COLLEGE TECHNICAL MATH 1A (NWTC)



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Detailed Licensing

Licensing

*A detailed breakdown of this resource's licensing can be found in **Back Matter/Detailed Licensing**.*

CHAPTER OVERVIEW

1: Operations with Real Numbers

1.1: Order of Operations
1.2: Negative Numbers
1.3: Decimals
1.4: Fractions
1.5: Multiplication Properties of Exponents
1.6: Division Properties of Exponents
1.7: Scientific and Engineering Notation

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1.1: Order of Operations

To evaluate an expression means to simplify it and find its value.

? Exercises 1.1.1

1. Evaluate by performing the addition first: $12 - 2 + 3$

2. Evaluate by performing the subtraction first: $12 - 2 + 3$

Answer

1. 7
2. 13

When we evaluate an expression, we want to have a single correct answer. It isn't very helpful for the answer to be "maybe 7, or maybe 13". Mathematicians have decided on an order of operations, which tells us which steps should be done before other steps. Think of them as the rules of the road.

Order of Operations: PEMDAS

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary inside the grouping symbols.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

? Exercises 1.1.1

3. $12 - (2 + 3)$

4. $12 - 2 + 3$

Answer

3. 7
4. 13

Based on Exercises 3 & 4, we can see that Exercise 1 told us to use the wrong order of operations. If there are no parentheses, we must evaluate $12 - 2 + 3$ by *first* performing the subtraction and *then* performing the addition.

Before we move on, you should be aware that there are a handful of ways to show multiplication. All of the following represent 3×4:

$$3 \cdot 4 \; 3 * 4 \; 3(4) \; (3)4 \; (3)(4)$$

In this textbook, you will most often see the dot, like $3 \cdot 4$, or parentheses directly next to a number, like $3(4)$. We tend to avoid using the 3×4 symbol because it can be mistaken for the letter x.

? Exercises 1.1.1

Simplify each expression.

5. $12 \div (3 \cdot 2)$

6. $12 \div 3 \cdot 2$

7. $5(1+3)-2$

8. $5(1)+(3-2)$

Answer

5. 2

6. 8

7. 18

8. 6

An ***exponent*** indicates repeated multiplication. For example, $6^2 = 6 \cdot 6 = 36$ and $4^3 = 4 \cdot 4 \cdot 4 = 64$. The exponent tells us how many factors of the base are being multiplied together.

? Exercises 1.1.1

Simplify each expression.

9. 3^2+4^2

10. $(3+4)^2$

11. $(7+3)(7-5)^3$

12. $7+3(7-5)^3$

Answer

9. 25

10. 49

11. 80

12. 31

In the next set of exercises, the only differences are the parentheses, but every exercise has a different answer.

? Exercises 1.1.1

Simplify each expression.

13. $39-7 \cdot 2+3$

14. $(39-7) \cdot 2+3$

15. $39-(7 \cdot 2+3)$

16. $39-7 \cdot (2+3)$

17. $(39-7) \cdot (2+3)$

Answer

13. 28

14. 67

15. 22

16. 4

17. 160

It is possible to have grouping symbols nested within grouping symbols; for example, $7+(5^2-(3(17-12\div 4)+2\cdot 5)\div 4)$.

To make it somewhat easier to match up the pairs of left and right parentheses, we can use square brackets instead: $7+(5^2-[3(17-12\div 4)+2\cdot 5]\div 4)$.

Exercises 1.1.1

Simplify the expression.

18. $7+(5^2-[3(17-12\div 4)+2\cdot 5]\div 4)$

Answer

18. 19

A fraction bar is another grouping symbol; it tells us to perform all of the steps on the top and separately perform all of the steps down below. The final step is to divide the top number by the bottom number.

Exercises 1.1.1

Simplify each expression.

19. $\dfrac{15-1}{6+1}$

20. $\dfrac{(7+2)\cdot 4}{18\div(3+3)}$

21. $\dfrac{5\cdot 4^2}{2}$

22. $\dfrac{(5\cdot 4)^2}{2}$

23. $\dfrac{(5-1)^2}{2+6}$

24. $(5-1)^2\div 2+6$

Answer

19. 2

20. 12

21. 40

22. 200

23. 2

24. 14

We will look at formulas in a later module, but let's finish by translating from words to a mathematical expression.

Exercises 1.1.1



https://math.libretexts.org/@go/page/142689

25. You can find the approximate Fahrenheit temperature by doubling the Celsius temperature and adding 30. If the temperature is 9°C, what is the approximate Fahrenheit temperature? Write an expression and simplify it.

26. You can find the approximate Celsius temperature by subtracting 30 from the Fahrenheit temperature and then dividing by 2. If the temperature is 72°F, what is the approximate Celsius temperature? Write an expression and simplify it.

Answer

25. $9 \cdot 2 + 30 = 48$°F

26. $(72 - 30) \div 2 = 21$°C

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1.2: Negative Numbers

Negative numbers are a fact of life, from winter temperatures to our bank accounts. Let's practice evaluating expressions involving negative numbers.



Absolute Value

The ***absolute value*** of a number is its distance from 0. You can think of it as the size of a number without identifying it as positive or negative. Numbers with the same absolute value but different signs, such as 3 and -3, are called ***opposites***. The absolute value of -3 is 3, and the absolute value of 3 is also 3.

We use a pair of straight vertical bars to indicate absolute value; for example, $|-3| = 3$ and $|3| = 3$.

? Exercises 1.2.1

Evaluate each expression.

1. $|-5|$

2. $|5|$

Answer

1. 5
2. 5

Adding Negative Numbers

To add two negative numbers, add their absolute values (i.e., ignore the negative signs) and make the final answer negative.

? Exercises 1.2.1

Perform each addition.

3. $-8 + (-7)$

4. $-13 + (-9)$

Answer

3. -15
4. -22

To add a positive number and a negative number, we *subtract* the smaller absolute value from the larger. If the positive number has the larger absolute value, the final answer is positive. If the negative number has the larger absolute value, the final answer is negative.

Exercises 1.2.1

Perform each addition.

5. $7+(-3)$

6. $-7+3$

7. $14+(-23)$

8. $-14+23$

9. The temperature at noon on a chilly Monday was -7°F. By the next day at noon, the temperature had risen 25°F. What was the temperature at noon on Tuesday?

Answer

5. 4

6. -4

7. -9

8. 9

9. 18°F

If an expression consists of only additions, we can break the rules for order of operations and add the numbers in whatever order we choose.

Exercises 1.2.1

Evaluate each expression using any shortcuts that you notice.

10. $-10+4+(-4)+3+10$

11. $-291+73+(-9)+27$

Answer

10. 3

11. -200

Subtracting Negative Numbers

The image below shows part of a paystub in which an $ 18 payment needed to be made, but the payroll folks wanted to track the payment in the deductions category. Of course, a positive number in the deductions will subtract money away from the paycheck. Here, though, a deduction of negative 18 dollars has the effect of *adding* 18 dollars to the paycheck. Subtracting a negative amount is equivalent to adding a positive amount.

Deductions after Federal Tax	
Faculty Union Dues	$27.00
Stipend for Part Time Faculty	-$18.00
Workers Compensation Hourly Assessment	$0.13

To subtract two signed numbers, we **add** the first number to the **opposite** of the second number.

Exercises 1.2.1

Perform each subtraction.

12. $5 - 2$

13. $2 - 5$

14. $-2 - 5$

15. $-5 - 2$

16. $2 - (-5)$

17. $5 - (-2)$

18. $-2 - (-5)$

19. $-5 - (-2)$

20. One day in February, the temperature in Portland, Oregon is 43°F, and the temperature in Portland, Maine is -12°F. What is the difference in temperature?

Answer

12. 3

13. -3

14. -7

15. -7

16. 7

17. 7

18. 3

19. -3

20. 55°F difference

Multiplying Negative Numbers

Suppose you spend 3 dollars on a coffee every day. We could represent spending 3 dollars as a negative number, -3 dollars. Over the course of a 5-day work week, you would spend 15 dollars, which we could represent as -15 dollars. This shows that $-3 \cdot 5 = -15$, or $5 \cdot -3 = -15$.



If two numbers with **opposite** signs are multiplied, the product is negative.

Exercise 1.2.1

Find each product.

21. $-4 \cdot 3$

22. $5(-8)$

Answer

21. -12

22. -40

Going back to our coffee example, we saw that $5(-3) = -15$. Therefore, the *opposite* of $5(-3)$ must be positive 15. Because -5 is the opposite of 5, this implies that $-5(-3) = 15$.

If two numbers with the **same** sign are multiplied, the product is positive.

WARNING! These rules are different from the rules for addition; be careful not to mix them up.

? Exercise 1.2.1

Find each product.

23. $-2(-9)$

24. $-3(-7)$

Answer

23. 18

24. 21

Recall that an exponent represents a repeated multiplication. Let's see what happens when we raise a negative number to an exponent.

? Exercises 1.2.1

Evaluate each expression.

25. $(-2)^2$

26. $(-2)^3$

27. $(-2)^4$

28. $(-2)^5$

Answer

25. 4

26. -8

27. 16

28. -32

If a negative number is raised to an **odd** power, the result is negative.
If a negative number is raised to an **even** power, the result is positive.

Dividing Negative Numbers

Let's go back to the coffee example we saw earlier: $-3 \cdot 5 = -15$. We can rewrite this fact using division and see that $-15 \div 5 = -3$; a negative divided by a positive gives a negative result. Also, $-15 \div -3 = 5$; a negative divided by a negative gives a positive result. This means that the rules for division work exactly like the rules for multiplication.

If two numbers with **opposite** signs are divided, the quotient is negative.
If two numbers with the **same** sign are divided, the quotient is positive.

Exercises 1.2.1

Find each quotient.

29. $-42 \div 6$

30. $32 \div (-8)$

31. $-27 \div (-3)$

32. $0 \div 4$

33. $0 \div (-4)$

34. $4 \div 0$

Answer

29. -7

30. -4

31. 9

32. 0

33. 0

34. undefined

Go ahead and check those last three exercises with a calculator. Any surprises?

- 0 divided by another number is 0.
- A number divided by 0 is undefined, or not a real number.

Here's a quick explanation of why $4 \div 0$ can't be a real number. Suppose that there is a mystery number, which we'll call n, such that $4 \div 0 = n$. Then we can rewrite this division as a related multiplication, $n \cdot 0 = 4$. But because 0 times any number is 0, the left side of this equation is 0, and we get the result that $0 = 4$, which doesn't make sense. Therefore, there is no such number n, and $4 \div 0$ cannot be a real number.

Order of Operations with Negative Numbers

P: Work inside of **parentheses** or grouping symbols, following the order PEMDAS as necessary.

E: Evaluate **exponents**.

MD: Perform **multiplications** and **divisions** from left to right.

AS: Perform **additions** and **subtractions** from left to right.

Let's finish up this module with some order of operations practice.

Exercises 1.2.1

Evaluate each expression using the order of operations.

35. $(2-5)^2 \cdot 2 + 1$

36. $2 - 5^2 \cdot (2+1)$

37. $[7(-2) + 16] \div 2$

38. $7(-2) + 16 \div 2$

39. $\dfrac{1-3^4}{2(5)}$

40. $\dfrac{(1-3)^4}{2} \cdot 5$

Answer

35. **19**

36. -73

37. 1

38. -6

39. -8

40. 40

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1.3: Decimals



Decimal notation is based on powers of 10: 0.1 is one tenth, 0.01 is one hundredth, 0.001 is one thousandth, and so on.

thousands	hundreds	tens	ones/units	.	tenths	hundredths	thousandths

? Exercises 1.3.1

Write each number.

1. ninety and twenty-three hundredths

2. seven and fifty-six thousandths

Answer

1. 90.23
2. 7.056

Adding & Subtracting Decimals

Before you add or subtract decimals, you must line up the decimal points.

? Exercises 1.3.1

Add each pair of numbers.

3. $3.75 + 12.8$

4. $71.085 + 112.93$

Answer

3. 16.55
4. 184.015

When subtracting, you may need to add zeros to the first number so you can borrow correctly.

? Exercises 1.3.1

Subtract each pair of numbers.

5. $46.57 - 38.29$

6. $82.78 - 67.024$

Answer

5. 8.28
6. 15.756

Multiplying Decimals

To multiply decimal numbers:

1. Temporarily ignore the decimal points.
2. Multiply the numbers as though they are whole numbers.
3. Add the total number of decimal digits in the two numbers you multiplied. The result will have that number of digits to the right of the decimal point.

Note: You do NOT need to line up the decimal points when you are multiplying.

? Exercises 1.3.1

Multiply each pair of numbers.

7. $13.5 \cdot 2.9$

8. $4.18 \cdot 3.7$

9. Evie worked 37.5 hours at a pay rate of $\$ 17.50$ per hour. How much did she earn in total?

Answer

7. 39.15
8. 15.466
9. $\$ 656.25$

Dividing Decimals

Let's review everyone's favorite topic, long division. The three parts of a division are named as follows: dividend ÷ divisor = quotient. When this is written with a long division symbol, the dividend is inside the symbol, the divisor is on the left, and the quotient is the answer we create on top.

$$\text{divisor}\overline{)\,\text{dividend}}\quad\text{quotient}$$

To divide by a decimal:

1. Write in long division form.
2. Move the decimal point of the divisor until it is a whole number.
3. Move the decimal point of the dividend the same number of places to the right.
4. Place the decimal point in the quotient directly above the decimal point in the dividend. Divide the numbers as though they are whole numbers.
5. If necessary, add zeros to the right of the last digit of the dividend to continue.

? Exercise 1.3.1

Divide each pair of numbers.

10. $97.4 \div 0.4$

11. $9.74 \div 0.04$

Answer

10. **243.5**
11. **2,435**

Rounding Numbers

It is often necessary to round a number to a specified place value. We will discuss this in much more depth in a future module, but let's practice rounding now.

Rounding a number:

1. Locate the **rounding digit** in the place to which you are rounding.
2. Look at the **test digit** directly to the right of the rounding digit.
3. If the test digit is 5 or greater, increase the rounding digit by 1 and drop all digits to its right. If the test digit is less than 5, keep the rounding digit the same and drop all digits to its right.

? Exercises 1.3.1

Round each number to the indicated place value.

12. **6,473** (thousands)

13. **6,473** (hundreds)

14. **6,473** (tens)

15. **0.7049** (tenths)

16. **0.7049** (hundredths)

17. **0.7049** (thousandths)

Answer

12. **6,000**
13. **6,500**
14. **6,470**
15. **0.7**
16. **0.70**
17. **0.705**

If a decimal answer goes on and on, it may be practical to round it off.

? Exercises 1.3.1

18. Jerry drove **257** miles using **11** gallons of gas. How many miles per gallon did his car get? Round your result to the nearest tenth.

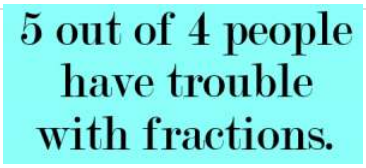
Answer

18. **23.4** miles per gallon

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1.4: Fractions



A fraction describes equal parts of a whole: $\frac{\text{part}}{\text{whole}}$

Using official math vocabulary: $\frac{\text{numerator}}{\text{denominator}}$

? Exercises 1.4.1

The month of April had 11 rainy days and 19 days that were not rainy.

1. What fraction of the days were rainy?

2. What fraction of the days were not rainy?

Answer

1. $\frac{11}{30}$
2. $\frac{19}{30}$

Simplifying Fractions

Two fractions are equivalent if they represent the same number. (The same portion of a whole.) To build an equivalent fraction, multiply the numerator and denominator by the same number.

? Exercises 1.4.1

3. Write $\frac{4}{5}$ as an equivalent fraction with a denominator of 15.

4. Write $\frac{2}{3}$ as an equivalent fraction with a denominator of 12.

Answer

3. $\frac{12}{15}$
4. $\frac{8}{12}$

Many fractions can be ***simplified***, or reduced. Here are four special cases.

? Exercises 1.4.1

Simplify each fraction, if possible.

5. $\frac{7}{1}$

6. $\frac{7}{7}$

7. $\frac{0}{7}$

8. $\frac{7}{0}$

Answer

5. 7

6. 1

7. 0

8. undefined

A fraction is completely reduced, or in simplest form, or in lowest terms, when the numerator and denominator have no common factors other than 1. To reduce a fraction, divide the numerator and denominator by the same number.

? Exercises 1.4.1

Reduce each fraction to simplest form.

9. $\frac{9}{12}$

10. $\frac{10}{6}$

Answer

9. $\frac{3}{4}$

10. $\frac{5}{3}$

Multiplying Fractions

To multiply fractions, multiply the numerators and multiply the denominators straight across. If possible, simplify your answer.

? Exercises 1.4.1

Multiply each pair of numbers. Be sure that each answer is in simplest form.

11. $8 \cdot \frac{1}{4}$

12. $\frac{6}{7} \cdot \frac{7}{12}$

13. $\frac{5}{8} \cdot \frac{2}{3}$

14. $\frac{6}{5} \cdot \frac{10}{12}$

Answer

11. 2

12. $\frac{1}{2}$

13. $\frac{5}{12}$

14. 1

To find a fraction **of** a number, multiply.

? Exercise 1.4.1

15. To pass his workplace training, Nathan must correctly answer at least $\frac{9}{10}$ of 50 questions. How many questions must he answer correctly to pass the training?

Answer

15. at least 45 questions

Dividing Fractions

To divide by a fraction, multiply by the reciprocal of the second number. (Flip the second fraction upside-down.)

Exercises 1.4.1

Divide. Be sure that each answer is in simplest form.

16. $12 \div \frac{3}{4}$

17. $\frac{3}{10} \div \frac{1}{2}$

18. Suppose you need to measure 2 cups of flour, but the only scoop you can find is $\frac{1}{3}$ cup. How many scoops of flour will you need?

Answer

16. 16

17. $\frac{3}{5}$

18. 6 scoops

Comparing Fractions

If two fractions have the same denominator, we can simply compare their numerators.

If two fractions have different denominators, we can rewrite them with a common denominator and then compare their numerators.

Exercise 1.4.1

19. Cookie recipe A requires $\frac{3}{4}$ cup of sugar, whereas cookie recipe B requires $\frac{2}{3}$ cup of sugar. Which recipe requires more sugar?



Answer

19. A requires $\frac{1}{12}$ cup more than B

Adding & Subtracting Fractions

To add or subtract two fractions with the same denominator, add or subtract the numerators and keep the common denominator.

Exercises 1.4.1

20. Jack ate $\frac{3}{8}$ of a pizza. Mack ate $\frac{1}{8}$ of the pizza. What fraction of the pizza did they eat together?

21. Tracy ate $\frac{5}{6}$ of a pizza. Stacy ate $\frac{1}{6}$ of the pizza. How much more of the pizza did Tracy eat?

Answer

20. $\frac{1}{2}$ of the pizza

21. $\frac{2}{3}$ more

To add or subtract two fractions with different denominators, first write them with a common denominator. Then add or subtract them.

? Exercises 1.4.1

A $\frac{3}{8}$-inch thick sheet of plywood is going to be laid onto a $\frac{1}{4}$-inch thick sheet of plywood.

22. What is the combined thickness of the two sheets?

23. What is the difference in thickness of the two sheets of plywood?

Jacqueline budgets $\frac{1}{4}$ of her monthly income for food and $\frac{1}{3}$ of her monthly income for rent.

24. What fraction of her monthly income does she budget for these two expenses combined?

25. What fraction more of her monthly income does she budget for her rent than for her food?

Answer

22. $\frac{5}{8}$ inches combined
23. $\frac{1}{8}$ inches difference
24. $\frac{7}{12}$ combined
25. $\frac{1}{12}$ more

Fractions and Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

A decimal that ends (eventually has a remainder of 0) is called a terminating decimal. Fun fact: If the denominator of a fraction has no prime factors other than 2's and 5's, the decimal will terminate. Also, the fraction can be built up to have a denominator of 10, or 100, or $1,000$…

? Exercises 1.4.1

Write each fraction as a decimal.

26. $\frac{11}{4}$

27. $\frac{7}{20}$

Answer

26. 2.75
27. 0.35

A decimal that continues a pattern of digits is called a repeating decimal. We can represent the repeating digits by using either an overbar or ellipsis (three dots)…

? Exercises 1.4.1

Write each fraction as a decimal.

28. $\frac{5}{9}$

29. $\frac{18}{11}$

Answer

28. $0.\overline{5}$ or $0.555\ldots$

29. $1.\overline{63}$ or $1.636363\ldots$

Mixed Numbers

A mixed number represents a **sum**. For example, $6\frac{2}{3}$ means $6+\frac{2}{3}$.

To write a mixed number as an improper fraction:

1. Multiply the whole number part by the denominator.
2. Add this result to the original numerator to get the new numerator.
3. Keep the same denominator.

? Exercises 1.4.1

Rewrite each mixed number as an improper fraction.

30. $2\frac{1}{5}$

31. $6\frac{2}{3}$

Answer

30. $\frac{11}{5}$

31. $\frac{20}{3}$

To write an improper fraction as a mixed number:

1. Divide the numerator by the denominator to get the whole number part.
2. The remainder after dividing is the new numerator.
3. Keep the same denominator.

? Exercises 1.4.1

Rewrite each improper fraction as a mixed number.

32. $\frac{23}{2}$

33. $\frac{14}{3}$

Answer

32. $11\frac{1}{2}$

33. $4\frac{2}{3}$

Adding or subtracting mixed numbers can be fairly simple or more complicated, depending on the numbers. If adding two mixed numbers would give you an improper fraction as part of your result, you'll need to carry; if subtracting two mixed numbers would give you a negative fraction as part of your result, you'll need to borrow.

Exercises 1.4.1

34. Add: $7\frac{1}{3}+2\frac{3}{4}$

35. Subtract: $7\frac{1}{3}-2\frac{3}{4}$

Answer

34. $10\frac{1}{12}$

35. $4\frac{7}{12}$

Multiplying or dividing mixed numbers is tricky. Change any mixed numbers into improper fractions before doing the calculation, then change the answer back to a mixed number if possible.

Exercises 1.4.1

36. Multiply: $3\frac{1}{2}\cdot 2\frac{1}{3}$

37. $5\frac{1}{2}$ cups of water will be divided equally into 3 jars. How much water will go into each jar?

Answer

36. $8\frac{1}{6}$

37. $1\frac{5}{6}$ cup

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1.5: Multiplication Properties of Exponents

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions using the Product Property for Exponents
- Simplify expressions using the Power Property for Exponents
- Simplify expressions using the Product to a Power Property
- Simplify expressions by applying several properties
- Multiply monomials

Note

Before you get started, take this readiness quiz.

1. Simplify: $\frac{3}{4} \cdot \frac{3}{4}$
   If you missed this problem, review Exercise 1.6.13.
2. Simplify: $(-2)(-2)(-2)$.
   If you missed this problem, review Exercise 1.5.13.

Simplify Expressions Using the Product Property for Exponents

You have seen that when you combine like terms by adding and subtracting, you need to have the same base with the same exponent. But when you multiply and divide, the exponents may be different, and sometimes the bases may be different, too.

We'll derive the properties of exponents by looking for patterns in several examples.

First, we will look at an example that leads to the Product Property.

	$x^2 \cdot x^3$
What does this mean? How many factors altogether?	$\underbrace{x \cdot x}_{2 \text{ factors}} \cdot \underbrace{x \cdot x \cdot x}_{3 \text{ factors}}$ $\underbrace{\qquad\qquad}_{5 \text{ factors}}$
So, we have	x^5
Notice that 5 is the sum of the exponents, 2 and 3.	$x^2 \cdot x^3$ is x^{2+3}, or x^5

We write:

$$\begin{aligned} &x^2 \cdot x^3 \\ &x^{2+3} \\ &x^5 \end{aligned} \tag{1.5.1}$$

The base stayed the same and we added the exponents. This leads to the **Product Property for Exponents**.

PRODUCT PROPERTY FOR EXPONENTS

If a is a real number, and m and n are counting numbers, then

$$a^m \cdot a^n = a^{m+n} \tag{1.5.2}$$

To multiply with like bases, add the exponents.

An example with numbers helps to verify this property.

$$\begin{aligned} 2^3 \cdot 2^2 &\stackrel{?}{=} 2^{2+3} \\ 4 \cdot 8 &\stackrel{?}{=} 2^5 \\ 32 &= 32\checkmark \end{aligned} \tag{1.5.3}$$

Example 1.5.7

Simplify: $y^5 \cdot y^6$

Answer

	$y^5 \cdot y^6$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	y^{5+6}
Simplify.	y^{11}

Example 1.5.8

Simplify: $b^9 \cdot b^8$

Answer

b^{17}

Example 1.5.9

Simplify: $x^{12} \cdot x^4$

Answer

x^{16}

Example 1.5.10

Simplify:

a. $2^5 \cdot 2^9$
b. $3 \cdot 3^4$

Answer

a.

	$2^5 \cdot 2^9$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	2^{5+9}
Simplify.	2^{14}

b.

	$3 \cdot 3^4$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	3^{1+4}
Simplify.	3^5

Example 1.5.11

Simplify:

a. $5 \cdot 5^5$
b. $4^9 \cdot 4^9$

Answer

a. 5^6
b. 4^{18}

Example 1.5.12

Simplify:

a. $7^6 \cdot 7^8$
b. $10 \cdot 10^{10}$

Answer

a. 7^{14}
b. 10^{11}

Example 1.5.13

Simplify:

a. $a^7 \cdot a$
b. $x^{27} \cdot x^{13}$

Answer

a.

	$a^7 \cdot a$
Rewrite, $a = a^1$	$a^7 \cdot a^1$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	a^{7+1}
Simplify.	a^8

b.

	$x^{27} \cdot x^{13}$
Notice, the bases are the same, so add the exponents.	x^{27+13}
Simplify.	x^{40}

Example 1.5.14

Simplify:

a. $p^5 \cdot p$
b. $y^{14} \cdot y^{29}$

Answer

a. p^6
b. y^{43}

Example 1.5.15

Simplify:

a. $z \cdot z^7$
b. $b^{15} \cdot b^{34}$

Answer

a. z^8
b. b^{49}

We can extend the Product Property for Exponents to more than two factors.

Example 1.5.16

Simplify: $d^4 \cdot d^5 \cdot d^2$

Answer

	$d^4 \cdot d^5 \cdot d^2$
Add the exponents, since bases are the same.	d^{4+5+2}
Simplify.	d^{11}

Example 1.5.17

Simplify: $x^6 \cdot x^4 \cdot x^8$

Answer

x^{18}

Example 1.5.18

Simplify: $b^5 \cdot b^9 \cdot b^5$

Answer

b^{19}

Simplify Expressions Using the Power Property for Exponents

Now let's look at an exponential expression that contains a power raised to a power. See if you can discover a general property.

	$(x^2)^3$
What does this mean? How many factors altogether?	$\underbrace{x^2 \cdot x^2 \cdot x^2}_{\underbrace{x \cdot x}_{2\text{ factors}} \cdot \underbrace{x \cdot x}_{2\text{ factors}} \cdot \underbrace{x \cdot x}_{2\text{ factors}}}$ 6 factors
So we have	x^6
Notice that 6 is the product of the exponents, 2 and 3.	$(x^2)^3$ is $x^{2\cdot3}$ or x^6

We write:

$$\begin{aligned} &(x^2)^3 \\ &x^{2\cdot3} \\ &x^6 \end{aligned} \tag{1.5.4}$$

We multiplied the exponents. This leads to the **Power Property for Exponents.**

POWER PROPERTY FOR EXPONENTS

If a is a real number, and m and n are whole numbers, then

$$(a^m)^n = a^{m\cdot n} \tag{1.5.5}$$

To raise a power to a power, multiply the exponents.

An example with numbers helps to verify this property.

$$\begin{aligned} (3^2)^3 &\stackrel{?}{=} 3^{2\cdot3} \\ (9)^3 &\stackrel{?}{=} 3^6 \\ 729 &= 729\checkmark \end{aligned} \tag{1.5.6}$$

Example 1.5.19

Simplify:

a. $(y^5)^9$
b. $(4^4)^7$

Answer

a.

	$(y^5)^9$
Use the power property, $(a^m)^n = a^{m\cdot n}$.	$y^{5\cdot9}$
Simplify.	y^{45}

b.

	$(4^4)^7$
Use the power property.	$4^{4 \cdot 7}$
Simplify.	4^{28}

Example 1.5.20

Simplify:

a. $(b^7)^5$
b. $(5^4)^3$

Answer

a. b^{35}
b. 5^{12}

Example 1.5.21

Simplify:

a. $(z^6)^9$
b. $(3^7)^7$

Answer

a. z^{54}
b. 3^{49}

Simplify Expressions Using the Product to a Power Property

We will now look at an expression containing a product that is raised to a power. Can you find this pattern?

What does this mean?	$(2x)^3$
We group the like factors together.	$2x \cdot 2x \cdot 2x$
How many factors of 2 and of x?	$2 \cdot 2 \cdot x^3$

Notice that each factor was raised to the power and $(2x)^3$ is $2^3 \cdot x^3$

We write: $(2x)^3$
$2^3 \cdot x^3$

The exponent applies to each of the factors! This leads to the **Product to a Power Property for Exponents.**

PRODUCT TO A POWER PROPERTY FOR EXPONENTS

If a and b are real numbers and m is a whole number, then

$$(ab)^m = a^m b^m \tag{1.5.7}$$

To raise a product to a power, raise each factor to that power.

An example with numbers helps to verify this property:

$$\begin{aligned} (2 \cdot 3)^2 &\stackrel{?}{=} 2^2 \cdot 3^2 \\ 6^2 &\stackrel{?}{=} 4 \cdot 9 \\ 36 &= 36\checkmark \end{aligned} \tag{1.5.8}$$

Example 1.5.22

Simplify:

a. $(-9d)^2$
b. $(3mn)^3$.

Answer

a.

	$(-9d)^2$
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(-9)^2 d^2$
Simplify.	$81d^2$

b.

	$(3mn)^3$
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(3)^3 m^3 n^3$
Simplify.	$27m^3 n^3$

Example 1.5.23

Simplify:

a. $(-12y)^2$
b. $(2wx)^5$

Answer

a. $144y^2$
b. $32w^5x^5$

Example 1.5.24

Simplify:

a. $(5wx)^3$
b. $(-3y)^3$

Answer

a. $125w^3x^3$
b. $-27y^3$

Simplify Expressions by Applying Several Properties

We now have three properties for multiplying expressions with exponents. Let's summarize them and then we'll do some examples that use more than one of the properties.

PROPERTIES OF EXPONENTS

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}	
Power Property	$(a^m)^n$	$=$	a^{mn}	(1.5.9)
Product to a Power	$(ab)^m$	$=$	$a^m b^m$	

All exponent properties hold true for any real numbers m and n. Right now, we only use whole number exponents.

? Example 1.5.25

Simplify:

a. $(y^3)^6(y^5)^4$
b. $(-6x^4y^5)^2$

Answer

a.

	$(y^3)^6(y^5)^4$
Use the Power Property.	$y^{18} \cdot y^{20}$
Add the exponents.	y^{38}

b.

	$(-6x^4y^5)^2$
Use the Product to a Power Property.	$(-6)^2(x^4)^2(y^5)^2$
Use the Power Property.	$(-6)^2\,(x^8)\,(y^{10})^2$
Simplify.	$36x^8y^{10}$

? Example 1.5.26

Simplify:

a. $(a^4)^5(a^7)^4$
b. $(-2c^4d^2)^3$

Answer

a. a^{48}
b. $-8c^{12}d^6$

? Example 1.5.27

Simplify:

a. $(-3x^6y^7)^4$
b. $(q^4)^5(q^3)^3$

Answer

a. $81x^{24}y^{28}$
b. q^{29}

? Example 1.5.28

Simplify:

a. $(5m)^2\,(3m^3)$
b. $(3x^2y)^4(2xy^2)^3$

Answer

		$(5m)^2(3m^3)$
	Raise $5m$ to the second power.	$5^2m^2 \cdot 3m^3$
a.	Simplify.	$25m^2 \cdot 3m^3$
	Use the Commutative Property.	$25 \cdot 3 \cdot m^2 \cdot m^3$
	Multiply the constants and add the exponents.	$75m^5$
		$(3x^2y)^4(2xy^2)^3$
	Use the Product to a Power Property.	$(3^4x^8y^4)(2^3x^3y^6)$
b.	Simplify.	$(81x^8y^4)(8x^3y^6)$
	Use the Commutative Property.	$81 \cdot 8 \cdot x^8 \cdot x^3 \cdot y^4 \cdot y^6$
	Multiply the constants and add the exponents.	$648x^{11}y^{10}$

? Example 1.5.29

Simplify:

a. $(5n)^2(3n^{10})$
b. $(c^4d^2)^5(3cd^5)^4$

Answer

a. $75n^{12}$
b. $81c^{24}d^{30}$

? Example 1.5.30

Simplify:

a. $(a^3b^2)^6(4ab^3)^4$
b. $(2x)^3(5x^7)$

Answer

a. $256a^{22}b^{24}$
b. $40x^{10}$

Multiply Monomials

Since a **monomial** is an algebraic expression, we can use the properties of exponents to multiply monomials.

? Example 1.5.31

Multiply: $(3x^2)(-4x^3)$

Answer

	$(3x^2)(-4x^3)$
Use the Commutative Property to rearrange the terms.	$3 \cdot (-4) \cdot x^2 \cdot x^3$
Multiply.	$-12x^5$

? Example 1.5.32

Multiply: $(5y^7)(-7y^4)$

Answer

$-35y^{11}$

Example 1.5.33

Multiply: $(-6b^4)(-9b^5)$

Answer

$54b^9$

Example 1.5.34

Multiply: $\left(\frac{5}{6}x^3y\right)(12xy^2)$

Answer

	$\left(\frac{5}{6}x^3y\right)(12xy^2)$
Use the Commutative Property to rearrange the terms.	$\frac{5}{6}\cdot 12\cdot x^3\cdot x\cdot y\cdot y^2$
Multiply.	$10x^4y^3$

Example 1.5.35

Multiply: $\left(\frac{2}{5}a^4b^3\right)(15ab^3)$

Answer

$6a^5b^6$

Example 1.5.36

Multiply: $\left(\frac{2}{3}r^5s\right)(12r^6s^7)$

Answer

$8r^{11}s^8$

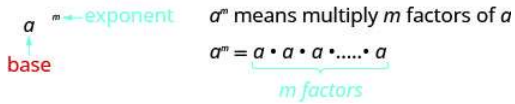
Note

Access these online resources for additional instruction and practice with using multiplication properties of exponents:

- Multiplication Properties of Exponents

Key Concepts

- **Exponential Notation**



- **Properties of Exponents**
  - If a and b are real numbers and m and n are whole numbers, then

$$\begin{array}{lrcl} \textbf{Product Property} & a^m\cdot a^n & = & a^{m+n} \\ \textbf{Power Property} & (a^m)^n & = & a^{mn} \\ \textbf{Product to a Power} & (ab)^m & = & a^mb^m \end{array} \quad (1.5.10)$$

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1.6: Division Properties of Exponents

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions using the Quotient Property for Exponents
- Simplify expressions with zero exponents
- Simplify expressions using the quotient to a Power Property
- Simplify expressions by applying several properties
- Divide monomials

Note

Before you get started, take this readiness quiz.

1. Simplify: $\frac{8}{24}$.
   If you missed this problem, review Example 1.6.4.
2. Simplify: $(2m^3)^5$.
   If you missed this problem, review Example 6.2.22.
3. Simplify: $\frac{12x}{12y}$
   If you missed this problem, review Example 1.6.10.

Simplify Expressions Using the Quotient Property for Exponents

Earlier in this chapter, we developed the properties of exponents for multiplication. We summarize these properties below.

SUMMARY OF EXPONENT PROPERTIES FOR MULTIPLICATION

If a and b are real numbers, and m and n are whole numbers, then

$$\begin{array}{ll} \textbf{Product Property} & a^m \cdot a^n = a^{m+n} \\ \textbf{Power Property} & (a^m)^n = a^{mn} \\ \textbf{Product to a Power} & (ab)^m = a^m b^m \end{array} \tag{1.6.1}$$

Now we will look at the exponent properties for division. A quick memory refresher may help before we get started. You have learned to simplify fractions by dividing out common factors from the numerator and denominator using the Equivalent Fractions Property. This property will also help you work with algebraic fractions—which are also quotients.

EQUIVALENT FRACTIONS PROPERTY

If a, b, and c are whole numbers where $b \neq 0, c \neq 0$.

$$\text{then} \quad \frac{a}{b} = \frac{a \cdot c}{b \cdot c} \quad \text{and} \quad \frac{a \cdot c}{b \cdot c} = \frac{a}{b} \tag{1.6.2}$$

As before, we'll try to discover a property by looking at some examples.

$$\begin{array}{lll} \textbf{Consider} & \frac{x^5}{x^2} & \text{and} \quad \frac{x^2}{x^3} \\ \textbf{What do they mean?} & \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} & \frac{x \cdot x}{x \cdot x \cdot x} \\ \textbf{Use the Equivalent Fractions Property.} & \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} & \frac{\cancel{x} \cdot \cancel{x} \cdot 1}{\cancel{x} \cdot \cancel{x} \cdot x} \\ \textbf{Simplify.} & x^3 & \frac{1}{x} \end{array} \tag{1.6.3}$$

Notice, in each case the bases were the same and we subtracted exponents.

When the larger exponent was in the numerator, we were left with factors in the numerator.

When the larger exponent was in the denominator, we were left with factors in the denominator—notice the numerator of 1.

We write:

$$\begin{array}{c|c} \frac{x^5}{x^2} & \frac{x^2}{x^3} \\ x^{5-2} & \frac{1}{x^{3-2}} \\ x^3 & \frac{1}{x} \end{array} \tag{1.6.4}$$

This leads to the *Quotient Property for Exponents*.

QUOTIENT PROPERTY FOR EXPONENTS

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \text{ and } \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m \tag{1.6.5}$$

A couple of examples with numbers may help to verify this property.

$$\begin{array}{rcl|rcl} \frac{3^4}{3^2} & = & 3^{4-2} & \frac{5^2}{5^3} & = & \frac{1}{5^{3-2}} \\ \frac{81}{9} & = & 3^2 & \frac{25}{125} & = & \frac{1}{5^1} \\ 9 & = & 9\checkmark & \frac{1}{5} & = & \frac{1}{5}\checkmark \end{array} \tag{1.6.6}$$

Example 1.6.1

Simplify:

1. $\frac{x^9}{x^7}$
2. $\frac{3^{10}}{3^2}$

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

1.

Since 9 > 7, there are more factors of *x* in the numerator.	$\frac{x^9}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$	x^{9-7}
Simplify.	x^2

2.

Since 10 > 2, there are more factors of *x* in the numerator.	$\frac{3^{10}}{3^2}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$	3^{10-2}
Simplify.	3^8

Notice that when the larger exponent is in the numerator, we are left with factors in the numerator.

Try It 1.6.2

Simplify:

a. $\frac{x^{15}}{x^{10}}$

b. $\frac{6^{14}}{6^5}$

Answer

1. x^5
2. 6^9

Try It 1.6.3

Simplify:

1. $\frac{y^{43}}{y^{37}}$
2. $\frac{10^{15}}{10^7}$

Answer

1. y^6
2. 10^8

Example 1.6.4

Simplify:

1. $\frac{b^8}{b^{12}}$
2. $\frac{7^3}{7^5}$

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

1.

Since 12 > 8, there are more factors of b in the denominator.	$\frac{b^8}{b^{12}}$
Use the Quotient Property, $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$	$\frac{1}{b^{12-8}}$
Simplify.	$\frac{1}{b^4}$

2.

Since 5 > 3, there are more factors of 3 in the denominator.	$\frac{7^3}{7^5}$
Use the Quotient Property, $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$	$\frac{1}{7^{5-3}}$
Simplify.	$\frac{1}{7^2}$
Simplify.	$\frac{1}{49}$

Notice that when the larger exponent is in the denominator, we are left with factors in the denominator.

Try It 1.6.5

Simplify:

1. $\frac{x^{18}}{x^{22}}$
2. $\frac{12^{15}}{12^{30}}$

Answer

1. $\frac{1}{x^4}$
2. $\frac{1}{12^{15}}$

Try It 1.6.6

Simplify:

1. $\frac{m^7}{m^{15}}$
2. $\frac{9^8}{9^{19}}$

Answer

1. $\frac{1}{m^8}$
2. $\frac{1}{9^{11}}$

Notice the difference in the two previous examples:

- If we start with more factors in the numerator, we will end up with factors in the numerator.
- If we start with more factors in the denominator, we will end up with factors in the denominator.

The first step in simplifying an expression using the Quotient Property for Exponents is to determine whether the exponent is larger in the numerator or the denominator.

Example 1.6.7

Simplify:

1. $\frac{a^5}{a^9}$
2. $\frac{x^{11}}{x^7}$

Solution

1. Is the exponent of a larger in the numerator or denominator? Since 9 > 5, there are more a's in the denominator and so we will end up with factors in the denominator.

	$\frac{a^5}{a^9}$
Use the Quotient Property, $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$	$\frac{1}{a^{9-5}}$
Simplify.	$\frac{1}{a^4}$

2. Notice there are more factors of xx in the numerator, since 11 > 7. So we will end up with factors in the numerator.

	$\frac{x^{11}}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n}=\frac{1}{a^{n-m}}$	x^{11-7}
Simplify.	x^4

Try It 1.6.8

Simplify:

1. $\frac{b^{19}}{b^{11}}$
2. $\frac{z^5}{z^{11}}$

Answer

1. b^8
2. $\frac{1}{z^6}$

Try It 1.6.9

Simplify:

1. $\frac{p^9}{p^{17}}$
2. $\frac{w^{13}}{w^9}$

Answer

1. $\frac{1}{p^8}$
2. w^4

Simplify Expressions with an Exponent of Zero

A special case of the Quotient Property is when the exponents of the numerator and denominator are equal, such as an expression like $\frac{a^m}{a^m}$. From your earlier work with fractions, you know that:

$$\frac{2}{2}=1 \quad \frac{17}{17}=1 \quad \frac{-43}{-43}=1 \tag{1.6.7}$$

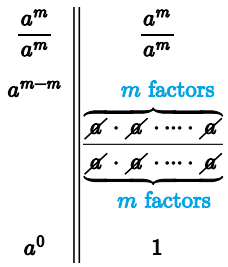
In words, a number divided by itself is 1. So, $\frac{x}{x}=1$, for any $x(x\neq 0)$, since any number divided by itself is 1.

The Quotient Property for Exponents shows us how to simplify $\frac{a^m}{a^n}$ when $m>n$ and when $n<m$ by subtracting exponents. What if $m=n$?

Consider $\frac{8}{8}$, which we know is 1.

	$\frac{8}{8}$	$=$	1
Write 8 as 2^3.	$\frac{2^3}{2^3}$	$=$	1
Subtract exponents.	2^{3-3}	$=$	1
Simplify.	2^0	$=$	1

Now we will simplify $\frac{a^m}{a^m}$ in two ways to lead us to the definition of the zero exponent. In general, for $a\neq 0$:



We see $\frac{a^m}{a^m}$ simplifies to a^0 and to 1. So $a^0 = 1$.

ZERO EXPONENT

If a is a non-zero number, then $a^0 = 1$.

Any nonzero number raised to the zero power is 1.

In this text, we assume any variable that we raise to the zero power is not zero.

Example 1.6.10

Simplify:

a. 9^0
b. n^0

Solution

The definition says any non-zero number raised to the zero power is 1.

a.	Use the definition of the zero exponent.	9^0 1
b.	Use the definition of the zero exponent.	n^0 1

Try It 1.6.11

Simplify:

1. 15^0
2. m^0

Answer

1. 1
2. 1

Try It 1.6.12

Simplify:

a. k^0
b. 29^0

Answer

a. 1
b. 1

Now that we have defined the zero exponent, we can expand all the Properties of Exponents to include whole number exponents.

What about raising an expression to the zero power? Let's look at $(2x)^0$. We can use the product to a power rule to rewrite this expression.

$$\begin{array}{ll} & (2x)^0 \\ \text{Use the product to a power rule.} & 2^0x^0 \\ \text{Use the zero exponent property.} & 1 \cdot 1 \\ \text{Simplify.} & 1 \end{array} \tag{1.6.8}$$

This tells us that any nonzero expression raised to the zero power is one.

Example 1.6.13

Simplify:

a. $(5b)^0$
b. $(-4a^2b)^0$.

Solution

a.	Use the definition of the zero exponent.	$(5b)^0$ 1
b.	Use the definition of the zero exponent.	$(-4a^2b)^0$ 1

Try It 1.6.14

Simplify:

a. $(11z)^0$
b. $(-11pq^3)^0$.

Answer

a. 1
b. 1

Try It 1.6.15

Simplify:

a. $(-6d)^0$
b. $(-8m^2n^3)^0$.

Answer

a. 1
b. 1

Simplify Expressions Using the Quotient to a Power Property

Now we will look at an example that will lead us to the Quotient to a Power Property.

$$\begin{array}{ll} & \left(\frac{x}{y}\right)^3 \\ \textbf{This means:} & \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y} \\ \textbf{Multiply the fractions.} & \frac{x\cdot x\cdot x}{y\cdot y\cdot y} \\ \textbf{Write with exponents.} & \frac{x^3}{y^3} \end{array} \tag{1.6.9}$$

Notice that the exponent applies to both the numerator and the denominator.

$$\begin{array}{ll} \textbf{We see that } \left(\frac{x}{y}\right)^3 \textbf{ is } \frac{x^3}{y^3} & \\ \textbf{We write:} & \left(\frac{x}{y}\right)^3 \\ & \frac{x^3}{y^3} \end{array} \tag{1.6.10}$$

This leads to the *Quotient to a Power Property for Exponents*.

QUOTIENT TO A POWER PROPERTY FOR EXPONENTS

If a and b are real numbers, $b \neq 0$, and m is a counting number, then

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \tag{1.6.11}$$

To raise a fraction to a power, raise the numerator and denominator to that power.

An example with numbers may help you understand this property:

$$\begin{array}{c} \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} \\ \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} = \frac{8}{27} \\ \frac{8}{27} = \frac{8}{27}\checkmark \end{array} \tag{1.6.12}$$

Example 1.6.16

Simplify:

a. $\left(\frac{3}{7}\right)^2$

b. $\left(\frac{b}{3}\right)^4$

c. $\left(\frac{k}{j}\right)^3$

Solution

1.

	$\left(\frac{3}{7}\right)^2$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{3^2}{7^2}$
Simplify.	$\frac{9}{49}$

2.

	$\left(\frac{b}{3}\right)^4$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{b^4}{3^4}$
Simplify.	$\frac{b^4}{81}$

3.

	$\left(\frac{k}{j}\right)^3$
Raise the numerator and denominator to the third power.	$\frac{k^3}{j^3}$

Try It 1.6.17

Simplify:

a. $\left(\frac{5}{8}\right)^2$
b. $\left(\frac{p}{10}\right)^4$
c. $\left(\frac{m}{n}\right)^7$

Answer

a. $\frac{25}{64}$
b. $\frac{p^4}{10,000}$
c. $\frac{m^7}{n^7}$

Try It 1.6.18

Simplify:

a. $\left(\frac{1}{3}\right)^3$
b. $\left(\frac{-2}{q}\right)^3$
c. $\left(\frac{w}{x}\right)^4$

Answer

a. $\frac{1}{27}$
b. $\frac{-8}{q^3}$
c. $\frac{w^4}{x^4}$

Simplify Expressions by Applying Several Properties

We'll now summarize all the properties of exponents so they are all together to refer to as we simplify expressions using several properties. Notice that they are now defined for whole number exponents.

SUMMARY OF EXPONENT PROPERTIES

If a and b are real numbers, and m and n are whole numbers, then

$$
\begin{array}{lrcll}
\textbf{Product Property} & a^m \cdot a^n & = & a^{m+n} & \\
\textbf{Power Property} & (a^m)^n & = & a^{m \cdot n} & \\
\textbf{Product to a Power} & (ab)^m & = & a^m b^m & \\
\textbf{Quotient Property} & \dfrac{a^m}{a^n} & = & a^{m-n}, & a \neq 0, m > n \\
 & \dfrac{a^n}{a^n} & = & 1, & a \neq 0, n > m \\
\textbf{Zero Exponent Definition} & a^0 & = & 1, & a \neq 0 \\
\textbf{Quotient to a Power Property} & \left(\dfrac{a}{b}\right)^m & = & \dfrac{a^m}{b^m}, & b \neq 0
\end{array}
\tag{1.6.13}
$$

Example 1.6.19

Simplify: $\dfrac{(y^4)^2}{y^6}$

Solution

	$\dfrac{(y^4)^2}{y^6}$
Multiply the exponents in the numerator.	$\dfrac{y^8}{y^6}$
Subtract the exponents.	y^2

Try It 1.6.20

Simplify: $\dfrac{(m^5)^4}{m^7}$

Answer

m^{13}

Try It 1.6.21

Simplify: $\dfrac{(k^2)^6}{k^7}$

Answer

k^5

Example 1.6.22

Simplify: $\dfrac{b^{12}}{(b^2)^6}$

Solution

	$\dfrac{b^{12}}{(b^2)^6}$
Multiply the exponents in the numerator.	$\dfrac{b^{12}}{b^{12}}$
Subtract the exponents.	b^0
Simplify	1

Notice that after we simplified the denominator in the first step, the numerator and the denominator were equal. So the final value is equal to 1.

Try It 1.6.23

Simplify $\frac{n^{12}}{(n^3)^4}$.

Answer

1

Try It 1.6.24

Simplify $\frac{x^{15}}{(x^3)^5}$.

Answer

1

Example 1.6.25

Simplify: $\left(\frac{y^9}{y^4}\right)^2$

Solution

	$\left(\frac{y^9}{y^4}\right)^2$
Remember parentheses come before exponents.	
Notice the bases are the same, so we can simplify inside the parentheses. Subtract the exponents.	$(y^5)^2$
Multiply the exponents.	y^{10}

Try It 1.6.26

Simplify: $\left(\frac{r^5}{r^3}\right)^4$

Answer

r^8

Try It 1.6.27

Simplify: $\left(\frac{v^6}{v^4}\right)^3$

Answer

v^6

Example 1.6.28

Simplify: $\left(\frac{j^2}{k^3}\right)^4$

Solution

Here we cannot simplify inside the parentheses first because the bases are not the same.	$\left(\frac{j^2}{k^3}\right)^4$
Raise the numerator and denominator to the 4th power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{(j^2)^4}{(k^3)^4}$
Use the Power Property and simplify.	$\frac{j^8}{k^{12}}$

Try It 1.6.29

Simplify: $\left(\frac{a^3}{b^2}\right)^4$

Answer

$\frac{a^{12}}{b^8}$

Try It 1.6.30

Simplify: $\left(\frac{q^7}{r^5}\right)^3$

Answer

$\frac{q^{21}}{r^{15}}$

Example 1.6.31

Simplify: $\left(\frac{2m^2}{5n}\right)^4$

Solution

	$\left(\frac{2m^2}{5n}\right)^4$
Raise the numerator and denominator to the 4^{th} power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\frac{(2m^2)^4}{(5n)^4}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$	$\frac{2^4(m^2)^4}{5^4 n^4}$
Use the Power Property and simplify.	$\frac{16m^8}{625n^4}$

Try It 1.6.32

Simplify: $\left(\frac{7x^3}{9y}\right)^2$

Answer

$\frac{49x^6}{81y^2}$

Try It 1.6.33

Simplify: $\left(\frac{3x^4}{7y}\right)^2$

Answer

$\frac{9x^8}{49y^2}$

Example 1.6.34

Simplify: $\frac{(x^3)^4(x^2)^5}{(x^6)^5}$

Solution

	$\frac{(x^3)^4(x^2)^5}{(x^6)^5}$
Use the Power Property, $(a^m)^n = a^{m \cdot n}$	$\frac{(x^{12})(x^{10})}{(x^{30})}$
Add the exponents in the numerator.	$\frac{x^{22}}{x^{30}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{1}{x^8}$

Try It 1.6.35

Simplify: $\frac{(a^2)^3(a^2)^4}{(a^4)^5}$

Answer

$\frac{1}{a^6}$

Try It 1.6.36

Simplify: $\frac{(p^3)^4(p^5)^3}{(p^7)^6}$

Answer

$\frac{1}{p^{15}}$

Example 1.6.37

Simplify: $\frac{(10p^3)^2}{(5p)^3(2p^5)^4}$

Solution

	$\frac{(10p^3)^2}{(5p)^3(2p^5)^4}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$	$\frac{(10)^2(p^3)^2}{(5)^3(p)^3(2)^4(p^5)^4}$
Use the Power Property, $(a^m)^n = a^{m \cdot n}$	$\frac{100p^6}{125p^3 \cdot 16p^{20}}$
Add the exponents in the denominator.	$\frac{100p^6}{125 \cdot 16p^{23}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$	$\frac{100}{125 \cdot 16p^{17}}$
Simplify.	$\frac{1}{20p^{17}}$

? Try It 1.6.38

Simplify: $\frac{(3r^3)^2(r^3)^7}{(r^3)^3}$

Answer

$9r^{18}$

? Try It 1.6.39

Simplify: $\frac{(2x^4)^5}{(4x^3)^2(x^3)^5}$

Answer

$\frac{2}{x}$

Divide Monomials

You have now been introduced to all the properties of exponents and used them to simplify expressions. Next, you'll see how to use these properties to divide monomials. Later, you'll use them to divide polynomials.

✓ Example 1.6.40

Find the quotient: $56x^7 \div 8x^3$

Solution

	$56x^7 \div 8x^3$
Rewrite as a fraction.	$\frac{56x^7}{8x^3}$
Use fraction multiplication.	$\frac{56}{8} \cdot \frac{x^7}{x^3}$
Simplify and use the Quotient Property.	$7x^4$

? Try It 1.6.41

Find the quotient: $42y^9 \div 6y^3$

Answer

$7y^6$

Try It 1.6.42

Find the quotient: $48z^8 \div 8z^2$

Answer

$6z^6$

Example 1.6.43

Find the quotient: $\frac{45a^2b^3}{-5ab^5}$

Solution

When we divide monomials with more than one variable, we write one fraction for each variable.

	$\frac{45a^2b^3}{-5ab^5}$
Use fraction multiplication.	$\frac{45}{-5} \cdot \frac{a^2}{a} \cdot \frac{b^3}{b^5}$
Simplify and use the Quotient Property.	$-9 \cdot a \cdot \frac{1}{b^2}$
Multiply.	$-\frac{9a}{b^2}$

Try It 1.6.44

Find the quotient: $\frac{-72a^7b^3}{8a^{12}b^4}$

Answer

$-\frac{9}{a^5b}$

Try It 1.6.45

Find the quotient: $\frac{-63c^8d^3}{7c^{12}d^2}$

Answer

$\frac{-9d}{c^4}$

Example 1.6.46

Find the quotient: $\frac{24a^5b^3}{48ab^4}$

Solution

	$\frac{24a^5b^3}{48ab^4}$
Use fraction multiplication.	$\frac{24}{48} \cdot \frac{a^5}{a} \cdot \frac{b^3}{b^4}$
Simplify and use the Quotient Property.	$\frac{1}{2} \cdot a^4 \cdot \frac{1}{b}$
Multiply.	$\frac{a^4}{2b}$

? Try It 1.6.47

Find the quotient: $\frac{16a^7b^6}{24ab^8}$

Answer

$\frac{2a^6}{3b^2}$

? Try It 1.6.48

Find the quotient: $\frac{27p^4q^7}{-45p^{12}q}$

Answer

$-\frac{3q^6}{5p^8}$

Once you become familiar with the process and have practiced it step by step several times, you may be able to simplify a fraction in one step.

✓ Example 1.6.49

Find the quotient: $\frac{14x^7y^{12}}{21x^{11}y^6}$

Solution

Be very careful to simplify $\frac{14}{21}$ by dividing out a common factor, and to simplify the variables by subtracting their exponents.

	$\frac{14x^7y^{12}}{21x^{11}y^6}$
Simplify and use the Quotient Property.	$\frac{2y^6}{3x^4}$

? Try It 1.6.50

Find the quotient: $\frac{28x^5y^{14}}{49x^9y^{12}}$

Answer

$\frac{4y^2}{7x^4}$

? Try It 1.6.51

Find the quotient: $\frac{30m^5n^{11}}{48m^{10}n^{14}}$

Answer

$\frac{5}{8m^5n^3}$

In all examples so far, there was no work to do in the numerator or denominator before simplifying the fraction. In the next example, we'll first find the product of two monomials in the numerator before we simplify the fraction. This follows the order of operations. Remember, a fraction bar is a grouping symbol.

Example 1.6.52

Find the quotient: $\frac{(6x^2y^3)(5x^3y^2)}{(3x^4y^5)}$

Solution

Answer

	$\frac{(6x^2y^3)(5x^3y^2)}{(3x^4y^5)}$
Simplify the numerator.	$\frac{30x^5y^5}{3x^4y^5}$
Simplify.	$10x$

Try It 1.6.53

Find the quotient: $\frac{(6a^4b^5)(4a^2b^5)}{12a^5b^8}$

Answer

$2ab^2$

Try It 1.6.54

Find the quotient: $\frac{(-12x^6y^9)(-4x^5y^8)}{-12x^{10}y^{12}}$

Answer

$-4xy^5$

Note

Access these online resources for additional instruction and practice with dividing monomials:

- Rational Expressions
- Dividing Monomials
- Dividing Monomials 2

Key Concepts

- **Quotient Property for Exponents:**
  - If a is a real number, $a \neq 0$, and m,n are whole numbers, then: $\frac{a^m}{a^n} = a^{m-n}, m > n$ and $\frac{a^m}{a^n} = \frac{1}{a^{m-n}}, n > m$
- **Zero Exponent**
  - If a is a non-zero number, then $a^0 = 1$.
- **Quotient to a Power Property for Exponents**:
  - If a and b are real numbers, $b \neq 0$, and mm is a counting number, then: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  - To raise a fraction to a power, raise the numerator and denominator to that power.
- **Summary of Exponent Properties**

- If a,b are real numbers and m,nm,n are whole numbers, then

Product Property	$a^m \cdot a^n$	$=$	a^{m+n}
Power Property	$(a^m)^n$	$=$	$a^{m \cdot n}$
Product to a Power	$(ab)^m$	$=$	$a^m b^m$
Quotient Property	$\frac{a^m}{a^n}$	$=$	$a^{m-n}, a \neq 0, m > n$
	$\frac{a^n}{a^n}$	$=$	$1, a \neq 0, n > m$
Zero Exponent Definition	a^0	$=$	$1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m$	$=$	$\frac{a^m}{b^m}, b \neq 0$

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1.7: Scientific and Engineering Notation



Powers of Ten

Decimal notation is based on powers of 10: 0.1 is $\frac{1}{10^1}$, 0.01 is $\frac{1}{10^2}$, 0.001 is $\frac{1}{10^3}$, and so on.

We represent these powers with negative exponents: $\frac{1}{10^1} = 10^{-1}$, $\frac{1}{10^2} = 10^{-2}$, $\frac{1}{10^3} = 10^{-3}$, etc.

Negative exponents: $\frac{1}{10^n} = 10^{-n}$

Note: This is true for any base, not just 10*, but we will focus only on* 10 *in this course.*

With our base 10 number system, any power of 10 can be written as a 1 in a certain decimal place.

10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
$10,000$	$1,000$	100	10	1	0.1	0.01	0.001	0.0001

If you haven't watched the video "Powers of Ten" from 1977 on YouTube, take ten minutes right now and check it out. Your mind will never be the same again.

Scientific Notation

Let's consider how we could rewrite some different numbers using these powers of 10.

Let's take $50,000$ as an example. $50,000$ is equal to $5 \times 10,000$ or 5×10^4.[1]

Looking in the other direction, a decimal such as 0.0007 is equal to 7×0.0001 or 7×10^{-4}.

The idea behind scientific notation is that we can represent very large or very small numbers in a more compact format: a number between 1 and 10, multiplied by a power of 10.

A number is written in scientific notation if it is written in the form $a \times 10^n$, where n is an integer and a is any real number such that $1 \leq a < 10$.

Note: An ***integer*** *is a number with no fraction or decimal part:* $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

? Exercise 1.7.1

1. The mass of the Earth is approximately $5,970,000,000,000,000,000,000,000$ kilograms. The mass of Mars is approximately $639,000,000,000,000,000,000,000$ kilograms. Can you determine which mass is larger?

Answer

Clearly, it is difficult to keep track of all those zeros. Let's rewrite those huge numbers using scientific notation.

Earth's mass is larger because it's a 25-digit number and Mars' mass is a 24-digit number, but it might take a lot of work counting the zeros to be sure.

? Exercise 1.7.2

2. The mass of the Earth is approximately 5.97×10^{24} kilograms. The mass of Mars is approximately 6.39×10^{23} kilograms. Can you determine which mass is larger?

Answer

Earth's mass is about ten times larger, because the power of 10 is 1 higher than that of Mars.

It is much easier to compare the powers of 10 and determine that the mass of the Earth is larger because it has a larger power of 10. You may be familiar with the term ***order of*** magnitude; this simply refers to the difference in the powers of 10 of the two numbers. Earth's mass is one order of magnitude larger because 24 is 1 more than 23.

We can apply scientific notation to small decimals as well.

? Exercise 1.7.3

3. The radius of a hydrogen atom is approximately 0.000000000053 meters. The radius of a chlorine atom is approximately 0.00000000018 meters. Can you determine which radius is larger?

Answer

A chlorine atom's radius is larger because it has 9 zeros before the significant digits begin, but a hydrogen atom's radius has 10 zeros before the significant digits begin. As above, counting the zeros is a pain in the neck.

Again, keeping track of all those zeros is a chore. Let's rewrite those decimal numbers using scientific notation.

? Exercise 1.7.4

4. The radius of a hydrogen atom is approximately 5.3×10^{-11} meters. The radius of a chlorine atom is approximately 1.8×10^{-10} meters. Can you determine which radius is larger?

Answer

The chlorine atom has a larger radius because its power of 10 is 1 higher than that of the hydrogen atom. (Remember that -10 is larger than -11 because -10 is farther to the right on a number line.)

The radius of the chlorine atom is larger because it has a larger power of 10; the digits 1 and 8 for chlorine begin in the tenth decimal place, but the digits 5 and 3 for hydrogen begin in the eleventh decimal place.

Scientific notation is very helpful for really large numbers, like the mass of a planet, or really small numbers, like the radius of an atom. It allows us to do calculations or compare numbers without going cross-eyed counting all those zeros.

? Exercises 1.7.5

Write each of the following numbers in scientific notation.

5. $1,234$

6. $10,200,000$

7. 0.00087

8. 0.0732

Convert the following numbers from scientific notation to standard decimal notation.

9. 3.5×10^4

10. 9.012×10^7

11. 8.25×10^{-3}

12. 1.4×10^{-5}

Answer

5. 1.234×10^3
6. 1.02×10^7
7. 8.7×10^{-4}
8. 7.32×10^{-2}
9. $35,000$
10. $90,120,000$
11. 0.00825
12. 0.000014

You may be familiar with a shortcut for multiplying numbers with zeros on the end; for example, to multiply $300 \times 4,000$, we can multiply the significant digits $3 \times 4 = 12$ and count up the total number of zeros, which is five, and write five zeros on the back end of the 12: $1,200,000$. This shortcut can be applied to numbers in scientific notation.

To multiply powers of 10, add the exponents: $10^m \cdot 10^n = 10^{m+n}$

? Exercises 1.7.6

Multiply each of the following and write the answer in scientific notation.

13. $(2 \times 10^3)(4 \times 10^4)$

14. $(5 \times 10^4)(7 \times 10^8)$

15. $(3 \times 10^{-2})(2 \times 10^{-3})$

16. $(8 \times 10^{-5})(6 \times 10^9)$

Answer

13. 8×10^7
14. 3.5×10^{13}
15. 6×10^{-5}
16. 4.8×10^5

When the numbers get messy, it's probably a good idea to use a calculator. If you are dividing numbers in scientific notation with a calculator, you may need to use parentheses carefully.

? Exercises 1.7.7

The mass of a proton is 1.67×10^{-27} kg. The mass of an electron is 9.11×10^{-31} kg.

17. Divide these numbers using a calculator to determine approximately how many times greater the mass of a proton is than the mass of an electron.

18. What is the approximate mass of one million protons? (Note: one million is 10^6.)

19. What is the approximate mass of one billion protons? (Note: one billion is 10^9.)

Answer

17. the proton's mass is roughly $1,830$ or 1.83×10^3 times larger

18. 1.67×10^{-21} kg

19. 1.67×10^{-18} kg

Engineering Notation

Closely related to scientific notation is engineering notation, which uses only multiples of $1,000$. This is the way large numbers are often reported in the news; if roughly $37,000$ people live in Oregon City, we say "thirty-seven thousand" and we might see it written as "37 thousand"; it would be unusual to think of it as $3.7 \times 10,000$ and report the number as "three point seven ten thousands".

One thousand = 10^3, one million = 10^6, one billion = 10^9, one trillion = 10^{12}, and so on.

In engineering notation, the power of 10 is always a multiple of 3, and the other part of the number must be between 1 and $1,000$.

A number is written in engineering notation if it is written in the form $a \times 10^n$, where n is a multiple of 3 and a is any real number such that $1 \leq a < 1,000$.

Note: Prefixes for large numbers such as kilo, mega, giga, and tera are essentially engineering notation, as are prefixes for small numbers such as micro, nano, and pico. We'll see these next.

? Exercises 1.7.8

Write each number in engineering notation, then in scientific notation.

20. The U.S. population is around 330.2 million people.[2]

21. The world population is around 7.68 billion people.[3]

22. The U.S. national debt is around 26.6 trillion dollars.[4]

Answer

20. 330.2×10^6; 3.302×10^8

21. 7.68×10^9; 7.68×10^9 \

22. (26.6 \times 10^{12}\); 2.66×10^{13}

Measurement Prefixes: Larger

Now let's turn our attention to converting units based on their prefixes. We'll start with large units of measure.

tera- (T)	giga- (G)	mega- (M)	kilo- (k)	**[base unit]**
trillion	billion	million	thousand	one
$1,000,000,000,000$	$1,000,000,000$	$1,000,000$	$1,000$	1
10^{12}	10^9	10^6	10^3	10^0

Notice that the powers of these units are multiples of 3, just as with the engineering notation we saw in a previous module. Each prefix is $1,000$ times the next smaller prefix, so moving one place in the chart means moving the decimal point three places. Also notice that capitalization is important; megagram (which is also called a metric ton) is Mg with a capital M, but milligram is mg with a lowercase m.

Using computer memory as an example:

1 kilobyte = $1,000$ bytes

1 megabyte = 1,000 kilobytes = 1,000,000 bytes

1 gigabyte = 1,000 megabytes = 1,000,000 kilobytes, etc.

1 terabyte = 1,000 gigabytes = 1,000,000 megabytes, etc.

Note: There can be inconsistencies with different people's understanding of these prefixes with regards to computer memory, which is counted in powers of 2, not 10. Computer engineers originally defined 1 kilobyte as 1,024 bytes because $2^{10} = 1,024$, which is very close to 1,000. However, we will consider these prefixes to be powers of 1,000, not 1,024. There is an explanation at https://physics.nist.gov/cuu/Units/binary.html.

? Exercises 1.7.1



23. A $5\frac{1}{4}$ inch floppy disk from the 1980s could store about 100 kB; a $3\frac{1}{2}$ inch floppy disk from the 1990s could store about 1.44 MB. By what factor was the storage capacity increased?

24. How many times greater is the storage capacity of a 2 TB hard drive than a 500 GB hard drive?

25. In an article describing small nuclear reactors that are designed to be retrofitted into coal plants, Dr. Jose Reyes of Oregon State University says "One module will produce 60 megawatts of electricity. That's enough for about 50 thousand homes."[5] How much electricity per home is this?

26. In the same article, Dr. Reyes says "a 60 megawatt module could produce about 60 million gallons of clean water per day using existing technologies in reverse osmosis." What is the rate of watts per gallon?

27. The destructive power of nuclear weapons is measured in kilotons (the equivalent of 1,000 tons of TNT) or megatons (the equivalent of 1,000,000 tons of TNT). The first nuclear device ever tested, the US's *Trinity*, was measured at roughly 20 kilotons on July 16, 1945. The largest thermonuclear weapon ever detonated, at 50 megatons, was the USSR's *Tsar Bomba*, on October 31, 1961.[6] (Video of *Tsar Bomba* was declassified almost 60 years later, in August 2020.) How many times more powerful was *Tsar Bomba* than *Trinity*?

