2: SETS, COUNTING, AND NUMERATION- CONNECTING EARLY MATH TO ADVANCED IDEAS



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2: SETS, COUNTING, AND NUMERATION- CONNECTING EARLY MATH TO ADVANCED IDEAS

Mathematics is all about finding patterns, organizing information, and understanding relationships. One of the most powerful tools for doing this is Set Theory (NCTM, 2020) But why should you, as a future elementary school teacher, care about set theory? The answer is simple: set theory is everywhere! It forms the foundation for many mathematical ideas and helps young learners develop essential thinking skills that will serve them throughout their education (Feike, Schwingendorf, and Gregg, 2018).

Set theory provides a structured way to classify, group, and analyze information. It gives us the tools to define and manipulate collections of objects, which is a crucial skill in problem-solving and logical reasoning. Whether we are talking about numbers, shapes, or real-world items, sets help us make sense of the world around us (Finnan, 2006). In fact, concepts from set theory are used in fields as diverse as computer science, probability, and logic (CDE, 2013).

Elementary teachers need to understand the big ideas and language of set theory to better grasp how young learners develop mathematical thinking (Burtob, 1984; NCTM 2020). Even before children formally learn about sets, they naturally engage in set-based reasoning by sorting objects by color, shape, or size—fundamental set operations. When they count apples in a basket or compare toy piles, they apply set concepts like cardinality (how many) and ordinality (order), building number sense and reasoning. In the classroom, sets help students learn sorting, classifying, and comparing numbers, concepts that also appear in science (grouping species), language arts (identifying vowels and consonants), and daily activities like organizing supplies. A strong foundation in set theory allows teachers to recognize these early learning experiences and support students in making deeper mathematical connections (Feike, Schwingendorf, and Gregg, 2018).

Set theory is not just theoretical—it has practical applications in daily life. More than a math topic, set theory is a way of thinking that helps us understand and navigate the world, shaping how you teach and how students build a strong mathematical foundation. By the end of the chapter we will explore each of the student learning objectives below.

Chapter Learning Outcomes

- **Essential Questions**:
  - How do different number systems shape the way we understand mathematics today?
  - In what ways can sets and whole numbers represent ideas in the world around us?
- **Learning that Transfers**:
  - **Beyond memorization:** Instead of just memorizing number facts, students will gain flexible reasoning skills that allow them to understand and explain mathematical structures.
  - **Real-world connections:** Understanding set theory and numeration systems will enable future teachers to guide students in recognizing mathematical patterns in everyday contexts (e.g., grouping in counting, understanding different number representations like Roman numerals on clocks).
  - **Problem-solving and adaptability:** By studying different number systems, students will develop the ability to approach unfamiliar problems with logical reasoning—an essential skill for both teaching and lifelong learning.

Student Learning Objectives

- SLO 2.1 Students will **demonstrate** an understanding of foundational set theory by defining and manipulating sets and using set operations (e.g., union, intersection, complement) to construct and analyze new sets.
- SLO 2.2 Students will **analyze** and **apply** the properties and functions of whole numbers, distinguishing between their various uses (cardinal, ordinal, and identification) and explaining relationships such as greater than and less than through set-based representations.
- SLO 2.3 Students will **compare** Egyptian, Roman, and Hindu-Arabic numeration systems, identifying differences in place value, positionality, and arithmetic operations, and evaluate each system's advantages and limitations.
- SLO 2.4 Students will **demonstrate** proficiency with the Hindu-Arabic numeration system by converting between bases, expanding numbers, and articulating the system's structure, base, and numeral naming conventions.

SET THEORY RAP

Audio

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Morgan and Grandpa's Treasure Hunt

Morgan woke up with a smile so wide,
A day at Grandpa's—she beamed with pride!
But Grandpa had a plan, oh what a treat,
A treasure hunt, with math to meet!

"To find the treasure, we must be wise,
We'll look for sets—so use your eyes!"
Grandpa said with a twinkle bright,
Morgan nodded—this felt just right!

"A set's a group, they share a trait,
Like round things here—oh won't this be great!"
Morgan searched and soon she found,
A soccer ball, an apple round.

"Now find me things that all are red,
Look around, use your head!"
Morgan grabbed a flower bright,
A ruby leaf, her backpack light.

"Now tell me dear, what things appear,
In both those sets? Look very near!"
Morgan thought, then gave a clap,
"The apple's round and red—oh snap!"

"That's an intersection, you see?"
Grandpa chuckled, filled with glee.
"Some sets don't meet, they stand apart,
Like noisy things and things with heart!"

They sorted, grouped, and laughed a lot,
Morgan learned with every spot.
She turned to Grandpa, her heart so light,
"This treasure hunt has been just right!"

Grandpa grinned, his arms stretched wide,
"The treasure's here, deep inside!
Math's not just numbers, dull and cold,
It helps us sort, it makes us bold!"

Morgan giggled, her mind so bright,
"Best treasure hunt, pure delight!"

Story Written by ChatGPT with thoughtful prompt engineering

2.1: The Language of Sets

2.1.1: Written Retrieval and Problem Solving Practice

2.1.2: Online Retrieval and Problem Solving Practice

2.1.3: Problem Solving with Sets

2.2: Whole Numbers and Numeration Systems

2.2.1: Written Retrieval and Problem Solving Practice

2.2.2: Online Retrieval and Problem Solving Practice

2.3: Hindu-Arabic Numeration System

2.3.1: Written Retreival and Problem Solving Practice

2.3.2: Online Retrieval and Problem Solving Practice

2.4: Chapter 2 Resources

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2.1: THE LANGUAGE OF SETS

As a future elementary school teacher, understanding set theory is essential to developing mathematical thinking and reasoning. Set theory provides a structured way to categorize and organize objects, making it a powerful tool for defining numbers and mathematical operations. The fundamental ideas of grouping, comparing, and relating collections of objects naturally align with how young children develop number sense (Finnan, 2006; Feike, Schwingendorf, and Gregg, 2018). By studying set theory, you gain insights into the logical foundations of arithmetic, helping you guide students through their early stages of mathematical understanding. Additionally, set notation offers a universal mathematical language that enhances problem-solving and communication across disciplines, equipping you with the tools to teach mathematics in a more structured and meaningful way (Feike et al., 2018).

One of the most accessible and visual applications of set theory is the use of Venn diagrams, which help illustrate relationships between different sets and make abstract concepts more tangible for learners. Understanding how to represent unions, intersections, and complements of sets graphically will enhance your ability to teach students how to classify, compare, and organize information—an essential skill not only in math but across subjects. Mastering these concepts will strengthen your ability to teach foundational math skills, foster deep mathematical thinking, and build problem-solving confidence in your students. Whether you're explaining counting, arithmetic operations, or logical reasoning, set theory will become an invaluable part of your instructional toolkit, helping you create engaging and effective learning experiences (Feike, Schwingendorf, and Gregg, 2018).

Before we jump into the formal language of Sets, let's take a glimpse of what some of these ideas look like for children. Use this video of UCLA Math Educator Dr. Megan Franke **(Supporting the Mathematical Brilliance of Young People)** sharing how early learners bring brilliant ideas into the classroom and how this looks when they start doing mathematics. View the video from 2:39 to 7:15, then answer Dr. Franke's question, "what do you notice that is brilliant?"

YouTube Video

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Make sure to hear what others found brilliant before moving on. We will begin to see how what we saw in the video emerges in this chapter.

Sets

The Language of Sets

A *collection of objects* is called a **set** and the objects are called **elements** or **members** of the set. Sets can be defined in three common ways: **(1)** a verbal description, **(2)** a listing of the elements separated by commas, with braces ("{" and "}") used to enclose the list of elements, and (**3**) set-builder notation. For example, consider the set of all primary colors. This set can be defined in multiple ways: (1) "the set of primary colors", (2) Listing: {Red, Blue, Yellow}, (2) Set-builder notation: C= {x | x is a primary color}.

Sets are usually denoted by capital letters such as A, B, C and so on. The symbols **"∈"** and **"∉"** are used to indicate that an object is or is not an element of a set, respectively. For example, if C represents the set of all primary colors, then Red ∈ C and Purple ∉ C. The set without elements is called the empty set (or null set) and is denoted by **{ }** or the symbol **∅.**

🧠**Let's Listen to Learn**❤️

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Two sets A and B are **equal,** written A = B, if and only if they have precisely the same elements.

Definition: One to One (1-1) Correspondence

A 1-1 correspondence between two sets A and B is a pairing of the elements of A with the elements of B so that each element of A corresponds to exactly one element of B, and vice versa. If there is a 1-1 correspondence between sets A and B, we write A ~ B and say that A and B are **equivalent** or **matching** sets.

The Birthday Party (Analogy)

Imagine you are organizing a birthday party with ten (10) children attending. You prepare ten party bags. If every child gets exactly one bag and no bags are left over, you have a **one-to-one correspondence** between children and hats. This means the number of children and the number of hats are the same.

Question: What scenario would cause us to have the sets (party bags and children) NOT be equivalent?

Think About and Discuss

- Think about the concept of **one-to-one correspondence**—when each element in one set is paired with exactly one element in another set, with no extras or missing pieces.
- Consider examples from your daily life where this idea appears. (Hint: Think about matching, pairing, or distributing things evenly.)
- Jot down at least **two examples** from real life that illustrate one-to-one correspondence.

In groups of **3-4 students**, share your examples with each other.

- How do your examples fit the definition of one-to-one correspondence?
- How does understanding this concept help in everyday life or in teaching math?

Definition: Subset

A set ***A*** is a ***subset*** of a set ***B*** if and only if every element of set ***A*** is an element of set ***B***. We write $A \subseteq B$**.**

*To show that one set is **not** a subset of another set, **there would have to be an element in the first set that is not in the second set.***

NOTE: If $A \subseteq B$ and B has an element that is not in A, we write $A \subset B$ and say that A is a **proper subset** of B (Figure 2.1.1). For example, {a, b}⊂{a, b, c}, since {a, b}⊆{a, b, c} and c is in the second set but not in the first.

🧠**Let's Listen to Learn**❤️

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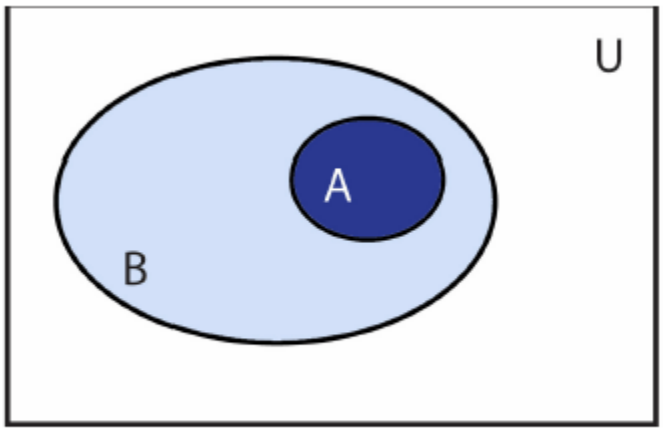


Figure 2.1.1: A is a **proper subset** of B.

There is one thing about sets that confuses a lot of people. By definition, we say that a set is a collection of objects. But in reality, a set can have no elements in it. A set containing no elements is called the ***empty set*** or ***null set***. Look at the definition of a subset very carefully. Is the null set is a subset of every set? I hope this was your conclusion: ***The null set is a subset of every set.*** [ANALOGY] Imagine a strict school rule posted on an empty classroom door: "Every student in this room wearing a neon-green cape must hop on one foot." Now, suppose you open the door and look inside—the room is completely empty. There are zero students. Can you find even one student wearing a neon-green cape who is refusing to hop? Of course not, because there are no students at all! Because you cannot find a single counterexample, the rule has not been broken. In logic, this is called being **vacuously true**—a condition is automatically satisfied simply because there are no items available to violate it.

Venn Diagrams

Relationships between sets can be visualized using Venn diagrams. Sets are represented by circles included in a rectangle that represents the universal set, i.e., the set of all elements being considered in a particular discussion. Figure 2.1.1 displays sets A and B inside a universal set U. Set A is comprised of everything inside circle A, and set B is comprised of everything inside circle B, including set A. We will come back in future sections to discuss and use Venn diagrams to provide a clear, intuitive way for students to classify and compare groups of objects or numbers. By comparing different sets, students can grasp the idea of greater than, less than, and equal to.

Example 2.1.1

Let A be the set of all counting numbers less than 8. This set is described by words (verbal description). The list of its elements provides another description A = {1, 2, 3, 4, 5, 6, 7}. Convert this set into Set-builder notation.

Answer available at the end of this section.

Example 2.1.2

A = {a, b, c, d, e} and B={a, c, f, e, b, g, d}. Is it true that $A \subseteq B$ **?** Is it true that $A \subset B$ **?**

Answer available at the end of this section.

Example 2.1.3

Are these statements TRUE for ANY set A?

a) $A \subset A$ **b)** $\emptyset \subset A$

Answer available at the end of this section.

Sets of Numbers

We work with different sets of numbers that play an important role in mathematics.

- Counting numbers are {1, 2, 3, 4, 5, 6, ...}
- Whole numbers include zero and counting numbers {0, 1, 2, 3, 4, 5, 6, ...}
- Even counting numbers include counting numbers that are multiples of two {2, 4, 6, 8, 10, ...}
- Odd counting numbers include counting numbers that are not multiples of two {1, 3, 5, 7, 9, ...}

Finite / Infinite Sets

Sets can be classified into two main types: **finite and infinite**. A set is considered finite if it contains no elements (an empty set) or if its elements can be counted in a way that eventually reaches an end. In contrast, an infinite set continues indefinitely without ever stopping. More precisely, a set is finite if (1) it is empty or (2) its elements can be placed in a one-to-one correspondence with the set {1, 2, 3, ..., n}, where *n* is some whole number. If a set does not meet these conditions, meaning it cannot be matched with a finite list of counting numbers, then it is classified as infinite.

