

COLLEGE MATHEMATICS - ABRIDGED EDITION



Cayla M Gaworecki
SUNY Schenectady County Community
College

College Mathematics - Abridged Edition

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College Mathematics

First Edition

Scottsdale Community College

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Licensing

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Development Team and Sources

Development Team

Tracey Haynie, Scottsdale Community College

Jenifer Bohart, Scottsdale Community College

Carla Stroud, Scottsdale Community College

Portions of this textbook adapted from the following OER sources:

Geometry, Dimensional Analysis, Percentages, Savings and Loans, Sets, Probability, Statistics, Describing Data - “Math in Society” by David Lippman

Geometry, Percentages – “Basic Arithmetic Student Workbook, 2nd Ed.” by Scottsdale Community College

Functions, Linear Functions – “Introductory Algebra Student Workbook, 6th Ed.” by Scottsdale Community College

Functions, Linear Functions, Exponential Functions, Regression – “Intermediate Algebra Student Workbook, 4th Ed.” by Scottsdale Community College

Odds, CK-12 Foundation, <http://www.ck12.org/>

Measurement and Geometry, The NROC Project, http://nrocmath.org/products/higher-ed/developmental-mathematics/#contents_tab

Statistics Practice Problems - “Introductory Statistics” by OpenStax College <https://openstax.org/details/introductory-statistics>

CHAPTER OVERVIEW

1: Measurement and Dimensional Analysis ..

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Learning Objectives

- Use dimensional analysis to convert between different units of measurement for length, mass, and capacity
- Use dimensional analysis to convert between units of measurement in the metric system
- Use dimensional analysis to convert between the metric and standard systems of measurement
- Use rates and unit rates to problem solve

Measurement is a number that describes the size or amount of something. You can measure many things like length, area, capacity, weight, temperature and time.

At times, you may need to convert between units of measurement. For example, you might want to express your height using feet and inches (5 feet 4 inches) or using only inches (64 inches). This process of converting from one unit to another unit is called **unit analysis** or **dimensional analysis**.

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1.1: Length

Length is the distance from one end of an object to the other end, or from one object to another. For example, the length of a letter-sized piece of paper is 11 inches. The system for measuring length in the United States is based on the four customary units of length: **inch**, **foot**, **yard**, and **mile**. You can use any of these four U.S. customary measurement units to describe the length of something, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the length of a rug in feet rather than miles, and to describe a marathon in miles rather than inches.

The table below shows equivalents and conversion factors for the four customary units of measurement of length.

Unit Equivalents	Conversion Factors (longer to shorter units of measurement)	Conversion Factors (shorter to longer units of measurement)
1 foot = 12 inches	12 inches 1 foot	1 foot 12 inches
1 yard = 3 feet	3 feet 1 yard	1 yard 3 feet
1 mile = 5,280 feet	5,280 feet 1 mile	1 mile 5,280 feet

You can use the conversion factors to convert a measurement, such as feet, to another type of measurement, such as inches. Note that each of these conversion factors is a ratio of equal values, so each conversion factor equals 1. Multiplying a measurement by a conversion factor does not change the size of the measurement at all since it is the same as multiplying by 1; it just changes the units that you are using to measure.

There are many more inches for a measurement than there are feet for the same measurement, as feet is a longer unit of measurement. So, you could use the conversion factor $\frac{12 \text{ inches}}{1 \text{ foot}}$.

If a length is measured in feet, and you'd like to convert the length to yards, you can think, "I am converting from a shorter unit to a longer one, so the length in yards will be less than the length in feet." You could use the conversion factor $\frac{1 \text{ yard}}{3 \text{ feet}}$.

Definition: Term

Dimensional Analysis: The Factor-Label Method

You can use the **factor label method** to convert a length from one unit of measure to another using the conversion factors. In the factor label method, you multiply by unit fractions to convert a measurement from one unit to another. Study the example below to see how the factor label method can be used to convert a measurement given in feet into an equivalent number of inches.

✓ Example 1.1.1

How many inches are in $3\frac{1}{2}$ feet?

Solution

$$3\frac{1}{2} \text{ feet} = \underline{\hspace{2cm}} \text{ inches}$$

Begin by reasoning about your answer. Since a foot is longer than an inch, this means the answer would be greater than $3\frac{1}{2}$.

$$3\frac{1}{2} \text{ feet} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = \underline{\hspace{2cm}} \text{ inches}$$

Find the conversion factor that compares inches and feet, with "inches" in the numerator, and multiply.

$$3\frac{1}{2} \text{ feet} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = \underline{\hspace{2cm}} \text{ inches}$$

Rewrite the mixed number as an improper fraction before multiplying.

$$\frac{7 \text{ feet}}{2} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = \underline{\hspace{2cm}} \text{ inches}$$

You can cancel similar units when they appear in the numerator *and* the denominator. So here, cancel the similar units “feet” and “foot.” This eliminates this unit from the problem.

$$\frac{7 \cdot 12 \text{ inches}}{2 \cdot 1} = \underline{\hspace{2cm}} \text{ inches}$$

$$\frac{7 \cdot 12 \text{ inches}}{2 \cdot 1} = \underline{\hspace{2cm}} \text{ inches}$$

Rewrite as multiplication of numerators and denominators.

$$\frac{84 \text{ inches}}{2} = 42 \text{ inches}$$

Divide

There are 42 inches in $3\frac{1}{2}$ feet.

Notice that by using the factor label method you can cancel the units out of the problem, just as if they were numbers. You can only cancel if the unit being cancelled is in both the numerator and denominator of the fractions you are multiplying. In the problem above, you cancelled *feet* and *foot* leaving you with *inches*, which is what you were trying to find.

What if you had used the wrong conversion factor? 

$$\frac{7 \text{ feet}}{2} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} = ? \quad (1.1.1)$$

You could not cancel the feet because the unit is not the same in *both* the numerator and the denominator. So if you complete the computation, you would still have both feet and inches in the answer and no conversion would take place.

✓ Example 1.1.2

An interior decorator needs border trim for a home she is wallpapering. She needs 15 feet of border trim for the living room, 30 feet of border trim for the bedroom, and 26 feet of border trim for the dining room. How many yards of border trim does she need?

Solution

$$15 \text{ feet} + 30 \text{ feet} + 26 \text{ feet} = 71 \text{ feet}$$

You need to find the total length of border trim that is needed for all three rooms in the house. Since the measurements for each room are given in feet, you can add the numbers.

$$71 \text{ feet} = \underline{\hspace{2cm}} \text{ yards}$$

How many yards is 71 feet?

Reason about the size of your answer. Since a yard is longer than a foot, there will be fewer yards. Expect your answer to be less than 71.

$$\frac{71 \text{ feet}}{1} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} = \underline{\hspace{2cm}} \text{ yards}$$

Use the conversion factor $\frac{\text{yard}}{3 \text{ feet}}$.

$$\frac{71 \text{ feet}}{1} \cdot \frac{1 \text{ yard}}{3 \text{ feet}} = \underline{\hspace{2cm}} \text{ yards}$$

Since “feet” is in the numerator and denominator, you can cancel this unit.

$$\frac{71 \cdot 1 \text{ yard}}{1 \cdot 3} = \underline{\hspace{2cm}} \text{ yards}$$

$$\frac{71 \cdot 1 \text{ yard}}{1 \cdot 3} = \underline{\hspace{2cm}} \text{ yards}$$

Multiply.

$$\frac{71 \text{ yards}}{3} = 23\frac{2}{3} \text{ yards}$$

Divide, and write as a mixed number.

The interior decorator needs $23\frac{2}{3}$ yards of border trim.

? You Try It 1.1.1

- a. Use the Factor-Label Method to determine the number of feet in 2 1 miles.
- b. A fence company is measuring a rectangular area in order to install a fence around its perimeter. If the length of the rectangular area is 130 yards and the width is 75 feet, what is the total length of the distance to be fenced?

Answer

- a. 13200 feet b. 930 feet or 310 yards

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1.2: Weight

You often use the word **weight** to describe how heavy or light an object or person is. Weight is measured in the U.S. customary system using three units: ounces, pounds, and tons. An **ounce** is the smallest unit for measuring weight, a **pound** is a larger unit, and a **ton** is the largest unit. You can use any of the customary measurement units to describe the weight of something, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the weight of a human being in pounds rather than tons. It makes more sense to describe the weight of a car in tons rather than ounces.

The following table shows the unit conversions and conversion factors that are used to make conversions between customary units of weight.

Unit Equivalents	Conversion Factors (heavier to lighter units of measurement)	Conversion Factors (lighter to heavier units of measurement)
1 pound = 16 ounces	$\frac{16 \text{ ounces}}{1 \text{ pound}}$	$\frac{1 \text{ pound}}{16 \text{ ounces}}$
1 ton = 2000 pounds	$\frac{2,000 \text{ pounds}}{1 \text{ ton}}$	$\frac{1 \text{ ton}}{2,000 \text{ pounds}}$

✓ Example 1.2.1

Use the Factor Label Method to determine the number of ounces in $2\frac{1}{4}$ pounds.

Solution

$2\frac{1}{4}$ pounds = _____ ounces	Begin by reasoning about your answer. Since a pound is heavier than an ounce, expect your answer to be a number greater than $2\frac{1}{4}$.
$2\frac{1}{4}$ pounds $\cdot \frac{16 \text{ ounces}}{1 \text{ pound}}$ = _____ ounces	Multiply by the conversion factor that relates ounces and pounds: $\frac{16 \text{ ounces}}{1 \text{ pound}}$
$\frac{9 \text{ pounds}}{4} \cdot \frac{16 \text{ ounces}}{1 \text{ pound}}$ = _____ ounces	Write the mixed number as an improper fraction. The common unit “pound” can be cancelled because it appears in both the numerator and denominator.
$\frac{9}{4} \cdot \frac{16 \text{ ounces}}{1}$ = _____ ounces	Multiply and simplify.
$\frac{9 \cdot 16 \text{ ounces}}{4 \cdot 1}$ = _____ ounces	
$\frac{144 \text{ ounces}}{4}$ = _____ ounces	
There are 36 ounces in $2\frac{1}{4}$ pounds.	

There are times when you need to perform calculations on measurements that are given in different units. To solve these problems, you need to convert one of the measurements to the same unit of measurement as the other measurement. Think about whether the unit you are

converting to is smaller or larger than the unit you are converting from. This will help you be sure that you are making the right computation. You can use the factor label method to make the conversion from one unit to another.

The following examples require converting between units of weight.

✓ Example 1.2.2

A municipal trash facility allows a person to throw away a maximum of 30 pounds of trash per week. Last week, 140 people threw away the maximum allowable trash. How many tons of trash did this equal?

Solution

$140 \cdot 30 \text{ pounds} = 4,200 \text{ pounds}$	Determine the total trash for the week expressed in pounds. If 140 people each throw away 30 pounds, you can find the total by multiplying.
$4,200 \text{ pounds} = \underline{\hspace{2cm}} \text{ tons}$	Then convert 4,200 pounds to tons. Reason about your answer. Since a ton is heavier than a pound, expect your answer to be a number less than 4,200.
$\frac{4,200 \text{ pounds}}{1} \cdot \frac{1 \text{ ton}}{2,000 \text{ pounds}} = \underline{\hspace{2cm}} \text{ tons}$	Find the conversion factor appropriate for the situation: $\frac{1 \text{ ton}}{2,000 \text{ pounds}}$
$\frac{4,200}{1} \cdot \frac{1 \text{ ton}}{2,000} = \underline{\hspace{2cm}} \text{ tons}$	
$\frac{4,200 \cdot 1 \text{ ton}}{1 \cdot 2,000} = \underline{\hspace{2cm}} \text{ tons}$	Multiply and simplify.
$\frac{4,200 \text{ ton}}{2,000} = 2\frac{1}{10} \text{ tons}$	
The total amount of trash generated is $2\frac{1}{10}$ tons.	

✓ Example 1.2.3

The post office charges \$0.44 to mail something that weighs an ounce or less. The charge for each additional ounce, or fraction of an ounce, of weight is \$0.17. At this rate, how much will it cost to mail a package that weighs 2 pounds 3 ounces?

Solution

$2 \text{ pounds } 3 \text{ ounces} = \underline{\hspace{2cm}} \text{ ounces}$	Since the pricing is for ounces, convert the weight of the package from pounds and ounces into just ounces.
$\frac{2 \text{ pounds}}{1} \cdot \frac{16 \text{ ounces}}{\text{pound}} = \underline{\hspace{2cm}} \text{ ounces}$	First use the factor label method to convert 2 pounds to ounces
$\frac{2 \text{ pounds}}{1} \cdot \frac{16 \text{ ounces}}{\text{pound}} = \underline{\hspace{2cm}} \text{ ounces}$	2 pounds = 32 ounces.
$\frac{2}{1} \cdot \frac{16 \text{ ounces}}{1} = 32 \text{ ounces}$	
$32 \text{ ounces} + 3 \text{ ounces} = 35 \text{ ounces}$	Add the additional 3 ounces to find the weight of the package. The package weighs 35 ounces. There are 34 additional ounces, since $35 - 1 = 34$.
$\$0.44 + \$0.17(34)$	Apply the pricing formula. \$0.44 for the first ounce and \$0.17 for each additional ounce.
$\$0.44 + \$5.78$	
$\$0.44 + \$5.78 = \$6.22$	
It will cost \$6.22 to mail a package that weighs 2 pounds 3 ounces.	

? You Try It 1.2.1

- a. How many pounds is 72 ounces?
- b. The average weight of a northern Bluefin tuna is 1,800 pounds. The average weight of a great white shark is $2\frac{1}{2}$ tons. On average, how much more does a great white shark weigh, in pounds, than a northern bluefin tuna?

Answer

- a. 4.5 pounds b. 3200 pounds

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1.3: Capacity

Capacity is the amount of liquid (or other pourable substance) that an object can hold when it's full. When a liquid, such as milk, is being described in gallons or quarts, this is a measure of capacity.

There are five main units for measuring capacity in the U.S. customary measurement system. The smallest unit of measurement is a **fluid ounce**. "Ounce" is also used as a measure of weight, so it is important to use the word "fluid" with ounce when you are talking about capacity. Sometimes the prefix "fluid" is not used when it is clear from the context that the measurement is capacity, not weight.

The other units of capacity in the customary system are the **cup, pint, quart, and gallon**. The table below describes each unit of capacity and provides an example to illustrate the size of the unit of measurement. You can use any of these five measurement units to describe the capacity of an object, but it makes more sense to use certain units for certain purposes. For example, it makes more sense to describe the capacity of a swimming pool in gallons and the capacity of an expensive perfume in fluid ounces.

The table below shows some of the most common equivalents and conversion factors for the five customary units of measurement of capacity.

Unit Equivalents	Conversion Factors (heavier to lighter units of measurement)	Conversion Factors (lighter to heavier units of measurement)
1 cup = 8 fluid ounces	$\frac{1 \text{ cup}}{8 \text{ fluid ounces}}$	$\frac{8 \text{ fluid ounces}}{1 \text{ cup}}$
1 pint = 2 cups	$\frac{1 \text{ pint}}{2 \text{ cups}}$	$\frac{2 \text{ cups}}{1 \text{ pint}}$
1 quart = 2 pints	$\frac{1 \text{ quart}}{2 \text{ pints}}$	$\frac{2 \text{ pints}}{1 \text{ quart}}$
1 quart = 4 cups	$\frac{1 \text{ gallon}}{4 \text{ quarts}}$	$\frac{4 \text{ cups}}{1 \text{ quart}}$
1 gallon = 4 quarts	$\frac{1 \text{ quart}}{4 \text{ cups}}$	$\frac{4 \text{ cups}}{1 \text{ quart}}$
1 gallon = 16 cups	$\frac{1 \text{ gallon}}{16 \text{ cups}}$	$\frac{16 \text{ cups}}{1 \text{ gallon}}$

As with converting units of length and weight, you can use the factor label method to convert from one unit of capacity to another.

✓ Example 1.3.1

Use the Factor Label Method to determine the number of pints in $2\frac{3}{4}$ gallons.

Solution

$2\frac{3}{4}$ gallons = _____ pints	Begin by reasoning about your answer. Since a gallon is larger than a pint, expect the answer in pints to be a number greater than $2\frac{3}{4}$ gallons.
$\frac{11 \text{ gallons}}{4} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} = \text{_____ pints}$	The table above does not contain a conversion factor for gallons and pints, so you cannot convert it in one step.
$\frac{11 \text{ gallons}}{4} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} = \text{_____ pints}$	However, you can use quarts as an intermediate unit, as shown here
$\frac{11}{4} \cdot \frac{4}{1} \cdot \frac{2 \text{ pints}}{1} = \text{_____ pints}$	Set up the equation so that two sets of labels cancel—gallons and quarts.

$$\frac{11 \cdot 4 \cdot 2 \text{ pints}}{4 \cdot 1 \cdot 1} = \text{_____ pints}$$

$$\frac{88 \text{ pints}}{4} = 22 \text{ pints}$$

$2\frac{3}{4}$ gallons is equivalent to 22 pints

✓ Example 1.3.2

Natasha is making lemonade to bring to the beach. She has two containers. One holds one gallon and the other holds 2 quarts. If she fills both containers, how many cups of lemonade will she have?

Solution

$$1 \text{ gallon} + 2 \text{ quarts} = \text{_____ cups}$$

This problem requires you to find the sum of the capacity of each container and then convert that sum to cups.

$$4 \text{ quarts} + 2 \text{ quarts} = 6 \text{ quarts}$$

First, find the sum in quarts. 1 gallon is equal to 4 quarts

$$\frac{6 \text{ quarts}}{1} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} = \text{_____ cups}$$

Since the problem asks for the capacity in cups, convert 6 quarts to cups.

$$\frac{6 \text{ quarts}}{1} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} = \text{_____ cups}$$

Cancel units that appear in both the numerator and denominator.

$$6 \cdot 2 \cdot 2 = 24 \text{ cups}$$

Multiply.

Natasha will have 24 cups of lemonade.

? You Try 1.3.1

Alan is making chili. He is using a recipe that makes 24 cups of chili. He has a 5-quart pot and a 2-gallon pot and is trying to determine whether the chili will all fit in one of these pots. Which of the pots will fit the chili?

- A) The chili will not fit into either of the pots.
- B) The chili can fit into either pot.
- C) The chili will fit into the 5-quart pot only.
- D) The chili will fit into the 2-gallon pot only.

Answer

- D) The chili will fit into the 2-gallon pot only.

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1.4: Converting Between Systems

We have spent the last several sections learning about the U.S. customary system of measurement, and the metric system. As you might guess, there are many applications where it is useful to be able to convert between measurements of length, mass, and volume (capacity) in the two systems.

The table below gives some useful conversions between U.S. and metric measurements:

Approximate Conversions Between Customary and Metric Systems	
Length	1 inch = 2.540 centimeters
	1 foot = 0.3048 m
	1 yard = 0.9144 meters
	1 mile = 1.6093 kilometers
Mass	1 pound = 0.4536 kilograms
	1 ounce = 28 grams
Volume	1 fluid ounce = 29.574 milliliters
	1 quart = 0.9464 liters
	1 gallon = 3.785 liters

✓ Example 1.4.1

An Olympic sprinter competes in the 400m dash. How many yards is the race?

Solution

$400 \text{ m} = \underline{\hspace{1cm}} \text{ y}$	A yard is less than a meter, so you expect your answer to be more than 400.
$\frac{400 \text{ m}}{1} \cdot \frac{1 \text{ yard}}{0.9144 \text{ m}}$	Using the factor label method, write 400 m as a fraction and use unit fractions to convert it to yards.
$\frac{400\cancel{\text{m}}}{1} \cdot \frac{1 \text{ yard}}{0.9144\cancel{\text{m}}} = \dots \text{ yards}$	Cancel similar units, multiply, and simplify.
$\frac{400}{1} \cdot \frac{1 \text{ yard}}{0.9144} = \frac{400}{0.9144} \text{ yards}$	
$\frac{400}{0.9144} = 437.45 \text{ yards}$	400 meters = 437.45 yards
The race is 437.45 yards	

✓ Example 1.4.2

A patient must be weighed prior to surgery so that the proper dosage of anesthesia can be given. A nurse weighs a patient and finds that he weighs 205 pounds. The anesthesiologist prefers weights in kilograms. What is the patient's weight in kilograms? Round to the nearest hundredth.

Solution

$205 \text{ lbs} = \underline{\hspace{1cm}} \text{ kg}$	Pounds are smaller than kilograms, so you expect your answer to be less than 205
$\frac{205\cancel{\text{lbs}}}{1} \cdot \frac{1 \text{ kg}}{2.2\cancel{\text{lbs}}}$	Using the factor label method, write 205 lbs as a fraction and use unit fractions to convert it to kilograms.

$$\frac{205\text{lbs}}{1} \cdot \frac{1\text{ kg}}{2.2\text{lbs}} =$$

Cancel similar units, multiply, and simplify.
205 lbs is equal to 93.18 kg
The patient's

$$\frac{205}{1} \cdot \frac{1\text{ kg}}{2.2} = \frac{205 \cdot 1}{1 \cdot 2.2} \text{ kg}$$

$$\frac{205}{2.2} = 93.18 \text{ kg}$$

205 lbs is equal to 93.18 kg
The

The patient's weight is 93.18 kg.

✓ Example 1.4.3

Your work is having its annual potluck, and you are asked to bring 5 gallons of lemonade. When you go to the store, each container of lemonade mix says it will make 6 liters. How many containers do you need to buy?

Solution

$$5 \text{ gal} = \text{_____ liters}$$

Liters are smaller than gallons, so you expect your answer to be more than 5.

$$\frac{5\text{gal}}{1} \cdot \frac{3.785 \text{ L}}{1\text{gal}}$$

Using the factor label method, write 5 gal as a fraction and use unit fractions to convert it to liters.

$$\frac{5\text{gal}}{1} \cdot \frac{3.785 \text{ L}}{1\text{gal}} = \text{_____ liters}$$

Cancel similar units, multiply, and simplify.

$$\frac{5}{1} \cdot \frac{3.785 \text{ L}}{1} = \frac{5 \cdot 3.785}{1} \text{ liters}$$

$$5 \cdot 3.785 = 18.925 \text{ liters}$$

$$5 \text{ gallons} = 18.925 \text{ liters}$$

$$\frac{18.925 \text{ L}}{1} \cdot \frac{1 \text{ container}}{6 \text{ L}}$$

You need 18.925 liters, and each container of mix will make 6 liters.

$$\frac{18.925}{6} = 3.15 \text{ containers}$$

You must buy 4 containers of mix (3 is not enough)

Temperature Conversions

There are three commonly used systems for measuring temperature. One such system is usually used in science, and is called the **Kelvin** scale. We will focus our attention on the other two scales for measuring temperature: Fahrenheit and Celsius. The United States usually uses the **Fahrenheit** system. In this system, water freezes to ice at 32°F and boils at 212°F. Another commonly used temperature scale is the **Celsius** scale, where water freezes to ice at 0°C and boils at 100°C. You might notice that in the Fahrenheit scale there is a 180 degree difference (212°F – 32°F) between the temperature where water boils and freezes, while in the Celsius scale there is a 100 degree difference (100°C – 0°C) between the temperature where water boils and freezes. Since neither of these two temperature scales has an absolute starting point (a lowest possible temperature) we cannot meaningfully compare temperatures in these scales using conversion factors.

Instead, we have temperature conversion formulas which allow us to convert temperatures back and forth between Fahrenheit and Celsius.

To Convert Between

Conversion Formula

Celsius and Fahrenheit

$$F = 1.8 C + 32$$

✓ Example 1.4.4

Your front yard is full of weeds, so you decide to spray it with weed killer. The bottle of weed killer that you purchase says "Do not apply in temperatures below 18 °C or above 32 °C. What are the corresponding temperatures in degrees Fahrenheit?

Solution

$$F = 1.8C + 32$$

Both

Both temperatures are in degrees Celsius and need to be converted into degrees Fahrenheit.

$$F = 1.8(18) + 32$$

$$F = 64.4^{\circ}\text{F}$$

The lower temperature corresponds to 64.4°F.

$$F = 1.8(32) + 32$$

$$F = 89.6^{\circ}\text{F}$$

The upper temperature corresponds to 89.6°F.

The weed killer should only be applied when the temperature is between 64.4 and 89.6°F.

✓ Example 1.4.5

When leaving the hospital with your sick child, you are told to return immediately if her temperature exceeds 101.5 °F. When you get home, you discover your thermometer will only measure temperature in degrees Celsius. At what temperature, in degrees Celsius, would you need to return your child to the hospital?

Solution

$$F = 1.8C + 32$$

The temperature is in degrees Fahrenheit and it needs to be converted into degrees Celsius.

$$101.5 = 1.8(C) + 32$$

$$101.5 - 32 = 1.8C$$

Subtract 32 from both sides of the equation.

$$69.5 = 1.8C$$

$$C = \frac{69.5}{1.8}$$

Divide both sides by 1.8

$$C = 38.61^{\circ}\text{C}$$

You should return your child to the hospital if her temperature exceeds 38.61°C.

? You Try It 1.4.6

What was the temperature in degrees Celsius, if the evening news reports that the high temperature in Phoenix, Arizona today was 115 °F?

Answer

$$46.1^{\circ}\text{C}$$

1.5: Problem Solving Using Rates and Dimensional Analysis .

A **rate** is the ratio of two quantities. A **unit rate** is a rate with a denominator of one.

✓ Example 1.5.1

Your car can drive 300 miles on a tank of 15 gallons.

Express this as a rate and as a unit rate, in miles per gallon.

Solution

Expressed as a rate, $\frac{300 \text{ miles}}{15 \text{ gallons}}$.

We can divide to find a unit rate: $\frac{20 \text{ miles}}{1 \text{ gallon}}$, which we could also write as $20 \frac{\text{miles}}{\text{gallon}}$, or just 20 miles per gallon.

? Exercise 1.5.1

Find the unit rates. If necessary, round your answers to the nearest hundredth. 6 pounds for \$5.29

_____ dollars per pound _____ pounds per dollar

Answer

\$0.88 per pound, 1.13 pounds per dollar

✓ Example 1.5.2

Compare the electricity consumption per capita in China to the rate in Japan.

Solution

To address this question, we will first need data. From the CIA ¹ website we can find the electricity consumption in 2011 for China was 4,693,000,000,000 KWH (kilowatt-hours), or 4.693 trillion KWH, while the consumption for Japan was 859,700,000,000 or 859.7 billion KWH. To find the rate per capita (per person), we will also need the population of the two countries. From the World Bank ², we can find the population of China is 1,344,130,000 or 1.344 billion, and the population of Japan is 127,817,277 or 127.8 million.

Computing the consumption per capita for each country:

China: $\frac{4,693,000,000,000 \text{ KWH}}{1,344,130,000 \text{ people}} \approx 3491.5 \text{ KWH per person}$

Japan: $\frac{859,700,000,000 \text{ KWH}}{127,817,277 \text{ people}} \approx 6726 \text{ KWH per person}$

While China uses more than 5 times the electricity of Japan overall, because the population of Japan is so much smaller, it turns out Japan uses almost twice the electricity per person compared to China.

Many problems can also be solved by multiplying a quantity by rates to change the units. This is the foundation of the Factor-Label process that we have been using already in this chapter.

✓ Example 1.5.3

Your car can drive 300 miles on a tank of 15 gallons.

- How far can it drive on 40 gallons?
- How many gallons are needed to drive 50 miles?

Solution

We earlier found that 300 miles on 15 gallons gives a rate of 20 miles per gallon.

a. If we multiply the given 40 gallon quantity by this rate, the gallons unit "cancels" and we're left with a number of miles:

$$40 \text{ gallons} \cdot \frac{20 \text{ miles}}{1 \text{ gallon}} = \frac{40 \text{ gallons}}{1} \cdot \frac{20 \text{ miles}}{\text{gallon}} = 800 \text{ miles}$$

Notice that this could also have been achieved using the rate given in the problem:

$$40 \text{ gallons} \cdot \frac{300 \text{ miles}}{15 \text{ gallons}} = \frac{40 \text{ gallons}}{1} \cdot \frac{300 \text{ miles}}{15 \text{ gallons}} = \frac{40}{1} \cdot \frac{300 \text{ miles}}{15} = \frac{1200}{15} \text{ miles} = 800 \text{ miles}$$

b. Notice if instead we were asked "how many gallons are needed to drive 50 miles?" we could answer this question by inverting the 20 mile per gallon rate so that the miles unit cancels and we're left with gallons:

$$50 \text{ miles} \cdot \frac{1 \text{ gallon}}{20 \text{ miles}} = \frac{50 \text{ miles}}{1} \cdot \frac{1 \text{ gallon}}{20 \text{ miles}} = \frac{50 \text{ gallons}}{20} = 2.5 \text{ gallons}$$

✓ Example 1.5.4

A bicycle is traveling at 15 miles per hour. How many feet will it cover in 20 seconds?

Solution

To answer this question, we need to convert 20 seconds into feet. If we know the speed of the bicycle in feet per second, this question would be simpler. Since we don't, we will need to do additional unit conversions. We will need to know that 5280 ft = 1 mile. We might start by converting the 20 seconds into hours:

$$20 \text{ seconds} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{1}{180} \text{ hour}$$

Now we can multiply by the 15 miles/hr

$$\frac{1}{180} \text{ hour} \cdot \frac{15 \text{ miles}}{1 \text{ hour}} = \frac{1}{12} \text{ mile}$$

Now we can convert to feet

$$\frac{1}{12} \text{ mile} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = 440 \text{ feet}$$

We could have also done this entire calculation in one long set of products:

$$20 \text{ seconds} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{15 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = 440 \text{ feet} \quad (1.5.1)$$

✓ Example 1.5.5

You are walking through a hardware store and notice two sales on tubing.

- 3 yards of Tubing A costs \$5.49.
- Tubing B sells for \$1.88 for 2 feet.

Either tubing is acceptable for your project. Which tubing is less expensive?

Find

Solution

Find the unit price for each tubing. This will make it easier to compare.

Tubing A: 3 yards = \$5.49

$$\frac{\$5.49 \div 3}{3 \text{ yards} \div 3} = \frac{\$1.83}{1 \text{ yard}}$$

Tubing B: 2 feet = \$1.88

Tubing A is sold by the yard.

Find the cost per yard of Tubing A by dividing the cost of 3 yards of the tubing by 3.

Tubing B is sold by the foot.

$$\frac{\$1.88 \div 2}{2 \text{ feet} \div 2} = \frac{\$0.94}{1 \text{ foot}}$$

Find the cost per foot by dividing \$1.88 by 2 feet.

To compare the prices, you need to have the same unit of measure. You can choose to use dollars per foot (like we have for Tubing B) or dollars per yard (like we have for Tubing A).

Either will work. For this example, we will go with dollars per yard.

Tubing A: \$1.83 per yard

Tubing B: $\frac{\$0.94}{1 \text{ foot}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}}$ Use the conversion factor $\frac{3 \text{ feet}}{1 \text{ yard}}$

$$\frac{\$0.94}{1 \text{ foot}} \cdot \frac{3 \text{ feet}}{1 \text{ yard}} = \frac{\$2.82}{1 \text{ yard}} \quad \text{Cancel and multiply.}$$

\$2.82 per yard

Compare prices for 1 yard of each tubing.

Tubing A: \$1.83 per yard Tubing B: \$2.82 per yard

Tubing A is less expensive than Tubing B.

✓ Example 1.5.6

The cost of gasoline in Arizona is about \$2.05 per gallon. When you travel over the border into Mexico, gasoline costs 14.81 pesos per liter. Where is gasoline more expensive?

Note: This problem requires a currency conversion factor. Currency conversions are constantly changing, but at the time of print \$1 = 18.68 pesos.

Solution

To answer this question, we need to convert from gallons to liters AND from U.S. dollars to Mexican pesos.

$$\begin{aligned} & \frac{\$2.05}{1 \text{ gallon}} \cdot \frac{1 \text{ gallon}}{3.8 \text{ liters}} \cdot \frac{18.68 \text{ pesos}}{\$1} \\ & \frac{\$2.05}{1 \text{ gallon}} \cdot \frac{1 \text{ gallon}}{3.8 \text{ liter s}} \cdot \frac{18.68 \text{ pesos}}{\$1} \\ & \frac{2.05}{1} \cdot \frac{1}{3.8 \text{ liters}} \cdot \frac{18.68 \text{ pesos}}{1} = \frac{2.05 \cdot 18.68 \text{ pesos}}{3.8 \text{ liters}} \quad (1.5.2) \\ & \frac{2.05 \cdot 18.68 \text{ pesos}}{3.8 \text{ liters}} = 10.08 \text{ pesos per liter} \end{aligned}$$

The price in Arizona of \$2.05 per gallon is equivalent to 10.08 pesos per liter. Since the actual price in Mexico is 14.81 pesos per liter, gasoline is more expensive in Mexico.

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1.6: Practice Problems

1. Use the Factor-Label Method to determine the number of inches in 8 feet.
2. Use the Factor-Label Method to determine the number of feet in 5 yards.
3. Use the Factor-Label Method to determine the number of inches in 6 yards.
4. Use the Factor-Label Method to determine the number of yards in 108 inches.
5. Use the Factor-Label Method to determine the number of yards in 203.1 miles. Round your answer to the nearest hundredth.
6. Use the Factor-Label Method to determine the number of inches in 275.2 miles.
7. The length of one student desk is 22 inches. If you line up 5 desks, what is the total length of the desks in feet?
8. You want to arrange your dorm room so that the length of your bed and the length of your desk are next to each other along the wall. Your bed is 80 inches long and your desk is 3 feet and 5 inches long. What is the total length of your bed and desk in feet?
9. Use the Factor-Label Method to determine the number of ounces in 4 pounds
10. Use the Factor-Label Method to determine the number of pounds in 38 ounces. Round your answer to the nearest hundredth.
11. Use the Factor-Label Method to determine the number of tons in 413 ounces. Round your answer to the nearest thousandth.
12. Amy bought 5 lbs. 5 oz. of turkey cold cuts and 3 lbs. 15 oz. of ham cold cuts. How much did she buy in total? (You should convert any ounces over 15 into pounds).
13. Leslie had twin girls – the first baby weighed 5 lbs. 14 oz. and the second baby weighed 6 lbs. 4 oz. How much did the twin baby girls weigh together? (You should convert any ounces over 15 into pounds).
14. Use the Factor-Label Method to determine the number of gallons in 48 cups.
15. Use the Factor-Label Method to determine the number of cups in 912 gallons.
16. Use the Factor-Label Method to determine the number of fluid ounces in 9 quarts.
17. Betty brought one gallon of iced tea to a potluck and Denis brought 3 quarts of iced tea to the potluck. How many cups of iced tea are at the potluck?
18. Natasha is heating up four 18-fluid ounce cans of soup on her stovetop. What is the smallest pot she can use if she has a 2-quart, 3-quart, and 5-quart saucepan?
19. Use the Factor-Label Method to determine the number of inches in 6 centimeters.
20. Use the Factor-Label Method to determine the number of centimeters in 4.5 inches.
21. Use the Factor-Label Method to determine the number of yards in 350 meters.
22. Use the Factor-Label Method to determine the number of kilometers in 7.2 miles.
23. Use the Factor-Label Method to determine the number of kilograms in 165 pounds.
24. Use the Factor-Label Method to determine the number of pounds in 0.8 kilograms.
25. Use the Factor-Label Method to determine the number of liters in 16 gallons.
26. An Olympic sprinter competes in the 400 meter dash. How many feet is the race?
27. Convert a temperature of 54°F to degrees Celsius.
28. Convert a temperature of 75°F to degrees Celsius.
29. Convert a temperature of -12°C to degrees Fahrenheit.
30. Convert a temperature of 37°C to degrees Fahrenheit.
31. You arrive in Paris with \$800. How many Euros do you have to spend? (\$1 = 0.9033 euros as of November, 2016)
32. A bicyclist is traveling at 540 feet in 30 seconds. Express this as a rate and as a unit rate, in feet per second.

