GEOMETRY

CK-12 Foundation

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Glossary

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CHAPTER OVERVIEW

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Thumbnail: A two-dimensional perspective projection of a sphere (CC BY-3.0; Geek3 via Wikipedia).

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1.1: Geometry Terms

Define and use terms, including points, lines, planes, space, and postulates.

Geometric Definitions

A **point** is an exact location in **space**. It describes a location, but has no size. Examples are shown below:

Figure **1.1.1**

Table 1.1.1

Label It	Say It
A	point A

A **line** is infinitely many points that extend forever in both directions. Lines have direction and location and are *always straight*.

Figure \(\PageIndex{2}\)

Table 1.1.2

Label It	*Say It*
line g	line g
$\overleftrightarrow{PQ}$	line PQ

A **plane** is a flat surface that contains infinitely many intersecting lines that extend forever in all directions. Think of a plane as a huge sheet of paper with no thickness that goes on forever.

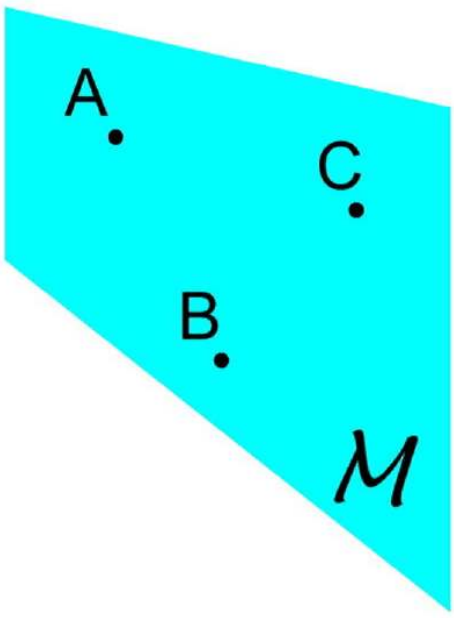


Figure **1.1.3**

Table 1.1.3

Label It	Say It
Plane M	Plane M
Plane ABC	Plane ABC

We can use **point**, **line**, and **plane** to define new terms.

Space is the set of all points extending in **three dimensions**. Think back to the plane. It extended in two dimensions, what we think of as up/down and left/right. If we add a third dimension, one that is perpendicular to the other two, we arrive at three-dimensional space.

Points that lie on the same line are **collinear**. P,Q,R,S, and T are collinear because they are all on line w. If a point U were located above or below line w, it would be **non-collinear**.

Figure **1.1.4**

Points and/or lines within the same plane are **coplanar**. Lines h and i, and points A,B,C,D,G, and K are **coplanar** in Plane J. Line $\overleftrightarrow{KF}$ and point E are **non-coplanar** with Plane J.

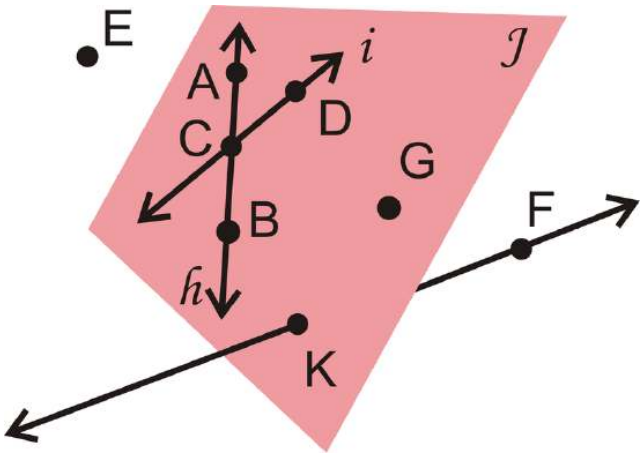


Figure **1.1.5**

An **endpoint** is a point at the end of a **line segment**. A **line segment** is a portion of a line with two endpoints. Or, it is a finite part of a line that stops at both ends. Line segments are labeled by their endpoints. Order does not matter.

Figure **1.1.6**

Table 1.1.4

Label It	Say It
$\overline{AB}$	Segment AB
$\overline{BA}$	Segment BA

A **ray** is a part of a line. It begins with an endpoint and extends forever away from the endpoint in one direction, perfectly straight. A ray is labeled by its endpoint and one other point on the ray. For rays, order does matter. When labeling, put the endpoint under the side WITHOUT the arrow.

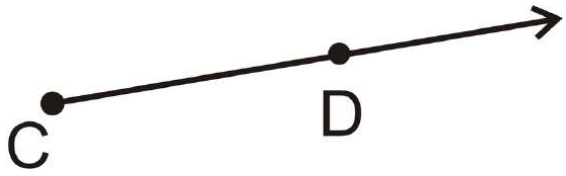


Figure **1.1.7**

Table 1.1.5

Label It	Say It
$\overrightarrow{CD}$	Ray CD
$\overleftarrow{DC}$	Ray CD

An **intersection** is a point or set of points where lines, planes, segments, or rays overlap.

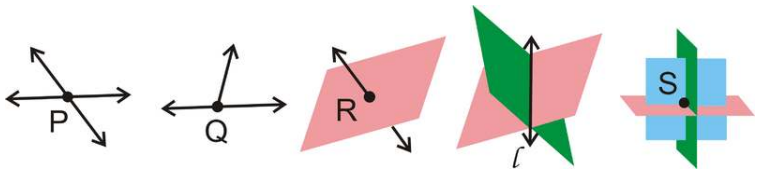


Figure **1.1.8**

Postulates

A **postulate** is a basic rule of geometry. Postulates are assumed to be true (rather than proven), much like definitions. The following is a list of some basic postulates.

Postulate #1: Given any two distinct points, there is exactly one (straight) line containing those two points.

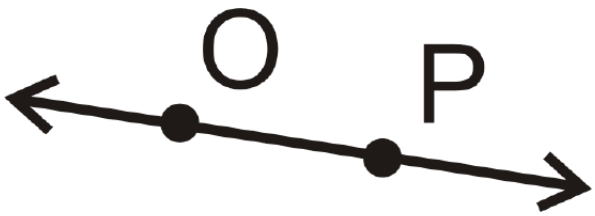


Figure **1.1.9**

Postulate #2: Given any three non-collinear points, there is exactly one plane containing those three points.

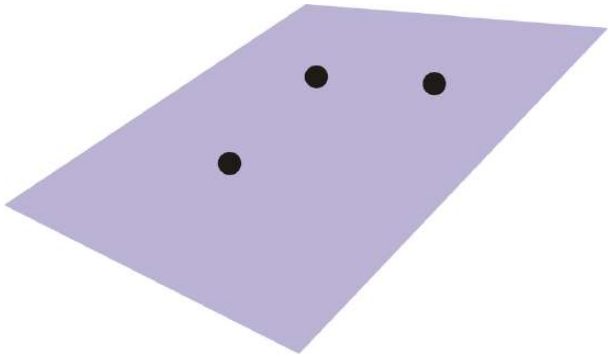


Figure **1.1.10**

Postulate #3: If a line and a plane share two points, then the entire line lies within the plane.

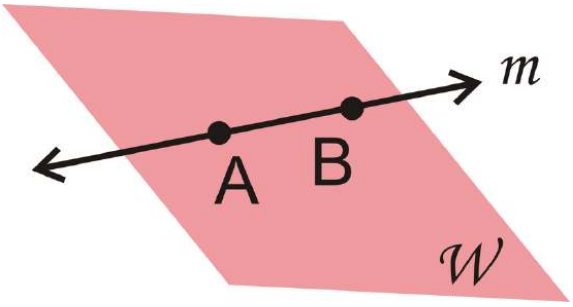


Figure **1.1.11**

Postulate #4: If two distinct lines intersect, the intersection will be one point.

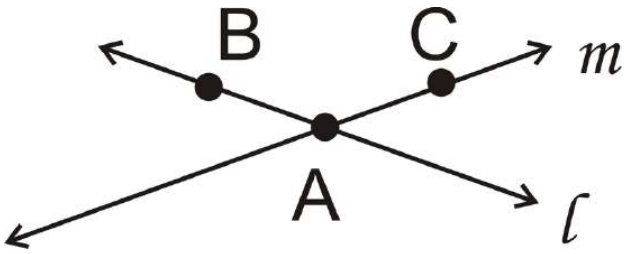


Figure 1.1.12

Lines l and m intersect at point A.

Postulate #5: If two distinct planes intersect, the intersection will be a line.

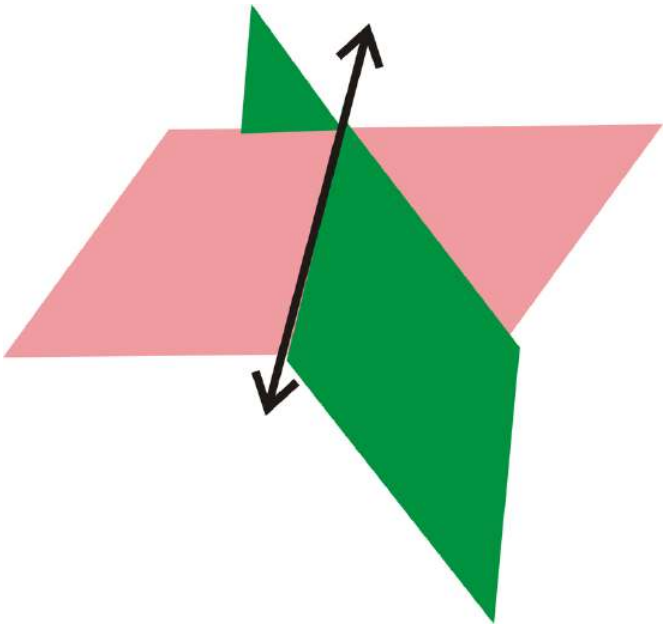


Figure 1.1.13

When making geometric drawings, be sure to be clear and label all points and lines.

What if you were given a picture of a figure or object, like a map with cities and roads marked on it? How could you explain that picture geometrically?

Example 1.1.1

What best describes San on a globe: point, line, or plane?

Solution

A city is usually labeled , on a globe.

Example 1.1.2

Use the picture below to answer these questions.

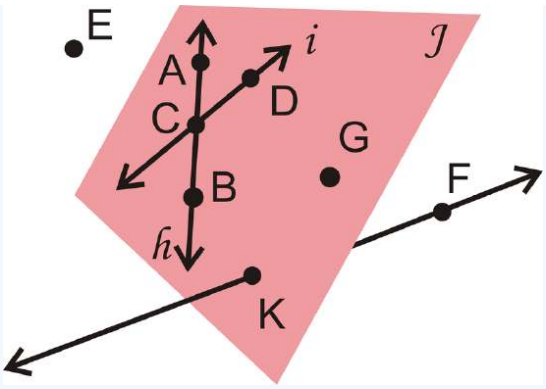


Figure \(\PageIndex{14}\)

a. List another way to label Plane J.
b. List another way to label line h.
c. Are K and F collinear?
d. Are E, B and F coplanar?

Solution

a. Plane BDG is one possibility. Any combination of three coplanar points that are not collinear would be correct.
b. $\overleftrightarrow{AB}$. Any combination of two of the letters A, B, or C would also be correct.
c. Yes, they both lie on $\overleftrightarrow{KF}$.
d. Yes, even though E is not in Plane J, any three points make a distinct plane. Therefore, the three points create Plane EBF.

Example 1.1.3

What best describes a straight road that begins in one city and stops in a second city: ray, line, segment, or plane?

Solution

The straight road connects two cities, which are like endpoints. The best term is segment.

Example 1.1.4

Answer the following questions about the picture.

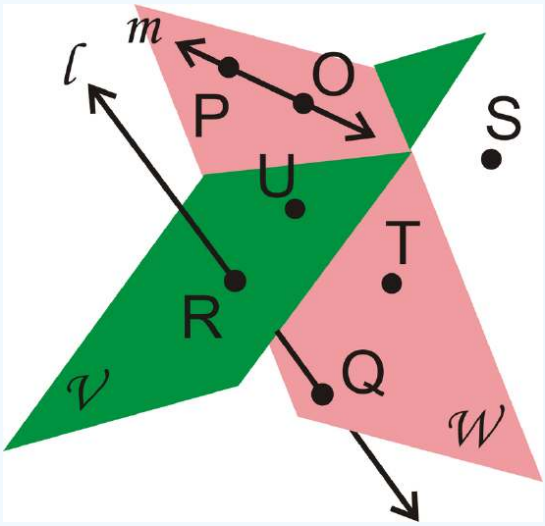


Figure **1.1.15**

a. Is line l coplanar with Plane V, Plane W, both, or neither?

b. Are R and Q collinear?
c. What point belongs to neither Plane V nor Plane W?
d. List three points in Plane W.

Solution

a. Neither
b. Yes
c. S
d. Any combination of P, O, T, and Q would work.

Example 1.1.5

Draw and label a figure matching the following description: Line $\overleftrightarrow{AB}$ and ray $\overrightarrow{CD}$ intersect at point \(C)\. Then, redraw so that the figure looks different but is still true to the description.

Solution

Neither the position of A or B on the line, nor the direction that $\overrightarrow{CD}$ points matter.

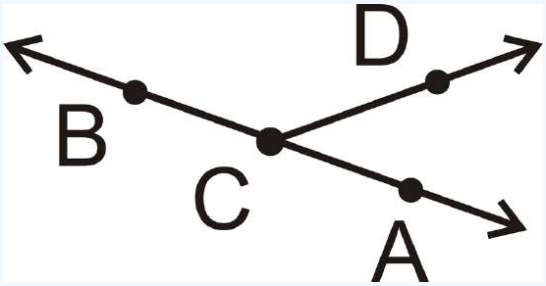


Figure **1.1.16**

For the second part, this is one way to draw the diagram differently:

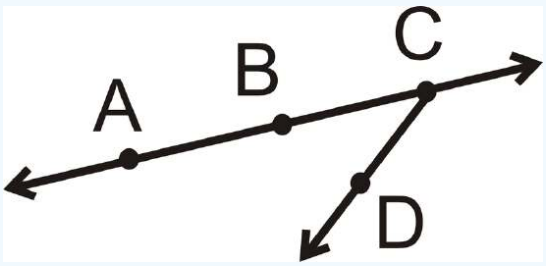


Figure **1.1.17**

Review

For questions 1-5, draw and label a figure to fit the descriptions.

1. $\overrightarrow{CD}$ intersecting $\overline{AB}$ and Plane P containing $\overline{AB}$ but not $\overrightarrow{CD}$.
2. Three collinear points A, B, and C. B is also collinear with points D and E.
3. $\overrightarrow{XY}$, $\overrightarrow{XZ}$, and $\overrightarrow{XW}$, such that $\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are coplanar, but $\overrightarrow{XW}$ is non-coplanar with both of the other rays.
4. Two intersecting planes, P and Q, with $\overline{GH}$, where G is in plane P and H is in plane Q.
5. Four non-collinear points I, J, K,and L, with line segments connecting all points to each other.
6. Name this line in five ways.
7. Name the geometric figure in three different ways.
8. Name the geometric figure in two different ways.
9. What is the best possible geometric model for a soccer field? Explain your answer.

10. List two examples of where you see rays in real life.
11. What type of geometric object is the intersection of a line and a plane? Draw your answer.
12. What is the difference between a postulate and a **theorem**?

For 13-16, use geometric notation to explain each picture in as much detail as possible.

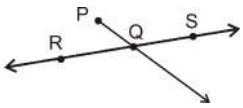
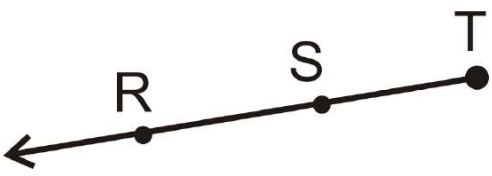
13. 

Figure **1.1.21**



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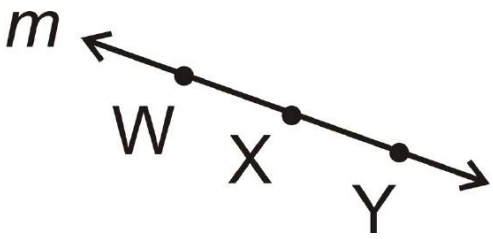
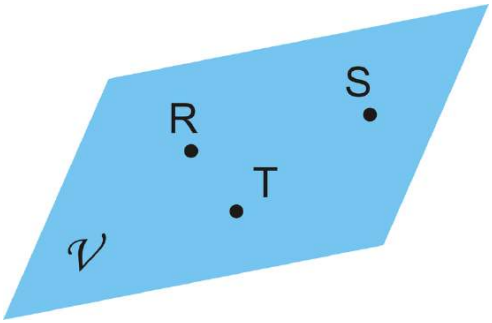


Figure **1.1.18**



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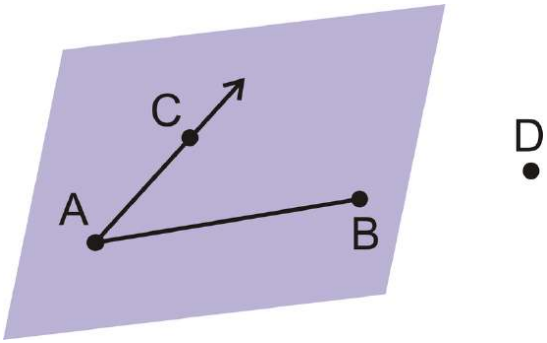
14. 

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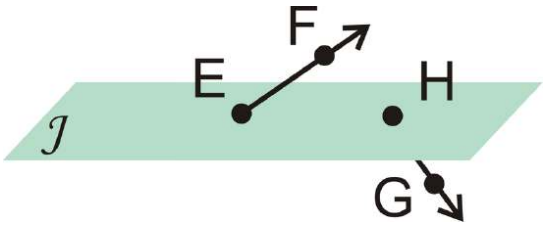
15. 

Figure **1.1.23**

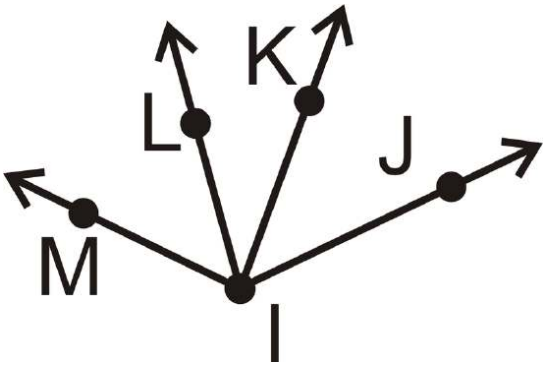
16. 

Figure **1.1.24**

For 17-25, determine if the following statements are true or false.

17. Any two points are collinear.
18. Any three points determine a plane.
19. A line is two rays with a common endpoint.
20. A line segment is infinitely many points between two endpoints.
21. A point takes up space.
22. A line is one-dimensional.
23. Any four points are coplanar.
24. $\overrightarrow{AB}$ could be read "ray AB" or "ray BA."
25. $\overleftrightarrow{AB}$ could be read "line AB" or "line BA."

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.1.

Vocabulary

Term	Definition
collinear	Three or more points are collinear when they lie on the same line.
coplanar	Points and/or lines located on the same plane are coplanar.
endpoint	An endpoint is a point at one end of a line segment or ray.
intersection	A point or set of points where lines, planes, segments, or rays cross.
line	Infinitely many points that extend forever in both directions.
line segment	A line segment is a part of a line that has two endpoints.
plane	A plane is a flat, two-dimensional surface. It can be conceptualized as a sheet of paper of infinite area.
Point	A point is a location in space that does not have size or shape.
postulate	A postulate is a statement that is accepted as true without proof.
ray	A part of a line with one endpoint that extends forever in the direction opposite that point.
space	Space is the set of all points extending in three dimensions.
Dimensions	Dimensions are the measurements that define the shape and size of a figure.
Non-collinear	A non-collinear point is located above or below a line.
Non-coplanar	A non-coplanar point is located above or below a plane.
theorem	A theorem is a statement that can be proven true using postulates, definitions, and other theorems that have already been proven.

Additional Resources

Video: Basic Geometric Definitions Principles - Basic

Activities: Basic Geometric Definitions Discussion Questions

Study Aids: Basics of Geometry Study Guide

Practice: Geometry Terms

Real World: Basic Geometric Definitions

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1.2: Shapes

2D Shapes

Overview of Lines, Curves, Vertices, and Sides of 2D Shapes

Video: Overview of Lines, Curves, Vertices, and Sides of 2D Shapes

Practice: 2D Shapes

Identify Shapes

Identifying rectangles, squares, circles, and triangles. Locate sets of shapes and identify shapes of common objects.

Video: Identifying Rectangles, Squares, Circles, and Triangles

Practice: Identify Shapes

Identify Less Common Shapes

Overview of ellipses, trapezoids, rhombi, and polygons. Match similar shapes.

Video: Overview of Ellipses, Trapezoids, Rhombi, and Polygons

Practice: Identify Less Common Shapes

Composite Shapes

Count Different 2D Shapes; rectangles, squares and triangles, within simple and within complex, composite figures.

Video: Identifying and Counting the Number of Rectangles, Squares,

Practice: Composite Shapes

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1.3: Definition of Line segments

Identify the Positive length between points.

Measuring Distance Between Two Points

Distance is the **measure** of length between two points. To **measure** is to determine how far apart two geometric objects are. The most common way to measure distance is with a ruler. Inch-rulers are usually divided up by eighth-inch (or 0.125 in) segments. Centimeter rulers are divided up by tenth-centimeter (or 0.1 cm) segments. Note that the distance between two points is the **absolute value** of the difference between the numbers shown on the ruler. This implies that you do not need to start measuring at "0", as long as you subtract the first number from the second.

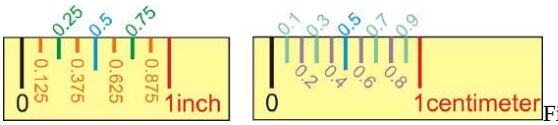


Figure 1.3.1

The **segment addition postulate** states that if A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

Figure 1.3.2

You can find the distances between points in the x–y plane if the lines are horizontal or vertical. If the line is vertical, find the change in the y−coordinates. If the line is horizontal, find the change in the x−coordinates.

Suppose you want to measure your height, but the measuring tape you have is old and the end is broken off. If the tape now starts at 6 cm and reads 138 cm from the floor to the top of your head, how tall are you?

Example 1.3.1

What is the distance ma low? The ruler is in centimeters.

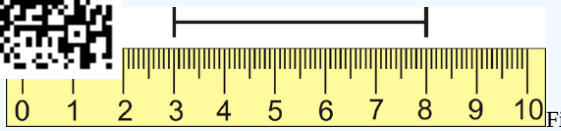


Figure 1.3.3

Solution

Subtract one endpoint from the other. The line segment spans from 3 cm to 8 cm.

$|8 - 3| = |5| = 5$

The line segment is 5 cm long. Notice that you also could have done

$|3 - 8| = |-5| = 5$.

Example 1.3.2

Make a sketch of $\overline{OP}$, where Q is between O and P.

Solution

Draw $\overline{OP}$ first, then place Q on the segment.

Figure 1.3.4

Example 1.3.3

What is the distance between the two points shown below?

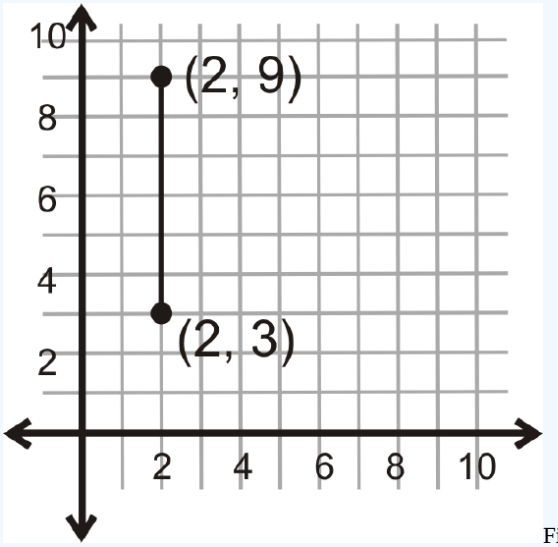


Figure 1.3.5

Solution

Because this line is vertical, look at the change in the y-coordinates.

$|9-3| = |6| = 6$

The distance between the two points is 6 units.

Example 1.3.4

In the picture from Example 2, if $OP = 17$ and $QP = 6$, what is OQ?

Solution

$$OQ + QP = OP$$

$$OQ + 6 = 17$$

$$OQ = 17 - 6$$

$$OQ = 11$$

Example 1.3.5

What is the distance between the two points shown below?

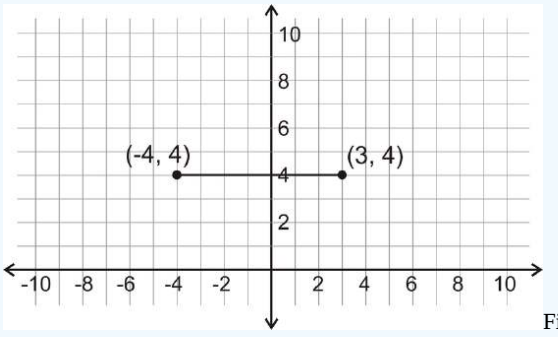


Figure 1.3.6

Solution

Because this line is horizontal, look at the change in the x-coordinates.

$$|(-4)-3| = |-7| = 7$$

The distance between the two points is 7 units.

Review

For 1-4, use the ruler in each picture to determine the length of the line segment.

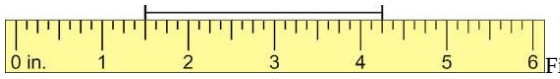
1. 

Figure 1.3.7

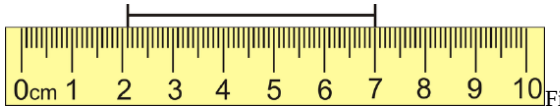
2. 

Figure 1.3.8

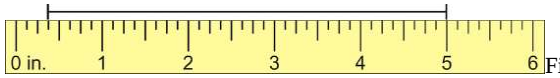
3. 

Figure 1.3.9

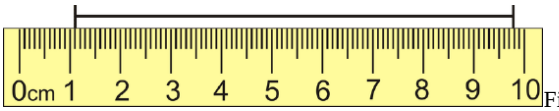
4. 

Figure 1.3.10

5. Make a sketch of $\overline{BT}$, with A between B and T.
6. If O is in the middle of $\overline{LT}$, where exactly is it located? If $LT = 16cm$, what is LO and OT?
7. For three collinear points, A between T and Q:
   a. Draw a sketch.
   b. Write the Segment Addition Postulate for your sketch.
   c. If $AT = 10$ in and $AQ = 5$ in, what is TQ?
8. For three collinear points, M between H and A:
   a. Draw a sketch.
   b. Write the Segment Addition Postulate for your sketch.
   c. If $HM = 18cm$ and $HA = 29cm$, what is AM?
9. For three collinear points, I between M and T:
   a. Draw a sketch.
   b. Write the Segment Addition Postulate for your sketch.
   c. If $IT = 6cm$ and $MT = 25cm$, what is AM?
10. Make a sketch that matches the description: B is between A and D. C is between B and D. $AB = 7cm$, $AC = 15cm$, and $AD = 32cm$. Find BC, BD, and CD.
11. Make a sketch that matches the description: $Eisbetween\backslash(F$ and G. H is between F and E. $FH = 4in$, $EG = 9in$, and $FH = HE$. Find FE, HG, and FG.

For 12 and 13, Suppose J is between H and K. Use the Segment Addition Postulate to solve for x. Then find the length of each segment.

12. $HJ = 4x + 9$, $JK = 3x + 3$, $KH = 33$
13. $HJ = 5x - 3$, $JK = 8x - 9$, $KH = 131$

For 14-17, determine the vertical or horizontal distance between the two points.

14.

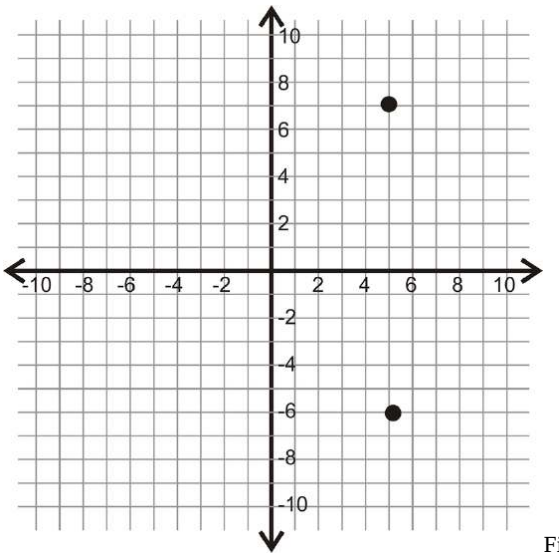


Figure 1.3.11

15.

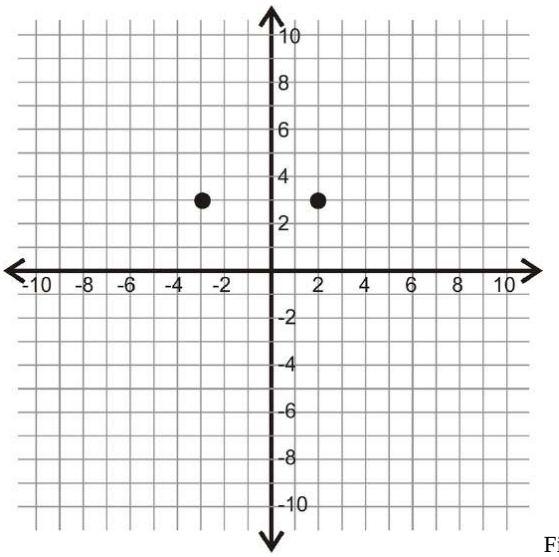


Figure 1.3.12

16.

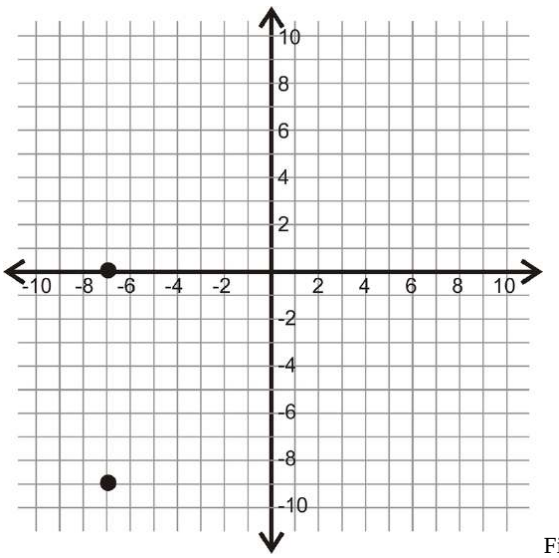
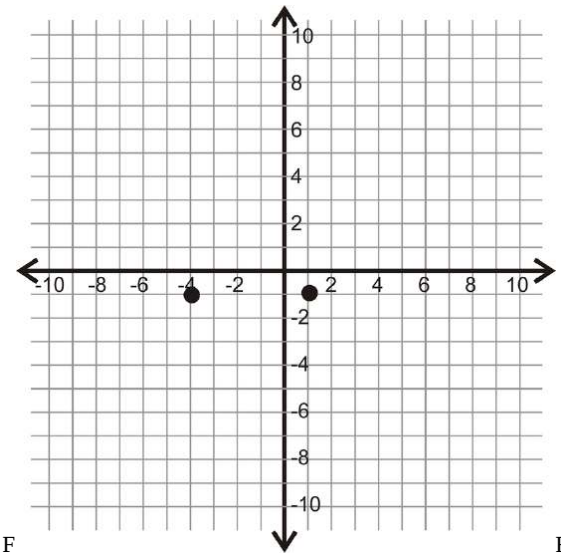


Figure 1.3.13



17. F Figure 1.3.14

18. Make a sketch of: S is between T and V. R is between S and T. $TR = 6$, $RV = 23$, and $TR = SV$.

19. Find SV, TS, RS and TV from #18.

20. For $\overline{HK}$, suppose that J is between H and K. If $HJ = 2x + 4$, $JK = 3x + 3$, and $KH = 22$, find x.

Vocabulary

Term	Definition
distance	Distance is the measure of length between two points.
Absolute Value	The absolute value of a number is the distance the number is from zero. Absolute values are never negative.
measure	To measure distance is to determine how far apart two geometric objects are by using a number line or ruler.

Additional Resources

Video: Ruler Postulate and the Segment Addition Postulate

Activities: Distance Between Two Points Discussion Questions

Study Aids: Segments Study Guide

Practice: Definition of Line Segment

Real World: Distance Between Two Points

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1.4: Midpoints and Segment Bisector

Use midpoints and bisectors to find the halfway mark between two coordinates.

When two segments are congruent, we indicate that they are congruent, or of equal length, with **segment markings**, as shown below:

Figure 1.4.1

A **midpoint** is a point on a line segment that divides it into two congruent segments.

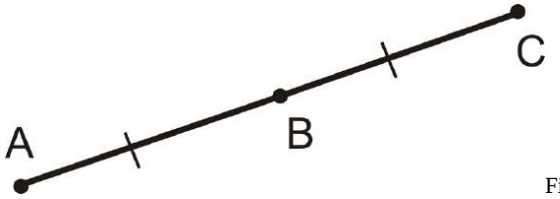

Figure 1.4.2

Because $AB = BC$, B is the midpoint of $\overline{AC}$. Any line segment will have exactly one midpoint.

When points are plotted in the coordinate plane, we can use a formula to find the midpoint between them.

Here are two points, $(-5, 6)$ and $(3, 2)$.

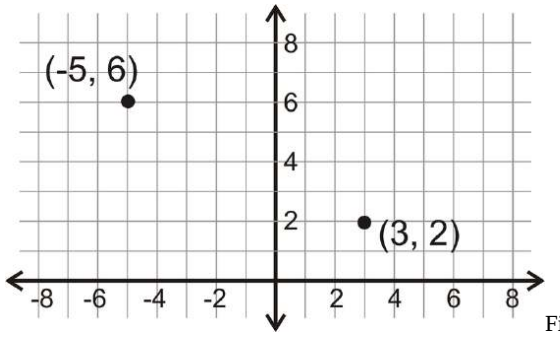

Figure 1.4.3

The midpoint should be halfway between the points on the segment connecting them. Just by looking, it seems like the midpoint is $(-1, 4)$.

Midpoint Formula: For two points, (x_1, y_1) and (x_2, y_2), the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Let's use the formula to make sure $(-1, 4)$ is the midpoint between $(-5, 6)$ and $(3, 2)$.

$$\left(\frac{-5 + 3}{2}, \frac{6 + 2}{2}\right) = (-22, 82) = (-1, 4)$$

A **segment bisector** cuts a line segment into two congruent parts and passes through the midpoint. A **perpendicular bisector** is a segment bisector that intersects the segment at a right angle.

$$\overline{AB} \cong \overline{BC}$$

$$\overline{AC} \perp \overleftrightarrow{DE}$$

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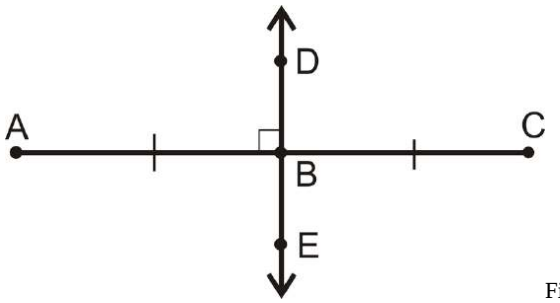

Figure 1.4.4

What if you were given the coordinates of two points and you wanted to find the point exactly in the middle of them? How would you find the coordinates of this third point?

Example 1.4.1

Write all equal segment

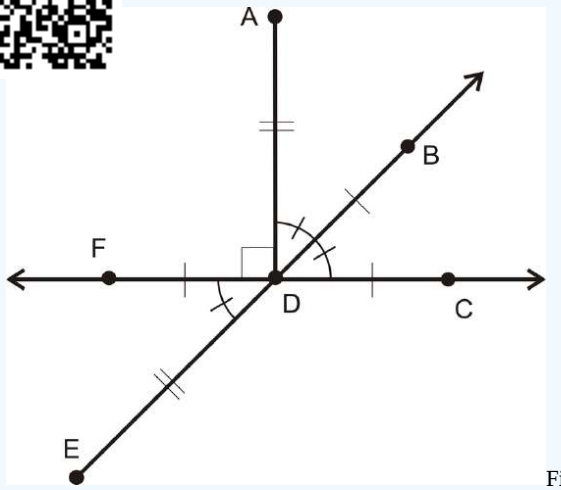

Figure 1.4.5

Solution

$$AD = DE$$

$$FD = DB = DC$$

Example 1.4.2

Is M a midpoint of $\overline{AB}$?

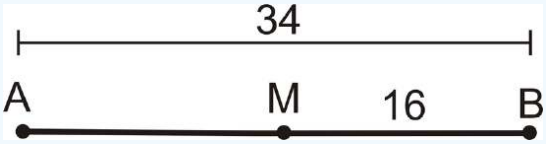

Figure 1.4.6

Solution

No, it is not $MB = 16$ and $AM = 34 - 16 = 18$. AM must equal MB in order for M to be the midpoint of $\overline{AB}$.

Example 1.4.3

Find the midpoint between $(9, -2)$ and $(-5, 14)$.

Solution

Plug the points into the formula.

$$\left(\frac{9+(-5)}{2}, \frac{-2+14}{2}\right) = \left(\frac{4}{2}, \frac{12}{2}\right) = (2, 6)$$

Example 1.4.4

Which line is the perpendicular bisector of $\overline{MN}$?

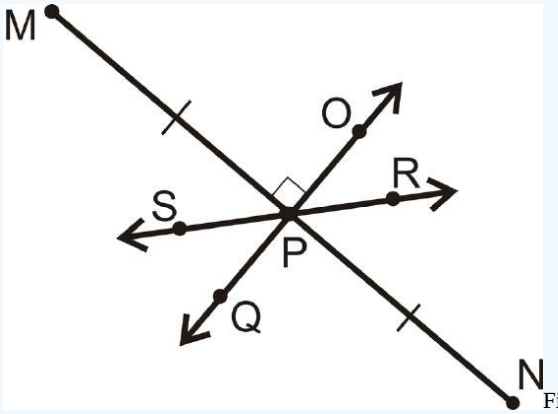

Figure 1.4.7

Solution

The perpendicular bisector must bisect $\overline{MN}$ and be perpendicular to it. Only $\overleftrightarrow{OQ}$ fits this description. $\overleftrightarrow{SR}$ is a bisector, but is not perpendicular.

Example 1.4.5

Find x and y.

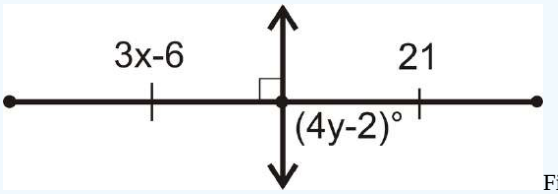

Figure 1.4.8

Solution

The line shown is the perpendicular bisector.

So, $3x - 6 = 21$

$3x = 27$

$x = 9$

And, $(4y - 2) = 90$

$4y = 92$

$y = 23$

Review

1. Copy the figure below and label it with the following information:

$\overline{AB} \cong \overline{AD}$

$\overline{CD} \cong \overline{BC}$

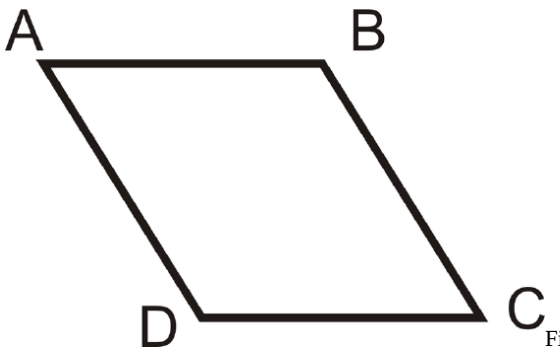

Figure 1.4.9

For 2-4, use the following picture to answer the questions.

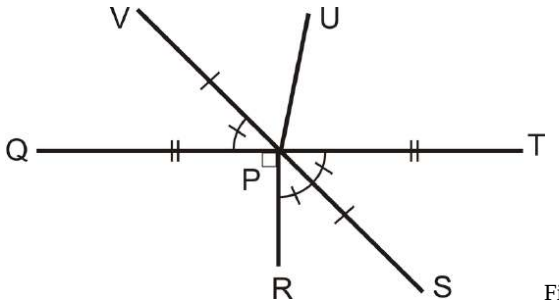

Figure 1.4.10

2. P is the midpoint of what two segments?
3. How does $\overline{VS}$ relate to $\overline{QT}$?
4. How does $\overline{QT}$ relate to $\overline{VS}$?

For exercise 5, use algebra to determine the value of x.

5.

Figure 1.4.11

For questions 6-10, find the midpoint between each pair of points.

6. (-2, -3) and (8, -7)
7. (9, -1) and (-6, -11)
8. (-4, 10) and (14, 0)
9. (0, -5) and (-9, 9)
10. (-3, -5) and (2, 1)

Given the midpoint (M) and either endpoint of $\overline{AB}$, find the other endpoint.

11. $A(-1, 2)$ and $M(3, 6)$
12. $B(-10, -7)$ and $M(-2, 1)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.4.

Vocabulary

Term	Definition
midpoint	The midpoint of a line segment is the point on the line segment that splits the segment into two congruent parts.
perpendicular bisector	A segment bisector that intersects the segment at a right angle.
segment bisector	A segment bisector is a line (or part of a line) that passes through the midpoint.
segment markings	When two segments are congruent, we indicate that they are congruent with ***segment markings***.
Midpoint Formula	The midpoint formula says that for endpoints (x_1, y_1) and (x_2, y_2), the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Additional Resources

Video: Midpoints and Segment Bisectors Examples - Basic

Activities: Midpoints and Segment Bisectors Discussion Questions

Study Aids: Segments Study Guide

Practice: Midpoints and Segment Bisectors

Real World: Midpoint And Segment Bisectors - Treasure Hunt Activity

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1.5: Midpoint Formula

Determine the midpoints of line segments by taking the average of x values and average of y values

Suppose a coordinate plane were transposed over a subway map, and the blue line went in a straight line from the point $(-9, 8)$ to the point $(1, -4)$. If you got on the blue line at the beginning of the line and traveled halfway to the end of the line, what would be your coordinates? How would you calculate these coordinates?

Consider the following situation: You live in Des Moines, Iowa and your grandparents live in Houston, Texas. You plan to visit them for the summer and your parents agree to meet your grandparents halfway to exchange you. How do you find this location?

By meeting something "halfway," you are finding the **midpoint** of the straight line connecting the two segments. In the above situation, the midpoint would be halfway between Des Moines and Houston.

The **midpoint** between two coordinate pairs represents the halfway point, or the **average**. It is the ordered pair (x_m, y_m) where:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Let's find the location that's halfway between Des Moines and Houston given the information from above:

Des Moines, Iowa has the coordinates $(41.59, 93.62)$.

Houston, Texas has the coordinates $(29.76, 95.36)$.

Find the coordinates of the midpoint between these two cities.

Decide which ordered pair will represent

(x_1, y_1) and which will represent (x_2, y_2).

$$(x_1, y_1) = (41.59, 93.62)$$

$$(x_2, y_2) = (29.76, 95.36)$$

Compute the midpoint using the formula

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(x_m, y_m) = \left(\frac{41.59 + 29.76}{2}, \frac{93.62 + 95.36}{2}\right)$$

$$(x_m, y_m) = (35.675, 94.49)$$

Using Google Maps, you can meet in the Ozark National Forest, halfway between the two cities.

Now, let's complete the following problems using the midpoint formula:

1. A segment with endpoints $(9, -2)$ and (x_1, y_1) has a midpoint of $(2, -6)$. Find (x_1, y_1).

Use the Midpoint Formula:

$$\frac{x_1 + x_2}{2} = x_m$$

$$2 = \frac{x_1 + 9}{2} \rightarrow 4 = x_1 + 9$$

$$x_1 = -5$$

Following the same procedure:

$$\frac{y_1 + (-2)}{2} = -6 \rightarrow y_1 + (-2) = -12$$

$$y_1 = -10$$

$$(x_1, y_1) = (-5, -10)$$

2. Find the values of x and y that make $(9.5, 6)$ the midpoint of $(3, 5)$ and (x, y).

Start with the formula, and solve for the variables:

The midpoint formula: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Substitute in the given values and variables: $(9.5, 6) = \left(\frac{3 + x}{2}, \frac{5 + y}{2}\right)$

This can be re-written as two equations:

$$9.5 = \frac{3 + x}{2}; \; 6 = \frac{5 + y}{2}$$

Multiply each side by 2: $19 = 3 + x; \; 12 = 5 + y$

Isolate the variables: $16 = x; \; 7 = y$

Example 1.5.1

Earlier, you were asked coordinates after traveling halfway to the end of the subway's blue line. You know that that the blue line go from $(-9, 8)$ to the point $(1, -4)$.

Solution

To calculate these coordinates, you need to calculate the midpoint between $(-9, 8)$ and $(1, -4)$.

To find the midpoint between $(-9, 8)$ and $(1, -4)$, substitute in the given values into the midpoint formula and solve.

The midpoint formula: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Substitute in the given values and variables: $midpoint = \left(\frac{-9 + 1}{2}, \frac{8 + (-2)}{2}\right)$

$midpoint = (-4, 2)$

Therefore, the coordinates of the subway train halfway down the blue line are $(-4, 2)$.

Example 1.5.2

On a hike, you and your friend decide to take different routes, but then meet up for lunch. You hike 3 miles north and 2 miles west. Starting from the same point, your friend hikes 4 miles east and 1 mile south. From these points, you each walk towards each other, meeting halfway for lunch. Where would your lunchtime meeting place be in reference to your starting point?

Solution

Think of the starting point as the origin of a Cartesian coordinate system. If you walk north 3 miles, that is walking straight up the graph 3 units. Walking west 2 miles is the same as walking left 2 units on the graph. Then you have arrived at the point $(-2, 3)$. For your friend, east is to the right (positive) and south is down (negative), so he/she arrives at $(4, -1)$. Now you need to find the midpoint:

$$(x_m, y_m) = \left(\frac{-2 + 4}{2}, \frac{3 + (-12)}{2}\right) = (22, 22) = (1, 1)$$

Your lunch time meeting place would be 1 mile north and 1 mile east of your starting point.

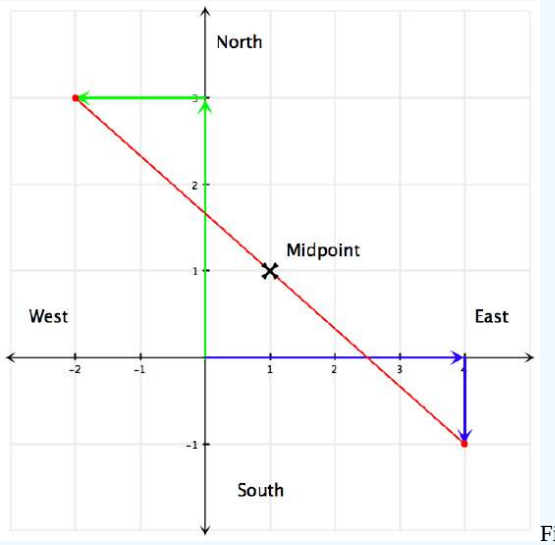

Figure 1.5.1

Review

In 1–10, find the midpoint of the line segment joining the two points.

1. (x_1, y_1) and (x_2, y_2)
2. $(7, 7)$ and $(-7, 7)$
3. $(-3, 6)$ and $(3, -6)$
4. $(-3, -1)$ and $(-5, -8)$
5. $(3, -4)$ and $(6, 1)$
6. $(2, -3)$ and $(2, 4)$
7. $(4, -5)$ and $(8, 2)$
8. $(1.8, -3.4)$ and $(-0.4, 1.4)$
9. $(5, -1)$ and $(-4, 0)$
10. $(10, 2)$ and $(2, -4)$
11. An endpoint of a line segment is $(4, 5)$ and the midpoint of the line segment is $(3, -2)$. Find the other endpoint.
12. An endpoint of a line segment is $(-10, -2)$ and the midpoint of the line segment is $(0, 4)$. Find the other endpoint.
13. Shawn lives six blocks west and ten blocks north of the center of town. Kenya lives fourteen blocks east and two blocks north of the center of town.
    a. How far apart are these two girls "as the crow flies"?
    b. Where is the halfway point between their houses?

Mixed Review

14. A population increases by 1.2% annually. The current population is 121,000.
    a. What will the population be in 13 years?
    b. Assuming this rate continues, when will the population reach 200,000?
15. Write $1.29651843 \cdot 10^5$ in standard form.
16. Is $4, 2, 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{8}, \ldots$ an example of a geometric sequence? Explain your answer.
17. Simplify $6x^3(4xy^2 + y^3z)$.
18. Suppose $0 = (x - 2)(x + 1)(x - 3)$. What are the x−intercepts?
19. Simplify $\sqrt{300}$.

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.8.

Vocabulary

Term	Definition
Midpoint Formula	The midpoint formula says that for endpoints (x_1, y_1) and (x_2, y_2), the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
midpoint	The midpoint of two vectors is the location in the center of their endpoints.

Additional Resource

Video: Midpoint Formula - Overview

Activities: Midpoint Formula Discussion Questions

Practice: Midpoint Formula

Real World: SF Map - PythThm

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1.6: Points that Partition Line Segments

Section formula finds coordinates of a point that splits a line segment in a given ratio.

Recall that a **median** of a triangle is a line segment that connects a vertex of the triangle to the **midpoint** of the side opposite the vertex. All triangles have three medians and these three medians intersect in one point called the **centroid**, shown below. *The centroid **partitions** each median in a 2:1 ratio.*

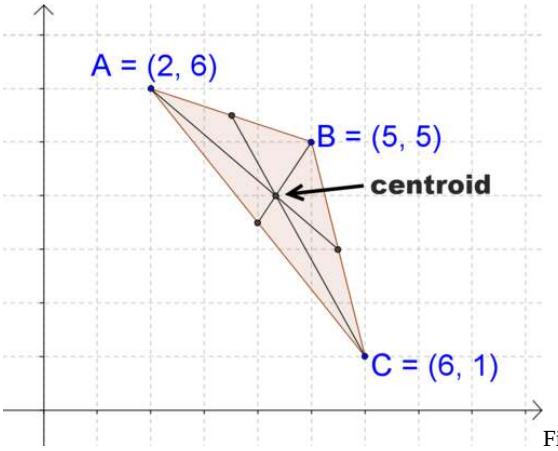

Figure **1.6.1**

Find the coordinates of the centroid, given the coordinates of the vertices of the triangle as shown.

Partitions

Suppose you have a line segment $\overline{AB}$. A point P divides this line segment into two parts such that $AP = mk$ and $PB = nk$. You can say that point P **partitions** segment AB in a $m : n$ ratio. (Note that $\frac{mk}{nk} = mn$, a ratio of $m : n$.)

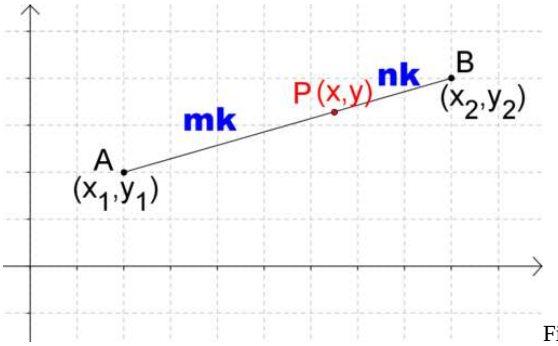

Figure **1.6.2**

A natural question to ask is, what are the coordinates of point P? It turns out that with the help of similar triangles and algebra, you can come up with a formula that will give you the coordinates of point P based on the coordinates of A, the coordinates of B, and the ratio $m : n$. This formula is sometimes referred to as the **section formula**.

Section Formula: Given $\overline{AB}$ with $A = (x_1, y_1)$ and $B = (x_2, y_2)$, if point P partitions $\overline{AB}$ in a $m : n$ ratio, then the coordinates of point P are:

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Proving Triangle Sim

For segment $\overline{AB}$ below, dı les, one with hypotenuse $\overline{AP}$ and one with hypotenuse $\overline{PB}$. Show that these triangles are similar.

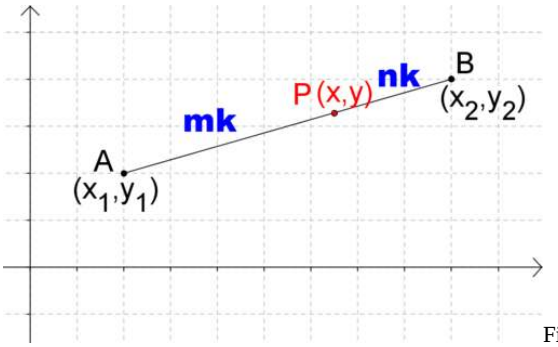

Figure 1.6.3

Start by drawing the right triangles. Below, the base and height of each triangle has been labeled in green.

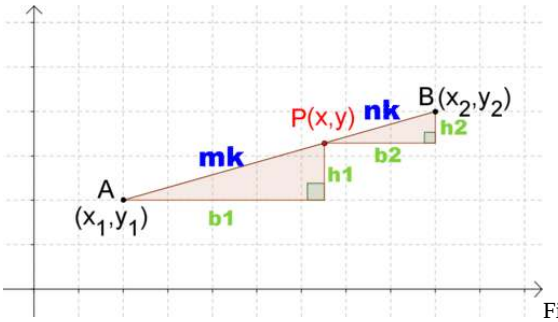

Figure 1.6.4

Clearly these triangles have one pair of congruent angles (the right angles). What other information do you have about the triangles? You know that for each triangle, the ratio $\frac{height}{base}$ is the slope of $\overline{AB}$. Because these two triangles are attached to the same line segment with the same slope, it means that $\frac{h_1}{b_1} = h_2 b_2$. This is equivalent to $\frac{b_2}{b_1} = \frac{h_2}{h_1}$. Two pairs of sides are in the same ratio.

Not only is there one pair of congruent angles, but there are also two pairs of corresponding sides with the same ratio. The triangles are similar by $SAS\sim$.

Finding Length and Height

Find the lengths of the bases and heights of each triangle. Use the fact that the triangles are similar to set up and solve proportions for x and then for y in order to find the coordinates of point P.

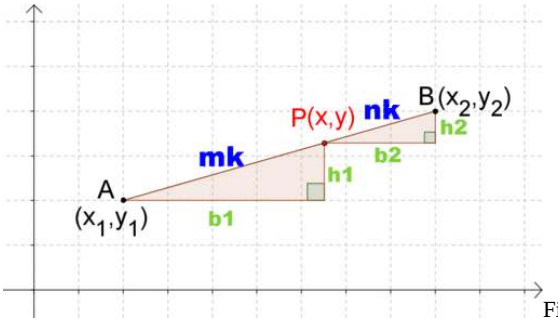

Figure 1.6.5

The bases and heights can be found in terms of x_1, y_1, x, y, x_2, y_2.

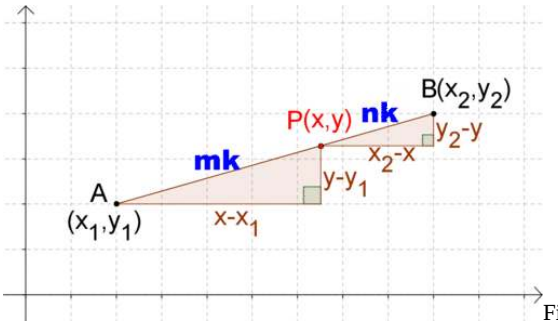

Figure **1.6.6**

Because the triangles are similar, the ratios between pairs of corresponding sides are equal. In particular, you know:

1. $\frac{mk}{nk} = \frac{x - x_1}{x_2 - x}$
2. $\frac{mk}{nk} = \frac{y - y_1}{y_2 - y}$

You can use algebra to solve the first equation for x and the second equation for y.

1. $\frac{mk}{nk} = \frac{x - x_1}{x_2 - x} \rightarrow \frac{m}{n} = \frac{x - x_1}{x_2 - x}$

$\rightarrow mx_2 - mx = nx - nx_1$

$\rightarrow mx_2 + mx_1 = mx + nx$

$\rightarrow mx_2 - nx_1 = x(m + n)$

$\rightarrow \frac{mx_2 - nx_1}{m + n} = \frac{x(m + n)}{m + n}$

$\rightarrow x = \frac{mx_2 + nx_1}{m + n}$

2. $\frac{mk}{nk} = \frac{y - y_1}{y_2 - y} \rightarrow \frac{m}{n} = \frac{y - y_1}{y_2 - y}$

$\rightarrow my_2 - my = ny - ny_1$

$\rightarrow my_2 + my_1 = my + ny$

$\rightarrow my_2 - ny_1 = y(m + n)$

$\rightarrow \frac{my_2 - ny_1}{m + n} = \frac{y(m + n)}{m + n}$

$\rightarrow x = \frac{my_2 + ny_1}{m + n}$

Point P is at:

$$P = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

Finding the Coordinates of a Point

Consider $\overline{AB}$ with $A = (10, 2)$ and $B = (4, 1)$. P partitions $\overline{AB}$ in a ratio of $2 : 3$. Find the coordinates of point P.

You can use the section formula with $(x_1, y_1) = (10, 2)$, $(x_2, y_2) = (4, 1)$, $m = 2$, $n = 3$.

$$P = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right) = \left(\frac{2 \cdot 4 + 3 \cdot 10}{2 + 3}, \frac{2 \cdot 1 + 3 \cdot 2}{2 + 3}\right) = (7.6, 1.6)$$

You can plot points A, B, and P to see if this answer is realistic.

Figure \(\
PageIndex
{7}\)

This does look like P partitions the segment from A to B in a ratio of $2:3$. Note that the answer would be different if you were looking for the point that partitioned the segment *from* B *to* A. The order of the letters and "direction" of the segment matters.

Example 1.6.1

Earlier, you were asked to find the coordinates of the centroid, given the coordinates of the vertices of the triangle as shown.

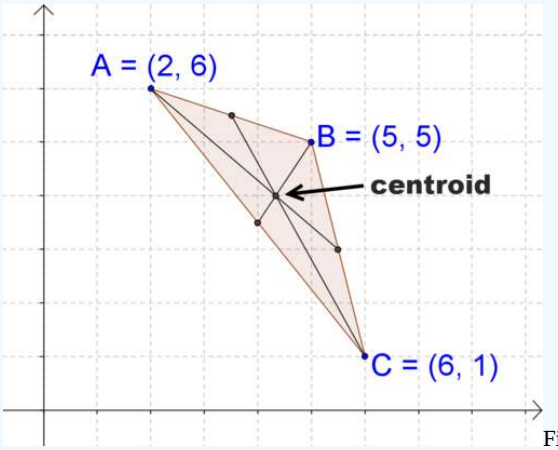

Figure **1.6.8**

Solution

One way to find the coordinates of the centroid is to use the section formula. You can focus on any of the three medians. Here, look at the median from point A. First, you will need to find the coordinates of the midpoint of $\overline{BC}$ *(the* ***midpoint formula****, a special case of the section formula, is derived in Guided Practice #1 and #2):*

$$\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)=\left(\frac{5+6}{2},\frac{5+1}{2}\right)=(5.5,3)$$

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)=\left(\frac{2\cdot 5.5+1\cdot 2}{2+1},\frac{2\cdot 3+1\cdot 6}{2+1}\right)=\left(\frac{13}{3},4\right)$$

Looking at the picture, these coordinates for the centroid are realistic.