Example 2.1.4

Determine whether the following sets are finite or infinite.

a) {100, 200, 300} b) {a, b, c, d, ... y, z} c) {5, 10, 15, 20, 25, ...}

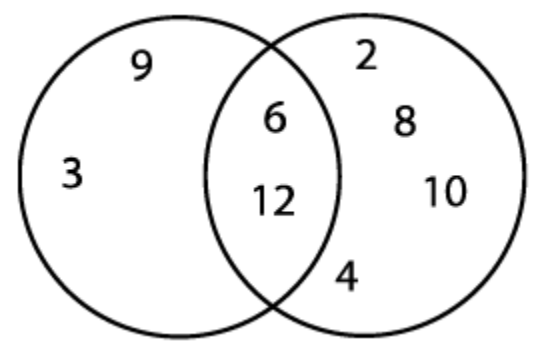
Answer available at the end of this section.

Cardinality

Something else to consider when working with sets is how many elements are in a given set. If we count the number of elements in a set, we call that number the **cardinality** or **cardinal number** of that set. To denote the cardinality of set **C={a, b, c}**, we write n(**C**) = 3. Think of the *n* as the ***number*** *of objects in the set*. When you are asked about the cardinality of a set, the answer is a number. We say that two sets are **equivalent** if they each have the *same* ***number*** *of elements* which is to say they have the *same cardinality*. We use the symbol, ~, to denote equivalence. For instance, if set **P** and **H** each contain four elements, we can write **P** ~ **H**. REMEMBER: Two sets are **equal** if they have the ***exact same elements***.

Wonder, Play, Grow

Where would you place the following numbers in the Venn diagram? 20, 21, 23, 24



Discuss with your peers where you would place the numbers, why. Be prepared to share your reasoning.

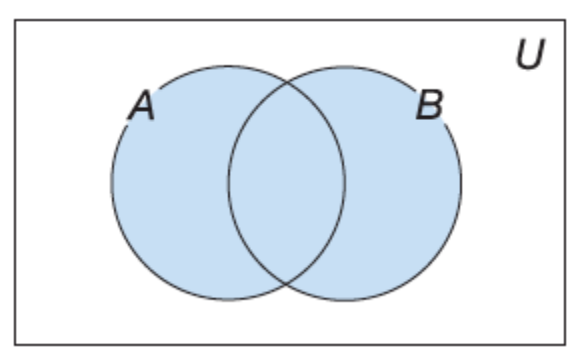
Answer available at the end of this section.

Operations of Sets

One of the most important ways we organize mathematical ideas is through **sets**. By studying set operations, we can see how math provides a precise way to describe relationships between groups of objects. This is one way that mathematics serves as a universal language—it allows us to clearly communicate ideas about numbers, patterns, and logical relationships. Let's explore some simple set operations and how they help us understand and organize information.

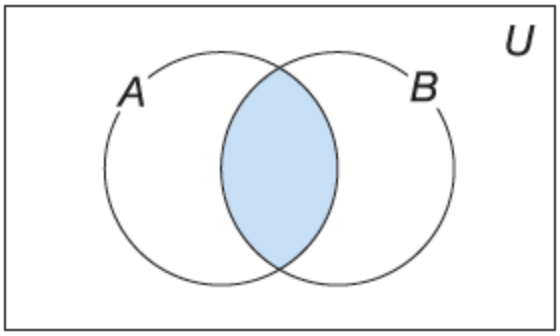
Definition: Union of Sets $A \cup B$

The union of two sets A and B, written $A \cup B$, is the set that consists of all elements belonging either to A or to B (or to both). Informally, $A \cup B$ is formed by putting all the elements of A and B together.



Definition: Intersection of Sets $A \cap B$

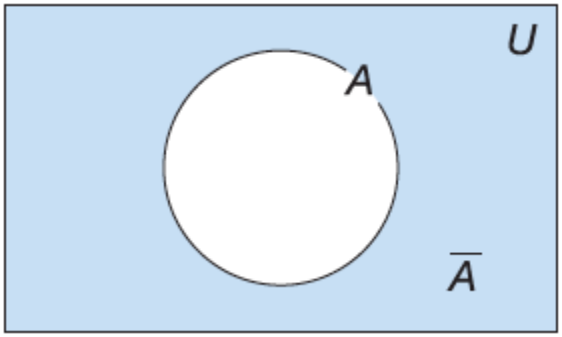
The intersection of sets A and B, written $A \cap B$, is the set of all elements common to sets A and B. Thus the intersection of A and B is the set of elements shared by A and B.



NOTE: Two sets are ***disjoint*** if their intersection is empty

Definition: Complement of Set A $\overline{A}$

The complement of a set A, written $\overline{A}$, is the set of all elements in the universe, U, that are not in A.

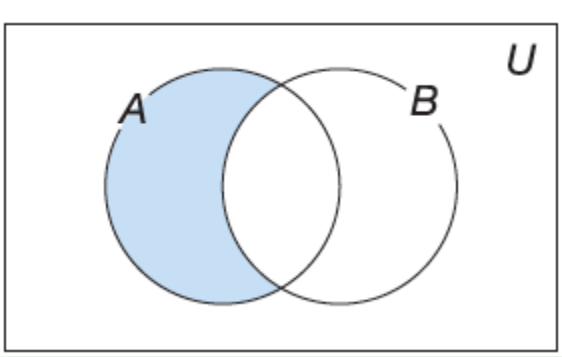


NOTE: The ***complement*** of a set **A**, can be written $\mathbf{A^c}$ or $\bar{A}$ or **A'**

For example, if **U**={1, 2, 3, 4, 5} and **H** = {3, 4}, then $\bar{H}$ = {1, 2, 5).

Definition: Difference of Sets A - B

The set difference (or relative complement) of set B from set A, written A – B, is the set of all elements in A that are not in B.



So if **F**= {1, 2, 3 ,4} and **G** = {1, 3, 5, 7}, then to find **F – G**, list the elements of **F** and cross off any that are in G: {~~1~~,2,~~3~~,4}. The answer is: {2, 4)

Audio

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Example 2.1.5

Try the following problems in which the universe is the first 9 counting numbers. If U represents the universe, we have U = {1, 2, 3, 4, *5,* 6, 7, 8, 9} Three subsets of U are defined as follows: **A** = {1, 2, 3, 4, 5} **B** = {2, 4, 6, 8} **C** = {3, 5, 7}

	a. **A** ∩ **B** = _____
b. **B** ∩ **C** = _____	c. **A** ∪ **B** = ____
d. **A** ∪ **C** = _____	e. **B** ∪ **C** = ____
f. $\mathbf{A}^C$ = _____	g. $\mathbf{B}^C$ = ____
h. $\mathbf{C}^C$ = _____	i. **A** – **B** = _____
j. **B** – **A** = ____	k. **A** – **C** = _____
l. **C** – **A** = ____	m. **B** – **C** = _____
n. **C** – **B** = _____	

*Did you notice that the order matters **for** difference, but not for union or intersection?*

Answer available at the end of this section.

Retrieval Practice

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Wonder, Play, Grow

Work with at least one peer to try these "more involved" problems. These will take more than one step to do. You might need scratch paper. **U** = {1, 2, 3, 4, 5, 6, 7, 8, 9}. The subsets are: **A**= {1, 2, 3, 4, 5} **B** = {2, 4, 6, 8} **C** = {3, 5, 7}

a. (**A** ∪ **C**) – **B** = ____	b. $(\mathbf{A} - \mathbf{C})^C$ = ____
c. (**A** ∩ **B**) ∪ **C** = ____	d. (**A** ∪ **C**) ∩ **B** = ____

e. $(\mathbf{A} \cup \mathbf{C})^C$ = ___	f. $\mathbf{A}^C \cap \mathbf{B}^C$ = ___

Answer available at the end of this section.

There is one more operation we will define on sets. So far, you've worked with union, intersection, difference and complement. Now we will learn what it means to take the Cartesian product of two sets, A and B. If you are finding the Cartesian product, the answer is a set that contains ordered pairs. The order matters! The first element must come from the set written to the left of the $\times$ and the second element must come from the set written to the right of the $\times$.

Cartesian product

The **Cartesian product** of set **A** with set **B**, which is written **A** $\times$ **B** and is read as '**A** cross **B**" is the set of all possible ordered pairs (a,b), where a $^{\text{int}}$ **A** and b $^{\text{int}}$ **B**.

NOTE: Using set builder notation, we can write ◇×◇={(a,b) | a∈◇ and b∈◇}.

More Info: Cartesian Products IRL

Example 2.1.6

If A = {x, y, z} and B = {a, b}, find A $\times$ B and B $\times$ A.

Answer available at the end of this section.

Wonder, Play, Grow

If a set **B** contains 5 elements (in other words, n(**B**) = 5) and a set **C** contains 7 elements (or n(**C**) = 7), then how many elements (where each element is an ordered pair) are in the Cartesian product for each of the following?

a. n(B $\times$ C) = ___	b. n(B $\times$ B) = ___	c. n(C $\times$ C) = ___

Answer available at the end of this section.

Answers

Example 2.1.1

A = {a is a counting number | a < 8}

Here, "a is a counting number" specifies that the set consists of numbers of a certain type and a < 8 specifies that we want to consider the numbers possessing this particular property.

Example 2.1.2

Both are true because all elements of A are in B (making A⊆B true), and B has extra elements not in A (making A⊂B true as a proper subset). Make sure to understand the distinction between the two. Remember, math is a language. In this section we will see the language of sets come to life.

Example 2.1.3

a) is False. a) Every element of A belongs to A, A cannot be a proper subset of itself and b) is True. The argument behind this statement is a little bit twisted. The empty set contains no elements. The definition of a subset requires that *every element* of the first set belongs to the second set.

Because the empty set has **no elements**, there is never an element you can point to that violates this rule. You can never find an element inside the empty set that isn't inside set A. Therefore, by default and logical necessity, the empty set is a subset of every set in existence, $\emptyset \subseteq \mathbf{A}$.

Example 2.1.4

a) finite since it can be matched with the set {1, 2, 3} b) finite since it can be matched with the set {1, 2, 3, ..., 26} c) infinite set

The **left circle** contains numbers **divisible by 3** (3, 9, 6, 12). The **right circle** contains numbers **even numbers** (2, 4, 6, 8, 10, 12).

20: Even number → Goes in the right circle. **21**: Divisible by 3 → Goes in the left circle. **23**: Neither even nor divisible by 3 → Stays outside the circles.

Example 2.1.5

a. {2, 4}
b. Ø
c. {1, 2, 3, 4, 5, 6, 8}
d. {1., 2, 3, 4, 5, 7}
e. {2, 3, 4, 5, 6, 7, 8}
f {6, 7, 8, 9}
g. {1, 3, 5, 7, 9}
h. {1, 2, 4, 6, 8, 9}
i. {1, 3, 5}
j. {6, 8}
k. {1, 2, 4}
l. {7}
m. {2, 4, 6, 8}
n. {3, 5, 7}

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a. {1, 3, 5, 7}
b. {3, 5, 6, 7, 8, 9}
c. {2, 3, 4, 5}
d. {2, 4, 6, 8}
e. {6, 8, 9}
f. { 7, 9}

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Example 2.1.6

A × B = {(x, a), (x, b), (y, a), (y, b), (z, a), (z, b)}
B × A = {(a, x), (b, x), (a, y), (b, y), (a, z), (b, z)}
Notice there were 3 elements in one set and two in the other. The Cartesian product has six elements in it, six ordered pairs.

a) 35 b) 25 c) 49

2.1.1: Written Retrieval and Problem Solving Practice

2.1.2: Online Retrieval and Problem Solving Practice

2.1.3: Problem Solving with Sets

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2.1.1: Written Retrieval and Problem Solving Practice

Student Learning Objectives

SLO 2.1 Students will demonstrate an understanding of foundational set theory by defining and manipulating sets and using set operations (e.g., union, intersection, complement) to construct and analyze new sets.

The mild assignment 🌶 is a shorter and less challenging assignment. The medium assignment🌶🌶 is right in the middle in terms of length and level of difficulty. Finally, the spicy assignment 🌶🌶🌶 is a longer assignment that is the most challenging.

(Spicy 🌶🌶🌶 Problems are OPTIONAL, but go for it when you have the time to challenge yourself)

Exercise 2.1.1.1 🌶

Which of the following sets are equal to {4, 5, 6}?

a) {5, 4, 6}

b) {7, 8}

c) Whole numbers less than 7.

d) Whole numbers greater than 3 and less than 7.

e) {e, f, g}

Answer available at the end of this section.

Exercise 2.1.1.2 🌶

True or False

a) $5^{\text{int}}\ \{5, 6, 7, 8\}$

b) $5 \notin \{2, 3, 4, 6\}$

c) $\{1, 2, 3\} \subseteq \{1, 2, 3\}$

d) $\{x, y, z\} \subset \{x, y, z\}$

Answer available at the end of this section.

Exercise 2.1.1.3 🌶

Determine which of the following sets are equivalent to the set {x, y, z}.

a) {w, x, y, z}

b) {1 , 5, 17}

c) {w, w, w}

d) {•, ♦, ►}

Answer available at the end of this section.

Exercise 2.1.1.4

a) List all of the subsets of {a, b, c}.

b) List all the proper subsets of {@, ©}

Answer available at the end of this section.

Exercise 2.1.1.5

True or False?

a) The set {10̸5, 11̸0, 115, 12̸0, ...} is an infinite set.

b) The set of whole numbers less than 6 is an infinite set.

c) The set of all books in your house is an finite set.

d) If A is infinite, and B is a subset of A, then B is always an infinite set.

Answer available at the end of this section.

Exercise 2.1.1.6

Show two different one-to-one correspondence between the sets {0̸, 1, 2, 3} and {w, x, y, z}.

Answer available at the end of this section.

Exercise 2.1.1.7

Given F = { 2, 4, 6, 8, 10̸, ...} and G = {4, 8, 12, 16, ...} answer the following questions.

a) Find $A \cap B$

b) Find $A \cup B$

c) Is G a subset of F? Explain your thinking.

d) Is F equivalent to G? Explain your thinking.

Answer available at the end of this section.

Exercise 2.1.1.8

Find the Cartesian Product for the following.

a) {a} x {b, c}

b) {5} x {x, y, z}

c) {2, 3} x {3, 2}

d) {x, y, z} x {5}

Answer available at the end of this section.