33. Your car can drive 380 miles on a tank of 22 gallons.
- How far can it drive on 50 gallons? Round your answer to the nearest hundredth.
 - How many gallons are needed to drive 650 miles? Round your answer to the nearest hundredth.
34. A bicycle is traveling at 16 miles per hour. How many feet will it cover in 40 seconds? Round your answer to the nearest tenth of a foot.
35. A car is traveling 33 miles per hour. How many feet will the car travel in 6 seconds? Round your answer to the nearest tenth of a foot.
36. A car is traveling at 60 kilometers per hour. How many meters does it travel in 4 seconds?
37. A car is driving at 100 kilometers per hour. How many meters does it travel in 2 seconds?
38. A chain weighs 10 pounds per foot. How many ounces will 4 inches weigh?
39. One 16-ounce can of vegetable soup costs \$1.34. One quart of vegetable soup costs \$3.49. Which option is less expensive?
40. An 8-foot length of 4 inch wide crown molding costs \$14. How much will it cost to buy 40 feet of crown molding?
41. A wire costs \$2 per meter. How much will 3 kilometers of wire cost?
42. A recipe for blueberry muffins states that it yields 12 muffins, with 270 calories per muffin. You instead decide to make mini-muffins, and the recipe yields 20 minimuffins. If you eat 6 mini-muffins, how many calories will you consume?
43. Four 3-megawatt wind turbines can supply enough electricity to power 3000 homes. How many turbines would be required to power 55,000 homes?
44. A highway had a landslide, where 3,000 cubic yards of material fell on the road, requiring 200 dump truck loads to clear. On another highway, a slide left 40,000 cubic yards on the road. How many dump truck loads would be needed to clear this slide?
45. You read online that a 15 -foot by 20 -foot brick patio would cost about \$2,275 to have professionally installed. Estimate the cost of having an 18 -foot by 22 -foot brick patio installed. Round your answer to the nearest dollar.
46. The store is selling lemons at 2 for \$1. Each lemon yields about 2 tablespoons of juice. How much will it cost to buy enough lemons to make a 9-inch lemon pie requiring half a cup of lemon juice? (1 cup = 16 tablespoons)
47. When Ibuprofen is given for fever to children 6 months of age up to 2 years, the usual dose is 5 milligrams (mg) per kilogram (kg) of body weight. How much medicine would be a usual dose for an 18 month old weighing 28 pounds? Round your answer to the nearest milligram. (1 kg = 2.2 pounds)
48. It is estimated that a driver takes, on average, 1.5 seconds from seeing an obstacle to reacting by applying the brake or swerving. How far will a car, traveling at 60 miles per hour, travel in feet before the driver reacts to an obstacle?
49. Sound travels about 750 miles per hour. If you stand in a canyon and sound a horn, you will hear an echo. Suppose it takes about 3 seconds to hear the echo. How far away is the canyon wall, in feet?
50. It takes a hose 2 minutes to fill a rectangular aquarium 9 inches long, 10 inches wide, and 13 inches tall. How long will it take the same hose to fill an aquarium measuring 21 inches by 27 inches by 35 inches? Round your answer to the nearest minute.
51. You want to put a 2-inch thick layer of topsoil for a new 20-foot by 30 -foot garden. The dirt store sells by the cubic yards. How many cubic yards will you need to order? Round you answer to the nearest tenth.
52. Gasoline in Paris costs 1.32 euros per liter. What is the price of gas in dollars per gallon? (\$1 = 0.9033 euros as of November, 2016)
53. A 0.75 liter bottle of wine costs 83 euros. What is the price of the wine in dollars per ounce? (\$1 = 0.9033 euros as of November, 2016)

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CHAPTER OVERVIEW

2: Linear Functions.

[2.1: Functions](#)

[2.2: Linear Functions .](#)

[2.3: Linear Regression](#)

[2.4: Correlation](#)

[2.5: Practice Problems](#)

Learning Objectives

- Use function notation to represent the relationship between inputs and outputs
- Read function inputs and outputs from a graph, table, equation, or word problem
- Recognize a linear function, and identify the slope and initial value
- Calculate a slope and interpret it as a rate of change
- Use technology to create a scatterplot and find a linear regression equation
- Interpret the meaning of the slope and initial value of a regression equation
- Use a regression equation to predict the output for a given input
- Use technology to calculate the correlation
- Interpret the correlation from a value of r , or by looking at a scatterplot

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2.1: Functions

The natural world is full of relationships between quantities that change. When we see these relationships, it is natural for us to ask “If I know one quantity, can I then determine the other?” This establishes the idea of an input quantity, or independent variable, and a corresponding output quantity, or dependent variable. From this we get the notion of a functional relationship in which the output can be determined from the input.

For some quantities, like height and age, there are certainly relationships between these quantities. Given a specific person and any age, it is easy enough to determine their height, but if we tried to reverse that relationship and determine age from a given height, that would be problematic, since most people maintain the same height for many years.

Definition: Term

A **function** is a rule for a relationship between an input, or independent, quantity and an output, or dependent, quantity in which each input value uniquely determines one output value. We say “the output is a function of the input.”

Note

To simplify writing out expressions and equations involving functions, a simplified notation is often used

FUNCTION NOTATION is used to indicate a functional relationship between two quantities as follows:

$$\text{Function Name (INPUT) = OUTPUT}$$

So, the statement $f(x) = y$ would refer to the function f , and correspond to the ordered pair (x, y) , where x is the input variable, and y is the output variable.

Rather than write “height is a function of age”, we could use the descriptive variable h to represent height and we could use the descriptive variable a to represent age.

“Height is a function of age”

If we name the function f we write “ h is f of a ” or more simply “ $h = f(a)$ ”

We could instead name the function h and write “ $h(a)$ ” which is read “ h of a ” and still means “height is a function of age”.

Remember we can use any variable to name the function; the notation $h(a)$ shows us that h depends on a . The value “ a ” must be put into the function “ h ” to get a result. Be careful - the parentheses indicate that age is input into the function (Note: do not confuse these parentheses with multiplication!).

Example 2.1.1

The number of days in a month is a function of the name of the month, so if we name the function f , we could write “days = $f(\text{month})$ ” or $d = f(m)$. If we simply name the function d , we could write $d(m)$

For example, $d(\text{March}) = 31$, since March has 31 days. The notation $d(m)$ reminds us that the number of days, d is dependent on the name of the month, m . Here the number of days is the output, and the name of the month is the input.

Example 2.1.2

A function $P(n)$ gives the number of police officers in a town n years after 2000. What does $P(5) = 300$ tell us?

Solution

When we read $P(5) = 300$, we see the value for the input quantity of the function is 5, meaning 5 years after 2000. This gives us the year $2000 + 5 = 2005$. The output value is 300, the number of police officers. So this tells us that in the year 2005 there were 300 police officers in the town.

When we work with functions, there are two typical things we do: evaluate and solve. **Evaluating a function** is what we do when we **know an input**, and use the function to determine the corresponding output. Evaluating will always produce one result, since

each input of a function corresponds to exactly one output.

Solving equations involving a function is what we do when we **know an output**, and use the function to determine the inputs that would produce that output. Solving a function could produce more than one solution, since different inputs can produce the same output.

✓ Example 2.1.3

Using the table shown,

n	1	2	3	4	5
g(n)	8	6	7	6	8

- Evaluate $g(3)$
- Determine n when $g(n) = 6$.

Solution

- Evaluating $g(3)$ (read: “g of 3”) means that we need to determine the output value of the function g given the input value of $n=3$. Looking at the table, we see the output corresponding to $n=3$ is $g(n)=7$, allowing us to conclude $g(3) = 7$.
- Determine n when $g(n) = 6$. Here, we need to determine what input (n) value(s) produce an output value of 6. Looking at the table we see there are two solutions: $n = 2$ and $n = 4$. When we input 2 into the function g , our output is 6. When we input 4 into the function g , our output is also 6.

? You Try It 2.1.1

The function $k(m)$ is given by the table below.

m	-2	0	1	4	8	15
k(m)	4	8	32	51	33	1

- Evaluate $k(4)$.
- Determine m when $k(m)=8$.

Answer

- 51
- 0

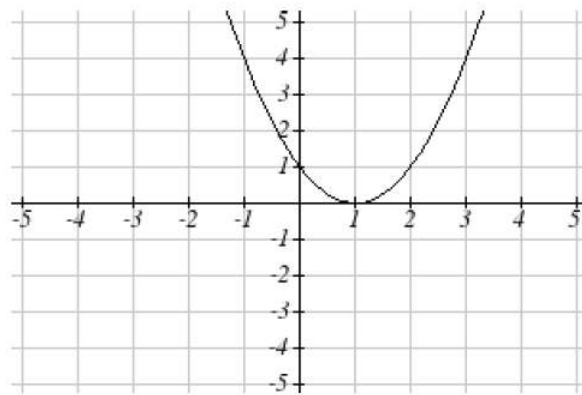
Graphs as Functions

Oftentimes a graph of a relationship can be used to define a function. By convention, graphs are typically created with the input quantity along the horizontal axis and the output quantity along the vertical axis. The most common graph has y on the vertical axis and x on the horizontal axis, and we say y is a function of x , or $y = f(x)$ when the function is named f .

Evaluating a function using a graph requires taking the given input and using the graph to look up the corresponding output. Solving a function equation using a graph requires taking the given output and looking on the graph to determine the corresponding input.

✓ Example 2.1.4

Given the graph below, a) Evaluate $f(2)$ b) Determine x when $f(x) = 4$

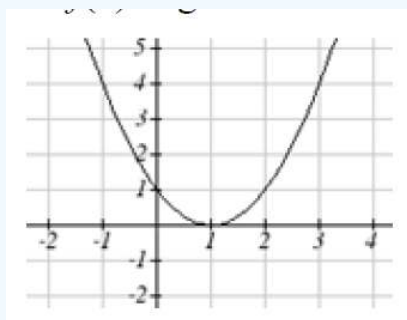


Solution

- To evaluate $f(2)$, we find the input of $x = 2$ on the horizontal axis. Moving up to the graph gives the point $(2, 1)$, giving an output of $y = 1$. So $f(2) = 1$
- To determine x when $f(x) = 4$, we find the value 4 on the vertical axis because if $f(x) = 4$ then 4 is the output. Moving horizontally across the graph gives two points with the output of 4: $(-1, 4)$ and $(3, 4)$. These give the two solutions to $f(x) = 4$: $x = -1$ or $x = 3$. This means $f(-1) = 4$ and $f(3) = 4$, or when the input is -1 or 3, the output is 4.

? You Try It 2.1.2

The function $f(x)$ is given below. a. Evaluate $f(1)$ b. Determine x when $f(x)=1$.



Answer

a. 0 b. 0 or 2

Formulas as Functions

When possible, it is very convenient to define relationships using formulas. If it is possible to express the output as a formula involving the input quantity, then we can define a function. As with tables and graphs, it is common to evaluate and solve functions involving formulas. Evaluating will require replacing the input variable in the formula with the value provided and calculating. Solving will require replacing the output variable in the formula with the value provided, and solving for the input(s) that would produce that output.

✓ Example 2.1.5

Given the function $k(t) = 2t + 5$

- Evaluate $k(8)$
- Determine t when $k(t) = 1$

Solution

- To evaluate $k(8)$, we plug in the input value 8 into the formula wherever we see the input variable t , then simplify

$$\begin{aligned}k(8) &= 2(8) + 5 \\k(8) &= 16 + 5 \\ \text{So } k(8) &= 21\end{aligned}\tag{2.1.1}$$

b) To determine t when $k(t) = 1$, we set the formula for $k(t)$ equal to 1, and solve for the input value that will produce that output

$$\begin{aligned}k(t) &= 1 && \text{substitute the original formula } k(t) = 2t + 5 \\2t + 5 &= 1 && \text{Solve for } t \\2t &= -4 \\t &= -2\end{aligned}\tag{2.1.2}$$

When solving an equation using formulas, you can check your answer by using your solution in the original equation to see if your calculated answer is correct.

We want to know if $k(t) = 1$ is true when $t = -2$.

$$\begin{aligned}k(-2) &= 2(-2) + 5 \\&= -4 + 5 \\&= 1\end{aligned}$$

(which was the desired result).

? You Try It 2.1.3

Given the function $g(m) = 5m + 17$

- Evaluate $g(9)$
- Determine m when $g(m) = 2$

Answer

a. 28 b. -3

Graphical Behavior of Functions

As part of exploring how functions change, it is interesting to explore the graphical behavior of functions.

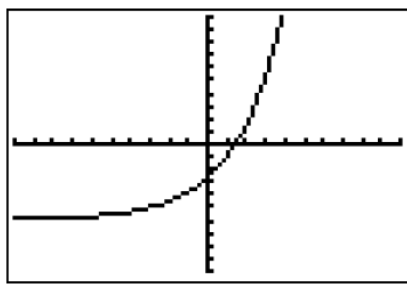
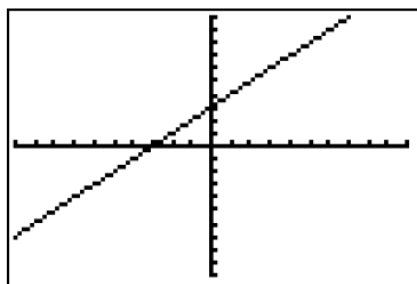
📌 Note

As we move from left to right (the inputs increase), we say a function is:

- INCREASING if the outputs get larger,
- DECREASING if the outputs get smaller,
- CONSTANT if the outputs do not change.

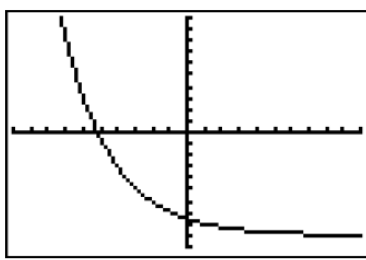
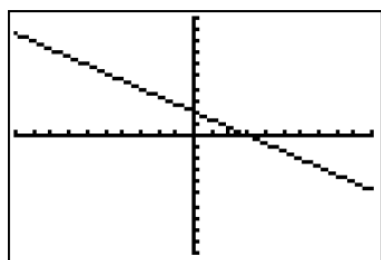
NOTE: We read graphs just like we read a book...from left to right.

The following functions are INCREASING



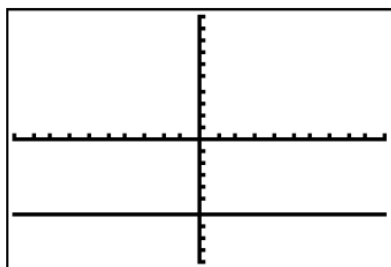
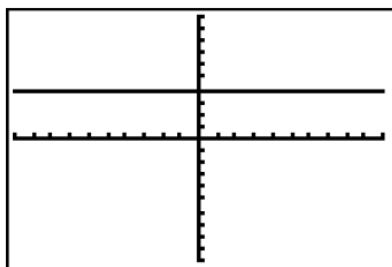
x	$f(x)$
0	4
1	6
2	12
3	24

The following functions are DECREASING



x	$g(x)$
0	10
1	5
2	0
3	-5

The following functions are CONSTANT



x	$h(x)$
0	4
1	4
2	4
3	4

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2.2: Linear Functions .

Definition: Linear Function

A linear function is a function whose graph produces a line.

Linear functions can always be written in the form:

$f(x) = b + mx$ or $f(x) = mx + b$ These are equivalent where b is the initial or starting value of the function (when input, $x = 0$), and m is the constant rate of change of the function.

Many people like to write linear functions in the form $f(x) = b + mx$ because it corresponds to the way we tend to speak: "The output starts at b and increases at a rate of m ."

For this reason alone we will use the form $f(x) = b + mx$ for many of the examples, but remember they are equivalent and can be written correctly both ways.

Here, m is the constant rate of change of the function (also called the slope). The slope determines if the function is an increasing function or a decreasing function.

If $m > 0$ then the function is increasing

If $m < 0$ then the function is decreasing

If $m = 0$, the rate of change is zero, and the function is a horizontal line

NOTE: A vertical line has a slope that is undefined. A vertical line is NOT a function.

✓ Example 2.2.1

Marcus currently owns 200 songs in his iTunes collection. Every month, he adds 15 new songs. Write a formula for the number of songs, N , in his iTunes collection as a function of the number of months, m . How many songs will he own in a year?

Solution

The initial value for this function is 200, since he currently owns 200 songs, so $N(0) = 200$. The number of songs increases by 15 songs per month, so the rate of change is 15 songs per month. With this information, we can write the formula:

$$N(m) = 200 + 15m. \quad (2.2.1)$$

$N(m)$ is an increasing linear function.

With this formula we can predict how many songs he will have in 1 year (12 months): $N(12) = 200 + 15(12) = 200 + 180 = 380$. Marcus will have 380 songs in 12 months.

? You Try It 2.2.1

You have just bought a new Sony 55" 3D television set for \$2300. The TV's value decreases at a rate of \$250 per year. Construct a linear function to represent this situation. Clearly indicate what your variables represent.

Answer

$V(n) = 2300 - 250n$, where $V(n)$ gives the value (in dollars) of the TV after n years.

Calculating Rate of Change

Given two values for the input x_1 and x_2 , and two corresponding values for the output, y_1 and y_2 , or a set of points, (x_1, y_1) and (x_2, y_2) , if we wish to find a linear function that contains both points we can calculate the rate of change, m :

$$m = \frac{\text{change in output}}{\text{change in input}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad (2.2.2)$$

Rate of change of a linear function is also called the slope of the line.

✓ Example 2.2.2

The population of a city increased from 23,400 to 27,800 between 2002 and 2006. Find the rate of change of the population during this time span.

Solution

The rate of change will relate the change in population to the change in time. The population increased by $27800 - 23400 = 4400$ people over the 4 year time interval. To find the rate of change, the number of people per year the population changed by:

$$\frac{4400 \text{ people}}{4 \text{ years}} = \frac{1100 \text{ people}}{1 \text{ year}} = 1100 \text{ people per year} \quad (2.2.3)$$

(Note: You should recognize the slope as a unit rate from chapter 2!)

Notice that we knew the population was increasing, so we would expect our value for m to be positive. This is a quick way to check to see if your value is reasonable.

? You Try It 2.2.2

In the year 1998, the surface elevation of Lake Powell was 3,843 feet above sea level. In the year 2001, the surface elevation of Lake Powell was 3,609 feet above sea level.

Determine the rate of change in this situation.

Answer

The surface elevation of Lake Powell is decreasing at a rate of 78 feet per year.

We can now find the rate of change given two input-output pairs, and can write an equation for a linear function once we have the rate of change and initial value. If we have two input-output pairs and they do not include the initial value of the function, then we will have to solve for it.

✓ Example 2.2.3

Working as an insurance salesperson, David earns a base salary and a commission on each new policy, so David's weekly income, I , depends on the number of new policies, n , he sells during the week. Last week he sold 3 new policies, and earned \$760 for the week. The week before, he sold 5 new policies, and earned \$920. Find an equation for $I(n)$, and interpret the meaning of the components of the equation.

Solution

The given information gives us two input-output pairs: $(3, 760)$ and $(5, 920)$. We start by finding the rate of change.

$$m = \frac{920 - 760}{5 - 3} = \frac{160}{2} = \frac{80}{1} \text{ (or just 80)} \quad (2.2.4)$$

Keeping track of units can help us interpret this quantity. Income increased by \$160 when the number of policies increased by 2, so the rate of change is $\frac{\$80}{1 \text{ policy}}$. David earns a commission of \$80 for each policy sold during the week.

We can then solve for the initial value

$$\begin{aligned} I(n) &= b + 80n && \text{then when } n = 3, I(3) = 760, \text{ giving} \\ 760 &= b + 80(3) && \text{this allows us to solve for } b \\ b &= 760 - 80(3) = 520 \end{aligned} \quad (2.2.5)$$

This value is the starting value for the function. This is David's income when $n = 0$, which means no new policies are sold. We can interpret this as David's base salary for the week, which does not depend upon the number of policies sold.

Writing the final equation: $I(n) = 520 + 80n$

Our final interpretation is: David's base salary is \$520 per week and he earns an additional \$80 commission for each policy sold during the week.

? You Try It 2.2.3

The balance in your college payment account, C , is a function of the number of quarters, q , you attend. Interpret the function $C(q) = 20000 - 4000q$ in words.

Answer

Your College account starts with \$20,000 in it and you withdraw \$4,000 each quarter (or your account contains \$20,000 and decreases by \$4000 each quarter.) You can pay for 5 quarters before the money in this account is gone.

When working with applied problems involving functions, use the following strategy.

📌 Note Problem Solving Strategy

- 1) Identify changing quantities, and then carefully and clearly define descriptive variables to represent those quantities. When appropriate, sketch a picture or define a coordinate system.
- 2) Carefully read the problem to identify important information. Look for information giving values for the variables, or values for parts of the functional model, like slope and initial value. Also identify what you are trying to find, identify, solve, or interpret.
- 3) Identify a solution pathway from the provided information to what we are trying to find. Often this will involve checking and tracking units, building a table or even finding a formula for the function being used to model the problem.
- 4) Solve or evaluate using the formula you found for the desired quantities.
- 5) Reflect on whether your answer is reasonable for the given situation and whether it makes sense mathematically.
- 6) Clearly convey your result using appropriate units, and answer in full sentences when appropriate.

✓ Example 2.2.4

Arielle saved up \$3500 for her summer visit to Seattle. She anticipates spending \$400 each week on rent, food, and fun. How long can she afford to stay in Seattle?

Solution

In the problem, there are two changing quantities: time and money. The amount of money she has remaining while on vacation depends on how long she stays. We can define our variables, including units.

Output: M , money remaining, in dollars Input: t , time, in weeks

Reading the problem, we identify two important values. The first, \$3500, is the initial value for M . The other value appears to be a rate of change - the units of dollars per week match the units of our output variable divided by our input variable. She is spending money each week, so you should recognize that the amount of money remaining is decreasing each week and the slope is negative.

To answer the question, it would be helpful to have an equation modeling this scenario. Using the intercept and slope provided in the problem, we can write the equation: $M(t) = 3500 - 400t$, where M is the amount of money remaining after t weeks.

To find out how long she can stay, we need to find out how long it will take for her to use up all of the money she has saved. So we are looking for the number of weeks, t , when $M = 0$. Set the output to zero, and solve for the input:

$$\begin{aligned} 0 &= 3500 - 400t \\ t &= \frac{3500}{400} = 8.75 \end{aligned} \quad (2.2.6)$$

Interpreting this, we could say: Arielle will have no money left after 8.75 weeks.

When modeling any real life scenario with functions, there is typically a limited domain over which that model will be valid - almost no trend continues indefinitely. In this case, it certainly doesn't make sense to talk about input values less than zero. It is also likely that this model is not valid after the horizontal intercept (unless Arielle is going to start using a credit card and go into debt).

The domain represents the set of input values and so the reasonable domain for this function is $0 \leq t \leq 8.75$.

However, in a real world scenario, the rental might be weekly or nightly. She may not be able to stay a partial week and so all options should be considered. Arielle could stay in Seattle for 0 to 8 full weeks (and a couple of days), but would have to go into debt to stay 9 full weeks, so restricted to whole weeks, a reasonable domain without going in to debt would be $0 \leq t \leq 8$, or $0 \leq t \leq 9$ if she went into debt to finish out the last week.

The range represents the set of output values and she starts with \$3500 and ends with \$0 after 8.75 weeks so the corresponding range is $0 \leq M(t) \leq 3500$.

If we limit the rental to whole weeks however, if she left after 8 weeks because she didn't have enough to stay for a full 9 weeks, she would have $M(8) = 3500 - 400(8) = \$300$ dollars left after 8 weeks, giving a range of $300 \leq M(t) \leq 3500$. If she wanted to stay the full 9 weeks she would be \$100 in debt giving a range of $-100 \leq M(t) \leq 3500$.

Most importantly remember that domain and range are tied together, and whatever you decide is most appropriate for the domain (the independent variable) will dictate the requirements for the range (the dependent variable).

✓ Example 2.2.5

Jamaal is choosing between two moving companies. The first, U-Haul, charges an upfront fee of \$20, then 59 cents per mile. The second, Budget, charges an up-front fee of \$16, then 63 cents per mile³. When will U-Haul be the better choice for Jamal?

Solution

The two important quantities in this problem are the cost, and the number of miles that are driven. Since we have two companies to consider, we will define two functions:

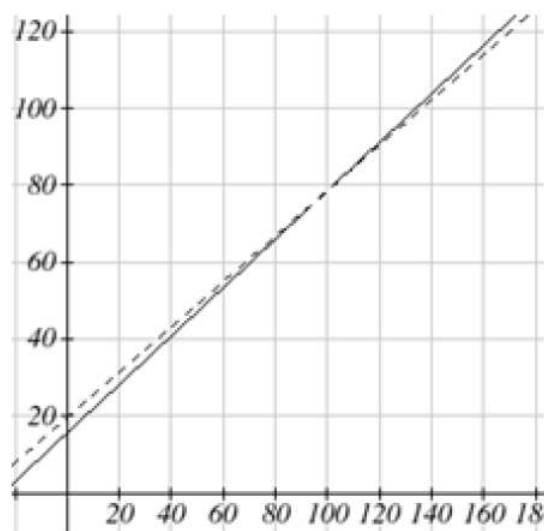
Input: m , miles driven
 Outputs: $Y(m)$: cost, in dollars, for renting from U-Haul
 $B(m)$: cost, in dollars, for renting from Budget

Reading the problem carefully, it appears that we were given an initial cost and a rate of change for each company. Since our outputs are measured in dollars but the costs per mile given in the problem are in cents, we will need to convert these quantities to match our desired units: \$0.59 a mile for U-Haul, and \$0.63 a mile for Budget. Looking to what we're trying to find, we want to know when U-Haul will be the better choice. Since all we have to make that decision from is the costs, we are looking for when U-Haul will cost less, or when $Y(m) < B(m)$.

Using the rates of change and initial charges, we can write the equations:

$$Y(m) = 20 + 0.59m \quad B(m) = 16 + 0.63m \quad (2.2.7)$$

These graphs are sketched below, with $Y(m)$ drawn dashed.



To find the intersection, we set the equations equal and solve: $Y(m) = B(m)$.

$$\begin{aligned} 20 + 0.59m &= 16 + 0.63m \\ -0.04m &= -4 \\ m &= 100 \end{aligned} \tag{2.2.8}$$

This tells us that the cost from the two companies will be the same if 100 miles are driven. Either by looking at the graph, or noting that $Y(m)$ is growing at a slower rate, we can conclude that U-Haul will be the cheaper price when more than 100 miles are driven.

✓ Example 2.2.6

A town's population has been growing linearly. In 2004 the population was 6,200. By 2009 the population had grown to 8,100. If this trend continues,

- Predict the population in 2013
- When will the population reach 15000?

Solution

The two changing quantities are the population and time.

Input: t , years since 2004 Output: $P(t)$, the town's population

The problem gives us two input-output pairs. Converting them to match our defined variables, the year 2004 would correspond to $t = 0$, giving the point $(0, 6200)$. Notice that through our clever choice of variable definition, we have "given" ourselves the vertical intercept of the function. The year 2009 would correspond to $t = 5$, giving the point $(5, 8100)$.

To predict the population in 2013 ($t = 9$), we would need an equation for the population. Likewise, to find when the population would reach 15000, we would need to solve for the input that would provide an output of 15000. Either way, we need an equation. To find it, we start by calculating the rate of change:

$$m = \frac{8100 - 6200}{5 - 0} = \frac{1900}{5} = \frac{380 \text{ people}}{1 \text{ year}} = 380 \text{ people per year} \tag{2.2.9}$$

Since we already know the vertical intercept of the line, we can immediately write the equation: $P(t) = 6200 + 380t$

To predict the population in 2013, we evaluate our function at $t = 9$ $P(9) = 6200 + 380(9) = 9620$

If the trend continues, our model predicts a population of 9,620 in 2013.

To find when the population will reach 15,000, we can set $P(t) = 15000$ and solve for t .

$$15000 = 6200 + 380t$$

$$8800 = 380t$$

$$t \approx 23.158$$

(2.2.10)

Our model predicts the population will reach 15,000 in a little more than 23 years after 2004 , or somewhere around the year 2027.

? You Try It (\PageIndex{4})

In 2004, a school population was 1001. By 2008 the population had grown to 1697. Assume the population is changing linearly.

- How much did the population grow between the year 2004 and 2008?
- How long did it take the population to grow from 1001 students to 1697 students?
- What is the average population growth per year?
- Find an equation for the population, P , of the school t years after 2004.
- Using your equation, predict the population of the school in 2011.

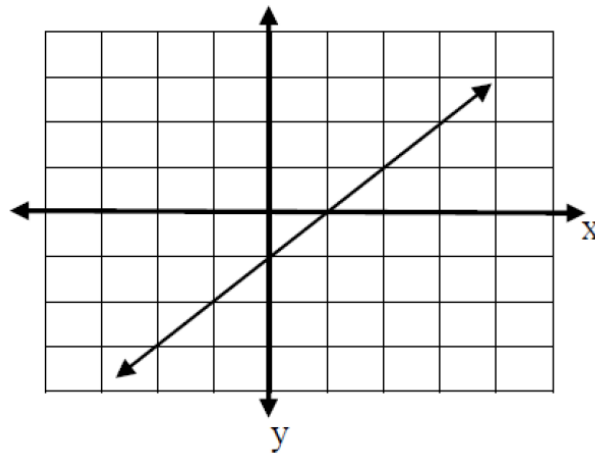
Answer

- 696 students
- 4 years
- 174 students per year
- $P(t) = 174t + 1001$
- 2219 students

2.2: Linear Functions . is shared under a [CC BY-NC-SA](#) license and was authored, remixed, and/or curated by LibreTexts.

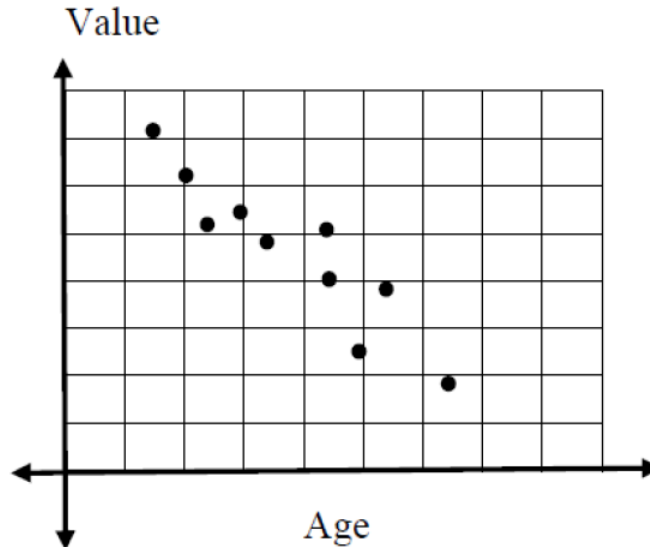
2.3: Linear Regression

In the last section we explored linear functions. Recall that linear functions have a constant rate of change, and can be written in the form $f(x) = b + mx$, or $f(x) = mx + b$. When graphed, a linear function is a straight line, where all points that satisfy the linear equation fall exactly on the graph of the line. This can be seen in the graph below:



This is the graph of $f(x) = -1 + x$. Some of the points on this line are: $(-2, -3)$, $(0, -1)$, $(1, 0)$, and $(3, 2)$. There are infinitely many other points on this line too.

Now consider the following graph that represents the relationship between the age of a vehicle (in years) and its value (in \$1000s) for 10 different vehicles.



Can we write the equation of a linear function that represents this set of points? No, not exactly.

There is no straight line that will go through ALL of these points, but there does appear to be a linear relationship between the two variables.

The graph that you see above is called a scatterplot. A scatterplot is used to show the relationship between two numerical variables. The independent (predictor) variable is plotted on the horizontal axis, and the dependent (response) variable is plotted on the vertical axis.

✓ Example 2.3.1

The data below show a person's body weight during a diet program.

Time, t , in weeks	0	1	2	3	4	5
Weight, W , in pounds	196	192	193	190	190	186

Draw a scatterplot of the data. Does the data appear linear?

Solution

Step 1: Enter the data into your calculator

Press STAT (Second Row of Keys)

Press ENTER to access 1:Edit under EDIT menu

Note: Be sure all data columns are cleared. To do so, use your arrows to scroll up to L1 or L2 then click CLEAR then scroll down. (DO NOT CLICK DELETE!)

Once your data columns are clear, enter the input data into L1 (press ENTER after each data value to get to the next row) then right arrow to L2 and enter the output data into L2. Your result should look like this when you are finished (for L1 and L2):

L1	L2	L3	1
0	196		
1	192		
2	193		
3	190		
4	190		
5	186		
L1(1) =			

L1	L2	L3	2
0	196		
1	192		
2	193		
3	190		
4	190		
5	186		
L2(6) = 186			

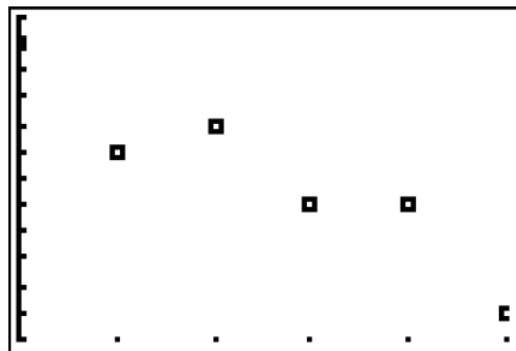
Step 2: Turn on your Stat Plot

- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should be highlighted as at left
- Clear out all entries below

Plot1	Plot2	Plot3
✓V1=		
✓V2=		
✓V3=		
✓V4=		
✓V5=		
✓V6=		
✓V7=		

Step 3: Graph the Data in an Appropriate Viewing Window

- Click the ZOOM key, and scroll down to Option 9: "ZoomStat"
- Once your cursor is on "ZoomStat", press the GRAPH key. A graph of your data should appear in an appropriate window so that all data points are clearly visible.



The scatterplot above shows that the data points seem to have a linear relationship.

From the previous scatterplot, it looks like there is a negative linear relationship between the time in weeks and the weight in pounds. We can see this linear relationship because the points are scattered around an imaginary straight line. More specifically, as the time increases, the weight seems to decrease by a fairly consistent amount.

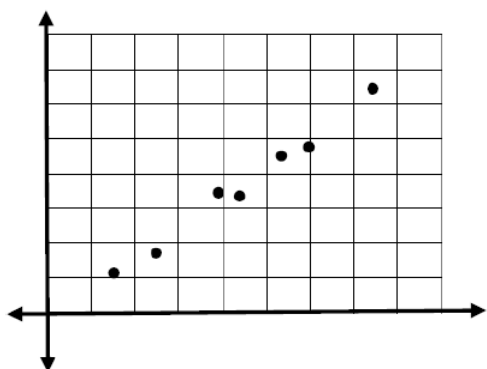
✓ You Try It 2.3.1

Consider the data set:

x	3	5	8	9	11	12	15
y	1.1	3.2	7.0	6.8	9.5	9.8	13.0

- Draw a scatterplot for the data set.
- Does the data set appear to be increasing or decreasing?
- Does the data set appear to have a linear relationship?

Solution



-
- Increasing
- Yes, there appears to be a linear trend

Just because data are not EXACTLY linear does not mean we cannot write an approximate linear model for the given data set. In fact, most data in the real world are NOT exactly linear and all we can do is write models that are close to the given values.

Once we have determined using a scatterplot that our two variables appear to have a roughly linear relationship, we can use a process called linear regression to find the equation of the line that best fits the data points. If you take a statistics class, you will learn a lot more about this process. In this class, you will be introduced to the basics. This process is also called “FINDING THE

LINE OF BEST FIT". This process is lengthy and calculation intensive by hand, but can be done quickly using our graphing calculator.

Our calculator will give us the best linear equation possible by taking into account ALL the given data points.

NOTE: Unless your data are exactly linear, the regression equation will not match all data points exactly. It is a model used to predict outcomes not provided in the data set.

✓ Example 2.3.2

Consider the data set from the previous problem:

Time, t , in weeks	0	1	2	3	4	5
Weight, W , in pounds	196	192	193	190	190	186

Use your graphing calculator to find the regression equation.

Solution

Step 1: Enter the Data into your Graphing Calculator

Step 1: Enter the Data into your Graphing Calculator

Press STAT then select option 1:Edit under EDIT menu. Clear lists, then enter the values.

```

30011 CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
  
```

L1	L2	L3	2
0	196	---	
1	192		
2	193		
3	190		
4	190		
5	186		
---	---		
L2(6) = 186			

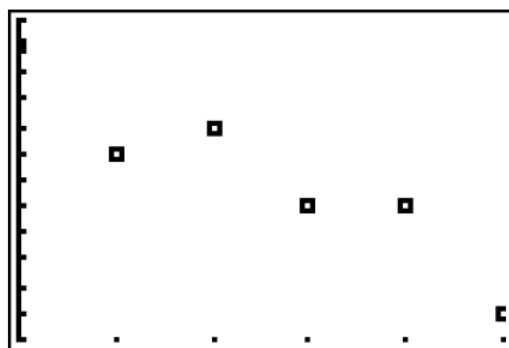
****NOTE** If you ever accidentally DELETE a column, then go to STAT, Option 5: SetUpEditor>ENTER. When you go back to STAT, your column should be restored.

Step 2: Turn on your Stat Plot and Graph the Data in an Appropriate Viewing Window

(Refer to previous example for help)

```

20011 Plot1 Plot2 Plot3
\Y1=
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
  
```



Note: Since we already graphed the scatterplot for this data set (and know the data looks linear), you could skip this step. Graphing the scatterplot is not required in order to calculate the regression equation, but you should always verify using a scatterplot that your data looks linear BEFORE doing a linear regression!

Step 3: Access the Linear Regression section of your calculator

- Press STAT
- Scroll to the right one place to CALC
- Scroll down to 4:LinReg(ax+b)
- Your screen should look as the one at left

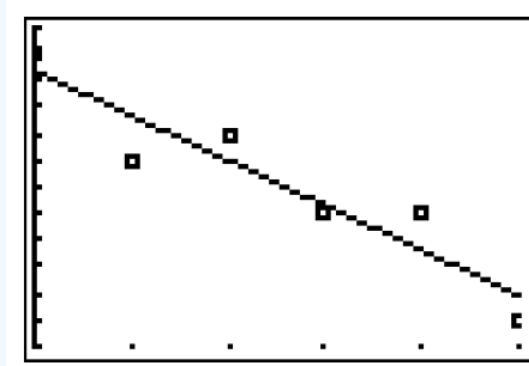
Step 4: Determine the linear regression equation

- Press ENTER twice in a row to view the screen at right
- The calculator computes values for slope (a) and y-intercept (b) in what is called the equation of best-fit for your data.
- Identify these values and round to the appropriate places. Let's say 2 decimals in this case. So, $a = -1.69$ and $b = 195.38$
- Now, replace the a and b in $y = ax + b$ with the rounded values to write the actual equation: $y = -1.69x + 195.38$
- To write the equation in terms of initial variables, we would write $W = -1.69t + 195.38$
- In function notation, $W(t) = -1.69t + 195.38$

```
LinReg
y=ax+b
a=-1.685714286
b=195.3809524
```

GRAPHING THE REGRESSION EQUATION ON TOP OF THE STAT PLOT

- Enter the Regression Equation with rounded values into Y=
- Press GRAPH
- You can see from the graph that the “best fit” line does not hit very many of the given data points. But, it will be the most accurate linear model for the overall data set.