Answer

23. the capacity increased by a factor of 14.4

24. 4 times greater

25. 1,200 megawatts per home

26. 1 watt per gallon

27. 2,500 times more powerful

Measurement Prefixes: Smaller

Now we'll go in the other direction and look at small units of measure.

[base unit]	milli- (m)	micro- (μ or mc)	nano- (n)	pico (p)
one	thousandth	millionth	billionth	trillionth
1	0.001	0.000001	0.000000001	0.000000000001
10^0	10^{-3}	10^{-6}	10^{-9}	10^{-12}

The symbol for micro- is the Greek letter μ (pronounced “myoo”). Because this character can be difficult to replicate, you may see the letter “u” standing in for “μ” in web-based or plaintext technical articles… or you may see the prefix “mc” instead.

Again, the powers are multiples of 3; each prefix gets smaller by a factor of $\frac{1}{1000}$. The negative exponents can sometime be complicated to work with, and it may help to think about things in reverse.

1 meter = 10^3 millimeters = 10^6 micrometers = 10^9 nanometers = 10^{12} picometers

1 second = 10^3 milliseconds = 10^6 microseconds = 10^9 nanoseconds = 10^{12} picoseconds

…and so on.

See https://physics.nist.gov/cuu/Units/prefixes.html for a list of more prefixes.

? Exercises 1.7.1

28. An article about network latency compares the following latency times: “So a 10 Mbps link adds 0.4 milliseconds to the RTT, a 100 Mbps link 0.04 ms and a 1 Gbps link just 4 microseconds.”[7] Rewrite these times so that they are all in terms of milliseconds, then rewrite them in terms of microseconds.

29. The wavelength of red light is around 700 nm. Infrared radiation has a wavelength of approximately 10 μm.[8] Find the ratio of these wavelengths.

30. Nuclear radiation is measured in units called Sieverts, but because this unit is too large to be practical when discussing people’s exposure to radiation, milliSieverts and microSieverts are more commonly used. In 1986, workers cleaning up the Chernobyl disaster were exposed to an estimated dose of 250 mSv.[9] A typical chest x-ray exposes a person to 100 μSv.[10] How many chest x-rays’ worth of radiation were the workers exposed to?

Answer

28. 0.4 ms, 0.04 ms, 0.004 ms; 400 μs, 40 μs, 4 μs the ratio of the wavelengths of red and infrared is 7 to 100;
29. the ratio of the wavelengths of infrared and red is around 14 to 1
30. this is equivalent to 2,500 chest x-rays

1. For some reason, although we generally try to avoid using the "x" shaped multiplication symbol, it is frequently used with scientific notation. ↵
2. August 27, 2020 estimate from *https://www.census.gov/popclock/*↵
3. August 27, 2020 estimate from *https://www.census.gov/popclock/*↵
4. August 27, 2020 data from *fiscaldata.treasury.gov/datasets/debt-to-the-penny/debt-to-the-penny*↵
5. https://www.kgw.com/article/news/local/oregon-company-get-approval-to-build-nuclear-power-plants/283-7b26b8cd-12d5-4116-928a-065731f7a0f6 ↵
6. https://en.Wikipedia.org/wiki/Nuclear_weapon_yield ↵
7. https://www.noction.com/blog/network-latency-effect-on-application-performance ↵
8. http://labman.phys.utk.edu/phys222core/modules/m6/The%20EM%20spectrum.html ↵
9. https://en.Wikipedia.org/wiki/Chernobyl_disaster ↵
10. https://www.cancer.org/treatment/understanding-your-diagnosis/tests/understanding-radiation-risk-from-imaging-tests.html ↵

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CHAPTER OVERVIEW

2: Working with and Converting Measurements

2.1: The US Measurement System

2.2: The Metric System

2.3: Converting Between Systems

2.4: Conversions with Time

2.5: Converting Units of Area

2.6: Converting Units of Volume

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2.1: The US Measurement System



Robin the cat has the spirit of ’76.

You may use a calculator throughout this module.

This system used to be called the English system, but now the U.S. has the dubious honor of being associated with the system that uses inches, feet, miles, ounces, pounds, cups, gallons, etc. To convert from one unit to another, we often have to perform messy calculations like dividing by 16 or multiplying by $5,280$.

We could solve these unit conversions using proportions, but there is another method than is more versatile, especially when a conversion requires more than one step. This method goes by various names, such as *dimensional analysis* or the *factor label method*. The basic idea is to begin with the measurement you know, then multiply it by a conversion ratio that will cancel the units you don’t want and replace it with the units you do want.

It’s okay if you don’t have the conversion ratios memorized; just be sure to have them available. If you discover other conversion ratios that aren’t provided here, go ahead and write them down!

U.S. System: Measurements of Length

1 foot = 12 inches

1 yard = 3 feet

1 mile = $5,280$ feet

? Exercises 2.1.1

1. How many inches are in 4.5 feet?

2. How many feet make up 18 yards?

3. 1 yard is equal to how many inches?

4. 1 mile is equivalent to how many yards?

5. How many feet is 176 inches?

6. 45 feet is what length in yards?

7. Convert $10,560$ feet into miles.

8. How many yards are the same as $1,080$ inches?

Answer

1. 54 in
2. 54 ft
3. 36 in
4. $1,760$ yd

5. $14\frac{2}{3}$ ft or 14 ft 8 in
6. 15 yd
7. 2 mi
8. 30 yd

Notice that Exercises 3 & 4 give us two more conversion ratios that we could add to our list.

U.S. System: Measurements of Weight or Mass

1 pound = 16 ounces

1 ton = 2,000 pounds

? Exercises 2.1.1

9. How many ounces are in 2.5 pounds?

10. How many pounds are equivalent to 1.2 tons?

11. Convert 300 ounces to pounds.

12. 1 ton is equivalent to what number of ounces?

Answer

9. 40 oz
10. 2,400 lb
11. 18.75 lb
12. 32,000 oz

U.S. System: Measurements of Volume or Capacity

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 gallon = 4 quarts

There are plenty of other conversions that could be provided, such as the number of fluid ounces in a gallon, but let's keep the list relatively short.

? Exercises 2.1.1

13. How many fluid ounces are in 6 cups?

14. How many pints are in 3.5 quarts?

15. 1 gallon is equal to how many pints?

16. How many cups equal 1.25 quarts?

17. Convert 20 cups into gallons.

18. How many fluid ounces are in one half gallon?

Answer

13. 48 fl oz

14. 7 pt
15. 8 pt
16. 5 c
17. 1.25 gal
18. 64 fl oz

U.S. System: Using Mixed Units of Measurement

Measurements are frequently given with mixed units, such as a person's height being given as 5 ft 7 in instead of 67 in, or a newborn baby's weight being given as 8 lb 3 oz instead of 131 oz. This can sometimes make the calculations more complicated, but if you can convert between improper fractions and mixed numbers, you can handle this.

Exercises 2.1.1

19. A bag of apples weighs 55 ounces. What is its weight in pounds and ounces?

20. A carton of orange juice contains 59 fluid ounces. Determine its volume in cups and fluid ounces.

21. A hallway is 182 inches long. Give its length in feet and inches.

22. The maximum loaded weight of a Ford F-150 pickup truck is 8,500 lb. Convert this weight into tons and pounds.

Answer

19. 3 lb 7 oz
20. 7 c 3 fl oz
21. 15 ft 2 in
22. 4 t 500 lb

We'll finish up this module by adding and subtracting with mixed units. Again, it may help to think of them as mixed numbers, with a whole number part and a fractional part.

Exercises 2.1.1



Comet weighs 8 lb 7 oz and Fred weighs 11 lb 9 oz.

23. Comet and Fred are being put into a cat carrier together. What is their combined weight?

24. How much heavier is Fred than Comet?

Two tables are 5 ft 3 in long and 3 ft 10 in long.

25. If the two tables are placed end to end, what is their combined length?

26. What is the difference in length between the two tables?

Answer

23. 20 lb or 20 lb 0 oz combined

24. 3 lb 2 oz heavier

25. 9 ft 1 in combined

26. 1 ft 5 in longer

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2.2: The Metric System



You will NOT need a calculator for this module.

The metric system was first implemented following the French Revolution; if we're overthrowing the monarchy, why should we use a unit of a "foot" that is based on the length of a king's foot?

The metric system was designed to be based on the natural world, and different units are related to each other by powers of 10 instead of weird numbers like 3, 12, 16, and $5,280$… This makes converting between metric units incredibly simple.

kilo- (k)	hecta- (h)	deka- (da)	**[base unit]**	deci- (d)	centi- (c)	milli- (m)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Notice that because deka- and deci- both start with d, the abbreviation for deka- is da.

Metric System: Measurements of Length

The base unit of length is the meter, which is a bit longer than a yard (three feet). Because the prefix kilo- means one thousand, 1 kilometer is $1,000$ meters. (One kilometer is around six tenths of a mile.) Similarly, because the prefix centi- means one hundredth, 1 centimeter is $\frac{1}{100}$ of a meter, or 1 meter is 100 centimeters. (One centimeter is roughly the thickness of a pen.) And because the prefix milli- means one thousandth, 1 millimeter is $\frac{1}{1,000}$ of a meter, or 1 meter is $1,000$ millimeters. (One millimeter is roughly the thickness of a credit card.)

? Exercises 2.2.1

From each of the four choices, choose the most reasonable measure.

1. The length of a car:
5 kilometers, 5 meters, 5 centimeters, 5 millimeters

2. The height of a notebook:
28 kilometers, 28 meters, 28 centimeters, 28 millimeters

3. The distance to the next town:
3.8 kilometers, 3.8 meters, 3.8 centimeters, 3.8 millimeters

4. An adult woman's height:
1.6 kilometers, 1.6 meters, 1.6 centimeters, 1.6 millimeters

5. An adult woman's height:
160 kilometers, 160 meters, 160 centimeters, 160 millimeters

6. The thickness of a pane of glass:
3 kilometers, **3** meters, **3** centimeters, **3** millimeters

Answer

1. **5** m
2. **28** cm
3. **3.8** km
4. **1.6** m
5. **160** cm
6. **3** mm

kilo- (km)	hecta- (hm)	deka- (dam)	**meter (m)**	deci- (dm)	centi- (cm)	milli- (mm)
1,000	**100**	**10**	**1**	**0.1**	**0.01**	**0.001**
1,000	**100**	**10**	**1**	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

To convert metric units, you can simply move the decimal point left or right the number of places indicated in the table above. No calculator required!

Exercises 2.2.1

7. Convert **3.7** meters to centimeters.

8. Convert **3.7** meters to millimeters.

9. Convert **2.45** kilometers to meters.

10. Convert **2.45** kilometers to centimeters.

11. Convert **342** millimeters to meters.

12. Convert **342** millimeters to centimeters.

13. Convert **528** meters to kilometers.

14. Convert **45** centimeters to meters.

Answer

7. **370** cm
8. **3,700** mm
9. **2,450** m
10. **245,000** cm
11. **0.342** m
12. **34.2** cm
13. **0.528** km
14. **0.45** m

Metric System: Measurements of Weight or Mass

The base unit for mass is the gram, which is approximately the mass of a paper clip. A kilogram is $1,000$ grams; as we'll see in the next module, this is around 2.2 pounds. The active ingredients in medicines may be measured using the milligram, or possibly the microgram, which we will come back to in a future module. For now, we will focus on the prefixes between kilo- and milli-.

Exercises 2.2.1

From each of the three choices, choose the most reasonable measure.

15. The mass of an apple:
100 kilograms, 100 grams, 100 milligrams

16. The mass of an adult man:
80 kilograms, 80 grams, 80 milligrams

17. The amount of active ingredient in a pain relief pill:
500 kilograms, 500 grams, 500 milligrams

18. The base vehicle weight of a GMC Sierra 1500:
$2,000$ kilograms, $2,000$ grams, $2,000$ milligrams

Answer

15. 100 g
16. 80 kg
17. 500 mg
18. $2,000$ kg

kilo- (kg)	hecta- (hg)	deka- (dag)	**gram (g)**	deci- (dg)	centi- (cg)	milli- (mg)
$1,000$	100	10	1	0.1	0.01	0.001
$1,000$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

This table is identical to the previous table; the only difference is that the base unit "meter" has been replaced by "gram". This means that converting metric units of mass is exactly the same process as converting metric units of length; just move the decimal point!

Exercises 2.2.1

19. Convert 0.813 grams to centigrams.

20. Convert 0.813 grams to milligrams.

21. Convert 1.25 kilograms to grams.

22. Convert 1.25 kilograms to milligrams.

23. Convert 960 milligrams to grams.

24. Convert 960 milligrams to centigrams.

25. Convert $1,350$ grams to dekagrams.

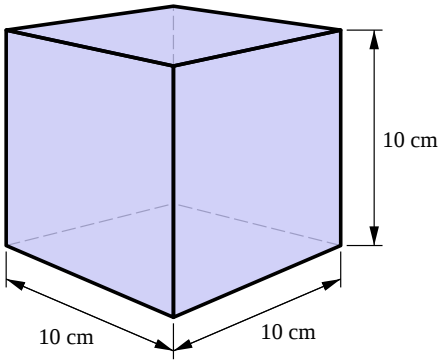
26. Convert 7.5 centigrams to grams.

Answer

19. 81.3 cg
20. 813 g
21. $1,250$ g
22. $1,250,000$ mg
23. 0.96 g
24. 96 cg
25. 135 dag
26. 0.075 g

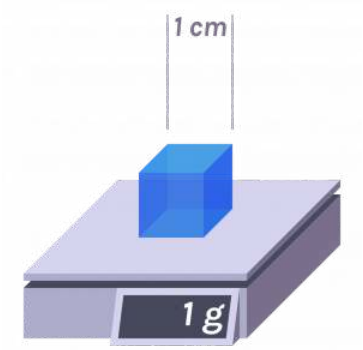
Metric System: Measurements of Volume or Capacity

The base unit of volume is the liter, which is slightly larger than one quart. The milliliter is also commonly used; of course, there are $1,000$ milliliters in 1 liter.



1 liter is equivalent to a cube with sides of 10 centimeters.

In case you were wondering, the units of volume, length, and mass are all connected; one cubic centimeter (a cube with each side equal to 1 cm) has the same volume as one milliliter, and one milliliter of water has a mass of one gram.



Exercises 2.2.1

From each of the two choices, choose the more reasonable measure.

27. The capacity of a car's gas tank: 50 liters, 50 milliliters

28. A dosage of liquid cough medicine: 30 liters, 30 milliliters

Answer

27. 50 L

28. 30 mL

kilo- (kL)	hecta- (hL)	deka- (daL)	**liter (L)**	deci- (dL)	centi- (cL)	milli- (mL)
1,000	100	10	1	0.1	0.01	0.001
1,000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Again, this table is identical to the previous tables; just move the decimal point left or right to convert the units.

? Exercises 2.2.1

29. Convert 2.8 liters to milliliters.

30. Convert 2.8 liters to deciliters.

31. Convert 15 dekaliters to liters.

32. Convert 0.75 deciliters to milliliters.

33. Convert 600 milliliters to centiliters.

34. Convert 600 milliliters to liters.

35. Convert 4.5 deciliters to liters.

36. Convert 550 centiliters to liters.

37. Flying on IcelandAir, you happen to notice that one mini bottle of booze is labeled 50 mL, but another mini bottle is labeled 5 cL. How do the two bottles compare in size?

38. How many 500-milliliter bottles of Coke[1] are equivalent to one 2-liter bottle?

39. The engine displacement of a Yamaha Majesty scooter is 125 cc (cubic centimeters), and the engine displacement of a Chevrolet Spark automobile is 1.4 L (liters). What is the approximate ratio of these engine displacements?

Answer

29. 2,800 mL
30. 28 dL
31. 150 L
32. 75 mL
33. 60 cL
34. 0.6 L
35. 0.45 L
36. 5.5 L
37. they are equal in size
38. 4 bottles
39. about 11 to 1

1. (Fun fact: in Spanish, a 500-milliliter bottle is called a *medio litro*.) ↵

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2.3: Converting Between Systems



You may use a calculator throughout this module.

Converting between the U.S. system and metric system is important in today's global economy; like it or not, the metric system is infiltrating our lives.

The numbers in these conversion ratios are usually difficult to work with, so we will use a calculator whenever necessary and pay attention to rounding. If you discover other conversion ratios that aren't provided here, write them down!

Converting Measurements of Length

You can use the conversion ratios in this table…

- 1 in = 2.54 cm
- 1 ft ≈ 0.305 m
- 1 yd ≈ 0.914 m
- 1 mi ≈ 1.61 km

…or the equivalent conversion ratios in this table.

- 1 cm ≈ 0.394 in
- 1 m ≈ 3.28 ft
- 1 m ≈ 1.09 yd
- 1 km ≈ 0.621 mi

Okay, full disclosure: these conversion ratios aren't exactly equivalent. For example, if we reverse the conversion ratio 1 in = 2.54 cm[1], we discover that 1 cm is actually $\frac{1}{2.54} \approx 0.3937$ inches. All of these conversion have been rounded to three significant figures, which means that we may get a slightly different result depending on which version of the conversion ratio we use. As long as we round our answer to three or fewer significant figures, though, we should be all right.

? Exercises 2.3.1

1. How many centimeters is 6 inches?

2. Convert 200 yards into meters.

3. 12 feet is equal to how many centimeters?

4. How many feet is 50 meters?

5. 15 meters is what length in yards?

6. Convert 75 centimeters into inches.

7. Is 21 kilometers equivalent to 13 miles? If not, what is the percent error?



8. Is 66 kilometers equivalent to 41 miles? If not, what is the percent error?



Answer

1. 15.2 cm
2. 183 m
3. 366 cm
4. 164 ft
5. 16.4 yd
6. 29.5 in
7. not exactly; the error is around 0.3%.
8. they are essentially the same; the error is around 0.015%.

Converting Measurements of Weight or Mass

You can use the conversion ratios in this table…

- 1 oz ≈ 28.35 g
- 1 lb ≈ 0.454 kg

…or the (almost) equivalent conversion ratios in this table.

- 1 g ≈ 0.0353 oz
- 1 kg ≈ 2.20 lb

You probably have noticed that some conversion problems involve multiplication but others involve division. This is based on which version of the conversion ratio you choose; any of these. If you prefer multiplying to dividing, see whether you can figure out a strategy for choosing your conversion ratio.

Exercises 2.3.1

9. Convert 4 ounces into grams.

10. How many kilograms are equivalent to 120 pounds?

11. Convert 50 grams to ounces.

12. Convert 5 kilograms to pounds.

13. How many grams is a half pound of ground beef?

14. In around 2010, the National Collector's Mint (not affiliated with the U.S. Mint) ran a TV commercial selling an imitation $\$\ 50$ gold coin modeled after the U.S. "buffalo" nickel. The commercial made the following claims. *This replica coin is coated in* 31 *milligrams of pure gold! And the price of gold keeps going up; gold is worth about* $\$\ 1,000$ *per ounce! But you can order these fake coins for only* $\$\ 19.95$ *apiece!* What is the approximate dollar value of the gold in one of these coins?

15. In 2020, the National Collector's Mint is still selling the imitation $\$\ 50$ gold "buffalo" nickel for $\$\ 19.95$. This version of the coin is coated in 14 milligrams of pure gold.[2] As of September 2, 2020, the price of gold is $\$\ 1,940$ per ounce.[3] What is the approximate dollar value of the gold in one of these coins?

Answer

9. 113 g
10. 54.5 kg
11. 1.76 oz
12. 11.0 lb
13. 227 g
14. about $\$\ 1.09$
15. about $\$\ 0.96$

Converting Measurements of Volume or Capacity

You can use the conversion ratios in this table…

- 1 fl oz ≈ 29.6 mL
- 1 qt ≈ 0.946 L
- 1 gal ≈ 3.79 L

…or the (almost) equivalent conversion ratios in this table. This table includes one extra entry.

- 1 mL ≈ 0.0338 fl oz
- 1 L ≈ 1.06 qt
- 1 L ≈ 0.264 gal
- 1 L ≈ 33.8 fl oz

Exercises 2.3.1

16. A toilet uses 3.5 gallons of water per flush. How many liters is this?

17. How many milliliters of drink are in a 12-ounce can?

18. How many fluid ounces are in a 50 milliliter mini-sized bottle of alcohol?

19. What part of a gallon is in a 2-liter bottle of soda pop?

Answer

16. 13.3 L

17. 355 mL

18. 1.69 fl oz

19. 0.53 gal

Converting Measurements: Extensions

Let's finish up with some rates that require conversions.

? Exercises 2.3.1

Maxine is driving across Canada. Her car has a 14.2-gallon gas tank and gets an average of 26 miles per gallon.

20. Approximately how many kilometers—actually, the Canadian spelling is kilometres. Approximately how many kilometres can she travel on a full tank of gas?

21. Of course, Canada measures gas in liters. Actually, litres. Convert Maxine's mileage rate, 26 miles per gallon, to kilometres per litre.

Answer

20. a bit less than 600 km

21. 11 km/L

---

1. An inch is defined to be exactly 2.54 cm, if you were curious. ↵
2. https://ncmint.com/2020-buffalo-tribute-proof/ ↵
3. https://markets.businessinsider.com/commodities/gold-price ↵

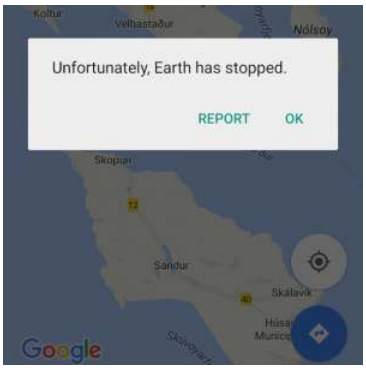
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2.4: Conversions with Time

You may use a calculator in this module as needed.

Converting Measurements of Time



You probably know all of the necessary conversions for time. When we get to units of time larger than weeks, however, we encounter problems because not all months have the same number of days, a year is not exactly 52 weeks, and the time it takes for the Earth to orbit the Sun is not exactly 365 days. Therefore, it doesn't make sense to expect an exact answer to a question like "how many minutes are in one month?" We will have to use our best judgment in situations such as these.

1 min = 60 sec

1 hr = 60 min

1 day (dy) = 24 hr

1 week (wk) = 7 dy

1 year (yr) = 365 dy

? Exercises 2.4.1

1. How many minutes is one standard 365-day year?[1]

2. Have you been alive for 1 billion seconds? Is this even possible?

Answer

1. $525,600$ min
2. this is roughly 31.7 years, which is indeed possible

Converting Rates

? Exercises 2.4.1

Usain Bolt holds the world record time for the 100-meter dash, 9.58 seconds.

3. What was his average speed in kilometers per hour?

4. What was his average speed in miles per hour?

Answer

3. 37.6 km/hr
4. 23.3 mi/hr

The more information we know, the more things we can figure out.

Exercises 2.4.1



An F-15 fighter jet can reach a sustained top speed of roughly Mach 2.3; this is 2.3 times the speed of sound, which is 770 miles per hour.[2]

5. What is the jet's top speed in miles per hour?

6. At this speed, how many miles would the jet travel in one minute?

The jet's range at this speed before it runs out of fuel is around 600 miles.

7. If the jet flies 600 miles at top speed, for how many minutes will it fly?

The jet's maximum fuel capacity is 3,475 gallons of fuel.

8. If the jet flies 600 miles and burns 3,475 gallons of fuel, find the jet's fuel efficiency, in miles per gallon.

9. Rewrite the jet's fuel efficiency, in gallons per mile.

10. How many gallons of fuel does the jet consume in one minute?

Answer

5. 1,770 mi/hr
6. 29.5 mi in 1 min
7. 20.3 min
8. 0.17 mi/gal
9. 5.8 gal/mi
10. 171 gal in 1 min

---

1. If you're familiar with the musical *Rent*, then you already know the answer. ↵
2. My sources for the following set of questions are a combination of former students in the Air National Guard and people who sound like they know what they're talking about on the internet, particularly in this Quora discussion. ↵

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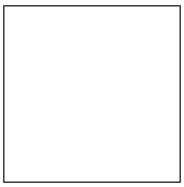
2.5: Converting Units of Area

You may use a calculator throughout this module.

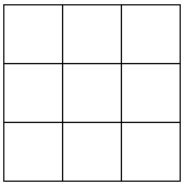
Converting between units of area requires us to be careful because square units behave differently than linear units.

U.S. System: Converting Measurements of Area

Consider a square yard; the area of a square with sides 1 yard long.



1 yard = 3 feet, so we can divide the square into three sections vertically and three sections horizontally to convert both dimensions of the square from yards to feet. This forms a 3 by 3 grid, which shows us visually that 1 square yard equals 9 square feet, not 3 square feet!



The linear conversion ratio of 1 to 3 means that that the conversion ratio for the areas is 1 to 3^2, or 1 to 9.

Here's another way to think about it without a diagram: $1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^2 = (3 \text{ ft})^2$. To remove the parentheses, we must square the number *and* square the units: $(3 \text{ ft})^2 = 3^2 \text{ ft}^2 = 9 \text{ ft}^2$.

More generally, we need to **square** the linear conversion factors when converting units of area. If the linear units have a ratio of 1 to n, the square units will have a ratio of 1 to n^2.

? Exercises 2.5.1

1. An acre is defined as the area of a 660 foot by 66 foot rectangle. (That's a furlong by a chain, if you were curious.) How many square feet are in 1 acre?

2. How many square yards are in 1 acre?

3. How many square inches equals 1 square foot?

Answer

1. $43,560 \text{ ft}^2$
2. $4,840 \text{yd}^2$
3. 144 in^2

It should be no surprise that this module will be full of conversion ratios. As always, if you discover other conversion ratios that aren't provided here, it would be a good idea to write them down so you can use them as needed.

- $1 \text{ ft}^2 = 144 \text{ in}^2$
- $1 \text{ yd}^2 = 9 \text{ ft}^2$
- $1 \text{ acre (ac)} = 43,560 \text{ ft}^2$
- $1 \text{ ac} = 4,840 \text{ yd}^2$
- $1 \text{ mi}^2 = 27,878,400 \text{ ft}^2$

- $1\ \text{mi}^2 = 3,097,600\ \text{yd}^2$
- $1\ \text{mi}^2 = 640\ \text{ac}$

An acre is defined as a unit of area; it would be wrong to say “acres squared” or put an exponent of 2 on the units.

? Exercises 2.5.1

4. A hallway is 9 yards long and 2 yards wide. How many square feet of linoleum are needed to cover the hallway?

5. A proposed site for an elementary school is 600 feet by 600 feet. Find its area, in acres.

Answer

4. $162\ \text{ft}^2$
5. $8.3\ \text{ac}$

Metric System: Converting Measurements of Area

- $1\ \text{cm}^2 = 100\ \text{mm}^2$
- $1\ \text{m}^2 = 1,000,000\ \text{mm}^2$
- $1\ \text{m}^2 = 10,000\ \text{cm}^2$
- $1\ \text{hectare (ha)} = 10,000\ \text{m}^2$
- $1\ \text{km}^2 = 1,000,000\ \text{m}^2$
- $1\ \text{km}^2 = 100\ \text{ha}$

A hectare is defined as a square with sides 100 meters long. Dividing a square kilometer into ten rows and ten columns will make a 10 by 10 grid of 100 hectares. As with acres, it would be wrong to say “hectares squared” or put an exponent of 2 on the units.

? Exercises 2.5.1

6. A hallway is 9 meters long and 2 meters wide. How many square centimeters of linoleum are needed to cover the hallway?

7. A proposed site for an elementary school is 200 meters by 200 meters. Find its area, in hectares.

Answer

6. $180,000\ \text{cm}^2$
7. $4\ \text{ha}$

Both Systems: Converting Measurements of Area

Converting between the U.S. and metric systems will involve messy decimal values. For example, because $1\ \text{in} = 2.54\ \text{cm}$, we can square both numbers and find that $(1\ \text{in})^2 = (2.54\ \text{cm})^2 = 6.4516\ \text{cm}^2$. The conversions are rounded to three or four significant digits in the table below.

- $1\ \text{in}^2 \approx 6.45\ \text{cm}^2 \leftrightarrow 1\ \text{cm}^2 \approx 0.155\ \text{in}^2$
- $1\ \text{in}^2 \approx 6.45\ \text{cm}^2 \leftrightarrow 1\ \text{cm}^2 \approx 0.155\ \text{in}^2$
- $1\ \text{yd}^2 \approx 0.836\ \text{m}^2 \leftrightarrow 1\ \text{m}^2 \approx 1.196\ \text{yd}^2$
- $1\ \text{mi}^2 \approx 2.59\ \text{km}^2 \leftrightarrow 1\ \text{km}^2 \approx 0.386\ \text{mi}^2$
- $1\ \text{ac} \approx 0.405\ \text{ha} \leftrightarrow 1\ \text{ha} \approx 2.47\ \text{ac}$

? Exercises 2.5.1

8. The area of Portland is $145\ \text{mi}^2$. Convert this area to square kilometers.

9. How many hectares is a $5,000$ acre ranch?

10. A sheet of paper measures 8.5 inches by 11 inches. What is the area in square centimeters?

11. A soccer field is **100** meters long and **70** meters wide. What is its area in square feet?

Answer

8. $376\ \text{km}^2$
9. $2,000\ \text{ha}$ (to one sig fig) or $2,\overline{0}00\ \text{ha}$ (to two sig figs)
10. $6\overline{0}0\ \text{cm}^2$ (to two sig figs)
11. $75,300\ \text{ft}^2$



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2.6: Converting Units of Volume

You may use a calculator throughout this module.



Just as we saw with area, converting between units of volume requires us to be careful because cubic units behave differently than linear units.

Quantities of mulch, dirt, or gravel are often measured by the cubic yard. How many cubic feet are in one cubic yard?

1 yard = 3 feet, so we can divide the length into three sections, the width into three sections, and the height into three sections to convert all three dimensions of the cube from yards to feet. This forms a 3 by 3 by 3 cube, which shows us that 1 cubic yard equals 27 cubic feet. The linear conversion ratio of 1 to 3 means that that the conversion ratio for the volumes is 1 to 3^3, or 1 to 27.

Here's another way to think about it without a diagram: $1 \text{ yd} = 3 \text{ ft}$, so $(1 \text{ yd})^3 = (3 \text{ ft})^3$. To remove the parentheses, we must cube the number *and* cube the units: $(3 \text{ ft})^3 = 3^3 \text{ ft}^3 = 27 \text{ ft}^3$.

More generally, we need to **cube** the linear conversion factors when converting units of volume. If the linear units have a ratio of 1 to n, the cubic units will have a ratio of 1 to n^3.

Exercises 2.6.1

1. Determine the number of cubic inches in 1 cubic foot.

2. Determine the number of cubic inches in 1 cubic yard.

3. Determine the number of cubic millimeters in 1 cubic centimeter.

4. Determine the number of cubic centimeters in 1 cubic meter.

Answer

1. $1,728 \text{ in}^3$
2. $46,656 \text{ in}^3$
3. $1,000 \text{ mm}^3$
4. $1,000,000 \text{ cm}^3$

U.S. System: Converting Measurements of Volume

$$1 \text{ ft}^3 = 1,728 \text{ in}^3$$

$$1 \text{ yd}^3 = 27 \text{ ft}^3$$

$$1 \text{ yd}^3 = 46,656 \text{ in}^3$$

Exercises 2.6.1

5. True story: A friend at the National Guard base gave us three long wooden crates to use as raised planting beds. (The crates probably carried some kind of weapons or ammunition, but our friend wouldn't say.) Henry, who was taking geometry in high school, was asked to measure the crates and figure out how much soil we needed. The inside dimensions of each crate were 112 inches long, 14 inches wide, and 14 inches deep. We wanted to fill them most of the way full with soil, leaving about 4 inches empty at the top. How many cubic yards of soil did we need to order from the supplier?

6. True story, continued: I decided to check our answer and did a rough estimate by rounding each dimension to the nearest foot, then figuring out the volume from there. Did this give the same result?

Answer

5. the result is very close to 1 cubic yard: $(112\text{ in} \cdot 14\text{ in} \cdot 10\text{ in}) \cdot 3\text{ crates} = 47,040\text{ in}^3 \approx 1.01\text{ yd}^3$
6. this estimate is also 1 cubic yard: $(9\text{ ft} \cdot 1\text{ ft} \cdot 1\text{ ft}) \cdot 3\text{ crates} = 27\text{ ft}^3 = 1\text{ yd}^3$

We can convert between units of volume and liquid capacity. As you might expect, the numbers are messy in the U.S. system.

$$1\text{ fl oz} \approx 1.805\text{ in}^3 \leftrightarrow 1\text{ in}^3 \approx 0.554\text{ fl oz}$$

$$1\text{ ft}^3 \approx 7.48\text{ gal} \leftrightarrow 1\text{ gal} \approx 0.1337\text{ ft}^3$$

? Exercises 2.6.1



7. A wading pool has a diameter of roughly 5 feet and a depth of 6 inches. How many gallons of water are required to fill it about 80% of the way full?

8. A standard U.S. soda pop can has a diameter of $2\frac{1}{2}$ inches and a height of $4\frac{3}{4}$ inches. Verify that the can is able to hold 12 fluid ounces of liquid.

Answer

7. around 60 gallons
8. yes, the can is able to hold 12 fluid ounces; the can's volume is roughly 23.3 cubic inches $\approx$ 12.9 fluid ounces.

Metric System: Converting Measurements of Volume

$$1\text{ cm}^3 = 1\text{ cc} = 1\text{ mL}$$

$$1\text{ cm}^3 = 1,000\text{ mm}^3$$

$$1\text{ m}^3 = 1,000,000\text{ cm}^3$$

$$1\text{ L} = 1,000\text{ cm}^3$$

$$1\text{ m}^3 = 1,000\text{ L}$$

It's no surprise that the metric conversion ratios are all powers of 10.

? Exercise 2.6.1

9. A can of Perrier mineral water has a diameter of 5.6 cm and a height of 14.7 cm. Verify that the can is able to hold 330 milliliters of liquid.

Answer

yes, the can is able to hold 330 milliliters; the can's volume is roughly 362 cubic centimeters, which is equivalent to 362 milliliters.

Both Systems: Converting Measurements of Volume

Converting between the U.S. and metric systems will of course involve messy decimal values. For example, because $1\text{ in} = 2.54\text{ cm}$, we can cube both numbers and find that $1\text{ in}^3 = (2.54\text{ cm})^3 \approx 16.387\text{ cm}^3$. The conversions are rounded to three or four significant figures in the table below.

$$1\text{ in}^3 \approx 16.39\text{ cm}^3 \leftrightarrow 1\text{ cm}^3 \approx 0.0612\text{ in}^3$$
$$1\text{ ft}^3 \approx 0.0284\text{ m}^3 \leftrightarrow 1\text{ m}^3 \approx 35.29\text{ ft}^3$$
$$1\text{ yd}^3 \approx 0.7646\text{ m}^3 \leftrightarrow 1\text{ m}^3 \approx 1.308\text{ yd}^3$$

? Exercises 2.6.1



10. A "two yard" dumpster has a volume of 2 cubic yards. Convert this to cubic meters.

11. Convert 240 in^3 to cm^3.

12. Convert 500 cm^3 to in^3.

13. Convert $1,000\text{ ft}^3$ to m^3.

14. Convert 45 m^3 to yd^3.

Answer

10. 1.53 m^3
11. $3,930\text{ cm}^3$
12. 30.5 in^3
13. 28.3 m^3
14. 59 yd^3

Density

The density of a material is its weight per volume such as pounds per cubic foot, or mass per volume such as grams per cubic centimeter. Multiplying the volume of an object by its density will give its weight or mass.

? Exercises 2.6.1

15. The standard size of a gold bar in the U.S. Federal Reserve is 7 inches by $3\frac{5}{8}$ inches by $1\frac{3}{4}$ inches.[1] The density of gold is 0.698 pounds per per cubic inch. How much does one gold bar weigh?

16. A cylindrical iron bar has a diameter of 3.0 centimeters and a length of 20.0 centimeters. The density of iron is 7.87 grams per cubic centimeter. What is the bar's mass, in kilograms?

Answer

15. 31 lb
16. 1.1 kg

1. https://www.usmint.gov/about/mint-tours-facilities/fort-knox ↵

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CHAPTER OVERVIEW

3: Solving Equations

3.1: The Language of Algebra
3.2: Solving Equations with Addition and Subtraction Properties of Equality
3.3: Solving Equations with the Division and Multiplication Properties of Equality
3.4: Solving Equations with Variables and Constants on Both Sides
3.5: General Strategy to Solve Linear Equations
3.6: Solving Equations with Fractions or Decimals
3.7: Solving a Formula for a Specific Variable

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3.1: The Language of Algebra

Learning Objectives

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Evaluate an expression
- Identify and combine like terms
- Translate an English phrase to an algebraic expression

Use Variables and Algebraic Symbols

Suppose this year Greg is **20** years old and Alex is **23**. You know that Alex is **3** years older than Greg. When Greg was **12**, Alex was **15**. When Greg is **35**, Alex will be **38**. No matter what Greg's age is, Alex's age will always be 3 years more, right? In the language of algebra, we say that Greg's age and Alex's age are variables and the **3** is a constant. The ages change ("vary") but the **3** years between them always stays the same ("constant"). Since Greg's age and Alex's age will always differ by **3** years, **3** is the constant. In algebra, we use letters of the alphabet to represent variables. So if we call Greg's age g, then we could use $g+3$ to represent Alex's age. See Table **3.1.1**.

Table **3.1.1**

Greg's age	Alex's age
12	15
20	23
35	38
g	$g+3$

The letters used to represent these changing ages are called variables. The letters most commonly used for variables are $x, y, a, b,$ and c.

Definition: VARIABLE

A **variable** is a letter that represents a number whose value may change.

Definition: CONSTANT

A **constant** is a number whose value always stays the same.

To write algebraically, we need some operation symbols as well as numbers and variables. There are several types of symbols we will be using.

There are four basic arithmetic **operations: addition, subtraction, multiplication,** and **division**. We'll list the symbols used to indicate these operations below (Table **3.1.2**). You'll probably recognize some of them.

Table **3.1.2**

Operation	Notation	Say:	The result is...
Addition	$a+b$	a plus b	the sum of a and b
Subtraction	$a-b$	a minus b	the difference of a and b
Multiplication	$a \cdot b, ab, (a)(b), (a)b, a(b)$	a times b	the product of a and b
Division	$a \div b, a/b, \frac{a}{b}, b\overline{)a}$	a divided by b	the quotient of a and b, a is called the dividend, and b is called the divisor

We perform these operations on two numbers. When translating from symbolic form to English, or from English to symbolic form, pay attention to the words “of” and “and.”

- The *difference* of 9 and 2 means subtract 9 and 2, in other words, 9 minus 2, which we write symbolically as $9-2$.
- The *product* of 4 and 8 means multiply 4 and 8, in other words 4 times 8, which we write symbolically as $4 \cdot 8$.

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ (‘three times y’) or $3 \cdot x \cdot y$ (three times x times y)? To make it clear, use $\cdot$ or parentheses for multiplication.

When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

EQUALITY SYMBOL

$a=b$ is read “a is equal to b”

The symbol "$=$" is called the **equal sign**.

On the number line, the numbers get larger as they go from left to right. The number line can be used to explain the symbols "$<$" and "$>$".

INEQUALITY

$a<b$ is read “a is less than b”

a is to the left of b on the number line

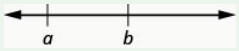


Figure 3.1.1

$a>b$ is read "a is greater than b”

a is to the right of b on the number line

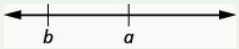


Figure 3.1.2

The expressions $a<b$ or $a>b$ can be read from left to right or right to left, though in English we usually read from left to right Table 3.1.3. In general, $a<b$ is equivalent to $b>a$. For example $7<11$ is equivalent to $11>7$. And $a>b$ is equivalent to $b<a$. For example $17>4$ is equivalent to $4<17$.

Table 3.1.3

Inequality Symbols	Words
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a \leq b$	a is less than or equal to b
$a > b$	a is greater than b
$a \geq b$	a is greater than or not equal to b

Exercise 3.1.1

Translate from algebra into English:

1. $17 \leq 26$
2. $8 \neq 17 - 3$
3. $12 > 27 \div 3$
4. $y + 7 < 19$

Answer

1. $17 \leq 26$, 17 is less than or equal to 26
2. $8 \neq 17 - 3$, 8 is not equal to 17 minus 3
3. $12 > 27 \div 3$, 12 is greater than 27 divided by 3
4. $y + 7 < 19$, y plus 7 is less than 19

Exercise 3.1.2

Translate from algebra into English:

1. $14 \leq 27$
2. $19 - 2 \neq 8$
3. $12 > 4 \div 2$
4. $x - 7 < 1$

Answer

1. 14 is less than or equal to 27
2. 19 minus 2 is not equal to 8
3. 12 is greater than 4 divided by 2
4. x minus 7 is less than 1

Exercise 3.1.3

Translate from algebra into English:

1. $19 \leq 15$
2. $7 = 12 - 5$
3. $15 \div 3 < 8$
4. $y + 3 < 6$

Answer

1. 19 is greater then or equal to 15
2. 7 is equal to 12 minus 5

3. 15 divided by 3 is less than 8
4. y plus 3 is greater than 6

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in English. They help to make clear which expressions are to be kept together and separate from other expressions. We will introduce three types now.

GROUPING SYMBOLS

Parentheses	()
Brackets	[]
Braces	{}

Here are some examples of expressions that include **grouping symbols**. We will simplify expressions like these later in this section.

$$8(14-8) \qquad 21-3[2+4(9-8)] \qquad 24\div\{13-2[1(6-5)+4]\}$$

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb. In algebra, we have expressions and *equations*.

EXPRESSION

An **expression** is a number, a variable, or a combination of numbers and variables using operation symbols.

An **expression** is like an English phrase. Here are some examples of expressions:

Table **3.1.4**

Expression	Words	English Phrase
$3+5$	3 plus 5	the sum of three and five
$n-1$	n minus one	the difference of n and one
$6\cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the English phrases do not form a complete sentence because the phrase does not have a verb. An **equation** is two expressions linked with an **equal sign**. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb.

Definition: EQUATION

An **equation** is two expressions connected by an equal sign.

Here are some examples of equations.

Table **3.1.5**

Equation	English Sentence
$3+5=8$	the sum of three and five is equal to eight
$n-1=14$	n minus one equals fourteen
$6\cdot 7=42$	The product of six ans seven is equal to forty-two
$x=53$	x is equal to fifty-three
$y+9=2y-3$	y plus nine is equal to two y minus three

? Exercise 3.1.4

Determine if each is an expression or an equation:

1. $2(x+3)=10$
2. $4(y-1)+1$
3. $x\div 25$
4. $y+8=40$

Answer

1. $2(x+3)=10$. This is an equation — two expressions are connected with an equal sign.
2. $4(y-1)+1$. This is an expression — no equal sign.
3. $x\div 25$. This is an expression — no equal sign.
4. $y+8=40$. This is an equation — two expressions are connected with an equal sign.

? Exercise 3.1.5

Determine if each is an expression or an equation:

1. $3(x-7)=27$
2. $5(4y-2)-7$

Answer

1. equation
2. expression

? Exercise 3.1.6

Determine if each is an expression or an equation:

1. $y^3\div 14$
2. $4x-6=22$

Answer

1. expression
2. equation

Evaluate an Expression

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

EVALUATE AN EXPRESSION

To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Exercise 3.1.19

Evaluate $7x - 4$, when

1. $x = 5$
2. $x = 1$

Answer

1.

when $x = 5$	$7x - 4$
	$7(5) - 4$
Multiply.	$35 - 4$
Subtract.	31

2.

when $x = 1$	$7x - 4$
	$7(1) - 4$
Multiply.	$7 - 4$
Subtract.	3

Exercise 3.1.20

Evaluate $8x - 3$, when

1. $x = 2$
2. $x = 1$

Answer

1. 13
2. 5

Exercise 3.1.21

Evaluate $4y - 4$, when

1. $y = 3$
2. $y = 5$

Answer

1. 8
2. 16

Exercise 3.1.22

Evaluate $x = 4$, when

1. x^2
2. 3^x

Answer

1.

	x^2
Replace x with 4.	$(4)^2$
Use definition of exponent.	$4 \cdot 4$
Simplify.	16

2.

	3^x
Replace x with 4.	\(3^ ParseError: invalid DekiScript (click for details) \)
Use definition of exponent.	$3 \cdot 3 \cdot 3 \cdot 3$
Simplify.	81

Exercise 3.1.23

Evaluate $x = 3$, when

1. x^2
2. 4^x

Answer

1. 9
2. 64

Exercise 3.1.24

Evaluate $x = 6$, when

1. x^3
2. 2^x

Answer

1. 216
2. 64

Exercise 3.1.25

Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Answer

	$2x^2 + 3x + 8$
Substitute $x = 4$.	$2x^2 + 3x + 8$ $2(4)^2 + 3(4) + 8$
Follow the order of operations.	$2(16) + 3(4) + 8$
	$32 + 12 + 8$
	52

Exercise 3.1.26

Evaluate $3x^2 + 4x + 1$ when $x = 3$.

Answer

40

Exercise 3.1.27

Evaluate $6x^2 - 4x - 7$ when $x = 2$.

Answer

9

Identify and Combine Like Terms

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

TERM

A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are $7, y, 5x^2, 9a$, and b^5.

The constant that multiplies the variable is called the **coefficient**.

COEFFICIENT

The **coefficient** of a term is the constant that multiplies the variable in a term.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x, the coefficient is 1, since $x = 1 \cdot x$.

Exercise 3.1.28

Identify the coefficient of each term:

1. $14y$
2. $15x^2$
3. a

Answer

1. The coefficient of $14y$ is 14
2. The coefficient of $15x^2$ is 15
3. The coefficient of a is 1 since $a = 1a$.

Exercise 3.1.29

Identify the coefficient of each term:

1. $17x$
2. $41b^2$
3. z

Answer

1. 14
2. 41
3. 1

Exercise 3.1.30

Identify the coefficient of each term:

1. $9p$
2. $13a^2$
3. y^3

Answer

1. 9
2. 13
3. 1

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?

$$5x \quad 7 \quad n^2 \quad 4 \quad 3x \quad 9n^2$$

The 7 and the 4 are both constant terms.

The $5x$ and the $3x$ are both terms with x.

The n^2 and the $9n^2$ are both terms with n^2.

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- x^2 and $9x^2$ are like terms.

LIKE TERMS

Terms that are either constants or have the same variables raised to the same powers are called **like terms**.

Exercise 3.1.31

Identify the like terms: $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$.

Answer

y^3 and $4y^3$ are like terms because both have y^3; the variable and the exponent match.

$7x^2$ and $5x^2$ are like terms because both have x^2; the variable and the exponent match.

14 and 23 are like terms because both are constants.

There is no other term like $9x$.

? Exercise 3.1.32

Identify the like terms: $9, 2x^3, y^2, 8x^3, 15, 9y, 11y^2$.

Answer

9 and 15, y^2 and $11y^2$, $2x^3$ and $8x^3$

? Exercise 3.1.33

Identify the like terms: $4x^3, 8x^2, 19, 3x^3, 24, 6x^3$.

Answer

19 and 24, $8x^2$ and $3x^2$, $4x^3$ and $6x^3$

Adding or subtracting terms forms an expression. In the expression $2x^2 + 3x + 8$, from Example, the three terms are $2x^2$, $3x$, and 8.

? Exercise 3.1.34

Identify the terms in each expression.

1. $9x^2 + 7x + 12$
2. $8x + 3y$

Answer

1. The terms of $9x^2 + 7x + 12$ are $9x^2, 7x$, and 12.
2. The terms of $8x + 3y$ are $8x$ and $3y$.

? Exercise 3.1.35

Identify the terms in the expression $4x^2 + 5x + 17$.

Answer

$4x^2, 5x, 17$

? Exercise 3.1.36

Identify the terms in the expression $5x + 2y$.

Answer

$5x, 2y$

If there are like terms in an expression, you can simplify the expression by combining the like terms. What do you think $4x + 7x + x$ would simplify to? If you thought $12x$, you would be right!

$$\begin{array}{c} 4x+7x+x \\ x+x+x+x \quad +x+x+x+x+x+x+x \quad +x \\ 12x \end{array} \tag{3.1.1}$$

Add the coefficients and keep the same variable. It doesn't matter what x is—if you have 4 of something and add 7 more of the same thing and then add 1 more, the result is 12 of them. For example, 4 oranges plus 7 oranges plus 1 orange is 12 oranges. We will discuss the mathematical properties behind this later.

Simplify: $4x+7x+x$

Add the coefficients. $12x$

? Exercise 3.1.37: How To Combine Like Terms

Simplify: $2x^2+3x+7+x^2+4x+5$

Answer

Step 1. Identify the like terms.	$2x^2+3x+7+x^2+4x+5$ $2x^2+3x+7+x^2+4x+5$
Step 2. Rearrange the expression so the like terms are together.	$2x^2+x^2+3x+4x+7+5$
Step 3. Combine like terms.	$3x^2+7x+12$

? Exercise 3.1.38

Simplify: $3x^2+7x+9+7x^2+9x+8$.

Answer

$10x^2+16x+17$

? Exercise 3.1.39

Simplify: $4y^2+5y+2+8y^2+4y+5$.

Answer

$12y^2+9y+7$

COMBINE LIKE TERMS.

1. Identify like terms.
2. Rearrange the expression so like terms are together.
3. Add or subtract the coefficients and keep the same variable for each group of like terms.

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra, then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. Table 3.1.7 summarizes them.

Table 3.1.7

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a+b$ (3.1.2)
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a-b$ (3.1.3)
Multiplication	a times b the product of a and b twice a	$a \cdot b, ab, a(b), (a)(b)$ (3.1.4) $2a$ (3.1.5)
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a \div b, a/b, \frac{a}{b}, b\overline{)a}$ (3.1.6)

Look closely at these phrases using the four operations:

the **sum** *of a and b*
the **difference** *of a and b*
the **product** *of a and b*
the **quotient** *of a and b*

Figure 3.1.5

Each phrase tells us to operate on two numbers. Look for the words *of* and *and* to find the numbers.

? Exercise 3.1.40

Translate each English phrase into an algebraic expression:

1. the difference of $17x$ and 5
2. the quotient of $10x^2$ and 7.

Answer

1. The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

   the *difference of* 17*x* *and* 5
   17*x* minus 5
   17*x* – 5

2. The key word is "quotient," which tells us the operation is division.

   the *quotient of* 10x^2 *and* 7
   divide 10x^2 by 7
   10x^2 ÷ 7

   This can also be written $10x^2/7$ or $\frac{10x^2}{7}$.

Exercise 3.1.41

Translate each English phrase into an algebraic expression:

1. the difference of $14x^2$ and 13
2. the quotient of $12x$ and 2.

Answer

1. $14x^2 - 13$
2. $12x \div 2$

Exercise 3.1.42

Translate each English phrase into an algebraic expression:

1. the sum of $17y^2$ and 19
2. the product of 7 and y.

Answer

1. $17y^2 + 19$
2. $7y$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight "more than" means 8 added to your present age. How old were you seven years ago? This is 7 years less than your age now. You subtract 7 from your present age. Seven "less than" means 7 subtracted from your present age.

Exercise 3.1.43

Translate the English phrase into an algebraic expression:

1. Seventeen more than y
2. Nine less than $9x^2$.

Answer

1. The key words are *more than*. They tell us the operation is addition. *More than* means "added to."

$$\begin{aligned} &\text{Seventeen more than } y \\ &\text{Seventeen added to } y \\ &y + 17 \end{aligned}$$

2. The key words are *less than*. They tell us to subtract. *Less than* means "subtracted from."

$$\begin{aligned} &\text{Nine less than } 9x^2 \\ &\text{Nine subtracted from } 9x^2 \\ &9x^2 - 9 \end{aligned}$$

Exercise 3.1.44

Translate the English phrase into an algebraic expression:

1. Eleven more than x
2. Fourteen less than $11a$.

Answer

1. $x + 11$

2. $11a - 14$

Exercise 3.1.45

Translate the English phrase into an algebraic expression:

1. 13 more than z
2. 18 less than $8x$.

Answer

1. $z + 13$
2. $8x - 18$

Exercise 3.1.46

Translate the English phrase into an algebraic expression:

1. five times the sum of m and n
2. the sum of five times m and n.

Answer

There are two operation words—*times* tells us to multiply and *sum* tells us to add.

1. Because we are multiplying 5 times the sum we need parentheses around the sum of m and n, $(m + n)$. This forces us to determine the sum first. (Remember the order of operations.)

$$\begin{array}{c} \text{five times the sum of } m \text{ and } n \\ 5(m + n) \end{array} \tag{3.1.7}$$

2. To take a sum, we look for the words "of" and "and" to see what is being added. Here we are taking the sum *of* five times m and \(n\.)

$$\begin{array}{c} \text{the sum of five times } m \text{ and } n \\ 5m + n \end{array} \tag{3.1.8}$$

Exercise 3.1.47

Translate the English phrase into an algebraic expression:

1. four times the sum of p and q
2. the sum of four times p and q.

Answer

1. $4(p + q)$
2. $4p + q$

Exercise 3.1.48

Translate the English phrase into an algebraic expression:

1. the difference of two times x and 8,
2. two times the difference of x and 8.

Answer

1. $2x - 8$

2. $2(x-8)$

Later in this course, we'll apply our skills in algebra to solving applications. The first step will be to translate an English phrase to an algebraic expression. We'll see how to do this in the next two examples.

Exercise 3.1.49

The length of a rectangle is 6 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Answer

Write a phrase about the length of the rectangle.	6 less than the width
Substitute w for 'the width.'	6 less then w
Rewrite 'less than' as 'subtracted from.'	6 subtracted from w
Translate the phrase into algebra.	$w-6$

(3.1.9)

Exercise 3.1.50

The length of a rectangle is 7 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Answer

$w-7$

Exercise 3.1.51

The width of a rectangle is 6 less than the length. Let l represent the length of the rectangle. Write an expression for the width of the rectangle.

Answer

$l-6$

Exercise 3.1.52

June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Answer

Write the phrase about the number of dimes.	three less than four times the number of quarters
Substitute q for the number of quarters.	3 less than 4 times q
Translate '4 times q .'	3 less than 4q
Translate the phrase into algebra.	4q - 3

(3.1.10)

Exercise 3.1.53

Geoffrey has dimes and quarters in his pocket. The number of dimes is eight less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Answer

$4q-8$

? Exercise 3.1.54

Lauren has dimes and nickels in her purse. The number of dimes is three more than seven times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Answer

$7n+3$

Key Concepts

- **Notation** — **The result is…**
  - $a+b$ — the sum of a and b
  - $a-b$ — the difference of a and b
  - $a \cdot b, ab, (a)(b), (a)b, a(b)$ — the product of a and b
  - $a \div b, a/b, \frac{a}{b}, b\overline{)a}$ — the quotient of a and b
- **Inequality**
  - $a < b$ is read 'a is less than b'' — a is to the left of b on the number line
  - $a > b$ is read 'a is greater than b'' — a is to the right of b on the number line
- **Inequality Symbols** — **Words**
  - $a \neq b$ — a is not equal to b
  - $a < b$ — a is less than b
  - $a \leq b$ — a is less than or equal to b
  - $a > b$ — a is greater than b
  - $a \geq b$ — a is greater than or equal to b
- **Combine Like Terms**
  1. Identify like terms.
  2. Rearrange the expression so like terms are together.
  3. Add or subtract the coefficients and keep the same variable for each group of like terms.

Glossary

coefficient

The coefficient of a term is the constant that multiplies the variable in a term.

constant

A constant is a number whose value always stays the same.

equality symbol

The symbol "$=$" is called the equal sign. We read $a=b$ as "a is equal to b."

equation

An equation is two expressions connected by an equal sign.

evaluate an expression

To evaluate an expression means to find the value of the expression when the variable is replaced by a given number.

expression

An expression is a number, a variable, or a combination of numbers and variables using operation symbols.

like terms

Terms that are either constants or have the same variables raised to the same powers are called like terms.

simplify an expression

To simplify an expression, do all operations in the expression.

term

A term is a constant or the product of a constant and one or more variables.

variable

A variable is a letter that represents a number whose value may change.

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3.2: Solving Equations with Addition and Subtraction Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Verify a solution of an equation
- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same – so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Definition: Solution of an Equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

TO DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is true (the left side is equal to the right side)
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

Exercise 3.2.1

Determine whether $x = \frac{3}{2}$ is a solution of $4x - 2 = 2x + 1$.

Answer

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

	$4x - 2 = 2x + 1$
Substitute $\frac{3}{2}$ for x.	$4\left(\frac{3}{2}\right) - 2 \stackrel{?}{=} 2\left(\frac{3}{2}\right) + 1$
Multiply.	$6 - 2 \stackrel{?}{=} 3 + 1$
Subtract.	$4 = 4\checkmark$

Since $x = \frac{3}{2}$ results in a true equation (4 is in fact equal to 4), $\frac{3}{2}$ is a solution to the equation $4x - 2 = 2x + 1$.

Exercise 3.2.2

Is $y = \frac{4}{3}$ a solution of $9y + 2 = 6y + 3$?

Answer

no

? Exercise 3.2.3

Is $y = \frac{7}{5}$ a solution of $5y + 3 = 10y - 4$?

Answer

yes

Solve Equations Using the Subtraction and Addition Properties of Equality

We are going to use a model to clarify the process of solving an equation. An envelope represents the variable – since its contents are unknown – and each counter represents one. We will set out one envelope and some counters on our workspace, as shown in Figure **3.2.1**. Both sides of the workspace have the same number of counters, but some counters are "hidden" in the envelope. Can you tell how many counters are in the envelope?

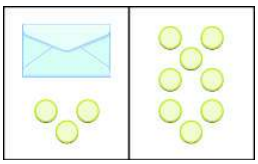


Figure **3.2.1**: The illustration shows a model of an equation with one variable. On the left side of the workspace is an unknown (envelope) and three counters, while on the right side of the workspace are eight counters.

What are you thinking? What steps are you taking in your mind to figure out how many counters are in the envelope?

Perhaps you are thinking: "I need to remove the 3 counters at the bottom left to get the envelope by itself. The 3 counters on the left can be matched with 3 on the right and so I can take them away from both sides. That leaves five on the right—so there must be 5 counters in the envelope." See Figure **3.2.2** for an illustration of this process.

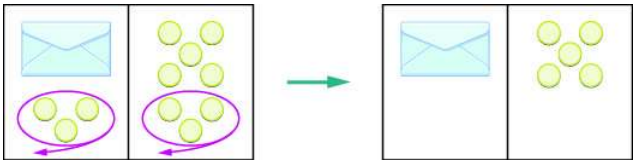


Figure **3.2.2**: The illustration shows a model for solving an equation with one variable. On both sides of the workspace remove three counters, leaving only the unknown (envelope) and five counters on the right side. The unknown is equal to five counters.

What algebraic equation would match this situation? In Figure **3.2.3** each side of the workspace represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x.

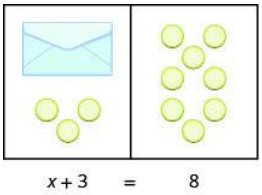


Figure **3.2.3**: The illustration shows a model for the equation $x + 3 = 8$.

Let's write algebraically the steps we took to discover how many counters were in the envelope:

Table **3.2.1**

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Check:

Five in the envelope plus three more does equal eight!

$$5+3=8 \tag{3.2.1}$$

Our model has given us an idea of what we need to do to solve one kind of equation. The goal is to isolate the variable by itself on one side of the equation. To solve equations such as these mathematically, we use the **Subtraction Property of Equality**.

SUBTRACTION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{aligned} &\textbf{If} \quad a=b \\ &\textbf{then} \quad a-c=b-c \end{aligned} \tag{3.2.2}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

Note

Doing the Manipulative Mathematics activity "Subtraction Property of Equality" will help you develop a better understanding of how to solve equations by using the **Subtraction Property of Equality**.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Exercise 3.2.4

Solve: $y+37=-13$.

Answer

To get y by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

	$y+37=-13$
Subtract 37 from each side to 'undo' the addition.	$y+37-37=-13-37$
Simplify.	$y=-50$
Check:	$y+37=-13$
Substitute $y=-50$	$-50+37=-13$
	$-13\stackrel{?}{=}-13$ ✓

Since y=−50 makes y+37=−13 a true statement, we have the solution to this equation.

Exercise 3.2.5

Solve: $x+19=-27$.

Answer

$x=-46$

Exercise 3.2.6

Solve: $x + 16 = -34$.

Answer

$x = -50$

What happens when an equation has a number subtracted from the variable, as in the equation $x - 5 = 8$? We use another property of equations to solve equations where a number is subtracted from the variable. We want to isolate the variable, so to 'undo' the subtraction we will add the number to both sides. We use the **Addition Property of Equality**.

ADDITION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{array}{ll} \textbf{If} & a = b \\ \textbf{then} & a + c = b + c \end{array} \tag{3.2.3}$$

When you add the same quantity from both sides of an equation, you still have equality.

In Exercise 3.2.4, 37 was added to the y and so we subtracted 37 to 'undo' the addition. In Exercise 3.2.7, we will need to 'undo' subtraction by using the **Addition Property of Equality**.

Exercise 3.2.7

Solve: $a - 28 = -37$.

Answer

		$a - 28 = -37$
Add 28 to each side to 'undo' the subtraction.		$a - 28 + 28 = -37 + 28$
Simplify.		$a = -9$
Check:	$a - 28 = -37$	
Substitute $a = -9$	$-9 - 28 = -37$	
	$-37 \stackrel{?}{=} -37$ ✓	
		The solution to $a - 28 = -37$ is $a = -9$.

Exercise 3.2.8

Solve: $n - 61 = -75$.

Answer

$n = -14$

Exercise 3.2.9

Solve: $p - 41 = -73$.

Answer

$p = -32$

Exercise 3.2.10

Solve: $x - \frac{5}{8} = \frac{3}{4}$

Answer

	$x - \frac{5}{8} = \frac{3}{4}$
Use the Addition Property of Equality.	$x - \frac{5}{8} + \frac{5}{8} = \frac{3}{4} + \frac{5}{8}$
Find the LCD to add the fractions on the right.	$x - \frac{5}{8} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8}$
Simplify.	$x = \frac{11}{8}$
Check:	$x - \frac{5}{8} = \frac{3}{4}$
Substitute $x = \frac{11}{8}$	$\frac{11}{8} - \frac{5}{8} \stackrel{?}{=} \frac{3}{4}$
Subtract.	$\frac{6}{8} \stackrel{?}{=} \frac{3}{4}$
Simplify.	$\frac{3}{4} = \frac{3}{4}$ ✓
	The solution to $x - \frac{5}{8} = \frac{3}{4}$ is $x = \frac{11}{8}$.

Exercise 3.2.11

Solve: $p - \frac{2}{3} = \frac{5}{6}$.

Answer

$p = \frac{9}{6} p = \frac{3}{2}$

Exercise 3.2.12

Solve: $q - \frac{1}{2} = \frac{5}{6}$.

Answer

$q = \frac{4}{3}$

The next example will be an equation with decimals.

Exercise 3.2.13

Solve: $n - 0.63 = -4.2$.

Answer

	$n - 0.63 = -4.2$
Use the Addition Property of Equality.	$n - 0.63 + 0.63 = -4.2 + 0.63$
Add.	$n = -3.57$
Check:	$n = -3.57$
Let $n = -3.57$.	$-3.57 - 0.63 \stackrel{?}{=} -4.2$
	$-4.2 = -4.2$ ✓

Exercise 3.2.14

Solve: $b - 0.47 = -2.1$.

Answer

$b = -1.63$

Exercise 3.2.15

Solve: $c - 0.93 = -4.6$.

Answer

$c = -3.67$

Solve Equations That Require Simplification

In the previous examples, we were able to isolate the variable with just one operation. Most of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality.

You should always simplify as much as possible before you try to isolate the variable. Remember that to simplify an expression means to do all the operations in the expression. Simplify one side of the equation at a time. Note that **simplification** is different from the process used to solve an equation in which we apply an operation to both sides.

Exercise 3.2.16: How to Solve Equations That Require Simplification

Solve: $9x - 5 - 8x - 6 = 7$.

Answer

Step 1. Simplify the expressions on each side as much as possible.	Rearrange the terms, using the Commutative Property of Addition. Combine like terms. Notice that each side is now simplified as much as possible.	$9x - 5 - 8x - 6 = 7$ $9x - 8x - 5 - 6 = 7$ $x - 11 = 7$
Step 2. Isolate the variable.	Now isolate x. Undo subtraction by adding 11 to both sides.	$x - 11 + 11 = 7 + 11$

Step 3. Simplify the expressions on both sides of the equation.		$x = 18$
Step 4. Check the solution.		**Check:** Substitute $x = 18$. $9x - 5 - 8x - 6 = 7$ $9(18) - 5 - 8(18) - 6 \stackrel{?}{=} 7$ $162 - 5 - 144 - 6 \stackrel{?}{=} 7$ $157 - 144 - 6 \stackrel{?}{=} 7$ $13 - 6 \stackrel{?}{=} 7$ $7 = 7$ ✓ The solution to $9x - 5 - 8x - 6 = 7$ is $x = 18$.

? Exercise 3.2.17

Solve: $8y - 4 - 7y - 7 = 4$.

Answer

$y = 15$

? Exercise 3.2.18

Solve: $6z + 5 - 5z - 4 = 3$.

Answer

$z = 2$

? Exercise 3.2.19

Solve: $5(n-4)-4n=-8$.

Answer

We simplify both sides of the equation as much as possible before we try to isolate the variable.

	$5(n-4)-4n=-8$
Distribute on the left.	$5n-20-4n=-8$
Use the Commutative Property to rearrange terms.	$5n-4n-20=-8$
Combine like terms.	$n-20=-8$
Each side is as simplified as possible. Next, isolate n.	
Undo subtraction by using the Addition Property of Equality.	$n-20+20=-8+20$
Add.	$n=12$
Check. Substitute n=12. $5(n-4)-4n=-8$ $5(12-4)-4(12)\stackrel{?}{=}-8z$ $5(8)-48\stackrel{?}{=}-8$ $40-48\stackrel{?}{=}-8$ $-8=-8\checkmark$	
	The solution to $5(n-4)-4n=-8$ is $n=12$.

Exercise 3.2.20

Solve: $5(p-3)-4p=-10$.

Answer

$p=5$

Exercise 3.2.21

Solve: $4(q+2)-3q=-8$.

Answer

$q=-16$

Exercise 3.2.22

Solve: $3(2y-1)-5y=2(y+1)-2(y+3)$.

Answer

We simplify both sides of the equation before we isolate the variable.

	$3(2y-1)-5y=2(y+1)-2(y+3)$
Distribute on both sides.	$6y-3-5y=2y+2-2y-6$
Use the Commutative Property of Addition.	$6y-5y-3=2y-2y+2-6$
Combine like terms.	$y-3=-4$
Each side is as simplified as possible. Next, isolate y.	
Undo subtraction by using the Addition Property of Equality.	$y-3+3=-4+3$
Add.	$y=-1$
Check. Let y=−1. $3(2y-1)-5y=2(y+1)-2(y+3)$ $3(2(-1)-1)-5(-1)\stackrel{?}{=}2(-1+1)-2(-1+3)$ $3(-2-1)+5\stackrel{?}{=}2(0)-2(2)$ $3(-3)+5\stackrel{?}{=}-4$ $-9+5\stackrel{?}{=}-4$ $-4=-4\checkmark$	
	The solution to $3(2y-1)-5y=2(y+1)-2(y+3)3(2y-1)-5y=2(y+1)-2(y+3)$ is $y=-1$.

? Exercise 3.2.23

Solve: $4(2h-3)-7h=6(h-2)-6(h-1)$.

Answer

$h=6$

? Exercise 3.2.24

Solve: $2(5x+2)-9x=3(x-2)-3(x-4)$.

Answer

$x=2$

Translate to an Equation and Solve

To solve applications algebraically, we will begin by translating from English sentences into equations. Our first step is to look for the word (or words) that would translate to the **equals sign**. Here are some of the words that are commonly used.

Equals =

- is
- is equal to
- is the same as
- the result is
- gives
- was

- will be

The steps we use to translate a sentence into an equation are listed below.

TRANSLATE AN ENGLISH SENTENCE TO AN ALGEBRAIC EQUATION

1. Locate the “equals” word(s). Translate to an equals sign (=).
2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

Exercise 3.2.25

Translate and solve: Eleven more than x is equal to 54.

