribute to put both equations in standard form, then solve by elimination.	$\begin{cases} 4s+4c=60 \\ 5s-5c=60 \end{cases}$
Multiply the top equation by 5 and the bottom equation by 4. Add the equations, then solve for s.	$\begin{array}{rl} 20s+20c=300 & \\ 20s-20c=240 & \\ \hline 40s & =540 \end{array}$
Substitute $s=13.5$ into one of the original equations.	$s=13.5$ $4(s+c)=60$
	$4(13.5+c)=60$
	$54+4c=60$
	$4c=6$
	$c=1.5$
Check the answer in the problem. The downstream rate would be 13.5 + 1.5 = 15 mph. In 4 hours, the ship would travel 15 · 4 = 60 miles. The upstream rate would be 13.5 − 1.5 = 12 mph. In 5 hours, the ship would travel 12 · 5 = 60 miles.	
Answer the question.	The rate of the ship is 13.5 mph, and the rate of the current is 1.5 mph.

Try It 5

Translate to a system of equations and then solve: Jason paddled his canoe 24 miles upstream for 4 hours. It took him 3 hours to paddle back. Find the speed of the canoe in still water and the speed of the river current.

Answer

The canoe's speed is 7 mph, and the speed of the current is 1 mph.

When we solved mixture applications with coins and tickets earlier, we began by creating a table to organize the information. For a coin example with nickels and dimes, the table looked like this:

Type	Number	• Value($)	= Total Value($)
nickels		0.05	
dimes		0.10	

Using one variable meant that we had to relate the number of nickels to the number of dimes. We had to decide whether to let n be the number of nickels and then express the number of dimes in terms of n, or let d be the number of dimes and express the number of nickels in terms of d.

Now that we know how to solve systems of equations with two variables, we'll just let n be the number of nickels and d be the number of dimes. We'll write one equation based on the total value column, like we did before, and the other equation will come from the number column.

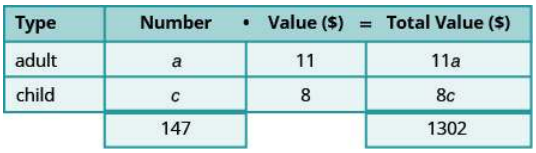
For the first example, we'll solve a ticket problem with ticket prices in whole dollars, so we won't need to use decimals just yet.

Example 6

Translate to a system of equations and solve:

The box office at a movie theater sold 147 tickets for the evening show, and receipts totaled $1,302. How many $11 adult tickets and how many $8 child tickets were sold?

Solution

Step 1. Read the problem.	We will create a table to organize the information.
Step 2. Identify what we are looking for.	We are looking for the number of adult tickets and the number of child tickets sold.
Step 3. Name what we are looking for.	Let a be the number of adult tickets and c the number of child tickets.
A table will help us organize the data. We have two types of tickets: adult and child.	Write a and c for the number of tickets.
Write the total number of tickets sold at the bottom of the Number column.	Altogether 147 were sold.
Write the value of each type of ticket in the Value column.	The value of each adult ticket is \$11. The value of each child tickets is \$8.
The number times the value gives the total value, so the total value of adult tickets is $a \cdot 11 = 11a$, and the total value of child tickets is $c \cdot 8 = 8c$.	
Altogether the total value of the tickets was \$1,302.	Fill in the Total Value column.
Step 4. Translate into a system of equations.	
The Number column and the Total Value column give us the system of equations. We will use the elimination method to solve this system.	$a + c = 147$ $11a + 8c = 1302$
Multiply the first equation by -8.	$-8(a + c) = -8(147)$ $11a + 8c = 1302$
Simplify and add, then solve for a.	$-8a - 8c = -1176$ $11a + 8c = 1302$ $3a = 126$
	$a = 42$
Substitute $a = 42$ into the first equation, then solve for c.	$42 + c = 147$
	$c = 105$
Step 5. Check the answer in the problem. 42 adult tickets at \$11 per ticket makes \$462 105 child tickets at \$8 per ticket makes \$840. The total receipts are \$1,302.✓	
Step 6. Answer the question.	The movie theater sold 42 adult tickets and 105 child tickets.

Try It 6

Translate to a system of equations and solve:

The zoo's ticket office sold 553 tickets in one day, totaling $3,936. How many $9 adult tickets and how many $6 child tickets were sold?

Answer

A total of 206 adult tickets and 347 children's tickets were sold.

Some mixture applications involve combining foods or drinks. Example situations might include combining raisins and nuts to make a trail mix or using two types of coffee beans to make a blend.

Example 7

Translate to a system of equations and solve:

Carson wants to make 20 pounds of trail mix using nuts and chocolate chips. His budget requires that the trail mix cost him $7.60 per pound. Nuts cost $9.00 per pound, and chocolate chips cost $2.00 per pound. How many pounds of nuts and how many pounds of chocolate chips should he use?

Solution

Step 1. Read the problem.	We will create a table to organize the information.
Step 2. Identify what we are looking for.	We are looking for the number of pounds of nuts and the number of pounds of chocolate chips.
Step 3. Name what we are looking for.	Let n be the number of pounds of nuts and c the number of pounds of chips.
Carson will mix nuts and chocolate chips to get trail mix. Write in n and c for the number of pounds of nuts and chocolate chips. There will be 20 pounds of trail mix. Put the price per pound of each item in the Value column. Fill in the last column using	

Type	Number of pounds	• Value ($)	= Total Value ($)
nuts	n	9.00	$9n$
chocolate chips	c	2.00	$2c$
trail mix	20	7.60	$7.60(20) = 152$

Number · Value = Total Value	
Step 4. Translate into a system of equations. We get the equations from the Number and Total Value columns.	$\begin{cases} n + c = 20 \\ 9n + 2c = 152 \end{cases}$
Step 5. Solve the system of equations We will use elimination to solve the system.	
Multiply the first equation by -2 to eliminate c.	$\begin{cases} -2(n + c) = -2(20) \\ 9n + 2c = 152 \end{cases}$
Simplify and add. Solve for /(n/).	$\begin{array}{r} -2n - 2c = -40 \\ 9n + 2c = 152 \\ \hline 7n \quad\quad = 112 \end{array}$
	$n = 16$
To find the number of pounds of chocolate chips, substitute $n = 16$ into the first equation, then solve for c.	$n + c = 20$ $16 + c = 20$
	$c = 4$
Step 6. Check the answer in the problem. $16 + 4 = 20$✓ $9 \cdot 16 + 2 \cdot 4 = 152$✓	
Step 7. Answer the question.	Carson should mix 16 pounds of nuts with 4 pounds of chocolate chips to create the trail mix.

? Try It 7

Translate to a system of equations and solve:

Sammy has most of the ingredients he needs to make a large batch of chili. The only items he lacks are beans and ground beef. He needs a total of 20 pounds of beans and ground beef, and he has a budget of $3 per pound. The price of beans is $1 per pound, and the price of ground beef is $5 per pound. How many pounds of beans and how many pounds of ground beef should he purchase?

Answer

Sammy should purchase 10 pounds of beans and 10 pounds of ground beef.

Another application of mixture problems relates to concentrated cleaning supplies, other chemicals, and mixed drinks. The concentration is given as a percent. For example, a 20% concentrated household cleanser means that 20% of the total amount is cleanser, and the rest is water. To make 35 ounces of a 20% concentration, you mix 7 ounces (20% of 35) of the cleanser with 28 ounces of water.

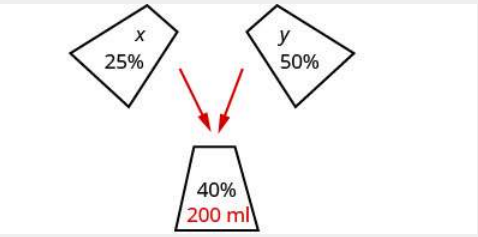
For these kinds of mixture problems, we'll use percent instead of value for one of the columns in our table.

Example 8

Translate to a system of equations and solve:

Sasheena is a lab assistant at her community college. She needs to make 200 milliliters of a 40% solution of sulfuric acid for a lab experiment. The lab has only 25% and 50% solutions in the storeroom. How much of the 25% and the 50% solutions should she mix to make the 40% solution?

Solution

Step 1. Read the problem.	A figure may help us visualize the situation, then we will create a table to organize the information.
Sasheena must mix some of the 25% solution and some of the 50% solution together to get 200 mL of the 40% solution.	
Step 2. Identify what we are looking for.	We are looking for how much of each solution she needs.
Step 3. Name what we are looking for.	Let x be mL of 25% solution and y mL of 50% solution.
A table will help us organize the data. She will mix x mL of 25% with y mL of 50% to get 200 mL of 40% solution. We write the percents as decimals in the chart. We multiply the number of units times the concentration to get the total amount of sulfuric acid in each solution.	

Type	Number of units	• Concentration %	= Amount
25%	x	0.25	$0.25x$
50%	y	0.50	$0.50y$
40%	200	0.40	$0.40(200)$

Step 4. Translate into a system of equations. We get the equations from the Number column and the Amount column.	
Now we have the system.	$\begin{cases} x+y=200 \\ 0.25x+0.50y=0.40(200) \end{cases}$
Step 5. Solve the system of equations. We will solve the system using the elimination method. Multiply the first equation by −0.5 to eliminate y.	$\begin{cases} -0.5(x+y)=-0.5(200) \\ 0.25x+0.50y=80 \end{cases}$
Simplify and add to solve for x.	$\begin{cases} -0.5x-0.5y=-100 \\ 0.25x+0.5y=80 \end{cases}$ $-0.25x = -20$ $x = 80$
To solve for y, substitute $x = 80$ into the first equation.	$x+y=200$
	$80+y=200$
	$y=120$
Step 6. Check the answer in the problem.	$80+120 = 120\checkmark$ $0.25(80)+0.50(120) = 80\checkmark$ Yes!

Step 7. Answer the question.	Sasheena should mix 80 mL of the 25% solution with 120 mL of the 50% solution to get the 200 ml of the 40% solution.

? Try It 8

Translate to a system of equations and solve:

Anatole needs to make 250 milliliters of a 25% solution of hydrochloric acid for a lab experiment. The lab only has 10% and 40% solutions in the storeroom. How much of the 10% and how much of the 40% solutions should he mix to make the 25% solution?

Answer

Anatole should mix 125 mL of the 10% solution with 125 mL of the 40% solution.

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Section 4.4P: Practice

Practice Makes Progress

Translate to a System of Equations

In the following exercises, translate to a system of equations and solve the system.

Exercise 1

A married couple together earns $75,000. One husband earns $15,000 more than five times what the other husband earns. How much does the lower-earning husband make?

Exercise 2

Daniela invested a total of $50,000, some in a certificate of deposit (CD) and the remainder in bonds. The amount invested in bonds was $5000, more than twice the amount she put into the CD. How much did she invest in each account?

Exercise 3

In her last two years in college, Marlene received $42,000 in loans. The first year, she received a loan that was $6,000 less than three times the amount of the second year's loan. What was the amount of her loan for each year?

Exercise 4

Alyssa is twelve years older than her sister, Bethany. The sum of their ages is forty-four. Find their ages.

Exercise 5

The age of Noelle's dad is six less than three times Noelle's age. The sum of their ages is seventy-four. Find their ages.

Exercise 6

Two containers of gasoline hold a total of fifty gallons. The big container can hold ten gallons less than twice the small container. How many gallons does each container hold?

Exercise 7

Shelly spent 10 minutes jogging and 20 minutes cycling and burned 300 calories. The next day, Shelly swapped times, doing 20 minutes of jogging and 10 minutes of cycling, and burned the same number of calories. How many calories were burned for each minute of jogging and how many for each minute of cycling?

Exercise 8

Troy and Lisa were shopping for school supplies. Each purchased different quantities of the same notebook and thumb drive. Troy bought four notebooks and five thumb drives for $116. Lisa bought two notebooks and three thumb drives for $68. Find the cost of each notebook and each thumb drive.

Exercise 9

Wayne is hanging a string of lights 45 feet long around the three sides of his rectangular patio, which is adjacent to his house. The length of his patio, the side along the house, is five feet longer than twice its width. Find the length and width of the patio.