Example 1.6.2

The midpoint of a line segment is the point exactly in the middle of the line segment. In what ratio does a midpoint partition a segment?

Solution

1:1, because the segments connecting the midpoint to each endpoint will be the same length.

Example 1.6.3

The midpoint formula is a special case of the section formula where m=n=1. Derive a formula that calculates the midpoint of the segment connecting (x_1,y_1) with (x_2,y_2).

Solution

For a midpoint, $m = n = 1$. The section formula becomes:

\(\left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)=\left(\dfrac{x_2+x_1}{2}, \dfrac{y_2+y_1}{2}\right)

This is the midpoint formula.

Example 1.6.4

Consider $\overline{BA}$ with $B = (4, 1)$ and $A = (10, 2)$. P partitions the segment in a ratio of $2 : 3$. Find the coordinates of point P. How and why is this answer different from the answer to Example C?

Solution

$(x_1, y_1) = (4, 1)$ and $(x_2, y_2) = (10, 2)$. $m = 2$ and $n = 3$.

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) = \left(\frac{2 \cdot 10 + 3 \cdot 4}{2+3}, \frac{2 \cdot 2 + 3 \cdot 1}{2+3}\right) = (6.4, 1.4)$$

This answer is different from the answer to Example C because in this case point P is partitioning the segment in a $2 : 3$ ratio starting from point B. In Example C, you were starting from point A.

Review

Find the midpoint of each of the following segments defined by the given endpoints.

1. $(2, 6)$ and $(-4, 8)$
2. $(1, 9)$ and $(-2, 5)$
3. $(11, 24)$ and $(8, 12)$
4. $(1, 3)$ is the midpoint of $\overline{AB}$ with $A = (-2, 1)$. Find the coordinates of B.
5. $(2, 4)$ is the midpoint of $\overline{CD}$ with $C = (-5, 9)$. Find the coordinates of D.
6. $(4, 23)$ is the midpoint of $\overline{EF}$ with $E = (7, 11)$. Find the coordinates of F.

Consider $A = (-9, 4)$ and $B = (11, 17)$.

7. Point P_1 partitions the segment from A to B in a $3 : 5$ ratio. Find the coordinates of point P_1.
8. Point P_2 partitions the segment from B to A in a $3 : 5$ ratio. Find the coordinates of point P_2.
9. Why are the answers to 7 and 8 different?
10. Find the length of AP_1 and P_2B. Why should these lengths be the same?

Consider $C = (-6, -1)$ and $D = (4, 8)$.

11. Point P_3 partitions the segment from A to B in a $1 : 2$ ratio. Find the coordinates of point P_3.
12. Point P_4 partitions the segment from B to A in a $4 : 5$ ratio. Find the coordinates of point P_4.
13. Point $P = (1, 2)$ partitions the segment from $E = (9, 6)$ to F in a $2 : 5$ ratio. Find the coordinates of point F.
14. Point $P = (-6, -4)$ partitions the segment from $G = (-4, 6)$ to H in a $5 : 3$ ratio. Find the coordinates of point H.
15. Point $P = (6, 8)$ partitions the segment from $I = (-2, 1)$ to J in a $6 : 7$ ratio. Find the coordinates of point J.
16. A triangle is defined by the points $(5, 6)$, $(9, 17)$, and $(-2, 1)$. Find the coordinates of the centroid of the triangle.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.6.

Vocabulary

Term	Definition
centroid	The centroid is the point of intersection of the medians in a triangle.
Median	The median of a triangle is the line segment that connects a vertex to the opposite side's midpoint.
midpoint	The midpoint of a line segment is the point on the line segment that splits the segment into two congruent parts.
Midpoint Formula	The midpoint formula says that for endpoints (x_1, y_1) and (x_2, y_2), the midpoint is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
Partitions	To partition is to divide into parts.
Section Formula	The section formula states how to find the coordinates of a point that partitions a line segment in a given ratio.

Additional Resource

Practice: Points that Partition Line Segments

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1.7: Introduction to Angles

Comparison and classification of angles, such as right or vertical, and their relation to lines and rays.

Line Segment and Ray

A line segment is a portion of a line with two endpoints. A ray is a portion of a line with one endpoint. Line segments are named by their endpoints and rays are named by their endpoint and another point. In each case, a segment or ray symbol is written above the points. Below, the line segment is $\overline{AB}$ and the ray is $\overrightarrow{AB}$.

Figure 1.7.1

Angles

- When two rays meet at their endpoints, they form an angle.
- Depending on the situation, an angle can be named with an angle symbol $\angle$ and its vertex, or by three letters.

If three letters are used, the middle letter should be the vertex. The angle below could be called $\angle B$ or $\angle ABC$ or $\angle CBA$. Use three letters to name an angle if using one letter would not make it clear what angle you are talking about.

Figure 1.7.2

The size of an angle is measured in degrees. Therefore, 'the measure of the angle ABC' refers to the size, or measure, of the angle in degrees, often written $m\angle ABC$. You can use a protractor or geometry software to measure angles. Remember that a full circle has $360°$.

Figure 1.7.3

Types of Angles

- An angle that is exactly $0°$ is called a zero angle.
- An angle that is less than $90°$ is called an acute angle.
- An angle that is exactly $90°$ (one quarter of a circle) is called a right angle. A right angle is noted with a little square at its vertex.
- An angle that is more than $90°$ but less than $180°$ is called an obtuse angle. An angle that is exactly $180°$ (one half of a circle) is called a straight angle.

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Figure 1.7.4

Identifying the Type of Angles

Name the angle below and classify it by its measure.

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Figure 1.7.5

$m\angle F$ or $m\angle DFE$ or $m\angle EFD$. It is _____ angle.

Complementary and Supplementary Angles

Two angles are complementary if the sum of their measures is $90°$.

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Figure 1.7.6

Two angles are supplementary if the sum of their measures is $180°$.

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Figure 1.7.7

Two angles that together form a straight angle will always be supplementary.

Finding an Unknown Angle

x and y are complementary angles with measure of $y = 20°$. What is the measure of x?

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Figure 1.7.8

The measure of x is _____.

Adjacent and Vertical Angles

When two lines intersect, many angles are formed, as shown below.

In the diagram above, $\angle AEC$ and $\angle AED$ are adjacent angles because they are next to each other and share a ray. They are also supplementary, because together they form a straight angle. $\angle AEC$ and $\angle DEB$ are called vertical angles. Vertical angles will always have the same measure.

Consider this example problem:

Let $m\angle AEC = x°$. Show that $m\angle DEB$ must also equal $x°$.

Figure **1.7.9**

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Figure **1.7.10**

This shows that vertical angles will always have the same measure.

Identifying Angles

Explain why you must use three letters to identify any of the angles in the diagram below.

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Figure **1.7.11**

All angles in this diagram have a vertex of E. Therefore $\angle$E is ambiguous because it could refer to many different angles. Use three letters with E as the middle letter to be clear about which angle you are referring to $\angle AEC$, $\angle AED$, $\angle DEB$, $\angle BEC$.

Example 1.7.1

Angles are formed by in ays. If you take any two lines or rays, will you form at least one angle?

Solution

As long as the lines or ast one angle will be formed. If the lines (or rays) are parallel, and therefore don't intersect, then no angles will be formed.

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Figure **1.7.11**

Example 1.7.2

Estimate the measure of angle $\angle DFE$. Use a protractor to confirm your answer.

Solution

Remember that exactly half of a right angle is $45°$. This angle looks to be more than half of a right angle. You might guess that it is approximately $55°$. Using a protractor, you can see that it is about $60°$.

Example 1.7.3

What are two lines that form a right angle called?

Solution

Perpendicular lines.

Review

1. What's the difference between a line segment, a line, and a ray?
2. Draw an example of a right angle.
3. Draw an example of an obtuse angle.
4. Draw an example of an acute angle.
5. Why are two angles that make a straight angle always supplementary?
6. If $m\angle ABC = (2x + 4)°$, $m\angle DEF = (3x - 5)°$, and $\angle ABC$ and $\angle DEF$ are complementary, what are the measures of the angles?
7. If $m\angle ABC = (2x + 4)°$, $m\angle DEF = (3x - 5)°$, and $\angle ABC$ and $\angle DEF$ are supplementary, what are the measures of the angles?

Use the diagram below for #8-#12.

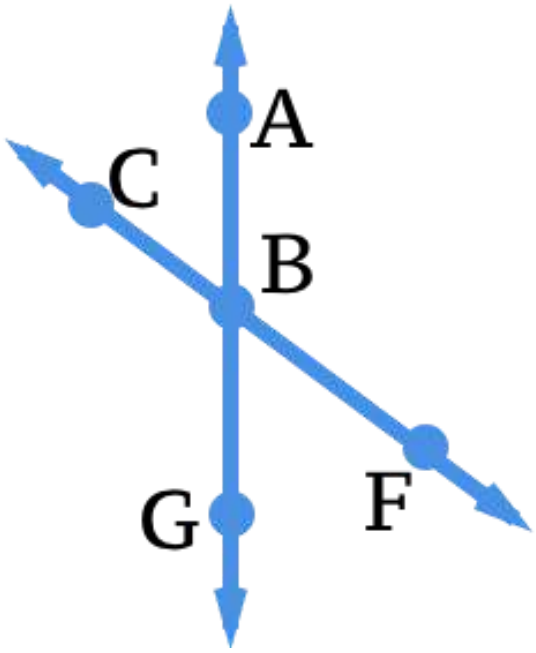

Figure 1.7.12

8. Give an example of vertical angles.
9. Give an example of a straight angle.
10. Give an example of supplementary angles.
11. If $m\angle ABC = 70°$, find $m\angle ABF$.
12. If $m\angle ABC = 70°$, find $m\angle FBG$.
13. What do you remember about perpendicular lines?

Use the angle in the image for #14-#15.

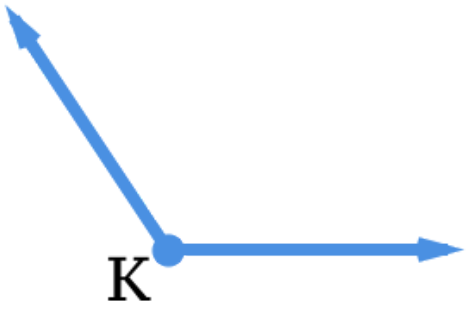

Figure **1.7.13**

14. Name the angle and classify it based on its size.

15. Estimate the measure of the angle. Use a protractor to confirm your answer.

16. Draw a diagram in which two angles are supplementary to the same angle. What must be true about the original two angles? Explain.

17. We use the term *complementary* to describe angles that sum to ___ and *supplementary* to describe angles that sum to ___. What about angles that sum to 360°? Invent a name for such angles and justify your choice.

18. Draw two vertical angles. How much must each ray of one of the vertical angles be rotated in order to match up with the other vertical angle? Explain.

19. Draw two angles of the same measure that are not vertical. Draw two angles that are supplementary but not adjacent. Draw two angles that are adjacent and have the same measure. What is the measure of each angle in the last drawing? Why?

Review (Answers)

To see the Review answers, click here.

Additional resources

Video: Comparing Angles

Practice: Introduction to Angles

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1.8: Identification of Angles by Vertex and Ray

Learn how to name angles using letters.

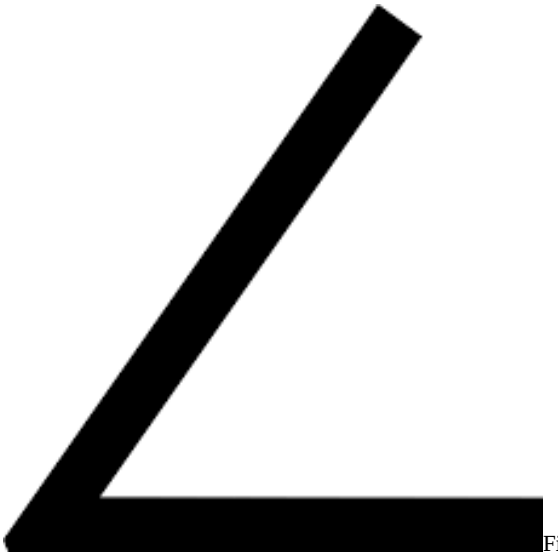
Figure 1.8.1

Donyell walks from his house to his school. Then, after school he walks to his friend's house. He notices that his journey forms an **angle**. He decides to name the angle. Donyell labels his house as **point** H, his school as point S and his friend's house as point F. Can you use the points to name of the angle formed by his journey?

In this concept, you will learn about the parts of an angle.

Identifying Angles by Vertex and Ray

An **angle** is one of the key geometric figures that you will be working with in geometry. An angle is created when two rays connect at a common point.

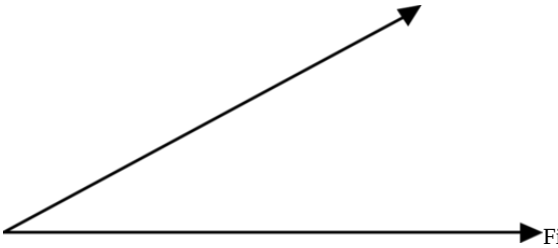
Figure 1.8.2

You can see that the two rays are connected at a common endpoint, called a **vertex**. This forms the angle. An angle is named by points on the rays.

Let's look at an example of an angle.

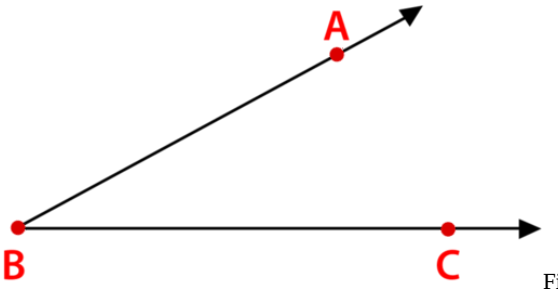

Figure 1.8.3

This is angle ABC. The vertex B is always in the middle. The symbol for angle looks like a small angle.

$\angle ABC$

Angle ABC is named with this symbol.

Example 1.8.1

Earlier, you were given a problem about Donyell and his walk to school and then to his friend's house.

Name the angle if his house is point H, his school is point S and his friend's house is point F.

Solution

First, determine the common endpoint.

S

Then, name the angle.

$\angle HSF$

The answer is $\angle HSF$.

Example 1.8.2

What is the common endpoint of the rays that form $\angle HIJ$?

Solution

First, determine the name of the common endpoint.

The vertex

Next, identify the location of the vertex in the angle name.

The middle point

Then, write the point.

I

The answer is I. The common endpoint of the rays that form $\angle HIJ$ is I.

Example 1.8.3

Name this angle. Be sure that the vertex is in the middle.

Figure 1.8.4

Solution

First, locate the common endpoint.

E

Then, name the angle.

$\angle DEF$

The answer is $\angle DEF$.

Example 1.8.4

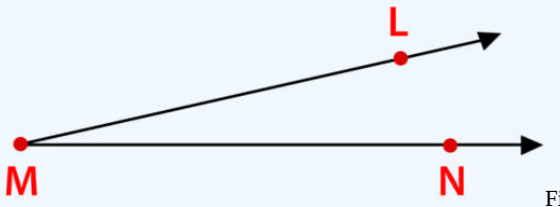

Figure 1.8.5

Solution

First, locate the common endpoint.

M

Then, name the angle.

$\angle LMN$

The answer is $\angle LMN$

Example 1.8.5

What is the common endpoint of the rays that form $\angle QRS$?

Solution

First, determine the name of the common endpoint.

The vertex

Next, identify the location of the vertex in the angle name.

The middle point

Then, write the point.

R

The answer is R. The common endpoint of the rays that form $\angle QRS$ is R.

Review

Draw a picture to illustrate each of the named geometric figures.

1. $\overrightarrow{AB}$
2. $\overleftrightarrow{CD}$
3. $\overleftrightarrow{DE}$
4. $\angle ABC$
5. $\angle LMN$
6. $\overline{XY}$
7. $\overrightarrow{PQ}$
8. $\overleftrightarrow{GH}$
9. $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$
10. $\overleftrightarrow{LM} \parallel \overleftrightarrow{DE}$
11. $\overleftrightarrow{RS} \parallel \overleftrightarrow{TU}$
12. $\overline{DF} \parallel \overline{XY}$

Define the following terms.

13. Ray
14. **Line**
15. Vertex
16. **Line Segment**

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.3.

Vocabulary

Term	Definition
Angle	A geometric figure formed by two rays that connect at a single point or vertex.
Intersecting lines	Intersecting lines are lines that cross or meet at some point.
line	A line is a straight one-dimensional figure that extends forever in opposite directions.
line segment	A line segment is a part of a line that has two endpoints.
Parallel	Two or more lines are parallel when they lie in the same plane and never intersect. These lines will always have the same slope.
Point	A point is a location in space that does not have size or shape.
Point of Intersection	A point of intersection is the point where two intersecting lines meet.
Ray	A ray is a part of a line that has one endpoint and continues indefinitely in one direction.
Vertex	A vertex is a point of intersection of the lines or rays that form an angle.

Additional Resource

Video: Introduction to Angles

Practice: Identification of Angles by Vertex and Ray

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1.9: Measuring Angles

Measurement of angles with protractors and the Angle Addition Postulate.

Angles

An **angle** is formed when two rays have the same endpoint. The **vertex** is the common endpoint of the two rays that form an angle. The **sides** are the two rays that form an angle.

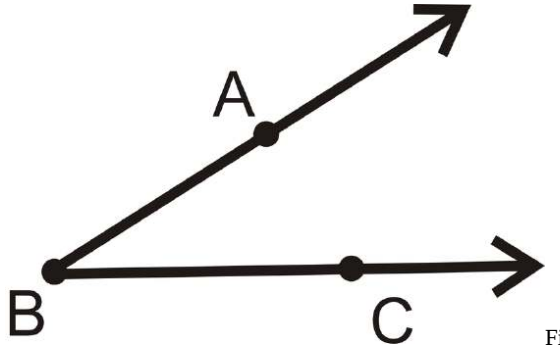


Figure **1.9.1**

Label It	*Say It*
$\angle ABC$	Angle ABC
$\angle CBA$	Angle CBA

The vertex is B and the sides are $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Always use three letters to name an angle, $\angle$ SIDE-VERTEX-SIDE.

Angles are measured with something called a **protractor**. A protractor is a measuring device that measures how "open" an angle is. Angles are measured in degrees and are labeled with a $^\circ$ symbol. For now, angles are always positive.

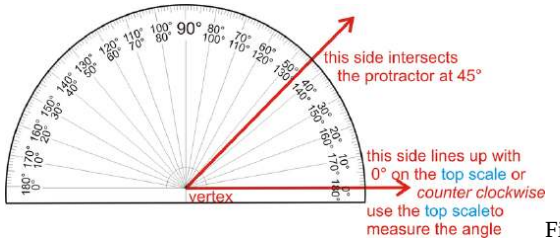


Figure **1.9.2**

There are two sets of measurements, one starting on the left and the other on the right side of the protractor. Both go around from **0°** to **180°**. When measuring angles, you can line up one side with **0°**, and see where the other side hits the protractor. The vertex lines up in the middle of the bottom line.

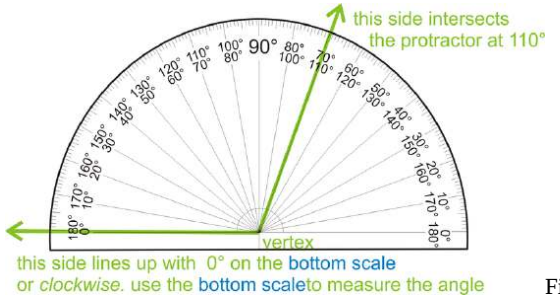


Figure **1.9.3**

Note that if you don't line up one side with **0°**, the angle's measure will be the difference of the degrees where the sides of the angle intersect the protractor.

Sometimes you will want to draw an angle that is a specific number of degrees. Follow the steps below to draw a $50°$ angle with a protractor:

1. Start by drawing a horizontal line across the page, 2 in long.

Figure 1.9.4

2. Place an endpoint at the left side of your line.
3. Place the protractor on this point, such that the line passes through the $0°$ mark on the protractor and the endpoint is at the center. Mark $50°$ on the appropriate scale.

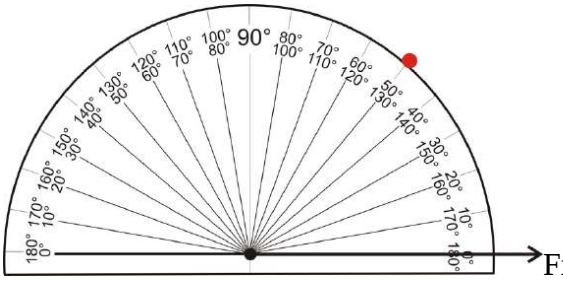


Figure \(\
PageIndex
{5}\)

4. Remove the protractor and connect the vertex and the $50°$ mark.

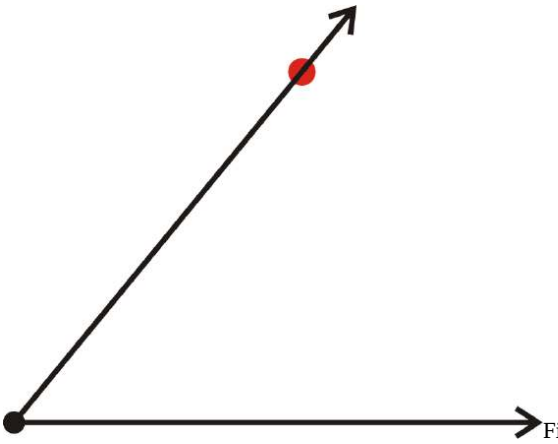


Figure 1.9.6

This process can be used to draw any angle between $0°$ and $180°$.

When two smaller angles [redacted]ger angle, the sum of the measures of the smaller angles will equal the measure of the larger angle. This is called [redacted]n **Postulate**. So, if B is on the interior of $\angle ADC$, then

$$\angle ADC = m\angle ADB + \angle BDC$$

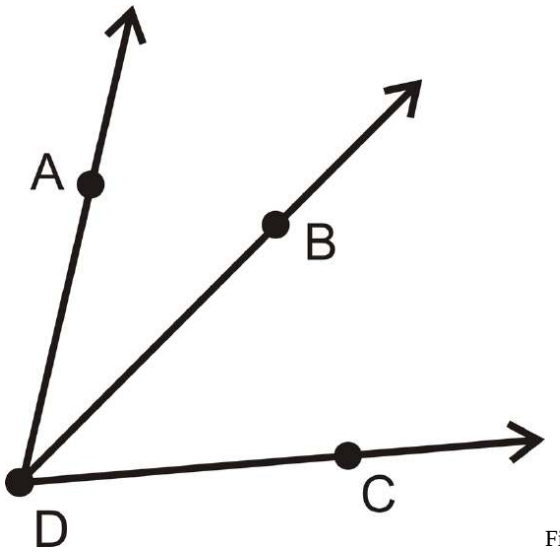


Figure 1.9.7

Example 1.9.1

How many angles are in the picture below? Label each one.

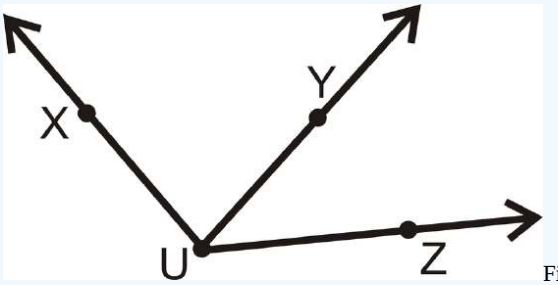


Figure 1.9.8

Solution

There are three angles with vertex U. It might be easier to see them all if we separate them.

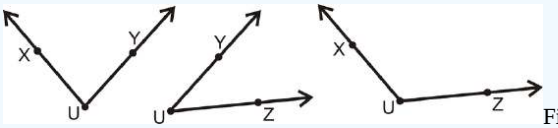


Figure 1.9.9

So, the three angles can be labeled, ($\angle XUY$ or $\angle YUX$), $\angle YUZ$ (or $\angle ZUY$), and $\angle XUZ$ (or $\angle ZUX$).

Example 1.9.2

Measure the three angles from Example 1, using a protractor.

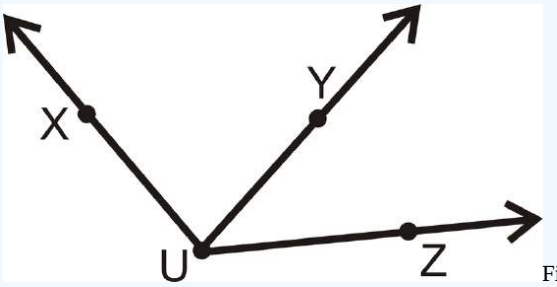


Figure 1.9.10

Solution

Just like in Example 1, it might be easier to measure these three angles if we separate them.

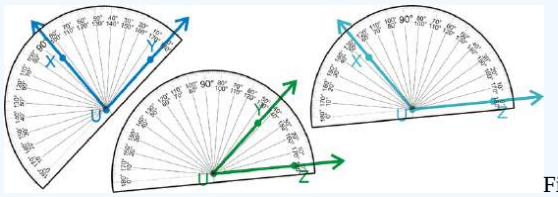


Figure 1.9.11

With measurement, we put an m in front of the $\angle$ sign to indicate measure. So, $m\angle XUY = 84°$, $m\angle YUZ = 42°$ and $m\angle XUZ = 126°$.

Example 1.9.3

What is the measure of the angle shown below?

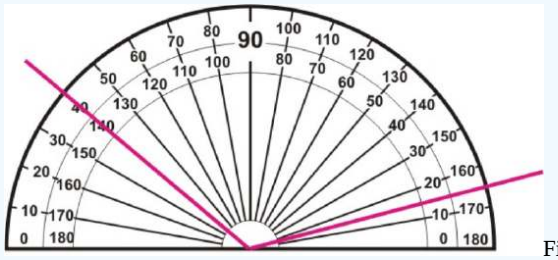


Figure 1.9.12

Solution

This angle is not lined up with $0\circ$, so use subtraction to find its measure. It does not matter which scale you use, as long as you are consistent.

Inner scale: $140° - 15° = 125°$

Outer scale: $165° - 40° = 125°$

Example 1.9.4

Use a protractor to measure $\angle RST$ below.

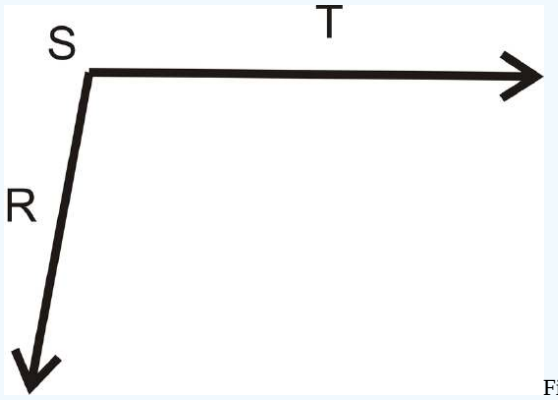


Figure 1.9.13

Solution

Lining up one side with $0°$ on the protractor, the other side hits $100°$.

Example 1.9.5

What is $m\angle QRT$ in the diagram below?

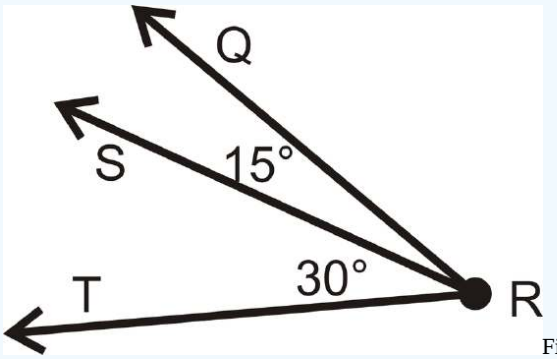


Figure 1.9.14

Solution

Using the Angle Addition Postulate, $m\angle QRT = 15^\circ + 30^\circ = 45^\circ$.

Review

1. What is $m\angle LMN$ if $m\angle LMO = 85^\circ$ and $m\angle NMO = 53^\circ$?

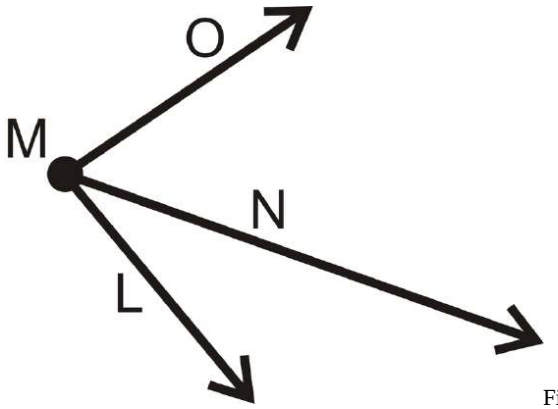


Figure 1.9.15

2. If $m\angle ABD = 100^\circ$, find x.

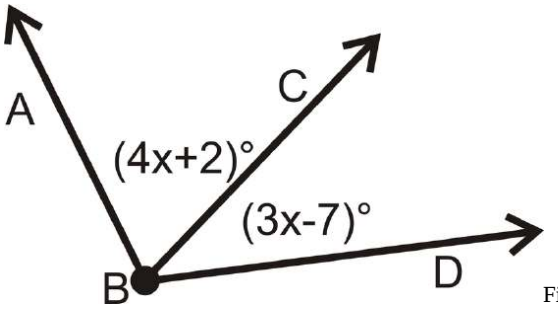


Figure 1.9.16

For questions 3-6, determine if the statement is true or false.

3. For an angle $\angle ABC$,C is the vertex.
4. For an angle $\angle ABC$, $\overline{AB}$ and $\overline{BC}$ are the sides.
5. The m in front of $m\angle ABC$ means measure.
6. The Angle Addition Postulate says that an angle is equal to the sum of the smaller angles around it.

For 7-12, draw the angle with the given degree, using a protractor and a ruler.

7. 55°
8. 92°
9. 178°
10. 5°

11. 120°
12. 73°

For 13-16, use a protractor to determine the measure of each angle.

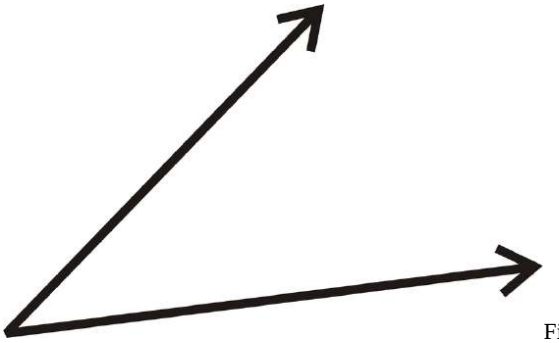
13. 

Figure 1.9.17

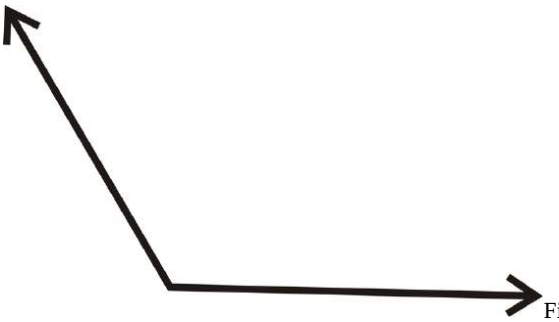
14. 

Figure 1.9.18

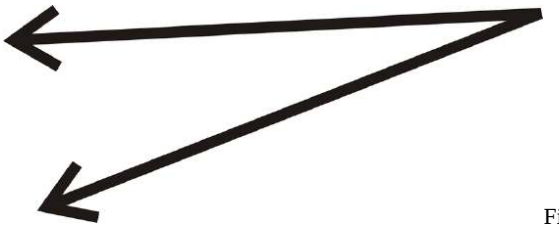
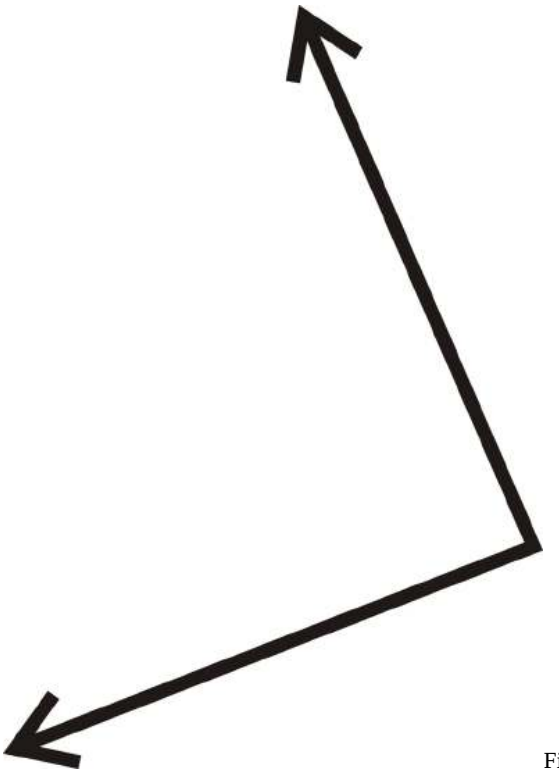
15. 

Figure 1.9.19



16. Figure 1.9.20

Solve for x.

17. $m\angle ADC = 56^\circ$

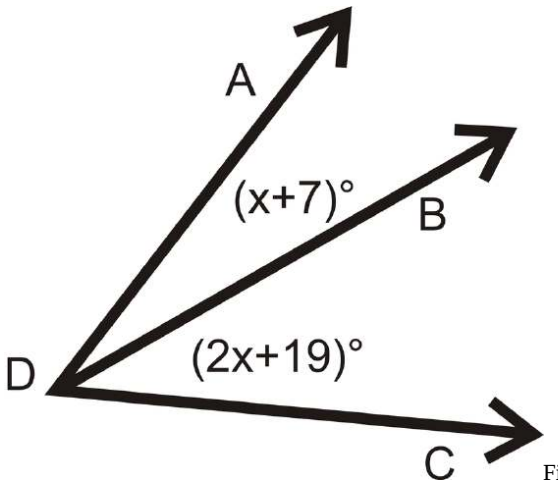


Figure 1.9.21

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.5.

Vocabulary

Term	Definition
Angle	A geometric figure formed by two rays that connect at a single point or vertex.
Protractor	A protractor is a tool used to measure an angle in terms of degrees.
Vertex	A vertex is a point of intersection of the lines or rays that form an angle.

Additional Resource

Video: Angle Basics

Activities: Angle Measurement Discussion Questions

Study Aids: Angles Study Guide

Practice: Measuring Angles

Real World: Angle Measurement

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1.10: Classifying Angles

Angle Classification

Figure 1.10.1

Mark wants to paint his bedroom door. The shapes on the door remind him of geometry. He decides to use a ruler to measure the actual lengths of the sides and a protractor to measure the angles. He notices the same angle repeating over and over again. What type of angle is formed by the corner of the door?

In this concept, you will learn how to classify angles.

Classifying Angles

An **angle** is the space created between two lines, line segments or rays connected at a common point.

Angles can be represented by using a symbol. Here is the symbol for an angle.

$\angle A$

This means "Angle A".

Angles are measured in **degrees. Degrees** measure the distance between the two lines. The o symbol means "degrees." The number of degrees tells how open or closed the angle is. The smaller the number of degrees, the smaller or more closed the angle.

Angles are classified by their size. Angle sizes can range from $0°$ to $360°$, a complete circle.

Here is a diagram that shows some angle measurements.

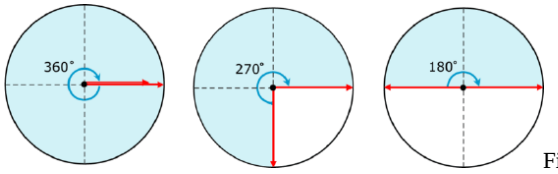

Figure 1.10.2

As you can see, an angle of $360°$ makes a complete circle. An angle of $270°$ is three-quarters of a circle, and an angle of $180°$ is half a circle. A $180°$ angle is a straight line.

Angles that measure between 0o and 180° are classified as **acute**, **right**, **obtuse**, or **straight**.

An **acute angle** measures less than 90°.

An **obtuse angle** has a measure greater than 90° and less than 180°.

Here are a few examples.

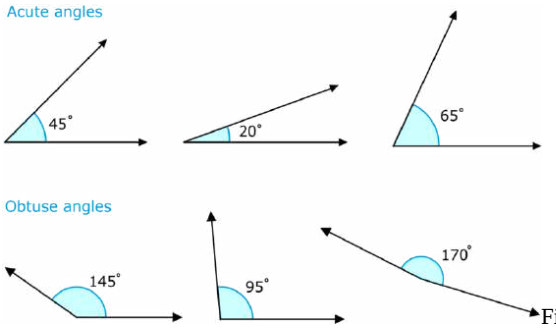

Figure 1.10.3

Most angles in the 0° and 180° range are either acute or obtuse. However, there are two special angles with exact measurements.

A **right angle** measures exactly 90°. Right angles are one of the most important concepts you need to know about geometry. Right angles are found in squares, rectangles, and triangles. They are everywhere in the real world too.

Here are a few examples.

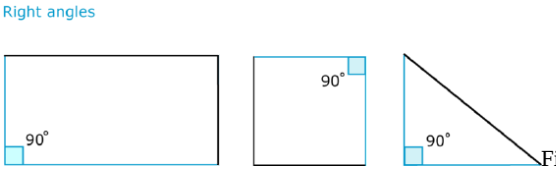

Figure 1.10.4

Notice that a small box is used when an angle is a right angle.

There are many places in the real world where you can see acute, obtuse and right angles.

Here are a few examples.

Figure 1.10.5

If you look at each of these pictures, you can see the right angles clearly. Also notice that the wires of the bridge stretch to create acute angles on each side of the center beam.

The other special angle is called a **straight angle**. A straight angle forms a line and measures exactly 180∘.

Let's look at an example.

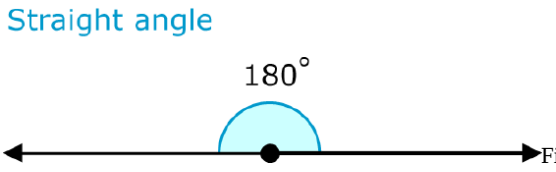

Figure 1.10.6

Now, let’s try classifying angles.

Classify each angle below.

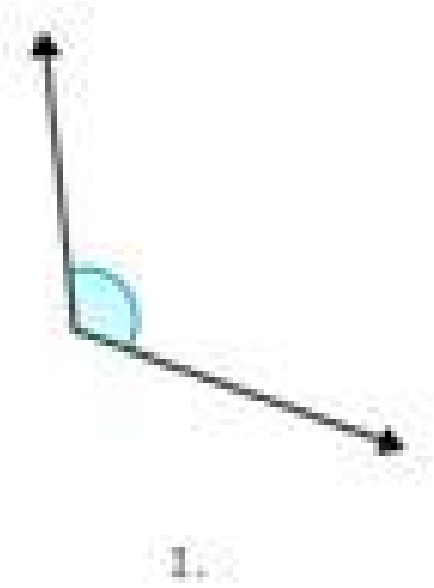

Figure 1.10.7

For each angle, it may help to ask yourself: “Is it bigger or smaller than a right angle?”

Remember, right angles always measure 90°, and are used to tell whether an angle is acute or obtuse.

First, understand that the angle in Figure 1.10.2 is not a right angle or straight angle.

The angle does not have a perfect corner to represent a right angle and the angle is not a straight line.

Next, determine if the angle in Figure 1.10.2 is larger or smaller than a right angle.

The angle is larger than a perfect corner, therefore, the angle is larger than a right angle.

Then, classify the angle.

Obtuse.

The answer is that Figure 1.10.2 shows an obtuse angle.

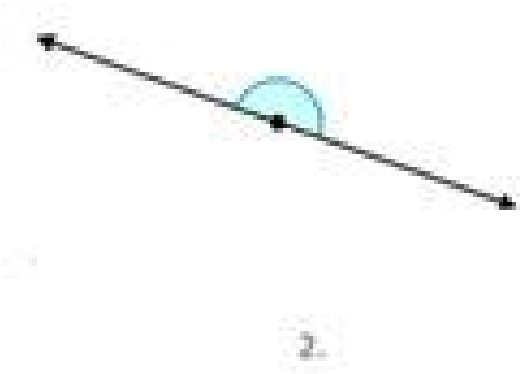

Figure 1.10.8

The angle in Figure 1.10.3 appears to be a straight line.

First, recognize that Figure 1.10.3 appears to be a straight line.

The line does not appear to bend.

Next, remember the meaning of a straight line.

A straight line is a special angle.

Then, classify the angle.

Straight.

The answer is that Figure **1.10.3** shows a straight angle.

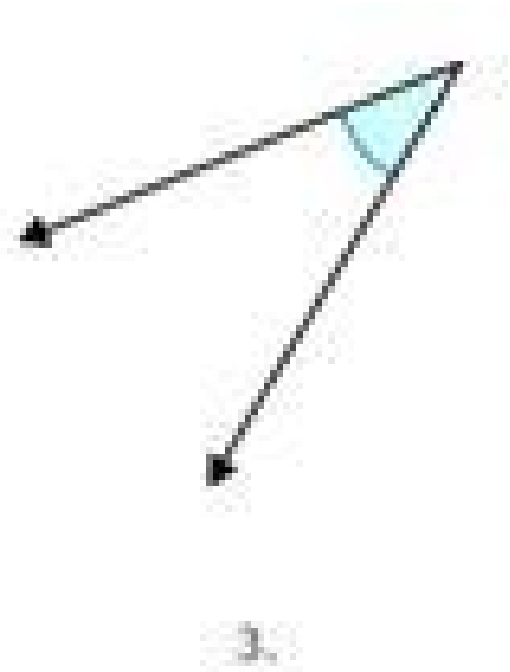

Figure **1.10.9**

Is Figure **1.10.4** larger or smaller than a right angle?

First, understand that the angle in Figure **1.10.4** is not a right angle or straight angle.

The angle does not have a perfect corner to represent a right angle and the angle is not a straight line.

Next, determine if the angle in Figure **1.10.4** is larger or smaller than a right angle.

The angle is smaller than a right angle.

Then, classify the angle.

Acute.

The answer is that Figure **1.10.4** shows an acute angle.

Figure 1.10.10

The angle in Figure 1.10.5 appears to be a special type of angle.

First, recognize the appearance of the angle in Figure 1.10.5.

The angle appears to form a perfect corner.

Next, remember what a perfect corner represents.

A perfect corner represents a special angle.

Then, classify the angle.

Right.

The answer is that Figure 1.10.5 shows a right angle.

Example 1.10.1

Earlier, you were given a problem about Mark and his bedroom door.

He notices the same angle on multiple different places on the door. What type of angle is formed by the corner of the door?

Solution

First, highlight the relevant angles.

Highlight each corner angle.

Next, list the angle options.

Acute, right, obtuse.

Then, label the corner angle.

Right.

The answer is that the corners of the door are right angles.

Example 1.10.2

True or false. An acute angle can also be a right angle.

Solution

First, recall the definition of an acute angle.

An acute angle is less than $90°$.

Next, remember that angles measuring exactly $90°$ are special angles.

Angles measuring exactly $90°$ are classified as right angles.

Then, state your conclusion.

An acute angle cannot be a right angle.

The answer is that the statement is false. An acute angle measures less than $90°$ while a right angle measures exactly $90°$, therefore, an acute angle cannot be a right angle.

Example 1.10.3

Identify each type of angle described.

An angle greater than $90°$ but less than $180°$.

Solution

First, note the size of the angle.

The angle is greater than $90°$ but less than $180°$.

Next, remember the rules for angles.

There is a name for angles greater than $90°$ but less than $180°$.

Then, classify the angle.

Obtuse.

The answer is that the angle is obtuse.

Example 1.10.4

Identify each type of angle described.

An angle that measures $15°$.

Solution

First, note the size of the angle.

The angle is less than $90°$.

Next, remember the rules for angles.

There is a name for angles that are less than $90°$.

Then, classify the angle.

Acute.

The answer is that the angle is acute.

Example 1.10.5

Identify each type of angle described.

An angle that measures exactly 90°

Solution

First, note the size of the angle.

The angle is exactly 90°.

Next, remember the rules for angles.

There is a name for angles that are exactly 90°.

Then classify the angle.

Right.

The answer is that the angle is a right angle.

Review

Label each angle as acute, obtuse, right, or straight.

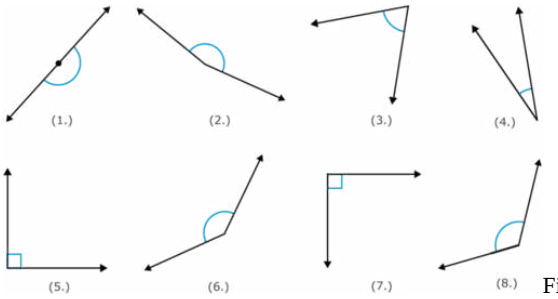

Figure 1.10.11

9. 55°
10. 102°
11. 90°
12. 180°
13. 10°
14. 87°
15. 134°

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.1.

Resources

Additional Resource

Video:Animation of Types

Activities: Angle Classific uestions

Study Aids: Angles Study

Practice: Classifying Angles

Real Worlds: Angle Classification

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1.11: Congruent Angles and Angle Bisectors

Angles of the same measure, and lines or portions of lines that divide angles into two equal halves.

When two geometric figures have the same shape and size (or the same angle measure in the case of angles) they are said to be **congruent**.

Label It	*Say It*
$\angle ABC \cong \angle DEF$	Angle ABC is congruent to angle DEF.

If two angles are congruent, then they are also equal. To label equal angles we use **angle markings**, as shown below:

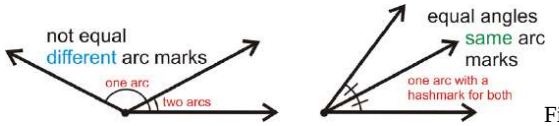


Figure 1.11.1

An **angle bisector** is a line, or a portion of a line, that divides an angle into two congruent angles, each having a measure exactly half of the original angle. Every angle has exactly one angle bisector.

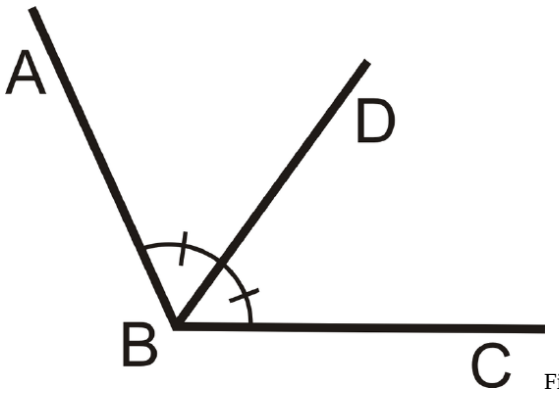


Figure 1.11.2

In the picture above, $\overline{BD}$ is the angle bisector of $\angle ABC$, so $\angle ABC \cong \angle DBC$ and $m\angle ABD \cong \frac{1}{2} m\angle ABC$.

What if you were told that a line segment divides an angle in half? How would you find the measures of the two new angles formed by that segment?

For Examples 1 and 2, cop and label it with the following information:

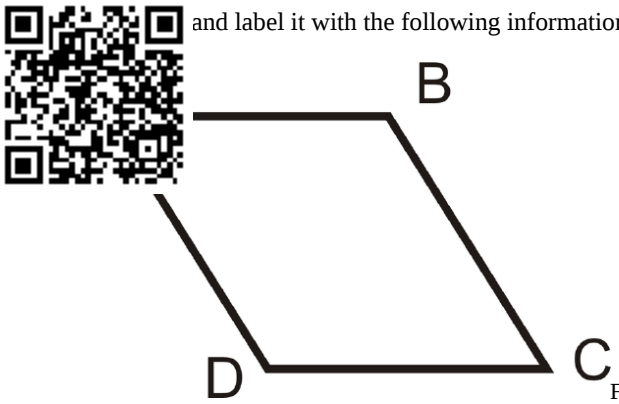


Figure 1.11.3

Example 1.11.1

$\angle A \cong \angle C$

Solution

You should have corresponding markings on $\angle A$ and $\angle C$.

Example 1.11.2

$\angle B \cong \angle D$

Solution

You should have corresponding markings on $\angle B$ and $\angle D$ (that look different from the markings you made in Example 1).

Example 1.11.3

Write all equal angle statements.

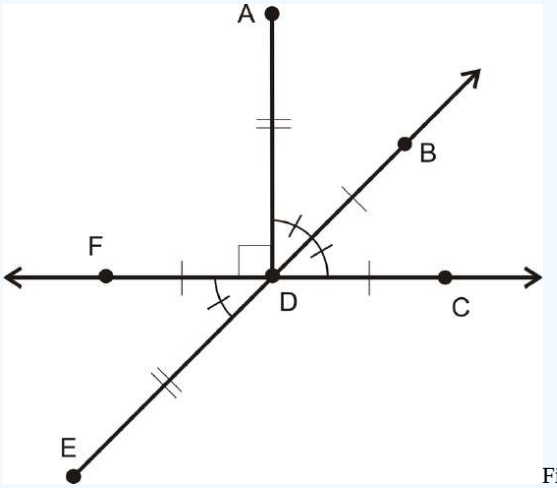


Figure 1.11.4

Solution

$$m\angle ADB = m\angle BDC = m\angle FDE = 45°$$

$$m\angle ADF = m\angle ADC = 90°$$

Example 1.11.4

What is the measure of each angle?

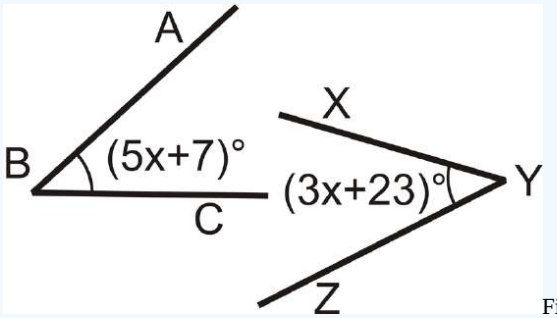


Figure 1.11.5

Solution

From the picture, we see that the angles are equal.

Set the angles equal to each other and solve.

$$(5x + 7)° = (3x + 23)°$$

$$(2x)° = 16°$$

$$x = 8$$

To find the measure of $\angle ABC$, plug in $x = 8$ to

$(5x + 7)^\circ \rightarrow (5(8) + 7)^\circ = (40 + 7)^\circ = 47^\circ$.

Because $m\angle ABC = m\angle XYZ$, $m\angle XYZ = 47^\circ$ too.

Example 1.11.5

Is $\overline{OP}$ the angle bisector of $\angle SOT$?

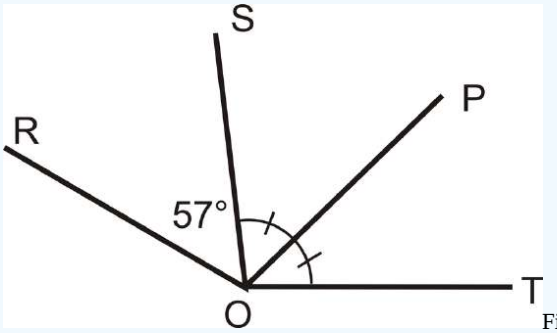


Figure 1.11.6

Solution

Yes, $\overline{OP}$ is the angle bisector of $\angle SOT$ from the markings in the picture.

Review

For 1-4, use the following picture to answer the questions.

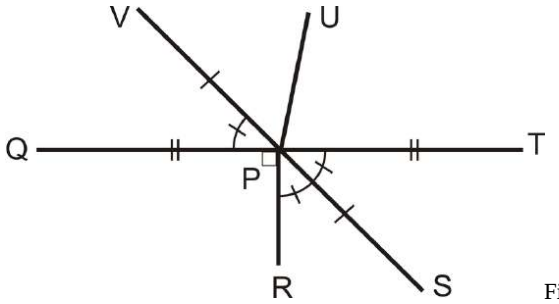


Figure 1.11.7

1. What is the angle bisector of $\angle TPR$?
2. What is $m\angle QPR$?
3. What is $m\angle TPS$?
4. What is $m\angle QPV$?

For 5-6, use algebra to determine the value of variable in each problem.

5.

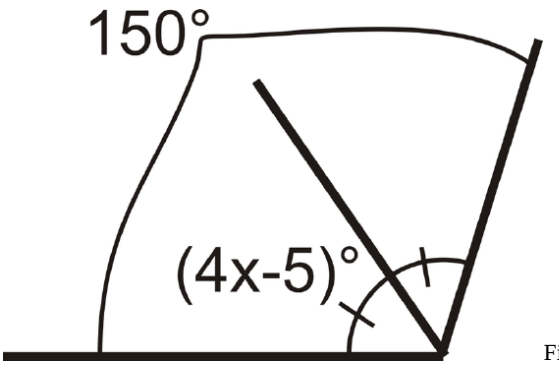


Figure 1.11.8

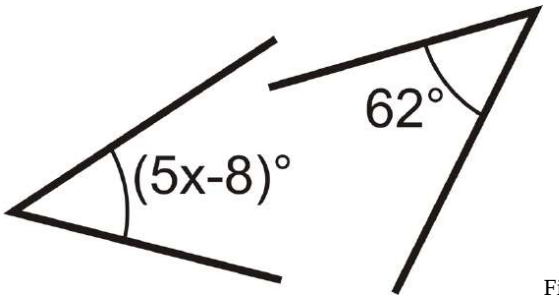
6. 

Figure **1.11.9**

For 7-10, decide if the statement is true or false.

7. Every angle has exactly one angle bisector.
8. Any marking on an angle means that the angle is **90°**.
9. An angle bisector divides an angle into three congruent angles.
10. Congruent angles have the same measure.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.3.

Vocabulary

Term	Definition
angle bisector	An angle bisector is a ray that splits an angle into two congruent, smaller angles.
Congruent	Congruent figures are identical in size, shape and measure.

Additional Resource

Video: Congruent Angles and Angle Bisectors Principles - Basic

Activities: Congruent Angles and Angle Bisectors Discussion Questions

Study Aids: Angles Study Guide

Practice: Congruent Angles and Angle Bisectors

Real World: Starry Night

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1.12: Constructions and Bisectors

Construct angles and line segments. Divide angles and line segments into two congruent parts.

Bisectors of Line Segments and Angles

Describe how to construct a $45°$ angle using a compass and a straightedge.

Bisectors

A **construction** is similar to a **drawing** in that it produces a visual outcome. However, while drawings are often just rough sketches that help to convey an idea, constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

To **bisect** a segment or an angle means to divide it into two congruent parts. A **bisector** of a line segment will pass through the midpoint of the line segment. A **perpendicular bisector** of a segment passes through the midpoint of the line segment and is perpendicular to the line segment. In order to construct bisectors of segments and angles, it's helpful to remember some relevant theorems:

Any point on the perpendicular bisector of a line segment will be equidistant from the endpoints of the line segment. *This means that one way to find the perpendicular bisector of a segment (such as $\overline{AB}$ below) is to find two points that are equidistant from the endpoints of the line segment (such as C and D below) and connect them.*

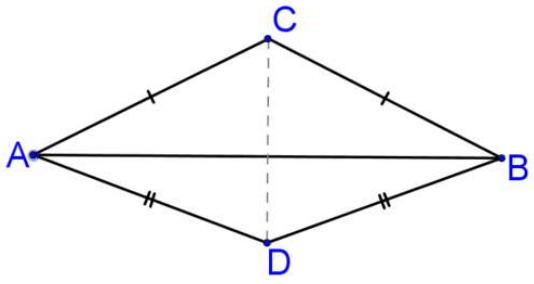


Figure 1.12.1

Any point on the angle bisector of an angle will be equidistant from the rays that create the angle. *This means that one way to find the angle bisector of an angle (such as $\angle BAC$ below) is to find two points that are equidistant from the rays that create the angle (such as points A and D below).*

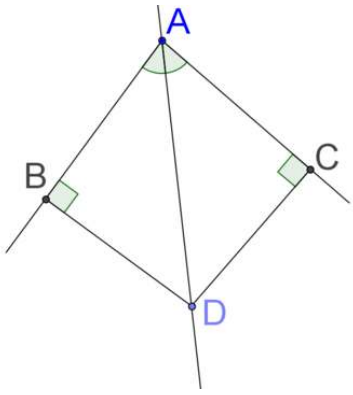


Figure 1.12.2

Let's take a look at differe perpendicular bisector of a line segment.

1. Draw a line segment lik elow on a piece of paper (the rectangle below represents the paper). Fold the paper in order to find the perpendic line segment.

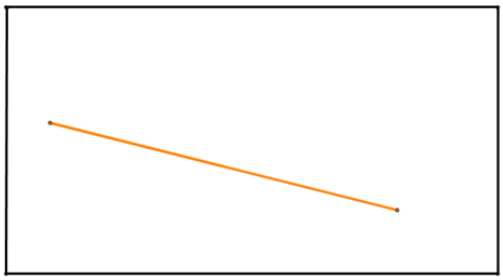


Figure 1.12.3

To find the perpendicular bisector, fold the paper so that the endpoints lie on top of each other. By doing this, you have matched the two halves of the line segment exactly.

Any point on the fold is now equidistant from both endpoints, because the endpoints are now in the same place! This means that the crease made by the fold will be the perpendicular bisector of the line segment.

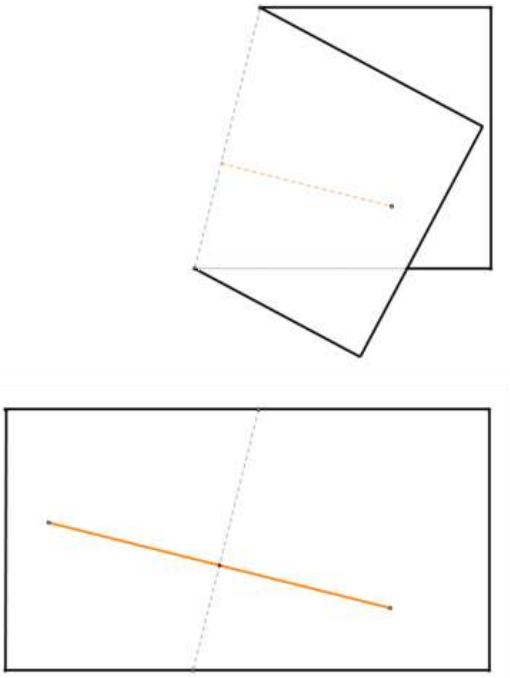


Figure 1.12.4

2. Draw a line segment like the orange one below on a piece of paper (the rectangle below represents the paper). Use a compass and straightedge to find the perpendicular bisector of the line segment.

