MATH 1043



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Detailed Licensing

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1.1: Find Multiples and Factors (Part 1)

Learning Objectives

- Identify multiples of numbers
- Use common divisibility tests
- Find all the factors of a number
- Identify prime and composite numbers

Be prepared!

Before you get started, take this readiness quiz.

1. Which of the following numbers are counting numbers (natural numbers)? $0, 4, 215$ If you missed this problem, review Example 1.1.1.
2. Find the sum of 3, 5, and 7. If you missed the problem, review Section 2.1.

Identify Multiples of Numbers

Annie is counting the shoes in her closet. The shoes are matched in pairs, so she doesn't have to count each one. She counts by twos: $2, 4, 6, 8, 10, 12$ She has 12 shoes in her closet.

The numbers $2, 4, 6, 8, 10, 12$ are called multiples of 2. Multiples of 2 can be written as the product of a counting number and 2. The first six multiples of 2 are given below.

$$\begin{aligned} 1 \cdot 2 &= 2 \\ 2 \cdot 2 &= 4 \\ 3 \cdot 2 &= 6 \\ 4 \cdot 2 &= 8 \\ 5 \cdot 2 &= 10 \\ 6 \cdot 2 &= 12 \end{aligned}$$

A **multiple of a number** is the product of the number and a counting number. So a multiple of 3 would be the product of a counting number and 3. Below are the first six multiples of 3.

$$\begin{aligned} 1 \cdot 3 &= 3 \\ 2 \cdot 3 &= 6 \\ 3 \cdot 3 &= 9 \\ 4 \cdot 3 &= 12 \\ 5 \cdot 3 &= 15 \\ 6 \cdot 3 &= 18 \end{aligned}$$

We can find the multiples of any number by continuing this process. Table $1.1.1$ shows the multiples of 2 through 9 for the first twelve counting numbers.

Table 1.1.1

Counting Number	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48

Counting Number	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108

Definition: Multiple of a Number

A number is a multiple of n if it is the product of a counting number and n.

Recognizing the patterns for multiples of 2, 5, 10, and 3 will be helpful to you as you continue in this course.

Figure $1.1.1$ shows the counting numbers from 1 to 50. Multiples of 2 are highlighted. Do you notice a pattern?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 1.1.1: Multiples of 2 between 1 and 50

The last digit of each highlighted number in Figure $1.1.1$ is either 0, 2, 4, 6, or 8. This is true for the product of 2 and any counting number. So, to tell if any number is a multiple of 2 look at the last digit. If it is 0, 2, 4, 6, or 8, then the number is a multiple of 2.

Example 1.1.1: multiples of 2

Determine whether each of the following is a multiple of 2:

a. 489
b. $3,714$

Solution

a.

Is 489 a multiple of 2?	
Is the last digit 0, 2, 4, 6, or 8?	No.
	489 is not a multiple of 2.

b.

Is 3,714 a multiple of 2?	
Is the last digit 0, 2, 4, 6, or 8?	Yes.
	3,714 is a multiple of 2.

Exercise 1.1.1

Determine whether each number is a multiple of 2:

a. 678
b. 21,493

Answer a

yes

Answer b

no

Exercise 1.1.2

Determine whether each number is a multiple of 2:

a. 979
b. 17,780

Answer a

no

Answer b

yes

Now let's look at multiples of 5. Figure 1.1.2 highlights all of the multiples of 5 between 1 and 50. What do you notice about the multiples of 5?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 1.1.2: Multiples of 5 between 1 and 50

All multiples of 5 end with either 5 or 0. Just like we identify multiples of 2 by looking at the last digit, we can identify multiples of 5 by looking at the last digit.

Example 1.1.2: multiples of 5

Determine whether each of the following is a multiple of 5:

a. 579
b. 880

Solution

a.

Is 579 a multiple of 5?	
Is the last digit 5 or 0?	No.
	579 is not a multiple of 5.

b.

Is 880 a multiple of 5?	

Is the last digit 5 or 0?	Yes.
	880 is not a multiple of 5.

Exercise 1.1.3

Determine whether each number is a multiple of 5.

a. 675
b. 1,578

Answer a

yes

Answer b

no

Exercise 1.1.4

Determine whether each number is a multiple of 5.

a. 421
b. 2,690

Answer a

no

Answer b

yes

Figure 1.1.3 highlights the multiples of 10 between 1 and 50. All multiples of 10 all end with a zero.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 1.1.3: Multiples of 10 between 1 and 50

Example 1.1.3: multiples of 10

Determine whether each of the following is a multiple of 10:

a. 425
b. 350

Solution

a.

Is 425 a multiple of 10?	
Is the last digit zero?	No.
	425 is not a multiple of 10.

b.

Is 350 a multiple of 10?	
Is the last digit zero?	Yes.
	350 is a multiple of 10.

Exercise 1.1.5

Determine whether each number is a multiple of 10:

a. 179
b. 3,540

Answer a

no

Answer b

yes

Exercise 1.1.6

Determine whether each number is a multiple of 10:

a. 110
b. 7,595

Answer a

yes

Answer b

no

Figure 1.1.4 highlights multiples of 3. The pattern for multiples of 3 is not as obvious as the patterns for multiples of 2, 5, and 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Figure 1.1.4: Multiples of 3 between 1 and 50

Unlike the other patterns we've examined so far, this pattern does not involve the last digit. The pattern for multiples of 3 is based on the sum of the digits. If the sum of the digits of a number is a multiple of 3, then the number itself is a multiple of 3. See Table 1.1.2.

Table 1.1.2

Multiple of 3	3	6	9	12	15	18	21	24
Sum of digits	3	6	9	1 + 2 3	1 + 5 6	1 + 8 9	2 + 1 3	2 + 4 6

Consider the number 42. The digits are 4 and 2, and their sum is $4+2=6$. Since 6 is a multiple of 3, we know that 42 is also a multiple of 3.

Example 1.1.4: multiples of 3

Determine whether each of the given numbers is a multiple of 3:

a. 645
b. 10,519

Solution

a. Is 645 a multiple of 3?

Find the sum of the digits.	6 + 4 + 5 = 15
Is 15 a multiple of 3?	Yes.
If we're not sure, we could add its digits to find out. We can check it by dividing 645 by 3.	645 ÷ 3
The quotient is 215.	3 • 215 = 645

b. Is 10,519 a multiple of 3?

Find the sum of the digits.	1 + 0 + 5 + 1 + 9 = 16
Is 15 a multiple of 3?	No.
So 10,519 is not a multiple of 3 either..	645 ÷ 3
We can check this by dividing by 10,519 by 3.	$3\overline{)10{,}519}$ $3{,}506\text{R}1$

When we divide 10,519 by 3, we do not get a counting number, so 10,519 is not the product of a counting number and 3. It is not a multiple of 3.

Exercise 1.1.7

Determine whether each number is a multiple of 3:

a. 954
b. 3,742

Answer a

yes

Answer b

no

Exercise 1.1.8

Determine whether each number is a multiple of 3:

a. 643
b. 8,379

Answer a

no

Answer b

yes

Look back at the charts where you highlighted the multiples of 2, of 5, and of 10. Notice that the multiples of 10 are the numbers that are multiples of both 2 and 5. That is because $10 = 2 \cdot 5$. Likewise, since $6 = 2 \cdot 3$, the multiples of 6 are the numbers that are multiples of both 2 and 3.

Use Common Divisibility Tests

Another way to say that 375 is a multiple of 5 is to say that 375 is divisible by 5. In fact, $375 \div 5$ is 75, so 375 is $5 \cdot 75$. Notice in Example 1.1.4 that 10,519 is not a multiple 3. When we divided 10,519 by 3 we did not get a counting number, so 10,519 is not divisible by 3.

Definition: Divisibility

If a number m is a multiple of n, then we say that m is divisible by n.

Since multiplication and division are inverse operations, the patterns of multiples that we found can be used as divisibility tests. Table 1.1.3 summarizes divisibility tests for some of the counting numbers between one and ten.

Table 1.1.3: Divisibility tests

A number is divisible by	
2	if the last digit is 0, 2, 4, 6, or 8
3	if the sum of the digits is divisible by 3
5	if the last digit is 5 or 0
6	if divisible by both 2 and 3
10	if the last digit is 0

Example 1.1.5: divisibility

Determine whether 1,290 is divisible by 2, 3, 5, and 10.

Solution

Table 1.1.4 applies the divisibility tests to 1,290. In the far right column, we check the results of the divisibility tests by seeing if the quotient is a whole number.

Table 1.1.4

Divisible by…?	Test	Divisible?	Check
2	Is last digit 0, 2, 4, 6, or 8?	yes	1290 ÷ 2 = 645
3	Is sum of digits divisible by 3? 1 + 2 + 9 + 0 = 12	yes	1290 ÷ 3 = 430
5	Is last digit 5 or 0?	yes	1290 ÷ 5 = 258
10	Is last digit 0?	yes	1290 ÷ 10 = 129

Thus, 1,290 is divisible by 2, 3, 5, and 10.

Exercise 1.1.9

Determine whether the given number is divisible by 2, 3, 5, and 10.

6240

Answer

Divisible by 2, 3, 5, and 10

? Exercise 1.1.10

Determine whether the given number is divisible by 2, 3, 5, and 10.

7248

Answer

Divisible by 2 and 3, not 5 or 10

✓ Example 1.1.6: divisibility

Determine whether 5,625 is divisible by 2, 3, 5, and 10.

Solution

Table 1.1.5 applies the divisibility tests to 5,625 and tests the results by finding the quotients.

Table 1.1.5

Divisible by…?	Test	Divisible?	Check
2	Is last digit 0, 2, 4, 6, or 8?	no	5625 ÷ 2 = 2812.5
3	Is sum of digits divisible by 3? 5 + 6 + 2 + 5 = 18	yes	5625 ÷ 3 = 1875
5	Is last digit 5 or 0?	yes	5625 ÷ 5 = 1125
10	Is last digit 0?	no	5625 ÷ 10 = 562.5

Thus, 5,625 is divisible by 3 and 5, but not 2, or 10.

? Exercise 1.1.11

Determine whether the given number is divisible by 2, 3, 5, and 10.

4962

Answer

Divisible by 2, 3, not 5 or 10

? Exercise 1.1.12

Determine whether the given number is divisible by 2, 3, 5, and 10.

3765

Answer

Divisible by 3 and 5

Find All the Factors of a Number

There are often several ways to talk about the same idea. So far, we've seen that if m is a multiple of n, we can say that m is divisible by n. We know that 72 is the product of 8 and 9, so we can say 72 is a multiple of 8 and 72 is a multiple of 9. We can also

say 72 is divisible by 8 and by 9. Another way to talk about this is to say that 8 and 9 are factors of 72. When we write $72 = 8 \cdot 9$ we can say that we have factored 72.

$$\underbrace{8 \cdot 9}_{\text{factors}} = \underbrace{72}_{\text{product}}$$

Definition: Factors

If $a \bullet b = m$, then a and b are factors of m, and m is the product of a and b.

In algebra, it can be useful to determine all of the factors of a number. This is called factoring a number, and it can help us solve many kinds of problems.

For example, suppose a choreographer is planning a dance for a ballet recital. There are 24 dancers, and for a certain scene, the choreographer wants to arrange the dancers in groups of equal sizes on stage.

In how many ways can the dancers be put into groups of equal size? Answering this question is the same as identifying the factors of 24. Table $1.1.6$ summarizes the different ways that the choreographer can arrange the dancers.

Table 1.1.6

Number of Groups	Dancers per Group	Total Dancers
1	24	1 • 24 = 24
2	12	2 • 12= 24
3	8	3 • 8= 24
4	6	4 • 6= 24
6	4	6 • 4= 24
8	3	8 • 3= 24
12	2	12 • 2= 24
24	1	24 • 1= 24

What patterns do you see in Table $1.1.6$? Did you notice that the number of groups times the number of dancers per group is always 24? This makes sense, since there are always 24 dancers.

You may notice another pattern if you look carefully at the first two columns. These two columns contain the exact same set of numbers—but in reverse order. They are mirrors of one another, and in fact, both columns list all of the factors of 24, which are:

$$1, 2, 3, 4, 6, 8, 12, 24$$

We can find all the factors of any counting number by systematically dividing the number by each counting number, starting with 1. If the quotient is also a counting number, then the divisor and the quotient are factors of the number. We can stop when the quotient becomes smaller than the divisor.

HOW TO: FIND ALL THE FACTORS OF A COUNTING NUMBER.

Step 1. Divide the number by each of the counting numbers, in order, until the quotient is smaller than the divisor.

- If the quotient is a counting number, the divisor and quotient are a pair of factors.
- If the quotient is not a counting number, the divisor is not a factor.

Step 2. List all the factor pairs.

Step 3. Write all the factors in order from smallest to largest.

Example 1.1.7: factors

Find all the factors of 72.

Solution

Divide 72 by each of the counting numbers starting with 1. If the quotient is a whole number, the divisor and quotient are a pair of factors.

Dividend	Divisor	Quotient	Factors
72	1	72	1, 72
72	2	36	2, 36
72	3	24	3, 24
72	4	18	4, 18
72	5	14.4	–
72	6	12	6, 12
72	7	~10.29	–
72	8	9	8, 9

The next line would have a divisor of 9 and a quotient of 8. The quotient would be smaller than the divisor, so we stop. If we continued, we would end up only listing the same factors again in reverse order. Listing all the factors from smallest to greatest, we have 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

Exercise 1.1.13

Find all the factors of the given number: 96

Answer

$1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96$

Exercise 1.1.14

Find all the factors of the given number: 80

Answer

$1, 2, 4, 5, 8, 10, 16, 20, 40, 80$

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1.2: Find Multiples and Factors (Part 2)

Identify Prime and Composite Numbers

Some numbers, like 72, have many factors. Other numbers, such as 7, have only two factors: 1 and the number. A number with only two factors is called a **prime number**. A number with more than two factors is called a **composite number**. The number 1 is neither prime nor composite. It has only one factor, itself.

Definition: Prime Numbers and Composite Numbers

A prime number is a counting number greater than 1 whose only factors are 1 and itself.

A composite number is a counting number that is not prime.

Figure 1.2.5 lists the counting numbers from 2 through 20 along with their factors. The highlighted numbers are prime, since each has only two factors.

Number	Factors	Prime or Composite?		Number	Factor	Prime or Composite?
2	1,2	Prime		12	1,2,3,4,6,12	Composite
3	1,3	Prime		13	1,13	Prime
4	1,2,4	Composite		14	1,2,7,14	Composite
5	1,5	Prime		15	1,3,5,15	Composite
6	1,2,3,6	Composite		16	1,2,4,8,16	Composite
7	1,7	Prime		17	1,17	Prime
8	1,2,4,8	Composite		18	1,2,3,6,9,18	Composite
9	1,3,9	Composite		19	1,19	Prime
10	1,2,5,10	Composite		20	1,2,4,5,10,20	Composite
11	1,11	Prime				

Figure 1.2.5: Factors of the counting numbers from 2 through 20, with prime numbers highlighted

The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. There are many larger prime numbers too. In order to determine whether a number is prime or composite, we need to see if the number has any factors other than 1 and itself. To do this, we can test each of the smaller prime numbers in order to see if it is a factor of the number. If none of the prime numbers are factors, then that number is also prime.

HOW TO: DETERMINE IF A NUMBER IS PRIME.

- Step 1. Test each of the primes, in order, to see if it is a factor of the number.
- Step 2. Start with 2 and stop when the quotient is smaller than the divisor or when a prime factor is found.
- Step 3. If the number has a prime factor, then it is a composite number. If it has no prime factors, then the number is prime.

Example 1.2.8: prime or composite

Identify each number as prime or composite:

a. 83
b. 77

Solution

a. Test each prime, in order, to see if it is a factor of 83, starting with 2, as shown. We will stop when the quotient is smaller than the divisor.

Prime	Test	Factor of 83?
2	Last digit of 83 is not 0, 2, 4, 6, or 8.	No.
3	8 + 3 = 11, and 11 is not divisible by 3.	No.

Prime	Test	Factor of 83?
5	The last digit of 83 is not 5 or 0.	No.
7	$83 \div 7 = 11.857\ldots$.	No.
11	$83 \div 11 = 7.545\ldots$	No.

We can stop when we get to 11 because the quotient ($7.545\ldots$) is less than the divisor. We did not find any prime numbers that are factors of 83, so we know 83 is prime.

b. Test each prime, in order, to see if it is a factor of 77.

Prime	Test	Factor of 77?
2	Last digit is not 0, 2, 4, 6, or 8.	No.
3	7 + 7 = 14, and 14 is not divisible by 3.	No.
5	The last digit is not 5 or 0.	No.
7	$77 \div 7 = 11$	Yes.

Since 77 is divisible by 7, we know it is not a prime number. It is composite.

? Exercise 1.2.15

Identify the number as prime or composite: 91

Answer

composite

? Exercise 1.2.16

Identify the number as prime or composite: 137

Answer

prime

Access Additional Online Resources

- Divisibility Rules
- Factors
- Ex 1: Determine Factors of a Number
- Ex 2: Determine Factors of a Number
- Ex 3: Determine Factors of a Number

Key Concepts

Divisibility Tests	
A number is divisible by	
2	if the last digit is 0, 2, 4, 6, or 8
3	if the sum of the digits is divisible by 3
5	if the last digit is 5 or 0

Divisibility Tests	
6	if divisible by both 2 and 3
10	if the last digit is 0

- Factors If $a \cdot b = m$, then a and b are factors of m, and m is the product of a and b.
- Find all the factors of a counting number.
  - Divide the number by each of the counting numbers, in order, until the quotient is smaller than the divisor.
    - If the quotient is a counting number, the divisor and quotient are a pair of factors.
    - If the quotient is not a counting number, the divisor is not a factor.
  - List all the factor pairs.
  - Write all the factors in order from smallest to largest.
- Determine if a number is prime.
  - Test each of the primes, in order, to see if it is a factor of the number.
  - Start with 2 and stop when the quotient is smaller than the divisor or when a prime factor is found.
  - If the number has a prime factor, then it is a composite number. If it has no prime factors, then the number is prime.

Glossary

multiple of a number

A number is a multiple of n if it is the product of a counting number and n.

divisibility

If a number m is a multiple of n, then we say that m is divisible by n.

prime number

A prime number is a counting number greater than 1 whose only factors are 1 and itself.

composite number

A composite number is a counting number that is not prime.

Practice Makes Perfect

Identify Multiples of Numbers

In the following exercises, list all the multiples less than 50 for the given number.

215. 2
216. 3
217. 4
218. 5
219. 6
220. 7
221. 8
222. 9
223. 10
224. 12

Use Common Divisibility Tests

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, 3, 4, 5, 6, and 10.

225. 84
226. 96
227. 75
228. 78

229. 168
230. 264
231. 900
232. 800
233. 896
234. 942
235. 375
236. 750
237. 350
238. 550
239. 1430
240. 1080
241. 22,335
242. 39,075

Find All the Factors of a Number

In the following exercises, find all the factors of the given number.

243. 36
244. 42
245. 60
246. 48
247. 144
248. 200
249. 588
250. 576

Identify Prime and Composite Numbers

In the following exercises, determine if the given number is prime or composite.

251. 43
252. 67
253. 39
254. 53
255. 71
256. 119
257. 481
258. 221
259. 209
260. 359
261. 667
262. 1771

Everyday Math

263. **Banking** Frank's grandmother gave him $100 at his high school graduation. Instead of spending it, Frank opened a bank account. Every week, he added $15 to the account. The table shows how much money Frank had put in the account by the end of each week. Complete the table by filling in the blanks.

Weeks after graduation	Total number of dollars Frank put in the account	Simplified Total
0	100	100
1	100 + 15	115
2	100 + 15 • 2	130

Weeks after graduation	Total number of dollars Frank put in the account	Simplified Total
3	100 + 15 • 3	
4	100 + 15 • []	
5	100 + []	
6		
20		
x		

264. **Banking** In March, Gina opened a Christmas club savings account at her bank. She deposited $75 to open the account. Every week, she added $20 to the account. The table shows how much money Gina had put in the account by the end of each week. Complete the table by filling in the blanks.

Weeks after opening the account	Total number of dollars Gina put in the account	Simplified Total
0	75	75
1	75 + 20	95
2	75 + 20 • 2	115
3	75 + 20 • 3	
4	75 + 20 • []	
5	75 + []	
6		
20		
x		

Writing Exercises

265. If a number is divisible by 2 and by 3, why is it also divisible by 6?
266. What is the difference between prime numbers and composite numbers?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify multiples of numbers.			
use common divisibility tests.			
find all the factors of a number.			
identify prime and composite numbers.			

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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SECTION OVERVIEW

1.3: Fractions

Often in life, whole amounts are not exactly what we need. A baker must use a little more than a cup of milk or part of a teaspoon of sugar. Similarly a carpenter might need less than a foot of wood and a painter might use part of a gallon of paint. In this chapter, we will learn about numbers that describe parts of a whole. These numbers, called fractions, are very useful both in algebra and in everyday life. You will discover that you are already familiar with many examples of fractions!

1.3.1: Visualize Fractions (Part 1)

1.3.2: Visualize Fractions (Part 2)

1.3.3: Multiply and Divide Fractions (Part 1)

1.3.4: Multiply and Divide Fractions (Part 2)

1.3.5: Multiply and Divide Mixed Numbers and Complex Fractions (Part 1)

1.3.6: Multiply and Divide Mixed Numbers and Complex Fractions (Part 2)

1.3.7: Add and Subtract Fractions with Common Denominators

1.3.8: Add and Subtract Fractions with Different Denominators (Part 1)

1.3.9: Add and Subtract Fractions with Different Denominators (Part 2)

1.3.10: Add and Subtract Mixed Numbers (Part 1)

1.3.11: Add and Subtract Mixed Numbers (Part 2)

1.3.12: Solve Equations with Fractions (Part 1)

1.3.13: Solve Equations with Fractions (Part 2)

1.3.E: Fractions (Exercises)

1.3.S: Fractions (Summary)

Figure 4.1 - Bakers combine ingredients to make delicious breads and pastries. (credit: Agustín Ruiz, Flickr)

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1.3.1: Visualize Fractions (Part 1)

Learning Objectives

- Understand the meaning of fractions
- Model improper fractions and mixed numbers
- Convert between improper fractions and mixed numbers
- Model equivalent fractions
- Find equivalent fractions
- Locate fractions and mixed numbers on the number line
- Order fractions and mixed numbers

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $5 \bullet 2+1$. If you missed this problem, review Example 2.1.8.
2. Fill in the blank with $<$ or $>$: $-2\_\_-5$. If you missed this problem, review Example 3.1.2.

Understand the Meaning of Fractions

Andy and Bobby love pizza. On Monday night, they share a pizza equally. How much of the pizza does each one get? Are you thinking that each boy gets half of the pizza? That's right. There is one whole pizza, evenly divided into two parts, so each boy gets one of the two equal parts. In math, we write $\frac{1}{2}$ to mean one out of two parts.

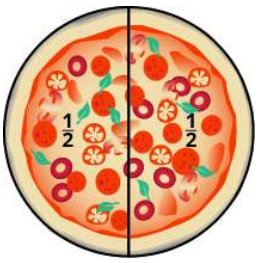


Figure 1.3.1.1

On Tuesday, Andy and Bobby share a pizza with their parents, Fred and Christy, with each person getting an equal amount of the whole pizza. How much of the pizza does each person get? There is one whole pizza, divided evenly into four equal parts. Each person has one of the four equal parts, so each has $\frac{1}{4}$ of the pizza.



Figure 1.3.1.2

On Wednesday, the family invites some friends over for a pizza dinner. There are a total of 12 people. If they share the pizza equally, each person would get $\frac{1}{12}$ of the pizza.



Figure 1.3.1.3

Definition: Fractions

A fraction is written $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In a fraction, a is called the numerator and b is called the denominator.

A fraction is a way to represent parts of a whole. The denominator b represents the number of equal parts the whole has been divided into, and the numerator a represents how many parts are included. The denominator, b, cannot equal zero because division by zero is undefined.

In Figure 1.3.1.4 the circle has been divided into three parts of equal size. Each part represents $\frac{1}{3}$ of the circle. This type of model is called a fraction circle. Other shapes, such as rectangles, can also be used to model fractions.

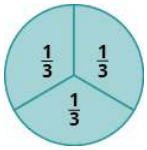


Figure 1.3.1.4

What does the fraction $\frac{2}{3}$ represent? The fraction $\frac{2}{3}$ means two of three equal parts.

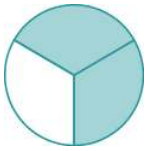
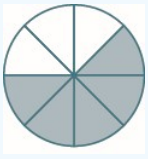
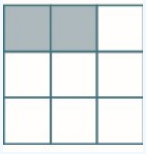


Figure 1.3.1.5

Example 1.3.1.1: name the fraction

Name the fraction of the shape that is shaded in each of the figures.

a. 

b. 

Solution

We need to ask two questions. First, how many equal parts are there? This will be the denominator. Second, of these equal parts, how many are shaded? This will be the numerator.

a.

How many equal parts are there?	There are eight equal parts.
How many are shaded?	Five parts are shaded.

Five out of eight parts are shaded. Therefore, the fraction of the circle that is shaded is $\frac{5}{8}$.

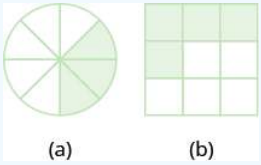
b.

How many equal parts are there?	There are nineequal parts.
How many are shaded?	Two parts are shaded.

Two out of nine parts are shaded. Therefore, the fraction of the square that is shaded is $\frac{2}{9}$.

Exercise 1.3.1.1

Name the fraction of the shape that is shaded in each figure:



Answer a

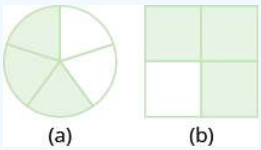
$\frac{3}{8}$

Answer b

$\frac{4}{9}$

Exercise 1.3.1.2

Name the fraction of the shape that is shaded in each figure:



Answer a

$\frac{3}{5}$

Answer b

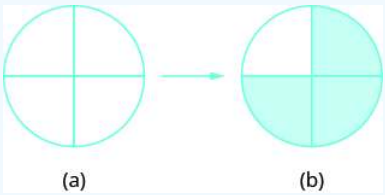
$\frac{3}{4}$

Example 1.3.1.2:

Shade $\frac{3}{4}$ of the circle.

Solution

The denominator is 4, so we divide the circle into four equal parts (a). The numerator is 3, so we shade three of the four parts (b).



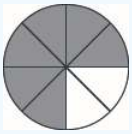
$\frac{3}{4}$ of the circle is shaded.

Exercise 1.3.1.3

Shade $\frac{6}{8}$ of the circle.



Answer



Exercise 1.3.1.4

Shade $\frac{2}{5}$ of the rectangle.

Answer



In Example 1.3.1.1 and Example 1.3.1.2, we used circles and rectangles to model fractions. Fractions can also be modeled as manipulatives called fraction tiles, as shown in Figure 1.3.1.6. Here, the whole is modeled as one long, undivided rectangular tile. Beneath it are tiles of equal length divided into different numbers of equally sized parts.

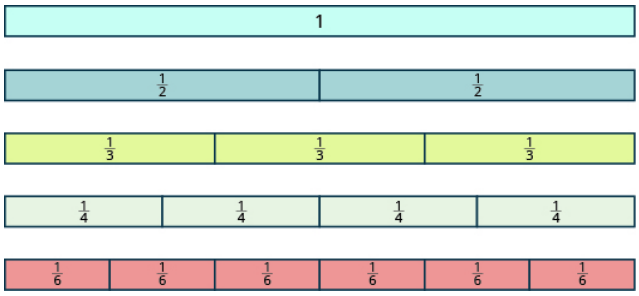


Figure 1.3.1.6

We'll be using fraction tiles to discover some basic facts about fractions. Refer to Figure 1.3.1.6 to answer the following questions:

How many $\frac{1}{2}$ tiles does it take to make one whole tile?	It takes two halves to make a whole, so two out of two is $\frac{2}{2} = 1$.
How many $\frac{1}{3}$ tiles does it take to make one whole tile?	It takes three thirds, so three out of three is $\frac{3}{3} = 1$.
How many $\frac{1}{4}$ tiles does it take to make one whole tile?	It takes four fourths, so four out of four is $\frac{4}{4} = 1$.
How many $\frac{1}{5}$ tiles does it take to make one whole tile?	It takes six sixths, so six out of six is $\frac{6}{6} = 1$.
What if the whole were divided into 24 equal parts? (We have not shown fraction tiles to represent this, but try to visualize it in your mind.) How many $\frac{1}{24}$ tiles does it take to make one whole tile?	It takes 24 twenty-fourths, so $\frac{24}{24} = 1$.

It takes 24 twenty-fourths, so $\frac{24}{24} = 1$. This leads us to the Property of One.

Definition: Property of One

Any number, except zero, divided by itself is one.

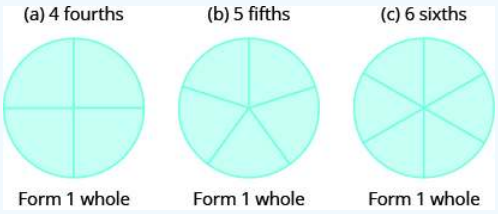
$$\frac{a}{a} = 1 \qquad (a \neq 0) \qquad (1.3.1.1)$$

Example 1.3.1.3: fraction circles to form wholes

Use fraction circles to make wholes using the following pieces:

a. 4 fourths
b. 5 fifths
c. 6 sixths

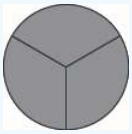
Solution



Exercise 1.3.1.5

Use fraction circles to make wholes with the following pieces: 3 thirds.

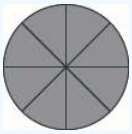
Answer



Exercise 1.3.1.6

Use fraction circles to make wholes with the following pieces: 8 eighths.

Answer



What if we have more fraction pieces than we need for 1 whole? We'll look at this in the next example.

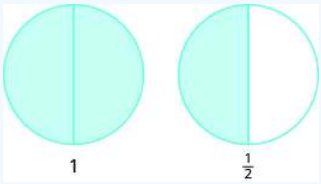
Example 1.3.1.4: fraction circles to form whole

Use fraction circles to make wholes using the following pieces:

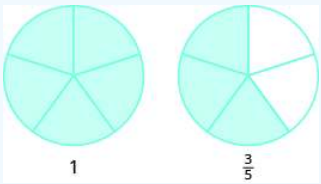
a. 3 halves
b. 8 fifths
c. 7 thirds

Solution

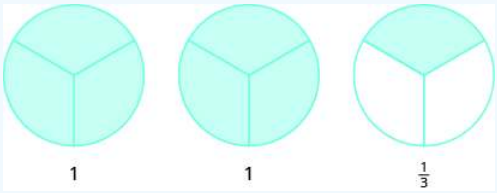
a. 3 halves make 1 whole with 1 half left over.



b. 8 fifths make 1 whole with 3 fifths left over.



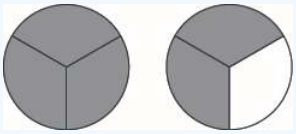
c. 7 thirds make 2 wholes with 1 third left over.



Exercise 1.3.1.7

Use fraction circles to make wholes with the following pieces: 5 thirds.

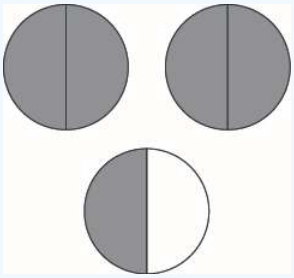
Answer



Exercise 1.3.1.8

Use fraction circles to make wholes with the following pieces: 5 halves.

Answer



Model Improper Fractions and Mixed Numbers

In Example 1.3.1.4*b* you had eight equal fifth pieces. You used five of them to make one whole, and you had three fifths left over. Let us use fraction notation to show what happened. You had eight pieces, each of them one fifth, $\frac{1}{5}$, so altogether you had eight fifths, which we can write as $\frac{8}{5}$. The fraction $\frac{8}{5}$ is one whole, 1, plus three fifths, $\frac{3}{5}$, or $1\frac{3}{5}$, which is read as *one and three-fifths*. The number $1\frac{3}{5}$ is called a mixed number. A mixed number consists of a whole number and a fraction.

Definition: Mixed Numbers

A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as follows.

$$a\frac{b}{c} \qquad c \neq 0 \tag{1.3.1.2}$$

Fractions such as $\frac{5}{4}$, $\frac{3}{2}$, $\frac{5}{5}$, and $\frac{7}{3}$ are called improper fractions. In an improper fraction, the numerator is greater than or equal to the denominator, so its value is greater than or equal to one. When a fraction has a numerator that is smaller than the denominator, it is called a proper fraction, and its value is less than one. Fractions such as $\frac{1}{2}$, $\frac{3}{7}$, and $\frac{11}{18}$ are proper fractions.

Definition: Proper and Improper Fractions

The fraction $\frac{a}{b}$ is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

Example 1.3.1.5: improper fraction

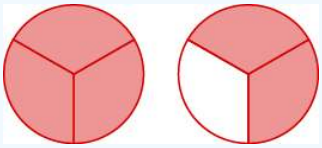
Name the improper fraction modeled. Then write the improper fraction as a mixed number.

Solution

Each circle is divided into three pieces, so each piece is $\frac{1}{3}$ of the circle. There are four pieces shaded, so there are four thirds or $\frac{4}{3}$. The figure shows that we also have one whole circle and one third, which is $1\frac{1}{3}$. So, $\frac{4}{3}=1\frac{1}{3}$.

Exercise 1.3.1.9

Name the improper fraction. Then write it as a mixed number.

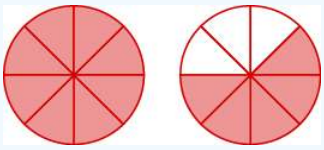


Answer

$\frac{5}{3}=1\frac{2}{3}$

Exercise 1.3.1.10

Name the improper fraction. Then write it as a mixed number.



Answer

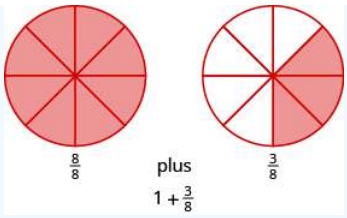
$\frac{13}{8}=1\frac{5}{8}$

Example 1.3.1.6: model a fraction

Draw a figure to model $\frac{11}{8}$.

Solution

The denominator of the improper fraction is 8. Draw a circle divided into eight pieces and shade all of them. This takes care of eight eighths, but we have 11 eighths. We must shade three of the eight parts of another circle.

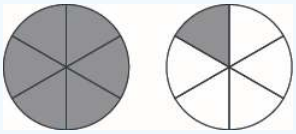


So, $\frac{11}{8} = 1\frac{3}{8}$.

Exercise1.3.1.11

Draw a figure to model $\frac{7}{6}$.

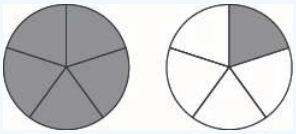
Answer



Exercise 1.3.1.12

Draw a figure to model $\frac{6}{5}$.

Answer



Example 1.3.1.7: model a fraction

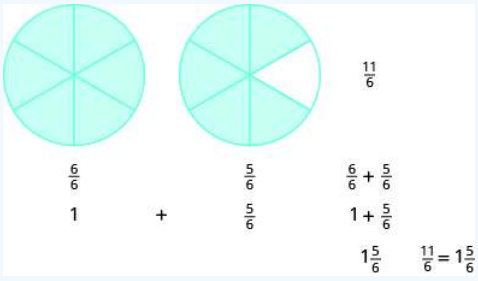
Use a model to rewrite the improper fraction $\frac{11}{6}$ as a mixed number.

Solution

We start with 11 sixths $\left(\frac{11}{6}\right)$. We know that six sixths makes one whole.

$$\frac{6}{6} = 1$$

That leaves us with five more sixths, which is $\frac{5}{6}$ (11 sixths minus 6 sixths is 5 sixths). So, $\frac{11}{6} = 1\frac{5}{6}$.



Exercise 1.3.1.13

Use a model to rewrite the improper fraction as a mixed number: $\frac{9}{7}$.

Answer

$1\frac{2}{7}$

Exercise 1.3.1.14

Use a model to rewrite the improper fraction as a mixed number: $\frac{7}{4}$.

Answer

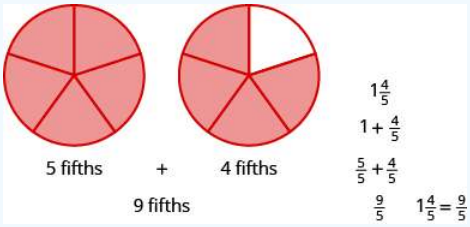
$1\frac{3}{4}$

Example 1.3.1.8: model a fraction

Use a model to rewrite the mixed number $1\frac{4}{5}$ as an improper fraction.

Solution

The mixed number $1\frac{4}{5}$ means one whole plus four fifths. The denominator is 5, so the whole is $\frac{5}{5}$. Together five fifths and four fifths equals nine fifths. So, $1\frac{4}{5} = \frac{9}{5}$.



Exercise 1.3.1.15

Use a model to rewrite the mixed number as an improper fraction: $1\frac{3}{8}$.

Answer

$\frac{11}{8}$

Exercise 1.3.1.16

Use a model to rewrite the mixed number as an improper fraction: $1\frac{5}{6}$.

Answer

$\frac{11}{6}$

Convert between Improper Fractions and Mixed Numbers

In Example 1.3.1.7, we converted the improper fraction $\frac{11}{6}$ to the mixed number $1\frac{5}{6}$ using fraction circles. We did this by grouping six sixths together to make a whole; then we looked to see how many of the 11 pieces were left. We saw that $\frac{11}{6}$ made one whole group of six sixths plus five more sixths, showing that $\frac{11}{6} = \frac{15}{6}$.

The division expression $\frac{11}{6}$ (which can also be written as $6\overline{)11}$) tells us to find how many groups of 6 are in 11. To convert an improper fraction to a mixed number without fraction circles, we divide.

Example 1.3.1.9:

Convert $\frac{11}{6}$ to a mixed number.

Solution

Divide the denominator into the numerator.	Remember $\frac{11}{6}$ means $11 \div 6$.
Identify the quotient, remainder and divisor.	$6\overline{)11}$ 1 ← quotient; divisor → 6; 5 ← remainder
Write the mixed number as $quotient\frac{remainder}{divisor}$.	$1\frac{5}{6}$

So, $\frac{11}{6} = 1\frac{5}{6}$.

Exercise 1.3.1.17

Convert the improper fraction to a mixed number: $\frac{13}{7}$.

Answer

$1\frac{6}{7}$

Exercise 1.3.1.18

Convert the improper fraction to a mixed number: $\frac{14}{9}$.

Answer

$1\frac{5}{9}$

HOW TO: CONVERT AN IMPROPER FRACTION TO A MIXED NUMBER

Step 1. Divide the denominator into the numerator.

Step 2. Identify the quotient, remainder, and divisor.

Step 3. Write the mixed number as $quotient\frac{remainder}{divisor}$.

Example 1.3.1.10:

Convert the improper fraction $\frac{33}{8}$ to a mixed number.

Solution

Divide the denominator into the numerator.	Remember, $\frac{33}{8}$ means $8\overline{)33}$.
Identify the quotient, remainder, and divisor.	$\begin{array}{r} 4 \leftarrow \text{quotient} \\ 8\overline{)33} \leftarrow \text{divisor} \\ \underline{32} \\ 1 \leftarrow \text{remainder} \end{array}$
Write the mixed number as $quotient\ \frac{remainder}{divisor}$.	$4\frac{1}{8}$

So, $\frac{33}{8} = 4\frac{1}{8}$.

Exercise 1.3.1.19

Convert the improper fraction to a mixed number: $\frac{23}{7}$.

Answer

$3\frac{2}{7}$

Exercise 1.3.1.20

Convert the improper fraction to a mixed number: $\frac{48}{11}$.

Answer

$4\frac{4}{11}$

In Example 1.3.1.8, we changed $1\frac{4}{5}$ to an improper fraction by first seeing that the whole is a set of five fifths. So we had five fifths and four more fifths.

$$\frac{5}{5} + \frac{4}{5} = \frac{9}{5}$$

Where did the nine come from? There are nine fifths—one whole (five fifths) plus four fifths. Let us use this idea to see how to convert a mixed number to an improper fraction.

Example 1.3.1.11: convert

Convert the mixed number $4\frac{2}{3}$ to an improper fraction.

Multiply the whole number by the denominator.	$4\frac{2}{3}$
The whole number is 4 and the denominator is 3.	$\frac{4 \cdot 3 + \square}{\square}$

Simplify.	$\frac{12+\square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{12+2}{\square}$
Simplify.	$\frac{14}{\square}$
Write the final sum over the original denominator.	
The denominator is 3.	$\frac{14}{3}$

? Exercise 1.3.1.21

Convert the mixed number to an improper fraction: $3\frac{5}{7}$.

Answer

$\frac{26}{7}$

? Exercise 1.3.1.22

Convert the mixed number to an improper fraction: $2\frac{7}{8}$.

Answer

$\frac{23}{8}$

HOW TO: CONVERT A MIXED NUMBER TO AN IMPROPER FRACTION

Step 1. Multiply the whole number by the denominator.

Step 2. Add the numerator to the product found in Step 1.

Step 3. Write the final sum over the original denominator.

✓ Example 1.3.1.12:

Convert the mixed number $10\frac{2}{7}$ to an improper fraction.

Multiply the whole number by the denominator.	$10\frac{2}{7}$
The whole number is 10 and the denominator is 7.	$\frac{10\cdot 7+\square}{\square}$
Simplify.	$\frac{70+\square}{\square}$
Add the numerator to the product.	
The numerator of the mixed number is 2.	$\frac{70+2}{\square}$

Simplify.	$\frac{72}{\square}$
Write the final sum over the original denominator.	
The denominator is 7.	$\frac{72}{7}$

Exercise 1.3.1.23

Convert the mixed number to an improper fraction: $4\frac{6}{11}$.

Answer

$\frac{50}{11}$

Exercise 1.3.1.24

Convert the mixed number to an improper fraction: $11\frac{1}{3}$.

Answer

$\frac{34}{3}$

Contributors and Attributions

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1.3.2: Visualize Fractions (Part 2)

Model Equivalent Fractions

Let's think about Andy and Bobby and their favorite food again. If Andy eats $\frac{1}{2}$ of a pizza and Bobby eats $\frac{2}{4}$ of the pizza, have they eaten the same amount of pizza? In other words, does $\frac{1}{2} = \frac{2}{4}$? We can use fraction tiles to find out whether Andy and Bobby have eaten *equivalent* parts of the pizza.

Definition: Equivalent Fractions

Equivalent fractions are fractions that have the same value.

Fraction tiles serve as a useful model of equivalent fractions. You may want to use fraction tiles to do the following activity. Or you might make a copy of Figure 4.3 and extend it to include eighths, tenths, and twelfths.

Start with a $\frac{1}{2}$ tile. How many fourths equal one-half? How many of the $\frac{1}{4}$ tiles exactly cover the $\frac{1}{2}$ tile?

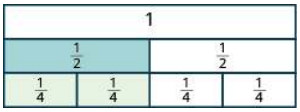


Figure 1.3.2.7

Since two $\frac{1}{4}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{2}{4}$ is the same as $\frac{1}{2}$, or $\frac{2}{4} = \frac{1}{2}$.

How many of the $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile?

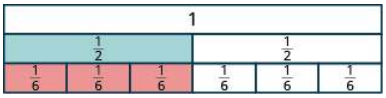


Figure 1.3.2.8

Since three $\frac{1}{6}$ tiles cover the $\frac{1}{2}$ tile, we see that $\frac{3}{6}$ is the same as $\frac{1}{2}$. So, $\frac{3}{6} = \frac{1}{2}$. The fractions are equivalent fractions.

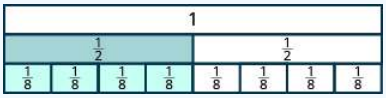
Example 1.3.2.13: equivalent fractions

Use fraction tiles to find equivalent fractions. Show your result with a figure.

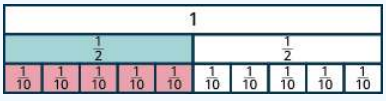
a. How many eighths equal one-half?
b. How many tenths equal one-half?
c. How many twelfths equal one-half?

Solution

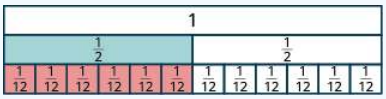
a. It takes four $\frac{1}{8}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{4}{8} = \frac{1}{2}$.



b. It takes five $\frac{1}{10}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{5}{10} = \frac{1}{2}$.



c. It takes six $\frac{1}{12}$ tiles to exactly cover the $\frac{1}{2}$ tile, so $\frac{6}{12} = \frac{1}{2}$.



Suppose you had tiles marked $\frac{1}{20}$. How many of them would it take to equal $\frac{1}{2}$? Are you thinking ten tiles? If you are, you're right, because $\frac{10}{20} = \frac{1}{2}$.

We have shown that $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$, and $\frac{10}{20}$ are all equivalent fractions.

? Exercise 1.3.2.25

Use fraction tiles to find equivalent fractions: How many eighths equal one-fourth?

Answer

2

? Exercise 1.3.2.26

Use fraction tiles to find equivalent fractions: How many twelfths equal one-fourth?

Answer

3

Find Equivalent Fractions

We used fraction tiles to show that there are many fractions equivalent to $\frac{1}{2}$. For example, $\frac{2}{4}, \frac{3}{6}$, and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$. When we lined up the fraction tiles, it took four of the $\frac{1}{8}$ tiles to make the same length as a $\frac{1}{2}$ tile. This showed that $\frac{4}{8} = \frac{1}{2}$. See Example 1.3.2.13

We can show this with pizzas, too. Figure $1.3.2.9a$ shows a single pizza, cut into two equal pieces with $\frac{1}{2}$ shaded. Figure $1.3.2.9b$ shows a second pizza of the same size, cut into eight pieces with $\frac{4}{8}$ shaded.

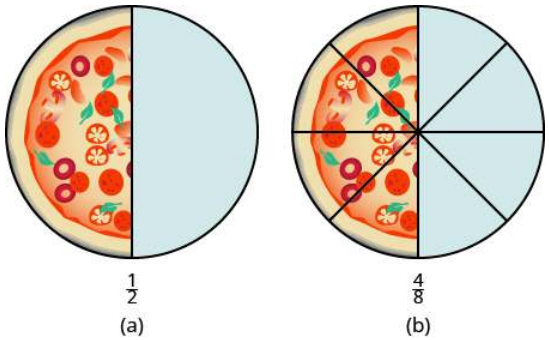


Figure 1.3.2.9

This is another way to show that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$. How can we use mathematics to change $\frac{1}{2}$ into $frac48$? How could you take a pizza that is cut into two pieces and cut it into eight pieces? You could cut each of the two larger pieces into four smaller

pieces! The whole pizza would then be cut into eight pieces instead of just two. Mathematically, what we've described could be written as:

$$\frac{1\cdot 4}{2\cdot 4}=\frac{4}{8}$$

These models lead to the Equivalent Fractions Property, which states that if we multiply the numerator and denominator of a fraction by the same number, the value of the fraction does not change.

Definition: Equivalent Fractions Property

If a, b, and c are numbers where $b\neq 0$ and $c\neq 0$, then

$$\frac{a}{b}=\frac{a\cdot c}{b\cdot c} \tag{1.3.2.1}$$

When working with fractions, it is often necessary to express the same fraction in different forms. To find equivalent forms of a fraction, we can use the Equivalent Fractions Property. For example, consider the fraction one-half.

$$\frac{1\cdot 3}{2\cdot 3}=\frac{3}{6} \text{ so } \frac{1}{2}=\frac{3}{6}$$
$$\frac{1\cdot 2}{2\cdot 2}=\frac{2}{4} \text{ so } \frac{1}{2}=\frac{2}{4}$$
$$\frac{1\cdot 10}{2\cdot 10}=\frac{10}{20} \text{ so } \frac{1}{2}=\frac{10}{20}$$

So, we say that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{10}{20}$ are equivalent fractions.

Example 1.3.2.14: equivalent fractions

Find three fractions equivalent to $\frac{2}{5}$.

Solution

To find a fraction equivalent to $\frac{2}{5}$, we multiply the numerator and denominator by the same number (but not zero). Let us multiply them by 2, 3, and 5.

$$\frac{2\cdot 2}{5\cdot 2}=\frac{4}{10} \qquad \frac{2\cdot 3}{5\cdot 3}=\frac{6}{15} \qquad \frac{2\cdot 5}{5\cdot 5}=\frac{10}{25}$$

So, $\frac{4}{10}$, $\frac{6}{15}$, and $\frac{10}{25}$ are equivalent to $\frac{2}{5}$.

Exercise 1.3.2.27

Find three fractions equivalent to $\frac{3}{5}$.

Answer

Correct answers include $\frac{6}{10}$, $\frac{9}{15}$ and $\frac{12}{20}$

Exercise 1.3.2.28

Find three fractions equivalent to $\frac{4}{5}$.

Answer

Correct answers include $\frac{8}{10}$, $\frac{12}{15}$ and $\frac{16}{20}$

Example 1.3.2.15: equivalent fractions

Find a fraction with a denominator of 21 that is equivalent to $\frac{2}{7}$.

Solution

To find equivalent fractions, we multiply the numerator and denominator by the same number. In this case, we need to multiply the denominator by a number that will result in 21.

Since we can multiply 7 by 3 to get 21, we can find the equivalent fraction by multiplying both the numerator and denominator by 3.

$$\frac{2}{7} = \frac{2 \cdot 3}{7 \cdot 3} = \frac{6}{21}$$

Exercise 1.3.2.29

Find a fraction with a denominator of 21 that is equivalent to $\frac{6}{7}$.

Answer

$\frac{18}{21}$

Exercise 1.3.2.30

Find a fraction with a denominator of 100 that is equivalent to $\frac{3}{10}$.

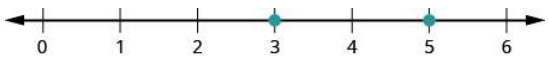
Answer

$\frac{30}{100}$

Locate Fractions and Mixed Numbers on the Number Line

Now we are ready to plot fractions on a number line. This will help us visualize fractions and understand their values.

Let us locate $\frac{1}{5}, \frac{4}{5}, 3, 3\frac{1}{3}, \frac{7}{4}, \frac{9}{2}, 5$, and $\frac{8}{3}$ on the number line. We will start with the whole numbers 3 and 5 because they are the easiest to plot.



The proper fractions listed are $\frac{1}{5}$ and $\frac{4}{5}$. We know proper fractions have values less than one, so $\frac{1}{5}$ and $\frac{1}{5}$ are located between the whole numbers 0 and 1. The denominators are both 5, so we need to divide the segment of the number line between 0 and 1 into five equal parts. We can do this by drawing four equally spaced marks on the number line, which we can then label as $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$, and $\frac{4}{5}$. Now plot points at $\frac{1}{5}$ and $\frac{4}{5}$.



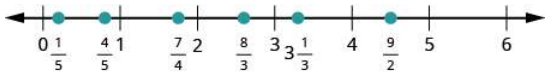
The only mixed number to plot is $3\frac{1}{3}$. Between what two whole numbers is $3\frac{1}{3}$? Remember that a mixed number is a whole number plus a proper fraction, so $3\frac{1}{3} > 3$. Since it is greater than 3, but not a whole unit greater, $3\frac{1}{3}$ is between 3 and 4. We need to divide the portion of the number line between 3 and 4 into three equal pieces (thirds) and plot $3\frac{1}{3}$ at the first mark.



Finally, look at the improper fractions $\frac{7}{4}$, $\frac{9}{2}$, and $\frac{8}{3}$. Locating these points will be easier if you change each of them to a mixed number.

$$\frac{7}{4} = 1\frac{3}{4}, \qquad \frac{9}{2} = 4\frac{1}{2}, \qquad \frac{8}{3} = 2\frac{2}{3}$$

Here is the number line with all the points plotted.

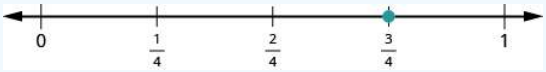


Example 1.3.2.16: locate and label

Locate and label the following on a number line: $\frac{3}{4}$, $\frac{4}{3}$, $\frac{5}{3}$, $4\frac{1}{5}$, and $\frac{7}{2}$.

Solution

Start by locating the proper fraction $\frac{3}{4}$. It is between 0 and 1. To do this, divide the distance between 0 and 1 into four equal parts. Then plot $\frac{3}{4}$.



Next, locate the mixed number $4\frac{1}{5}$. It is between 4 and 5 on the number line. Divide the number line between 4 and 5 into five equal parts, and then plot $4\frac{1}{5}$ one-fifth of the way between 4 and 5.



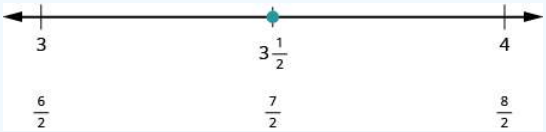
Now locate the improper fractions $\frac{4}{3}$ and $\frac{5}{3}$. It is easier to plot them if we convert them to mixed numbers first.

$$\frac{4}{3} = 1\frac{1}{3}, \qquad \frac{5}{3} = 1\frac{2}{3}$$

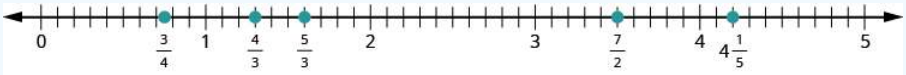
Divide the distance between 1 and 2 into thirds.



Next let us plot $\frac{7}{2}$. We write it as a mixed number, $\frac{7}{2} = 3\frac{1}{2}$. Plot it between 3 and 4.



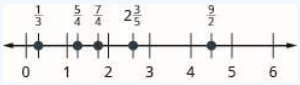
The number line shows all the numbers located on the number line.



Exercise 1.3.2.31

Locate and label the following on a number line: $\frac{1}{3}, \frac{5}{4}, \frac{7}{4}, 2\frac{3}{5}, \frac{9}{2}$.

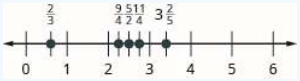
Answer



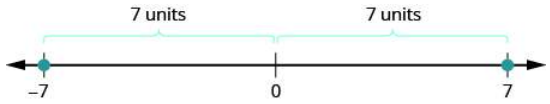
Exercise 1.3.2.32

Locate and label the following on a number line: $\frac{2}{3}, \frac{5}{2}, \frac{9}{4}, \frac{11}{4}, 3\frac{2}{5}$.

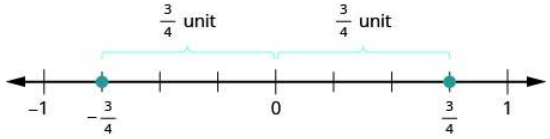
Answer



In Introduction to Integers, we defined the opposite of a number. It is the number that is the same distance from zero on the number line but on the opposite side of zero. We saw, for example, that the opposite of 7 is -7 and the opposite of -7 is 7.



Fractions have opposites, too. The opposite of $\frac{3}{4}$ is $-\frac{3}{4}$. It is the same distance from 0 on the number line, but on the opposite side of 0.



Thinking of negative fractions as the opposite of positive fractions will help us locate them on the number line. To locate $-\frac{15}{8}$ on the number line, first think of where $\frac{15}{8}$ is located. It is an improper fraction, so we first convert it to the mixed number $1\frac{7}{8}$ and see that it will be between 1 and 2 on the number line. So its opposite, $-\frac{15}{8}$, will be between -1 and -2 on the number line.



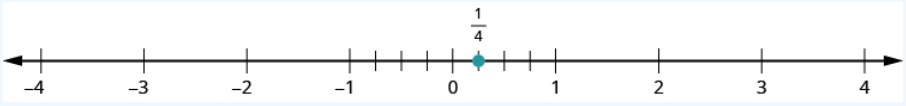
Example 1.3.2.17: locate and label

Locate and label the following on the number line: $\frac{1}{4}, -\frac{1}{4}, 1\frac{1}{3}, -1\frac{1}{3}, \frac{5}{2}$, and $-\frac{5}{2}$.

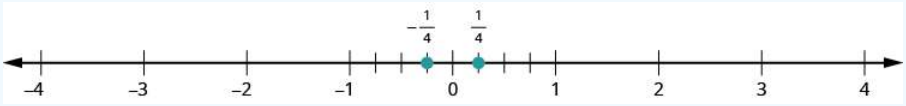
Solution

Draw a number line. Mark 0 in the middle and then mark several units to the left and right.

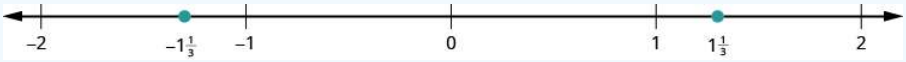
To locate $\frac{1}{4}$, divide the interval between 0 and 1 into four equal parts. Each part represents one-quarter of the distance. So plot $\frac{1}{4}$ at the first mark.



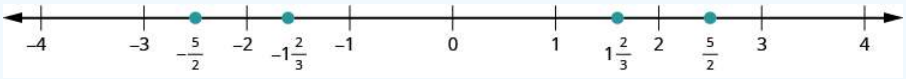
To locate $-\frac{1}{4}$, divide the interval between 0 and -1 into four equal parts. Plot $-\frac{1}{4}$ at the first mark to the left of 0.



Since $1\frac{1}{3}$ is between 1 and 2, divide the interval between 1 and 2 into three equal parts. Plot $1\frac{1}{3}$ at the first mark to the right of 1. Then since $-1\frac{1}{3}$ is the opposite of $1\frac{1}{3}$ it is between -1 and -2. Divide the interval between -1 and -2 into three equal parts. Plot $-1\frac{1}{3}$ at the first mark to the left of -1.



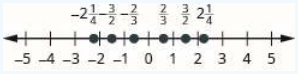
To locate $\frac{5}{2}$ and $-\frac{5}{2}$, it may be helpful to rewrite them as the mixed numbers $2\frac{1}{2}$ and $-2\frac{1}{2}$. Since $2\frac{1}{2}$ is between 2 and 3, divide the interval between 2 and 3 into two equal parts. Plot $\frac{5}{2}$ at the mark. Then since $-2\frac{1}{2}$ is between -2 and -3, divide the interval between -2 and -3 into two equal parts. Plot $-\frac{5}{2}$ at the mark.



Exercise 1.3.2.33

Locate and label each of the given fractions on a number line: $\frac{2}{3}, -\frac{2}{3}, 2\frac{1}{4}, -2\frac{1}{4}, \frac{3}{2}, -\frac{3}{2}$

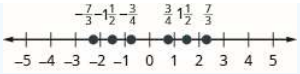
Answer



Exercise 1.3.2.34

Locate and label each of the given fractions on a number line: $\frac{3}{4}, -\frac{3}{4}, 1\frac{1}{2}, -1\frac{1}{2}, \frac{7}{3}, -\frac{7}{3}$

Answer



Order Fractions and Mixed Numbers

We can use the inequality symbols to order fractions. Remember that $a > b$ means that a is to the right of b on the number line. As we move from left to right on a number line, the values increase.

Example 1.3.2.18: order

Order each of the following pairs of numbers, using $<$ or $>$:

a. $-\frac{2}{3}\_\_\_\_-1$

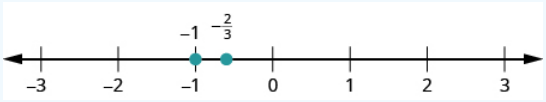
b. $-3\frac{1}{2}\_\_\_\_-3$

c. $-\frac{3}{7}\_\_\_\_-\frac{3}{8}$

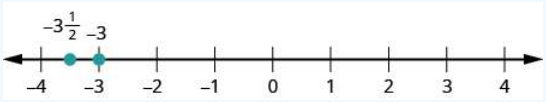
d. $-2\_\_\_\_-\frac{16}{9}$

Solution

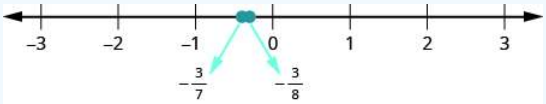
a. $-\frac{2}{3} > -1$



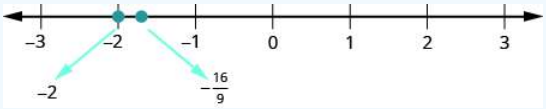
b. $-3\frac{1}{2} < -3$



c. $-\frac{3}{7} < -\frac{3}{8}$



d. $-2 < -\frac{16}{9}$



Exercise 1.3.2.35

Order each of the following pairs of numbers, using $<$ or $>$:

a. $-\frac{1}{3}\_\_-1$

b. $-1\frac{1}{2}\_\_-2$

c. $-\frac{2}{3}\_\_-\frac{1}{3}$

d. $-3 \_\_ -\frac{7}{3}$

Answer a

$>$

Answer b

$>$

Answer c

$<$

Answer d

$<$

Exercise 1.3.2.36

Order each of the following pairs of numbers, using $<$ or $>$:

a. $-3 \_\_ -\frac{17}{5}$

b. $-2\frac{1}{4} \_\_ -2$

c. $-\frac{3}{5} \_\_ -\frac{4}{5}$

d. $-4 \_\_ -\frac{10}{3}$

Answer a

$>$

Answer b

$<$

Answer c

$>$

Answer d

$<$

Access Additional Online Resources

- Introduction to Fractions
- Identify Fractions Using Pattern Blocks

Key Concepts

- Property of One
  - Any number, except zero, divided by itself is one.
    $\frac{a}{a} = 1$, where $a \neq 0$.
- Mixed Numbers
  - A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$.
  - It is written as follows: $a\frac{b}{c}$ $c \neq 0$
- Proper and Improper Fractions

- The fraction $\frac{a}{b}$ is a proper fraction if $a < b$ and an improper fraction if $a \geq b$.

- Convert an improper fraction to a mixed number.
  1. Divide the denominator into the numerator.
  2. Identify the quotient, remainder, and divisor.
  3. Write the mixed number as $\frac{\text{remainder}}{\text{divisor}}$.
- Convert a mixed number to an improper fraction.
  1. Multiply the whole number by the denominator.
  2. Add the numerator to the product found in Step 1.
  3. Write the final sum over the original denominator.
- Equivalent Fractions Property
  - If a, b and c are numbers where $b \neq 0$, $c \neq 0$, then \(\dfrac{a}{b} = \dfrac{a\cdot c}{b\cdot c}\]).

Glossary

equivalent fractions

Equivalent fractions are two or more fractions that have the same value.

fraction

A fraction is written $\frac{a}{b}$. in a fraction, a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.

mixed number

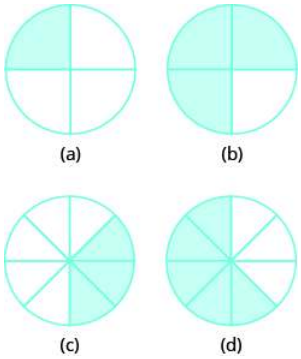
A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where $c \neq 0$. It is written as $a\frac{b}{c}$, where $c \neq 0$.

proper and improper fractions

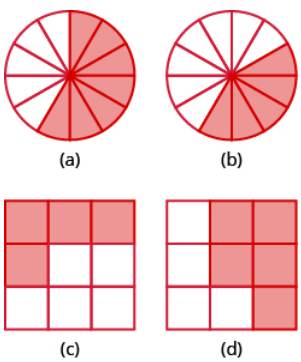
The fraction $\frac{a}{b}$ is proper if $a < b$ and improper if $a > b$.

Practice Makes Perfect

1.



2.



In the following exercises, shade parts of circles or squares to model the following fractions.

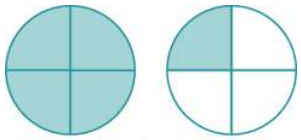
3. $\frac{1}{2}$
4. $\frac{1}{3}$
5. $\frac{3}{4}$
6. $\frac{2}{5}$
7. $\frac{5}{6}$
8. $\frac{7}{8}$
9. $\frac{5}{8}$
10. $\frac{7}{10}$

In the following exercises, use fraction circles to make wholes, if possible, with the following pieces.

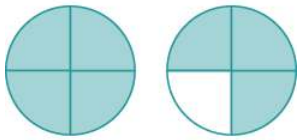
11. 3 thirds
12. 8 eighths
13. 7 sixths
14. 4 thirds
15. 7 fifths
16. 7 fourths

In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

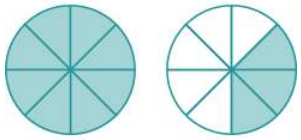
17.



(a)

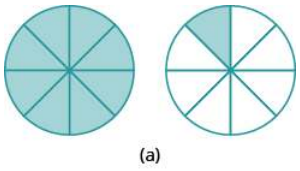


(b)

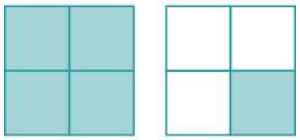


(c)

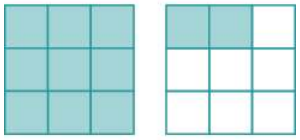
18.



(a)

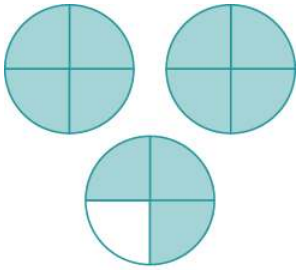


(b)

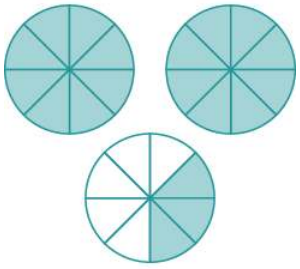


(c)

19.



(a)



(b)

In the following exercises, draw fraction circles to model the given fraction.

20. $\frac{3}{3}$
21. $\frac{4}{4}$
22. $\frac{7}{4}$
23. $\frac{5}{3}$
24. $\frac{11}{6}$
25. $\frac{13}{8}$
26. $\frac{10}{3}$
27. $\frac{9}{4}$

In the following exercises, rewrite the improper fraction as a mixed number.

28. $\frac{3}{2}$
29. $\frac{5}{3}$
30. $\frac{11}{4}$
31. $\frac{13}{5}$
32. $\frac{25}{6}$
33. $\frac{28}{9}$
34. $\frac{42}{13}$
35. $\frac{47}{15}$

In the following exercises, rewrite the mixed number as an improper fraction.

36. $1\frac{2}{3}$
37. $1\frac{2}{5}$
38. $2\frac{1}{4}$
39. $2\frac{5}{6}$
40. $2\frac{7}{9}$
41. $2\frac{5}{7}$
42. $3\frac{4}{7}$
43. $3\frac{5}{9}$

In the following exercises, use fraction tiles or draw a figure to find equivalent fractions.

44. How many sixths equal one-third?
45. How many twelfths equal one-third?
46. How many eighths equal three-fourths?
47. How many twelfths equal three-fourths?
48. How many fourths equal three-halves?
49. How many sixths equal three-halves?

In the following exercises, find three fractions equivalent to the given fraction. Show your work, using figures or algebra.

50. $\frac{1}{4}$
51. $\frac{1}{3}$
52. $\frac{3}{8}$
53. $\frac{5}{6}$
54. $\frac{2}{7}$
55. $\frac{5}{9}$

In the following exercises, plot the numbers on a number line.

56. $\frac{2}{3}, \frac{5}{4}, \frac{12}{5}$

57. $\frac{1}{3}, \frac{7}{4}, \frac{13}{5}$
58. $\frac{1}{4}, \frac{9}{5}, \frac{11}{3}$
59. $\frac{7}{10}, \frac{5}{2}, \frac{13}{8}, 3$
60. $2\frac{1}{3}, -2\frac{1}{3}$
61. $1\frac{3}{4}, -1\frac{3}{5}$
62. $\frac{3}{4}, -\frac{3}{4}, 1\frac{2}{3}, -1\frac{2}{3}, \frac{5}{2}, -\frac{5}{2}$
63. $\frac{2}{5}, -\frac{2}{5}, 1\frac{3}{4}, -1\frac{3}{4}, \frac{8}{3}, -\frac{8}{3}$

In the following exercises, order each of the following pairs of numbers, using < or >.

64. $-1 \_\_ -\frac{1}{4}$
65. $-1 \_\_ -\frac{1}{3}$
66. $-2\frac{1}{2} \_\_ -3$
67. $-1\frac{3}{4} \_\_ -2$
68. $-\frac{5}{12} \_\_ -\frac{7}{12}$
69. $-\frac{9}{10} \_\_ -\frac{3}{10}$
70. $-3 \_\_ -\frac{13}{5}$
71. $-4 \_\_ -\frac{23}{6}$

Everyday Math

72. **Music Measures** A choreographed dance is broken into counts. A $\frac{1}{1}$ count has one step in a count, a $\frac{1}{2}$ count has two steps in a count and a 1 3 count has three steps in a count. How many steps would be in a $\frac{1}{5}$ count? What type of count has four steps in it?
73. **Music Measures** Fractions are used often in music. In 4 4 time, there are four quarter notes in one measure.
    a. How many measures would eight quarter notes make?
    b. The song “Happy Birthday to You” has 25 quarter notes. How many measures are there in “Happy Birthday to You?”
74. **Baking** Nina is making five pans of fudge to serve after a music recital. For each pan, she needs 1 2 cup of walnuts.
    a. How many cups of walnuts does she need for five pans of fudge?
    b. Do you think it is easier to measure this amount when you use an improper fraction or a mixed number? Why?

Writing Exercises

75. Give an example from your life experience (outside of school) where it was important to understand fractions.
76. Explain how you locate the improper fraction $\frac{21}{4}$ on a number line on which only the whole numbers from 0 through 10 are marked.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
understand the meaning of fractions.			
model improper fractions and mixed numbers.			
convert between improper fractions and mixed numbers.			
model equivalent fractions.			
find equivalent fractions.			
locate fractions and mixed numbers on the number line.			
order fractions and mixed numbers.			

(b) If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Contributors and Attributions

- Lynn Marecek (Santa Ana College) and MaryAnne Anthony-Smith (Formerly of Santa Ana College). This content is licensed under Creative Commons Attribution License v4.0 "Download for free at http://cnx.org/contents/fd53eae1-fa2...49835c3c@5.191."

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1.3.3: Multiply and Divide Fractions (Part 1)

Learning Objectives

- Simplify fractions
- Multiply fractions
- Find reciprocals
- Divide fractions

be prepared!

Before you get started, take this readiness quiz.

1. Find the prime factorization of 48. If you missed this problem, review Example 2.5.1.
2. Draw a model of the fraction $\frac{3}{4}$. If you missed this problem, review Example 4.1.2.
3. Find two fractions equivalent to $\frac{5}{6}$. Answers may vary. Acceptable answers include $\frac{10}{12}$, $\frac{15}{18}$, $\frac{50}{60}$, etc. If you missed this problem, review Example 4.1.14.

Simplify Fractions

In working with equivalent fractions, you saw that there are many ways to write fractions that have the same value, or represent the same part of the whole. How do you know which one to use? Often, we'll use the fraction that is in *simplified* form.

A fraction is considered simplified if there are no common factors, other than 1, in the numerator and denominator. If a fraction does have common factors in the numerator and denominator, we can reduce the fraction to its simplified form by removing the common factors.

Definition: Simplified Fraction

A fraction is considered simplified if there are no common factors in the numerator and denominator.

For example,

- $\frac{2}{3}$ is simplified because there are no common factors of 2 and 3.
- $\frac{10}{15}$ is not simplified because 5 is a common factor of 10 and 15.

The process of simplifying a fraction is often called *reducing the fraction*. In the previous section, we used the Equivalent Fractions Property to find equivalent fractions. We can also use the Equivalent Fractions Property in reverse to simplify fractions. We rewrite the property to show both forms together.

Definition: Equivalent Fractions Property

If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.

Notice that c is a common factor in the numerator and denominator. Anytime we have a common factor in the numerator and denominator, it can be removed.

HOW TO: SIMPLIFY A FRACTION

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.

Step 2. Simplify, using the equivalent fractions property, by removing common factors.

Step 3. Multiply any remaining factors.

Example 1.3.3.1: simplify

Simplify: $\frac{10}{15}$.

Solution

To simplify the fraction, we look for any common factors in the numerator and the denominator.

Notice that 5 is a factor of both 10 and 15.	$\frac{10}{15}$
Factor the numerator and denominator.	$\frac{2 \cdot 5}{3 \cdot 5}$
Remove the common factors.	$\frac{2 \cdot \cancel{5}}{3 \cdot \cancel{5}}$
Simplify.	$\frac{2}{3}$

Exercise 1.3.3.1

Simplify: $\frac{8}{12}$.

Answer

$\frac{2}{3}$

Exercise 1.3.3.2

Simplify: $\frac{12}{16}$.

Answer

$\frac{3}{4}$

To simplify a negative fraction, we use the same process as in Example 1.3.3.1 Remember to keep the negative sign.

Example 1.3.3.2: simplify

Simplify: $-\frac{18}{24}$.

Solution

We notice that 18 and 24 both have factors of 6.	$-\frac{18}{24}$
Rewrite the numerator and denominator showing the common factor.	$-\frac{3 \cdot 6}{4 \cdot 6}$
Remove common factors.	$-\frac{3 \cdot \cancel{6}}{4 \cdot \cancel{6}}$
Simplify.	$-\frac{3}{4}$

Exercise 1.3.3.3

Simplify: $-\frac{21}{28}$.

Answer

$-\frac{3}{4}$

Exercise 1.3.3.4

Simplify: $-\frac{16}{24}$.

Answer

$-\frac{2}{3}$

After simplifying a fraction, it is always important to check the result to make sure that the numerator and denominator do not have any more factors in common. Remember, the definition of a simplified fraction: *a fraction is considered simplified if there are no common factors in the numerator and denominator*.

When we simplify an improper fraction, there is no need to change it to a mixed number.

Example 1.3.3.3:

Simplify: $-\frac{56}{32}$.

Solution

	$-\frac{56}{32}$
Rewrite the numerator and denominator, showing the common factors, 8.	$-\frac{7\cdot 8}{4\cdot 8}$
Remove common factors.	$-\frac{7\cdot \cancel{8}}{4\cdot \cancel{8}}$
Simplify.	$-\frac{7}{4}$

Exercise 1.3.3.5

Simplify: $-\frac{54}{42}$.

Answer

$-\frac{9}{7}$

Exercise 1.3.3.6

Simplify: $-\frac{81}{45}$.

Answer

$-\frac{9}{5}$

HOW TO: SIMPLIFY A FRACTION

Step 1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.

Step 2. Simplify, using the equivalent fractions property, by removing common factors.

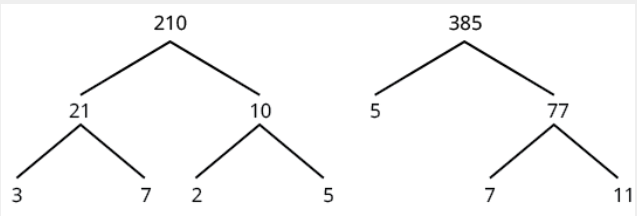
Step 3. Multiply any remaining factors.

Sometimes it may not be easy to find common factors of the numerator and denominator. A good idea, then, is to factor the numerator and the denominator into prime numbers. (You may want to use the factor tree method to identify the prime factors.) Then divide out the common factors using the Equivalent Fractions Property.

Example 1.3.3.4: simplify

Simplify: $\frac{210}{385}$.

Solution

	$\frac{210}{385}$
Use factor trees to factor the numerator and denominator.	
Rewrite the numerator and denominator as the product of the primes.	$\frac{210}{385} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{5 \cdot 7 \cdot 11}$
Remove the common factors.	$\frac{2 \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}{\cancel{5} \cdot \cancel{7} \cdot 11}$
Simplify.	$\frac{2 \cdot 3}{11}$
Multiply any remaining factors.	$\frac{6}{11}$

Exercise 1.3.3.7

Simplify: $\frac{69}{120}$.

Answer

$\frac{23}{40}$

Exercise 1.3.3.8

Simplify: $\frac{120}{192}$.

Answer

$\frac{5}{8}$

We can also simplify fractions containing variables. If a variable is a common factor in the numerator and denominator, we remove it just as we do with an integer factor.

Example 1.3.3.5: simplify

Simplify: $\frac{5xy}{15x}$.

Solution

	$\frac{5xy}{15x}$
Rewrite numerator and denominator showing common factors.	$\frac{5 \cdot x \cdot y}{3 \cdot 5 \cdot x}$
Remove common factors.	$\frac{\cancel{5} \cdot \cancel{x} \cdot y}{3 \cdot \cancel{5} \cdot \cancel{x}}$
Simplify.	$\frac{y}{3}$

Exercise 1.3.3.9

Simplify: $\frac{7x}{7y}$.

Answer

$\frac{x}{y}$

Exercise 1.3.3.10

Simplify: $\frac{9a}{9b}$.

Answer

$\frac{a}{b}$

Multiply Fractions

A model may help you understand multiplication of fractions. We will use fraction tiles to model $\frac{1}{2} \cdot \frac{3}{4}$. To multiply $\frac{1}{2}$ and $\frac{3}{4}$, think $\frac{1}{2}$ of $\frac{3}{4}$.

Start with fraction tiles for three-fourths. To find one-half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three $\frac{1}{4}$ tiles evenly into two parts, we exchange them for smaller tiles.

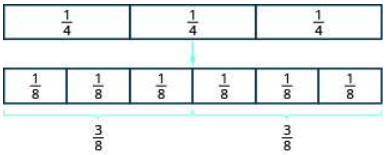


Figure 1.3.3.1

We see $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Taking half of the six $\frac{1}{8}$ tiles gives us three $\frac{1}{8}$ tiles, which is $\frac{3}{8}$. Therefore,

$$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$$

Example 1.3.3.6: model a fraction

Use a diagram to model $\frac{1}{2} \cdot \frac{3}{4}$.

Solution

First shade in $\frac{3}{4}$ of the rectangle.

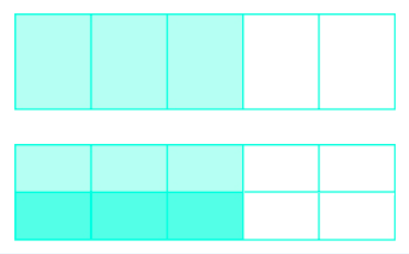
We will take $\frac{1}{2}$ of this $\frac{3}{4}$, so we heavily shade $\frac{1}{2}$ of the shaded region.

Notice that 3 out of the 8 pieces are heavily shaded. This means that $\frac{3}{8}$ of the rectangle is heavily shaded. Therefore, $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{4}$, or $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

Exercise 1.3.3.11

Use a diagram to model: $\frac{1}{2} \cdot \frac{3}{5}$.

Answer

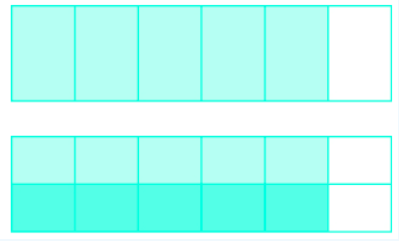


$\frac{3}{10}$

Exercise 1.3.3.12

Use a diagram to model: $\frac{1}{2} \cdot \frac{5}{6}$.

Answer



$\frac{5}{12}$

Look at the result we got from the model in Example 1.3.3.6 We found that $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$. Do you notice that we could have gotten the same answer by multiplying the numerators and multiplying the denominators?

	$\frac{1}{2} \cdot \frac{3}{4}$
Multiply the numerators, and multiply the denominators.	$\frac{1}{2} \cdot \frac{3}{4}$
Simplify.	$\frac{3}{8}$

This leads to the definition of fraction multiplication. To multiply fractions, we multiply the numerators and multiply the denominators. Then we write the fraction in simplified form.

Definition: Fraction Multiplication

If a, b, c, and d are numbers where $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \tag{1.3.3.1}$$

Example 1.3.3.7: multiply

Multiply, and write the answer in simplified form: $\frac{3}{4} \cdot \frac{1}{5}$.

Solution

	$\frac{3}{4} \cdot \frac{1}{5}$
Multiply the numerators, and multiply the denominators.	$\frac{3 \cdot 1}{4 \cdot 5}$
Simplify.	$\frac{3}{20}$

There are no common factors, so the fraction is simplified.

Exercise 1.3.3.13

Multiply, and write the answer in simplified form: $\frac{1}{3} \cdot \frac{2}{5}$.

Answer

$\frac{2}{15}$

Exercise 1.3.3.14

Multiply, and write the answer in simplified form: $\frac{3}{5} \cdot \frac{7}{8}$.

Answer

$\frac{21}{40}$

When multiplying fractions, the properties of positive and negative numbers still apply. It is a good idea to determine the sign of the product as the first step. In Example 1.3.3.8we will multiply two negatives, so the product will be positive.

Example 1.3.3.8: multiply

Multiply, and write the answer in simplified form: $-\frac{5}{8}\left(-\frac{2}{3}\right)$.

Solution

	$-\frac{5}{8}\left(-\frac{2}{3}\right)$
The signs are the same, so the product is positive. Multiply the numerators, multiply the denominators.	$\frac{5 \cdot 2}{8 \cdot 3}$
Simplify.	$\frac{10}{24}$
Look for common factors in the numerator and denominator. Rewrite showing common factors.	$\frac{5 \cdot \cancel{2}}{12 \cdot \cancel{2}}$
Remove common factors.	$\frac{5}{12}$

Another way to find this product involves removing common factors earlier.

	$-\frac{5}{8}\left(-\frac{2}{3}\right)$
Determine the sign of the product. Multiply.	$\frac{5 \cdot 2}{8 \cdot 3}$
Show common factors and then remove them.	$\frac{5 \cdot \cancel{2}}{12 \cdot \cancel{2}}$
Multiply remaining factors.	$\frac{5}{12}$

We get the same result.

Exercise 1.3.3.15

Multiply, and write the answer in simplified form: $-\frac{4}{7}\left(-\frac{5}{8}\right)$.

Answer

$\frac{5}{14}$

Exercise 1.3.3.16

Multiply, and write the answer in simplified form: $-\frac{7}{12}\left(-\frac{8}{9}\right)$.

Answer

$\frac{14}{27}$

Example 1.3.3.9: multiply

Multiply, and write the answer in simplified form: $-\frac{14}{15}\cdot\frac{20}{21}$.

Solution

	$-\frac{14}{15}\cdot\frac{20}{21}$
Determine the sign of the product; multiply.	$-\frac{14}{15}\cdot\frac{20}{21}$
Are there any common factors in the numerator and the denominator? We know that 7 is a factor of 14 and 21, and 5 is a factor of 20 and 15.	
Rewrite showing common factors.	$-\frac{2\cdot\cancel{7}\cdot 4\cdot\cancel{5}}{3\cdot\cancel{5}\cdot 3\cdot\cancel{7}}$
Remove the common factors.	$-\frac{2\cdot 4}{3\cdot 3}$
Multiply the remaining factors.	$-\frac{8}{9}$

Exercise 1.3.3.17

Multiply, and write the answer in simplified form: $-\frac{10}{28}\cdot\frac{8}{15}$.

Answer

$-\frac{4}{21}$

Exercise 1.3.3.18

Multiply, and write the answer in simplified form: $-\frac{9}{20}\cdot\frac{5}{12}$.

Answer

$-\frac{3}{16}$

When multiplying a fraction by an integer, it may be helpful to write the integer as a fraction. Any integer, a, can be written as $\frac{a}{1}$. So, $3=\frac{3}{1}$, for example.

Example 1.3.3.10:

Multiply, and write the answer in simplified form:

a. $\frac{1}{7} \cdot 56$

b. $\frac{12}{5}(-20x)$

Solution

a.

	$\frac{1}{7} \cdot 56$
Write 56 as a fraction.	$\frac{1}{7} \cdot \frac{56}{1}$
Determine the sign of the product; multiply.	$\frac{56}{7}$
Simplify.	8

b.

	$\frac{12}{5}(-20x)$
Write −20x as a fraction.	$\frac{12}{5}\left(\frac{-20x}{1}\right)$
Determine the sign of the product; multiply.	$-\frac{12 \cdot 20 \cdot x}{5 \cdot 1}$
Show common factors and then remove them.	$-\frac{12 \cdot 4 \cdot \cancel{5}x}{\cancel{5} \cdot 1}$
Multiply remaining factors; simplify.	$-48x$

Exercise 1.3.3.19

Multiply, and write the answer in simplified form:

a. $\frac{1}{8} \bullet 72$

b. $\frac{11}{3}(-9a)$

Answer a

9

Answer b

$-33a$

Exercise 1.3.3.20

Multiply, and write the answer in simplified form:

a. $\frac{3}{8} \bullet 64$

b. $16x \bullet \frac{11}{12}$

Answer a

24

Answer b

$\frac{44x}{3}$

Contributors and Attributions

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1.3.4: Multiply and Divide Fractions (Part 2)

Find Reciprocals

The fractions $\frac{2}{3}$ and $\frac{3}{2}$ are related to each other in a special way. So are $-\frac{10}{7}$ and $-\frac{7}{10}$. Do you see how? Besides looking like upside-down versions of one another, if we were to multiply these pairs of fractions, the product would be 1.

$$\frac{2}{3}\cdot\frac{3}{2}=1 \quad and \quad -\frac{10}{7}\left(-\frac{7}{10}\right)=1$$

Such pairs of numbers are called reciprocals.

Definition: Reciprocal

The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$, where $a\neq 0$ and $b\neq 0$.

A number and its reciprocal have a product of 1.

$$\frac{a}{b}\cdot\frac{b}{a}=1 \tag{4.2.54}$$

To find the reciprocal of a fraction, we invert the fraction. This means that we place the numerator in the denominator and the denominator in the numerator.

To get a positive result when multiplying two numbers, the numbers must have the same sign. So reciprocals must have the same sign.

$$\frac{a}{b}\cdot\frac{b}{a}=1 \text{ positive}$$

$$3\cdot\frac{1}{3}=1 \quad \text{and} \quad -3\cdot\left(-\frac{1}{3}\right)=1$$

both positive both negative

To find the reciprocal, keep the same sign and invert the fraction. The number zero does not have a reciprocal. Why? A number and its reciprocal multiply to 1. Is there any number r so that $0\cdot r=1$? No. So, the number 0 does not have a reciprocal.

Example 1.3.4.11: reciprocal

Find the reciprocal of each number. Then check that the product of each number and its reciprocal is 1.

a. $\frac{4}{9}$
b. $-\frac{1}{6}$
c. $-\frac{14}{5}$
d. 7

Solution

To find the reciprocals, we keep the sign and invert the fractions.

a.

Find the reciprocal of $\frac{4}{9}$.	The reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$.

Check:

Multiply the number and its reciprocal.	$\frac{4}{9}\cdot\frac{9}{4}$

Multiply numerators and denominators.	$\frac{36}{36}$
Simplify.	1 ✓

b.

Find the reciprocal of $-\frac{1}{6}$.	The reciprocal of $-\frac{1}{6}$ is $\frac{6}{1}$.
Simplify.	-6
Check.	$-\frac{1}{6} \cdot (-6) = 1$ ✓

c.

Find the reciprocal of $-\frac{14}{5}$.	$-\frac{5}{14}$
Check.	$-\frac{14}{5} \cdot \left(-\frac{5}{14}\right) = \frac{70}{70} = 1$ ✓

d.

Find the reciprocal of 7.	
Write 7 as a fraction.	$\frac{7}{1}$
Write the reciprocal of $\frac{7}{1}$.	$\frac{1}{7}$
Check.	$7 \cdot \left(\frac{1}{7}\right) = 1$ ✓

Exercise 1.3.4.21

Find the reciprocal:

a. $\frac{5}{7}$
b. $-\frac{1}{8}$
c. $-\frac{11}{4}$
d. 14

Answer a

$\frac{7}{5}$

Answer b

-8

Answer c

$-\frac{4}{11}$

Answer d

$\frac{1}{14}$

Exercise 1.3.4.22

Find the reciprocal:

a. $\frac{3}{7}$
b. $-\frac{1}{12}$
c. $-\frac{14}{9}$
d. 21

Answer a

$\frac{7}{3}$

Answer b

-12

Answer c

$-\frac{9}{14}$

Answer d

$\frac{1}{21}$

In a previous chapter, we worked with opposites and absolute values. Table 1.3.4.1 compares opposites, absolute values, and reciprocals.

Table 1.3.4.1

Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

Example 1.3.4.12: fractions

Fill in the chart for each fraction in the left column:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$			
$\frac{1}{2}$			
$\frac{9}{5}$			
-5			

Solution

To find the opposite, change the sign. To find the absolute value, leave the positive numbers the same, but take the opposite of the negative numbers. To find the reciprocal, keep the sign the same and invert the fraction.

Number	Opposite	Absolute Value	Reciprocal

Number	Opposite	Absolute Value	Reciprocal
$-\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$-\frac{8}{3}$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	2
$\frac{9}{5}$	$-\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$
-5	5	5	$-\frac{1}{5}$

Exercise 1.3.4.23

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$			
$\frac{1}{4}$			
$\frac{8}{3}$			
-8			

Answer

Number	Opposite	Absolute Value	Reciprocal
$-\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$-\frac{8}{5}$
$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	4
$\frac{8}{3}$	$-\frac{8}{3}$	$\frac{8}{3}$	$\frac{3}{8}$
-8	8	8	$-\frac{1}{8}$

Exercise 1.3.4.24

Fill in the chart for each number given:

Number	Opposite	Absolute Value	Reciprocal
$-\frac{4}{7}$			
$\frac{1}{8}$			
$\frac{9}{4}$			
-1			

Answer

Number	Opposite	Absolute Value	Reciprocal
$-\frac{4}{7}$	$\frac{4}{7}$	$\frac{4}{7}$	$-\frac{7}{4}$
$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	8
$\frac{9}{4}$	$-\frac{9}{4}$	$\frac{9}{4}$	$\frac{4}{9}$
-1	1	1	$-\frac{1}{1}$

Divide Fractions

Why is $12 \div 3 = 4$? We previously modeled this with counters. How many groups of 3 counters can be made from a group of 12 counters?

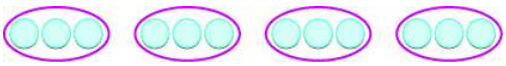


Figure 1.3.4.2

There are 4 groups of 3 counters. In other words, there are four 3s in 12. So, $12 \div 3 = 4$.

What about dividing fractions? Suppose we want to find the quotient: $\frac{1}{2} \div \frac{1}{6}$. We need to figure out how many $\frac{1}{6}$s there are in $\frac{1}{2}$. We can use fraction tiles to model this division. We start by lining up the half and sixth fraction tiles as shown in Figure 1.3.4.3 Notice, there are three $\frac{1}{6}$ tiles in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{6} = 3$.

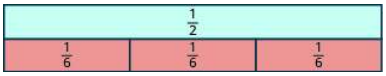


Figure 1.3.4.3

Example 1.3.4.13: model

Model: $\frac{1}{4} \div \frac{1}{8}$.

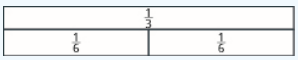
Solution

We want to determine how many $\frac{1}{8}$s are in $\frac{1}{4}$. Start with one $\frac{1}{4}$ tile. Line up $\frac{1}{8}$ tiles underneath the $\frac{1}{4}$ tile.

Exercise 1.3.4.25

Model: $\frac{1}{3} \div \frac{1}{6}$.

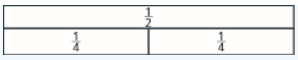
Answer



Exercise 1.3.4.26

Model: $\frac{1}{2} \div \frac{1}{4}$.

Answer

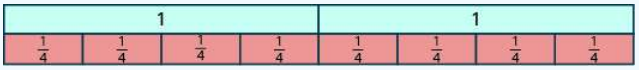


Example 1.3.4.14: model

Model: $2 \div \frac{1}{4}$.

Solution

We are trying to determine how many $\frac{1}{4}$s there are in 2. We can model this as shown.

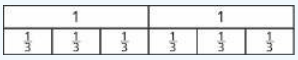


Because there are eight $\frac{1}{4}$s in 2, $2 \div \frac{1}{4} = 8$.

Exercise 1.3.4.27

Model: $2 \div \frac{1}{3}$

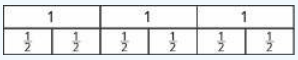
Answer



Exercise 1.3.4.28

Model: $3 \div \frac{1}{2}$

Answer



Let's use money to model $2 \div \frac{1}{4}$ in another way. We often read $\frac{1}{4}$ as a 'quarter', and we know that a quarter is one-fourth of a dollar as shown in Figure 1.3.4.4 So we can think of $2 \div \frac{1}{4}$ as, "How many quarters are there in two dollars?" One dollar is 4 quarters, so 2 dollars would be 8 quarters. So again, $2 \div \frac{1}{4} = 8$.



Figure 1.3.4.4The U.S. coin called a quarter is worth one-fourth of a dollar.

Using fraction tiles, we showed that $\frac{1}{2} \div \frac{1}{6} = 3$. Notice that $\frac{1}{2} \cdot \frac{6}{1} = 3$ also. How are $\frac{1}{6}$ and $\frac{6}{1}$ related? They are reciprocals. This leads us to the procedure for fraction division.

Definition: Fraction Division

If a, b, c, and d are numbers where $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \tag{1.3.4.1}$$

To divide fractions, multiply the first fraction by the reciprocal of the second.

We need to say $b \neq 0$, $c \neq 0$ and $d \neq 0$ to be sure we don't divide by zero.

Example 1.3.4.15: divide

Divide, and write the answer in simplified form: \(\dfrac{2}{5} \div \left(- \dfrac{3}{7}\right).

Solution

Multiply the first fraction by the reciprocal of the second.	$\frac{2}{5}\left(-\frac{7}{3}\right)$
Multiply. The product is negative.	$-\frac{14}{15}$

Exercise 1.3.4.29

Divide, and write the answer in simplified form: $\frac{3}{7} \div \left(-\frac{2}{3}\right)$.

Answer

$-\frac{9}{14}$

Exercise 1.3.4.30

Divide, and write the answer in simplified form: $\frac{2}{3} \div \left(-\frac{7}{5}\right)$.

Answer

$-\frac{10}{21}$

Example 1.3.4.16: divide

Divide, and write the answer in simplified form: $\frac{2}{3} \div \frac{n}{5}$.

Solution

Multiply the first fraction by the reciprocal of the second.	$\frac{2}{3} \div \frac{5}{n}$
Multiply.	$\frac{10}{3n}$

Exercise 1.3.4.31

Divide, and write the answer in simplified form: $\frac{3}{5} \div \frac{p}{7}$.

Answer

$\frac{21}{5p}$

Exercise 1.3.4.32

Divide, and write the answer in simplified form: $\frac{5}{8} \div \frac{q}{3}$.

Answer

$\frac{15}{8q}$

Example 1.3.4.17: divide

Divide, and write the answer in simplified form: $-\frac{3}{4} \div \left(-\frac{7}{8}\right)$.

Solution

Multiply the first fraction by the reciprocal of the second.	$-\frac{3}{4} \cdot \left(-\frac{8}{7}\right)$
Multiply. Remember to determine the sign first.	$\frac{3 \cdot 8}{4 \cdot 7}$
Rewrite to show common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 7}$
Remove common factors and simplify.	$\frac{6}{7}$

Exercise 1.3.4.33

Divide, and write the answer in simplified form: $-\frac{2}{3} \div \left(-\frac{5}{6}\right)$.

Answer

$\frac{4}{5}$

Exercise 1.3.4.34

Divide, and write the answer in simplified form: $-\frac{5}{6} \div \left(-\frac{2}{3}\right)$.

Answer

$\frac{5}{4}$

Example 1.3.4.18: divide

Divide, and write the answer in simplified form: $\frac{7}{18} \div \frac{14}{27}$.

Solution

Multiply the first fraction by the reciprocal of the second.	$\frac{7}{18} \cdot \frac{27}{14}$
Multiply.	$\frac{7 \cdot 27}{18 \cdot 14}$

Rewrite showing common factors.	$\frac{\cancel{7}\cdot\cancel{9}\cdot 3}{\cancel{9}\cdot\cancel{7}\cdot 2}$
Remove common factors.	$\frac{3}{2\cdot 2}$
Simplify.	$\frac{3}{4}$

? Exercise 1.3.4.35

Divide, and write the answer in simplified form: $\frac{7}{27} \div \frac{35}{36}$.

Answer

$\frac{4}{15}$

? Exercise 1.3.4.36

Divide, and write the answer in simplified form: $\frac{5}{14} \div \frac{15}{28}$.

Answer

$\frac{2}{3}$

Access Additional Online Resources

- Simplifying Fractions
- Multiplying Fractions (Positive Only)
- Multiplying Signed Fractions
- Dividing Fractions (Positive Only)
- Dividing Signed Fractions

Key Concepts

- **Equivalent Fractions Property**
- If a, b, c are numbers where $b \neq 0, c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$
- **Simplify a fraction.**
  1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
  2. Simplify, using the equivalent fractions property, by removing common factors.
  3. Multiply any remaining factors.
- **Fraction Multiplication**
  - If a, b, c, and dd are numbers where $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$
- **Reciprocal**
  - A number and its reciprocal have a product of 1. $\frac{a}{b} \cdot \frac{b}{a} = 1$
Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

- **Fraction Division**
  - If a, b, c, and d are numbers where $b \neq 0$, $c \neq 0$, and $d \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$
  - To divide fractions, multiply the first fraction by the reciprocal of the second.

Glossary

reciprocal

The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ where $a \neq 0$ and $b \neq 0$.

simplified fraction

A fraction is considered simplified if there are no common factors in the numerator and denominator.

Practice Makes Perfect

Simplify Fractions

In the following exercises, simplify each fraction. Do not convert any improper fractions to mixed numbers.

77. $\frac{7}{21}$
78. $\frac{8}{24}$
79. $\frac{15}{20}$
80. $\frac{12}{18}$
81. $-\frac{40}{88}$
82. $-\frac{63}{99}$
83. $-\frac{108}{63}$
84. $-\frac{104}{48}$
85. $\frac{120}{252}$
86. $\frac{182}{294}$
87. $-\frac{168}{192}$
88. $-\frac{140}{224}$
89. $\frac{11x}{11y}$
90. $\frac{15a}{15b}$
91. $-\frac{3x}{12y}$
92. $-\frac{4x}{32y}$
93. $\frac{14x^2}{21y}$
94. $\frac{24a}{32b^2}$

Multiply Fractions

In the following exercises, use a diagram to model.

95. $\frac{1}{2} \cdot \frac{2}{3}$
96. $\frac{1}{2} \cdot \frac{5}{8}$
97. $\frac{1}{3} \cdot \frac{5}{6}$
98. $\frac{1}{3} \cdot \frac{2}{5}$

In the following exercises, multiply, and write the answer in simplified form.

99. $\frac{2}{5} \cdot \frac{1}{3}$
100. $\frac{1}{2} \cdot \frac{3}{8}$
101. $\frac{3}{4} \cdot \frac{9}{10}$
102. $\frac{4}{5} \cdot \frac{2}{7}$
103. $-\frac{2}{3}\left(-\frac{3}{8}\right)$
104. $-\frac{3}{4}\left(-\frac{4}{9}\right)$
105. $-\frac{5}{9} \cdot \frac{3}{10}$
106. $-\frac{3}{8} \cdot \frac{4}{15}$
107. $-\frac{7}{12}\left(-\frac{8}{21}\right)$
108. $\frac{5}{12}\left(-\frac{8}{15}\right)$
109. $\left(-\frac{14}{15}\right)\left(\frac{9}{20}\right)$
110. $\left(-\frac{9}{10}\right)\left(\frac{25}{33}\right)$
111. $\left(-\frac{63}{84}\right)\left(-\frac{44}{90}\right)$
112. $\left(-\frac{33}{60}\right)\left(-\frac{40}{88}\right)$
113. $4 \cdot \frac{5}{11}$
114. $5 \cdot \frac{8}{3}$
115. $\frac{3}{7} \cdot 21n$
116. $\frac{5}{6} \cdot 30m$
117. $-28p\left(-\frac{1}{4}\right)$
118. $-51q\left(-\frac{1}{3}\right)$
119. $-8\left(\frac{17}{4}\right)$
120. $\frac{14}{5}(-15)$
121. $-1\left(-\frac{3}{8}\right)$
122. $(-1)\left(-\frac{6}{7}\right)$

123. $\left(\frac{2}{3}\right)^3$
124. $\left(\frac{4}{5}\right)^2$
125. $\left(\frac{6}{5}\right)^4$
126. $\left(\frac{4}{7}\right)^4$

Find Reciprocals In the following exercises, find the reciprocal.

127. $\frac{3}{4}$
128. $\frac{2}{3}$
129. $-\frac{5}{17}$
130. $-\frac{6}{19}$
131. $\frac{11}{8}$
132. -13
133. -19
134. -1
135. 1
136. Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{7}{11}$			
$\frac{4}{5}$			
$\frac{10}{7}$			
-8			

137. Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{3}{13}$			
$\frac{9}{14}$			
$\frac{15}{7}$			
-9			

Divide Fractions

In the following exercises, model each fraction division.

138. $\frac{1}{2} \div \frac{1}{4}$
139. $\frac{1}{2} \div \frac{1}{8}$
140. $2 \div \frac{1}{5}$

141. $3 \div \frac{1}{4}$

In the following exercises, divide, and write the answer in simplified form.

142. $\frac{1}{2} \div \frac{1}{4}$
143. $\frac{1}{2} \div \frac{1}{8}$
144. $\frac{3}{4} \div \frac{2}{3}$
145. $\frac{4}{5} \div \frac{3}{4}$
146. $-\frac{4}{5} \div \frac{4}{7}$
147. $-\frac{3}{4} \div \frac{3}{5}$
148. $-\frac{7}{9} \div \left(-\frac{7}{9}\right)$
149. $-\frac{5}{6} \div \left(-\frac{5}{6}\right)$
150. $\frac{3}{4} \div \frac{x}{11}$
151. $\frac{2}{5} \div \frac{y}{9}$
152. $\frac{5}{8} \div \frac{a}{10}$
153. $\frac{5}{6} \div \frac{c}{15}$
154. $\frac{5}{18} \div \left(-\frac{15}{24}\right)$
155. $\frac{7}{18} \div \left(-\frac{14}{27}\right)$
156. $\frac{7p}{12} \div \frac{21p}{8}$
157. $\frac{5q}{12} \div \frac{15q}{8}$
158. $\frac{8u}{15} \div \frac{12v}{25}$
159. $\frac{12r}{25} \div \frac{18s}{35}$
160. $-5 \div \frac{1}{2}$
161. $-3 \div \frac{1}{4}$
162. $\frac{3}{4} \div (-12)$
163. $\frac{2}{5} \div (-10)$
164. $-18 \div \left(-\frac{9}{2}\right)$
165. $-15 \div \left(-\frac{5}{3}\right)$
166. $\frac{1}{2} \div \left(-\frac{3}{4}\right) \div \frac{7}{8}$
167. $\frac{11}{2} \div \frac{7}{8} \cdot \frac{2}{11}$

Everyday Math

168. **Baking** A recipe for chocolate chip cookies calls for 3 4 cup brown sugar. Imelda wants to double the recipe.
    a. How much brown sugar will Imelda need? Show your calculation. Write your result as an improper fraction and as a mixed number.
    b. Measuring cups usually come in sets of $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Imelda could measure the brown sugar needed to double the recipe.
169. **Baking** Nina is making 4 pans of fudge to serve after a music recital. For each pan, she needs 2 3 cup of condensed milk.
    a. How much condensed milk will Nina need? Show your calculation. Write your result as an improper fraction and as a mixed number.
    b. Measuring cups usually come in sets of $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Nina could measure the condensed milk she needs.
170. **Portions** Don purchased a bulk package of candy that weighs 5 pounds. He wants to sell the candy in little bags that hold $\frac{1}{4}$ pound. How many little bags of candy can he fill from the bulk package?
171. **Portions** Kristen has $\frac{3}{4}$ yards of ribbon. She wants to cut it into equal parts to make hair ribbons for her daughter's 6 dolls. How long will each doll's hair ribbon be?

Writing Exercises

172. Explain how you find the reciprocal of a fraction.
173. Explain how you find the reciprocal of a negative fraction.
174. Rafael wanted to order half a medium pizza at a restaurant. The waiter told him that a medium pizza could be cut into 6 or 8 slices. Would he prefer 3 out of 6 slices or 4 out of 8 slices? Rafael replied that since he wasn't very hungry, he would prefer 3 out of 6 slices. Explain what is wrong with Rafael's reasoning.
175. Give an example from everyday life that demonstrates how $\frac{1}{2} \cdot \frac{2}{3}$ is $\frac{1}{3}$.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify fractions.			
multiply fractions.			
find reciprocals.			
divide fractions.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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1.3.5: Multiply and Divide Mixed Numbers and Complex Fractions (Part 1)

Learning Objectives

- Multiply and divide mixed numbers
- Translate phrases to expressions with fractions
- Simplify complex fractions
- Simplify expressions written with a fraction bar

be prepared!

Before you get started, take this readiness quiz.

1. Divide and reduce, if possible: $(4+5) \div (10-7)$. If you missed this problem, review Example 3.2.8.
2. Multiply and write the answer in simplified form: $\frac{1}{8} \cdot \frac{2}{3}$. If you missed this problem, review Example 4.2.7.
3. Convert $2\frac{3}{5}$ into an improper fraction. If you missed this problem, review Example 4.1.11.

Multiply and Divide Mixed Numbers

In the previous section, you learned how to multiply and divide fractions. All of the examples there used either proper or improper fractions. What happens when you are asked to multiply or divide mixed numbers? Remember that we can convert a mixed number to an improper fraction. And you learned how to do that in Visualize Fractions.

Example 1.3.5.1: multiply

Multiply: $3\frac{1}{3} \cdot \frac{5}{8}$

Solution

Convert $3\frac{1}{3}$ to an improper fraction.	$\frac{10}{3} \cdot \frac{5}{8}$
Multiply.	$\frac{10 \cdot 5}{3 \cdot 8}$
Look for common factors.	$\frac{\cancel{2} \cdot 5 \cdot 5}{3 \cdot \cancel{2} \cdot 4}$
Remove common factors.	$\frac{5 \cdot 5}{3 \cdot 4}$
Simplify.	$\frac{25}{12}$

Notice that we left the answer as an improper fraction, $\frac{25}{12}$, and did not convert it to a mixed number. In algebra, it is preferable to write answers as improper fractions instead of mixed numbers. This avoids any possible confusion between $2\frac{1}{12}$ and $2 \cdot \frac{1}{12}$.

Exercise 1.3.5.1

Multiply, and write your answer in simplified form: $5\frac{2}{3} \cdot \frac{6}{17}$.

Answer

2

Exercise 1.3.5.2

Multiply, and write your answer in simplified form: $\frac{3}{7} \cdot 5\frac{1}{4}$.

Answer

$\frac{9}{4}$

HOW TO: MULTIPLY OR DIVIDE MIXED NUMBERS

Step 1. Convert the mixed numbers to improper fractions.

Step 2. Follow the rules for fraction multiplication or division.

Step 3. Simplify if possible.

Example 1.3.5.2:

Multiply, and write your answer in simplified form: $2\frac{4}{5}\left(-1\frac{7}{8}\right)$.

Solution

Convert mixed numbers to improper fractions.	$\frac{14}{5}\left(-1\frac{7}{8}\right)$
Multiply.	$-\frac{14 \cdot 15}{5 \cdot 8}$
Look for common factors.	$-\frac{\cancel{2} \cdot 7 \cdot \cancel{5} \cdot 3}{\cancel{5} \cdot \cancel{2} \cdot 4}$
Remove common factors.	$-\frac{7 \cdot 3}{4}$
Simplify.	$-\frac{21}{4}$

Exercise 1.3.5.3

Multiply, and write your answer in simplified form. $5\frac{5}{7}\left(-2\frac{5}{8}\right)$.

Answer

-15

Exercise 1.3.5.4

Multiply, and write your answer in simplified form. $-3\frac{2}{5} \cdot 4\frac{1}{6}$.

Answer

$-\frac{85}{6}$

Example 1.3.5.3: divide

Divide, and write your answer in simplified form: $3\frac{4}{7} \div 5$.

Solution

Convert mixed numbers to improper fractions.	$\frac{25}{7} \div \frac{5}{1}$
Multiply the first fraction by the reciprocal of the second.	$\frac{25}{7} \cdot \frac{1}{5}$
Multiply.	$\frac{25 \cdot 1}{7 \cdot 5}$
Look for common factors.	$\frac{\cancel{5} \cdot 5 \cdot 1}{7 \cdot \cancel{5}}$
Remove common factors.	$\frac{5 \cdot 1}{7}$
Simplify.	$\frac{5}{7}$

Exercise 1.3.5.5

Divide, and write your answer in simplified form: $4\frac{3}{8} \div 7$.

Answer

$\frac{5}{8}$

Exercise 1.3.5.6

Divide, and write your answer in simplified form: $2\frac{5}{8} \div 3$.

Answer

$\frac{7}{8}$

Example 1.3.5.4: divide

Divide: $2\frac{1}{2} \div 1\frac{1}{4}$.

Solution

Convert mixed numbers to improper fractions.	$\frac{5}{2} \div \frac{5}{4}$
Multiply the first fraction by the reciprocal of the second.	$\frac{5}{2} \cdot \frac{4}{5}$
Multiply.	$\frac{5 \cdot 4}{2 \cdot 5}$
Look for common factors.	$\frac{\cancel{5} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot 1 \cdot \cancel{5}}$

Remove common factors.	$\frac{2}{1}$
Simplify.	2

Exercise 1.3.5.7

Divide, and write your answer in simplified form: $2\frac{2}{3} \div 1\frac{1}{3}$.

Answer

2

Exercise 1.3.5.8

Divide, and write your answer in simplified form: $3\frac{3}{4} \div 1\frac{1}{2}$.

Answer

$\frac{5}{2}$

Translate Phrases to Expressions with Fractions

The words *quotient* and *ratio* are often used to describe fractions. In Subtract Whole Numbers, we defined quotient as the result of division. The quotient of a and b is the result you get from dividing a by b, or $\frac{a}{b}$. Let's practice translating some phrases into algebraic expressions using these terms.

Example 1.3.5.5: translate

Translate the phrase into an algebraic expression: "the quotient of $3x$ and 8."

Solution

The keyword is *quotient*; it tells us that the operation is division. Look for the words *of* and *and* to find the numbers to divide.

The quotient **of** $3x$ **and** 8.

This tells us that we need to divide $3x$ by 8. $\frac{3x}{8}$

Exercise 1.3.5.9

Translate the phrase into an algebraic expression: the quotient of $9s$ and 14.

Answer

$\frac{9s}{14}$

Exercise 1.3.5.10

Translate the phrase into an algebraic expression: the quotient of $5y$ and 6.

Answer

$\frac{5y}{6}$

Example 1.3.5.6:

Translate the phrase into an algebraic expression: the quotient of the difference of m and n, and p.

Solution

We are looking for the *quotient* of the *difference* of m and n, and p. This means we want to divide the difference of m and n by p.

$$\frac{m-n}{p}$$

Exercise 1.3.5.11

Translate the phrase into an algebraic expression: the quotient of the difference of a and b, and cd.

Answer

$$\frac{a-b}{cd}$$

Exercise 1.3.5.12

Translate the phrase into an algebraic expression: the quotient of the sum of p and q, and r.

Answer

$$\frac{p+q}{r}$$

Simplify Complex Fractions

Our work with fractions so far has included proper fractions, improper fractions, and mixed numbers. Another kind of fraction is called **complex fraction**, which is a fraction in which the numerator or the denominator contains a fraction. Some examples of complex fractions are:

$$\frac{\frac{6}{7}}{3} \quad \frac{\frac{3}{4}}{\frac{5}{8}} \quad \frac{\frac{x}{2}}{\frac{5}{6}}$$

To simplify a complex fraction, remember that the fraction bar means division. So the complex fraction $\frac{\frac{3}{4}}{\frac{5}{8}}$ can be written as $\frac{3}{4} \div \frac{5}{8}$.

Example 1.3.5.7: simplify

Simplify: $\frac{\frac{3}{4}}{\frac{5}{8}}$.

Solution

Rewrite as division.	$\frac{3}{4} \div \frac{5}{8}$

Multiply the first fraction by the reciprocal of the second.	$\frac{3}{4} \cdot \frac{8}{5}$
Multiply.	$\frac{3 \cdot 8}{4 \cdot 5}$
Look for common factors.	$\frac{3 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 5}$
Remove common factors and simplify.	$\frac{6}{5}$

Exercise 1.3.5.13

Simplify: $\frac{\frac{2}{3}}{\frac{5}{6}}$.

Answer

$\frac{4}{5}$

Exercise 1.3.5.14

Simplify: $\frac{\frac{3}{7}}{\frac{6}{11}}$.

Answer

$\frac{11}{14}$

HOW TO: SIMPLIFY A COMPLEX FRACTION

Step 1. Rewrite the complex fraction as a division problem.

Step 2. Follow the rules for dividing fractions.

Step 3. Simplify if possible.

Example 1.3.5.8: simplify

Simplify: $\frac{-\frac{6}{7}}{3}$.

Solution

Rewrite as division.	$-\frac{6}{7} \div 3$
Multiply the first fraction by the reciprocal of the second.	$-\frac{6}{7} \cdot \frac{1}{3}$
Multiply; the product will be negative.	$-\frac{6 \cdot 1}{7 \cdot 3}$

Look for common factors.	$-\frac{\cancel{3}\cdot 2\cdot 1}{7\cdot \cancel{3}}$
Remove common factors and simplify.	$-\frac{2}{7}$

? Exercise 1.3.5.15

Simplify: $\frac{-\frac{8}{7}}{4}$.

Answer

$-\frac{2}{7}$

? Exercise 1.3.5.16

Simplify: $-\frac{3}{\frac{9}{10}}$.

Answer

$-\frac{10}{3}$

✓ Example 1.3.5.9: simplify

Simplify: $\frac{\frac{x}{2}}{\frac{xy}{6}}$.

Solution

Rewrite as division.	$\frac{x}{2}\div\frac{xy}{6}$
Multiply the first fraction by the reciprocal of the second.	$\frac{x}{2}\cdot\frac{6}{xy}$
Multiply.	$\frac{x\cdot 6}{2\cdot xy}$
Look for common factors.	$\frac{\cancel{x}\cdot 3\cdot \cancel{2}}{\cancel{2}\cdot \cancel{x}\cdot y}$
Remove common factors and simplify.	$\frac{3}{y}$

? Exercise 1.3.5.17

Simplify: $\frac{\frac{a}{8}}{\frac{ab}{6}}$.

Answer

$$\frac{3}{4b}$$

Exercise 1.3.5.18

Simplify: $\dfrac{\frac{p}{2}}{\frac{pq}{8}}$.

Answer

$$\frac{4}{q}$$

Example 1.3.5.10: simplify

Simplify: $\dfrac{2\frac{3}{4}}{\frac{1}{8}}$.

Solution

Rewrite as division.	$2\frac{3}{4} \div \frac{1}{8}$
Change the mixed number to an improper fraction.	$\frac{11}{4} \div \frac{1}{8}$
Multiply the first fraction by the reciprocal of the second.	$\frac{11}{4} \cdot \frac{8}{1}$
Multiply.	$\frac{11 \cdot 8}{4 \cdot 1}$
Look for common factors.	$\frac{11 \cdot \cancel{4} \cdot 2}{\cancel{4} \cdot 1}$
Remove common factors and simplify.	22

Exercise 1.3.5.19

Simplify: $\dfrac{\frac{5}{7}}{1\frac{2}{5}}$.

Answer

$$\frac{25}{49}$$

Exercise 1.3.5.20

Simplify: $\dfrac{\frac{8}{5}}{3\frac{1}{5}}$.

Answer

$\frac{1}{2}$

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1.3.6: Multiply and Divide Mixed Numbers and Complex Fractions (Part 2)

Simplify Expressions with a Fraction Bar

Where does the negative sign go in a fraction? Usually, the negative sign is placed in front of the fraction, but you will sometimes see a fraction with a negative numerator or denominator. Remember that fractions represent division. The fraction $-\frac{1}{3}$ could be the result of dividing $\frac{-1}{3}$, a negative by a positive, or of dividing $\frac{1}{-3}$, a positive by a negative. When the numerator and denominator have different signs, the quotient is negative.

$$\frac{-1}{3}=-\frac{1}{3} \quad \frac{negative}{positive}=negative \quad \frac{1}{-3}=-\frac{1}{3} \quad \frac{positive}{negative}=negative$$

If *both* the numerator and denominator are negative, then the fraction itself is positive because we are dividing a negative by a negative.

$$\frac{-1}{-3}=\frac{1}{3} \qquad \frac{negative}{negative}=positive$$

Placement of Negative Sign in a Fraction

For any positive numbers a and b,

$$\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b} \tag{1.3.6.1}$$

Example 1.3.6.11: equivalent fractions

Which of the following fractions are equivalent to $\frac{7}{-8}$?

$$\frac{-7}{-8}, \frac{-7}{8}, \frac{7}{8}, -\frac{7}{8}$$

Solution

The quotient of a positive and a negative is a negative, so $\frac{7}{-8}$ is negative. Of the fractions listed, $\frac{-7}{8}$ and $-\frac{7}{8}$ are also negative.

Exercise 1.3.6.21

Which of the following fractions are equivalent to $\frac{-3}{5}$?

$$\frac{-3}{-5}, \frac{3}{5}, -\frac{3}{5}, \frac{3}{-5}$$

Answer

$$-\frac{3}{5}, \frac{3}{-5}$$

Exercise 1.3.6.22

Which of the following fractions are equivalent to $-\frac{2}{7}$?

$$\frac{-2}{-7}, \frac{-2}{7}, \frac{2}{7}, \frac{2}{-7}$$

Answer

$\frac{-2}{7}, \frac{2}{-7}$

Fraction bars act as grouping symbols. The expressions above and below the fraction bar should be treated as if they were in parentheses. For example, $\frac{4+8}{5-3}$ means $(4+8) \div (5-3)$. The order of operations tells us to simplify the numerator and the denominator first—as if there were parentheses—before we divide.

We'll add fraction bars to our set of grouping symbols from Use the Language of Algebra to have a more complete set here.

Grouping Symbols

Parentheses	( )
Brackets	[ ]
Braces	{ }
Absolute value	| |
Fraction Bar	$\frac{\square}{\square}$

HOW TO: SIMPLIFY AN EXPRESSION WITH A FRACTION BAR

Step 1. Simplify the numerator.

Step 2. Simplify the denominator.

Step 3. Simplify the fraction.

Example 1.3.6.12:

Simplify: $\frac{4+8}{5-3}$.

Solution

Simplify the expression in the numerator.	$\frac{12}{5-3}$
Simplify the expression in the denominator.	$\frac{12}{2}$
Simplify the fraction.	6

Exercise 1.3.6.23

Simplify: $\frac{4+6}{11-2}$.

Answer

$\frac{10}{9}$

Exercise 1.3.6.24

Simplify: $\frac{3+5}{18-2}$.

Answer

$\frac{1}{2}$

Example 1.3.6.13:

Simplify: $\frac{4-2(3)}{2^2+2}$.

Solution

Use the order of operations. Multiply in the numerator and use the exponent in the denominator.	$\frac{4-6}{4+2}$
Simplify the numerator and the denominator.	$\frac{-2}{6}$
Simplify the fraction.	$-\frac{1}{3}$

Exercise 1.3.6.25

Simplify: $\frac{6-3(5)}{3^2+3}$.

Answer

$\frac{-3}{4}$

Exercise 1.3.6.26

Simplify: $\frac{4-4(6)}{3^3+3}$.

Answer

$-\frac{2}{3}$

Example 1.3.6.14: simplify

Simplify: $\frac{(8-4)^2}{8^2-4^2}$.

Solution

Use the order of operations (parentheses first, then exponents).	$\frac{(4)^2}{64-16}$
Simplify the numerator and denominator.	$\frac{16}{48}$
Simplify the fraction.	$\frac{1}{3}$

Exercise 1.3.6.27

Simplify: $\frac{(11-7)^2}{11^2-7^2}$.

Answer

$\frac{2}{9}$

Exercise 1.3.6.28

Simplify: $\frac{(6+2)^2}{6^2-2^2}$.

Answer

$\frac{8}{5}$

Example 1.3.6.15: simplify

Simplify: $\frac{4(-3)+6(-2)}{-3(2)-2}$.

Solution

Multiply.	$\frac{-12+(-12)}{-6-2}$
Simplify.	$\frac{-24}{-8}$
Divide.	3

Exercise 1.3.6.29

Simplify: $\frac{8(-2)+4(-3)}{-5(2)+3}$.

Answer

4

Exercise 1.3.6.30

Simplify: $\frac{7(-1)+9(-3)}{-5(3)+2}$.

Answer

2

Access Additional Online Resources

- Division Involving Mixed Numbers
- Evaluate a Complex Fraction

Key Concepts

- Multiply or divide mixed numbers.
  1. Convert the mixed numbers to improper fractions.
  2. Follow the rules for fraction multiplication or division.
  3. Simplify if possible.
- Simplify a complex fraction.
  1. Rewrite the complex fraction as a division problem.
  2. Follow the rules for dividing fractions.
  3. Simplify if possible.
- Placement of negative sign in a fraction.
  - For any positive numbers a and b, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
- Simplify an expression with a fraction bar.
  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Simplify the fraction.

Glossary

complex fraction

A complex fraction is a fraction in which the numerator or the denominator contains a fraction.

Practice Makes Perfect

Multiply and Divide Mixed Numbers

In the following exercises, multiply and write the answer in simplified form.

176. $4\frac{3}{8} \cdot \frac{7}{10}$
177. $2\frac{4}{9} \cdot \frac{6}{7}$
178. $\frac{15}{22} \cdot 3\frac{3}{5}$
179. $\frac{25}{36} \cdot 6\frac{3}{10}$
180. $4\frac{2}{3}(-1\frac{1}{8})$
181. $2\frac{2}{5}(-2\frac{2}{9})$
182. $-4\frac{4}{9} \cdot 5\frac{13}{16}$
183. $-1\frac{7}{20} \cdot 2\frac{11}{12}$

In the following exercises, divide, and write your answer in simplified form.

184. $5\frac{1}{3} \div 4$
185. $13\frac{1}{2} \div 9$
186. $-12 \div 3\frac{3}{11}$
187. $-7 \div 5\frac{1}{4}$
188. $6\frac{3}{8} \div 2\frac{1}{8}$
189. $2\frac{1}{5} \div 1\frac{1}{10}$

190. $-9\frac{3}{5} \div (-1\frac{3}{5})$
191. $-18\frac{3}{4} \div (-3\frac{3}{4})$

Translate Phrases to Expressions with Fractions

In the following exercises, translate each English phrase into an algebraic expression.

192. the quotient of 5u and 11
193. the quotient of 7v and 13
194. the quotient of p and q
195. the quotient of a and b
196. the quotient of r and the sum of s and 10
197. the quotient of A and the difference of 3 and B

Simplify Complex Fractions

In the following exercises, simplify the complex fraction.

198. $\frac{\frac{2}{3}}{\frac{8}{9}}$
199. $\frac{\frac{4}{5}}{\frac{8}{15}}$
200. $\frac{-\frac{8}{21}}{\frac{12}{35}}$
201. $\frac{-\frac{9}{16}}{\frac{33}{40}}$
202. $\frac{-\frac{4}{5}}{2}$
203. $\frac{-\frac{9}{10}}{3}$
204. $\frac{2}{\frac{5}{8}}$
205. $\frac{\frac{5}{3}}{10}$
206. $\frac{\frac{m}{3}}{n}$
207. $\frac{\frac{r}{2}}{\frac{s}{3}}$
208. $\frac{-\frac{x}{6}}{-\frac{8}{9}}$

209. $\dfrac{-\frac{3}{8}}{-\frac{y}{12}}$

210. $\dfrac{2\frac{4}{5}}{\frac{1}{10}}$

211. $\dfrac{4\frac{2}{3}}{\frac{1}{6}}$

212. $\dfrac{\frac{7}{9}}{-2\frac{4}{5}}$

213. $\dfrac{\frac{3}{8}}{-6\frac{3}{4}}$

Simplify Expressions with a Fraction Bar

In the following exercises, identify the equivalent fractions.

214. Which of the following fractions are equivalent to $\dfrac{5}{-11}$?

$$\frac{-5}{-11}, \frac{-5}{11}, \frac{5}{11}, -\frac{5}{11} \tag{1.3.6.2}$$

215. Which of the following fractions are equivalent to $\dfrac{-4}{9}$?

$$\frac{-4}{-9}, \frac{-4}{9}, \frac{4}{9}, -\frac{4}{9} \tag{1.3.6.3}$$

216. Which of the following fractions are equivalent to $-\dfrac{11}{3}$?

$$\frac{-11}{3}, \frac{11}{3}, \frac{-11}{-3}, \frac{11}{-3} \tag{1.3.6.4}$$

217. Which of the following fractions are equivalent to $-\dfrac{13}{6}$?

$$\frac{13}{6}, \frac{13}{-6}, \frac{-13}{-6}, \frac{-13}{6} \tag{1.3.6.5}$$

In the following exercises, simplify.

218. $\dfrac{4+11}{8}$

219. $\dfrac{9+3}{7}$

220. $\dfrac{22+3}{10}$

221. $\dfrac{19-4}{6}$

222. $\dfrac{48}{24-15}$

223. $\dfrac{46}{4+4}$

224. $\dfrac{-6+6}{8+4}$

225. $\dfrac{-6+3}{17-8}$
226. $\dfrac{22-14}{19-13}$
227. $\dfrac{15+9}{18+12}$
228. $\dfrac{5\cdot 8}{-10}$
229. $\dfrac{3\cdot 4}{-24}$
230. $\dfrac{4\cdot 3}{6\cdot 6}$
231. $\dfrac{6\cdot 6}{9\cdot 2}$
232. $\dfrac{4^2-1}{25}$
233. $\dfrac{7^2+1}{60}$
234. $\dfrac{8\cdot 3+2\cdot 9}{14+3}$
235. $\dfrac{9\cdot 6-4\cdot 7}{22+3}$
236. $\dfrac{15\cdot 5-5^2}{2\cdot 10}$
237. $\dfrac{12\cdot 9-3^2}{3\cdot 18}$
238. $\dfrac{5\cdot 6-3\cdot 4}{4\cdot 5-2\cdot 3}$
239. $\dfrac{8\cdot 9-7\cdot 6}{5\cdot 6-9\cdot 2}$
240. $\dfrac{5^2-3^2}{3-5}$
241. $\dfrac{6^2-4^2}{4-6}$
242. $\dfrac{2+4(3)}{-3-2^2}$
243. $\dfrac{7+3(5)}{-2-3^2}$
244. $\dfrac{7\cdot 4-2(8-5)}{9\cdot 3-3\cdot 5}$
245. $\dfrac{9\cdot 7-3(12-8)}{8\cdot 7-6\cdot 6}$
246. $\dfrac{9(8-2)-3(15-7)}{6(7-1)-3(17-9)}$
247. $\dfrac{8(9-2)-4(14-9)}{7(8-3)-3(16-9)}$

Everyday Math

248. **Baking** A recipe for chocolate chip cookies calls for $2\dfrac{1}{4}$ cups of flour. Graciela wants to double the recipe.
    a. How much flour will Graciela need? Show your calculation. Write your result as an improper fraction and as a mixed number.
    b. Measuring cups usually come in sets with cups for $\dfrac{1}{8}$, $\dfrac{1}{4}$, $\dfrac{1}{3}$, $\dfrac{1}{2}$, and 1 cup. Draw a diagram to show two different ways that Graciela could measure out the flour needed to double the recipe.

249. **Baking** A booth at the county fair sells fudge by the pound. Their award winning "Chocolate Overdose" fudge contains $2\frac{2}{3}$ cups of chocolate chips per pound.
   a. How many cups of chocolate chips are in a half-pound of the fudge?
   b. The owners of the booth make the fudge in 10 -pound batches. How many chocolate chips do they need to make a 10 -pound batch? Write your results as improper fractions and as a mixed numbers.

Writing Exercises

250. Explain how to find the reciprocal of a mixed number.
251. Explain how to multiply mixed numbers.
252. Randy thinks that $3\frac{1}{2} \cdot 5\frac{1}{4}$ is $15\frac{1}{8}$. Explain what is wrong with Randy's thinking.
253. Explain why $-\frac{1}{2}$, $\frac{-1}{2}$, and $\frac{1}{-2}$ are equivalent.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply and divide mixed numbers.			
translate phrases to expressions with fractions.			
simplify complex fractions.			
simplify expressions written with a fraction bar.			

(b) What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Contributors and Attributions

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1.3.7: Add and Subtract Fractions with Common Denominators

Learning Objectives

- Model fraction addition
- Add fractions with a common denominator
- Model fraction subtraction
- Subtract fractions with a common denominator

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $2x+9+3x-4$. If you missed this problem, review Example 2.2.10.
2. Draw a model of the fraction $\frac{3}{4}$. If you missed this problem, review Example 4.1.2.
3. Simplify: $\frac{3+2}{6}$. If you missed this problem, review Example 4.3.12.

Model Fraction Addition

How many quarters are pictured? One quarter plus 2 quarters equals 3 quarters.

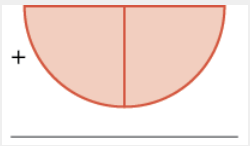
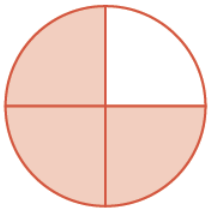


Figure 1.3.7.1

Remember, quarters are really fractions of a dollar. Quarters are another way to say fourths. So the picture of the coins shows that

$$\begin{matrix} \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \\ one\ quarter + two\ quarters & = & three\ quarters \end{matrix}$$

Let's use fraction circles to model the same example, $\frac{1}{4}+\frac{2}{4}$.

Start with one $\frac{1}{4}$ piece.		$\frac{1}{4}$
Add two more $\frac{1}{4}$ pieces.		$+\frac{2}{4}$
The result is $\frac{3}{4}$.		$\frac{3}{4}$

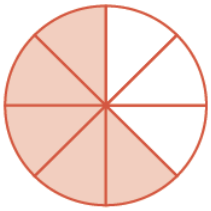
So again, we see that

$$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$$

Example 1.3.7.1: addition

Use a model to find the sum $\frac{3}{8}+\frac{2}{8}$.

Solution

Start with three $\frac{1}{8}$ pieces.		$\frac{3}{8}$
Add two $\frac{1}{8}$ pieces.		$+\frac{2}{8}$
How many $\frac{1}{8}$ pieces are there?		$\frac{5}{8}$

There are five $\frac{1}{8}$ pieces, or five-eighths. The model shows that $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$.

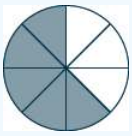
Exercise 1.3.7.1

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{8}+\frac{4}{8}$$

Answer

$\frac{5}{8}$



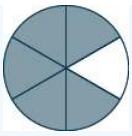
Exercise 1.3.7.2

Use a model to find each sum. Show a diagram to illustrate your model.

$$\frac{1}{6}+\frac{4}{6}$$

Answer

$\frac{5}{6}$



Add Fractions with a Common Denominator

Example 1.3.7.1 shows that to add the same-size pieces—meaning that the fractions have the same denominator—we just add the number of pieces.

Definition: Fraction Addition

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \tag{1.3.7.1}$$

To add fractions with a common denominator, add the numerators and place the sum over the common denominator.

Example 1.3.7.2: addition

Find the sum: $\frac{3}{5} + \frac{1}{5}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{3+1}{5}$
Simplify.	$\frac{4}{5}$

Exercise 1.3.7.3

Find each sum: $\frac{3}{6} + \frac{2}{6}$.

Answer

$\frac{5}{6}$

Exercise 1.3.7.4

Find each sum: $\frac{3}{10} + \frac{7}{10}$.

Answer

1

Example 1.3.7.3: addition

Find the sum: $\frac{x}{3} + \frac{2}{3}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{x+2}{3}$

Note that we cannot simplify this fraction any more. Since x and 2 are not like terms, we cannot combine them.

? Exercise 1.3.7.5

Find the sum: $\frac{x}{4}+\frac{3}{4}$.

Answer

$\frac{x+3}{4}$

? Exercise 1.3.7.6

Find the sum: $\frac{y}{8}+\frac{5}{8}$.

Answer

$\frac{y+5}{8}$

✓ Example 1.3.7.4: addition

Find the sum: $-\frac{9}{d}+\frac{3}{d}$.

Solution

We will begin by rewriting the first fraction with the negative sign in the numerator.

$$-\frac{a}{b}=\frac{-a}{b}$$

Rewrite the first fraction with the negative in the numerator.	$\frac{-9}{d}+\frac{3}{d}$
Add the numerators and place the sum over the common denominator.	$\frac{-9+3}{d}$
Simplify the numerator.	$\frac{-6}{d}$
Rewrite with negative sign in front of the fraction.	$-\frac{6}{d}$

? Exercise 1.3.7.7

Find the sum: $-\frac{7}{d}+\frac{8}{d}$.

Answer

$\frac{1}{d}$

Exercise 1.3.7.8

Find the sum: $-\frac{6}{m}+\frac{9}{m}$.

Answer

$\frac{3}{m}$

Example 1.3.7.5: addition

Find the sum: $\frac{2n}{11}+\frac{5n}{11}$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{2n+5n}{11}$
Combine like terms.	$\frac{7n}{11}$

Exercise 1.3.7.9

Find the sum: $\frac{3p}{8}+\frac{6p}{8}$.

Answer

$\frac{9p}{8}$

Exercise 1.3.7.10

Find the sum: $\frac{2q}{5}+\frac{7q}{5}$.

Answer

$\frac{9q}{5}$

Example 1.3.7.6: addition

Find the sum: $-\frac{3}{12}+\left(-\frac{5}{12}\right)$.

Solution

Add the numerators and place the sum over the common denominator.	$\frac{-3+(-5)}{12}$
Add.	$\frac{-8}{12}$
Simplify the fraction.	$-\frac{2}{3}$

Exercise 1.3.7.11

Find each sum: $-\frac{4}{15}+\left(-\frac{6}{15}\right)$.

Answer

$-\frac{2}{3}$

Exercise 1.3.7.12

Find each sum: $-\frac{5}{21}+\left(-\frac{9}{21}\right)$.

Answer

$-\frac{2}{3}$

Model Fraction Subtraction

Subtracting two fractions with common denominators is much like adding fractions. Think of a pizza that was cut into 12 slices. Suppose five pieces are eaten for dinner. This means that, after dinner, there are seven pieces (or $\frac{7}{12}$ of the pizza) left in the box. If Leonardo eats 2 of these remaining pieces (or $\frac{2}{12}$ of the pizza), how much is left? There would be 5 pieces left (or $\frac{5}{12}$ of the pizza).

$$\frac{7}{12}-\frac{2}{12}=\frac{5}{12}$$

Let's use fraction circles to model the same example, $\frac{7}{12}-\frac{2}{12}$. Start with seven $\frac{1}{12}$ pieces. Take away two $\frac{1}{12}$ pieces. How many twelfths are left?

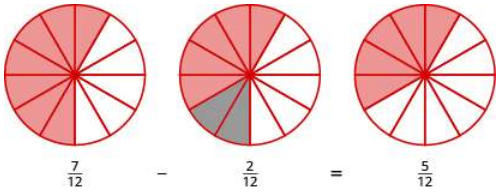


Figure 1.3.7.2

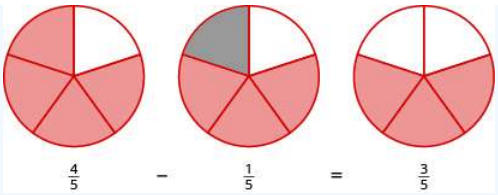
Again, we have five twelfths, $\frac{5}{12}$.

Example 1.3.7.7: difference

Use fraction circles to find the difference: $\frac{4}{5}-\frac{1}{5}$.

Solution

Start with four $\frac{1}{5}$ pieces. Take away one $\frac{1}{5}$ piece. Count how many fifths are left. There are three $\frac{1}{5}$ pieces left.



Exercise 1.3.7.13

Use a model to find each difference. Show a diagram to illustrate your model. $\frac{7}{8}-\frac{4}{8}$

Answer

$\frac{3}{8}$, models may differ.

Exercise 1.3.7.14

Use a model to find each difference. Show a diagram to illustrate your model. $\frac{5}{6}-\frac{4}{6}$

Answer

$\frac{1}{6}$, models may differ.

Subtract Fractions with a Common Denominator

We subtract fractions with a common denominator in much the same way as we add fractions with a common denominator.

Definition: Fraction Subtraction

If a, b, and c are numbers where $c \neq 0$, then

$$\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c} \tag{1.3.7.2}$$

To subtract fractions with a common denominator, we subtract the numerators and place the difference over the common denominator.

Example 1.3.7.8: difference

Find the difference: $\frac{23}{24}-\frac{14}{24}$.

Solution

Subtract the numerators and place the difference over the common denominator.	$\frac{23-14}{24}$
Simplify the numerator.	$\frac{9}{24}$
Simplify the fraction by removing common factors.	$\frac{3}{8}$

Exercise 1.3.7.15

Find the difference: $\frac{19}{28} - \frac{7}{28}$.

Answer

$\frac{3}{7}$

Exercise 1.3.7.16

Find the difference: $\frac{27}{32} - \frac{11}{32}$.

Answer

$\frac{1}{2}$

Example 1.3.7.9: difference

Find the difference: $\frac{y}{6} - \frac{1}{6}$.

Solution

Subtract the numerators and place the difference over the common denominator.	$\frac{y-1}{6}$

The fraction is simplified because we cannot combine the terms in the numerator.

Exercise 1.3.7.17

Find the difference: $\frac{x}{7} - \frac{2}{7}$.

Answer

$\frac{x-2}{7}$

Exercise 1.3.7.18

Find the difference: $\frac{y}{14} - \frac{13}{14}$.

Answer

$\frac{y-13}{14}$

Example 1.3.7.10: difference

Find the difference: $-\frac{10}{x} - \frac{4}{x}$.

Solution

Remember, the fraction $-\frac{10}{x}$ can be written as $\frac{-10}{x}$.

Subtract the numerators.	$\frac{-10-4}{x}$
Simplify.	$\frac{-14}{x}$
Rewrite with the negative sign in front of the fraction.	$-\frac{14}{x}$

Exercise 1.3.7.19

Find the difference: $-\frac{9}{x}-\frac{7}{x}$.

Answer

$-\frac{16}{x}$

Exercise 1.3.7.20

Find the difference: $-\frac{17}{a}-\frac{5}{a}$.

Answer

$-\frac{22}{a}$

Now lets do an example that involves both addition and subtraction.

Example 1.3.7.11: simplify

Simplify: $\frac{3}{8}+\left(-\frac{5}{8}\right)-\frac{1}{8}$.

Solution

Combine the numerators over the common denominator.	$\frac{3+(-5)-1}{8}$
Simplify the numerator, working left to right.	$\frac{-2-1}{8}$
Subtract the terms in the numerator.	$\frac{-3}{8}$
Rewrite with the negative sign in front of the fraction.	$-\frac{3}{8}$

Exercise 1.3.7.21

Simplify: $\frac{2}{5}+\left(-\frac{4}{5}\right)-\frac{3}{5}$.

Answer

-1

Exercise 1.3.7.22

Simplify: $\frac{5}{9}+\left(-\frac{4}{9}\right)-\frac{7}{9}$.

Answer

$-\frac{2}{3}$

Access Additional Online Resources

- Adding Fractions With Pattern Blocks
- Adding Fractions With Like Denominators
- Subtracting Fractions With Like Denominators

Key Concepts

- Fraction Addition
  - If $a, b,$, and c are numbers where $c \neq 0$, then $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$
  - To add fractions, add the numerators and place the sum over the common denominator.
- Fraction Subtraction
  - If $a, b,$, and c are numbers where $c \neq 0$, then $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.

Practice Makes Perfect

Model Fraction Addition

In the following exercises, use a model to add the fractions. Show a diagram to illustrate your model.

254. $\frac{2}{5}+\frac{1}{5}$
255. $\frac{3}{10}+\frac{4}{10}$
256. $\frac{1}{6}+\frac{3}{6}$
257. $\frac{3}{8}+\frac{3}{8}$

Add Fractions with a Common Denominator

In the following exercises, find each sum.

258. $\frac{4}{9}+\frac{1}{9}$
259. $\frac{2}{9}+\frac{5}{9}$
260. $\frac{6}{13}+\frac{7}{13}$
261. $\frac{9}{15}+\frac{7}{15}$
262. $\frac{x}{4}+\frac{3}{4}$
263. $\frac{y}{3}+\frac{2}{3}$
264. $\frac{7}{p}+\frac{9}{p}$

265. $\frac{8}{q}+\frac{6}{q}$
266. $\frac{8b}{9}+\frac{3b}{9}$
267. $\frac{5a}{7}+\frac{4a}{7}$
268. $\frac{-12y}{8}+\frac{3y}{8}$
269. $\frac{-11x}{5}+\frac{7x}{5}$
270. $-\frac{1}{8}+\left(-\frac{3}{8}\right)$
271. $-\frac{1}{8}+\left(-\frac{5}{8}\right)$
272. $-\frac{3}{16}+\left(-\frac{7}{16}\right)$
273. $-\frac{5}{16}+\left(-\frac{9}{16}\right)$
274. $-\frac{8}{17}+\frac{15}{17}$
275. $-\frac{9}{19}+\frac{17}{19}$
276. $-\frac{6}{13}+\left(-\frac{10}{13}\right)+\left(-\frac{12}{13}\right)$
277. $-\frac{5}{12}+\left(-\frac{7}{12}\right)+\left(-\frac{11}{12}\right)$

Model Fraction Subtraction

In the following exercises, use a model to subtract the fractions. Show a diagram to illustrate your model.

278. $\frac{5}{8}-\frac{2}{8}$
279. $\frac{5}{6}-\frac{2}{6}$

Subtract Fractions with a Common Denominator

In the following exercises, find the difference.

280. $\frac{4}{5}-\frac{1}{5}$
281. $\frac{4}{5}-\frac{3}{5}$
282. $\frac{11}{15}-\frac{7}{15}$
283. $\frac{9}{13}-\frac{4}{13}$
284. $\frac{11}{12}-\frac{5}{12}$
285. $\frac{7}{12}-\frac{5}{12}$
286. $\frac{4}{21}-\frac{19}{21}$
287. $-\frac{8}{9}-\frac{16}{9}$
288. $\frac{y}{17}-\frac{9}{17}$
289. $\frac{x}{19}-\frac{8}{19}$
290. $\frac{5y}{8}-\frac{7}{8}$

291. $\frac{11z}{13} - \frac{8}{13}$
292. $-\frac{8}{d} - \frac{3}{d}$
293. $-\frac{7}{c} - \frac{7}{c}$
294. $-\frac{23}{u} - \frac{15}{u}$
295. $-\frac{29}{v} - \frac{26}{v}$
296. $-\frac{6c}{7} - \frac{5c}{7}$
297. $-\frac{12d}{11} - \frac{9d}{11}$
298. $\frac{-4r}{13} - \frac{5r}{13}$
299. $\frac{-7s}{3} - \frac{7s}{3}$
300. $-\frac{3}{5} - \left(-\frac{4}{5}\right)$
301. $-\frac{3}{7} - \left(-\frac{5}{7}\right)$
302. $-\frac{7}{9} - \left(-\frac{5}{9}\right)$
303. $-\frac{8}{11} - \left(-\frac{5}{11}\right)$

Mixed Practice

In the following exercises, perform the indicated operation and write your answers in simplified form.

304. $-\frac{5}{18} \cdot \frac{9}{10}$
305. $-\frac{3}{14} \cdot \frac{7}{12}$
306. $\frac{n}{5} - \frac{4}{5}$
307. $\frac{6}{11} - \frac{s}{11}$
308. $-\frac{7}{24} - \frac{2}{24}$
309. $-\frac{5}{18} - \frac{1}{18}$
310. $\frac{8}{15} \div \frac{12}{5}$
311. $\frac{7}{12} \div \frac{9}{28}$

Everyday Math

312. **Trail Mix** Jacob is mixing together nuts and raisins to make trail mix. He has $\frac{6}{10}$ of a pound of nuts and $\frac{3}{10}$ of a pound of raisins. How much trail mix can he make?
313. **Baking** Janet needs $\frac{5}{8}$ of a cup of flour for a recipe she is making. She only has $\frac{3}{8}$ of a cup of flour and will ask to borrow the rest from her next-door neighbor. How much flour does she have to borrow?

Writing Exercises

314. Greg dropped his case of drill bits and three of the bits fell out. The case has slots for the drill bits, and the slots are arranged in order from smallest to largest. Greg needs to put the bits that fell out back in the case in the empty slots. Where do the three bits go? Explain how you know.

Bits in case: $\frac{1}{16}, \frac{1}{8}, \_\_\_, \_\_\_, \frac{5}{16}, \frac{3}{8}, \_\_\_, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}$.

Bits that fell out: $\frac{7}{16}, \frac{3}{16}, \frac{1}{4}$.

315. After a party, Lupe has $\frac{5}{12}$ of a cheese pizza, $\frac{4}{12}$ of a pepperoni pizza, and $\frac{4}{12}$ of a veggie pizza left. Will all the slices fit into 1 pizza box? Explain your reasoning.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			
find the least common denominator (LCD).			
convert fractions to equivalent fractions with the LCD.			

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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1.3.8: Add and Subtract Fractions with Different Denominators (Part 1)

Learning Objectives

- Find the least common denominator (LCD)
- Convert fractions to equivalent fractions with the LCD
- Add and subtract fractions with different denominators
- Identify and use fraction operations
- Use the order of operations to simplify complex fractions
- Evaluate variable expressions with fractions

be prepared!

Before you get started, take this readiness quiz.

1. Find two fractions equivalent to $\frac{5}{6}$. If you missed this problem, review Example 4.1.14.
2. Simplify: $\frac{1+5\cdot 3}{2^2+4}$. If you missed this problem, review Example 4.3.13.

Find the Least Common Denominator

In the previous section, we explained how to add and subtract fractions with a common denominator. But how can we add and subtract fractions with unlike denominators?

Let's think about coins again. Can you add one quarter and one dime? You could say there are two coins, but that's not very useful. To find the total value of one quarter plus one dime, you change them to the same kind of unit—cents. One quarter equals 25 cents and one dime equals 10 cents, so the sum is 35 cents. See Figure 1.3.8.1.



Figure 1.3.8.1: Together, a quarter and a dime are worth 35 cents, or $\frac{35}{100}$ of a dollar.

Similarly, when we add fractions with different denominators we have to convert them to equivalent fractions with a common denominator. With the coins, when we convert to cents, the denominator is 100. Since there are 100 cents in one dollar, 25 cents is $\frac{25}{100}$ and 10 cents is $\frac{10}{100}$. So we add $\frac{25}{100}+\frac{10}{100}$ to get $\frac{35}{100}$, which is 35 cents.

You have practiced adding and subtracting fractions with common denominators. Now let's see what you need to do with fractions that have different denominators.

First, we will use fraction tiles to model finding the common denominator of $\frac{1}{2}$ and $\frac{1}{3}$. We'll start with one $\frac{1}{2}$ tile and $\frac{1}{3}$ tile. We want to find a common fraction tile that we can use to match *both* $\frac{1}{2}$ and $\frac{1}{3}$ exactly. If we try the $\frac{1}{4}$ pieces, 2 of them exactly match the $\frac{1}{2}$ piece, but they do not exactly match the $\frac{1}{3}$ piece.

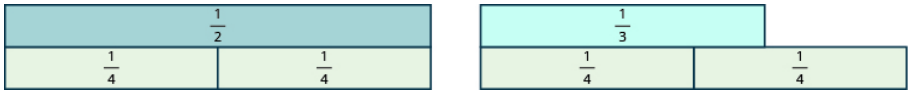


Figure 1.3.8.2

If we try the $\frac{1}{5}$ pieces, they do not exactly cover the $\frac{1}{2}$ piece or the $\frac{1}{3}$ piece.

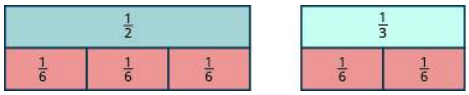


Figure 1.3.8.3

If we were to try the $\frac{1}{12}$ pieces, they would also work.

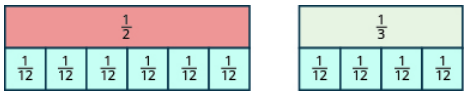


Figure 1.3.8.4

Even smaller tiles, such as $\frac{1}{24}$ and $\frac{1}{48}$, would also exactly cover the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece. The denominator of the largest piece that covers both fractions is the **least common denominator** (LCD) of the two fractions. So, the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$ is 6.

Notice that all of the tiles that cover $\frac{1}{2}$ and $\frac{1}{3}$ have something in common: Their denominators are common multiples of 2 and 3, the denominators of $\frac{1}{2}$ and $\frac{1}{3}$. The least common multiple (LCM) of the denominators is 6, and so we say that 6 is the least common denominator (LCD) of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

Definition: Least Common Denominator

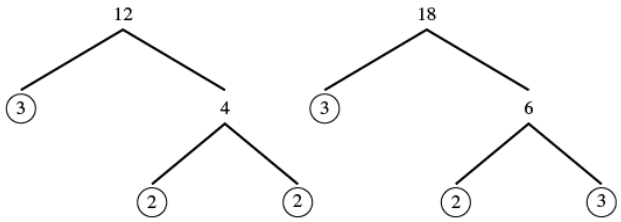
The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.

To find the LCD of two fractions, we will find the LCM of their denominators. We follow the procedure we used earlier to find the LCM of two numbers. We only use the denominators of the fractions, not the numerators, when finding the LCD.

Example 1.3.8.1: lcd

Find the LCD for the fractions $\frac{7}{12}$ and $\frac{5}{18}$.

Solution

Factor each denominator into its primes.	
List the primes of 12 and the primes of 18 lining them up in columns when possible.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$
Bring down the columns.	$12 = 2 \cdot 2 \cdot 3$ $18 = 2 \cdot \quad 3 \cdot 3$ $\text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3$
Multiply the factors. The product is the LCM.	LCM = 36
The LCM of 12 and 18 is 36, so the LCD of $\frac{7}{12}$ and $\frac{5}{18}$ is 36.	LCD of $\frac{7}{12}$ and $\frac{5}{18}$ is 36.

Exercise 1.3.8.1

Find the least common denominator for the fractions: $\frac{7}{12}$ and $\frac{11}{15}$.

Answer

60

Exercise 1.3.8.2

Find the least common denominator for the fractions: $\frac{13}{15}$ and $\frac{17}{5}$.

Answer

15

To find the LCD of two fractions, find the LCM of their denominators. Notice how the steps shown below are similar to the steps we took to find the LCM.

HOW TO: FIND THE LEAST COMMON DENOMINATOR (LCD) OF TWO FRACTIONS

Step 1. Factor each denominator into its primes.

Step 2. List the primes, matching primes in columns when possible.

Step 3. Bring down the columns.

Step 4. Multiply the factors. The product is the LCM of the denominators.

Step 5. The LCM of the denominators is the LCD of the fractions.

Example 1.3.8.2:

Find the least common denominator for the fractions $\frac{8}{15}$ and $\frac{11}{24}$.

Solution

To find the LCD, we find the LCM of the denominators. Find the LCM of 15 and 24.

$$\begin{aligned} 15 &= 3 \cdot 5 \\ 24 &= 2 \cdot 2 \cdot 2 \cdot 3 \\ \hline \text{LCD} &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\ \text{LCD} &= 120 \end{aligned}$$

The LCM of 15 and 24 is 120. So, the LCD of $\frac{8}{15}$ and $\frac{11}{24}$ is 120.

Exercise 1.3.8.3

Find the least common denominator for the fractions: $\frac{13}{24}$ and $\frac{17}{32}$.

Answer

96

Exercise 1.3.8.4

Find the least common denominator for the fractions: $\frac{9}{28}$ and $\frac{21}{32}$.

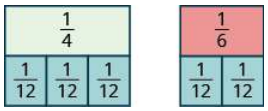
Answer

224

Convert Fractions to Equivalent Fractions with the LCD

Earlier, we used fraction tiles to see that the LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12. We saw that three $\frac{1}{12}$ pieces exactly covered $\frac{1}{4}$ and two $\frac{1}{12}$ pieces exactly covered $\frac{1}{6}$, so

$$\frac{1}{4}=\frac{3}{12} \quad and \quad \frac{1}{6}=\frac{2}{12}.$$



We say that $\frac{1}{4}$ and $\frac{3}{12}$ are equivalent fractions and also that $\frac{1}{6}$ and $\frac{2}{12}$ are equivalent fractions.

We can use the Equivalent Fractions Property to algebraically change a fraction to an equivalent one. Remember, two fractions are equivalent if they have the same value. The Equivalent Fractions Property is repeated below for reference.

Definition: Equivalent Fractions Property

If a, b, c are whole numbers where $b \neq 0$, $c \neq 0$, then

$$\frac{a}{b}=\frac{a \cdot c}{b \cdot c} \quad and \quad \frac{a \cdot c}{b \cdot c}=\frac{a}{b} \tag{1.3.8.1}$$

To add or subtract fractions with different denominators, we will first have to convert each fraction to an equivalent fraction with the LCD. Let's see how to change $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12 without using models.

Example 1.3.8.3: conversion

Convert $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with denominator 12, their LCD.

Solution

Find the LCD.	The LCD of $\frac{1}{4}$ and $\frac{1}{6}$ is 12.
Find the number to multiply 4 to get 12.	$4 \cdot 3 = 12$
Find the number to multiply 6 to get 12.	$6 \cdot 2 = 12$
Use the Equivalent Fractions Property to convert each fraction to an equivalent fraction with the LCD, multiplying both the numerator and denominator of each fraction by the same number.	$\frac{1}{4}$ $\quad$ $\frac{1}{6}$ $\frac{1 \cdot 3}{4 \cdot 3}$ $\quad$ $\frac{1 \cdot 2}{6 \cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{12}$ $\quad$ $\frac{2}{12}$

We do not reduce the resulting fractions. If we did, we would get back to our original fractions and lose the common denominator.

Exercise 1.3.8.5

Change to equivalent fractions with the LCD: $\frac{3}{4}$ and $\frac{5}{6}$, $LCD = 12$

Answer

$\frac{9}{12}, \frac{10}{12}$

Exercise 1.3.8.6

Change to equivalent fractions with the LCD: $-\frac{7}{12}$ and $\frac{11}{15}$, $LCD = 60$

Answer

$-\frac{35}{60}, \frac{44}{60}$

HOW TO: CONVERT TWO FRACTIONS TO EQUIVALENT FRACTIONS WITH THEIR LCD AS THE COMMON DENOMINATOR

Step 1. Find the LCD.

Step 2. For each fraction, determine the number needed to multiply the denominator to get the LCD.

Step 3. Use the Equivalent Fractions Property to multiply both the numerator and denominator by the number you found in Step 2.

Step 4. Simplify the numerator and denominator.

Example 1.3.8.4: conversion

Convert $\frac{8}{15}$ and $\frac{11}{24}$ to equivalent fractions with denominator 120, their LCD.

Find the number that must multiply 15 to get 120.	$15 \cdot 8 = 120$
Find the number that must multiply 24 to get 120.	$24 \cdot 5 = 120$
Use the Equivalent Fractions Property.	$\frac{8 \cdot 8}{15 \cdot 8}$ $\frac{11 \cdot 5}{24 \cdot 5}$
Simplify the numerators and denominators.	$\frac{64}{120}$ $\frac{55}{120}$

Exercise 1.3.8.7

Change to equivalent fractions with the LCD: $\frac{13}{24}$ and $\frac{17}{32}$, LCD 96

Answer

$\frac{52}{96}, \frac{51}{96}$

Exercise 1.3.8.8

Change to equivalent fractions with the LCD: $\frac{9}{28}$ and $\frac{27}{32}$, LCD 224

Answer

$\frac{72}{224}, \frac{189}{224}$

Add and Subtract Fractions with Different Denominators

Once we have converted two fractions to equivalent forms with common denominators, we can add or subtract them by adding or subtracting the numerators.

HOW TO: ADD OR SUBTRACT FRACTIONS WITH DIFFERENT DENOMINATORS

Step 1. Find the LCD.

Step 2. Convert each fraction to an equivalent form with the LCD as the denominator.

Step 3. Add or subtract the fractions.

Step 4. Write the result in simplified form.

Example 1.3.8.5: add

Add: $\frac{1}{2}+\frac{1}{3}$.

Solution

Find the LCD of 2, 3.	$2=2$ $3=\quad 3$ $\text{LCD}=2\cdot 3$ $\text{LCD}=6$
Change into equivalent fractions with the LCD 6.	$\frac{1\cdot 3}{2\cdot 3}+\frac{1\cdot 2}{3\cdot 2}$
Simplify the numerators and denominators.	$\frac{3}{6}+\frac{2}{6}$
Add.	$\frac{5}{6}$

Remember, always check to see if the answer can be simplified. Since 5 and 6 have no common factors, the fraction $\frac{5}{6}$ cannot be reduced.

Exercise 1.3.8.9

Add: $\frac{1}{4}+\frac{1}{3}$.

Answer

$\frac{7}{12}$

Exercise 1.3.8.10

Add: $\frac{1}{2}+\frac{1}{5}$.

Answer

$\frac{7}{10}$

Example 1.3.8.6: subtract

Subtract: $\frac{1}{2}-\left(-\frac{1}{4}\right)$.

Solution

Find the LCD of 2 and 4.	$\begin{array}{l} 2=2 \\ 4=2\cdot 2 \\ \hline \text{LCD}=2\cdot 2 \\ \text{LCD}=4 \end{array}$
Rewrite as equivalent fractions using the LCD 4.	$\frac{1\cdot 2}{2\cdot 2}-\left(-\frac{1}{4}\right)$
Simplify the first fraction.	$\frac{2}{4}-\left(-\frac{1}{4}\right)$
Subtract.	$\frac{2-(-1)}{4}$
Simplify.	$\frac{3}{4}$

One of the fractions already had the least common denominator, so we only had to convert the other fraction.

Exercise 1.3.8.11

Subtract: $\frac{1}{2}-\left(-\frac{1}{8}\right)$.

Answer

$\frac{5}{8}$

Exercise 1.3.8.12

Subtract: $\frac{1}{3}-\left(-\frac{1}{6}\right)$.

Answer

$\frac{1}{2}$

Example 1.3.8.7: add

Add: $\frac{7}{12}+\frac{5}{18}$.

Solution

Find the LCD of 12 and 18.	$\begin{array}{l} 12 = 2 \cdot 2 \cdot 3 \\ 18 = 2 \cdot \quad 3 \cdot 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCD} = 36 \end{array}$
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 3}{12 \cdot 3} + \frac{5 \cdot 2}{18 \cdot 2}$
Simplify the numerators and denominators.	$\frac{21}{36} + \frac{10}{36}$
Add.	$\frac{31}{36}$

Because 31 is a prime number, it has no factors in common with 36. The answer is simplified.

Exercise 1.3.8.13

Add: $\frac{7}{12} + \frac{11}{15}$.

Answer

$\frac{79}{60}$

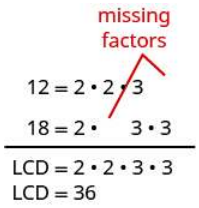
Exercise 1.3.8.14

Add: $\frac{13}{15} + \frac{17}{20}$.

Answer

$\frac{103}{60}$

When we use the Equivalent Fractions Property, there is a quick way to find the number you need to multiply by to get the LCD. Write the factors of the denominators and the LCD just as you did to find the LCD. The "missing" factors of each denominator are the numbers you need.



The LCD, 36, has 2 factors of 2 and 2 factors of 3. Twelve has two factors of 2, but only one of 3 —so it is 'missing' one 3. We multiplied the numerator and denominator of $\frac{7}{12}$ by 3 to get an equivalent fraction with denominator 36. Eighteen is missing one factor of 2 —so you multiply the numerator and denominator $\frac{5}{18}$ by 2 to get an equivalent fraction with denominator 36. We will apply this method as we subtract the fractions in the next example.

Example 1.3.8.8: subtract

Subtract: $\frac{7}{15} - \frac{19}{24}$.

Solution

Find the LCD. 15 is 'missing' three factors of 2 24 is 'missing' a factor of 5	$\begin{array}{l} 15 = \quad\quad 3 \cdot 5 \\ 24 = 2 \cdot 2 \cdot 2 \cdot 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\ \text{LCD} = 120 \end{array}$
Rewrite as equivalent fractions with the LCD.	$\frac{7 \cdot 8}{15 \cdot 8} - \frac{19 \cdot 5}{24 \cdot 5}$
Simplify each numerator and denominator.	$\frac{56}{120} - \frac{95}{120}$
Subtract.	$-\frac{39}{120}$
Rewrite showing the common factor of 3.	$-\frac{13 \cdot 3}{40 \cdot 3}$
Remove the common factor to simplify.	$-\frac{13}{40}$

? Exercise 1.3.8.15

Subtract: $\frac{13}{24} - \frac{17}{32}$.

Answer

$\frac{1}{96}$

? Exercise 1.3.8.16

Subtract: $\frac{21}{32} - \frac{9}{28}$.

Answer

$\frac{75}{224}$

✓ Example 1.3.8.9: add

Add: $-\frac{11}{30} + \frac{23}{42}$.

Solution

Find the LCD.	$\begin{array}{l} 30 = 2 \cdot 3 \cdot 5 \\ 42 = 2 \cdot 3 \cdot \quad 7 \\ \hline \text{LCD} = 2 \cdot 3 \cdot 5 \cdot 7 \\ \text{LCD} = 210 \end{array}$
Rewrite as equivalent fractions with the LCD.	$-\frac{11 \cdot 7}{30 \cdot 7} + \frac{23 \cdot 5}{42 \cdot 5}$
Simplify each numerator and denominator.	$-\frac{77}{210} + \frac{115}{210}$
Add.	$\frac{38}{210}$
Rewrite showing the common factor of 2.	$\frac{19 \cdot 2}{105 \cdot 2}$
Remove the common factor to simplify.	$\frac{19}{105}$

Exercise 1.3.8.17

Add: $-\frac{13}{42}+\frac{17}{35}$.

Answer

$\frac{37}{210}$

Exercise 1.3.8.18

Add: $-\frac{19}{24}+\frac{17}{32}$.

Answer

$-\frac{25}{96}$

In the next example, one of the fractions has a variable in its numerator. We follow the same steps as when both numerators are numbers.

Example 1.3.8.10: add

Add: $\frac{3}{5}+\frac{x}{8}$.

Solution

The fractions have different denominators.

Find the LCD.	$\begin{array}{l} 5= \quad 5 \\ 8=2\cdot 2\cdot 2 \\ \hline \text{LCD}=2\cdot 2\cdot 2\cdot 5 \\ \text{LCD}=40 \end{array}$
Rewrite as equivalent fractions with the LCD.	$\frac{3\cdot 8}{5\cdot 8}+\frac{x\cdot 5}{8\cdot 5}$
Simplify the numerators and denominators.	$\frac{24}{40}+\frac{5x}{40}$
Add.	$\frac{24+5x}{40}$

We cannot add 24 and $5x$ since they are not like terms, so we cannot simplify the expression any further.

Exercise 1.3.8.19

Add: $\frac{y}{6}+\frac{7}{9}$.

Answer

$\frac{3y+14}{18}$

Exercise 1.3.8.20

Add: $\frac{x}{6}+\frac{7}{15}$.

Answer

$$\frac{5x+14}{30}$$

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1.3.9: Add and Subtract Fractions with Different Denominators (Part 2)

Identify and Use Fraction Operations

By now in this chapter, you have practiced multiplying, dividing, adding, and subtracting fractions. The following table summarizes these four fraction operations. Remember: You need a common denominator to add or subtract fractions, but not to multiply or divide fractions.

Summary of Fraction Operations

Fraction multiplication: Multiply the numerators and multiply the denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \tag{1.3.9.1}$$

Fraction division: Multiply the first fraction by the reciprocal of the second.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \tag{1.3.9.2}$$

Fraction addition: Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \tag{1.3.9.3}$$

Fraction subtraction: Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD.

$$\frac{a}{c} - \frac{a}{c} = \frac{a-b}{c} \tag{1.3.9.4}$$

Example 1.3.9.11: simplify

Simplify:

a. $-\frac{1}{4} + \frac{1}{6}$

b. $-\frac{1}{4} \div \frac{1}{6}$

Solution

First we ask ourselves, “What is the operation?”

a. The operation is addition. Do the fractions have a common denominator? No.

Find the LCD.	$\begin{array}{l} 4 = 2 \cdot 2 \\ 6 = 2 \cdot \quad 3 \\ \hline \text{LCD} = 2 \cdot 2 \cdot 3 \\ \text{LCD} = 12 \end{array}$
Rewrite each fraction as an equivalent fraction with the LCD.	$-\frac{1 \cdot 3}{4 \cdot 3} + \frac{1 \cdot 2}{6 \cdot 2}$
Simplify the numerators and denominators.	$-\frac{3}{12} + \frac{2}{12}$
Add the numerators and place the sum over the common denominator.	$-\frac{1}{12}$
Check to see if the answer can be simplified. It cannot.	

b. The operation is division. We do not need a common denominator.

To divide fractions, multiply the first fraction by the reciprocal of the second.	$-\frac{1}{4} \cdot \frac{6}{1}$
Multiply.	$-\frac{6}{4}$
Simplify.	$-\frac{3}{2}$

Exercise 1.3.9.21

Simplify:

a. $-\frac{3}{4} - \frac{1}{6}$

b. $-\frac{3}{4} \cdot \frac{1}{6}$

Answer a

$-\frac{11}{12}$

Answer b

$-\frac{1}{8}$

Exercise 1.3.9.22

Simplify:

a. $\frac{5}{6} \div \left(-\frac{1}{4}\right)$

b. $\frac{5}{6} - \left(-\frac{1}{4}\right)$

Answer a

$-\frac{10}{3}$

Answer b

$\frac{13}{12}$

Example 1.3.9.12: simplify

Simplify:

a. $\frac{5x}{6} - \frac{3}{10}$

b. $\frac{5x}{6} \cdot \frac{3}{10}$

Solution

a. The operation is subtraction. The fractions do not have a common denominator.

Rewrite each fraction as an equivalent fraction with the LCD, 30.	$\frac{5x \cdot 5}{6 \cdot 5} - \frac{3 \cdot 3}{10 \cdot 3}$

	$\frac{25x}{30}-\frac{9}{30}$
Subtract the numerators and place the difference over the common denominator.	$\frac{25x-9}{30}$

b. The operation is multiplication; no need for a common denominator.

To multiply fractions, multiply the numerators and multiply the denominators.	$\frac{5x\cdot 3}{6\cdot 10}$
Rewrite, showing common factors.	$\frac{\cancel{5}\cdot x\cdot \cancel{3}}{2\cdot \cancel{3}\cdot 2\cdot \cancel{5}}$
Remove common factors to simplify.	$\frac{x}{4}$

? Exercise 1.3.9.23

Simplify:

a. $\frac{3a}{4}-\frac{8}{9}$

b. $\frac{3a}{4}\cdot\frac{8}{9}$

Answer a

$\frac{27a-32}{36}$

Answer b

$\frac{2a}{3}$

? Exercise 1.3.9.24

Simplify:

a. $\frac{4k}{5}+\frac{5}{6}$

b. $\frac{4k}{5}\div\frac{5}{6}$

Answer a

$\frac{24k+25}{30}$

Answer b

$\frac{24k}{25}$

Use the Order of Operations to Simplify Complex Fractions

In Multiply and Divide Mixed Numbers and Complex Fractions, we saw that a complex fraction is a fraction in which the numerator or denominator contains a fraction. We simplified complex fractions by rewriting them as division problems. For example,

$$\frac{\frac{3}{4}}{\frac{5}{8}} = \frac{3}{4} \div \frac{5}{8}$$

Now we will look at complex fractions in which the numerator or denominator can be simplified. To follow the order of operations, we simplify the numerator and denominator separately first. Then we divide the numerator by the denominator.

HOW TO: SIMPLIFY COMPLEX FRACTIONS

Step 1. Simplify the numerator.

Step 2. Simplify the denominator.

Step 3. Divide the numerator by the denominator.

Step 4. Simplify if possible.

Example 1.3.9.13: simplify

Simplify: $\frac{\left(\frac{1}{2}\right)^2}{4+3^2}$.

Solution

Simplify the numerator.	$\frac{\frac{1}{4}}{4+3^2}$
Simplify the term with the exponent in the denominator.	$\frac{\frac{1}{4}}{4+9}$
Add the terms in the denominator.	$\frac{\frac{1}{4}}{13}$
Divide the numerator by the denominator.	$\frac{1}{4} \div 13$
Rewrite as multiplication by the reciprocal.	$\frac{1}{4} \cdot \frac{1}{13}$
Multiply.	$\frac{1}{52}$

Exercise 1.3.9.25

Simplify: $\frac{\left(\frac{1}{3}\right)^2}{2^3+2}$.