Exercise 2.1.1.9

A = {1, 2, 3, 4} , B = {4, 5, 6, 7}, and C = {1, 2, 7, 8}. List elements of the sets...

a) $A \cap B$

b) $A \cup B$

c) $(A \cup B) \cap C$

d) $(A \cup C) - B$

Answer available at the end of this section.

Exercise 2.1.1.10

Given A = {0, 1, 2, 3, 4, 5}, B={0, 2, 4, 6, 8, 10}, and C = {0, 4, 8}, find each of the following.

a) $B \cap C$

b) B - C

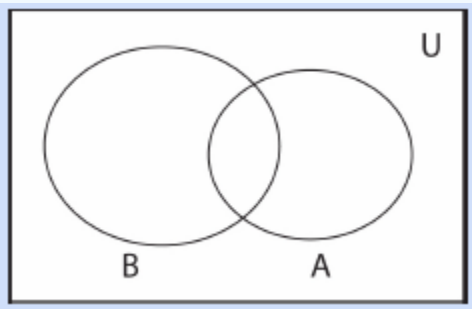
c) $(A \cup B) \cap C$

d) $A \cap \emptyset$

Answer available at the end of this section.

Exercise 2.1.1.11

Draw Venn diagrams of the sets A and B. Shade the area that represents the result of operations.



a) $\overline{A} \cap B$ b) $\overline{A} \cup \overline{B}$ c) $B - (A \cap B)$

Answer available at the end of this section.

Exercise 2.1.1.12 🌶🌶

Kayla says that if two sets are equivalent, they must be equal. Christina says that if two sets are equal, then they must be equivalent. Who is correct? Explain.

Answer available at the end of this section.

Answers

Exercise 2.1.1.1🌶

a) equal b) NOT equal c) Not equal d) equal e) Not equal

Exercise 2.1.1.2🌶

a) True b) True c) True d) False

Exercise 2.1.1.3🌶

a) No b) Yes c) No d) yes

Exercise 2.1.1.4🌶🌶

a) {a}, {a, b}, {a, b, c}, {a, c}, {b, c}, {b}, {c}, {}

b) {@} , {©}, {}

Exercise 2.1.1.5🌶🌶

a) True b) False c) True d) false

Exercise 2.1.1.6🌶🌶

Here are two different one-to-one correspondences between the sets {0, 1, 2, 3} and {w, x, y, z}:

Correspondence 1:

0 ↔ w 1 ↔ x 2 ↔ y 3 ↔ z

Correspondence 2:

0 ↔ x 1 ↔ w 2 ↔ z 3 ↔ y

Exercise 2.1.1.7🌶🌶

a) {4, 8, 12, 16, ...} = G

b) {2, 4, 6, 8, 10, ...} = F

c) Yes, all of the elements of G are an element of F.

d) Yes - because we can match each number in the first set to exactly one number in the second set, without missing any. This means they have the **same size**, even though both are infinite! In elementary set theory, **two sets are considered *equivalent*** if there is a **one-to-one correspondence. T**hat is, the elements can be matched up **exactly**, with nothing left over in either set.

Exercise 2.1.1.8🌶🌶

a) {(a,b),(a,c)} b) {(5,x),(5,y),(5,z)} c) Ordered pairs: {(2,3),(2,2),(3,3),(3,2)} d) {(x,5),(y,5),(z,5)}

NOTE: Sets resulting from b) and d) are NOT equal. Each set contains three ordered pairs, but **the order of elements in the pairs is different**.

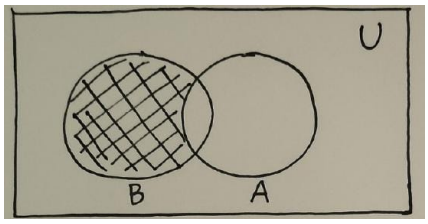
Exercise 2.1.1.9🌶🌶

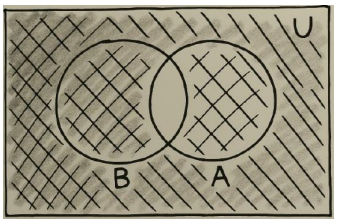
a) {4} b) {1, 2, 3, 4, 5, 6, 7} c) {1, 2, 7} d) {1, 2, 3, 8}

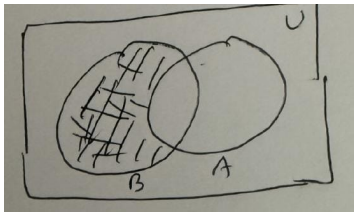
Exercise 2.1.1.10🌶🌶

a) {0, 4, 8} b) {2, 6, 10} c) {0, 4, 8} d) ∅

Exercise 2.1.1.11🌶🌶

a) 

The Logic: "Show me things that are NOT in A **AND** are in B." b) 

The Logic: This is a famous rule called **De Morgan's Law**. It means "Show me everything that is not in the shared middle." c) 

The Logic: "Take all of Circle B, then subtract the shared middle part."

Exercise 2.1.1.12🌶🌶

Kayla is incorrect, but **Christina is correct**.

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2.1.2: Online Retrieval and Problem Solving Practice

ADAPT 2.1.2.1

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2.1.3: Problem Solving with Sets

Problem solving with Venn diagrams combines the principles of productive problem solving, Pólya's four-step process, and the visual strengths of set theory. Productive problem solving emphasizes sense-making, reasoning, perseverance, and using multiple strategies. When applied to set problems, these ideas often involve using Venn diagrams to represent and explore relationships like union, intersection, and complement. Venn diagrams help students visualize how sets overlap or differ, organize information, and identify unknowns. This visual and strategic approach not only supports accurate solutions but also fosters deeper conceptual understanding.

Student Learning Objectives

SLO 2.1 Students will demonstrate an understanding of foundational set theory by defining and manipulating sets and using set operations (e.g., union, intersection, complement) to construct and analyze new sets.

Problem Solving Problems

Here are some problems for you to try. Let's start with you playing with representations and notation.

Problem 1

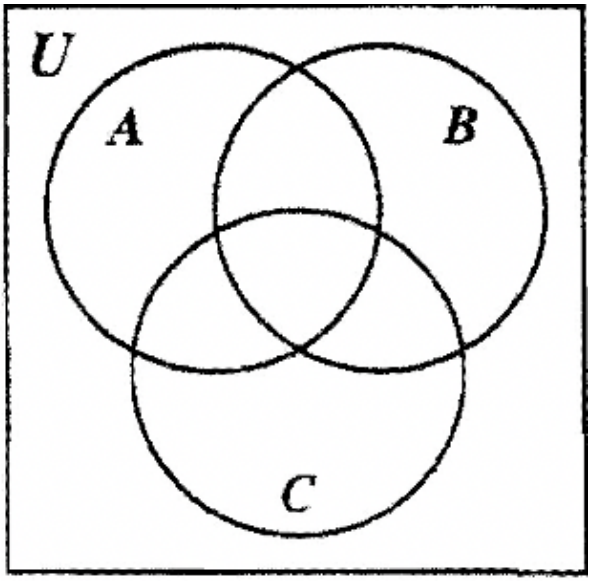
Click on the numbers in the Venn diagram to shade those particular area in order to match the prompted set notation.

_____

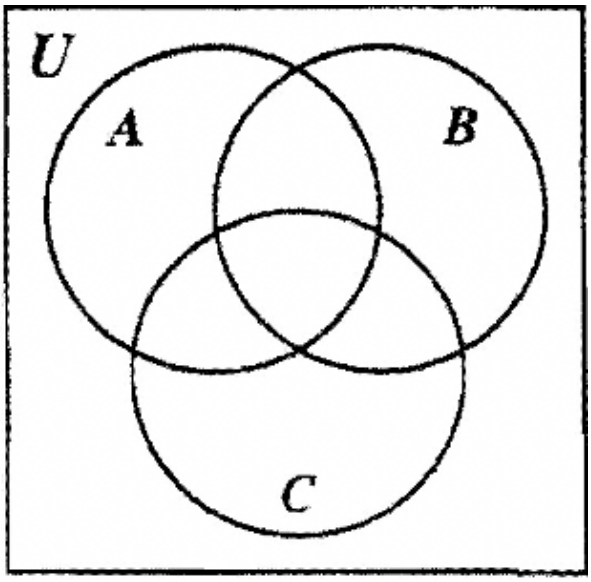
Problem 2

Shade the appropriate areas.

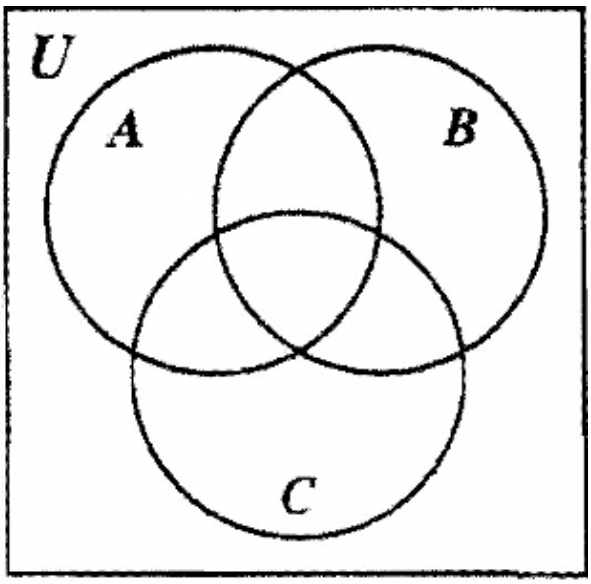
a) (**A** ∩ **B**) ∩ **C**



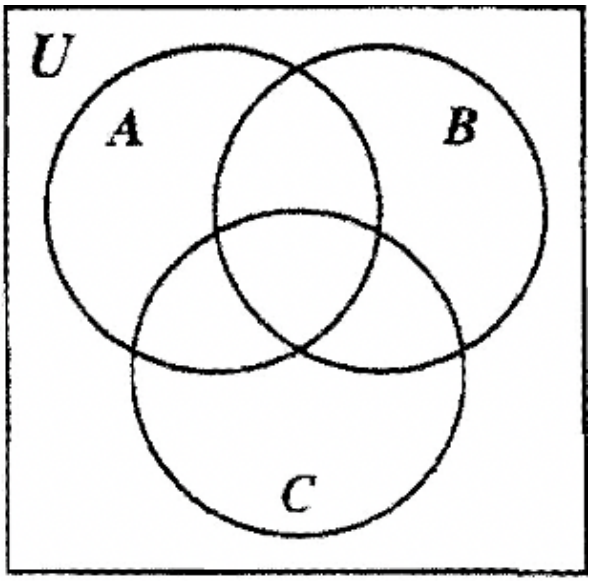
b) (**A** ∪ (**B** ∪ **C**))



c) $(\mathbf{C} \cap \mathbf{A})^c$



d) $(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C})^c$



YouTube Video

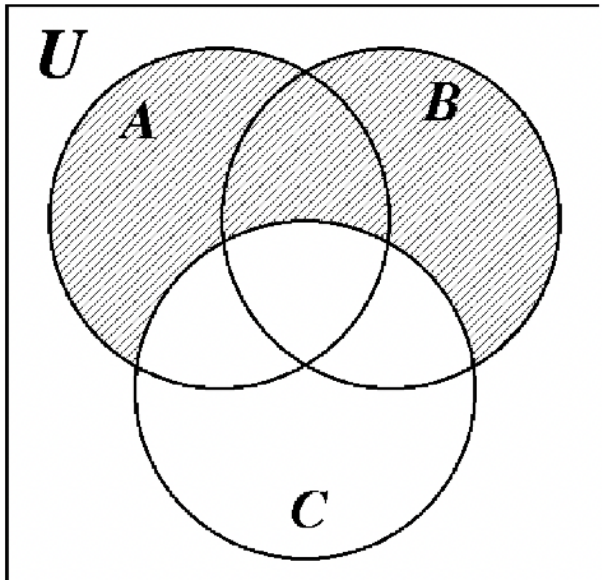
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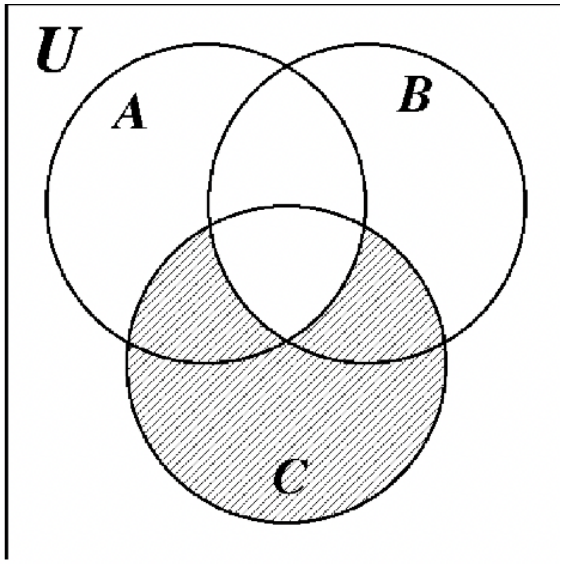
Problem 3

Identify the proper set notation for each Venn Diagram.