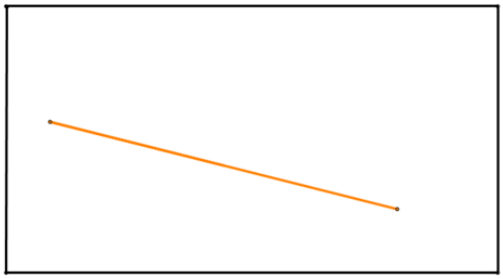


Figure 1.12.5

Use your compass to draw a circle centered at each endpoint with the **same radius**. Make sure the radius of the circles is large enough so that the circles will intersect. *If the circles are too big to fit on the paper, draw the portion of the circle that fits on the paper.* Here is the first circle:

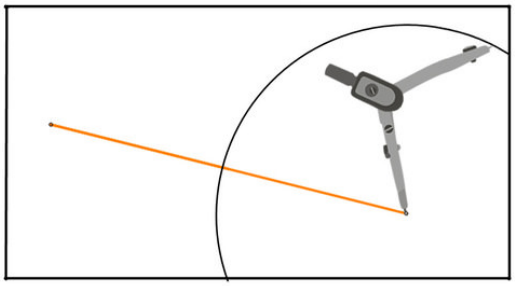


Figure 1.12.6

Here is the second circle:

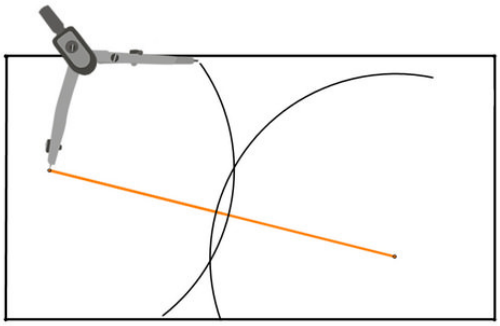


Figure 1.12.7

The points of interest are the two points where the circles intersect. Because the radius of each circle is the same, each of these points are equidistant from the endpoints of the line segment.

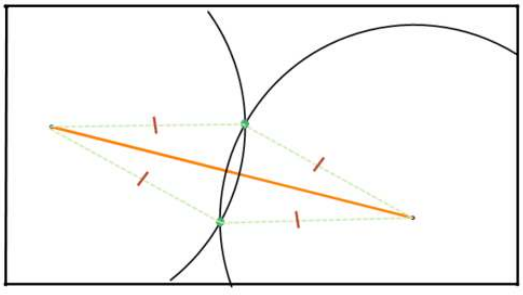


Figure 1.12.8

Therefore, each of these points lies on the perpendicular bisector of the line segment. Use your straightedge to draw a line connecting the two intersection points. This is the perpendicular bisector of the line segment.

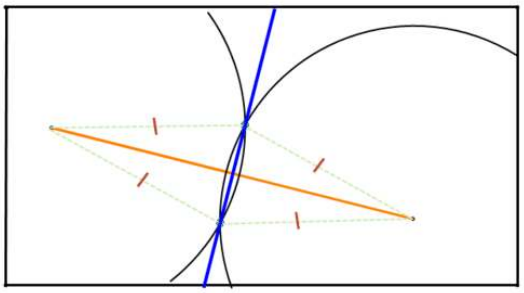


Figure 1.12.9

How can you use a compass and straightedge to find the midpoint of a line segment?

One way is to use the process from Example B to construct the perpendicular bisector. The midpoint of the line segment will be the point where the perpendicular bisector intersects the line segment.

Example 1.12.1

Earlier, you were asked how to construct a $45°$ angle using a compass and a straightedge.

One way to construct a $45°$ angle is to:

- Draw a segment and construct its perpendicular bisector. This will give you $90°$ angles.
- Construct the angle bisector of one of those $90°$ angles. This will produce two $45°$ angles. *Note: Steps for constructing an angle bisector are explored in the guided practice questions.*

Can you think of another way to construct a perfect $45°$ angle with just a compass and a straightedge?

Example 1.12.2

Draw an angle like the one below on a piece of paper (the rectangle below represents the paper). Note that the two segments creating the angle do NOT need to be the same length. Fold the paper to create the bisector of the angle.

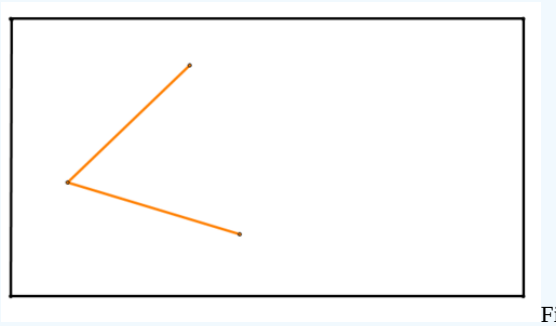
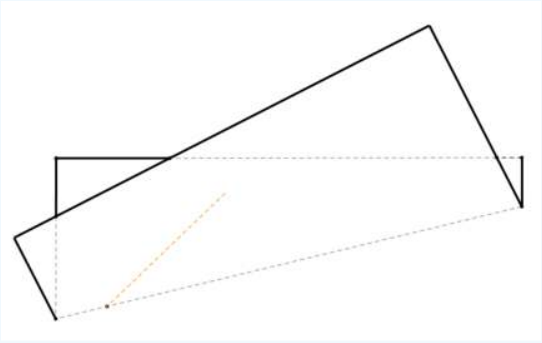


Figure 1.12.10

Solution

Fold the paper so that one segment overlaps the other segment. The crease will be the angle bisector.



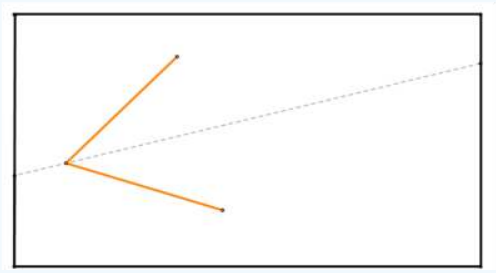


Figure 1.12.11

Example 1.12.3

Draw an angle like the one below on a piece of paper (the rectangle below represents the paper). Note that the two segments creating the angle do NOT need to be the same length. Use a compass and straightedge to find the bisector of the angle.

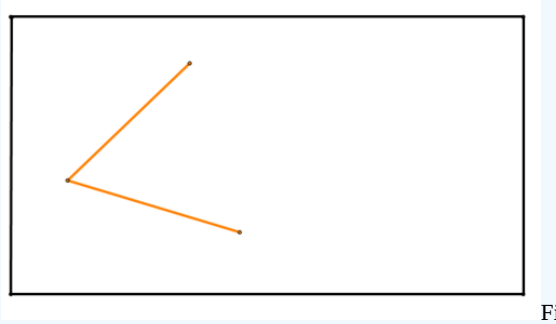


Figure 1.12.12

Solution

Use your compass to create a portion of a circle centered at the vertex of the angle that passes through both segments creating the angle:

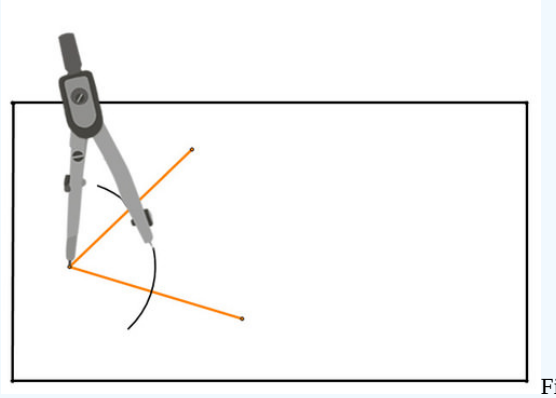


Figure 1.12.13

Create two circles with the same radius centered at each intersection point of the arc and the two segments. Make sure the radius is big enough so that the circles will overlap. Here is the first circle:

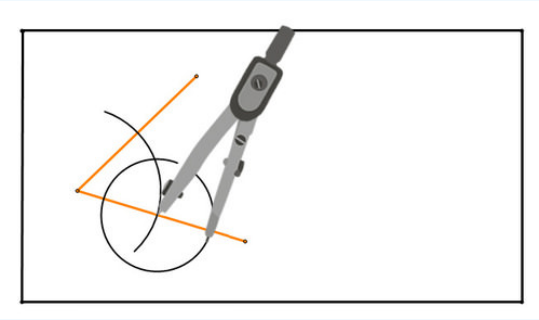


Figure 1.12.14

Here is the second circle:

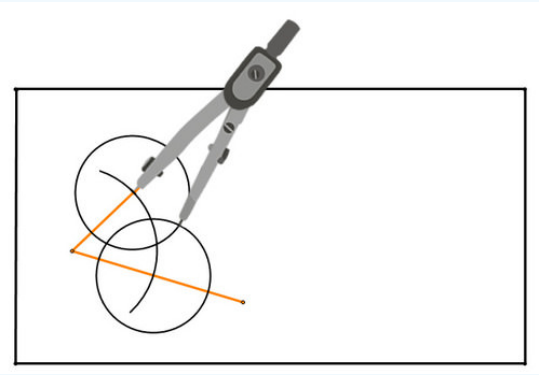


Figure 1.12.15

The points where the circles intersect are equidistant from the segments creating the angle. Therefore, they define the bisector of the angle. Connect those intersection points to create the angle bisector:

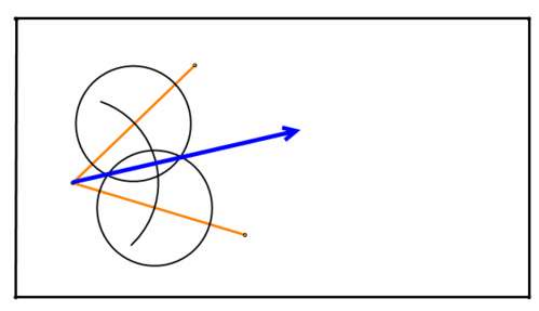


Figure 1.12.16

Example 1.12.4

Prove that the angle bisector created using the method in #2 is actually the angle bisector.

Consider the construction from #2 below. Points have been labeled and two additional segments have been drawn in (shown in green):

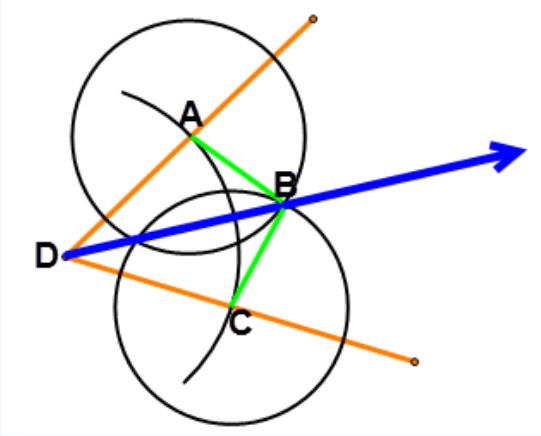


Figure \(\

PageIndex
{17}\)

Solution

In this picture, \overline$AD \cong \overline{CD}$because they are both radiuses of the same partial circle centered at point D. $\overline{AB} \cong$ \overline$CB$ because they are both radiuses of congruent circles centered at A and C respectively. $\overline{BD} \cong \overline{BD}$ by the reflexive property. Therefore, $\Delta ABD \cong \Delta CBD$ by $SSS \cong$. $\angle ADB \cong \angle BDC$ because they are corresponding parts of congruent triangles. Therefore, $\overrightarrow{BD}$ must be the angle bisector of $\angle ADC$.

Review

1. What does it mean to *bisect* a segment or an angle?
2. Describe the steps for finding the perpendicular bisector of a line segment.
3. Describe how to use the perpendicular bisector of a line segment to find the midpoint of the line segment.
4. What's the difference between a bisector and a perpendicular bisector? How can you construct a non-perpendicular bisector of a line segment?
5. Draw a line segment on your paper and construct the perpendicular bisector of the segment.
6. Draw another line segment on your paper and construct the perpendicular bisector of that segment using another method.
7. Draw an angle on your paper and construct the bisector of the angle.
8. Draw another angle on your paper and construct the bisector of that angle using another method.
9. Use your straightedge to draw a triangle on your paper. Construct the angle bisector of each angle. What point have you found?
10. Use your straightedge to draw a triangle on your paper. Construct the perpendicular bisector of each side of the triangle. What point have you found?
11. Use the triangle and your work from #10. Construct the medians of the triangle. What point have you found?
12. Compare and contrast the two methods for finding a bisector-paper folding vs. compass and straightedge.
13. Construct a 45° angle (look at the concept problem for help). Then, construct a 22.5° angle.
14. Construct an isosceles right triangle. *Hint: Start by creating a right angle by constructing a perpendicular bisector*.
15. If possible, extend your construction of an isosceles right triangle to construct a square. Describe your steps.

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.2.

Vocabulary

Term	Definition
perpendicular bisector	A perpendicular bisector of a line segment passes through the midpoint of the line segment and intersects the line segment at $90°$.
Ray	A ray is a part of a line that has one endpoint and continues indefinitely in one direction.
Bisector	A bisector divides a line segment or angle into two congruent parts.

Additional Resource

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1.13: Angle Properties and Theorems

Find angles and line segments, and determine if shapes are congruent and lines are parallel. Understand complementary angles as angles whose sum is 90 degrees and supplementary angles as angles whose sum is 180 degrees.

Measures of Angle Pairs

Figure **1.13.1**

The foul lines of a baseball diamond intersect at home plate to form a right **angle**. A baseball is hit from home plate and forms an angle of **36°** with the third base foul line. What is the measure of the angle between the first base foul line and the bath of the baseball?

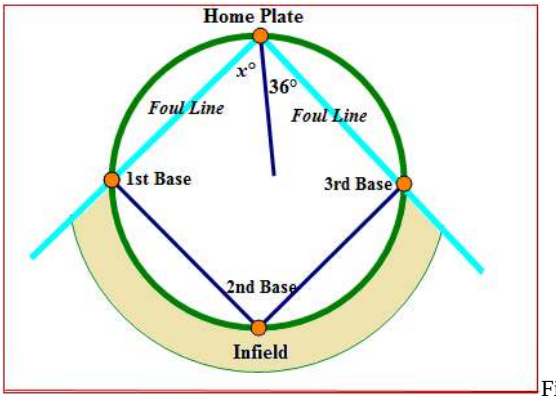

Figure **1.13.2**

How can you use your knowledge of angles to figure out the measure of the angle?

In this concept, you will learn measure of angle pairs.

Measuring Angle Pairs

There are different types of angle pairs. **Vertical angles** are an angle pair formed by **intersecting lines** such that they are never adjacent. They have a common vertex and never share a common side. Vertical angles are equal in measure. The following diagram shows vertical angle pairs.

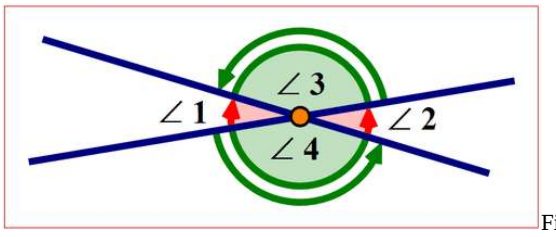

Figure **1.13.3**

$\angle 1$ and $\angle 2$ are vertical angles. $m\angle 1 = m\angle 2$

$\angle 3$ and $\angle 4$ are vertical angles. $m\angle 3 = m\angle 4$

Adjacent angles are an angle pair also formed by two intersecting lines. **Adjacent angles** are side by side, have a common vertex and share a common side. The following diagram shows pairs of adjacent angles.

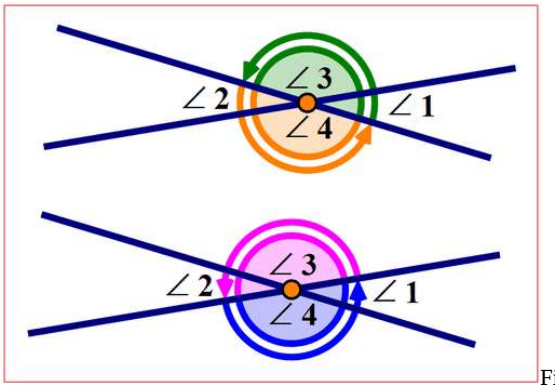

Figure 1.13.4

Each pair of adjacent angles forms a **straight angle**. Therefore the sum of any two adjacent angles equals $180°$.

$$m\angle 1 + m\angle 3 = 180°$$

$$m\angle 2 + m\angle 4 = 180°$$

$$m\angle 2 + m\angle 3 = 180°$$

$$m\angle 1 + m\angle 4 = 180°$$

If the sum two angles is $180°$ then the angles are called **supplementary angles**. The following diagram shows two supplementary angles.

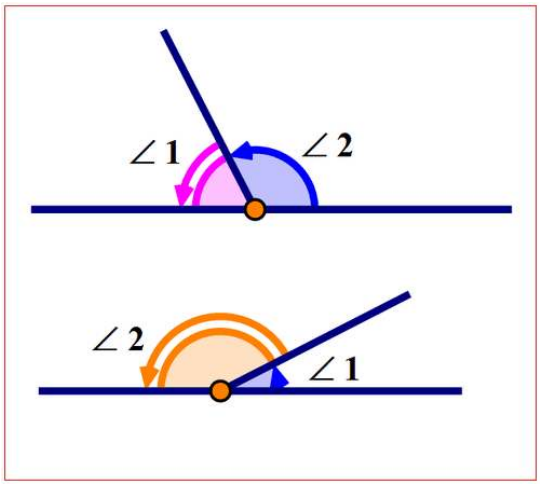

Figure 1.13.5

In both diagrams, $m\angle 1 + m\angle 2 = 180°$.

If the sum of two angles equals 90° then the angles are called **complementary angles**. The following diagram shows two complementary angles.

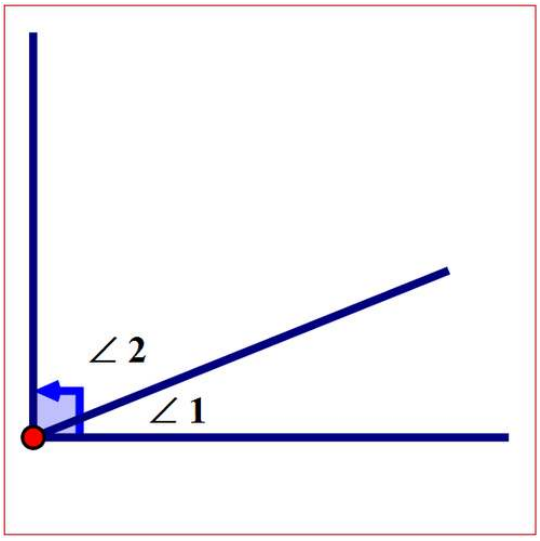

Figure 1.13.6

$m\angle 1 + m\angle 2 = 90°$

Let's apply all this information about angles and their measure to determine the measure of $\angle a$, $\angle b$, $\angle c$ in the following diagram.

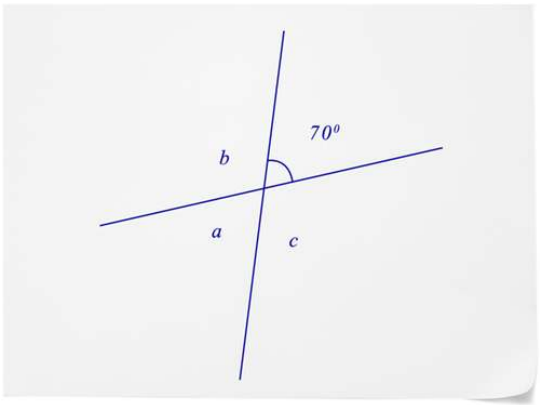

Figure 1.13.7

There are four angles formed by intersecting lines. The measure of one of the angles is $70°$.

First, state the relationship between the angle of $70°$ and $\angle b$.

The angle of $70°$. is adjacent to $\angle b$ and the two angles form a straight angle.

Next, express the relationship using symbols.

$$\angle b + 70° = 180°$$

Next, subtract 70° from both sides of the equation.

$$\angle b + 70° = 180°$$

$$\angle b + 70° - 70° = 180° - 70°$$

Then, simplify both sides of the equation.

$$\angle b + 70° - 70° = 180° - 70°$$

$$\angle b = 110°$$

The answer is $110°$.

$$m\angle b = 110°$$

First, state the relationship between the angle of $70°$ and $\angle a$.

The angle of $70°$ and $\angle a$ are vertical angles and are equal in measure.

Next, express the relationship using symbols.

$$m\angle a = 70°$$

The answer is $70°$.

$$m\angle a = 70°$$

First state the relationship between the angle of $70°$ and $\angle c$.

The angle of $70°$ is adjacent to $\angle c$ and the two angles form a straight angle.

Next, express the relationship using symbols.

$$\angle c + 70° = 180°$$

Next, subtract $70°$ from both sides of the equation.

$$\angle c + 70° = 180°$$

$$\angle c + 70° - 70° = 180° - 70°$$

Then, simplify both sides of the equation.

$$\angle c + 70° - 70° = 180° - 70°$$

$$\angle c = 110°$$

The answer is $110°$.

$$m\angle c = 110°$$

Example 1.13.1

Earlier, you were given a problem about the baseball field and the foul lines.

The angle between the path of the ball and the first base foul line needs to be figured out. This can be done using complementary angles.

Solution

First, draw a diagram to model the problem.

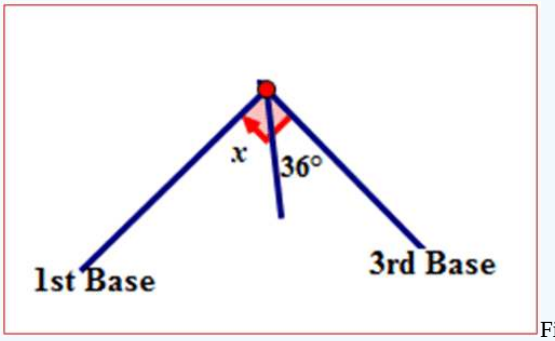

Figure 1.13.8

Next, state the relationship between $36°$ and $\angle x$.

$36°$ and $\angle x$ are complementary angles. The sum of the angles is $90°$.

Next, express the relationship using symbols.

$$36° + \angle x = 90°$$

Next, subtract 36° from both sides of the equation.

$$36° + \angle x = 90°$$

$$36^\circ - 36^\circ + \angle x = 90^\circ - 36^\circ$$

Then, simplify both sides of the equation.

$$36^\circ - 36^\circ + \angle x = 90^\circ - 36^\circ$$

$$\angle x = 54^\circ$$

The answer is 54°.

An angle of 54° is made between the first base foul line and the path of the baseball.

Example 1.13.2

If the following angles are complementary, find the measure of the missing angle.

$\angle A = 37^\circ$ then $\angle B =$?

Solution

First, draw a diagram to model the problem.

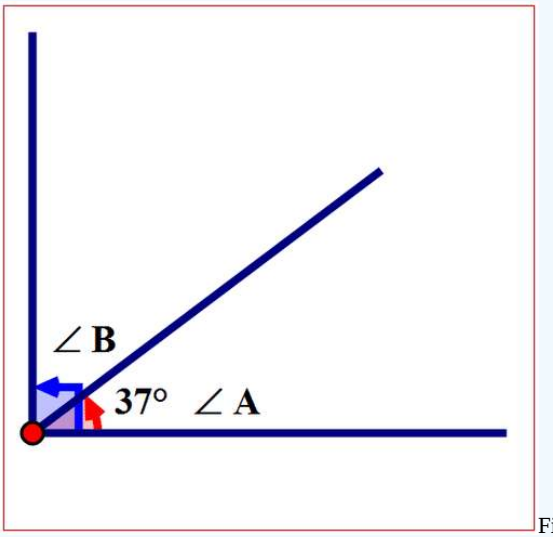

Figure 1.13.9

Next, state the relationship between $\angle A$ and $\angle B$.

$\angle A$ and $\angle B$ are complementary angles. The sum of the angles is 90°.

Next, express the relationship using symbols.

$$\angle A + \angle B = 90^\circ$$

Next, substitute the measure of $\angle A$ into the equation.

$$37^\circ + \angle B = 90^\circ$$

Next, subtract 37° from both sides of the equation.

$$37^\circ + \angle B = 90^\circ$$

$$37^\circ - 37^\circ + \angle B = 90^\circ - 37^\circ$$

Then, simplify both sides of the equation.

$$37^\circ - 37^\circ + \angle B = 90^\circ - 37^\circ$$

$$\angle B = 53^\circ$$

The answer is 53°.

$$m\angle B = 53^\circ$$

Example 1.13.3

If the following angles are supplementary, find the measure of the missing angle.

$\angle A = 102^\circ$ then $\angle B =$?

Solution

First, draw a diagram to model the problem.

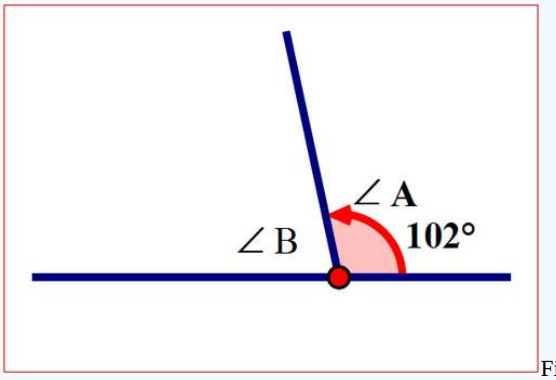

Figure 1.13.10

Next, state the relationship between $\angle A$ and $\angle B$.

$\angle A$ and $\angle B$ are supplementary angles. The sum of the angles is 180°.

Next, express the relationship using symbols.

$$\angle A + \angle B = 180^\circ$$

Next, substitute the measure of $\angle A$ into the equation.

$$102^\circ + \angle B = 180^\circ$$

Next, subtract 102° from both sides of the equation.

$$102^\circ + \angle B = 180^\circ$$

$$102^\circ - 102^\circ + \angle B = 180^\circ - 102^\circ$$

Then, simplify both sides of the equation.

$$102^\circ - 102^\circ + \angle B = 180^\circ - 102^\circ$$

$$\angle B = 78^\circ$$

The answer is 78°.

$$m\angle B = 78^\circ$$

Example 1.13.4

Using the following diagram, determine the measures of the missing angles.

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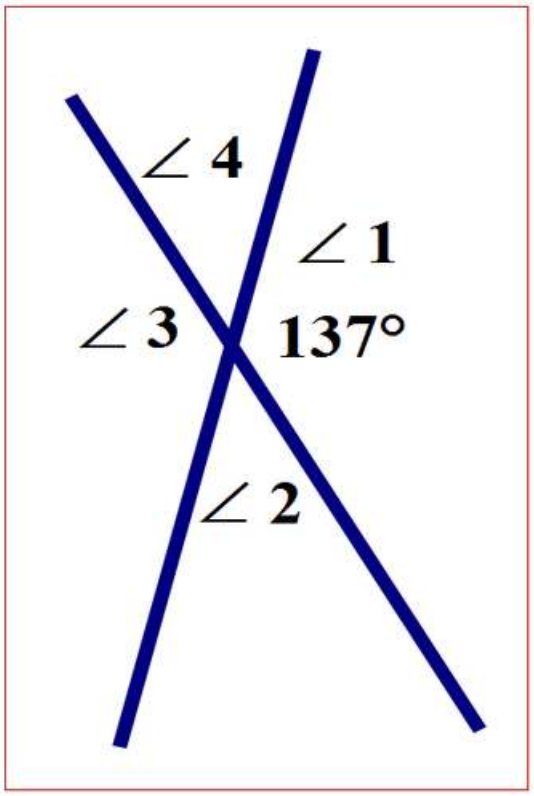

Figure 1.13.11

Solution

First, state the relationship between the angle of $\angle 1$ and $\angle 3$.

$\angle 1$ and $\angle 3$ are vertical angles and are equal in measure.

Next, express the relationship using symbols.

$$m\angle 1 = m\angle 3$$

Next, substitute the measure of $\angle 1$ into the equation.

$$m\angle 1 = m\angle 3$$

$$137^\circ = m\angle 3$$

The answer is 137°.

$$m\angle 3 = 137^\circ$$

First, state the relationship between the angle of $\angle 1$ and $\angle 2$.

$\angle 1$ is adjacent to $\angle 2$ and the two angles form a straight angle.

Next, express the relationship using symbols.

$$\angle 1 + \angle 2 = 180^\circ$$

Next, substitute the measure of $\angle 1$ into the equation.

$$137^\circ + \angle 2 = 180^\circ$$

Next, subtract 137° from both sides of the equation.

$$137^\circ + \angle 2 = 180^\circ$$

$$137^\circ - 137^\circ + \angle 2 = 180^\circ - 137^\circ$$

Then, simplify both sides of the equation.

$$137^\circ - 137^\circ + \angle 2 = 180^\circ - 137^\circ$$

$$\angle 2 = 43^\circ$$

The answer is 43°.

$$m\angle 2 = 43^\circ$$

First, state the relationship between the angle of $\angle 2$ and $\angle 4$.

$\angle 2$ and $\angle 4$ are vertical angles and are equal in measure.

Next, express the relationship using symbols.

$$m\angle 2 = m\angle 4$$

Next, substitute the measure of $\angle 2$ into the equation.

$$m\angle 2 = m\angle 4$$

$$43^\circ = m\angle 4$$

The answer is 43°.

$$m\angle 4 = 43^\circ$$

Review

If the following angle pairs are complementary, then what is the measure of the missing angle?

1. If $\angle A = 45^\circ$ then $\angle B =$?
2. If $\angle C = 83^\circ$ then $\angle D =$?
3. If $\angle E = 33^\circ$ then $\angle F =$?
4. If $\angle G = 53^\circ$ then $\angle H =$?

If the following angle pairs are supplementary, then what is the measure of the missing angle?

5. If $\angle A = 40^\circ$ then $\angle B =$?
6. If $\angle A = 75^\circ$ then $\angle B =$?
7. If $\angle C = 110^\circ$ then $\angle F =$?
8. If $\angle D = 125^\circ$ then $\angle E =$?
9. If $\angle M = 10^\circ$ then $\angle N =$?
10. If $\angle O = 157^\circ$ then $\angle P =$?

Define the following types of angle pairs.

11. Vertical angles
12. Adjacent angles
13. Complementary angles
14. Supplementary angles
15. Interior angles

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.4.

Resources

Vocabulary

Term	Definition
Adjacent Angles	Two angles are adjacent if they share a side and vertex. The word 'adjacent' means 'beside' or 'next-to'.
Angle	A geometric figure formed by two rays that connect at a single point or vertex.
Intersecting lines	Intersecting lines are lines that cross or meet at some point.
Parallel	Two or more lines are parallel when they lie in the same plane and never intersect. These lines will always have the same slope.
Perpendicular lines	Perpendicular lines are lines that intersect at a $90°$ angle.
Straight angle	A straight angle is a straight line equal to $180°$.

Additional Resources

Video: Complementary, Supplementary, and Vertical Angles

Practice:Angle Properties and Theorems

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1.14: Complementary Angles

Two angles that add up to 90 degrees.

Two angles are **complementary** if they add up to $90°$. Complementary angles **do not** have to be congruent or adjacent.

What if you were given two angles of unknown size and were told they are complementary? How would you determine their angle measures?

Example 1.14.1

Find the measure of an a mentary to $\angle ABC$ if $m\angle ABC$ is $82°$.

Solution

$90° - 82° = 8°$.

Example 1.14.2

Find the measure of an angle that is complementary to $\angle ABC$ if $m\angle ABC$ is $12°$.

Solution

$90° - 12° = 78°$.

Example 1.14.3

The two angles below are complementary. $m\angle GHI = x$. What is x?

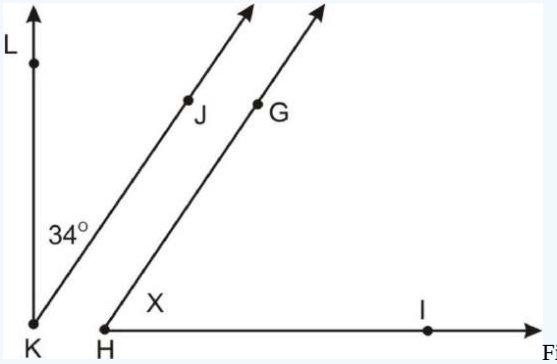

Figure 1.14.1

Solution

Because the two angles are complementary, they add up to $90°$. Make an equation.

$$x + 34° = 90°$$

$$x = 56°$$

Example 1.14.4

The two angles below are complementary. Find the measure of each angle.

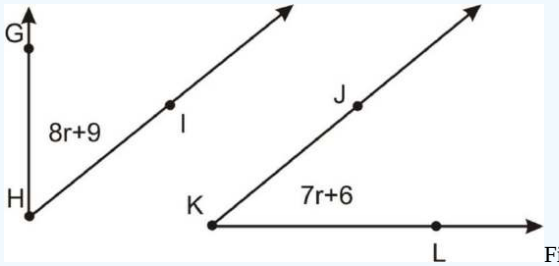

Figure \(\PageIndex{2}\

Solution

The two angles add up to 90°. Make an equation.

$(8r+9)+(7r+6)=90^\circ$

$(15r+15)=90^\circ$

$15r=75^\circ$

$r=5^\circ$

However, you need to find each angle. Plug r back into each expression.

(m\angle GHI=8(5^{\circ})+9^{\circ}=49^{\circ}\)

$m\angle JKL=7(5^\circ)+6^\circ=41^\circ$

Example 1.14.5

Find the measure of an angle that is complementary to $\angle MRS$ if $m\angle MRS$ is 70°.

Solution

Because complementary angles have to add up to 90°, the other angle must be $90^\circ-70^\circ=20^\circ$.

Review

Find the measure of an angle that is complementary to $\angle ABC$ if $m\angle ABC$ is:

1. 4°
2. 89°
3. 54°
4. 32°
5. 27°
6. $(x+y)^\circ$
7. z°

Use the diagram below for exercises 8-9. Note that $\overline{NK}\perp\overleftrightarrow{IL}$.

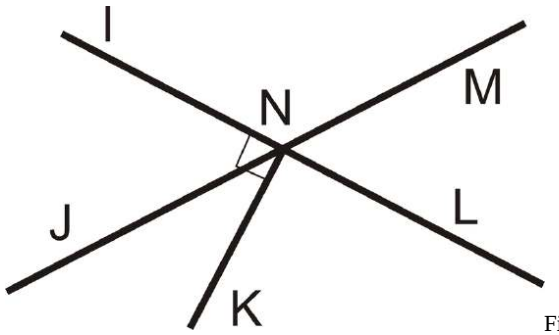

Figure 1.14.3

8. Name two complementary angles.
9. If $m\angle INJ=63^\circ$, find $m\angle KNJ$.

For 10-11, determine if the statement is true or false.

10. Complementary angles add up to 180°.
11. Complementary angles are always 45°.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.7.

Resource

Vocabulary

Term	Definition
complementary angles	Two angles are ***complementary*** if they add up to **90°**.

Additional Resource

Video: Complementary, Supplementary, and Vertical Angles

Activities: Complementary Angles Discussion Questions

Study Aids: Angles Study Guide

Practice: Complementary Angles

Real World: Complementary Angles

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1.15: Supplementary Angles

Two angles that add to 180 degrees and when adjacent form a straight line.

Linear Pairs

Two angles are **adjacent** if they have the same vertex, share a side, and do not overlap. $\angle PSQ$ and $\angle QSR$ are adjacent.

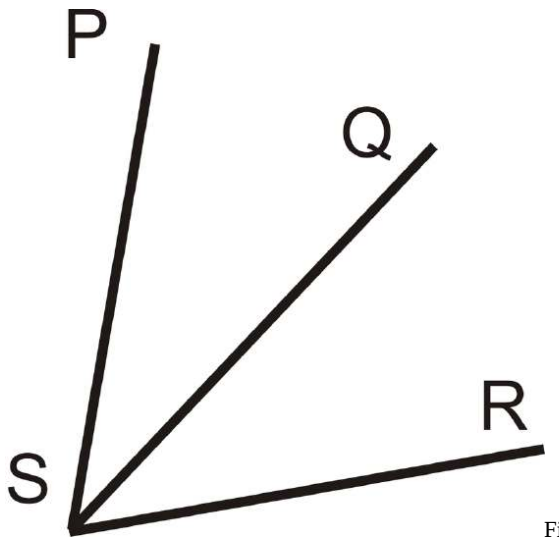


Figure 1.15.1

A **linear pair** is two angles that are adjacent and whose non-common sides form a straight line. If two angles are a linear pair, then they are supplementary (add up to $180°$). $\angle PSQ$ and $\angle QSR$ are a linear pair.

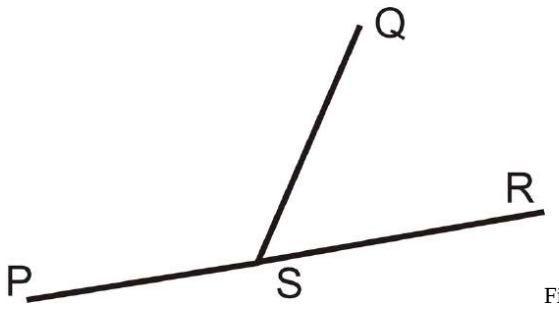


Figure 1.15.2

What if you were given two angles of unknown size and were told they form a linear pair? How would you determine their angle measures?

For Example 1.15.1 and 1.15.2 **m** below. Note that $\overline{NK} \perp \overleftrightarrow{IL}$.

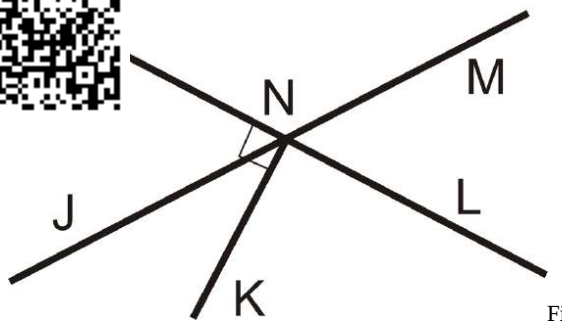


Figure 1.15.3

Example 1.15.1

Name one linear pair of angles.

Solution

$\angle MNL$ and $\angle LNJ$

Example 1.15.2

What is $m\angle INL$?

Solution

180°

Example 1.15.3

What is the measure of each angle?

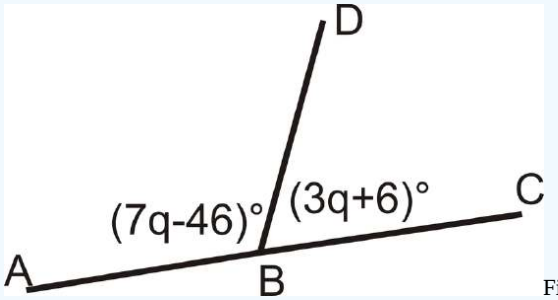


Figure 1.15.4

Solution

These two angles are a linear pair, so they add up to 180°.

$$(7q-46)^\circ + (3q+6)^\circ = 180^\circ$$

$$10q - 40^\circ = 220^\circ$$

$$10q = 180^\circ$$

$$q = 22^\circ$$

Plug in q to get the measure of each angle.

$$m\angle ABD = 7(22^\circ) - 46^\circ = 108^\circ$$

$$m\angle DBC = 180^\circ - 108^\circ = 72^\circ$$

Example 1.15.4

Are $\angle CDA$ and $\angle DAB$ a linear pair? Are they supplementary?

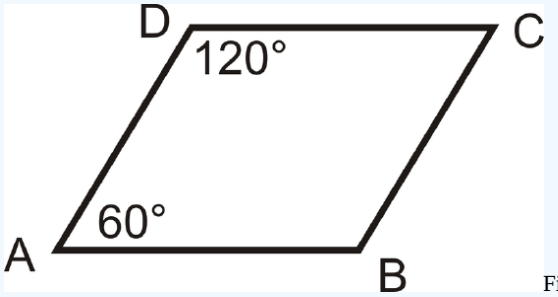


Figure 1.15.5

Solution

The two angles are not a linear pair because they do not have the same vertex. They are supplementary because they add up to $180^\circ : 120^\circ + 60^\circ = 180^\circ$.

Example 1.15.5

Find the measure of an angle that forms a linear pair with $\angle MRS$ if $m\angle MRS$ is 150°.

Solution

Because linear pairs have to add up to 180°, the other angle must be $180^\circ - 150^\circ = 30^\circ$.

Review

For 1-5, determine if the statement is true or false.

1. Linear pairs are congruent.
2. **Adjacent angles** share a vertex.
3. Adjacent angles overlap.
4. Linear pairs are supplementary.
5. Supplementary angles form linear pairs.

For exercise 6, find the value of x.

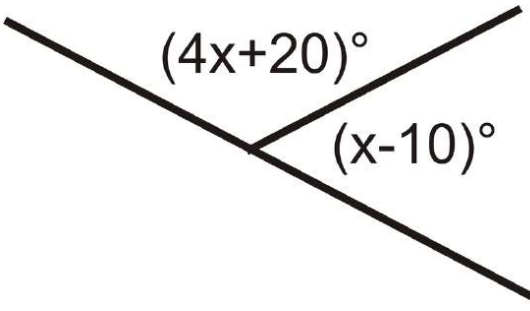
6. 

Figure 1.15.6

Find the measure of an angle that forms a linear pair with $\angle MRS$ if $m\angle MRS$ is:

7. 61°
8. 23°
9. 114°
10. 7°
11. 179°
12. z°

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.9.

Vocabulary

Term	Definition
Adjacent Angles	Two angles are adjacent if they share aside and vertex. The word 'adjacent' means 'beside' or 'next-to'.
linear pair	Two angles form a linear pair if they are supplementary and adjacent.
Diagram	A diagram is a drawing used to represent a mathematical problem.

Additional Resources

Video: Complementary, Supplementary, and Vertical Angles

Activities: Supplementary Angles Discussion Questions

Study Aids: Angles Study Guide

Practice: Supplementary Angles

Real World: Supplementary Angles

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1.16: Missing Measures of Complementary and Supplementary Angles

Find missing angle measures for supplementary or complementary angles.

Let's Think About It

Figure 1.16.1

Marco is building a house. He bought lots of wood to make the frame of the house. He wants right angles for his corners. If he uses a piece of wood that is cut at a 55° angle, what must be the angle measure of the other piece of wood that he uses to complete the corner?

In this concept, you will learn how reasoning can help you figure out the measures of missing angles.

Guidance

Some special angle pairs are identified by their sum. If you know the measure of one angle, you can calculate the measure of the second angle. For instance, **complementary angles** always add up to 90°. Let's look at an example.

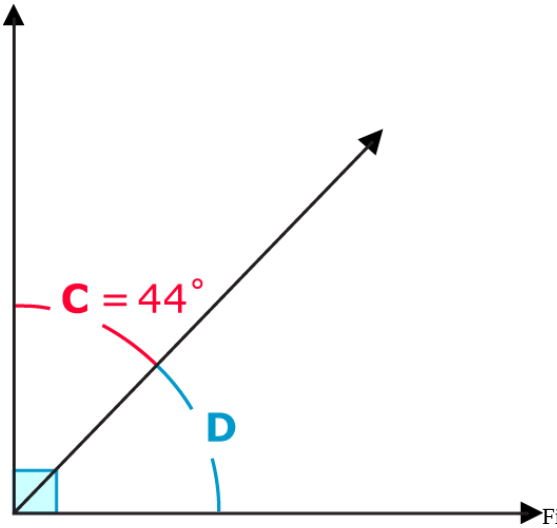

Figure 1.16.2

Together, C and D form a right angle. Therefore they are complementary, and they add up to 90°. C has a measure of 44°.

To find the measurement of angle D, simply subtract the measure of angle C from 90°.

$$\angle C + \angle D = 90^\circ$$

$$44^\circ + \angle D = 90^\circ$$

$$\angle D = 90 - 44$$

$$\angle D = 46^\circ$$

Angle D therefore measures 46°. You can check the calculation by adding angles C and D. The sum must be equal to 90°.

$44° + 46° = 90°$

The same process can be used to find the unknown angle in a pair of supplementary angles. Let's look at another example.

Angles P and Q are **supplementary angles**. If angle P measures $112°$, what is the measure of angle Q?

Supplementary angles have a total of $180°$. Subtract the measurement of P, from $180°$ to find the measure of angle Q.

$$\angle P + \angle Q = 180°$$

$$112° + \angle Q = 180°$$

$$\angle Q = 180 - 112$$

$$\angle Q = 68°$$

Angle Q is $68°$. You can check the calculation by adding angles P and Q. Remember, in order to be supplementary angles, their sum must equal $180°$.

$$68° + 112° = 180°$$

This process can often be used to find the measure of unknown angles. Use logical reasoning to interpret the information in order to find the unknown measure.

Take a look at the diagram below.

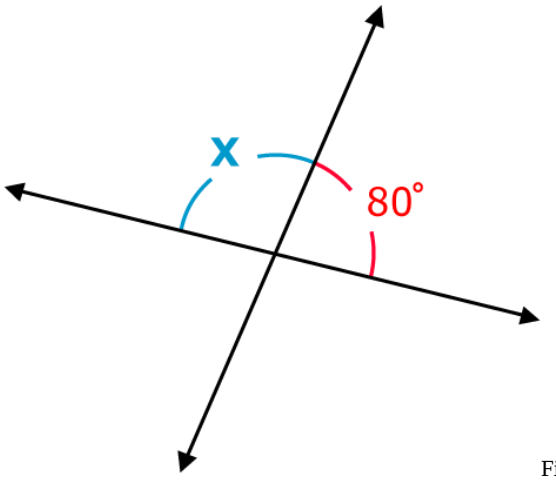

Figure **1.16.3**

Guided Practice

Solve the following problem.

What is the measure of angle R?

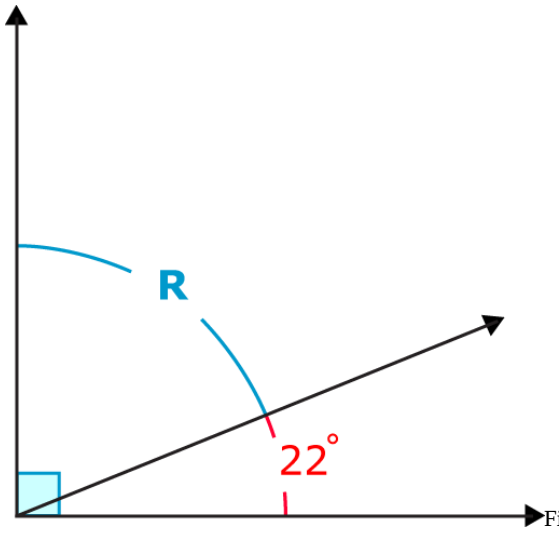

Figure 1.16.4

First, set up an equation that represents the relationship between the two angles.

$R + 22 = 90$

Next, subtract the given angle from the sum of the two angles.

$R = 90° - 22°$

Then, calculate the difference.

The difference is $68°$.

The answer is angle $R = 68°$.

The measure of the unknown angle is $68°$. You can check your answer by putting this value in for R in the equation.

$68 + 22 = 90°$

Find the complement or supplement in each example.

Example 1.16.1

Angles A and B are complementary. Angle A is $33°$. Find the measure of angle B.

Solution

First, set up an equation that represents the relationship between the angles.

$33° + B = 90°$

Next, subtract the given angle from the sum of the two angles.

$B = 90° - 33°$

Then, calculate the difference.

The difference is $57°$.

The answer is angle $B = 57°$.

Example 1.16.2

Angles C and D are supplementary. Angle C is $59°$. Find the measure of angle D.

Solution

First, set up an equation that represents the relationship between the angles.

$59° + D = 180°$

Next, subtract the given angle from the sum of the two angles.

$D = 180° - 59°$

Then, calculate the difference.

The difference is $121°$.

The answer is angle $D = 121°$

Example 1.16.3

Angles A and B are supplementary. Angle A is 169^{\circ}. Find the measure of angle B.

Solution

First, set up an equation that represents the relationship between the angles.

$169° + B = 180°$

Next, subtract the given angle from the sum of the angles.

$B = 180° - 169°$

Then, calculate the difference.

The difference is $11°$.

The answer is angle $B = 11°$

Follow Up

Figure 1.16.5

Remember Marco and his house? If one piece of wood has an angled cut that is $55°$, what is the measure of the angled cut for the second piece of wood?

First, set up an equation that represents the relationship between the two angles.

$55° + M = 90°$

Next, subtract the given angle from the sum of the two angles.

$M = 90° - 55°$

Then calculate the difference.

The difference is $35°$.

The answer is that the second piece of wood is cut at a $35°$ angle.

Video Review

Explore More

Find the measure of missi ir of complementary or supplementary angles.

1. Angles A and B are com A is $63°$. Find the measure of angle B.
2. Angles A and B are com A is $83°$. Find the measure of angle B.
3. Angles A and B are complementary. Angle A is $3°$. Find the measure of angle B.
4. Angles A and B are complementary. Angle A is $23°$. Find the measure of angle B.
5. Angles A and B are complementary. Angle A is $70°$. Find the measure of angle B.
6. Angles A and B are complementary. Angle A is $29°$. Find the measure of angle B.
7. Angles A and B are complementary. Angle A is $66°$. Find the measure of angle B.
8. Angles A and B are complementary. Angle A is $87°$. Find the measure of angle B.
9. Angles A and B are supplementary. Angle A is $33°$. Find the measure of angle B.
10. Angles A and B are supplementary. Angle A is $103°$. Find the measure of angle B.
11. Angles A and B are supplementary. Angle A is $73°$. Find the measure of angle B.
12. Angles A and B are supplementary. Angle A is $78°$. Find the measure of angle B.
13. Angles A and B are supplementary. Angle A is $99°$. Find the measure of angle B.
14. Angles A and B are supplementary. Angle A is $110°$. Find the measure of angle B.
15. Angles A and B are supplementary. Angle A is $127°$. Find the measure of angle B.

Vocabulary

Term	Definition
Acute Angle	An acute angle is an angle with a measure of less than 90 degrees.
Complementary angles	Complementary angles are a pair of angles with a sum of $90\circ$.
Obtuse angle	An obtuse angle is an angle greater than 90 degrees but less than 180 degrees.
Straight angle	A straight angle is a straight line equal to $180\circ$.
Supplementary angles	Supplementary angles are two angles whose sum is 180 degrees.

Additional Resources

Video: Complementary, Supplementary, and Vertical Angles

Practice: Missing Measures of Complementary and Supplementary Angles

Real World: Leaning Towers

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1.17: Vertical Angles

Understand that vertical angles are opposite each other and have the same measure.

Vertical angles are two non-adjacent angles formed by intersecting lines. $\angle 1$ and $\angle 3$ are vertical angles and $\angle 2$ and $\angle 4$ are vertical angles.

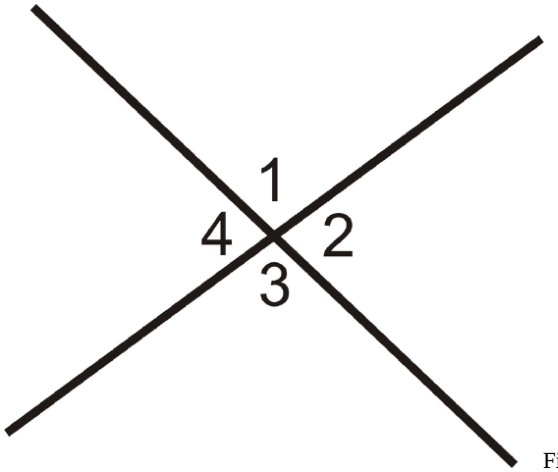

Figure 1.17.1

The **Vertical Angles Theorem** states that if two angles are vertical angles, then they are congruent.

What if you were given two angles of unknown size and were told they are vertical angles? How would you determine their angle measures?

Example 1.17.1

Find the value of x.

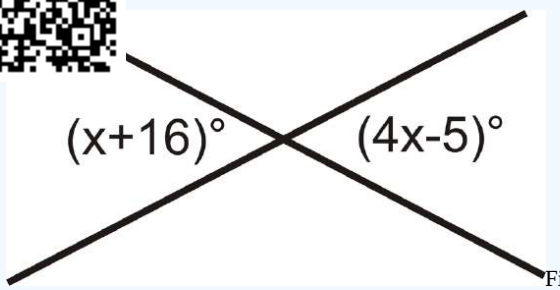

Figure 1.17.2

Solution

Vertical angles are congruent, so set the angles equal to each other and solve for x.

$$x + 16 = 4x - 5$$

$$3x = 21$$

$$x = 7°$$

Example 1.17.2

Find the value of y.

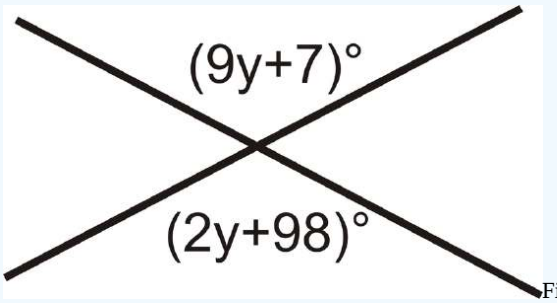

Figure 1.17.3

Solution

Vertical angles are congruent, so set the angles equal to each other and solve for y.

$$9y + 7 = 2y + 98$$

$$7y = 91$$

$$y = 13^\circ$$

Example 1.17.3

Find $m\angle 1$.

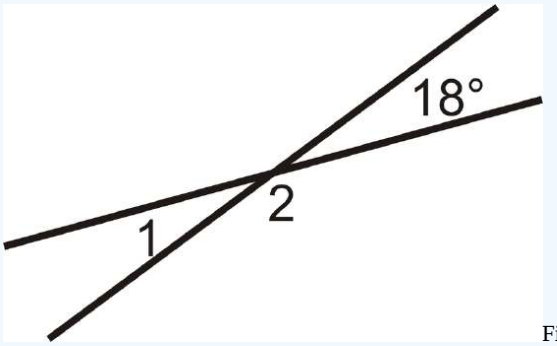

Figure 1.17.4

Solution

$\angle 1$ is vertical angles with 18°, so \(m\angle 1=18^{\circ}.

Example 1.17.4

If \angle ABC and \angle DEF are vertical angles and \(m\angle ABC=(4x+10)^{\circ} and \(m\angle DEF=(5x+2)^{\circ}, what is the measure of each angle?

Solution

Vertical angles are congruent, so set the angles equal to each other and solve for x. Then go back to find the measure of each angle.

$$4x + 10 = 5x + 2$$

$$x = 8$$

So, $m\angle ABC = m\angle DEF = (4(8) + 10)^\circ = 42^\circ$

Example 1.17.5

True or false: vertical angles are always less than 90^{\circ}.

Solution

This is false, you can have vertical angles that are more than 90^{\circ}. Vertical angles are less than 180^{\circ}.

Review

Use the diagram below for exercises 1-2. Note that $\overline{NK} \perp \overleftrightarrow{IL}$.

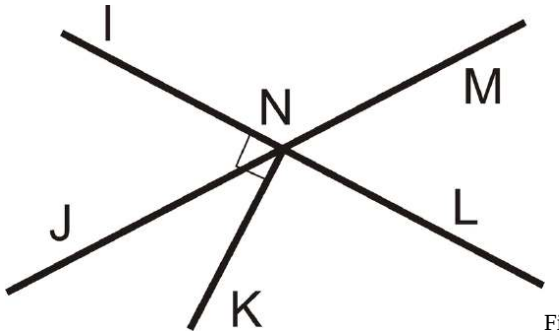

Figure 1.17.5

1. Name one pair of vertical angles.
2. If $m\angle INJ = 63°$, find $m\angle MNL$.

For exercise 3, determine if the statement is true or false.

3. Vertical angles have the same vertex.
4. If $\angle ABC$ and $\angle DEF$ are vertical angles and $m\angle ABC = (9x + 1)°$ and $m\angle DEF = (5x + 29)°$, what is the measure of each angle?
5. If $\angle ABC$ and $\angle DEF$ are vertical angles and $m\angle ABC = (8x + 2)°$ and $m\angle DEF = (2x + 32)°$, what is the measure of each angle?
6. If $\angle ABC$ and $\angle DEF$ are vertical angles and $m\angle ABC = (x + 22)°$ and $m\angle DEF = (5x + 2)°$, what is the measure of each angle?
7. If $\angle ABC$ and $\angle DEF$ are vertical angles and $m\angle ABC = (3x + 12)°$ and $m\angle DEF = (7x)°$, what is the measure of each angle?
8. If $\angle ABC$ and $\angle DEF$ are vertical angles and $m\angle ABC = (5x + 2)°$ and $m\angle DEF = (x + 26)°$, what is the measure of each angle?
9. If $\angle ABC$ and $\angle DEF$are vertical angles and $m\angle ABC = (3x + 1)°$ and $m\angle DEF = (2x + 2)°$, what is the measure of each angle?
10. If $\angle ABC$and $\angle DEF$ are vertical angles and $m\angle ABC = (6x - 3)°$ and $m\angle DEF = (5x + 1)°$, what is the measure of each angle?

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.10.

Resources

Vocabulary

Term	Definition
Vertical Angles	Vertical angles are a pair of opposite angles created by intersecting lines.
Vertical Angles Theorem	The Vertical Angles Theorem states that if two angles are vertical, then they are congruent.

Additional Resources

Video: Complementary, Supplementary, and Vertical Angles

Activities: Vertical Angles Discussion Questions

Study Aids: Angles Study Guide

Practice: Vertical Angles

Real World: Vertical Angles

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1.18: Classify Polygons

Categories of polygons based on the number of sides, and whether they are concave or convex.

Polygons

A **polygon** is any closed, 2-dimensional figure that is made entirely of line segments that intersect at their endpoints. Polygons can have any number of sides and angles, but the sides can never be curved. The segments are called the **sides** of the polygons, and the points where the segments intersect are called **vertices**.

Polygons can be either **convex** or **concave**. The term concave refers to a cave, or the polygon is "caving in". All stars are concave polygons.

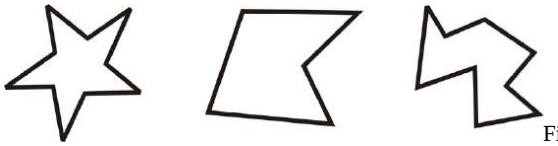


Figure **1.18.1**

A convex polygon does not cave in. Convex polygons look like:

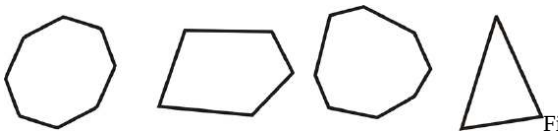


Figure **1.18.2**

A **diagonal** is a non-side line segment that connects two vertices of a convex polygon.