Answer

$\frac{1}{90}$

Exercise 1.3.9.26

Simplify: $\dfrac{1+4^2}{\left(\frac{1}{4}\right)^2}$.

Answer

272

Example 1.3.9.14: simplify

Simplify: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}-\frac{1}{6}}$.

Solution

Rewrite numerator with the LCD of 6 and denominator with LCD of 12.	$\dfrac{\frac{3}{6}+\frac{4}{6}}{\frac{9}{12}-\frac{2}{12}}$
Add in the numerator. Subtract in the denominator.	$\dfrac{\frac{7}{6}}{\frac{7}{12}}$
Divide the numerator by the denominator.	$\frac{7}{6} \div \frac{7}{12}$
Rewrite as multiplication by the reciprocal.	$\frac{7}{6} \cdot \frac{12}{7}$
Rewrite, showing common factors.	$\dfrac{\cancel{7} \cdot \cancel{6} \cdot 2}{\cancel{6}\cancel{7} \cdot 1}$
Simplify.	2

Exercise 1.3.9.27

Simplify: $\dfrac{\frac{1}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{3}}$.

Answer

2

Exercise 1.3.9.28

Simplify: $\dfrac{\frac{2}{3}-\frac{1}{2}}{\frac{1}{4}+\frac{1}{3}}$.

Answer

$\frac{2}{7}$

Evaluate Variable Expressions with Fractions

We have evaluated expressions before, but now we can also evaluate expressions with fractions. Remember, to evaluate an expression, we substitute the value of the variable into the expression and then simplify.

Example 1.3.9.15: evaluate

Evaluate $x+\frac{1}{3}$ when

a. $x=-\frac{1}{3}$

b. $x=-\frac{3}{4}$

Solution

a. To evaluate $x+\frac{1}{3}$ when $x=-\frac{1}{3}$, substitute $-\frac{1}{3}$ for x in the expression.

Substitute $-\frac{1}{3}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Simplify.	0

b. To evaluate $x+\frac{1}{3}$ when $x=-\frac{3}{4}$, we substitute $-\frac{3}{4}$ for x in the expression.

Substitute $-\frac{3}{4}$ for x.	$-\frac{1}{3}+\frac{1}{3}$
Rewrite as equivalent fractions with the LCD, 12.	$-\frac{3\cdot 3}{4\cdot 3}+\frac{1\cdot 4}{3\cdot 4}$
Simplify the numerators and denominators.	$-\frac{9}{12}+\frac{4}{12}$
Add.	$-\frac{5}{12}$

Exercise 1.3.9.29

Evaluate $x+\frac{3}{4}$ when:

a. $x=-\frac{7}{4}$

b. $x=-\frac{5}{4}$

Answer a

-1

Answer b

$-\frac{1}{2}$

Exercise 1.3.9.30

Evaluate $y+\frac{1}{2}$ when:

a. $y=\frac{2}{3}$

b. $y=-\frac{3}{4}$

Answer a

$\frac{7}{6}$

Answer b

$-\frac{1}{4}$

Example 1.3.9.16: evaluate

Evaluate $y-\frac{5}{6}$ when $y=-\frac{2}{3}$.

Solution

We substitute $-\frac{2}{3}$ for y in the expression.

Substitute $-\frac{2}{3}$ for y.	$-\frac{2}{3}-\frac{5}{6}$
Rewrite as equivalent fractions with the LCD, 6.	$-\frac{4}{6}-\frac{5}{6}$
Subtract.	$-\frac{9}{6}$
Simplify.	$-\frac{3}{2}$

Exercise 1.3.9.31

Evaluate $y-\frac{1}{2}$ when $y=-\frac{1}{4}$.

Answer

$-\frac{3}{4}$

Exercise 1.3.9.32

Evaluate $x-\frac{3}{8}$ when $x=-\frac{5}{2}$.

Answer

$-\frac{23}{8}$

Example 1.3.9.17:

Evaluate $2x^2y$ when $x = \frac{1}{4}$ and $y = -\frac{2}{3}$.

Solution

Substitute the values into the expression. In $2x^2y$, the exponent applies only to x.

Substitute $\frac{1}{4}$ for x and $-\frac{2}{3}$ for y.	$2\left(\frac{1}{4}\right)^2\left(-\frac{2}{3}\right)$
Simplify exponents first.	$2\left(\frac{1}{16}\right)\left(-\frac{2}{3}\right)$
Multiply. The product will be negative.	$-\frac{2}{1}\cdot\frac{1}{16}\cdot\frac{2}{3}$
Simplify.	$-\frac{4}{48}$
Remove the common factors.	$-\frac{1\cdot\cancel{4}}{\cancel{4}\cdot 12}$
Simplify.	$-\frac{1}{12}$

Exercise 1.3.9.33

Evaluate: $3ab^2$ when $a = -\frac{2}{3}$ and $b = -\frac{1}{2}$.

Answer

$-\frac{1}{2}$

Exercise 1.3.9.34

Evaluate: $4c^3d$ when $c = -\frac{1}{2}$ and $d = -\frac{4}{3}$.

Answer

$\frac{2}{3}$

Example 1.3.9.18: evaluate

Evaluate: $\frac{p+q}{r}$ when $p = -4$, $q = -2$, and $r = 8$.

Solution

We substitute the values into the expression and simplify.

Substitute -4 for p, -2 for q and 8 for r.	$\frac{-4+(-2)}{8}$
Add in the numerator first.	$-\frac{6}{8}$
Simplify.	$-\frac{3}{4}$

Exercise 1.3.9.35

Evaluate: $\frac{a+b}{c}$ when $a=-8$, $b=-7$, and $c=6$.

Answer

$-\frac{5}{2}$

Exercise 1.3.9.36

Evaluate: $\frac{x+y}{z}$ when $x=9$, $y=-18$, and $z=-6$.

Answer

$\frac{3}{2}$

Practice Makes Perfect

Find the Least Common Denominator (LCD)

In the following exercises, find the least common denominator (LCD) for each set of fractions.

316. $\frac{2}{3}$ and $\frac{3}{4}$
317. $\frac{3}{4}$ and $\frac{2}{5}$
318. $\frac{7}{12}$ and $\frac{5}{8}$
319. $\frac{9}{16}$ and $\frac{7}{12}$
320. $\frac{13}{30}$ and $\frac{25}{42}$
321. $\frac{23}{30}$ and $\frac{5}{48}$
322. $\frac{21}{35}$ and $\frac{39}{56}$
323. $\frac{18}{35}$ and $\frac{33}{49}$
324. $\frac{2}{3}$, $\frac{1}{6}$ and $\frac{3}{4}$
325. $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{3}{5}$

Convert Fractions to Equivalent Fractions with the LCD

In the following exercises, convert to equivalent fractions using the LCD.

326. $\frac{1}{3}$ and $\frac{1}{4}$, LCD = 12
327. $\frac{1}{4}$ and $\frac{1}{5}$, LCD = 20
328. $\frac{5}{12}$ and $\frac{7}{8}$, LCD = 24
329. $\frac{7}{12}$ and $\frac{5}{8}$, LCD = 24
330. $\frac{13}{16}$ and $-\frac{11}{12}$, LCD = 48
331. $\frac{11}{16}$ and $-\frac{5}{12}$, LCD = 48

332. $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$, LCD = 12
333. $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{3}{5}$, LCD = 60

Add and Subtract Fractions with Different Denominators

In the following exercises, add or subtract. Write the result in simplified form.

334. $\frac{1}{3}+\frac{1}{5}$
335. $\frac{1}{4}+\frac{1}{5}$
336. $\frac{1}{2}+\frac{1}{7}$
337. $\frac{1}{3}+\frac{1}{8}$
338. $\frac{1}{3}-\left(-\frac{1}{9}\right)$
339. $\frac{1}{4}-\left(-\frac{1}{8}\right)$
340. $\frac{1}{5}-\left(-\frac{1}{10}\right)$
341. $\frac{1}{2}-\left(-\frac{1}{6}\right)$
342. $\frac{2}{3}+\frac{3}{4}$
343. $\frac{3}{4}+\frac{2}{5}$
344. $\frac{7}{12}+\frac{5}{8}$
345. $\frac{5}{12}+\frac{3}{8}$
346. $\frac{7}{12}-\frac{9}{16}$
347. $\frac{7}{16}-\frac{5}{12}$
348. $\frac{11}{12}-\frac{3}{8}$
349. $\frac{5}{8}-\frac{7}{12}$
350. $\frac{2}{3}-\frac{3}{8}$
351. $\frac{5}{6}-\frac{3}{4}$
352. $-\frac{11}{30}+\frac{27}{40}$
353. $-\frac{9}{20}+\frac{17}{30}$
354. $-\frac{13}{30}+\frac{25}{42}$
355. $-\frac{23}{30}+\frac{5}{48}$
356. $-\frac{39}{56}-\frac{22}{35}$
357. $-\frac{33}{49}-\frac{18}{35}$
358. $-\frac{2}{3}-\left(-\frac{3}{4}\right)$
359. $-\frac{3}{4}-\left(-\frac{4}{5}\right)$

360. $-\frac{9}{16}-\left(-\frac{4}{5}\right)$
361. $-\frac{7}{20}-\left(-\frac{5}{8}\right)$
362. $1+\frac{7}{8}$
363. $1+\frac{5}{6}$
364. $1-\frac{5}{9}$
365. $1-\frac{3}{10}$
366. $\frac{x}{3}+\frac{1}{4}$
367. $\frac{y}{2}+\frac{2}{3}$
368. $\frac{y}{4}-\frac{3}{5}$
369. $\frac{x}{5}-\frac{1}{4}$

Identify and Use Fraction Operations

In the following exercises, perform the indicated operations. Write your answers in simplified form.

370. (a) $\frac{3}{4}+\frac{1}{6}$ (b) $\frac{3}{4}\div\frac{1}{6}$
371. (a) $\frac{2}{3}+\frac{1}{6}$ (b) $\frac{2}{3}\div\frac{1}{6}$
372. (a) $-\frac{2}{5}-\frac{1}{8}$ (b) $-\frac{2}{5}\cdot\frac{1}{8}$
373. (a) $-\frac{4}{5}-\frac{1}{8}$ (b) $-\frac{4}{5}\cdot\frac{1}{8}$
374. (a) $\frac{5n}{6}\div\frac{8}{15}$ (b) $\frac{5n}{6}-\frac{8}{15}$
375. (a) $\frac{3a}{8}\div\frac{7}{12}$ (b) $\frac{3a}{8}-\frac{7}{12}$
376. (a) $\frac{9}{10}\cdot\left(-\frac{11d}{12}\right)$ (b) $\frac{9}{10}+\left(-\frac{11d}{12}\right)$
377. (a) $\frac{4}{15}\cdot\left(-\frac{5}{q}\right)$ (b) $\frac{4}{15}+\left(-\frac{5}{q}\right)$
378. $-\frac{3}{8}\div\left(-\frac{3}{10}\right)$
379. $-\frac{5}{12}\div\left(-\frac{5}{9}\right)$
380. $-\frac{3}{8}+\frac{5}{12}$
381. $-\frac{1}{8}+\frac{7}{12}$
382. $\frac{5}{6}-\frac{1}{9}$
383. $\frac{5}{9}-\frac{1}{6}$
384. $\frac{3}{8}\cdot\left(-\frac{10}{21}\right)$
385. $\frac{7}{12}\cdot\left(-\frac{8}{35}\right)$
386. $-\frac{7}{15}-\frac{y}{4}$
387. $-\frac{3}{8}-\frac{x}{11}$

388. $\frac{11}{12a} \cdot \frac{9a}{16}$

389. $\frac{10y}{13} \cdot \frac{8}{15y}$

Use the Order of Operations to Simplify Complex Fractions

In the following exercises, simplify.

390. $\frac{\left(\frac{1}{5}\right)^2}{2+3^2}$

391. $\frac{\left(\frac{1}{3}\right)^2}{5+2^2}$

392. $\frac{2^3+4^2}{\left(\frac{2}{3}\right)^2}$

393. $\frac{3^3-3^2}{\left(\frac{3}{4}\right)^2}$

394. $\frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{7}\right)^2}$

395. $\frac{\left(\frac{3}{4}\right)^2}{\left(\frac{5}{8}\right)^2}$

396. $\frac{2}{\frac{1}{3}+\frac{1}{5}}$

397. $\frac{5}{\frac{1}{4}+\frac{1}{3}}$

398. $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{2}{3}}$

399. $\frac{\frac{3}{4}+\frac{1}{2}}{\frac{5}{6}-\frac{2}{3}}$

400. $\frac{\frac{7}{8}-\frac{2}{3}}{\frac{1}{2}+\frac{3}{8}}$

401. $\frac{\frac{3}{4}-\frac{3}{5}}{\frac{1}{4}+\frac{2}{5}}$

Mixed Practice

In the following exercises, simplify.

402. $\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{12}$
403. $\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{4}$
404. $1-\frac{3}{5}\div\frac{1}{10}$
405. $1-\frac{5}{6}\div\frac{1}{12}$
406. $\frac{2}{3}+\frac{1}{6}+\frac{3}{4}$
407. $\frac{2}{3}+\frac{1}{4}+\frac{3}{5}$
408. $\frac{3}{8}-\frac{1}{6}+\frac{3}{4}$
409. $\frac{2}{5}+\frac{5}{8}-\frac{3}{4}$
410. $12\left(\frac{9}{20}-\frac{4}{15}\right)$
411. $8\left(\frac{15}{16}-\frac{5}{6}\right)$
412. $\dfrac{\frac{5}{8}+\frac{1}{6}}{\frac{19}{24}}$
413. $\dfrac{\frac{1}{6}+\frac{3}{10}}{\frac{14}{30}}$
414. $\left(\frac{5}{9}+\frac{1}{6}\right)\div\left(\frac{2}{3}-\frac{1}{2}\right)$
415. $\left(\frac{3}{4}+\frac{1}{6}\right)\div\left(\frac{5}{8}-\frac{1}{3}\right)$

In the following exercises, evaluate the given expression. Express your answers in simplified form, using improper fractions if necessary.

416. $x+\frac{1}{2}$ when
    a. $x=-\frac{1}{8}$
    b. $x=-\frac{1}{2}$
417. $x+\frac{2}{3}$ when
    a. $x=-\frac{1}{6}$
    b. $x=-\frac{5}{3}$
418. $x+\left(-\frac{5}{6}\right)$ when
    a. $x=\frac{1}{3}$
    b. $x=-\frac{1}{6}$
419. $x+\left(-\frac{11}{12}\right)$ when
    a. $x=\frac{11}{12}$

b. $x = \frac{3}{4}$

420. $x - \frac{2}{5}$ when

a. $x = \frac{3}{5}$

b. $x = -\frac{3}{5}$

421. $x - \frac{1}{3}$ when

a. $x = \frac{2}{3}$

b. $x = -\frac{2}{3}$

422. $\frac{7}{10} - w$ when

a. $w = \frac{1}{2}$

b. $w = -\frac{1}{2}$

423. $\frac{5}{12} - w$ when

a. $w = \frac{1}{4}$

b. $w = -\frac{1}{4}$

424. $4p^2q$ when $p = -\frac{1}{2}$ and $q = \frac{5}{9}$

425. $5m^2n$ when $m = -\frac{2}{5}$ and $n = \frac{1}{3}$

426. $2x^2y^3$ when $x = -\frac{2}{3}$ and $y = -\frac{1}{2}$

427. $8u^2v^3$ when $u = -\frac{3}{4}$ and $v = -\frac{1}{2}$

428. $\frac{u+v}{w}$ when $u = -4$, $v = -8$, $w = 2$

429. $\frac{m+n}{p}$ when $m = -6$, $n = -2$, $p = 4$

430. $\frac{a+b}{a-b}$ when $a = -3$, $b = 8$

431. $\frac{r-s}{r+s}$ when $r = 10$, $s = -5$

Everyday Math

432. **Decorating** Laronda is making covers for the throw pillows on her sofa. For each pillow cover, she needs $\frac{3}{16}$ yard of print fabric and $\frac{3}{8}$ yard of solid fabric. What is the total amount of fabric Laronda needs for each pillow cover?

433. **Baking** Vanessa is baking chocolate chip cookies and oatmeal cookies. She needs $1\frac{1}{4}$ cups of sugar for the chocolate chip cookies, and $1\frac{1}{8}$ cups for the oatmeal cookies How much sugar does she need altogether?

Writing Exercises

434. Explain why it is necessary to have a common denominator to add or subtract fractions.

435. Explain how to find the LCD of two fractions.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			
evaluate variable expressions with fractions.			

(b) After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Contributors and Attributions

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1.3.10: Add and Subtract Mixed Numbers (Part 1)

Learning Objectives

- Model addition of mixed numbers with a common denominator
- Add mixed numbers with a common denominator
- Model subtraction of mixed numbers
- Subtract mixed numbers with a common denominator
- Add and subtract mixed numbers with different denominators

be prepared!

Before you get started, take this readiness quiz.

1. Draw figure to model $\frac{7}{3}$. If you missed this problem, review Example 4.1.6.
2. Change $\frac{11}{4}$ to a mixed number. If you missed this problem, review Example 4.1.9.
3. Change $3\frac{1}{2}$ to an improper fraction. If you missed this problem, review Example 4.1.11.

Model Addition of Mixed Numbers with a Common Denominator

So far, we've added and subtracted proper and improper fractions, but not mixed numbers. Let's begin by thinking about addition of mixed numbers using money.

If Ron has 1 dollar and 1 quarter, he has $1\frac{1}{4}$ dollars. If Don has 2 dollars and 1 quarter, he has $2\frac{1}{4}$ dollars. What if Ron and Don put their money together? They would have 3 dollars and 2 quarters. They add the dollars and add the quarters. This makes $3\frac{2}{4}$ dollars. Because two quarters is half a dollar, they would have 3 and a half dollars, or $3\frac{1}{2}$ dollars.

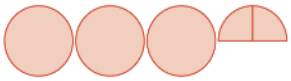
$$\begin{array}{r} 1\frac{1}{4} \\ +2\frac{1}{4} \\ \hline \end{array}$$

$$3\frac{2}{4}=3\frac{1}{2}$$

When you added the dollars and then added the quarters, you were adding the whole numbers and then adding the fractions.

$$1\frac{1}{4}+2\frac{1}{4}$$

We can use fraction circles to model this same example:

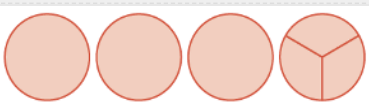
Start with $1\frac{1}{4}$.	one whole and one $\frac{1}{4}$ pieces		$1\frac{1}{4}$
Add $2\frac{1}{4}$ more.	two wholes and one $\frac{1}{4}$ pieces		$+2\frac{1}{4}$
The sum is:	three wholes and two $\frac{1}{4}$'s		$3\frac{2}{4}=3\frac{1}{2}$

Example 1.3.10.1: model

Model $2\frac{1}{3}+1\frac{2}{3}$ and give the sum.

Solution

We will use fraction circles, whole circles for the whole numbers and $\frac{1}{3}$ pieces for the fractions.

two wholes and one $\frac{1}{3}$		$2\frac{1}{3}$
plus one whole and two $\frac{1}{3}$s		$+1\frac{2}{3}$
sum is three wholes and three $\frac{1}{3}$s		$3\frac{3}{3}=4$

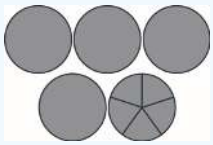
This is the same as 4 wholes. So, $2\frac{1}{3}+1\frac{2}{3}=4$.

Exercise 1.3.10.1

Use a model to add the following. Draw a picture to illustrate your model. $1\frac{2}{5}+3\frac{3}{5}$

Answer

5

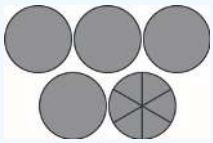


Exercise 1.3.10.2

Use a model to add the following. Draw a picture to illustrate your model. $2\frac{1}{6}+2\frac{5}{6}$

Answer

5



Example 1.3.10.2: model

Model $1\frac{3}{5}+2\frac{3}{5}$ and give the sum as a mixed number.

Solution

We will use fraction circles, whole circles for the whole numbers and $\frac{1}{5}$ pieces for the fractions.

one whole and three $\frac{1}{5}$s		$1\frac{3}{5}$
plus two wholes and three $\frac{1}{5}$s		$+2\frac{3}{5}$
sum is three wholes and six $\frac{1}{5}$s		$3\frac{6}{5}=4\frac{1}{5}$

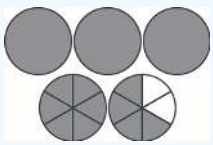
Adding the whole circles and fifth pieces, we got a sum of $3\frac{6}{5}$. We can see that $\frac{6}{5}$ is equivalent to $1\frac{1}{5}$, so we add that to the 3 to get $4\frac{1}{5}$.

Exercise 1.3.10.3

Model, and give the sum as a mixed number. Draw a picture to illustrate your model. $2\frac{5}{6}+1\frac{5}{6}$

Answer

$4\frac{2}{3}$

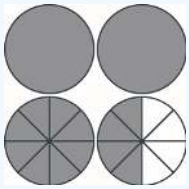


Exercise 1.3.10.4

Model, and give the sum as a mixed number. Draw a picture to illustrate your model. $1\frac{5}{8}+1\frac{7}{8}$

Answer

$3\frac{1}{2}$



Add Mixed Numbers

Modeling with fraction circles helps illustrate the process for adding mixed numbers: We add the whole numbers and add the fractions, and then we simplify the result, if possible.

HOW TO: ADD MIXED NUMBERS WITH A COMMON DENOMINATOR

Step 1. Add the whole numbers.

Step 2. Add the fractions.

Step 3. Simplify, if possible.

Example 1.3.10.3: add

Add: $3\frac{4}{9}+2\frac{2}{9}$.

Solution

Add the whole numbers.	$\begin{array}{r} 3\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 5 \end{array}$
Add the fractions.	$\begin{array}{r} 3\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 5\frac{6}{9} \end{array}$
Simplify the fraction.	$\begin{array}{r} 3\frac{4}{9} \\ +2\frac{2}{9} \\ \hline 5\frac{6}{9}=5\frac{2}{3} \end{array}$

Exercise 1.3.10.5

Find the sum: $4\frac{4}{7}+1\frac{2}{7}$.

Answer

$5\frac{6}{7}$

Exercise 1.3.10.6

Find the sum: $2\frac{3}{11}+5\frac{6}{11}$.

Answer

$7\frac{9}{11}$

In Example 1.3.10.3 the sum of the fractions was a proper fraction. Now we will work through an example where the sum is an improper fraction.

Example 1.3.10.4: add

Find the sum: $9\frac{5}{9}+5\frac{7}{9}$.

Solution

Add the whole numbers and then add the fractions.	$\begin{array}{r} 9\frac{5}{9} \\ +5\frac{7}{9} \\ \hline 14\frac{12}{9} \end{array}$
Rewrite $\frac{12}{9}$ as an improper fraction.	$14+1\frac{3}{9}$
Add.	$15\frac{3}{9}$
Simplify.	$15\frac{1}{3}$

Exercise 1.3.10.7

Find the sum: $8\frac{7}{8}+7\frac{5}{8}$.

Answer

$16\frac{1}{2}$

Exercise 1.3.10.8

Find the sum: $6\frac{7}{9}+8\frac{5}{9}$.

Answer

$15\frac{1}{3}$

An alternate method for adding mixed numbers is to convert the mixed numbers to improper fractions and then add the improper fractions. This method is usually written horizontally.

Example 1.3.10.5: add

Add by converting the mixed numbers to improper fractions: $3\frac{7}{8}+4\frac{3}{8}$.

Solution

Convert to improper fractions.	$\frac{31}{8}+\frac{35}{8}$
Add the fractions.	$\frac{31+35}{8}$
Simplify the numerator.	$\frac{66}{8}$
Rewrite as a mixed number.	$8\frac{2}{8}$
Simplify the fraction.	$8\frac{1}{4}$

Since the problem was given in mixed number form, we will write the sum as a mixed number.

Exercise 1.3.10.9

Find the sum by converting the mixed numbers to improper fractions: $5\frac{5}{9}+3\frac{7}{9}$

Answer

$9\frac{1}{3}$

Exercise 1.3.10.10

Find the sum by converting the mixed numbers to improper fractions: $3\frac{7}{10}+2\frac{9}{10}$

Answer

$6\frac{3}{5}$

Table 1.3.10.1 compares the two methods of addition, using the expression $3\frac{2}{5}+6\frac{4}{5}$ as an example. Which way do you prefer?

Table 1.3.10.1

Mixed Numbers	Improper Fractions
$3\frac{2}{5}$ $+6\frac{4}{5}$ $9\frac{6}{5}$	$3\frac{2}{5}+6\frac{4}{5}$
$9+\frac{6}{5}$	$\frac{17}{5}+\frac{34}{5}$
$9+1\frac{1}{5}$	$\frac{51}{5}$
$10\frac{1}{5}$	$10\frac{1}{5}$

Model Subtraction of Mixed Numbers

Let's think of pizzas again to model subtraction of mixed numbers with a common denominator. Suppose you just baked a whole pizza and want to give your brother half of the pizza. What do you have to do to the pizza to give him half? You have to cut it into at least two pieces. Then you can give him half.

We will use fraction circles (pizzas!) to help us visualize the process. Start with one whole.

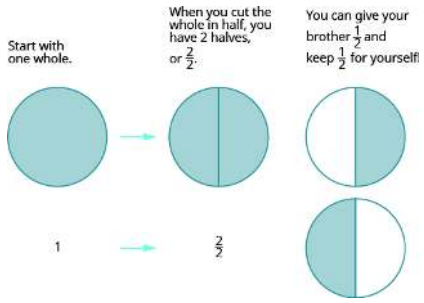


Figure 1.3.10.1

Algebraically, you would write:

$$\begin{array}{r} 1 \\ -\frac{1}{2} \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} \frac{2}{2} \\ -\frac{1}{2} \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} \frac{2}{2} \\ -\frac{1}{2} \\ \hline \frac{1}{2} \end{array}$$

Example 1.3.10.6: subtract

Use a model to subtract: $1-\frac{1}{3}$.

Solution

	Model	Math Notation
Rewrite vertically. Start with one whole.		$\begin{array}{r} 1 \\ -\frac{1}{3} \\ \hline \end{array}$
Since $\frac{1}{3}$ has denominator 3, cut the whole into 3 pieces. The 1 whole becomes $\frac{3}{3}$.		$\begin{array}{r} \frac{3}{3} \\ -\frac{1}{3} \\ \hline \end{array}$
Take away $\frac{1}{3}$. There are $\frac{2}{3}$ left.		$\begin{array}{r} \frac{3}{3} \\ -\frac{1}{3} \\ \hline \frac{2}{3} \end{array}$

Exercise 1.3.10.11

Use a model to subtract: $1-\frac{1}{4}$.

Answer

$\frac{3}{4}$

Exercise 1.3.10.12

Use a model to subtract: $1 - \frac{1}{5}$.

Answer

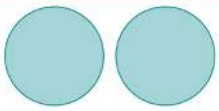
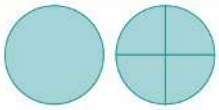
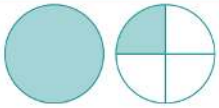
$\frac{4}{5}$

What if we start with more than one whole? Let's find out.

Example1.3.10.7: subtract

Use a model to subtract: $2 - \frac{3}{4}$.

Solution

	Model	Math Notation
Rewrite vertically. Start with two wholes.		$\begin{array}{r} 2 \\ -\frac{3}{4} \\ \hline \end{array}$
Since $\frac{3}{4}$ has denominator 4, cut one of the wholes into 4 pieces. You have 1 whole and $\frac{4}{4}$.		$\begin{array}{r} 1\frac{4}{4} \\ -\frac{3}{4} \\ \hline \end{array}$
Take away $\frac{3}{4}$. There is $1\frac{1}{4}$ left.		$\begin{array}{r} 1\frac{4}{4} \\ -\frac{3}{4} \\ \hline 1\frac{1}{4} \end{array}$

Exercise 1.3.10.13

Use a model to subtract: $2 - \frac{1}{5}$.

Answer

$\frac{9}{5}$

Exercise 1.3.10.14

Use a model to subtract: $2 - \frac{1}{3}$.

Answer

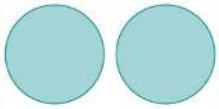
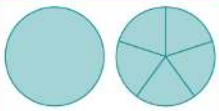
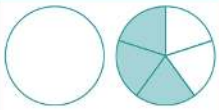
$\frac{5}{3}$

In the next example, we'll subtract more than one whole.

Example 1.3.10.8: subtract

Use a model to subtract: $2 - 1\frac{2}{5}$.

Solution

	Model	Math Notation
Rewrite vertically. Start with two wholes.		$\begin{array}{r} 2 \\ -1\frac{2}{5} \\ \hline \end{array}$
Since $\frac{2}{5}$ has denominator 5, cut one of the wholes into 5 pieces. You have 1 whole and $\frac{5}{5}$.		$\begin{array}{r} 1\frac{5}{5} \\ -1\frac{2}{5} \\ \hline \end{array}$
Take away $1\frac{2}{5}$. There is $\frac{3}{5}$ left.		$\begin{array}{r} 1\frac{5}{5} \\ -1\frac{2}{5} \\ \hline \frac{3}{5} \end{array}$

Exercise 1.3.10.15

Use a model to subtract: $2-1\frac{1}{3}$.

Answer

$\frac{2}{3}$

Exercise 1.3.10.16

Use a model to subtract: $2-1\frac{1}{4}$.

Answer

$\frac{3}{4}$

What if you start with a mixed number and need to subtract a fraction? Think about this situation: You need to put three quarters in a parking meter, but you have only a $1 bill and one quarter. What could you do? You could change the dollar bill into 4 quarters. The value of 4 quarters is the same as one dollar bill, but the 4 quarters are more useful for the parking meter. Now, instead of having a $1 bill and one quarter, you have 5 quarters and can put 3 quarters in the meter.

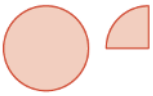
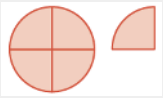
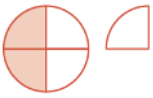
This models what happens when we subtract a fraction from a mixed number. We subtracted three quarters from one dollar and one quarter.

We can also model this using fraction circles, much like we did for addition of mixed numbers.

Example 1.3.10.9: subtract

Use a model to subtract: $1\frac{1}{4}-\frac{3}{4}$

Solution

Rewrite vertically. Start with one whole and one fourth.		$\begin{array}{r} 1\frac{1}{4} \\ -\frac{3}{4} \\ \hline \end{array}$
Since the fractions have denominator 4, cut the whole into 4 pieces. You now have $\frac{4}{4}$ and $\frac{1}{4}$ which is $\frac{5}{4}$.		$\begin{array}{r} \frac{5}{4} \\ -\frac{3}{4} \\ \hline \end{array}$
Take away $\frac{3}{4}$. There is $\frac{1}{2}$ left.		\begin{split} & \dfrac{5}{4}\\\textcolor{red}{- & \dfrac{3}{4}}\\ \hline \\

Exercise 1.3.10.17

Use a model to subtract. Draw a picture to illustrate your model. $1\frac{1}{3}-\frac{2}{3}$

Answer



Exercise 1.3.10.18

Use a model to subtract. Draw a picture to illustrate your model. $1\frac{1}{5}-\frac{4}{5}$

Answer



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1.3.11: Add and Subtract Mixed Numbers (Part 2)

Subtract Mixed Numbers with a Common Denominator

Now we will subtract mixed numbers without using a model. But it may help to picture the model in your mind as you read the steps.

HOW TO: Subtract mixed numbers with common denominators

Step 1. Rewrite the problem in vertical form.

Step 2. Compare the two fractions.

- If the top fraction is larger than the bottom fraction, go to Step 3.
- If not, in the top mixed number, take one whole and add it to the fraction part, making a mixed number with an improper fraction.

Step 3. Subtract the fractions.

Step 4. Subtract the whole numbers.

Step 5. Simplify, if possible.

Example 1.3.11.10: subtract

Find the difference: $5\frac{3}{5}-2\frac{4}{5}$.

Solution

Rewrite the problem in vertical form.	$5\frac{3}{5}$ $-2\frac{4}{5}$
Since $\frac{3}{5}$ is less than $\frac{4}{5}$, take 1 from the 5 and add it to the $\frac{3}{5}$: $\left(\frac{5}{5}+\frac{3}{5}=\frac{8}{5}\right)$	$5\frac{3}{5} \rightarrow 4\frac{8}{5}$ $-2\frac{4}{5} \quad -2\frac{4}{5}$
Subtract the fractions.	$4\frac{8}{5}$ $-2\frac{4}{5}$ $\frac{4}{5}$
Subtract the whole parts. The result is in simplest form.	$4\frac{8}{5}$ $-2\frac{4}{5}$ $2\frac{4}{5}$

Since the problem was given with mixed numbers, we leave the result as mixed numbers.

Exercise 1.3.11.19

Find the difference: $6\frac{4}{9}-3\frac{7}{9}$.

Answer

$2\frac{2}{3}$

Exercise 1.3.11.20

Find the difference: $4\frac{4}{7}-2\frac{6}{7}$.

Answer

$1\frac{5}{7}$

Just as we did with addition, we could subtract mixed numbers by converting them first to improper fractions. We should write the answer in the form it was given, so if we are given mixed numbers to subtract we will write the answer as a mixed number.

HOW TO: SUBTRACT MIXED NUMBERS WITH COMMON DENOMINATORS AS IMPROPER FRACTIONS

Step 1. Rewrite the mixed numbers as improper fractions.

Step 2. Subtract the numerators.

Step 3. Write the answer as a mixed number, simplifying the fraction part, if possible.

Example 1.3.11.11: subtract

Find the difference by converting to improper fractions: $9\frac{6}{11}-7\frac{10}{11}$.

Solution

Rewrite as improper fractions.	$\frac{105}{11}-\frac{87}{11}$
Subtract the numerators.	$\frac{18}{11}$
Rewrite as a mixed number.	$1\frac{7}{11}$

Exercise 1.3.11.21

Find the difference by converting to improper fractions: $6\frac{4}{9}-3\frac{7}{9}$.

Answer

$2\frac{2}{3}$

Exercise 1.3.11.22

Find the difference by converting to improper fractions: $4\frac{4}{7}-2\frac{6}{7}$.

Answer

$1\frac{5}{7}$

Add and Subtract Mixed Numbers with Different Denominators

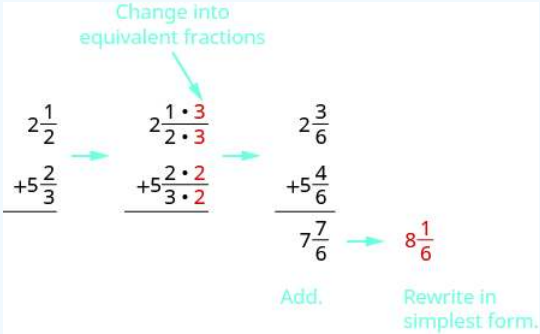
To add or subtract mixed numbers with different denominators, we first convert the fractions to equivalent fractions with the LCD. Then we can follow all the steps we used above for adding or subtracting fractions with like denominators.

Example 1.3.11.12: add

Add: $2\frac{1}{2}+5\frac{2}{3}$.

Solution

Since the denominators are different, we rewrite the fractions as equivalent fractions with the LCD, 6. Then we will add and simplify.



We write the answer as a mixed number because we were given mixed numbers in the problem.

Exercise 1.3.11.23

Add: $1\frac{5}{6}+4\frac{3}{4}$.

Answer

$6\frac{7}{12}$

Exercise 1.3.11.24

Add: $3\frac{4}{5}+8\frac{1}{2}$.

Answer

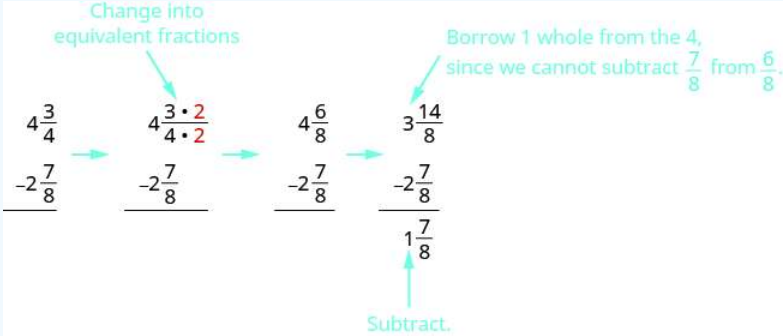
$12\frac{3}{10}$

Example 1.3.11.13: subtract

Subtract: $4\frac{3}{4}-2\frac{7}{8}$.

Solution

Since the denominators of the fractions are different, we will rewrite them as equivalent fractions with the LCD 8. Once in that form, we will subtract. But we will need to borrow 1 first.



We were given mixed numbers, so we leave the answer as a mixed number.

Exercise 1.3.11.25

Find the difference: $8\frac{1}{2}-3\frac{4}{5}$.

Answer

$4\frac{7}{10}$

Exercise 1.3.11.26

Find the difference: $4\frac{3}{4}-1\frac{5}{6}$.

Answer

$2\frac{11}{12}$

Example 1.3.11.14:

Subtract: $3\frac{5}{11}-4\frac{3}{4}$.

Solution

We can see the answer will be negative since we are subtracting 4 from 3. Generally, when we know the answer will be negative it is easier to subtract with improper fractions rather than mixed numbers.

Change to equivalent fractions with the LCD.	$3\frac{5\cdot 4}{11\cdot 4}-4\frac{3\cdot 11}{4\cdot 11}$ $3\frac{20}{44}-4\frac{33}{44}$
Rewrite as improper fractions.	$\frac{152}{44}-\frac{209}{44}$
Subtract.	$-\frac{57}{44}$

Rewrite as a mixed number.	$-1\frac{13}{44}$

Exercise 1.3.11.27

Subtract: $1\frac{3}{4}-6\frac{7}{8}$.

Answer

$-\frac{41}{8}$

Exercise 1.3.11.28

Subtract: $10\frac{3}{7}-22\frac{4}{9}$.

Answer

$-\frac{757}{63}$

Access Additional Online Resources

- Adding Mixed Numbers
- Subtracting Mixed Numbers

Practice Makes Perfect

Model Addition of Mixed Numbers

In the following exercises, use a model to find the sum. Draw a picture to illustrate your model.

436. $1\frac{1}{5}+3\frac{1}{5}$
437. $2\frac{1}{3}+1\frac{1}{3}$
438. $1\frac{3}{8}+1\frac{7}{8}$
439. $1\frac{5}{6}+1\frac{5}{6}$

Add Mixed Numbers with a Common Denominator

In the following exercises, add.

440. $5\frac{1}{3}+6\frac{1}{3}$
441. $2\frac{4}{9}+5\frac{1}{9}$
442. $4\frac{5}{8}+9\frac{3}{8}$
443. $7\frac{9}{10}+3\frac{1}{10}$
444. $3\frac{4}{5}+6\frac{4}{5}$
445. $9\frac{2}{3}+1\frac{2}{3}$
446. $6\frac{9}{10}+8\frac{3}{10}$

447. $8\frac{4}{9}+2\frac{8}{9}$

Model Subtraction of Mixed Numbers

In the following exercises, use a model to find the difference. Draw a picture to illustrate your model.

448. $1\frac{1}{6}-\frac{1}{6}$
449. $1\frac{1}{8}-\frac{1}{8}$

Subtract Mixed Numbers with a Common Denominator

In the following exercises, find the difference.

450. $2\frac{7}{8}-1\frac{3}{8}$
451. $2\frac{7}{12}-1\frac{5}{12}$
452. $8\frac{3}{7}-4\frac{4}{7}$
453. $19\frac{13}{15}-13\frac{7}{15}$
454. $8\frac{3}{7}-4\frac{4}{7}$
455. $5\frac{2}{9}-3\frac{4}{9}$
456. $2\frac{5}{8}-1\frac{7}{8}$
457. $2\frac{5}{12}-1\frac{7}{12}$

Add and Subtract Mixed Numbers with Different Denominators

In the following exercises, write the sum or difference as a mixed number in simplified form.

458. $3\frac{1}{4}+6\frac{1}{3}$
459. $2\frac{1}{6}+5\frac{3}{4}$
460. $1\frac{5}{8}+4\frac{1}{2}$
461. $7\frac{2}{3}+8\frac{1}{2}$
462. $2\frac{5}{12}-1\frac{7}{12}$
463. $6\frac{4}{5}-1\frac{1}{4}$
464. $2\frac{2}{3}-3\frac{1}{2}$
465. $2\frac{7}{8}-4\frac{1}{3}$

Mixed Practice

In the following exercises, perform the indicated operation and write the result as a mixed number in simplified form.

466. $2\frac{5}{8}\cdot 1\frac{3}{4}$
467. $1\frac{2}{3}\cdot 4\frac{1}{6}$
468. $\frac{2}{7}+\frac{4}{7}$
469. $\frac{2}{9}+\frac{5}{9}$

470. $1\frac{5}{12} \div \frac{1}{12}$
471. $2\frac{3}{10} \div \frac{1}{10}$
472. $13\frac{5}{12} - 9\frac{7}{12}$
473. $15\frac{5}{8} - 6\frac{7}{8}$
474. $\frac{5}{9} - \frac{4}{9}$
475. $\frac{11}{15} - \frac{7}{15}$
476. $4 - \frac{3}{4}$
477. $6 - \frac{2}{5}$
478. $\frac{9}{20} \div \frac{3}{4}$
479. $\frac{7}{24} \div \frac{14}{3}$
480. $9\frac{6}{11} + 7\frac{10}{11}$
481. $8\frac{5}{13} + 4\frac{9}{13}$
482. $3\frac{2}{5} + 5\frac{3}{4}$
483. $2\frac{5}{6} + 4\frac{1}{5}$
484. $\frac{8}{15} \cdot \frac{10}{19}$
485. $\frac{5}{12} \cdot \frac{8}{9}$
486. $6\frac{7}{8} - 2\frac{1}{3}$
487. $6\frac{5}{9} - 4\frac{2}{5}$
488. $5\frac{2}{9} - 4\frac{4}{5}$
489. $4\frac{3}{8} - 3\frac{2}{3}$

Everyday Math

490. **Sewing** Renata is sewing matching shirts for her husband and son. According to the patterns she will use, she needs $2\frac{3}{8}$ yards of fabric for her husband's shirt and $1\frac{1}{8}$ yards of fabric for her son's shirt. How much fabric does she need to make both shirts?
491. **Sewing** Pauline has $3\frac{1}{4}$ yards of fabric to make a jacket. The jacket uses $2\frac{2}{3}$ yards. How much fabric will she have left after making the jacket?
492. **Printing** Nishant is printing invitations on his computer. The paper is $8\frac{1}{2}$ inches wide, and he sets the print area to have a $1\frac{1}{2}$-inch border on each side. How wide is the print area on the sheet of paper?
493. **Framing a picture** Tessa bought a picture frame for her son's graduation picture. The picture is 8 inches wide. The picture frame is $2\frac{5}{8}$ inches wide on each side. How wide will the framed picture be?

Writing Exercises

494. Draw a diagram and use it to explain how to add $1\frac{5}{8} + 2\frac{7}{8}$.
495. Edgar will have to pay $3.75 in tolls to drive to the city.

a. Explain how he can make change from a $10 bill before he leaves so that he has the exact amount he needs.

b. How is Edgar's situation similar to how you subtract $10 - 3\frac{3}{4}$?

496. Add $4\frac{5}{12} + 3\frac{7}{8}$ twice, first by leaving them as mixed numbers and then by rewriting as improper fractions. Which method do you prefer, and why?

497. Subtract $3\frac{7}{8} - 4\frac{5}{12}$ twice, first by leaving them as mixed numbers and then by rewriting as improper fractions. Which method do you prefer, and why?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model addition of mixed numbers common with a denominator.			
add mixed numbers with a common denominator.			
model subtraction of mixed numbers.			
subtract mixed numbers with a common denominator.			
add and subtract mixed numbers with different denominators.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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1.3.12: Solve Equations with Fractions (Part 1)

Learning Objectives

- Determine whether a fraction is a solution of an equation
- Solve equations with fractions using the Addition, Subtraction, and Division Properties of Equality
- Solve equations using the Multiplication Property of Equality
- Translate sentences to equations and solve

be prepared!

Before you get started, take this readiness quiz. If you miss a problem, go back to the section listed and review the material.

1. Evaluate $x+4$ when $x=-3$ If you missed this problem, review Example 3.2.10.
2. Solve: $2y-3=9$. If you missed this problem, review Example 3.5.2.
3. Solve: $y-3=-9$ If you missed this problem, review Example 4.2.10.

Determine Whether a Fraction is a Solution of an Equation

As we saw in Solve Equations with the Subtraction and Addition Properties of Equality and Solve Equations Using Integers; The Division Property of Equality, a solution of an equation is a value that makes a true statement when substituted for the variable in the equation. In those sections, we found whole number and integer solutions to equations. Now that we have worked with fractions, we are ready to find fraction solutions to equations.

The steps we take to determine whether a number is a solution to an equation are the same whether the solution is a whole number, an integer, or a fraction.

HOW TO: DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

Step 1. Substitute the number for the variable in the equation.

Step 2. Simplify the expressions on both sides of the equation.

Step 3. Determine whether the resulting equation is true. If it is true, the number is a solution. If it is not true, the number is not a solution.

Example 1.3.12.1: find solution

Determine whether each of the following is a solution of $x-\frac{3}{10}=\frac{1}{2}$.

a. $x=1$

b. $x=\frac{4}{5}$

c. $x=-\frac{4}{5}$

Solution

a.

Substitute 1 for x.	$1-\frac{3}{10}\stackrel{?}{=}\frac{1}{2}$
Change to fractions with a LCD of 10.	$\frac{10}{10}-\frac{3}{10}\stackrel{?}{=}\frac{5}{10}$
Subtract.	$\frac{7}{10}\stackrel{?}{=}\frac{5}{10}$

Since $x=1$ does not result in a true equation, 1 is not a solution to the equation.

b.

Substitute $\color{red}{\frac{4}{5}}$ for x.	$\color{red}{\frac{4}{5}} - \frac{3}{10} \stackrel{?}{=} \frac{1}{2}$
	$\color{red}{\frac{8}{10}} - \frac{3}{10} \stackrel{?}{=} \frac{5}{10}$
Subtract.	$\frac{5}{10} = \frac{5}{10}$ ✓

Since $x = \frac{4}{5}$ results in a true equation, $\frac{4}{5}$ is a solution to the equation $x - \frac{3}{10} = \frac{1}{2}$.

c.

Substitute $\color{red}{-\frac{4}{5}}$ for x.	$\color{red}{-\frac{4}{5}} - \frac{3}{10} \stackrel{?}{=} \frac{1}{2}$
	$\color{red}{-\frac{8}{10}} - \frac{3}{10} \stackrel{?}{=} \frac{5}{10}$
Subtract.	$\frac{11}{10} \neq \frac{5}{10}$

Since $x = -\frac{4}{5}$ does not result in a true equation, $-\frac{4}{5}$ is not a solution to the equation.

Exercise 1.3.12.1

Determine whether each number is a solution of the given equation. $x - \frac{2}{3} = \frac{1}{6}$

a. $x = 1$
b. $x = \frac{5}{6}$
c. $x = -\frac{5}{6}$

Answer a

no

Answer b

yes

Answer c

no

Exercise 1.3.12.2

Determine whether each number is a solution of the given equation. $y - \frac{1}{4} = \frac{3}{8}$

a. $y = 1$
b. $y = -\frac{5}{8}$
c. $y = \frac{5}{8}$

Answer a

no

Answer b

no

Answer c

yes

Solve Equations with Fractions using the Addition, Subtraction, and Division Properties of Equality

In Solve Equations with the Subtraction and Addition Properties of Equality and Solve Equations Using Integers; The Division Property of Equality, we solved equations using the Addition, Subtraction, and Division Properties of Equality. We will use these same properties to solve equations with fractions.

Addition, Subtraction, and Division Properties of Equality

For any numbers a, b, and c,

Table 1.3.12.1

if $a = b$, then $a + c = b + c$.	Addition Property of Equality
if $a = b$, then $a - c = b - c$.	Subtraction Property of Equality
if $a = b$, then $ac = bc$, $c \neq 0$.	Division Property of Equality

In other words, when you add or subtract the same quantity from both sides of an equation, or divide both sides by the same quantity, you still have equality.

Example 1.3.12.2: solve

Solve: $y + \frac{9}{16} = \frac{5}{16}$.

Solution

Subtract $\frac{9}{16}$ from each side to undo the addition.	$y + \frac{9}{16} - \frac{9}{16} = \frac{5}{16} - \frac{9}{16}$
Simplify on each side of the equation.	$y + 0 = -\frac{4}{16}$
Simplify the fraction.	$y = -\frac{1}{4}$

Check:

Substitute y = (− \dfrac{1}{4}\).	$-\frac{1}{4} + \frac{9}{16} \stackrel{?}{=} \frac{5}{16}$
Rewrite as fractions with the LCD.	\textcolor{red}{- \dfrac{4}{16}} + \dfrac{9}{16} \stackrel{?}{=} \dfrac{5}{16}\)
Add.	$\frac{5}{16} \stackrel{?}{=} \frac{5}{16}$ ✓

Since $y = -\frac{1}{4}$ makes $y + \frac{9}{16} = \frac{5}{16}$ a true statement, we know we have found the solution to this equation.

Exercise 1.3.12.3

Solve: $y + \frac{11}{12} = \frac{5}{12}$.

Answer

$-\frac{1}{2}$

Exercise 1.3.12.4

Solve: $y+\frac{8}{15}=\frac{4}{15}$.

Answer

$-\frac{4}{15}$

We used the Subtraction Property of Equality in Example 1.3.12.2 Now we'll use the Addition Property of Equality.

Example 1.3.12.3: solve

Solve: a $-\frac{5}{9}=-\frac{8}{9}$.

Solution

Add $\frac{5}{9}$ from each side to undo the addition.	$a-\frac{5}{9}+\frac{5}{9}=-\frac{8}{9}+\frac{5}{9}$
Simplify on each side of the equation.	$a+0=-\frac{3}{9}$
Simplify the fraction.	$a=-\frac{1}{3}$

Check:

Substitute a = $-\frac{1}{3}$.	$-\frac{1}{3}-\frac{5}{9}\stackrel{?}{=}-\frac{8}{9}$
Change to common denominator.	$-\frac{3}{9}-\frac{5}{9}\stackrel{?}{=}-\frac{8}{9}$
Subtract.	$-\frac{8}{9}=-\frac{8}{9}$ ✓

Since $a=-\frac{1}{3}$ makes the equation true, we know that $a=-\frac{1}{3}$ is the solution to the equation.

Exercise 1.3.12.5

Solve: $a-\frac{3}{5}=-\frac{8}{5}$.

Answer

-1

Exercise 1.3.12.6

Solve: $n-\frac{3}{7}=-\frac{9}{7}$.

Answer

$-\frac{6}{7}$

The next example may not seem to have a fraction, but let's see what happens when we solve it.

Example 1.3.12.4: solve

Solve: $10q = 44$.

Solution

Divide both sides by 10 to undo the multiplication.	$\frac{10q}{10} = \frac{44}{10}$
Simplify.	$q = \frac{22}{5}$

Check:

Substitute $q = \frac{22}{5}$ into the original equation.	$10\left(\frac{22}{5}\right) \stackrel{?}{=} 44$
Simplify.	$\cancel{10}^{2}\left(\frac{22}{\cancel{5}}\right) \stackrel{?}{=} 44$
Multiply.	$44 = 44$ ✓

The solution to the equation was the fraction $\frac{22}{5}$. We leave it as an improper fraction.

Exercise 1.3.12.7

Solve: $12u = -76$.

Answer

$-\frac{19}{3}$

Exercise 1.3.12.8

Solve: $8m = 92$.

Answer

$\frac{23}{2}$

Solve Equations with Fractions Using the Multiplication Property of Equality

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to "undo" the division, we will need to multiply by 4. The *Multiplication Property of Equality* will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

Definition: The Multiplication Property of Equality

For any numbers a, b, and c, if $a = b$, then $ac = bc$. If you multiply both sides of an equation by the same quantity, you still have equality.

Let's use the Multiplication Property of Equality to solve the equation $\frac{x}{7} = -9$.

Example 1.3.12.5: solve

Solve: $\frac{x}{7} = -9$.

Solution

Use the Multiplication Property of Equality to multiply both sides by 7 . This will isolate the variable.	$7 \cdot \frac{x}{7} = 7(-9)$
Multiply.	$\frac{7x}{7} = -63$
Simplify.	$x = -63$
Check. Substitute -63 for x in the original equation.	$\frac{-63}{7} \stackrel{?}{=} -9$
The equation is true.	$-9 = -9$; ✓

Exercise 1.3.12.9

Solve: $\frac{f}{5} = -25$.

Answer

-125

Exercise 1.3.12.10

Solve: $\frac{h}{9} = -27$.

Answer

-243

Example 1.3.12.6:solve

Solve: $\frac{p}{-8} = -40$.

Solution

Here, p is divided by -8. We must multiply by -8 to isolate p.

Multiply both sides by −8.	$-8\left(\frac{p}{-8}\right) = -8(-40)$
Multiply.	$\frac{-8p}{-8} = 320$
Simplify.	$p = 320$

Check:

Substitute p = 320.	$\frac{320}{-8} \stackrel{?}{=} -40$
The equation is true.	$-40 = -40$ ✓

Exercise 1.3.12.11

Solve: $\frac{c}{-7} = -35$.

Answer

245

Exercise 1.3.12.12

Solve: $\frac{x}{-11} = -12$.

Answer

132

Solve Equations with a Coefficient of -1

Look at the equation $-y = 15$. Does it look as if y is already isolated? But there is a negative sign in front of y, so it is not isolated.

There are three different ways to isolate the variable in this type of equation. We will show all three ways in Example 1.3.12.7.

Example 1.3.12.7: solve

Solve: $-y = 15$.

Solution

One way to solve the equation is to rewrite $-y$ as $-1y$, and then use the Division Property of Equality to isolate y.

Rewrite −y as −1y.	$-1y = 15$
Divide both sides by −1.	$\frac{-1y}{-1} = \frac{15}{-1}$
Simplify each side.	$y = -15$

Another way to solve this equation is to multiply both sides of the equation by −1.

Multiply both sides by −1.	$-1(-y) = -1(15)$
Simplify each side.	$y = -15$

The third way to solve the equation is to read $-y$ as "the opposite of y." What number has 15 as its opposite? The opposite of 15 is -15. So $y = -15$.

For all three methods, we isolated y is isolated and solved the equation.

Check:

Substitute y = −15.	$-(15) \stackrel{?}{=} (15)$
Simplify. The equation is true.	$15 = 15$ ✓

Exercise 1.3.12.13

Solve: $-y = 48$.

Answer

-48

Exercise 1.3.12.14

Solve: $-c = -23$.

Answer

23

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1.3.13: Solve Equations with Fractions (Part 2)

Solve Equations with a Fraction Coefficient

When we have an equation with a fraction coefficient we can use the Multiplication Property of Equality to make the coefficient equal to 1. For example, in the equation:

$$\frac{3}{4}x = 24$$

The coefficient of x is $\frac{3}{4}$. To solve for x, we need its coefficient to be 1. Since the product of a number and its reciprocal is 1, our strategy here will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$. We will do this in Example 1.3.13.1.

Example 1.3.13.8: solve

Solve: $\frac{3}{4}x = 24$.

Solution

Multiply both sides by the reciprocal of the coefficient.	$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 24$
Simplify.	$1x = \frac{4}{3} \cdot \frac{24}{1}$
Multiply.	$x = 32$

Check:

Substitute x = 32.	$\frac{3}{4} \cdot 32 \stackrel{?}{=} 24$
Rewrite 32 as a fraction.	$\frac{3}{4} \cdot \frac{32}{1} \stackrel{?}{=} 24$
Multiply. The equation is true.	$24 = 24$ ✓

Notice that in the equation $\frac{3}{4}x = 24$, we could have divided both sides by $\frac{3}{4}$ to get x by itself. Dividing is the same as multiplying by the reciprocal, so we would get the same result. But most people agree that multiplying by the reciprocal is easier.

Exercise 1.3.13.15

Solve: $\frac{2}{5}n = 14$.

Answer

35

Exercise 1.3.13.16

Solve: $\frac{5}{6}y = 15$.

Answer

18

Example 1.3.13.9: solve

Solve: $-\frac{3}{8}w = 72$.

Solution

The coefficient is a negative fraction. Remember that a number and its reciprocal have the same sign, so the reciprocal of the coefficient must also be negative.

Multiply both sides by the reciprocal of $-\frac{3}{8}$.	$-\frac{8}{3}\left(-\frac{3}{8}w\right) = \left(-\frac{8}{3}\right)72$
Simplify; reciprocals multiply to one.	$1w = -\frac{8}{3}\cdot\frac{72}{1}$
Multiply.	$w = -192$

Check:

Let w = −192.	$-\frac{3}{8}(-192) \stackrel{?}{=} 72$
Multiply. It checks.	$72 = 72$ ✓

Exercise 1.3.13.17

Solve: $-\frac{4}{7}a = 52$.

Answer

-91

Exercise 1.3.13.18

Solve: $-\frac{7}{9}w = 84$.

Answer

-108

Translate Sentences to Equations and Solve

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

Table 1.3.13.2

Subtraction Property of Equality: For any real numbers a, b, and c, if a = b, then a − c = b − c.	**Addition Property of Equality**: For any real numbers a, b, and c, if a = b, then a + c = b + c.
Division Property of Equality: For any numbers a, b, and c, where c ≠ 0 if a = b, then $\frac{a}{c} = \frac{b}{c}$.	**Multiplication Property of Equality**: For any real numbers a, b, and c if a = b, then ac = bc.

When you add, subtract, multiply or divide the same quantity from both sides of an equation, you still have equality. In the next few examples, we'll translate sentences into equations and then solve the equations. It might be helpful to review the translation table in Evaluate, Simplify, and Translate Expressions.

Example 1.3.13.10: solve

Translate and solve: n divided by 6 is -24.

Solution

Translate.	$\underbrace{\frac{n}{6}}_{n \text{ divided by } 6} \underbrace{=}_{\text{is}} \underbrace{-24}_{24}$
Multiply both sides by 6.	$6 \cdot \frac{n}{6} = 6(-24)$
Simplify.	$n = -144$
Check:	Is −144 divided by 6 equal to −24?
Translate.	$\frac{-144}{6} \stackrel{?}{=} -24$
Simplify. It checks.	$-24 = -24$ ✓

Exercise 1.3.13.19

Translate and solve: n divided by 7 is equal to -21.

Answer

$\frac{n}{7} = -21$; $n = -147$

Exercise 1.3.13.20

Translate and solve: n divided by 8 is equal to -56.

Answer

$\frac{n}{8} = -56$; $n = -448$

Example 1.3.13.11: solve

Translate and solve: The quotient of q and -5 is 70.

Solution

Translate.	$\underbrace{\frac{q}{-5}}_{\text{The quotient of } q \text{ and } -5} \underbrace{=}_{\text{is}} \underbrace{70}_{70}$
Multiply both sides by −5.	$-5\left(\frac{q}{-5}\right) = -5(70)$
Simplify.	$q = -350$
Check:	Is the quotient of −350 and −5 equal to 70?
Translate.	$\frac{-350}{-5} \stackrel{?}{=} 70$
Simplify. It checks.	$70 = 70$ ✓

Exercise 1.3.13.21

Translate and solve: The quotient of q and -8 is 72.

Answer

$\frac{q}{-8} = 72; q = -576$

Exercise 1.3.13.22

Translate and solve: The quotient of p and -9 is 81.

Answer

$\frac{p}{-9} = 81; p = -729$

Example 1.3.13.12: solve

Translate and solve: Two-thirds of f is 18.

Solution

Translate.	$\underbrace{\frac{2}{3}f}_{\text{Two-thirds of } f} \quad \underbrace{=}_{\text{is}} \quad \underbrace{18}_{18}$
Multiply both sides by $\frac{3}{2}$.	$\frac{3}{2} \cdot \frac{2}{3}f = \frac{3}{2} \cdot 18$
Simplify.	$f = 27$
Check:	Is two-thirds of 27 equal to 18?
Translate.	$\frac{2}{3}(27) \stackrel{?}{=} 18$
Simplify. It checks.	$18 = 18$ ✓

Exercise 1.3.13.23

Translate and solve: Two-fifths of f is 16.

Answer

$\frac{2}{5}f = 16; f = 40$

Exercise 1.3.13.24

Translate and solve: Three-fourths of f is 21.

Answer

$\frac{3}{4}f = 21; f = 28$

Example 1.3.13.13: solve

Translate and solve: The quotient of m and $\frac{5}{6}$ is $\frac{3}{4}$.

Solution

Translate.	$\frac{\frac{m}{5}}{6} = \frac{3}{4}$
Multiply both sides by $frac56$ to isolate m.	$\frac{5}{6}\left(\frac{\frac{m}{5}}{6}\right) = \frac{5}{6}\left(\frac{3}{4}\right)$
Simplify.	$m = \frac{5 \cdot 3}{6 \cdot 4}$
Remove common factors and multiply.	$m = \frac{5}{8}$

Check:

Is the quotient of $\frac{5}{8}$ and $\frac{5}{6}$ equal to $\frac{3}{4}$?	$\frac{\frac{5}{8}}{\frac{5}{6}} \stackrel{?}{=} \frac{3}{4}$
Rewrite as division.	$\frac{5}{8} \div \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$
Multiply the first fraction by the reciprocal of the second.	$\frac{5}{8} \cdot \frac{6}{5} \stackrel{?}{=} \frac{3}{4}$
Simplify.	$\frac{3}{4} = \frac{3}{4}$ ✓

Our solution checks.

Exercise 1.3.13.25

Translate and solve. The quotient of n and $\frac{2}{3}$ is $\frac{5}{12}$.

Answer

$\frac{\frac{n}{2}}{3} = \frac{5}{12}$; $n = \frac{5}{18}$

Exercise 1.3.13.26

Translate and solve. The quotient of c and $\frac{3}{8}$ is $\frac{4}{9}$.

Answer

$\frac{\frac{c}{3}}{8} = \frac{4}{9}$; $c = \frac{1}{6}$

Example 1.3.13.14: solve

Translate and solve: The sum of three-eighths and x is three and one-half.

Solution

Translate.	The sum of three-eighths and x is three and one-half $\frac{3}{8}+x = 3\frac{1}{2}$
Use the Subtraction Property of Equality to subtract $\frac{3}{8}$ from both sides.	$\frac{3}{8}+x-\frac{3}{8}=3\frac{1}{2}-\frac{3}{8}$
Combine like terms on the left side.	$x=3\frac{1}{2}-\frac{3}{8}$
Convert mixed number to improper fraction.	$x=3\frac{1}{2}-\frac{3}{8}$
Convert to equivalent fractions with LCD of 8.	$x=\frac{7}{2}-\frac{3}{8}$
Subtract.	$x=\frac{25}{8}$
Write as a mixed number.	$x=3\frac{1}{8}$

We write the answer as a mixed number because the original problem used a mixed number. Check: Is the sum of three-eighths and $3\frac{1}{8}$ equal to three and one-half?

Add.	$3\frac{4}{8} \stackrel{?}{=} 3\frac{1}{2}$
Simplify.	$3\frac{1}{2}=3\frac{1}{2}$

The solution checks.

Exercise 1.3.13.27

Translate and solve: The sum of five-eighths and x is one-fourth.

Answer

$\frac{5}{8}+x=\frac{1}{4}$; $x=-\frac{3}{8}$

Exercise 1.3.13.28

Translate and solve: The difference of one-and-three-fourths and x is five-sixths.

Answer

$1\frac{3}{4}-x=\frac{5}{6}$; $x=\frac{11}{12}$

Access Additional Online Resources

- Solve One Step Equations With Fractions
- Solve One Step Equations With Fractions by Adding or Subtracting

- Solve One Step Equations With Fraction by Multiplying

Practice Makes Perfect

Determine Whether a Fraction is a Solution of an Equation

In the following exercises, determine whether each number is a solution of the given equation.

498. $x - \frac{2}{5} = \frac{1}{10}$:
   a. $x = 1$
   b. $x = \frac{1}{2}$
   c. $x = -\frac{1}{2}$

499. $y - \frac{1}{2} = \frac{5}{12}$:
   a. $y = 1$
   b. $y = \frac{3}{4}$
   c. $y = -\frac{3}{4}$

500. $h + \frac{3}{4} = \frac{2}{5}$:
   a. $h = 1$
   b. $h = \frac{7}{20}$
   c. $h = -\frac{7}{20}$

501. $k + \frac{2}{5} = \frac{5}{6}$:
   a. $k = 1$
   b. $k = \frac{13}{30}$
   c. $k = -\frac{13}{30}$

Solve Equations with Fractions using the Addition, Subtraction, and Division Properties of Equality

In the following exercises, solve.

502. $y + \frac{1}{3} = \frac{4}{3}$

503. $m + \frac{3}{8} = \frac{7}{8}$

504. $f + \frac{9}{10} = \frac{2}{5}$

505. $h + \frac{5}{6} = \frac{1}{6}$

506. $a - \frac{5}{8} = -\frac{7}{8}$

507. $c - \frac{1}{4} = -\frac{5}{4}$

508. $x - \left(-\frac{3}{20}\right) = -\frac{11}{20}$

509. $z - \left(-\frac{5}{12}\right) = -\frac{7}{12}$

510. $n - \frac{1}{6} = \frac{3}{4}$

511. $p - \frac{3}{10} = \frac{5}{8}$

512. $s + \left(-\frac{1}{2}\right) = -\frac{8}{9}$
513. $k + \left(-\frac{1}{3}\right) = -\frac{4}{5}$
514. $5j = 17$
515. $7k = 18$
516. $-4w = 26$
517. $-9v = 33$

Solve Equations with Fractions Using the Multiplication Property of Equality

In the following exercises, solve.

518. $\frac{f}{4} = -20$
519. $\frac{b}{3} = -9$
520. $\frac{y}{7} = -21$
521. $\frac{x}{8} = -32$
522. $\frac{p}{-5} = -40$
523. $\frac{q}{-4} = -40$
524. $\frac{r}{-12} = -6$
525. $\frac{s}{-15} = -3$
526. $-x = 23$
527. $-y = 42$
528. $-h = -\frac{5}{12}$
529. $-k = -\frac{17}{20}$
530. $\frac{4}{5}n = 20$
531. $\frac{3}{10}p = 30$
532. $\frac{3}{8}q = -48$
533. $\frac{5}{2}m = -40$
534. $-\frac{2}{9}a = 16$
535. $-\frac{3}{7}b = 9$
536. $-\frac{6}{11}u = -24$
537. $-\frac{5}{12}v = -15$

Mixed Practice

In the following exercises, solve.

538. $3x = 0$
539. $8y = 0$
540. $4f = \frac{4}{5}$
541. $7g = \frac{7}{9}$

542. $p + \frac{2}{3} = \frac{1}{12}$
543. $q + \frac{5}{6} = \frac{1}{12}$
544. $\frac{7}{8}m = \frac{1}{10}$
545. $\frac{1}{4}n = \frac{7}{10}$
546. $-\frac{2}{5} = x + \frac{3}{4}$
547. $-\frac{2}{3} = y + \frac{3}{8}$
548. $\frac{11}{20} = -f$
549. $\frac{8}{15} = -d$

Translate Sentences to Equations and Solve

In the following exercises, translate to an algebraic equation and solve.

550. n divided by eight is −16.
551. n divided by six is −24.
552. m divided by −9 is −7.
553. m divided by −7 is −8.
554. The quotient of f and −3 is −18.
555. The quotient of f and −4 is −20.
556. The quotient of g and twelve is 8.
557. The quotient of g and nine is 14.
558. Three-fourths of q is 12.
559. Two-fifths of q is 20.
560. Seven-tenths of p is −63.
561. Four-ninths of p is −28.
562. m divided by 4 equals negative 6.
563. The quotient of h and 2 is 43.
564. Three-fourths of z is the same as 15.
565. The quotient of a and $\frac{2}{3}$ is $\frac{3}{4}$.
566. The sum of five-sixths and x is $\frac{1}{2}$.
567. The sum of three-fourths and x is $\frac{1}{8}$.
568. The difference of y and one-fourth is $-\frac{1}{8}$.
569. The difference of y and one-third is $-\frac{1}{6}$.

Everyday Math

570. **Shopping** Teresa bought a pair of shoes on sale for $48. The sale price was $\frac{2}{3}$ of the regular price. Find the regular price of the shoes by solving the equation $\frac{2}{3}p = 48$
571. **Playhouse** The table in a child's playhouse is $\frac{3}{5}$ of an adult-size table. The playhouse table is 18 inches high. Find the height of an adult-size table by solving the equation $\frac{3}{5}h = 18$.

Writing Exercises

572. Example 4.100 describes three methods to solve the equation $-y = 15$. Which method do you prefer? Why?

573. Richard thinks the solution to the equation $\frac{3}{4}x = 24$ is 16. Explain why Richard is wrong.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether a fraction is a solution of an equation.			
solve equations with fractions using the addition, subtraction, and division properties of equality.			
solve equations using the multiplication property of equality.			
translate sentences to equations and solve.			

(b) Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Contributors and Attributions

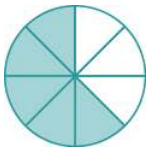
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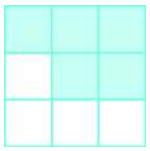
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1.3.E: Fractions (Exercises)

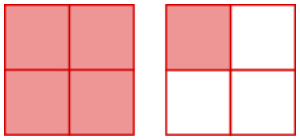
4.1 - Visualize Fractions

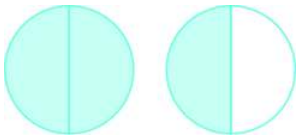
In the following exercises, name the fraction of each figure that is shaded.

574. 

575. 

In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

576. 

577. 

In the following exercises, convert the improper fraction to a mixed number.

578. $\frac{58}{15}$

579. $\frac{63}{11}$

In the following exercises, convert the mixed number to an improper fraction.

580. $12\frac{1}{4}$

581. $9\frac{4}{5}$

582. Find three fractions equivalent to $\frac{2}{5}$. Show your work, using figures or algebra.

583. Find three fractions equivalent to $-\frac{4}{3}$. Show your work, using figures or algebra.

In the following exercises, locate the numbers on a number line.

584. $\frac{5}{8}, \frac{4}{3}, 3\frac{3}{4}, 4$

585. $\frac{1}{4}, -\frac{1}{4}, 1\frac{1}{3}, -1\frac{1}{3}, \frac{7}{2}, -\frac{7}{2}$

In the following exercises, order each pair of numbers, using < or >.

586. $-1 \_\_\_ -\frac{2}{5}$

587. $-2\frac{1}{2} \_\_\_ -3$

4.2 - Multiply and Divide Fractions

In the following exercises, simplify.

588. $-\frac{63}{84}$
589. $-\frac{90}{120}$
590. $-\frac{14a}{14b}$
591. $-\frac{8x}{8y}$

In the following exercises, multiply.

592. $\frac{2}{5} \cdot \frac{8}{13}$
593. $-\frac{1}{3} \cdot \frac{12}{7}$
594. $\frac{2}{9} \cdot \left(-\frac{45}{32}\right)$
595. $6\text{m} \cdot \frac{4}{11}$
596. $-\frac{1}{4}\ (-32)$
597. $3\frac{1}{5} \cdot 1\frac{7}{8}$

In the following exercises, find the reciprocal.

598. $\frac{2}{9}$
599. $\frac{15}{4}$
600. 3
601. $-\frac{1}{4}$
602. Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{5}{13}$			
$\frac{3}{10}$			
$\frac{9}{4}$			
-12			

In the following exercises, divide.

603. $\frac{2}{3} \div \frac{1}{6}$
604. $\left(-\frac{3x}{5}\right) \div \left(-\frac{2y}{3}\right)$
605. $\frac{4}{5} \div 3$
606. $8 \div 2\frac{2}{3}$
607. $8\frac{2}{3} \div 1\frac{1}{12}$

4.3 - Multiply and Divide Mixed Numbers and Complex Fractions

In the following exercises, perform the indicated operation.

608. $3\frac{1}{5} \cdot 1\frac{7}{8}$
609. $-5\frac{7}{12} \cdot 4\frac{4}{11}$
610. $8 \div 2\frac{2}{3}$
611. $8\frac{2}{3} \div 1\frac{1}{12}$

In the following exercises, translate the English phrase into an algebraic expression.

612. the quotient of 8 and y
613. the quotient of V and the difference of h and 6

In the following exercises, simplify the complex fraction.

614. $\dfrac{\frac{5}{8}}{\frac{4}{5}}$
615. $\dfrac{\frac{8}{9}}{-4}$
616. $\dfrac{\frac{n}{4}}{\frac{3}{8}}$
617. $\dfrac{-1\frac{5}{6}}{-\frac{1}{12}}$

In the following exercises, simplify.

618. $\dfrac{5+16}{5}$
619. $\dfrac{8 \cdot 4 - 5^2}{3 \cdot 12}$
620. $\dfrac{8 \cdot 7 + 5(8-10)}{9 \cdot 3 - 6 \cdot 4}$

4.4 - Add and Subtract Fractions with Common Denominators

In the following exercises, add.

621. $\frac{3}{8} + \frac{2}{8}$
622. $\frac{4}{5} + \frac{1}{5}$
623. $\frac{2}{5} + \frac{1}{5}$
624. $\frac{15}{32} + \frac{9}{32}$
625. $\frac{x}{10} + \frac{7}{10}$

In the following exercises, subtract.

626. $\frac{8}{11} - \frac{6}{11}$
627. $\frac{11}{12} - \frac{5}{12}$
628. $\frac{4}{5} - \frac{y}{5}$

629. $-\frac{31}{30}-\frac{7}{30}$
630. $\frac{3}{2}-\left(\frac{3}{2}\right)$
631. $\frac{11}{15}-\frac{5}{15}-\left(-\frac{2}{15}\right)$

4.5 - Add and Subtract Fractions with Different Denominators

In the following exercises, find the least common denominator.

632. $\frac{1}{3}$ and $\frac{1}{12}$
633. $\frac{1}{3}$ and $\frac{4}{5}$
634. $\frac{8}{15}$ and $\frac{11}{20}$
635. $\frac{3}{4}$, $\frac{1}{6}$, and $\frac{5}{10}$

In the following exercises, change to equivalent fractions using the given LCD.

636. $\frac{1}{3}$ and $\frac{1}{5}$, LCD = 15
637. $\frac{3}{8}$ and $\frac{5}{6}$, LCD = 24
638. $-\frac{9}{16}$ and $\frac{5}{12}$, LCD = 48
639. $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$, LCD = 60

In the following exercises, perform the indicated operations and simplify.

640. $\frac{1}{5}+\frac{2}{3}$
641. $\frac{11}{12}-\frac{2}{3}$
642. $-\frac{9}{10}-\frac{3}{4}$
643. $-\frac{11}{36}-\frac{11}{20}$
644. $-\frac{22}{25}+\frac{9}{40}$
645. $\frac{y}{10}-\frac{1}{3}$
646. $\frac{2}{5}+\left(-\frac{5}{9}\right)$
647. $\frac{4}{11}\div\frac{2}{7d}$
648. $\frac{2}{5}+\left(-\frac{3n}{8}\right)\left(-\frac{2}{9n}\right)$
649. $\frac{\left(\frac{2}{3}\right)^2}{\left(\frac{5}{8}\right)^2}$
650. $\left(\frac{11}{12}+\frac{3}{8}\right)\div\left(\frac{5}{6}-\frac{1}{10}\right)$

In the following exercises, evaluate.

651. y − $\frac{4}{5}$ when (a) y = $-\frac{4}{5}$ (b) y = $\frac{1}{4}$

652. $6mn^2$ when m = $\frac{3}{4}$ and n = $-\frac{1}{3}$

4.6 - Add and Subtract Mixed Numbers

In the following exercises, perform the indicated operation.

653. $4\frac{1}{3}+9\frac{1}{3}$

654. $6\frac{2}{5}+7\frac{3}{5}$

655. $5\frac{8}{11}+2\frac{4}{11}$

656. $3\frac{5}{8}+3\frac{7}{8}$

657. $9\frac{13}{20}-4\frac{11}{20}$

658. $2\frac{3}{10}-1\frac{9}{10}$

659. $2\frac{11}{12}-1\frac{7}{12}$

660. $8\frac{6}{11}-2\frac{9}{11}$

4.7 - Solve Equations with Fractions

In the following exercises, determine whether the each number is a solution of the given equation.

661. x − $\frac{1}{2}$ = $\frac{1}{6}$:
   a. x = 1
   b. x = $\frac{2}{3}$
   c. x = $-\frac{1}{3}$

662. y + $\frac{3}{5}$ = $\frac{5}{9}$:
   a. y = $\frac{1}{2}$
   b. y = $\frac{52}{45}$
   c. y = $-\frac{2}{45}$

In the following exercises, solve the equation.

663. n + $\frac{9}{11}$ = $\frac{4}{11}$

664. x − $\frac{1}{6}$ = $\frac{7}{6}$

665. h − $\left(-\frac{7}{8}\right)$ = $-\frac{2}{5}$

666. $\frac{x}{5}$ = −10

667. −z = 23

In the following exercises, translate and solve.

668. The sum of two-thirds and n is $-\frac{3}{5}$.

669. The difference of q and one-tenth is $\frac{1}{2}$.

670. The quotient of p and −4 is −8.

671. Three-eighths of y is 24.

PRACTICE TEST

Convert the improper fraction to a mixed number.

672. $\frac{19}{5}$

Convert the mixed number to an improper fraction.

673. $3\frac{2}{7}$

Locate the numbers on a number line.

674. $\frac{1}{2}, 1\frac{2}{3}, -2\frac{3}{4}$, and $\frac{9}{4}$

In the following exercises, simplify.

675. $\frac{5}{20}$

676. $\frac{18r}{27s}$

677. $\frac{1}{3} \cdot \frac{3}{4}$

678. $\frac{3}{5} \cdot 15$

679. $-36\text{u}\left(-\frac{4}{9}\right)$

680. $-5\frac{7}{12} \cdot 4\frac{4}{11}$

681. $-\frac{5}{6} \div \frac{5}{12}$

682. $\frac{7}{11} \div \left(-\frac{7}{11}\right)$

683. $\frac{9a}{10} \div \frac{15a}{8}$

684. $-6\frac{2}{5} \div 4$

685. $\left(-15\frac{5}{6}\right) \div \left(-3\frac{1}{6}\right)$

686. $\dfrac{\frac{-6}{6}}{\frac{11}{}}$

687. $\dfrac{\frac{p}{2}}{\frac{q}{5}}$

688. $\dfrac{-\frac{4}{15}}{-2\frac{2}{3}}$

689. $\frac{9^2 - 4^2}{9 - 4}$

690. $\frac{2}{d} + \frac{9}{d}$

691. $-\frac{3}{13} + \left(-\frac{4}{13}\right)$

692. $-\frac{22}{25} + \frac{9}{40}$

693. $\frac{2}{5} + \left(-\frac{7}{5}\right)$

694. $-\frac{3}{10}+\left(-\frac{5}{8}\right)$

695. $-\frac{3}{4} \div \frac{x}{3}$

696. $\frac{2^3-2^2}{\left(\frac{3}{4}\right)^2}$

697. $\frac{\frac{5}{14}+\frac{1}{8}}{\frac{9}{56}}$

Evaluate.

698. x + $\frac{1}{3}$ when (a) x = $\frac{2}{3}$ (b) x = $-\frac{5}{6}$

In the following exercises, solve the equation.

699. y + $\frac{3}{5}$ = $\frac{7}{5}$

700. a − $\frac{3}{10}$ = $-\frac{9}{10}$

701. f + $\left(-\frac{2}{3}\right)$ = $\frac{5}{12}$

702. $\frac{m}{-2}$ = −16

703. $-\frac{2}{3}$c = 18

704. Translate and solve: The quotient of p and −4 is −8. Solve for p.

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1.3.S: Fractions (Summary)

Key Terms

complex fraction	A fraction in which the numerator or the denominator contains a fraction.
equivalent fractions	Two or more fractions that have the same value.
fraction	A fraction is written $\frac{a}{b}$. in a fraction, a is the numerator and b is the denominator. A fraction represents parts of a whole. The denominator b is the number of equal parts the whole has been divided into, and the numerator a indicates how many parts are included.
least common denominator (LCD)	The least common denominator (LCD) of two fractions is the least common multiple (LCM) of their denominators.
mixed number	A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where c ≠ 0. It is written as $a\frac{b}{c}$, where c ≠ 0.
proper and improper fractions	The fraction $\frac{a}{b}$ is proper if a < b and improper if a > b.
reciprocal	The reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$ where a ≠ 0 and b ≠ 0.
simplified fraction	A fraction is considered simplified if there are no common factors in the numerator and denominator.

Key Concepts

4.1 - Visualize Fractions

- **Property of One**
  - Any number, except zero, divided by itself is one. $\frac{a}{a}$ = 1, where a ≠ 0.
- **Mixed Numbers**
  - A mixed number consists of a whole number a and a fraction $\frac{b}{c}$ where c ≠ 0.
  - It is written as follows: $a\frac{b}{c} \quad c \neq 0$
- **Proper and Improper Fractions**
  - The fraction $\frac{a}{b}$ is a proper fraction if a < b and an improper fraction if a ≥ b .
- **Convert an improper fraction to a mixed number**.
  1. Divide the denominator into the numerator.
  2. Identify the quotient, remainder, and divisor.
  3. Write the mixed number as $quotient\frac{remainder}{divisor}$.
- **Convert a mixed number to an improper fraction**.
  1. Multiply the whole number by the denominator.
  2. Add the numerator to the product found in Step 1.
  3. Write the final sum over the original denominator.
- **Equivalent Fractions Property**: If a, b, and c are numbers where b ≠ 0, c ≠ 0, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$.

4.2 - Multiply and Divide Fractions

- **Equivalent Fractions Property**
  - If a, b, c are numbers where b ≠ 0, c ≠ 0, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$.
- **Simplify a fraction**.
  1. Rewrite the numerator and denominator to show the common factors. If needed, factor the numerator and denominator into prime numbers.
  2. Simplify, using the equivalent fractions property, by removing common factors.
  3. Multiply any remaining factors.
- **Fraction Multiplication**
  - If a, b, c, and d are numbers where b ≠ 0 and d ≠ 0, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.
- **Reciprocal**
  - A number and its reciprocal have a product of 1. $\frac{a}{b} \cdot \frac{b}{a} = 1$.
  - Table 4.98

Opposite	Absolute Value	Reciprocal
has opposite sign	is never negative	has same sign, fraction inverts

- **Fraction Division**
  - If a, b, c, and d are numbers where b ≠ 0, c ≠ 0, and d ≠ 0, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.
  - To divide fractions, multiply the first fraction by the reciprocal of the second.

4.3 - Multiply and Divide Mixed Numbers and Complex Fractions

- **Multiply or divide mixed numbers**.
  1. Convert the mixed numbers to improper fractions.
  2. Follow the rules for fraction multiplication or division.
  3. Simplify if possible.
- **Simplify a complex fraction**.
  1. Rewrite the complex fraction as a division problem.
  2. Follow the rules for dividing fractions.
  3. Simplify if possible.
- **Placement of negative sign in a fraction**.
  - For any positive numbers a and b, $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.
- **Simplify an expression with a fraction bar**.
  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Simplify the fraction.

4.4 - Add and Subtract Fractions with Common Denominators

- **Fraction Addition**
  - If a, b, and c are numbers where c ≠ 0, then $\frac{a}{c} + \frac{b}{c} = \frac{a+c}{c}$.
  - To add fractions, add the numerators and place the sum over the common denominator.
- **Fraction Subtraction**
  - If a, b, and c are numbers where c ≠ 0, then $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.
  - To subtract fractions, subtract the numerators and place the difference over the common denominator.

4.5 - Add and Subtract Fractions with Different Denominators

- **Find the least common denominator (LCD) of two fractions**.
  1. Factor each denominator into its primes.
  2. List the primes, matching primes in columns when possible.
  3. Bring down the columns.
  4. Multiply the factors. The product is the LCM of the denominators.
  5. The LCM of the denominators is the LCD of the fractions.
- **Equivalent Fractions Property**
  - If a, b, and c are whole numbers where b ≠ 0, c ≠ 0 then $\frac{a}{b}=\frac{a\cdot c}{b\cdot c}$ and $\frac{a\cdot c}{b\cdot c}=\frac{a}{b}$.
- **Convert two fractions to equivalent fractions with their LCD as the common denominator**.
  1. Find the LCD.
  2. For each fraction, determine the number needed to multiply the denominator to get the LCD.
  3. Use the Equivalent Fractions Property to multiply the numerator and denominator by the number from Step 2.
  4. Simplify the numerator and denominator.
- **Add or subtract fractions with different denominators**.
  1. Find the LCD.
  2. Convert each fraction to an equivalent form with the LCD as the denominator.
  3. Add or subtract the fractions.
  4. Write the result in simplified form.
- **Summary of Fraction Operations**
  - *Fraction multiplication*: Multiply the numerators and multiply the denominators. $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$.
  - *Fraction division*: Multiply the first fraction by the reciprocal of the second. $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}$.
  - *Fraction addition*: Add the numerators and place the sum over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD. $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$.
  - *Fraction subtraction*: Subtract the numerators and place the difference over the common denominator. If the fractions have different denominators, first convert them to equivalent forms with the LCD. $\frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}$.
- **Simplify complex fractions**.
  1. Simplify the numerator.
  2. Simplify the denominator.
  3. Divide the numerator by the denominator.
  4. Simplify if possible.

4.6 - Add and Subtract Mixed Numbers

- **Add mixed numbers with a common denominator**.
  1. Add the whole numbers.
  2. Add the fractions.
  3. Simplify, if possible.
- **Subtract mixed numbers with common denominators**.
  1. Rewrite the problem in vertical form.
  2. Compare the two fractions. If the top fraction is larger than the bottom fraction, go to Step 3. If not, in the top mixed number, take one whole and add it to the fraction part, making a mixed number with an improper fraction.
  3. Subtract the fractions.
  4. Subtract the whole numbers.
  5. Simplify, if possible.
- **Subtract mixed numbers with common denominators as improper fractions.**
  1. Rewrite the mixed numbers as improper fractions.

2. Subtract the numerators.
3. Write the answer as a mixed number, simplifying the fraction part, if possible.

4.7 - Solve Equations with Fractions

- Determine whether a number is a solution to an equation.
  1. Substitute the number for the variable in the equation.
  2. Simplify the expressions on both sides of the equation.
  3. Determine whether the resulting equation is true. If it is true, the number is a solution. If it is not true, the number is not a solution.
- Addition, Subtraction, and Division Properties of Equality: For any numbers a, b, and c,
  - if a = b, then a + c = b + c. *Addition Property of Equality*
  - if a = b, then a - c = b - c. *Subtraction Property of Equality*
  - if a = b, then $\frac{a}{c} = \frac{b}{c}$, c ≠ 0. *Division Property of Equality*
- **The Multiplication Property of Equality**
  - For any numbers ab and c, a = b, then ac = bc.
  - If you multiply both sides of an equation by the same quantity, you still have equality.

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SECTION OVERVIEW

1.4: Decimals

Gasoline price changes all the time. They might go down for a period of time, but then they usually rise again. One thing that stays the same is that the price is not usually a whole number. Instead, it is shown using a decimal point to describe the cost in dollars and cents. We use decimal numbers all the time, especially when dealing with money. In this chapter, we will explore decimal numbers and how to perform operations using them.

Topic hierarchy

1.4.1: Decimals (Part 1)

1.4.2: Decimals (Part 2)

1.4.3: Decimal Operations (Part 1)

1.4.4: Decimal Operations (Part 2)

1.4.5: Decimals and Fractions (Part 1)

1.4.6: Decimals and Fractions (Part 2)

1.4.7: Solve Equations with Decimals

1.4.8: Averages and Probability (Part 1)

1.4.9: Averages and Probability (Part 2)

1.4.10: Ratios and Rate (Part 1)

1.4.11: Ratios and Rate (Part 2)

1.4.12: Simplify and Use Square Roots (Part 1)

1.4.13: Simplify and Use Square Roots (Part 2)

1.4.E: Decimals (Exercises)

1.4.S: Decimals (Summary)

Figure 5.1 - The price of a gallon of gasoline is written as a decimal number. (credit: Mark Turnauckus, Flickr)

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1.4.1: Decimals (Part 1)

Learning Objectives

- Name decimals
- Write decimals
- Convert decimals to fractions or mixed numbers
- Locate decimals on the number line
- Order decimals
- Round decimals

be prepared!

Before you get started, take this readiness quiz.

1. Name the number 4,926,015 in words. If you missed this problem, review Example 1.1.4.
2. Round 748 to the nearest ten. If you missed this problem, review Example 1.2.9.
3. Locate $\frac{3}{10}$ on a number line. If you missed this problem, review Example 4.2.16.

Name Decimals

You probably already know quite a bit about decimals based on your experience with money. Suppose you buy a sandwich and a bottle of water for lunch. If the sandwich costs \$3.45, the bottle of water costs \$1.25, and the total sales tax is \$0.33, what is the total cost of your lunch?

$$\begin{array}{rl} \$3.45 & Sandwich \\ \$1.25 & Water \\ +\$0.33 & Tax \\ \hline \$5.03 & Total \end{array}$$

The total is \$5.03. Suppose you pay with a \$5 bill and 3 pennies. Should you wait for change? No, \$5 and 3 pennies is the same as \$5.03.

Because 100 pennies = \$1, each penny is worth $\frac{1}{100}$ of a dollar. We write the value of one penny as \$0.01, since 0.01 = $\frac{1}{100}$.

Writing a number with a decimal is known as decimal notation. It is a way of showing parts of a whole when the whole is a power of ten. In other words, decimals are another way of writing fractions whose denominators are powers of ten. Just as the counting numbers are based on powers of ten, decimals are based on powers of ten. Table 1.4.1.1shows the counting numbers.

Table 1.4.1.1

Counting number	Name
1	One
10 = 10	Ten
10 • 10 = 100	One hundred
10 • 10 • 10 = 1000	One thousand
10 • 10 • 10 • 10 = 10,000	Ten thousand

How are decimals related to fractions? Table 1.4.1.2shows the relation.

Table 1.4.1.2

Decimal	Fraction	Name

Decimal	Fraction	Name
0.1	$\frac{1}{10}$	One tenth
0.01	$\frac{1}{100}$	One hundredth
0.001	$\frac{1}{1000}$	One thousandth
0.0001	$\frac{1}{10,000}$	One ten-thousandth

When we name a whole number, the name corresponds to the place value based on the powers of ten. In Whole Numbers, we learned to read 10,000 as *ten thousand*. Likewise, the names of the decimal places correspond to their fraction values. Notice how the place value names in Figure 1.4.1.1relate to the names of the fractions from Table 1.4.1.2

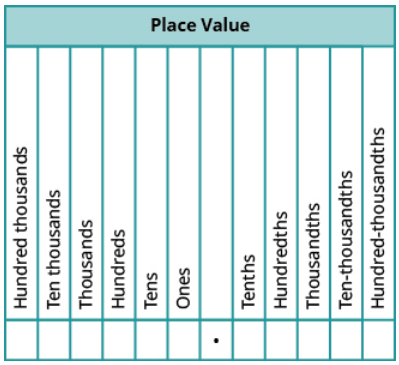


Figure 1.4.1.1- This chart illustrates place values to the left and right of the decimal point.

Notice two important facts shown in Figure 1.4.1.1.

- The “th” at the end of the name means the number is a fraction. “One thousand” is a number larger than one, but “one thousandth” is a number smaller than one.
- The tenths place is the first place to the right of the decimal, but the tens place is two places to the left of the decimal.

Remember that $5.03 lunch? We read $5.03 as *five dollars and three cents*. Naming decimals (those that don’t represent money) is done in a similar way. We read the number 5.03 as *five and three hundredths*.

We sometimes need to translate a number written in decimal notation into words. As shown in Figure 1.4.1.2 we write the amount on a check in both words and numbers.



Figure 1.4.1.2- When we write a check, we write the amount as a decimal number as well as in words. The bank looks at the check to make sure both numbers match. This helps prevent errors.

Let’s try naming a decimal, such as 15.68.	
We start by naming the number to the left of the decimal.	fifteen_____
We use the word “and” to indicate the decimal point.	fifteen and_____

Then we name the number to the right of the decimal point as if it were a whole number.	fifteen and sixty-eight_____
Last, name the decimal place of the last digit.	fifteen and sixty-eight hundredths

The number 15.68 is read *fifteen and sixty-eight hundredths*.

HOW TO: NAME A DECIMAL NUMBER

- Name the number to the left of the decimal point.
- Write “and” for the decimal point.
- Name the “number” part to the right of the decimal point as if it were a whole number.
- Name the decimal place of the last digit.

Example 1.4.1.1:

Name each decimal: (a) 4.3 (b) 2.45 (c) 0.009 (d) −15.571.

Solution

(a) 4.3

Name the number to the left of the decimal point.	four_____
Write "and" for the decimal point.	four and_____
Name the number to the right of the decimal point as if it were a whole number.	four and three_____
Name the decimal place of the last digit.	four and three tenths

(b) 2.45

Name the number to the left of the decimal point.	two_____
Write "and" for the decimal point.	two and_____
Name the number to the right of the decimal point as if it were a whole number.	two and forty-five_____
Name the decimal place of the last digit.	two and forty-five hundredths

(c) 0.009

Name the number to the left of the decimal point.	Zero is the number to the left of the decimal; it is not included in the name.
Name the number to the right of the decimal point as if it were a whole number.	nine_____
Name the decimal place of the last digit.	nine thousandths

(d) −15.571

Name the number to the left of the decimal point.	negative fifteen
Write "and" for the decimal point.	negative fifteen and_____
Name the number to the right of the decimal point as if it were a whole number.	negative fifteen and five hundred seventyone_____

Name the decimal place of the last digit.	negative fifteen and five hundred seventyone thousandths

? Exercise 1.4.1.1:

Name each decimal: (a) 6.7 (b) 19.58 (c) 0.018 (d) −2.053

Answer a

six and seven tenths

Answer b

nineteen and fifty-eight hundredths

Answer c

eighteen thousandths

Answer d

negative two and fifty-three thousandths

? Exercise 1.4.1.2:

Name each decimal: (a) 5.8 (b) 3.57 (c) 0.005 (d) −13.461

Answer a

five and eight tenths three and fifty-seven hundredths

Answer b

three and fifty-seven hundredths

Answer c

five thousandths

Answer d

negative thirteen and four hundred sixty-one thousandths

Write Decimals

Now we will translate the name of a decimal number into decimal notation. We will reverse the procedure we just used. Let's start by writing the number six and seventeen hundredths:

The word *and* tells us to place a decimal point.	___.___
The word before *and* is the whole number; write it to the left of the decimal point.	6._____
The decimal part is seventeen hundredths. Mark two places to the right of the decimal point for hundredths.	6._ _
Write the numerals for seventeen in the places marked.	6.17

Example 1.4.1.2:

Write fourteen and thirty-seven hundredths as a decimal.

Solution

Place a decimal point under the word ‘and’.	_____. _____
Translate the words before ‘and’ into the whole number and place it to the left of the decimal point.	14. _____
Mark two places to the right of the decimal point for “hundredths”.	14.__ __
Translate the words after “and” and write the number to the right of the decimal point.	14.37

Fourteen and thirty-seven hundredths is written 14.37.

Exercise 1.4.1.3:

Write as a decimal: thirteen and sixty-eight hundredths.

Answer

13.68

Exercise 1.4.1.4:

Write as a decimal: five and eight hundred ninety-four thousandths.

Answer

5.894

HOW TO: WRITE A DECIMAL NUMBER FROM ITS NAME

Step 1. Look for the word “and”—it locates the decimal point.

Step 2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.

- Place a decimal point under the word “and.” Translate the words before “and” into the whole number and place it to the left of the decimal point.
- If there is no “and,” write a “0” with a decimal point to its right.

Step 3. Translate the words after “and” into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.

Step 4. Fill in zeros for place holders as needed.

The second bullet in Step 2 is needed for decimals that have no whole number part, like ‘nine thousandths’. We recognize them by the words that indicate the place value after the decimal – such as ‘tenths’ or ‘hundredths.’ Since there is no whole number, there is no ‘and.’ We start by placing a zero to the left of the decimal and continue by filling in the numbers to the right, as we did above.

Example 1.4.1.3:

Write twenty-four thousandths as a decimal.

Solution

Look for the word "and".	There is no "and" so start with 0.
To the right of the decimal point, put three decimal places for thousandths.	0. _ _ _ tenths hundredths thousandths
Write the number 24 with the 4 in the thousandths place.	0. _ 2 4 tenths hundredths thousandths
Put zeros as placeholders in the remaining decimal places.	0.024

So, twenty-four thousandths is written 0.024

Exercise 1.4.1.5:

Write as a decimal: fifty-eight thousandths.

Answer

0.058

Exercise 1.4.1.6:

Write as a decimal: sixty-seven thousandths.

Answer

0.067

Before we move on to our next objective, think about money again. We know that $1 is the same as $1.00. The way we write $1 (or $1.00) depends on the context. In the same way, integers can be written as decimals with as many zeros as needed to the right of the decimal.

$$\begin{aligned} 5 &= 5.0 \qquad -2 = -2.0 \\ 5 &= 5.00 \qquad -2 = -2.00 \\ 5 &= 5.000 \qquad -2 = -2.000 \end{aligned}$$

$$and\ so\ on\ldots$$

Convert Decimals to Fractions or Mixed Numbers

We often need to rewrite decimals as fractions or mixed numbers. Let's go back to our lunch order to see how we can convert decimal numbers to fractions. We know that $5.03 means 5 dollars and 3 cents. Since there are 100 cents in one dollar, 3 cents means $\frac{3}{100}$ of a dollar, so 0.03 = $\frac{3}{100}$.

We convert decimals to fractions by identifying the place value of the farthest right digit. In the decimal 0.03, the 3 is in the hundredths place, so 100 is the denominator of the fraction equivalent to 0.03.

$$0.03 = \frac{3}{100}$$

For our $5.03 lunch, we can write the decimal 5.03 as a mixed number.

$$5.03 = 5\frac{3}{100}$$

Notice that when the number to the left of the decimal is zero, we get a proper fraction. When the number to the left of the decimal is not zero, we get a mixed number.

HOW TO: CONVERT A DECIMAL NUMBER TO A FRACTION OR MIXED NUMBER

Step 1. Look at the number to the left of the decimal.

- If it is zero, the decimal converts to a proper fraction.
- If it is not zero, the decimal converts to a mixed number.
  - Write the whole number.

Step 2. Determine the place value of the final digit.

Step 3. Write the fraction.

- numerator—the 'numbers' to the right of the decimal point
- denominator—the place value corresponding to the final digit

Step 4. Simplify the fraction, if possible.

Example 1.4.1.4:

Write each of the following decimal numbers as a fraction or a mixed number: (a) 4.09 (b) 3.7 (c) −0.286

Solution

(a) 4.09

There is a 4 to the left of the decimal point. Write "4" as the whole number part of the mixed number.	$4\frac{\square}{\square}$
Determine the place value of the final digit.	4. 0 9 tenths hundredths
Write the fraction. Write 9 in the numerator as it is the number to the right of the decimal point.	$4\frac{9}{\square}$
Write 100 in the denominator as the place value of the final digit, 9, is hundredth.	$4\frac{9}{100}$
The fraction is in simplest form.	So, $4.09 = 4\frac{9}{100}$

Did you notice that the number of zeros in the denominator is the same as the number of decimal places?

(b) 3.7

There is a 3 to the left of the decimal point. Write "3" as the whole number part of the mixed number.	$3\frac{\square}{\square}$
Determine the place value of the final digit.	3. 7 tenths
Write the fraction. Write 7 in the numerator as it is the number to the right of the decimal point.	$3\frac{7}{\square}$
Write 10 in the denominator as the place value of the final digit, 7, is tenths.	$3\frac{7}{10}$
The fraction is in simplest form.	So, $3.7 = 3\frac{7}{10}$

(c) −0.286

There is a 0 to the left of the decimal point. Write a negative sign before the fraction.	$-\frac{\square}{\square}$
Determine the place value of the final digit and write it in the denominator.	$-0.\underset{\text{tenths}}{2}\underset{\text{hundredths}}{8}\underset{\text{thousandths}}{6}$
Write the fraction. Write 286 in the numerator as it is the number to the right of the decimal point. Write 1,000 in the denominator as the place value of the final digit, 6, is thousandths.	$-\frac{286}{1000}$
We remove a common factor of 2 to simplify the fraction.	$-\frac{143}{500}$

? Exercise 1.4.1.7:

Write as a fraction or mixed number. Simplify the answer if possible. (a) 5.3 (b) 6.07 (c) −0.234

Answer a

$5\frac{3}{10}$

Answer b

$6\frac{7}{100}$

Answer c

$-\frac{234}{1000}$

? Exercise 1.4.1.8:

Write as a fraction or mixed number. Simplify the answer if possible. (a) 8.7 (b) 1.03 (c) −0.024

Answer a

$8\frac{7}{10}$

Answer b

$1\frac{3}{100}$

Answer c

$-\frac{24}{1000}$

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1.4.2: Decimals (Part 2)

Locate Decimals on the Number Line

Since decimals are forms of fractions, locating decimals on the number line is similar to locating fractions on the number line.

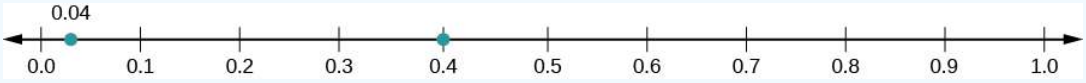
Example 1.4.2.5:

Locate 0.4 on a number line.

Solution

The decimal 0.4 is equivalent to $\frac{4}{10}$, so 0.4 is located between 0 and 1. On a number line, divide the interval between 0 and 1 into 10 equal parts and place marks to separate the parts.

Label the marks 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0. We write 0 as 0.0 and 1 as 1.0, so that the numbers are consistently in tenths. Finally, mark 0.4 on the number line.



Exercise 1.4.2.9:

Locate 0.6 on a number line.

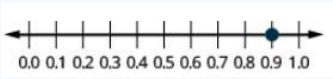
Answer



Exercise 1.4.2.10:

Locate 0.9 on a number line.

Answer

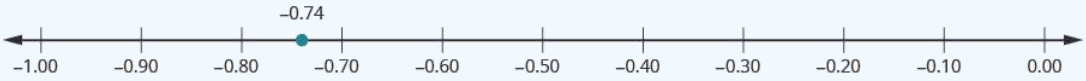


Example 1.4.2.6:

Locate −0.74 on a number line.

Solution

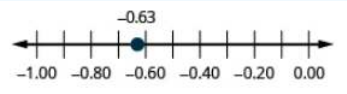
The decimal −0.74 is equivalent to $-\frac{74}{100}$, so it is located between 0 and −1. On a number line, mark off and label the multiples of -0.10 in the interval between 0 and −1 (−0.10, −0.20, etc.) and mark −0.74 between −0.70 and −0.80, a little closer to −0.70.



Exercise 1.4.2.11:

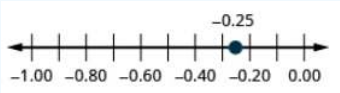
Locate −0.63 on a number line.

Answer



Exercise 1.4.2.12:

Locate −0.25 on a number line.



Answer

Order Decimals

Which is larger, 0.04 or 0.40?

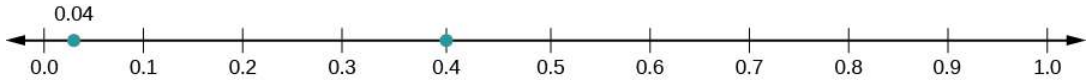
If you think of this as money, you know that $0.40 (forty cents) is greater than $0.04 (four cents). So, 0.40 > 0.04.

In previous chapters, we used the number line to order numbers.

a < b: 'a is less than b' when a is to the left of b on the number line

a > b: 'a is greater than b' when a is to the right of b on the number line

Where are 0.04 and 0.40 located on the number line?



We see that 0.40 is to the right of 0.04. So we know 0.40 > 0.04.

How does 0.31 compare to 0.308? This doesn't translate into money to make the comparison easy. But if we convert 0.31 and 0.308 to fractions, we can tell which is larger.

	0.31		0.308	
Convert to fractions.	$\frac{31}{100}$	(1.4.2.1)	$\frac{308}{1000}$	(1.4.2.2)
We need a common denominator to compare them.	$\frac{31 \cdot 10}{100 \cdot 10}$	(1.4.2.3)	$\frac{308}{1000}$	(1.4.2.4)
	$\frac{310}{1000}$	(1.4.2.5)	$\frac{308}{1000}$	(1.4.2.6)

Because 310 > 308, we know that $\frac{310}{1000} > \frac{308}{1000}$. Therefore, 0.31 > 0.308.

Notice what we did in converting 0.31 to a fraction—we started with the fraction $\frac{31}{100}$ and ended with the equivalent fraction $\frac{310}{1000}$. Converting $\frac{310}{1000}$ back to a decimal gives 0.310. So 0.31 is equivalent to 0.310. Writing zeros at the end of a decimal does

not change its value.

$$\frac{31}{100} = \frac{310}{1000} \quad and \quad 0.31 = 0.310 \tag{1.4.2.7}$$

If two decimals have the same value, they are said to be equivalent decimals.

$$0.31 = 0.310 \tag{1.4.2.8}$$

We say 0.31 and 0.310 are equivalent decimals.

HOW TO: ORDER DECIMALS

Step 1. Check to see if both numbers have the same number of decimal places. If not, write zeros at the end of the one with fewer digits to make them match.

Step 2. Compare the numbers to the right of the decimal point as if they were whole numbers.

Step 3. Order the numbers using the appropriate inequality sign.

Example 1.4.2.7:

Order the following decimals using < or >: (a) 0.64 __0.6 (b) 0.83 __0.803

Solution

(a) 0.64 __0.6

Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.6.	0.64 __0.60
Compare the numbers to the right of the decimal point as if they were whole numbers.	64 > 60
Order the numbers using the appropriate inequality sign.	0.64 > 0.60 0.64 > 0.6

(b) 0.83 __0.803

Check to see if both numbers have the same number of decimal places. They do not, so write one zero at the right of 0.83.	0.830 __0.803
Compare the numbers to the right of the decimal point as if they were whole numbers.	830 > 803
Order the numbers using the appropriate inequality sign.	0.830 > 0.803 0.83 > 0.803

Exercise 1.4.2.13:

Order each of the following pairs of numbers, using < or >: (a) 0.42__0.4 (b) 0.76__0.706

Answer a

>

Answer b

>

Exercise 1.4.2.14:

Order each of the following pairs of numbers, using < or >: (a) 0.1__0.18 (b) 0.305__0.35

Answer a

<

Answer b

<

When we order negative decimals, it is important to remember how to order negative integers. Recall that larger numbers are to the right on the number line. For example, because −2 lies to the right of −3 on the number line, we know that −2 > −3. Similarly, smaller numbers lie to the left on the number line. For example, because −9 lies to the left of −6 on the number line, we know that −9 < −6.



If we zoomed in on the interval between 0 and −1, we would see in the same way that −0.2 > −0.3 and −0.9 < −0.6.

Example 1.4.2.8:

Use < or > to order. −0.1__−0.8.

Solution

Write the numbers one under the other, lining up the decimal points.	−0.1 −0.8
They have the same number of digits.	
Since −1 > −8, −1 tenth is greater than −8 tenths.	−0.1 > −0.8

Exercise 1.4.2.15:

Order each of the following pairs of numbers, using < or >: −0.3___−0.5

Answer

>

Exercise 1.4.2.16:

Order each of the following pairs of numbers, using < or >: −0.6___−0.7

Answer

>

Round Decimals

In the United States, gasoline prices are usually written with the decimal part as thousandths of a dollar. For example, a gas station might post the price of unleaded gas at $3.279 per gallon. But if you were to buy exactly one gallon of gas at this price, you would pay $3.28 , because the final price would be rounded to the nearest cent. In Whole Numbers, we saw that we round numbers to get an approximate value when the exact value is not needed. Suppose we wanted to round $2.72 to the nearest dollar. Is it closer to $2 or to $3? What if we wanted to round $2.72 to the nearest ten cents; is it closer to $2.70 or to $2.80? The number lines in Figure 1.4.2.3can help us answer those questions.

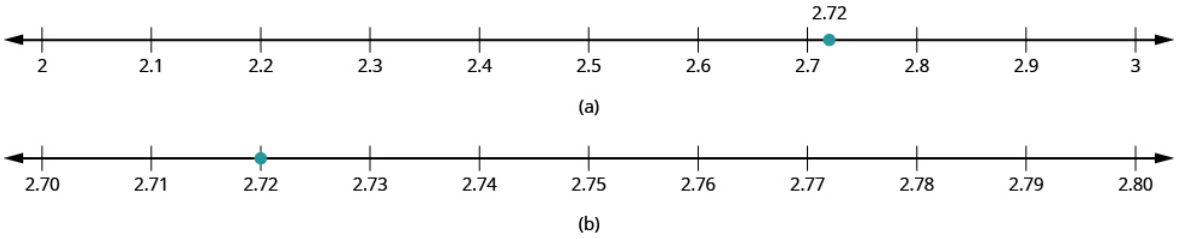


Figure 1.4.2.3- (b) We see that 2.72 is closer to 3 than to 2. So, 2.72 rounded to the nearest whole number is 3. (b) We see that 2.72 is closer to 2.70 than 2.80. So we say that 2.72 rounded to the nearest tenth is 2.7.

Can we round decimals without number lines? Yes! We use a method based on the one we used to round whole numbers.

HOW TO: ROUND A DECIMAL

Step 1. Locate the given place value and mark it with an arrow.

Step 2. Underline the digit to the right of the given place value.

Step 3. Is this digit greater than or equal to 5?

- Yes - add 1 to the digit in the given place value.
- No - do not change the digit in the given place value

Step 4. Rewrite the number, removing all digits to the right of the given place value.

Example 1.4.2.9:

Round 18.379 to the nearest hundredth.

Solution

Locate the hundredths place and mark it with an arrow.	hundredths place ↓ 18.379
Underline the digit to the right of the 7.	hundredths place ↓ 18.379
Because 9 is greater than or equal to 5, add 1 to the 7.	18.379 (add 1 / delete)
Rewrite the number, deleting all digits to the right of the hundredths place.	18.38

18.38 is 18.379 rounded to the nearest hundredth.

Exercise 1.4.2.17:

Round to the nearest hundredth: 1.047.

Answer

1.05

Exercise 1.4.2.18:

Round to the nearest hundredth: 9.173.

Answer

9.17

Example 1.4.2.10:

Round 18.379 to the nearest (a) tenth (b) whole number.

Solution

(a) Round 18.379 to the nearest tenth.

Locate the tenths place and mark it with an arrow.	
Underline the digit to the right of the tenths digit.	
Because 7 is greater than or equal to 5, add 1 to the 3.	
Rewrite the number, deleting all digits to the right of the tenths place.	18.4

So, 18.379 rounded to the nearest tenth is 18.4.

(b) Round 18.379 to the nearest whole number.

Locate the ones place and mark it with an arrow.	
Underline the digit to the right of the ones place.	
Since 3 is not greater than or equal to 5, do not add 1 to the 8.	
Rewrite the number, deleting all digits to the right of the ones place.	18

So 18.379 rounded to the nearest whole number is 18.

Exercise 1.4.2.19:

Round 6.582 to the nearest (a) hundredth (b) tenth (c) whole number.

Answer a

6.58

Answer b

6.6

Answer c

7

Exercise 1.4.2.20:

Round 15.2175 to the nearest (a) thousandth (b) hundredth (c) tenth.

Answer a

15.218

Answer b

15.22

Answer c

15.2

ACCESS ADDITIONAL ONLINE RESOURCES

Introduction to Decimal Notation

Write a Number in Decimal Notation from Words

Identify Decimals on the Number Line

Rounding Decimals

Writing a Decimal as a Simplified Fraction

Practice Makes Perfect

Name Decimals In the following exercises, name each decimal.

1. 5.5
2. 7.8
3. 5.01
4. 14.02
5. 8.71
6. 2.64
7. 0.002
8. 0.005
9. 0.381
10. 0.479
11. −17.9
12. −31.4

Write Decimals

In the following exercises, translate the name into a decimal number.

13. Eight and three hundredths
14. Nine and seven hundredths
15. Twenty-nine and eighty-one hundredths
16. Sixty-one and seventy-four hundredths
17. Seven tenths
18. Six tenths

19. One thousandth
20. Nine thousandths
21. Twenty-nine thousandths
22. Thirty-five thousandths
23. Negative eleven and nine ten-thousandths
24. Negative fifty-nine and two ten-thousandths
25. Thirteen and three hundred ninety-five ten thousandths
26. Thirty and two hundred seventy-nine thousandths

Convert Decimals to Fractions or Mixed Numbers

In the following exercises, convert each decimal to a fraction or mixed number.

27. 1.99
28. 5.83
29. 15.7
30. 18.1
31. 0.239
32. 0.373
33. 0.13
34. 0.19
35. 0.011
36. 0.049
37. −0.00007
38. −0.00003
39. 6.4
40. 5.2
41. 7.05
42. 9.04
43. 4.006
44. 2.008
45. 10.25
46. 12.75
47. 1.324
48. 2.482
49. 14.125
50. 20.375

Locate Decimals on the Number Line

In the following exercises, locate each number on a number line.

51. 0.8
52. 0.3
53. −0.2
54. −0.9
55. 3.1
56. 2.7
57. −2.5
58. −1.6

Order Decimals

In the following exercises, order each of the following pairs of numbers, using < or >.

59. 0.9__0.6
60. 0.7__0.8
61. 0.37__0.63

62. 0.86__0.69
63. 0.6__0.59
64. 0.27__0.3
65. 0.91__0.901
66. 0.415__0.41
67. −0.5__−0.3
68. −0.1_−0.4
69. −0.62_−0.619
70. −7.31_−7.3

Round Decimals

In the following exercises, round each number to the nearest tenth.

71. 0.67
72. 0.49
73. 2.84
74. 4.63

In the following exercises, round each number to the nearest hundredth.

75. 0.845
76. 0.761
77. 5.7932
78. 3.6284
79. 0.299
80. 0.697
81. 4.098
82. 7.096

In the following exercises, round each number to the nearest (a) hundredth (b) tenth (c) whole number.

83. 5.781
84. 1.638
85. 63.479
86. 84.281

Everyday Math

87. **Salary Increase** Danny got a raise and now makes $58,965.95 a year. Round this number to the nearest: (a) dollar (b) thousand dollars (c) ten thousand dollars.
88. **New Car Purchase** Selena's new car cost $23,795.95. Round this number to the nearest: (a) dollar (b) thousand dollars (c) ten thousand dollars.
89. **Sales Tax** Hyo Jin lives in San Diego. She bought a refrigerator for $1624.99 and when the clerk calculated the sales tax it came out to exactly $142.186625. Round the sales tax to the nearest (a) penny (b) dollar.
90. **Sales Tax** Jennifer bought a $1,038.99 dining room set for her home in Cincinnati. She calculated the sales tax to be exactly $67.53435. Round the sales tax to the nearest (a) penny (b) dollar.

Writing Exercises

91. How does your knowledge of money help you learn about decimals?
92. Explain how you write "three and nine hundredths" as a decimal.
93. Jim ran a 100-meter race in 12.32 seconds. Tim ran the same race in 12.3 seconds. Who had the faster time, Jim or Tim? How do you know?
94. Gerry saw a sign advertising postcards marked for sale at "10 for 0.99¢." What is wrong with the advertised price?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
name decimals.			
write decimals.			
convert decimals to fractions or mixed numbers.			
locate decimals on the number line.			
order decimals.			
round decimals.			

(b) If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

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1.4.3: Decimal Operations (Part 1)

Learning Objectives

- Add and subtract decimals
- Multiply decimals
- Divide decimals
- Use decimals in money applications

be prepared!

Before you get started, take this readiness quiz.

1. Simplify $\frac{70}{100}$. If you missed this problem, review Example 4.3.1.
2. Multiply $\frac{3}{10} \cdot \frac{9}{10}$. If you missed this problem, review Example 4.3.7.
3. Divide $-36 \div (-9)$. If you missed this problem, review Example 3.7.3.

Add and Subtract Decimals

Let's take one more look at the lunch order from the start of Decimals, this time noticing how the numbers were added together.

$$\begin{array}{rl} \$3.45 & Sandwich \\ \$1.25 & Water \\ +\$0.33 & Tax \\ \hline \$5.03 & Total \end{array}$$

All three items (sandwich, water, tax) were priced in dollars and cents, so we lined up the dollars under the dollars and the cents under the cents, with the decimal points lined up between them. Then we just added each column, as if we were adding whole numbers. By lining up decimals this way, we can add or subtract the corresponding place values just as we did with whole numbers.

HOW TO: ADD OR SUBTRACT DECIMALS

Step 1. Write the numbers vertically so the decimal points line up.

Step 2. Use zeros as place holders, as needed.

Step 3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.

Example 1.4.3.1:

Add: 3.7 + 12.4.

Solution

Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 3.7 \\ +12.4 \\ \hline \end{array}$
Place holders are not needed since both numbers have the same number of decimal places.	

Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} {\scriptstyle 1} \\ 3.7 \\ +12.4 \\ \hline 16.1 \end{array}$

Exercise 1.4.3.1:

Add: 5.7 + 11.9.

Answer

17.6

Exercise 1.4.3.2:

Add: 18.32 + 14.79.

Answer

13.11

Example 1.4.3.2:

Add: 23.5 + 41.38.

Solution

Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 23.5 \\ +41.38 \\ \hline \end{array}$
Place 0 as a place holder after the 5 in 23.5, so that both numbers have two decimal places.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline \end{array}$
Add the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} 23.50 \\ +41.38 \\ \hline 64.88 \end{array}$

Exercise 1.4.3.3:

Add: 4.8 + 11.69.

Answer

16.49

Exercise 1.4.3.4:

Add: 5.123 + 18.47.

Answer

23.593

How much change would you get if you handed the cashier a $20 bill for a $14.65 purchase? We will show the steps to calculate this in the next example.

Example 1.4.3.3:

Subtract: $20 - 14.65$.

Solution

Write the numbers vertically so the decimal points line up. Remember 20 is a whole number, so place the decimal point after the 0.	$\begin{array}{r} 20. \\ -14.65 \\ \hline \end{array}$
Place two zeros after the decimal point in 20, as place holders so that both numbers have two decimal places.	$\begin{array}{r} 20.00 \\ -14.65 \\ \hline \end{array}$
Subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.	$\begin{array}{r} {\scriptstyle 1}\,{\scriptstyle 9}\,{\scriptstyle 9}\,{\scriptstyle 9} \\ 1\,\cancel{10}\,\cancel{10}\,\cancel{10} \\ \cancel{2}\,\cancel{0}.\,\cancel{0}\,\cancel{0} \\ -1\;4\;.\;6\;5 \\ \hline 5\;.\;3\;5 \end{array}$

Exercise 1.4.3.5:

Subtract: $10 - 9.58$.

Answer

0.42

Exercise 1.4.3.6:

Subtract: $50 - 37.42$.

Answer

12.58

Example 1.4.3.4:

Subtract: $2.51 - 7.4$.

Solution

If we subtract 7.4 from 2.51, the answer will be negative since $7.4 > 2.51$. To subtract easily, we can subtract 2.51 from 7.4. Then we will place the negative sign in the result.

Write the numbers vertically so the decimal points line up.	$\begin{array}{r} 7.4 \\ -2.51 \\ \hline \end{array}$
Place zero after the 4 in 7.4 as a place holder, so that both numbers have two decimal places.	$\begin{array}{r} 7.40 \\ -2.51 \\ \hline \end{array}$

Subtract and place the decimal in the answer.	$\begin{array}{r} 7.40 \\ -2.51 \\ \hline 4.89 \end{array}$
Remember that we are really subtracting 2.51 − 7.4 so the answer is negative.	2.51 − 7.4 = − 4.89

? Exercise 1.4.3.7:

Subtract: 4.77 − 6.3.

Answer

-1.53

? Exercise 1.4.3.8:

Subtract: 8.12 − 11.7.

Answer

-3.58

Multiply Decimals

Multiplying decimals is very much like multiplying whole numbers—we just have to determine where to place the decimal point. The procedure for multiplying decimals will make sense if we first review multiplying fractions.

Do you remember how to multiply fractions? To multiply fractions, you multiply the numerators and then multiply the denominators. So let's see what we would get as the product of decimals by converting them to fractions first. We will do two examples side-by-side in Table 5.22. Look for a pattern.

Table 1.4.3.1

	A	B
	(0.3)(0.7)	(0.2)(0.46)
Convert to fractions.	$\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)$ (1.4.3.1)	$\left(\frac{2}{10}\right)\left(\frac{46}{100}\right)$ (1.4.3.2)
Multiply.	$\frac{21}{100}$ (1.4.3.3)	$\frac{92}{1000}$ (1.4.3.4)
Convert back to decimals	0.21	0.092

There is a pattern that we can use. In A, we multiplied two numbers that each had one decimal place, and the product had two decimal places. In B, we multiplied a number with one decimal place by a number with two decimal places, and the product had three decimal places.

How many decimal places would you expect for the product of (0.01)(0.004)? If you said "five", you recognized the pattern. When we multiply two numbers with decimals, we count all the decimal places in the factors—in this case two plus three—to get the number of decimal places in the product—in this case five.

$$(0.01)\ (0.004) = 0.00004$$
$$\text{2 places}\quad \text{3 places}\quad \text{5 places}$$

$$\left(\frac{1}{100}\right)\left(\frac{4}{1000}\right) = \frac{4}{100{,}000}$$

Once we know how to determine the number of digits after the decimal point, we can multiply decimal numbers without converting them to fractions first. The number of decimal places in the product is the sum of the number of decimal places in the factors.

The rules for multiplying positive and negative numbers apply to decimals, too, of course.

Definition: Multiplying Two Numbers

When multiplying two numbers,

- if their signs are the same, the product is positive.
- if their signs are different, the product is negative.

When you multiply signed decimals, first determine the sign of the product and then multiply as if the numbers were both positive. Finally, write the product with the appropriate sign.

HOW TO: MULTIPLY DECIMAL NUMBERS

Step 1. Determine the sign of the product.

Step 2. Write the numbers in vertical format, lining up the numbers on the right.

Step 3. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.

Step 4. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors. If needed, use zeros as placeholders.

Step 5. Write the product with the appropriate sign.

Example 1.4.3.5:

Multiply: (3.9)(4.075).

Solution

Determine the sign of the product. The signs are the same.	The product will be positive.
Write the numbers in vertical format, lining up the numbers on the right.	$\begin{array}{r} 4.075 \\ \times 3.9 \\ \hline \end{array}$
Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.	$\begin{array}{r} 4.075 \\ \times 3.9 \\ \hline 36675 \\ 12225 \\ \hline 158925 \end{array}$
Place the decimal point. Add the number of decimal places in the factors (1 + 3). Place the decimal point 4 places from the right.	$\begin{array}{rl} 4.075 & 3\ places \\ \times 3.9 & 1\ place \\ \hline 36675 & \\ 12225 & \\ \hline 158925 & 4\ places \end{array}$
The product is positive.	(3.9)(4.075) = 15.8925

Exercise 1.4.3.9:

Multiply: 4.5(6.107).

Answer

27.4815

Exercise 1.4.3.10:

Multiply: 10.79(8.12).

Answer

87.6148

Example 1.4.3.6:

Multiply: (−8.2)(5.19).

Solution

The signs are different.	The product will be negative.
Write in vertical format, lining up the numbers on the right.	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline \end{array}$
Multiply.	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline 1038 \\ 4152 \\ \hline 42558 \end{array}$
Place the decimal point 3 places from the right. (−8.2)(5.19) 1 place 2 places	$\begin{array}{r} 5.19 \\ \times 8.2 \\ \hline 1038 \\ 4152 \\ \hline 42.558 \end{array}$
The product is negative.	(−8.2)(5.19) = −42.558

Exercise 1.4.3.11:

Multiply: (4.63)(−2.9).

Answer

−13.427

Exercise 1.4.3.12:

Multiply: (−7.78)(4.9).

Answer

-38.122

In the next example, we'll need to add several placeholder zeros to properly place the decimal point.

Example 1.4.3.7:

Multiply: (0.03)(0.045).

Solution

The product is positive.	(0.03)(0.045)
Write in vertical format, lining up the numbers on the right.	0.045 $\times 0.03$
Multiply.	0.045 $\times 0.03$ 135
The decimal point must be 5 places from the right. (0.03)(0.045) 2 places 3 places Add zeros as needed to get the 5 places.	0.045 $\times 0.03$ 0.00135
The product is positive.	(0.03)(0.045) = 0.00135

Exercise 1.4.3.13:

Multiply: (0.04)(0.087).

Answer

0.00348

Exercise 1.4.3.14:

Multiply: (0.09)(0.067).

Answer

0.00603

Multiply by Powers of 10

In many fields, especially in the sciences, it is common to multiply decimals by powers of 10. Let's see what happens when we multiply 1.9436 by some powers of 10.

1.9436(10)	1.9436(100)	1.9436(1000)
1.9436 × 10 19.4360	1.9436 × 100 194.3600	1.9436 × 1000 1943.6000

Look at the results without the final zeros. Do you notice a pattern?

$$1.9436(10) = 19.436$$
$$1.9436(100) = 194.36$$
$$1.9436(1000) = 1943.6$$

The number of places that the decimal point moved is the same as the number of zeros in the power of ten. Table 5.26 summarizes the results.

Table 1.4.3.2

Multiply by	Number of zeros	Number of places decimal point moves
10	1	1 place to the right
100	2	2 places to the right
1,000	3	3 places to the right
10,000	4	4 places to the right

We can use this pattern as a shortcut to multiply by powers of ten instead of multiplying using the vertical format. We can count the zeros in the power of 10 and then move the decimal point that same of places to the right. So, for example, to multiply 45.86 by 100, move the decimal point 2 places to the right.

$$45.86 \times 100 = 4586.$$

Sometimes when we need to move the decimal point, there are not enough decimal places. In that case, we use zeros as placeholders. For example, let's multiply 2.4 by 100. We need to move the decimal point 2 places to the right. Since there is only one digit to the right of the decimal point, we must write a 0 in the hundredths place.

$$2.4 \times 100 = 240.$$

HOW TO: MULTIPLY A DECIMAL BY A POWER OF 10

Step 1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.

Step 2. Write zeros at the end of the number as placeholders if needed.

Example 1.4.3.8:

Multiply 5.63 by factors of (a) 10 (b) 100 (c) 1000.

Solution

By looking at the number of zeros in the multiple of ten, we see the number of places we need to move the decimal to the right.

(a) 5.63(10)

There is 1 zero in 10, so move the decimal point 1 place to the right.	5.63
	56.3

(b) 5.63(100)

There are 2 zeros in 100, so move the decimal point 2 places to the right.	5.63
	563

(c) 5.63(1000)

There are 3 zeros in 1000, so move the decimal point 3 places to the right.	5.63

A zero must be added at the end.	5,630

Exercise 1.4.3.15:

Multiply 2.58 by factors of (a) 10 (b) 100 (c) 1000.

Answer a

25.8

Answer b

258

Answer c

$2,580$

Exercise 1.4.3.16:

Multiply 14.2 by factors of (a) 10 (b) 100 (c) 1000.

Answer a

142

Answer b

$1,420$

Answer c

$14,200$

Contributors and Attributions

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1.4.4: Decimal Operations (Part 2)

Divide Decimals

Just as with multiplication, division of decimals is very much like dividing whole numbers. We just have to figure out where the decimal point must be placed.

To understand decimal division, let's consider the multiplication problem

$$(0.2)(4) = 0.8 \tag{1.4.4.1}$$

Remember, a multiplication problem can be rephrased as a division problem. So we can write 0.8 ÷ 4 = 0.2 We can think of this as "If we divide 8 tenths into four groups, how many are in each group?" Figure 1.4.4.1 shows that there are four groups of two-tenths in eight-tenths. So 0.8 ÷ 4 = 0.2.

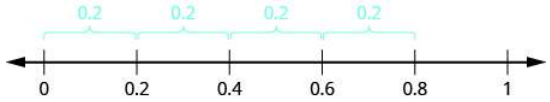


Figure 1.4.4.1

Using long division notation, we would write

$$4\overline{)0.8}\quad 0.2$$

Notice that the decimal point in the quotient is directly above the decimal point in the dividend.

To divide a decimal by a whole number, we place the decimal point in the quotient above the decimal point in the dividend and then divide as usual. Sometimes we need to use extra zeros at the end of the dividend to keep dividing until there is no remainder.

HOW TO: DIVIDE A DECIMAL BY A WHOLE NUMBER

Step 1. Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.

Step 2. Divide as usual.

Example 1.4.4.9:

Divide: 0.12 ÷ 3.

Solution

Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.	$3\overline{)0.12}$
Divide as usual. Since 3 does not go into 0 or 1 we use zeros as placeholders.	$3\overline{)0.12}\quad 0.04$ 12 0

0.12 ÷ 3 = 0.04

Exercise 1.4.4.17:

Divide: 0.28 ÷ 4.

Answer

0.07

Exercise 1.4.4.18:

Divide: 0.56 ÷ 7.

Answer

0.08

In everyday life, we divide whole numbers into decimals—money—to find the price of one item. For example, suppose a case of 24 water bottles cost $3.99. To find the price per water bottle, we would divide $3.99 by 24, and round the answer to the nearest cent (hundredth).

Example 1.4.4.10:

Divide: $3.99 ÷ 24.

Solution

Place the decimal point in the quotient above the decimal point in the dividend.	$24\overline{)3.99}$
Divide as usual. When do we stop? Since this division involves money, we round it to the nearest cent (hundredth). To do this, we must carry the division to the thousandths place.	$\begin{array}{r} 0.166 \\ 24\overline{)3.990} \\ \underline{24} \\ 159 \\ \underline{144} \\ 150 \\ \underline{144} \\ 6 \end{array}$
Round to the nearest cent.	$\$0.166 \approx \$0.17$ (1.4.4.2)

$3.99 ÷ 24 ≈ $0.17

This means the price per bottle is 17 cents.

Exercise 1.4.4.19:

Divide: $6.99 ÷ 36.

Answer

$0.19

Exercise 1.4.4.20:

Divide: $4.99 ÷ 12.

Answer

$0.42

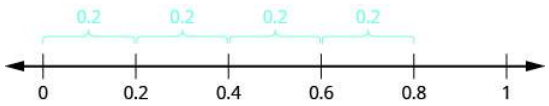
Divide a Decimal by Another Decimal

So far, we have divided a decimal by a whole number. What happens when we divide a decimal by another decimal? Let's look at the same multiplication problem we looked at earlier, but in a different way.

$$(0.2)(4) = 0.8 \quad (1.4.4.3)$$

Remember, again, that a multiplication problem can be rephrased as a division problem. This time we ask, “Ho w many times does 0.2 go into 0.8?” Because (0.2)(4) = 0.8, we can say that 0.2 goes into 0.8 four times. This means that 0.8 divided by 0.2 is 4.

$$0.8 \div 0.2 = 4 \tag{1.4.4.4}$$



We would get the same answer, 4, if we divide 8 by 2, both whole numbers. Why is this so? Let’s think about the division problem as a fraction.

$$\frac{0.8}{0.2} \tag{1.4.4.5}$$

$$\frac{(0.8)10}{(0.2)10} \tag{1.4.4.6}$$

$$\frac{8}{2} \tag{1.4.4.7}$$

$$4 \tag{1.4.4.8}$$

We multiplied the numerator and denominator by 10 and ended up just dividing 8 by 2. To divide decimals, we multiply both the numerator and denominator by the same power of 10 to make the denominator a whole number. Because of the Equivalent Fractions Property, we haven’t changed the value of the fraction. The effect is to move the decimal points in the numerator and denominator the same number of places to the right.

We use the rules for dividing positive and negative numbers with decimals, too. When dividing signed decimals, first determine the sign of the quotient and then divide as if the numbers were both positive. Finally, write the quotient with the appropriate sign. It may help to review the vocabulary for division:

$a \div b$ — a dividend, b divisor

$\frac{a}{b}$ — a dividend, b divisor

$b\overline{)a}$ — b divisor, a dividend

HOW TO: DIVIDE DECIMAL NUMBERS

Step 1. Determine the sign of the quotient.

Step 2. Make the divisor a whole number by moving the decimal point all the way to the right. Move the decimal point in the dividend the same number of places to the right, writing zeros as needed.

Step 3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.

Step 4. Write the quotient with the appropriate sign.

Example 1.4.4.11:

Divide: −2.89 ÷ (3.4).

Solution

Determine the sign of the quotient.	The quotient will be negative.
Make the divisor the whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places to the right.	$3.4\overline{)2.89}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend. Add zeros as needed until the remainder is zero.	$34.\overline{)28.90}$ → 0.85; 272; 170; 170; 0

Write the quotient with the appropriate sign.	$-2.89 \div (3.4) = -0.85$

Exercise 1.4.4.21:

Divide: $-1.989 \div 5.1$.

Answer

-0.39

Exercise 1.4.4.22:

Divide: $-2.04 \div 5.1$.

Answer

-0.4

Example 1.4.4.12:

Divide: $-25.65 \div (-0.06)$.

Solution

The signs are the same.	The quotient is positive.
Make the divisor a whole number by 'moving' the decimal point all the way to the right. 'Move' the decimal point in the dividend the same number of places.	$0.06\overline{)25.65}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 427.5 \\ 006.\overline{)2565.0} \\ \underline{-24} \\ 16 \\ \underline{-12} \\ 45 \\ \underline{-42} \\ 30 \\ \underline{30} \end{array}$
Write the quotient with the appropriate sign.	$-25.65 \div (-0.06) = 427.5$

Exercise 1.4.4.23:

Divide: $-23.492 \div (-0.04)$.

Answer

587.3

Exercise 1.4.4.24:

Divide: $-4.11 \div (-0.12)$.

Answer

34.25

Now we will divide a whole number by a decimal number.

Example 1.4.4.13

Divide: 4 ÷ (0.05).

Solution

The signs are the same.	The quotient is positive.
Make the divisor a whole number by 'moving' the decimal point all the way to the right. Move the decimal point in the dividend the same number of places, adding zeros as needed.	$0.05\overline{)4.00}$
Divide. Place the decimal point in the quotient above the decimal point in the dividend.	$\begin{array}{r} 80. \\ 5\overline{)400.} \\ \underline{40} \\ 00 \\ \underline{00} \end{array}$
Write the quotient with the appropriate sign.	4 ÷ 0.05 = 80

We can relate this example to money. How many nickels are there in four dollars? Because 4 ÷ 0.05 = 80, there are 80 nickels in $4.

Exercise 1.4.4.25:

Divide: 6 ÷ 0.03.

Answer

200

Exercise 1.4.4.26:

Divide: 7 ÷ 0.02

Answer

350

Use Decimals in Money Applications

We often apply decimals in real life, and most of the applications involving money. The Strategy for Applications we used in The Language of Algebra gives us a plan to follow to help find the answer. Take a moment to review that strategy now.

Strategy for Applications

1. Identify what you are asked to find.
2. Write a phrase that gives the information to find it.
3. Translate the phrase to an expression.
4. Simplify the expression.
5. Answer the question with a complete sentence.

Example 1.4.4.14:

Paul received $50 for his birthday. He spent $31.64 on a video game. How much of Paul's birthday money was left?

Solution

What are you asked to find?	How much did Paul have left?
Write a phrase.	$50 less $31.64
Translate.	$50 - 31.64$
Simplify.	18.36
Write a sentence.	Paul has $18.36 left.

Exercise 1.4.4.27:

Nicole earned $35 for babysitting her cousins, then went to the bookstore and spent $18.48 on books and coffee. How much of her babysitting money was left?

Answer

$16.52

Exercise 1.4.4.28:

Amber bought a pair of shoes for $24.75 and a purse for $36.90. The sales tax was $4.32. How much did Amber spend?

Answer

$65.97

Example 1.4.4.15:

Jessie put 8 gallons of gas in her car. One gallon of gas costs $3.529. How much does Jessie owe for the gas? (Round the answer to the nearest cent.)

Solution

What are you asked to find?	How much did Jessie owe for all the gas?
Write a phrase.	8 times the cost of one gallon of gas
Translate.	$8($3.529)$
Simplify.	$28.232
Round to the nearest cent.	$28.23
Write a sentence.	Jessie owes $28.23 for her gas purchase.

Exercise 1.4.4.29:

Hector put 13 gallons of gas into his car. One gallon of gas costs $3.175. How much did Hector owe for the gas? Round to the nearest cent.

Answer

$41.28

Exercise 1.4.4.30:

Christopher bought 5 pizzas for the team. Each pizza cost $9.75. How much did all the pizzas cost?

Answer

$48.75

Example 1.4.4.16:

Four friends went out for dinner. They shared a large pizza and a pitcher of soda. The total cost of their dinner was $31.76. If they divide the cost equally, how much should each friend pay?

Solution

What are you asked to find?	How much should each friend pay?
Write a phrase.	$31.76 divided equally among the four friends.
Translate to an expression.	$31.76 ÷ 4
Simplify.	$7.94
Write a sentence.	Each friend should pay $7.94 for his share of the dinner.

Exercise 1.4.4.31:

Six friends went out for dinner. The total cost of their dinner was $92.82. If they divide the bill equally, how much should each friend pay?

Answer

$15.47

Exercise 1.4.4.32:

Chad worked 40 hours last week and his paycheck was $570. How much does he earn per hour?

Answer

$14.25

Be careful to follow the order of operations in the next example. Remember to multiply before you add.

Example 1.4.4.17:

Marla buys 6 bananas that cost $0.22 each and 4 oranges that cost $0.49 each. How much is the total cost of the fruit?

Solution

What are you asked to find?	How much is the total cost of the fruit?
Write a phrase.	6 times the cost of each banana plus 4 times the cost of each orange
Translate to an expression.	6($0.22) + 4($0.49)
Simplify.	$1.32 + $1.96
Add.	$3.28

Write a sentence.	Marla's total cost for the fruit is $3.28.

Exercise 1.4.4.33:

Suzanne buys 3 cans of beans that cost $0.75 each and 6 cans of corn that cost $0.62 each. How much is the total cost of these groceries?

Answer

$5.97

Exercise 1.4.4.34:

Lydia bought movie tickets for the family. She bought two adult tickets for $9.50 each and four children's tickets for $6.00 each. How much did the tickets cost Lydia in all?

Answer

$43.00

ACCESS ADDITIONAL ONLINE RESOURCES

Adding and Subtracting Decimals

Multiplying Decimals

Multiplying by Powers of Ten

Dividing Decimals

Dividing by Powers of Ten

Practice Makes Perfect

Add and Subtract Decimals

In the following exercises, add or subtract.

95. 16.92 + 7.56
96. 18.37 + 9.36
97. 256.37 − 85.49
98. 248.25 − 91.29
99. 21.76 − 30.99
100. 15.35 − 20.88
101. 37.5 + 12.23
102. 38.6 + 13.67
103. −16.53 − 24.38
104. −19.47 − 32.58
105. −38.69 + 31.47
106. −29.83 + 19.76
107. −4.2 + (− 9.3)
108. −8.6 + (− 8.6)
109. 100 − 64.2
110. 100 − 65.83
111. 72.5 − 100
112. 86.2 − 100
113. 15 + 0.73

114. 27 + 0.87
115. 2.51 + 40
116. 9.38 + 60
117. 91.75 − (− 10.462)
118. 94.69 − (− 12.678)
119. 55.01 − 3.7
120. 59.08 − 4.6
121. 2.51 − 7.4
122. 3.84 − 6.1

Multiply Decimals

In the following exercises, multiply.

123. (0.3)(0.4)
124. (0.6)(0.7)
125. (0.24)(0.6)
126. (0.81)(0.3)
127. (5.9)(7.12)
128. (2.3)(9.41)
129. (8.52)(3.14)
130. (5.32)(4.86)
131. (−4.3)(2.71)
132. (− 8.5)(1.69)
133. (−5.18)(− 65.23)
134. (− 9.16)(− 68.34)
135. (0.09)(24.78)
136. (0.04)(36.89)
137. (0.06)(21.75)
138. (0.08)(52.45)
139. (9.24)(10)
140. (6.531)(10)
141. (55.2)(1,000)
142. (99.4)(1,000)

Divide Decimals

In the following exercises, divide.

143. 0.15 ÷ 5
144. 0.27 ÷ 3
145. 4.75 ÷ 25
146. 12.04 ÷ 43
147. $8.49 ÷ 12
148. $16.99 ÷ 9
149. $117.25 ÷ 48
150. $109.24 ÷ 36
151. 0.6 ÷ 0.2
152. 0.8 ÷ 0.4
153. 1.44 ÷ (− 0.3)
154. 1.25 ÷ (− 0.5)
155. −1.75 ÷ (− 0.05)
156. −1.15 ÷ (− 0.05)
157. 5.2 ÷ 2.5
158. 6.5 ÷ 3.25
159. 12 ÷ 0.08

160. 5 ÷ 0.04
161. 11 ÷ 0.55
162. 14 ÷ 0.35

Mixed Practice

In the following exercises, simplify.

163. 6(12.4 − 9.2)
164. 3(15.7 − 8.6)
165. $24(0.5) + (0.3)^2$
166. $35(0.2) + (0.9)^2$
167. 1.15(26.83 + 1.61)
168. 1.18(46.22 + 3.71)
169. \$45 + 0.08(\$45)
170. \$63 + 0.18(\$63)
171. 18 ÷ (0.75 + 0.15)
172. 27 ÷ (0.55 + 0.35)
173. (1.43 + 0.27) ÷ (0.9 − 0.05)
174. (1.5 − 0.06) ÷ (0.12 + 0.24)
175. [\$75.42 + 0.18(\$75.42)] ÷ 5
176. [\$56.31 + 0.22(\$56.31)] ÷ 4

Use Decimals in Money Applications

In the following exercises, use the strategy for applications to solve.

177. **Spending money** Brenda got \$40 from the ATM. She spent \$15.11 on a pair of earrings. How much money did she have left?
178. **Spending money** Marissa found \$20 in her pocket. She spent \$4.82 on a smoothie. How much of the \$20 did she have left?
179. **Shopping** Adam bought a t-shirt for \$18.49 and a book for \$8.92 The sales tax was \$1.65. How much did Adam spend?
180. **Restaurant** Roberto's restaurant bill was \$20.45 for the entrée and \$3.15 for the drink. He left a \$4.40 tip. How much did Roberto spend?
181. **Coupon** Emily bought a box of cereal that cost \$4.29. She had a coupon for \$0.55 off, and the store doubled the coupon. How much did she pay for the box of cereal?
182. **Coupon** Diana bought a can of coffee that cost \$7.99. She had a coupon for \$0.75 off, and the store doubled the coupon. How much did she pay for the can of coffee?
183. **Diet** Leo took part in a diet program. He weighed 190 pounds at the start of the program. During the first week, he lost 4.3 pounds. During the second week, he had lost 2.8 pounds. The third week, he gained 0.7 pounds. The fourth week, he lost 1.9 pounds. What did Leo weigh at the end of the fourth week?
184. **Snowpack** On April 1, the snowpack at the ski resort was 4 meters deep, but the next few days were very warm. By April 5, the snow depth was 1.6 meters less. On April 8, it snowed and added 2.1 meters of snow. What was the total depth of the snow?
185. **Coffee** Noriko bought 4 coffees for herself and her coworkers. Each coffee was \$3.75. How much did she pay for all the coffees?
186. **Subway Fare** Arianna spends \$4.50 per day on subway fare. Last week she rode the subway 6 days. How much did she spend for the subway fares? 187. Income Mayra earns \$9.25 per hour. Last week she worked 32 hours. How much did she earn?
187. **Income** Peter earns \$8.75 per hour. Last week he worked 19 hours. How much did he earn?
188. **Hourly Wage** Alan got his first paycheck from his new job. He worked 30 hours and earned \$382.50. How much does he earn per hour?
189. **Hourly Wage** Maria got her first paycheck from her new job. She worked 25 hours and earned \$362.50. How much does she earn per hour?
190. **Restaurant** Jeannette and her friends love to order mud pie at their favorite restaurant. They always share just one piece of pie among themselves. With tax and tip, the total cost is \$6.00. How much does each girl pay if the total number sharing the mud pie is (a) 2? (b) 3? (c) 4? (d) 5? (e) 6?
191. **Pizza** Alex and his friends go out for pizza and video games once a week. They share the cost of a \$15.60 pizza equally. How much does each person pay if the total number sharing the pizza is (a) 2? (b) 3? (c) 4? (d) 5? (e) 6?

192. **Fast Food** At their favorite fast food restaurant, the Carlson family orders 4 burgers that cost $3.29 each and 2 orders of fries at $2.74 each. What is the total cost of the order?
193. **Home Goods** Chelsea needs towels to take with her to college. She buys 2 bath towels that cost $9.99 each and 6 washcloths that cost $2.99 each. What is the total cost for the bath towels and washcloths?
194. **Zoo** The Lewis and Chousmith families are planning to go to the zoo together. Adult tickets cost $29.95 and children's tickets cost $19.95. What will the total cost be for 4 adults and 7 children?
195. **Ice Skating** Jasmine wants to have her birthday party at the local ice skating rink. It will cost $8.25 per child and $12.95 per adult. What will the total cost be for 12 children and 3 adults?

Everyday Math

197. **Paycheck** Annie has two jobs. She gets paid $14.04 per hour for tutoring at City College and $8.75 per hour at a coffee shop. Last week she tutored for 8 hours and worked at the coffee shop for 15 hours. (a) How much did she earn? (b) If she had worked all 23 hours as a tutor instead of working both jobs, how much more would she have earned?
198. **Paycheck** Jake has two jobs. He gets paid $7.95 per hour at the college cafeteria and $20.25 at the art gallery. Last week he worked 12 hours at the cafeteria and 5 hours at the art gallery. (a) How much did he earn? (b) If he had worked all 17 hours at the art gallery instead of working both jobs, how much more would he have earned?

Writing Exercises

199. In the 2010 winter Olympics, two skiers took the silver and bronze medals in the Men's Super-G ski event. The silver medalist's time was 1 minute 30.62 seconds and bronze medalist's time was 1 minute 30.65 seconds. Whose time was faster? Find the difference in their times and then write the name of that decimal.
200. Find the quotient of 0.12 ÷ 0.04 and explain in words all the steps taken.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
add and subtract decimals.			
multiply decimals.			
divide decimals.			
use decimals in money applications.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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1.4.5: Decimals and Fractions (Part 1)

Learning Objectives

- Convert fractions to decimals
- Order decimals and fractions
- Simplify expressions using the order of operations
- Find the circumference and area of circles

be prepared!

Before you get started, take this readiness quiz.

1. Divide: 0.24 ÷ 8. If you missed this problem, review Example 5.4.9.
2. Order 0.64__0.6 using < or >. If you missed this problem, review Example 5.2.7.
3. Order −0.2__−0.1 using < or >. If you missed this problem, review Example 5.2.8.

Convert Fractions to Decimals

In Decimals, we learned to convert decimals to fractions. Now we will do the reverse—convert fractions to decimals. Remember that the fraction bar indicates division. So $\frac{4}{5}$ can be written $4 \div 5$ or $5\overline{)4}$. This means that we can convert a fraction to a decimal by treating it as a division problem.

Note: Convert a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.

Example 1.4.5.1:

Write the fraction $\frac{3}{4}$ as a decimal.

Solution

A fraction bar means division, so we can write the fraction 3 4 using division.	$4\overline{)3}$
Divide.	$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$

So the fraction $\frac{3}{4}$ is equal to 0.75.

Exercise 1.4.5.1:

Write the fraction as a decimal: $\frac{1}{4}$.

Answer

0.25

Exercise 1.4.5.2:

Write the fraction as a decimal: $\frac{3}{8}$.

Answer

0.375

Example 1.4.5.2:

Write the fraction $-\frac{7}{2}$ as a decimal.

Solution

The value of this fraction is negative. After dividing, the value of the decimal will be negative. We do the division ignoring the sign, and then write the negative sign in the answer.	$-\frac{7}{2}$	(1.4.5.1)
Divide 7 by 2.	$\begin{array}{r} 3.5 \\ 2\overline{)7.0} \\ \underline{6} \\ 10 \\ \underline{10} \\ 0 \end{array}$	

So, $-\frac{7}{2} = -3.5$.

Exercise 1.4.5.3:

Write the fraction as a decimal: $-\frac{9}{4}$.

Answer

-2.25

Exercise 1.4.5.4:

Write the fraction as a decimal: $-\frac{11}{2}$.

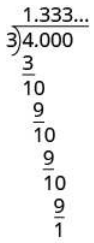
Answer

-5.5

Repeating Decimals

So far, in all the examples converting fractions to decimals the division resulted in a remainder of zero. This is not always the case. Let's see what happens when we convert the fraction $\frac{4}{3}$ to a decimal. First, notice that $\frac{4}{3}$ is an improper fraction. Its value is greater than 1. The equivalent decimal will also be greater than 1.

We divide 4 by 3.



No matter how many more zeros we write, there will always be a remainder of 1, and the threes in the quotient will go on forever. The number 1.333… is called a repeating decimal. Remember that the "…" means that the pattern repeats.

Definition: Repeating Decimal

A repeating decimal is a decimal in which the last digit or group of digits repeats endlessly.

How do you know how many 'repeats' to write? Instead of writing 1.333 … we use a shorthand notation by placing a line over the digits that repeat. The repeating decimal 1.333 … is written $1.\overline{3}$. The line above the 3 tells you that the 3 repeats endlessly. So $1.333\ldots = 1.\overline{3}$. For other decimals, two or more digits might repeat. Table 1.4.5.1 shows some more examples of repeating decimals.

Table 1.4.5.1

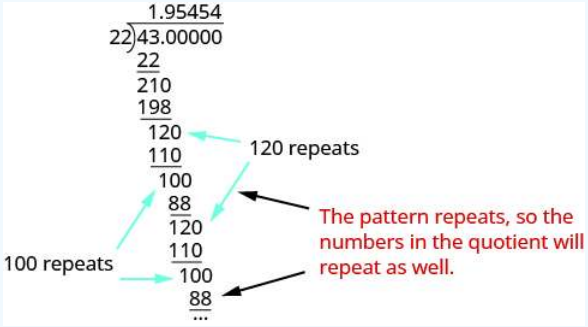
$1.333\ldots = 1.\overline{3}$	3 is the repeating digit
$4.1666\ldots = 4.1\overline{6}$	6 is the repeating digit
$4.161616\ldots = 4.\overline{16}$	16 is the repeating block
$0.271271271\ldots = 0.\overline{271}$	271 is the repeating block

Example 1.4.5.3:

Write $\frac{43}{22}$ as a decimal.

Solution

Divide 43 by 22.



Notice that the differences of 120 and 100 repeat, so there is a repeat in the digits of the quotient; 54 will repeat endlessly. The first decimal place in the quotient, 9, is not part of the pattern. So,

$$\frac{43}{22} = 1.9\overline{54} \tag{1.4.5.2}$$

Exercise 1.4.5.5:

Write as a decimal: $\frac{27}{11}$.

Answer

$2.\overline{45}$

Exercise 1.4.5.6:

Write as a decimal: $\frac{51}{22}$.

Answer

$2.3\overline{18}$

It is useful to convert between fractions and decimals when we need to add or subtract numbers in different forms. To add a fraction and a decimal, for example, we would need to either convert the fraction to a decimal or the decimal to a fraction.

Example 1.4.5.4:

Simplify: $\frac{7}{8} + 6.4$.

Solution

Change $\frac{7}{8}$ to a decimal.	$8\overline{)7.000}$ → 0.875; 64; 60; 56; 40; 40; 0	$0.875 + 6.4$
Add.		7.275

Exercise 1.4.5.7:

Simplify: $\frac{3}{8} + 4.9$.

Answer

5.275

Exercise 1.4.5.8:

Simplify: $5.7 + \frac{13}{20}$.

Answer

6.35

Order Decimals and Fractions

In Decimals, we compared two decimals and determined which was larger. To compare a decimal to a fraction, we will first convert the fraction to a decimal and then compare the decimals.

Example 1.4.5.5:

Order $\frac{3}{8}$__0.4 using < or >.

Solution

Convert $\frac{3}{8}$ to a decimal.	0.375__0.4
Compare 0.375 to 0.4	0.375 < 0.4
Rewrite with the original fraction.	$\frac{3}{8}$ < 0.4

Exercise 1.4.5.9:

Order each of the following pairs of numbers, using < or >.

$$\frac{17}{20}\_\_\ 0.82 \tag{1.4.5.3}$$

Answer

>

Exercise 1.4.5.10:

Order each of the following pairs of numbers, using < or >.

$$\frac{3}{4}\_\_\ 0.785 \tag{1.4.5.4}$$

Answer

<

When ordering negative numbers, remember that larger numbers are to the right on the number line and any positive number is greater than any negative number.

Example 1.4.5.6:

Order −0.5___$-\frac{3}{4}$ using < or >.

Solution

Convert $-\frac{3}{4}$ to a decimal.	−0.5___−0.75
Compare −0.5 to −0.75.	−0.5 > −0.75
Rewrite the inequality with the original fraction.	$-0.5 > -\frac{3}{4}$

Exercise 1.4.5.11:

Order each of the following pairs of numbers, using < or >:

$$-\frac{5}{8}\_\_-0.58 \tag{1.4.5.5}$$

Answer

$<$

Exercise 1.4.5.12:

Order each of the following pairs of numbers, using < or >:

$$-0.53\_\_-\frac{11}{20} \tag{1.4.5.6}$$

Answer

$>$

Example 1.4.5.7:

Write the numbers $\frac{13}{20}$, 0.61, $\frac{11}{16}$ in order from smallest to largest.

Solution

Convert the fractions to decimals.	0.65, 0.61, 0.6875
Write the smallest decimal number first.	0.61, ____, _____
Write the next larger decimal number in the middle place.	0.61, 0.65, _____
Write the last decimal number (the larger) in the third place.	0.61, 0.65, 0.6875
Rewrite the list with the original fractions.	0.61, $\frac{13}{20}$, $\frac{11}{16}$

Exercise 1.4.5.13:

Write each set of numbers in order from smallest to largest: $\frac{7}{8}$, $\frac{4}{5}$, 0.82.

Answer

$\frac{4}{5}$, 0.82 $\frac{7}{8}$

Exercise 1.4.5.14:

Write each set of numbers in order from smallest to largest: 0.835, $\frac{13}{16}$, $\frac{3}{4}$.

Answer

$\frac{3}{4}$, $\frac{13}{16}$, 0.835

Simplify Expressions Using the Order of Operations

The order of operations introduced in Use the Language of Algebra also applies to decimals. Do you remember what the phrase "Please excuse my dear Aunt Sally" stands for?

Example 1.4.5.8:

Simplify the expressions: (a) $7(18.3 - 21.7)$ (b) $\frac{2}{3}(8.3 - 3.8)$

Solution

(a) $7(18.3 - 21.7)$

Simplify inside parentheses.	$7(-3.4)$
Multiply.	-23.8

(b) $\frac{2}{3}(8.3 - 3.8)$

Simplify inside parentheses.	$\frac{2}{3}(4.5)$	(1.4.5.7)
Write 4.5 as a fraction.	$\frac{2}{3}\left(\frac{4.5}{1}\right)$	(1.4.5.8)
Multiply.	$\frac{9}{3}$	(1.4.5.9)
Simplify.	3	(1.4.5.10)

Exercise 1.4.5.15:

Simplify: (a) $8(14.6 - 37.5)$ (b) $\frac{3}{5}(9.6 - 2.1)$.

Answer a

-183.2

Answer b

4.5

Exercise 1.4.5.16:

Simplify: (a) $25(25.69 - 56.74)$ (b) $\frac{2}{7}(11.9 - 4.2)$.

Answer a

-776.25

Answer b

2.2

Example 1.4.5.9:

Simplify each expression: (a) $6 \div 0.6 + (0.2)4 - (0.1)^2$ (b) $\left(\frac{1}{10}\right)^2 + (3.5)(0.9)$

Solution

(a) $6 \div 0.6 + (0.2)4 - (0.1)^2$

Simplify exponents.	$6 \div 0.6 + (0.2)4 - 0.01$
Divide.	$10 + (0.2)4 - 0.01$
Multiply.	$10 + 0.8 - 0.01$
Add.	$10.8 - 0.01$
Subtract.	10.79

(b) $\left(\frac{1}{10}\right)^2 + (3.5)(0.9)$

Simplify exponents.	$\frac{1}{100} + (3.5)(0.9)$
Multiply.	$\frac{1}{100} + 3.15$
Convert $\frac{1}{100}$ to a decimal.	$0.01 + 3.15$
Add.	3.16

Exercise 1.4.5.17:

Simplify: $9 \div 0.9 + (0.4)3 - (0.2)^2$.

Answer

11.16

Exercise 1.4.5.18:

Simplify: $\left(\frac{1}{2}\right)^2 + (0.3)(4.2)$.

Answer

1.51

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1.4.6: Decimals and Fractions (Part 2)

Find the Circumference and Area of Circles

The properties of circles have been studied for over 2,000 years. All circles have exactly the same shape, but their sizes are affected by the length of the **radius**, a line segment from the center to any point on the circle. A line segment that passes through a circle's center connecting two points on the circle is called a **diameter**. The diameter is twice as long as the radius. See Figure 1.4.6.1.

The size of a circle can be measured in two ways. The distance around a circle is called its **circumference**.

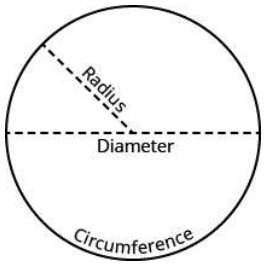
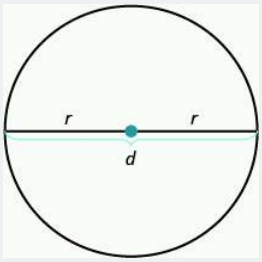


Figure 1.4.6.1

Archimedes discovered that for circles of all different sizes, dividing the circumference by the diameter always gives the same number. The value of this number is pi, symbolized by Greek letter π (pronounced pie). However, the exact value of π cannot be calculated since the decimal never ends or repeats (we will learn more about numbers like this in The Properties of Real Numbers.)

If we want the exact circumference or area of a circle, we leave the symbol π in the answer. We can get an approximate answer by substituting 3.14 as the value of π. We use the symbol ≈ to show that the result is approximate, not exact.

Note: Properties of Circles



r is the length of the radius.

d is the length of the diameter.

The circumference is $2\pi r$.

$$C = 2\pi r \tag{1.4.6.1}$$

The area is πr^2.

$$A = \pi r^2 \tag{1.4.6.2}$$

Since the diameter is twice the radius, another way to find the circumference is to use the formula $C = \pi d$.

Suppose we want to find the exact area of a circle of radius 10 inches. To calculate the area, we would evaluate the formula for the area when r = 10 inches and leave the answer in terms of π.

$$A = \pi r^2$$
$$A = \pi(10^2)$$
$$A = \pi \cdot 100$$

We write π after the 100. So the exact value of the area is A = 100π square inches. To approximate the area, we would substitute $\pi \approx 3.14$.

$$\begin{aligned} A &= 100\pi \\ &\approx 100 \cdot 3.14 \\ &\approx 314 \ square\ inches \end{aligned}$$

Remember to use square units, such as square inches, when you calculate the area.

Example 1.4.6.10:

A circle has radius 10 centimeters. Approximate its (a) circumference and (b) area.

Solution

(a) Find the circumference when r = 10.

Write the formula for circumference.	$C = 2\pi r$
Substitute 3.14 for π and 10 for r.	$C \approx 2(3.14)(10)$
Multiply.	C ≈ 62.8 centimeters

(b) Find the area when r = 10.

Write the formula for area.	$A = \pi r^2$
Substitute 3.14 for π and 10 for r.	$A \approx (3.14)(10)^2$
Multiply.	A ≈ 314 square centimeters

Exercise 1.4.6.19:

A circle has radius 50 inches. Approximate its (a) circumference and (b) area.

Answer a

314 in.

Answer b

7850 sq. in.

Exercise 1.4.6.20:

A circle has radius 100 feet. Approximate its (a) circumference and (b) area.

Answer a

628 ft.

Answer b

31,400 sq. ft.

Example 1.4.6.11:

A circle has radius 42.5 centimeters. Approximate its (a) circumference and (b) area.

Solution

(a) Find the circumference when r = 42.5.

Write the formula for circumference.	$C = 2\pi r$
Substitute 3.14 for π and 42.5 for r.	$C \approx 2(3.14)(42.5)$
Multiply.	C ≈ 266.9 centimeters

(b) Find the area when r = 42.5.

Write the formula for area.	$A = \pi r^2$
Substitute 3.14 for π and 42.5 for r.	$A \approx (3.14)(42.5)^2$
Multiply.	A ≈ 5671.625 square centimeters

? Exercise 1.4.6.21:

A circle has radius 51.8 centimeters. Approximate its (a) circumference and (b) area.

Answer a

325.304 cm

Answer b

8425.3736 sq. cm

? Exercise 1.4.6.22:

A circle has radius 26.4 meters. Approximate its (a) circumference and (b) area.

Answer a

165.792 m

Answer b

2188.4544 sq. m

Approximate π with a Fraction

Convert the fraction $\frac{22}{7}$ to a decimal. If you use your calculator, the decimal number will fill up the display and show 3.14285714. But if we round that number to two decimal places, we get 3.14, the decimal approximation of π. When we have a circle with radius given as a fraction, we can substitute $\frac{22}{7}$ for π instead of 3.14. And, since $\frac{22}{7}$ is also an approximation of π, we will use the ≈ symbol to show we have an approximate value.

✓ Example 1.4.6.12:

A circle has radius $\frac{14}{15}$ meter. Approximate its (a) circumference and (b) area.

Solution

(a) Find the circumference when r = $\frac{14}{15}$

Write the formula for circumference.	$C = 2\pi r$

Substitute $\frac{22}{7}$ for π and $\frac{14}{15}$ for r .	$C \approx 2\left(\frac{22}{7}\right)\left(\frac{14}{15}\right)$	(1.4.6.3)
Multiply.	C ≈ $\frac{88}{15}$ meters	

.(b) Find the area when r = $\frac{14}{15}$.

Write the formula for area.	$A = \pi r^2$	
Substitute $\frac{22}{7}$ for π and $\frac{14}{15}$ for r.	$A \approx \left(\frac{22}{7}\right)\left(\frac{14}{15}\right)^2$	(1.4.6.4)
Multiply.	A ≈ $\frac{616}{225}$ square meters	

? Exercise 1.4.6.23:

A circle has radius $\frac{5}{21}$ meters. Approximate its (a) circumference and (b) area.

Answer a

$\frac{220}{147}$ m

Answer b

$\frac{550}{3087}$ sq. m

? Exercise 1.4.6.24:

A circle has radius $\frac{10}{33}$ inches. Approximate its (a) circumference and (b) area.

Answer a

$\frac{40}{21}$ in.

Answer b

$\frac{200}{693}$ sq. in.

ACCESS ADDITIONAL ONLINE RESOURCES

Converting a Fraction to a Decimal - Part 2

Convert a Fraction to a Decimal (repeating)

Compare Fractions and Decimals using Inequality Symbols

Determine the Area of a Circle

Determine the Circumference of a Circle

Practice Makes Perfect

Convert Fractions to Decimals

In the following exercises, convert each fraction to a decimal.

201. $\frac{2}{5}$
202. $\frac{4}{5}$
203. $-\frac{3}{8}$
204. $-\frac{5}{8}$
205. $\frac{17}{20}$
206. $\frac{13}{20}$
207. $\frac{11}{4}$
208. $\frac{17}{4}$
209. $-\frac{310}{25}$
210. $-\frac{284}{25}$
211. $\frac{5}{9}$
212. $\frac{2}{9}$
213. $\frac{15}{11}$
214. $\frac{18}{11}$
215. $\frac{15}{111}$
216. $\frac{25}{111}$

In the following exercises, simplify the expression.

217. $\frac{1}{2}$ + 6.5
218. $\frac{1}{4}$ + 10.75
219. 2.4 + $\frac{5}{8}$
220. 3.9 + $\frac{9}{20}$
221. 9.73 + $\frac{17}{20}$
222. 6.29 + $\frac{21}{40}$

Order Decimals and Fractions

In the following exercises, order each pair of numbers, using < or >.

223. $\frac{1}{8}$ ___ 0.8
224. $\frac{1}{4}$ ___ 0.4
225. $\frac{2}{5}$ ___ 0.25
226. $\frac{3}{5}$ ___ 0.35

227. $0.725 \_\_\_ \frac{3}{4}$
228. $0.92 \_\_\_ \frac{7}{8}$
229. $0.66 \_\_\_ \frac{2}{3}$
230. $0.83 \_\_\_ \frac{5}{6}$
231. $-0.75 \_\_\_ -\frac{4}{5}$
232. $-0.44 \_\_\_ -\frac{9}{20}$
233. $-\frac{3}{4} \_\_\_ -0.925$
234. $-\frac{2}{3} \_\_\_ -0.632$

In the following exercises, write each set of numbers in order from least to greatest.

235. $\frac{3}{5}, \frac{9}{16}, 0.55$
236. $\frac{3}{8}, \frac{7}{20}, 0.36$
237. $0.702, \frac{13}{20}, \frac{5}{8}$
238. $0.15, \frac{3}{16}, \frac{1}{5}$
239. $-0.3, -\frac{1}{3}, -\frac{7}{20}$
240. $-0.2, -\frac{3}{20}, -\frac{1}{6}$
241. $-\frac{3}{4}, -\frac{7}{9}, -0.7$
242. $-\frac{8}{9}, -\frac{4}{5}, -0.9$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify.

243. $10(25.1 - 43.8)$
244. $30(18.1 - 32.5)$
245. $62(9.75 - 4.99)$
246. $42(8.45 - 5.97)$
247. $\frac{3}{4}(12.4 - 4.2)$
248. $\frac{4}{5}(8.6 + 3.9)$
249. $\frac{5}{12}(30.58 + 17.9)$
250. $\frac{9}{16}(21.96 - 9.8)$
251. $10 \div 0.1 + (1.8)4 - (0.3)^2$
252. $5 \div 0.5 + (3.9)6 - (0.7)^2$
253. $(37.1 + 52.7) \div (12.5 \div 62.5)$
254. $(11.4 + 16.2) \div (18 \div 60)$
255. $\left(\frac{1}{5}\right)^2 + (1.4)(6.5)$
256. $\left(\frac{1}{2}\right)^2 + (2.1)(8.3)$
257. $-\frac{9}{10} \cdot \frac{8}{15} + 0.25$

258. $-\frac{3}{8} \cdot \frac{14}{15} + 0.72$

Mixed Practice

In the following exercises, simplify. Give the answer as a decimal.

259. $3\frac{1}{4} - 6.5$
260. $5\frac{2}{5} - 8.75$
261. $10.86 \div \frac{2}{3}$
262. $5.79 \div \frac{3}{4}$
263. $\frac{7}{8}(103.48) + 1\frac{1}{2}(361)$
264. $\frac{5}{16}(117.6) + 2\frac{1}{3}(699)$
265. $3.6\left(\frac{9}{8} - 2.72\right)$
266. $5.1\left(\frac{12}{5} - 3.91\right)$

Find the Circumference and Area of Circles

In the following exercises, approximate the (a) circumference and (b) area of each circle. If measurements are given in fractions, leave answers in fraction form.

267. radius = 5 in.
268. radius = 20 in.
269. radius = 9 ft.
270. radius = 4 ft.
271. radius = 46 cm
272. radius = 38 cm
273. radius = 18.6 m
274. radius = 57.3 m
275. radius = $\frac{7}{10}$ mile
276. radius = $\frac{7}{11}$ mile
277. radius = $\frac{3}{8}$ yard
278. radius = $\frac{5}{12}$ yard
279. diameter = $\frac{5}{6}$ m
280. diameter = $\frac{3}{4}$ m

Everyday Math

281. Kelly wants to buy a pair of boots that are on sale for $\frac{2}{3}$ of the original price. The original price of the boots is $84.99. What is the sale price of the shoes?
282. An architect is planning to put a circular mosaic in the entry of a new building. The mosaic will be in the shape of a circle with radius of 6 feet. How many square feet of tile will be needed for the mosaic? (Round your answer up to the next whole number.)

Writing Exercises

283. Is it easier for you to convert a decimal to a fraction or a fraction to a decimal? Explain.
284. Describe a situation in your life in which you might need to find the area or circumference of a circle.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section

I can...	Confidently	With some help	No-I don't get it!
convert fractions to decimals.			
order decimals and fractions.			
simplify expressions using the order of operations.			
find the circumference and area of circles.			