Answer

Translate.	Eleven more than x is equal to 54 $x+11 = 54$
Subtract 11 from both sides.	$x+11-11 = 54-11$
Simplify.	$x = 43$
Check: Is 54 eleven more than 43? $43+11 \stackrel{?}{=} 54$ $54 = 54\checkmark$ (3.2.4)	

Exercise 3.2.26

Translate and solve: Ten more than x is equal to 41.

Answer

$x+10=41; x=31$

Exercise 3.2.27

Translate and solve: Twelve less than x is equal to 51.

Answer

y−12=51;y=63

Exercise 3.2.28

Translate and solve: The difference of 12t and 11t is −14.

Answer

Translate.	The difference of $12t$ and $11t$ is -14 $12t - 11t = -14$
Simplify.	$t = -14$
Check: $12(-14) - 11(-14) \stackrel{?}{=} -14$ $-168 + 154 \stackrel{?}{=} -14$ (3.2.5) $-14 = -14$✓	

? Exercise 3.2.29

Translate and solve: The difference of 4x and 3x is 14.

Answer

$4x - 3x = 14; x = 14$

? Exercise 3.2.30

Translate and solve: The difference of 7a and 6a is −8.

Answer

$7a - 6a = -8; a = -8$

Translate and Solve Applications

Most of the time a question that requires an algebraic solution comes out of a real life question. To begin with that question is asked in English (or the language of the person asking) and not in math symbols. Because of this, it is an important skill to be able to translate an everyday situation into algebraic language.

We will start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for. For example, you might use q for the number of quarters if you were solving a problem about coins.

? Exercise 3.2.31: How to Solve Translate and Solve Applications

The MacIntyre family recycled newspapers for two months. The two months of newspapers weighed a total of 57 pounds. The second month, the newspapers weighed 28 pounds. How much did the newspapers weigh the first month?

Answer

Step 1. Read the problem. Make sure all the words and ideas are understood.	The problem is about the weight of newspapers.	
Step 2. Identify what we are asked to find.	What are we asked to find?	"How much did the newspapers weigh the 2nd month?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Choose a variable.	Let w = weight of the newspapers the 1st month

Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.	Restate the problem. We know the weight of the newspapers the second month is 28 pounds. Translate into an equation, using the variable w.	Weight of newspapers the 1st month plus the weight of the newspapers the 2nd month equals 57 pounds. Weight from 1st month plus 28 equals 57. $w + 28 = 57$
Step 5. Solve the equation using good algebra techniques.	Solve.	$w + 28 - 28 = 57 - 28$ $w = 29$
Step 6. Check the answer in the problem and make sure it makes sense.	Does 1st month's weight plus 2nd month's weight equal 57 pounds?	**Check:** Does 1st month's weight plus 2nd month's weight equal 57 pounds? $29 + 28 \stackrel{?}{=} 57$ $57 = 57$ ✓
Step 7. Answer the question with a complete sentence.	Write a sentence to answer "How much did the newspapers weigh the 2nd month?"	The 2nd month the newspapers weighed 29 pounds.

Exercise 3.2.32

Translate into an algebraic equation and solve:

The Pappas family has two cats, Zeus and Athena. Together, they weigh 23 pounds. Zeus weighs 16 pounds. How much does Athena weigh?

Answer

7 pounds

Exercise 3.2.33

Translate into an algebraic equation and solve:

Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Answer

42 books

SOLVE AN APPLICATION.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with the important information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Exercise 3.2.34

Randell paid $28,675 for his new car. This was $875 less than the sticker price. What was the sticker price of the car?

Answer

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	"What was the sticker price of the car?"
Step 3. Name what we are looking for. Choose a variable to represent that quantity.	Let s = the sticker price of the car.
Step 4. Translate into an equation. Restate the problem in one sentence.	$28675 is $875 less than the sticker price
	$28675 is $875 less than s
Step 5. Solve the equation.	$28675 = s - 875$ $28675 + 875 = s - 875 + 875$ $29550 = s$
Step 6. Check the answer. Is $875 less than $29550 equal to $28675?	$29550 - 875 \stackrel{?}{=} 28675$ $28675 = 28675$✓
Step 7. Answer the question with a complete sentence.	The sticker price of the car was $29550.

Exercise 3.2.35

Translate into an algebraic equation and solve:

Eddie paid $19875 for his new car. This was $1025 less than the sticker price. What was the sticker price of the car?

Answer

$20900

Exercise 3.2.36

Translate into an algebraic equation and solve:

The admission price for the movies during the day is $7.75. This is $3.25 less the price at night. How much does the movie cost at night?

Answer

$11.00

Key Concepts

- **To Determine Whether a Number is a Solution to an Equation**
  1. **Substitute the number in for the variable in the equation.**
  2. **Simplify the expressions on both sides of the equation.**
  3. **Determine whether the resulting statement is true.**
     - If it is true, the number is a solution.

- If it is not true, the number is not a solution.

- **Addition Property of Equality**
  - For any numbers *a*, *b*, and *c*, if a=b, then a+c=b+c.
- **Subtraction Property of Equality**
  - For any numbers *a*, *b*, and *c*, if a=b, then a−c=b−c.
- **To Translate a Sentence to an Equation**
  1. Locate the “equals” word(s). Translate to an equal sign (=).
  2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
  3. Translate the words to the right of the “equals” word(s) into an algebraic expression.
- **To Solve an Application**
  1. Read the problem. Make sure all the words and ideas are understood.
  2. Identify what we are looking for.
  3. Name what we are looking for. Choose a variable to represent that quantity.
  4. Translate into an equation. It may be helpful to restate the problem in one sentence with the important information.
  5. Solve the equation using good algebra techniques.
  6. Check the answer in the problem and make sure it makes sense.
  7. Answer the question with a complete sentence.

Glossary

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

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3.3: Solving Equations with the Division and Multiplication Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that require simplification
- Translate to an equation and solve
- Translate and solve applications

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in Figure **3.3.1**.

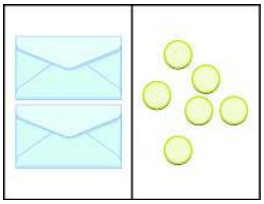


Figure **3.3.1**: The illustration shows a model of an equation with one variable multiplied by a constant. On the left side of the workspace are two instances of the unknown (envelope), while on the right side of the workspace are six counters.

In the illustration there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are "hidden" in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in Figure **3.3.2**? There are two envelopes, and each contains xx counters. Together, the two envelopes must contain a total of 6 counters.

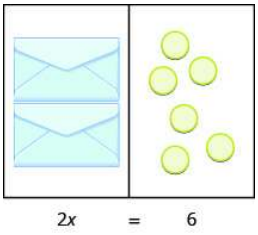


Figure **3.3.2**: The illustration shows a model of the equation $2x = 6$.

	$2x = 6$
If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2} = \frac{6}{2}$
we get:	$x = 3$

We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works! Three counters in each of two envelopes does equal six!

This example leads to the **Division Property of Equality**.

THE DIVISION PROPERTY OF EQUALITY

For any numbers a, b, and c, and $c \neq 0$,

$$\begin{array}{rcl} \text{If} & a & = & b \\ \text{then} & \frac{a}{c} & = & \frac{b}{c} \end{array} \tag{3.3.1}$$

When you divide both sides of an equation by any non-zero number, you still have equality.

Note

Doing the Manipulative Mathematics activity "**Division Property of Equality**" will help you develop a better understanding of how to solve equations by using the Division Property of Equality.

The goal in solving an equation is to 'undo' the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to 'undo' the multiplication.

Exercise 3.3.1

Solve: $5x = -27$.

Answer

To isolate x, "undo" the multiplication by 5.		$5x = -27$
Divide to 'undo' the multiplication.		$\frac{5x}{5} = -\frac{27}{5}$
Simplify.		$x = -\frac{27}{5}$
Check:	$5x = -27$	
Substitute $-\frac{27}{5}$ for x.	$5\left(-\frac{27}{5}\right) \stackrel{?}{=} -27$	
	$-27 = -27$ ✓	
		Since this is a true statement, $x = -\frac{27}{5}$ is the solution to $5x = -27$.

Exercise 3.3.2

Solve: $3y = -41$.

Answer

$y = -\frac{41}{3}$

Exercise 3.3.3

Solve: $4z = -55$.

Answer

$y = -\frac{55}{4}$

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The **Multiplication Property of Equality** will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

THE MULTIPLICATION PROPERTY OF EQUALITY

For any numbers a, b, and c,

$$\begin{array}{lll} \text{If} & a & = & b \\ \text{then} & ac & = & bc \end{array} \qquad (3.3.2)$$

If you multiply both sides of an equation by the same number, you still have equality.

Exercise 3.3.4

Solve: $\frac{y}{-7} = -14$

Answer

Here y is divided by −7. We must multiply by −7 to isolate y.

	$\frac{y}{-7} = -14$
Multiply both sides by −7.	$-7\left(\frac{y}{-7}\right) = -7(-14)$
Multiply.	$\frac{-7y}{7} = 98$
Simplify.	$y = 98$
Check: $\frac{y}{-7} = -14$	
Substitute y=98.	$\frac{98}{-7} \stackrel{?}{=} -14$
Divide.	$-14 = -14$ ✓

Exercise 3.3.5

Solve: $\frac{a}{-7} = -42$

Answer

$a = 294$

Exercise 3.3.6

Solve: $\frac{b}{-6} = -24$

Answer

$b = 144$

Exercise 3.3.7

Solve: $-n = 9$

Answer

	$-n = 9$
Remember −n is equivalent to −1n.	$-1n = 9$
Divide both sides by −1.	$\frac{-1n}{-1} = \frac{9}{-1}$
Divide.	$n = -9$

Notice that there are two other ways to solve −n=9. We can also solve this equation by multiplying both sides by −1 and also by taking the opposite of both sides.

Check:	$-n = 9$
Substitute n=−9.	$-(-9) \stackrel{?}{=} 9$
Simplify.	$9 = 9$ ✓

? Exercise 3.3.8

Solve: $-k = 8$.

Answer

$k = -8$

? Exercise 3.3.9

Solve: $-g = 3$.

Answer

$g = -3$

? Exercise 3.3.10

Solve: $\frac{3}{4}x = 12$

Answer

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

	$\frac{3}{4}x = 12$
Multiply by the reciprocal of $\frac{3}{4}$.	$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 12$
Reciprocals multiply to 1.	$1x = \frac{4}{3} \cdot \frac{12}{1}$
Multiply.	$x = 16$
Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x. While this would work, most people would find multiplying by the reciprocal easier.	
Check:	$\frac{3}{4}x = 12$
Substitute $x = 16$.	$\frac{3}{4} \cdot 16 \stackrel{?}{=} 12$
	$12 = 12$ ✓

Exercise 3.3.11

Solve: $\frac{2}{5}n = 14$.

Answer

$n = 35$

Exercise 3.3.12

Solve: $\frac{5}{6}y = 15$.

Answer

$y = 18$

In the next example, all the variable terms are on the right side of the equation. As always, our goal in solving the equation is to isolate the variable.

Exercise 3.3.13

Solve: $\frac{8}{15} = -\frac{4}{5}x$

Answer

	$\frac{8}{15}=-\frac{4}{5}x$
Multiply by the reciprocal of $-\frac{4}{5}$.	$\left(-\frac{5}{4}\right)\left(\frac{8}{15}\right)=\left(-\frac{5}{4}\right)\left(-\frac{4}{5}x\right)$
Reciprocals multiply to 1.	$-\frac{5\cdot 4\cdot 2}{4\cdot 3\cdot 5}=1x$
Multiply.	$-\frac{2}{3}=x$
Check:	$\frac{8}{15}=-\frac{4}{5}x$
Let $x=-\frac{2}{3}$.	$\frac{8}{15}=-\frac{4}{5}\left(-\frac{2}{3}\right)$
	$\frac{8}{15}=\frac{8}{15}$ ✓

Exercise 3.3.14

Solve: $\frac{9}{25}=-\frac{4}{5}z$

Answer

$z=-\frac{9}{5}$

Exercise 3.3.15

$\frac{5}{6}=-\frac{8}{3}r$

Answer

$r=-\frac{5}{16}$

Solve Equations That Require Simplification

Many equations start out more complicated than the ones we have been working with.

With these more complicated equations the first step is to simplify both sides of the equation as much as possible. This usually involves combining like terms or using the distributive property.

Exercise 3.3.16

Solve: $14-23=12y-4y-5y$.

Answer

Begin by simplify each side of the equation.

	$14-23\stackrel{?}{=}-36+12+15$
Simplify each side.	$-9=-9$ ✓
Divide both sides by 3 to isolate y.	$14-23=12y-4y-5y$
Divide.	$-9=3y$
Check:	$14-23=12y-4y-5y$
Substitute $y=-3$.	$14-23\stackrel{?}{=}12(-3)-4(-3)-5(-3)$
	$14-23\stackrel{?}{=}-36+12+15$
	$-9=-9$ ✓

Exercise 3.3.17

Solve: $18-27=15c-9c-3c$.

Answer

$c=-3$

Exercise 3.3.18

Solve: $18-22=12x-x-4x$.

Answer

$x=-\frac{4}{7}$

Exercise 3.3.19

Solve: $-4(a-3)-7=25$.

Answer

Here we will simplify each side of the equation by using the distributive property first.

	$-4(a-3)-7=25$
Distribute.	$-4a+12-7=25$
Simplify.	$-4a+5=25$
Simplify.	$-4a=20$
Divide both sides by -4 to isolate a.	$\frac{-4a}{-4}=\frac{20}{-4}$
Divide.	$a=-5$
Check:	$-4(a-3)-7=25$
Substitute $a=-5$	$-4(-5-3)-7\stackrel{?}{=}25$
	$-4(-8)-7\stackrel{?}{=}25$
	$32-7\stackrel{?}{=}25$
	$25=25$ ✓

? Exercise 3.3.20

Solve: $-4(q-2)-8=24$.

Answer

$q=-6$

? Exercise 3.3.21

Solve: $-6(r-2)-12=30$.

Answer

$r=-5$

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

PROPERTIES OF EQUALITY

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Subtraction Property of Equality For any real numbers a, b, and c, if $a=b$, then $a-c=b-c$	**Addition Property of Equality** For any real numbers a, b, and c, if $a=b$, then $a+c=b+c$	(3.3.3)
Divition Property of Equality For any real numbers a, b, and c, if $a=b$, then $a-c=b-c$	**Multiplication Property of Equality** For any real numbers a, b, and c, if $a=b$, then $a+c=b+c$	

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Translate to an Equation and Solve

In the next few examples, we will translate sentences into equations and then solve the equations. You might want to review the translation table in the previous chapter.

Exercise 3.3.22

Translate and solve: The number 143 is the product of −11 and y.

Answer

Translate.	The number 143 is the product of −11 and y. $143 = -11y$
Divide by −11.	$\frac{143}{-11} = \frac{-11y}{-11}$
Simplify.	$-13 = y$
Check: $143 = -11y$ $143 \stackrel{?}{=} -11(-13)$ (3.3.4) $143 = 143\checkmark$	

Exercise 3.3.23

Translate and solve: The number 132 is the product of -12 and y.

Answer

132=−12y;y=−11

Exercise 3.3.24

Translate and solve: The number 117 is the product of -13 and z.

Answer

117=−13z;z=−9

Exercise 3.3.25

Translate and solve: n divided by 8 is −32.

Answer

Begin by translating the sentence into an equation. Translate.		n divided by 8 is –32. $\frac{n}{8} = -32$
Multiple both sides by 8.		$8 \cdot \frac{n}{8} = 8(-32)$
Simplify.		$n = -256$
Check:	Is nn divided by 8 equal to −32?	
Let $n = -256$.	Is −256 divided by 88 equal to −32?	
Translate.	$\frac{-256}{8} \stackrel{?}{=} -32$	
Simplify.	$-32 = -32$✓	

Exercise 3.3.26

Translate and solve: nn divided by 7 is equal to −21.

Answer

$\frac{n}{7} = -21; n = -147$

Exercise 3.3.27

Translate and solve: n divided by 8 is equal to −56.

Answer

$\frac{n}{8} = -56; n = -448$

Exercise 3.3.28

Translate and solve: The quotient of yy and −4 is 68.

Answer

Begin by translating the sentence into an equation.

Translate.		The quotient of y and –4 is 68. $\frac{y}{-4} = 68$
Multiply both sides by -4.		$-4\left(\frac{y}{-4}\right) = -4(68)$
Simplify.		$y = -272$
Check:	Is the quotient of y and −4 equal to 68?	
Let y=−272.	Is the quotient of −272 and −4 equal to 68?	
Translate.	$\frac{-272}{-4} \stackrel{?}{=} 68$	
Simplify.	$68 = 68$✓	

Exercise 3.3.29

Translate and solve: The quotient of q and −8 is 72.

Answer

$\frac{q}{-8} = 72; q = -576$

Exercise 3.3.30

Translate and solve: The quotient of p and −9 is 81.

Answer

$\frac{p}{-9} = 81; p = -729$

Exercise 3.3.31

Translate and solve: Three-fourths of p is 18.

Answer

Begin by translating the sentence into an equation. Remember, “of” translates into multiplication.

Translate.	Three-fourths of p is 18. $\frac{3}{4}p = 18$
Multiply both sides by $\frac{4}{3}$.	$\frac{4}{3} \cdot \frac{3}{4}p = \frac{4}{3} \cdot 18$
Simplify.	$p = 24$
Check:	Is three-fourths of p equal to 18?
Let p = 24.	Is three-fourths of 24 equal to 18?
Translate.	$\frac{3}{4} \cdot 24 \stackrel{?}{=} 18$
Simplify.	$18 = 18$✓

Exercise 3.3.32

Translate and solve: Two-fifths of f is 16.

Answer

$\frac{2}{5}f = 16; f = 40$

Exercise 3.3.33

Translate and solve: Three-fourths of f is 21.

Answer

$\frac{3}{4}f = 21; f = 28$

Exercise 3.3.34

Translate and solve: The sum of three-eighths and x is one-half.

Answer

Begin by translating the sentence into an equation.

Translate.	The sum of three – eighths and x is $\frac{1}{2}$ $\frac{3}{8} + x = \frac{1}{2}$
Subtract $\frac{3}{8}$ from each side.	$\frac{3}{8} - \frac{3}{8} + x = \frac{1}{2} - \frac{3}{8}$
Simplify and rewrite fractions with common denominators.	$x = \frac{4}{8} - \frac{3}{8}$
Simplify.	$x = \frac{1}{8}$
Check:	Is the sum of three-eighths and x equal to one-half?
Let $x = \frac{1}{8}$.	Is the sum of three-eighths and one-eighth equal to one-half?
Translate.	$\frac{3}{8} + \frac{1}{8} \stackrel{?}{=} \frac{1}{2}$
Simplify.	$\frac{4}{8} \stackrel{?}{=} \frac{1}{2}$
Simplify.	$\frac{1}{2} = \frac{1}{2}$ ✓

Exercise 3.3.35

Translate and solve: The sum of five-eighths and x is one-fourth.

Answer

$\frac{5}{8} + x = \frac{1}{4}; x = -\frac{3}{8}$

Exercise 3.3.36

Translate and solve: The sum of three-fourths and x is five-sixths.

Answer

$\frac{3}{4} + x = \frac{5}{6}; x = \frac{1}{12}$

Translate and Solve Applications

To solve applications using the Division and Multiplication Properties of Equality, we will follow the same steps we used in the last section. We will restate the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve.

Exercise 3.3.37

Denae bought 6 pounds of grapes for $10.74. What was the cost of one pound of grapes?

Answer

What are you asked to find?	The cost of 1 pound of grapes	
Assign a variable.	Let c = the cost of one pound.	
Write a sentence that gives the information to find it.	The cost of 6 pounds is $10.74	
Translate into an equation.	$6c = 10.74$	
Solve.	$\frac{6c}{c} = \frac{10.74}{6}$ $c = 1.79$	(3.3.5)
	The grapes cost $1.79 per pound.	
Check: If one pound costs $1.79, *do* 6 pounds cost $10.74?		
$6(1.79) \stackrel{?}{=} 10.74$ $10.74 = 10.74$✓		

? Exercise 3.3.38

Translate and solve:

Arianna bought a 24-pack of water bottles for $9.36. What was the cost of one water bottle?

Answer

$0.39

? Exercise 3.3.39

Translate and solve:

At JB's Bowling Alley, 6 people can play on one lane for $34.98. What is the cost for each person?

Answer

$5.83

? Exercise 3.3.40

Andreas bought a used car for $12,000. Because the car was 4-years old, its price was $\frac{3}{4}$ of the original price, when the car was new. What was the original price of the car?

Answer

What are you asked to find?	The original price of the car	
Assign a variable.	Let p = the original price.	
Write a sentence that gives the information to find it.	\$12000 is $\frac{3}{4}$ of the original price.	
Translate into an equation.	$12000 = \frac{3}{4}p$	(3.3.6)
	$\frac{3}{4}(12000) = \frac{4}{3} \cdot \frac{3}{4}p$	
	$16000 = p$	
Solve.		
	The original cost of the car was \$16000.	
Check: Is $\frac{3}{4}$ of \$16000 equal to \$12000		
$\frac{3}{4} \cdot 16000 \stackrel{?}{=} 12000$ $12000 = 12000$✓		

? Exercise 3.3.41

Translate and solve:

The annual property tax on the Mehta's house is \$1,800, calculated as $\frac{15}{1000}$ of the assessed value of the house. What is the assessed value of the Mehta's house?

Answer

\$120000

? Exercise 3.3.42

Translate and solve:

Stella planted 14 flats of flowers in $\frac{2}{3}$ of her garden. How many flats of flowers would she need to fill the whole garden?

Answer

21 flats

Key Concepts

- **The Division Property of Equality**—For any numbers a, b, and c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
  When you divide both sides of an equation by any non-zero number, you still have equality.
- **The Multiplication Property of Equality**—For any numbers a, b, and c, if $a = b$, then $ac = bc$.
  If you multiply both sides of an equation by the same number, you still have equality.

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3.4: Solving Equations with Variables and Constants on Both Sides

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides

Solve Equations with Constants on Both Sides

In all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we will learn to solve equations in which the variable terms, or constant terms, or both are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the "variable side", and the other side of the equation to be the "constant side." Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Exercise 3.4.1

Solve: $7x + 8 = -13$.

Answer

In this equation, the variable is found only on the left side. It makes sense to call the left side the "variable" side. Therefore, the right side will be the "constant" side. We will write the labels above the equation to help us remember what goes where.

variable constant
$7x + 8 = -13$

Since the left side is the "xx", or variable side, the 8 is out of place. We must "undo" adding 8 by subtracting 8, and to keep the equality we must subtract 8 from both sides.

	variable constant $7x + 8 = -13$
Use the Subtraction Property of Equality.	$7x + 8 - 8 = -13 - 8$
Simplify.	$7x = -21$
Now all the variables are on the left and the constant on the right. The equation looks like those you learned to solve earlier.	
Use the Division Property of Equality.	$\frac{7x}{7} = \frac{-21}{7}$
Simplify.	$x = -3$
Check:	$7x + 8 = -13$
Let x=−3.	$7(-3) + 8 \stackrel{?}{=} -13$
	$-21 + 8 \stackrel{?}{=} -13$ $-13 = -13$ ✓

Exercise 3.4.2

Solve: $3x + 4 = -8$.

Answer

$x = -4$

Exercise 3.4.3

Solve: $5a + 3 = -37$.

Answer

$a = -8$

Exercise 3.4.4

Solve: $8y - 9 = 31$.

Answer

Notice, the variable is only on the left side of the equation, so we will call this side the "variable" side, and the right side will be the "constant" side. Since the left side is the "variable" side, the 9 is out of place. It is subtracted from the 8y, so to "undo" subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

	variable constant $8y - 9 = 31$
Add 9 to both sides.	$8y - 9 + 9 = 31 + 9$
Simplify.	$8y = 40$
	The variables are now on one side and the constants on the other. We continue from here as we did earlier.
Divide both sides by 8.	$\frac{8y}{8} = \frac{40}{8}$
Simplify.	$y = 5$
Check:	$8y - 9 = 31$
Let y=5.	$8 \cdot 5 - 9 \stackrel{?}{=} 31$
	$40 - 9 \stackrel{?}{=} 31$
	$31 = 31$ ✓

Exercise 3.4.5

Solve: $5y - 9 = 16$.

Answer

$y = 5$

Exercise 3.4.6

Solve: $3m - 8 = 19$.

Answer

$m = 9$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Exercise 3.4.7

Solve: $9x = 8x - 6$.

Answer

Here the variable is on both sides, but the constants only appear on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

	variable constant $9x = 8x - 6$
We don't want any x's on the right, so subtract the 8x from both sides.	$9x - 8x = 8x - 8x - 6$
Simplify.	$x = -6$
We succeeded in getting the variables on one side and the constants on the other, and have obtained the solution.	
Check:	$9x = 8x - 6$
Let x=−6.	$9(-6) \stackrel{?}{=} 8(-6) - 6$
	$-54 \stackrel{?}{=} -48 - 6$
	$-54 = -54$ ✓

Exercise 3.4.8

Solve: $6n = 5n - 10$.

Answer

$n = -10$

Exercise 3.4.9

Solve: $-6c = -7c - 1$

Answer

$c = -1$

Exercise 3.4.10

Solve: $5y - 9 = 8y$

Answer

The only constant is on the left and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

	constant variable $5y-9=8y$
Subtract 5y from both sides.	$5y-5y-9=8y-5y$
Simplify.	$-9=3y$
We have the y's on the right and the constants on the left. Divide both sides by 3.	$\frac{-9}{3}=\frac{3y}{3}$
Simplify.	$-3=y$

Check:	$5y-9=8y$
Let $y=-3$.	$5(-3)-9\stackrel{?}{=}8(-3)$
	$-15-9\stackrel{?}{=}-24$
	$-24=-24$ ✓

Exercise 3.4.11

Solve: $3p-14=5p$.

Answer

$p=-7$

Exercise 3.4.12

Solve: $8m+9=5m$

Answer

$m=-3$

Exercise 3.4.13

Solve: $12x=-x+26$

Answer

The only constant is on the right, so let the left side be the "variable" side.

	variable constant $12x=-x+26$
Remove the −x from the right side by adding x to both sides.	$12x+x=-x+x+26$
Simplify.	$13x=26$
All the x's are on the left and the constants are on the right. Divide both sides by 13.	$\frac{13x}{13}=\frac{26}{13}$
Simplify.	$x=2$

Exercise 3.4.14

Solve: $12j = -4j + 32$

Answer

$j = 2$

Exercise 3.4.15

Solve: $8h = -4h + 12$

Answer

$h = 1$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Exercise 3.4.16: How to Solve Equations with Variables and Constants on Both Sides

Solve: $7x + 5 = 6x + 2$

Answer

Step 1. Choose which side will be the "variable" side—the other side will be the "constant" side.	The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the "x" side. The right side will be the "constant" side.	variable constant $7x + 5 = 6x + 2$
Step 2. Collect the variable terms to the "variable" side of the equation, using the addition or subtraction property of equality.	With the right side as the "constant" side, the $6x$ is out of place, so subtract $6x$ from both sides. Combine like terms. Now, the variable is only on the left side!	$7x - 6x + 5 = 6x - 6x + 2$ $x + 5 = 2$
Step 3. Collect all the constants to the other side of the equation, using the addition or subtraction property of equality.	The right side is the "constant" side, so the 5 is out of place. Subtract 5 from both sides. Simplify.	$x + 5 - 5 = 2 - 5$ $x = -3$
Step 4. Make the coefficient of the variable equal 1, using the multiplication or division property of equality.	The coefficient of x is one. The equation is solved.	
Step 5. Check.	Let $x = -3$ Simplify. Add.	**Check:** $7x + 6 = 6x + 2$ $(-3) + 5 = 6(-3) + 2$ $-21 + 5 = -18 + 2$ $-16 = -16$ ✓

Exercise 3.4.17

Solve: $12x + 8 = 6x + 2$.

Answer

$x = -1$

Exercise 3.4.18

Solve: $9y + 4 = 7y + 12$.

Answer

$y = 4$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

BEGINNING STRATEGY FOR SOLVING EQUATIONS WITH VARIABLES AND CONSTANTS ON BOTH SIDES OF THE EQUATION.

1. Choose which side will be the "variable" side—the other side will be the "constant" side.
2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the "variable" side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

Exercise 3.4.19

Solve: $8n - 4 = -2n + 6$.

Answer

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since $8 > -2$, make the left side the "variable" side.	variable constant $8n - 4 = -2n + 6$
We don't want variable terms on the right side—add 2n to both sides to leave only constants on the right.	$8n + 2n - 4 = -2n + 2n + 6$
Combine like terms.	$10n - 4 = 6$
We don't want any constants on the left side, so add 4 to both sides.	$10n - 4 + 4 = 6 + 4$
Simplify.	$10n = 10$
The variable term is on the left and the constant term is on the right. To get the coefficient of nn to be one, divide both sides by 10.	$\frac{10n}{10} = \frac{10}{10}$
Simplify.	$n = 1$
Check:	$8n - 4 = -2n + 6$
Let n=1.	$8 \cdot 1 - 4 \stackrel{?}{=} -2 \cdot 1 + 6$
	$8 - 4 \stackrel{?}{=} -2 + 6$
	$4 = 4 \checkmark$

Exercise 3.4.20

Solve: $8q - 5 = -4q + 7$

Answer

$q = 1$

Exercise 3.4.21

Solve: $7n - 3 = n + 3$

Answer

$n = 1$

Exercise 3.4.22

Solve: $7a - 3 = 13a + 7$

Answer

In the first step, choose the variable side by comparing the coefficients of the variables on each side.

Since 13>7, make the right side the “variable” side and the left side the “constant” side.

	constant variable $7a-3=13a+7$
Subtract 7a from both sides to remove the variable term from the left.	$7a-7a-3=13a-7a+7$
Combine like terms.	$-3=6a+7$
Subtract 7 from both sides to remove the constant from the right.	$-3-7=6a+7-7$
Simplify.	$-10=6a$
Divide both sides by 6 to make 1 the coefficient of a.	$\frac{-10}{6}=\frac{6a}{6}$
Simplify.	$-\frac{5}{3}=a$
Check:	$7a-3=13a+7$
Let $a=-\frac{5}{3}$.	$7\left(-\frac{5}{3}\right)-3\stackrel{?}{=}13\left(-\frac{5}{3}\right)+7$
	$-\frac{35}{3}-\frac{9}{3}\stackrel{?}{=}-\frac{65}{3}+\frac{21}{3}$
	$-\frac{54}{3}=-\frac{54}{3}$ ✓

? Exercise 3.4.23

Solve: $2a-2=6a+18$

Answer

$a=-5$

? Exercise 3.4.24

Solve: $4k-1=7k+17$

Answer

$k=-6$

In the last example, we could have made the left side the “variable” side, but it would have led to a negative coefficient on the variable term. (Try it!) While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

? Exercise 3.4.25

Solve: $\frac{4}{5}x+6=\frac{1}{4}x-2$

Answer

Since $\frac{5}{4} > \frac{1}{4}$, make the left side the "variable" side and the right side the "constant" side.

	variable constant $\frac{5}{4}x+6=\frac{1}{4}x-2$
Subtract $\frac{1}{4}x$ from both sides.	$\frac{5}{4}x-\frac{1}{4}x+6=\frac{1}{4}x-\frac{1}{4}x-2$
Combine like terms.	$x+6=-2$
Subtract 6 from both sides.	$x+6-6=-2-6$
Simplify.	$x=-8$
Check: Let $x=-8$	$\frac{5}{4}x+6 = \frac{1}{4}x-2$ $\frac{5}{4}(-8)+6 \stackrel{?}{=} \frac{1}{4}(-8)-2$ $-10+6 \stackrel{?}{=} -2-2$ $-4 = -4\checkmark$

Exercise 3.4.26

Solve: $\frac{7}{8}x-12=-\frac{1}{8}x-2$

Answer

$x=10$

Exercise 3.4.27

Solve: $\frac{7}{6}x+11=\frac{1}{6}y+8$

Answer

$y=-3$

We will use the same strategy to find the solution for an equation with decimals.

Exercise 3.4.28

Solve: $7.8x+4=5.4x-8$.

Answer

Since $7.8 > 5.4$, make the left side the "variable" side and the right side the "constant" side.

	variable side constant side $7.8x + 4 = 5.4x - 8$
Subtract 5.4x from both sides.	$7.8x - 5.4x + 4 = 5.4x - 5.4x - 8$
Combine like terms.	$2.4x + 4 = -8$
Subtract 4 from both sides.	$2.4x + 4 - 4 = -8 - 4$
Simplify.	$2.4x = -12$
Use the Division Property of Equality.	$\frac{2.4x}{2.4} = \frac{-12}{2.4}$
Simplify.	$x = -5$
Check:	$7.8x + 4 = 5.4x - 5$
Let $x = -5$	$7.8(-5) + 4 = 5.4(-5) - 8$
	$-39 + 4 \stackrel{?}{=} -27 - 8$
	$-35 = -35$ ✓

? Exercise 3.4.29

Solve: $2.8x + 12 = -1.4x - 9$

Answer

$x = -5$

? Exercise 3.4.30

Solve: $3.6y + 8 = 1.2y - 4$

Answer

$y = -5$

Key Concepts

- **Beginning Strategy for Solving an Equation with Variables and Constants on Both Sides of the Equation**
  1. Choose which side will be the "variable" side—the other side will be the "constant" side.
  2. Collect the variable terms to the "variable" side of the equation, using the Addition or Subtraction Property of Equality.
  3. Collect all the constants to the other side of the equation, using the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
  5. Check the solution by substituting it into the original equation.

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3.5: General Strategy to Solve Linear Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

Exercise 3.5.1: How to Solve Linear Equations Using the General Strategy

Solve: $-6(x+3)=24$.

Answer

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x+3)=24$ $-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do – all x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right, add 18 to each side. Simplify.	$-6x-18+18=24+18$ $-6x=42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by –6. Simplify.	$\frac{-6x}{-6}=\frac{42}{-6}$ $x=-7$
Step 5. Check the solution.	Let $x=-7$ Simplify. Multiply.	**Check:** $-6(x+3)=24$ $-6(-7+3)\stackrel{?}{=}24$ $-6(-4)\stackrel{?}{=}24$ $24=24$ ✓

Exercise 3.5.2

Solve: $5(x+3)=35$

Answer

$x=4$

Exercise 3.5.3

Solve: $6(y-4)=-18$

Answer

$y=1$

GENERAL STRATEGY FOR SOLVING LINEAR EQUATIONS.

1. **Simplify each side of the equation as much as possible.**
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. **Collect all the variable terms on one side of the equation.**
   Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
   Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term to equal to 1.**
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

Exercise 3.5.4

Solve: $-(y+9)=8$

Answer

	$-(y+9)=8$
Simplify each side of the equation as much as possible by distributing.	$-y-9=8$
The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y-9+9=8+9$
Simplify.	$-y=17$
Rewrite −y as −1y.	$-1y=17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by −1.	$\frac{-1y}{-1}=\frac{17}{-1}$
Simplify.	$y=-17$
Check:	$-(y+9)=8$
Let y=−17.	$-(-17+9)\stackrel{?}{=}8$
	$-(-8)\stackrel{?}{=}8$
	$8=8\checkmark$

Exercise 3.5.5

Solve: $-(y+8)=-2$

Answer

$y = -6$

Exercise 3.5.6

Solve: $-(z+4) = -12$

Answer

$z = 8$

Exercise 3.5.7

Solve: $5(a-3)+5 = -10$

Answer

	$5(a-3)+5=-10$
Simplify each side of the equation as much as possible.	
Distribute.	$5a-15+5=-10$
Combine like terms.	$5a-10=-10$
The only a term is on the left side, so all variable terms are on one side of the equation.	
Add 10 to both sides to get all constant terms on the other side of the equation.	$5a-10+10=-10+10$
Simplify.	$5a=0$
Make the coefficient of the variable term to equal to 11 by dividing both sides by 55.	$\frac{5a}{5}=\frac{0}{5}$
Simplify.	$a=0$
Check:	$5(a-3)+5=-10$
Let a=0.	$5(0-3)+5\stackrel{?}{=}-10$
	$5(-3)+5\stackrel{?}{=}-10$
	$-15+5\stackrel{?}{=}-10$
	$-10=-10$ ✓

Exercise 3.5.8

Solve: $2(m-4)+3 = -1$

Answer

$m = 2$

Exercise 3.5.9

Solve:$7(n-3)-8 = -15$

Answer

$n = 2$

Exercise 3.5.10

Solve: $\frac{2}{3}(6m - 3) = 8 - m$

Answer

	$\frac{2}{3}(6m-3) = 8-m$
Distribute.	$4m - 2 = 8 - m$
Add m to get the variables only to the left.	$4m + m - 2 = 8 - m + m$
Simplify.	$5m - 2 = 8$
Add 2 to get constants only on the right.	$5m - 2 + 2 = 8 + 2$
Simplify.	$5m = 10$
Divide by 5.	$\frac{5m}{5} = \frac{10}{5}$
Simplify.	$m = 2$
Check:	$\frac{2}{3}(6m-3) = 8-m$
Let m=2.	$\frac{2}{3}(6 \cdot 2 - 3) \stackrel{?}{=} 8 - 2$
	$\frac{2}{3}(12-3) \stackrel{?}{=} 6$
	$\frac{2}{3}(9) \stackrel{?}{=} 6$
	$6 = 6$ ✓

Exercise 3.5.11

Solve: $\frac{1}{3}(6u + 3) = 7 - u$

Answer

$u = 2$

Exercise 3.5.12

Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$

Answer

$x = 4$

Exercise 3.5.13

Solve: $8 - 2(3y + 5) = 0$

Answer

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by -6−6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$
Check: Let y=−13. $8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0$ ✓	

Exercise 3.5.14

Solve: $12-3(4j+3)=-17$

Answer

$j=\frac{5}{3}$

Exercise 3.5.15

Solve: $-6-8(k-2)=-10$

Answer

$k=\frac{5}{2}$

Exercise 3.5.16

Solve: $4(x-1)-2=5(2x+3)+6$

Answer

	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4x-4-2=10x+15+6$
Combine like terms.	$4x-6=10x+21$
Subtract 4x to get the variables only on the right side since $10>4$.	$4x-4x-6=10x-4x+21$
Simplify.	$-6=6x+21$
Subtract 21 to get the constants on left.	$-6-21=6x+21-21$
Simplify.	$-27=6x$
Divide by 6.	$\frac{-27}{6}=\frac{6x}{6}$
Simplify.	$-\frac{9}{2}=x$
Check:	$4(x-1)-2=5(2x+3)+6$
Let $x=-\frac{9}{2}$.	$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$
	$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$
	$-22-2\stackrel{?}{=}5(-6)+6$
	$-24\stackrel{?}{=}-30+6$
	$-24=-24$ ✓

? Exercise 3.5.17

Solve: $6(p-3)-7=5(4p+3)-12$

Answer

$p=-2$

? Exercise 3.5.18

Solve: $8(q+1)-5=3(2q-4)-1$

Answer

$q=-8$

? Exercise 3.5.19

Solve: $10[3-8(2s-5)]=15(40-5s)$

Answer

		$10[3-8(2s-5)]=15(40-5s)$
Simplify from the innermost parentheses first.		$10[3-16s+40]=15(40-5s)$
Combine like terms in the brackets.		$10[43-16s]=15(40-5s)$
Distribute.		$430-160s=600-75s$
Add 160s to get the s's to the right.		$430-160s+160s=600-75s+160s$
Simplify.		$430=600+85s$
Subtract 600 to get the constants to the left.		$430-600=600+85s-600$
Simplify.		$-170=85s$
Divide.		$\frac{-170}{85}=\frac{85s}{85}$
Simplify.		$-2=s$
Check:	$10[3-8(2s-5)]=15(40-5s)$	
Substitute s=−2.	$10[3-8(2(-2)-5)]\stackrel{?}{=}15(40-5(-2))$	
	$10[3-8(-4-5)]\stackrel{?}{=}15(40+10)$	
	$10[3-8(-9)]\stackrel{?}{=}15(50)$	
	$10[3+72]\stackrel{?}{=}750$	
	$10[75]\stackrel{?}{=}750$	
	$750=750$ ✓	

? Exercise 3.5.20

Solve: $6[4-2(7y-1)]=8(13-8y)$.

Answer

$y=-\frac{17}{5}$

? Exercise 3.5.21

Solve: $12[1-5(4z-1)]=3(24+11z)$.

Answer

$z=0$

? Exercise 3.5.22

Solve: $0.36(100n+5)=0.6(30n+15)$.

Answer

		$0.36(100n + 5) = 0.6(30n + 15)$
Distribute.		$36n + 1.8 = 18n + 9$
Subtract 18n to get the variables to the left.		$36n - 18n + 1.8 = 18n - 18n + 9$
Simplify.		$18n + 1.8 = 9$
Subtract 1.8 to get the constants to the right.		$18n + 1.8 - 1.8 = 9 - 1.8$
Simplify.		$18n = 7.2$
Divide.		$\frac{18n}{18} = \frac{7.2}{18}$
Simplify.		$n = 0.4$
Check:	$0.36(100n + 5) = 0.6(30n + 15)$	
Let n=0.4.	$0.36(100(0.4) + 5) \stackrel{?}{=} 0.6(30(0.4) + 15)$	
	$0.36(40 + 5) \stackrel{?}{=} 0.6(12 + 15)$	
	$0.36(45) \stackrel{?}{=} 0.6(27)$	
	$16.2 = 16.2$ ✓	

? Exercise 3.5.23

Solve: $0.55(100n + 8) = 0.6(85n + 14)$.

Answer

$n = 1$

? Exercise 3.5.24

Solve: $0.15(40m - 120) = 0.5(60m + 12)$.

Answer

$m = -1$

Classify Equations

Consider the equation we solved at the start of the last section, 7x+8=−13. The solution we found was x=−3. This means the equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3. We showed this when we checked the solution x=−3 and evaluated 7x+8=−13 for x=−3.

$$7(-3) + 8 \stackrel{?}{=} -13$$
$$-21 + 8 \stackrel{?}{=} -13$$
$$-13 = -13 \checkmark$$

If we evaluate 7x+8 for a different value of *x*, the left side will not be −13.

The equation 7x+8=−13 is true when we replace the variable, *x*, with the value −3, but not true when we replace *x* with any other value. Whether or not the equation 7x+8=−13 is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

CONDITIONAL EQUATION

An equation that is true for one or more values of the variable and false for all other values of the variable is a **conditional equation**.

Now let's consider the equation 2y+6=2(y+3). Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for *y*.

	$2y + 6 = 2(y + 3)$
Distribute.	$2y + 6 = 2y + 6$
Subtract 2y to get the y's to one side.	$2y - 2y + 6 = 2y - 2y + 6$
Simplify—the y's are gone!	$6 = 6$

But 6=6 is true.

This means that the equation 2y+6=2(y+3) is true for any value of *y*. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an **identity**.

IDENTITY

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is all real numbers.

What happens when we solve the equation 5z=5z−1?

	$5z = 5z - 1$
Subtract 5z to get the constant alone on the right.	$5z - 5z = 5z - 5z - 1$
Simplify—the z's are gone!	$0 \neq -1$

But $0 \neq -1$.

Solving the equation 5z=5z−1 led to the false statement 0=−1. The equation 5z=5z−1 will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a **contradiction**.

CONTRADICTION

An equation that is false for all values of the variable is called a **contradiction**.

A contradiction has no solution.

Exercise 3.5.25

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$6(2n - 1) + 3 = 2n - 8 + 5(2n + 1)$

Answer

$6(2n-1)+3=2n-8+5(2n+1)$	
Distribute.	$12n-6+3=2n-8+10n+5$
Combine like terms.	$12n-3=12n-3$
Subtract 12n to get the nn's to one side.	$12n-12n-3=12n-12n-3$
Simplify.	$-3=-3$
This is a true statement.	The equation is an identity. The solution is all real numbers.

? Exercise 3.5.26

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4+9(3x-7)=-42x-13+23(3x-2)$

Answer

identity; all real numbers

? Exercise 3.5.27

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$8(1-3x)+15(2x+7)=2(x+50)+4(x+3)+1$

Answer

identity; all real numbers

? Exercise 3.5.28

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10+4(p-5)=0$

Answer

	$10 + 4(p - 5) = 0$
Distribute.	$10 + 4p - 20 = 0$
Combine like terms.	$4p - 10 = 0$
Add 10 to both sides.	$4p - 10 + 10 = 0 + 10$
Simplify.	$4p = 10$
Divide.	$\frac{4p}{4} = \frac{10}{4}$
Simplify.	$p = \frac{5}{2}$
The equation is true when $p = frac52$.	This is a conditional equation. The solution is $p = frac52$.

? Exercise 3.5.29

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11(q + 3) - 5 = 19$

Answer

conditional equation; \(q = \frac{9}{11}\

? Exercise 3.5.30

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6 + 14(k - 8) = 95$

Answer

conditional equation; $k = \frac{193}{14}$

? Exercise 3.5.31

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$5m + 3(9 + 3m) = 2(7m - 11)$

Answer

	$5m + 3(9 + 3m) = 2(7m - 11)$
Distribute.	$5m + 27 + 9m = 14m - 22$
Combine like terms.	$14m + 27 = 14m - 22$
Subtract 14m from both sides.	$14m + 27 - 14m = 14m - 22 - 14m$
Simplify.	$27 \neq -22$
But $27 \neq -22$.	The equation is a contradiction. It has no solution.

Exercise 3.5.32

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$12c+5(5+3c)=3(9c-4)$

Answer

contradiction; no solution

Exercise 3.5.33

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4(7d+18)=13(3d-2)-11d$

Answer

contradiction; no solution

Type of equation	What happens when you solve it?	Solution
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**
  1. Simplify each side of the equation as much as possible.
     Use the Distributive Property to remove any parentheses.
     Combine like terms.
  2. Collect all the variable terms on one side of the equation.
     Use the Addition or Subtraction Property of Equality.
  3. Collect all the constant terms on the other side of the equation.
     Use the Addition or Subtraction Property of Equality.
  4. Make the coefficient of the variable term to equal to 1.
     Use the Multiplication or Division Property of Equality.
     State the solution to the equation.
  5. Check the solution.
     Substitute the solution into the original equation.

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3.6: Solving Equations with Fractions or Decimals

Learning Objectives

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Solve Equations with Fraction Coefficients

Let's use the general strategy for solving linear equations introduced earlier to solve the equation, $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$.

Table **3.6.1**

	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x+\frac{1}{2}-\frac{1}{2}=\frac{1}{4}-\frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x=\frac{1}{4}-\frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x=\frac{1}{4}-\frac{2}{4}$
Subtract.	$\frac{1}{8}x=-\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1}\cdot\frac{1}{8}x=\frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x=-2$

This method worked fine, but many students do not feel very confident when they see all those fractions. So, we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of all the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but without fractions. This process is called "clearing" the equation of fractions.

Let's solve a similar equation, but this time use the method that eliminates the fractions.

Exercise 3.6.1: How to Solve Equations with Fraction Coefficients

Solve: $\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$

Answer

Step 1. Find the least common denominator of *all* the fractions in the equation.	What is the LCD of $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{5}{6}$?	$\frac{1}{6}y-\frac{1}{3}=\frac{5}{6}$ LCD $=6$
Step 2. Multiply both sides of the equation by that LCD. This clears the fractions.	Multiply both sides of the equation by the LCD 6.	$6\left(\frac{1}{6}y-\frac{1}{3}\right)=6\left(\frac{5}{6}\right)$
	Use the Distributive Property.	$6\cdot\frac{1}{6}y-6\cdot\frac{1}{3}=6\cdot\frac{5}{6}$
	Simplify – and notice, no more fractions!	$y-2=5$

Step 3. Solve using the General Strategy for Solving Linear Equations.	To isolate the "y" term, add 2. Simplify.	$y-2+2=5+2$ $y=7$

? Exercise 3.6.2

Solve: $\frac{1}{4}x+\frac{1}{2}=\frac{5}{8}$

Answer

$x=\frac{1}{2}$

? Exercise 3.6.3

Solve: $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$

Answer

$x=-2$

Notice in Exercise 3.6.1, once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

STRATEGY TO SOLVE EQUATIONS WITH FRACTION COEFFICIENTS.

1. Find the least common denominator of *all* the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

? Exercise 3.6.4

Solve: $6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$

Answer

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the LCD of all fractions in the equation.		$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$
The LCD is 20.		
Multiply both sides of the equation by 20.		$20(6)=20\cdot\left(\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v\right)$
Distribute.		$20(6)=20\cdot\frac{1}{2}v+20\cdot\frac{2}{5}v-20\cdot\frac{3}{4}v$
Simplify—notice, no more fractions!		$120=10v+8v-15v$
Combine like terms.		$120=3v$
Divide by 3.		$\frac{120}{3}=\frac{3v}{3}$
Simplify.		$40=v$
Check:	$6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v$	
Let v=40.	$6\stackrel{?}{=}\frac{1}{2}(40)+\frac{2}{5}(40)-\frac{3}{4}(40)$	
	$6\stackrel{?}{=}20+16-30$	
	$6=6\checkmark$	

Exercise 3.6.5

Solve: $7=\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$

Answer

$x=12$

Exercise 3.6.6

Solve: $-1=\frac{1}{2}u+\frac{1}{4}u-\frac{2}{3}u$

Answer

$u=-12$

In the next example, we again have variables on both sides of the equation.

Exercise 3.6.7

Solve: $a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$

Answer

	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Find the LCD of all fractions in the equation. The LCD is 8.	
Multiply both sides by the LCD.	$8\left(a+\frac{3}{4}\right)=8\left(\frac{3}{8}a-\frac{1}{2}\right)$
Distribute.	$8\cdot a+8\cdot\frac{3}{4}=8\cdot\frac{3}{8}a-8\cdot\frac{1}{2}$
Simplify—no more fractions.	$8a+6=3a-4$
Subtract 3a3a from both sides.	$8a-3a+6=3a-3a-4$
Simplify.	$5a+6=-4$
Subtract 6 from both sides.	$5a+6-6=-4-6$
Simplify.	$5a=-10$
Divide by 5.	$\frac{5a}{5}=\frac{-10}{5}$
Simplify.	$a=-2$
Check:	$a+\frac{3}{4}=\frac{3}{8}a-\frac{1}{2}$
Let a=−2.	$-2+\frac{3}{4}\stackrel{?}{=}\frac{3}{8}(-2)-\frac{1}{2}$
	$-\frac{8}{4}+\frac{3}{4}\stackrel{?}{=}-\frac{16}{8}-\frac{4}{8}$
	$-\frac{5}{4}=-\frac{10}{8}$
	$-\frac{5}{4}=-\frac{5}{4}\checkmark$

? Exercise 3.6.8

Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$

Answer

$x=-1$

? Exercise 3.6.9

Solve: $c+\frac{3}{4}=\frac{1}{2}c-\frac{1}{4}$

Answer

$c=-2$

In the next example, we start by using the Distributive Property. This step clears the fractions right away.

? Exercise 3.6.10

Solve: $-5=\frac{1}{4}(8x+4)$

Answer

	$-5=\frac{1}{4}(8x+4)$
Distribute.	$-5=\frac{1}{4}\cdot 8x+\frac{1}{4}\cdot 4$
Simplify. Now there are no fractions.	$-5=2x+1$
Subtract 1 from both sides.	$-5-1=2x+1-1$
Simplify.	$-6=2x$
Divide by 2.	$\frac{-6}{2}=\frac{2x}{2}$
Simplify.	$-3=x$

Check:	$-5=\frac{1}{4}(8x+4)$
Let x=−3.	$-5\stackrel{?}{=}\frac{1}{2}(4(-3)+2)$
	$-5\stackrel{?}{=}\frac{1}{2}(-12+2)$
	$-5\stackrel{?}{=}\frac{1}{2}(-10)$
	$-5=-5$ ✓

Exercise 3.6.11

Solve: $-11=\frac{1}{2}(6p+2)$

Answer

$p=-4$

Exercise 3.6.12

Solve: $8=\frac{1}{3}(9q+6)$

Answer

$q=2$

In the next example, even after distributing, we still have fractions to clear.

Exercise 3.6.13

Solve: $\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$

Answer

	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2}\cdot y-\frac{1}{2}\cdot 5=\frac{1}{4}\cdot y-\frac{1}{4}\cdot 1$
Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, 4.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the variables to the left.	$2y-y-10=y-y-1$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$
Check:	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Let y=9.	$\frac{1}{2}(9-5)\stackrel{?}{=}\frac{1}{4}(9-1)$
Finish the check on your own.	

? Exercise 3.6.14

Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$

Answer

$n=2$

? Exercise 3.6.15

Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$

Answer

$m=-1$

? Exercise 3.6.16

Solve: $\frac{5x-3}{4}=\frac{x}{2}$

Answer

	$\frac{5x-3}{4}=\frac{x}{2}$
Multiply by the LCD, 4.	$4\left(\frac{5x-3}{4}\right)=4\left(\frac{x}{2}\right)$
Simplify.	$5x-3=2x$
Collect the variables to the right.	$5x-5x-3=2x-5x$
Simplify.	$-3=-3x$
Divide.	$\frac{-3}{-3}=\frac{-3x}{-3}$
Simplify.	$1=x$
Check:	$\frac{5x-3}{4}=\frac{x}{2}$
Let x=1.	$\frac{5(1)-3}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{2}{4}\stackrel{?}{=}\frac{1}{2}$
	$\frac{1}{2}=\frac{1}{2}$✓

Exercise 3.6.17

Solve: $\frac{4y-7}{3}=\frac{y}{6}$

Answer

$y=2$

Exercise 3.6.18

Solve: $\frac{-2z-5}{4}=\frac{z}{8}$

Answer

$z=-2$

Exercise 3.6.19

Solve: $\frac{a}{6}+2=\frac{a}{4}+3$

Answer

	$\frac{a}{6}+2=\frac{a}{4}+3$
Multiply by the LCD, 12.	$12\left(\frac{a}{6}+2\right)=12\left(\frac{a}{4}+3\right)$
Distribute.	$12\cdot\frac{a}{6}+12\cdot 2=12\cdot\frac{a}{4}+12\cdot 3$
Simplify.	$2a+24=3a+36$
Collect the variables to the right.	$2a-2a+24=3a-2a+36$
Simplify.	$24=a+36$
Collect the constants to the left.	$24-36=a+36-36$
Simplify.	$a=-12$
Check:	$\frac{a}{6}+2=\frac{a}{4}+3$
Let a=−12.	$\frac{-12}{6}+2\stackrel{?}{=}\frac{-12}{4}+3$
	$-2+2\stackrel{?}{=}-3+3$
	$0=0\checkmark$

Exercise 3.6.20

Solve: $\frac{b}{10}+2=\frac{b}{4}+5$

Answer

$b=-20$

Exercise 3.6.21

Solve: $\frac{c}{6}+3=\frac{c}{3}+4$

Answer

$c=-6$

Exercise 3.6.22

Solve: $\frac{4q+3}{2}+6=\frac{3q+5}{4}$

Answer

	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Multiply by the LCD, 4.	$4\left(\frac{4q+3}{2}+6\right)=4\left(\frac{3q+5}{4}\right)$
Distribute.	$4\left(\frac{4q+3}{2}\right)+4\cdot 6=4\cdot\left(\frac{3q+5}{4}\right)$
Simplify.	$2(4q+3)+24=3q+5$ $8q+6+24=3q+5$ $8q+30=3q+5$
Collect the variables to the left.	$8q-3q+30=3q-3q+5$
Simplify.	$5q+30=5$
Collect the constants to the right.	$5q+30-30=5-30$
Simplify.	$5q=-25$
Divide by 5.	$\frac{5q}{5}=\frac{-25}{5}$
Simplify.	$q=-5$
Check:	$\frac{4q+3}{2}+6=\frac{3q+5}{4}$
Let q=−5.	$\frac{4(-5)+3}{2}+6\stackrel{?}{=}\frac{3(-5)+5}{4}$
Finish the check on your own.	

? Exercise 3.6.23

Solve: $\frac{3r+5}{6}+1=\frac{4r+3}{3}$

Answer

$r=1$

? Exercise 3.6.24

Solve: $\frac{2s+3}{2}+1=\frac{3s+2}{4}$

Answer

$s=-8$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money or percentages. But decimals can also be expressed as fractions. For example, $0.3=\frac{3}{10}$ and $0.17=\frac{17}{100}$. So, with an equation with decimals, we can use the same method we used to clear fractions—multiply both sides of the equation by the least common denominator.

? Exercise 3.6.25

Solve: $0.06x+0.02=0.25x-1.5$

Answer

Look at the decimals and think of the equivalent fractions.

$0.06 = \frac{6}{100}$ $0.02 = \frac{2}{100}$ $0.25 = \frac{25}{100}$ $1.5 = 1\frac{5}{10}$

Notice, the LCD is 100.

By multiplying by the LCD, we will clear the decimals from the equation.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both side by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now we have no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the variables to the right.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let x=8 $0.06(8) + 0.02 \stackrel{?}{=} 0.25(8) - 1.5$ $0.48 + 0.02 \stackrel{?}{=} 2.00 - 1.5$ $0.50 = 0.50$ ✓	

? Exercise 3.6.26

Solve: $0.14h + 0.12 = 0.35h - 2.4$

Answer

$h = 12$

? Exercise 3.6.27

Solve: $0.65k - 0.1 = 0.4k - 0.35$

Answer

$k = -1$

The next example uses an equation that is typical of the money applications in the next chapter. Notice that we distribute the decimal before we clear all the decimals.

? Exercise 3.6.28

Solve: $0.25x + 0.05(x + 3) = 2.85$

Answer

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check it yourself by substituting x=9 into the original equation.	

? Exercise 3.6.29

Solve: $0.25n + 0.05(n + 5) = 2.95$

Answer

$n = 9$

? Exercise 3.6.30

Solve: $0.10d + 0.05(d - 5) = 2.15$

Answer

$d = 16$

Key Concepts

- **Strategy to Solve an Equation with Fraction Coefficients**
  1. Find the least common denominator of all the fractions in the equation.
  2. Multiply both sides of the equation by that LCD. This clears the fractions.
  3. Solve using the General Strategy for Solving Linear Equations.

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3.7: Solving a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

DISTANCE, RATE, AND TIME

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \quad \text{where} \quad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned} \tag{3.7.1}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

SOLVE AN APPLICATION (WITH A FORMULA).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

Exercise 3.7.1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let d = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$
	$d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check	
Does 42 miles make sense?	
Jamal rides:	
12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable 48 miles in 4 hours.	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Exercise 3.7.2

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Answer

330 miles

Exercise 3.7.3

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Answer

7 miles

Exercise 3.7.4

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Answer

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let t = time.
	$d = 520$ miles $r = 65$ mph $t = ?$ hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	
$d = rt$ $520 \stackrel{?}{=} 65 \cdot 8$ $520 = 520$✓	
Step 7. Answer the question with a complete sentence. Rey's trip will take 8 hours.	

? Exercise 3.7.5

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Answer

11 hours

? Exercise 3.7.6

Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Answer

56 mph

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In Exercise **3.7.1** and Exercise **3.7.4**, we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of r and t, the rate and time. But in Exercise **3.7.4**, we had to find the value of t. We substituted in values of d and r and then used algebra to solve for tt. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of d and r. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

Exercise 3.7.7

Solve the formula d=rt for t:

1. when d=520 and r=65
2. in general

Answer

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

1. when d=520 and r=65		2. in general	
Write the formula.	$d = rt$	Write the formula.	$d = rt$
Substitute.	$520 = 65t$		
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$	Divide, to isolate tt.	$\frac{d}{r} = \frac{rt}{t}$
Simplify.	$8 = t$	Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

Exercise 3.7.8

Solve the formula $d = rt$ for r:

1. when d=180 and t=4
2. in general

Answer

1. $r = 45$
2. $r = \frac{d}{t}$

Exercise 3.7.9

Solve the formula $d = rt$ for r:

1. when d=780 and t=12
2. in general

Answer

1. $r = 65$
2. $r = \frac{d}{t}$

Exercise 3.7.10

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 90$ and $b = 15$
2. in general

Answer

1. when $A = 90$ and $b = 15$		2. in general	
Write the formula.	$A = \frac{1}{2}bh$	Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2}15h$	Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$	Simplify.	$2A = bh$
Solve for h.	$12 = h$	Solve for hh.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$

? Exercise 3.7.11

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 170$ and $b = 17$
2. in general

Answer

1. $h = 20$
2. $h = \frac{2A}{b}$

? Exercise 3.7.12

Solve the formula $A = \frac{1}{2}bh$ for h:

1. when $A = 62$ and $h = 31$
2. in general

Answer

1. $b = 4$
2. $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, I, for a principal, P, invested at rate, r, for t years.

? Exercise 3.7.13

Solve the formula I=Prt to find the principal, P:

1. when I=$5,600, r=4% ,t=7years
2. in general

Answer

1. I=$5,600, r=4% ,t=7years		2. in general	
Write the formula.	$I = Prt$	Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$		
Simplify.	$5600 = P(0.28)$	Simplify.	$I = P(rt)$
Divide, to isolate P.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	Divide, to isolate P.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$	Simplify.	$\frac{I}{rt} = P$
The principal is	$20,000		$P = \frac{I}{rt}$

? Exercise 3.7.14

Solve the formula I=Prt to find the principal, P:

1. when I=$2160, r=6% ,t=3 years
2. in general

Answer

1. $12000
2. $P = \frac{I}{rt}$

? Exercise 3.7.15

Solve the formula I=Prt to find the principal, P:

1. when I=$5400, r= 12% ,t=5 years
2. in general

Answer

1. $9000
2. $P = \frac{I}{rt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually x and y. You might be given an equation that is solved for y and need to solve it for x, or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y.

? Exercise 3.7.16

Solve the formula 3x+2y=18 for y:

1. when x=4
2. in general

Answer

1. when x=4		2. in general	
	$3x+2y=18$		$3x+2y=18$
Substitute.	$3(4)+2y=18$		
Subtract to isolate the y-term.	$12-12+2y=18-12$	Subtract to isolate the y-term.	$3x-3x+2y=18-3x$
Divide.	$\frac{2y}{2}=\frac{6}{2}$	Divide.	$\frac{2y}{2}=\frac{18}{2}-\frac{3x}{2}$
Simplify.	$y=3$	Simplify.	$y=-\frac{3x}{2}+9$

Exercise 3.7.17

Solve the formula 3x+4y=10 for *y*:

1. when $x=\frac{14}{3}$
2. in general

Answer

1. $y=-1$
2. $y=\frac{10-3x}{4}$

Exercise 3.7.18

Solve the formula 5x+2y=18 for *y*:

1. when $x=4$
2. in general

Answer

1. $y=-1$
2. $y=\frac{18-5x}{2}$

In Exercise 3.7.7 through Exercise 3.7.18 we used the numbers in part 1 as a guide to solving in general in part 2. Now we will solve a formula in general without using numbers as a guide.

Exercise 3.7.19

Solve the formula P=a+b+c for a.

Answer

We will isolate aa on one side of the equation.	$P=a+b+c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P-b-c=a+b+c-b-c$
Simplify.	$P-b-c=a$ $a=P-b-c$

Exercise 3.7.20

Solve the formula P=a+b+c for *b*.

Answer

b=P−a−c

Exercise 3.7.21

Solve the formula P=a+b+c for *c*.

Answer

c=P−a−b

Exercise 3.7.22

Solve the formula 6x+5y=13 for *y*.

Answer

	$6x + 5y = 13$
Subtract 6x from both sides to isolate the term with y.	$6x - 6x + 5y = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13 - 6x}{5}$
Simplify.	$y = \frac{13 - 6x}{5}$

The fraction is simplified. We cannot divide 13−6x by 5.

Exercise 3.7.23

Solve the formula 4x+7y=9 for *y*.

Answer

$y = \frac{9-4x}{7}$

Exercise 3.7.24

Solve the formula 5x+8y=1 for *y*.

Answer

$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.

5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: d=rt where d = distance, r = rate, t = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

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CHAPTER OVERVIEW

4: Percents

4.1: Basics of Percents

4.2: Percents Problems and Applications of Percent

4.3: Percents Problems and Applications Part 2

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4.1: Basics of Percents

Percent Basics

Percent means “per one hundred”. A percent is a ratio or fraction with a denominator of 100.

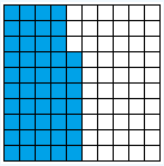
Exercises 4.1.1



During Super Bowl XLIX between the Seahawks and Patriots, 89 out of 100 television sets in Seattle were tuned to the game.[1]

1. What percent of the television sets were tuned to the game?

2. What percent of the television sets were not tuned to the game?



3. What percent of the squares are shaded?

4. What percent of the squares are not shaded?

Answer

1. 89%
2. 11%
3. 47%
4. 53%

To write a percent as a fraction: drop the percent sign, write the number over 100, and simplify if possible.

Tip: If a percent is greater than 100%, the fraction will be greater than 1. If a percent is less than 1%, the fraction will be less than $\frac{1}{100}$.

Exercises 4.1.1

Write each percent as a fraction, and simplify if possible.

5. About 71% of Earth’s surface is covered by water.[2]

6. About 1.3% of Earth’s land surface is permanent cropland.[3]

7. About 0.04% of Earth’s atmosphere is carbon dioxide.[4]

8. The worldwide number of active Facebook users in the fourth quarter of 2018 was approximately 102% of the number of users in the third quarter of 2018.[5]

Answer

5. $\frac{71}{100}$

6. $\frac{1.3}{100} = \frac{13}{1000}$
7. $\frac{0.04}{100} = \frac{1}{2500}$
8. $\frac{102}{100} = \frac{51}{50}$

To write a percent as a decimal: drop the percent sign and move the decimal point two places to the left.

? Exercises 4.1.1

Write each percent from Exercises 5 through 8 as a decimal.

9. 71%

10. 1.3%

11. 0.04%

12. 102%

Answer

9. 0.71
10. 0.013
11. 0.0004
12. 1.02

To write a decimal as a percent: move the decimal point two places to the right and insert a percent sign.

? Exercises 4.1.1

Write each decimal number as a percent.

13. 0.23

14. 0.07

15. 0.085

16. 2.5

Answer

13. 23%
14. 7%
15. 8.5%
16. 250%

To write a fraction as a percent, write the fraction as a decimal by dividing the numerator by the denominator, then move the decimal point two places to the right and insert a percent sign.

Alternate method: Recall from the fractions module that if the denominator of a fraction has no prime factors other than 2's and 5's, then the fraction can be built up to have a denominator of 10, or 100, or $1,000$…

Exercises 4.1.1

17. 7 out of 25 students were tardy on Wednesday. Write $\frac{7}{25}$ as a percent.

18. A package of 24 m&m's contained 3 orange m&m's. Write $\frac{3}{24}$ as a percent.

Answer

17. 28%
18. 12.5%

Solving Percent Problems: Finding the Amount

You may use a calculator throughout the remainder of this module.

We often use the words ***amount*** and ***base*** in a percent problem. The ***amount*** is the answer we get after finding the percent of the original number. The ***base*** is the original number, the number we find the percent of. (You may also think of the amount as the part, and the base as the whole.) We can call the percent the ***rate***.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

Be sure to change the percent to a decimal before multiplying.

Exercises 4.1.1

19. What is 9% of 350?

20. 30% of 75 is what number?

21. Find 13.5% of 500.

22. 125% of 80 is equal to what amount?

23. What number is 40% of 96.5?

24. Calculate 0.5% of 450.

Suppose you buy an electric drill with a retail price of $\$ 109.97$ in a city with 8.5% sales tax.

25. Find the amount of the tax. Round to the nearest cent, if necessary.

26. How much do you pay in total?

Answer

19. 31.5
20. 22.5
21. 67.5
22. 100
23. 38.6
24. 2.25
25. $\$ 9.35$
26. $\$ 119.32$

1. https://twitter.com/darrenrovell/status/562258101337067521 ↵
2. https://en.Wikipedia.org/wiki/Earth#Surface ↵

3. https://en.Wikipedia.org/wiki/Earth#Surface ↵
4. https://en.Wikipedia.org/wiki/Atmosphere_of_Earth#Composition ↵
5. https://www.statista.com/statistics/264810/number-of-monthly-active-facebook-users-worldwide/ ↵

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4.2: Percents Problems and Applications of Percent

You may use a calculator throughout this module.



Recall: The amount is the answer we get after finding the percent of the original number. The base is the original number, the number we find the percent of. We can call the percent the rate.

When we looked at percents in a previous module, we focused on finding the amount. In this module, we will learn how to find the percentage rate and the base.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

We can translate from words into algebra.

- "is" means equals
- "of" means multiply
- "what" means a variable

Solving Percent Problems: Finding the Rate

Suppose you earned 56 points on a 60-point quiz. To figure out your grade as a percent, you need to answer the question "56 is what percent of 60?" We can translate this sentence into the equation $56 = R \cdot 60$.