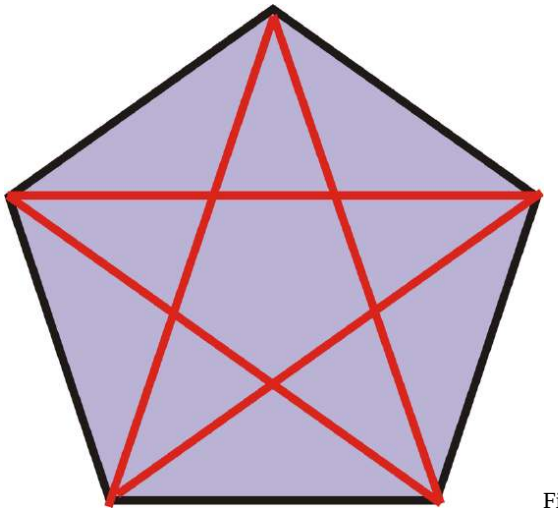
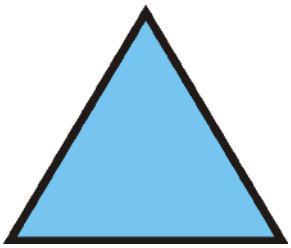
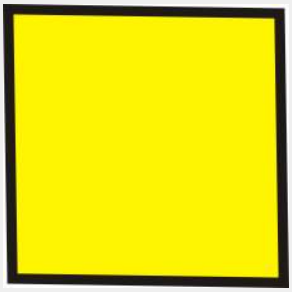
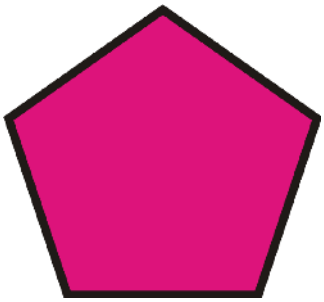
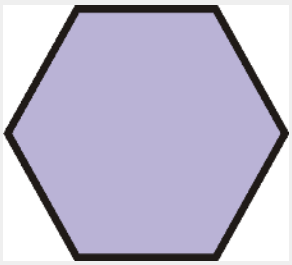
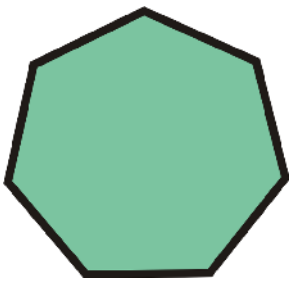
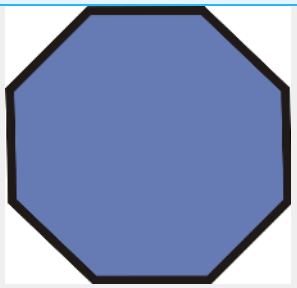
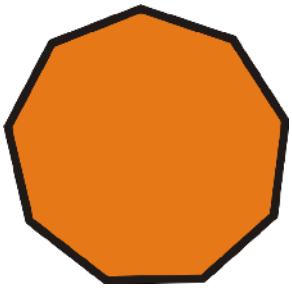
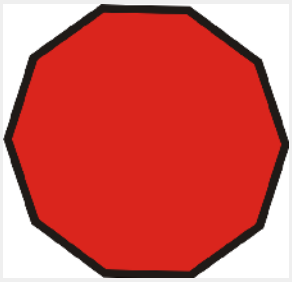
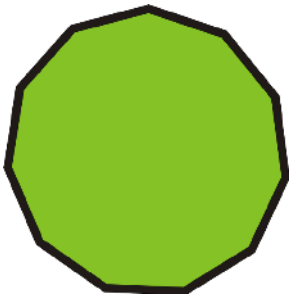
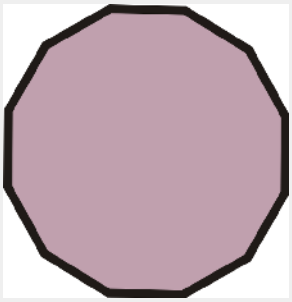


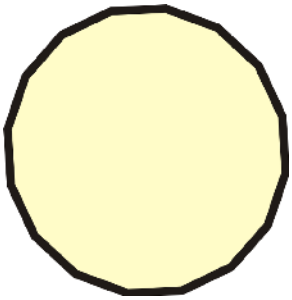
Figure **1.18.3**

The red line segments are all diagonals. This pentagon has 5 diagonals.

Whether a polygon is convex or concave, it is always named by the number of sides. Explore the relationship between the number of sides of a convex polygon and its diagonals. Can you complete the table?

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
Triangle	**3**	**0**	 Figure **1.18.4**
Quadrilateral	**4**	**2**	 Figure **1.18.5**
Pentagon	**5**	**5**	 Figure **1.18.6**
Hexagon	**6**	**9**	 Figure **1.18.7**
Heptagon	**7**	**?**	 Figure **1.18.8**
Octagon	**8**	**?**	

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
			 Figure 1.18.9
Nonagon	**9**	**?**	 Figure 1.18.10
Decagon	**10**	**?**	 Figure 1.18.11
Undecagon or hendecagon	**11**	**?**	 Figure 1.18.12
Dodecagon	**12**	**?**	 Figure 1.18.13

Polygon Name	Number of Sides	Number of Diagonals	Convex Example
n-gon	**n (where n>12)**	**?**	 Figure **1.18.14**

What if you were told how many sides a polygon has? How would you describe the polygon based on that information?

Example 1.18.1

Which of the figures bel

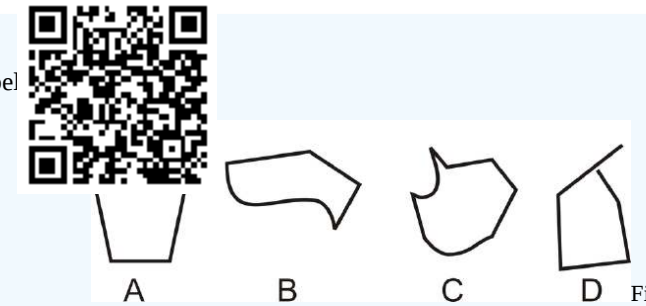


Figure **1.18.15**

Solution

The easiest way to identify the polygon is to identify which shapes are **not** polygons. B and C each have at least one curved side, so they are not be polygons. D has all straight sides, but one of the vertices is not at the endpoint, so it is not a polygon. A is the only polygon.

Example 1.18.2

Determine if the shapes below are convex or concave.

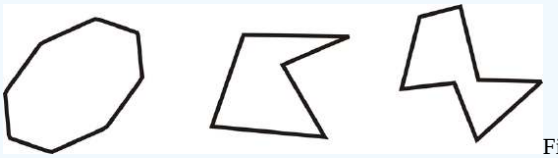


Figure **1.18.16**

Solution

To see if a polygon is concave, look at the polygons and see if any angle "caves in" to the interior of the polygon. The first polygon does not do this, so it is convex. The other two do, so they are concave.

Example 1.18.3

Name the three polygons below by their number of sides and if it is convex or concave.

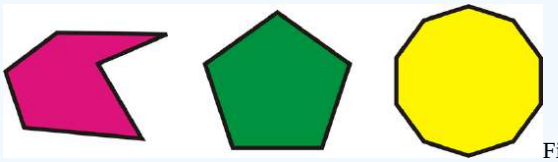


Figure **1.18.17**

Solution

The pink polygon is a concave hexagon (6 sides).

The green polygon convex pentagon (5 sides).

The yellow polygon is a convex decagon (10 sides).

Example 1.18.4

Draw a 7-sided polygon, also called a heptagon. How many diagonals does a heptagon have?

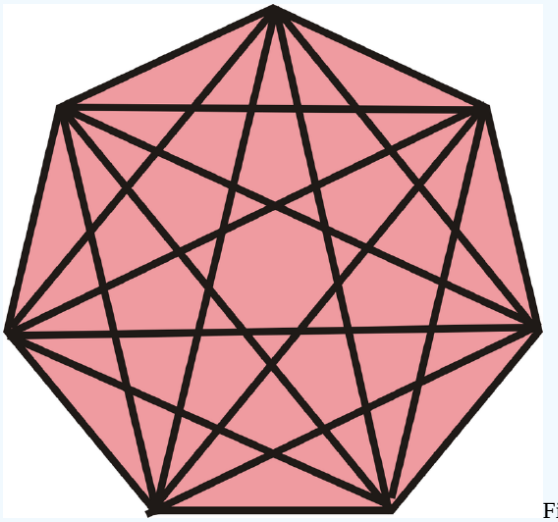


Figure **1.18.18**

Solution

First, draw the heptagon. Drawing in all the diagonals and counting them, we see there are 14.

Example 1.18.5

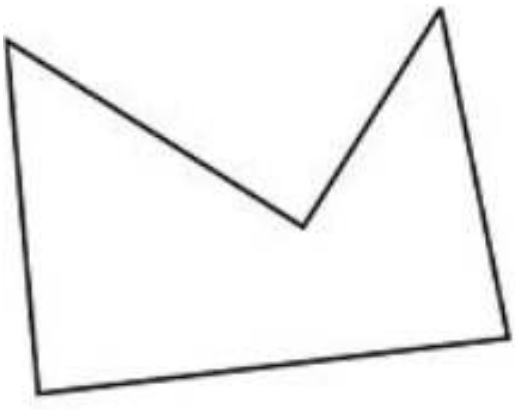
True or false: A quadrilateral is always a square.

Solution

False. Only quadrilaterals with four congruent sides and four right angles will be squares. There are many quadrilaterals (such as rectangles, kites, parallelograms, trapezoids, etc.) that are not necessarily squares.

Review

In problems 1-6, name each polygon in as much detail as possible.



1.

Figure **1.18.19**

2.

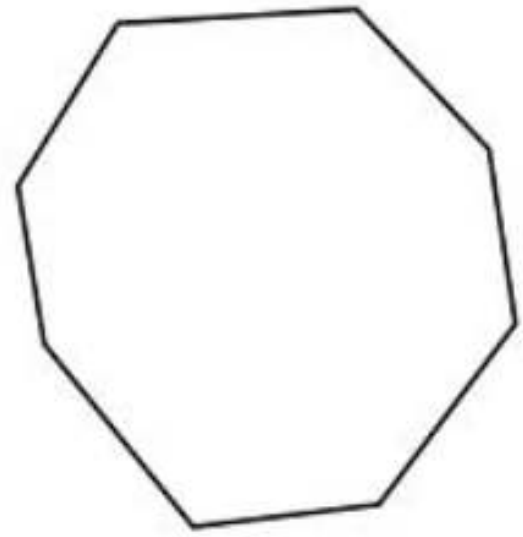


Figure 1.18.20

3.

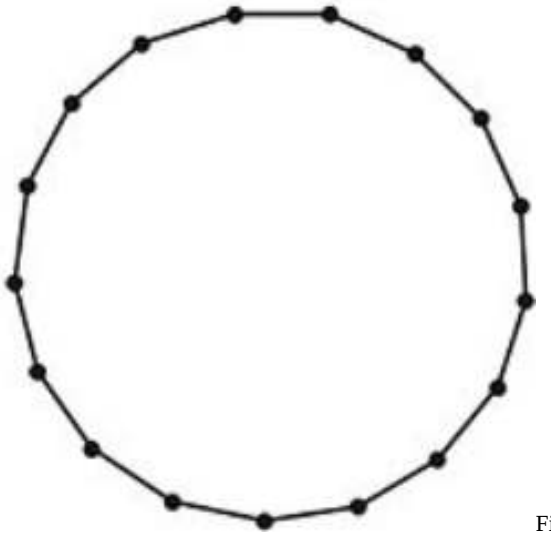


Figure 1.18.21

4.

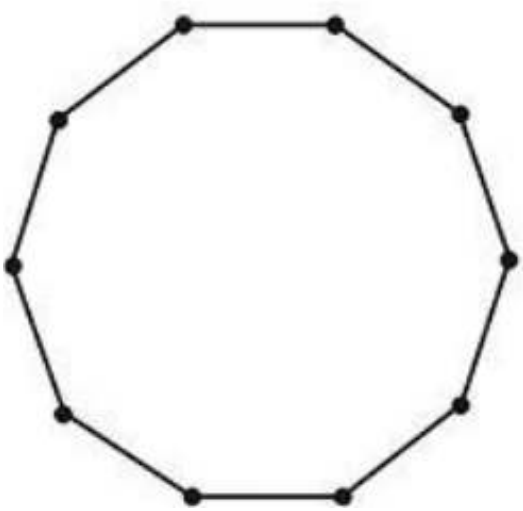


Figure 1.18.22

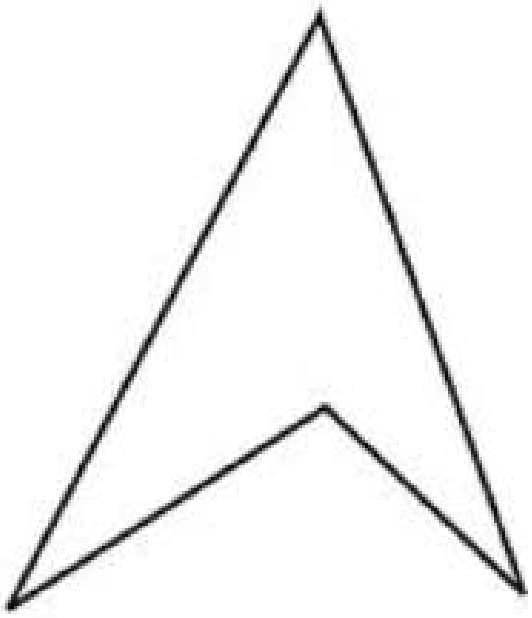
5. 

Figure 1.18.23

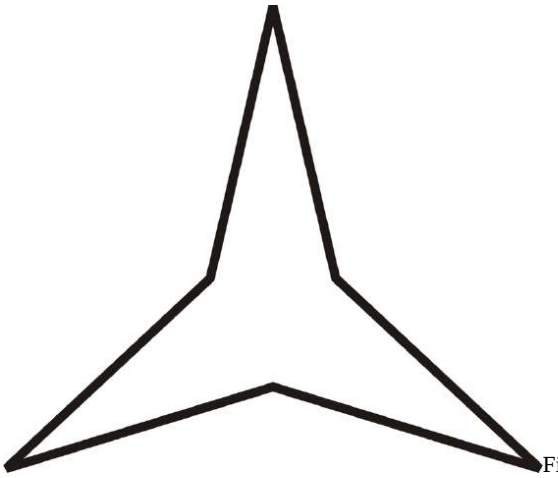
6. 

Figure 1.18.24

7. Explain why the following figures are NOT polygons:

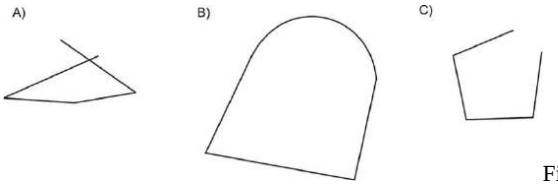


Figure 1.18.25

8. How many diagonals can you draw from **one vertex** of a pentagon? Draw a sketch of your answer.
9. How many diagonals can you draw from **one vertex** of an octagon? Draw a sketch of your answer.
10. How many diagonals can you draw from **one vertex** of a dodecagon?
11. Determine the number of total diagonals for an octagon, nonagon, decagon, undecagon, and dodecagon.

For 12-14, determine if the statement is true or false.

12. A polygon must be enclosed.
13. A star is a convex polygon.
14. A 5-point star is a decagon.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.12.

Resource

Vocabulary

Term	Definition
diagonal (of a polygon)	A non-side line segment that connect two vertices of a convex polygon.
Polygon	A polygon is a simple closed figure with at least three straight sides.
Concave	A concave polygon has at least one interior angle greater than 180 degrees. A common way to identify a concave polygon is to look for a "caved-in" side of the polygon.
Convex	A convex polygon contains no interior angles greater than 180 degrees.
Diagonal	A diagonal is a line segment in a polygon that connects nonconsecutive vertices
Exterior angles	An exterior angle is the angle formed by one side of a polygon and the extension of the adjacent side.
Interior angles	Interior angles are the angles inside a figure.
Regular Polygon	A regular polygon is a polygon with all sides the same length and all angles the same measure.
Vertices	Vertices are points where line segments intersect.
Equilateral	A polygon is equilateral if all of its sides are the same length.

Additional Resource

Video: Sum of Interior Angles of a Polygon

Activities: Polygon Classification Discussion Questions

Study Aids: Polygons Study Guide

Practice: Classify Polygons

Real World: Nature's Polygons

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1.19: Vertices and Sides

Count the angles/vertices and sides in a closed figure.

Video: Counting the Angles in a Closed Figure

Practice: Vertices and Sides

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1.20: Polygon Classification in the Coordinate Plane

Figure **1.20.1**

Parker is taking her first drawing class, though she doesn't have high hopes. When she walks into class the first day, her art teacher pulls up a picture of a cityscape and tells them that they will all be able to draw this by the end of the week. Her teacher gives them each a copy of this picture and tells them that their first assignment is to break the picture into its underlying shapes. Parker is utterly confused and overwhelmed. How can Parker figure out how to break the picture into shapes?

In this concept, you will learn how to graph geometric figures given **coordinates** of vertices and identify graphed figures.

Classifying Polygons in the Coordinate Plane

A **coordinate grid** is a grid in which points are graphed. It usually has two or more intersecting lines which divide a plane into **quadrants**, and in which ordered pairs, or coordinates, are defined. It usually has four **quadrants,** or sections, to it.

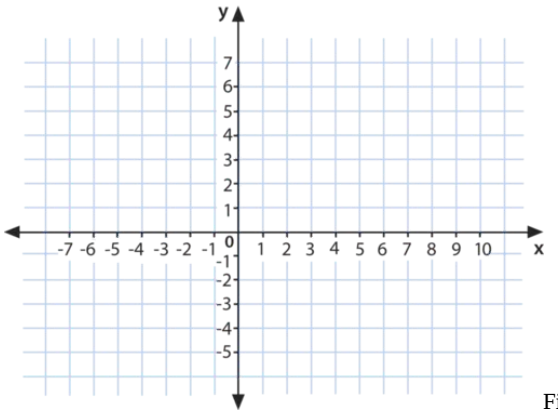

Figure **1.20.2**

The **origin** is the place where the two lines intersect. Its coordinates are defined as $(0, 0)$.

The x-**axis** is the line running from left to right that has the numbers defined on it and is usually labeled with an "x". The x-coordinate of an **ordered pair** is found with relation to it. All the points located on the x-axis have a y-coordinate of 0.

The y-**axis** is the central line that runs up-down and is labeled with a "y". Y-coordinates are plotted in reference to this axis. Again, all the x-coordinates of points located on the y-axis are 0.

An **ordered pair** is a list of two numbers in parenthesis, separated by a comma like this: $(5, -3)$. It tells where a point is located on the coordinate plane. The first number is the x-coordinate. It tells you where to go on the x-axis. If it is positive, you go to the right. If it is negative, you go to the left. The second number is the y-coordinate. It tells you where to go on the y-axis. If it is positive, you go up. If it is negative, you go down.

The **vertex** of a shape is the place where two sides of the shape come together. In general, when a shape is defined inside of a coordinate plane, it is defined by the vertices, and then the lines are drawn to connect them.

A **polygon** is any shape made up of rectilineal, or straight, lines. The smallest polygon is a **triangle**, which has three sides. A five-sided figure is a **pentagon**. And many polygons with more sides than five are also named.

A **right angle** is an angle that looks like where the axes on the coordinate plane meet.

Parallel lines are lines that will go on forever but will never converge.

The category of four-sided polygons includes:

- the **square**, which has four sides of equal length and its angles are all right angles;
- the **rhombus** which, like the square, has four equal sides, but is "tilty";
- the **rectangle** which has two pairs of sides which are equal and all its angles are right angles;
- the **trapezoid** which may have no equal sides, but it has two lines which are parallel; and
- the **parallelogram** which has two sets of parallel lines which are equal in length to each other (but like the rhombus, it is "tilty").

In order to graph a figure in the coordinate plane, you just graph each of the vertices and then connect them with straight lines so that none of the lines cross. The number of sides you have is the same as the number of vertices. So a triangle, for example, is defined with three vertices.

Here is an example.

Graph a figure with the coordinates $A(-4, 3) B(2, 3) C(2, -1) D(-4, -1)$. When finished, name the figure that has been drawn on the grid.

First, plot each point on the coordinate grid and then connect the lines.

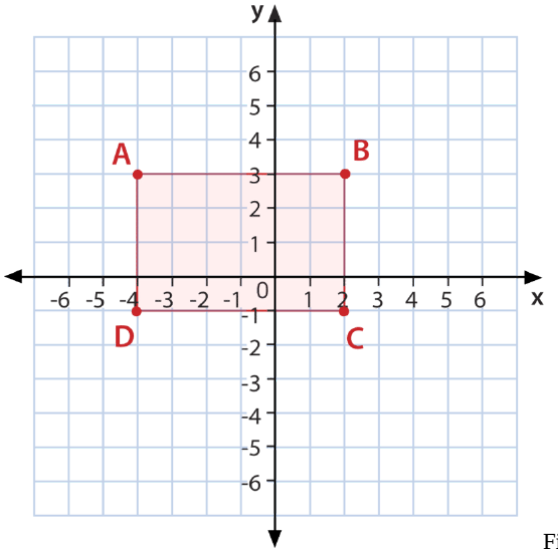

Figure 1.20.3

Next, in order to determine what kind of shape it is, first count the number of vertices.

This figure has four, so it is one of the four-sided shapes.

Then, look to see how many of the sides are equal.

In this case, side $AB = DC$ and $AD = BC$. Since there are two sets of equal sides, this is either a rectangle or a parallelogram.

Finally, check the angles.

In this shape, the angles are right angles, so this is a rectangle.

Here is another example.

Graph and name the following figure with these coordinates $D(1, 3) E(5, 3) F(7, -1) G(1, -1)$.

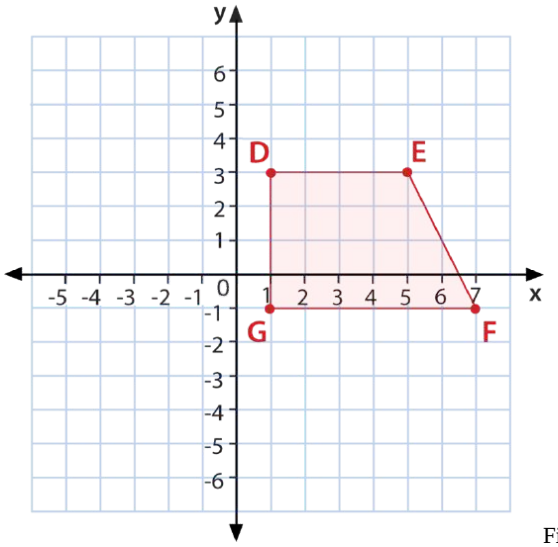

Figure 1.20.4

First, plot the vertices and connect them.

Next, count the number of vertices.

There are four, so this is a four-sided figure, which you can see from its shape.

Then, look at the sides.

None of the sides are the same length, so this shape must be a trapezoid.

Example 1.20.1

Earlier, you were given a problem about Parker and her math-art panic attack.

Her art teacher gives her a picture of buildings in perspective and tells her to break it into its constituent shapes.

Solution

First, she draws a y-axis down the center of the back building in the picture, which looks about halfway across the picture. Then, she draws the x-axis as the horizon line.

Next, she squints at the picture and tries to see it not as buildings, but as shapes, instead. On the right side of the picture, she realizes that the buildings make a triangle with one vertex starting at the origin. And on the left of the y-axis, the other side of buildings make another triangle, also with a vertex at the origin. Even the street, she realizes, makes its own triangle, again with one vertex at the origin. And then the sky finishes it with a forth triangle with a vertex at the origin.

Then, she sees that the rest of the picture is just made up of rectangles. There is one below the street triangle, and one on the left buildings. And one where the tiny back building is.

Parker concludes that she might be able to learn to draw after all.

Example 1.20.2

Determine the shape given by the following vertices.

$(-4, 6)(4, 6)(0, -6)$

Solution

First, count the number of vertices.

In this case, there are three vertices. If there are three vertices, then the shape has three sides.

Next, determine which shapes are possible.

There is only one closed shape with three sides, and that is a triangle.

The answer is a triangle.

Example 1.20.3

Determine the shape given by the following vertices.

$(0, 2)(2, 0)(0, -3)$

Solution

First, count the number of vertices.

In this case, there are three vertices. If there are three vertices, then there are three sides.

Next, determine which shapes are possible.

There is only one closed shape with three sides, and that is a triangle.

The answer is a triangle.

Example 1.20.4

Determine the shape given by the following vertices.

$(-1, -3)(-5, -3)(-7, 1)(-1, 1)$

Solution

First, count the number of vertices.

In this case, there are four vertices. If there are four vertices, then the shape has four sides.

Next, determine which shapes are possible.

The four-sided figures are: square, rhombus, trapezoid, rectangle, and parallelogram.

Then, plot the vertices on a coordinate plane.

Then, determine the properties of the shape.

In this case, the shape has two parallel lines and two non-parralel lines. The only four sides figure in which that is the case is the trapezoid.

The answer is a trapezoid.

Example 1.20.5

Determine the shape given by the following vertices.

$(3, 3)(0, 3)(0, 0)(3, 0)$

Solution

First, count the number of vertices.

In this case, there are four vertices. If there are four vertices, then the shape has four sides.

Next, determine which shapes are possible.

The four-sided figures are: square, rhombus, trapezoid, rectangle, and parallelogram.

Then, plot the vertices on a coordinate plane.

Then, determine the properties of the shape.

In this case, the shape has two sets of parallel lines and and all the sides are the same length. Additionally, the shape has four right angles. The square is the shape that meets these criterion.

The answer is a square.

Review

Graph each figure using the vertices. Then name the graphed figure.

1. $A(-2,2); B(2,2); C(2,-2); D(-2,-2)$
2. $D(-4,3); E(-1,1); F(-4,1)$
3. $(1,1)(2,3)(4,1)$
4. $(-1,3)(-5,3)(-1,0)(-5,0)$
5. $(0,5)(3,5)(0,9)(3,9)$
6. $(0,6)(2,6)(0,10)(2,10)$
7. $(-3,6)(6,0)(9,0)$
8. $(-1,6)(-1,8)(-9,6)(-9,8)$
9. $(0,-8)(1,-5)(5,-5)(4,-8)$
10. $(12,0)(12,6)(7,0)$

For 11 - 15 draw five of your own figures on a coordinate grid. Write out each set of coordinates and work with a partner to identify each figure using only the coordinates.

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.15.

Vocabulary

Term	Definition
x-axis	The x-axis is the horizontal axis in the coordinate plane, commonly representing the value of the input or independent variable.
y axis	The y-axis is the vertical number line of the Cartesian plane.
Coordinates	The coordinates of a point represent the point's location on the Cartesian plane. Coordinates are written in ordered pairs: $x, y)$.
Ordered Pair	An ordered pair, $x, y)$, describes the location of a point on a coordinate grid.
Origin	The origin is the point of intersection of the x and y axes on the Cartesian plane. The coordinates of the origin are $(0,0)$.
Quadrants	A quadrant is one-fourth of the coordinate plane. The four quadrants are numbered using Roman Numerals I, II, III, and IV, starting in the top-right, and increasing counter-clockwise.

Additional Resources

Video: Plotting Points on the Coordinate Plane

Practice: Polygon Classification in the Coordinate Plane

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CHAPTER OVERVIEW

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2.5: Inductive Reasoning from Patterns
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Thumbnail: One of the oldest surviving fragments of Euclid's Elements. (Public Domain; Euclid via Wikipedia)

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2.1: Basic Visual Patterns

Describe non-arithmetic sequences by finding a rule. Identify visual and geometric patterns

Figure **2.1.1**

Whitney is on a school trip to Italy. The fifth day there her group visits Venice where she gets to see the Doge's Palace. She notices a visual pattern in the architecture and snapped the photo above. How could Whitney describe this visual pattern? What type of visual pattern is it?

In this concept, you will learn to recognize, describe, and extend visual patterns.

Visual Patterns

A **visual pattern** is a sequence of pictures or geometric objects that have been created based on some rule. There are two categories of visual patterns that you will learn about here.

1. Some visual patterns are made up of a sequence of pictures or geometric objects that repeat over and over. These types of patterns are called **repeating patterns**. The sequence of pictures that repeat is called the **pattern unit**.
2. Some visual patterns are based on a picture or geometric object that keeps increasing or decreasing in size in a specific way. These types of patterns have a **pattern rule** that help to describe how the picture or object is growing or shrinking.

Here is an example of a visual pattern that is a repeating pattern.

Find the pattern unit for the pattern below.

Figure **2.1.2**

First, notice that this visual pattern is made up of circles, squares, and triangles.

Next, look carefully to see how the shapes have been placed. After each circle is a square, after each square is two triangles, and after each pair of triangles is another circle. This is a repeating pattern because the same set of four pictures repeats over and over.

The pattern unit for this visual pattern is the set of pictures that repeats.

The answer is that the pattern unit is "circle, square, triangle, triangle":

Figure **2.1.3**

Once you understand the pattern and know the pattern unit, you can extend the pattern. The next picture in our sequence of pictures would be a triangle.

Here is an example of an increasing visual pattern.

Find a pattern rule to describe the pattern below. What will the next step in the pattern look like?

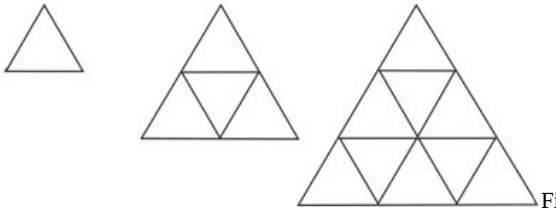
Figure 2.1.4

As you can see, this pattern is different from a repeating pattern because the geometric objects keep getting bigger and bigger. There is not a pattern unit that repeats, but you can find a pattern rule to describe what is happening.

To find the pattern rule, look to see how the steps in the pattern are related. Each new big triangle is made up of more and more smaller triangles. In fact, each new big triangle is made up of the previous big triangle plus an additional row of small triangles.

- The first step in the pattern has just one row of 1 triangle.
- The second step in the pattern has two rows of 1 triangle, then 3 triangles.
- The third step in the pattern has three rows of 1 triangle, then 3 triangles, then 5 triangles.

You can see that the number of rows of triangles corresponds to the step in the pattern. Each time a new row is added, it has 2 more triangles than the row above it.

The answer for the pattern rule is: to get the next step in the pattern, take the previous step and add another row of small triangles to the bottom with 2 more triangles than the row above it.

Now that you have the pattern rule, you can extend the pattern.

You'd expect the next step in the pattern to look like the third step in the pattern except with an additional row of triangles.

The answer is that the fourth step in the pattern will have four rows of 1 triangle, then 3 triangles, then 5 triangles, then 7 triangles:

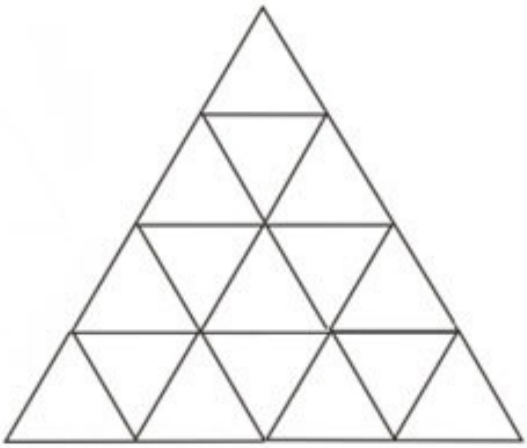
Figure 2.1.5

Example 2.1.1

Earlier, you were given a problem about Whitney and her school trip to Italy.

She got to see the Doge's Palace in Venice where she noticed an interesting visual pattern in the architecture.

Solution

First, notice that this is a repeating pattern. Objects are being repeated as opposed to increasing/decreasing in size.

As this is a repeating pattern, next try to describe the pattern unit.

The same column with railing is being repeated over and over. This is the pattern unit.

The answer is that this is a repeating pattern with a pattern unit that is a column/railing pair.

Example 2.1.2

What will the next step in the pattern below look like?

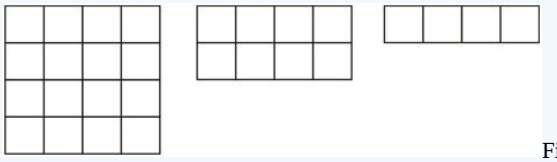

Figure 2.1.6

Solution

First, notice that this is a decreasing visual pattern.

To find the pattern rule, look to see how the steps in the pattern are related. Each new rectangle is made up of fewer and fewer squares. In fact, each new rectangle is made up of half as many squares as the previous rectangle.

- The first step in the pattern has 16 squares.
- The second step in the pattern has 8 squares.
- The third step in the pattern has 4 squares.

You can see that if you take the number of squares in one step of the pattern and divide by 2, you will get the number of squares in the next step of the pattern.

The answer is that the pattern rule is to get the number of squares in the next step in the pattern, take the number of squares in the previous step and divide by 2.

Now that you have the pattern rule, you can extend the pattern.

You'd expect the next step in the pattern to have half as many squares as the third step. The third step has 4 squares, so the fourth step will have two squares.

The answer is that the next step in the pattern will look like:

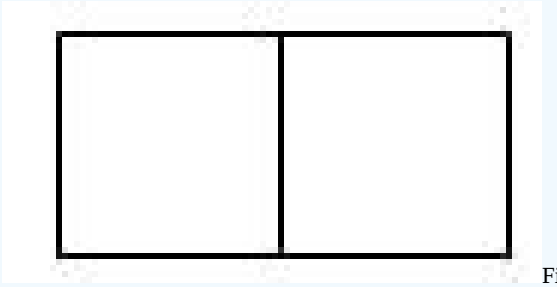

Figure 2.1.7

Example 2.1.3

Describe the following pattern. What is its pattern unit or pattern rule?

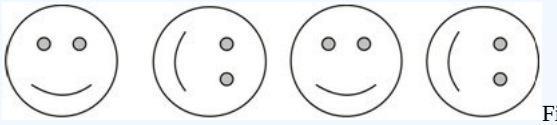

Figure 2.1.8

Solution

First notice that this is a repeating pattern. Images are being repeated as opposed to increasing/decreasing in size.

Next, look carefully to see how the images have been placed. After each smiley face with the mouth on the bottom is a smiley face with the mouth on the left. The same set of two pictures repeats over and over.

The answer is that this is a repeating pattern. The pattern unit is:

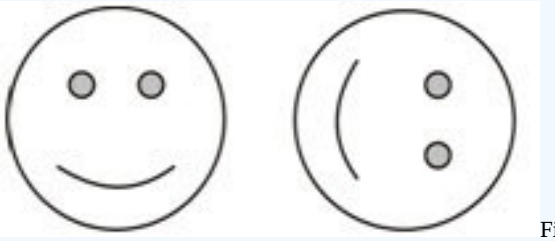

Figure **2.1.9**

Example 2.1.4

Describe the following pattern. What is its pattern unit or pattern rule?

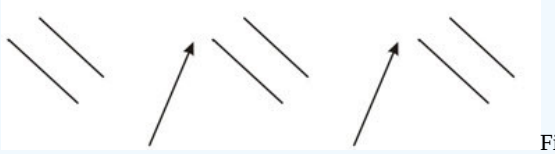

Figure **2.1.10**

Solution

First notice that this is a repeating pattern. Images are being repeated as opposed to increasing/decreasing in size.

Next, look carefully to see how the images have been placed. After each set of lines is an arrow. The same set of two pictures repeats over and over.

The answer is that this is a repeating pattern. The pattern unit is:

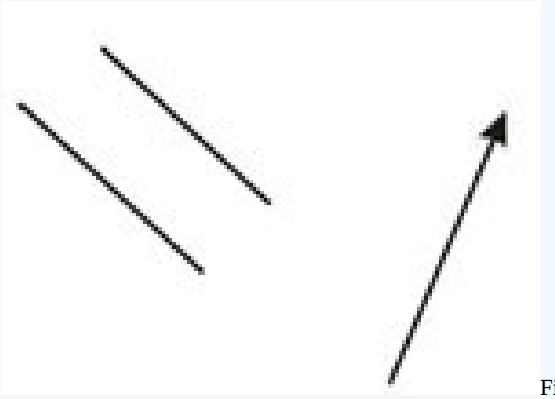

Figure **2.1.11**

Example 2.1.5

Continue the described pattern with the next two shapes.

Triangle, star, triangle, star, square, square, triangle, star, triangle

Solution

First notice that this is a repeating pattern. The images haven't been displayed, but they have been described. The images are being repeated as opposed to increasing/decreasing in size.

Next, look carefully to see how the shapes have been ordered. After "triangle, star, triangle, star" is "square, square". After "square, square" the "triangle, star, ..." sequence seems to repeat. It appears that the same set of six shapes will repeat over and over.

The pattern unit is "triangle, star, triangle, star, square, square".

Now that you have the pattern unit, you can extend the pattern. After the "triangle, star, triangle" at the end of the pattern will be another star, then two squares, and then back to the beginning of the pattern unit with "triangle, star, triangle, star, square, square...".

> The answer is that the next two shapes will be "star, square".

Review

Write a rule for each number pattern.

1. $4, 7, 13, 25$
2. $216, 196, 176, 156$
3. $1, 7, 19, 43$
4. $10,000, 1,000, 100, 10$

Write the next number in each pattern.

5. Missing open brace for subscript
6. Missing open brace for subscript
7. Missing open brace for subscript
8. Missing open brace for subscript

Write down the next image in each pattern based on the description.

9. A hexagon, a pentagon, a square, _____
10. Five dots, ten dots, fifteen dots, _____
11. Fifteen dots, fourteen dots, twelve dots, _____
12. Two triangles, four triangles, three triangles, six triangles, five triangles, _____
13. An octagon, a hexagon, an octagon, a pentagon, an octagon, ____
14. One square, 1.5 squares, 2 squares, 2.5 squares, _____
15. Eighteen circles, nine circles, four and a half circles _____

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.3.

Additional Resources

Video: Geometric Sequences (Introduction)

Practice: Basic Visual Patterns

Real World: The Math Behind It All

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2.2: Identify Basic Pattern Type

Video: Describing Patterns Using Letters

Practice: Identify Basic Pattern Type

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2.3: Number Patterns

Identify missing values in numerical patterns.

Extend Numerical Patterns

Figure **2.3.1**

Nicholas is on his way to his cousin's new house. He found the street and is just looking for the house. His cousin lives at number 1644. Nicholas notices that the numbers of the houses on the right side of the street seem to follow a pattern:

1574, 1584, 1594, ...

Number 1574 was the first house Nicholas saw on the right. If he assumes that the pattern of how the houses are numbered will continue, how can Nicholas extend the pattern to figure out how many houses away his cousin's house is?

In this concept, you will learn how to extend numerical patterns.

Numerical Patterns

A **numerical pattern** is a sequence of numbers that has been created based on a rule called a **pattern rule**. Pattern rules can use one or more mathematical operations to describe the relationship between consecutive numbers in the sequence.

Knowing the pattern rule will help you to **extend** the pattern. To **extend** the pattern means to use the pattern rule to write the numbers that would come next in the sequence.

Here is an example.

Find the next two numbers in the following sequence: 3, 6, 9, 12,__, __.

First, figure out the pattern rule. This is an ascending pattern so the rule likely involves addition or multiplication.

Next, come up with potential pattern rules. Think: "What could you do to 3 to get 6?"

- You could add 3
- You could multiply by 2.
- You could do a combination of two or more operations.

Now, check if any of these potential pattern rules work with the rest of the sequence.

Consider 6 and 9.

- If you add 3 to 6 you get 9. So the pattern rule "add 3" seems to work.

Make sure "add 3" works throughout the whole sequence.

"Add 3" works for the whole sequence.

Now, extend the pattern. Apply the pattern rule of "add 3" to the 12 at the end of the pattern.

The next number in the pattern will be 15.

"Add 3" one more time to get the sixth number in the sequence.

The answer is that the extended pattern is 3, 6, 9, 12, \(\underline{15}\), \(\underiline{18}\).

Sometimes you will be interested in a particular term in the pattern. In order to figure out a particular term, keep extending the pattern until you've reached the term you are looking for.

Let's look at an example.

What is the seventh number in the sequence: $1, 3, 9, 27, ...$?

First, figure out the pattern rule. This is an ascending pattern so the rule likely involves addition or multiplication.

Next, come up with potential pattern rules. Think: "What could you do to 1 to get 3?"

- You could add 2.
- You could multiply by 3.
- You could do a combination of two or more operations.

Now, check if any of these potential pattern rules work with the rest of the sequence.

Consider 1 and 3.

- If you add 2 to 3 you get 5, not 9. So the pattern rule is not "add 2."
- If you multiply 3 by 3 you get 9. So the pattern rule "multiply by 3" seems to work.

Make sure "multiply by 3" works throughout the whole sequence.

"Multiply by 3" works for the whole sequence.

Finally, you can extend the pattern to find the seventh number. Keep **multiplying by 3** until you reach the seventh number in the sequence.

The first seven numbers in the sequence are $1, 3, 9, 27, 81, 243, 729$.

The answer is that the seventh number in the sequence is 729.

Example 2.3.1

Earlier, you were given a problem about Nicholas, who is going to his cousin's house.

He just passed house number 1574 and noticed that the house numbers seem to follow a pattern:

$1574, 1584, 1594$

His cousin lives at number 1644. Nicholas wants to extend the pattern to predict how many houses away his cousin's house is.

Solution

First, figure out the pattern rule. This is an ascending pattern so the rule likely involves multiplication or addition.

Next, look at how the numbers are related. The difference between the numbers is a constant 10. This means that the pattern rule is "add 10." Double check that this pattern rule works for the three given numbers in the sequence.

The pattern rule "add 10" works for the three given numbers in the sequence.

Now, extend the pattern. Apply the pattern rule "add 10" at the end of the pattern until you hit 1644.

Next, write out the extended sequence.

Notice that 1644 is the eighth number in the sequence.

The answer is that Nicholas can expect his cousin's house to be the eighth house on the right.

Example 2.3.2

Find the next two numbers in the following sequence: 24, 14, 9, __, __.

Solution

First, figure out the pattern rule. This is a descending pattern so the rule likely involves division or subtraction.

Next, look at how the numbers are related. The difference between the numbers decreases as you move through the sequence. To get from 24 to 14 you have to subtract 10, but to get from 14 to 9 you have to subtract 5. This means division is involved. Since there is no whole number you can divide 24 by to get 14, the pattern rule is likely division with either addition or subtraction.

Now, consider possible pattern rules that involve division and addition or subtraction. Think: "What could you do to 24 to get 14?"

- You could divide by 2 and add 2.
- You could divide by 3 and add 6.
- You could do some other combination of two or more operations.

Next, look back at the rest of the sequence. Consider 14 and 9.

- If you divide 14 by 2 and add 2 you get 9. So the pattern rule "divide by 2 and add 2" works for the three given numbers in the sequence.

Now, extend the pattern. Apply the pattern rule of "divide by 2 and add 2" at the end of the pattern two times.

The next two numbers in the sequence will be 6.5 and 5.25.

The answer is that the extended pattern is 24, 14, 9, \(\underline{6.5}\), **5.25**.

Example 2.3.3

Find the next two numbers in the following sequence: 9,17,33, __, __.

Solution

First, figure out the pattern rule. This is an ascending pattern so the rule likely involves multiplication or addition.

Next, look at how the numbers are related. The difference between the numbers increases as you move through the sequence. This means multiplication is involved. Since there is no whole number you can multiply 9 by to get 17, the pattern rule is likely multiplication with either addition or subtraction.

Now, consider possible pattern rules that involve multiplication and addition or subtraction. Think: "What could you do to 9 to get 17?"

- You could multiply by 2 and subtract 1.
- You could multiply by 3 and subtract 10.
- You could do some other combination of two or more operations.

Next, look back at the rest of the sequence. Consider 17 and 33.

- If you multiply 17 by 2 and subtract 1 you get 33. So the pattern rule multiply by 2 and subtract 1 works for the three given numbers in the sequence.

Now, extend the pattern. Apply the pattern rule of "multiply by 2 and subtract 1" at the end of the pattern two times.

The next two numbers in the pattern will be 65 and 129.

The answer is that the extended pattern is 9, 17, 33, **65**, \(\underline{129}\).

Example 2.3.4

Find the next two numbers in the following sequence: 3, 10, 31, __, __.

Solution

First, figure out the pattern rule. This is an ascending pattern so the rule likely involves multiplication or addition.

Next, look at how the numbers are related. The difference between the numbers increases as you move through the sequence. This means multiplication is involved. Since there is no whole number you can multiply 3 by to get 10, the pattern rule is likely multiplication with either addition or subtraction.

Now, consider possible pattern rules that involve multiplication and addition or subtraction. Think: "What could you do to 3 to get 10?"

- You could multiply by 2 and add 4.
- You could multiply by 3 and add 1.
- You could do some other combination of two or more operations.

Next, look back at the rest of the sequence. Consider 10 and 31.

- If you multiply 10 by 3 and add 1 you get 31. So the pattern rule "multiply by 3 and add 1" works for the three given numbers in the sequence.

Now, extend the pattern. Apply the pattern rule "multiply by 3 and add 1" at the end of the pattern two times.

The next two numbers in the pattern will be 94 and 283.

The answer is that the extended pattern is 3, 10, 31, \(\underline{94}\), \(\underline{283}\).

Example 2.3.5

Find the sixth number in the following sequence: $4, 17, 56, \ldots$

Solution

First, figure out the pattern rule. This is an ascending pattern so the rule likely involves multiplication or addition.

Next, look at how the numbers are related. The difference between the numbers increases as you move through the sequence. This means multiplication is involved. Since there is no whole number you can multiply 4 by to get 17, the pattern rule is likely multiplication with either addition or subtraction.

Now, consider possible pattern rules that involve multiplication and addition or subtraction. Think: "What could you do to 4 to get 17?"

- You could multiply by 2 and add 9.
- You could multiply by 3 and add 5.
- You could multiply by 4 and add 1.
- You could do some other combination of two or more operations.

Next, look back at the rest of the sequence. Consider 17 and 56.

- If you multiply 17 by 3 and add 5 you get 56. So the pattern rule "multiply by 3 and add 5" works for the three given numbers in the sequence.

Now, extend the pattern. Apply the pattern rule of "multiply by 3 and add 5" at the end of the pattern three times in order to get to the sixth number in the sequence.

The extended pattern is $4, 17, 56, 173, 524, 1577$.

The answer is that the sixth number in the sequence is 1577.

Review

Extend each numerical pattern by filling in the blanks.

1. 2, 3, 4, 5, __, __
2. 2, 4, 6, 8, __, __
3. 2, 5, 11, 23, __, __
4. 3, 6, 9, __, __
5. 64, 16, 4, __, __
6. 150, 100, 50, __, __
7. 10, 20, 30, 40,__, __
8. 15, 30, 45,__, __

9. 100, 112, 124,__, __
10. 4, 18, 74,__, __
11. 40, 120, 360,__, __
12. 2.5, 6, 13,__, __
13. 50, 25, 12.5,__, __
14. 3, 4.5, 6, 7.5, 9,__

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.2.

Additional Resources

Video: Patterns in Sequences 2

Practice: Number Patterns

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2.4: Reasoning Types

Deductive versus Inductive Reasoning. Deductive is from the general to the specific. Inductive is the reverse. It is impossible for the premises to be true and the conclusion to be false.

Inductive and Deductive Reasoning

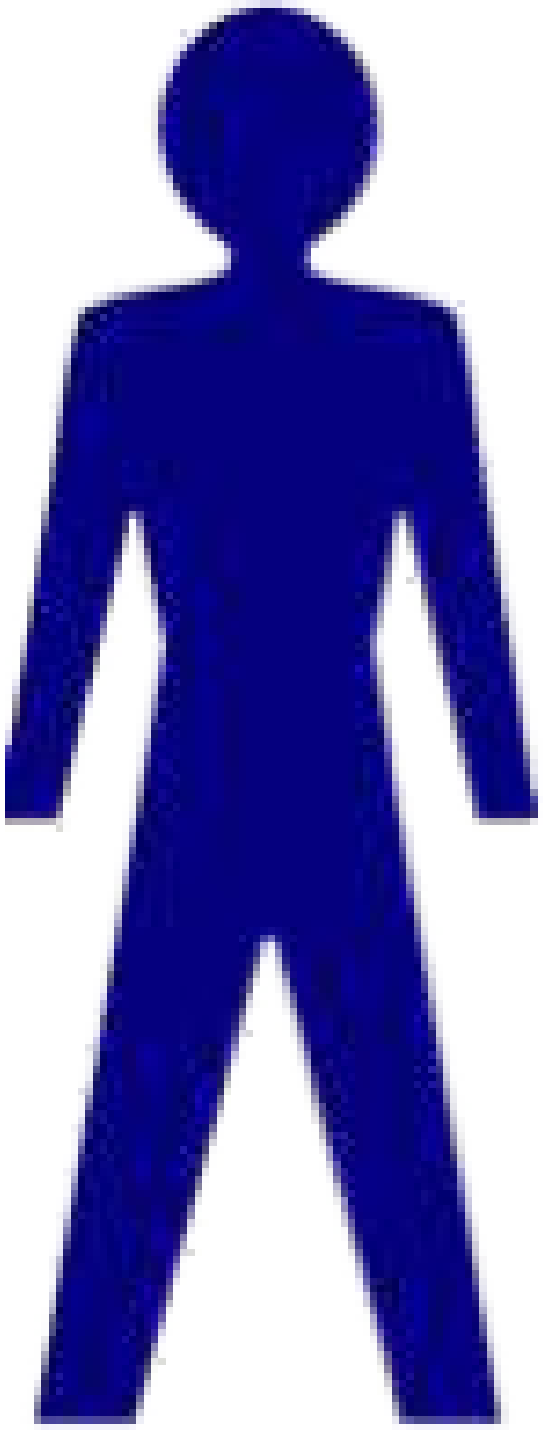

Figure **2.4.1**

Suppose you were given the task of collecting data from each class in your school on the ratio between male and female students. After reviewing the M:F ratios of each classroom, would you use inductive reasoning or deductive reasoning to come up with a

hypothesis regarding an average M:F ratio for the school? What kind of reasoning would be involved if your friend asked you to review your data to see if her theory about ratios being different in different grades was supported by your observations?

Figure **2.4.2**

Inductive and Deductive Reasoning

One of the primary uses of probability and statistics is to learn about parameters of a population, and to do that, one must be able to reason from a sample to a population. Either a person observes something and tries to explain it by collecting and distilling data into a conclusion, or else he/she begins with a hypothesis and seeks data to support or renounce it. In this lesson, we will discuss these two types of reasoning, Inductive and Deductive.

- **Deductive Reasoning** – Begins with the question or theory and works toward specific examples or evidences to support or renounce it.
  - Every morning, I eat eggs for breakfast. Every day, I am not hungry again until lunchtime. This morning if I eat eggs for breakfast, I will not be hungry until lunchtime.
- **Inductive Reasoning** – Begins with specific observations or data and works toward a general statement to explain it.
  - This morning I ate eggs for breakfast and was not hungry until lunchtime. As long as I eat eggs for breakfast, I'll never be hungry until lunchtime.

In scientific study, both sorts of reasoning are used, often in conjunction and to support each other. However, as you will see over the next few lessons, there are a lot of ways to make errors in reasoning (called **fallacies**), and knowing what type of reasoning you are using will help you to learn which fallacies to watch out for!

Choosing the Applicable Reasoning

1. What sort of reasoning is applicable to finding the solution to a five-step linear equation such as the one below?

$$2(x+3)-7=x+4$$

$$2(x+3)=x+11$$

$$2x+6=11$$

$$2x-x=11-6$$

$$x=5$$

This is deductive reasoning, since we started with a statement or theory: $2(x+3)-7=x+4$, and used a step-by-step process to find a specific example supporting it, namely that if $x=5$, then $2(5+3)-7=5+4$, so the original statement is supported by a specific example.

Since we progressed from general to specific, this was deductive reasoning.

2. Assuming the sequence below, what type of reasoning would you use to conjecture the next number in the sequence?

$1, 4, 10, 19, 31, 46, 64, \ldots$

This is an example of inductive reasoning, since we started with a number of specific observations, namely the $1^{st}, 2^{rd}, 3^{rd}, 4^{nd}$, and so on numbers in a sequence, and use the observations to make the statement that the pattern is to add 3n, where n is the count, to each number to get the next: $1+3(1)=4, 4+3(2)=10, 10+3(3)=19, 19+3(4)=31$, and so on. That tells us that the next number in the series should be: $64+3(7)=85$.

Since we progressed from specific to general, this was inductive reasoning.

Determining What Type of Reasoning is Expressed

What sort of reasoning is expressed in the following statements?

Chloe took her umbrella to work today, and it rained.

Every time Chloe takes her umbrella, it will rain.

Figure **2.4.3**

This is inductive reasoning, beginning with the specific statement about a specific day and action, and progressing to a general statement about *all* days with the same action.

Earlier Problem Revisited

Suppose you were given the task of collecting data from each class in your school on the ratio between male and female students. After reviewing the M:F ratios of each classroom, would you use inductive reasoning or deductive reasoning to come up with a hypothesis regarding an average M:F ratio for the school? What kind of reasoning would be involved if your friend asked you to review your data to see if her theory about ratios being different in different grades was supported by your observations?

First, you begin with specific examples of the ratios of males and females and use them to create a general statement about the ratio of the entire school. That was inductive reasoning: specific to general. Second, you started with the general statement that the ratios are different in different grades, and considered the specific data to support or not support the statement. That was deductive reasoning: general to specific.

For Examples **2.4.1**-**2.4.4**, describe the type of reasoning demonstrated in each passage.

Example 2.4.1

Scott leaves for school at 8:15 in the morning every day, it takes him 15 minutes to get to school, and he arrives on time. If Scott leaves at 8:15 this morning, he will arrive at school on time.

Solution

This is deductive reasoning, starting with a general statement about Scott's actions everyday and progressing to the specific occurrence of today.

Example 2.4.2

On Monday, Sophie went to lunch at the local fast-food joint on her lunch break and arrived back at school in time for class. On Tuesday, she did the same thing and was on time again. If Sophie goes to the same fast-food place for lunch on every day, she will be back in time for class.

Solution

This is inductive reasoning, starting with specific examples of actions and progressing to a general statement about every similar action.

Example 2.4.3

$3(x-4)-7=6x$, therefore, $x=-6.3\overline{3}$.

Solution

Deductive reasoning, from a general statement to a specific example of the statement being true.

Example 2.4.4

If $y = 7$, and $x = 4$, therefore $x \times \frac{7}{4} = y$.

Solution

Inductive reasoning, from specific stated values of x and y to a general statement about them both.

Review

For each question, state whether the reasoning is an example of inductive or deductive logic.

1. All housecats are felines. All felines have claws. Therefore all housecats have claws.
2. My dog has fleas. My neighbor's dog has fleas. Therefore all dogs must have fleas.
3. All cows like hay. My cow will like hay.
4. My Mac laptop is fast. All Mac laptops are fast.
5. My tennis shoes are comfortable. My friend's tennis shoes are comfortable. All tennis shoes are comfortable.
6. The scalloped potatoes I took from the oven were cheesy. The enchiladas I took from the oven were cheesy. If I take cookies from the oven, they will be cheesy.
7. Everything cooked on the stove gets hot. If I cook macaroni on the stove, it will get hot.
8. iPads are popular. iPhones are popular. Every phone or tablet is popular.
9. Roses are red. Tomatoes are red. All red things come from plants.
10. Rock music is loud. Sayber listens to rock music. Sayber's music is loud.
11. Milk is good with cookies. Snicker doodles are cookies. Milk is good with snicker doodles.
12. Hummers use a lot of gas. Suburbans use a lot of gas. Large SUV's use a lot of gas.
13. My garden has pumpkins. My dad's garden has pumpkins. All gardens have pumpkins.
14. Prob and Stats students are smart. You are a Prob and Stats student. You are smart.
15. Students who study hard get good grades. You are a student who studies hard. You will get good grades.

Vocabulary

Term	Definition
fallacies	Fallacies are errors in reasoning.

Additional Resource

Practice: Reasoning Types

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2.5: Inductive Reasoning from Patterns

Inductive Reasoning

One type of reasoning is **inductive reasoning**. Inductive reasoning entails making conclusions based upon examples and patterns. Visual patterns and **number patterns** provide good examples of inductive reasoning. Let's look at some patterns to get a feel for what inductive reasoning is.

What if you were given a pattern of three numbers or shapes and asked to determine the sixth number or shape that fit that pattern?

Example 2.5.1

A dot pattern is shown b ots would there be in the 4th figure? How many dots would be in the 6th figure?

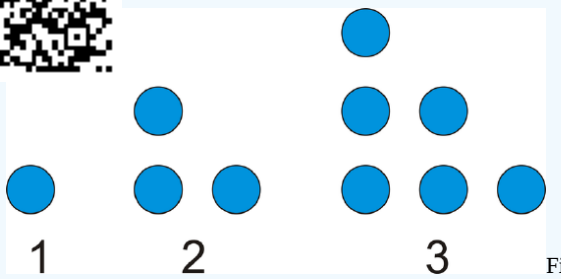


Figure 2.5.1

Solution

Draw a picture. Counting the dots, there are $4+3+2+1=10$ dots.

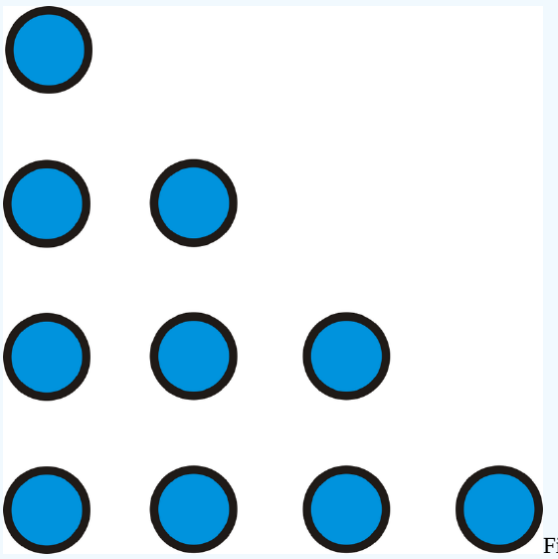


Figure 2.5.2

For the 6th figure, we can use the same pattern, $6+5+4+3+2+1$. There are 21 dots in the 6th figure.

Example 2.5.2

How many *triangles* would be in the 10th figure?

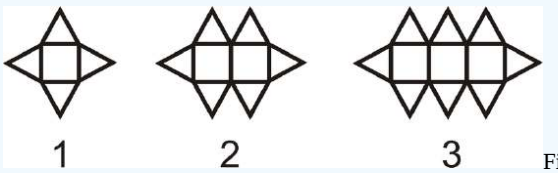


Figure 2.5.3

Solution

There would be 10 squares in the 10th figure, with a triangle above and below each one. There is also a triangle on each end of the figure. That makes $10 + 10 + 2 = 22$ triangles in all.

Example 2.5.3

Look at the pattern $2, 4, 6, 8, 10, \ldots$ What is the 19th term in the pattern?

Solution

Each term is 2 more than the previous term.