(b) What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.4.7: Solve Equations with Decimals

Learning Objectives

- Determine whether a decimal is a solution of an equation
- Solve equations with decimals
- Translate to an equation and solve

be prepared!

Before you get started, take this readiness quiz.

1. Evaluate $x + \frac{2}{3}$ when $x = -\frac{1}{4}$. If you missed this problem, review Example 4.9.15.
2. Evaluate $15 - y$ when $y = -5$. If you missed this problem, review Example 3.6.12.
3. Solve $\frac{n}{-7} = 42$. If you missed this problem, review Example 4.12.5.

Determine Whether a Decimal is a Solution of an Equation

Solving equations with decimals is important in our everyday lives because money is usually written with decimals. When applications involve money, such as shopping for yourself, making your family's budget, or planning for the future of your business, you'll be solving equations with decimals.

Now that we've worked with decimals, we are ready to find solutions to equations involving decimals. The steps we take to determine whether a number is a solution to an equation are the same whether the solution is a whole number, an integer, a fraction, or a decimal. We'll list these steps here again for easy reference.

HOW TO: DETERMINE WHETHER A NUMBER IS A Solution TO AN EQUATION

- Step 1. Substitute the number for the variable in the equation.
- Step 2. Simplify the expressions on both sides of the equation.
- Step 3. Determine whether the resulting equation is true.
  - If so, the number is a solution.
  - If not, the number is not a solution.

Example 1.4.7.1:

Determine whether each of the following is a solution of $x - 0.7 = 1.5$: (a) $x = 1$ (b) $x = -0.8$ (c) $x = 2.2$

Solution

(a) $x - 0.7 = 1.5$

Substitute 1 for x.	$1 - 0.7 \stackrel{?}{=} 1.5$	(1.4.7.1)
Subtract.	$0.3 \neq 1.5$	(1.4.7.2)

Since $x = 1$ does not result in a true equation, 1 is not a solution to the equation.

(b) $x - 0.7 = 1.5$

Substitute -0.8 for x.	$-0.8 - 0.7 \stackrel{?}{=} 1.5$	(1.4.7.3)
Subtract.	$-1.5 \neq 1.5$	(1.4.7.4)

Since x = −0.8 does not result in a true equation, −0.8 is not a solution to the equation.

(c) x - 0.7 = 1.5

Substitute 2.2 for x.	$2.2 - 0.7 \stackrel{?}{=} 1.5$	(1.4.7.5)
Subtract.	$1.5 = 1.5$ ✓	(1.4.7.6)

Since x = 2.2 results in a true equation, 2.2 is a solution to the equation.

Exercise 1.4.7.1:

Determine whether each value is a solution of the given equation. x − 0.6 = 1.3: (a) x = 0.7 (b) x = 1.9 (c) x = −0.7

Answer a

no

Answer b

yes

Answer c

no

Exercise 1.4.7.2:

Determine whether each value is a solution of the given equation. y − 0.4 = 1.7: (a) y = 2.1 (b) y = 1.3 (c) −1.3

Answer a

yes

Answer b

no

Answer c

no

Solve Equations with Decimals

In previous chapters, we solved equations using the Properties of Equality. We will use these same properties to solve equations with decimals.

Properties of Equality

Subtraction Property of Equality For any numbers a, b, and c, If a = b, then a − c = b − c.	**Addition Property of Equality** For any numbers a, b, and c, If a = b, then a + c = b + c.
The Division Property of Equality For any numbers a, b, and c, and c ≠ 0 If a = b, then $\frac{a}{c} = \frac{b}{c}$	**The Multiplication Property of Equality** For any numbers a, b, and c, If a = b, then ac = bc

When you add, subtract, multiply or divide the same quantity from both sides of an equation, you still have equality.

Example 1.4.7.2:

Solve: y + 2.3 = −4.7.

Solution

We will use the Subtraction Property of Equality to isolate the variable.

Subtract 2.3 from each side, to undo the addition.	$y+2.3-2.3=-4.7-2.3$	(1.4.7.7)
Simplify.	$y=-7$	(1.4.7.8)

Check:

Substitute y = −7.	$-7+2.3\stackrel{?}{=}-4.7$	(1.4.7.9)
Simplify.	$-4.7=-4.7$ ✓	(1.4.7.10)

Since y = −7 makes y + 2.3 = −4.7 a true statement, we know we have found a solution to this equation.

Exercise 1.4.7.3:

Solve: y + 2.7 = −5.3.

Answer

$y=-8$

Exercise 1.4.7.4:

Solve: y + 3.6 = −4.8.

Answer

$y=-8.4$

Example 1.4.7.3:

Solve: a − 4.75 = −1.39.

Solution

We will use the Addition Property of Equality.

Add 4.75 to each side, to undo the subtraction.	$a-4.75+4.75=-1.39+4.75$	(1.4.7.11)
Simplify.	$a=3.36$	(1.4.7.12)

Check:

Substitute a = 3.36.	$3.36-4.75\stackrel{?}{=}-1.39$	(1.4.7.13)

	$-1.39 = -1.39$ ✓	(1.4.7.14)

Since the result is a true statement, a = 3.36 is a solution to the equation.

? Exercise 1.4.7.5:

Solve: a − 3.93 = −2.86.

Answer

$a = 1.07$

? Exercise 1.4.7.6:

Solve: n − 3.47 = −2.64.

Answer

$n = 0.83$

✓ Example 1.4.7.4:

Solve: −4.8 = 0.8n.

Solution

We will use the Division Property of Equality. Use the Properties of Equality to find a value for n.

We must divide both sides by 0.8 to isolate n.	$\frac{-4.8}{0.8} = \frac{0.8n}{08}$	(1.4.7.15)
Simplify.	$-6 = n$	(1.4.7.16)

Check:

Substitute n = -6.	$-4.8 \stackrel{?}{=} 0.8(-6)$	(1.4.7.17)
	$-4.8 = -4.8$ ✓	(1.4.7.18)

Since n = −6 makes −4.8 = 0.8n a true statement, we know we have a solution.

? Exercise 1.4.7.7:

Solve: −8.4 = 0.7b.

Answer

$b = -12$

Exercise 1.4.7.8:

Solve: −5.6 = 0.7c.

Answer

$c = -8$

Example 1.4.7.5:

Solve: $\frac{p}{-1.8} = -6.5$.

Solution

We will use the Multiplication Property of Equality.

Here, p is divided by −1.8. We must multiply by −1.8 to isolate p.	$-1.8\left(\frac{p}{-1.8}\right) = -1.8(-6.5)$	(1.4.7.19)
Multiply.	$p = 11.7$	(1.4.7.20)

Check:

Substitute p = 11.7.	$\frac{11.7}{-1.8} \stackrel{?}{=} -6.5$	(1.4.7.21)
	$-6.5 = -6.5$ ✓	(1.4.7.22)

A solution to $\frac{p}{-1.8} = -6.5$ is p = 11.7.

Exercise 1.4.7.9:

Solve: $\frac{c}{-2.6} = -4.5$.

Answer

$c = -11.7$

Exercise 1.4.7.10:

Solve: $\frac{b}{-1.2} = -5.4$.

Answer

$b = 6.48$

Translate to an Equation and Solve

Now that we have solved equations with decimals, we are ready to translate word sentences to equations and solve. Remember to look for words and phrases that indicate the operations to use.

Example 1.4.7.6:

Translate and solve: The difference of n and 4.3 is 2.1.

Solution

Translate.	$\underbrace{\text{The difference of } n \text{ and } 4.3}_{n-4.3}\ \underbrace{\text{is}}_{=}\ \underbrace{2.1.}_{2.1}$	
Add 4.3 to both sides of the equation.	$n-4.3+4.3=2.1+4.3$	(1.4.7.23)
Simplify.	$n=6.4$	(1.4.7.24)

Check:

Let n = 6.4.	Is the difference of 6.4 and 4.3 equal to 2.1?	
Translate.	$6.4-4.3\stackrel{?}{=}2.1$	(1.4.7.25)
Simplify.	$2.1=2.1$ ✓	(1.4.7.26)

Exercise 1.4.7.11:

Translate and solve: The difference of y and 4.9 is 2.8.

Answer

$y-4.9=2.8; y=7.7$

Exercise 1.4.7.12:

Translate and solve: The difference of z and 5.7 is 3.4.

Answer

$z-5.7=3.4; z=9.1$

Example 1.4.7.7:

Translate and solve: The product of −3.1 and x is 5.27.

Solution

Translate.	$\underbrace{\text{The product of } 3.1 \text{ and } x}_{-3.1x}\ \underbrace{\text{is}}_{=}\ \underbrace{5.27.}_{5.27}$	
Divide both sides by −3.1.	$\frac{-3.1x}{-3.1}=\frac{5.27}{-3.1}$	(1.4.7.27)
Simplify.	$x=-1.7$	(1.4.7.28)

Check:

Let x = −1.7.	Is the product of −3.1 and −1.7 equal to 5.27?	
Translate.	$-3.1(-1.7) \stackrel{?}{=} 5.27$	(1.4.7.29)
Simplify.	$5.27 = 5.27$ ✓	(1.4.7.30)

? Exercise 1.4.7.13:

Translate and solve: The product of −4.3 and x is 12.04.

Answer

$-4.3x = 12.04; x = -2.8$

? Exercise 1.4.7.14:

Translate and solve: The product of −3.1 and m is 26.66.

Answer

$-3.1m = 26.66; m = -8.6$

✓ Example 1.4.7.8:

Translate and solve: The quotient of p and −2.4 is 6.5.

Solution

Translate.	The quotient of p and −2.4 is 6.5. $\frac{p}{-2.4} = 6.5$	
Multiply both sides by −2.4.	$-2.4\left(\frac{p}{-2.4}\right) = -2.4(6.5)$	(1.4.7.31)
Simplify.	$p = -15.6$	(1.4.7.32)

Check:

Let p = −15.6.	Is the quotient of −15.6 and −2.4 equal to 6.5?	
Translate.	$\frac{-15.6}{-2.4} \stackrel{?}{=} 6.5$	(1.4.7.33)
Simplify.	$6.5 = 6.5$ ✓	(1.4.7.34)

? Exercise 1.4.7.15:

Translate and solve: The quotient of q and −3.4 is 4.5.

Answer

$\frac{1}{-3.4} = 4.5; q = -15.3$

Exercise 1.4.7.16:

Translate and solve: The quotient of r and −2.6 is 2.5.

Answer

$\frac{r}{-2.6} = 2.5; r = -6.5$

Example 1.4.7.9:

Translate and solve: The sum of n and 2.9 is 1.7.

Solution

Translate.	$\underbrace{\text{The sum of } n \text{ and } 2.9}_{n+2.9}\ \underbrace{\text{is}}_{=}\ \underbrace{1.7.}_{1.7}$	
Subtract 2.9 from each side.	$n + 2.9 - 2.9 = 1.7 - 2.9$	(1.4.7.35)
Simplify.	$n = -1.2$	(1.4.7.36)

Check:

Let n = −1.2.	Is the sum −1.2 and 2.9 equal to 1.7?	
Translate.	$-1.2 + 2.9 \stackrel{?}{=} 1.7$	(1.4.7.37)
Simplify.	$1.7 = 1.7$ ✓	(1.4.7.38)

Exercise 1.4.7.17:

Translate and solve: The sum of j and 3.8 is 2.6.

Answer

$j + 3.8 = 2.6; j = -1.2$

Exercise 1.4.7.18:

Translate and solve: The sum of k and 4.7 is 0.3.

Answer

$k + 4.7 = 0.3; k = -4.4$

ACCESS ADDITIONAL ONLINE RESOURCES

Solving One Step Equations Involving Decimals

Solve a One Step Equation With Decimals by Adding and Subtracting

Solve a One Step Equation With Decimals by Multiplying

Solve a One Step Equation With Decimals by Dividing

Practice Makes Perfect

Determine Whether a Decimal is a Solution of an Equation

In the following exercises, determine whether each number is a solution of the given equation.

285. $x - 0.8 = 2.3$
   a. $x = 2$
   b. $x = -1.5$
   c. $x = 3.1$

286. $y + 0.6 = -3.4$
   a. $y = -4$
   b. $y = -2.8$
   c. $y = 2.6$

287. $\frac{h}{1.5} = -4.3$
   a. $h = 6.45$
   b. $h = -6.45$
   c. $h = -2.1$

288. $0.75k = -3.6$
   a. $k = -0.48$
   b. $k = -4.8$
   c. $k = -2.7$

Solve Equations with Decimals

In the following exercises, solve the equation.

289. $y + 2.9 = 5.7$
290. $m + 4.6 = 6.5$
291. $f + 3.45 = 2.6$
292. $h + 4.37 = 3.5$
293. $a + 6.2 = -1.7$
294. $b + 5.8 = -2.3$
295. $c + 1.15 = -3.5$
296. $d + 2.35 = -4.8$
297. $n - 2.6 = 1.8$
298. $p - 3.6 = 1.7$
299. $x - 0.4 = -3.9$
300. $y - 0.6 = -4.5$
301. $j - 1.82 = -6.5$
302. $k - 3.19 = -4.6$
303. $m - 0.25 = -1.67$
304. $q - 0.47 = -1.53$
305. $0.5x = 3.5$
306. $0.4p = 9.2$
307. $-1.7c = 8.5$
308. $-2.9x = 5.8$
309. $-1.4p = -4.2$
310. $-2.8m = -8.4$
311. $-120 = 1.5q$
312. $-75 = 1.5y$
313. $0.24x = 4.8$
314. $0.18n = 5.4$
315. $-3.4z = -9.18$

316. −2.7u = −9.72
317. $\frac{a}{0.4}$ = −20
318. $\frac{b}{0.3}$ = −9
319. $\frac{x}{0.7}$ = −0.4
320. $\frac{y}{0.8}$ = −0.7
321. $\frac{p}{-5}$ = −1.65
322. $\frac{q}{-4}$ = −5.92
323. $\frac{r}{-1.2}$ = −6
324. $\frac{s}{-1.5}$ = −3

Mixed Practice

In the following exercises, solve the equation. Then check your solution.

325. x − 5 = −11
326. $-\frac{2}{5}$ = x + (\dfrac{3}{4}\)
327. p + 8 = −2
328. p + $\frac{2}{3} = \frac{1}{12}$
329. −4.2m = −33.6
330. q + 9.5 = −14
331. q + $\frac{5}{6} = \frac{1}{12}$
332. $\frac{8.6}{15}$ = − d
333. $\frac{7}{8}m = \frac{1}{10}$
334. $\frac{j}{-6.2}$ = −3
335. $-\frac{2}{3}$ = y + $\frac{3}{8}$
336. s − 1.75 = −3.2
337. $\frac{11}{20}$ = − f
338. −3.6b = 2.52
339. −4.2a = 3.36
340. −9.1n = −63.7
341. r − 1.25 = −2.7
342. $\frac{1}{4}n = \frac{7}{10}$
343. $\frac{h}{-3}$ = −8
344. y − 7.82 = −16

Translate to an Equation and Solve

In the following exercises, translate and solve.

345. The difference of n and 1.9 is 3.4.
346. The difference n and 1.5 is 0.8.
347. The product of −6.2 and x is −4.96.
348. The product of −4.6 and x is −3.22.
349. The quotient of y and −1.7 is −5.

350. The quotient of z and −3.6 is 3.
351. The sum of n and −7.3 is 2.4.
352. The sum of n and −5.1 is 3.8.

Everyday Math

353. Shawn bought a pair of shoes on sale for $78. Solve the equation 0.75p = 78 to find the original price of the shoes, p.
354. Mary bought a new refrigerator. The total price including sales tax was $1,350. Find the retail price, r, of the refrigerator before tax by solving the equation 1.08r = 1,350.

Writing Exercises

355. Think about solving the equation 1.2y = 60, but do not actually solve it. Do you think the solution should be greater than 60 or less than 60? Explain your reasoning. Then solve the equation to see if your thinking was correct.
356. Think about solving the equation 0.8x = 200, but do not actually solve it. Do you think the solution should be greater than 200 or less than 200? Explain your reasoning. Then solve the equation to see if your thinking was correct.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether a decimal is a solution of an equation.			
solve equations with decimals.			
translate to an equation and solve.			

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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1.4.8: Averages and Probability (Part 1)

Learning Objectives

- Calculate the mean of a set of numbers
- Find the median of a set of numbers
- Find the mode of a set of numbers
- Apply the basic definition of probability

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $\frac{4+9+2}{3}$. If you missed this problem, review Example 4.6.12.
2. Simplify: 4(8) + 6(3). If you missed this problem, review Example 2.2.8.
3. Convert $\frac{5}{2}$ to a decimal. If you missed this problem, review Example 5.5.1.

One application of decimals that arises often is finding the *average* of a set of numbers. What do you think of when you hear the word *average*? Is it your grade point average, the average rent for an apartment in your city, the batting average of a player on your favorite baseball team? The average is a typical value in a set of numerical data. Calculating an average sometimes involves working with decimal numbers. In this section, we will look at three different ways to calculate an average.

Calculate the Mean of a Set of Numbers

The mean is often called the arithmetic average. It is computed by dividing the sum of the values by the number of values. Students want to know the mean of their test scores. Climatologists report that the mean temperature has, or has not, changed. City planners are interested in the mean household size.

Suppose Ethan's first three test scores were 85, 88, and 94. To find the mean score, he would add them and divide by 3.

$$\frac{85+88+94}{3} \tag{1.4.8.1}$$

$$\frac{267}{3} \tag{1.4.8.2}$$

$$89 \tag{1.4.8.3}$$

His mean test score is 89 points.

Definition: The Mean

The mean of a set of n numbers is the arithmetic average of the numbers.

$$mean = \frac{sum\ of\ values\ in\ data\ set}{n} \tag{1.4.8.4}$$

HOW TO: CALCULATE THE MEAN OF A SET OF NUMBERS

Step 1. Write the formula for the mean

$$mean = \frac{sum\ of\ values\ in\ data\ set}{n} \tag{1.4.8.5}$$

Step 2. Find the sum of all the values in the set. Write the sum in the numerator.

Step 3. Count the number, n, of values in the set. Write this number in the denominator.

Step 4. Simplify the fraction.

Step 5. Check to see that the mean is reasonable. It should be greater than the least number and less than the greatest number in the set.

Example 1.4.8.1:

Find the mean of the numbers 8, 12, 15, 9, and 6.

Solution

Write the formula for the mean.	$mean = \frac{sum\ of\ values\ in\ data\ set}{n}$ (1.4.8.6)
Write the sum of the numbers in the numerator.	$mean = \frac{8+12+15+9+6}{n}$ (1.4.8.7)
Count how many numbers are in the set. There are 5 numbers in the set, so n = 5.	$mean = \frac{8+12+15+9+6}{5}$ (1.4.8.8)
Add the numbers in the numerator.	$mean = \frac{50}{5}$ (1.4.8.9)
Then divide.	mean = 10
Check to see that the mean is 'typical': 10 is neither less than 6 nor greater than 15.	The mean is 10.

Exercise 1.4.8.1:

Find the mean of the numbers: 8, 9, 7, 12, 10, 5.

Answer

8.5

Exercise 1.4.8.2:

Find the mean of the numbers: 9, 13, 11, 7, 5.

Answer

9

Example 1.4.8.2:

The ages of the members of a family who got together for a birthday celebration were 16, 26, 53, 56, 65, 70, 93, and 97 years. Find the mean age.

Solution

Write the formula for the mean.	$mean = \frac{sum\ of\ values\ in\ data\ set}{n}$ (1.4.8.10)
Write the sum of the numbers in the numerator.	$mean = \frac{16+26+53+56+65+70+93+97}{n}$ (1.4.8
Count how many numbers are in the set. Call this n and write it in the denominator.	$mean = \frac{16+26+53+56+65+70+93+97}{8}$ (1.4.8
Simplify the fraction.	$mean = \frac{476}{5}$ (1.4.8.13)
	mean = 59.5

Is 59.5 ‘typical’? Y es, it is neither less than 16 nor greater than 97. The mean age is 59.5 years.

Exercise 1.4.8.3:

The ages of the four students in Ben’s carpool are 25, 18, 21, and 22. Find the mean age of the students.

Answer

21.5 years

Exercise 1.4.8.4:

Yen counted the number of emails she received last week. The numbers were 4, 9, 15, 12, 10, 12, and 8. Find the mean number of emails

Answer

10

Did you notice that in the last example, while all the numbers were whole numbers, the mean was 59.5, a number with one decimal place? It is customary to report the mean to one more decimal place than the original numbers. In the next example, all the numbers represent money, and it will make sense to report the mean in dollars and cents.

Example 1.4.8.3:

For the past four months, Daisy’s cell phone bills were $42.75, $50.12, $41.54, $48.15. Find the mean cost of Daisy’s cell phone bills.

Solution

Write the formula for the mean.	$$mean = \frac{sum\ of\ all\ the\ numbers}{n}$$	(1.4.8.14)
Write the sum of the numbers in the numerator.	$$mean = \frac{sum\ of\ all\ the\ numbers}{4}$$	(1.4.8.15)
Count how many numbers are in the set. Call this n and write it in the denominator.	$$mean = \frac{42.75 + 50.12 + 41.54 + 48.15}{4}$$	(1.4.8.16)
Simplify the fraction.	$$mean = \frac{182.56}{4}$$	(1.4.8.17)
	mean = 45.64	

Does $45.64 seem ‘typical’ of this set of numbers? Yes, it is neither less than $41.54 nor greater than $50.12. The mean cost of her cell phone bill was $45.64.

Exercise 1.4.8.5:

Last week Ray recorded how much he spent for lunch each workday. He spent $6.50, $7.25, $4.90, $5.30, and $12.00. Find the mean of how much he spent each day.

Answer

$7.19

Exercise 1.4.8.6:

Lisa has kept the receipts from the past four trips to the gas station. The receipts show the following amounts: $34.87, $42.31, $38.04, and $43.26. Find the mean.

Answer

$39.62

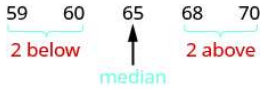
Find the Median of a Set of Numbers

When Ann, Bianca, Dora, Eve, and Francine sing together on stage, they line up in order of their heights. Their heights, in inches, are shown in Table 5.70.

Table 5.70

Ann	Bianca	Dora	Eve	Francine
59	60	65	68	70

Dora is in the middle of the group. Her height, 65″, is the *median* of the girls' heights. Half of the heights are less than or equal to Dora's height, and half are greater than or equal. The median is the middle value.



Definition: Median

The median of a set of data values is the middle value.

- Half the data values are less than or equal to the median.
- Half the data values are greater than or equal to the median.

What if Carmen, the pianist, joins the singing group on stage? Carmen is 62 inches tall, so she fits in the height order between Bianca and Dora. Now the data set looks like this:

$$59, 60, 62, 65, 68, 70 \tag{1.4.8.18}$$

There is no single middle value. The heights of the six girls can be divided into two equal parts.

$$\underbrace{59, 60, 62} \quad \underbrace{65, 68, 70} \tag{1.4.8.19}$$

Statisticians have agreed that in cases like this the median is the mean of the two values closest to the middle. So the median is the mean of 62 and 65, $\frac{62+65}{2}$. The median height is 63.5 inches.



Notice that when the number of girls was 5, the median was the third height, but when the number of girls was 6, the median was the mean of the third and fourth heights. In general, when the number of values is odd, the median will be the one value in the middle, but when the number is even, the median is the mean of the two middle values.

HOW TO: FIND THE MEDIAN OF A SET OF NUMBERS

Step 1. List the numbers from smallest to largest.

Step 2. Count how many numbers are in the set. Call this n.

Step 3. Is n odd or even?

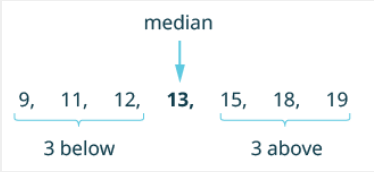
- If n is an odd number, the median is the middle value.

- If n is an even number, the median is the mean of the two middle values.

Example 1.4.8.4:

Find the median of 12, 13, 19, 9, 11, 15, and 18.

Solution

List the numbers in order from smallest to largest.	9, 11, 12, 13, 15, 18, 19
Count how many numbers are in the set. Call this n.	n = 7
Is n odd or even?	odd
The median is the middle value.	
The middle is the number in the 4th position.	So the median of the data is 13.

Exercise 1.4.8.7:

Find the median of the data set: 43, 38, 51, 40, 46.

Answer

43

Exercise 1.4.8.8:

Find the median of the data set: 15, 35, 20, 45, 50, 25, 30.

Answer

30

Example 1.4.8.5:

Kristen received the following scores on her weekly math quizzes: 83, 79, 85, 86, 92, 100, 76, 90, 88, and 64. Find her median score.

Solution

List the numbers in order from smallest to largest.	64, 76, 79, 83, 85, 86, 88, 90, 92, 100
Count the number of data values in the set. Call this n.	n = 10
Is n odd or even?	even
The median is the mean of the two middle values, the 5th and 6th numbers.	64, 76, 79, 83, **85,** **86,** 88, 90, 92, 100 (5 numbers / 5 numbers)
Find the mean of 85 and 86.	mean = 85.5

Kristen's median score is 85.5.

Exercise 1.4.8.9:

Find the median of the data set: 8, 7, 5, 10, 9, 12.

Answer

8.5

Exercise 1.4.8.10:

Find the median of the data set: 21, 25, 19, 17, 22, 18, 20, 24.

Answer

20.5

Identify the Mode of a Set of Numbers

The *average* is one number in a set of numbers that is somehow typical of the whole set of numbers. The mean and median are both often called the average. Yes, it can be confusing when the word average refers to two different numbers, the mean and the median! In fact, there is a third number that is also an average. This average is the **mode**. The mode of a set of numbers is the number that occurs the most. The **frequency**, is the number of times a number occurs. So the mode of a set of numbers is the number with the highest frequency.

Definition: mode

The mode of a set of numbers is the number with the highest frequency.

Suppose Jolene kept track of the number of miles she ran since the start of the month, as shown in Figure 5.7.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1 2 mi New Year's day	2	3 15 mi
4 8 mi	5	6 3 mi	7 8 mi	8	9 5 mi	10 8 mi
11	12	13	14	15	16	17

Figure 5.7

If we list the numbers in order it is easier to identify the one with the highest frequency.

$$2, 3, 5, 8, 8, 8, 15 \tag{1.4.8.20}$$

Jolene ran 8 miles three times, and every other distance is listed only once. So the mode of the data is 8 miles.

HOW TO: IDENTIFY THE MODE OF A SET OF NUMBERS

Step 1. List the data values in numerical order.

Step 2. Count the number of times each value appears.

Step 3. The mode is the value with the highest frequency.

Example 1.4.8.6:

The ages of students in a college math class are listed below. Identify the mode.

18, 18, 18, 18, 19, 19, 19, 20, 20, 20, 20, 20, 20, 20, 21, 21, 22, 22, 22, 22, 22, 23, 24, 24, 25, 29, 30, 40, 44

Solution

The ages are already listed in order. We will make a table of frequencies to help identify the age with the highest frequency.

Age	18	19	20	21	22	23	24	25	29	30	40	44
Frequency	4	3	7	2	5	1	2	1	1	1	1	1

Now look for the highest frequency. The highest frequency is 7, which corresponds to the age 20. So the mode of the ages in this class is 20 years.

Exercise 1.4.8.11:

The number of sick days employees used last year: 3, 6, 2, 3, 7, 5, 6, 2, 4, 2. Identify the mode.

Answer

2

Exercise 1.4.8.12:

The number of handbags owned by women in a book club: 5, 6, 3, 1, 5, 8, 1, 5, 8, 5. Identify the mode.

Answer

5

Example 1.4.8.7:

The data lists the heights (in inches) of students in a statistics class. Identify the mode.

56	61	63	64	65	66	67	67
60	62	63	64	65	66	67	70
60	63	63	64	66	66	67	74
61	63	64	65	66	67	67	

Solution

List each number with its frequency.

Number	56	60	61	62	63	64	65	66	67	70	74
Frequency	1	2	2	1	5	4	3	5	6	1	1

Now look for the highest frequency. The highest frequency is 6, which corresponds to the height 67 inches. So the mode of this set of heights is 67 inches.

Exercise 1.4.8.13:

The ages of the students in a statistics class are listed here: 19, 20, 23, 23, 38, 21, 19, 21, 19, 21, 20, 43, 20, 23, 17, 21, 21, 20, 29, 18, 28. What is the mode?

Answer

21

Exercise 1.4.8.14:

Students listed the number of members in their household as follows: 6, 2, 5, 6, 3, 7, 5, 6, 5, 3, 4, 4, 5, 7, 6, 4, 5, 2, 1, 5. What is the mode?

Answer

5

Some data sets do not have a mode because no value appears more than any other. And some data sets have more than one mode. In a given set, if two or more data values have the same highest frequency, we say they are all modes.

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1.4.9: Averages and Probability (Part 2)

Use the Basic Definition of Probability

The probability of an event tells us how likely that event is to occur. We usually write probabilities as fractions or decimals. For example, picture a fruit bowl that contains five pieces of fruit - three bananas and two apples.

If you want to choose one piece of fruit to eat for a snack and don't care what it is, there is a $\frac{3}{5}$ probability you will choose a banana, because there are three bananas out of the total of five pieces of fruit. The probability of an event is the number of favorable outcomes divided by the total number of outcomes.

$$\text{Probability of an event} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

$$\text{Probability of choosing a banana} = \frac{3}{5} \begin{array}{l} \leftarrow \text{There are 3 bananas.} \\ \leftarrow \text{There are 5 pieces of fruit.} \end{array}$$

Definition: Probability

The probability of an event is the number of favorable outcomes divided by the total number of outcomes possible.

$$Probability = \frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes} \tag{1.4.9.1}$$

Converting the fraction $\frac{3}{5}$ to a decimal, we would say there is a 0.6 probability of choosing a banana.

$$\text{Probability of choosing a banana} = \frac{3}{5}$$

$$\text{Probability of choosing a banana} = 0.6$$

This basic definition of probability assumes that all the outcomes are equally likely to occur. If you study probabilities in a later math class, you'll learn about several other ways to calculate probabilities.

Example 1.4.9.8:

The ski club is holding a raffle to raise money. They sold 100 tickets. All of the tickets are placed in a jar. One ticket will be pulled out of the jar at random, and the winner will receive a prize. Cherie bought one raffle ticket. (a) Find the probability she will win the prize. (b) Convert the fraction to a decimal.

Solution

(a)

What are you asked to find?	The probability Cherie wins the prize.
What is the number of favorable outcomes?	1, because Cherie has 1 ticket.
Use the definition of probability.	$Probability\ of\ an\ event = \frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes}$
Substitute into the numerator and denominator.	Probability Cherie wins = $\frac{1}{100}$

(b)

Write the probability as a fraction.	Probability = $\frac{1}{100}$
Convert the fraction to a decimal.	Probability = 0.01

Exercise 1.4.9.15:

Ignaly is attending a fashion show where the guests are seated at tables of ten. One guest from each table will be selected at random to receive a door prize. (a) Find the probability Ignaly will win the door prize for her table. (b) Convert the fraction to a decimal.

Answer a

$\frac{1}{10}$

Answer b

0.1

Exercise 1.4.9.16:

Hoang is among 20 people available to sit on a jury. One person will be chosen at random from the 20. (a) Find the probability Hoang will be chosen. (b) Convert the fraction to a decimal.

Answer a

$\frac{1}{20}$

Answer b

0.05

Example 1.4.9.9:

Three women and five men interviewed for a job. One of the candidates will be offered the job. (a) Find the probability the job is offered to a woman. (b) Convert the fraction to a decimal.

Solution

What are you asked to find?	The probability the job is offered to a woman.
What is the number of favorable outcomes?	3, because there are three women.
What are the total number of outcomes?	8, because 8 people interviewed.
Use the definition of probability.	$Probability\ of\ an\ event = \frac{number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes}$
Substitute into the numerator and denominator.	Probability = $\frac{3}{8}$

(b)

Write the probability as a fraction.	Probability = $\frac{3}{8}$
Convert the fraction to a decimal.	Probability = 0.375

Exercise 1.4.9.17:

A bowl of Halloween candy contains 5 chocolate candies and 3 lemon candies. Tanya will choose one piece of candy at random.(a) Find the probability Tanya will choose a chocolate candy.(b) Convert the fraction to a decimal.

Answer a

$\frac{5}{8}$

Answer b

0.625

Exercise 1.4.9.18:

Dan has 2 pairs of black socks and 6 pairs of blue socks. He will choose one pair at random to wear tomorrow. (a) Find the probability Dan will choose a pair of black socks (b) Convert the fraction to a decimal.

Answer a

$\frac{2}{8}$

Answer b

0.25

ACCESS ADDITIONAL ONLINE RESOURCES

Mean, Median, and Mode

Find the Mean of a Data Set

Find the Median of a Data Set

Find the Mode of a Data Set

Practice Makes Perfect

Calculate the Mean of a Set of Numbers

In the following exercises, find the mean.

357. 3, 8, 2, 2, 5
358. 6, 1, 9, 3, 4, 7
359. 65, 13, 48, 32, 19, 33
360. 34, 45, 29, 61, and 41
361. 202, 241, 265, 274
362. 525, 532, 558, 574
363. 12.45, 12.99, 10.50, 11.25, 9.99, 12.72
364. 28.8, 32.9, 32.5, 27.9, 30.4, 32.5, 31.6, 32.7
365. Four girls leaving a mall were asked how much money they had just spent. The amounts were $0, $14.95, $35.25, and $25.16. Find the mean amount of money spent.
366. Juan bought 5 shirts to wear to his new job. The costs of the shirts were $32.95, $38.50, $30.00, $17.45, and $24.25. Find the mean cost.
367. The number of minutes it took Jim to ride his bike to school for each of the past six days was 21, 18, 16, 19, 24, and 19. Find the mean number of minutes.
368. Norris bought six books for his classes this semester. The costs of the books were $74.28, $120.95, $52.40, $10.59, $35.89, and $59.24. Find the mean cost.
369. The top eight hitters in a softball league have batting averages of .373, .360, .321, .321, .320, .312, .311, and .311. Find the mean of the batting averages. Round your answer to the nearest thousandth.
370. The monthly snowfall at a ski resort over a six-month period was 60.3, 79.7, 50.9, 28.0, 47.4, and 46.1 inches. Find the mean snowfall.

Find the Median of a Set of Numbers

In the following exercises, find the median.

371. 24, 19, 18, 29, 21
372. 48, 51, 46, 42, 50
373. 65, 56, 35, 34, 44, 39, 55, 52, 45
374. 121, 115, 135, 109, 136, 147, 127, 119, 110
375. 4, 8, 1, 5, 14, 3, 1, 12
376. 3, 9, 2, 6, 20, 3, 3, 10
377. 99.2, 101.9, 98.6, 99.5, 100.8, 99.8
378. 28.8, 32.9, 32.5, 27.9, 30.4, 32.5, 31.6, 32.7
379. Last week Ray recorded how much he spent for lunch each workday. He spent $6.50, $7.25, $4.90, $5.30, and $12.00 . Find the median.
380. Michaela is in charge of 6 two-year olds at a daycare center. Their ages, in months, are 25, 24, 28, 32, 29, and 31. Find the median age.
381. Brian is teaching a swim class for 6 three-year olds. Their ages, in months, are 38, 41, 45, 36, 40, and 42. Find the median age.
382. Sal recorded the amount he spent for gas each week for the past 8 weeks. The amounts were $38.65, $32.18, $40.23, $51.50, $43.68, $30.96, $41.37, and $44.72. Find the median amount.

Identify the Mode of a Set of Numbers

In the following exercises, identify the mode.

383. 2, 5, 1, 5, 2, 1, 2, 3, 2, 3, 1
384. 8, 5, 1, 3, 7, 1, 1, 7, 1, 8, 7
385. 18, 22, 17, 20, 19, 20, 22, 19, 29, 18, 23, 25, 22, 24, 23, 22, 18, 20, 22, 20
386. 42, 28, 32, 35, 24, 32, 48, 32, 32, 24, 35, 28, 30, 35, 45, 32, 28, 32, 42, 42, 30
387. The number of children per house on one block: 1, 4, 2, 3, 3, 2, 6, 2, 4, 2, 0, 3, 0.
388. The number of movies watched each month last year: 2, 0, 3, 0, 0, 8, 6, 5, 0, 1, 2, 3.

389. The number of units being taken by students in one class: 12, 5, 11, 10, 10, 11, 5, 11, 11, 11, 10, 12.
390. The number of hours of sleep per night for the past two weeks: 8, 5, 7, 8, 8, 6, 6, 6, 6, 9, 7, 8, 8, 8.

Use the Basic Definition of Probability

In the following exercises, express the probability as both a fraction and a decimal. (Round to three decimal places, if necessary.)

391. Josue is in a book club with 20 members. One member is chosen at random each month to select the next month's book. Find the probability that Josue will be chosen next month.
392. Jessica is one of eight kindergarten teachers at Mandela Elementary School. One of the kindergarten teachers will be selected at random to attend a summer workshop. Find the probability that Jessica will be selected.
393. There are 24 people who work in Dane's department. Next week, one person will be selected at random to bring in doughnuts. Find the probability that Dane will be selected. Round your answer to the nearest thousandth.
394. Monica has two strawberry yogurts and six banana yogurts in her refrigerator. She will choose one yogurt at random to take to work. Find the probability Monica will choose a strawberry yogurt.
395. Michel has four rock CDs and six country CDs in his car. He will pick one CD to play on his way to work. Find the probability Michel will pick a rock CD.
396. Noah is planning his summer camping trip. He can't decide among six campgrounds at the beach and twelve campgrounds in the mountains, so he will choose one campground at random. Find the probability that Noah will choose a campground at the beach.
397. Donovan is considering transferring to a 4-year college. He is considering 10 out-of state colleges and 4 colleges in his state. He will choose one college at random to visit during spring break. Find the probability that Donovan will choose an out-of-state college.
398. There are 258,890,850 number combinations possible in the Mega Millions lottery. One winning jackpot ticket will be chosen at random. Brent chooses his favorite number combination and buys one ticket. Find the probability Brent will win the jackpot. Round the decimal to the first digit that is not zero, then write the name of the decimal.

Everyday Math

399. Joaquin gets paid every Friday. His paychecks for the past 8 Fridays were $315, $236.25, $236.25, $236.25 $315, $315, $236.25, $393.75. Find the (a) mean (b) median, and (c) mode.
400. The cash register receipts each day last week at a coffee shop were $1,845, $1,520, $1,438, $1,682, $1,850, $2,721, $2,539. Find the (a) mean, (b) median, and (c) mode.

Writing Exercises

401. Explain in your own words the difference between the mean, median, and mode of a set of numbers.
402. Make an example of probability that relates to your life. Write your answer as a fraction and explain what the numerator and denominator represent.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
calculate the mean of a set of numbers.			
find the median of a set of numbers.			
find the mode of a set of numbers.			
use the basic definition of probability.			

(b) After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

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1.4.10: Ratios and Rate (Part 1)

Learning Objectives

- Write a ratio as a fraction
- Write a rate as a fraction
- Find unit rates
- Find unit price
- Translate phrases to expressions with fractions

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $\frac{16}{24}$. If you missed this problem, review Example 4.3.1.
2. Divide: 2.76 ÷ 11.5. If you missed this problem, review Example 5.4.9.
3. Simplify: $\frac{1\frac{1}{2}}{2\frac{3}{4}}$. If you missed this problem, review Example 4.5.7.

Write a Ratio as a Fraction

When you apply for a mortgage, the loan officer will compare your total debt to your total income to decide if you qualify for the loan. This comparison is called the debt-to-income ratio. A **ratio** compares two quantities that are measured with the same unit. If we compare a and b , the ratio is written as a to b, $\frac{a}{b}$, or a:b.

Definition: ratios

A ratio compares two numbers or two quantities that are measured with the same unit. The ratio of a to b is written a to b, $\frac{a}{b}$, or a:b.

In this section, we will use the fraction notation. When a ratio is written in fraction form, the fraction should be simplified. If it is an improper fraction, we do not change it to a mixed number. Because a ratio compares two quantities, we would leave a ratio as $\frac{4}{1}$ instead of simplifying it to 4 so that we can see the two parts of the ratio.

Example 1.4.10.1:

Write each ratio as a fraction: (a) 15 to 27 (b) 45 to 18.

Solution

(a) 15 to 27

Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{15}{27}$	(1.4.10.1)
Simplify the fraction.	$\frac{5}{9}$	(1.4.10.2)

(b) 45 to 18

Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{45}{18}$	(1.4.10.3)
Simplify.	$\frac{5}{2}$	(1.4.10.4)

We leave the ratio in (b) as an improper fraction.

? Exercise 1.4.10.1:

Write each ratio as a fraction: (a) 21 to 56 (b) 48 to 32.

Answer a

$\frac{3}{8}$

Answer b

$\frac{3}{2}$

? Exercise 1.4.10.2:

Write each ratio as a fraction: (a) 27 to 72 (b) 51 to 34.

Answer a

$\frac{1}{1}$

Answer b

$\frac{3}{2}$

Ratios Involving Decimals

We will often work with ratios of decimals, especially when we have ratios involving money. In these cases, we can eliminate the decimals by using the Equivalent Fractions Property to convert the ratio to a fraction with whole numbers in the numerator and denominator.

For example, consider the ratio 0.8 to 0.05. We can write it as a fraction with decimals and then multiply the numerator and denominator by 100 to eliminate the decimals.

$$\frac{0.8}{0.05} \tag{1.4.10.5}$$

$$\frac{(0.8)100}{(0.05)100} \tag{1.4.10.6}$$

$$\frac{80}{5} \tag{1.4.10.7}$$

Do you see a shortcut to find the equivalent fraction? Notice that 0.8 = $\frac{8}{10}$ and 0.05 = $\frac{5}{100}$. The least common denominator of $\frac{8}{10}$ and 5 100 is 100. By multiplying the numerator and denominator of $\frac{0.8}{0.05}$ by 100, we 'moved' the decimal two places to the right to get the equivalent fraction with no decimals. Now that we understand the math behind the process, we can find the fraction with no decimals like this:

$$\frac{0.80}{0.05}$$

"Move" the decimal 2 places.	$\frac{80}{5}$	(1.4.10.8)
Simplify.	$\frac{16}{1}$	(1.4.10.9)

You do not have to write out every step when you multiply the numerator and denominator by powers of ten. As long as you move both decimal places the same number of places, the ratio will remain the same.

Example 1.4.10.2:

Write each ratio as a fraction of whole numbers: (a) 4.8 to 11.2 (b) 2.7 to 0.54

Solution

(a) 4.8 to 11.2

Write as a fraction.	$\frac{4.8}{11.2}$	(1.4.10.10)
Rewrite as an equivalent fraction without decimals, by moving both decimal points 1 place to the right.	$\frac{48}{112}$	(1.4.10.11)
Simplify.	$\frac{3}{7}$	(1.4.10.12)

So 4.8 to 11.2 is equivalent to $\frac{3}{7}$.

(b) 2.7 to 0.54

Write as a fraction.	$\frac{2.7}{0.54}$	(1.4.10.13)
The numerator has one decimal place and the denominator has 2. To clear both decimals we need to move the decimal 2 places to the right.	$\frac{270}{54}$	(1.4.10.14)
Simplify.	$\frac{5}{1}$	(1.4.10.15)

So 2.7 to 0.54 is equivalent to $\frac{5}{1}$.

Exercise 1.4.10.3:

Write each ratio as a fraction: (a) 4.6 to 11.5 (b) 2.3 to 0.69.

Answer a

$\frac{2}{5}$

Answer b

$\frac{10}{3}$

Exercise 1.4.10.4:

Write each ratio as a fraction: (a) 3.4 to 15.3 (b) 3.4 to 0.68.

Answer a

$\frac{2}{9}$

Answer b

$\frac{5}{1}$

Some ratios compare two mixed numbers. Remember that to divide mixed numbers, you first rewrite them as improper fractions.

Example 1.4.10.3:

Write the ratio of $1\frac{1}{4}$ to $2\frac{3}{8}$ as a fraction.

Solution

Write as a fraction.	$\frac{1\frac{1}{4}}{2\frac{3}{8}}$	(1.4.10.16)
Convert the numerator and denominator to improper fractions.	$\frac{\frac{5}{4}}{\frac{19}{8}}$	(1.4.10.17)
Rewrite as a division of fractions.	$\frac{5}{4} \div \frac{19}{8}$	(1.4.10.18)
Invert the divisor and multiply.	$\frac{5}{4} \cdot \frac{8}{19}$	(1.4.10.19)
Simplify.	$\frac{10}{19}$	(1.4.10.20)

Exercise 1.4.10.5:

Write each ratio as a fraction: $1\frac{3}{4}$ to $2\frac{5}{8}$.

Answer

\(\dfrac{2}{3}\

Exercise 1.4.10.6:

Write each ratio as a fraction: $1\frac{1}{8}$ to $2\frac{3}{4}$.

Answer

$\frac{9}{22}$

Applications of Ratios

One real-world application of ratios that affects many people involves measuring cholesterol in blood. The ratio of total cholesterol to HDL cholesterol is one way doctors assess a person's overall health. A ratio of less than 5 to 1 is considered good.

Example 1.4.10.4:

Hector's total cholesterol is 249 mg/dl and his HDL cholesterol is 39 mg/dl. (a) Find the ratio of his total cholesterol to his HDL cholesterol. (b) Assuming that a ratio less than 5 to 1 is considered good, what would you suggest to Hector?

Solution

(a) First, write the words that express the ratio. We want to know the ratio of Hector's total cholesterol to his HDL cholesterol.

Write as a fraction.	$\frac{total\ cholesterol}{HDL\ cholesterol}$	(1.4.10.21)
Substitute the values.	$\frac{249}{39}$	(1.4.10.22)
Simplify.	$\frac{83}{13}$	(1.4.10.23)

(b) Is Hector's cholesterol ratio ok? If we divide 83 by 13 we obtain approximately 6.4, so $\frac{83}{13} \approx \frac{6.4}{1}$. Hector's cholesterol ratio is high! Hector should either lower his total cholesterol or raise his HDL cholesterol.

Exercise 1.4.10.7:

Find the patient's ratio of total cholesterol to HDL cholesterol using the given information. Total cholesterol is 185 mg/dL and HDL cholesterol is 40 mg/dL.

Answer

\(\dfrac{37}{8}\

Exercise 1.4.10.8:

Find the patient’s ratio of total cholesterol to HDL cholesterol using the given information. Total cholesterol is 204 mg/dL and HDL cholesterol is 38 mg/dL.

Answer

\(\dfrac{102}{19}\

Ratios of Two Measurements in Different Units

To find the ratio of two measurements, we must make sure the quantities have been measured with the same unit. If the measurements are not in the same units, we must first convert them to the same units.

We know that to simplify a fraction, we divide out common factors. Similarly in a ratio of measurements, we divide out the common unit.

Example 1.4.10.5:

The Americans with Disabilities Act (ADA) Guidelines for wheel chair ramps require a maximum vertical rise of 1 inch for every 1 foot of horizontal run. What is the ratio of the rise to the run?

Solution

In a ratio, the measurements must be in the same units. We can change feet to inches, or inches to feet. It is usually easier to convert to the smaller unit, since this avoids introducing more fractions into the problem. Write the words that express the ratio.

Write the ratio as a fraction.	$\frac{rise}{run}$	(1.4.10.24)
Substitute in the given values.	$\frac{1\ inch}{1\ foot}$	(1.4.10.25)
Convert 1 foot to inches.	$\frac{1\ inch}{12\ inches}$	(1.4.10.26)
Simplify, dividing out common factors and units.	$\frac{1}{12}$	(1.4.10.27)

So the ratio of rise to run is 1 to 12. This means that the ramp should rise 1 inch for every 12 inches of horizontal run to comply with the guidelines.

Exercise 1.4.10.9:

Find the ratio of the first length to the second length: 32 inches to 1 foot.

Answer

\(\dfrac{8}{3}\

Exercise 1.4.10.10:

Find the ratio of the first length to the second length: 1 foot to 54 inches.

Answer a

\(\dfrac{2}{9}\

Write a Rate as a Fraction

Frequently we want to compare two different types of measurements, such as miles to gallons. To make this comparison, we use a **rate**. Examples of rates are 120 miles in 2 hours, 160 words in 4 minutes, and $5 dollars per 64 ounces.

Definition: rate

A rate compares two quantities of different units. A rate is usually written as a fraction.

When writing a fraction as a rate, we put the first given amount with its units in the numerator and the second amount with its units in the denominator. When rates are simplified, the units remain in the numerator and denominator.

Example 1.4.10.6:

Bob drove his car 525 miles in 9 hours. Write this rate as a fraction.

Solution

Write as a fraction, with 525 miles in the numerator and 9 hours in the denominator.	$\frac{525\ miles}{9\ hours}$	(1.4.10.28)
	$\frac{175\ miles}{3\ hours}$	(1.4.10.29)

So 525 miles in 9 hours is equivalent to $\frac{175\ miles}{3\ hours}$.

Exercise 1.4.10.11:

Write the rate as a fraction: 492 miles in 8 hours.

Answer

$\frac{123\ miles}{2\ hours}$

Exercise 1.4.10.12:

Write the rate as a fraction: 242 miles in 6 hours.

Answer

$\frac{121\ miles}{3\ hours}$

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1.4.11: Ratios and Rate (Part 2)

Find Unit Rates

In the last example, we calculated that Bob was driving at a rate of $\frac{175\ miles}{3\ hours}$. This tells us that every three hours, Bob will travel 175 miles. This is correct, but not very useful. We usually want the rate to reflect the number of miles in one hour. A rate that has a denominator of 1 unit is referred to as a **unit rate**.

Definition: Unit rate

A unit rate is a rate with denominator of 1 unit.

Unit rates are very common in our lives. For example, when we say that we are driving at a speed of 68 miles per hour we mean that we travel 68 miles in 1 hour. We would write this rate as 68 miles/hour (read 68 miles per hour). The common abbreviation for this is 68 mph. Note that when no number is written before a unit, it is assumed to be 1. So 68 miles/hour really means 68 miles/1 hour.

Two rates we often use when driving can be written in different forms, as shown:

Example	Rate	Write	Abbreviate	Read
68 miles in 1 hour	$\frac{68\ miles}{1\ hour}$	(1.4.11.2) 68 miles/hour	68 mph	68 miles per hour
36 miles to 1 gallon	$\frac{36\ miles}{1\ gallon}$	(1.4.11.3) 36 miles/gallon	36 mpg	36 miles per gallon

Another example of unit rate that you may already know about is hourly pay rate. It is usually expressed as the amount of money earned for one hour of work. For example, if you are paid $12.50 for each hour you work, you could write that your hourly (unit) pay rate is $12.50/hour (read $12.50 per hour.)

To convert a rate to a unit rate, we divide the numerator by the denominator. This gives us a denominator of 1.

Example 1.4.11.7:

Anita was paid $384 last week for working 32 hours. What is Anita's hourly pay rate?

Solution

Start with a rate of dollars to hours. Then divide.	$384 last week for 32 hours	
Write as a rate.	$\frac{\$384}{32\ hours}$	(1.4.11.3)
Divide the numerator by the denominator.	$\frac{\$12}{1\ hour}$	(1.4.11.4)
Rewrite as a rate.	$12 / hour	

Anita's hourly pay rate is $12 per hour.

Exercise 1.4.11.13:

Find the unit rate: $630 for 35 hours.

Answer

$18 / hour

Exercise 1.4.11.14:

Find the unit rate: $684 for 36 hours.

Answer

$19 / hour

Example 1.4.11.8:

Sven drives his car 455 miles, using 14 gallons of gasoline. How many miles per gallon does his car get?

Solution

Start with a rate of miles to gallons. Then divide.

Write as a rate.	$\frac{455\ miles}{14\ gallons}$	(1.4.11.5)
Divide 455 by 14 to get the unit rate.	$\frac{32.5\ miles}{1\ gallon}$	(1.4.11.6)

Sven's car gets 32.5 miles/gallon, or 32.5 mpg.

Exercise 1.4.11.15:

Find the unit rate: 423 miles to 18 gallons of gas.

Answer

23.5 mpg

Exercise 1.4.11.16:

Find the unit rate: 406 miles to 14.5 gallons of gas.

Answer

28 mpg

Find Unit Price

Sometimes we buy common household items 'in bulk', where several items are packaged together and sold for one price. To compare the prices of different sized packages, we need to find the unit price. To find the unit price, divide the total price by the number of items. A **unit price** is a unit rate for one item.

Definition: unit price

A unit price is a unit rate that gives the price of one item.

Example 1.4.11.9:

The grocery store charges $3.99 for a case of 24 bottles of water. What is the unit price?

Solution

What are we asked to find? We are asked to find the unit price, which is the price per bottle.

Write as a rate.	$\frac{\$3.99}{24\ bottles}$	(1.4.11.7)
Divide to find the unit price.	$\frac{\$0.16625}{1\ bottle}$	(1.4.11.8)
Round the result to the nearest penny.	$\frac{\$0.17}{1\ bottle}$	(1.4.11.9)

The unit price is approximately $0.17 per bottle. Each bottle costs about $0.17.

Exercise 1.4.11.17:

Find the unit price. Round your answer to the nearest cent if necessary: 24-pack of juice boxes for $6.99

Answer

$\frac{\$0.29}{1\ box}$

Exercise 1.4.11.18:

Find the unit price. Round your answer to the nearest cent if necessary: 24-pack of bottles of ice tea for $12.72

Answer

$\frac{\$0.53}{1\ bottle}$

Unit prices are very useful if you comparison shop. The *better buy* is the item with the lower unit price. Most grocery stores list the unit price of each item on the shelves.

Example 1.4.11.10:

Paul is shopping for laundry detergent. At the grocery store, the liquid detergent is priced at $14.99 for 64 loads of laundry and the same brand of powder detergent is priced at $15.99 for 80 loads. Which is the better buy, the liquid or the powder detergent?

Solution

To compare the prices, we first find the unit price for each type of detergent.

	Liquid	Powder

Write as a rate.	$\frac{\$14.99}{64\ loads}$ (1.4.11.10)	$\frac{\$15.99}{80\ loads}$ (1.4.11.11)
Find the unit price.	$\frac{\$0.234\ldots}{1\ load}$ (1.4.11.12)	$\frac{\$0.199\ldots}{1\ load}$ (1.4.11.13)
Round to the nearest cent.	\$0.23/load (23 cents per load.)	\$0.20/load (20 cents per load)

Now we compare the unit prices. The unit price of the liquid detergent is about \$0.23 per load and the unit price of the powder detergent is about \$0.20 per load. The powder is the better buy.

? Exercise 1.4.11.19:

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand A Storage Bags, \$4.59 for 40 count, or Brand B Storage Bags, \$3.99 for 30 count

Answer

Brand A costs \$0.12 per bag. Brand B costs \$0.13 per bag. Brand A is the better buy

? Exercise 1.4.11.20:

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand C Chicken Noodle Soup, \$1.89 for 26 ounces, or Brand D Chicken Noodle Soup, \$0.95 for 10.75 ounces

Answer

Brand C costs \$0.07 per ounce. Brand D costs \$0.09 per ounce. Brand C is the better buy

Notice in Example 1.4.11.10that we rounded the unit price to the nearest cent. Sometimes we may need to carry the division to one more place to see the difference between the unit prices.

Translate Phrases to Expressions with Fractions

Have you noticed that the examples in this section used the comparison words *ratio of, to, per, in, for, on*, and *from*? When you translate phrases that include these words, you should think either ratio or rate. If the units measure the same quantity (length, time, etc.), you have a ratio. If the units are different, you have a rate. In both cases, you write a fraction.

✓ Example 1.4.11.11:

Translate the word phrase into an algebraic expression: (a) 427 miles per h hours (b) x students to 3 teachers (c) y dollars for 18 hours

Solution

(a) 427 miles per h hours

Write as a rate.	$\frac{427\ miles}{h\ hours}$ (1.4.11.14)

(b) x students to 3 teachers

Write as a rate.	$\frac{x\ students}{3\ teachers}$	(1.4.11.15)

(c) y dollars for 18 hours

Write as a rate.	$\frac{\$y}{18\ hours}$	(1.4.11.16)

Exercise 1.4.11.21:

Translate the word phrase into an algebraic expression. (a) 689 miles per h hours (b) y parents to 22 students (c) d dollars for 9 minutes

Answer a

$$\frac{689\ mi}{h\ hours}$$

Answer b

$$\frac{y\ parents}{22\ students}$$

Answer c

$$\frac{\$d}{9\ min}$$

Exercise 1.4.11.22:

Translate the word phrase into an algebraic expression. (a) m miles per 9 hours (b) x students to 8 buses (c) y dollars for 40 hours

Answer a

$$\frac{m\ mi}{9\ h}$$

Answer b

$$\frac{x\ students}{8\ buses}$$

Answer c

$$\frac{\$y}{40\ h}$$

ACCESS ADDITIONAL ONLINE RESOURCES

Ratios

Write Ratios as a Simplified Fractions Involving Decimals and Fractions

Write a Ratio as a Simplified Fraction

Rates and Unit Rates

Unit Rate for Cell Phone Plan

Practice Makes Perfect

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction.

403. 20 to 36
404. 20 to 32
405. 42 to 48
406. 45 to 54
407. 49 to 21
408. 56 to 16
409. 84 to 36
410. 6.4 to 0.8
411. 0.56 to 2.8
412. 1.26 to 4.2
413. $1\frac{2}{3}$ to $2\frac{5}{6}$
414. $1\frac{3}{4}$ to $2\frac{5}{8}$
415. $4\frac{1}{6}$ to $3\frac{1}{3}$
416. $5\frac{3}{5}$ to $3\frac{3}{5}$
417. \$18 to \$63
418. \$16 to \$72
419. \$1.21 to \$0.44
420. \$1.38 to \$0.69
421. 28 ounces to 84 ounces
422. 32 ounces to 128 ounces
423. 12 feet to 46 feet
424. 15 feet to 57 feet
425. 246 milligrams to 45 milligrams
426. 304 milligrams to 48 milligrams
427. total cholesterol of 175 to HDL cholesterol of 45
428. total cholesterol of 215 to HDL cholesterol of 55
429. 27 inches to 1 foot 430. 28 inches to 1 foot

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction.

431. 140 calories per 12 ounces
432. 180 calories per 16 ounces
433. 8.2 pounds per 3 square inches
434. 9.5 pounds per 4 square inches
435. 488 miles in 7 hours
436. 527 miles in 9 hours
437. \$595 for 40 hours
438. \$798 for 40 hours

Find Unit Rates

In the following exercises, find the unit rate. Round to two decimal places, if necessary.

439. 140 calories per 12 ounces
440. 180 calories per 16 ounces
441. 8.2 pounds per 3 square inches
442. 9.5 pounds per 4 square inches

443. 488 miles in 7 hours
444. 527 miles in 9 hours
445. $595 for 40 hours
446. $798 for 40 hours
447. 576 miles on 18 gallons of gas
448. 435 miles on 15 gallons of gas
449. 43 pounds in 16 weeks
450. 57 pounds in 24 weeks
451. 46 beats in 0.5 minute
452. 54 beats in 0.5 minute
453. The bindery at a printing plant assembles 96,000 magazines in 12 hours. How many magazines are assembled in one hour?
454. The pressroom at a printing plant prints 540,000 sections in 12 hours. How many sections are printed per hour?

Find Unit Price

In the following exercises, find the unit price. Round to the nearest cent.

455. Soap bars at 8 for $8.69
456. Soap bars at 4 for $3.39
457. Women's sports socks at 6 pairs for $7.99
458. Men's dress socks at 3 pairs for $8.49
459. Snack packs of cookies at 12 for $5.79
460. Granola bars at 5 for $3.69
461. CD-RW discs at 25 for $14.99
462. CDs at 50 for $4.49
463. The grocery store has a special on macaroni and cheese. The price is $3.87 for 3 boxes. How much does each box cost?
464. The pet store has a special on cat food. The price is $4.32 for 12 cans. How much does each can cost?

In the following exercises, find each unit price and then identify the better buy. Round to three decimal places.

465. Mouthwash, 50.7-ounce size for $6.99 or 33.8-ounce size for $4.79
466. Toothpaste, 6 ounce size for $3.19 or 7.8-ounce size for $5.19
467. Breakfast cereal, 18 ounces for $3.99 or 14 ounces for $3.29
468. Breakfast Cereal, 10.7 ounces for $2.69 or 14.8 ounces for $3.69
469. Ketchup, 40-ounce regular bottle for $2.99 or 64-ounce squeeze bottle for $4.39
470. Mayonnaise, 15-ounce regular bottle for $3.49 or 22-ounce squeeze bottle for $4.99
471. Cheese, $6.49 for 1 lb. block or $3.39 for $\frac{1}{2}$ lb. block
472. Candy, $10.99 for a 1 lb. bag or $2.89 for $\frac{1}{4}$ lb. of loose candy

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.

473. 793 miles per p hours
474. 78 feet per r seconds
475. $3 for 0.5 lbs.
476. j beats in 0.5 minutes
477. 105 calories in x ounces
478. 400 minutes for m dollars
479. the ratio of y and 5x
480. the ratio of 12x and y

Everyday Math

481. One elementary school in Ohio has 684 students and 45 teachers. Write the student-to-teacher ratio as a unit rate.
482. The average American produces about 1,600 pounds of paper trash per year (365 days). How many pounds of paper trash does the average American produce each day? (Round to the nearest tenth of a pound.)

483. A popular fast food burger weighs 7.5 ounces and contains 540 calories, 29 grams of fat, 43 grams of carbohydrates, and 25 grams of protein. Find the unit rate of (a) calories per ounce (b) grams of fat per ounce (c) grams of carbohydrates per ounce (d) grams of protein per ounce. Round to two decimal places.
484. A 16-ounce chocolate mocha coffee with whipped cream contains 470 calories, 18 grams of fat, 63 grams of carbohydrates, and 15 grams of protein. Find the unit rate of (a) calories per ounce (b) grams of fat per ounce (c) grams of carbohydrates per ounce (d) grams of protein per ounce.

Writing Exercises

485. Would you prefer the ratio of your income to your friend's income to be 3/1 or 1/3? Explain your reasoning.
486. The parking lot at the airport charges $0.75 for every 15 minutes. (a) How much does it cost to park for 1 hour? (b) Explain how you got your answer to part (a). Was your reasoning based on the unit cost or did you use another method?
487. Kathryn ate a 4-ounce cup of frozen yogurt and then went for a swim. The frozen yogurt had 115 calories. Swimming burns 422 calories per hour. For how many minutes should Kathryn swim to burn off the calories in the frozen yogurt? Explain your reasoning.
488. Mollie had a 16-ounce cappuccino at her neighborhood coffee shop. The cappuccino had 110 calories. If Mollie walks for one hour, she burns 246 calories. For how many minutes must Mollie walk to burn off the calories in the cappuccino? Explain your reasoning.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
write a ratio as a fraction.			
write a rate as a fraction.			
find unit rates.			
find unit price.			
translate phrases to expressions with fractions.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

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1.4.12: Simplify and Use Square Roots (Part 1)

Learning Objectives

- Simplify expressions with square roots
- Estimate square roots
- Approximate square roots
- Simplify variable expressions with square roots
- Use square roots in applications

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $(-9)^2$. If you missed this problem, review Example 3.7.6.
2. Round 3.846 to the nearest hundredth. If you missed this problem, review Example 5.2.9.
3. Evaluate 12d for d = 80. If you missed this problem, review Example 2.3.2.

Simplify Expressions with Square Roots

To start this section, we need to review some important vocabulary and notation. Remember that when a number n is multiplied by itself, we can write this as n^2, which we read aloud as "n squared." For example, 8^2 is read as "8 squared." We call 64 the *square* of 8 because $8^2 = 64$. Similarly, 121 is the square of 11, because $11^2 = 121$.

Definition: square of a number

If $n^2 = m$, then m is the square of n.

Modeling Squares

Do you know why we use the word *square*? If we construct a square with three tiles on each side, the total number of tiles would be nine.

This is why we say that the square of three is nine.

$$3^2 = 9 \qquad (1.4.12.1)$$

The number 9 is called a perfect square because it is the square of a whole number.

The chart shows the squares of the counting numbers 1 through 15. You can refer to it to help you identify the perfect squares.

Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Definition: perfect squares

A perfect square is the square of a whole number.

What happens when you square a negative number?

$$\begin{aligned}(-8)^2 &= (-8)(-8)\\ &= 64\end{aligned}$$

When we multiply two negative numbers, the product is always positive. So, the square of a negative number is always positive. The chart shows the squares of the negative integers from −1 to −15.

Number	n	−1	−2	−3	−4	−5	−6	−7	−8	−9	−10	−11	−12	−13	−14	−15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Did you notice that these squares are the same as the squares of the positive numbers?

Square Roots

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We can also say that 10 is a square root of 100.

Definition: Square Root of a Number

A number whose square is m is called a square root of m. If $n^2 = m$, then n is a square root of m.

Notice $(-10)^2 = 100$ also, so −10 is also a square root of 100. Therefore, both 10 and −10 are square roots of 100. So, every positive number has two square roots: one positive and one negative.

What if we only want the positive square root of a positive number? The *radical sign*, $\sqrt{}$, stands for the positive square root. The positive square root is also called the **principal square root**.

Definition: Square Root Notation

$\sqrt{m}$ is read as "the square root of m." If $m = n^2$, then $m = n$ for $n \geq 0$.



We can also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root. The chart shows the square roots of the first 15 perfect square numbers.

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Example 1.4.12.1:

Simplify: (a) $\sqrt{25}$ (b) $\sqrt{121}$.

Solution

(a) $\sqrt{25}$

Since $5^2 = 25$	5

(b) $\sqrt{121}$

Since $11^2 = 121$	11

Exercise 1.4.12.1:

Simplify: (a) $\sqrt{36}$ (b) $\sqrt{169}$.

Answer a

6

Answer b

13

Exercise 1.4.12.2:

Simplify: (a) $\sqrt{16}$ (b) $\sqrt{196}$.

Answer a

4

Answer b

14

Every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$.

Example 1.4.12.2:

Simplify. (a) $-\sqrt{9}$ (b) $-\sqrt{144}$.

Solution

(a) $-\sqrt{9}$

The negative is in front of the radical sign.	-3

(b) $-\sqrt{144}$

The negative is in front of the radical sign.	-12

Exercise 1.4.12.3:

Simplify: (a) $-\sqrt{4}$ (b) $-\sqrt{225}$.

Answer a

-2

Answer b

-15

Exercise 1.4.12.4:

Simplify: (a) $-\sqrt{81}$ (b) $-\sqrt{64}$.

Answer a

-9

Answer b

-8

Square Root of a Negative Number

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25?

$$(\ )^2 = -25? \tag{1.4.12.2}$$

None of the numbers that we have dealt with so far have a square that is −25. Why? Any positive number squared is positive, and any negative number squared is also positive. In the next chapter we will see that all the numbers we work with are called the real numbers. So we say there is no real number equal to $\sqrt{-25}$. If we are asked to find the square root of any negative number, we say that the solution is not a real number.

Example 1.4.12.3:

Simplify: (a) $\sqrt{-169}$ (b) $-\sqrt{121}$.

Solution

(a) There is no real number whose square is −169. Therefore, $\sqrt{-169}$ is not a real number.

(b) The negative is in front of the radical sign, so we find the opposite of the square root of 121.

The negative is in front of the radical.	-11

Exercise 1.4.12.5:

Simplify: (a) $\sqrt{-196}$ (b) $-\sqrt{81}$.

Answer a

not a real number

Answer b

-9

Exercise 1.4.12.6:

Simplify: (a) $-\sqrt{49}$ (b) $\sqrt{-121}$.

Answer a

-7

Answer b

not a real number

Square Roots and the Order of Operations

When using the order of operations to simplify an expression that has square roots, we treat the radical sign as a grouping symbol. We simplify any expressions under the radical sign before performing other operations.

Example 1.4.12.4:

Simplify: (a) $\sqrt{25}+\sqrt{144}$ (b) $\sqrt{25+144}$.

Solution

(a)

Use the order of operations.	$\sqrt{25}+\sqrt{144}$	(1.4.12.3)
Simplify each radical.	5 + 12	
Add.	17	

(b)

Use the order of operations.	$\sqrt{25+144}$	(1.4.12.4)
Add under the radical sign.	$\sqrt{169}$	(1.4.12.5)
Simplify.	13	

? Exercise 1.4.12.7:

Simplify: (a) $\sqrt{9}+\sqrt{16}$ (b) $\sqrt{9+16}$.

Answer a

7

Answer b

5

? Exercise 1.4.12.8:

Simplify: (a) $\sqrt{64+225}$ (b) $\sqrt{64}+\sqrt{225}$.

Answer a

17

Answer b

23

Notice the different answers in parts (a) and (b) of Example 1.4.12.4 It is important to follow the order of operations correctly. In (a), we took each square root first and then added them. In (b), we added under the radical sign first and then found the square root.

Estimate Square Roots

So far we have only worked with square roots of perfect squares. The square roots of other numbers are not whole numbers.

Number	Square root
4	$\sqrt{4}=2$
5	$\sqrt{5}$
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	$\sqrt{9}=3$

We might conclude that the square roots of numbers between 4 and 9 will be between 2 and 3, and they will not be whole numbers. Based on the pattern in the table above, we could say that $\sqrt{5}$ is between 2 and 3. Using inequality symbols, we write

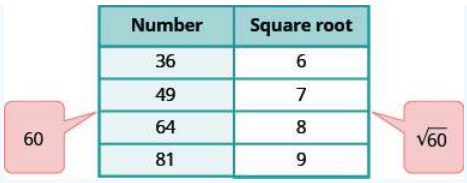
$$2<\sqrt{5}<3 \qquad (1.4.12.6)$$

✓ Example 1.4.12.5:

Estimate $\sqrt{60}$ between two consecutive whole numbers.

Solution

Think of the perfect squares closest to 60. Make a small table of these perfect squares and their squares roots.



Locate 60 between two consecutive perfect squares.	$49 < 60 < 64$	
$\sqrt{60}$ is between their square roots.	$7 < \sqrt{60} < 8$	$(1.4.12.7)$

? Exercise 1.4.12.9:

Estimate $\sqrt{38}$ between two consecutive whole numbers.

Answer

$6 < \sqrt{38} < 7$

? Exercise 1.4.12.10:

Estimate $\sqrt{84}$ between two consecutive whole numbers.

Answer

$9 < \sqrt{84} < 10$

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1.4.13: Simplify and Use Square Roots (Part 2)

Approximate Square Roots with a Calculator

There are mathematical methods to approximate square roots, but it is much more convenient to use a calculator to find square roots. Find the $\sqrt{\ }$ or $\sqrt{x}$ key on your calculator. You will to use this key to approximate square roots. When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact number. It is an approximation, to the number of digits shown on your calculator's display. The symbol for an approximation is ≈ and it is read *approximately*.

Suppose your calculator has a 10-digit display. Using it to find the square root of 5 will give 2.236067977. This is the approximate square root of 5. When we report the answer, we should use the "approximately equal to" sign instead of an equal sign.

$$\sqrt{5} \approx 2.236067978 \tag{1.4.13.1}$$

You will seldom use this many digits for applications in algebra. So, if you wanted to round $\sqrt{5}$ to two decimal places, you would write

$$\sqrt{5} \approx 2.24 \tag{1.4.13.2}$$

How do we know these values are approximations and not the exact values? Look at what happens when we square them.

$$2.236067978^2 = 5.000000002$$
$$2.24^2 = 5.0176$$

The squares are close, but not exactly equal, to 5.

Example 1.4.13.6:

Round $\sqrt{17}$ to two decimal places using a calculator.

Solution

Use the calculator square root key.	4.123105626	
Round to two decimal places.	4.12	
	$\sqrt{17} \approx 4.12$	(1.4.13.3)

Exercise 1.4.13.11:

Round $\sqrt{11}$ to two decimal places.

Answer

≈ 3.32

Exercise 1.4.13.12:

Round $\sqrt{13}$ to two decimal places

Answer

≈ 3.61

Simplify Variable Expressions with Square Roots

Expressions with square root that we have looked at so far have not had any variables. What happens when we have to find a square root of a variable expression?

Consider $\sqrt{9x^2}$, where x ≥ 0. Can you think of an expression whose square is $9x^2$?

$$(?)^2 = 9x^2$$
$$(3x)^2 = 9x^2$$
$$so\ \sqrt{9x^2} = 3x$$

When we use a variable in a square root expression, for our work, we will assume that the variable represents a nonnegative number. In every example and exercise that follows, each variable in a square root expression is greater than or equal to zero.

Example 1.4.13.7:

Simplify: x^2.

Solution

Think about what we would have to square to get x^2. Algebraically, $(?)^2 = x^2$

Since $(x)^2 = x^2$	x

Exercise 1.4.13.13:

Simplify: $\sqrt{y^2}$.

Answer

y

Exercise 1.4.13.14:

Simplify: $\sqrt{m^2}$.

Answer

m

Example 1.4.13.8:

Simplify: $\sqrt{16x^2}$.

Solution

Since $(4x)^2 = 16x^2$	4x

Exercise 1.4.13.15:

Simplify: $\sqrt{64x^2}$.

Answer

$8x$

Exercise 1.4.13.16:

Simplify: $\sqrt{169y^2}$.

Answer

$13y$

Example 1.4.13.9:

Simplify: $-\sqrt{81y^2}$.

Solution

Since $(9y)^2 = 81y^2$	-9y

Exercise 1.4.13.17:

Simplify: $-\sqrt{121y^2}$.

Answer

$-11y$

Exercise 1.4.13.18:

Simplify: $-\sqrt{100p^2}$.

Answer

$-10p$

Example 1.4.13.10:

Simplify: $\sqrt{36x^2y^2}$.

Solution

Since $(6xy)^2 = 36x^2 y^2$	6xy

Exercise 1.4.13.19:

Simplify: $\sqrt{100a^2b^2}$.

Answer

$10ab$

Exercise 1.4.13.20:

Simplify: $\sqrt{225m^2n^2}$.

Answer

$10mn$

Use Square Roots in Applications

As you progress through your college courses, you'll encounter several applications of square roots. Once again, if we use our strategy for applications, it will give us a plan for finding the answer!

HOW TO: USE A STRATEGY FOR APPLICATIONS WITH SQUARE ROOTS

Step 1. Identify what you are asked to find.

Step 2. Write a phrase that gives the information to find it.

Step 3. Translate the phrase to an expression.

Step 4. Simplify the expression.

Step 5. Write a complete sentence that answers the question.

Square Roots and Area

We have solved applications with area before. If we were given the length of the sides of a square, we could find its area by squaring the length of its sides. Now we can find the length of the sides of a square if we are given the area, by finding the square root of the area.

If the area of the square is A square units, the length of a side is A units. See Table 1.4.13.1.

Table 1.4.13.1

Area (square units)	Length of side (units)	
9	$\sqrt{9}=3$	(1.4.13.4)
144	$\sqrt{144}=12$	(1.4.13.5)
A	$\sqrt{A}$	(1.4.13.6)

Example 1.4.13.11:

Mike and Lychelle want to make a square patio. They have enough concrete for an area of 200 square feet. To the nearest tenth of a foot, how long can a side of their square patio be?

Solution

We know the area of the square is 200 square feet and want to find the length of the side. If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.

What are you asked to find?	The length of each side of a square patio	
Write a phrase.	The length of a side	
Translate to an expression.	$\sqrt{A}$	(1.4.13.7)
Evaluate $\sqrt{A}$ when A = 200.	$\sqrt{200}$	(1.4.13.8)
Use your calculator.	14.142135...	
Round to one decimal place.	14.1 feet	
Write a sentence.	Each side of the patio should be 14.1 feet.	

Exercise 1.4.13.21:

Katie wants to plant a square lawn in her front yard. She has enough sod to cover an area of 370 square feet. To the nearest tenth of a foot, how long can a side of her square lawn be?

Answer

19.2 feet

Exercise 1.4.13.22:

Sergio wants to make a square mosaic as an inlay for a table he is building. He has enough tile to cover an area of 2704 square centimeters. How long can a side of his mosaic be?

Answer

52 centimeters

Square Roots and Gravity

Another application of square roots involves gravity. On Earth, if an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$. For example, if an object is dropped from a height of 64 feet, we can find the time it takes to reach the ground by evaluating $\frac{\sqrt{64}}{4}$.

Take the square root of 64.	$\frac{8}{4}$	(1.4.13.9)
Simplify the fraction.	2	

It would take 2 seconds for an object dropped from a height of 64 feet to reach the ground.

Example 1.4.13.12:

Christy dropped her sunglasses from a bridge 400 feet above a river. How many seconds does it take for the sunglasses to reach the river?

Solution

What are you asked to find?	The number of seconds it takes for the sunglasses to reach the river	
Write a phrase.	The time it will take to reach the river	
Translate to an expression.	$\frac{\sqrt{h}}{4}$	(1.4.13.10)
Evaluate $\frac{\sqrt{h}}{4}$ when h = 400.	$\frac{\sqrt{400}}{4}$	(1.4.13.11)
Find the square root of 400.	14.142135...	
Simplify.	14.1 feet	
Write a sentence.	Each side of the patio should be 14.1 feet.	

Exercise 1.4.13.23:

A helicopter drops a rescue package from a height of 1296 feet. How many seconds does it take for the package to reach the ground?

Answer

9 seconds

Exercise 1.4.13.24:

A window washer drops a squeegee from a platform 196 feet above the sidewalk. How many seconds does it take for the squeegee to reach the sidewalk?

Answer

3.5 seconds

Square Roots and Accident Investigations

Police officers investigating car accidents measure the length of the skid marks on the pavement. Then they use square roots to determine the speed, in miles per hour, a car was going before applying the brakes. According to some formulas, if the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$.

Example 1.4.13.13:

After a car accident, the skid marks for one car measured 190 feet. To the nearest tenth, what was the speed of the car (in mph) before the brakes were applied?

Solution

What are you asked to find?	The speed of the car before the brakes were applied	
Write a phrase.	The speed of the car	
Translate to an expression.	$\sqrt{24d}$	(1.4.13.12)
Evaluate $\sqrt{24d}$ when d = 190.	$\sqrt{24 \cdot 190}$	(1.4.13.13)
Multiply.	$\sqrt{4,560}$	(1.4.13.14)
Use your calculator.	67.527772...	
Round to tenths.	67.5	
Write a sentence.	The speed of the car was approximately 67.5 miles per hour.	

Exercise 1.4.13.25:

An accident investigator measured the skid marks of a car and found their length was 76 feet. To the nearest tenth, what was the speed of the car before the brakes were applied?

Answer

42.7 mph

Exercise 1.4.13.26:

The skid marks of a vehicle involved in an accident were 122 feet long. To the nearest tenth, how fast had the vehicle been going before the brakes were applied?

Answer

54.1 mph

ACCESS ADDITIONAL ONLINE RESOURCES

Introduction to Square Roots

Estimating Square Roots with a Calculator

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.

489. $\sqrt{36}$
490. $\sqrt{4}$
491. $\sqrt{64}$
492. $\sqrt{144}$
493. $-\sqrt{4}$
494. $-\sqrt{100}$
495. $-\sqrt{1}$
496. $-\sqrt{121}$
497. $\sqrt{-121}$
498. $\sqrt{-36}$
499. $\sqrt{-9}$
500. $\sqrt{-49}$
501. $\sqrt{9+16}$
502. $\sqrt{25+144}$
503. $\sqrt{9}+\sqrt{16}$
504. $\sqrt{25}+\sqrt{144}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

505. $\sqrt{70}$
506. $\sqrt{5}$
507. $\sqrt{200}$
508. $\sqrt{172}$

Approximate Square Roots with a Calculator

In the following exercises, use a calculator to approximate each square root and round to two decimal places.

509. $\sqrt{19}$
510. $\sqrt{21}$
511. $\sqrt{53}$
512. $\sqrt{47}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

513. $\sqrt{y^2}$
514. $\sqrt{b^2}$
515. $\sqrt{49x^2}$
516. $\sqrt{100y^2}$
517. $-\sqrt{64a^2}$
518. $-\sqrt{25x^2}$
519. $\sqrt{144x^2y^2}$
520. $\sqrt{196a^2b^2}$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

521. **Landscaping** Reed wants to have a square garden plot in his backyard. He has enough compost to cover an area of 75 square feet. How long can a side of his garden be?
522. **Landscaping** Vince wants to make a square patio in his yard. He has enough concrete to pave an area of 130 square feet. How long can a side of his patio be?
523. **Gravity** An airplane dropped a flare from a height of 1,024 feet above a lake. How many seconds did it take for the flare to reach the water?
524. **Gravity** A hang glider dropped his cell phone from a height of 350 feet. How many seconds did it take for the cell phone to reach the ground?
525. **Gravity** A construction worker dropped a hammer while building the Grand Canyon skywalk, 4,000 feet above the Colorado River. How many seconds did it take for the hammer to reach the river?
526. **Accident investigation** The skid marks from a car involved in an accident measured 54 feet. What was the speed of the car before the brakes were applied?
527. **Accident investigation** The skid marks from a car involved in an accident measured 216 feet. What was the speed of the car before the brakes were applied?
528. **Accident investigation** An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 175 feet. What was the speed of the vehicle before the brakes were applied?
529. **Accident investigation** An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 117 feet. What was the speed of the vehicle before the brakes were applied?

Everyday Math

530. **Decorating** Denise wants to install a square accent of designer tiles in her new shower. She can afford to buy 625 square centimeters of the designer tiles. How long can a side of the accent be?
531. **Decorating** Morris wants to have a square mosaic inlaid in his new patio. His budget allows for 2,025 tiles. Each tile is square with an area of one square inch. How long can a side of the mosaic be?

Writing Exercises

532. Why is there no real number equal to $\sqrt{-64}$?
533. What is the difference between 9^2 and $\sqrt{9}$?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			
simplify variable expressions with square roots.			
use square roots in applications.			

(b) Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

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1.4.E: Decimals (Exercises)

5.1 - Decimals

Name Decimals

In the following exercises, name each decimal.

534. 0.8
535. 0.375
536. 0.007
537. 5.24
538. −12.5632
539. −4.09

Write Decimals

In the following exercises, write as a decimal.

540. three tenths
541. nine hundredths
542. twenty-seven hundredths
543. ten and thirty-five thousandths
544. negative twenty and three tenths
545. negative five hundredths

Convert Decimals to Fractions or Mixed Numbers

In the following exercises, convert each decimal to a fraction. Simplify the answer if possible.

546. 0.43
547. 0.825
548. 9.7
549. 3.64

Locate Decimals on the Number Line

550. (a) 0.6 (b) −0.9 (c) 2.2 (d) −1.3

Order Decimals

In the following exercises, order each of the following pairs of numbers, using < or >.

551. 0.6___0.8
552. 0.2___0.15
553. 0.803____0.83
554. −0.56____−0.562

Round Decimals

In the following exercises, round each number to the nearest: (a) hundredth (b) tenth (c) whole number.

555. 12.529
556. 4.8447
557. 5.897

5.2 - Decimal Operations

Add and Subtract Decimals

In the following exercises, add or subtract.

558. 5.75 + 8.46
559. 32.89 − 8.22

560. 24 − 19.31
561. 10.2 + 14.631
562. −6.4 + (−2.9)
563. 1.83 − 4.2

Multiply Decimals

In the following exercises, multiply.

564. (0.3)(0.7)
565. (−6.4)(0.25)
566. (−3.35)(−12.7)
567. (15.4)(1000)

Divide Decimals

In the following exercises, divide.

568. 0.48 ÷ 6
569. 4.32 ÷ 24
570. $6.29 ÷ 12
571. (−0.8) ÷ (−0.2)
572. 1.65 ÷ 0.15
573. 9 ÷ 0.045

Use Decimals in Money Applications

In the following exercises, use the strategy for applications to solve.

574. Miranda got $40 from her ATM. She spent $9.32 on lunch and $16.99 on a book. How much money did she have left? Round to the nearest cent if necessary.
575. Jessie put 8 gallons of gas in her car. One gallon of gas costs $3.528. How much did Jessie owe for all the gas?
576. A pack of 16 water bottles cost $6.72. How much did each bottle cost?
577. Alice bought a roll of paper towels that cost $2.49. She had a coupon for $0.35 off, and the store doubled the coupon. How much did Alice pay for the paper towels?

5.3 - Decimals and Fractions

Convert Fractions to Decimals

In the following exercises, convert each fraction to a decimal.

578. $\frac{3}{5}$
579. $\frac{7}{8}$
580. $-\frac{19}{20}$
581. $-\frac{21}{4}$
582. $\frac{1}{3}$
583. $\frac{6}{11}$

Order Decimals and Fractions

In the following exercises, order each pair of numbers, using < or >.

584. $\frac{1}{2}$ ___0.2
585. $\frac{3}{5}$ ___0.

586. $-\frac{7}{8}$ ___ −0.84

587. $-\frac{5}{12}$ ___ −0.42

588. 0.625___ $\frac{13}{20}$

589. 0.33___ $\frac{5}{16}$

In the following exercises, write each set of numbers in order from least to greatest.

590. $\frac{2}{3}$, $\frac{17}{20}$, 0.65

591. $\frac{7}{9}$, 0.75, $\frac{11}{15}$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify.

592. 4(10.3 − 5.8)

593. $\frac{3}{4}$(15.44 − 7.4)

594. 30 ÷ (0.45 + 0.15)

595. 1.6 + $\frac{3}{8}$

596. 52(0.5) + $(0.4)^2$

597. $-\frac{2}{5} \cdot \frac{9}{10}$ + 0.14

Find the Circumference and Area of Circles

In the following exercises, approximate the (a) circumference and (b) area of each circle.

598. radius = 6 in.

599. radius = 3.5 ft.

600. radius = 7 33 m

601. diameter = 11 cm

5.4 - Solve Equations with Decimals

Determine Whether a Decimal is a Solution of an Equation

In the following exercises, determine whether the each number is a solution of the given equation.

602. x − 0.4 = 2.1

a. x = 1.7

b. x = 2.5

603. y + 3.2 = −1.5

a. y = 1.7

b. y = −4.7

604. $\frac{u}{2.5}$ = −12.5

a. u = −5

b. u = −31.25

605. 0.45v = −40.5

a. v = −18.225

b. v = −90

Solve Equations with Decimals

In the following exercises, solve.

606. m + 3.8 = 7.5
607. h + 5.91 = 2.4
608. a + 2.26 = −1.1
609. p − 4.3 = −1.65
610. x − 0.24 = −8.6
611. j − 7.42 = −3.7
612. 0.6p = 13.2
613. −8.6x = 34.4
614. −22.32 = −2.4z
615. $\frac{a}{0.3} = -24$
616. $\frac{p}{-7} = -4.2$
617. $\frac{s}{-2.5} = -10$

Translate to an Equation and Solve

In the following exercises, translate and solve.

618. The difference of n and 15.2 is 4.4.
619. The product of −5.9 and x is −3.54.
620. The quotient of y and −1.8 is −9.
621. The sum of m and −4.03 is 6.8.

5.5 - Averages and Probability

Find the Mean of a Set of Numbers

In the following exercises, find the mean of the numbers.

622. 2, 4, 1, 0, 1, and 1
623. $270, $310.50, $243.75, and $252.15
624. Each workday last week, Yoshie kept track of the number of minutes she had to wait for the bus. She waited 3, 0, 8, 1, and 8 minutes. Find the mean
625. In the last three months, Raul's water bills were $31.45, $48.76, and $42.60. Find the mean.

Find the Median of a Set of Numbers

In the following exercises, find the median.

626. 41, 45, 32, 60, 58
627. 25, 23, 24, 26, 29, 19, 18, 32
628. The ages of the eight men in Jerry's model train club are 52, 63, 45, 51, 55, 75, 60, and 59. Find the median age.
629. The number of clients at Miranda's beauty salon each weekday last week were 18, 7, 12, 16, and 20. Find the median number of clients.

Find the Mode of a Set of Numbers

In the following exercises, identify the mode of the numbers.

630. 6, 4, 4, 5, 6, 6, 4, 4, 4, 3, 5
631. The number of siblings of a group of students: 2, 0, 3, 2, 4, 1, 6, 5, 4, 1, 2, 3

Use the Basic Definition of Probability

In the following exercises, solve. (Round decimals to three places.)

632. The Sustainability Club sells 200 tickets to a raffle, and Albert buys one ticket. One ticket will be selected at random to win the grand prize. Find the probability Albert will win the grand prize. Express your answer as a fraction and as a decimal.

633. Luc has to read 3 novels and 12 short stories for his literature class. The professor will choose one reading at random for the final exam. Find the probability that the professor will choose a novel for the final exam. Express your answer as a fraction and as a decimal.

5.6 - Ratios and Rate

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction. Simplify the answer if possible.

634. 28 to 40
635. 56 to 32
636. 3.5 to 0.5
637. 1.2 to 1.8
638. $1\frac{3}{4}$ to $1\frac{5}{8}$
639. $2\frac{1}{3}$ to $5\frac{1}{4}$
640. 64 ounces to 30 ounces
641. 28 inches to 3 feet

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction. Simplify the answer if possible.

642. 180 calories per 8 ounces 643. 90 pounds per 7.5 square inches
643. 126 miles in 4 hours 645. $612.50 for 35 hours

Find Unit Rates

In the following exercises, find the unit rate.

646. 180 calories per 8 ounces
647. 90 pounds per 7.5 square inches
648. 126 miles in 4 hours
649. $612.50 for 35 hours

Find Unit Price

In the following exercises, find the unit price.

650. T-shirts: 3 for $8.97
651. Highlighters: 6 for $2.52
652. An office supply store sells a box of pens for $11. The box contains 12 pens. How much does each pen cost?
653. Anna bought a pack of 8 kitchen towels for $13.20. How much did each towel cost? Round to the nearest cent if necessary.

In the following exercises, find each unit price and then determine the better buy.

654. Shampoo: 12 ounces for $4.29 or 22 ounces for $7.29?
655. Vitamins: 60 tablets for $6.49 or 100 for $11.99?

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.

656. 535 miles per h hours
657. a adults to 45 children
658. the ratio of 4y and the difference of x and 10
659. the ratio of 19 and the sum of 3 and n

5.7 - Simplify and Use Square Roots

Simplify Expressions with Square Roots

In the following exercises, simplify.

660. $\sqrt{64}$
661. $\sqrt{144}$
662. $-\sqrt{25}$
663. $-\sqrt{81}$
664. $-\sqrt{9}$
665. $\sqrt{-36}$
666. $\sqrt{64}+\sqrt{225}$
667. $\sqrt{64+225}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

668. $\sqrt{28}$
669. $\sqrt{155}$

Approximate Square Roots

In the following exercises, approximate each square root and round to two decimal places.

670. $\sqrt{15}$
671. $\sqrt{57}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

672. $\sqrt{q^2}$
673. $\sqrt{64b^2}$
674. $-\sqrt{121a^2}$
675. $\sqrt{225m^2n^2}$
676. $-\sqrt{100q^2}$
677. $\sqrt{49y^2}$
678. $\sqrt{4a^2b^2}$
679. $\sqrt{121c^2d^2}$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

680. **Art** Diego has 225 square inch tiles. He wants to use them to make a square mosaic. How long can each side of the mosaic be?
681. **Landscaping** Janet wants to plant a square flower garden in her yard. She has enough topsoil to cover an area of 30 square feet. How long can a side of the flower garden be?
682. **Gravity** A hiker dropped a granola bar from a lookout spot 576 feet above a valley. How long did it take the granola bar to reach the valley floor?
683. **Accident investigation** The skid marks of a car involved in an accident were 216 feet. How fast had the car been going before applying the brakes?

PRACTICE TEST

684. Write six and thirty-four thousandths as a decimal.
685. Write 1.73 as a fraction.
686. Write 5 8 as a decimal.
687. Round 16.749 to the nearest (a) tenth (b) hundredth (c) whole number
688. Write the numbers $\frac{4}{5}$, −0.1, 0.804, $\frac{2}{9}$, −7.4, 0.21 in order from smallest to largest.

In the following exercises, simplify each expression.

689. 15.4 + 3.02
690. 20 − 5.71
691. (0.64)(0.3)
692. (−4.2)(100)
693. 0.96 ÷ (−12)
694. −5 ÷ 0.025
695. −0.6 ÷ (−0.3)
696. (0.7) 2
697. 24 ÷ (0.1 + 0.02)
698. 4(10.3 − 5.8)
699. 1.6 + $\frac{3}{8}$
700. $\frac{2}{3}$(14.65 − 4.6)

In the following exercises, solve.

701. m + 3.7 = 2.5
702. $\frac{h}{0.5}$ = 4.38
703. −6.5y = −57.2
704. 1.94 = a − 2.6
705. Three friends went out to dinner and agreed to split the bill evenly. The bill was $79.35. How much should each person pay?
706. A circle has radius 12. Find the (a) circumference and (b) area. [Use 3.14 for π.]
707. The ages, in months, of 10 children in a preschool class are: 55, 55, 50, 51, 52, 50, 53, 51, 55, 49. Find the (a) mean (b) median (c) mode
708. Of the 16 nurses in Doreen's department, 12 are women and 4 are men. One of the nurses will be assigned at random to work an extra shift next week. (a) Find the probability a woman nurse will be assigned the extra shift. (b) Convert the fraction to a decimal.
709. Find each unit price and then the better buy. Laundry detergent: 64 ounces for $10.99 or 48 ounces for $8.49

In the following exercises, simplify.

710. $\sqrt{36+64}$
711. $\sqrt{144n^2}$
712. Estimate $\sqrt{54}$ to between two whole numbers.
713. Yanet wants a square patio in her backyard. She has 225 square feet of tile. How long can a side of the patio be?

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1.4.S: Decimals (Summary)

Key Terms

circumference of a circle	The distance around a circle
diameter of a circle	A line segment that passes through a circle's center connecting two points on the circle.
equivalent decimals	Two decimals are equivalent decimals if they convert to equivalent fractions.
mean	The mean of a set of n numbers is the arithmetic average of the numbers. $$mean = \frac{sum\ of\ values\ in\ data\ set}{n} \quad (1.4.S.1)$$
median	The median of a set of data values is the middle value. Half the data values are less than or equal to the median. Half the data values are greater than or equal to the median.
mode	The mode of a set of numbers is the number with the highest frequency.
radius of a circle	A line segment from the center to any point on the circle.
rate	A comparison of two quantities with different units. A rate is usually written as a fraction.
ratio	A comparison of two numbers or two quantities that are measured with the same unit. The ratio of a to b is written a to b, $\frac{a}{b}$, or a : b.
repeating decimal	A decimal in which the last digit or group of digits repeats endlessly.
unit price	A unit rate that gives the price of one item.
unit rate	A rate with denominator of 1 unit.

Key Concepts

5.1 - Decimals

- **Name a decimal number.**
  1. Name the number to the left of the decimal point.
  2. Write "and" for the decimal point.
  3. Name the "number" part to the right of the decimal point as if it were a whole number.
  4. Name the decimal place of the last digit.
- **Write a decimal number from its name.**
  1. Look for the word "and"—it locates the decimal point. Place a decimal point under the word "and." Translate the words before "and" into the whole number and place it to the left of the decimal point. If there is no "and," write a "0" with a decimal point to its right.
  2. Mark the number of decimal places needed to the right of the decimal point by noting the place value indicated by the last word.
  3. Translate the words after "and" into the number to the right of the decimal point. Write the number in the spaces—putting the final digit in the last place.
  4. Fill in zeros for place holders as needed.
- **Convert a decimal number to a fraction or mixed number.**

1. Look at the number to the left of the decimal. If it is zero, the decimal converts to a proper fraction. If it is not zero, the decimal converts to a mixed number. Write the whole number.
2. Determine the place value of the final digit.
3. Write the fraction. numerator—the ‘numbers’ to the right of the decimal point denominator—the place value corresponding to the final digit
4. Simplify the fraction, if possible.

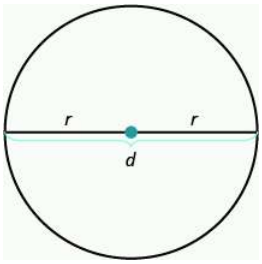
- **Order decimals.**
  1. Check to see if both numbers have the same number of decimal places. If not, write zeros at the end of the one with fewer digits to make them match.
  2. Compare the numbers to the right of the decimal point as if they were whole numbers.
  3. Order the numbers using the appropriate inequality sign.
- Round a decimal.
  1. Locate the given place value and mark it with an arrow.
  2. Underline the digit to the right of the given place value.
  3. Is this digit greater than or equal to 5? Yes - add 1 to the digit in the given place value. No - do not change the digit in the given place value
  4. Rewrite the number, removing all digits to the right of the given place value.

5.2 - Decimal Operations

- **Add or subtract decimals.**
  1. Write the numbers vertically so the decimal points line up.
  2. Use zeros as place holders, as needed.
  3. Add or subtract the numbers as if they were whole numbers. Then place the decimal in the answer under the decimal points in the given numbers.
- **Multiply decimal numbers.**
  1. Determine the sign of the product.
  2. Write the numbers in vertical format, lining up the numbers on the right.
  3. Multiply the numbers as if they were whole numbers, temporarily ignoring the decimal points.
  4. Place the decimal point. The number of decimal places in the product is the sum of the number of decimal places in the factors. If needed, use zeros as placeholders.
  5. Write the product with the appropriate sign.
- **Multiply a decimal by a power of 10.**
  1. Move the decimal point to the right the same number of places as the number of zeros in the power of 10.
  2. Write zeros at the end of the number as placeholders if needed.
- **Divide a decimal by a whole number.**
  1. Write as long division, placing the decimal point in the quotient above the decimal point in the dividend.
  2. Divide as usual.
- **Divide decimal numbers.**
  1. Determine the sign of the quotient.
  2. Make the divisor a whole number by moving the decimal point all the way to the right. Move the decimal point in the dividend the same number of places to the right, writing zeros as needed.
  3. Divide. Place the decimal point in the quotient above the decimal point in the dividend.
  4. Write the quotient with the appropriate sign.
- **Strategy for Applications**
  1. Identify what you are asked to find.
  2. Write a phrase that gives the information to find it.
  3. Translate the phrase to an expression.
  4. Simplify the expression.
  5. Answer the question with a complete sentence.

5.3 - Decimals and Fractions

- **Convert a Fraction to a Decimal**: To convert a fraction to a decimal, divide the numerator of the fraction by the denominator of the fraction.
- **Properties of Circles**



r is the length of the radius

d is the length of the diameter

The circumference is $2\pi r$.

$$C = 2\pi r \tag{1.4.S.2}$$

The area is πr^2.

$$A = \pi r^2 \tag{1.4.S.3}$$

5.4 - Solve Equations with Decimals

- **Determine whether a number is a solution to an equation.**
  - Substitute the number for the variable in the equation.
  - Simplify the expressions on both sides of the equation.
  - Determine whether the resulting equation is true. If so, the number is a solution. If not, the number is not a solution.
- **Properties of Equality**

Table 1.4.S.1

Subtraction Property of Equality For any numbers a, b, and c, if a = b then a − c = b − c.	**Addition Property of Equality** For any numbers a, b, and c, if a = b then a + c = b + c.
Division of Property of Equality For any numbers a, b, and c ≠ 0, if a = b then \(\dfrac{a}{c} = \dfrac{b}{c}).	**Multiplication Property of Equality** For any numbers a, b, and c, if a = b then a • c = b • c.

5.5 - Averages and Probability

- **Calculate the mean of a set of numbers.**
  1. Write the formula for the mean

$$mean = \frac{sum\ of\ values\ in\ data\ set}{n} \tag{1.4.S.4}$$

  2. Find the sum of all the values in the set. Write the sum in the numerator.
  3. Count the number, n, of values in the set. Write this number in the denominator.
  4. Simplify the fraction.
  5. Check to see that the mean is reasonable. It should be greater than the least number and less than the greatest number in the set.
- **Find the median of a set of numbers.**
  1. List the numbers from least to greatest.
  2. Count how many numbers are in the set. Call this n.
  3. Is n odd or even? If n is an odd number, the median is the middle value. If n is an even number, the median is the mean of the two middle values.

- **Identify the mode of a set of numbers.**
  1. List the data values in numerical order.
  2. Count the number of times each value appears.
  3. The mode is the value with the highest frequency.

5.7 - Simplify and Use Square Roots

- **Square Root Notation**: $\sqrt{m}$ is read 'the square root of m '
  - If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.



- **Use a strategy for applications with square roots.**
  - Identify what you are asked to find.
  - Write a phrase that gives the information to find it.
  - Translate the phrase to an expression.
  - Simplify the expression.
  - Write a complete sentence that answers the question.

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SECTION OVERVIEW

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1.5.1: Unit 1 - Chapter 1- Introduction to Expressions

1.5.2: Unit 1 - Chapter 2- The Order of Operations

1.5.3: Unit 1 - Chapter 3- Evaluating Expressions

1.5.4: Unit 1 - Chapter 6- Roots and Radicals

1.5.4.1: Simplify Expressions with Square Roots

1.5.4.1E: Exercises

1.5.4.2: Simplify Radical Expressions

1.5.4.2E: Exercises

1.5.4.3: Simplify Rational Exponents

1.5.4.3E: Exercises

1.5.4.4: Add, Subtract, and Multiply Radical Expressions

1.5.4.4E: Exercises

1.5.4.5: Divide Radical Expressions

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1.5.4.6: Complex Numbers

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1.5.5: Unit 1 - Chapter 7- Exponential and Logarithmic Expressions

1.5.5.1: Evaluate Exponential Expressions

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1.5.5.3: Use the Properties of Logarithms

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1.5.5.4: Chapter 7 Review Exercises

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1.5.5.3: Use the Properties of Logarithms

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SECTION OVERVIEW

1.6: Percents

When you deposit money in a savings account at a bank, it earns additional money. Figuring out how your money will grow involves understanding and applying concepts of percents. In this chapter, we will find out what percents are and how we can use them to solve problems.

1.6.1: Understand Percent

1.6.2: Solve General Applications of Percent

1.6.3: Solve Sales Tax, Commission, and Discount Applications

1.6.4: Solve Simple Interest Applications

1.6.5: Solve Proportions and their Applications (Part 1)

1.6.6: Solve Proportions and their Applications (Part 2)

1.6.E: Percents (Exercises)

1.6.S: Percents (Summary)

Figure 6.1 - Banks provide money for savings and charge money for loans. The interest on savings and loans is usually given as a percent. (credit: Mike Mozart, Flickr)

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1.6.1: Understand Percent

Learning Objectives

- Use the definition of percent
- Convert percents to fractions and decimals
- Convert decimals and fractions to percents

be prepared!

Before you get started, take this readiness quiz.

1. Translate “the ratio of 33 to 5” into an algebraic expression. If you missed this problem, review Table 2.4.3.
2. Write 3 5 as a decimal. If you missed this problem, review Example 5.5.1.
3. Write 0.62 as a fraction. If you missed this problem, review Example 5.1.4.

Use the Definition of Percent

How many cents are in one dollar? There are 100 cents in a dollar. How many years are in a century? There are 100 years in a century. Does this give you a clue about what the word “per cent” means? It is really two words, “per cent,” and means per one hundred. A **percent** is a ratio whose denominator is 100. We use the percent symbol %, to show percent.

Definition: Percent

A percent is a ratio whose denominator is 100.

According to data from the American Association of Community Colleges (2015), about 57% of community college students are female. This means 57 out of every 100 community college students are female, as Figure 1.6.1.1shows. Out of the 100 squares on the grid, 57 are shaded, which we write as the ratio $\frac{57}{100}$.

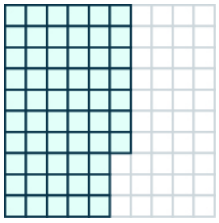


Figure 1.6.1.1- Among every 100 community college students, 57 are female.

Similarly, 25% means a ratio of $\frac{25}{100}$, 3% means a ratio of $\frac{3}{100}$ and 100% means a ratio of $\frac{100}{100}$. In words, "one hundred percent" means the total 100% is $\frac{100}{100}$, and since $\frac{100}{100} = 1$, we see that 100% means 1 whole.

Example 1.6.1.1:

According to the Public Policy Institute of California (2010), 44% of parents of public school children would like their youngest child to earn a graduate degree. Write this percent as a ratio.

Solution

The amount we want to convert is 44%.	44%	
Write the percent as a ratio. Remember that percent means per 100.	$\frac{44}{100}$	(1.6.1.1)

Exercise 1.6.1.1:

Write the percent as a ratio. According to a survey, 89% of college students have a smartphone.

Answer

$\frac{89}{100}$

Exercise 1.6.1.2:

Write the percent as a ratio. A study found that 72% of U.S. teens send text messages regularly.

Answer

$\frac{72}{100}$

Example 1.6.1.2:

In 2007, according to a U.S. Department of Education report, 21 out of every 100 first-time freshmen college students at 4-year public institutions took at least one remedial course. Write this as a ratio and then as a percent.

Solution

The amount we want to convert is 21 out of 100.	21 out of 100	
Write as a ratio.	$\frac{21}{100}$	(1.6.1.2)
Convert the 21 per 100 to percent.	21%	

Exercise 1.6.1.3:

Write as a ratio and then as a percent: The American Association of Community Colleges reported that 62 out of 100 full-time community college students balance their studies with full-time or part time employment.

Answer

$\frac{62}{100}$, 62%

Exercise 1.6.1.4:

Write as a ratio and then as a percent: In response to a student survey, 41 out of 100 Santa Ana College students expressed a goal of earning an Associate's degree or transferring to a four-year college.

Answer

$\frac{41}{100}$, 41%

Convert Percents to Fractions and Decimals

Since percents are ratios, they can easily be expressed as fractions. Remember that percent means per 100, so the denominator of the fraction is 100.

HOW TO: CONVERT A PERCENT TO A FRACTION

Step 1. Write the percent as a ratio with the denominator 100.

Step 2. Simplify the fraction if possible.

Example 1.6.1.3:

Convert each percent to a fraction: (a) 36% (b) 125%

Solution

(a) 36%

Write as a ratio with denominator 100.	$\frac{36}{100}$	(1.6.1.3)
Simplify.	$\frac{9}{25}$	(1.6.1.4)

(b) 125%

Write as a ratio with denominator 100.	$\frac{125}{100}$	(1.6.1.5)
Simplify.	$\frac{5}{4}$	(1.6.1.6)

Exercise 1.6.1.5:

Convert each percent to a fraction: (a) 48% (b) 110%

Answer a

$\frac{12}{25}$

Answer b

$\frac{11}{10}$

Exercise 1.6.1.6:

Convert each percent to a fraction: (a) 64% (b) 150%

Answer a

$\frac{16}{25}$

Answer b

$\frac{3}{2}$

The previous example shows that a percent can be greater than 1. We saw that 125% means $\frac{125}{100}$, or $\frac{5}{4}$. These are improper fractions, and their values are greater than one.

Example 1.6.1.4:

Convert each percent to a fraction: (a) 24.5% (b) $33\frac{1}{3}\%$

Solution

(a) 24.5%

Write as a ratio with denominator 100.	$\frac{24.5}{100}$	(1.6.1.7)
Clear the decimal by multiplying numerator and denominator by 10.	$\frac{24.5(10)}{100(10)}$	(1.6.1.8)
Multiply.	$\frac{245}{1000}$	(1.6.1.9)
Rewrite showing common factors.	$\frac{5 \cdot 49}{5 \cdot 200}$	(1.6.1.10)
Simplify.	$\frac{49}{200}$	(1.6.1.11)

(b) $33\frac{1}{3}\%$

Write as a ratio with denominator 100.	$\frac{33\frac{1}{3}}{100}$	(1.6.1.12)
Write the numerator as an improper fraction.	$\frac{\frac{100}{3}}{100}$	(1.6.1.13)
Rewrite as fraction division, replacing 100 with $\frac{100}{1}$.	$\frac{100}{3} \div \frac{100}{1}$	(1.6.1.14)
Multiply by the reciprocal.	$\frac{100}{3} \cdot \frac{1}{100}$	(1.6.1.15)
Simplify.	$\frac{1}{3}$	(1.6.1.16)

Exercise 1.6.1.7:

Convert each percent to a fraction: (a) 64.4% (b) $66\frac{2}{3}\%$

Answer a

$\frac{161}{250}$

Answer b

$\frac{2}{3}$

Exercise 1.6.1.8:

Convert each percent to a fraction: (a) 42.5% (b) $8\frac{3}{4}\%$

Answer a

$\frac{113}{250}$

Answer b

$\frac{7}{80}$

In Decimals, we learned how to convert fractions to decimals. To convert a percent to a decimal, we first convert it to a fraction and then change the fraction to a decimal.

HOW TO: CONVERT A PERCENT TO A DECIMAL

Step 1. Write the percent as a ratio with the denominator 100.

Step 2. Convert the fraction to a decimal by dividing the numerator by the denominator.

Example 1.6.1.5:

Convert each percent to a decimal: (a) 6% (b) 78%

Solution

Because we want to change to a decimal, we will leave the fractions with denominator 100 instead of removing common factors.

(a) 6%

Write as a ratio with denominator 100.	$\frac{6}{100}$	(1.6.1.17)
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06	

(b) 78%

Write as a ratio with denominator 100.	$\frac{78}{100}$	(1.6.1.18)
Change the fraction to a decimal by dividing the numerator by the denominator.	0.78	

Exercise 1.6.1.9:

Convert each percent to a decimal: (a) 9% (b) 87%

Answer a

0.09

Answer b

0.87

Exercise 1.6.1.10:

Convert each percent to a decimal: (a) 3% (b) 91%

Answer a

0.03

Answer b

0.91

Example 1.6.1.6:

Convert each percent to a decimal: (a) 135% (b) 12.5%

Solution

Write as a ratio with denominator 100.	$\frac{135}{100}$	(1.6.1.19)
Change the fraction to a decimal by dividing the numerator by the denominator.	1.35	

(b) 12.5%

Write as a ratio with denominator 100.	$\frac{12.5}{100}$	(1.6.1.20)
Change the fraction to a decimal by dividing the numerator by the denominator.	0.125	

Exercise 1.6.1.11:

Convert each percent to a decimal: (a) 115% (b) 23.5%

Answer a

1.15

Answer b

0.235

Exercise 1.6.1.12:

Convert each percent to a decimal: (a) 123% (b) 16.8%

Answer a

1.23

Answer b

0.168

Let's summarize the results from the previous examples in Table 1.6.1.1, and look for a pattern we could use to quickly convert a percent number to a decimal number.

Table 1.6.1.1

Percent	Decimal
6%	0.06
78%	0.78
135%	1.35
12.5%	0.125

Do you see the pattern? To convert a percent number to a decimal number, we move the decimal point two places to the left and remove the % sign. (Sometimes the decimal point does not appear in the percent number, but just like we can think of the integer 6 as 6.0, we can think of 6% as 6.0%.) Notice that we may need to add zeros in front of the number when moving the decimal to the left.

Figure 1.6.1.2uses the percents in Table 1.6.1.1and shows visually how to convert them to decimals by moving the decimal point two places to the left.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 1.6.1.2

Example 1.6.1.7:

Among a group of business leaders, 77% believe that poor math and science education in the U.S. will lead to higher unemployment rates. Convert the percent to: (a) a fraction (b) a decimal

Solution

(a)

Write as a ratio with denominator 100.	$\frac{77}{100}$	(1.6.1.21)

(b)

Change the fraction to a decimal by dividing the numerator by the denominator.	0.77

Exercise 1.6.1.13:

Twitter's share of web traffic jumped 24% when one celebrity tweeted live on air. Convert the percent to: (a) a fraction and (b) a decimal.

Answer a

$\frac{6}{25}$

Answer b

0.24

Exercise 1.6.1.14:

The U.S. Census estimated that in 2013, 44% of the population of Boston age 25 or older have a bachelor's or higher degrees. Convert the percent to: (a) a fraction and (b) a decimal.

Answer a

$\frac{22}{50}$

Answer b

0.44

Example 1.6.1.8:

There are four suits of cards in a deck of cards—hearts, diamonds, clubs, and spades. The probability of randomly choosing a heart from a shuffled deck of cards is 25%. Convert the percent to: (a) a fraction (b) a decimal.



Figure 1.6.1.3- (credit: Riles32807, Wikimedia Commons)

Solution

(a)

Write as a ratio with denominator 100.	$\frac{25}{100}$	(1.6.1.22)
Simplify.	$\frac{1}{4}$	(1.6.1.23)

(b)

Change the fraction to a decimal by dividing the numerator by the denominator.	0.25

Exercise 1.6.1.15:

The probability that it will rain Monday is 30%. Convert the percent to: (a) a fraction, and (b) a decimal.

Answer a

$\frac{3}{10}$

Answer b

0.3

Exercise 1.6.1.16:

The probability of getting heads three times when tossing a coin three times is 12.5%. Convert the percent to: (a) a fraction, and (b) a decimal.

Answer a

$\frac{12.5}{100}$

Answer b

0.125

Convert Decimals and Fractions to Percents

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

HOW TO: CONVERT A DECIMAL TO A PERCENT

Step 1. Write the decimal as a fraction.

Step 2. If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.

Step 3. Write this ratio as a percent.

Example 1.6.1.9:

Convert each decimal to a percent: (a) 0.05 (b) 0.83

Solution

(a) 0.05

Write as a fraction. The denominator is 100.	$\frac{5}{100}$	(1.6.1.24)
Write this ratio as a percent.	5%	

(b)

The denominator is 100.	$\frac{83}{100}$	(1.6.1.25)
Write this ratio as a percent.	83%	

Exercise 1.6.1.17:

Convert each decimal to a percent: (a) 0.01 (b) 0.17.

Answer a

1%

Answer b

17%

Exercise 1.6.1.18:

Convert each decimal to a percent: (a) 0.04 (b) 0.41.

Answer a

4%

Answer b

41%

To convert a mixed number to a percent, we first write it as an improper fraction.

Example 1.6.1.10:

Convert each decimal to a percent: (a) 1.05 (b) 0.075

Solution

(a) 1.05

Write as a fraction.	$1\frac{5}{100}$	(1.6.1.26)
Write as an improper fraction. The denominator is 100.	$\frac{105}{100}$	(1.6.1.27)
Write this ratio as a percent.	105%	

Notice that since 1.05 > 1, the result is more than 100%.

(b) 0.075

Write as a fraction.	$\frac{75}{1,000}$	(1.6.1.28)
Divide the numerator and denominator by 10, so that the denominator is 100.	$\frac{7.5}{100}$	(1.6.1.29)
Write this ratio as a percent.	7.5%	

Exercise 1.6.1.19:

Convert each decimal to a percent: (a) 1.75 (b) 0.0825

Answer a

175%

Answer b

8.25%

Exercise 1.6.1.20:

Convert each decimal to a percent: (a) 2.25 (b) 0.0925

Answer a

225%

Answer b

9.25%

Let's summarize the results from the previous examples in Table 6.20 so we can look for a pattern.

Table 1.6.1.2

Decimal	Percent
0.05	5%
0.83	83%
1.05	105%
0.075	7.5%

Do you see the pattern? To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.

Figure 1.6.1.4 uses the decimal numbers in Table 1.6.1.2 and shows visually to convert them to percents by moving the decimal point two places to the right and then writing the % sign.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 1.6.1.4

In Decimals, we learned how to convert fractions to decimals. Now we also know how to change decimals to percents. So to convert a fraction to a percent, we first change it to a decimal and then convert that decimal to a percent.

HOW TO: CONVERT A FRACTION TO A PERCENT

Step 1. Convert the fraction to a decimal.

Step 2. Convert the decimal to a percent.

Example 1.6.1.11:

Convert each fraction or mixed number to a percent: (a) $\frac{3}{4}$ (b) $\frac{11}{8}$ (c) $2\frac{1}{5}$

Solution

To convert a fraction to a decimal, divide the numerator by the denominator.

(a)

Change to a decimal.	$\frac{3}{4}$	(1.6.1.30)

Write as a percent by moving the decimal two places.	0.75	
	75%	

(b)

Change to a decimal.	$\frac{11}{8}$	(1.6.1.31)
Write as a percent by moving the decimal two places.	1.375	
	137.5%	

(c)

Write as an improper fraction.	$2\frac{1}{5}$	(1.6.1.32)
Change to a decimal.	$\frac{11}{5}$	(1.6.1.33)
Write as a percent.	2.20	
	220%	

Notice that we needed to add zeros at the end of the number when moving the decimal two places to the right.

? Exercise 1.6.1.21:

Convert each fraction or mixed number to a percent: (a) $\frac{5}{8}$ (b) $\frac{11}{4}$ (c) $3\frac{2}{5}$

Answer a

62.5%

Answer b

275%

Answer c

340%

? Exercise 1.6.1.22:

Convert each fraction or mixed number to a percent: (a) $\frac{7}{8}$ (b) $\frac{9}{4}$ (c) $1\frac{3}{5}$

Answer a

87.5%

Answer b

225%

Answer c

160%

Sometimes when changing a fraction to a decimal, the division continues for many decimal places and we will round off the quotient. The number of decimal places we round to will depend on the situation. If the decimal involves money, we round to the hundredths place. For most other cases in this book we will round the number to the nearest thousandth, so the percent will be rounded to the nearest tenth.

Example 1.6.1.12:

Convert $\frac{5}{7}$ to a percent.

Solution

To change a fraction to a decimal, we divide the numerator by the denominator.

Change to a decimal—rounding to the nearest thousandth.	0.714
Write as a percent.	71.4%

Exercise 1.6.1.23:

Convert the fraction to a percent: $\frac{3}{7}$

Answer

42.9%

Exercise 1.6.1.24:

Convert the fraction to a percent: $\frac{4}{7}$

Answer

57.1%

When we first looked at fractions and decimals, we saw that fractions converted to a repeating decimal. When we converted the fraction $\frac{4}{3}$ to a decimal, we wrote the answer as $1.\overline{3}$. We will use this same notation, as well as fraction notation, when we convert fractions to percents in the next example.

Example 1.6.1.13:

An article in a medical journal claimed that approximately $\frac{1}{3}$ of American adults are obese. Convert the fraction $\frac{1}{3}$ to a percent.

Solution

Change to a decimal.	$3\overline{)1.00}\ 0.33\ldots$ $\underline{9}$ 10 $\underline{9}$ 1
Write as a repeating decimal.	0.333 …
Write as a percent.	$33\frac{1}{3}\%$

We could also write the percent as $33.\overline{3}\%$.

Exercise 1.6.1.25:

According to the U.S. Census Bureau, about $\frac{1}{9}$ of United States housing units have just 1 bedroom. Convert the fraction to a percent.

Answer

$11.\overline{1}\%$, or $11\frac{1}{9}\%$

Exercise 1.6.1.26:

According to the U.S. Census Bureau, about $\frac{1}{6}$ of Colorado residents speak a language other than English at home. Convert the fraction to a percent.

Answer

$16.\overline{6}\%$, or $16\frac{2}{3}\%$

Practice Makes Perfect

Use the Definition of Percents

In the following exercises, write each percent as a ratio.

1. In 2014, the unemployment rate for those with only a high school degree was 6.0%.
2. In 2015, among the unemployed, 29% were long-term unemployed.
3. The unemployment rate for those with Bachelor's degrees was 3.2% in 2014.
4. The unemployment rate in Michigan in 2014 was 7.3%.

In the following exercises, write as (a) a ratio and (b) a percent.

5. 57 out of 100 nursing candidates received their degree at a community college.
6. 80 out of 100 firefighters and law enforcement officers were educated at a community college.
7. 42 out of 100 first-time freshmen students attend a community college.
8. 71 out of 100 full-time community college faculty have a master's degree.

Convert Percents to Fractions and Decimals

In the following exercises, convert each percent to a fraction and simplify all fractions.

9. 4%
10. 8%
11. 17%
12. 19%
13. 52%
14. 78%
15. 125%
16. 135%
17. 37.5%
18. 42.5%
19. 18.4%
20. 46.4%
21. $9\frac{1}{2}\%$

22. $8\frac{1}{2}$ %
23. $5\frac{1}{3}$ %
24. $6\frac{2}{3}$ %

In the following exercises, convert each percent to a decimal.

25. 5%
26. 9%
27. 1%
28. 2%
29. 63%
30. 71%
31. 40%
32. 50%
33. 115%
34. 125%
35. 150%
36. 250%
37. 21.4%
38. 39.3%
39. 7.8%
40. 6.4%

In the following exercises, convert each percent to (a) a simplified fraction and (b) a decimal.

41. In 2010, 1.5% of home sales had owner financing. (Source: Bloomberg Businessweek, 5/ 23–29/2011)
42. In 2000, 4.2% of the United States population was of Asian descent. (Source: www.census.gov)
43. According to government data, in 2013 the number of cell phones in India was 70.23% of the population.
44. According to the U.S. Census Bureau, among Americans age 25 or older who had doctorate degrees in 2014, 37.1% are women.
45. A couple plans to have two children. The probability they will have two girls is 25%.
46. Javier will choose one digit at random from 0 through 9. The probability he will choose 3 is 10%.
47. According to the local weather report, the probability of thunderstorms in New York City on July 15 is 60%.
48. A club sells 50 tickets to a raffle. Osbaldo bought one ticket. The probability he will win the raffle is 2%.

Convert Decimals and Fractions to Percents

In the following exercises, convert each decimal to a percent.

49. 0.01
50. 0.03
51. 0.18
52. 0.15
53. 1.35
54. 1.56
55. 3
56. 4
57. 0.009
58. 0.008
59. 0.0875
60. 0.0625
61. 1.5
62. 2.2
63. 2.254
64. 2.317

In the following exercises, convert each fraction to a percent.

65. $\frac{1}{4}$
66. $\frac{1}{5}$
67. $\frac{3}{8}$
68. $\frac{5}{8}$
69. $\frac{7}{4}$
70. $\frac{9}{8}$
71. $6\frac{4}{5}$
72. $5\frac{1}{4}$
73. $\frac{5}{12}$
74. $\frac{11}{12}$
75. $2\frac{2}{3}$
76. $1\frac{2}{3}$
77. $\frac{3}{7}$
78. $\frac{6}{7}$
79. $\frac{5}{9}$
80. $\frac{4}{9}$

In the following exercises, convert each fraction to a percent.

81. $\frac{1}{4}$ of washing machines needed repair.
82. $\frac{1}{5}$ of dishwashers needed repair.

In the following exercises, convert each fraction to a percent.

83. According to the National Center for Health Statistics, in 2012, 7 20 of American adults were obese.
84. The U.S. Census Bureau estimated that in 2013, 85% of Americans lived in the same house as they did 1 year before.

In the following exercises, complete the table.

85. Table 6.26

Fraction	Decimal	Percent
$\frac{1}{2}$ $(1.6.1.34)$		
	0.45	
		18%
$\frac{1}{3}$ $(1.6.1.35)$		
	0.0008	

Fraction	Decimal	Percent
2		

86. Table 6.27

Fraction	Decimal	Percent
$\frac{1}{4}$ (1.6.1.36)		
	0.65	
		22%
$\frac{2}{3}$ (1.6.1.37)		
	0.0004	
3		

Everyday Math

87. Sales tax Felipa says she has an easy way to estimate the sales tax when she makes a purchase. The sales tax in her city is 9.05%. She knows this is a little less than 10%.
    a. Convert 10% to a fraction.
    b. Use your answer from (a) to estimate the sales tax Felipa would pay on a $95 dress.
88. Savings Ryan has 25% of each paycheck automatically deposited in his savings account.
    a. Write 25% as a fraction.
    b. Use your answer from (a) to find the amount that goes to savings from Ryan's $2,400 paycheck.
89. Amelio is shopping for textbooks online. He found three sellers that are offering a book he needs for the same price, including shipping. To decide which seller to buy from he is comparing their customer satisfaction ratings. The ratings are given in the chart.

Seller	Rating
A	4/5
B	3.5/4
C	85%

90. Write seller C's rating as a fraction and a decimal.
91. Write seller B's rating as a percent and a decimal.
92. Write seller A's rating as a percent and a decimal.
93. Which seller should Amelio buy from and why?

Writing Exercises

94. Convert 25%, 50%, 75%, and 100% to fractions. Do you notice a pattern? Explain what the pattern is.
95. Convert $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}$, and $\frac{9}{10}$ to percents. Do you notice a pattern? Explain what the pattern is.
96. When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.
97. According to cnn.com, cell phone use in 2008 was 600% of what it had been in 2001. Explain what 600% means in this context.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the definition of percent.			
convert percents to fractions and decimals.			
convert decimals and fractions to percents.			

(b) If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Contributors and Attributions

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1.6.2: Solve General Applications of Percent

Learning Objectives

- Translate and solve basic percent equations
- Solve applications of percent
- Find percent increase and percent decrease

be prepared!

Before you get started, take this readiness quiz.

1. Translate and solve: $\frac{3}{4}$ of x is 24. If you missed this problem, review Example 4.13.11.
2. Simplify: (4.5)(2.38). If you missed this problem, review Example 5.3.5.
3. Solve: 3.5 = 0.7n. If you missed this problem, review Example 5.7.4.

Translate and Solve Basic Percent Equations

We will solve percent equations by using the methods we used to solve equations with fractions or decimals. In the past, you may have solved percent problems by setting them up as proportions. That was the best method available when you did not have the tools of algebra. Now as a prealgebra student, you can translate word sentences into algebraic equations, and then solve the equations.

We'll look at a common application of percent—tips to a server at a restaurant—to see how to set up a basic percent application.

When Aolani and her friends ate dinner at a restaurant, the bill came to \$80. They wanted to leave a 20% tip. What amount would the tip be? To solve this, we want to find what *amount* is 20% of \$80. The \$80 is called the *base*. The amount of the tip would be 0.20(80), or \$16 See Figure 1.6.2.1. To find the amount of the tip, we multiplied the percent by the base.

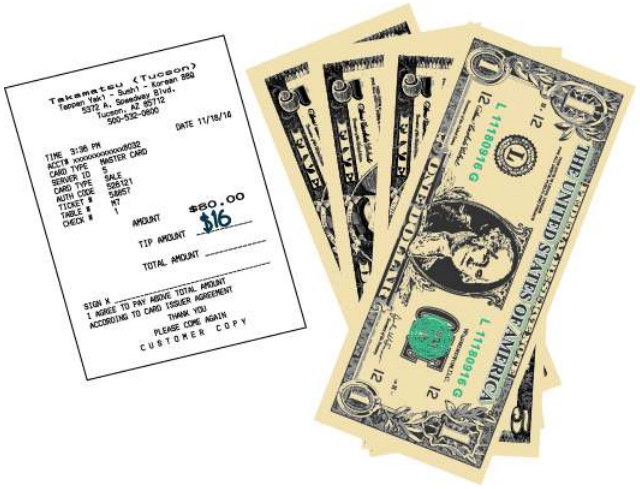


Figure 1.6.2.1- A 20% tip for an \$80 restaurant bill comes out to \$16.

In the next examples, we will find the amount. We must be sure to change the given percent to a decimal when we translate the words into an equation.

Example 1.6.2.1:

What number is 35% of 90?

Solution

Translate into algebra. Let n = the number.	$\underbrace{\text{What number}}_{n} \quad \underbrace{\text{is}}_{=} \quad \underbrace{35\%}_{0.35} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{90?}_{90}$
Multiply.	n = 31.5

31.5 is 35% of 90

? Exercise 1.6.2.1:

What number is 45% of 80?

Answer

36

? Exercise 1.6.2.2:

What number is 55% of 60?

Answer

33

✓ Example 1.6.2.2:

125% of 28 is what number?

Solution

Translate into algebra. Let a = the number.	$\underbrace{125\%}_{1.25} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{28}_{28} \quad \underbrace{\text{is}}_{=} \quad \underbrace{\text{what number?}}_{a}$
Multiply.	35 = a

125% of 28 is 35

Remember that a percent over 100 is a number greater than 1. We found that 125% of 28 is 35, which is greater than 28.

? Exercise 1.6.2.3:

150% of 78 is what number?

Answer

117

? Exercise 1.6.2.4:

175% of 72 is what number?

Answer

126

In the next examples, we are asked to find the base.

Example 1.6.2.3:

Translate and solve: 36 is 75% of what number?

Solution

Translate. Let b = the number.	$\underbrace{36}_{36}\ \underbrace{\text{is}}_{=}\ \underbrace{75\%}_{0.75}\ \underbrace{\text{of}}_{.}\ \underbrace{\text{what number?}}_{b}$
Divide both sides by 0.75.	$\frac{36}{0.75}=\frac{0.75b}{0.75}$ (1.6.2.1)
Simplify.	48 = b

36 is 75% of 48

Exercise 1.6.2.5:

17 is 25% of what number?

Answer

68

Exercise 1.6.2.6:

40 is 62.5% of what number?

Answer

64

Example 1.6.2.4:

6.5% of what number is $1.17?

Solution

Translate. Let b = the number.	$\underbrace{6.5\%}_{0.065}\ \underbrace{\text{of}}_{.}\ \underbrace{\text{what number}}_{b}\ \underbrace{\text{is}}_{=}\ \underbrace{\$1.17?}_{1.17}$
Divide both sides by 0.065.	$\frac{0.065n}{0.065}=\frac{1.17}{0.065}$ (1.6.2.2)
Simplify.	n = 18

6.5% of $18 is $1.17.

Exercise 1.6.2.7:

7.5% of what number is $1.95?

Answer

$26

Exercise 1.6.2.8:

8.5% of what number is $3.06?

Answer

$36

In the next examples, we will solve for the percent.

Example 1.6.2.5:

What percent of 36 is 9?

Solution

Translate into algebra. Let p = the percent.	$\underbrace{p}_{\text{What percent}} \underbrace{\cdot}_{\text{of}} \underbrace{36}_{36} \underbrace{=}_{\text{is}} \underbrace{9}_{9?}$
Divide by 36.	$\frac{36p}{36} = \frac{9}{36}$ (1.6.2.3)
Simplify.	$p = \frac{1}{4}$ (1.6.2.4)
Convert to decimal form.	p = 0.25
Convert to percent.	p = 25%

25% of 36 is 9.

Exercise 1.6.2.9:

What percent of 76 is 57?

Answer

75%

Exercise 1.6.2.10:

What percent of 120 is 96?

Answer

80%

Example 1.6.2.6:

144 is what percent of 96?

Solution

Translate into algebra. Let p = the percent.	$\underbrace{144}_{144} \underbrace{=}_{\text{is}} \underbrace{p}_{\text{what percent}} \underbrace{\cdot}_{\text{of}} \underbrace{96}_{96?}$
Divide by 96.	$\frac{144}{96} = \frac{96p}{96}$ (1.6.2.5)

Simplify.	1.5 = p
Convert to percent.	150% = p

144 is 150% of 96.

Exercise 1.6.2.11:

110 is what percent of 88?

Answer

125%

Exercise 1.6.2.12:

126 is what percent of 72?

Answer

175%

Solve Applications of Percent

Many applications of percent occur in our daily lives, such as tips, sales tax, discount, and interest. To solve these applications we'll translate to a basic percent equation, just like those we solved in the previous examples in this section. Once you translate the sentence into a percent equation, you know how to solve it.

We will update the strategy we used in our earlier applications to include equations now. Notice that we will translate a sentence into an equation.

HOW TO: SOLVE AN APPLICATION

- Step 1. Identify what you are asked to find and choose a variable to represent it.
- Step 2. Write a sentence that gives the information to find it.
- Step 3. Translate the sentence into an equation.
- Step 4. Solve the equation using good algebra techniques.
- Step 5. Check the answer in the problem and make sure it makes sense.
- Step 6. Write a complete sentence that answers the question.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we'll solve involve everyday situations, you can rely on your own experience.

Example 1.6.2.7:

Dezohn and his girlfriend enjoyed a dinner at a restaurant, and the bill was $68.50. They want to leave an 18% tip. If the tip will be 18% of the total bill, how much should the tip be?

Solution

What are you asked to find?	the amount of the tip
Choose a variable to represent it.	Let t = amount of tip.
Write a sentence that give the information to find it.	The tip is 18% of the total bill.

Translate the sentence into an equation.	$\underbrace{t}_{\text{The tip}} = \underbrace{0.18}_{\text{is 18\%}} \cdot \underbrace{\$68.50}_{\text{of \$68.50}}$
Multiply.	t = 12.33
Check. Is this answer reasonable?	If we approximate the bill to \$70 and the percent to 20%, we would have a tip of \$14. So a tip of \$12.33 seems reasonable.
Write a complete sentence that answers the question.	The couple should leave a tip of \$12.33.

? Exercise 1.6.2.13:

Cierra and her sister enjoyed a special dinner in a restaurant, and the bill was \$81.50. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Answer

\$14.67

? Exercise 1.6.2.14:

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was \$14.40, how much will she leave for the tip?

Answer

\$2.16

✔ Example 1.6.2.8:

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Nutrition Facts

Serving Size: 1 cup (47g)
Servings Per Container: About 7

Amount Per Serving	Cereal	With Milk
Calories	180	230
Calories from Fat	10	20
	% Daily Value*	
Total Fat 1g	**2%**	**2%**
Saturated Fat 0g	**0%**	**0%**
Trans Fat 0g		
Polyunsaturated Fat 0.5g		
Monounsaturated Fat 0.5g		
Cholesterol 0mg	**0%**	**0%**
Sodium 190mg	**8%**	**11%**
Potassium 85mg	**2%**	**8%**
Total Carbohydrate 40g	**13%**	**15%**
Dietary Fiber 1g	**4%**	**4%**
Sugars 8g		
Protein 3g		

Solution

What are you asked to find?	the total amount of potassium recommended
Choose a variable to represent it.	Let a = total amount of potassium.

Write a sentence that gives the information to find it.	85 mg is 2% of the total amount.
Translate the sentence into an equation.	$\underbrace{85}_{85}\ \underbrace{\text{mg}}\ \underbrace{\text{is}}_{=}\ \underbrace{2\%}_{0.02}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{a}_{a}\ ?$
Divide both sides by 0.02.	$\frac{85}{0.02} = \frac{0.02a}{0.02}$ (1.6.2.6)
Simplify.	4,250 =a
Check: Is this answer reasonable?	Yes. 2% is a small percent and 85 is a small part of 4,250.
Write a complete sentence that answers the question.	The amount of potassium that is recommended is 4250 mg.

? Exercise 1.6.2.15:

One serving of wheat square cereal has 7 grams of fiber, which is 29% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.1 grams

? Exercise 1.6.2.16:

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

✓ Example 1.6.2.9:

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution

What are you asked to find?	the percent of the total calories from fat
Choose a variable to represent it.	Let p = percent from fat.
Write a sentence that gives the information to find it.	What percent of 480 is 240?
Translate the sentence into an equation.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{480}_{480}\ \underbrace{\text{is}}_{=}\ \underbrace{240?}_{240}$
Divide both sides by 480.	$\frac{p \cdot 480}{480} = \frac{240}{480}$ (1.6.2.7)
Simplify.	p = 0.5
Convert to percent form.	p = 50%
Check. Is this answer reasonable?	Yes. 240 is half of 480, so 50% makes sense.
Write a complete sentence that answers the question.	Of the total calories in each brownie, 50% is fat.

Exercise 1.6.2.17:

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat? (Round to the nearest whole percent.)

Answer

26%

Exercise 1.6.2.18:

The brownie mix Ricardo plans to use says that each brownie will be 190 calories, and 70 calories are from fat. What percent of the total calories are from fat?

Answer

37%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the **percent increase**, first we find the amount of increase, which is the difference between the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

how to: find percent increase

Step 1. Find the amount of increase.

- increase = new amount − original amount

Step 2. Find the percent increase as a percent of the original amount.

Example 1.6.2.10:

In 2011, the California governor proposed raising community college fees from $26 per unit to $36 per unit. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution

What are you asked to find?	the percent increase
Choose a variable to represent it.	Let p = percent.
Find the amount of increase.	$\underbrace{36}_{\text{new amount}} - \underbrace{26}_{\text{original amount}} = \underbrace{10}_{\text{increase}}$
Find the percent increase.	The increase is what percent of the original amount?
Translate to an equation.	$\underbrace{10}_{10} \underbrace{\text{ is }}_{=} \underbrace{\text{ what percent }}_{p} \underbrace{\text{ of }}_{\cdot} \underbrace{26?}_{26}$
Divide both sides by 26.	$\frac{10}{26} = \frac{26p}{26}$ (1.6.2.8)
Round to the nearest thousandth.	0.384 = p
Convert to percent form.	38.4% = p

Write a complete sentence.	The new fees represent a 38.4% increase over the old fees.

Exercise 1.6.2.19:

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

8.8%

Exercise 1.6.2.20:

In 1995, the standard bus fare in Chicago was $1.50. In 2008, the standard bus fare was $2.25. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

50%

Finding the **percent decrease** is very similar to finding the percent increase, but now the amount of decrease is the difference between the original amount and the final amount. Then we find what percent the amount of decrease is of the original amount.

HOW TO: FIND PERCENT DECREASE

Step 1. Find the amount of decrease.

- decrease = original amount − new amount

Step 2. Find the percent decrease as a percent of the original amount.

Example 1.6.2.11:

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

Solution

The average price of a gallon of gas in one city in June 2014 was $3.71. The average price in that city in July was $3.64. Find the percent decrease.

What are you asked to find?	the percent decrease
Choose a variable to represent it.	Let p = percent.
Find the amount of decrease.	$\underbrace{3.71}_{\text{original amount}} - \underbrace{3.64}_{\text{new amount}} = \underbrace{0.07}_{\text{increase}}$
Find the percent of decrease.	The decrease is what percent of the original amount?
Translate to an equation.	$\underbrace{0.07}_{0.07} \underbrace{\text{is}}_{=} \underbrace{\text{what percent}}_{p} \underbrace{\text{of}}_{\cdot} \underbrace{3.71}_{3.71}$
Divide both sides by 3.71.	$\frac{0.07}{3.71} = \frac{3.71p}{3.71}$ (1.6.2.9)
Round to the nearest thousandth.	0.019 = p

Convert to percent form.	1.9% = p
Write a complete sentence.	The price of gas decreased 1.9%.

Exercise 1.6.2.21:

The population of one city was about 672,000 in 2010. The population of the city is projected to be about 630,000 in 2020. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

6.3%

Exercise 1.6.2.22:

Last year Sheila's salary was $42,000. Because of furlough days, this year her salary was $37,800. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

10%

ACCESS ADDITIONAL ONLINE RESOURCES

Percent Increase and Percent Decrease Visualization

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

98. What number is 45% of 120?
99. What number is 65% of 100?
100. What number is 24% of 112?
101. What number is 36% of 124?
102. 250% of 65 is what number?
103. 150% of 90 is what number?
104. 800% of 2,250 is what number?
105. 600% of 1,740 is what number?
106. 28 is 25% of what number?
107. 36 is 25% of what number?
108. 81 is 75% of what number?
109. 93 is 75% of what number?
110. 8.2% of what number is $2.87?
111. 6.4% of what number is $2.88?
112. 11.5% of what number is $108.10?
113. 12.3% of what number is $92.25?
114. What percent of 260 is 78?
115. What percent of 215 is 86?
116. What percent of 1,500 is 540?
117. What percent of 1,800 is 846?
118. 30 is what percent of 20?
119. 50 is what percent of 40?
120. 840 is what percent of 480?

121. 790 is what percent of 395?

Solve Applications of Percents

In the following exercises, solve the applications of percents.

122. Geneva treated her parents to dinner at their favorite restaurant. The bill was $74.25. She wants to leave 16% of the total bill as a tip. How much should the tip be?
123. When Hiro and his co-workers had lunch at a restaurant the bill was $90.50. They want to leave 18% of the total bill as a tip. How much should the tip be?
124. Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $2,165. How much money was deposited to Trong's savings account?
125. Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $1,485. How much did Cherise deposit into her retirement account?
126. One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?
127. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?
128. A bacon cheeseburger at a popular fast food restaurant contains 2,070 milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?
129. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?
130. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?
131. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?
132. Emma gets paid $3,000 per month. She pays $750 a month for rent. What percent of her monthly pay goes to rent?
133. Dimple gets paid $3,200 per month. She pays $960 a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, find the percent increase or percent decrease.

134. Tamanika got a raise in her hourly pay, from $15.50 to $17.55. Find the percent increase.
135. Ayodele got a raise in her hourly pay, from $24.50 to $25.48. Find the percent increase.
136. Annual student fees at the University of California rose from about $4,000 in 2000 to about $9,000 in 2014. Find the percent increase.
137. The price of a share of one stock rose from $12.50 to $50. Find the percent increase.
138. According to Time magazine (7/19/2011) annual global seafood consumption rose from 22 pounds per person in 1960 to 38 pounds per person today. Find the percent increase. (Round to the nearest tenth of a percent.)
139. In one month, the median home price in the Northeast rose from $225,400 to $241,500. Find the percent increase. (Round to the nearest tenth of a percent.)
140. A grocery store reduced the price of a loaf of bread from $2.80 to $2.73. Find the percent decrease.
141. The price of a share of one stock fell from $8.75 to $8.54. Find the percent decrease.
142. Hernando's salary was $49,500 last year. This year his salary was cut to $44,055. Find the percent decrease.
143. From 2000 to 2010, the population of Detroit fell from about 951,000 to about 714,000. Find the percent decrease. (Round to the nearest tenth of a percent.)
144. In one month, the median home price in the West fell from $203,400 to $192,300. Find the percent decrease. (Round to the nearest tenth of a percent.)
145. Sales of video games and consoles fell from $1,150 million to $1,030 million in one year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Everyday Math

146. **Tipping** At the campus coffee cart, a medium coffee costs $1.65. MaryAnne brings $2.00 with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?

147. **Late Fees** Alison was late paying her credit card bill of $249. She was charged a 5% late fee. What was the amount of the late fee?

Writing Exercises

148. Without solving the problem “44 is 80% of what number”, think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.
149. Without solving the problem “What is 20% of 300?” think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.
150. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.
151. Because of road construction in one city, commuters were advised to plan their Monday morning commute to take 150% of their usual commuting time. Explain what this means.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
translate and solve basic percent equations.			
solve applications of percent.			
find percent increase and percent decrease.			

(b) After reviewing this checklist, what will you do to become confident for all objectives?

Contributors and Attributions

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1.6.3: Solve Sales Tax, Commission, and Discount Applications

Learning Objectives

- Solve sales tax applications
- Solve commission applications
- Solve discount applications
- Solve mark-up applications

be prepared!

Before you get started, take this readiness quiz.

1. Solve 0.0875(720) through multiplication. If you missed this problem, review Example 5.3.7.
2. Solve 12.96 ÷ 0.04 through division. If you missed this problem, review Example 5.4.12.

Solve Sales Tax Applications

Sales tax and commissions are applications of percent in our everyday lives. To solve these applications, we will follow the same strategy we used in the section on decimal operations. We show it again here for easy reference.

HOW TO: Solve an application

- Step 1. Identify what you are asked to find and choose a variable to represent it.
- Step 2. Write a sentence that gives the information to find it.
- Step 3. Translate the sentence into an equation.
- Step 4. Solve the equation using good algebra techniques.
- Step 5. Check the answer in the problem and make sure it makes sense.
- Step 6. Write a complete sentence that answers the question.

Remember that whatever the application, once we write the sentence with the given information (Step 2), we can translate it to a percent equation and then solve it.

Do you pay a tax when you shop in your city or state? In many parts of the United States, sales tax is added to the purchase price of an item. See Figure 1.6.3.1. The sales tax is determined by computing a percent of the purchase price.

To find the sales tax multiply the purchase price by the sales tax rate. Remember to convert the sales tax rate from a percent to a decimal number. Once the sales tax is calculated, it is added to the purchase price. The result is the total cost—this is what the customer pays.



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Figure 1.6.3.1- The sales tax is calculated as a percent of the purchase price.

Definition: sales tax

The sales tax is a percent of the purchase price.

$$\text{Sales Tax} = \text{Tax Rate} \cdot \text{Purchase Price}$$

$$\text{Total Cost} = \text{Purchase Price} + \text{Sales Tax}$$

Example 1.6.3.1:

Cathy bought a bicycle in Washington, where the sales tax rate was 6.5% of the purchase price. What was (a) the sales tax and (b) the total cost of a bicycle if the purchase price of the bicycle was $392?

Solution

(a)

Identify what you are asked to find.	What is the sales tax?
Choose a variable to represent it.	Let t = sales tax.
Write a sentence that gives the information to find it.	The sales tax is 6.5% of the purchase price.
Translate into an equation. (Remember to change the percent to a decimal).	$\underbrace{\text{The sales tax}}_{t} \ \underbrace{\text{is}}_{=} \ \underbrace{6.5\%}_{0.065} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{\text{the \$392 purchase price.}}_{392}$
Simplify.	t = 25.48
Check: Is this answer reasonable?	Yes, because the sales tax amount is less than 10% of the purchase price.
Write a complete sentence that answers the question.	The sales tax is $25.48.

(b)

Identify what you are asked to find.	What is the total cost of the bicycle?
Choose a variable to represent it.	Let c = total cost of bicycle.
Write a sentence that gives the information to find it.	The total cost is the purchase price plus the sales tax.

Translate into an equation. (Remember to change the percent to a decimal).	$\underbrace{c}_{\text{The total cost}} = \underbrace{392}_{\$392} + \underbrace{25.48}_{\$25.48}$ (is, plus)
Simplify.	c = 417.48
Check: Is this answer reasonable?	Yes, because the total cost is a little more than the purchase price.
Write a complete sentence that answers the question.	The total cost of the bicycle is $417.48.

Exercise 1.6.3.1:

Find (a) the sales tax and (b) the total cost: Alexandra bought a television set for $724 in Boston, where the sales tax rate was 6.25% of the purchase price.

Answer a

$45.25

Answer b

$769.25

Exercise 1.6.3.2:

Find (a) the sales tax and (b) the total cost: Kim bought a winter coat for $250 in St. Louis, where the sales tax rate was 8.2% of the purchase price.

Answer a

$20.50

Answer b

$270.50

Example 1.6.3.2:

Evelyn bought a new smartphone for $499 plus tax. She was surprised when she got the receipt and saw that the tax was $42.42. What was the sales tax rate for this purchase?

Solution

Identify what you are asked to find.	What is the sales tax rate?
Choose a variable to represent it.	Let r = sales tax.
Write a sentence that gives the information to find it.	What percent of the price is the sales tax?
Translate into an equation.	$\underbrace{r}_{\text{What percent}} \cdot \underbrace{499}_{\text{the \$499 price}} = \underbrace{42.42}_{\text{the \$42.42 tax?}}$ (of, is)
Divide.	$\frac{499r}{499} = \frac{42.42}{499}$ (1.6.3.1)
Simplify	r = 0.085

Check. Is this answer reasonable?	Yes, because 8.5% is close to 10%. 10% of $500 is $50, which is close to $42.42.
Write a complete sentence that answers the question.	The sales tax rate is 8.5%.

Exercise 1.6.3.3:

Diego bought a new car for $26,525. He was surprised that the dealer then added $2,387.25. What was the sales tax rate for this purchase?

Answer

9%

Exercise 1.6.3.4:

What is the sales tax rate if a $7,594 purchase will have $569.55 of sales tax added to it?

Answer

7.5%

Solve Commission Applications

Sales people often receive a **commission**, or percent of total sales, for their sales. Their income may be just the commission they earn, or it may be their commission added to their hourly wages or salary. The commission they earn is calculated as a certain percent of the price of each item they sell. That percent is called the **rate of commission**.

Definition: commission

A commission is a percentage of total sales as determined by the rate of commission.

$$\text{commission} = \text{rate of commission} \cdot \text{total sales}$$

To find the commission on a sale, multiply the rate of commission by the total sales. Just as we did for computing sales tax, remember to first convert the rate of commission from a percent to a decimal.

Example 1.6.3.3:

Helene is a realtor. She receives 3% commission when she sells a house. How much commission will she receive for selling a house that costs $260,000?

Solution

Identify what you are asked to find.	What is the commission?
Choose a variable to represent it.	Let c = the commission.
Write a sentence that gives the information to find it.	The commission is 3% of the price.
Translate into an equation.	$\underbrace{c}_{\text{The commission}} = \underbrace{0.03}_{3\%} \underbrace{\cdot}_{\text{of}} \underbrace{260{,}000}_{\text{the \$260,000 price.}}$
Simplify.	c = 7800
Change to percent form.	r = 8.5%
Check. Is this answer reasonable?	Yes. 1% of $260,000 is $2,600, and $7,800 is three times $2,600.

Write a complete sentence that answers the question.	Helene will receive a commission of $7,800.

Exercise 1.6.3.5:

Bob is a travel agent. He receives 7% commission when he books a cruise for a customer. How much commission will he receive for booking a $3,900 cruise?

Answer

$273

Exercise 1.6.3.6:

Fernando receives 18% commission when he makes a computer sale. How much commission will he receive for selling a computer for $2,190?

Answer

$394.20

Example 1.6.3.4:

Rikki earned $87 commission when she sold a $1,450 stove. What rate of commission did she get?

Solution

Identify what you are asked to find.	What is the rate of commission?
Choose a variable to represent it.	Let r = the rate of commission.
Write a sentence that gives the information to find it.	The commission is what percent of the sale?
Translate into an equation.	$\underbrace{87}_{\text{The \$87 commission}} \underbrace{=}_{\text{is}} \underbrace{r}_{\text{what percent}} \underbrace{\cdot}_{\text{of}} \underbrace{1450}_{\text{the \$1450 sale?}}$
Divide.	$\frac{87}{1450} = \frac{1450r}{1450}$ (1.6.3.2)
Simplify.	0.06 = r
Change to percent form.	r = 6%
Check if this answer is reasonable.	Yes. A 10% commission would have been $145. The 6% commission, $87, is a little more than half of that.
Write a complete sentence that answers the question.	The commission was 6% of the price of the stove.

Exercise 1.6.3.7:

Homer received $1,140 commission when he sold a car for $28,500. What rate of commission did he get?

Answer

4%

Exercise 1.6.3.8:

Bernice earned $451 commission when she sold an $8,200 living room set. What rate of commission did she get?

Answer

5.5%

Solve Discount Applications

Applications of discount are very common in retail settings Figure 6.8. When you buy an item on sale, the **original price** of the item has been reduced by some dollar amount. The **discount rate**, usually given as a percent, is used to determine the amount of the discount. To determine the **amount of discount**, we multiply the discount rate by the original price. We summarize the discount model in the box below.



Figure 6.8 - Applications of discounts are common in everyday life. (credit: Charleston's TheDigitel, Flickr)

Definition: discount

An amount of discount is a percent off the original price.

$$\text{amount of discount} = \text{discount rate} \cdot \text{original price}$$

$$\text{sale price} = \text{original price} - \text{discount}$$

The sale price should always be less than the original price. In some cases, the amount of discount is a fixed dollar amount. Then we just find the sale price by subtracting the amount of discount from the original price.

Example 1.6.3.5:

Jason bought a pair of sunglasses that were on sale for $10 off. The original price of the sunglasses was $39. What was the sale price of the sunglasses?

Solution

Identify what you are asked to find.	What is the sale price?
Choose a variable to represent it.	Let s = the sale price.
Write a sentence that gives the information to find it.	The sale price is the original price minus the discount.
Translate into an equation.	$\underbrace{s}_{\text{The sale price}} \quad \underbrace{=}_{\text{is}} \quad \underbrace{39}_{\text{the original \$39 price}} \quad \underbrace{-}_{\text{minus}} \quad \underbrace{10}_{\text{the \$10 discount.}}$
Simplify.	s = 29
Check if this answer is reasonable.	Yes. The sale price, $29, is less than the original price, $39.

Write a complete sentence that answers the question.	The sale price of the sunglasses was $29.

? Exercise 1.6.3.9:

Marta bought a dishwasher that was on sale for $75 off. The original price of the dishwasher was $525. What was the sale price of the dishwasher?

Answer

$450

? Exercise 1.6.3.10:

Orlando bought a pair of shoes that was on sale for $30 off. The original price of the shoes was $112. What was the sale price of the shoes?

Answer

$82

In Example 6.29, the amount of discount was a set amount, $10. In Example 6.30 the discount is given as a percent of the original price.

✓ Example 1.6.3.6:

Elise bought a dress that was discounted 35% off of the original price of $140. What was (a) the amount of discount and (b) the sale price of the dress?

Solution

(a) Before beginning, you may find it helpful to organize the information in a list. Original price = $140, Discount rate = 35%, Amount of discount = ?

Identify what you are asked to find.	What is the amount of discount?
Choose a variable to represent it.	Let d = the amount of discount.
Write a sentence that gives the information to find it.	The discount is 35% of the original price.
Translate into an equation.	$\underbrace{\text{The discount}}_{s}\ \underbrace{\text{is}}_{=}\ \underbrace{35\%}_{0.35}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$140 original price.}}_{140}$
Simplify.	d = 49
Check if this answer is reasonable.	Yes. A $49 discount is reasonable for a $140 dress.
Write a complete sentence that answers the question.	The amount of discount was $49.

(b) Original price = $140, Amount of discount = $49, Sale price = ?

Identify what you are asked to find.	What is the sale price of the dress?
Choose a variable to represent it.	Let s = the sale price.
Write a sentence that gives the information to find it.	The sale price is the original price minus the discount.
Translate into an equation.	$\underbrace{\text{The sale price}}_{s}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the \$140}}_{140}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{the \$49 discount.}}_{49}$

Simplify.	s = 91
Check if this answer is reasonable.	Yes. The sale price, $91, is less than the original price, $140.
Write a complete sentence that answers the question.	The sale price of the dress was $91.

Exercise 1.6.3.11:

Find (a) the amount of discount and (b) the sale price: Sergio bought a belt that was discounted 40% from an original price of $29.

Answer a

$11.60

Answer b

$17.40

Exercise 1.6.3.12:

Find (a) the amount of discount and (b) the sale price: Oscar bought a barbecue grill that was discounted 65% from an original price of

$395.

Answer a

$256.75

Answer b

$138.25

There may be times when you buy something on sale and want to know the discount rate. The next example will show this case.

Example 1.6.3.7:

Jeannette bought a swimsuit at a sale price of $13.95. The original price of the swimsuit was $31. Find the (a) amount of discount and (b) discount rate.

Solution

(a) Before beginning, you may find it helpful to organize the information in a list. Original price = $31, Amount of discount = ?, Sale price = $13.95

Identify what you are asked to find.	What is the amount of discount?
Choose a variable to represent it.	Let d = the amount of discount.
Write a sentence that gives the information to find it.	The discount is the original price minus the sale price.
Translate into an equation.	$\underbrace{\text{The discount}}_{s}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the \$31 original price}}_{31}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{the \$13.95 sale price.}}_{13.95}$
Simplify.	d = 17.05
Check if this answer is reasonable.	Yes. The $17.05 discount is less than the original price.
Write a complete sentence that answers the question.	The amount of discount was $17.05.

(b) Before beginning, you may find it helpful to organize the information in a list. Original price = $31, Amount of discount = $17.05, Discount rate = ?

Identify what you are asked to find.	What is the amount of discount?
Choose a variable to represent it.	Let r = the discount rate.
Write a sentence that gives the information to find it.	The discount is what percent of the original price?
Translate into an equation.	$\underbrace{\text{The discount of \$17.05}}_{s}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{r}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$31 original price.}}_{31}$
Divide.	$\frac{17.05}{31} = \frac{r(31)}{31}$ (1.6.3.3)
Simplify.	0.55 = r
Check if this answer is reasonable.	The rate of discount was a little more than 50% and the amount of discount is a little more than half of $31.
Write a complete sentence that answers the question.	The rate of discount was 55%.

? Exercise 1.6.3.13:

Find (a) the amount of discount and (b) the discount rate: Lena bought a kitchen table at the sale price of $375.20. The original price of the table was $560.

Answer a

$184.80

Answer b

33%

? Exercise 1.6.3.14:

Find (a) the amount of discount and (b) the discount rate: Nick bought a multi-room air conditioner at a sale price of $340. The original price of the air conditioner was $400.

Answer a

$60

Answer b

15%

Solve Mark-up Applications

Applications of mark-up are very common in retail settings. The price a retailer pays for an item is called the **wholesale price**. The retailer then adds a **mark-up** to the wholesale price to get the **list price**, the price he sells the item for. The mark-up is usually calculated as a percent of the wholesale price. The percent is called the **mark-up rate**. To determine the amount of mark-up, multiply the mark-up rate by the wholesale price. We summarize the mark-up model in the box below.

Definition: mark-up

The mark-up is the amount added to the wholesale price.

$$\text{amount of mark-up} = \text{mark-up rate} \cdot \text{wholesale price}$$

$$\text{list price} = \text{wholesale price} + \text{mark up}$$

The list price should always be more than the wholesale price.

Example 1.6.3.8:

Adam's art gallery bought a photograph at the wholesale price of $250. Adam marked the price up 40%. Find the (a) amount of mark-up and (b) the list price of the photograph.

Solution

(a)

Identify what you are asked to find.	What is the amount of mark-up?
Choose a variable to represent it.	Let m = the amount of each mark-up.
Write a sentence that gives the information to find it.	The mark-up is 40% of the wholesale price.
Translate into an equation.	$\underbrace{\text{The mark-up}}_{m}\ \underbrace{\text{is}}_{=}\ \underbrace{40\%}_{0.40}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{the \$250 wholesale price.}}_{250}$
Simplify.	m = 100
Check if this answer is reasonable.	Yes. The markup rate is less than 50% and $100 is less than half of $250.
Write a complete sentence that answers the question.	The mark-up on the photograph was $100.

(b)

Identify what you are asked to find.	What is the list price?
Choose a variable to represent it.	Let p = the list price.
Write a sentence that gives the information to find it.	The list price is the wholesale price plus the mark-up.
Translate into an equation.	$\underbrace{\text{The list price}}_{p}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{the \$250 wholesale price}}_{250}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{the \$100 mark-up.}}_{100}$
Simplify.	p = 350
Check if this answer is reasonable.	Yes. The list price, $350, is more than the wholesale price, $250.
Write a complete sentence that answers the question.	The list price of the photograph was $350.

Exercise 1.6.3.15:

Jim's music store bought a guitar at wholesale price $1,200. Jim marked the price up 50%. Find the (a) amount of mark-up and (b) the list price.

Answer a

600%

Answer b

$1,800

Exercise 1.6.3.16:

The Auto Resale Store bought Pablo's Toyota for $8,500. They marked the price up 35%. Find the (a) amount of mark-up and (b) the list price.

Answer a

$2,975

Answer b

$11,475

Practice Makes Perfect

Solve Sales Tax Applications

In the following exercises, find (a) the sales tax and (b) the total cost.

152. The cost of a pair of boots was $84. The sales tax rate is 5% of the purchase price.
153. The cost of a refrigerator was $1,242. The sales tax rate is 8% of the purchase price.
154. The cost of a microwave oven was $129. The sales tax rate is 7.5% of the purchase price.
155. The cost of a tablet computer is $350. The sales tax rate is 8.5% of the purchase price.
156. The cost of a file cabinet is $250. The sales tax rate is 6.85% of the purchase price.
157. The cost of a luggage set $400. The sales tax rate is 5.75% of the purchase price.
158. The cost of a 6 -drawer dresser $1,199. The sales tax rate is 5.125% of the purchase price.
159. The cost of a sofa is $1,350. The sales tax rate is 4.225% of the purchase price.

In the following exercises, find the sales tax rate.

160. Shawna bought a mixer for $300. The sales tax on the purchase was $19.50.
161. Orphia bought a coffee table for $400. The sales tax on the purchase was $38.
162. Bopha bought a bedroom set for $3,600. The sales tax on the purchase was $246.60.
163. Ruth bought a washer and dryer set for $2,100. The sales tax on the purchase was $152.25.

Solve Commission Applications

In the following exercises, find the commission.

164. Christopher sold his dinette set for $225 through an online site, which charged him 9% of the selling price as commission. How much was the commission?
165. Michele rented a booth at a craft fair, which charged her 8% commission on her sales. One day her total sales were $193. How much was the commission?
166. Farrah works in a jewelry store and receives 12% commission when she makes a sale. How much commission will she receive for selling a $8,125 ring?
167. Jamal works at a car dealership and receives 9% commission when he sells a car. How much commission will he receive for selling a $32,575 car?
168. Hector receives 17.5% commission when he sells an insurance policy. How much commission will he receive for selling a policy for $4,910?
169. Denise receives 10.5% commission when she books a tour at the travel agency. How much commission will she receive for booking a tour with total cost $7,420?

In the following exercises, find the rate of commission.

170. Dontay is a realtor and earned $11,250 commission on the sale of a $375,000 house. What is his rate of commission?
171. Nevaeh is a cruise specialist and earned $364 commission after booking a cruise that cost $5,200. What is her rate of commission?

172. As a waitress, Emily earned $420 in tips on sales of $2,625 last Saturday night. What was her rate of commission?
173. Alejandra earned $1,393.74 commission on weekly sales of $15,486 as a salesperson at the computer store. What is her rate of commission?
174. Maureen earned $7,052.50 commission when she sold a $45,500 car. What was the rate of commission?
175. Lucas earned $4,487.50 commission when he brought a $35,900 job to his office. What was the rate of commission?

Solve Discount Applications

In the following exercises, find the sale price.

176. Perla bought a cellphone that was on sale for $50 off. The original price of the cellphone was $189.
177. Sophie saw a dress she liked on sale for $15 off. The original price of the dress was $96.
178. Rick wants to buy a tool set with original price $165. Next week the tool set will be on sale for 40% off.
179. Angelo's store is having a sale on TV sets. One set, with an original price of $859, is selling for $125 off.

In the following exercises, find (a) the amount of discount and (b) the sale price.

180. Janelle bought a beach chair on sale at 60% off. The original price was $44.95
181. Errol bought a skateboard helmet on sale at 40% off. The original price was $49.95.
182. Kathy wants to buy a camera that lists for $389. The camera is on sale with a 33% discount.
183. Colleen bought a suit that was discounted 25% from an original price of $245.
184. Erys bought a treadmill on sale at 35% off. The original price was $949.95.
185. Jay bought a guitar on sale at 45% off. The original price was $514.75.

In the following exercises, find (a) the amount of discount and (b) the discount rate. (Round to the nearest tenth of a percent if needed.)

186. Larry and Donna bought a sofa at the sale price of $1,344. The original price of the sofa was $1,920.
187. Hiroshi bought a lawnmower at the sale price of $240. The original price of the lawnmower is $300.
188. Patty bought a baby stroller on sale for $301.75. The original price of the stroller was $355.
189. Bill found a book he wanted on sale for $20.80. The original price of the book was $32.
190. Nikki bought a patio set on sale for $480. The original price was $850.
191. Stella bought a dinette set on sale for $725. The original price was $1,299.

Solve Mark-up Applications

In the following exercises, find (a) the amount of the mark-up and (b) the list price.

192. Daria bought a bracelet at wholesale cost $16 to sell in her handicraft store. She marked the price up 45%.
193. Regina bought a handmade quilt at wholesale cost $120 to sell in her quilt store. She marked the price up 55%.
194. Tom paid $0.60 a pound for tomatoes to sell at his produce store. He added a 33% mark-up.
195. Flora paid her supplier $0.74 a stem for roses to sell at her flower shop. She added an 85% mark-up.
196. Alan bought a used bicycle for $115. After re-conditioning it, he added 225% mark-up and then advertised it for sale.
197. Michael bought a classic car for $8,500. He restored it, then added 150% mark-up before advertising it for sale.

Everyday Math

198. Coupons Yvonne can use two coupons for the same purchase at her favorite department store. One coupon gives her $20 off and the other gives her 25% off. She wants to buy a bedspread that sells for $195.
    a. Calculate the discount price if Yvonne uses the $20 coupon first and then takes 25% off.
    b. Calculate the discount price if Yvonne uses the 25% off coupon first and then uses the $20 coupon.
    c. In which order should Yvonne use the coupons?
199. Cash Back Jason can buy a bag of dog food for $35 at two different stores. One store offers 6% cash back on the purchase plus $5 off his next purchase. The other store offers 20% cash back.
    a. Calculate the total savings from the first store, including the savings on the next purchase.
    b. Calculate the total savings from the second store.
    c. Which store should Jason buy the dog food from? Why?

Writing Exercises

200. Priam bought a jacket that was on sale for 40% off. The original price of the jacket was $150. While the sales clerk figured the price by calculating the amount of discount and then subtracting that amount from $150, Priam found the price faster by calculating 60% of $150.
    a. Explain why Priam was correct.
    b. Will Priam's method work for any original price?
201. Roxy bought a scarf on sale for 50% off. The original price of the scarf was $32.90. Roxy claimed that the price she paid for the scarf was the same as the amount she saved. Was Roxy correct? Explain.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve sales tax applications.			
solve commission applications.			
solve discount applications.			
solve mark-up applications.			

(b) What does this checklist tell you about your mastery of this section? What steps will you take to improve?

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1.6.4: Solve Simple Interest Applications

Learning Objectives

- Use the simple interest formula
- Solve simple interest applications

Be prepared!

Before you get started, take this readiness quiz.

1. Solve 0.6y = 45. If you missed this problem, review Example 5.7.4.
2. Solve $\frac{n}{1.45}$ = 4.6. If you missed this problem, review Example 5.7.5.

Use the Simple Interest Formula

Do you know that banks pay you to let them keep your money? The money you put in the bank is called the **principal**, P, and the bank pays you **interest**, I. The interest is computed as a certain percent of the principal; called the **rate of interest**, r. The rate of interest is usually expressed as a percent per year, and is calculated by using the decimal equivalent of the percent. The variable for time, t, represents the number of years the money is left in the account.

Definition: simple interest

If an amount of money, P, the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I, earned is

$$I = Prt$$

where

- I = interest
- P = principal
- r = rate
- t = time

Interest earned according to this formula is called simple interest.

The formula we use to calculate simple interest is I = Prt. To use the simple interest formula we substitute in the values for variables that are given, and then solve for the unknown variable. It may be helpful to organize the information by listing all four variables and filling in the given information.

Example 1.6.4.1

Find the simple interest earned after 3 years on $500 at an interest rate of 6%.

Solution

Organize the given information in a list.

I = ?, P = $500, r = 6%, t = 3 years

We will use the simple interest formula to find the interest.

Write the formula.	I = Prt
Substitute the given information. Remember to write the percent in decimal form.	I = (500)(0.06)(3)
Simplify.	I = 90

Check your answer. Is $90 a reasonable interest earned on $500 in 3 years?	In 3 years the money earned 18%. If we rounded to 20%, the interest would have been 500(0.20) or $100. Yes, $90 is reasonable.
Write a complete sentence that answers the question.	The simple interest is $90.

? Exercise 1.6.4.1:

Find the simple interest earned after 4 years on $800 at an interest rate of 5%.

Answer

$160

? Exercise 1.6.4.2:

Find the simple interest earned after 2 years on $700 at an interest rate of 4%.

Answer

$56

In the next example, we will use the simple interest formula to find the principal.

✓ Example 1.6.4.2:

Find the principal invested if $178 interest was earned in 2 years at an interest rate of 4%.

Solution

Organize the given information in a list.

$$I = \$178,\ P = ?,\ r = 4\%,\ t = 2 \text{ years}$$

We will use the simple interest formula to find the principal.

Write the formula.	$I = Prt$	
Substitute the given information.	$178 = P(0.04)(2)$	
Divide.	$\frac{178}{0.08} = \frac{0.08P}{0.08}$	(1.6.4.1)
Simplify.	$2{,}225 = P$	
Check your answer. Is it reasonable that $2,225 would earn $178 in 2 years?	$178 \stackrel{?}{=} 2,225(0.04)(2)$	(1.6.4.2)
	$178 = 178$ ✓	(1.6.4.3)
Write a complete sentence that answers the question.	The principal is $2,225.	

? Exercise 1.6.4.3:

Find the principal invested if $495 interest was earned in 3 years at an interest rate of 6%.

Answer

$2,750

Exercise 1.6.4.4

Find the principal invested if $1,246 interest was earned in 5 years at an interest rate of 7%.

Answer

$3,560

Now we will solve for the rate of interest.

Example 1.6.4.3

Find the rate if a principal of $8,200 earned $3,772 interest in 4 years.

Solution

Organize the given information.

$$I = \$3{,}772,\ P = \$8{,}200,\ r = ?,\ t = 4 \text{ years}$$

We will use the simple interest formula to find the rate.

Write the formula.	$I = Prt$	
Substitute the given information.	$3{,}772 = 8{,}200r(4)$	
Multiply.	$3{,}772 = 32{,}800r$	
Divide.	$\frac{3,772}{32,800} = \frac{32,800r}{32,800}$	(1.6.4.4)
Simplify.	$0.115 = r$	
Write as a percent.	$11.5\% = r$	
Check your answer. Is 11.5% a reasonable rate if $3,772 was earned in 4 years?	$3,772 \stackrel{?}{=} 8,200(0.115)(4)$	(1.6.4.5)
	$3,772 = 3,772$ ✓	(1.6.4.6)
Write a complete sentence that answers the question.	The rate was 11.5%.	

Exercise 1.6.4.5:

Find the rate if a principal of $5,000 earned $1,350 interest in 6 years.

Answer

4.5%

Exercise 1.6.4.6:

Find the rate if a principal of $9,000 earned $1,755 interest in 3 years.