Exercise 10

A frame around a rectangular family portrait has a perimeter of 60 inches. The length is fifteen less than twice the width. Find the length and width of the frame.

Exercise 11

Sarah left Minneapolis heading east on the interstate at a speed of 60 mph. Her sister followed her on the same route, leaving two hours later and driving at a rate of 70 mph. How long will it take for Sarah's sister to catch up to Sarah?

Exercise 12

At the end of spring break, Lucy left the beach and drove back towards home, driving at a rate of 40 mph. Lucy's friend left the beach for home 30 minutes (half an hour) later and drove 50 mph. How long did it take Lucy's friend to catch up to Lucy?

Exercise 13

The Jones family took a 12-mile canoe ride down the Indian River in two hours. After lunch, the return trip up the river took three hours. Find the rate of the canoe in still water and the rate of the current.

Exercise 14

A motorboat traveled 18 miles down a river in two hours, but going back upstream, it took 4.5 hours due to the current. Find the rate of the motor boat in still water and the rate of the current.

Exercise 15

A small jet can fly 1,072 miles in 4 hours with a tailwind but only 848 miles in 4 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Exercise 16

A commercial jet can fly 868 miles in 2 hours with a tailwind, but only 792 miles in 2 hours into a headwind. Find the speed of the jet in still air and the speed of the wind.

Exercise 17

Tickets to a Broadway show cost $35 for adults and $15 for children. The total receipts for 1650 tickets at one performance were $47,150. How many adult and how many child tickets were sold?

Exercise 18

Tickets for a train cost $10 for children and $22 for adults. Josie paid $1,200 for a total of 72 tickets. How many children's tickets and how many adult tickets did Josie buy?

Exercise 19

Tickets for a dance recital cost $15 for adults and $7 for children. The dance company sold 253 tickets, and the total receipts were $2,771. How many adult tickets and how many child tickets were sold?

Exercise 20

Marissa wants to blend candy selling for $1.80 per pound with candy costing $1.20 per pound to get a mixture that costs her $1.40 per pound to make. She wants to make 90 pounds of the candy blend. How many pounds of each type of candy should she use?

Exercise 21

Hannah has to make twenty-five gallons of punch for a potluck. The punch is made of soda and a fruit drink. The cost of the soda is $1.79 per gallon, and the cost of the fruit drink is $2.49 per gallon. Hannah's budget requires that the punch cost $2.21 per gallon. How many gallons of soda and how many gallons of fruit drink does she need?

Exercise 22

Julia and her husband own a coffee shop. They experimented with mixing a City Roast Colombian coffee that cost $7.80 per pound with French Roast Colombian coffee that cost $8.10 per pound to make a 20-pound blend. Their blend should cost them $7.92 per pound. How much of each type of coffee should they buy?

Exercise 23

Jotham needs 70 liters of a 50% alcohol solution. He has a 30% and an 80% solution available. How many liters of the 30% solution and how many liters of the 80% solution should he mix to make the 50% solution?

Exercise 24

A scientist needs 65 liters of a 15% alcohol solution. She has available a 25% and a 12% solution. How many liters of the 25% solution and how many liters of the 12% solution should she mix to make the 15% solution?

Exercise 25

A 40% antifreeze solution is to be mixed with a 70% antifreeze solution to get 240 liters of a 50% solution. How many liters of the 40% and how many liters of the 70% solution will be used?

Everyday Math

Exercise 26

Laurie was completing the treasurer's report for her son's Boy Scout troop at the end of the school year. She didn't remember how many boys had paid the $15 full-year registration fee and how many had paid the $10 partial-year fee. She knew that the number of boys who paid for a whole year was ten more than the number who paid for a partial year. If $250 was collected for all the registrations, how many boys had paid the full-year fee, and how many had paid the partial-year fee?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
translate to a system of equations			
solve direct translation applications			
solve geometry applications			
solve uniform motion applications			
solve mixture applications			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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Section 4.5: Graphing Systems of Linear Inequalities

Be Prepared

We will rely heavily on these skills throughout this section.

- Graph $x > 2$ on a number line
- Determine whether $1, 1$ is a solution to $y > x - 2$
- Solve $2a < 5a + 12$
- Determine whether $(3, \frac{1}{2})$ is a solution to the system $\begin{cases} x + 2y = 4 \\ y = 6x \end{cases}$.

Learning Objectives

Motivating Problem

You're planning a school event and have a budget of $500. You're choosing between two options:

- Renting tables: $25 each
- Renting chairs: $10 each

You need at least 10 tables, but you want to see all the combinations of tables and chairs that will keep you within budget.

Fun Fact

The idea of using shaded regions to represent solutions dates back to the 1800s, when economists began using graphs to illustrate constraints on supply and demand, long before the advent of spreadsheets or apps.

The Goal

In this section, we will learn how to graph two separate 2-variable linear inequalities by plotting the boundary line, determining whether to use a solid or dashed line, and shading the solution region.

Determine Whether an Ordered Pair is a Solution of a System of Linear Inequalities

The definition of a system of linear inequalities is similar to that of a system of linear equations.

SYSTEM OF LINEAR INEQUALITIES

Two or more linear inequalities grouped together form a **system of linear inequalities**.

A system of linear inequalities resembles a system of linear equations, but it contains inequalities instead of equations. A system of two linear inequalities is shown below.

$$\begin{cases} x + 4y \geq 10 \\ 3x - 2y < 12 \end{cases}$$

To solve a system of linear inequalities, we will find values of the variables that are solutions to both inequalities. We solve the system by using the graphs of each inequality and show the solution as a graph. We will find the region on the plane that contains all ordered pairs (x, y) that make *both* inequalities true.

SolutionS OF A SYSTEM OF LINEAR INEQUALITIES

Solutions of a system of linear inequalities are the values of the variables that make all the inequalities true.

The solution of a system of linear inequalities is represented as a shaded region in the x-y coordinate system, encompassing all points whose ordered pairs satisfy the inequalities.

To determine if an ordered pair is a solution to a system of two inequalities, we substitute the values of the variables into each inequality. If the ordered pair satisfies both inequalities, it is a solution to the system.

Example 1

Determine whether the ordered pair is a solution to the system. $\begin{cases} x+4y \geq 10 \\ 3x-2y < 12 \end{cases}$

1. $(-2, 4)$
2. $(3, 1)$

Solution

1. Is the ordered pair $(-2, 4)$ a solution?

We substitute $x = -2$ and $y = 4$ into both inequalities.

$x+4y \geq 10$	$3x-2y < 12$
$-2+4(4) \stackrel{?}{\geq} 10$	$3(-2)-2(4) \stackrel{?}{<} 12$
$14 \geq 10$ true	$-14 < 12$ true

The ordered pair $(-2, 4)$ satisfies both inequalities. Therefore, $(-2, 4)$ is a solution to this system.

2. Is the ordered pair $(3, 1)$ a solution?

We substitute $x = 3$ and $y = 1$ into both inequalities.

$x+4y \geq 10$	$3x-2y < 12$
$3+4(1) \stackrel{?}{\geq} 10$	$3(3)-2(1) \stackrel{?}{<} 12$
$7 \geq 10$ false	$7 < 12$ true

The ordered pair $(3, 1)$ made one inequality true, but the other one false. Therefore, $(3, 1)$ is not a solution to this system.

Try It 1

Determine whether the ordered pair is a solution to the system. $\begin{cases} x-5y > 10 \\ 2x+3y > -2 \end{cases}$

a. $(3, -1)$
b. $(6, -3)$

Answer

a. no
b. yes

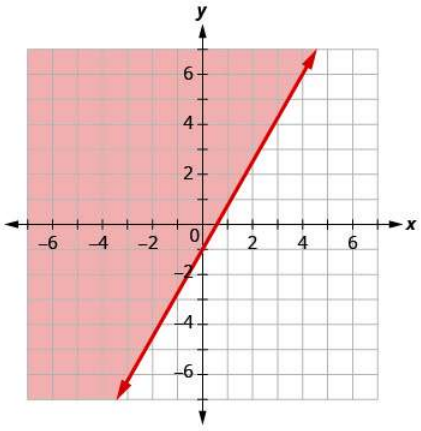
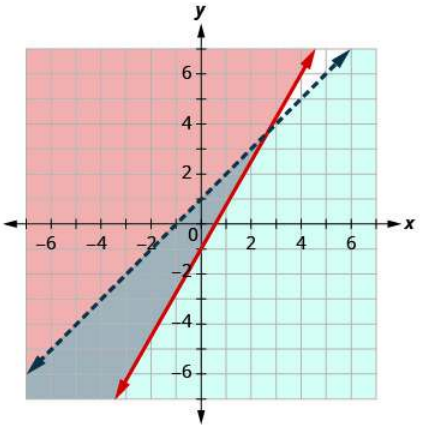
Solve a System of Linear Inequalities by Graphing

The solution to a single linear inequality is the region on one side of the boundary line that contains all the points that make the inequality true. The solution to a system of two linear inequalities is a region that contains the solutions to both inequalities. We will graph each inequality separately to find this region and then locate the region where both are true. The solution is always shown as a graph.

Example 2

Solve the system by graphing. $\begin{cases} y \geq 2x-1 \\ y < x+1 \end{cases}$

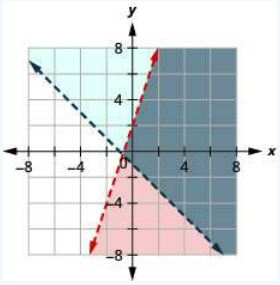
Solution

Step 1. Graph the first inequality. Graph the boundary line. Shade in the side of the boundary line where the inequality is true.	We will graph $y \geq 2x - 1$. We graph the line $y = 2x - 1$. It is a solid line because the inequality sign is $\geq$. We choose (0,0) as a test point. It is a solution to $y \geq 2x - 1$, so we shade in the left side of the boundary line.	$\begin{cases} y \geq 2x - 1 \\ y < x + 1 \end{cases}$ 
Step 2. On the same grid, graph the second inequality. Graph the boundary line. Shade in the side of that boundary line where the inequality is true.	We will graph $y < x + 1$ on the same grid. We graph the line $y = x + 1$. It is a dashed line because the inequality sign is $<$. Again, we use (0,0) as a test point. It is a solution so we shade in that side of the line $y = x + 1$.	
Step 3. The solution is the region where the shading overlaps.	The point where the boundary lines intersect is not a solution because it is not a solution to $y < x + 1$.	The solution is all points in the darker shaded region.
Step 4. Check by choosing a test point.	We'll use (–1, –1) as a test point.	Is (–1, –1) a solution to $y \geq 2x - 1$? $-1 \stackrel{?}{\geq} 2(-1) - 1$ $-1 \geq -3$ true Is (–1, –1) a solution to $y < x + 1$? $-1 \stackrel{?}{<} -1 + 1$ $-1 < 0$ true The region containing (–1, –1) is the solution to this system.

? Try It 2

Solve the system by graphing. $\begin{cases} y < 3x + 2 \\ y > -x - 1 \end{cases}$

Answer



SOLVE A SYSTEM OF LINEAR INEQUALITIES BY GRAPHING.

1. Graph the first inequality.
   - Graph the boundary line.
   - Shade the side of the boundary line where the inequality is true.
2. On the same grid, graph the second inequality.
   - Graph the boundary line.
   - Shade in the side of that boundary line where the inequality is true.
3. The solution is the region where the shading overlaps.
4. Check by choosing a test point.

Example 3

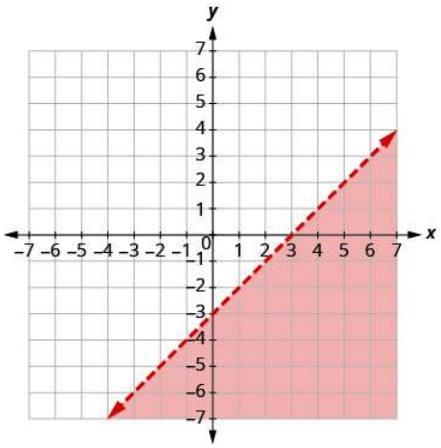
Solve the system by graphing. $\begin{cases} x - y > 3 \\ y < -\frac{1}{5}x + 4 \end{cases}$

Solution

Graph $x - y > 3$, by graphing $x - y = 3$ and testing a point.