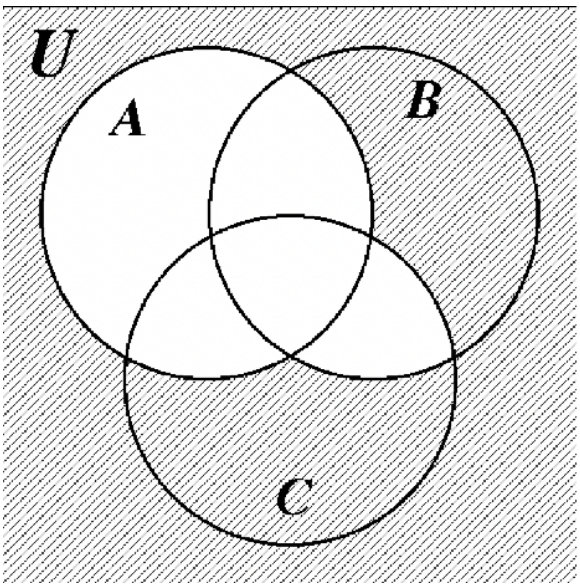
a)



b)



c)



Answer

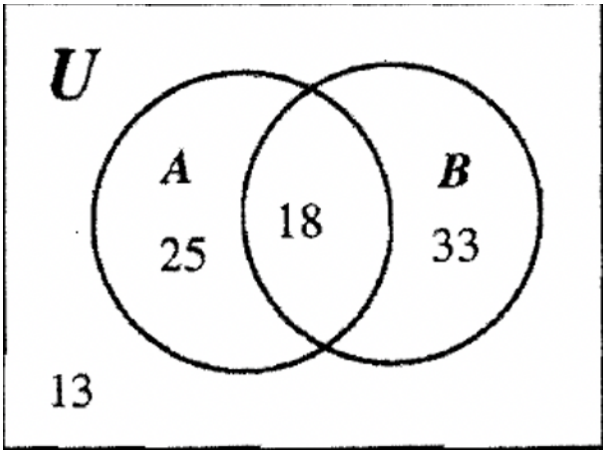
a) (**A** ∪ **B**) – **C**

b) C – (**A** ∩ **B**)

c) A^c – (**B** ∩ **C**)

Problem 4

Look at the Venn diagram below and answer the questions about the cardinality of the regions indicated.



a. $n(\mathbf{A})$ = ____	b. $n(\mathbf{B})$ = ____
c. $n(\mathrm{A}^c)$ = ____	d. $n(\mathrm{B}^c)$ = ____
e. $n(\mathrm{A} \cap \mathrm{B})$ = ____	f. $n(\mathbf{A - B})$ = ____
g. $n((\mathrm{A} \cup \mathrm{B})^c)$ = ____	h. $n(\mathbf{U})$ = ____

Imagine that the above problem was really a model for what kind of fruit a group of people packed in their lunches. **A** stood for those who packed apples and **B** stood for those who packed bananas. Assume apples and bananas are the only kind of fruit available. Then the questions you answered were these:

a. How many packed apples?
b. How many packed bananas?
c. How many didn't pack apples?
d. How many didn't pack bananas?
e. How many packed both kinds of fruit?
f. How many packed apples only?
g. How many didn't pack either fruit?
h. How many people were surveyed?

Look to see if you would have looked in the same regions if the questions were asked in this way.

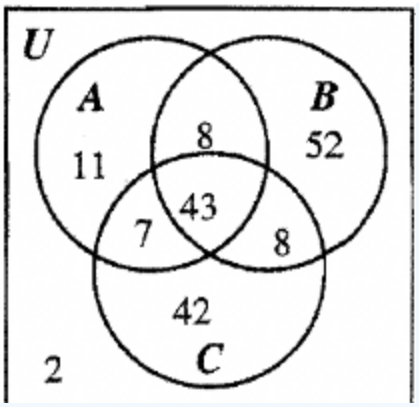
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Problem 5

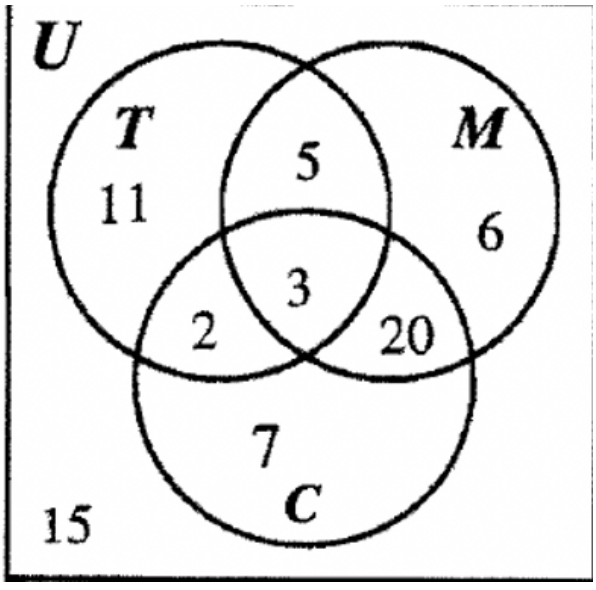
Let's see how well you do working with a three set Venn diagram. The Venn diagram below shows the cardinality of each region. A survey of college students showed the grades they received on their last report card. **A** stands for getting an A, **B** stands for getting a B and **C** stands for getting a C. Some people took only one course whereas others took more. Look at the Venn diagram and answer the following questions. *Extra Credit: Instead of just writing the number, write down what the question is asking* ***symbolically****. In other words, if the question is "How many received both A's and B's?" you would write* $n(\mathrm{A} \cap \mathrm{B}) = 51$.



a. How many got all A's? ___
b. How many got A's? ___
c. How many got B's and C's? ___
d. How many got only A's or B's? ____
e. How many didn't get a C? ____
f. How many got B's or C's? ____
g. How many got B's, but not C's? ___
h. How many got at least two different grades of A, B or C? ___
i. How many got exactly two different grades of A, B or C? ___

Problem 6

In another survey, questions were asked to see how many people were in their teens, how many were married and how many had children. **T** represents teens, **M** represents those who were married and **C** represents those having children. Look at the Venn diagram which shows the cardinality of each region to answer the following questions:

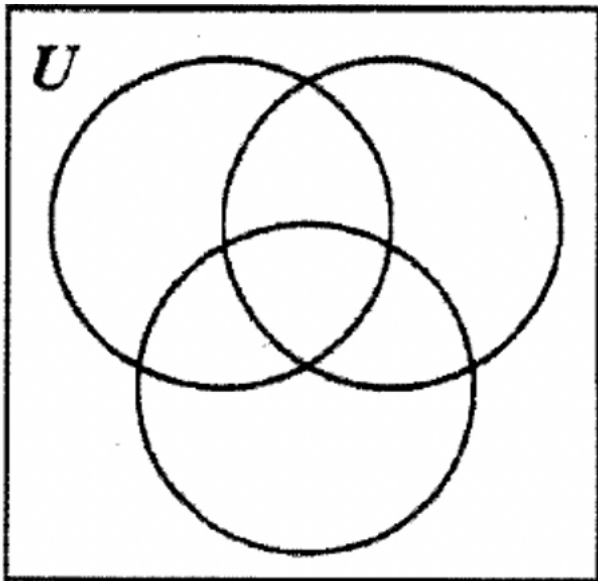


a. How many people were married? ___
b. How many teens had children? ___

c. How many people were in the survey? ___
d. How many people did not have children? ___
e. How many teens weren't married? ___
f. How many teens were married with children? ___
g. How many teens were in the survey? ___
h. How many people were unmarried with children? ___
i. How many people were married with children? ___

Problem 7

A survey of 60 college students was taken asking about which meals they regularly ate at school. Thirty students usually ate breakfast at school, 26 usually ate lunch there and 23 usually ate dinner at school. Six students said they ate all three meals at school. Eleven ate both breakfast and lunch there whereas 15 only ate breakfast at school and 12 only ate lunch there. Label the sets and fill in the Venn diagram below representing this information and answer the following questions:



a. How many students ate both lunch and dinner at school?
b. How many didn't eat any meals at school?
c. How many only ate dinner at school?
d. How many ate breakfast and/or dinner at school?
e. How many ate exactly one meal at school?
f. How many ate at least two meals at school?
g. How many ate both lunch and dinner at school, but not breakfast?

Problem 8

At a computer repair shop, 50 laptops were inspected. Suppose that 23 laptops needed a new screen and 34 laptops needed a new keyboard.

a) What is the least number of laptops that could have needed both?

b) What is the greatest number of laptops that could have needed both?

c) What is the greatest number of laptops that could have needed neither?

Audio

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Answer

a) Least that needed both: 7
b) Greatest that needed both: 23
c) Greatest that needed neither: 16

Go For It! 🌶️🌶️🌶️

Let **U**={1, 2, 3, 4, 5, 6, 7, 8, 9} with these subsets: **A**={6, 7, 8}, **B**={2, 3}, **C**={3, 4}, **D**={5}, **E**={1, 2, 3, 4, 5, 6}. Arrange these sets in a Venn diagram that clearly shows their relationship to each other. It's a bit tricky because there are five subsets, but you can do it!

Interactive APP for Problems 7 and 8

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2.2: WHOLE NUMBERS AND NUMERATION SYSTEMS

From Sets to Whole Numbers

Mathematics begins with understanding how we group and organize objects—this is where set theory comes in. As we recall, a set is simply a collection of objects, and this basic idea helps us define numbers and operations in a logical way. When young children count, they naturally group objects into sets (Feike, Schwingendorf, and Gregg, 2018). For example, when they see two apples and three bananas, they are subconsciously working with sets. Mathematicians formalize this idea by defining numbers as the sizes of sets—what we call cardinal numbers.

A key concept in counting is one-to-one correspondence, where each object in one set is paired with exactly one object in another set. This forms the basis for understanding numbers and operations like addition and subtraction. When we combine two sets, we are performing addition; when we take objects away, we are performing subtraction. Our numeration systems, such as the Hindu-Arabic system, build on these foundational ideas to represent numbers efficiently (NCTM, 2020). Understanding the connection between sets and whole numbers gives us a strong framework for learning mathematical operations and problem-solving (Feike, Schwingendorf, and Gregg, 2018).

Think About and Discuss

What do you remember about learning about numbers or how to count? Have you ever helped to teach a young child how to count? If so, what was it like? How long do you think it takes for someone to learn how to count for the first time?

Task 1:

- Now take a look at the video below from 25:52-27:32 (youtube: courtesy of UCLA Professor Dr. Megan Franke) to see how this 4-year old counts the bears.
- **Pause at 27:32.**

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(https://youtu.be/5Bl7SX7l6Qs?si=IUN7...h-jKI_f&t=1552)

Task 2

- What does she know? Discuss with your peers before playing the video to hear what the audience thinks.
- Play the video 27:32-29:31
- How does what the audience and Dr. Franke discuss compare to what you noticed?
- What are you wondering about now?

With all of this brilliant thinking in mind, lets learn about numbers.

One difficulty with the concept of numbers as we use them today is that *they are concepts* and not necessarily used only for counting objects. The number four, for instance, is used in many ways –four years old, room 4, four feet high, 4 children, four hours, four minutes after eight, mail station 4, 4 fingers, a grade of 4 on a quiz, count to 4, four times bigger than something, 4th place, four wishes, fourth in line, call 444-4444, code 44, etc.

So, what is a number?

Well, a **number** is an abstract idea that represents a quantity. You can't see or touch a number. The symbols that we see and touch *that are used to represent numbers* are actually *numerals*. A **numeral** is made up of one or more symbols. In the Hindu-Arabic system we use, the numerals are made up of one or more of these ten symbols –0,1,2,3,4,5,6,7,8 and 9. For instance, in the Hindu-Arabic system, the number three is represented by writing the numeral three which we write like this: 3 and the number two million is represented by writing the numeral two million which we write like this: 2,000,000. Notice, it took one symbol to write the number for three and seven symbols to write the number for two million (we don't count the commas as symbols because they aren't essential to writing the numeral). Although it took seven symbols to write the numeral two million, note that there are *only two different symbols* used to write that numeral –a two (2) and a zero (0).

There are three common uses of numbers. Describing how many elements (quantity) are in a finite set is the most common use of the counting numbers. When used this way, the number is referred to as a **cardinal number.** If a 1-1 correspondence between two sets exists, what does that imply about the cardinality of the two sets? Another way numbers are used concerns order. For instance, someone might be the fifth place winner or you might be eighth in line. When used in this way, numbers are called **ordinal numbers**. There are also numbers used for identification such as your phone number, zip code, social security number or credit card number. These are called **identification numbers** and it is the actual order of the symbols that is important –not the value of the number.

🧠**Let's Listen to Learn**❤️

Audio

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More Information: How we use Numbers

Ordering Whole Numbers

The relationship between sets, one-to-one correspondence, and ordering whole numbers is essential for understanding fundamental mathematics. Sets serve as collections of objects, and their size, referred to as a cardinal number, helps define numerical values. The concept of one-to-one correspondence establishes equivalence between sets by pairing each element of one set with exactly one element of another. This principle is crucial in understanding whole numbers, as it allows for meaningful comparisons between different quantities. Additionally, ordering whole numbers emerges from counting and comparing sets—if one set corresponds to a proper subset of another, it represents a smaller number. For instance, recognizing that a set of two elements is a subset of a set of five elements illustrates why 2 is less than 5. Ultimately, set theory provides the foundation for defining numbers, one-to-one correspondence enables comparisons, and these comparisons lead to the logical ordering of whole numbers.

Definition: Ordering Whole Numbers

Let a = n(A) and b = n(B). Then a < b (read "a is less than b") or b > a (read "b is greater than a") if A is equivalent to a proper subset of B.

Recall: n(A) is the cardinality of set A, the number of elements in set A

There are several ways to compare whole numbers. Three common methods include:

- **Counting Chant:** Whole numbers can be ordered by reciting the counting sequence ("one, two, three..."). For example, **2 is less than 5** because "two" comes before "five" when counting.
- **One-to-One Correspondence:** Comparing numbers by matching objects in sets. For example, **2 is less than 5** because a set with two elements corresponds to a proper subset of a set with five elements.

- **Whole-Number Line:** Numbers are arranged in order on a number line, starting from **0** on the left and increasing by one as you move to the right. (Musser, 2011)

Example 2.2.1

Determine the greater of the two numbers 3 and 4 in three different ways.

Answer available at the end of this section.

Numeration Systems

Today, most but not all civilizations around the world use the Hindu-Arabic system of counting and numerals 1,2,3,4,5,6,7,8,9,10,11,12,... There are many advantages to this number system. While you might think this common, widely used system of counting the obvious choice, you might consider both the advantages and disadvantages of Hindu-Arabic as we learn about other numeration systems.

Consider the **STROKE** system where there was only one symbol, a simple vertical stroke, to express a number. A vertical stroke like this, |, expressed one. To express the number seven, you would write seven strokes: ||||||| .