Answer

6.5%

Solve Simple Interest Applications

Applications with simple interest usually involve either investing money or borrowing money. To solve these applications, we continue to use the same strategy for applications that we have used earlier in this chapter. The only difference is that in place of translating to get an equation, we can use the simple interest formula.

We will start by solving a simple interest application to find the interest.

Example 1.6.4.4:

Nathaly deposited $12,500 in her bank account where it will earn 4% interest. How much interest will Nathaly earn in 5 years?

Solution

We are asked to find the Interest, I. Organize the given information in a list.

$$I = ?,\ P = \$12{,}500,\ r = 4\%,\ t = 5 \text{ years}$$

Write the formula.	$I = Prt$
Substitute the given information.	$I = (12{,}500)(0.04)(5)$
Simplify.	$I = 2{,}500$
Check your answer. Is $2,500 a reasonable interest on $12,500 over 5 years?	At 4% interest per year, in 5 years the interest would be 20% of the principal. Is 20% of $12,500 equal to $2,500? Yes.
Write a complete sentence that answers the question.	The interest is $2,500.

Exercise 1.6.4.7:

Areli invested a principal of $950 in her bank account with interest rate 3%. How much interest did she earn in 5 years?

Answer

$142.50

Exercise 1.6.4.8:

Susana invested a principal of $36,000 in her bank account with interest rate 6.5%. How much interest did she earn in 3 years?

Answer

$7,020

There may be times when you know the amount of interest earned on a given principal over a certain length of time, but you don't know the rate. For instance, this might happen when family members lend or borrow money among themselves instead of dealing with a bank. In the next example, we'll show how to solve for the rate.

Example 1.6.4.5:

Loren lent his brother $3,000 to help him buy a car. In 4 years his brother paid him back the $3,000 plus $660 in interest. What was the rate of interest?

Solution

We are asked to find the rate of interest, r. Organize the given information.

$$I = 660,\ P = \$3{,}000,\ r = ?,\ t = 4 \text{ years}$$

Write the formula.	$I = Prt$

Substitute the given information.	660 = (3,000)r(4)	
Multiply.	660 = (12,000)r	
Divide.	$\frac{660}{12,000} = \frac{(12,000)r}{12,000}$	(1.6.4.7)
Simplify.	0.055 = r	
Change to percent form.	5.5% = r	
Check your answer. Is 5.5% a reasonable interest rate to pay your brother?	$660 \stackrel{?}{=} (3,000)(0.055)(4)$	(1.6.4.8)
	$660 = 660$ ✓	(1.6.4.9)
Write a complete sentence that answers the question.	The rate of interest was 5.5%.	

? Exercise 1.6.4.9:

Jim lent his sister $5,000 to help her buy a house. In 3 years, she paid him the $5,000, plus $900 interest. What was the rate of interest?

Answer

6%

? Exercise 1.6.4.10:

Hang borrowed $7,500 from her parents to pay her tuition. In 5 years, she paid them $1,500 interest in addition to the $7,500 she borrowed. What was the rate of interest?

Answer

4%

There may be times when you take a loan for a large purchase and the amount of the principal is not clear. This might happen, for instance, in making a car purchase when the dealer adds the cost of a warranty to the price of the car. In the next example, we will solve a simple interest application for the principal.

✓ Example 1.6.4.6:

Eduardo noticed that his new car loan papers stated that with an interest rate of 7.5%, he would pay $6,596.25 in interest over 5 years. How much did he borrow to pay for his car?

Solution

We are asked to find the principal, P. Organize the given information.

I = 6,596.25, P = ?, r = 7.5%, t = 5 years

Write the formula.	I = Prt
Substitute the given information.	6,596.25 = P(0.075)(5)
Multiply.	6,596.25 = 0.375P

Divide.	$\frac{6,596.25}{0.375} = \frac{0.375P}{0.375}$	(1.6.4.10)
Simplify.	17,590 = P	
Check your answer. Is $17,590 a reasonable amount to borrow to buy a car?	$6,596.25 \stackrel{?}{=} (17,590)(0.075)(5)$	(1.6.4.11)
	$6,596.25 = 6,596.25$ ✓	(1.6.4.12)
Write a complete sentence that answers the question.	The amount borrowed was $17,590.	

? Exercise 1.6.4.11:

Sean's new car loan statement said he would pay $4,866.25 in interest from an interest rate of 8.5% over 5 years. How much did he borrow to buy his new car?

Answer

$11,450

? Exercise 1.6.4.12:

In 5 years, Gloria's bank account earned $2,400 interest at 5%. How much had she deposited in the account?

Answer

$9,600

In the simple interest formula, the rate of interest is given as an annual rate, the rate for one year. So the units of time must be in years. If the time is given in months, we convert it to years.

✓ Example 1.6.4.7:

Caroline got $900 as graduation gifts and invested it in a 10-month certificate of deposit that earned 2.1% interest. How much interest did this investment earn?

Solution

We are asked to find the interest, I. Organize the given information.

I = ?, P = $900, r = 2.1%, t = 10 months

Write the formula.	I = Prt	
Substitute the given information, converting 10 months to $\frac{10}{12}$ of a year.	$I = \$900(0.021)\left(\frac{10}{12}\right)$	(1.6.4.13)
Multiply.	I = 15.75	
Check your answer. Is $15.75 a reasonable amount of interest?	If Caroline had invested the $900 for a full year at 2% interest, the amount of interest would have been $18. Yes, $15.75 is reasonable.	
Write a complete sentence that answers the question.	The interest earned was $15.75.	

Exercise 1.6.4.13:

Adriana invested $4,500 for 8 months in an account that paid 1.9% interest. How much interest did she earn?

Answer

$57.00

Exercise 1.6.4.14:

Milton invested $2,460 for 20 months in an account that paid 3.5% interest How much interest did he earn?

Answer

$143.50

Practice Makes Perfect

Use the Simple Interest Formula

In the following exercises, use the simple interest formula to fill in the missing information.

202.

Table 1.6.4.1

Interest	Principal	Rate	Time (years)
	$1200	3%	5

203.

Table 1.6.4.2

Interest	Principal	Rate	Time (years)
	$1500	2%	4

204.

Table 1.6.4.3

Interest	Principal	Rate	Time (years)
$4410		4.5%	7

205.

Table 1.6.4.4

Interest	Principal	Rate	Time (years)
$2212		3.2%	6

206.

Table 1.6.4.5

Interest	Principal	Rate	Time (years)
$577.08	$4580		2

207.

Table 1.6.4.6

Interest	Principal	Rate	Time (years)
$528.12	$3260		3

In the following exercises, solve the problem using the simple interest formula.

208. Find the simple interest earned after 5 years on $600 at an interest rate of 3%.
209. Find the simple interest earned after 4 years on $900 at an interest rate of 6%.
210. Find the simple interest earned after 2 years on $8,950 at an interest rate of 3.24%.
211. Find the simple interest earned after 3 years on $6,510 at an interest rate of 2.85%.
212. Find the simple interest earned after 8 years on $15,500 at an interest rate of 11.425%.
213. Find the simple interest earned after 6 years on $23,900 at an interest rate of 12.175%.
214. Find the principal invested if $656 interest was earned in 5 years at an interest rate of 4%.
215. Find the principal invested if $177 interest was earned in 2 years at an interest rate of 3%.
216. Find the principal invested if $70.95 interest was earned in 3 years at an interest rate of 2.75%.
217. Find the principal invested if $636.84 interest was earned in 6 years at an interest rate of 4.35%.
218. Find the principal invested if $15,222.57 interest was earned in 6 years at an interest rate of 10.28%.
219. Find the principal invested if $10,953.70 interest was earned in 5 years at an interest rate of 11.04%.
220. Find the rate if a principal of $5,400 earned $432 interest in 2 years.
221. Find the rate if a principal of $2,600 earned $468 interest in 6 years.
222. Find the rate if a principal of $11,000 earned $1,815 interest in 3 years.
223. Find the rate if a principal of $8,500 earned $3,230 interest in 4 years.

Solve Simple Interest Applications

In the following exercises, solve the problem using the simple interest formula.

224. Casey deposited $1,450 in a bank account with interest rate 4%. How much interest was earned in 2 years?
225. Terrence deposited $5,720 in a bank account with interest rate 6%. How much interest was earned in 4 years?
226. Robin deposited $31,000 in a bank account with interest rate 5.2%. How much interest was earned in 3 years?
227. Carleen deposited $16,400 in a bank account with interest rate 3.9%. How much interest was earned in 8 years?
228. Hilaria borrowed $8,000 from her grandfather to pay for college. Five years later, she paid him back the $8,000, plus $1,200 interest. What was the rate of interest?
229. Kenneth lent his niece $1,200 to buy a computer. Two years later, she paid him back the $1,200, plus $96 interest. What was the rate of interest?
230. Lebron lent his daughter $20,000 to help her buy a condominium. When she sold the condominium four years later, she paid him the $20,000, plus $3,000 interest. What was the rate of interest?
231. Pablo borrowed $50,000 to start a business. Three years later, he repaid the $50,000, plus $9,375 interest. What was the rate of interest?
232. In 10 years, a bank account that paid 5.25% earned $18,375 interest. What was the principal of the account?
233. In 25 years, a bond that paid 4.75% earned $2,375 interest. What was the principal of the bond?
234. Joshua's computer loan statement said he would pay $1,244.34 in interest for a 3 year loan at 12.4%. How much did Joshua borrow to buy the computer?
235. Margaret's car loan statement said she would pay $7,683.20 in interest for a 5 year loan at 9.8%. How much did Margaret borrow to buy the car?
236. Caitlin invested $8,200 in an 18-month certificate of deposit paying 2.7% interest. How much interest did she earn form this investment?
237. Diego invested $6,100 in a 9-month certificate of deposit paying 1.8% interest. How much interest did he earn form this investment?
238. Airin borrowed $3,900 from her parents for the down payment on a car and promised to pay them back in 15 months at a 4% rate of interest. How much interest did she owe her parents?
239. Yuta borrowed $840 from his brother to pay for his textbooks and promised to pay him back in 5 months at a 6% rate of interest. How much interest did Yuta owe his brother?

Everyday Math

240. **Interest on savings** Find the interest rate your local bank pays on savings accounts.
    a. What is the interest rate?
    b. Calculate the amount of interest you would earn on a principal of $8,000 for 5 years.

241. **Interest on a loan** Find the interest rate your local bank charges for a car loan.
    a. What is the interest rate?
    b. Calculate the amount of interest you would pay on a loan of $8,000 for 5 years.

Writing Exercises

242. Why do banks pay interest on money deposited in savings accounts?
243. Why do banks charge interest for lending money?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the simple interest formula.			
solve simple interest applications.			

(b) On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

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1.6.5: Solve Proportions and their Applications (Part 1)

Learning Objectives

- Use the definition of proportion
- Solve proportions
- Solve applications using proportions
- Write percent equations as proportions
- Translate and solve percent proportions

be prepared!

Before you get started, take this readiness quiz.

1. Simplify: $\frac{\frac{1}{3}}{4}$. If you missed this problem, review Example 4.5.8.
2. Solve: $\frac{x}{4} = 20$. If you missed this problem, review Example 4.12.5.
3. Write as a rate: Sale rode his bike 24 miles in 2 hours. If you missed this problem, review Example 5.10.6.

Use the Definition of Proportion

In the section on Ratios and Rates we saw some ways they are used in our daily lives. When two ratios or rates are equal, the equation relating them is called a **proportion**.

Definition: proportion

A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where b ≠ 0, d ≠ 0.

The proportion states two ratios or rates are equal. The proportion is read "a is to b, as c is to d".

The equation $\frac{1}{2} = \frac{4}{8}$ is a proportion because the two fractions are equal. The proportion $\frac{1}{2} = \frac{4}{8}$ is read "1 is to 2 as 4 is to 8".

If we compare quantities with units, we have to be sure we are comparing them in the right order. For example, in the proportion $\frac{20\ students}{1\ teacher} = \frac{60\ students}{3\ teachers}$ we compare the number of students to the number of teachers. We put students in the numerators and teachers in the denominators.

Example 1.6.5.1:

Write each sentence as a proportion: (a) 3 is to 7 as 15 is to 35. (b) 5 hits in 8 at bats is the same as 30 hits in 48 at-bats. (c) $1.50 for 6 ounces is equivalent to $2.25 for 9 ounces.

Solution

(a) 3 is to 7 as 15 is to 35

Write as a proportion.	$\frac{3}{7} = \frac{15}{35}$	(1.6.5.1)

(b) 5 hits in 8 at bats is the same as 30 hits in 48 at-bats

Write each fraction to compare hits to at-bats.	$\frac{hits}{at-bats} = \frac{hits}{at-bats}$	(1.6.5.2)

Write as a proportion.	$\frac{5}{8}=\frac{30}{48}$	(1.6.5.3)

(c) $1.50 for 6 ounces is equivalent to $2.25 for 9 ounces

Write each fraction to compare dollars to ounces.	$\frac{\$}{ounces}=\frac{\$}{ounces}$	(1.6.5.4)
Write as a proportion.	$\frac{1.50}{6}=\frac{2.25}{9}$	(1.6.5.5)

? Exercise 1.6.5.1:

Write each sentence as a proportion: (a) 5 is to 9 as 20 is to 36. (b) 7 hits in 11 at-bats is the same as 28 hits in 44 at-bats. (c) $2.50 for 8 ounces is equivalent to $3.75 for 12 ounces.

Answer a

$\frac{5}{9}=\frac{20}{36}$

Answer b

$\frac{7}{11}=\frac{28}{44}$

Answer c

$\frac{2.50}{8}=\frac{3.75}{12}$

? Exercise 1.6.5.2:

Write each sentence as a proportion: (a) 6 is to 7 as 36 is to 42. (b) 8 adults for 36 children is the same as 12 adults for 54 children. (c) $3.75 for 6 ounces is equivalent to $2.50 for 4 ounces.

Answer a

$\frac{6}{7}=\frac{36}{42}$

Answer b

$\frac{8}{36}=\frac{12}{54}$

Answer c

$\frac{3.75}{6}=\frac{2.50}{4}$

Look at the proportions $\frac{1}{2}=\frac{4}{8}$ and $\frac{2}{3}=\frac{6}{9}$. From our work with equivalent fractions we know these equations are true. But how do we know if an equation is a proportion with equivalent fractions if it contains fractions with larger numbers? To determine if a proportion is true, we find the **cross products** of each proportion. To find the cross products, we multiply each denominator with the opposite numerator (diagonally across the equal sign). The results are called a cross products because of the cross formed. The cross products of a proportion are equal.

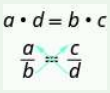
$8 \cdot 1 = 8 \quad 2 \cdot 4 = 8 \qquad 9 \cdot 2 = 18 \quad 3 \cdot 6 = 18$

$\frac{1}{2} = \frac{4}{8} \qquad \frac{2}{3} = \frac{6}{9}$

Definition: Cross Products of a Proportion

For any proportion of the form $\frac{a}{b} = \frac{c}{d}$, where b ≠ 0, d ≠ 0, its cross products are equal.

$$a \cdot d = b \cdot c$$



Cross products can be used to test whether a proportion is true. To test whether an equation makes a proportion, we find the cross products. If they are the equal, we have a proportion.

Example 1.6.5.2:

Determine whether each equation is a proportion: (a) $\frac{4}{9} = \frac{12}{28}$ (b) $\frac{17.5}{37.5} = \frac{7}{15}$

Solution

To determine if the equation is a proportion, we find the cross products. If they are equal, the equation is a proportion.

(a) $\frac{4}{9} = \frac{12}{28}$

Find the cross products.	$\frac{4}{9} \times \frac{12}{28}$ $28 \cdot 4 = 112 \qquad 9 \cdot 12 = 108 \qquad (1.6.5.6)$

Since the cross products are not equal, 28 · 4 ≠ 9 · 12, the equation is not a proportion.

(b) $\frac{17.5}{37.5} = \frac{7}{15}$

Find the cross products.	$\frac{17.5}{37.5} \times \frac{7}{15}$ $15 \cdot 17.5 = 262.5 \qquad 37.5 \cdot 7 = 262.5 \qquad (1.6.5.7)$

Since the cross products are equal, 15 • 17.5 = 37.5 • 7, the equation is a proportion.

Exercise 1.6.5.3:

Determine whether each equation is a proportion: (a) $\frac{7}{9} = \frac{54}{72}$ (b) $\frac{24.5}{45.5} = \frac{7}{13}$

Answer a

no

Answer b

yes

Exercise 1.6.5.4:

Determine whether each equation is a proportion: (a) $\frac{8}{9} = \frac{56}{73}$ (b) $\frac{28.5}{52.5} = \frac{8}{15}$

Answer a

no

Answer b

no

Solve Proportions

To solve a proportion containing a variable, we remember that the proportion is an equation. All of the techniques we have used so far to solve equations still apply. In the next example, we will solve a proportion by multiplying by the Least Common Denominator (LCD) using the Multiplication Property of Equality.

Example 1.6.5.3:

Solve: $\frac{x}{63}=\frac{4}{7}$.

Solution

To isolate x, multiply both sides by the LCD, 63.	$63\left(\frac{x}{63}\right)=63\left(\frac{4}{7}\right)$	(1.6.5.8)
Simplify.	$x=\frac{9\cdot \cancel{7}\cdot 4}{\cancel{7}}$	(1.6.5.9)
Divide the common factors.	$x=36$	(1.6.5.10)

Check: To check our answer, we substitute into the original proportion.

Substitute x = 36	$\frac{36}{63}\stackrel{?}{=}\frac{4}{7}$	(1.6.5.11)
Show common factors.	$\frac{4\cdot 9}{7\cdot 9}\stackrel{?}{=}\frac{4}{7}$	(1.6.5.12)
Simplify.	$\frac{4}{7}=\frac{4}{7}$ ✓	(1.6.5.13)

Exercise 1.6.5.5:

Solve the proportion: $\frac{n}{84}=\frac{11}{12}$.

Answer

77

Exercise 1.6.5.6:

Solve the proportion: $\frac{y}{96}=\frac{13}{12}$.

Answer

104

When the variable is in a denominator, we'll use the fact that the cross products of a proportion are equal to solve the proportions.

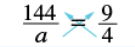
We can find the cross products of the proportion and then set them equal. Then we solve the resulting equation using our familiar techniques.

Example 1.6.5.4:

Solve: $\frac{144}{a} = \frac{9}{4}$.

Solution

Notice that the variable is in the denominator, so we will solve by finding the cross products and setting them equal.



Find the cross products and set them equal.	4 • 144 = a • 9	
Simplify.	576 = 9a	
Divide both sides by 9.	$\frac{576}{9} = \frac{9a}{9}$	(1.6.5.14)
Simplify.	$64 = a$	(1.6.5.15)

Check your answer.

Substitute a = 64	$\frac{144}{64} \stackrel{?}{=} \frac{9}{4}$	(1.6.5.16)
Show common factors.	$\frac{9 \cdot 16}{4 \cdot 16} \stackrel{?}{=} \frac{9}{4}$	(1.6.5.17)
Simplify.	$\frac{9}{4} = \frac{9}{4}$ ✓	(1.6.5.18)

Another method to solve this would be to multiply both sides by the LCD, 4a. Try it and verify that you get the same solution.

Exercise 1.6.5.7:

Solve the proportion: $\frac{91}{b} = \frac{7}{5}$.

Answer

65

Exercise 1.6.5.8:

Solve the proportion: $\frac{39}{c} = \frac{13}{8}$.

Answer

24

Example 1.6.5.5:

Solve: $\frac{52}{91} = \frac{-4}{y}$

Solution

Find the cross products and set them equal.	$\frac{52}{91} \times \frac{-4}{y}$	
	y • 52 = 91(-4)	
Simplify.	52y = -364	
Divide both sides by 52.	$\frac{52y}{52} = \frac{-364}{52}$	(1.6.5.19)
Simplify.	$y = -7$	(1.6.5.20)

Check:

Substitute y = -7	$\frac{52}{91} \stackrel{?}{=} \frac{-4}{-7}$	(1.6.5.21)
Show common factors.	$\frac{13 \cdot 4}{13 \cdot 7} \stackrel{?}{=} \frac{-4}{-7}$	(1.6.5.22)
Simplify.	$\frac{4}{7} = \frac{4}{7}$ ✓	(1.6.5.23)

Exercise 1.6.5.9:

Solve the proportion: $\frac{84}{98} = \frac{-6}{x}$.

Answer

-7

Exercise 1.6.5.10:

Solve the proportion: $\frac{-7}{y} = \frac{105}{135}$.

Answer

-9

Solve Applications Using Proportions

The strategy for solving applications that we have used earlier in this chapter, also works for proportions, since proportions are equations. When we set up the proportion, we must make sure the units are correct—the units in the numerators match and the units in the denominators match.

Example 1.6.5.6:

When pediatricians prescribe acetaminophen to children, they prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of the child's weight. If Zoe weighs 80 pounds, how many milliliters of acetaminophen will her doctor prescribe?

Solution

Identify what you are asked to find.	How many ml of acetaminophen the doctor will prescribe?
Choose a variable to represent it.	Let a = ml of acetaminophen.
Write a sentence that gives the information to find it.	If 5 ml is prescribed for every 25 pounds, how much will be prescribed for 80 pounds?
Translate into a proportion.	$\frac{ml}{pounds} = \frac{ml}{pounds}$ (6.5.24)
Substitute given values—be careful of the units.	$\frac{5}{25} = \frac{a}{80}$ (6.5.25)
Multiply both sides by 80.	$80 \cdot \frac{5}{25} = 80 \cdot \frac{a}{80}$ (6.5.26)
Multiply and show common factors.	$\frac{16 \cdot 5 \cdot 5}{5 \cdot 5} = \frac{80a}{80}$ (6.5.27)
Simplify.	$16 = a$ (6.5.28)
Check if the answer is reasonable.	Yes. Since 80 is about 3 times 25, the medicine should be about 3 times 5.
Write a complete sentence.	The pediatrician would prescribe 16 ml of acetaminophen to Zoe.

You could also solve this proportion by setting the cross products equal.

Exercise 1.6.5.11:

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Emilia, who weighs 60 pounds?

Answer

12 ml

Exercise 1.6.5.12:

For every 1 kilogram (kg) of a child's weight, pediatricians prescribe 15 milligrams (mg) of a fever reducer. If Isabella weighs 12 kg, how many milligrams of the fever reducer will the pediatrician prescribe?

Answer

180 mg

Example 1.6.5.7:

One brand of microwave popcorn has 120 calories per serving. A whole bag of this popcorn has 3.5 servings. How many calories are in a whole bag of this microwave popcorn?

Solution

Identify what you are asked to find.	How many calories are in a whole bag of microwave popcorn?
Choose a variable to represent it.	Let c = number of calories.
Write a sentence that gives the information to find it.	If there are 120 calories per serving, how many calories are in a whole bag with 3.5 servings?
Translate into a proportion.	$\frac{calories}{serving} = \frac{calories}{serving}$ (6.5.29)
Substitute given values.	$\frac{120}{1} = \frac{c}{3.5}$ (6.5.30)
Multiply both sides by 3.5.	$(3.5)\left(\frac{120}{1}\right) = (3.5)\left(\frac{c}{3.5}\right)$ (6.5.31)
Multiply.	$420 = c$ (6.5.32)
Check if the answer is reasonable.	Yes. Since 3.5 is between 3 and 4, the total calories should be between 360 (3 • 120) and 480 (4 • 120).
Write a complete sentence.	The whole bag of microwave popcorn has 420 calories.

Exercise 1.6.5.13:

Marissa loves the Caramel Macchiato at the coffee shop. The 16 oz. medium size has 240 calories. How many calories will she get if she drinks the large 20 oz. size?

Answer

300

Exercise 1.6.5.14:

Yaneli loves Starburst candies, but wants to keep her snacks to 100 calories. If the candies have 160 calories for 8 pieces, how many pieces can she have in her snack?

Answer

5

Example 1.6.5.8:

Josiah went to Mexico for spring break and changed $325 dollars into Mexican pesos. At that time, the exchange rate had $1 U.S. is equal to 12.54 Mexican pesos. How many Mexican pesos did he get for his trip?

Solution

Identify what you are asked to find.	How many Mexican pesos did Josiah get?

Choose a variable to represent it.	Let p = number of pesos.
Write a sentence that gives the information to find it.	If $1 U.S. is equal to 12.54 Mexican pesos, then $325 is how many pesos?
Translate into a proportion.	$\frac{\$}{pesos} = \frac{\$}{pesos}$ (6.5.33)
Substitute given values.	$\frac{1}{12.54} = \frac{325}{p}$ (6.5.34)
The variable is in the denominator, so find the cross products and set them equal.	$p \cdot 1 = 12.54(325)$ (6.5.35)
Simplify.	$c = 4,075.5$ (6.5.36)
Check if the answer is reasonable.	Yes, $100 would be $1,254 pesos. $325 is a little more than 3 times this amount.
Write a complete sentence.	Josiah has 4075.5 pesos for his spring break trip.

? Exercise 1.6.5.15:

Yurianna is going to Europe and wants to change $800 dollars into Euros. At the current exchange rate, $1 US is equal to 0.738 Euro. How many Euros will she have for her trip?

Answer

590 Euros

? Exercise 1.6.5.16:

Corey and Nicole are traveling to Japan and need to exchange $600 into Japanese yen. If each dollar is 94.1 yen, how many yen will they get?

Answer

56,460 yen

Contributors and Attributions

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1.6.6: Solve Proportions and their Applications (Part 2)

Write Percent Equations As Proportions

Previously, we solved percent equations by applying the properties of equality we have used to solve equations throughout this text. Some people prefer to solve percent equations by using the proportion method. The proportion method for solving percent problems involves a percent proportion. A **percent proportion** is an equation where a percent is equal to an equivalent ratio.

For example, 60% = $\frac{60}{100}$ and we can simplify $\frac{60}{100} = \frac{3}{5}$. Since the equation $\frac{60}{100} = \frac{3}{5}$ shows a percent equal to an equivalent ratio, we call it a **percent proportion**. Using the vocabulary we used earlier:

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$
$$\frac{3}{5} = \frac{60}{100}$$

Definition: Percent Proportion

The amount is to the base as the percent is to 100.

$$\frac{amount}{base} = \frac{percent}{100} \tag{6.5.37}$$

If we restate the problem in the words of a proportion, it may be easier to set up the proportion:

The amount is to the base as the percent is to one hundred.

We could also say:

The amount out of the base is the same as the percent out of one hundred.

First we will practice translating into a percent proportion. Later, we'll solve the proportion.

Example 1.6.6.9:

Translate to a proportion. What number is 75% of 90?

Solution

If you look for the word "of", it may help you identify the base.

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}}$ is $\underbrace{75\%}_{\text{percent}}$ of $\underbrace{90?}_{\text{base}}$
Restate as a proportion.	$What\ number\ out\ of\ 90\ is\ the\ same\ as\ 75\ out\ of\ 100?$ (1.6
Set up the proportion. Let n = number.	$\frac{n}{90} = \frac{75}{100}$ (6.5.38)

Exercise 1.6.6.17

Translate to a proportion: What number is 60% of 105?

Answer

$\frac{n}{105} = \frac{60}{100}$

Exercise 1.6.6.18

Translate to a proportion: What number is 40% of 85?

Answer

$\frac{n}{85} = \frac{40}{100}$

Example 1.6.6.10

Translate to a proportion. 19 is 25% of what number?

Solution

Identify the parts of the percent proportion.	$\underbrace{19}_{\text{amount}}\ \text{is}\ \underbrace{25\%}_{\text{percent}}\ \text{of}\ \underbrace{\text{what number?}}_{\text{base}}$
Restate as a proportion.	$19\ out\ of\ what\ number\ is\ the\ same\ as\ 25\ out\ of\ 100?$ (1.6.
Set up the proportion. Let n = number.	$\frac{19}{n}=\frac{25}{100}$ (6.5.39)

Exercise 1.6.6.19:

Translate to a proportion: 36 is 25% of what number?

Answer

$\frac{36}{n}=\frac{25}{100}$

Exercise 1.6.6.20:

Translate to a proportion: 27 is 36% of what number?

Answer

$\frac{27}{n}=\frac{36}{100}$

Example 1.6.6.11:

Translate to a proportion. What percent of 27 is 9?

Solution

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}\ \text{of}\ \underbrace{27}_{\text{base}}\ \text{is}\ \underbrace{9?}_{\text{amount}}$
Restate as a proportion.	$9\ out\ of\ 27\ is\ the\ same\ as\ what\ number\ out\ of\ 100$ (1.6.6.
Set up the proportion. Let p = percent.	$\frac{9}{27}=\frac{p}{100}$ (6.5.40)

Exercise 1.6.6.21:

Translate to a proportion: What percent of 52 is 39?

Answer

$\frac{n}{100}=\frac{39}{52}$

Exercise 1.6.6.22:

Translate to a proportion: What percent of 92 is 23?

Answer

$\frac{n}{100}=\frac{23}{92}$

Translate and Solve Percent Proportions

Now that we have written percent equations as proportions, we are ready to solve the equations.

Example 1.6.6.12:

Translate and solve using proportions: What number is 45% of 80?

Solution

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}}$ is $\underbrace{45\%}_{\text{percent}}$ of $\underbrace{80?}_{\text{base}}$
Restate as a proportion.	$What\ number\ out\ of\ 80\ is\ the\ same\ as\ 45\ out\ of\ 100?$ (1.6
Set up the proportion. Let n = number.	$\frac{n}{80} = \frac{45}{100}$ (6.5.41)
Find the cross products and set them equal.	$100 \cdot n = 80 \cdot 45$ (6.5.42)
Simplify.	$100n = 3,600$ (6.5.43)
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,600}{100}$ (6.5.44)
Simplify.	$n = 36$ (6.5.45)
Check if the answer is reasonable.	Yes. 45 is a little less than half of 100 and 36 is a little less than half 80.
Write a complete sentence that answers the question.	36 is 45% of 80.

Exercise 1.6.6.23:

Translate and solve using proportions: What number is 65% of 40?

Answer

26

Exercise 1.6.6.24:

Translate and solve using proportions: What number is 85% of 40?

Answer

34

In the next example, the percent is more than 100, which is more than one whole. So the unknown number will be more than the base.

Example 1.6.6.13:

Translate and solve using proportions: 125% of 25 is what number?

Solution

Identify the parts of the percent proportion.	$\underbrace{125\%}_{\text{percent}}$ is $\underbrace{25}_{\text{base}}$ of $\underbrace{\text{what number?}}_{\text{amount}}$
Restate as a proportion.	$What\ number\ out\ of\ 25\ is\ the\ same\ as\ 125\ out\ of\ 100?$ (1.
Set up the proportion. Let n = number.	$\frac{n}{25} = \frac{125}{100}$ (6.5.46)

Find the cross products and set them equal.	$100 \cdot n = 25 \cdot 125$	(6.5.47)
Simplify.	$100n = 3,125$	(6.5.48)
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,125}{100}$	(6.5.49)
Simplify.	$n = 31.25$	(6.5.50)
Check if the answer is reasonable.	Yes. 125 is more than 100 and 31.25 is more than 25.	
Write a complete sentence that answers the question.	125% of 25 is 31.25.	

? Exercise 1.6.6.25:

Translate and solve using proportions: 125% of 64 is what number?

Answer

80

? Exercise 1.6.6.26:

Translate and solve using proportions: 175% of 84 is what number?

Answer

147

Percents with decimals and money are also used in proportions.

✓ Example 1.6.6.14:

Translate and solve: 6.5% of what number is $1.56?

Solution

Identify the parts of the percent proportion.	$\underbrace{6.5\%}_{\text{percent}}$ of $\underbrace{\text{what number}}_{\text{base}}$ is $\underbrace{\$1.56?}_{\text{amount}}$	
Restate as a proportion.	$\$1.56 \; out \; of \; what \; number \; is \; the \; same \; as \; 6.5 \; out \; of \; 100?$	(
Set up the proportion. Let n = number.	$\frac{1.56}{n} = \frac{6.5}{100}$	(6.5.51)
Find the cross products and set them equal.	$100(1.56) = n \cdot 6.5$	(6.5.52)
Simplify.	$156 = 6.5n$	(6.5.53)
Divide both sides by 6.5 to isolate the variable.	$\frac{156}{6.5} = \frac{6.5n}{6.5}$	(6.5.54)
Simplify.	$24 = n$	(6.5.55)
Check if the answer is reasonable.	Yes. 6.5% is a small amount and $1.56 is much less than $24.	
Write a complete sentence that answers the question.	6.5% of $24 is $1.56.	

Exercise 1.6.6.27:

Translate and solve using proportions: 8.5% of what number is $3.23?

Answer

38

Exercise 1.6.6.28:

Translate and solve using proportions: 7.25% of what number is $4.64?

Answer

64

Example 1.6.6.15:

Translate and solve using proportions: What percent of 72 is 9?

Solution

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}\ \text{of}\ \underbrace{72}_{\text{base}}\ \text{is}\ \underbrace{9?}_{\text{amount}}$	
Restate as a proportion.	$9\ out\ of\ 72\ is\ the\ same\ as\ what\ number\ out\ of\ 100?$	(1.6.6
Set up the proportion. Let n = number.	$\frac{9}{72} = \frac{n}{100}$	(6.5.56)
Find the cross products and set them equal.	$72 \cdot n = 100 \cdot 9$	(6.5.57)
Simplify.	$72n = 900$	(6.5.58)
Divide both sides by 72.	$\frac{72n}{72} = \frac{900}{72}$	(6.5.59)
Simplify.	$n = 12.5$	(6.5.60)
Check if the answer is reasonable.	Yes. 9 is $\frac{1}{8}$ of 72 and $\frac{1}{8}$ is 12.5%.	
Write a complete sentence that answers the question.	12.5% of 72 is 9.	

Exercise 1.6.6.29:

Translate and solve using proportions: What percent of 72 is 27?

Answer

37.5%

Exercise 1.6.6.30:

Translate and solve using proportions: What percent of 92 is 23?

Answer

25%

Practice Makes Perfect

Use the Definition of Proportion

In the following exercises, write each sentence as a proportion.

244. 4 is to 15 as 36 is to 135.
245. 7 is to 9 as 35 is to 45.
246. 12 is to 5 as 96 is to 40.
247. 15 is to 8 as 75 is to 40.
248. 5 wins in 7 games is the same as 115 wins in 161 games.
249. 4 wins in 9 games is the same as 36 wins in 81 games.
250. 8 campers to 1 counselor is the same as 48 campers to 6 counselors.
251. 6 campers to 1 counselor is the same as 48 campers to 8 counselors.
252. \$9.36 for 18 ounces is the same as \$2.60 for 5 ounces.
253. \$3.92 for 8 ounces is the same as \$1.47 for 3 ounces.
254. \$18.04 for 11 pounds is the same as \$4.92 for 3 pounds.
255. \$12.42 for 27 pounds is the same as \$5.52 for 12 pounds.

In the following exercises, determine whether each equation is a proportion.

256. $\frac{7}{15}=\frac{56}{120}$
257. $\frac{5}{12}=\frac{45}{108}$
258. $\frac{11}{6}=\frac{21}{16}$
259. $\frac{9}{4}=\frac{39}{34}$
260. $\frac{12}{18}=\frac{4.99}{7.56}$
261. $\frac{9}{16}=\frac{2.16}{3.89}$
262. $\frac{13.5}{8.5}=\frac{31.05}{19.55}$
263. $\frac{10.1}{8.4}=\frac{3.03}{2.52}$

Solve Proportions

In the following exercises, solve each proportion.

264. $\frac{x}{56}=\frac{7}{8}$
265. $\frac{n}{91}=\frac{8}{13}$
266. $\frac{49}{63}=\frac{z}{9}$
267. $\frac{56}{72}=\frac{y}{9}$
268. $\frac{5}{a}=\frac{65}{117}$
269. $\frac{4}{b}=\frac{64}{144}$
270. $\frac{98}{154}=\frac{-7}{p}$
271. $\frac{72}{156}=\frac{-6}{q}$
272. $\frac{a}{-8}=\frac{-42}{48}$
273. $\frac{b}{-7}=\frac{-30}{42}$
274. $\frac{2.6}{3.9}=\frac{c}{3}$
275. $\frac{2.7}{3.6}=\frac{d}{4}$
276. $\frac{2.7}{j}=\frac{0.9}{0.2}$
277. $\frac{2.8}{k}=\frac{2.1}{1.5}$

278. $\frac{\frac{1}{2}}{1} = \frac{m}{8}$
279. $\frac{\frac{1}{3}}{3} = \frac{9}{n}$

Solve Applications Using Proportions

In the following exercises, solve the proportion problem.

280. Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?
281. Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child's weight. How many milligrams will the doctor prescribe?
282. At the gym, Carol takes her pulse for 10 sec and counts 19 beats. How many beats per minute is this? Has Carol met her target heart rate of 140 beats per minute?
283. Kevin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds. How many beats per minute is this? Has Kevin met his target heart rate?
284. A new energy drink advertises 106 calories for 8 ounces. How many calories are in 12 ounces of the drink?
285. One 12 ounce can of soda has 150 calories. If Josiah drinks the big 32 ounce size from the local mini-mart, how many calories does he get?
286. Karen eats \(\dfrac{1}{2}) cup of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?
287. An oatmeal cookie recipe calls for $\frac{1}{2}$ cup of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?
288. Janice is traveling to Canada and will change $250 US dollars into Canadian dollars. At the current exchange rate, $1 US is equal to $1.01 Canadian. How many Canadian dollars will she get for her trip?
289. Todd is traveling to Mexico and needs to exchange $450 into Mexican pesos. If each dollar is worth 12.29 pesos, how many pesos will he get for his trip?
290. Steve changed $600 into 480 Euros. How many Euros did he receive per US dollar?
291. Martha changed $350 US into 385 Australian dollars. How many Australian dollars did she receive per US dollar?
292. At the laundromat, Lucy changed $12.00 into quarters. How many quarters did she get?
293. When she arrived at a casino, Gerty changed $20 into nickels. How many nickels did she get?
294. Jesse's car gets 30 miles per gallon of gas. If Las Vegas is 285 miles away, how many gallons of gas are needed to get there and then home? If gas is $3.09 per gallon, what is the total cost of the gas for the trip?
295. Danny wants to drive to Phoenix to see his grandfather. Phoenix is 370 miles from Danny's home and his car gets 18.5 miles per gallon. How many gallons of gas will Danny need to get to and from Phoenix? If gas is $3.19 per gallon, what is the total cost for the gas to drive to see his grandfather?
296. Hugh leaves early one morning to drive from his home in Chicago to go to Mount Rushmore, 812 miles away. After 3 hours, he has gone 190 miles. At that rate, how long will the whole drive take?
297. Kelly leaves her home in Seattle to drive to Spokane, a distance of 280 miles. After 2 hours, she has gone 152 miles. At that rate, how long will the whole drive take?
298. Phil wants to fertilize his lawn. Each bag of fertilizer covers about 4,000 square feet of lawn. Phil's lawn is approximately 13,500 square feet. How many bags of fertilizer will he have to buy?
299. April wants to paint the exterior of her house. One gallon of paint covers about 350 square feet, and the exterior of the house measures approximately 2000 square feet. How many gallons of paint will she have to buy?

Write Percent Equations as Proportions

In the following exercises, translate to a proportion.

300. What number is 35% of 250?
301. What number is 75% of 920?
302. What number is 110% of 47?
303. What number is 150% of 64?
304. 45 is 30% of what number?
305. 25 is 80% of what number?
306. 90 is 150% of what number?
307. 77 is 110% of what number?
308. What percent of 85 is 17?
309. What percent of 92 is 46?
310. What percent of 260 is 340?

311. What percent of 180 is 220?

Translate and Solve Percent Proportions

In the following exercises, translate and solve using proportions.

312. What number is 65% of 180?
313. What number is 55% of 300?
314. 18% of 92 is what number?
315. 22% of 74 is what number?
316. 175% of 26 is what number?
317. 250% of 61 is what number?
318. What is 300% of 488?
319. What is 500% of 315?
320. 17% of what number is $7.65?
321. 19% of what number is $6.46?
322. $13.53 is 8.25% of what number?
323. $18.12 is 7.55% of what number?
324. What percent of 56 is 14?
325. What percent of 80 is 28?
326. What percent of 96 is 12?
327. What percent of 120 is 27?

Everyday Math

328. **Mixing a concentrate** Sam bought a large bottle of concentrated cleaning solution at the warehouse store. He must mix the concentrate with water to make a solution for washing his windows. The directions tell him to mix 3 ounces of concentrate with 5 ounces of water. If he puts 12 ounces of concentrate in a bucket, how many ounces of water should he add? How many ounces of the solution will he have altogether?
329. **Mixing a concentrate** Travis is going to wash his car. The directions on the bottle of car wash concentrate say to mix 2 ounces of concentrate with 15 ounces of water. If Travis puts 6 ounces of concentrate in a bucket, how much water must he mix with the concentrate?

Writing Exercises

330. To solve “what number is 45% of 350” do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.
331. To solve “what percent of 125 is 25” do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use definition of proportion.			
solve proportions.			
solve applications using proportions.			
write percent equations as proportions.			
translate and solve percent proportions.			

(b) Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

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1.6.E: Percents (Exercises)

6.1 - Understand Percent

In the following exercises, write each percent as a ratio.

332. 32% admission rate for the university
333. 53.3% rate of college students with student loans

In the following exercises, write as a ratio and then as a percent.

334. 13 out of 100 architects are women.
335. 9 out of every 100 nurses are men.

In the following exercises, convert each percent to a fraction.

336. 48%
337. 175%
338. 64.1%
339. $8\frac{1}{4}\%$

In the following exercises, convert each percent to a decimal.

340. 6%
341. 23%
342. 128%
343. 4.9%

In the following exercises, convert each percent to (a) a simplified fraction and (b) a decimal.

344. In 2012, 13.5% of the United States population was age 65 or over. (Source: www.census.gov)
345. In 2012, 6.5% of the United States population was under 5 years old. (Source: www.census.gov)
346. When a die is tossed, the probability it will land with an even number of dots on the top side is 50%.
347. A couple plans to have three children. The probability they will all be girls is 12.5%.

In the following exercises, convert each decimal to a percent.

348. 0.04
349. 0.15
350. 2.82
351. 3
352. 0.003
353. 1.395

In the following exercises, convert each fraction to a percent.

354. 3 4
355. 11 5
356. 3 5 8
357. 2 9
358. According to the Centers for Disease Control, $\frac{2}{5}$ of adults do not take a vitamin or supplement.
359. According to the Centers for Disease Control, among adults who do take a vitamin or supplement, $\frac{3}{4}$ take a multivitamin.

In the following exercises, translate and solve.

360. What number is 46% of 350?
361. 120% of 55 is what number?
362. 84 is 35% of what number?
363. 15 is 8% of what number?
364. 200% of what number is 50?

365. 7.9% of what number is $4.74?
366. What percent of 120 is 81.6?
367. What percent of 340 is 595?

6.2 - Solve General Applications of Percents

In the following exercises, solve.

368. When Aurelio and his family ate dinner at a restaurant, the bill was $83.50. Aurelio wants to leave 20% of the total bill as a tip. How much should the tip be?
369. One granola bar has 2 grams of fiber, which is 8% of the recommended daily amount. What is the total recommended daily amount of fiber?
370. The nutrition label on a package of granola bars says that each granola bar has 190 calories, and 54 calories are from fat. What percent of the total calories is from fat?
371. Elsa gets paid $4,600 per month. Her car payment is $253. What percent of her monthly pay goes to her car payment?

In the following exercises, solve.

372. Jorge got a raise in his hourly pay, from $19.00 to $19.76. Find the percent increase.
373. Last year Bernard bought a new car for $30,000. This year the car is worth $24,000. Find the percent decrease.

6.3 - Solve Sales Tax, Commission, and Discount Applications

In the following exercises, find (a) the sales tax (b) the total cost.

374. The cost of a lawn mower was $750. The sales tax rate is 6% of the purchase price.
375. The cost of a water heater is $577. The sales tax rate is 8.75% of the purchase price.

In the following exercises, find the sales tax rate.

376. Andy bought a piano for $4,600. The sales tax on the purchase was $333.50.
377. Nahomi bought a purse for $200. The sales tax on the purchase was $16.75.

In the following exercises, find the commission.

378. Ginny is a realtor. She receives 3% commission when she sells a house. How much commission will she receive for selling a house for $380,000?
379. Jackson receives 16.5% commission when he sells a dinette set. How much commission will he receive for selling a dinette set for $895?

In the following exercises, find the rate of commission.

380. Ruben received $675 commission when he sold a $4,500 painting at the art gallery where he works. What was the rate of commission?
381. Tori received $80.75 for selling a $950 membership at her gym. What was her rate of commission?

In the following exercises, find the sale price.

382. Aya bought a pair of shoes that was on sale for $30 off. The original price of the shoes was $75.
383. Takwanna saw a cookware set she liked on sale for $145 off. The original price of the cookware was $312.

In the following exercises, find (a) the amount of discount and (b) the sale price.

384. Nga bought a microwave for her office. The microwave was discounted 30% from an original price of $84.90.
385. Jarrett bought a tie that was discounted 65% from an original price of $45.

In the following exercises, find (a) the amount of discount (b) the discount rate. (Round to the nearest tenth of a percent if needed.)

386. Hilda bought a bedspread on sale for $37. The original price of the bedspread was $50.
387. Tyler bought a phone on sale for $49.99. The original price of the phone was $79.99.

In the following exercises, find (a) the amount of the mark-up (b) the list price.

388. Manny paid $0.80 a pound for apples. He added 60% markup before selling them at his produce stand. What price did he charge for the apples?

389. It cost Noelle $17.40 for the materials she used to make a purse. She added a 325% markup before selling it at her friend's store. What price did she ask for the purse?

6.4 - Solve Simple Interest Applications

In the following exercises, solve the simple interest problem.

390. Find the simple interest earned after 4 years on $2,250 invested at an interest rate of 5%.
391. Find the simple interest earned after 7 years on $12,000 invested at an interest rate of 8.5%.
392. Find the principal invested if $660 interest was earned in 5 years at an interest rate of 3%.
393. Find the interest rate if $2,898 interest was earned from a principal of $23,000 invested for 3 years.
394. Kazuo deposited $10,000 in a bank account with interest rate 4.5%. How much interest was earned in 2 years?
395. Brent invested $23,000 in a friend's business. In 5 years the friend paid him the $23,000 plus $9,200 interest. What was the rate of interest?
396. Fresia lent her son $5,000 for college expenses. Three years later he repaid her the $5,000 plus $375 interest. What was the rate of interest?
397. In 6 years, a bond that paid 5.5% earned $594 interest. What was the principal of the bond?

6.5 - Solve Proportions and their Applications

In the following exercises, write each sentence as a proportion.

398. 3 is to 8 as 12 is to 32.
399. 95 miles to 3 gallons is the same as 475 miles to 15 gallons.
400. 1 teacher to 18 students is the same as 23 teachers to 414 students.
401. $7.35 for 15 ounces is the same as $2.94 for 6 ounces.

In the following exercises, determine whether each equation is a proportion.

402. $\frac{5}{13}=\frac{30}{78}$
403. $\frac{16}{7}=\frac{48}{23}$
404. $\frac{12}{18}=\frac{6.99}{10.99}$
405. $\frac{11.6}{9.2}=\frac{37.12}{29.44}$

In the following exercises, solve each proportion.

406. $\frac{x}{36}=\frac{5}{9}$
407. $\frac{7}{a}=\frac{-6}{84}$
408. $\frac{1.2}{1.8}=\frac{d}{6}$
409. $\frac{\frac{1}{2}}{2}=\frac{m}{20}$

In the following exercises, solve the proportion problem.

410. The children's dosage of acetaminophen is 5 milliliters (ml) for every 25 pounds of a child's weight. How many milliliters of acetaminophen will be prescribed for a 60 pound child?
411. After a workout, Dennis takes his pulse for 10 sec and counts 21 beats. How many beats per minute is this?
412. An 8 ounce serving of ice cream has 272 calories. If Lavonne eats 10 ounces of ice cream, how many calories does she get?
413. Alma is going to Europe and wants to exchange $1,200 into Euros. If each dollar is 0.75 Euros, how many Euros will Alma get?
414. Zack wants to drive from Omaha to Denver, a distance of 494 miles. If his car gets 38 miles to the gallon, how many gallons of gas will Zack need to get to Denver?
415. Teresa is planning a party for 100 people. Each gallon of punch will serve 18 people. How many gallons of punch will she need?

In the following exercises, translate to a proportion.

416. What number is 62% of 395?
417. 42 is 70% of what number?
418. What percent of 1,000 is 15?
419. What percent of 140 is 210?

In the following exercises, translate and solve using proportions.

420. What number is 85% of 900?
421. 6% of what number is $24?
422. $3.51 is 4.5% of what number?
423. What percent of 3,100 is 930?

PRACTICE TEST

In the following exercises, convert each percent to (a) a decimal (b) a simplified fraction.

424. 24%
425. 5%
426. 350%

In the following exercises, convert each fraction to a percent. (Round to 3 decimal places if needed.)

427. $\frac{7}{8}$
428. $\frac{1}{3}$
429. $\frac{11}{12}$

In the following exercises, solve the percent problem.

430. 65 is what percent of 260?
431. What number is 27% of 3,000?
432. 150% of what number is 60?
433. Yuki's monthly paycheck is $3,825. She pays $918 for rent. What percent of her paycheck goes to rent?
434. The total number of vehicles on one freeway dropped from 84,000 to 74,000. Find the percent decrease (round to the nearest tenth of a percent).
435. Kyle bought a bicycle in Denver where the sales tax was 7.72% of the purchase price. The purchase price of the bicycle was $600. What was the total cost?
436. Mara received $31.80 commission when she sold a $795 suit. What was her rate of commission?
437. Kiyoshi bought a television set on sale for $899. The original price was $1,200. Find: (a) the amount of discount (b) the discount rate (round to the nearest tenth of a percent)
438. Oxana bought a dresser at a garage sale for $20. She refinished it, then added a 250% markup before advertising it for sale. What price did she ask for the dresser?
439. Find the simple interest earned after 5 years on $3000 invested at an interest rate of 4.2%.
440. Brenda borrowed $400 from her brother. Two years later, she repaid the $400 plus $50 interest. What was the rate of interest?
441. Write as a proportion: 4 gallons to 144 miles is the same as 10 gallons to 360 miles.
442. Solve for a: $\frac{12}{a} = \frac{-15}{65}$
443. Vin read 10 pages of a book in 12 minutes. At that rate, how long will it take him to read 35 pages?

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1.6.S: Percents (Summary)

Key Terms

commission	A percentage of total sales as determined by the rate of commission
discount	A percent off the original price, determined by the discount rate
mark-up	The amount added to the wholesale price, determined by the mark-up rate
percent	A ratio whose denominator is 100
percent decrease	The percent the amount of decrease is of the original amount
percent increase	The percent the amount of increase is of the original amount
proportion	An equation of the form $\frac{a}{b} = \frac{c}{d}$, where b ≠ 0, d ≠ 0.The proportion states two ratios or rates are equal. The proportion is read " a is to b, as c is to d".
sales tax	A percent of the purchase price
simple interest	If an amount of money, P, the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I, earned is I = Prt. Interest earned according to this formula is called simple interest.

Key Concepts

6.1 - Understand Percent

- **Convert a percent to a fraction.**
  1. Write the percent as a ratio with the denominator 100.
  2. Simplify the fraction if possible.
- **Convert a percent to a decimal.**
  1. Write the percent as a ratio with the denominator 100.
  2. Convert the fraction to a decimal by dividing the numerator by the denominator.
- **Convert a decimal to a percent.**
  1. Write the decimal as a fraction.
  2. If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.
  3. Write this ratio as a percent.
- **Convert a fraction to a percent.**
  1. Convert the fraction to a decimal.
  2. Convert the decimal to a percent.

6.2 - Solve General Applications of Percent

- **Solve an application.**
  1. Identify what you are asked to find and choose a variable to represent it.
  2. Write a sentence that gives the information to find it.
  3. Translate the sentence into an equation.
  4. Solve the equation using good algebra techniques.
  5. Write a complete sentence that answers the question.
  6. Check the answer in the problem and make sure it makes sense.
- **Find percent increase.**

1. Find the amount of increase: increase = new amount − original amount
2. Find the percent increase as a percent of the original amount.

- **Find percent decrease.**
  1. Find the amount of decrease. decrease = original amount − new amount
  2. Find the percent decrease as a percent of the original amount.

6.3 - Solve Sales Tax, Commission, and Discount Applications

- **Sales Tax**: The sales tax is a percent of the purchase price.
  - sales tax = tax rate • purchase price
  - total cost = purchase price + sales tax
- **Commission**: A commission is a percentage of total sales as determined by the rate of commission.
  - commission = rate of commission • original price
- **Discount**: An amount of discount is a percent off the original price, determined by the discount rate.
  - amount of discount = discount rate • original price
  - sale price = original price – discount
- **Mark-up**: The mark-up is the amount added to the wholesale price, determined by the mark-up rate.
  - amount of mark-up = mark-up rate wholesale price
  - list price = wholesale price + mark up

6.4 - Solve Simple Interest Applications

- **Simple interest**
  - If an amount of money, P, the principal, is invested for a period of t years at an annual interest rate r, the amount of interest, I, earned is I = Prt
  - Interest earned according to this formula is called simple interest.

6.5 - Solve Proportions and their Applications

- **Proportion**
  - A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where b ≠ 0, d ≠ 0.The proportion states two ratios or rates are equal. The proportion is read " a is to b, as c is to d".
- **Cross Products of a Proportion**
  - For any proportion of the form $\frac{a}{b} = \frac{c}{d}$, where b ≠ 0, its cross products are equal: a • d = b • c.
- **Percent Proportion**
  - The amount is to the base as the percent is to 100. $\frac{amount}{base} = \frac{percent}{100}$

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SECTION OVERVIEW

1.7: Decimals Fractions and Percents

1.7.1: Comparing Fractions, Decimals, and Percents

1.7.2: Converting Between Fractions, Decimals and Percents

1.7.3: Decimals- Rounding and Scientific Notation

1.7.4: Using Fractions, Decimals and Percents to Describe Charts

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1.7.1: Comparing Fractions, Decimals, and Percents

Learning Outcomes

1. Compare two fractions
2. Compare two numbers given in different forms

In this section, we will go over techniques to compare two numbers. These numbers could be presented as fractions, decimals or percents and may not be in the same form. For example, when we look at a histogram, we can compute the fraction of the group that occurs the most frequently. We might be interested in whether that fraction is greater than 25% of the population. By the end of this section we will know how to make this comparison.

Comparing Two Fractions

Whether you like fractions or not, they come up frequently in statistics. For example, a probability is defined as the number of ways a sought after event can occur over the total number of possible outcomes. It is commonly asked to compare two such probabilities to see if they are equal, and if not, which is larger. There are two main approaches to comparing fractions.

Approach 1: Change the fractions to equivalent fractions with a common denominator and then compare the numerators

The procedure of approach 1 is to first find the common denominator and then multiply the numerator and the denominator by the same whole number to make the denominators common.

Example 1.7.1.1

Compare: $\frac{2}{3}$ and $\frac{5}{7}$

Solution

A common denominator is the product of the two: $3 \times 7 = 21$. We convert:

$$\frac{2}{3}\frac{7}{7} = \frac{14}{21}$$

and

$$\frac{5}{7}\frac{3}{3} = \frac{15}{21}$$

Next we compare the numerators and see that $14 < 15$, hence

$\frac{2}{3} < \frac{5}{7}$

Example 1.7.1.2

In statistics, we say that two events are independent if the probability of the second occurring is equal to the probability of the second occurring given that the first occurs. The probability of rolling two dice and having the sum equal to 7 is $\frac{6}{36}$. If you know that the first die lands on a 4, then the probability that the sum of the two dice is a 7 is $\frac{1}{6}$. Are these events independent?

Solution

We need to compare $\frac{6}{36}$ and $\frac{1}{6}$. The common denominator is 36. We convert the second fraction to

$$\frac{1}{6}\frac{6}{6} = \frac{6}{36}$$

Now we can see that the two fractions are equal, so the events are independent.

Approach 2: Use a calculator or computer to convert the fractions to decimals and then compare the decimals

If it is easy to build up the fractions so that we have a common denominator, then Approach 1 works well, but often the fractions are not simple, so it is easier to make use of the calculator or computer.

Example 1.7.1.3

In computing probabilities for a uniform distribution, fractions come up. Given that the number of ounces in a medium sized drink is uniformly distributed between 15 and 26 ounces, the probability that a randomly selected medium sized drink is less than 22 ounces is $\frac{7}{11}$. Given that the weight of in a medium sized American is uniformly distributed between 155 and 212 pounds, the probability that a randomly selected medium sized American is less than 195 pounds is $\frac{40}{57}$. Is it more likely to select a medium sized drink that is less than 22 ounces or to select a medium sized American who is less than 195 pounds?

Solution

We could get a common denominator and build the fractions, but it is much easier to just turn both fractions into decimal numbers and then compare. We have:

$$\frac{7}{11} \approx 0.6364$$

and

$$\frac{40}{57} \approx 0.7018$$

Notice that

$$0.6364 < 0.7018$$

Hence, we can conclude that it is less likely to pick the medium sized 22 ounce or less drink than to pick the 195 pound or lighter medium sized person.

Exercise

If you guess on 10 true or false questions, the probability of getting at least 9 correct is $\frac{11}{1024}$. If you guess on six multiple choice questions with three choices each, then the probability of getting at least five of the six correct is $\frac{7}{729}$. Which of these is more likely?

Comparing Fractions, Decimals and Percents

When you want to compare a fraction to a decimal or a percent, it is usually easiest to convert to a decimal number first, and then compare the decimal numbers.

Example 1.7.1.4

Compare 0.52 and $\frac{7}{13}$.

Solution

We first convert $\frac{7}{13}$ to a decimal by dividing to get 0.5385. Now notice that

$$0.52 < 0.5385$$

Thus

$$0.52 < \frac{7}{13}$$

Example 1.7.1.5

When we preform a hypothesis test in statistics, We have to compare a number called the p-value to another number called the level of significance. Suppose that the p-value is calculated as 0.0641 and the level of significance is 5%. Compare these two numbers.

Solution

We first convert the level of significance, 5%, to a decimal number. Recall that to convert a percent to a decimal, we move the decimal over two places to the right. This gives us 0.05. Now we can compare the two decimals:

$$0.0641 > 0.05$$

Therefore, the p-value is greater than the level of significance.



This is an application of comparing fractions to probability.

- Example: Comparing Fractions with Different Denominators using Inequality Symbols
- Ex: Compare Fractions and Decimals using Inequality Symbols
- https://youtu.be/lSzNkQjcfEU

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1.7.2: Converting Between Fractions, Decimals and Percents

Learning Outcomes

1. Given a decimal, convert it to a percent
2. Given a percent, convert it to a decimal
3. Convert a fraction to a decimal and percent

In this section, we will convert from decimals to percents and back. We will also start with a fraction and convert it to a decimal and a percent. In statistics we are often given a number as a percent and have to do calculations on it. To do so, we must first convert it to a percent. Also, the computer or calculator shows numbers as decimals, but for presentations, percents are friendlier. It is also much easier to compare decimals than fractions, thus converting to a decimal is helpful.

For example, we often want to see if a probability is greater than 5%. A computer will display the probability as a decimal such as 0.04836. To make the comparison we will first change it to a percent and then compare it to 5%.

Transforming a Decimal to a Percent

We have all heard of percents before. "You only have a 20% chance of winning the game", "Just 38% of all Americans approve of Congress", and "I am 95% confident that my answer is correct" are just a few of the countless examples of percents as they come up in statistics.

Defintion: Percent

Percent means Parts Per Hundred

Thus if we are given a decimal and want to convert it to a percent, we multiply the decimal by 100. In practice, this means we move the decimal point two places to the right.

Example 1.7.2.1

Convert the number 0.1738 to a percent.

Solution

We move the decimal over two to the right as shown below.



We get: 17.38% for the answer.

Example 1.7.2.2

Convert 0.7 to a percent.

Solution

We want to move the decimal two places to the right, but there is only one digit to the right of the decimal place. The good news is that we can always add a 0 to the right of the last digit. We write:

$$0.7 = 0.70$$

Now move the decimal place two digits to the right to get 70%.

Example 1.7.2.3

In regression analysis, an important number that is calculated is called R-Squared. It helps us determine how helpful one variable is in predicting another variable. The computer and calculator always display it as a decimal, but it is more meaningful

as a percent. Suppose that the R-Squared value that relates the amount of studying students do to prepare for a final exam and the score on the exam is: $r^2 = 0.8971$. Convert this to a percent rounded to the nearest whole number percent.

Solution

We move the decimal 0.8971 two places to the right to get 89.71%

Now round to the nearest whole number percent. Note that the digit to the left of the whole number is $7 \geq 5$. Thus we add 1 to the whole number, 89. This gives us 90%.

Exercise

A standard goal in statistics is to come up with a range of values that a population proportion is likely to lie. This range is called a confidence interval. Suppose that we want to interpret a confidence interval for the percent of patients who experience side effects from an experimental cancer treatment. The computer calculates it as the decimal range: [0.023,0.029]. What is the likely range for the percent of patients who experience side effects from the experimental cancer treatment?

Transforming a Percent to a Decimal

To convert a decimal to a percent, we multiply the decimal by 100 which is equivalent to moving the decimal two places to the right. Not surprisingly, to convert a percent to a decimal, we do exactly the opposite. We divide the number by 100 which is equivalent to moving the decimal two places to the left.

Example 1.7.2.4

Convert the percent 89.4% to a decimal.

Solution

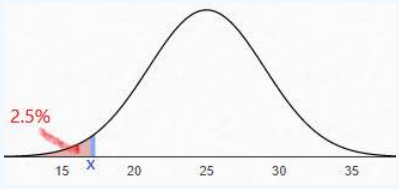
We move the decimal over two to the left as shown below.



We get: 0.894 for the answer.

Example 1.7.2.5

Suppose that you want to find the value of x such that 2.5% of the entire area under the Normal curve lies to the left of x. The first step will be to convert the 2.5% to a decimal. What decimal is equivalent to 2.5%?



Solution

We want to move the decimal 2.5 two places to the left, but since there is only one digit to the left of the decimal, we add a zero first: 02.5. Now move the decimal two places to the left to get 0.025.

Converting a Fraction to a Decimal and a Percent

Often in probability it is natural to represent probabilities as fractions, but it is easier to make comparisons as decimals. Thus, we need to be able to convert fractions to decimals. To do so we just divide.

Example 1.7.2.6

Convert the fraction $\frac{4}{7}$ to a decimal, rounding to the nearest hundredth.

Solution

We use long division:

$$\begin{array}{r} .571 \\ \hline 7)4.000 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \end{array} \tag{1.7.2.1}$$

Next round to the nearest hundredth to get 0.57.

Although everyone's favorite thing to do is to perform long division by hand, in most statistics classes you will have a calculator or computer to use. Thus you just have to remember to perform the division with the calculator or computer and then round.

Example 1.7.2.7

In statistics we need to find basic probabilities and create a table for them. Suppose that you roll two six-sided dice, what percent of the time will the sum equal to a 4? Round to the nearest whole number percent.

Solution

First, notice that there are 36 total possibilities for rolling the dice, since there are 6 faces on the first die and for each value of the first die roll, there are 6 possibilities for the second die roll. Multiplying: 6 x 6 = 36. This will be the denominator. To find the numerator, we list all the possible outcome where the sum is 4:

(1,3), (2,2), and (3,1)

There are three possible outcomes with the sum equaling a 4. Thus:

$$P(\text{sum} = 4) = 3/36$$

Now we divide:

$$\frac{3}{36} = 0.08333...$$

Next to convert this decimal to a percent, we move the decimal two places to the right to get: 8.333...%

We are asked to round to the nearest whole number percent. The digit to the right or the whole number (8) is a 3. Since 3 < 5, we can just erase everything to the left of the 8 and leave the 8 unchanged to get 8%. Thus there is an 8% chance of getting a sum of 4 if you roll two six sided dice.

- Convert Percentages to Decimals
- Relating Fractions, Decimals, and Percents
- Statistics Application of Converting Decimals to Percents

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1.7.3: Decimals- Rounding and Scientific Notation

Learning Outcomes

1. Understand what it means to have a number rounded to a certain number of decimal places.
2. Round a number to a fixed number of digits.
3. Convert from scientific notation to decimal notation and back.

In this section, we will go over how to round decimals to the nearest whole number, nearest tenth, nearest hundredth, etc. In most statistics applications that you will encounter, the numbers will not come out evenly, and you will need to round the decimal. We will also look at how to read scientific notation. A very common error that statistics students make is not noticing that the calculator is giving an answer in scientific notation.

For example, suppose that you used a calculator to find the probability that a randomly selected day in July will have a high temperature of over 90 degrees. Your calculator gives the answer: 0.4987230156. This is far too many digits for practical use, so it makes sense to round to just a few digits. By the end of this section you will be able to perform the rounding that is necessary to make unmanageable numbers manageable.

Brief Review of Decimal Language

Consider the decimal number: 62.5739. There is a defined way to refer to each of the digits.

- The digit 6 is in the "Tens Place"
- The digit 2 is in the "Ones Place"
- The digit 5 is in the "Tenths Place"
- The digit 7 is in the "Hundredths Place"
- The digit 3 is in the "Thousandths Place"
- The digit 9 is in the "Ten-thousandths Place"
- We also say that 62 is the "Whole Number" part.



Keeping this example in mind will help you when you are asked to round to a specific place value.

Example 1.7.3.1

It is reported that the mean number of classes that college students take each semester is 3.2541. Then the digit in the *hundredths place* is 5.

Rules of Rounding

Now that we have reviewed place values of numbers, we are ready to go over the process of rounding to a specified place value. When asked to round to a specified place value, the answer will erase all the digits after the specified digit. The process to deal with the other digits is best shown by examples.

Example 1.7.3.2: Case 1 - The Test Digit is Less Than 5

Round 3.741 to the nearest tenth.

Solution



Since the test digit (4) is less than 5, we just erase everything to the right of the tenths digit, 7. The answer is: 3.7.

Example 1.7.3.3: Case 2 - The Test Digit is 5 or Greater

Round 8.53792 to the nearest hundredth.

Solution



Since the test digit (6) is 5 or greater, we add one to the hundredths digit and erase everything to the right of the hundredths digit, 3. Thus the 3 becomes a 4. The answer is: 8.54.

Example 1.7.3.4: Case 3 - The Test Digit is 5 or Greater and the rounding position digit is a 9

Round 0.014952 to four decimal places.

Solution



The test digit is 5, so we must round up. The rounding position is a 9 and adding 1 gives 10, which is not a single digit number. Instead look at the two digits to the left of the test digit: 49. If we add 1 to 49, we get 50. Thus the answer is 0.0150.

Applications

Rounding is used in most areas of statistics, since the calculator or computer will produce numerical answers with far more digits than are useful. If you are not told how many decimal places to round to, then you often want to think about the smallest number of decimals to keep so that no important information is lost. For example suppose you conducted a sample to find the proportion of college students who receive financial aid and the calculator presented 0.568429314. You could turn this into a percent at 56.8429314%. There are no applications where keeping this many decimal places is useful. If, for example, you wanted to present this finding to the student government, you might want to round to the nearest whole number. In this case the ones digit is 6 and the test digit is 8. Since $8 \geq 5$, you add 1 to the ones digit. You can tell the student government that 57% of all college students receive financial aid.

Example 1.7.3.5

Suppose that you found out that the probability that a randomly selected person with who has misused prescription opioids will transition to heroin is 0.04998713. Round this number to four decimal places.

Solution

The first four decimal places are 0.0499 and the test digit is 8. Since $8 \geq 5$, we would like to add 1 to the fourth digit. Since this is a 9, we go to the next digit to the left. This is also a 9, so we go to the next one which is a 4. We can think of adding 0499 +

1 = 0500. Thus the answer is 0.0500. Note that we keep the last two 0's after the 5 to emphasize that this is accurate to the fourth decimal place.

Rounding and Arithmetic

Many times, we have to do arithmetic on numbers with several decimal places and want the answer rounded to a smaller number of decimal places. One question you might ask is should you round before you perform the arithmetic or after. For the most accurate result, you should always round after you preform the arithmetic if possible.

> *When asked to do arithmetic and present you answer rounded to a fixed number of decimal places, only round after performing the arithmetic.*

Example 1.7.3.6

Suppose you pick three cards from a 52 card deck with replacement and want to find the probability of the event, A, that none of the three cards will be a 2 through 7 of hearts. This probability is:

$$P(A) = (0.8846)^3$$

Round the answer to 2 decimal places.

Solution

Note that we have to first perform the arithmetic. With a computer or calculator we get:

$$0.8846^3 = 0.69221467973$$

Now we round to two decimal places. Notice that the hundredths digit is a 9 and the test digit is a 2. Thus the 9 remains unchanged and everything to the right of the 9 goes away. the result is

$$P(A) \approx 0.69$$

If we mistakenly rounded 0.8846 to two decimal places (0.88) and then cubed the answer we would have gotten 0.68 which is not the correct answer.

Scientific Notation

When a calculator presents a number in scientific notation, we must pay attention to what this represents. The standard way of writing a number in scientific notation is writing the number as a product of a number greater than or equal 1 but less than 10 followed by a power of 10. For example:

$$602,000,000,000,000,000,000,000 = 6.02 \times 10^{23}$$

The main purpose of scientific notation is to allow us to write very large numbers or numbers very close to 0 without having to use so many digits. Most calculators and computers use a different notation for scientific notation, most likely because the superscript is difficult to render on a screen. For example, with a calculator:

$$0.00000032 = 3.2E-7$$

Notice that to arrive at 3.2, the decimal needed to be moved 7 places to the right.

Example 1.7.3.7

A calculator displays:

$$2.0541E6$$

Write this number in decimal form.

Solution

Notice that the number following E is 6. This means move the decimal over 6 places to the right. The first 4 moves is natural, but for the last 2 moves, there are no numbers to move the decimal place past. We can always add extra zeros after the last

number to the right of the decimal place:

$$2.0541E6 = 2.054100E6$$

Now we can move the decimal place to the right 6 places to get

$$2.0541E6 = 2.054100E6 = 2,054,100$$

Example 1.7.3.8

If you use a calculator or computer to find the probability of flipping a coin 27 times and getting all heads, then it will display:

$$7.45E-9$$

Write this number in decimal form.

Solution

Many students will forget to look for the "E" and just write that the probability is 7.45, but probabilities can never be bigger than 1. You can not have a 745% chance of it occurring. Notice that the number following E is −9. Since the power is negative, this means move the decimal to the left, and in particular 9 places to the left. There is only one digit to the left of the decimal place, so we need to insert 8 zeros:

$$7.45E-9 = 000000007.45E-9$$

Now we can move the decimal place to the right 9 places to the left to get

$$7.45E-9 = 000000007.45E-9 = 0.00000000745$$

- Application of Rounding Decimal Numbers
- Here is a video that explains rounding.

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1.7.4: Using Fractions, Decimals and Percents to Describe Charts

Learning Outcomes

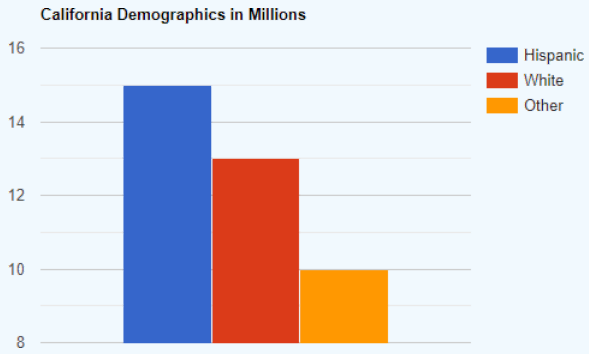
1. Interpret bar charts using fractions, decimals and percents
2. Interpret pie charts using fractions, decimals and percents

Charts, such as bar charts and pie charts are visual ways of presenting data. You can think of each slice of the pie or each bar as a part of the whole. The numerical versions of this are a list of fractions, decimals and percents. By the end of this section we will be able to look at one of these charts and produce the corresponding fractions, decimals, and percents.

Reading a Bar Chart

Bar charts occur frequently and it is definitely required to understand how to read them and interpret them in statistics. Often we want to convert the information of a bar chart to information shown numerically. We need fractions and/or percents to do this.

Example 1.7.4.1



The above bar chart shows the demographics of California in 2019 where the numbers represent millions of people. Here are some questions that might come up in a statistics class.

A. What fraction of Californians was Hispanic in 2019?
B. What proportion of all Californians was White in 2019? Write your answer as a decimal number rounded to four decimal places.
C. What percent of Californians who were neither Hispanic nor White in 2019? Round your answer to the nearest percent.

Solution

A. To find the fraction of California that was Hispanic in 2019, the numerator will be the total number of Hispanics and the denominator will be the total number of people in California in 2019. The height of the bar that represents Hispanics is 15. Therefore the numerator is 15. To find the total number of people in California, we add up the heights of the three bars:

$$15+13+10 = 38$$

Now we can just write down the fraction:

$$\frac{15}{38}$$

To find the proportion of Californians who were White in 2019, we start in the same way. The numerator will be the number of Whites: 13. The denominator will be the total number of Californians which we already computed as 38. Therefore the fraction of Californians who were White is:

$$\frac{13}{38}$$

To convert this to a decimal, we use a calculator to get:

$$\frac{13}{38} \approx 0.342105$$

Next round to four decimal places. Since the digit to the right of the fourth decimal place is $0 < 5$, we round down to:

$$0.3421$$

B. To find the percent of Californians who were neither Hispanic nor White in 2019, we first find the fraction who were neither. The numerator will be the number of "Other" which is: 10. The denominator will be the total which is 38. Thus the fraction is:

$$\frac{10}{38}$$

Next, use a calculator to divide these numbers to get:

$$\frac{10}{38} \approx 0.263158$$

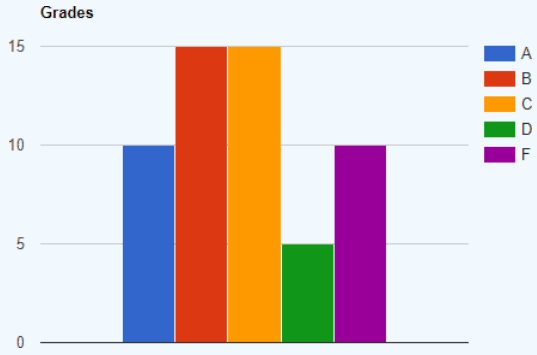
To convert this to a percent we multiply by 100% by moving the decimal two places to the right:

$$0.263158 \times 100\% = 26.3158\%$$

Finally we round to the nearest whole number. Noting that $3 < 5$, we round down to get: 26%

Exercise

The bar chart below shows the grade distribution for a math class.



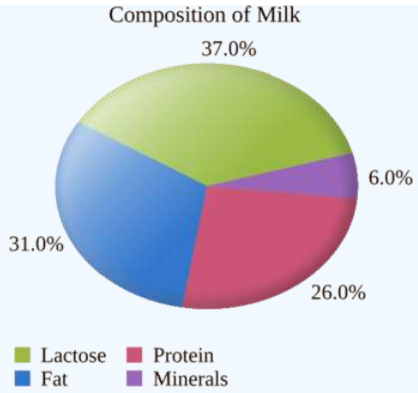
A. Find the fraction of students who received a "C" grade.
B. Find the proportion of grades below a "C". Write your answer as a decimal number rounded to the nearest hundredth.
C. What percent of the students received an "A" grade? Round your answer to the nearest whole number percent.

Reading a Pie Chart

Another important chart that is used to display the components of a whole is a pie chart. With a pie chart, it is very easy to determine the percent of each item.

Example 1.7.4.2

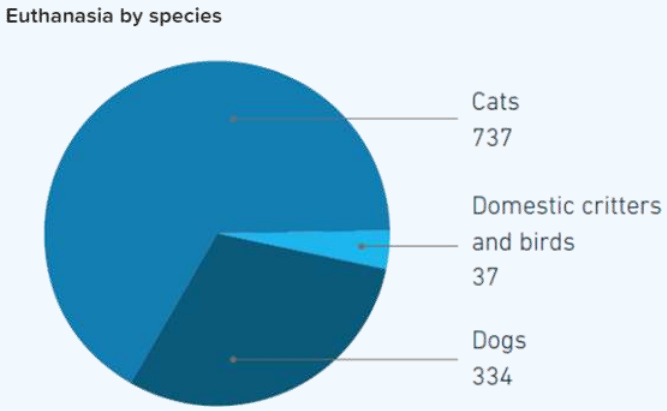
The pie chart below shows the makeup of milk. Write the proportion of fat contained in milk as a decimal.



Solution

We see that 31% of milk is fat. To convert a percent to a decimal, we just move the decimal over two places to the left. Thus, 31% becomes 0.31.

Example 1.7.4.3



The pie chart above shows the number of pets of each type that had to be euthanized by the humane society due to incurable illnesses.

A. What fraction of the euthanized pets were dogs?
B. What percent of the euthanized pets were cats? Round to the nearest whole number percent.

Solution

A. We take the number of dogs over the total. There were 334 euthanized dogs. To find the total we add:

$$737+37+334 \;=\; 1108$$

Therefore, the fraction of euthanized dogs is

$$\frac{334}{1108}$$

B. To find the percent of euthanized cats, we first find the fraction. There were 737 cats over a total of 1108 pets. The fraction is

$$\frac{737}{1108}$$

Next use a calculator to get the decimal number: 0.66516. Now multiply by 100% by moving the decimal place two digits to the right to get: 66.516%. Finally, we need to round to the nearest whole number percent. Since $5 \geq 5$, we round up.

> Thus the percent of euthanized cats is 67%.

- Finding Fractions, Decimals and Percents from a Bar Chart
- Ex: Find the a Percent of a Total Using an Amount in Pie Chart

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1.8: Rounding Whole Numbers

Learning Objectives

- understand that rounding is a method of approximation
- be able to round a whole number to a specified position

Rounding as an Approximation

A primary use of whole numbers is to keep count of how many objects there are in a collection. Sometimes we're only interested in the approximate number of objects in the collection rather than the precise number. For example, there are *approximately* 20 symbols in the collection below.



The *precise* number of symbols in the above collection is 18.

Definition: Rounding

We often approximate the number of objects in a collection by mentally seeing the collection as occurring in groups of tens, hundreds, thousands, etc. This process of approximation is called **rounding**. Rounding is very useful in estimation. We will study estimation in Chapter 8.

When we think of a collection as occurring in groups of tens, we say we're *rounding to the nearest ten*. When we think of a collection as occurring in groups of hundreds, we say we're *rounding to the nearest hundred*. This idea of rounding continues through thousands, ten thousands, hundred thousands, millions, etc.

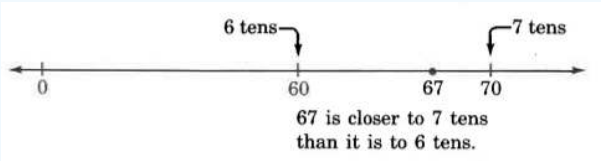
The process of rounding whole numbers is illustrated in the following examples.

Example 1.8.1

Round 67 to the nearest ten.

Solution

On the number line, 67 is more than halfway from 60 to 70. The digit immediately to the right of the tens digit, the round-off digit, is the indicator for this.



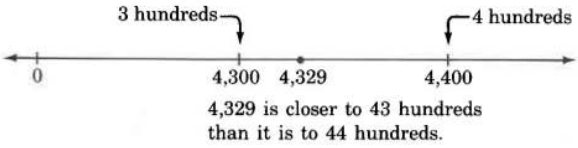
Thus, 67, rounded to the nearest ten, is 70.

Example 1.8.2

Round 4,329 to the nearest hundred.

Solution

On the number line, 4,329 is less than halfway from 4,300 to 4,400. The digit to the immediate right of the hundreds digit, the round-off digit, is the indicator.



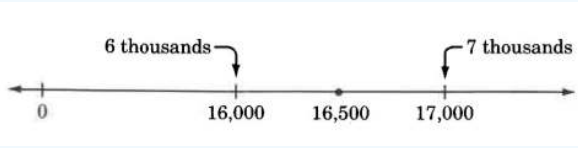
Thus, 4,329, rounded to the nearest hundred is 4,300.

Example 1.8.3

Round 16,500 to the nearest thousand.

Solution

On the number line, 16,500 is exactly halfway from 16,000 to 17,000.



By convention, when the number to be rounded is *exactly halfway* between two numbers, it is rounded to the *higher* number.

Thus, 16,500, rounded to the nearest thousand, is 17,000.

Example 1.8.4

A person whose salary is $41,450 per year might tell a friend that she makes $41,000 per year. She has rounded 41,450 to the nearest thousand. The number 41,450 is closer to 41,000 than it is to 42,000.

The Method of Rounding Whole Numbers

From the observations made in the preceding examples, we can use the following method to **round a whole number** to a particular position.

1. Mark the position of the round-off digit.
2. Note the digit to the immediate right of the round-off digit.
   a. If it is less than 5, replace it and all the digits to its right with zeros. Leave the round-off digit unchanged.
   b. If it is 5 or larger, replace it and all the digits to its right with zeros. Increase the round-off digit by 1.

Sample Set A

Use the method of rounding whole numbers to solve the following problems.

Round 3,426 to the nearest ten.

Solution

- We are rounding to the tens position. Mark the digit in the tens position

  3,426
  ↑
  tens position

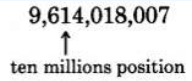
- Observe the digit immediately to the right of the tens position. It is 6. Since 6 is greater than 5, we *round up* by replacing 6 with 0 and adding 1 to the digit in the tens position (the round-off position): 2+1=32+1=3 .
  3,430

Thus, 3,426 rounded to the nearest ten is 3,430.

Sample Set A

Round 9,614,018,007 to the nearest ten million.

Solution

1. We are rounding to the nearest ten million.
   

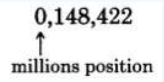
2. Observe the digit immediately to the right of the ten millions position. It is 4. Since 4 is less than 5, we *round down* by replacing 4 and all the digits to its right with zeros.
   9,610,000,000

Thus, 9,614,018,007 rounded to the nearest ten million is 9,610,000,000.

Sample Set A

Round 148,422 to the nearest million.

Solution

1. Since we are rounding to the nearest million, we'll have to *imagine* a digit in the millions position. We'll write 148,422 as 0,148,422.
   

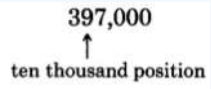
2. The digit immediately to the right is 1. Since 1 is less than 5, we'll *round down* by replacing it and all the digits to its right with zeros.
   0,000,000
   This number is 0.

Thus, 148,422 rounded to the nearest million is 0.

Sample Set A

Round 397,000 to the nearest ten thousand.

Solution

- We are rounding to the nearest ten thousand.
  

- The digit immediately to the right of the ten thousand position is 7. Since 7 is greater than 5, we round up by replacing 7 and all the digits to its right with zeros and adding 1 to the digit in the ten thousands position. But 9+1=109+1=10 and we must carry the 1 to the next (the hundred thousands) position.
  400,000

Thus, 397,000 rounded to the nearest ten thousand is 400,000.

Practice Set A

Use the method of rounding whole numbers to solve each problem.

Round 3387 to the nearest hundred.

Answer

3400

Practice Set A

Round 26,515 to the nearest thousand.

Answer

27,000

Practice Set A

Round 30,852,900 to the nearest million.

Answer

31,000,000

Practice Set A

Round 39 to the nearest hundred.

Answer

0

Practice Set A

Round 59,600 to the nearest thousand.

Answer

60,000

Exercises

For the following problems, complete the table by rounding each number to the indicated positions.

Exercise 1.8.1

1,642

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
1,600	2000	0	0

Exercise 1.8.2

5,221

hundred	thousand	ten thousand	million

Exercise 1.8.3

91,803

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
91,800	92,000	90,000	0

Exercise 1.8.4

106,007

hundred	thousand	ten thousand	million

Exercise 1.8.5

208

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
200	0	0	0

Exercise 1.8.6

199

hundred	thousand	ten thousand	million

Exercise 1.8.7

863

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
900	1,000	0	0

Exercise 1.8.8

794

hundred	thousand	ten thousand	million

Exercise 1.8.9

925

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
900	1,000	0	0

Exercise 1.8.10

909

hundred	thousand	ten thousand	million

Exercise 1.8.11

981

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
1,000	1,000	0	0

Exercise 1.8.12

965

hundred	thousand	ten thousand	million

Exercise 1.8.13

551,061,285

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
551,061,300	551,061,000	551,060,000	551,000,000

Exercise 1.8.14

23,047,991,521

hundred	thousand	ten thousand	million

Exercise 1.8.15

106,999,413,206

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
106,999,413,200	106,999,413,000	106,999,410,000	106,999,000,000

Exercise 1.8.16

5,000,000

hundred	thousand	ten thousand	million

Exercise 1.8.17

8,006,001

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
8,006,000	8,006,000	8,010,000	8,000,000

Exercise 1.8.18

94,312

hundred	thousand	ten thousand	million

Exercise 1.8.19

33,486

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
33,500	33,000	30,000	0

Exercise 1.8.20

560,669

hundred	thousand	ten thousand	million

Exercise 1.8.21

388,551

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
388,600	389,000	390,000	0

Exercise 1.8.22

4,752

hundred	thousand	ten thousand	million

Exercise 1.8.23

8,209

hundred	thousand	ten thousand	million

Answer

hundred	thousand	ten thousand	million
8,200	8,000	10,000	0

Exercise 1.8.24

In 1950, there were 5,796 cases of diphtheria reported in the United States. Round to the nearest hundred.

Exercise 1.8.25

In 1979, 19,309,000 people in the United States received federal food stamps. Round to the nearest ten thousand.

Answer

19,310,000

Exercise 1.8.26

In 1980, there were 1,105,000 people between 30 and 34 years old enrolled in school. Round to the nearest million.

Exercise 1.8.27

In 1980, there were 29,100,000 reports of aggravated assaults in the United States. Round to the nearest million.

Answer

29,000,000

For the following problems, round the numbers to the position you think is most reasonable for the situation.

Exercise 1.8.28

In 1980, for a city of one million or more, the average annual salary of police and firefighters was $16,096.

Exercise 1.8.29

The average percentage of possible sunshine in San Francisco, California, in June is 73%.

Answer

70% or 75%

Exercise 1.8.30

In 1980, in the state of Connecticut, $3,777,000,000 in defense contract payroll was awarded.

Exercise 1.8.31

In 1980, the federal government paid $5,463,000,000 to Viet Nam veterans and dependants.

Answer

$5,500,000,000

Exercise 1.8.32

In 1980, there were 3,377,000 salespeople employed in the United States.

Exercise 1.8.33

In 1948, in New Hampshire, 231,000 popular votes were cast for the president.

Answer

230,000

Exercise 1.8.34

In 1970, the world production of cigarettes was 2,688,000,000,000.

Exercise 1.8.35

In 1979, the total number of motor vehicle registrations in Florida was 5,395,000.

Answer

5,400,000

Exercise 1.8.36

In 1980, there were 1,302,000 registered nurses the United States.

Exercises for Review

Exercise 1.8.37

There is a term that describes the visual displaying of a number. What is the term?

Answer

graphing

Exercise 1.8.38

What is the value of 5 in 26,518,206?

Exercise 1.8.39

Write 42,109 as you would read it.

Answer

Forty-two thousand, one hundred nine

Exercise 1.8.40

Write "six hundred twelve" using digits.

Exercise 1.8.41

Write "four billion eight" using digits.

Answer

4,000,000,008

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1.9: Scientific Notation

Standard Form to Scientific Form

Very large numbers such as $43,000,000,000,000,000,000,000$ (the number of different possible configurations of Rubik's cube) and very small numbers such as $0.000000000000000000000034$ (the mass of the amino acid tryptophan) are extremely inconvenient to write and read. Such numbers can be expressed more conveniently by writing them as part of a power of 10.

To see how this is done, let us start with a somewhat smaller number such as 2480. Notice that

$$\begin{aligned} \underbrace{2480}_{\text{Standard form}} &= 248.0 \times 10^1 \\ &= 24.80 \times 10^2 \\ &= \underbrace{2.480 \times 10^3}_{\text{Scientific form}} \end{aligned}$$

Scientific Form

The last form is called the **scientific form** of the number. There is **one** nonzero digit to the left of the decimal point and the absolute value of the exponent on 10 records the number of places the original decimal point was moved to the **left**.

$$\begin{aligned} 0.00059 &= \frac{0.0059}{10} = \frac{0.0059}{10^1} = 0.0059 \times 10^{-1} \\ &= \frac{0.059}{100} = \frac{0.059}{10^2} = 0.059 \times 10^{-2} \\ &= \frac{0.59}{1000} = \frac{0.59}{10^3} = 0.59 \times 10^{-3} \\ &= \frac{5.9}{10,000} = \frac{5.9}{10^4} = 5.9 \times 10^{-4} \end{aligned}$$

There is **one** nonzero digit to the left of the decimal point and the absolute value of the exponent of 10 records the number of places the original decimal point was moved to the **right**.

Scientific Notation

Numbers written in scientific form are also said to be written using scientific notation. In **scientific notation**, a number is written as the product of a number between and including 1 and 10 (1 is included,10 is not) and some power of 10.

Writing a Number in Scientific Notation

To write a number in scientific notation:

1. Move the decimal point so that there is one nonzero digit to its left.
2. Multiply the result by a power of 10 using an exponent whose absolute value is the number of places the decimal point was moved. Make the exponent positive if the decimal point was moved to the left and negative if the decimal point was moved to the right.

Sample Set A

Write the numbers in scientific notation.

Example 1.9.1

981

The number 981 is actually $981.$, and it is followed by a decimal point. In integers, the decimal point at the end is usually omitted.

$981 = 981. = 9.81 \times 10^2$

The decimal point is now two places to the left of its original position, and the power of 10 is 2.

Example 1.9.2

$54.066 = 5.4066 \times 10^1 = 5.4066 \times 10$

The decimal point is one place to the left of its original position, and the power of 10 is 1

Example 1.9.3

$0.000000000004632 = 4.632 \times 10^{-12}$

The decimal point is twelve places to the right of its original position, and the power of 10 is -12.

Example 1.9.4

$0.027 = 2.7 \times 10^{-2}$

The decimal point is two places to the right of its original position, and the power of 10 is -2

Practice Set A

Write the following numbers in scientific notation.

Practice Problem 1.9.1

346

Answer

3.46×10^2

Practice Problem 1.9.2

72.33

Answer

7.233×10

Practice Problem 1.9.3

5387.7965

Answer

5.3877965×10^3

Practice Problem 1.9.4

$87,000,000$

Answer

8.7×10^7

Practice Problem 1.9.5

$179,000,000,000,000,000,000$

Answer

1.79×10^{20}

Practice Problem 1.9.6

$100,000$

Answer

1.0×10^{5}

Practice Problem 1.9.7

$1,000,000$

Answer

1.0×10^{6}

Practice Problem 1.9.8

0.0086

Answer

8.6×10^{-3}

Practice Problem 1.9.9

0.000098001

Answer

9.8001×10^{-5}

Practice Problem 1.9.10

0.000000000000000054

Answer

5.4×10^{-17}

Practice Problem 1.9.11

0.0000001

Answer

1.0×10^{-7}

Practice Problem 1.9.12

0.00000001

Answer

1.0×10^{-8}

Scientific Form to Standard Form

A number written in scientific notation can be converted to standard form by reversing the process shown in Sample Set A.

Converting from Scientific Notation

To convert a number written in scientific notation to a number in standard form, move the decimal point the number of places prescribed by the exponent on the 10.

Positive Exponent Negative Exponent

Move the decimal point to the right when you have a positive exponent, and move the decimal point to the left when you have a negative exponent.

Sample Set B

Example 1.9.5

4.63×10^4

The exponent of 10 is 4 so we must move the decimal point to the right 4 places (adding 0's if necessary).

$4.6730 \times 10^4 = 46730$

Example 1.9.5

2.9×10^7

The exponent of 10 is 7 so we must move the decimal point to the right 7 places (adding 0's if necessary).

$2.9 \times 10^7 = 29000000$

Example 1.9.6

1×10^{27}

The exponent of 10 is 27 so we must move the decimal point to the right 27 places (adding 0's if necessary).

$1 \times 10^{27} = 1,000,000,000,000,000,000,000,000,000$

Example 1.9.7

4.21×10^{-5}

The exponent of 10 is -5 so we must move the decimal point to the left 5 places (adding 0's if necessary).

$4.21 \times 10^{-5} = 0.0000421$

Example 1.9.8

1.006×10^{-18}

The exponent of 10 is -18 so we must move the decimal point to the left 18 places (adding 0's if necessary).

$1.006 \times 10^{-18} = 0.00000000000000001006$

Practice Set B

Convert the following numbers to standard form.

Practice Problem 1.9.13

9.25×10^{2}

Answer

925

Practice Problem 1.9.14

4.01×10^{5}

Answer

401000

Practice Problem 1.9.15

1.2×10^{-1}

Answer

0.12

Practice Problem 1.9.16

8.88×10^{-5}

Answer

0.0000888

Working with Numbers in Scientific Notation

Multiplying Numbers Using Scientific Notation

There are many occasions (particularly in the sciences) when it is necessary to find the product of two numbers written in scientific notation. This is accomplished by using two of the basic rules of algebra.

Suppose we wish to find \((a \times 10^n)(b \times 10^m). Since the only operation is multiplication, we can use the commutative property of multiplication to rearrange the numbers.

\((a \times 10^n)(b \times 10^m) = (a \times b)(10^n \times 10^m)

Then, by the rules of exponents, $10^{n} \times 10^{m} = 10^{n+m}$. Thus,

$(a \times 10^{n})(b \times 10^{m}) = (a \times b) \times 10^{n+m}$

The product of $(a \times b)$ may not be between 1 and 10, so $(a \times b) \times 10^{n+m}$ may not be in scientific form. The decimal point in $(a \times b)$ may have to be moved. An example of this situation is in Sample Set C, example 3.8.10.

Sample Set C

Example 1.9.9

$$\begin{aligned}\left(2\times10^{3}\right)\left(4\times10^{8}\right) &= (2\times4)\left(10^{3}\times10^{8}\right)\\ &= 8\times10^{3+8}\\ &= 8\times10^{11}\end{aligned}$$

Example 1.9.10

$$\begin{aligned}\left(5\times10^{17}\right)\left(8.1\times10^{-22}\right) &= (5\times8.1)\left(10^{17}\times10^{-22}\right)\\ &= 40.5\times10^{17-22}\\ &= 40.5\times10^{-5}\end{aligned}$$

We need to move the decimal point one place to the left to put this number in scientific notation.
Thus, we must also change the exponent of 10.

40.5×10^{-5}
$4.05\times10^{1}\times10^{-5}$
$4.05\times\left(10^{1}\times10^{-5}\right)$
$4.05\times\left(10^{1-5}\right)$
4.05×10^{-4}

Thus,
$\left(5\times10^{17}\right)\left(8.1\times10^{-22}\right)=4.05\times10^{-4}$

Practice Set C

Perform each multiplication.

Practice Problem 1.9.17

$(3\times10^{5})(2\times10^{12})$

Answer

6×10^{17}

Practice Problem 1.9.18

$(1\times10^{-4})(6\times10^{24}$

Answer

6×10^{20}

Practice Problem 1.9.19

$(5\times10^{18})(3\times10^{6})$

Answer

1.5×10^{25}

Practice Problem 1.9.20

$(2.1 \times 10^{-9})(3 \times 10^{-11})$

Answer

6.3×10^{-20}

Exercises

Convert the numbers used in the following problems to scientific notation.

Exercise 1.9.1

Mount Kilimanjaro is the highest mountain in Africa. It is 5890 meters high.

Answer

5.89×10^{3}

Exercise 1.9.2

The planet Mars is about 222,900,000,000 meters from the sun.

Exercise 1.9.3

There is an irregularly shaped galaxy, named NGC 4449, that is about 250,000,000,000,000,000,000,000 meters from earth.

Answer

2.5×10^{23}

Exercise 1.9.4

The farthest object astronomers have been able to see (as of 1981) is a quasar named 3C427. There seems to be a haze beyond this quasar that appears to mark the visual boundary of the universe. Quasar 3C427 is at a distance of 110,000,000,000,000,000,000,000,000 meters from the earth.

Exercise 1.9.5

The smallest known insects are about the size of a typical grain of sand. They are about 0.0002 meters in length (2 ten-thousandths of a meter).

Answer

2×10^{-4}

Exercise 1.9.6

Atoms such as hydrogen, carbon, nitrogen, and oxygen are about 0.0000000001 meter across.

Exercise 1.9.7

The island of Manhattan, in New York, is about 57,000 square meters in area.

Answer

5.7×10^4

Exercise 1.9.8

The second largest moon of Saturn is Rhea. Rhea has a surface area of about 735,000 square meters, roughly the same surface area as Australia.

Exercise 1.9.9

A star, named Epsilon Aurigae B, has a diameter (distance across) of 2,800,000,000,000 meters. This diameter produces a surface area of about 24,630,000,000,000,000,000,000,000 square meters. This star is what astronomers call a red giant and it is the largest red giant known. If Epsilon Aurigae were placed at the sun's position, its surface would extend out to the planet Uranus.

Answer

2.8×10^{12}, 2.463×10^{25}

Exercise 1.9.10

The volume of the planet Venus is 927,590,000,000,000,000,000 cubic meters.

Exercise 1.9.11

The average mass of a newborn American female is about 3360 grams.

Answer

3.36×10^3

Exercise 1.9.12

The largest brain ever measured was that of a sperm whale. It had a mass of 9200 grams.

Exercise 1.9.13

The mass of the Eiffel tower in Paris, France, is 8,000,000 grams.

Answer

8×10^6

Exercise 1.9.14

In 1981, a Japanese company built the largest oil tanker to date. The ship has a mass of about 510,000,000,000 grams. This oil tanker is more than 6 times as massive as the U.S. aircraft carrier, U.S.S. Nimitz.

Exercise 1.9.15

In the constellation of Virgo, there is a cluster of about 2500 galaxies. The combined mass of these galaxies is 150,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000 grams.

Answer

1.5×10^{62}

Exercise 1.9.16

The mass of an amoeba is about 0.000004 gram.

Exercise 1.9.17

Cells in the human liver have masses of about 0.000000008 gram.

Answer

8×10^{-9}

Exercise 1.9.18

The human sperm cell has a mass of about 0.000000000017 gram.

Exercise 1.9.19

The principal protein of muscle is myosin. Myosin has a mass of 0.00000000000000000103 gram.

Answer

1.03×10^{-18}

Exercise 1.9.20

Amino acids are molecules that combine to make up protein molecules. The amino acid tryptophan has a mass of 0.0000000000000000000000340 gram.

Exercise 1.9.21

An atom of the chemical element bromine has 35 electrons. The mass of a bromine atom is 0.000000000000000000000000031 gram.

Answer

3.1×10^{-26}

Exercise 1.9.22

Physicists are performing experiments that they hope will determine the mass of a small particle called a neutrino. It is suspected that neutrinos have masses of about 0.00000000000000000000000000000001 gram.

Exercise 1.9.23

The approximate time it takes for a human being to die of asphyxiation is 316 seconds.

Answer

3.16×10^{2}

Exercise 1.9.24

On the average, the male housefly lives 1,468,800 seconds (17 days).

Exercise 1.9.25

Aluminum-26 has a half-life of 740,000 years.

Answer

7.4×10^5

Exercise 1.9.26

Manganese-53 has a half-life of 59,918,000,000,000 seconds (1,900,000 years).

Exercise 1.9.27

In its orbit around the sun, the earth moves a distance one and one half feet in about 0.0000316 second.

Answer

3.16×10^{-5}

Exercise 1.9.28

A pi-meson is a subatomic particle that has a half-life of about 0.0000000261 second.

Exercise 1.9.29

A subatomic particle called a neutral pion has a half-life of about 0.0000000000000001 second.

Answer

1×10^{-16}

Exercise 1.9.30

Near the surface of the earth, the speed of sound is 1195 feet per second.

For the following problems, convert the numbers from scientific notation to standard decimal form.

Exercise 1.9.31

The sun is about 1×10^8 meteres from earth.

Answer

100,000,000

Exercise 1.9.32

The mass of the earth is about 5.98×10^{27} grams.

Exercise 1.9.33

Light travels about 5.866×10^{12} miles in one year.

Answer

5,866,000,000,000

Exercise 1.9.34

One year is about 3×10^7 seconds.

Exercise 1.9.35

Rubik's cube has about 4.3×10^{19} different configurations.

Answer

43,000,000,000,000,000,000

Exercise 1.9.36

A photon is a particle of light. A 100-watt light bulb emits 1×10^{20} photons every second.

Exercise 1.9.37

There are about 6×10^7 cells in the retina of the human eye.

Answer

60,000,000

Exercise 1.9.38

A car traveling at an average speed will travel a distance about equal to the length of the smallest fingernail in 3.16×10^{-4} seconds.

Exercise 1.9.39

A ribosome of **E. coli** has a mass of about 4.7×10^{-19} grams.

Answer

0.00000000000000000047

Exercise 1.9.40

A mitochondrion is the energy-producing element of a cell. A mitochondrion is about 1.5×10^{-6} meters in diameter.

Exercise 1.9.41

There is a species of frogs in Cuba that attain a length of at most 1.25×10^{-2} meters.

Answer

0.0125

Perform the following operations.

Exercise 1.9.42

$(2 \times 10^4)(3 \times 10^5)$

Exercise 1.9.43

$(4 \times 10^2)(8 \times 10^6)$

Answer

3.2×10^9

Exercise 1.9.44

$(6 \times 10^{14})(6 \times 10^{-10})$

Exercise 1.9.45

$(3 \times 10^{-5})(8 \times 10^7)$

Answer

2.4×10^3

Exercise 1.9.46

$(2 \times 10^{-1})(3 \times 10^{-5})$

Exercise 1.9.47

$(9 \times 10^{-5})(1 \times 10^{-11})$

Answer

9×10^{-16}

Exercise 1.9.48

$(3.1 \times 10^4)(3.1 \times 10^{-6})$

Exercise 1.9.49

$4.2 \times 10^{-12})(3.6 \times 10^{-20})$

Answer

1.512×10^{-31}

Exercise 1.9.50

$(1.1 \times 10^6)^2$

Exercises for Review

Exercise 1.9.51

What integers can replace x so that the statement $-6 < x < -2$ is true?

Answer

$-5, -4, -3$

Exercise 1.9.52

Simplify $(5x^2y^4)(2xy^5)$

Exercise 1.9.53

Determine the value of $-[-(-|-5|)]$.

Answer

-5

Exercise 1.9.54

Write \(\dfrac{x^3y^{-5}}{z^{-4}\) so that only positive exponents appear.

Exercise 1.9.55

Write $(2z+1)^3(2z+1)^{-5}$ so that only positive exponents appear.

Answer

$$\frac{1}{(2z+1)^2}$$

FFFF

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CHAPTER OVERVIEW

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SECTION OVERVIEW

2.1: The Integers

Today, much as we take for granted the fact that there exists a number zero, denoted by 0, such that $a+0 = a$ for any whole number a, we similarly take for granted that for any whole number a there exists a unique number $-a$, called the "negative" or "opposite" of a, so that $a + (-a) = 0$.

In a natural way, or so it seems to modern-day mathematicians, this easily introduces the concept of a *negative* number. However, history teaches us that the concept of negative numbers was not embraced wholeheartedly by mathematicians until somewhere around the 17th century.

In his work *Arithmetica* (c. 250 AD), the Greek mathematician Diophantus (c. 200-284 AD), who some call the "Father of Algebra," described the equation $4 = 4x + 20$ as "absurd," for how could one talk about an answer less than nothing? Girolamo Cardano (1501-1576), in his seminal work *Ars Magna* (c. 1545 AD) referred to negative numbers as "numeri ficti," while the German mathematician Michael Stifel (1487-1567) referred to them as "numeri absurdi." John Napier (1550-1617) (the creator of logarithms) called negative numbers "defectivi," and Rene Descartes (1596-1650) (the creator of analytic geometry) labeled negative solutions of algebraic equations as "false roots."

On the other hand, there were mathematicians whose treatment of negative numbers resembled somewhat our modern notions of the properties held by negative numbers. The Indian mathematician Brahmagupta described arithmetical rules in terms of fortunes (positive number) and debts (negative numbers). Indeed, in his work *Brahmasphutasiddhanta*, he writes "a fortune subtracted from zero is a debt," which in modern notation would resemble $0 - 4 = -4$.

If you find the study of the integers somewhat difficult, do not be discouraged, as centuries of mathematicians have struggled mightily with the topic. With this thought it mind, let's begin the study of the integers.

2.1.1: An Introduction to the Integers

2.1.2: Adding Integers

2.1.3: Subtracting Integers

2.1.4: Multiplication and Division of Integers

2.1.5: Order of Operations

2.1.6: Solving Equations

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2.1.1: An Introduction to the Integers

As we saw in the introduction to the chapter, negative numbers have a rich and storied history. One of the earliest applications of negative numbers had to do with credits and debits. For example, if \$5 represents a credit or profit, then −\$5 represents a debit or loss. Of course, the ancients had a different monetary system than ours, but you get the idea. Note that if a vendor experiences a profit of \$5 on a sale, then a loss of −\$5 on a second sale, the vendor breaks even. That is, the sum of \$5 and −\$5 is zero. In much the same way, every whole number has an *opposite* or *negative* counterpart.

The Opposite or Negative of a Whole Number

For every whole number a, there is a unique number $-a$, called the opposite or negative of a, such that $a + (-a) = 0$.

The opposite or negative of any whole number is easily located on the number line.

Number Line Locations

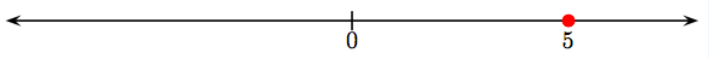
To locate the opposite (or negative) of any whole number, first locate the whole number on the number line. The opposite is the reflection of the whole number through the origin (zero).

Example 1

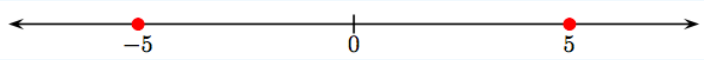
Locate the whole number 5 and its opposite (negative) on the number line.

Solution

Draw a number line, then plot the whole number 5 on the line as a shaded dot.



To find its opposite, reflect the number 5 through the origin. This will be the location of the opposite (negative) of the whole number 5, which we indicate by the symbol −5.



Note the symmetry. The whole number 5 is located five units to the right of zero. Its negative is located five units to the left of zero.

Exercise

Locate the number -7 and its opposite on the number line.

Important Pronunciation

The symbol −5 is pronounced in one of two ways: (1) “negative five,” or (2) “the opposite of five.”

In similar fashion, we can locate the opposite or negative of any whole number by reflecting the whole number through the origin (zero), which leads to the image shown in Figure 2.1.

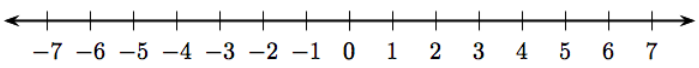


Figure 2.1: The opposite (negative) of any whole number is a reflection of that number through the origin (zero).

The Integers

The collection of numbers arranged on the number line in Figure 2.1 extend indefinitely to the right, and because the numbers on the left are reflections through the origin, the numbers also extend indefinitely to the left. This collection of numbers is called the set of *integers*.

The Integers

The infinite collection of numbers

$$\{..., -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, ...\}$$

is called the **set of integers.**

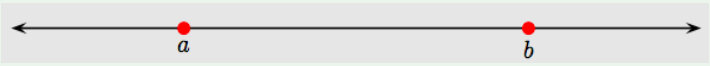
The ellipsis . . . at each end of this infinite collection means “etcetera,” as the integers continue indefinitely to the right and left. Thus, for example, both 23,456 and −117, 191 are elements of this set and are therefore integers.

Ordering the Integers

As we saw with the whole numbers, as you move to the right on the number line, the numbers get larger; as you move to the left, the numbers get smaller.

Order on the Number Line

Let a and b be integers located on the number line so that the point representing the integer a lies to the left of the point representing the integer b.



Then the integer a is “less than” the integer b and we write

$$a < b$$

Alternatively, we can also say that the integer b is “larger than” the integer a and write

$$b > a.$$

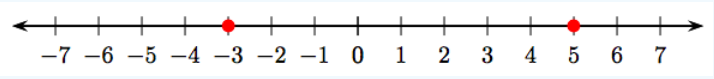
Example 3

Replace each shaded box with < (less than) or > (greater than) so the resulting inequality is a true statement.

$$-3 \quad \square \quad 5 \qquad\qquad -2 \quad \square \quad -4$$

Solution

For the first case, locate −3 and 5 on the number line as shaded dots.

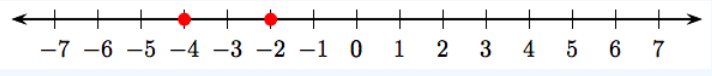


Note that −3 lies to the left of 5, so:

$$-3 \quad < \quad 5$$

That is, −3 is “less than” 5.

In the second case, locate −2 and −4 as shaded dots on the number line.



Note that −2 lies to the right of −4, so:

$$-2 \quad > \quad -4$$

That is, −2 is “greater than” −4.

Exercise

Compare −12 and −11.

Answer

−12 < −11

Important Observation

In Example 3, note that the "pointy end" of the inequality symbol always points towards the smaller number.

We stated earlier that every integer has a unique number called its "opposite" or "negative." Thus, the integer −5 is the opposite (negative) of the integer 5. Thus, we can say that the pair −5 and 5 are opposites. Each is the opposite of the other. Logically, this leads us to the conclusion that the opposite of −5 is 5. In symbols, we would write

$$-(-5) = 5.$$

Opposites of Opposites

Let a be an integer. Then the "opposite of the opposite of a is a." In symbols, we write

$$-(-a) = a.$$

We can also state that the "negative of a negative a is a.

Example 4

Simplify −(−13) and −(−119).

Solution

The opposite of the opposite of a number returns the original number. That is,

−(−13) = 13 and − (−119) = 11.

Exercise

Simplify: −(−50).

Answer

50

Positive and Negative

We now define the terms *positive integer* and *negative integer*.

Positive Integer

If a is an integer that lies to the right of zero (the origin) on the number line, then a is a **positive** integer. This means that a is a positive integer if and only if $a > 0$.

Thus, 2, 5, and 117 are positive integers.

Negative Integer

If a is an integer that lies to the left of zero (the origin) on the number line, then a is a **negative** integer. This means that a is a negative integer if and only if $a < 0$.

Thus, −4, −8, and −1, 123 are negative integers.

Zero

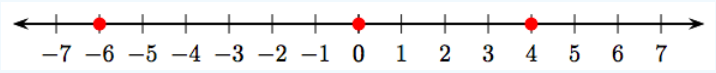
The integer zero is neither positive nor negative.

Example 5

Classify each of the following numbers as negative, positive, or neither: 4, −6, and 0.

Solution

Locate 4, −6, and 0 on the number line.



Thus:

- 4 lies to the right of zero. That is, $4 > 0$, making 4 a positive integer.
- −6 lies to the left of zero. That is, $-6 < 0$, making −6 a negative integer.
- The number 0 is neutral. It is neither negative nor positive.

Exercise

Classify −11 as positive, negative, or neither.

Answer

Negative

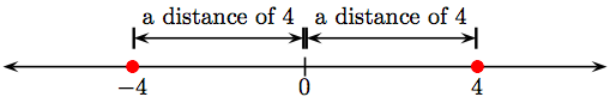
Absolute Value

We define the absolute value of an integer.

Absolute Value

The absolute value of an integer is defined as its distance from the origin (zero).

It is important to note that distance is always a nonnegative quantity (not negative); i.e., distance is either positive or zero. As an example, we've shaded the integers −4 and 4 on a number line.



The number line above shows two cases:

- The integer −4 is 4 units from zero. Because absolute value measures the distance from zero, $|-4| = 4$.
- The integer 4 is also 4 units from zero. Again, absolute value measures the distance from zero, so $|4| = 4$.

Let's look at another example.

Example 6

Determine the value of each expression: a) $|-7|$, b) $|3|$, and c) $|0|$.

Solution

The absolute value of any integer is equal to the distance that number is from the origin (zero) on the number line. Thus:

a) The integer -7 is 7 units from the origin; hence, $|-7| = 7$.

b) The integer 3 is 3 units from the origin; hence, $|3| = 3$.

c) The integer 0 is 0 units from the origin; hence, $|0| = 0$.

Exercise

Simplify: $|-33|$.

Answer

33

Example 7

Determine the value of each expression: a) $-(-8)$ and b) $-|-8|$

Solution

These are distinctly different problems.

a) The opposite of -8 is 8. That is, $-(-8) = 8$.

b) However, in this case, we take the absolute value of -8 first, which is 8, then the opposite of that result to get -8. That is,

$$\begin{aligned} -|-8| &= -(8) \quad \text{First: } |-8| = 8. \\ &= -8 \quad \text{Second: The opposite of } 8 \text{ is } -8. \end{aligned}$$

Exercise

Simplify: $-|-50|$.

Answer

50

Applications

There are a number of applications that benefit from the use of integers.

Example 8

Profits and losses for the first six months of the fiscal year for a small business are shown in Table 2.1. Profits and losses are measured in thousands of dollars. A positive number represents a profit, while a negative number represents a loss. Create a bar graph representing the profits and losses for this small business for each month of the first half of the fiscal year.

Month	Jan	Feb	Mar	Apr	May	Jun
Profit/Loss	10	12	7	-2	-4	5

Table 2.1: Profit and loss are measured in thousands of dollars.

Solution

Start by labeling the horizontal axis with the months of the fiscal year. Once you've completed that task, scale the vertical axis to accommodate the Profit/Loss values recorded in Table 2.1. Finally, starting at the 0 level on the horizontal axis, sketch bars having heights equal to the profit and loss for each month.

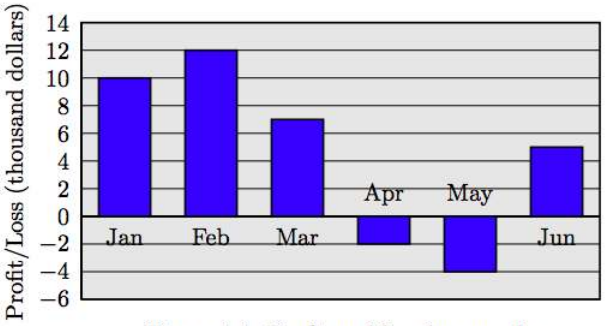


Figure 2.2: Profit and loss bar graph.

Figure 2.2 for the months January, February, March, and June have heights greater than zero, representing a profit in each of those months. The bars for the months April and May have heights that are less than zero, representing a loss for each of those months.

Exercise

The following table contains record low temperatures (degrees Fahrenheit) for Jackson Hole, Wyoming for the indicated months.

Month	Temp
Sept.	14
Oct.	2
Nov.	−27
Dec.	−49
Jan.	−50

Create a bar graphs of temperature versus months.

Example 9

Table 2.2 contains the low temperature recordings (degrees Fahrenheit) on five consecutive days in Fairbanks, Alaska, 1995. Create a line graph for the data in Table 2.2.

Date	Jan 21	Jan 22	Jan 23	Jan 24	Jan 25
Low Temp	1	10	5	−20	−28

Solution

Start by labeling the horizontal axis with the days in January that the temperatures occurred. Scale the vertical axis to accommodate the temperatures given in Table 2.2. Finally, plot points on each day at a height that equals the temperature for that given day. Connect consecutive pairs of points with line segments to produce the line graph shown in Figure 2.3.

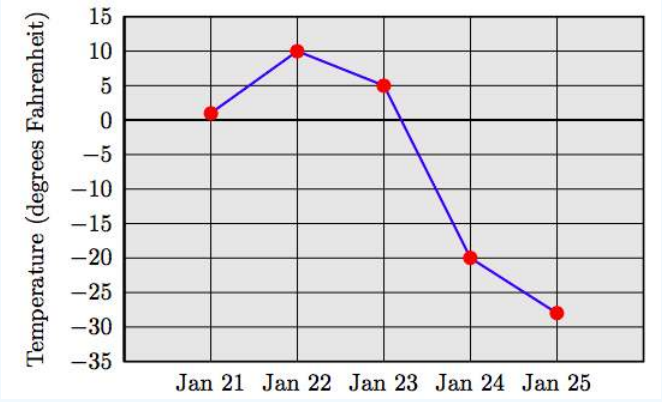


Figure 2.3: Plotting daily low temps versus the date they occurred.

Note that the points in Figure 2.3 have heights greater than zero for January 21-23, representing temperatures greater than zero. The points representing January 24-45 have negative heights corresponding to the negative temperatures of Table 2.2.

? Exercise

A man stands on the roof of a multistory building and throws a baseball vertically upward. The height (in feet) of the ball above the edge of the roof at measured times (in seconds) is given in the following table.

Time	Height
0	0
1	24
2	16
3	−24
4	−96

Create a line graph of the height of the ball versus time in the air.

Exercises

In Exercises 1-12, for each of the following questions, provide a number line sketch with your answer.

1. What number lies three units to the left of 4 on the number line?
2. What number lies three units to the left of 1 on the number line?
3. What number lies three units to the left of 6 on the number line?
4. What number lies four units to the left of −2 on the number line?
5. What number lies two units to the right of 0 on the number line?
6. What number lies four units to the right of −2 on the number line?
7. What number lies two units to the right of 1 on the number line?
8. What number lies two units to the right of −4 on the number line?
9. What number lies four units to the left of 6 on the number line?
10. What number lies two units to the left of 0 on the number line?
11. What number lies two units to the right of −5 on the number line?
12. What number lies three units to the right of −6 on the number line?

In Exercises 13-24, for each of the following sets of integers, perform the following tasks:

1. Plot each of the integers on a numberline.
2. List the numbers in order, from smallest to largest.

13. 6, 1, −3, and −5
14. 5, −3, −5, and 2
15. 5, −6, 0, and 2
16. 4, 2, 6, and −4
17. −3, −5, 3, and 5
18. −4, 5, 2, and −6
19. −5, 4, 2, and −3
20. 6, 1, −3, and −1
21. 3, 5, −5, and −1
22. −4, 6, −2, and 3
23. −2, −4, 3, and −6
24. 2, −6, −4, and 5

In Exercises 25-36, in each of the following exercises, enter the inequality symbol < or the symbol > in the box in order that the resulting inequality is a true statement.

25. $-4\square 0$
26. $-4\square 3$
27. $-2\square -1$
28. $3\square 0$
29. $-3\square -1$
30. $6\square 5$
31. $3\square 6$
32. $-4\square -2$
33. $-3\square -6$
34. $0\square -3$
35. $-1\square 4$
36. $1\square -4$

In Exercises 37-52, simplify each of the following expressions.

37. $-(-4)$.
38. $-(-6)$.
39. $|7|$.
40. $|1|$.
41. $|5|$.
42. $|3|$.
43. $-|-11|$.
44. $-|-1|$.

45. $|-5|$.

46. $|-1|$.

47. $-|-20|$.

48. $-|-8|$.

49. $|-4|$.

50. $|-3|$.

51. $-(-2)$.

52. $-(-17)$.

In Exercises 53-64, for each of the following exercises, provide a number line sketch with your answer.

53. Find two integers on the number line that are 2 units away from the integer 2.

54. Find two integers on the number line that are 2 units away from the integer -3.

55. Find two integers on the number line that are 4 units away from the integer -3.

56. Find two integers on the number line that are 2 units away from the integer -2.

57. Find two integers on the number line that are 3 units away from the integer -2.

58. Find two integers on the number line that are 4 units away from the integer 1.

59. Find two integers on the number line that are 2 units away from the integer 3.

60. Find two integers on the number line that are 3 units away from the integer 3.

61. Find two integers on the number line that are 3 units away from the integer 0.

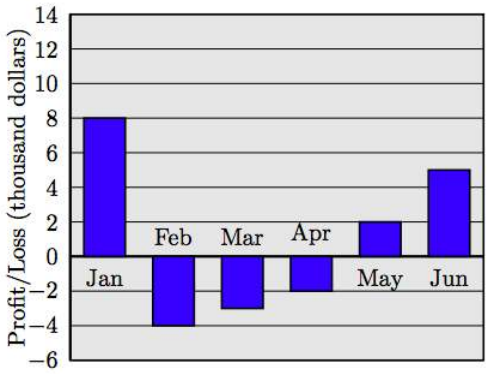
62. Find two integers on the number line that are 4 units away from the integer 2.

63. Find two integers on the number line that are 2 units away from the integer 0.

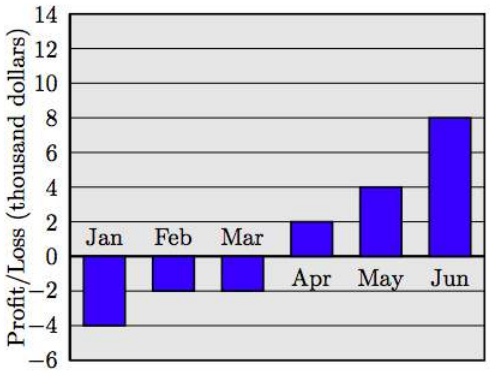
64. Find two integers on the number line that are 3 units away from the integer 1.

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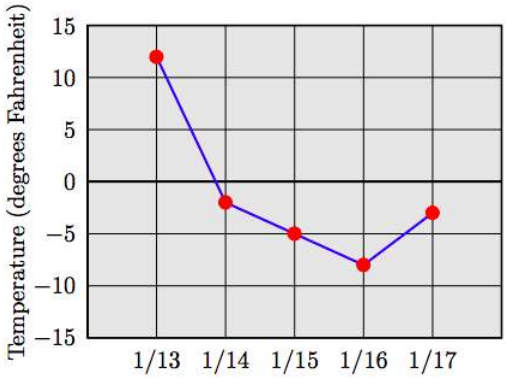
65. Dam. Utah's lowest point of elevation is 2,350 feet above sea level and occurs at Beaver Dam Wash. Express the height as an integer. 66. Underwater Glider. The National Oceanic and Atmospheric Administration's underwater glider samples the bottom of the Atlantic Ocean at 660 feet below sea level. Express that depth as an integer. Associated Press Times-Standard 4/19/09 67. Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart.



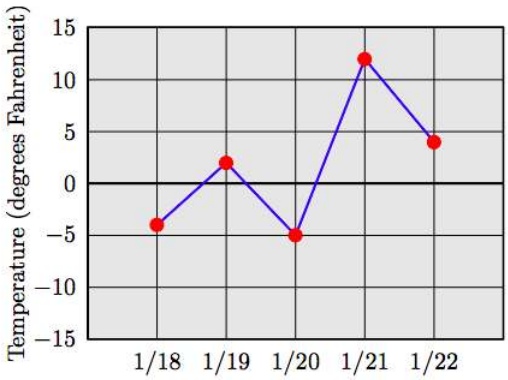
Create a table showing the profit and loss for each month. Use positive integers for the profit and negative integers for the loss. Create a line graph using the data in your table. 68. Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart.



Create a table showing the profit and loss for each month. Use positive integers for the profit and negative integers for the loss. Create a line graph using the data in your table. 69. The following line graph displays the low temperature recordings (degrees Fahrenheit) on five consecutive days in Big Bear, California. What was the lowest temperature reading for the week and on what date did it occur?



70. The following line graph displays the low temperature recordings (degrees Fahrenheit) on five consecutive days in Ogden, Utah. What was the lowest temperature reading for the week and on what date did it occur?



71. The following table contains the low temperature recordings (degrees Fahrenheit) on five consecutive days in Littletown, Ohio. Create a line graph for the data.

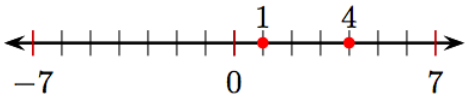
Date	Feb 11	Feb 12	Feb 13	Feb 14	Feb 15
Low Temp	10	-2	-5	-12	8

72. The following table contains the low temperature recordings (degrees Fahrenheit) on five consecutive days in MyTown, Ottawa. Create a line graph for the data.

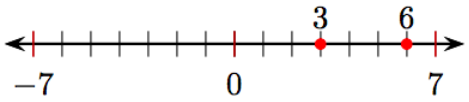
Date	Apr 20	Apr 21	Apr 22	Apr 23	Apr 24
Low Temp	-10	-2	8	5	-5

Answers

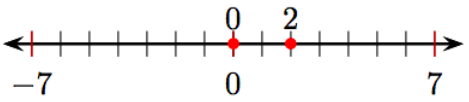
1.



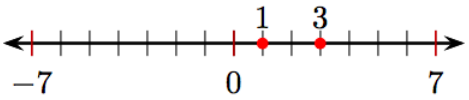
3.



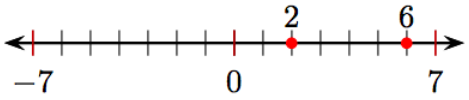
5.



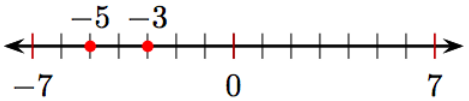
7.



9.

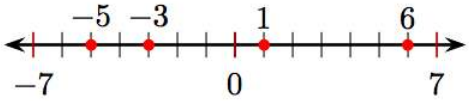


11.



13.

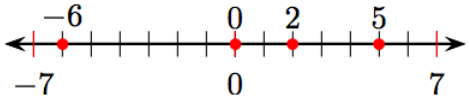
i) Arrange the integers 6, 1, −3, and −5 on a number line.



ii) −5, −3, 1, 6

15.

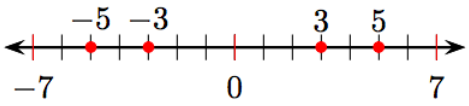
i) Arrange the integers 5, −6, 0, and 2 on a number line.



ii) −6, 0, 2, 5

17.

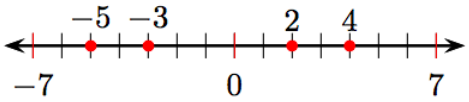
i) Arrange the integers −3, −5, 3, and 5 on a number line.



ii) −5, −3, 3, 5

19.

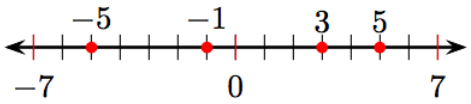
i) Arrange the integers −5, 4, 2, and −3 on a number line.



ii) −5, −3, 2, 4

21.

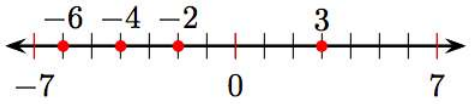
i) Arrange the integers 3, 5, −5 and −1 on a number line.



ii) −5, −1, 3, 5

23.

i) Arrange the integers −2, −4, 3, and −6 on a number line.



ii) −6, −4, −2, 3

25. −4 <0

27. −2 < −1

29. −3 < −1

31. 3 < 6

33. −3 > −6

35. −1 < 4

37. 4

39. 7

41. 5

43. −11

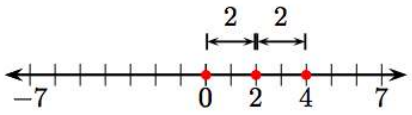
45. 5

47. −20

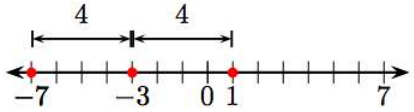
49. 4

51. 2

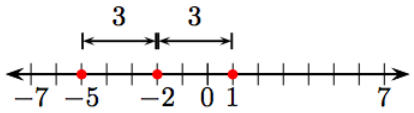
53. 0 and 4.



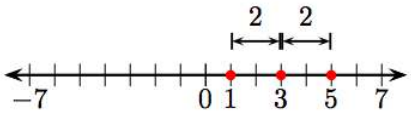
55. −7 and 1.



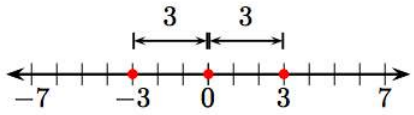
57. −5 and 1.



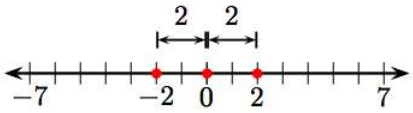
59. 1 and 5.



61. −3 and 3.



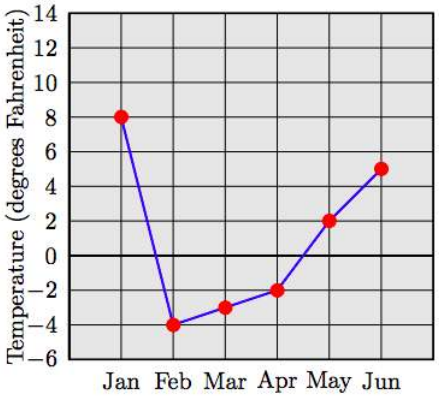
63. −2 and 2.



65. 2,350 feet

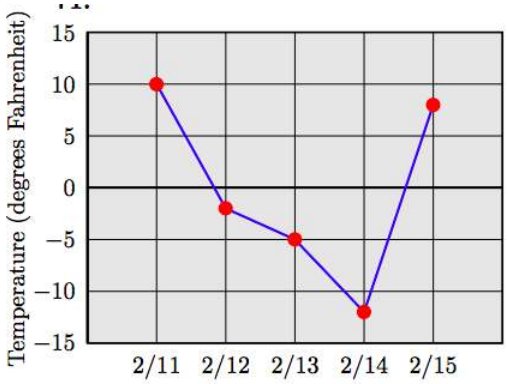
67.

Month	Profit/Loss
Jan	8
Feb	−4
Mar	−3
Apr	−2
May	2
Jun	5



69. Approximately -8° Fahrenheit on January 16.

71.



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2.1.2: Adding Integers

Like our work with the whole numbers, addition of integers is best explained through the use of number line diagrams. However, before we start, let's take a moment to discuss the concept of a vector.

Vectors

A *vector* is a mathematical object that possesses two import qualities: (1) magnitude or length, and (2) directon.

Vectors are a fundamental problem solving tool in mathematics, science, and engineering. In physics, vectors are used to represent forces, position, velocity, and acceleration, while engineers use vectors to represent both internal and external forces on structures, such as bridges and buildings. In this course, and in this particular section, we will concentrate on the use of vectors to help explain addition of integers.

Vectors on the Number Line

Consider the number line in Figure 2.1.2.1.

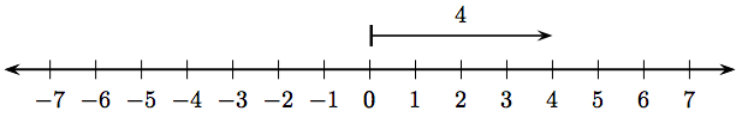


Figure 2.1.2.1: A vector representing positive four.

Above the line we've drawn a vector with tail starting at the integer 0 and arrowhead finishing at the integer 4. There are two important things to note about this vector:

1. The magnitude (length) of the vector in Figure 2.1.2.1is four.
2. The vector in Figure 2.1.2.1points to the right.

We will agree that the vector in Figure 2.1.2.1 represents positive four. It is not important that the vector start at the origin. Consider, for example, the vector pictured in Figure 2.1.2.2

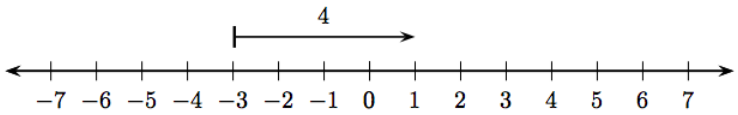


Figure 2.1.2.2: A vector representing positive four.

Again, there are two important observations to be made:

1. The magnitude (length) of the vector in Figure 2.1.2.2is four.
2. The vector in Figure 2.1.2.2points to the right.

Hopefully, you have the idea. Any vector that has length 4 and points to the right will represent positive four, regardless of its starting or finishing point. Conversely, consider the vector in Figure 2.1.2.3 which starts at the integer 4 and finishes at the integer −3.

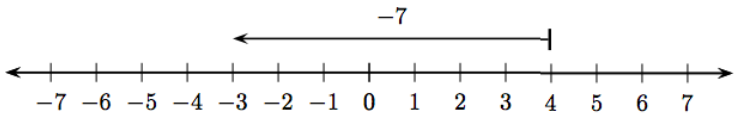


Figure 2.1.2.3: A vector representing negative seven.

Two observations:

1. The magnitude (length) of the vector in Figure 2.1.2.3is seven.
2. The vector in Figure 2.1.2.3points to the left.

We will agree that the vector in Figure 2.1.2.3represents negative seven. We could select different starting and finishing points for our vector, but as long as the vector has length seven and points to the left, it represents the integer −7.

Important Observation

A vector pointing to the right represents a **positive** number. A vector pointing to the left represents a **negative** number.

Magnitude and Absolute Value

In Figure 2.1.2.1and Figure 2.1.2.2 the vectors pictured represent the integer positive four. Note that the absolute value of four is four; that is, $|4| = 4$. Note also that this absolute value is the magnitude or length of the vectors representing the integer positive four in Figure 2.1.2.1and Figure 2.1.2.2

In Figure 2.1.2.3 the vector pictured represents the integer -7. Note that $|-7| = 7$. This shows that the absolute value represents the magnitude or length of the vector representing -7.

Magnitude and Absolute Value

If a is an integer, then $|a|$ gives the magnitude or length of the vector that represents the integer a.

Adding Integers with Like Signs

Because the positive integers are also whole numbers, we've already seen how to add to them in Section 1.2.

Example 1

Find the sum $3 + 4$.

Solution

To add the positive integers 3 and 4, proceed as follows.

1. Start at the integer 0, then draw a vector 3 units in length pointing to the right, as shown in Figure 2.1.2.4 This arrow has magnitude (length) three and represents the positive integer 3.
2. Draw a second vector of length four that points to the right, starting at the end of the first vector representing the positive integer 3. This arrow has magnitude (length) four and represents the positive integer 4.
3. The sum of the positive integers 3 and 4 could be represented by a vector that starts at the integer 0 and ends at the positive integer 7. However, we prefer to mark this sum on the number line as a solid dot at the positive integer 7. This integer represents the sum of the positive integers 3 and 4.

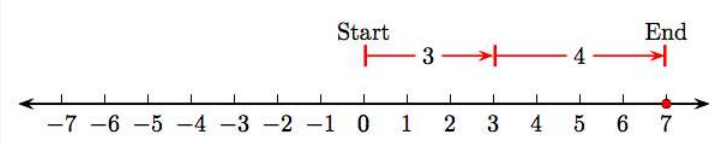


Figure 2.1.2.4: Adding two positive integers on the number line.

Thus, $3 + 4 = 7$

Exercise

Use a number line diagram to show the sum $5 + 7$.

Answer

12

Negative integers are added in a similar fashion.

Example 2

Find the sum −3+(−4).

Solution

To add the negative integers −3 and −4, proceed as follows.

1. Start at the integer 0, then draw a vector 3 units in length pointing to the left, as shown in Figure 2.1.2.5 This arrow has magnitude (length) three and represents the negative integer −3.
2. Draw a second vector of length four that points to the left, starting at the end of the first vector representing the negative integer −3. This arrow has magnitude (length) four and represents the negative integer −4.
3. The sum of the negative integers −3 and −4 could be represented by a vector that starts at the integer 0 and ends at the negative integer −7. However, we prefer to mark this sum on the number line as a solid dot at the negative integer −7. This integer represents the sum of the negative integers −3 and −4.

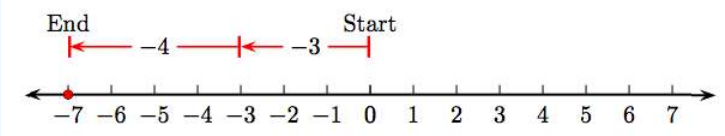


Figure 2.1.2.5: Adding two positive integers on the number line.

Thus, −3+(−4) = −7.

Drawing on Physical Intuition. Imagine that you are "walking the number line" in Figure 2.1.2.5 You start at the origin (zero) and take 3 paces *to the left*. Next, you walk an additional four paces *to the left*, landing at the number −7

Exercise

Use a number line diagram to show the sum −7+(−3).

Answer

−10

It should come as no surprise that the procedure used to add two negative integers comprises two steps.

Adding Two Negative Integers

To add two negative integers, proceed as follows:

1. Add the magnitudes of the integers.
2. Prefix the common negative sign

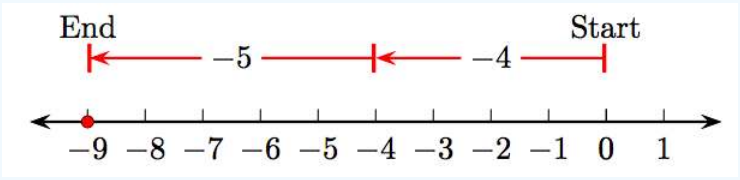
Example 3

Find the sums: (a) −4+(−5), (b) −12 + (−9), and (c) −2+(−16).

Solution

We'll examine three separate but equivalent approaches, as discussed in the narrative above.

a) The number line schematic



shows that (−4) + (−5) = −9.

b) Drawing on physical intuition, start at zero, walk 12 units to the left, then an additional 9 units to the left. You should find yourself 21 units to the left of the origin (zero). Hence, −12 + (−9) = −21.

c) Following the algorithm above in "Adding Two Negative Integers," first add the magnitudes of −2 and −16; that is, 2+16 = 18. Now prefix the common sign. Hence, −2+(−16) = −18.

? Exercise

Find the sum: −5+(−9).

Answer

−14

Adding Integers with Unlike Signs

Adding integers with unlike signs is no harder than adding integers with like signs.

✔ Example 4

Find the sum −8 + 4.

Solution

To find the sum −8 + 4, proceed as follows:

1. Start at the integer 0, then draw a vector eight units in length pointing to the left, as shown in Figure 2.1.2.6 This arrow has magnitude (length) eight and represents the negative integer −8.
2. Draw a second vector of length four that points to the right, starting at the end of the first vector representing the negative integer −8. This arrow (also shown in Figure 2.1.2.6) has magnitude (length) four and represents the positive integer 4.
3. The sum of the negative integers −8 and 4 could be represented by a vector that starts at the integer 0 and ends at the negative integer −4. However, we prefer to mark this sum on the number line as a solid dot at the negative integer −4. This integer represents the sum of the integers −8 and 4.

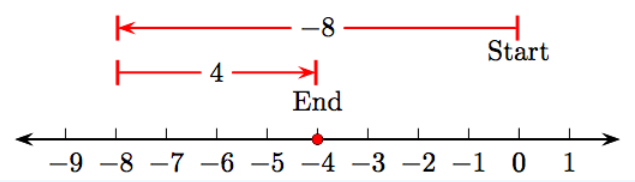


Figure 2.1.2.6: Adding −8 and 4 on the number line.

Thus, −8+4= −4.

Drawing on Physical Intuition. Imagine you are "walking the number line in Figure 2.1.2.6 You start at the origin (zero) and walk eight paces *to the left*. Next, turn around and walk four paces *to the right*, landing on the number −4.

? Exercise

Use a number line diagram to show the sum −9 + 2.

Answer

−7

Note that adding integers with unlike signs is a *subtractive process*. This is due to the reversal of direction experienced in drawing Figure 2.1.2.6in Example 4.

Adding Two Integers with Unlike Signs

To add two integers with unlike signs, proceed as follows:

1. Subtract the smaller magnitude from the larger magnitude.
2. Prefix the sign of the number with the larger magnitude.

For example, to find the sum −8 + 4 of Example 4, we would note that the integers −8 and 4 have magnitudes 8 and 4, respectively. We would then apply the process outlined in “Adding Two Integers with Unlike Signs.”

1. Subtract the smaller magnitude from the larger magnitude; that is, 8 − 4 = 4.
2. Prefix the sign of the number with the larger magnitude. Because −8 has the larger magnitude and its sign is negative, we prefix a negative sign to the difference of the magnitudes. Thus, −8+4= −4.

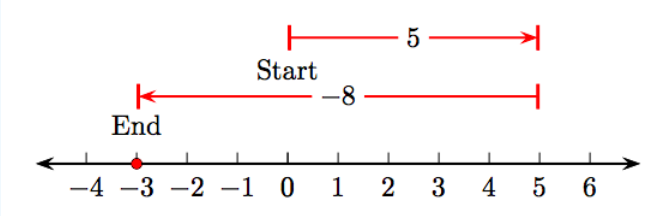
Example 5

Find the sums: (a) 5+ (−8), (b) −12+ 16, and (c) −117+115.

Solution

We’ll examine three separate but equivalent approaches, as discussed in the narrative above.

a) The number line schematic



shows that 5 + (−8) = −3.

b) Drawing on physical intuition, start at zero, walk 12 units to the left, then turn around and walk 16 units to the right. You should find yourself 4 units to the right of the origin (zero). Hence, −12 + 16 = 4.

c) Following the algorithm in “Adding Two Integers with Unlike Signs,” subtract the smaller magnitude from the larger magnitude, thus 117−115 = 2. Because −117 has the larger magnitude and its sign is negative, we prefix a negative sign to the difference of the magnitudes. Thus, −117 + 115 = −2.

Exercise

Use a number line diagram to show the sum 5 + (−11).

Answer

−6

Properties of Addition of Integers

You will be pleased to learn that the properties of addition for whole numbers also apply to addition of integers.

The Commutative Property of Addition

Let a and b represent two integers. Then,

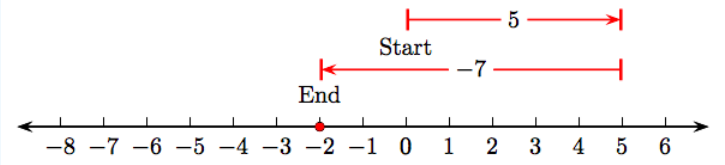
$$a+b=b+a.$$

Example 6

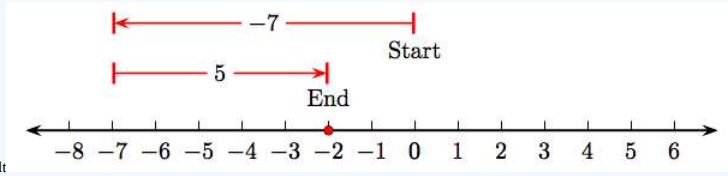
Show that 5 + (−7) = −7 + 5.

Solution

The number line schematic



shows that 5 + (−7) = −2. On the other hand, the number line schematic



shows that −7+5= −2. Therefore, 5 + (−7) = −7 + 5.

Exercise

Use a number line diagram to show that −8 + 6 is the same as 6 + (−8).

Addition of integers is also associative.

The Associative Property of Addition

Let a, b, and c represent integers. Then,

$$(a+b)+c=a+(b+c)$$

.

Example 7

Show that (−9 + 6) + 2 = −9 + (6 + 2).

Solution

On the left, the grouping symbols demand that we add −9 and 6 first. Thus,

$$\begin{aligned}(-9+6)+2 &= -3+2\\ &= -1.\end{aligned}$$

On the right, the grouping symbols demand that we add 6 and 2 first. Thus,

$$\begin{aligned}-9+(6+2) &= -9+8\\ &= -1.\end{aligned}$$

Both sides simplify to −1. Therefore, (−9 + 6) + 2 = −9 + (6 + 2).

Exercise

Show that the expression (−8 + 5) + 3 is the same as −8 + (5 + 3) by simplifying each of the two expressions independently.

The Additive Identity Property

The integer zero is called the *additive identity*. If a is any integer, then

$$a + 0 = a \text{ and } 0 + a = a.$$

Thus, for example, $-8+0 = -8$ and $0 + (-113) = -113$.

Finally, every integer has a unique opposite, called its *additive inverse*.

The Additive Inverse Property

Let a represent any integer. Then there is a unique integer $-a$, called the **opposite** or **additive inverse** of a, such that

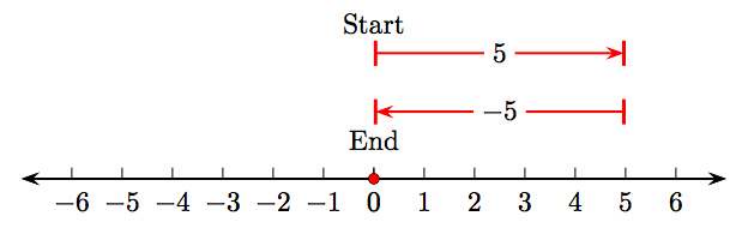
$$a + (-a) = 0 \text{ and } -a + a = 0.$$

Example 8

Show that $5 + (-5) = 0$.

Solution

The number line schematic



clearly shows that $5 + (-5) = 0$.

Exercise

Use a number line diagram to show that $9 + (-9) = 0$.

Important Observation

We have used several equivalent phrases to pronounce the integer $-a$. We've used "the opposite of a," "negative a," and "the additive inverse of a." All are equivalent pronunciations.

Grouping for Efficiency

Order of operations require that we perform all additions as they occur, working from left to right.

Example 9

Simplify $-7+8+(-9) + 12$.

Solution

We perform the additions as they occur, working left to right.

$$\begin{aligned} &= -7+8+(-9)+12 = 1+(-9)+12 && \text{Working left to right, } -7+8=1. \\ &= -8+12 && \text{Working left to right, } 1+(-9=-8. \\ &= 4 && -8+12=4 \end{aligned}$$

Thus, $-7+8+(-9) + 12 = 4$.

Exercise

Simplify: −8+9+(−4) + 2.

Answer

−1

The commutative property of addition tells us that changing the order of addition does not change the answer. The associative property of addition tells us that a sum is not affected by regrouping. Let's work Example 9 again, first grouping positive and negative numbers together.

Example 10

Simplify −7+8+(−9) + 12.

Solution

The commutative and associative properties allows us to change the order of addition and regroup.

$$\begin{aligned} -7+8+(-9)+12 &= -7+(-9)+8+12 && \text{Use the commutative property to change the order.} \\ &= [-7+(-9)]+[8+12] && \text{Use the associative property to regroup.} \\ &= -16+20 && \text{Add the negatives. Add the positives.} \\ &= 4 && \text{One final addition.} \end{aligned}$$

Thus, −7+8+(−9) + 12 = 4.

Exercise

Simplify: −11 + 7 + (−12) + 3.

Answer

−13

At first glance, there seems to be no advantage in using the technique in Example 10 over the technique used in Example 9. However, the technique in Example 10 is much quicker in practice, particularly if you eliminate some of the explanatory steps.

Efficient Grouping

When asked to find the sum of a number of integers, it is most efficient to first add all the positive integers, then add the negatives, then add the results.

Example 11

Simplify −7+8+(−9) + 12.

Solution

Add the positive integers first, then the negatives, then add the results.

$$\begin{aligned} -7+8+(-9)+12 &= 20+(-16) && \text{Add the positives: } 8+12=20. \\ & && \text{Add the negatives: } -7+(-9)=-16. \\ &= 4 && \text{Add the results: } 20+(-16)=4. \end{aligned}$$

Thus, −7+8+(−9) + 12 = 4.

Exercise

Simplify: −11 + 3 + (−2) + 7.

Answer

−7

Using Correct Notation

Never write +−! That is, the notation

$$9 + -4 \text{ and } -8 + -6$$

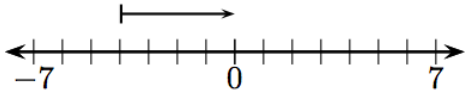
should not be used. Instead, use grouping symbols as follows:

$$9+(-4) \text{ and } -8+(-6)$$

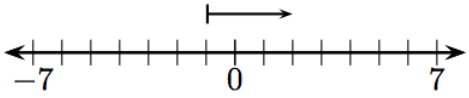
Exercises

In Exercises 1-12, what integer is represented by the given vector?

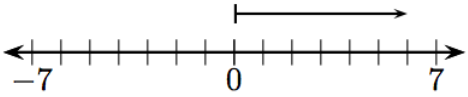
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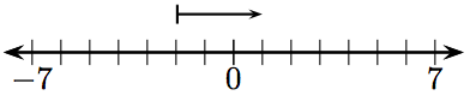
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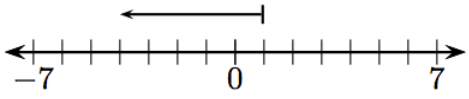
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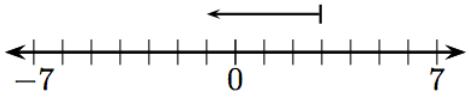
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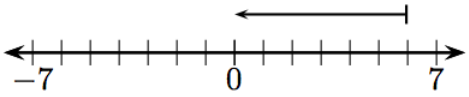
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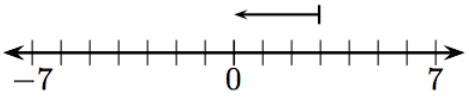
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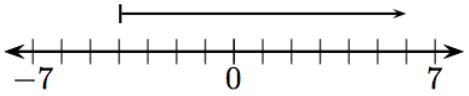
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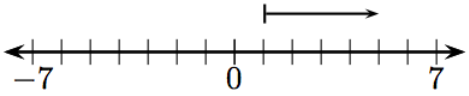
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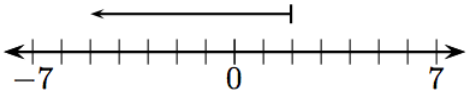
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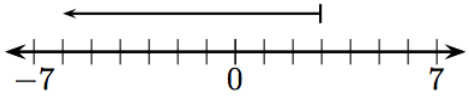
10.



11.



12.



In Exercises 13-36, find the sum of the given integers.

13. −15 + 1
14. −1 + 18
15. 18 + (−10)
16. 2+(−19)
17. −10 + (−12)
18. −1+(−7)
19. 5 + 10
20. 1 + 12

21. 2+5

22. 14 + 1

23. 19 + (−15)

24. 20 + (−17)

25. −2+(−7)

26. −14 + (−6)

27. −6 + 16

28. −2 + 14

29. −11 + (−6)

30. −7+(−8)

31. 14 + (−9)

32. 5+(−15)

33. 10 + 11

34. 14 + 11

35. −13 + 1

36. −8+2

In Exercises 37-52, state the property of addition depicted by the given identity.

37. −1 + (3 + (−8)) = (−1 + 3) + (−8)

38. −4 + (6 + (−5)) = (−4 + 6) + (−5)

39. 7+(−7) = 0

40. 14 + (−14) = 0

41. 15 + (−18) = −18 + 15

42. 14 + (−8) = −8 + 14

43. −15 + 0 = −15

44. −11 + 0 = −11

45. −7 + (1 + (−6)) = (−7 + 1) + (−6)

46. −4 + (8 + (−1)) = (−4 + 8) + (−1)

47. 17 + (−2) = −2 + 17

48. 5+(−13) = −13 + 5

49. −4+0= −4

50. −7+0= −7

51. 19 + (−19) = 0

52. 5+(−5) = 0

In Exercises 53-64, state the additive inverse of the given integer.

53. 18

54. 10

55. 12

56. 15

57. −16

58. −4

59. 11

60. 13

61. −15

62. −19

63. −18

64. −9

In Exercises 65-80, find the sum of the given integers.

65. 6+(−1) + 3 + (−4)

66. 6+(−3) + 2 + (−7)

67. 15 + (−1) + 2

68. 11 + (−16) + 16

69. −17 + 12 + 3

70. −5+(−3) + 2

71. 7 + 20 + 19

72. 14 + (−14) + (−20)

73. 4+(−8) + 2 + (−5)

74. 6+(−3) + 7 + (−2)

75. 7+(−8) + 2 + (−1)

76. 8+(−9) + 5 + (−3)

77. 9+(−3) + 4 + (−1)

78. 1+(−9) + 7 + (−6)

79. 9 + 10 + 2

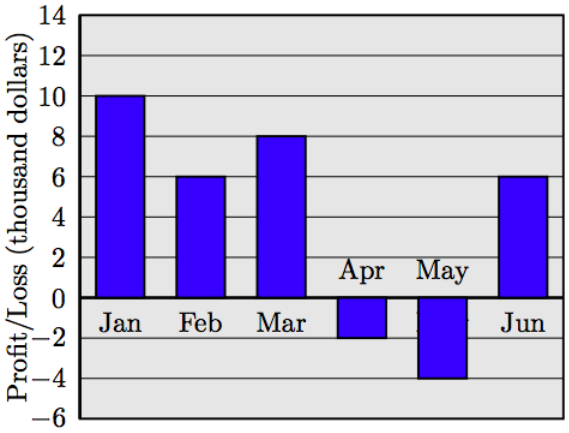
80. −6 + 15 + (−18)

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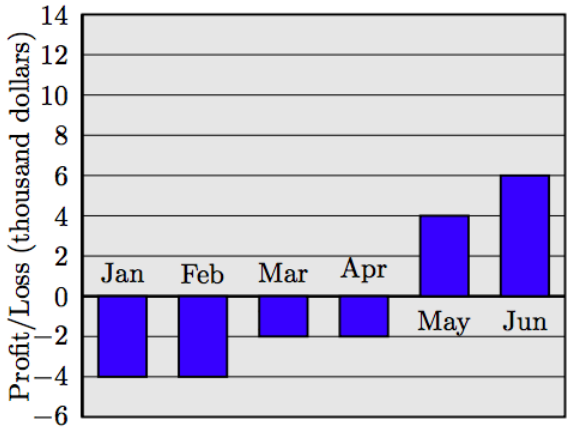
81. **Bank Account.** Gerry opened a new bank account, depositing a check for $215. He then made several withdraws of $40, $75, and $20 before depositing another check for $185. How much is in Gerry's account now?

82. **Dead Sea Sinking.** Due to tectonic plate movement, the Dead Sea is sinking about 1 meter each year. If it's currently −418 meters now, what will Dead Sea elevation be in 5 years? Write an expression that models this situation and compute the result.

83. **Profit and Loss.** Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart. Sum the profits and losses from each month. Was there a net profit or loss over the six-month period? How much?



84. **Profit and Loss.** Profits and losses for the first six months of the fiscal year for a small business are shown in the following bar chart. Sum the profits and losses from each month. Was there a net profit or loss over the six-month period? How much?



Answers

1. 4
3. 6
5. −5
7. −6
9. 10
11. −7
13. −14
15. 8
17. −22
19. 15
21. 7
23. 4
25. −9
27. 10

29. −17

31. 5

33. 21

35. −12

37. Associative property of addition

39. Additive inverse property

41. Commutative property of addition

43. Additive identity property

45. Associative property of addition

47. Commutative property of addition

49. Additive identity property

51. Additive inverse property

53. −18

55. −12

57. 16

59. −11

61. 15

63. 18

65. 4

67. 16

69. −2

71. 46

73. −7

75. 0

77. 9

79. 21

81. $265

83. Net Profit: $24,000

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2.1.3: Subtracting Integers

In Section 1.2, we stated that "Subtraction is the opposite of addition." Thus, to subtract 4 from 7, we walked seven units to the right on the number line, but then walked 4 units in the opposite direction (to the left), as shown in Figure 2.1.3.1.

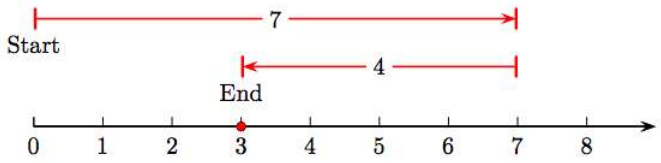


Figure 2.1.3.1: Subtraction requires that we reverse direction.

Thus, 7 − 4 = 3. The key phrase is "add the opposite." Thus, the subtraction 7 − 4 becomes the addition 7 + (−4), which we would picture on the number line as shown in Figure 2.1.3.2

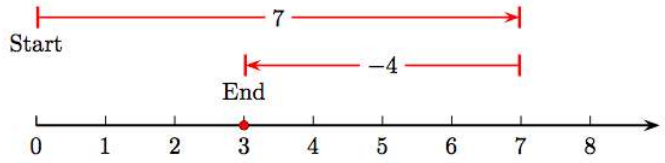


Figure 2.1.3.2: Subtraction means add the opposite.

Figure 2.1.3.1 and Figure 2.1.3.2 provide ample evidence that the subtraction 7−4 is identical to the addition 7+(−4). Again, subtraction means "add the opposite." That is, 7 − 4=7+(−4).

Defining Subtraction

Subtraction means "add the opposite." That is, if a and b are any integers, then

$$a - b = a + (-b).$$

Thus, for example, −123−150 = −123+(−150) and −57−(−91) = −57+91. In each case, subtraction means "add the opposite." In the first case, subtracting 150 is the same as adding −150. In the second case, subtracting−91 is the same as adding 91.

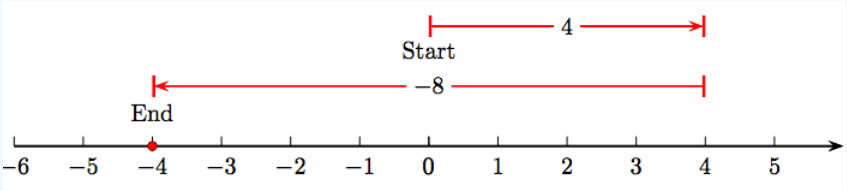
Example 1

Find the differences: (a) 4 − 8, (b) −15 − 13, and (c) −117 − (−115).

Solution

In each case, subtraction means "add the opposite."

a) Change the subtraction to addition with the phrase "subtraction means add the opposite." That is, 4−8 = 4+(−8). We can now perform this addition on the number line.



Thus, 4 − 8=4+(−8) = −4.

b) First change the subtraction into addition by "adding the opposite." That is, −15 − 13 = −15 + (−13). We can now use physical intuition to perform the addition. Start at the origin (zero), walk 15 units to the left, then an additional 13 units to the left, arriving at the answer −28. That is,

$$\begin{aligned} -15 - 13 &= -15 + (-13) \\ &= -28. \end{aligned}$$

c) First change the subtraction into addition by "adding the opposite." That is, −117 − (−115) = −117 + 115. Using "Adding Two Integers with Unlike Signs" from Section 2.2, first subtract the smaller magnitude from the larger magnitude; that is, 117 − 115 = 2. Because −117 has the larger magnitude and its sign is negative, prefix a negative sign to the difference in magnitudes. Thus,

$$\begin{aligned} -117-(-115) &= -117+115 \\ &= -2. \end{aligned}$$

? Exercise

Use each of the techniques in parts (a), (b), and (c) of Example 1 to evaluate the difference −11 − (−9).

Answer

−2

Order of Operations

We will now apply the "Rules Guiding Order of Operations" from Section 1.5 to a number of example exercises.

✔ Example 2

Simplify −5 − (−8) − 7.

Solution

We work from left to right, changing each subtraction by "adding the opposite."

$$\begin{aligned} -5-(-8)-7 &= -5+8+(-7) && \text{Add the opposite of } -8, \text{ which is } 8. \\ & && \text{Add the opposite of } 7, \text{ which is } -7. \\ &= 3+(-7) && \text{Working left to right, } -5+8=3. \\ &= -4 && 3+(-7)=-4. \end{aligned}$$

? Exercise

Simplify: −3 − (−9) − 11.

Answer

−5

Grouping symbols say "do me first."

✔ Example 2.1.3.1

Simplify −2 − (−2 − 4).

Solution

Parenthetical expressions must be evaluated first.

$$\begin{aligned} -2(-2-4) &= -2-(-2+(-4)) && \text{Simplify the parenthetical expression} \\ & && \text{first. Add the opposite of } 4, \text{ which is } -4. \\ &= -2-(-6) && \text{Inside the parentheses, } -2+(-4)=-6. \\ &= -2+6 && \text{Subtracting a } -6 \text{ is the same as adding a } 6. \\ &= 4 && \text{Add: } -2+6=4. \end{aligned}$$

Exercise

Simplify: $-3-(-3-3)$.

Answer

3

Change as a Difference

Suppose that when I leave my house in the early morning, the temperature outside is 40° Fahrenheit. Later in the day, the temperature measures 60° Fahrenheit. How do I measure the change in the temperature?

The Change in a Quantity

To measure the change in a quantity, always subtract the *former* measurement from the *latter* measurement. That is:

$$\text{Change in a Quantity} = \text{Latter Measurement} - \text{Former Measurement}$$

Thus, to measure the change in temperature, I perform a subtraction as follows:

$$\begin{aligned} \text{Change in Temperature} &= \text{Latter Measurement} && - \text{Former Measurement} \\ &= 60^\circ\text{F} && -40^\circ\text{F} \\ &= 20^\circ\text{F} \end{aligned}$$

Note that the positive answer is in accord with the fact that the temperature has increased.

Example 4

Suppose that in the afternoon, the temperature measures 65° Fahrenheit, then late evening the temperature drops to 44° Fahrenheit. Find the change in temperature.

Solution

To measure the change in temperature, we must subtract the former measurement from the latter measurement.

$$\begin{aligned} \text{Change in Temperature} &= \text{Latter Measurement} && - \text{Former Measurement} \\ &= 44^\circ\text{F} && -65^\circ\text{F} \\ &= -11^\circ\text{F} \end{aligned}$$

Note that the negative answer is in accord with the fact that the temperature has decreased. There has been a "change" of -11° Fahrenheit.

Exercise

Marianne awakes to a morning temperature of 54° Fahrenheit. A storm hits, dropping the temperature to 43° Fahrenheit. Find the change in temperature.

Answer

-11° Fahrenheit

Example 5

Sometimes a bar graph is not the most appropriate visualization for your data. For example, consider the bar graph in Figure 2.1.3.3 depicting the Dow Industrial Average for seven consecutive days in March of 2009. Because the bars are of almost equal height, it is difficult to detect fluctuation or change in the Dow Industrial Average.

Let's determine the change in the Dow Industrial average on a day-to-day basis. Remember to subtract the latter measurement minus the former (current day minus former day). This gives us the following changes.

Consecutive Days	Change in Dow Industrial Average
Sun-Mon	6900 - 7000 = -100
Mon-Tues	6800 - 6900 = -100
Tues-Wed	6800 - 6800 = 0
Wed-Thu	7000 - 6800 = 200
Thu-Fri	7100 - 7000 = 100
Fri-Sat	7200 - 7100 = 100

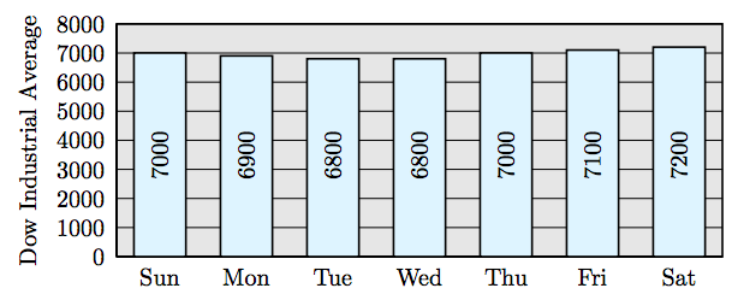


Figure 2.1.3.3: Profit and loss bar graph.

We will use the data in the table to construct a line graph. On the horizontal axis, we place the pairs of consecutive days (see Figure 2.1.3.4). On the vertical axis we place the Change in the Industrial Dow Average. At each pair of days we plot a point at a height equal to the change in Dow Industrial Average as calculated in our table.

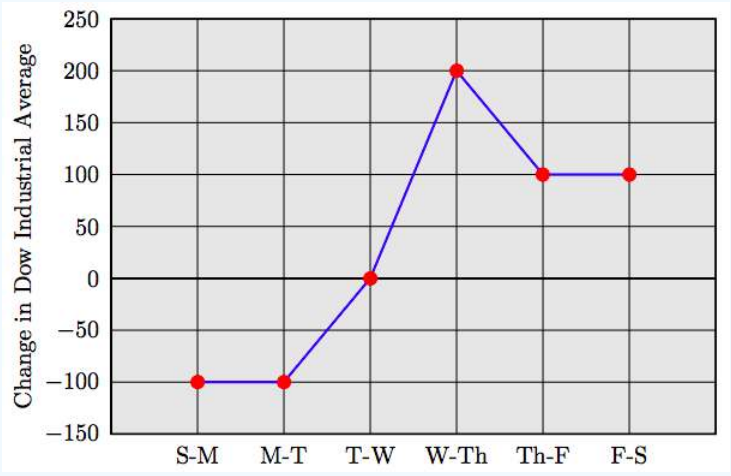


Figure 2.1.3.4: Plotting change in Dow versus consecutive days.

Note that the data as displayed by Figure 2.1.3.4 more readily shows the changes in the Dow Industrial Average on a day-to-day basis. For example, it is now easy to pick the day that saw the greatest increase in the Dow (from Wednesday to Thursday, the Dow rose 200 points).

Exercises

In Exercises 1-24, find the difference.

1. 16 − 20
2. 17 − 2

3. $10 - 12$

4. $16 - 8$

5. $14 - 11$

6. $5 - 8$

7. $7 - (-16)$

8. $20 - (-10)$

9. $-4 - (-9)$

10. $-13 - (-3)$

11. $8 - (-3)$

12. $14 - (-20)$

13. $2 - 11$

14. $16 - 2$

15. $-8 - (-10)$

16. $-14 - (-2)$

17. $13 - (-1)$

18. $12 - (-13)$

19. $-4 - (-2)$

20. $-6 - (-8)$

21. $7 - (-8)$

22. $13 - (-14)$

23. $-3 - (-10)$

24. $-13 - (-9)$

In Exercises 25-34, simplify the given expression.

25. $14 - 12 - 2$

26. $-19 - (-7) - 11$

27. $-20 - 11 - 18$

28. $7 - (-13) - (-1)$

29. $5 - (-10) - 20$

30. $-19 - 12 - (-8)$

31. $-14 - 12 - 19$

32. $-15 - 4 - (-6)$

33. $-11 - (-7) - (-6)$

34. $5 - (-5) - (-14)$

In Exercises 35-50, simplify the given expression.

35. $-2 - (-6 - (-5))$

36. $6 - (-14 - 9)$

37. $(-5 - (-8)) - (-3 - (-2))$

38. $(-6 - (-8)) - (-9 - 3)$

39. $(6 - (-9)) - (3 - (-6))$

40. $(-2 - (-3)) - (3 - (-6))$

41. $-1 - (10 - (-9))$

42. $7 - (14 - (-8))$

43. $3 - (-8 - 17)$

44. $1 - (-1 - 4)$

45. $13 - (16 - (-1))$

46. $-7 - (-3 - (-8))$

47. $(7 - (-8)) - (5 - (-2))$

48. $(6 - 5) - (7 - 3)$

49. $(6 - 4) - (-8 - 2)$

50. $(2 - (-6)) - (-9 - (-3))$

---

51. The first recorded temperature is 42°F. Four hours later, the second temperature is 65°F. What is the change in temperature?

52. The first recorded temperature is 79°F. Four hours later, the second temperature is 46°F. What is the change in temperature?

53. The first recorded temperature is 30°F. Four hours later, the second temperature is 51°F. What is the change in temperature?

54. The first recorded temperature is 109°F. Four hours later, the second temperature is 58°F. What is the change in temperature?

55. Typical temperatures in Fairbanks, Alaska in January are −2 degrees Fahrenheit in the daytime and −19 degrees Fahrenheit at night. What is the change in temperature from day to night?

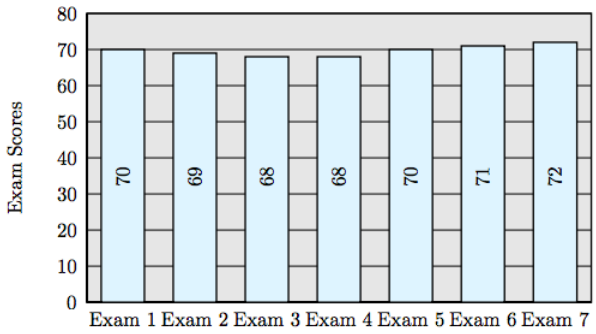
56. Typical summertime temperatures in Fairbanks, Alaska in July are 79 degrees Fahrenheit in the daytime and 53 degrees Fahrenheit at night. What is the change in temperature from day to night?

57. **Communication.** A submarine 1600 feet below sea level communicates with a pilot flying 22,500 feet in the air directly above the submarine. How far is the communique traveling?

58. **Highest to Lowest.** The highest spot on earth is on Mount Everest in Nepal-Tibet at 8,848 meters. The lowest point on the earth's crust is the Mariana's Trench in the North Pacific Ocean at 10,923 meters below sea level. What is the distance between the highest and the lowest points on earth? *Wikipedia* http://en.Wikipedia.org/wiki/Extremes_on_Earth

59. **Lowest Elevation.** The lowest point in North America is Death Valley, California at -282 feet. The lowest point on the entire earth's landmass is on the shores of the Dead Sea along the Israel-Jordan border with an elevation of -1,371 feet. How much lower is the Dead Sea shore from Death Valley?

60. **Exam Scores.** Freida's scores on her first seven mathematics exams are shown in the following bar chart. Calculate the differences between consecutive exams, then create a line graph of the differences on each pair of consecutive exams. Between which two pairs of consecutive exams did Freida show the most improvement?