The intercepts are $x = 3$ and $y = -3$ and the boundary line will be dashed.

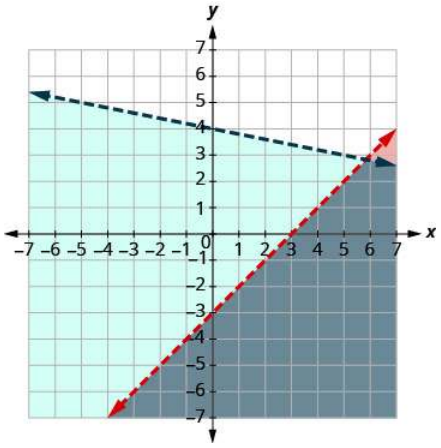
Test $(0, 0)$. It makes the inequality false. So, shade the side that does not contain $(0, 0)$ red.



Graph $y < -15x + 4$ by graphing $y = -15x + 4$ using the slope $m = -15$ and y-intercept $b = 4$. The boundary line will be dashed.

Test $(0, 0)$. It makes the inequality true, so shade the side that contains $(0, 0)$ blue.

Select a test point in the solution and verify that it satisfies both inequalities.

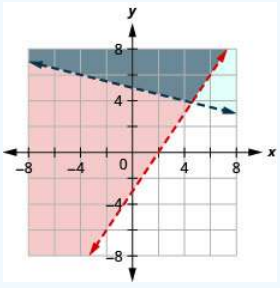


The point of intersection of the two lines is not included, as both boundary lines were dashed. The solution is the area shaded twice, which is the darker-shaded region.

? Try It 3

Solve the system by graphing. $\begin{cases} 3x - 2y \leq 6 \\ y > -\frac{1}{4}x + 5 \end{cases}$

Answer



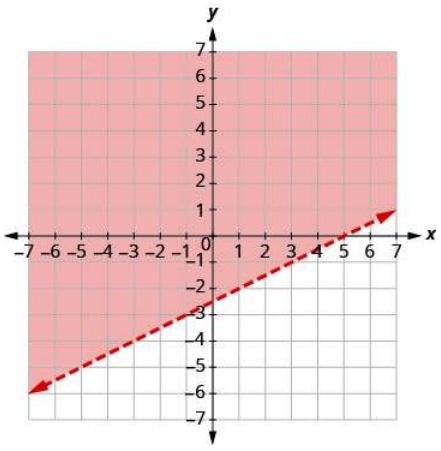
Example 4

Solve the system by graphing. $\begin{cases} x-2y<5 \\ y>-4 \end{cases}$

Solution

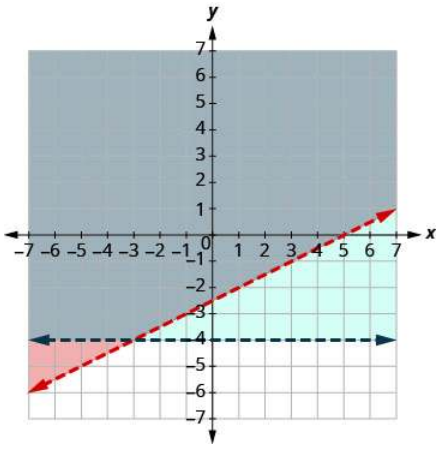
Graph $x-2y<5$, by graphing $x-2y=5$ and testing a point. The intercepts are $x=5$ and $y=-2.5$, and the boundary line will be dashed.

Test $(0,0)$. It makes the inequality true. So, shade the side that contains $(0,0)$ red.



Graph $y>-4$, by graphing $y=-4$ and recognizing that it is a horizontal line through $y=-4$. The boundary line will be dashed.

Test $(0,0)$. It makes the inequality true. So, shade (blue) the side that contains $(0,0)$ blue.



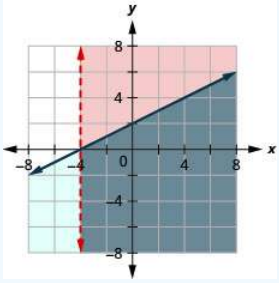
The point $(0,0)$ is in the solution, and we have already found it to be a solution of each inequality. The point of intersection of the two lines is not included, as both boundary lines were dashed.

The solution is the area shaded twice, which is the darker-shaded region.

Try It 4

Solve the system by graphing. $\begin{cases} x>-4 \\ x-2y\leq -4 \end{cases}$

Answer



Systems of linear inequalities with parallel boundary lines might have no solution. This will be illustrated in the following example.

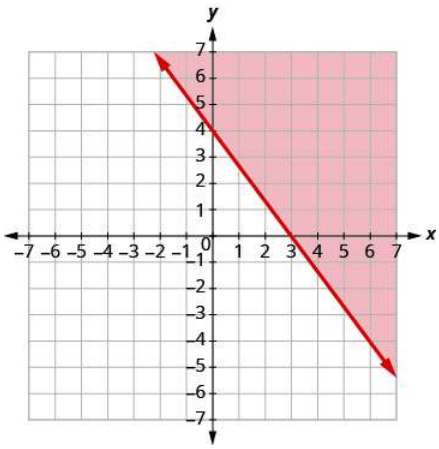
Example 5

Solve the system by graphing. $\begin{cases} 4x+3y \geq 12 \\ y < -\frac{4}{3}x+1 \end{cases}$

Solution

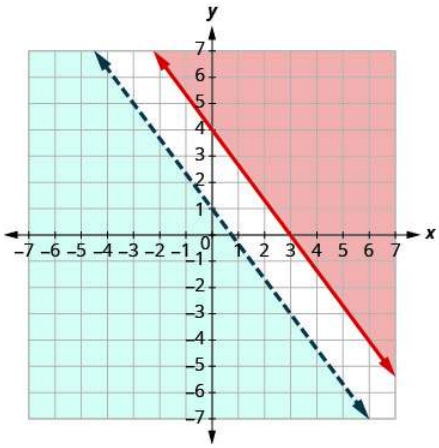
Graph $4x+3y \geq 12$, by graphing $4x+3y=12$ and testing a point.
The intercepts are $x=3$ and $y=4$ and the boundary line will be solid.

Test $(0,0)$. It makes the inequality false. So,
shade the side that does not contain $(0,0)$ red.



Graph $y < -\frac{4}{3}x+1$ by graphing $y=-\frac{4}{3}x+1$ using the slope $m=\frac{4}{3}$ and the y-intercept $b=1$. The boundary line will be dashed.

Test $(0,0)$. It makes the inequality true. So,
shade the side that contains $(0,0)$ blue.



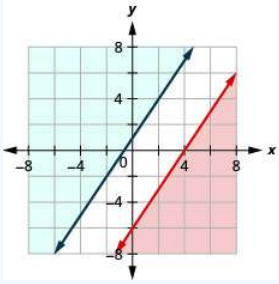
There is no point of intersection between the two shaded regions, so the system has no solution. This system has no solution.

Try It 5

Solve the system by graphing. $\begin{cases} 3x-2y \leq 12 \\ y \geq \frac{3}{2}x+1 \end{cases}$

Answer

no solution



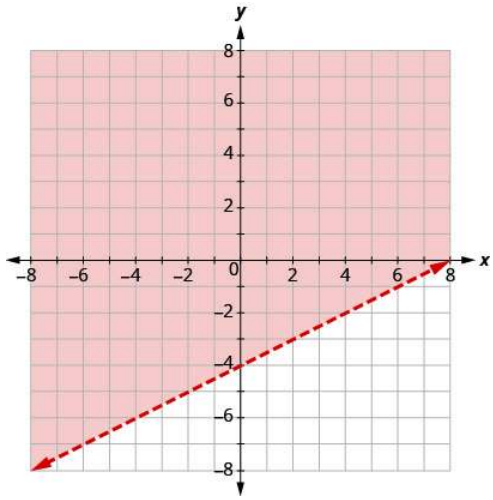
However, as the following example will show, sometimes linear inequalities with parallel boundary lines have infinite solutions.

Example 6

Solve the system by graphing. $\begin{cases} y > \frac{1}{2}x - 4 \\ x - 2y < -4 \end{cases}$

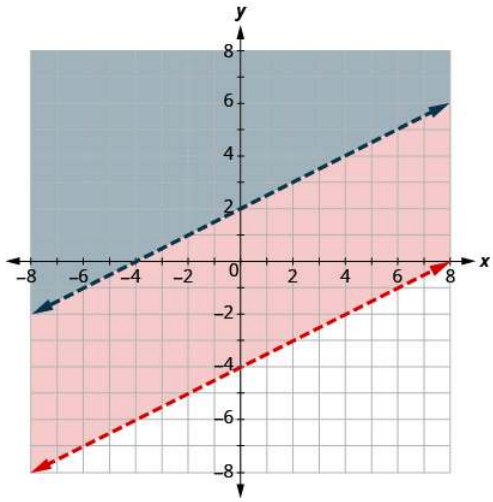
Solution

Graph $y > \frac{1}{2}x - 4$ by graphing $y = \frac{1}{2}x - 4$ using the slope $m = \frac{1}{2}$ and the intercept $b = -4$. The boundary line will be dashed. Test $(0, 0)$. It makes the inequality true. So, shade the side that contains $(0, 0)$ red.



Graph $x - 2y < -4$ by graphing $x - 2y = -4$ and testing a point. The intercepts are $x = -4$ and $y = 2$, and the boundary line will be dashed.

Choose a test point in the solution and verify that it is a solution to both inequalities.



No point on the boundary lines is included in the solution as both lines are dashed.

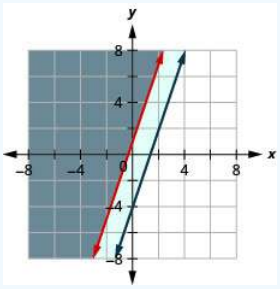
The solution is the region that is shaded twice, which is also the solution to $x - 2y < -4$.

? Try It 6

Solve the system by graphing. $\begin{cases} y \geq 3x + 1 \\ -3x + y \geq -4 \end{cases}$

Answer

$y \geq 3x + 1$



Solve Applications of Systems of Inequalities

To solve applications of systems of inequalities, we first need to translate each condition into an inequality. Then, we graph the system as we did above to see the region that contains the solutions. Many situations will be realistic only if both variables are positive, so their graphs show only Quadrant I.

✓ Example 7

Christy sells her photographs at a booth at a street fair. At the start of the day, she wants to have at least 25 photos to display at her booth. Each small photo she displays costs her $4, and each large photo costs her $10. She doesn't want to spend more than $200 on photos to display.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could she display 15 small and 5 large photos?
4. Could she display 3 large and 22 small photos?

Solution

1.Let x be the number of small photos and y the number of large photos.

To find the system of inequalities, translate the information.

She wants to have at least 25 photos.

The number of small plus the number of large should be at least 25.

$$x + y \geq 25$$

$4 for each small and $10 for each large must be no more than $200

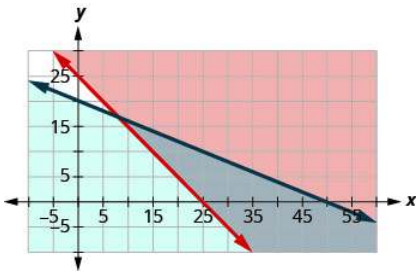
$$4x + 10y \leq 200$$

We have our system of inequalities. $\begin{cases} x + y \geq 25 \\ 4x + 10y \leq 200 \end{cases}$

2.

To graph $x + y \geq 25$, graph $x + y = 25$ as a solid line.
Choose $(0, 0)$ as a test point. Since it does not make the inequality
true, shade the side that does not include the point $(0, 0)$ red.

To graph $4x + 10y \leq 200$, graph $4x + 10y = 200$ as a solid line.
Choose $(0, 0)$ as a test point. Since it does not make the inequality
true, shade the side that includes the point $(0, 0)$ blue.