Think About and Discuss

a. Show how you would write the numeral eleven in STROKE:

b. Describe in words how to write the numerals five hundred twelve ; and two million in STROKE. Don't attempt to write the actual strokes!! We only have a semester!

c. Name some advantages and disadvantages of the STROKE system, compared with the Hindu-Arabic system.

Wonder, Play, Grow

The tally system improved the STROKE system by introducing the concept of **grouping**. You are probably familiar with this system where the fifth stroke was placed across the previous four so it would be easier to read since you can count by fives. STROKE is a single symbol system, but Tally can be thought of as a two symbol system, although the second symbol is really just made up of five strokes. Note the difference in writing the numeral twenty-eight in the two systems.

STROKE system:	*Tally system:*
||||||||||||||||||||||||||||	

a. Write the numeral for 17 in STROKE.

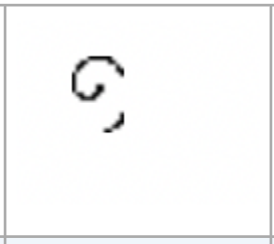
b. Write the numeral for 17 in Tally.

c. If you think of a single stroke (|) as one symbol and the 5-stroke tally () as a separate symbol, then it takes eight symbols to express the numeral for 28. In STROKE, it took 28 symbols to write the same number. How many symbols does it take to express the numeral for 172 in TALLY?

Egyptian Numeral System

The Old **Egyptian Numeral System** which was developed around 3400 B.C., employs an **additive system** of counting, where

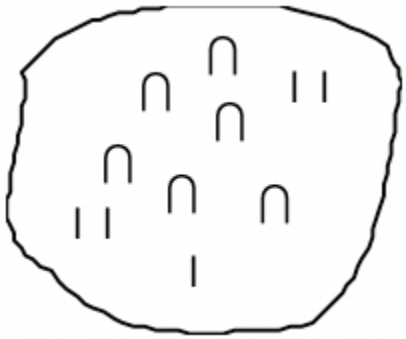
the symbols of the numeral do not have to be in any special order or even on a line going from left to right. The seven symbols in this ancient system are listed below, along with their Hindu-Arabic equivalents.

					
Staff	Heel bone	Scroll	Lotus flower	Pointing finger	Polliwog
One	Ten	Hundred	Thousand	Ten thousand	Hundred thousand
1	10	100	1,000	10,000	100,000

Here are three different ways to write the numeral *sixty five* in the Old Egyptian system:

	or	

or as shown below with all the symbols enclosed in a visual set



You simply add up the values of each of the symbols to get the answer.

Definition: Additive Number System

In an **additive number system,** the value of a written number is the sum of the face values of the symbols that make up the number. The only symbol necessary for an additive number system is a symbol for 1, however many additive number systems contain other symbols.

Remember: The order in which the symbols are written is irrelevant in an additive system.

In addition to being able to write the symbols of the numeral in any order, you may also use a *different combination* of symbols used to represent the same number. Here are two different ways to write the number for *one hundred twenty three*:



Example 2.2.2

Express each Egyptian numeral below as a Hindu-Arabic numeral. Show work.

a) 

b) 

c) 

Answer available at the end of this section.

Roman Numerals

Even though **Roman Numerals** are rare in today's society, they are still used and expected to be understood. They are taught in grades three through five, depending on the district. They can be seen in clocks, the Super Bowl, Film Credits for the copyright date like MCMLXII, preface of textbooks and others like Star Wars Episode VI and WWII.

2.2.1: Basic Table of Roman Numerals

Value	Symbol
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

2.2.2: The Complete Table of Roman Numerals

	1	2	3	4	5	6	7	8	9
ONES	I	II	III	IV	V	VI	VII	VIII	IX
TENS	X	XX	XXX	XL	L	LX	LXX	LXXX	XC
HUNDREDS	C	CC	CCC	CD	D	DC	DCC	DCCC	CM
THOUSANDS	M	MM	MMM	$\overline{\text{IV}}$	$\overline{\text{V}}$	$\overline{\text{VI}}$	$\overline{\text{VII}}$	$\overline{\text{VIII}}$	$\overline{\text{IX}}$
TEN THOUSANDS	$\bar{X}$	$\overline{XX}$	$\overline{XXX}$	$\overline{XL}$	$\bar{L}$	$\overline{LX}$	$\overline{LXX}$	$\overline{LXXX}$	$\overline{XC}$
HUNDRED THOUSANDS	$\bar{C}$	$\overline{CC}$	$\overline{CCC}$	$\overline{CD}$	$\bar{D}$	$\overline{DC}$	$\overline{DCC}$	$\overline{DCCC}$	$\overline{CM}$

The **Roman numeral system** is an ancient numeration system that represents numbers using combinations of letters from the Latin alphabet (I, V, X, L, C, D, M). This system is primarily **additive**, meaning values are determined by summing the symbols (e.g., **VII = 5 + 2 = 7**). However, it also incorporates a **subtractive** principle, where a smaller numeral placed before a larger one indicates subtraction (e.g., **IV = 5 - 1 = 4** and **IX = 10 - 1 = 9**). Additionally, Roman numerals rely on **grouping**, as repeated symbols represent multiples of a value (e.g., **XXX = 10 + 10 + 10 = 30**). Unlike place-value systems, Roman numerals do not use zero, and their structure makes arithmetic operations more complex compared to modern numeration systems.

Example 2.2.3

For all numbers except 4 and 9, we **ADD** the Roman Numerals together, in order from left to right, greatest value to lowest value.

$$
\begin{aligned}
11 &= 10 + 1 = XI \\
8 &= 5 + 1 + 1 + 1 = VIII \\
123 &= 100 + 10 + 10 + 1 + 1 + 1 = CXXIII \\
3816 &= 1000 + 1000 + 1000 + 500 + 100 + 100 + 100 + 10 + 5 + 1 = MMMDCCCXVI \\
7002 &= 5000 + 1000 + 1000 + 1 + 1 = \overline{V}IIII
\end{aligned}
$$

Example 2.2.4

For any number that includes a 4 or a 9, we subtract. When we are looking at a Roman Number expression and we see a Roman character **OUT OF ORDER**, which is the clue to **SUBTRACT**!

$$
\begin{aligned}
9 &= 10 - 1 = IX \\
4 &= 5 - 1 = IV \\
1400 &= 1000 + (500 - 100) = MCD \\
29452 &= 10000 + 10000 + 10000 - 1000 + (500 - 100) + 50 + 1 + 1 = \overline{IXXX}CDLII
\end{aligned}
$$

Retreival Practice

Practice converting Roman Numerals into Hindu-Arabic numeral until you feel comfortable with the process.

Interactive Author:chris cambré - https://www.geogebra.org/m/ttsxcbzn#material/aJ5q37Bs

Wonder, Play Grow

Write each Hindu-Arabic numeral as a Roman numeral using the least amount of symbols:

a. 32:	b. 561:
c. 708:	d. 2053:

Write each Roman numeral as a Hindu-Arabic numeral and show work:

a. MMDCLXXXVII: ___	b. MCCXXXII:____

The Roman numeration system developed and incorporated changes over time. One limitation of a simple additive system is that you eventually run out of symbols for very large numbers. The Romans devised a clever way to take care of this problem. If a bar was placed over a symbol or set of symbols, it indicated that number was to be multiplied by a thousand. More than one bar could be used on any given symbol or set of numerals. For example, two bars indicated the number was to be multiplied by 1000 1000 (or one million). Simply put, symbols with one bar over them are in the thousands, those with two bars over them are in the millions, those with three bars over them are in the billions and so on. Thus, the Roman numeration system is ***multiplicative*** as well as ***additive.***

Example 2.2.5

For each of the numeration systems we've learned about so far, state how many different symbols a person has to memorize to understand the system. For Roman, do not consider symbols with bars over them as different symbols.

a. Hindu-Arabic: _____	b. STROKE: ___	c. Tally: ___	d. Egyptian: ___	e. Roman: ___

Let's write the numeral 67 in the different systems and note how many total symbols (not necessarily different symbols) it takes to express the number 67 in each system.

Hindu-Arabic:	We write: 67	2 symbols
STROKE:	We write 67 strokes, shown below ||||||||||||||||||||||||||||||||||||||||||||||	67 symbols
Tally:	WE write 13 tally groups and 2 strokes, shown below 	15 symbols
Egyptian:	We write:	13 symbols
Roman:	We write: LXVII	5 symbols

Answer available at the end of this section.

Mayan Numerals

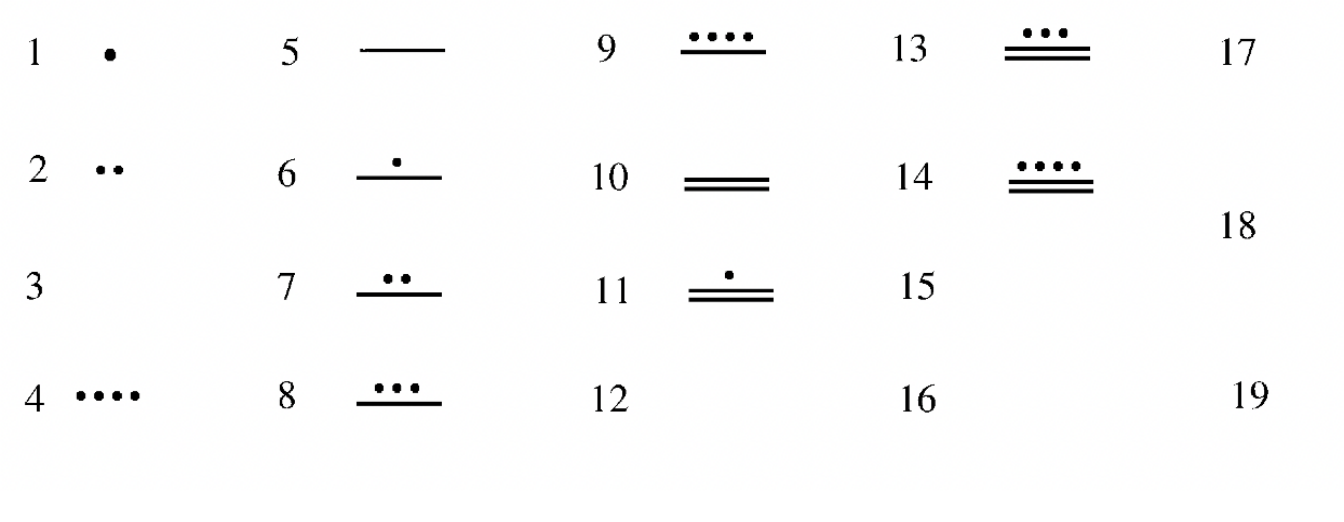
Mayan Numerals were developed by the Mayan priests of southern Mexico and Central America around 300B.C. It is believed to be the earliest **positional numeration system** incorporating a zero and using it for a placeholder. The Mayan numbers are based on three symbols:a dot, a bar, and a symbol for zero, or completion, usually a shell. It has a vertical place-value system.

Positional Number System

A **positional number system** is a method of representing numbers in which the position of each numeral (symbol) affects its value.

Example 2.2.6

Some Mayan Numerals are shown below. Try to figure out the pattern and then fill in the missing numerals.



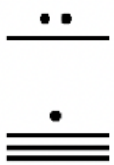
Answer available at the end of this section.

From what you have seen of this system so far, it might look like a simple additive system. One might guess that the numeral for 20 would be four line segments and that the numeral for 103 would be twenty line segments and three dots. At first glance, it is very similar to the tally system. However, the Mayans used a vertical ***positional* system.** The bottom level represented how many units (or ones), the second level up represented how many 20's, the third level up represented how many 360's (20 18), the fourth level up represented how many 7200's (20 18 20), the fifth level up represented how many 144,000's (20 18 20 20), etc. Except between the second and third level, each place value increased by a multiple of 20. It is almost a Base Twenty system except for that strange third level. Why the third level is 18 times the second level is explained later. Looking at the chart to the left, which shows the first four place values, may help you understand the system.

7200's
360's
20's
1's

To try and make sense of all of this, we will look at some Mayan numerals that have more than one position now. The numerals from one to nineteen only utilize the bottom level so it isn't apparent that the Mayan system is positional until you count past nineteen.

To the right is a two-level Mayan numeral. There is a 16 in the bottom level, representing 16 ones, or 16 (16×1), plus there is a 7 in the second level up, representing 7 groups of twenty, or 140 (7×20). We add the values of each level, 16 + 140, so the numeral you see represents 156.



You have seen two basic symbols in this system so far –a dot ( ), which represents the number one and a line segment ( ), which represents the number five. As previously mentioned, there needs to be a symbol for zero to incorporate the use of place value. For that, the Mayans used a shell that looked something like this:

To the right is a two level Mayan numeral with a zero in the bottom level, representing zero ones, or 0, plus a 13 in the second level up representing 13 groups of twenty, or 260 (13×20). So after adding the values together (0 + 260), the numeral you see represents the number 260.



🧠Let's Listen to Learn❤️

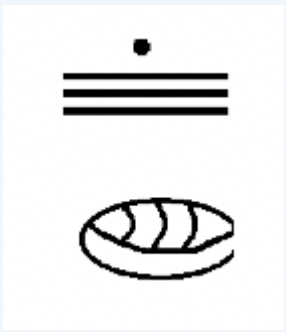
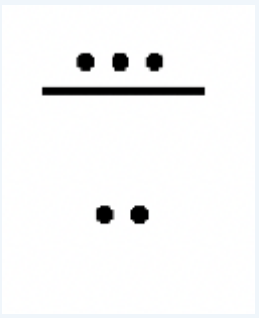
Audio

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Example 2.2.7

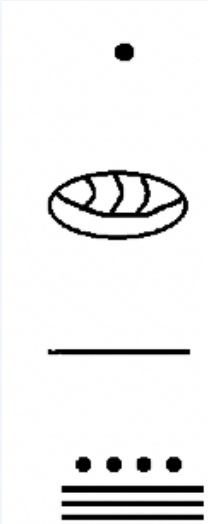
State the Hindu-Arabic equivalent of each Mayan numeral. Show how you obtained your answers. Note that proper space should be left between each place value. Otherwise, someone might incorrectly conclude the number shown for a. represents 14, which would be the answer if there was no space.

a. ____	b. ____	c. ____
		

Answer available at the end of this section.