IMPORTANT NOTE: When you are finished graphing your data, TURN OFF YOUR PLOT1. Otherwise, you will encounter an INVALID DIMENSION error when trying to graph other functions. To do this:

- Press Y=
- Use your arrow keys to scroll up to Plot1
- Press ENTER
- Scroll down and Plot1 should be UNhighlighted

✓ You Try It 2.3.1

Consider the data set:

n	0	2	4	6	8	10	12
f(n)	23.76	24.78	25.93	26.24	26.93	27.04	27.93

Use your graphing calculator to find the linear regression equation for this data set.

Solution

$$y = 0.32x + 24.16$$

Once you have a linear regression equation, you can then use that equation to provide you with information about how your two variables change together, and to make predictions about other values of your variables of interest.

✓ Example 2.3.3

Consider again the data set from the previous problem

Time, t , in weeks	0	1	2	3	4	5
Weight, W , in pounds	196	192	193	190	190	186

- Interpret the meaning of the vertical intercept.
- Interpret the meaning of the slope.
- Use your regression equation to predict the weight in pounds after 2.5 weeks.
- Use your regression equation to predict the weight in pounds after 6 weeks.

Solution

Recall, the regression equation for this data set was

$$y = -1.69x + 195.38$$

To write the equation in terms of initial variables, we would write $W = -1.69t + 195.38$

- The vertical intercept is the value of W when $t = 0$. Based on the regression equation, we know that when $t = 0$, $W = 195.38$. This tells us that when first starting the weight loss program, the individual weighed approximately 195.38 pounds.

Note: In this particular case, we had a data point for time 0 weeks of 196 pounds. In general, the regression equation gives an estimate (or predicted value) of the response variable. Output from the regression equation will rarely equal the exact data values themselves.

- The slope of the regression equation is -1.69 . Recall that the slope measures $\frac{\text{change in } y}{\text{change in } x}$, or in our problem, $\frac{\text{change in weight } (W)}{\text{change in time } (t)}$. We can write the slope for this equation of -1.69 as a fraction by writing it over 1:

$$\frac{\text{change in weight } (W)}{\text{change in time } (t)} = \frac{-1.69}{1} \quad (2.3.1)$$

To interpret the meaning of the slope, we would say that for each additional week, the weight decreases by approximately 1.69 pounds.

- To predict the weight in pounds after 2.5 weeks, plug in $t = 2.5$ to the regression equation, and solve for W .

$$\begin{aligned} W &= -1.69(2.5) + 195.38 \\ W &= 191 \text{ pounds} \end{aligned} \quad (2.3.2)$$

- To predict the weight in pounds after 6 weeks, plug in $t = 6$ to the regression equation, and solve for W .

$$\begin{aligned} W &= -1.69(6) + 195.38 \\ W &= 185 \text{ pounds} \end{aligned} \quad (2.3.3)$$

In the previous problem, we used our regression equation to tell us how our two variables change together, and to make predictions about other values. You may have noticed that we made predictions about the values of our dependent variable only for values of our independent variable that were close to the actual values in our data set. In other words, our original data set included values of

time from 0-5 weeks. Based on that information, we created our model. It would not be appropriate to try to make predictions for values of time that are far outside our original range of time values (from 0 – 5 weeks). This is called **extrapolation** and can result in very poor predictions. Consider the following example.

✓ Example 2.3.4

Consider again the data set from the previous problem:

Time, t , in weeks	0	1	2	3	4	5
Weight, W , in pounds	196	192	193	190	190	186

Would it be reasonable to use your regression equation to predict the weight in pounds after 100 weeks? Why or why not?

Solution

Recall, the regression equation for this data set was

$$W = -1.69t + 195.38 \quad (2.3.4)$$

Our model was created based off of data from 0 to 5 weeks into a weight loss program. Is it reasonable to assume that the same pattern of weight loss that held in weeks 0 – 5 would be present in week 100? It certainly doesn't seem so. This is an example of extrapolation. Let's see what would happen if we tried to use the model to predict the weight in pounds after 100 weeks. We plug in $t = 100$ to the regression equation, and solve for W .

$$\begin{aligned} W &= -1.69(100) + 195.38 \\ W &= 26 \text{ pounds} \end{aligned} \quad (2.3.5)$$

If this pattern of weight loss continued, the individual would weigh only 26 pounds

✓ You Try It 2.3.2

The following table gives the total number of live Christmas trees sold, in millions, in the United States from 2004 to 2011. (Source: Statista.com).

Year	2004	2006	2008	2010	2011
$C(t)$ = Total number of Christmas trees sold in the US (in millions)	27.10	28.60	28.20	27	30.80

a) Use your calculator to determine the equation of the regression line, $C(t)$ where t represents the number of years since 2004. Start by entering new t values for the table below based upon the number of years since 2004. The first few are done for you:

t = Number of years since 2004	0	2			
$C(t)$ = Total number of Christmas trees sold in the US (in millions)	27.10	28.60	28.20	27	30.80

- b) Identify the slope of the regression equation and explain its meaning in the context of this problem.
- c) Use the regression equation to predict the number of Christmas trees that will be sold in the year 2013. Write your answer as a complete sentence.
- d) Should you use your regression equation to predict the number of Christmas trees that will be sold in the year 2030 ? Why or why not?

Solution

- a. $C(t) = 0.279t + 27.28$
- b. For each additional year since 2004, the number of Christmas trees sold in the U.S. will increase by approximately 279,000 trees.
- c. 29.79 million Christmas trees
- d. No. The original data is for years 2004 through 2011. The year 2030 is too far from the original data set.

2.3: Linear Regression is shared under a [CC BY-NC-SA](#) license and was authored, remixed, and/or curated by LibreTexts.

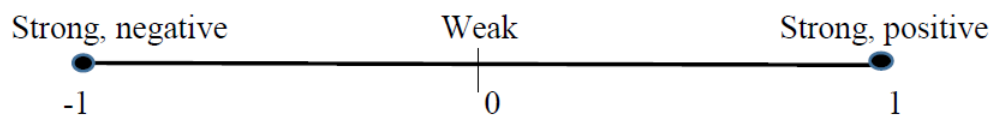
2.4: Correlation

In the last section we learned about relationships between two variables that appeared to be linear. In some cases, the scatterplot shows very clearly that the two variables are linearly related, but in many cases it can be a bit challenging to tell exactly how strong the linear relationship is between the two variables of interest. The linear correlation (r), also called the correlation coefficient, is a way to numerically measure the strength and the direction of the linear relationship between two variables.

Properties of Correlation:

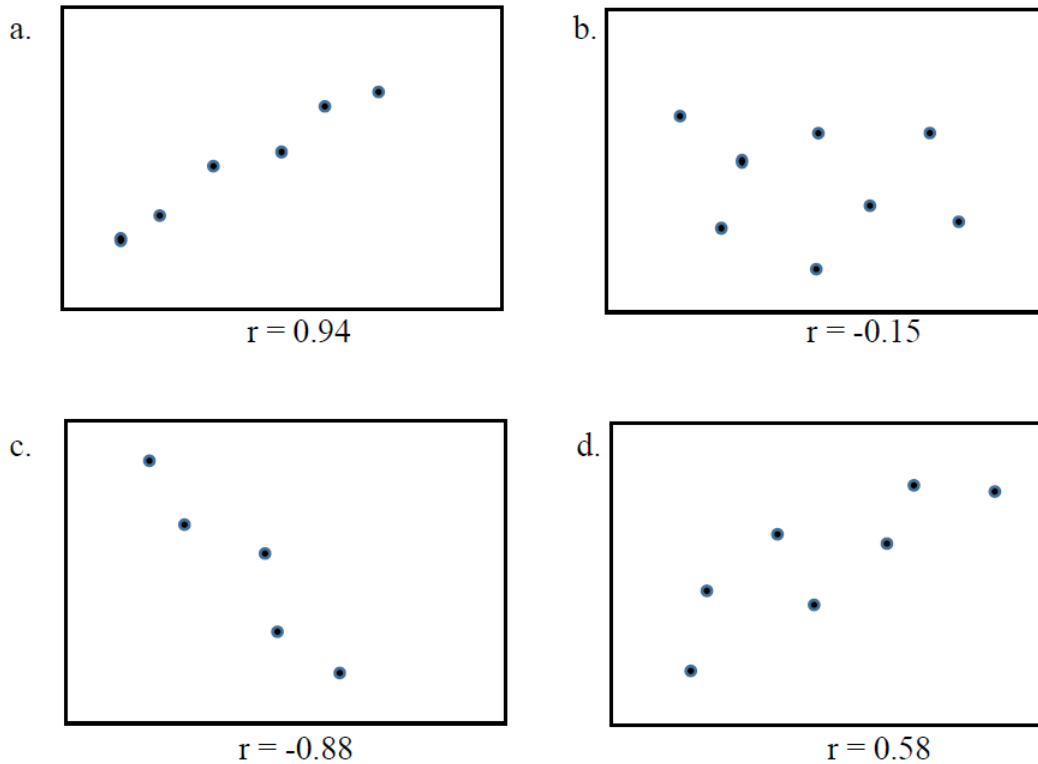
- We use " r " to represent the correlation
- $-1 \leq r \leq 1$
- The sign of r indicates the direction of the linear relationship between the 2 variables
 - If $r < 0$, there is a negative linear relationship between the 2 variables (slope of the regression equation is negative).
 - If $r > 0$, there is a positive linear relationship between the 2 variables (slope of the regression equation is positive).
- The magnitude of r indicates the strength of the linear relationship between the 2 variables
 - If $r \approx 0$, then there is no linear relationship between the 2 variables
 - If $|r| \approx 1$, then there is a very strong linear relationship between the 2 variables
 - If $r = 1$ or -1 , then there is an exact linear relationship between the 2 variables (every data point falls exactly on the regression line)

These properties of correlation can be summarized in the graphic below:



✓ Example 2.4.1

Consider the following scatterplots, and their corresponding correlation values.



Classify the strength and direction of the linear relationship between the 2 variables in each scatterplot

Solution

- Here $r = 0.94$, which is close to positive 1. We would say these 2 variables have a strong, positive linear relationship.
- Here $r = -0.15$, which is negative and close to 0. We could say these 2 variables have a very weak, negative linear relationship.
- Here $r = -0.88$, which is close to negative 1. We would say these 2 variables have a strong, negative linear relationship.
- Here $r = 0.58$, which is positive, but not particularly close to 0 or 1. We would say these 2 variables have a moderate, positive linear relationship.

Note: There is no “cutoff” value for which values of r are considered “strong” and which are considered “weak”. The interpretation of r can be fairly subjective. What you should know is that the farther the value of r is from 0, the stronger the linear relationship between the 2 variables is considered to be.

Now that we have a basic understanding of interpreting the correlation, we can discuss how to find it. As with linear regression, the calculations for finding correlation by hand are rather time consuming. We will again rely on the graphing calculator to find the correlation for us.

✓ Example 2.4.1

Consider again the data set from the weight loss problem:

Time, t , in weeks	0	1	2	3	4	5
Weight, W , in pounds	196	192	193	190	190	186

- Use your graphing calculator to find the correlation between time and weight.
- Interpret the meaning of the correlation you found in part a.

Solution

To find the correlation on the graphing calculator, press the "2nd" key and then "Catalog". "Catalog" is a 2nd function on the zero key. Use the arrow keys to scroll down to "DiagnosticOn". Hit ENTER. On the main screen, your calculator should display DiagnosticOn followed by a blinking cursor. Hit ENTER. The calculator should display "Done". Once you have turned on the DiagnosticOn, you do not have to do it again. Also, it will not interfere with any other calculator functions, so you can just leave it on.

Once you have turned on "DiagnosticOn" you can follow the instructions for finding the linear regression equation from section 3.3. When the calculator displays the linear regression equation, it will also display the correlation value, r .

- Enter your data into L1 and L2, then press STAT, scroll over to CALC, and select option 4: LinReg($ax + b$). Your output should consist of the values of a and b (just like we did in section 3.3) AND two new values: r^2 and r . The value of $r = -0.94$. (NOTE: The quantity r^2 is the coefficient of determination, which we do not cover in this course).
- The value of $r = -0.94$. This means that time and weight have a strong, negative linear relationship. This means that as time increases, weight tends to decrease. This can be confirmed by a scatterplot of the data which showed a negative slope to the data points.

? You Try It 2.4.1

Recall, the following table gives the total number of live Christmas trees sold, in millions, in the United States from 2004 to 2011. (Source: Statista.com).

t= number of years since 2004	0	2	4	6	7
C(t) - Total number of Christmas trees sold in the US (in millions)	27.10	28.60	28.20	27	30.80

- Use your graphing calculator to find the numerical value of the correlation.
- Interpret the meaning of the correlation from part a).

Answer

- $r = 0.52$
- There is a moderate, positive linear relationship between the time in years since 2004 and the number of Christmas trees sold in the U.S.

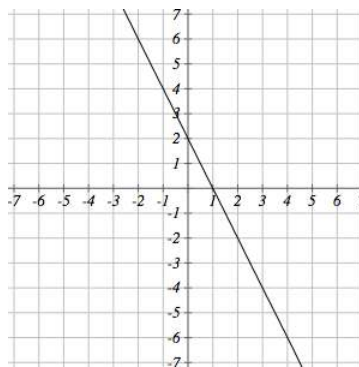
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2.5: Practice Problems

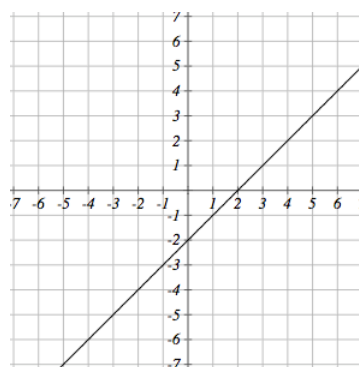
1. A function $P(n)$ gives the wholesale price in dollars of a package of cookies after n packages have been sold. Interpret the meaning of $P(124)=0.8$.
2. The function $C(g)$ represents the cost to produce g gallons of chocolate chunk ice cream. Interpret the meaning of $C(751)=160$.
3. The function $D(t)$ can be used to approximate the total average credit card debt in a U.S. household (in thousands of dollars) t years after 1995. Interpret the meaning of $D(29)=21.5$.
4. A function $d(h)$ gives the number of miles Billy has driven in his car from his house h hours after leaving his house.
 - a. Interpret $d(4)=160$
 - b. Interpret $d(0.1)=2$
5. The function $g(t)$ is given by the table below.

t	0	1	2	3	4	5	6
$g(t)$	12	24	3	24	27	0	3

- a. Evaluate $g(1)$
 - b. Evaluate $g(6)$
 - c. Determine t when $g(t)=24$
6. The function $f(x)$ is represented below as a graph. Use $f(x)$ to answer the following questions:



- a. Evaluate $f(-1)$
 - b. Evaluate $f(3)$
 - c. Determine x when $f(x)=0$
 - d. Determine x when $f(x)=6$
7. The function $f(x)$ is represented below as a graph. Use $f(x)$ to answer the following questions:



- a. Evaluate $f(0)$
- b. Evaluate $f(4)$

- c. Determine x when $f(x)=-3$
 - d. Determine x when $f(x)=1$
8. Given the function $g(x)=x+2$
 - a. Evaluate $g(-5)$
 - b. Evaluate $g(0)$
 - c. Determine x when $g(x)=-2$
 9. Given the function $k(t)=-8t+9$
 - a. Evaluate $k(3)$
 - b. Evaluate $k(0)$
 - c. Determine t when $k(t)=-39$
 10. Identify the rate of change and initial value for the function $f(x)=3x+7$
 11. Identify the rate of change and initial value for the function $f(x)=-9-10x$
 12. Identify the rate of change and initial value for the function $f(x)=-5x$
 13. Identify the rate of change and initial value for the function $f(x)=1$
 14. Calculate the rate of change of the linear function that contains the points (1,8) and (4,17).
 15. Calculate the rate of change of the linear function that contains the points (-15,30) and (7,-14).
 16. Calculate the rate of change of the linear function that contains the points (3,2) and (10,2).
 17. Determine the rate of change and initial value of the linear function that generates the following table of values.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	-70	-49	-28	-7	14	35	56	77	98

18. Determine the rate of change and initial value of the linear function that generates the following table of values.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	102	75	48	21	-6	-33	-60	-87	-114

19. Determine the rate of change and initial value of the linear function that generates the following table of values.

x	-4	-2	0	2	4
f(x)	-90	-14	62	138	214

20. The function $V(t)=-5.4t+34.2$ gives the value (in thousands of dollars) of an investment after t years. Interpret the rate of change in this situation.
21. The function $V(x)=2.5x+25.4$ gives the value (in thousands of dollars) of an investment after x months. Interpret the rate of change in this situation.
22. Paul is planning to sell bottled water at the local carnival. Paul's profit (in dollars) from selling b bottles of water is given by the formula $P(b)=1.4b-338$. Interpret the rate of change in this situation.
23. When a new charter school opened in 1996, there were 440 students enrolled. Write a formula for the function $N(t)$, representing the number of students attending this charter school t years after 1996, assuming that the student population:
 - a. Increased by 44 students per year
 - b. Decreased by 32 students per year
 - c. Remained constant (did not change)
24. A town's population has been growing linearly. In 2003, the population was 59,000 and the population has been growing by 1,700 people each year.
 - a. Write a formula for the population x years after 2003.

- b. What will the town's population be in 2007?
c. In what year will the population be 77,700 people?
25. Last year, Pinwheel Industries introduced a new toy. It cost \$1,300 to develop the toy and \$30 to manufacture each toy.
a. Write a formula for the total cost to produce n of these toys.
b. What is the total cost to produce 4,400 toys?
c. How many total toys can be produced with \$124,300?
26. In the year 1999, the surface elevation of Lake Powell was 3848 feet above sea level. In the year 2007, the surface elevation of Lake Powell was 3,227.2 feet above sea level. Find the rate of change of the surface elevation during this time span.
27. In the year 1987, an investment was worth \$29,800. In the year 1997, this investment was worth \$43,800.
a. Find the rate of change of the investment during this time span.
b. Is the investment increasing or decreasing?
28. In the year 1982, an investment was worth \$26,200. In the year 1990, this investment was worth \$39,000.
a. Find the rate of change of the investment during this time span.
b. Is the investment increasing or decreasing?
29. In 1995, the cost of tuition at a large Midwestern university was \$170 per credit hour. In 2000, tuition had risen to \$235 per credit hour.
a. Find the rate of change of the cost of tuition during this time span.
b. Write a formula for the cost of tuition as a function of the number of years since 1990.
c. What will the tuition be in 2007?
d. What year will the tuition be \$417 per credit hour?
30. Robert has \$100 in his school lunch account. If the school charges \$3.50 per lunch, how many lunches can he purchase? How much money will remain in his account?
31. You are interested in starting a gym membership and have narrowed your selection to two gyms. The first, Phoenix Fitness, charges a one-time membership fee of \$99 and charges \$35 per month. The second, Slim Gym, charges a one-time membership fee of \$10 and charges \$40 per month.
a. Which gym is the more affordable choice if you are going to commit to a 1-year membership?
b. Which gym is the more affordable choice if you are going to commit to a 2-year membership?
- 32 a. Use your graphing calculator to create of scatterplot of the data set shown below.

	1	3	4	6	7	9	10
	437	901	1155	1358	1768	2103	2437

- b. Does the data set appear to be increasing or decreasing?
c. Does the data set appear to be roughly linear?
33. a. Use your graphing calculator to create of scatterplot of the data set shown below.

	2	9	14	23	33	42
	-60.2	-130.1	-243.7	-328.9	-580.5	-643.8

- b. Does the data set appear to be increasing or decreasing?
c. Does the data set appear to be roughly linear?
34. a. Use your graphing calculator to create of scatterplot of the data set shown below.

	8	12	14	23	24	29	33
	10	20	15	24	35	30	28

b. Does the data set appear to be increasing or decreasing?

c. Does the data set appear to be roughly linear?

35. a. Use your graphing calculator to create of scatterplot of the data set shown below.

	20	24	27	32	48	60	68
	12	20	28	41	24	15	11

b. Does the data set appear to be increasing or decreasing?

c. Does the data set appear to be roughly linear?

36. Calculate the linear regression equation for the data from problem 32 :

	1	3	4	6	7	9	10
	437	901	1155	1358	1768	2103	2437

37 Calculate the linear regression equation for the data from problem 33 :

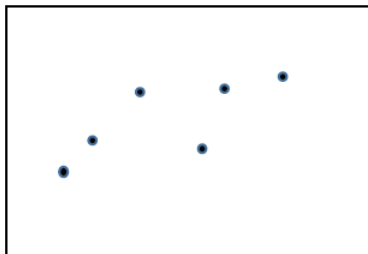
	2	9	14	23	33	42
	-60.2	-130.1	-243.7	-328.9	-580.5	-643.8

38 Calculate the linear regression equation for the data from problem 34 :

	8	12	14	23	24	29	33
	10	20	15	24	35	30	28

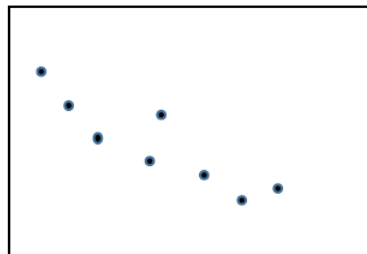
39. Consider the following 4 scatterplots. Match each scatterplot with the most appropriate value from the 4 correlations listed below.

a.



$r =$ _____

b.



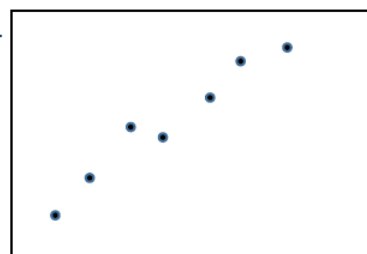
$r =$ _____

c.



$r =$ _____

d.



$r =$ _____

$r = 0.38$ $r = 0.94$ $r = -0.15$ $r = -0.85$

40. Find and interpret the linear correlation for the data from problem 32:

x	1	3	4	6	7	9	10
y	437	901	1155	1358	1768	2103	2437

41. Find and interpret the linear correlation for the data from problem 33:

x	2	9	14	23	33	42
y	-60.2	-130.1	-243.7	-328.9	-580.5	-643.8

42. Find and interpret the linear correlation for the data from problem 34:

x	8	12	14	23	24	29	33
y	10	20	15	24	35	30	28

43. Find and interpret the linear correlation for the data from problem 35:

x	20	24	27	32	48	60	68
y	12	20	28	41	24	15	11

44. The following table shows the number of newspaper subscriptions in Middletown, USA where t represents the number of years since 2002 ($t = 0$ in 2002) and $S(t)$ represents the total subscriptions each year measured in thousands.

t (year)	0	2	4	6	8
S(t) (total subscriptions in 1000s)	448	372	198	145	45

Find and interpret the linear correlation.

45. Scott is hiking the Appalachian Trail from Georgia to Maine. The distance of his hike is 2200 miles. It took Scott 123 days to complete the hike. The data below represent the distance, D , he had hiked t days after the start of his trip.

t (days hiking)	0	32	47	73	99	123
D(t) distance in miles	0	590	912	1212	1876	2200

Find and interpret the linear correlation.

46. Which implies a stronger linear relationship, a correlation of -0.84 or -0.47? Explain.

47. Which implies a stronger linear relationship, a correlation of -0.73 or 0.51? Explain.

48. The following table shows the number of newspaper subscriptions in Middletown, USA where t represents the number of years since 2002 ($t = 0$ in 2002) and $S(t)$ represents the total subscriptions each year measured in thousands.

t (year)	0	2	4	6	8
S(t) (total subscriptions in 1000s)	448	372	198	145	45

a) Use your graphing calculator to create of scatterplot of the data set.

b) Based on your graph above, do the data appear to be approximately linear?

- c) Use your calculator to determine the regression equation in $S(t) = at + b$ form. (Round to 2 decimal places)
- d) What is the slope of your regression model for $S(t)$ and what is its meaning in the context of this problem?
- e) What is the initial value of your linear regression model for $S(t)$ and what is its meaning in the context of the problem.
- f) Use your linear regression equation to estimate the total number of subscriptions in 2007 (i.e. when $t = 5$). Show your computations here and your final result.
- g) Use your linear regression equation to estimate the total number of subscriptions in 2004. How does this value compare to the data value in the table?
- h) Should you use your linear regression equation to estimate the circulation in the year 2030? Why or why not?

49. Scott is hiking the Appalachian Trail from Georgia to Maine. The distance of his hike is 2200 miles. It took Scott 123 days to complete the hike. The data below represent the distance, D , he had hiked t days after the start of his trip.

t (days hiking)	0	32	47	73	99	123
D(t) distance in miles	0	590	912	1212	1876	2200

- a) Use your graphing calculator to create of scatterplot of the data set.
- b) Based on your graph above, do the data appear to be approximately linear?
- c) Use your calculator to determine the regression equation in $D(t) = at + b$ form. (Round to 2 decimal places)
- d) What is the slope of your regression model for $D(t)$ and what is its meaning in the context of this problem?
- e) Use your linear regression equation to estimate the total number of miles Scott has hiked in 50 days. Show your computations here and your final result.
- f) Should you use your linear regression equation to estimate when Scott has hiked 2500 miles? Why or why not?

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CHAPTER OVERVIEW

3: Quadratic Functions

[3.1: Graphs of Quadratic Functions](#)

[3.2: Quadratic Equations by Graphing](#)

[3.3: Linear, Exponential and Quadratic Models](#)

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3.1: Graphs of Quadratic Functions

Learning Objectives

- Graph quadratic functions.
- Compare graphs of quadratic functions.
- Graph quadratic functions in intercept form.
- Analyze graphs of real-world quadratic functions.

Introduction

The graphs of quadratic functions are curved lines called **parabolas**. You don't have to look hard to find parabolic shapes around you. Here are a few examples:

- The path that a ball or a rocket takes through the air.
- Water flowing out of a drinking fountain.
- The shape of a satellite dish.
- The shape of the mirror in car headlights or a flashlight.
- The cobbles in a suspension bridge.

Graphing Quadratic Functions

Let's see what a parabola looks like by graphing the simplest quadratic function, $y = x^2$.

We'll graph this function by making a table of values. Since the graph will be curved, we need to plot a fair number of points to make it accurate.

1.1. Graphs of Quadratic Functions

x	$y = x^2$
-3	$(-3)^2 = 9$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$
3	$(3)^2 = 9$

Here are the points plotted on a coordinate graph:

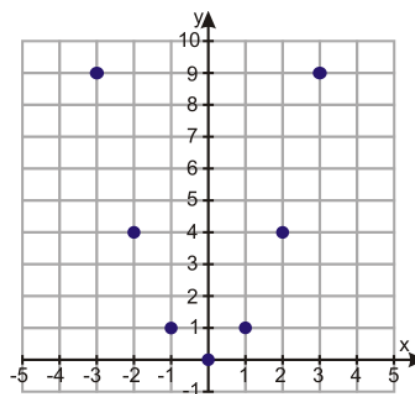


Figure 3.1.1

To draw the parabola, draw a smooth curve through all the points. (Do not connect the points with straight lines).

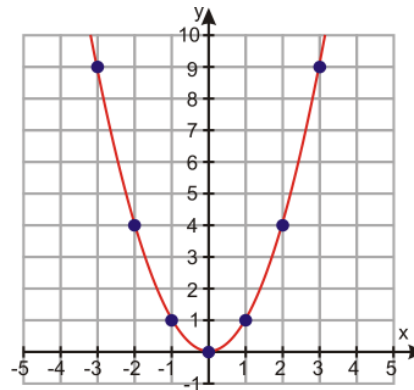


Figure 3.1.2

Let's graph a few more examples.

✓ Example 3.1.1

Graph the following parabolas.

- $y = 2x^2 + 4x + 1$
- $y = -x^2 + 3$
- $y = x^2 - 8x + 3$

Solution

a) $y = 2x^2 + 4x + 1$

Make a table of values:

x	$y = 2x^2 + 4x + 1$
-3	$2(-3)^2 + 4(-3) + 1 = 7$
-2	$2(-2)^2 + 4(-2) + 1 = 1$
-1	$2(-1)^2 + 4(-1) + 1 = -1$
0	$2(0)^2 + 4(0) + 1 = 1$
1	$2(1)^2 + 4(1) + 1 = 7$
2	$2(2)^2 + 4(2) + 1 = 17$
3	$2(3)^2 + 4(3) + 1 = 31$

Notice that the last two points have very large y - values. Since we don't want to make our y - scale too big, we'll just skip graphing those two points. But we'll plot the remaining points and join them with a smooth curve.

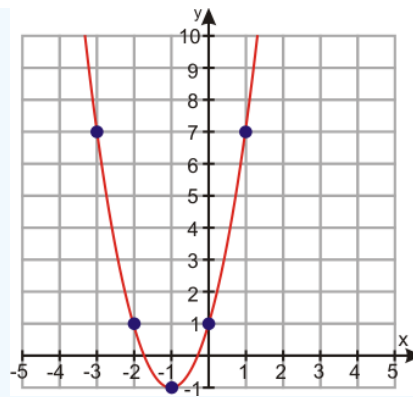


Figure 3.1.3

b) $y = -x^2 + 3$

Make a table of values:

x	$y = -x^2 + 3$
-3	$-(-3)^2 + 3 = -6$
-2	$-(-2)^2 + 3 = -1$
-1	$-(-1)^2 + 3 = 2$
0	$-(0)^2 + 3 = 3$
1	$-(1)^2 + 3 = 2$
2	$-(2)^2 + 3 = -1$
3	$-(3)^2 + 3 = -6$

Plot the points and join them with a smooth curve.

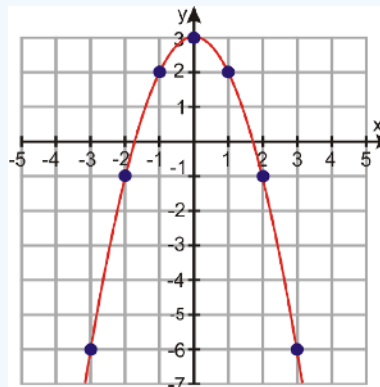


Figure 3.1.4

Notice that this time we get an “upside down” parabola. That’s because our equation has a negative sign in front of the x^2 term. The sign of the coefficient of the x^2 term determines whether the parabola turns up or down: the parabola turns up if it’s positive and down if it’s negative.

c) $y = x^2 - 8x + 3$

Make a table of values:

x	$y = x^2 - 8x + 3$
-3	$(-3)^2 - 8(-3) + 3 = 36$

-2	$(-2)^2 - 8(-2) + 3 = 23$
-1	$(-1)^2 - 8(-1) + 3 = 12$
0	$(0)^2 - 8(0) + 3 = 3$
1	$(1)^2 - 8(1) + 3 = -4$
2	$(2)^2 - 8(2) + 3 = -9$
3	$(3)^2 - 8(3) + 3 = -12$

Let's not graph the first two points in the table since the values are so big. Plot the remaining points and join them with a smooth curve.

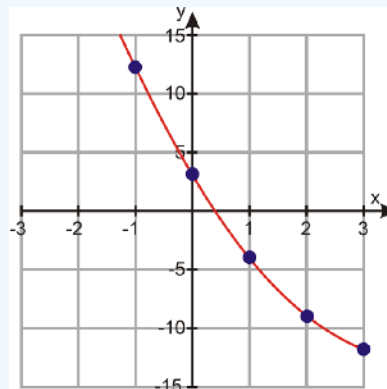


Figure 3.1.5

Wait—this doesn't look like a parabola. What's going on here?

Maybe if we graph more points, the curve will look more familiar. For negative values of x it looks like the values of y are just getting bigger and bigger, so let's pick more positive values of x beyond $x = 3$.

x	$y = x^2 - 8x + 3$
-1	$(-1)^2 - 8(-1) + 3 = 12$
0	$(0)^2 - 8(0) + 3 = 3$
1	$(1)^2 - 8(1) + 3 = -4$
2	$(2)^2 - 8(2) + 3 = -9$
3	$(3)^2 - 8(3) + 3 = -12$
4	$(4)^2 - 8(4) + 3 = -13$
5	$(5)^2 - 8(5) + 3 = -12$
6	$(6)^2 - 8(6) + 3 = -9$
7	$(7)^2 - 8(7) + 3 = -4$
8	$(8)^2 - 8(8) + 3 = 3$

Plot the points again and join them with a smooth curve.

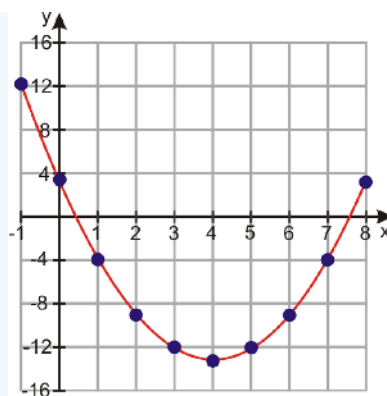


Figure 3.1.6

Now we can see the familiar parabolic shape. And now we can see the drawback to graphing quadratics by making a table of values—if we don’t pick the right values, we won’t get to see the important parts of the graph.

In the next couple of lessons, we’ll find out how to graph quadratic equations more efficiently—but first we need to learn more about the properties of parabolas.

Compare Graphs of Quadratic Functions

The **general form** (or **standard form**) of a quadratic function is:

$$y = ax^2 + bx + c$$

Here a , b and c are the **coefficients**. Remember, a coefficient is just a number (a constant term) that can go before a variable or appear alone.

Although the graph of a quadratic equation in standard form is always a parabola, the shape of the parabola depends on the values of the coefficients a , b and c . Let’s explore some of the ways the coefficients can affect the graph

Dilation

Changing the value of a makes the graph “fatter” or “skinnier”. Let’s look at how graphs compare for different positive values of a . The plot on the left shows the graphs of

$y = x^2$ and $y = 3x^2$. The plot on the right shows the graphs of $y = x^2$ and $\frac{1}{3}x^2$

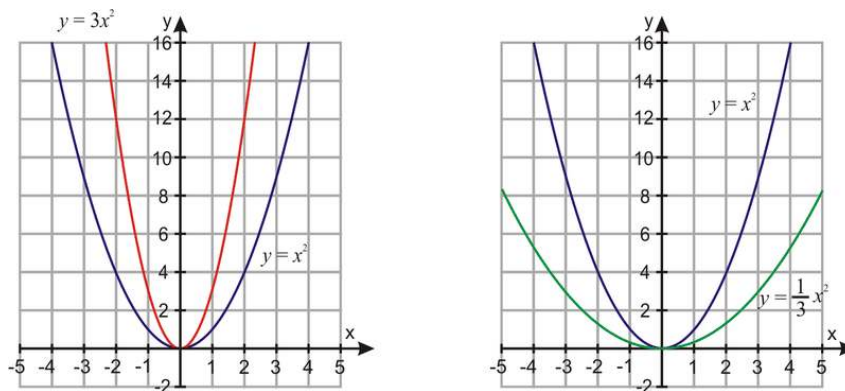


Figure 3.1.7

Notice that the larger the value of a is, the skinnier the graph is – for example, in the first plot, the graph of $y = 3x^2$ is skinnier than the graph of $y = x^2$. Also, the smaller a is, the fatter the graph is – for example, in the second plot, the graph of $y = \frac{1}{3}x^2$ is fatter than the graph of $y = x^2$. This might seem counterintuitive, but if you think about it, it should make sense. Let’s look at a table of values of these graphs and see if we can explain why this happens.

x	$y = x^2$	$y = 3x^2$	$y = \frac{1}{3}x^2$
-5	25	75	8.33
-4	16	48	5.33
-3	9	27	3
-2	4	12	1.33
-1	1	3	0.33
0	0	0	0
1	1	3	0.33
2	4	12	1.33
3	9	27	3
4	16	48	5.33
5	25	75	8.33

-3	$(-3)^2 = 9$	$3(-3)^2 = 27$	$\frac{-3^2}{3} = 3$
-2	$(-2)^2 = 4$	$3(-2)^2 = 12$	$\frac{-2^2}{3} = \frac{4}{3}$
-1	$(-1)^2 = 1$	$3(-1)^2 = 3$	$\frac{-1^2}{3} = \frac{1}{3}$
0	$(0)^2 = 0$	$3(0)^2 = 0$	$\frac{-0^2}{3} = 0$
1	$(1)^2 = 1$	$3(1)^2 = 3$	$\frac{1^2}{3} = \frac{1}{3}$
2	$(2)^2 = 4$	$3(2)^2 = 12$	$\frac{2^2}{3} = \frac{4}{3}$
3	$(3)^2 = 9$	$3(3)^2 = 27$	$\frac{3^2}{3} = 3$

From the table, you can see that the values of $y = 3x^2$ are bigger than the values of $y = x^2$. This is because each value of y gets multiplied by 3. As a result the parabola will be skinnier because it grows three times faster than $y = x^2$. On the other hand, you can see that the values of $y = \frac{1}{3}x^2$ are smaller than the values of $y = x^2$, because each value of y gets divided by 3. As a result the parabola will be fatter because it grows at one third the rate of $y = x^2$.

Orientation

As the value of a gets smaller and smaller, then, the parabola gets wider and flatter. What happens when a gets all the way down to zero? What happens when it's negative?

Well, when $a=0$, the x^2 term drops out of the equation entirely, so the equation becomes linear and the graph is just a straight line. For example, we just saw what happens to $y = ax^2$ when we change the value of a ; if we tried to graph $y = 0x^2$, we would just be graphing $y=0$, which would be a horizontal line.