? Exercises 4.2.1

1. 56 is what percent of 60?

2. What percent of 120 is 45?

Answer

1. 93% or 93.3%
2. 37.5%

Be aware that this method gives us the answer in decimal form and we must move the decimal point to convert the answer to a percent.

Also, if the instructions don't explicitly tell you how to round your answer, use your best judgment: to the nearest whole percent or nearest tenth of a percent, to two or three significant figures, etc.

Solving Percent Problems: Finding the Base

Suppose you earn 2% cash rewards for the amount you charge on your credit card. If you want to earn $\$ 50$ in cash rewards, how much do you need to charge on your card? To figure this out, you need to answer the question "50 is 2% of what number?" We can translate this into the equation $50 = 0.02 \cdot B$.

? Exercises 4.2.1

3. $ 50 is 2% of what number?

4. 5% of what number is 36?

Answer

3. $ 2,500

4. 720

? Exercises 4.2.1

5. An 18% tip will be added to a dinner that cost $ 107.50. What is the amount of the tip?

6. The University of Oregon women's basketball team made 13 of the 29 three-points shots they attempted during a game against UNC. What percent of their three-point shots did the team make?

7. 45% of the people surveyed answered "yes" to a poll question. If 180 people answered "yes", how many people were surveyed altogether?

Answer

5. $ 19.35

6. 44.8% or 45%

7. 400 people were surveyed

Solving Percent Problems: Percent Increase

When a quantity changes, it is often useful to know by what percent it changed. If the price of a candy bar is increased by 50 cents, you might be annoyed because it's it's a relatively large percentage of the original price. If the price of a car is increased by 50 cents, though, you wouldn't care because it's such a small percentage of the original price.

To find the percent of increase:

1. Subtract the two numbers to find the amount of increase.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Notice that we always use the ***original*** number for the base, the number that occurred earlier in time. In the case of a percent increase, this is the smaller of the two numbers.

? Exercises 4.2.1

8. The price of a candy bar increased from $ 0.89 to $ 1.39. By what percent did the price increase?

9. The population of Portland in 2010 was 583,793. The estimated population in 2019 was 654,741. Find the percent of increase in the population.[1]

Answer

8. 56.2% increase

9. 12.2% increase

Solving Percent Problems: Percent Decrease

Finding the percent decrease in a number is very similar.

To find the percent of decrease:

1. Subtract the two numbers to find the amount of decrease.
2. Using this result as the amount and the **original** number as the base, find the unknown percent.

Again, we always use the ***original*** number for the base, the number that occurred earlier in time. For a percent decrease, this is the larger of the two numbers.

? Exercises 4.2.1

10. During a sale, the price of a candy bar was reduced from $\$ 1.39$ to $\$ 0.89$. By what percent did the price decrease?

11. The number of students enrolled at Clackamas Community College decreased from $7,439$ in Summer 2019 to $4,781$ in Summer 2020. Find the percent of decrease in enrollment.

Answer

10. 36.0% decrease
11. 35.7% decrease

Relative Error

In an earlier module, we said that a measurement will always include some error, no matter how carefully we measure. It can be helpful to consider the size of the error relative to the size of what is being measured. As we saw in the examples above, a difference of 50 cents is important when we're pricing candy bars but insignificant when we're pricing cars. In the same way, an error of an eighth of an inch could be a deal-breaker when you're trying to fit a screen into a window frame, but an eighth of an inch is insignificant when you're measuring the length of your garage.

The ***expected outcome*** is what the number would be in a perfect world. If a window screen is supposed to be exactly 25 inches wide, we call this the expected outcome, and we treat it as though it has infinitely many significant digits. In theory, the expected outcome is $25.000000\ldots$

To find the ***absolute error***, we subtract the measurement and the expected outcome. Because we always treat the expected outcome as though it has unlimited significant figures, the absolute error should have the same precision (place value) as the *measurement*, not the expected outcome.

To find the ***relative error***, we divide the absolute error by the expected outcome. We usually express the relative error as a percent. In fact, the procedure for finding the relative error is identical to the procedures for finding a percent increase or percent decrease!

To find the relative error:

1. Subtract the two numbers to find the absolute error.
2. Using the **absolute error** as the amount and the **expected outcome** as the base, find the unknown percent.

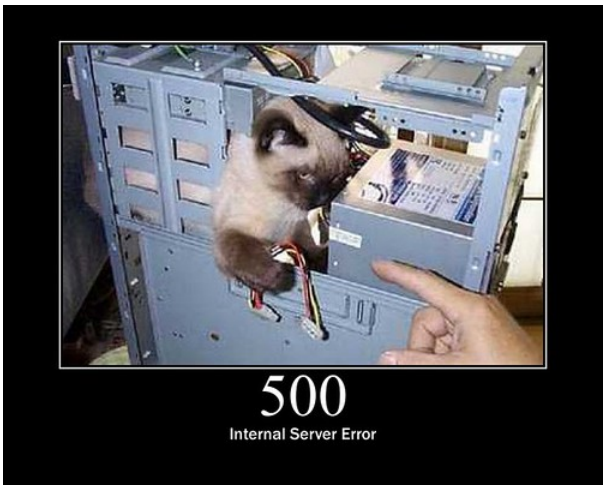
? Exercisew 4.2.1

12. A window screen is measured to be $25\frac{3}{16}$ inches wide instead of the advertised 25 inches. Determine the relative error, rounded to the nearest tenth of a percent.

13. The contents of a box of cereal are supposed to weigh 10.8 ounces, but they are measured at 10.67 ounces. Determine the relative error, rounded to the nearest tenth of a percent.

Answer

12. $0.1875 \div 25 \approx 0.8\%$
13. $0.13 \div 10.8 \approx 1.2\%$



Tolerance

The ***tolerance*** is the maximum amount that a measurement is allowed to differ from the expected outcome. For example, the U.S. Mint needs its coins to have a consistent size and weight so that they will work in vending machines. A dime (10 cents) weighs 2.268 grams, with a tolerance of ± 0.091 grams.[2] This tells us that the minimum acceptable weight is $2.268 - 0.091 = 2.177$ grams, and the maximum acceptable weight is $2.268 + 0.091 = 2.359$ grams. A dime with a weight outside of the range $2.177 \leq \text{weight} \leq 2.359$ would be unacceptable.



? Exercises 4.2.1

A U.S. nickel (5 cents) weighs 5.000 grams with a tolerance of ± 0.194 grams.

14. Determine the lowest acceptable weight and highest acceptable weight of a nickel.

15. Determine the relative error of a nickel that weighs 5.21 grams.

A U.S. quarter (25 cents) weighs 5.670 grams with a tolerance of ± 0.227 grams.

16. Determine the lowest acceptable weight and highest acceptable weight of a quarter.

17. Determine the relative error of a quarter that weighs 5.43 grams.

Answer

14. 4.806 g; 5.194 g
15. $0.21 \div 5.000 = 4.2\%$
16. 5.443 g; 5.897 g
17. $0.24 \div 5.670 \approx 4.2\%$

1. www.census.gov/quickfacts/fact/table/portlandcityoregon,OR,US/PST045219 ↵

2. https://www.usmint.gov/learn/coin-and-medal-programs/coin-specifications and https://www.thesprucecrafts.com/how-much-do-coins-weigh-4171330↵

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4.3: Percents Problems and Applications Part 2



You may use a calculator throughout this module.

There is one more situation involving percents that often trips people up: working backwards from the result of a percent change to find the original value.

$$\text{Amount} = \text{Rate} \cdot \text{Base}$$

$$A = R \cdot B$$

Finding the Base After Percent Increase

Suppose a 12% tax is added to a price; what percent of the original is the new amount?

Well, the original number is 100% of itself, so the new amount must be $100\% + 12\% = 112\%$ of the original.

As an equation, $A = 1.12 \cdot B$.

If a number is **increased** by a percent, add that percent to 100% and use that result for R.

The most common error in solving this type of problem is applying the percent to the new number instead of the original. For example, consider this question: "After a 12% increase, the new price of a computer is $\$\ 1,120$. What was the original price?"

People often work this problem by finding 12% of $\$\ 1,120$ and subtracting that away: 12% of $1,120$ is 134.40, and $1,120 - 134.40 = 985.60$. It appears that the original price was $\$\ 985.60$, but if we check this result, we find that the numbers don't add up. 12% of 985.60 is 118.272, and $985.60 + 118.272 = 1,103.872$, not $1,120$.

The correct way to think about this is $1,120 = 1.12 \cdot B$. Dividing $1,120$ by 1.12 gives us the answer $1,000$, which is clearly correct because we can find that 12% of $1,000$ is 120, making the new amount $1,120$. The original price was $\$\ 1,000$.

To summarize, we cannot subtract 12% from the new amount; we must instead ***divide*** the new amount by 112%.

? Exercises 4.3.1

1. A sales tax of 8% is added to the selling price of a lawn tractor, making the total price $\$\ 1,402.92$. What is the selling price of the lawn tractor without tax?

2. The U.S. population in 2018 was estimated to be 327.2 million, which represents a 7.6% increase from 2008. What was the U.S. population in 2008?

Answer

1. $\$\ 1,299.00$
2. 304.1 million

Finding the Base After Percent DEcrease

Suppose a 12% discount is applied to a price; what percent of the original is the new amount?

As above, the original number is 100% of itself, so the new amount must be $100\% - 12\% = 88\%$ of the original.

As an equation, $A = 0.88 \cdot B$.

If a number is **decreased** by a percent, subtract that percent from 100% and use that result for R.

As above, the most common error in solving this type of problem is applying the percent to the new number instead of the original. For example, consider this question: "After a 12% decrease, the new price of a computer is $\$ 880$. What was the original price?"

People often work this problem by finding 12% of 880 and adding it on: 12% of 880 is 105.60, and $880 + 105.60 = 985.60$. It appears that the original price was $\$ 985.60$, but if we check this result, we find that the numbers don't add up. 12% of 985.60 is 118.272, and $985.60 - 118.272 = 867.328$, not 880.

The correct way to think about this is $880 = 0.88 \cdot B$. Dividing 880 by 0.88 gives us the answer $1,000$, which is clearly correct because we can find that 12% of $1,000$ is 120, making the new amount 880. The original price was $\$ 1,000$.

To summarize, we cannot add 12% to the new amount; we must instead ***divide*** the new amount by 88%.

? Exercises 4.3.1

3. A city department's budget was cut by 5% this year. If this year's budget is $\$ 3.04$ million, what was last year's budget?

4. CCC's enrollment in Summer 2019 was $9,116$ students, which was a decrease of 2.17% from Summer 2018. What was the enrollment in Summer 2018? (Round to the nearest whole number.)[1]

5. An educational website claims that by purchasing access for $\$ 5$, you'll save 69% off the standard price. What was the standard price? (Use you best judgment when rounding your answer.)

Answer

3. $\$ 3.20$ million
4. $9,318$ students
5. $\$ 16.13$, or more likely, just $\$ 16$.

1. These enrollment numbers don't match those in Percents Part 2, which makes me wonder how accurate the yearly reports are. Or maybe I inadvertently grabbed data from two different ways that enrollment was being counted. ↵

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CHAPTER OVERVIEW

5: Proportions and Variation

5.1: Ratios, Rates, Proportions

5.2: Applications of Proportionality

5.3: Use Direct and Inverse Variation

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5.1: Ratios, Rates, Proportions

Ratios & Rates

A ratio is the quotient of two numbers or the quotient of two quantities with the same units.

When writing a ratio as a fraction, the first quantity is the numerator and the second quantity is the denominator.

Exercise 5.1.1

1. Find the ratio of 45 minutes to 2 hours. Simplify the fraction, if possible.



Answer

1. $\frac{3}{8}$

A rate is the quotient of two quantities with different units. You must include the units.

When writing a rate as a fraction, the first quantity is the numerator and the second quantity is the denominator. Simplify the fraction, if possible. Include the units in the fraction.

Exercise 5.1.1

2. A car travels 105 miles in 2 hours. Write the rate as a fraction.

Answer

2. $\frac{105 \text{ mi}}{2 \text{ hr}}$

A unit rate has a denominator of 1. If necessary, divide the numerator by the denominator and express the rate as a mixed number or decimal.

Exercise 5.1.1

3. A car travels 105 miles in 2 hours. Write as a unit rate.

Answer

3. $\frac{52.5 \text{ mi}}{1 \text{ hr}}$ or 52.5 miles per hour

A unit price is a rate with the price in the numerator and a denominator equal to 1. The unit price tells the cost of one unit or one item. You can also simply divide the cost by the size or number of items.

Exercises 5.1.1

4. An 18-ounce box of cereal costs $ 3.59. Find the unit price.

5. A 12-ounce box of cereal costs $ 2.99. Find the unit price.

6. Which box has a lower unit price?



Answer

4. $ 0.199/oz, or around 20 cents per ounce
5. $ 0.249/oz, or around 25 cents per ounce
6. the 18-ounce box has the lower unit price

Proportions

A proportion says that two ratios (or rates) are equal.

Exercises 5.1.1

Determine whether each proportion is true or false by simplifying each fraction.

7. $\frac{6}{8} = \frac{21}{28}$

8. $\frac{10}{15} = \frac{16}{20}$

Answer

7. $\frac{3}{4} = \frac{3}{4}$; true
8. $\frac{2}{3} \neq \frac{4}{5}$; false

A common method of determining whether a proportion is true or false is called cross-multiplying or finding the ***cross products***. We multiply diagonally across the equal sign. In a true proportion, the cross products are equal.

$$\frac{a}{b} = \frac{c}{d} \rightarrow a \cdot d = b \cdot c$$

Exercises 5.1.1

Determine whether each proportion is true or false by cross-multiplying.

9. $\frac{6}{8} = \frac{21}{28}$

10. $\frac{10}{15} = \frac{16}{20}$
11. $\frac{14}{4} = \frac{15}{5}$
12. $\frac{0.8}{4} = \frac{5}{25}$

Answer

9. $168 = 168$; true
10. $200 \neq 240$; false
11. $70 \neq 60$; false
12. $20 = 20$; true

As we saw in a previous module, we can use a variable to stand for a missing number. If a proportion has a missing number, we can use cross multiplication to solve for the missing number. This is as close to algebra as we get in this textbook.

To solve a proportion for a variable:

1. Set the cross products equal to form an equation of the form $a \cdot d = b \cdot c$.
2. Isolate the variable by rewriting the multiplication equation as a division equation.
3. Check the solution by substituting the answer into the original proportion and finding the cross products.

You may discover slightly different methods that you prefer.[1] If you think "Hey, can't I do this a different way?", you may be correct.

Exercises 5.1.1

Solve for the variable.

13. $\frac{8}{10} = \frac{x}{15}$
14. $\frac{3}{2} = \frac{7.5}{n}$
15. $\frac{3}{k} = \frac{18}{24}$
16. $\frac{w}{6} = \frac{15}{9}$
17. $\frac{5}{4} = \frac{13}{x}$
18. $\frac{3.2}{7.2} = \frac{m}{4.5}$ (calculator recommended)

Answer

13. $x = 12$
14. $n = 5$
15. $k = 4$
16. $w = 10$
17. $x = 10.4$
18. $m = 2.0$

Now, let's try some with more than one variable in parts of the proportion.

Example 5.1.3

$\frac{n}{n+14}=\frac{5}{7}$.

		$\frac{n}{n+14}=\frac{5}{7}$
Multiply both sides by the LCD.		$7(n+14)\left(\frac{n}{n+14}\right)=7(n+14)\left(\frac{5}{7}\right)$
Remove common factors on each side.		$7n=5(n+14)$
Simplify.		$7n=5n+70$
Solve for n.		$2n=70$
		$n=35$
Check.		
	$\frac{n}{n+14}=\frac{5}{7}$	
Substitute $n=35$.	$\frac{35}{35+14}\stackrel{?}{=}\frac{5}{7}$	
Simplify.	$\frac{35}{49}\stackrel{?}{=}\frac{5}{7}$	
Show common factors.	$\frac{5\cdot 7}{7\cdot 7}\stackrel{?}{=}\frac{5}{7}$	
Simplify.	$\frac{5}{7}=\frac{5}{7}$ ✓	

TRy it 5.1.5

$\frac{y}{y+55}=\frac{3}{8}$.

Answer

33

TRy it 5.1.6

$\frac{z}{z-84}=-\frac{1}{5}$.

Answer

14

Example 5.1.4

$\frac{p+12}{9}=\frac{p-12}{6}$.

		$\frac{p+12}{9}=\frac{p-12}{6}$
Multiply both sides by the LCD, 18.		$18\left(\frac{p+12}{9}\right)=18\left(\frac{p-12}{6}\right)$
Simplify.		$2(p+12)=3(p-12)$
Distribute.		$2p+24=3p-36$
Solve fpr p.		$60=p$
Check.		
	$\frac{p+12}{9}=\frac{p-12}{6}$	
Substitute $p=60$.	$\frac{60+12}{9}\stackrel{?}{=}\frac{60-12}{6}$	
Simplify.	$\frac{72}{9}\stackrel{?}{=}\frac{48}{6}$	
Divide.	$8=8$ ✓	

Try it 5.1.7

$\frac{v+30}{8}=\frac{v+66}{12}$.

Answer

42

Try it 5.1.8

$\frac{2x+15}{9}=\frac{7x+3}{15}$.

Answer

6

1. The steps in the box are designed to avoid mentioning the algebraic step of dividing both sides of the equation by a number. If you are comfortable with basic algebra, then you would phrase step 2 differently. ↵

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5.2: Applications of Proportionality

In the previous section, we studied proportions, and used them to solve problems involving ratios. In this section, we continue our study of proportions, and investigate two different types of proportionality.

In this section, you will learn to:

Recognize direct proportionality relationships, and use them to answer questions involving direct proportionality

- Compute and interpret the constant of proportionality in context
- Recognize inverse proportionality relationships, and use them to answer questions involving inverse proportionality

There are two main types of proportionality. We will learn about the more common one first.

5.2.1: Direct Proportionality

Definition: Directly Proportional

Two quantities are **directly proportional** if, as one quantity increases, the other quantity also increases at the same rate.

Direct proportionality describes all of the proportion problems we've seen before. Here is another example that shows how direct proportionality works, and introduces the next important notion.

Example 5.2.1

At an hourly wage job, you work for 5 hours, and get paid $\$83.75$. How much money will you earn if you work 7 hours? (Assume that you are not making overtime pay, or any other sort of special pay rate.)

Solution

This is a situation described by direct proportionality. Since this is an hourly wage job, and we're told that there is no overtime pay or other special pay rate, we can safely assume that we make the same amount per hour in this job. In other words, the rate of pay will be the same no matter how many hours we work. That means that if the number of hours worked increases, the amount you're paid increases at the same rate, no matter the number of hours worked.

This is a situation described by direct proportionality. Since this is an hourly wage job, and we're told that there is no overtime pay or other special pay rate, we can safely assume that we make the same amount per hour in this job. In other words, the rate of pay will be the same no matter how many hours we work. That means that if the number of hours worked increases, the amount you're paid increases at the same rate, no matter the number of hours worked.

We can solve this using a proportion, using the same techniques as in the last section. We will set up the following proportion equation:

$$\frac{\$83.75}{5 \text{ hours}} = \frac{\$x}{7 \text{ hours}} \tag{5.2.1}$$

Notice that we have picked a variable, x, to denote the answer we are trying to find -- the number of dollars earned for working 7 hours. Also notice how we've labeled our units, and have made sure that the corresponding quantities are together. If you do this every time -- label your units, and make sure corresponding quantities stay together -- you can solve any direct proportion problem.

Now for the process that will actually help us solve for x. First, rewrite the equation without labels:

$$\frac{83.75}{5} = \frac{x}{7} \tag{5.2.2}$$

Next, apply Cross Multiplication

$$5x = 83.75 \times 7 \tag{5.2.3}$$

Next, simplify the right side (using a calculator):

$$5x = 586.25 \tag{5.2.4}$$

Finally, apply Division undoes Multiplication to find x:

$$x = \frac{586.25}{5} = 117.25 \tag{5.2.5}$$

That means that if you work 7 hours, you will make $\$117.25$. Think for a moment to see if that's reasonable: it's more, but not too much more, than you made working for 5 hours. So, it seems like a sensible answer.

The approach above works just fine to find the desired answer. However, what if you wanted to know how much you'd make working for 3 hours? Or 4 hours? Or 10 hours? You could just reproduce the work above each time, you wanted. But you may also find the following approach quicker:

Definition: Constant of Proportionality

In a situation involving directly proportional quantities, the **constant of proportionality** is the common ratio that describes the comparison of any two corresponding quantities. In other words, it is the constant rate of change between the two quantities.

Let's see how to find a constant of proportionality, and how to interpret it.

Example 5.2.1

You're in the same situation as the previous example: you work for 5 hours, and earn $\$83.75$. What is the constant of proportionality in this example, and what does it mean in context?

Solution

The constant of proportionality is the common ratio that describes the comparison of any two corresponding quantities. In this situation, we actually have two sets of corresponding quantities, one of which we found in the previous example. You know that you'll make $\$83.75$ working for 5 hours, and we calculated above that you'll make $\$117.25$ if you work for 7 hours. Let's look at these two ratios:

$$\frac{\$83.75}{5 \text{ hours}} \quad \text{and} \quad \frac{\$117.25}{7 \text{ hours}} \tag{5.2.6}$$

Both of these ratios are fractions, and we can simply divide the top by the bottom to reduce them to a single number. Using a calculator, we can see that

$$\frac{\$83.75}{5 \text{ hours}} = \$83.75 \div 5 \text{ hours} = \$16.75 \text{ per hour} \tag{5.2.7}$$

and

$$\frac{\$117.25}{7 \text{ hours}} = \$117.25 \div 7 \text{ hours} = \$16.75 \text{ per hour} \tag{5.2.8}$$

These are the same answer! Do you see why? We originally found our value for x in the previous example by setting the two ratios equal. So, they must give the same answer when divided.

This shared rate -- $\$16.75$ per hour -- is the *constant of proportionality* in this situation. It is the shared value of all ratios described by this problem, where the top of the ratio is money earned, and the bottom is hours worked.

What does this constant of proportionality mean in this situation? In this case, the constant of proportionality is your hourly pay rate. *In other words, it's how much you make per hour.*

A few more comments on the example above: now that you know this rate, it's quite simple to find how much money you'll make if you work $3, 4$, or 10 hours. You just multiply your hourly rate by the number of hours worked. For example, if you worked 4 hours, you could calculate:

$$\underset{\text{hours}}{4} \times \$16.75 \text{ per hour} = \$67.00 \tag{5.2.9}$$

This means you would make $67.00 working for 4 hours. Notice that if we reverse the process -- in other words, if we try to extract the constant of proportionality knowing that we make $67.00 in 4 hours, we get:

$$\frac{\$67.00}{4 \text{ hours}} = \$67.00 \div 4 \text{ hours} = \$16.75 \text{ per hour} \tag{5.2.10}$$

It's the same rate we found before. This is why it's called a *constant* of proportionality -- it stays the same, even as the corresponding quantities change.

Constants of proportionality will change in meaning depending on the context of the problem. For example, you might ask: If 5 people eat a total of 10 slices of pizza, how many slices would 7 people eat? In this case, the constant of proportionality could be found by:

$$\frac{10 \text{ slices}}{5 \text{ people}} = 2 \text{ slices per person} \tag{5.2.11}$$

In this case, a correct interpretation would be: "The constant of proportionality is 2 slices per person, which means that each person eats 2 slices of pizza." When asked for an interpretation, you should write a sentence similar to the previous -- your goal is the explain the meaning of the constant of proportionality in context of the situation. You will need to read carefully and use critical thinking to deduce a meaningful interpretation. As with many questions in this class, there are multiple good answers to these types of questions!

Problems that involve rates, ratios, scale models, etc. can be solved with proportions. When solving a real-world problem using a proportion, be consistent with the units.

5.2.2: Inverse Proportionality

Let's start this section with a question to illustrate the main concept. Before reading ahead, try to answer this question on your own:

Example 5.2.2

Suppose it takes 6 sanitation workers, all working simultaneously, 4 hours to pick up the trash and recycling in a given neighborhood. How many sanitation workers would it take to pick up the trash and recycling in the same neighborhood in 3 hours? (Assume that all sanitation workers work at the same rate, and can work independently.)

Did you try to answer the question yourself? What did you come up with? If you're like many students who carefully read the previous sections, you might have written this down:

$$\frac{6 \text{ sanitation workers}}{4 \text{ hours}} = \frac{x \text{ sanitation workers}}{3 \text{ hours}} \tag{5.2.12}$$

Then you'd apply Cross Multiplication to get:

$$4x = 18 \tag{5.2.13}$$

and then you'd use Division which undoes Multiplication to get

$$x = \frac{18}{4} = 4.5 \tag{5.2.14}$$

Now, the numerical answer 4.5 is a bit nonsensical, because it's talking about a number of people. So you could round up to 5, and say "It would take 5 sanitation workers to pick up the trash and recycling in 3 hours."

But wait a second: this does not make sense! Think about it: if it takes 6 workers 4 hours to accomplish this task, shouldn't it take 5 workers *more* time than 4 hours? After all, there is the same amount of work to be done, but fewer people to do it! So the answer "5 workers" cannot possibly be correct. We expect a number of workers that is *larger* than 6 to get the task done in a shorter amount of time.

We can learn two things from the previous discussion:

- It is important to evaluate whether or not an answer to a question makes sense in context by asking: What sort of answer would I expect to get? Does my answer seem reasonable?
- Not all problems can be solved using direct proportionality!

The good news is that this type of problem can be solved in a relatively simple way. We define the main concept in this section to see how these problems work.

Definition: Inversely Proportional

Two quantities are **inversely proportional** if, as one quantity increases, the other quantity decreases at the same rate.

Note how similar this definition is to the previous definition of direct proportionality. The only difference here is that one quantity increases while the other decreases:

Direct Proportionality	Inverse Proportionality
As one quantity *increases*	As one quantity *increases*
the other quantity *increases*	the other quantity *decreases*

In order to determine what type of problem you're working on, you'll need to think critically about the quantities involved, and use clues from your experience and the context of the problem to determine how the quantities are related. Things like the previous problem -- when a group of people are working together to accomplish a specific task -- are one of the primary examples of inverse proportionality. Let's see the same example again, and this time find the correct answer.

Example 5.2.2

Suppose it takes 6 sanitation workers, all working simultaneously, 4 hours to pick up the trash and recycling in a given neighborhood. How many sanitation workers would it take to pick up the trash and recycling in the same neighborhood in 3 hours? (Assume that all sanitation workers work at the same rate, and can work independently.)

Solution

The way to approach this is to find the number of *worker hours* needed to accomplish the task of picking up the trash and recycling in this neighborhood. A worker hour is defined to be an hour of work done by a worker, and that number will remain constant no matter the number of workers used.

To find the number of worker hours needed for this particular neighborhood, we simply multiply the known number of workers by the known number of hours:

$$6 \underset{\text{workers}}{} \times 4 \underset{\text{hours}}{} = 24 \underset{\text{worker hours}}{} \tag{5.2.15}$$

That means that it will require 24 *worker hours* to pick up the trash and recycling in this neighborhood.

To find the number of workers needed to pick up the trash and recycling in 3 hours, we divide the number of worker hours by the number of hours to find the number of workers:

$$\frac{24 \text{ worker hours}}{3 \text{ hours}} = 24 \underset{\text{worker hours}}{} \div 3 \underset{\text{hours}}{} = 8 \underset{\text{workers}}{} \tag{5.2.16}$$

This means it will take 8 workers 3 hours to pick up the trash and recycling. This makes sense -- it's larger than 6, which was the number of workers needed to accomplish the task in 4 hours.

All inverse proportionality problems work this way -- multiply the two known corresponding quantities, and then divide to find the answer. As always, label your units, and check to see if your answers make sense!

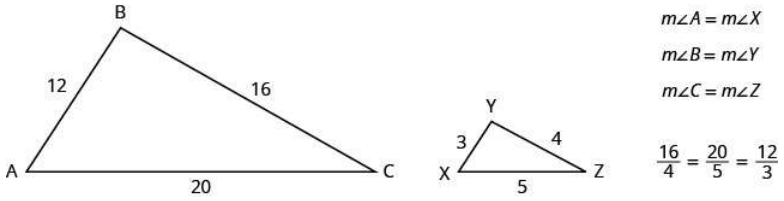
5.2.3: Solve Similar Figure Applications

When you shrink or enlarge a photo on a phone or tablet, figure out a distance on a map, or use a pattern to build a bookcase or sew a dress, you are working with **similar figures**. If two figures have exactly the same shape, but different sizes, they are said to be *similar.* One is a scale model of the other. All their corresponding angles have the same measures and their corresponding sides are in the same ratio.

Definition: SIMILAR FIGURES

Two figures are similar if the measures of their corresponding angles are equal and their corresponding sides are in the same ratio.

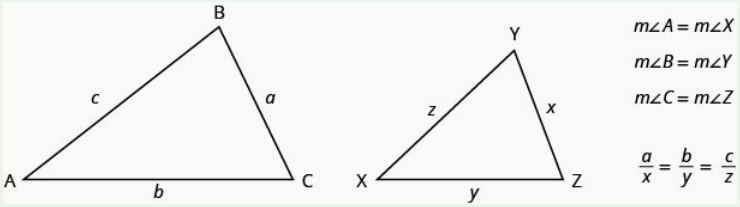
For example, the two triangles in Figure are similar. Each side of ΔABC is 4 times the length of the corresponding side of ΔXYZ.



This is summed up in the Property of Similar Triangles.

Definition: PROPERTY OF SIMILAR TRIANGLES

If ΔABC is similar to ΔXYZ



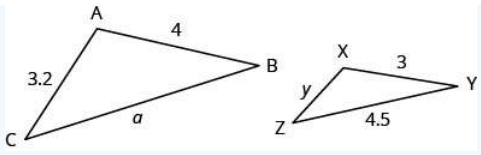
To solve applications with similar figures we will follow the Problem-Solving Strategy for Geometry Applications we used earlier.

Definition: SOLVE GEOMETRY APPLICATIONS.

1. **Read** the problem and make all the words and ideas are understood. Draw the figure and label it with the given information.
2. **Identify** what we are looking for.
3. **Name** what we are looking for by choosing a variable to represent it.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve the equation** using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example 5.2.8

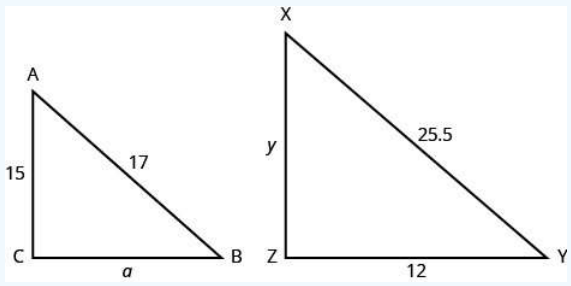
ΔABC is similar to ΔXYZ



Step 1. Read the problem. Draw the figure and label it with the given information.	Figure is given.
Step 2. Identify what we are looking for.	the length of the sides of similar triangles.
Step 3. Name the variables.	Let a= length of the third side of ΔABC. y= length of the third side of ΔXYZ.
Step 4. Translate.	Since the triangles are similar, the corresponding sides are proportional.
We need to write an equation that compares the side we are looking for to a known ratio. Since the side $AB = 4$ corresponds to the side $XY = 3$ we know $\frac{AB}{XY} = \frac{4}{3}$. So we write equations with $\frac{AB}{XY}$ to find the sides we are looking for. Be careful to match up corresponding sides correctly.	$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$. To find a: To find y: sides of large triangle → $\frac{AB}{XY} = \frac{BC}{YZ}$ $\frac{AB}{XY} = \frac{AC}{XZ}$ sides of small triangle →
Substitute.	$\frac{4}{3} = \frac{a}{4.5}$ $\frac{4}{3} = \frac{3.2}{y}$
Step 5. Solve the equation.	$3a = 4(4.5)$ Figure_08_07_030f_img_new.jpg" />
	$a = 6$ $y = 2.4$
Step 6. Check.	
Step 7. Answer the question.	The third side of ΔABC is 6 and the third side of ΔXYZ is 2.4.

TRy it 5.2.15

ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure.



Find the length of side a

Answer

8

TRy it 5.2.16

ΔABC is similar to ΔXYZ. The lengths of two sides of each triangle are given in the figure.

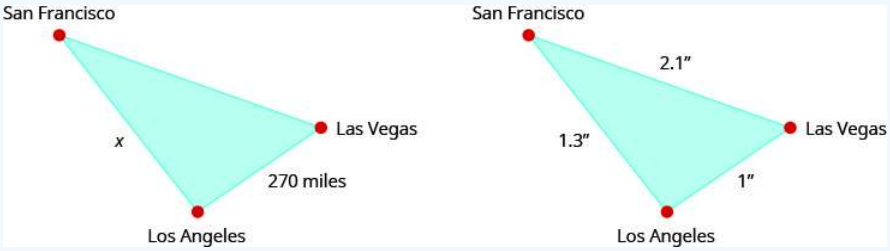
Answer

22.5

The next example shows how similar triangles are used with maps.

Example 5.2.9

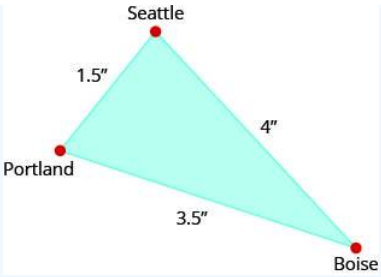
On a map, San Francisco, Las Vegas, and Los Angeles form a triangle whose sides are shown in the figure below. If the actual distance from Los Angeles to Las Vegas is 270 miles find the distance from Los Angeles to San Francisco.



Read the problem. Draw the figures and label with the given information.	The figures are shown above.
Identify what we are looking for.	The actual distance from Los Angeles to San Francisco.
Name the variables.	Let x= distance from Los Angeles to San Francisco.
Translate into an equation. Since the triangles are similar, the corresponding sides are proportional. We'll make the numerators "miles" and the denominators "inches."	$\frac{x \text{ miles}}{1.3 \text{ inches}} = \frac{270 \text{ miles}}{1 \text{ inch}}$
	$1.3\left(\frac{x}{1.3}\right) = 1.3\left(\frac{270}{1}\right)$
Solve the equation.	$x = 351$
Check.	
On the map, the distance from Los Angeles to San Francisco is more than the distance from Los Angeles to Las Vegas. Since 351 is more than 270 the answer makes sense.	
Check $x = 351$ in the original proportion. Use a calculator. $\frac{x \text{ miles}}{1.3 \text{ inches}} = \frac{270 \text{ miles}}{1 \text{ inch}}$ $\frac{351 \text{ miles}}{1.3 \text{ inches}} \stackrel{?}{=} \frac{270 \text{ miles}}{1 \text{ inch}}$ $\frac{270 \text{ miles}}{1 \text{ inch}} = \frac{270 \text{ miles}}{1 \text{ inch}}$ ✓	
Answer the question.	The distance from Los Angeles to San Francisco is 351 miles.

TRY IT 5.2.17

On the map, Seattle, Portland, and Boise form a triangle whose sides are shown in the figure below. If the actual distance from Seattle to Boise is 400 miles, find the distance from Seattle to Portland.



Answer

150 miles

? Try it 5.2.18

Using the map above, find the distance from Portland to Boise.

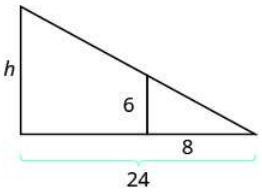
Answer

350 miles

We can use **similar figures** to find heights that we cannot directly measure.

? Example 5.2.10

Tyler is 6 feet tall. Late one afternoon, his shadow was 8 feet long. At the same time, the shadow of a tree was 24 feet long. Find the height of the tree.

Read the problem and draw a figure.	
We are looking for h, the height of the tree.	
We will use similar triangles to write an equation.	
The small triangle is similar to the large triangle.	$\frac{h}{24}=\frac{6}{8}$
Solve the proportion.	$24\left(\frac{6}{8}\right)=24\left(\frac{h}{24}\right)$
Simplify.	$18=h$
Check.	
Tyler's height is less than his shadow's length so it makes sense that the tree's height is less than the length of its shadow.	
Check $h=18$ in the original proportion. $\frac{6}{8}=\frac{h}{24}$ $\frac{6}{8}\stackrel{?}{=}\frac{18}{24}$ $\frac{3}{4}=\frac{3}{4}$ ✓	

? Try it 5.2.19

A telephone pole casts a shadow that is 50 feet long. Nearby, an 8 foot tall traffic sign casts a shadow that is 10 feet long. How tall is the telephone pole?

Answer

40 feet

? Try it 5.2.20

A pine tree casts a shadow of 80 feet next to a 30-foot tall building which casts a 40 feet shadow. How tall is the pine tree?

Answer

60 feet

5.2.4: Exercises

Make sure that when you are asked to *interpret* something, you write a complete sentence describing the meaning of your numerical answer in the context of the problem.

1. Your car uses 10 gallons of gas to go 300 miles.
   - How many gallons of gas will you need to go 400 miles?
   - Is this situation described by direct or inverse proportionality, and why? Give a one-sentence answer.
   - What is the constant of proportionality in this situation, and how would you interpret it?
2. It takes 2 math professors a total of 6 hours to grade the exams for a large math class. Assume all professor grade at the same rate.
   - How many professors would it take to grade the same exams in 4 hours?
   - Is this situation described by direct or inverse proportionality, and why? Give a one-sentence answer.
3. A family drinks 2 gallons of milk every 9 days. How many gallons of milk will they use in 2 weeks? Be careful with units here! (Round to one decimal place.)
4. At a rate of 30 miles per hour, a certain trip takes 2 hours. How long would the same trip take at 40 miles per hour? (Round to one decimal place or give a fractional answer.)
5. Think of a real-world example of direct proportionality that is different than ones we've covered in this section. Give a 2-3 sentence description of the quantities involved, and why you think they are directly proportional. If you use a source, please cite it by providing a URL.
6. Think of a real-world example of inverse proportionality that is different than ones we've covered in this section. Give a 2-3 sentence description of the quantities involved, and why you think they are inversely proportional. If you use a source, please cite it by providing a URL.
7. Tonisha drove her car 320 miles and used 12.5 gallons of gas. At this rate, how far could she drive using 10 gallons of gas?
8. Marcus worked 14 hours and earned $\$ 210$. At the same rate of pay, how long would he have to work to earn $\$ 300$?
9. A picture of your author appearing on *Jeopardy!* that is 375 pixels high and 475 pixels wide needs to be reduced in size so that it is 150 pixels high. If the height and width are kept proportional, what is the width of the picture after it has been reduced?





10. At a fast-food restaurant, a 22-ounce chocolate shake has 850 calories. How many calories are in their 12-ounce chocolate shake? Round your answer to nearest whole number.
11. Yurianna is going to Europe and wants to change $800 dollars into Euros. At the current exchange rate, $1 US is equal to 0.738 Euro. How many Euros will she have for her trip? **Answer**

7. **256** miles
8. **20** hours
9. **190** pixels wide
10. 464 calories
11. 590.4 Euros

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5.3: Use Direct and Inverse Variation

Learning Objectives

By the end of this section, you will be able to:

- Solve direct variation problems
- Solve inverse variation problems

Note

Before you get started, take this readiness quiz.

If you miss a problem, go back to the section listed and review the material.

1. Find the multiplicative inverse of −8
   If you missed this problem, review Exercise 1.10.13.
2. Solve for n: 45=20n
   If you missed this problem, review Exercise 2.2.1.
3. Evaluate $5x^2$ when x=10
   If you missed this problem, review Exercise 1.3.25.

When two quantities are related by a proportion, we say they are *proportional* to each other. Another way to express this relation is to talk about the *variation* of the two quantities. We will discuss direct variation and inverse variation in this section.

Solve Direct Variation Problems

Lindsay gets paid $15 per hour at her job. If we let *s* be her salary and *h* be the number of hours she has worked, we could model this situation with the equation

s=15h

Lindsay's salary is the product of a constant, 15, and the number of hours she works. We say that Lindsay's salary *varies directly* with the number of hours she works. Two variables vary directly if one is the product of a constant and the other.

Definition: DIRECT VARIATION

For any two variables *x* and *y*, *y* varies directly with *x* if

y=kx, where $n \neq 0$

In applications using direct variation, generally we will know values of one pair of the variables and will be asked to find the equation that relates *x* and *y*. Then we can use that equation to find values of *y* for other values of *x*.

How to Solve Direct Variation Problems

Example 5.3.1

If y varies directly with x and y=20 when x=8, find the equation that relates *x* and *y*.

Answer

Step 1. Write the formula for direct variation.	The direct variation formula is $y = kx$.	$y = kx$	**Step 2.** Substitute the given values for the variables.	We are given $y = 20$, $x = 8$.	$20 = k \cdot 8$
Step 3. Solve for the constant of variation.	Divide both sides of the equation by 8, then simplify.	$\frac{20}{8} = k$ $k = 2.5$	**Step 4.** Write the equation that relates *x* and *y*.	Rewrite the general equation with the value we found for *k*.	$y = 2.5x$

Example 5.3.2

If y varies directly as x and y=3, when x=10, find the equation that relates *x* and *y*.

Answer

$y = \frac{3}{10}x$

Example 5.3.3

If y varies directly as x and y=12 when x=4, find the equation that relates *x* and *y*.

Answer

y=3x

We'll list the steps below.

Definition: SOLVE DIRECT VARIATION PROBLEMS

1. Write the formula for direct variation.
2. Substitute the given values for the variables.
3. Solve for the constant of variation.
4. Write the equation that relates x and y.

Now we'll solve a few applications of direct variation.

Example 5.3.4

When Raoul runs on the treadmill at the gym, the number of calories, c, he burns varies directly with the number of minutes, m, he uses the treadmill. He burned 315 calories when he used the treadmill for 18 minutes.

1. Write the equation that relates c and m.
2. How many calories would he burn if he ran on the treadmill for 25 minutes?

Answer

1.

	The number of calories, c varies directly with the number of minutes, m, on the treadmill, and c=315 when m=18
Write the formula for direct variation.	$y = kx$
We will use c in place of y and mm in place of x.	$c = km$
Substitute the given values for the variables.	$315 = k \cdot 18$
Solve for the constant of variation.	$\frac{315}{18} = \frac{k \cdot 18}{18}$
	$17.5 = k$
Write the equation that relates c and m.	$c = km$
Substitute in the constant of variation.	$c = 17.5m$

2.

	Find c when m=25.
Write the equation that relates c and m.	$c = 17.5m$
Substitute the given value for m.	$c = 17.5(25)$
Simplify.	$c = 437.5$
	Raoul would burn 437.5 calories if he used the treadmill for 25 minutes.

Example 5.3.5

The number of calories, c, burned varies directly with the amount of time, t, spent exercising. Arnold burned 312 calories in 65 minutes exercising.

1. Write the equation that relates c and t.
2. How many calories would he burn if he exercises for 90 minutes?

Answer

1. c=4.8t
2. 432 calories

Example 5.3.6

The distance a moving body travels, d, varies directly with time, t, it moves. A train travels 100 miles in 2 hours

1. Write the equation that relates d and t.
2. How many miles would it travel in 5 hours?

Answer

1. d=50t
2. 250 miles

In the previous example, the variables c and m were named in the problem. Usually that is not the case. We will have to name the variables in the next example as part of the solution, just like we do in most applied problems.

Example 5.3.7

The number of gallons of gas Eunice's car uses varies directly with the number of miles she drives. Last week she drove 469.8 miles and used 14.5 gallons of gas.

1. Write the equation that relates the number of gallons of gas used to the number of miles driven.
2. How many gallons of gas would Eunice's car use if she drove 1000 miles?

Answer

1.

	The number of gallons of gas varies directly with the number of miles driven.
First we will name the variables.	Let g= number of gallons of gas. m= number of miles driven.
Write the formula for direct variation.	$y = kx$
We will use g in place of y and m in place of x.	$g = km$
Substitute the given values for the variables	$g = 14.5$ when $m = 469.8$
	$14.5 = k(469.8)$
Solve for the constant of variation.	$\frac{14.5}{469.8} = \frac{k(469.8)}{469.8}$
We will round to the nearest thousandth.	$0.031 = k$
Write the equation that relates g and m.	$g = km$
Substitute in the constant of variation.	$g = 0.031m$

2.

	Find g when m=1000.
Write the equation that relates g and m.	g=0.031m
Substitute the given value for m.	g=0.031(1000)
Simplify.	g=31
	Eunice's car would use 31 gallons of gas if she drove it 1,000 miles.

Notice that in this example, the units on the constant of variation are gallons/mile. In everyday life, we usually talk about miles/gallon.

Example 5.3.8

The distance that Brad travels varies directly with the time spent traveling. Brad travelled 660 miles in 12 hours,

1. Write the equation that relates the number of miles travelled to the time.
2. How many miles could Brad travel in 4 hours?

Answer

1. m=55h
2. 220 miles

Example 5.3.9

The weight of a liquid varies directly as its volume. A liquid that weighs 24 pounds has a volume of 4 gallons.

1. Write the equation that relates the weight to the volume.
2. If a liquid has volume 13 gallons, what is its weight?

Answer

1. w=6v
2. 78 pounds

In some situations, one variable varies directly with the square of the other variable. When that happens, the equation of direct variation is $y = kx^2$.

Example 5.3.10

The maximum load a beam will support varies directly with the square of the diagonal of the beam's cross-section. A beam with diagonal 4" will support a maximum load of 75 pounds.

1. Write the equation that relates the maximum load to the cross-section.
2. What is the maximum load that can be supported by a beam with diagonal 8"?

Answer

1.

	The maximum load varies directly with the square of the diagonal of the cross-section.
Name the variables.	Let L= maximum load. c= the diagonal of the cross-section
Write the formula for direct variation, where y varies directly with the square of x.	$y = kx^2$
We will use L in place of y and c in place of x.	$L = kc^2$
Substitute the given values for the variables.	$L = 75$ when $c = 4$
	$75 = k \cdot 4^2$
Solve for the constant of variation.	$\frac{75}{16} = \frac{k \cdot 16}{16}$
	$4.6875 = k$
Write the equation that relates L and c.	$L = kc^2$
Substitute in the constant of variation.	$L = 4.6875c^2$

2.

	Find L when c=8.
Write the equation that relates L and c.	$L = 4.6875c^2$
Substitute the given value for c.	$L = 4.6875(8)^2$
Simplify.	L=300
	A beam with diagonal 8” could supporta maximum load of 300 pounds.

Example 5.3.11

The distance an object falls is directly proportional to the square of the time it falls. A ball falls 144 feet in 3 seconds.

1. Write the equation that relates the distance to the time.
2. How far will an object fall in 4 seconds?

Answer

1. $d = 16t^2$
2. 256 feet

Example 5.3.12

The area of a circle varies directly as the square of the radius. A circular pizza with a radius of 6 inches has an area of 113.04 square inches.

1. Write the equation that relates the area to the radius.
2. What is the area of a pizza with a radius of 9 inches?

Answer

1. $A = 3.14r^2$
2. 254.34 square inches

Solve Inverse Variation Problems

Many applications involve two variable that *vary inversely*. As one variable increases, the other decreases. The equation that relates them is $y = \frac{k}{x}$.

Definition: INVERSE VARIATION

For any two variables x and y, y varies inversely with x if

$y = \frac{k}{x}$, where $k \neq 0$

The word ‘inverse’ in inverse variation refers to the multiplicative inverse. The multiplicative inverse of x is $\frac{1}{x}$.

We solve inverse variation problems in the same way we solved direct variation problems. Only the general form of the equation has changed. We will copy the procedure box here and just change ‘direct’ to ‘inverse’.

Definition: SOLVE INVERSE VARIATION PROBLEMS

1. Write the formula for inverse variation.
2. Substitute the given values for the variables.
3. Solve for the constant of variation.
4. Write the equation that relates x and y.

Example 5.3.13

If y varies inversely with x and y=20 when x=8 x and y.

Answer

Write the formula for inverse variation.	$y = \frac{k}{x}$
Substitute the given values for the variables.	$y = 20$ when $x = 8$
	$20 = \frac{k}{8}$
Solve for the constant of variation.	$8(20) = 8\left(\frac{k}{8}\right)$
	$160 = k$
Write the equation that relates x and y.	$y = \frac{k}{x}$
Substitute in the constant of variation.	$y = \frac{160}{x}$

Example 5.3.14

If p varies inversely with q and p=30 when q=12 find the equation that relates p and q.

Answer

$p = \frac{360}{q}$

Example 5.3.15

If y varies inversely with x and y=8 when x=2 find the equation that relates x and y.

Answer

$y = \frac{16}{x}$

Example 5.3.16

The fuel consumption (mpg) of a car varies inversely with its weight. A car that weighs 3100 pounds gets 26 mpg on the highway.

1. Write the equation of variation.
2. What would be the fuel consumption of a car that weighs 4030 pounds?

Answer

1.

	The fuel consumption varies inversely with the weight.
First we will name the variables.	Let f= fuel consumption. w= weight.
Write the formula for inverse variation.	$y = \frac{k}{x}$
We will use f in place of y and w in place of x.	$f = \frac{k}{w}$
Substitute the given values for the variables.	$f = 26$ when $w = 3100$
	$26 = \frac{k}{3100}$
Solve for the constant of variation.	$3100(26) = 3100\left(\frac{k}{3100}\right)$
	$80{,}600 = k$
Write the equation that relates f and w.	$f = \frac{k}{w}$
Substitute in the constant of variation.	$f = \frac{80{,}600}{w}$

2.

	Find f when w=4030.
Write the equation that relates f and w.	$f = \frac{80{,}600}{w}$
Substitute the given value for w.	$f = \frac{80{,}600}{4030}$
Simplify.	f=20
	A car that weighs 4030 pounds would have fuel consumption of 20 mpg.

Example 5.3.17

A car's value varies inversely with its age. Elena bought a two-year-old car for $20,000.

1. Write the equation of variation.
2. What will be the value of Elena's car when it is 5 years old?

Answer

1. $v = \frac{40,000}{a}$
2. $8,000

Example 5.3.18

The time required to empty a pool varies inversely as the rate of pumping. It took Lucy 2.5 hours to empty her pool using a pump that was rated at 400 gpm (gallons per minute).

1. Write the equation of variation.
2. How long will it take her to empty the pool using a pump rated at 500 gpm?

Answer

1. $t = \frac{1000}{r}$
2. 2 hours

Example 5.3.19

The frequency of a guitar string varies inversely with its length. A 26" long string has a frequency of 440 vibrations per second.

1. Write the equation of variation.
2. How many vibrations per second will there be if the string's length is reduced to 20" by putting a finger on a fret?

Answer

1.

	The frequency varies inversely with the length.
Name the variables.	Let f= frequency. L=length.
Write the formula for inverse variation.	$y = \frac{k}{x}$
We will use f in place of y and L in place of x	$f = \frac{k}{L}$
Substitute the given values for the variables.	$f = 440$ when $L = 26$
	$440 = \frac{k}{26}$
Solve for the constant of variation.	$26(440) = 26\left(\frac{k}{26}\right)$
	$11,440 = k$
Write the equation that relates f and L.	$f = \frac{k}{L}$
Substitute in the constant of variation.	$f = \frac{11,440}{L}$

2.

	Find f when L=20.
Write the equation that relates f and L.	$f = \frac{11,440}{L}$
Substitute the given value for L.	$f = \frac{11,440}{20}$
Simplify.	f=572
	A 20" guitar string has frequency 572 vibrations per second.

Example 5.3.20

The number of hours it takes for ice to melt varies inversely with the air temperature. Suppose a block of ice melts in 2 hours when the temperature is 65 degrees.

1. Write the equation of variation.
2. How many hours would it take for the same block of ice to melt if the temperature was 78 degrees?

Answer

1. $h = \frac{130}{t}$
2. $1\frac{2}{3}$ hours

? Example 5.3.21

The force needed to break a board varies inversely with its length. Richard uses 24 pounds of pressure to break a 2-foot long board.

1. Write the equation of variation.
2. How many pounds of pressure is needed to break a 5-foot long board?

Answer

1. $F = \frac{48}{l}$
2. 9.6 pounds

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CHAPTER OVERVIEW

6: Graphing Linear Equations

6.1: The Rectangular Coordinate System
6.2: Relations and Functions
6.3: Graph Linear Equations in Two Variables
6.4: Graph with Intercepts
6.5: Understanding the Slope of a Line
6.6: Slope–Intercept Form of an Equation of a Line
6.7: Find the Equation of a Line

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6.1: The Rectangular Coordinate System

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the 'coordinate plane'.

The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis*. The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See Figure **6.1.1**.

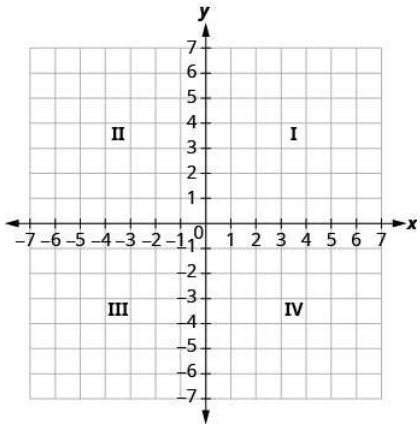


Figure **6.1.1**: 'Quadrant' has the root 'quad,' which means 'four.'

In the **rectangular coordinate system**, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

ORDERED PAIR

An **ordered pair**, (x,y)(x,y), gives the coordinates of a point in a rectangular coordinate system.

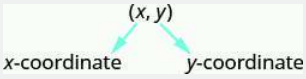


Figure **6.1.2**

The first number is the *x*-coordinate.

The second number is the *y*-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0,0)$. The point $(0,0)$ has a special name. It is called the **origin**.

THE ORIGIN

The point $(0,0)$ is called the **origin**. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1,3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through x=1x=1. Then, locate 3 on the *y*-axis and sketch a horizontal line through y=3y=3. Now, find the point where these two lines meet—that is the point with coordinates $(1,3)$.

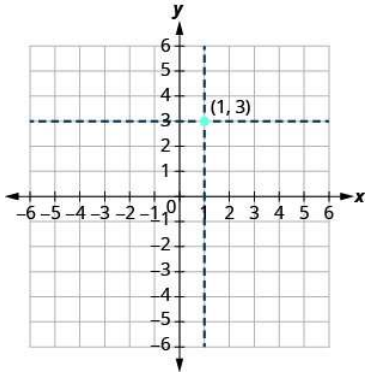


Figure **6.1.3**

Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1,3)$.

Exercise 6.1.1

Plot each point in the rectangular coordinate system and identify the **quadrant** in which the point is located:

1. (−5,4)
2. (−3,−4)
3. (2,−3)
4. (−2,3)
5. $(3, \frac{5}{2})$

Answer

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

1. Since x=−5, the point is to the left of the *y*-axis. Also, since y=4, the point is above the *x*-axis. The point (−5,4) is in Quadrant II.
2. Since x=−3, the point is to the left of the *y*-axis. Also, since y=−4, the point is below the *x*-axis. The point (−3,−4) is in Quadrant III.
3. Since x=2, the point is to the right of the *y*-axis. Since y=−3, the point is below the *x*-axis. The point (2,−3) is in Quadrant lV.
4. Since x=−2, the point is to the left of the *y*-axis. Since y=3, the point is above the *x*-axis. The point (−2,3) is in Quadrant II.
5. Since x=3, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $(3, \frac{5}{2})$ is in Quadrant I.

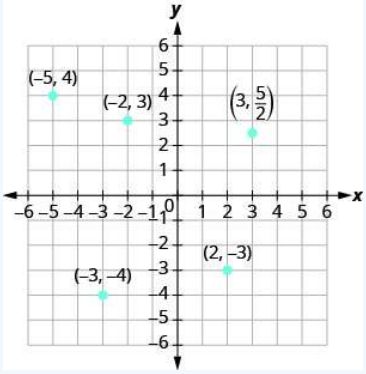


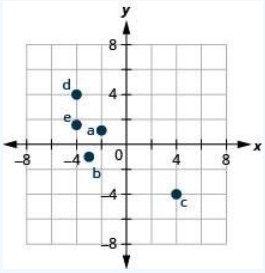
Figure 6.1.4

Exercise 6.1.2

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

1. (−2,1)
2. (−3,−1)
3. (4,−4)
4. (−4,4)
5. $(-4, \frac{3}{2})$

Answer

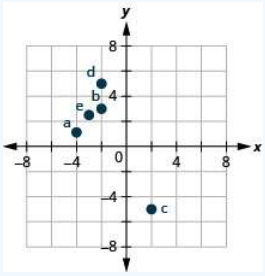


Exercise 6.1.3

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located:

1. (−4,1)
2. (−2,3)
3. (2,−5)
4. (−2,5)
5. $(-3, \frac{5}{2})$

Answer



How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

For the point in Figure **6.1.4** in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point (−2,5) is located? In which quadrant is (2,−5) located?

QUADRANTS

We can summarize sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV	(6.1.1)
(x, y)	(x, y)	(x, y)	(x, y)	
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$	

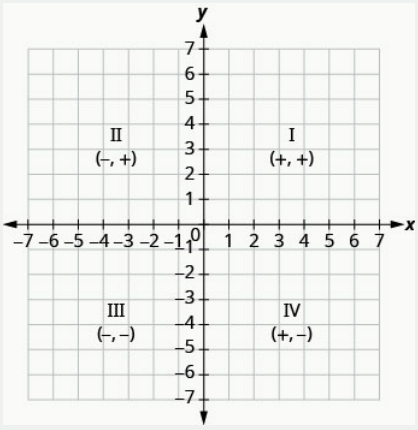


Figure **6.1.5**

What if one coordinate is zero as shown in Figure **6.1.6**? Where is the point (0,4) located? Where is the point (−2,0) located?

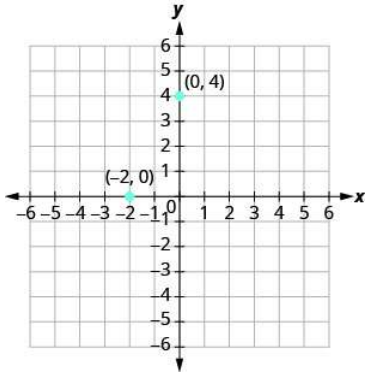


Figure **6.1.6**

The point (0,4) is on the *y*-axis and the point (−2,0) is on the *x*-axis.

POINTS ON THE AXES

Points with a ***y*-coordinate** equal to 0 are on the *x*-axis, and have coordinates (a,0).

Points with an ***x*-coordinate** equal to 0 are on the *y*-axis, and have coordinates (0,b).

Exercise 6.1.4

Plot each point:

1. (0,5)
2. (4,0)
3. (−3,0)
4. (0,0)
5. (0,−1)

Answer

1. Since x=0, the point whose coordinates are (0,5) is on the *y*-axis.
2. Since y=0, the point whose coordinates are (4,0) is on the *x*-axis.
3. Since y=0, the point whose coordinates are (−3,0) is on the *x*-axis.
4. Since x=0 and y=0, the point whose coordinates are (0,0) is the origin.
5. Since x=0, the point whose coordinates are (0,−1) is on the *y*-axis.

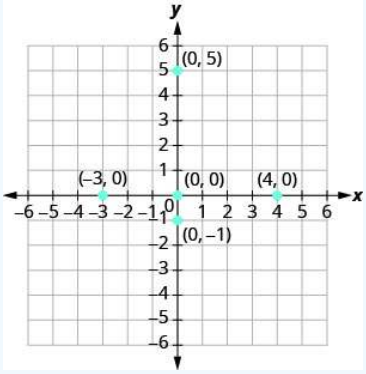


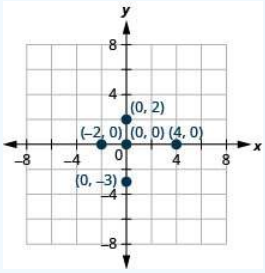
Figure **6.1.7**

Exercise 6.1.5

Plot each point:

1. (4,0)
2. (−2,0)
3. (0,0)
4. (0,2)
5. (0,−3).

Answer

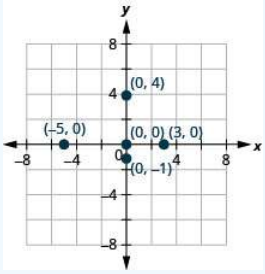


Exercise 6.1.6

Plot each point:

1. (−5,0)
2. (3,0)
3. (0,0)
4. (0,−1)
5. (0,4).

Answer



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the **ordered pair** use the correct order, (x,y).

Exercise 6.1.7

Name the ordered pair of each point shown in the rectangular coordinate system.

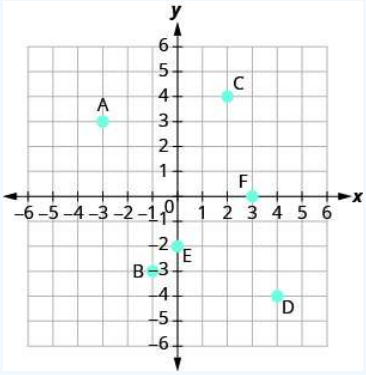


Figure **6.1.8**

Answer

Point A is above −3 on the *x*-axis, so the *x*-coordinate of the point is −3.

The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.

The coordinates of the point are (−3,3).

Point B is below −1 on the *x*-axis, so the *x*-coordinate of the point is −1.

The point is to the left of −3 on the *y*-axis, so the *y*-coordinate of the point is −3.

The coordinates of the point are (−1,−3).

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2.

The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.

The coordinates of the point are (2,4).

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4.

The point is to the right of −4 on the *y*-axis, so the *y*-coordinate of the point is −4.

The coordinates of the point are (4,−4).

Point E is on the *y*-axis at y=−2. The coordinates of point E are (0,−2).

Point F is on the *x*-axis at x=3. The coordinates of point F are (3,0).

Exercise 6.1.8

Name the ordered pair of each point shown in the rectangular coordinate system.

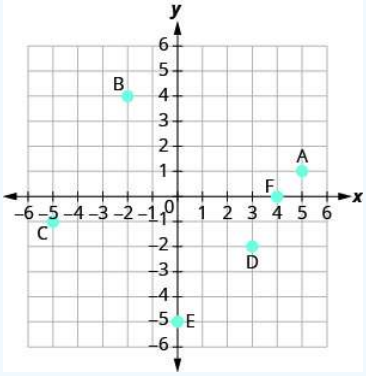


Figure **6.1.9**

Answer

A: (5,1) B: (−2,4) C: (−5,−1) D: (3,−2) E: (0,−5) F: (4,0)

Exercise 6.1.9

Name the ordered pair of each point shown in the rectangular coordinate system.

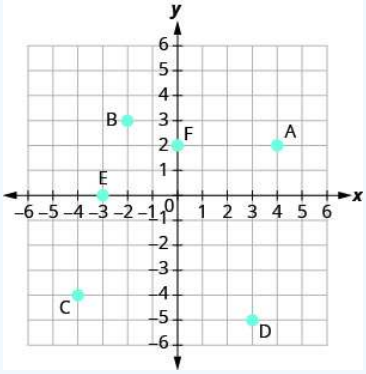


Figure **6.1.10**

Answer

A: (4,2) B: (−2,3) C: (−4,−4) D: (3,−5) E: (−3,0) F: (0,2)

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like x=4. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x+5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned} \tag{6.1.2}$$

But equations can have more than one variable. Equations with two variables may be of the form Ax+By=C. Equations of this form are called **linear equations in two variables**.

LINEAR EQUATION

An equation of the form Ax+By=C, where A and B are not both zero, is called a **linear equation in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$Ax+By=C$$
$$x+4y=8$$
$$A=1,\ B=4,\ C=8$$

Figure **6.1.11**

The equation y=−3x+5 is also a **linear equation**. But it does not appear to be in the form Ax+By=C. We can use the Addition Property of Equality and rewrite it in Ax+By=C form.

	y	$=$	$-3x+5$
Add to both sides.	$y+3x$	$=$	$-3x+5+3x$
Simplify.	$y+3x$	$=$	5
Use the Commutative Property to put it in $Ax+By=C$ form.	$3x+y$	$=$	5

By rewriting y=−3x+5 as 3x+y=5, we can easily see that it is a linear equation in two variables because it is of the form Ax+By=C. When an equation is in the form Ax+By=C, we say it is in *standard form*.

STANDARD FORM OF LINEAR EQUATION

A linear equation is in standard form when it is written Ax+By=C.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x,y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution OF A LINEAR EQUATION IN TWO VARIABLES

An **ordered pair** (x,y) is a **solution** of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

? Exercise 6.1.10

Determine which ordered pairs are solutions to the equation x+4y=8.

(a) (0,2)

(b) (2,−4)

(c) (−4,3)

Answer

Substitute the x- and y-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \stackrel{?}{=} 8$	$2 + 4(-4) \stackrel{?}{=} 8$	$-4 + 4 \cdot 3 \stackrel{?}{=} 8$
$0 + 8 \stackrel{?}{=} 8$	$2 + (-16) \stackrel{?}{=} 8$	$-4 + 12 \stackrel{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

Exercise 6.1.11

Which of the following ordered pairs are solutions to 2x+3y=6?

1. (3,0)
2. (2,0)
3. (6,−2)

Answer

1, 3

Exercise 6.1.12

Which of the following ordered pairs are solutions to the equation 4x−y=8?

1. (0,8)
2. (2,0)
3. (1,−4)

Answer

2, 3

Exercise 6.1.13

Which of the following ordered pairs are solutions to the equation y=5x−1?

(a) (0,−1)

(b) (1,4)

(c) (−2,−7)

Answer

Substitute the x- and y-values from each **ordered pair** into the equation and determine if it results in a true statement.

(a)	(b)	(c)
(0, –1)	(1, 4)	(–2, –7)
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
(0, –1) is a solution.	(1, 4) is a solution.	(–2, –7) is not a solution.

Exercise 6.1.14

Which of the following ordered pairs are solutions to the equation $y=4x-3$?

1. (0,3)
2. (1,1)
3. $(-1,-1)$

Answer

2

Exercise 6.1.15

Which of the following ordered pairs are solutions to the equation $y=-2x+6$?

1. (0,6)
2. (1,4)
3. $(-2,-2)$

Answer

1, 2

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*- and *y*-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for xx and then solve the equation for yy. Or, pick a value for yy and then solve for xx.

We'll start by looking at the solutions to the equation $y = 5x-1$ that we found in Exercise **6.1.13**. We can summarize this information in a table of solutions, as shown in Table **6.1.1**.

Table **6.1.1**

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)

To find a third solution, we'll let $x=2$ and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

Figure **6.1.12**

The ordered pair (2,9) is a solution to $y=5x-1$. We will add it to Table **6.1.2**.

Table 6.1.2

y=5x−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	4	(1,4)
2	9	(2,9)

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

Exercise 6.1.16

Complete Table to find three solutions to the equation y=4x−2.

Table 6.1.3

y=4x−2		
x	**y**	**(x,y)**
0		
−1		
2		

Answer

Substitute x=0, x=−1, and x=2 into y=4x−2.

$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = 0 - 2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in Table 6.1.4.

Table 6.1.4

y=4x−2		
x	**y**	**(x,y)**
0	−2	(0,−2)
−1	−6	(−1,−6)
2	6	(2,6)

Exercise 6.1.17

Complete the table to find three solutions to this equation: y=3x−1.

Table 6.1.5

y=3x−1		
x	**y**	**(x,y)**
0		
−1		
2		

Answer

Table 6.1.6

y=3x−1		
x	**y**	**(x,y)**
0	-1	(0, -1)
−1	-4	(-1, -4)
2	5	(2, 5)

Exercise 6.1.18

Complete the table to find three solutions to this equation: y=6x+1.

Table 6.1.7

y=6x+1		
x	**y**	**(x,y)**
-2		

Answer

Table 6.1.8

y=6x+1		
x	**y**	**(x,y)**
0	1	(0,1)
1	7	(1,7)
−2	−11	(−2,−11)

Exercise 6.1.19

Complete Table 6.1.9 to find three solutions to the equation 5x−4y=20.

Table **6.1.9**

5x−4y=20		
x	**y**	**(x,y)**
	0	
	5	

Answer

Substitute the given value into the equation 5x−4y=20 and solve for the other variable. Then, fill in the values in the table.

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in Table **6.1.10**.

Table **6.1.10**

5x−4y=20		
x	**y**	**(x,y)**
0	−5	(0,−5)
4	0	(4,0)
8	5	(8,5)

? Exercise 6.1.20

Complete the table to find three solutions to this equation: 2x−5y=20.