Now, we'll go on to some three and four level Mayan numeral. Remember that the place value for the third level up is 360 and the place value for the fourth level up is 7200. See if you can figure these out on your own first.

Example 2.2.8

a. ___	b. ___	c. ___	d. ___	e. ___
				

Answer available at the end of this section.

Wonder, Play, Grow

It's a little trickier to start with a Hindu-Arabic numeral and convert it to Mayan, but with a little patience and practice, you'll be doing it quickly and accurately! Before showing a method for doing this, try the following exercise. Think about the place values of the various levels in the Mayan system.

Work with your peers to try and figure out (Problem Solve) how to write these Mayan Numerals for each of the following numbers.

a. 36	b. 387	c. 1234	d. 7654

To convert a number to a Mayan numeral, the first thing you'll have to determine is how many levels the numeral will have. Remember the levels: 1, 20, 360, 7200, 144000, ...

Check Your Answer: Online Mayan Numeral Converter

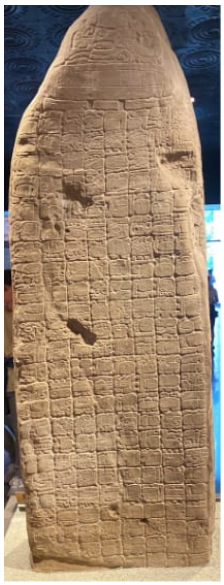
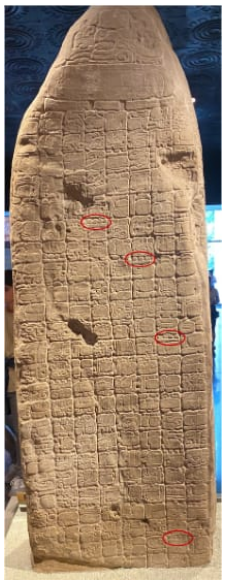
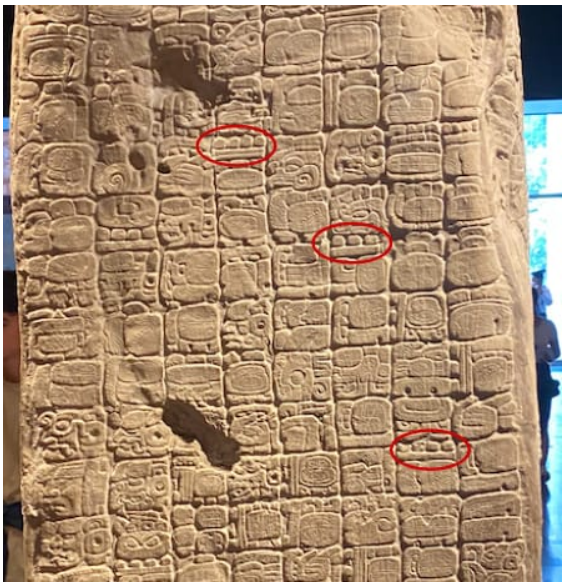
Mayan History

Another New World culture that utilized a place value numeration system was the ancient Maya. This civilization occupied a vast region encompassing southern Mexico and parts of present-day Guatemala, El Salvador, and Honduras. The Mayan civilization thrived for over 2,000 years, reaching its peak between 300 and 900 A.D. One of its most remarkable achievements was the development of an intricate hieroglyphic writing system consisting of approximately 1,000 glyphs. These glyphs could be either sound-based or meaning-based, though the majority of those that have survived remain undeciphered.

Following 900 A.D., the Mayan civilization experienced a sudden and dramatic decline—often referred to as The Great Collapse—during which its once-thriving cities were abandoned. The reasons behind this mass exodus remain a mystery, with theories ranging from natural disasters and epidemic diseases to warfare. Despite the Spanish Conquest, which began in the early 1500s, elements of Mayan culture persisted. The struggle against Spanish rule was marked by relentless brutality and lasted

nearly a century before the last independent Mayan kingdom was finally subdued in 1597.

The Mayan calendar year consisted of 365 days, divided into 18 months of 20 days each, with an additional 5-day period. This structure led to the adoption of a base-20, or vigesimal, counting system. Numbers were represented in two symbolic forms. The priestly class used elaborate glyphs depicting grotesque deity faces to signify the numbers 1 through 19, which were commonly used in carved stone inscriptions marking significant events. Meanwhile, the common people recorded numbers using a combination of bars and dots, where a short horizontal bar signified 5 and a dot represented 1. A distinctive feature of this system was the inclusion of a stylized shell symbol to denote zero—one of the earliest known uses of a symbol for that number (Boyer, 2011; Burton, 1984). Below are real photos of Mayan Glyhphs found at the **Museo Nacional de Antropología in Mexico City.** See if you can find more Mayan Numerals on the pictures - you may need to zoom in.



Photos Courtesy of Marty Romero

Number System Attributes

System	Additive	Subtractive	Multiplicative	Positional	Place Value	Zero
Tally	Yes	No	No	No	No	No
Egyptian	Yes	No	No	No	No	No
Roman	Yes	Yes	Yes	No	No	No
Mayan	Yes	No	Yes	Yes	Yes	Yes

Mayan Number Web App Exploration and Retrieval Practice

Answers

Example 2.2.1

1) "one", "two", "three", "four" - Chant Counting - "four" comes after "three" or "three" comes before "four", 3 < 4.

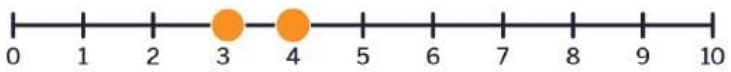
2) **Create two sets**:

Set A has **3 objects** (e.g., 🟦🟦🟦)

Set B has **4 objects** (e.g., 🟥🟥🟥🟥)

Using one-to-one correspondence, we determine that **4 is greater than 3**, because after matching each object from Set A with Set B, one object is left unmatched in Set B.

3) One a number line, three is the left of four, so 3 < 4.



Example 2.2.2

a) 10 + 10 + 1000 + 100000 + 100000 + 100000 = 301020

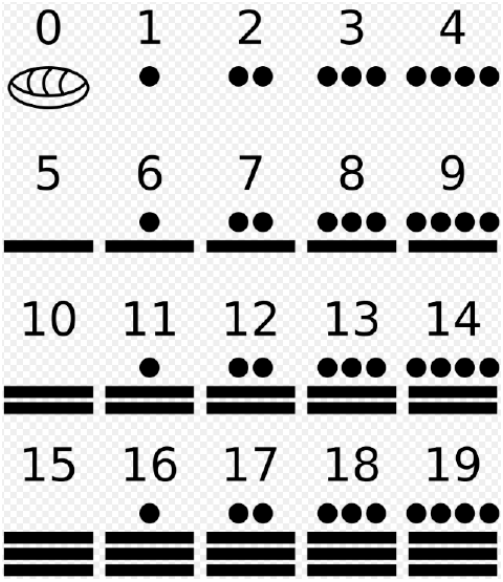
b) 10000 + 10 + 10 + 100000 = 110020

c) 4(1000000) + 2(100) + 7 + 10000 + 3(100) = 4010507

Example 2.2.5

a) 10 different symbols b) 1 symbol c) 2 symbols d) 7 different symbols e) 7 different symbols

Example 2.2.6



Example 2.2.7

a) 10(1) + 4(20) = 90 b) 0 + 16(20) = 320 c) 2(1) + 8(20) = 162

Example 2.2.8

a) 14(1) + 0 + 11(360) = 3974 b) 6(1) + 7(20) + 5(360) = 1946 c) 0 + 3(20) + 9(360) = 3300
d) 14(1) + 2(20) + 10(360) + 4(7200) = 32454 e) 19(1) + 5(20) + 0 + 1(7200) = 7319

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2.2.1: Written Retrieval and Problem Solving Practice

2.2.2: Online Retrieval and Problem Solving Practice

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2.2.1: Written Retrieval and Problem Solving Practice

Student Learning Objectives

SLO 2.2 Students will analyze and apply the properties and functions of whole numbers, distinguishing between their various uses (cardinal, ordinal, and identification) and explaining relationships such as greater than and less than through set-based representations.

SLO 2.3 Students will compare Egyptian, Roman, and Hindu-Arabic numeration systems, identifying differences in place value, positionality, and arithmetic operations, and evaluate each system's advantages and limitations.

The mild assignment 🌶 is a shorter and less challenging assignment. The medium assignment🌶🌶 is right in the middle in terms of length and level of difficulty. Finally, the spicy assignment 🌶🌶🌶 is a longer assignment that is the most challenging.

(Spicy 🌶🌶🌶 Problems are OPTIONAL, but go for it when you have the time to challenge yourself)

Exercise 2.2.1.1 🌶

Identify how the number is used in the sentence: cardinal, ordinal, or identification.

a) Sammy came in first place.

b) A cat is an animal with four legs.

c) I work in suite 201.

Answer available at the end of this section.

Exercise 2.2.1.2 🌶🌶

Explain how we know that 6 > 3.

Answer available at the end of this section.

Exercise 2.2.1.3 🌶🌶

Change to Roman Numerals.

a) 14

b) 52

c) 299

Answer available at the end of this section.

Exercise 2.2.1.4 🌶🌶

Change to Mayan Numerals.

a) 24

b) 152

c) 760

Answer available at the end of this section.

Exercise 2.2.1.5

Change to Hindu-Arabic Numerals.

a) MCMXCI

b) CMLXXVI

c) MMMCCXLV

Answer available at the end of this section.

Exercise 2.2.1.6

Change to Hindu-Arabic Numerals.

a) 

b) 

Answer available at the end of this section.

Exercise 2.2.1.7

Change to Hindu-Arabic Numerals.

a) 

b) 

c) 

Answer available at the end of this section.

Exercise 2.2.1.8 🌶🌶

Write sentences that show the number 33 used in each of the following ways: As a cardinal number, as an ordinal number, and as an identification number.

Answer available at the end of this section.

Exercise 2.2.1.9 🌶🌶🌶

Some children go through reversal stage; that is they confuse 13 and 31, 27 and 72, 59 and 95. What numerals would give Roman children similar difficulties?

Answer available at the end of this section.

Exercise 2.2.1.10 🌶🌶🌶

Which way is a child most likely to first learn the ordering of whole numbers less than 10: i) using the counting chant, ii) using the set method, iii) using the number line method? Explain.

Answer available at the end of this section.

Problem Solving Practice

Design a **new numeration system** using symbols of your choice. Your system must:

- Represent the numbers 1–50 clearly
- Use at least 5 different symbols
- Explain how your system is similar to or different from one of the systems we have studied so far.

You must also **justify**:

- Is your system positional or non-positional?
- What are the pros/cons of using your system compared to others?

Answers

Exercise 2.2.1.1 🌶

a) ordinal b) cardinal c) identification

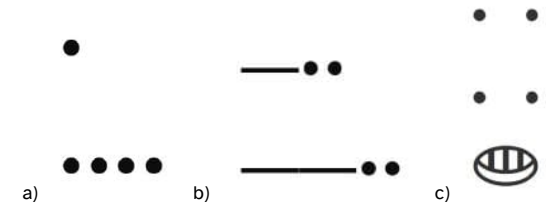
🌶🌶**Exercise** 2.2.1.2

1) Via chant counting, 6 comes after 3. On a number line, 6 is to the right of 3. One-to-One correspondence, there are 3 more objects in a set of 6 coparted to a set of 3 objects.

🌶🌶**Exercise** 2.2.1.3

a) XIV b) LII c) CCXCIX

🌶🌶**Exercise** 2.2.1.4



a) b) c)

🌶🌶**Exercise** 2.2.1.5

a) 1991 b) 976 c) 3245

🌶🌶**Exercise** 2.2.1.6

a) 2,342 b) 13,026

🌶🌶**Exercise** 2.2.1.7

a) 0 + 18(20) + 6(360) = 2,520 b) 7 + 14(20) + 0 + 6(7200) = 43487 c) 4 + 6(20) + 0 + 14(7200) = 100,942

🌶🌶**Exercise** 2.2.1.8

Cardinal number (used for counting):
◈ *There are **33 students** in Mrs. Rivera's class this semester.*

Ordinal number (used for order or position):
◈ *Lina finished in **33rd place** in the citywide spelling bee.*

Identification number (used as a label or code):
◈ *The player wearing jersey number **33** scored the winning goal.*

🌶🌶🌶**Exercise** 2.2.1.9

VI (6) and **IV (4) -** One is 5 + 1, the other is 5 - 1.

XI (11) and **IX (9)** - One is 10 + 1, the other is 10 - 1.

These confusions happen because Roman numerals rely heavily on **symbol order** to determine whether values are added or subtracted—just like place value in our Hindu-Arabic system.

🌶🌶🌶**Exercise** 2.2.1.10

A child is **most likely to first learn the ordering of whole numbers less than 10 using the counting chant** (**i**). Children often know how to "count to 10" before they can accurately count objects or compare sets. **ii) Set method (likely later)** This involves understanding that **larger sets have more objects** than smaller ones (e.g., 6 apples is more than 4 apples). It requires **conceptual understanding of quantity and one-to-one correspondence**, which comes **after** memorizing the counting sequence.

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2.2.2: Online Retrieval and Problem Solving Practice

ADAPT 2.2.2.1

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2.3: HINDU-ARABIC NUMERATION SYSTEM

The Hindu-Arabic Number System

The Hindu-Arabic numeral system, also known as the decimal or base-ten system, is the most widely used numeration system in the world today. It was developed by Indian scholars as early as 800 BCE and later transmitted to the Western world by Arabic mathematicians. The system's efficiency in arithmetic operations and its adaptability to complex mathematical computations contributed to its widespread adoption over other ancient systems like Roman numerals and Egyptian hieroglyphs.