The solution of the system is the region of the graph that is double shaded and therefore is shaded darker.

3. To determine if 10 small and 20 large photos would work, we see if the point $(10, 20)$ is in the solution region. It is not. Christy would not display 10 small and 20 large photos.

4. To determine if 20 small and 10 large photos would work, we see if the point $(20, 10)$ is in the solution region. It is. Christy could choose to display 20 small and 10 large photos.

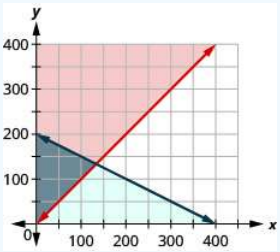
Notice that we could also test the possible solutions by substituting the values into each inequality.

Try It 7

Mary needs to purchase supplies of answer sheets and pencils for a standardized test to be given to the juniors at her high school. The number of answer sheets needed is at least five more than the number of pencils. The pencils cost $2 and the answer sheets cost $1. Mary's budget for these supplies allows for a maximum cost of $400.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could Mary purchase 100 pencils and 100 answer sheets?
4. Could Mary purchase 150 pencils and 150 answer sheets?

Answer

1. $\begin{cases} a \geq p + 5 \\ a + 2p \leq 400 \end{cases}$
2. 

3. no
4. no

Example 8

Omar needs to eat at least 800 calories before going to his team practice. All he wants is hamburgers and cookies, and he doesn't want to spend more than $5. At the hamburger restaurant near his college, each hamburger has 240 calories and costs $1.40. Each cookie has 160 calories and costs $0.50.

1. Write a system of inequalities to model this situation.
2. Graph the system.
3. Could he eat one cookie and three hamburgers?
4. Could he eat four cookies and two hamburgers?

Solution

1.

Let h be the number of hamburgers and c the number of cookies
To find the system of inequalities, translate the information.
The calories from hamburgers at 240 calories each, plus the calories from cookies at 160 calories each, must be more than 800.

$$240h + 160c \geq 800$$

The amount spent on hamburgers at \$1.40 each, plus the amount spent on cookies at \$0.50 each, must be no more than \$5.00.

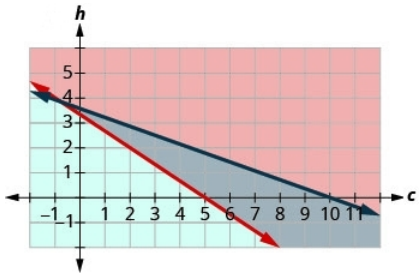
$$1.40h + 0.50c \leq 5$$

We have our system of inequalities. $\begin{cases} 240h + 160c \geq 800 \\ 1.40h + 0.50c \leq 5 \end{cases}$

2.

To graph $240h + 160c \geq 800$ graph $240h + 160c = 800$ as a solid line.
Choose $(0, 0)$ as a test point. it does not make the inequality true.
So, shade (red) the side that does not include the point $(0, 0)$.

To graph $1.40h + 0.50c \leq 5$, graph $1.40h + 0.50c = 5$ as a solid line.
Choose $(0, 0)$ as a test point. It makes the inequality true. So, shade
(blue) the side that includes the point.



The solution of the system is the region of the graph that is double shaded and so is shaded darker.

3. To determine if 1 cookie and 3 hamburgers would meet Omar's criteria, we see if the point $(1, 3)$ is in the solution region. It is not.
4. To determine if 4 cookies and 2 hamburgers would meet Omar's criteria, we see if the point $(4, 2)$ is in the solution region. It is. He might choose to eat 4 cookies and 2 hamburgers.

We could also test the possible solutions by substituting the values into each inequality.

Try It 8

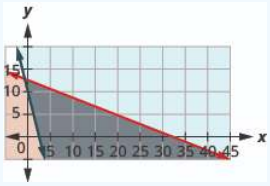
Tension needs to eat at least an extra 1,000 calories a day to prepare for running a marathon. He has only \$25 to spend on the extra food he needs and will spend it on \$0.75 donuts, which have 360 calories each, and \$2 energy drinks, which have 110 calories.

1. Write a system of inequalities that models this situation.
2. Graph the system.
3. Can he buy 8 donuts and 4 energy drinks?
4. Can he buy 1 donut and 3 energy drinks?

Answer

1. $\begin{cases} 0.75d + 2e \leq 25 \\ 360d + 110e \geq 1000 \end{cases}$

2.



3. yes

4. no

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Section 4.5P: Practice

Practice Makes Progress

Determine Whether an Ordered Pair is a Solution of a System of Linear Inequalities

In the following exercises, determine whether each ordered pair is a solution to the system.

Exercise 1

$$\begin{cases} 3x + y > 5 \\ 2x - y \leq 10 \end{cases}$$

a. (3, −3)
b. (7, 1)

Exercise 2

$$\begin{cases} y > \frac{2}{3}x - 5 \\ x + \frac{1}{2}y \leq 4 \end{cases}$$

a. (6, −4)
b. (3, 0)

Exercise 3

$$\begin{cases} 2x + 3y \geq 2 \\ 4x - 6y < -1 \end{cases}$$

a. $\left(\frac{3}{2}, \frac{4}{3}\right)$
b. $\left(\frac{1}{4}, \frac{7}{6}\right)$

Solve a System of Linear Inequalities by Graphing

In the following exercises, solve each system by graphing.

Exercise 4

$$\begin{cases} y \leq 3x + 2 \\ y > x - 1 \end{cases}$$

Exercise 5

$$\begin{cases} x - y > 1 \\ y < -\frac{1}{4}x + 3 \end{cases}$$

Exercise 6

$$\begin{cases} 2x - 5y < 10 \\ 3x + 4y \geq 12 \end{cases}$$

Exercise 7

$$\begin{cases} x - 2y < 3 \\ y \leq 1 \end{cases}$$

Exercise 8

$$\begin{cases} y \geq -\frac{1}{2}x - 3 \\ x \leq 2 \end{cases}$$

Exercise 9

$$\begin{cases} y \geq \frac{3}{4}x - 2 \\ y < 2 \end{cases}$$

Exercise 10

$$\begin{cases} 3x - 4y < 8 \\ x < 1 \end{cases}$$

Exercise 11

$$\begin{cases} x \geq 3 \\ y \leq 2 \end{cases}$$

Exercise 12

$$\begin{cases} 2x + 4y > 4 \\ y \leq -\frac{1}{2}x - 2 \end{cases}$$

Exercise 13

$$\begin{cases} -2x + 6y < 0 \\ 6y > 2x + 4 \end{cases}$$

Exercise 14

$$\begin{cases} y \geq -3x + 2 \\ 3x + y > 5 \end{cases}$$

Exercise 15

$$\begin{cases} y \leq -\frac{1}{4}x - 2 \\ x + 4y < 6 \end{cases}$$

Exercise 16

$$\begin{cases} 3y > x + 2 \\ -2x + 6y > 8 \end{cases}$$

Solve Applications of Systems of Inequalities

In the following exercises, translate to a system of inequalities and solve.

Exercise 17

Caitlyn sells her drawings at the county fair. She wants to sell at least 60 drawings, and has portraits and landscapes. She sells the portraits for $15 and the landscapes for $10. She needs to sell at least $800 worth of drawings to earn a profit.

a. Write a system of inequalities to model this situation.
b. Graph the system.

c. Will she make a profit if she sells 20 portraits and 35 landscapes?
d. Will she make a profit if she sells 50 portraits and 20 landscapes?

Exercise 18

Reiko needs to mail her Christmas cards and packages and wants to keep her mailing costs to no more than $500. The number of cards is at least 4 more than twice the number of packages. The cost of mailing a card (with pictures enclosed) is $3, and for a package, the cost is $7.

a. Write a system of inequalities to model this situation.
b. Graph the system.
c. Can she mail 60 cards and 26 packages?
d. Can she mail 90 cards and 40 packages?

Exercise 19

Jocelyn is pregnant and needs to eat at least 500 more calories a day than usual. When buying groceries one day with a budget of $15 for the extra food, she buys bananas that have 90 calories each and chocolate granola bars that have 150 calories each. The bananas cost $0.35 each, and the granola bars cost $2.50 each.

a. Write a system of inequalities to model this situation.
b. Graph the system.
c. Could she buy 5 bananas and 6 granola bars?
d. Could she buy 3 bananas and 4 granola bars?

Exercise 20

Jocelyn desires to increase both her protein consumption and caloric intake. She desires to have at least 35 more grams of protein each day and no more than an additional 200 calories daily. An ounce of cheddar cheese has 7 grams of protein and 110 calories. An ounce of parmesan cheese has 11 grams of protein and 22 calories.

a. Write a system of inequalities to model this situation.
b. Graph the system.
c. Could she eat 1 ounce of cheddar cheese and 3 ounces of Parmesan cheese?
d. Could she eat 2 ounces of cheddar cheese and 1 ounce of Parmesan cheese?

Everyday Math

Exercise 21

Tickets for an American Baseball League game for three adults and three children cost less than $75, while tickets for two adults and four children cost less than $62.

a. Write a system of inequalities to model this problem.
b. Graph the system.
c. Could the tickets cost $20 for adults and $8 for children?
d. Could the tickets cost $15 for adults and $5 for children?

Writing Exercises

Exercise 22

Graph the inequality $x - y \geq 3$. How do you know which side of the line $x - y = 3$ should be shaded?

Self Check

Use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an ordered pair is a solution of a system of linear inequalities.			
solve a system of linear inequalities by graphing.			
solve applications of systems of inequalities.			

If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is crucial to establish a strong foundation before proceeding. Who can you ask for help? Your classmates, tutors, and class instructor are good resources.

…no - I don't get it! This is a warning sign, and you must not ignore it. You should get help right away, or you will quickly be overwhelmed. Meet with your instructor during their drop-in hours as soon as you can to discuss your situation. Together, you can devise a plan to get the help you need.

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CHAPTER OVERVIEW

Unit 5: Practice Problem Answers

This unit provides the answers to all the practice problems from the textbook. It's here to help you confirm whether your solutions are correct and to support your learning by giving you immediate feedback. But remember: knowing the final answer is only part of doing math. You're still responsible for showing clear, step-by-step work that demonstrates your thinking and understanding.

With great power comes great responsibility—so use these answers wisely. They are meant to *check* your work, not *replace* it. If you find a mistake, go back, try again, and figure out where things went off track. That process is how real learning happens.

Section 5.1: Unit 1 Practice Answers

Section 5.2: Unit 2 Practice Answers

Section 5.3: Unit 3 Practice Answers

Section 5.4: Unit 4 Practice Answers

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Section 5.1: Unit 1 Practice Answers

Unit 1 Answers

Section 1.1P: Language of Algebra

1. 16 minus 9, the difference of sixteen and nine
2. 28 divided by 4, the quotient of twenty-eight and four
3. 2 times 7, the product of two and seven
4. Fourteen is less than twenty-one
5. Thirty-six is greater than or equal to nineteen
6. y minus 1 is greater than 6, the difference of y and one is greater than six
7. 2 is less than or equal to 18 divided by 6, or 2 is less than or equal to the quotient of eighteen and six
8. equation
9. expression
10. expression
11. He will pay $750, and the insurance company will pay $1,350.
12. Answers will vary.

Section 1.2P: Order of Operations

1. 125
2. 256
3. 43
4. 55
5. 5
6. 34
7. 58
8. 6
9. 4
10. 35
11. 58
12. 149
13. 50

Section 1.3P: Exploring Expressions

1. 22
2. 144
3. 32
4. 21
5. \72\)
6. 73
7. 54
8. 8
9. 5
10. x^3 and $8x^3$, 14 and 5
11. 16 and 4, $16b^2$ and $9b^2$
12. $15x^2, 6x, 2$
13. $10y^3, y, 2$
14. $13x$
15. $7c$
16. $10u + 3$

17. $22a + 1$
18. $17x^2 + 20x + 16$
19. $14 - 9$
20. $9 \cdot 7$
21. $36 \div 9$
22. $8x + 3x$
23. $\frac{y}{3}$
24. $8(y - 9)$
25. $c + 3$
26. $2n - 7$
27. Answers will vary.