So as a gets smaller and smaller, the graph of $y = ax^2$ gets flattened all the way out into a horizontal line. Then, when a becomes negative, the graph of $y = ax^2$ starts to curve again, only it curves downward instead of upward. This fits with what you've already learned: the graph opens upward if a is positive and downward if a is negative.

For example, here are the graphs of $y = ax^2$ and $y = -ax^2$. You can see that the parabola has the same shape in both graphs, but the graph of $y = ax^2$ is right-side-up and the graph of $y = -ax^2$ is upside-down.

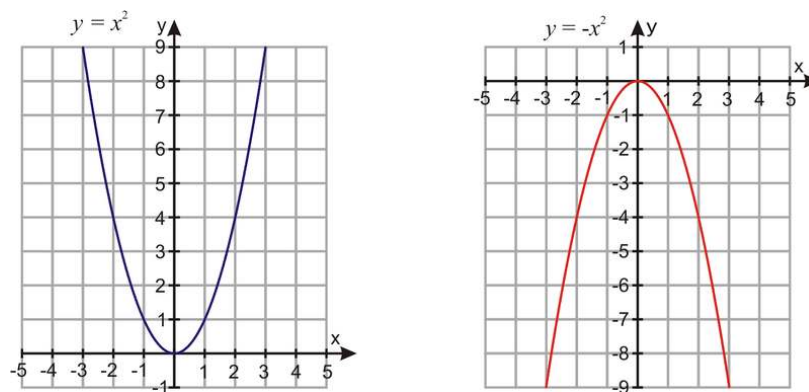


Figure 3.1.8

Vertical Shift

Changing the constant c just shifts the parabola up or down. The following plot shows the graphs of $y = x^2$, $y = x^2 + 1$, $y = x^2 - 1$, $y = x^2 + 2$, and $y = x^2 - 2$.

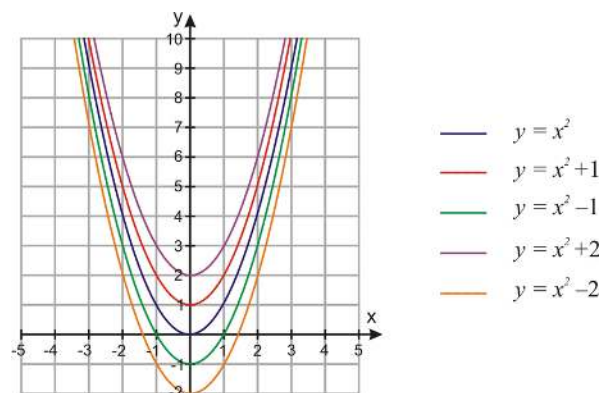


Figure 3.1.9

You can see that when c is positive, the graph shifts up, and when c is negative the graph shifts down; in either case, it shifts by $|c|$ units. In one of the later sections we'll learn about **horizontal shift** (i.e. moving to the right or to the left). Before we can do that, though, we need to learn how to rewrite quadratic equations in different forms.

Meanwhile, if you want to explore further what happens when you change the coefficients of a quadratic equation, the page at <http://www.analyzemath.com/quadraticg/quadraticg.htm> has an applet you can use. Click on the "Click here to start" button in section A, and then use the sliders to change the values of a , b , and c .

Graph Quadratic Functions in Intercept Form

Now it's time to learn how to graph a parabola without having to use a table with a large number of points.

Let's look at the graph of $y = x^2 - 6x + 8$.

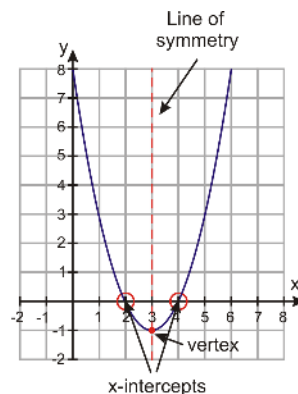


Figure 3.1.10

There are several things we can notice:

- The parabola crosses the x -axis at two points: $x=2$ and $x=4$. These points are called the x - **intercepts** of the parabola.
- The lowest point of the parabola occurs at $(3, -1)$.
 - This point is called the **vertex** of the parabola.
 - The vertex is the lowest point in any parabola that turns upward, or the highest point in any parabola that turns downward.
 - The vertex is **exactly halfway between the two x -intercepts**. This will always be the case, and you can find the vertex using that property.
- The parabola is **symmetric**. If you draw a vertical line through the vertex, you see that the two halves of the parabola are mirror images of each other. This vertical line is called the **line of symmetry**.

We said that the general form of a quadratic function is $y=ax^2+bx+c$. When we can factor a quadratic expression, we can rewrite the function in **intercept form**:

$$y = a(x - m)(x - n)$$

This form is very useful because it makes it easy for us to find the x -intercepts and the vertex of the parabola. The x -intercepts are the values of x where the graph crosses the x -axis; in other words, they are the values of x when $y=0$. To find the x -intercepts from

the quadratic function, we set $y=0$ and solve:

$$0 = a(x - m)(x - n) \quad (3.1.1)$$

Since the equation is already factored, we use the zero-product property to set each factor equal to zero and solve the individual linear equations:

$$\begin{array}{rcl} x - m = 0 & x - n & = 0 \\ x \text{ or } & x & = n \end{array}$$

So the x -intercepts are at points $(m, 0)$ and $(n, 0)$.

Once we find the x -intercepts, it's simple to find the vertex. The x value of the vertex is halfway between the two x -intercepts, so we can find it by taking the average of the two values: $\frac{m+n}{2}$. Then we can find the y -value by plugging the value of x back into the equation of the function.

✓ Example 3.1.2

Find the x -intercepts and the vertex of the following quadratic functions:

a) $y = x^2 - 8x + 15$

b) $y = 3x^2 + 6x - 24$

Solution

a) $y = x^2 - 8x + 15$

Write the quadratic function in intercept form by factoring the right hand side of the equation. Remember, to factor we need two numbers whose product is 15 and whose sum is -8. These numbers are -5 and -3.

The function in intercept form is $y = (x - 5)(x - 3)$

We find the x -intercepts by setting $y = 0$.

We have:

$$\begin{array}{rcl} 0 = (x - 5)(x - 3) & & \\ x - 5 = 0 & \text{or} & x - 3 = 0 \\ x = 5 & & x = 3 \end{array} \quad (3.1.2)$$

So the x -intercepts are $(5, 0)$ and $(3, 0)$.

The vertex is halfway between the two x -intercepts. We find the x -value by taking the average of the two x -intercepts:

$$x = \frac{5+3}{2} = 4$$

We find the y -value by plugging the x -value we just found into the original equation:

$$y = x^2 - 8x + 15 \Rightarrow y = 4^2 - 8(4) + 15 = 16 - 32 + 15 = -1 \quad (3.1.3)$$

So the vertex is $(4, -1)$.

b) $y = 3x^2 + 6x - 24$

Re-write the function in intercept form.

Factor the common term of 3 first: $y = 3(x^2 + 2x - 8)$

Then factor completely: $y = 3(x + 4)(x - 2)$

Set $y = 0$ and solve:

$$\begin{aligned}
 x + 4 &= 0 \\
 x - 2 &= 0 \\
 0 &= 3(x + 4)(x - 2) \Rightarrow \\
 x &= -4 \\
 x &= 2
 \end{aligned}
 \tag{3.1.4}$$

The x -intercepts are $(-4, 0)$ and $(2, 0)$.

For the vertex,

$$\begin{aligned}
 x &= \frac{-4+2}{2} = -1 \text{ and} \\
 y &= 3(-1)^2 + 6(-1) - 24 = 3 - 6 - 24 = -27
 \end{aligned}
 \tag{3.1.5}$$

The vertex is: $(-1, -27)$

Knowing the vertex and x -intercepts is a useful first step toward being able to graph quadratic functions more easily. Knowing the vertex tells us where the middle of the parabola is. When making a table of values, we can make sure to pick the vertex as a point in the table. Then we choose just a few smaller and larger values of x . In this way, we get an accurate graph of the quadratic function without having to have too many points in our table.

✓ Example 3.1.3

Find the x -intercepts and vertex. Use these points to create a table of values and graph each function.

- a) $y = x^2 - 4$
b) $y = -x^2 + 14x - 48$

Solution

a) $y = x^2 - 4$

Let's find the x -intercepts and the vertex:

Factor the right-hand side of the function to put the equation in intercept form:

$$y = (x - 2)(x + 2) \tag{3.1.6}$$

Set $y = 0$ and solve:

$$\begin{aligned}
 0 &= (x - 2)(x + 2) \\
 x - 2 &= 0 & \text{or} & & x + 2 = 0 \\
 x &= 2 & & & x &= -2
 \end{aligned}
 \tag{3.1.7}$$

The x -intercepts are $(2, 0)$ and $(-2, 0)$.

Find the vertex:

$$x = \frac{2-2}{2} = 0 \quad y = (0)^2 - 4 = -4 \tag{3.1.8}$$

The vertex is $(0, -4)$.

Make a table of values using the vertex as the middle point. Pick a few values of x smaller and larger than $x = 0$. Include the x -intercepts in the table.

--

<https://math.libretexts.org/@go/page/141501>

x	$y = -x^2 + 14x - 48$
4	$y = -(4)^2 + 14(4) - 48 = -8$
5	$y = -(5)^2 + 14(5) - 48 = -3$
6	$y = -(6)^2 + 14(6) - 48 = 0$
7	$y = -(7)^2 + 14(7) - 48 = 1$
8	$y = -(8)^2 + 14(8) - 48 = 0$
9	$y = -(9)^2 + 14(9) - 48 = -3$
10	$y = -(10)^2 + 14(10) - 48 = -8$

Then plot the graph:

Figure 3.1.12

Analyze Graphs of Real-World Quadratic Functions.

As we mentioned at the beginning of this section, parabolic curves are common in real-world applications. Here we will look at a few graphs that represent some examples of real-life application of quadratic functions.

✓ Example 3.1.4

Andrew has 100 feet of fence to enclose a rectangular tomato patch. What should the dimensions of the rectangle be in order for the rectangle to have the greatest possible area?

Solution

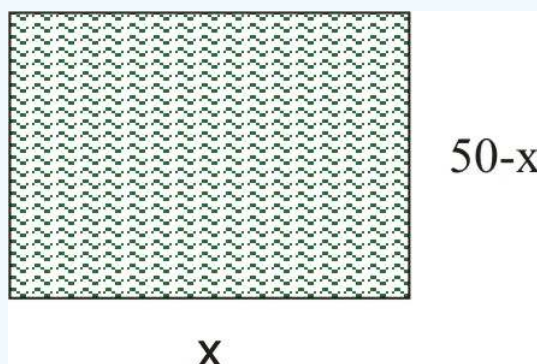


Figure 3.1.13

If the length of the rectangle is x , then the width is $50 - x$. (The length and the width add up to 50, not 100, because two lengths and two widths together add up to 100.)

If we let y be the area of the triangle, then we know that the area is length $\times$ width, so $y = x(50 - x) = 50x - x^2$.

Here's the graph of that function, so we can see how the area of the rectangle depends on the length of the rectangle:

Figure 3.1.14

We can see from the graph that the highest value of the area occurs when the length of the rectangle is 25. The area of the rectangle for this side length equals 625. (Notice that the width is also 25, which makes the shape a square with side length 25.)

This is an example of an optimization problem. These problems show up often in the real world, and if you ever study calculus, you'll learn how to solve them without graphs.

✓ Example 3.1.5

Anne is playing golf. On the 4th tee, she hits a slow shot down the level fairway. The ball follows a parabolic path described by the equation $y = x - 0.04x^2$, where y is the ball's height in the air and x is the horizontal distance it has traveled from the tee. The distances are measured in feet. How far from the tee does the ball hit the ground? At what distance from the tee does the ball attain its maximum height? What is the maximum height?

Solution

Let's graph the equation of the path of the ball:

Figure
3.1.15

$x(1 - 0.04x) = 0$ has solutions $x = 0$ and $x = 25$.

From the graph, we see that the ball hits the ground **25** feet from the tee. (The other x -intercept, $x = 0$, tells us that the ball was also on the ground when it was on the tee!)

We can also see that the ball reaches its maximum height of about 6.25 feet when it is **12.5** feet from the tee.

Review Questions

Rewrite the following functions in intercept form. Find the x -intercepts and the vertex.

1. $y = x^2 - 2x - 8$
2. $y = -x^2 + 10x - 21$
3. $y = 2x^2 + 6x + 4$
4. $y = 3(x + 5)(x - 2)$

Does the graph of the parabola turn up or down?

5. $y = -2x^2 - 2x - 3$
6. $y = 3x^2$
7. $y = 16 - 4x^2$
8. $y = 3x^2 - 2x - 4x^2 + 3$

The vertex of which parabola is higher?

9. $y = x^2 + 4$ or $y = x^2 + 1$
10. $y = -2x^2$ or $y = -2x^2 - 2$
11. $y = 3x^2 - 3$ or $y = 3x^2 - 6$
12. $y = 5 - 2x^2$ or $y = 8 - 2x^2$

Which parabola is wider?

13. $y = x^2$ or $y = 4x^2$
14. $y = 2x^2 + 4$ or $y = \frac{1}{2}x^2 + 4$
15. $y = -2x^2 - 2$ or $y = -x^2 - 2$
16. $y = x^2 + 3x^2$ or $y = x^2 + 3$

Graph the following functions by making a table of values. Use the vertex and x -intercepts to help you pick values for the table.

17. $y = 4x^2 - 4$
18. $y = -x^2 + x + 12$
19. $y = 2x^2 + 10x + 8$
20. $y = \frac{1}{2}x^2 - 2x$
21. $y = x - 2x^2$
22. $y = 4x^2 - 8x + 4$

23. Nadia is throwing a ball to Peter. Peter does not catch the ball and it hits the ground. The graph shows the path of the ball as it flies through the air. The equation that describes the path of the ball is $y = 4 + 2x - 0.16x^2$. Here y is the height of the ball and x is the horizontal distance from Nadia. Both distances are measured in feet.

a. How far from Nadia does the ball hit the ground?

- b. At what distance x from Nadia, does the ball attain its maximum height?
- c. What is the maximum height?

24. Jasreel wants to enclose a vegetable patch with 120 feet of fencing. He wants to put the vegetable against an existing wall, so he only needs fence for three of the sides. The equation for the area is given by $A = 120x - x^2$. From the graph, find what dimensions of the rectangle would give him the greatest area.

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3.2: Quadratic Equations by Graphing

Learning Objectives

- Identify the number of solutions of a quadratic equation.
- Solve quadratic equations by graphing.
- Analyze quadratic functions using a graphing calculator.
- Solve real-world problems by graphing quadratic functions.

Introduction

Solving a quadratic equation means finding the x -values that will make the quadratic function equal zero; in other words, it means finding the points where the graph of the function crosses the x -axis. The solutions to a quadratic equation are also called the roots or zeros of the function, and in this section we'll learn how to find them by graphing the function.

Identify the Number of Solutions of a Quadratic Equation

Three different situations can occur when graphing a quadratic function:

Case 1: The parabola crosses the x -axis at two points. An example of this is $y = x^2 + x - 6$:

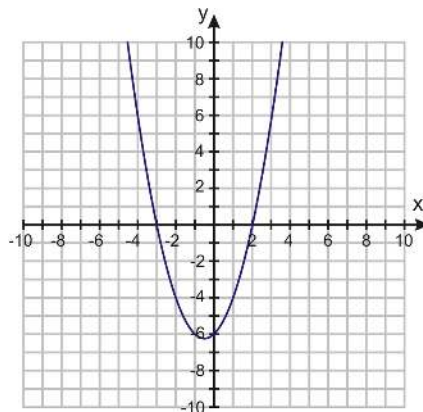


Figure 3.2.1

Looking at the graph, we see that the parabola crosses the x -axis at $x = -3$ and $x = 2$.

We can also find the solutions to the equation $x^2 + x - 6 = 0$ by setting $y = 0$. We solve the equation by factoring:

$$(x + 3)(x - 2) = 0, \text{ so } x = -3 \text{ or } x = 2. \quad (3.2.1)$$

When the graph of a quadratic function crosses the x -axis at two points, we get two distinct solutions to the quadratic equation.

Case 2: The parabola touches the x -axis at one point. An example of this is $y = x^2 - 2x + 1$:

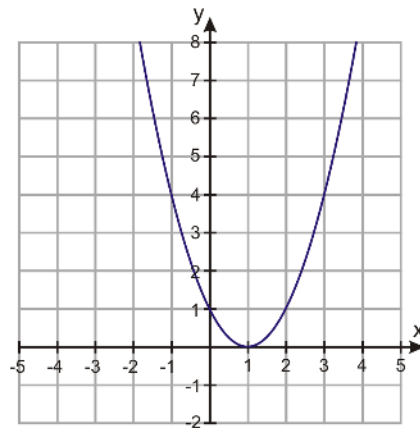


Figure 3.2.2

We can see that the graph touches the x -axis at $x = 1$.

We can also solve this equation by factoring. If we set $y = 0$ and factor, we obtain $(x - 1)^2 = 0$, so $x = 1$. Since the quadratic function is a perfect square, we get only one solution for the equation—it's just the same solution repeated twice over.

When the graph of a quadratic function touches the x -axis at one point, the quadratic equation has one solution and the solution is called a double root.

Case 3: The parabola does not cross or touch the x -axis. An example of this is $y = x^2 + 4$:

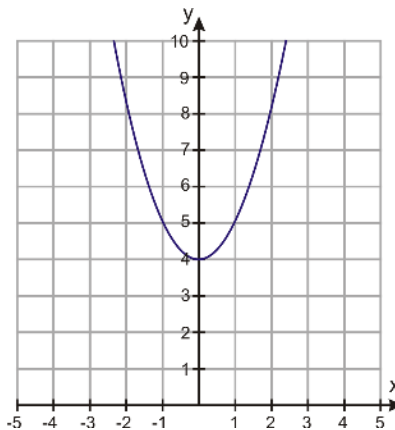


Figure 3.2.3

If we set $y = 0$ we get $x^2 + 4 = 0$. This quadratic polynomial does not factor.

When the graph of a quadratic function does not cross or touch the x -axis, the quadratic equation has no real solutions.

Solve Quadratic Equations by Graphing

So far we've found the solutions to quadratic equations using factoring. However, in real life very few functions factor easily. As you just saw, graphing a function gives a lot of information about the solutions. We can find exact or approximate solutions to a quadratic equation by graphing the function associated with it.

✓ Example 3.2.1

Find the solutions to the following quadratic equations by graphing.

- a) $-x^2 + 3 = 0$
- b) $2x^2 + 5x - 7 = 0$
- c) $-x^2 + x - 3 = 0$
- d) $y = -x^2 + 4x - 4$

Solution

Since we can't factor any of these equations, we won't be able to graph them using intercept form (if we could, we wouldn't need to use the graphs to find the intercepts!) We'll just have to make a table of arbitrary values to graph each one.

a)

x	$y = -x^2 + 3$
-3	$y = -(-3)^2 + 3 = -6$
-2	$y = -(-2)^2 + 3 = -1$
-1	$y = -(-1)^2 + 3 = 2$
0	$y = -(0)^2 + 3 = 3$
1	$y = -(1)^2 + 3 = 2$
2	$y = -(2)^2 + 3 = -1$
3	$y = -(3)^2 + 3 = -6$

We plot the points and get the following graph:

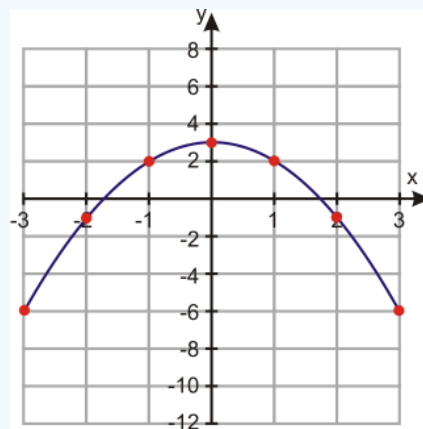


Figure 3.2.4

From the graph we can read that the x-intercepts are approximately $x=1.7$ and $x=-1.7$. These are the solutions to the equation.

b)

X	$y = 2x^2 + 5x - 7$
-5	$y = 2(-5)^2 + 5(-5) - 7 = 18$
-4	$y = 2(-4)^2 + 5(-4) - 7 = 5$
-3	$y = 2(-3)^2 + 5(-3) - 7 = -4$
-2	$y = 2(-2)^2 + 5(-2) - 7 = -9$
-1	$y = 2(-1)^2 + 5(-1) - 7 = -10$
0	$y = 2(0)^2 + 5(0) - 7 = -7$
1	$y = 2(1)^2 + 5(1) - 7 = 0$
2	$y = 2(2)^2 + 5(2) - 7 = 11$
3	$y = 2(3)^2 + 5(3) - 7 = 26$

We plot the points and get the following graph:

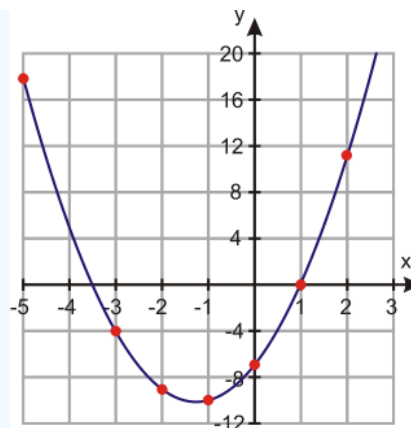


Figure 3.2.5

From the graph we can read that the x-intercepts are $x=1$ and $x=-3.5$. These are the solutions to the equation.

c)

X	$y = -x^2 + x - 3$
-3	$y = -(-3)^2 + (-3) - 3 = -15$
-2	$y = -(-2)^2 + (-2) - 3 = -9$
-1	$y = -(-1)^2 + (-1) - 3 = -5$
0	$y = -(0)^2 + (0) - 3 = -3$
1	$y = -(1)^2 + (1) - 3 = -3$
2	$y = -(2)^2 + (2) - 3 = -5$
3	$y = -(3)^2 + (3) - 3 = -9$

We plot the points and get the following graph:

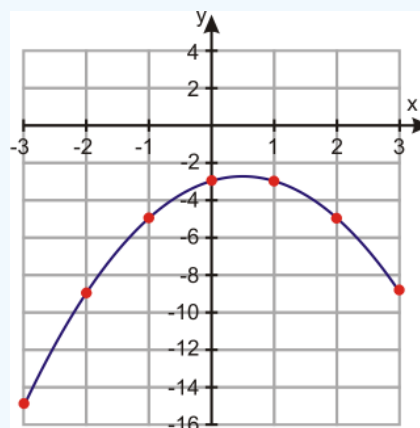


Figure 3.2.6

The graph curves up toward the x-axis and then back down without ever reaching it. This means that the graph never intercepts the x-axis, and so the corresponding equation has no real solutions.

d)

x	$y = -x^2 + 4x - 4$
-3	$y = -(-3)^2 + 4(-3) - 4 = -25$
-2	$y = -(-2)^2 + 4(-2) - 4 = -16$

-1	$y = -(-1)^2 + 4(-1) - 4 = -9$
0	$y = -(0)^2 + 4(0) - 4 = -4$
1	$y = -(1)^2 + 4(1) - 4 = -1$
2	$y = -(2)^2 + 4(2) - 4 = 0$
3	$y = -(3)^2 + 4(3) - 4 = -1$
4	$y = -(4)^2 + 4(4) - 4 = -4$
5	$y = -(5)^2 + 4(5) - 4 = -9$

Here is the graph of this function:

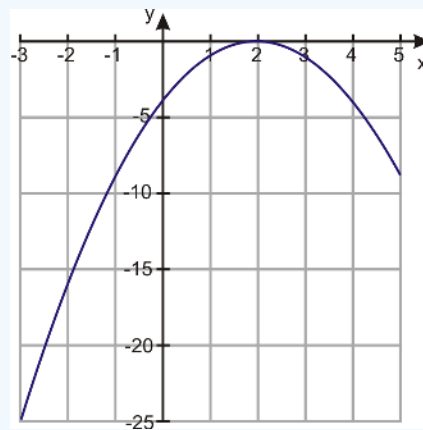


Figure 3.2.7

The graph just touches the x-axis at $x=2$, so the function has a double root there. $x=2$ is the only solution to the equation.

Analyze Quadratic Functions Using a Graphing Calculator

A graphing calculator is very useful for graphing quadratic functions. Once the function is graphed, we can use the calculator to find important information such as the roots or the vertex of the function.

✓ Example 3.2.2

Use a graphing calculator to analyze the graph of $y = x^2 - 20x + 35$.

Solution

1. Graph the function.

Press the [Y=] button and enter " $x^2 - 20x + 35$ " next to $[Y_1 =]$. Press the [GRAPH] button. This is the plot you should see:

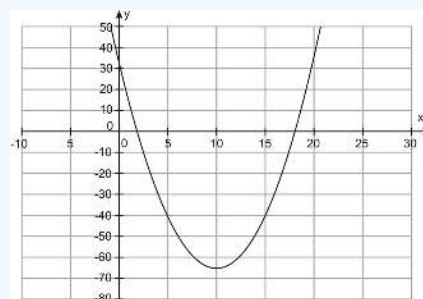


Figure 3.2.8

[Figure 8]

If this is not what you see, press the [WINDOW] button to change the window size. For the graph shown here, the x-values should range from -10 to 30 and the y-values from -80 to 50 .

2. Find the roots.

There are at least three ways to find the roots:

Use [TRACE] to scroll over the x-intercepts. The approximate value of the roots will be shown on the screen. You can improve your estimate by zooming in.

OR

Use [TABLE] and scroll through the values until you find values of y equal to zero. You can change the accuracy of the solution by setting the step size with the [TBLSET] function.

OR

Use [2nd] [TRACE] (i.e. 'calc' button) and use option 'zero'.

Move the cursor to the left of one of the roots and press [ENTER]

Move the cursor to the right of the same root and press [ENTER].

Move the cursor close to the root and press [ENTER].

The screen will show the value of the root. Repeat the procedure for the other root.

Whichever technique you use, you should get about $x=1.9$ and $x=18$ for the two roots.

3. Find the **vertex**.

There are three ways to find the vertex:

Use [TRACE] to scroll over the highest or lowest point on the graph. The approximate value of the roots will be shown on the screen.

OR

Use [TABLE] and scroll through the values until you find values the lowest or highest value of y. You can change the accuracy of the solution by setting the step size with the [TBLSET] function.

OR

Use [2nd] [TRACE] and use the option 'maximum' if the vertex is a maximum or 'minimum' if the vertex is a minimum.

Move the cursor to the left of the vertex and press [ENTER].

Move the cursor to the right of the vertex and press [ENTER].

Move the cursor close to the vertex and press [ENTER].

The screen will show the x - and y-values of the vertex.

Whichever method you use, you should find that the vertex is at **(10, -65)**.

Solve Real-World Problems by Graphing Quadratic Functions

Here's a real-world problem we can solve using the graphing methods we've learned.

✓ Example 3.2.3

Andrew is an avid archer. He launches an arrow that takes a parabolic path. The equation of the height of the ball with respect to time is $y = -4.9t^2 + 48t$, where y is the height of the arrow in meters and t is the time in seconds since Andrew shot the arrow. Find how long it takes the arrow to come back to the ground.

Solution

Let's graph the equation by making a table of values.

$$y = -4.9t^2 + 48t$$

x	$y = -4.9t^2 + 48t$
0	$y = -4.9(0)^2 + 48(0) = 0$
1	$y = -4.9(1)^2 + 48(1) = 43.1$
2	$y = -4.9(2)^2 + 48(2) = 76.4$
3	$y = -4.9(3)^2 + 48(3) = 99.9$
4	$y = -4.9(4)^2 + 48(4) = 113.6$
5	$y = -4.9(5)^2 + 48(5) = 117.6$
6	$y = -4.9(6)^2 + 48(6) = 111.6$
7	$y = -4.9(7)^2 + 48(7) = 95.9$
8	$y = -4.9(8)^2 + 48(8) = 70.4$
9	$y = -4.9(9)^2 + 48(9) = 35.1$
10	$y = -4.9(10)^2 + 48(10) = -10$

Here's the graph of the function:

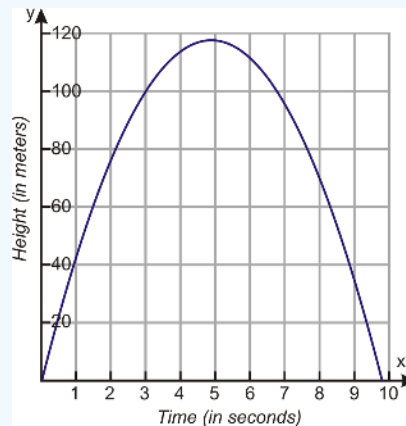


Figure 3.2.9

The roots of the function are approximately $x=0$ sec and $x=9.8$ sec. The first root tells us that the height of the arrow was 0 meters when Andrew first shot it. The second root says that it takes approximately **9.8 seconds** for the arrow to return to the ground.

Further Practice

Now that you've learned how to solve quadratic equations by graphing them, you can sharpen your skills even more by learning how to find an equation from the graph alone. Go to the page linked in the previous section, <http://www.anlyzemath.com/quadratic/quadratic.htm> (<http://www.anlyzemath.com/quadratic/quadratic.htm>), and scroll down to section E. Read the example there to learn how to find the equation of a quadratic function by reading off a few key values from the graph; then click the "Click here to start" button to try a problem yourself. The "New graph" button will give you a new problem when you finish the first one.

Review Questions

Find the solutions of the following equations by graphing.

- $x^2 + 3x + 6 = 0$
- $-2x^2 + x + 4 = 0$
- $x^2 - 9 = 0$
- $x^2 + 6x + 9 = 0$

5. $10x - 3x^2 = 0$
6. $\frac{1}{2}x^2 - 2x + 3 = 0$

Find the roots of the following quadratic functions by graphing.

7. $y = -3x^2 + 4x - 1$
8. $y = 9 - 4x^2$
9. $y = x^2 + 7x + 2$
10. $y = -x^2 - 10x - 25$
11. $y = 2x^2 - 3x$
12. $y = x^2 - 2x + 5$

Using your graphing calculator, find the roots and the vertex of each polynomial.

13. $y = x^2 + 12x + 5$
14. $y = x^2 + 3x + 6$
15. $y = -x^2 - 3x + 9$
16. $y = -x^2 + 4x - 12$
17. $y = 2x^2 - 4x + 8$
18. $y = -5x^2 - 3x + 2$

19. Graph the equations $y = 2x^2 - 4x + 8$ and $y = x^2 - 2x + 4$ on the same screen. Find their roots and vertices.

- a. What is the same about the graphs? What is different?
b. How are the two equations related to each other? (Hint: factor them.)
c. What might be another equation with the same roots? Graph it and see.

20. Graph the equations $y = x^2 - 2x + 2$ and $y = x^2 - 2x + 4$ on the same screen. Find their roots and vertices.

- a. What is the same about the graphs? What is different?
b. How are the two equations related to each other?

21. Phillip throws a ball and it takes a parabolic path. The equation of the height of the ball with respect to time is $y = -16t^2 + 60t$, where y is the height in feet and t is the time in seconds. Find how long it takes the ball to come back to the ground.

22. Use your graphing calculator to solve Ex. 3. You should get the same answers as we did graphing by hand, but a lot quicker!

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3.3: Linear, Exponential and Quadratic Models

Learning Objectives

- Identify functions using differences and ratios.
- Write equations for functions.
- Perform exponential and quadratic regressions with a graphing calculator.
- Solve real-world problems by comparing function models.

Introduction

In this course we've learned about three types of functions, linear, quadratic and exponential.

- Linear functions take the form $y=mx+b$.
- Quadratic functions take the form $y=ax^2+bx+c$.
- Exponential functions take the form $y=a \cdot bx$.

In real-world applications, the function that describes some physical situation is not given; it has to be found before the problem can be solved. For example, scientific data such as observations of planetary motion are often collected as a set of measurements given in a table. Part of the scientist's job is to figure out which function best fits the data. In this section, you'll learn some methods that are used to identify which function describes the relationship between the variables in a problem.

Identify Functions Using Differences or Ratios

One method for identifying functions is to look at the difference or the ratio of different values of the dependent variable. For example, **if the difference between values of the dependent variable is the same each time we change the independent variable by the same amount, then the function is linear.**

✓ Example 3.3.1

Determine if the function represented by the following table of values is linear.

x	y
-2	-4
-1	-1
0	2
1	5
2	8

Solution

If we take the difference between consecutive y-values, we see that each time the x-value increases by one, the y-value always increases by 3.

Since the difference is always the same, **the function is linear.**

When we look at the difference of the y-values, we have to make sure that we examine entries for which the x-values increase by the same amount.

For example, examine the values in this table:

x	y
0	5
1	10

x	y
3	20
4	25
6	35

At first glance this function might not look linear, because the difference in the y-values is not always the same. But if we look closer, we can see that when the y-value increases by 10 instead of 5, it's because the x-value increased by 2 instead of 1. Whenever the x-value increases by the *same* amount, the y-value does too, so the function is linear.

Another way to think of this is in mathematical notation. We can say that a function is linear if $\frac{y_2 - y_1}{x_2 - x_1}$ is always the same for any two pairs of x- and y-values. Notice that the expression we used here is simply the definition of the slope of a line.

Differences can also be used to identify quadratic functions. **For a quadratic function, when we increase the x-values by the same amount, the difference between y-values will not be the same. However, the difference of the differences of the y-values will be the same.**

Here are some examples of quadratic relationships represented by tables of values:

x	y = x ²	difference of y-values	difference of differences
0	0	1 - 0 = 1	$3 - 1 = 2$ $5 - 3 = 2$ $7 - 5 = 2$ $9 - 7 = 2$ $11 - 9 = 2$
1	1	4 - 1 = 3	
2	4	9 - 4 = 5	
3	9	16 - 9 = 7	
4	16	25 - 16 = 9	
5	25	36 - 25 = 11	
6	36		

In this quadratic function, $(y=x^2)$, when we increase the x-value by one, the value of y increases by different values. However, it increases at a constant rate, so the difference of the difference is always 2.

x	y = 2x ² - 3x + 1	difference of y-values	difference of differences
0	0	0 - 1 = -1	$3 + 1 = 4$ $7 - 3 = 4$ $11 - 7 = 4$ $15 - 11 = 4$ $19 - 15 = 4$
1	1	3 - 0 = 3	
2	3	10 - 3 = 7	
3	10	21 - 10 = 11	
4	21	36 - 21 = 15	
5	36	55 - 36 = 19	
6	55		

In this quadratic function, $(y=2x^2-3x+1)$, when we increase the x-value by one, the value of y increases by different values. However, the increase is constant: the difference of the difference is always 4.

To identify exponential functions, we use ratios instead of differences. **If the ratio between values of the dependent variable is the same each time we change the independent variable by the same amount, then the function is exponential.**

✓ Example 3.3.2

Determine if the function represented by each table of values is exponential.

x	y	ratio of y - values
0	4	$\frac{12}{4} = 3$
1	12	
2	36	$\frac{36}{12} = 3$
3	108	
4	324	$\frac{108}{36} = 3$

a)

x	y	ratio of y - values
0	240	$\frac{120}{240} = \frac{1}{2}$
1	120	
2	60	$\frac{60}{120} = \frac{1}{2}$
3	30	
4	15	$\frac{30}{60} = \frac{1}{2}$

b)

Solution

- a) If we take the ratio of consecutive y-values, we see that each time the x-value increases by one, the y-value is multiplied by 3. Since the ratio is always the same, **the function is exponential**.
- b) If we take the ratio of consecutive y-values, we see that each time the x-value increases by one, the y-value is multiplied by 12. Since the ratio is always the same, **the function is exponential**.

Write Equations for Functions

Once we identify which type of function fits the given values, we can write an equation for the function by starting with the general form for that type of function.

✓ Example 3.3.3

Determine what type of function represents the values in the following table.

x	y
0	5
1	1
2	-3
3	-7
4	-11

Solution

Let's first check the difference of consecutive values of y .

x	y	difference of y - values
0	5	
1	1	
2	-3	
3	-7	
4	-11	

$$1 - 5 = -4$$

$$-3 - 1 = -4$$

$$-7 + 3 = -4$$

$$-11 + 7 = -4$$

If we take the difference between consecutive y -values, we see that each time the x -value increases by one, the y -value always decreases by 4. Since the difference is always the same, **the function is linear**.

To find the equation for the function, we start with the general form of a linear function: $y=mx+b$. Since m is the slope of the line, it's also the quantity by which y increases every time the value of x increases by one. The constant b is the value of the function when $x=0$. Therefore, the function is $y=-4x+5$.

✓ Example 3.3.4

Determine what type of function represents the values in the following table.

x	y
0	0
1	5
2	20
3	45
4	80
5	125
6	180

Solution

Here, the difference between consecutive y -values isn't constant, so the function is not linear. Let's look at those differences more closely.

x	y	
0	0	
1	5	$5-0=5$
2	20	$20-5=15$
3	45	$45-20=25$
4	80	$80-45=35$
5	125	$125-80=45$

6

180

 $180 - 125 = 55$

When the x -value increases by one, the difference between the y -values increases by 10 every time. Since the difference of the differences is constant, the function describing this set of values is **quadratic**.

To find the equation for the function that represents these values, we start with the general form of a quadratic function:
 $y = ax^2 + bx + c$.

We need to use the values in the table to find the values of the constants: a , b and c .

The value of c represents the value of the function when $x=0$, so $c=0$.