Table **6.1.11**

2x−5y=20		
x	**y**	**(x,y)**
-5		

Answer

Table 6.1.12

2x−5y=20		
x	**y**	**(x,y)**
0	−4	(0,−4)
10	0	(10,0)
−5	−6	(−5,−6)

Exercise 6.1.21

Complete the table to find three solutions to this equation: 3x−4y=12.

Table 6.1.13

3x−4y=12		
x	**y**	**(x,y)**
-4		

Answer

Table 6.1.14

3x−4y=12		
x	**y**	**(x,y)**
0	−3	(0,−3)
4	0	(4,0)
−4	−6	(−4,−6)

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

Exercise 6.1.22

Find three solutions to the equation y=−3x+2.

Answer

We can substitute any value we want for x or any value for y. Since the equation is in *y*-form, it will be easier to substitute

in values of x. Let's pick x=0, x=1, and x=−1.

	$x=0$	$x=1$	$x=-1$
	$y=-3x+2$	$y=-3x+2$	$y=-3x+2$
Substitute the value into the equation.	$y=-3\cdot 0+2$	$y=-3\cdot 1+2$	$y=-3(-1)+2$
Simplify.	$y=0+2$	$y=-3+2$	$y=3+2$
Simplify.	$y=2$	$y=-1$	$y=5$
Write the ordered pair.	(0, 2)	(1, -1)	(-1, 5)
Check.			
y=−3x+2	y=−3x+2	y=−3x+2	
$2\stackrel{?}{=}-3\cdot 0+2$	$-1\stackrel{?}{=}-3\cdot 1+2$	$5\stackrel{?}{=}-3(-1)+2$	
$2\stackrel{?}{=}0+2$	$-1\stackrel{?}{=}-3+2$	$5\stackrel{?}{=}-3+2$	
$2=2\checkmark$	$-1=-1\checkmark$	$5=5\checkmark$	

Table **6.1.15**

So, (0,2), (1,−1) and (−1,5) are all solutions to y=−3x+2. We show them in Table **6.1.16**.

Table **6.1.16**

y=−3x+2		
x	**y**	**(x,y)**
0	2	(0,2)
1	−1	(1,−1)
−1	5	(−1,5)

? Exercise 6.1.23

Find three solutions to this equation: y=−2x+3.

Answer

Answers will vary.

? Exercise 6.1.24

Find three solutions to this equation: y=−4x+1.

Answer

Answers will vary.

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when x=0 find a second solution when y=0, and then find a third solution.

Exercise 6.1.25

Find three solutions to the equation 3x+2y=6.

Answer

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first x=0, then y=0, and then find a third point.

Table **6.1.17**

$x=0$	$y=0$	$x=1$			
			$3x+2y=6$	$3x+2y=6$	$3x+2y=6$
Substitute the value into the equation.			$3(0)+2y=6$	$3x+2(0)=6$	$3(1)+2y=6$
Simplify.			$0+2y=6$	$3x+0=6$	$3+2y=6$
Solve.			$2y=6$	$3x=6$	$2y=3$
			$y=3$	$x=2$	$y=\frac{3}{2}$
Write the ordered pair.			(0, 3)	(2, 0)	$(1, \frac{3}{2})$
Check.					
3x+2y=6	3x+2y=6	3x+2y=6			
$3\cdot 0+2\cdot 3\stackrel{?}{=}6$	$3\cdot 2+2\cdot 0\stackrel{?}{=}6$	$3\cdot 1+2\cdot \frac{3}{2}\stackrel{?}{=}6$			
$0+6\stackrel{?}{=}6$	$6+0\stackrel{?}{=}6$	$3+3\stackrel{?}{=}6$			
$6=6$✓	$6=6$✓	$6=6$✓			

So (0,3), (2,0), and $(1, \frac{3}{2})$ are all solutions to the equation 3x+2y=6. We can list these three solutions in Table **6.1.18**.

Table **6.1.18**

3x+2y=63x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

Exercise 6.1.26

Find three solutions to the equation 2x+3y=6.

Answer

Answers will vary.

Exercise 6.1.27

Find three solutions to the equation 4x+2y=8.

Answer

Answers will vary.

Key Concepts

- **Sign Patterns of the Quadrants**

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- **Points on the Axes**
  - On the *x*-axis, y=0. Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates (a,0).
  - On the *y*-axis, x=0. Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates (0,b).
- **Solution of a Linear Equation**
  - An ordered pair (x,y) is a solution of the linear equation Ax+By=C, if the equation is a true statement when the *x*- and *y*-values of the ordered pair are substituted into the equation.

Glossary

linear equation

A linear equation is of the form Ax+By=C, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x,y) gives the coordinates of a point in a rectangular coordinate system.

origin

The point (0,0)(0,0) is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

quadrant

The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.

rectangular coordinate system

A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane’.

***x*-coordinate**

The first number in an ordered pair (x,y).

***y*-coordinate**

The second number in an ordered pair (x,y).

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6.2: Relations and Functions

Learning Objectives

By the end of this section, you will be able to:

- Find the domain and range of a relation
- Determine if a relation is a function
- Find the value of a function

Find the Domain and Range of a Relation

As we go about our daily lives, we have many data items or quantities that are paired to our names. Our social security number, student ID number, email address, phone number and our birthday are matched to our name. There is a relationship between our name and each of those items.

When your professor gets her class roster, the names of all the students in the class are listed in one column and then the student ID number is likely to be in the next column. If we think of the correspondence as a set of ordered pairs, where the first element is a student name and the second element is that student's ID number, we call this a **relation**.

Misplaced '#'

The set of all the names of the students in the class is called the **domain** of the relation and the set of all student ID numbers paired with these students is the range of the relation.

There are many similar situations where one variable is paired or matched with another. The set of ordered pairs that records this matching is a relation.

Definition: Relation

A **relation** is any set of ordered pairs, (x, y). All the x-values in the ordered pairs together make up the **domain**. All the y-values in the ordered pairs together make up the **range**.

Example 6.2.1

For the relation $(1,1), (2,4), (3,9), (4,16), (5,25)$:

a. Find the domain of the relation.
b. Find the range of the relation.

Answer

$\{(1,1), (2,4), (3,9), (4,16), (5,25)\}$

ⓐ The domain is the set of all x-values of the relation. $\{1,2,3,4,5\}$

ⓑ The range is the set of all y-values of the relation. $\{1,4,9,16,25\}$

Example 6.2.2

For the relation $\{(1,1), (2,8), (3,27), (4,64), (5,125)\}$:

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{1, 8, 27, 64, 125\}$

Example 6.2.3

For the relation $\{(1, 3), (2, 6), (3, 9), (4, 12), (5, 15)\}$:

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1, 2, 3, 4, 5\}$

Answer b

$\{3, 6, 9, 12, 15\}$

MAPPING

A **mapping** is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

Example 6.2.4

Use the **mapping** of the relation shown to

a. list the ordered pairs of the relation,
b. find the domain of the relation, and
c. find the range of the relation.



Answer

ⓐ The arrow shows the matching of the person to their birthday. We create ordered pairs with the person's name as the *x*-value and their birthday as the *y*-value.

{(Alison, April 25), (Penelope, May 23), (June, August 2), (Gregory, September 15), (Geoffrey, January 12), (Lauren, May 10), (Stephen, July 24), (Alice, February 3), (Liz, August 2), (Danny, July 24)}

ⓑ The domain is the set of all *x*-values of the relation.

{Alison, Penelope, June, Gregory, Geoffrey, Lauren, Stephen, Alice, Liz, Danny}

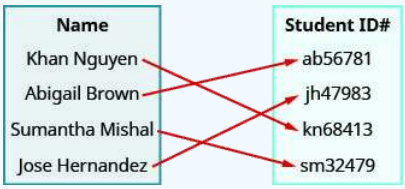
ⓒ The range is the set of all *y*-values of the relation.

{January 12, February 3, April 25, May 10, May 23, July 24, August 2, September 15}

Example 6.2.5

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Khanh Nguyen, kn68413), (Abigail Brown, ab56781), (Sumantha Mishal, sm32479), (Jose Hern and ez, jh47983)

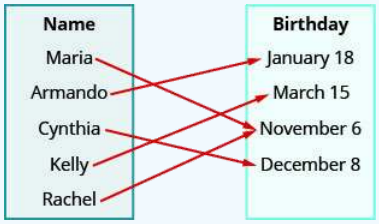
ⓑ {Khanh Nguyen, Abigail Brown, Sumantha Mishal, Jose Hern and ez}

ⓒ {kn68413, ab56781, sm32479, jh47983}

Example 6.2.6

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Maria, November 6), (Arm and o, January 18), (Cynthia, December 8), (Kelly, March 15), (Rachel, November 6)

ⓑ {Maria, Arm and o, Cynthia, Kelly, Rachel}

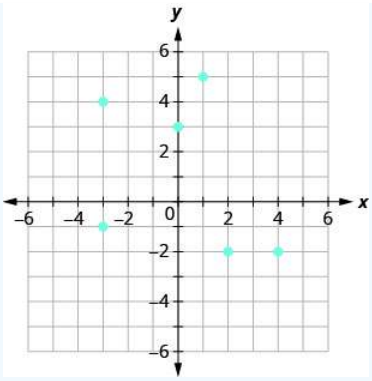
ⓒ{November 6, January 18, December 8, March 15}

A graph is yet another way that a relation can be represented. The set of ordered pairs of all the points plotted is the relation. The set of all x-coordinates is the domain of the relation and the set of all y-coordinates is the range. Generally we write the numbers in ascending order for both the domain and range.

Example 6.2.7

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ The ordered pairs of the relation are:

$$\{(1,5),(-3,-1),(4,-2),(0,3),(2,-2),(-3,4)\}.$$

ⓑ The domain is the set of all x-values of the relation: $\{-3,0,1,2,4\}$.

Notice that while -3 repeats, it is only listed once.

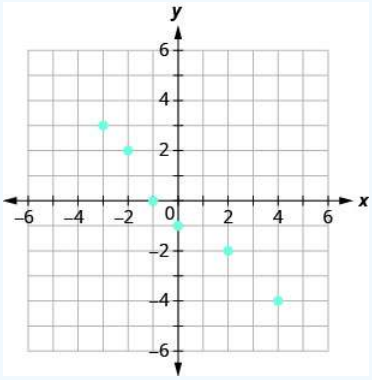
ⓒ The range is the set of all y-values of the relation: $\{-2,-1,3,4,5\}$.

Notice that while -2 repeats, it is only listed once.

Example 6.2.8

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

a. $(-3,3),(-2,2),(-1,0),,\ (0,-1),(2,-2),(4,-4)$

b. $\{-3,-2,-1,0,2,4\}$

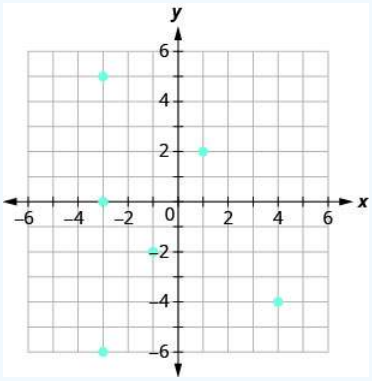
c. $\{3,2,0,-1,-2,-4\}$

Example 6.2.9

Use the graph of the relation to

a. list the ordered pairs of the relation

b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ $(-3,0),(-3,5),(-3,-6),(-1,-2),(1,2),(4,-4)$
ⓑ $\{-3,-1,1,4\}$
ⓒ $\{-6,0,5,-2,2,-4\}$

Determine if a Relation is a Function

A special type of relation, called a **function**, occurs extensively in mathematics. A function is a relation that assigns to each element in its domain exactly one element in the range. For each ordered pair in the relation, each *x*-value is matched with only one *y*-value.

Definition: Function

A **function** is a relation that assigns to each element in its domain exactly one element in the range.

The birthday example from Example helps us understand this definition. Every person has a birthday but no one has two birthdays. It is okay for two people to share a birthday. It is okay that Danny and Stephen share July 24th as their birthday and that June and Liz share August 2nd. Since each person has exactly one birthday, the relation in Example is a function.

The relation shown by the graph in Example includes the ordered pairs $(-3,-1)$ and $(-3,4)$. Is that okay in a function? No, as this is like one person having two different birthdays.

Example 6.2.10

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$
b. $\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

Answer

ⓐ $\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$

(i) Each *x*-value is matched with only one *y*-value. So this relation is a function.

(ii) The domain is the set of all *x*-values in the relation.
The domain is: $\{-3,-2,-1,0,1,2,3\}$.

(iii) The range is the set of all *y*-values in the relation. Notice we do not list range values twice.
The range is: $\{27,8,1,0\}$.

ⓑ $\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

(i) The x-value 9 is matched with two y-values, both 3 and -3. So this relation is not a function.

(ii) The domain is the set of all x-values in the relation. Notice we do not list domain values twice.
The domain is: $\{0, 1, 2, 4, 9\}$.

(iii) The range is the set of all y-values in the relation.
The range is: $\{-3, -2, -1, 0, 1, 2, 3\}$.

Example 6.2.11

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the function.

a. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4), (3, 6)\}$
b. $\{(8, -4), (4, -2), (2, -1), (0, 0), (2, 1), (4, 2), (8, 4)\}$

Answer

ⓐ Yes; $\{-3, -2, -1, 0, 1, 2, 3\}$;
$\{-6, -4, -2, 0, 2, 4, 6\}$
ⓑ No; $\{0, 2, 4, 8\}$;
$\{-4, -2, -1, 0, 1, 2, 4\}$

Example 6.2.12

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(27, -3), (8, -2), (1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$
b. $\{(7, -3), (-5, -4), (8, -0), (0, 0), (-6, 4), (-2, 2), (-1, 3)\}$

Answer

ⓐ No; $\{0, 1, 8, 27\}$;
$\{-3, -2, -1, 0, 2, 2, 3\}$
ⓑ Yes; $\{7, -5, 8, 0, -6, -2, -1\}$;
$\{-3, -4, 0, 4, 2, 3\}$

Example 6.2.13

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation

c. find the range of the relation.

NAME	PHONE
Lydia	321-549-3327 427-658-2314
Eugene	321-964-7324
Janet	684-358-7961
Rick	
Marty	684-369-7231 798-367-8541

Answer

ⓐ Both Lydia and Marty have two phone numbers. So each x-value is not matched with only one y-value. So this relation is not a function.

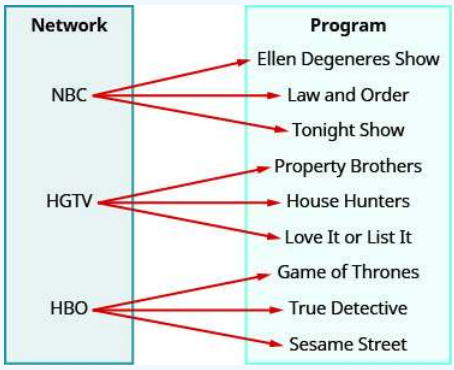
ⓑ The domain is the set of all x-values in the relation. The domain is: {Lydia, Eugene, Janet, Rick, Marty}

ⓒ The range is the set of all y-values in the relation. The range is:

$$\{321-549-3327, 427-658-2314, 321-964-7324, 684-358-7961, 684-369-7231, 798-367-8541\}$$

Example 6.2.14

Use the mapping to ⓐ determine whether the relation is a function ⓑ find the domain of the relation ⓒ find the range of the relation.



Answer

ⓐ no ⓑ {NBC, HGTV, HBO} ⓒ {Ellen Degeneres Show, Law and Order, Tonight Show, Property Brothers, House Hunters, Love it or List it, Game of Thrones, True Detective, Sesame Street}

Example 6.2.15

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation

c. find the range of the relation.

Neal	123-567-4839 work 231-378-5941 cell
Krystal	743-469-9731 cell
Kelvin	567-534-2970 work
George	684-369-7231 cell
Christa	684-369-7231 cell
Mike	639-847-6971 cell

Answer

ⓐ No ⓑ {Neal, Krystal, Kelvin, George, Christa, Mike} ⓒ {123-567-4839 work, 231-378-5941 cell, 743-469-9731 cell, 567-534-2970 work, 684-369-7231 cell, 798-367-8541 cell, 639-847-6971 cell}

In algebra, more often than not, functions will be represented by an equation. It is easiest to see if the equation is a function when it is solved for *y*. If each value of *x* results in only one value of *y*, then the equation defines a function.

? Example 6.2.16

Determine whether each equation is a function.

a. $2x + y = 7$
b. $y = x^2 + 1$
c. $x + y^2 = 3$

Answer

ⓐ $2x + y = 7$

For each value of *x*, we multiply it by -2 and then add 7 to get the *y*-value

	$y = -2x + 7$
For example, if $x = 3$:	$y = -2 \cdot 3 + 7$
	$y = 1$

We have that when $x = 3$, then $y = 1$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓑ $y = x^2 + 1$

For each value of *x*, we square it and then add 1 to get the *y*-value.

	$y = x^2 + 1$
For example, if $x = 2$:	$y = 2^2 + 1$
	$y = 5$

We have that when $x = 2$, then $y = 5$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓒ

	$x+y^2=3$
Isolate the y term.	$y^2=-x+3$
Let's substitute $x=2$.	$y^2=-2+3$
	$y^2=1$
This give us two values for y.	$y=1\ y=-1$

We have shown that when $x=2$, then $y=1$ and $y=-1$. It would work similarly for any value of x. Since each value of x does not corresponds to only one value of y the equation does not define a function.

? Example 6.2.17

Determine whether each equation is a function.

a. $4x+y=-3$
b. $x+y^2=1$
c. $y-x^2=2$

Answer

ⓐ yes ⓑ no ⓒ yes

? Example 6.2.18

Determine whether each equation is a function.

a. $x+y^2=4$
b. $y=x^2-7$
c. $y=5x-4$

Answer

ⓐ no ⓑ yes ⓒ yes

Find the Value of a Function

It is very convenient to name a function and most often we name it f, g, h, F, G, or H. In any function, for each x-value from the domain we get a corresponding y-value in the range. For the function f, we write this range value y as $f(x)$. This is called **function notation** and is read f of x or the value of f at x. In this case the parentheses does not indicate multiplication.

Definition: Function Notation

For the function $y=f(x)$

f is the name of the function
x is the domain value
$f(x)$ is the range value y corresponding to the value x

We read $f(x)$ as f of x or the value of f at x.

We call x the independent variable as it can be any value in the domain. We call y the dependent variable as its value depends on x.

INDEPENDENT AND DEPENDENT VARIABLES

For the function $y = f(x)$,

x is the independent variable as it can be any value in the domain
y the dependent variable as its value depends on x

Much as when you first encountered the variable *x*, function notation may be rather unsettling. It seems strange because it is new. You will feel more comfortable with the notation as you use it.

Let's look at the equation $y = 4x - 5$. To find the value of *y* when $x = 2$, we know to substitute $x = 2$ into the equation and then simplify.

	$y = 4x - 5$
Let x=2.	$y = 4 \cdot 2 - 5$
	$y = 3$

The value of the function at $x = 2$ is 3.

We do the same thing using function notation, the equation $y = 4x - 5$ can be written as $f(x) = 4x - 5$. To find the value when $x = 2$, we write:

	$f(x) = 4x - 5$
Let x=2.	$f(2) = 4 \cdot 2 - 5$
	$f(2) = 3$

The value of the function at $x = 2$ is 3.

This process of finding the value of $f(x)$ for a given value of *x* is called *evaluating the function.*

? Example 6.2.19

For the function $f(x) = 2x^2 + 3x - 1$, evaluate the function.

a. $f(3)$
b. $f(-2)$
c. $f(a)$

Answer

ⓐ

	$f(x) = 2x^2 + 3x - 1$
To evaluate $f(3)$, substitute 3 for *x*.	$f(3) = 2(3)^2 + 3 \cdot 3 - 1$
Simplify.	$f(3) = 2 \cdot 9 + 3 \cdot 3 - 1$
	$f(3) = 18 + 9 - 1$
	$f(3) = 26$

ⓑ

		$f(x) = 2x^2 + 3x - 1$
To evaluate $f(-2)$, substitute -2 for x.		$f(-2) = 2(-2)^2 + 3(-2) - 1$
Simplify.		$f(-2) = 2 \cdot 4 + (-6) - 1$
		$f(-2) = 8 + (-6) - 1$
		$f(-2) = 1$

ⓒ

	$f(x) = 2x^2 + 3x - 1$
To evaluate f(a),f(a), substitute a for x.	$f(a) = 2(a)^2 + 3 \cdot a - 1$
Simplify.	$f(a) = 2a^2 + 3a - 1$

? Example 6.2.20

For the function $f(x) = 3x^2 - 2x + 1$, evaluate the function.

a. $f(3)$
b. $f(-1)$
c. $f(t)$

Answer

ⓐ $f(3) = 22$ ⓑ $f(-1) = 6$ ⓒ $f(t) = 3t^2 - 2t - 1$

? Example 6.2.21

For the function $f(x) = 2x^2 + 4x - 3$, evaluate the function.

a. $f(2)$
b. $f(-3)$
c. $f(h)$

Answer

ⓐ $(2) = 13$ ⓑ $f(-3) = 3$
ⓒ $f(h) = 2h2 + 4h - 3$

In the last example, we found $f(x)$ for a constant value of x. In the next example, we are asked to find $g(x)$ with values of x that are variables. We still follow the same procedure and substitute the variables in for the x.

? Example 6.2.22

For the function $g(x) = 3x - 5$, evaluate the function.

a. $g(h^2)$
b. $g(x + 2)$
c. $g(x) + g(2)$

Answer

ⓐ

			$g(x) = 3x - 5$
To evaluate $g(h^2)$, substitute h^2 for x.			$g(h^2) = 3h^2 - 5$
			$g(h^2) = 3h^2 - 5$

ⓑ

	$g(x) = 3x - 5$
To evaluate $g(x+2)$, substitute $x+2$ for x.	$g(x+2) = 3(x+2) - 5$
Simplify.	$g(x+2) = 3x + 6 - 5$
	$g(x+2) = 3x + 1$

ⓒ

		$g(x) = 3x - 5$
To evaluate $g(x) + g(2)$, first find $g(2)$.		$g(2) = 3 \cdot 2 - 5$
		$g(2) = 1$
Now find $g(x) + g(2)$		$g(x) + g(2) = 3x - 5 + 1$ $g(x) \quad g(2)$
Simplify.		$g(x) + g(2) = 3x - 5 + 1$
		$g(x) + g(2) = 3x - 4$

Notice the difference between part ⓑ and ⓒ. We get $g(x+2) = 3x + 1$ and $g(x) + g(2) = 3x - 4$. So we see that $g(x+2) \neq g(x) + g(2)$.

Example 6.2.23

For the function $g(x) = 4x - 7$, evaluate the function.

a. $g(m^2)$
b. $g(x-3)$
c. $g(x) - g(3)$

Answer

ⓐ $4m^2 - 7$ ⓑ $4x - 19$
ⓒ $x - 12$

Example 6.2.24

For the function $h(x) = 2x + 1$, evaluate the function.

a. $h(k^2)$
b. $h(x+1)$
c. $h(x) + h(1)$

Answer

ⓐ $2k^2+1$ ⓑ $2x+3$
ⓒ $2x+4$

Many everyday situations can be modeled using functions.

? Example 6.2.25

The number of unread emails in Sylvia's account is 75. This number grows by 10 unread emails a day. The function $N(t)=75+10t$ represents the relation between the number of emails, N, and the time, t, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(5)$. Explain what this result means.

Answer

ⓐ The number of unread emails is a function of the number of days. The number of unread emails, N, depends on the number of days, t. Therefore, the variable N, is the dependent variable and the variable tt is the independent variable.

ⓑ Find $N(5)$. Explain what this result means.

	$N(t)=75+10t$
Substitute in t=5.t=5.	$N(5)=75+10\cdot 5$
Simplify.	$N(5)=75+50$
	$N(5)=125$

Since 5 is the number of days, $N(5)$, is the number of unread emails after 5 days. After 5 days, there are 125 unread emails in the account.

? Example 6.2.26

The number of unread emails in Bryan's account is 100. This number grows by 15 unread emails a day. The function $N(t)=100+15t$ represents the relation between the number of emails, N, and the time, t, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(7)$]. Explain what this result means.

Answer

ⓐ t IND; N DEP ⓑ 205; the number of unread emails in Bryan's account on the seventh day.

? Example 6.2.27

The number of unread emails in Anthony's account is 110. This number grows by 25 unread emails a day. The function $N(t)=110+25t$ represents the relation between the number of emails, N, and the time, t, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(14)$. Explain what this result means.

Answer

ⓐ t IND; N DEP ⓑ 460; the number of unread emails in Anthony's account on the fourteenth day

Access this online resource for additional instruction and practice with relations and functions.

- Introduction to Functions

Key Concepts

- **Function Notation:** For the function $y = f(x)$
  - f is the name of the function
  - x is the domain value
  - $f(x)$ is the range value y corresponding to the value x
    We read $f(x)$ as f of x or the value of f at x.
- **Independent and Dependent Variables:** For the function $y = f(x)$,
  - x is the independent variable as it can be any value in the domain
  - y is the dependent variable as its value depends on x

Glossary

domain of a relation

The domain of a relation is all the x-values in the ordered pairs of the relation.

function

A function is a relation that assigns to each element in its domain exactly one element in the range.

mapping

A mapping is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

range of a relation

The range of a relation is all the y-values in the ordered pairs of the relation.

relation

A relation is any set of ordered pairs,(x,y).(x,y). All the x-values in the ordered pairs together make up the domain. All the y-values in the ordered pairs together make up the range.

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6.3: Graph Linear Equations in Two Variables

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relationship between the solutions of an equation and its graph.
- Graph a linear equation by plotting points.
- Graph vertical and horizontal lines.

Recognize the Relationship Between the Solutions of an Equation and its Graph

In the previous section, we found several solutions to the equation $3x + 2y = 6$. They are listed in Table **6.3.1**. So, the ordered pairs (0,3), (2,0), and $(1, \frac{3}{2})$ are some solutions to the equation $3x + 2y = 6$. We can plot these solutions in the rectangular coordinate system as shown in Figure **6.3.1**.

Table **6.3.1**

3x+2y=6		
x	**y**	**(x,y)**
0	3	(0,3)
2	0	(2,0)
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

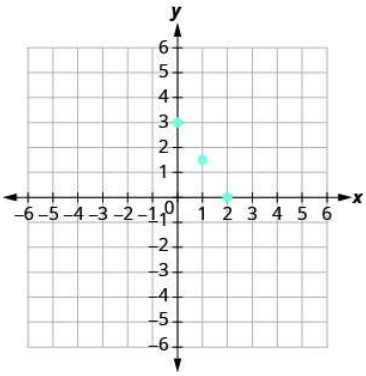


Figure **6.3.1**

Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation 3x+2y=6. See Figure **6.3.2**. Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.

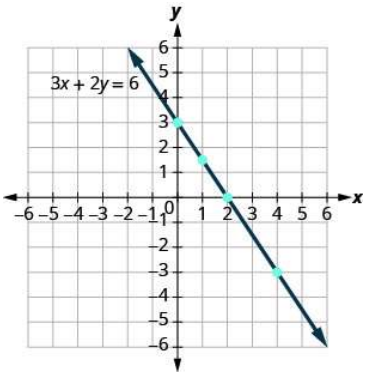


Figure **6.3.2**

Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are (−2,6) is on the line shown in Figure **6.3.3**. If you substitute x=−2 and y=6 into the equation, you find that it is a solution to the equation.

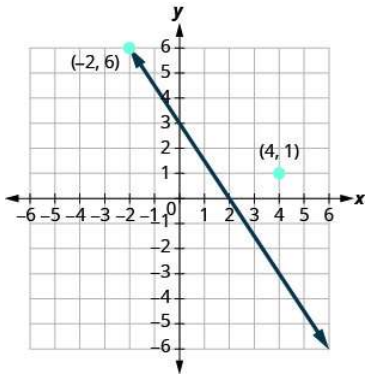


Figure **6.3.3**

Test $(-2, 6)$

$$3x + 2y = 6$$

$$3(-2) + 2(6) = 6$$

$$-6 + 12 = 6$$

$$6 = 6 \checkmark$$

So the point (−2,6) is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are (−2,6)" is often shortened to "the point (−2,6).")

What about $(4, 1)$?

$$3x + 2y = 6$$

$$3 \cdot 4 + 2 \cdot 1 = 6$$

$$12 + 2 \stackrel{?}{=} 6$$

$$14 \neq 6$$

Figure **6.3.3**. This is an example of the saying, "A picture is worth a thousand words." The line shows you *all* the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

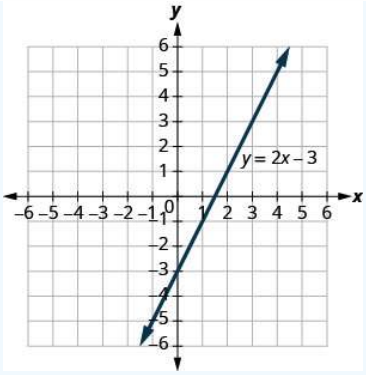
GRAPH OF A LINEAR EQUATION

The **graph of a linear equation** Ax+By=C is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Exercise 6.3.1

The graph of y=2x−3 is shown.



For each ordered pair, decide:

1. Is the ordered pair a solution to the equation?
2. Is the point on the line?

A (0,−3) B (3,3) C (2,−3) D (−1,−5)

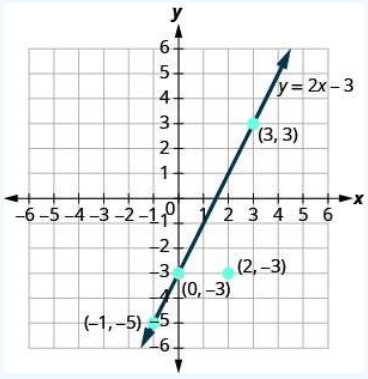
Answer

Substitute the *x*- and *y*- values into the equation to check if the ordered pair is a solution to the equation.

1.

A: (0, –3)	B: (3, 3)	C: (2, –3)	D: (–1, –5)
$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$
$-3 \stackrel{?}{=} 2(0) - 3$	$3 \stackrel{?}{=} 2(3) - 3$	$-3 \stackrel{?}{=} 2(2) - 3$	$-5 \stackrel{?}{=} 2(-1) - 3$
$-3 = -3$ ✓	$3 = 3$ ✓	$-3 \neq 1$	$-5 = -5$ ✓
(0, –3) is a solution.	(3, 3) is a solution.	(2, –3) is not a solution.	(–1, –5) is a solution.

2. Plot the points A (0,3), B (3,3), C (2,−3), and D (−1,−5).



The points that are solutions to y=2x−3 are on the line, but the point that is not a solution is not on the line.

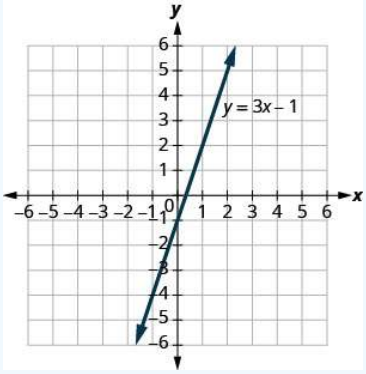
The points (0,3), (3,3), and (−1,−5) are on the line y=2x−3, and the point (2,−3) is not on the line.

? Exercise 6.3.2

Use the graph of y=3x−1 to decide whether each ordered pair is:

- a solution to the equation.
- on the line.

1. (0,−1)
2. (2,5)



Answer

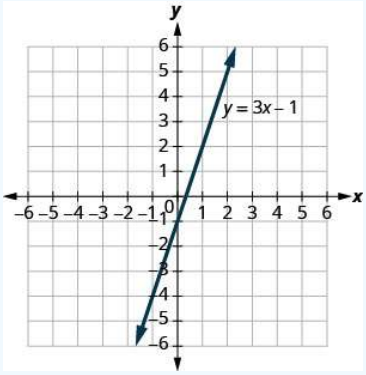
1. yes, yes
2. yes, yes

? Exercise 6.3.3

Use graph of y=3x−1 to decide whether each ordered pair is:

- a solution to the equation
- on the line

1. (3,−1)
2. (−1,−4)



Answer

1. no, no
2. yes, yes

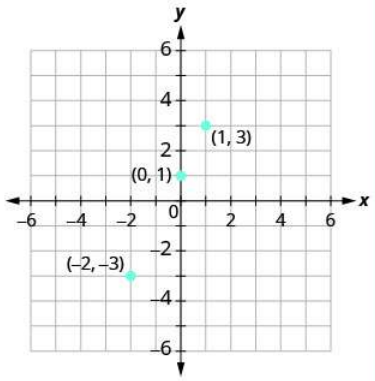
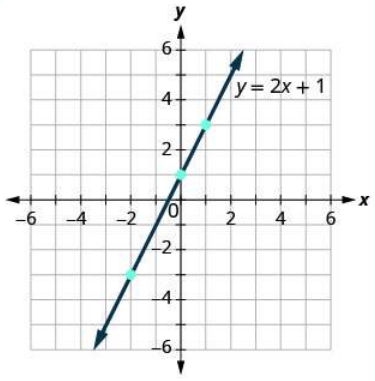
Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The method we used to graph 3x+2y=6 is called plotting points, or the Point–Plotting Method.

? Exercise 6.3.4: How To Graph an Equation By Plotting Points

Graph the equation y=2x+1 by plotting points.

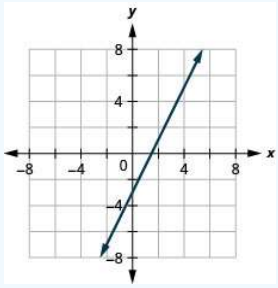
Answer

Step 1. Find three points whose coordinates are solutions to the equation.	You can choose any values for x or y. In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.	$y = 2x + 1$ $x = 0$ $y = 2x + 1$ $y = 2 \cdot 0 + 1$ $y = 0 + 1$ $y = 1$ $x = 1$ $y = 2x + 1$ $y = 2 \cdot 1 + 1$ $y = 2 + 1$ $y = 3$ $x = -2$ $y = 2x + 1$ $y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$
Organize the solutions in a table.	Put the three solutions in a table.	$y = 2x + 1$ x | y | (x, y) 0 | 1 | (0, 1) 1 | 3 | (1, 3) –2 | –3 | (–2, –3)
Step 2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!	Plot: (0, 1), (1, 3), (–2, –3). Do the points line up? Yes, the points line up.	
Step 3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.	This line is the graph of $y = 2x + 1$.	

Exercise 6.3.5

Graph the equation by plotting points: $y=2x-3$.

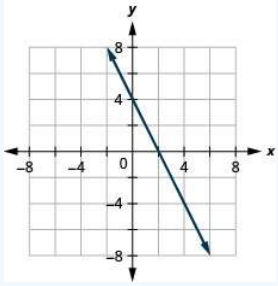
Answer



Exercise 6.3.6

Graph the equation by plotting points: $y=-2x+4$.

Answer



The steps to take when graphing a linear equation by plotting points are summarized below.

GRAPH A LINEAR EQUATION BY PLOTTING POINTS

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between part (a) and part (b) in Figure **6.3.4**.

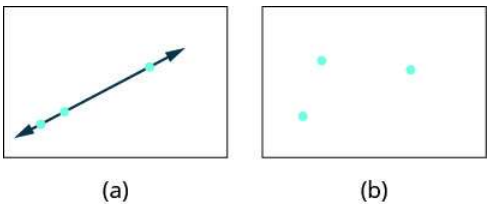


Figure **6.3.4**

Let’s do another example. This time, we’ll show the last two steps all on one grid.

Exercise 6.3.7

Graph the equation y=−3x.

Answer

Find three points that are solutions to the equation. Here, again, it's easier to choose values for x. Do you see why?

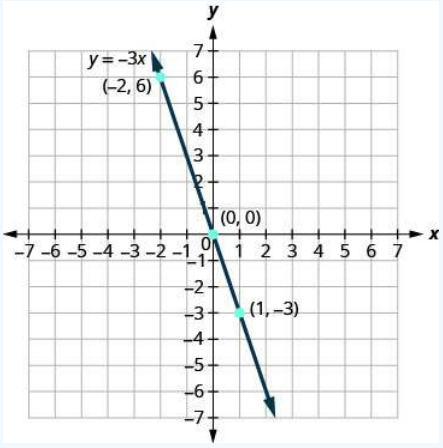
$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3x$	$y = -3x$
$y = -3 \cdot 0$	$y = -3 \cdot 1$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$

We list the points in Table **6.3.2**.

Table **6.3.2**

y=−3x		
x	**y**	**(x,y)**
0	0	(0,0)
1	−3	(1,−3)
−2	6	(−2,6)

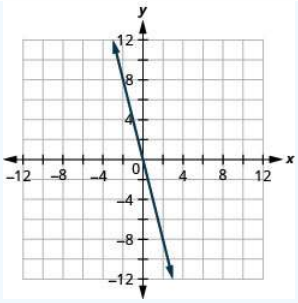
Plot the points, check that they line up, and draw the line.



Exercise 6.3.8

Graph the equation by plotting points: y=−4x.

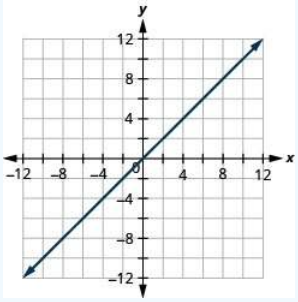
Answer



Exercise 6.3.9

Graph the equation by plotting points: y=x.

Answer



When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for x. But the math is easier if we make 'good' choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

Exercise 6.3.10

Graph the equation $y = \frac{1}{2}x + 3$.

Answer

Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?

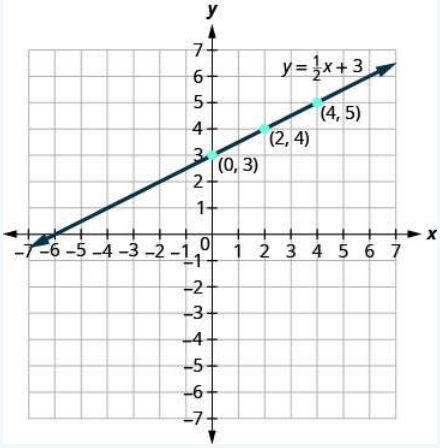
$x = 0$	$x = 2$	$x = 4$
$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$
$y = \frac{1}{2}(0) + 3$	$y = \frac{1}{2}(2) + 3$	$y = \frac{1}{2}(4) + 3$
$y = 0 + 3$	$y = 1 + 3$	$y = 2 + 3$
$y = 3$	$y = 4$	$y = 5$

The points are shown in Table 6.3.3.

Table 6.3.3

y=12x+3		
x	**y**	**(x,y)**
0	3	(0,3)
2	4	(2,4)
4	5	(4,5)

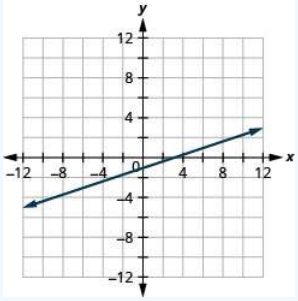
Plot the points, check that they line up, and draw the line.



Exercise 6.3.11

Graph the equation $y = \frac{1}{3}x - 1$.

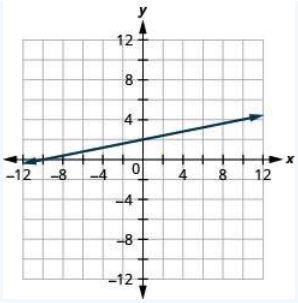
Answer



Exercise 6.3.12

Graph the equation $y = \frac{1}{4}x + 2$.

Answer



So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation 2x+y=3. If y=0 what is the value of x?

$$\begin{aligned} y &= 0 \\ 2x + y &= 3 \\ 2x + 0 &= 3 \\ 2x &= 3 \\ x &= \frac{3}{2} \\ &\left(\frac{3}{2}, 0\right) \end{aligned}$$

This point has a fraction for the *x*- coordinate and, while we could graph this point, it is hard to be precise graphing fractions. Remember in the example y=12x+3, we carefully chose values for x so as not to graph fractions at all. If we solve the equation 2x+y=3 for y, it will be easier to find three solutions to the equation.

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned} \tag{6.3.1}$$

The solutions for x=0, x=1, and x=−1 are shown in the Table **6.3.4**. The graph is shown in Figure **6.3.5**.

Table **6.3.4**

2x+y=3		
x	**y**	**(x,y)**
0	3	(0,3)
1	1	(1,1)
−1−1	5	(−1,5)

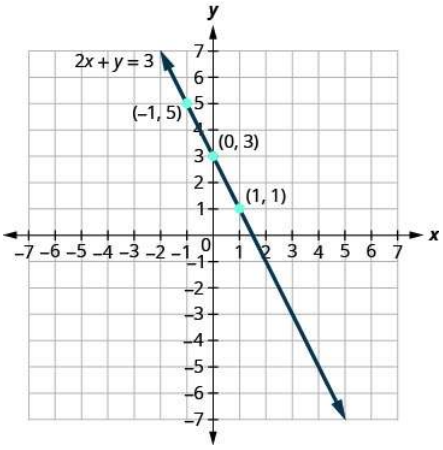


Figure **6.3.5**

Can you locate the point $(\frac{3}{2}, 0)$ which we found by letting y=0, on the line?

Exercise 6.3.13

Graph the equation 3x+y=−1.

Answer

Find three points that are solutions to the equation. $3x + y = -1$
First solve the equation for y. $y = -3x - 1$

We'll let x be 0, 1, and −1 to find 3 points. The ordered pairs are shown in Table **6.3.5**. Plot the points, check that they line up, and draw the line. See Figure **6.3.6**.

Table **6.3.5**

3x+y=−1		
x	**y**	**(x,y)**
0	−1	(0,−1)
1	−4	(1,−4)
−1	2	(−1,2)

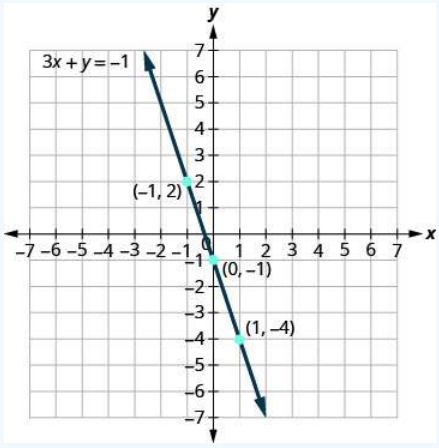
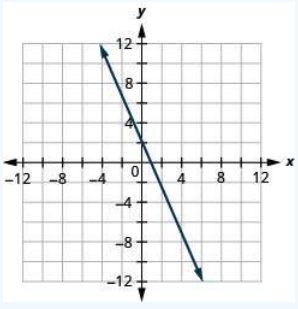


Figure **6.3.6**

Exercise 6.3.14

Graph the equation 2x+y=2.

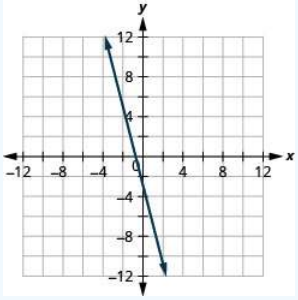
Answer



Exercise 6.3.15

Graph the equation 4x+y=−3.

Answer



If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the *x*- and *y*-axis are the same, the graphs match!

The equation in Exercise **6.3.13** was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting x=0 and solving for y. We can plot a second point by letting y=0 and then solving for x. Then we will plot a third point by using some other value for x or y.

Exercise 6.3.16

Graph the equation $2x - 3y = 6$.

Answer

Find three points that are solutions to the equation.	$2x - 3y = 6$
First let $x = 0$.	$2x - 3y = 6$
Solve for y.	$2(0) - 3y = 6$
	$-3y = 6$
	$y = -2$
Now let $y = 0$.	$2x - 3(0) = 6$
Solve for x.	$2x = 6$
	$x = 3$
We need a third point. Remember, we can choose any value for x or y. We'll let x = 6.	$2(6) - 3y = 6$
Solve fory.	$12 - 3y = 6$
	$-3y = -6$
	$y = 2$

We list the ordered pairs in Table **6.3.6**. Plot the points, check that they line up, and draw the line. See Figure **6.3.7**.

Table **6.3.6**

2x−3y=6		
x	**yT**	**(x,y)**
0	−2	(0,−2)
3	0	(3,0)
6	2	(6,2)

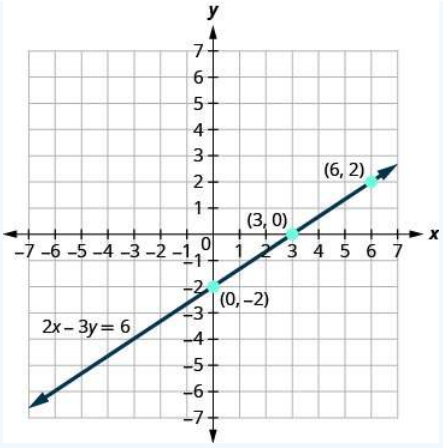
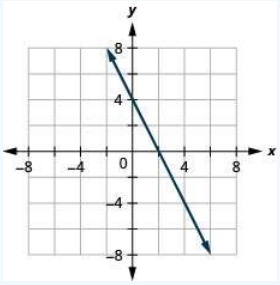


Figure **6.3.7**

? Exercise 6.3.17

Graph the equation $4x + 2y = 8$.

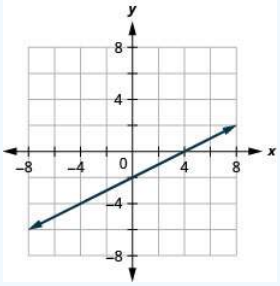
Answer



? Exercise 6.3.18

Graph the equation $2x - 4y = 8$.

Answer



Graph Vertical and Horizontal Lines

Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let's consider the equation x=−3. This equation has only one variable, x. The equation says that x is *always* equal to −3, so its value does not depend on y. No matter what y is, the value of x is always −3.

So to make a table of values, write −3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we'll use 1, 2, and 3 for the *y*-coordinates. See Table **6.3.7**

Table **6.3.7**

x=−3		
x	**y**	**(x,y)**
−3	1	(−3,1)
−3	2	(−3,2)
−3	3	(−3,3)

Plot the points from Table **6.3.7** and connect them with a straight line. Notice in Figure **6.3.8** that we have graphed a *vertical line*.

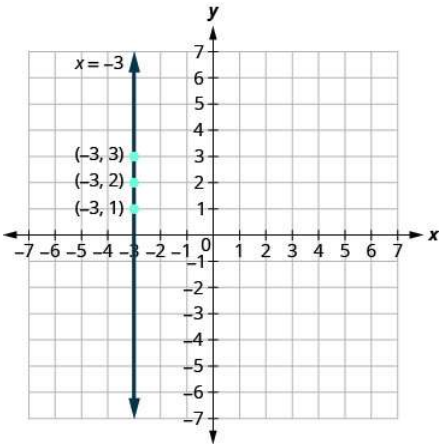


Figure **6.3.8**

VERTICAL LINE

A **vertical line** is the graph of an equation of the form x=a.

The line passes through the *x*-axis at (a,0).

Exercise 6.3.19

Graph the equation x=2.

Answer

The equation has only one variable, x, and x is always equal to 2. We create Table **6.3.8** where x is always 2 and then put in any values for y. The graph is a vertical line passing through the *x*-axis at 2. See Figure **6.3.9**.

Table **6.3.8**

x=2		
x	**y**	**(x,y)**
2	1	(2,1)
2	2	(2,2)
2	3	(2,3)

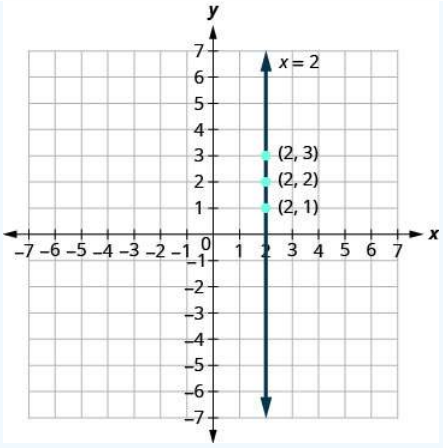
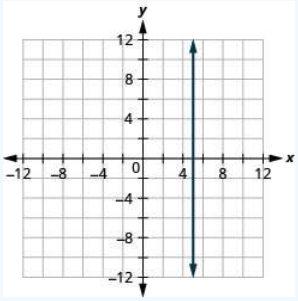


Figure **6.3.9**

Exercise 6.3.20

Graph the equation x=5.

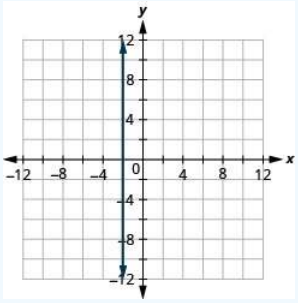
Answer



Exercise 6.3.21

Graph the equation x=−2.

Answer



What if the equation has y but no x? Let's graph the equation y=4. This time the *y*- value is a constant, so in this equation, y does not depend on xx. Fill in 4 for all the y's in Table **6.3.9** and then choose any values for x. We'll use 0, 2, and 4 for the *x*-coordinates.

Table 6.3.9

y=4		
x	**y**	**(x,y)**
0	4	(0,4)
2	4	(2,4)
4	4	(4,4)

The graph is a horizontal line passing through the *y*-axis at 4. See Figure 6.3.10.

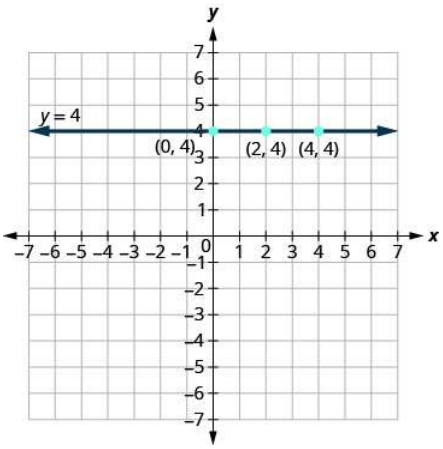


Figure 6.3.10

HORIZONTAL LINE

A **horizontal line** is the graph of an equation of the form y=b.

The line passes through the *y*-axis at (0,b).

Exercise 6.3.22

Graph the equation y=−1.

Answer

The equation y=−1y=−1 has only one variable, y. The value of y is constant. All the ordered pairs in Table 6.3.10 have the same *y*-coordinate. The graph is a horizontal line passing through the *y*-axis at −1−1, as shown in Figure 6.3.11.

Table 6.3.10

y=−1		
x	**y**	**(x,y)**
	Ta−1	(0,−1)
	−1	(3,−1)
−3	−1	(−3,−1)

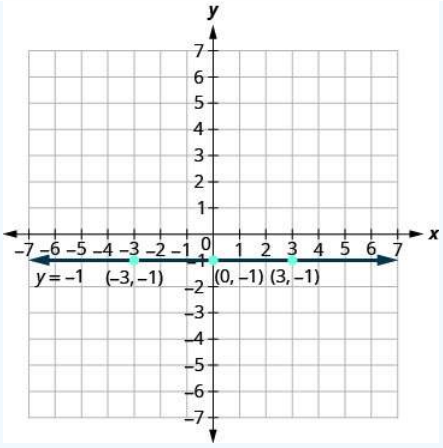
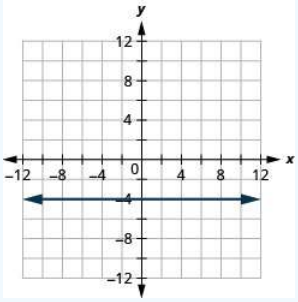


Figure 6.3.11

? Exercise 6.3.23

Graph the equation y=−4.

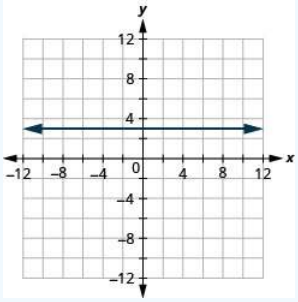
Answer



? Exercise 6.3.24

Graph the equation y=3.

Answer



The equations for vertical and horizontal lines look very similar to equations like y=4x. What is the difference between the equations y=4x and y=4?

The equation y=4x has both x and y. The value of y depends on the value of x. The *y*-coordinate changes according to the value of x. The equation y=4 has only one variable. The value of y is constant. The *y*-coordinate is always 4. It does not depend on the value of x. See Table **6.3.11**.

Table 6.3.11

y=4x				y=4		
x	**y**	**(x,y)**		**x**	**y**	**(x,y)**
0	0	(0,0)		0	4	(0,4)
1	4	(1,4)		1	4	(1,4)
2	8	(2,8)		2	4	(2,4)

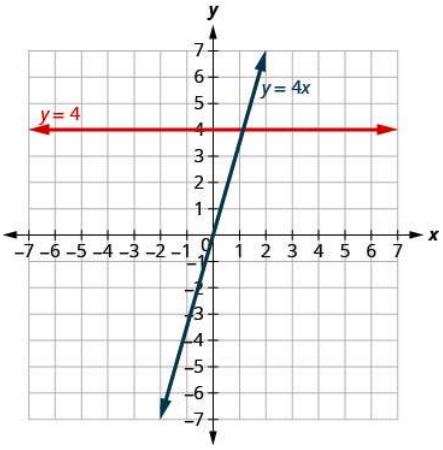


Figure 6.3.12

Notice, in Figure 6.3.12, the equation y=4x gives a slanted line, while y=4 gives a horizontal line.

? Exercise 6.3.25

Graph y=−3x and y=−3 in the same rectangular coordinate system.

Answer

Notice that the first equation has the variable x, while the second does not. See Table 6.3.12. The two graphs are shown in Figure 6.3.13.

Table 6.3.12

y=−3x				y=−3		
x	**y**	**(x,y)**		**x**	**y**	**(x,y)**
		(0,0)			−3	(0,−3)
	−3	(1,−3)			−3	(1,−3)
	−6	(2,−6)			−3	(2,−3)

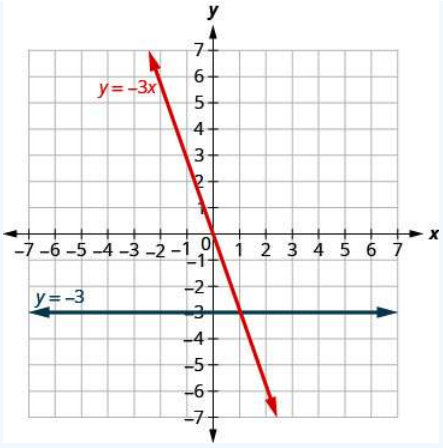
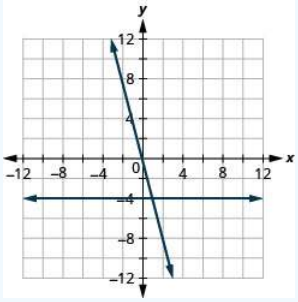


Figure 6.3.13

Exercise 6.3.26

Graph y=−4x and y=−4 in the same rectangular coordinate system.

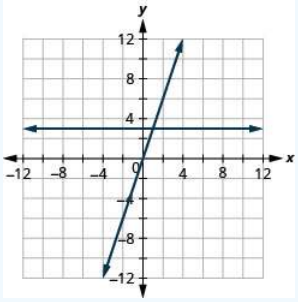
Answer



Exercise 6.3.27

Graph y=3 and y=3x in the same rectangular coordinate system.

Answer



Key Concepts

- **Graph a Linear Equation by Plotting Points**
  1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
  2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!
  3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

Glossary

graph of a linear equation

The graph of a linear equation Ax+By=C is a straight line. Every point on the line is a solution of the equation. Every solution of this equation is a point on this line.

horizontal line

A horizontal line is the graph of an equation of the form y=b. The line passes through the *y*-axis at (0,b).

vertical line

A vertical line is the graph of an equation of the form x=a. The line passes through the *x*-axis at (a,0).

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6.4: Graph with Intercepts

Learning Objectives

By the end of this section, you will be able to:

- Identify the x- and y- intercepts on a graph
- Find the x- and y- intercepts from an equation of a line
- Graph a line using the intercepts

Identify the *x*- and *y*- Intercepts on a Graph

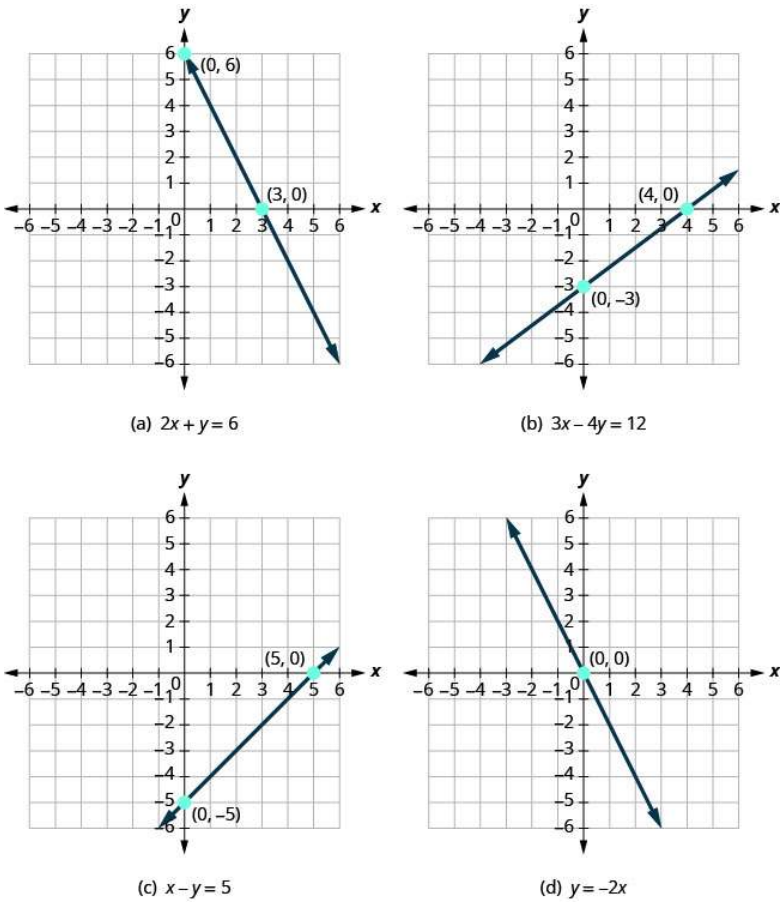
Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the *x*- axis and the *y*- axis. These points are called the *intercepts* of the line.

INTERCEPTS OF A LINE

The points where a line crosses the *x*- axis and the *y*- axis are called the **intercepts of a line**.

Let's look at the graphs of the lines in Figure 6.4.1.



(a) $2x + y = 6$ (b) $3x - 4y = 12$

(c) $x - y = 5$ (d) $y = -2x$

Figure 6.4.1: Examples of graphs crossing the x-negative axis.

First, notice where each of these lines crosses the x negative axis. See Figure **6.4.1**.

Table **6.4.1**

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	3	(3,0)
Figure (b)	4	(4,0)
Figure (c)	5	(5,0)
Figure (d)	0	(0,0)

Do you see a pattern?

For each row, the *y*- coordinate of the point where the line crosses the *x*- axis is zero. The point where the line crosses the *x*- axis has the form (a,0) and is called the ***x*- intercept of a line**. The *x*- intercept occurs when y is zero. Now, let's look at the points where these lines cross the *y*- axis. See Table **6.4.2**.

Table **6.4.2**

Figure	The line crosses the x-axis at:	Ordered pair of this point
Figure (a)	6	(0,6)
Figure (b)	−3	(0,−3)
Figure (c)	−5	(0,5)
Figure (d)	0	(0,0)

What is the pattern here?

In each row, the *x*- coordinate of the point where the line crosses the *y*- axis is zero. The point where the line crosses the *y*- axis has the form (0,b) and is called the *y*- *intercept* of the line. The *y*- intercept occurs when x is zero.

X- INTERCEPT AND Y- INTERCEPT OF A LINE

The *x*- intercept is the point (a,0) where the line crosses the *x*- axis.

The *y*- intercept is the point (0,b) where the line crosses the *y*- axis.

	x	*y*
• The *x*-intercept occurs when *y* is zero.	*a*	0
• The *y*-intercept occurs when *x* is zero.	0	*b*

Figure **6.4.2**

Exercise 6.4.1

Find the *x*- and *y*- intercepts on each graph.

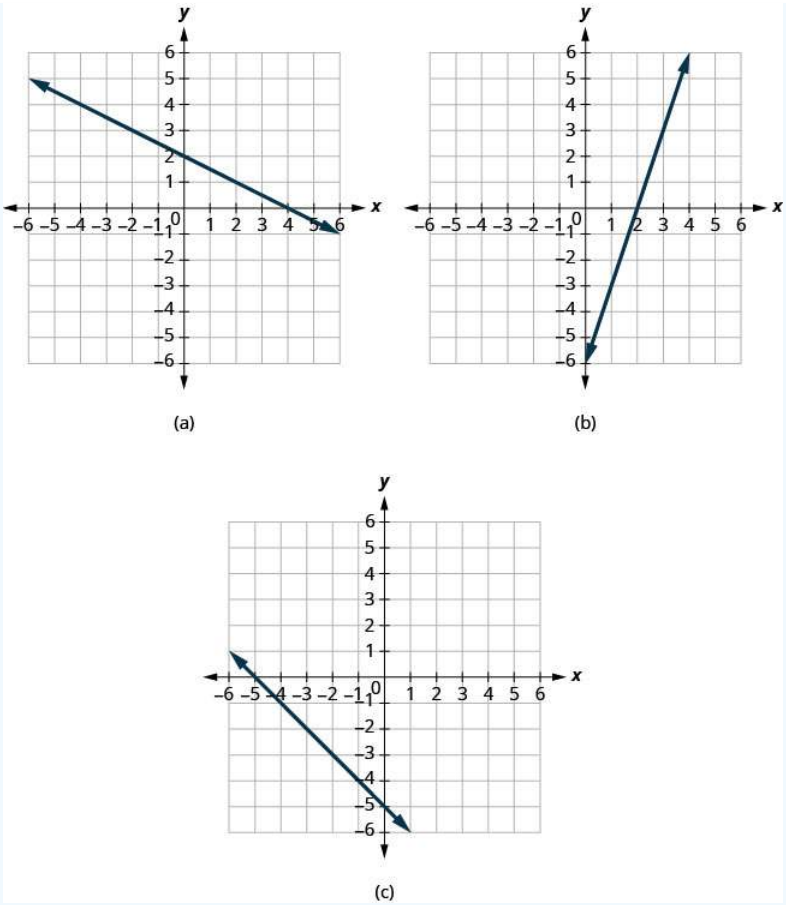


Figure 6.4.3

Answer

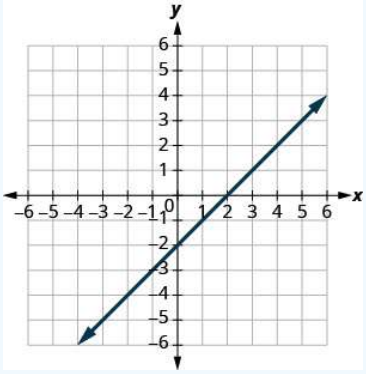
(a) The graph crosses the x- axis at the point (4,0). The x- intercept is (4,0).
The graph crosses the y- axis at the point (0,2). The y- intercept is (0,2).

(b) The graph crosses the x- axis at the point (2,0). The x- intercept is (2,0)
The graph crosses the y- axis at the point (0,−6). The y- intercept is (0,−6).

(c) The graph crosses the x- axis at the point (−5,0). The x- intercept is (−5,0).
The graph crosses the y- axis at the point (0,−5). The y- intercept is (0,−5).

? Exercise 6.4.2

Find the x- and y- intercepts on the graph.

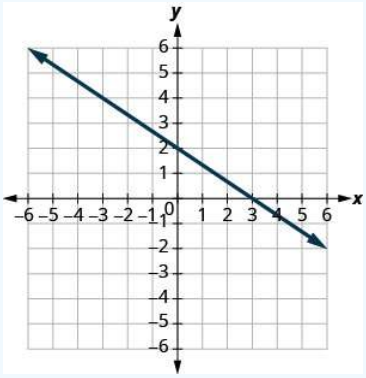


Answer

x- intercept: (2,0); *y*- intercept: (0,−2)

Exercise 6.4.3

Find the *x*- and *y*- intercepts on the graph.



Answer

x- intercept: (3,0), *y*- intercept: (0,2)

Find the *x*- and *y*- Intercepts from an Equation of a Line

Recognizing that the ***x*- intercept** occurs when *y* is zero and that the *y*- intercept occurs when *x* is zero, gives us a method to find the intercepts of a line from its equation. To find the *x*- intercept, let y=0 and solve for *x*. To find the ***y*- intercept**, let x=0 and solve for *y*.

X- AND Y- INTERCEPTS FROM THE EQUATION OF A LINE

Use the equation of the line. To find:

- the *x*- intercept of the line, let y=0 and solve for x.
- the *y*- intercept of the line, let x=0 and solve for y.

Exercise 6.4.4

Find the intercepts of 2x+y=6.

Answer

We will let y=0 to find the *x*- intercept, and let x=0 to find the *y*- intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$	
x	y
	0
0	

x-intercept

y-intercept

To find the *x*- intercept, let y=0.

Table **6.4.3**

	$2x+y=6$
Let $y = 0$.	$2x+0=6$
Simplify.	$2x=6$
	$x=3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$2x+y=6$
Let $x = 0$.	$2 \cdot 0+y=6$
Simplify.	$0+y=6$
	$y=6$
The *y*-intercept is	(0, 6)

The intercepts are the points (3,0) and (0,6) as shown in Table **6.4.4**.

Table **6.4.4**

2x+y=6	
x	**y**
3	0
0	6

? Exercise 6.4.5

Find the intercepts of 3x+y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,12)

Exercise 6.4.6

Find the intercepts of x+4y=8.

Answer

x- intercept: (8,0), *y*- intercept: (0,2)

Exercise 6.4.7

Find the intercepts of 4x–3y=12.

Answer

To find the *x*-intercept, let $y = 0$.	
	$4x - 3y = 12$
Let $y = 0$.	$4x - 3 \cdot 0 = 12$
Simplify.	$4x - 0 = 12$
	$4x = 12$
	$x = 3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let $x = 0$.	
	$4x - 3y = 12$
Let $x = 0$.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The *y*-intercept is	(0, −4)

Table **6.4.5**

The intercepts are the points (3, 0) and (0, −4) as shown in the following table.

Table **6.4.6**

4x−3y=12	
x	**y**
3	0
0	−4

Exercise 6.4.8

Find the intercepts of 3x–4y=12.

Answer

x- intercept: (4,0), *y*- intercept: (0,−3)

Exercise 6.4.9

Find the intercepts of 2x–4y=8.

Answer

x- intercept: (4,0), *y*- intercept: (0,−2)

Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the *x*- and *y*- intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

Exercise 6.4.10: How to Graph a Line Using Intercepts

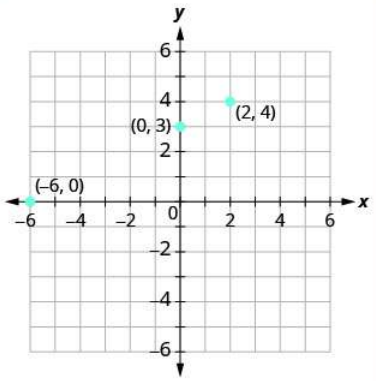
Graph –x+2y=6 using the intercepts.

Answer

Step 1. Find the *x*- and *y*-intercepts of the line. Let $y = 0$ and solve for x. Let $x = 0$ and solve for y.	Find the *x*-intercept.	Let $y = 0$ $-x + 2y = 6$ $-x + 2(0) = 6$ $-x = 6$ $x = -6$ The *x*-intercept is (–6, 0).
	Find the *y*-intercept.	Let $x = 0$ $-x + 2y = 6$ $-0 + 2y = 6$ $2y = 6$ $y = 3$ The *y*-intercept is (0, 3).
Step 2. Find another solution to the equation.	We'll use $x = 2$.	Let $x = 2$ $-x + 2y = 6$ $-2 + 2y = 6$ $2y = 8$ $y = 4$ A third point is (2, 4).

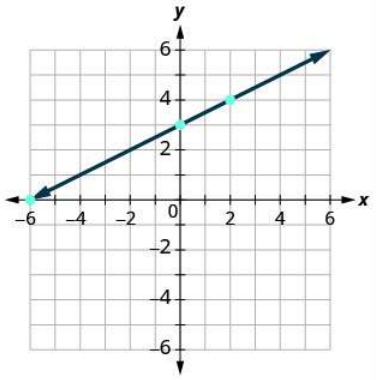
Step 3. Plot the three points. Check that the points line up.

x	y	(x, y)
–6	0	(–6, 0)
0	3	(0, 3)
2	4	(2, 4)



Step 4. Draw the line.