Section 1.4P: Integers

1. $>$
2. $>$
3. $>$
4. $>$
5. $=$
6. $=$
7. $=$
8. 8
9. 15
10. 1
11. 32
12. -22
13. 29
14. 6
15. -6
16. 22
17. 45
18. 0
19. 4
20. 6
21. -32
22. -65
23. -4
24. 13
25. -4
26. -12
27. 64
28. -64
29. -47
30. 90
31. 9
32. 41
33. -9
34. -29
35. 1
36. -3
37. 5

38. 121
39. -56
40. 6
41. -5
42. 28
43. $-\frac{6}{a+b}$
44. 96
45. 21
46. Answers will vary.
47. Answers will vary.

Section 1.5P: Fractions

1. $-\frac{12}{7}$
2. $\frac{2x^2}{3y}$
3. $-\frac{21a^2}{11b^2}$
4. $\frac{1}{3}$
5. $-\frac{21}{50}$
6. $9n$
7. $\frac{33}{4x}$
8. $\frac{10u}{9v}$
9. $-\frac{1}{16}$
10. $-\frac{10}{9}$
11. $-\frac{2}{5}$
12. $\frac{2m}{3n}$
13. $\frac{1}{48}$
14. $-\frac{53}{40}$
15. $\frac{1}{12}$
16. $\frac{4x+3}{12}$
17. $\frac{-8x-15}{18}$
18. $-\frac{10k}{27}$
19. $\frac{-5(a+1)}{3}$
20. a
21. $\frac{9}{7}$
22. -8
23. $\frac{5}{2}$
24. 54
25. $\frac{49}{25}$
26. $\frac{15}{4}$
27. $\frac{5}{21}$
28. $\frac{5}{4}$
29. $\frac{-28-15y}{60}$
30. $\frac{33}{64}$

31. $\frac{7}{9}$
32. -5
33. $\frac{23}{24}$
34. 1
35. $\frac{13}{3}$
36. $-\frac{1}{9}$
37. $-\frac{5}{11}$
38. Answers will vary.
39. Answers will vary.

Section 1.6P: Dimensional Analysis

1. 72 inches
2. $53\frac{1}{3}$ yards
3. 90 feet
4. 9,200 pounds
5. $17\frac{1}{2}$ tons
6. 5,400 s
7. 224 ounces
8. $1\frac{1}{4}$ gallons
9. 76 in.
10. 65 days
11. 115 ounces
12. 8 lbs. 13 oz.
13. 3.05 hours
14. 10 ft. 2 in.
15. 4 yards
16. 5,000 meters
17. 155 centimeters
18. 3.072 kilometers
19. 1.5 grams
20. 91,600 milligrams
21. 0.75 liters
22. 91 centimeters
23. 855 grams
24. 2.1 kilograms
25. 1.6 liters
26. 190.5 centimeters
27. 21.9 meters
28. 3.1 miles
29. 44 pounds
30. 53.2 liters
31. 14.6 kilograms
32. Answers will vary.
33. Answers will vary.

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Section 5.2: Unit 2 Practice Answers

Unit 2 Answers

Section 2.1P: Solving Equations - Building Blocks

1. Yes
2. $x = 11$
3. $b = \frac{1}{2}$
4. $x = \frac{7}{3}$
5. $q = -\frac{1}{4}$
6. $c = 25$
7. $y = 24$
8. $n = -50$
9. $a = -\frac{11}{4}$
10. $x + 9 = 52; x = 43$
11. $m - 10 = -14; m = -4$
12. $y + (-30) = 40; y = 70$
13. $9x - 8x = 107; 107$
14. 7 years old
15. $121.19
16. $c = -11$
17. $z = 13$
18. $x = 140$
19. $q = 100$
20. $y = -6$
21. $y = 72$
22. $a = -4$
23. $u = \frac{1}{4}$
24. $n = 88$
25. $q = -11$
26. $133 = -19n; n = -7$
27. $\frac{r}{12} = -48; r = -576$
28. $\frac{b}{-6} = 18; b = -108$
29. $\frac{3}{10}x = 15; x = 50$
30. $\frac{2}{5} + f = \frac{1}{2}; f = \frac{1}{10}$
31. $q - \frac{1}{8} = \frac{3}{4}; q = \frac{7}{8}$
32. 6 balloons
33. $1.08
34. 36 mpg
35. $d = \frac{13}{24}$ inch
36. Answers will vary.
37. No. Justifications will vary.

Section 2.2P: Solving Equations - Refining the Process

1. $k = -11$
2. $b = -3$
3. $x = -\frac{3}{4}$
4. $x = 19$
5. $q = -5$
6. $x = 2$

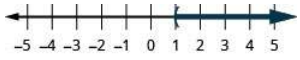
7. $m = -6$
8. $a = 7$
9. $p = 15$
10. $w = 60$
11. $n = -2$
12. $t = -9$
13. $m = 6$
14. $t = 1$
15. $x = 5$
16. $r = -7$
17. $y = -4$
18. $a = 4$
19. $m = 3$
20. $x = 34$
21. $n = -1$
22. $m = -4$
23. $k = \frac{3}{5}$
24. $v = 1$
25. $m = -4$
26. identity; all real numbers
27. conditional equation; $n = 7$
28. contradiction; no solution
29. contradiction; no solution
30. identity; all real numbers
31. 30 feet
32. 8 nickels
33. Answers will vary.
34. Answers will vary.

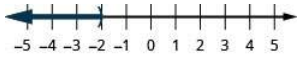
Section 2.3P: Solving Equations - Advanced Techniques and Formulas

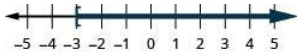
1. $x = 1$
2. $b = -2$
3. $n = -24$
4. $z = -2$
5. $x = -6$
6. $q = 7$
7. $x = -\frac{5}{2}$
8. $p = -4$
9. $d = -24$
10. $v = 4$
11. $k = 2$
12. $x = 22$
13. $n = 19$
14. $q = 15$
15. 290 miles
16. 30 miles
17. 5 hours
18. 11.25 hours
19. 75 mph
20. 75 mph

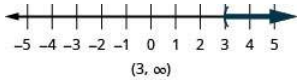
21. $t = 4$
22. $h = \frac{2A}{b}$
23. $h = 16$
24. $c = 180 - a - b$
25. $y = -1 + 5x$
26. $y = \frac{11-3x}{2}$
27. $y = 3 + x$
28. $W = \frac{P-2L}{2}$
29. $\pi = \frac{C}{d}$
30. $H = \frac{V}{LW}$
31. $30°\text{C}$
32. $-17.8°\text{C}$
33. $14°\text{F}$
34. $71.6°\text{F}$
35. $s = 35$
36. $10°\text{C}$
37. Answers will vary.
38. 100. Justifications will vary.

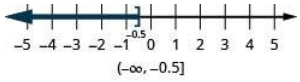
Section 2.4P: Linear Inequalities

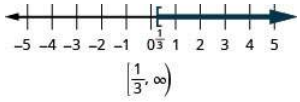
1. 

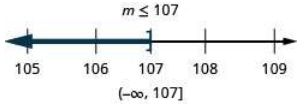
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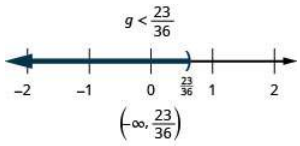
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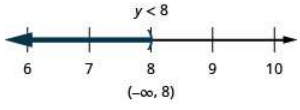
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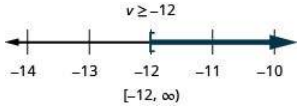
5. 

6. 

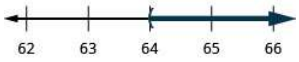
7. 

8. 

9. 

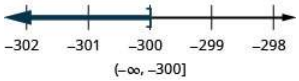
10. 

11. $k > 64$



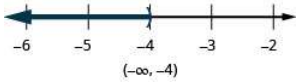
$(64, \infty)$

12. $b \leq -300$



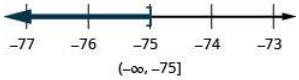
$(-\infty, -300]$

13. $s < -4$



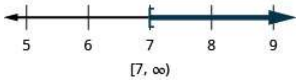
$(-\infty, -4)$

14. $x \leq -75$



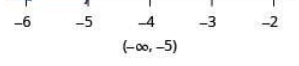
$(-\infty, -75]$

15. $u \geq 7$



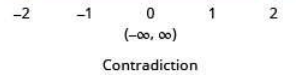
$[7, \infty)$

16. $y < -5$



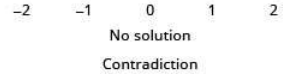
$(-\infty, -5)$

17. identity



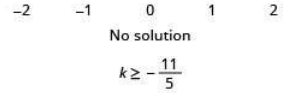
$(-\infty, \infty)$

18. Contradiction



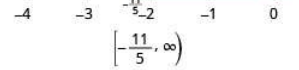
No solution

19. Contradiction



No solution

20. $k \geq -\frac{11}{5}$



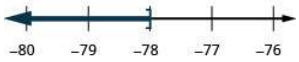
$\left[-\frac{11}{5}, \infty\right)$

21. $y \geq \frac{10}{49}$



$\left[\frac{10}{49}, \infty\right)$

22. $n \leq -78$

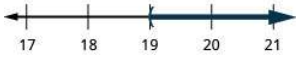


$(-\infty, -78]$

23. $k + 6 > 25$

$k > 19$

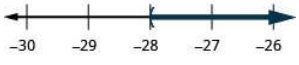
$(19, \infty)$

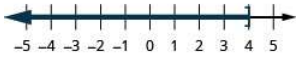


24. $-2s < 56$

$s > -28$

$(-28, \infty)$



25. $a - 15 \geq -7$
$a \geq 8$
$[8, \infty)$


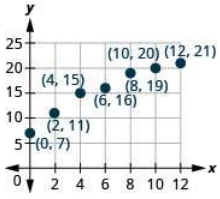
26. five drinks
27. $15,000
28. 16 ccf
29. 48 cut & styles
30. 49 cars
31. $h \leq 77$
32. 169 guests
33. Answers will vary.
34. Answers will vary.

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Section 5.3: Unit 3 Practice Answers

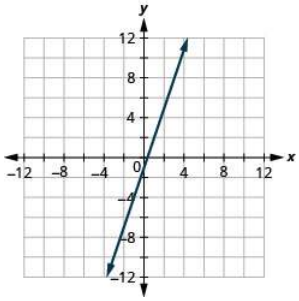
Section 3.1P: Cartesian Plane

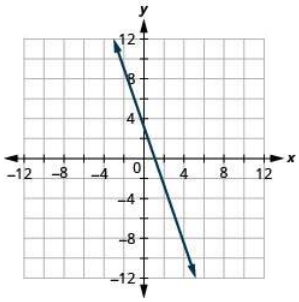
1. $(-4,2)$
2. $(-1,-2)$
3. $(3,-5)$
4. $(-3,5)$
5. $(\frac{5}{3},2)$
6. $(-2,0)$
7. $(-3,0)$
8. $(0,0)$
9. $(0,4)$
10. $(0,2)$
11. 4
12. 2
13. 2
14. 3
15. 1
16. $A:(-4,1)\quad B:(-3,-4)\quad C:(1,-3)\quad D:(4,3)$
17. $A:(0,-2)\quad B:(-2,0)\quad C:(0,5)\quad D:(5,0)$
18. yes
19. yes
20. no
21. yes
22. no
23. yes
24. no
25. yes
26. yes
27. $y=-6;\ (-1,-6)$
28. $y=2;\ (3,2)$
29. $y=3;\ (6,3)$
30. $x=-2;\ (-2,1)$
31. $x=6;\ (6,0)$
32. $x=\frac{25}{4};\ (\frac{25}{4},\frac{1}{2})$
33. 

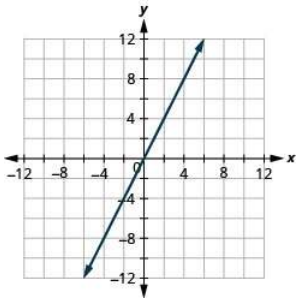
34. Answers will vary.
35. Answers will vary.