Answers

1. −4

3. −2

5. 3

7. 23

9. 5 11. 11 13. −9 15. 2 17. 14 19. −2 21. 15 23. 7 25. 0 27. −49 29. −5 31. −45 33. 2 35. −1 37. 4 39. 6 41. −20 43. 28 45. −4 47. 8 49. 12 51. 23◦ F 53. 21◦ F 55. −17 degrees Fahrenheit 57. 24,100 feet 59. 1,089 feet lower

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2.1.4: Multiplication and Division of Integers

Before we begin, let it be known that the integers satisfy the same properties of multiplication as do the whole numbers.

Integer Properties of Multiplication

Commutative Property. If *a* and *b* are integers, then their product commutes. That is,

$$a \cdot b = b \cdot a.$$

Associative Property. If a, b, and c are integers, then their product is associative. That is,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

Multiplicative Identity Property. If *a* is any integer, then

$$a \cdot 1 = a \text{ and } 1 \cdot a = a.$$

Because multiplying any integer by 1 returns the identical integer, the integer 1 is called the **multiplicative identity**.

In Section 1.3, we learned that multiplication is equivalent to *repeated addition*. For example,

$$3 \cdot 4 = \underbrace{4+4+4}_{\text{three fours}}$$

On the number line, three sets of four is equivalent to walking three sets of four units to the right, starting from zero, as shown in Figure 2.1.4.1

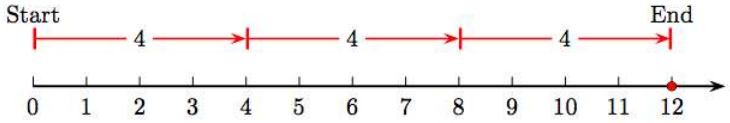


Figure 2.1.4.1: Note that 3 · 4 = 4 + 4 + 4. That is, 3 · 4 = 12.

This example and a little thought should convince readers that the product of two positive integers will always be a positive integer.

The Product of Two Positive Integers

If *a* and *b* are two positive integers, then their product *ab* is also a positive integer.

For example, 2 · 3 = 6 and 13 · 117 = 1521. In each case, the product of two positive integers is a positive integer.

The Product of a Positive Integer and a Negative Integer

If we continue with the idea that multiplication is equivalent to repeated addition, then it must be that

$$3 \cdot (-4) = \underbrace{-4+(-4)+(-4)}_{\text{three negative fours}}.$$

Pictured on the number line, 3 · (−4) would then be equivalent to walking three sets of negative four units (to the left), starting from zero, as shown in Figure 2.1.4.2

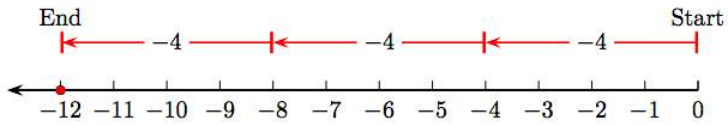


Figure 2.1.4.2: Note that 3 · (−4) = −4+(−4) + (−4). That is, 3 · (−4) = −12.

Note, at least in this particular case, that the product of a positive integer and a negative integer is a negative integer.

We've shown that 3 · (−4) = −12. However, integer multiplication is commutative, so it must also be true that −4 · 3 = −12. That is, the product of a negative integer and a positive integer is also a negative integer. Although not a proof, this argument motivates the

following fact about integer multiplication.

The Product of a Positive Integer and a Negative Integer

Two facts are true:

1. If a is a positive integer and b is a negative integer, then the product ab is a negative integer.
2. If a is a negative integer and b is a positive integer, then the product ab is a negative integer.

Thus, for example, $5 \cdot (-12) = -60$ and $-13 \cdot 2 = -26$. In each case the answer is negative because we are taking a product where one of the factors is positive and the other is negative.

The Distributive Property

The integers satisfy the distributive property.

The Distributive Property

Let a, b, and c be integers. Then,

$$a \cdot (b + c) = a \cdot b + a \cdot c.$$

We say that "multiplication is distributive with respect to addition."

Note how the a is "distributed." The a is multiplied times each term in the parentheses.

For example, consider the expression $3 \cdot (4 + 5)$. We can evaluate this expression according to the order of operations, simplifying the expression inside the parentheses first.

$$\begin{aligned} 3 \cdot (4+5) &= 3 \cdot 9 \\ &= 27 \end{aligned}$$

But we can also use the distributive property, multiplying each term inside the parentheses by three, then simplifying the result.

$$\begin{aligned} &\text{Distribute the 3: } 3(4+5) \\ &\text{Perform multiplications first: } = 3 \cdot 4 + 3 \cdot 5 \\ &\text{Add: } = 12 + 15 \\ &\text{Sum: } = 27 \end{aligned}$$

Note that evaluating $3 \cdot (4 + 5)$ using the distributive property provides the same result as the evaluation (2.1) using the order of operations.

The Multiplicative Property of Zero

The distributive property can be used to provide proofs of a number of important properties of integers. One important property is the fact that if you multiply an integer by zero, the product is zero. Here is a proof of that fact that uses the distributive property.

Let a be any integer. Then,

$$\begin{aligned} a \cdot 0 &= a \cdot (0+0) && \text{Additive Identity Property: } = 0 + 0 = 0. \\ a \cdot 0 &= a \cdot 0 + a \cdot 0 && \text{Distribute } a \text{ times each zero in the parentheses.} \end{aligned}$$

Next, to "undo" the effect of adding $a \cdot 0$, subtract $a \cdot 0$ from both sides of the equation.

$$\begin{aligned} a \cdot 0 - a \cdot 0 &= a \cdot 0 + a \cdot 0 + a \cdot 0 - a \cdot 0 && \text{Subtract } a \cdot 0 \text{ from both sides.} \\ 0 &= a \cdot 0 && a \cdot 0 - a \cdot 0 = 0 \text{ on each side.} \end{aligned}$$

Multiplicative Property of Zero

Let a represent any integer. Then

$$a \cdot 0 = 0 \text{ and } 0 \cdot a = 0.$$

Thus, for example, $-18 \cdot 0 = 0$ and $0 \cdot 122 = 0$.

Multiplying by Minus One

Here is another useful application of the distributive property.

$$\begin{aligned}(-1)a + a &= (-1)a + 1a && \text{Replace } a \text{ with } 1a.\\ &= (-1+1)a && \text{Use the distributive property to factor out } a.\\ &= 0a && \text{Replace } -1+1 \text{ with } 0.\\ &= 0 && \text{Replace } 0a \text{ with } 0.\end{aligned}$$

Thus, $(-1)a + a = 0$. That is, if you add $(-1)a$ to a you get zero. However, the Additive Inverse Property says that $-a$ is the *unique* number that you add to a to get zero. The conclusion must be that $(-1)a = -a$.

Multiplying by Minus One

If a is any integer, then

$$(-1)a = -a.$$

Thus, for example, $-1(4) = -4$ and $-1(-4) = -(-4) = 4$.

This property is rather important, as we will see in future work. Not only does it tell us that $(-1)a = -a$, but it also tells us that if we see $-a$, then it can be interpreted to mean $(-1)a$.

The Product of Two Negative Integers

We can employ the multiplicative property of -1, that is, $(-1)a = -a$ to find the product of two negative numbers.

$$\begin{aligned}(-4)(-3) &= [(-1)(4)](-3) && \text{Replace } -4 \text{ with } (-1)(4).\\ &= (-1)[(4)(-3)] && \text{Use the associative property to regroup.}\\ &= (-1)(-12) && \text{We know: } (4)(-3) = -12.\\ &= -(-12) && (-1)a = -a. \text{ Here } (-1)(-12) = -(-12).\\ &= 12 && -(-a) = a. \text{ Here } -(-12) = 12.\end{aligned}$$

Thus, at least in the case of $(-4)(-3)$, the product of two negative integers is a positive integer. This is true in general.

The Product of Two Negative Integers

If both a and b are negative integers, then their product ab is a positive integer.

Thus, for example, $(-5)(-7) = 35$ and $(-112)(-6) = 672$. In each case the answer is positive, because the product of two negative integers is a positive integer.

Memory Device

Here's a simple memory device to help remember the rules for finding the product of two integers.

Like and Unlike Signs

There are two cases:

Unlike Signs. The product of two integers with unlike signs is negative. That is:

(+)(−) = −

(−)(+) = −

Like Signs. The product of two integers with like signs is positive. That is:

(+)(+) = +

(−)(−)=+

Example 1

Simplify: (a) (−3)(−2), (b) (4)(−10), and (c) (12)(−3).

Solution

In each example, we use the “like” and “unlike” signs approach.

a) Like signs gives a positive result. Hence, (−3)(−2) = 6.

b) Unlike signs gives a negative result. Hence, (4)(−10) = −40.

c) Unlike signs gives a negative result. Hence, (12)(−3) = −36.

Exercise

Simplify: (a) (−12)(4) and (b) (−3)(−11).

Answer

(a) −48, (b) 33

Example 2

Simplify (−3)(2)(−4)(−2).

Solution

Order of operations demands that we work from left to right.

$$\begin{aligned}(-3)(2)(-4)(-2) &= (-6)(-4)(-2) && \text{Work left to right: } (-3)(2) = -6.\\ &= (24)(-2) && \text{Work left to right: } (-6)(-4) = 24.\\ &= -48 && \text{Multiply: } (24)(-2) = -48.\end{aligned}$$

Hence, (−3)(2)(−4)(−2) = −48.

Exercise

Simplify: (−2)(−3)(4)(−1).

Answer

−24.

Example 3

Simplify: (a)$(-2)^3$ and (c) $(-3)^4$.

Solution

In each example, use

$$a^m = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{m \text{ times}},$$

then work left to right with the multiplication.

a) Use the definition of an exponent, then order of operations.

$$\begin{aligned}(-2)^3 &= (-2)(-2)(-2) && \text{Write } -2 \text{ as a factor three times.}\\ &= 4(-2) && \text{Work left to right: } (-2)(-2) = 4.\\ &= -8\end{aligned}$$

b) Use the definition of an exponent, then order of operations.

$$\begin{aligned}(-3)^4 &= (-3)(-3)(-3)(-3) && \text{Write } -3 \text{ as a factor four times.}\\ &= 9(-3)(-3) && \text{Work left to right: } (-3)(-3)=9.\\ &= -27(-3) && \text{Work left to right: } 9(-3)=-27.\\ &= 81\end{aligned}$$

? Exercise

Simplify: (a) $(-2)^2$ and (b) -2^2.

Answer

(a) 4 and (b) −4

Example 3 motivates the following fact.

Even and Odd

Two facts are apparent.

1. If a product contains an odd number of negative factors, then the product is negative.
2. If a product contains an even number of negative factors, then the product is positive.

Thus, for example,

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$$

quickly evaluates as −32 as it has an odd number of negative factors. On the other hand,

$$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$$

quickly evaluates as 64 as it has an even number of negative factors.

Division of Integers

Consider that

$\frac{12}{3} = 4$ because $3(4) = 12$ and $\frac{-12}{-3} = 4$ because $-3(4) = -12$.

In like manner,

$\frac{12}{-3} = -4$ because -3(-4) = 12 and $\frac{-12}{3} = -4$ because $3(-4) = -12$.

Thus, the rules for dividing integers are the same as the rules for multiplying integers.

Like and Unlike Signs

There are two cases:

Unlike Signs. The quotient of two integers with *unlike* signs is negative. That is,

$$\begin{aligned}(+)(-) &= -\\ \frac{(-)}{(+)} &= -\end{aligned}$$

Like Signs. The quotient of two integers with like signs is positive. That is,

$$\begin{aligned}(+)(+) &= +\\ \frac{(-)}{(-)} &= +\end{aligned}$$

Thus, for example, 12/(−6) = −2 and −44/(−4) = 11. In the first case, unlike signs gives a negative quotient. In the second case, like signs gives a positive quotient.

One final reminder.

Division by Zero is Undefined

If a is any integer, the quotient

$$\frac{a}{0}$$

is undefined. Division by zero is meaningless.

See the discussion in Section 1.3 for a discussion on division by zero.

Example 4

Simplify: (a) −12/(−4), (b) 6/(−3), and (c) −15/0.

Solution

In each example, we use the “like” and “unlike” signs approach.

a) Like signs gives a positive result. Hence,

$$\frac{-12}{-4} = 3.$$

b) Unlike signs gives a negative result. Hence,

$$\frac{6}{-3} = -2.$$

c) Division by zero is undefined. Hence,

$$\frac{-15}{0}$$

is undefined.

Exercise

Simplify: (a) −24/4 and (b) −28/(−7).

Answer

(a) −6, (b) 4

Exercises

In Exercises 1-16, state the property of multiplication depicted by the given identity.

1. (−2) (−16)(13) = (−2)(−16) (13)
2. (10) (−15)(−6) = (10)(−15) (−6)
3. (−17)(−10) = (−10)(−17)
4. (−5)(3) = (3)(−5)
5. (4)(11) = (11)(4)
6. (−5)(−11) = (−11)(−5)
7. 16 · (8 + (−15) = 16 · 8 + 16 · (−15)

8. $1 \cdot (-16 + (-6) = 1 \cdot (-16) + 1 \cdot (-6)$

9. $(17)\ (20)(11) = (17)(20)\ (11)$

10. $(14)\ (-20)(-18) = (14)(-20)\ (-18)$

11. $-19 \cdot 1 = -19$

12. $-17 \cdot 1 = -17$

13. $8 \cdot 1=8$

14. $-20 \cdot 1 = -20$

15. $14 \cdot (-12 + 7 = 14 \cdot (-12) + 14 \cdot 7$

16. $-14 \cdot (-3+6 = -14 \cdot (-3) + (-14) \cdot 6$

In Exercises 17-36, simplify each given expression.

17. $4 \cdot 7$

18. $4 \cdot 2$

19. $3 \cdot (-3)$

20. $7 \cdot (-9)$

21. $-1 \cdot 10$

22. $-1 \cdot 11$

23. $-1 \cdot 0$

24. $-8 \cdot 0$

25. $-1 \cdot (-14)$

26. $-1 \cdot (-13)$

27. $-1 \cdot (-19)$

28. $-1 \cdot (-17)$

29. $2 \cdot 0$

30. $-6 \cdot 0$

31. $-3 \cdot 8$

32. $7 \cdot (-3)$

33. $7 \cdot 9$

34. $6 \cdot 3$

35. $-1 \cdot 5$

36. $-1 \cdot 2$

In Exercises 37-48, simplify each given expression.

37. $(-7)(-1)(3)$

38. $(10)(6)(3)$

39. $(-7)(9)(10)(-10)$

40. $(-8)(-5)(7)(-9)$

41. $(6)(5)(8)$

42. $(7)(-1)(-9)$

43. $(-10)(4)(-3)(8)$

44. $(8)(-2)(-5)(2)$

45. $(6)(-3)(-8)$

46. $(-5)(-4)(1)$

47. $(2)(1)(3)(4)$

48. $(7)(5)(1)(4)$

In Exercises 49-60, compute the exact value.

49. $(-4)^4$

50. $(-3)^4$

51. $(-5)^4$

52. $(-2)^2$

53. $(-5)^2$

54. $(-3)^3$

55. $(-6)^2$

56. $(-6)^4$

57. $(-4)^5$

58. $(-4)^2$

59. $(-5)^3$

60. $(-3)^2$

In Exercises 61-84, simplify each given expression.

61. $-16 \div (-8)$

62. $-33 \div (-3)$

63. $\frac{-8}{1}$

64. $\frac{40}{-20}$

65. $\frac{-1}{0}$

66. \(\frac{2}{0}\

67. $-3 \div 3$

68. $-58 \div 29$

69. $\frac{56}{-28}$

70. $\frac{60}{-12}$

71. $0 \div 15$

72. $0 \div (-4)$

73. $\frac{63}{21}$

74. $\frac{-6}{-1}$

75. $\frac{78}{13}$

76. $\frac{-84}{-14}$

77. $0 \div 5$

78. $0 \div (-16)$

79. $\frac{17}{0}$

80. $\frac{-20}{0}$

81. $-45 \div 15$

82. $-28 \div 28$

83. $12 \div 3$

84. $-22 \div (-22)$

---

85. **Scuba**. A diver goes down 25 feet. A second diver then dives down 5 times further than the first diver. Write the final depth of the second diver as an integer.

86. **Investing Loss**. An investing club of five friends has lost $4400 on a trade. If they share the loss equally, write each members' loss as an integer.

Answers

1. Associative property of multiplication

3. Commutative property of multiplication

5. Commutative property of multiplication

7. Distributive property

9. Associative property of multiplication

11. Multiplicative identity property

13. Multiplicative identity property

15. Distributive property

17. 28

19. −9

21. −10

23. 0

25. 14

27. 19

29. 0

31. −24

33. 63

35. −5

37. 21

39. 6300

41. 240

43. 960

45. 144

47. 24

49. 256

51. 625

53. 25

55. 36

57. −1024

59. −125

61. 2

63. −8

65. Division by zero is undefined.

67. −1

69. −2

71. 0

73. 3

75. 6

77. 0

79. Division by zero is undefined.

81. −3 83. 4

85. −125 feet

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2.1.5: Order of Operations

For convenience, we repeat the "Rules Guiding Order of Operations" first introduced in Section 1.5.

Rules Guiding Order of Operations

When evaluating expressions, proceed in the following order.

1. Evaluate expressions contained in grouping symbols first. If grouping symbols are nested, evaluate the expression in the innermost pair of grouping symbols first.
2. Evaluate all exponents that appear in the expression.
3. Perform all multiplications and divisions in the order that they appear in the expression, moving left to right.
4. Perform all additions and subtractions in the order that they appear in the expression, moving left to right.

Let's look at a number of examples that require the use of these rules.

Example 1

Simplify: (a) $(-3)^2$ and (b) -3^2

Solution

Recall that for any integer a, we have $(-1)a = -a$. Because negating is equivalent to multiplying by -1, the "Rules Guiding Order of Operations" require that we address grouping symbols and exponents before negation.

a) Because of the grouping symbols, we negate first, then square. That is,

$$\begin{aligned}(-3)^2 &= (-3)(-3)\\ &= 9.\end{aligned}$$

b) There are no grouping symbols in this example. Thus, we must square first, then negate. That is,

$$\begin{aligned}-3^2 &= -(3 \cdot 3)\\ &= -9.\end{aligned}$$

Exercise

Simplify: -2^2.

Answer

-4

Example 2

Simplify: $-2 - 3(5 - 7)$.

Solution

Grouping symbols first, then multiplication, then subtraction.

$$\begin{aligned}-2-3(5-7) &= -2-3(-2) && \text{Perform subtraction within parentheses.}\\ &= 2-2-(-6) && \text{Multiply: } 3(-2) = -6.\\ &= -2+6 && \text{Add the opposite.}\\ &= 4\end{aligned}$$

Exercise

Simplify: $-3-2(6-8)$.

Answer

1

Example 3

Simplify: $-2(2-4)^2-3(3-5)^3$.

Solution

Grouping symbols first, then multiplication, and subtraction, in that order.

\[\begin{aligned} -2(2-4)^2 -3(3-5)^3 = -2(-2)^2 -3(-2)^3 ~ & \textcolor{red}{ \text{ Perform subtraction within parentheses first.} } \\ =2 (4) -3(-8) ~ & \textcolor{red}{ \text{ Exponents are next.} } \\ =-8-(-24) ~ & \textcolor{red}{ \text{ Multiplications are next.} } \\ =-8+24 ~ & \textcolor{red}{ \text{ Add the opposite.} } \\ =16 \end{aligned}\nonumber \]

Exercise

Simplify: $-2(5-6)^3-3(5-7)^2$

Answer

-10

Example 4

Simplify: $-24 \div 8(-3)$.

Solution

Division has no preference over multiplication, or vice versa. Divisions and multiplications must be performed in the order that they occur, moving left to right.

$$\begin{aligned} -24 \div 8(-3) &= -3(-3) \quad \text{Division first: } -24 \div 8 = -3. \\ &= 9 \end{aligned}$$

Note that if you multiply first, which would be incorrect, you would get a completely different answer.

Exercise

Simplify: $-48 \div 6(-2)$.

Answer

16

Example 5

Simplify: $(-2)(-3)(-2)^3$.

Solution

Exponents first, then multiplication in the order that it occurs, moving left to right.

$$\begin{aligned} (-2)(-3)(-2)^3 &= (-2)(-3)(-8) \quad \text{Exponent first: } (-2)^3 = -8. \\ &= 6(-8) \quad \text{Multiply from left to right: } (-2)(-3) = 6. \\ &= -48 \end{aligned}$$

? You try it!

Simplify: $(-4)(-2)^2(-1)^3$.

Answer

16

Evaluating Fractions

If a fraction bar is present, evaluate the numerator and denominator separately according to the "Rules Guiding Order of Operations," then perform the division in the final step.

✔ Example 6

Simplify:

$$\frac{-5-5(2-4)^3}{-22-3(-5)}$$

Solution

Evaluate numerator and denominator separately, then divide.

$$\begin{aligned}
\frac{-5-5(2-4)^3}{-22-3(-5)} &= \frac{-5-5(-2)^3}{-22-(-15)} && \text{Numerator: parentheses first.}\\
&= \frac{-5-5(-8)}{-22+15} && \text{Denominator: multiply } 3(-5)=-15.\\
&= \frac{-5-(-40)}{-7} && \text{Numerator: exponent } (-2)^3=-8.\\
&= \frac{-5+40}{-7} && \text{Denominator: add the opposite.}\\
&= \frac{35}{-7} && \text{Numerator: multiply } 5(-8)=-40.\\
&= -5 && \text{Denominator: add } -22+15=-7.\\
& && \text{Numerator: add the opposite.}\\
& && \text{Numerator: } -5+40=35.\\
& && \text{Divide: } 35/-7=-5.
\end{aligned}$$

? Exercise

Simplify:

$$\frac{6-2(-6)}{-2-(-2)^2}$$

Answer

-3

Absolute Value

Absolute value calculates the magnitude of the vector associated with an integer, which is equal to the distance between the number and the origin (zero) on the number line. Thus, for example, |4| = 4 and | − 5| = 5.

But absolute value bars also act as grouping symbols, and according to the "Rules Guiding Order of Operations," you should evaluate the expression inside a pair of grouping symbols first.

Example 7

Simplify: (a) $-(-3)$ and (b) $-|-3|$.

Solution

There is a huge difference between simple grouping symbols and absolute value.

a) This is a case of $-(-a) = a$. Thus, $-(-3) = 3$.

b) This case is much different. The absolute value of -3 is 3, and then the negative of that is -3. In symbols,

$$-|-3| = -3$$

Exercise

Simplify: $-|-8|$.

Answer

-8

Example 8

Simplify: $-3 - 2|5 - 7|$.

Solution

Evaluate the expression inside the absolute value bars first. Then multiply, then subtract.

$$-3-2|5-7| = -3-2|-2|$$

Exercise

Simplify: $-2 - 4|6 - 8|$.

Answer

-10

Exercises

In Exercises 1-40, compute the exact value of the given expression.

1. $7 - \frac{-14}{2}$
2. $-2 - \frac{-16}{4}$
3. $-7 - \frac{-18}{9}$
4. $-6 - \frac{-7}{7}$
5. -54
6. -33
7. $9 - 1(-7)$
8. $85 - 8(9)$
9. -63
10. -35
11. $3 + 9(4)$

12. $6 + 7(-1)$
13. $10 - 72 \div 6 \cdot 3+8$
14. $8 - 120 \div 5 \cdot 6+7$
15. $6 + \frac{14}{2}$
16. $16 + \frac{8}{2}$
17. -34
18. -22
19. $3 - 24 \div 4 \cdot 3+4$
20. $4 - 40 \div 5 \cdot 4+9$
21. $64 \div 4 \cdot 4$
22. $18 \div 6 \cdot 1$
23. $-2 - 3(-5)$
24. $64 - 7(7)$
25. $15 \div 1 \cdot 3$
26. $30 \div 3 \cdot 5$
27. $8 + 12 \div 6 \cdot 1 - 5$
28. $9 + 16 \div 2 \cdot 4 - 9$
29. $32 \div 4 \cdot 4$
30. $72 \div 4 \cdot 6$
31. $-11 + \frac{16}{16}$
32. $4 + \frac{-20}{4}$
33. -52
34. -43
35. $10 + 12(-5)$
36. $4 + 12(4)$
37. $2+6 \div 1 \cdot 6 - 1$
38. $1 + 12 \div 2 \cdot 2 - 6$
39. $40 \div 5 \cdot 4$
40. $30 \div 6 \cdot 5$

In Exercises 41-80, simplify the given expression.

41. $-11 + |-1 - (-6)^2|$
42. $13 + |-21 - (-4)^2|$
43. $|0(-4)| - 4(-4)$
44. $|10(-3)| - 3(-1)$
45. $(2 + 3 \cdot 4) - 8$
46. $(11 + 5 \cdot 2) - 10$
47. $(8 - 1 \cdot 12) + 4$
48. $(9 - 6 \cdot 1) + 3$

49. $(6 + 10 \cdot 4) - 6$

50. $(8 + 7 \cdot 6) - 12$

51. $10 + (6 - 4)^3 - 3$

52. $5 + (12 - 7)^2 - 6$

53. $(6 - 8)^2 - (4 - 7)^3$

54. $(3 - 8)^2 - (4 - 9)^3$

55. $|0(-10)| + 4(-4)$

56. $|12(-5)| + 7(-5)$

57. $|8(-1)| - 8(-7)$

58. $|6(-11)| - 7(-1)$

59. $3 + (3 - 8)^2 - 7$

60. $9 + (8 - 3)^3 - 6$

61. $(4 - 2)^2 - (7 - 2)^3$

62. $(1 - 4)^2 - (3 - 6)^3$

63. $8 - |- 25 - (-4)^2|$

64. $20 - |- 22 - 4^2|$

65. $-4 - |30 - (-5)^2|$

66. $-8 - |- 11 - (-6)^2|$

67. $(8 - 7)^2 - (2 - 6)^3$

68. $(2 - 7)^2 - (4 - 7)^3$

69. $4 - (3 - 6)^3 + 4$

70. $6 - (7 - 8)^3 + 2$

71. $-3 + | - 22 - 5^2|$

72. $12 + |23 - (-6)^2|$

73. $(3 - 4 \cdot 1) + 6$

74. $(12 - 1 \cdot 6) + 4$

75. $1 - (1 - 5)^2 + 11$

76. $9 - (3 - 1)^3 + 10$

77. $(2 - 6)^2 - (8 - 6)^3$

78. $(2 - 7)^2 - (2 - 4)^3$

79. $|9(-3)| + 12(-2)$

80. $|0(-3)| + 9(-7)$

In Exercises 81-104, simplify the given expression.

81. $\frac{4(-10)-5}{-9}$

82. $\frac{-4 \cdot 6-(-8)}{-4}$

83. $\frac{10^2-4^2}{2 \cdot 6-10}$

84. $\frac{3^2-9^2}{2 \cdot 7-5}$

85. $\frac{3^2+6^2}{5-1\cdot 8}$

86. $\frac{10^2+4^2}{1-6\cdot 5}$

87. $\frac{-8-4}{7-13}$

88. $\frac{13-1}{8-4}$

89. $\frac{2^2+6^2}{11-4\cdot 4}$

90. $\frac{7^2+3^2}{10-8\cdot 1}$

91. $\frac{1^2-5^2}{9\cdot 1-5}$

92. $\frac{5^2-7^2}{2\cdot 2-12}$

93. $\frac{4^2-8^2}{6\cdot 3-2}$

94. $\frac{7^2-6^2}{6\cdot 3-5}$

95. $\frac{10^2+2^2}{10-2\cdot 7}$

96. $\frac{2^2+10^2}{10-2\cdot 7}$

97. $\frac{16-(-2)}{19-1}$

98. $\frac{-8-20}{-15-(-17}$

99. $\frac{15-(-15)}{13-(-17)}$

100. $\frac{7-(-9)}{-1-1}$

101. $\frac{4\cdot 5-(-19)}{3}$

102. $\frac{10\cdot 7-(-11)}{-3}$

103. $\frac{-6\cdot 9-(-4)}{2}$

104. $\frac{-6\cdot 2-10}{-11}$

Answers

1. 14

3. −5

5. −625

7. 16

9. −216

11. 39

13. −18

15. 13

17. −81

19. −11

21. 64

23. 13

25. 45

27. 5

29. 32

31. −10

33. −25

35. −50

37. 37

39. 32

41. 26

43. 16

45. 6

47. 0

49. 40

51. 15

53. 31

55. −16

57. 64

59. 21

61. −121

63. −33

65. −9

67. 65

69. 35

71. 44

73. 5

75. −4

77. 8

79. 3

81. 5

83. 42

85. −15

87. 2

89. −8

91. −6

93. −3

95. −8

97. 1

99. 1

101. 13

103. −25

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2.1.6: Solving Equations

Recall (see Section 1.6) that a variable is a symbol (usually a letter) that stands for a value that varies. If a variable in an equation is replaced by a number and a true statement results, then that number is called a solution of the equation.

Example 1

Is −6 a solution of the equation 2x +5= −7?

Solution

Substitute −6 for x in the equation.

$$\begin{aligned} 2x+5 &= 7 && \text{Original equation.} \\ 2(-6)+5 &= -7 && \text{Substitute } -6 \text{ for } x. \\ -12+5 &= -7 && \text{On the left, multiply first.} \\ -7 &= -7 && \text{On the left, add.} \end{aligned}$$

Because the last statement is a true statement, −6 is a solution of the equation.

Exercise

Is −4 a solution of 8 − 2x = 5?

Answer

No

Adding or Subtracting the Same Amount

Two equations having the same set of solutions are *equivalent*. For example, 2x+5 = −7 and x = −6 have the same solutions. Therefore, they are equivalent equations. Certain algebraic operations produce equivalent equations.

Producing Equivalent Equations

Adding the Same Quantity to Both Sides of an Equation. If we start with the equation

$$a = b,$$

then adding c to both sides of the equation produces the equivalent equation

$$a + c = b + c.$$

Subtracting the Same Quantity from Both Sides of an Equation. If we start with the equation

$$a = b,$$

then subtracting c from both sides of the equation produces the equivalent equation

$$a - c = b - c.$$

That is, adding or subtracting the same amount from both sides of an equation will not change the solutions of the equation.

Example 2

Solve for x: x + 3= −7.

Solution

To undo the effect of adding 3, subtract 3 from both sides of the equation.

$$\begin{aligned} x+3 &= -7 && \text{Original equation.} \\ x+3-3 &= -7-3 && \text{Subtract 3 from both sides.} \\ x &= -7+(-3) && \text{Simplify the left hand side. On the right,} \\ & && \text{express subtractions as adding the opposite.} \\ x &= -10 \end{aligned}$$

To check the solution, substitute −10 for *x* in the original equation and simplify.

$$\begin{aligned} x+3 &= -7 && \text{Original equation.} \\ -10+3 &= -7 && \text{Substitute } -10 \text{ for } x. \\ &= 7 = -7 && \text{Simplify both sides.} \end{aligned}$$

Since the last line of the check is a true statement, this confirms that −10 is a solution.

? Exercise

Solve for *x*: *x* + 9 = -11.

Answer

x = −20

✔ Example 3

Solve for *x*: *x* − 8 = −11.

Solution

To undo the effect of subtracting 8, add 8 to both sides of the equation.

$$\begin{aligned} x-8 &= -11 && \text{Original equation.} \\ x-8+8 &= -11+8 && \text{Add 8 to both sides.} \\ x &= -3 && \text{Simplify both equations.} \end{aligned}$$

To check the solution, substitute −3 for *x* in the original equation and simplify.

$$\begin{aligned} x-8 &= -11 && \text{Original equation.} \\ -3-8 &= -11 && \text{Substitute } -3 \text{ for } x. \\ -11 &= -11 && \text{Simplify both sides.} \end{aligned}$$

Since the last line of the check is a true statement, this confirms that −3 is a solution.

? Exercise

Solve for *x*: *x* − 2 = −7

Answer

x = −5

Sometimes a bit of simplification is in order before you start the solution process.

✔ Example 4

Solve for *y*: −8+2= *y* − 11(−4).

Solution

First, simplify both sides of the equation.

$$\begin{aligned} -8+2 &= y-11(-4) && \text{Original equation.} \\ -6 &= y-(-44) && \text{Simplify. On the left, } -8+2=-6. \text{ On the right, } 11(-4)=-44. \\ -6 &= y+44-44 && \text{Subtract 44 from both sides of the equation.} \\ -6+(-44) &= y && \text{Express subtraction as addition. Simplify on the right.} \\ -50 &= y \end{aligned}$$

To check the solution, substitute −50 for *y* in the original equation and simplify.

$$\begin{aligned} -8+2 &= y-11(-4) && \text{Original equation.} \\ -8+2 &= -50-11(-4) && \text{Substitute } -50 \text{ for } y. \\ -6 &= -50-(-44) && \text{Express subtraction on the right as addition.} \\ -6 &= -6 && \text{On the right, add: } -50+44=-6. \end{aligned}$$

Since the last line of the check is a true statement, this confirms that −50 is a solution.

Exercise

Solve for *y*: *y* + 2(−4) = −8+6

Answer

y = 6

Multiplying or Dividing by the Same Amount

Adding and subtracting is not the only way to produce an equivalent equation.

Producing Equivalent Equations

Multiplying Both Sides of an Equation by the Same Quantity. If we start with the equation

$$a = b,$$

then multiplying both sides of the equation by *c* produces the equivalent equation

$$a \cdot c = b \cdot c, \text{ or equivalently, } ac = bc,$$

provided c ≠ 0.

Dividing Both Sides of an Equation by the Same Quantity. If we start with the equation

$$a = b,$$

then dividing both sides of the equation by c produces the equivalent equation

$$\frac{a}{c} = \frac{b}{c},$$

provided c ≠ 0.

That is, multiplying or dividing both sides of an equation by the same amount will not change the solutions of the equation.

Example 5

Solve for *x*: −3*x* = 30.

Solution

To undo the effect of multiplying by −3, divide both sides of the equation by −3.

$$\begin{aligned} -3x &= 30 && \text{Original equation.} \\ \frac{-3x}{-3} &= \frac{30}{-3} && \text{Divide both sides by } -3. \\ x &= -10 && \text{On the left, } -3 \text{ times } x, \text{ divided by } -3 \text{ is } x. \\ & && \text{On the right, } 30/(-3) = -10. \end{aligned}$$

To check the solution, substitute −10 for x in the original equation and simplify.

$$\begin{aligned} -3x &= 30 && \text{Original equation.} \\ -3(-10) &= 30 && \text{Substitute } -10 \text{ for } x. \\ 30 - 30 & && \text{Simplify.} \end{aligned}$$

Because the last line of the check is a true statement, this confirms that −10 is a solution.

? Exercise

Solve for z: −4z = −28

Answer

z = 7

✔ Example 6

Solve for x: $\frac{x}{-2} = -20$.

Solution

To undo the effect of dividing by −2, multiply both sides of the equation by −2.

$$\begin{aligned} \frac{x}{-2} &= -20 && \text{Original equation.} \\ -2\left(\frac{x}{-2}\right) &= -2(-20) && \text{Multiply both sides by } -2. \\ x &= 40 && \text{On the left, } x \text{ divided by } -2, \text{ multiplied by } -2, \\ & && \text{the result is } x. \text{ On the right, } -2(-20) = 40. \end{aligned}$$

To check the solution, substitute 40 for x in the original equation and simplify.

$$\begin{aligned} \frac{x}{-2} &= -20 && \text{Original equation.} \\ \frac{40}{-2} &= -20 && \text{Substitute 40 for } x. \\ -20 &= -20 && \text{Simplify both sides.} \end{aligned}$$

Because the last line of the check is a true statement, this confirms that 40 is a solution.

Combining Operations

Recall the “Wrap” and “Unwrap” discussion from Section 1.6. To wrap a present we: (1) put the gift paper on, (2) put the tape on, and (3) put the decorative bow on. To unwrap the gift, we must “undo” each of these steps in inverse order. Hence, to unwrap the gift we: (1) take off the decorative bow, (2) take off the tape, and (3) take off the gift paper.

Now, imagine a machine that takes its input, then: (1) multiplies the input by 2, and (2) adds 3 to the result. This machine is pictured on the left in Figure 2.16.

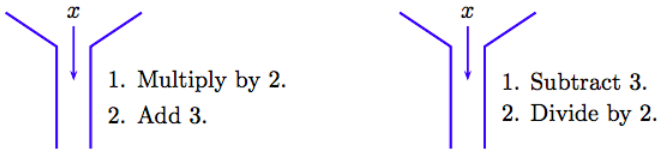


Figure 2.16: The second machine "unwraps" the first machine.

To "unwrap" the effect of the machine on the left, we need a machine that will "undo" each of the steps of the first machine, but in inverse order. The "unwrapping" machine is pictured on the right in Figure 2.16. It will first subtract three from its input, then divide the result by 2. Note that each of these operations "undoes" the corresponding operation of the first machine, but in inverse order.

For example, drop the integer 7 into the first machine on the left in Figure 2.16. First, we double 7, then add 3 to the result. The result is 2(7) + 3 = 17.

Now, to "unwrap" this result, we drop 17 into the second machine. We first subtract 3, then divide by 2. The result is (17 − 3)/2 = 7, the original integer input into the first machine.

Now, consider the equation

$$2x+3=7.$$

On the left, order of operations demands that we first multiply x by 2, then add 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract three from both sides of the equation, then (2) divide both sides of the resulting equation by 2.

$$\begin{aligned} 2x+3-3 &= 7-3 && \text{Subtract 3 from both sides.} \\ 2x &= 4 && \text{Simplify both sides.} \\ \frac{2x}{2} &= \frac{4}{2} && \text{Divide both sides by 2.} \\ x &= 2 && \text{Simplify both sides.} \end{aligned}$$

Readers should check this solution in the original equation.

Example 7

Solve for x: $\frac{x}{4}-3=-7$.

Solution

On the left, order of operations demands that we first divide x by 4, then subtract 3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will (1) add 3 to both sides of the equation, then (2) multiply both sides of the resulting equation by 4.

$$\begin{aligned} \frac{x}{4}-3 &= -7 && \text{Original equation.} \\ \frac{x}{4}-3+3 &= -7+3 && \text{Add 3 to both sides.} \\ \frac{x}{4} &= -4 && \text{Simplify both sides.} \\ 4\left(\frac{x}{4}\right) &= 4(-4) && \text{Multiply both sides by 4.} \\ x &= -16 && \text{Simplify both sides.} \end{aligned}$$

Check

Substitute −16 for x in the original equation.

$$\begin{aligned} \frac{x}{4}-3 &= 7 && \text{Original equation.} \\ \frac{-16}{4}-3 &= -7 && \text{Substitute } -16 \text{ for } x. \\ -4-3 &= -7 && \text{Divide first: } -16/4 = -4. \\ -7 &= -7 && \text{Subtract: } -4-3 = -7. \end{aligned}$$

Because the last line of the check is a true statement, −16 is a solution of the original equation.

? Exercise

Solve for *x*:

$$\frac{x}{2}+6=4$$

Answer

x = -4

✔ Example 8

Solve for *t*: 0=8 − 2*t*.

Solution

On the right, order of operations demands that we first multiply t by −2, then add 8. To solve this equation for t, we must "undo" each of these operations in inverse order. Thus, we will (1) subtract 8 from both sides of the equation, then (2) divide both sides of the resulting equation by −2.

$$\begin{aligned} 0 &= 8-2t && \text{Original equation.} \\ 0-8 &= 8-2t-8 && \text{Subtract 8 from both sides.} \\ -8 &= -2t && \text{Simplify both sides.} \\ \frac{-8}{-2} &= \frac{-2t}{-2} && \text{Divide both sides by 2.} \\ 4 &= t && \text{Simplify both sides.} \end{aligned}$$

Check

Substitute 4 for t in the original equation.

$$\begin{aligned} 0 &= 8-2t && \text{Original equation.} \\ 0 &= 8-2(4) && \text{Substitute 4 for } t. \\ 0 &= 8-8 && \text{Multiply first: } 2(4)=8. \\ 0 &= 0 && \text{Subtract: } 8-8=0. \end{aligned}$$

Because the last line in the check is a true statement, 4 is a solution of the original equation.

? Exercise

Solve for *r*: 0 = 9 + 3*r*

Answer

r = -3

Example 9

Solve for p: $-12+3=-8+4+\frac{p}{-3}$.

Solution

Always simplify when possible.

$$\begin{aligned} -12+3 &= -8+4+\frac{p}{-3} && \text{Origianl equation.} \\ -9 &= -4+\frac{p}{-3} && \text{Simplify both sides.} \end{aligned}$$

On the right, order of operations demands that we first divide p by -3, then add -4. To solve this equation for p, we must "undo" each of these operations in inverse order. Thus, we will (1) add a positive 4 to both sides of the equation, then (2) multiply both sides of the resulting equation by -3.

$$\begin{aligned} -9+-4 &= -4+\frac{p}{-3}+4 && \text{Add 4 to both sides.} \\ -5 &= \frac{p}{-3} && \text{Simplify both sides.} \\ -3(-5) &= -3\left(\frac{p}{-3}\right) && \text{Multiply both sides by } -3. \\ 15 &= p && \text{Simplify both sides.} \end{aligned}$$

Check

Substitute 15 for p in the original equation.

$$\begin{aligned} -12+3 == 8+4+\frac{p}{-3} && \text{Original equation.} \\ -12+3 = -8+4+\frac{15}{-3} && \text{Substitute 15 for } p. \\ -9 = -8+4+(-5) && \text{On the left, add: } -12+3=-9. \text{ On the right, divide: } 15/(-3)=-5. \\ -9 = -4+(-5) && \text{On the right, } -8+4=-4. \\ -9 = -9 && \text{On the right, add: } -4+(-5)=-9. \end{aligned}$$

Because the last line in the check is a true statement, 15 is a solution of the original equation.

Exercise

Solve for q:

$$\frac{q}{-2}-9=-8+3$$

Answer

$q = -8$

Applications

Let's look at some applications of equations involving integers. First, we remind readers that a solution of a word problem must incorporate each of the following steps.

Requirements for Word Problem Solutions

1. **Set up a Variable Dictionary.** You must let your readers know what each variable in your problem represents. This can be accomplished in a number of ways:
    1. Statements such as "Let P represent the perimeter of the rectangle."

2. Labeling unknown values with variables in a table.
3. Labeling unknown quantities in a sketch or diagram.

2. **Set up an Equation.** Every solution to a word problem must include a carefully crafted equation that accurately describes the constraints in the problem statement.
3. **Solve the Equation.** You must always solve the equation set up in the previous step.
4. **Answer the Question.** This step is easily overlooked. For example, the problem might ask for Jane's age, but your equation's solution gives the age of Jane's sister Liz. Make sure you answer the original question asked in the problem. Your solution should be written in a sentence with appropriate units.
5. **Look Back.** It is important to note that this step does not imply that you should simply check your solution in your equation. After all, it's possible that your equation incorrectly models the problem's situation, so you could have a valid solution to an incorrect equation. The important question is: "Does your answer make sense based on the words in the original problem statement."

Example 10

A student's bank account is overdrawn. After making a his account, Allen finds that he is overdrawn by $15. What was his account balance before his withdrawal? deposit of $120, he finds that his account is still overdrawn by an amount of $75. What was his balance before he made his deposit?

Solution

In our solution, we address each step of the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary*. In this case, the unknown is the original balance in the student's account. Let B represent this original balance.

2. *Set up an Equation.* A positive integer represents a healthy balance, while a negative number represents an account that is overdrawn. After the student's deposit, the account is still overdrawn by $75. We will say that this balance is −$75. Thus,

Original Balance	plus	Student Deposit	equals	Current Balance
B	$+$	$\$120$	$=$	$-\$75$

3. *Solve the Equation.* To "undo" the addition, subtract 120 from both sides of the equation.

$$\begin{aligned} B+120 &= -75 && \text{Original equation.} \\ B+120-120 &= -75-120 && \text{Subtract 120 from both sides.} \\ B &= -195 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The original balance was overdrawn to the tune of $195.

5. *Look Back.* If the original balance was overdrawn by $195, then we let −$195 represent this balance. The student makes a deposit of $120. Add this to the original balance to get −$195 + $120 = −$75, the correct current balance.

Exercise

After withdrawing $125 from his account, Allen finds that he is overdrawn by $15. What was his account balance before his withdrawal?

Answer

$110

Example 11

Three more than twice a certain number is −11. Find the unknown number.

Solution

In our solution, we address each step of the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary.* Let x represent the unknown number. 2. Set up an Equation. “Three more than twice a certain number” becomes:

Three	more than	Twice a Certain Number	is	-11
3	$+$	$2x$	$=$	11

3. *Solve the Equation.* On the left, order of operations requires that we first multiply x by 2, then add 3. To solve this equation for x, we must “undo” each of these operations in inverse order. Thus, we will (1) subtract 3 from both sides of the equation, then (2) divide both sides of the resulting equation by 2.

$$\begin{aligned} 3+2x &= -11 && \text{Original equation.} \\ 3+2x-3 &= -11-3 && \text{Subtract 3 from both sides.} \\ 2x &= -14 && \text{Simplify both sides.} \\ \frac{2x}{2} &= \frac{-14}{2} && \text{Divide both sides by 2.} \\ x &= -7 && \text{Simplify both sides.} \end{aligned}$$

4. *Answer the Question.* The unknown number is −7.

5. *Look Back.* Does the answer satisfy the problem constraints? Three more than twice −7 is three more than −14, or −11. So the solution is correct.

? Exercise 2.1.6.1

Five less than twice a certain number is −7. Find the unknown number.

Answer

−1

Exercises

1. Is −11 a solution of 2x +3= −19?
2. Is −8 a solution of 2x +7= −9?
3. Is 6 a solution of 3x + 1 = 19?
4. Is −6 a solution of 2x +7= −5?
5. Is 12 a solution of 4x +5= −8?
6. Is −8 a solution of −3x + 8 = 18?
7. Is 15 a solution of 2x +6= −9?
8. Is 3 a solution of −4x +1= −20?
9. Is −15 a solution of −3x +6= −17?
10. Is −18 a solution of −3x +9= −9?
11. Is −6 a solution of −2x + 3 = 15?
12. Is 7 a solution of −3x +5= −16?

In Exercises 13-28, solve the given equation for x.

13. x − 13 = 11
14. x − 6 = 12
15. x − 3=6

16. $x - 3 = -19$
17. $x + 10 = 17$
18. $x + 3 = 9$
19. $x - 6 = 1$
20. $x - 10 = 12$
21. $x - 15 = -12$
22. $x - 2 = 13$
23. $x + 11 = -19$
24. $x + 3 = 17$
25. $x + 2 = 1$
26. $x + 2 = -20$
27. $x + 5 = -5$
28. $x + 14 = -15$

In Exercises 29-44, solve the given equation for x.

29. $-x = -20$
30. $5x = -35$
31. $\frac{x}{-7} = 10$
32. $\frac{x}{-6} = -20$
33. $\frac{x}{-10} = 12$
34. $\frac{x}{2} = 11$
35. $\frac{x}{9} = -16$
36. $\frac{x}{-3} = -7$
37. $-10x = 20$
38. $-17x = -85$
39. $14x = 84$
40. $-10x = -40$
41. $-2x = 28$
42. $-14x = 42$
43. $\frac{x}{-10} = 15$
44. $\frac{x}{-8} = -1$

In Exercises 45-68, solve the given equation for x.

45. $-4x - 4 = 16$
46. $-6x - 14 = 4$
47. $4x - 4 = 76$
48. $-5x - 15 = 45$
49. $5x - 14 = -79$
50. $15x - 2 = 43$

51. $-10x - 16 = 24$

52. $2x - 7 = -11$

53. $9x + 5 = -85$

54. $8x + 8 = -16$

55. $7x + 15 = -55$

56. $2x + 2 = -38$

57. $-x + 8 = 13$

58. $-5x + 20 = -50$

59. $12x - 15 = -3$

60. $-19x - 17 = -36$

61. $4x - 12 = -56$

62. $7x - 16 = 40$

63. $19x + 18 = 113$

64. $-6x + 20 = -64$

65. $-14x + 12 = -2$

66. $-9x + 5 = 104$

67. $14x + 16 = 44$

68. $-14x + 10 = -60$

---

69. Two less than eight times an unknown number is -74. Find the unknown number.

70. Six less than three times an unknown number is 21. Find the unknown number.

71. Eight more than two times an unknown number is 0. Find the unknown number.

72. Five more than eight times an unknown number is -35. Find the unknown number.

73. The number -6 is 2 more than an unknown number. Find the unknown number.

74. The number -4 is 7 more than an unknown number. Find the unknown number.

75. Three more than eight times an unknown number is -29. Find the unknown number.

76. Four more than nine times an unknown number is 85. Find the unknown number.

77. Alan's scores on his first three exams are 79, 61, and 54. What must Alan score on his next exam to average 71 for all four exams?

78. Benny's scores on his first three exams are 54, 68, and 54. What must Benny score on his next exam to average 61 for all four exams?

79. The quotient of two integers is 5. One of the integers is -2. Find the other integer.

80. The quotient of two integers is 3. One of the integers is -7. Find the other integer.

81. The quotient of two integers is 9. One of the integers is -8. Find the other integer.

82. The quotient of two integers is 9. One of the integers is -2. Find the other integer.

83. The number -5 is 8 more than an unknown number. Find the unknown number.

84. The number -6 is 8 more than an unknown number. Find the unknown number.

85. A student's bank account is overdrawn. After making a deposit of \$260, he finds that his account is still overdrawn by an amount of \$70. What was his balance before he made his deposit?

86. A student’s bank account is overdrawn. After making a deposit of $300, he finds that his account is still overdrawn by an amount of $70. What was his balance before he made his deposit?

87. A student’s bank account is overdrawn. After making a deposit of $360, he finds that his account is still overdrawn by an amount of $90. What was his balance before he made his deposit?

88. A student’s bank account is overdrawn. After making a deposit of $260, he finds that his account is still overdrawn by an amount of $50. What was his balance before he made his deposit?

89. The number −10 is −5 times larger than an unknown number. Find the unknown number.

90. The number −3 is −3 times larger than an unknown number. Find the unknown number. 91. The number −15 is −5 times larger than an unknown number. Find the unknown number.

92. The number −16 is 4 times larger than an unknown number. Find the unknown number.

93. Two less than nine times an unknown number is 7. Find the unknown number.

94. Four less than two times an unknown number is 8. Find the unknown number.

95. Mark’s scores on his first three exams are 79, 84, and 71. What must Mark score on his next exam to average 74 for all four exams?

96. Alan’s scores on his first three exams are 85, 90, and 61. What must Alan score on his next exam to average 77 for all four exams?

Answers

1. Yes

3. Yes

5. No

7. No

9. No

11. Yes

13. 24

15. 9

17. 7

19. 7

21. 3

23. −30

25. −1

27. −10

29. 20

31. −70

33. −120

35. −144

37. −2

39. 6

41. −14

43. −150

45. −5
47. 20
49. −13
51. −4
53. −10
55. −10
57. −5
59. 1
61. −11
63. 5
65. 1
67. 2
69. −9
71. −4
73. −8
75. −4
77. 90
79. −10
81. −72
83. −13
85. −$330
87. −$450
89. 2
91. 3
93. 1
95. 62

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SECTION OVERVIEW

2.2: The Fundamentals of Algebra

In this section we'll learn how to manipulate symbols in order to combine like terms and restore and balance equations.

2.2.1: Prelude to the Fundamentals of Algebra

2.2.2: Mathematical Expressions

2.2.3: Evaluating Algebraic Expressions

2.2.4: Simplifying Algebraic Expressions

2.2.5: Combining Like Terms

2.2.6: Solving Equations Involving Integers II

2.2.7: Applications

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2.2.1: Prelude to the Fundamentals of Algebra

As his name portends, Abu Jafr Muhammad ibn Musa al-Khwarizmi was one of the greatest Arab mathematicians of his time. While living in Baghdad during the ninth century AD he became the Chief Librarian at the House of Wisdom, a library and major center of intellectual study. In the year 820 CE, al-Khwarizmi wrote *Al-Kitab al-mukhtasar ti Hisab al-jabr w'al-muqabala*, translated to, *The Compendious Book on Calculation by Restoration and Reduction*, the first book to generalize solving equations using the principles of equality. In fact, the word algebra itself comes from *al-jabr*, meaning reunion or completion.



Figure 2.2.1.1: Pages from a 14th-century Arabic copy of the book, showing geometric solutions to two quadratic equations. (Public Domain; Muḥammad ibn Mūsā al-Khwārizmī via The Bodleian Library, University of Oxford)

Many earlier cultures had employed what we might call algebra in the service of business, land management, inheritance, and trade. The Bablyonians were solving quadratic equations in 2000 CE, but only specific equations, with specific numbers. Hindus on the Indian continent were also developing algebra along side their invention of the symbol for zero 0. But like al-Khwarizmi and the Moslem Arabs of the first millenium, to write equations these early cultures did not use symbols like x or y, or even equal signs = that we use today. al-Khwarizmi wrote absolutely everything in words! 42 would be forty-two!

Early algebra written all with words is called **rhetorical algebra**, and a thousand years ago, each mathematician had their own way of expressing it. Algebra was a language with many different dialects, and communicating it from one mathematician to another was difficult as they had to explain themselves with words. It was not until well after the Gutenberg printing press was invented in 1436 and print became standardized, that Rene Descartes, a Frenchman began to develop a modern symbolic algebra.

In this section we'll learn how to manipulate symbols in order to al-muqabalah (combine like terms) and al-jabr (restore and balance equations). But we'll use modern notation to make it easier!

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2.2.2: Mathematical Expressions

Recall the definition of a *variable* presented in Section 1.6.

Definition: Variable

A variable is a symbol (usually a letter) that stands for a value that may vary.

Let's add the definition of a *mathematical expression*.

Definition: Mathematical Expression

When we combine numbers and variables in a valid way, using operations such as addition, subtraction, multiplication, division, exponentiation, and other operations and functions as yet unlearned, the resulting combination of mathematical symbols is called a *mathematical expression*.

Thus,

$$2a,\ x + 5, \text{ and } y^2,$$

being formed by a combination of numbers, variables, and mathematical operators, are valid mathematical expressions. A mathematical expression must be *well-formed*. For example,

$$2 + \div 5x$$

is *not a valid expression* because there is no term following the plus sign (it is not valid to write +÷ with nothing between these operators). Similarly,

$$2 + 3(2$$

is not well-formed because parentheses are not balanced.

Translating Words into Mathematical Expressions

In this section we turn our attention to translating word phrases into mathematical expressions. We begin with phrases that translate into *sums*. There is a wide variety of word phrases that translate into sums. Some common examples are given in Table 2.2.2.1*a*, though the list is far from complete. In like manner, a number of phrases that translate into differences are shown in Table 2.2.2.1*b*

Table 2.2.2.1: Translating words into symbols.

Phrase	Translates to:	Phrase	Translates to:
sum of x and 12	$x + 12$	difference of x and 12	$x - 12$
4 greater than b	$b + 4$	4 less than b	$b - 4$
6 more than y	$y + 6$	7 subtracted from y	$y - 7$
44 plus r	$44 + r$	44 minus r	$44 - r$
3 larger than z	$z + 3$	3 smaller than z	$z - 3$
a) Phrases that are sums		b) Phrases that are differences	

Let's look at some examples, some of which translate into expressions involving sums, and some of which translate into expressions involving differences.

Example 1

Translate the following phrases into mathematical expressions:

a. "12 larger than x,"
b. "11 less than y," and

c. "r decreased by 9."

Solution

Here are the translations.

a. “12 larger than x” becomes $x + 12$.
b. “11 less than y” becomes $y - 11$.
c. “r decreased by 9” becomes $r - 9$.

Exercise

Translate the following phrases into mathematical expressions:

a. "13 more than x" and
b. "12 fewer than y".

Answer

(a) $x + 13$ and

(b) $y - 12$

Example 2

Let W represent the width of the rectangle. The length of a rectangle is 4 feet longer than its width. Express the length of the rectangle in terms of its width W.

Solution

We know that the width of the rectangle is W. Because the length of the rectangle is 4 feet longer that the width, we must add 4 to the width to find the length.

Length	is	4	more than	the width
Length	$=$	4	$+$	W

Thus, the length of the rectangle, in terms of its width W, is $4 + W$.

Exercise

The width of a rectangle is 5 inches shorter than its length L. Express the width of the rectangle in terms of its length L.

Answer

$L - 5$

Example 3

A string measures 15 inches is cut into two pieces. Let x represent the length of one of the resulting pieces. Express the length of the second piece in terms of the length x of the first piece.

Solution

The string has original length 15 inches. It is cut into two pieces and the first piece has length x. To find the length of the second piece, we must subtract the length of the first piece from the total length.

Length of the second piece	is	Total length	minus	the length of the first piece
Length of the second piece	$=$	15	$-$	x

Thus, the length of the second piece, in terms of the length x of the first piece, Answer: $12 + x$ is $15 - x$.

Exercise

A string is cut into two pieces, the first of which measures 12 inches. Express the total length of the string as a function of x, where x represents the length of the second piece of string.

Answer

$12 + x$

There is also a wide variety of phrases that translate into products. Some examples are shown in Table 3.2(a), though again the list is far from complete. In like manner, a number of phrases translate into quotients, as shown in Table 3.2(b).

Table 2.2.2.2: Translating words into symbols.

Phrase	Translates to:	Phrase	Translates to:
product of x and 12	$12x$	quotient of x and 12	$x/12$
4 times b	$4b$	4 divided by b	$4/b$
twice r	$2r$	the ratio of 44 to r	$44/r$
a) Phrases that are products.		b) Phrases that are differences.	

Let's look at some examples, some of which translate into expressions involving products, and some of which translate into expressions involving quotients.

Example 4

Translate the following phrases into mathematical expressions: (a) "11 times x," (b) "quotient of y and 4," and (c) "twice a."

Solution

Here are the translations. a) "11 times x" becomes $11x$. b) "quotient of y and 4" becomes $y/4$, or equivalently, $\frac{y}{4}$. c) "twice a" becomes $2a$.

Exercise

Translate into mathematical symbols: (a) "the product of 5 and x" and (b) "12 divided by y."

Answer

(a) $5x$ and (b) $12/y$.

Example 5

A plumber has a pipe of unknown length x. He cuts it into 4 equal pieces. Find the length of each piece in terms of the unknown length x.

Solution

The total length is unknown and equal to x. The plumber divides it into 4 equal pieces. To find the length of each pieces, we must divide the total length by 4.

Length of each piece	is	Total length	divided by	4
Length of each piece	$=$	x	$\div$	4

Thus, the length of each piece, in terms of the unknown length x, is $x/4$, or equivalently, $\frac{x}{4}$.

Exercise

A carpenter cuts a board of unknown length L into three equal pieces. Express the length of each piece in terms of L.

Answer

L/3

Example 6

Mary invests A dollars in a savings account paying 2% interest per year. She invests five times this amount in a certificate of deposit paying 5% per year. How much does she invest in the certificate of deposit, in terms of the amount A in the savings account?

Solution

The amount in the savings account is A dollars. She invests five times this amount in a certificate of deposit.

Amount in CD	is	5	times	Amount in savings
Amount in CD	$=$	5	$\cdot$	A

Thus, the amount invested in the certificate of deposit, in terms of the amount A in the savings account, is $5A$.

Exercise

David invests K dollars in a savings account paying 3% per year. He invests half this amount in a mutual fund paying 4% per year. Express the amount invested in the mutual fund in terms of K, the amount invested in the savings account.

Answer

$\frac{1}{2}K$

Combinations

Some phrases require combinations of the mathematical operations employed in previous examples.

Example 7

Let the first number equal x. The second number is 3 more than twice the first number. Express the second number in terms of the first number x.

Solution

The first number is x. The second number is 3 more than twice the first number.

Second number	is	3	more than	twice the first number
Second number	$=$	$3+$		$2x$

Therefore, the second number, in terms of the first number x, is $3 + 2x$.

Exercise

A second number is 4 less than 3 times a first number. Express the second number in terms of the first number y.

Answer

$3y - 4$

Example 8

The length of a rectangle is L. The width is 15 feet less than 3 times the length. What is the width of the rectangle in terms of the length L?

Solution

The length of the rectangle is L. The width is 15 feet less than 3 times the length.

Width	is	3 times the length	less	15
Width $=$		$3L-$		15

Therefore, the width, in terms of the length L, is $3L - 15$.

Exercise

The width of a rectangle is W. The length is 7 inches longer than twice the width. Express the length of the rectangle in terms of its length L.

Answer

$2W + 7$

Exercises

In Exercises 1-20, translate the phrase into a mathematical expression involving the given variable.

1. “8 times the width n ”
2. “2 times the length z ”
3. “6 times the sum of the number n and 3”
4. “10 times the sum of the number n and 8”
5. “the demand b quadrupled”
6. “the supply y quadrupled”
7. “the speed y decreased by 33”
8. “the speed u decreased by 30”
9. “10 times the width n ”
10. “10 times the length z ”
11. “9 times the sum of the number z and 2”
12. “14 times the sum of the number n and 10”
13. “the supply y doubled”
14. “the demand n quadrupled”
15. “13 more than 15 times the number p ”
16. “14 less than 5 times the number y ”
17. “4 less than 11 times the number x ”
18. “13 less than 5 times the number p ”
19. “the speed u decreased by 10”
20. “the speed w increased by 32”

---

21. Representing Numbers. Suppose n represents a whole number.

i) What does n + 1 represent?

ii) What does n + 2 represent?

iii) What does n − 1 represent?

22. Suppose 2n represents an even whole number. How could we represent the next even number after 2n?

23. Suppose 2n + 1 represents an odd whole number. How could we represent the next odd number after 2n + 1?

24. There are b bags of mulch produced each month. How many bags of mulch are produced each year?

25. Steve sells twice as many products as Mike. Choose a variable and write an expression for each man's sales.

26. Find a mathematical expression to represent the values.

i) How many quarters are in d dollars?

ii) How many minutes are in h hours?

iii) How many hours are in d days?

iv) How many days are in y years?

v) How many months are in y years?

vi) How many inches are in f feet?

vii) How many feet are in y yards?

Answers

1. 8n

3. 6(n + 3)

5. 4b

7. y − 33

9. 10n

11. 9(z + 2)

13. 2y

15. 15p + 13 17.

11x − 4

19. u − 10

21.

i) n+1 represents the next whole number after n.

ii) n+2 represents the next whole number after n + 1, or, two whole numbers after n.

iii) n − 1 represents the whole number before n.

23. 2n + 3

25. Let Mike sell p products. Then Steve sells 2p products.

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2.2.3: Evaluating Algebraic Expressions

In this section we will evaluate *algebraic expressions* for given values of the variables contained in the expressions. Here are some simple tips to help you be successful.

Tips for Evaluating Algebraic Expressions

1. Replace all occurrences of variables in the expression with open parentheses. Leave room between the parentheses to substitute the given value of the variable.
2. Substitute the given values of variables in the open parentheses prepared in the first step.
3. Evaluate the resulting expression according to the Rules Guiding Order of Operations.

Let's begin with an example.

Example 1

Evaluate the expression $x^2 - 2xy + y^2$ at $x = -3$ and $y = 2$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $x^2 - 2xy + y^2$ with open parentheses.

$$x^2 - 2xy + y^2 = (\,)^2 - 2(\,)(\,) + (\,)^2$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned} & x^2 - 2xy + y^2 && \text{Original expression.} \\ = {} & (-3)^2 - 2(-3)(2) + (2)^2 && \text{Substitute } -3 \text{ for } x \text{ and } 2 \text{ for } y. \\ = {} & 9 - 2(-3)(2) + 4 && \text{Evaluate exponents first.} \\ = {} & 9 - (-6)(2) + 4 && \text{Left to right, multiply } 2(-3) = -6. \\ = {} & 9 - (-12) + 4 && \text{Left to right, multiply: } (-6)(2) = -12. \\ = {} & 9 + 12 + 4 && \text{Add the opposite.} \\ = {} & 25 && \text{Add.} \end{aligned}$$

Exercise

If $x = -2$ and $y = -1$, evaluate $x^3 - y^3$.

Answer

-7

Example 2

Evaluate the expression $(a - b)^2$ If $a = 3$ and $b = -5$, at $a = 3$ and $b = -5$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $(a - b)^2$ with open parentheses.

$$(a - b)^2 = ((\,) - (\,))^2$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}(a-b)^2 &= ((3)-(-5))^2 && \text{Substitute 3 for } a \text{ and } -5 \text{ for } b.\\ &= (3+5)^2 && \text{Add the opposite: } (3)-(-5)=3+5\\ &= 8^2 && \text{Simplify inside parentheses: } 3+5=8\\ &= 64 && \text{Evaluate exponent: } 8^2=64\end{aligned}$$

? Exercise

If $a = 3$ and $b = -5$, evaluate $a^2 - b^2$.

Answer

−16

✓ Example 3

Evaluate the expression |a|−|b| at a = 5 and b = −7.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression |a|−|b| with open parentheses.

$$|a|-|b| = |(\,)|-|(\,)|$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}|a|-|b| = |(5)| &= |(-7)| && \text{Substitute 5 for } a \text{ and } -7 \text{ for } b.\\ &= 5-7 && \text{Absolute values first: } |(5)|=5 \text{ and } |(-7)|=7|\\ &= 5+(-7) && \text{Add the opposites: } 5-7=5+(-7).\\ &= -2 && \text{Add: } 5+(-7)=-2.\end{aligned}$$

? Exercise

If $a = 5$ and $b = -7$, evaluate $2|a| - 3|b|$.

Answer

−11

✓ Example 4

Evaluate the expression $|a - b|$ at $a = 5$ and $b = -7$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression $|a - b|$ with open parentheses.

$$|a-b| = |(\,)-(\,)|$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}|a-b| &= |(5)-(-7)| && \text{Substitute 5 for } a \text{ and } -7 \text{ for } b.\\ &= |5+7| && \text{Add the opposite: } 5-(-7)=5+7.\\ &= |12| && \text{Add: } 5+7=12.\\ &= 12 && \text{Take the absolute value: } |12|=12.\end{aligned}$$

Exercise

If $a = 5$ and $b = -7$, evaluate $|2a - 3b|$.

Answer

31

Example 5

Evaluate the expression

$$\frac{ad-bc}{a+b}$$

at $a = 5$, $b = -3$, $c = 2$, and $d = -4$.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of variables in the expression with open parentheses.

$$\frac{ad-bc}{a+b} = \frac{(\,)(\,)-(\,)(\,)}{(\,)+(\,)}$$

Secondly, replace each variable with its given value, and thirdly, follow the "Rules Guiding Order of Operations" to evaluate the resulting expression.

$$\begin{aligned}
\frac{ad-bc}{a+b} &= \frac{(5)-(-3)(2)}{(5)+(-3)} && \text{Substitute: } 5 \text{ for } a,\ -3 \text{ for } b,\ 2 \text{ for } c,\ -4 \text{ for } d.\\
&= \frac{-20-(-6)}{2} && \text{Numerator: } (5)(=4) = -20,\ (-3)(2) = -6.\\
&&& \text{Denominator: } 5+(-3) = 2.\\
&= \frac{-20+6}{2} && \text{Numerator: Add the opposite.}\\
&= \frac{-14}{2} && \text{Numerator: } -20+6 = -14.\\
&= -7 && \text{Divide.}
\end{aligned}$$

Exercise

If a = −7, b = −3, c = −15, 15, and $d = -14$, evaluate:

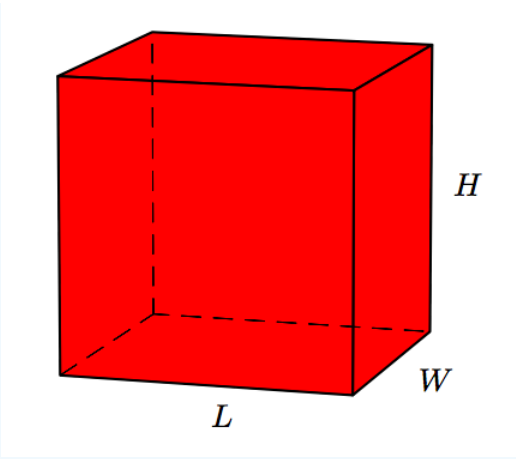
$$\frac{a^2+b^2}{c+d}$$

Answer

−2

Example 6

Pictured below is a rectangular prism.



The volume of the rectangular prism is given by the formula

$$V = LWH,$$

where L is the length, W is the width, and H is the height of the rectangular prism. Find the volume of a rectangular prism having length 12 feet, width 4 feet, and height 6 feet.

Solution

Following "Tips for Evaluating Algebraic Expressions," first replace all occurrences of of L, W, and H in the formula

$$V = LWH$$

with open parentheses.

$$V = (\,)(\,)(\,)$$

Next, substitute 12 ft for L, 4 ft for W, and 6 ft for H and simplify.

$$\begin{aligned} V &= (12\text{ft})(4\text{ft})(6\text{ft}) \\ &= 288\text{ft}^3 \end{aligned}$$

Hence, the volume of the rectangular prism is 288 cubic feet.

? Exercise

The surface area of the prism pictured in this example is given by the following formula:

$$S = 2(WH + LH + LW)$$

If L = 12, W = 4, and H = 6 feet, respectively, calculate the surface area.

Answer

288 square feet

Exercises

In Exercises 1-12, evaluate the expression at the given value of x.

1. $-3x^2 - 6x + 3$ at $x = 7$
2. $7x^2 - 7x + 1$ at $x = -8$
3. $-6x - 6$ at $x = 3$
4. $6x - 1$ at $x = -10$
5. $5x^2 + 2x + 4$ at $x = -1$

6. $4x^2 - 9x + 4$ at $x = -3$

7. $-9x - 5$ at $x = -2$

8. $-9x + 12$ at $x = 5$

9. $4x^2 + 2x + 6$ at $x = -6$

10. $-3x^2 + 7x + 4$ at $x = -7$

11. $12x + 10$ at $x = -12$

12. $-6x + 7$ at $x = 11$

In Exercises 13-28, evaluate the expression at the given values of x and y.

13. $|x|-|y|$ at $x = -5$ and $y = 4$

14. $|x|-|y|$ at $x = -1$ and $y = -2$

15. $-5x^2 + 2y^2$ at $x = 4$ and $y = 2$

16. $-5x^2 - 4y^2$ at $x = -2$ and $y = -5$

17. $|x|-|y|$ at $x = 0$ and $y = 2$

18. $|x|-|y|$ at $x = -2$ and $y = 0$

19. $|x - y|$ at $x = 4$ and $y = 5$

20. $|x - y|$ at $x = -1$ and $y = -4$

21. $5x^2 - 4xy + 3y^2$ at $x = 1$ and $y = -4$

22. $3x^2 + 5xy + 3y^2$ at $x = 2$ and $y = -1$

23. $|x - y|$ at $x = 4$ and $y = 4$

24. $|x - y|$ at $x = 3$ and $y = -5$

25. $-5x^2 - 3xy + 5y^2$ at $x = -1$ and $y = -2$

26. $3x^2 - 2xy - 5y^2$ at $x = 2$ and $y = 5$

27. $5x^2 + 4y^2$ at $x = -2$ and $y = -2$

28. $-4x^2 + 2y^2$ at $x = 4$ and $y = -5$

In Exercises 29-40, evaluate the expression at the given value of x.

29. $\frac{9+9x}{-x}$ at $x = -3$

30. $\frac{9-2x}{-x}$ at $x = -1$

31. $\frac{-8x+9}{-9+x}$ at $x = 10$

32. $\frac{2x+4}{1+x}$ at $x = 0$

33. $\frac{-4+9x}{7x}$ at $x = 2$

34. $\frac{-1-9x}{x}$ at $x = -1$

35. $\frac{-12-7x}{x}$ at $x = -1$

36. $\frac{12+11x}{3x}$ at $x = -6$

37. $\frac{6x-10}{5}$ + x at $x = -6$

38. $\frac{11x+11}{-4}$ + x at $x = 5$

39. $\frac{10x+11}{5}$ + x at $x = -4$

40. $\frac{6x+12}{-3}$ + x at x = 2

41. The formula

$$d = 16t^2$$

gives the distance (in feet) that an object falls from rest in terms of the time t that has elapsed since its release. Find the distance d (in feet) that an object falls in t = 4 seconds.

42. The formula

$$d = 16t^2$$

gives the distance (in feet) that an object falls from rest in terms of the time t that has elapsed since its release. Find the distance d (in feet) that an object falls in t = 24 seconds.

43. The formula

$$C = \frac{5(F-32)}{9}$$

gives the Celcius temperature C in terms of the Fahrenheit temperature F. Use the formula to find the Celsius temperature (◦ C) if the Fahrenheit temperature is F = 230◦ F.

44. The formula

$$C = \frac{5(F-32)}{9}$$

gives the Celcius temperature C in terms of the Fahrenheit temperature F. Use the formula to find the Celsius temperature (°C) if the Fahrenheit temperature is F = 95 °F.

45. The Kelvin scale of temperature is used in chemistry and physics. Absolute zero occurs at 0° K, the temperature at which molecules have zero kinetic energy. Water freezes at 273° K and boils at K = 373° K. To change Kelvin temperature to Fahrenheit temperature, we use the formula

$$F = \frac{9(K-273)}{5} + 32.$$

Use the formula to change 28° K to Fahrenheit.

46. The Kelvin scale of temperature is used in chemistry and physics. Absolute zero occurs at 0° K, the temperature at which molecules have zero kinetic energy. Water freezes at 273° K and boils at K = 373° K. To change Kelvin temperature to Fahrenheit temperature, we use the formula

$$F = \frac{9(K-273)}{5} + 32.$$

Use the formula to change 248° K to Fahrenheit.

47. A ball is thrown vertically upward. Its velocity t seconds after its release is given by the formula

$$v = v0 - gt,$$

where v_0 is its initial velocity, g is the acceleration due to gravity, and v is the velocity of the ball at time t. The acceleration due to gravity is g = 32 feet per second per second. If the initial velocity of the ball is v_0 = 272 feet per second, find the speed of the ball after t = 6 seconds.

48. A ball is thrown vertically upward. Its velocity t seconds after its release is given by the formula

$$v = v_0 - gt,$$

where v_0 is its initial velocity, g is the acceleration due to gravity, and v is the velocity of the ball at time t. The acceleration due to gravity is g = 32 feet per second per second. If the initial velocity of the ball is v_0 = 470 feet per second, find the speed of the ball after t = 4 seconds.

49. **Even numbers**. Evaluate the expression 2n for the following values:

i) n = 1

ii) n = 2

iii) n = 3

iv) n = −4

v) n = −5

vi) Is the result always an even number? Explain.

50. **Odd numbers**. Evaluate the expression 2n + 1 for the following values:

i) n = 1

ii) n = 2

iii) n = 3

iv) n = −4

v) n = −5

vi) Is the result always an odd number? Explain.

Answers

1. −186
3. −24
5. 7
7. 13
9. 138
11. −134
13. 1
15. −72
17. −2
19. 1
21. 69
23. 0
25. 9
27. 36
29. −6
31. −71
33. 1
35. 5
37. 46
39. −29
41. 256 feet
43. 110 degrees
45. −409° F

47. 80 feet per second

49.

i) 2

ii) 4

iii) 6

iv) −8

v) −10

vi) Yes, the result will always be an even number because 2 will always be a factor of the product 2n.

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2.2.4: Simplifying Algebraic Expressions

Recall the commutative and associative properties of multiplication.

The Commutative Property of Multiplication. If a and b are any integers, then

$$a \cdot b = b \cdot a, \text{ or equivalently, } ab = ba.$$

The Associative Property of Multiplication. If a, b, and c are any integers, then

$$(a \cdot b) \cdot c = a \cdot (b \cdot c), \text{ or equivalently, } (ab)c = a(bc).$$

The commutative property allows us to change the order of multiplication without affecting the product or answer. The associative property allows us to regroup without affecting the product or answer.

Example 1

Simplify: $2(3x)$.

Solution

Use the associative property to regroup, then simplify.

$$\begin{aligned} 2(3x) &= (2 \cdot 3)x \quad \text{Regrouping with the associative property.} \\ &= 6x \quad \text{Simplify: } 2 \cdot 3 = 6. \end{aligned}$$

Exercise

Simplify: $-5(7y)$

Answer

$-35y$

The statement $2(3x)=6x$ is an *identity*. That is, the left-hand side and right-hand side of $2(3x)=6x$ are the same for all values of x. Although the derivation in Example 1 should be the proof of this statement, it helps the intuition to check the validity of the statement for one or two values of x.

If $x = 4$, then

$$\begin{aligned} 2(3x) &= 2(3(4)) \\ &= 2(12) \\ &= 24 \end{aligned} \quad \text{and} \quad \begin{aligned} 6x &= 6(4) \\ &= 24 \end{aligned}$$

If $x = -5$, then

$$\begin{aligned} 2(3x) &= 2(3(-5)) \\ &= 2(-15) \\ &= -30 \end{aligned} \quad \text{and} \quad \begin{aligned} 6x &= 6(-5) \\ &= -30 \end{aligned}$$

The above calculations show that $2(3x)=6x$ for both $x = 4$ and $x = -5$. Indeed, the statement $2(3x)=6x$ is true, regardless of what is substituted for x.

Example 2

Simplify: $(-3t)(-5)$.

Solution

In essence, we are multiplying three numbers, −3, *t*, and −5, but the grouping symbols ask us to multiply the −3 and the t first. The associative and commutative properties allow us to change the order and regroup.

$$\begin{aligned}(-3t)(-5) &= ((-3)(-5))t \quad \text{Change the order and regroup.}\\ &= 15t \quad \text{Multiply: } (-3)(-5) = 15.\end{aligned}$$

? Exercise

Simplify: (−8*a*)(5)

Answer

−40*a*

✓ Example 3

Simplify: (−3*x*)(−2*y*)

Solution

In essence, we are multiplying four numbers, −3, x, −2, and y, but the grouping symbols specify a particular order. The associative and commutative properties allow us to change the order and regroup.

$$\begin{aligned}(-3x)(-2y) &= ((-3)(-2))(xy) \quad \text{Change the order and regroup.}\\ &= 6xy \quad \text{Multiply: } (-3)(-2) = 6.\end{aligned}$$

? Exercise

Simplify: (−4*a*)(5*b*)

Answer

−20*ab*

Speeding Things Up

The meaning of the expression $2 \cdot 3 \cdot 4$ is clear. Parentheses and order of operations are really not needed, as the commutative and associative properties explain that it doesn't matter which of the three numbers you multiply together first.

- You can multiply 2 and 3 first:

$$\begin{aligned}2 \cdot 3 \cdot 4 &= (2 \cdot 3) \cdot 4\\ &= 6 \cdot 4\\ &= 24.\end{aligned}$$

- Or you can multiply 3 and 4 first:

$$\begin{aligned}2 \cdot 3 \cdot 4 &= 2 \cdot (3 \cdot 4)\\ &= 2 \cdot 12\\ &= 24.\end{aligned}$$

- Or you can multiply 2 and 4 first:

$$\begin{aligned}2 \cdot 3 \cdot 4 &= (2 \cdot 4) \cdot 3\\ &= 8 \cdot 3\\ &= 24.\end{aligned}$$

So, it does not matter which two factors you multiply first.

Of course, this would not be the case if there were a mixture of multiplication and other operators (division, addition, subtraction). Then we would have to strictly follow the "Rules Guiding Order of Operations." But if the only operator is multiplication, the order

of multiplication is irrelevant.

Thus, when we see 2(3x), as in Example 1, we should think "It's all multiplication and it doesn't matter which two numbers I multiply together first, so I'll multiply the 2 and the 3 and get 2(3x)=6x."

Our comments apply equally well to a product of four or more factors. It simply doesn't matter how you group the multiplication. So, in the case of (−3x)(−2y), as in Example 3, find the product of −2 and −3 and multiply the result by the product of x and y. That is, (−3x)(−2y)=6xy.

Example 4

Simplify: (2a)(3b)(4c).

Solution

The only operator is multiplication, so we can order and group as we please. So, we'll take the product of 2, 3, and 4, and multiply the result by the product of a, b, and c. That is,

$$(2a)(3b)(4c) = 24abc$$

Exercise

Simplify: (−3x)(−2y)(−4z)

Answer

−24xyz.

The Distributive Property

Multiplication is distributive with respect to addition.

The Distributive Property

If a, b, and c are any integers, then

$a \cdot (b + c) = a \cdot b + a \cdot c$, or equivalently, $a(b + c) = ab + ac$.

For example, if we follow the "Rules Guiding Order of Operations" and first evaluate the expression inside the parentheses, then

$$\begin{aligned} 3(4+5) &= 3(9) && \text{Parentheses first: } 4+5=9. \\ &= 27. && \text{Multiply: } 3(9)=27. \end{aligned}$$

But if we "distribute" the 3, we get the same answer.

$$\begin{aligned} 3(4+5) &= 3(4+5) && \text{Each number in parentheses is multiplied by the number 3 outside the parentheses.} \\ &= 3(4)+3(5) \\ &= 12+15 && \text{Multiply first: } 3(4)=12,\ 3(5)=15. \\ &= 27 && \text{Add.} \end{aligned}$$

Example 5

Use the distributive property to simplify: 3(4x + 5).

Solution

Distribute the 3.

$$\begin{aligned} 3(4x+5) &= 3(4x)+3(5) && \text{Each number in parentheses is multiplied by the number 3 outside the parentheses.} \\ &= 12x+15 && \text{Multiply first: } 3(4x)=12x,\ 3(5)=15. \end{aligned}$$

Exercise

Use the distributive property to simplify: 2(5z+7).

Answer

10z + 14

Multiplication is also distributive with respect to subtraction.

The Distributive Property

If *a*, *b*, and *c* are any integers, then

$a \cdot (b - c) = a \cdot b - a \cdot c$, or equivalently, $a(b - c) = ab - ac$.

The application of this form of the distributive property is identical to the first, the only difference being the subtraction symbol.

Example 6

Use the distributive property to simplify: 5(3*x* − 2).

Solution

Distribute the 5.

$$\begin{aligned} 5(3x-2) &= 5(3x)-5(2) && \text{Each number in parentheses is multiplied by the number 5 outside the parentheses.} \\ &= 15x-10 && \text{Multiply first: } 5(3x)=15x, 5(2)=10. \end{aligned}$$

Exercise

Use the distributive property to simplify: 7(4*a* − 5).

Answer

28*a* − 35

Example 7

Remove parentheses: (a) −9(2*t* + 7), and (b) −5(4 − 3*y*).

Solution

a) Use the distributive property.

$$\begin{aligned} -9(2t+7) &= -9(2t)+(-9)(7) && \text{Distribute multiplication by } -9. \\ &= -18t+(-63) && \text{Multiply: } -9(2t)=-18t \text{ and } -9(7)=-63. \\ &= -18t-63 && \text{Write the answer in simpler form.} \\ & && \text{Adding } -63 \text{ is the same as subtracting } 63. \end{aligned}$$

b) Use the distributive property.

$$\begin{aligned} -5(4-3y) &= -5(4)-(-5)(3y) && \text{Distribute multiplication by } -5. \\ &= -20-(-15y) && \text{Multiply: } -5(4)=-20 \text{ and } -5(3y)=-15y. \\ &= -18t-63 && \text{Write the answer in simpler form.} \\ & && \text{Subtracting } -15y \text{ is the same as adding } 15y. \end{aligned}$$

Exercise

Remove parentheses: $-3(4t - 11)$.

Answer

$-12t + 33$

Writing Mathematics

Example 7 stresses the importance of using as few symbols as possible to write your final answer. Hence, $-18t - 63$ is favored over $-18t + (-63)$ and $-20 + 15y$ is favored over $-20 - (-15y)$. You should always make these final simplifications.

Moving a Bit Quicker

Once you've applied the distributive property to a number of problems, showing all the work as in Example 7, you should try to eliminate some of the steps. For example, consider again Example 7(a). It's not difficult to apply the distributive property without writing down a single step, getting:

$$-9(2t+7) = -18t - 63.$$

Here's the thinking behind this technique:

1. First, multiply -9 times $2t$, getting $-18t$.
2. Second, multiply -9 times $+7$, getting -63.

Note that this provides exactly the same solution found in Example 7(a).

Let try this same technique on Example 7(b).

$$-5(4 - 3y) = -20 + 15y$$

Here's the thinking behind this technique.

1. First, multiply -5 times 4, getting -20.
2. Second, multiply -5 times $-3y$, getting $+15y$.

Note that this provides exactly the same solution found in Example 7(b).

Extending the Distributive Property

Suppose that we add an extra term inside the parentheses.

Distributive Property

If a, b, c, and d are any integers, then

$$a(b + c + d) = ab + ac + ad.$$

Note that we "distributed" the a times each term inside the parentheses. Indeed, if we added still another term inside the parentheses, we would "distribute" a times that term as well.

Example 8

Remove parentheses: $-5(2x - 3y + 8)$.

Solution

We will use the "quicker" technique, "distributing" -5 times each term in the parentheses mentally.

$$-5(2x - 3y + 8) = -10x + 15y - 40$$

Here is our thought process:

1. First, multiply −5 times 2x, getting −10x.
2. Second, multiply −5 times −3y, getting +15y.
3. Third, multiply −5 times +8, getting −40.

? Exercise

Remove parentheses: −3(4a − 5b + 7)

Answer

−12a + 15b − 21

✓ Example 9

Remove parentheses: −4(−3a + 4b − 5c + 12).

Solution

We will use the "quicker" technique, "distributing" −4 times each term in the parentheses mentally.

$$-4(-3a+4b-5c+12)=12a-16b+20c-48$$

Here is our thought process:

1. First, multiply −4 times −3a, getting 12a.
2. Second, multiply −4 times +4b, getting −16b.
3. Third, multiply −4 times −5c, getting +20c.
4. Fourth, multiply −4 times +12, getting −48.

? Exercise

Remove parentheses: −2(−2x + 4y − 5z − 11).

Answer

4x − 8y + 10z + 22

Distributing a Negative

It is helpful to recall that negating is equivalent to multiplying by −1.

Multiplying by −1

Let a be any integer, then

$$(-1)a = -a \text{ and } -a = (-1)a.$$

We can use this fact, combined with the distributive property, to negate a sum.

✓ Example 10

Remove parentheses: −(a + b).

Solution

Change the negative symbol into multiplying by −1, then distribute the −1.

$$\begin{aligned} -(a+b) &= (-1)(a+b) && \text{Negating is equivalent to multiplying by } -1. \\ &= -a-b && \text{Distribute the } -1. \end{aligned}$$

We chose to use the "quicker" technique of "distributing" the −1. Here is our thinking:

1. Multiply -1 times a, getting $-a$.
2. Multiply -1 times $+b$, getting $-b$.

Exercise

Remove parentheses: $-(4a - 3c)$

Answer

$-4a + 3c$

The results in Example 10 and Example 11 show us how to negate a sum: Simply negate each term of the sum. Positive terms change to negative, negative terms turn to positive.

Negating a Sum

To negate a sum, simply negate each term of the sum. For example, if a and b are integers, then

$$-(a + b) = -a - b \text{ and } -(a - b) = -a + b.$$

Example 12

Remove parentheses: $-(5 - 7u + 3t)$.

Solution

Simply negate each term in the parentheses.

$$-(5 - 7u + 3t) = -5+7u - 3t$$

Exercise

Remove parentheses: $-(5 - 2x + 4y - 5z)$

Answer

$-5+2x - 4y + 5z$

Exercises

In Exercises 1-20, use the associative and commutative properties of multiplication to simplify the expression.

1. $10(-4x)$
2. $7(-8x)$
3. $(-10x)(-3)$
4. $(-5x)(-8)$
5. $-5(3x)$
6. $9(6x)$
7. $(-4x)10$
8. $(-10x)(-6)$
9. $(5x)3$
10. $(3x)3$
11. $(5x)10$
12. $(-2x)(-10)$

13. −9(−7x)
14. −10(5x)
15. 6(2x)
16. 3(−10x)
17. −8(−9x)
18. 3(−3x)
19. (6x)7
20. (−8x)(−5)

In Exercises 21-44, simplify the expression.

21. 8(7x + 8)
22. −2(5x + 5)
23. 9(−2 + 10x)
24. −9(4 + 9x)
25. −(−2x + 10y − 6)
26. −(−6y + 9x − 7)
27. 2(10 + x)
28. 2(10 − 6x)
29. 3(3 + 4x)
30. 3(4 + 6x)
31. −(−5 − 7x + 2y)
32. −(4x − 8 − 7y)
33. 4(−6x + 7)
34. 6(4x + 9)
35. 4(8x − 9)
36. 10(−10x + 1)
37. −(4 − 2x − 10y)
38. −(−4x + 6 − 8y)
39. −(−5x +1+9y)
40. −(−10 − 5x − 4y)
41. −(6x + 2 − 10y)
42. −(6x + 4 − 10y)
43. −(−3y − 4+4x)
44. −(−7 − 10x + 7y)

Answers

1. −40x
3. 30x
5. −15x

7. −40x

9. 15x

11. 50x

13. 63x

15. 12x

17. 72x

19. 42x

21. 56x + 64

23. −18 + 90x

25. 2x − 10y + 6

27. 20 + 2x

29. 9 + 12x

31. 5+7x − 2y

33. −24x + 28

35. 32x − 36

37. −4+2x + 10y

39. 5x − 1 − 9y

41. −6x − 2 + 10y

43. 3y + 4 − 4x

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2.2.5: Combining Like Terms

We begin our discussion with the definition of a *term*.

Definition: Term

A **term** is a single number or variable, or it can be the product of a number (called its coefficient) and one or more variables (called its variable part). The terms in an algebraic expression are separated by addition symbols.

Example 1

Identify the terms in the algebraic expression

$$3x^2 + 5xy + 9y^2 + 12$$

For each term, identify its coefficient and variable part.

Solution

In tabular form, we list each term of the expression 3x^2 + 5xy + 9y^2 + 12, its coefficient, and its variable part.

Term	Coefficient	Variable Part
$3x^2$	3	x^2
$5xy$	5	xy
$9y^2$	9	y^2
12	12	None

Exercise

How many terms are in the algebraic expression $3x^2 + 2xy - 3y^2$?

Answer

3

Example 2

Identify the terms in the algebraic expression

$$a^3 - 3a^2b + 3ab^2 - b^3$$

For each term, identify its coefficient and variable part.

Solution

The first step is to write each difference as a sum, because the terms of an expression are defined above to be those items separated by addition symbols.

$$a^3 + (-3a^2b) + 3ab^2 + (-b^3)$$

In tabular form, we list each term of the expression $a^3 + (-3a^2b) + 3ab^2 + (-b^3)$, its coefficient, and its variable part.

Term	Coefficient	Variable Part
a^3	1	a^3
$-3a^2b$	-3	a^2b

Term	Coefficient	Variable Part
$3ab^2$	3	ab^2
$-b^3$	-1	b^3

Exercise

How many terms are in the algebraic expression $11 - a^2 - 2ab + 3b^2a$?

Answer

4

Like Terms

We define what is meant by “like terms” and “unlike terms.”

Definition

Like and Unlike Terms. The variable parts of two terms determine whether the terms are like terms or unlike terms.

Like Terms. Two terms are called like terms if they have identical variable parts, which means that the terms must contain the same variables raised to the same exponential powers.

Unlike Terms. Two terms are called unlike terms if their variable parts are different.

Example 3

Classify each of the following pairs as either *like terms* or *unlike terms*: (a) $3x$ and $-7x$, (b) $2y$ and $3y^2$, (c) $-3t$ and $5u$, and (d) $-4a^3$ and $3a^3$.

Solution

Like terms must have *identical* variable parts.

a. $3x$ and $-7x$ have identical variable parts. They are “like terms.”
b. $2y$ and $3y^2$ do not have identical variable parts (the exponents differ). They are “unlike terms.”
c. $-3t$ and $5u$ do not have identical variable parts (different variables). They are “unlike terms.”
d. $-4a^3$ and $3a^3$ have identical variable parts. They are “like terms.”

Exercise

Are $-3xy$ and $11xy$ *like* or *unlike* terms?

Answer

Like terms

Combining Like Terms

When using the distributive property, it makes no difference whether the multiplication is on the left or the right, one still distributes the multiplication times each term in the parentheses.

Distributive Property

If a, b, and c are integers, then

$$a(b + c) = ab + ac \text{ and } (b + c)a = ba + ca.$$

In either case, you distribute a times each term of the sum.

"Like terms" can be combined and simplified. The tool used for combining like terms is the distributive property. For example, consider the expression $3y + 7y$, composed of two "like terms" with a common variable part. We can use the distributive property and write

$$3y + 7y = (3 + 7)y$$

Note that we are using the distributive property in reverse, "factoring out" the common variable part of each term. Checking our work, note that if we redistribute the variable part y times each term in the parentheses, we are returned to the original expression $3y + 7y$.

Example 4

Use the distributive property to combine like terms (if possible) in each of the following expressions: (a) $-5x^2 - 9x^2$, (b) $-5ab + 7ab$, (c) $4y^3 - 7y^2$, and (d) $3xy^2 - 7xy^2$.

Solution

If the terms are "like terms," you can use the distributive property to "factor out" the common variable part.

a) Factor out the common variable part x^2.

$$\begin{aligned} -5x^2 - 9x^2 &= (-5 - 9)x^2 \quad \text{Use the distributive property.} \\ &= -14x^2 \quad \text{Simplify: } -5 - 9 = -5 + (-9) = -14. \end{aligned}$$

b) Factor out the common variable part ab.

$$\begin{aligned} -5ab + 7ab &= (-5 + 7)ab \quad \text{Use the distributive property.} \\ &= 2ab \quad \text{Simplify: } -5 + 7 = 2. \end{aligned}$$

c) The terms in the expression $4y^3 - 7y^2$ have different variable parts (the exponents are different). These are "unlike terms" and cannot be combined.

d) Factor out the common variable part xy^2.

$$\begin{aligned} 3xy^2 - 7xy^2 &= (3 - 7)xy^2 \quad \text{Use the distributive property.} \\ &= -4xy^2 \quad \text{Simplify: } 3 - 7 = 3 + (-7) = -4. \end{aligned}$$

Exercise

Simplify: $-8z - 11z$

Answer

$-19z$

Speeding Things Up a Bit

Once you've written out all the steps for combining like terms, like those shown in Example 4, you can speed things up a bit by following this rule:

Combining Like Terms

To combine like terms, simply add their coefficients and keep the common variable part.

Thus for example, when presented with the sum of two like terms, such as in $5x + 8x$, simply add the coefficients and repeat the common variable part; that is, $5x + 8x = 13x$.

Example 5

Combine like terms:

a. $-9y - 8y$,
b. $-3y^5 + 4y^5$ and
c. $-3u^2 + 2u^2$.

Solution

a) Add the coefficients and repeat the common variable part. Therefore,

$$-9y - 8y = -17y.$$

b) Add the coefficients and repeat the common variable part. Therefore,

$$-3y^5 + 4y^5 = 1y^5.$$

However, note that $1y^5 = y^5$. Following the rule that the final answer should use as few symbols as possible, a better answer is $-3y^5 + 4y^5 = y^5$.

c) Add the coefficients and repeat the common variable part. Therefore,

$$-3u^2 + 2u^2 = (-1)u^2.$$

However, note that $(-1)u^2 = -u^2$. Following the rule that the final answer should use as few symbols as possible, a better answer is $-3u^2 + 2u^2 = -u^2$.

Exercise

Combine: $-3x$, and $-4x^2$

Answer

$-7x^2$

Simplify

A frequently occurring instruction asks the reader to *simplify* an expression.

Simplify

The instruction simplify is a generic term that means "try to write the expression in its most compact form, using the fewest symbols possible."

One way you can accomplish this goal is by combining like terms when they are present.

Example 6

Simplify: $2x + 3y - 5x + 8y$.

Solution

Use the commutative property to reorder terms and the associative and distributive properties to regroup and combine like terms.

$$\begin{aligned} 2x + 3y - 5x + 8y &= (2x - 5x) + (3y + 8y) && \text{Reorder and regroup.} \\ &= -3x + 11y && \text{Combine like terms:} \\ & && 2x - 5x = -3x \text{ and } 3y + 8y = 11y. \end{aligned}$$

Alternate solution

Of course, you do not need to show the regrouping step. If you are more comfortable combining like terms in your head, you are free to present your work as follows:

$$2x+3y-5x+8y=-3x+11y.$$

Exercise

Simplify: −3*a* + 4*b* − 7*a* − 9*b*

Answer

−10*a* − 5*b*

Example 7

Simplify: −2*x* − 3 − (3*x* + 4).

Solution

First, distribute the negative sign.

$$-2x-3-(3x+4)=-2x-3-3x-4 \quad -(3x+4)=-3x-4.$$

Next, use the commutative property to reorder, then the associative property to regroup. Then combine like terms.

$$\begin{aligned} &=(-2x-3x)+(-3-4) && \text{Reorder and regroup.}\\ &=-5x+(-7) && \text{Combine like terms:}\\ & && -2x-3x=-5x.\\ &=-5x-7 && \text{Simplify:}\\ & && -5x+(-7)=-5x-7. \end{aligned}$$

Alternate solution

You may skip the second step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$\begin{aligned} -2x-3-(3x+4)&=-2x-3-3x-4 && \text{Distribute negative sign.}\\ &=-5x-7 && \text{Combine like terms.} \end{aligned}$$

Exercise

Simplify: −9*a* − 4 − (4*a* − 8)

Answer

−13*a* + 4

Example 8

Simplify: 2(5 − 3*x*) − 4(*x* + 3).

Solution

Use the distributive property to expand, then use the commutative and associative properties to group the like terms and combine them.

\[\begin{aligned} 2(5-3x)-4(x+3) = 10-6x-4x-12 ~ & \textcolor{red}{ \text{ Use the distributive property.}} \\ =(-6x-4x)+(10-12) ~ & \textcolor{red}{ \text{ Group like terms.}} \\ =-10x-2 ~ & \textcolor{red}{ \text{ Combine like terms: }} \\ ~ & \textcolor{red}{-6x-4x=-10x \text{ and}} \\ ~ & \textcolor{red}{10-12=-2.} \end{aligned}\nonumber \]

Alternate solution

You may skip the second step if you wish, simply combining like terms mentally. That is, it is entirely possible to order your work as follows:

$$\begin{aligned} 2(5-3x)-4(x+3) &= 10-6x-4x-12 \quad \text{Distribute.} \\ &= -10x-2 \quad \text{Combine like terms.} \end{aligned}$$

Exercise

Simplify: $-2(3a-4)-2(5-a)$

Answer

$-4a-2$

Example 9

Simplify: $-8(3x^2y-9xy)-8(-7x^2-8xy)$

Solution

We will proceed a bit quicker with this solution, using the distributive property to expand, then combining like terms mentally.

$$\begin{aligned} -8(3x^2y-9xy)-8(-7x^2y-8xy) &= -24x^2y+72xy+56x^2y+64xy \\ &= 32x^2y+136xy \end{aligned}$$

Exercise

Simplify: $(a^2-2ab)-2(3ab+a^2)$

Answer

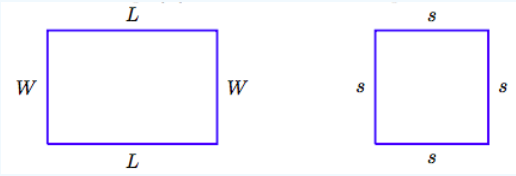
$-a^2-8ab$

Applications

We can simplify a number of useful formulas by combining like terms.

Example 10

Find the perimeter P of the (a) rectangle and (b) square pictured below. Simplify your answer as much as possible.



Solution

The perimeter of any polygonal figure is the sum of the lengths of its sides.

a) To find the perimeter P of the rectangle, sum its four sides.

$$P = L+W+L+W.$$

Combine like terms.

$$P = 2L+2W.$$

b) To find the perimeter P of the square, sum its four sides.

$$P = s + s + s + s.$$

Combine like terms.

$$P = 4s.$$

? Exercise

A regular hexagon has six equal sides, each with length *x*. Find its perimeter in terms of *x*.

Answer

P = 6*x*

Sometimes it is useful to replace a variable with an expression containing another variable.

✔ Example 11

The length of a rectangle is three feet longer than twice its width. Find the perimeter *P* of the rectangle in terms of its width alone.

Solution

From the previous problem, the perimeter of the rectangle is given by

$$P = 2L + 2W,$$

where *L* and *W* are the length and width of the rectangle, respectively. This equation gives the perimeter in terms of its length and width, but we're asked to get the perimeter in terms of the width alone. However, we're also given the fact that the length is three feet longer than twice the width.

$$\begin{array}{rcrcr} cyanLength & \text{is} & \text{Three Feet} & \text{longer than} & \text{Twice the Width} \\ L & = & 3 & + & 2W \end{array}$$

Because *L* = 3+2*W*, we can replace *L* with 3+2*W* in the perimeter equation 3.1.

$$P = 2L + 2W$$

$$P = 2(3 + 2W) + 2W$$

Use the distributive property, then combine like terms.

$$P = 6 + 4W + 2W$$

$$P = 6 + 6W.$$

This last equation gives the perimeter *P* in terms of the width *W* alone.

? Exercise

The length *L* of a rectangle is 5 meters longer than twice its width *W*. Find the perimeter *P* of the rectangle in terms of its width *W*.

Answer

Answer: *P* = 6*W* + 10

Example 12

The width of a rectangle is two feet less than its length. Find the perimeter P of the rectangle in terms of its length alone.

Solution

Again, the perimeter of a rectangle is given by the equation

$$P = 2L + 2W,$$

where L and W are the length and width of the rectangle, respectively. This equation gives the perimeter in terms of its length and width, but we're asked to get the perimeter in terms of the length alone.

However, we're also given the fact that the width is two feet less than the length.

Width	is	Length	minus	Two feet
W	$=$	L	$-$	2

Because $W = L-2$, we can replace W with $L-2$ in the perimeter equation 3.2.

$$P = 2L + 2W$$

$$P = 2L + 2(L - 2)$$

Use the distributive property, then combine like terms.

$$P = 2L + 2L - 4$$

$$P = 4L - 4.$$

This last equation gives the perimeter P in terms of the length L alone.

Exercise

The width W of a rectangle is 5 feet less than twice its width L. Find the perimeter P of the rectangle in terms of its length L.

Answer

$P = 6L - 10$

Exercises

In Exercises 1-16, combine like terms by first using the distributive property to factor out the common variable part, and then simplifying.

1. $17xy^2 + 18xy^2 + 20xy^2$
2. $13xy - 3xy + xy$
3. $-8xy^2 - 3xy^2 - 10xy^2$
4. $-12xy - 2xy + 10xy$
5. $4xy - 20xy$
6. $-7y^3 + 15y^3$
7. $12r - 12r$
8. $16s - 5s$
9. $-11x - 13x + 8x$
10. $-9r - 10r + 3r$
11. $-5q + 7q$
12. $17n + 15n$

13. $r - 13r - 7r$

14. $19m + m + 15m$

15. $3x^3 - 18x^3$

16. $13x^2y + 2x^2y$

In Exercises 17-32, combine like terms by first rearranging the terms, then using the distributive property to factor out the common variable part, and then simplifying.

17. $-8 + 17n + 10 + 8n$

18. $11 + 16s - 14 - 6s$

19. $-2x^3 - 19x^2y - 15x^2y + 11x^3$

20. $-9x^2y - 10y^3 - 10y^3 + 17x^2y$

21. $-14xy - 2x^3 - 2x^3 - 4xy$

22. $-4x^3 + 12xy + 4xy - 12x^3$

23. $-13 + 16m + m + 16$

24. $9 - 11x - 8x + 15$

25. $-14x^2y - 2xy^2 + 8x^2y + 18xy^2$

26. $-19y^2 + 18y^3 - 5y^2 - 17y^3$

27. $-14x^3 + 16xy + 5x^3 + 8xy$

28. $-16xy + 16y^2 + 7xy + 17y^2$

29. $9n + 10 + 7 + 15n$

30. $-12r + 5 + 17 + 17r$

31. $3y + 1 + 6y + 3$

32. $19p + 6 + 8p + 13$

In Exercises 33-56, simplify the expression by first using the distributive property to expand the expression, and then rearranging and combining like terms mentally.

33. $-4(9x^2y + 8) + 6(10x^2y - 6)$

34. $-4(-4xy + 5y^3) + 6(-5xy - 9y^3)$

35. $3(-4x^2 + 10y^2) + 10(4y^2 - x^2)$

36. $-7(-7x^3 + 6x^2) - 7(-10x^2 - 7x^3)$

37. $-s + 7 - (-1 - 3s)$

38. $10y - 6 - (-10 - 10y)$

39. $-10q - 10 - (-3q + 5)$

40. $-2n + 10 - (7n - 1)$

41. $7(8y + 7) - 6(8 - 7y)$

42. $-6(-5n - 4) - 9(3 + 4n)$

43. $7(10x^2 - 8xy^2) - 7(9xy^2 + 9x^2)$

44. $10(8x^2y - 10xy^2) + 3(8xy^2 + 2x^2y)$

45. $-2(6 + 4n) + 4(-n - 7)$

46. $-6(-2 - 6m) + 5(-9m + 7)$

47. $8 - (4 + 8y)$

48. $-1 - (8 + s)$

49. $-8(-n + 4) - 10(-4n + 3)$

50. $3(8r - 7) - 3(2r - 2)$

51. $-5 - (10p + 5)$

52. $-1 - (2p - 8)$

53. $7(1 + 7r) + 2(4 - 5r)$

54. $(5 - s) + 10(9 + 5s)$

55. $-2(-5 - 8x^2) - 6(6)$

56. $8(10y^2 + 3x^3) - 5(-7y^2 - 7x^3)$

---

57. The length L of a rectangle is 2 feet longer than 6 times its width W. Find the perimeter of the rectangle in terms of its width alone.

58. The length L of a rectangle is 7 feet longer than 6 times its width W. Find the perimeter of the rectangle in terms of its width alone.

59. The width W of a rectangle is 8 feet shorter than its length L. Find the perimeter of the rectangle in terms of its length alone.

60. The width W of a rectangle is 9 feet shorter than its length L. Find the perimeter of the rectangle in terms of its length alone.

61. The length L of a rectangle is 9 feet shorter than 4 times its width W. Find the perimeter of the rectangle in terms of its width alone.

62. The length L of a rectangle is 2 feet shorter than 6 times its width W. Find the perimeter of the rectangle in terms of its width alone.

Answers

1. $55xy^2$

3. $-21xy^2$

5. $-16xy$

7. 0

9. $-16x$

11. $2q$

13. $-19r$

15. $-15x^3$

17. $2 + 25n$

19. $9x^3 - 34x^2y$

21. $-18xy - 4x^3$

23. $3 + 17m$

25. $-6x^2y + 16xy^2$

27. $-9x^3 + 24xy$

29. $24n + 17$

31. $9y + 4$

33. $24x^2y - 68$

35. $-22x^2 + 70y^2$

37. $2s + 8$

39. $-7q - 15$

41. $98y + 1$

43. $7x^2 - 119xy^2$

45. $-40 - 12n$

47. $4 - 8y$

49. $48n - 62$

51. $-10 - 10p$

53. $15 + 39r$

55. $-26 + 16x^2$

57. $4 + 14W$

59. $4L - 16$

61. $10W - 18$

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2.2.6: Solving Equations Involving Integers II

We return to solving equations involving integers, only this time the equations will be a bit more advanced, requiring the use of the distributive property and skill at combining like terms. Let's begin.

Example 1

Solve for x: $7x - 11x = 12$.

Solution

Combine like terms.

$$\begin{aligned} 7x - 11x &= 12 \quad \text{Original equation.} \\ = 4x &= 12 \quad \text{Combine like terms: } 7x - 11x = -4x. \end{aligned}$$

To undo the effect of multiplying by −4, divide both sides of the last equation by −4.

$$\begin{aligned} \frac{-4x}{-4} &= \frac{12}{-4} \quad \text{Divide both sides by } -4. \\ x &= -3 \quad \text{Simplify: } 12/(-4) = -3. \end{aligned}$$

Check

Substitute −3 for x in the original equation.

$$\begin{aligned} 7x - 11x &= 12 \quad \text{Original equation.} \\ 7(-3) - 11(-3) &= 12 \quad \text{Substitute } -3 \text{ for } x. \\ -21 + 33 &= 12 \quad \text{On the left, multiply first.} \\ 12 &= 12 \quad \text{On the left, add.} \end{aligned}$$

Because the last line of the check is a true statement, −3 is a solution of the original equation.

Exercise

Solve for x: $-6x - 5x = 22$.

Answer

$x = -2$

Example 2

Solve for x: $12 = 5x - (4 + x)$.

Solution

To take the negative of a sum, negate each term in the sum (change each term to its opposite). Thus, $-(4 + x) = -4 - x$.

$$\begin{aligned} 12 &= 5x - (4 + x) \quad \text{Original equation.} \\ 12 &= 5x - 4 - x \quad -(4 + x) = -4 - x. \\ 12 &= 4x - 4 \quad \text{Combine like terms: } 5x - x = 4x. \end{aligned}$$

To undo the effect of subtracting 4, add 4 to both sides of the last equation.

$$\begin{aligned} 12 + 4 &= 4x - 4 + 4 \quad \text{Add 4 to both sides.} \\ 16 &= 4x \quad \text{Simplify both sides.} \end{aligned}$$

To undo the effect of multiplying by 4, divide both sides of the last equation by 4.

\[\begin{aligned} \frac{16}{4} = \frac{4x}{4} ~ & \textcolor{red}{ \text{ Divide both sides by 4.}} \\ 4 = x ~ & \textcolor{red}{ \text{ Simplify: 16/4=4.} } \end{aligned}]

Check

Substitute 4 for x in the original equation.

$$\begin{aligned} 12 &= 5x - (4 + x) \quad \text{Original equation.} \\ 12 &= 5(4) - (4 + 4) \quad \text{Substitute 4 for } x. \\ 12 &= 20 - 8 \quad \text{On the right, } 5(4) = 20 \text{ and evaluate} \\ & \qquad \text{parentheses: } 4 + 4 = 8. \\ 12 &= 12 \quad \text{Simplify.} \end{aligned}$$

Because the last line of the check is a true statement, 4 is a solution of the original equation.

Exercise

Solve for x: $11 = 3x - (1 - x)$

Answer

$x = 3$

Variables on Both Sides

Variables can occur on both sides of the equation.

Goal

Isolate the terms containing the variable you are solving for on one side of the equation.

Example 3

Solve for x: $5x = 3x - 18$.

Solution

To isolate the variables on one side of the equation, subtract $3x$ from both sides of the equation and simplify.

$$\begin{aligned} 5x &= 3x - 18 && \text{Original equation.} \\ 5x - 3x &= 3x - 18 - 3x && \text{Subtract } 3x \text{ from both sides.} \\ 2x &= -18 && \text{Combine like terms: } 5x - 3x = 2x \\ & && \text{and } 3x - 3x = 0. \end{aligned}$$

Note that the variable is now isolated on the left-hand side of the equation. To undo the effect of multiplying by 2, divide both sides of the last equation by 2.

$$\begin{aligned} \frac{2x}{2} &= \frac{-18}{2} && \text{Divide both sides by 2.} \\ x &= -9 && \text{Simplify: } -18/2 == -9. \end{aligned}$$

Check

Substitute -9 for x in the original equation.

$$\begin{aligned} 5x &= 3x - 18 && \text{Original equation.} \\ 5(-9) &= 3(-9) - 18 && \text{Substitute } -9 \text{ for } x. \\ -45 &= -27 - 18 && \text{Multiply first on both sides.} \\ -45 &= -45 && \text{Subtract on the right: } -27 - 18 = -45. \end{aligned}$$

Because the last line of the check is a true statement, -9 is a solution of the original equation.

Exercise

Solve for x: $4x - 3 = x$

Answer

$x = 1$

Example 4

Solve for x: $5x = 3+6x$.

Solution

To isolate the variables on one side of the equation, subtract $6x$ from both sides of the equation and simplify.

$$\begin{aligned} 5x &= 3 + 6x && \text{Original equation.} \\ 5x - 6x &= 3 + 6x - 6x && \text{Subtract } 6x \text{ from both sides.} \\ -x &= 3 && \text{Combine like terms: } 5x - 6x = -x \\ & && \text{and } 6x - 6x = 0. \end{aligned}$$

Note that the variable is now isolated on the left-hand side of the equation.

There are a couple of ways we can finish this solution. Remember, $-x$ is the same as $(-1)x$, so we could undo the effects of multiplying by -1 by dividing both sides of the equation by -1. Multiplying both sides of the equation by -1 will work equally well. But perhaps the easiest way to proceed is to simply negate both sides of the equation.

$$\begin{aligned} -(-x) &= -3 && \text{Negate both sides.} \\ x &= -3 && \text{Simplify: } -(-x) = x. \end{aligned}$$

Check

Substitute -3 for x in the original equation.

$$\begin{aligned} 5x &= 3 + 6x && \text{Original equation.} \\ 5(-3) &= 3 + 6(-3) && \text{Substitute } -3 \text{ for } x. \\ -15 &= 3 - 18 && \text{Multiply first on both sides.} \\ -15 &= -15 && \text{Subtract on the right: } 3 - 8 = -15. \end{aligned}$$

Because the last line of the check is a true statement, -3 is a solution of the original equation.

Exercise

Solve for x: $7x = 18 + 9x$

Answer

$x = -9$

Dealing with −x.

If your equation has the form

$$-x = c,$$

where c is some integer, note that this is equivalent to the equation $(-1)x = c$. Therefore, dividing both sides by -1 will produce a solution for x. Multiplying both sides by -1 works equally well. However, perhaps the easiest thing to do is negate each side, producing

$-(-x) = -c$, which is equivalent to $x = -c$.

Example 5

Solve for x: 6x − 5 = 12x + 19.

Solution

To isolate the variables on one side of the equation, subtract 12x from both sides of the equation and simplify.

$$\begin{aligned} 6x-5 &= 12x+19 && \text{Original equation.} \\ 6x-5-12x &= 12x+19-12x && \text{Subtract } 12x \text{ from both sides.} \\ -6x-5 &= 19 && \text{Combine like terms: } 6x-12x=-6x \\ & && \text{and } 12x-12x=0. \end{aligned}$$

Note that the variable is now isolated on the left-hand side of the equation. Next, to "undo" subtracting 5, add 5 to both sides of the equation.

$$\begin{aligned} -6x-5+5 &= 19+5 && \text{Add 5 to both sides.} \\ -6x &= 24 && \text{Simplify: } -5+5=0 \text{ and } 19+5=24. \end{aligned}$$

Finally, to "undo" multiplying by −6, divide both sides of the equation by −6.

$$\begin{aligned} \frac{-6x}{-6} &= \frac{24}{-6} && \text{Divide both sides by } -6. \\ x &= -4 && \text{Simplify: } 24/(-6)=-4. \end{aligned}$$

Check. Substitute −4 for x in the original equation

$$\begin{aligned} 6x-5 &= 12x+19 && \text{Original equation.} \\ 6(-4)-5 &= 12(-4)+19 && \text{Substitute } -4 \text{ for } x. \\ -24-5 &= -48+19 && \text{Multiply first on bo} \end{aligned}$$

Because the last line of the check is a true statement, −4 is a solution of the original equation.

Exercise

Solve for x: 2x + 3 = 18 − 3x

Answer

x = 3

Example 6

Solve for x: 2(3x + 2) − 3(4 − x) = x + 8.

Solution

Use the distributive property to remove parentheses on the left-hand side of the equation.

$$\begin{aligned} 2(3x+2)-3(4-x) &= x+8 && \text{Original equation.} \\ 6x+4-12+3x &= x+8 && \text{Use the distributive property.} \\ 9x-8 &= x+8 && \text{Combine like terms: } 6x+3x=9x \\ & && \text{and } 4-12=-8. \end{aligned}$$

Isolate the variables on the left by subtracting x from both sides of the equation

$$\begin{aligned} 9x-8-x &= x+8-x && \text{Subtract } x \text{ from both sides.} \\ 8x-8 &= 8 && \text{Combine like terms: } 9x-x=8x \\ & && \text{and } x-x=0. \end{aligned}$$

Note that the variable is now isolated on the left-hand side of the equation. Next, to "undo" subtracting 8, add 8 to both sides of the equation.

$$\begin{aligned} 8x-8+8 &= 8+8 && \text{Add 8 to both sides.} \\ 8x &= 16 && \text{Simplify: } -8+8=0 \text{ and } 8+8=16. \end{aligned}$$

Finally, to "undo" multiplying by 8, divide both sides of the equation by 8.

$$\begin{aligned} \frac{8x}{8} &= \frac{16}{8} && \text{Divide both sides by 8.} \\ x &= 2 && \text{Simplify: } 16/8=2. \end{aligned}$$

Check. Substitute 2 for x in the original equation.

Because the last line of the check is a true statement, 2 is a solution of the original equation.

Exercise

Solve for x: 3(2x − 4) − 2(5 − x) = 18

Answer

x = 5

Exercises

In Exercises 1-16, solve the equation.

1. −9x + x = −8
2. 4x − 5x = −3
3. −4=3x − 4x
4. −6 = −5x + 7x
5. 27x + 51 = −84
6. −20x + 46 = 26

7. $9=5x + 9 - 6x$
8. $-6 = x + 3 - 4x$
9. $0= -18x + 18$
10. $0= -x + 71$
11. $41 = 28x + 97$
12. $-65 = -x - 35$
13. $8x - 8 - 9x = -3$
14. $6x + 7 - 9x = 4$
15. $-85x + 85 = 0$
16. $17x - 17 = 0$

In Exercises 17-34, solve the equation.

17. $-6x = -5x - 9$
18. $-5x = -3x - 2$
19. $6x - 7=5x$
20. $3x +8= -5x$
21. $4x - 3=5x - 1$
22. $x - 2=9x - 2$
23. $-3x +5=3x - 1$
24. $-5x +9= -4x - 3$
25. $-5x = -3x + 6$
26. $3x = 4x - 6$
27. $2x - 2=4x$
28. $6x - 4=2x$
29. $-6x +8= -2x$
30. $4x - 9=3x$
31. $6x = 4x - 4$
32. $-8x = -6x + 8$
33. $-8x +2= -6x + 6$
34. $-3x +6= -2x - 5$

In Exercises 35-52, solve the equation.

35. $1 - (x - 2) = -3$
36. $1 - 8(x - 8) = 17$
37. $-7x + 6(x + 8) = -2$
38. $-8x + 4(x + 7) = -12$
39. $8(-6x - 1) = -8$
40. $-7(-2x - 4) = -14$
41. $-7(-4x - 6) = -14$
42. $-2(2x + 8) = -8$
43. $2 - 9(x - 5) = -16$
44. $7 - 2(x + 4) = -1$
45. $7x + 2(x + 9) = -9$
46. $-8x + 7(x - 2) = -14$
47. $2(-x + 8) = 10$
48. $2(-x - 2) = 10$
49. $8 + 2(x - 5) = -4$
50. $-5 + 2(x + 5) = -5$
51. $9x - 2(x + 5) = -10$
52. $-8x - 5(x - 3) = 15$

In Exercises 53-68, solve the equation.

53. $4(-7x + 5) + 8 = 3(-9x - 1) - 2$
54. $-4(-x + 9) + 5 = -(-5x - 4) - 2$
55. $-8(-2x - 6) = 7(5x - 1) - 2$
56. $5(-4x - 8) = -9(-6x + 4) - 4$
57. $2(2x - 9) + 5 = -7(-x - 8)$
58. $-6(-4x - 9) + 4 = -2(-9x - 8)$
59. $6(-3x + 4) - 6 = -8(2x + 2) - 8$

60. $-5(5x-9)-3=-4(2x+5)-6$
61. $2(-2x-3)=3(-x+2)$
62. $-2(7x+1)=-2(3x-7)$
63. $-5(-9x+7)+7=-(-9x-8)$
64. $7(-2x-6)+1=9(-2x+7)$
65. $5(5x-2)=4(8x+1)$
66. $5(-x-4)=-(-x+8)$
67. $-7(9x-6)=7(5x+7)-7$
68. $-8(2x+1)=2(-9x+8)-2$

Answers

1. 1
3. 4
5. −5
7. 0
9. 1
11. −2
13. −5
15. 1
17. 9
19. 7
21. −2
23. 1
25. −3
27. −1
29. 2
31. −2
33. −2
35. 6
37. 50
39. 0
41. −2
43. 7
45. −3
47. 3
49. −1
51. 0
53. 33
55. 3
57. −23
59. 21
61. −12
63. 1
65. −2
67. 0

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2.2.7: Applications

Because we've increased our fundamental ability to simplify algebraic expressions, we're now able to tackle a number of more advanced applications. Before we begin, we remind readers of required steps that must accompany solutions of applications.

Requirements for Word Problem Solutions

1. **Set up a Variable Dictionary.** You must let your readers know what each variable in your problem represents. This can be accomplished in a number of ways:
    1. Statements such as "Let P represent the perimeter of the rectangle."
    2. Labeling unknown values with variables in a table.
    3. Labeling unknown quantities in a sketch or diagram.
2. **Set up an Equation.** Every solution to a word problem must include a carefully crafted equation that accurately describes the constraints in the problem statement.
3. **Solve the Equation.** You must always solve the equation set up in the previous step.
4. **Answer the Question.** This step is easily overlooked. For example, the problem might ask for Jane's age, but your equation's solution gives the age of Jane's sister Liz. Make sure you answer the original question asked in the problem.
5. **Look Back.** It is important to note that this step does not imply that you should simply check your solution in your equation. After all, it's possible that your equation incorrectly models the problem's situation, so you could have a valid solution to an incorrect equation. The important question is: "Does your answer make sense based on the words in the original problem statement."

Consecutive Integers

The integers are *consecutive*, in the sense that one follows right after another. For example, 5 and 6 are a pair of consecutive integers. The important relation to notice is the fact that the second integer of this pair is one larger than its predecessor. That is, 6 = 5 + 1.

Definition: Consecutive Integers

Let k represent an integer. The next consecutive integer is the integer $k + 1$.

Thus, if k is an integer, then $k + 1$ is the next integer, $k + 2$ is the next integer after that, and so on.

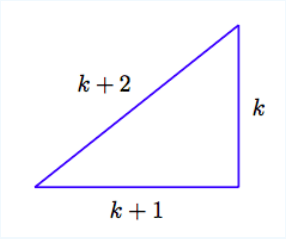
Example 1

The three sides of a triangle are consecutive integers and the perimeter is 72 inches. Find the measure of each side of the triangle.

Solution

We follow the Requirements for Word Problem Solutions.

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.



In our schematic diagram, we've labeled the three sides of the triangle with expressions representing the consecutive integers k, k+ 1, and k+ 2.

2. *Set up an Equation.* To find the perimeter P of the triangle, sum the three sides.

$$P = k + (k+1) + (k+2)$$

However, we're given the fact that the perimeter is 72 inches. Thus,

$$72 = k + (k+1) + (k+2)$$

3. *Solve the Equation.* On the right, regroup and combine like terms.

$$72 = 3k + 3$$

Now, solve.

$$\begin{aligned} 72 - 3 &= 3k + 3 - 3 && \text{Subtract 3 from both sides.} \\ 69 &= 3k && \text{Simplify.} \\ \frac{69}{3} &= \frac{3k}{3} && \text{Divide both sides by 3.} \\ 23 &= k && \text{Simplify.} \end{aligned}$$

4. *Answer the Question.* We've only found one side, but the question asks for the measure of all three sides. However, the remaining two sides can be found by substituting 23 for k into the expressions k + 1 and k + 2.

k + 1 = 23 + 1 and k + 2 = 23 + 2

= 24 = 25

Hence, the three sides measure 23 inches, 24 inches, and 25 inches.

5. *Look Back.* Does our solution make sense? Well, the three sides are certainly consecutive integers, and their sum is 23 inches + 24 inches + 25 inches = 72 inches, which was the given perimeter. Therefore, our solution is correct.

? Exercise

The three sides of a triangle are consecutive integers and the perimeter is 57 centimeters. Find the measure of each side of the triangle.

Answer

18, 19, and 20 cm

Consecutive Odd Integers

The integer pair 19 and 21 are an example of a pair of *consecutive odd integers*. The important relation to notice is the fact that the second integer of this pair is two larger than its predecessor. That is, 21 = 19 + 2.

Definition: Consecutive Odd Integers

Let k represent an odd integer. The next consecutive odd integer is k + 2.

Thus, if k is an odd integer, then k + 2 is the next odd integer, k + 4 is the next odd integer after that, and so on.

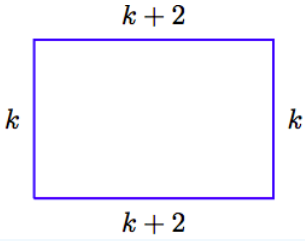
✓ Example 2

The length and width of a rectangle are consecutive odd integers and the perimeter is 168 centimeters. Find the length and width of the rectangle.

Solution

We follow the Requirements for Word Problem Solutions.

1. *Set up a Variable Dictionary.* In this case, a carefully labeled diagram is the best way to indicate what the unknown variable represents.



In our schematic diagram, if the width k is an odd integer, then the length k + 2 is the next consecutive odd integer.

2. *Set up an Equation*. To find the perimeter of the rectangle, sum the four sides.

$$P = k + (k+2) + k + (k+2)$$

However, we're given the fact that the perimeter is 168 centimeters. Thus,

$$168 = k + (k+2) + k + (k+2)$$

3. *Solve the Equation*. On the right, regroup and combine like terms.

$$168 = 4k + 4$$

Now, solve.

$$\begin{aligned} 168 - 4 &= 4k + 4 - 4 && \text{Subtract 4 from both sides.} \\ 164 &= 4k && \text{Simplify.} \\ \frac{164}{4} &= \frac{4k}{4} && \text{Divide both sides by 4.} \\ 41 &= k && \text{Simplify.} \end{aligned}$$

4. *Answer the Question.* We've only found the width, but the question asks for the measure of both the width and the length. However, the length can be found by substituting 41 for k into the expression k + 2.

$$\text{k + 2 = 41 + 2}$$

$$\text{= 43}$$

Hence, the width is 41 centimeters and the length is 43 centimeters.

5. *Look Back.* Does our solution make sense? Well, the width is 41 cm and the length is 43 cm, certainly consecutive odd integers. Further, the perimeter would be 41 cm + 43 cm + 41 cm + 43 cm = 168 cm, so our solution is correct.

? Exercise

The length and width of a rectangle are consecutive odd integers and the perimeter is 120 meters. Find the length and width of the rectangle.

Answer

W = 29 cm, L = 31 cm

Tables

In the remaining applications in this section, we will strive to show how tables can be used to summarize information, define variables, and construct equations to help solve the application.

✔ Example 3

Hue inherits $10,000 and decides to invest in two different types of accounts, a savings account paying 2% interest, and a certificate of deposit paying 4% interest. He decides to invest $1,000 more in the certificate of deposit than in savings. Find the amount invested in each account.

Solution

We follow the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let S represent the amount Hue invests in the savings account. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting S represent the amount invested in savings is far better than letting x represent the amount invested in savings.*

Account Type	Amount Deposited
Savings Account (2%)	S
Certificate of Deposit (4%)	S + 1000
Totals	10000

Because S represents the investment in savings, and we're told that the investment in the certificate of deposit (CD) is $1,000 more than the investment in savings, the investment in the CD is therefore S + 1000, as indicated in the table.

2. *Set up an Equation.* The second column of the table reveals that the sum of the individual investments in the CD and savings totals $10,000. Hence, the equation that models this application is

$$(S+1000)+S=10000.$$

3. *Solve the Equation*. On the left, regroup and combine like terms.

$$2S+1000=10000$$

Now, solve.

$$\begin{aligned} 2S+1000-1000&=1000-1000 && \text{Subtract 1000 from both sides.}\\ 2S&=9000 && \text{Simplify.}\\ \frac{2S}{2}&=\frac{9000}{2} && \text{Divide both sides by 2.}\\ S&=4500 && \text{Simplify.}\end{aligned}$$

4. *Answer the Question*. We've only found the investment in savings, but the question also asks for the amount invested in the CD. However, the investment in the CD is easily found by substituting 4500 for S in the expression S + 1000.

S + 1000 = 4500 + 1000

= 5500.

Hence, the investment in savings is $4,500 and the investment in the CD is $5,500.

5. *Look Back*. Does our solution make sense? Well, the amount invested in the CD is $5,500, which is certainly $1,000 more than the $4,500 invested in savings. Secondly, the two investments total $5, 500 + $4, 500 = $10, 000, so our solution is correct.

? Exercise

Dylan invests a total of $2,750 in two accounts, a savings account paying 3% interest, and a mutual fund paying 5% interest. He invests $250 less in the mutual fund than in savings. Find the amount invested in each account.

Answer

$1,500 in savings, $1,250 in the mutual fund

Example 4

Jose cracks open his piggy bank and finds that he has $3.25 (325 cents), all in nickels and dimes. He has 10 more dimes than nickels. How many dimes and nickels does Jose have?

Solution

We follow the *Requirements for Word Problem Solutions*.

1. *Set up a Variable Dictionary.* We're going to use a table to summarize information and declare variables. In the table that follows, we let N represent the number of nickels from the piggy bank. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting N represent the number of nickels is far better than letting x represent the number of nickels.*

Coins	Number of Coins	Value (cents)
Nickels (5 cents apiece)	N	$5N$
Dimes (10 cents apiece)	$N + 10$	$10(N + 10)$
Totals	-	325

Because there are 10 more dimes than nickels, the number of dimes is N + 10, recorded in the second column. In the third column, N nickels, worth 5 cents apiece, have a value of 5N cents. Next, N + 10 dimes, worth 10 cents apiece, have a value of 10(N + 10) cents. The final entry in the column gives the total value of the coins as 325 cents.

2. *Set up an Equation.* The third column of the table reveals that the sum of the coin values is 325 cents. Hence, the equation that models this application is

$$5N + 10(N + 10) = 325,$$

which sums the value of the nickels and the value of the dimes to a total of 325 cents.

3. *Solve the Equation*. On the left, use the distributive property to remove parentheses.

$$5N + 10N + 100 = 325$$

Combine like terms.

$$15N + 100 = 325$$

Now, solve.

$$\begin{aligned} 15N + 100 - 100 &= 325 - 100 && \text{Subtract 100 from both sides.} \\ 15N &= 225 && \text{Simplify.} \\ \frac{15N}{15} &= \frac{225}{15} && \text{Divide both sides by 15.} \\ N &= 15 && \text{Simplify.} \end{aligned}$$

4. *Answer the Question.* We've only found the number of nickels, but the question also asks for the number of dimes. However, the number of dimes is easily found by substituting 15 for N in the expression N + 10.

$$\text{N + 10 = 15 + 10}$$

$$\text{= 25.}$$

Hence, Jose has 15 nickels and 25 dimes.

5. *Look Back.* Does our solution make sense? Well, the number of dimes is 25, which is certainly 10 more than 15 nickels. Also, the monetary value of 15 nickels is 75 cents and the monetary value of 25 dimes is 250 cents, a total of 325 cents, or $3.25, so our solution is correct.

Exercise

David keeps his change in a bowl made by his granddaughter. There is $1.95 in change in the bowl, all in dimes and quarters. There are two fewer quarters than dimes. How many dimes and quarters does he have in the bowl?

Answer

7 dimes, 5 quarters

Example 5

A large children's organization purchases tickets to the circus. The organization has a strict rule that every five children must be accompanied by one adult guardian. Hence, the organization orders five times as many child tickets as it does adult tickets. Child tickets are three dollars and adult tickets are six dollars. If the total cost of tickets is $4,200, how many child and adult tickets were purchased?

Solution

We follow the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary*. We're going to use a table to summarize information and declare variables. In the table that follows, we let A represent the number of adult tickets purchased. *Using a variable letter that "sounds like" the quantity it represents is an excellent strategy. Thus, in this case, letting A represent the number of adult tickets is far better than letting x represent the number of adult tickets.*

	Number of Tickets	Cost (dollars)
Adults ($6 apiece)	A	$6A$
Children ($3 apiece)	$5A$	$3(5A)$
Totals	-	4200

Because there are 5 times as many children's tickets purchased than adult tickets, the number of children's tickets purchased is 5A, recorded in the second column. In the third column, 5A children's tickets at $3 apiece will cost 3(5A) dollars, and A adult tickets at $6 apiece will cost 6A dollars. The final entry in the column gives the total cost of all tickets as $4,200.

2. *Set up an Equation*. The third column of the table reveals that the sum of the costs for both children and adult tickets is $4,200. Hence, the equation that models this application is

$$6A + 3(5A) = 4200$$

which sums the cost of children and adult tickets at $4,200.

3. *Solve the Equation*. On the left, use the associative property to remove parentheses.

$$6A + 15A = 4200$$

Combine like terms.

$$21A = 4200$$

Now, solve.

$$\frac{21A}{21} = \frac{4200}{21} \quad \text{Divide both sides by 21.}$$
$$A = 200 \quad \text{Simplify.}$$

4. *Answer the Question*. We've only found the number of adult tickets, but the question also asks for the number of children's tickets. However, the number of children's tickets is easily found by substituting 200 for A in the expression 5A.

$$5A = 5(200)$$

= 1000.

Hence, 1000 children's tickets and 200 adult tickets were purchased.

5. *Look Back*. Does our solution make sense? Well, the number of children's tickets purchased is 1000, which is certainly 5 times more than the 200 adult tickets purchased. Also, the monetary value of 1000 children's tickets at $3 apiece is $3,000, and the monetary value of 200 adult tickets at $6 apiece is $1,200, a total cost of $4,200. Our solution is correct.

? Exercise

Emily purchase tickets to the IMAX theater for her family. An adult ticket cost $12 and a child ticket costs $4. She buys two more child tickets than adult tickets and the total cost is $136. How many adult and child tickets did she buy?

Answer

8 adult and 10 child tickets

Exercises

1. The three sides of a triangle are consecutive odd integers. If the perimeter of the triangle is 39 inches, find the lengths of the sides of the triangle.

2. The three sides of a triangle are consecutive odd integers. If the perimeter of the triangle is 51 inches, find the lengths of the sides of the triangle.

3. The width and length of a rectangle are consecutive integers. If the perimeter of the rectangle is 142 inches, find the width and length of the rectangle.

4. The width and length of a rectangle are consecutive integers. If the perimeter of the rectangle is 166 inches, find the width and length of the rectangle.

5. The three sides of a triangle are consecutive even integers. If the perimeter of the triangle is 240 inches, find the lengths of the sides of the triangle.

6. The three sides of a triangle are consecutive even integers. If the perimeter of the triangle is 30 inches, find the lengths of the sides of the triangle.

7. The width and length of a rectangle are consecutive integers. If the perimeter of the rectangle is 374 inches, find the width and length of the rectangle.

8. The width and length of a rectangle are consecutive integers. If the perimeter of the rectangle is 318 inches, find the width and length of the rectangle.

9. The width and length of a rectangle are consecutive odd integers. If the perimeter of the rectangle is 208 inches, find the width and length of the rectangle.

10. The width and length of a rectangle are consecutive odd integers. If the perimeter of the rectangle is 152 inches, find the width and length of the rectangle.

11. The width and length of a rectangle are consecutive even integers. If the perimeter of the rectangle is 76 inches, find the width and length of the rectangle.

12. The width and length of a rectangle are consecutive even integers. If the perimeter of the rectangle is 300 inches, find the width and length of the rectangle.

13. The three sides of a triangle are consecutive even integers. If the perimeter of the triangle is 144 inches, find the lengths of the sides of the triangle.

14. The three sides of a triangle are consecutive even integers. If the perimeter of the triangle is 198 inches, find the lengths of the sides of the triangle.

15. The three sides of a triangle are consecutive integers. If the perimeter of the triangle is 228 inches, find the lengths of the sides of the triangle.

16. The three sides of a triangle are consecutive integers. If the perimeter of the triangle is 216 inches, find the lengths of the sides of the triangle.

17. The width and length of a rectangle are consecutive even integers. If the perimeter of the rectangle is 92 inches, find the width and length of the rectangle.

18. The width and length of a rectangle are consecutive even integers. If the perimeter of the rectangle is 228 inches, find the width and length of the rectangle.

19. The three sides of a triangle are consecutive integers. If the perimeter of the triangle is 105 inches, find the lengths of the sides of the triangle.

20. The three sides of a triangle are consecutive integers. If the perimeter of the triangle is 123 inches, find the lengths of the sides of the triangle.

21. The width and length of a rectangle are consecutive odd integers. If the perimeter of the rectangle is 288 inches, find the width and length of the rectangle.

22. The width and length of a rectangle are consecutive odd integers. If the perimeter of the rectangle is 352 inches, find the width and length of the rectangle.

23. The three sides of a triangle are consecutive odd integers. If the perimeter of the triangle is 165 inches, find the lengths of the sides of the triangle.

24. The three sides of a triangle are consecutive odd integers. If the perimeter of the triangle is 99 inches, find the lengths of the sides of the triangle.

25. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 8 children must be accompanied by one adult guardian. Hence, the organization orders 8 times as many child tickets as it does adult tickets. Child tickets are $7 and adult tickets are $19. If the total cost of tickets is $975, how many adult tickets were purchased?

26. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 2 children must be accompanied by one adult guardian. Hence, the organization orders 2 times as many child tickets as it does adult tickets. Child tickets are $6 and adult tickets are $16. If the total cost of tickets is $532, how many adult tickets were purchased?

27. Judah cracks open a piggy bank and finds $3.30 (330 cents), all in nickels and dimes. There are 15 more dimes than nickels. How many nickels does Judah have?

28. Texas cracks open a piggy bank and finds $4.90 (490 cents), all in nickels and dimes. There are 13 more dimes than nickels. How many nickels does Texas have?

29. Steve cracks open a piggy bank and finds $4.00 (400 cents), all in nickels and dimes. There are 7 more dimes than nickels. How many nickels does Steve have?

30. Liz cracks open a piggy bank and finds $4.50 (450 cents), all in nickels and dimes. There are 15 more dimes than nickels. How many nickels does Liz have?

31. Jason inherits $20,300 and decides to invest in two different types of accounts, a savings account paying 2.5% interest, and a certificate of deposit paying 5% interest. He decides to invest $7,300 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

32. Trinity inherits $24,300 and decides to invest in two different types of accounts, a savings account paying 2% interest, and a certificate of deposit paying 5.75% interest. She decides to invest $8,500 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

33. Gina cracks open a piggy bank and finds $4.50 (450 cents), all in nickels and dimes. There are 15 more dimes than nickels. How many nickels does Gina have?

34. Dylan cracks open a piggy bank and finds $4.05 (405 cents), all in nickels and dimes. There are 6 more dimes than nickels. How many nickels does Dylan have?

35. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 2 children must be accompanied by one adult guardian. Hence, the organization orders 2 times as many child tickets as it does adult tickets. Child tickets are $4 and adult tickets are $10. If the total cost of tickets is $216, how many adult tickets were purchased?

36. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 2 children must be accompanied by one adult guardian. Hence, the organization orders 2 times as many child tickets as it does adult tickets. Child tickets are $7 and adult tickets are $11. If the total cost of tickets is $375, how many adult tickets were purchased?

37. Connie cracks open a piggy bank and finds $3.70 (370 cents), all in nickels and dimes. There are 7 more dimes than nickels. How many nickels does Connie have?

38. Don cracks open a piggy bank and finds $3.15 (315 cents), all in nickels and dimes. There are 3 more dimes than nickels. How many nickels does Don have?

39. Mary inherits $22,300 and decides to invest in two different types of accounts, a savings account paying 2% interest, and a certificate of deposit paying 4% interest. She decides to invest $7,300 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

40. Amber inherits $26,000 and decides to invest in two different types of accounts, a savings account paying 2.25% interest, and a certificate of deposit paying 4.25% interest. She decides to invest $6,200 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

41. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 8 children must be accompanied by one adult guardian. Hence, the organization orders 8 times as many child tickets as it does adult tickets. Child tickets are $6 and adult tickets are $16. If the total cost of tickets is $1024, how many adult tickets were purchased?

42. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 3 children must be accompanied by one adult guardian. Hence, the organization orders 3 times as many child tickets as it does adult tickets. Child tickets are $3 and adult tickets are $18. If the total cost of tickets is $351, how many adult tickets were purchased?

43. Alan inherits $25,600 and decides to invest in two different types of accounts, a savings account paying 3.5% interest, and a certificate of deposit paying 6% interest. He decides to invest $6,400 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

44. Mercy inherits $27,100 and decides to invest in two different types of accounts, a savings account paying 3% interest, and a certificate of deposit paying 4% interest. She decides to invest $8,700 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

45. Tony inherits $20,600 and decides to invest in two different types of accounts, a savings account paying 2% interest, and a certificate of deposit paying 4% interest. He decides to invest $9,200 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

46. Connie inherits $17,100 and decides to invest in two different types of accounts, a savings account paying 2% interest, and a certificate of deposit paying 5.5% interest. She decides to invest $6,100 more in the certificate of deposit than in savings. Find the amount invested in the savings account.

47. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 2 children must be accompanied by one adult guardian. Hence, the organization orders 2 times as many child tickets as it does adult tickets. Child tickets are $2 and adult tickets are $14. If the total cost of tickets is $234, how many adult tickets were purchased?

48. A large children's organization purchases tickets to the circus. The organization has a strict rule that every 8 children must be accompanied by one adult guardian. Hence, the organization orders 8 times as many child tickets as it does adult tickets. Child tickets are $8 and adult tickets are $13. If the total cost of tickets is $1078, how many adult tickets were purchased?

Answers

1. 11 in., 13 in., 15 in.
3. 35 in., 36 in.
5. 78 in., 80 in., 82 in.
7. 93 in., 94 in.
9. 51 in., 53 in.
11. 18 in., 20 in.
13. 46 in., 48 in., 50 in.
15. 75 in., 76 in., 77 in.
17. 22 in., 24 in.
19. 34 in., 35 in., 36 in.
21. 71 in., 73 in.
23. 53 in., 55 in., 57 in.
25. 13 adult tickets
27. 12 nickels
29. 22 nickels
31. $6, 500
33. 20 nickels
35. 12 child tickets
37. 20 nickels
39. $7, 500
41. 16 child tickets
43. $9, 600
45. $5, 700
47. 13 child tickets

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2.3: Literal equations

A literal equation is synonymous with a formula and similar to solving general linear equations because we apply the same method. We say, methods never change, just the problems. The only difference is we have several variables in the equation and we will attempt to solve for one specific variable of the formula. For example, we may have a formula such as $A = \pi r^2 + \pi r s$, the formula for surface area of a right circular cone, and we may be interested in solving for the variable s. This means we want to isolate the variable s so the equation has s isolated on one side, and everything else on the other. This looks like

$$s = \frac{A - \pi r^2}{\pi r}$$

This second equation gives the same information as the first, meaning they are algebraically equivalent. However, the original formula gives area, while the other gives s, the slant height of the cone. In this section, we discuss the process in which we start from the first equation and result in the second equation.

Example 2.3.1

Let's take a look at these two examples below, side by side. The left equation is a familiar one-step equation and the right equation is also a one-step equation, this time a literal equation (or formula).

$3x = 12$	$wx = z$	Both have coefficients
		Multiply by the reciprocal of 3 and w, respectively
$\frac{1}{3} \cdot 3x = \frac{1}{3} \cdot 12$	$\frac{1}{w} \cdot wx = z \cdot \frac{1}{w}$	Simplify
$x = 4$	$x = \frac{z}{w}$	Solution

We used the same process for solving $3x = 12$ for x as we did for solving $wx = z$ for x. Because we are solving for x, we treat all the other variables the same way we would treat numbers or coefficients. Thus, we applied the multiplication property and multiplied by the reciprocal of 3 and w to isolate x.

Solving for a Variable with One and Two-Step Equations

Example 2.3.2

Solve the equation $m + n = p$ for n.

Solution

$m + n = p$	Add the opposite of m
$m + n + (-m) = p + (-m)$	Simplify
$n = p - m$	Solution

Since p and m are not like terms, they cannot be combined. Hence, $n = p - m$.

Example 2.3.3

Solve the equation $a(x - y) = b$ for x.

Solution

$a(x - y) = b$	Distribute
$ax - ay = b$	Add the opposite of ay
$ax + ay + (-ay) = b + (-ay)$	Simplify
$ax = b - ay$	Isolate x by multiplying by the reciprocal of a
$\frac{1}{a} \cdot ax = (b - ay) \cdot \frac{1}{a}$	Simplify
$x = \frac{b - ay}{a}$	Solution

Equivalently, x can be written as $\frac{b}{a} - y$ by simplifying the fraction. However, it is common practice to leave it as one fraction.

Example 2.3.4

Solve the equation $y = mx + b$ for m.

Solution

$$\begin{aligned} y &= mx + b && \text{Isolate the variable term by adding the opposite of } b \\ y + (-b) &= mx + b + (-b) && \text{Simplify} \\ y - b &= mx && \text{Isolate } m \text{ by multiplying by the reciprocal of } x \\ \tfrac{1}{x} \cdot (y - b) &= mx \cdot \tfrac{1}{x} && \text{Simplify} \\ \tfrac{y-b}{x} &= m && \text{Rewrite with } m \text{ on the left side} \\ m &= \tfrac{y-b}{x} && \text{Solution} \end{aligned}$$

Solving for a Variable in Multiple Steps

Example 2.3.5

Solve the equation $A = \pi r^2 + \pi r s$ for s. *This should remind you of the equation in the beginning of the section.*

Solution

$$\begin{aligned} A &= \pi r^2 + \pi r s && \text{Isolate the variable term by adding the opposite of } \pi r^2 \\ A + (-\pi r^2) &= \pi r^2 + \pi r s + (-\pi r^2) && \text{Simplify} \\ A - \pi r^2 &= \pi r s && \text{Isolate } s \text{ by multiplying by the reciprocal of } \pi r \\ \tfrac{1}{\pi r} \cdot (A - \pi r^2) &= \pi r s \cdot \tfrac{1}{\pi r} && \text{Simplify} \\ \tfrac{A - \pi r^2}{\pi r} &= s && \text{Rewrite with } s \text{ on the left side} \\ s &= \tfrac{A - \pi r^2}{\pi r} && \text{Solution} \end{aligned}$$

Solving for a Variable with Fractions

Formulas often include fractions and we can solve with the same method as used previously. First, identify the LCD, and then multiply each term by the LCD. After we clear denominators, we obtain a general equation and solve as usual.

Example 2.3.6

Solve the equation $h = \frac{2m}{n}$ for m.

Solution

$$\begin{aligned} h &= \tfrac{2m}{n} && \text{Multiply by the LCD}=n \\ n \cdot h &= \tfrac{2m}{n} \cdot n && \text{Simplify} \\ nh &= 2m && \text{Multiply by the reciprocal of } 2 \\ \tfrac{1}{2} \cdot nh &= 2m \cdot \tfrac{1}{2} && \text{Simplify} \\ \tfrac{nh}{2} &= m && \text{Rewrite with } m \text{ on the left side} \\ m &= \tfrac{nh}{2} && \text{Solution} \end{aligned}$$

Example 2.3.7

Solve the equation $\frac{a}{b} + \frac{c}{b} = e$ for a.

Solution

$\frac{a}{b}+\frac{c}{b}=e$	Multiply each term by the LCD=b
$b\cdot\frac{a}{b}+b\cdot\frac{c}{b}=e\cdot b$	Simplify
$a+c=eb$	Add the opposite of c
$a+c+(-c)=eb+(-c)$	Simplify
$a=eb-c$	Solution

Example 2.3.8

Solve the equation $a=\frac{A}{2-b}$ for b.

Solution

$a=\frac{A}{2-b}$	Multiply each term by the LCD $=(2-b)$
$(2-b)\cdot a=\frac{A}{2-b}\cdot(2-b)$	Simplify
$a(2-b)=A$	Distribute
$2a-2b=A$	Isolate the variable term by adding the opposite of $2a$
$2a-2b+(-2a)=A+(-2a)$	Simplify
$-2b=A-2a$	Multiply by the reciprocal of -2
$-\frac{1}{2}\cdot-2b=(A-2a)\cdot-\frac{1}{2}$	Simplify
$b=-\frac{(A-2a)}{2}$	Distribute the negative
$b=\frac{-A+2a}{2}$	Solution

Note, we could also write the solution as $b=\frac{2a-A}{2}$, where the positive term is written first in the numerator. It's not necessary, but for aesthetic reasons, we can write b this way.

Note

The father of algebra, Persian mathematician, *Muhammad ibn Musa Khwarizmi*, introduced the fundamental idea of balancing by subtracting the same term from the other side of the equation. He called this process *al-jabr*, which later became the world Algebra.

Literal Equations Homework

Solve each of the following equations for the indicated variable.

Exercise 2.3.1

$ab=c$ for b

Exercise 2.3.2

$\frac{f}{g}x=b$ for x

Exercise 2.3.3

$3x=\frac{a}{b}$ for x

Exercise 2.3.4

$E=mc^2$ for m

Exercise 2.3.5

$V = \frac{4}{3}\pi r^3$ for π

Exercise 2.3.6

$a + c = b$ for c

Exercise 2.3.7

$c = \frac{4y}{m+n}$ for y

Exercise 2.3.8

$V = \frac{\pi D n}{12}$ for D

Exercise 2.3.9

$P = n(p - c)$ for n

Exercise 2.3.10

$T = \frac{D-d}{L}$ for D

Exercise 2.3.11

$L = L_0(1 + at)$ for L_0

Exercise 2.3.12

$2m + p = 4m + q$ for m

Exercise 2.3.13

$\frac{k-m}{r} = q$ for k

Exercise 2.3.14

$h = vt - 16t^2$ for v

Exercise 2.3.15

$Q_1 = P(Q_2 - Q_1)$ for Q_2

Exercise 2.3.16

$R = \frac{kA(T_1+T_2)}{d}$ for T_1

Exercise 2.3.17

$ax + b = c$ for a

Exercise 2.3.18

$lwh = V$ for w

Exercise 2.3.19

$\frac{1}{a} + b = \frac{c}{a}$ for a

Exercise 2.3.20

$at - bw = s$ for t

Exercise 2.3.21

$ax + bx = c$ for a

Exercise 2.3.22

$x + 5y = 3$ for y

Exercise 2.3.23

$3x + 2y = 7$ for y

Exercise 2.3.24

$5a - 7b = 4$ for b

Exercise 2.3.25

$4x - 5y = 8$ for y

Exercise 2.3.26

$g = \frac{h}{i}$ for h

Exercise 2.3.27

$p = \frac{3y}{q}$ for y

Exercise 2.3.28

$\frac{ym}{b} = \frac{c}{d}$ for y

Exercise 2.3.29

$DS = ds$ for D

Exercise 2.3.30

$E = \frac{mv^2}{2}$ for m

Exercise 2.3.31

$x - f = g$ for x

Exercise 2.3.32

$\frac{rs}{a-3} = k$ for r

Exercise 2.3.33

$F = k(R - L)$ for k

Exercise 2.3.34

$S = L + 2B$ for L

Exercise 2.3.35

$I = \frac{E_a - E_q}{R}$ for E_a

Exercise 2.3.36

$ax + b = c$ for x

Exercise 2.3.37

$q = 6(L - p)$ for L

Exercise 2.3.38

$R = aT + b$ for T

Exercise 2.3.39

$S = \pi rh + \pi r^2$ for h

Exercise 2.3.40

$L = \pi(r_1 + r_2) + 2d$ for r_1

Exercise 2.3.41

$P = \frac{V_1(V_2 - V_1)}{g}$ for V_2

Exercise 2.3.42

$rt = d$ for r

Exercise 2.3.43

$V = \frac{\pi r^2 h}{3}$ for h

Exercise 2.3.44

$\frac{1}{a}+b=\frac{c}{a}$ for b

Exercise 2.3.45

$at-bw=s$ for w

Exercise 2.3.46

$x+5y=3$ for x

Exercise 2.3.47

$3x+2y=7$ for x

Exercise 2.3.48

$5a-7b=4$ for a

Exercise 2.3.49

$4x-5y=8$ for x

Exercise 2.3.50

$C=\frac{5}{9}(F-32)$ for F

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CHAPTER OVERVIEW

3: Ratios, Rates, Proportions and Percent

3.1: Ratios and Rates

3.1.1: Ratios and Rates

3.1.2: Proportions

3.1.3: Applications of Proportions

3.1.4: Percent

3.1.5: Fractions of One Percent

3.1.6: Applications of Percents

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SECTION OVERVIEW

3.1: Ratios and Rates

3.1.1: Ratios and Rates

3.1.2: Proportions

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3.1.1: Ratios and Rates

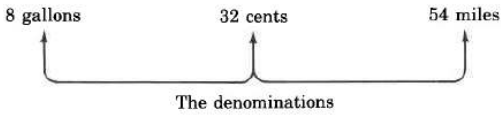
Learning Objectives

- be able to distinguish between denominate and pure numbers and between ratios and rates

Denominate Numbers and Pure Numbers

Denominate Numbers, Like and Unlike Denominate Numbers

It is often necessary or convenient to compare two quantities**. Denominate numbers** are numbers together with some specified unit. If the units being compared are alike, the denominate numbers are called **like denominate numbers**. If units are not alike, the numbers are called **unlike denominate numbers**. Examples of denominate numbers are shown in the diagram:



Pure Numbers

Numbers that exist purely as numbers and do *not* represent amounts of quantities are called **pure numbers**. Examples of pure numbers are 8, 254, 0, $21\frac{5}{8}$, $\frac{2}{5}$, and 0.07.

Numbers can be *compared* in two ways: subtraction and division.

Comparing Numbers by Subtraction and Division

Comparison of two numbers by subtraction indicates how *much more* one number is than another.
Comparison by division indicates how *many times* larger or smaller one number is than another.

Comparing Pure or Like Denominate Numbers by Subtraction

Numbers can be compared by subtraction if and only if they both are like denominate numbers or both pure numbers.

Sample Set A

Compare 8 miles and 3 miles by subtraction.

Solution

$8 \text{ miles} - 3 \text{ miles} = 5 \text{ miles}$

This means that 8 miles is 5 miles more than 3 miles.

Examples of use: I can now jog 8 miles whereas I used to jog only 3 miles. So, I can now jog 5 miles more than I used to.

Sample Set A

Compare 12 and 5 by subtraction.

Solution

$12 - 5 = 7$

This means that 12 is 7 more than 5.

Sample Set A

Comparing 8 miles and 5 gallons by subtraction makes no sense.

Solution

$8 \text{ miles} - 5 \text{ gallons} = ?$

Sample Set A

Compare 36 and 4 by division.

Solution

$36 \div 4 = 9$

This means that 36 is 9 times as large as 4. Recall that $36 \div 4 = 9$ can be expressed as $\frac{36}{4} = 9$.

Sample Set A

Compare 8 miles and 2 miles by division.

Solution

$$\frac{8 \text{ miles}}{2 \text{ miles}} = 4$$

This means that 8 miles is 4 times as large as 2 miles.

Example of use: I can jog 8 miles to your 2 miles. Or, for every 2 miles that you jog, I jog 8. So, I jog 4 times as many miles as you jog.

Notice that when like quantities are being compared by division, we drop the units. Another way of looking at this is that the units divide out (cancel).

Sample Set A

Compare 30 miles and 2 gallons by division.

Solution

$$\frac{30 \text{ miles}}{2 \text{ gallons}} = \frac{15 \text{ miles}}{1 \text{ gallon}}$$

Example of use: A particular car goes 30 miles on 2 gallons of gasoline. This is the same as getting 15 miles to 1 gallon of gasoline.

Notice that when the quantities being compared by division are unlike quantities, we do not drop the units.

Practice Set A

Make the following comparisons and interpret each one.

Compare 10 diskettes to 2 diskettes by

a. subtraction:
b. division:

Answer

a. 8 diskettes; 10 diskettes is 8 diskettes more than 2 diskettes.

b. 5; 10 diskettes is 5 times as many diskettes as 2 diskettes.

Practice Set A

Compare, if possible, 16 bananas and 2 bags by

a. subtraction:
b. division:

Answer

a. Comparison by subtraction makes no sense.

b. $\frac{16 \text{ bananas}}{2 \text{ bags}} = \frac{8 \text{ bananas}}{\text{bag}}$, 8 bananas per bag.

Ratios and Rates

Definition: Ratio

A comparison, by division, of two pure numbers or two like denominate numbers is a **ratio**.

The comparison by division of the pure numbers $\frac{36}{4}$ and the like denominate numbers $\frac{8 \text{ miles}}{2 \text{ miles}}$ are examples of ratios.

Definition: Rate

A comparison, by division, of two unlike denominate numbers is a **rate**.

The comparison by division of two unlike denominate numbers, such as

$\frac{55 \text{ miles}}{1 \text{ gallon}}$ and $\frac{40 \text{ dollars}}{5 \text{ tickets}}$

are examples of rates.

Let's agree to represent two numbers (pure or denominate) with the letters a and b. This means that we're letting a represent some number and b represent some, perhaps different, number. With this agreement, we can write the ratio of the two numbers a and b as

$\frac{a}{b}$ or $\frac{b}{a}$

The ratio $\frac{a}{b}$ is read as "a to b."

The ratio $\frac{b}{a}$ is read as "b to a."

Since a ratio or a rate can be expressed as a fraction, it may be reducible.

Sample Set B

The ratio 30 to 2 can be expressed as $\frac{30}{2}$. Reducing, we get $\frac{15}{1}$.

The ratio 30 to 2 is *equivalent* to the ratio 15 to 1.

Sample Set B

The rate "4 televisions to 12 people" can be expressed as $\frac{4 \text{ televisions}}{12 \text{ people}}$. The meaning of this rate is that "for every 4 televisions, there are 12 people."

Reducing, we get $\frac{1 \text{ television}}{3 \text{ people}}$. The meaning of this rate is that "for every 1 television, there are 3 people."

Thus, the rate of "4 televisions to 12 people" is the *same* as the rate of "1 television to 3 people."

Practicee Set B

Write the following ratios and rates as fractions.

3 to 2

Answer

$\frac{3}{2}$

Practicee Set B

1 to 9

Answer

$\frac{1}{9}$

Practicee Set B

5 books to 4 people

Answer

$\frac{5 \text{ books}}{4 \text{ people}}$

Practicee Set B

120 miles to 2 hours

Answer

$\frac{60 \text{ miles}}{1 \text{ hour}}$

Practicee Set B

8 liters to 3 liters

Answer

$\frac{8}{3}$

Write the following ratios and rates in the form "a to b." Reduce when necessary.

Practicee Set B

$\frac{9}{5}$

Answer

9 to 5

Practicee Set B

$\frac{1}{3}$

Answer

1 to 3

Practicee Set B

$\frac{25 \text{ miles}}{2 \text{ gallons}}$

Answer

25 miles to 2 gallons

Practicee Set B

$\frac{2 \text{ mechanics}}{4 \text{ wrenches}}$

Answer

1 mechanic to 2 wrenches

Practicee Set B

$\frac{15 \text{ video tapes}}{18 \text{ video tapes}}$

Answer

5 to 6

Exercises

For the following 9 problems, complete the statements.

Exercise 3.1.1.1

Two numbers can be compared by subtraction if and only if .

Answer

They are pure numbers or like denominate numbers.

Exercise 3.1.1.2

A comparison, by division, of two pure numbers or two like denominate numbers is called a .

Exercise 3.1.1.3

A comparison, by division, of two unlike denominate numbers is called a .

Answer

rate

Exercise 3.1.1.4

$\frac{6}{11}$ is an example of a . (ratio/rate)

Exercise 3.1.1.5

$\frac{5}{12}$ is an example of a . (ratio/rate)

Answer

ratio

Exercise 3.1.1.6

$\frac{7 \text{ erasers}}{12 \text{ pencils}}$ is an example of a . (ratio/rate)

Exercise 3.1.1.7

$\frac{20 \text{ silver coins}}{35 \text{ gold coins}}$ is an example of a . (ratio/rate)

Answer

rate

Exercise 3.1.1.8

$\frac{3 \text{ sprinklers}}{5 \text{ sprinklers}}$ is an example of a . (ratio/rate)

Exercise 3.1.1.9

$\frac{18 \text{ exhaust valves}}{11 \text{ exhaust valves}}$ is an example of a . (ratio/rate)

Answer

ratio

For the following 7 problems, write each ratio or rate as a verbal phrase.

Exercise 3.1.1.10

$\frac{8}{3}$

Exercise 3.1.1.11

$\frac{2}{5}$

Answer

two to five

Exercise 3.1.1.12

$\frac{8 \text{ feet}}{3 \text{ seconds}}$

Exercise 3.1.1.13

$\frac{29 \text{ miles}}{2 \text{ gallons}}$

Answer

29 mile per 2 gallons or $14\frac{1}{2}$ miles per 1 gallon

Exercise 3.1.1.14

$\frac{30{,}000 \text{ stars}}{300 \text{ stars}}$

Exercise 3.1.1.15

$\frac{5 \text{ yards}}{2 \text{ yards}}$

Answer

5 to 2

Exercise 3.1.1.16

$\frac{164 \text{ trees}}{28 \text{ trees}}$

For the following problems, write the simplified fractional form of each ratio or rate.

Exercise 3.1.1.17

12 to 5

Answer

$\frac{12}{5}$

Exercise 3.1.1.18

81 to 19

Exercise 3.1.1.19

42 plants to 5 homes

Answer

$\frac{42 \text{ plants}}{5 \text{ homes}}$

Exercise 3.1.1.20

8 books to 7 desks

Exercise 3.1.1.21

16 pints to 1 quart

Answer

$$\frac{16 \text{ pints}}{1 \text{ quart}}$$

Exercise 3.1.1.22

4 quarts to 1 gallon

Exercise 3.1.1.23

2.54 cm to 1 in

Answer

$$\frac{2.54 \text{ cm}}{1 \text{ inch}}$$

Exercise 3.1.1.24

80 tables to 18 tables

Exercise 3.1.1.25

25 cars to 10 cars

Answer

$$\frac{5}{2}$$

Exercise 3.1.1.26

37 wins to 16 losses

Exercise 3.1.1.27

105 hits to 315 at bats

Answer

$$\frac{1 \text{ hit}}{3 \text{ at bats}}$$

Exercise 3.1.1.28

510 miles to 22 gallons

Exercise 3.1.1.29

1,042 characters to 1 page

Answer

$$\frac{1{,}042 \text{ characters}}{1 \text{ page}}$$

Exercise 3.1.1.30

1,245 pages to 2 books

Exercises for Review

Exercise 3.1.1.31

Convert $\frac{16}{3}$ to a mixed number.

Answer

$5\frac{1}{3}$

Exercise 3.1.1.32

$1\frac{5}{9}$ of $2\frac{4}{7}$ is what number?

Exercise 3.1.1.33

Find the difference. $\frac{11}{28} - \frac{7}{45}$.

Answer

$\frac{299}{1260}$

Exercise 3.1.1.34

Perform the division. If no repeating patterns seems to exist, round the quotient to three decimal places: $22.35 \div 17$

Exercise 3.1.1.35

Find the value of $1.85 + \frac{3}{8} \cdot 4.1$

Answer

3.3875

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3.1.2: Proportions

Learning Objectives

- be able to describe proportions and find the missing factor in a proportion
- be able to work with proportions involving rates

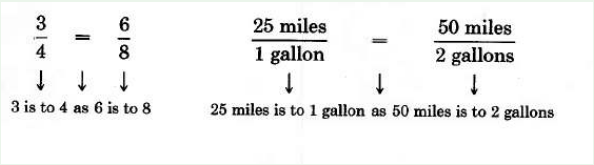
Ratios, Rates, and Proportions

Definition: Ratio, Rate

We have defined a **ratio** as a comparison, by division, of two pure numbers or two *like* denominate numbers. We have defined a **rate**as a comparison, by division, of two *unlike* denominate numbers.

Definition: Proportion

A **proportion** is a statement that two ratios or rates are equal. The following two examples show how to read proportions.



Sample Set A

Write or read each proportion.

$\frac{3}{5}=\frac{12}{20}$

Solution

3 is to 5 as 12 is to 20

Sample Set A

$\frac{10 \text{ items}}{5 \text{ dollars}}=\frac{2 \text{ items}}{1 \text{ dollar}}$

Solution

10 items is to 5 dollars as 2 items is to 1 dollar

Sample Set A

8 is to 12 as 16 is to 24.

Solution

$\frac{8}{12}=\frac{16}{24}$

Sample Set A

50 milligrams of vitamin C is to 1 tablet as 300 milligrams of vitamin C is to 6 tablets.

Solution

$\frac{50}{1}=\frac{300}{6}$

Practice Set A

Write or read each proportion.

$\frac{3}{8} = \frac{6}{16}$

Answer

3 is to 8 as 6 is to 16

Practice Set A

$\frac{2 \text{ people}}{1 \text{ window}} = \frac{10 \text{ people}}{5 \text{ windows}}$

Answer

2 people are to 1 window as 10 people are to 5 windows

Practice Set A

15 is to 4 as 75 is to 20.

Answer

$\frac{15}{4} = \frac{75}{20}$

Practice Set A

2 plates are to 1 tray as 20 plates are to 10 trays.

Answer

\(\dfrac{\text{2 plates}}{\text{1 tray}} = \dfrac{\text{20 plates}}{\text{10 trays}}

Finding the Missing Factor in a Proportion

Many practical problems can be solved by writing the given information as proportions. Such proportions will be composed of three specified numbers and one unknown number. It is customary to let a letter, such as x, represent the unknown number. An example of such a proportion is

$\frac{x}{4} = \frac{20}{16}$

This proportion is read as "x is to 4 as 20 is to 16."

There is a method of solving these proportions that is based on the equality of fractions. Recall that two fractions are equivalent if and only if their cross products are equal. For example,

$\frac{3}{4} = \frac{6}{8}$ since $3 \cdot 8 = 6 \cdot 4$, $24 = 24$

Notice that in a proportion that contains three specified numbers and a letter representing an unknown quantity, that regardless of where the letter appears, the following situation always occurs.

$\underbrace{(\text{number}) \cdot (\text{letter}) = (\text{number}) \cdot (\text{number})}$

We recognize this as a multiplication statement. Specifically, it is a missing factor statement. (See [link] for a discussion of multiplication statements.) For example,

$\frac{x}{4}=\frac{20}{16}$ means that $16\cdot x=4\cdot 20$

$\frac{4}{x}=\frac{16}{20}$ means that $4\cdot 20=16\cdot x$

$\frac{5}{4}=\frac{x}{16}$ means that $5\cdot 16=4\cdot x$

$\frac{5}{4}=\frac{20}{x}$ means that $5\cdot x=4\cdot 20$

Each of these statements is a multiplication statement. Specifically, each is a missing factor statement. (The letter used here is x, whereas M was used in [link].)

Finding the Missing Factor in a Proportion

The missing factor in a missing factor statement can be determined by dividing the product by the known factor, that is, if x represents the missing factor, then

$x=(\text{product})\div(\text{known factor})$

Sample Set B

Find the unknown number in each proportion.

$\frac{x}{4}=\frac{20}{16}$. Find the cross product.

Solution

$$\begin{aligned} 16\cdot x &= 20\cdot 4 \\ 16\cdot x &= 80 \qquad \text{Divide the product 80 by the known factor 16.} \\ x &= \frac{80}{16} \\ x &= 5 \qquad \text{The unkown number is 5.} \end{aligned}$$

This mean that $\frac{5}{4}=\frac{20}{16}$, or 5 is to 4 as 20 is to 16.

Sample Set B

$\frac{5}{x}=\frac{20}{16}$. Find the cross product.

Solution

$$\begin{aligned} 5\cdot 16 &= 20\cdot x \\ 80 &= 20\cdot x \qquad \text{Divide the product 80 by the known factor 20.} \\ \frac{80}{20} &= x \\ 4 &= x \qquad \text{The unkown number is 4.} \end{aligned}$$

This mean that $\frac{5}{4}=\frac{20}{16}$, or 5 is to 4 as 20 is to 16.

Sample Set B

$\frac{16}{3}=\frac{64}{x}$. Find the cross product.

Solution

$$\begin{aligned} 16 \cdot x &= 64 \cdot 3 \\ 16 \cdot x &= 192 \quad \text{Divide 192 by 16.} \\ x &= \frac{192}{16} \\ x &= 12 \quad \text{The unkown number is 12.} \end{aligned}$$

This mean that $\frac{16}{3} = \frac{64}{12}$, or 16 is to 3 as 64 is to 12.

Sample Set B

$\frac{9}{8} = \frac{x}{40}$. Find the cross product.

Solution

$$\begin{aligned} 9 \cdot 40 &= 8 \cdot x \\ 360 &= 8 \cdot x \quad \text{Divide 360 by 8.} \\ \frac{360}{8} &= x \\ 45 &= x \quad \text{The unkown number is 45.} \end{aligned}$$

Practice Set B

Find the unknown number in each proportion.

$\frac{x}{8} = \frac{12}{32}$

Answer

$x = 3$

Practice Set B

$\frac{7}{x} = \frac{14}{10}$

Answer

$x = 5$

Practice Set B

$\frac{9}{11} = \frac{x}{55}$

Answer

$x = 45$

Practice Set B

$\frac{1}{6} = \frac{8}{x}$

Answer

$x = 48$

Proportions Involving Rates

Recall that a rate is a comparison, by division, of unlike denominate numbers. We must be careful when setting up proportions that involve rates. The *form* is important. For example, if a rate involves two types of units, say unit type 1 and unit type 2, we can write

$$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 1}}{\text{unit type 2}}$$

(Same units appear on same side.; Same units appear on same side.)

or

$$\frac{\text{unit type 1}}{\text{unit type 1}} = \frac{\text{unit type 2}}{\text{unit type 2}}$$

(Same units appear on same side.; Same units appear on same side.)