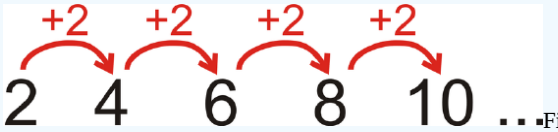


Figure 2.5.4

You could count out the pattern until the 19th term, but that could take a while. Notice that the 1st term is $2 \cdot 1$, the 2nd term is $2 \cdot 2$, the 3rd term is $2 \cdot 3$, and so on. So, the 19th term would be $2 \cdot 19$ or 38.

Example 2.5.4

Look at the pattern: $3, 6, 12, 24, 48, \ldots$

What is the next term in the pattern? What is the 10th term?

Solution

Each term is *multiplied* by 2 to get the next term.

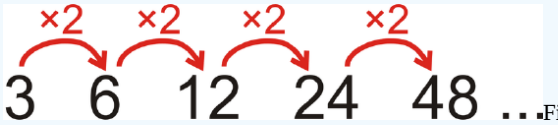


Figure 2.5.5

Therefore, the next term will be $48 \cdot 2$ or 96.

To find the 10th term, continue to multiply by 2, or $3 \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{2^9} = 1536$.

Example 2.5.5

Find the 8th term in the list of numbers: $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25} \ldots$

Solution

First, change 2 into a fraction, or $\frac{2}{1}$. So, the pattern is now $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25} \ldots$ The top is $2, 3, 4, 5, 6$. It increases by 1 each time, so the 8th term's numerator is 9. The denominators are the square numbers, so the 8th term's denominator is 8^2 or 64. The 8th term is $\frac{9}{64}$.

Review

For questions 1-3, determine how many dots there would be in the 4th and the 10th pattern of each figure below.

1.

Figure 2.5.6

2.

Figure 2.5.7

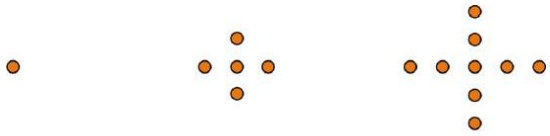
3. 

Figure 2.5.8

4. Use the pattern below to answer the questions.

Figure 2.5.9

   1. Draw the next figure in the pattern.
   2. How does the number of points in each star relate to the figure number?

5. Use the pattern below to answer the questions. All the triangles are equilateral triangles.

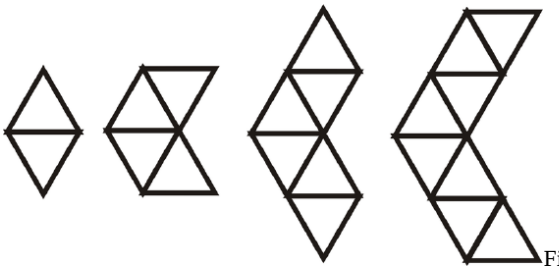


Figure 2.5.10

   1. Draw the next figure in the pattern. How many triangles does it have?
   2. Determine how many triangles are in the 24th figure.

For questions 6-13, determine: the next three terms in the pattern.

6. $5, 8, 11, 14, 17, \ldots$
7. $6, 1, -4, -9, -14, \ldots$
8. $2, 4, 8, 16, 32, \ldots$
9. $67, 56, 45, 34, 23, \ldots$
10. $9, -4, 6, -8, 3, \ldots$
11. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$
12. $\frac{2}{3}, \frac{4}{7}, \frac{6}{11}, \frac{8}{15}, \frac{10}{19}, \ldots$
13. $-1, 5, -9, 13, -17, \ldots$

For questions 14-17, determine the next two terms **and** describe the pattern.

14. $3, 6, 11, 18, 27, \ldots$
15. $3, 8, 15, 24, 35, \ldots$
16. $1, 8, 27, 64, 125, \ldots$
17. $1, 1, 2, 3, 5, \ldots$

Review (Answers)

To see the Review answers, open this PDF file and look for section 2.1.

Vocabulary

Term	Definition
Inductive Reasoning	Inductive reasoning is a type of reasoning where one draws conclusions from patterns and previous examples.
Equilateral Triangle	An equilateral triangle is a triangle in which all three sides are the same length.

Additional Resources

Video: Inductive Reasoning

Activities: Inductive Reasoning from Patterns Discussion Questions

Study Aids: Types of Reasoning Study Guide

Practice: Inductive Reasoning from Patterns

Real World: The Science of Induction

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2.6: Conjectures and Counterexamples

Educated guesses and examples that disprove them.

A **conjecture** is an "educated guess" that is based on examples in a pattern. A **counterexample** is an example that disproves a conjecture.

Suppose you were given a mathematical pattern like $h = \frac{-16}{t^2}$. What if you wanted to make an educated guess, or conjecture, about h?

Use the following informa . and 2:

A car salesman sold 5 use ent couples. He noticed that each couple was under 30 years old. The following day, he sold a new, luxury car t 0's. The salesman determined that only younger couples by used cars.

Example 2.6.1

Is the salesman's conjecture logical? Why or why not?

Solution

It is logical based on his experiences, but is not true.

Example 2.6.2

Can you think of a counterexample?

Solution

A counterexample would be a couple that is 30 years old or older buying a used car.

Example 2.6.3

Here's an algebraic equation and a table of values for n and t.

$t = (n-1)(n-2)(n-3)$

n	$(n-1)(n-2)(n-3)$	t
1	$(0)(-1)(-2)$	0
2	$(1)(0)(-1)$	0
3	$(2)(1)(0)$	0

Solution

After looking at the table, Pablo makes this conjecture:

The value of $(n-1)(n-2)(n-3)$ is 0 for any number n.

Is this a true conjecture?

This is not a valid conjecture. If Pablo were to continue the table to n=4, he would have see that $(n-1)(n-2)(n-3) = (4-1)(4-2)(4-3) = (3)(2)(1) = 6$

In this example $n = 4$ is the counterexample.

Example 2.6.4

Arthur is making figures for an art project. He drew polygons and some of their diagonals.

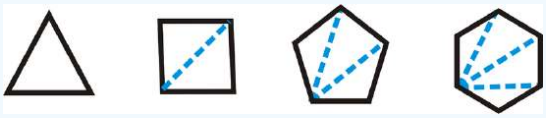

Figure 2.6.1

From these examples, Arthur made this conjecture:

If a convex polygon has n sides, then there are $n-2$ triangles formed when diagonals are drawn from any vertex of the polygon.

Is Arthur's conjecture correct? Or, can you find a counterexample?

Solution

The conjecture appears to be correct. If Arthur draws other polygons, in every case he will be able to draw $n-2$ triangles if the polygon has n sides.

Notice that we have **not proved** *Arthur's conjecture, but only found several examples that hold true. So, at this point, we say that the conjecture is true.*

Example 2.6.5

Give a counterexample to this statement: Every prime number is an odd number.

Solution

The only counterexample is the number 2: an even number (not odd) that is prime.

Review

Give a counterexample for each of the following statements.

1. If n is a whole number, then $n^2 > n$.
2. All numbers that end in 1 are prime numbers.
3. All positive fractions are between 0 and 1.
4. Any three points that are coplanar are also collinear.
5. All girls like ice cream.
6. All high school students are in choir.
7. For any angle there exists a complementary angle.
8. All teenagers can drive.
9. If n is an integer, then $n > 0$.
10. All equations have integer solutions.

Review (Answers)

To see the Review answers, open this PDF file and look for section 2.5.

Resources

Vocabulary

Term	Definition
conjecture	A conjecture is an educated guess that is based on examples in a pattern.
counterexample	A counterexample is an example that disproves a conjecture.
fraction	A fraction is a part of a whole. A fraction is written mathematically as one value on top of another, separated by a fraction bar. It is also called a *rational number*.

Additional Resources

Video: Inductive Reasoning

Activities: Conjectures and Counterexamples Discussion Questions

Study Aids: Types of Reasoning Study Guide

Practice: Conjectures and Counterexamples

Real World: Conjectures And Counterexamples

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2.7: Deductive Reasoning

Drawing conclusions from facts.

Deductive reasoning entails drawing conclusion from facts. When using deductive reasoning there are a few laws that are helpful to know.

Law of Detachment: If $p \to q$ is true, and p is true, then q is true. See the example below.

Here are two true statements:

1. *If a number is odd (p), then it is the sum of an even and odd number (q).*
2. *5 is an odd number (a specific example of p).*

The conclusion must be that *5 is the sum of an even and an odd number (q).*

Law of Contrapositive: If $p \to q$ is true and $\sim q$ is true, then you can conclude $\sim p$. See the example below.

Here are two true statements:

1. *If a student is in Geometry (p), then he or she has passed Algebra I (q).*
2. *Daniel has not passed Algebra I (a specific example of $\sim q$).*

The conclusion must be that *Daniel is not in Geometry ($\sim q$).*

Law of Syllogism: If $p \to q$ and $q \to r$ are true, then $p \to r$ is true. See the example below.

Here are three true statements:

1. *If Pete is late (p), Mark will be late (q).*
2. *If Mark is late (q), Karl will be late (r).*
3. *Pete is late (p).*

Notice how each “then” becomes the next “if” in a chain of statements. If Pete is late, this starts a domino effect of lateness. Mark will be late and Karl will be late too. So, if Pete is late, then *Karl will be late (r)*, is the logical conclusion.

What if you were given a fact like "If you are late for class, you will get a detention"? What conclusions could you draw from this fact?

Example 2.7.1

Suppose Bea makes the ts, which are known to be true.

If Central High School ! go to the regional tournament. Central High School won today.

What is the logical conc

Solution

These are true statements that we can take as facts. The conclusion is: *Central High School will go to the regional tournament.*

Example 2.7.2

Here are two true statements.

If $\angle A$ and $\angle B$ are a linear pair, then $m\angle A + m\angle B = 180°$.

$\angle ABC$ and $\angle CBD$ are a linear pair.

What conclusion can you draw from this?

Solution

This is an example of the Law of Detachment, therefore:

$m\angle ABC + m\angle CBD = 180°$

Example 2.7.3

Determine the conclusion from the true statements below.

Babies wear diapers.

My little brother does not wear diapers.

Solution

The second statement is the equivalent of $\sim q$. Therefore, the conclusion is $\sim p$, or: *My little brother is not a baby*.

Example 2.7.4

Here are two true statements.

If $\angle A$ *and* $\angle B$ *are a linear pair, then* $m\angle A + m\angle B = 180°$.

$m\angle 1 = 90°$ *and* $m\angle 2 = 90°$.

What conclusion can you draw from these two statements?

Solution

Here there is NO conclusion. These statements are in the form:

$p \rightarrow q$

q

We ***cannot*** conclude that $\angle 1$ and $\angle 2$ are a linear pair.

Here are two counterexamples:

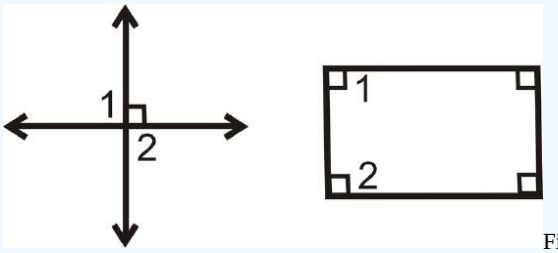

Figure 2.7.1

Example 2.7.5

Determine the conclusion from the true statements below.

If you are not in Chicago, then you can't be on the L.

Sally is on the L.

Solution

If we were to rewrite this symbolically, it would look like:

$\sim p \rightarrow \sim q$

q

Even though it looks a little different, this is an example of the Law of Contrapositive. Therefore, the logical conclusion is: *Sally is in Chicago*.

Review

Determine the logical conclusion and state which law you used (Law of Detachment, Law of Contrapositive, or Law of Syllogism). If no conclusion can be drawn, write "no conclusion."

1. People who vote for Jane Wannabe are smart people. I voted for Jane Wannabe.
2. If Rae is the driver today then Maria is the driver tomorrow. Ann is the driver today.
3. All equiangular triangles are equilateral. δABC is equiangular.
4. If North wins, then West wins. If West wins, then East loses.
5. If $z > 5$, then $x > 3$. If $x > 3$, then $y > 7$.
6. If I am cold, then I wear a jacket. I am not wearing a jacket.
7. If it is raining outside, then I need an umbrella. It is not raining outside.
8. If a shape is a circle, then it never ends. If it never ends, then it never starts. If it never starts, then it doesn't exist. If it doesn't exist, then we don't need to study it.
9. If you text while driving, then you are unsafe. You are a safe driver.
10. If you wear sunglasses, then it is sunny outside. You are wearing sunglasses.
11. If you wear sunglasses, then it is sunny outside. It is cloudy.
12. I will clean my room if my mom asks me to. I am not cleaning my room.
13. Write the symbolic representation of #8. Include your conclusion. Does this argument make sense?
14. Write the symbolic representation of #10. Include your conclusion.
15. Write the symbolic representation of #11. Include your conclusion.

Resources

Vocabulary

Term	Definition
Deductive Reasoning	When using deductive reasoning, one would draw conclusions that are based on facts.

Additional Resources

Video: Types of Reasoning: Deductive Principles - Basic

Activities: Deductive Reasoning Discussion Questions

Study Aids: Types of Reasoning Study Guide

Practice: Deductive Reasoning

Real World: Deductive Reasoning

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2.8: Truth Tables

So far we know these symbols for logic:

- $\sim$ not (negation)
- $\rightarrow$ if-then
- $\therefore$ therefore

Two more symbols are:

- $\wedge$ and
- $\vee$ or

We would write "p and q" as $p \wedge q$ and "p or q" as $p \vee q$.

Truth tables use these symbols and are another way to analyze logic. First, let's relate p and \sim p. To make it easier, set p as: *An even number.* Therefore, \sim p is *An odd number*. Make a truth table to find out if they are both true. Begin with all the "truths" of p, true (T) or false (F).

p	
T	
F	

Next we write the corresponding truth values for $\sim p$. $\sim p$ has the opposite truth values of p. So, if p is true, then $\sim p$ is false and vise versa.

p	\sim p
T	F
F	T

To Recap:

- Start truth tables with all the possible combinations of truths. For 2 variables there are 4 combinations for 3 variables there are 8. **You always start a truth table this way.**
- Do any negations on the any of the variables.
- Do any combinations in parenthesis.
- Finish with completing what the problem was asking for.

Drawing a Truth Table

1. Draw a truth table for p, q, $p \wedge q$

First, make columns for p nns with all the possible true and false combinations for the two.

p	q	
T	T	
T	F	
F	T	
F	F	

Notice all the combinations of p and q. **Anytime we have truth tables with two variables, this is <u>always</u> how we fill out the first two columns.**

Next, we need to figure out when $p \wedge q$ is true, based upon the first two columns. p \wedge q **can only be true if BOTH p and q are true.** So, the completed table looks like this:

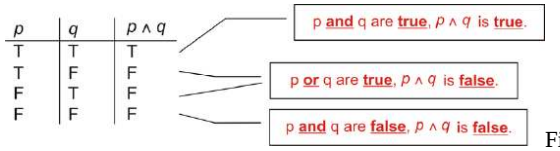

Figure **2.8.1**

This is how a truth table with two variables and their "and" column is always filled out.

2. Draw a truth table for p, q and $p \vee q$.

First, make columns for $p \vee q$ and q, just like Example A.

p	q	
T	T	
T	F	
F	T	
F	F	

Next, we need to figure out when $p \vee q$ is true, based upon the first two columns. $p \vee q$ **is true if** p **OR** q **are true, or both are true.** So, the completed table looks like this:

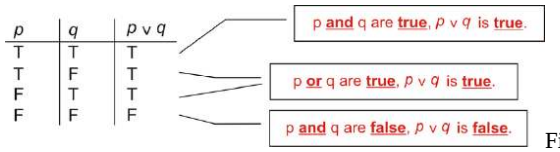

Figure **2.8.2**

The difference between $p \wedge q$ and $p \vee q$ is the second and third rows. For "and" both p and q have to be true, but for "or" only one has to be true.

Determining the Truths

Determine the truths for $p \wedge$

First, there are three varial ıg to need all the combinations of their truths. **For three variables, there are always 8 possible combinations.**

p	q	r			
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Next, address the $\sim q$. It will just be the opposites of the q column.

p	q	r	$\sim q$		
T	T	T	F		
T	T	F	F		
T	F	T	T		
T	F	F	T		
F	T	T	F		
F	T	F	F		
F	F	T	T		
F	F	F	T		

Now, let's do what's in the parenthesis, $\sim q \vee r$. Remember, for "or" only $\sim q$ OR r has to be true. Only use the $\sim q$ and r columns to determine the values in this column.

p	q	r	$\sim q$	$\sim q \vee r$	
T	T	T	F	T	
T	T	F	F	F	
T	F	T	T	T	
T	F	F	T	T	
F	T	T	F	T	
F	T	F	F	F	
F	F	T	T	T	
F	F	F	T	T	

Finally, we can address the entire problem, $p \wedge (\sim q \vee r)$. Use the p and $\sim q \vee r$ to determine the values. Remember, for "and" both p and $\sim q \vee r$ must be true.

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

Write a truth table for the

Example 2.8.1

$p \wedge \sim p$

Solution

First, make columns for p, then add in $\sim p$ and finally, evaluate $p \wedge \sim p$.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Example 2.8.2

$\sim p \vee \sim q$

Solution

First, make columns for p and q, then add in $\sim p$ and $\sim q$. Finally, evaluate $\sim p \vee \sim q$.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
$p \vee \sim q$	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Example 2.8.3

$p \wedge (q \vee \sim q)$

Solution

First, make columns for p and q, then add in $\sim q$ and $q \vee \sim q$. Finally, evaluate $p \wedge (q \vee \sim q)$.

p	q	$\sim q$	$q \vee \sim q$	$p \wedge (q \vee \sim q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

Review

Write a truth table for the following variables.

1. $(p \wedge q) \vee \sim r$
2. $p \vee (\sim q \vee r)$
3. $p \wedge (q \vee \sim r)$
4. The only difference between #1 and #3 is the placement of the parenthesis. How do the truth tables differ?
5. When is $p \vee q \vee r$ true?
6. $p \vee q \vee r$

7. $(p \vee q) \vee \sim r$
8. $(\sim p \wedge \sim q) \wedge r$
9. $(\sim p \vee \sim q) \wedge r$

Is the following a valid argument? If so, what law is being used? HINT: Statements could be out of order.

10.

$$p \rightarrow q$$

$$r \rightarrow p$$

$$\therefore r \rightarrow q$$

11.

$$p \rightarrow q$$

$$r \rightarrow q$$

$$\therefore p \rightarrow r$$

12.

$$p \rightarrow \sim r$$

$$r$$

$$\therefore \sim p$$

13.

$$\sim q \rightarrow r$$

$$q$$

$$\therefore \sim r$$

14.

$$p \rightarrow (r \rightarrow s)$$

$$p$$

$$\therefore r \rightarrow s$$

15.

$$r \rightarrow q$$

$$r \rightarrow s$$

$$\therefore q \rightarrow s$$

Additional Resources

Video: Truth Tables Principles

Practice: Truth Tables

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2.9: And and Or Statements

Truth tables for conjunctions and disjunctions.

The words “and” and “or” are common in everyday language. In mathematics, there are some subtle differences that you need to watch out for, especially considering the word “or”.

It will rain or it will snow.

When is this statement true and when is it false?

Introduction to Logic

- $P = It\ is\ snowing.$
- $Q = I\ am\ cold.$

The **truth value** of a statement is whether the statement is true or false. As a mathematician, your job is to determine when a logical statement is true and when it is false. If you don’t have enough information to determine whether the original statements are true or false, you can build a truth table to organize all the possible cases.

Consider the atomic statement P joined with the atomic statement Q. The following sentence can be written using the symbol “ $\vee$ ” for the logical connective “or”.

$$It\ is\ snowing\ or\ I\ am\ cold.$$

$$P \vee Q$$

This statement is a little strange because it seems to imply that it is always the case that one or both of those atomic statements is happening. Your common sense may dictate that this statement isn’t true because of course there are times when it is sunny and you are warm. It’s important to remember that not all statements are true! Your job is to determine what has to be true for the above statement to be true. To organize your work, you should construct a truth table. A truth table considers all possible combinations of the original atomic statements being true or false, and then uses logic to deduce the truth value of the compound statement in each case.

Here is the truth table for OR:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Notice that there are four possible truth combinations of P and Q (both true, first true/second false, first false/second true, both false). Only one of these combinations yields a false statement for $P \vee Q$. What this means is that the statement “It is snowing or I am cold” is only false if “it is snowing” is false and “I am cold” is false. Note that if “it is snowing” is true and “I am cold” is also true, then “It is snowing or I am cold” is true. In mathematics, the word “or” does not mean exactly one or the other. It means “one or the other or both”.

Next consider the truth table for the following statement that uses the connective “and”. The following sentence can be written using the symbol “ $\wedge$ ” for the logical connective “and”.

$$It\ is\ snowing\ and\ I\ am\ cold.$$

$$P \wedge Q$$

Here is the truth table for AND:

P	Q	P \wedge Q
T	*T*	*T*
T	*F*	*F*
F	*T*	*F*
F	*F*	*F*

Notice that a compound statement using “and” is true only if each atomic statement is individually true.

Watch the portion of this video focusing on truth tables of **conjunction** and **disjunction**:

Example 2.9.1

Earlier, you were asked [illegible] "It will rain or it will snow" will be true and false. In English, most people use the word “or” to mean excl[illegible]vere told “you can have a brownie or a cookie for dessert”, you would assume you had to choose just one [illegible]oth the brownie and the cookie. In mathematics, the word “or” means “one or the other or both”. Therefore in logic, “or” includes the case when both atomic parts of the state are true.

$P = It\ will\ rain.$

$Q = It\ will\ snow.$

$P \vee Q$

Solution

P	Q	$P \vee Q$
T	*T*	*T*
T	*F*	*T*
F	*T*	*T*
F	*F*	*F*

The statement is only false when both parts of the statement are false. In other words, the statement is only false if “it will rain” is false and “it will snow” is also false. When one or both parts of an “or” statement are true then the whole statement is true.

Example 2.9.2

Identify the atomic statements in the following compound sentence. Then, use logical connectives to rewrite the sentence with symbols.

I am tired and hungry and I want a burger or a nap.

- $P = I\ am\ tired.$
- $Q = I\ am\ hungry.$
- $R = I\ want\ a\ burger.$
- $S = I\ want\ a\ nap.$

Solution

The sentence could be rewritten with symbols as: $(P \wedge Q) \wedge (R \vee S)$

Example 2.9.3

Identify the atomic statements in the following compound sentence. Then, use logical connectives to rewrite the sentence with symbols.

For lunch you had a ham and cheese sandwich and an apple or an orange.

- $A = You\ had\ a\ ham\ and\ cheese\ sandwich\ for\ lunch.$
- $B = You\ had\ an\ apple\ for\ lunch.$
- $C = You\ had\ an\ orange\ for\ lunch.$

Solution

The sentence could be rewritten with symbols as: $A \wedge (B \vee C)$.

Note that each statement A,B,C contains the words “you had” and “for lunch” and is a complete sentence.

Example 2.9.4

Diagram the sentence from Guided Practice #1 using the logical connectives “$\vee$ ” for “or” and “ $\wedge$ ” for “and”.

Solution

The hardest part in diagramming the logical connectives is often determining which parts of the sentence should be grouped together. In this case there is a clear separation between the three positive outcomes and with the two negative outcomes:

$(M \wedge B \wedge D) \vee (W \wedge J)$

Example 2.9.5

Use a truth table to identify all cases when the statement in Guided Practice #1 is true or false.

Solution

Truth tables of complex sentences can be overwhelming, especially since 5 atomic statements means that there should be 25 rows in the truth table to account for all of the T/F combinations. To save time and space you can note that the statement $M \wedge B \wedge D$ is only true when M,B, and D are all true and $W \wedge J$ is only true when both W and J are true. This means that you now only need 4 rows in the truth table.

$M \wedge B \wedge D$	$W \wedge J$	(M \wedge B \wedge D) \lor (W \wedge J)
T	T	T
F	T	T
T	F	T
F	F	F

The statement is true if:

1. M,B, and D are all true.
2. W and J are both true.
3. M,B,D,W, and J are all true.

The statement is false if:

1. Not all of M,B, and D are true and not both of W and J are true.

Review

I go to school and do my work or stay home and play games.

1. Identify the atomic statements in the above compound sentence.

2. Use logical connectives to rewrite the sentence with symbols.

I have macaroni and cheese or steak and green beans or potatoes.

3. Identify the atomic statements in the above compound sentence.

4. Use logical connectives to rewrite the sentence with symbols.

I wear flip flops and either shorts and a t-shirt or a dress.

5. Identify the atomic statements in the above compound sentence.

6. Use logical connectives to rewrite the sentence with symbols.

It is dark outside and I light a candle.

7. Identify the atomic statements in the above compound sentence.

8. Use logical connectives to rewrite the sentence with symbols.

We will go to the beach and have a picnic or go to the movies and eat popcorn.

9. Identify the atomic statements in the above compound sentence.

10. Use logical connectives to rewrite the sentence with symbols.

Make a truth table for each of the following statements.

11. $(P \wedge Q) \vee R$

12. $P \wedge (Q \vee R)$

13.$(P \vee Q) \vee R$

14. $P \vee (Q \vee R)$

15. How does the placement of parentheses affect the truth values of compound statements?

Vocabulary

Term	Definition
∧	A conjunction is an "and" statement, which is a statement that combines two logical statements and is only true when both statements are true. The symbol for “and” is “∧ ”.
∨	A disjunction is an "or" statement, which is a statement that combines two logical statements and is only false when both statements are false. The symbol for “or” is “∨”.
atomic statement	An atomic statement is a declarative statement without logical connectives that has a truth value.
conjunction	A conjunction is an "and" statement, which is a statement that combines two logical statements and is only true when both statements are true. The symbol for “and” is “∧”.
disjunction	A disjunction is an "or" statement that combines two logical statements, and is only false when both statements are false. The symbol for “or” is “∨”.
truth value	The truth value of a statement is whether the statement is true or false.

Additional Resources

Practice: And and Or Statements

Real World: Binary Logic

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2.10: Negative Statements

Truth values for negation of conditional statements, conjunctions, and disjunctions.

In everyday speech, negative statements are often ambiguous or unclear. Mathematically, you need a precise way to negate statements so that you can accurately determine whether statements are true or false. When is the **negation** of the following sentence true?

If I am not cold then it is not snowing.

Negations

While in everyday language the opposite of “dog” might be “cat”, in mathematics the opposite of “dog” is “not a dog”. Using the word “not” is the basic way to negate an atomic sentence. An atomic sentence is a logical statement without logical connectives that has a truth value.

- Original sentence (D): That thing is a dog.
- Negation of sentence ($\sim D$): That thing is not a dog.

The box below represents the universe of all things. This universe can be separated into things that are dogs and things that are not dogs.

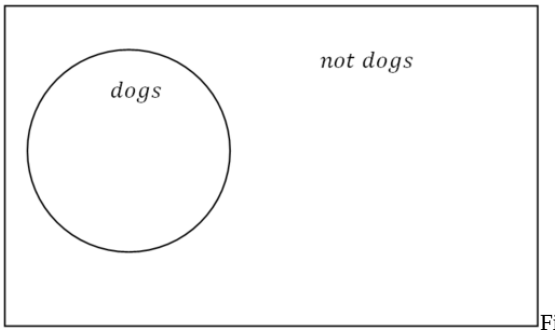

Figure **2.10.1**

To negate complex statements that involve logical connectives like or, and, or if-then, you should start by constructing a truth table and noting that negation completely switches the truth value.

The negation of a conditional statement is only true when the original if-then statement is false.

P	Q	$P \rightarrow Q$	$\sim (P \rightarrow Q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

The negation of a **conjunction** is only false when the original two statements are both true. A conjunction is two statements that are joined by an "and".

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

The negation of a **disjunction** is only true when both of the original statements are false. A disjunction is two statements that are joined by an "or".

P	Q	$P \vee Q$	$\sim (P \vee Q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Set theory circles can also be used to interpret the negation of a disjunction or a conjunction.

The shaded portion in the box represents the area that is within P or within Q. Recall that in mathematical logic, this is written as $P \vee Q$. In set theory this area is represented similarly as $P \cup Q$ where the symbol $\cup$ stands for union. While the notation is slightly different, the reasoning about the relationships and implications is identical.

$P \vee Q$

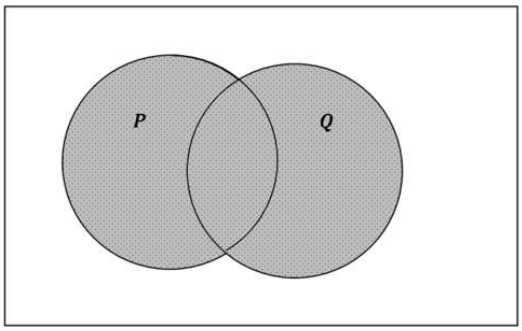

Figure **2.10.2**

When you negate the statement, you completely switch what is shaded.

$\sim (P \vee Q)$

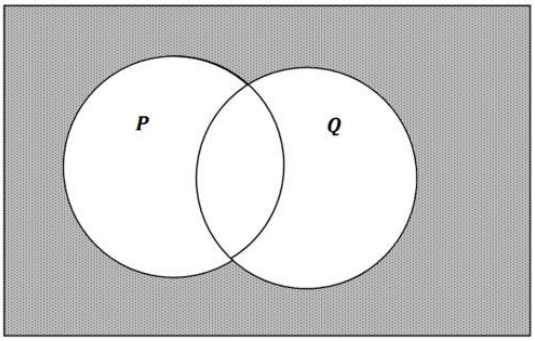

Figure **2.10.3**

A different way to think about this shaded region is that it is the region that is not in P and also not in **Q**. This means that $\sim (P \vee Q)$ is equivalent to $\sim P \wedge \sim Q$.

$\sim (P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$

The symbol "$\Leftrightarrow$" works in mathematical logic and set theory the same way "$=$" works in arithmetic and algebra. In this case, the negative appears to distribute throughout the or statement by negating each statement individually and changes the or statement to an and statement. This is called **De Morgan's Law**.

Negating a conjunction gives a similar result.

The shaded portion in the box represents the area that is within **P** and within Q. In mathematical logic this is written as $P \wedge Q$. In set theory this area is represented similarly as $P \cap Q$ where the symbol $\cap$ stands for intersection. As before, the notation between mathematical logic and set theory is slightly different. However, the reasoning about the relationships and the logical implications are identical.

$P \wedge Q$

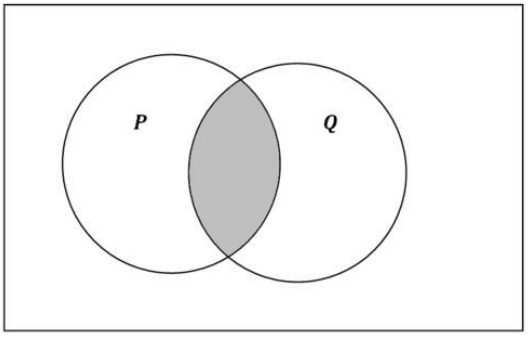

Figure **2.10.4**

When you negate the statement, you completely switch what is shaded.

$\sim (P \wedge Q)$

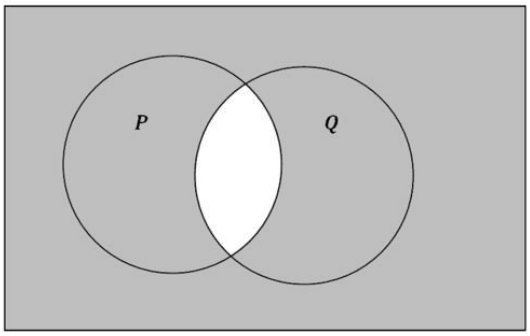

Figure **2.10.5**

A different way to think about this shaded region is to consider that it represents everything that isn't in P or isn't in Q.

$\sim (P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$

This is a second representation of De Morgan's Law.

As mathematical sentences become more complex with additional connectives, truth tables and set theory circles are good ways to interpret when the statements are true and when the statements are false.

Example 2.10.1

Earlier, you were asked when the negation of the following statement is true.

If I am not cold, then it is not snowing.

The statement already has several negative parts, so it is incorrect to simply switch one or both of the negations.

- $P = I\ am\ cold.$
- $Q = It\ is\ snowing.$

Solution

P	Q	$\sim P$	$\sim Q$	$\sim P \rightarrow \sim Q$	$\sim (\sim P \rightarrow \sim Q)$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	F

Start by building up to the original statement. Then, add a column that completely negates the original statement. Notice that there is only one row where the final negated statement is true. That is when P is false and Q is true. Therefore, the negation of the original sentence is true when “I am not cold” and “it is snowing”.

Example 2.10.2

Use set theory circles to interpret the negation of a conditional statement and explain how the negation of a conditional can be written in a different way.

Solution

The shaded portion in the box represents the area that makes the following statement true.

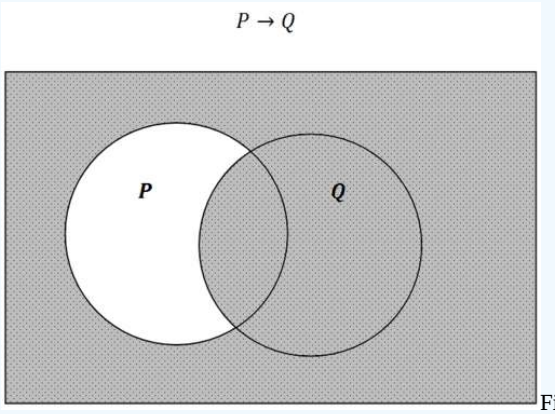

Figure 2.10.6

Notice that there are four different regions in the Venn Diagram which correspond to the four different rows of the conditional truth table. The only space that does not make the statement true is when P is true (inside circle P) and Q is false (outside circle Q).

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

To negate this statement, you switch the values in the truth table and switch the shaded region in the Venn Diagram.

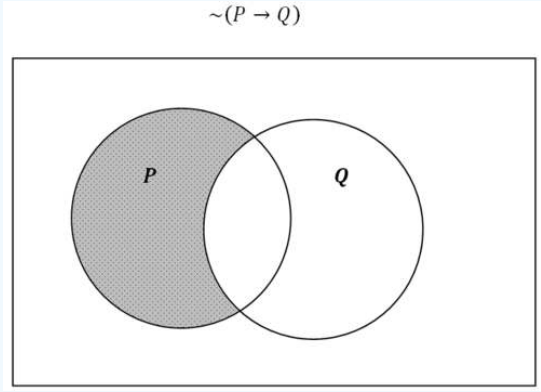

Figure 2.10.7

A different way to think about this shaded region is to see it as the space that is in P but not in Q.

$\sim (P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$

Example 2.10.3

Show that the following statements are equivalent in a truth table. The symbol ≡ means equivalent. To be equivalent in this case means to be true at the same time and false at the same time.

$P \wedge Q \equiv \sim (\sim P \vee \sim Q)$

Solution

While a truth table is not a proof, it can help you recognize when two statements have the same truth values.

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$\sim (\sim P \vee \sim Q)$	$P \wedge Q$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

Notice that the final two columns are identical.

Example 2.10.4

Write a sentence in two different ways illustrating the mathematical statement in Example 3

Solution

$P \wedge Q$: I like movies and I like TV. $\sim (\sim P \vee \sim Q)$: It is not the case that either I don't like movies or I don't like TV.

Example 2.10.5

A **tautology** is a logical statement that is always true. Demonstrate the following tautology in a truth table.

$B \rightarrow (A \vee \sim A)$

Solution

A	$\sim A$	B	$A \vee \sim A$	$B \rightarrow (A \vee \sim A)$
T	F	F	T	T
T	F	T	T	T
F	T	T	T	T
F	T	F	T	T

Notice that the last column is all T's. This means that this statement is always true and therefore is a tautology.

Review

I'm either going to go skiing or snowboarding next weekend.

1. Identify the atomic statements in the above sentence and use logical connectives to rewrite the sentence with symbols.

2. Write the negation of the sentence with symbols and write the negation of the sentence in words in a natural way.

Mike and John both ate lunch with me.

3. Identify the atomic statements in the above sentence and use logical connectives to rewrite the sentence with symbols.

4. Write the negation of the sentence with symbols and write the negation of the sentence in words in a natural way.

Neither my brother nor my sister wants to play with me.

5. Identify the atomic statements in the above sentence and use logical connectives to rewrite the sentence with symbols.

6. Write the negation of the sentence with symbols and write the negation of the sentence in words in a natural way.

Write negations for the following statements.

7. All dogs go to heaven.

8. My teacher is seldom wrong.

9. Everyone likes pizza.

Make truth tables for each of the following.

10. $(P \wedge Q) \vee \sim Q$

11. $P \wedge (Q \vee \sim Q)$

12. $(P \vee Q) \vee \sim R$

13. $(\sim P \wedge \sim Q) \vee \sim R$

14. What is the simplest statement that is equivalent to #11: $P \wedge (Q \vee \sim Q)$?

15. Use De Morgan's Law to find a statement equivalent to the following statement: $\sim (Q \vee \sim Q)$

16. Use De Morgan's Law to find a statement equivalent to the following statement: $\sim (P \vee Q) \vee \sim R$

Vocabulary

Term	Definition
atomic statement	An atomic statement is a declarative statement without logical connectives that has a truth value.
conjunction	A conjunction is an "and" statement, which is a statement that combines two logical statements and is only true when both statements are true. The symbol for “and” is “ $\wedge$ ”.
De Morgan's law	De Morgan’s law transforms a conjunction into a disjunction using negation.
disjunction	A disjunction is an "or" statement that combines two logical statements, and is only false when both statements are false. The symbol for “or” is “ $\vee$”.
negation	The negation of a statement is the opposite of the statement. If the original statement is D, then the negation of the statement is represented by $\sim D$. A statement and its negation will always have opposite truth values.
tautology	A tautology is a logical statement that is always true. A tautology is a type of basic theorem.

Additional Resource

Practice: Negative Statements

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2.11: If Then Statements

Hypothesis followed by a conclusion in a conditional statement.

Conditional Statements

A **conditional statement** (also called an **if-then statement**) is a statement with a **hypothesis** followed by a **conclusion**. The **hypothesis** is the first, or "if," part of a conditional statement. The **conclusion** is the second, or "then," part of a conditional statement. The conclusion is the result of a hypothesis.

Figure 2.11.1

If-then statements might not always be written in the "if-then" form. Here are some examples of conditional statements:

- **Statement 1:** If you work overtime, then you'll be paid time-and-a-half.
- **Statement 2:** I'll wash the car if the weather is nice.
- **Statement 3:** If 2 divides evenly into x, then x is an even number.
- **Statement 4:** I'll be a millionaire when I win the lottery.
- **Statement 5:** All equiangular triangles are equilateral.

Statements 1 and 3 are written in the "if-then" form. The hypothesis of Statement 1 is "you work overtime." The conclusion is "you'll be paid time-and-a-half." **Statement 2** has the hypothesis after the conclusion. If the word "if" is in the middle of the statement, then the hypothesis is after it. The statement can be rewritten: *If the weather is nice, then I will wash the car.* **Statement 4** uses the word "when" instead of "if" and is like Statement 2. It can be written: *If I win the lottery, then I will be a millionaire.* **Statement 5** "if" and "then" are not there. It can be rewritten: *If a triangle is equiangular, then it is equilateral.*

What if you were given a statement like "All squares are rectangles"? How could you determine the hypothesis and conclusion of this statement?

Example 2.11.1

Determine the hypothes 'll bring an umbrella if it rains.

Solution

Hypothesis: "It rains." C ıg an umbrella."

Example 2.11.2

Determine the hypothesis and conclusion: All right angles are $90°$.

Solution

Hypothesis: "An **angle** is right." Conclusion: "It is $90°$."

Example 2.11.3

Use the statement: *I will graduate when I pass Calculus.*

Rewrite in if-then form and determine the hypothesis and conclusion.

Solution

This statement can be rewritten as *If I pass Calculus, then I will graduate.* The hypothesis is "I pass Calculus," and the conclusion is "I will graduate."

Example 2.11.4

Use the statement: *All prime numbers are odd.*

Rewrite in if-then form, determine the hypothesis and conclusion, and determine whether this is a true statement.

Solution

This statement can be rewritten as *If a number is prime, then it is odd.* The hypothesis is "a number is prime" and the conclusion is "it is odd". This is not a true statement (remember that not all conditional statements will be true!) since 2 is a prime number but it is not odd.

Example 2.11.5

Determine the hypothesis and conclusion: Sarah will go to the store if Riley does the laundry.

Solution

The statement can be rewritten as "If Riley does the laundry then Sarah will go to the store." The hypothesis is "Riley does the laundry" and the conclusion is "Sarah will go to the store."

Review

Determine the hypothesis and the conclusion for each statement.

1. If 5 divides evenly into x, then x ends in 0 or 5.
2. If a triangle has three congruent sides, it is an equilateral triangle.
3. Three points are coplanar if they all lie in the same plane.
4. If $x = 3$, then $x^2 = 9$.
5. If you take yoga, then you are relaxed.
6. All baseball players wear hats.
7. I'll learn how to drive when I am 16 years old.
8. If you do your homework, then you can watch TV.
9. Alternate interior angles are congruent if lines are parallel.
10. All kids like ice cream.

Resource

Vocabulary

Term	Definition
Conditional Statement	A conditional statement (or 'if-then' statement) is a statement with a hypothesis followed by a conclusion.
Angle	A geometric figure formed by two rays that connect at a single point or vertex.
antecedent	The antecedent is the first, or “if,” part of a conditional statement.
apodosis	The “then” part of an if-then statement is called the conclusion, consequent or apodosis.
conclusion	The conclusion of a conditional statement is the result of the hypothesis.
consequent	The “then” part of an if-then statement is called the conclusion, consequent or apodosis.
hypothesis	The hypothesis is the first, or “if,” part of a conditional statement.
if-then statement	An if-then statement is another name for a conditional statement.
protasis	The protasis is the first, or “if,” part of a conditional statement.
set	A set is a collection of numbers, letters or anything.
set theory	Set theory studies the relationships of sets and subsets.
Subset	A subset is a collection of numbers or objects within a larger set.

Additional Resources

Video: If-Then Statements Principles - Basic

Activities: If-Then Statements Discussion Questions

Study Aids: Conditional Statements Study Guide

Practice: If Then Statements

Real World: If Then Statements

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2.12: Converse, Inverse, and Contrapositive Statements

Conditional statements drawn from an if-then statement.

Converse, Inverse, and Contrapositive

Consider the statement: *If the weather is nice, then I'll wash the car.* We can rewrite this statement using letters to represent the hypothesis and conclusion.

$$p = \textit{the weather is nice} \qquad q = \textit{I'll wash the car}$$

Now the statement is: if p, then q, which can also be written as $p \to q$.

We can also make the negations, or "nots," of p and q. The symbolic version of "not p" is \(\sim p.

$$\sim p = \textit{the weather is not nice} \qquad \sim q = \textit{I won't wash the car}$$

Using these "nots" and switching the order of p and q, we can create three new statements.

$$\textit{Converse} \quad q \to p \quad \underbrace{\textit{If I wash the car}}_{\text{q}}, \underbrace{\textit{then the weather is nice}}_{\text{p}}.$$

$$\textit{Inverse} \quad \sim p \to \sim q \quad \underbrace{\textit{If the weather is not nice}}_{\text{p}}, \underbrace{\textit{then I won't wash the car}}_{\text{q}}.$$

$$\textit{Contrapositive} \quad \sim q \to \sim p \quad \underbrace{\textit{If I don't wash the car}}_{\text{q}}, \underbrace{\textit{thentheweatherisnotnice}}_{\text{p}}.$$

If the "if-then" statement is true, then the contrapositive is also true. The contrapositive is ***logically equivalent*** to the original statement. The converse and inverse may or may not be true. When the original statement and converse are both true then the statement is a **biconditional statement**. In other words, if $p \to q$ is true and $q \to p$ is true, then $p \leftrightarrow q$ (said "p if and only if q").

What if you were given a **conditional statement** like "If I walk to school, then I will be late"? How could you rearrange and/or negate this statement to form new conditional statements?

Example 2.12.1

If $n > 2$, then $n^2 > 4$.

Find the converse, inve itive. Determine if each resulting statement is true or false. If it is false, find a counterexample.

Solution

The original statement is true.

$\underline{\textit{Converse}}$: If $n^2 > 4$, then $n > 2$.

False. If $n^2 = 9$, $n = -3$ *or* 3. $(-3)^2 = 9$

$\underline{\textit{Inverse}}$: If $n \leq 2$, then $n^2 \leq 4$.

False. If $n = -3$, then $n^2 = 9$.

$\underline{\textit{Contrapositive}}$: If $n^2 \leq 4$, then $n \leq 2$.

True. The only $n^2 \leq 4$ is 1 or 4. $\sqrt{1} = \pm 1$ and $\sqrt{4} = \pm 2$, which are both less than or equal to 2.

Example 2.12.2

If I am at Disneyland, then I am in California.

Find the converse, inverse, and contrapositive. Determine if each resulting statement is true or false. If it is false, find a counterexample.

Solution

The original statement is true.

Converse: If I am in California, then I am at Disneyland.

False. I could be in San Francisco.

Inverse: If I am not at Disneyland, then I am not in California.

False. Again, I could be in San Francisco.

Contrapositive: If I am not in California, then I am not at Disneyland.

True. If I am not in the state, I couldn't be at Disneyland.

Notice for the converse and inverse we can use the same counterexample.

Example 2.12.3

Rewrite as a biconditional statement: Any two points are collinear.

Solution

This statement can be rewritten as:

Two points are on the same line if and only if they are collinear. Replace the "if-then" with "if and only if" in the middle of the statement.

Example 2.12.4

Any two points are collinear.

Find the converse, inverse, and contrapositive. Determine if each resulting statement is true or false. If it is false, find a counterexample.

Solution

First, change the statement into an "if-then" statement:

If two points are on the same line, then they are collinear.

Converse: If two points are collinear, then they are on the same line. True.

Inverse: If two points are not on the same line, then they are not collinear. True.

Contrapositive: If two points are not collinear, then they do not lie on the same line. True.

Example 2.12.5

The following is a true statement:

$m\angle ABC > 90°$ if and only if $\angle ABC$ is an obtuse angle.

Determine the two true statements within this biconditional.

Solution

Statement 1: If $m\angle ABC > 90°$, then $\angle ABC$ is an obtuse angle.

Statement 2: If $\angle ABC$ is an obtuse angle, then $m\angle ABC > 90°$.

Review

For questions 1-4, use the statement:

If $AB = 5$ and $BC = 5$, then B is the midpoint of $\overline{AC}$.

1. Is this a true statement? If not, what is a counterexample?

2. Find the converse of this statement. Is it true?
3. Find the inverse of this statement. Is it true?
4. Find the contrapositive of this statement. Which statement is it the same as?

Find the converse of each true if-then statement. If the converse is true, write the biconditional statement.

5. An acute angle is less than $90°$.
6. If you are at the beach, then you are sun burnt.
7. If $x > 4$, then $x + 3 > 7$.

For questions 8-10, determine the two true conditional statements from the given biconditional statements.

8. A U.S. citizen can vote if and only if he or she is 18 or more years old.
9. A whole number is prime if and only if its factors are 1 and itself.
10. $2x = 18$ if and only if $x = 9$.

Review (Answers)

To see the Review answers, open this PDF file and look for section 2.4.

Resources

Vocabulary

Term	Definition
biconditional statement	A statement is biconditional if the original conditional statement and the converse statement are both true.
Conditional Statement	A conditional statement (or 'if-then' statement) is a statement with a hypothesis followed by a conclusion.
contrapositive	If a conditional statement is $p \to q$ (if p then q), then the contrapositive is $\sim q \to \sim p$ (if *not* q then *not* p).
converse	If a conditional statement is $p \to q$ (if p, then q), then the converse is $q \to p$ (if q, then p. Note that the converse of a statement is not true just because the original statement is true.
inverse	If a conditional statement is $p \to q$, then the inverse is $\sim p \to \sim q$.
Logically Equivalent	A statement is logically equivalent if the "if-then" statement and the contrapositive statement are both true.
premise	A premise is a starting statement that you use to make logical conclusions.

Additional Resources

Video: Converse, Inverse and Contrapositive of a Conditional Statement Principles - Basic

Activities: Converse, Inverse, and Contrapositive Discussion Questions

Study Aids: Conditional Statements Study Guide

Practice: Converse, Inverse, and Contrapositive Statements

Real World: Converse Inverse Contrapositive

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2.13: Introduction to Proofs

Use two column proofs to assert and prove the validity of a statement by writing formal arguments of mathematical statements. Also learn about paragraph and flow diagram proof formats.

Two-Column Proofs

A **two-column proof** is one common way to organize a proof in geometry. Two-column proofs always have two columns: one for statements and one for reasons. The best way to understand two-column proofs is to read through examples.

When writing your own two-column proof, keep these things in mind:

- **Number each step.**
- **Start with the given information.**
- **Statements with the same reason can be combined into one step**. It is up to you.
- **Draw a picture and mark it with the given information.**
- **You must have a reason for EVERY statement.**
- **The order of the statements in the proof is not always fixed, but make sure the order makes logical sense.**
- **Reasons will be definitions, postulates, properties and previously proven theorems.** "Given" is only used as a reason if the information in the statement column was given in the problem.
- **Use symbols and abbreviations for words within proofs.** For example, ≅ can be used in place of the word *congruent*. You could also use \(\angle for the word angle.

Suppose you are told that $\angle XYZ$ is a right angle and that $\overrightarrow{YW}$ bisects $\angle XYZ$. You are then asked to prove $\angle XYW \cong \angle WYZ$.

Example 2.13.1

Write a two-column pro :

If A,B,C, and D are poir given order, and $AB = CD$, then $AC = BD$.

Solution

When the statement is given in this way, the "if" part is the given and the "then" part is what we are trying to prove.

Always start with drawing a picture of what you are given.

Plot the points in the order A,B,C,D on a line.

Figure 2.13.1

Add the given, $AB = CD$.

Figure 2.13.2

Draw the two-column proof and start with the given information.

Statement	*Reason*
1. A,B,C, and D are collinear, in that order.	1. Given
2. $AB = CD$	2. Given
3. $BC = BC$	3. Reflexive PoE
4. $AB + BC = BC + CD$	4. Addition PoE
5. $AB + BC = AC$ $BC + CD = BD$	5. Segment Addition Postulate
6. $AC = BD$	6. Substitution or Transitive PoE

Example 2.13.2

Write a two-column proof.

Given: $\overrightarrow{BF}$ bisects $\angle ABC$; $\angle ABD \cong \angle CBE$

Prove: $\angle DBF \cong \angle EBF$

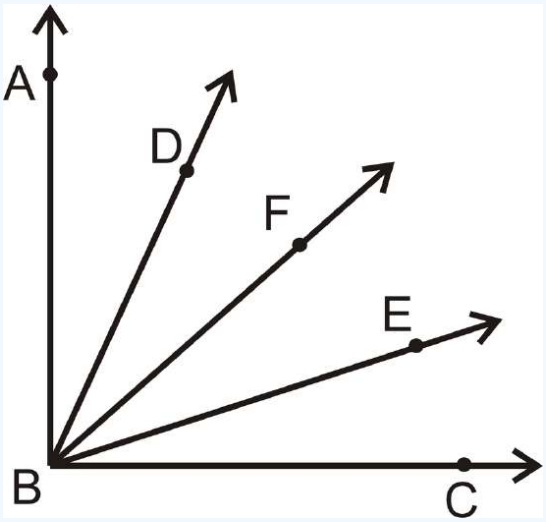

Figure \(\
PageIndex
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Solution

First, put the appropriate markings on the picture. Recall, that bisect means “to cut in half.” Therefore, $m\angle ABF = m\angle FBC$.

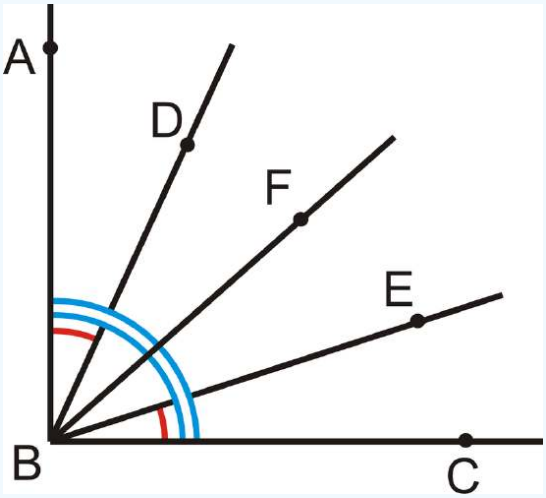

Figure **2.13.4**

Statement	*Reason*
1. $\overrightarrow{BF}$ bisects $\angle ABC$, $\angle ABD \cong \angle CBE$	1. Given
2. $m\angle ABF = m\angle FBC$	2. Definition of an Angle Bisector
3. $m\angle ABD = m\angle CBE$	3. If angles are ≅, then their measures are equal.
4. $m\angle ABF = m\angle ABD + m\angle DBF$ $m\angle FBC = m\angle EBF + m\angle CBE$	4. Angle Addition Postulate
5. $m\angle ABD + m\angle DBF = m\angle EBF + m\angle CBE$	5. Substitution PoE
6. $m\angle ABD + m\angle DBF = m\angle EBF + m\angle ABD$	6. Substitution PoE
7. $m\angle DBF = m\angle EBF$	7. Subtraction PoE
8. $\angle DBF \cong \angle EBF$	8. If measures are equal, the angles are \(\cong .

Example 2.13.3

The **Right Angle Theorem** states that if two angles are right angles, then the angles are congruent. Prove this theorem.

To prove this theorem, set up your own drawing and name some angles so that you have specific angles to talk about.

Given: $\angle A$ and $\angle B$ are right angles

Prove: $\angle A \cong \angle B$

Solution

Statement	*Reason*
1. $\angle A$ and $\angle B$ are right angles	1. Given
2. $m\angle A = 90°$ and $m\angle B = 90°$	2. Definition of right angles
3. $m\angle A = m\angle B$	3. Transitive PoE
4. $\angle A \cong \angle B$	4. ≅ angles have = measures

Any time right angles are mentioned in a proof, you will need to use this theorem to say the angles are congruent.

Example 2.13.4

The **Same Angle Supplements Theorem** states that if two angles are supplementary to the same angle then the two angles are congruent. Prove this theorem.

Given: $\angle A$ and \angle B are supplementary angles. $\angle B$ and $\angle C$ are supplementary angles.

Prove: $\angle A \cong \angle C$

Solution

Statement	*Reason*
1. $\angle A$ and $\angle B$ are supplementary $\angle B$ and $\angle C$ are supplementary	1. Given
2. \(m\angle A+m\angle B=180^{\circ} \(m\angle B+m\angle C=180^{\circ}	2. Definition of supplementary angles
3. $m\angle A + m\angle B = m\angle B + m\angle C$	3. Substitution PoE
4. $m\angle A = m\angle C$	4. Subtraction PoE
5. $\angle A \cong \angle C$	5. $\cong$ angles have = measures

Example 2.13.5

The **Vertical Angles Theorem** states that vertical angles are congruent. Prove this theorem.

Given: Lines k and m intersect.

Prove: $\angle 1 \cong \angle 3$

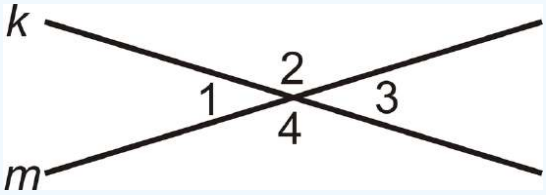

Figure 2.13.5

Solution

Statement	*Reason*
1. Lines k and m intersect	1. Given
2. $\angle 1$ and $\angle 2$ are a **linear pair** \angle 2 and $\angle 3$ are a linear pair	2. Definition of a Linear Pair
3. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	3. Linear Pair Postulate
4. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 2 + m\angle 3 = 180^\circ$	4. Definition of Supplementary Angles
5. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	5. Substitution PoE
6. $m\angle 1 = m\angle 3$	6. Subtraction PoE
7. $\angle 1 \cong \angle 3$	7. $\cong$ angles have = measures

Example 2.13.6

$\angle 1 \cong \angle 4$ and $\angle C$ and $\angle F$ are right angles.

Which angles are congruent and why?

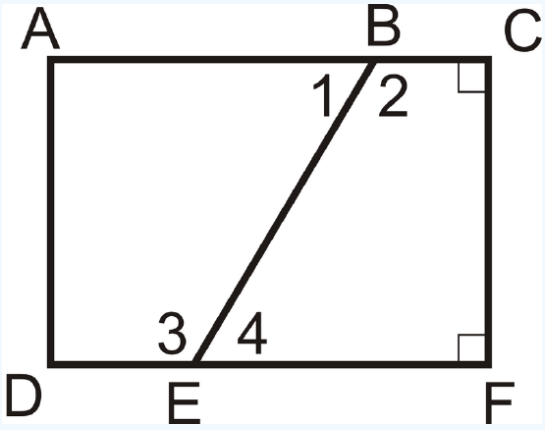

Figure 2.13.6

Solution

By the Right Angle Theorem, $\angle C \cong \angle F$. Also, $\angle 2 \cong \angle 3$ by the Same Angles Supplements Theorem because $\angle 1 \cong \angle 4$ and they are linear pairs with these congruent angles.

Review

Fill in the blanks in the proofs below.

1. Given: $\angle ABC \cong \angle DEF$ and $\angle GHI \cong \angle JKL$

Prove: $m\angle ABC + m\angle GHI = m\angle DEF + m\angle JKL$

Statement	*Reason*
1.	1. Given
2. $m\angle ABC = m\angle DEF$ $m\angle GHI = m\angle JKL$	2.
3.	3. Addition PoE
4. $m\angle ABC + m\angle GHI = m\angle DEF + m\angle JKL$	4.

2. Given: M is the midpoint of $\overline{AN}$. N is the midpoint $\overline{MB}$

Prove: $AM = NB$

Statement	*Reason*
1.	Given
2.	Definition of a midpoint
3. $AM = NB$	

3. Given: $\overline{AC} \perp \overline{BD}$ and $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

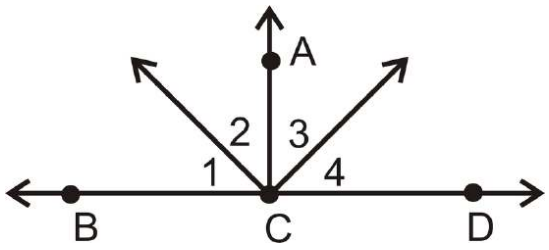

Figure **2.13.7**

Statement	*Reason*
1. \(\overline{AC}\perp \overline{BD}, \(\angle 1\cong \angle 4	1.
2. $m\angle 1 = m\angle 4$	2.
3.	3. \(\perp lines create right angles
4. $m\angle ACB = 90°$ $m\angle ACD = 90°$	4.
5. $m\angle 1 + m\angle 2 = m\angle ACB$ $m\angle 3 + m\angle 4 = m\angle ACD$	5.
6.	6. Substitution
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	7.
8.	8. Substitution
9.	9.Subtraction PoE
10. $\angle 2 \cong \angle 3$	10.