Plug in the point (1,5):	$5 = a + b$
Plug in the point (2,20):	$20 = 4a + 2b \Rightarrow 10 = 2a + b$
To find a and b , we solve the system of equations:	$5 = a + b$
	$10 = 2a + b$
Solve the first equation for b :	$5 = a + b \Rightarrow b = 5 - a$
Plug the first equation into the second:	$10 = 2a + 5 - a$
Solve for a and b :	$a = 5$ and $b = 0$

Therefore the equation of the quadratic function is $y = 5x^2$.

✓ Example 3.3.5

Determine what type of function represents the values in the following table.

x	y
0	400
1	500
2	25
3	6.25
4	1.5625

Solution

The differences between consecutive y -values aren't the same, and the differences between those differences aren't the same either. So let's check the ratios instead.

x	y	ratio of y -values
0	400	$\frac{100}{400} = \frac{1}{4}$
1	100	
2	25	$\frac{25}{100} = \frac{1}{4}$
3	6.25	
4	1.5625	$\frac{1.5625}{6.25} = \frac{1}{4}$

Each time the x -value increases by one, the y -value is multiplied by $1/4$. Since the ratio is always the same, **the function is exponential**.

To find the equation for the function that represents these values, we start with the general form of an exponential function, $y = a \cdot bx^2$.

Here b is the ratio between the values of y each time x is increased by one. The constant a is the value of the function when $x=0$. Therefore, the function is $y=400 \left(\frac{1}{4}\right)^x$.

Perform Exponential and Quadratic Regressions with a Graphing Calculator

Earlier, you learned how to perform linear regression with a graphing calculator to find the equation of a straight line that fits a linear data set. In this section, you'll learn how to perform exponential and quadratic regression to find equations for curves that fit non-linear data sets.

✓ Example 3.3.6

The following table shows how many miles per gallon a car gets at different speeds.

Speed (mph)	Miles per gallon
30	18
35	20
40	23
45	25
50	28
55	30
60	29
65	25
70	25

Using a graphing calculator:

- Draw the scatterplot of the data.
- Find the quadratic function of best fit.
- Draw the quadratic function of best fit on the scatterplot.
- Find the speed that maximizes the miles per gallon.
- Predict the miles per gallon of the car if you drive at a speed of 48 mph.

Solution

Step 1: Input the data.

Press **[STAT]** and choose the **[EDIT]** option.

Input the values of x in the first column (L1) and the values of y in the second column (L2). (**Note:** in order to clear a list, move the cursor to the top so that L1 or L2 is highlighted. Then press **[CLEAR]** and then **[ENTER]**.)

Step 2: Draw the scatterplot.

First press **[Y=]** and clear any function on the screen by pressing **[CLEAR]** when the old function is highlighted.

Press **[STATPLOT]** **[STAT]** and **[Y=]** and choose option 1.

Choose the ON option; after TYPE, choose the first graph type (scatterplot) and make sure that the X list and Y list names match the names on top of the columns in the input table.

Press **[GRAPH]** and make sure that the window is set so you see all the points in the scatterplot. In this case, the settings should be $30 \leq x \leq 80$ and $0 \leq y \leq 40$. You can set the window size by pressing the **[WINDOW]** key at the top.

Step 3: Perform quadratic regression.

Press **[STAT]** and use the right arrow to choose **[CALC]**.

Choose Option 5 (QuadReg) and press **[ENTER]**. You will see “QuadReg” on the screen.

Type in L1,L2 after ‘QuadReg’ and press **[ENTER]**. The calculator shows the quadratic function: $y = -0.017x^2 + 1.9x - 25$

Step 4: Graph the function.

Press **[Y=]** and input the function you just found.

Press **[GRAPH]** and you will see the curve fit drawn over the data points.

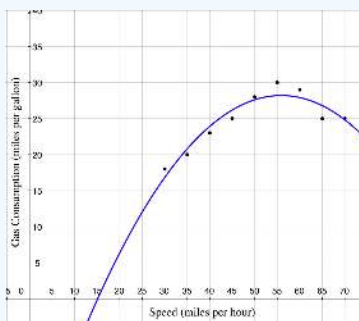


Figure 3.3.1:

To find the speed that maximizes the miles per gallon, use **[TRACE]** and move the cursor to the top of the parabola. You can also use **[CALC]** **[2nd]** **[TRACE]** and option 4:Maximum, for a more accurate answer. The speed that maximizes miles per gallon is **56 mph**.

Finally, plug $x=48$ into the equation you found: $y = -0.017(48)^2 + 1.9(48) - 25 = 27.032$ miles per gallon.

Note: The image above shows our function plotted on the same graph as the data points from the table. One thing that is clear from this graph is that predictions made with this function won’t make sense for all values of x . For example, if $x < 15$, this graph predicts that we will get negative mileage, which is impossible. Part of the skill of using regression on your calculator is being aware of the strengths and limitations of this method of fitting functions to data.

✓ Example 3.3.7

The following table shows the amount of money an investor has in an account each year for 10 years.

Year	Value of account
1996	\$5000
1997	\$5400
1998	\$5800
1999	\$6300
2000	\$6800
2001	\$7300
2002	\$7900
2003	\$8600
2004	\$9300
2005	\$10000

Year	Value of account
2006	\$11000

Using a graphing calculator:

- Draw a scatterplot of the value of the account as the dependent variable, and the number of years since 1996 as the independent variable.
- Find the exponential function that fits the data.
- Draw the exponential function on the scatterplot.
- What will be the value of the account in 2020?

Solution

Step 1: Input the data.

Press **[STAT]** and choose the **[EDIT]** option.

Input the values of x in the first column (L_1) and the values of y in the second column (L_2).

Step 2: Draw the scatterplot.

First press **[Y=]** and clear any function on the screen.

Press **[GRAPH]** and choose Option 1.

Choose the ON option and make sure that the X list and Y list names match the names on top of the columns in the input table.

Press **[GRAPH]** make sure that the window is set so you see all the points in the scatterplot. In this case the settings should be $0 \leq x \leq 10$ and $0 \leq y \leq 11000$.

Step 3: Perform exponential regression.

Press **[STAT]** and use the right arrow to choose **[CALC]**.

Choose Option 0 and press **[ENTER]**. You will see “ExpReg” on the screen.

Press **[ENTER]**. The calculator shows the exponential function: $y = 4975.7(1.08)^x$

Step 4: Graph the function.

Press **[Y=]** and input the function you just found. Press **[GRAPH]**.

Finally, plug $x = 2020 - 1996 = 24$ into the function: $y = 4975.7(1.08)^{24} = \$31551.81$

In 2020, **the account will have a value of \$31551.81.**

Note: The function above is the curve that comes *closest* to all the data points. It won’t return y-values that are exactly the same as in the data table, but they will be close. It is actually more accurate to use the curve fit values than the data points.

If you don’t have a graphing calculator, there are resources available on the Internet for finding lines and curves of best fit. For example, the applet at <http://science.kennesaw.edu/~plaval/applets/LRegression.html> does linear regression on a set of data points; the one at <http://science.kennesaw.edu/~plaval/applets/QRegression.html> does quadratic regression; and the one at <http://science.kennesaw.edu/~plaval/applets/ERegression.html> does exponential regression. Also, programs like Microsoft Office or OpenOffice have the ability to create graphs and charts that include lines and curves of best fit.

Solve Real-World Problems by Comparing Function Models

✓ Example 3.3.8

The following table shows the number of students enrolled in public elementary schools in the US (source: US Census Bureau). Make a scatterplot with the number of students as the dependent variable, and the number of years since 1990 as the independent variable. Find which curve fits this data the best and predict the school enrollment in the year 2007.

Year	Number of students (millions)
1990	26.6
1991	26.6
1992	27.1
1993	27.7
1994	28.1
1995	28.4
1996	28.1
1997	29.1
1998	29.3
2003	32.5

Solution

We need to perform linear, quadratic and exponential regression on this data set to see which function represents the values in the table the best.

Step 1: Input the data.

Input the values of x in the first column (L_1) and the values of y in the second column (L_2).

Step 2: Draw the scatterplot.

Set the window size: $0 \leq x \leq 10$ and $20 \leq y \leq 40$.

Here is the scatterplot:

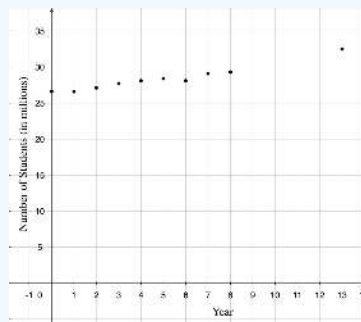


Figure 3.3.2:

Step 3: Perform Regression.

Linear Regression

The function of the line of best fit is $y = 0.51x + 26.1$. Here is the graph of the function on the scatterplot:

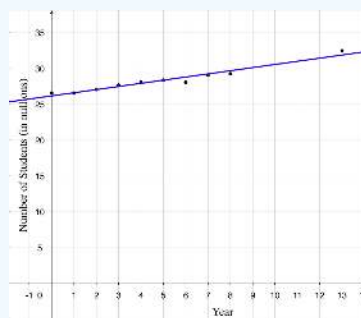


Figure 3.3.3:

Quadratic Regression

The quadratic function of best fit is $y = 0.064x^2 - .067x + 26.84$. Here is the graph of the function on the scatterplot:

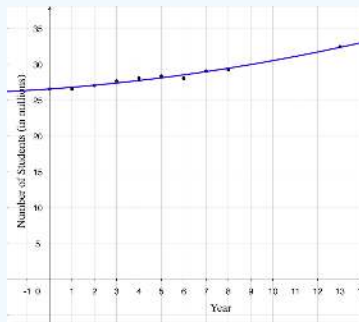


Figure 3.3.4:

Exponential Regression

The exponential function of best fit is $y=26.2(1.018)^x$. Here is the graph of the function on the scatterplot:

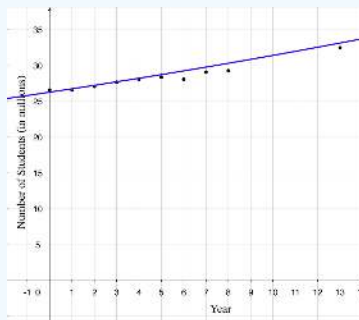


Figure 3.3.5:

From the graphs, it looks like the quadratic function is the best fit for this data set. We'll use this function to predict school enrollment in 2007.

Plug in $x=2007-1990=17$: $y=0.064(17)^2-.067(17)+26.84=44.2$ million students.

Review Questions

Determine whether the data in the following tables can be represented by a linear function.

1.

x	y
-4	10
-3	7
-2	4
-1	1
0	-2
1	-5

2.

x	y
-2	4
-1	3

x	y
0	2
1	3
2	6
3	11

3.

x	y
0	50
1	75
2	100
3	125
4	150
5	175

Determine whether the data in the following tables can be represented by a quadratic function.

4.

x	y
-10	10
-5	2.5
0	0
5	2.5
10	10
15	22.5

5.

x	y
1	4
2	6
3	6
4	4
5	0
6	-6

6.

x	y
-3	-27

x	y
-2	-8
-1	-1
0	0
1	1
2	8
3	27

Determine whether the data in the following tables can be represented by an exponential function.

7.

x	y
0	200
1	300
2	1800
3	8300
4	25800
5	62700

8.

x	y
0	120
1	180
2	270
3	405
4	607.5
5	911.25

9.

x	y
0	4000
1	2400
2	1440
3	864
4	518.4
5	311.04

Determine what type of function represents the values in the following tables and find the equation of each function.

10.

x	y
0	400
1	500
2	625
3	781.25
4	976.5625

11.

x	y
-9	-3
-7	-2
-5	-1
-3	0
-1	1
1	2

12.

x	y
-3	14
-2	4
-1	-2
0	-4
1	-2
2	4
3	14

13. As a ball bounces up and down, the maximum height that the ball reaches continually decreases from one bounce to the next.
For a given bounce, this table shows the height of the ball with respect to time:

Time (seconds)	Height (inches)
2	2
2.2	16
2.4	24
2.6	33
2.8	38
3.0	42
3.2	36

Time (seconds)	Height (inches)
3.4	30
3.6	28
3.8	14
4.0	6

- Draw the scatterplot of the data
- Find the quadratic function of best fit
- Draw the quadratic function of best fit on the scatterplot
- Find the maximum height the ball reaches on the bounce
- Predict how high the ball is at time $t=2.5$ seconds

14. A chemist has a 250 gram sample of a radioactive material. She records the amount of radioactive material remaining in the sample every day for a week and obtains the data in the following table:

Day	Weight (grams)
0	250
1	208
2	158
3	130
4	102
5	80
6	65
7	50

15. The following table shows the rate of pregnancies (per 1000) for US women aged 15 to 19. (source: US Census Bureau).
- Make a scatterplot with the rate of pregnancies as the dependent variable and the number of years since 1990 as the independent variable.
 - Find which type of curve fits this data best.
 - Predict the rate of teen pregnancies in the year 2010.

Year	Rate of pregnancy (per 1000)
1990	116.9
1991	115.3
1992	111.0
1993	108.0
1994	104.6
1995	99.6
1996	95.6
1997	91.4
1998	88.7

Year	Rate of pregnancy (per 1000)
1999	85.7
2000	83.6
2001	79.5
2002	75.4

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CHAPTER OVERVIEW

4: Percentages

[4.1: Percentage Basics](#)

[4.2: Applications Involving Percentages](#)

[4.3: Exponential Functions](#)

[4.4: Practice Problems](#)

Learning Objectives

- Convert between fractions, decimals, and percentages
- Solve for part, whole, or percent given any two of the three values
- Calculate and interpret both absolute and relative change
- Find percent increase and percent decrease
- Recognize an exponential function and identify both the growth factor and initial value
- Determine the growth or decay rate from an exponential function
- Model real world problems using exponential functions

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4.1: Percentage Basics

Percent literally means "per 100," or "parts per hundred." When we write 40%, this is equivalent to the fraction $\frac{40}{100}$ or the decimal 0.40. Notice that 80 out of 200 and 10 out of 25 are also 40%, since $\frac{80}{200} = \frac{10}{25} = \frac{40}{100}$.

✓ Example 4.1.1

Write each as a percent:

- a) $\frac{1}{4}$
- b) 0.02
- c) 2.35

Solution

- a) $\frac{1}{4} = 0.25 = 25\%$
- b) $0.02 = 2\%$
- c) $2.35 = 235\%$

? You Try It 4.1.1

Fraction	Decimal	Percent
$\frac{3}{5}$		
	0.02	
	0.72	72 %
	0.025	
$4\frac{1}{2}$		

Answer

Fraction	Decimal	Percent
$\frac{3}{5}$	0.6	60%
$\frac{2}{100} = \cancel{\left(\frac{1}{50}\right)}$	0.02	2%
$\frac{72}{100} = \cancel{\left(\frac{18}{25}\right)}$	0.72	72 %
$\frac{25}{1000} = \cancel{\left(\frac{1}{40}\right)}$	0.025	2.5%
$4\frac{1}{2}$	4.5	450%

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4.2: Applications Involving Percentages

Percent

If we have a part that is some percent of a whole, then $\text{percent} = \frac{\text{part}}{\text{whole}}$, or equivalently, $\text{part} = \text{percent} \cdot \text{whole}$. To do the calculations, we write the percent as a decimal.

✓ Example 4.2.2

243 people out of 400 state that they like dogs. What percent is this?

Solution

$$\frac{243}{400} = 0.6075 = \frac{60.75}{100}. \text{ This is } 60.75\%.$$

Notice that the percent can be found from the equivalent decimal by moving the decimal point two places to the right

? YouTry It 4.2.2

To win the election as president of the United States of America, a person must obtain 270 out of 538 possible votes from the electoral college. What percentage of the overall electoral votes is this? Round your answer to the nearest tenth of a percent.

Answer

50.2%

✓ Example 4.2.3

The sales tax in a town is 9.4%. How much tax will you pay on a \$140 purchase?

Solution

Here, \$140 is the whole, and we want to find 9.4% of \$140. We start by writing the percent as a decimal by moving the decimal point two places to the left (which is equivalent to dividing by 100). We can then compute: $\text{Tax} = 0.094(140) = \$13.16$

? You Try It 4.2.3

In a recent poll, 28% of the 750 individuals polled indicated that they would vote purely Democratic in the next election. How many of the individuals would vote a straight Democratic ticket?

Answer

210

✓ Example 4.2.4

A lender requires a minimum down payment of 12% of the value of the home. You have \$22,020 cash available to use as a down payment toward a home. Determine the maximum home value that you can finance.

Solution

To compute the maximum home value, we need to understand what the \$22,020 represents. This is the down payment; we need to find the value of the home. In this case, the down payment is the “part” and the home value is the “whole.” Recall that $\text{part} = \text{percent} \times \text{whole}$

Let V represent the value of the home. Since we know that the down payment is 12% of the value of the home, we can write the equation: $22,020 = 0.12V$.

Solving this equation for V, we get $V = \frac{22,020}{0.12} = 183,500$

So, with \$22,020 cash available to use as a down payment, you can afford to finance a home worth at most \$183,500.

? You Try It 4.2.3

One banana contains about 425mg of potassium. That is about 13% of the recommended daily amount of potassium. How much potassium should be consumed daily?

Answer

3,269mg, or 3.269 g

When working with percentages, it is very important to understand the quantities being compared. Consider the examples below.

✓ Example 4.2.5

In the 2004 vice-presidential debates, Edwards's claimed that US forces have suffered "90% of the coalition casualties" in Iraq. Cheney disputed this, saying that in fact Iraqi security forces and coalition allies "have taken almost 50 percent" of the casualties. Who is correct?

Solution

Without more information, it is hard for us to judge who is correct, but we can easily conclude that these two percentages are talking about different things, so one does not necessarily contradict the other. Edward's claim was a percent of coalition forces, while Cheney's claim was a percent with both coalition and Iraqi security forces. It turns out both statistics are in fact fairly accurate.

✓ Example 4.2.6

Over the basketball season, Isaac scores on 40% of 2-point field goal attempts, and on 30% of 3-point of field goal attempts. Find Isaac's overall field goal percentage.

Solution

It is very tempting to average these values, and claim the overall average is 35%, but this is likely not correct, since most players make many more 2-point attempts than 3-point attempts. We don't actually have enough information to answer the question. Suppose Isaac attempted 200 2-point field goals and 100 3-point field goals. Then he made $200(0.40) = 80$ 2-point shots and $100(0.30) = 30$ 3-point shots. Overall, Isaac made 110shots out of 300, for a $\frac{110}{300} = 0.367 = 36.7$ overall field goal percentage.

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4.3: Exponential Functions

India is the second most populous country in the world, with a population in 2008 of about 1.14 billion people. The population is growing by about 1.34% each year⁵. We might ask if we can find a formula to model the population, P , as a function of time, t , in years after 2008, if the population continues to grow at this rate.

In **linear growth**, we had a constant rate of change – a constant number that the output increased for each increase in input. For example, in the equation $f(x) = 3x + 4$, the slope tells us the output increases by three each time the input increases by one.

This population scenario is different – we have a percent rate of change rather than a constant number of people as our rate of change.

To see the significance of this difference consider these two companies:

- Company A has 100 stores, and expands by opening 50 new stores a year
- Company B has 100 stores, and expands by increasing the number of stores by 50% of their total each year.

Looking at a few years of growth for these companies:

Year	Stores, company	Stores, company	Starting with 100 each
0	100	100	Starting with 100 each
1	$100 + 50 = 150$	$100 + 50\% \text{ of } 100$ $100 + 0.50(100) = 150$	They both grow by 50 stores in the first year.
2	$150 + 50 = 200$	$150 + 50\% \text{ of } 150$ $150 + 0.50(150) = 225$	Store A grows by 50, Store B grows by 75
3	$200 + 50 = 250$	$225 + 50\% \text{ of } 225$ $225 + 0.50(225) = 337.5$	Store A grows by 50, Store B grows by 112.5

Notice that with the percent growth, each year the company grows by 50% of the current year's total, so as the company grows larger, the number of stores added in a year grows as well. To try to simplify the calculations, notice that after 1 year the number of stores for company B was: $100 + 0.50(100)$ or equivalently by factoring $100(1 + 0.50) = 150$. We can think of this as "the new number of stores is the original 100% plus another 50%".

After 2 years, the number of stores was:

- $150 + 0.50(150)$ or equivalently by factoring
- $150(1 + 0.50)$ now recall the 150 came from $100(1 + 0.50)$. Substituting that,
- $100(1 + 0.50)(1 + 0.50) = 100(1 + 0.50)^2 = 225$

After 3 years, the number of stores was:

- $225 + 0.50(225)$ or equivalently by factoring
- $225(1 + 0.50)$ now recall the 225 came from $100(1 + 0.50)^2$. Substituting that, $100(1 + 0.50)^2(1 + 0.50) = 100(1 + 0.50)^3 = 337.5$

From this, we can generalize, noticing that to show a 50% increase, each year we multiply by a factor of $(1 + 0.50)$, so after n years, our equation would be:

- $B(n) = 100(1 + 0.50)^n$

In this equation, the 100 represented the initial quantity, and the 0.50 was the percent growth rate. Generalizing further, we arrive at the general form of exponential functions.

Exponential Function

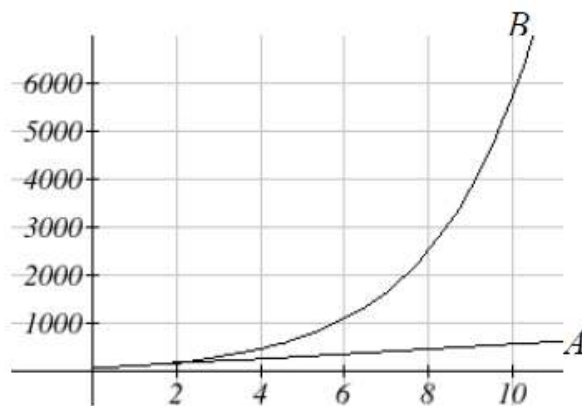
An exponential growth or decay function is a function that grows or shrinks at a constant percent growth rate. An exponential function has the form $f(x) = ab^x$ where a is the initial or starting value of the function b is the growth factor (or decay factor)

- If $b > 1$, then the function is increasing (growth).
- If $0 < b < 1$, then the function is decreasing (decay).

If r is the percent growth or decay rate (in decimal form), the exponential function can be written as follows: Growth: $f(x) = a(1 + r)^x$ or Decay: $f(x) = a(1 - r)^x$

To see more clearly the difference between exponential and linear growth, compare the two tables and graphs below, which illustrate the growth of company A and B described above over a longer time frame if the growth patterns were to continue.

Years	Company	Company
2	200	225
4	300	506
6	400	1139
8	500	2563
10	600	5767



✓ Example 4.3.1

Complete the table below.

Function	Initial Value	Growth or Decay	Rate (as a %)
$f(x)=430(1.15)^x$			
$f(x)=23(0.8)^x$			
	1.87	Growth	5 %
	52	Decay	10%

Solution

Function	Initial Value	Growth or Decay	Rate (as a %)
$f(x)=430(1.15)^x$	430	Growth	$1.15 - 1 = 0.15$ Growth Rate: 15%
$f(x)=23(0.8)^x$	23	Decay	$1 - 0.8 = 0.2$ Decay Rate = 20%
$f(x)=1.87(1+0.05)^x$ $f(x)=1.87(1.05)^x$	1.87	Growth	5 %
$f(x)=52(1-0.10)^x$ $f(x)=52(0.9)^x$	52	Decay	10%

✓ Example 4.3.2

$T(q)$ represents the total number of Android smart phone contracts, in thousands, held by a certain Verizon store region measured q quarters since January 1, 2010.

Interpret all of the parts of the equation $T(2) = 86(1.64)^2 = 231.3056$.

Interpreting

Solution

Interpreting this from the basic exponential form, we know that 86 is our initial value. This means that on Jan. 1, 2010 this region had 86,000 Android smart phone contracts.

Since $b = 1 + r = 1.64$, we know that every quarter the number of smart phone contracts grows by 64%.

$T(2) = 231.3056$ means that in the 2nd quarter (or at the end of the second quarter) there were approximately 231,306 Android smart phone contracts.

? You Try It 4.3.1

a. Given the three statements below, identify which represent exponential functions.

1. The cost of living allowance for state employees increases salaries by 3.1% each year
2. The value of an investment is increasing by \$300 each year
3. Tuition costs have increased by 2.8% each year for the last 3 years.

b. Looking at the following two equations that represent the balance in two different savings accounts, which account is growing faster, and which account will have a higher balance after 3 years?

$$A(t) = 1,000(1.05)^t \quad B(t) = 900(1.075)^t$$

c. Complete the table. The first one has been done for you.

Function	Initial Value	Growth or Decay	Rate (as a %)
$f(x)=615(0.79)^x$	615	Decay	21 %
$f(x)=500(1.03)^x$			
$f(x)=3,200(1.2)^x$			
$f(x)=19,611(0.98)^x$			
	50	Growth	12 %
	377	Decay	8 %

Answer

- 1&3 are exponential functions; they grow by a % not a constant number.
- $B(t)$ is growing faster, but after 3 years $A(t)$ still has a higher account balance
- c.

$f(x) = 500(1.03)^x$	500	Growth	
$(f(x)=3,200(1.2)^x)$	3,200	Growth	
$f(x) = 19,611(0.98)^x$	19,611	Decay	
$f(x) = 50(1.12)^x$	50	Growth	
$f(x) = 377(0.92)^x$	377	Decay	

✓ Example 4.3.3

In the year 2008, India's population was 1.14 billion and growing at a rate of 1.34% per year. Write an exponential function for India's population, and use it to predict the population in 2020.

Solution

Using 2008 as our starting time ($t = 0$), our initial population will be 1.14 billion. Since the percent growth rate was 1.34%, our value for r is 0.0134. Using the basic formula for exponential growth $f(x) = a(1 + r)^x$ we can write the formula,

$$P(t) = 1.14(1 + 0.0134)^t = 1.14(1.0134)^t, \quad (4.3.1)$$

where $P(t)$ is the population of India (measured in billions) t years after 2008.

To estimate the population in 2020, we evaluate the function at $t = 12$, since 2020 is 12 years after 2008.

$$P(12) = 1.14(1.0134)^{12} \approx 1.337 \text{ billion people in 2020} \quad (4.3.2)$$

✓ Example 4.3.2

Bismuth-210 is an isotope that radioactively decays by about 13% each day, meaning 13% of the remaining Bismuth-210 transforms into another atom (polonium-210 in this case) each day. If you begin with 100 mg of Bismuth-210, how much remains after one week?

Solution

With radioactive decay, instead of the quantity increasing at a percent rate, the quantity is decreasing at a percent rate. Our initial quantity is $a = 100\text{mg}$, and our decay rate is 13%, so $r = 0.13$.

This gives the equation: $Q(d) = 100(1 - 0.13)^d = 100(0.87)^d$ where $Q(d)$ is the amount (in mg) of Bismuth-210 remaining after d days.

This can also be explained by recognizing that if 13% decays, then 87% remains.

After one week, 7 days, the quantity remaining would be $Q(7) = 100(0.87)^7 = 37.73\text{mg}$ of Bismuth-210 remains.

✓ Example 4.3.3

An investment is initially worth \$10,000. Write a formula for the value of this investment for each situation described below.

- The value increases by 5% every year.
- The value increases by 5% every 3 years.
- The value increases by 5% every 6 months.

Solution

In this problem, we are looking for three exponential functions of the form $V(t) = ab^t$, where $V(t)$ represents the value of the investment (in dollars) after t years. Since the initial value is \$10,000 we know that $a = 10,000$. In each case, the value of the investment is increasing by 5%, so $b = 1 + 0.05 = 1.05$. The only difference in these three cases is the time period for which the 5% increase occurs.

- Since the 5% increase occurs every year, we have $V(t) = 10,000(1.05)^t$

t	$V(t) = 10,000(1.05)^t$
0	$10,000(1.05)^0 = 10,000$
1	$10,000(1.05)^1 = 10,500$
2	$10,000(1.05)^2 = 11,025$
3	$10,000(1.05)^3 = 11,576$

- Since the 5% increase occurs every three years, we need to multiply by the growth factor, 1.05, only every three years. When $t = 3$ we multiply by the growth factor once. If $t = 6$, we multiply by the growth factor twice, and so on. The formula would then be $V(t) = 10,000(1.05)^{t/3}$

t	$V(t) = 10,000(1.05)^{t/3}$
0	$10,000(1.05)^0/3 = 10,000(1.05)^0 = 10,000$
3	$10,000(1.05)^3/3 = 10,000(1.05)^1 = 10,500$
6	$10,000(1.05)^6/3 = 10,000(1.05)^2 = 11,025$
9	$10,000(1.05)^9/3 = 10,000(1.05)^3 = 11,576$

c) Since the 5% increase occurs every six months, or twice each year, we need to multiply by the growth factor, 1.05, two times each year. The formula would then be $V(t) = 10,000(1.05)^{2t}$

t	$V(t) = 10,000(1.05)^{2t}$
0	$10,000(1.05)^2(0) = 10,000(1.05)^0 = 10,000$
1/2	$10,000(1.05)^2(1/2) = 10,000(1.05)^1 = 10,500$
1	$10,000(1.05)^2(1) = 10,000(1.05)^2 = 11,025$
3/2	$10,000(1.05)^2(3/2) = 10,000(1.05)^3 = 11,576$

To verify that these formulas are correct, we can compare the following tables of values

	a) Every year		b) Every 3 years		c) Every 6 months
t	$V(t) = 10,000(1.05)^t$	t	$V(t) = 10,000(1.05)^{t/3}$	t	$V(t) = 10,000(1.05)^{2t}$
0	10,000	0	10,000	0	10,000
1	10,500	1	10,164	1	10,500
2	11,025	2	10,331	2	11,025
3	11,576	3	10,500	3	11,576
4	12,155	4	10,672	4	12,155
5	12,763	5	10,847	5	12,763
6	13,401	6	11,025	6	13,401

✓ Example \PageIndex{4}

Consider the following situations:

1. The population of a town is 83.6 thousand in the year 2005 and grows at a rate of 11.5% every 8 years.
2. A company's sales are \$14.2 million at the beginning of 2011, and have been decreasing at a rate of about 3.2% per quarter.

Write a formula for each of these situations using time, in years, as the input quantity

Solution

1. In this situation, let $P(t)$ represent the population of the town (in thousands) t years after 2005. Since the population is increasing at a rate of 14% every 8 years, the formula would then be:

$$\begin{aligned} P(t) &= 83.6(1 + 0.115)^{t/8} \\ P(t) &= 83.6(1.115)^{t/8} \end{aligned} \quad (4.3.3)$$

2. In this situation, let $S(t)$ represent the sales (in millions of dollars) t years after 2011. Since sales are decreasing at a rate of 3.2% per quarter, or 4 times each year, the formula for $S(t)$ would then be:

$$\begin{aligned}S(t) &= 14.2(1 - 0.032)^{4t} \\S(t) &= 14.2(0.968)^{4t}\end{aligned}\tag{4.3.4}$$

✓ You Try It 4.3.2

An investment is initially worth \$84,300. Write a formula for the function $V(t)$, representing the value of this investment after t years in each of the following situations.

- a) The value increases by 6.5% every year.
- b) The value decreases by 3% every year.
- c) The value increases by \$600 every year.
- d) The value decreases by \$1400 every year.
- e) The value increases by 26% every 2 years.
- f) The value decreases by 13% every month.

Answer

- a $V(t) = 84,300(1.065)^t$
- b. $V(t) = 84,300(0.97)^t$
- c. $V(t) = 84,300 + 600t$
- d. $V(t) = 84,300 - 1,400t$
- e. $V(t) = 84,300(1.26)^{t/2}$
- f. $V(t) = 84,300(0.87)^{12t}$

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4.4: Practice Problems

1. Write each as a percent:

- a. $\frac{3}{4}$
- b. $\frac{4}{5}$
- c. $\frac{7}{10}$

2. The fraction $\frac{5}{7}$ is equivalent to what percent? Round your answer to the nearest tenth of a percent.

3. The fraction $\frac{7}{9}$ is equivalent to what percent? Round your answer to the nearest tenth of a percent.

4. The fraction $\frac{2}{11}$ is equivalent to what percent? Round your answer to the nearest tenth of a percent.

5. Write each as a percent:

- a. 0.23
- b. 0.215
- c. 1.09

6. Write each as a decimal:

- a. 15%
- b. 4%
- c. 7.5%

7. 20% is equivalent to what fraction in reduced terms?

8. 86% is equivalent to what fraction in reduced terms?

9. 58% is equivalent to what fraction in reduced terms?

10. What number is 70% of 40 ?

11. What number is 45% of 155 ?

12. 105 is what percent of 225 ?

13. 170 is what percent of 245 ?

14. 18 is 30% of what number?

15. 130 is 65% of what number?

16. Valerie paid \$170 for an item that was originally priced at \$550. What percent of the original price did Valerie pay? Round your answer to the nearest tenth of a percent.

17. Trader Joe's sold 8,132 bags of tortilla chips last month. If 7,687 of these bags were fat free, find the percent that were fat free. Round your answer to the nearest percent.

18. 76% of the questions on a student's test were correct. There were 50 questions. How many of the questions were correct?

19. At a restaurant, the bill comes to \$35. If you decide to leave a 15% tip, how much is the tip? Round your answer to the nearest cent.

20. What is the sales tax on a suit priced at \$295 if the sales tax is 7% ?

21. What is the sales tax on a suit priced at \$1266 if the sales tax is 6% ?

22. A lender requires a minimum down payment of 16% of the value of the home. You have \$39,000 cash available to use as a down payment toward a home. Determine the maximum home value that you can finance.

23. The population of a town increased from 3,850 in 2005 to 5,900 in 2012 . By what percent did the population increase?

24. The price of a latte dropped from \$3.12 to \$3.00. Find the absolute change and relative change of the price of a latte.

25. Theresa's annual income increased from \$31,000 to \$48,000. Find the absolute change and relative change of Joe's annual income.

26. A shirt that was originally marked as \$24.95 rings up at the register as \$19.96. What is the percent discount on the shirt?
27. The U.S. Weather Bureau has a station on Mauna Loa in Hawaii that has measured carbon dioxide levels since 1959. At that time, there were 307 parts per million of carbon dioxide in the atmosphere. In 2005, the figure was 391 parts per million. Find the increase in carbon dioxide levels and the percent of increase. Round to the nearest tenth of a percent.
28. Anthony and his son Johnny are exercising more regularly and keeping track of their weight loss. Both Anthony and Johnny lost 10 pounds.
- From this information alone, can you determine if they both lost the same percentage of weight?
 - Suppose Anthony's starting weight is 190 pounds and Johnny's starting weight is 155. Who lost a larger percent of their starting weight?
29. A company calls to notify you that the interest rate on your credit card will increase from 16% to 18%.
- Determine the number of percentage points the interest rate increased by.
 - Determine the percent increase of the interest rate.
30. The sales tax rate for the state of Washington was 5.7%. If the sticker price of a car in Washington is \$4,100, what will be the final cost of the car, including tax?
31. A store is having a 35% off sale. If the original price of the shirt is \$18.00, what is the price of the shirt after the discount is applied (not including tax)?
32. Hank currently makes \$13.00 per hour and is going to receive a 7% raise. What will be Hank's new hourly wage?
33. At a restaurant, the bill comes to \$50. If you decide to leave a 15% tip, what is the total amount you paid?
34. At a restaurant, the bill comes to \$32.20. If you decide to leave an 18% tip, what is the total amount you paid?
35. Sam lost 11% of his body weight after including healthier foods in his diet and exercising regularly. If his original weight was 218 lbs, what is his current weight?
36. A sales clerk received an increase in her annual salary, which is now \$27,300. If she received a 5% raise, what was her starting salary? Round your answer to the nearest cent.
37. Joyce paid \$84.50 for an item at the store that was 35% off the original price. What was the original price? Round your answer to the nearest cent.
38. You purchased a new car for a total cost of \$19,241.94 which includes the 13% sales tax. What was the original price of the car? Round your answer to the nearest cent.
39. Given the function $f(x) = 24(0.72)^x$, evaluate the following.
- $f(-3)$
 - $f(0)$
 - $f(2)$
40. Given the function $f(x) = 5(1.08)^x$, evaluate the following.
- $f(-1)$
 - $f(0)$
 - $f(3)$
41. For the function $f(x) = 170(0.8)^x$
- Identify the initial value
 - Identify the function's behavior as growth or decay
 - Identify the growth/decay factor
 - Identify the growth/decay rate (as a percent)
42. For the function $f(x) = 5(2)^x$
- Identify the initial value
 - Identify the function's behavior as growth or decay

- c. Identify the growth/decay factor
 - d. Identify the growth/decay rate (as a percent)
43. For the function $h(t) = 110(1.3)^t$
- a. Identify the initial value
 - b. Identify the functions behavior as growth or decay
 - c. Identify the growth/decay factor
 - d. Identify the growth/decay rate (as a percent)
44. For the function $f(x) = 60(0.7)^x$
- a. Identify the initial value
 - b. Identify the functions behavior as growth or decay
 - c. Identify the growth/decay factor
 - d. Identify the growth/decay rate (as a percent)
45. Write the exponential function with an initial value of 110 and a growth rate of 3.5%.
46. Write the exponential function with an initial value of 1504 and a growth rate of 12%.
47. Write the exponential function with an initial value of 63 and a decay rate of 28%.
48. Write the exponential function with an initial value of 545 and a decay rate of 0.4%.
49. Write the exponential function with an initial value of 79 and a growth rate of 119%.
50. Given the three statements below, identify which represent exponential functions.
- a. The number of pennies in your piggy bank doubles every year.
 - b. The value of a car depreciates at a rate of 22% per year.
 - c. The temperature is decreasing at the rate of 3°F per hour.
51. When a new charter school opened in 2010 , there were 970 students enrolled. Using function notation, write a formula representing the number of students attending this charter school t years after 2010, assuming that the student population:
- a. Increases by 73 students per year
 - b. Decreases by 36 students per year
 - c. Decreases by 9.4% per year
 - d. Increases by 6.1% per year
 - e. Remains constant (does not change)
52. A population numbers 1,944 organisms initially and increases by 5.2% each year.
- a. Write an exponential function for the population of the organisms.
 - b. Predict the number of organisms after 7 years.
53. A population numbers 17,000 organisms initially and decreases by 8% each year.
- a. Write an exponential function for the population of the organisms.
 - b. Predict the number of organisms after 2 years.
54. In 2014 Rose invested \$16, 000 in a savings account for her newborn son. The account pays 3.6% interest each year. Determine the total value of the account in the year 2032, when her son will go to college. Round your answer the nearest cent.
55. You have saved \$1, 000 to invest in a savings account. Write an exponential function for the value of this investment for each situation that follows:
- a. The value increases by 4% every year.
 - b. The value increases by 4% every 2 years.
 - c. The value increases by 4% every 6 months.
 - d. The value increases by 4% every 3 months.
56. An investment is initially worth \$7, 400. Write an exponential function representing the value of this investment after t years in each of the following situations.