Both cross products produce a statement of the type

$$(\text{unit type 1}) \cdot (\text{unit type 2}) = (\text{unit type 1}) \cdot (\text{unit type 2})$$

which we take to mean the comparison

(unit type 1) is to (unit type 2) as (unit type 1) is to (unit type 2)

Comparison of type 1 with type 2 — Comparison of type 1 with type 2

Same overall type

Examples of correctly expressed proportions are the following:

1. $\frac{\text{mi}}{\text{hr}} = \frac{\text{mi}}{\text{hr}}$ Same units appear on the same side
2. $\frac{\text{mi}}{\text{mi}} = \frac{\text{hr}}{\text{hr}}$ Same units appear on the same side.

However, if we write the same type of units on different sides, such as,

$$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 2}}{\text{unit type 1}}$$

the cross product produces a statement of the form

(unit type 1) · (unit type 1) = (unit type 2) · (unit type 2)

Comparison of type 1 with type 1 — Comparison of type 2 with type 2

Different overall types

We can see that this is an incorrect comparison by observing the following example: It is *incorrect* to write

$$\frac{2 \text{ hooks}}{3 \text{ poles}} = \frac{6 \text{ poles}}{4 \text{ hooks}}$$

for two reason.

The cross product is numerically wrong: $(2 \cdot 4 \neq 3 \cdot 6)$
The cross product produces the statement “hooks are to hooks as poles are to poles,” which makes no sense.

Exercises

Exercise 3.1.2.1

A statement that two ratios or are equal is called a .

Answer

rates, proportion

For the following 9 problems, write each proportion in fractional form.

Exercise 3.1.2.2

3 is to 7 as 18 is to 42.

Exercise 3.1.2.3

1 is to 11 as 3 is to 33.

Answer

$$\frac{1}{11} = \frac{3}{33}$$

Exercise 3.1.2.4

9 is to 14 as 27 is to 42.

Exercise 3.1.2.5

6 is to 90 as 3 is to 45.

Answer

$$\frac{6}{90} = \frac{3}{45}$$

Exercise 3.1.2.6

5 liters is to 1 bottle as 20 liters is to 4 bottles.

Exercise 3.1.2.7

18 grams of cobalt is to 10 grams of silver as 36 grams of cobalt is to 20 grams of silver.

Answer

$$\frac{18 \text{ gr cobalt}}{10 \text{ gr silver}} = \frac{36 \text{ gr cobalt}}{20 \text{ gr silver}}$$

Exercise 3.1.2.8

4 cups of water is to 1 cup of sugar as 32 cups of water is to 8 cups of sugar.

Exercise 3.1.2.9

3 people absent is to 31 people present as 15 people absent is to 155 people present.

Answer

$$\frac{3 \text{ people absent}}{31 \text{ people present}} = \frac{15 \text{ people absent}}{155 \text{ people present}}$$

Exercise 3.1.2.10

6 dollars is to 1 hour as 90 dollars is to 15 hours.

For the following 10 problems, write each proportion as a sentence.

Exercise 3.1.2.11

$$\frac{3}{4} = \frac{15}{20}$$

Answer

3 is to 4 as 15 is to 20

Exercise 3.1.2.12

$$\frac{1}{8} = \frac{5}{40}$$

Exercise 3.1.2.13

$$\frac{3 \text{ joggers}}{100 \text{ feet}} = \frac{6 \text{ joggers}}{200 \text{ feet}}$$

Answer

3 joggers are to 100 feet as 6 joggers are to 200 feet

Exercise 3.1.2.14

$$\frac{12 \text{ marshmallows}}{3 \text{ sticks}} = \frac{36 \text{ marshmallows}}{9 \text{ sticks}}$$

Exercise 3.1.2.15

$$\frac{40 \text{ miles}}{80 \text{ miles}} = \frac{2 \text{ gallons}}{4 \text{ gallons}}$$

Answer

40 miles are to 80 miles as 2 gallons are to 4 gallons

Exercise 3.1.2.16

$$\frac{4 \text{ couches}}{10 \text{ couches}} = \frac{2 \text{ houses}}{5 \text{ houses}}$$

Exercise 3.1.2.17

$$\frac{1 \text{ person}}{1 \text{ job}} = \frac{8 \text{ people}}{8 \text{ jobs}}$$

Answer

1 person is to 1 job as 8 people are to 8 jobs

Exercise 3.1.2.18

$$\frac{1 \text{ popsicle}}{2 \text{ children}} = \frac{\frac{1}{2}8 \text{ popsicle}}{1 \text{ child}}$$

Exercise 3.1.2.19

$$\frac{2{,}000 \text{ pounds}}{1 \text{ ton}} = \frac{60{,}000 \text{ pounds}}{30 \text{ tons}}$$

Answer

2,000 pounds are to 1 ton as 60,000 pounds are to 30 tons

Exercise 3.1.2.20

$$\frac{1 \text{ table}}{5 \text{ tables}} = \frac{2 \text{ people}}{10 \text{ people}}$$

For the following 10 problems, solve each proportion.

Exercise 3.1.2.21

$$\frac{x}{5} = \frac{6}{15}$$

Answer

$x = 2$

Exercise 3.1.2.22

$$\frac{x}{10} = \frac{28}{40}$$

Exercise 3.1.2.23

$$\frac{5}{x} = \frac{10}{16}$$

Answer

$x = 8$

Exercise 3.1.2.24

$$\frac{13}{x} = \frac{39}{60}$$

Exercise 3.1.2.25

$$\frac{1}{3} = \frac{x}{24}$$

Answer

$x = 8$

Exercise 3.1.2.26

$\frac{7}{12} = \frac{x}{60}$

Exercise 3.1.2.27

$\frac{8}{3} = \frac{72}{x}$

Answer

$x = 27$

Exercise 3.1.2.28

$\frac{16}{1} = \frac{48}{x}$

Exercise 3.1.2.29

$\frac{x}{25} = \frac{200}{125}$

Answer

$x = 40$

Exercise 3.1.2.30

$\frac{65}{30} = \frac{x}{60}$

For the following 5 problems, express each sentence as a proportion then solve the proportion.

Exercise 3.1.2.31

5 hats are to 4 coats as x hats are to 24 coats.

Answer

$x = 30$

Exercise 3.1.2.32

x cushions are to 2 sofas as 24 cushions are to 16 sofas.

Exercise 3.1.2.33

1 spacecraft is to 7 astronauts as 5 spacecraft are to x astronauts.

Answer

$x = 35$

Exercise 3.1.2.34

56 microchips are to x circuit boards as 168 microchips are to 3 circuit boards.

Exercise 3.1.2.35

18 calculators are to 90 calculators as x students are to 150 students.

Answer

$x = 30$

Exercise 3.1.2.36

x dollars are to \$40,000 as 2 sacks are to 1 sack.

Indicate whether the proportion is true or false.

Exercise 3.1.2.37

$$\frac{3}{16} = \frac{12}{64}$$

Answer

true

Exercise 3.1.2.38

$$\frac{2}{15} = \frac{10}{75}$$

Exercise 3.1.2.39

$$\frac{1}{9} = \frac{3}{30}$$

Answer

false

Exercise 3.1.2.40

$$\frac{6 \text{ knives}}{7 \text{ forks}} = \frac{12 \text{ knives}}{15 \text{ forks}}$$

Exercise 3.1.2.41

$$\frac{33 \text{ miles}}{1 \text{ gallon}} = \frac{99 \text{ miles}}{3 \text{ gallons}}$$

Answer

true

Exercise 3.1.2.42

$$\frac{320 \text{ feet}}{5 \text{ seconds}} = \frac{65 \text{ feet}}{1 \text{ second}}$$

Exercise 3.1.2.43

$$\frac{35 \text{ students}}{70 \text{ students}} = \frac{1 \text{ class}}{2 \text{ classes}}$$

Answer

true

Exercise 3.1.2.44

$$\frac{9 \text{ ml chloride}}{45 \text{ ml chloride}} = \frac{1 \text{ test tube}}{7 \text{ test tubes}}$$

Exercises for Review

Exercise 3.1.2.43

([link]) Use the number 5 and 7 to illustrate the commutative property of addition.

Answer

$5+7=12$
$7+5=12$

Exercise 3.1.2.44

([link]) Use the numbers 5 and 7 to illustrate the commutative property of multiplication.

Exercise 3.1.2.43

([link]) Find the difference. $\frac{5}{14} - \frac{3}{22}$.

Answer

$\frac{17}{77}$

Exercise 3.1.2.44

([link]) Find the product. $8.06129 \cdot 1,000$

Exercise 3.1.2.43

([link]) Write the simplified fractional form of the rate “sixteen sentences to two paragraphs.”

Answer

$\frac{8 \text{ sentences}}{1 \text{ paragraph}}$

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3.1.3: Applications of Proportions

Learning Objectives

- solve proportion problems using the five-step method

The Five-Step Method

In [link] we noted that many practical problems can be solved by writing the given information as proportions. Such proportions will be composed of three specified numbers and one unknown number represented by a letter.

The first and most important part of solving a proportion problem is to determine, by careful reading, what the unknown quantity is and to represent it with some letter.

The Five-Step Method
The five-step method for solving proportion problems:

1. By careful reading, determine what the unknown quantity is and represent it with some letter. There will be only one unknown in a problem.
2. Identify the three specified numbers.
3. Determine which comparisons are to be made and set up the proportion.
4. Solve the proportion (using the methods of [link]).
5. Interpret and write a conclusion in a sentence with the appropriate units of measure.

Step 1 is extremely important. Many problems go unsolved because time is not taken to establish what quantity is to be found.

When solving an applied problem, **always** begin by determining the unknown quantity and representing it with a letter.

Problem Solving

Sample Set A

On a map, 2 inches represents 25 miles. How many miles are represented by 8 inches?

Solution

Step 1: The unknown quantity is miles.
Let $x =$ number of miles represented by 8 inches

Step 2: The three specified numbers are
2 inches
25 miles
8 inches

Step 3: The comparisons are
2 inches to 25 miles $\rightarrow \dfrac{2 \text{ inches}}{25 \text{ miles}}$
8 inches to x miles $\rightarrow \dfrac{8 \text{ inches}}{\text{x miles}}$
Proportions involving ratios and rates are more readily solved by suspending the units while doing the computations. $\dfrac{2}{25} = \dfrac{8}{x}$

Step 4: $\dfrac{2}{25} = \dfrac{8}{x}$ Perform the cross multiplication.

$$\begin{aligned} 2 \cdot x &= 8 \cdot 25 \\ 2 \cdot x &= 200 \quad \text{Divide 200 by 2.} \\ x &= \frac{200}{2} \\ x &= 100 \end{aligned}$$

In step 1, we let x represent the number of miles. So, x represents 100 miles.

Step 5: If 2 inches represents 25 miles, then 8 inches represents 100 miles.

Sample Set A

An acid solution is composed of 7 parts water to 2 parts acid. How many parts of water are there in a solution composed of 20 parts acid?

Solution

Step 1: The unknown quantity is the number of parts of water.
Let $x =$ number of parts of water.

Step 2: The three specified numbers are
7 parts water
2 parts acid
20 parts acid

Step 3: The comparisons are
7 parts water to 2 parts acid $\rightarrow \dfrac{7}{2}$
n parts water to 20 parts acid $\rightarrow \dfrac{n}{20}$
$\dfrac{7}{2} = \dfrac{n}{20}$

Step 4: $\dfrac{7}{2} = \dfrac{n}{20}$ Perform the cross multiplication.

$$\begin{aligned} 7 \cdot 20 &= 2 \cdot n \\ 140 &= 2 \cdot n \quad \text{Divide 140 by 2.} \\ \frac{140}{2} &= n \\ 70 &= n \end{aligned}$$

In step 1, we let n represent the number of parts of water. So, n represents 70 parts of water.

Step 5: 7 parts water to 2 parts acid indicates 70 parts water to 20 parts acid.

Sample Set A

A 5-foot girl casts a $3\dfrac{1}{3}$ foot shadow at a particular time of the day. How tall is a person who casts a 3-foot shadow at the same time of the day?

Solution

Step 1: The unknown quantity is the height of the person.
Let $h =$ height of the preson.

Step 2: The three specified numbers are
5 feet (height of girl)
$3\dfrac{1}{3}$ feet (length of shadow)
3 feet (length of shadow)

Step 3: The comparisons are
5-foot girl is to $3\dfrac{1}{3}$ foot shadow $\rightarrow \dfrac{5}{3\frac{1}{3}}$
h-foot person is to 3-foot shadow $\rightarrow \dfrac{h}{3}$
$\dfrac{5}{3\frac{1}{3}} = \dfrac{h}{3}$

Step 4:

$$\begin{aligned} \frac{5}{3\frac{1}{3}} &= \frac{h}{3} \\ 5 \cdot 3 &= 3\frac{1}{3} \cdot h \\ 15 &= \frac{10}{3} \cdot h \quad \text{Divide 15 by } \frac{10}{3} \\ \frac{15}{\frac{10}{3}} &= h \end{aligned}$$

$$\begin{aligned} \frac{\cancel{15}^{3}}{1} \cdot \frac{3}{\cancel{10}_{2}} &= h \\ \frac{9}{2} &= h \\ h &= 4\frac{1}{2} \end{aligned}$$

Step 5: A person who casts a 3-foot shadow at this particular time of the day is $4\frac{1}{2}$ feet tall.

Sample Set A

The ratio of men to women in a particular town is 3 to 5. How many women are there in the town if there are 19,200 men in town?

Solution

Step 1: The unknown quantity is the number of women in town.
Let $x =$ number of women in town.

Step 2: The three specified numbers are
3
5
19,200

Step 3: The comparisons are 3 men to 5 women $\rightarrow \frac{3}{5}$
19,200 men to x women $\rightarrow \frac{19,200}{x}$
$\frac{3}{5} = \frac{19,200}{x}$

Step 4:

$$\begin{aligned} \frac{3}{5} &= \frac{19,200}{x} \\ 3 \cdot x &= 19,200 \cdot 5 \\ 3 \cdot x &= 96,000 \\ x &= \frac{96,000}{3} \\ x &= 32,000 \end{aligned}$$

Step 5: There are 32,000 women in town.

Sample Set A

The rate of wins to losses of a particular baseball team is $\frac{9}{2}$. How many games did this team lose if they won 63 games?

Solution

Step 1: The unknown quantity is the number of games lost.
Let $n =$ number of games lost.

Step 2: Since $\frac{9}{2} \rightarrow$ means 9 wins to 2 losses, the three specified numbers are
9 (wins)
2 (losses)
63 (wins)

Step 3: The comparisons are
9 wins to 2 losses $\rightarrow \frac{9}{2}$
63 wins to n losses $\rightarrow \frac{63}{n}$
$\frac{9}{2} = \frac{63}{n}$

Step 4:

$$\begin{aligned} \frac{9}{2} &= \frac{63}{n} \\ 9 \cdot n &= 2 \cdot 63 \\ 9 \cdot n &= 126 \\ n &= \frac{126}{9} \\ n &= 14 \end{aligned}$$

Step 5: This team had 14 losses.

Practice Set A

Solve each problem.

On a map, 3 inches represents 100 miles. How many miles are represented by 15 inches?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answer

500 miles

Practice Set A

An alcohol solution is composed of 14 parts water to 3 parts alcohol. How many parts of alcohol are in a solution that is composed of 112 parts water?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answer

24 parts of alcohol

Practice Set A

A $5\frac{1}{2}$-foot woman casts a 7-foot shadow at a particular time of the day. How long of a shadow does a 3-foot boy cast at that same time of day?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answer

$3\frac{9}{11}$ feet

Practice Set A

The rate of houseplants to outside plants at a nursery is 4 to 9. If there are 384 houseplants in the nursery, how many outside plants are there?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answer

864 outside plants

Practice Set A

The odds for a particular event occurring are 11 to 2. (For every 11 times the event does occur, it will not occur 2 times.) How many times does the event occur if it does not occur 18 times?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answer

The event occurs 99 times.

Practice Set A

The rate of passing grades to failing grades in a particular chemistry class is 7272 . If there are 21 passing grades, how many failing grades are there?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answer

6 failing grades

Exercises

For the following 20 problems, use the five-step method to solve each problem.

Exercise 3.1.3.1

On a map, 4 inches represents 50 miles. How many inches represent 300 miles?

Answer

24

Exercise 3.1.3.2

On a blueprint for a house, 2 inches represents 3 feet. How many inches represent 10 feet?

Exercise 3.1.3.3

A model is built to $\frac{2}{15}$ scale. If a particular part of the model measures 6 inches, how long is the actual structure?

Answer

45 inches

Exercise 3.1.3.4

An acid solution is composed of 5 parts acid to 9 parts of water. How many parts of acid are there in a solution that contains 108 parts of water?

Exercise 3.1.3.5

An alloy contains 3 parts of nickel to 4 parts of silver. How much nickel is in an alloy that contains 44 parts of silver?

Answer

33 parts

Exercise 3.1.3.6

The ratio of water to salt in a test tube is 5 to 2. How much salt is in a test tube that contains 35 ml of water?

Exercise 3.1.3.7

The ratio of sulfur to air in a container is $\frac{4}{45}$. How many ml of air are there in a container that contains 207 ml of sulfur?

Answer

2328.75

Exercise 3.1.3.8

A 6-foot man casts a 4-foot shadow at a particular time of the day. How tall is a person that casts a 3-foot shadow at that same time of the day?

Exercise 3.1.3.9

A $5\frac{1}{2}$-foot woman casts a $1\frac{1}{2}$-foot shadow at a particular time of the day. How long a shadow does her $3\frac{1}{2}$-foot niece cast at the same time of the day?

Answer

$\frac{21}{22}$ feet

Exercise 3.1.3.10

A man, who is 6 feet tall, casts a 7-foot shadow at a particular time of the day. How tall is a tree that casts an 84-foot shadow at that same time of the day?

Exercise 3.1.3.11

The ratio of books to shelves in a bookstore is 350 to 3. How many books are there in a store that has 105 shelves?

Answer

12,250

Exercise 3.1.3.12

The ratio of algebra classes to geometry classes at a particular community college is 13 to 2. How many geometry classes does this college offer if it offers 13 algebra classes?

Exercise 3.1.3.13

The odds for a particular event to occur are 16 to 3. If this event occurs 64 times, how many times would you predict it does not occur?

Answer

12

Exercise 3.1.3.14

The odds against a particular event occurring are 8 to 3. If this event does occur 64 times, how many times would you predict it does not occur?

Exercise 3.1.3.15

The owner of a stationery store knows that a 1-inch stack of paper contains 300 sheets. The owner wishes to stack the paper in units of 550 sheets. How many inches tall should each stack be?

Answer

$1\frac{5}{6}$

Exercise 3.1.3.16

A recipe that requires 6 cups of sugar for 15 servings is to be used to make 45 servings. How much sugar will be needed?

Exercise 3.1.3.17

A pond loses $7\frac{1}{2}$ gallons of water every 2 days due to evaporation. How many gallons of water are lost, due to evaporation, in $\frac{1}{2}$ day?

Answer

$1\frac{7}{8}$

Exercise 3.1.3.18

A photograph that measures 3 inches wide and $4\frac{1}{2}$ inches high is to be enlarged so that it is 5 inches wide. How high will it be?

Exercise 3.1.3.19

If 25 pounds of fertilizer covers 400 square feet of grass, how many pounds will it take to cover 500 square feet of grass?

Answer

$31\frac{1}{4}$

Exercise 3.1.3.20

Every $\frac{1}{2}$ teaspoons of a particular multiple vitamin, in granular form, contains 0.65 the minimum daily requirement of vitamin C. How many teaspoons of this vitamin are required to supply 1.25 the minimum daily requirement?

Exercises for Review

Exercise 3.1.3.21

Find the product, $818 \cdot 0$.

Answer

0

Exercise 3.1.3.22

Determine the missing numerator: $\frac{8}{15} = \frac{N}{90}$.

Exercise 3.1.3.23

Find the value of $\frac{\frac{3}{10} + \frac{4}{12}}{\frac{19}{20}}$.

Answer

$\frac{2}{3}$

Exercise 3.1.3.24

Subtract 0.249 from the sum of 0.344 and 0.612.

Exercise 3.1.3.25

Solve the proportion: $\frac{6}{x} = \frac{36}{30}$.

Answer

5

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3.1.4: Percent

Learning Objectives

- understand the relationship between ratios and percents
- be able to make conversions between fractions, decimals, and percents

Ratios and Percents

Ratio, Percent

We defined a **ratio** as a comparison, by division, of two pure numbers or two like denominate numbers. A most convenient number to compare numbers to is 100. Ratios in which one number is compared to 100 are called **percents**. The word *percent* comes from the Latin word "per centum." The word "per" means "for each" or "for every," and the word "centum" means "hundred." Thus, we have the following definition.

Percent means “for each hundred," or "for every hundred."

The symbol % is used to represent the word percent.

Sample Set A

The ratio 26 to 100 can be written as 26%. We read 26% as "twenty-six percent."

Sample Set A

The ratio $\frac{165}{100}$ can be written as 165%.

We read 165% as "one hundred sixty-five percent."

Sample Set A

The percent 38% can be written as the fraction $\frac{38}{100}$.

Sample Set A

The percent 210% can be written as the fraction $\frac{210}{100}$ or the mixed number $2\frac{1)}{100}$ or 2.1.

Sample Set A

Since one dollar is 100 cents, 25 cents is $\frac{25}{100}$ of a dollar. This implies that 25 cents is 25% of one dollar.

Practice Set A

Write the ratio 16 to 100 as a percent.

Answer

16%

Practice Set A

Write the ratio 195 to 100 as a percent.

Answer

195%

Practice Set A

Write the percent 83% as a ratio in fractional form.

Answer

$\frac{83}{100}$

Practice Set A

Write the percent 362% as a ratio in fractional form.

Answer

$\frac{362}{100}$ or $\frac{181}{50}$

The Relationship Between Fractions, Decimals, and Percents – Making Conversions

Since a percent is a ratio, and a ratio can be written as a fraction, and a fraction can be written as a decimal, any of these forms can be converted to any other.

Before we proceed to the problems in Sample Set B and Practice Set B, let's summarize the conversion techniques.

Conversion Techniques – Fractions, Decimals, Percents

To Convert a Fraction	To Convert a Decimal	To Convert a Percent
To a decimal: Divide the numerator by the denominator	To a fraction: Read the decimal and reduce the resulting fraction	To a decimal: Move the decimal point 2 places to the left and drop the % symbol
To a percent: Convert the fraction first to a decimal, then move the decimal point 2 places to the right and affix the % symbol.	To a percent: Move the decimal point 2 places to the right and affix the % symbol	To a fraction: Drop the % sign and write the number "over" 100. Reduce, if possible.

Sample Set B

Convert 12% to a decimal.

Solution

$12\% = \frac{12}{100} = 0.12$

Note that

$12\% = 12.\% = 0.12$

The % symbol is dropped, and the decimal point moves 2 places to the left.

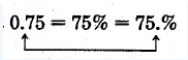
Sample Set B

Convert 0.75 to a percent.

Solution

$0.75 = \frac{75}{100} = 75\%$

Note that



The % symbol is affixed, and the decimal point moves 2 units to the right.

Sample Set B

Convert $\frac{3}{5}$ to a percent.

Solution

We see in Example above that we can convert a decimal to a percent. We also know that we can convert a fraction to a decimal. Thus, we can see that if we first convert the fraction to a decimal, we can then convert the decimal to a percent.

$$\frac{3}{5} \rightarrow \begin{array}{r} .6 \\ 5\overline{)3.0} \\ \underline{3\,0} \\ 0 \end{array} \text{ or } \frac{3}{5} = 0.6 = \frac{6}{10} = \frac{60}{100} = 60\%$$

Sample Set B

Convert 42% to a fraction.

Solution

$$42\% = \frac{42}{100} = \frac{21}{50}$$

or

$$42\% = 0.42 = \frac{42}{100} = \frac{21}{50}$$

Practice Set B

Convert 21% to a decimal.

Answer

0.21

Practice Set B

Convert 461% to a decimal.

Answer

4.61

Practice Set B

Convert 0.55 to a percent.

Answer

55%

Practice Set B

Convert 5.64 to a percent.

Answer

564%

Practice Set B

Convert $\frac{3}{20}$ to a percent.

Answer

15%

Practice Set B

Convert $\frac{11}{8}$ to a percent.

Answer

137.5%

Practice Set B

Convert $\frac{3}{11}$ to a percent.

Answer

$27.\overline{27}\%$

Exercises

For the following 12 problems, convert each decimal to a percent.

Exercise 3.1.4.1

0.25

Answer

25%

Exercise 3.1.4.2

0.36

Exercise 3.1.4.3

0.48

Answer

48%

Exercise 3.1.4.4

0.343

Exercise 3.1.4.5

0.771

Answer

77.1%

Exercise 3.1.4.6

1.42

Exercise 3.1.4.7

2.58

Answer

258%

Exercise 3.1.4.8

4.976

Exercise 3.1.4.9

16.1814

Answer

1,618.14%

Exercise 3.1.4.10

533.01

Exercise 3.1.4.11

2

Answer

200%

Exercise 3.1.4.12

14

For the following 10 problems, convert each percent to a decimal.

Exercise 3.1.4.13

15%

Answer

0.15

Exercise 3.1.4.14

43%

Exercise 3.1.4.15

16.2%

Answer

0.162

Exercise 3.1.4.16

53.8%

Exercise 3.1.4.17

5.05%

Answer

0.0505

Exercise 3.1.4.18

6.11%

Exercise 3.1.4.19

0.78%

Answer

0.0078

Exercise 3.1.4.20

0.88%

Exercise 3.1.4.21

0.09%

Answer

0.0009

Exercise 3.1.4.22

0.001%

For the following 14 problems, convert each fraction to a percent.

Exercise 3.1.4.23

$\frac{1}{5}$

Answer

20%

Exercise 3.1.4.24

$\frac{3}{5}$

Exercise 3.1.4.25

$\frac{5}{8}$

Answer

62.5%

Exercise 3.1.4.26

$\frac{1}{16}$

Exercise 3.1.4.27

$\frac{7}{25}$

Answer

28%

Exercise 3.1.4.28

$\frac{16}{45}$

Exercise 3.1.4.29

$\frac{27}{55}$

Answer

$49.\overline{09}\%$

Exercise 3.1.4.30

$\frac{15}{8}$

Exercise 3.1.4.31

$\frac{41}{25}$

Answer

164%

Exercise 3.1.4.32

$6\frac{4}{5}$

Exercise 3.1.4.33

$9\frac{9}{20}$

Answer

945%

Exercise 3.1.4.34

$\frac{1}{200}$

Exercise 3.1.4.35

$\frac{6}{11}$

Answer

$54.\overline{54}\%$

Exercise 3.1.4.36

$\frac{35}{27}$

For the following 14 problems, convert each percent to a fraction.

Exercise 3.1.4.37

80%

Answer

$\frac{4}{5}$

Exercise 3.1.4.38

60%

Exercise 3.1.4.37

25%

Answer

$\frac{1}{4}$

Exercise 3.1.4.38

75%

Exercise 3.1.4.37

65%

Answer

$\frac{13}{20}$

Exercise 3.1.4.38

18%

Exercise 3.1.4.37

12.5%

Answer

$\frac{1}{8}$

Exercise 3.1.4.38

37.5%

Exercise 3.1.4.37

512.5%

Answer

$\frac{41}{8}$ or $5\frac{1}{8}$

Exercise 3.1.4.38

937.5%

Exercise 3.1.4.37

$9.\overline{9}\%$

Answer

$\frac{1}{10}$

Exercise 3.1.4.38

$55.\overline{5}\%$

Exercise 3.1.4.37

$22.\overline{2}\%$

Answer

$\frac{2}{9}$

Exercise 3.1.4.38

$63.\overline{6}\%$

Exercises for Review

Exercise 3.1.4.39

Find the quotient. $\frac{40}{54} \div 8\frac{7}{21}$.

Answer

$\frac{2}{9}$

Exercise 3.1.4.40

$\frac{3}{8}$ of what number is $2\frac{2}{3}$?

Exercise 3.1.4.41

Find the value of $\frac{28}{15} + \frac{7}{10} - \frac{5}{12}$.

Answer

$\frac{129}{60}$ or $2\frac{9}{60} = 2\frac{3}{20}$

Exercise 3.1.4.42

Round 6.99997 to the nearest ten thousandths.

Exercise 3.1.4.43

On a map, 3 inches represent 40 miles. How many inches represent 480 miles?

Answer

36 inches

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3.1.5: Fractions of One Percent

Learning Objectives

- understand the meaning of a fraction of one percent
- be able to make conversions involving fractions of one percent

Conversions Involving Fractions of One Percent

Percents such as $\frac{1}{2}\%$, $\frac{3}{5}\%$, $\frac{5}{8}\%$, and $\frac{7}{11}\%$. where 1% has not been attained, are fractions of 1%. This implies that

$$\frac{1}{2}\% = \frac{1}{2} \text{ of } 1\%$$
$$\frac{3}{5}\% = \frac{3}{5} \text{ of } 1\%$$
$$\frac{5}{8}\% = \frac{5}{8} \text{ of } 1\%$$
$$\frac{7}{11}\% = \frac{7}{11} \text{ of } 1\%$$

Since "percent" means "for each hundred," and "of" means "times," we have

$$\frac{1}{2}\% = \frac{1}{2} \text{ of } 1\% = \frac{1}{2} \cdot \frac{1}{100} = \frac{1}{200}$$
$$\frac{3}{5}\% = \frac{3}{5} \text{ of } 1\% = \frac{3}{5} \cdot \frac{1}{100} = \frac{3}{500}$$
$$\frac{5}{8}\% = \frac{5}{8} \text{ of } 1\% = \frac{5}{8} \cdot \frac{1}{100} = \frac{5}{800}$$
$$\frac{7}{11}\% = \frac{7}{11} \text{ of } 1\% = \frac{7}{11} \cdot \frac{1}{100} = \frac{7}{1100}$$

Sample Set A

Convert $\frac{2}{3}\%$ to a fraction

Solution

$$\begin{aligned} \frac{2}{3}\% = \frac{2}{3} \text{ of } 1\% &= \frac{\overset{1}{\cancel{2}}}{3} \cdot \frac{1}{\underset{50}{\cancel{100}}} \\ &= \frac{1 \cdot 1}{3 \cdot 50} \\ &= \frac{1}{150} \end{aligned}$$

Sample Set A

Convert $\frac{5}{8}\%$ to a decimal

Solution

$$\begin{aligned} \frac{5}{8}\% = \frac{5}{8} \text{ of } 1\% &= \frac{5}{8} \cdot \frac{1}{100} \\ &= 0.625 \cdot 0.01 \\ &= 0.00625 \end{aligned}$$

Practice Set A

Convert $\frac{1}{4}\%$ to a fraction

Answer

$\frac{1}{400}$

Practice Set A

Convert $\frac{3}{8}\%$ to a fraction

Answer

$\frac{3}{800}$

Practice Set A

Convert $3\frac{1}{3}\%$ to a fraction

Answer

$\frac{1}{30}$

Conversions Involving Nonterminating Fractions

We must be careful when changing a fraction of 1% to a decimal. The number $\frac{2}{3}$, as we know, has a nonterminating decimal representation. Therefore, it cannot be expressed exactly as a decimal.

When converting nonterminating fractions of 1% to decimals, it is customary to express the fraction as a rounded decimal with at least three decimal places.

Converting a Nonterminating Fraction to a Decimal
To convert a nonterminating fraction of 1% to a decimal:

1. Convert the fraction as a rounded decimal.
2. Move the decimal point two digits to the left and remove the percent sign.

Sample Set B

Convert $\frac{2}{3}\%$ to a three-place decimal.

Solution

1. Convert $\frac{2}{3}$ to a decimal.
   Since we wish the resulting decimal to have three decimal digits, and removing the percent sign will account for two of them, we need to round $\frac{2}{3}$ to one place (2 + 1 = 3).
   $\frac{2}{3}\% = 0.7\%$ to one decimal place. $(\frac{2}{3} = 0.6666\ldots)$
2. Move the decimal point two digits to the left and remove the % sign. We'll need to add zeros to locate the decimal point in the correct location.
   $\frac{2}{3}\% = 0.007$ to 3 decimal places

Sample Set B

Convert $5\frac{4}{11}\%$ to a four-place decimal.

Solution

1. Since we wish the resulting decimal to have four decimal places, and removing the percent sign will account for two, we to round $\frac{4}{11}$ to two places.
   $5\frac{4}{11}\% = 5.36\%$ to two decimal places. $(\frac{4}{11} = 0.3636\ldots)$
2. Move the decimal point two places to the left and drop the percent sign.
   $5\frac{4}{11}\% = 0.0536$ to four decimal places.

Sample Set B

Convert $28\frac{5}{9}\%$ to a decimal rounded to ten thousandths.

Solution

1. Since we wish the resulting decimal to be rounded to ten thousandths (four decimal places), and removing the percent sign will account for two, we need to round $\frac{5}{9}$ to two places.
   $28\frac{5}{9}\% = 28.56\%$ to two decimal places. $(\frac{5}{9} = 0.5555\ldots)$
2. Move the decimal point to the left two places and drop the percent sign.
   $28\frac{5}{9}\% = 0.2856$ correct to ten thousandths.

Practice Set B

Convert $\frac{7}{9}\%$ to a three-place decimal.

Answer

0.008

Practice Set B

Convert $51\frac{5}{11}\%$ to a decimal rounded to ten thousandths.

Answer

0.5145

Exercises

Make the conversions as indicated.

Exercise 3.1.5.1

Convert $\frac{3}{4}\%$ to a fraction

Answer

$\frac{3}{400}$

Exercise 3.1.5.2

Convert $\frac{5}{6}\%$ to a fraction

Exercise 3.1.5.3

Convert $\frac{1}{9}\%$ to a fraction

Answer

$\frac{1}{900}$

Exercise 3.1.5.4

Convert $\frac{15}{19}\%$ to a fraction

Exercise 3.1.5.5

Convert $\frac{5}{4}\%$ to a fraction

Answer

$\frac{5}{400}$ or $\frac{1}{80}$

Exercise 3.1.5.6

Convert $\frac{7}{3}\%$ to a fraction

Exercise 3.1.5.7

Convert $1\frac{6}{7}\%$ to a fraction

Answer

$\frac{13}{700}$

Exercise 3.1.5.8

Convert $2\frac{5}{16}\%$ to a fraction

Exercise 3.1.5.9

Convert $25\frac{1}{4}\%$ to a fraction

Answer

$\frac{101}{400}$

Exercise 3.1.5.10

Convert $50\frac{5}{6}\%$ to a fraction

Exercise 3.1.5.11

Convert $72\frac{3}{5}\%$ to a fraction

Answer

$\frac{363}{500}$

Exercise 3.1.5.12

Convert $99\frac{1}{8}\%$ to a fraction

Exercise 3.1.5.13

Convert $136\frac{2}{3}\%$ to a fraction

Answer

$\frac{41}{30}$

Exercise 3.1.5.14

Convert $521\frac{3}{4}\%$ to a fraction

Exercise 3.1.5.15

Convert $10\frac{1}{5}\%$ to a decimal.

Answer

$\frac{51}{500} = 0.102$

Exercise 3.1.5.16

Convert $12\frac{3}{4}\%$ to a decimal.

Exercise 3.1.5.17

Convert $3\frac{7}{8}\%$ to a decimal.

Answer

$\frac{31}{800} = 0.03875$

Exercise 3.1.5.18

Convert $7\frac{1}{16}$% to a decimal.

Exercise 3.1.5.19

Convert $\frac{3}{7}$% to a three-place decimal.

Answer

0.004

Exercise 3.1.5.20

Convert $\frac{1}{9}$% to a three-place decimal.

Exercise 3.1.5.21

Convert $6\frac{3}{11}$% to a four-place decimal.

Answer

0.0627

Exercise 3.1.5.22

Convert $9\frac{2}{7}$% to a four-place decimal.

Exercise 3.1.5.23

Convert $24\frac{5}{21}$% to a three-place decimal.

Answer

0.242

Exercise 3.1.5.24

Convert $45\frac{8}{27}$% to a three-place decimal.

Exercise 3.1.5.25

Convert $11\frac{16}{17}$% to a four-place decimal.

Answer

0.1194

Exercise 3.1.5.26

Convert $5\frac{1}{7}$% to a three-place decimal.

Exercises for Review

Exercise 3.1.5.27

Write $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ using exponents.

Answer

8^5

Exercise 3.1.5.28

Convert $4\frac{7}{8}$ to an improper fraction.

Exercise 3.1.5.29

Find the sum. $\frac{7}{10} + \frac{2}{21} + \frac{1}{7}$.

Answer

$\frac{197}{210}$

Exercise 3.1.5.30

Find the product. (4.21)(0.006).

Exercise 3.1.5.31

Convert 8.062 to a percent.

Answer

806.2%

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3.1.6: Applications of Percents

Learning Objectives

- be able to distinguish between base, percent, and percentage
- be able to find the percentage, the percent, and the base

Base, Percent, and Percentage

There are three basic types of percent problems. Each type involves a base, a percent, and a percentage, and when they are translated from words to mathematical symbols *each becomes a multiplication statement*. Examples of these types of problems are the following:

1. What number is 30% of 50? (Missing product statement.)
2. 15 is what percent of 50? (Missing factor statement.)
3. 15 is 30% of what number? (Missing factor statement.)

In problem 1, the product is missing. To solve the problem, we represent the missing product with P.

$P = 30\% \cdot 50$

Definition: Percentage

The missing product P is called the **percentage**. Percentage means *part*, or *portion*. In $P = 30\% \cdot 50$. P represents a particular *part* of 50.

In problem 2, one of the factors is missing. Here we represent the missing factor with Q.

$15 = Q \cdot 50$

Percent

The missing factor is the **percent**. Percent, we know, means *per* 100, or *part* of 100. In $15 = Q \cdot 50$. Q indicates what part of 50 is being taken or considered. Specifically, $15 = Q \cdot 50$ means that if 50 was to be divided into 100 equal parts, then Q indicates 15 are being considered.

In problem 3, one of the factors is missing. Represent the missing factor with B.

$15 = 30\% \cdot B$

Base

The missing factor is the **base**. Some meanings of base are a *source of supply*, or a *starting place*. In $15 = 30\% \cdot B$, B indicates the amount of supply. Specifically, $15 = 30\% \cdot B$ indicates that 15 represents 30% of the total supply.

Each of these three types of problems is of the form

$(\text{percentage}) = (\text{percent}) \cdot (\text{base})$

We can determine any one of the three values given the other two using the methods discussed in [link].

Finding the Percentage

Sample Set A

$$\begin{array}{lllll} \underbrace{\text{What number}} & \text{is} & 30\% & \text{of} & 50? \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ P & = & 30\% & \cdot & 50 \\ P & = & .30 & \cdot & 50 \\ P & = & 15 & & \end{array}$$

Missing product statement.

Convert 30% to a decimal.

Multiply.

Thus, 15 is 30% of 50.

Sample Set A

$$\begin{array}{lcccll} \underbrace{\text{What number}} & \text{is} & 36\% & \text{of} & 95? & \text{Missing product statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ P & = & 36\% & \cdot & 95 & \text{Convert } 36\% \text{ to a decimal.} \\ P & = & .36 & \cdot & 95 & \text{Multiply.} \\ P & = & 34.2 & & & \end{array}$$

Thus, 34.2 is 36% of 95.

Sample Set A

A salesperson, who gets a commission of 12% of each sale she makes, makes a sale of $8,400.00. How much is her commission?

Solution

We need to determine what part of $8,400.00 is to be taken. What *part* indicates *percentage*.

$$\begin{array}{lcccll} \underbrace{\text{What number}} & \text{is} & 12\% & \text{of} & 8,400.00? & \text{Missing product statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ P & = & 12\% & \cdot & 8,400.00 & \text{Convert to decimals.} \\ P & = & .12 & \cdot & 8,400.00 & \text{Multiply.} \\ P & = & 1008.00 & & & \end{array}$$

Thus, the salesperson's commission is $1,008.00.

Sample Set A

A girl, by practicing typing on her home computer, has been able to increase her typing speed by 110%. If she originally typed 16 words per minute, by how many words per minute was she able to increase her speed?

Solution

We need to determine what part of 16 has been taken. What *part* indicates *percentage*.

$$\begin{array}{lcccll} \underbrace{\text{What number}} & \text{is} & 110\% & \text{of} & 16? & \text{Missing product statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ P & = & 110\% & \cdot & 16 & \text{Convert to decimals.} \\ P & = & 1.10 & \cdot & 16 & \text{Multiply.} \\ P & = & 17.6 & & & \end{array}$$

Thus, the girl has increased her typing speed by 17.6 words per minute. Her new speed is $16 + 17.6 = 33.6$ words per minute.

Sample Set A

A student who makes $125 a month working part-time receives a 4% salary raise. What is the student's new monthly salary?

Solution

With a 4% raise, this student will make 100% of the original salary + 4% of the original salary. This means the new salary will be 104% of the original salary. We need to determine what part of $125 is to be taken. What *part* indicates *percentage*.

What number	is	104%	of	125	Missing product statement.
↓	↓	↓	↓	↓	
(percentage)	=	(percent)	·	(base)	
↓	↓	↓	↓	↓	
P	=	104%	·	125	Convert to decimals.
P	=	1.04	·	125	Multiply.
P	=	130			

Thus, this student's new monthly salary is \$130.

Sample Set A

An article of clothing is on sale at 15% off the marked price. If the marked price is \$24.95, what is the sale price?

Solution

Since the item is discounted 15%, the new price will be $100\% - 15\% = 85\%$ of the marked price. We need to determine what part of 24.95 is to be taken. What *part* indicates *percentage*.

What number	is	85%	of	\$24.95	Missing product statement.
↓	↓	↓	↓	↓	
(percentage)	=	(percent)	·	(base)	
↓	↓	↓	↓	↓	
P	=	85%	·	24.95	Convert to decimals.
P	=	.85	·	24.95	Multiply.
P	=	21.2075			Since this number represents money, we'll round to 2 decimal places
P	=	21.21			

Thus, the sale price of the item is \$21.21.

Practice Set A

What number is 42% of 85?

Answer

35.7

Practice Set A

A sales person makes a commission of 16% on each sale he makes. How much is his commission if he makes a sale of \$8,500?

Answer

\$1,360

Practice Set A

An assembly line worker can assemble 14 parts of a product in one hour. If he can increase his assembly speed by 35%, by how many parts per hour would he increase his assembly of products?

Answer

4.9

Practice Set A

A computer scientist in the Silicon Valley makes \$42,000 annually. What would this scientist's new annual salary be if she were to receive an 8% raise?

Answer

$45,360

Finding the Percent

Sample Set B

$$\begin{array}{cccccc} 15 & \text{is} & \underbrace{\text{What number}} & \text{of} & 50? & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 15 & = & Q & \cdot & 50 & \end{array}$$

Recall that (missing factor) = (product) ÷ (known factor).

$$\begin{array}{rcll} Q & = & 15 \div 50 & \text{Divide.} \\ Q & = & 0.3 & \text{Convert to a percent} \\ Q & = & 30\% & \end{array}$$

Thus, 15 is 30% of 50.

Sample Set B

$$\begin{array}{cccccc} 4.32 & \text{is} & \underbrace{\text{What percent}} & \text{of} & 72? & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 4.32 & = & Q & \cdot & 72 & \end{array}$$

$$\begin{array}{rcll} Q & = & 4.32 \div 72 & \text{Divide.} \\ Q & = & 0.06 & \text{Convert to a percent} \\ Q & = & 6\% & \end{array}$$

Thus, 4.32 is 6% of 72.

Sample Set B

On a 160 question exam, a student got 125 correct answers. What percent is this? Round the result to two decimal places.

Solution

We need to determine the percent.

$$\begin{array}{cccccc} 125 & \text{is} & \underbrace{\text{What percent}} & \text{of} & 160? & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 125 & = & Q & \cdot & 160 & \end{array}$$

$$\begin{array}{rcll} Q & = & 125 \div 160 & \text{Divide.} \\ Q & = & 0.78125 & \text{Round to two decimal places} \\ Q & = & .78 & \end{array}$$

Thus, this student received a 78% on the exam.

Sample Set B

A bottle contains 80 milliliters of hydrochloric acid (HCl) and 30 milliliters of water. What percent of HCl does the bottle contain? Round the result to two decimal places.

Solution

We need to determine the percent. The total amount of liquid in the bottle is

$80 \text{ milliliters} + 30 \text{ milliliters} = 110 \text{ milliliters}$

$$\begin{array}{ccccccc} 80 & \text{is} & \underbrace{\text{What percent}} & \text{of} & 110? & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 80 & = & Q & \cdot & 110 & \end{array}$$

$$\begin{array}{rcll} Q & = & 80 \div 110 & \text{Divide.} \\ Q & = & 0.727272\ldots & \text{Round to two decimal places} \\ Q & \approx & 73\% & \text{The symbol ” } \approx \text{ ” is read as ”approximately.”} \end{array}$$

Thus, this bottle contains approximately 73% HCl.

Sample Set B

Five years ago a woman had an annual income of $19,200. She presently earns $42,000 annually. By what percent has her salary increased? Round the result to two decimal places.

Solution

We need to determine the percent.

$$\begin{array}{ccccccc} 42,000 & \text{is} & \underbrace{\text{What percent}} & \text{of} & 19,200? & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 42,000 & = & Q & \cdot & 19,200 & \end{array}$$

$$\begin{array}{rcll} Q & = & 42,000 \div 19,200 & \text{Divide.} \\ Q & = & 2.1875 & \text{Round to two decimal places} \\ Q & = & 2.19 & \text{Convert to a percent.} \\ Q & = & 219\% & \text{Convert to a percent.} \end{array}$$

Thus, this woman's annual salary has increased 219%.

Practice Set B

99.13 is what percent of 431?

Answer

23%

Practice Set B

On an 80 question exam, a student got 72 correct answers. What percent did the student get on the exam?

Answer

90%

Practice Set B

A bottle contains 45 milliliters of sugar and 67 milliliters of water. What fraction of sugar does the bottle contain? Round the result to two decimal places (then express as a percent).

Answer

40%

Sample Set C

$$\begin{array}{cccccc} 15 & \text{is} & 30\% & \text{of} & \underbrace{\text{What number?}} & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 15 & = & 30\% & \cdot & B & \text{Convert to decimals.} \\ 15 & = & .30 & \cdot & B & [\text{(missing factor)} = \text{(product)} \div \text{(known factor)}] \end{array}$$

$$\begin{aligned} B &= 15 \div .30 \\ B &= 50 \end{aligned}$$

Thus, 15 is 30% of 50.

Sample Set C

$$\begin{array}{cccccc} 56.43 & \text{is} & 33\% & \text{of} & \underbrace{\text{What number?}} & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 56.43 & = & 33\% & \cdot & B & \text{Convert to decimals.} \\ 56.43 & = & .33 & \cdot & B & \text{Divide.} \end{array}$$

$$\begin{aligned} B &= 56.43 \div .33 \\ B &= 171 \end{aligned}$$

Thus, 56.43 is 33% of 171.

Sample Set C

Fifteen milliliters of water represents 2% of a hydrochloric acid (HCl) solution. How many milliliters of solution are there?

Solution

We need to determine the total supply. The word *supply* indicates *base*.

$$\begin{array}{cccccc} 15 & \text{is} & 2\% & \text{of} & \underbrace{\text{What number?}} & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 15 & = & 2\% & \cdot & B & \text{Convert to decimals.} \\ 15 & = & .02 & \cdot & B & \text{Divide.} \end{array}$$

$$\begin{aligned} B &= 15 \div .02 \\ B &= 750 \end{aligned}$$

Thus, there are 750 milliliters of solution in the bottle.

Sample Set C

In a particular city, a sales tax of $6\frac{1}{2}$ % is charged on items purchased in local stores. If the tax on an item is \$2.99, what is the price of the item?

Solution

We need to determine the price of the item. We can think of *price* as the *starting place*. *Starting place* indicates *base*. We need to determine the base.

$$\begin{array}{ccccccc} 2.99 & \text{is} & 6\frac{1}{2}\% & \text{of} & \underbrace{\text{What number?}} & & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 2.99 & = & 6\frac{1}{2}\% & \cdot & B & & \text{Convert to decimals.} \\ 2.99 & = & 6.5\% & \cdot & B & & \\ 2.99 & = & 0.065 & \cdot & B & & [(\text{missing factor}) = (\text{product}) \div (\text{known factor})] \end{array}$$

$$\begin{array}{lll} B & = & 2.99 \div .065 \quad \text{Divide.} \\ B & = & 46 \end{array}$$

Thus, the price of the item is \$46.00.

Sample Set C

A clothing item is priced at \$20.40. This marked price includes a 15% discount. What is the original price?

Solution

We need to determine the original price. We can think of the original price as the *starting place*. *Starting place* indicates *base*. We need to determine the base. The new price, \$20.40, represents $100\% - 15\% = 85\%$ of the original price.

$$\begin{array}{ccccccc} 20.40 & \text{is} & 85\% & \text{of} & \underbrace{\text{What number?}} & & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 20.40 & = & 85\% & \cdot & B & & \text{Convert to decimals.} \\ 20.40 & = & 0.85 & \cdot & B & & [(\text{missing factor}) = (\text{product}) \div (\text{known factor})] \end{array}$$

$$\begin{array}{lll} B & = & 20.40 \div .85 \quad \text{Divide.} \\ B & = & 24 \end{array}$$

Thus, the original price of the item is \$24.00.

Practice Set C

1.98 is 2% of what number?

Answer

99

Practice Set C

3.3 milliliters of HCl represents 25% of an HCl solution. How many milliliters of solution are there?

Answer

13.2ml

Practice Set C

A salesman, who makes a commission of $18\frac{1}{4}\%$ on each sale, makes a commission of \$152.39 on a particular sale. Rounded to the nearest dollar, what is the amount of the sale?

Answer

\$835

Practice Set C

At "super-long play," $2\frac{1}{2}$ hours of play of a video cassette recorder represents 31.25% of the total playing time. What is the total playing time?

Answer

8 hours

Exercises

For the following 25 problems, find each indicated quantity.

Exercise 3.1.6.1

What is 21% of 104?

Answer

21.84

Exercise 3.1.6.2

What is 8% of 36?

Exercise 3.1.6.3

What is 98% of 545?

Answer

534.1

Exercise 3.1.6.4

What is 143% of 33?

Exercise 3.1.6.5

What is $10\frac{1}{2}$% of 20?

Answer

2.1

Exercise 3.1.6.6

3.25 is what percent of 88?

Exercise 3.1.6.7

22.44 is what percent of 44?

Answer

51

Exercise 3.1.6.8

0.0036 is what percent of 0.03?

Exercise 3.1.6.9

31.2 is what percent of 26?

Answer

120

Exercise 3.1.6.10

266.4 is what percent of 74?

Exercise 3.1.6.11

0.0101 is what percent of 0.0505?

Answer

20

Exercise 3.1.6.12

2.4 is 24% of what number?

Exercise 3.1.6.13

24.19 is 41% of what number?

Answer

59

Exercise 3.1.6.14

61.12 is 16% of what number?

Exercise 3.1.6.15

82.81 is 91% of what number?

Answer

91

Exercise 3.1.6.16

115.5 is 20% of what number?

Exercise 3.1.6.17

43.92 is 480% of what number?

Answer

9.15

Exercise 3.1.6.18

What is 85% of 62?

Exercise 3.1.6.19

29.14 is what percent of 5.13?

Answer

568

Exercise 3.1.6.20

0.6156 is what percent of 5.13?

Exercise 3.1.6.21

What is 0.41% of 291.1?

Answer

1.19351

Exercise 3.1.6.22

26.136 is 121% of what number?

Exercise 3.1.6.23

1,937.5 is what percent of 775?

Answer

250

Exercise 3.1.6.24

1 is what percent of 2,000?

Exercise 3.1.6.25

0 is what percent of 59?

Answer

0

Exercise 3.1.6.26

An item of clothing is on sale for 10% off the marked price. If the marked price is $14.95, what is the sale price? (Round to two decimal places.)

Exercise 3.1.6.27

A grocery clerk, who makes $365 per month, receives a 7% raise. How much is her new monthly salary?

Answer

390.55

Exercise 3.1.6.28

An item of clothing which originally sells for $55.00 is marked down to $46.75. What percent has it been marked down?

Exercise 3.1.6.29

On a 25 question exam, a student gets 21 correct. What percent is this?

Answer

84

Exercise 3.1.6.30

On a 45 question exam, a student gets 40%. How many questions did this student get correct?

Exercise 3.1.6.31

A vitamin tablet, which weighs 250 milligrams, contains 35 milligrams of vitamin C. What percent of the weight of this tablet is vitamin C?

Answer

14

Exercise 3.1.6.32

Five years ago a secretary made $11,200 annually. The secretary now makes $17,920 annually. By what percent has this secretary's salary been increased?

Exercise 3.1.6.33

A baseball team wins $48\frac{3}{4}$% of all their games. If they won 78 games, how many games did they play?

Answer

160

Exercise 3.1.6.34

A typist was able to increase his speed by 120% to 42 words per minute. What was his original typing speed?

Exercise 3.1.6.35

A salesperson makes a commission of 12% on the total amount of each sale. If, in one month, she makes a total of $8,520 in sales, how much has she made in commission?

Answer

$1,022.40

Exercise 3.1.6.36

A salesperson receives a salary of $850 per month plus a commission of $8\frac{1}{2}$ % of her sales. If, in a particular month, she sells $22,800 worth of merchandise, what will be her monthly earnings?

Exercise 3.1.6.37

A man borrows $1150.00 from a loan company. If he makes 12 equal monthly payments of $130.60, what percent of the loan is he paying in interest?

Answer

36.28%

Exercise 3.1.6.38

The distance from the sun to the earth is approximately 93,000,000 miles. The distance from the sun to Pluto is approximately 860.2% of the distance from the sun to the Earth. Approximately, how many miles is Pluto from the sun?

Exercise 3.1.6.39

The number of people on food stamps in Maine in 1975 was 151,000. By 1980, the number had decreased to 59,200. By what percent did the number of people on food stamps decrease? (Round the result to the nearest percent.)

Answer

61

Exercise 3.1.6.40

In Nebraska, in 1960, there were 734,000 motor-vehicle registrations. By 1979, the total had increased by about 165.6%. About how many motor-vehicle registrations were there in Nebraska in 1979?

Exercise 3.1.6.41

From 1973 to 1979, in the United States, there was an increase of 166.6% of Ph.D. social scientists to 52,000. How many were there in 1973?

Answer

19,500

Exercise 3.1.6.42

In 1950, in the United States, there were 1,894 daily newspapers. That number decreased to 1,747 by 1981. What percent did the number of daily newspapers decrease?

Exercise 3.1.6.43

A particular alloy is 27% copper. How many pounds of copper are there in 55 pounds of the alloy?

Answer

14.85

Exercise 3.1.6.44

A bottle containing a solution of hydrochloric acid (HCl) is marked 15% (meaning that 15% of the HCl solution is acid). If a bottle contains 65 milliliters of solution, how many milliliters of water does it contain?

Exercise 3.1.6.45

A bottle containing a solution of HCl is marked 45%. A test shows that 36 of the 80 milliliters contained in the bottle are hydrochloric acid. Is the bottle marked correctly? If not, how should it be remarked?

Answer

Marked correctly

Exercises For Review

Exercise 3.1.6.46

Use the numbers 4 and 7 to illustrate the commutative property of multiplication.

Exercise 3.1.6.47

Convert $\frac{14}{5}$ to a mixed number.

Answer

$2\frac{4}{5}$

Exercise 3.1.6.48

Arrange the numbers $\frac{7}{12}$, $\frac{5}{9}$ and $\frac{4}{7}$ in increasing order.

Exercise 3.1.6.49

Convert 4.006 to a mixed number.

Answer

$4\frac{3}{500}$

Exercise 3.1.6.50

Convert $\frac{7}{8}\%$ to a fraction.

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3.1.7: Summary of Key Concepts

Summary of Key Concepts

Denominate Numbers
Numbers that appear along with units are **denominate numbers**. The amounts 6 dollars and 4 pints are examples of denominate numbers.

Like and Unlike Denominate Numbers
Like denominate numbers are denominate numbers with like units. If the units are not the same, the numbers are **unlike denominate numbers**.

Pure Numbers
Numbers appearing without a unit are **pure numbers**.

Comparing Numbers by Subtraction and Division
Comparison of two numbers by subtraction indicates how much more one number is than another. Comparison by division indicates how many times larger or smaller one number is than another.

Comparing Pure or Like Denominate Numbers by Subtraction
Numbers can be compared by subtraction if and only if they are pure numbers or like denominate numbers.

Ratio Rate
A comparison, by division, of two like denominate numbers is a **ratio**. A comparison, by division, of two unlike denominate numbers is a **rate**.

Proportion
A **proportion** is a statement that two ratios or rates are equal.
$\frac{3 \text{ people}}{2 \text{ jobs}} = \frac{6 \text{ people}}{4 \text{ jobs}}$ is a proportion.

Solving a Proportion
To **solve a proportion** that contains three known numbers and a letter that represents an unknown quantity, perform the cross multiplication, then divide the product of the two numbers by the number that multiplies the letter.

Proportions Involving Rates
When writing a proportion involving rates it is very important to write it so that the same type of units appears on the same side of either the equal sign or the fraction bar.
$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 1}}{\text{unit type 2}}$ or $\frac{\text{unit type 1}}{\text{unit type 1}} = \frac{\text{unit type 2}}{\text{unit type 2}}$

Five-Step Method for Solving Proportions

1. By careful reading, determine what the unknown quantity is and represent it with some letter. There will be only one unknown in a problem.
2. Identify the three specified numbers.
3. Determine which comparisons are to be made and set up the proportion.
4. Solve the proportion.
5. Interpret and write a conclusion.

When solving applied problems, ALWAYS begin by determining the unknown quantity and representing it with a letter.

Percents
A ratio in which one number is compared to 100 is a **percent**. Percent means "for each hundred."

Conversion of Fractions, Decimals, and Percents
It is possible to convert decimals to percents, fractions to percents, percents to decimals, and percents to fractions.

Applications of Percents:
The three basic types of percent problems involve a **base**, a **percentage**, and a **percent**.

Base
The **base** is the number used for comparison.

Percentage
The **percentage** is the number being compared to the base.

Percent
By its definition, **percent** means *part of*.

Solving Problems

$$\text{Percentage} = (\text{percent}) \times (\text{base})$$

$$\text{Percent} = \frac{\text{percentage}}{\text{base}}$$

$$\text{Base} = \frac{\text{percentage}}{\text{percent}}$$

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3.1.8: Exercise Supplement

Ratios and Rates

Exercise 3.1.8.1

Compare 250 watts to 100 watts by subtraction.

Answer

250 watts are 150 watts more than 100 watts

Exercise 3.1.8.2

Compare 126 and 48 by subtraction.

Exercise 3.1.8.3

Compare 98 radishes to 41 radishes by division.

Answer

98 radishes are 2.39 times as many radishes as 41 radishes

Exercise 3.1.8.4

Compare 144 to 9 by division.

Exercise 3.1.8.5

Compare 100 tents to 5 tents by division.

Answer

100 tents are 20 times as many tents as 5 tents

Exercise 3.1.8.6

Compare 28 feet to 7 feet by division.

Exercise 3.1.8.7

Comparison, by division, of two pure numbers or two like denominate numbers is called a .

Answer

ratio

Exercise 3.1.8.8

A comparison, by division, of two unlike denominate numbers is called a .

For problems 9-12, express each ratio or rate as a fraction.

Exercise 3.1.8.9

15 to 5

Answer

$\frac{3}{1}$

Exercise 3.1.8.10

72 to 12

Exercise 3.1.8.11

8 millimeters to 5 milliliters

Answer

$\frac{8\text{ml}}{5\text{ml}}$

Exercise 3.1.8.12

106 tablets to 52 tablets

For problems 13-16, write each ratio in the form "a to b".

Exercise 3.1.8.13

$\frac{9}{16}$

Answer

9 to 16

Exercise 3.1.8.14

$\frac{5}{11}$

Exercise 3.1.8.15

$\frac{1\text{ diskette}}{8\text{ diskettes}}$

Answer

1 diskette to 8 diskettes

Exercise 3.1.8.16

$\frac{5\text{ papers}}{3\text{ pens}}$

For problems 17-21, write each ratio or rate using words.

Exercise 3.1.8.17

$$\frac{9}{16} = \frac{18}{32}$$

Answer

9 is to 16 as 18 is to 32

Exercise 3.1.8.18

$$\frac{1}{4} = \frac{12}{48}$$

Exercise 3.1.8.19

$$\frac{8 \text{ items}}{4 \text{ dollars}} = \frac{2 \text{ items}}{1 \text{ dollar}}$$

Answer

8 items are to 4 dollars as 2 items are to 1 dollar

Exercise 3.1.8.20

150 milligrams of niacin is to 2 tablets as 300 milligrams of niacin is to 4 tablets.

Exercise 3.1.8.21

20 people is to 4 seats as 5 people is to 1 seat.

Answer

$$\frac{20}{4} = \frac{5}{1}$$

20 people are to 4 seats as 5 people are to 1 seat

Proportions

For problems 22-27, determine the missing number in each proportion.

Exercise 3.1.8.22

$$\frac{x}{3} = \frac{24}{9}$$

Exercise 3.1.8.23

$$\frac{15}{7} = \frac{60}{x}$$

Answer

28

Exercise 3.1.8.24

$$\frac{1}{1} = \frac{x}{44}$$

Exercise 3.1.8.25

$$\frac{3}{x} = \frac{15}{50}$$

Answer

10

Exercise 3.1.8.26

$$\frac{15 \text{ bats}}{16 \text{ balls}} = \frac{\text{x bats}}{128 \text{ balls}}$$

Exercise 3.1.8.27

$$\frac{36 \text{ rooms}}{29 \text{ fans}} = \frac{504 \text{ rooms}}{\text{x fans}}$$

Answer

406

Applications of Proportions

Exercise 3.1.8.28

On a map, 3 inches represents 20 miles. How many miles does 27 inches represent?

Exercise 3.1.8.29

A salt solution is composed of 8 parts of salt to 5 parts of water. How many parts of salt are there in a solution that contains 50 parts of water?

Answer

80

Exercise 3.1.8.30

A model is built to $\frac{4}{15}$ scale. If a particular part of the model measures 8 inches in length, how long is the actual structure?

Exercise 3.1.8.31

The ratio of ammonia to air in a container is $\frac{3}{40}$. How many milliliters of air should be in a container that contains 8 milliliters of ammonia?

Answer

$\frac{320}{3}$ or $106\frac{2}{3}$

Exercise 3.1.8.32

A 4-foot girl casts a 9-foot shadow at a particular time of the day. How tall is a pole that casts a 144-foot shadow at the same time of the day?

Exercise 3.1.8.33

The odds that a particular event will occur are 11 to 2. If this event occurs 55 times, how many times would you predict it does not occur?

Answer

10

Exercise 3.1.8.34

Every $1\frac{3}{4}$ teaspoon of a multiple vitamin, in granular form, contains 0.85 the minimum daily requirement of vitamin A. How many teaspoons of this vitamin are required to supply 2.25 the minimum daily requirement?

Percent and Fractions of One Percent ([link],[link])

For problems 35-39, convert each decimal to a percent.

Exercise 3.1.8.35

0.16

Answer

16%

Exercise 3.1.8.36

0.818

Exercise 3.1.8.37

5.3536

Answer

535.36%

Exercise 3.1.8.38

0.50

Exercise 3.1.8.39

3

Answer

300%

For problems 40-48, convert each percent to a decimal.

Exercise 3.1.8.40

62%

Exercise 3.1.8.41

1.58%

Answer

0.0158

Exercise 3.1.8.42

9.15%

Exercise 3.1.8.43

0.06%

Answer

0.0006

Exercise 3.1.8.44

0.003%

Exercise 3.1.8.45

$5\frac{3}{11}$% to a three-place decimal

Answer

0.053

Exercise 3.1.8.46

$\frac{9}{13}$% to a three-place decimal

Exercise 3.1.8.47

$82\frac{25}{29}$% to a four-place decimal

Answer

0.8286

Exercise 3.1.8.48

$18\frac{1}{7}$% to a four-place decimal

For problems 49-55, convert each fraction or mixed number to a percent.

Exercise 3.1.8.49

$\frac{3}{5}$

Answer

60%

Exercise 3.1.8.50

$\frac{2}{10}$

Exercise 3.1.8.51

$\frac{5}{16}$

Answer

31.25%

Exercise 3.1.8.52

$\frac{35}{8}$

Exercise 3.1.8.53

$\frac{105}{16}$

Answer

656.25%

Exercise 3.1.8.54

$45\frac{1}{11}$

Exercise 3.1.8.55

$6\frac{278}{9}$

Answer

$3688.\overline{8}\%$

For problems 56-64, convert each percent to a fraction or mixed number.

Exercise 3.1.8.54

95%

Exercise 3.1.8.55

12%

Answer

$\frac{3}{25}$

Exercise 3.1.8.54

83%

Exercise 3.1.8.55

38.125%

Answer

$\frac{61}{160}$

Exercise 3.1.8.54

$61.\overline{2}\%$

Exercise 3.1.8.55

$\frac{5}{8}\%$

Answer

$\frac{1}{160}$

Exercise 3.1.8.54

$6\frac{9}{20}\%$

Exercise 3.1.8.55

$15\frac{3}{22}\%$

Answer

$\frac{2977}{19800}$

Exercise 3.1.8.54

$106\frac{19}{45}\%$

Applications of Percents

For problems 65-72, find each solution.

Exercise 3.1.8.55

What is 16% of 40?

Answer

6.4

Exercise 3.1.8.54

29.4 is what percent of 105?

Exercise 3.1.8.55

$3\frac{21}{50}$ is 547.2% of what number?

Answer

0.625 or $\frac{5}{8}$

Exercise 3.1.8.54

0.09378 is what percent of 52.1?

Exercise 3.1.8.55

What is 680% of 1.41?

Answer

9.588

Exercise 3.1.8.54

A kitchen knife is on sale for 15% off the marked price. If the marked price is $ 39.50, what is the sale price?

Exercise 3.1.8.55

On an 80 question geology exam, a student gets 68 correct. What percent is correct?

Answer

85

Exercise 3.1.8.54

A salesperson makes a commission of 18% of her monthly sales total. She also receives a monthly salary of $1,600.00. If, in a particular month, she sells $4,000.00 worth of merchandise, how much will she make that month?

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3.1.9: Proficiency Exam

Proficiency Exam

Exercise 3.1.9.1

Compare 4 cassette tapes to 7 dollars.

Answer

$$\frac{4 \text{ cassette tapes}}{7 \text{ dollars}}$$

Exercise 3.1.9.2

What do we call a comparison, by division, of two unlike denominate numbers?

Answer

Rate

For problems 3 and 4, express each ratio or rate as a fraction.

Exercise 3.1.9.3

11 to 9

Answer

$$\frac{11}{9}$$

Exercise 3.1.9.4

5 televisions to 2 radios

Answer

$$\frac{5 \text{ televisions}}{2 \text{ radios}}$$

For problems 5 and 6, write each ratio or rate in the form "a to b".

Exercise 3.1.9.5

$$\frac{8 \text{ maps}}{3 \text{ people}}$$

Answer

8 maps to 3 people

Exercise 3.1.9.6

$$\frac{2 \text{ psychologists}}{75 \text{ people}}$$

Answer

two psychologists to seventy-five people

For problems 7-9, solve each proportion.

Exercise 3.1.9.7

$$\frac{8}{x} = \frac{48}{90}$$

Answer

15

Exercise 3.1.9.8

$$\frac{x}{7} = \frac{4}{28}$$

Answer

1

Exercise 3.1.9.9

$$\frac{3 \text{ computers}}{8 \text{ students}} = \frac{24 \text{ computers}}{\text{x students}}$$

Answer

64

Exercise 3.1.9.10

On a map, 4 inches represents 50 miles. How many miles does 3 inches represent?

Answer

$37\frac{1}{2}$

Exercise 3.1.9.11

An acid solution is composed of 6 milliliters of acid to 10 milliliters of water. How many milliliters of acid are there in an acid solution that is composed of 3 milliliters of water?

Answer

1.8

Exercise 3.1.9.12

The odds that a particular event will occur are 9 to 7. If the event occurs 27 times, how many times would you predict it will it not occur?

Answer

21

For problems 13 and 14, convert each decimal to a percent.

Exercise 3.1.9.13

0.82

Answer

82%

Exercise 3.1.9.14

$3.\overline{7}$

Answer

$377\frac{7}{9}\%$

For problems 15 and 16, convert each percent to a decimal.

Exercise 3.1.9.15

2.813%

Answer

0.02813

Exercise 3.1.9.16

0.006%

Answer

0.00006

For problems 17-19, convert each fraction to a percent.

Exercise 3.1.9.17

$\frac{42}{5}$

Answer

840%

Exercise 3.1.9.18

$\frac{1}{8}$

Answer

12.5%

Exercise 3.1.9.19

$\frac{800}{80}$

Answer

1,000%

For problems 20 and 21, convert each percent to a fraction.

Exercise 3.1.9.20

15%

Answer

$\frac{3}{20}$

Exercise 3.1.9.21

$\frac{4}{27}\%$

Answer

$\frac{4}{2,700}$ or $\frac{1}{675}$

For problems 22-25, find each indicated quantity.

Exercise 3.1.9.22

What is 18% of 26?

Answer

4.68

Exercise 3.1.9.23

0.618 is what percent of 0.3?

Answer

206

Exercise 3.1.9.24

0.1 is 1.1% of what number?

Answer

$9.\overline{09}$

Exercise 3.1.9.25

A salesperson makes a monthly salary of $1,000.00. He also gets a commission of 12% of his total monthly sales. If, in a particular month, he sells $5,500.00 worth of merchandise, what is his income that month?

Answer

$1,660

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CHAPTER OVERVIEW

4: Measurement

4.1: Unit Conversion - American System

4.2: Unit Conversion- Metric System

4.3: American Units to Metric Units and Vice-Versa

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4.1: Unit Conversion - American System

In this section we will develop a technique for converting units used in the American system. We begin with a discussion of common measurements of length in the United States.

Units of Length

The most common units of length are the inch, foot, yard, and mile. Our focus will be on the technique used to convert from one unit of length to another.

American Units of Length

Facts relating common units of length.

- 1 foot (ft) = 12 inches (in)
- 1 yard (yd) = 3 feet (ft)
- 1 mile (mi) = 5280 feet (ft)

Take for example, the fact that there are 3 feet in 1 yard, which can be stated as an equation, using the common abbreviations for feet (ft) and yards (yd).

$$3 \text{ ft} = 1 \text{ yd}$$

If we divide both sides of the equation by 3 ft,

$$\frac{3 \text{ ft}}{3 \text{ ft}} = \frac{1 \text{ yd}}{3 \text{ ft}},$$

or equivalently,

$$1 = \frac{1 \text{ yd}}{3 \text{ ft}}.$$

The key observation is the fact that the ratio 1 yd/3 ft equals the number 1. Consequently, multiplying by the "conversion factor" 1 yd/3 ft is equivalent to multiplying by 1. This can be used to change a measurement in feet to yards.

Example 1

Change 36 feet to yards.

Solution

Multiply by the conversion factor 1 yd/3 ft.

$$\begin{aligned} 36 \text{ ft} &= 36 \text{ ft} \cdot 1 && \text{Multiplicative Identity Property.} \\ &= 36 \text{ ft} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} && \text{Replace 1 with 1 yd/3 ft.} \\ &= 36 \ \cancel{\text{ft}} \cdot \frac{1 \text{ yd}}{3 \ \cancel{\text{ft}}} && \text{Cancel common unit.} \\ &= \frac{36 \cdot 1}{3} \text{ yd} && \text{Multiply fractions.} \\ &= \frac{36}{3} \text{ yd} && \text{Simplify.} \\ &= 12 \text{ yd} && \text{Divide.} \end{aligned}$$

Hence, 36 feet equals 12 yards.

Exercise

Change 81 feet to yards.

Answer

27 yards

On the other hand, we can start again with

$$3 \text{ ft} = 1 \text{ yd}$$

and divide both sides of the equation by 1 yd.

$$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{1 \text{ yd}}{1 \text{ yd}}$$

This gives the conversion factor

$$\frac{3 \text{ ft}}{1 \text{ yd}} = 1.$$

The key observation is the fact that the ratio 3 ft/1 yd equals the number 1. Consequently, multiplying by the "conversion factor" 3 ft/1 yd is equivalent to multiplying by 1. This can be used to change a measurement in yards to feet.

Example 2

Change 18 yards to feet.

Solution

Multiply by the conversion factor 3 ft/1 yd.

\[\begin{aligned} 18 \text{ yd} = 18 \text{ yd} \cdot 1 ~ & \textcolor{red}{ \text{ Multiplicative Identity Property.}} \\ = 18 \text{ yd} \cdot \frac{1 \text{ ft}}{1 \text{ yd}} ~ & \textcolor{red}{ \text{ Replace 1 with 3 ft/1 yd.} } \\ = 18 \cancel{ \text{ yd}} \cdot \frac{3 \text{ ft}}{1 \cancel{ \text{ yd}}} ~ & \textcolor{red}{ \text{ Cancel common unit.}} \\ = \frac{18 \cdot 3}{1} \text{ ft} ~ & \textcolor{red}{ \text{ Multiply fractions.}} \\ = 54 \text{ ft} ~ & \textcolor{red}{ \text{ Simplify.}} \end{aligned}\nonumber \]

Hence, 18 yards equals 54 feet.

Exercise

Change 15 yards to feet.

Answer

45 feet

Another common comparison is the fact that there are 12 inches in 1 foot. This can be represented as an equation using the common abbreviation for inches (in) and feet (ft).

$$12 \text{ in} = 1 \text{ ft}$$

Dividing both sides by 12 in

$$\frac{12 \text{ in}}{12 \text{ in}} = \frac{1 \text{ ft}}{12 \text{ in}},$$

yields the conversion factor

$$1 = \frac{1 \text{ ft}}{12 \text{ in}}.$$

The key observation is the fact that the ratio 1 ft/12 in equals the number 1. Consequently, multiplying by the "conversion factor" 1 ft/12 in is equivalent to multiplying by 1. This can be used to change a measurement in inches to feet.

Example 3

Change 24 inches to feet.

Solution

Multiply by the conversion factor 1 ft/12 in.

$$\begin{aligned} 24 \text{ in} &= 24 \text{ in} \cdot 1 && \text{Multiplicative Identity Property.} \\ &= 24 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} && \text{Replace 1 with 1 ft/12 in.} \\ &= 24 \ \cancel{\text{in}} \cdot \frac{1 \text{ in}}{12 \ \cancel{\text{in}}} && \text{Cancel common unit.} \\ &= \frac{24 \cdot 1}{12} \text{ ft} && \text{Multiply fractions.} \\ &= 2 \text{ ft} && \text{Simplify.} \end{aligned}$$

Hence, 24 inches equals 2 feet.

Exercise

Change 48 inches to feet.

Answer

4 feet

We provide a summary of conversion factors for units of length in Table 6.1.

Table 6.1: Conversion factors for units of length.

Convert	Conversion Factor	Convert	Conversion Factor
feet to inches	12 in/1 ft	inches to feet	1 ft/12 in
yards to feet	3 ft/1 yd	feet to yards	1 yd/3 ft
miles to feet	5280 ft/1 mi	feet to miles	1 mi/5280 ft

Some conversions require more than one application of a conversion factor.

Example 4

Change 4 yards to inches.

Solution

We multiply by a chain of conversion factors, the first to change yards to feet, the second to change feet to inches.

$$\begin{aligned} 4 \text{ yd} &= 4 \text{ yd} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} && \textcolor{red}{\text{Multiply by conversion factors.}} \\ &= 4 \ \cancel{\text{yd}} \cdot \frac{3 \ \cancel{\text{ft}}}{1 \ \cancel{\text{yd}}} \cdot \frac{12 \text{ in}}{1 \ \cancel{\text{ft}}} && \textcolor{red}{\text{Cancel common units.}} \\ &= \frac{4 \cdot 3 \cdot 12}{1 \cdot 1} \text{ in} && \textcolor{red}{\text{Multiply fractions.}} \\ &= 144 \text{ in} && \textcolor{red}{\text{Simplify.}} \end{aligned}$$

Hence, 4 yards equals 144 inches.

Exercise

Change 8 yards to inches.

Answer

288 inches

Example 5

Change 2 miles to yards.

Solution

We multiply by a chain of conversion factors, the first to change miles to feet, the second to change feet to yards.

$$2 \text{ mi} = 2 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} \quad \text{Multiply by conversion factors.}$$

$$= 2 \ \cancel{\text{mi}} \cdot \frac{5280 \ \cancel{\text{ft}}}{1 \ \cancel{\text{mi}}} \cdot \frac{1 \text{ yd}}{3 \ \cancel{\text{ft}}} \quad \text{Cancel common units.}$$

$$= \frac{2 \cdot 5280 \cdot 1}{1 \cdot 3} \text{ yd} \quad \text{Multiply fractions.}$$

$$= 3520 \text{ yd} \quad \text{Simplify.}$$

Hence, 2 miles equals 3,520 yards.

Exercise

Change 5 miles to yards.

Answer

8,800 yards

Units of Weight

The most common units of weight are the ounce, pound, and ton. Our focus will remain on how to convert from one unit to another.

American Units of Weight

Facts relating common units of weight.

- 1 pound (lb) = 16 ounces (oz)
- 1 ton = 2000 pounds (lb)

The above facts lead to the conversion factors in Table 6.2.

Table 6.2: Conversion factors for units of weight.

Convert	Conversion Factor	Convert	Conversion Factor
pounds to ounces	16 oz/1 lb	ounces to pounds	1 lb/16 oz
tons to pounds	2000 lb/1 ton	pounds to tons	1 ton/2000 lb

Example 6

Change $2\frac{1}{2}$ pounds to ounces.

Solution

Multiply by the appropriate conversion factor.

$$\begin{aligned} 2\frac{1}{2}\text{ lb} &= 2\frac{1}{2}\text{ lb}\cdot\frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factor.}\\ &= 2\frac{1}{2}\ \cancel{\text{lb}}\cdot\frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.}\\ &= \left(2\frac{1}{2}\cdot 16\right)\text{ oz} && \text{Multiply fractions.}\\ &= \left(\frac{5}{2}\cdot 16\right)\text{ oz} && \text{Mixed to improper fraction.}\\ &= \frac{80}{2}\text{ oz} && \text{Multiply.}\\ &= 40\text{ oz} && \text{Divide.} \end{aligned}$$

Hence, $2\frac{1}{2}$ pounds equals 40 ounces.

Exercise

Change $6\frac{1}{4}$ pounds to ounces.

Answer

100 ounces

Example 7

Change 3.2 tons to ounces.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$\begin{aligned} 3.2\text{ ton} &= 3.2\text{ ton}\cdot\frac{2000\text{ lb}}{1\text{ ton}}\cdot\frac{16\text{ oz}}{1\text{ lb}} && \text{Multiply by conversion factors.}\\ &= 3.2\ \cancel{\text{ton}}\cdot\frac{2000\ \cancel{\text{lb}}}{1\ \cancel{\text{ton}}}\cdot\frac{16\text{ oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.}\\ &= \frac{3.2\cdot 2000\cdot 16}{1\cdot 1}\text{ oz} && \text{Multiply fractions.}\\ &= 102,400\text{ oz} && \text{Simplify.} \end{aligned}$$

Hence, 3.2 tons equals 102,400 ounces.

Exercise

Change 4.1 tons to ounces.

Answer

128,000 ounces

Units of Volume

The most common units of volume are fluid ounces, cups, pints, quarts, and gallons. We will focus on converting from one unit to another.

American Units of Volume

Facts relating common units of volume.

- 1 cup (c) = 8 fluid ounces (fl oz)
- 1 pint (pt) = 2 cups (c)
- 1 quart (qt) = 2 pints (pt)
- 1 gallon (gal) = 4 quarts (qt)

These facts lead to the conversion factors listed in Table 6.3.

Table 6.3: Conversion factors for units of volume.

Convert	Conversion Factor	Convert	Conversion Factor
cups to ounces	8 fl oz/1 c	ounces to cups	1c/8 fl oz
pints to cups	2 c/1 pt	cups to pints	1 pt/2 c
quarts to pints	2 pt/1 qt	pints to quarts	1 qt/2 pt
gallons to quarts	4 qt/1 gal	quarts to gallons	1 gal/4 qt

Example 8

Change 5.6 gallons to pints.

Solution

This exercise requires multiplying by a chain of conversion factors.

$$5.6\ \text{gal} = 5.6\ \text{gal} \cdot \frac{4\ \text{qt}}{1\ \text{gal}} \cdot \frac{2\ \text{pt}}{1\ \text{qt}} \quad \text{Multiply by conversion factors.}$$

$$= 5.6\ \cancel{\text{gal}} \cdot \frac{4\ \cancel{\text{qt}}}{1\ \cancel{\text{gal}}} \cdot \frac{2\ \text{pt}}{1\ \cancel{\text{qt}}} \quad \text{Cancel common units.}$$

$$= \frac{5.6 \cdot 4 \cdot 2}{1 \cdot 1}\ \text{pt} \quad \text{Multiply fractions.}$$

$$= 44.8\ \text{pt} \quad \text{Simplify.}$$

Hence, 5.6 gallons equals 44.8 pints.

Exercise

Change 3.2 gallons to pints.

Answer

25.6 pints

Units of Time

The most common units of time are seconds, minutes, hours, days, and years.

American Units of Time

Facts relating common units of time.

- 1 minute (min) = 60 seconds (s)
- 1 hour (hr) = 60 minutes (min)
- 1 day (day) = 24 hours (hr)
- 1 year (yr) = 365 days (day)

These facts lead to the conversion factors in Table 6.4.

Table 6.4: Conversion factors for units of time.

Convert	Conversion Factor	Convert	Conversion Factor
minutes to seconds	60 s/1 min	seconds to minutes	1 min/60 s
hours to minutes	60 min/1 hr	minutes to hours	1 hr/60 min
days to hours	24 hr/1 day	hours to days	1 day/24 hr
years to days	365 day/1 yr	days to years	1 yr/365 day

Example 9

How many seconds in a year?

Solution

A chain of conversion factors is needed.

$$1\text{ yr} = 1\text{ yr} \cdot \frac{365\text{ day}}{1\text{ yr}} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{60\text{ s}}{1\text{ min}} \quad \text{Conversion factors.}$$

$$= 1\ \cancel{\text{yr}} \cdot \frac{365\ \cancel{\text{day}}}{1\ \cancel{\text{yr}}} \cdot \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \cdot \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \cdot \frac{60\text{ s}}{1\ \cancel{\text{min}}} \quad \text{Cancel common units.}$$

$$= \frac{1 \cdot 365 \cdot 24 \cdot 60 \cdot 60}{1 \cdot 1 \cdot 1 \cdot 1}\text{ s} \quad \text{Multiply fractions.}$$

$$= 31,536,000\text{ s} \quad \text{Simplify.}$$

Thus, 1 year equals 31,536,000 seconds.

Exercise

How many seconds in a day?

Answer

86,400 seconds

Converting Units of Speed

Ever wonder how fast a baseball is moving?

Example 10

A professional pitcher can throw a baseball at 95 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Solution

There are 5280 feet in a mile, 60 minutes in an hour, and 60 seconds in a minute.

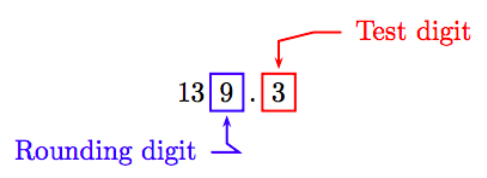
$$95\frac{\text{mi}}{\text{h}} \approx 95\frac{\text{mi}}{\text{h}} \cdot \frac{5280\ \text{ft}}{1\ \text{mi}} \cdot \frac{1\ \text{h}}{60\ \text{min}} \cdot \frac{1\ \text{min}}{60\ \text{s}} \quad \text{Conversion factors.}$$

$$\approx 95\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{5280\ \text{ft}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \cdot \frac{1\ \cancel{\text{min}}}{60\ \text{s}} \quad \text{Cancel common units.}$$

$$\approx \frac{95 \cdot 5280 \cdot 1 \cdot 1}{1 \cdot 60 \cdot 60}\frac{\text{ft}}{\text{s}} \quad \text{Multiply fractions.}$$

$$\approx 139.3\frac{\text{ft}}{\text{s}} \quad \text{Multiply and divide.}$$

To round to the nearest foot per second, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, to the nearest foot per second, the speed is approximately 139 feet per second.

Whew! Since the batter stands at home plate, which is about 60 feet from where the pitch is delivered, the batter has less than 1/2 a second to react to the pitch!

? Exercise

A women's softball pitcher can throw her fastball at 60 miles per hour. How fast is this in feet per second? Round your answer to the nearest foot per second.

Answer

88 feet per second

Exercises

1. Change 8 yards to feet.
2. Change 60 yards to feet.
3. Change 261 feet to yards.
4. Change 126 feet to yards.
5. Change 235 inches to yards. Round your answer to the nearest tenth of a yard.
6. Change 244 inches to yards. Round your answer to the nearest tenth of a yard.
7. Change 141 feet to yards.
8. Change 78 feet to yards.
9. Change 2.8 miles to feet.
10. Change 4.9 miles to feet.
11. Change 104 inches to yards. Round your answer to the nearest tenth of a yard.
12. Change 101 inches to yards. Round your answer to the nearest tenth of a yard.
13. Change 168,372 inches to miles, correct to the nearest tenth of a mile.
14. Change 198,550 inches to miles, correct to the nearest tenth of a mile.
15. Change 82 feet to inches.
16. Change 80 feet to inches.

17. Change 2.9 yards to inches. Round your answer to the nearest inch.
18. Change 4.5 yards to inches. Round your answer to the nearest inch.
19. Change 25,756 feet to miles. Round your answer to the nearest tenth of a mile.
20. Change 19,257 feet to miles. Round your answer to the nearest tenth of a mile.
21. Change 5 yards to feet.
22. Change 20 yards to feet.
23. Change 169,312 inches to miles, correct to the nearest tenth of a mile.
24. Change 162,211 inches to miles, correct to the nearest tenth of a mile.
25. Change 1.5 yards to inches. Round your answer to the nearest inch.
26. Change 2.1 yards to inches. Round your answer to the nearest inch.
27. Change 360 inches to feet.
28. Change 768 inches to feet.
29. Change 48 inches to feet.
30. Change 528 inches to feet.
31. Change 15,363 feet to miles. Round your answer to the nearest tenth of a mile.
32. Change 8,540 feet to miles. Round your answer to the nearest tenth of a mile.
33. Change 1.7 miles to inches.
34. Change 4.7 miles to inches.
35. Change 3.1 miles to inches.
36. Change 1.8 miles to inches.
37. Change 3.6 miles to feet.
38. Change 3.1 miles to feet.
39. Change 18 feet to inches.
40. Change 33 feet to inches.

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41. Change $5\frac{1}{8}$ pounds to ounces.
42. Change $3\frac{1}{16}$ pounds to ounces.
43. Change 2.4 tons to ounces.
44. Change 3.4 tons to ounces.
45. Change 34 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
46. Change 78 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
47. Change 2.2 tons to pounds.
48. Change 4.8 tons to pounds.
49. Change 70 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
50. Change 20 ounces to pounds. Express your answer as a fraction reduced to lowest terms.
51. Change 9,560 pounds to tons. Round your answer to the nearest tenth of a ton.
52. Change 9,499 pounds to tons. Round your answer to the nearest tenth of a ton.
53. Change $2\frac{1}{2}$ pounds to ounces.
54. Change $7\frac{1}{16}$ pounds to ounces.

55. Change 5.9 tons to pounds.
56. Change 2.1 tons to pounds.
57. Change 2.5 tons to ounces.
58. Change 5.3 tons to ounces.
59. Change 8,111 pounds to tons. Round your answer to the nearest tenth of a ton.
60. Change 8,273 pounds to tons. Round your answer to the nearest tenth of a ton.

---

61. Change 4.5625 pints to fluid ounces.
62. Change 2.9375 pints to fluid ounces.
63. Change 32 fluid ounces to pints.
64. Change 160 fluid ounces to pints.
65. Change 3.7 gallons to pints.
66. Change 2.4 gallons to pints.
67. Change 216 pints to gallons.
68. Change 96 pints to gallons.
69. Change 544 fluid ounces to pints.
70. Change 432 fluid ounces to pints.
71. Change 112 pints to gallons.
72. Change 200 pints to gallons.
73. Change 7.7 gallons to pints.
74. Change 5.7 gallons to pints.
75. Change 3.875 pints to fluid ounces.
76. Change 3 pints to fluid ounces.

---

77. Change 7.8 years to hours.
78. Change 4.7 years to hours.
79. Change 7.6 years to hours.
80. Change 6.6 years to hours.
81. Change 4,025,005 seconds to days. Round your answer to the nearest tenth of a day.
82. Change 4,672,133 seconds to days. Round your answer to the nearest tenth of a day.
83. Change 37,668 hours to years.
84. Change 40,296 hours to years.
85. Change 22,776 hours to years.
86. Change 29,784 hours to years.
87. Change 96 days to seconds.
88. Change 50 days to seconds.
89. Change 40 days to seconds.
90. Change 10 days to seconds.
91. Change 3,750,580 seconds to days. Round your answer to the nearest tenth of a day.
92. Change 4,493,469 seconds to days. Round your answer to the nearest tenth of a day.

93. Change 367 feet per second to miles per hour. Round your answer to the nearest mile per hour.

94. Change 354 feet per second to miles per hour. Round your answer to the nearest mile per hour.

95. Change 442 feet per second to miles per hour. Round your answer to the nearest mile per hour.

96. Change 388 feet per second to miles per hour. Round your answer to the nearest mile per hour.

97. Change 30 miles per hour to feet per second. Round your answer to the nearest foot per second.

98. Change 99 miles per hour to feet per second. Round your answer to the nearest foot per second.

99. Change 106 miles per hour to feet per second. Round your answer to the nearest foot per second.

100. Change 119 miles per hour to feet per second. Round your answer to the nearest foot per second.

101. **Strong man.** Famed strongman Joe Rollino, who was still bending quarters with his fingers at age 104, once lifted 3, 200 pounds at Coney Island Amusement Park. How many tons did Joe lift that day? *Associated Press-Times-Standard 01/12/10 NYC amusement park strongman, 104, killed by van.*

102. **Earth day.** The amount of time it takes the Earth to rotate once around its axis is one day. How many seconds is that?

103. **Water break.** "The average age of Washington, DC's water pipes is 76 years, and they are not alone. Every two minutes, somewhere in the country, a pipe breaks." How many pipes break each year in the US? *New York Times 03/14/10 Saving U.S. water and sewer systems could be costly.*

Answers

1. 24 feet

3. 87 yards

5. 6.5 yards

7. 47 yards

9. 14,784 feet

11. 2.9 yards

13. 2.7 miles

15. 984 inches

17. 104 inches

19. 4.9 miles

21. 15 feet

23. 2.7 miles

25. 54 inches

27. 30 feet

29. 4 feet

31. 2.9 miles

33. 107,712 inches

35. 196,416 inches

37. 19,008 feet

39. 216 inches

41. 82 ounces

43. 76,800 ounces

45. $2\frac{1}{8}$ pounds

47. 4,400 pounds

49. $4\frac{3}{8}$ pounds

51. 4.8 tons

53. 40 ounces

55. 11,800 pounds

57. 80,000 ounces

59. 4.1 tons

61. 73 fluid ounces

63. 2 pints

65. 29.6 pints

67. 27 gallons

69. 34 pints

71. 14 gallons

73. 61.6 pints

75. 62 fluid ounces

77. 68,328 hours

79. 66,576 hours

81. 46.6 days

83. 4.3 years

85. 2.6 years

87. 8,294,400 seconds

89. 3,456,000 seconds

91. 43.4 days

93. 250 mi/hr

95. 301 mi/hr

97. 44 ft/s

99. 155 ft/s

101. 1.6 tons

103. 262, 800

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4.2: Unit Conversion- Metric System

The metric system of units is the standard system of units preferred by scientists. It is based on the base ten number system and its decimal format is more friendly to users of this system. There is a common set of prefixes adopted by the metric system to indicate a power of ten to apply to the base unit.

Metric System Preferences

This is a list of standard prefixes for the metric system and their meanings.

$$\begin{array}{ll} \text{deka} = 10 & \text{deci} = 1/10 \\ \text{hecto} = 100 & \text{centi} = 1/100 \\ \text{kilo} = 1000 & \text{milli} = 1/1000 \end{array}$$

Thus, for example, a decameter is 10 meters, a hectoliter is 100 liters, and a kilogram is 1000 grams. Similarly, a decimeter is 1/10 of a meter, a centiliter is 1/100 of a liter, and a milligram is 1/1000 of a gram.

Units of Length

The standard measure of length in the metric system is the meter.

> *Historically, the meter was defined by the French Academy of Sciences as the length between two marks on a platinum-iridium bar, which was designed to represent 110,000,000 of the distance from the equator to the north pole through Paris. In 1983, it was redefined by the International Bureau of Weights and Measures (BIPM) as the distance travelled by light in free space in 1299,792,458 of a second. (Wikipedia)*

We can apply the standard prefixes to get the following result.

Metric Units of Length

These units of length are used in the metric system.

Unit Length	Unit Abbreviation
1 kilometer = 1000 meters	km
1 hectometer = 100 meters	hm
1 dekameter = 10 meters	dam
1 meter	m
1 decimeter = $\frac{1}{10}$ meter	dm
1 centimeter = $\frac{1}{100}$ meter	cm
1 millimeter = $\frac{1}{1000}$ meter	mm

We can use these facts to build conversion factors as we did in Section 6.3. For example, because

$$1 \text{ km} = 1000 \text{ m},$$

we can divide both sides by 1000 m to produce the conversion factor

$$1 = \frac{1 \text{ km}}{1000 \text{ m}}.$$

This conversion factor can help change meters into kilometers.

Before using this conversion factor in an example, we repeat here the rules for multiplying and dividing by powers of ten. We will be making heavy use of these rules in this section.

Multiplying and Dividing by Powers of Ten

- Multiplying a decimal number by 10^n will move the decimal point n places to the right. For example, $3.2567 \cdot 10^2 = 3.2567 \cdot 100 = 325.67$.
- Dividing a decimal number by 10^n will move the decimal point n places to the left. For example, $3.2567/10^2 = 3.2567/100 = 0.032567$.

And now the example.

Example 1

Change 2,326 meters to kilometers.

Solution

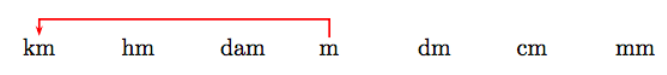
Multiply by the conversion factor 1 km/1000m.

$$\begin{aligned} 2326\ \text{m} &= 2326\ \text{m} \cdot \frac{1\ \text{km}}{1000\ \text{m}} && \text{Apply conversion factor.} \\ &= 2326\ \cancel{\text{m}} \cdot \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} && \text{Cancel common units.} \\ &= \frac{2326 \cdot 1}{1000}\ \text{km} && \text{Multiply fractions.} \\ &= 2.326\ \text{km} && \text{Simplify.} \end{aligned}$$

In the last step, note that dividing by 1000 moves the decimal point three places to the left. Thus, 2326 meters is equal to 2.326 kilometers.

Alternate Solution

A second solution depends upon the fact that multiplying or dividing by a power of ten will move the decimal point right or left a number of places equal to the number of zeros present in the multiplier or divisor. Thus, as we saw above, dividing by 1000 moved the decimal point 3 places to the left. Suppose that we arrange the metric units of length in order, from largest to smallest, as shown below.



Note that we must move 3 places left to move from the meters (m) abbreviation to the kilometers (km) abbreviation. In like manner, if we write 2,326 meters as 2,326.0meters, then we can convert to kilometers by moving the decimal 3 places to the left.

$$2,326.0\,\text{m} = 2.236\,0\,\text{km} = 2.326\,\text{km}$$

Exercise

Change 1,156 meters to kilometers

Answer

1.156 kilometers

Example 2

Change 537 centimeters to meters.

Solution

We know that

$$1\text{ cm} = \frac{1}{100}\text{ m},$$

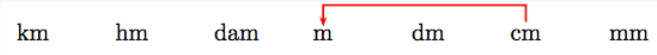
or multiplying both sides of this result by 100,

$$100\text{ cm} = 1\text{ m}.$$

Dividing both sides of this last result by 100 cm, we obtain the conversion factor 1 m/100 cm.

$$\begin{aligned} 537\text{ cm} &= 537\text{ cm} \cdot \frac{1\text{ m}}{100\text{ cm}} && \text{Apply conversion factor.} \\ &= 537\ \cancel{\text{cm}} \cdot \frac{1\text{ m}}{100\ \cancel{\text{cm}}} && \text{Cancel common units.} \\ &= \frac{537 \cdot 1}{100}\text{ m} && \text{Multiply fractions.} \\ &= 5.37\text{ m} \end{aligned}$$

In the last step, note that dividing by 100 moves the decimal point two places to the left. Alternately, we can set up our ordered list of units.



Note that we must move 2 places left to move from the centimeters (cm) abbreviation to the meters (m) abbreviation. In like manner, if we write 537 centimeters as 537.0 centimeters, then we can convert to meters by moving the decimal 2 places to the left.



Sometimes more than one conversion factor is needed.

Exercise

Change 276 centimeters to meters.

Answer

2.76 meters

Example 3

Change 10.2 dekameters to centimeters.

Solution

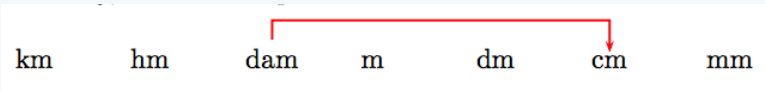
We have two facts:

- 1 dam=10 m, which yields the conversion factor 10 m/1 dam.
- 1 cm=(1/100) m or 100 cm=1 m, which yields the conversion factor 100 cm/1 m.

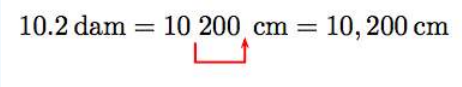
$$
\begin{aligned}
10.2\ \text{dam} &= 10.2\ \text{dam} \cdot \frac{10\ \text{m}}{1\ \text{dam}} \cdot \frac{100\ \text{cm}}{1\ \text{m}} && \text{Apply conversion factor.} \\
&= 10.2\ \cancel{\text{dam}} \cdot \frac{10\ \cancel{\text{m}}}{1\ \cancel{\text{dam}}} \cdot \frac{100\ \text{cm}}{1\ \cancel{\text{m}}} && \text{Cancel common units.} \\
&= \frac{10.2 \cdot 10 \cdot 100}{1}\ \text{cm} && \text{Multiply fractions.} \\
&= 10,200\ \text{cm}
\end{aligned}
$$

In the last step, note that multiplying by 10, then by 100, moves the decimal point three places to the right.

Alternately, we can set up our ordered list of units.



Note that we must move 3 places right to move from the dekameters (dam) abbreviation to the centimeters (cm) abbreviation. In like manner, we can convert 10.2 dekameters to centimeters by moving the decimal 3 places to the right.



Exercise

Change 13.5 dekameters to centimeters

Answer

13,500 centimeters

Units of Mass

The fundamental unit of mass in the metric system is called a gram. Originally, it was defined to be equal to one cubic centimeter of water measured at the temperature of melting ice. Now it is simply defined as 1/1000 of a kilogram, which is defined by a physical prototype preserved by the International Bureau of Weights and Measures (Wikipedia). The mass of an object is not the same as an object's weight, but rather a resistance to motion when an external force is applied.

The same metric system prefixes apply.

Metric Units of Mass

These units of mass are used in the metric system.

Units of Mass	Unit Abbreviation
1 kilogram = 1000 grams	kg
1 hectogram = 100 grams	hg
1 dekagram = 10 grams	dag
1 gram	g
1 decigram = $\frac{1}{10}$ gram	dg
1 centigram = $\frac{1}{100}$ gram	cg
1 milligram = $\frac{1}{1000}$ gram	mg

Example 4

Convert 0.025 dekagrams to milligrams.

Solution

We'll use two conversion factors:

- 1 dag=10 g, which yields the conversion factor 10 g/1 dag.
- 1 mg=(1/1000) g, which yields the conversion factor 1000 mg/1 g.

$$0.025\ \text{dag} = 0.025\ \text{dag} \cdot \frac{10\ \text{g}}{1\ \text{dag}} \cdot \frac{1000\ \text{mg}}{1\ \text{g}} \quad \text{Apply conversion factor.}$$

$$= 0.025\ \cancel{\text{dag}} \cdot \frac{10\ \cancel{\text{g}}}{1\ \cancel{\text{dag}}} \cdot \frac{1000\ \text{mg}}{1\ \cancel{\text{g}}} \quad \text{Cancel common units.}$$

$$= \frac{0.025 \cdot 10 \cdot 1000}{1}\ \text{mg} \quad \text{Multiply fractions.}$$

$$= 250\ \text{mg}$$

Alternately, we can set up our ordered list of units.

kg hg dag g dg cg mg

Note that we must move 4 places right to move from the dekagrams (dag) abbreviation to the milligrams (mg) abbreviation. In like manner, we can convert 0.025 dekagrams to milligrams by moving the decimal 4 places to the right.

$$0.0250\ \text{dag} = 0\ 0250.\ \text{mg} = 250\ \text{mg}$$

Exercise

Convert 0.05 dekagrams to milligrams.

Answer

500 milligrams

Units of Volume

The fundamental unit of volume in the metric system is called a litre. Originally, one litre was defined as the volume of one kilogram of water measured at 4° C at 760 millimeters of mercury (Wikipedia). Currently, 1 litre is defined as 1 cubic decimeter (imagine a cube with each edge 1/10 of a meter).

The same metric system prefixes apply.

Metric Units of Volume

These units of volume are used in the metric system.

Unit of Volume	Unit Abbreviation
1 kilolitre = 1000 litres	kL
1 hectolitre = 100 litres	hL
1 dekalitre = 10 litres	daL
1 litre	L
1 decilitre = $\frac{1}{10}$ litre	dL

1 centilitre = $\frac{1}{100}$ litre	cL
1 millilitre = $\frac{1}{1000}$ litre	mL

Example 5

Convert 11,725 millilitres to dekalitres.

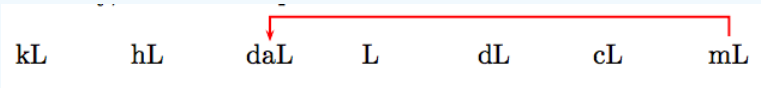
Solution

We'll use two conversion factors:

- 1 daL=10 L, which yields the conversion factor 1 daL/10 L.
- 1 mL=(1/1000) L, which yields the conversion factor 1 L/1000 mL.

$$\begin{aligned} 11,725\ \text{mL} &= 11,725\ \text{mL} \cdot \frac{1\ \text{L}}{1000\ \text{mL}} \cdot \frac{1\ \text{daL}}{10\ \text{L}} && \text{Apply conversion factors.} \\ &= 11,725\ \cancel{\text{mL}} \cdot \frac{1\ \cancel{\text{L}}}{1000\ \cancel{\text{mL}}} \cdot \frac{1\ \text{daL}}{10\ \cancel{\text{L}}} && \text{Cancel common units.} \\ &= \frac{11,725 \cdot 1 \cdot 1}{1000 \cdot 10}\ \text{daL} && \text{Multiply fractions.} \\ &= 1.1725\ \text{daL} \end{aligned}$$

Alternately, we can set up our ordered list of units.



Note that we must move 4 places left to move from the millitres (mL) abbreviation to the dekalitres (daL) abbreviation. In like manner, we can convert 11,725 millilitres to dekalitres by moving the decimal 4 places to the left.

$$11,725.0\ \text{mL} = 1.1725\ 0\ \text{daL} = 1.1725\ \text{daL}$$

Exercise

Convert 5,763 millilitres to dekalitres.

Answer

0.5763 dekalitres

Exercises

1. What is the meaning of the metric system prefix centi?
2. What is the meaning of the metric system prefix deka?
3. What is the meaning of the metric system prefix hecto?
4. What is the meaning of the metric system prefix kilo?
5. What is the meaning of the metric system prefix deci?
6. What is the meaning of the metric system prefix milli?

---

7. What is the meaning of the metric system abbreviation mg?
8. What is the meaning of the metric system abbreviation g?
9. What is the meaning of the metric system abbreviation m?

10. What is the meaning of the metric system abbreviation km?
11. What is the meaning of the metric system abbreviation kL?
12. What is the meaning of the metric system abbreviation daL?
13. What is the meaning of the metric system abbreviation hm?
14. What is the meaning of the metric system abbreviation dm?
15. What is the meaning of the metric system abbreviation dam?
16. What is the meaning of the metric system abbreviation cm?
17. What is the meaning of the metric system abbreviation dL?
18. What is the meaning of the metric system abbreviation L?
19. What is the meaning of the metric system abbreviation hg?
20. What is the meaning of the metric system abbreviation kg?
21. What is the meaning of the metric system abbreviation dg?
22. What is the meaning of the metric system abbreviation dag?
23. What is the meaning of the metric system abbreviation hL?
24. What is the meaning of the metric system abbreviation cL?

---

25. Change 5,490 millimeters to meters.
26. Change 8,528 millimeters to meters.
27. Change 64 meters to millimeters.
28. Change 65 meters to millimeters.
29. Change 4,571 millimeters to meters.
30. Change 8,209 millimeters to meters.
31. Change 15 meters to centimeters.
32. Change 12 meters to centimeters.
33. Change 569 centimeters to meters.
34. Change 380 centimeters to meters.
35. Change 79 meters to centimeters.
36. Change 60 meters to centimeters.
37. Change 7.6 kilometers to meters.
38. Change 4.9 kilometers to meters.
39. Change 861 centimeters to meters.
40. Change 427 centimeters to meters.
41. Change 4,826 meters to kilometers.
42. Change 1,929 meters to kilometers.
43. Change 4,724 meters to kilometers.
44. Change 1,629 meters to kilometers.
45. Change 6.5 kilometers to meters.
46. Change 7.9 kilometers to meters.
47. Change 17 meters to millimeters.

48. Change 53 meters to millimeters.

---

49. Change 512 milligrams to centigrams.
50. Change 516 milligrams to centigrams.
51. Change 541 milligrams to centigrams.
52. Change 223 milligrams to centigrams.
53. Change 70 grams to centigrams.
54. Change 76 grams to centigrams.
55. Change 53 centigrams to milligrams.
56. Change 30 centigrams to milligrams.
57. Change 83 kilograms to grams.
58. Change 70 kilograms to grams.
59. Change 8,196 grams to kilograms.
60. Change 6,693 grams to kilograms.
61. Change 564 centigrams to grams.
62. Change 884 centigrams to grams.
63. Change 38 grams to centigrams.
64. Change 88 grams to centigrams.
65. Change 77 centigrams to milligrams.
66. Change 61 centigrams to milligrams.
67. Change 5,337 grams to kilograms.
68. Change 4,002 grams to kilograms.
69. Change 15 kilograms to grams.
70. Change 45 kilograms to grams.
71. Change 833 centigrams to grams.
72. Change 247 centigrams to grams.

---

73. Change 619,560 centilitres to kilolitres.
74. Change 678,962 centilitres to kilolitres.
75. Change 15.2 litres to millilitres.
76. Change 9.7 litres to millilitres.
77. Change 10,850 centilitres to litres.
78. Change 15,198 centilitres to litres.
79. Change 10.7 litres to millilitres.
80. Change 17.3 litres to millilitres.
81. Change 15,665 millilitres to litres.
82. Change 12,157 millilitres to litres.
83. Change 6.3 kilolitres to centilitres.
84. Change 8.3 kilolitres to centilitres.
85. Change 4.5 kilolitres to centilitres.

86. Change 6.2 kilolitres to centilitres.
87. Change 10.6 litres to centilitres.
88. Change 16.6 litres to centilitres.
89. Change 14,383 centilitres to litres.
90. Change 11,557 centilitres to litres.
91. Change 9.9 litres to centilitres.
92. Change 19.5 litres to centilitres.
93. Change 407,331 centilitres to kilolitres.
94. Change 827,348 centilitres to kilolitres.
95. Change 14,968 millilitres to litres.
96. Change 18,439 millilitres to litres.

Answers

1. 1/100
3. 100
5. 1/10
7. milligram
9. meter
11. kilolitre or kiloliter
13. hectometer
15. dekameter
17. decilitre or deciliter
19. hectogram
21. decigram
23. hectolitre or hectoliter
25. 5.49 meters
27. 64,000 millimeters
29. 4.571 meters
31. 1,500 centimeters
33. 5.69 meters
35. 7,900 centimeters
37. 7,600 meters
39. 8.61 meters
41. 4.826 kilometers
43. 4.724 kilometers
45. 6,500 meters
47. 17,000 millimeters
49. 51.2 centigrams
51. 54.1 centigrams

53. 7,000 centigrams
55. 530 milligrams
57. 83,000 grams
59. 8.196 kilograms
61. 5.64 grams
63. 3,800 centigrams
65. 770 milligrams
67. 5.337 kilograms
69. 15,000 grams
71. 8.33 grams
73. 6.1956 kilolitres
75. 15,200 millilitres
77. 108.5 litres
79. 10,700 millilitres
81. 15.665 litres
83. 630,000 centilitres
85. 450,000 centilitres
87. 1,060 centilitres
89. 143.83 litres
91. 990 centilitres
93. 4.07331 kilolitres
95. 14.968 litres

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4.3: American Units to Metric Units and Vice-Versa

We often need to convert from the American system of units to the metric system of units or vice-versa (imagine traveling to a European country using the metric system). That will be our focus in this section.

Converting Units of Length

One meter is slightly longer than one yard. Indeed,

$$1\text{ m} \approx 1.0936\text{ yd}. \tag{4.3.1}$$

If we divide both sides of this equation by 1.0936, then

$$\frac{1\text{ m}}{1.0936} \approx \frac{1.0936\text{ yd}}{1.0936}$$
$$0.9144\text{ m} \approx 1\text{ yd}$$

Further conversions can be made. For example, to change meters to feet, we make the following conversions.

$$\begin{aligned} 1\text{ m} &\approx 1\text{ m} \cdot 1\frac{\text{yd}}{0.9144\text{ m}} \cdot \frac{3\text{ ft}}{1\text{ yd}} && \text{Apply conversion factors.} \\ &\approx 1\ \cancel{\text{m}} \cdot \frac{1\ \cancel{\text{yd}}}{0.9144\ \cancel{\text{m}}} \cdot \frac{3\text{ ft}}{1\ \cancel{\text{yd}}} && \text{Cancel common units.} \\ &\approx \frac{1 \cdot 1 \cdot 3}{0.9144 \cdot 1}\text{ ft} && \text{Multiply fractions.} \\ &\approx 3.2808\text{ ft} \end{aligned}$$

Table 6.5 shows some of the most common conversions between American units of length and metric units of length.

Table 6.5: Length conversions: American — Metric.

American to Metric	Metric to American
1 in ≈ 2.54 cm	1 cm ≈ 0.3937 in
1 ft ≈ 0.3048 m	1 m ≈ 3.2808 ft
1 yd ≈ 0.9144 m	1 m ≈ 1.0936 yd
1 mi ≈ 1.6093 km	1 km ≈ 0.6214 mi

Example 1

A car's speedometer shows that a family has currently traveled 154 miles in route to their vacation destination. Convert this distance to kilometers.

Solution

Choose 1 mi = 1.6093 km from Table 6.5.

$$\begin{aligned} 154\text{ mi} &\approx 154\text{ mi} \cdot \frac{1.6093\text{ km}}{1\text{ mi}} && \text{Apply conversion factor.} \\ &\approx 154\ \cancel{\text{mi}} \cdot \frac{1.6093\text{ km}}{1\ \cancel{\text{mi}}} && \text{Cancel common units.} \\ &\approx \frac{154 \cdot 1.6093}{1}\text{ km} && \text{Multiply fractions.} \\ &\approx 247.8\text{ km} \end{aligned}$$

Hence, 154 miles is approximately 247.8 kilometers.

Alternate Solution

Note that this would work equally well if we chose 1 km ≈ 0.6214 mi from Table 6.5.

$$\begin{aligned} 154 \text{ mi} &\approx 154 \text{ mi} \cdot \frac{1 \text{ km}}{0.6214 \text{ mi}} && \text{Apply conversion factor.} \\ &\approx 154 \ \cancel{\text{mi}} \cdot \frac{1 \text{ km}}{0.6214 \ \cancel{\text{mi}}} && \text{Cancel common units.} \\ &\approx \frac{154 \cdot 1}{0.6214} \text{ km} && \text{Multiply fractions.} \\ &\approx 247.8 \text{ km} \end{aligned}$$

? Exercise

Change 227 miles to kilometers. Round to the nearest tenth of a kilometer.

Answer

365.3 kilometers

Converting Units of Weight and Mass

It is known that

$$1 \text{ kg} \approx 2.2 \text{ lb}. \tag{4.3.2}$$

Dividing both sides of this equation by 2.2,

$$\frac{1 \text{ kg}}{2.2} \approx \frac{2.2 \text{ lb}}{2.2}$$

$$0.454 \text{ kg} \approx 1 \text{lb}$$

A summary of the more common conversion factors regarding mass and weight are given in Table 6.6.

Table 6.6: Mass—Weight conversions: American — Metric.

American to Metric	Metric to American
1 oz ≈ 28.35 g	1 g ≈ 0.035 oz
1 lb ≈ 0.454 kg	1 kg ≈ 2.2 lb

✓ Example 2

Change 2.3 kilograms to ounces.

Solution

One kilogram weighs 2.2 pounds and there are 16 ounces in a pound.

$$\begin{aligned} 2.3 \text{ kg} &\approx 2.3 \text{ kg} \cdot \frac{2.2 \text{ lb}}{1 \text{ kg}} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3 \ \cancel{\text{kg}} \cdot \frac{2.2 \ \cancel{\text{lb}}}{1 \ \cancel{\text{kg}}} \cdot \frac{16 \text{ oz}}{1 \ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2 \cdot 16}{1 \cdot 1} \text{ oz} && \text{Multiply fractions.} \\ &\approx 80.96 \text{ oz} \end{aligned}$$

Hence, 2.3 kilograms weighs 80.96 ounces.

Alternate Solution

Another approach uses the facts that 1 kilogram equals 1000 grams and 1 ounce equals 28.35 grams.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{1000\text{ g}}{1\text{ kg}} \cdot \frac{1\text{ oz}}{28.35\text{ g}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \cdot \frac{1\text{ oz}}{28.35\ \cancel{\text{g}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 1000 \cdot 1}{1 \cdot 28.35}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

Roundoff Error

Why the discrepancy in answers? This difference in approximations is due to something called *round-off error*. Indeed, in the first calculation, we used a conversion factor that is rounded to the nearest tenth; i.e., 1 kg ≈ 2.2 lb. If we use a more accurate conversion factor to change kilograms to pounds, namely 1 kg ≈ 2.2046 lb, we get the following result.

$$\begin{aligned} 2.3\text{ kg} &\approx 2.3\text{ kg} \cdot \frac{2.2046\text{ lb}}{1\text{ kg}} \cdot \frac{16\text{ oz}}{1\text{ lb}} && \text{Apply conversion factors.} \\ &\approx 2.3\ \cancel{\text{kg}} \cdot \frac{2.2046\ \cancel{\text{lb}}}{1\ \cancel{\text{kg}}} \cdot \frac{16\text{oz}}{1\ \cancel{\text{lb}}} && \text{Cancel common units.} \\ &\approx \frac{2.3 \cdot 2.2046 \cdot 16}{1 \cdot 1}\text{ oz} && \text{Multiply fractions.} \\ &\approx 81.13\text{ oz} \end{aligned}$$

Note that this result is in better agreement with the second result above.

? Exercise

Change 5.7 kilograms to ounces. Round to the nearest ounce.

Answer

201 ounces

Important Observation

To obtain better approximations, you need to use a conversion factor that is more accurate. Any time you feel you need more accuracy, you might try an online conversion utility, such as the one at: www.france-property-and-information.com/metric_conversion_table.htm

Converting Units of Volume

It is known that

$$1\text{ qt} \approx 0.946\text{ L}.$$

Dividing both sides of this equation by 0.946,

$$1\text{ L} \approx 1.06\text{ qt}.$$

Again, these conversion factors for volume have been well worked out by scientists. The more common conversion factors for volume are shown in Table 6.7.

American to Metric	Metric to American
1 fl oz ≈ 0.030 L	1 L ≈ 33.8 fl oz
1 pt ≈ 0.473 L	1 L ≈ 2.1 pt
1 qt ≈ 0.946 L	1 L ≈ 1.06 qt
1 gal ≈ 3.785 L	1 L ≈ 0.264 gal

Table 6.7: Volume conversions: American — Metric.

Example 3

Change 2.5 dekalitres to gallons.

Solution

Recall that 1 daL = 10 L and 1 L = 0.264 gal.

$$\begin{aligned} 2.5 \text{ daL} &\approx 2.5 \text{ daL} \cdot \frac{10 \text{ L}}{1 \text{ daL}} \cdot \frac{0.264 \text{ gal}}{1 \text{ L}} && \text{Apply conversion factors.} \\ &\approx 2.5 \ \cancel{\text{daL}} \cdot \frac{10 \ \cancel{\text{L}}}{1 \ \cancel{\text{daL}}} \cdot \frac{0.264 \text{ gal}}{1 \ \cancel{\text{L}}} && \text{Cancel common units.} \\ &\approx \frac{2.5 \cdot 10 \cdot 0.264}{1 \cdot 1} \text{ gal} && \text{Multiply fractions.} \\ &\approx 6.6 \text{ gal} \end{aligned}$$

Hence, 2.5 dekalitres is approximately equal to 6.6 gallons.

Exercise

Change 3.2 dekalitres to gallons.Round your answer to the nearest tenth of a gallon.

Answer

0.8 gallons

Converting Units of Speed

Modern speedometers often show a car's speed in both miles per hour and kilometers per hour.

Example 4

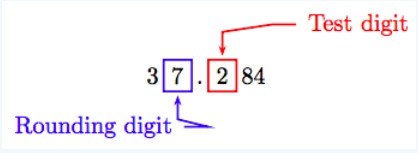
A car's speedometer is showing it speeding along at 60 kilometers per hour. How fast is it traveling in miles per hour? Round your answer to the nearest mile per hour.

Solution

From Table 6.5, 1 km ≈ 0.6214 mi.

$$\begin{aligned} 60 \frac{\text{km}}{\text{h}} &\approx 60 \frac{\text{km}}{\text{h}} \cdot \frac{0.6214 \text{ mi}}{1 \text{ km}} && \text{Apply conversion factor.} \\ &\approx 60 \frac{\cancel{\text{km}}}{\text{h}} \cdot \frac{0.6214 \text{ mi}}{1 \ \cancel{\text{km}}} && \text{Cancel common units.} \\ &\approx \frac{60 \cdot 0.6214}{1} \frac{\text{mi}}{\text{h}} && \text{Multiply fractions.} \\ &\approx 37.284 \frac{\text{mi}}{\text{h}} \end{aligned}$$

To round to the nearest mile per hour, identify the rounding and test digits.



Because the test digit is less than 5, leave the rounding digit alone and truncate. Thus, to the nearest mile per hour, the speed is approximately 37 miles per hour.

Exercise

A car's speedometer registers 45 kilometers per hour. Change this speed to miles per hour. Round your answer to the nearest mile per hour.

Answer

28 miles per hour

Exercises

1. Use the conversion 1 in = 2.54 cm to convert 68 inches to centimeters, rounded to the nearest tenth of a centimeter.
2. Use the conversion 1 in = 2.54 cm to convert 42 inches to centimeters, rounded to the nearest tenth of a centimeter.
3. Use the conversion 1 in = 2.54 cm to convert 44 centimeters to inches, rounded to the nearest tenth of a inch.
4. Use the conversion 1 in = 2.54 cm to convert 22 centimeters to inches, rounded to the nearest tenth of a inch.
5. Use the conversion 1 mi = 1.6093 km to convert 79 miles to kilometers, rounded to the nearest tenth of a kilometer.
6. Use the conversion 1 mi = 1.6093 km to convert 39 miles to kilometers, rounded to the nearest tenth of a kilometer.
7. Use the conversion 1 yd = 0.9144 m to convert 1489 centimeters to yards, rounded to the nearest tenth of a yard.
8. Use the conversion 1 yd = 0.9144 m to convert 1522 centimeters to yards, rounded to the nearest tenth of a yard.
9. Use the conversion 1 yd = 0.9144 m to convert 28 yards to centimeters, rounded to the nearest tenth of a centimeter.
10. Use the conversion 1 yd = 0.9144 m to convert 34 yards to centimeters, rounded to the nearest tenth of a centimeter.
11. Use the conversion 1 m = 3.2808 ft to convert 8.6 meters to inches, rounded to the nearest tenth of a inch.
12. Use the conversion 1 m = 3.2808 ft to convert 8.3 meters to inches, rounded to the nearest tenth of a inch.
13. Use the conversion 1 in = 2.54 cm to convert 60 inches to centimeters, rounded to the nearest tenth of a centimeter.
14. Use the conversion 1 in = 2.54 cm to convert 75 inches to centimeters, rounded to the nearest tenth of a centimeter.
15. Use the conversion 1 m = 3.2808 ft to convert 208 inches to meters, rounded to the nearest tenth of a inch.
16. Use the conversion 1 m = 3.2808 ft to convert 228 inches to meters, rounded to the nearest tenth of a inch.
17. Use the conversion 1 m = 1.0936 yd to convert 20 yards to meters, rounded to the nearest tenth of a meter.
18. Use the conversion 1 m = 1.0936 yd to convert 44 yards to meters, rounded to the nearest tenth of a meter.
19. Use the conversion 1 mi = 1.6093 km to convert 29 miles to kilometers, rounded to the nearest tenth of a kilometer.
20. Use the conversion 1 mi = 1.6093 km to convert 15 miles to kilometers, rounded to the nearest tenth of a kilometer.
21. Use the conversion 1 m = 1.0936 yd to convert 8.2 meters to yards, rounded to the nearest tenth of a yard.
22. Use the conversion 1 m = 1.0936 yd to convert 6.9 meters to yards, rounded to the nearest tenth of a yard.
23. Use the conversion 1 mi = 1.6093 km to convert 4.9 kilometers to miles, rounded to the nearest tenth of a mile.
24. Use the conversion 1 mi = 1.6093 km to convert 4.2 kilometers to miles, rounded to the nearest tenth of a mile.
25. Use the conversion 1 m = 1.0936 yd to convert 25 yards to meters, rounded to the nearest tenth of a meter.
26. Use the conversion 1 m = 1.0936 yd to convert 2 yards to meters, rounded to the nearest tenth of a meter.
27. Use the conversion 1 in = 2.54 cm to convert 47 centimeters to inches, rounded to the nearest tenth of a inch.
28. Use the conversion 1 in = 2.54 cm to convert 19 centimeters to inches, rounded to the nearest tenth of a inch.
29. Use the conversion 1 mi = 1.6093 km to convert 8.3 kilometers to miles, rounded to the nearest tenth of a mile.
30. Use the conversion 1 mi = 1.6093 km to convert 4.8 kilometers to miles, rounded to the nearest tenth of a mile.

31. Use the conversion 1 yd = 0.9144 m to convert 41 yards to centimeters, rounded to the nearest tenth of a centimeter.
32. Use the conversion 1 yd = 0.9144 m to convert 20 yards to centimeters, rounded to the nearest tenth of a centimeter.
33. Use the conversion 1 m = 3.2808 ft to convert 3.7 meters to inches, rounded to the nearest tenth of a inch.
34. Use the conversion 1 m = 3.2808 ft to convert 7.9 meters to inches, rounded to the nearest tenth of a inch.
35. Use the conversion 1 yd = 0.9144 m to convert 1323 centimeters to yards, rounded to the nearest tenth of a yard.
36. Use the conversion 1 yd = 0.9144 m to convert 1715 centimeters to yards, rounded to the nearest tenth of a yard.
37. Use the conversion 1 m = 1.0936 yd to convert 8.4 meters to yards, rounded to the nearest tenth of a yard.
38. Use the conversion 1 m = 1.0936 yd to convert 7.3 meters to yards, rounded to the nearest tenth of a yard.
39. Use the conversion 1 m = 3.2808 ft to convert 289 inches to meters, rounded to the nearest tenth of a inch.
40. Use the conversion 1 m = 3.2808 ft to convert 251 inches to meters, rounded to the nearest tenth of a inch.

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41. Use the conversion 1 kg = 2.2 lb to convert 15.8 kilograms to pounds, rounded to the nearest tenth of a pound.
42. Use the conversion 1 kg = 2.2 lb to convert 19.4 kilograms to pounds, rounded to the nearest tenth of a pound.
43. Use the conversion 1 oz = 28.35 g to convert 35 ounces to grams, rounded to the nearest tenth of a gram.
44. Use the conversion 1 oz = 28.35 g to convert 33 ounces to grams, rounded to the nearest tenth of a gram.
45. Use the conversion 1 kg = 2.2 lb to convert 2.48 kilograms to ounces, rounded to the nearest tenth of a ounce.
46. Use the conversion 1 kg = 2.2 lb to convert 3.74 kilograms to ounces, rounded to the nearest tenth of a ounce.
47. Use the conversion 1 kg = 2.2 lb to convert 2.35 kilograms to ounces, rounded to the nearest tenth of a ounce.
48. Use the conversion 1 kg = 2.2 lb to convert 3.57 kilograms to ounces, rounded to the nearest tenth of a ounce.
49. Use the conversion 1 kg = 2.2 lb to convert 15 pounds to kilograms, rounded to the nearest tenth of a kilogram.
50. Use the conversion 1 kg = 2.2 lb to convert 27 pounds to kilograms, rounded to the nearest tenth of a kilogram.
51. Use the conversion 1 kg = 2.2 lb to convert 10.4 kilograms to pounds, rounded to the nearest tenth of a pound.
52. Use the conversion 1 kg = 2.2 lb to convert 17.7 kilograms to pounds, rounded to the nearest tenth of a pound.
53. Use the conversion 1 kg = 2.2 lb to convert 352 ounces to kilograms, rounded to the nearest tenth of a kilogram.
54. Use the conversion 1 kg = 2.2 lb to convert 326 ounces to kilograms, rounded to the nearest tenth of a kilogram.
55. Use the conversion 1 oz = 28.35 g to convert 96 grams to ounces, rounded to the nearest tenth of a ounce.
56. Use the conversion 1 oz = 28.35 g to convert 100 grams to ounces, rounded to the nearest tenth of a ounce.
57. Use the conversion 1 oz = 28.35 g to convert 14 ounces to grams, rounded to the nearest tenth of a gram.
58. Use the conversion 1 oz = 28.35 g to convert 29 ounces to grams, rounded to the nearest tenth of a gram.
59. Use the conversion 1 kg = 2.2 lb to convert 54 pounds to kilograms, rounded to the nearest tenth of a kilogram.
60. Use the conversion 1 kg = 2.2 lb to convert 82 pounds to kilograms, rounded to the nearest tenth of a kilogram.
61. Use the conversion 1 oz = 28.35 g to convert 92 grams to ounces, rounded to the nearest tenth of a ounce.
62. Use the conversion 1 oz = 28.35 g to convert 103 grams to ounces, rounded to the nearest tenth of a ounce.
63. Use the conversion 1 kg = 2.2 lb to convert 388 ounces to kilograms, rounded to the nearest tenth of a kilogram.
64. Use the conversion 1 kg = 2.2 lb to convert 395 ounces to kilograms, rounded to the nearest tenth of a kilogram.

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65. Use the conversion 1 qt = 0.946 L to convert 55.1 litres to quarts, rounded to the nearest tenth of a quart.
66. Use the conversion 1 qt = 0.946 L to convert 50.3 litres to quarts, rounded to the nearest tenth of a quart.
67. Use the conversion 1 L = 33.8 fl oz to convert 72073 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.
68. Use the conversion 1 L = 33.8 fl oz to convert 56279 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.

69. Use the conversion 1 L = 33.8 fl oz to convert 2.5 kilolitres to fluid ounces.

70. Use the conversion 1 L = 33.8 fl oz to convert 4.5 kilolitres to fluid ounces.

71. Use the conversion 1 qt = 0.946 L to convert 24 quarts to litres, rounded to the nearest tenth of a litre.

72. Use the conversion 1 qt = 0.946 L to convert 21 quarts to litres, rounded to the nearest tenth of a litre.

73. Use the conversion 1 qt = 0.946 L to convert 30 quarts to litres, rounded to the nearest tenth of a litre.

74. Use the conversion 1 qt = 0.946 L to convert 22 quarts to litres, rounded to the nearest tenth of a litre.

75. Use the conversion 1 gal = 3.785 L to convert 11.8 gallons to litres, rounded to the nearest tenth of a litre.

76. Use the conversion 1 gal = 3.785 L to convert 13.5 gallons to litres, rounded to the nearest tenth of a litre.

77. Use the conversion 1 gal = 3.785 L to convert 50.5 litres to gallons, rounded to the nearest tenth of a gallon.

78. Use the conversion 1 gal = 3.785 L to convert 55.9 litres to gallons, rounded to the nearest tenth of a gallon.

79. Use the conversion 1 L = 33.8 fl oz to convert 8.3 kilolitres to fluid ounces.

80. Use the conversion 1 L = 33.8 fl oz to convert 5.3 kilolitres to fluid ounces.

81. Use the conversion 1 qt = 0.946 L to convert 42.4 litres to quarts, rounded to the nearest tenth of a quart.

82. Use the conversion 1 qt = 0.946 L to convert 55.4 litres to quarts, rounded to the nearest tenth of a quart.

83. Use the conversion 1 gal = 3.785 L to convert 17.2 gallons to litres, rounded to the nearest tenth of a litre.

84. Use the conversion 1 gal = 3.785 L to convert 19.6 gallons to litres, rounded to the nearest tenth of a litre.

85. Use the conversion 1 L = 33.8 fl oz to convert 51274 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.

86. Use the conversion 1 L = 33.8 fl oz to convert 82164 fluid ounces to kilolitres, rounded to the nearest tenth of a kilolitre.

87. Use the conversion 1 gal = 3.785 L to convert 55.6 litres to gallons, rounded to the nearest tenth of a gallon.

88. Use the conversion 1 gal = 3.785 L to convert 59.2 litres to gallons, rounded to the nearest tenth of a gallon.

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89. Change 60 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

90. Change 56 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

91. Change 77 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

92. Change 57 miles per hour to kilometers per hour. Round your answer to the nearest kilometer per hour.

93. Change 42 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

94. Change 56 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

95. Change 62 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

96. Change 63 kilometers per hour to miles per hour. Round your answer to the nearest mile per hour.

---

97. **Tallest tower.** The world's tallest tower in Dubai has 160 floors at a height of 2, 717 feet. Convert the height of the tower to the nearest tenth of a meter. *Associated Press-Times-Standard 02/09/10 World's tallest tower closed a month after opening.*

98. **High peaks.** For the first time, foreigners will be allowed to climb nearly 100 high-altitude Himalayan peaks on the Indian side of Kashmir, peaks ranging from 9, 840 feet to nearly 26, 246 feet. Convert the highest of the peaks to the nearest tenth of a meter. *Associated Press-TimesStandard 04/11/10 India opens Himalayan peaks to foreigners.*

99. **Ancient find.** In the southern Egyptian town of Luxor, a 3, 400-year-old 4-meter statue of Thoth, the ancient Egyptian god of Wisdom and Magic, was unearthed. Convert the height of the statue to the nearest tenth of a foot. *Associated Press-Times-Standard 03/17/10 3,400-year-old statues unearthed in Egypt.*

100. **Arctic wind.** Blizzard condition winds in the Arctic blew 80 miles per hour. Find the wind speed to the nearest kilometer per hour. *Associated Press-Times-Standard 12/31/09 After 562-mile ski trek, seven women reach the South Pole.*

101. **Solar plane.** The Solar Impulse lifted off from a military airport at a speed no faster than 28 miles per hour. Convert the speed of the solar-powered plane to the nearest kilometer per hour. *Associated Press-Times-Standard 04/09/10 Solar-powered plane makes successful maiden flight.*

Answers

1. 172.7 centimeters
3. 17.3 inches
5. 127.1 kilometers
7. 16.3 yards
9. 2560.3 centimeters
11. 338.6 inches
13. 152.4 centimeters
15. 5.3 meters
17. 18.3 meters
19. 46.7 kilometers
21. 9.0 yards
23. 3.0 miles
25. 22.9 meters
27. 18.5 inches
29. 5.2 miles
31. 3749.0 centimeters
33. 145.7 inches
35. 14.5 yards
37. 9.2 yards
39. 7.3 meters
41. 34.8 pounds
43. 992.3 grams
45. 87.3 ounces
47. 82.7 ounces
49. 6.8 kilograms
51. 22.9 pounds
53. 10.0 kilograms
55. 3.4 ounces
57. 396.9 grams
59. 24.5 kilograms
61. 3.2 ounces
63. 11.0 kilograms
65. 58.2 quarts
67. 2.1 kilolitres

69. 84500 ounces

71. 22.7 litres

73. 28.4 litres

75. 44.7 litres

77. 13.3 gallons

79. 280540 ounces

81. 44.8 quarts

83. 65.1 litres

85. 1.5 kilolitres

87. 14.7 gallons

89. 97 km/hr

91. 124 km/hr

93. 26 mi/hr

95. 39 mi/hr

97. 828.2 meters

99. 13.1 feet

101. 45 km/hr

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CHAPTER OVERVIEW

5: Polynomials

5.1: Introduction to Polynomials
5.2: Adding and Subtracting Polynomials
5.3: Multiplying Polynomials
5.4: Dividing Polynomials

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5.1: Introduction to Polynomials

Learning Objectives

- Identify a polynomial and determine its degree.
- Evaluate a polynomial for given values of the variables.
- Evaluate a polynomial using function notation.

Definitions

A polynomial is a special algebraic expression with terms that consist of real number coefficients and variable factors with whole number exponents.

Examples of polynomials :

$$3x^2 \quad 7xy+5 \quad \tfrac{3}{2}x^3+3x^2-\tfrac{1}{2}x+1 \quad 6x^2y-4xy^3-4xy^3+7$$

Polynomials do not have variables in the denominator of any term.

Examples that are not polynomials :

$$\tfrac{2x^2}{y} \quad 5\sqrt{x}+5 \quad 5x^2+3x^{-2}+7 \quad \tfrac{2}{x}-\tfrac{5}{y}=3$$

The degree of a term in a polynomial is defined to be the exponent of the variable, or if there is more than one variable in the term, the degree is the sum of their exponents. Recall that $x^0=1$; any constant term can be written as a product of x^0 and itself. Hence the degree of a constant term is 0.

Table 5.1.1

Term	Degree
$3x^2$	2
$6x^2y$	$2+1=3$
$7a^2b^3$	$2+3=5$
8	0, since $8=8x^0$
$2x$	1, since $x=x^1$

The degree of a polynomial is the largest degree of all of its terms.

Table 5.1.2

Polynomial	Degree
$4x^5-3x^3+2x-1$	5
$6x^2y-5xy^3+7$	4, because $5xy^3$ has degree 4.
$12x+54$	1, because $x=x^1$

We classify polynomials by the number of terms and the degree as follows:

Table 5.1.3

Expression	Classification	Degree
$5x^7$	Monomial (one term)	7
$8x^6-1$	Binomial (two terms)	6
$-3x^2+x-1$	Trinomial (three terms)	2
$5x^3-2x^2+3x-6$	Polynomial (many terms)	3

In this text, we will call polynomials with four or more terms simply polynomials.

Example 5.1.1

Classify and state the degree:

$$7x^2 - 4x^5 - 1 \, .$$

Solution:

Here there are three terms. The highest variable exponent is 5. Therefore, this is a trinomial of degree 5.

Answer:

Trinomial; degree 5

Example 5.1.2

Classify and state the degree:

$$12a^5bc^3.$$

Solution:

Since the expression consists of only multiplication, it is one term, a monomial. The variable part can be written as $a^5b^1c^3$; hence its degree is $5+1+3=9$.

Answer:

Monomial; degree 9

Example 5.1.3

Classify and state the degree:

$$4x^2y - 6xy^4 + 5x^3y^3 + 4 \, .$$

Solution:

The term $4x^2y$ has degree 3; $-6xy^4$ has degree 5; $5x^3y^3$ has degree 6; and the constant term 4 has degree 0. Therefore, the polynomial has 4 terms with degree 6.

Answer:

Polynomial; degree 6

Of particular interest are polynomials with one variable, where each term is of the form a_nx^n. Here a_n is any real number and n is any whole number. Such polynomials have the standard form

$$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 \tag{5.1.1}$$

Typically, we arrange terms of polynomials in descending order based on the degree of each term. The leading coefficient is the coefficient of the variable with the highest power, in this case, a_n.

Example 5.1.4

Write in standard form:

$$3x - 4x^2 + 5x^3 + 7 - 2x^4 \ .$$

Solution:

Since terms are separated by addition, write the following:

$$\begin{aligned} &3x - 4x^2 + 5x^3 + 7 - 2x^4 \\ &= 3x + (-4x^2) + 5x^3 + 7 + (-2x^4) \end{aligned}$$

In this form, we can see that the subtraction in the original corresponds to negative coefficients. Because addition is commutative, we can write the terms in descending order based on the degree of each term as follows:

$$\begin{aligned} &= (-2x^4) + 5x^3 + (-4x^2) + 3x + 7 \\ &= -2x^4 + 5x^3 - 4x^2 + 3x + 7 \end{aligned}$$

Answer:

$$-2x^4 + 5x^3 - 4x^2 + 3x + 7$$

We can further classify polynomials with one variable by their degree as follows:

Table 5.1.4

Polynomial	Name
5	**Constant** (degree 0)
$2x + 1$	**Linear** (degree 1)
$3x^2 + 5x - 3$	**Quadratic** (degree 2)
$x^3 + x^2 + x + 1$	**Cubic** (degree 3)
$7x^4 + 3x^3 - 7x + 8$	Fourth-degree polynomial

In this text, we call any polynomial of degree $n \geq 4$ an nth-degree polynomial. In other words, if the degree is 4, we call the polynomial a fourth-degree polynomial. If the degree is 5, we call it a fifth-degree polynomial, and so on.

Evaluating Polynomials

Given the values for the variables in a polynomial, we can substitute and simplify using the order of operations.

Example 5.1.5

Evaluate:

$$3x - 1, \text{ where } x = -\tfrac{3}{2}.$$

Solution:

First, replace the variable with parentheses and then substitute the given value.

Answer:

$-\frac{11}{2}$

Example 5.1.6

Evaluate:

$$3x^2 + 2x - 1 \text{ , where } x = -1.$$

Solution:

Answer:

0

Example 5.1.7

Evaluate:

$$-2a^2b + ab^2 - 7 \text{ , where } a = 3 \text{ and } b = -2.$$

Solution:

Answer:

41

Example 5.1.8

The volume of a sphere in cubic units is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. Calculate the volume of a sphere with radius $r = \frac{3}{2}$ meters.

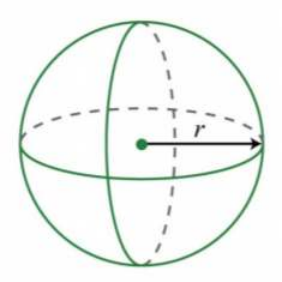


Figure 5.1.1

Solution:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 \\ &= \frac{4}{3}\pi\frac{3^3}{2^3} \\ &= \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\pi\frac{27}{\underset{2}{\cancel{8}}} \\ &= \frac{9}{2}\pi \end{aligned}$$

Answer:

$\frac{9}{2}\pi$ cubic meters

Exercise 5.1.1

Evaluate:

$$x^3 - x^2 + 4x - 2 \text{ , where } x = -3.$$

Answer

-50

Polynomial Functions

Polynomial functions with one variable are functions that can be written in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0 \qquad (5.1.2)$$

,

where a_n is any real number and n is any whole number. Some examples of the different classes of polynomial functions are listed below:

Table 5.1.5

Polynomial function	Name
$f(x)=5$	Constant function (degree 0)
$f(x)=-2x+1$	Linear function (degree 1)
$f(x)=5x^2+4x-3$	Quadratic function (degree 2)
$f(x)=x^3-1$	Cubic function (degree 3)
$f(x)=4x^5+3x^4-7$	**Polynomial function**

Since there are no restrictions on the values for x, the domain of any polynomial function consists of all real numbers.

Example 5.1.9

Calculate:

$$f(5), \text{ given } f(x)=-2x^2+5x+10\,.$$

Solution:

Recall that the function notation $f(5)$ indicates we should evaluate the function when $x=5$. Replace every instance of the variable x with the value 5.

Answer:

$f(5)=-15$

Example 5.1.10

Calculate:

$$f(-1), \text{ given } f(x)=-x^3+2x^2-4x+1\ .$$

Solution:

Replace the variable x with -1.

$$\begin{aligned} f(-1) &= -(-1)^3+2(-1)^2-4(-1)+1 \\ &= -(-1)+2\cdot 1+4+1 \\ &= 1+2+4+1 \\ &= 8 \end{aligned}$$

Answer:

$$f(-1)=8$$

Exercise 5.1.2

Given $g(x)=x^3-2x^2-x-4$, calculate $g(-1)$.

Answer

$g(-1)=-6$

Key Takeaways

- Polynomials are special algebraic expressions where the terms are the products of real numbers and variables with whole number exponents.
- The degree of a polynomial with one variable is the largest exponent of the variable found in any term.
- The terms of a polynomial are typically arranged in descending order based on the degree of each term.

- When evaluating a polynomial, it is a good practice to replace all variables with parentheses and then substitute the appropriate values.
- All polynomials are functions.

Exercise 5.1.3 Definitions

Classify the given polynomial as linear, quadratic, or cubic.

1. $2x+1$
2. x^2+7x+2
3. $2-3x^2+x$
4. $4x$
5. x^2-x^3+x+1
6. $5-10x^3$

Answer

1. Linear

3. Quadratic

5. Cubic

Exercise 5.1.4 Definitions

Classify the given polynomial as a monomial, binomial, or trinomial and state the degree.

1. x^3-1
2. x^2y^2
3. $x-x^5+1$
4. x^2+3x-1
5. $5ab^4$
6. $13x-12$
7. $-5x^3+2x+1$
8. $8x^2-9$
9. $4x^5-5x^3+6x$
10. $8x^4-x^5+2x-3$
11. $9x+7$
12. $x^5+x^4+x^3+x^2-x+1$
13. $6x-1+5x^4-8$
14. $4x-3x^2+3$
15. 7
16. x^2
17. $4x^2y-3x^3y^3+xy^3$
18. a^3b^2-6ab
19. a^3b^3
20. x^2y-y^2x
21. $xy-3$
22. $a^5bc^2+3a^9-5a^4b^3c$
23. $-3x^{10}y^2z-xy^{12}z+9x^{13}+30$
24. $7x^0$

Answer

1. Binomial; degree 3

3. Trinomial; degree 5

5. Monomial; degree 5

7. Trinomial; degree 3

9. Trinomial; degree 5

11. Binomial; degree 1

13. Not a polynomial

15. Monomial; degree 0

17. Trinomial; degree 6

19. Monomial; degree 6

21. Binomial; degree 2

23. Polynomial; degree 14

Exercise 5.1.5 Definitions

Write the following polynomials in standard form.

1. $1-6x+7x^2$
2. $x-9x^2-8$
3. $7-x^3+x^7-x^2+x-5x^5$
4. $a^3-a^9+6a^5-a+3-a^4$

Answer

1. $7x^2-6x+1$
3. $x^7-5x^5-x^3-x^2+x+7$

Exercise 5.1.6 Evaluating Polynomials

1. Fill in the following chart:

x	x^2
-3	
-2	
0	
2	
3	

***Figure* 5.1.2**

2. Fill in the following chart:

x	x^3
-3	
-2	
0	
2	
3	

***Figure* 5.1.3**

Answer

1.

x	x^2
-3	9
-2	4
0	0
2	4
3	9

Figure 5.1.4

? Exercise 5.1.7 Evaluating Polynomials

Evaluate.

1. $2x-3$, where $x=3$
2. x^2-3x+5 , where $x=-2$
3. $-12x+13$, where $x=-13$
4. $-x^2+5x-1$, where $x=-12$
5. $-2x^2+3x-5$, where $x=0$
6. $8x^5-27x^3+81x-17$, where $x=0$
7. y^3-2y+1 , where $y=-2$
8. y^4+2y^2-32 , where $y=2$
9. a^3+2a^2+a-3 , where $a=-3$
10. x^3-x^2 , where $x=5$
11. $34x^2-12x+36$, where $x=-23$
12. $58x^2-14x+12$, where $x=4$
13. x^2y+xy^2, where $x=2$ and $y=-3$
14. $2a^5b-ab^4+a^2b^2$, where $a=-1$ and $b=-2$
15. a^2-b^2 , where $a=5$ and $b=-6$
16. a^2-b^2 , where $a=34$ and $b=-14$
17. a^3-b^3 , where $a=-2$ and $b=3$

18. $a^3 + b^3$, where $a = 5$ and $b = -5$

Answer

1. 3
3. $\frac{1}{2}$
5. -5
7. -3
9. -15
11. $\frac{7}{6}$
13. 6
15. -11
17. -35

Exercise 5.1.8 Evaluating Polynomials

For each problem, evaluate $b^2 - 4ac$, given the following values.

1. $a = -1, b = 2$, and $c = -1$
2. $a = 2, b = -2$, and $c = 12$
3. $a = 3, b = -5, c = 0$
4. $a = 1, b = 0$, and $c = -4$
5. $a = 14, b = -4$, and $c = 2$
6. $a = 1, b = 5$, and $c = 6$

Answer

1. 0
3. 25
5. 14

Exercise 5.1.9 Evaluating Polynomials

The volume of a sphere in cubic units is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. For each problem, calculate the volume of a sphere given the following radii.

1. $r = 3$ centimeters
2. $r = 1$ centimeter
3. $r = \frac{1}{2}$ feet
4. $r = \frac{3}{2}$ feet
5. $r = 0.15$ in
6. $r = 1.3$ inches

Answer

1. 36π cubic centimeters
3. $\frac{\pi}{6}$ cubic feet
5. 0.014cubic inches

Exercise 5.1.10 Evaluating Polynomials

The height in feet of a projectile launched vertically from the ground with an initial velocity v_0 in feet per second is given by the formula $h = -16t^2 + v_0 t$, where t represents time in seconds. For each problem, calculate the height of the projectile given the following initial velocity and times.

1. $v_0 = 64$ feet/second, at times $t = 0, 1, 2, 3, 4$ seconds
2. $v_0 = 80$ feet/second, at times $t = 0, 1, 2, 2.5, 3, 4, 5$seconds

Answer

1.

Table 5.1.6

Time	Height
$t = 0$ seconds	$h = 0$ feet
$t = 1$ second	$h = 48$ feet
$t = 2$ seconds	$h = 64$ feet
$t = 3$ seconds	$h = 48$ feet
$t = 4$ seconds	$h = 0$ feet

Exercise 5.1.11 Evaluating Polynomials

The stopping distance of a car, taking into account an average reaction time, can be estimated with the formula $d = 0.05v^2 + 1.5$, where d is in feet and v is the speed in miles per hour. For each problem, calculate the stopping distance of a car traveling at the given speeds.

1. 20 miles per hour
2. 40 miles per hour
3. 80 miles per hour
4. 100 miles per hour

Answer

1. 21.5feet

3. 321.5feet

Exercise 5.1.12 Polynomial Functions

Given the linear function $f(x) = \frac{2}{3}x + 6$, evaluate each of the following.

1. $f(-6)$
2. $f(-3)$
3. $f(0)$
4. $f(3)$
5. Find x when $f(x) = 10$.
6. Find x when $f(x) = -4$.

Answer

1. 2

3. 6

5. $x = 6$

Exercise 5.1.13 Polynomial Functions

Given the quadratic function $f(x) = 2x^2 - 3x + 5$, evaluate each of the following.

1. $f(-2)$
2. $f(-1)$
3. $f(0)$
4. $f(2)$

Answer

1. 19

3. 5

Exercise 5.1.14 Polynomial Functions

Given the cubic function $g(x) = x^3 - x^2 + x - 1$, evaluate each of the following.

1. $g(-2)$
2. $g(-1)$
3. $g(0)$
4. $g(1)$

Answer

1. -15

3. -1

Exercise 5.1.15 Polynomial Functions

The height in feet of a projectile launched vertically from the ground with an initial velocity of 128 feet per second is given by the function $h(t) = -16t^2 + 128t$, where t is in seconds. Calculate and interpret the following.

1. $h(0)$
2. $h(12)$
3. $h(1)$
4. $h(3)$
5. $h(4)$
6. $h(5)$
7. $h(7)$
8. $h(8)$

Answer

1. The projectile is launched from the ground.

3. The projectile is 112 feet above the ground 1 second after launch.

5. The projectile is 256 feet above the ground 4 seconds after launch.

7. The projectile is 112 feet above the ground 7 seconds after launch.

Exercise 5.1.16 Discussion Board Topics

1. Find and share some graphs of polynomial functions.
2. Explain how to convert feet per second into miles per hour.
3. Find and share the names of fourth-degree, fifth-degree, and higher polynomials.

Answer

1. Answers may vary

3. Answers may vary

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5.2: Adding and Subtracting Polynomials

Learning Objectives

- Add polynomials.
- Subtract polynomials.
- Add and subtract polynomial functions

Adding Polynomials

Recall that we combine like terms, or terms with the same variable part, as a means to simplify expressions. To do this, add the coefficients of the terms to obtain a single term with the same variable part. For example,

$$5x^2+8x^2=13x^2 \tag{5.2.1}$$

Notice that the variable part, x^2, does not change. This, in addition to the commutative and associative properties of addition, allows us to add polynomials.

Example 5.2.1

Add:

$$3x+(4x-5).$$

Solution:

The property $+(a+b)=a+b$, which was derived using the distributive property, allows us to remove the parentheses so that we can add like terms.

Answer:

$7x-5$

Example 5.2.2

Add:

$$(3x^2+3x+5)+(2x^2-x-2)\ .$$

Solution:

Remove the parentheses and then combine like terms.

Answer:

$5x^2+2x+3$

Example 5.2.3

Add:

$$(-5x^2y-2xy^2+7xy)+(4x^2y+7xy^2-3xy)\,.$$

Solution:

Remember that the variable parts have to be exactly the same before we can add the coefficients.

$$\begin{aligned}&(-5x^2y-2xy^2+7xy)+(4x^2y+7xy^2-3xy)\\&=\underline{-5x^2y}\underline{-2xy^2}+\underline{\underline{7xy}}+\underline{4x^2y}+\underline{7xy^2}-\underline{\underline{3xy}}\\&=-x^2y+5xy^2+4xy\end{aligned}$$

Answer:

$$-x^2y+5xy^2+4xy$$

It is common practice to present the terms of the simplified polynomial expression in descending order based on their degree. In other words, we typically present polynomials in standard form, with terms in order from highest to lowest degree.

Example 5.2.4

Add:

$$(a-4a^3+a^5-8)+(-9a^5+a^4-7a+5+a^3) \quad .$$

Solution:

Answer:

$-8a^5+a^4-3a^3-6a-3$

Exercise 5.2.1

Add:

$$(6-5x^3+x^2-x)+(x^2+x+6x^3-1) \quad .$$

Answer

x^3+2x^2+5

Subtracting Polynomials

When subtracting polynomials, we see that the parentheses become very important. Recall that the distributive property allowed us to derive the following:

$$-(a+b)=-a-b \tag{5.2.2}$$

In other words, when subtracting an algebraic expression, we remove the parentheses by subtracting each term.

Example 5.2.5

Subtract:

$$10x-(3x+5).$$

Solution:

Subtract each term within the parentheses and then combine like terms.

Answer:

$7x-5$

Subtracting a quantity is equivalent to multiplying it by -1.

Example 5.2.6

Subtract:

$$(3x^2+3x+5)-(2x^2-x-2) \quad .$$

Solution:

Distribute the -1, remove the parentheses, and then combine like terms.

$$\begin{aligned}&\left(3x^2+3x+5\right)-\left(2x^2-x-2\right)\\&=3x^2+3x+5-2x^2+x+2\\&=x^2+4x+7\end{aligned}$$

Figure 5.2.1

Answer:

$$x^2+4x+7$$

Multiplying the terms of a polynomial by -1 changes all the signs.

Example 5.2.7

Subtract:

$$(-5x^3-2x^2+7)-(4x^3+7x^2-3x+2) \quad .$$

Solution:

Distribute the -1, remove the parentheses, and then combine like terms.

Answer:

$-9x^3-9x^2+3x+5$

Example 5.2.8

Subtract $6x^2-3x-1$ from $2x^2+5x-2$.

Solution

Since subtraction is not commutative, we must take care to set up the difference correctly. First, write the quantity $(2x^2+5x-2)$; from this, subtract the quantity $(6x^2-3x-1)$.

Answer:

$-4x^2+8x-1$

Example 5.2.9

Simplify:

$$(2x^2-3x+5)-(x^2-3x+1)+(5x^2-4x-8) \quad .$$

Solution:

Apply the distributive property, remove the parentheses, and then combine like terms.

Answer:

$6x^2-4x-4$

Exercise 5.2.2

Subtract:

$$(8x^2y-5xy^2+6)-(x^2y+2xy^2-1) \quad .$$

Answer

$7x^2y-7xy^2+7$

Adding and Subtracting Polynomial Functions

We use function notation to indicate addition and subtraction of functions as follows:

Table 5.2.1

Addition of functions:	$(f+g)(x)=f(x)+g(x)$
Subtraction of functions:	$(f-g)(x)=f(x)-g(x)$

When using function notation, be careful to group the entire function and add or subtract accordingly.

Example 5.2.10

Calculate:

$$(f+g)(x), \text{ given } f(x)=-x^2-3x+5 \text{ and } g(x)=3x^2+2x+1\ .$$

Solution:

The notation $(f+g)(x)$ indicates that you should add the functions $f(x)+g(x)$ and collect like terms.

$$\begin{aligned} f(x)+g(x) &= (-x^2-3x+5)+(3x^2+2x+1) \\ &= -x^2-3x+5+3x^2+2x+1 \\ &= 2x^2-x+6 \end{aligned}$$

Answer:

$$(f+g)(x)=2x^2-x+6$$

Example 5.2.11

Calculate:

$$(f-g)(x), \text{ given } f(x)=2x-3 \text{ and } g(x)=-2x^2+2x+5\ .$$

Solution:

The notation $(f-g)(x)$ indicates that you should subtract the functions $f(x)-g(x)$:

Answer:

$(f-g)(x)=2x^2-8$

We may be asked to evaluate the sum or difference of two functions. We have the option to first find the sum or difference and use the resulting function to evaluate for the given variable, or to first evaluate each function and then find the sum or difference.

Example 5.2.12

Calculate:

$$(f-g)(5), \text{ given } f(x)=x^2+x-7 \text{ and } g(x)=4x+10\,.$$

Solution:

First, find $(f-g)(x)=f(x)-g(x)$.

Therefore,

$(f-g)(x)=x^2-3x-17$

Next, substitute 5 for the variable x.

Answer:

$(f-g)(5)=-7$

Alternate Solution:

Since $(f-g)(5)=f(5)-g(5)$, we can find $f(5)$ and $g(5)$ and then subtract the results.

$$\begin{array}{l|l} f(x)=x^2+x-7 & g(x)=4x+10 \\ f(5)=(5)^2+(5)-7 & g(5)=4(5)+10 \\ \quad =25+5-7 & \quad =20+10 \\ \quad =23 & \quad =30 \end{array}$$

Therefore, we have

Answer:

$(f-g)(5)=-7$

Key Takeaways

- When adding polynomials, remove the associated parentheses and then combine like terms.
- When subtracting polynomials, distribute the -1 and subtract all the terms before removing the parentheses and combining like terms.
- The notation $(f+g)(x)$ indicates that you add the functions.
- The notation $(f-g)(x)$ indicates that you subtract the functions

? Exercise 5.2.3 Addition of Polynomials

Add.

1. $(2x+1)+(-x+7)$
2. $(-6x+5)+(3x-1)$
3. $(\frac{2}{3}x+\frac{1}{2})+(\frac{1}{3}x-2)$
4. $(\frac{1}{3}x-\frac{3}{4})+(\frac{5}{6}x+\frac{1}{8})$
5. $(2x+1)+(x-3)+(5x-2)$
6. $(2x-8)+(-3x^2+7x-5)$
7. $(x^2-3x+7)+(3x^2-8x-5)$
8. $(-5x^2-1+x)+(-x+7x^2-9)$
9. $(\frac{1}{2}x^2-\frac{1}{3}x+\frac{1}{6})+(-\frac{3}{2}x^2+\frac{2}{3}x-1)$
10. $(-\frac{3}{5}x2+\frac{1}{4}x-6)+(2x^2-\frac{3}{8}x+\frac{5}{2})$
11. $(x^2+5)+(3x^2-2x+1)+(x^2+x-3)$
12. $(a^3-a^2+a-8)+(a^3+a^2+6a-2)$
13. $(a^3-8)+(-3a^3+5a^2-2)$
14. $(4a^5+5a^3-a)+(3a^4-2a^2+7)$
15. $(2x^2+5x-12)+(7x-5)$
16. $(3x+5)+(x^2-x+1)+(x^3+2x^2-3x+6)$
17. $(6x^5-7x^3+x^2-15)+(x^4+2x^3-6x+12)$
18. $(1+7x-5x^3+4x^4)+(-3x^3+5-x^2+x)$
19. $(x^2y^2-7xy+7)+(4x^2y^2-3xy-8)$
20. $(x^2+xy-y^2)+(7x^2-5xy+2y^2)$
21. $(2x^2+3xy-7y^2)+(-5x^2-3xy+8y^2)$
22. $(a^2b^2-100)+(2a^2b^2-3ab+20)$
23. $(ab^2-3a^2b+ab-3)+(-2a^2b+ab^2-7ab-1)$
24. $(10a^2b-7ab+8ab^2)+(6a^2b-ab+5ab^2)$
25. Find the sum of $2x+8$ and $7x-1$.
26. Find the sum of $13x-15$ and $16x+110$.
27. Find the sum of $x^2-10x+8$ and $5x^2-2x-6$.
28. Find the sum of $a^2-5a+10$ and $-9a^2+7a-11$.
29. Find the sum of x^2y^2-xy+6 and x^2y^2+xy-7.
30. Find the sum of $x^2-9xy+7y^2$ and $-3x^2-3xy+7y^2$.

Answer

1. $x+8$
3. $x-\frac{3}{2}$
5. $8x-4$
7. $4x^2-11x+2$
9. $-x^2+\frac{1}{3}x-\frac{5}{6}$
11. $5x^2-x+3$
13. $-2a^3+5a^2-10$
15. $2x^2+12x-17$
17. $6x^5+x^4-5x^3+x^2-6x-3$
19. $5x^2y^2-10xy-1$
21. $-3x^2+y^2$
23. $-5a^2b+2ab^2-6ab-4$
25. $9x+7$
27. $6x^2-12x+2$
29. $2x^2y^2-1$

Exercise 5.2.4 Subtraction of Polynomials

Subtract.

1. $(5x-3)-(2x-1)$
2. $(-4x+1)-(7x+10)$
3. $(\frac{1}{4}x-\frac{3}{4})-(\frac{3}{4}x+\frac{1}{8})$
4. $(-\frac{3}{5}x+\frac{3}{7})-(\frac{2}{5}x-\frac{3}{2})$
5. $(x^2+7x-5)-(4x^2-5x+1)$
6. $(-6x^2+3x-12)-(-6x^2+3x-12)$
7. $(-3x^3+4x-8)-(-x^2+4x+10)$
8. $(\frac{1}{2}x^2+\frac{1}{3}x-\frac{3}{4})-(\frac{3}{2}x^2-\frac{1}{6}x+\frac{1}{2})$
9. $(\frac{5}{9}x^2+\frac{1}{5}x-\frac{1}{3})-(\frac{1}{3}x^2+\frac{3}{10}x+\frac{5}{9})$
10. $(a^3-4a^2+3a-7)-(7a^3-2a^2-6a+9)$
11. $(3a^3+5a^2-2)-(a^3-a+8)$
12. $(5x^5+4x^3+x^2-6)-(4x^4-3x^3-x+3)$
13. $(3-5x-x^3+5x^4)-(-5x^3+2-x^2-7x)$
14. $(x^5-6x^3+9x)-(4x^4+2x^2-5)$
15. $(2x^2y^2-4xy+9)-(3x^2y^2-3xy-5)$
16. $(x^2+xy-y^2)-(x^2+xy-y^2)$
17. $(2x^2+3xy-7y^2)-(-5x^2-3xy+8y^2)$
18. $(ab^2-3a^2b+ab-3)-(-2a^2b+ab^2-7ab-1)$
19. $(10a^2b-7ab+8ab^2)-(6a^2b-ab+5ab^2)$
20. $(10a^2b^2+5ab-6)-(5a^2b^2+5ab-6)$
21. Subtract $3x+1$ from $5x-9$.
22. Subtract $x^2-5x+10$ from x^2+5x-5.
23. Find the difference of $3x-7$ and $8x+6$.
24. Find the difference of $2x^2+3x-5$ and x^2-9.

25. The cost in dollars of producing customized coffee mugs with a company logo is given by the formula $C = 150 + 0.10x$, where x is the number of cups produced. The revenue from selling the cups in the company store is given by $R = 10x - 0.05x^2$, where x is the number of units sold.
    a. Find a formula for the profit. (profit = revenue − cost)
    b. Find the profit from producing and selling 100 mugs in the company store.
26. The cost in dollars of producing sweat shirts is given by the formula $C = 10q + 1200$, where C is the cost and q represents the quantity produced. The revenue generated by selling the sweat shirts for $\$37$ each is given by $R = 37q$, where q represents the quantity sold. Determine the profit generated if 125 sweat shirts are produced and sold.
27. The outer radius of a washer is 3 times the radius of the hole.

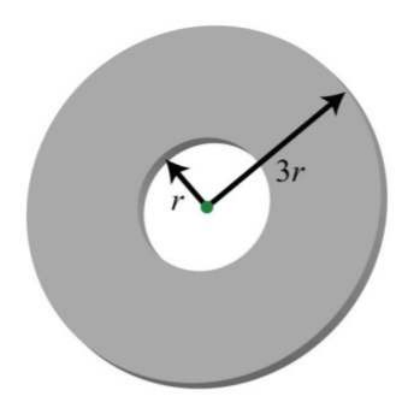


***Figure* 5.2.2**

    a. Derive a formula for the area of the face of the washer.
    b. What is the area of the washer if the hole has a diameter of 10 millimeters?
28. Derive a formula for the surface area of the following rectangular solid.

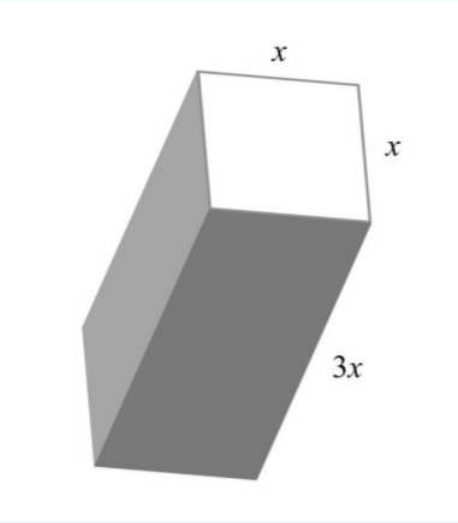


***Figure* 5.2.3**

Answer

1. $3x - 2$
3. $-\frac{1}{2}x - \frac{7}{8}$
5. $-3x^2 + 12x - 6$
7. $-3x^3 + x^2 - 18$
9. $\frac{2}{9}x^2 - \frac{1}{10}x - \frac{8}{9}$
11. $2a^3 + 5a^2 + a - 10$

13. $5x^4+4x^3+x^2+2x+1$
15. $-x^2y^2-xy+14$
17. $7x^2+6xy-15y^2$
19. $4a^2b+3ab^2-6ab$
21. $2x-10$
23. $-5x-13$
25. a. $P=-0.05x^2+9.9x-150$; b. \$340
27. a. $A=8\pi r^2$; b. 628.32square millimeters

Exercise 5.2.5 Addition and Subtraction of Polynomial

Simplify.

1. $(2x+3)-(5x-8)+(x-7)$
2. $(3x-5)-(7x-11)-(5x+2)$
3. $(3x-2)-(4x-1)+(x+7)$
4. $(5x-3)-(2x+1)-(x-1)$
5. $(5x^2-3x+2)-(x^2+x-4)+(7x^2-2x-6)$
6. $(-2x^3+x^2-8)-(3x^2+x-6)-(2x-1)$
7. $(2x-7)-(x^2+3x-7)+(6x-1)$
8. $(6x^2-10x+13)+(4x^2-9)-(9-x^2)$
9. $(a^2-b^2)-(2a^2+3ab-4b^2)+(5ab-1)$
10. $(a^2-3ab+b^2)-(a^2+\ b^2)-(3ab-5)$
11. $(\frac{1}{2}x^2-\frac{3}{4}x+\frac{1}{4})-(\frac{3}{2}x-\frac{3}{4})+(\frac{5}{4}x-\frac{1}{2})$
12. $(\frac{9}{5}x^2-\frac{1}{3}x+2)-(\frac{3}{10}x^2-\frac{4}{5})-(x+\frac{5}{2})$

Answer

1. $-2x+4$
3. 6
5. $11x^2-6x$
7. $-x^2+5x-1$
9. $-a^2+2ab+3b^2-1$
11. $12x^2-x+12$

Exercise 5.2.6 Addition and Subtraction of Polynomial Functions

Find $(f+g)(x)$ and $(f-g)(x)$, given the following functions.

1. $f(x)=4x-1$ and $g(x)=-3x+1$
2. $f(x)=-x+5$ and $g(x)=2x-3$
3. $f(x)=3x^2-5x+7$ and $g(x)=-2x^2+5x-1$
4. $f(x)=x^3+2x^2-6x+2$ and $g(x)=2x^3+2x^2-5x-1$
5. $f(x)=\frac{1}{2}x+\frac{1}{3}$ and $g(x)=\frac{1}{5}x^2-\frac{3}{2}x+\frac{1}{6}$
6. $f(x)=x^2-5x+\frac{1}{3}$ and $g(x)=\frac{2}{3}x^2-x-\frac{1}{2}$

Answer

1. $(f+g)(x)=x$ and $(f-g)(x)=7x-2$
3. $(f+g)(x)=x^2+6$ and $(f-g)(x)=5x^2-10x+8$

5. $(f+g)(x)=\frac{1}{5}x^2-x+\frac{1}{2}$ and $(f-g)(x)=-\frac{1}{5}x^2+2x+\frac{1}{6}$

Exercise 5.2.7 Addition and Subtraction of Polynomial Functions

Given $f(x)=2x-3$ and $g(x)=x^2+3x-1$, find the following.

1. $(f+g)(x)$
2. $(g+f)(x)$
3. $(f-g)(x)$
4. $(g-f)(x)$
5. $(g+g)(x)$
6. $(f+g)(3)$
7. $(f+g)(-2)$
8. $(f+g)(0)$
9. $(f-g)(0)$
10. $(f-g)(-2)$
11. $(g-f)(-2)$
12. $(g-f)(12)$

Answer

1. $(f+g)(x)=x^2+5x-4$
3. $(f-g)(x)=-x^2-x-2$
5. $(g+g)(x)=2x^2+6x-2$
7. $(f+g)(-2)=-10$
9. $(f-g)(0)=-2$
11. $(g-f)(-2)=4$

Exercise 5.2.8 Addition and Subtraction of Polynomial Functions

Given $f(x)=5x^2-3x+2$ and $g(x)=2x^2+6x-4$, find the following.

1. $(f+g)(x)$
2. $(g+f)(x)$
3. $(f-g)(x)$
4. $(g-f)(x)$
5. $(f+g)(-2)$
6. $(f-g)(-2)$
7. $(f+g)(0)$
8. $(f-g)(0)$

Answer

1. $(f+g)(x)=7x^2+3x-2$
3. $(f-g)(x)=3x^2-9x+6$
5. $(f+g)(-2)=20$
7. $(f+g)(0)=-2$

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5.3: Multiplying Polynomials

Learning Objectives

- Multiply a polynomial by a monomial.
- Multiply a polynomial by a binomial.
- Multiply a polynomial by any size polynomial.
- Recognize and calculate special products.
- Multiply polynomial functions.