Key Features:

1. **Digits:** The system uses ten symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) that can be combined to represent all possible numbers.
2. **Grouping by Ten:** Numbers are grouped in sets of ten, making it a base-ten system, likely influenced by the human ten fingers.
3. **Place Value (Positional Number System):** The value of a digit depends on its position in the numeral. For example, in the number 5984, the 5 represents thousands, the 9 represents hundreds, the 8 represents tens, and the 4 represents ones.
4. **Use of Zero:** A significant innovation of this system is the concept of zero as a placeholder, which allows for efficient representation of large numbers.
5. **Expanded Form (Additive and Multiplicative Number System):** Numbers can be written as the sum of each digit multiplied by its place value (e.g., 5984 = 5×1000 + 9×100 + 8×10 + 4).

Think About and Discuss

Before we proceed to learn about our base-10 number system (Hindu-Arabic) lets review what we know about the early learning of math for those you will be teaching.

Task 1: Considering what we learned so far, what are some of the first concepts of number and counting that children learn? Discuss with others and be ready to share with the class.

Task 2: Now view this video from Math Educator Graham Fletchy about his thoughts on early learning of number and counting. This video explores the progression of early number concepts and counting.

Task 3: Based on the progression of early number sense, what surprised you the most about how children develop mathematical understanding? What questions do you have now? Discuss with others and be ready to share with the class.

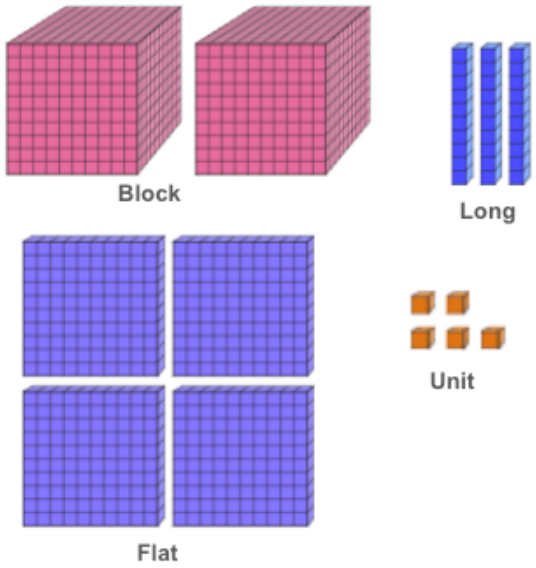
The progression of early number sense—subitizing, more/less comparisons, counting, cardinality, and conservation—lays the foundation for understanding **place value and the base-ten system.**

Models for Our Number System

The Hindu-Arabic numeration system operates on a base-ten structure, grouping numbers into sets of ten due to our natural inclination to count on our fingers. This fundamental principle replaces ten ones with one ten, ten tens with one hundred, and so forth. This concept is crucial for children developing number sense, and manipulatives such as Unifix cubes and Base Ten Blocks serve as effective teaching tools.



Unifix cubes, also known as linker cubes or multi-links, are particularly beneficial for young learners, allowing them to physically stack ten individual cubes together to form one ten, reinforcing the concept of regrouping in addition and subtraction. For instance, when solving 27 + 38, children can visually and physically group ten ones into a new ten, illustrating the process of carrying or trading. Similarly, in subtraction, breaking a ten into ten ones supports the understanding of place value. These hands-on experiences enable children to see ten both as a collection of individual units and as a single unit, fostering a flexible grasp of number relationships. (CML)



Base Ten Blocks provide another visual and tactile way for students to grasp place value, with different-sized blocks representing units (1), longs (10), flats (100), and blocks (1,000). Base Ten Blocks are common in elementary classrooms. They remain valuable for small-group instruction and remediation, particularly in helping students understand multi-digit operations and the borrowing and carrying processes in addition and subtraction. (CML)

Example 2.3.1

Express the number 5,302 in expanded form.

Answer available at the end of this section.

Numbers up to the Thousands Place

Use the interactive below to create numbers expressed in expanded form.

Interactive

Word Names for Hindu-Arabic numerals

Associated with each Hindu–Arabic numeral is a word name. Some of the English names are below.

0 zero 10 ten

1 one 11 eleven

2 two 12 twelve

3 three 13 thirteen (three plus ten)

4 four 14 fourteen (four plus ten)

5 five 24 twenty-four (2 tens plus 4 ones)

6 six 67 sixty-seven (6 tens plus seven ones)

7 seven 305 two hundred five (3 hundreds plus five ones)

8 eight 1,274 one thousand two hundred seventy-four

9 nine 25,100 twenty-five thousand one hundred

In numerals containing more than three digits, groups of three digits are sets off by commas. For example, the number 864,456,221,653,127,851 is read: ”eight hundred sixty four quadrillion four hundred fifty six trillion two hundred twenty one billion six hundred fifty three million one hundred twenty seven thousand eight hundred fifty one”. Notice that the word and does not appear in any of these names: it is reserved to separate the decimal portion of a numeral from the whole-number portion.

Numbers as Language

Many children initially perceive multi-digit numbers as a sequence of individual digits rather than understanding their place value (Fuson, 2004). For instance, they may see 435 as "four, three, five" rather than recognizing it as four hundreds, three tens, and five ones. The position of the number means something in the language of mathematics. To address this, elementary curricula introduce expanded notation, such as $576 = (5 \times 100) + (7 \times 10) + (6 \times 1)$, aiming to help students grasp the meaning behind each digit. True place value comprehension extends beyond knowing that the seven in 576 represents seven tens; it includes understanding that there are 57 tens within 576. Despite significant instructional time devoted to naming place values, many students struggle with the broader concept because they focus on labels rather than the relationships between numbers. Connected to students learning the names of numbers, they put together the three distinct ideas, number (concept) - numeral (symbol) - name (word) in order to understand the Hindu–Arabic numeration system. Figure 2.3.1

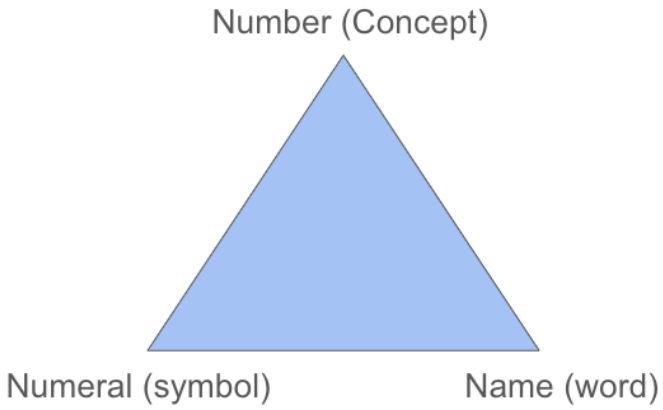


Figure 2.3.1

Wonder, Play, Grow

Task 1: Discuss with your peers- "Why do you think our number system is called 'base 10'?" Be prepared to share your answer with the class.

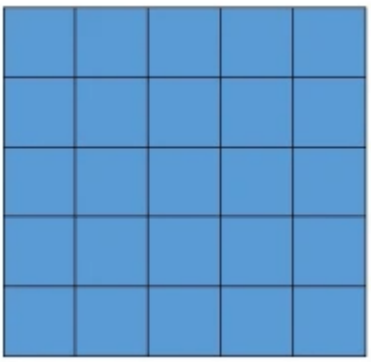
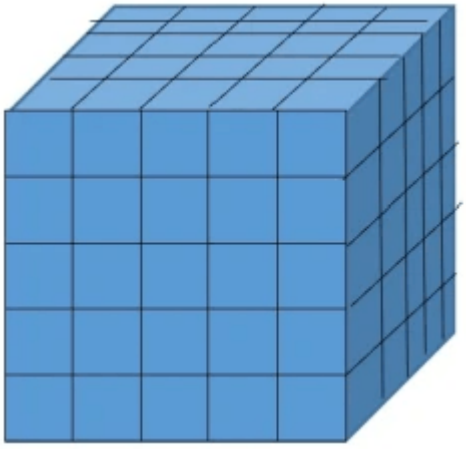
Task 2: Represent 342 in expanded form and with base-10 blocks.

Task 3: Calculate 10^3, 10^2, 10^1, 10^0. How do these values help explain why we call our number system base-10? Be prepared to share your answer with the class.

5,302 in base-10 expanded form. available at the end of this section.

Nondecimal Numeration Systems

Our Hindu-Arabic numeration system is structured around grouping by tens. To gain a deeper understanding of this system and the challenges children may encounter when learning it, it can be helpful to examine alternative systems that use different bases. For instance, imagine a numeration system based on a single hand (five fingers). In this case, numbers would be grouped in sets of five. If objects such as sticks were used, they would be bundled in groups of five. For example, the number 17 in base ten would be represented as three bundles of five with two left over in base five, written as 32_5 (or 32_{five}) . It is crucial to read this correctly as “three two base five,” rather than “thirty-two,” since the place values differ from those in base ten. When considering the language aspect of base 5, we see that the digit 3 and digit 2 represent something different because the subscript symbol 5 is included in the numeral. When we see the subscript symbol, it tells us how to assign a place value for each digit.

Base 5 Blocks	**Units are $5^0=1$**	**Longs are $5^1=5$**	**Flats are $5^2=25$**	**Blocks are $5^3=125$**
				

🧠Let's Listen to Learn❤️

Audio

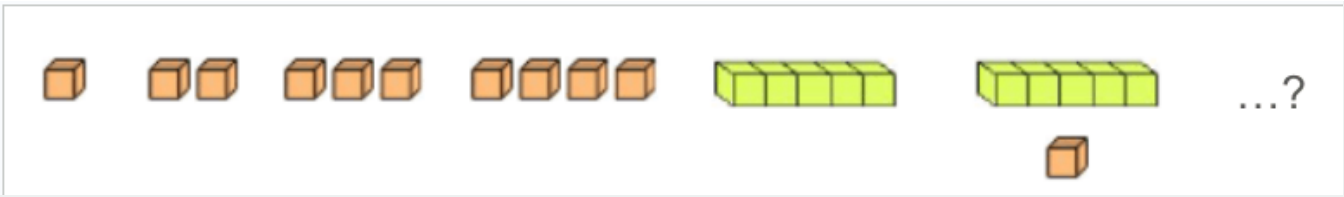
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Think About and Discuss

Considering that base-10 has nine digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to represent all of our numbers, how many and which digits would we use for a base-5 number system? Once you and your peers agree on that then discuss how to count in base-5.

Keep in mind that counting using base five names differs from counting in base ten. What are the first 10 base-5 numerals?



Base-5 Counting available at the end of this section.

Example 2.3.2

a) Make 213_5 using base five blocks, then convert into base ten.

b) Make 2142_5 using base five blocks, then convert into base ten.

Answer available at the end of this section.

Play, Wonder, Grow

Convert the number 81 into base 5, using base 5 blocks. It may also be a good problem solving strategy to consider a place value table to organize your thinking.

25	5	1
?	?	?

Answer available at the end of this section.

Example 2.3.3

Convert the number 341 into base 5

Answer available at the end of this section.

🧠Let's Listen to Learn❤️

Audio

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Other Bases

Base-10 and Base-5 are not the only systems that can represent numbers. The **base-2 system**, also known as the **binary system**, is a number system that uses only two digits: **0 and 1**. Unlike our familiar **base-10 (decimal) system,** where each place value represents a power of 10, the binary system is based on **powers of 2**. The binary system is **fundamental to computing and digital electronics** because modern computers and electronic devices operate using electrical circuits that have **two states**:

- **ON (1)**
- **OFF (0)**

Since binary numbers correspond directly to these two states, they are the most efficient way for computers to **store, process, and transmit data**.

When you see a Base Two numeral like 11011_{two}, you need to understand what each place value stands for. *In all bases*, the rightmost place value is the units or 1's place. Next, the number name "two" to the right of the numeral tells you this is a Base Two numeral, which means each place value as you move left increases by a multiple of 2. Below is a chart showing the first eight place values in Base Two. They are written in Base Ten.

128	64	32	16	8	4	2	1

To check that the Base Two numeral 11011_{two} really is 27_{ten}, put it into a chart with just five place values showing (since it's a five-digit numeral) and check the total value of the numeral. Look at the numeral written to the right. The place value for each digit is written below each digit of the numeral. The numeral is written in bold so it does not get confused with the place values. You check this exactly the way you did for the Mayan numerals:

$$\begin{aligned}(1 \text{ group of } 16) + (1 \text{ group of } 8) + (0 \text{ groups of } 4) + (1 \text{ group of } 2) + (1 \text{ group of } 1) &= (\mathbf{1} \times 16) + (\mathbf{1} \times 8) + (\mathbf{0} \times 4) + (\mathbf{1} \times 2) + (\mathbf{1} \times 1) \\ &= 16 + 8 + 2 + 1 \\ &= \mathbf{27}.\end{aligned}$$

All right! It really works! In fact, it's really easy to check the arithmetic for a Base Two numeral because for each place value containing a 1, that place value gets added to the total, whereas those place values containing a zero do not get added to the total. There's no multiplication you have to do –you can skip directly to the addition: 16 + 8 + 2 + 1 = **27**.

$$\boxed{\begin{array}{ccccc}\mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \hline 16 & 8 & 4 & 2 & 1\end{array}\ \text{two}}$$

Let's convert $10\ 011\ 010_{\text{two}}$ to Base Ten. Again, we'll write it with the place values shown under each digit and add up the place values containing a 1. This numeral converts to 128 + 16 + 8 + 2 = **154**.

$$\boxed{\begin{array}{cccccccc}\mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \hline 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1\end{array}\ \text{two}}$$

Example 2.3.4

Convert each Base Two numeral to Base Ten.

a) $10\ 011_{\text{two}}$	b) $1\ 000\ 001_{\text{two}}$	c) $111\ 111_{\text{two}}$

Answer available at the end of this section.

Wonder, Play, Grow

Convert the following numerals to Base Ten. Work with your peers to figure it out.

a. $10\text{Ø}2_3$ = ____
b. $10\text{Ø}23_4$ = ____
c. You make up a number in base-6 and have a peer convert it to base-10.