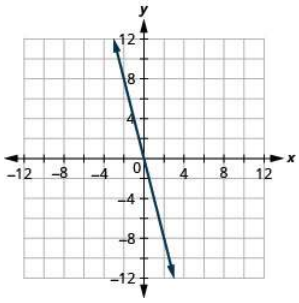
Section 3.2P: Graph Linear Equations in Two Variables

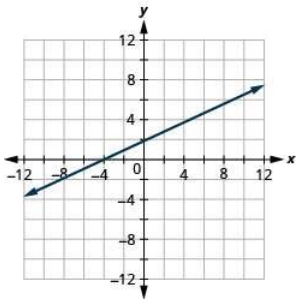
1. $a:$ yes; no $\quad b:$ no; no $\quad c:$ yes; yes $\quad d:$ yes; yes
2. $a:$ yes; yes $\quad b:$ yes; yes $\quad c:$ yes; yes $\quad d:$ no; no

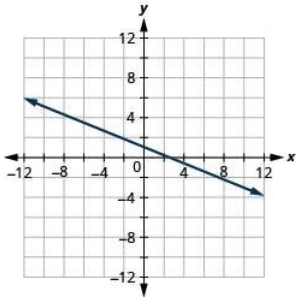
3. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

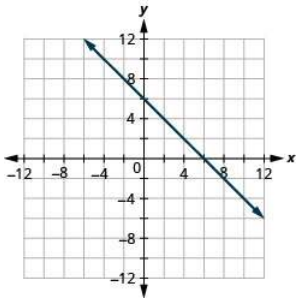
4. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

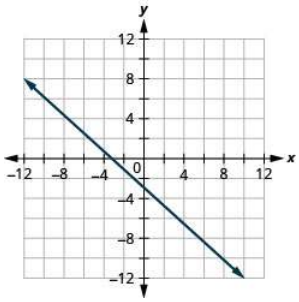
5. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

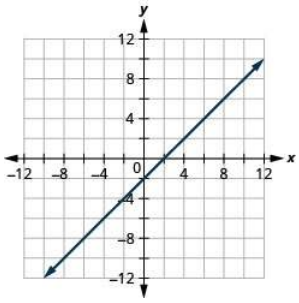
6. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

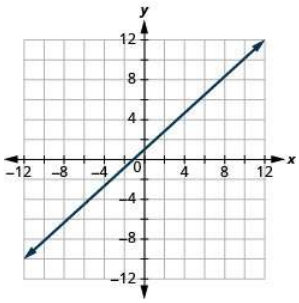
7. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

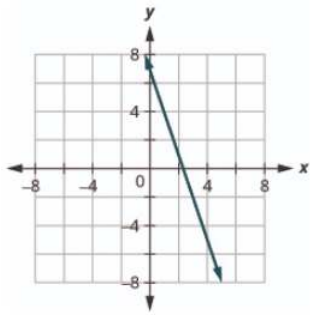
8. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

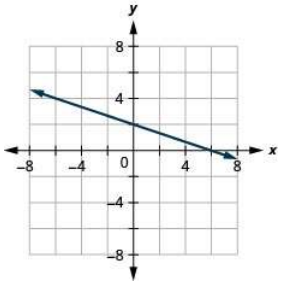
9. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

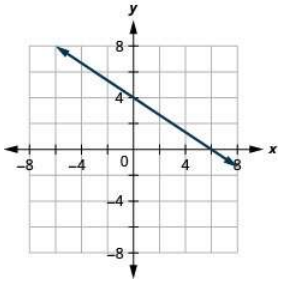
10. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

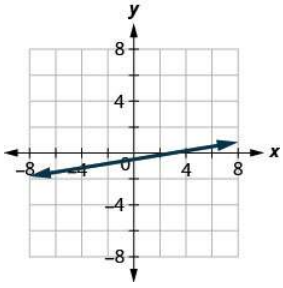
11. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

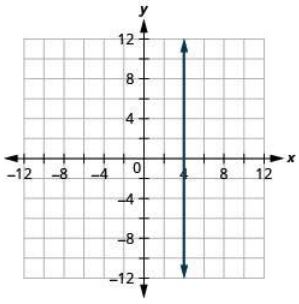
12. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

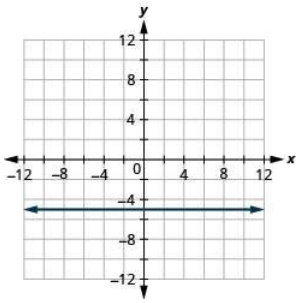
13. 
y
8
4
x
–8
–4
0
4
8
–4
–8

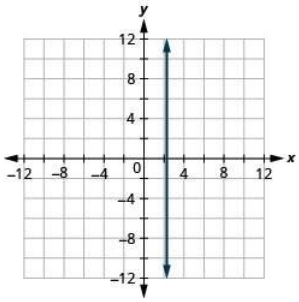
14. 
y
8
4
x
–8
–4
0
4
8
–4
–8

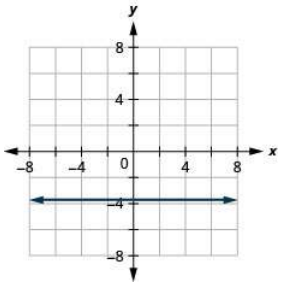
15. 
y
8
4
x
–8
–4
0
4
8
–4
–8

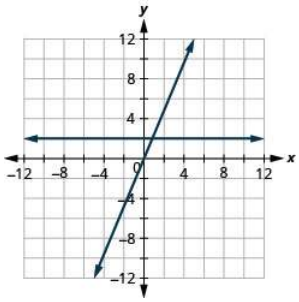
16. 
y
8
4
x
–8
–4
0
4
8
–4
–8

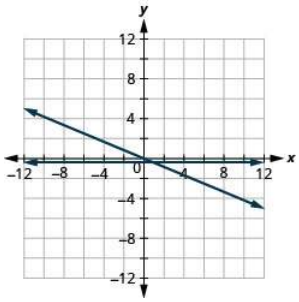
17. 
y
12
8
4
x
–12
–8
–4
0
4
8
12
–4
–8
–12

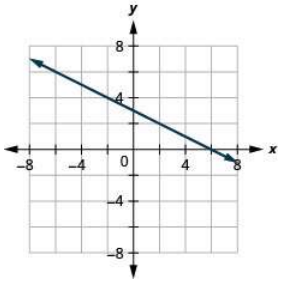
18. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

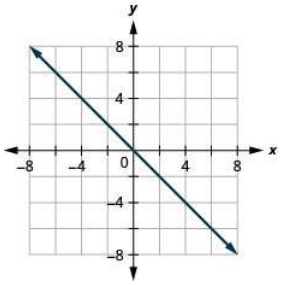
19. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

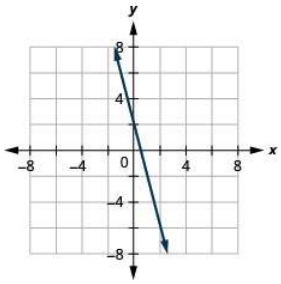
20. 
y
x
8
4
0
−4
−8
−8
−4
4
8

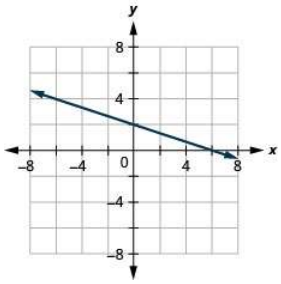
21. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

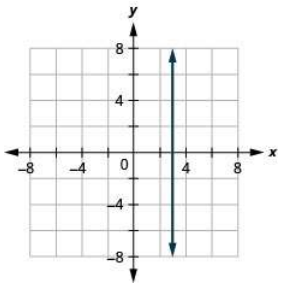
22. 
y
x
12
8
4
0
−4
−8
−12
−12
−8
−4
4
8
12

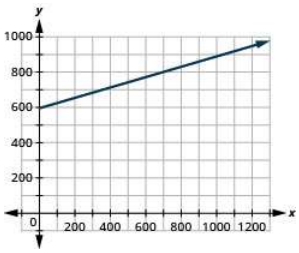
23. 

24. 

25. 

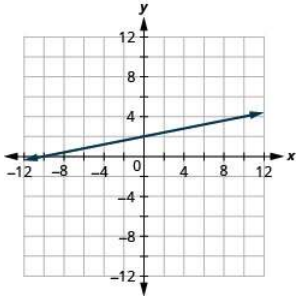
26. 

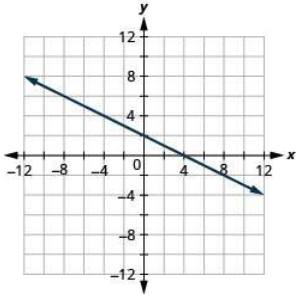
27. 

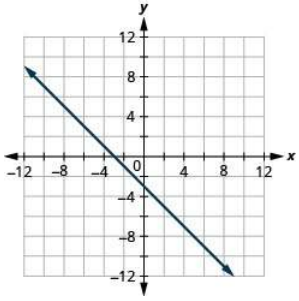
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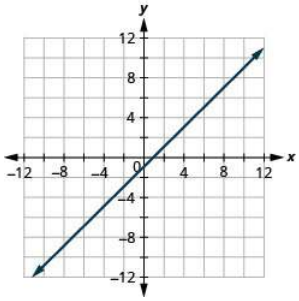
29. Answers will vary.

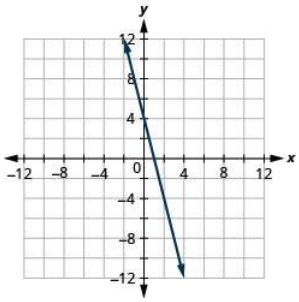
Section 3.3P: Intercepts

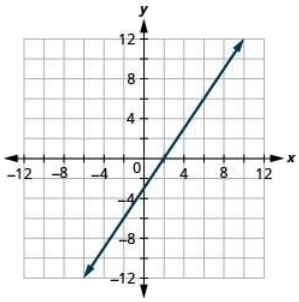
1. (3,0), (0,3)
2. (5,0), (0,−5)
3. (−2,0), (0,−2)
4. (−1,0), (0,1)
5. (6,0), (0,3)
6. (0,0)
7. (4,0), (0,4)
8. (−2,0), (0,−2)
9. (5,0), (0,−5)
10. (−3,0), (0,3)
11. (2,0), (0,6)
12. (12,0), (0,−4)
13. (5,0), (0,2)
14. (-3,0), (0,1)
15. (−10,0), (0,2)
16. (0,0)
17. (0,0)
18. 

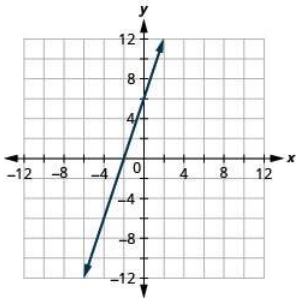
19. 

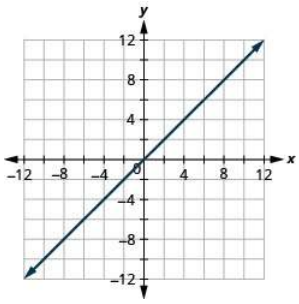
20. 

21. 

22. 

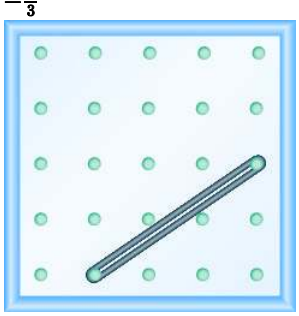
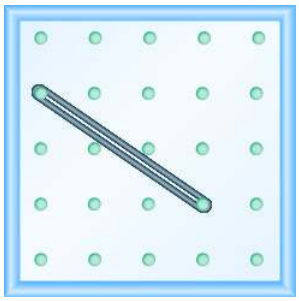
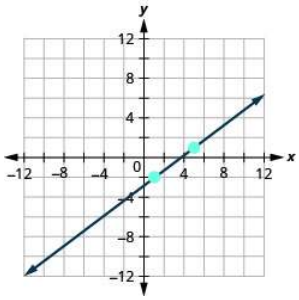
23. 