Multiplying by a Monomial

Recall the product rule for exponents: if m and n are positive integers, then

$$x^m \cdot x^n = x^{m+n} \tag{5.3.1}$$

In other words, when multiplying two expressions with the same base, add the exponents. This rule applies when multiplying a monomial by a monomial. To find the product of monomials, multiply the coefficients and add the exponents of variable factors with the same base. For example,

$$\begin{aligned} 3x \cdot 5x^2 &= 3 \cdot 5 \cdot x^1 \cdot x^2 && \textit{Commutative property} \\ &= 15x^{1+2} && \textit{Product rule for exponents} \\ &= 15x^3 \end{aligned}$$

To multiply a polynomial by a monomial, apply the distributive property and then simplify each term.

Example 5.3.1

Multiply:

$$-5x(4x-2).$$

Solution:

In this case, multiply the monomial, $-5x$, by the binomial, $4x-2$. Apply the distributive property and then simplify.

$$\begin{aligned} &-5x(4x-2) \\ &= (-5x)\cdot 4x + (-5x)\cdot(-2) \\ &= -20x^2 + 10x \end{aligned}$$

Figure 5.3.1

Answer:

$$-20x^2+10x$$

Example 5.3.2

Multiply:

$$2x^2(3x^2-5x+1).$$

Solution:

Apply the distributive property and then simplify.

$$2x^2(3x^2-5x+1)$$
$$=2x^2\cdot 3x^2-2x^2\cdot 5x+2x^2\cdot 1$$
$$=6x^4-10x^3+2x^2$$

Figure 5.3.2

Answer:

$$6x^4-10x^3+2x^2$$

Example 5.3.3

Multiply:

$$-3ab^2(a^2b^3+2a^3b-6ab-4)\ .$$

Solution:

Answer:

To summarize, multiplying a polynomial by a monomial involves the distributive property and the product rule for exponents. Multiply all of the terms of the polynomial by the monomial. For each term, multiply the coefficients and add exponents of variables where the bases are the same.

Exercise 5.3.1

Multiply:

$$-5x^2y(2xy^2-3xy+6x^2y-1)\,.$$

Answer

$$-10x^3y^3+15x^3y^2-30x^4y^2+5x^2y$$

Multiplying by a Binomial

In the same way that we used the distributive property to find the product of a monomial and a binomial, we will use it to find the product of two binomials.

$$\begin{aligned}(a+b)(c+d)&\\ &=(a+b)\cdot c+(a+b)\cdot d\\ &=ac+bc+ad+bd\\ &=ac+ad+bc+bd\end{aligned}$$

Here we apply the distributive property multiple times to produce the final result. This same result is obtained in one step if we apply the distributive property to a and b separately as follows:

$$(a+b)(c+d)=ac+ad+bc+bd$$

Figure 5.3.3

This is often called the FOIL method. We add the products of the first terms of each binomial ac, the $o$uter terms ad, the $i$nner terms bc, and finally the last terms bd. This mnemonic device only works for products of binomials; hence it is best to just remember that the distributive property applies.

Example 5.3.4

Multiply:

$$(2x+3)(5x-2).$$

Solution:

Distribute $2x$ and then distribute 3.

Simplify by combining like terms.

$=10x^2+11x-6$

Answer:

$$10x^2+11x-6$$

Example 5.3.5

Multiply:

$$\left(\tfrac{1}{2}x-\tfrac{1}{4}\right)\left(\tfrac{1}{2}x+\tfrac{1}{4}\right).$$

Solution:

Distribute $\frac{1}{2}x$ and then distribute $-\frac{1}{4}$.

$$\begin{aligned}\left(\frac{1}{2}x-\frac{1}{4}\right)\left(\frac{1}{2}x+\frac{1}{4}\right) &= \frac{1}{2}x\cdot\frac{1}{2}x+\frac{1}{2}x\cdot\frac{1}{4}+\left(-\frac{1}{4}\right)\cdot\frac{1}{2}x+\left(-\frac{1}{4}\right)\cdot\frac{1}{4}\\ &= \frac{1}{4}x^2+\frac{1}{8}x-\frac{1}{8}x-\frac{1}{16}\\ &= \frac{1}{4}x^2-\frac{1}{16}\end{aligned}$$

Answer:

$$\tfrac{1}{4}x^2-\tfrac{1}{16}$$

Example 5.3.6

Multiply:

$$(3y^2-1)(2y+1).$$

Solution:

Answer:

$6y^3+3y^2-2y-1$

After applying the distributive property, combine any like terms.

Example 5.3.7

Multiply:

$$(x^2-5)(3x^2-2x+2).$$

Solution:

After multiplying each term of the trinomial by x^2 and -5, simplify.

Answer:

$3x^4-2x^3-13x^2+10x-10$

Example 5.3.8

Multiply:

$$(2x-1)^3.$$

Solution:

Perform one product at a time.

$$\begin{aligned}(2x-1)^3 &= (2x-1)\underbrace{(2x-1)(2x-1)}_{\textit{multiply first}}\\ &= (2x-1)\underbrace{(4x^2-2x-2x+1)}_{\textit{simplify}}\\ &= \underbrace{(2x-1)(4x^2-4x+1)}_{\textit{multiply second}}\\ &= 8x^3-8x^2+2x-4x^2+4x-1\\ &= 8x^3-12x^2+6x-1\end{aligned}$$

Figure 5.3.4

Answer:

$$8x^3-12x^2+6x-1$$

At this point, it is worth pointing out a common mistake:

$$(2x-1)^3 \neq (2x)^3-(1)^3$$

The confusion comes from the product to a power rule of exponents, where we apply the power to all factors. Since there are two terms within the parentheses, that rule does not apply. Care should be taken to understand what is different in the following two examples:

$$(xy)^2 = x^2y^2 \quad \checkmark$$
$$(x+y)^2 \neq x^2+y^2 \quad x$$

Exercise 5.3.2

Multiply:

$$(2x-3)(7x^2-5x+4)\,.$$

Answer

$$14x^3-31x^2+23x-12$$

Product of Polynomials

When multiplying polynomials, we apply the distributive property many times. Multiply all of the terms of each polynomial and then combine like terms.

Example 5.3.9

Multiply:

$$(2x^2+x-3)(x^2-2x+5)\ .$$

Solution:

Multiply each term of the first trinomial by each term of the second trinomial and then combine like terms.

Aligning like terms in columns, as we have here, aids in the simplification process

Answer:

$2x^4-3x^3+5x^2+11x-15$

Notice that when multiplying a trinomial by a trinomial, we obtain nine terms before simplifying. In fact, when multiplying an n-term polynomial by an m-term polynomial, we will obtain $n \times m$ terms. In the previous example, we were asked to multiply and found that

$$(2x^2+x-3)(x^2-2x+5)=2x^4-3x^3+5x^2+11x-15$$

Because it is easy to make a small calculation error, it is a good practice to trace through the steps mentally to verify that the operations were performed correctly. Alternatively, we can check by evaluating any value for x in both expressions to verify that the results are the same. Here we choose $x=2$:

$$\begin{aligned}(2x^2+x-3)(x^2-2x+5) &= (2(2)^2+(2)-3)((2)^2-2(2)+5)\\ &= (8+2-3)(4-4+5)\\ &= (7)(5)\\ &= 35\end{aligned}$$

Because the results could coincidentally be the same, a check by evaluating does not necessarily prove that we have multiplied correctly. However, after verifying a few values, we can be fairly confident that the product is correct.

Multiply:

$$(x^2-2x-3)^2\ .$$

Answer

$x^4-4x^3-2x^2+12x+9$

Special Products

In this section, the goal is to recognize certain special products that occur often in our study of algebra. We will develop three formulas that will be very useful as we move along. The three should be memorized. We begin by considering the following two calculations:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+ab+ab+b^2\\ &= a^2+2ab+b^2\end{aligned} \qquad \begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a^2-ab-ba+b^2\\ &= a^2-ab-ab+b^2\\ &= a^2-2ab+b^2\end{aligned}$$

This leads us to two formulas that describe perfect square trinomials:

$$(a+b)^2=a^2+2ab+b^2 \tag{5.3.2}$$

$$(a-b)^2=a^2-2ab+b^2 \tag{5.3.3}$$

We can use these formulas to quickly square a binomial.

Example 5.3.10

Multiply:

$$(3x+5)^2.$$

Solution:

Here $a=3x$ and $b=5$. Apply the formula:

$$\begin{array}{rl} (a+b)^2 = & a^2 \quad +2 \quad a \quad b \quad + \quad b^2 \\ & \downarrow \qquad\quad \downarrow \quad \downarrow \qquad \downarrow \\ (3x+5)^2 & = (3x)^2 + 2\cdot(3x)(5) + (5)^2 \\ & = 9x^2 + 30x + 25 \end{array}$$

Answer:

$$9x^2 + 30x + 25$$

This process should become routine enough to be performed mentally.

Example 5.3.11

Multiply:

$$(x-4)^2.$$

Solution:

Here $a = x$ and $b = 4$. Apply the appropriate formula as follows:

$$\begin{array}{rl} (a-b)^2 = & a^2 \quad -2 \quad a \quad b \quad + b^2 \\ & \downarrow \qquad\quad \downarrow \quad \downarrow \qquad \downarrow \\ (x-4)^2 & = (x)^2 - 2\cdot(x)(4) + (4)^2 \\ & = x^2 - 8x + 16 \end{array}$$

Answer:

$$x^2 - 8x + 16$$

Our third special product follows:

$$\begin{aligned} (a+b)(a-b) &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2 \end{aligned}$$

This product is called difference of squares:

$$(a+b)(a-b) = a^2 - b^2 \tag{5.3.4}$$

The binomials $(a+b)$ and $(a-b)$ are called conjugate binomials. Therefore, when conjugate binomials are multiplied, the middle term eliminates, and the product is itself a binomial.

Example 5.3.12

Multiply:

$$(7x+4)(7x-4).$$

Solution:

Answer:

$49x^2 - 16$

Exercise 5.3.4

Multiply:

$$(-5x+2)^2.$$

Answer

$25x^2 - 20x + 4$

Multiplying Polynomial Functions

We use function notation to indicate multiplication as follows:

Table 5.3.1

Multiplication of functions:	$(f \cdot g)(x) = f(x) \cdot g(x)$

Example 5.3.13

Calculate:

$$(f \cdot g)(x), \text{ given } f(x) = 5x^2 \text{ and } g(x) = -x^2 + 2x - 3 \ .$$

Solution:

Multiply all terms of the trinomial by the monomial function $f(x)$.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= 5x^2 \cdot (-x^2 + 2x - 3) \\ &= -5x^4 + 10x^3 - 15x^2\end{aligned}$$

Answer:

$$(f \cdot g)(x) = -5x^4 + 10x^3 - 15x^2$$

Example 5.3.14

Calculate:

$$(f \cdot g)(-1), \text{ given } f(x) = -x + 3 \text{ and } g(x) = 4x^2 - 3x + 6 \ .$$

Solution:

First, determine $(f \cdot g)(x)$.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (-x + 3)(4x^2 - 3x + 6) \\ &= -4x^3 + 3x^2 - 6x + 12x^2 - 9x + 18 \\ &= -4x^3 + 15x^2 - 15x + 18\end{aligned}$$

We have

\((f\cdot g)(x) = -4x^{3}+15x^{2}-15x+18

Next, substitute -1 for the variable x.

$$\begin{aligned}(f \cdot g)(-1) &= -4(-1)^3 + 15(-1)^2 - 15(-1) + 18 \\ &= -4 \cdot (-1) + 15 \cdot 1 + 15 + 18 \\ &= 4 + 15 + 15 + 18 \\ &= 52\end{aligned}$$

Answer:

$$(f \cdot g)(-1) = 52$$

Because $(f \cdot g)(-1) = f(-1) \cdot g(-1)$, we could alternatively calculate $f(-1)$ and $g(-1)$ separately and then multiply the results (try this as an exercise). However, if we were asked to evaluate multiple values for the function $(f \cdot g)(x)$, it would be best to first determine the general form, as we have in the previous example.

Key Takeaways

- To multiply a polynomial by a monomial, apply the distributive property and then simplify each of the resulting terms.

- To multiply polynomials, multiply each term in the first polynomial with each term in the second polynomial. Then combine like terms.
- The product of an n-term polynomial and an m-term polynomial results in an $m \times n$ term polynomial before like terms are combined.
- Check results by evaluating values in the original expression and in your answer to verify that the results are the same.
- Use the formulas for special products to quickly multiply binomials that occur often in algebra.

? Exercise 5.3.5 Product of a Monomial and a Polynomial

Multiply.

1. $5x(-3x^2y)$
2. $(-2x^3y^2)(-3xy^4)$
3. $\frac{1}{2}(4x-3)$
4. $-\frac{3}{4}(\frac{2}{3}x-6)$
5. $3x(5x-2)$
6. $-4x(2x-1)$
7. $x^2(3x+2)$
8. $-6x^2(5x+3)$
9. $2ab(4a-2b)$
10. $5a^2b(a^2-b^2)$
11. $6x^2y^3(-3x^3y+xy^2)$
12. $3ab^3(-5ab^3+6a^2b)$
13. $-\frac{1}{2}x^2y(4xy-10)$
14. $-3x^4y^2(3x^8y^3)$
15. $2x^2(-5x^3)(3x^4)$
16. $4ab(a^2b^3c)(a^4b^2c^4)$
17. $-2(5x^2-3x+4)$
18. $\frac{4}{5}(25x^2-50xy+5y^2)$
19. $3x(5x^2-2x+3)$
20. $-x(x^2+x-1)$
21. $x^2(3x^2-5x-7)$
22. $x^3(-4x^2-7x+9)$
23. $\frac{1}{4}x^4(8x^3-2x^2+\frac{1}{2}x-5)$
24. $-\frac{1}{3}x^3(\frac{3}{2}x^5-\frac{2}{3}x^3+\frac{9}{2}x-1)$
25. $a^2b(a^2-3ab+b^2)$
26. $6a^2bc^3(2a-3b+c^2)$
27. $\frac{2}{3}xy^2(9x^3y-27xy+3xy^3)$
28. $-3x^2y^2(12x^2-10xy-6y^2)$
29. Find the product of $3x$ and $2x^2-3x+5$.
30. Find the product of $-8y$ and $y^2-2y+12$.
31. Find the product of $-4x$ and $x^4-3x^3+2x^2-7x+8$.
32. Find the product of $3xy^2$ and $-2x^2y+4xy-xy^2$.

Answer

1. $-15x^3y$
3. $2x-\frac{3}{2}$
5. $15x^2-6x$
7. $3x^3+2x^2$
9. $8a^2b-4ab^2$
11. $-18x^5y^4+6x^3y^5$

13. $-2x^3y^2+5x^2y$
15. $-30x^9$
17. $-10x^2+6x-8$
19. $15x^3-6x^2+9x$
21. $3x^4-5x^3-7x^2$
23. $2x^7-\frac{1}{2}x^6+\frac{1}{8}x^5-\frac{5}{4}x^4$
25. $a^4b-3a^3b^2+a^2b^3$
27. $6x^4y^3-18x^2y^3+2x^2y^5$
29. $6x^3-9x^2+15x$
31. $-4x^5+12x^4-8x^3+28x^2-32x$

Exercise 5.3.6 Product of a Binomial and a Polynomial

Multiply.

1. $(3x-2)(x+4)$
2. $(x+2)(x-3)$
3. $(x-1)(x+1)$
4. $(3x-1)(3x+1)$
5. $(2x-5)(x+3)$
6. $(5x-2)(3x+4)$
7. $(-3x+1)(x-1)$
8. $(x+5)(-x+1)$
9. $(y-\frac{2}{3})(y+\frac{2}{3})$
10. $(\frac{1}{2}x+\frac{1}{3})(\frac{3}{2}x-\frac{2}{3})$
11. $(\frac{3}{4}x+\frac{1}{5})(\frac{1}{4}x+\frac{2}{5})$
12. $(\frac{1}{5}x+\frac{3}{10})(\frac{3}{5}x-\frac{5}{2})$
13. $(y^2-2)(y+2)$
14. $(y^3-1)(y^2+2)$
15. $(a^2-b^2)(a^2+b^2)$
16. $(a^2-3b)^2$
17. $(x-5)(2x^2+3x+4)$
18. $(3x-1)(x^2-4x+7)$
19. $(2x-3)(4x^2+6x+9)$
20. $(5x+1)(25x^2-5x+1)$
21. $(x-\frac{1}{2})(3x^2+4x-1)$
22. $(\frac{1}{3}x-\frac{1}{4})(3x^2+9x-3)$
23. $(x+3)^3$
24. $(x-2)^3$
25. $(3x-1)^3$
26. $(2x+y)^3$
27. $(5x-2)(2x^3-4x^2+3x-2)$
28. $(x^2-2)(x^3-2x^2+x+1)$

Answer

1. $3x^2+10x-8$
3. x^2-1
5. $2x^2+x-15$

7. $-3x^2+4x-1$
9. $y^2-\frac{4}{9}$
11. $\frac{3}{16}x^2+\frac{7}{20}x+\frac{2}{25}$
13. y^3+2y^2-2y-4
15. a^4-b^4
17. $2x^3-7x^2-11x-20$
19. $8x^3-27$
21. $3x^3+\frac{5}{2}x^2-3x+12$
23. $x^3+9x^2+27x+27$
25. $27x^3-27x^2+9x-1$
27. $10x^4-24x^3+23x^2-16x+4$

Exercise 5.3.7 Product of Polynomials

Multiply.

1. $(x^2-x+1)(x^2+2x+1)$
2. $(3x^2-2x-1)(2x^2+3x-4)$
3. $(2x^2-3x+5)(x^2+5x-1)$
4. $(a+b+c)(a-b-c)$
5. $(a+2b-c)^2$
6. $(x+y+z)^2$
7. $(x-3)^4$
8. $(x+y)^4$
9. Find the volume of a rectangular solid with sides measuring x, $x+2$, and $x+4$ units.
10. Find the volume of a cube where each side measures $x-5$ units.

Answer

1. x^4+x^3+x+1
3. $2x^4+7x^3-12x^2+28x-5$
5. $a^2+4ab-2ac+4b^2-4bc+c^2$
7. $x^4-12x^3+54x^2-108x+81$
9. x^3+6x^2+8x

Exercise 5.3.8 Special Products

Multiply.

1. $(x+2)^2$
2. $(x-3)^2$
3. $(2x+5)^2$
4. $(3x-7)^2$
5. $(-x+2)^2$
6. $(-9x+1)^2$
7. $(a+6)^2$
8. $(2a-3b)^2$
9. $(\frac{2}{3}x+\frac{3}{4})^2$

10. $(\frac{1}{2}x-\frac{3}{5})^2$
11. $(x^2+2)^2$
12. $(x^2+y^2)^2$
13. $(x+4)(x-4)$
14. $(2x+1)(2x-1)$
15. $(5x+3)(5x-3)$
16. $(\frac{1}{5}x-\frac{1}{3})(\frac{1}{5}x+\frac{1}{3})$
17. $(\frac{3}{2}x+\frac{2}{5})(\frac{3}{2}x-\frac{2}{5})$
18. $(2x-3y)(2x+3y)$
19. $(4x-y)(4x+y)$
20. $(a^3-b^3)(a^3+b^3)$
21. A box is made by cutting out the corners and folding up the edges of a square piece of cardboard. A template for a cardboard box with a height of 2 inches is given. Find a formula for the volume, if the initial piece of cardboard is a square with sides measuring x inches.

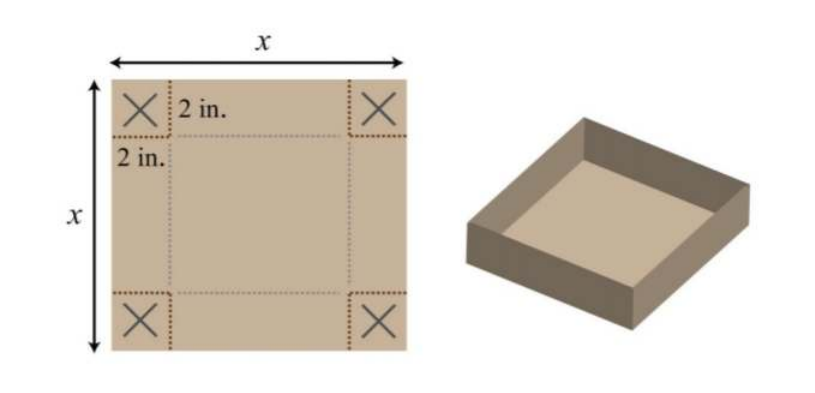


***Figure* 5.3.5**

22. A template for a cardboard box with a height of x inches is given. Find a formula for the volume, if the initial piece of cardboard is a square with sides measuring 12 inches.

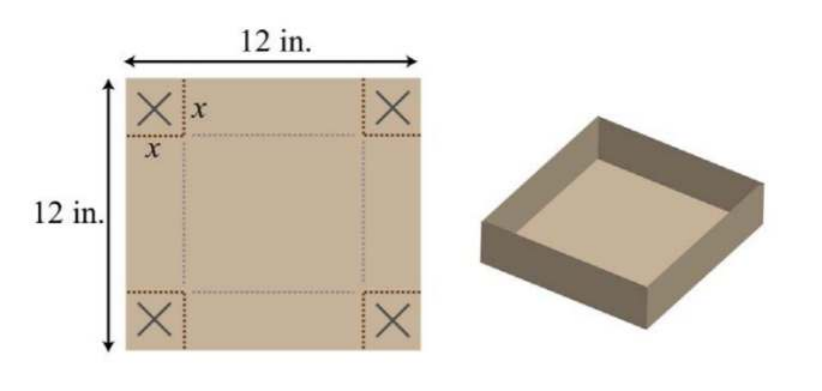


***Figure* 5.3.6**

Answer

1. x^2+4x+4
3. $4x^2+20x+25$
5. x^2-4x+4
7. $a^2+12a+36$
9. $\frac{4}{9}x^2+x+\frac{9}{16}$
11. x^4+4x^2+4

13. $x^2 - 16$
15. $25x^2 - 9$
17. $\frac{9}{4}x^2 - \frac{4}{25}$
19. $16x^2 - y^2$
21. $V = 2x^2 - 16x + 32$ cubic inches

Exercise 5.3.9 Multiplying Polynomial Functions

For each problem, calculate $(f \cdot g)(x)$, given the functions.

1. $f(x) = 8x$ and $g(x) = 3x - 5$
2. $f(x) = x^2$ and $g(x) = -5x + 1$
3. $f(x) = x - 7$ and $g(x) = 6x - 1$
4. $f(x) = 5x + 3$ and $g(x) = x^2 + 2x - 3$
5. $f(x) = x^2 + 6x - 3$ and $g(x) = 2x^2 - 3x + 5$
6. $f(x) = 3x^2 - x + 1$ and $g(x) = -x^2 + 2x - 1$

Answer

1. $(f \cdot g)(x) = 24x^2 - 40x$
3. $(f \cdot g)(x) = 6x^2 - 43x + 7$
5. $(f \cdot g)(x) = 2x^4 + 9x^3 - 19x^2 + 39x - 15$

Exercise 5.3.10 Multiplying Polynomial Functions

Given $f(x) = 2x - 3$ and $g(x) = 3x - 1$, find the following

1. $(f \cdot g)(x)$
2. $(g \cdot f)(x)$
3. $(f \cdot g)(0)$
4. $(f \cdot g)(-1)$
5. $(f \cdot g)(1)$
6. $(f \cdot g)(\frac{1}{2})$

Answer

1. $(f \cdot g)(x) = 6x^2 - 11x + 3$
3. $(f \cdot g)(0) = 3$
5. $(f \cdot g)(1) = -2$

Exercise 5.3.11 Multiplying Polynomial Functions

Given $f(x) = 5x - 1$ and $g(x) = 2x^2 - 4x + 5$, find the following.

1. $(f \cdot g)(x)$
2. $(g \cdot f)(x)$
3. $(f \cdot g)(0)$
4. $(f \cdot g)(-1)$
5. $(f \cdot g)(1)$
6. $(f \cdot g)(\frac{1}{2})$
7. $(f \cdot f)(x)$
8. $(g \cdot g)(x)$

Answer

1. $(f \cdot g)(x) = 10x^3 - 22x^2 + 29x - 5$
3. $(f \cdot g)(0) = -5$
5. $(f \cdot g)(1) = 12$
7. $(f \cdot f)(x) = 25x^2 - 10x + 1$

Exercise 5.3.12 Discussion Board Topics

1. Explain why $(x + y)^2 \neq x^2 + y2$.
2. Explain how to quickly multiply a binomial with its conjugate. Give an example.
3. What are the advantages and disadvantages of using the mnemonic device FOIL?

Answer

1. Answers may vary
3. Answers may vary

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5.4: Dividing Polynomials

Learning Objectives

- Divide by a monomial.
- Divide by a polynomial using the division algorithm.
- Divide polynomial functions.

Dividing by a Polynomial

Recall the quotient rule for exponents: if x is nonzero and m and n are positive integers, then

$$\frac{x^m}{x^n} = x^{m-n} \tag{5.4.1}$$

In other words, when dividing two expressions with the same base, subtract the exponents. This rule applies when dividing a monomial by a monomial. In this section, we will assume that all variables in the denominator are nonzero.

Example 5.4.1

Divide:

$$\frac{28y^3}{7y}.$$

Solution:

Divide the coefficients and subtract the exponents of the variable y.

$$\begin{aligned}\frac{28y^3}{7y} &= \frac{28}{7}y^{3-1} \\ &= 4y^2\end{aligned}$$

Answer:

$$4y^2$$

Example 5.4.2

Divide:

$$\frac{24x^7y^5}{8x^3y^2}.$$

Solution:

Divide the coefficients and apply the quotient rule by subtracting the exponents of the like bases.

$$\begin{aligned}\frac{24x^7y^5}{8x^3y^2} &= \frac{24}{8}x^{7-3}y^{5-2} \\ &= 3x^4y^3\end{aligned}$$

Answer:

$$3x^4y^3$$

When dividing a polynomial by a monomial, we may treat the monomial as a common denominator and break up the fraction using the following property:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \tag{5.4.2}$$

Applying this property results in terms that can be treated as quotients of monomials.

Example 5.4.3

Divide:

$$\frac{-5x^4+25x^3-15x^2}{5x^2}.$$

Solution:

Break up the fraction by dividing each term in the numerator by the monomial in the denominator and then simplify each term.

Answer:

$-x^2+5x-3\cdot 1$

Check your division by multiplying the answer, the quotient, by the monomial in the denominator, the divisor, to see if you obtain the original numerator, the dividend.

$$\frac{dividend}{divisor}=quotient \tag{5.4.3}$$

or

$$dividend=divisor\cdot quotient \tag{5.4.4}$$

$$\begin{aligned} 5x^2\cdot(-x^2+5x-3) &= 5x^2\cdot(-x^2)+5x^2\cdot 5x-5x^2\cdot 3 \\ &= -5x^4+25x^3-15x^2 \quad \checkmark \end{aligned}$$

Example 5.4.4

Divide:

$$\frac{9a^4b-7a^3b^2+3a^2b}{-3a^2b}.$$

Solution:

Answer:

$-3a^2+\frac{7}{3}ab-1$. The check is optional and is left to the reader.

Exercise 5.4.1

$(16x^5-8x^4+5x^3+2x^2)\div(2x^2)$.

Answer

$8x^3-4x^2+\frac{5}{2}x+1$

Dividing by a Polynomial

The same technique outlined for dividing by a monomial ***does not*** work for polynomials with two or more terms in the denominator. In this section, we will outline a process called polynomial long division, which is based on the division algorithm for real numbers. For the sake of clarity, we will assume that all expressions in the denominator are nonzero.

Example 5.4.5

Divide:

$$\frac{x^3+3x^2-8x-4}{x-2}.$$

Solution:

Here $x-2$ is the divisor and x^3+3x^2-8x-4 is the dividend.

Step 1: To determine the first term of the quotient, divide the leading term of the dividend by the leading term of the divisor.

$$x-2\overline{)\,x^3+3x^2-8x-4}\qquad \begin{array}{l}\textit{Begin by dividing the leading terms.}\\ x^3\div x=x^2\end{array}$$

Figure 5.4.1

Step 2: Multiply the first term of the quotient by the divisor, remembering to distribute, and line up like terms with the dividend.

$$\begin{array}{r} x^2 \\ x-2\overline{)\,x^3+3x^2-8x-4} \\ \underline{x^3-2x^2} \end{array}\qquad \begin{array}{l}\textit{Distribute and line up like terms.}\\ x^2\cdot(x-2)=x^3-2x^2\end{array}$$

Figure 5.4.2

Step 3: Subtract the resulting quantity from the dividend. Take care to subtract both terms.

$$\begin{array}{r} x^2 \\ x-2\overline{)\,x^3+3x^2-8x-4} \\ \underline{-(x^3-2x^2)} \\ 5x^2 \end{array}\qquad \begin{array}{l}\textit{Subtract.}\\ -(x^3-2x^2)=-x^3+2x^2\end{array}$$

Figure 5.4.3

Step 4: Bring down the remaining terms and repeat the process from step 1.

$$\begin{array}{r} x^2\quad ? \\ x-2\overline{)\,x^3+3x^2-8x-4} \\ \underline{-(x^3-2x^2)} \\ 5x^2-8x-4 \end{array}\qquad \textit{Bring down the remaining terms.}$$

Figure 5.4.4

Notice that the leading term is eliminated and that the result has a degree that is one less than the dividend. The complete process is illustrated below:

$$
\begin{array}{r}
9x^2-12x+16 \\
3x+4\overline{)\ 27x^3+0x^2+0x+64} \\
-(27x^3+36x^2) \\
\hline
-36x^2+\ 0x+64 \\
-(-36x^2-48x) \\
\hline
48x+64 \\
-(48x+64) \\
\hline
0
\end{array}
$$

Figure 5.4.5

Polynomial long division ends when the degree of the remainder is less than the degree of the divisor. Here the remainder is 0. Therefore, the binomial divides the polynomial evenly and the answer is the quotient shown above the division line.

$$\frac{x^3+3x^2-8x-4}{x-2} = x^2+5x+2$$

To check the answer, multiply the divisor by the quotient to see if you obtain the dividend:

$$x^3+3x^2-8x-4 = (x-2)(x^2+5x+2)$$

Answer:

$$x^2+5x+2$$

Next, we demonstrate the case where there is a nonzero remainder.

$$
\begin{array}{r}
quotient \\
divisor\overline{)dividend} \\
\vdots \\
\hline
remainder
\end{array}
$$

Figure 5.4.6

Just as with real numbers, the final answer adds the fraction where the remainder is the numerator and the divisor is the denominator to the quotient. In general, when dividing we have

$$\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor} \tag{5.4.5}$$

If we multiply both sides by the divisor we obtain

$$dividend = quotient \times divisor + remainder \tag{5.4.6}$$

Example 5.4.6

Divide:

$$\frac{6x^2-5x+3}{2x-1}$$

Solution:

Since the denominator is a binomial, begin by setting up polynomial long division.

$$2x-1\overline{)\,6x^2-5x+3}\qquad ?$$

Figure 5.4.7

To start, determine what monomial times $2x-1$ results in a leading term $6x^2$. This is the quotient of the given leading terms: $(6x^2)\div(2x)=3x$. Multiply $3x$ times the divisor $2x-1$ and line up the result with like terms of the dividend.

$$\begin{array}{r} 3x \\ 2x-1\overline{)\,6x^2-5x+3} \\ 6x^2-3x \end{array}$$

Figure 5.4.8

Subtract the result from the dividend and bring down the constant term $+3$.

$$\begin{array}{r} 3x\quad ? \\ 2x-1\overline{)\,6x^2-5x+3} \\ \underline{-(6x^2-3x)} \\ -2x+3 \end{array}$$

Figure 5.4.9

Subtracting eliminates the leading term and $-5x-(-3x)=-5x+3x=-2x$. The quotient of $-2x$ and $2x$ is -1. Multiply $2x-1$ by -1 and line up the result.

$$\begin{array}{r} 3x-1 \\ 2x-1\overline{)\,6x^2-5x+3} \\ \underline{-(6x^2-3x)} \\ -2x+3 \\ -2x+1 \end{array}$$

Figure 5.4.10

Subtract again and notice that we are left with a remainder.

$$\begin{array}{r} 3x-1 \\ 2x-1\overline{)\,6x^2-5x+3} \\ \underline{-(6x^2-3x)} \\ -2x+3 \\ \underline{-(-2x+1)} \\ 2 \end{array}$$

Figure 5.4.11

The constant term 2 has degree 0, and thus the division ends. We may write

$$\frac{6x^2-5x+3}{2x-1}=3x-1+\frac{2}{2x-1}$$

Answer:

$3x-1+\frac{2}{2x-1}$. To check that this result is correct, we multiply as follows:

$$\begin{aligned} quotient \times divisor + remainder &= (3x-1)(2x-1)+2 \\ &= 6x^2-3x-2x+1+2 \\ &= 6x^2-5x+3 = dividend \quad \checkmark \end{aligned}$$

Occasionally, some of the powers of the variables appear to be missing within a polynomial. This can lead to errors when lining up like terms. Therefore, when first learning how to divide polynomials using long division, fill in the missing terms with zero coefficients, called placeholders.

Example 5.4.7

Divide:

$$\frac{27x^3+64}{3x+4}.$$

Solution:

Notice that the binomial in the numerator does not have terms with degree 2 or 1. The division is simplified if we rewrite the expression with placeholders:

$$27x^3+64 = 27x^3+0x^2+0x+64$$

Set up polynomial long division:

$$3x+4\overline{)\,27x^3+0x^2+0x+64}\qquad ?$$

Figure 5.4.12

We begin with $27x^3 \div 3x = 9x^2$ and work the rest of the division algorithm.

$$\begin{array}{r} 9x^2-12x+16 \\ 3x+4\overline{)\;27x^3+0x^2+0x+64} \\ \underline{-\left(27x^3+36x^2\right)} \\ -36x^2+\;\;0x+64 \\ \underline{-\left(-36x^2-48x\right)} \\ 48x+64 \\ \underline{-\left(48x+64\right)} \\ 0 \end{array}$$

Figure 5.4.13

Answer:

$$9x^2-12x+16$$

Example 5.4.8

Divide:

$$\frac{3x^4-2x^3+6x^2+23x-7}{x^2-2x+5}.$$

Solution:

$$x^2-2x+5\overline{)3x^4-2x^3+6x^2+23x-7}$$

Figure 5.4.14

Begin the process by dividing the leading terms to determine the leading term of the quotient $3x^4 \div x^2 = 3x^2$. Take care to distribute and line up the like terms. Continue the process until the remainder has a degree less than 2.

$$\begin{array}{r} 3x^2+4x-1 \\ x^2-2x+5\overline{)3x^4-2x^3+6x^2+23x-7} \\ \underline{-(3x^4-6x^3+15x^2)} \\ 4x^3-9x^2+23x-7 \\ \underline{-(4x^3-8x^2+20x)} \\ -x^2+3x-7 \\ \underline{-(-x^2+2x-5)} \\ x-2 \end{array}$$

Figure 5.4.15

The remainder is $x-2$. Write the answer with the remainder:

$$\frac{3x^4-2x^3+6x^2+23x-7}{x^2-2x+5} = 3x^2+4x-1+\frac{x-2}{x^2-2x+5}$$

Answer:

$$3x^2+4x-1+\frac{x-2}{x^2-2x+5}$$

Polynomial long division takes time and practice to master. Work lots of problems and remember that you may check your answers by multiplying the quotient by the divisor (and adding the remainder if present) to obtain the dividend.

? Exercise 5.4.2

Divide:

$$\frac{20x^4-32x^3+7x^2+8x-10}{5x-3}.$$

Answer

$4x^3-4x^2-x+1-\frac{7}{5}x-3$

Dividing Polynomial Functions

We may use function notation to indicate division as follows:

Table 5.4.1

Division of functions:	$(f/g)(x) = \frac{f(x)}{g(x)}$

The quotient of two polynomial functions does not necessarily have a domain of all real numbers. The values for x that make the function in the denominator 0 are restricted from the domain. This will be discussed in more detail at a later time. For now, assume all functions in the denominator are nonzero.

Example 5.4.9

Calculate:

$$(f/g)(x) \text{ given } f(x) = 6x^5 - 36x^4 + 12x^3 - 6x^2 \text{ and } g(x) = -6x^2 .$$

Solution:

The notation indicates that we should divide:

$$\begin{aligned}(f/g)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{6x^5 - 36x^4 + 12x^3 - 6x^2}{-6x^2} \\ &= \frac{6x^5}{-6x^2} - \frac{36x^4}{-6x^2} + \frac{12x^3}{-6x^2} - \frac{6x^2}{-6x^2} \\ &= -1x^{5-2} + 6x^{4-2} - 2x^{3-2} + 1x^{2-2} \\ &= -x^3 + 6x^2 - 2x + 1\end{aligned}$$

Answer:

$$(f/g)(x) = -x^3 + 6x^2 - 2x + 1$$

Example 5.4.10

Calculate:

$$(f/g)(-1), \text{ given } f(x) = -3x^3 + 7x^2 - 11x - 1 \text{ and } g(x) = 3x - 1 .$$

Solution:

First, determine $(f/g)(x)$.

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{-3x^3+7x^2-11x-1}{3x-1}$$

$$\begin{array}{r} -x^2 + 2x - 3 \\ 3x-1\overline{)\,-3x^3 + 7x^2 - 11x - 1} \\ \underline{-(-3x^3 + x^2)} \\ 6x^2 - 11x - 1 \\ \underline{-(6x^2 - 2x)} \\ -9x - 1 \\ \underline{-(-9x + 3)} \\ -4 \end{array}$$

Figure 5.4.16

Therefore,

$$(f/g)(x) = -x^2 + 2x - 3 - \frac{4}{3x-1}$$

Substitute -1 for the variable x.

Answer:

$(f/g)(-1) = -5$

Key Takeaways

- When dividing by a monomial, divide all terms in the numerator by the monomial and then simplify each term. To simplify each term, divide the coefficients and apply the quotient rule for exponents.
- When dividing a polynomial by another polynomial, apply the division algorithm.
- To check the answer after dividing, multiply the divisor by the quotient and add the remainder (if necessary) to obtain the dividend.
- It is a good practice to include placeholders when performing polynomial long division.

? Exercise 5.4.3 Dividing by a Monomial

Divide.

1. $\frac{81y^5}{9y^2}$
2. $\frac{36y^9}{9y^3}$
3. $\frac{52x^2y}{4xy}$
4. $\frac{24xy^5}{2xy^4}$
5. $\frac{25x^2y^5z^3}{5xyz}$
6. $\frac{-77x^4y^9z^2}{2x^3y^3z}$
7. $\frac{125a^3b^2c}{-10abc}$
8. $\frac{36a^2b^3c^5}{-6a^2b^2c^3}$
9. $\frac{9x^2+27x-3}{3}$
10. $\frac{10x^3-5x^2+40x-15}{5}$
11. $\frac{20x^3-10x^2+30x}{2x}$
12. $\frac{10x^4+8x^2-6x}{24x}$
13. $\frac{-6x^5-9x^3+3x}{-3x}$
14. $\frac{36a^{12}-6a^9+12a^5}{-12a^5}$
15. $\frac{-12x^5+18x^3-6x^2}{-6x^2}$
16. $\frac{-49a^8+7a^5-21a^3}{7a^3}$
17. $\frac{9x^7-6x^4+12x^3-x^2}{3x^2}$
18. $\frac{8x^9+16x^7-24x^4+8x^3}{-8x^3}$
19. $\frac{16a^7-32a^6+20a^5-a^4}{4a^4}$
20. $\frac{5a^6+2a^5+6a^3-12a^2}{3a^2}$
21. $\frac{-4x^2y^3+16x^7y^8-8x^2y^5}{-4x^2y^3}$
22. $\frac{100a^{10}b^{30}c^5-50a^{20}b^5c^{40}+20a^5b^{20}c^{10}}{10a^5b^5c^5}$
23. Find the quotient of $-36x^9y^7$ and $2x^8y^5$.
24. Find the quotient of $144x^3y^{10}z^2$ and $-12x^3y^5z$.
25. Find the quotient of $3a^4-18a^3+27a^2$ and $3a^2$.
26. Find the quotient of $64a^2bc^3-16a^5bc^7$ and $4a^2bc^3$.

Answer

1. $9y^3$
3. $13x$

5. $5xy^4z^2$

7. $-\frac{25}{2}a^2b$

9. $3x^2+9x-1$

11. $10x^2-5x+15$

13. $2x^4+3x^2-1$

15. $2x^3-3x+1$

17. $3x^5-2x^2+4x-\frac{1}{3}$

19. $4a^3-8a^2+5a-\frac{1}{4}$

21. $-4x^5y^5+2y^2+1$

23. $-18xy^2$

25. a^2-6a+9

? Exercise 5.4.4 Dividing by a Polynomial

Divide.

1. $(2x^2-5x-3)\div(x-3)$
2. $(3x^2+5x-2)\div(x+2)$
3. $(6x^2+11x+3)\div(3x+1)$
4. $(8x^2-14x+3)\div(2x-3)$
5. $\frac{x^3-x^2-2x-12}{x-3}$
6. $\frac{2x^3+11x^2+4x-5}{x+5}$
7. $\frac{2x^3-x^2-4x+3}{2x+3}$
8. $\frac{-15x^3-14x^2+23x-6}{5x-2}$
9. $\frac{14x^4-9x^3+22x^2+4x-1}{7x-1}$
10. $\frac{8x^5+16x^4-8x^3-5x^2-21x+10}{2x+5}$
11. $\frac{x^2+8x+17}{x+5}$
12. $\frac{2x^2-5x+5}{x-2}$
13. $\frac{6x^2-13x+9}{-2x+1}$
14. $\frac{-12x^2+x+1}{3x+2}$
15. $\frac{x^3+9x^2+19x+1}{x+4}$
16. $\frac{2x^3-13x^2+17x-11}{x-5}$
17. $\frac{9x^3-12x^2+16x-15}{3x-2}$
18. $\frac{3x^4-8x^3+5x^2-5x+9}{x-2}$
19. $(6x^5-13x^4+4x^3-3x^2+13x-2)\div(3x+1)$
20. $(8x^5-22x^4+19x^3-20x^2+23x-3)\div(2x-3)$
21. $\frac{5x^5+12x^4+12x^3-7x^2-19x+3}{x^2+2x+3}$
22. $\frac{6x^5-17x^4+5x^3+16x^2-7x-3}{2x^2-3x-1}$
23. $\frac{x^5+7x^4-x^3-7x^2-49x+9}{x^2+7x-1}$
24. $\frac{5x^6-6x^4-4x^2+x+2}{5x^2-1}$
25. $\frac{x^3-27}{x-3}$
26. $\frac{8x^3+125}{2x+5}$
27. $(15x^5-9x^4-20x^3+12x^2+15x-9)\div(5x-3)$

28. $(2x^6-5x^5-4x^4+10x^3+6x^2-17x+5)\div(2x-5)$
29. $\frac{x^5-2x^3+3x-1}{x-1}$
30. $\frac{x^4-3x^2+5x-1}{3x+2}$
31. $\frac{a^2-4}{a+2}$
32. $\frac{a^5+1}{a^5+1}$
33. $\frac{a^6-1}{a-1}$
34. $\frac{x^5-1}{x-1}$
35. $\frac{x^5+x^4+6x^3+12x^2-4}{x^2+x-1}$
36. $\frac{50x^6-30x^5-5x^4+15x^3-5x+1}{5x^2-3x+2}$
37. $\frac{5x^5-15x^3+25x^2-5}{5x}$
38. $\frac{-36x^6+12x^4-6x^2}{6x^2}$
39. $\frac{150x^5y^2z^{15}-10x^3y^6z^5+4x^3y^2z^4}{10x^3y^2z^5}$
40. $\frac{27m^6+9m^4-81m^2+1}{9m^2}$
41. Divide $3x^6-2x^5+27x^4-18x^3-6x^2+7x-10$ by $3x-2$.
42. Divide $8x^6+4x^5-14x^4-5x^3+x^2-2x-3$ by $2x+1$.

Answer

1. $2x+1$
3. $2x+3$
5. x^2+2x+4
7. x^2-2x+1
9. $2x^3-x^2+3x+1$
11. $x+3+\frac{2}{x+5}$
13. $-3x+5+\frac{4}{-2x+1}$
15. $x^2+5x-1+\frac{5}{x+4}$
17. $3x^2-2x+4-\frac{7}{3x-2}$
19. $2x^4-5x^3+3x^2-2x+5-\frac{7}{3x+1}$
21. $5x^3+2x^2-7x+1$
23. $x^3-7+\frac{2}{x^2+7x-1}$
25. x^2+3x+9
27. $3x^4-4x^2+3$
29. $x^4+x^3-x^2-x+2+\frac{1}{x-1}$
31. $a-2$
33. $a^5+a^4+a^3+a^2+a+1$
35. $x^3+7x+5+\frac{2x+1}{x^2+x-1}$
37. $x^4-3x^2+5x-\frac{1}{x}$
39. $15x^2z^{10}-y^4+\frac{2}{5z}$
41. $x^5+9x^3-2x+1-\frac{8}{3x-2}$

Exercise 5.4.5 Dividing Polynomial Functions

Calculate $(f/g)(x)$, given the functions.

1. $f(x)=40x^8$ and $g(x)=10x^5$
2. $f(x)=54x^5$ and $g(x)=9x^3$
3. $f(x)=12x^2+24x-15$ and $g(x)=2x+5$
4. $f(x)=-8x^2+30x-7$ and $g(x)=2x-7$
5. $f(x)=18x^2-36x+5$ and $g(x)=3x-5$
6. $f(x)=-7x^2+29x-6$ and $g(x)=7x-1$
7. $f(x)=10x^3-9x^2+27x-10$ and $g(x)=5x-2$
8. $f(x)=15x^3+28x^2-11x+56$ and $g(x)=3x+8$
9. $f(x)=2x^4+5x^3-11x^2-19x+20$ and $g(x)=x^2+x-5$
10. $f(x)=4x^4-12x^3-20x^2+26x-3$ and $g(x)=2x^2+2x-3$

Answer

1. $(f/g)(x)=4x^3$
3. $(f/g)(x)=6x-3$
5. $(f/g)(x)=6x-2-\frac{5}{3x-5}$
7. $(f/g)(x)=2x^2-x+5$
9. $(f/g)(x)=2x^2+3x-4$

Exercise 5.4.6 Dividing Polynomial Functions

Given $f(x)=6x^3+4x^2-11x+3$ and $g(x)=3x-1$, find the following.

1. $(f/g)(x)$
2. $(f/g)(-1)$
3. $(f/g)(0)$
4. $(f/g)(1)$

Answer

1. $(f/g)(x)=2x^2+2x-3$
3. $(f/g)(0)=-3$

Exercise 5.4.7 Dividing Polynomial Functions

Given $f(x)=5x^3-13x^2+7x+3$ and $g(x)=x-2$, find the following.

1. $(f/g)(x)$
2. $(f/g)(-3)$
3. $(f/g)(0)$
4. $(f/g)(7)$

Answer

1. $(f/g)(x)=5x^2-3x+1+\frac{5}{x-2}$
3. $(f/g)(0)=-\frac{3}{2}$

Exercise 5.4.8 Discussion Board Topics

1. How do you use the distributive property when dividing a polynomial by a monomial?
2. Compare long division of real numbers with polynomial long division. Provide an example of each.

Answer

1. Answers may vary

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CHAPTER OVERVIEW

6: Functional Analysis

6.1: Rectangular Coordinate System

6.2: Relations and Functions

6.2E: Exercises

6.3: The Graph of a Function

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6.1: Rectangular Coordinate System

Learning Objectives

- Plot points using the rectangular coordinate system.
- Calculate the distance between any two points in the rectangular coordinate plane.
- Determine the midpoint between any two points.

Rectangular Coordinate System

The rectangular coordinate system consists of two real number lines that intersect at a right angle. The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. These two number lines define a flat surface called a plane, and each point on this plane is associated with an ordered pair of real numbers (x, y). The first number is called the **x-coordinate**, and the second number is called the **y-coordinate**. The intersection of the two axes is known as the origin, which corresponds to the point $(0, 0)$.

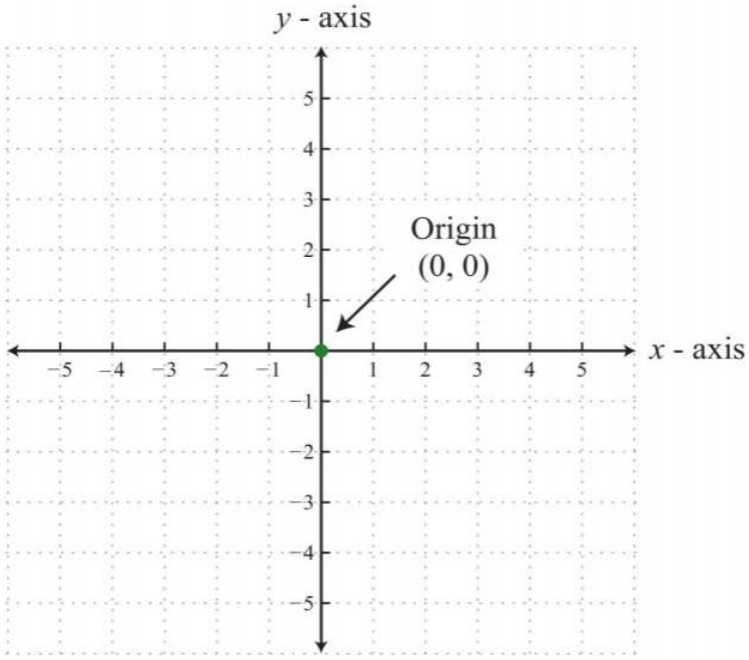


Figure 6.1.1

An ordered pair (x, y) represents the position of a point relative to the origin. The x-coordinate represents a position to the right of the origin if it is positive and to the left of the origin if it is negative. The y-coordinate represents a position above the origin if it is positive and below the origin if it is negative. Using this system, every position (point) in the plane is uniquely identified. For example, the pair $(2, 3)$ denotes the position relative to the origin as shown:

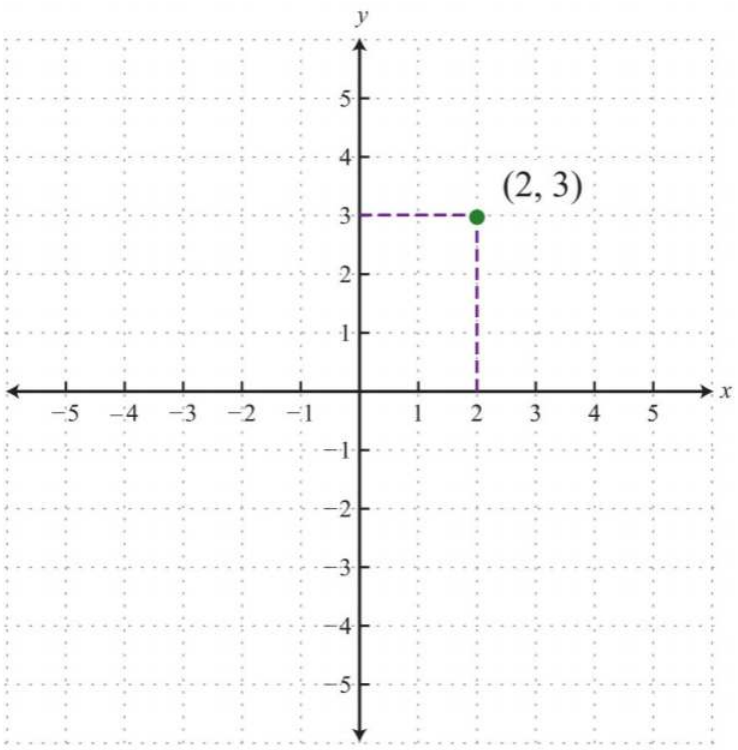


Figure 6.1.2

This system is often called the Cartesian coordinate system, named after the French mathematician René Descartes (1596– 1650).

The x- and y-axes break the plane into four regions called quadrants, named using roman numerals I, II, III, and IV, as pictured. In quadrant I, both coordinates are positive. In quadrant II, the x-coordinate is negative and the y-coordinate is positive. In quadrant III, both coordinates are negative. In quadrant IV, the x-coordinate is positive and the y-coordinate is negative.

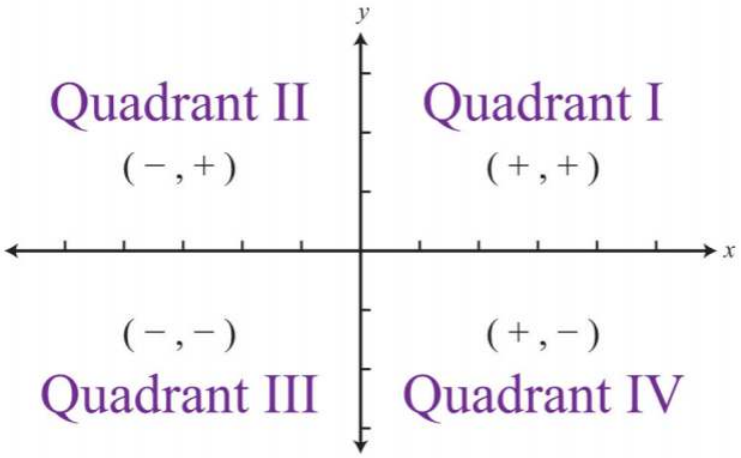


Figure 6.1.3

Example 6.1.1

Plot the ordered pair $(-3, 5)$ and determine the quadrant in which it lies.

Solution:

The coordinates $x = -3$ and $y = 5$ indicate a point 3 units to the left of and 5 units above the origin.

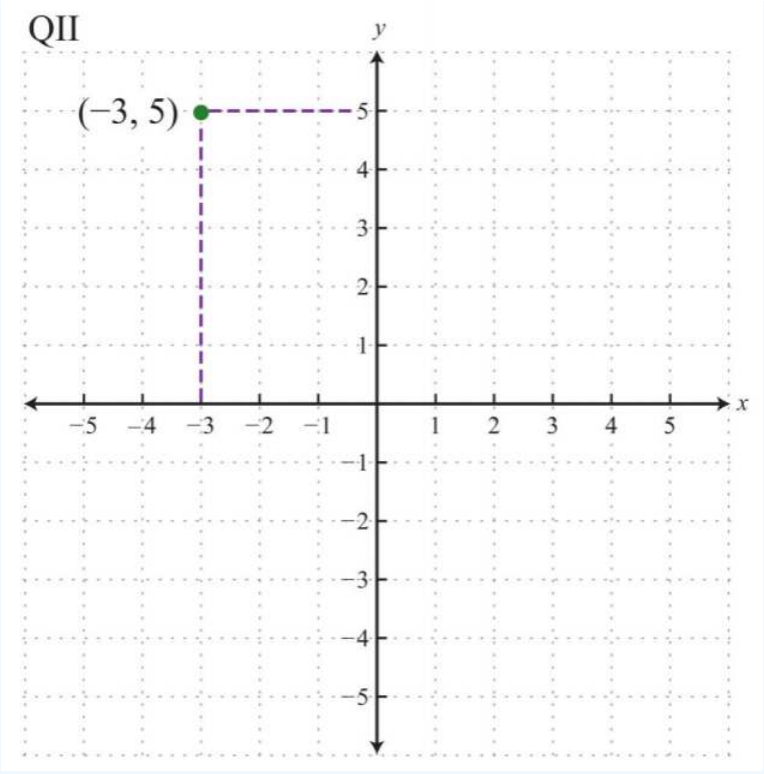


Figure 6.1.4

Answer:

The point is plotted in quadrant II (QII) because the x-coordinate is negative and the y-coordinate is positive.

Ordered pairs with 0 as one of the coordinates do not lie in a quadrant; these points are on one axis or the other (or the point is the origin if both coordinates are 0). Also, the scale indicated on the x-axis may be different from the scale indicated on the y-axis. Choose a scale that is convenient for the given situation.

Example 6.1.2

Plot this set of ordered pairs:

$$\{(4,0),(-6,0),(0,3),(-2,6),(-4,-6)\}$$

Solution:

Each tick mark on the x-axis represents 2 units and each tick mark on the y-axis represents 3 units.

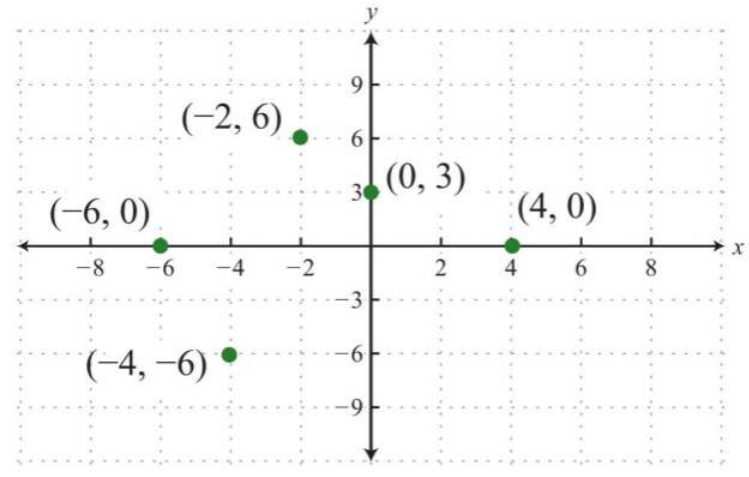


Figure 6.1.5

Example 6.1.3

Plot this set of ordered pairs:

$$\{(-6,-5),(-3,-3),(0,-1),(3,1),(6,3)\}$$

Solution:

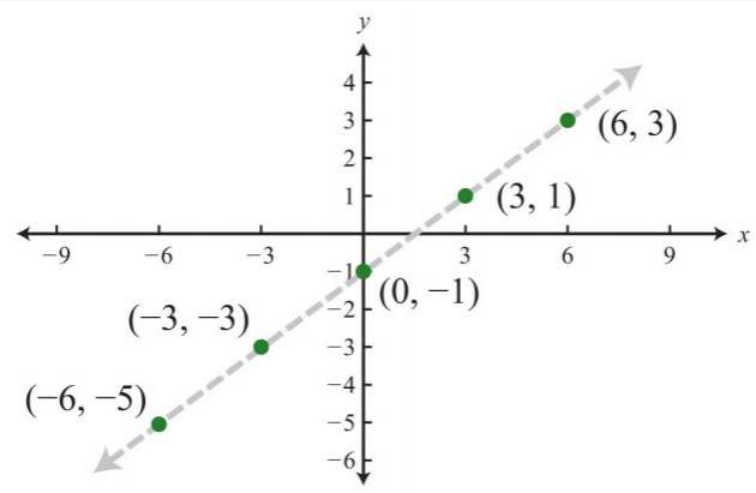


Figure 6.1.6

In this example, the points appear to be collinear, or to lie on the same line. The entire chapter focuses on finding and expressing points with this property.

Exercise 6.1.1

Plot the set of points $\{(5,3),(-3,2),(-2,-4),(4,-3)\}$and indicate in which quadrant they lie.

Answer

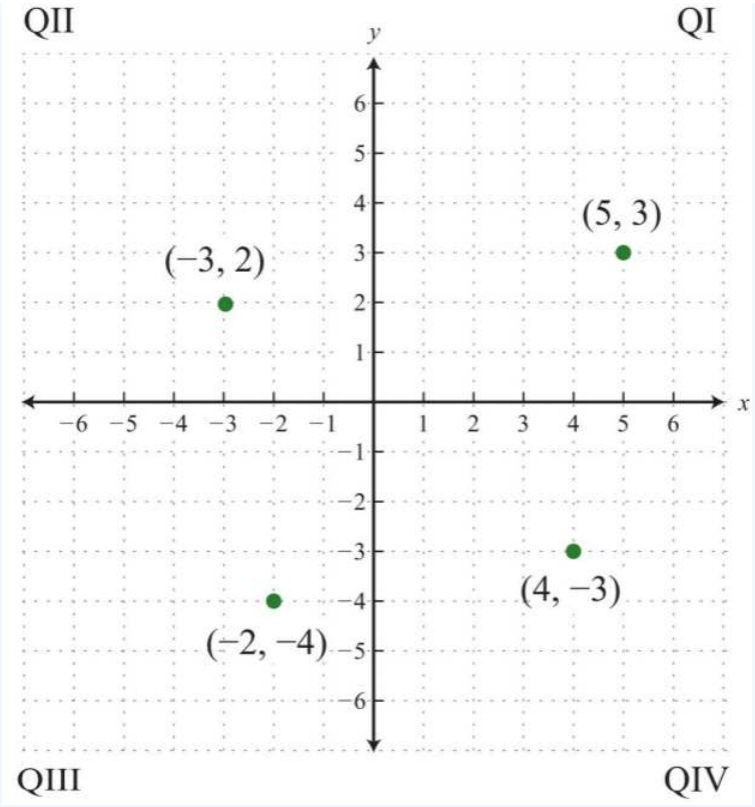


Figure 6.1.7

Graphs are used in everyday life to display data visually. A line graph consists of a set of related data values graphed on a coordinate plane and connected by line segments. Typically, the independent quantity, such as time, is displayed on the x-axis and the dependent quantity, such as distance traveled, on the y-axis.

Example 6.1.4

The following line graph shows the number of mathematics and statistics bachelor's degrees awarded in the United States each year since 1970.



Figure 6.1.8*: Source: Digest of Education Statistics.*

a. How many mathematics and statistics bachelor's degrees were awarded in 1975?

b. In which years were the number of mathematics and statistics degrees awarded at the low of $11,000$?

Solution:

a. The scale on the x-axis represents time since 1970, so to determine the number of degrees awarded in 1975, read the y-value of the graph at $x = 5$.

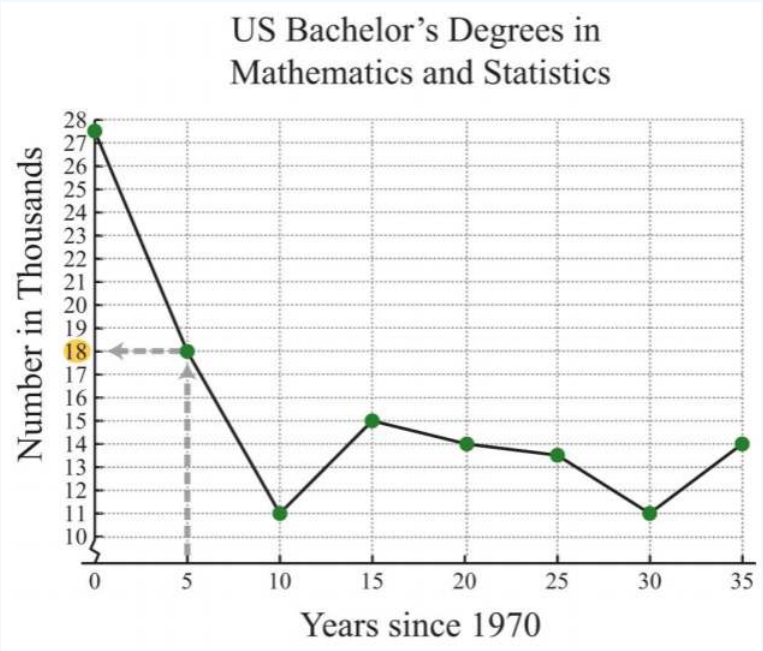


Figure 6.1.9*: Source: Digest of Education Statistics.*

The y-value corresponding to $x = 5$ is 18. The graph indicates that this is in thousands; there were $18,000$ mathematics and statistics degrees awarded in 1975.

b. To find the year a particular number of degrees was awarded, first look at the y-axis. In this case, $11,000$ degrees is represented by 11 on the y-axis; look to the right to see in which years this occurred.

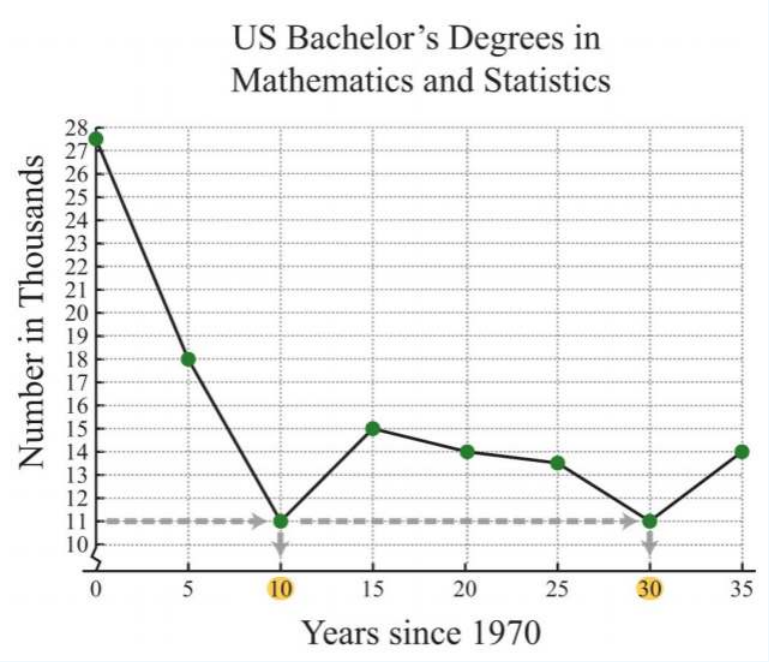


Figure 6.1.10*: Source: Digest of Education Statistics.*

The y-value of 11 occurs at two data points, one where $x = 10$ and the other where $x = 30$. These values correspond to the years 1980 and 2000, respectively.

Answers:

a. In the year 1975, $18,000$ mathematics and statistics degrees were awarded.
b. In the years 1980 and 2000, the lows of $11,000$ mathematics and statistics degrees were awarded.

Distance Formula

Frequently you need to calculate the distance between two points in a plane. To do this, form a right triangle using the two points as vertices of the triangle and then apply the Pythagorean theorem. Recall that the **Pythagorean theorem** states that if given any right triangle with legs measuring a and b units, then the square of the measure of the hypotenuse c is equal to the sum of the squares of the legs: $a^2 + b^2 = c^2$. In other words, the hypotenuse of any right triangle is equal to the square root of the sum of the squares of its legs.

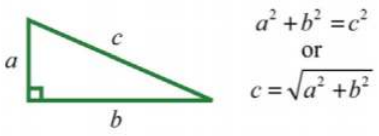


Figure 6.1.11

Example 6.1.5

Find the distance between $(-1, 2)$ and $(3, 5)$.

Solution:

Form a right triangle by drawing horizontal and vertical lines through the two points. This creates a right triangle as shown below:

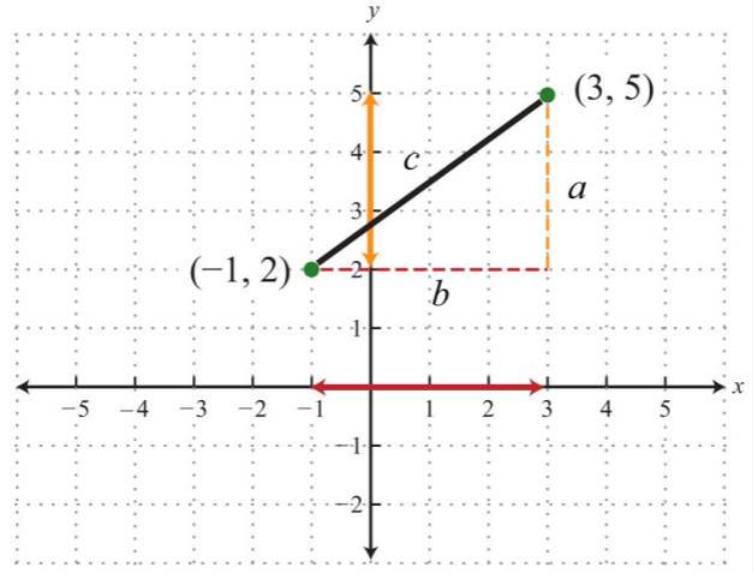


Figure 6.1.12

The length of leg b is calculated by finding the distance between the x-values of the given points, and the length of leg a is calculated by finding the distance between the given y-values.

$$a = 5 - 2 = 3 \text{units}$$
$$b = 3 - (-1) = 3 + 1 = 4 \text{units}$$

Next, use the Pythagorean theorem to find the length of the hypotenuse.

$$\begin{aligned} c &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \text{ units} \end{aligned}$$

Answer:

The distance between the two points is 5 units.

Generalize this process to produce a formula that can be used to algebraically calculate the distance between any two given points.

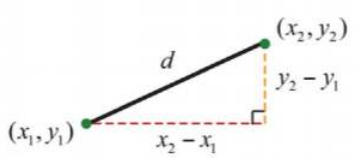


Figure 6.1.13

Given two points, (x_1, y_1) and (x_2, y_2), then the distance, d, between them is given by the distance formula:

$$\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2} \tag{6.1.1}$$

Example 6.1.6

Calculate the distance between $(-3, -1)$ and $(-2, 4)$.

Solution:

Use the distance formula.

x_1, y_1	x_2, y_2
$(-3, -1)$	$(-2, 4)$

It is a good practice to include the formula in its general form as a part of the written solution before substituting values for the variables. This improves readability and reduces the chance for errors.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2-(-3))^2+(4-(-1))^2} \\ &= \sqrt{(-2+3)^2 + (4+1)^2} \\ &= \sqrt{(1)^2 + (5)^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26} \end{aligned}$$

Answer:

$\sqrt{26}$ units

Exercise 6.1.2

Calculate the distance between $(-7, 5)$ and $(-1, 13)$.

Answer

10 units

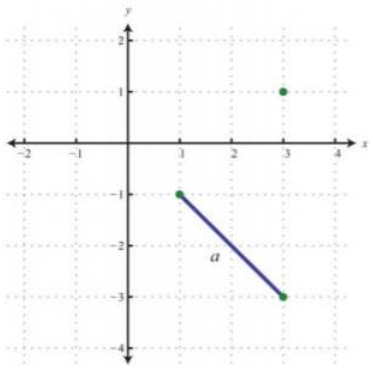
Example 6.1.7

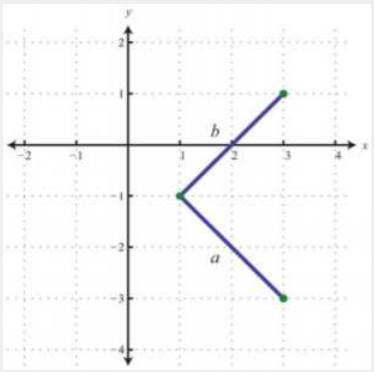
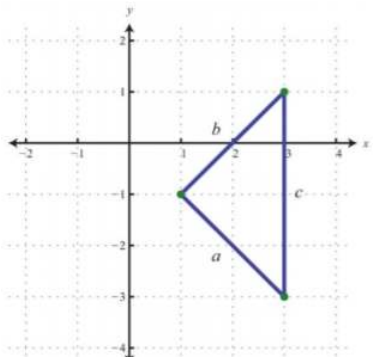
Do the three points $(1, -1)$, $(3, -3)$, and $(3, 1)$ form a right triangle?

Solution:

The Pythagorean theorem states that having side lengths that satisfy the property $a^2 + b^2 = c^2$ is a necessary and sufficient condition of right triangles. In other words, if you can show that the sum of the squares of the leg lengths of the triangle is equal to the square of the length of the hypotenuse, then the figure must be a right triangle. First, calculate the length of each side using the distance formula.

Table 6.1.1

Geometry	Calculation
 Figure 6.1.14	Points: $(1, -1)$ and $(3, -3)$ $$\begin{aligned} a &= \sqrt{(3-1)^2 + (-3-(-1))^2} \\ &= \sqrt{(a)^2 + (-3+1)^2} \\ &= \sqrt{4+(-2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \end{aligned}$$

Geometry	Calculation
 Figure 6.1.15	Points: $(1,-1)$ and $(3,1)$ $$\begin{aligned} b &= \sqrt{(3-1)^2+(1-(-1))^2} \\ &= \sqrt{2^2+(1+1)^2} \\ &= \sqrt{4+(2)^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \end{aligned}$$
 Figure 6.1.16	Points: $(3,-3)$ and $(3,1)$ $$\begin{aligned} c &= \sqrt{(3-3)^2+(1-(-3))^2} \\ &= \sqrt{(0)^2+(1+3)^2} \\ &= \sqrt{0+(4)^2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

Now we check to see if $a^2+b^2=c^2$.

$$\begin{aligned} a^2+b^2 &= c^2 \\ (\sqrt{8})^2+(\sqrt{8})^2 &= (4)^2 \\ \sqrt{64}+\sqrt{64} &= 16 \\ 8+8 &= 16 \\ 16 &= 16 \quad \checkmark \end{aligned}$$

Answer:

Yes, the three points form a right triangle. In fact, since two of the legs are equal in length, the points form an isosceles right triangle.

Midpoint Formula

The point that bisects the line segment formed by two points, (x_1,y_1) and (x_2,y_2), is called the midpoint and is given by the following formula:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) \tag{6.1.2}$$

The midpoint is an ordered pair formed by finding the average of the x-values and the average of the y-values of the given points.

Example 6.1.8

Calculate the midpoint between $(-1,-2)$ and $(7,4)$.

Solution:

First, calculate the average of the x- and y-values of the given points.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ (-1, -2) & (7, 4) \end{array}$$

$$\frac{x_1 + x_2}{2} = \frac{-1 + 7}{2} = \frac{6}{2} = 3$$
$$\frac{y_1 + y_2}{2} = \frac{-2 + 4}{2} = \frac{2}{2} = 1$$

Next, form the midpoint as an ordered pair using the averaged coordinates.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(3, 1)$$

Answer:

$$(3, 1)$$

To verify that this is indeed the midpoint, calculate the distance between the two given points and verify that the result is equal to the sum of the two equal distances from the endpoints to this midpoint. This verification is left to the reader as an exercise.

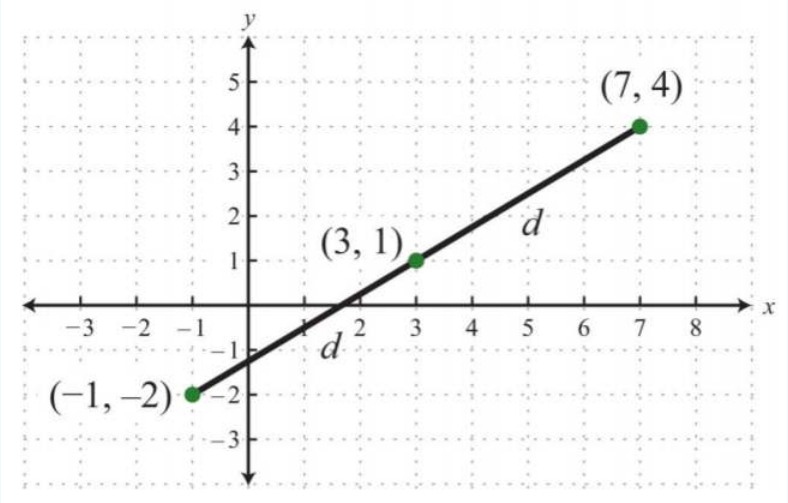


Figure 6.1.17

? Exercise 6.1.3

Find the midpoint between $(-6, 5)$ and $(6, -11)$.

Answer

$(0, -3)$

Key Takeaways

- Use the rectangular coordinate system to uniquely identify points in a plane using ordered pairs (x, y). Ordered pairs indicate position relative to the origin. The x-coordinate indicates position to the left and right of the origin. The y-coordinate indicates position above or below the origin.
- The scales on the x-axis and y-axis may be different. Choose a scale for each axis that is appropriate for the given problem.
- Graphs are used to visualize real-world data. Typically, independent data is associated with the x-axis and dependent data is associated with the y-axis.
- The Pythagorean theorem gives us a necessary and sufficient condition of right triangles. Given a right triangle, then the measures of the sides satisfy $a^2 + b^2 = c^2$. Conversely, if the sides satisfy $a^2 + b^2 = c^2$, then the triangle must be a right triangle.
- The distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, is derived from the Pythagorean theorem and gives us the distance between any two points, (x_1, y_1) and (x_2, y_2), in a rectangular coordinate plane.

- The midpoint formula, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, is derived by taking the average of each coordinate and forming an ordered pair.

Exercise 6.1.4 Ordered Pairs

Give the coordinates of points A, B, C, D, and E.

1.

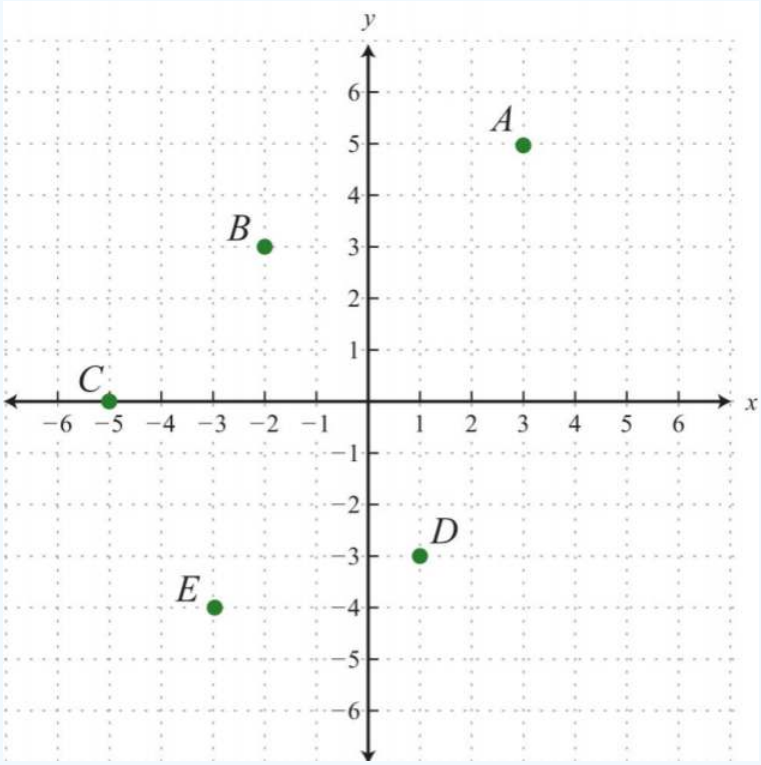


Figure 6.1.18

2.

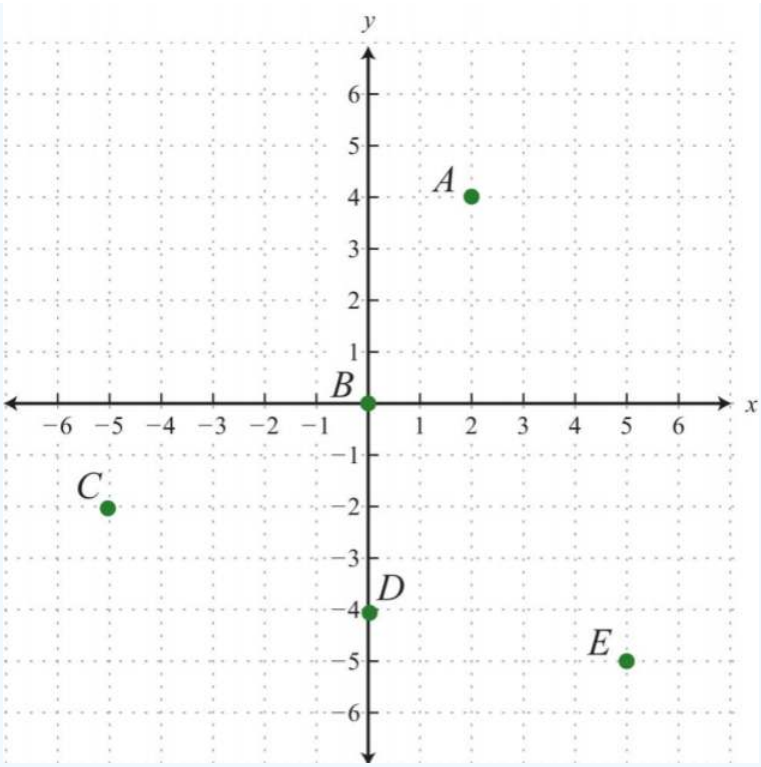


Figure 6.1.19

3.

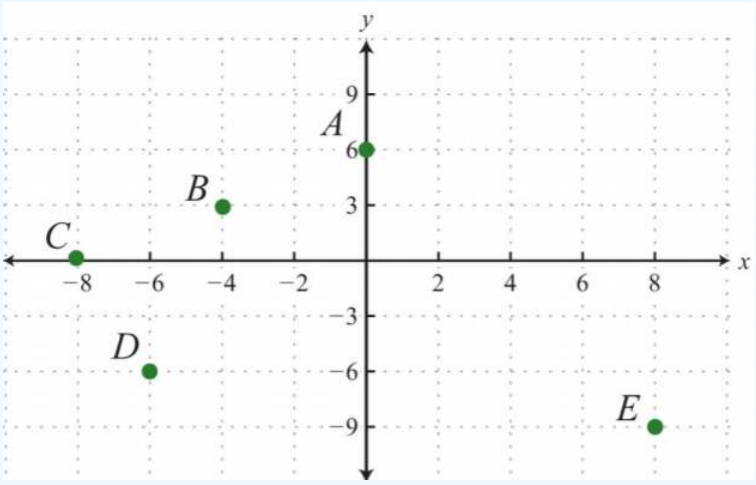


Figure 6.1.20

4.

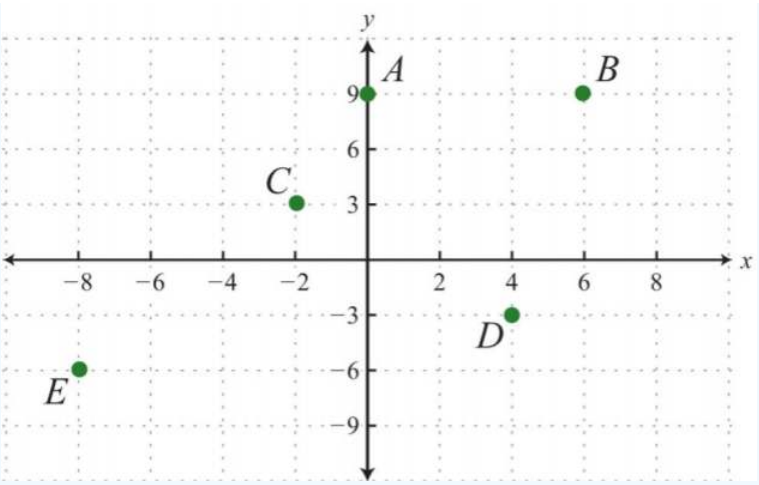


Figure 6.1.21

5.

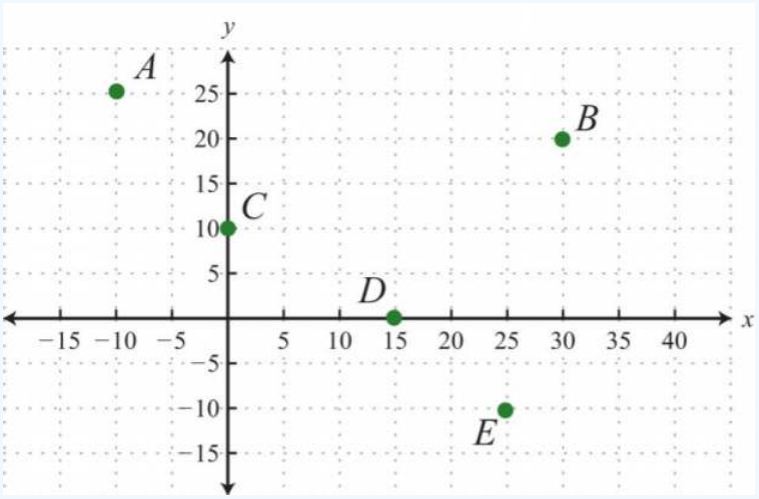


Figure 6.1.22

6.

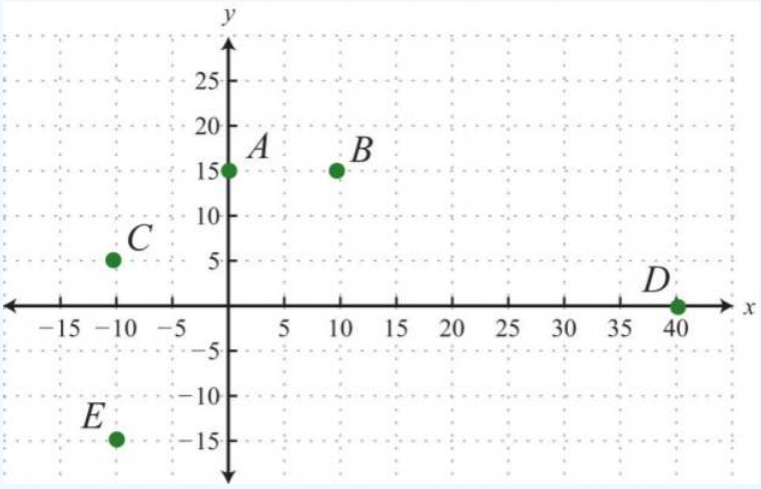


Figure 6.1.23

Answer

1. $A:(3,5); B:(-2,3); C:(-5,0); D:(1,-3); E:(-3,-4)$

3. $A:(0,6); B:(-4,3); C:(-8,0); D:(-6,-6); E:(8,-9)$
5. $A:(-10,25); B:(30,20); C:(0,10); D:(15,0); E:(25,-10)$

Exercise 6.1.5 Ordered Pairs

Graph the given set of ordered pairs.

1. $\{(-4,5),(-1,1),(-3,-2),(5,-1)\}$
2. $\{(-15,-10),(-5,10),(15,10),(5,-10)\}$
3. $\{(-2,5),(10,0),(2,-5),(6,-10)\}$
4. $\{(-8,3),(-4,6),(0,-6),(6,9)\}$
5. $\{(-10,5),(20,-10),(30,15),(50,0)\}$
6. $\{(-\frac{5}{3},-\frac{1}{2}),(-\frac{1}{3},\frac{1}{2}),(\frac{2}{3},-1),(\frac{5}{3},1)\}$
7. $\{(-\frac{3}{5},-\frac{4}{3}),(\frac{2}{5},\frac{4}{3}),(1,-\frac{2}{3}),(0,1)\}$
8. $\{(-3.5,0),(-1.5,2),(0,1.5),(2.5,-1.5)\}$
9. $\{(-0.8,0.2),(-0.2,-0.4),(0,-1),(0.6,-0.4)\}$
10. $\{(-1.2,-1.2),(-0.3,-0.3),(0,0),(0.6,0.6),(1.2,1.2)\}$

Answer

1.

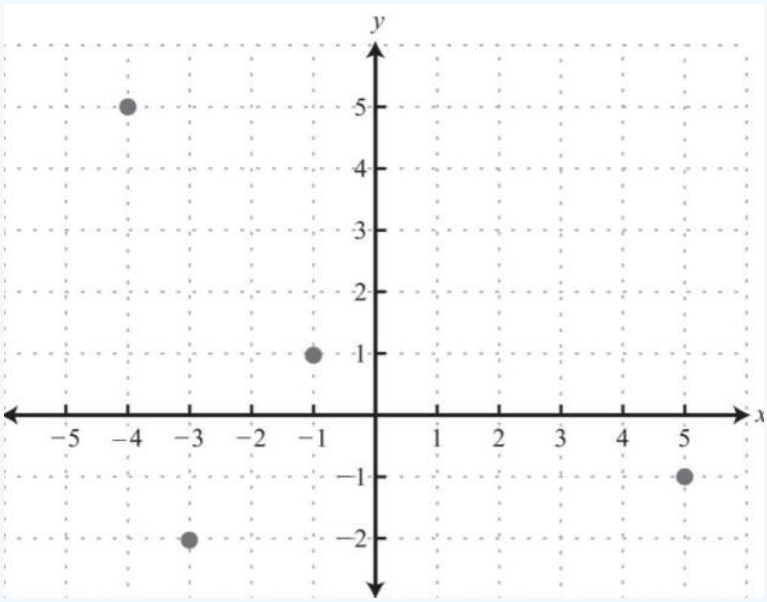


Figure 6.1.24

3.

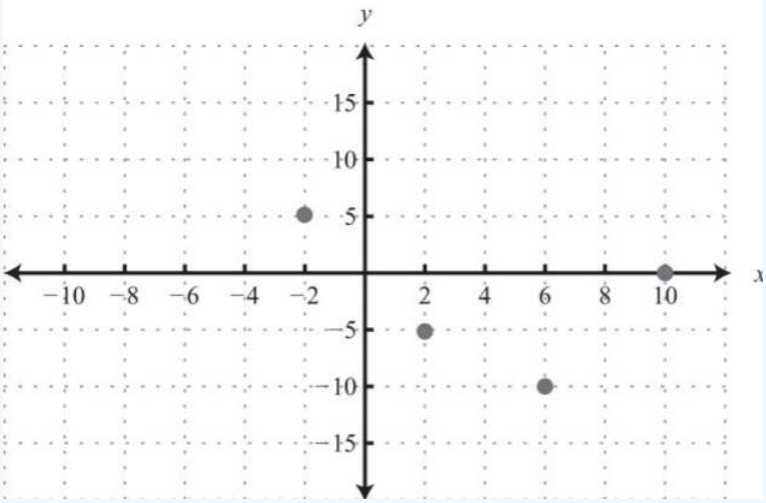


Figure 6.1.25

5.

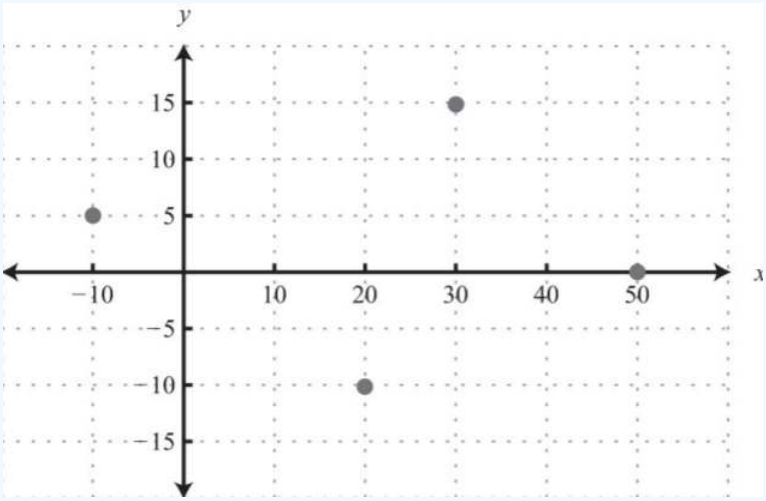


Figure 6.1.26

7.

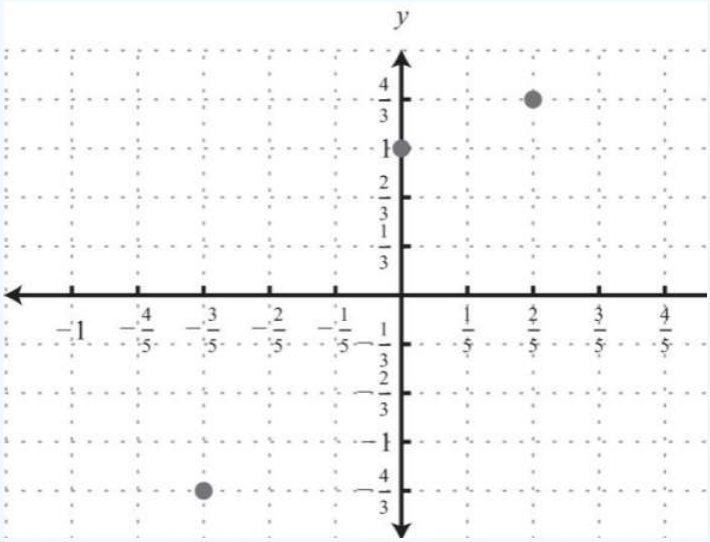


Figure 6.1.27

9.

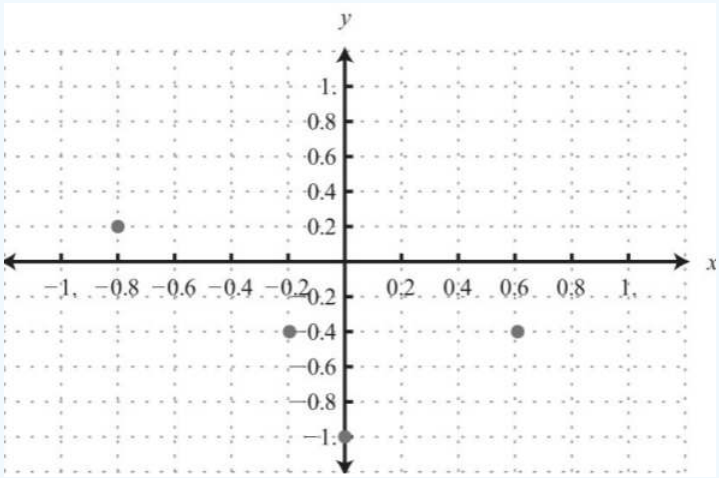


Figure 6.1.28

Exercise 6.1.6 Ordered Pairs

State the quadrant in which the given point lies.

1. $(-3, 2)$
2. $(5, 7)$
3. $(-12, -15)$
4. $(7, -8)$
5. $(-3.8, 4.6)$
6. $(17.3, 1.9)$
7. $(-\frac{1}{8}, -\frac{5}{8})$
8. $(\frac{3}{4}, -\frac{1}{4})$
9. $x > 0$ and $y < 0$
10. $x < 0$ and $y < 0$
11. $x < 0$ and $y > 0$
12. $x > 0$ and $y > 0$

Answer

1. QII
3. QIII
5. QII
7. QIII
9. QIV
11. QII

Exercise 6.1.7 Ordered Pairs

The average price of a gallon of regular unleaded gasoline in US cities is given in the following line graph. Use the graph to answer the following questions.

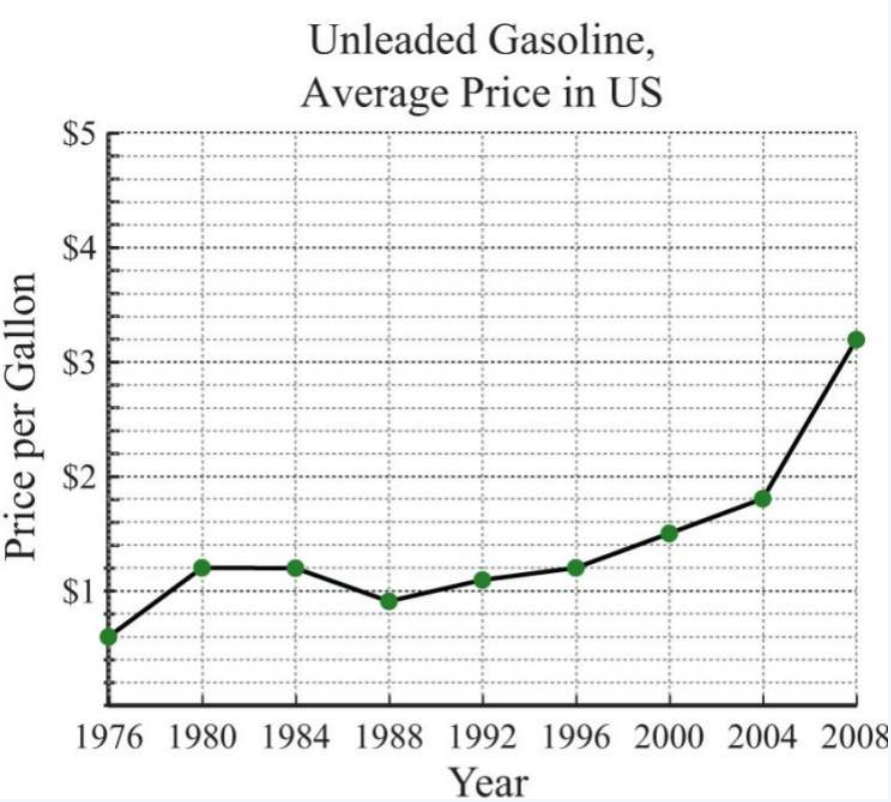


Figure 6.1.29*: Source: Bureau of Labor Statistics*

1. What was the average price of a gallon of unleaded gasoline in 2004?
2. What was the average price of a gallon of unleaded gasoline in 1976?
3. In which years were the average price of a gallon of unleaded gasoline $1.20?
4. What is the price increase of a gallon of gasoline from 1980 to 2008?
5. What was the percentage increase in the price of a gallon of unleaded gasoline from 1976 to 1980?
6. What was the percentage increase in the price of a gallon of unleaded gasoline from 2000 to 2008?

Answer

1. $1.80

3. 1980 to 1984, 1996

3. 100%

? Exercise 6.1.8 Ordered Pairs

The average price of all-purpose white flour in US cities from 1980 to 2008 is given in the following line graph. Use the graph to answer the questions that follow.

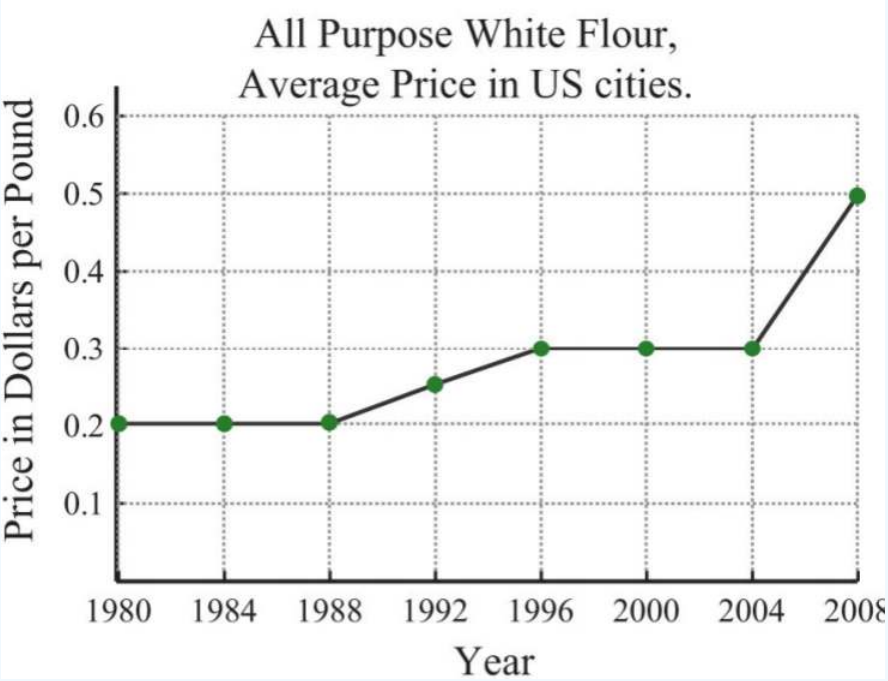


Figure 6.1.30*: Source: Bureau of Labor Statistics*

1. What was the average price per pound of all-purpose white flour in 2000?
2. What was the average price per pound of all-purpose white flour in 2008?
3. In which year did the price of flour average $0.25 per pound?
4. In which years did the price of flour average $0.20 per pound?
5. What was the percentage increase in flour from the year 2000 to 2008?
6. What was the percentage increase in flour from the year 1992 to 2000?

Answer

1. $0.30
3. 1992
5. 67%

? Exercise 6.1.9 Ordered Pairs

Given the following data, create a line graph.

1. The percentage of total high school graduates who enrolled in college.

Table 6.1.2*: Source: Digest of Education Statistics*

Year	Percentage
1969	36%
1979	40%
1989	47%
1999	42%

2. The average daily temperature given in degrees Fahrenheit in May.

Table 6.1.3

Exam	Temperature

Exam	Temperature
8:00 am	60
12:00 pm	72
4:00 pm	75
8:00 pm	67
12:00 am	60
4:00 am	55

Answer

1.

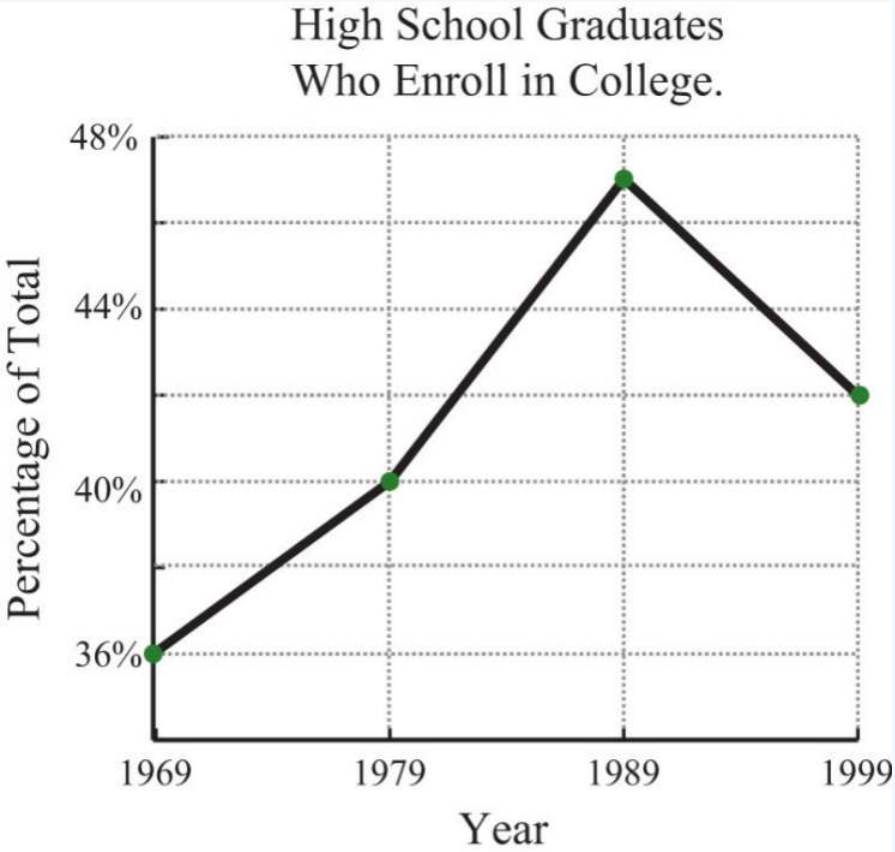


Figure 6.1.31

Exercise 6.1.10 Ordered Pairs

Calculate the area of the shape formed by connecting the following set of vertices.

1. $\{(0,0),(0,3),(5,0),(5,3)\}$
2. $\{(-1,-1),(-1,1),(1,-1),(1,1)\}$
3. $\{(-2,-1),(-2,3),(5,3),(5,-1)\}$
4. $\{(-5,-4),(-5,5),(3,5),(3,-4)\}$
5. $\{(0,0),(4,0),(2,2)\}$
6. $\{(-2,-2),(2,-2),(0,2)\}$
7. $\{(0,0),(0,6),(3,4)\}$
8. $\{(-2,0),(5,0),(3,-3)\}$

Answer

1. 15 square units
3. 28 square units
5. 4 square units
7. 9 square units

Exercise 6.1.11 Distance Formula

Calculate the distance between the given two points.

1. $(-5, 3)$ and $(-1, 6)$
2. $(6, -2)$ and $(-2, 4)$
3. $(0, 0)$ and $(5, 12)$
4. $(-6, -8)$ and $(0, 0)$
5. $(-7, 8)$ and $(5, -1)$
6. $(-1, -2)$ and $(9, 22)$
7. $(-1, 2)$ and $(-\frac{7}{2}, -4)$
8. $(-\frac{1}{2}, \frac{1}{3})$ and $(\frac{5}{2}, -\frac{11}{3})$
9. $(-\frac{1}{3}, \frac{2}{3})$ and $(1, -\frac{1}{3})$
10. $(\frac{1}{2}, -\frac{3}{4})$ and $(\frac{3}{2}, \frac{1}{4})$
11. $(1, 2)$ and $(4, 3)$
12. $(2, -4)$ and $(-3, -2)$
13. $(-1, 5)$ and $(1, -3)$
14. $(1, -7)$ and $(5, -1)$
15. $(-7, -3)$ and $(-1, 6)$
16. $(0, 1)$ and $(1, 0)$
17. $(-0.2, -0.2)$ and $(1.8, 1.8)$
18. $(1.2, -3.3)$ and $(2.2, -1.7)$

Answer

1. 5 units
3. 13 units
5. 15 units
7. $\frac{13}{2}$ units
9. $\frac{5}{3}$ units
11. $\sqrt{10}$ units
13. $2\sqrt{17}$ units
15. $3\sqrt{13}$ units
17. 2.8 units

Exercise 6.1.12 Distance Formula

For each problem, show that the three points form a right triangle.

1. $(-3, -2), (0, -2)$, and $(0, 4)$
2. $(7, 12), (7, -13)$,and $(-5, -4)$
3. $(-1.4, 0.2), (1, 2)$,and $(1, -3)$
4. $(2, -1), (-1, 2)$, and $(6, 3)$

5. $(-5, 2)$, $(-1, -2)$, and $(-2, 5)$
6. $(1, -2)$, $(2, 3)$, and $(-3, 4)$

Answer

1. Proof
3. Proof
5. Proof

Exercise 6.1.13 Distance Formula

Isosceles triangles have two legs of equal length. For each problem, show that the following points form an isosceles triangle.

1. $(1, 6)$, $(-1, 1)$, and $(3, 1)$
2. $(-6, -2)$, $(-3, -5)$, and $(-9, -5)$
3. $(-3, 0)$, $(0, 3)$, and $(3, 0)$
4. $(0, -1)$, $(0, 1)$, and $(1, 0)$

Answer

1. Proof
3. Proof

Exercise 6.1.14 Distance Formula

Calculate the area and the perimeter of the triangles formed by the following set of vertices.

1. $\{(-4, -5), (-4, 3), (2, 3)\}$
2. $\{(-1, 1), (3, 1), (3, -2)\}$
3. $\{(-3, 1), (-3, 5), (1, 5)\}$
4. $\{(-3, -1), (-3, 7), (1, -1)\}$

Answer

1. Perimeter: 24 units; area: 24 square units
3. Perimeter: $8 + 4\sqrt{2}$ units; area: 8 square units

Exercise 6.1.15 Midpoint Formula

Find the midpoint between the given two points.

1. $(-1, 6)$ and $(-7, -2)$
2. $(8, 0)$ and $(4, -3)$
3. $(-10, 0)$ and $(10, 0)$
4. $(-3, -6)$ and $(-3, 6)$
5. $(-10, 5)$ and $(14, -5)$
6. $(0, 1)$ and $(2, 2)$
7. $(5, -3)$ and $(4, -5)$
8. $(0, 0)$ and $(1, 1)$
9. $(-1, -1)$ and $(4, 4)$
10. $(3, -5)$ and $(3, 5)$
11. $(-\frac{1}{2}, -\frac{1}{3})$ and $(\frac{3}{2}, \frac{7}{3})$
12. $(\frac{3}{4}, -\frac{2}{3})$ and $(\frac{1}{8}, -\frac{1}{2})$
13. $(\frac{5}{3}, \frac{1}{4})$ and $(-\frac{1}{6}, -\frac{3}{2})$

14. $(-\frac{1}{5}, -\frac{5}{2})$ and $(\frac{7}{10}, -\frac{1}{4})$
15. Given the right triangle formed by the vertices $(0, 0)$, $(6, 0)$, and $(6, 8)$, show that the midpoints of the sides form a right triangle.
16. Given the isosceles triangle formed by the vertices $(-10, -12)$, $(0, 12)$, and $(10, -12)$, show that the midpoints of the sides also form an isosceles triangle.
17. Calculate the area of the triangle formed by the vertices $(-4, -3)$, $(-1, 1)$, and $(2, -3)$. (Hint: The vertices form an isosceles triangle.)
18. Calculate the area of the triangle formed by the vertices $(-2, 1)$, $(4, 1)$, and $(1, -5)$.

Answer

1. $(-4, 2)$
3. $(0, 0)$
5. $(2, 0)$
7. $(\frac{9}{2}, -4)$
9. $(\frac{3}{2}, \frac{3}{2})$
11. $(\frac{1}{2}, 1)$
13. $(\frac{3}{4}, -\frac{5}{8})$
15. 12 square units

? Exercise 6.1.16 Discussion Board Topics

1. Research and discuss the life and contributions to mathematics of René Descartes.
2. Research and discuss the history of the right triangle and the Pythagorean theorem.
3. What is a Pythagorean triple? Provide some examples.
4. Explain why you cannot use a ruler to calculate distance on a graph.
5. How do you bisect a line segment with only a compass and a straightedge?

Answer

1. Answers may vary
3. Answers may vary
5. Answers may vary

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6.2: Relations and Functions

Learning Objectives

By the end of this section, you will be able to:

- Find the domain and range of a relation
- Determine if a relation is a function
- Find the value of a function

Before you get started, take this readiness quiz.

1. Evaluate $3x-5$ when $x=-2$.
   If you missed this problem, review [link].
2. Evaluate $2x^2-x-3$ when $x=a$.
   If you missed this problem, review [link].
3. Simplify: $7x-1-4x+5$.
   If you missed this problem, review [link].

Find the Domain and Range of a Relation

As we go about our daily lives, we have many data items or quantities that are paired to our names. Our social security number, student ID number, email address, phone number and our birthday are matched to our name. There is a relationship between our name and each of those items.

When your professor gets her class roster, the names of all the students in the class are listed in one column and then the student ID number is likely to be in the next column. If we think of the correspondence as a set of ordered pairs, where the first element is a student name and the second element is that student's ID number, we call this a **relation**.

$$(\text{Student name},\ \text{Student ID }\#)$$

The set of all the names of the students in the class is called the **domain** of the relation and the set of all student ID numbers paired with these students is the range of the relation.

There are many similar situations where one variable is paired or matched with another. The set of ordered pairs that records this matching is a relation.

Definition: Relation

A **relation** is any set of ordered pairs, (x,y). All the x-values in the ordered pairs together make up the **domain**. All the y-values in the ordered pairs together make up the **range**.

Example 6.2.1

For the relation $(1,1),(2,4),(3,9),(4,16),(5,25)$:

a. Find the domain of the relation.
b. Find the range of the relation.

Answer

$\{(1,1),(2,4),(3,9),(4,16),(5,25)\}$

ⓐ The domain is the set of all x-values of the relation. $\{1,2,3,4,5\}$

ⓑ The range is the set of all y-values of the relation. $\{1,4,9,16,25\}$

Example 6.2.2

For the relation $\{(1,1),(2,8),(3,27),(4,64),(5,125)\}$

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{1,8,27,64,125\}$

Example 6.2.3

For the relation $\{(1,3),(2,6),(3,9),(4,12),(5,15)\}$

a. Find the domain of the relation.
b. Find the range of the relation.

Answer a

$\{1,2,3,4,5\}$

Answer b

$\{3,6,9,12,15\}$

MAPPING

A **mapping** is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

Example 6.2.4

Use the **mapping** of the relation shown to

a. list the ordered pairs of the relation,
b. find the domain of the relation, and
c. find the range of the relation.



Answer

ⓐ The arrow shows the matching of the person to their birthday. We create ordered pairs with the person's name as the *x*-value and their birthday as the *y*-value.

{(Alison, April 25), (Penelope, May 23), (June, August 2), (Gregory, September 15), (Geoffrey, January 12), (Lauren, May 10), (Stephen, July 24), (Alice, February 3), (Liz, August 2), (Danny, July 24)}

ⓑ The domain is the set of all x-values of the relation.

{Alison, Penelope, June, Gregory, Geoffrey, Lauren, Stephen, Alice, Liz, Danny}

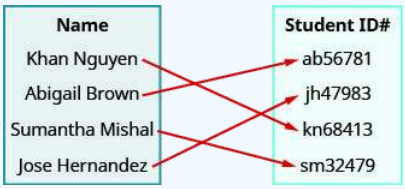
ⓒ The range is the set of all y-values of the relation.

{January 12, February 3, April 25, May 10, May 23, July 24, August 2, September 15}

Example 6.2.5

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Khanh Nguyen, kn68413), (Abigail Brown, ab56781), (Sumantha Mishal, sm32479), (Jose Hern and ez, jh47983)

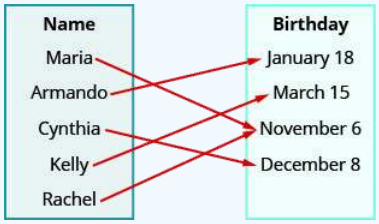
ⓑ {Khanh Nguyen, Abigail Brown, Sumantha Mishal, Jose Hern and ez}

ⓒ {kn68413, ab56781, sm32479, jh47983}

Example 6.2.6

Use the mapping of the relation shown to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ (Maria, November 6), (Arm and o, January 18), (Cynthia, December 8), (Kelly, March 15), (Rachel, November 6)

ⓑ {Maria, Arm and o, Cynthia, Kelly, Rachel}

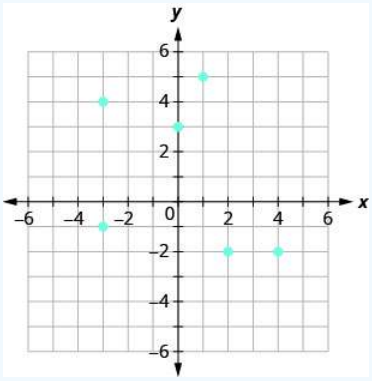
ⓒ{November 6, January 18, December 8, March 15}

A graph is yet another way that a relation can be represented. The set of ordered pairs of all the points plotted is the relation. The set of all x-coordinates is the domain of the relation and the set of all y-coordinates is the range. Generally we write the numbers in ascending order for both the domain and range.

Example 6.2.7

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ The ordered pairs of the relation are:

$$\{(1,5),(-3,-1),(4,-2),(0,3),(2,-2),(-3,4)\}.$$

ⓑ The domain is the set of all x-values of the relation: $\{-3,0,1,2,4\}$

Notice that while -3 repeats, it is only listed once.

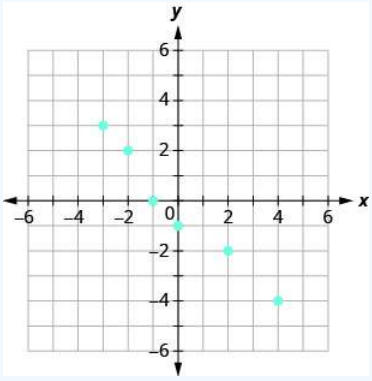
ⓒ The range is the set of all y-values of the relation: $\{-2,-1,3,4,5\}$

Notice that while -2 repeats, it is only listed once.

Example 6.2.8

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

a. $(-3,3),(-2,2),(-1,0),(0,-1),(2,-2),(4,-4)$

b. $\{-3,-2,-1,0,2,4\}$

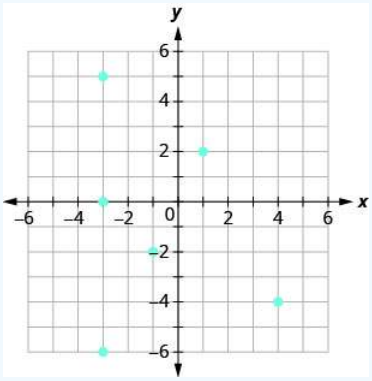
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c. $\{3, 2, 0, -1, -2, -4\}$

Example 6.2.9

Use the graph of the relation to

a. list the ordered pairs of the relation
b. find the domain of the relation
c. find the range of the relation.



Answer

ⓐ $(-3, 0), (-3, 5), (-3, -6), (-1, -2), (1, 2), (4, -4)$
ⓑ $\{-3, -1, 1, 4\}$
ⓒ $\{-6, 0, 5, -2, 2, -4\}$

Determine if a Relation is a Function

A special type of relation, called a **function**, occurs extensively in mathematics. A function is a relation that assigns to each element in its domain exactly one element in the range. For each ordered pair in the relation, each *x*-value is matched with only one *y*-value.

Definition: Function

A **function** is a relation that assigns to each element in its domain exactly one element in the range.

The birthday example from Example helps us understand this definition. Every person has a birthday but no one has two birthdays. It is okay for two people to share a birthday. It is okay that Danny and Stephen share July 24th as their birthday and that June and Liz share August 2nd. Since each person has exactly one birthday, the relation in Example is a function.

The relation shown by the graph in Example includes the ordered pairs $(-3, -1)$ and $(-3, 4)$. Is that okay in a function? No, as this is like one person having two different birthdays.

Example 6.2.10

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$
b. $\{(9, -3), (4, -2), (1, -1), (0, 0), (1, 1), (4, 2), (9, 3)\}$

Answer

ⓐ $\{(-3, 27), (-2, 8), (-1, 1), (0, 0), (1, 1), (2, 8), (3, 27)\}$

(i) Each x-value is matched with only one y-value. So this relation is a function.

(ii) The domain is the set of all x-values in the relation.
The domain is: $\{-3, -2, -1, 0, 1, 2, 3\}$

(iii) The range is the set of all y-values in the relation. Notice we do not list range values twice.
The range is: $\{27, 8, 1, 0\}$

ⓑ $\{(9, -3), (4, -2), (1, -1), (0, 0), (1, 1), (4, 2), (9, 3)\}$

(i) The x-value 9 is matched with two y-values, both 3 and -3. So this relation is not a function.

(ii) The domain is the set of all x-values in the relation. Notice we do not list domain values twice.
The domain is: $\{0, 1, 2, 4, 9\}$

(iii) The range is the set of all y-values in the relation.
The range is: $\{-3, -2, -1, 0, 1, 2, 3\}$

Example 6.2.11

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the function.

a. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4), (3, 6)\}$
b. $\{(8, -4), (4, -2), (2, -1), (0, 0), (2, 1), (4, 2), (8, 4)\}$

Answer

ⓐ Yes; $\{-3, -2, -1, 0, 1, 2, 3\}$
$\{-6, -4, -2, 0, 2, 4, 6\}$
ⓑ No; $\{0, 2, 4, 8\}$
$\{-4, -2, -1, 0, 1, 2, 4\}$

Example 6.2.12

Use the set of ordered pairs to (i) determine whether the relation is a function (ii) find the domain of the relation (iii) find the range of the relation.

a. $\{(27, -3), (8, -2), (1, -1), (0, 0), (1, 1), (8, 2), (27, 3)\}$
b. $\{(7, -3), (-5, -4), (8, -0), (0, 0), (-6, 4), (-2, 2), (-1, 3)\}$

Answer

ⓐ No; $\{0, 1, 8, 27\}$
$\{-3, -2, -1, 0, 2, 2, 3\}$
ⓑ Yes; $\{7, -5, 8, 0, -6, -2, -1\}$
$\{-3, -4, 0, 4, 2, 3\}$

Example 6.2.13

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation
c. find the range of the relation.

NAME	PHONE

Lydia	321-549-3327 427-658-2314
Eugene	321-964-7324
Janet	684-358-7961
Rick	
Marty	684-369-7231 798-367-8541

Answer

ⓐ Both Lydia and Marty have two phone numbers. So each x-value is not matched with only one y-value. So this relation is not a function.

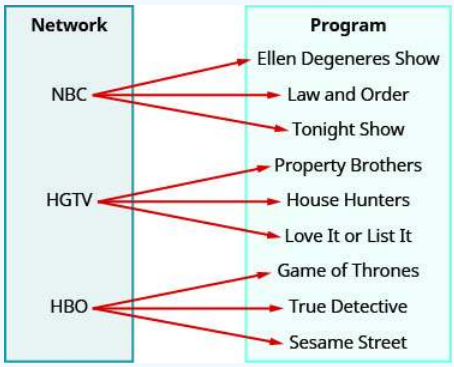
ⓑ The domain is the set of all x-values in the relation. The domain is: {Lydia, Eugene, Janet, Rick, Marty}

ⓒ The range is the set of all y-values in the relation. The range is:

$\{321-549-3327, 427-658-2314, 321-964-7324, 684-358-7961, 684-369-7231, 798-367-8541\}$

Example 6.2.14

Use the mapping to ⓐ determine whether the relation is a function ⓑ find the domain of the relation ⓒ find the range of the relation.



Answer

ⓐ no ⓑ {NBC, HGTV, HBO} ⓒ {Ellen Degeneres Show, Law and Order, Tonight Show, Property Brothers, House Hunters, Love it or List it, Game of Thrones, True Detective, Sesame Street}

Example 6.2.15

Use the mapping to

a. determine whether the relation is a function
b. find the domain of the relation
c. find the range of the relation.

Neal	123-567-4839 work 231-378-5941 cell
Krystal	743-469-9731 cell
Kelvin	567-534-2970 work
George	684-369-7231 cell
Christa	684-369-7231 cell
Mike	639-847-6971 cell

Answer

ⓐ No ⓑ {Neal, Krystal, Kelvin, George, Christa, Mike} ⓒ {123-567-4839 work, 231-378-5941 cell, 743-469-9731 cell, 567-534-2970 work, 684-369-7231 cell, 798-367-8541 cell, 639-847-6971 cell}

In algebra, more often than not, functions will be represented by an equation. It is easiest to see if the equation is a function when it is solved for *y*. If each value of *x* results in only one value of *y*, then the equation defines a function.

? Example 6.2.16

Determine whether each equation is a function.

a. $2x+y=7$
b. $y=x^2+1$
c. $x+y^2=3$

Answer

ⓐ $2x+y=7$

For each value of *x*, we multiply it by -2 and then add 7 to get the *y*-value

	$y=-2x+7$
For example, if $x=3$:	$y=-2\cdot 3+7$
	$y=1$

We have that when $x=3$, then $y=1$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓑ $y=x^2+1$

For each value of *x*, we square it and then add 1 to get the *y*-value.

	$y=x^2+1$
For example, if $x=2$:	$y=2^2+1$
	$y=5$

We have that when $x=2$, then $y=5$. It would work similarly for any value of *x*. Since each value of *x*, corresponds to only one value of *y* the equation defines a function.

ⓒ

	$x+y^2=3$
Isolate the y term.	$y^2=-x+3$
Let's substitute $x=2$.	$y^2=-2+3$
	$y^2=1$
This give us two values for y.	$y=1\ y=-1$

We have shown that when $x=2$, then $y=1$ and $y=-1$. It would work similarly for any value of x. Since each value of x does not corresponds to only one value of y the equation does not define a function.

Example 6.2.17

Determine whether each equation is a function.

a. $4x+y=-3$
b. $x+y^2=1$
c. $y-x^2=2$

Answer

ⓐ yes ⓑ no ⓒ yes

Example 6.2.18

Determine whether each equation is a function.

a. $x+y^2=4$
b. $y=x^2-7$
c. $y=5x-4$

Answer

ⓐ no ⓑ yes ⓒ yes

Find the Value of a Function

It is very convenient to name a function and most often we name it f, g, h, F, G, or H. In any function, for each x-value from the domain we get a corresponding y-value in the range. For the function f, we write this range value y as $f(x)$. This is called **function notation** and is read f of x or the value of f at x. In this case the parentheses does not indicate multiplication.

Definition: Function Notation

For the function $y=f(x)$

f is the name of the function
x is the domain value
$f(x)$ is the range value y corresponding to the value x

We read $f(x)$ as f of x or the value of f at x.

We call x the independent variable as it can be any value in the domain. We call y the dependent variable as its value depends on x.

INDEPENDENT AND DEPENDENT VARIABLES

For the function $y = f(x)$,

x is the independent variable as it can be any value in the domain
y the dependent variable as its value depends on x

Much as when you first encountered the variable *x*, function notation may be rather unsettling. It seems strange because it is new. You will feel more comfortable with the notation as you use it.

Let's look at the equation $y = 4x - 5$. To find the value of *y* when $x = 2$, we know to substitute $x = 2$ into the equation and then simplify.

	$y = 4x - 5$
Let x=2.	$y = 4 \cdot 2 - 5$
	$y = 3$

The value of the function at $x = 2$ is 3.

We do the same thing using function notation, the equation $y = 4x - 5$ can be written as $f(x) = 4x - 5$. To find the value when $x = 2$, we write:

	$f(x) = 4x - 5$
Let x=2.	$f(2) = 4 \cdot 2 - 5$
	$f(2) = 3$

The value of the function at $x = 2$ is 3.

This process of finding the value of $f(x)$ for a given value of *x* is called *evaluating the function.*

Example 6.2.19

For the function $f(x) = 2x^2 + 3x - 1$, evaluate the function.

a. $f(3)$
b. $f(-2)$
c. $f(a)$

Answer

ⓐ

	$f(x) = 2x^2 + 3x - 1$
To evaluate $f(3)$, substitute 3 for *x*.	$f(3) = 2(3)^2 + 3 \cdot 3 - 1$
Simplify.	$f(3) = 2 \cdot 9 + 3 \cdot 3 - 1$
	$f(3) = 18 + 9 - 1$
	$f(3) = 26$

ⓑ

		$f(x) = 2x^2 + 3x - 1$
To evaluate $f(-2)$, substitute -2 for *x*.		$f(-2) = 2(-2)^2 + 3(-2) - 1$

Simplify.		$f(-2) = 2 \cdot 4 + (-6) - 1$
		$f(-2) = 8 + (-6) - 1$
		$f(-2) = 1$

ⓒ

	$f(x) = 2x^2 + 3x - 1$
To evaluate f(a),f(a), substitute a for x.	$f(a) = 2(a)^2 + 3 \cdot a - 1$
Simplify.	$f(a) = 2a^2 + 3a - 1$

Example 6.2.20

For the function $f(x) = 3x^2 - 2x + 1$, evaluate the function.

a. $f(3)$
b. $f(-1)$
c. $f(t)$

Answer

ⓐ $f(3) = 22$ ⓑ $f(-1) = 6$ ⓒ $f(t) = 3t^2 - 2t - 1$

Example 6.2.21

For the function $f(x) = 2x^2 + 4x - 3$, evaluate the function.

a. $f(2)$
b. $f(-3)$
c. $f(h)$

Answer

ⓐ $(2) = 13$ ⓑ $f(-3) = 3$
ⓒ $f(h) = 2h2 + 4h - 3$

In the last example, we found $f(x)$ for a constant value of x. In the next example, we are asked to find $g(x)$ with values of x that are variables. We still follow the same procedure and substitute the variables in for the x.

Example 6.2.22

For the function $g(x) = 3x - 5$, evaluate the function.

a. $g(h^2)$
b. $g(x + 2)$
c. $g(x) + g(2)$

Answer

ⓐ

			$g(x) = 3x - 5$
To evaluate $g(h^2)$, substitute h^2 for x.			$g(h^2) = 3h^2 - 5$

			$g(h^2) = 3h^2 - 5$

ⓑ

	$g(x) = 3x - 5$
To evaluate $g(x+2)$, substitute $x+2$ for x.	$g(x+2) = 3(x+2) - 5$
Simplify.	$g(x+2) = 3x + 6 - 5$
	$g(x+2) = 3x + 1$

ⓒ

		$g(x) = 3x - 5$
To evaluate $g(x) + g(2)$, first find $g(2)$.		$g(2) = 3 \cdot 2 - 5$
		$g(2) = 1$
Now find $g(x) + g(2)$		$g(x) + g(2) = 3x - 5 + 1$ $g(x) \quad g(2)$
Simplify.		$g(x) + g(2) = 3x - 5 + 1$
		$g(x) + g(2) = 3x - 4$

Notice the difference between part ⓑ and ⓒ. We get $g(x+2) = 3x+1$ and $g(x)+g(2) = 3x-4$. So we see that $g(x+2) \neq g(x)+g(2)$.

Example 6.2.23

For the function $g(x) = 4x - 7$, evaluate the function.

a. $g(m^2)$
b. $g(x-3)$
c. $g(x) - g(3)$

Answer

ⓐ $4m^2 - 7$ ⓑ $4x - 19$
ⓒ $x - 12$

Example 6.2.24

For the function $h(x) = 2x + 1$, evaluate the function.

a. $h(k^2)$
b. $h(x+1)$
c. $h(x) + h(1)$

Answer

ⓐ $2k^2 + 1$ ⓑ $2x + 3$
ⓒ $2x + 4$

Many everyday situations can be modeled using functions.

Example 6.2.25

The number of unread emails in Sylvia's account is 75. This number grows by 10 unread emails a day. The function $N(t) = 75 + 10t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(5)$. Explain what this result means.

Answer

ⓐ The number of unread emails is a function of the number of days. The number of unread emails, *N*, depends on the number of days, *t*. Therefore, the variable *N*, is the dependent variable and the variable tt is the independent variable.

ⓑ Find $N(5)$. Explain what this result means.

	$N(t) = 75 + 10t$
Substitute in t=5.t=5.	$N(5) = 75 + 10 \cdot 5$
Simplify.	$N(5) = 75 + 50$
	$N(5) = 125$

Since 5 is the number of days, $N(5)$, is the number of unread emails after 5 days. After 5 days, there are 125 unread emails in the account.

Example 6.2.26

The number of unread emails in Bryan's account is 100. This number grows by 15 unread emails a day. The function $N(t) = 100 + 15t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(7)]$. Explain what this result means.

Answer

ⓐ *t* IND; *N* DEP ⓑ 205; the number of unread emails in Bryan's account on the seventh day.

Example 6.2.27

The number of unread emails in Anthony's account is 110. This number grows by 25 unread emails a day. The function $N(t) = 110 + 25t$ represents the relation between the number of emails, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.
b. Find $N(14)$. Explain what this result means.

Answer

ⓐ *t* IND; *N* DEP ⓑ 460; the number of unread emails in Anthony's account on the fourteenth day

Access this online resource for additional instruction and practice with relations and functions.

- Introduction to Functions

Key Concepts

- **Function Notation:** For the function $y = f(x)$
  - f is the name of the function
  - x is the domain value

- $f(x)$ is the range value y corresponding to the value x
  We read $f(x)$ as f of x or the value of f at x.

- **Independent and Dependent Variables:** For the function $y = f(x)$,
  - x is the independent variable as it can be any value in the domain
  - y is the dependent variable as its value depends on x

Glossary

domain of a relation

The domain of a relation is all the x-values in the ordered pairs of the relation.

function

A function is a relation that assigns to each element in its domain exactly one element in the range.

mapping

A mapping is sometimes used to show a relation. The arrows show the pairing of the elements of the domain with the elements of the range.

range of a relation

The range of a relation is all the y-values in the ordered pairs of the relation.

relation

A relation is any set of ordered pairs,(x,y).(x,y). All the x-values in the ordered pairs together make up the domain. All the y-values in the ordered pairs together make up the range.

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6.2E: Exercises

Practice Makes Perfect

Find the Domain and Range of a Relation

In the following exercises, for each relation a. find the domain of the relation b. find the range of the relation.

1. $\{(1,4),(2,8),(3,12),(4,16),(5,20)\}$

Answer

a. $\{1,2,3,4,5\}$b. $\{4,8,12,16,20\}$

2. $\{(1,-2),(2,-4),(3,-6),(4,-8),(5,-10)\}$

3. $\{(1,7),(5,3),(7,9),(-2,-3),(-2,8)\}$

Answer

a. $\{1,5,7,-2\}$b. $\{7,3,9,-3,8\}$

4. $\{(11,3),(-2,-7),(4,-8),(4,17),(-6,9)\}$

In the following exercises, use the mapping of the relation to a. list the ordered pairs of the relation, b. find the domain of the relation, and c. find the range of the relation.

5.



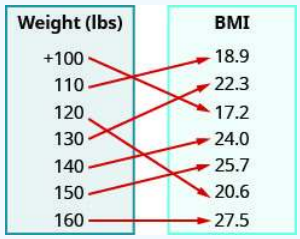
Answer

a. (Rebecca, January 18), (Jennifer, April 1), (John, January 18), (Hector, June 23), (Luis, February 15), (Ebony, April 7), (Raphael, November 6), (Meredith, August 19), (Karen, August 19), (Joseph, July 30)
b. {Rebecca, Jennifer, John, Hector, Luis, Ebony, Raphael, Meredith, Karen, Joseph}
c. {January 18, April 1, June 23, February 15, April 7, November 6, August 19, July 30}

6.



7. For a woman of height $5'4''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$is considered healthy.



Answer

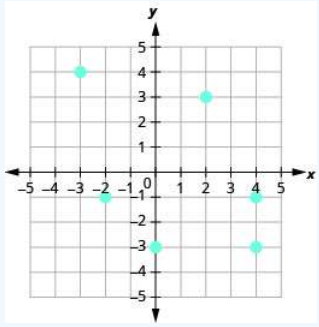
a. $(+100, 17.2), (110, 18.9), (120, 20.6), (130, 22.3), (140, 24.0), (150, 25.7), (160, 27.5)$ b. $\{+100, 110, 120, 130, 140, 150, 160\}$c. $\{17.2, 18.9, 20.6, 22.3, 24.0, 25.7, 27.5\}$

8. For a man of height $5'11''$ the mapping below shows the corresponding Body Mass Index (BMI). The body mass index is a measurement of body fat based on height and weight. A BMI of $18.5–24.9$is considered healthy.



In the following exercises, use the graph of the relation to a. list the ordered pairs of the relation b. find the domain of the relation c. find the range of the relation.

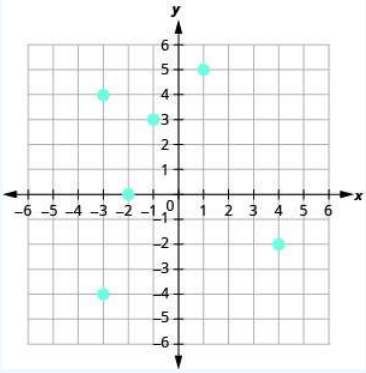
9.



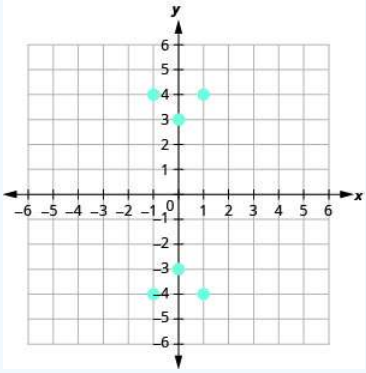
Answer

a. $(2, 3), (4, -3), (-2, -1), (-3, 4), (4, -1), (0, -3)$b. $\{-3, -2, 0, 2, 4\}$
c. $\{-3, -1, 3, 4\}$

10.



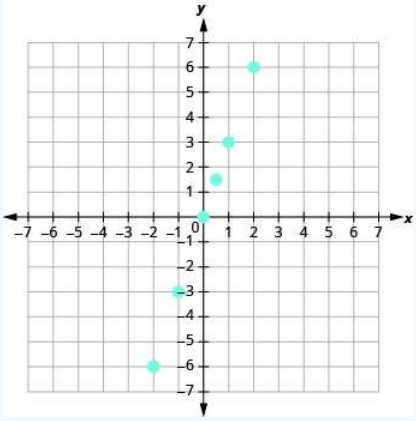
11.



Answer

a. $(1,4),(1,-4),(-1,4),(-1,-4),(0,3),(0,-3)$b. $\{-1,0,1\}$c. $\{-4,-3,3,4\}$

12.



Determine if a Relation is a Function

In the following exercises, use the set of ordered pairs to a. determine whether the relation is a function, b. find the domain of the relation, and c. find the range of the relation.

13. $\{(-3,9),(-2,4),(-1,1),(0,0),(1,1),(2,4),(3,9)\}$

Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{9,4,1,0\}$

14. $\{(9,-3),(4,-2),(1,-1),(0,0),(1,1),(4,2),(9,3)\}$

15. $\{(-3,27),(-2,8),(-1,1),(0,0),(1,1),(2,8),(3,27)\}$

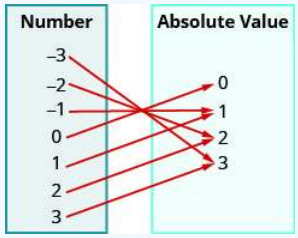
Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{0,1,8,27\}$

16. $\{(-3,-27),(-2,-8),(-1,-1),(0,0),(1,1),(2,8),(3,27)\}$

In the following exercises, use the mapping to a. determine whether the relation is a function, b. find the domain of the function, and c. find the range of the function.

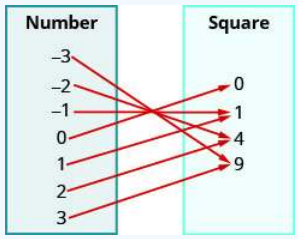
17.



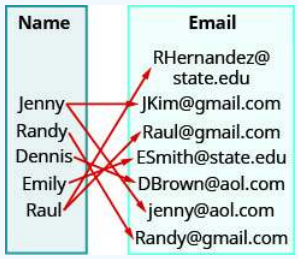
Answer

a. yes b. $\{-3,-2,-1,0,1,2,3\}$c. $\{0,1,2,3\}$

18.



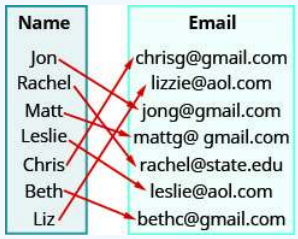
19.



Answer

a. no b. {Jenny, R and y, Dennis, Emily, Raul} c. {RHern and ez@state.edu, JKim@gmail.com, Raul@gmail.com, ESmith@state.edu, DBroen@aol.com, jenny@aol.cvom, R and y@gmail.com}

20.



In the following exercises, determine whether each equation is a function.

21. a. $2x+y=-3$
b. $y=x^2$
c. $x+y^2=-5$

Answer

a. yes b. yes c. no

22. a. $y=3x-5$
b. $y=x^3$
c. $2x+y^2=4$

23. a. $y-3x^3=2$
b. $x+y^2=3$
c. $3x-2y=6$

Answer

a. yes b. no c. yes

24. a. $2x-4y=8$
b. $-4=x^2-y$
c. $y^2=-x+5$

Find the Value of a Function

In the following exercises, evaluate the function: a. $f(2)$ b. $f(-1)$ c. $f(a)$.

25. $f(x)=5x-3$

Answer

a. $f(2)=7$ b. $f(-1)=-8$ c. $f(a)=5a-3$

26. $f(x)=3x+4$

27. $f(x)=-4x+2$

Answer

a. $f(2)=-6$ b. $f(-1)=6$ c. $f(a)=-4a+2$

28. $f(x)=-6x-3$

29. $f(x)=x^2-x+3$

Answer

a. $f(2)=5$ b. $f(-1)=5$
c. $f(a)=a^2-a+3$

30. $f(x)=x^2+x-2$

31. $f(x)=2x^2-x+3$

Answer

a. $f(2)=9$ b. $f(-1)=6$
c. $f(a)=2a^2-a+3$

32. $f(x)=3x^2+x-2$

In the following exercises, evaluate the function: a. $g(h^2)$ b. $g(x+2)$ c. $g(x)+g(2)$.

33. $g(x)=2x+1$

Answer

a. $g(h^2)=2h^2+1$
b. $g(x+2)=4x+5$
c. $g(x)+g(2)=2x+6$

34. $g(x)=5x-8$

35. $g(x)=-3x-2$

Answer

a. $g(h^2)=-3h^2-2$
b. $g(x+2)=-3x-8$
c. $g(x)+g(2)=-3x-10$

36. $g(x)=-8x+2$

37. $g(x)=3-x$

Answer

a. $g(h^2)=3-h^2$
b. $g(x+2)=1-x$
c. $g(x)+g(2)=4-x$

38. $g(x)=7-5x$

In the following exercises, evaluate the function.

39. $f(x)=3x^2-5x;\ f(2)$

Answer

2

40. $g(x) = 4x^2 - 3x$; $g(3)$

41. $F(x) = 2x^2 - 3x + 1$; $F(-1)$

Answer

6

42. $G(x) = 3x^2 - 5x + 2$; $G(-2)$

43. $h(t) = 2|t - 5| + 4$; $f(-4)$

Answer

22

44. $h(y) = 3|y - 1| - 3$; $h(-4)$

45. $f(x) = x + 2x - 1$; $f(2)$

Answer

4

46. $g(x) = x - 2x + 2$; $g(4)$

In the following exercises, solve.

47. The number of unwatched shows in Sylvia's DVR is 85. This number grows by 20 unwatched shows per week. The function $N(t) = 85 + 20t$ represents the relation between the number of unwatched shows, *N*, and the time, *t*, measured in weeks.

a. Determine the independent and dependent variable.

b. Find $N(4)$. Explain what this result means

Answer

a. *t* IND; *N* DEP
b. $N(4) = 165$ the number of unwatched shows in Sylvia's DVR at the fourth week.

48. Every day a new puzzle is downloaded into Ken's account. Right now he has 43 puzzles in his account. The function $N(t) = 43 + t$ represents the relation between the number of puzzles, *N*, and the time, *t*, measured in days.

a. Determine the independent and dependent variable.

b. Find $N(30)$. Explain what this result means.

49. The daily cost to the printing company to print a book is modeled by the function $C(x) = 3.25x + 1500$ where *C* is the total daily cost and *x* is the number of books printed.

a. Determine the independent and dependent variable.

b. Find $N(0)$. Explain what this result means.

c. Find $N(1000)$. Explain what this result means.

Answer

a. x IND; C DEP
b. $N(0) = 1500$ the daily cost if no books are printed
c. $N(1000) = 4750$the daily cost of printing 1000 books

50. The daily cost to the manufacturing company is modeled by the function $C(x) = 7.25x + 2500$ where $C(x)$ is the total daily cost and x is the number of items manufactured.

a. Determine the independent and dependent variable.

b. Find $C(0)$. Explain what this result means.

c. Find $C(1000)$. Explain what this result means.

Writing Exercises

51. In your own words, explain the difference between a relation and a function.

52. In your own words, explain what is meant by domain and range.

53. Is every relation a function? Is every function a relation?

54. How do you find the value of a function?

Self Check

a. After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the domain and range of a relation.			
determine if a relation is a function.			
find the value of a function.			

b. After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

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6.3: The Graph of a Function

Rene Descartes (1596-1650) was a French philosopher and mathematician who is well known for the famous phrase “cogito ergo sum” (I think, therefore I am), which appears in his Discours de la methode pour bien conduire sa raison, et chercher la verite dans les sciences (Discourse on the Method of Rightly Conducting the Reason, and Seeking Truth in the Sciences). In that same treatise, Descartes introduces his coordinate system, a method for representing points in the plane via pairs of real numbers. Indeed, the Cartesian plane of modern day is so named in honor of Rene Descartes, who some call the “Father of Modern Mathematics.”

Descartes’ work, which forever linked geometry and algebra, was continued in an appendix to Discourse on Method, entitled La Geometrie, which some consider the beginning of modern mathematics. Certainly both Newton and Leibniz, in developing the Calculus, built upon the foundation provided in this work by Descartes.

A Cartesian Coordinate System consists of a pair of axes, usually drawn at right angles to one another in the plane, one horizontal (labeled x) and one vertical (labeled y), as shown in the Figure 6.3.1. The quadrants are numbered I, II, III, and IV, in counterclockwise order, and samples of ordered pairs of the form (x, y) are shown in each quadrant of the Cartesian coordinate system in Figure 6.3.1.

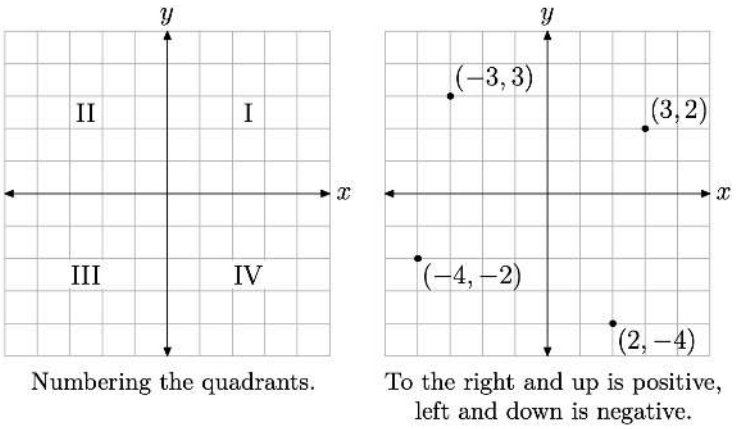


Figure 6.3.1. The Cartesian coordinate system.

Now, suppose that we have a relation

$$R = \{(1, 2), (3, 1), (3, 4), (4, 3)\}$$

Recall that relation is the name given to a collection of ordered pairs. In Figure 6.3.2(b) we’ve plotted each of the ordered pairs in the relation R. This is called the graph of the relation R.

Definition

The graph of a relation is the collection of all ordered pairs of the relation. These are usually represented as points in a Cartesian coordinate system.

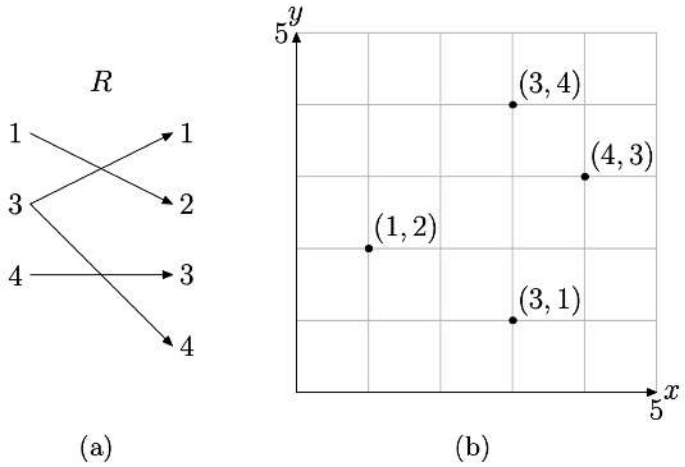


Figure 6.3.2 A mapping diagram and its graph.

In Figure 6.3.2(a), we've created a mapping diagram of the ordered pairs. Note that the domain object 3 is paired with two range elements, namely 1 and 4. Hence the relation R is not a function. It is interesting to note that there are two points in the graph of R in Figure 6.3.2(b) that have the same first coordinate, namely (3, 1) and (3, 4). This is a signal that the graph of the relation R is not a function. In the next section we will discuss the Vertical Line Test, which will use this dual use of the first coordinate to determine when a relation is a not a function.

Creating the Graph of a Function

Some texts will speak of the graph of an equation, such as "Draw the graph of the equation $y = x^2$." This instruction raises a number of difficulties.

- First, the instruction provides no direction to the reader; that is, what does the instruction mean? It's not very helpful.
- Secondly, the instruction is incorrect. You don't draw the graphs of equations. Rather, you draw the graphs of relations and/or functions. A graph is just another way of representing a function, a relation that pairs each element in its domain with exactly one element in its range.

So, what is the proper instruction? First, we will provide the formal definition of the graph of a function, then we will break it down by means of examples.

Definition

The graph of a function is the collection of all ordered pairs of the function. These are usually represented as points in a Cartesian coordinate system.

As an example, consider the function

$$f = \{(1, 2), (2, 4), (3, 1), (4, 3)\}$$

Readers will note that each object in the domain is paired with one and only one object in the range, as seen in the mapping diagram of Figure 6.3.3(a).

Thus, we have two representations of the function f, the collection of ordered pairs (3), and the mapping diagram of in Figure 6.3.3(a). A third representation of the function f is the graph of the ordered pairs of the function, shown in the Cartesian plane in Figure 6.3.3(b).

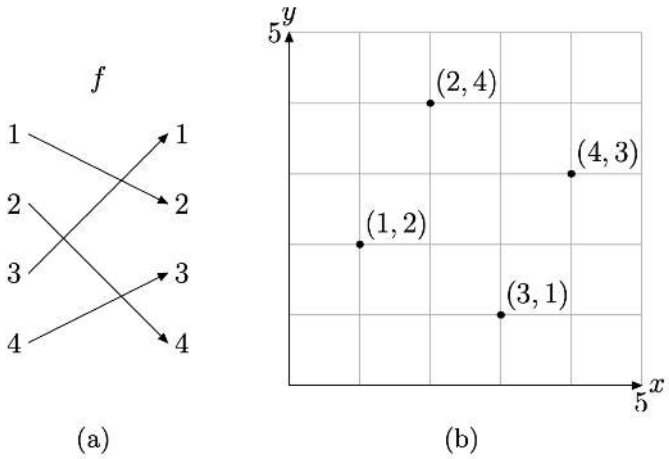


Figure 6.3.3 A mapping diagram and its graph.

When the function is represented by an equation or formula, then we adjust our definition of its graph somewhat.

Definition

The graph of f is the set of all ordered pairs $(x, f(x))$ so that x is in the domain of f. In symbols,

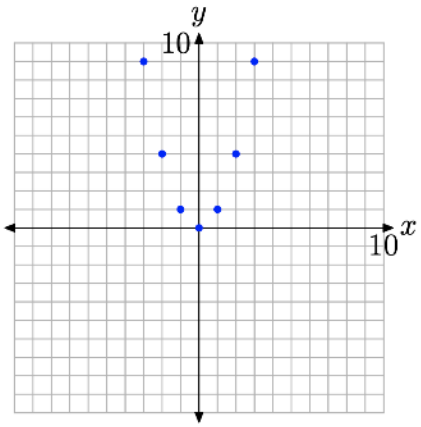
$$\text{Graph of } f = \{(x, f(x)) : x \text{ is in the domain of } f.\}$$

This last definition is most easily explained by example. So, let's define a function f that maps any real number x to the real number x^2; that is, let $f(x) = x^2$. Now, according to Definition, the graph of f is the set of all points $(x, f(x))$, such that x is in the domain of f.

The way is now clear. We begin by creating a table of points $(x, f(x))$, where x is in the domain of the function f defined by $f(x) = x^2$. The choice of x is both subjective and experimental, so we begin by choosing integer values of x between −3 and 3. We then evaluate the function at each of these x-values (e.g., $f(-3) = (-3)^2 = 9$). The results are shown in the table in Figure 6.3.4(a). We then plot the points in our table in the Cartesian plane as shown in Figure 6.3.4(b).

x	$f(x) = x^2$	$(x, f(x))$
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)

(a)



(b)

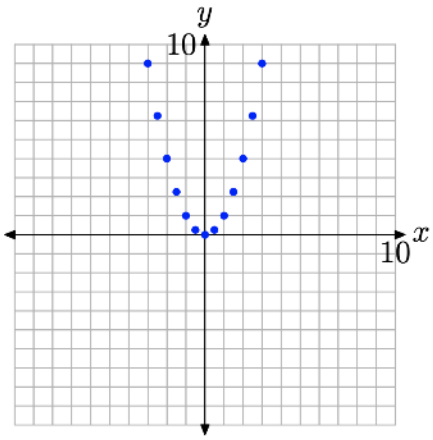
Figure 6.3.4. Plotting pairs satisfying the functional relationship defined by the equation $f(x) = x^2$.

Although this is a good start, the graph in Figure 6.3.4(b) is far from complete. Definition requires that we plot the ordered pairs $(x, f(x))$ for every value of x that is in the domain of f. We've only plotted seven such points, so we're not done. Let's add more points to the graph of f. We'll evaluate the function at each of the x-values shown in the table in Figure 6.3.5(a), then plot the

additional pairs $(x, f(x))$ from the table in the Cartesian plane, as shown in Figure 6.3.5(b).

x	$f(x) = x^2$	$(x, f(x))$
$-5/2$	$25/4$	$(-5/2, 25/4)$
$-3/2$	$9/4$	$(-3/2, 9/4)$
$-1/2$	$1/4$	$(-1/2, 1/4)$
$1/2$	$1/4$	$(1/2, 1/4)$
$3/2$	$9/4$	$(3/2, 9/4)$
$5/2$	$25/4$	$(5/2, 25/4)$

(a)



(b)

Figure 6.3.5. Plotting additional pairs $(x, f(x))$ defined by the equation $f(x) = x^2$.

We're still not finished, because we've only plotted 13 pairs $(x, f(x))$, such that $f(x) = x^2$. Definition 4 requires that we plot the ordered pairs $(x, f(x))$ for every value of x in the domain of f.

However, a pattern is certainly establishing itself, as seen in Figure 6.3.5(b). At some point, we need to "make a leap of faith," and plot all ordered pairs $(x, f(x))$, such that x is in the domain of f. This is done in Figure 6.3.6.

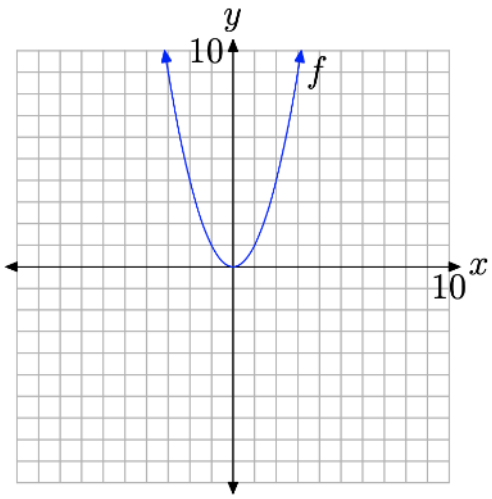


Figure 6.3.6. Plotting all pairs $(x, f(x))$ so that x is in the domain of f.

There are several important points we need to make about the final result in Figure 6.3.6.

- When we draw a smooth curve, such as that shown in Figure 6.3.6, it is important to understand that this is a simply a shortcut for plotting all pairs (x, f(x)), where $f(x) = x^2$ and x is in the domain of f.
- It is important to understand that we are NOT "connecting the dots," neither with a ruler nor with curved segments. Rather, the curve in Figure 6.3.6 is the result of plotting all of the individual pairs $(x, f(x))$.
- The "arrows" at each end of the curve have an important meaning. Much as the ellipsis at the end of the progression $2, 4, 6, \ldots$ mean "et-cetera," the arrows at each end of the curve have a similar meaning. The arrow at the end of the left-half of the curve indicates that the graph continues opening upward and to the left, while the arrow at the end of the right-half of the curve indicates that the graph continues opening upward and to the right.

Creating Graphs by Hand

We're going to look at several basic graphs, which we'll create by employing the strategy used to create the graph of $f(x) = x^2$. First, let's summarize that process.

Summary

If a function is defined by an equation, you can create the graph of the function as follows.

1. Select several values of x in the domain of the function f.
2. Use the selected values of x to create a table of pairs (x, f(x)) that satisfy the equation that defines the function f.
3. Create a Cartesian coordinate system on a sheet of graph paper. Label and scale each axis, then plot the pairs (x, f(x)) from your table on your coordinate system.
4. If the plotted pairs (x, f(x)) provide enough of a pattern for you to intuit the shape of the graph of f, make the "leap of faith" and plot all pairs that satisfy the equation defining f by drawing a smooth curve on your coordinate system. Of course, this curve should contain all previously plotted pairs.
5. If your plotted pairs do not provide enough of a pattern to determine the final shape of the graph of f, then add more pairs to your table and plot them on your Cartesian coordinate system. Continue in this manner until you are confident in the shape of the graph of f.

Let's look at an example.

Example 6.3.1

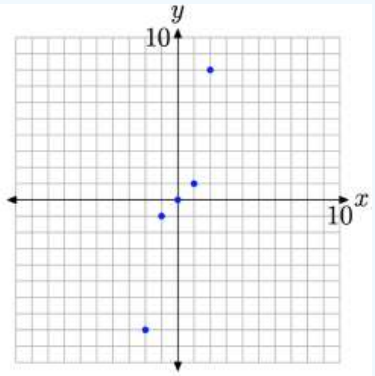
Sketch the graph of the function defined by the equation $f(x) = x^3$.

Solution

We'll start with x-values $-2, -1, 0, 1$, and 2, then use the equation $f(x) = x^3$ to determine pairs (x, f(x)) (e.g., $f(-2) = (-2)^3 = -8$). These are listed in the table in Figure 6.3.7(a). We then plot the points from the table on a Cartesian coordinate system, as shown in Figure 6.3.7(b).

x	$f(x) = x^3$	$(x, f(x))$
-2	-8	$(-2, -8)$
-1	-1	$(-1, -1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	8	$(2, 8)$

(a)



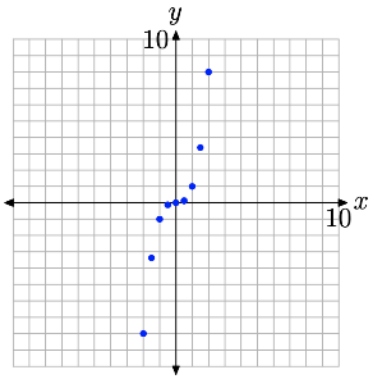
(b)

Figure 6.3.7. Plotting pairs (x, f(x)) defined by the equation $f(x) = x^3$.

We're a bit unsure of the shape of the graph of f, so we'll add a few more pairs to our table and plot them. This is shown in Figures 6.3.8(a) and (b).

x	$f(x) = x^3$	$(x, f(x))$
$-3/2$	$-27/8$	$(-3/2, -27/8)$
$-1/2$	$-1/8$	$(-1/2, -1/8)$
$1/2$	$1/8$	$(1/2, 1/8)$
$3/2$	$27/8$	$(3/2, 27/8)$

(a)



(b)

Figure 6.3.8. Plotting additional pairs (x, f(x)) defined by the equation $f(x) = x^3$.

The additional pairs fill in the shape of f in Figure 6.3.8(b) a bit better than those in Figure 6.3.7(b), enough so that we're confident enough to make a "leap of faith" and draw the final shape of the graph of $f(x) = x^3$ in Figure 6.3.9.

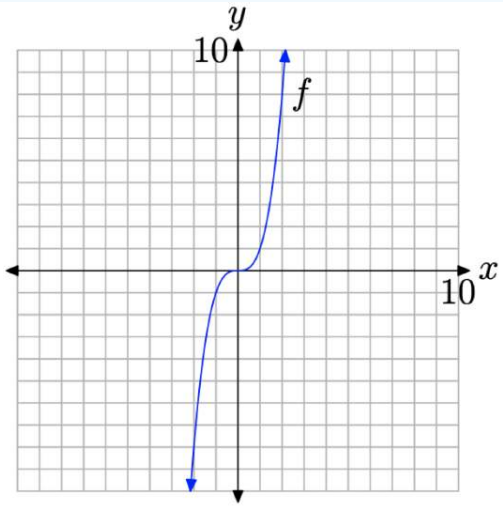


Figure 6.3.9. The final graph of $f(x) = x^3$.

Let's look at another example.

Example 6.3.2

Sketch the graph of $f(x) = \sqrt{x}$

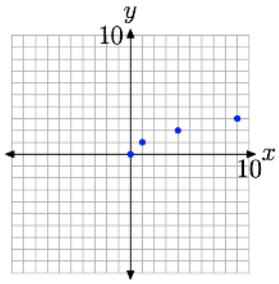
Solution

Again, we'll start by selecting several values of x in the domain of f. In this case, $f(x) = \sqrt{x}$, and it's not possible to take the square root of a negative number. Also, if we're creating a table of pairs by hand, it's good strategy to select known squares. Thus, we'll use x = 0, 1, 4, and 9 for starters.

https://math.libretexts.org/@go/page/116792

x	$f(x)=\sqrt{x}$	$(x, f(x))$
0	0	(0, 0)
1	1	(1, 1)
4	2	(4, 2)
9	3	(9, 3)

(a)



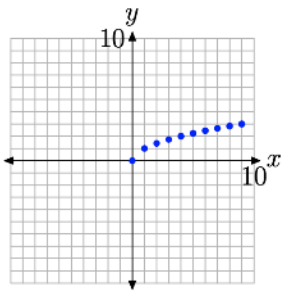
(b)

Figure 6.3.10 Plotting pairs (x, f(x)) defined by the equation $f(x)=\sqrt{x}$.

Some might be ready to make a “leap of faith” based on these initial results. Others might want to use a calculator to compute decimal approximations for additional square roots. The resulting pairs are shown in the table in Figure 6.3.11(a) and the additional pairs are plotted in Figure 6.3.11(b).

x	$f(x)=\sqrt{x}$	$(x, f(x))$
2	1.4	(2, 1.4)
3	1.7	(3, 1.7)
5	2.2	(5, 2.2)
6	2.4	(6, 2.4)
7	2.6	(7, 2.6)
8	2.8	(8, 2.8)

(a)



(b)

Figure 6.3.11 Plotting additional pairs (x, f(x)) defined by the equation $f(x)=\sqrt{x}$

The pattern in Figure 6.3.11(b) is clear enough to make a “leap of faith” and complete the graph as shown in Figure 6.3.12

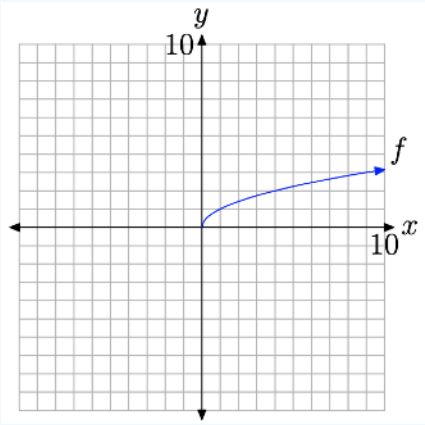


Figure 6.3.12 The graph of f defined by the equation $f(x)=\sqrt{x}$.

Using the Table Feature of the Graphing Calculator

The TABLE feature on your graphing calculator can be of immense help when creating tables of points that satisfy the equation defining the function f. Let’s look at an example

Example 6.3.3

Sketch the graph of $f(x)=|x|$

Solution

Enter the function $f(x) = |x|$ in the Y = menu as follows.

1. Press the Y= button on your calculator. This will open the Y= menu as shown in Figure 6.3.13(a). Use the arrow keys and the CLEAR button on your calculator to delete any existing functions.
2. Press the MATH button to open the menu shown in Figure 6.3.13(b).
3. Press the right-arrow on your calculator to select the NUM submenu as shown in Figure 6.3.13(c).
4. Select 1:abs(, then enter X and close the parentheses, as shown in Figure 6.3.13(d).

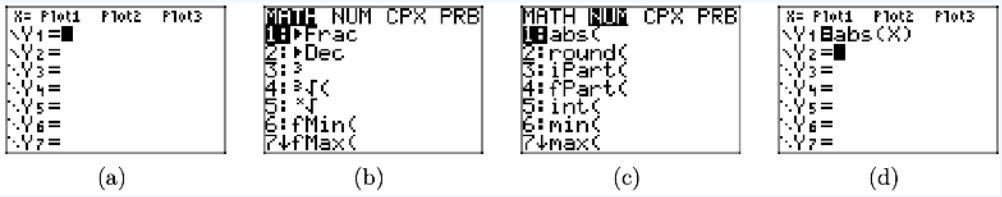


Figure 6.3.13. Entering $f(x) = |x|$ in the Y = menu.

We will now use the TABLE feature of the graphing calculator to help create a table of pairs (x, f(x)) satisfying the equation $f(x) = |x|$. Proceed as follows.

1. Select 2nd TBLSET (i.e., push the 2nd button followed by TBLSET), which is located over the WINDOW button. Enter TblStart=-3, ΔTbl = 1, and set the independent and dependent variables to Auto (this is done by highlighting Auto and pressing the Enter button), as shown in Figure 6.3.14(a).
2. Press 2nd TABLE, which is located above the GRAPH button, to produce the table of pairs (x, f(x)) shown in Figure 6.3.14(b).

We've plotted the pairs directly from the calculator onto a Cartesian coordinate system on graph paper in Figure 6.3.14(c).

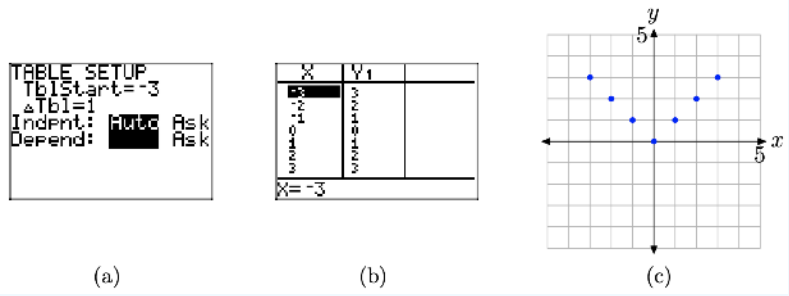


Figure 6.3.14. Creating a table with the TABLE feature of the graphing calculator.

Based on what we see in Figure 6.3.14(c), we're ready to make a "leap of faith" and draw the graph of f shown in Figure 6.3.15.

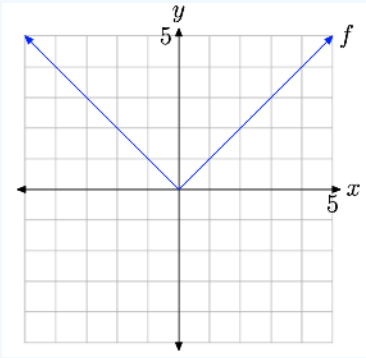


Figure 6.3.15. The graph of f defined by $f(x) = |x|$.

Alternatively, or as a check, we can have the graphing calculator draw the graph for us. Push the ZOOM button, then select 6:ZStandard (shown in Figure 6.3.16(a)) to produce the graph shown in Figure 6.3.16(b).

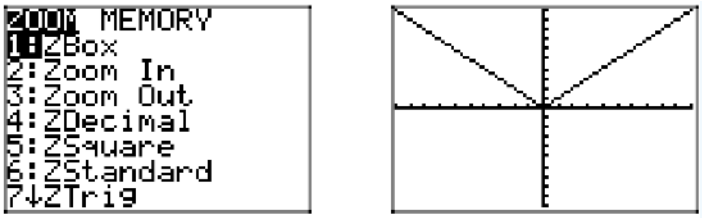


Figure 6.3.16. Creating the graph of $f(x) = |x|$ with the graphing calculator.

Adjusting the Viewing Window

In Example 6.3.3, we used the graphing calculator to draw the graph of the function defined by the equation $f(x) = |x|$. For the functions we've encountered thus far, drawing their graphs using the graphing calculator is pretty trivial. Simply enter the equation in the Y= menu, then press the ZOOM button and select 6:ZStandard. However, if the graph of a function doesn't fit (or even appear) in the "standard" viewing window, it can be quite challenging to find optimal view settings so that the important features of the graph are visible.

Indeed, as one might not even know what "important" features to look for, setting the viewing window is usually highly subjective and experimental by nature. Let's look at some examples.

Example 6.3.4

Use a graphing calculator to sketch the graph of $f(x) = 56 - x - x^2$. Experiment with the WINDOW settings until you feel you have a viewing window that exhibits the important features of the graph.

Solution

First, start by entering the function in the Y= menu, as shown in Figure 6.3.17(a). The caret ^ on the keyboard is used for exponents. Press the ZOOM button and select 6:ZStandard to produce the graph shown in Figure 6.3.17(b).



Figure 6.3.17. The graph of $f(x) = 56 - x - x^2$ in the "standard" viewing window.

As the graph draws, observe that the graph rises from the bottom of the screen, leaves the top of the screen, then returns, falling from the top of the screen and leaving again at the bottom of the screen. This would indicate that there must be some sort of "turning point" that is not visible at the top of the screen.

Press the WINDOW button to reveal the "standard viewing window" settings shown in Figure 6.3.18(a). The following legend explains each of the WINDOW parameters in Figure 6.3.18(a).

\(\begin{array} { lll} Xmin & =& x-value of left edge of viewing window \\

Xmax &=& x-value of right edge of viewing window \\

Xscl &=& x-axis tick increment \\

Y min &=& y-value of bottom edge of viewing window \\

Y max &=& y-value of top edge of viewing window \\

Y scl &=& y-axis tick increment \end{array}\)

It is easy to evaluate the function $f(x) = 56 - x - x^2$ at $x = 0$. Indeed, $f(0) = 56 - 0 - 0^2 = 56$. This indicates that the graph of f must pass through the point (0, 56). This gives us a clue at how we should set the upper bound on our viewing

window. Set Ymax = 60, as shown in Figure 6.3.18(b), then press the GRAPH button to produce the graph and viewing window shown in Figure 6.3.18(c).

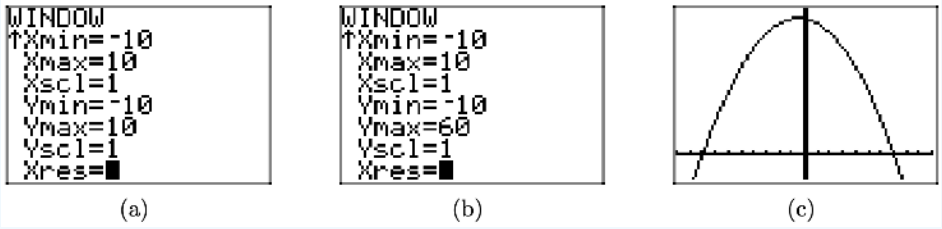


Figure 6.3.18. Changing the viewing window.

Although the viewing window in Figure 6.3.18(c) shows the "turning point" of the graph of f, we will make some additional changes to the window settings, as shown in Figure 6.3.19(a). First, we "widen" the viewing window a bit, setting Xmin = -15 and Xmax = 15, then we set tick marks on the x-axis every 5 units with Xscl = 5. Next, to create a little room at the top of the screen, we set Ymax = 100, then we "balance" this setting with Ymin = -100. Finally, we set tick marks on the y-axis every 10 units with Yscl = 10.