- a. The value increases by 3.6% every year.
- b. The value decreases by 2% every year.
- c. The value increases by 11% every 3 years.
- d. The value decreases by 5% every 6 months.

57. A new home is purchased for \$150,000 The value of the home increases by 11% every 2 years. Write an exponential function for the value of the home using time in years as the input quantity.

58. A new home is purchased for \$200,000 The value of the home increases by 1.5% every 5 years. Write an exponential function for the value of the home using time in years as the input quantity.

59. A new home is purchased for \$195,000 The value of the home decreases by 4% every 3 years. Write an exponential function for the value of the home using time in years as the input quantity.

60. A new home is purchased for \$420,000 The value of the home decreases by 8% every 4 years. Write an exponential function for the value of the home using time in years as the input quantity.

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CHAPTER OVERVIEW

5: Savings .

[5.1: Simple Interest](#)

[5.2: Compound Interest](#)

[5.3: Practice Problems](#)

Learning Objectives

- Recognize that simple interest grows linearly
- Be able to correctly apply the simple interest formula to solve for A , r , t , or P_0
- Recognize that compound interest grows exponentially
- Correctly apply the compound interest formula to solve for A , r , or P_0
- Recognize that the compound interest formula is used for solving when you have a one-time, lump sum deposit
- Calculate the APY for an account earning compound interest

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5.1: Simple Interest

Discussing interest starts with the principal, or amount your account starts with initially. This could be a starting investment, or the starting amount of a loan. Interest, in its most simple form, is calculated as a percent of the principal.

Definition: APR - Annual Percentage Rate

APR – Annual Percentage Rate Interest rates are usually given as an annual percentage rate (APR) – the total interest that will be paid in the year.

For most long term, simple interest loans, it is common for interest to be paid on an annual basis. In that case, interest would be earned each year on the principal.

For example, suppose you borrowed \$10,000 from a friend and agree to repay it with 3% annual interest, in 5 years. You would not only repay your friend the \$10,000 you borrowed. You would also pay simple interest for each year you had borrowed the money.

Year	Starting balance	Interest earned	Ending Balance
1	10,000	300.00	10300
2	10300	300.00	10,600
3	10,600	300.00	10,900
4	10,900	300.00	11,200
5	11,200	300.00	11,500

The total amount you would repay your friend would be \$11,500.00, which is the original principal plus the interest over 5 years.

This process can be generalized with the following formulas:

Simple Interest Over Time

$$I = P_0rt \quad (5.1.1)$$

$$A = P_0 + I = P_0 + P_0rt = P_0(1 + rt)$$

I is the dollar amount of interest A is the balance in the account after t years P_0 is the balance in the account at the beginning (starting amount, or principal). r is the annual interest rate (APR) in decimal form (Example: $5\% = 0.05$) t is the number of years we plan to leave the money in the account

Bonds are essentially a loan made to the bond issuer (a company or government) by you, the bond holder. In return for the loan, the issuer agrees to pay interest, often annually. Bonds have a maturity date, at which time the issuer pays back the original bond value.

Caution

Bonds are issued with a wide variety of conditions for repayment and interest, often not with simple interest. Be careful to understand the repayment and conditions and interest rules for any bond you consider buying.

✓ Example 5.1.1

Suppose your city is building a new park, and issues bonds to raise the money to build it. You obtain a \$5,000 bond that pays 4.5% interest annually that matures in 5 years.

Solution

When the bond matures, you would receive back the \$5,000 you originally paid, plus \$1125 in interest, leaving you with a total of \$6125.

? Exercise 5.1.1

Maria invests \$16,000 at 3% simple interest for 19 years.

- How much interest will she earn?
- How much is in the account at the end of the 19 year period?

Answer

At the end of the 19 year period, she will have earned \$9120 in interest. The balance in her account will be \$25120.

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5.2: Compound Interest

With simple interest, we were assuming that we pocketed the interest when we received it. In a standard bank account, any interest we earn is automatically added to our balance, and we earn interest on that interest in future years. This reinvestment of interest is called **compounding**.

Suppose that we deposit \$1,000 in a bank account offering 3% interest, compounded monthly. How will our money grow?

The 3% interest is an annual percentage rate (APR) - the total interest to be paid during the year. Since our interest is being paid monthly, each month we will earn $\frac{1}{12}$ of the 3% annual interest, or $\frac{3\%}{12} = 0.25\%$ per month.

In the first month,

$$\begin{aligned} P_0 &= \$1,000 \\ r &= 0.0025(0.25\%) \\ I &= \$1,000(0.0025) = \$2.50 \\ A &= \$1,000 + \$2.50 = \$1,002.50 \end{aligned} \tag{5.2.1}$$

In the first month, we will earn \$2.50 in interest, raising our account balance to \$1002.50

In the second month,

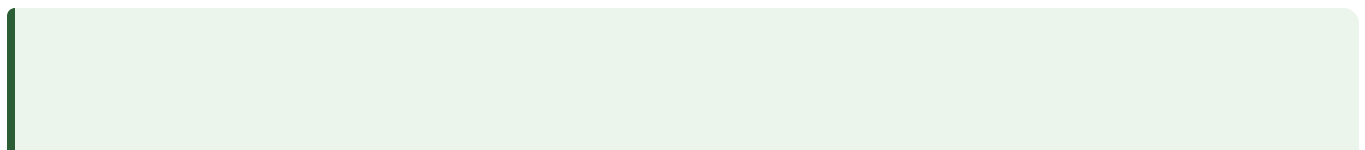
$$\begin{aligned} P_0 &= \$1,002.50 \\ I &= \$1,002.50(0.0025) = \$2.51 \text{ (rounded)} \\ A &= \$1,002.50 + \$2.51 = \$1,005.01 \end{aligned} \tag{5.2.2}$$

Notice that in the second month we earned more interest than we did in the first month. This is because we earned interest not only on the original \$1000 we deposited, but we also earned interest on the \$2.50 of interest we earned the first month. This is the key advantage that compounding of interest gives us.

Calculating out a few more months (rounding to the nearest cent):

Month	Starting balance	Interest earned	Ending Balance
1	1000.00	2.50	1002.50
2	1002.50	2.51	1005.01
3	1005.01	2.51	1007.52
4	1007.52	2.52	1010.04
5	1010.04	2.53	1012.57
6	1012.57	2.53	1015.10
7	1015.10	2.54	1017.64
8	1017.64	2.54	1020.18
9	1020.18	2.55	1022.73
10	1022.73	2.56	1025.29
11	1025.29	2.56	1027.85
12	1027.85	2.57	1030.42

The standard formula for compound interest is as follows:



Compound Interest

$$A = P_0 \left(1 + \frac{r}{n}\right)^{nt} \quad (5.2.3)$$

A is the balance in the account after t years

t is the number of years we plan to leave the money in the account

P_0 is the balance in the account at the beginning (starting amount, or principal).

r is the annual interest rate (APR) in decimal form (Example: $5\% = 0.05$)

n is the number of compounding periods in one year.

- If the compounding is done annually (once a year), $n = 1$.
- If the compounding is done quarterly, $n = 4$.
- If the compounding is done monthly, $n = 12$.
- If the compounding is done daily, $n = 365$.

✓ Example 5.2.1

If you invest \$3,000 in an investment account paying 3% interest compounded quarterly, how much will the account be worth in 10 years?

Solution

Since we are starting with \$3,000, $P_0 = 3,000$

Our interest rate is 3%, so $r = 0.03$

Since we are compounding quarterly, we are compounding 4 times per year, so $n = 4$. We want to know the value of the account in 10 years, so we are looking for the ending value, A , when $t = 10$.

$$A = 3,000 \left(1 + \frac{0.03}{4}\right)^{4(10)} = \$4,045.05 \quad (5.2.4)$$

The account will be worth \$4,045.05 in 10 years.

? Exercise 5.2.1

If you invest \$3,000 in an investment account paying 3% interest compounded weekly, how much will the account be worth in 10 years?

Answer

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✓ Example 5.2.2

A certificate of deposit (CD) is a savings instrument that many banks offer. It usually gives a higher interest rate, but you cannot access your investment for a specified length of time. Suppose you deposit \$3,000 in a CD paying 6% interest, compounded monthly. How much will you have in the account after 20 years?

Solution

In this example,

$P_0 = \$3,000$	the initial deposit	
$r = 0.06$	6% annual rate	
$n = 12$	12 months in 1 year	
$t = 20$	since we're looking for how much we'll have after 20 years	(5.2.5)

years

So $A = 3,000 \left(1 + \frac{0.06}{12}\right)^{12 \times 20} = \$9,930.61$ (round your answer to the nearest penny)

Let us compare the amount of money earned from compounding against the amount you would earn from simple interest

Years	Simple Interest (\$15 per month)	6% compounded monthly = 0.5% each month
5	\$3,900	\$4,046.55
10	\$4,800	\$5,458.19
15	\$5,700	\$7,362.28
20	\$6,600	\$9,930.61
25	\$7,500	\$13,394.91
30	\$8,400	\$18,067.73
35	\$9,300	\$24,370.65

As you can see, over a long period of time, compounding makes a large difference in the account balance. You may recognize this as the difference between linear growth and exponential growth.

✓ Example 3

A 529 plan is a college savings plan in which a relative can invest money to pay for a child's later college tuition, and the account grows tax free. If Lily wants to set up a 529 account for her new granddaughter, wants the account to grow to \$40,000 over 18 years, and she believes the account will earn 6% compounded semi-annually (twice a year), how much will Lily need to invest in the account now?

Solution

Since the account is earning 6%, $r = 0.06$

Since interest is compounded twice a year, $n = 2$

In this problem, we don't know how much we are starting with, so we will be solving for P_0 , the initial amount needed. We do know we want the end amount, A , to be \$40,000 so we will be looking for the value of P_0 so that $A = 40,000$.

$$\begin{aligned}
 40,000 &= P_0 \left(1 + \frac{0.06}{2}\right)^{2(18)} \\
 40,000 &= P_0 (2.898278328) \\
 P_0 &= \frac{40,000}{2.898278328} \approx \$13,801.30
 \end{aligned}
 \tag{5.2.6}$$

Lily will need to invest \$13,801.30 to have \$40,000 in 18 years.

📌 Note on Rounding

It is important to be very careful about rounding when performing calculations. If possible, enter the entire calculation in one step into your calculator to avoid rounding error. If this is not possible, you want to keep as many decimals during calculations as you can. Try calculating example 2 using 2.898 instead of 2.898278328, and compare your answer with the example.

🚫 Caution

Over several compounding periods rounding can be manipulated to favor bank or the customer depending on the conditions of the loan. Rounding down will favor the bank, rounding down to the dollar amount in the account also. Rounding compounds

also.

? Exercise 5.2.1

If Lily wants to set up a 529 account for her new granddaughter, wants the account to grow to \$40,000 over 18 years, and she believes the account will earn 6% compounded daily, how much will Lily need to invest in the account now?

Answer

Lily will need to invest \$13,693.20 to have \$40,000 in 18 years.

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5.3: Practice Problems

1. Janelle invests \$18,000 at 8% simple interest for 41 years. How much is in the account at the end of the 41-year period?
2. Carlos invests \$15,000 at 2% simple interest for 29 years. How much is in the account at the end of the 29-year period?
3. Kristina invests \$13,000 at 7% simple interest for 32 years. How much is in the account at the end of the 32-year period?
4. How much would you need to deposit in an account now in order to have \$3,000 in the account in 5 years? Assume the account earns 2% simple interest.
5. How much would you need to deposit in an account now in order to have \$2,000 in the account in 15 years? Assume the account earns 6% simple interest.
6. You invest \$9600 in a savings account that pays 3.6% simple interest. How long will it take for this investment to double in value? Round your answer to the nearest cent.
7. Jenelle wants to invest \$500 in a savings account that pays 4.3% simple interest. How long will it take for this investment to triple in value? Round your answer to the nearest cent.
8. Brian invests \$1200 in a savings account that pays 5.1% simple interest. How long will it take for this investment to double in value? Round your answer to the nearest cent.
9. You deposit \$300 in an account earning 5% interest compounded annually. How much will you have in the account in 10 years?
10. How much will \$1,000 deposited in an account earning 7% interest compounded annually be worth in 20 years?
11. You deposit \$9,400 in an account earning 4% interest compounded semi-annually. How much will you have in the account in 5 years?
12. You deposit \$2,100 in an account earning 2.5% interest compounded semi-annually. How much will you have in the account in 7 years?
13. You deposit \$4,000 in an account earning 8% interest compounded monthly. How much will you have in the account in 10 years?
14. You deposit \$200 in an account earning 5.2% interest compounded monthly. How much will you have in the account in 15 years?
15. You deposit \$2,000 in an account earning 3% interest compounded monthly.
 - a. How much money will you have in the account in 20 years?
 - b. How much interest will you earn?
16. You deposit \$10,000 in an account earning 4% interest compounded monthly.
 - a. How much money will you have in the account in 25 years?
 - b. How much interest will you earn?
17. You deposit \$5,300 in an account earning 5% interest compounded quarterly. How much will you have in the account in 6 years?
18. You deposit \$7,000 in an account earning 3.5% interest compounded quarterly. How much will you have in the account in 8 years?
19. You deposit \$3,000 in an account earning 8% interest compounded quarterly. How much will you have in the account in 10 years?
20. How much would you need to deposit in an account now in order to have \$4,000 in the account in 10 years? Assume the account earns 3% interest compounded monthly.
21. How much would you need to deposit in an account now in order to have \$2,500 in the account in 5 years? Assume the account earns 2.5% interest compounded monthly.
22. How much would you need to deposit in an account now in order to have \$6,000 in the account in 8 years? Assume the account earns 6% interest compounded monthly.

23. How much would you need to deposit in an account now in order to have \$20,000 in the account in 10 years? Assume the account earns 7% interest compounded quarterly.
24. How much would you need to deposit in an account now in order to have \$3,000 in the account in 15 years? Assume the account earns 2% interest compounded quarterly.
25. How much would you need to deposit in an account now in order to have \$7,000 in the account in 10 years? Assume the account earns 3% interest compounded annually.
26. You deposit \$23,000 in an account. Find the value of the investment at the end of 10 years if the account earns
- 7% interest compounded annually
 - 7% simple interest
27. You deposit \$4,000 in an account. Find the value of the investment at the end of 10 years if the account earns
- 5% interest compounded annually
 - 5% simple interest
28. A bank features a savings account that has an annual percentage rate of 2.8% with interest compounded monthly. Devin deposits \$9,500 into the account.
- How much money will Devin have in the account in 1 year?
 - What is the annual percentage yield (APY) for the savings account?
29. A bank features a savings account that has an annual percentage rate of 5.1% with interest compounded quarterly. Miranda deposits \$3,000 into the account.
- How much money will Miranda have in the account in 1 year?
 - What is the annual percentage yield (APY) for the savings account?

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CHAPTER OVERVIEW

6: Introduction to Probability

[6.1: Types of Probability](#)

[6.2: Basic Concepts](#)

[6.3: Complementary Events .](#)

[6.4: Practice Problems](#)

Learning Objectives

- Distinguish between subjective, experimental, and theoretical probabilities
- Be familiar with probability terminology and properties of probabilities
- Use the complement formula to find probabilities
- Be able to calculate odds for and/or against an event
- Create tables to calculate expected values for real world applications including insurance and product warranties

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6.1: Types of Probability

The probability of a specified event is the chance or likelihood that it will occur. There are several ways of viewing probability. One would be subjective in nature, in other words an educated guess. If someone asked you the probability that the Seattle Mariners would win their next baseball game, it would be impossible to conduct an experiment where the same two teams played each other repeatedly, each time with the same starting lineup and starting pitchers, each starting at the same time of day on the same field under the precisely the same conditions. Since there are so many variables to take into account, someone familiar with baseball and with the two teams involved might make an educated guess that there is a 75% chance they will win the game; that is, if the same two teams were to play each other repeatedly under identical conditions, the Mariners would win about three out of every four games. But this is just a guess, with no way to verify its accuracy, and depending upon how educated the educated guesser is, a subjective probability may not be worth very much. Let's consider two other types of probabilities.

Another view of probability would be experimental in nature, where we repeatedly conduct an experiment. Suppose we flipped a coin over and over and over again and it came up heads about half of the time; we would expect that in the future whenever we flipped the coin it would turn up heads about half of the time. When a weather reporter says "there is a 10% chance of rain tomorrow," she is basing that on prior evidence; that out of all days with similar weather patterns, it has rained on 1 out of 10 of those days.

We will return to the subjective and experimental probabilities from time to time, but in this course we will mostly be concerned with theoretical probability, which is defined as follows:

Definition: Theoretical Probability

Suppose there is a situation with n equally likely possible outcomes and that m of those n outcomes correspond to a particular event; then the probability of that event is defined as $\frac{m}{n}$.

$$\text{Probability of an event} = \frac{\text{number of ways the event can occur}}{\text{total number of possible outcomes}} \quad (6.1.1)$$

Let's return to the coin flipping example. Instead of calculating the probability of the coin landing on heads by flipping a coin many times and observing the number of heads (which is an experimental probability) we could instead use what we know about coins. A fair coin has 2 equally likely possible outcomes: heads or tails. If we are interested in calculating the theoretical probability of a coin landing on heads, we could do so using the formula above: $\frac{\text{number of ways a head can occur}}{\text{number of possible outcomes}} = \frac{1}{2}$.

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6.2: Basic Concepts

If you roll a die, pick a card from deck of playing cards, or randomly select a person and observe their hair color, we are executing an experiment or procedure. In probability, we look at the likelihood of different outcomes. We begin with some terminology.

Definition: Events and Outcomes

The result of an experiment is called an **outcome**.

An **event** is any particular outcome or group of outcomes.

A **simple event** is an event that cannot be broken down further.

The **sample space** is the set of all possible outcomes.

✓ Example 6.2.1

If we roll a standard 6-sided die, describe the sample space and some simple events.

Solution

Some examples of simple events:

- We roll a 1
- We roll a 5

Some examples of events:

- We roll a number bigger than 4



Basic Probability

Given that all outcomes are equally likely, we can compute the probability of an event E using this formula:

$$P(E) = \frac{\text{Number of outcomes corresponding to the event } E}{\text{Total number of equally likely outcomes}} \quad (6.2.1)$$

✓ Example 6.2.2

If we roll a 6-sided die, calculate

- P(rolling a 1)
- P(rolling a number bigger than 4)

Solution

Recall that the sample space is $\{1, 2, 3, 4, 5, 6\}$

- There is one outcome corresponding to "rolling a 1", so the probability is $\frac{1}{6}$
- There are two outcomes bigger than a 4, so the probability is $\frac{2}{6} = \frac{1}{3}$

? You Try It 6.2.1

If we roll a 6-sided die, calculate P(rolling an odd number).

Answer

$$P(\text{rolling an odd}) = 3/6 = 1/2$$

Probabilities are essentially fractions, and can be reduced to lower terms like fractions.

✓ Example 6.2.3

Let's say you have a bag with 20 cherries, 14 sweet and 6 sour. If you pick a cherry at random, what is the probability that it will be sweet?

Solution

There are 20 possible cherries that could be picked, so the number of possible outcomes is 20. Of these 20 possible outcomes, 14 are favorable (sweet), so the probability that the cherry will be sweet is $\frac{14}{20} = \frac{7}{10}$.

There is one potential complication to this example, however. It must be assumed that the probability of picking any of the cherries is the same as the probability of picking any other. This wouldn't be true if (let us imagine) the sweet cherries are smaller than the sour ones. (The sour cherries would come to hand more readily when you sampled from the bag.) Let us keep in mind, therefore, that when we assess probabilities in terms of the ratio of favorable to all potential cases, we rely heavily on the assumption of equal probability for all outcomes.

? You Try It 6.2.2

At some random moment, you look at your clock and note the minutes reading.

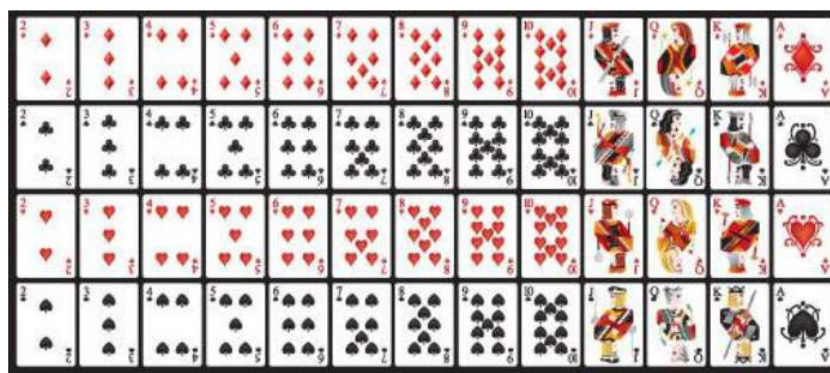
- What is probability the minutes reading is 15?
- What is the probability the minutes reading is 15 or less?

Answer

a. 1/60 b. 16/60

♠ Cards

A standard deck of 52 playing cards consists of four suits (hearts, spades, diamonds and clubs). Spades and clubs are black while hearts and diamonds are red. Each suit contains 13 cards, each of a different rank: an Ace (which in many games functions as both a low card and a high card), cards numbered 2 through 10, a Jack, a Queen and a King. The Jack, Queen, and King are called face cards.



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✓ Example 6.2.4

Compute the probability of randomly drawing one card from a deck and getting an Ace.

Solution

There are 52 cards in the deck and 4 Aces so $P(Ace) = \frac{4}{52} = \frac{1}{13} \approx 0.0769$

We can also think of probabilities as percents: There is a 7.69% chance that a randomly selected card will be an Ace.

Notice that the smallest possible probability is 0 (there are no outcomes that correspond with the event). The largest possible probability is 1 (all possible outcomes correspond with the event).

Definition: Certain and Impossible Events

An **impossible** event has a probability of 0 .

A **certain** event has a probability of 1 .

The probability of any event must be $0 \leq P(E) \leq 1$

Note

In the course of this chapter, if you compute a probability and get an answer that is negative or greater than 1, you have made a mistake and should check your work.

✓ Example 6.2.5

Find the probability of drawing a single card from a deck and getting a red spade.

Solution

There are 52 cards in the deck, but all spades are black. It is impossible to draw a single card and have that card be a red spade. This is an impossible event, so

$$\mathbb{P}(\text{redspade}) = \frac{0}{52} = 0 \quad (6.2.2)$$

✓ Example 6.2.6

You wake up on a cloudy, drizzly Seattle day to the weatherman saying the chance of rain today is 100%. Find the probability that it will rain today.

Solution

Since we are told the chance of rain is 100%, we know that this is a certain event.

$$\mathbb{P}(\text{rain}) = \frac{100}{100} = 1 \quad (6.2.3)$$

? You Try It 6.2.3

Your neighbor tells you that he heard on the radio that the probability of an earthquake of magnitude 7.0 or higher hitting California in the next 10 years is 1.2. How do you know that this is incorrect?

Answer

Probabilities must be between 0 and 1, so a probability of 1.2 is impossible.

6.3: Complementary Events .

Now let us examine the probability that an event does not happen. As in the previous section, consider the situation of rolling a six-sided die and first compute the probability of rolling a six: the answer is $P(\text{six}) = 1/6$. Now consider the probability that we do not roll a six: there are 5 outcomes that are not a six, so the answer is $P(\text{not a six}) = \frac{5}{6}$.

Notice that $P(\text{six}) + P(\text{not a six}) = \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1$ This is not a coincidence.

Definition: Complement of an Event

The **complement** of an event is the event " E does NOT happen".

The notation $\bar{E}$ is used for the complement of event E . You should recognize this notation from the last chapter.

For any event E , $P(E) + P(\bar{E}) = 1$

We can compute the probability of the complement using $P(\bar{E}) = 1 - P(E)$

Notice also that $P(E) = 1 - P(\bar{E})$

✓ Example 6.3.1

If you pull a random card from a deck of playing cards, what is the probability it is not a heart?

Solution

There are 13 hearts in the deck, so $P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$.

The probability of not drawing a heart is the complement:

$$P(\text{not heart}) = 1 - P(\text{heart}) = 1 - \frac{1}{4} = \frac{3}{4} \quad (6.3.1)$$

✓ Example 6.3.2

A jar contains 28 marbles, 12 of which are red. If you pick 1 marble out of the jar, what is the probability that it is not red?

Solution

There are 12 red marbles out of 28 total marbles, so $P(\text{red}) = \frac{12}{28} = \frac{3}{7}$.

The probability of not drawing a red marble is the complement:

$$P(\text{not red}) = 1 - \frac{3}{7} = \frac{4}{7} \quad (6.3.2)$$

? Exercise 6.3.1

Your favorite basketball player is an 84% free throw shooter. Find the probability that they do NOT make their next free throw.

Answer

$$1 - 0.84 = 0.16$$

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6.4: Practice Problems

1. Suppose you roll a six-sided die. Determine the following:

- $P(\text{rolling a 5})$
- $P(\text{rolling an even number})$
- $P(\text{rolling a number bigger than 2})$

2. Compute the probability of tossing a six-sided die and getting a 7.

3. Compute the probability of tossing a six-sided die and getting a number less than 3.

4. Compute the probability of rolling a 12-sided die and getting a number other than 8.

5. A test was given to a group of students. The grades and gender are summarized below.

	A	B	C	Total
Male	8	18	13	39
Female	10	4	12	26
Total	18	22	25	65

- Find the probability that a student chosen at random was female.
- Find the probability that a student chosen at random did NOT earn a C?

6. The table below shows the number of credit cards owned by a group of individuals.

	Zero	One	Two or more	Total
Male	9	5	19	33
Female	18	10	20	48
Total	27	15	39	81

- Find the probability that a person chosen at random had no credit cards.
- Find the probability that a person chosen at random has at least one credit card?

7. A ball is drawn randomly from a jar that contains 6 red balls, 2 white balls, and 5 yellow balls. Find the probability of the given event.

- A red ball is drawn
- A white ball is drawn

8. A bag contains 2 gold marbles, 10 silver marbles, and 25 black marbles. You randomly select one marble from the bag. What is the probability that you select a gold marble?

9. Suppose you write each letter of the alphabet on a different slip of paper and put the slips into a hat. What is the probability of drawing one slip of paper from the hat at random and getting:

- A consonant
- A vowel

10. A group of people were asked if they had run a red light in the last year. 150 responded "yes", and 185 responded "no". Find the probability that if a person is chosen at random, they have run a red light in the last year.

11. In a survey, 205 people indicated they prefer cats, 160 indicated they prefer dogs, and 40 indicated they don't enjoy either pet. Find the probability that if a person is chosen at random, they prefer cats.

12. If you pick one card at random from a standard deck of 52 cards, what is the probability it will be a King?

13. If you pick one card at random from a standard deck of 52 cards, what is the probability it will be a Diamond?

14. If you pick one card at random from a standard deck of 52 cards, what is the probability it is not the Ace of Spades?
15. If you pick one card at random from a standard deck of 52 cards, what is the probability it is not a heart?
16. If you pick one card at random from a standard deck of 52 cards, what is the probability it will be an 8 ?
17. If you pick one card at random from a standard deck of 52 cards, what is the probability it is not a 6 ?
18. If you pick one card at random from a standard deck of 52 cards, what is the probability it will be a face card?
19. If the probability of a certain event is $\frac{24}{61}$, what is the probability of the event not happening?
20. If the probability of a certain event not happening is $\frac{38}{45}$, what is the probability of the event happening?
21. Find the odds for rolling a die and getting a 1.
22. Find the odds for rolling a die and getting an even number.
23. If you pick one card at random from a standard deck of 52 cards, what are the odds it will be an ace?
24. If you pick one card at random from a standard deck of 52 cards, what are the odds it will be a face card?
25. If the probability of a certain event is $\frac{51}{92}$, what are the odds against the event happening?
26. If the odds against a certain event are 2 to 7 , what is the probability of the event occurring?
27. If you pick one card at random from a standard deck of 52 cards, what are the odds it will not be a heart?
28. Find the odds for rolling a die and not getting a 6 .

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7: Probability and Counting

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Learning Objectives

- Be able to calculate “and”/“or” probabilities
- Determine if two or more events are independent
- Determine if two or more events are mutually exclusive
- Use tables and tree diagrams to model possible outcomes in counting problems
- Use permutations and combinations for counting
- Be able to find probabilities using permutations and combinations

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7.1: Independent Events

✓ Example 7.1.1

Suppose we flipped a coin and rolled a die, and wanted to know the probability of getting a head on the coin and a 6 on the die.

Solution

We could list all possible outcomes: {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}. Notice there are $2 \cdot 6 = 12$ total outcomes. Out of these, only 1 is the desired outcome, so the probability is $P(\text{head}) = \frac{1}{12}$.

Definition: Independent Events

Events A and B are **independent events** if the probability of Event B occurring is the same whether or not Event A occurs.

✓ Example 7.1.2

Are these events independent?

- A fair coin is tossed two times. The two events are (1) first toss is a head and (2) second toss is a head.
- You draw a card from a deck, then draw a second card without replacing the first.

Solution

- The probability that a head comes up on the second toss is $1/2$ regardless of whether or not a head came up on the first toss, so these events are independent.
- The probability of the first card being red is $P(\text{first card is red}) = \frac{26}{52} = \frac{1}{2}$. Since the first card is not returned to the deck, the probability of the second card being red is not the same as the probability that the first card is red. After drawing the first card, there are only 51 cards remaining in the deck. If the first card was red, then there are only 25 red cards remaining, and the probability that the second card would be red is $P(\text{second card is red}) = \frac{25}{51}$. If the first card was not red, then the probability that the second card would be red is $P(\text{second card is red}) = \frac{26}{51}$. These events are NOT independent.

When two events are independent, the probability of both occurring is just the product of the probabilities of the individual events.

P(A and B) for Independent Events

If events A and B are independent, then the probability of both A and B occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B) \quad (7.1.1)$$

where $P(A \text{ and } B)$ is the probability of events A and B both occurring, $P(A)$ is the probability of event A occurring, and $P(B)$ is the probability of event B occurring

If you look back at the coin and die example from earlier, you can see how the number of outcomes of the first event multiplied by the number of outcomes in the second event multiplied to equal the total number of possible outcomes in the combined event.

✓ Example 7.1.1

In Alex's drawer she has 10 pairs of socks, 6 of which are white, and 7 tee shirts, 3 of which are white. If she randomly reaches in and pulls out a pair of socks and a tee shirt, what is the probability that both are white?

Solution

The probability of choosing a white pair of socks is $\frac{6}{10}$.

The probability of choosing a white tee shirt is $\frac{3}{7}$.

The probability of both being white is $\frac{6}{10} \cdot \frac{3}{7} = \frac{18}{70} = \frac{9}{35}$

? Exercise 7.1.1

A couple has two children. Find the probability that both children are girls.

Answer

1/4

✓ Example 7.1.4

You roll two die. Find the probability that the first die lands on an even number, and the second die lands on 5.

Solution

Since the outcome of the first die has no effect on the outcome of the second die, the events are independent.

$$P(\text{first die is even}) = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{second die lands on 5}) = \frac{1}{6}$$

$$P(\text{first die is even AND second die lands on 5}) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

As a check, it may be helpful to consider all possible outcomes in the experiment of rolling a pair of dice:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Notice there are 3 outcomes (out of 36 possible outcomes) where the first die is even and the second die lands on 5. We could have found this probability directly as $\frac{3}{36} = \frac{1}{12}$.

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7.2: Conditional Probability

Often it is required to compute the probability of an event given that another event has occurred.

✓ Example 7.2.1

What is the probability that two cards drawn at random from a deck of playing cards will both be aces?

Solution

It might seem that you could use the formula for the probability of two independent events and simply multiply $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$. This would be incorrect, however, because the two events are not independent. Once the first card is drawn, there are only 51 cards remaining in the deck. If the first card drawn is an ace, then the probability that the second card is also an ace would be $\frac{3}{51}$ because there would only be three aces left in the deck.

Once the first card chosen is an ace, the probability that the second card chosen is also an ace is called the *conditional probability* of drawing an ace. In this case, the "condition" is that the first card is an ace. Symbolically, we write this as:

$P(\text{ace on second draw} \mid \text{an ace on the first draw})$.

The vertical bar "|" is read as "given," so the above expression is short for "The probability that an ace is drawn on the second draw given that an ace was drawn on the first draw." What is this probability? After an ace is drawn on the first draw, there are 3 aces out of 51 total cards left. This means that the conditional probability of drawing an ace after one ace has already been drawn is $\frac{3}{51} = \frac{1}{17}$.

Thus, the probability of both cards being aces is $\frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$.

Definition: Conditional Probability

The probability that event B occurs, given that event A has happened, is represented as $P(B \mid A)$. This is read as "the probability of B given A."

✓ Example 7.2.2

Find the probability that a die rolled shows a 6, given that a flipped coin shows a head.

Solution

These are two independent events, so the probability of the die rolling a 6 is $\frac{1}{6}$, regardless of the result of the coin flip.

✓ Example 7.2.3

The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their cars. Find the probability that a randomly chosen person:

- Has a speeding ticket given they have a red car
- Has a red car given they have a speeding ticket

	Speeding Ticket	No Speeding Ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

Solution

a) Since we know the person has a red car, we are only considering the 150 people in the first row of the table. Of those, 15 have a speeding ticket, so

$$P(\text{ticket} \mid \text{red car}) = \frac{15}{150} = \frac{1}{10} = 0.1 \quad (7.2.1)$$

b) Since we know the person has a speeding ticket, we are only considering the 60 people in the first column of the table. Of those, 15 have a red car, so

$$P(\text{red car} \mid \text{ticket}) = \frac{15}{60} = \frac{1}{4} = 0.25. \quad (7.2.2)$$

Notice from the last example that $P(B \mid A)$ is not equal to $P(A \mid B)$.

These kinds of conditional probabilities are what insurance companies use to determine your insurance rates. They look at the conditional probability of you having an accident, given your age, your car, your car color, your driving history, etc., and price your policy based on that likelihood.

P(A and B) for dependent events

If Events A and B are **dependent** (not independent), then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

✓ Example 7.2.4

If you pull 2 cards out of a deck, what is the probability that both are spades?

Solution

The probability that the first card is a spade is $\frac{13}{52}$.

The probability that the second card is a spade, given the first was a spade, is $\frac{12}{51}$, since there is one less spade in the deck, and one less total cards.

The probability that both cards are spades is $\frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} \approx 0.0588$

? You Try It 7.2.1

If you draw 2 cards from a standard deck, what is the probability that both cards are red kings?

Answer

0.00075

✓ Example 7.2.5

A home pregnancy test was given to women, then pregnancy was verified through blood tests. The following table shows the home pregnancy test results. Find

a) $P(\text{not pregnant} \mid \text{positive test result})$

b) $P(\text{positive test result} \mid \text{not pregnant})$

	Positive test	Negative test	Total
Pregnant	70	4	74
Not Pregnant	5	14	19

Total	75	18	93
-------	----	----	----

Solution

a) Since we know the test result was positive, we're limited to the 75 women in the first column, of which 5 were not pregnant.

$$P(\text{not pregnant} \mid \text{positive test result}) = \frac{5}{75} \approx 0.067 \quad (7.2.3)$$

.

b) Since we know the woman is not pregnant, we are limited to the 19 women in the second row, of which 5 had a positive test.

$$P(\text{positive test result} \mid \text{not pregnant}) = \frac{5}{19} \approx 0.263 \quad (7.2.4)$$

The second result is what is usually called a false positive: A positive result when the woman is not actually pregnant.

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7.3: "Or" Probabilities

The previous examples looked at the probability of both events occurring. Now we will look at the probability of either event occurring.

✓ Example 7.3.1

Suppose we flipped a coin and rolled a die, and wanted to know the probability of getting a head on the coin or a 6 on the die.

Solution

Here, there are still 12 possible outcomes: {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

By simply counting, we can see that 7 of the outcomes have a head on the coin or a 6 on the die or both - we use or inclusively here (these 7 outcomes are H1, H2, H3, H4, H5, H6, T6), so the probability is $\frac{7}{12}$. How could we have found this from the individual probabilities?

As we would expect, $\frac{1}{2}$ of these outcomes have a head, and $\frac{1}{6}$ of these outcomes have a 6 on the die. If we add these, $\frac{1}{2} + \frac{1}{6} = \frac{6}{12} + \frac{2}{12} = \frac{8}{12}$, which is not the correct probability. Looking at the outcomes we can see why: the outcome H6 would have been counted twice, since it contains both a head and a 6; the probability of both a head and rolling a 6 is $\frac{1}{12}$.