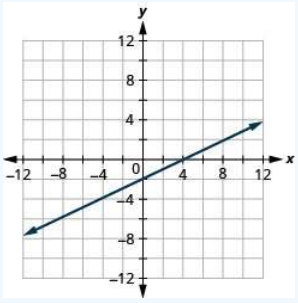
See the graph.



Exercise 6.4.11

Graph x–2y=4 using the intercepts.

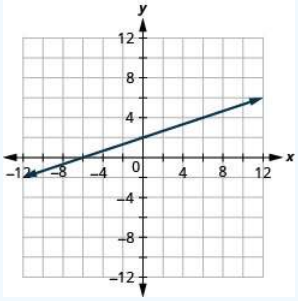
Answer



Exercise 6.4.12

Graph –x+3y=6 using the intercepts.

Answer



The steps to graph a linear equation using the intercepts are summarized below.

GRAPH A LINEAR EQUATION USING THE INTERCEPTS.

1. Find the *x*- and *y*- intercepts of the line.
   - Let y=0 and solve for x
   - Let x=0 and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

Exercise 6.4.13

Graph 4x–3y=12 using the intercepts.

Answer

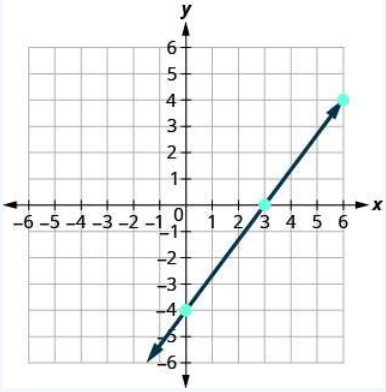
Find the intercepts and a third point.

x-intercept, let $y = 0$	*y*-intercept, let $x = 0$	third point, let $y = 4$
$4x - 3y = 12$	$4x - 3y = 12$	$4x - 3y = 12$
$4x - 3(0) = 12$	$4(0) - 3y = 12$	$4x - 3(4) = 12$
$4x = 12$	$-3y = 12$	$4x - 12 = 12$
$x = 3$	$y = -4$	$4x = 24$
		$x = 6$

We list the points in Table 6.4.7 and show the graph below.

4x−3y=12		
x	**y**	**(x,y)**
3	0	(3,0)
0	−4	(0,−4)
6	4	(6,4)

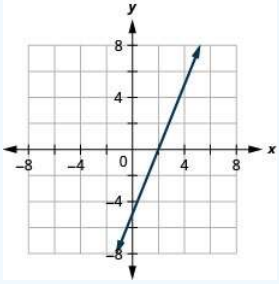
Table 6.4.7



Exercise 6.4.14

Graph 5x–2y=10 using the intercepts.

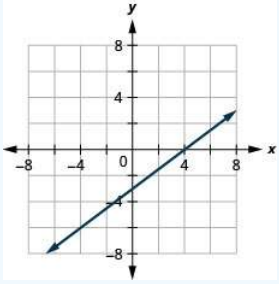
Answer



Exercise 6.4.15

Graph 3x–4y=12 using the intercepts.

Answer



Exercise 6.4.16

Graph y=5x using the intercepts.

Answer

x-intercept	y-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
$(0, 0)$	$(0, 0)$

This line has only one intercept. It is the point (0,0).

To ensure accuracy we need to plot three points. Since the x- and y- intercepts are the same point, we need *two* more points to graph the line.

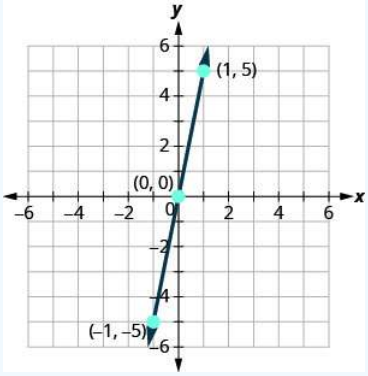
Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

See Table **6.4.8**.

y=5x		
x	**y**	**(x,y)**
		(0,0)
		(1,5)
−1	−5	(−1,−5)

Table **6.4.8**

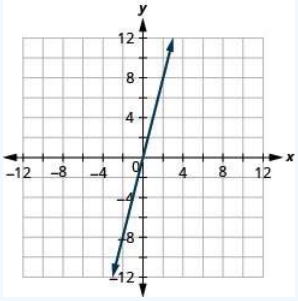
Plot the three points, check that they line up, and draw the line.



Exercise 6.4.17

Graph y=4x using the intercepts.

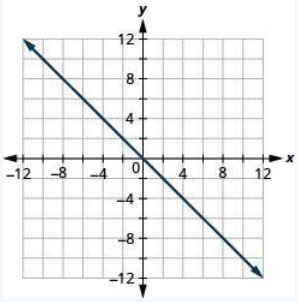
Answer



? Exercise 6.4.18

Graph y=−x the intercepts.

Answer



Key Concepts

- **Find the *x*- and *y*- Intercepts from the Equation of a Line**
  - Use the equation of the line to find the *x*- intercept of the line, let y=0 and solve for *x*.
  - Use the equation of the line to find the *y*- intercept of the line, let x=0 and solve for *y*.
- **Graph a Linear Equation using the Intercepts**
  1. Find the *x*- and *y*- intercepts of the line.
     Let y=0 and solve for *x*.
     Let x=0 and solve for *y*.
  2. Find a third solution to the equation.
  3. Plot the three points and then check that they line up.
  4. Draw the line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
  - Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    x=a is a vertical line passing through the *x*- axis at a
    y=b is a horizontal line passing through the *y*- axis at b.
  - If *y* is isolated on one side of the equation, graph by plotting points.
  - Choose any three values for *x* and then solve for the corresponding *y*- values.
  - If the equation is of the form ax+by=c, find the intercepts. Find the *x*- and *y*- intercepts and then a third point.

Glossary

intercepts of a line

The points where a line crosses the *x*- axis and the *y*- axis are called the intercepts of the line.

***x*- intercept**

The point (a,0) where the line crosses the *x*- axis; the *x*- intercept occurs when y is zero.

***y*-intercept**

The point (0,b) where the line crosses the *y*- axis; the *y*- intercept occurs when x is zero.

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6.5: Understanding the Slope of a Line

Learning Objectives

- By the end of this section, you will be able to:
- Use geoboards to model slope
- Use $m = \frac{\text{rise}}{\text{run}}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

In this section, we will explore the concept of slope.

Use Geoboards to Model Slope

A **geoboard** is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

Doing the Manipulative Mathematics activity "Exploring Slope" will help you develop a better understanding of the slope of a line. (Graph paper can be used instead of a geoboard, if needed.)

We'll start by stretching a rubber band between two pegs as shown in Figure **6.5.1**.

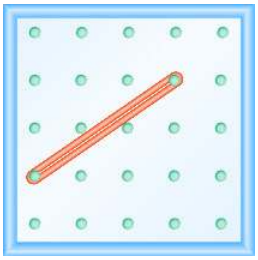


Figure **6.5.1**

Doesn't it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in Figure **6.5.2**

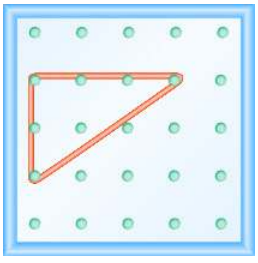


Figure **6.5.2**

We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the **rise** and the horizontal distance is called the **run**, as shown in Figure **6.5.3**.

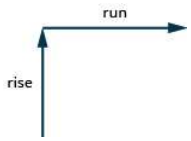


Figure **6.5.3**

If our **geoboard** and rubber band look just like the one shown in Figure **6.5.4**, the rise is 2. The rubber band goes up 2 units. (Each space is one unit.)

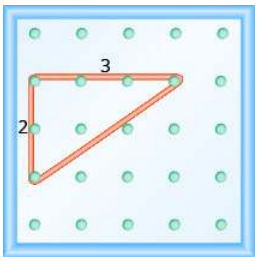


Figure **6.5.4**

The rise on this geoboard is 2, as the rubber band goes up two units.

What is the run?

The rubber band goes across 3 units. The run is 3 (see Figure **6.5.4**).

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

SLOPE OF A LINE

The **slope of a line** of a line is $m = \frac{\text{rise}}{\text{run}}$.

The **rise** measures the vertical change and the **run** measures the horizontal change between two points on the line.

What is the slope of the line on the geoboard in Figure **6.5.4**?

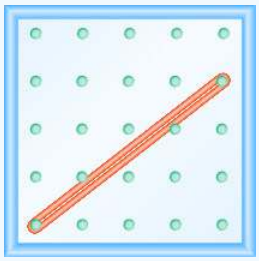
$$m = \frac{\text{rise}}{\text{run}} \qquad (6.5.1)$$
$$m = \frac{2}{3}$$

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up it is positive and if it goes down it is negative. The run will go from left to right and be positive.

Exercise 6.5.1

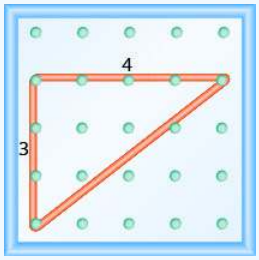
What is the slope of the line on the geoboard shown?



Answer

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the spaces up and to the right to reach the second peg.

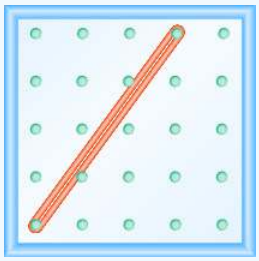


$$\begin{aligned} &\text{The rise is 3.} && m = \tfrac{3}{\text{run}} \\ &\text{The run is 4.} && m = \tfrac{3}{4} \\ & && \text{The slope is } \tfrac{3}{4}\ . \end{aligned} \tag{6.5.2}$$

This means that the line rises 3 units for every 4 units of run.

Exercise 6.5.2

What is the slope of the line on the geoboard shown?

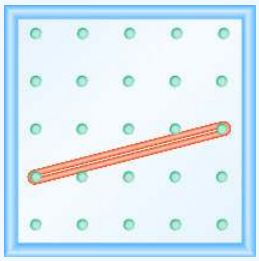


Answer

$\frac{4}{3}$

Exercise 6.5.3

What is the slope of the line on the geoboard shown?

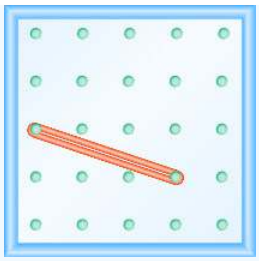


Answer

$\frac{1}{4}$

Exercise 6.5.4

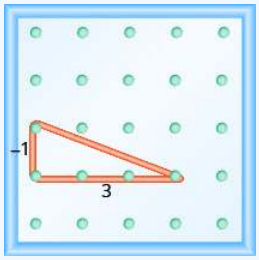
What is the slope of the line on the geoboard shown?



Answer

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg.

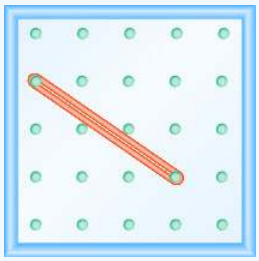


$$\begin{aligned} &\text{The rise is } -1. && m = \tfrac{-1}{\text{run}} \\ &\text{The run is } 3. && m = \tfrac{-1}{3} \\ & && m = -\tfrac{1}{3} \\ & && \text{The slope is } -\tfrac{1}{3} \end{aligned} \tag{6.5.3}$$

This means that the line drops 1 unit for every 3 units of run.

Exercise 6.5.5

What is the slope of the line on the geoboard?

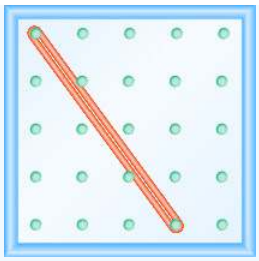


Answer

$-\frac{2}{3}$

Exercise 6.5.6

What is the slope of the line on the geoboard?



Answer

$-\frac{4}{3}$

Notice that in Exercise **6.5.1** the slope is positive and in Exercise **6.5.4** the slope is negative. Do you notice any difference in the two lines shown in Figure(a) and Figure(b)?

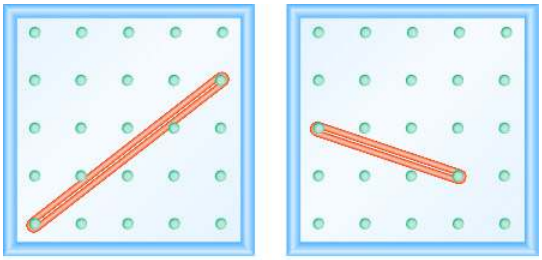


Figure **6.5.5**

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line in Figure(a) is going up; it has **positive slope**. The line in Figure(b) is going down; it has **negative slope**.

POSITIVE AND NEGATIVE SLOPES

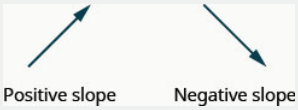


Figure **6.5.6**

Exercise 6.5.7

Use a geoboard to model a line with slope $\frac{1}{2}$.

Answer

To model a line on a geoboard, we need the rise and the run.

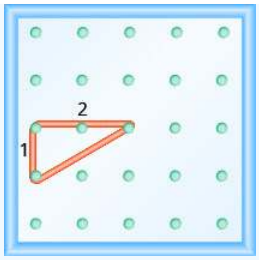
Use the slope formula. $m = \frac{\text{rise}}{\text{run}}$

Replace m with $\frac{1}{2}$. $\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

So, the rise is 1 and the run is 2.

Start at a peg in the lower left of the geoboard.

Stretch the rubber band up 1 unit, and then right 2 units.

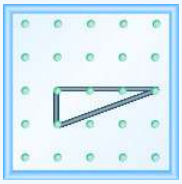


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{1}{2}$.

Exercise 6.5.8

Model the slope $m = \frac{1}{3}$. Draw a picture to show your results.

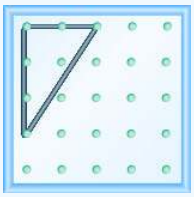
Answer



Exercise 6.5.9

Model the slope $m = \frac{3}{2}$. Draw a picture to show your results.

Answer



Exercise 6.5.10

Use a geoboard to model a line with slope $\frac{-1}{4}$.

Answer

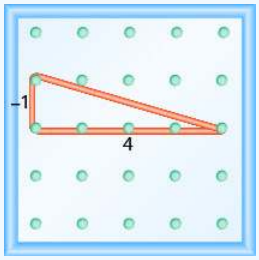
Use the slope formula. $m = \frac{\text{rise}}{\text{run}}$

Replace m with $\frac{-1}{4}$. $\frac{-1}{4} = \frac{\text{rise}}{\text{run}}$

So, the rise is -1 and the run is 4 .

Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down.

We stretch the rubber band down 1 unit, and then right 4 units.

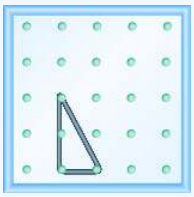


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{-1}{4}$.

Exercise 6.5.11

Model the slope $m = \frac{-2}{3}$. Draw a picture to show your results.

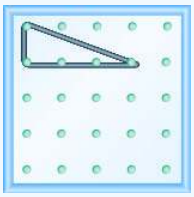
Answer



Exercise 6.5.12

Model the slope $m = \frac{-1}{3}$. Draw a picture to show your results.

Answer



Use $m = \frac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

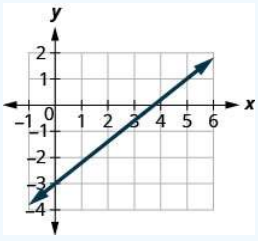
Now, we'll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method will be very similar to what we just modeled on our geoboards.

To find the slope, we must count out the rise and the run. But where do we start?

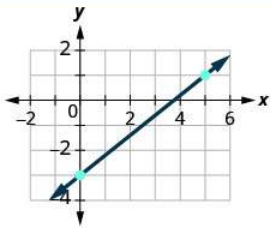
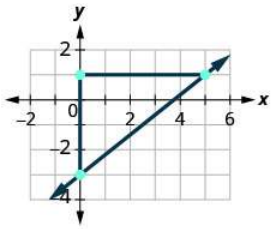
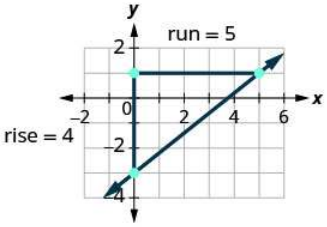
We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

Exercise 6.5.13:

Find the slope of the line shown.

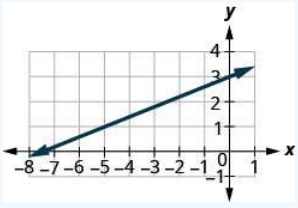


Answer

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	
Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.
Step 4. Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$	Use the slope formula. Substitute the values of the rise and run.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

Exercise 6.5.14

Find the slope of the line shown.



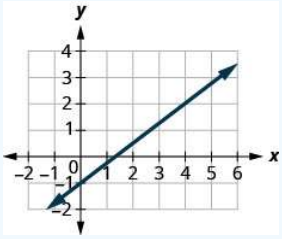
Answer

$\frac{2}{5}$

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Exercise 6.5.15

Find the slope of the line shown.



Answer

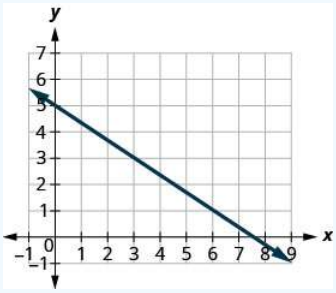
$\frac{3}{4}$

FIND THE SLOPE OF A LINE FROM ITS GRAPH

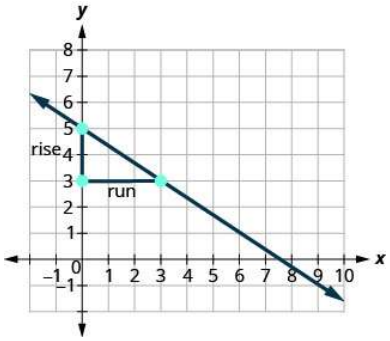
1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$.

Exercise 6.5.16

Find the slope of the line shown.

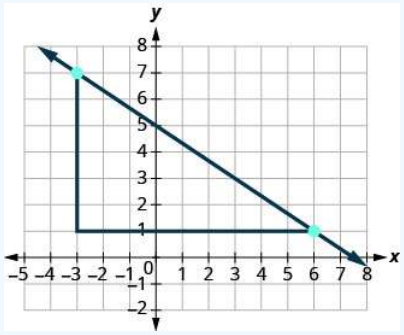


Answer

Locate two points on the graph whose coordinates are integers.	(0,5) and (3,3)
Which point is on the left?	(0,5)
Starting at (0,5), sketch a right triangle to (3,3).	
Count the rise—it is negative.	The rise is −2.
Count the run.	The run is 3.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y increases by 3 units as xx decreases by 2 units.

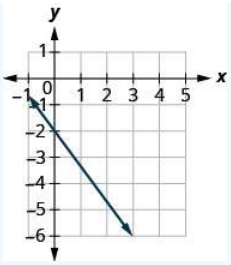
What if we used the points (−3,7) and (6,1) to find the slope of the line?



The rise would be −6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$. Remember, it does not matter which points you use—the slope of the line is always the same.

? Exercise 6.5.17

Find the slope of the line shown.

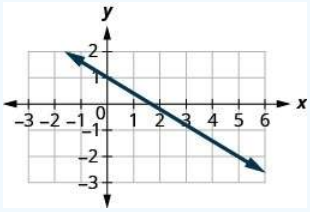


Answer

$-\frac{4}{3}$

Exercise 6.5.18

Find the slope of the line shown.



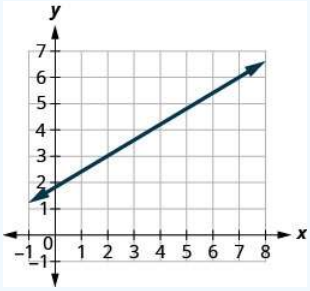
Answer

$-\frac{3}{5}$

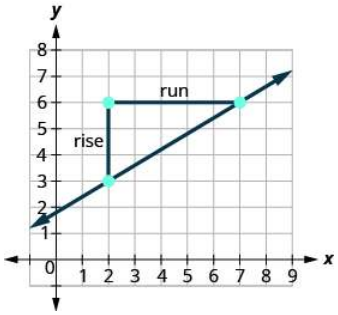
In the last two examples, the lines had *y*-intercepts with integer values, so it was convenient to use the *y*-intercept as one of the points to find the slope. In the next example, the *y*-intercept is a fraction. Instead of using that point, we'll look for two other points whose coordinates are integers. This will make the slope calculations easier.

Exercise 6.5.19

Find the slope of the line shown.



Answer

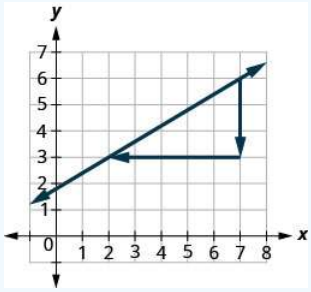
Locate two points on the graph whose coordinates are integers.	(2,3) and (7,6)
Which point is on the left?	(2,3)
Starting at (2,3), sketch a right triangle to (7,6).	
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

This means that y increases 5 units as x increases 3 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let's use the points (2,3) and (7,6) again, but now we'll start at (7,6).

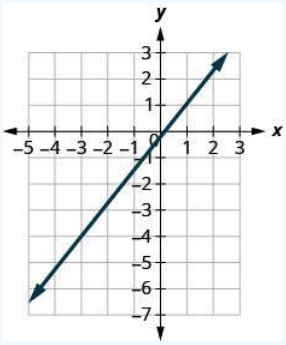


Count the rise.	**The rise is −3.**
Count the run. It goes from right to left, so it is negative.	**The run is−5.**
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5}$
	The slope of the line is $\frac{3}{5}$

It does not matter where you start—the slope of the line is always the same.

Exercise 6.5.20

Find the slope of the line shown.

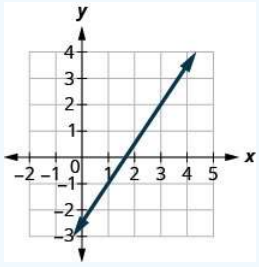


Answer

$\frac{5}{4}$

Exercise 6.5.21

Find the slope of the line shown.



Answer

$\frac{3}{2}$

Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $y = b$	**Vertical line** $x = a$	(6.5.4)
y **-coordinates are the same.**	x **-coordinates are the same.**	

So how do we find the slope of the horizontal line y=4y=4? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.

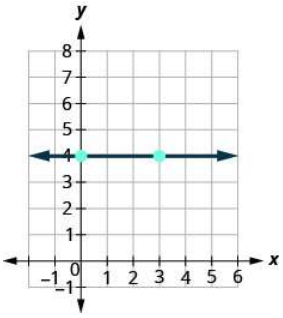


Figure 6.5.7

What is the rise?	The rise is 0.
What is the run?	The run is 3.
	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{0}{3}$
What is the slope?	$m = 0$
	The slope of the horizontal line $y = 4$ is 0.

All horizontal lines have slope 0. When the *y*-coordinates are the same, the rise is 0.

SLOPE OF A HORIZONTAL LINE

The slope of a horizontal line, y=b, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.

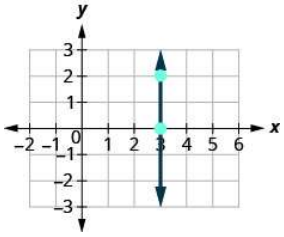


Figure 6.5.8

What is the rise?	The rise is 2.
What is the run?	The run is 0.
	$m = \frac{\text{rise}}{\text{run}}$
What is the slope?	$m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line x=3x=3 is undefined.

The slope of any vertical line is undefined. When the *x*-coordinates of a line are all the same, the run is 0.

SLOPE OF A VERTICAL LINE

The slope of a vertical line, x=a, is undefined.

Exercise 6.5.22

Find the slope of each line:

ⓐ x=8 ⓑ y=−5.

Answer

ⓐ x=8
This is a vertical line.
Its slope is undefined.

ⓑ y=−5
This is a horizontal line.
It has slope 0.

? Exercise 6.5.23

Find the slope of the line: x=−4.

Answer

undefined

? Exercise 6.5.24

Find the slope of the line: y=7.

Answer

0

QUICK GUIDE TO THE SLOPES OF LINES

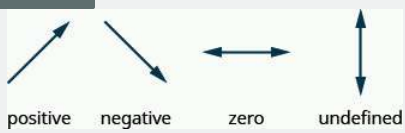


Figure 6.5.9

Remember, we 'read' a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Doing the Manipulative Mathematics activity "Slope of Lines Between Two Points" will help you develop a better understanding of how to find the slope of a line between two points.

Sometimes we'll need to find the slope of a line between two points when we don't have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we'll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x,y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x,y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

$$\begin{array}{ll}(x_1, y_1) & \text{read } `x \text{ sub } 1, y \text{ sub } 1'\\ (x_2, y_2) & \text{read } `x \text{ sub } 2, y \text{ sub } 2'\end{array} \tag{6.5.5}$$

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points (2,3) and (7,6).

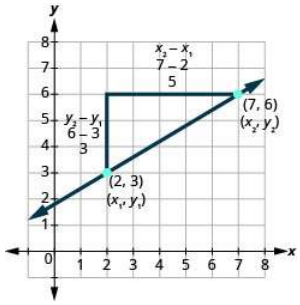


Figure 6.5.10

Since we have two points, we will use subscript notation, $\begin{pmatrix} x_1, y_1 \\ 2, 3 \end{pmatrix} \begin{pmatrix} x_2, y_2 \\ 7, 6 \end{pmatrix}$.

On the graph, we counted the rise of 3 and the run of 5.

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3.

$$3 = 6 - 3 \tag{6.5.6}$$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2.

$$5 = 7 - 2 \tag{6.5.7}$$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$

But 6 is y2, the *y*-coordinate of the second point and 3 is y1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y2-y1}{7-2}$

Also, 7 is x2, the *x*-coordinate of the second point and 2 is x1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y2-y1}{x2-x1}$

We've shown that $m = \frac{y2-y1}{x2-x1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

SLOPE FORMULA

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{6.5.8}$$

This is the **slope formula**.

The slope is:

$$\begin{gathered} y \text{ of the second point minus } y \text{ of the first point} \\ \text{over} \\ x \text{ of the second point minus } x \text{ of the first point.} \end{gathered} \tag{6.5.9}$$

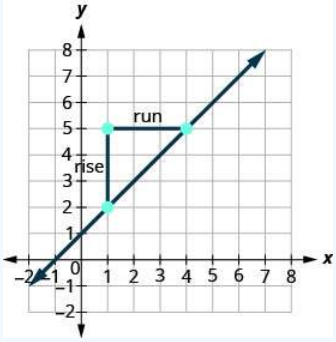
Exercise 6.5.25

Use the **slope formula** to find the slope of the line between the points (1,2) and (4,5).

Answer

Misplaced '#'

Let's confirm this by counting out the slope on a graph using $m = \frac{\text{rise}}{\text{run}}$.



It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

Exercise 6.5.26

Use the slope formula to find the slope of the line through the points: (8,5) and (6,3).

Answer

1

Exercise 6.5.27

Use the slope formula to find the slope of the line through the points: (1,5) and (5,9).

Answer

1

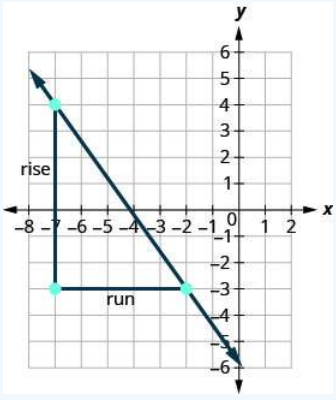
Exercise 6.5.28

Use the slope formula to find the slope of the line through the points (−2,−3) and (−7,4).

Answer

Misplaced '#'

Let's verify this slope on the graph shown.



$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{-7}{5} \tag{6.5.10}$$
$$m = -\frac{7}{5}$$

? Exercise 6.5.29

Use the slope formula to find the slope of the line through the points: (−3,4) and (2,−1).

Answer

-1

? Exercise 6.5.30

Use the slope formula to find the slope of the line through the pair of points: (−2,6) and (−3,−4).

Answer

10

Graph a Line Given a Point and the Slope

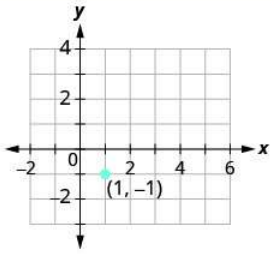
Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

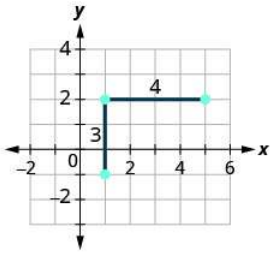
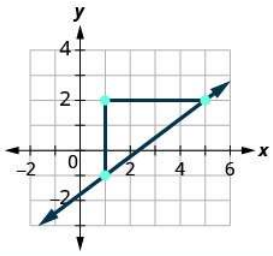
One other method we can use to graph lines is called the **point–slope method**. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

? Exercise 6.5.31

Graph the line passing through the point (1,−1) whose slope is $m = \frac{3}{4}$.

Answer

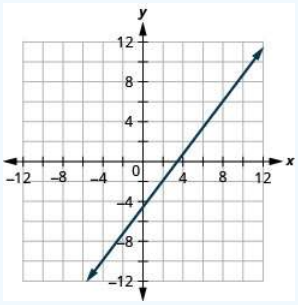
Step 1. Plot the given point.	Plot (1, –1).	
Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ rise = 3 run = 4

Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	
Step 4. Connect the points with a line.	Connect the two points with a line.	

Exercise 6.5.32

Graph the line passing through the point $(2,-2)$ with the slope $m = \frac{4}{3}$.

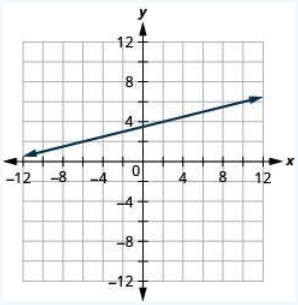
Answer



Exercise 6.5.33

Graph the line passing through the point $(-2,3)$ with the slope $m = \frac{1}{4}$.

Answer



GRAPH A LINE GIVEN A POINT AND THE SLOPE.

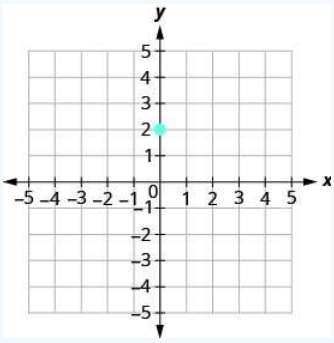
1. Plot the given point.
2. Use the slope formula $m = \frac{\text{rise}}{\text{rise}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

Exercise 6.5.34

Graph the line with *y*-intercept 2 whose slope is $m = -\frac{2}{3}$.

Answer

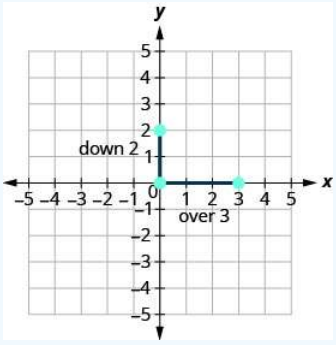
Plot the given point, the *y*-intercept, (0,2).



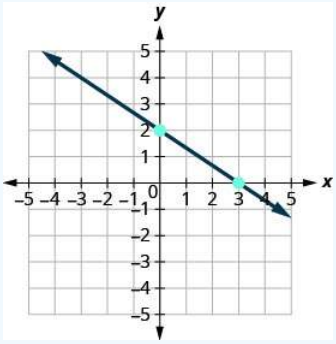
Identify the rise and the run. $m = -\frac{2}{3}$

$$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$$
$$\text{rise} = -2$$
$$\text{run} = 3$$

Count the rise and the run. Mark the second point.



Connect the two points with a line.

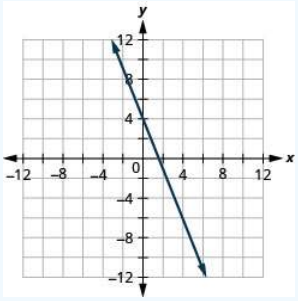


You can check your work by finding a third point. Since the slope is $m = -\frac{2}{3}$, it can be written as $m = \frac{2}{-3}$. Go back to (0,2) and count out the rise, 2, and the run, −3.

Exercise 6.5.35

Graph the line with the *y*-intercept 4 and slope $m = -\frac{5}{2}$.

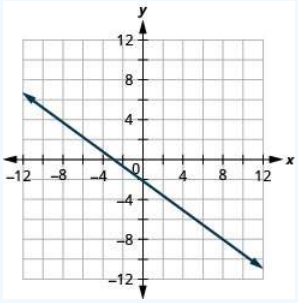
Answer



Exercise 6.5.36

Graph the line with the *x*-intercept −3 and slope $m = -\frac{3}{4}$.

Answer

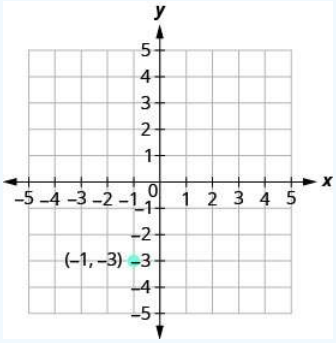


Exercise 6.5.37

Graph the line passing through the point (−1,−3) whose slope is m=4.

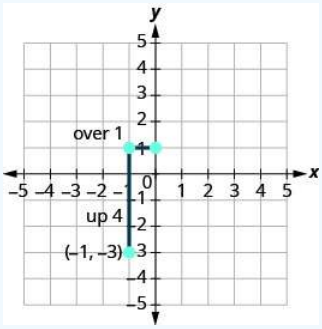
Answer

Plot the given point.

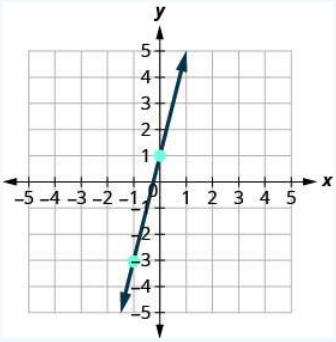


Identify the rise and the run.	$m = 4$
Write 4 as a fraction.	$\frac{\text{rise}}{\text{run}} = \frac{4}{1}$
	$\text{rise} = 4 \quad \text{run} = 3$

Count the rise and run and mark the second point.



Connect the two points with a line.

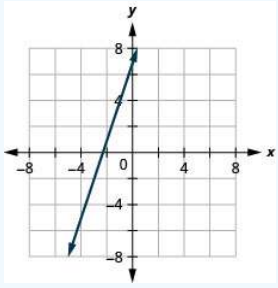


You can check your work by finding a third point. Since the slope is m=4, it can be written as $m = \frac{-4}{-1}$. Go back to (−1,−3) and count out the rise, −4, and the run, −1.

Exercise 6.5.38

Graph the line with the point (−2,1) and slope m=3.

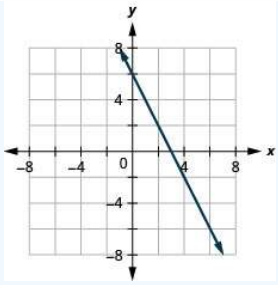
Answer



Exercise 6.5.39

Graph the line with the point (4,−2) and slope m=−2.

Answer

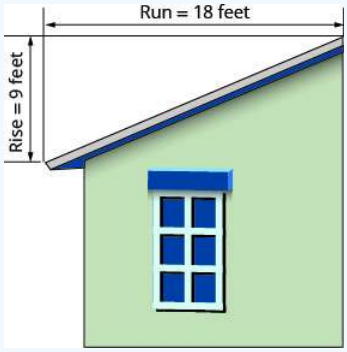


Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

? Exercise 6.5.40

The 'pitch' of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Answer

Use the slope formula.	$m = \frac{\text{rise}}{\text{rise}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$

The slope of the roof is $\frac{1}{2}$.

The roof rises 1 foot for every 2 feet of horizontal run.

? Exercise 6.5.41

Use Exercise **6.5.40**, substituting the rise = 14 and run = 24.

Answer

$\frac{7}{12}$

? Exercise 6.5.42

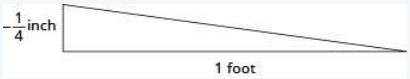
Use Exercise **6.5.40**, substituting rise = 15 and run = 36.

Answer

$\frac{5}{12}$

Exercise 6.5.43

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Answer

Use the slope formula. $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{-\frac{1}{4}\text{ inch}}{1\text{ foot}}$

$m = \frac{-\frac{1}{4}\text{ inch}}{12\text{ inches}}$

Simplify. $m = -\frac{1}{48}$

The slope of the pipe is $-\frac{1}{48}$

The pipe drops 1 inch for every 48 inches of horizontal run.

Exercise 6.5.44

Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot.

Answer

$-\frac{1}{36}$

Exercise 6.5.45

Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard.

Answer

$-\frac{1}{48}$

Access these online resources for additional instruction and practice with understanding slope of a line.

- Practice Slope with a Virtual Geoboard
- Small, Medium, and Large Virtual Geoboards
- Explore Area and Perimeter with a Geoboard

Key Concepts

- **Find the Slope of a Line from its Graph using $m = \frac{\text{rise}}{\text{run}}$**
  1. Locate two points on the line whose coordinates are integers.
  2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
  3. Count the rise and the run on the legs of the triangle.
  4. Take the ratio of rise to run to find the slope.
- **Graph a Line Given a Point and the Slope**
  1. Plot the given point.
  2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  3. Starting at the given point, count out the rise and run to mark the second point.
  4. Connect the points with a line.
- **Slope of a Horizontal Line**
  - The slope of a horizontal line, y=b, is 0.

- **Slope of a vertical line**
  - The slope of a vertical line, x=a, is undefined

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6.6: Slope–Intercept Form of an Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In **Graph Linear Equations in Two Variables**, we graphed the line of the equation $y = 12x + 3$ by plotting points. See Figure 6.6.1. Let's find the slope of this line.

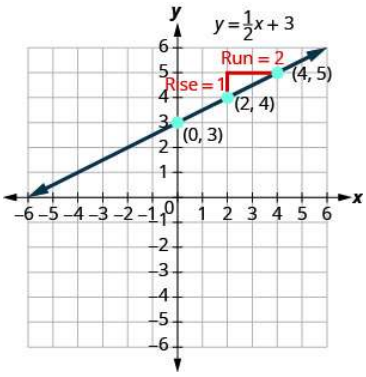


Figure 6.6.1

The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

$$m = \frac{\text{rise}}{\text{rise}} \qquad (6.6.1)$$
$$m = \frac{1}{2}$$

What is the y-intercept of the line? The y-intercept is where the line crosses the y-axis, so y-intercept is $(0, 3)$. The equation of this line is:

$y = \frac{1}{2}x + 3$

Notice, the line has:

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

When a linear equation is solved for y, the coefficient of the x-term is the slope and the constant term is the y-coordinate of the y-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$m = \frac{1}{2}$; y-intercept is $(0, 3)$

$y = \frac{1}{2}x + 3$

$y = mx + b$

SLOPE-INTERCEPT FORM OF AN EQUATION OF A LINE

The **slope–intercept form** of an equation of a line with slope mm and y-intercept, $(0, b)$ is,

$$y = mx + b \tag{6.6.2}$$

Sometimes the slope–intercept form is called the "y-form."

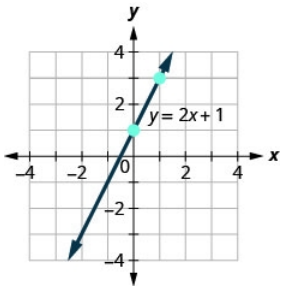
Exercise 6.6.1

Use the graph to find the slope and y-intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Answer

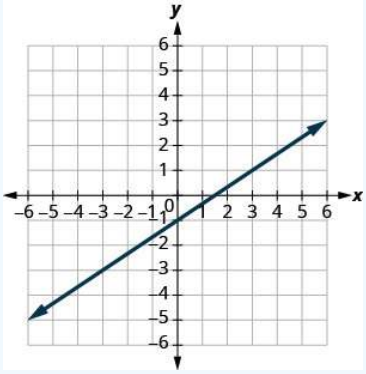
To find the slope of the line, we need to choose two points on the line. We'll use the points $(0, 1)$ and $(1, 3)$.

	
Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the y-intercept of the line.	The y-intercept is the point $(0, 1)$.
We found slope $m = 2$ and y-intercept $(0, 1)$.	$y = 2x + 1$ $y = mx + b$

The slope is the same as the coefficient of x and the y-coordinate of the y-intercept is the same as the constant term.

Exercise 6.6.2

Use the graph to find the slope and y-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.

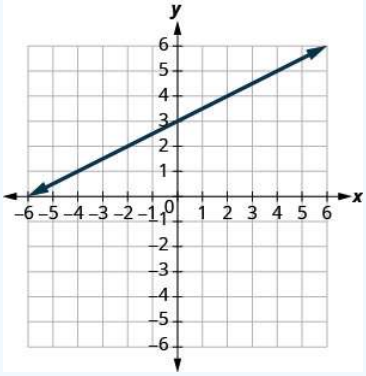


Answer

slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

Exercise 6.6.3

Use the graph to find the slope and y-intercept of the line $y = \frac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.



Answer

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In **Understand Slope of a Line**, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the y-intercept as the point, and then count out the slope from there. Let's practice finding the values of the slope and y-intercept from the equation of a line.

Exercise 6.6.4

Identify the slope and y-intercept of the line with equation $y = -3x + 5$.

Answer

We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the y-intercept.	y-intercept is $(0, 5)$

? Exercise 6.6.5

Identify the slope and y-intercept of the line $y = \frac{2}{5}x - 1$.

Answer

$\frac{2}{5}$; $(0,-1)$

? Exercise 6.6.6

Identify the slope and y-intercept of the line $y = -\frac{4}{3}x + 1$.

Answer

$-\frac{4}{3}$; $(0,1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

? Exercise 6.6.7

Identify the slope and y-intercept of the line with equation $x + 2y = 6$.

Answer

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for y.

Solve for y.	$x + 2y = 6$
Subtract x from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x+6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
(Remember: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$)	
Simplify.	$y = -\frac{1}{2}x + 3$
Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the y-intercept.	y-intercept is $(0, 3)$

Exercise 6.6.8

Identify the slope and y-intercept of the line $x + 4y = 8$.

Answer

$-\frac{1}{4}$;(0,2)

Exercise 6.6.9

Identify the slope and y-intercept of the line $3x + 2y = 12$.

Answer

$-\frac{2}{3}$;(0,6)

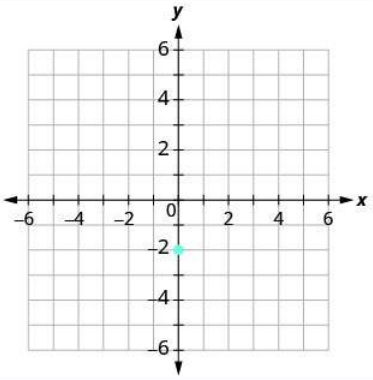
Graph a Line Using its Slope and Intercept

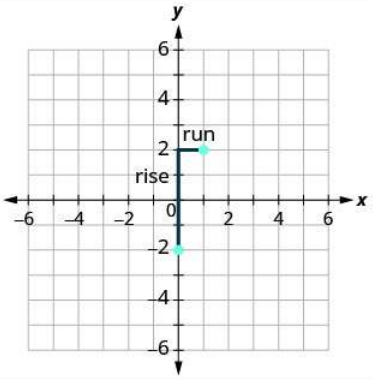
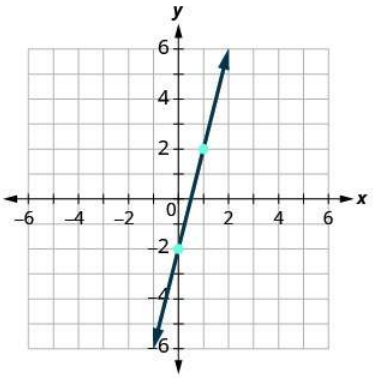
Now that we know how to find the slope and y-intercept of a line from its equation, we can graph the line by plotting the y-intercept and then using the slope to find another point.

Exercise 6.6.10: How to Graph a Line Using its Slope and Intercept

Graph the line of the equation $y = 4x - 2$ using its slope and y-intercept.

Answer

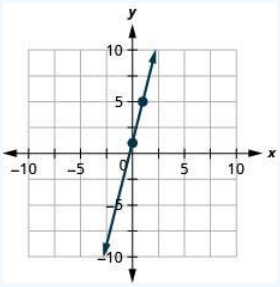
Step 1. Find the slope–intercept form of the equation.	This equation is in slope–intercept form.	$y = 4x - 2$
Step 2. Identify the slope and y-intercept.	Use $y = mx + b$ Find the slope. Find the y-intercept.	$y = mx + b$ $y = 4x + (-2)$ $m = 4$ $b = -2$, $(0, -2)$
Step 3. Plot the y-intercept.	Plot $(0, -2)$.	
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = 4$ $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$ rise $= 4$ run $= 1$

Step 5. Starting at the *y*-intercept, count out the rise and run to mark the second point.	Start at (0, –2) and count the rise and the run. Up 4, right 1.	
Step 6. Connect the points with a line.	Connect the two points with a line.	

Exercise 6.6.11

Graph the line of the equation $y = 4x + 1$ using its slope and y-intercept.

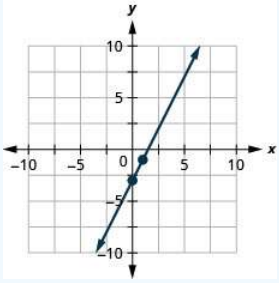
Answer



Exercise 6.6.12

Graph the line of the equation $y = 2x - 3$ using its slope and y-intercept.

Answer



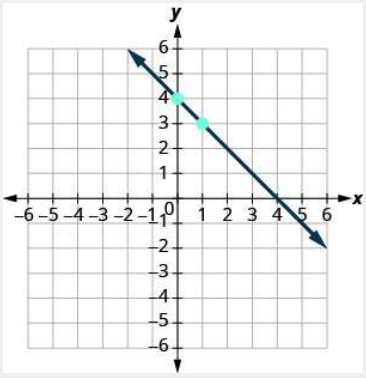
GRAPH A LINE USING ITS SLOPE AND y-INTERCEPT.

1. Find the slope-intercept form of the equation of the line.
2. Identify the slope and y-intercept.
3. Plot the y-intercept.
4. Use the slope formula $\frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
5. Starting at the y-intercept, count out the rise and run to mark the second point.
6. Connect the points with a line.

Exercise 6.6.13

Graph the line of the equation $y = -x + 4$ using its slope and y-intercept.

Answer

	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and y-intercept.	$m = -1$
	y-intercept is $(0, 4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise -1, run 1
Draw the line.	
To check your work, you can find another point on the line and make sure it is a solution of the equation. In the graph we see the line goes through $(4, 0)$.	

check.

$y = -x + 4$

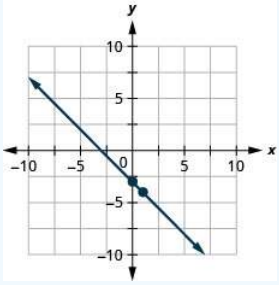
$0 \stackrel{?}{=} -4 + 4$

$0 = 0 \checkmark$

Exercise 6.6.14

Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.

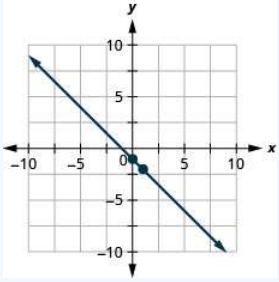
Answer



Exercise 6.6.15

Graph the line of the equation $y = -x - 1$ using its slope and y-intercept.

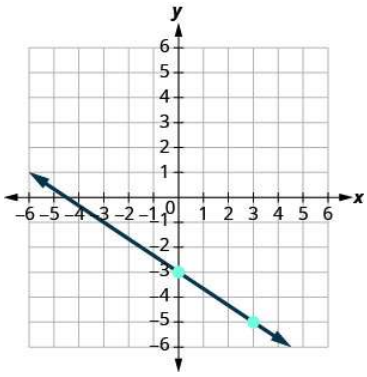
Answer



Exercise 6.6.16

Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and y-intercept.

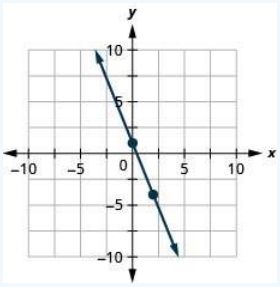
Answer

	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and y-intercept.	$m = -\frac{2}{3}$; y-intercept is $(0, -3)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	
Draw the line.	

Exercise 6.6.17

Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and y-intercept.

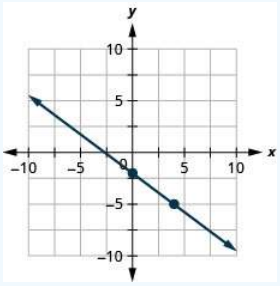
Answer



Exercise 6.6.18

Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and y-intercept.

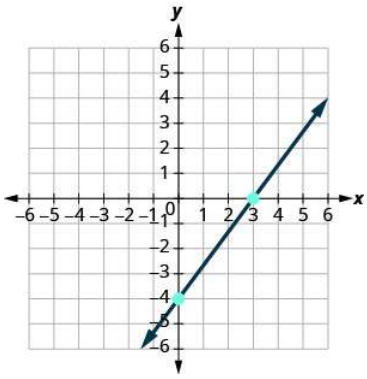
Answer



Exercise 6.6.19

Graph the line of the equation $4x - 3y = 12$ using its slope and y-intercept.

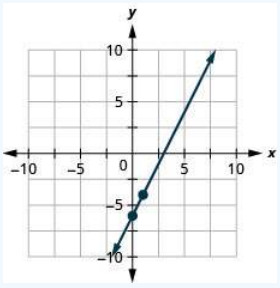
Answer

	$4x - 3y = 12$
Find the slope–intercept form of the equation.	$-3y = -4x + 12$
	$-\frac{3y}{3} = \frac{-4x+12}{-3}$
The equation is now in slope–intercept form.	$y = \frac{4}{3}x - 4$
Identify the slope and y-intercept.	$m = \frac{4}{3}$
	y-ntercept is $(0, -4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

Exercise 6.6.20

Graph the line of the equation $2x - y = 6$ using its slope and y-intercept.

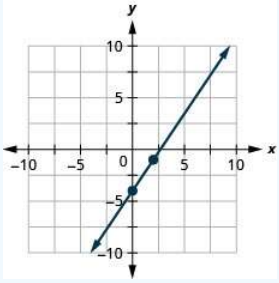
Answer



Exercise 6.6.21

Graph the line of the equation $3x - 2y = 8$ using its slope and y-intercept.

Answer



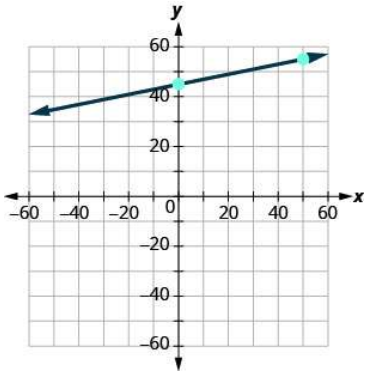
We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

Exercise 6.6.22

Graph the line of the equation $y = 0.2x + 45$ using its slope and y-intercept.

Answer

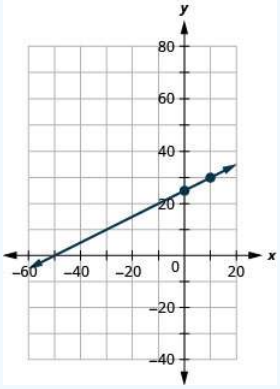
We'll use a grid with the axes going from about -80 to 80.

	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and y-intercept.	$m = 0.2$
	The y-intercept is $(0, 45)$
Plot the y-intercept.	See graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

Exercise 6.6.23

Graph the line of the equation $y = 0.5x + 25$ using its slope and y-intercept.

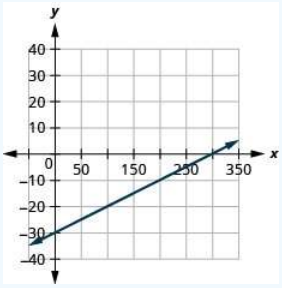
Answer



Exercise 6.6.24

Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Answer



Now that we have graphed lines by using the slope and y-intercept, let's summarize all the methods we have used to graph lines. See Figure 6.6.2.

Methods to Graph Lines			
Point Plotting	**Slope–Intercept**	**Intercepts**	**Recognize Vertical and Horizontal Lines**
x | y	$y = mx + b$	x | y 0 | | 0	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Figure 6.6.2

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope–intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in sections 4.3, 4.4, and earlier in this section. Let's look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

Misplaced '#'

Equations #2 and #3 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #4 and #5, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the x-intercept and $x = 0$ to find the y-intercept, and then found a third point by choosing another value for x or y.

Equations #6 and #7 are written in slope–intercept form. After identifying the slope and y-intercept from the equation we used them to graph the line.

This leads to the following strategy.

STRATEGY FOR CHOOSING THE MOST CONVENIENT METHOD TO GRAPH A LINE

Consider the form of the equation.

- If it only has one variable, it is a vertical or horizontal line.
  - $x = a$ is a vertical line passing through the x-axis at a.
  - $y = b$ is a horizontal line passing through the y-axis at b.
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
  - Identify the slope and y-intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
  - Find the x- and y-intercepts, a third point, and then graph.

Exercise 6.6.25

Determine the most convenient method to graph each line.

1. $y = -6$
2. $5x - 3y = 15$
3. $x = 7$
4. $y = \frac{2}{5}x - 1$.

Answer

1. $y = -6$
   This equation has only one variable, y. Its graph is a horizontal line crossing the y-axis at -6.
2. $5x - 3y = 15$
   This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.
3. $x = 7$
   There is only one variable, x. The graph is a vertical line crossing the x-axis at 7.
4. $y = \frac{2}{5}x - 1$
   Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and y-intercept.

Exercise 6.6.26

Determine the most convenient method to graph each line:

1. $3x + 2y = 12$
2. $y = 4$
3. $y = \frac{1}{5}x - 4$
4. $x = -7$

Answer

1. intercepts
2. horizontal line
3. slope–intercept
4. vertical line

Exercise 6.6.27

Determine the most convenient method to graph each line:

1. $x = 6$
2. $y = -\frac{3}{4}x + 1$
3. $y = -8$
4. $4x - 3y = -1$

Answer

1. vertical line
2. slope–intercept
3. horizontal line
4. intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually when a linear equation models a real-world situation, different letters are used for the variables, instead of x and y. The variable names remind us of what quantities are being measured.

Exercise 6.6.28

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

1. Find the Fahrenheit temperature for a Celsius temperature of 0.
2. Find the Fahrenheit temperature for a Celsius temperature of 20.
3. Interpret the slope and F-intercept of the equation.
4. Graph the equation.

Answer

1. | | |
   |---|---|
   | Find the Fahrenheit temperature for a Celsius temperature of 0. | $F = \frac{9}{5}C + 32$ |
   | Find F when $C = 0$. | $F = \frac{9}{5}(0) + 32$ |
   | Simplify. | $F = 32$ |

2. | | |
   |---|---|
   | Find the Fahrenheit temperature for a Celsius temperature of 20. | $F = \frac{9}{5}C + 32$ |
   | Find F when $C = 20$. | $F = \frac{9}{5}(20) + 32$ |
   | Simplify. | $F = 36 + 32$ |
   | Simplify. | $F = 68$ |

   (6.6.3)

3. Interpret the slope and F-intercept of the equation.

   Even though this equation uses F and C, it is still in slope–intercept form.

$y = mx + b$

$F = mC + b$

$F = \frac{9}{5}C + 32$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees.

The F-intercept means that when the temperature is $0°$ on the Celsius scale, it is $32°$ on the Fahrenheit scale.

4. Graph the equation.

We'll need to use a larger scale than our usual. Start at the F-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See Figure $6.6.3$.

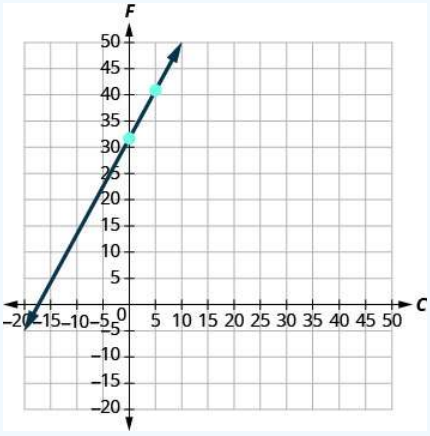


Figure $6.6.3$

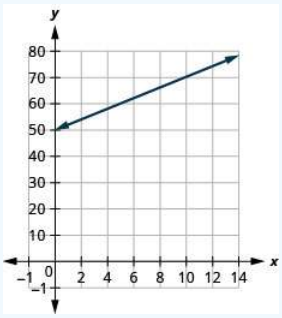
Exercise 6.6.29

The equation $h = 2s + 50$ is used to estimate a woman's height in inches, h, based on her shoe size, s.

1. Estimate the height of a child who wears women's shoe size 0.
2. Estimate the height of a woman with shoe size 8.
3. Interpret the slope and h-intercept of the equation.
4. Graph the equation.

Answer

1. 50 inches
2. 66 inches
3. The slope, 2, means that the height, h, increases by 2 inches when the shoe size, s, increases by 1. The h-intercept means that when the shoe size is 0, the height is 50 inches.
4.

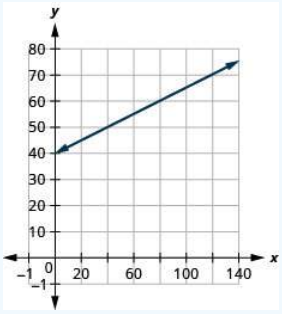


Exercise 6.6.30

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, T, based on the number of cricket chirps, n, in one minute.

1. Estimate the temperature when there are no chirps.
2. Estimate the temperature when the number of chirps in one minute is 100.
3. Interpret the slope and T-intercept of the equation.
4. Graph the equation.

Answer

1. 40 degrees
2. 65 degrees
3. The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (F) increases 1 degree when the number of chirps, n, increases by 4. The T-intercept means that when the number of chirps is 0, the temperature is $40°$.
4. 

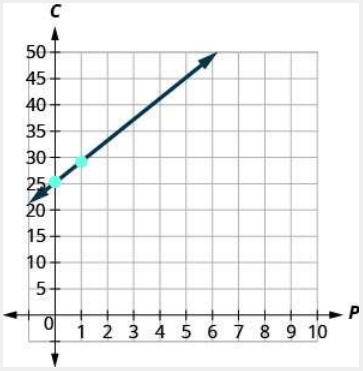
The cost of running some types business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labor needed to produce each item.

Exercise 6.6.31

Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C, in dollars and the number of pizzas, p, that she sells.

1. Find Stella's cost for a week when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is $25 when she sells no pizzas.
2. Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are $85 when she sells 15 pizzas.
3. Interpret the slope and C-intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by $4 for each pizza Stella sells. The C-intercept means that even when Stella sells no pizzas, her costs for the week are $25.
4. Graph the equation. We'll need to use a larger scale than our usual. Start at the C-intercept $(0, 25)$ then count out the rise of 4 and the run of 1 to get a second point.	

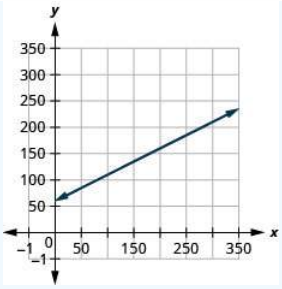
Exercise 6.6.32

Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, C, in dollars and the number of miles, m, that he drives.

1. Find Sam's cost for a week when he drives 0 miles.
2. Find the cost for a week when he drives 250 miles.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $60
2. $185
3. The slope, 0.5, means that the weekly cost, C, increases by $0.50 when the number of miles driven, n, increases by 1. The C-intercept means that when the number of miles driven is 0, the weekly cost is $60.
4.



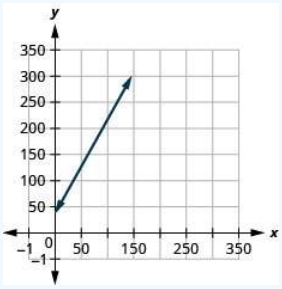
? Exercise 6.6.33

Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, C, in dollars and the number of wedding invitations, n, that she writes.

1. Find Loreen's cost for a week when she writes no invitations.
2. Find the cost for a week when she writes 75 invitations.
3. Interpret the slope and C-intercept of the equation.
4. Graph the equation.

Answer

1. $35
2. $170
3. The slope, 1.8, means that the weekly cost, C, increases by $1.80 when the number of invitations, n, increases by 1.80. The C-intercept means that when the number of invitations is 0, the weekly cost is $35.
4.



Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.

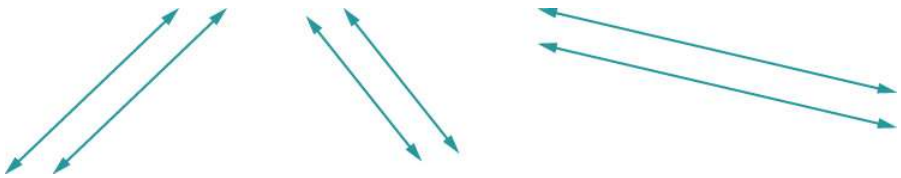


Figure 6.6.4.

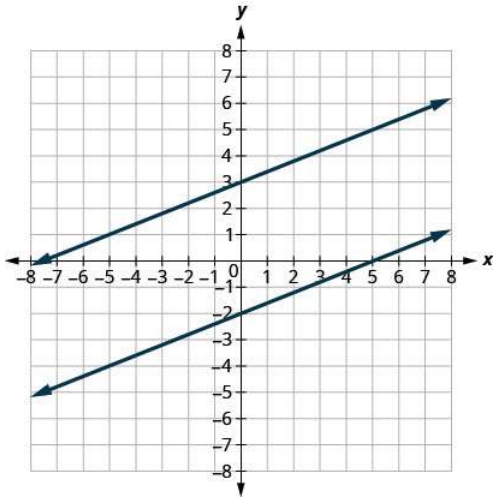


Figure **6.6.4**: Verify that both lines have the same slope, $m = \frac{2}{5}$, and different y-intercepts.

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different x-intercepts are parallel. See Figure **6.6.5**.

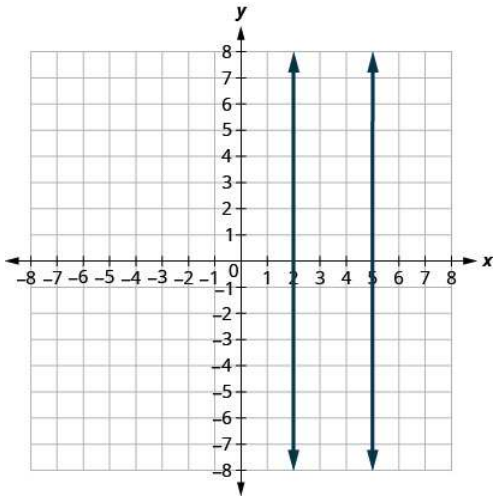


Figure **6.6.5**: Vertical lines with different *x*-intercepts are parallel.

PARALLEL LINES

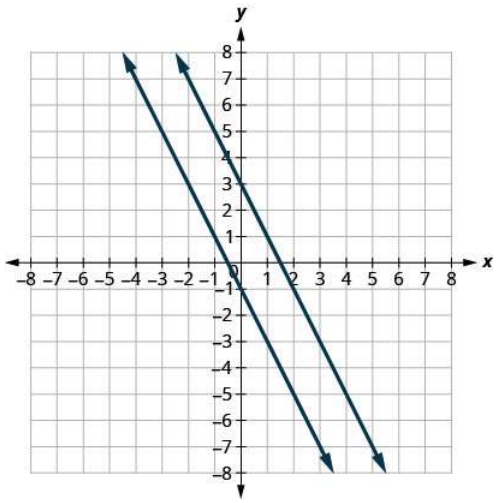
Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope–intercept form: $y = -2x + 3$. We solve the second equation for y:

$$\begin{aligned} 2x + y &= -1 \\ y &= -2x - 1 \end{aligned} \tag{6.6.4}$$

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the y-intercept of each line?

$$\begin{array}{lll} y & = mx + b & y = mx + b \\ y & = -2x + 3 & y = -2x - 1 \\ m & = -2 & m = -2 \\ b & = 3, (0, 3) & b = -1, (0, -1) \end{array} \tag{6.6.5}$$

The slopes of the lines are the same and the y-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different y-intercepts, we can now just look at the slope–intercept form of the equations of lines and decide if the lines are parallel.

? Exercise 6.6.34

Use slopes and y-intercepts to determine if the lines $3x - 2y = 6$ and $y = \frac{3}{2}x + 1$ are parallel.

Answer

Solve the first equation for y.

$$\begin{aligned} 3x - 2y &= 6 \\ \tfrac{-2y}{-2} &= -3x + 6 \\ \tfrac{-2y}{-2} &= \tfrac{-3x+6}{-2} \\ y &= \tfrac{3}{2}x - 3 \end{aligned}$$

The equation is now in slope–intercept form.

The equation of the second line is already in slope–intercept form.

Identify the slope and y-intercept of both lines.

$y = \frac{3}{2}x + 1$	$y = \frac{3}{2}x - 3$
$y = mx + b$	$y = mx + b$
$m = \frac{3}{2}$	$m = \frac{3}{2}$
y-intercept is $(0, 1)$	y-intercept is $(0, -3)$

The lines have the same slope and different y-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

? Exercise 6.6.35

Use slopes and y-intercepts to determine if the lines $2x + 5y = 5$ and $y = -\frac{2}{5}x - 4$ are parallel.

Answer

parallel

Exercise 6.6.36

Use slopes and y-intercepts to determine if the lines $4x - 3y = 6$ and $y = \frac{4}{3}x - 1$ are parallel.

Answer

parallel

Exercise 6.6.37

Use slopes and y-intercepts to determine if the lines $y = -4$ and $y = 3$ are parallel.

Answer

Write each equation in slope-intercept form.	$y = -4$	**and**	$y = 3$
Since there is no x term we write $0x$.	$y = 0x - 4$		$y = 0x + 3$
Identify the slope and y-intercept of both lines.	$y = mx + b$		$y = mx + b$
	$m = 0$		$m = 0$
	y-intercept is $(0, 4)$		**y-intercept is $(0, 3)$**

The lines have the same slope and different y-intercepts and so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y-axis at $y = -4$ and at $y = 3$, we know the y-intercepts are $(0, -4)$ and $(0, 3)$. The lines have the same slope and different y-intercepts and so they are parallel.

Exercise 6.6.38

Use slopes and y-intercepts to determine if the lines $y = 8$ and $y = -6$ are parallel.

Answer

parallel

Exercise 6.6.39

Use slopes and y-intercepts to determine if the lines $y = 1$ and $y = -5$ are parallel.

Answer

parallel

Exercise 6.6.40

Use slopes and y-intercepts to determine if the lines $x = -2$ and $x = -5$ are parallel.

Answer

$$x = -2 \text{ and } x = -5 \tag{6.6.6}$$

Since there is no y, the equations cannot be put in slope–intercept form. But we recognize them as equations of vertical lines. Their x-intercepts are -2 and -5. Since their x-intercepts are different, the vertical lines are parallel.

Exercise 6.6.41

Use slopes and y-intercepts to determine if the lines $x = 1$ and $x = -5$ are parallel.

Answer

parallel

Exercise 6.6.42

Use slopes and y-intercepts to determine if the lines $x = 8$ and $x = -6$ are parallel.

Answer

parallel

Exercise 6.6.43

Use slopes and y-intercepts to determine if the lines $y = 2x - 3$ and $-6x + 3y = -9$ are parallel. You may want to graph these lines, too, to see what they look like.

Answer

The first equation is already in slope-intercept form. $y = 2x - 3$

Solve the second equation for y

$$\begin{aligned} -6x + 3y &= -9 \\ 3y &= 6x - 9 \\ \tfrac{3y}{3} &= \tfrac{6x-9}{3} \\ y &= 2x - 3 \end{aligned}$$

The second equation is now in slope–intercept form as well.

Identify the slope and y-intercept of both lines.

$y = 2x - 3$	$y = 2x - 3$
$y = mx + b$	$y = mx + b$
$m = 2$	$m = 2$
The y-intercept is $(0, -3)$	The y-intercept is $(0, -3)$

The lines have the same slope, but they also have the same y-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Exercise 6.6.44

Use slopes and y-intercepts to determine if the lines $y = -\frac{1}{2}x - 1$ and $x + 2y = 2$ are parallel.