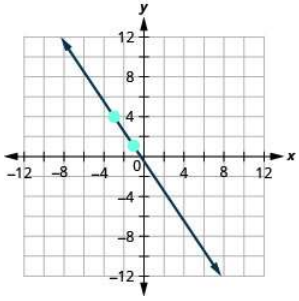
24. 

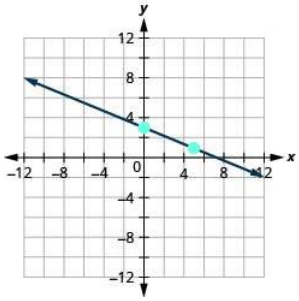
25. 

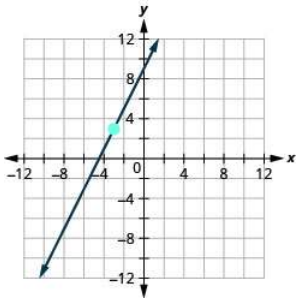
26. At (0,1000), he has been gone 0 hours and has 1000 miles left to go; At (15,0), he has been gone 15 hours and has 0 miles left to go.
27. Answers will vary.
28. Answers will vary.

Section 3.4P: Understanding the Slope of a Line

1. $\frac{1}{4}$
2. $\frac{-3}{2} = -\frac{3}{2}$
3. $-\frac{2}{3}$
4. 
5. 
6. $\frac{2}{5}$
7. $-\frac{1}{3}$
8. $\frac{3}{4}$
9. $-\frac{5}{2}$
10. $\frac{1}{4}$
11. 0
12. undefined
13. $\frac{5}{2}$
14. $-\frac{8}{7}$
15. $\frac{7}{3}$
16. -1
17. 

18. 

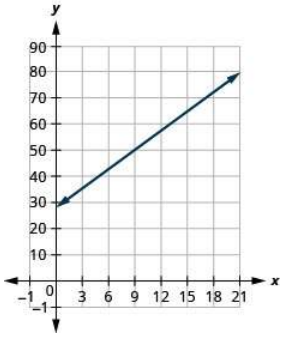
19. 

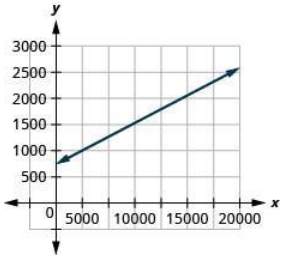
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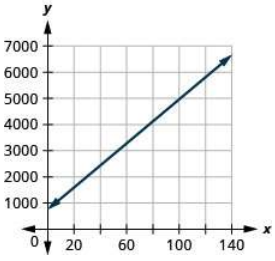
21. 4 12 pitch or 4-in-12 pitch
22. $\frac{3}{50}$; rise $= 3$, run $= 50$
23. 288 inches (24 feet)
24. When the slope is a positive number, the line goes up from left to right. When the slope is a negative number, the line goes down from left to right.
25. A vertical line has 0 run, and since division by 0 is undefined, the slope is undefined.

Section 3.5P: Slope-Intercept Form of Line

1. slope $m = 4$ and y-intercept $(0, -2)$
2. slope $m = -\frac{2}{5}$ and y-intercept $(0, 3)$
3. $m = 4$; y-intercept: $(0, -10)$
4. $m = -\frac{8}{3}$; y-intercept: $(0, 4)$
5. graph of $y = x + 4$
6. graph of $y = -x + 3$
7. graph of $y = -\frac{2}{5}x - 3$
8. graph of $4x - 3y = 6$
9. horizontal line
10. slope–intercept
11. slope–intercept
12. horizontal line
13. intercepts
14. not parallel
15. parallel
16. parallel
17. parallel
18. not parallel
19. not parallel
20. not parallel
21. perpendicular
22. not perpendicular
23. not perpendicular
24. perpendicular
25. perpendicular

26. a. $28
    b. $66.10
    c. The slope, 2.54, means that Randy's payment, P, increases by $\$2.54$ when the number of units of water he used, w, increases by 1. The P-intercept means that if the number of units of water Randy used was 0, the payment would be $\$28$.
    d. 

27. a. $750
    b. $2418.60
    c. The slope, 0.09, means that Patel's salary, S, increases by $\$0.09$ for every $\$1$ increase in his sales. The S-intercept means that when his sales are $\$0$, his salary is $\$750$.
    d. 

28. a. $2850
    b. $4950
    c. The slope, 42, means that the cost, C, increases by $\$42$ when the number of guests increases by 1. The C-intercept means that when the number of guests is 0, the cost would be $\$750$.
    d. 

29. a. For every increase of one degree Fahrenheit, the number of chirps increases by four.
    b. There would be -160 chirps when the Fahrenheit temperature is $0°$. (This does not make sense; the model cannot be used for all temperatures.)

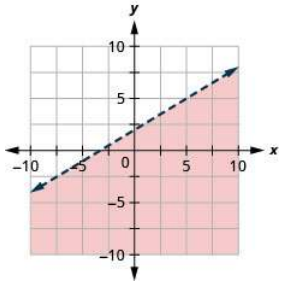
30. Answers will vary.

Section 3.6P: Line Equations

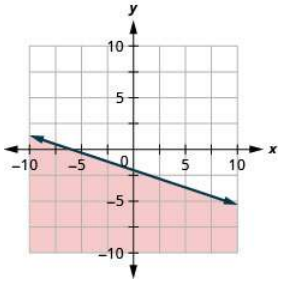
1. $y = 8x - 6$
2. $y = -3x - 1$
3. $y = -\frac{2}{3}x - 3$
4. $y = -4x$
5. $y = \frac{3}{4}x + 2$
6. $y = 6$

7. $y = \frac{5}{6}x + 2$
8. $y = -\frac{1}{3}x - 11$
9. $y = -7$
10. $y = -4x - 11$
11. $y = -4x + 13$
12. $y = -\frac{1}{3}x - \frac{14}{3}$
13. $y = -\frac{6}{7}x + \frac{4}{7}$
14. $x = 4$
15. $y = 2$
16. $y = 4x$
17. $y = 5$
18. $y = -3x + 3$
19. $y = -\frac{2}{3}x + 5$
20. $x = 4$
21. $y = -1$
22. $y = -\frac{3}{2}x - 1$
23. $y = \frac{5}{4}x$
24. $x = -5$
25. $y = 1$
26. $y = x + 5$
27. $y = \frac{5}{6}x + 2$
28. $y = -\frac{2}{3}x + 5$
29. $y = 2$
30. $y = x + 2$
31. $x = -3$
32. $y = \frac{3}{4}x$
33. $y = 1.2x + 5.2$
34. Answers will vary.

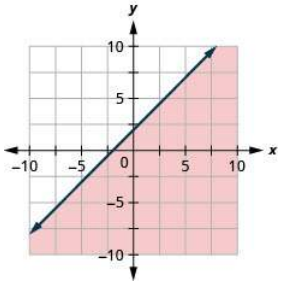
Section 3.7P: Linear Inequalities in Two Variables

1. a.) yes b.) no c.) no d.) yes e.) no
2. a.) yes b.) no c.) no d.) yes e.) yes
3. a.) no b.) no c.) no d.) yes e.) yes
4. $y < 2x - 4$
5. $y \leq -1/3x - 2$
6. $x + y \geq 3$
7. $x + 2y \geq -2$
8. $2x - y < 4$
9. $4x - 3y > 12$
10. 

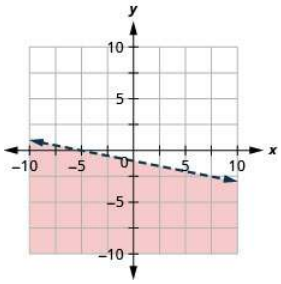
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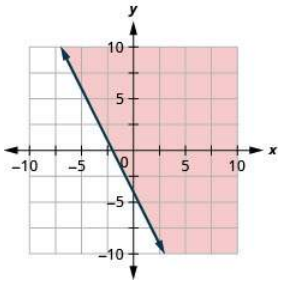
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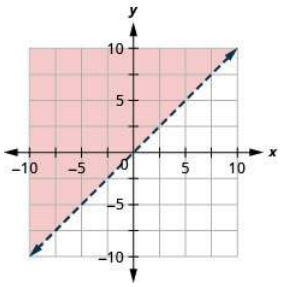
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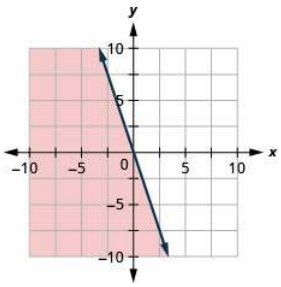
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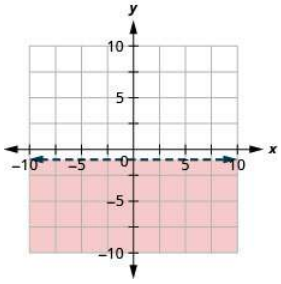


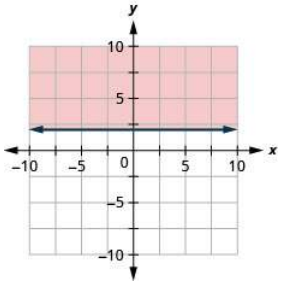
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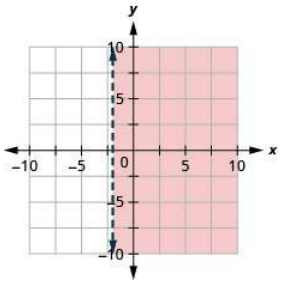


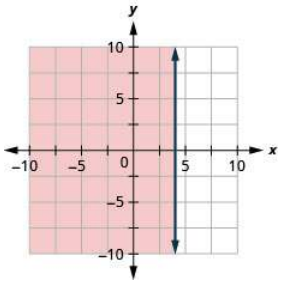
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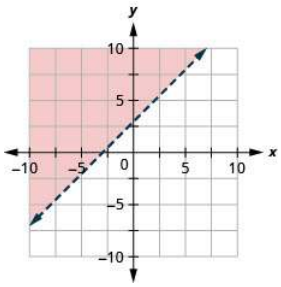


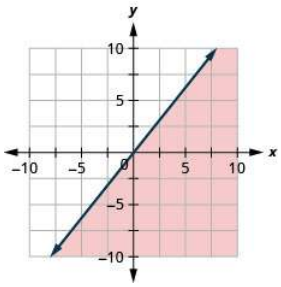
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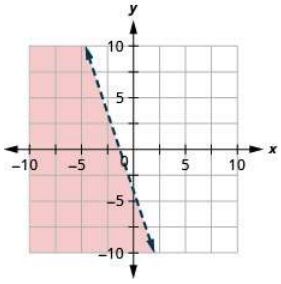
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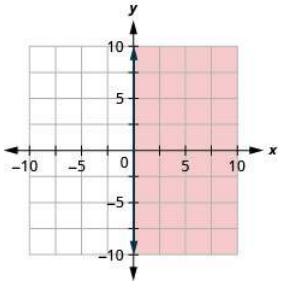
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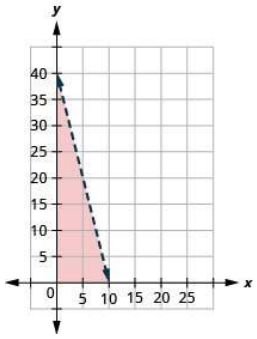
20. 

21. 

22. 

23. 

24. 

25. 

a.)

b.) Answers will vary.