If we subtract out this double count, we have the correct probability: $\frac{8}{12} - \frac{1}{12} = \frac{7}{12}$.

P(A or B)

The probability of either A or B occurring (or both) is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

✓ Example 7.3.2

Suppose we draw one card from a standard deck. What is the probability that we get a Queen or a King?

Solution

Half the cards are red, so $P(\text{red}) = \frac{26}{52}$

There are four kings, so $P(\text{King}) = \frac{4}{52}$

There are two red kings, so $P(\text{Red and King}) = \frac{2}{52}$

We can then calculate

$$P(\text{Red or King}) = P(\text{Red}) + P(\text{King}) - P(\text{Red and King}) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$$

In the last example, the events were **mutually exclusive**, so $P(A \text{ or } B) = P(A) + P(B)$.

✓ Example 7.3.3

Suppose we draw one card from a standard deck. What is the probability that we get a red card or a King?

Solution

Half the cards are red, so $P(\text{red}) = \frac{26}{52}$

There are four kings, so $P(\text{King}) = \frac{4}{52}$

There are two red kings, so $P(\text{Red and King}) = \frac{2}{52}$

We can then calculate

$$P(\text{Red or King}) = P(\text{Red}) + P(\text{King}) - P(\text{Red and King}) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$$

? Exercise 7.3.4

Consider again that we roll pair of dice. What is the probability that the sum of the pair of dice is odd or one of the die is a 1?

Answer

Below is a list of all possible outcomes in the experiment of rolling 2 die:

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Note: The table gives the values of each die, not the sum of the two die.

Since it is not possible for the sum to be both odd and 4, these events are **mutually exclusive**.

$$\begin{aligned}
 P(\text{sum is odd}) &= \frac{18}{36} = \frac{1}{2} \\
 P(\text{sum is 4}) &= \frac{3}{36} = \frac{1}{12} \\
 P(\text{sum is odd and 4}) &= \frac{0}{36} = 0 \\
 P(\text{sum is odd or 4}) &= P(\text{sum is odd}) + P(\text{sum is 4}) - P(\text{sum is odd and 4}) \\
 &= \frac{18}{36} + \frac{3}{36} - \frac{0}{36} = \frac{21}{36} \\
 &= \frac{7}{12}
 \end{aligned}
 \tag{7.3.1}$$

? You Try It 7.3.1

Consider again that we roll a pair of dice. What is the probability that the sum of the two die is odd or one of the die is a 1?

Answer

23/36

✓ Example 7.3.5

The table below shows the number of survey subjects who have received and not received a speeding ticket in the last year, and the color of their car. Find the probability that a randomly chosen person:

- Has a red car and got a speeding ticket

b) Has a red car or got a speeding ticket.

	Speeding ticket	No speeding ticket	Total
Red car	15	135	150
Not red car	45	470	515
Total	60	605	665

Solution

a) We can see that 15 people of the 665 surveyed had both a red car and got a speeding ticket, so the probability is

$$P(\text{red car and speeding ticket}) = \frac{15}{665} \approx 0.0226.$$

Notice that having a red car and getting a speeding ticket are not independent events, so the probability of both of them occurring is not simply the product of probabilities of each one occurring.

b) We could answer this question by simply adding up the numbers: 15 people with red cars and speeding tickets + 135 with red cars but no ticket + 45 with a ticket but no red car = 195 people. So the probability is

$$P(\text{red car or speeding ticket}) = \frac{195}{665}.$$

We also could have found this probability by:

$$P(\text{red car}) + P(\text{speeding ticket}) - P(\text{red car and speeding ticket})$$

$$P(\text{red car or speeding ticket}) = \frac{150}{665} + \frac{60}{665} - \frac{15}{665} = \frac{195}{665} \approx 0.2932.$$

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7.4: Practice Problems

1. A six-sided die is rolled twice. What is the probability of showing a 2 on the first roll and an odd number on the second roll?
2. A six-sided die is rolled three times. What is the probability of showing an even number on all three rolls?
3. A six-sided die is rolled twice. What is the probability of showing a 6 on both rolls?
4. A fair coin is flipped twice. What is the probability of showing heads on both flips?
5. A die is rolled twice. What is the probability of showing a 5 on the first roll and an even number on the second roll?
6. A couple has three children. What is the probability that all three are girls?
7. Suppose that 21% of people own dogs. If you pick two people at random, what is the probability that they both own a dog?
8. In your drawer you have 5 pairs of socks, 4 of which are white, and 11 tee shirts, 2 of which are white. If you randomly reach in and pull out a pair of socks and a tee shirt, what is the probability that both are white?
9. Bert and Ernie each have a well-shuffled standard deck of 52 cards. They each draw one card from their own deck. Compute the probability that:
 - a. Bert and Ernie both draw an Ace.
 - b. Bert draws an Ace but Ernie does not.
 - c. Neither Bert nor Ernie draws an Ace.
 - d. Bert and Ernie both draw a heart.
 - e. Bert gets a card that is not a Jack and Ernie draws a card that is not a heart.
10. Bert has a well-shuffled standard deck of 52 cards, from which he draws one card; Ernie has a 12-sided die, which he rolls at the same time Bert draws a card. Compute the probability that:
 - a. Bert gets a Jack and Ernie rolls a five.
 - b. Bert gets a heart and Ernie rolls a number less than six.
 - c. Bert gets a face card (Jack, Queen or King) and Ernie rolls an even number.
 - d. Bert gets a red card and Ernie rolls a fifteen.
 - e. Bert gets a card that is not a Jack and Ernie rolls a number that is not twelve.
11. A jar contains 5 red marbles numbered 1 to 5 and 8 blue marbles numbered 1 to 8 . A marble is drawn at random from the jar. Find the probability the marble is
 - a. Even-numbered given that the marble is red.
 - b. Red given that the marble is even-numbered.
12. A jar contains 4 red marbles numbered 1 to 4 and 8 blue marbles numbered 1 to 8 . A marble is drawn at random from the jar. Find the probability the marble is
 - a. Odd-numbered given that the marble is blue.
 - b. Blue given that the marble is odd-numbered.
13. Compute the probability of flipping a coin and getting heads, given that the previous flip was tails.
14. Find the probability of rolling a " 1 " on a fair die, given that the last 3 rolls were all ones.
15. Suppose a math class contains 25 students, 14 females (three of whom speak French) and 11 males (two of whom speak French).
 - a. Compute the probability that a randomly selected student speaks French, given that the student is female.
 - b. Compute the probability that a randomly selected student is male, given that the student speaks French.
16. A test was given to a group of students. The grades and gender are summarized below.

	A	B	C	Total
Male	10	12	2	24
Female	16	6	9	31

Total	26	18	11	55
-------	----	----	----	----

Suppose a student is chosen at random:

- Find the probability that the student was male given they earned an A.
- Find the probability that the student was male given they earned a C.
- Find the probability that the student was female given they earned a B.

17. A test was given to a group of students. The grades and gender are summarized below.

	A	B	C	Total
Male	7	6	4	17
Female	9	2	10	21
Total	16	8	14	38

Suppose a student is chosen at random:

- Find the probability that the student earned a B given they are male.
- Find the probability that the student earned a B given they are female.
- Find the probability that the student earned a C given they are male.

18. A certain virus infects one in every 400 people. A test used to detect the virus in a person is positive 90% of the time if the person has the virus and 10% of the time if the person does not have the virus. Let A be the event "the person is infected" and B be the event "the person tests positive".

- Find the probability that a person has the virus given that they have tested positive, i.e. find $P(A | B)$.
- Find the probability that a person does not have the virus given that they test negative, i.e. find $P(\text{not } A | \text{not } B)$.

19. A certain virus infects one in every 2000 people. A test used to detect the virus in a person is positive 96% of the time if the person has the virus and 4% of the time if the person does not have the virus. Let A be the event "the person is infected" and B be the event "the person tests positive".

- Find the probability that a person has the virus given that they have tested positive, i.e. find $P(A | B)$.
- Find the probability that a person does not have the virus given that they test negative, i.e. find $P(\text{not } A | \text{not } B)$.

20. Two cards are drawn from a standard deck of cards. What is the probability of drawing a King and then drawing a Queen.

21. Two cards are drawn from a standard deck of cards. What is the probability of both cards being red?

22. Two cards are drawn from a standard deck of cards. What is the probability of drawing a Jack and then drawing an Ace?

23. Five cards are drawn from a standard deck of cards. What is the probability of all cards being black?

24. Tony buys a bag of cookies that contains 4 chocolate chip cookies, 9 peanut butter cookies, 7 sugar cookies and 8 oatmeal cookies. What is the probability that Tony reaches in the bag and randomly selects a peanut butter cookie from the bag, eats it, then reaches back in the bag and randomly selects a sugar cookie?

25. Suppose a jar contains 17 red marbles and 32 blue marbles. If you reach in the jar and pull out 2 marbles at random, find the probability that both are red.

26. Suppose you write each letter of the alphabet on a different slip of paper and put the slips into a hat. If you pull out two slips at random, find the probability that both are vowels.

27. A math class consists of 25 students, 14 female and 11 male. Two students are selected at random to participate in a probability experiment. Compute the probability that

- a male is selected, then a female.
- a female is selected, then a male.
- two males are selected.

- d. two females are selected.
- e. no males are selected.

28. A math class consists of 25 students, 14 female and 11 male. Three students are selected at random to participate in a probability experiment. Compute the probability that

- a. a male is selected, then two females.
- b. a female is selected, then two males.
- c. two females are selected, then one male.
- d. three males are selected.
- e. three females are selected.

29. A test was given to a group of students. The grades and gender are summarized below.

	A	B	C	Total
Male	8	18	13	39
Female	10	4	12	26
Total	18	22	25	65

Suppose a student is chosen at random:

- a. Find the probability that the student is female and earned an A.
- b. Find the probability that the student is male and earned an A.
- c. Find the probability that the student is male or earned a C.
- d. Find the probability that the student is female or earned a B.

30. The table below shows the number of credit cards owned by a group of individuals.

	Zero	One	Two or more	Total
Male	9	5	19	33
Female	18	10	20	48
Total	27	15	39	81

Suppose a person is chosen at random:

- a. Find the probability that the person is male and has two or more credit cards.
- b. Find the probability that the person is female and has one credit card.
- c. Find the probability that the person is male or has no credit cards.
- d. Find the probability that the person is female or has two or more credit cards.

31. Suppose we draw one card from a standard deck. What is the probability of drawing the King of hearts or a Queen?

32. Suppose you roll a six-sided die. What is the probability that you roll an even number or a five?

33. Suppose we draw one card from a standard deck. What is the probability of drawing a King or a heart?

34. Suppose we draw one card from a standard deck. What is the probability of drawing a face card or a diamond?

35. Suppose we draw one card from a standard deck. What is the probability of drawing a Jack or a black card?

36. Two six-sided dice are rolled. What is the probability of getting a sum of either 11 or 12 ?

37. A jar contains 6 red marbles numbered 1 to 6 and 8 blue marbles numbered 1 to 8 . A marble is drawn at random from the jar. Find the probability the marble is red or odd-numbered.

38. A jar contains 4 red marbles numbered 1 to 4 and 10 blue marbles numbered 1 to 10. A marble is drawn at random from the jar. Find the probability the marble is blue or even-numbered.

39. A jar contains 4 red marbles numbered 1 to 4 and 12 blue marbles numbered 1 to 12. A marble is drawn at random from the jar. Find the probability of the given event.

- a. The marble is red or odd-numbered
- b. The marble is blue and even-numbered

40. Given the following information, determine $P(A \text{ or } B)$.

$$\begin{aligned}P(A) &= 0.56 \\P(B) &= 0.53 \\P(A \text{ and } B) &= 0.42 \\P(B | A) &= 0.3\end{aligned}\tag{7.4.1}$$

41. Given the following information, determine $P(A \text{ or } B)$.

$$\begin{aligned}P(A) &= 0.76 \\P(B) &= 0.3 \\P(A \text{ and } B) &= 0.28 \\P(B | A) &= 0.2\end{aligned}\tag{7.4.2}$$

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8: Statistics

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Learning Objectives

- Categorize data as either qualitative or quantitative
- Classify the type of experimental study
- Use bar graphs and pie charts to graphically display data
- Use a histogram to graphically display data

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8.1: Categorizing Data

Once we have gathered data, we might wish to classify it. Roughly speaking, data can be classified as categorical data or quantitative data.

Quantitative and Categorical (Qualitative) Data

Categorical (qualitative) data are pieces of information that allow us to classify the objects under investigation into various categories.

Quantitative data are responses that are numerical in nature and with which we can perform meaningful arithmetic calculations.

✓ Example 8.1.1

We might conduct a survey to determine the name of the favorite movie that each person in a math class saw in a movie theater.

Solution

When we conduct such a survey, the responses would look like: *Finding Nemo*, *The Hulk*, or *Terminator 3: Rise of the Machines*. We might count the number of people who give each answer, but the answers themselves do not have any numerical values: we cannot perform computations with an answer like "*Finding Nemo*." This would be categorical data.

✓ Example 8.1.2

A survey could ask the number of movies you have seen in a movie theater in the past 12 months (0, 1, 2, 3, 4, ...)

Solution

This would be quantitative data.

Other examples of quantitative data would be the running time of the movie you saw most recently (104 minutes, 137 minutes, 104 minutes, ...) or the amount of money you paid for a movie ticket the last time you went to a movie theater (\$5.50, \$7.75, \$9, ...).

Sometimes, determining whether or not data is categorical or quantitative can be a bit trickier.

✓ Example 8.1.3

Suppose we gather respondents' ZIP codes in a survey to track their geographical location.

Solution

ZIP codes are numbers, but we can't do any meaningful mathematical calculations with them (it doesn't make sense to say that 98036 is "twice" 49018 — that's like saying that Lynnwood, WA is "twice" Battle Creek, MI, which doesn't make sense at all), so ZIP codes are really categorical data.

✓ Example 8.1.4

A survey about the movie you most recently attended includes the question "How would you rate the movie you just saw?" with these possible answers:

1 - it was awful 2 - it was just OK 3 - I liked it 4 - it was great 5 - best movie ever!

Solution

Again, there are numbers associated with the responses, but we can't really do any calculations with them: a movie that rates a 4 is not necessarily twice as good as a movie that rates a 2, whatever that means; if two people see the movie and one of them thinks it stinks and the other thinks it's the best ever it doesn't necessarily make sense to say that "on average they liked it."

As we study movie-going habits and preferences, we shouldn't forget to specify the population under consideration. If we survey 3-7 year-olds the runaway favorite might be *Finding Nemo*. 13-17 year-olds might prefer *Terminator 3*. And 33-37 year-olds might prefer...well, *Finding Nemo*.

? Exercise 8.1.1

Classify each measurement as categorical or quantitative

- a. Eye color of a group of people
- b. Daily high temperature of a city over several weeks
- c. Annual income

Answer

- a. Categorical. b. Quantitative c. Quantitative

Once we have collected data from surveys or experiments, we need to summarize and present the data in a way that will be meaningful to the reader. We will begin with graphical presentations of data then explore numerical summaries of data.

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8.2: Presenting Categorical Data Graphically

Categorical, or qualitative, data are pieces of information that allow us to classify the objects under investigation into various categories. We usually begin working with categorical data by summarizing the data into a **frequency table**.

Frequency Table

A frequency table is a table with two columns. One column lists the possible values of the variable, and the other lists the corresponding frequency for each value (how many items fit into each category).

Many times a 3rd column is added, called the relative frequency column. The relative frequency expresses the frequency of each possible value, relative to the whole. Relative frequencies can be written as fractions, decimals, or percentages.

✓ Example 8.2.1

An insurance company determines vehicle insurance premiums based on known risk factors. If a person is considered a higher risk, their premiums will be higher. One potential factor is the color of your car. The insurance company believes that people with some color cars are more likely to get in accidents. To research this, they examine police reports for recent total-loss collisions. The data is summarized in the frequency table below.

Color	Frequency
Blue	25
Green	52
Red	41
White	36
Black	39
Grey	23

Solution

When you add together the frequencies for each possible value of the variable, you end up with the total number of observed values. In this case, that total is 216.

If we wanted to add a relative frequency column, we would find the relative frequency for each color by dividing the corresponding frequency by the total number of observations. The new table would be as follows:

Color	Frequency	Relative Frequency
Blue	25	$\frac{25}{216} = .1157 = 11.57\%$
Green	52	$\frac{52}{216} = .2407 = 24.07\%$
Red	41	$\frac{41}{216} = .1898 = 18.98\%$
White	36	$\frac{36}{216} = .1667 = 16.67\%$
Black	39	$\frac{39}{216} = .1806 = 18.06\%$
Grey	23	$\frac{23}{216} = .1065 = 10.65\%$

Note: The relative frequency column should always sum to 1 (or 100%) since you are representing the entire data set.

Sometimes we need an even more intuitive way of displaying data. This is where charts and graphs come in. There are many, many ways of displaying data graphically, but we

will concentrate on one very useful type of graph called a bar graph. In this section we will work with bar graphs that display categorical data; the next section will be devoted to histograms that display quantitative data.

Bar Graph

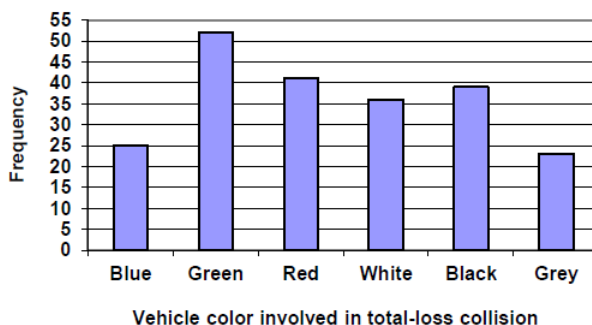
A **bar graph** is a graph that displays a bar for each category with the length of each bar indicating the frequency of that category.

To construct a bar graph, we need to draw a vertical axis and a horizontal axis. The vertical direction will have a scale and measure the frequency of each category; the horizontal axis has no scale in this instance, and represents the possible values of the variable of interest. The construction of a bar chart is most easily described by use of an example.

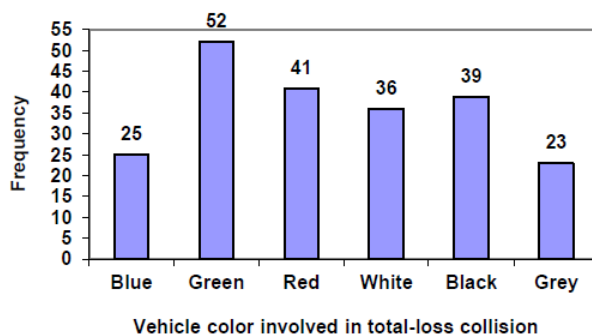
✓ Example 8.2.2

Using our car data from above, note the highest frequency is 52, so our vertical axis needs to go from 0 to 52, but we might as well use 0 to 55, so that we can put a hash mark every 5 units:

Solution



Notice that the height of each bar is determined by the frequency of the corresponding color. The horizontal gridlines are a nice touch, but not necessary. In practice, you will find it useful to draw bar graphs using graph paper, so the gridlines will already be in place, or using technology. Instead of gridlines, we might also list the frequencies at the top of each bar, like this:



In this case, our chart might benefit from being reordered from largest to smallest frequency values. This arrangement can make it easier to compare similar values in the chart, even without gridlines. When we arrange the categories in decreasing frequency order like this, it is called a **Pareto chart**.

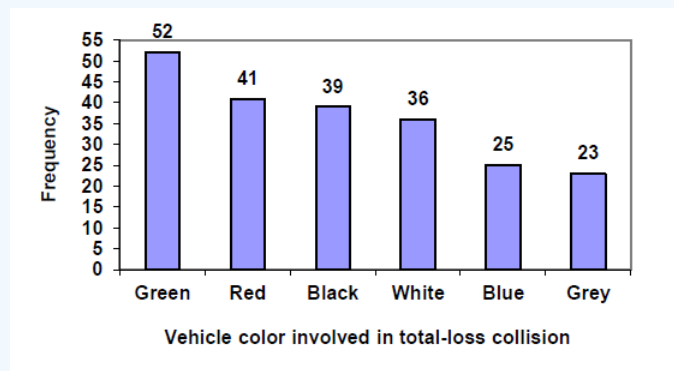
Pareto Chart

A **Pareto chart** is a bar graph ordered from highest to lowest frequency

✓ Example 8.2.3

Transforming our bar graph from earlier into a Pareto chart, we get:

Solution



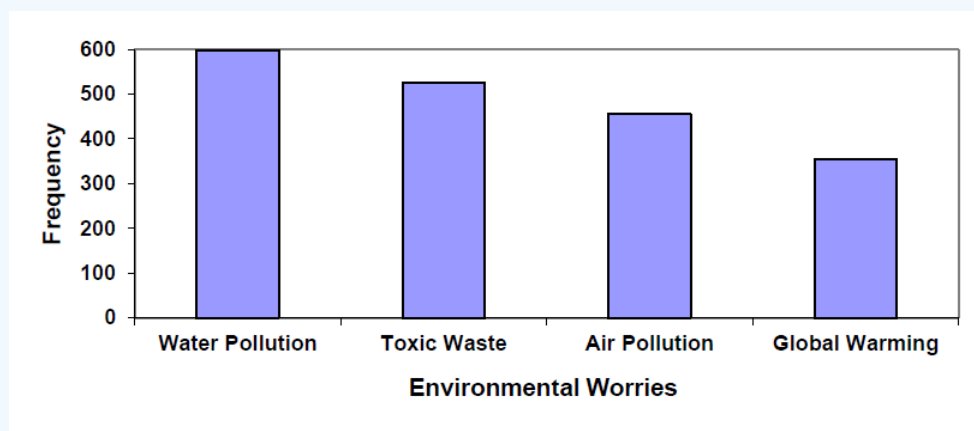
✓ Example 8.2.4

In a survey adults were asked whether they personally worried about a variety of environmental concerns. The numbers (out of 1012 surveyed) who indicated that they worried “a great deal” about some selected concerns are summarized below.

Environmental Issue	Frequency
Pollution of drinking water	597
Contamination of soil and water by toxic waste	526
Air pollution	455
Global warming	354

Solution

This data could be shown graphically in a bar graph:

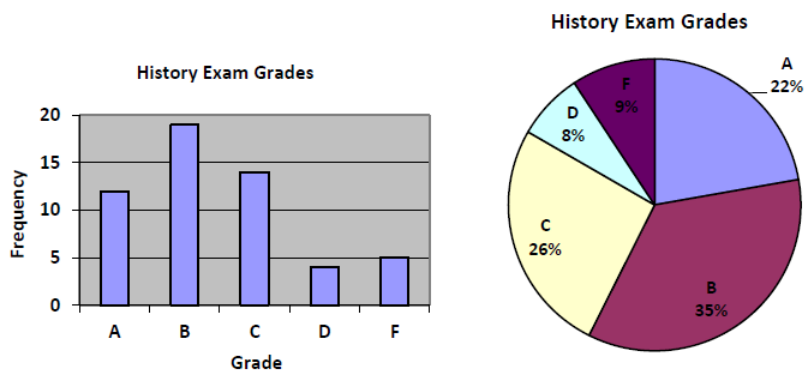


? Exercise 8.2.5

Create a frequency table and bar graph to illustrate the grades on a history exam below.

A: 12 students, B: 19 students, C: 14 students, D: 4 students, F: 5 students

Answer



To show relative sizes, it is common to use a **pie chart**.

Pie Chart

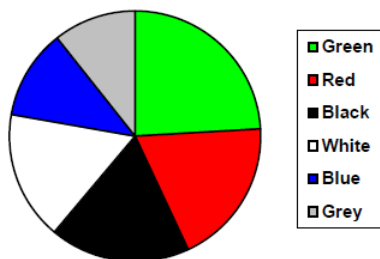
A **pie chart** is a circle with wedges cut of varying sizes marked out like slices of pie or pizza. The relative sizes of the wedges correspond to the relative frequencies of the categories. These relative frequencies are usually expressed as a percentage.

✓ Example 8.2.6

For our vehicle color data, a pie chart might look like this:

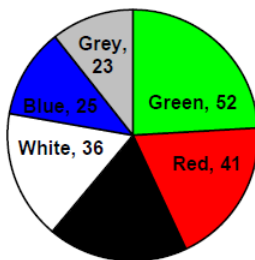
Solution

Vehicle color involved in total-loss collisions



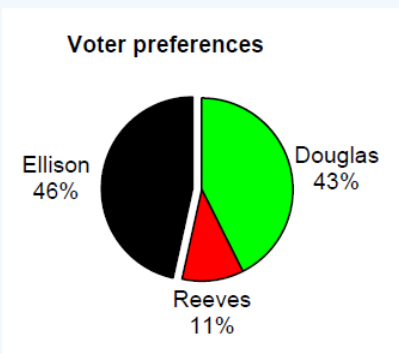
Pie charts can often benefit from including frequencies or relative frequencies (percentages) in the chart next to the pie slices. Often having the category names next to the pie slices also makes the chart clearer.

Vehicle color involved in total-loss collisions



✓ Example 8.2.7

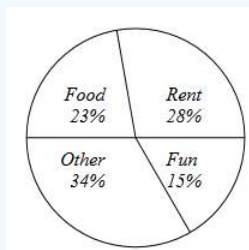
The pie chart below shows the percentage of voters supporting each candidate running for a local senate seat. If there are 20,000 voters in the district, the pie chart shows that about 11% of those, about 2,200 voters, support Reeves.



Pie charts look nice, but are harder to draw by hand than bar charts since to draw them accurately we would need to compute the angle each wedge cuts out of the circle, then measure the angle with a protractor. Computers are much better suited to drawing pie charts. Common software programs like Microsoft Word or Excel, OpenOffice.org, Write or Calc, or Google Docs are able to create bar graphs, pie charts, and other graph types. There are also numerous online tools that can create graphs.

? Exercise 8.2.8

Logan categorized his spending for this month into four categories: Rent, Food, Fun, and Other. The percentages he spent in each category are pictured here. If Logan spent a total of \$2400 this month, how much did he spend on Food?



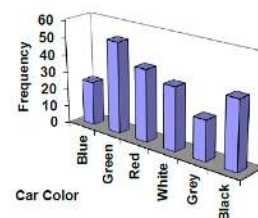
Answer

$$\$2400(.23) = \$552$$

Don't get fancy with graphs! People sometimes add features to graphs that don't help to convey their information.

For example, 3-dimensional bar charts like the one shown below are usually not as effective as their two-dimensional counterparts.

Here is another way that fanciness can lead to trouble. Instead of plain bars, it is tempting to substitute meaningful images. This type of graph is called a **pictogram**.



Pictogram

A **pictogram** is a statistical graphic in which the size of the picture is intended to represent the frequencies or size of the values being represented.

✓ Example 8.2.9

A labor union might produce the graph to the right to show the difference between the average manager salary and the average worker salary.

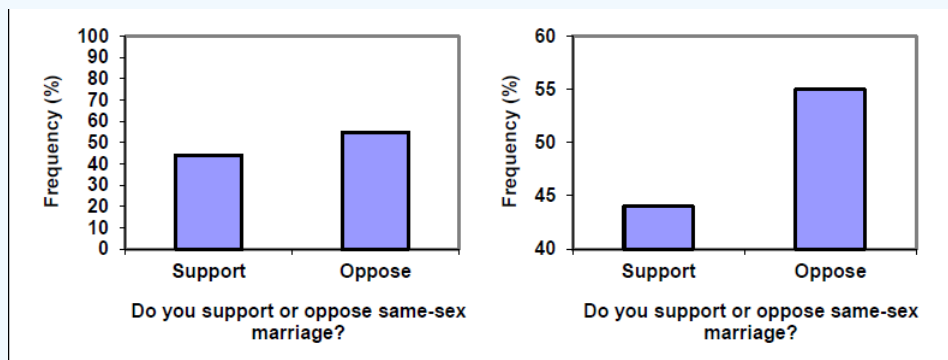
Looking at the picture, it would be reasonable to guess that the manager salaries is 4 times as large as the worker salaries – the area of the bag looks about 4 times as large. However, the manager salaries are in fact only twice as large as worker salaries, which were reflected in the picture by making the manager bag twice as tall.



Another distortion in bar charts results from setting the baseline to a value other than zero. The baseline is the bottom of the vertical axis, representing the least number of cases that could have occurred in a category. Normally, this number should be zero.

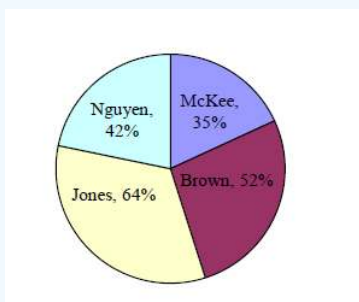
✓ Example 8.2.10

Compare the two graphs below showing support for same-sex marriage rights from a poll taken in December, 2008¹⁴. The difference in the vertical scale on the first graph suggests a different story than the true differences in percentages; the second graph makes it look like twice as many people oppose marriage rights as support it.



? Exercise 8.2.1

A poll was taken asking people if they agreed with the positions of the 4 candidates for a county office. Does the pie chart present a good representation of this data? Explain.



Answer

While the pie chart accurately depicts the relative size of the people agreeing with each candidate, the chart is confusing, since usually percentages on a pie chart represent the percentage of the pie that the slice represents

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8.3: Presenting Quantitative Data Graphically

Quantitative, or numerical, data can also be summarized into frequency tables.

✓ Example 8.3.1

A teacher records scores on a 20-point quiz for the 30 students in his class.

The scores are:

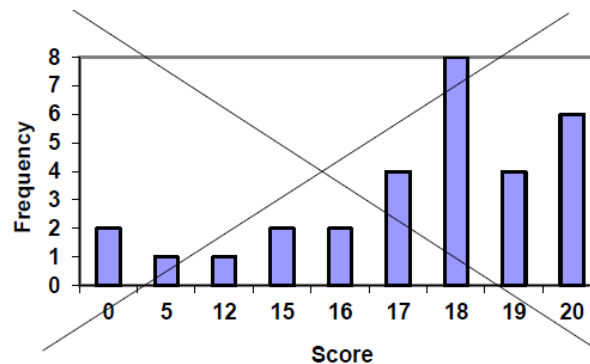
19 20 18 18 17 18 19 17 20 18 20 16 20 15 17 12 18 19 18 19 17 20 18 16 15 18 20 5 0 0

These scores could be summarized into a frequency table by grouping like values:

Solution

Score	Frequency
0	2
5	1
12	1
15	2
16	2
17	4
18	8
19	4
20	6

Using this table, it seems like we could create a standard bar chart from this summary, like we did for categorical data:



However, since the scores are numerical values, the bar graph above doesn't really make sense; the first and second bars are five values apart, while the later bars are only one value apart. It would be more correct to treat the horizontal axis as a number line. This type of graph is called a **histogram**.

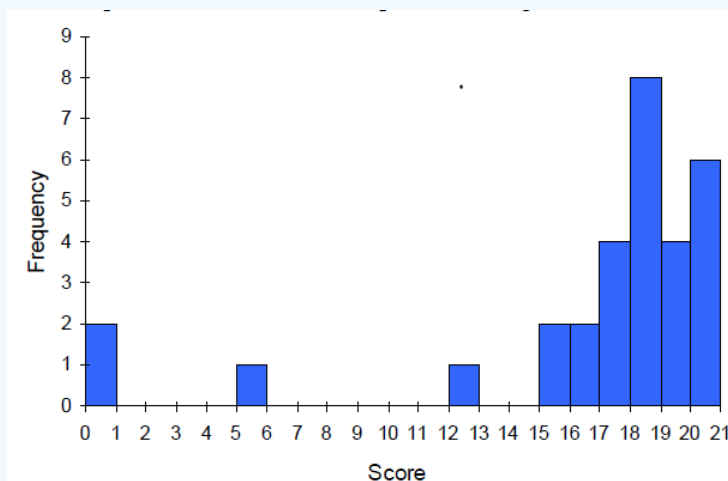
Histogram

A **histogram** is like a bar graph, but where the horizontal axis is a number line.

The vertical axis still represents frequency, and the horizontal axis still represents the variable of interest.

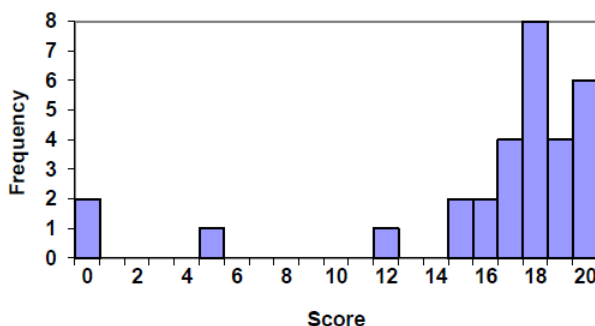
✓ Example 8.3.2

For the quiz score values in example 9, a histogram would look like:



Notice that in the histogram, a bar represents values on the horizontal axis from that on the left hand-side of the bar up to, but not including, the value on the right hand side of the bar. Some people choose to have bars start at $\frac{1}{2}$ values to avoid this ambiguity.

Unfortunately, not a lot of common software packages can correctly graph a histogram. Fortunately, your graphing calculator can quickly create an accurate histogram using the STAT PLOT option.



If we have a large number of widely varying data values, creating a frequency table that lists every possible value as a category would lead to an exceptionally long frequency table, and probably would not reveal any patterns. For this reason, it is common with quantitative data to group data into **class intervals**.

Class Intervals

Class intervals are groupings of the data. In general, we define class intervals so that:

- Each interval is equal in size. For example, if the first class contains values from 120-129, the second class should include values from 130-139.
- We have somewhere between 5 and 20 classes, typically, depending upon the number of data we're working with.

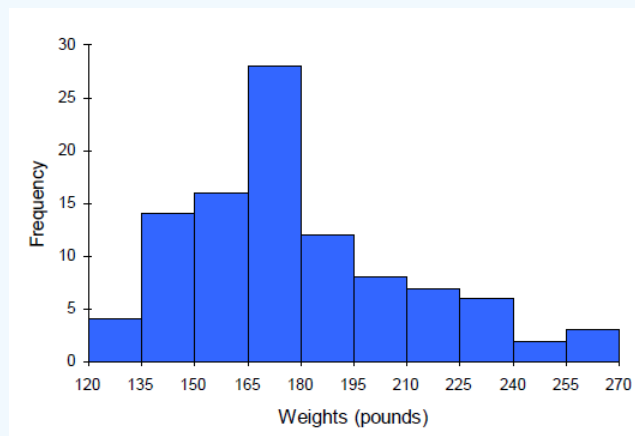
✓ Example 8.3.3

Suppose that we have collected weights from 100 male subjects as part of a nutrition study. For our weight data, we have values ranging from a low of 121 pounds to a high of 263 pounds, giving a total span of $263 - 121 = 142$. We could create 7 intervals with a width of around 20, 14 intervals with a width of around 10, or somewhere in between. Often time we have to

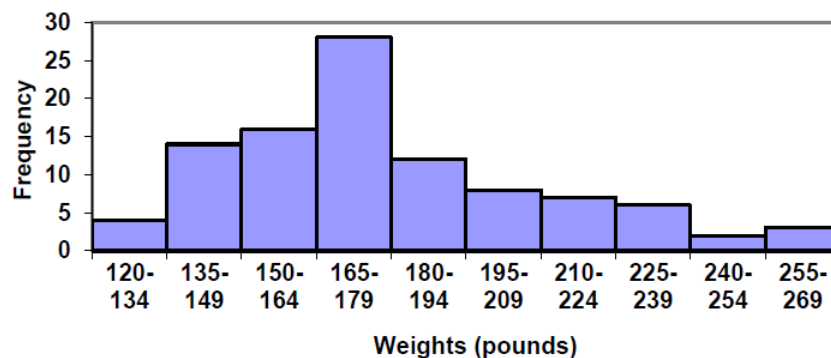
experiment with a few possibilities to find something that represents the data well. Let us try using an interval width of 15. We could start at 121, or at 120 since it is a nice round number.

Interval	Frequency
120 - 134	4
135 - 149	14
150 - 164	16
165 - 179	28
180 - 194	12
195 - 209	8
210 - 224	7
225 - 239	6
240 - 254	2
255 - 269	3

A histogram of this data would look like:



In many software packages, you can create a graph similar to a histogram by putting the class intervals as the labels on a bar chart.



? Exercise 8.3.1

The total cost of textbooks for the term was collected from 36 students. Create a histogram for this data.

\$140 \$160 \$160 \$165 \$180 \$220 \$235 \$240 \$250 \$260 \$280 \$285
\$285 \$285 \$290 \$300 \$300 \$305 \$310 \$310 \$315 \$315 \$320 \$320
\$330 \$340 \$345 \$350 \$355 \$360 \$360 \$380 \$395 \$420 \$460 \$460

Solution

Calculator Instructions for Drawing a Histogram Using a TI-83/84:

1. Turn on the calculator
2. Press the “STAT” key
3. Hit “Enter” on option 1: “Edit”

This will bring you to a screen that contains lists: L1, L2, L3, etc.

3. Enter the data values (one value per row) into L1. For any negative values you need to use the (-) key, not the subtraction key. Continue until all data is entered into L1.
4. Press the 2nd key and then “STAT PLOT” (top left of calculator)
5. Press “Enter” on 1: “Plot 1”
6. Use the arrows to “ON” to turn on Plot 1. Hit “Enter”.
7. Use the arrows to go to the histogram picture and hit “Enter”.
8. Make sure that Xlist displays L1
9. Press the 2nd key and then “STAT PLOT” again.
Make sure to turn off all other stat plots, and clear all equations before graphing the histogram.
10. Press the ZOOM menu.
11. Scroll down to option 9: “ZoomStat” and hit “Enter”.