Answer

not parallel; same line

Exercise 6.6.45

Use slopes and y-intercepts to determine if the lines $y = \frac{3}{4}x - 3$ and $3x - 4y = 12$ are parallel.

Answer

not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y = \frac{1}{4}x - 1$ and $y = -4x + 2$, shown in Figure 6.6.5.

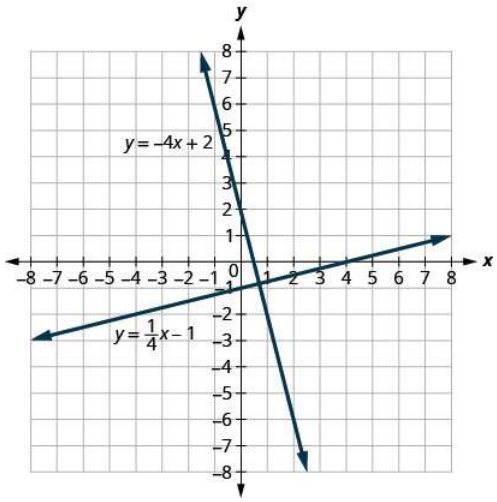


Figure 6.6.5

These lines lie in the same plane and intersect in right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y = 14x - 1$ rises, so its slope is positive. The line $y = -4x + 2$ drops from left to right, so it has a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1 = 14$, and the slope of the second line, $m_2 = -4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

$$\begin{array}{c} m_1 \cdot m_2 \\ \frac{1}{4}(-4) \\ -1 \end{array} \tag{6.6.7}$$

This is always true for **perpendicular lines** and leads us to this definition.

PERPENDICULAR LINES

Perpendicular lines are lines in the same plane that form a right angle.

If m1 and m2 are the slopes of two perpendicular lines, then:

$$m_1 \cdot m_2 = -1 \text{ and } m_1 = \frac{-1}{m_2} \tag{6.6.8}$$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope–intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope–intercept form of the equation, and then see if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same y-intercepts.

Exercise 6.6.46

Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Answer

The first equation is already in slope–intercept form: $y = -5x - 4$

Solve the second equation for y.

$$\begin{aligned} x - 5y &= 5 \\ -5y &= -x + 5 \\ \tfrac{-5y}{-5} &= \tfrac{-x+5}{-5} \\ y &= \tfrac{1}{5}x - 1 \end{aligned}$$

The second equation is now in slope-intercept form as well.

Identify the slope of each line.

$$\begin{aligned} y &= -5x - 4 & y &= \tfrac{1}{5}x - 1 \\ y &= mx + b & y &= mx + b \\ m_1 &= -5 & m_2 &= \tfrac{1}{5} \end{aligned}$$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

$$\begin{aligned} & m_1 \cdot m_2 \\ & -5\left(\tfrac{1}{5}\right) \\ & -1\checkmark \end{aligned} \tag{6.6.9}$$

Exercise 6.6.47

Use slopes to determine if the lines $y = -3x + 2$ and $x - 3y = 4$ are perpendicular.

Answer

perpendicular

Exercise 6.6.48

Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Answer

perpendicular

Exercise 6.6.49

Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Answer

Solve the equations for y.

$$\begin{aligned} 7x + 2y &= 3 & 2x + 7y &= 5 \\ 2y &= -7x + 3 & 7y &= -2x + 5 \\ \tfrac{2y}{2} &= \tfrac{-7x+3}{2} & \tfrac{7y}{7} &= \tfrac{-2x+5}{7} \\ y &= -\tfrac{7}{2}x + \tfrac{3}{2} & y &= \tfrac{-2}{7}x + \tfrac{5}{7} \end{aligned}$$

Identify the slope of each line.

$$\begin{aligned} y &= mx + b & y &= mx + b \\ m_1 &= -\tfrac{7}{2} & m_2 &= -\tfrac{2}{7} \end{aligned}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Exercise 6.6.50

Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Answer

not perpendicular

Exercise 6.6.51

Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Answer

not perpendicular

Note

Access this online resource for additional instruction and practice with graphs.

- Explore the Relation Between a Graph and the Slope–Intercept Form of an Equation of a Line

Key Concepts

- The slope–intercept form of an equation of a line with slope mm and y-intercept, $(0, b)$ is, $y = mx + b$.
- **Graph a Line Using its Slope and y-Intercept**
  1. Find the slope-intercept form of the equation of the line.
  2. Identify the slope and y-intercept.
  3. Plot the y-intercept.
  4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  5. Starting at the y-intercept, count out the rise and run to mark the second point.
  6. Connect the points with a line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    $x = a$ is a vertical line passing through the x-axis at a.
    $y = b$ is a horizontal line passing through the y-axis at b.
  - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
    Identify the slope and y-intercept and then graph.
  - If the equation is of the form $Ax + By = C$, find the intercepts.
    Find the x- and y-intercepts, a third point, and then graph.
- Parallel lines are lines in the same plane that do not intersect.
  - Parallel lines have the same slope and different y-intercepts.
  - If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
  - Parallel vertical lines have different x-intercepts.
- Perpendicular lines are lines in the same plane that form a right angle.
  - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and $m_1 = \frac{-1}{m_2}$.
  - Vertical lines and horizontal lines are always perpendicular to each other.

Glossary

parallel lines

Lines in the same plane that do not intersect.

perpendicular lines

Lines in the same plane that form a right angle.

slope-intercept form of an equation of a line

The slope–intercept form of an equation of a line with slope mm and y-intercept, $(0, b)$ is, $y = mx + b$.

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6.7: Find the Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Find an equation of the line given the slope and y-intercept
- Find an equation of the line given the slope and a point
- Find an equation of the line given two points
- Find an equation of a line parallel to a given line
- Find an equation of a line perpendicular to a given line

How do online retailers know that 'you may also like' a particular item based on something you just ordered? How can economists know how a rise in the minimum wage will affect the unemployment rate? How do medical researchers create drugs to target cancer cells? How can traffic engineers predict the effect on your commuting time of an increase or decrease in gas prices? It's all mathematics.

You are at an exciting point in your mathematical journey as the mathematics you are studying has interesting applications in the real world.

The physical sciences, social sciences, and the business world are full of situations that can be modeled with linear equations relating two variables. Data is collected and graphed. If the data points appear to form a straight line, an equation of that line can be used to predict the value of one variable based on the value of the other variable.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and *y*-Intercept

We can easily determine the slope and intercept of a line if the equation was written in slope–intercept form, y=mx+b. Now, we will do the reverse—we will start with the slope and *y*-intercept and use them to find the equation of the line.

Exercise 6.7.1

Find an equation of a line with slope −7 and *y*-intercept (0,−1).

Answer

Since we are given the slope and *y*-intercept of the line, we can substitute the needed values into the slope–intercept form, y=mx+b.

Name the slope.	$m=-7$
Name the *y*-intercept.	*y*-intercept $(0,-1)$
Substitute the values into y=mx+b.	$y=mx+b$
	$y=-7x+(-1)$
	$y=-7x+-1$

Exercise 6.7.2

Find an equation of a line with slope $\frac{2}{5}$ and *y*-intercept (0,4).

Answer

$y = \frac{2}{5}x + 4$

Exercise 6.7.3

Find an equation of a line with slope −1 and y-intercept (0,−3).

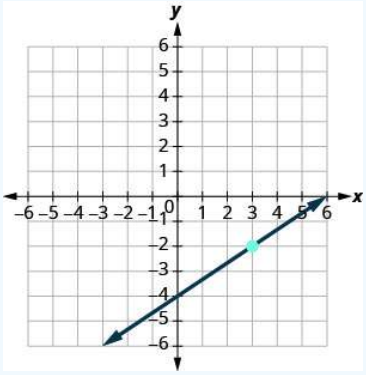
Answer

$y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

Exercise 6.7.4

Find the equation of the line shown.



Answer

We need to find the slope and y-intercept of the line from the graph so we can substitute the needed values into the slope–intercept form, y=mx+by=mx+b.

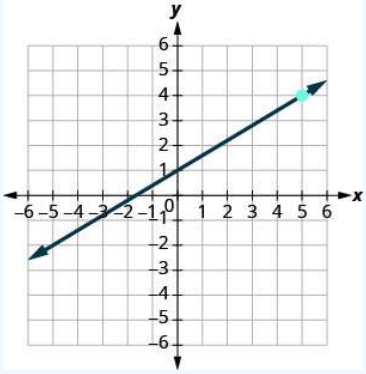
To find the slope, we choose two points on the graph.

The y-intercept is (0,−4) and the graph passes through (3,−2).

Find the slope by counting the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{3}$
Find the y-intercept.	y-intercept (0, −4)
Substitute the values into y=mx+b.	$y = mx + b$
	$y = \frac{2}{3}x - 4$

Exercise 6.7.5

Find the equation of the line shown in the graph.

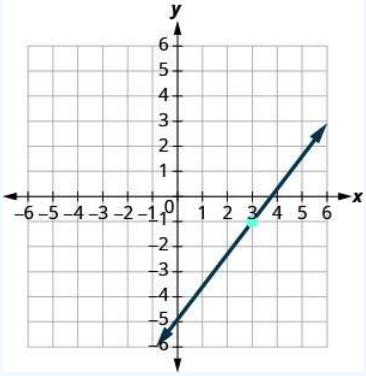


Answer

$y = \frac{3}{5}x + 1$

Exercise 6.7.6

Find the equation of the line shown in the graph.



Answer

$y = \frac{4}{3}x - 5$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope–intercept form of the equation works well when you are given the slope and *y*-intercept or when you read them off a graph. But what happens when you have another point instead of the *y*-intercept?

We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope mm and that contains some specific point (x_1, y_1) and some other point, which we will just call (x,y). We can write the slope of this line and then change it to a different form.

	m	$= \frac{y-y_1}{x-x_1}$
Multiply both sides of the equation by $x - x_1$.	$m(x - x_1)$	$= \left(\frac{y-y_1}{x-x_1}\right)(x - x_1)$
Simplify.	$m(x - x_1)$	$= y - y_1$
Rewrite the equation with the y terms on the left.	$y - y_1$	$= m(x - x_1)$

This format is called the point–slope form of an equation of a line.

POINT–SLOPE FORM OF AN EQUATION OF A LINE

The **point–slope form** of an equation of a line with slope mm and containing the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

We can use the point–slope form of an equation to find an equation of a line when we are given the slope and one point. Then we will rewrite the equation in slope–intercept form. Most applications of linear equations use the the slope–intercept form.

Exercise 6.7.7: Find an Equation of a Line Given the Slope and a Point

Find an equation of a line with slope $m = \frac{2}{5}$ that contains the point (10,3). Write the equation in slope–intercept form.

Answer

Step 1. Identify the slope.	The slope is given.	$m = \frac{2}{5}$
Step 2. Identify the point.	The point is given.	$\underset{10,}{\overset{x_1,}{(}}\ \underset{3)}{\overset{y_1}{}}$
Step 3. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.		$y - y_1 = m(x - x_1)$ $y - 3 = \frac{2}{5}(x - 10)$
	Simplify.	$y - 3 = \frac{2}{5}x - 4$
Step 4. Write the equation in slope–intercept form.		$y = \frac{2}{5}x - 1$

Exercise 6.7.8

Find an equation of a line with slope $m = \frac{5}{6}$ and containing the point (6,3).

Answer

$y = \frac{5}{6}x - 2$

Exercise 6.7.9

Find an equation of a line with slope $m = \frac{2}{3}$ and containing the point (9,2).

Answer

$y = \frac{2}{3}x - 4$

FIND AN EQUATION OF A LINE GIVEN THE SLOPE AND A POINT.

1. Identify the slope.
2. Identify the point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Exercise 6.7.10

Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point (6,−4). Write the equation in slope–intercept form.

Answer

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1$

$= m(x - x_1)$.

Identify the slope.	$m = -\frac{1}{3}$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ 6, & -4 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - (-4) = -\frac{1}{3}(x - 6)$
Simplify.	$y + 4 = -\frac{1}{3}x + 2$
Write in slope–intercept form.	$y = -\frac{1}{3}x - 2$

Exercise 6.7.11

Find an equation of a line with slope $m = -\frac{2}{5}$ and containing the point (10,−5).

Answer

$y = -\frac{2}{5}x - 1$

Exercise 6.7.12

Find an equation of a line with slope $m = -\frac{3}{4}$, and containing the point (4,−7).

Answer

$y = -\frac{3}{4}x - 4$

Exercise 6.7.13

Find an equation of a horizontal line that contains the point (−1,2). Write the equation in slope–intercept form.

Answer

Every horizontal line has slope 0. We can substitute the slope and points into the point–slope form, $y - y_1 = m(x - x_1)$.

Identify the slope.	$m = 0$
Identify the point.	$\begin{matrix} x_1, & y_1 \\ -1, & 2 \end{matrix}$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y - 2 = 0(x - (-1))$
Simplify.	$y - 2 = 0(x + 1)$
	$y - 2 = 0$
	$y = 2$
Write in slope–intercept form.	It is in *y*-form, but could be written y=0x+2.

Did we end up with the form of a horizontal line, y=a?

Exercise 6.7.14

Find an equation of a horizontal line containing the point (−3,8).

Answer

y = 8

Exercise 6.7.15

Find an equation of a horizontal line containing the point (−1,4).

Answer

y = 4

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example we'll see how to find an equation of a line when just two points are given.

We have two options so far for finding an equation of a line: slope–intercept or point–slope. Since we will know two points, it will make more sense to use the point–slope form.

But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation.

Exercise 6.7.16: Find an Equation of a Line Given Two Points

Find an equation of a line that contains the points (5,4) and (3,6). Write the equation in slope–intercept form.

Answer

Step 1. Find the slope using the given points.	To use the point-slope form, we first find the slope.	$m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{6-4}{3-5}$ $m=\frac{2}{-2}$ $m=-1$
Step 2. Choose one point.	Choose either point.	$(\overset{x_1}{5},\ \overset{y_1}{4})$
Step 3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-4=-1(x-5)$ $y-4=-1x+5$
Step 4. Write the equation in slope–intercept form.		$y=-1x+9$

Use the point (3,6) and see that you get the same equation.

Exercise 6.7.17

Find an equation of a line containing the points (3,1) and (5,6).

Answer

$y = \frac{5}{2}x - \frac{13}{2}$

Exercise 6.7.18

Find an equation of a line containing the points (1,4) and (6,2).

Answer

$y = -\frac{2}{5}x + \frac{22}{5}$

FIND AN EQUATION OF A LINE GIVEN TWO POINTS.

1. Find the slope using the given points.
2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
4. Write the equation in slope–intercept form.

Exercise 6.7.19

Find an equation of a line that contains the points (−3,−1) and (2,−2). Write the equation in slope–intercept form.

Answer

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

Find the slope of the line through (−3, −1) and (2, −2).	$m = \frac{y_2 - y_1}{x_2 - x_1}$
	$m = \frac{-2-(-1)}{2-(-3)}$
	$m = \frac{-1}{5}$
	$m = -\frac{1}{5}$
Choose either point.	(x_1, y_1) $(2, -2)$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$
	$y-(-2) = -\frac{1}{5}(x-2)$
	$y + 2 = -\frac{1}{5}x + \frac{2}{5}$
Write in slope–intercept form.	$y = -\frac{1}{5}x - \frac{8}{5}$

Exercise 6.7.20

Find an equation of a line containing the points (−2,−4) and (1,−3).

Answer

$y = \frac{1}{3}x - \frac{10}{3}$

Exercise 6.7.21

Find an equation of a line containing the points (−4,−3) and (1,−5).

Answer

$y = -\frac{2}{5}x - \frac{23}{5}$

Exercise 6.7.22

Find an equation of a line that contains the points (−2,4) and (−2,−3). Write the equation in slope–intercept form.

Answer

Again, the first step will be to find the slope.

Find the slope of the line through $(-2, 4)$ and $(-2, -3)$

$$\begin{aligned} m &= \frac{y_2 - x_1}{x_2 - x_1} \\ m &= \frac{-3-4}{-2-(-2)} \\ m &= \frac{-7}{0} \end{aligned}$$

The slope is undefined.

This tells us it is a vertical line. Both of our points have an x-coordinate of −2. So our equation of the line is x=−2. Since there is no yy, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

Exercise 6.7.23

Find an equation of a line containing the points (5,1) and (5,−4).

Answer

x = 5

Exercise 6.7.24

Find an equation of a line containing the points (−4,4) and (−4,3).

Answer

x=−4

We have seen that we can use either the slope–intercept form or the point–slope form to find an equation of a line. Which form we use will depend on the information we are given. This is summarized in Table **6.7.1**.

Table 6.7.1

To Write an Equation of a Line		
If given:	Use:	Form:
Slope and *y*-intercept	slope–intercept	y=mx+b
Slope and a point	point–slope	$y-y_1=m(x-x_1)$
Two points	point–slope	$y-y_1=m(x-x_1)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope—just what we need to use the point–slope equation.

First let's look at this graphically.

The graph shows the graph of y=2x−3. We want to graph a line parallel to this line and passing through the point (−2,1).

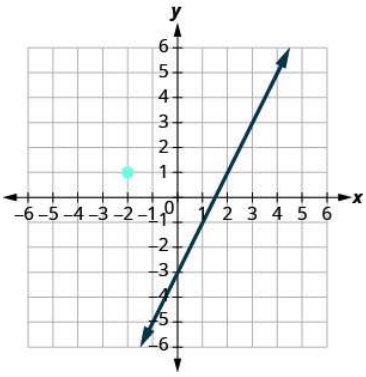


Figure 6.7.1

We know that parallel lines have the same slope. So the second line will have the same slope as y=2x−3. That slope is $m_{\parallel}=2$. We'll use the notation $m_{\parallel}$ to represent the slope of a line parallel to a line with slope m. (Notice that the subscript ∥ looks like two parallel lines.)

The second line will pass through (−2,1) and have m=2. To graph the line, we start at (−2,1) and count out the rise and run. With m=2 (or $m=\frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.

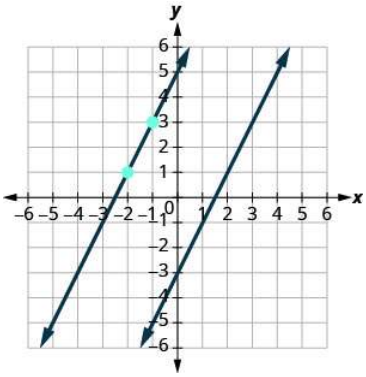


Figure 6.7.2

Do the lines appear parallel? Does the second line pass through (−2,1)?

Now, let's see how to do this algebraically.

We can use either the slope–intercept form or the point–slope form to find an equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

Exercise 6.7.25: How to Find an Equation of a Line Parallel to a Given Line

Find an equation of a line parallel to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Answer

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_{\parallel} = 2$
Step 3. Identify the point.	The given point is, (–2, 1).	$\begin{pmatrix} x_1, & y_1 \\ -2, & 1 \end{pmatrix}$
Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Does this equation make sense? What is the *y*-intercept of the line? What is the slope?

Exercise 6.7.26

Find an equation of a line parallel to the line y=3x+1 that contains the point (4,2). Write the equation in slope–intercept form.

Answer

y=3x−10

Exercise 6.7.27

Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point (6,4).

Answer

$y = \frac{1}{2}x + 1$

FIND AN EQUATION OF A LINE PARALLEL TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the parallel line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line passing through a specific point and which is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of y=2x−3. Now, we want to graph a line perpendicular to this line and passing through (−2,1).

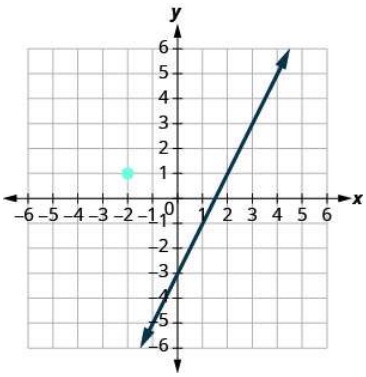


Figure 6.7.3

We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_{\perp}$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $\perp$ looks like the right angles made by two perpendicular lines.)

$$\begin{array}{ll} y = 2x - 3 & \text{perpendicular line} \\ m = 2 & m_{\perp} = -\frac{1}{2} \end{array} \tag{6.7.1}$$

We now know the perpendicular line will pass through (−2,1) with $m_{\perp} = -\frac{1}{2}$.

To graph the line, we will start at (−2,1) and count out the rise −1 and the run 2. Then we draw the line.

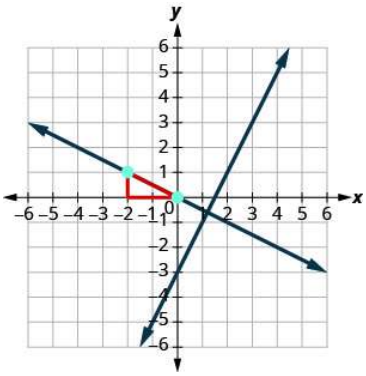


Figure 6.7.4

Do the lines appear perpendicular? Does the second line pass through (−2,1)?

Now, let's see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find an equation of a line. In this example we know one point, and can find the slope, so we will use the point–slope form.

? Exercise 6.7.28

Find an equation of a line perpendicular to y=2x−3 that contains the point (−2,1). Write the equation in slope–intercept form.

Answer

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_{\perp} = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, (–2, 1)	(x_1, y_1) $(-2, 1)$

Step 4. Substitute the values into the point–slope form, $y-y_1=m(x-x_1)$.	Simplify.	$y-y_1=m(x-x_1)$ $y-1=-\frac{1}{2}(x-(-2))$ $y-1=-\frac{1}{2}(x+2)$ $y-1=-\frac{1}{2}x-1$
Step 5. Write the equation in slope–intercept form.		$y=-\frac{1}{2}x$

? Exercise 6.7.29

Find an equation of a line perpendicular to the line y=3x+1 that contains the point (4,2). Write the equation in slope–intercept form.

Answer

$y=-\frac{1}{3}x+\frac{10}{3}$

? Exercise 6.7.30

Find an equation of a line perpendicular to the line $y=\frac{1}{2}x-3$ that contains the point (6,4).

Answer

y=−2x+16

FIND AN EQUATION OF A LINE PERPENDICULAR TO A GIVEN LINE.

1. Find the slope of the given line.
2. Find the slope of the perpendicular line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y-y_1=m(x-x_1)$.
5. Write the equation in slope–intercept form.

? Exercise 6.7.31

Find an equation of a line perpendicular to x=5 that contains the point (3,−2). Write the equation in slope–intercept form.

Answer

Again, since we know one point, the point–slope option seems more promising than the slope–intercept option. We need the slope to use this form, and we know the new line will be perpendicular to x=5. This line is vertical, so its perpendicular will be horizontal. This tells us the $m_\perp=0$.

Identify the point.	$(3$	,	$-2)$
Identify the slope of the perpendicular line.	$m_\perp$	=	0
Substitute the values into $y-y_1=m(x-x_1)$.	$y-y_1$	=	$m(x-x_1)$
	$y-(-2)$	=	$0(x-3)$
Simplify.	$y+2$	=	0
	y	=	-2

Sketch the graph of both lines. Do they appear to be perpendicular?

Exercise 6.7.32

Find an equation of a line that is perpendicular to the line x=4 that contains the point (4,−5). Write the equation in slope–intercept form.

Answer

y=−5

Exercise 6.7.33

Find an equation of a line that is perpendicular to the line x=2 that contains the point (2,−1). Write the equation in slope–intercept form.

Answer

y=−1

In Exercise **6.7.31**, we used the point–slope form to find the equation. We could have looked at this in a different way.

We want to find a line that is perpendicular to x=5 that contains the point (3,−2). The graph shows us the line x=5 and the point (3,−2).

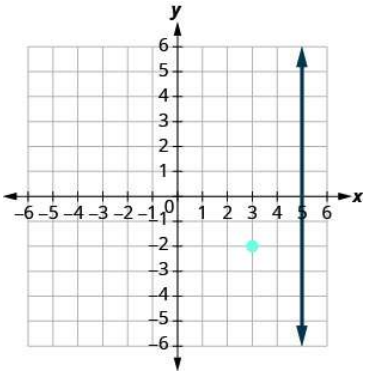


Figure **6.7.5**

We know every line perpendicular to a vertical line is horizontal, so we will sketch the horizontal line through (3,−2).

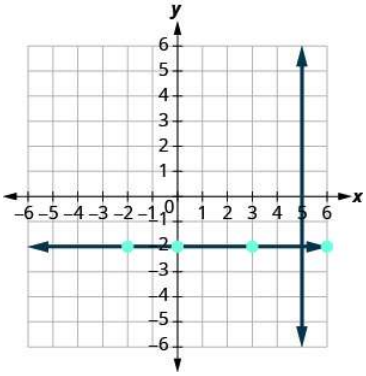


Figure **6.7.6**

Do the lines appear perpendicular?

If we look at a few points on this horizontal line, we notice they all have *y*-coordinates of −2. So, the equation of the line perpendicular to the vertical line x=5 is y=−2.

Exercise 6.7.34

Find an equation of a line that is perpendicular to y=−4 that contains the point (−4,2). Write the equation in slope–intercept form.

Answer

The line y=−4 is a horizontal line. Any line perpendicular to it must be vertical, in the form x=a. Since the perpendicular line is vertical and passes through (−4,2), every point on it has an *x*-coordinate of −4. The equation of the perpendicular line is x=−4. You may want to sketch the lines. Do they appear perpendicular?

Exercise 6.7.35

Find an equation of a line that is perpendicular to the line y=1 that contains the point (−5,1). Write the equation in slope–intercept form.

Answer

x=−5

Exercise 6.7.36

Find an equation of a line that is perpendicular to the line y=−5 that contains the point (−4,−5).

Answer

x=−4

Note

Access this online resource for additional instruction and practice with finding the equation of a line.

- Use the Point-Slope Form of an Equation of a Line

Key Concepts

- **To Find an Equation of a Line Given the Slope and a Point**
  1. Identify the slope.
  2. Identify the point.
  3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  4. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Given Two Points**
  1. Find the slope using the given points.
  2. Choose one point.
  3. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  4. Write the equation in slope-intercept form.
- **To Write and Equation of a Line**
  - If given slope and y-intercept, use slope–intercept form $y=mx+b$.
  - If given slope and a point, use point–slope form $y-y_1=m(x-x_1)$.
  - If given two points, use point–slope form $y-y_1=m(x-x_1)$.
- **To Find an Equation of a Line Parallel to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the parallel line.

3. Identify the point.
4. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
5. Write the equation in slope-intercept form.

- **To Find an Equation of a Line Perpendicular to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the perpendicular line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y-y_1=m(x-x_1)$.
  5. Write the equation in slope-intercept form.

Glossary

point–slope form

The point–slope form of an equation of a line with slope mm and containing the point (x_1, y_1) is $y-y_1=m\,(x-x_1)$.

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CHAPTER OVERVIEW

7: Geometry

7.1: Perimeter and Circumference
7.2: Area of Polygons and Circles
7.3: Composite Figures
7.4: Surface Area of Common Solids
7.5: Volume of Common Solids
7.6: Pyramids and Cones

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7.1: Perimeter and Circumference

You may use a calculator throughout this module if needed.

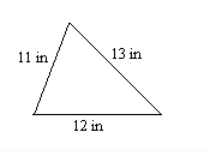
Perimeter

A polygon is a closed geometric figure with straight sides. Common polygons include triangles, squares, rectangles, parallelograms, trapezoids, pentagons, hexagons, octagons… The perimeter of a polygon is the distance around the outside. In general, to find the perimeter of a polygon, you can add up the lengths of all of its sides.

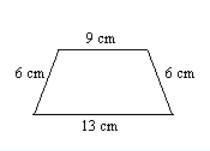
Also, if you haven't already, now is the time to get in the habit of including units in your answers.

Exercises 7.1.1

1. Find the perimeter of the triangle.



2. Find the perimeter of the trapezoid.



Answer

1. **36** in
2. **34** cm

If we know that some of the sides of a polygon are equal, we can use a formula as an alternative to adding up all of the lengths individually. The first formula shown below uses the variable s for the side of a square. The rectangle formulas use l for length and w for width, or b for base and h for height; these terms are interchangeable.

Perimeter Formulas

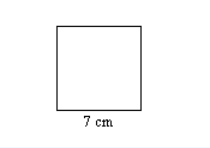
Square: $P = 4s$

Rectangle: $P = 2l + 2w$ or $P = 2b + 2h$

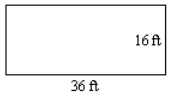
Rectangle: $P = 2(l + w)$ or $P = 2(b + h)$

Exercises 7.1.1

3. Find the perimeter of the square.



4. Find the perimeter of the rectangle.



5. A storage area, which is a rectangle that is 45 feet long and 20 feet wide, needs to be fenced around all four sides. How many feet of fencing is required? (To keep it simple, ignore any gates or other complications.)

6. Giancarlo is putting crown molding around the edge of the ceiling of his living room. If the room is a 12-foot by 16-foot rectangle, how much crown molding does he need?

Answer

3. 28 cm

4. 104 ft

5. 130 ft

6. 56 ft

The sides of a regular polygon are all equal in length. Therefore, multiplying the length of a side by the number of sides will give us the perimeter.

Perimeter Formula

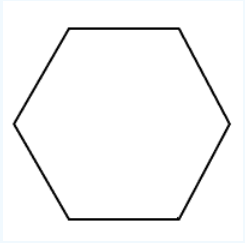
Regular Polygon with n sides of length s:

$$P = n \cdot s$$

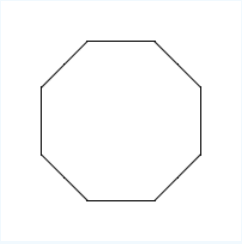
Exercises 7.1.1

Find the perimeter of each regular polygon.

7. Each side of the hexagon is 4 inches long.



8. Each side of the octagon is 2.5 centimeters long.



Answer

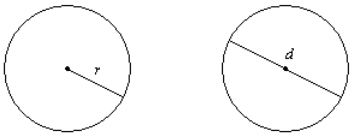
7. 24 in

8. 20 cm

Circumference

Instead of calling it the perimeter, the distance around the outside of circle is called the ***circumference***. Let's review some circle vocabulary before moving on.

Every point on a circle is the same distance from its center. This distance from the center to the edge of the circle is called the ***radius***. The distance from one edge to another, through the center of the circle, is called the ***diameter***. As you can see, the diameter is twice the length of the radius.



Throughout history, different civilizations have discovered that the circumference of a circle is slightly more than 3 times the length of its diameter. By the year 2000 BCE, the Babylonians were using the value $3\frac{1}{8} = 3.125$ and the Egyptians were using the value $3\frac{13}{81} \approx 3.1605$.[1] The value $3\frac{1}{7} \approx 3.1429$ is an even better approximation for the ratio of the circumference to the diameter. However, the actual value cannot be written as an exact fraction. It is the irrational number π, pronounced "pie", which is approximately 3.14159.

Circumference Formulas

$$C = \pi d$$

$$C = 2\pi r$$

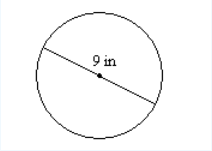
Any scientific calculator will have a π key; using this will give you the most accurate result, although you should be sure to round your answer appropriately. (See this module if you need a refresher on rounding with multiplying or dividing.) Many people use 3.14 as an approximation for π, but this can lead to round-off error; if you must use an approximation, 3.1416 is better than 3.14.

Sometimes we bend the rules in this textbook and ask you to round to a certain place value instead of rounding to a certain number of significant digits.

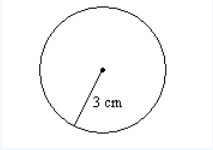
? Exercises 7.1.1

Calculate the circumference of each circle. Round to the nearest tenth.

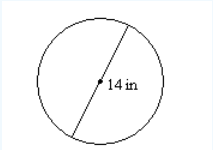
9.

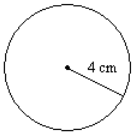


10.



11.



12. 

Answer

9. 28.3 in
10. 18.8 cm
11. 44.0 in
12. 25.1 cm

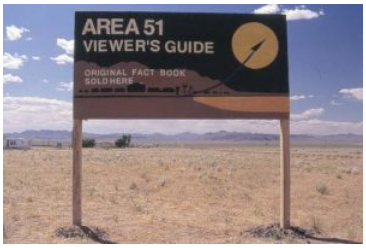
1. This information comes from Chapter 1 of the book *A History of Pi* by Petr Beckmann. It is a surprisingly interesting read. ↵

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7.2: Area of Polygons and Circles

You may use a calculator for most of this module as needed.

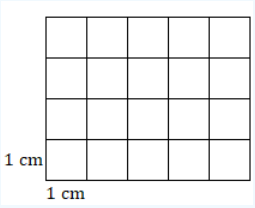


We have seen that the perimeter of a polygon is the distance around the outside. Perimeter is a length, which is one-dimensional, and so it is measured in linear units (feet, centimeters, miles, etc.). The area of a polygon is the amount of two-dimensional space inside the polygon, and it is measured in square units: square feet, square centimeters, square miles, etc.

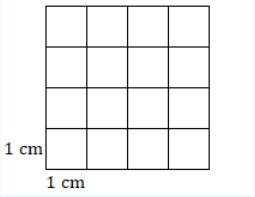
You can always think of area as the number of squares required to completely fill in the shape.

Exercises 7.2.1

1. Find the area of this rectangle.



2. Find the area of this square.



Answer

1. $20\ \text{cm}^2$
2. $16\ \text{cm}^2$

Rectangles and Squares

There are of course formulas for finding the areas of rectangles and squares; we don't have to count little squares.

Area of a Rectangle

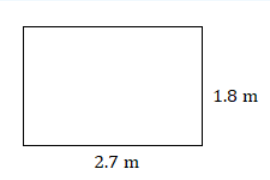
$A = lw$[1] or $A = bh$

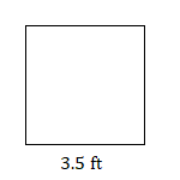
Area of a Square

$$A = s^2$$

Exercises 7.2.1

Find the area of each figure.

3. 

4. 

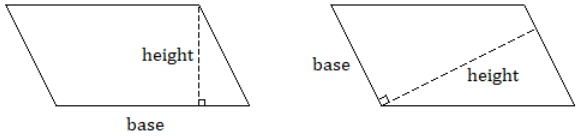
Answer

3. $4.86\ \text{m}^2$

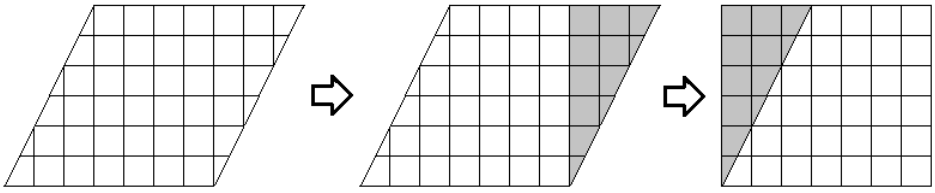
4. $12.25\ \text{ft}^2$

Parallelograms

Another common polygon is the ***parallelogram***, which looks like a tilted rectangle. As the name implies, the pairs of opposite sides are parallel and have the same length. Notice that, if we label one side as a base of the parallelogram, we have a perpendicular height which is *not* the length of the other sides.



The following set of diagrams shows that we can cut off part of a parallelogram and rearrange the pieces into a rectangle with the same base and height as the original parallelogram. A parallelogram with a base of **7** units and a vertical height of **6** units is transformed into a **7** by **6** rectangle, with an area of **42** square units.



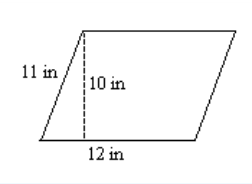
Therefore, the formula for the area of a parallelogram is identical to the formula for the area of a rectangle, provided that we are careful to use the base and the height, which must be perpendicular.

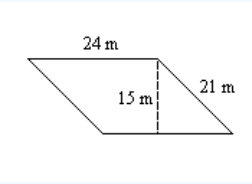
Area of a Parallelogram

$A = bh$

Exercises 7.2.1

Find the area of each parallelogram.

5. 

6. 

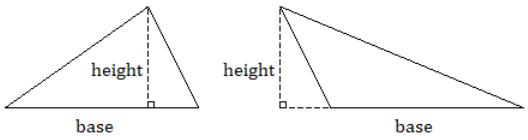
Answer

5. **120 in^2**

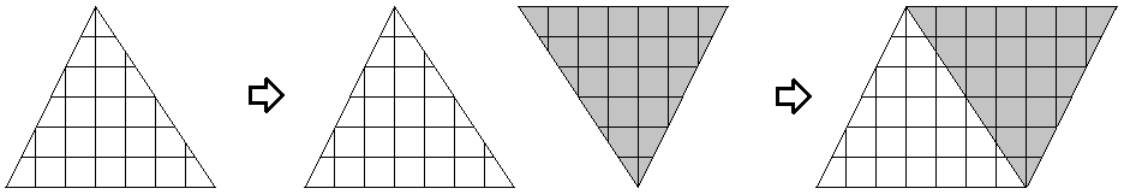
6. **360 m^2**

Triangles

When we need to find the area of a triangle, we need to identify a base and a height that is perpendicular to that base. If the triangle is obtuse, you may have to imagine the height outside of the triangle and extend the base line to meet it.



As shown below, any triangle can be doubled to form a parallelogram. Therefore, the area of a triangle is one half the area of a parallelogram with the same base and height.



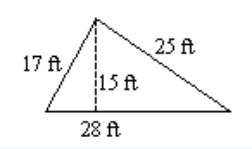
Area of a Triangle

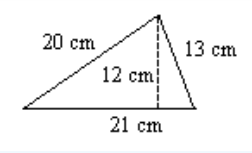
$A = \frac{1}{2}bh$ or $A = bh \div 2$

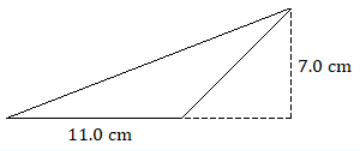
As with a parallelogram, remember that the height must be perpendicular to the base.

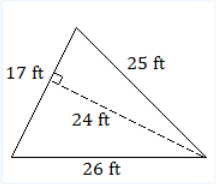
Exercises 7.2.1

Find the area of each triangle.

7. 

8. 

9. 

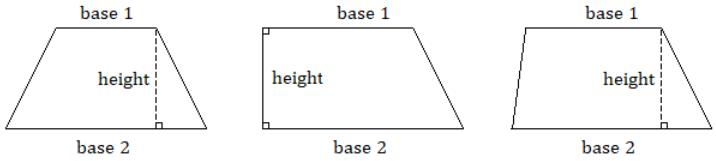
10. 

Answer

7. $210\ \text{ft}^2$
8. $126\ \text{cm}^2$
9. $38.5\ \text{cm}^2$
10. $204\ \text{ft}^2$

Trapezoids

A somewhat less common quadrilateral is the ***trapezoid***, which has exactly one pair of parallel sides, which we call the bases. The first example shown below is called an isosceles trapezoid because, like an isosceles triangle, its two nonparallel sides have equal lengths.



There are a number of ways to show where the area formula comes from, but the explanations are better in video because they can be animated.[2][3][4]

Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

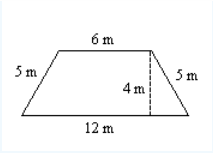
or

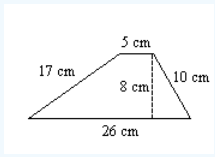
$$A = (b_1 + b_2)h \div 2$$

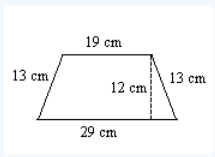
Don't be intimidated by the subscripts on b_1 and b_2; it's just a way to name two different measurements using the same letter for the variable. (Many people call the bases a and b instead; feel free to write it whichever way you prefer.) Whatever you call them, you just add the two bases, multiply by the height, and take half of that.

? Exercises 7.2.1

Find the area of each trapezoid.

11. 

12. 

13. 

Answer

11. $36\ \text{m}^2$
12. $124\ \text{cm}^2$
13. $288\ \text{cm}^2$

Circles

The area of a circle is π times the square of the radius: $A = \pi r^2$. The units are still square units, even though a circle is round. (Think of the squares on a round waffle.) Because we can't fit a whole number of squares—or an exact fraction of squares—inside the circle, the area of a circle will be an approximation.



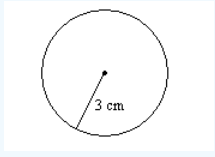
Area of a Circle

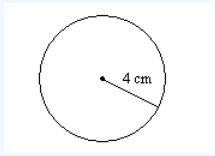
\[A=\pi{r^2}\]

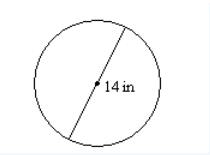
Remember that $\pi \approx 3.1416$.

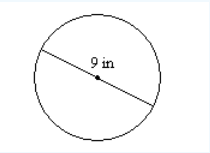
Exercises 7.2.1

Find the area of each circle. Round to the nearest tenth or to three significant figures, whichever seems appropriate.

14. 

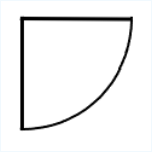
15. 

16. 

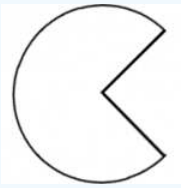
17. 

Each figure is a fraction of a circle. Calculate each area.

18. The radius of the quarter circle is **5** meters.



19. A quarter circle has been removed from a circle with a diameter of **7** feet.



Answer

14. **28.3 cm^2**
15. **50.3 cm^2**
16. **153.9 in^2**
17. **63.6 in^2**
18. **19.6 m^2**
19. **28.9 ft^2**

1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1". ↵
2. https://youtu.be/yTnYRpcZA9c ↵
3. https://youtu.be/WZtO3oERges ↵
4. https://youtu.be/uLHc6Br2veg ↵

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7.3: Composite Figures



This sunburst pattern is composed of **276** geometric tiles.

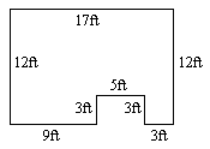
You may use a calculator throughout this module as needed.

Many objects have odd shapes made up of simpler shapes. A composite figure is a geometric figure which is formed by—or *composed* of—two or more basic geometric figures. We will look at a handful of fairly simple examples, but this concept can of course be extended to much more complicated figures.

To find the area of a composite figure, it is generally a good idea to divide it into simpler shapes and either add or subtract their areas as necessary.

? Exercises 7.3.1

A floor plan of a room is shown. The room is a **12**-foot by **17**-foot rectangle, with a **3**-foot by **5**-foot rectangle cut out of the south side.



1. Determine the amount of molding required to go around the perimeter of the room.

2. Determine the amount of flooring required to cover the entire area.

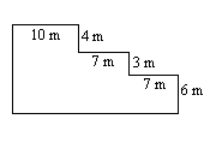
Answer

1. **64 ft**
2. **189 ft^2**

You may need to figure out some unmarked dimensions.

? Exercises 7.3.1

A plan for an irregular parking lot is shown.



3. Calculate the perimeter.

4. Calculate the area.

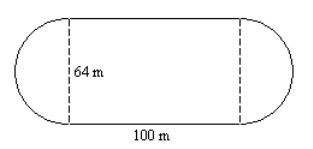
Answer

3. **74 m**
4. **235 m^2**

Sometimes, composite figures involve parts of circles coupled with polygons.

Exercises 7.3.1

A high school is building a track at its athletic fields. The track, which is formed by two straight sides and two semicircles as shown in the plans below, is supposed to have a total length of 400 meters.



5. Determine the distance around the track. Will the track be the right length?

6. After the new track is built, landscapers need to lay sod on the field inside the track. What is the area of the field inside the track?

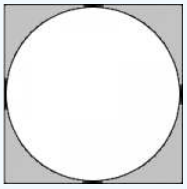
Answer

5. Based on the stated measurements, the distance around the track will be 401 meters, which appears to be 1 meter too long. In real life, precision would be very important here, and you might ask for the measurements to be given to the nearest tenth of a meter.

6. around $9,620\ \text{m}^2$

If we need to determine a fraction or percent out of the whole, we may be able to solve the problem without knowing any actual measurements.

Exercise 7.3.1



7. Circular disks are being cut from squares of sheet metal, with the remainder around the corners being discarded. Assuming that the circles are made as large as possible, what percent of the sheet metal will be discarded?

Answer

around 21.5% (Hint: Make up an easy number for the side of the square, like 2 or 10.)

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7.4: Surface Area of Common Solids

You may use a calculator throughout this module.

We will now turn our attention from two-dimensional figures to three-dimensional figures, which we often call ***solids***, even if they are hollow inside. In this module, we will look the surface areas of some common solids. (We will look at volume in a later module.) Surface area is what it sounds like: it's the sum of the areas of all of the outer surfaces of the solid. When you are struggling to wrap a present because your sheet of wrapping paper isn't quite big enough, you are dealing with surface area.

There are two different kinds of surface area that are important: the ***lateral surface area*** (LSA) and ***total surface area*** (TSA). To visualize the difference between LSA and TSA, consider a can of soup. The lateral surface area would be used to measure the size of the paper label around the can. The total surface area would be used to measure the amount of sheet metal needed to make the can. In other words, the total surface area includes the top and bottom, whereas the lateral surface area does not.



Rectangular Solids

A ***rectangular solid*** looks like a rectangular box. It has three pairs of equally sized rectangles on the front and back, on the left and right, and on the top and bottom. A ***cube*** is a special rectangular solid with equally-sized squares for all six faces.

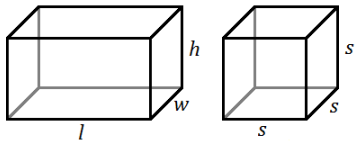


Figure **7.4.1**: (left) A rectangular solid and (right) a cube.

The lateral surface area is the combined total area of the four vertical faces of the solid, but not the top and bottom. If you were painting the four walls of a room, you would be thinking about the lateral surface area.

The total surface area is the combined total area of all six faces of the solid. If you were painting the four walls, the floor, and the ceiling of a room, you would be thinking about the total surface area.

For a rectangular solid with length l, width w, and height h…[1]

$LSA = 2lh + 2wh$

$TSA = 2lh + 2wh + 2lw$

$TSA = LSA + 2lw$

For a cube with side length s…

$LSA = 4s^2$

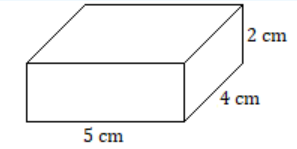
$TSA = 6s^2$

Note: These dimensions are sometimes called base, depth, and height.

Exercises 7.4.1

1. Find the lateral surface area of this rectangular solid.

2. Find the total surface area of this rectangular solid.

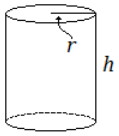


Answer

1. $36\ \text{cm}^2$
2. $76\ \text{cm}^2$

Cylinders

As mentioned earlier in this module, the lateral surface area of a soup can is the paper label, which is a rectangle. Therefore, the lateral surface area of a ***cylinder*** is a rectangle; its width is equal to the circumference of the circle, $2\pi r$, and its height is the height of the cylinder.



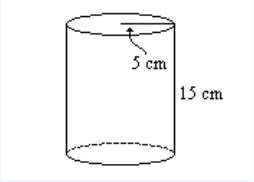
Since a cylinder has equal-sized circles at the top and bottom, its total surface area is equal to the lateral surface area plus twice the area of one of the circles.

For a cylinder with radius r and height h…

- $LSA = 2\pi rh$
- $TSA = 2\pi rh + 2\pi r^2$
- $TSA = LSA + 2\pi r^2$

Be aware that if you are given the diameter of the cylinder, you will need to cut it in half before using these formulas.

Exercises 7.4.1



3. Find the lateral surface area of this cylinder.

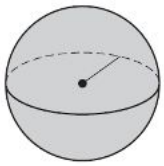
4. Find the total surface area of this cylinder.

Answer

3. $471\ \text{cm}^2$
4. $628\ \text{cm}^2$

Spheres

The final solid of this module is the ***sphere***, which can be thought of as a circle in three dimensions: every point on the surface of a sphere is the same distance from the center. Because of this, a sphere has only one important measurement: its radius. Of course, its diameter could be important also, but the idea is that a sphere doesn't have different dimensions such as length, width, and height. A sphere has the same radius (or diameter) in every direction.



We would need to use calculus to derive the formula for the surface area of a sphere, so we'll just assume it's true and get on with the business at hand. Notice that, because a sphere doesn't have top or bottom faces, we don't need to worry about finding the lateral surface area. The only surface area is the total surface area.

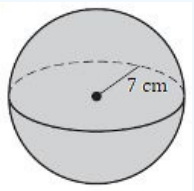
For a sphere with radius r or diameter d…

$SA = 4\pi r^2$ or $SA = \pi d^2$

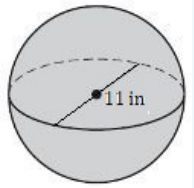
Coincidentally, the surface area of a sphere is 4 times the area of the cross-sectional circle at the sphere's widest part. You may find it interesting to try to visualize this, or head to the kitchen for a demonstration: if you cut an orange into four quarters, the peel on one of those quarter oranges has the same area as the circle formed by the first cut.

? Exercises 7.4.1

5. Find the surface area of this sphere.



6. Find the surface area of this sphere.



Answer

5. $616\ \text{cm}^2$
6. $380\ \text{cm}^2$

---

1. You might choose to use capital letters for the variables here because a lowercase letter "l" can easily be mistaken for a number "1". ↵

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7.5: Volume of Common Solids



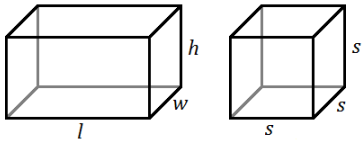
Wanda Ortiz of The Iron Maidens turns up the volume.

You may use a calculator throughout this module.

Note: We will not necessarily follow the rules for rounding (precision and accuracy) in this module. Many of these figures have dimensions with only one significant figure, but we would lose a lot of information if we rounded the results to only one sig fig.

In the answer key, we will often round to the nearest whole number, or to the nearest tenth, or to two or three significant figures as we deem appropriate.

The surface area of a solid is the sum of the areas of all its faces; therefore, surface area is two-dimensional and measured in square units. The volume is the amount of space inside the solid. Volume is three-dimensional, measured in cubic units. You can imagine the volume as the number of cubes required to completely fill up the solid.



Volume of a Rectangular Solid

For a rectangular solid with length l, width w, and height h:

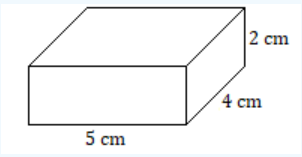
$$V = lwh$$

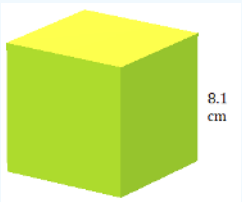
For a cube with side length s:

$$V = s^3$$

Exercises 7.5.1

Find the volume of each solid.

1. 

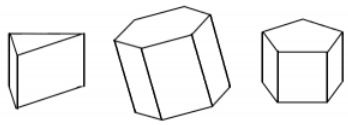
2. 

Answer

1. $40\ \text{cm}^3$

2. $531\ cm^3$

A solid with two equal sized polygons as its bases and rectangular lateral faces is called a right-angle prism. Some examples are shown below. We will refer to them simply as ***prisms*** in this textbook. (We will not be working with oblique prisms, which have parallelograms for the lateral faces.)



If you know the area of one of the bases, multiplying it by the height gives you the volume of the prism. In the formula below, we are using a capital B to represent the *area* of the base.

Volume of a Prism

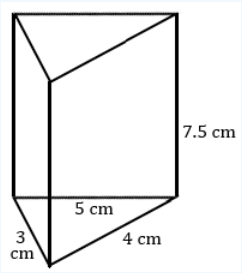
For a prism with base area B and height h:

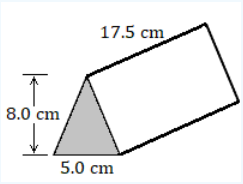
$$V = Bh$$

If the prism is lying on its side, the "height" will look like a length. No matter how the prism is oriented, the height is the dimension that is perpendicular to the planes of the two parallel bases.

? Exercises 7.5.1

Find the volume of each prism.

3. 

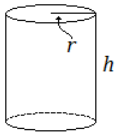
4. 

Answer

3. $45\ cm^3$

4. $350\ cm^3$

A cylinder can be thought of as a prism with bases that are circles, rather than polygons. Just as with a prism, the volume is the area of the base multiplied by the height.



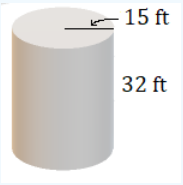
Volume of a Cylinder

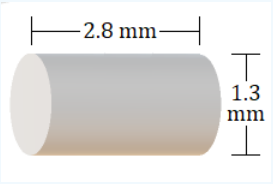
For a cylinder with radius r and height h:

$$V = \pi r^2 h$$

Exercises 7.5.1

Find the volume of each cylinder.

6. 

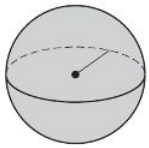
7. 

Answer

6. $22,600\ \text{ft}^3$

7. $3.7\ \text{mm}^3$

As with surface area, we would need to use calculus to derive the formula for the volume of a **sphere**. Just believe it. ¯_(ツ)_/¯



Volume of a Sphere

For a sphere with radius r:

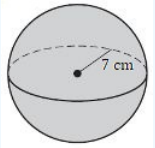
$$V = \frac{4}{3}\pi r^3$$

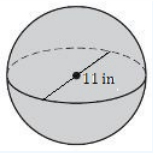
or

$$V = 4\pi r^3 \div 3$$

Exercises 7.5.1

Find the volume of each sphere.

8. 

9. 

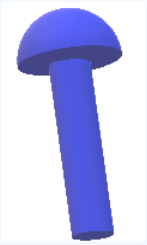
Answer

8. $1,440\ \text{cm}^3$
9. $700\ \text{in}^3$

Composite Solids

Of course, not every three-dimensional object is a prism, cylinder, or sphere. A composite solid is made up of two or more simpler solids. As with two-dimensional composite figures, breaking the figure into recognizable solids is a good first step.

? Exercise 7.5.1



10. A rivet is formed by topping a cylinder with a hemisphere. The width of the cylindrical part (the rivet pin) is **1.6** cm and the length is **7** cm. The width of the hemisphere-shaped top (the rivet head) is **3.2** cm. Find the rivet's volume.

Answer

$31.2\ \text{cm}^3$ (the cylinder's volume $\approx 14.07\ \text{cm}^3$ and the hemisphere's volume $\approx 17.16\ \text{cm}^3$.)

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7.6: Pyramids and Cones

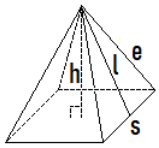
You may use a calculator throughout this module.

Note: We will not necessarily follow the rules for rounding (precision and accuracy) in this module. Many of these figures have dimensions with only one significant figure, but we would lose a lot of information if we rounded the results to only one sig fig.

In the answer key, we will often round to the nearest whole number, or to the nearest tenth, or to two or three significant figures as we deem appropriate.

Pyramids

A **pyramid** is a geometric solid with a polygon base and triangular faces with a common vertex (called the *apex* of the pyramid). Pyramids are named according to the shape of their bases. The most common pyramids have a square or another regular polygon for a base, making all of the faces identical isosceles triangles. The height, h, is the distance from the apex straight down to the center of the base. Two other measures used with pyramids are the edge length e, the sides of the triangular faces, and the slant height l, the height of the triangular faces.



Volume of a Pyramid

In general, the volume of a pyramid with base of area B and height h is

$$V = \frac{1}{3}Bh$$

or

$$V = Bh \div 3$$

If the base is a square with side length s, the volume is

$$V = \frac{1}{3}s^2h$$

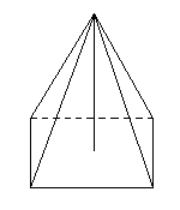
or

$$V = s^2h \div 3$$

Interestingly, the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.

? Exercises 7.6.1

1. A pyramid has a square base with sides **16** centimeters long, and a height of **15** centimeters. Find the volume of the pyramid.



2. The Great Pyramid at Giza in Egypt has a height of 137 meters and a square base with sides 230 meters long.[1] Find the volume of the pyramid.



Answer

1. $1,280\ \text{cm}^3$
2. $2.4\ \text{million m}^3$

The lateral surface area (LSA) of a pyramid is found by adding the area of each triangular face.

Lateral Surface Area of a Pyramid

If the base of a pyramid is a regular polygon with n sides each of length s, and the slant height is l, then

$$LSA = \frac{1}{2}nsl$$

or

$$LSA = nsl \div 2$$

If the base is a square, then

$$LSA = 2sl$$

The total surface area (TSA) is of course found by adding the area of the base B to the lateral surface area. If the base is a regular polygon, you will need to use the techniques we studied in a previous module.

Total Surface Area of a Pyramid

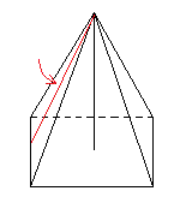
$$TSA = LSA + B$$

If the base is a square, then

$$TSA = 2sl + s^2$$

? Exercises 7.6.1

3. A pyramid has a square base with sides 16 centimeters long, and a slant height of 17 centimeters. Find the lateral surface area and total surface area of the pyramid.



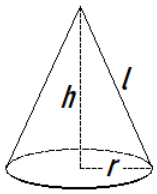
4. The Great Pyramid at Giza has a slant height of 179 meters and a square base with sides 230 meters long. Find the lateral surface area of the pyramid.

Answer

3. $544\ \text{cm}^2$; $800\ \text{cm}^2$
4. $82,300\ \text{m}^2$; $135,000\ \text{m}^2$

Cones

A cone is like a pyramid with a circular base.



You may be able to determine the height h of a cone (the altitude from the apex, perpendicular to the base), or the slant height l (which is the length from the apex to the edge of the circular base). Note that the height, radius, and slant height form a right triangle with the slant height as the hypotenuse. We can use the Pythagorean theorem to determine the following equivalences.

The slant height$l$, height h, and radius r of a cone are related as follows:

$$l = \sqrt{r^2 + h^2}$$

$$h = \sqrt{l^2 - r^2}$$

$$r = \sqrt{l^2 - h^2}$$

Just as the volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height, the volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height.

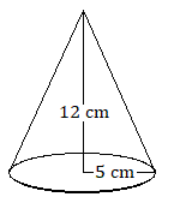
Volume of a Cone

The volume of a cone with a base radius r and height h is

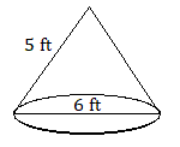
$$V = \frac{1}{3}\pi r^2 h \)or\ (V = \pi r^2 h \div 3$$

? Exercises 7.6.1

5. The base of a cone has a radius of 5 centimeters, and the vertical height of the cone is 12 centimeters. Find the volume of the cone.



6. The base of a cone has a diameter of 6 feet, and the slant height of the cone is 5 feet. Find the volume of the cone.



Answer

5. $314\ \text{cm}^3$
6. $37.7\ \text{ft}^3$

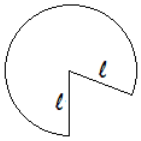
For the surface area of a cone, we have the following formulas.

Surface Area of a Cone

$$LSA = \pi rl$$

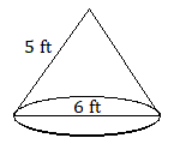
$$TSA = LSA + \pi r^2 = \pi rl + \pi r^2$$

It's hard to explain the justification for the LSA formula in words, but here goes. The lateral surface of a cone, when flattened out, is a circle with radius l that is missing a wedge. The circumference of this partial circle, because it matched the circumference of the circular base, is $2\pi r$. The circumference of the entire circle with radius l would be $2\pi l$, so the part we have is just a fraction of the entire circle. To be precise, the fraction is $\frac{2\pi r}{2\pi l}$, which reduces to $\frac{r}{l}$. The area of the entire circle with radius l would be πl^2. Because the partial circle is the fraction $\frac{r}{l}$ of the entire circle, the area of the partial circle is $\pi l^2 \cdot \frac{r}{l} = \pi rl$.

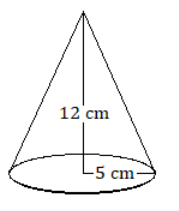


? Exercises 7.6.1

7. The base of a cone has a diameter of **6** feet, and the slant height of the cone is **5** feet. Find the lateral surface area and total surface area of the cone.



8. The base of a cone has a radius of **5** centimeters, and the vertical height of the cone is **12** centimeters. Find the lateral surface area and total surface area of the cone.



Answer

7. **47.1 ft^2; 75.4 ft^2**

8. **204 cm^2; 283 cm^2**

Now that we have looked at the five major solids—prism, cylinder, sphere, pyramid, cone—you should be able to handle composite solids made from these shapes. Just remember to take them in pieces.

? Exercise 7.6.1

A **250**-gallon propane tank is roughly in the shape of a cylinder with a hemisphere on each end. The length of the cylindrical part is **6** feet long, and the cross-sectional diameter of the tank is **2.5** feet.



9. Calculate the volume of the tank in cubic feet.

10. Verify that the tank can hold 250 gallons of liquid propane.

Answer

9. $37.6\ \text{ft}^3$ (the cylinder's volume $\approx 29.45\ \text{ft}^3$ and the two hemispheres' combined volume $\approx 8.18\ \text{ft}^3$)

10. $37.6\ \text{ft}^3 \approx 281.5\ \text{gal}$, which is more than $250\ \text{gal}$.

1. https://en.Wikipedia.org/wiki/Great_Pyramid_of_Giza ↵

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CHAPTER OVERVIEW

8: Trigonometry

8.1: Angles
8.2: Triangles
8.3: Right Triangle Trigonometry

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8.1: Angles

You will need a calculator near the end of this module.



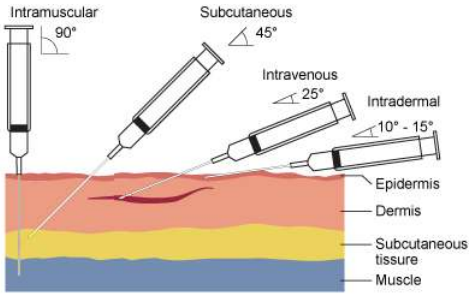
an angle grinder

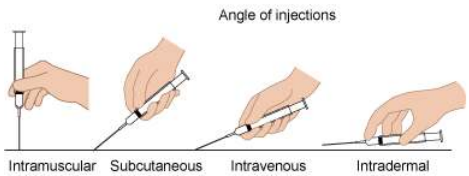
Angle measurement is important in construction, surveying, physical therapy, and many other fields. We can visualize an angle as the figure formed when two line segments share a common endpoint. We can also think about an angle as a measure of rotation. A full rotation or a full circle is $360°$, so a half rotation or U-turn is $180°$, and a quarter turn is $90°$.

We often classify angles by their size.

- **Acute Angle:** between $0°$ and $90°$
- **Right Angle:** exactly $90°$
- **Obtuse Angle:** between $90°$ and $180°$
- **Straight Angle:** exactly $180°$
- **Reflexive Angle:** between $180°$ and $360°$

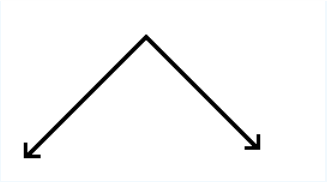
Lines that form a $90°$ angle are called ***perpendicular***. As shown below, the needle should be perpendicular to the body surface for an intramuscular injection.

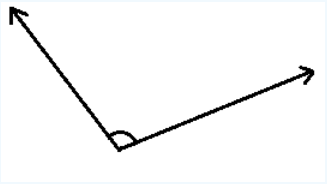


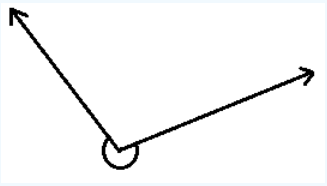


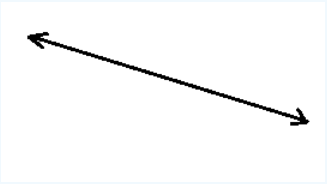
Exercises 8.1.1

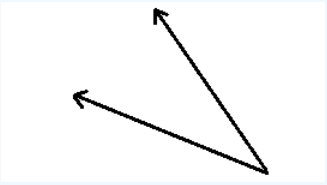
Identify each angle shown below as acute, right, obtuse, straight, or reflexive.

1. 

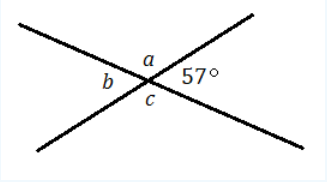
2. 

3. 

4. 

5. 

Find the measure of each unknown angle.

6. 

Answer

1. right angle
2. obtuse angle
3. reflexive angle
4. straight angle
5. acute angle
6. $a = 123°$; $b = 57°$; $c = 123°$

Angles in Triangles

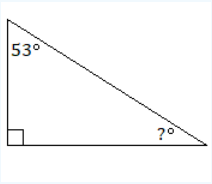
If you need to find the measures of the angles in a triangle, there are a few rules that can help.

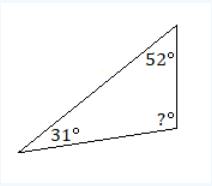
The sum of the angles of every triangle is $180°$.

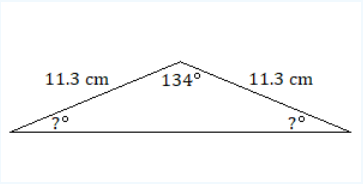
If any sides of a triangle have equal lengths, then the angles opposite those sides will have equal measures.

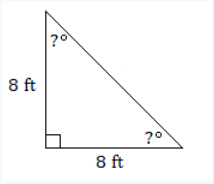
Exercises 8.1.1

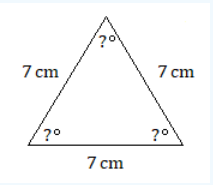
Find the measures of the unknown angles in each triangle.

7. 

8. 

9. 

10. 

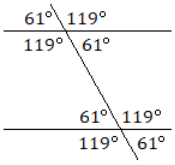
11. 

Answer

7. **37°**
8. **97°**
9. **23°** each
10. **45°** each
11. **60°** each

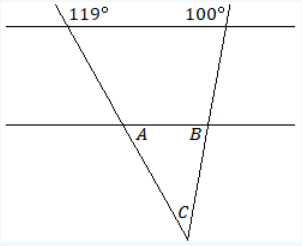
Angles and Parallel Lines

Two lines that point in the exact same direction and will never cross are called parallel lines. If two parallel lines are crossed by a third line, sets of equally-sized angles will be formed, as shown in the following diagram. All four acute angles will be equal in measure, all four obtuse angles will be equal in measure, and any acute angle and obtuse angle will have a combined measure of **180°**.



Example 8.1.1

Find the measures of angles A, B, and C.



Solution

A. 61°

B. 80°

C. 39°

Degrees, Minutes, Seconds

It is possible to have angle measures that are not a whole number of degrees. It is common to use decimals in these situations, but the older method—called the ***degrees-minutes-seconds*** or DMS system—divides a degree using fractions out of 60: a minute is $\frac{1}{60}$ of a degree, and a second is $\frac{1}{60}$ of a minute, which means a second is $\frac{1}{3,600}$ of a degree. (Fortunately, these conversions work exactly like time; think of 1 degree as 1 hour.) For example, $2.5^\circ = 2^\circ 30'$.



We will look at the procedure for converting between systems, but there are online calculators such as the one at https://www.fcc.gov/media/radio/dms-decimal which will do the conversions for you.

If you have latitude and longitude in DMS, like N $18^\circ 54' 40''$ W $155^\circ 40' 51''$, and need to convert it to decimal degrees, the process is fairly simple with a calculator.

Converting from DMS to Decimal Degrees

Enter $\text{degrees} + \text{minutes} \div 60 + \text{seconds} \div 3600$ in your calculator. Round the result to the fourth decimal place, if necessary.[1]

? Exercises 8.1.1

Convert each angle measurement from degrees-minutes-seconds into decimal form. Round to the nearest ten-thousandth, if necessary.

13. $67°48'54''$

14. $19°37'25''$

15. $34°14'12''$

Answer

13. $67.815°$
14. $19.6236°$
15. $34.2367°$

Going from decimal degrees to DMS is a more complicated process.

Converting from Decimal Degrees to DMS

1. The whole-number part of the angle measurement gives the number of degrees.
2. Multiply the decimal part by 60. The whole number part of this result is the number of minutes.
3. Multiply the decimal part *of the minutes* by 60. This gives the number of seconds (including any decimal part of seconds).

For example, let's convert $15.374°$.