Did You Know

DID YOU KNOW: In Base Eleven, you need eleven different symbols (or digits) for possible placeholders. The problem we encounter is that we only have these ten recognizable digits –0,1,2,3,4,5,6,7,8 and 9. *Remember that 10 is made up of two separate digits –it isn't a single symbol!!* So we need to introduce a new symbol to represent ten in Base Eleven and higher. The convention is to use the letter "T". Similarly, we use the letter "E" to represent the number eleven in Base Twelve and higher and the letter "W" to represent the number twelve in Base Thirteen and higher. Since the second place value from the right represents the base you are working in, you never need a separate symbol to represent the value of the base or any number higher than the base. That's why there are only 0's and 1's and *no 2*'s in Base Two and why there are only 0's, 1's, 2's, 3's and 4's and *no 5*'s in Base Five, etc.

Let's convert the numeral $TE5_{\text{twelve}}$ to Base Ten. To remember the place values, we might want to draw a Base Twelve place value chart for three digits as shown to the right.

Then, $TE5_{\text{twelve}} = 10 \times 144 + 11 \times 12 + 5 \times 1 = 1440 + 132 + 5 = \mathbf{1577}$.

Pay close attention to the difference between $2T9_{\text{eleven}}$ and 2109_{eleven}.

Example 2.3.5

Convert each of the following to a Base Ten numeral. Show work

a. 110_{two} =	e. 110_{six} =
b. 110_{three} =	f. 110_{seven} =
c. 110_{four} =	g. 110_{eight} =
d. 110_{five} =	h. 110_{nine} =

Answer available at the end of this section.

Base 5 and Base 2 Conversion Retrieval Practice

Let's see what Dr. Howie Hua has to say about this.

YouTube Video

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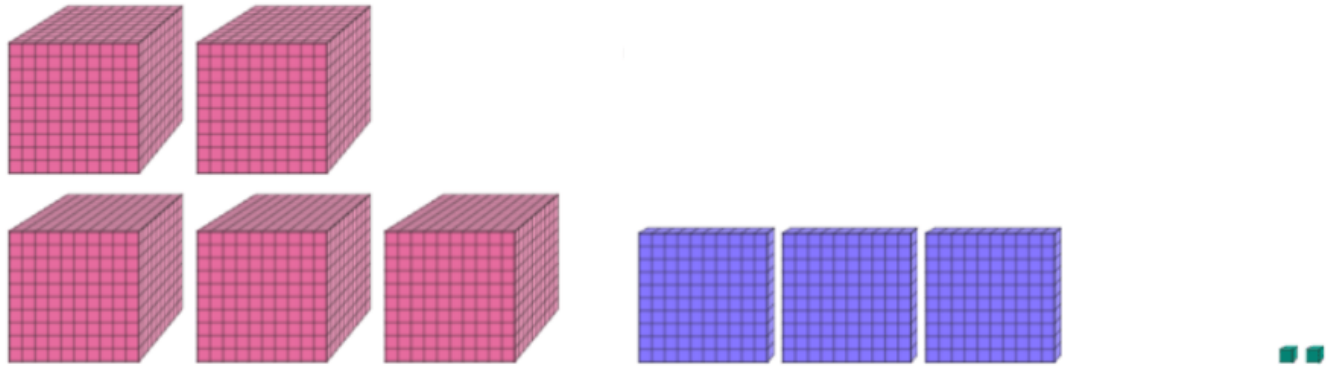
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Answers

Example 2.3.1

5(1000) + 3(100)+ 0(10) + 2(1) = 5,302

This is like saying we have 5 groups of 1000 (block), 3 groups of 100 (flat), 0 groups of 10 (long) , and 2 group 1 (unit).



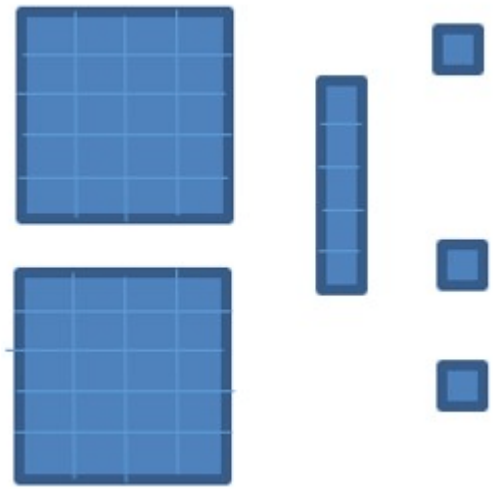
Notice that we didn't represent 5,302 using "unifix" cubes. :)

$5(10^3) + 3(10^2) + 0(10^1) + 1(10^0) = 5(1000) + 5(100) + 0(10) + 2(1) = 5,302$

Think About and Discuss

$1_5, 2_5, 3_5, 4_5, 10_5, 11_5, 12_5, 13_5, 14_5, 20_5$

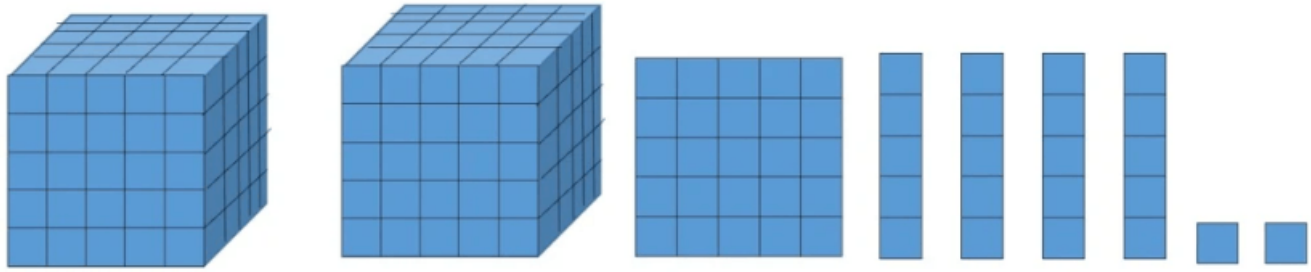
Example 2.3.2



a)

$2(5^2) + 2(5^1) + 3(5^0) = 2(25) + 1(5) + 3(1) = 58$

b)



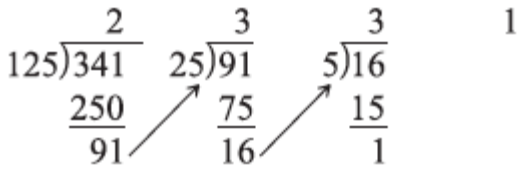
$2(5^3) + 1(5^2) + + 4(5^1) + 2(5^0) = 2(125) + 1(25) + 4(5) + 2(1) = 422$

311_5

Example 2.3.3

2331_5

You can use base blocks to model this situation, but you may also want to consider a more systematic method can be used to convert 341 to its base five numeral. First, find the highest power of 5 that will divide into 341 (the "how many" approach); that is, which is the greatest among 1, 5, 25, 125, 625, 3125, and so on, that will divide into 341? The answer in this case is 125. The rest of that procedure uses long division, each time dividing the remainder by the next smaller place value.



Remainder Approach:

🔁 **Step 1: Divide 341 by 5**

341 ÷ 5 = 68 with a remainder of **1**

Write down the **remainder**.

🔁 **Step 2: Divide 68 by 5**

68 ÷ 5 = 13 with a remainder of **3**

Write down the **remainder**.

🔁 **Step 3: Divide 13 by 5**

13 ÷ 5 = 2 with a remainder of **3**

Write down the **remainder**.

🔁 **Step 4: Divide 2 by 5**

2 ÷ 5 = 0 with a remainder of **2**

Write down the **remainder**.

📌 **Step 5: Read the remainders from bottom to top**

From last to first: **2 3 3 1**

So, **341 in base 5 is: 2331**

Example 2.3.4

a) = $(1\times2^4 + 0\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0) = (16 + 0 + 0 + 2 + 1) = 19$

b) = $(1\times2^6 + 0\times2^5 + 0\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0) = (64 + 0 + 0 + 0 + 0 + 0 + 1) = 65$

c) = $(1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0) = (32 + 16 + 8 + 4 + 2 + 1) = 63$

Example 2.3.5

TO convert each of the numbers 110 from different bases into **Base Ten (Decimal)**. We'll use this formula:

$110_{base}=1\times base^2+1\times base^1+0\times base^0$

a) 6 b) 12 c) 20 d) 30 e) 42 f) 56 g) 72 h) 90

2.3.1: Written Retreival and Problem Solving Practice

2.3.2: Online Retrieval and Problem Solving Practice

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2.3.1: Written Retreival and Problem Solving Practice

Student Learning Objectives

2.4 Students will demonstrate proficiency with the Hindu-Arabic numeration system by converting between bases, expanding numbers, and articulating the system's structure, base, and numeral naming conventions.

The mild assignment 🌶 is a shorter and less challenging assignment. The medium assignment🌶🌶 is right in the middle in terms of length and level of difficulty. Finally, the spicy assignment 🌶🌶🌶 is a longer assignment that is the most challenging.

(Spicy 🌶🌶🌶 Problems are OPTIONAL, but go for it when you have the time to challenge yourself)

Exercise 2.3.1.1 🌶

Write each of the following numbers in expanded notation.

a) 75 b) 302 c) 6814 d) 50,004,030

Answer available at the end of this section.

Exercise 2.3.1.2 🌶

Write each of the following expressions in standard place value form.

a) $2(1000) + 3(100) + 4(10) + 5(1)$

b) $6(100{,}000) + 5(100) + 7(1)$

c) $9(10^6) + 1(10^4) + 2(10^2) + 5(10^0)$

d) $2(10^9) + 4(10^6) + 6(10^5) + 8(10^1)$

Answer available at the end of this section.

Exercise 2.3.1.3 🌶

State the place value of the digit 2 in each numeral.

a) 7234 b) 4142 c) 2168

Answer available at the end of this section.

Exercise 2.3.1.4 🌶

The following numbers are written in words. Rewrite each one using Hindu–Arabic numerals.

a) Six million five hundred three thousand fifty-nine

b) Three hundred six billion four hundred fifty-three thousand

Answer available at the end of this section.

Exercise 2.3.1.5

Write each of the following numerals in expanded notation.

a) 14_5 b) 26_7 c) 321_4 d) 10101_2

Answer available at the end of this section.

Exercise 2.3.1.6

Convert the following base 2 numerals into base ten numerals.

a) 101_2 b) 1111_2 c) 10101_2

Answer available at the end of this section.

Exercise 2.3.1.7

Convert the following base five numerals into base ten numerals.

a) 12_5 b) 204_5 c) 1322_5

Answer available at the end of this section.

Extra Help 2.3.8 - "Does it Fit" Method

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Exercise 2.3.1.8

Convert each base ten numeral into a numeral in the base requested.

a) 126 in base 5

b) 400 in base 5

b) 17 in base 2

d) 71 in base 2

Answer available at the end of this section.

Exercise 2.3.1.9

a. Write out the base five numerals in order from 1 to 100_5.

b. Write out the base two numerals in order from 1 to 10000_2.

Answer available at the end of this section.

Exercise 2.3.1.10

What is wrong with the numeral 24_3?

Answer available at the end of this section.

Exercise 2.3.1.11

Write out the first 10 base four numerals.

Answer available at the end of this section.

Exercise 2.3.1.12

The base twelve numeration system has the following twelve symbols:0,1,2,3,4,5,6,7,8,9,T,E.

Change each of the following numerals to base ten numerals.

a) 142_{12} b) $T9_{12}$ c) $ETET_{12}$

Answer available at the end of this section.

Problem Solving Practice

1) Directions: Using the digits 0 through 9 at most one time each, place a digit in each box to make a two-digit number that has a value between given numbers.

$$\square\square, 35, \square\square, 41, \square\square, \square\square, 88, \square\square, 107$$

Courtesy of Open Middle

Answer

One Potential Answer : 20,35,37,41,45,76,88,89,107

2) Find the number.

I am greater than 52 tens.

I am less than half of 1200.

If you round me to the nearest 100, I become 500.

I am even.

Who am I?

Answer

The number can be **any even number between 520 and 548**, such as **524, 526, 528, 530, ..., 548**.

Converting Base Into Base 5 and 2

Interactive Content

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Answers

Exercise 2.3.1.1

a) 7(10) + 5(1)

b) 3(100) + 0(10) + 2(1)

c) 6(1000) + 8(100) + 1(10) + 4(1)

d) $5(10^7) + 4(10^3) + 3(10^1)$

Exercise 2.3.1.2🌶

a) 2,345 b) 600,507 c) 9,010,205 d) 2,004,500,080

Exercise 2.3.1.3🌶

a) hundreds b) ones c) thousands

Exercise 2.3.1.4🌶

a) 6,503,059 b) 306,000,453,000

Exercise 2.3.1.5🌶

a) 1(5) + 4(1) b) 2(7) + 6(1) c) 3(16) + 2(4) + 1(1) d) 1(16) + 1(4) + 1(1)

Exercise 2.3.1.6🌶🌶

a) 4 + 1 = 5 b) 8 + 4 + 2 + 1 = 15 c) 16 + 4 + 1 = 21

Exercise 2.3.1.7🌶🌶

a) 1(5) + 2(1) = 7 b) 2(25) + 4(1) = 54 c) 1(125) + 3(25) + 2(5) + 2(1) = 212

Exercise 2.3.1.8🌶🌶🌶

a) 1001_5 b) 3100_5 c) 10001_2 d) 1000111_2

Exercise 2.3.1.9🌶🌶

a) 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 100

Exercise 2.3.1.10🌶🌶

In base 3, we only use the symbols 0, 1, 2.

Exercise 2.3.1.11🌶🌶🌶

1, 2, 3, 10, 11, 12, 13, 20, 21, 22

Exercise 2.3.1.12🌶🌶🌶

a) $1(12^2) + 4(12^1) + 2(12^0) = 194$

b) $10(12^1) + 9(12^0) = 129$

c) $11(12^3) + 10(12^2) + 11(12^1) + 10(12^0) = 20590$

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2.3.2: Online Retrieval and Problem Solving Practice

Base 5 and Base 2 Conversion Retrieval Practice

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2.4: Chapter 2 Resources

Chapter Example Videos

Chapter 2.1

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Chapter 2.3

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Chapter Extras

Set Operation Practice

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