Push the GRAPH button to view the effects of these changes to the WINDOW parameters in Figure 6.3.19(b). Note that these settings are highly subjective, and what one reader might find quite pleasing will not necessarily find favor with other readers.

However, what is important is the fact that we've captured the "important features" of the graph of $f(x) = 56 - x - x^2$. Note that this is a very controversial statement. If one is just beginning to learn about the graphs of functions, how is one to determine what are the "important features" of the graph? Unfortunately, the answer to this question is, "through experience." Undoubtedly, this is a very frustrating phrase for readers to hear, but at least it's truthful. The more graphs that you draw, the more you will learn how to look for "turning points," "end-behavior," "x- and y-intercepts," and the like.

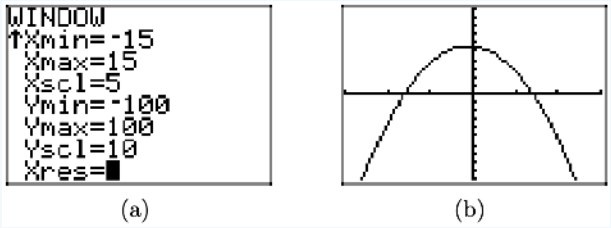


Figure 6.3.19. Improving the WINDOW settings.

For example, how do we know that the WINDOW settings in Figure 6.3.19(a) determine a viewing window (Figure 6.3.19(b)) that reveals all "important features" of the graph? The answer at this point is, "we don't, not without further experiment." For example, the careful reader might want to try the window settings Xmin=-50, Xmax=50, Xscl=10, Ymin=-500, Ymax=500, and Yscl=100 to see if any unexpected behavior crops up.

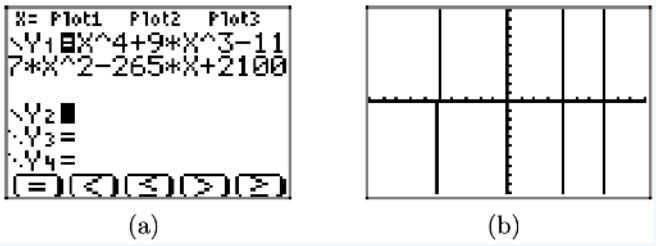
Let's look at one last example.

Example 6.3.5

Sketch the graph of the function f defined by the equation $f(x) = x^4 + 9x^3 - 117x^2 - 265x + 2100$.

Solution

Load the function into the Y= menu (shown in Figure 6.3.20(a)) and select 6:ZStandard to produce the graph shown in Figure 6.3.20(b).

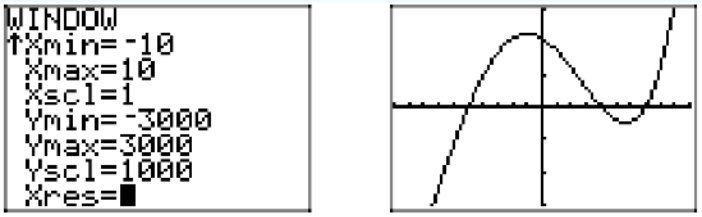


(a) (b)

Figure 6.3.20 Sketching the graph of $f(x) = x^4 + 9x^3 - 117x^2 - 265x + 2100$.

As the graph draws, observe that it rises form the bottom of the viewing window, leaves the top of the viewing window, then returns to fall off the bottom of the viewing window, then returns again and rises off the top of the viewing window.

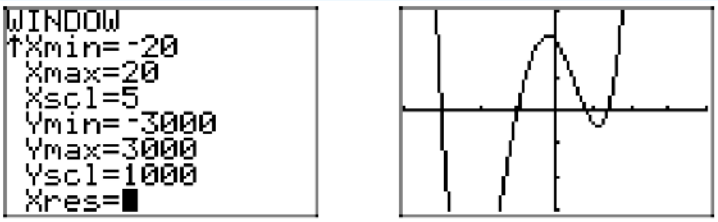
We notice that f(0) = 2100, so we'll need to set the top of the viewing window to that value or higher. With this thought in mind, we'll set Ymax=3000, then set Ymin=-3000 for balance, then to avoid a million little tick marks, we'll set Yscl=1000, all shown in Figure 6.3.21(a). Pressing the GRAPH button then produces the image shown in Figure 6.3.21(b).



(a) (b)

Figure 6.3.21 Adjusting the viewing window.

Does it appear that we have all of the "important features" of this graph displayed in our viewing window? Note that we did not experiment very much. Perhaps we should try expanding the window a bit more to see if we have missed any important behavior. With that thought in mind, we set Xmin=-20, Xmax=20, and to avoid a ton of tick marks, Xscl=5, as shown in Figure 6.3.22(a). Pushing the GRAPH button produces the image in Figure 6.3.22(b).



(a) (b)

Figure 6.3.22 Adjusting the viewing window again reveals behavior not seen.

Note that the viewing window in Figure 6.3.22(b) reveals behavior not seen in the viewing window of Figure 6.3.21(b). If we had not experimented further, if we had not expanded the viewing window, we would not have seen this new behavior. This is an important lesson.

Note that one of the "turning points" of the graph in Figure 6.3.22(b) lies off the bottom of the viewing window. We'll make one more adjustment to include this important feature. Set Ymin=-10000, Ymax=10000, and Yscl=5000, as shown in Figure 6.3.23(a), then push the GRAPH button to produce the image shown in Figure 6.3.23(b).

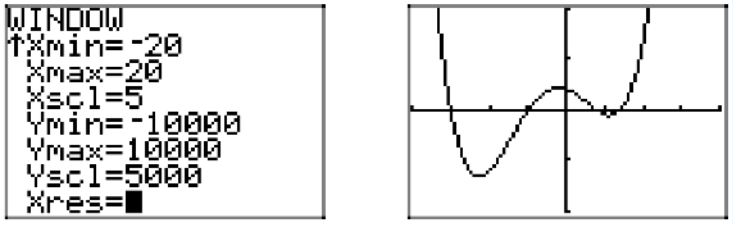


Figure 6.3.23: Adjusting the viewing window again reveals behavior not seen.

The graph in Figure 6.3.23(b) shows all of the "important features" of the graph of f, but the careful reader will continue to experiment, expanding the viewing window to ascertain the truth of this statement.

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CHAPTER OVERVIEW

7: Geometry

7.1: Radicals
7.2: Triangles
7.3: Triangle CIassifications
7.4: The Pythagorean Theorem

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7.1: Radicals

Learning Objectives

- Find square roots.
- Find cube roots.
- Find *n* th roots.
- Simplify expressions using the product and quotient rules for radicals.

Square Roots

The square root of a number is that number that when multiplied by itself yields the original number. For example, 4 is a square root of 16, because $4^2 = 16$. Since $(-4)^2 = 16$, we can say that -4 is a square root of 16 as well. Every positive real number has two square roots, one positive and one negative. For this reason, we use the radical sign $\sqrt{}$ to denote the principal (nonnegative) square root and a negative sign in front of the radical $-\sqrt{}$ to denote the negative square root.

Zero is the only real number with one square root.

$$\sqrt{0} = 0 \quad \text{because} \quad 0^2 = 0 \tag{7.1.1}$$

If the radicand, the number inside the radical sign, is nonnegative and can be factored as the square of another nonnegative number, then the square root of the number is apparent. In this case, we have the following property:

$$\sqrt{a^2} = a, \quad \text{if} \quad a \geq 0 \tag{7.1.2}$$

Example 7.1.1

Find the square root.

a. $\sqrt{36}$
b. $\sqrt{144}$
c. $\sqrt{0.04}$
d. $\sqrt{\frac{1}{9}}$

Solution:

a. $\sqrt{36} = \sqrt{6^2} = 6$
b. $\sqrt{144} = \sqrt{12^2} = 12$
c. $\sqrt{0.04} = \sqrt{0.02^2} = 0.02$
d. $\sqrt{\frac{1}{9}} = \sqrt{(\frac{1}{3})^2} = \frac{1}{3}$

Example 7.1.2

Find the negative square root.

a. $-\sqrt{4}$
b. $-\sqrt{1}$

Solution:

a. $-\sqrt{4} = -\sqrt{2^2} = -2$
b. $-\sqrt{1} = -\sqrt{1^2} = -1$

The radicand may not always be a perfect square. If a positive integer is not a perfect square, then its square root will be irrational. For example, $\sqrt{2}$ is an irrational number and can be approximated on most calculators using the square root button.

$$\sqrt{2} \approx 1.414 \quad \text{because} \quad 1.414\text{^}2 \approx 2$$

Next, consider the square root of a negative number. To determine the square root of −9, you must find a number that when squared results in −9:

$$\sqrt{-9} = ? \quad \text{or} \quad (?)^2 = -9$$

However, any real number squared always results in a positive number:

$$(3)^2 = 9 \quad \text{and} \quad (-3)^2 = 9$$

The square root of a negative number is currently left undefined. For now, we will state that $\sqrt{-9}$ is not a real a number.

Cube Roots

The cube root of a number is that number that when multiplied by itself three times yields the original number. Furthermore, we denote a cube root using the symbol $\sqrt[3]{}$, where 3 is called the index. For example,

$$\sqrt[3]{125} = 5, \quad \text{because} \quad 5^3 = 125$$

The product of three equal factors will be positive if the factor is positive and negative if the factor is negative. For this reason, any real number will have only one real cube root. Hence the technicalities associated with the principal root do not apply. For example,

$$\sqrt[3]{-125} = -5, \quad \text{because} \quad (-5)^3 = -125$$

In general, given any real number ***a***, we have the following property:

$$\sqrt[3]{a^3} = a \tag{7.1.3}$$

When simplifying cube roots, look for factors that are perfect cubes.

Example 7.1.3

Find the cube root.

a. $\sqrt[3]{27}$
b. $\sqrt[3]{64}$
c. $\sqrt[3]{0}$
d. $\sqrt[3]{\frac{1}{8}}$

Solution:

a. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
b. $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$
c. $\sqrt[3]{0} = \sqrt[3]{0^3} = 0$
d. $\sqrt[3]{\frac{1}{8}} = \sqrt[3]{(\frac{1}{2})^3} = \frac{1}{2}$

Example 7.1.4

Find the cube root.

a. $\sqrt[3]{-8}$
b. $\sqrt[3]{-1}$
c. $\sqrt[3]{-\frac{1}{27}}$

Solution:

a. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$
b. $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = -1$
c. $\sqrt[3]{-\frac{1}{27}} = \sqrt[3]{(-\frac{1}{3})^3} = -\frac{1}{3}$

It may be the case that the radicand is not a perfect cube. If an integer is not a perfect cube, then its cube root will be irrational. For example, $\sqrt[3]{2}$ is an irrational number which can be approximated on most calculators using the root button. Depending on the

calculator, we typically type in the index prior to pushing the button and then the radicand as follows:

$$3 \ \sqrt[x]{y} \ 2 \ =$$

Therefore we have

$$\sqrt[3]{2} \approx 1.260, \quad \text{because} \quad 1.260\hat{}3 \approx 2$$

n th Roots

For any integer n≥2, we define the ***n*** th root of a positive real number as that number that when raised to the ***n*** th power yields the original number. Given any nonnegative real number ***a***, we have the following property:

$$\sqrt[n]{a^n} = a, \quad \text{if} \quad a \geq 0 \tag{7.1.4}$$

Here *n* is called the **index** and a^n is called the **radicand**. Furthermore, we can refer to the entire expression \sqrt[n]{a}\) as a radical. When the index is an integer greater than 3, we say “fourth root”, “fifth root”, and so on. The ***n*** th root of any number is apparent if we can write the radicand with an exponent equal to the index.

Example 7.1.5

Find the ***n*** th root.

a. $\sqrt[4]{81}$
b. $\sqrt[5]{32}$
c. $\sqrt[7]{1}$
d. $\sqrt[4]{\frac{1}{16}}$

Solution:

a. $\sqrt[4]{81} = \sqrt[4]{3^4} = 3$
b. $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$
c. $\sqrt[7]{1} = \sqrt[7]{1^7} = 1$
d. $\sqrt[4]{\frac{1}{16}} = \sqrt[4]{(\frac{1}{2})^4} = \frac{1}{2}$

If the index is n=2, then the radical indicates a square root and it is customary to write the radical without the index, as illustrated below:

$$\sqrt[2]{a} = \sqrt{a} \tag{7.1.5}$$

We have already taken care to define the principal square root of a number. At this point, we extend this idea to ***n*** th roots when ***n*** is even. For example, 3 is a fourth root of 81, because $3^4 = 81$. And since $(-3)^4 = 81$, we can say that −3 is a fourth root of 81 as well. Hence we use the radical sign $\sqrt[n]{}$ to denote the principal (nonnegative) ***n*** th root when ***n*** is even. In this case, for any real number ***a***, we use the following property:

$$\sqrt[n]{a^n} = |a| \qquad When\ n\ is\ even \tag{7.1.6}$$

For example,

$$\sqrt[4]{81} = \sqrt[4]{3^4} = |3| = 3$$
$$\sqrt[4]{81} = \sqrt[4]{(-3)^4} = |-3| = 3$$

The negative ***n*** th root, when ***n*** is even, will be denoted using a negative sign in front of the radical $-\sqrt[n]{}$.

$$-\sqrt[4]{81} = -\sqrt[4]{3^4} = -3$$

We have seen that the square root of a negative number is not real because any real number, when squared, will result in a positive number. In fact, a similar problem arises for any even index:

$$\sqrt[4]{-81} = ? \quad \text{or} \quad (?)^4 = -81$$

Here the fourth root of −81 is not a real number because the fourth power of any real number is always positive.

$$\left.\begin{array}{l}\sqrt{-4}\\ \sqrt[4]{-81}\end{array}\right\} \quad \textit{These radicals are not real numbers.}$$

Example 7.1.6

Simplify

a. $\sqrt[4]{-16}$
b. $-\sqrt[4]{16}$

Solution:

a. The radicand is negative and the index is even. Therefore, there is no real number that when raised to the fourth power is -16.

$$\sqrt[4]{-16} \qquad \textit{Not a real number}$$

b. Here the radicand is positive. Furthermore, $16 = 2^4$, and we can simplify as follows:

$$-\sqrt[4]{16} = -\sqrt[4]{2^4} = -2$$

When ***n*** is odd, the same problems do not occur. The product of an odd number of positive factors is positive and the product of an odd number of negative factors is negative. Hence when the index ***n*** is odd, there is only one real ***n*** th root for any real number ***a***. And we have the following property:

$$\sqrt[n]{a^n} = a \qquad \textit{When } n \textit{ is odd} \tag{7.1.7}$$

Example 7.1.7

Find the ***n*** th root.

a. $\sqrt[5]{-32}$
b. $\sqrt[7]{-1}$

Solution:

a. $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$

b. $\sqrt[7]{-1} = \sqrt[7]{(-1)^7} = -1$

Exercise 7.1.1

Find the fourth root:

$$\sqrt[4]{625}$$

Answer

5

Summary

When ***n* is odd**, the nth root is **positive or negative** depending on the sign of the radicand.

When ***n* is even**, the ***n*** th root is **positive or not real** depending on the sign of the radicand.

$\sqrt[4]{16} = \sqrt[4]{(-2)^4} = |-2| = 2$

$$\sqrt[4]{-16} \qquad \textit{The radical is not a real number.}$$

Simplifying Using the Product and Quotient Rule for Radicals

It will not always be the case that the radicand is a perfect power of the given index. If not, we use the following two properties to simplify them. If ***a*** and ***b*** represent positive real numbers, then we have

Table 7.1.1

Product rule for radicals:	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	(7.1.8)
Quotient rule for radicals:	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	(7.1.9)

A radical is simplified if it does not contain any factor that can be written as a perfect power of the index.

Example 7.1.8

Simplify:

$$\sqrt{12}$$

Solution:

Here 12 can be written as $4 \cdot 3$, where 4 is a perfect square.

$$\begin{aligned} \sqrt{12} &= \sqrt{4 \cdot 3} && \textit{Apply the product rule for radicals.} \\ &= \sqrt{4} \cdot \sqrt{3} && \textit{Simplify.} \\ &= 2 \cdot \sqrt{3} \end{aligned}$$

We can verify our answer on a calculator:

$$\sqrt{12} \approx 3.46 \quad \text{and} \quad 2 \cdot \sqrt{3} \approx 3.46$$

Also, it is worth noting that

$$3.46\text{^}2 \approx 12$$

Answer:

$2\sqrt{3}$

Example 7.1.9

Simplify:

$$\sqrt{135}$$

Solution:

Begin by finding the largest perfect square factor of 135.

$$\begin{aligned} 135 &= 3^3 \cdot 5 \\ &= 3^2 \cdot 3 \cdot 5 \\ &= 9 \cdot 15 \end{aligned}$$

Therefore,

$$\begin{aligned} \sqrt{135} &= \sqrt{9 \cdot 15} && \textit{Apply the product rule for radicals.} \\ &= \sqrt{9} \cdot \sqrt{15} && \textit{Simplify.} \\ &= 3 \cdot \sqrt{15} \end{aligned}$$

Answer:

$3\sqrt{15}$

Example 7.1.10

Simplify:

$$\sqrt{\frac{50}{121}}$$

Solution:

Begin by finding the prime factorizations of both 50 and 121. This will enable us to easily determine the largest perfect square factors.

$$50 = 5^2 \cdot 2$$
$$121 = 11^2$$

Therefore,

$$\begin{aligned} \sqrt{\frac{50}{121}} &= \sqrt{\frac{5^2 \cdot 2}{11^2}} && \textit{Apply the product and quotient rule for radicals.} \\ &= \frac{\sqrt{5^2} \cdot \sqrt{2}}{\sqrt{11^2}} && \textit{Simplify.} \\ &= \frac{5 \cdot \sqrt{2}}{11} \end{aligned}$$

Answer:

$$\frac{5 \cdot \sqrt{2}}{11}$$

Example 7.1.11

Simplify:

$$\sqrt[3]{162}$$

Solution:

Use the prime factorization of 162 to find the largest perfect cube factor:

$$\begin{aligned} 162 &= 3^4 \cdot 2 \\ &= 3^3 \cdot 3 \cdot 2 \end{aligned}$$

Replace the radicand with this factorization and then apply the product rule for radicals.

$$\begin{aligned} \sqrt[3]{162} &= \sqrt[3]{3^3 \cdot 3 \cdot 2} && \textit{Apply the product rule for radicals.} \\ &= \sqrt[3]{3^3} \cdot \sqrt[3]{3 \cdot 2} && \textit{Simplify.} \\ &= 3 \cdot \sqrt[3]{6} \end{aligned}$$

We can verify our answer on a calculator.

$$\sqrt[3]{162} \approx 5.451 \quad \text{and} \quad 3 \cdot \sqrt[3]{6} \approx 5.451$$

Answer:

$$3\sqrt[3]{6}$$

Exercise 7.1.2

Simplify:

$$2\sqrt[3]{96}$$

Answer

$4\sqrt[3]{12}$

Example 7.1.12

Simplify:

$$\sqrt[5]{-96}$$

Solution:

Here we note that the index is odd and the radicand is negative; hence the result will be negative. We can factor the radicand as follows:

Then simplify:

Answer:

Simplify:

$$\sqrt[3]{-\frac{8}{64}}$$

Solution:

In this case, consider the equivalent fraction with $-8=(-2)^3$ in the numerator and then simplify.

$$\begin{aligned}\sqrt[3]{-\frac{8}{64}} &= \sqrt[3]{\frac{-8}{64}} && \textit{Apply the quotient rule for radicals.}\\ &= \frac{\sqrt[3]{(-2)^3}}{\sqrt[3]{4^3}} && \textit{Simplify.}\\ &= \frac{-2}{4}\\ &= -\frac{1}{2}\end{aligned}$$

Answer:

$$-\tfrac{1}{2}$$

? Exercise 7.1.3

Simplify:

$$\sqrt[-3]{108}$$

Answer

$-3\sqrt[3]{4}$

Key Takeaways

- The square root of a number is that number that when multiplied by itself yields the original number. When the radicand *a* is positive, \(\sqrt{a^{2}=a\). When the radicand is negative, the result is not a real number.
- The cube root of a number is that number that when used as a factor with itself three times yields the original number. The cube root may be positive or negative depending on the sign of the radicand. Therefore, for any real number *a*, we have the property $\sqrt[3]{a^3}=a$.
- When working with *n* th roots, *n* determines the definition that applies. We use $\sqrt[n]{a^n}=a$ when *n* is odd and $\sqrt[n]{a^n}=|a|$ when *n* is even. When *n* is even, the negative *n* th root is denoted with a negative sign in front of the radical sign.
- To simplify square roots, look for the largest perfect square factor of the radicand and then apply the product or quotient rule for radicals.
- To simplify cube roots, look for the largest perfect cube factor of the radicand and then apply the product or quotient rule for radicals.
- To simplify *n* th roots, look for the factors that have a power that is equal to the index *n* and then apply the product or quotient rule for radicals. Typically, the process is streamlined if you work with the prime factorization of the radicand.

Exercise 7.1.4 radicals

Simplify.

1. $\sqrt{81}$
2. $\sqrt{100}$
3. $\sqrt{64}$
4. $\sqrt{121}$
5. $\sqrt{0}$
6. $\sqrt{1}$
7. $\sqrt{0.25}$
8. $\sqrt{0.01}$
9. $\sqrt{1.21}$
10. $\sqrt{2.25}$
11. $\sqrt{14}$
12. $\sqrt{136}$
13. $\sqrt{\frac{25}{16}}$
14. $\sqrt{\frac{9}{25}}$
15. $\sqrt{-25}$
16. $\sqrt{-9}$
17. $-\sqrt{36}$
18. $-\sqrt{81}$
19. $-\sqrt{100}$
20. $-\sqrt{1}$
21. $\sqrt[3]{27}$
22. $\sqrt[3]{125}$
23. $\sqrt[3]{64}$
24. $\sqrt[3]{8}$
25. $\sqrt[3]{\frac{1}{1}}$
26. $\sqrt[3]{\frac{1}{64}}$
27. $\sqrt[3]{\frac{8}{27}}$
28. $\sqrt[3]{\frac{64}{125}}$
29. $\sqrt[3]{0.001}$
30. $\sqrt[3]{1,000}$
31. $\sqrt[3]{-1}$
32. $\sqrt[3]{-8}$
33. $\sqrt[3]{-27}$
34. $\sqrt[3]{-64}$
35. $\sqrt[3]{-18}$
36. $-\sqrt[3]{\frac{27}{64}}$
37. $-\sqrt[3]{\frac{8}{27}}$
38. $-\sqrt[3]{\frac{1}{125}}$
39. $\sqrt[4]{81}$
40. $\sqrt[4]{625}$
41. $\sqrt[4]{16}$
42. $\sqrt[4]{10,000}$
43. $\sqrt[5]{32}$
44. $\sqrt[5]{1}$

45. $\sqrt[5]{243}$
46. $\sqrt[5]{100,000}$
47. $-\sqrt[4]{16}$
48. $-\sqrt[6]{1}$
49. $\sqrt[5]{-32}$
50. $\sqrt[5]{-1}$
51. $\sqrt{-1}$
52. $\sqrt[4]{-16}$
53. $-5\sqrt[3]{-27}$
54. $-2\sqrt[3]{-8}$
55. $5\sqrt[3]{-1,000}$
56. $3\sqrt[5]{-243}$
57. $10\sqrt[4]{-16}$
58. $2\sqrt[6]{-64}$
59. $\sqrt{325}$
60. $\sqrt{64}$
61. $2\sqrt[3]{27}$
62. $8\sqrt[3]{243}$
63. $-7\sqrt[3]{8}$
64. $-4\sqrt[4]{625}$
65. $6\sqrt[5]{100,000}$
66. $5\sqrt[7]{128}$

Answer

1. 9
3. 8
5. 0
7. 0.5
9. 1.1
11. $\frac{1}{2}$
13. $\frac{5}{4}$
15. Not a real number
17. −6
19. −10
21. 3
23. 4
25. $\frac{1}{2}$
27. $\frac{2}{3}$
29. 0.1
31. −1
33. −3
35. $-\frac{1}{2}$
37. $-\frac{2}{3}$
39. 3

41. 2

43. 2

45. 3

47. −2

49. −2

51. Not a real number

53. 15

55. −50

57. Not a real number

59. 15

61. 6

63. −14

65. 60

Exercise 7.1.5 simplifying radicals

Simplify.

1. $\sqrt{32}$
2. $\sqrt{250}$
3. $\sqrt{80}$
4. $\sqrt{150}$
5. $\sqrt{160}$
6. \(\sqrt{60}\
7. $\sqrt{175}$
8. $\sqrt{216}$
9. $\sqrt{5112}$
10. $\sqrt{10135}$
11. $\sqrt{\frac{50}{49}}$
12. $-\sqrt{2120}$
13. $-3\sqrt{162}$
14. $\sqrt{89}$
15. $\sqrt{\frac{45}{121}}$
16. $\sqrt{\frac{9}{681}}$
17. $\sqrt[3]{54}$
18. $\sqrt[3]{24}$
19. $\sqrt[3]{48}$
20. $\sqrt[3]{81}$
21. $\sqrt[3]{40}$
22. $\sqrt[3]{120}$
23. $\sqrt[3]{162}$
24. $\sqrt[3]{500}$
25. $\sqrt[3]{\frac{54}{125}}$
26. $\sqrt[3]{\frac{40}{343}}$
27. $5\sqrt[3]{-48}$
28. $2\sqrt[3]{-108}$

29. $8\sqrt[4]{96}$
30. $7\sqrt[4]{162}$
31. $\sqrt[5]{160}$
32. $\sqrt[5]{486}$
33. $\sqrt[5]{\frac{224}{243}}$
34. $\sqrt[5]{532}$

Answer

1. $4\sqrt{2}$
3. $4\sqrt{5}$
5. $4\sqrt{10}$
7. $5\sqrt{7}$
9. $6\sqrt{142}$
11. $\frac{5\sqrt{2}}{7}$
13. $-27\sqrt{2}$
15. $\frac{3\sqrt{5}}{11}$
17. $3\sqrt[3]{2}$
19. $2\sqrt[3]{6}$
21. $2\sqrt[3]{5}$
23. $3\sqrt[3]{6}$
25. $\frac{3\sqrt[3]{2}}{5}$
27. $-10\sqrt[3]{6}$
29. $16\sqrt[4]{6}$
31. $2\sqrt[5]{5}$
33. $\frac{2\sqrt[5]{7}}{3}$

? Exercise 7.1.6 simplifying radicals

Simplify. Give the exact answer and the approximate answer rounded to the nearest hundredth.

1. $\sqrt{8}$
2. $\sqrt{200}$
3. $\sqrt{45}$
4. $\sqrt{72}$
5. $3\sqrt{4}$
6. $\sqrt{59}$
7. $\sqrt{3225}$
8. $\sqrt{4849}$
9. $\sqrt[3]{80}$
10. $\sqrt[3]{320}$
11. $\sqrt[3]{48}$
12. $\sqrt[3]{270}$

Answer

1. $2\sqrt{2} \approx 2.83$

3. $3\sqrt{5} \approx 6.71$
5. $3\sqrt{2} \approx 0.87$
7. $\frac{4\sqrt{2}}{5} \approx 1.13$
9. $2\sqrt[3]{10} \approx 4.31$
11. $2\sqrt[3]{6} \approx 3.63$

Exercise 7.1.7 simplifying radicals

Rewrite the following as a radical expression with coefficient 1.

1. $2\sqrt{15}$
2. $3\sqrt{7}$
3. $5\sqrt{10}$
4. $10\sqrt{3}$
5. $2\sqrt[3]{7}$
6. $3\sqrt[3]{6}$
7. $2\sqrt[4]{5}$
8. $3\sqrt[4]{2}$
9. The formula for the area A of a square is $A = s^2$. If the area is 18 square units, then what is the length of each side?
10. Calculate the length of a side of a square with an area of 60 square centimeters.
11. The formula for the volume V of a cube is $V = s^3$. If the volume of a cube is 112 cubic units, then what is the length of each side?
12. Calculate the length of a side of a cube with a volume of 54 cubic centimeters.

Answer

1. $\sqrt{60}$
3. $\sqrt{250}$
5. $\sqrt[3]{56}$
7. $\sqrt[4]{80}$
9. $3\sqrt{2}$ units
11. $2\sqrt[3]{14}$ units

Exercise 7.1.8 discussion board

1. Explain why there are two square roots for any nonzero real number.
2. Explain why there is only one cube root for any real number.
3. What is the square root of 1, and what is the cube root of 1? Explain why.
4. Explain why $\sqrt{-1}$ is not a real number and why $\sqrt[3]{-1}$ is a real number.

Answer

1. Answers may vary
3. Answers may vary

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7.2: Triangles

A **triangle** is formed when three straight line segments bound a portion of the plane. The line segments are called the **sides of the triangle**. A point where two sides meet is called a **vertex of the triangle**, and the angle formed is called an **angle of the triangle.** The symbol for triangle is $\triangle$.

The triangle in Figure $7.2.1$ is denoted by $\triangle ABC$ (or $\triangle BCA$ or $\triangle CAB$, etc.).

- Its sides are AB, AC, and BC.
- Its vertices are A, B, and C.
- Its angles are $\angle A$, $\angle B$, and $\angle C$.

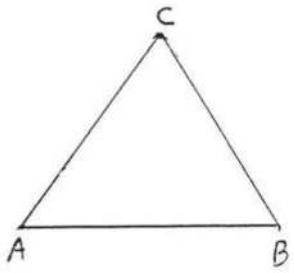


Figure $7.2.1$: Triangle ABC.

The triangle is the most important figure in plane geometry, This is because figures with more than three sides can always be divided into triangles (Figure $7.2.2$). If we know the properties of a triangle, we can extend this knowledge to the study of other figures as well.

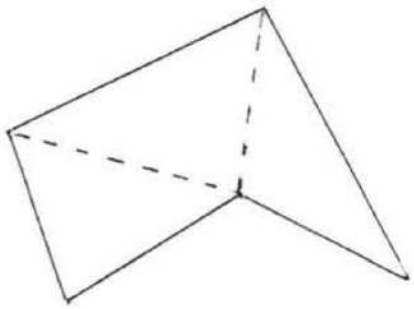


Figure $7.2.2$: A closed figure formed by more than three straight lines can be divided into triangles.

A fundamental property of triangles is the following:

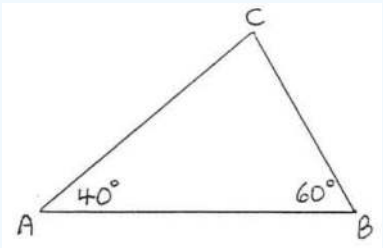
Theorem 7.2.1

The sum of the angles of a triangle is 180°.

In $\triangle ABC$ of Figure $7.2.1$, $\angle A + \angle B + \angle C = 180^\circ$.

Example 7.2.1

Find $\angle C$:



Solution

$$\begin{aligned} \angle A+\angle B+\angle C &= 180^\circ \\ 40^\circ+60^\circ+\angle C &= 180^\circ \\ 100^\circ+\angle C &= 180^\circ \\ \angle C &= 180^\circ-100^\circ \\ \angle C &= 80^\circ \end{aligned}$$

Answer: $\angle C = 80^\circ$

Proof of Theorem 7.2.1: Through C draw DE parallel to AB (see Figure 7.2.3). Note that we are using the parallel postulate here, $\angle 1 = \angle A$ and $\angle 3 = \angle B$ because they are alternate interior angles of parallel lines, Therefore $\angle A+\angle B+\angle C = \angle 1+\angle 3+\angle 2 = 180^\circ$.

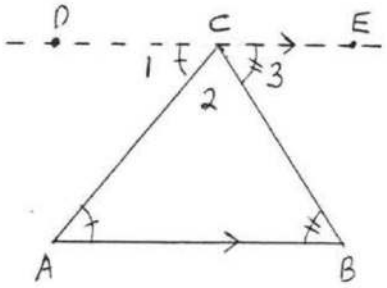
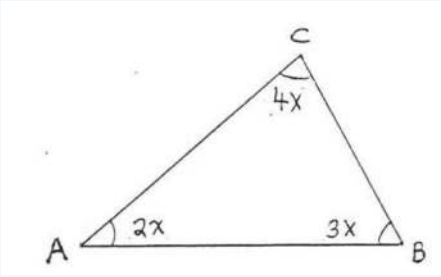


Figure 7.2.3: Through C draw DE parallel to AB.

We may verify Theorem 7.2.1 by measuring the angles of a triangle with a protractor and taking the sum, However no measuring instrument is perfectly accurate, It is reasonable to expect an answer such as 179°, 182°, 180.5°, etc. The purpose of our mathematical proof is to assure us that the sum of the angles of every triangle must be exactly \(180^{\circ}\).

Example 7.2.2

Find x:



Solution

$$\begin{aligned} \angle A+\angle B+\angle C &= 180^\circ \\ 2x+3x+4x &= 180 \\ 9x &= 180 \\ x &= 20 \end{aligned}$$

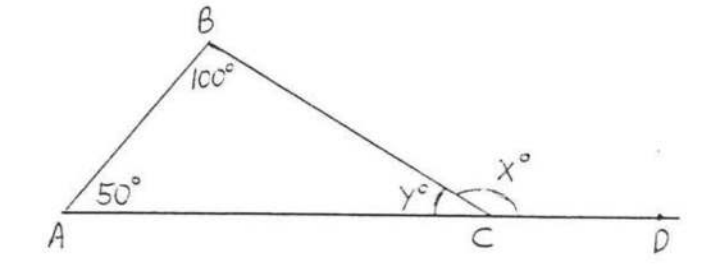
Check:

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ 2x + 3x + 4x & \\ 2(20) + 3(20) + 4(20) & \\ 40^\circ + 60^\circ + 80^\circ & \\ 180^\circ & \end{aligned}$$

Answer: $x = 20$.

Example 7.2.3

Find y and x:



Solution

$$\begin{aligned} 50 + 100 + y &= 180 \\ 150 + y &= 180 \\ y &= 180 - 150 \\ y &= 30 \end{aligned} \tag{7.2.1}$$

$$x = 180 - 30 = 150$$

Answer: $y = 30$, $x = 150$.

In Figure 7.2.4, $\angle x$ is called an **exterior angle** of $\triangle ABC$, $\angle A$, $\angle B$, and $\angle y$ are called the **interior angles** of $\triangle ABC$. $\angle A$ and $\angle B$ are said to be the interior angles **remote** from the exterior angle $\angle x$.

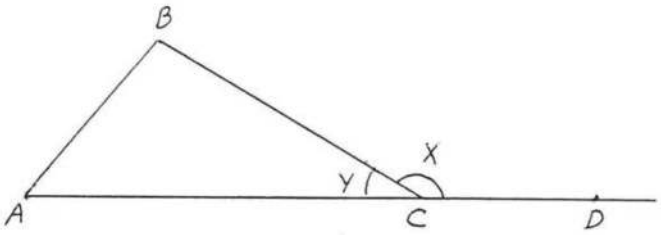


Figure 7.2.4: $\angle x$ is an exterior angle of $\triangle ABC$.

The results of Example 7.2.3 suggest the following theorem.

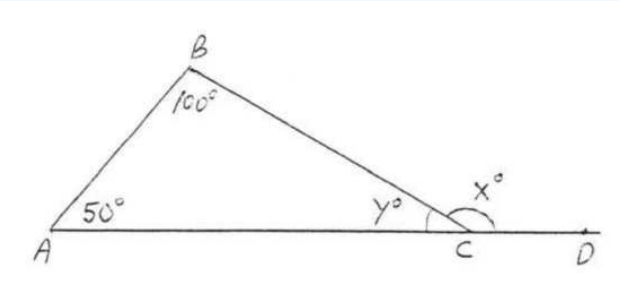
Theorem 7.2.2

An exterior angle is equal to the sum of the two remote interior angles,

In Figure 7.2.4, $\angle x = \angle A + \angle B$.

Example 7.2.3 (repeated)

Find x:



Solution

Using Theorem 7.2.2, $x^\circ = 100^\circ + 50^\circ = 150^\circ$.

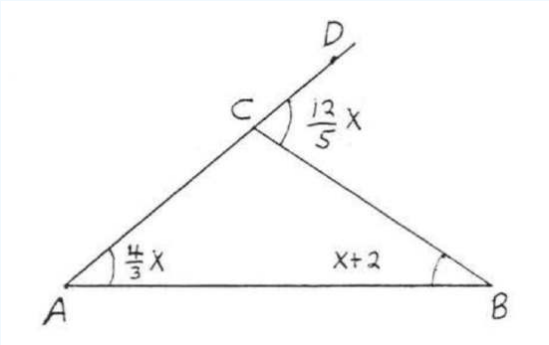
Answer: $x = 150$.

Proof of Theorem 7.2.2: We present this proof in **double-column form**, with statements in the left column and the reason for each statement in the right column. The last statement is the theorem we wish to prove.

Statements	Reasons
1. $\angle A + \angle B + \angle y = 180^\circ$	1. The sum of the angles of a triangle is 180°.
2. $\angle A + \angle B = 180^\circ - \angle y$	2. Subtract $\angle y$ from both sides of the equation, statement 1.
3. $\angle x = 180^\circ - \angle y$.	3. $\angle x$ and $\angle y$ are supplementary.
4. $\angle x = \angle A + \angle B$.	4. Both $\angle x$ (statement 3) and $\angle A + \angle B$ (statement 2) equal $180^\circ - \angle y$.

Example 7.2.4

Find x:



Solution

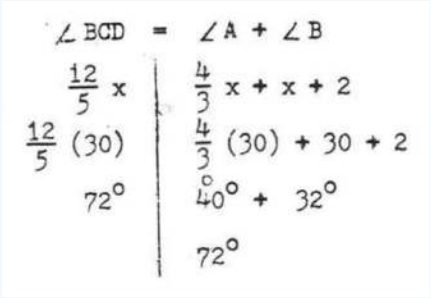
$\angle BCD$ is an exterior angle with remote interior angles $\angle A$ and $\angle B$. By Theorem 7.2.2,

$$\begin{aligned} \angle BCD &= \angle A + \angle B \\ \frac{12}{5}x &= \frac{4}{3}x + x + 2 \end{aligned} \tag{7.2.2}$$

The least common denominator (1, c, d) is 15.

$$
\begin{aligned}
(15)\frac{12}{5}x &= (15)\frac{4}{3}x+(15)x+(15)(2)\\
36x &= 20x+15x+30\\
36x &= 35x+30\\
36x-35x &= 30\\
x &= 30
\end{aligned}
$$

Check:



Answer: $x = 30$.

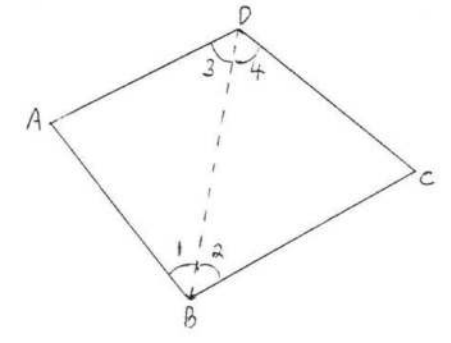
Our work on the sum of the angles of a triangle can easily be extended to other figures:

Example 7.2.5

Find the sum of the angles of a quadrilateral (four sided figure),

Solution

Divide the quadrilateral into two triangles as illustrated,



$$
\begin{aligned}
\angle A+\angle B+\angle C+\angle D &= \angle A+\angle 1+\angle 3+\angle 2+\angle 4+\angle C\\
&= 180^\circ+180^\circ\\
&= 360^\circ
\end{aligned}
$$

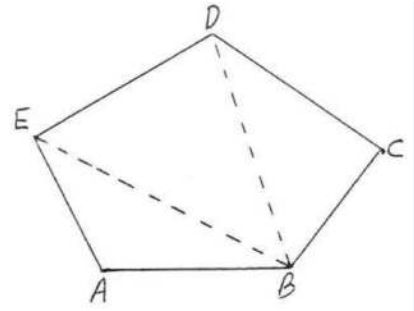
Answer: 360°.

Example 7.2.6

Find the sum of the angles of a pentagon (five-sided figure).

Solution

Divide the pentagon into three triangles as illustrated, The sum is equal to the sum of the angles of the three triangle = $(3)(180^\circ) = 540^\circ$.



Answer: 540°.

There is one more simple principle which we will derive from Theorem 7.2.1, Consider the two triangles in Figure 7.2.5.

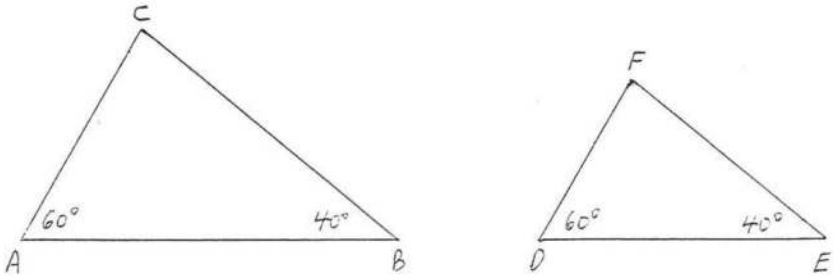


Figure 7.2.5: Each triangle has an angle of 60° and 40°.

We are given that $\angle A = \angle D = 60^\circ$ and $\angle B = \angle E = 40^\circ$. A short calculation shows that we must also have $\angle C = \angle F = 80^\circ$. This suggests the following theorem:

Theorem 7.2.3

If two angles of one triangle are equal respectively to two angles of another triangle, then their remaining angles are also equal.

In Figure 7.2.6, if $\angle A = \angle D$ and $\angle B = \angle E$ then $\angle C = \angle F$.

Proof

$\angle C = 180^\circ - (\angle A + \angle B) = 180^\circ - (\angle D + \angle E) = \angle F$.

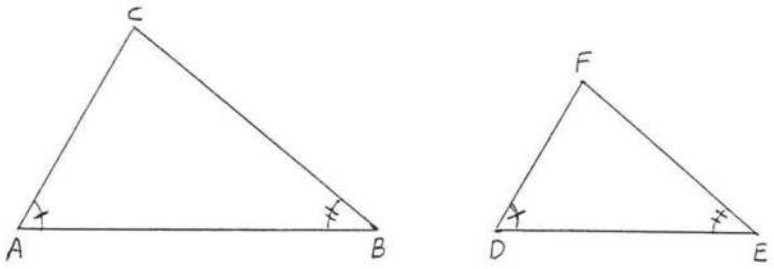


Figure 7.2.6. $\angle A = \angle D$ and $\angle B = \angle E$.

Historical Note

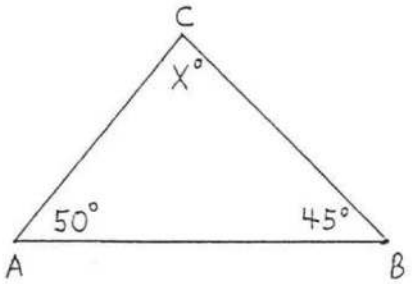
Our Theorem 7.2.1, which states that the sum of the angles of a triangle is 180°, is one of the most important consequences of the parallel postulate, Therefore, one way of testing the truth of the parallel postulate (see the **Historical Note** in Section 1.4) is to test the truth of Theorem 7.2.1, This was actually tried by the German mathematician, astronomer, and physicist, Karl Friedrich Gauss (1777 - 1855). (This is the same Gauss whose name is used as a unit of measurement in the theory of magnetism), Gauss measured the sum of the angles of the triangle formed by three mountain peaks in Germany, He found the

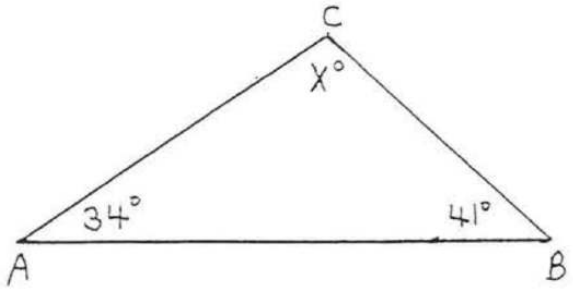
sum of the angles to be 14.85 seconds more than 180° (60 seconds 1 minute, 60 minutes = 1 degree). However this small excess could have been due to experimental error, so the sum might actually have been 180°.

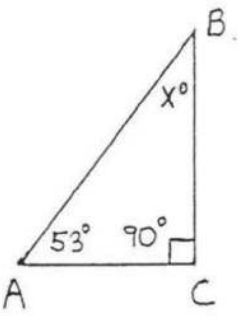
Aside from experimental error, there is another difficulty involved in verifying the angle sum theorem. According to the non-Euclidean geometry of Lobachevsky, the sum of the angles of a triangle is always **less** than 180°. In the non-Euclidean geometry of Riemann, the sum of the angles is always more than 180°, However in both cases the difference from 180° is insignificant unless the triangle is very large, Neither theory tells us exactly how large such a triangle should be, Even if we measured the angles of a very large triangle, like one formed by three stars, and found the sum to be indistinguishable from 180°, we could only say that the angle sum theorem and parallel postulate are apparently true for these large distances, These distances still might be too small to enable us to determine whichgeometric system best describes the universe as a whole,

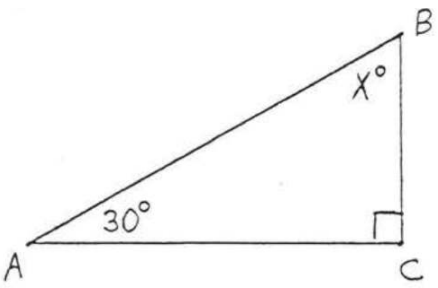
Problems

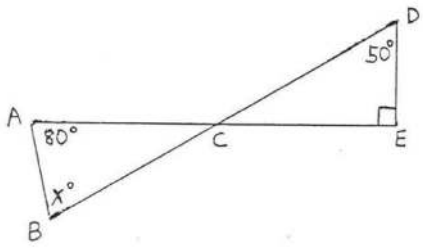
1 - 12. Find x and all the missing angles of each triangle:

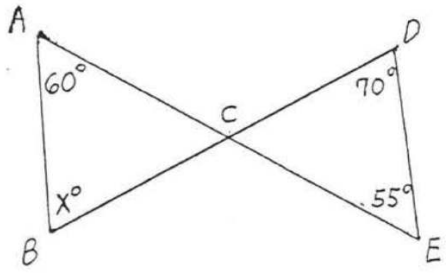
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2. 

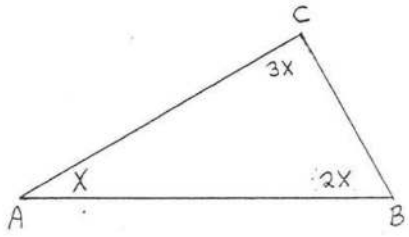
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4. 

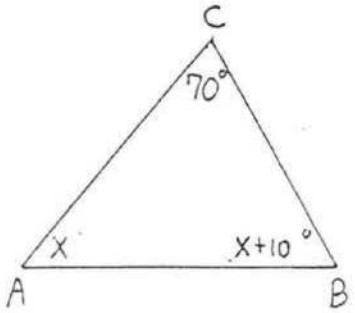
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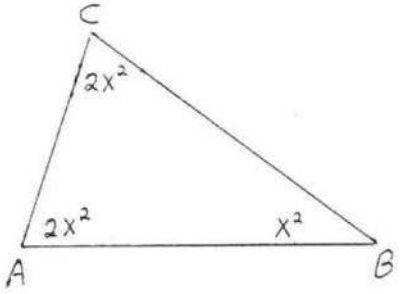
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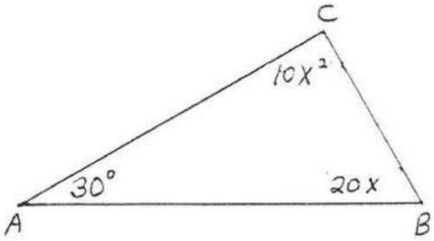
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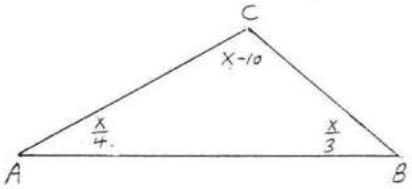
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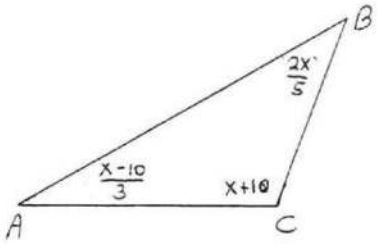
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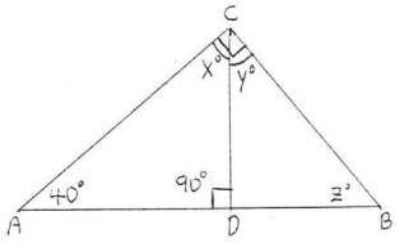


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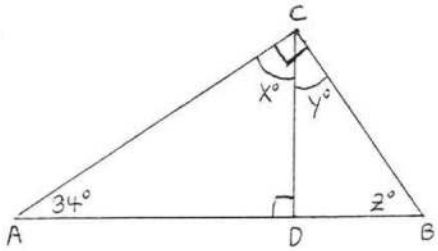


13 - 14. Find x, y, and z:

13.

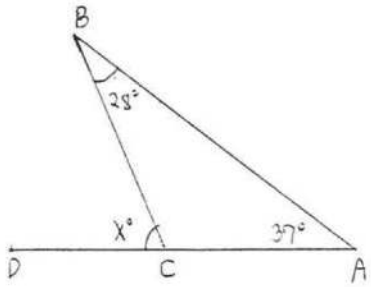


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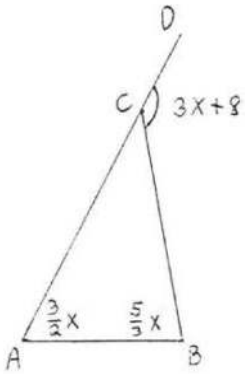


15 - 20. Find x:

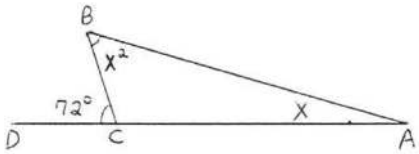
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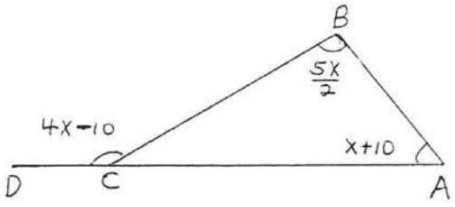
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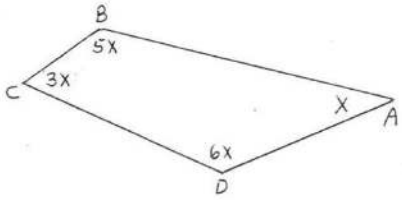
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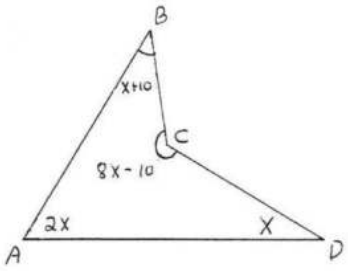
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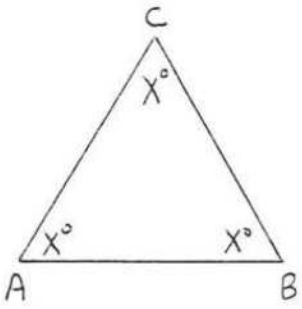


21. Find the sum of the angles of a hexagon (6-sided figure).

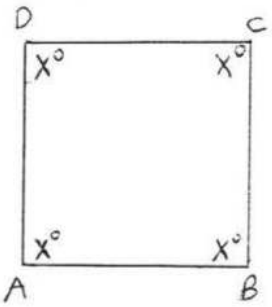
22. Find the sum of the angles of an octagon (8-sided figure).

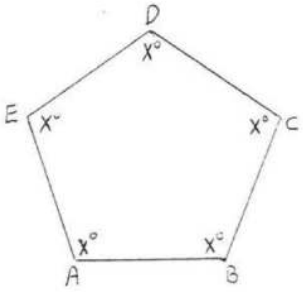
23 - 26. Find x:

23.

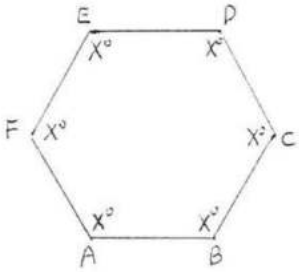


24.





25.



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7.3: Triangle Classifications

Triangles may be classified according to the relative lengths of their sides:

- An **equilateral** triangle has three equal sides,
- An **isosceles** triangle has two equal sides.
- A **scalene** triangle has no equal sides.

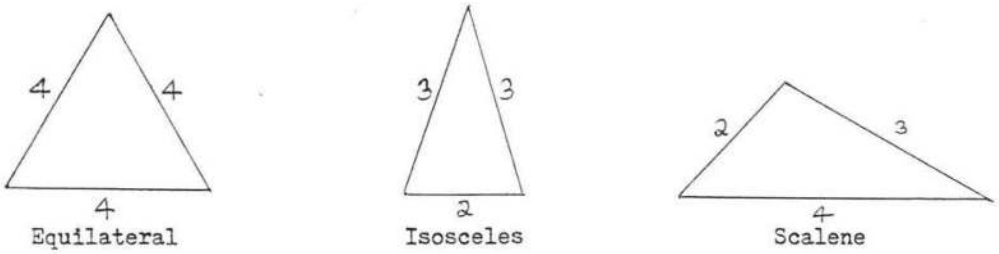


Figure 7.3.1: Triangles classified according to their sides,

Triangles may also be classified according to the measure of their angles:

- An **acute** triangle is a triangle with three acute angles.
- An **obtuse** triangle is a triangle with one obtuse angle.
- An **equiangular** triangle is a triangle with three equal angles,

Each angle of an equiangular triangle must be 60°, We will show in section 2, 5 that equiangular triangle are the same as equilateral triangles,

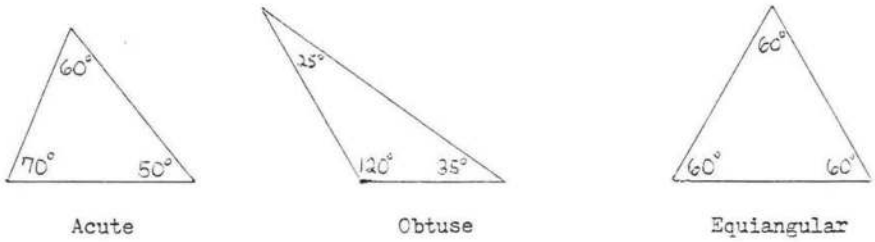


Figure 7.3.2: Triangles classified according to their angles,

A **right** triangle is a triangle with one right angle, The sides of the right angle are called the **legs** of the triangle and the remaining side is called the **hypotenuse**,

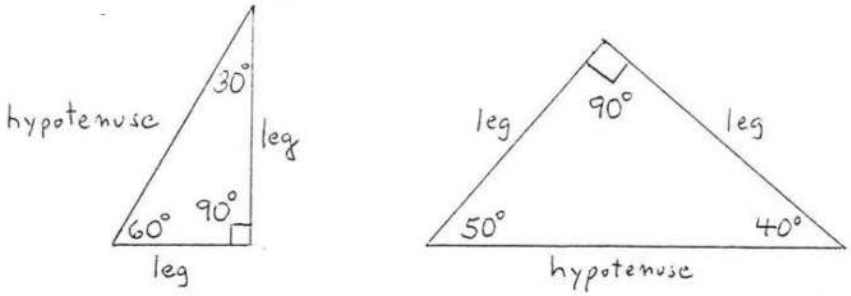
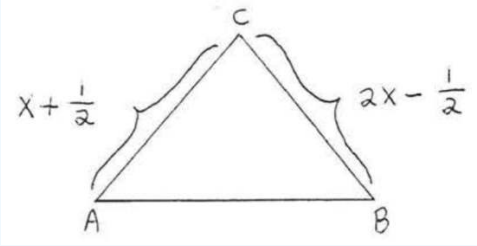


Figure 7.3.3: Right triangles.

Example 7.3.1

Find x if $\triangle ABC$ is isosceles with $AC = BC$:



Solution

$$\begin{aligned} x+\frac{1}{2} &= 2x-\frac{1}{2} \\ (2)(x+\frac{1}{2}) &= (2)(2x-\frac{1}{2}) \\ 2x+2(\frac{1}{2}) &= (2)(2x)-(2)(\frac{1}{2}) \\ 2x+1 &= 4x-1 \\ 1+1 &= 4x-2x \\ 2 &= 2x \\ 1 &= x \end{aligned} \tag{7.3.1}$$

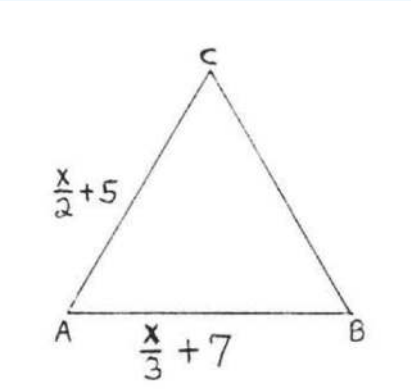
Check:

$x+\frac{1}{2}=$	$2x-\frac{1}{2}$
$1+\frac{1}{2}$	$2(1)-\frac{1}{2}$
$1\frac{1}{2}$	$2-\frac{1}{2}$
	$1\frac{1}{2}$

Answer: $x=1$.

Example 7.3.2

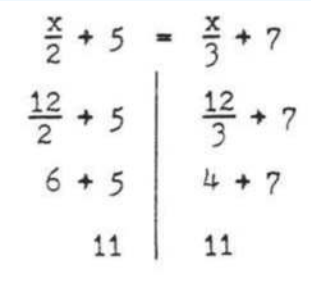
$\triangle ABC$ is equilateral. Find x:



Solution

$$\begin{aligned} \frac{x}{2}+5 &= \frac{x}{3}+7 \\ (6)(\frac{x}{2}+5) &= (6)(\frac{x}{3}+7) \\ (6)(\frac{x}{2}+(6)(5) &= (6)(\frac{x}{3}+(6)(7) \\ 3x+30 &= 2x+42 \\ 3x-2x &= 42-30 \\ x &= 12 \end{aligned} \tag{7.3.2}$$

Check:



Answer: $x = 12$.

An **altitude** of a triangle is a line segment from a vertex perpendicular to the opposite·side, In Figure 4, CD and GH are altitudes, Note that altitude GH lies outside $\triangle EFG$ and side EF must be extended to meet it.

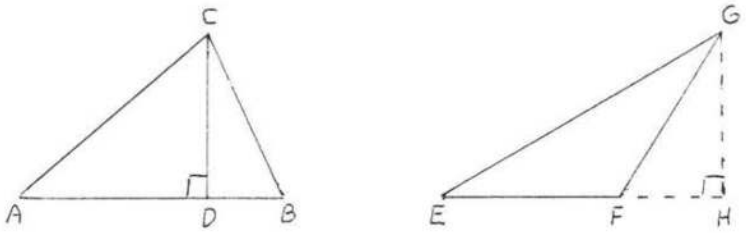


Figure 7.3.4. CD and GH are altitudes.

A **median** of a triangle is a line segment from a vertex to the midpoint of the opposite side, In Figure 5, CD is a median,

An **angle bisector** is a ray which divides an angle into two eaual angles. In Figure 7.3.6, $\overrightarrow{CD}$ is an angle bisector.

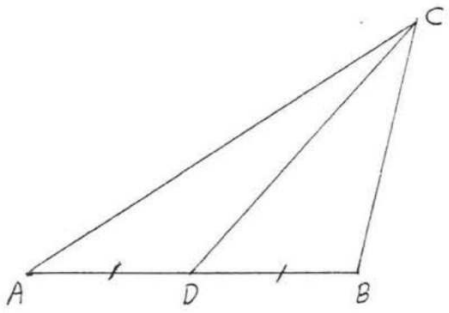


Figure 7.3.5: CD is a median.

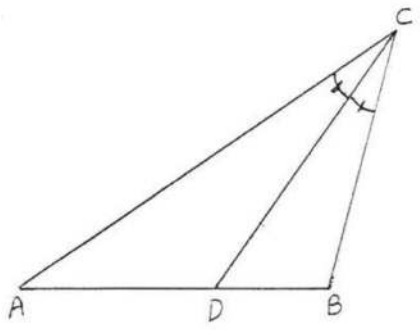
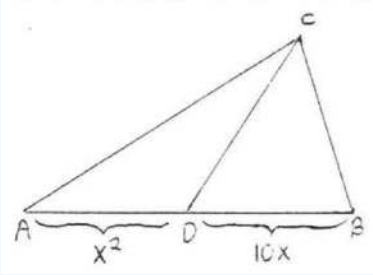


Figure 7.3.6. $\overrightarrow{CD}$ is an angle bisector of $\angle ACB$.

Example 7.3.3

Find AB if CD is a median:



Solution

$$\begin{aligned} AD &= DB \\ x^2 &= 10x \\ x^2 - 10x &= 0 \\ (x)(x-10) &= 0 \\ x = 0 \quad \text{or} \quad x - 10 &= 0 \\ x &= 10 \end{aligned} \tag{7.3.3}$$

Check, $x = 0$:

AD	=	DB
x^2		10x
0^2		10(0)
0		0

Check, $x = 10$:

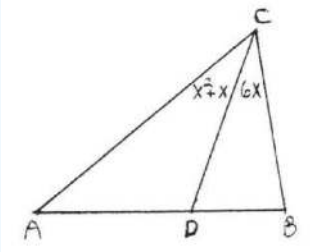
AD	=	DB
x^2		10x
10^2		10(10)
100		100

We reject the answer $x=0$ because the length of a line segment must be greater than 0, Therefore $AB=AD+DB=100+100=200$.

Answer: $AB=200$.

Example 7.3.4

Find $\angle ACB$ if $\overrightarrow{CD}$ is an angle bisector:



Solution

$$\begin{aligned} \anlge ACD &= \angle BCD \\ x^2+x &= 6x \\ x^2+x-6x &= 0 \\ x^2-5x &= 0 \\ (x)(x-5) &= 0 \end{aligned} \qquad (7.3.4)$$

$$x = 0 \quad \text{or} \quad x-5 = 0$$
$$x = 5$$

Check, $x=0$:

$\angle ACD$	=	$\angle BCD$
x^2+x		$6x$
0^2+0		$6(0)$
0		0

Check, $x=5$:

$\angle ACD$	=	$\angle BCD$
x^2+x		$6x$
5^2+5		$6(5)$
30		30

We reject the answer $x=0$ because the measures of $\angle ACD$ and $\angle BCD$ must be greater than 0°. Therefore $\angle ACB=\angle ACD+\angle BCD=30^\circ+30^\circ=60^\circ$.

Answer: $\angle ACB=60^\circ$.

The **perimeter** of a triangle is the sum of the lengths of the sides. The perimeter of $\triangle ABC$ in Figure 7.3.7 is $3+4+5=12$.

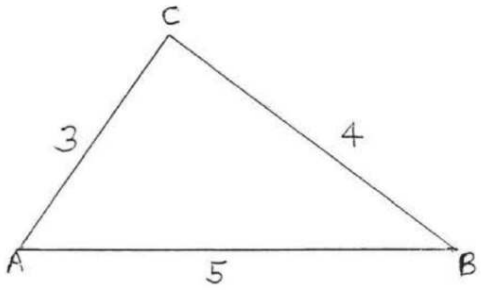


Figure 7.3.7. The perimeter of $\triangle ABC$ is 12.

Theorem 7.3.1

The sum of any two sides of a triangle is greater than the remaining side.

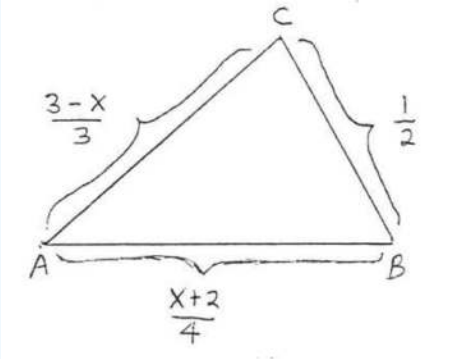
For example, in Figure 7.3.7, $AC + BC = 3 + 4 > AB = 5$.

Proof

This follows from the postulate that the shortest distance between two noints is along a straight line, For example, in Figure 7.3.7, the length AB (a straight line segment) must be less than the combined lengths of AC and CB (not on a straight line from A to B),

Example 7.3.5

Find the perimeter of the triangle in terms of x, Then find the perimeter if $x = 1$:



Solution

$$\begin{aligned} &= \\ &= \\ &= \\ &= \\ &= \end{aligned} \tag{7.3.5}$$

If $x = 1$, $\dfrac{24 - x}{12} = \dfrac{24 - 1}{12} = \dfrac{23}{12}$.

Check:

https://math.libretexts.org/@go/page/116796

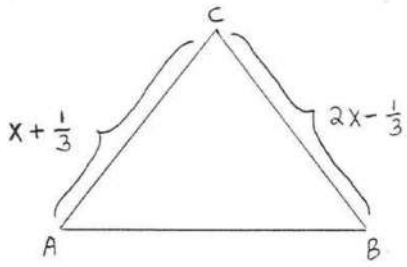
$$\frac{3-x}{3}+\frac{x+2}{4}+\frac{1}{2}=\frac{24-x}{12}$$
$$\frac{3-1}{3}+\frac{1+2}{4}+\frac{1}{2}\quad\Big|\quad\frac{24-1}{12}$$
$$\frac{2}{3}+\frac{3}{4}+\frac{1}{2}\quad\Big|\quad\frac{23}{12}$$
$$\frac{8}{12}+\frac{9}{12}+\frac{6}{12}$$
$$\frac{23}{12}$$

Answer: $\frac{24-x}{12}$, $\frac{23}{12}$.

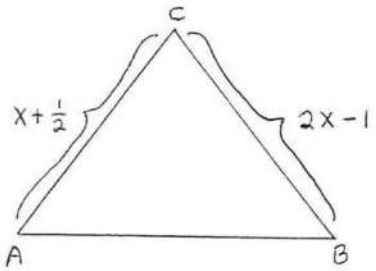
Problems

1 - 2. Find x if $\triangle ABC$ is isosceles with $AC = BC$:

1.

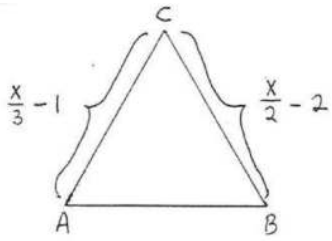


2.

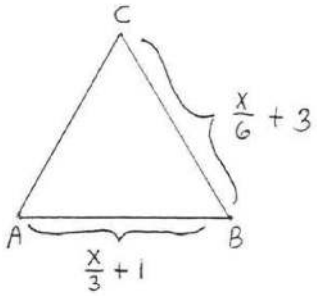


3 - 4. Find x if $\triangle ABC$ is equilateral:

3.

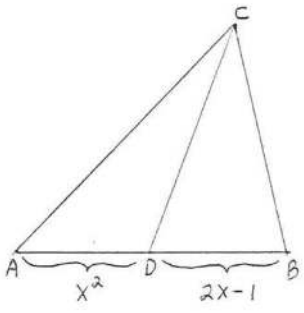


4.

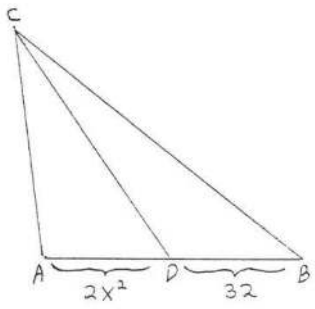


5 - 6. Find AB if CD is a median:

5.

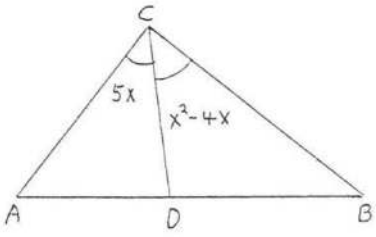


6.

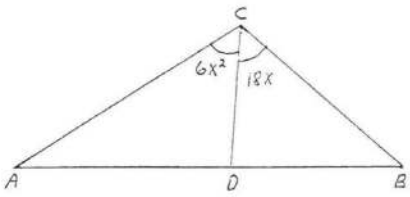


7 - 8. Find $\angle ACB$ if $\overrightarrow{CD}$ is an angle bisector:

7.

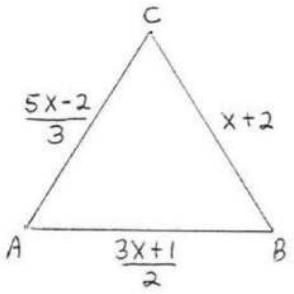


8.

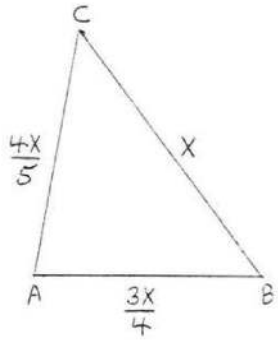


9 - 10. Find the perimeter of the triangle in terms of x, Then find the perimeter if $x = 4$:

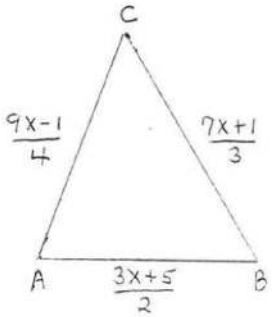
9.



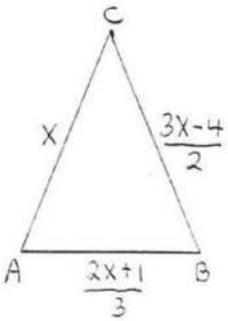
10.



11. Find x if the perimeter of $\triangle ABC$ is 33.



12. Find x if the perimeter of $\triangle ABC$ is 11.



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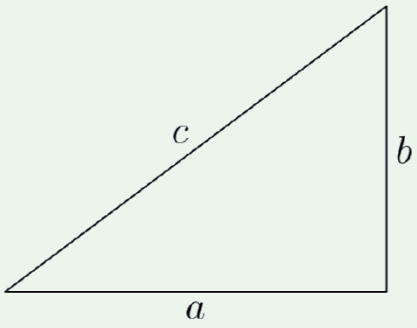
7.4: The Pythagorean Theorem

Pythagoras was a Greek mathematician and philosopher, born on the island of Samos (ca. 582 BC). He founded a number of schools, one in particular in a town in southern Italy called Crotone, whose members eventually became known as the Pythagoreans. The inner circle at the school, the Mathematikoi, lived at the school, rid themselves of all personal possessions, were vegetarians, and observed a strict vow of silence. They studied mathematics, philosophy, and music, and held the belief that numbers constitute the true nature of things, giving numbers a mystical or even spiritual quality.

Today, nothing is known of Pythagoras's writings, perhaps due to the secrecy and silence of the Pythagorean society. However, one of the most famous theorems in all of mathematics does bear his name, the Pythagorean Theorem.

PYTHAGOREAN THEOREM

Let c represent the length of the **hypotenuse**, the side of a right triangle directly opposite the right angle (a right angle measures 90º) of the triangle. The remaining sides of the right triangle are called the **legs** of the right triangle, whose lengths are designated by the letters a and b.



The relationship involving the legs and hypotenuse of the right triangle, given by

$$a^2 + b^2 = c^2 \tag{7.4.1}$$

is called the **Pythagorean Theorem**.

Note that the Pythagorean Theorem can only be applied to right triangles. Let's look at a simple application of the Pythagorean Theorem (Equation 7.4.1).

Example 7.4.2

Given that the length of one leg of a right triangle is 4 centimeters and the hypotenuse has length 8 centimeters, find the length of the second leg.

Let's begin by sketching and labeling a right triangle with the given information. We will let *x* represent the length of the missing leg.

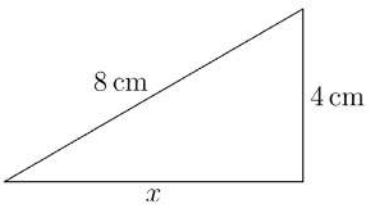


Figure 1. A sketch make things a bit easier

Here is an important piece of advice.

TIP 7.4.3: Hypotenuse

The hypotenuse is the longest side of the right triangle. It is located directly opposite the right angle of the triangle. Most importantly, it is the quantity that is **isolated** by itself in the Pythagorean Theorem (Equation 7.4.1). Always isolate the quantity representing the hypotenuse on one side of the equation. The "legs" go on the other side of the equation.

So, taking the tip to heart, and noting the lengths of the legs and hypotenuse in **Figure 1**, we write

$$4^2 + x^2 = 8^2 \ .$$

Square, then isolate x on one side of the equation.

$$16 + x^2 = 64$$

$$x^2 = 48$$

Normally, we would take plus or minus the square root in solving this equation, but *x* represents the length of a leg, which must be a positive number. Hence, we take just the positive square root of 48.

$$x = \sqrt{48}$$

Of course, place your answer in simple radical form.

$$x = \sqrt{16}\sqrt{3}$$

$$x = 4\sqrt{3}$$

If need be, you can use your graphing calculator to approximate this length. To the nearest hundredth of a centimeter, $x \approx 6.93$ centimeters.

Proof of the Pythagorean Theorem

It is not known whether Pythagoras was the first to provide a proof of the Pythagorean Theorem. Many mathematical historians think not. Indeed, it is not even known if Pythagoras crafted a proof of the theorem that bears his name, let alone was the first to provide a proof.

There is evidence that the ancient Babylonians were aware of the Pythagorean Theorem over a 1000 years before the time of Pythagoras. A clay tablet, now referred to as Plimpton 322 (see **Figure 2**), contains examples of Pythagorean Triples, sets of three numbers that satisfy the Pythagorean Theorem (such as 3, 4, 5).



Figure 2. A photograph of Plimpton 322.

One of the earliest recorded proofs of the Pythagorean Theorem dates from the Han dynasty (206 BC to AD 220), and is recorded in the Chou Pei Suan Ching (see **Figure 3**). You can see that this figure specifically addresses the case of the 3, 4, 5 right triangle. Mathematical historians are divided as to whether or not the image was meant to be part of a general proof or was just devised to address this specific case. There is also disagreement over whether the proof was provided by a more modern commentator or dates back further in time.

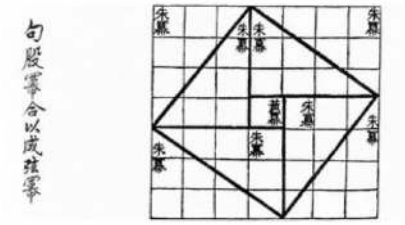


Figure 3. A figure from the Chou Pei Suan Ching

However, **Figure 3** does suggest a path we might take on the road to a proof of the Pythagorean Theorem. Start with an arbitrary right triangle having legs of lengths a and b, and hypotenuse having length c, as shown in **Figure 4**(a).

Next, make four copies of the triangle shown in **Figure 4**(a), then rotate and translate them into place as shown in **Figure 4**(b). Note that this forms a big square that is c units on a side.

Further, the position of the triangles in **Figure 4**(b) allows for the formation of a smaller, unshaded square in the middle of the larger square. It is not hard to calculate the length of the side of this smaller square. Simply subtract the length of the smaller leg from the larger leg of the original triangle. Thus, the side of the smaller square has length b − a.

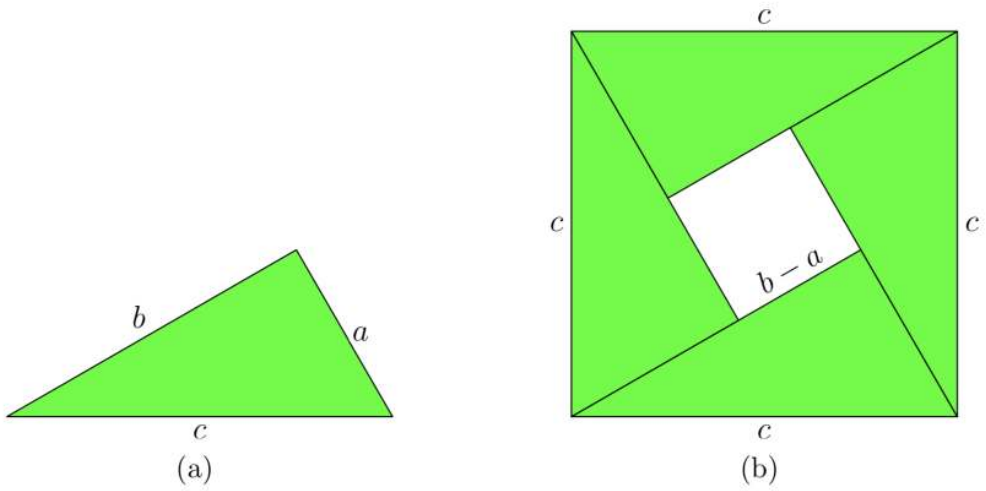


Figure 4. Proof of the Pythagorean Theorem.

Now, we will calculate the area of the large square in **Figure 4**(b) in two separate ways.

- First, the large square in **Figure 4**(b) has a side of length c. Therefore, the area of the large square is

$$Area = c^2 .$$

- Secondly, the large square in **Figure 4**(b) is made up of 4 triangles of the same size and one smaller square having a side of length b−a. We can calculate the area of the large square by summing the area of the 4 triangles and the smaller square.

  1. The area of the smaller square is $(b-a)^2$.
  2. The area of each triangle is $\frac{ab}{2}$. Hence, the area of four triangles of equal size is four times this number;

  i.e., $4(\frac{ab}{2})$. Thus, the area of the large square is

  Area = Area of small square + 4· Area of triangle

  $=(b-a)^2+4(\frac{ab}{2})$.

  We calculated the area of the larger square twice. The first time we got c^2; the second time we got $(b-a)^2+4(\frac{ab}{2})$. Therefore, these two quantities must be equal.

$$c^2 = (b-a)^2+4(\frac{ab}{2}) .$$

  Expand the binomial and simplify.

$$c^2 = b^2 - 2ab + a^2 + 2ab$$
$$c^2 = b^2 + a^2$$

That is,

$$a^2 + b^2 = c^2 \ ,$$

and the Pythagorean Theorem is proven.

Applications of the Pythagorean Theorem

In this section we will look at a few applications of the Pythagorean Theorem, one of the most applied theorems in all of mathematics. Just ask your local carpenter.

The ancient Egyptians would take a rope with 12 equally spaced knots like that shown in **Figure 5**, and use it to square corners of their buildings. The tool was instrumental in the construction of the pyramids.

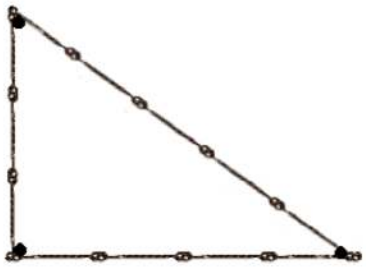


Figure 5. A basic 3-4-5 right triangle for squaring corners.

The Pythagorean theorem is also useful in surveying, cartography, and navigation, to name a few possibilities.

Let's look at a few examples of the Pythagorean Theorem in action.

? Example 7.4.4

One leg of a right triangle is 7 meters longer than the other leg. The length of the hypotenuse is 13 meters. Find the lengths of all sides of the right triangle.

Let *x* represent the length of one leg of the right triangle. Because the second leg is 7 meters longer than the first leg, the length of the second leg can be represented by the expression *x* + 7, as shown in **Figure 6**, where we've also labeled the length of the hypotenuse (13 meters).

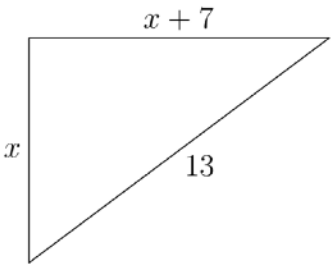


Figure 6. The second leg is 7 meters longer than the first.

Remember to isolate the length of the hypotenuse on one side of the equation representing the Pythagorean Theorem. That is,

$$x^2 + (x+7)^2 = 13^2 \ .$$

Note that the legs go on one side of the equation, the hypotenuse on the other. Square and simplify. Remember to use the squaring a binomial pattern.

$$x^2 + x^2 + 14x + 49 = 169$$
$$2x^2 + 14x + 49 = 169$$

This equation is nonlinear, so make one side zero by subtracting 169 from both sides of the equation.

$$2x^2 + 14x + 49 - 169 = 0$$

$$2x^2 + 14x - 120 = 0$$

Note that each term on the left-hand side of the equation is divisible by 2. Divide both sides of the equation by 2.

$$x^2 + 7x - 60 = 0$$

Let's use the quadratic formula with a = 1, b = 7, and c = −60.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4(1)(-60)}}{2(1)}$$

Simplify.

$$x = \frac{-7 \pm \sqrt{289}}{2}$$

Note that 289 is a perfect square ($17^2 = 289$). Thus,

$$x = \frac{-7 \pm 17}{2}.$$

Thus, we have two solutions,

x = 5 or x = −12.

Because length must be a positive number, we eliminate −12 from consideration. Thus, the length of the first leg is x = 5 meters. The length of the second leg is x+7, or 12 meters.

Check. Checking is an easy matter. The legs are 5 and 12 meters, respectively, and the hypotenuse is 13 meters. Note that the second leg is 7 meters longer than the first. Also,

$$5^2 + 12^2 = 25 + 144 = 169\ ,$$

which is the square of 13.

The integral sides of the triangle in the previous example, 5, 12, and 13, are an example of a Pythagorean Triple.

PYTHAGOREAN TRIPLE

A set of positive integers a, b, and c, is called a Pythagorean Triple if they satisfy the Pythagorean Theorem; that is, if $a^2 + b^2 = c^2$.

If the greatest common factor of a, b, and c is 1, then the triple (a, b, c) is called a **primitive** Pythagorean Triple.

Thus, for example, the Pythagorean Triple (5, 12, 13) is primitive. Let's look at another example.

Example 7.4.5

If (a,b,c) is a Pythagorean Triple, show that any positive integral multiple is also a Pythagorean Triple.

Thus, if the positive integers (a, b, c) is a Pythagorean Triple, we must show that(ka, kb, kc), where k is a positive integer, is also a Pythagorean Triple.

However, we know that

$$a^2 + b^2 = c^2\ .$$

Multiply both sides of this equation by k^2.

$$k^2a^2 + k^2b^2 = k^2c^2$$

This last result can be written

$$(ka)^2 + (kb)^2 = (kc)^2\ .$$

Hence, (ka, kb, kc) is a Pythagorean Triple.

Hence, because (3, 4, 5) is a Pythagorean Triple, you can double everything to get another triple (6, 8, 10). Note that $6^2+8^2=10^2$ is easily checked. Similarly, tripling gives another triple (9, 12, 15), and so on.

In **Example 5**, we showed that (5, 12, 13) was a triple, so we can take multiples to generate other Pythagorean Triples, such as (10, 24, 26) or (15, 36, 39), and so on.

Formulae for generating Pythagorean Triples have been know since antiquity.

? Example 7.4.6

The following formula for generating Pythagorean Triples was published in Euclid's (325–265 BC) Elements, one of the most successful textbooks in the history of mathematics. If m and n are positive integers with m > n, show

$a=m^2-n^2$,

b = 2mn, (7)

$c=m^2+n^2$,

generates Pythagorean Triples.

We need only show that the formulae for *a*, *b*, and *c* satisfy the Pythagorean Theorem. With that is mind, let's first compute \ (*a*^2+*b*^2\).

$$a^2+b^2=(m^2-n^2)^2+(2mn)^2$$
$$=m^4-2m^2n^2+n^4+4m^2n^2$$
$$=m^4+2m^2n^2+n^4$$

On the other hand,

$$c^2=(m^2+n^2)^2$$
$$=m^4+2m^2n^2+n^4 .$$

Hence, $a^2+b^2=c^2$, and the expressions for a, b, and c form a Pythagorean Triple.

It is both interesting and fun to generate Pythagorean Triples with the formulae from **Example 6**. Choose m = 4 and n = 2, then

$$a=m^2-n^2=(4)^2-(2)^2=12 \ ,$$
$$b=2mn=2(4)(2)=16 ,$$
$$c=m^2+n^2=(4)^2+(2)^2=20 \ .$$

It is easy to check that the triple (12, 16, 20) will satisfy $12^2+16^2=20^2$. Indeed, note that this triple is a multiple of the basic (3, 4, 5) triple, so it must also be a Pythagorean Triple.

It can also be shown that if m and n are relatively prime, and are not both odd or both even, then the formulae in **Example 6** will generate a **primitive** Pythagorean Triple. For example, choose m = 5 and n = 2. Note that the greatest common divisor of m = 5 and n = 2 is one, so m and n are relatively prime. Moreover, m is odd while n is even. These values of m and n generate

$$a=m^2-n^2=(5)^2-(2)^2=21 \ ,$$
$$b=2mn=2(5)(2)=20 ,$$
$$c=m^2+n^2=(5)^2+(2)^2=29 \ .$$

Note that

$$21^2+20^2=441+400=841=29^2 .$$

Hence, (21, 20, 29) is a Pythagorean Triple. Moreover, the greatest common divisor of 21, 20, and 29 is one, so (21, 20, 29) is primitive.
The practical applications of the Pythagorean Theorem are numerous.

? Example 7.4.8

A painter leans a 20 foot ladder against the wall of a house. The base of the ladder is on level ground 5 feet from the wall of the house. How high up the wall of the house will the ladder reach?

Consider the triangle in **Figure 7**. The hypotenuse of the triangle represents the ladder and has length 20 feet. The base of the triangle represents the distance of the base of the ladder from the wall of the house and is 5 feet in length. The vertical leg of the triangle is the distance the ladder reaches up the wall and the quantity we wish to determine.

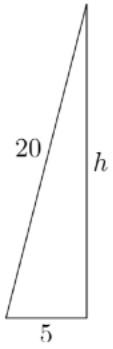


Figure 7. A ladder leans against the wall of a house.

Applying the Pythagorean Theorem,

$$5^2 + h^2 = 20^2 \ .$$

Again, note that the square of the length of the hypotenuse is the quantity that is isolated on one side of the equation.

Next, square, then isolate the term containing h on one side of the equation by subtracting 25 from both sides of the resulting equation.

$$25 + h^2 = 400$$

$$h^2 = 375$$

We need only extract the positive square root.

$$h = \sqrt{375}$$

We could place the solution in simple form, that is, $h = 5\sqrt{15}$, but the nature of the problem warrants a decimal approximation. Using a calculator and rounding to the nearest tenth of a foot,

$$h \approx 19.4.$$

Thus, the ladder reaches about 19.4 feet up the wall.

The Distance Formula

We often need to calculate the distance between two points P and Q in the plane. Indeed, this is such a frequently recurring need, we'd like to develop a formula that will quickly calculate the distance between the given points P and Q. Such a formula is the goal of this last section.

Let P(x1, y1) and Q(x2, y2) be two arbitrary points in the plane, as shown in **Figure 8**(a) and let d represent the distance between the two points.

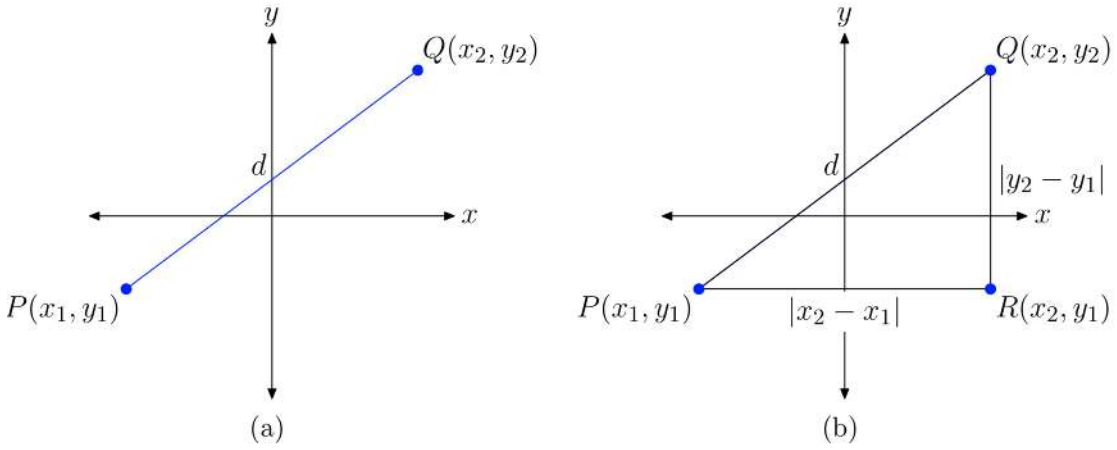


Figure 8. Finding the distance between the points *P* and *Q*.

To find the distance d, first draw the right triangle △PQR, with legs parallel to the axes, as shown in **Figure 8**(b). Next, we need to find the lengths of the legs of the right triangle △PQR.

- The distance between P and R is found by subtracting the x coordinate of P from the x-coordinate of R and taking the absolute value of the result. That is, the distance between P and R is $|x_2 - x_1|$.
- The distance between R and Q is found by subtracting the y-coordinate of R from the y-coordinate of Q and taking the absolute value of the result. That is, the distance between R and Q is $|y_2 - y_1|$.

We can now use the Pythagorean Theorem to calculate d. Thus,

$$d^2 = (|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 \quad .$$

However, for any real number a,

$$(|a|)^2 = |a| \cdot |a| = |a^2| = a^2 \ ,$$

because a2 is nonnegative. Hence, $(|x_2 - x_1|)^2 = (x_2 - x_1)^2 and (|y_2 - y_1|)^2 = (y_2 - y_1)^2$ and we can write

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \quad .$$

Taking the positive square root leads to the Distance Formula.

THE DISTANCE FORMULA

Let P (x1, y1) and Q(x2, y2) be two arbitrary points in the plane. The distance d between the points P and Q is given by the formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (9)

The direction of subtraction is unimportant. Because you square the result of the subtraction, you get the same response regardless of the direction of subtraction (e.g.\((5 − 2)^2 = (2 − 5)^2\)). Thus, it doesn't matter which point you designate as the point P, nor does it matter which point you designate as the point *Q*. Simply subtract *x*- coordinates and square, subtract *y*-coordinates and square, add, then take the square root.

Let's look at an example.

? Example 7.4.10

Find the distance between the points P (−4, −2) and Q (4, 4).

It helps the intuition if we draw a picture, as we have in **Figure 9**. One can now take a compass and open it to the distance between points P and Q. Then you can place your compass on the horizontal axis (or any horizontal gridline) to estimate the distance between the points P and Q. We did that on our graph paper and estimate the distance $d \approx 10$.

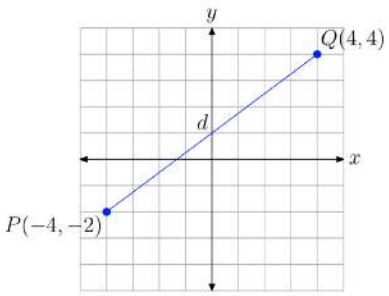


Figure 9. Graphing the distance between P (−4, −2) and Q (4, 4).

Let's now use the distance formula to obtain an exact value for the distance d. With (x_1, y_1) = P (−4, −2) and (x_2, y_2) = Q (4, 4),

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - (-4))^2 + (4 - (-2))^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

= 10.

It's not often that your exact result agrees with your approximation, so never worry if you're off by just a little bit.

Exercise

In **Exercises 1-8**, state whether or not the given triple is a Pythagorean Triple. Give a reason for your answer.

? Exercise 7.4.1

(8, 15, 17)

Answer

Yes, because $8^2 + 15^2 = 17^2$

? Exercise 7.4.2

(7, 24, 25)

? Exercise 7.4.3

(8, 9, 17)

Answer

No, because $8^2 + 9^2 \neq 17^2$

? Exercise 7.4.4

(4, 9, 13)

? Exercise 7.4.5

(12, 35, 37)

Answer

Yes, because $12^2 + 35^2 = 37^2$

Exercise 7.4.6

(12, 17, 29)

Exercise 7.4.7

(11, 17, 28)

Answer

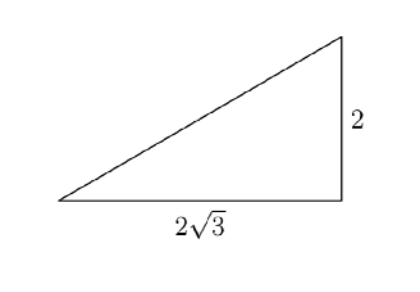
No, because $11^2 + 17^2 \neq 28^2$

Exercise 7.4.8

(11, 60, 61)

In **Exercises 9-16**, set up an equation to model the problem constraints and solve. Use your answer to find the missing side of the given right triangle. Include a sketch with your solution and check your result.

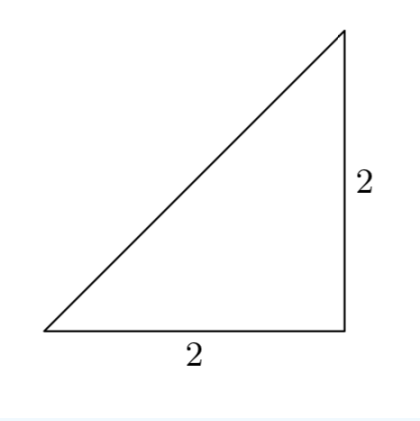
Exercise 7.4.9



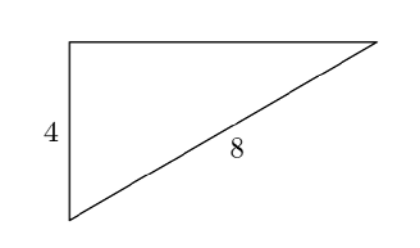
Answer

4

Exercise 7.4.10



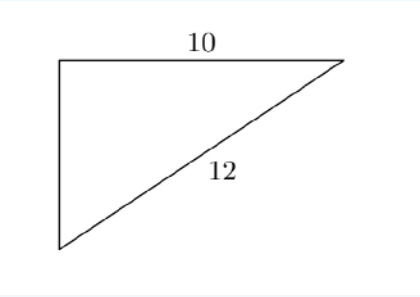
Exercise 7.4.11



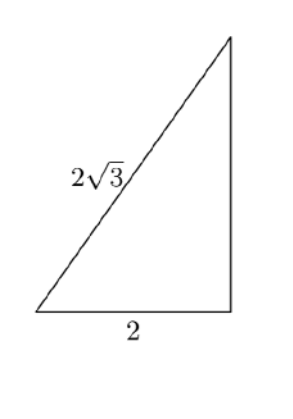
Answer

$4\sqrt{2}$

Exercise 7.4.12



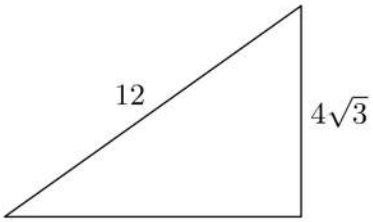
Exercise 7.4.13



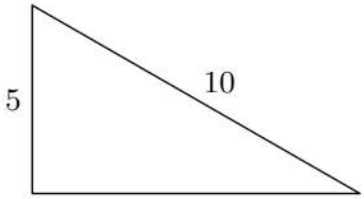
Answer

$2\sqrt{2}$

Exercise 7.4.14



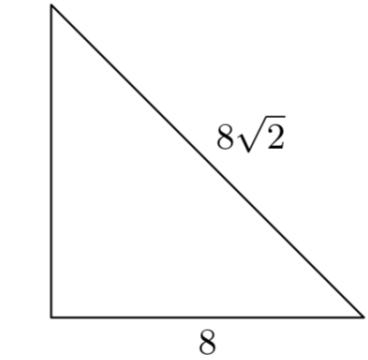
Exercise 7.4.15



Answer

$5\sqrt{3}$

Exercise 7.4.16



In **Exercises 17-20**, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

Exercise 7.4.17

The legs of a right triangle are consecutive positive integers. The hypotenuse has length 5. What are the lengths of the legs?

Answer

The legs have lengths 3 and 4.

? Exercise 7.4.18

The legs of a right triangle are consecutive even integers. The hypotenuse has length 10. What are the lengths of the legs?

? Exercise 7.4.19

One leg of a right triangle is 1 centimeter less than twice the length of the first leg. If the length of the hypotenuse is 17 centimeters, find the lengths of the legs.

Answer

The legs have lengths 8 and 15 centimeters.

? Exercise 7.4.20

One leg of a right triangle is 3 feet longer than 3 times the length of the first leg. The length of the hypotenuse is 25 feet. Find the lengths of the legs.

? Exercise 7.4.21

Pythagoras is credited with the following formulae that can be used to generate Pythagorean Triples.

a = m

$b = \frac{m^2-1}{2}$,

$c = \frac{m^2+1}{2}$

Use the technique of Example 6 to demonstrate that the formulae given above will generate Pythagorean Triples, provided that m is an odd positive integer larger than one. Secondly, generate at least 3 instances of Pythagorean Triples with Pythagoras's formula.

Answer

(3, 4, 5), (5, 12, 13), and (7, 24, 25), with m = 3, 5, and 7, respectively.

? Exercise 7.4.22

Plato (380 BC) is credited with the following formulae that can be used to generate Pythagorean Triples.

$a = 2m$

$b = m^2 - 1$,

$c = m^2 + 1$

Use the technique of Example 6 to demonstrate that the formulae given above will generate Pythagorean Triples, provided that m is a positive integer larger than 1. Secondly, generate at least 3 instances of Pythagorean Triples with Plato's formula.

In **Exercises 23-28**, set up an equation that models the problem constraints. Solve the equation and use the result to answer. Look back and check your answer.

? Exercise 7.4.23

Fritz and Greta are planting a 12-foot by 18-foot rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Approximate your answer to within 0.1 feet.

Answer

21.63 ft

Exercise 7.4.24

Angelina and Markos are planting a 20-foot by 28-foot rectangular garden, and are laying it out using string. They would like to know the length of a diagonal to make sure that right angles are formed. Find the length of a diagonal. Approximate your answer to within 0.1 feet.

Exercise 7.4.25

The base of a 36-foot long guy wire is located 16 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Approximate your answer to within 0.1.

Answer

32.25 ft

Exercise 7.4.26

The base of a 35-foot long guy wire is located 10 feet from the base of the telephone pole that it is anchoring. How high up the pole does the guy wire reach? Approximate your answer to within 0.1 feet.

Exercise 7.4.27

A stereo receiver is in a corner of a 13-foot by 16-foot rectangular room. Speaker wire will run under a rug, diagonally, to a speaker in the far corner. If 3 feet of slack is required on each end, how long a piece of wire should be purchased? Approximate your answer to within 0.1 feet.

Answer

26.62 ft

Exercise 7.4.28

A stereo receiver is in a corner of a 10-foot by 15-foot rectangular room. Speaker wire will run under a rug, diagonally, to a speaker in the far corner. If 4 feet of slack is required on each end, how long a piece of wire should be purchased? Approximate your answer to within 0.1 feet.

In **Exercises 29-38**, use the distance formula to find the exact distance between the given points.

Exercise 7.4.29

$(-8, -9)$ and $(6, -6)$

Answer

$\sqrt{205}$

Exercise 7.4.30

$(1, 0)$ and $(-9, -2)$

Exercise 7.4.31

$(-9, 1)$ and $(-8, 7)$

Answer

$\sqrt{37}$

Exercise 7.4.32

(0, 9) and (3, 1)

Exercise 7.4.33

(6, −5) and (−9, −2)

Answer

$\sqrt{234} = 3\sqrt{26}$

Exercise 7.4.34

(−5, 6) and (1, 4)

Exercise 7.4.35

(−7, 7) and (−3, 6)

Answer

$\sqrt{17}$

Exercise 7.4.36

(−7, −6) and (−2, −4)

Exercise 7.4.37

(4, −3) and (−9, 6)

Answer

$\sqrt{250} = 5\sqrt{10}$

Exercise 7.4.38

(−7, −1) and (4, −5)

In **Exercises 39-42**, set up an equation that models the problem constraints. Solve the equation and use the result to answer the question. Look back and check your result.

Exercise 7.4.39

Find k so that the point (4, k) is $2\sqrt{2}$ units away from the point (2, 1).

Answer

k = 3, −1.

Exercise 7.4.40

Find k so hat the point (k, 1) is $2\sqrt{2}$ units away from the point (0, −1)

Exercise 7.4.41

Find k so hat the point (k, 1) is $\sqrt{17}$ units away from the point (2, −3)

Answer

k = 1, 3.

Exercise 7.4.42

Find k so that the point (−1, k) is $\sqrt{13}$ units away from the point (−4, −3).

Exercise 7.4.43

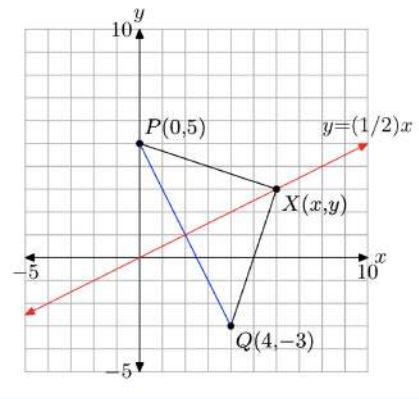
Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Plot the points P (0, 5) and Q (4, −3) on your coordinate system.

a) Plot several points that are equidistant from the points P and Q on your coordinate system. What graph do you get if you plot all points that are equidistant from the points P and Q? Determine the equation of the graph by examining the resulting image on your coordinate system.

b) Use the distance formula to find the equation of the graph of all points that are equidistant from the points P and Q. Hint: Let (x, y) represent an arbitrary point on the graph of all points equidistant from points P and Q. Calculate the distances from the point (x, y) to the points P and Q separately, then set them equal and simplify the resulting equation. Note that this analytical approach should provide an equation that matches that found by the graphical approach in part (a).

Answer

a) In the figure that follows, XP = XQ.



b) $y = \frac{1}{2}x$

Exercise 7.4.44

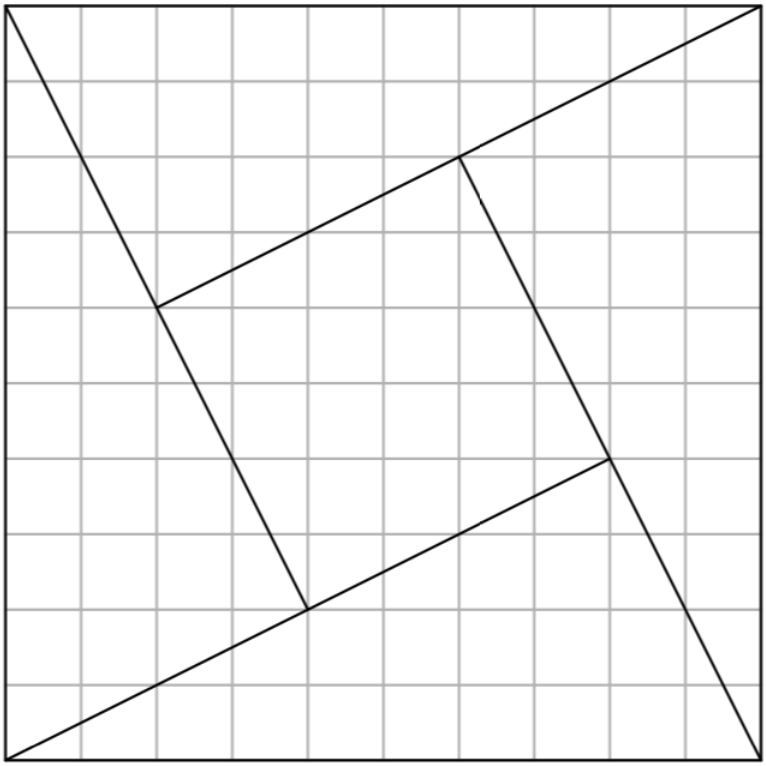
Set up a coordinate system on a sheet of graph paper. Label and scale each axis. Plot the point P (0, 2) and label it with its coordinates. Draw the line y = −2 and label it with its equation.

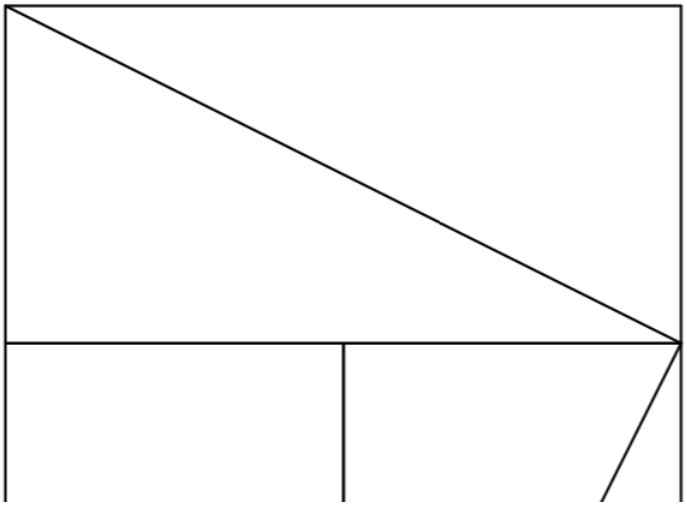
a) Plot several points that are equidistant from the point P and the line y = −2 on your coordinate system. What graph do you get if you plot **all** points that are equidistant from the points P and the line y = −2.

b) Use the distance formula to find the equation of the graph of all points that are equidistant from the points P and the line y = −2. Hint: Let(x, y) represent an arbitrary point on the graph of all points equidistant from points P and the line y = −2. Calculate the distances from the point(x,y) to the points P and the line y = −2 separately, then set them equal and simplify the resulting equation.

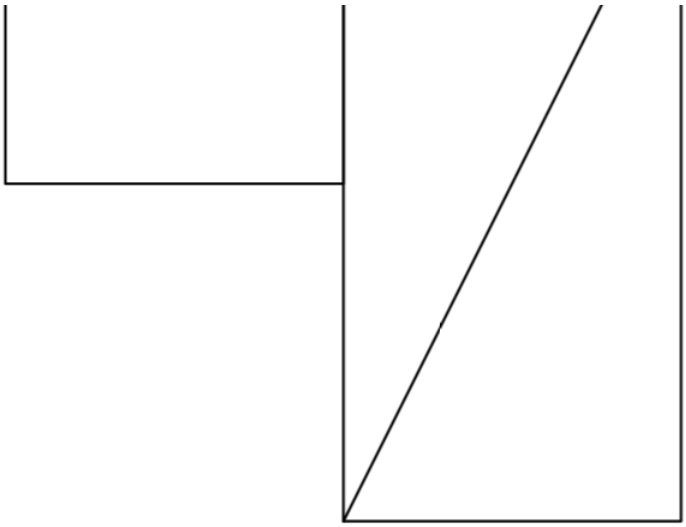
Exercise 7.4.45

Copy the following figure onto a sheet of graph paper. Cut the pieces of the first figure out with a pair of scissors, then rearrange them to form the second figure. Explain how this proves the Pythagorean Theorem.



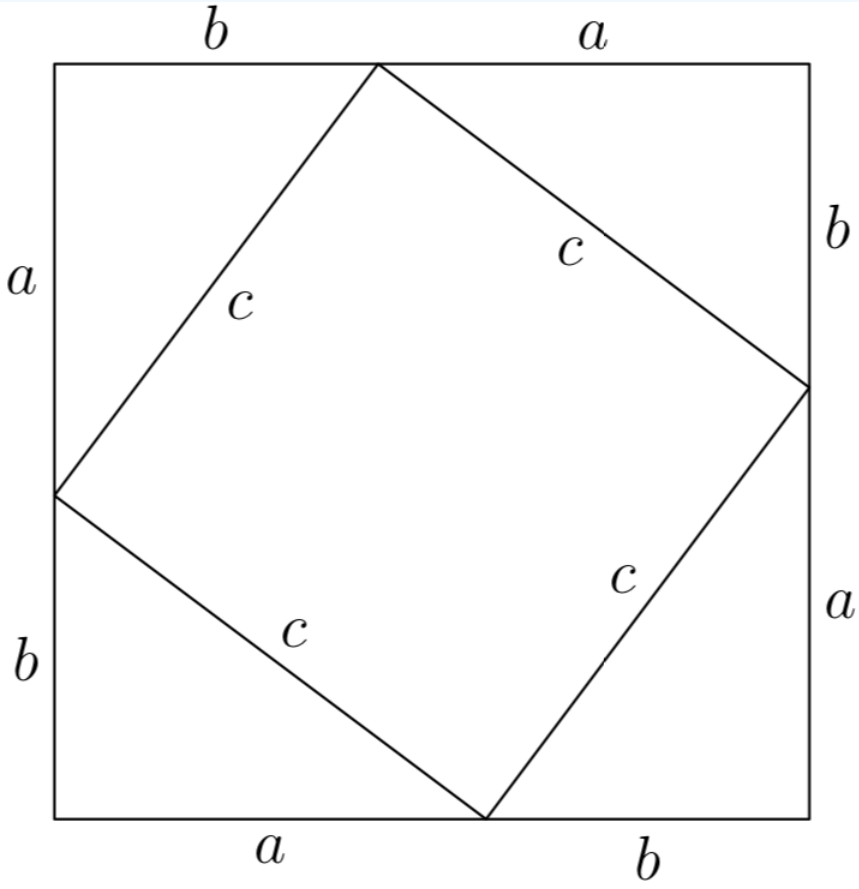


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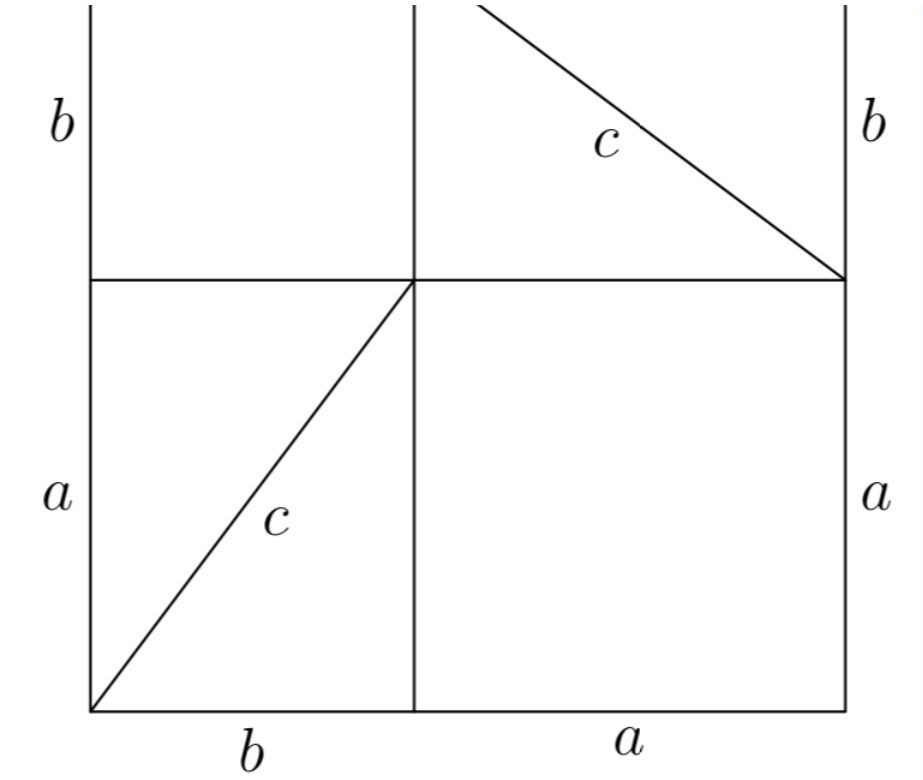


Exercise 7.4.46

Compare this image to the one that follows and explain how this proves the Pythagorean Theorem.







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CHAPTER OVERVIEW

8: Exponents and Radicals

8.1: Rules of Exponents
8.2: Negative Exponents
8.3: Exponents and Roots

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8.1: Rules of Exponents

Learning Objectives

- Simplify expressions using the rules of exponents.
- Simplify expressions involving parentheses and exponents.
- Simplify expressions involving 0 as an exponent.

Product, Quotient, and Power Rule for Exponents

If a factor is repeated multiple times, then the product can be written in exponential form x_n. The positive integer **exponent** n indicates the number of times the **base** x is repeated as a factor

$$x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ times}}$$

For example,

$$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$$

Here the base is 5 and the exponent is 4. Exponents are sometimes indicated with the caret (^) symbol found on the keyboard: 5^ $4 = 5 * 5 * 5 * 5$.

Next consider the product of 2^3 and 2^5,

$$2^3 \cdot 2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{8 \text{ factors of } 2} = 2^8$$

Expanding the expression using the definition produces multiple factors of the base, which is quite cumbersome, particularly when n is large. For this reason, we will develop some useful rules to help us simplify expressions with exponents. In this example, notice that we could obtain the same result by adding the exponents.

$$2^3 \cdot 2^5 = 2^{3+5} = 2^8$$

In general, this describes the product rule for exponents. If m and n are positive integers, then

$$x^m \cdot x^n = x^{m+n} \tag{8.1.1}$$

In other words, when multiplying two expressions with the same base, add the exponents.

Example 8.1.1

Simplify: $10^5 \cdot 10^{18}$.

Solution:

$$\begin{aligned} 10^5 \cdot 10^{18} &= 10^{5+18} \\ &= 10^{23} \end{aligned}$$

Answer:

$$10^{23}$$

In the previous example, notice that we did not multiply the base 10 times itself. When applying the product rule, add the exponents and leave the base unchanged.

Example 8.1.2

Simplify: $x^6 \cdot x^{12} \cdot x$.

Solution:

Recall that the variable x is assumed to have an exponent of 1 : $x = x^1$.

$$\begin{aligned} x^6 \cdot x^{12} \cdot x &= x^6 \cdot x^{12} \cdot x^1 \\ &= x^{6+12+1} \\ &= x^{19} \end{aligned}$$

Answer:

$$x^{19}$$

The base could be any algebraic expression.

Example 8.1.3

Simplify: $(x+y)^9(x+y)^{13}$.

Solution:

Treat the expression $(x+y)$ as the base.

$$\begin{aligned} (x+y)^9(x+y)^{13} &= (x+y)^{9+13} \\ &= (x+y)^{22} \end{aligned}$$

Answer:

$$(x+y)^{22}$$

The commutative property of multiplication allows us to use the product rule for exponents to simplify factors of an algebraic expression.

Example 8.1.4

Simplify: $2x^8y \cdot 3x^4y^7$.

Solution:

Multiply the coefficients and add the exponents of variable factors with the same base.

$$\begin{aligned} 2x^8y \cdot 3x^4y^7 &= 2 \cdot 3 \cdot x^8 \cdot x^4 \cdot y^1 \cdot y^7 && \textit{Commutative property} \\ &= 6 \cdot x^{8+4} \cdot y^{1+7} && \textit{Power rule for exponents} \\ &= 6x^{12}y^8 \end{aligned}$$

Answer:

$$6x^{12}y^8$$

Next, we will develop a rule for division by first looking at the quotient of 2^7 and 2^3.

$$\frac{2^7}{2^3} = \frac{\overbrace{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2}^{7\ factors\ of\ 2}}{\underbrace{2\cdot2\cdot2}_{3\ factors\ of\ 2}} = \frac{\cancel{2\cdot2\cdot2}\cdot2\cdot2\cdot2\cdot2}{\cancel{2\cdot2\cdot2}} = \frac{2\cdot2\cdot2\cdot2}{1} = 2^4$$

Figure 8.1.3

Here we can cancel factors after applying the definition of exponents. Notice that the same result can be obtained by subtracting the exponents.

$$\frac{2^7}{2^3} = 2^{7-3} = 2^4$$

This describes the quotient rule for exponents. If m and n are positive integers and $x \neq 0$, then

$$\frac{x^m}{x^n} = x^{m-n}$$

In other words, when you divide two expressions with the same base, subtract the exponents.

Example 8.1.5

Simplify: $\frac{12y^{15}}{4y^7}$.

Solution:

Divide the coefficients and subtract the exponents of the variable y.

$$\begin{aligned}\frac{12y^{15}}{4y^7} &= \frac{12}{4} \cdot y^{15-7} \\ &= 3y^8\end{aligned}$$

Answer:

$$3y^8$$

Example 8.1.6

Simplify: $\frac{20x^{10}(x+5)^6}{10x^9(x+5)^2}$

Solution:

$$\begin{aligned}\frac{20x^{10}(x+5)^6}{10x^9(x+5)^2} &= \frac{20}{10} \cdot x^{10-9} \cdot (x+5)^{6-2} \\ &= 2x^1(x+5)^4\end{aligned}$$

Answer:

$$2x(x+5)^4$$

Now raise 2^3 to the fourth power as follows:

$$\begin{aligned}\left(2^3\right)^4 &= \underbrace{2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3}_{\text{4 factors of } 2^3} \\ &= \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{12 factors of 2}} \\ &= 2^{12}\end{aligned}$$

After writing the base 2^3 as a factor four times, expand to obtain 12 factors of 2. We can obtain the same result by multiplying the exponents.

$$\left(2^3\right)^4 = 2^{3 \cdot 4} = 2^{12}$$

In general, this describes the power rule for exponents. Given positive integers m and n, then

$$\left(x^m\right)^n = x^{m \cdot n} \tag{8.1.2}$$

In other words, when raising a power to a power, multiply the exponents.

Example 8.1.7

Simplify: $(y^6)^7 = y^{6 \cdot 7}$

Solution:

$$\begin{aligned}(y^6)^7 &= y^{6 \cdot 7} \\ &= y^{42}\end{aligned}$$

Answer:

$$y^{42}$$

To summarize, we have developed three very useful rules of exponents that are used extensively in algebra. If given positive integers m and n, then

- Product rule:

$$x^m \cdot x^n = x^{m+n} \tag{8.1.3}$$

- Quotient rule:

$$\frac{x^m}{x^n} = x^{m-n}, x \neq 0 \tag{8.1.4}$$

- Power rule:

$$(x^m)^n = x^{m \cdot n} \tag{8.1.5}$$

Exercise 8.1.1

Simplify: $y^5 \cdot (y^4)^6$.

Answer

y^{29}

Power Rules for Products and Quotients

Now we consider raising grouped products to a power. For example,

$$\begin{aligned}(xy)^4 &= xy \cdot xy \cdot xy \cdot xy \\ &= x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \quad \textit{Commutative property} \\ &= x^4 \cdot y^4\end{aligned}$$

After expanding, we have four factors of the product xy. This is equivalent to raising each of the original factors to the fourth power. In general, this describes the power rule for a product. If n is a positive integer, then

$$(xy)^n = x^n y^n \tag{8.1.6}$$

Example 8.1.8

Simplify: $(2ab)^7 = 2^7 a^7 b^7$.

Solution:

We must apply the exponent 7 to all the factors, including the coefficient, 2.

$$\begin{aligned}(2ab)^7 &= 2^7 a^7 b^7 \\ &= 128 a^7 b^7\end{aligned}$$

If a coefficient is raised to a relatively small power, then present the real number equivalent, as we did in this example: $2^7 = 128$.

Answer:

$$128a^7b^7$$

In many cases, the process of simplifying expressions involving exponents requires the use of several rules of exponents.

Example 8.1.9

Simplify: $(3xy^3)^4$.

Solution:

$$\begin{aligned}(3xy^3)^4 &= 3^4 \cdot x^4 \cdot (y^3)^4 && \textit{Power rule for products}\\ &= 3^4x^4y^{3\cdot 4} && \textit{Power rule for exponents}\\ &= 81x^4y^{12}\end{aligned}$$

Answer:

$$81x^4y^{12}$$

Example 8.1.10

Simplify: $(4x^2y^5z)^3$.

Solution:

$$\begin{aligned}(4x^2y^5z)^3 &= 4^3 \cdot (x^2)^3 \cdot (y^5)^3 \cdot z^3\\ &= 64x^6y^{15}z^3\end{aligned}$$

Answer:

$$64x^6y^{15}z^3$$

Example 8.1.11

Simplify: $[5(x+y)^3]^3$

Solution:

$$\begin{aligned}{}^3 &= 5^3 \cdot (x+y)^9\\ &= 125(x+y)^9\end{aligned}$$

Answer:

$$125(x+y)^9$$

Next, consider a quotient raised to a power.

$$\begin{aligned}\left(\frac{x}{y}\right)^4 &= \frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y}\cdot\frac{x}{y}\\ &= \frac{x\cdot x\cdot x\cdot x}{y\cdot y\cdot y\cdot y}\\ &= \frac{x^4}{y^4}\end{aligned}$$

Here we obtain four factors of the quotient, which is equivalent to the numerator and the denominator both raised to the fourth power. In general, this describes the power rule for a quotient. If n is a positive integer and $y \neq 0$, then

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \tag{8.1.7}$$

In other words, given a fraction raised to a power, we can apply that exponent to the numerator and the denominator. This rule requires that the denominator is nonzero. We will make this assumption for the remainder of the section.

Example 8.1.12

Simplify: $\left(\frac{3a}{b}\right)^3$

Solution:

First, apply the power rule for a quotient and then the power rule for a product.

$$\begin{aligned}\left(\frac{3a}{b}\right)^3 &= \frac{(3a)^3}{b^3} && \textit{Power rule for a quotient}\\ &= \frac{3^3 \cdot a^3}{b^3} && \textit{Power rule for a product}\\ &= \frac{27a^3}{b^3}\end{aligned}$$

Answer:

$\frac{27a^3}{b^3}$

In practice, we often combine these two steps by applying the exponent to all factors in the numerator and the denominator.

Example 8.1.13

Simplify: $\left(\frac{ab^2}{2c^3}\right)^5$

Solution:

Apply the exponent 5 to all of the factors in the numerator and the denominator.

$$\begin{aligned}\left(\frac{ab^2}{2c^3}\right)^5 &= \frac{a^5(b^2)^5}{2^5(c^3)^5}\\ &= \frac{a^5b^{10}}{32c^{15}}\end{aligned}$$

Answer:

$\frac{a^5b^{10}}{32c^{15}}$

Example 8.1.14

Simplify: $\left(\frac{5x^5(2x-1)^4}{3y^7}\right)^2$

Solution:

$$\begin{aligned}\left(\frac{5x^5(2x-1)^4}{3y^7}\right)^2 &= \frac{(5x^5(2x-1)^4)^2}{(3y^7)^2} && \textit{Power rule for a quotient}\\ &= \frac{5^2 \cdot (x^5)^2 \cdot [(2x-1)^4]^2}{3^2 \cdot (y^7)^2} && \textit{Power rule for products}\\ &= \frac{25x^{10}(2x-1)^8}{9y^{14}} && \textit{Power rule for exponents}\end{aligned}$$

Answer:

$\frac{25x^{10}(2x-1)^8}{9y^{14}}$

It is a good practice to simplify within parentheses before using the power rules; this is consistent with the order of operations.

Example 8.1.15

Simplify: $\left(\frac{-2x^3y^4z}{xy^2}\right)^4$

Solution:

$$\begin{aligned}\left(\frac{-2x^3y^4z}{xy^2}\right)^4 &= (-2\cdot x^{3-1}\cdot y^{4-2}\cdot z)^4 && \textit{Simplify within the parentheses first.}\\ &= (-2\cdot x^2\cdot y^2\cdot z)^4 && \textit{Apply the power rule for a product.}\\ &= (-2)^4\cdot (x^2)^4\cdot (y^2)^4\cdot z^4 && \textit{Apply the power rule for exponents.}\\ &= 16x^8y^8z^4\end{aligned}$$

Answer:

$$16x^8y^8z^4$$

To summarize, we have developed two new rules that are useful when grouping symbols are used in conjunction with exponents. If given a positive integer n, where y is a nonzero number, then

- Power rule for a product:

$$(xy)^n = x^n y^n \tag{8.1.8}$$

- Power rule for a quotient:

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \tag{8.1.9}$$

Exercise 8.1.2

Simplify: $\left(\frac{4x^2(x-y)^3}{3yz^5}\right)^3$

Answer

$\frac{64x^6(x-y)^9}{27y^3z^{15}}$

Zero as an Exponent

Using the quotient rule for exponents, we can define what it means to have 0 as an exponent. Consider the following calculation:

\(\color{Cerulean}{1}\color{black}{=\frac{8}{8}=\frac{2^{3}}{2^{3}}=2^{3-3}=}\color{Cerulean}{2^{0}}

Eight divided by 8 is clearly equal to 1, and when the quotient rule for exponents is applied, we see that a 0 exponent results. This leads us to the definition of zero as an exponent, where $x \neq 0$:

$$x^0 = 1 \tag{8.1.10}$$

It is important to note that 0^0 is undefined. If the base is negative, then the result is still $+1$. In other words, any nonzero base raised to the 0 power is defined to be 1. In the following examples, assume all variables are nonzero.

Example 8.1.16

Simplify:

a. $(-5)^0$
b. -5^0

Solution:

a. Any nonzero quantity raised to the 0 power is equal to 1.

$$(-5)^0 = 1$$

b.In the example -5^0, the base is 5, not -5.

Answer:

1. 1
2. -1

Example 8.1.17

Simplify:

$$(5x^3y^0z^2)^2.$$

Solution:

It is good practice to simplify within the parentheses first.

$$\begin{aligned}(5x^3y^0z^2)^2 &= (5x^3 \cdot 1 \cdot z^2)^2 \\ &= (5x^3z^2)^2 \\ &= 5^2x^{3\cdot 2}z^{2\cdot 2} \\ &= 25x^6z^4\end{aligned}$$

Answer:

$$25x^6z^4$$

Example 8.1.18

Simplify:

$$\left(-\frac{8a^{10}b^5}{5c^{12}d^{14}}\right)^0.$$

Solution:

$$\left(-\frac{8a^{10}b^5}{5c^{12}d^{14}}\right)^0 = 1$$

Answer:

$$1$$

Exercise 8.1.3

Simplify:

$$5x^0 \text{ and } (5x)^0$$

Answer

$5x^0 = 5$ and $(5x)^0 = 1$

Key Takeaways

- The rules of exponents allow you to simplify expressions involving exponents.
- When multiplying two quantities with the same base, add exponents: $x^m \cdot x^n = x^{m+n}$.
- When dividing two quantities with the same base, subtract exponents: $\frac{x^m}{x^n} = x^{m-n}$.
- When raising powers to powers, multiply exponents: $(x^m)^n = x^{m\cdot n}$.
- When a grouped quantity involving multiplication and division is raised to a power, apply that power to all of the factors in the numerator and the denominator: $(xy)^n = x^n y^n$ and $(\frac{x}{y})^n = \frac{x^n}{y^n}$.
- Any nonzero quantity raised to the 0 power is defined to be equal to 1 : $x^0 = 1$.

Exercise 8.1.4 Product, Quotient, and Power Rule for Exponents

Write each expression using exponential form.

1. $(2x)(2x)(2x)(2x)(2x)$
2. $(-3y)(-3y)(-3y)$
3. $-10 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$
4. $12 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$
5. $-6 \cdot (x-1)(x-1)(x-1)$
6. $(9ab)(9ab)(9ab)(a^2-b)(a^2-b)$

Answer

1. $(2x)^5$
3. $-10a^7$
5. $-6(x-1)^3$

Exercise 8.1.5 Product, Quotient, and Power Rule for Exponents

Simplify.

1. $2^7 \cdot 2^5$
2. $3^9 \cdot 3$
3. -2^4
4. $(-2)^4$
5. -3^3
6. $(-3)^4$
7. $10^{13} \cdot 10^5 \cdot 10^4$
8. $10^8 \cdot 10^7 \cdot 10$
9. $\frac{5^{12}}{5^2}$
10. $\frac{10^7}{10^{10}}$
11. $\frac{10^{12}}{10^9}$
12. $(7^3)^5$
13. $(4^8)^4$
14. $10^6 \cdot (10^5)^4$

Answer

1. 2^{12}
3. -16
5. -27
7. 10^{22}
9. 5^{10}
11. 10^3
13. 4^{32}

Exercise 8.1.6 Product, Quotient, and Power Rule for Exponents

Simplify.

1. $(-x)^6$

2. $a^5 \cdot (-a)^2$
3. $x^3 \cdot x^5 \cdot x$
4. $y^5 \cdot y^4 \cdot y^2$
5. $(a^5)^2 \cdot (a^3)^4 \cdot a$
6. $(x+1)^4 (y^5)^4 \cdot y^2$
7. $(x+1)^5 (x+1)^8$
8. $(2a-b)^{12} (2a-b)^9$
9. $\frac{(3x-1)^5}{(3x-1)^2}$
10. $\frac{(a-5)^{37}}{(a-5)^{13}}$
11. $xy^2 \cdot x^2 y$
12. $3x^2 y^3 \cdot 7xy^5$
13. $-8a^2 b \cdot 2ab$
14. $-3ab^2 c^3 \cdot 9a^4 b^5 c^6$
15. $2a^2 b^4 c(-3abc)$
16. $5a^2 (b^3)^3 c^3 \cdot (-2) 2a^3 (b^2)^4$
17. $2x^2 (x+y)^5 \cdot 3x^5 (x+y)^4$
18. $-5xy^6 (2x-1)^6 \cdot x^5 y (2x-1)^3$
19. $x^2 y \cdot xy^3 \cdot x^5 y^5$
20. $-2x^{10} y \cdot 3x^2 y^{12} \cdot 5xy^3$
21. $3^2 x^4 y^2 z \cdot 3xy^4 z^4$
22. $(-x^2)^3 (x^3)^2 (x^4)^3$
23. $a^{10} \cdot \frac{(a^6)^3}{a^3}$
24. $\frac{10x^9 (x^3)^5}{2x^5}$
25. $\frac{a^6 b^3}{a^2 b^2}$
26. $\frac{m^{10} n^7}{m^3 n^4}$
27. $\frac{20x^5 y^{12} z^3}{10x^2 y^{10} z}$
28. $\frac{-24a^{16} b^{12} c^3}{6a^6 b^{11} c}$
29. $\frac{16x^4 (x+2)^3}{4x(x+2)}$
30. $\frac{50y^2 (x+y)^{20}}{10y(x+y)^{17}}$

Answer

1. x^6
3. x^9
5. a^{23}
7. $(x+1)^{13}$
9. $(3x-1)^3$
11. $x^3 y^3$
13. $-16a^3 b^2$
15. $-6a^3 b^5 c^2$
17. $6x^7 (x+y)^9$
19. $x^8 y^9$
21. $27x^5 y^6 z^5$
23. a^{25}

25. a^4b

27. $2x^3y^2z^2$

29. $4x^3(x+2)^2$

Exercise 8.1.7 Power Rules for Products and Quotients

Simplify.

1. $(2x)^5$
2. $(-3y)^4$
3. $(-xy)^3$
4. $(5xy)^3$
5. $(-4abc)^2$
6. $\left(\frac{7}{2x}\right)^2$
7. $-\left(\frac{5}{3y}\right)^3$
8. $(3abc)^3$
9. $\left(\frac{-2xy}{3z}\right)^4$
10. $\left(\frac{5y}{(2x-1)x}\right)^3$
11. $(3x^2)^3$
12. $(-2x^3)^2$
13. $(xy^5)^7$
14. $(x^2y^{10})^2$
15. $\left(\frac{3x^2}{y}\right)^3$
16. $(2x^2y^3z^4)^5$
17. $\left(\frac{-7ab^4}{c^2}\right)^2$
18. $[x^5y^4(x+y)^4]^5$
19. $[2y(x+1)^5]^3$
20. $(ab^3)^3$
21. $\left(\frac{5a^2}{3b}\right)^4$
22. $\left(\frac{-2x^3}{3y^2}\right)^2$
23. $\left(\frac{-x^2}{y^3}\right)^3$
24. $\left(\frac{ab^2}{3c^3d^2}\right)^4$
25. $\left(\frac{2x^7y}{(x-1)^3z^5}\right)^6$
26. $(2x^4)^3\cdot(x^5)^2$
27. $(x^3y)^2\cdot(xy^4)^3$
28. $(-2a^2b^3)^2\cdot(2a^5b)^4$
29. $(-a^2b)^3(3ab^4)^4$
30. $(2x^3(x+y)^4)^5\cdot(2x^4(x+y)^2)^3$
31. $\left(\frac{-3x^5y^4}{xy^2}\right)^3$
32. $\left(\frac{-3x^5y^4}{xy^2}\right)^2$
33. $\left(\frac{-25x^{10}y^{15}}{5x^5y^{10}}\right)^3$

34. $\left(\frac{10x^3y^5}{5xy^2}\right)^2$
35. $\left(\frac{-24ab^3}{6bc}\right)^5$
36. $\left(\frac{-2x^3y^{16}}{x^2y}\right)^2$
37. $\left(\frac{30ab^3}{3abc}\right)^3$
38. $\left(\frac{3s^3t^2}{2s^2t}\right)^3$
39. $\left(\frac{6xy^5(x+y)^6}{3y^2z(x+y)^2}\right)^5$
40. $\left(\frac{-64a^5b^{12}c^2(2ab-1)^{14}}{32a^2b^{10}c^2(2ab-1)^7}\right)^4$
41. The probability of tossing a fair coin and obtaining n heads in a row is given by the formula $P=(12)^n$. Determine the probability, as a percent, of tossing 5 heads in a row.
42. The probability of rolling a single fair six-sided die and obtaining n of the same faces up in a row is given by the formula $P=(16)^n$. Determine the probability, as a percent, of obtaining the same face up two times in a row.
43. If each side of a square measures $2x^3$ units, then determine the area in terms of the variable x.
44. If each edge of a cube measures $5x^2$ units, then determine the volume in terms of the variable x.

Answer

1. $32x^5$
3. $-x^3y^3$
5. $16a^2b^2c^2$
7. $-\frac{125}{27y^3}$
9. $\frac{16x^4y^4}{81z^4}$
11. $27x^6$
13. x^7y^{35}
15. $\frac{27x^6}{y^3}$
17. $\frac{49a^2b^8}{c^4}$
19. $8y^3(x+1)^{15}$
21. $\frac{625a^8}{81b^4}$
23. $-\frac{x^6}{y^9}$
25. $\frac{64x^{42}y^6}{(x-1)^{18}z^{30}}$
27. x^9y^{14}
29. $-81a^{10}b^{19}$
31. $-27x^{12}y^6$
33. $-125x^{15}y^{15}$
35. $\frac{-1024a^5b^{10}}{c^5}$
37. $\frac{1000b^6}{c^3}$
39. $\frac{32x^5y^{15}(x+y)^{20}}{z^5}$

41. $3\frac{1}{8}\%$

43. $A = 4x^6$

Exercise 8.1.8 Zero Exponents

Simplify. (Assume variables are nonzero.)

1. 7^0
2. $(-7)^0$
3. -10^0
4. $-3^0 \cdot (-7)^0$
5. 8675309^0
6. $5^2 \cdot 3^0 \cdot 2^3$
7. $-3^0 \cdot (-2)^2 \cdot (-3)^0$
8. $\frac{5x^0}{y^2}$
9. $(-3)^2 x^2 y^0 z^5$
10. $-3^2 (x^3)^2 y^2 (z^3)^0$
11. $2x^3 y^0 z \cdot 3x^0 y^3 z^5$
12. $-3ab^2 c^0 \cdot 3a^2 (b^3 c^2)^0$
13. $(-8xy^2)^0$
14. $(2x^2 y^3)^0$
15. $\frac{9x^0 y^4}{3y^3}$

Answer

1. 1

3. -1

5. 1

7. -4

9. $9x^2 z^5$

11. $6x^3 y^3 z^6$

13. 1

15. $3y$

Exercise 8.1.9 Discussion Board Topics

1. René Descartes (1637) established the usage of exponential form: a^2, a^3, and so on. Before this, how were exponents denoted?
2. Discuss the accomplishments accredited to Al-Karismi.
3. Why is 0^0 undefined?
4. Explain to a beginning student why $3^4 \cdot 3^2 \neq 9^6$.

Answer

1. Answers may vary

3. Answers may vary

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8.2: Negative Exponents

Learning Objectives

- Simplify expressions with negative integer exponents.
- Work with scientific notation.

Negative Exponents

In this section, we define what it means to have negative integer exponents. We begin with the following equivalent fractions:

$$\frac{1}{8} = \frac{4}{32}$$

Notice that $4, 8$, and 32 are all powers of 2. Hence we can write $4 = 2^2, 8 = 2^3, and 32 = 2^5$.

$$\frac{1}{2^3} = \frac{1}{8} = \frac{4}{32} = \frac{2^2}{2^5}$$

If the exponent of the term in the denominator is larger than the exponent of the term in the numerator, then the application of the quotient rule for exponents results in a negative exponent. In this case, we have the following:

$$\frac{1}{2^3} = \frac{1}{8} = \frac{4}{32} = \frac{2^2}{2^5} = 2^{2-5} = 2^{-3}$$

We conclude that $2^{-3} = \frac{1}{2}^3$. This is true in general and leads to the definition of negative exponents. Given any integer n and $x \neq 0$, then

$$x^{-n} = \frac{1}{x^n} \tag{8.2.1}$$

Here $x \neq 0$ because \frac{1}{0}\) is undefined. For clarity, in this section, assume all variables are nonzero.

Simplifying expressions with negative exponents requires that we rewrite the expression with positive exponents.

Example 8.2.1

Simplify:

$$10^{-2}.$$

Solution:

$$\begin{aligned} 10^{-2} &= \frac{1}{10^2} \\ &= \frac{1}{100} \end{aligned}$$

Answer:

$$\frac{1}{100}$$

Example 8.2.2

Simplify:

$$(-3)^{-1}.$$

Solution:

$$\begin{aligned} (-3)^{-1} &= \frac{1}{(-3)^1} \\ &= -\frac{1}{3} \end{aligned}$$

Answer:

$$-\frac{1}{3}$$

Example 8.2.3

Simplify:

$$\frac{1}{y^{-3}}.$$

Solution:

$$\begin{aligned}\frac{1}{y^{-3}} &= \frac{1}{\frac{1}{y^3}} \\ &= 1 \cdot \frac{y^3}{1} \\ &= y^3\end{aligned}$$

Answer:

$$y^3$$

At this point we highlight two very important examples,

Simplify $(2x)^{-1}$	*Simplify* $2x^{-1}$
$(2x)^{-1} = \frac{1}{(2x)^1}$	$2x^{-1} = 2 \cdot \frac{1}{x^1}$
$= \frac{1}{2x}$	$= \frac{2}{x}$

Figure 8.2.1

If the grouped quantity is raised to a negative exponent, then apply the definition and write the entire grouped quantity in the denominator. If there is no grouping, then apply the definition only to the base preceding the exponent.

Example 8.2.4

Simplify:

$$(2ab)^{-3}.$$

Solution:

First, apply the definition of −3 as an exponent and then apply the power of a product rule.

$$\begin{aligned}(2ab)^{-3} &= \frac{1}{(2ab)^3} && \textit{Apply the negative exponent.} \\ &= \frac{1}{2^3a^3b^3} && \textit{Apply the power rule for a product.} \\ &= \frac{1}{8a^3b^3}\end{aligned}$$

Answer:

$$\frac{1}{8a^3b^3}$$

Example 8.2.5

Simplify:

$$(-3xy^3)^{-2}.$$

Solution:

$$\begin{aligned}(-3xy^3)^{-2} &= \frac{1}{(-3xy^3)^2} \\ &= \frac{1}{(-3)^2x^2(y^3)^2} \\ &= \frac{1}{9x^2y^6}\end{aligned}$$

Answer:

$\frac{1}{9x^2y^6}$

Example 8.2.6

Simplify:

$\frac{x^{-3}}{y^{-4}}$.

Solution:

$$\frac{x^{-3}}{y^{-4}} = \frac{\frac{1}{x^3}}{\frac{1}{y^4}} = \frac{1}{x^3} \cdot \frac{y^4}{1} = \frac{y^4}{x^3}$$

Answer:

$\frac{y^4}{x^3}$

The previous example suggests a property of quotients with negative exponents. If given any integers m and n, where $x \neq 0$ and $y \neq 0$, then

$$\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n} \tag{8.2.2}$$

In other words, negative exponents in the numerator can be written as positive exponents in the denominator, and negative exponents in the denominator can be written as positive exponents in the numerator.

Example 8.2.7

Simplify:

$\frac{-2x^{-5}y^3}{z^{-2}}$.

Solution:

Take care with the coefficient -2; recognize that this is the base and that the exponent is actually $+1$: $-2 = (-2)^1$. Hence the rules of negative exponents do not apply to this coefficient; leave it in the numerator.

$$\begin{aligned}\frac{-2x^{-5}y^3}{z^{-2}} &= \frac{-2x^{-5}y^3}{z^{-2}} \\ &= \frac{-2y^3z^2}{x^5}\end{aligned}$$

Answer:

$\frac{-2y^3z^2}{x^5}$

Example 8.2.8

Simplify:

$\frac{(-3x^{-4})^{-3}}{y^{-2}}$.

Solution:

$$\begin{aligned}\frac{(-3x^{-4})^{-3}}{y^{-2}} &= \frac{(-3)^{-3}(x^{-4})^{-3}}{y^{-2}} && \textit{Apply the product to a power rule.}\\ &= \frac{(-3)^{-3}x^{12}}{y^{-2}} && \textit{Power rule}\\ &= \frac{x^{12}y^2}{(-3)^3} && \textit{Negative exponents}\\ &= \frac{x^{12}y^2}{-27}\\ &- \frac{x^{12}y^2}{27}\end{aligned}$$

Answer:

$$-\frac{x^{12}y^2}{27}$$

Example 8.2.9

Simplify:

$$\frac{(3x^2)^{-4}}{(-2y^{-1}z^3)^{-2}}.$$

Solution:

$$\begin{aligned}\frac{(3x^2)^{-4}}{(-2y^{-1}z^3)^{-2}} &= \frac{3^{-4}(x^2)^{-4}}{(-2)^{-2}(y^{-1})^{-2}(z^3)^{-2}} && \textit{Product to a power rule}\\ &= \frac{3^{-4}x^{-8}}{(-2)^{-2}y^2z^{-6}} && \textit{Power rule}\\ &= \frac{(-2)^2z^6}{3^4x^8y^2} && \textit{Negative exponents}\\ &= \frac{4z^6}{81x^8y^2}\end{aligned}$$

Answer:

$$\frac{4z^6}{81x^8y^2}$$

Example 8.2.10

Simplify:

$$\frac{(5x^2y)^3}{x^{-5}y^{-3}}.$$

Solution:

First, apply the power of a product rule and then the quotient rule.

$$\frac{(5x^2)^3}{x^{-5}y^{-3}} = \frac{5^3x^6y^3}{x^{-5}y^{-3}} = 5^3x^{6-(-5)}y^{3-(-3)} = 5^3x^{6+5}y^{3+3} = 125x^{11}y^6$$

Answer:

$$125x^{11}y^6$$

To summarize, we have the following rules for negative integer exponents with nonzero bases:

Table 8.2.1

Negative exponents:	$x^{-n} = \frac{1}{x^n}$

Quotients with negative exponents:	$\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n}$

? Exercise 8.2.1

Simplify:

$$\frac{(-5xy^{-3})^{-2}}{5x^4y^{-4}}.$$

Answer

$\frac{y^{10}}{125x^6}$

Scientific Notation

Real numbers expressed in scientific notation have the form

$$a \times 10^n$$

where n is an integer and $1 \leq a < 10$. This form is particularly useful when the numbers are very large or very small. For example,

$$9,460,000,000,000,000m = 9.46 \times 10^{15}m \qquad \textit{One light year}$$
$$0.000000000025m = 2.5 \times 10^{-11}m \qquad \textit{Radius of a hydrogen atom}$$

It is cumbersome to write all the zeros in both of these cases. Scientific notation is an alternative, compact representation of these numbers. The factor 10^n indicates the power of 10 to multiply the coefficient by to convert back to decimal form:

$$9.46\times10^{15} = 9.46\times1,\overbrace{000,000,000,000,000}^{15\ zeros} = 9,460,000,000,000,000$$

Figure 8.2.2

This is equivalent to moving the decimal in the coefficient fifteen places to the right. A negative exponent indicates that the number is very small:

$$2.5\times10^{-11} = 2.5\times\frac{1}{10^{11}} = \frac{2.5}{\underbrace{100,000,000,000}_{11\ zeros}} = 0.000000000025$$

Figure 8.2.3

This is equivalent to moving the decimal in the coefficient eleven places to the left.

Converting a decimal number to scientific notation involves moving the decimal as well. Consider all of the equivalent forms of $0.00563$with factors of 10 that follow:

$$\begin{aligned} 0.00563 &= 0.0563 \times 10^{-1} \\ &= 0.563 \times 10^{-2} \\ &= 5.63 \times 10^{-3} \\ &= 56.3 \times 10^{-4} \\ &= 563 \times 10^{-5} \end{aligned}$$

While all of these are equal, 5.63×10^{-3} is the only form considered to be expressed in scientific notation. This is because the coefficient 5.63 is between 1 and 10 as required by the definition. Notice that we can convert 5.63×10^{-3} back to decimal form, as a check, by moving the decimal to the left three places.

Example 8.2.11

Write $1,075,000,000,000$ using scientific notation.

Solution:

Here we count twelve decimal places to the left of the decimal point to obtain the number 1.075.

$$1,075,000,000,000 = 1.075 \times 10^{12}$$

Answer:

$$1.075 \times 10^{12}$$

Example 8.2.12

Write 0.000003045 using scientific notation.

Solution:

Here we count six decimal places to the right to obtain 3.045.

$$0.000003045 = 3.045 \times 10^{-6}$$

Answer:

$$3.045 \times 10^{-6}$$

Often we will need to perform operations when using numbers in scientific notation. All the rules of exponents developed so far also apply to numbers in scientific notation.

Example 8.2.13

Multiply:

$$(4.36 \times 10^{-5})(5.3 \times 10^{12}).$$

Solution:

Use the fact that multiplication is commutative and apply the product rule for exponents.

$$\begin{aligned}(4.36 \times 10^{-5})(5.3 \times 10^{12}) &= (4.36 \cdot 5.30) \times (10^{-5} \cdot 10^{12}) \\ &= 23.108 \times 10^{-5+12} \\ &= 2.3108 \times 10^{1} \times 10^{7} \\ &= 2.3108 \times 10^{1+7} \\ &= 2.3108 \times 10^{8}\end{aligned}$$

Answer:

$$2.3108 \times 10^{8}$$

Example 8.2.14

Divide:

$$(3.24 \times 10^{8}) \div (9.0 \times 10^{-3}).$$

Solution:

$$\begin{aligned}\frac{(3.24\times10^{8})}{(9.0\times10^{-3})} &= \left(\frac{3.24}{9.0}\right)\times\left(\frac{10^{8}}{10^{-3}}\right)\\ &= 0.36\times10^{8-(-3)}\\ &= 0.36\times10^{8+3}\\ &= 3.6\times10^{-1}\times10^{11}\\ &= 3.6\times10^{-1+11}\\ &= 3.6\times10^{10}\end{aligned}$$

Answer:

$$3.6\times10^{10}$$

Example 8.2.15

The speed of light is approximately 6.7×10^{8} miles per hour. Express this speed in miles per second.

Solution:

A unit analysis indicates that we must divide the number by $3,600$.

$$\begin{aligned}6.7\times10^{8}\ mph &= \frac{6.7\times10^{8} miles}{1\ \cancel{hour}}\cdot\left(\frac{1\ \cancel{hour}}{60\ \cancel{minutes}}\right)\cdot\left(\frac{1\ \cancel{minutes}}{60 seconds}\right)\\ &= \frac{6.7\times10^{8} miles}{3600 seconds}\\ &= \left(\frac{6.7}{3600}\right)\times10^{8}\\ &\approx 0.0019\times10^{8} \qquad Rounded\ to\ two\ significant\ digits\\ &= 1.9\times10^{-3}\times10^{8}\\ &= 1.9\times10^{-3+8}\\ &= 1.9\times10^{5}\end{aligned}$$

Answer:

The speed of light is approximately 1.9×10^{5} miles per second.

Example 8.2.16

By what factor is the radius of the sun larger than the radius of earth?

$$\begin{aligned}6,300,000m &= 6.3\times10^{6}m \qquad Radius\ of\ Earth\\ 700,000,000m &= 7.0\times10^{8}m \qquad Radius\ of\ the\ Sun\end{aligned}$$

Solution:

We want to find the number that when multiplied times the radius of earth equals the radius of the sun.

$$\begin{aligned}n\cdot radius\ of\ the\ Earth &= radius\ of\ the\ Sun\\ n &= \frac{radius\ of\ the\ Sun}{radius\ of\ the\ Earth}\end{aligned}$$

Therefore,

$$\begin{aligned}n &= \frac{7.0\times10^{8}m}{6.3\times10^{6}m}\\ &= \frac{7.0}{6.3}\times\frac{10^{8}}{10^{6}}\\ &\approx 1.1\times10^{8-6}\\ &= 1.1\times10^{2}\\ &= 110\end{aligned}$$

? Exercise 8.2.2

Divide:

$$(6.75 \times 10^{-8}) \div (9 \times 10^{-17}) \ .$$

Answer

7.5×10^{8}

Key Takeaways

- Expressions with negative exponents in the numerator can be rewritten as expressions with positive exponents in the denominator.
- Expressions with negative exponents in the denominator can be rewritten as expressions with positive exponents in the numerator.
- Take care to distinguish negative coefficients from negative exponents.
- Scientific notation is particularly useful when working with numbers that are very large or very small.

? Exercise 8.2.3 Negative Exponents

Simplify. (Assume variables are nonzero.)

1. 5^{-1}
2. 5^{-2}
3. $(-7)^{-1}$
4. -7^{-1}
5. $\frac{1}{2}^{-3}$
6. $\frac{5}{3}^{-2}$
7. $(\frac{3}{5})^{-2}$
8. $(\frac{1}{2})^{-5}$
9. $(-\frac{2}{3})^{-4}$
10. $(-\frac{1}{3})^{-3}$
11. x^{-4}
12. y^{-1}
13. $3x^{-5}$
14. $(3x)^{-5}$
15. $\frac{1}{y^{-3}}$
16. $\frac{5}{2}x^{-1}$
17. $\frac{x^{-1}}{y^{-2}}$
18. $\frac{1}{(x-y)^{-4}}$
19. $\frac{x^2y^{-3}}{z^{-5}}$
20. $\frac{x}{y^{-3}}$
21. $(ab)^{-1}$
22. $\frac{1}{(ab)^{-1}}$
23. $-5x^{-3}y^2z^{-4}$
24. $\frac{3}{-2x^3y^{-5}z}$
25. $3x^{-4}y^2 \cdot 2x^{-1}y^3$
26. $-10a^2b^3 \cdot 2a^{-8}b^{-10}$
27. $(2a^{-3})^{-2}$
28. $(-3x^2)^{-1}$
29. $(5a^2b^{-3}c)^{-2}$

30. $(7r^3s^{-5}t)^{-3}$
31. $(-2r^2s^0t^{-3})^{-1}$
32. $(2xy^{-3}z^2)^{-3}$
33. $(-5a^2b^{-3}c^0)^4$
34. $(-x^{-2}y^3z^{-4})^{-7}$
35. $(\frac{1}{2}x^{-3})^{-5}$
36. $(2xy^2)^{-2}$
37. $(x^2y^{-1})^{-4}$
38. $(-3a^2bc^5)^{-5}$
39. $(\frac{20x^{-3}y^2}{5yz^{-1}})^{-1}$
40. $(\frac{4r^5s^{-3}t^4}{2r^3st^0})^{-3}$
41. $(\frac{2xy^3z^{-1}}{y^2z^3})^{-3}$
42. $(-\frac{3a^2bc}{ab^0c^4})^2$
43. $(\frac{-xyz}{x^4y^{-2}z^3})^{-4}$
44. $(-\frac{125x^{-3}y^4z^{-5}}{5x^2y^4(x+y)^3})^0$
45. $(x^n)^{-2}$
46. $(x^ny^n)^{-2}$

Answer

1. $\frac{1}{5}$
3. $-\frac{1}{7}$
5. 8
7. $\frac{25}{9}$
9. $\frac{81}{16}$
11. $\frac{1}{x^4}$
13. $3x^5$
15. y^3
17. $\frac{y^2}{x}$
19. $\frac{x^2z^5}{y^3}$
21. $\frac{1}{ab}$
23. $\frac{-5y^2}{x^3z^4}$
25. $\frac{6y^5}{x^5}$
27. $\frac{a^6}{4}$
29. $\frac{b^6}{25a^4c^2}$
31. $-\frac{t^3}{2r^2}$
33. $\frac{625a^8}{b^{12}}$
35. $32x^{15}$
37. $\frac{y^4}{x^8}$

39. $\frac{x^3}{4yz}$

41. $\frac{z^{12}}{8x^3y^3}$

43. $\frac{x^{12}z^8}{y^{12}}$

45. $\frac{1}{x^{2n}}$

Exercise 8.2.4 Negative Exponents

The value in dollars of a new MP3 player can be estimated by using the formula $V = 100(t+1)^{-1}$, where t is the number of years after purchase.

1. How much was the MP3 player worth new?
2. How much will the MP3 player be worth in 1 year?
3. How much will the MP3 player be worth in 4 years?
4. How much will the MP3 player be worth in 9 years?
5. How much will the MP3 player be worth in 99 years?
6. According to the formula, will the MP3 ever be worthless? Explain.

Answer

1. \$100

3. \$20

5. \$1

Exercise 8.2.5 Scientific Notation

Convert to a decimal number.

1. 9.3×10^9
2. 1.004×10^4
3. 6.08×10^{10}
4. 3.042×10^7
5. 4.01×10^{-7}
6. 1.0×10^{-10}
7. 9.9×10^{-3}
8. 7.0011×10^{-5}

Answer

1. $9,300,000,000$

3. $60,800,000,000$

5. 0.000000401

7. 0.0099

Exercise 8.2.6 Scientific Notation

Rewrite using scientific notation.

1. $500,000,000$
2. $407,300,000,000,000$
3. $9,740,000$
4. $100,230$

5. 0.0000123
6. 0.000012
7. 0.000000010034
8. 0.99071

Answer

1. 5×10^{8}
3. 9.74×10^{6}
5. 1.23×10^{-5}
7. 1.0034×10^{-8}

? Exercise 8.2.7 Scientific Notation

Perform the indicated operations.

1. $(3 \times 10^{5})(9 \times 10^{4})$
2. $(8 \times 10^{-22})(2 \times 10^{-12})$
3. $(2.1 \times 10^{-19})(3.0 \times 10^{8})$
4. $(4.32 \times 10^{7})(1.50 \times 10^{-18})$
5. $9.12 \times 10^{-9} 3.2 \times 10^{10}$
6. $1.15 \times 10^{9} 2.3 \times 10^{-11}$
7. $1.004 \times 10^{-8} 2.008 \times 10^{-14}$
8. $3.276 \times 10^{25} 5.2 \times 10^{15}$
9. $59,000,000,000,000 \times 0.000032$
10. $0.0000000000432 \times 0.0000000000673$
11. $1,030,000,000,000,000,000 \div 2,000,000$
12. $6,045,000,000,000,000 \div 0.00000005$
13. The population density of earth refers to the number of people per square mile of land area. If the total land area on earth is 5.751×10^{7} square miles and the population in 2007 was estimated to be 6.67×10^{9} people, then calculate the population density of earth at that time.
14. In 2008 the population of New York City was estimated to be 8.364 million people. The total land area is 305 square miles. Calculate the population density of New York City.
15. The mass of earth is 5.97×10^{24} kilograms and the mass of the moon is 7.35×10^{22} kilograms. By what factor is the mass of earth greater than the mass of the moon?
16. The mass of the sun is 1.99×10^{30} kilograms and the mass of earth is 5.97×10^{24} kilograms. By what factor is the mass of the sun greater than the mass of earth? Express your answer in scientific notation.
17. The radius of the sun is 4.322×10^{5} miles and the average distance from earth to the moon is 2.392×10^{5} miles. By what factor is the radius of the sun larger than the average distance from earth to the moon?
18. One light year, 9.461×10^{15} meters, is the distance that light travels in a vacuum in one year. If the distance to the nearest star to our sun, Proxima Centauri, is estimated to be 3.991×10^{16} meters, then calculate the number of years it would take light to travel that distance.
19. It is estimated that there are about 1 million ants per person on the planet. If the world population was estimated to be 6.67 billion people in 2007, then estimate the world ant population at that time.
20. The sun moves around the center of the galaxy in a nearly circular orbit. The distance from the center of our galaxy to the sun is approximately $26,000$ light years. What is the circumference of the orbit of the sun around the galaxy in meters?
21. Water weighs approximately 18 grams per mole. If one mole is about 6×10^{23} molecules, then approximate the weight of each molecule of water.
22. A gigabyte is 1×10^{9} bytes and a megabyte is 1×10^{6} bytes. If the average song in the MP3 format consumes about 4.5 megabytes of storage, then how many songs will fit on a 4-gigabyte memory card?

Answer

1. 2.7×10^{10}

3. 6.3×10^{-11}

5. 2.85×10^{-19}

7. 5×10^{5}

9. 1.888×10^{9}

11. 5.15×10^{11}

13. About 116 people per square mile

15. 81.2

17. 1.807

19. 6.67×10^{15} ants

21. 3×10^{-23} grams

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8.3: Exponents and Roots

Learning Objectives

- understand and be able to read exponential notation
- understand the concept of root and be able to read root notation
- be able to use a calculator having the y^x key to determine a root

Exponential Notation

Definition: Exponential Notation

We have noted that multiplication is a description of repeated addition. **Exponential notation** is a description of repeated multiplication.

Suppose we have the repeated multiplication

$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

Definition: Exponent

The factor 8 is repeated 5 times. Exponential notation uses a *superscript* for the number of times the factor is repeated. The superscript is placed on the repeated factor, 8^5, in this case. The superscript is called an **exponent**.

Definition: The Function of an Exponent

An **exponent** records the number of identical factors that are repeated in a multiplication.

Sample Set A

Write the following multiplication using exponents.

$3 \cdot 3$. Since the factor 3 appears 2 times, we record this as

Solution

3^2

Sample Set A

$62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62$. Since the factor 62 appears 9 times, we record this as

Solution

62^9

Expand (write without exponents) each number.

Sample Set A

12^4. The exponent 4 is recording 4 factors of 12 in a multiplication. Thus,

Solution

$12^4 = 12 \cdot 12 \cdot 12 \cdot 12$

Sample Set A

706^3. The exponent 3 is recording 3 factors of 706 in a multiplication. Thus,

Solution

$706^3 = 706 \cdot 706 \cdot 706$

Practice Set A

Write the following using exponents.

$37 \cdot 37$

Answer

37^2

Practice Set A

$16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$

Answer

16^5

Practice Set A

$9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Answer

9^{10}

Write each number without exponents.

Practice Set A

85^3

Answer

$85 \cdot 85 \cdot 85$

Practice Set A

4^7

Answer

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Practice Set A

$1,739^2$

Answer

$1,739 \cdot 1,739$

Reading Exponential Notation

In a number such as 8^5.

Base
8 is called the **base**.

Exponent, Power
5 is called the **exponent**, or **power**. 8^5 is read as "eight to the fifth power," or more simply as "eight to the fifth," or "the fifth power of eight."

Squared
When a whole number is raised to the second power, it is said to be **squared**. The number 5^2 can be read as

5 to the second power, or
5 to the second, or
5 squared.

Cubed
When a whole number is raised to the third power, it is said to be **cubed**. The number 5^3 can be read as

5 to the third power, or
5 to the third, or
5 cubed.

When a whole number is raised to the power of 4 or higher, we simply say that that number is raised to that particular power. The number 5^8 can be read as

5 to the eighth power, or just
5 to the eighth.

Roots

In the English language, the word "root" can mean a source of something. In mathematical terms, the word "root" is used to indicate that one number is the source of another number through repeated multiplication.

Square Root
We know that $49 = 7^2$, that is, $49 = 7 \cdot 7$. Through repeated multiplication, 7 is the source of 49. Thus, 7 is a root of 49. Since two 7's must be multiplied together to produce 49, the 7 is called the second or **square root** of 49.

Cube Root
We know that $8 = 2^3$, that is, $8 = 2 \cdot 2 \cdot 2$. Through repeated multiplication, 2 is the source of 8. Thus, 2 is a root of 8. Since three 2's must be multiplied together to produce 8, 2 is called the third or **cube root** of 8.

We can continue this way to see such roots as fourth roots, fifth roots, sixth roots, and so on.

Reading Root Notation

There is a symbol used to indicate roots of a number. It is called the radical sign $\sqrt[n]{}$

The Radical Sign $\sqrt[n]{}$

The symbol $\sqrt[n]{}$ is called a **radical sign** and indicates the nth root of a number.

We discuss *particular roots* using the radical sign as follows:

Square Root
$\sqrt[2]{\text{number}}$ indicates the **square root** of the number under the radical sign. It is customary to drop the 2 in the radical sign when discussing square roots. The symbol $\sqrt{}$ is understood to be the square root radical sign.
$\sqrt{49} = 7$ since $7 \cdot 7 = 7^2 = 49$

Cube Root
$\sqrt[3]{\text{number}}$ indicates the **cube root** of the number under the radical sign.
$\sqrt[3]{8} = 2$ since $2 \cdot 2 \cdot 2 = 2^3 = 8$

Fourth Root

$\sqrt[4]{\text{number}}$ indicates the **fourth root** of the number under the radical sign.

$\sqrt[4]{81} = 3$ since $3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = 81$

In an expression such as $\sqrt[5]{32}$

Radical Sign

$\sqrt{}$ is called the **radical sign**.

Index

5 is called the **index**. (The index describes the indicated root.)

Radicand

32 is called the **radicand**.

Radical

$\sqrt[5]{32}$ is called a **radical** (or radical expression).

Sample Set B

Find each root.

$\sqrt{25}$ To determine the square root of 25, we ask, "What whole number squared equals 25?" From our experience with multiplication, we know this number to be 5. Thus,

Solution

$\sqrt{25} = 5$

Check: $5 \cdot 5 = 5^2 = 25$

Sample Set B

$\sqrt[5]{32}$ To determine the fifth root of 32, we ask, "What whole number raised to the fifth power equals 32?" This number is 2.

Solution

$\sqrt[5]{32} = 2$

Check: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

Practice Set B

Find the following roots using only a knowledge of multiplication.

$\sqrt{64}$

Answer

8

Practice Set B

$\sqrt{100}$

Answer

10

Practice Set B

$\sqrt[3]{64}$

Answer

4

Practice Set B

$\sqrt[6]{64}$

Answer

2

Calculators

Calculators with the $\sqrt{x}$, y^x, and $1/x$ keys can be used to find or approximate roots.

Sample Set C

Use the calculator to find $\sqrt{121}$

Solution

		Display Reads
Type	121	121
Press	$\sqrt{x}$	11

Sample Set C

Find $\sqrt[7]{2187}$.

Solution

		Display Reads
Type	2187	2187
Press	y^x	2187
Type	7	7
Press	$1/x$	.14285714
Press	=	3

$\sqrt[7]{2187} = 3$ (which means that $3^7 = 2187$)

Practice Set C

Use a calculator to find the following roots.

$\sqrt[3]{729}$

Answer

9

Practice Set C

$\sqrt[4]{8503056}$

Answer

54

Practice Set C

$\sqrt{53361}$

Answer

231

Practice Set C

$\sqrt[12]{16777216}$

Answer

4

Exercises

For the following problems, write the expressions using exponential notation.

Exercise 8.3.1

$4 \cdot 4$

Answer

4^2

Exercise 8.3.2

$12 \cdot 12$

Exercise 8.3.3

$9 \cdot 9 \cdot 9 \cdot 9$

Answer

9^4

Exercise 8.3.4

$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Exercise 8.3.5

$826 \cdot 826 \cdot 826$

Answer

826^3

Exercise 8.3.6

$3,021 \cdot 3,021 \cdot 3,021 \cdot 3,021 \cdot 3,021$

Exercise 8.3.7

$\underbrace{6 \cdot 6 \cdot \cdots \cdot 6}_{85 \text{ factors of } 6}$

Answer

6^{85}

Exercise 8.3.8

$\underbrace{2 \cdot 2 \cdot \cdots \cdot 2}_{112 \text{ factors of } 2}$

Exercise 8.3.9

$\underbrace{1 \cdot 1 \cdot \cdots \cdot 1}_{3,008 \text{ factors of } 1}$

Answer

1^{3008}

For the following problems, expand the terms. (Do not find the actual value.)

Exercise 8.3.10

5^3

Exercise 8.3.11

7^4

Answer

$7 \cdot 7 \cdot 7 \cdot 7$

Exercise 8.3.12

15^2

Exercise 8.3.13

117^5

Answer

$117 \cdot 117 \cdot 117 \cdot 117 \cdot 117$

Exercise 8.3.14

61^6

Exercise 8.3.15

30^2

Answer

$30 \cdot 30$

For the following problems, determine the value of each of the powers. Use a calculator to check each result.

Exercise 8.3.16

3^2

Exercise 8.3.17

4^2

Answer

$4 \cdot 4 = 16$

Exercise 8.3.18

1^2

Exercise 8.3.19

10^2

Answer

$10 \cdot 10 = 100$

Exercise 8.3.20

11^2

Exercise 8.3.21

12^2

Answer

$12 \cdot 12 = 144$

Exercise 8.3.22

13^2

Exercise 8.3.23

15^2

Answer

$15 \cdot 15 = 225$

Exercise 8.3.24

1^4

Exercise 8.3.25

3^4

Answer

$3 \cdot 3 \cdot 3 \cdot 3 = 81$

Exercise 8.3.26

7^3

Exercise 8.3.27

10^3

Answer

$10 \cdot 10 \cdot 10 = 1000$

Exercise 8.3.28

100^2

Exercise 8.3.29

8^3

Answer

$8 \cdot 8 \cdot 8 = 512$

Exercise 8.3.30

5^5

Exercise 8.3.31

9^3

Answer

$9 \cdot 9 \cdot 9 = 729$

Exercise 8.3.32

6^2

Exercise 8.3.33

7^1

Answer

$7^1 = 7$

Exercise 8.3.34

1^{28}

Exercise 8.3.35

2^7

Answer

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

Exercise 8.3.36

0^5

Exercise 8.3.37

8^4

Answer

$8 \cdot 8 \cdot 8 \cdot 8 = 4,096$

Exercise 8.3.38

5^8

Exercise 8.3.39

6^9

Answer

$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 10,077,696$

Exercise 8.3.40

25^3

Exercise 8.3.41

42^2

Answer

$42 \cdot 42 = 1,764$

Exercise 8.3.42

31^3

Exercise 8.3.43

15^5

Answer

$15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 = 759,375$

Exercise 8.3.44

2^{20}

Exercise 8.3.45

816^2

Answer

$816 \cdot 816 = 665,856$

For the following problems, find the roots (using your knowledge of multiplication). Use a calculator to check each result.

Exercise 8.3.46

$\sqrt{9}$

Exercise 8.3.47

$\sqrt{16}$

Answer

4

Exercise 8.3.48

$\sqrt{36}$

Exercise 8.3.49

$\sqrt{64}$

Answer

8

Exercise 8.3.50

$\sqrt{121}$

Exercise 8.3.51

$\sqrt{144}$

Answer

12

Exercise 8.3.52

$\sqrt{169}$

Exercise 8.3.53

$\sqrt{225}$

Answer

15

Exercise 8.3.54

$\sqrt[3]{27}$

Exercise 8.3.55

$\sqrt[5]{32}$

Answer

2

Exercise 8.3.56

$\sqrt[7]{1}$

Exercise 8.3.57

$\sqrt{400}$

Answer

20

Exercise 8.3.58

$\sqrt{900}$

Exercise 8.3.59

$\sqrt{10,000}$

Answer

100

Exercise 8.3.60

$\sqrt{324}$

Exercise 8.3.61

$\sqrt{3,600}$

Answer

60

For the following problems, use a calculator with the keys $\sqrt{x}$, y^x, and $1/x$ to find each of the values.

Exercise 8.3.62

$\sqrt{676}$

Exercise 8.3.63

$\sqrt{1,156}$

Answer

34

Exercise 8.3.64

$\sqrt{46,225}$

Exercise 8.3.65

$\sqrt{17,288,964}$

Answer

4,158

Exercise 8.3.66

$\sqrt[3]{3,375}$

Exercise 8.3.67

$\sqrt[4]{331,776}$

Answer

24

Exercise 8.3.68

$\sqrt[8]{5,764,801}$

Exercise 8.3.69

$\sqrt[12]{16,777,216}$

Answer

4

Exercise 8.3.70

$\sqrt[8]{16,777,216}$

Exercise 8.3.71

$\sqrt[10]{9,765,625}$

Answer

5

Exercise 8.3.72

$\sqrt[4]{160,000}$

Exercise 8.3.73

$\sqrt[3]{531,441}$

Answer

81

Exercises for Review

Exercise 8.3.74

Use the numbers 3, 8, and 9 to illustrate the associative property of addition.

Exercise 8.3.75

In the multiplication $8 \cdot 4 = 32$, specify the name given to the numbers 8 and 4.

Answer

81

Exercise 8.3.76

Does the quotient $15 \div 0$ exist? If so, what is it?

Exercise 8.3.77

Does the quotient $0 \div 15$ exist? If so, what is it?

Answer

Yes; 0

Exercise 8.3.78

Use the numbers 4 and 7 to illustrate the commutative property of multiplication.

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