4. Given: $\angle MLN \cong \angle OLP$

Prove: $\angle MLO \cong \angle NLP$

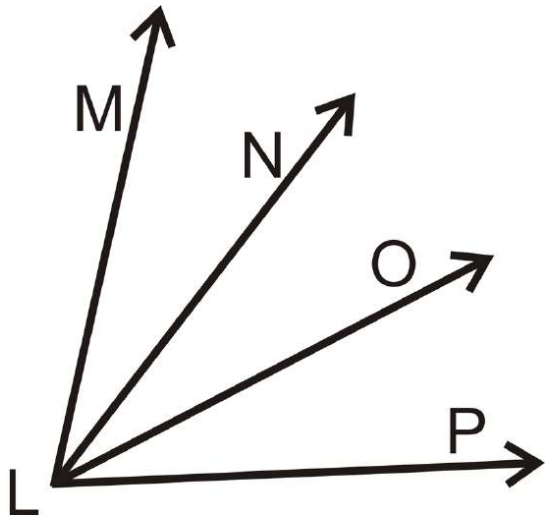

Figure **2.13.8**

Statement	*Reason*
1.	1.
2.	2. ≅ angles have = measures
3.	3. Angle Addition Postulate
4.	4. Substitution
5. $m\angle MLO = m\angle NLP$	5.
6.	6. ≅ angles have = measures

5. Given: $\underline{AE} \perp \underline{EC}$ and $\underline{BE} \perp \underline{ED}$

Prove: $\angle 1 \cong \angle 3$

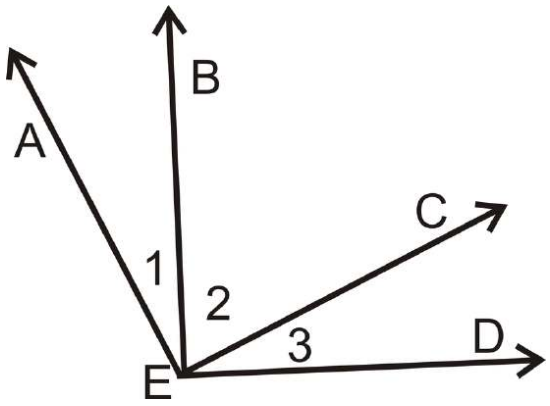

Figure **2.13.9**

Statement	*Reason*
1.	1.
2.	2. ⊥ lines create right angles
3. $m\angle BED = 90°$ $m\angle AEC = 90°$	3.
4.	4. Angle Addition Postulate
5.	5. Substitution
6. $m\angle 2 + m\angle 3 = m\angle 1 + m\angle 3$	6.
7.	7. Subtraction PoE
8.	8. ≅ angles have = measures

6. Given: $\angle L$ is supplementary to $\angle M$ and $\angle P$ is supplementary to $\angle O$ and $\angle L \cong \angle O$

Prove: $\angle P \cong \angle M$

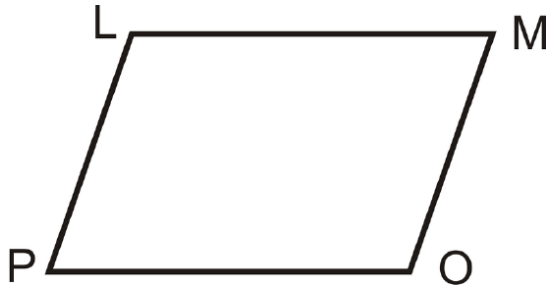

Figure **2.13.10**

Statement	*Reason*
1.	1.
2. $m\angle L = m\angle O$	2.
3.	3. Definition of supplementary angles
4.	4. Substitution
5.	5. Substitution
6.	6. Subtraction PoE
7. $\angle M \cong \angle P$	7.

7. Given: $\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 3$

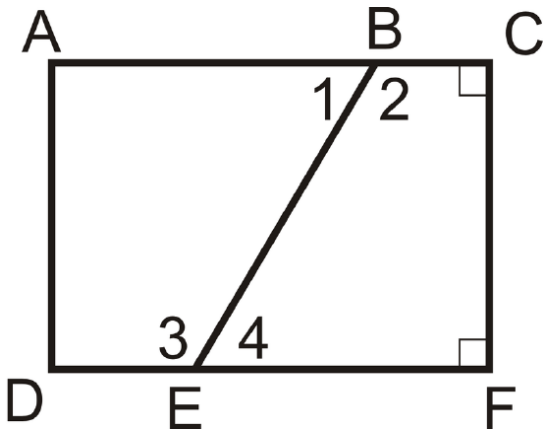

Figure **2.13.11**

Statement	*Reason*
1.	1.
2. $m\angle 1 = m\angle 4$	2.
3.	3. Definition of a Linear Pair
4. $\angle 1$ and $\angle 2$ are supplementary $\angle 3$ and $\angle 4$ are supplementary	4.
5.	5. Definition of supplementary angles
6. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	6.
7. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	7.
8. $m\angle 2 = m\angle 3$	8.
9. $\angle 2 \cong \angle 3$	9.

8. Given: $\angle C$ and $\angle F$ are right angles

Prove: $m\angle C + m\angle F = 180°$

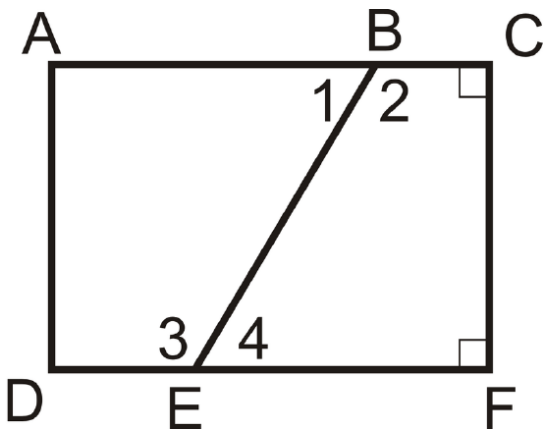

Figure **2.13.12**

Statement	*Reason*
1.	1.
2. $m\angle C = 90°, \(m\angle F = 90°$	2.
3. $90° + 90° = 180°$	3.
4. $m\angle C + m\angle F = 180°$	4.

9. Given: $l \perp m$

Prove: $\angle 1 \cong \angle 2$

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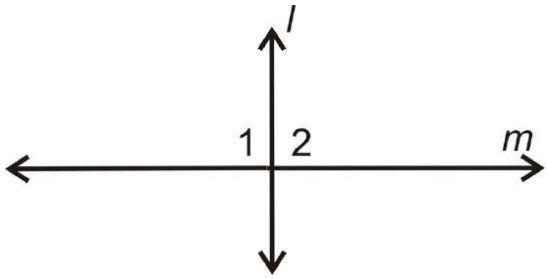

Figure **2.13.13**

Statement	*Reason*
1. $l \perp m$	1.
2. $\angle 1$ and $\angle 2$ are right angles	2.
3.	3.

10. Given: $m\angle 1 = 90°$

Prove: $m\angle 2 = 90°$

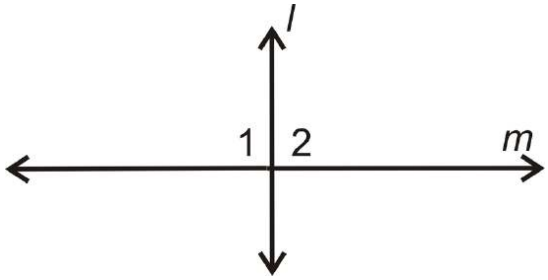

Figure **2.13.14**

Statement	*Reason*
1.	1.
2. $\angle 1$ and $\angle 2$ are a linear pair	2.
3.	3. Linear Pair Postulate
4.	4. Definition of supplementary angles
5.	5. Substitution
6. $m\angle 2 = 90°$	6.

11. Given: $l \perp m$

Prove: $\angle 1$ and $\angle 2$ are complements

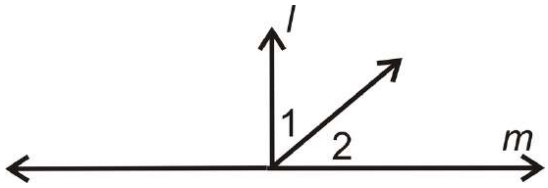

Figure **2.13.15**

Statement	*Reason*
1.	1.
2.	2. $\perp$ lines create right angles
3. $m\angle 1 + m\angle 2 = 90°$	3.
4. $\angle 1$ and $\angle 2$ are complementary	4.

12. Given: $l \perp m$ and $\angle 2 \cong \angle 6$

Prove: $\angle 6 \cong \angle 5$

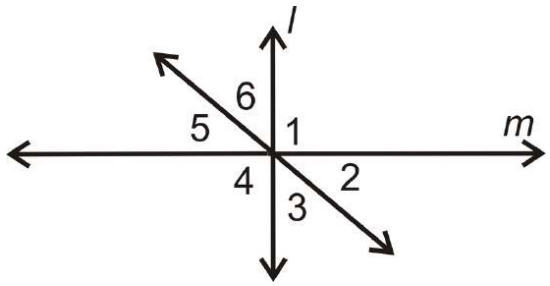

Figure 2.13.16

Statement	*Reason*
1.	1.
2. $m\angle 2 = m\angle 6$	2.
3. $\angle 5 \cong \angle 2$	3.
4. $m\angle 5 = m\angle 2$	4.
5. $m\angle 5 = m\angle 6$	5.

Review (Answers)

To see the Review answers, open this PDF file and look for section 2.7.

Vocabulary

Term	Definition
two column proof	A common way to organize a proof in geometry. Two column proofs always have two columns- statements and reasons.
linear pair	Two angles form a linear pair if they are supplementary and adjacent.

Additional Resources

Video: Two Column Proofs Principles - Basic

Activities: Two-Column Proofs Discussion Questions

Study Aids: Proofs Study Guide

Practice: Introduction to Proofs

Real World: Give Me One Reason

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2.14: Properties of Equality and Congruence

Logical rules involving equality and congruence that allow equations to be manipulated and solved.

Properties of Equality and Congruence

The basic properties of equality were introduced to you in Algebra I. Here they are again:

- **Reflexive Property of Equality**: $AB = AB$
- **Symmetric Property of Equality**: If $m\angle A = m\angle B$, then $m\angle B = m\angle A$
- **Transitive Property of Equality**: If $AB = CD$ and $CD = EF$, then $AB = EF$
- **Substitution Property of Equality**: If $a = 9$ and $a - c = 5$, then $9 - c = 5$
- **Addition Property of Equality**: If $2x = 6$, then $2x + 5 = 6 + 5$ or $2x + 5 = 11$
- **Subtraction Property of Equality**: If $m\angle x + 15° = 65°$, then $m\angle x + 15° - 15° = 65° - 15°$ or $m\angle x = 50°$
- **Multiplication Property of Equality**: If $y = 8$, then $5 \cdot y = 5 \cdot 8$ or $5y = 40$
- **Division Property of Equality**: If $3b = 18$, then $\frac{3b}{3} = \frac{18}{3}$ or $b = 6$
- **Distributive Property**: $5(2x - 7) = 5(2x) - 5(7) = 10x - 35$

Just like the properties of equality, there are properties of congruence. These properties hold for figures and shapes.

- **Reflexive Property of Congruence**: $\overline{AB} \cong \overline{AB}$ or $\angle B \cong \angle B$
- **Symmetric Property of Congruence**: If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. Or, if $\angle ABC \cong \angle DEF$, then $\angle DEF \cong \angle ABC$
- **Transitive Property of Congruence**: If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. Or, if $\angle ABC \cong \angle DEF$ and $\angle DEF \cong \angle GHI$, then $\angle ABC \cong \angle GHI$

When you solve equations in algebra you use properties of equality. You might not write out the property for each step, but you should know that there is an equality property that justifies that step. We will abbreviate "Property of Equality" "*PoE*" and "Property of Congruence" "*PoC*" when we use these properties in proofs.

Suppose you know that a circle measures 360 degrees and you want to find what kind of angle one-quarter of a circle is.

For Examples 1 and 2, use of equality to fill in the blank. x and y are real numbers.

Example 2.14.1

Distributive: If $4(3x - 8)$ ____.

Solution

$12x - 32$

Example 2.14.2

Transitive: If $y = 12$ and $x = y$, then _____

Solution

$x = 12$

Example 2.14.3

Solve $2(3x - 4) + 11 = x - 27$ and write the property for each step (also called "to justify each step").

Solution

$$
\begin{aligned}
2(3x-4)+11 &= x-27 & \\
6x-8+11 &= x-27 & \textit{Distributive Property} \\
6x+3 &= x-27 & \textit{Combine like terms} \\
6x+3-3 &= x-27-3 & \textit{Subtraction PoE} \\
6x &= x-30 & \textit{Simplify} \\
6x-x &= x-x-30 & \textit{Subtraction PoE} \\
5x &= -30 & \textit{Simplify} \\
\frac{5x}{5} &= \frac{-30}{5} & \textit{Division PoE} \\
x &= -6 & \textit{Simplify}
\end{aligned}
$$

Example 2.14.4

$AB = 8$, $BC = 17$, and $AC = 20$. Are points A, B, and C collinear?

Solution

Set up an equation using the Segment Addition Postulate.

$$
\begin{aligned}
AB + BC &= AC & \textit{Segment Addition Postulate} \\
8 + 17 &= 20 & \textit{Substitution PoE} \\
25 &\neq 20 & \textit{Combine like terms}
\end{aligned}
$$

Because the two sides of the equation are not equal, A, B, and $C$are not collinear.

Example 2.14.5

If $m\angle A + m\angle B = 100^\circ$ and $m\angle B = 40^\circ$, prove that $m\angle A$ is an acute angle.

Solution

We will use a 2-column format, with statements in one column and their reasons next to it, just like Example A.

$$
\begin{aligned}
m\angle A + m\angle B &= 100^\circ & \textit{Given Information} \\
m\angle B &= 40^\circ & \textit{Given Information} \\
m\angle A + 40^\circ &= 100^\circ & \textit{Substitution PoE} \\
m\angle A &= 60^\circ & \textit{Subtraction PoE} \\
\angle A \textit{ is an acute angle} & & \textit{Definition of an acute angle}, m\angle A < 90^\circ
\end{aligned}
$$

Review

For questions 1-8, solve each equation and justify each step.

1. $3x + 11 = -16$
2. $7x - 3 = 3x - 35$
3. $\frac{2}{3}g + 1 = 19$
4. $\frac{1}{2}MN = 5$
5. $5m\angle ABC = 540^\circ$
6. $10b - 2(b + 3) = 5b$
7. $\frac{1}{4}y + \frac{5}{6} = \frac{1}{3}$
8. $\frac{1}{4}AB + \frac{1}{3}AB = 12 + \frac{1}{2}AB$

For questions 9-11, use the given property or properties of equality to fill in the blank. x, y, and z are real numbers.

9. Symmetric: If $x + y = y + z$, then _____.
10. Transitive: If $AB = 5$ and $AB = CD$, then _____.
11. Substitution: If $x = y - 7$ and $x = z + 4$, then _____.

Review (Answers)

To see the Review answers, open this PDF file and look for section 2.6.

Resources

Vocabulary

Term	Definition
properties of equality	Together with ***properties of congruence***, the logical rules that allow equations to be manipulated and solved.
Addition Property of Inequality	You can add a quantity to both sides of an inequality and it does not change the sense of the inequality. If $x > 3$, then $x + 2 > 3 + 2$.
distributive property	The distributive property states that the product of an expression and a sum is equal to the sum of the products of the expression and each term in the sum. For example, $a(b + c) = ab + ac$.
Division Property of Inequality	The division property of inequality states that two unequal values divided by a positive number retain the same relationship. Two unequal values divided by a negative number result in a reversal of the relationship.
Multiplication Property of Equality	The multiplication property of equality states that if the same constant is multiplied to both sides of the equation, the equality holds true.
Real Number	A real number is a number that can be plotted on a number line. Real numbers include all rational and irrational numbers.
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$ or $\angle B \cong \angle B$
Reflexive Property of Equality	Any algebraic or geometric item is equal in value to itself.
Right Angle Theorem	The Right Angle Theorem states that if two angles are right angles, then the angles are congruent.
Same Angle Supplements Theorem	The Same Angle Supplements Theorem states that if two angles are supplementary to the same angle then the two angles are congruent.
Substitution Property of Equality	If a variable is equal to a specified amount, that amount can be directly substituted into an equation for the given variable.
Subtraction Property of Equality	The subtraction property of equality states that you can subtract the same quantity from both sides of an equation and it will still balance.
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. Or, if $\angle ABC \cong \angle DEF$, then $\angle DEF \cong \angle ABC$
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. Or, if $\angle ABC \cong \angle DEF$ and $\angle DEF \cong \angle GHI$, then $\angle ABC \cong \angle GHI$
Transitive Property of Equality	If $a = 5$, and $b = 5$, then $a = b$.
Vertical Angles Theorem	The Vertical Angles Theorem states that if two angles are vertical, then they are congruent.

Additional Resources

Video: Properties of Equality and Congruence Principles - Basic

Activities: Properties of Equality and Congruence Discussion Questions

Study Aids: Proofs Study Guide

Practice: Properties of Equality and Congruence

Real World: Properties Of Equality And Congruence

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2.15: Proofs- Angle Pairs and Segments

Vertical angles, perpendicular bisectors, and other theorems based on intersecting lines or parallel lines and a transversal.

Theorems about Lines and Angles

Consider line l and a point P that is not on line l. How many lines exist that are parallel to l and pass through point P?

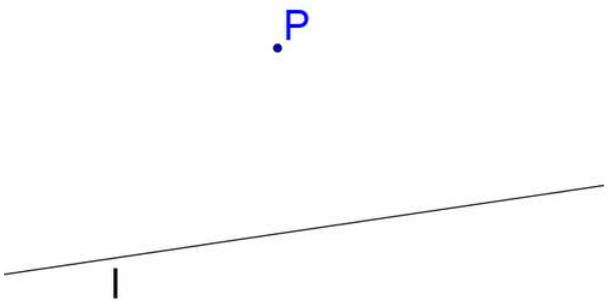


Figure 2.15.1

Line and Angle Theorems

Consider two parallel lines that are intersected by a third line. (*Remember that tick marks (≫) can be used to indicate that two lines are parallel.)*

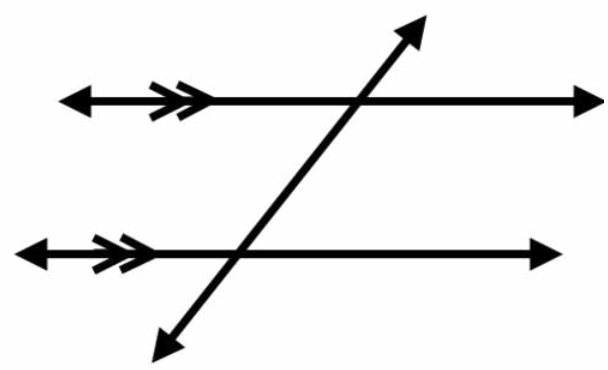


Figure 2.15.2

This third line is called a **transversal**. Note that four angles are created where the transversal intersects each line. Each angle created by the transversal and the top line has a **corresponding angle** with an angle create by the transversal and the bottom line. These corresponding angle pairs are shown color-coded below. How do you think these **corresponding angles** are related?

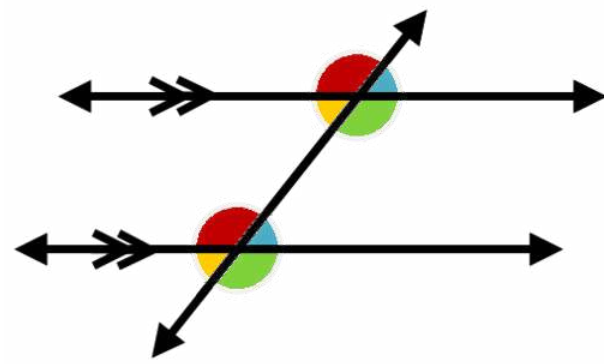


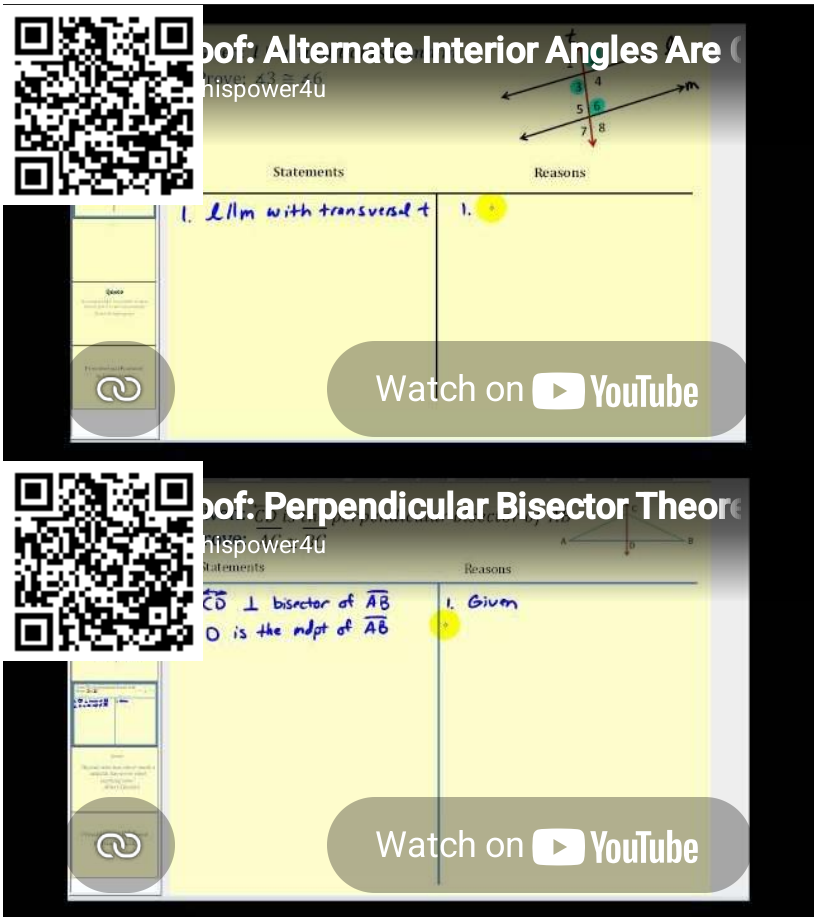
Figure 2.15.3

Your intuition and knowledge of translations might suggest that these angles are congruent. Imagine translating one of the angles along the transversal until it meets the second parallel line. It will match its corresponding angle exactly. This is known as the corresponding angle **postulate**:

If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

Remember that a postulate is a statement that is accepted as true without proof. Your knowledge of translations should convince you that this postulate is true.

Resources



Let's take a look at some example problems.

1. Recall that vertical angles are a pair of opposite angles created by intersecting lines. Prove that **vertical angles are congruent.**

For this proof, you are not given a specific picture. When not given a picture, it helps to create a generic picture to reference in your proof. It's important that the picture does not include any information that you cannot assume. Below is a generic picture of intersecting lines with angles numbered for reference.

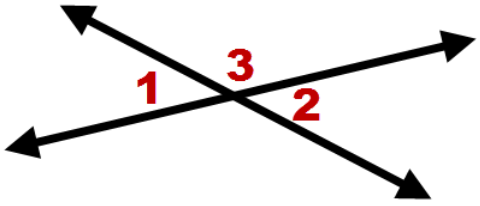


Figure **2.15.4**

In this picture, $\angle 1$ and $\angle 2$ are vertical angles. Your job is to prove that $\angle 1 \cong \angle 2$. You can use any style of proof you prefer. Here is a two-column proof.

Statements	Reasons
$m\angle 1 + m\angle 3 = 180°$ and $m\angle 2 + m\angle 3 = 180°$	Two angles that form a line are supplementary
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	Algebraic substitution
$m\angle 1 = m\angle 2$	Subtraction property of equality
$\angle 1 \cong \angle 2$	If two angles have the same measure, they are congruent.

Vertical angles are congruent is a **theorem**. Now that it has been proven, you can use it in future proofs without proving it again.

2. When two parallel lines are cut by a transversal, two pairs of **alternate interior angles** are formed. In the diagram below, $\angle 3$ and $\angle 5$ are **alternate interior angles**. Similarly, $\angle 4$ and $\angle 6$ are alternate interior angles.

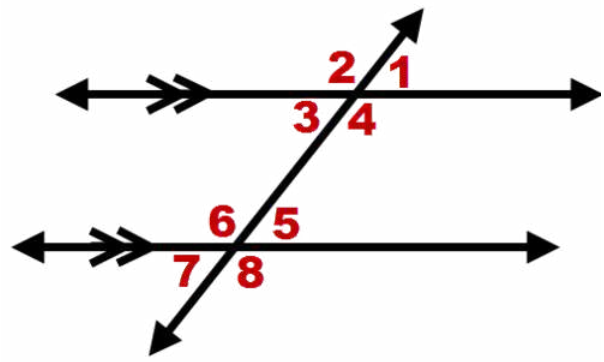


Figure 2.15.5

Prove that **if two parallel lines are cut by a transversal, alternate interior angles are congruent.**

Use the diagram above, and prove that $\angle 3 \cong \angle 5$. (The same exact proof would show that $\angle 4 \cong \angle 6$). Again, in general you can use any style of proof that you prefer. Here is a paragraph proof.

$\angle 1 \cong \angle 3$ because they are vertical angles and vertical angles are always congruent. $\angle 1 \cong \angle 5$ because they are corresponding angles created by parallel lines and corresponding angles are congruent when lines are parallel. $\angle 3 \cong \angle 5$ because if two angles are congruent to the same angle, they are congruent to each other by the transitive property.

*Note: The transitive property states that if two objects are equal/congruent to the same third object, then they are equal/congruent to each other. The transitive property is a form of substitution. You can use it in any proof.

3. The statement "*if two parallel lines are cut by a transversal, then alternate interior angles are congruent*" is a **theorem.** Now that it has been proven, you can use it in future proofs without proving it again.

Prove that **points on the perpendicular bisector of a line segment are equidistant from the endpoints of the line segment**.

Start by exploring this claim. Draw a picture of a line segment and a perpendicular bisector of this segment. Remember that a perpendicular bisector is **perpendicular** to the line segment (meets it at a right angle) and **bisects** the line segment (cuts it in half).

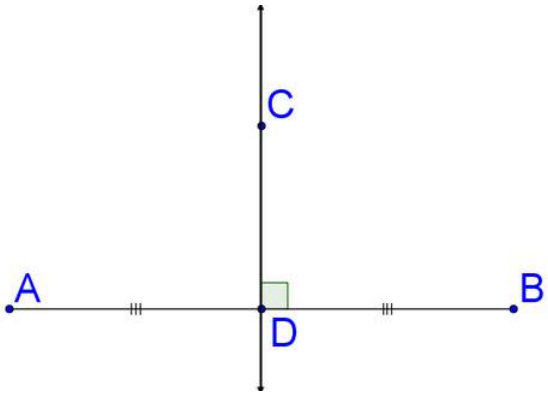


Figure **2.15.6**

The claim is that any point on the perpendicular bisector (such as point C), is the same distance away from each endpoint. In other words, the claim is that for some generic point C on the perpendicular bisector, $\overline{AC} \cong \overline{BC}$.

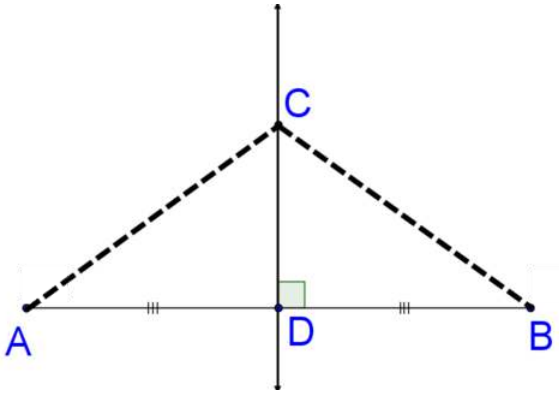


Figure **2.15.7**

To prove the original statement, it will suffice to prove that if $\overleftrightarrow{CD}$ is the perpendicular bisector to $\overline{AB}$ with D on $\overline{AB}$, then $\overline{AC} \cong \overline{BC}$ using the above diagram as a reference. Remember, in general you can use any style of proof that you prefer. Here is a flow diagram. *To prove this statement, you will show that the two triangles are congruent and then that the $\overline{AC}$ and $\overline{BC}$ are corresponding parts so must be congruent.*

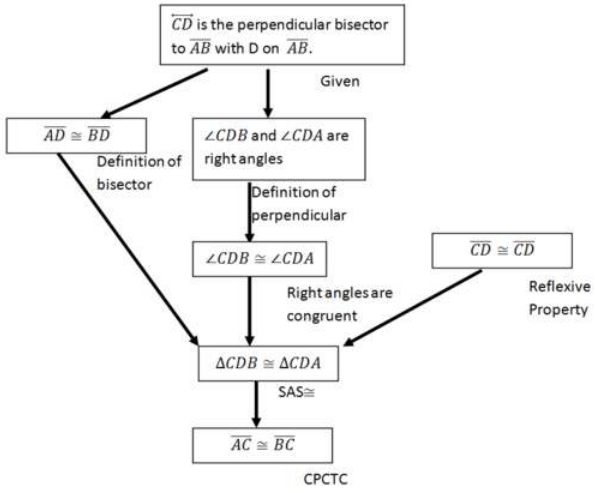


Figure **2.15.8**

The statement "*points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints*" is a **theorem**. Now that it has been proven, you can use it in future proofs without proving it again.

Example 2.15.1

Earlier, you were asked how many lines exist that are parallel to l and pass through point P.

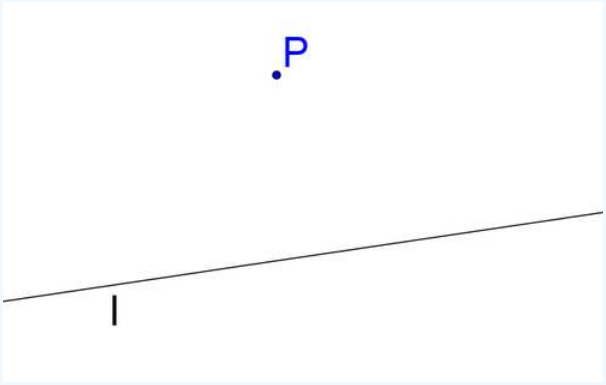


Figure 2.15.9

Solution

Common sense should tell you that there is only **one** line through P that is parallel to l.

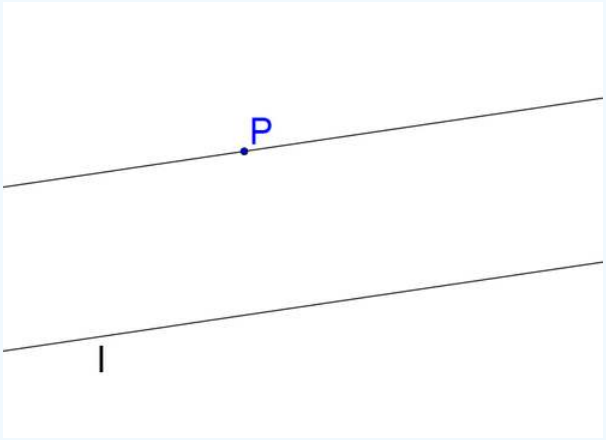


Figure 2.15.10

Interestingly, for hundreds of years people have attempted to **prove** this statement from simpler statements with no luck. Eventually, it was accepted that it simply must be a **postulate**, a statement that is assumed to be true without proof. This is known as the **parallel postulate.**

Example 2.15.2

If corresponding angles are congruent, then are lines parallel?

Solution

This is known as the **converse** of the corresponding angles postulate. The original postulate said:

Original: If **lines are parallel**, then **corresponding angles are congruent.**

Here, the "if" part of the statement (known as the hypothesis) is switched with the "then" part of the statement (known as the conclusion).

Converse: If **corresponding angles are congruent**, then **lines are parallel.**

In general, just because a statement is true doesn't necessarily mean its converse is true. In this case, the converse does happen to be true. The only way for corresponding angles to be congruent is for the lines to be parallel. The **corresponding angles converse** is also a **postulate**, which means it is accepted as true without proof.

Example 2.15.3

Prove that if lines are parallel, then same side interior angles (such as $\angle 3$ and $\angle 6$) are supplementary.

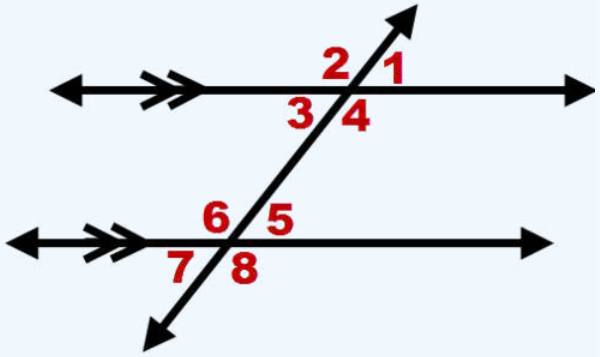


Figure 2.15.11

Solution

In general you can use any style of proof that you prefer. Here, use a paragraph proof.

$\angle 3 \cong \angle 5$ because they are alternate interior angles created by parallel lines and alternate interior angles are congruent when lines are parallel. $m\angle 3 = m\angle 5$ because congruent angles have the same measure. $m\angle 5 + m\angle 6 = 180°$ because two angles that form a line are supplementary. By substitution, $m\angle 3 + m\angle 6 = 180°$. $\angle 3$ and $\angle 6$ are supplementary because two angles with measures that add to $180°$ are supplementary.

The statement "*if two parallel lines are cut by a transversal, then same side interior angles are supplementary*" is a **theorem.** Now that it has been proven, you can use it in future proofs without proving it again.

Example 2.15.4

Prove that if alternate interior angles are congruent, then lines are parallel.

Solution

This is the converse of the alternate interior angles theorem.

Original: If **lines are parallel**, then **alternate interior angles are congruent.**

Here, the "if" part of the statement (known as the hypothesis) is switched with the "then" part of the statement (known as the conclusion).

Converse: If **alternate interior angles are congruent**, then **lines are parallel.**

To prove this statement, start with a picture of alternate interior angles that are assumed to be congruent, but don't assume the lines are parallel. In the picture below, assume $\angle 1 \cong \angle 2$. Prove that $m \parallel n$ (two parallel bars indicate parallel lines).

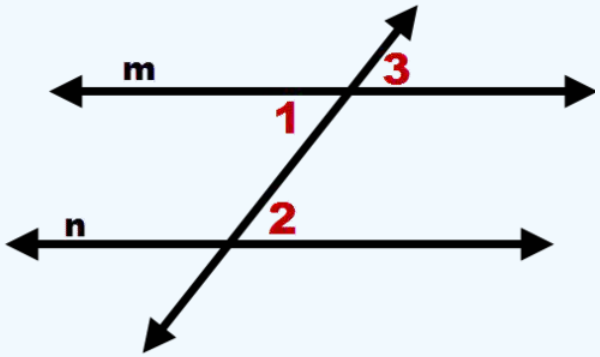


Figure 2.15.12

Statements	Reasons
$\angle 1 \cong \angle 2$	Given
$\angle 1 \cong \angle 3$	Vertical angles are congruent
$\angle 2 \cong \angle 3$	Transitive property of congruence
$m \parallel n$	If corresponding angles are congruent then lines are parallel.

Review

1. In problem 1 from the Line and Angle Theorems section above, the theorem “vertical angles are congruent” was proved with a two-column proof. Rewrite this proof in a paragraph format.

2. In problem 2 from the Line and Angle Theorems section above, the theorem “if lines are parallel then alternate interior angles are congruent” was proved with a paragraph proof. Rewrite this proof with a flow diagram.

3. In problem 3 from the Line and Angle Theorems section above, the theorem “points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints” was proved with a flow diagram. Rewrite this proof in a two-column format.

4. In Example 3, the theorem “if lines are parallel then same side interior angles are supplementary” was proved with a paragraph proof. Rewrite this proof in a two-column format.

5. In Example 4, the theorem “if alternate interior angles are congruent then lines are parallel” was proved with a two-column proof. Rewrite this proof with a flow diagram.

6. Alternate exterior angles are outside a pair of lines and on opposite sides of a transversal. $\angle 2$ and $\angle 8$ are an example of alternate exterior angles. \angle 1 and \angle 7 are another example of alternate exterior angles.

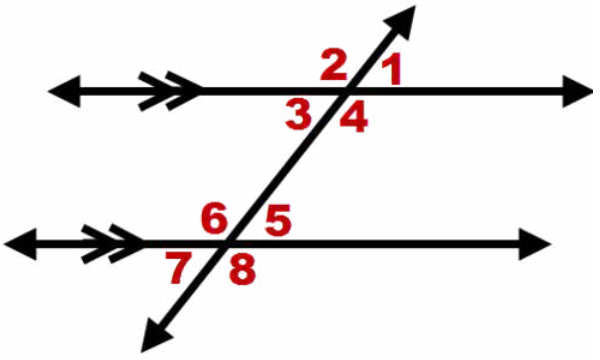


Figure 2.15.13

The theorem "**if lines are parallel then alternate exterior angles are congruent**" is partially proved below. Fill in the blanks to complete the proof. *Note, the angles referenced are from the above picture.*

Statements	Reasons
Two parallel lines are cut by a transversal	_____
$\angle 2 \cong \angle 6$	_____
$\angle 6 \cong$ ___	Vertical angles are congruent.
_____	Transitive property.

7. What is the **converse** of the theorem "if lines are parallel, then same side interior angles are supplementary"?

8. Prove the converse that you wrote in #7. Use any style of proof you prefer.

9. In problem 3 from the Line and Angle Theorems section above, the theorem "**points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints**" was proved. This theorem could be rewritten as "**if a point is on the perpendicular bisector of a line segment, then the point is equidistant from the endpoints of the line segment**". What is the **converse** of this theorem? *To check your answer, look at #10.*

10. The converse of the theorem in #9 is "**if a point is equidistant from the endpoints of a line segment, then the point is on the perpendicular bisector of the line segment**". To prove this new theorem you can use the picture below.

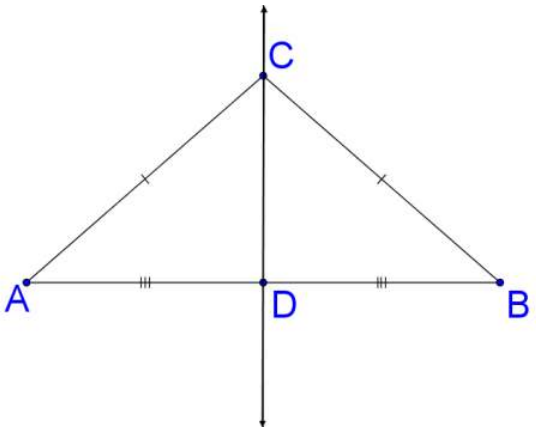


Figure **2.15.14**

Assume point C is a random point that is equidistant from endpoints A and B. Point D is the midpoint of line segment $\overline{AB}$. Your goal is to show that $\overline{CD}$ must be ***perpendicular to*** $\overline{AB}$ ($\overline{CD} \perp \overline{AB}$). This new theorem is partially proved below. Fill in the blanks to complete the proof.

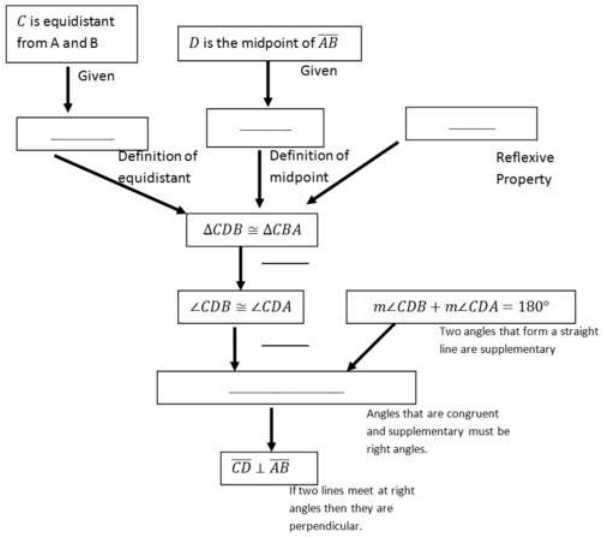


Figure **2.15.15**

In 11-13, you will prove that **if two angles are complementary to the same angle, then the two angles are congruent.**

11. Draw a generic picture of this situation and label the three angles.

12. What are the "givens" from your picture? What are you trying to prove?

13. Write a proof of the statement using whatever proof style you prefer.

14. Using your work from 11-13 to help, prove that **if two angles are supplementary to the same angle, then the two angles are congruent.**

15. Give at least 3 methods for proving that lines are parallel.

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.2.

Vocabulary

Term	Definition
alternate interior angles	Alternate interior angles are two angles that are on the interior of two different lines, but on the opposite sides of the transversal.
Corresponding Angles	Corresponding angles are two angles that are in the same position with respect to the transversal, but on different lines.
perpendicular bisector	A perpendicular bisector of a line segment passes through the midpoint of the line segment and intersects the line segment at $90°$.
postulate	A postulate is a statement that is accepted as true without proof.
transversal	A transversal is a line that intersects two other lines.

Additional Resources

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2.16: Parallelogram Proofs

Apply theorems to show if a quadrilateral has two pairs of parallel sides.

Quadrilaterals that are Parallelograms

Recall that a **parallelogram** is a quadrilateral with two pairs of parallel sides. Even if a quadrilateral is not marked with having two pairs of sides, it still might be a parallelogram. The following is a list of theorems that will help you decide if a quadrilateral is a parallelogram or not.

1. **Opposite Sides Theorem Converse:** If both pairs of opposite sides of a quadrilateral are congruent, then the figure is a parallelogram.

If

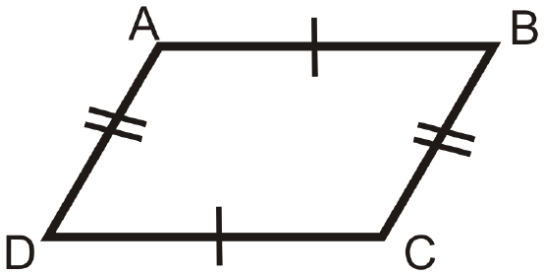

Figure **2.16.1**

then

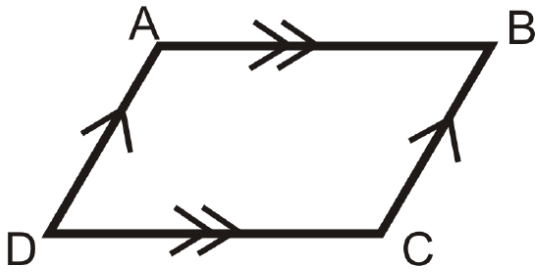

Figure **2.16.2**

2. **Opposite Angles Theorem Converse:** If both pairs of opposite angles of a quadrilateral are congruent, then the figure is a parallelogram.

If

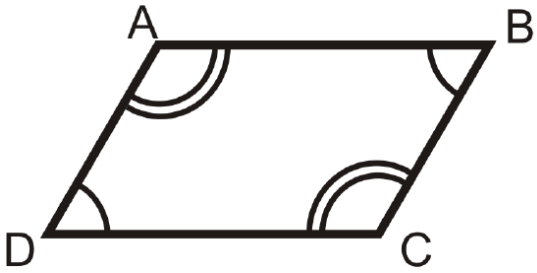

Figure **2.16.3**

then

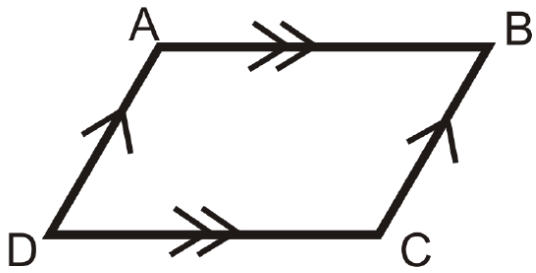

Figure **2.16.4**

3. **Parallelogram Diagonals Theorem Converse:** If the diagonals of a quadrilateral bisect each other, then the figure is a parallelogram.

If

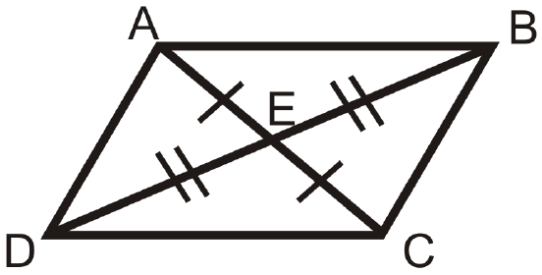

Figure **2.16.5**

then

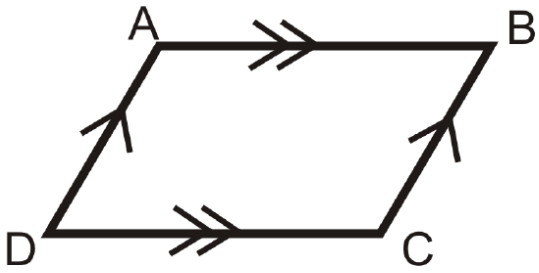

Figure **2.16.6**

4. **Parallel Congruent Sides Theorem:** If a quadrilateral has one set of parallel lines that are also congruent, then it is a parallelogram.

If

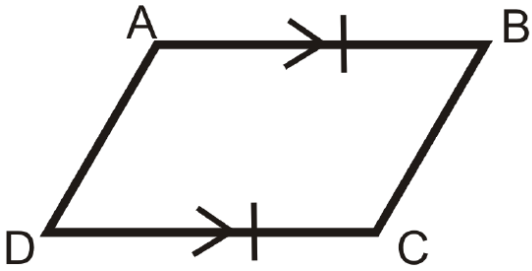

Figure **2.16.7**

then

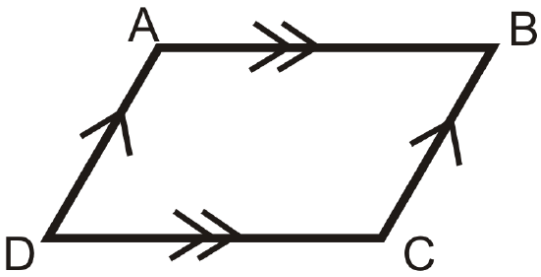

Figure **2.16.8**

You can use any of the above theorems to help show that a quadrilateral is a parallelogram. If you are working in the x−y plane, you might need to know the formulas shown below to help you use the theorems.

- The Slope Formula, $\frac{y_2 - y_1}{x_2 - x_1}$. (Remember that if slopes are the same then lines are parallel).
- The Distance Formula, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. (This will help you to show that two sides are congruent).
- The Midpoint Formula, $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$. (If the midpoints of the diagonals are the same then the diagonals bisect each other).

What if you were given four pairs of coordinates that form a quadrilateral? How could you determine if that quadrilateral is a parallelogram?

Example 2.16.1

Prove the Parallel Cong n.

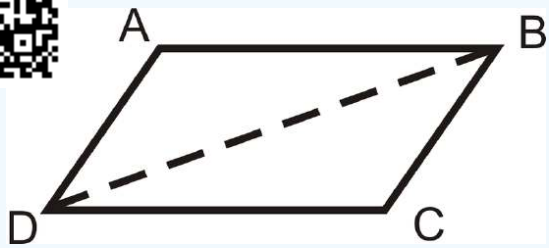

Figure **2.16.9**

Given: $\overline{AB} \parallel \overline{DC}$, and $\overline{AB} \cong \overline{DC}$

Prove: $ABCD$ is a parallelogram

Solution

Statement	*Reason*
1. $\overline{AB} \parallel \overline{DC}$, and $\overline{AB} \cong \overline{DC}$	1. Given
2. $\angle ABD \cong \angle BDC$	2. Alternate Interior Angles
3. $\overline{DB} \cong \overline{DB}$	3. Reflexive *PoC*
4. $\Delta ABD \cong \Delta CDB$	4. SAS
5. $\overline{AD} \cong \overline{BC}$\)	5. CPCTC
6. $ABCD$ is a parallelogram	6. Opposite Sides Converse

Example 2.16.2

What value of x would make $ABCD$ a parallelogram?

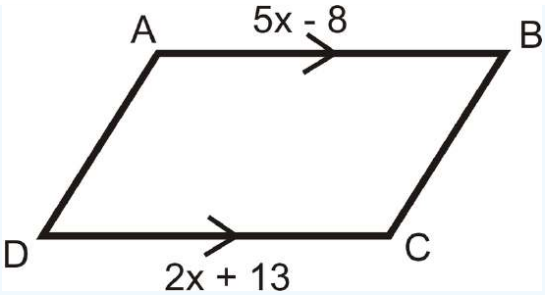

Figure **2.16.10**

Solution

$\overline{AB} \parallel \overline{DC}$. By the Parallel Congruent Sides Theorem, $ABCD$ would be a parallelogram if $AB = DC$.

$$5x - 8 = 2x + 13$$
$$3x = 21$$
$$x = 7$$

Example 2.16.3

Prove the Opposite Sides Theorem Converse.

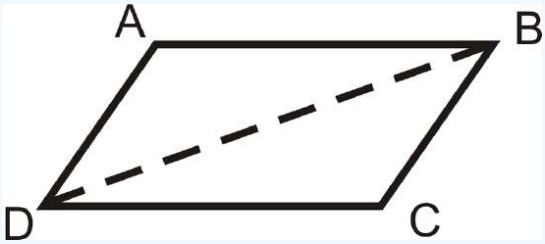

Figure **2.16.11**

Given: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

Prove: $ABCD$ is a parallelogram

Solution

Statement	*Reason*
1. $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$	1.Given
2. $\overline{DB} \cong \overline{DB}$	2. Reflexive *PoC*
3. $\Delta ABD \cong \Delta CDB$	3. SSS
4. $\angle ABD \cong \angle BDC, \angle ADB \cong \angle DBC$	4. *CPCTC*
5. $\overline{AB} \parallel \overline{DC}, \overline{AD} \parallel \overline{BC}$	5. Alternate Interior Angles Converse
6. $ABCD$ is a parallelogram	6. Definition of a parallelogram

Example 2.16.4

Is quadrilateral $EFGH$ a parallelogram? How do you know?

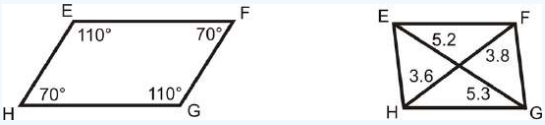

Figure **2.16.12**

Solution

By the Opposite Angles Theorem Converse, $EFGH$ is a parallelogram.

$EFGH$ is not a parallelogram because the diagonals do not bisect each other.

Example 2.16.5

Is the quadrilateral $ABCD$ a parallelogram?

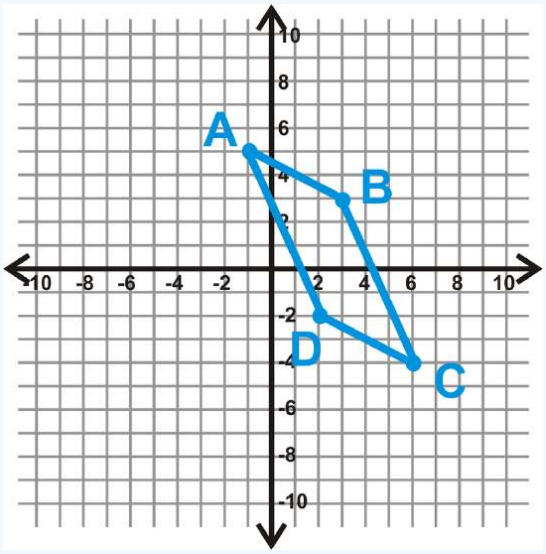

Figure **2.16.13**

Solution

Let's use the Parallel Congruent Sides Theorem to see if $ABCD$ is a parallelogram. First, find the length of AB and CD using the distance formula.

$$AB = \sqrt{(-1-3)^2+(5-3)^2} = \sqrt{(-4)^2+2^2} = \sqrt{16+4} = \sqrt{20}$$

$$CD = \sqrt{(2-6)^2+(-2+4)^2} = \sqrt{(-4)^2+2^2} = \sqrt{16+4} = \sqrt{20}$$

Next find the slopes to check if the lines are parallel.

$$\text{Slope } AB = \frac{5-3}{-1-3} = \frac{2}{-4} = -\frac{1}{2} \qquad \text{Slope } CD = \frac{-2+4}{2-6} = \frac{2}{-4} = -\frac{1}{2}$$

$AB = CD$ and the slopes are the same (implying that the lines are parallel), so $ABCD$ is a parallelogram.

Review

For questions 1-12, determine if the quadrilaterals are parallelograms.

1.

Figure **2.16.14**

2.

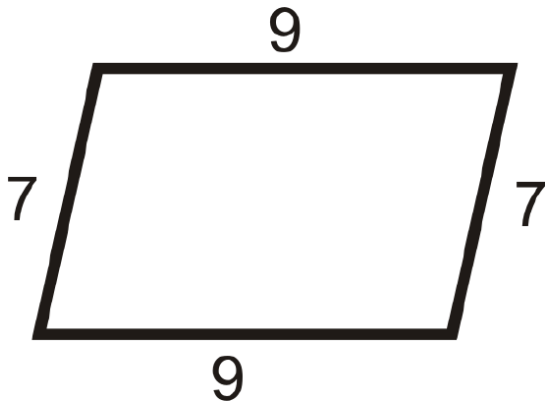

Figure **2.16.15**

3.

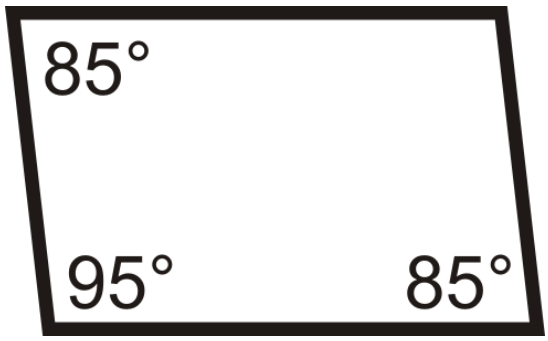

Figure **2.16.16**

4.

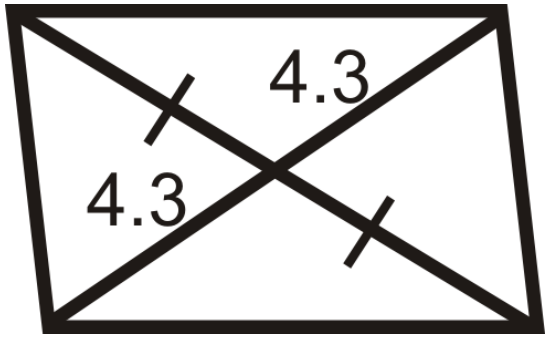

Figure **2.16.17**

5.

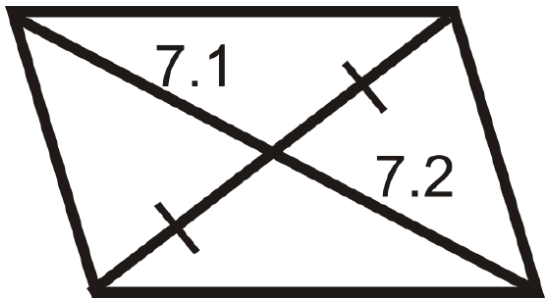

Figure **2.16.18**

6.

Figure **2.16.19**

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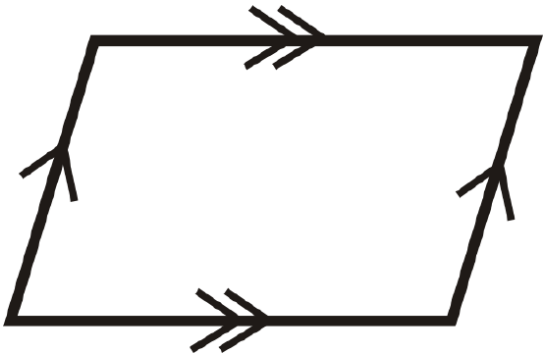

Figure **2.16.20**

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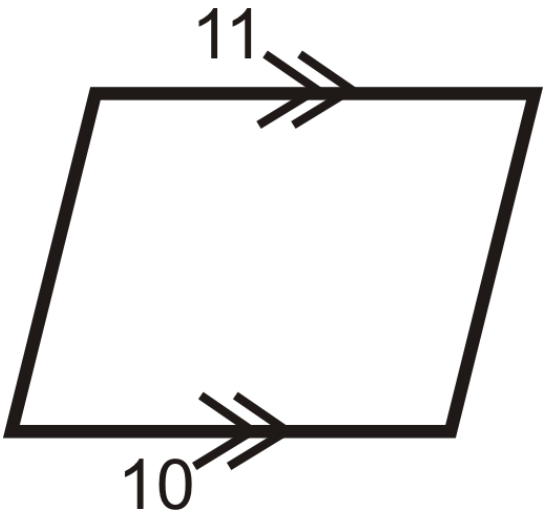

Figure **2.16.21**

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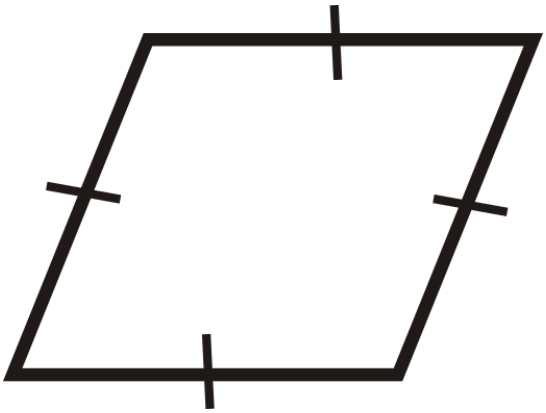

Figure **2.16.22**

10.

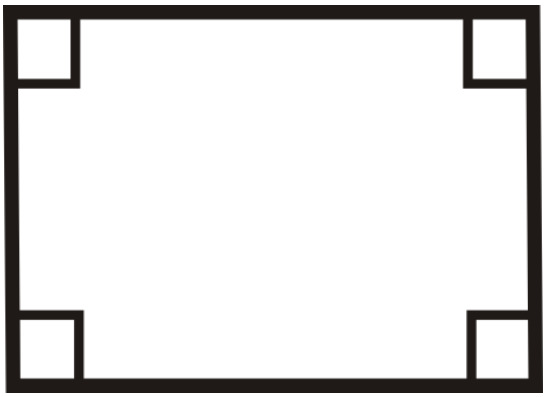

Figure **2.16.23**

11.