Answer

Answers will vary depending on how the histogram is built

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8.4: Practice Problems

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CHAPTER OVERVIEW

9: Describing Data .

[9.1: Describing a Distribution](#)

[9.2: Measures of Central Tendency](#)

[9.3: Measures of Variation .](#)

[9.4: Practice Problems](#)

Learning Objectives

- Identify distribution shapes from histograms
- Calculate and interpret the mean, median, and mode
- Compute and interpret the range and standard deviation
- Calculate and interpret the quartiles, 5 number summary, and boxplots

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9.1: Describing a Distribution

We will now begin to discuss how to describe a particular variable of interest. Of particular importance is the variable's distribution.

Distribution

The distribution of a variable tells you all possible values the variable can take on, and the frequency that each of the possible values occurs.

In the last chapter, we looked at frequency tables and histograms, which are two ways of showing a variable's distribution. Distributions can also be shown by fitting smooth curves to a graphical display of a data set, or using a formula.

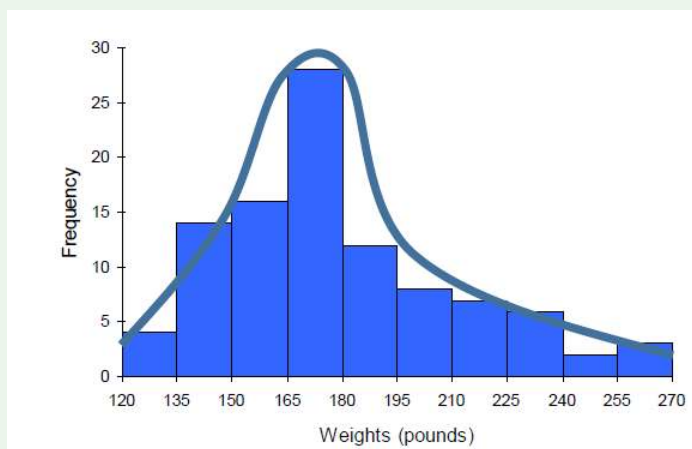
In statistics, when we consider differences between distributions, we usually look at several characteristics of the distribution:

Shape
Outliers
Center
Spread

Together we refer to these as the “SOCS” of the distribution

Distribution Shape

The shape of a distribution is frequently estimated by drawing a smooth curve to a histogram, as shown in the following diagram:



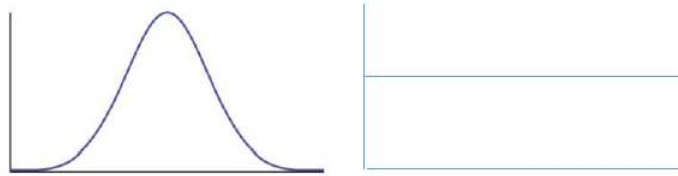
In the histogram above, a smooth curve has been drawn to roughly approximate the shape of the distribution for the variable “Weights”. This particular distribution is right-skewed.

The previous example showed just one of the many possible distribution shapes that can be observed. There are several common distribution shapes, each with unique combinations of number of peaks, symmetry, center, and spread. The most commonly used distribution curve in elementary statistics is a bell-shaped distribution curve, but it is important to realize that there are many possible distribution shapes. Some are symmetric, some are right-skewed, left-skewed, unimodal (one peak), or bimodal (two peaks). A few of the most common shapes are shown below.

Symmetric distribution

A symmetric distribution is a distribution shape in which the left and right sides of the distribution are roughly mirror images of one another.

Symmetric Distribution Shapes:



Bell Shaped Uniform

Skewed distribution

Definition text

Skewed Distribution Shapes:



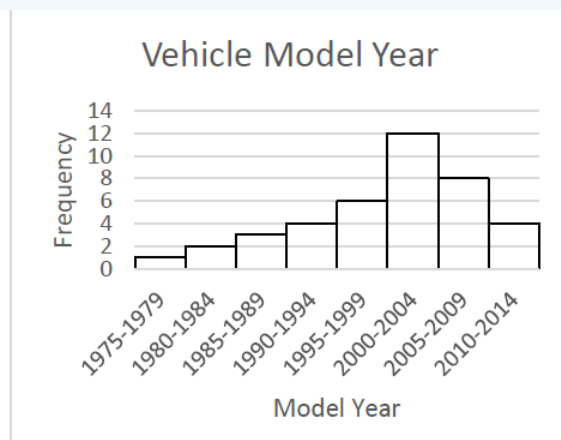
Right-skewed Left-skewed
(tail extends to right) (tail extends to left)

Note

In practice, there will be variables whose distribution shapes do not exactly fit any of the shapes described above.
Example

✓ Example 9.1.1

The histogram below shows the “Model Year” for a sample of 40 vehicles.



- Is the distribution symmetric or skewed?
- Classify the shape of the distribution as bell shaped, uniform, right-skewed, or left-skewed.

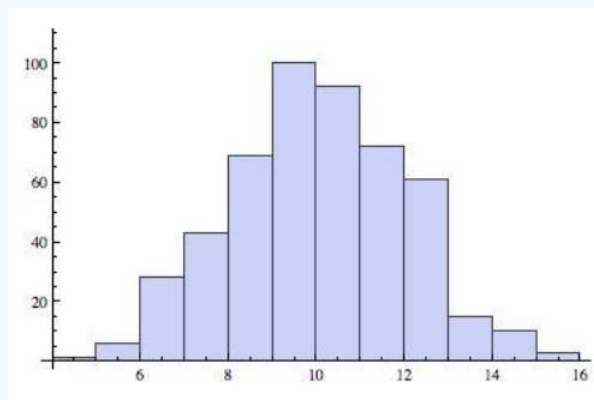
Solution

- The distribution is a skewed distribution, as the data clusters on the right side and has a longer tail on the left side.

b) This distribution is left-skewed, as the longer tail extends to the left.

✓ Example 9.1.2

Given the histogram below:



- Is the distribution symmetric or skewed?
- Classify the shape of the distribution as bell shaped, uniform, right-skewed, or left-skewed.

Solution

- The distribution is a symmetric distribution, as the data clusters fairly evenly on both the left and right sides.
- This distribution is bell-shaped.

✎ Outliers

An **outlier** is a data value that is “extreme” when compared to the rest of the values in the data set. It may be extreme in either direction, meaning it may be very small when compared to the rest of the values in the data set, or it may be very large when compared to the rest of the values in the data set.

While there are several formal methods for determining if a data value is an outlier (or a potential outlier), the determination of outliers can be somewhat subjective. This means that it can be left up to the researcher to determine which observations are “extreme”.

✓ Example 9.1.3

Consider the data set below which consists of “Exam Scores” for a sample of 20 Statistics students:

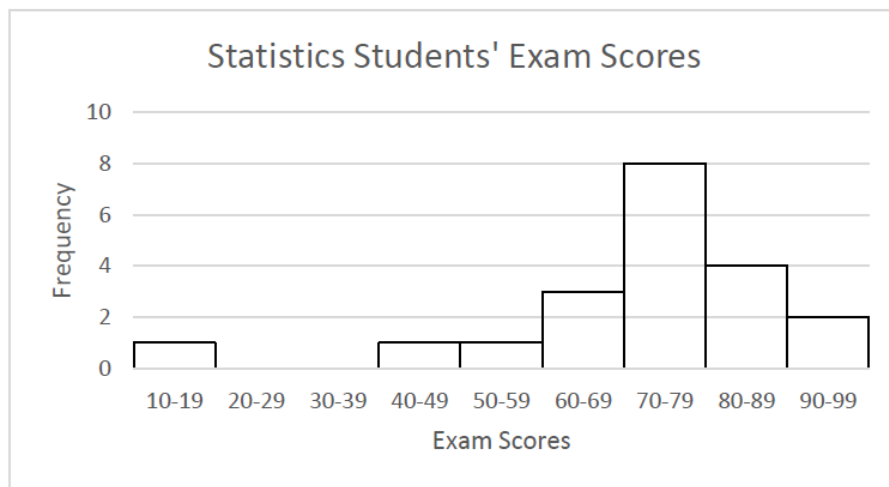
89	73	76	95	72	61	70	18	65	57
77	93	85	81	72	48	69	75	83	70

Do there appear to be any outliers in the data set? Why or why not?

Solution

It can be difficult to look at a list of data and try to identify any “extreme” or unusual values. These outliers are easier to identify by looking at a graph of the data.

Create a histogram, and look for any outliers. They will be values that appear to fall far outside the normal values.



Notice the graph shows one bar that is “far” below the other bars in the graph. When looking more closely, we see that this bar represents 1 test score. If we look back at the data set, we can see this bar represents 1 student who scored an 18 on the Exam. The value 18 appears to be an outlier, because it is “far” outside the original group of data values. It is important to recognize that there is an outlier in the data set, as the outlier may affect the descriptive measures of the mean and standard deviation (coming up in the next sections).

Since outliers can affect the descriptive measures we use to describe a data set (specifically the mean as a measure of center and the standard deviation as a measure of spread), it is important to visually inspect a data set to see if there appear to be any outliers. We will come back to this idea in the next several sections.

Recall, we started this chapter by defining a distribution, and describing the “SOCS” (Shape, Outliers, Center, and Spread) of a distribution. Now that we have spent time introducing the “Shape” of a distribution, and briefly discussing “Outliers”, we will move on to measuring the “Center”. We will discuss “Spread” later on in Sections 8.3 and 8.4.

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9.2: Measures of Central Tendency

The "Center" is an important aspect of a distribution. When we refer to the "Center" we are trying to get an idea of the location of the middle of the data set. There are three common measures of central tendency. They are the mean, median, and mode.

Let's begin by trying to find the most "typical" value of a data set.

Note that we just used the word "typical" although in many cases you might think of using the word "average." We need to be careful with the word "average" as it means different things to different people in different contexts. One of the most common uses of the word "average" is what mathematicians and statisticians call the **arithmetic mean**, or just plain old **mean** for short. "Arithmetic mean" sounds rather fancy, but you have likely calculated a mean many times without realizing it; the mean is what most people think of when they use the word "average".

Mean

The mean of a set of data is the sum of the data values divided by the number of values.

✓ Example 9.2.1

Marci's exam scores for her last math class were: 79, 86, 82, 94. The mean of these values would be:

Solution

$$\frac{79 + 86 + 82 + 94}{4} = 85.25.$$
 Typically we round means to one more decimal place than the original data had. In this case, we would round 85.25 to 85.3 .

✓ Example 9.2.2

The number of touchdown (TD) passes thrown by each of the 31 teams in the National Football League in the 2000 season are shown below.

37	33	33	32	29	28	28	23	22	22	22	21	21	21	20	
20	19	19	18	18	18	18	16	15	14	14	14	12	12	9	6

Solution

Adding these values, we get 634 total TDs. Dividing by 31, the number of data values, we get $634/31 = 20.4516$. It would be appropriate to round this to 20.5.

It would be most correct for us to report that "The mean number of touchdown passes thrown in the NFL in the 2000 season was 20.5 passes," but it is not uncommon to see the more casual word "average" used in place of "mean."

? You Try It 9.2.1

The price of a jar of peanut butter at 5 stores was: \$3.29, \$3.59, \$3.79, \$3.75, and \$3.99. Find the mean price.

Answer

The range of the data is \$0.70.

✓ Example 9.2.3

The one hundred families in a particular neighborhood are asked their annual household income, to the nearest \$5 thousand dollars. The results are summarized in a frequency table below.

Calculating the mean by hand could get tricky if we try to type in all 100 values:

$$\frac{\overbrace{15 + \dots + 15}^{6 \text{ temms}} + \overbrace{20 + \dots + 20}^{8 \text{ tems}} + \overbrace{25 + \dots + 25}^{11 \text{ tems}} + \dots}{100} \quad (9.2.1)$$

We could calculate this more easily by noticing that adding 15 to itself six times is the same as $15 \cdot 6 = 90$. Using this simplification, we get

$$\frac{15 \cdot 6 + 20 \cdot 8 + 25 \cdot 11 + 30 \cdot 17 + 35 \cdot 19 + 40 \cdot 20 + 45 \cdot 12 + 50 \cdot 7}{100} = \frac{3390}{100} = 33.9 \quad (9.2.2)$$

The mean household income of our sample is 33.9 thousand dollars (\$33,900)

✓ Example 9.2.4

Extending off the last example, suppose a new family moves into the neighborhood example that has a household income of \$5 million (\$5000 thousand).

Income (thousands of dollars)	Frequency
15	6
20	8
25	11
30	17
35	19
40	20
45	12
50	7

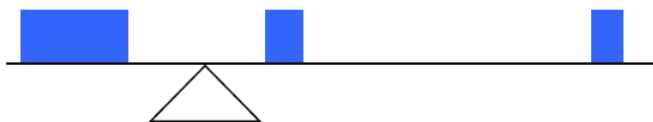
Solution

Adding this to our sample, our mean is now:

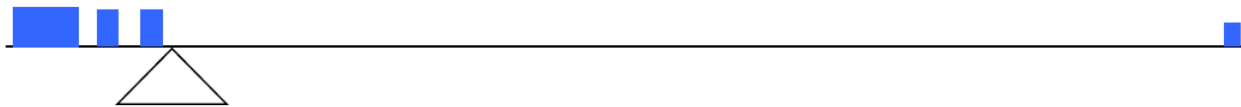
$$\frac{15 \cdot 6 + 20 \cdot 8 + 25 \cdot 11 + 30 \cdot 17 + 35 \cdot 19 + 40 \cdot 20 + 45 \cdot 12 + 50 \cdot 7 + 5000 \cdot 1}{101} = \frac{8390}{101} = 83.069 \quad (9.2.3)$$

While 83.1 thousand dollars (\$83,069) is the correct mean household income, it no longer represents a "typical" value.

Imagine the data values on a see-saw or balance scale. The mean is the value that keeps the data in balance, like in the picture below.



If we graph our household data, the \$5 million data value is so far out to the right that the mean has to adjust up to keep things in balance



For this reason, when working with data that have **outliers** – as described in the previous section - it is common to use a different measure of center, the **median**.

Median

The **median** of a set of data is the value in the middle when the data is in order

To find the median, begin by listing the data in order from smallest to largest.

If the number of data values, n , is odd, then the median is the middle data value. This position in the ordered list of this value can be found by rounding $n/2$ up to the next whole number.

If the number of data values is even, there is no one middle value, so we find the mean of the two middle values. The two middle numbers will be in the $n/2$ and the $(n/2) + 1$ positions in the ordered list.

✓ Example 9.2.5

Returning to the football touchdown data, we would start by listing the data in order from largest to smallest (we could have listed them from smallest to largest, it makes no difference to the answer)

Solution

37	33	33	32	29	28	28	23	22	22	22	21	21	21	20	
20	19	19	18	18	18	18	16	15	14	14	14	12	12	9	6

Solution

Since there are 31 data values, an odd number, the median will be the middle number, the 16th data value ($31/2 = 15.5$, round up to 16, leaving 15 values below and 15 above). The 16th data value is 20, so the median number of touchdown passes in the 2000 season was 20 passes. Notice that for this data, the median is fairly close to the mean we calculated earlier, 20.5.

✓ Example 9.2.6

Find the median of these quiz scores: 15 20 18 16 14 18 12 15 17 17.

Solution

We start by listing the data in order: 12 14 15 15 16 17 17 18 18 20

Since there are 10 data values, an even number, there is no one middle number. The median will be the mean of the numbers in positions $n/2$ and $(n/2) + 1$.

Since $n = 10$, $n/2 = 5$ and $(n/2) + 1 = 6$, so we need the mean of the 5th and 6th numbers in the ordered list.

The 5th number in the ordered list is 16, and the 6th number in the ordered list is 17. So we find the mean of the two middle numbers, 16 and 17, and get $(16+17)/2 = 16.5$.

The median quiz score was 16.5.

? You Try It 9.2.2

The price of a jar of peanut butter at 5 stores was: \$3.29, \$3.59, \$3.79, \$3.75, and \$3.99. Find the median price.

Answer

First we put the data in order: \$3.29, \$3.59, \$3.75, \$3.79, \$3.99. Since there are an odd number of data, the median will be the middle value, \$3.75.

✓ Example 9.2.7

Let us return now to our original household income data

Income (thousands of dollars)	Frequency
15	6
20	8
25	11
30	17
35	19
40	20
45	12
50	7

Solution

There are 6 data values of \$15, so	→ Values 1 to 6 are \$15 thousand
The next 8 data values are \$20, so	→ Values 7 to $(6+8)=14$ are \$20 thousand
The next 11 data values are \$25, so	→ Values 15 to $(14+11)=25$ are \$25 thousand
The next 17 data values are \$30, so	→ Values 26 to $(25+17)=42$ are \$30 thousand
The next 19 data values are \$35, so	→ Values 43 to $(42+19)=61$ are \$35 thousand

From this we can tell that values 50 and 51 will be \$35 thousand, and the mean of these two values is \$35 thousand. The median income in this neighborhood is \$35 thousand.

✓ Example 9.2.8

If we add in the new neighbor with a \$5 million household income, then there will be 101 data values, and the 51st value will be the median. As we discovered in the last example, the 51st value is \$35 thousand. Notice that the new neighbor did not affect the median in this case. The median is not swayed as much by outliers as the mean is.

In addition to the mean and the median, there is one other common measurement of the "typical" value of a data set: the **mode**.

Mode

The **mode** is the element of the data set that occurs most frequently.

The mode is fairly useless with data like weights or heights where there are a large number of possible values. The mode is most commonly used for categorical data, for which median and mean cannot be computed.

✓ Example 9.2.9

In our vehicle color survey, we collected the data

Color	Frequency

	Color	Frequency
	Blue	3
	Green	5
	Red	4
	White	3
	Black	2
	Grey	3

Solution

For this data, Green is the mode, since it is the data value that occurred the most frequently.

It is possible for a data set to have more than one mode if several categories have the same frequency, or no modes if each every category occurs only once.

? You Try It 9.2.3

Reviewers were asked to rate a product on a scale of 1 to 5. Find

	Rating	Frequency
	1	4
	2	8
	3	7
	4	3
	5	1

- The mean rating
- The median rating
- The mode rating

Answer

There are 23 ratings.

- The mean is $\frac{1 \cdot 4 + 2 \cdot 8 + 3 \cdot 7 + 4 \cdot 3 + 5 \cdot 1}{23} \approx 2.5$
- There are 23 data values, so the median will be the 12th data value. Ratings of 1 are the first 4 values, while a rating of 2 are the next 8 values, so the 12th value will be a rating of 2. The median is 2 .
- The mode is the most frequent rating. The mode rating is 2 .

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9.3: Measures of Variation .

Another component of describing a data set is how much “Spread” there is in the data set. In other words, how much the data in the distribution vary from one another. It may seem like once we know the center of a data set, we know everything there is to know. The first example will demonstrate why we need measures of variation (or spread).

There are several ways to measure this “Spread” of the data. The three most common measures are the range, standard deviation, and quartiles. In this section we will learn about the range and standard deviation. We will discuss quartiles in the following section.

We will focus first on the simplest measure of spread, called the **range**.

Range

The range is the difference between the maximum value and the minimum value of the data set.

✓ Example 9.3.1

Consider these three sets of quiz scores:

Section A	5	5	5	5	5	5	5	5	5	5	
Section B	0	0	0	0	0	10	10	10	10	10	10
Section C	4	4	4	5	5	5	5	6	6	6	

Solution

All three of these sets of data have a mean of 5 and median of 5 . If we only calculated a measure of center for each set of scores, we would say the three sets are all identical, yet the sets of scores are clearly quite different. Calculating a measure of variability (or spread) will help identify how they are different.

For section A, the range is 0 since both maximum and minimum are 5 and $5 - 5 = 0$

For section B, the range is 10 since $10 - 0 = 10$

For section C, the range is 2 since $6 - 4 = 2$

? You Try It 9.3.1

The price of a jar of peanut butter at 5 stores was: \$3.29, \$3.59, \$3.79, \$3.75, and \$3.99. Find the range of the prices.

Answer

The range of the data is \$0.70.

In example 1, the range seems to be revealing how spread out the data is. However, suppose we add a fourth section, Section D,.

This section also has a mean and median of 5. The range is 10, yet this data set is quite different than Section B. To better illuminate the differences, we’ll have to turn to more sophisticated measures of variation.

Standard Deviaion

The standard deviation is a measure of variation based on measuring how far, on average, each data value deviates, or is different, from the mean. A few important characteristics:

- Standard deviation is always positive. Standard deviation will be zero if all the data values are equal, and will get larger as the data spreads out.
- Standard deviation has the same units as the original data.
- Standard deviation, like the mean, can be highly influenced by outliers.

Using the data from Section D: 0 5 5 5 5 5 5 5 5 5 10,

Section D	0	5	5	5	5	5	5	5	5	10	
-----------	---	---	---	---	---	---	---	---	---	----	--

we could compute for each data value the difference between the data value and the mean. This will give us an idea of “how far” each value in the data set lies away from the mean.

data value	deviation: data value - mean
0	$0-5 = -5$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
5	$5-5 = 0$
10	$10-5 = 5$

We would like to get an idea of the "average" deviation from the mean, but if we find the average of the values in the second column the negative and positive values cancel each other out (this always happens), so instead we square every value in the second column:

data value	deviation: data value - mean	deviation squared
0	$0-5 = -5$	$(-5)^2 = 25$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
5	$5-5 = 0$	$0^2 = 0$
10	$10-5 = 5$	$(5)^2 = 25$

We then add the squared deviations up to get $25 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 25 = 50$. Ordinarily we would then divide by the number of scores, n , (in this case, 10) to find the mean of the deviations. But we only do this if the data set represents a

population; if the data set represents a sample (as it almost always does), we instead divide by $n - 1$ (in this case, $10 - 1 = 9$)

Note

The reason we do this is highly technical, but we can see how it might be useful by considering the case of a small sample from a population that contains an outlier, which would increase the average deviation: the outlier very likely won't be included in the sample, so the mean deviation of the sample would underestimate the mean deviation of the population; thus we divide by a slightly smaller number to get a slightly bigger average deviation.

So in our example, we would have $50/10 = 5$ if section D represents a population and $50/9 =$ about 5.56 if section D represents a sample. These values (5 and 5.56) are called, respectively, the population variance and the sample variance for section D.

Variance can be a useful statistical concept, but note that the units of variance in this instance would be points-squared since we squared all of the deviations. What are points-squared? Good question. We would rather deal with the units we started with (points in this case), so to convert back we take the square root and get:

$$\text{population standard deviation} = \sqrt{\frac{50}{10}} = \sqrt{5} \approx 2.2$$

or

$$\text{sample standard deviation} = \sqrt{\frac{50}{9}} \approx 2.4$$

If we are unsure whether the data set is a sample or a population, we will usually assume it is a sample, and we will round answers to one more decimal place than the original data, as we have done above.

To Compute Standard Deviation

To Compute Standard Deviation:

1. Find the deviation of each data from the mean. In other words, subtract the mean from the data value.
2. Square each deviation.
3. Add the squared deviations.
4. Divide by n , the number of data values, if the data represents a whole population; divide by $n - 1$ if the data is from a sample.
5. Compute the square root of the result.

✓ Example 9.3.2

Computing the standard deviation for Section B above, we first calculate that the mean is

5. Using a table can help keep track of your computations for the standard deviation:

Solution

data value	deviation: data value - mean	deviation squared
0	$0-5 = -5$	$(-5)^2 = 25$
0	$0-5 = -5$	$(-5)^2 = 25$
0	$0-5 = -5$	$(-5)^2 = 25$
0	$0-5 = -5$	$(-5)^2 = 25$
0	$0-5 = -5$	$(-5)^2 = 25$
10	$10-5=5$	$(5)^2 = 25$
10	$10-5=5$	$(5)^2 = 25$
10	$10-5=5$	$(5)^2 = 25$
10	$10-5=5$	$(5)^2 = 25$

data value	deviation: data value - mean	deviation squared
10	10-5=5	(5) ² = 25

Computing the standard deviation for Section B above, we first calculate that the mean is 5. Using a table can help keep track of your computations for the standard deviation:

$$\sqrt{\frac{25 + 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25}{10}} = \sqrt{\frac{250}{10}} = 5 \quad (9.3.1)$$

Notice that the standard deviation of this data set is much larger than that of section D since the data in this set is more spread out.

For comparison, the standard deviations of all four sections are:

Section A	5	5	5	5	5	5	5	5	5	5		Standard deviation: 0
Section B	0	0	0	0	0	10	10	10	10	10	10	Standard deviation: 5
Section C	4	4	4	5	5	5	5	6	6	6		Standard deviation: 0.8
Section D	0	5	5	5	5	5	5	5	5	10		Standard deviation: 2.2

? You Try It 9.3.2

The price of a jar of peanut butter at 5 stores were: \$3.29, \$3.59, \$3.79, \$3.75, and \$3.99. Find the standard deviation of the prices.

Answer

The standard deviation of the data is \$0.26.

Calculator Instructions for Finding Summary Statistics Using TI-83/84:

1. Turn on the calculator
2. Press the “STAT” key
3. Hit “Enter” on option 1: “Edit” This will bring you to a screen that contains lists: L1, L2, L3, etc.
4. Enter the data values (one value per row) into L1. For any negative values you need to use the (-) key, not the subtraction key.
Continue until all data is entered into L1.
5. Press the “STAT” key again
6. Use the arrow key to scroll over to “CALC”.
7. Select option 1: “1-Var Stats”
8. Indicate that the data is in L1
9. Scroll down to “Calculate” and hit “Enter”

The summary statistics should now be displayed. You may scroll down with your arrow key to get remaining statistics. Using “1-Var Stats” you can get the sample mean, sample standard deviation, population standard deviation, and 5 number summary.

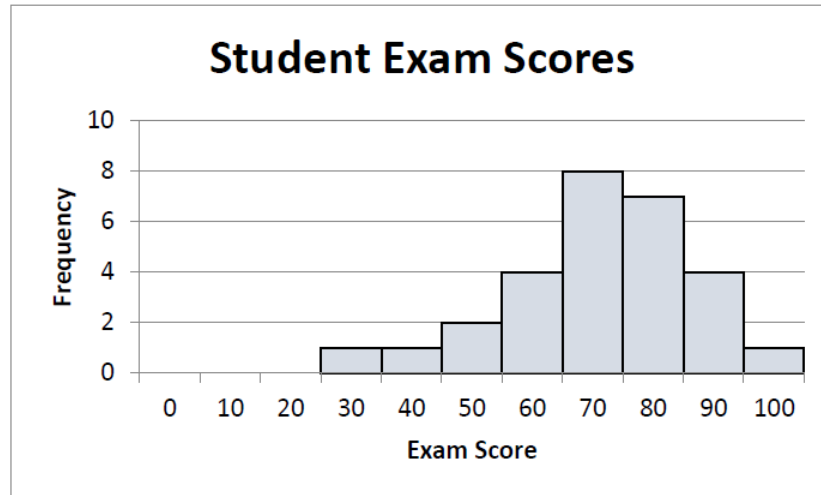
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9.4: Practice Problems

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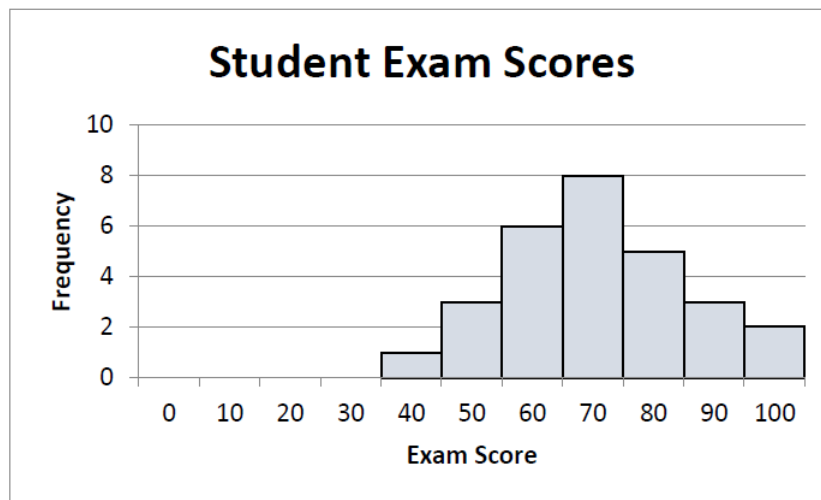
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1. The histogram below shows the “Exam Scores” for a sample of 28 students.



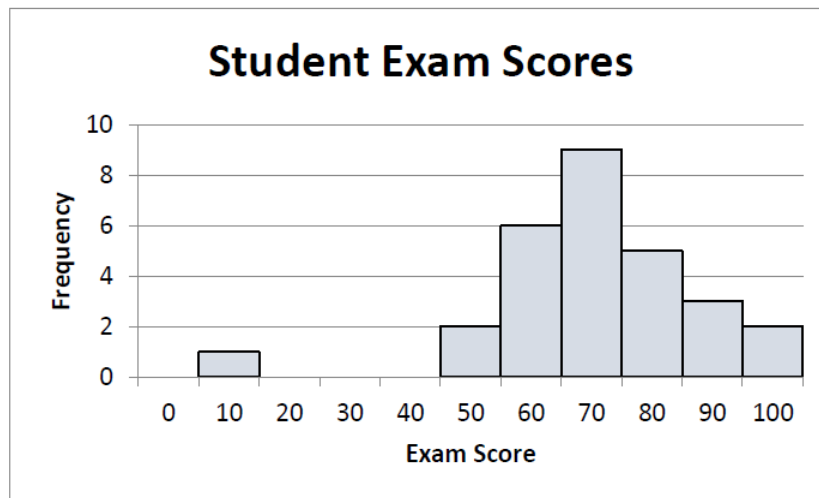
- Is the distribution symmetric or skewed?
- Classify the shape of the distribution as bell shaped, uniform, right-skewed, or left-skewed.
- Do there appear to be any outliers in the data set? Why or why not?

2. The histogram below shows the “Exam Scores” for a sample of 28 students.



- Is the distribution symmetric or skewed?
- Classify the shape of the distribution as bell shaped, uniform, right-skewed, or left-skewed.
- Do there appear to be any outliers in the data set? Why or why not?

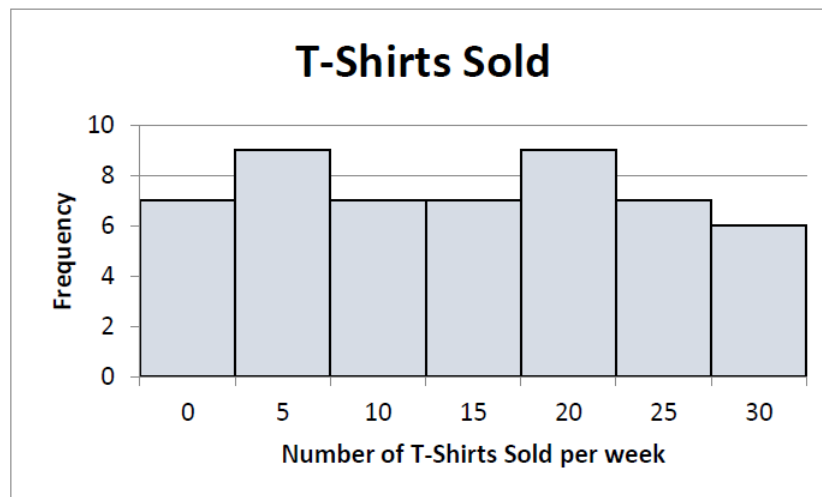
3. The histogram below shows the "Exam Scores" for a sample of 28 students.



- Is the distribution symmetric or skewed?
- Classify the shape of the distribution as bell shaped, uniform, rightskewed, or left-skewed.
- Do there appear to be any outliers in the data set? Why or why not?

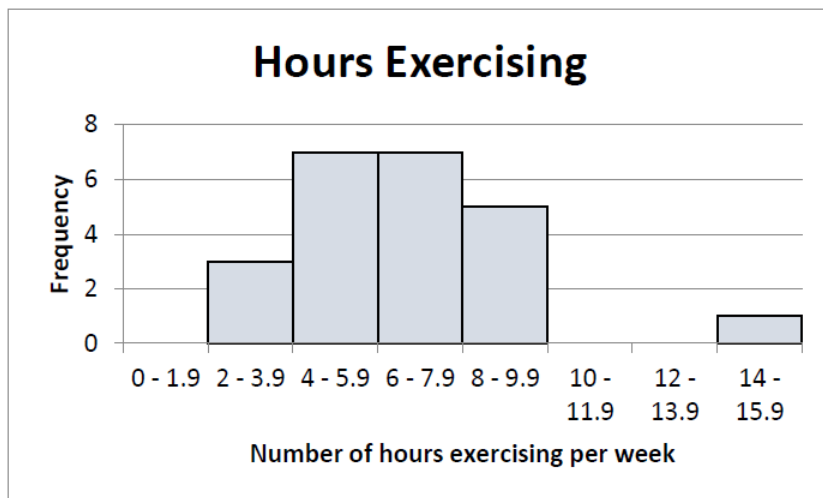
4. A student started an online T-shirt business. The histogram below shows the "Number of T-Shirts Sold" over a period of 52 weeks.

T-Shirts Sold

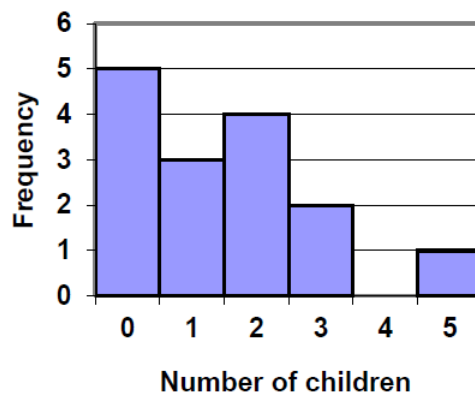


- Is the distribution symmetric or skewed?
- Classify the shape of the distribution as bell shaped, uniform, rightskewed, or left-skewed.
- Do there appear to be any outliers in the data set? Why or why not?

5. A survey was administered at a gym asking people how many hours they spend exercising in one week. The histogram below shows the "Number of Hours Exercising" for a sample of 15 people.



- a. Is the distribution symmetric or skewed?
 - b. Classify the shape of the distribution as bell shaped, uniform, rightskewed, or left-skewed.
 - c. Do there appear to be any outliers in the data set? Why or why not?
6. A group of diners were asked how much they would pay for a meal. Their responses were: \$7.50, \$8.25, \$9.00, \$8.00, \$7.25, \$7.50, \$8.00, \$7.00
- a. Find the mean.
 - b. Find the median.
 - c. Find the standard deviation.
7. You recorded the time in seconds it took for 8 participants to solve a puzzle. The times were: 15.2, 18.8, 19.3, 19.7, 20.2, 21.8, 22.1, 29.4
- a. Find the mean.
 - b. Find the median.
 - c. Find the standard deviation.
8. Five real estate exam scores are 430, 430, 480, and 495. Find the mode(s).
9. Refer back to the type of pet(s) owned by a sample of individuals:
- Dog Dog Dog Cat Bird Cat Turtle Dog Cat Cat Bird Hamster Fish Bird Dog Dog Cat Turtle Hamster Dog Cat Dog
Dog Bird
- a. What is the mode for type of pet(s) owned?
 - b. Why is the mode the best measure of center for this data set?
10. Refer to the histogram:



- What is the mode of the number of children for the group surveyed?
- Compute the mean number of children for the group surveyed.
- Compute the median number of children for the group surveyed.

11. Refer to the histogram:



- What is the mode of the number of shipping days?
- Compute the mean number of shipping days.
- Compute the median number of shipping days.

12. Suppose that in a small town of 50 people, one person earns \$5,000,000 per year and the other 49 each earn \$30,000. Calculate both the mean and the median. Which is the better measure of the "center": the mean or the median?

13. The following table provides the tuition rates (in \$ per credit hour) at a sample of 15 colleges and universities.

175	416	87	203	180
257	182	111	315	504
163	218	744	221	1300

- Find the mean tuition rate.
- Find the standard deviation of the tuition rate.

14. A survey of enrollment at 35 community colleges across the United States yielded the following figures:

6414; 1550; 2109; 9350; 21828; 4300; 5944; 5722; 2825; 2044; 5481; 5200;
 5853; 2750; 10012; 6357; 27000; 9414; 7681; 3200; 17500; 9200; 7380; 18314;
 6557; 13713; 17768; 7493; 2771; 2861; 1263; 7285; 28165; 5080; 11622

(9.4.1)

- a) Organize the data into a chart with five intervals of equal width. Label the two columns "Enrollment" and "Frequency."
- b) Construct a histogram of the data.
- c) Calculate the sample mean.
- d) Calculate the sample standard deviation.

15. On a baseball team, the ages of each of the players are as follows:

21; 21; 22; 23; 24; 24; 25; 25; 28; 29; 29; 31; 32; 33; 33; 34; 35; 36; 36; 36; 36;
38; 38; 38; 40 (9.4.2)

- a) Organize the data into a chart with five intervals of equal width. Label the two columns "Age" and "Frequency."
- b) Construct a histogram of the data.
- c) Calculate the sample mean.
- d) Calculate the sample standard deviation.

16. Following are the published weights (in pounds) of all of the team members of the San Francisco 49ers from a previous year.

177; 205; 210; 210; 232; 205; 185; 185; 178; 210; 206; 212; 184; 174; 185; 242; 188; 212; 215; 247; 241; 223; 220; 260; 245; 259;
278; 270; 280; 295; 275; 285; 290; 272; 273; 280; 285; 286; 200; 215; 185; 230; 250; 241; 190; 260; 250; 302; 265; 290; 276; 228;
265

- a) Find the mean of the data.
- b) Find the standard deviation of the data.

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