26. Answers will vary.

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Section 5.4: Unit 4 Practice Answers

Section 4.1P: Solve Systems of Equations by Graphing

1. yes
2. yes
3. no
4. no
5. yes
6. Solution Point: $(-2, 3)$
7. Solution Point: $(1, 2)$
8. Solution Point: $(0, 2)$
9. Solution Point: $(2, 4)$
10. Solution Point: $(2, -1)$
11. Solution Point: $(1, 2)$
12. Solution Point: $(3, 2)$
13. Solution Point: $(1, 1)$
14. Solution Point: $(-1, -4)$
15. infinitely many solutions with solution set: $\{(x, y) \mid 2x + y = 6\}$
16. Solution Point: $(3, 3)$
17. no solution
18. Solution Point: $(-5, 6)$
19. Solution Point: $(6, -2)$
20. Solution Point: $(3, 2)$
21. no solution
22. Solution Point: $(1, 3)$
23. Solution Point: $(-3, 1)$
24. no solution
25. infinitely many solutions with solution set: $\{(x, y) \mid x + 3y = -6\}$
26. Solution Point: $(2, 2)$
27. infinitely many solutions
28. 0 solutions
29. consistent, 1 solution
30. infinitely many solutions
31. Molly needs 16 ounces of strawberry juice and 48 ounces of water.
32. Enrique needs 8 ounces of nuts and 16 ounces of water.
33. Leo should plant 50 tulips and 300 daffodils.
34. Given that it is only known that the slopes of both linear equations are the same, there are either no solutions (the graphs of the equations are parallel) or infinitely many.

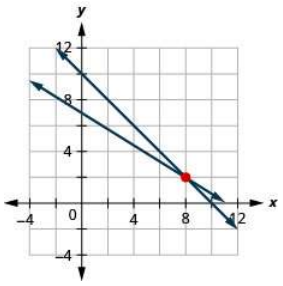
Section 4.2P: Solve Systems of Equations by Substitution

1. $\left(\frac{1}{2}, -\frac{3}{4}\right)$
2. $\left(\frac{1}{2}, 3\right)$
3. $(3, -1)$
4. $(-1, 7)$
5. $(6, 6)$
6. $(-2, 7)$
7. No solution
8. $(5, 0)$
9. $(-3, 5)$

10. Infinitely many solutions. The two equations represent the same line. The solution set is: $\{(x, y) \mid 2x + 16y = 8\}$
11. $(7, 6)$
12. $(10, 12)$
13. $(0, 3)$
14. Infinitely many solutions. The two equations represent the same line. The solution set is: $\{(x, y) \mid y = 3x\}$
15. No solution
16. $(-5, 2)$
17. $(-2, 0)$
18. $(6, -3)$
19. The numbers are 6 and 9.
20. The numbers are −7 and −19.
21. The length is 20 and the width is 10.
22. The length is 34 and the width is 8.
23. The measures are $16°$ and $74°$.
24. The measures are $45°$ and $45°$.
25. 80 cable packages would need to be sold.
26. Mitchell would need to sell 120 stoves.
27. Answers will vary.

Section 4.3P: Solve Systems of Equations by Elimination

1. (6, 9)
2. (-2, 1)
3. infinitely many solutions
4. (-7, -1)
5. (-2, -4)
6. inconsistent, no solution
7. (-1, 3)
8. (-1, 2)
9. (-5, 9)
10. (6, 1)
11. (-2, 10)
12. (2, 3)
13. inconsistent, no solution
14. (-7, 6)
15. (-9, 3)
16. infinitely many solutions
17. (9, 5)
18. (-3, 2)
19. (-2, 3)
20. infinitely many solutions
21. The numbers are 20 and 45.
22. The numbers are 16 and −43.
23. elimination
24. substitution
25. substitution
26. elimination
27. A shirt costs $16 and a sweater costs $33.
28. There are 860 mg in a hot dog. There are 1,000 mg in a cup of cottage cheese.
29. a. (8, 2)

b. 

c. Answers will vary.

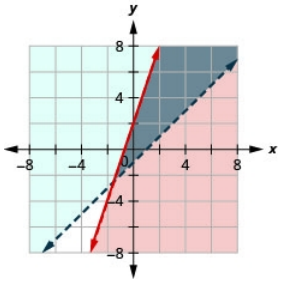
Section 4.4P: Solve Applications with Systems of Equations

1. He earns $10,000 per year.
2. She put $15,000 into a CD and $35,000 into bonds.
3. The amount of the first year's loan was $30,000, and the amount of the second year's loan was $12,000.
4. Bethany is 16 years old, and Alyssa is 28 years old.
5. Noelle is 20 years old, and her dad is 54 years old.
6. The small container holds 20 gallons, and the large container holds 30 gallons.
7. There were 10 calories burned jogging and 10 calories burned cycling.
8. Notebooks are $4, and thumb drives are $20.
9. The width is 10 feet and the length is 25 feet.
10. The width is 15 feet and the length is 15 feet.
11. It took Sarah's sister 12 hours.
12. It took Lucy's friend 2 hours.
13. The canoe rate is 5 mph, and the current rate is 1 mph.
14. The boat rate is 6.5 mph, and the current rate is 2.5 mph.
15. The jet rate is 240 mph, and the wind speed is 28 mph.
16. The jet rate is 415 mph, and the wind speed is 19 mph.
17. There are 1120 adult tickets and 530 child tickets sold.
18. Josie bought 40 adult tickets and 32 children's tickets.
19. There were 125 adult tickets and 128 children's tickets sold.
20. Marissa should use 60 pounds of the $1.20/lb candy and 30 pounds of the $1.80/lb candy.
21. Hannah needs 10 gallons of soda and 15 gallons of fruit drink.
22. Julia and her husband should buy 12 pounds of City Roast Colombian coffee and 8 pounds of French Roast Colombian coffee.
23. Jotham should mix 42 liters of the 30% solution and 28 liters of the 80% solution.
24. The scientist should mix 15 liters of the 25% solution and 50 liters of the 12% solution.
25. 160 liters of the 40% solution and 80 liters of the 70% solution will be used.
26. 14 boys paid the full-year fee, 4 boys paid the partial-year fee

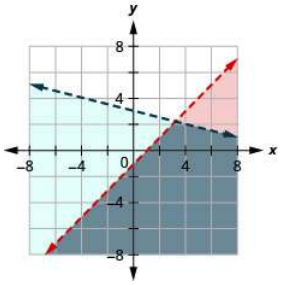
Section 4.5P: Graphing Systems of Linear Equations

1. a. true
   b. false
2. a. false
   b. true
3. a. true
   b. true

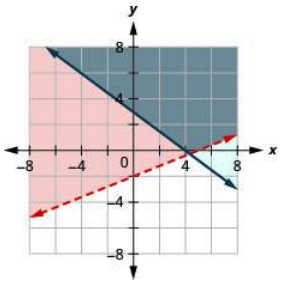
4.


y
x
8
4
−8
−4
0
4
8
−4
−8

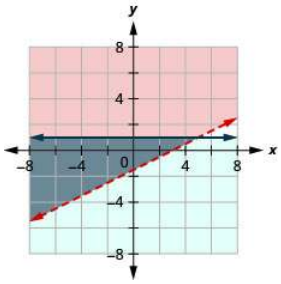
5.


y
x
8
4
−8
−4
0
4
8
−4
−8

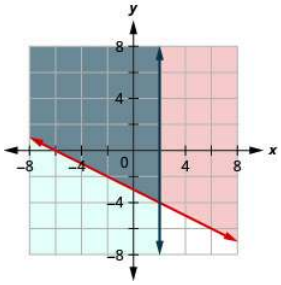
6.


y
x
8
4
−8
−4
0
4
8
−4
−8

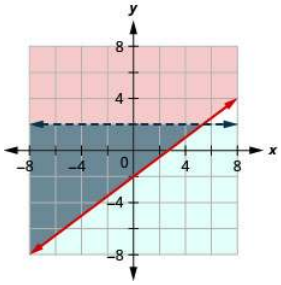
7.

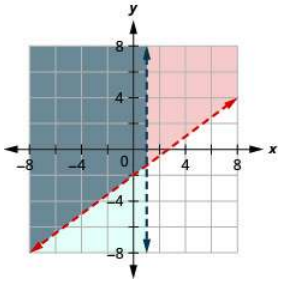

y
x
8
4
−8
−4
0
4
8
−4
−8

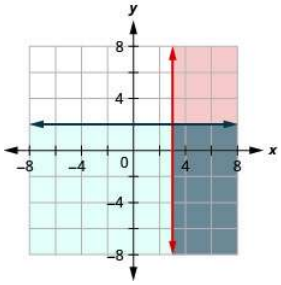
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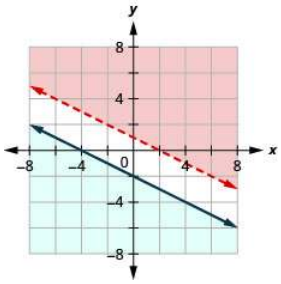

y
x
8
4
−8
−4
0
4
8
−4
−8

9.

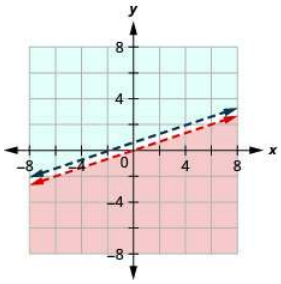

y
x
8
4
−8
−4
0
4
8
−4
−8

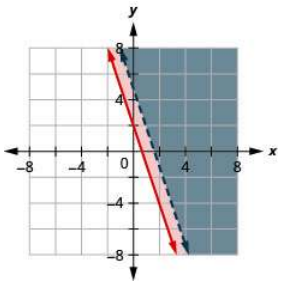
10. 

11. 

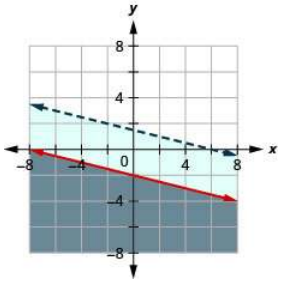
12. 

13. No solution

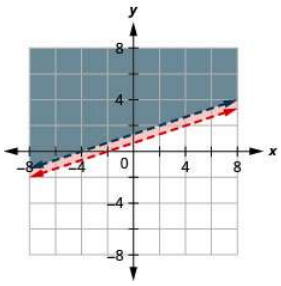


14. 

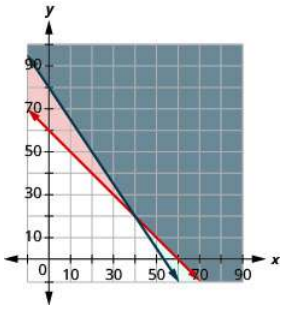
15. $x + 4y < 6$



16. $-2x + 6y > 8$



17. a. $\begin{cases} p + l \geq 60 \\ 15p + 10l \geq 800 \end{cases}$

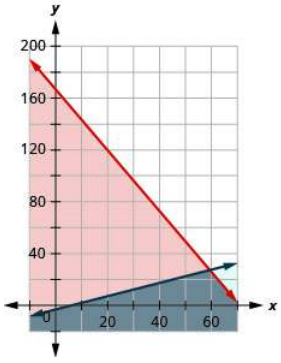


b.

c. No

d. Yes

18. a. $\begin{cases} 7p + 3c \leq 500 \\ p \geq 2c + 4 \end{cases}$

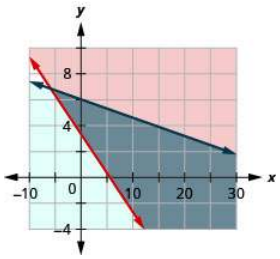


b.

c. Yes

d. No

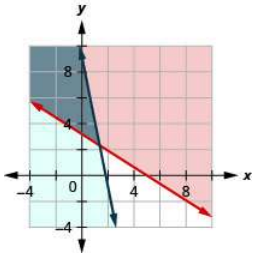
19. a. $\begin{cases} 90b + 150g \geq 500 \\ 0.35b + 2.50g \leq 15 \end{cases}$

b. 

c. No

d. Yes

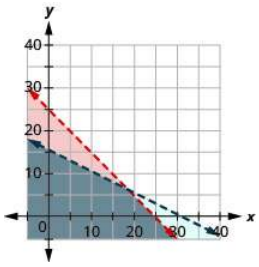
20. a. $\begin{cases} 7c + 11p \geq 35 \\ 110c + 22p \leq 200 \end{cases}$

b. 

c. Yes

d. No

21. a. $\begin{cases} 3a + 3c < 75 \\ 2a + 4c < 62 \end{cases}$

b. 

c. No

d. Yes

22. Answers will vary.

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