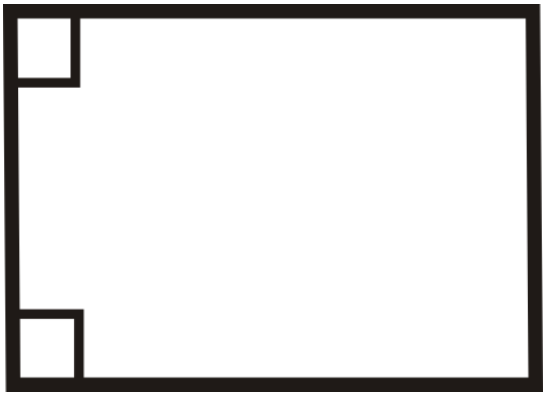

Figure **2.16.24**

12.

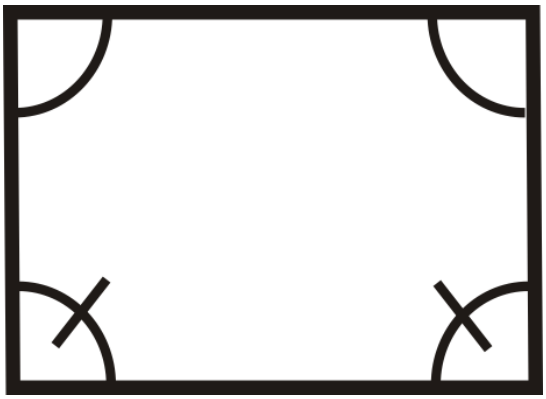

Figure **2.16.25**

For questions 13-18, determine the value of x and y that would make the quadrilateral a parallelogram.

13.

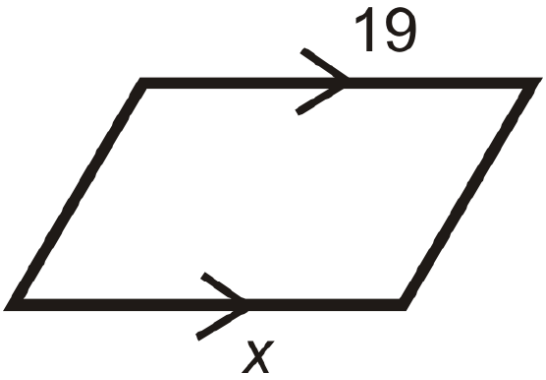

Figure **2.16.26**

14.

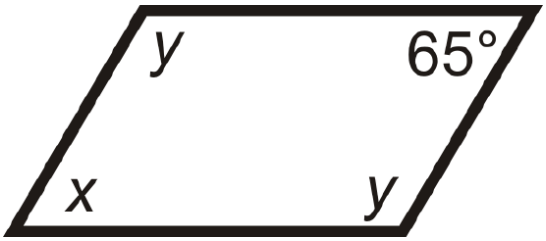

Figure **2.16.27**

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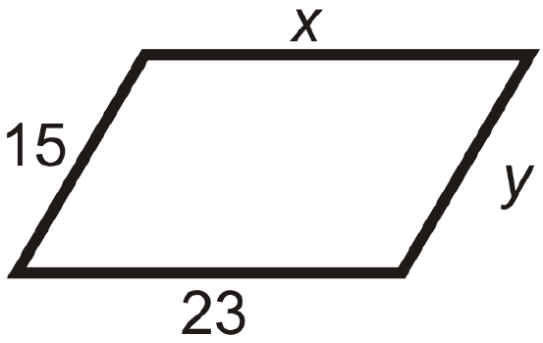

Figure **2.16.28**

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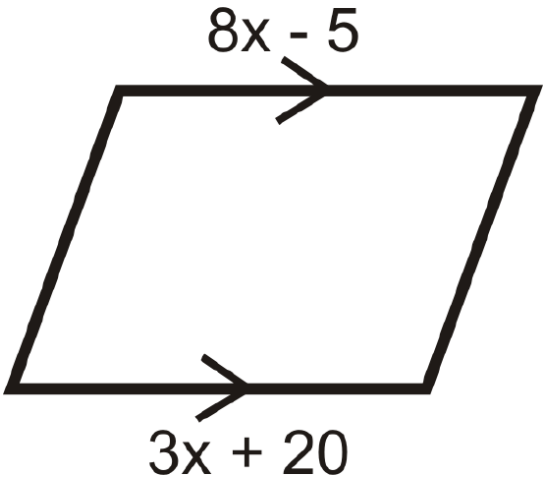

Figure **2.16.29**

17.

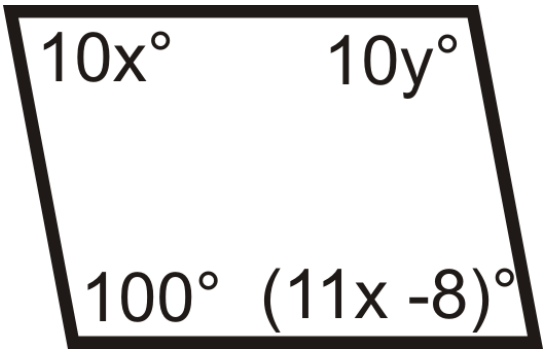

Figure **2.16.30**

18.

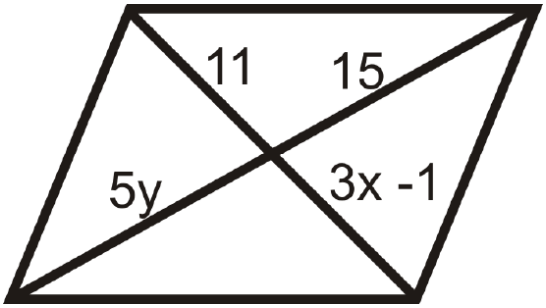

Figure **2.16.31**

For questions 19-22, determine if $ABCD$ is a parallelogram.

19. $A(8,-1)$, $B(6,5)$, $C(-7,2)$, $D(-5,-4)$
20. $A(-5,8)$, $B(-2,9)$, $C(3,4)$, $D(0,3)$
21. $A(-2,6)$, $B(4,-4)$, $C(13,-7)$, $D(4,-10)$
22. $A(-9,-1)$, $B(-7,5)$, $C(3,8)$, $D(1,2)$

Fill in the blanks in the proofs below.

23. ***Opposite Angles Theorem Converse***

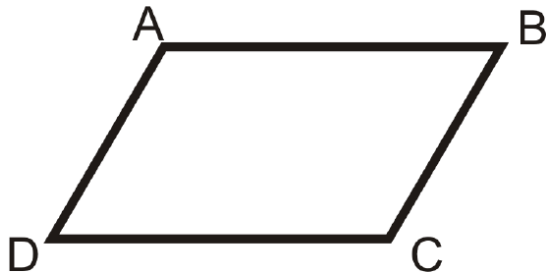

Figure **2.16.32**

Given: $\angle A \cong \angle C$, $\angle D \cong \angle B$

Prove: $ABCD$ is a parallelogram

Statement	*Reason*
1.	1.
2. $m\angle A = m\angle C$, $m\angle D = m\angle B$	2.
3.	3. Definition of a quadrilateral
4. $m\angle A + m\angle A + m\angle B + m\angle B = 360°$	4.
5.	5. Combine Like Terms
6.	6. Division *PoE*
7. $\angle A$ and $\angle B$ are supplementary $\angle A$ and $\angle D$ are supplementary	7.
8.	8. Consecutive Interior Angles Converse
9. $ABCD$ is a parallelogram	9.

24. ***Parallelogram Diagonals Theorem Converse***

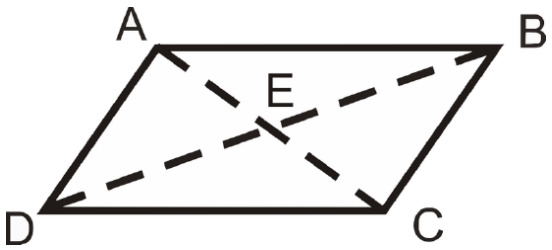

Figure **2.16.33**

Given: $\overline{AE} \cong \overline{EC}$, $\overline{DE} \cong \overline{EB}$

Prove: $ABCD$ is a parallelogram

Statement	*Reason*
1.	1.
2.	2. Vertical Angles Theorem
3. $\Delta AED \cong \Delta CEB$ $\Delta AEB \cong \Delta CED$	3.
4.	4.
5. $ABCD$ is a parallelogram	5.

25. Given: $\angle ADB \cong \angle CBD$, $\overline{AD} \cong \overline{BC}$

Prove: $ABCD$ is a parallelogram

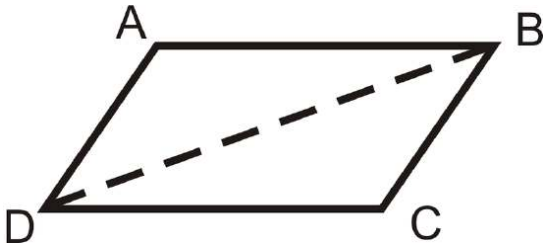

Figure **2.16.34**

Statement	*Reason*
1.	1.
2. $\overline{AD} \parallel \overline{BC}$	2.
3. $ABCD$ is a parallelogram	3.

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.4.

Additional Resources

Video: Proving a Quadrilateral is a Parallelogram Principles - Basic

Activities: Quadrilaterals that are Parallelograms Discussion Questions

Study Aids: Parallelograms Study Guide

Practice: Parallelogram Proofs

Real World: Quadrilaterals That Are Parallelograms

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2.17: Proofs Involving Triangles

Prove theorems about the sum of angles, base angles of isosceles triangles, and exterior and interior angles.

Theorems about Triangles

Recall that a triangle is a shape with exactly three sides. Triangles can be classified by their sides and by their angles.

When classifying a triangle by its sides, you should look to see if any of the sides are the same length.

- If no sides are the same length, then it is a scalene triangle.
- If two sides are the same length, then it is an isosceles triangle.
- If all three sides are the same length, then it is an equilateral triangle.

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Figure **2.17.1**

When classifying a triangle by its angles, you should look at the size of the angles:

- If a triangle has a right angle, then it is a right triangle.
- If the measures of all angles in a triangle are less than $90°$, then it is an acute triangle.
  - A special case of an acute triangle is when all three angles are equal. In that case, all three angles are 60 degrees, and they form an **equiangular triangle**.
- If the measure of one angle in a triangle is greater than $90°$, then it is an obtuse triangle.

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Figure **2.17.2**

Triangle Sum Theorem

Prove that the interior angles of a triangle sum to $180°$.

This is a property of triangles that you have heard of and used before, but you may not have ever seen a proof for why it is true. Here is a paragraph format proof that relies on parallel lines and alternate interior angles.

Consider the generic triangle below.

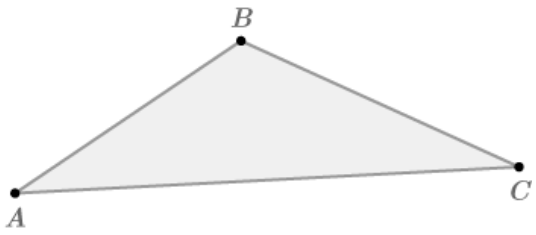

Figure **2.17.3**

By the parallel postulate, there exists exactly one line parallel to \overline{AC}\) through B\). Draw this line.

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Figure 2.17.4

$\angle DBA \cong \angle A$ because they are alternate interior angles and alternate interior angles are congruent when lines are parallel. Therefore, $m\angle DBA = m\angle A$. Similarly, $\angle EBC \cong \angle C$ because they are also alternate interior angles, and so $m\angle EBC = m\angle$. $m\angle DBA + m\angle ABC + m\angle EBC = 180°$ because these three angles form a straight line. By substitution, $m\angle A + m\angle ABC + m\angle C = 180°$.

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Figure 2.17.5

The statement "*the sum of the measures of the interior angles of a triangle is* 180°" is known as the **Triangle Sum Theorem**. Now that it has been proven, you can use it in future proofs without proving it again.

Click the small blue arrow next to the image below and then drag the orange vertices to reshape the triangle. Observe that the sum of the measures of the interior angles of a triangle is always 180°.

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Figure 2.17.6

Isosceles Triangle Th

Prove that the base angles ıgle are congruent.

The base angles of an isos e angles opposite the congruent sides. Below, the base angles are marked for isosceles ΔABC.

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Figure 2.17.7

You need to prove that $\angle B \cong \angle C$ given that $\overline{AB} \cong \overline{AC}$. Here is a proof in the two-column format, that relies on angle bisectors and congruent triangles. The proof will reference the picture below.

Figure **2.17.8**

Statements	**Reasons**
Isosceles ΔABC	Given
$\overline{AB} \cong \overline{AC}$	Definition of isosceles triangle
Construct $\overrightarrow{AD}$, the angle bisector of $\angle A$, with F the intersection of $\overline{BC}$ and $\overrightarrow{AD}$	An angle has only one angle bisector
$\overline{AF} \cong \overline{AF}$	Reflexive Property
$\angle BAF \cong \angle CAF$	Definition of angle bisector
$\Delta ABF \cong \Delta ACF$	$SAS \cong$
$\angle B \cong \angle C$	$CPCTC$

The statement "*the base angles of an isosceles triangle are congruent*" is the **Isosceles Triangle Theorem**. Now that it has been proven, you can use it in future proofs without proving it again.

Click the small blue arrow next to the image below and then drag the orange vertices to reshape the triangle. Observe that the base angles of an isosceles triangle are always congruent.

Figure **2.17.9**

Exterior Angles Theorem

Prove that the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

An exterior angle of a triangle is an angle outside of a triangle created by extending one of the sides of the triangles. Below, $\angle ACD$ is an exterior angle. For exterior angle $\angle ACD$, the angles $\angle A$ and $\angle B$ are the remote interior angles, because they are the interior angles that are not adjacent to the exterior angle.

Figure **2.17.10**

Here is a flow diagram proof of this theorem.

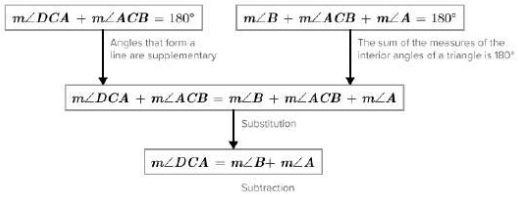

Figure **2.17.11**

The statement *"the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles*" is the **Exterior Angles Theorem**. Now that it has been proven, you can use it in future proofs without proving it again.

Click the small blue arrow next to the image below and then drag the orange vertices to reshape the triangle. Choose two remote interior angles of the triangle and observe how measure of an exterior angle remains equal to the sum of the remote interior angles.

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Figure **2.17.12**

In the interactive below, move the red points to change the shape of the triangle. Move the blue points to compare angles \angle 1\), \angle 2\), and \angle 3\).

Notice that because they are vertical angles, the angle pairs $m\angle 1 = m\angle 4$, $m\angle 2 = m\angle 5$, and $m\angle 3 = m\angle 6$.

Triangle Midsegment Theorem

In the triangle below, point D is a midpoint of $\overline{AC}$ and point E is the midpoint of $\overline{BC}$. Make a conjecture about how $\overline{DE}$ relates to $\overline{AB}$.

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Figure **2.17.13**

A conjecture is a guess about something that might be true. After making a conjecture, usually you will try to prove it. Two possible conjectures are:

1. $\overline{DE} \parallel \overline{AB}$
2. The length of $\overline{DE}$ is half the length of $\overline{AB}$

Consider the picture below.

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Figure **2.17.14**

To prove the conjecture \(DE\parallel AB, we need to prove that \(\Delta FEB\cong \Delta DEC which we then use to prove that \(BF\parallel AC. We then continue with the proof to prove that \(\Delta ADB\cong \Delta FBD which then leads us to prove that \(DE\parallel AB.

Statements	Reasons	
1.	$\overline{DC}\cong\overline{AD}$, $\overline{CE}\cong\overline{EB}$, $\overline{DE}\cong\overline{EF}$	Given
	$\angle CED\cong\angle FEB$	Vertical angles are congruent
	$\Delta FEB\cong\Delta DEC$	$SAS\cong$
2.	$\angle FBE\cong\angle ECD$	$CPCTC$
	$\overline{BF}\parallel\overline{AC}$	If alternate interior angles are congruent then lines are parallel.
3.	$\angle ADB\cong\angle DBF$	If lines are parallel then alternate interior angles are congruent.
	$\overline{DB}\cong\overline{DB}$	Reflexive property
	$\overline{BF}\cong\overline{DC}$	$CPCTC$
	$\overline{BF}\cong\overline{AD}$	Substitution
	$\Delta ADB\cong\Delta FBD$	$SAS\cong$
4.	$\angle ABD\cong\angle FDB$	$CPCTC$
	$\overline{DE}\parallel\overline{AB}$	If alternate interior angles are congruent then lines are parallel.

Here is the figure again, for reference:

Figure **2.17.15**

Now prove the conjecture that the length of $DEishalfthelengthof$\(AB.

	Statements	Reasons
5.	$\overline{DF}\parallel\overline{AB}$	$\overline{DE}$ produced at F
	$\overline{BF}\parallel\overline{AD}$	D is a midpoint of AC
	$ABFD$ is a parallelogram	Definition of a parallelogram
	$\overline{DE}=\frac{1}{2}\overline{DF}$	$\overline{DE}\cong\overline{EF}$
	$\overline{DE}=\frac{1}{2}\overline{AB}$	Definition of a parallelogram

Thus, the line segment joining the midpoints of any two sides of a triangles is parallel to the third side and equal to half of it, this is the **Triangle Midsegment Theorem**. Note that there are other ways to prove that the two segments are parallel. One method relies on similar triangles, which will be explored in another concept.

Click on the small blue arrow next to the image below and then drag the orange vertices to reshape the triangle. Observe that the line segment joining the mid-points of the two sides of a triangle is parallel to the third side and equal to half of it.

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Figure **2.17.16**

CK-12 PLIX Interactive

Review

1. Earlier, you proved that the sum of the interior angles of a triangle is $180°$ (the triangle sum theorem) using a paragraph proof. Now, rewrite this proof in the two-column format.

2. Rewrite the proof of the Triangle Sum Theorem again, this time in the flow diagram format.

3. Earlier, you proved that the base angles of an isosceles triangle are congruent using a two-column proof. Now, rewrite this proof in the paragraph format.

4. Rewrite the isosceles triangle theorem again, this time in the flow diagram format.

5. Earlier, you proved that the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles using the flow diagram format. Now, rewrite this proof in the paragraph format.

6. Rewrite the exterior angles proof again, this time in the two-column format.

7. Recall that the segment joining midpoints of two sides of a triangle is parallel to the third side of the triangle. Given the diagram below and that $\Delta ADB \cong \Delta FBD$ as proved earlier in the lesson, prove that $DE = \frac{1}{2}AB$.

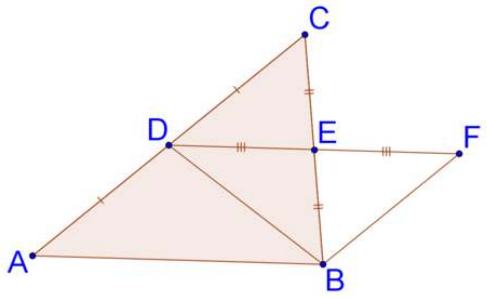

Figure **2.17.17**

8. What is the converse of: "if a triangle is isosceles, the base angles are congruent?" Why do you think the converse is also true or is not true?

9. Prove that if two angles of a triangle are congruent, then the triangle is isosceles. Use the diagram and two-column proof below and fill in the blanks to complete the proof.

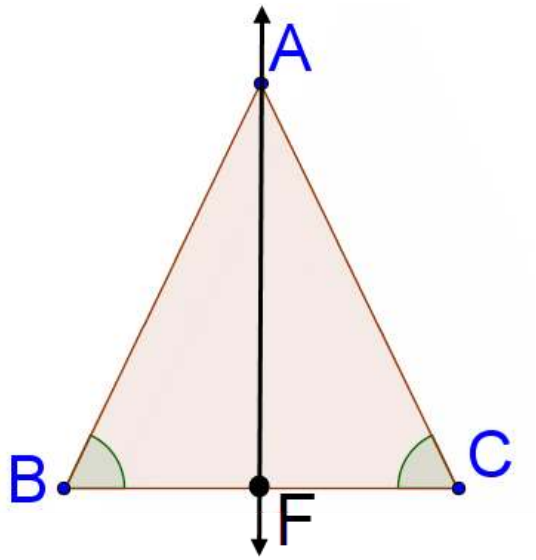

Figure **2.17.18**

Statements	**Reasons**
$\angle B \cong \angle C$	_____
Construct $\overrightarrow{AF}$, the angle bisector of $\angle A$, with F the intersection of $\overline{BC}$ *and* $\overrightarrow{AF}$	An angle has only one angle bisector
_____	Definition of angle bisector
_____	Reflexive Property
$\Delta ABF \cong \Delta ACF$	_____
_____	$CPCTC$

10. Rewrite the proof that a triangle with two congruent angles is isosceles in the flow diagram format.

11. Rewrite the isosceles triangle proof in the paragraph format.

12. Given that $\Delta ABC \cong \Delta BAD$, prove that ΔAEB is isosceles.

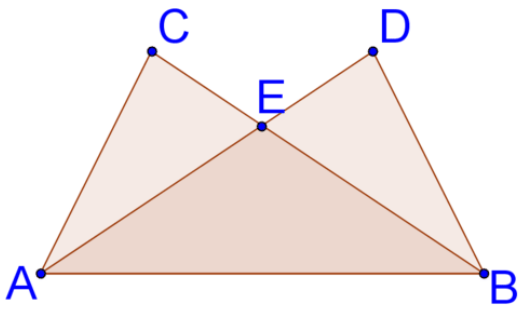

Figure **2.17.19**

13. Given the markings in the picture below, explain why $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.

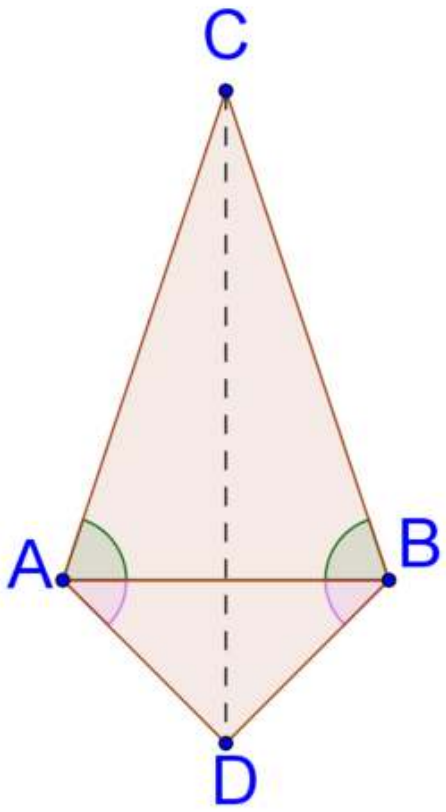

Figure **2.17.20**

14. In the picture below, $\Delta ABC is isosceles with \(\overline{AC} \cong \overline{CB}$. E is the midpoint of $\overline{AC}$ and D is the midpoint of $\overline{CB}$. Prove that $\Delta EAB \cong \Delta DBA$.

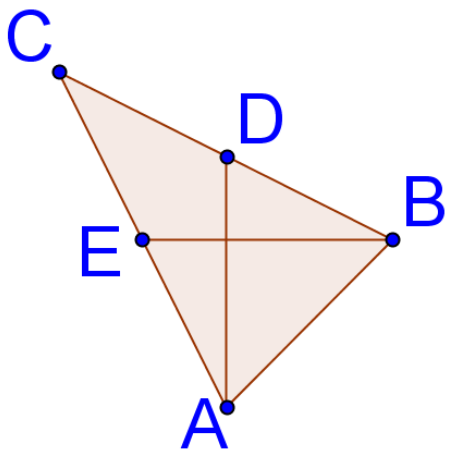

Figure **2.17.21**

15. Explain why knowing that \DeltaABC is isosceles is not enough information to prove that \DeltaABD ≅ \DeltaCBD.

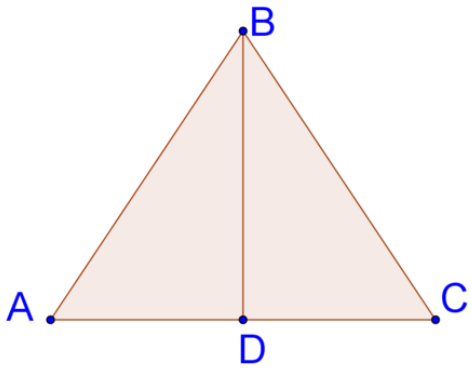

Figure **2.17.22**

16 Given: $\angle CBD \cong \angle EFD$; $\overline{CB} \cong \overline{EF}$

Prove: $\angle DBF \cong \angle DFB$

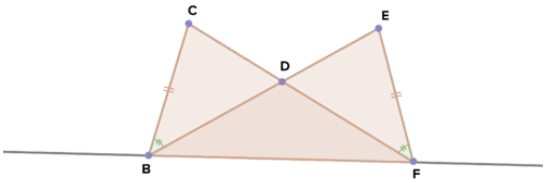

Figure **2.17.23**

17. Given: $\overline{BC} \cong \overline{EF}$; $\overline{CF} \cong \overline{EB}$

Prove: ΔBDF is isosceles

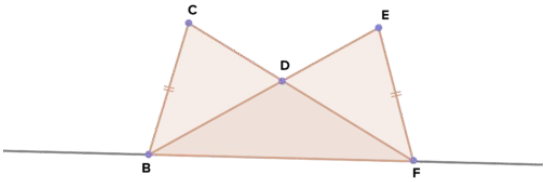

Figure **2.17.23**

18. Given: $\overline{DE}$ midsegment of ΔABC; $\overline{GF} \cong \overline{JI}$; $\overline{FH} \cong \overline{IK}$

Prove: $\Delta GFH \cong \Delta JIK$

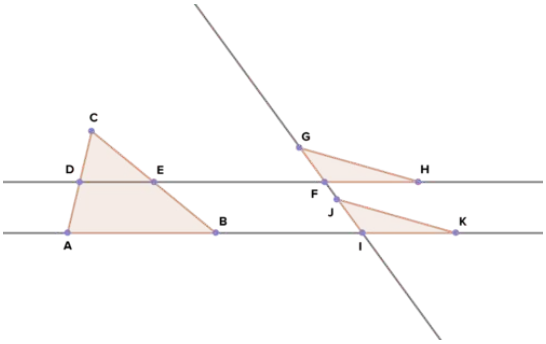

Figure **2.17.24**

Review (Answers)

To see the Review answers, click here.

Additional Resources

Practice: Proofs Involving Triangles

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2.18: Indirect Proof in Algebra and Geometry

Proof by contradiction, beginning with the assumption that the conclusion is false.

Indirect Proofs

Most likely, the first type of formal proof you learned was a direct proof using direct reasoning. Most of the proofs done in geometry are done in the two-column format, which is a direct proof format. Another common type of reasoning is indirect reasoning, which you have likely done outside of math class. Below we will formally learn what an **indirect proof** is and see some examples in both algebra and geometry.

Indirect Proof or Proof by Contradiction: When the conclusion from a hypothesis is assumed false (or opposite of what it states) and then a contradiction is reached from the given or deduced statements.

In other words, if you are trying to show that something is true, show that if it was not true there would be a contradiction (something else would not make sense).

The steps to follow when proving indirectly are:

- Assume the ***opposite*** of the conclusion (second half) of the statement.
- Proceed as if this assumption is true to find the ***contradiction.***
- Once there is a contradiction, the original statement is true.
- ***DO NOT use specific examples.*** Use variables so that the contradiction can be generalized.

The easiest way to understand indirect proofs is by example.

What if you wanted to prove a statement was true without a two-column proof? How might you go about doing so?

Example 2.18.1

If $x = 2$, then $3x - 5 \neq 10$ ent is true by contradiction.

Solution

Remember that in an in st thing you do is assume the conclusion of the statement is ***false.*** In this case, we will assume the ***opposite*** of "If $x = 2$, then $3x - 5 \neq 10$":

If $x = 2$, then $3x - 5 = 10$.

Take this statement as true and solve for x.

$$\begin{aligned} 3x - 5 &= 10 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

But $x = 5$ ***contradicts*** the given statement that $x = 2$. Hence, our ***assumption is incorrect*** and $3x - 5 \neq 10$ is ***true***.

Example 2.18.2

If \Delta ABC is isosceles, then the measure of the base angles cannot be $92°$. Prove this indirectly.

Solution

Remember, to start assume the ***opposite*** of the conclusion.

The measure of the base angles are $92°$.

If the base angles are $92°$, then they add up to $184°$. This ***contradicts*** the **Triangle Sum Theorem** that says the three angle measures of all triangles add up to $180°$. Therefore, the base angles cannot be $92°$.

Example 2.18.3

If $\angle A$ and $\angle B$ are complementary then $\angle A \leq 90°$. Prove this by contradiction.

Solution

Assume the ***opposite*** of the conclusion.

$\angle A > 90°$.

Consider first that the measure of \angle B cannot be negative. So if $\angle A > 90°$ this contradicts the definition of complementary, which says that two angles are complementary if they add up to $90°$. Therefore, $\angle A \leq 90°$.

Example 2.18.4

If n is an integer and n^2 is odd, then n is odd. Prove this is true indirectly.

Solution

First, assume the ***opposite*** of "n is odd."

n is even.

Now, square n and see what happens.

If n is even, then $n = 2a$, where a is any integer.

$n^2 = (2a)^2 = 4a^2$

This means that n^2 is a multiple of 4. No odd number can be divided evenly by an even number, so this ***contradicts our assumption*** that n is even. Therefore, n must be odd if n^2 is odd.

Example 2.18.5

Prove the SSS Inequality Theorem is true by contradiction. (The SSS Inequality Theorem says: "If two sides of a triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle's two congruent sides is greater in measure than the included angle of the second triangle's two congruent sides.")

Solution

First, assume the opposite of the conclusion.

The included angle of the first triangle is less than or equal to the included angle of the second triangle.

If the included angles are equal then the two triangles would be congruent by SAS and the third sides would be congruent by CPCTC. This contradicts the hypothesis of the original statement "the third side of the first triangle is longer than the third side of the second." Therefore, the included angle of the first triangle must be larger than the included angle of the second.

Review

Prove the following statements true indirectly.

1. If n is an integer and n^2 is even, then n is even.
2. If $m\angle A \neq m\angle B$ in ΔABC, then ΔABC is not equilateral.
3. If $x > 3$, then $x^2 > 9$.
4. The base angles of an isosceles triangle are congruent.
5. If x is even and y is odd, then $x + y$ is odd.
6. In ΔABE, if $\angle A$ is a right angle, then $\angle B$ cannot be obtuse.
7. If A, B, and C are collinear, then $AB + BC = AC$ (Segment Addition Postulate).
8. If ΔABC is equilateral, then the measure of the base angles cannot be $72°$.
9. If $x = 11$ then $2x - 3 \neq 21$.

10. If ΔABC is a right triangle, then it cannot have side lengths 3, 4, and 6.

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.8.

Additional Resources

Video: Indirect Proof in Algebra and Geometry Examples - Basic

Activities: Indirect Proof in Algebra and Geometry Discussion Questions

Study Aids: Types of Reasoning Study Guide

Practice: Indirect Proof in Algebra and Geometry

Real World: Contradictory Evidence

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CHAPTER OVERVIEW

3: Lines

3.1: Line Types
3.2: Parallel and Skew Lines
3.3: Parallel Lines and Transversals
3.4: Corresponding Angles
3.5: Alternate Interior Angles
3.6: Alternate Exterior Angles
3.7: Same Side Interior Angles
3.8: Angles and Perpendicular Lines
3.9: Parallel Lines in the Coordinate Plane
3.10: Perpendicular Lines in the Coordinate Plane
3.11: Line Construction

Thumbnail: Angles A and B are adjacent. (Public Domain; Limaner via Wikipedia)

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3.1: Line Types

Identify parallel, perpendicular and intersecting lines.

Parallel and Perpendicular Lines

Figure 3.1.1

Victoria is on a class field trip. While riding over a bridge, she notices that the road and the structure of the bridge make a variety of lines. She believes that the posts of the bridge are perpendicular to the road. What can she say to prove this to her classmates?

In this concept, you will learn how to identify **parallel**, intersecting and **perpendicular lines.**

Identifying Parallel and Perpendicular Lines

Lines exist in space. Two lines **intersect** when they cross each other. Because all lines are straight, intersecting lines can only cross each other once. Look at the examples below. Imagine the lines extend beyond the picture, on forever.

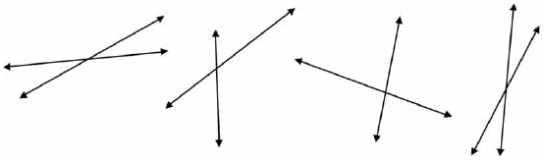


Figure 3.1.2

Two lines that form right angles when they intersect are **perpendicular lines.** All four angles formed by the perpendicular lines measure 90°. If any of the four angles is marked with the small box, the lines are perpendicular. Take a look at the perpendicular lines below.

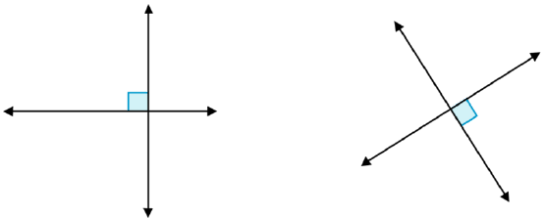


Figure 3.1.3

Lines that never intersect are **parallel lines**. Parallel lines do not form any angles. Parallel lines look like railroad tracks: they are always the same distance apart, running next to each other.

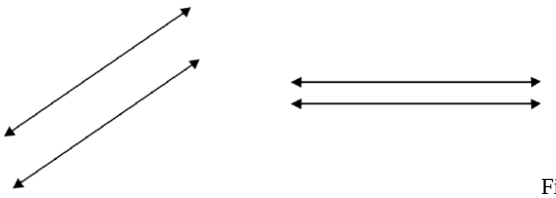


Figure 3.1.4

Identify whether the pairs of lines are parallel, perpendicular, or intersecting.

1.

Figure **3.1.5**

First, look to see if the lines intersect.

The lines do not intersect.

Next, determine if the lines would intersect if they were extended.

No

Then, label the lines as parallel, perpendicular or intersecting.

The lines are parallel.

The answer is that the lines are parallel.

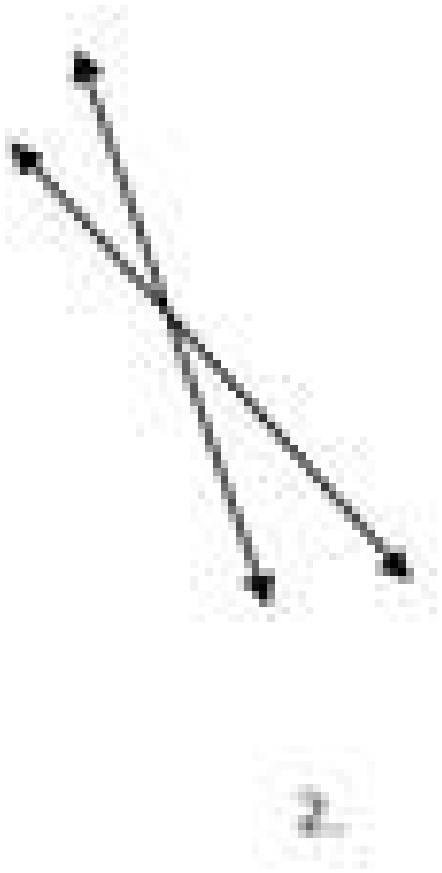


Figure **3.1.6**

First, look to see if the lines intersect.

The lines do intersect.

Next, determine if the lines intersect at a right angle.

The lines do not intersect at a right angle.

Then, label the lines as parallel, perpendicular or neither.

The lines are intersecting.

The answer is that the lines are intersecting but not perpendicular.

Example 3.1.1

Earlier, you were given a problem about Victoria and the bridge.

As she rides across the bridge, she notices the different lines. What can she say to prove that the posts are perpendicular to the road?

Solution

First, treat the posts and the road like lines. Look to see if the lines intersect.

They do intersect.

Next, determine if the lines intersect the road at a right angle.

The lines do intersect at a right angle.

Then, label the lines as parallel, perpendicular or neither.

The lines are perpendicular.

The answer is that the lines are perpendicular.

Example 3.1.2

Identify whether the pair of lines are parallel, perpendicular or intersecting.

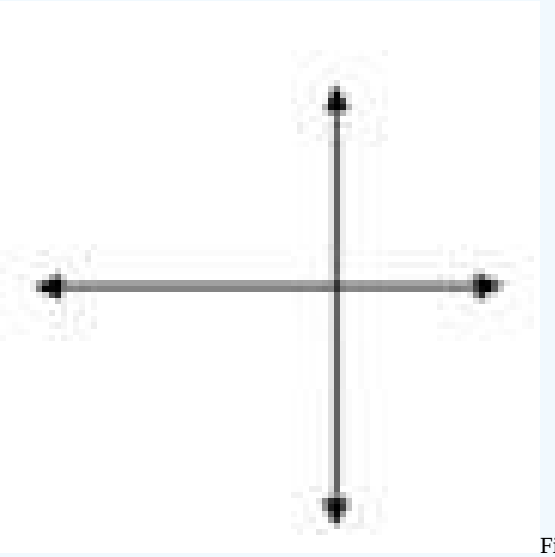


Figure 3.1.7

Solution

First, look to see if the lines intersect.

The lines do intersect.

Next, determine if the lines intersect at a right angle.

Yes

Then, label the lines as parallel, perpendicular or intersecting.

Perpendicular.

The answer is that the lines are perpendicular.

Example 3.1.3

Identify each pair of lines as parallel, perpendicular or neither.

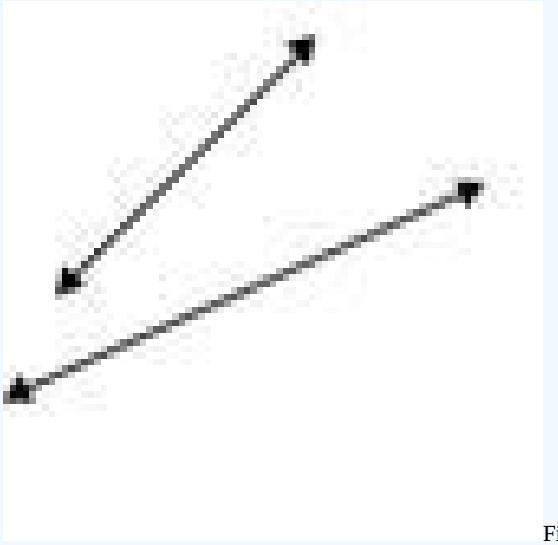


Figure 3.1.8

Solution

First, look to see if the lines intersect.

The lines will intersect if they are extended.

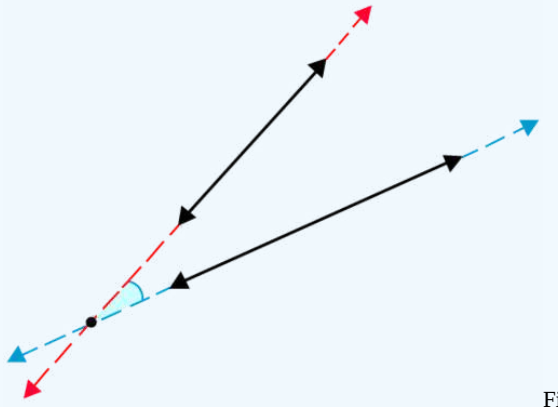


Figure 3.1.9

Next, determine if the lines intersect at a right angle.

The lines do not intersect at a right angle.

Then label the lines as parallel, perpendicular or neither.

Intersecting.

The answer is that the lines are intersecting but not perpendicular.

Example 3.1.4

Figure 3.1.10

Do the two white bars form parallel lines, perpendicular lines or intersecting?

Solution

First, look to see if the bars intersect.

The bars do not intersect.

Next, determine if the bars would intersect if they were extended.

No.

Then, label the lines as parallel, perpendicular or intersecting.

The lines are parallel.

The answer is that the lines are parallel.

Example 3.1.5

Jeff traces the left side of a standard book and the right side of the book. When he removes the book, what type of lines will be have: parallel, perpendicular or intersecting?

Solution

First, think about a standard book. Do these two sides of a standard book intercept?

The sides do not intersect.

Next, determine if the lines will intersect if they are extended.

No, the lines will not intersect if they are extended.

Then, label the lines as parallel, perpendicular or neither.

The lines are parallel.

The answer is that the lines are parallel.

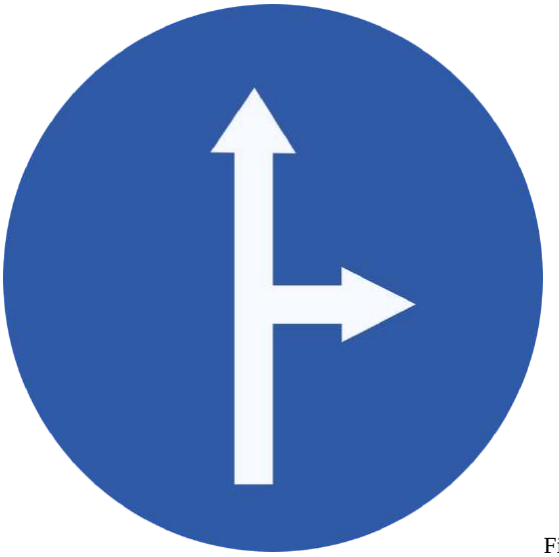
Review

Use what you have learned to answer each question true or false.

1. Adjacent angles are also supplementary angles.
2. Vertical angles are complementary angles.
3. If one adjacent angle is $100°$, then its angle pair is also $100°$.
4. Vertical angles have the same measure.
5. Vertical angles and corresponding angles are located in the same position.
6. You can have corresponding angles when you only have two intersecting lines.
7. Corresponding angles are in the same place given the intersection.

8. Parallel lines will never intersect.
9. Perpendicular lines intersect at a 90 degree angle.

Tell whether each picture shows parallel or intersecting lines.



10. Figure 3.1.11

11. Figure 3.1.12

Think about each example described below and determine whether the lines would be intersecting or parallel.

12. Telephone wires
13. The yellow lines down a highway
14. Stitches on a sweater
15. The sides of a ramp

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.5.

Resources

Vocabulary

Term	Definition
Corresponding Angles	Corresponding angles are two angles that are in the same position with respect to the transversal, but on different lines.
Parallel	Two or more lines are parallel when they lie in the same plane and never intersect. These lines will always have the same slope.
Perpendicular lines	Perpendicular lines are lines that intersect at a 90° angle.

Additional Resource

Interactive Element

Video: Perpendicular Lines - Example 1

Practice: Line Types

Real World: A Road Map to Healing

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3.2: Parallel and Skew Lines

Lines that never intersect.

Parallel lines are two or more lines that lie in the same plane and never intersect. To show that lines are parallel, arrows are used.

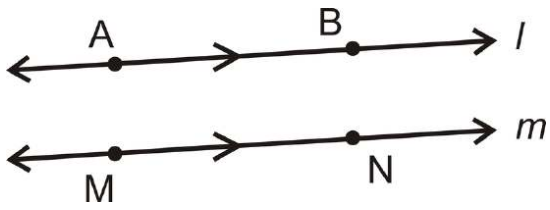

Figure 3.2.1

Label It	*Say It*
$\overleftrightarrow{AB} \parallel \overleftrightarrow{MN}$	Line AB is parallel to line MN
$l \parallel m$	Line l is parallel to line m.

In the definition of parallel the word “line” is used. However, line segments, rays and planes can also be parallel. The image below shows two parallel planes, with a third blue plane that is perpendicular to both of them.

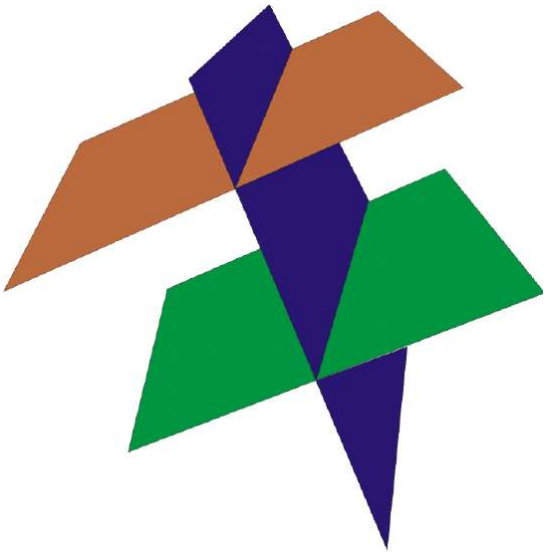

Figure 3.2.2

Skew lines are lines that are in different planes and never intersect. They are different from **parallel lines** because parallel lines lie in the SAME plane. In the cube below, $\overline{AB}$ and $\overline{FH}$ are skew and $\overline{AC}$ and $\overline{EF}$ are skew.

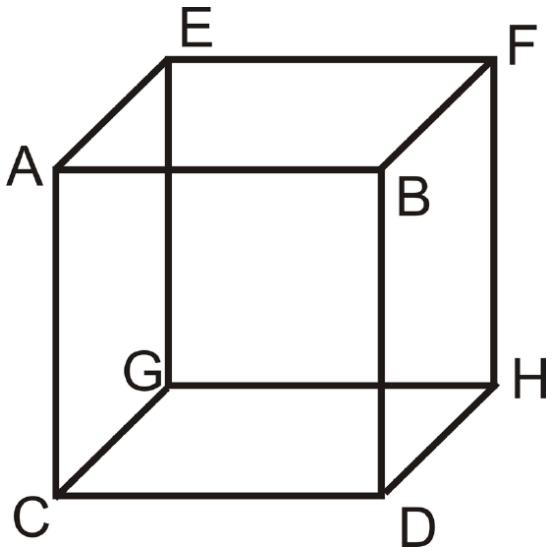

Figure **3.2.3**

Basic Facts About Parallel Lines

Property: If lines $l \parallel m$ and $m \parallel n$, then $l \parallel n$.

If

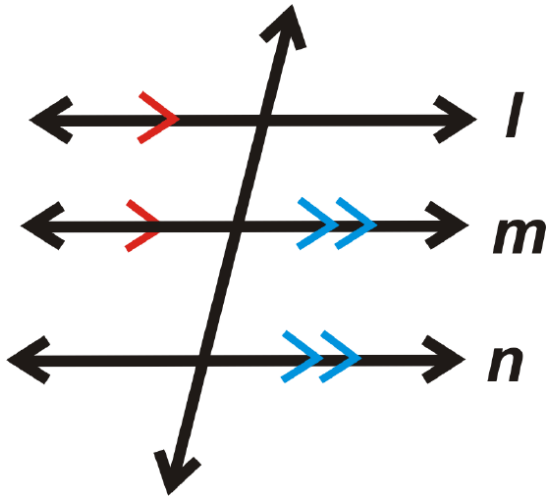

Figure **3.2.4**

then

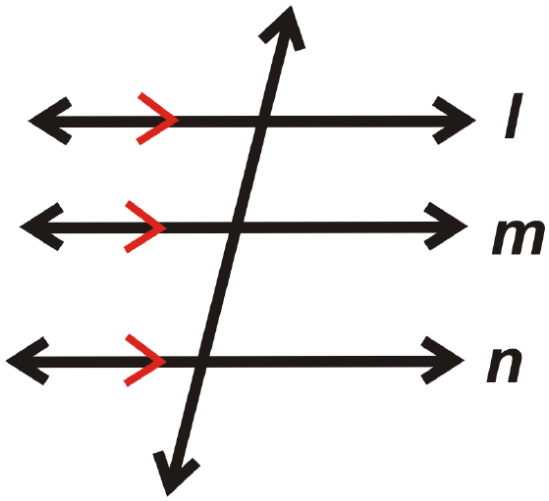

Figure **3.2.5**

Postulate: For any line and a point ***not*** on the line, there is one line parallel to this line through the point. There are infinitely many lines that go through A, but only ***one*** that is parallel to l.

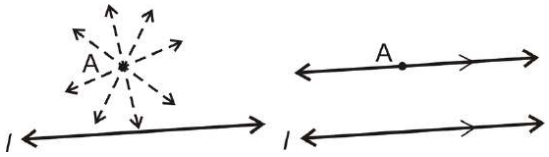

Figure **3.2.6**

A **transversal** is a line that intersects two other lines. The area *between* l and m is the *interior*. The area *outside* l and m is the *exterior*.

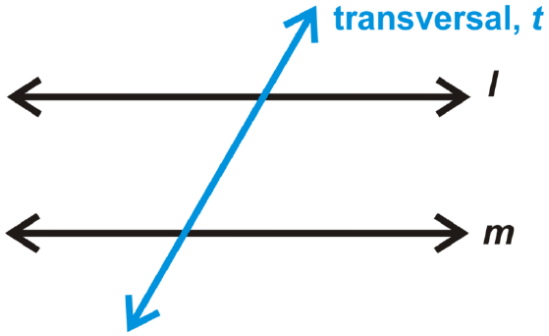

Figure **3.2.7**

What if you were given a pair of lines that never intersect and were asked to describe them? What terminology would you use?

Use the figure below for E 21 **3.2.2**. The two pentagons are parallel and all of the rectangular sides are perpendicular to both of them.

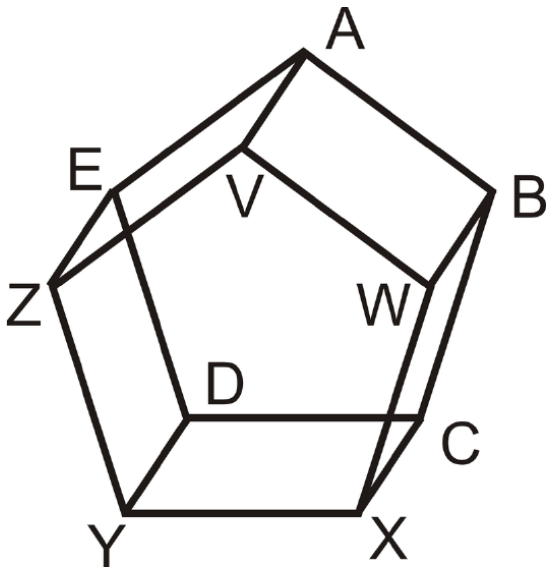

Figure **3.2.8**

Example 3.2.1

Find two pairs of skew lines.

Solution

$\overline{ZV}$ and $\overline{WB}$. $\overline{YD}$ and $\overline{VW}$

Example 3.2.2

For $\overline{XY}$, how many parallel lines would pass through point D? Name this/these line(s).

Solution

One line, $\overline{CD}$

Example 3.2.3

True or false: some pairs of skew lines are also parallel.

Solution

This is false, by definition skew lines are in **different** planes and parallel lines are in the **same** plane. Two lines could be skew or parallel (or neither), but never both.

Example 3.2.4

Using the cube below, list a pair of parallel lines.

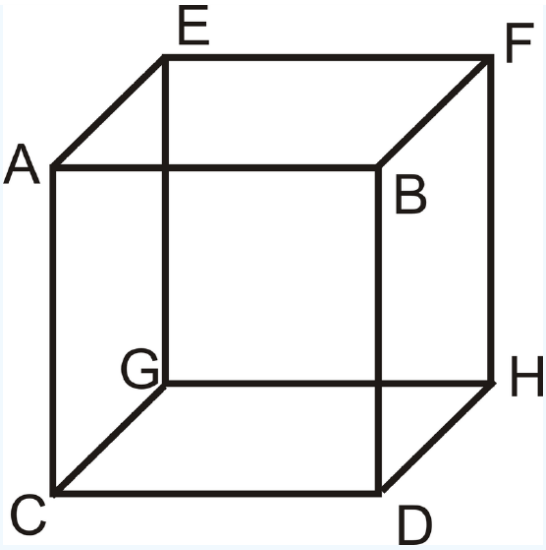

Figure **3.2.9**

Solution

One possible answer is lines $\overline{AB}$ and $\overline{EF}$.

Example 3.2.5

Using the cube below, list a pair of skew lines.

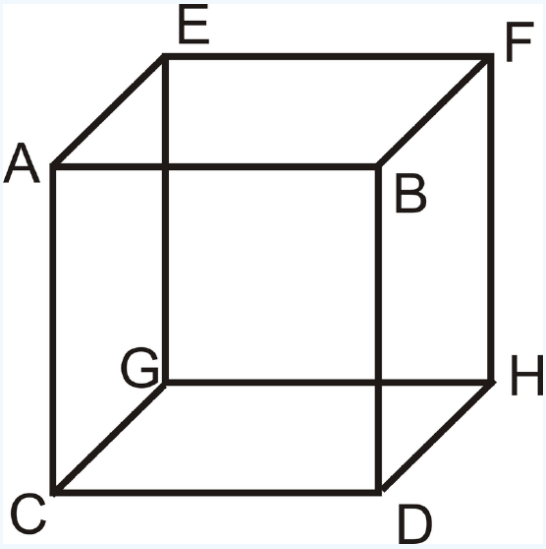

Figure **3.2.10**

Solution

One possible answer is $\overline{BD}$ and $\overline{CG}$.

Review

1. Which of the following is the best example of parallel lines?
    1. Railroad Tracks
    2. Lamp Post and a Sidewalk
    3. Longitude on a Globe
    4. Stonehenge (the stone structure in Scotland)
2. Which of the following is the best example of skew lines?

1. Roof of a Home
2. Northbound Freeway and an Eastbound Overpass
3. Longitude on a Globe
4. The Golden Gate Bridge

Use the picture below for questions 3-5.

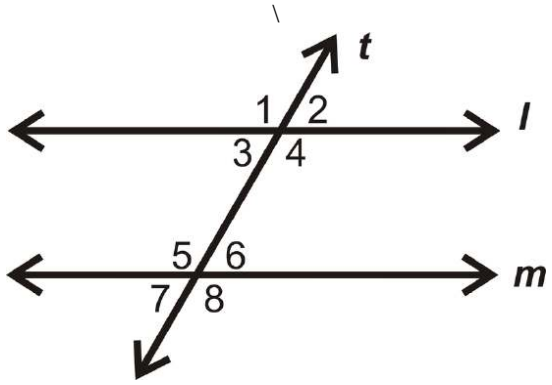

Figure **3.2.11**

3. If $m\angle 2 = 55°$, what other angles do you know?
4. If $m\angle 5 = 123°$, what other angles do you know?
5. Is $l \parallel m$? Why or why not?

For 6-10, determine whether the statement is true or false.

6. If $p \parallel q$and $q \parallel r$, then $p \parallel r$.
7. Skew lines are never in the same plane.
8. Skew lines can be perpendicular.
9. Planes can be parallel.
10. Parallel lines are never in the same plane.

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.1.

Resources

Vocabulary

Term	Definition
parallel lines	Two or more lines that lie in the same plane and never intersect. Parallel lines will always have the same **slope**.
Skew lines	Skew lines are lines that are in different planes and never intersect.
transversal	A transversal is a line that intersects two other lines.
Parallel	Two or more lines are parallel when they lie in the same plane and never intersect. These lines will always have the same slope.
Skew	To skew a given set means to cause the trend of data to favor one end or the other

Additional Resources

Video: Proving Lines Parallel

Practice: Parallel and Skew Lines Discussion Questions

Study Aids: Lines and Angles Study Guide

Pratice: Parallel and Skew Lines

Real World: Short Circuits: How Parallel Circuits Work

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3.3: Parallel Lines and Transversals

Evaluate parallel lines cut by a traversal.

Video: Visually Evaluating Parallel Lines Cut by a Traversal

Practice: Parallel Lines and Transversals

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3.4: Corresponding Angles

Identify corresponding angles as matching angles.

Corresponding angles are two angles that are in the "same place" with respect to the transversal but on different lines. Imagine sliding the four angles formed with line l down to line m. The angles which match up are corresponding.

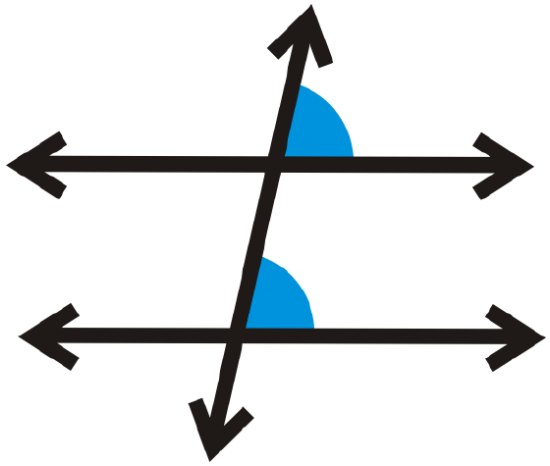

Figure 3.4.1

Corresponding Angles Postulate: If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

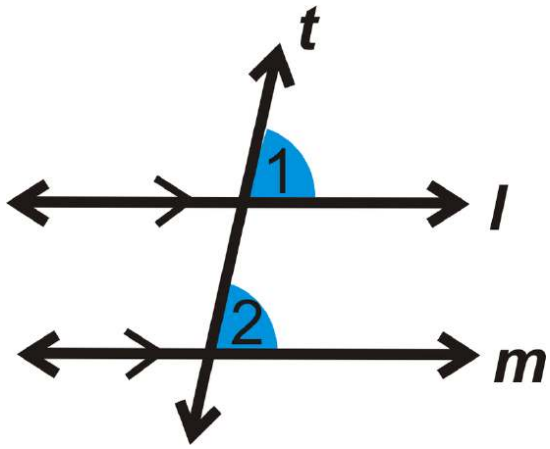

Figure 3.4.2

If $l \parallel m$, then $\angle 1 \cong \angle 2$.

Converse of Corresponding Angles Postulate: If corresponding angles are congruent when two lines are cut by a transversal, then the lines are parallel.

If

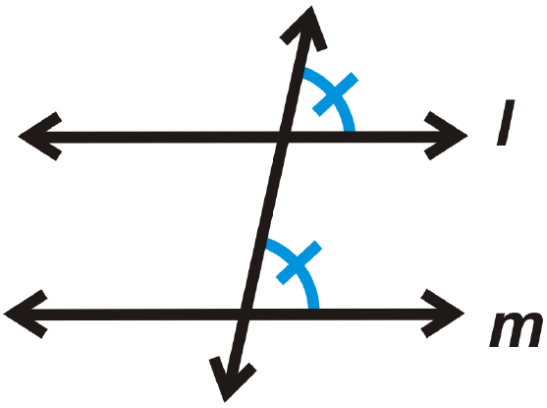

Figure 3.4.3

then $l \parallel m$.

What if you were presented with two angles that are in the same place with respect to the transversal but on different lines? How would you describe these angles and what could you conclude about their measures?

Example 3.4.1

If $m\angle 2 = 76°$, what is $m\angle 6$?