1. The *degrees* part of our answer will be 15.
2. The decimal part times 60 is $0.374 \cdot 60 = 22.44$ minutes. The *minutes* part of our answer will be 22.
3. The decimal part times 60 is $0.44 \cdot 60 = 26.4$ seconds. The *seconds* part of our answer will be 26.4.

So $15.374° = 15°22'26.4''$.

? Exercises 8.1.1

Convert each angle measurement from decimal into degrees-minutes-seconds form.

16. $26.785°$

17. $58.216°$

18. $41.13°$

Answer

16. $26°47'6''$
17. $58°12'57.6''$
18. $41°7'48''$

---

1. We round to four decimal places because 1 second of angle is $\frac{1}{3,600}$ of a degree. This is a smaller fraction than $\frac{1}{1,000}$ so our precision is slightly better than the thousandths place. ↵

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8.2: Triangles

You may use a calculator throughout this module as needed.



Classifying Triangles

We can classify triangles into three categories based on the lengths of their sides.

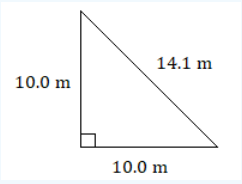
- Equilateral triangle: all three sides have the same length
- Isosceles triangle: exactly two sides have the same length
- Scalene triangle: all three sides have different lengths

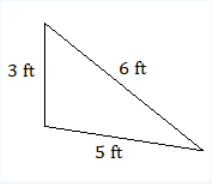
We can also classify triangles into three categories based on the measures of their angles.

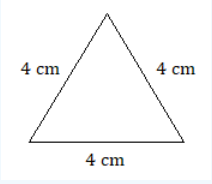
- Obtuse triangle: one of the angles is an obtuse angle
- Right triangle: one of the angles is a right angle
- Acute triangle: all three of the angles are acute

Exercises 8.2.1

Classify each triangle by angle and side. For example, “acute scalene”.)

1. 

2. 

3. 

Answer

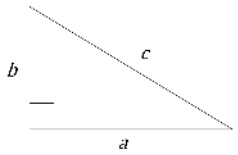
1. right isosceles triangle
2. obtuse scalene triangle

3. acute equilateral triangle (yes, an equilateral triangle will always be acute)

Right Triangles

In a right triangle, the two sides that form the right angle are called the ***legs***. The side opposite the right angle, which will always be the longest side, is called the ***hypotenuse***.

The Pythagorean theorem says that the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.



Theorem 8.2.1: The Pythagorean Theorem

In a right triangle with legs a and b and hypotenuse c,

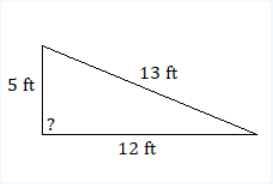
$$a^2 + b^2 = c^2$$

If you know the lengths of all three sides of a triangle, you can use the Pythagorean theorem to verify whether the triangle is a right triangle or not. The ancient Egyptians used this method for surveying when they needed to redraw boundaries after the yearly flooding of the Nile washed away their previous markings.[1]

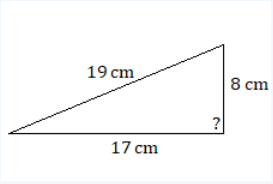
Exercises 8.2.1

Use the Pythagorean theorem to determine whether either of the following triangles is a right triangle.

8.



9.



Answer

8. right triangle, because $5^2 + 12^2 = 13^2$

9. not a right triangle, because $8^2 + 17^2 \neq 19^2$

Before we continue, we need to briefly discuss square roots. Calculating a square root is the opposite of squaring a number. For example, $\sqrt{49} = 7$ because $7^2 = 49$. If the number under the square root symbol is not a perfect square like 49, then the square root will be an irrational decimal that we will round off as necessary.

Exercises 8.2.1

Use a calculator to find the value of each square root. Round to the hundredths place.

10. $\sqrt{50}$

11. $\sqrt{296}$

12. $\sqrt{943}$

Answer

10. **7.07**

11. **17.20**

12. **30.71**

We most often use the Pythagorean theorem to calculate the length of a missing side of a right triangle. Here are three different versions of the Pythagorean theorem arranged to find a missing side, so you don't have to use algebra with $a^2 + b^2 = c^2$.

The Pythagorean Theorem, three other versions

$c = \sqrt{a^2 + b^2}$

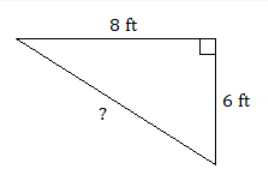
$b = \sqrt{c^2 - a^2}$

$a = \sqrt{c^2 - b^2}$

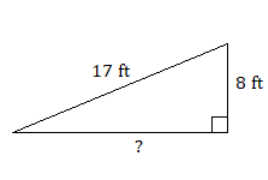
Exercises 8.2.1

Find the length of the missing side for each of these right triangles. Round to the nearest tenth, if necessary.

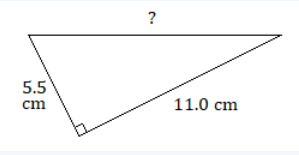
13.



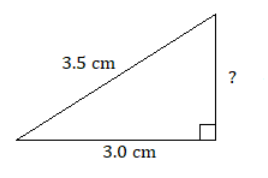
14.



15.



16.



Answer

13. **10 ft**

14. 15 ft
15. 12.3 cm
16. 1.8 cm

1. The surveyors were called "rope-stretchers" because they used a loop of rope 12 units long with 12 equally-spaced knots. Three rope-stretchers each held a knot, forming a triangle with lengths 3, 4, and 5 units. When the rope was stretched tight, they knew that the angle between the 3-unit and 4-unit sides was a right angle because $3^2 + 4^2 = 5^2$. From *Discovering Geometry: an Inductive Approach* by Michael Serra, Key Curriculum Press, 1997. ↵

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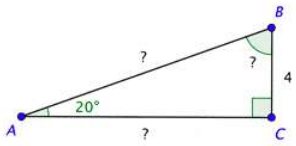
8.3: Right Triangle Trigonometry

Learning Objectives

- Find the missing lengths and angles of a right triangle.
- Solve applied problems using right-triangle trigonometry.

Introduction

Suppose you have to build a ramp and don't know how long it needs to be. You know certain angle measurements and side lengths, but you need to find the missing pieces of information.



There are six trigonometric functions, or ratios, that you can use to compute what you don't know. You will now learn how to use these six functions to solve right-triangle application problems.

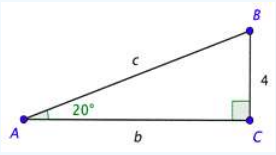
Solving Right Triangles

Determining all of the side lengths and angle measures of a right triangle is known as *solving* a right triangle. Let's look at how to do this when you're given one side length and one acute angle measure. Once you learn how to solve a right triangle, you'll be able to solve many real world applications, such as the ramp problem at the beginning of this lesson, and the only tools you'll need are the definitions of the trigonometric functions, the Pythagorean Theorem, and a calculator.

Example 8.3.2

Problem: You need to build a ramp with the following dimensions. Solve the right triangle shown below. Use the approximations $\sin 20° \approx 0.342$ and $\tan 20° \approx 0.364$, and give the lengths to the nearest tenth.

Answer



Remember that the acute angles in a right triangle are complementary, which means their sum is $90°$. Since $m\angle A = 20°$, it follows that $m\angle B = 90° - 20° = 70°$.

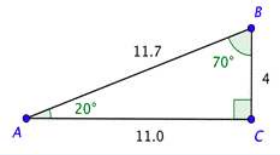
You can use the definition of sine to find c. Substitute the measure of the angle on the left side of the equation and use the triangle to set up the ratio on the right. Solving the equation and rounding to the nearest tenth gives you $c \approx 11.7$.

$$\sin 20° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$0.342 \approx \frac{4}{c}$$
$$c(0.342) \approx 4$$
$$c \approx \frac{4}{0.342} \approx 11.695$$

In a similar way, you can use the definition of tangent and the measure of the angle to find b. Solving the equation and rounding to the nearest tenth gives you $b \approx 11.0$.

$$\tan 20^\circ = \frac{\text{opposite}}{\text{adjacent}}$$
$$0.364 \approx \frac{4}{b}$$
$$0.364b \approx 4$$
$$b \approx \frac{4}{0.364} \approx 10.989$$

The ramp needs to be 11.7 feet long.

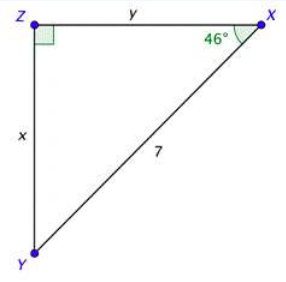


In the problem above, you were given the values of the trigonometric functions. In the next problem, you'll need to use the trigonometric function keys on your calculator to find those values.

✔ Example ✔ 8.3.3

Problem: Solve the right triangle shown below. Give the lengths to the nearest tenth.

Answer



The acute angles are complementary, which means their sum is 90°. Since Since $m\angle X = 46^\circ$, it follows that $m\angle Y = 90^\circ - 46^\circ = 44^\circ$.

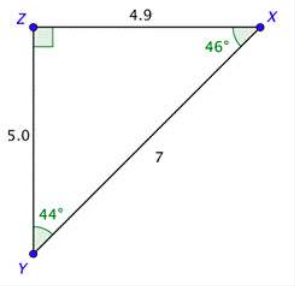
You can use the definition of sine to find x. Use your calculator to find the value of $\sin 46^\circ$ and the triangle to set up the ratio on the right. Solving the equation and rounding to the nearest tenth gives you $x \approx 5.0$.

$$\sin 46^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$0.72 \approx \frac{x}{7}$$
$$7(0.72) \approx x$$
$$5.04 \approx x$$

To find y, you can either use another trigonometric function (such as cosine) or you can use the Pythagorean Theorem. Solving the equation and rounding to the nearest tenth gives you $y \approx 4.9$.

$$5.0^2 + y^2 \approx 7^2$$
$$25 + y^2 \approx 49$$
$$y^2 \approx 24$$
$$y \approx \sqrt{24}$$
$$y \approx 4.899$$

We now know all three sides and all three angles. Their values are shown in the drawing.

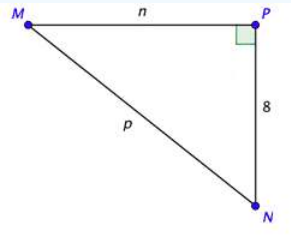


Sometimes you may be given enough information about a right triangle to solve the triangle, but that information may not include the measures of the acute angles. In this situation, you will need to use the inverse trigonometric function keys on your calculator to solve the triangle.

Example 8.3.4

Problem: Solve the right triangle shown below, given that $\tan N = \frac{5}{4}$. Find the exact side lengths and approximate the angles to the nearest degree.

Answer



You are not given an angle measure, but you can use the definition of tangent to find the value of n.

Use the ratio you are given on the left side and the information from the triangle on the right side. Cross-multiply and solve for n.

$$\tan N = \frac{\text{opposite}}{\text{adjacent}}$$
$$\frac{5}{4} = \frac{n}{8}$$
$$40 = 4n$$
$$n = 10$$

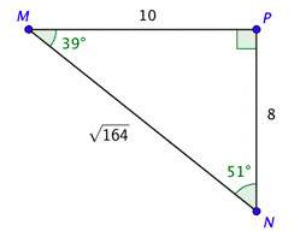
Use the Pythagorean Theorem to find the value of p.

$$10^2 + 8^2 = p^2$$
$$100 + 64 = p^2$$
$$164 = p^2$$
$$\sqrt{164} = p$$

We can use the triangle to find a value of the tangent and the inverse tangent key on your calculator to find the angle that yields that value. Rounding to the nearest degree, $\angle M$ is approximately $39°$, $M \approx 39°$. Subtract $39°$, from $90°$ to get $N \approx 51°$.

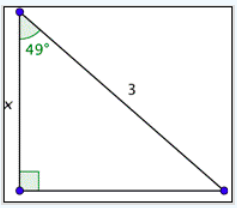
$$\tan M = \frac{8}{10} = 0.8$$
$$M = \tan^{-1} 0.8$$
$$M \approx 38.7°$$

We now know all three sides and all three angles. Their values are shown in the drawing.



? Try It 8.3.1

What is the value of x to the nearest hundredth?



A. 4.57
B. 1.97
C. 0.90
D. 0.22

Answer

A. 4.57. Incorrect. You probably set up the ratio incorrectly, equating $\cos 49°$ and $\frac{3}{x}$. A correct way to set up the equation is $\cos 49° = \frac{x}{3}$. The correct answer is 1.97.

B. 1.97. Correct. One way to set up a correct equation is to use the definition of cosine. This will give you $\cos 49° = \frac{x}{3}$. The solution to this equation is: $x = 3 \cdot \cos 49° \approx 3 \cdot (0.656) = 1.968$. This rounds to 1.97.

C. 0.90. Incorrect. You probably set up the correct equation, $\cos 49° = \frac{x}{3}$, and solved it correctly. However, your calculator was not set to degrees. The correct answer is 1.97.

D. 0.22. Incorrect. You may have correctly set up your equation as $\cos 49° = \frac{x}{3}$, but then incorrectly solved it as $x = \frac{\cos 49°}{3}$. The correct answer is 1.97.

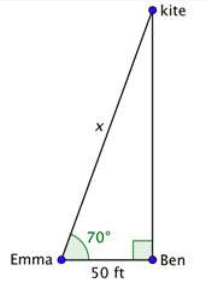
Using Trigonometry in Real-World Problems

There are situations in the real world, such as building a ramp for a loading dock, in which you have a right triangle with certain information about the sides and angles, and you wish to find unknown measures of sides or angles. This is where understanding trigonometry can help you.

✔ Example ✔ 8.3.5

Problem: Ben and Emma are out flying a kite. Emma can see that the kite string she is holding is making a $70°$ angle with the ground. The kite is directly above Ben, who is standing 50 feet away. To the nearest foot, how many feet of string has Emma let out?

Answer



We want to find the length of string let out. It is the hypotenuse of the right triangle shown.

Since the 50-foot distance measures the adjacent side to the $70°$ angle, you can use the cosine function to find x.

$$\cos 70° = \frac{50}{x}$$

Solve the equation for x. Use a calculator to find a numerical value. The answer rounds to 146.

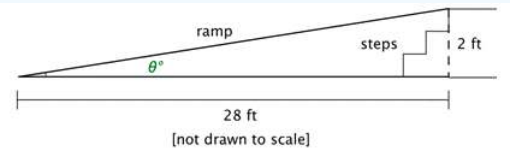
$$\begin{aligned} x \cdot \cos 70° &= 50 \\ x &= \frac{50}{\cos 70°} \\ x &= \frac{50}{0.342...} \\ x &= 146.19... \end{aligned}$$

Emma has let out approximately 146 feet of string.

In the example above, you were given one side and an acute angle. In the next one, you're given two sides and asked to find an angle. Finding an angle will usually involve using an inverse trigonometric function. The Greek letter theta, θ, is commonly used to represent an unknown angle. In this example, θ represents the angle of elevation.

✔ Example ✔ 8.3.6

Problem: A wheelchair ramp is placed over a set of stairs so that one end is 2 feet off the ground. The other end is at a point that is a horizontal distance of 28 feet away, as shown in the diagram. What is the angle of elevation to the nearest tenth of a degree?



Answer

The angle of elevation is labeled $\theta°$ in the diagram. The lengths given are the sides opposite and adjacent to this angle, so you can use the tangent function to find θ.

You want to find the measure of an angle that gives you a certain tangent value. This means that you need to find the inverse tangent. Remember that you have to use the keys 2ND and TAN on your calculator. Look at the hundredths place to round to the nearest tenth.

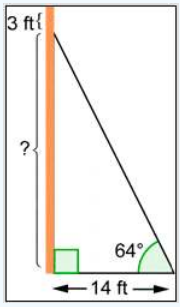
$$\begin{aligned} \tan \theta° &= \frac{2}{28} \\ \tan \theta° &= \frac{1}{14} \\ \theta° &= \tan^{-1}\left(\frac{1}{14}\right) \\ \theta &= 4.0856... \end{aligned}$$

The angle of elevation is approximately $4.1°$.

Sometimes the right triangle can be part of a bigger picture.

Try It 8.3.2

A guy wire is attached to a telephone pole 3 feet below the top of the pole, as shown below. The guy wire is anchored 14 feet from the telephone pole and makes a $64°$ angle with the ground. How high up the pole is the guy wire attached? Round your answer to the nearest tenth of a foot.



A. $14 \cdot \sin 64° \approx 12.6$ feet
B. $14 \cdot \tan 64° \approx 28.7$ feet
C. $14 \cdot \tan 64° + 3 \approx 31.7$ feet
D. $\frac{14}{\cos 64°} \approx 31.9$ feet

Answer

A. $14 \cdot \sin 64° \approx 12.6$ feet . Incorrect. You may have been confused as to which ratio corresponds to which trigonometric function. You need to solve the equation $\tan 64° = \frac{x}{14}$, where x represents the vertical distance from the base of the telephone pole up to where the guy wire is attached. The correct answer is $14 \cdot \tan 64° \approx 28.7$ feet .

B. $14 \cdot \tan 64° \approx 28.7$ feet . Correct. Let x represent the vertical distance from the base of the telephone pole up to where the guy wire is attached. Then $\tan 64° = \frac{x}{14}$. Solving this equation for x gives you $x = 14 \cdot \tan 64° \approx 28.7$ feet .

C. $14 \cdot \tan 64° + 3 \approx 31.7$ feet . Incorrect. It looks like you set up and solved the correct equation to find the unknown length. However, you misread the problem. When you added the 3 you found the height of the entire pole. The correct answer is $14 \cdot \tan 64° \approx 28.7$ feet .

D. $\frac{14}{\cos 64°} \approx 31.9$ feet . Incorrect. It looks like you set up and solved an equation to find the length of the wire (the hypotenuse of the triangle). You need to solve the equation $\tan 64° = \frac{x}{14}$, where x represents the vertical distance from the base of the telephone pole up to where the guy wire is attached. The correct answer is $14 \cdot \tan 64° \approx 28.7$ feet .

Summary

There are many ways to find the missing side lengths or angle measures in a right triangle. Solving a right triangle can be accomplished by using the definitions of the trigonometric functions and the Pythagorean Theorem. This process is called solving a right triangle. Being able to solve a right triangle is useful in solving a variety of real-world problems such as the construction of a wheelchair ramp.

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CHAPTER OVERVIEW

9: Vectors

9.1: Trigonometric Functions of Any Angle
9.2: Vectors

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9.1: Trigonometric Functions of Any Angle

To define the trigonometric functions of *any* angle - including angles less than $0°$ or greater than $360°$ - we need a more general definition of an angle. We say that an **angle** is formed by rotating a ray $\overrightarrow{OA}$ about the endpoint O (called the **vertex**), so that the ray is in a new position, denoted by the ray $\overrightarrow{OB}$. The ray $\overrightarrow{OA}$ is called the **initial side** of the angle, and $\overrightarrow{OB}$ is the **terminal side** of the angle (see Figure 1.4.1(a)).

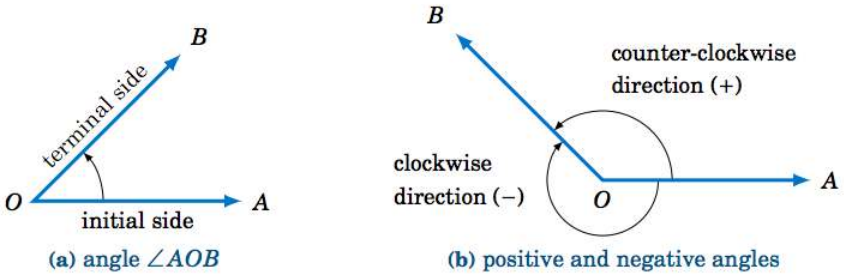


Figure 1.4.1 Definition of a general angle

We denote the angle formed by this rotation as $\angle AOB$, or simply $\angle O$, or even just O. If the rotation is counter-clockwise then we say that the angle is **positive**, and the angle is **negative** if the rotation is clockwise (see Figure 1.4.1(b)).

One full counter-clockwise rotation of $\overrightarrow{OA}$ back onto itself (called a **revolution**), so that the terminal side coincides with the initial side, is an angle of $360°$; in the clockwise direction this would be $-360°$. Not rotating $\overrightarrow{OA}$ constitutes an angle of $0°$. More than one full rotation creates an angle greater than $360°$. For example, notice that $30°$ and $390°$ have the same terminal side in Figure 1.4.2, since $30 + 360 = 390$.

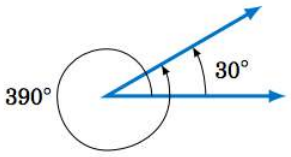


Figure 1.4.2 Angle greater than $360°$

We can now define the trigonometric functions of any angle in terms of **Cartesian coordinates**. Recall that the **xy-coordinate plane** consists of points denoted by pairs (x, y) of real numbers. The first number, x, is the point's **x coordinate**, and the second number, y, is its **y coordinate**. The x and y coordinates are measured by their positions along the **x-axis** and **y-axis**, respectively, which determine the point's position in the plane. This divides the xy-coordinate plane into four **quadrants** (denoted by QI, QII, QIII, QIV), based on the signs of x and y (see Figure 1.4.3(a)-(b)).

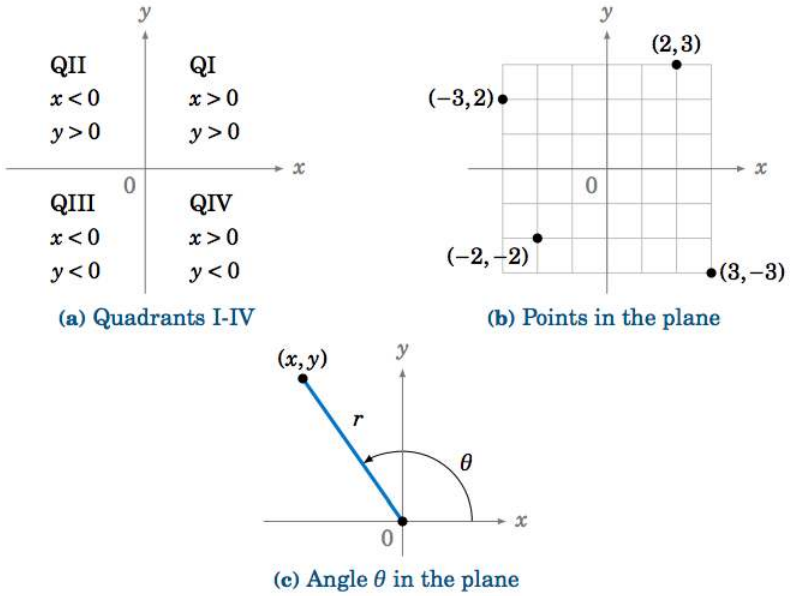


Figure 1.4.3 xy-coordinate plane

Now let θ be any angle. We say that θ is in **standard position** if its initial side is the positive x-axis and its vertex is the origin $(0,0)$. Pick *any* point (x,y) on the terminal side of θ a distance $r>0$ from the origin (see Figure 1.4.3(c)). (Note that $r=\sqrt{x^2+y^2}$. Why?) We then define the trigonometric functions of θ as follows:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \tag{9.1.1}$$

As in the acute case, by the use of similar triangles these definitions are well-defined (i.e. they do not depend on which point (x,y) we choose on the terminal side of θ). Also, notice that $|\sin\theta| \le 1$ and $|\cos\theta| \le 1$, since $|y| \le r$ and $|x| \le r$ in the above definitions.

Notice that in the case of an acute angle these definitions are equivalent to our earlier definitions in terms of right triangles: draw a right triangle with angle θ such that $x=$ **adjacent side**, $y=$ **opposite side**, and $r=$ **hypotenuse**. For example, this would give us $\sin\theta = \frac{y}{r} = \frac{\text{opposite}}{\text{hypotenuse}}$ and $\cos\theta = \frac{x}{r} = \frac{\text{adjacent}}{\text{hypotenuse}}$, just as before (see Figure 1.4.4(a)).

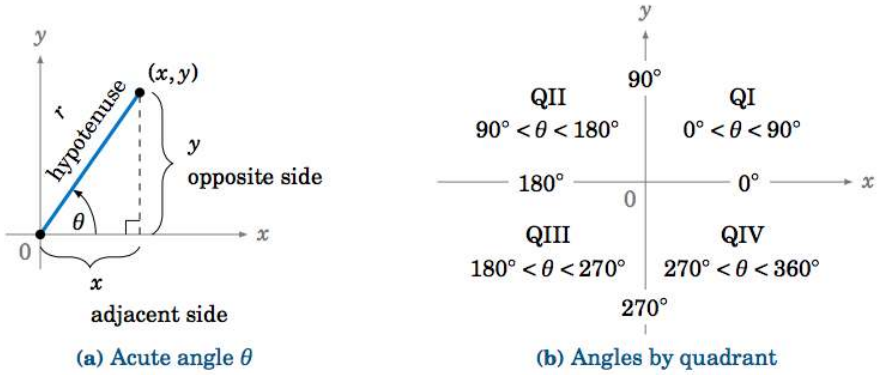


Figure 1.4.4

Since $360°$ represents one full revolution, the trigonometric function values repeat every $360°$. For example, $\sin 360° = \sin 0°$, $\cos 390° = \cos 30°$, $\tan 540° = \tan 180°$, $\sin(-45°) = \sin 315°$, etc. In general, if two angles differ by an integer multiple of $360°$ then each trigonometric function will have equal values at both angles. Angles such as these, which have the same initial and terminal sides, are called **coterminal**.

In Examples 1.20-1.22, we saw how the values of trigonometric functions of an angle θ larger than $90°$ were found by using a certain acute angle as part of a right triangle. That acute angle has a special name: if θ is a nonacute angle then we say that the

reference angle for θ is the acute angle formed by the terminal side of θ and either the positive or negative x-axis. So in Example 1.20, we see that $60°$ is the reference angle for the nonacute angle $\theta = 120°$; in Example 1.21, $45°$ is the reference angle for $\theta = 225°$; and in Example 1.22, $30°$ is the reference angle for $\theta = 330°$.

Example 1.24

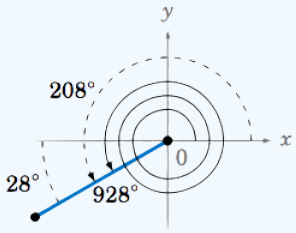


Figure 1.4.7

a. Which angle between $0°$ and $360°$ has the same terminal side (and hence the same trigonometric function values) as θ?
b. What is the reference angle for θ?

Solution

(a) Since $928° = 2 \times 360° + 208°$, then θ has the same terminal side as $208°$, as in Figure 1.4.7.

(b) $928°$ and $208°$ have the same terminal side in QIII, so the reference angle for $\theta = 928°$ is $208° - 180° = 28°$.

Example 1.25

Suppose that $\cos\ \theta = -\frac{4}{5}$. Find the exact values of $\sin\ \theta$ and $\tan\ \theta$. Also find θ in each quadrant.

Solution

We can use a method similar to the one used to solve Example 1.8 in Section 1.2. That is, draw a right triangle and interpret $\cos\ \theta$ as the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ of two of its sides. Since $\cos\ \theta = -\frac{4}{5}$, we can use 4 as the length of the adjacent side and 5 as the length of the hypotenuse. By the Pythagorean Theorem, the length of the opposite side must then be 3. Since $\cos\ \theta$ is negative, we know from Figure 1.4.5 that θ must be in either QII or QIII. Thus, we have *two* possibilities, as shown in Figure 1.4.8 below:

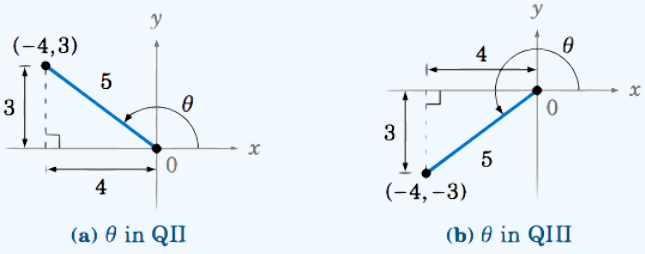


Figure 1.4.8 $\cos\theta = -\frac{4}{5}$

When θ is in QII, we see from Figure 1.4.8(a) that the point $(-4, 3)$ is on the terminal side of θ, and so we have $x = -4$, $y = 3$, and $r = 5$. Thus, $\sin\ \theta = \frac{y}{r} = \frac{3}{5}$ and $\tan\ \theta = \frac{y}{x} = \frac{3}{-4}$.

When θ is in QIII, we see from Figure 1.4.8(b) that the point $(-4, -3)$ is on the terminal side of θ, and so we have $x = -4$, $y = -3$, and $r = 5$. Thus, $\sin\ \theta = \frac{y}{r} = \frac{-3}{5}$ and $\tan\ \theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$.

Thus, either $\boxed{\sin\ \theta = \frac{3}{5} \text{ and } \tan\ \theta = -\frac{3}{4}}$ or $\boxed{\sin\ \theta = -\frac{3}{5} \text{ and } \tan\ \theta = \frac{3}{4}}$.

If $\cos\ \theta = -\frac{4}{5}$ were solved we would get θ = 143.14, which lies in quadrant II. The reference angle is 36.87. To determine the angle in quadrant III, θ = 180 + 36.87 = 216.87.

For an angle θ in standard position and a point (x, y) on its terminal side:

a. $\sin\ \theta$ has the same sign as y

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b. $\cos\ \theta$ has the same sign as x
c. $\tan\ \theta$ is positive when x and y have the same sign
d. $\tan\ \theta$ is negative when x and y have opposite signs

Contributors

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9.2: Vectors

Learning Objectives

- View vectors geometrically.
- Find magnitude and direction.
- Perform vector addition and scalar multiplication.
- Find the component form of a vector.
- Find the unit vector in the direction of v.
- Perform operations with vectors in terms of i and j.
- Find the dot product of two vectors.

An airplane is flying at an airspeed of 200 miles per hour headed on a SE bearing of $140°$. A north wind (from north to south) is blowing at 16.2 miles per hour, as shown in Figure $9.2.1$. What are the ground speed and actual bearing of the plane?

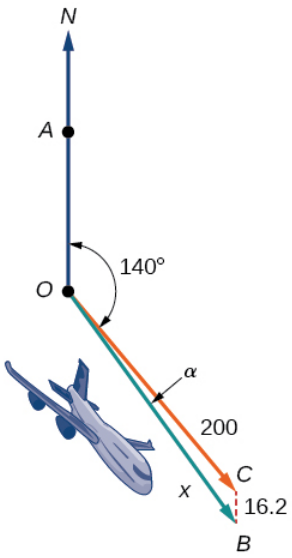


***Figure* 9.2.1**

Ground speed refers to the speed of a plane relative to the ground. Airspeed refers to the speed a plane can travel relative to its surrounding air mass. These two quantities are not the same because of the effect of wind. In an earlier section, we used triangles to solve a similar problem involving the movement of boats. Later in this section, we will find the airplane's ground speed and bearing, while investigating another approach to problems of this type. First, however, let's examine the basics of vectors.

A Geometric View of Vectors

A vector is a specific quantity drawn as a line segment with an arrowhead at one end. It has an **initial point**, where it begins, and a **terminal point**, where it ends. A vector is defined by its magnitude, or the length of the line, and its direction, indicated by an arrowhead at the terminal point. Thus, a vector is a directed line segment. There are various symbols that distinguish vectors from other quantities:

- Lower case, boldfaced type, with or without an arrow on top such as u, w, $\vec{v}$, $\vec{u}$, $\vec{w}$.
- Given initial point P and terminal point Q, a vector can be represented as $\overrightarrow{PQ}$. The arrowhead on top is what indicates that it is not just a line, but a directed line segment.
- Given an initial point of $(0,0)$ and terminal point (a,b), a vector may be represented as $\langle a,b\rangle$.

This last symbol $\langle a,b\rangle$ has special significance. It is called the standard position. The position vector has an initial point $(0,0)$ and a terminal point $\langle a,b\rangle$. To change any vector into the position vector, we think about the change in the x-coordinates and the change in

the y-coordinates. Thus, if the initial point of a vector $\overrightarrow{CD}$ is $C(x_1, y_1)$ and the terminal point is $D(x_2, y_2)$, then the position vector is found by calculating

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle a, b \rangle\end{aligned}$$

In Figure 9.2.2, we see the original vector $\overrightarrow{CD}$ and the position vector $\overrightarrow{AB}$.

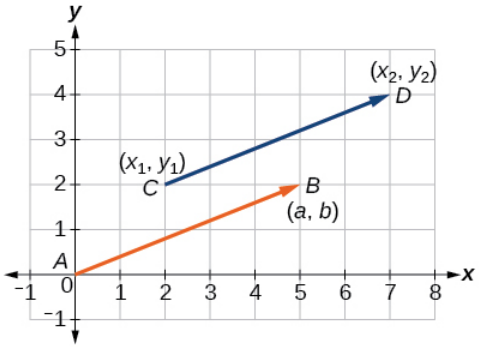


***Figure* 9.2.2**

PROPERTIES OF VECTORS

A vector is a directed line segment with an initial point and a terminal point. Vectors are identified by magnitude, or the length of the line, and direction, represented by the arrowhead pointing toward the terminal point. The position vector has an initial point at $(0, 0)$ and is identified by its terminal point $\langle a, b \rangle$.

Example 9.2.1*A*: Find the Position Vector

Consider the vector whose initial point is $P(2, 3)$ and terminal point is $Q(6, 4)$. Find the position vector.

Solution

The position vector is found by subtracting one x-coordinate from the other x-coordinate, and one y-coordinate from the other y-coordinate. Thus

$$\begin{aligned}v &= \langle 6 - 2, 4 - 3 \rangle \\ &= \langle 4, 1 \rangle\end{aligned}$$

The position vector begins at $(0, 0)$ and terminates at $(4, 1)$. The graphs of both vectors are shown in Figure 9.2.3.

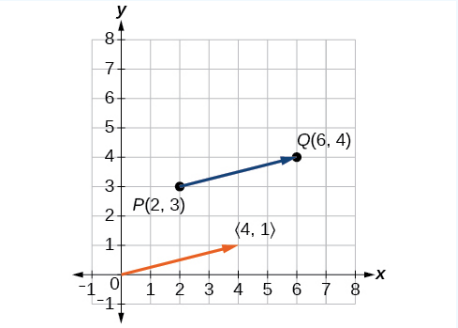


***Figure* 9.2.3**

We see that the position vector is $\langle 4, 1 \rangle$.

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Example 9.2.1B: Drawing a Vector with the Given Criteria and Its Equivalent Position Vector

Find the position vector given that vector v has an initial point at $(-3, 2)$ and a terminal point at $(4, 5)$, then graph both vectors in the same plane.

Solution

The position vector is found using the following calculation:

$$\begin{aligned} v &= \langle 4-(-3), 5-2 \rangle \\ &= \langle 7, 3 \rangle \end{aligned}$$

Thus, the position vector begins at $(0, 0)$ and terminates at $(7, 3)$. See Figure 9.2.4.

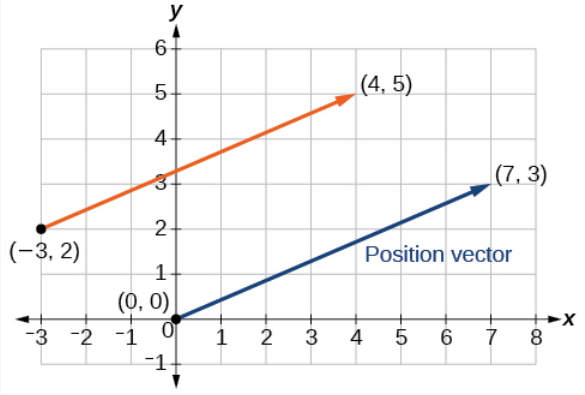


***Figure* 9.2.4**

Exercise 9.2.1

Draw a vector $\vec{v}$ that connects from the origin to the point $(3, 5)$.

Answer

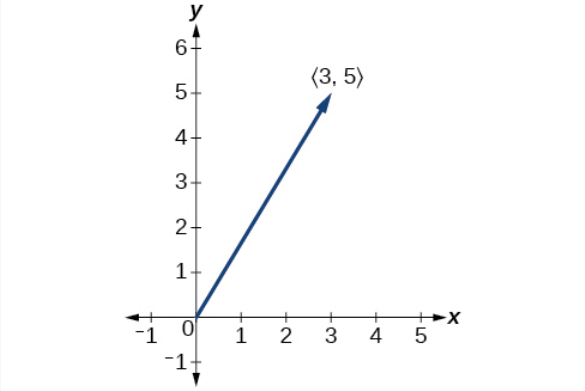


***Figure* 9.2.5**

Finding Magnitude and Direction

To work with a vector, we need to be able to find its magnitude and its direction. We find its magnitude using the Pythagorean Theorem or the distance formula, and we find its direction using the inverse tangent function.

MAGNITUDE AND DIRECTION OF A VECTOR

Given a position vector $\vec{v} = \langle a, b \rangle$,the magnitude is found by $|v| = \sqrt{a^2 + b^2}$.The direction is equal to the angle formed with the x -axis, or with the y-axis, depending on the application. For a position vector, the direction is found by $\tan\theta = \left(\frac{b}{a}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$, as illustrated in Figure 9.2.6.

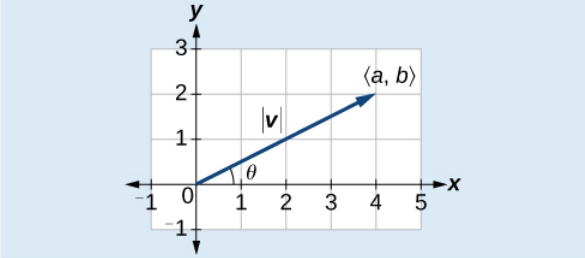


***Figure* 9.2.6**

Two vectors $\vec{v}$ and $\vec{u}$ are considered equal if they have the same magnitude and the same direction. Additionally, if both vectors have the same position vector, they are equal.

Example 9.2.2*A*: Finding the Magnitude and Direction of a Vector

Find the magnitude and direction of the vector with initial point $P(-8, 1)$ and terminal point $Q(-2, -5)$.Draw the vector.

Solution

First, find the position vector.

$$\begin{aligned} u &= \langle -2, -(-8), -5 - 1 \rangle \\ &= \langle 6, -6 \rangle \end{aligned}$$

We use the Pythagorean Theorem to find the magnitude.

$$\begin{aligned} |u| &= \sqrt{(6)^2 + (-6)^2} \\ &= \sqrt{72} \\ &= \sqrt{62} \end{aligned}$$

The direction is given as

$$\begin{aligned} \tan\theta = \frac{-6}{6} = -1 \rightarrow \theta &= \tan^{-1}(-1) \\ &= -45° \end{aligned}$$

However, the angle terminates in the fourth quadrant, so we add $360°$ to obtain a positive angle. Thus, $-45° + 360° = 315°$. See Figure 9.2.7.

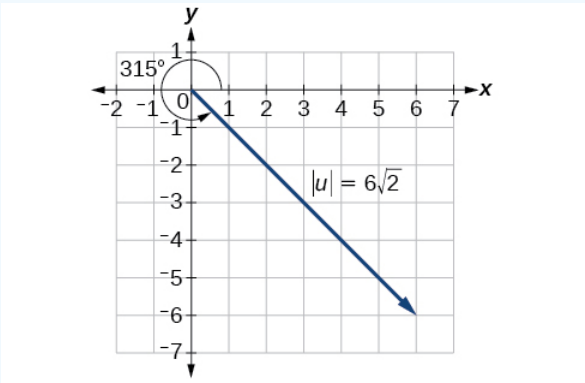


Figure 9.2.7

Example $9.2.2B$: Showing That Two Vectors Are Equal

Show that vector $\vec{v}$ with initial point at $(5,-3)$ and terminal point at $(-1,2)$ is equal to vector $\vec{u}$ with initial point at $(-1,-3)$ and terminal point at $(-7,2)$. Draw the position vector on the same grid as $\vec{v}$ and $\vec{u}$. Next, find the magnitude and direction of each vector.

Solution

As shown in Figure 9.2.8, draw the vector $\vec{v}$ starting at initial $(5,-3)$ and terminal point $(-1,2)$. Draw the vector $\vec{u}$ with initial point $(-1,-3)$ and terminal point $(-7,2)$. Find the standard position for each.

Next, find and sketch the position vector for $\vec{v}$ and $\vec{u}$. We have

$$\begin{aligned} v &= \langle -1-5, 2-(-3)\rangle \\ &= \langle -6,5\rangle u \\ &= \langle -7-(-1), 2-(-3)\rangle \\ &= \langle -6,5\rangle \end{aligned}$$

Since the position vectors are the same, $\vec{v}$ and $\vec{u}$ are the same.

An alternative way to check for vector equality is to show that the magnitude and direction are the same for both vectors. To show that the magnitudes are equal, use the Pythagorean Theorem.

$$\begin{aligned} |v| &= \sqrt{(-1-5)^2+(2-(-3))^2} \\ &= \sqrt{(-6)^2+(5)^2} \\ &= \sqrt{36+25} \\ &= \sqrt{61} \\ |u| &= \sqrt{(-7-(-1))^2+(2-(-3))^2} \\ &= \sqrt{(-6)^2+(5)^2} \\ &= \sqrt{36+25} \\ &= \sqrt{61} \end{aligned}$$

As the magnitudes are equal, we now need to verify the direction. Using the tangent function with the position vector gives

$$\begin{aligned} \tan\theta = -\frac{5}{6} \Rightarrow \theta &= \tan^{-1}\left(-\frac{5}{6}\right) \\ &= -39.8° \end{aligned}$$

However, we can see that the position vector terminates in the second quadrant, so we add $180°$. Thus, the direction is $-39.8° + 180° = 140.2°$.

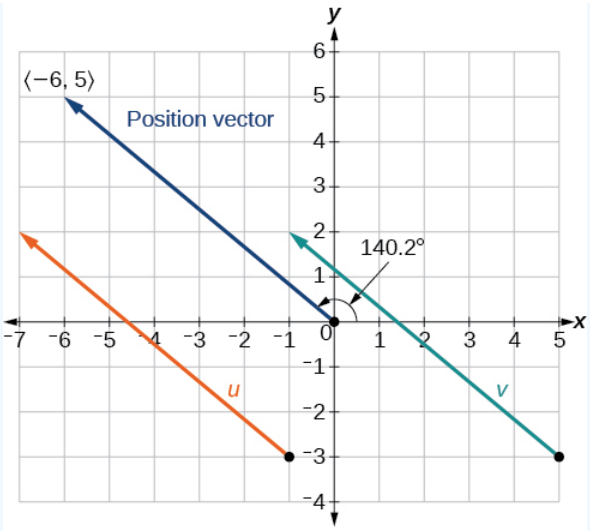


***Figure* 9.2.8**

Performing Vector Addition and Scalar Multiplication

Now that we understand the properties of vectors, we can perform operations involving them. While it is convenient to think of the vector $u = \langle x, y \rangle$ as an arrow or directed line segment from the origin to the point (x, y), vectors can be situated anywhere in the plane. The sum of two vectors $\vec{u}$ and $\vec{v}$, or vector addition, produces a third vector $\overrightarrow{u+v}$, the resultant vector.

To find $\overrightarrow{u+v}$, we first draw the vector $\vec{u}$, and from the terminal end of $\vec{u}$, we drawn the vector $\vec{v}$. In other words, we have the initial point of $\vec{v}$ meet the terminal end of $\vec{u}$. This position corresponds to the notion that we move along the first vector and then, from its terminal point, we move along the second vector. The sum $\overrightarrow{u+v}$ is the resultant vector because it results from addition or subtraction of two vectors. The resultant vector travels directly from the beginning of $\vec{u}$ to the end of $\vec{v}$ in a straight path, as shown in Figure **9.2.9**.

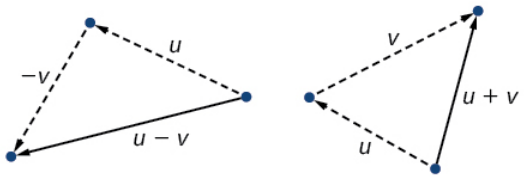


***Figure* 9.2.9**

Vector subtraction is similar to vector addition. To find $\overrightarrow{u-v}$, view it as $\overrightarrow{u+(-v)}$. Adding $\overrightarrow{-v}$ is reversing direction of $\vec{v}$ and adding it to the end of $\vec{u}$. The new vector begins at the start of $\vec{u}$ and stops at the end point of $\overrightarrow{-v}$. See Figure **9.2.10** for a visual that compares vector addition and vector subtraction using parallelograms.

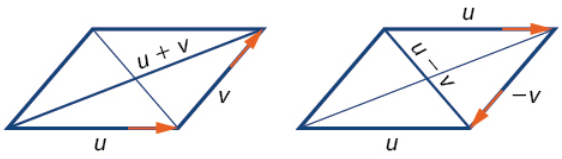


***Figure* 9.2.10**

✔ Example 9.2.3: Adding and Subtracting Vectors

Given $u = \langle 3, -2 \rangle$ and $v = \langle -1, 4 \rangle$, find two new vectors $\overrightarrow{u+v}$, and $\overrightarrow{u-v}$.

Solution

To find the sum of two vectors, we add the components. Thus,

$$\begin{aligned} u+v &= \langle 3,-2\rangle + \langle -1,4\rangle \\ &= \langle 3+(-1),-2+4\rangle \\ &= \langle 2,2\rangle \end{aligned}$$

See Figure 9.2.11*a*.

To find the difference of two vectors, add the negative components of $\vec{v}$ to $\vec{u}$. Thus,

$$\begin{aligned} u+(-v) &= \langle 3,-2\rangle + \langle 1,-4\rangle \\ &= \langle 3+1,-2+(-4)\rangle \\ &= \langle 4,-6\rangle \end{aligned}$$

See Figure 9.2.11*b*.

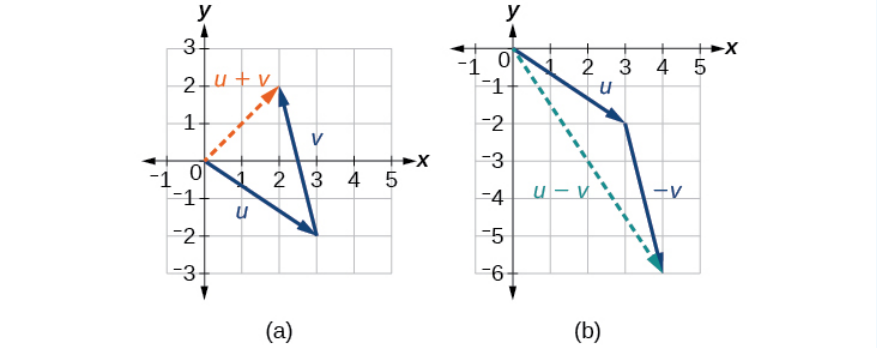


***Figure* 9.2.11:** *(a) Sum of two vectors (b) Difference of two vectors*

Multiplying By a Scalar

While adding and subtracting vectors gives us a new vector with a different magnitude and direction, the process of multiplying a vector by a **scalar**, a constant, changes only the magnitude of the vector or the length of the line. Scalar multiplication has no effect on the direction unless the scalar is negative, in which case the direction of the resulting vector is opposite the direction of the original vector.

SCALAR MULTIPLICATION

Scalar multiplication involves the product of a vector and a scalar. Each component of the vector is multiplied by the scalar. Thus, to multiply $v = \langle a, b\rangle$ by k, we have

$$kv = \langle ka, kb\rangle$$

Only the magnitude changes, unless k is negative, and then the vector reverses direction.

Example 9.2.4: Performing Scalar Multiplication

Given vector $\vec{v} = \langle 3,1\rangle$, find $3\vec{v}$, $\frac{1}{2}$, and $-\vec{v}$.

Solution

See Figure 9.2.12 for a geometric interpretation. If $\vec{v} = \langle 3,1\rangle$, then

$$\begin{aligned} 3v &= \langle 3\cdot 3, 3\cdot 1\rangle \\ &= \langle 9,3\rangle \\ \frac{1}{2}v &= \langle \frac{1}{2}\cdot 3, \frac{1}{2}\cdot 1\rangle \\ &= \langle \frac{3}{2}, \frac{1}{2}\rangle \\ -v &= \langle -3,-1\rangle \end{aligned}$$

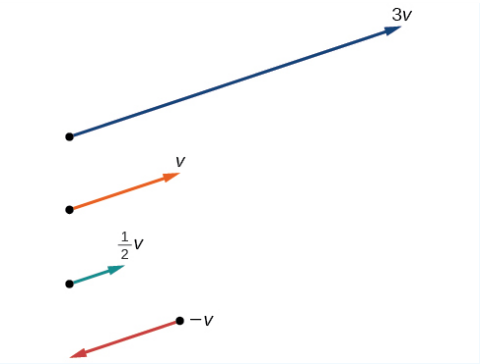


Figure 9.2.12

Analysis

Notice that the vector $3\vec{v}$ is three times the length of $\vec{v}$, $\frac{1}{2}\vec{v}$ is half the length of $\vec{v}$, and $-\vec{v}$ is the same length of $\vec{v}$, but in the opposite direction.

Exercise 9.2.2

Find the scalar multiple $3u$ given $\vec{u} = \langle 5, 4 \rangle$.

Answer

$3u = \langle 15, 12 \rangle$

Example 9.2.5

Find a linear equation to solve for the following unknown quantities: One number exceeds another number by 17 and their sum is 31. Find the two numbers.

Solution

First, we must multiply each vector by the scalar.

$$\begin{aligned} 3u &= 3\langle 3, -2 \rangle \\ &= \langle 9, -6 \rangle \\ 2v &= 2\langle -1, 4 \rangle \\ &= \langle -2, 8 \rangle \end{aligned}$$

Then, add the two together.

$$\begin{aligned} w &= 3u + 2v \\ &= \langle 9, -6 \rangle + \langle -2, 8 \rangle \\ &= \langle 9 - 2, -6 + 8 \rangle \\ &= \langle 7, 2 \rangle \end{aligned}$$

So, $w = \langle 7, 2 \rangle$.

Finding Component Form

In some applications involving vectors, it is helpful for us to be able to break a vector down into its components. Vectors are comprised of two components: the horizontal component is the x direction, and the vertical component is the y direction. For example, we can see in the graph in Figure 9.2.13 that the position vector $\langle 2, 3 \rangle$ comes from adding the vectors v_1 and v_2. We have v_2 with initial point $(0, 0)$ and terminal point $(2, 0)$.

$$v_1 = \langle 2-0, 0-0 \rangle$$
$$= \langle 2, 0 \rangle$$

We also have v_2 with initial point $(0, 0)$ and terminal point $(0, 3)$.

$$v_2 = \langle 0-0, 3-0 \rangle$$
$$= \langle 0, 3 \rangle$$

Therefore, the position vector is

$$v = \langle 2+0, 3+0 \rangle$$
$$= \langle 2, 3 \rangle$$

Using the Pythagorean Theorem, the magnitude of v_1 is 2, and the magnitude of v_2 is 3. To find the magnitude of v, use the formula with the position vector.

$$|v| = \sqrt{|v_1|^2 + |v_2|^2}$$
$$= \sqrt{2^2 + 3^2}$$
$$= \sqrt{13}$$

The magnitude of v is $\sqrt{13}$. To find the direction, we use the tangent function $\tan\theta = \frac{y}{x}$.

$$\tan\theta = \frac{v_2}{v_1}$$
$$\tan\theta = \frac{3}{2}$$
$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$$

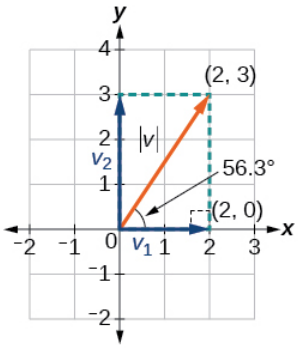


***Figure* 9.2.13**

Thus, the magnitude of $\vec{v}$ is $\sqrt{13}$ and the direction is $56.3°$ off the horizontal.

Example 9.2.6: Finding the Components of the Vector

Find the components of the vector $\vec{v}$ with initial point $(3, 2)$ and terminal point $(7, 4)$.

Solution

First find the standard position.

$$v = \langle 7-3, 4-2 \rangle$$
$$= \langle 4, 2 \rangle$$

See the illustration in Figure 9.2.14.

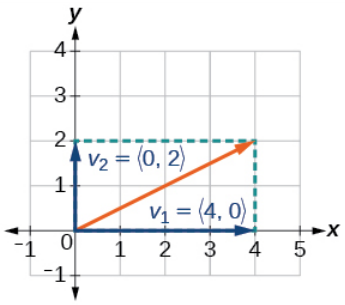


***Figure* 9.2.14**

The horizontal component is $\vec{v_1} = \langle 4, 0 \rangle$ and the vertical component is $\vec{v_2} = \langle 0, 2 \rangle$.

Finding the Unit Vector in the Direction of v

In addition to finding a vector's components, it is also useful in solving problems to find a vector in the same direction as the given vector, but of magnitude 1. We call a vector with a magnitude of 1 a unit vector. We can then preserve the direction of the original vector while simplifying calculations.

Unit vectors are defined in terms of components. The horizontal unit vector is written as $\vec{i} = \langle 1, 0 \rangle$ and is directed along the positive horizontal axis. The vertical unit vector is written as $\vec{j} = \langle 0, 1 \rangle$ and is directed along the positive vertical axis. See Figure 9.2.15.

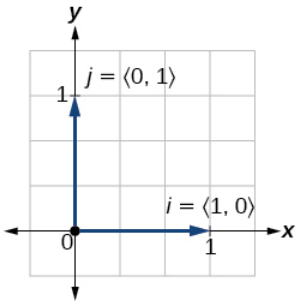


***Figure* 9.2.15**

THE UNIT VECTORS

If $\vec{v}$ is a nonzero vector, then $\frac{v}{|v|}$ is a unit vector in the direction of v. Any vector divided by its magnitude is a unit vector. Notice that magnitude is always a scalar, and dividing by a scalar is the same as multiplying by the reciprocal of the scalar.

Example 9.2.7: Finding the Unit Vector in the Direction of v

Find a unit vector in the same direction as $v = \langle -5, 12 \rangle$.

Solution

First, we will find the magnitude.

$$\begin{aligned} |v| &= \sqrt{(-5)^2 + (12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

Then we divide each component by $|v|$, which gives a unit vector in the same direction as $\vec{v}$:

$$\frac{v}{|v|} = -\frac{5}{13}i + \frac{12}{13}j$$

or, in component form

$$\frac{v}{|v|} = \left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle$$

See Figure 9.2.16.

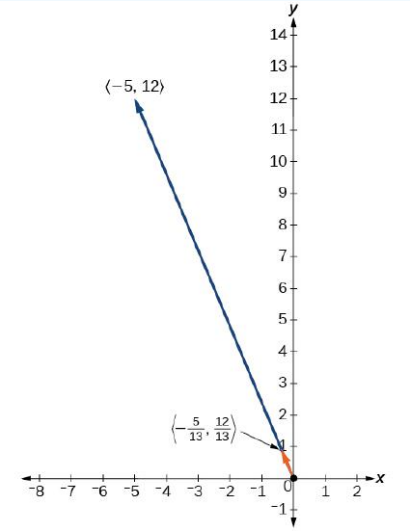


***Figure* 9.2.16**

Verify that the magnitude of the unit vector equals 1. The magnitude of $-\frac{5}{13}i + \frac{12}{13}j$ is given as

$$\begin{aligned}\sqrt{\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} &= \sqrt{\frac{25}{169} + \frac{144}{169}} \\ &= \sqrt{\frac{169}{169}} \\ &= 1\end{aligned}$$

The vector $u = \frac{5}{13}i + \frac{12}{13}j$ is the unit vector in the same direction as $v = \langle -5, 12 \rangle$.

Performing Operations with Vectors in Terms of i and j

So far, we have investigated the basics of vectors: magnitude and direction, vector addition and subtraction, scalar multiplication, the components of vectors, and the representation of vectors geometrically. Now that we are familiar with the general strategies used in working with vectors, we will represent vectors in rectangular coordinates in terms of i and j.

VECTORS IN THE RECTANGULAR PLANE

Given a vector $\vec{v}$ with initial point $P = (x_1, y_1)$ and terminal point $Q = (x_2, y_2)$, $\vec{v}$ is written as

$$v = (x_2 - x_1)i + (y_1 - y_2)j \tag{9.2.1}$$

The position vector from $(0, 0)$ to (a, b), where $(x_2 - x_1) = a$ and $(y_2 - y_1) = b$, is written as $\vec{v} = \vec{a}i + \vec{b}j$. This vector sum is called a linear combination of the vectors $\vec{i}$ and $\vec{j}$.

The magnitude of $\vec{v} = \vec{a}i + \vec{b}j$ is given as $|v| = \sqrt{a^2 + b^2}$. See Figure 9.2.17.

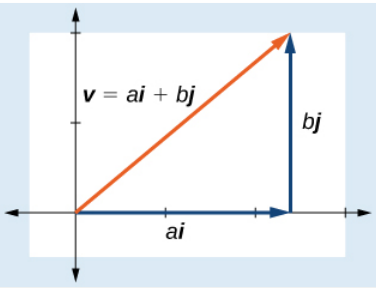


Figure 9.2.17

Example 9.2.8A: Writing a Vector in Terms of i and j

Given a vector $\vec{v}$ with initial point $P = (2, -6)$ and terminal point $Q = (-6, 6)$, write the vector in terms of $\vec{i}$ and $\vec{j}$.

Solution

Begin by writing the general form of the vector. Then replace the coordinates with the given values.

$$\begin{aligned} v &= (x_2 - x_1)i + (y_2 - y_1)j \\ &= (-6 - 2)i + (6 - (-6))j \\ &= -8i + 12j \end{aligned}$$

Example 9.2.8B: Writing a Vector in Terms of i and j Using Initial and Terminal Points

Given initial point $P_1 = (-1, 3)$ and terminal point $P_2 = (2, 7)$, write the vector $\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$.

Solution

Begin by writing the general form of the vector. Then replace the coordinates with the given values.

$$\begin{aligned} v &= (x_2 - x_1)i + (y_2 - y_1)j \\ v &= (2 - (-1))i + (7 - 3)j \\ &= 3i + 4j \end{aligned}$$

Exercise 9.2.3

Write the vector $\vec{u}$ with initial point $P = (-1, 6)$ and terminal point $Q = (7, -5)$ in terms of $\vec{i}$ and $\vec{j}$.

Answer

$u = 8i - 11j$

Performing Operations on Vectors in Terms of i and j

When vectors are written in terms of i and j, we can carry out addition, subtraction, and scalar multiplication by performing operations on corresponding components.

ADDING AND SUBTRACTING VECTORS IN RECTANGULAR COORDINATES

Given $v = ai + bj$ and $u = ci + dj$, then

$$\begin{aligned} v + u &= (a + c)i + (b + d)j \\ v - u &= (a - c)i + (b - d)j \end{aligned}$$

Example 9.2.9: Finding the Sum of the Vectors

Find the sum of $v_1 = 2i - 3j$ and $v_2 = 4i + 5j$.

Solution

$$\begin{aligned} v_1 + v_2 &= (2+4)i + (-3+5)j \\ &= 6i + 2j \end{aligned}$$

Calculating the Component Form of a Vector: Direction

We have seen how to draw vectors according to their initial and terminal points and how to find the position vector. We have also examined notation for vectors drawn specifically in the Cartesian coordinate plane using i and j. For any of these vectors, we can calculate the magnitude. Now, we want to combine the key points, and look further at the ideas of magnitude and direction.

Calculating direction follows the same straightforward process we used for polar coordinates. We find the direction of the vector by finding the angle to the horizontal. We do this by using the basic trigonometric identities, but with $|v|$ replacing r.

VECTOR COMPONENTS IN TERMS OF MAGNITUDE AND DIRECTION

Given a position vector $v = \langle x, y \rangle$ and a direction angle θ,

$$\cos\theta = \frac{x}{|v|} \text{ and } \sin\theta = y|v|$$
$$x = |v|\cos\theta$$
$$y = |v|\sin\theta$$

Thus, $v = xi + yj = |v|\cos\theta i + |v|\sin\theta j$, and magnitude is expressed as $|v| = \sqrt{x^2 + y^2}$.

Example 9.2.10: Writing a Vector in Terms of Magnitude and Direction

Write a vector with length 7 at an angle of $135°$ to the positive x-axis in terms of magnitude and direction.

Solution

Using the conversion formulas $x = |v|\cos\theta i$ and $y = |v|\sin\theta j$, we find that

$$\begin{aligned} x &= 7\cos(135°)i \\ &= -\frac{7\sqrt{2}}{2} \\ y &= 7\sin(135°)j \\ &= \frac{7\sqrt{2}}{2} \end{aligned}$$

This vector can be written as $v = 7\cos(135°)i + 7\sin(135°)j$ or simplified as

$$v = -\frac{7\sqrt{2}}{2}i + \frac{7\sqrt{2}}{2}j$$

Exercise 9.2.4

A vector travels from the origin to the point $(3, 5)$. Write the vector in terms of magnitude and direction.

Answer

$v = \sqrt{34}\cos(59°)i + \sqrt{34}\sin(59°)j$

Magnitude = 34

$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.04°$

Finding the Dot Product of Two Vectors

As we discussed earlier in the section, scalar multiplication involves multiplying a vector by a scalar, and the result is a vector. As we have seen, multiplying a vector by a number is called scalar multiplication. If we multiply a vector by a vector, there are two possibilities: the *dot product* and the *cross product*. We will only examine the dot product here; you may encounter the cross product in more advanced mathematics courses.

The dot product of two vectors involves multiplying two vectors together, and the result is a scalar.

DOT PRODUCT

The dot product of two vectors $v = \langle a, b \rangle$ and $u = \langle c, d \rangle$ is the sum of the product of the horizontal components and the product of the vertical components.

$$v \cdot u = ac + bd \tag{9.2.2}$$

To find the angle between the two vectors, use the formula below.

$$\cos\theta = \frac{v}{|v|} \cdot \frac{u}{|u|} \tag{9.2.3}$$

Example 9.2.11A: Finding the Dot Product of Two Vectors

Find the dot product of $v = \langle 5, 12 \rangle$ and $u = \langle -3, 4 \rangle$.

Solution

Using the formula, we have

$$\begin{aligned} v \cdot u &= \langle 5, 12 \rangle \cdot \langle -3, 4 \rangle \\ &= 5 \cdot (-3) + 12 \cdot 4 \\ &= -15 + 48 \\ &= 33 \end{aligned}$$

Example 9.2.11B: Finding the Dot Product of Two Vectors and the Angle between Them

Find the dot product of $v_1 = 5i + 2j$ and $v_2 = 3i + 7j$. Then, find the angle between the two vectors.

Solution

Finding the dot product, we multiply corresponding components.

$$\begin{aligned} v_1 \cdot v_2 &= \langle 5, 2 \rangle \cdot \langle 3, 7 \rangle \\ &= 5 \cdot 3 + 2 \cdot 7 \\ &= 15 + 14 \\ &= 29 \end{aligned}$$

To find the angle between them, we use the formula $\cos\theta = \frac{v}{|v|} \cdot \frac{u}{|u|}$.

$$\begin{aligned} \frac{v}{|v|} \cdot \frac{u}{|u|} &= \left\langle \frac{5}{\sqrt{29}} + \frac{2}{\sqrt{29}} \right\rangle \cdot \left\langle \frac{3}{\sqrt{58}} + \frac{7}{\sqrt{58}} \right\rangle \\ &= \frac{5}{\sqrt{29}} \cdot \frac{3}{\sqrt{58}} + \frac{2}{\sqrt{29}} \cdot \frac{7}{\sqrt{58}} \\ &= \frac{15}{\sqrt{1682}} + \frac{14}{\sqrt{1682}} \\ &= \frac{29}{\sqrt{1682}} \\ &= 0.707107 \end{aligned}$$

$$\cos^{-1}(0.707107) = 45°$$

See Figure 9.2.18.

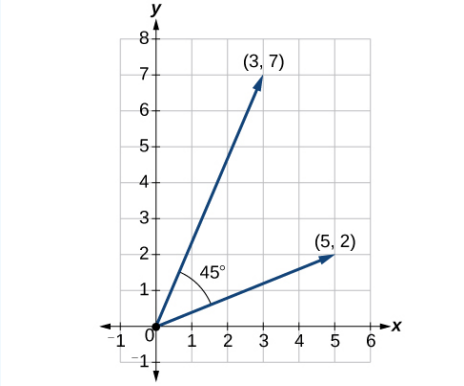


***Figure* 9.2.18**

Example 9.2.11C: Finding the Angle between Two Vectors

Find the angle between $u = \langle -3, 4 \rangle$ and $v = \langle 5, 12 \rangle$.

Solution

Using the formula, we have

$$\begin{aligned} \theta &= \cos^{-1}\left(\frac{u}{|u|} \cdot \frac{v}{|v|}\right) \\ \left(\frac{u}{|u|} \cdot \frac{v}{|v|}\right) &= \frac{-3i + 4j}{5} \cdot \frac{5i + 12j}{13} \\ &= \left(-\frac{3}{5} \cdot \frac{5}{13}\right) + \left(\frac{4}{5} \cdot \frac{12}{13}\right) \\ &= -\frac{15}{65} + \frac{48}{65} \\ &= \frac{33}{65} \\ \theta &= \cos^{-1}\left(\frac{33}{65}\right) \\ &= 59.5^\circ \end{aligned}$$

See Figure 9.2.19.

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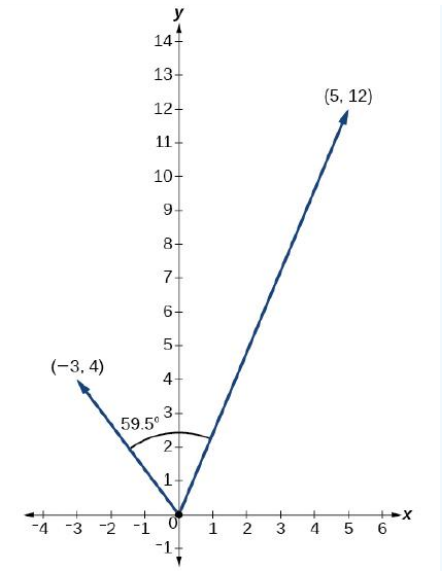


***Figure* 9.2.19**

Media: Access these online resources for additional instruction and practice with vectors.

- Introduction to Vectors
- Vector Operations
- The Unit Vector

Key Concepts

- The position vector has its initial point at the origin. See Example **9.2.1**.
- If the position vector is the same for two vectors, they are equal. See Example **9.2.2**.
- Vectors are defined by their magnitude and direction. See Example **9.2.3**.
- If two vectors have the same magnitude and direction, they are equal. See Example **9.2.4**.
- Vector addition and subtraction result in a new vector found by adding or subtracting corresponding elements. See Example **9.2.5**.
- Scalar multiplication is multiplying a vector by a constant. Only the magnitude changes; the direction stays the same. See Example **9.2.6** and Example **9.2.7**.
- Vectors are comprised of two components: the horizontal component along the positive x-axis, and the vertical component along the positive y-axis. See Example **9.2.8**.
- The unit vector in the same direction of any nonzero vector is found by dividing the vector by its magnitude.
- The magnitude of a vector in the rectangular coordinate system is $|v| = \sqrt{a^2 + b^2}$. See Example **9.2.9**.
- In the rectangular coordinate system, unit vectors may be represented in terms of ii and jj where i represents the horizontal component and j represents the vertical component. Then, $v = ai + bj$ is a scalar multiple of v by real numbers a and b. See Example **9.2.10** and Example **9.2.11**.
- Adding and subtracting vectors in terms of i and j consists of adding or subtracting corresponding coefficients of i and corresponding coefficients of j. See Example **9.2.12**.
- A vector $v = ai + bj$ is written in terms of magnitude and direction as $v = |v| \cos\theta i + |v| \sin\theta j$. See Example **9.2.13**.
- The dot product of two vectors is the product of the i terms plus the product of the j terms. See Example **9.2.14**.
- We can use the dot product to find the angle between two vectors. Example **9.2.15** and Example **9.2.16**.
- Dot products are useful for many types of physics applications. See Example **9.2.17**.

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7.4: Surface Area of Common Solids
surface area
7.4: Surface Area of Common Solids

T

total surface area
7.4: Surface Area of Common Solids
triangles
8.2: Triangles

V

vector magnitude
9.2: Vectors
vectors
9.2: Vectors
volume
7.5: Volume of Common Solids
volume of a pyramid
7.6: Pyramids and Cones

Glossary

Sample Word 1 | Sample Definition 1

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