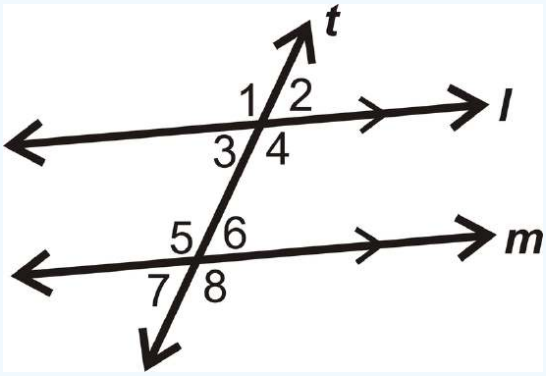

Figure 3.4.4

Solution

$\angle 2$ and $\angle 6$ are corresponding angles and l||m from the arrows in the figure. $\angle 2 \cong \angle 6$ by the Corresponding Angles Postulate, which means that $m\angle 6 = 76°$.

Example 3.4.2

Using the measures of \angle 2 and \angle 6 from Example 1, find all the other angle measures.

Solution

If $m\angle 2 = 76°$, then $m\angle 1 = 180° - 76° = 104°$ (linear pair). \angle 3\cong \angle 2 \)(vertical angles), so $m\angle 3 = 76°$. $m\angle 4 = 104°$ (vertical angle with \angle 1\)).

By the Corresponding Angles Postulate, we know $\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$, so $m\angle 5 = 104°$, $m\angle 6 = 76°$, $m\angle 7 = 76°$, and $m\angle 8 = 104°$.

Example 3.4.3

Find the value of y:

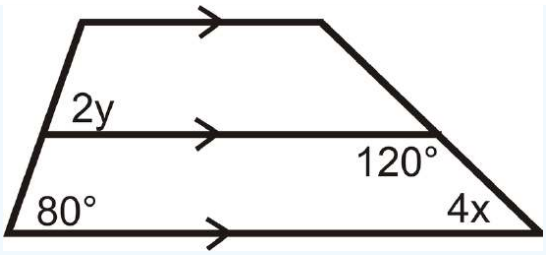

Figure 3.4.5

Solution

The horizontal lines are marked parallel and the angle marked $2y$ is corresponding to the angle marked 80 so these two angles are congruent. This means that $2y = 80$ and therefore $y = 40$.

Example 3.4.4

If a||b, which pairs of angles are congruent by the Corresponding Angles Postulate?

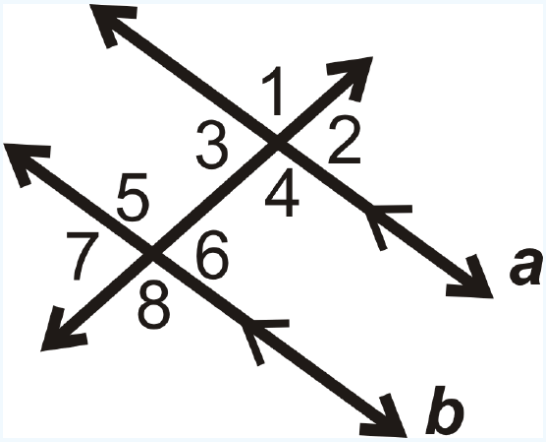

Figure 3.4.6

Solution

There are 4 pairs of congruent corresponding angles:

$\angle 1 \cong \angle 5$, $\angle 2 \cong \angle 6$, $\angle 3 \cong \angle 7$, and $\angle 4 \cong \angle 8$.

Example 3.4.5

If $m\angle 8 = 110°$ and $m\angle 4 = 110°$, then what do we know about lines l and m?

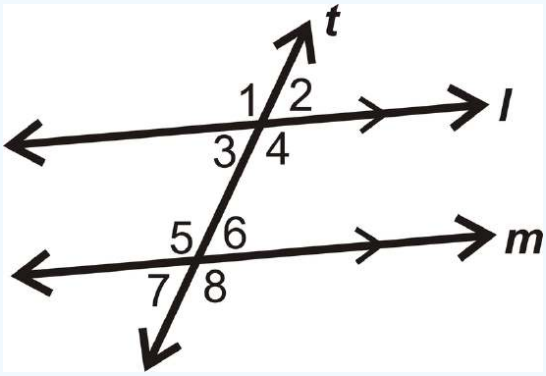

Figure 3.4.7

Solution

$\angle 8$ and $\angle 4$ are corresponding angles. Since $m\angle 8 = m\angle 4$, we can conclude that l||m.

Review

1. Determine if the angle pair $\angle 4$ and $\angle 2$ is congruent, supplementary or neither:

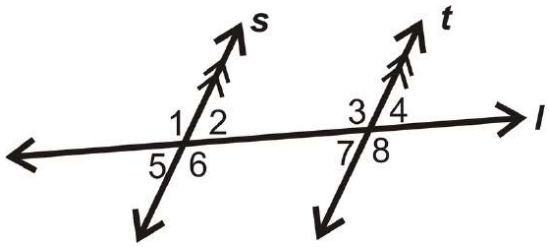

Figure 3.4.8

2. Give two examples of corresponding angles in the diagram:

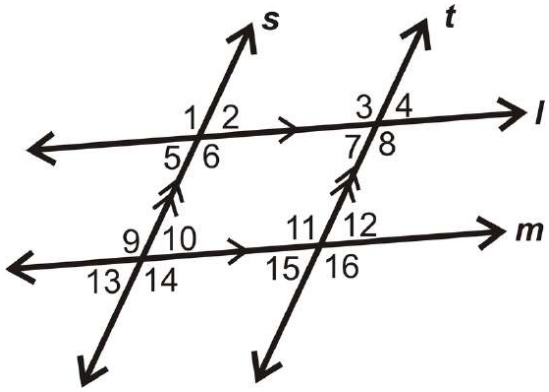

Figure 3.4.9

3. Find the value of x:

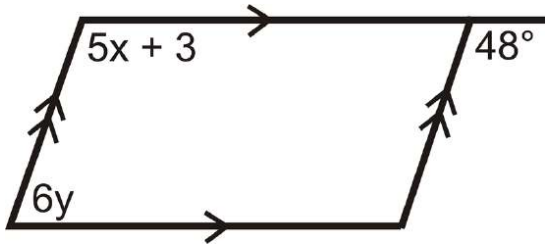

Figure 3.4.10

4. Are the lines parallel? Why or why not?

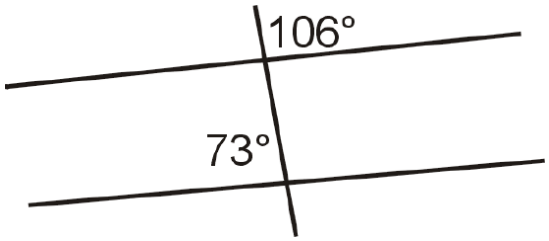

Figure 3.4.11

5. Are the lines parallel? Justify your answer.

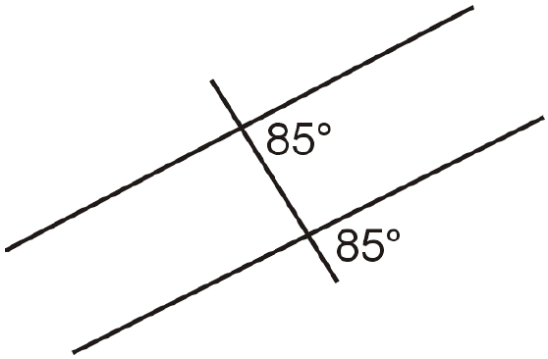

Figure **3.4.12**

For 6-10, what does the value of x have to be to make the lines parallel?

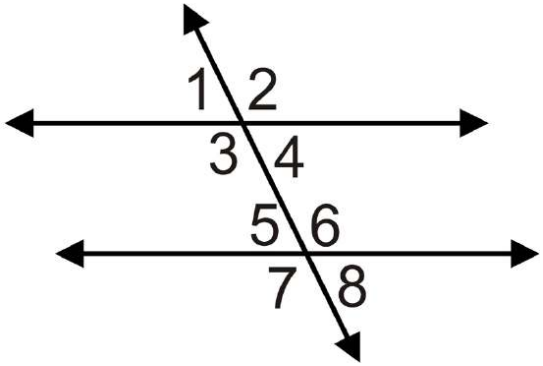

Figure **3.4.13**

6. If $m\angle 1 = (6x - 5)^\circ$ and $m\angle 5 = (5x + 7)^\circ$.
7. If $m\angle 2 = (3x - 4)^\circ$ and $m\angle 6 = (4x - 10)^\circ$.
8. If $m\angle 3 = (7x - 5)^\circ$ and $m\angle 7 = (5x + 11)^\circ$.
9. If $m\angle 4 = (5x - 5)^\circ$ and $m\angle 8 = (3x + 15)^\circ$.
10. If $m\angle 2 = (2x + 4)^\circ$ and $m\angle 6 = (5x - 2)^\circ$.

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.3.

Resources

Vocabulary

Term	Definition
Corresponding Angles	Corresponding angles are two angles that are in the same position with respect to the transversal, but on different lines.

Additional Resource

Video: Corresponding Angles Principles - Basic

Activities: Corresponding Angles Discussion Questions

Study Aids: Angles and Transversals Study Guide

Practice: Corresponding Angles

Real World: Corresponding Angles

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3.5: Alternate Interior Angles

Angles on opposite sides of a transversal, but inside the lines it intersects.

Alternate interior angles are two angles that are on the interior of l and m, but on opposite sides of the transversal.

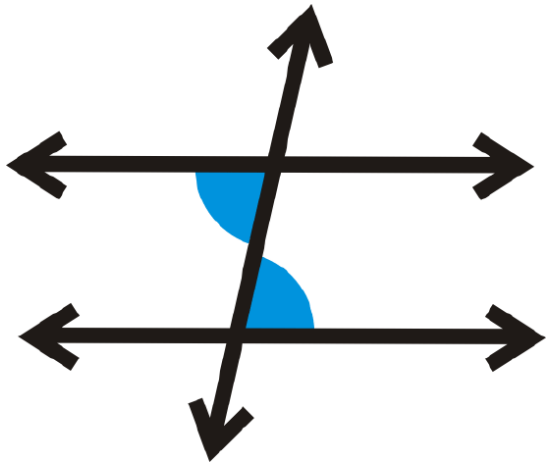

Figure 3.5.1

Alternate Interior Angles Theorem: If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.

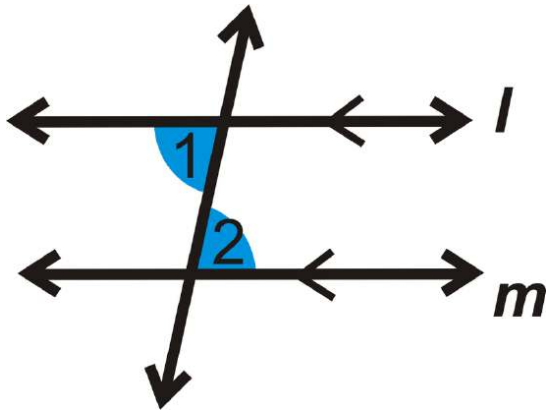

Figure 3.5.2

If $l \parallel m$, then $\angle 1 \cong \angle 2$

Converse of Alternate Interior Angles Theorem: If two lines are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.

If

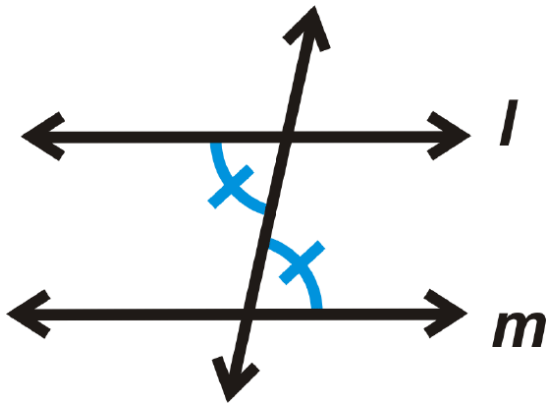

Figure 3.5.3

then $l \parallel m$.

What if you were presented with two angles that are on the interior of two parallel lines cut by a transversal but on opposite sides of the transversal? How would you describe these angles and what could you conclude about their measures?

For Examples **3.5.1** and **3.5.2** information to determine which lines are parallel. If there are none, write none. Consider each question in

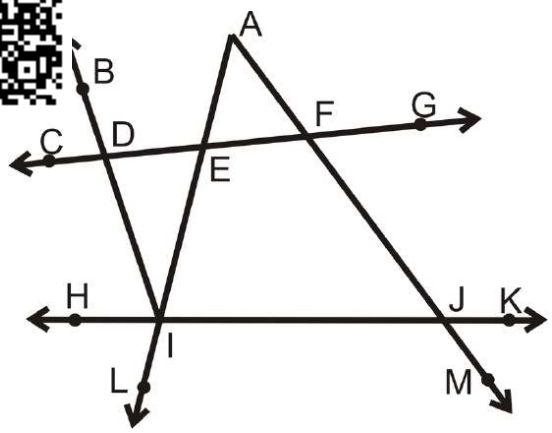

Figure **3.5.4**

Example 3.5.1

$\angle EAF \cong \angle FJI$

Solution

None

Example 3.5.2

$\angle EFJ \cong \angle FJK$

Solution

$\overleftarrow{CG} \parallel \overleftarrow{HK}$

Example 3.5.3

Find the value of x.

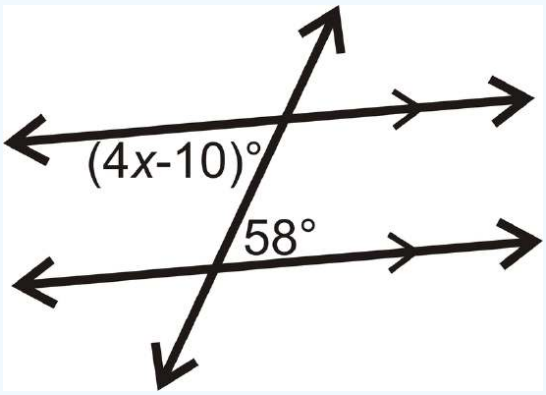

Figure **3.5.5**

Solution

The two given angles are alternate interior angles and equal.

$$(4x - 10)^\circ = 58^\circ$$
$$4x = 68$$
$$x = 17$$

Example 3.5.4

True or false: alternate interior angles are always congruent.

Solution

This statement is false, but is a common misconception. Remember that alternate interior angles are only congruent when the lines are parallel.

Example 3.5.5

The angles are alternate interior angles, and must be equal for $a \parallel b$. Set the expressions equal to each other and solve.

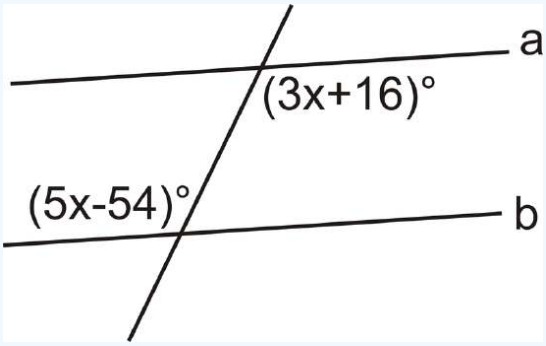

Figure 3.5.6

Solution

$$3x + 16^\circ = 5x - 54^\circ$$
$$70 = 2x$$
$$35 = x$$

To make $a \parallel b$, $x = 35$.

Review

1. Is the angle pair $\angle 6$ and $\angle 3$ congruent, supplementary or neither?

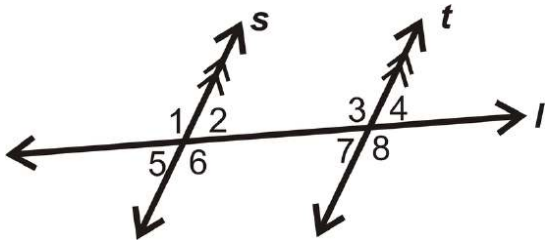

Figure 3.5.7

2. Give two examples of alternate interior angles in the diagram:

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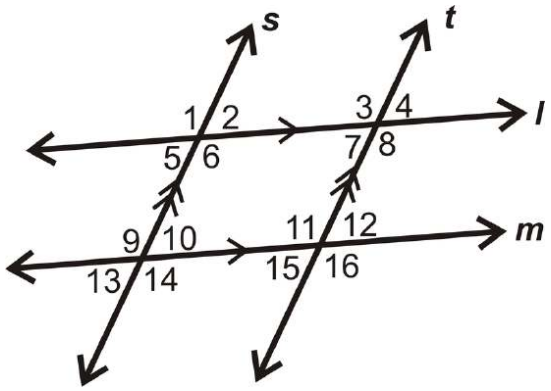

Figure **3.5.8**

For 3-4, find the values of x.

3.

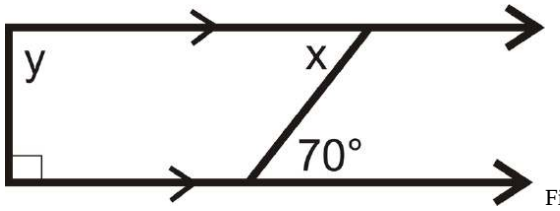

Figure **3.5.9**

4.

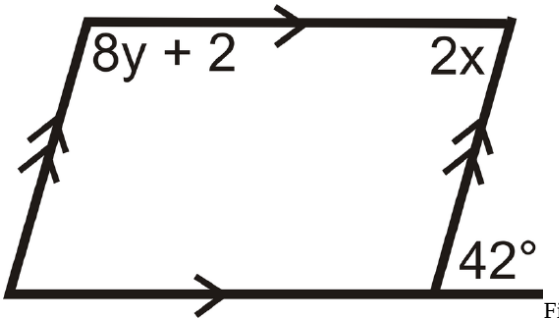

Figure **3.5.10**

For question 5, use the picture below. Find the value of x.

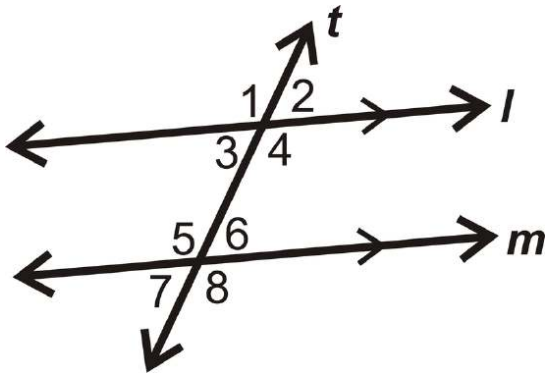

Figure **3.5.11**

5. $m\angle 4 = (5x - 33)^\circ$, $m\angle 5 = (2x + 60)^\circ$

6. Are lines l and m parallel? If yes, how do you know?

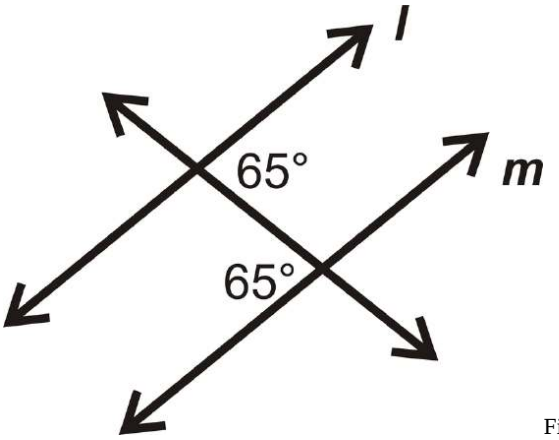

Figure 3.5.12

For 7-10, what does the value of x have to be to make the lines parallel?

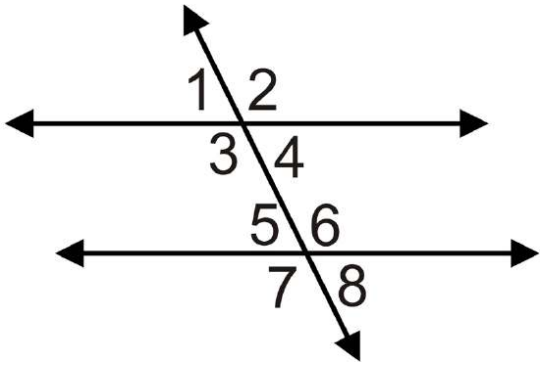

Figure 3.5.13

7. $m\angle 4 = (3x - 7)^\circ$ and $m\angle 5 = (5x - 21)^\circ$
8. $m\angle 3 = (2x - 1)^\circ$ and $m\angle 6 = (4x - 11)^\circ$
9. $m\angle 3 = (5x - 2)^\circ$ and $m\angle 6 = (3x)^\circ$
10. $m\angle 4 = (x - 7)^\circ$ and $m\angle 5 = (5x - 31)^\circ$

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.4.

Resources

Vocabulary

Term	Definition
alternate interior angles	Alternate interior angles are two angles that are on the interior of two different lines, but on the opposite sides of the transversal.
alternate exterior angles	Alternate exterior angles are two angles that are on the exterior of two different lines, but on the opposite sides of the transversal.

Additional Resources

Video: Alternate Interior Angles Principles - Basic

Activities: Alternate Interior Angles Discussion Questions

Study Aids: Angles and Transversals Study Guide

Practice: Alternate Interior Angles

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3.6: Alternate Exterior Angles

Angles on opposite sides of a transversal, but outside the lines it intersects.

Alternate exterior angles are two angles that are on the exterior of l and m, but on opposite sides of the transversal.

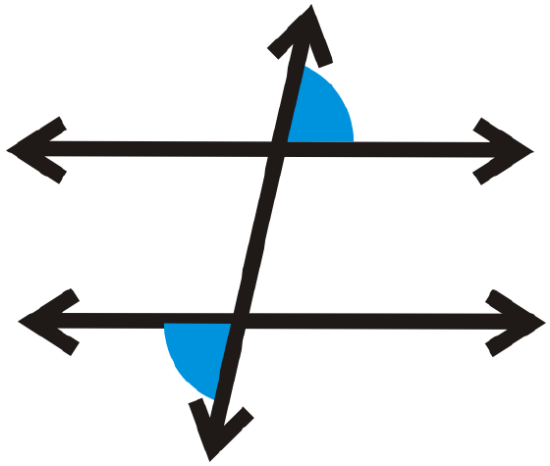

Figure **3.6.1**

Alternate Exterior Angles Theorem: If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.

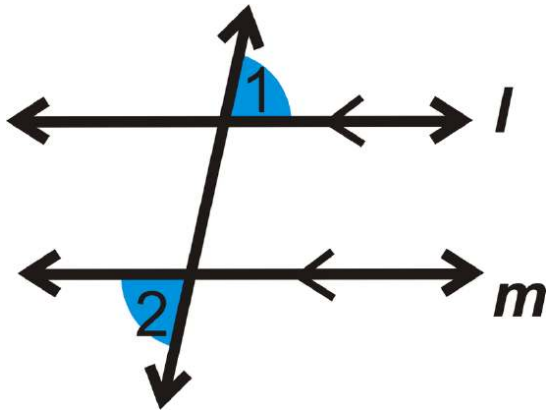

Figure **3.6.2**

If $l \parallel m$, then $\angle 1 \cong \angle 2$.

Converse of the Alternate Exterior Angles Theorem: If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel.

If

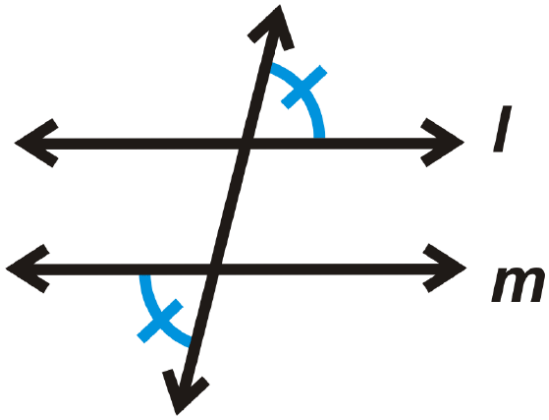

Figure 3.6.3

then $l \parallel m$.

What if you were presented with two angles that are on the exterior of two parallel lines cut by a transversal but on opposite sides of the transversal? How would you describe these angles and what could you conclude about their measures?

For Examples 3.6.1 and 3.6.2 g diagram:

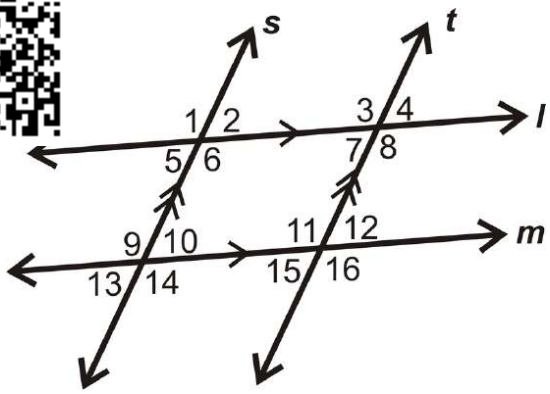

Figure 3.6.4

Example 3.6.1

Give an example of a pair of alternate exterior angles.

Solution

$\angle 1$ and $\angle 14$ (many other possibilities)

Example 3.6.2

Give another example of a pair of alternate exterior angles.

Solution

$\angle 2$ and $\angle 13$ (many other possibilities, must be different than answer to Example 1)

Example 3.6.3

Find the measure of each angle and the value of \)y\).

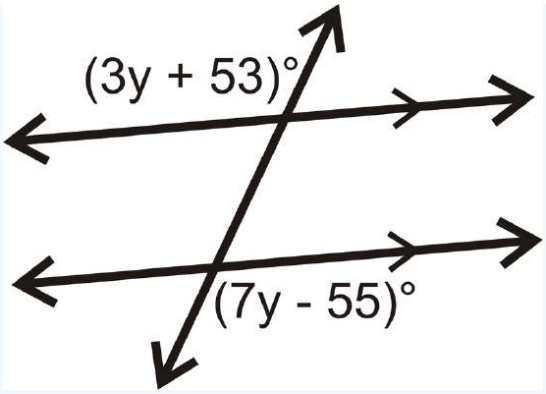

Figure 3.6.5

Solution

The angles are alternate exterior angles. Because the lines are parallel, the angles are equal.

$$(3y + 53)^\circ = (7y - 55)^\circ$$
$$108 = 4y$$
$$27 = y$$

If $y = 27, the\ each\ angle\ is\ [3(27) + 53]^\circ = 134^\circ$.

Example 3.6.4

The map below shows three roads in Julio's town.

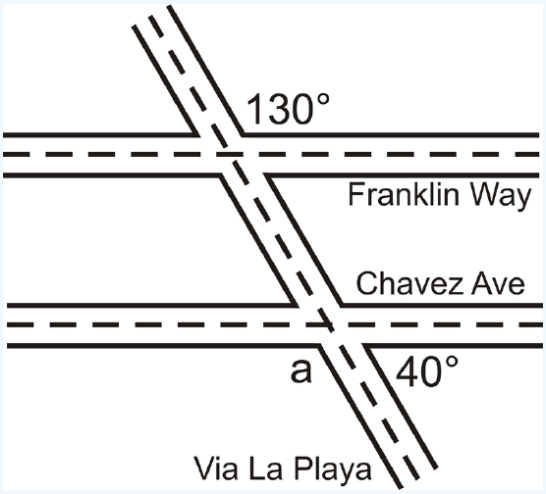

Figure 3.6.6

Solution

Julio used a surveying tool to measure two angles at the intersections in this picture he drew (NOT to scale). Julio wants to know if Franklin Way is parallel to Chavez Avenue.

The 130° angle and \angle a are alternate exterior angles. If $m\angle a = 130^\circ$, then the lines are parallel.

$$\angle a + 40^\circ = 180^\circ \qquad \textit{by the Linear Pair Postulate}$$
$$\angle a = 140^\circ$$

$140^\circ \neq 130^\circ$, so Franklin Way and Chavez Avenue are not parallel streets.

Example 3.6.5

Which lines are parallel if $\angle AFG \cong \angle IJM$?

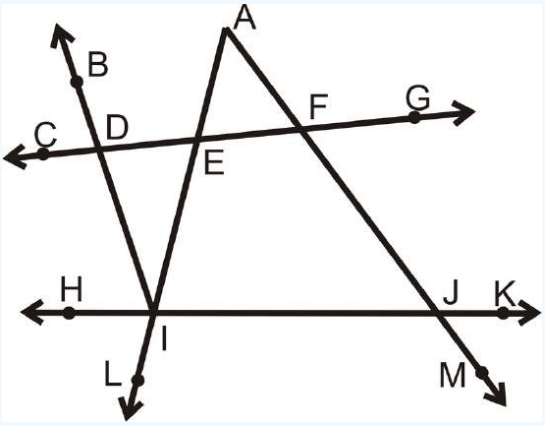

Figure 3.6.7

Solution

These two angles are alternate exterior angles so if they are congruent it means that $\overleftrightarrow{CG} \parallel \overleftrightarrow{HK}$.

Review

1. Find the value of x if $m\angle 1 = (4x + 35)°$, $m\angle 8 = (7x - 40)°$:

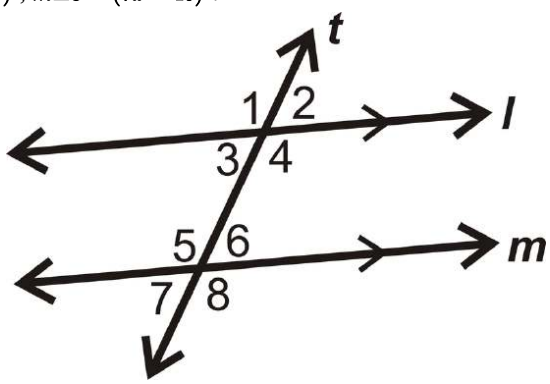

Figure 3.6.8

2. Are lines 1 and 2 parallel? Why or why not?

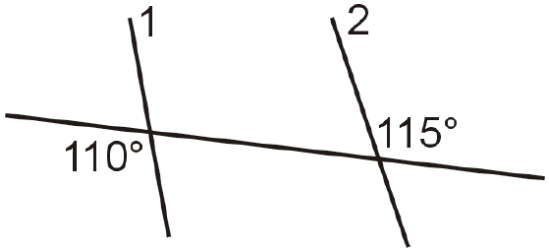

Figure 3.6.9

For 3-6, what does the value of x have to be to make the lines parallel?

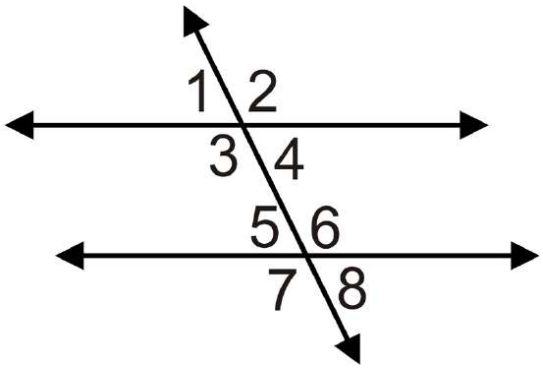

Figure **3.6.10**

3. $m\angle 2 = (8x)^\circ$ and $m\angle 7 = (11x - 36)^\circ$
4. $m\angle 1 = (3x + 5)^\circ$ and $m\angle 8 = (4x - 3)^\circ$
5. $m\angle 2 = (6x - 4)^\circ$ and $m\angle 7 = (5x + 10)^\circ$
6. $m\angle 1 = (2x - 5)^\circ$ and $m\angle 8 = (x)^\circ$

For 7-10, determine whether the statement is true or false.

7. Alternate exterior angles are always congruent.
8. If alternate exterior angles are congruent then lines are parallel.
9. Alternate exterior angles are on the interior of two lines.
10. Alternate exterior angles are on opposite sides of the transversal.

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.5.

Resources

Vocabulary

Term	Definition
alternate exterior angles	Alternate exterior angles are two angles that are on the exterior of two different lines, but on the opposite sides of the transversal.

Additional Resource

Video: Alternate Exterior Angles Principles - Basic

Activities: Alternate Exterior Angles Discussion Questions

Study Aids: Angles and Transversals Study Guide

Practice: Alternate Exterior Angles

Real World: Alternate Exterior Angles

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3.7: Same Side Interior Angles

Angles on the same side of a transversal and inside the lines it intersects.

Same side interior angles are two angles that are on the same side of the **transversal** and on the interior of (between) the two lines.

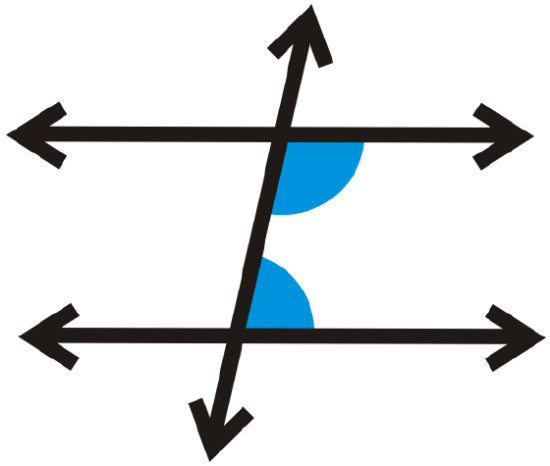

Figure 3.7.1

Same Side Interior Angles Theorem: If two parallel lines are cut by a transversal, then the same side interior angles are supplementary.

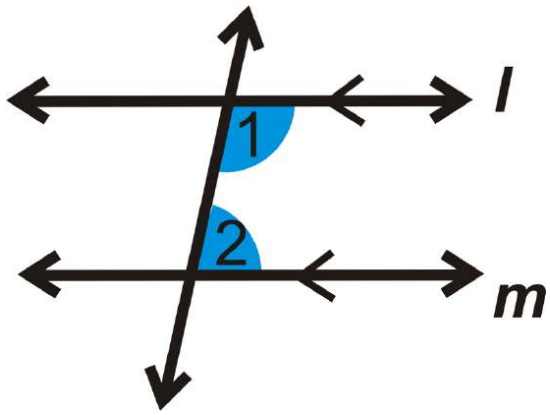

Figure 3.7.2

If $l \parallel m$, then $m\angle 1 + m\angle 2 = 180°$.

Converse of the Same Side Interior Angles Theorem: If two lines are cut by a transversal and the same side interior angles are supplementary, then the lines are parallel.

If

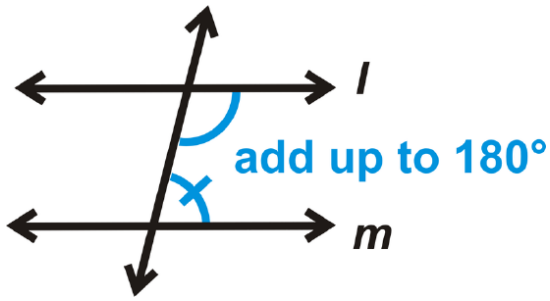

Figure 3.7.3

then $l \parallel m$

Suppose you were presented with two angles that are on the same side of a transversal and between the two parallel lines crossed by the transversal. How would you describe these angles and what could you conclude about their measures?

Example 3.7.1

Is $l \parallel m$? How do you kn

Solution

These angles are Same S . So, if they add up to $180°$, then $l \parallel m$.

$130° + 67° = 197°$, therefore the lines are not parallel.

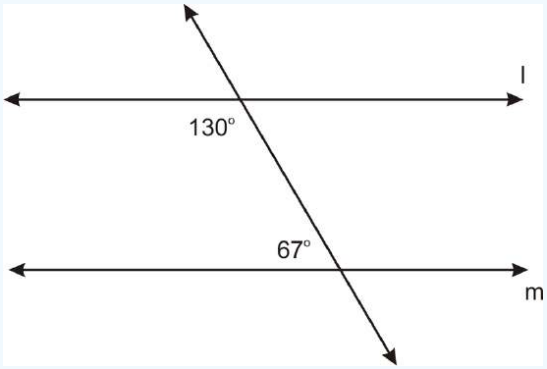

Figure 3.7.4

Example 3.7.2

Give two examples of same side interior angles in the diagram:

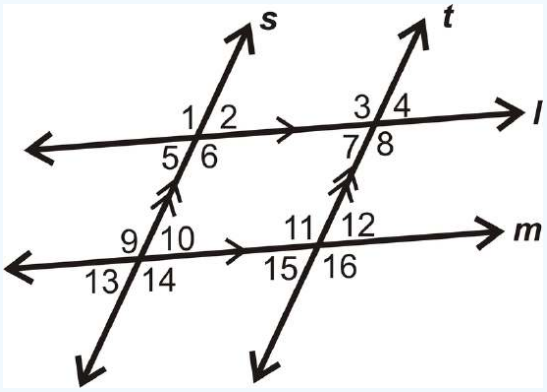

Figure 3.7.5

Solution

There are MANY examples of same side interior angles in the diagram. Two are $\angle 6$ and $\angle 10$, and $\angle 8$ and $\angle 12$.

Example 3.7.3

Find the value of x.

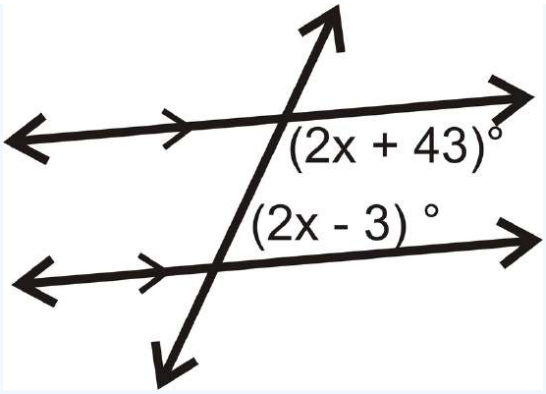

Figure 3.7.6

Solution

The given angles are same side interior angles. Because the lines are parallel, the angles add up to $180°$.

$$\begin{aligned}(2x+43)°+(2x-3)°&=180°\\(4x+40)°&=180°\\4x&=140\\x&=35\end{aligned}$$

Example 3.7.4

Find the value of y.

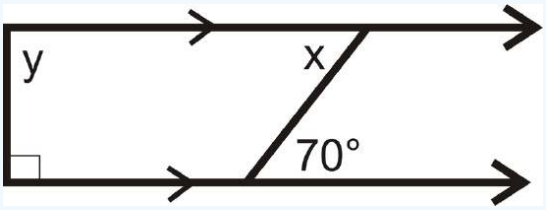

Figure 3.7.7

Solution

y is a same side interior angle with the marked right angle. This means that $90° + y = 180$ so $y = 90$.

Example 3.7.5

Find the value of x if $m\angle 3 = (3x + 12)°$ and $m\angle 5 = (5x + 8)°$.

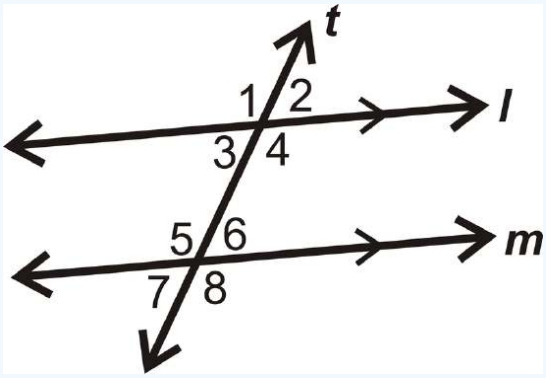

Figure 3.7.8

Solution

These are same side interior angles, so set up an equation and solve for x. Remember that same side interior angles add up to $180°$.

$$(3x+12)^\circ + (5x+8)^\circ = 180^\circ$$
$$(8x+20)^\circ = 180^\circ$$
$$8x = 160$$
$$x = 20$$

Review

For questions 1-2, use the diagram to determine if each angle pair is congruent, supplementary or neither.

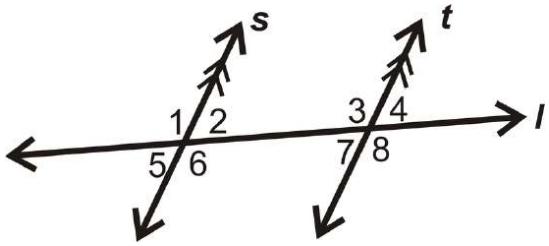

Figure **3.7.9**

1. $\angle 5$ and $\angle 8$
2. $\angle 2$ and $\angle 3$
3. Are the lines parallel? Justify your answer.

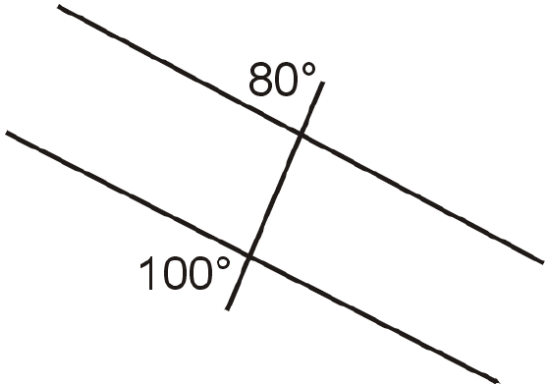

Figure **3.7.10**

In 4-5, use the given information to determine which lines are parallel. If there are none, write none. Consider each question individually.

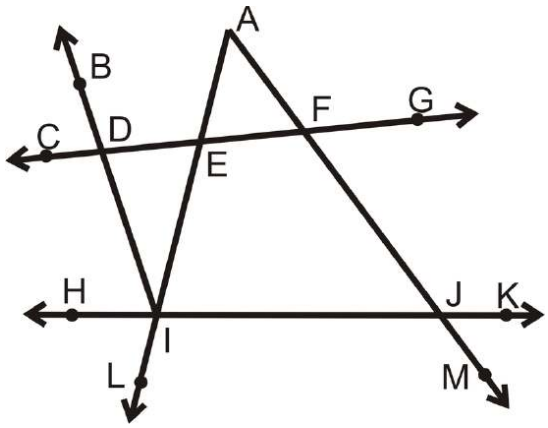

Figure **3.7.11**

4. $\angle AFD$ *and* $\angle BDF$ are supplementary
5. $\angle DIJ$ *and* $\angle FJI$ are supplementary

For 6-8, what does the value of x have to be to make the lines parallel?

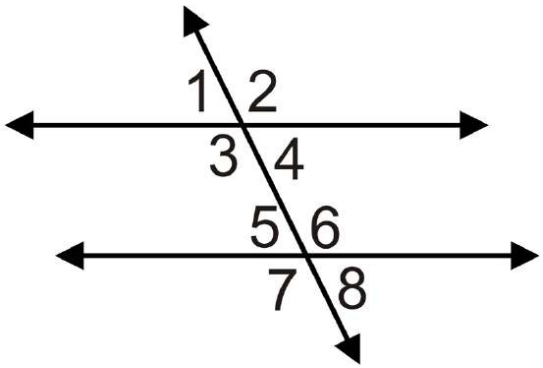

Figure 3.7.12

6. $m\angle 3 = (3x + 25)^\circ$ and $m\angle 5 = (4x - 55)^\circ$
7. $m\angle 4 = (2x + 15)^\circ$ and $m\angle 6 = (3x - 5)^\circ$
8. $m\angle 3 = (x + 17)^\circ$ and $m\angle 5 = (3x - 5)^\circ$

For 9-10, determine whether the statement is true or false.

9. Same side interior angles are on the same side of the transversal.
10. Same side interior angles are congruent when lines are parallel.

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.6.

Vocabulary

Term	Definition
same side interior angles	Same side interior angles are two angles that are on the same side of the transversal and on the interior of the two lines.
supplementary angles	Two angles that add up to 180°.
transversal	A line that intersects two other lines.

Additional Resource

Video: Same Side Interior Angles Principles - Basic

Activities: Same Side Interior Angles Discussion Questions

Study Aids: Angles and Transversals Study Guide

Practice: Same Side Interior Angles

Real World: Alternate Exterior Angles

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3.8: Angles and Perpendicular Lines

Lines that intersect at a 90 degree or right angle.

Two lines are **perpendicular** when they intersect to form a 90° **angle**. Below, $l \perp \overline{AB}$.

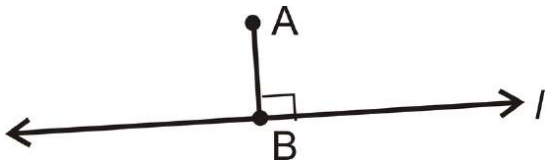

Figure **3.8.1**

In the definition of perpendicular the word “line” is used. However, line segments, rays and planes can also be perpendicular. The image below shows two parallel planes, with a third blue plane that is perpendicular to both of them.

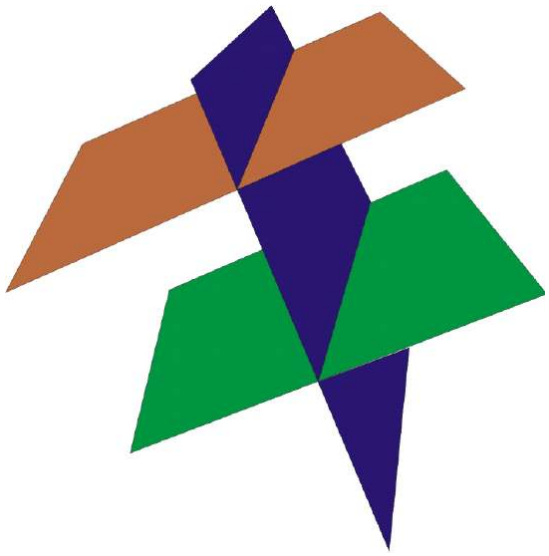

Figure **3.8.2**

Basic Facts about Perpendicular Lines

Theorem #1: If $l \parallel m$ and $n \perp l$, then $n \perp m$.

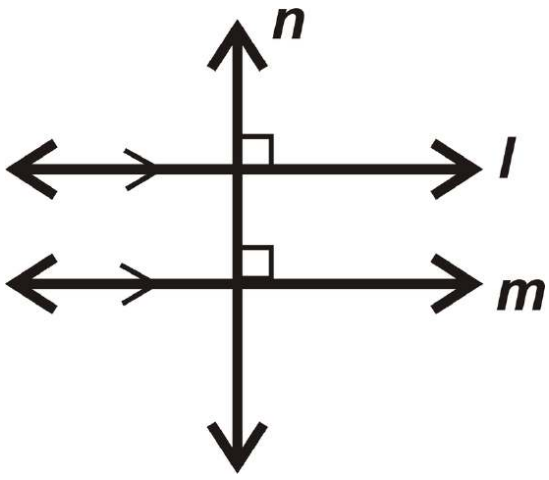

Figure **3.8.3**

Theorem #2: If $l \perp n$ and $n \perp m$, then $l \parallel m$.

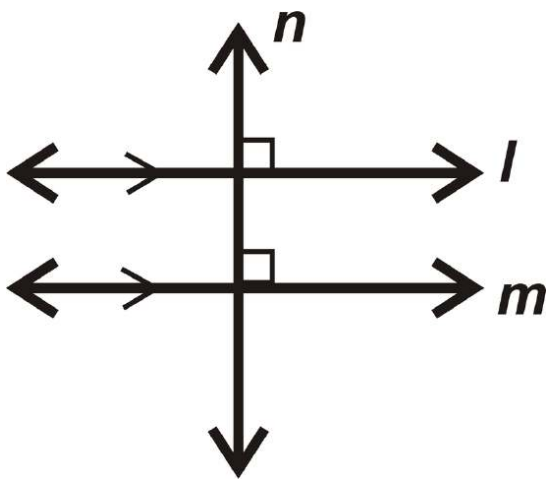

Figure 3.8.4

Postulate: For any line and a point ***not*** on the line, there is one line perpendicular to this line passing through the point. There are infinitely many lines that pass through ***A***, but only ***one*** that is perpendicular to ***l***.

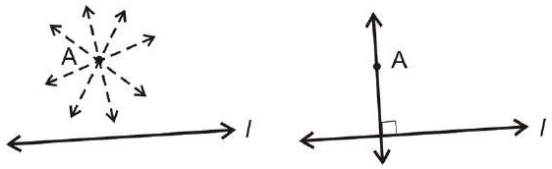

Figure 3.8.5

What if you were given a pair of lines that intersect each other at a 90° angle? What terminology would you use to describe such lines?

Example 3.8.1

Determine the measure

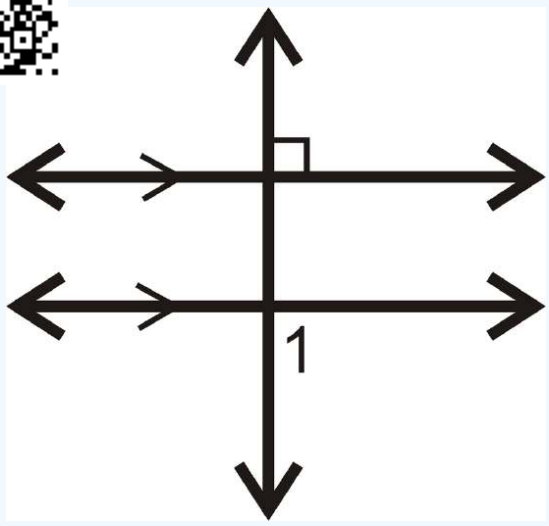

Figure 3.8.6

Solution

We know that both parallel lines are perpendicular to the transversal.

$m\angle 1 = 90°$.

Example 3.8.2

Find $m\angle 1$.

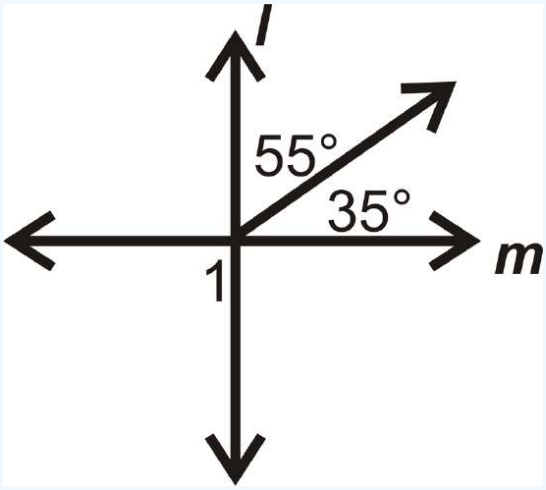

Figure 3.8.7

Solution

The two adjacent angles add up to $90°$, so $l \perp m$.

$$m\angle 1 = 90°$$

because it is a vertical angle to the pair of adjacent angles and vertical angles are congruent.

Example 3.8.3

Which of the following is the best example of perpendicular lines: Latitude on a Globe, Opposite Sides of a Picture Frame, Fence Posts, or Adjacent Sides of a Picture Frame?

Solution

The best example would be adjacent sides of a picture frame. Remember that adjacent means next to and sharing a vertex. The adjacent sides of a picture frame meet at a $90°$ angle and so these sides are perpendicular.

Example 3.8.4

Is $\overleftrightarrow{SO} \perp \overrightarrow{GD}$?

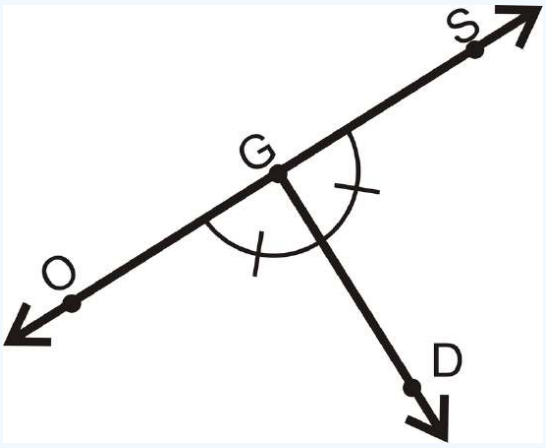

Figure 3.8.8

Solution

$\angle OGD \cong \angle SGD$ and the angles form a linear pair. This means both angles are $90°$, so the lines are perpendicular.

Example 3.8.5

Write a 2-column proof to prove Theorem #1. *Note: You need to understand* corresponding angles *in order to understand this proof. If you have not yet learned corresponding angles, be sure to check out that concept first, or skip this example for now.*

Given: $l \parallel m, l \perp n$

Prove: $n \perp m$

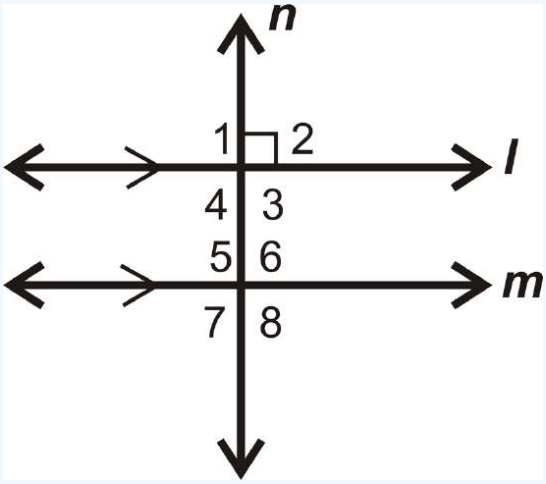

Figure 3.8.9

Solution

Statement	*Reason*
1. $l \parallel m, l \perp n$	1. Given
2. $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4 are right angles$	2. Definition of perpendicular lines
3. $m\angle 1 = 90°$	3. Definition of a right angle
4. $m\angle 1 = m\angle 5$	4. Corresponding Angles Postulate
5. $m\angle 5 = 90°$	5. Transitive *PoE*
6. $m\angle 6 = m\angle 7 = 90°$	6. Congruent Linear Pairs
7. $m\angle 8 = \(90°$	7. Vertical Angles Theorem
8. $\angle 5$, $\angle 6$, $\angle 7$, and $\angle 8$ are right angles	8. Definition of right angle
9. $n \perp m$	9. Definition of perpendicular lines

Review

Use the figure below to answer questions 1-2. The two pentagons are parallel and all of the rectangular sides are perpendicular to both of them.

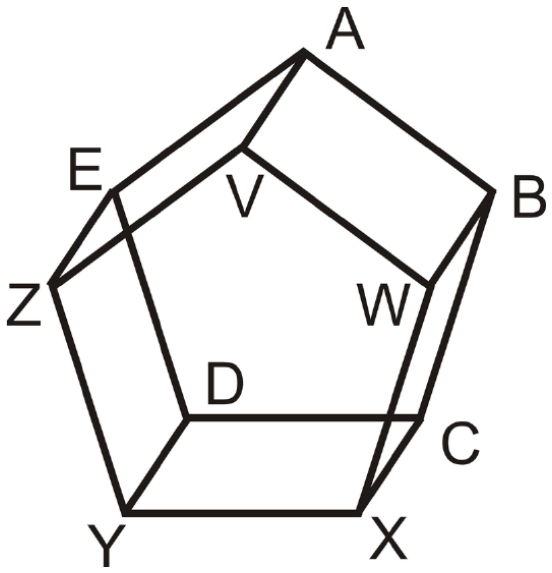

Figure **3.8.10**

1. List a pair of perpendicular lines.
2. For $\overline{AB}$, how many perpendicular lines would pass through point V? Name this/these line(s).

Use the picture below for question 3.

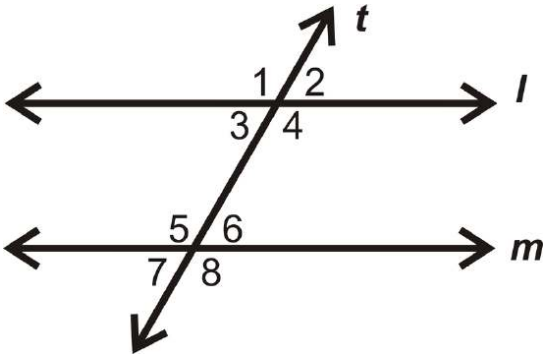

Figure **3.8.11**

3. If $t \perp l$, is $t \perp m$? Why or why not?

Find the measure of $\angle 1$ for each problem below.

4.

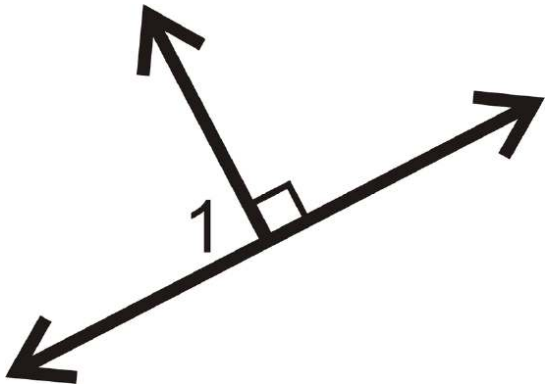

Figure **3.8.12**

5.

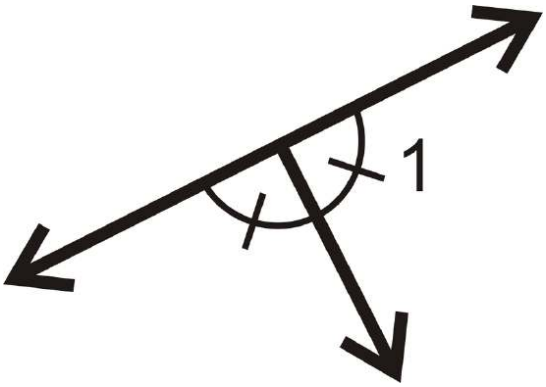

Figure **3.8.13**

6.

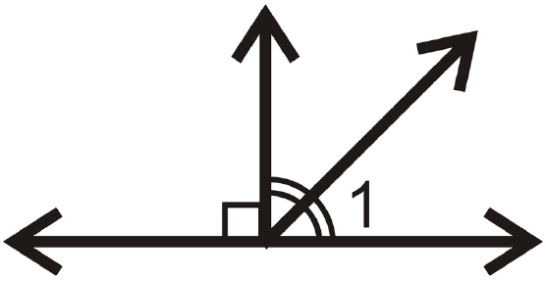

Figure **3.8.14**

7.

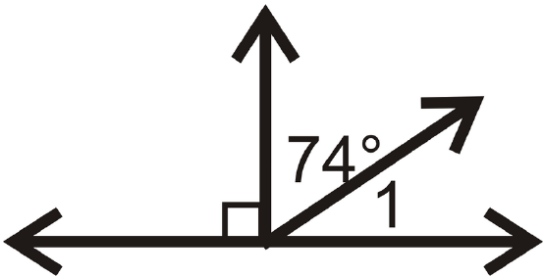

Figure **3.8.15**

8.

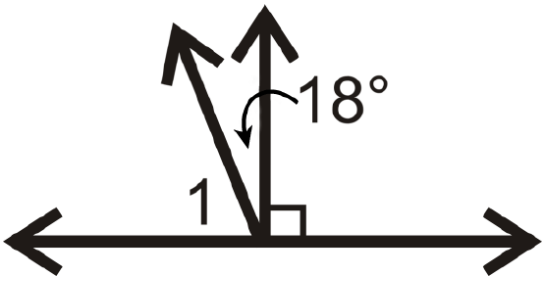

Figure **3.8.16**

9.

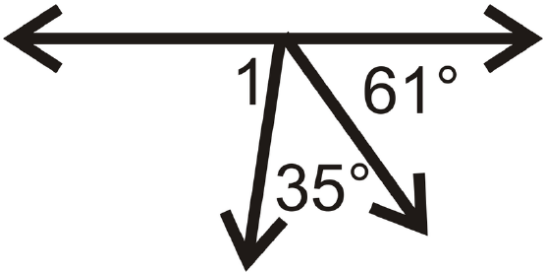

Figure **3.8.17**

10.

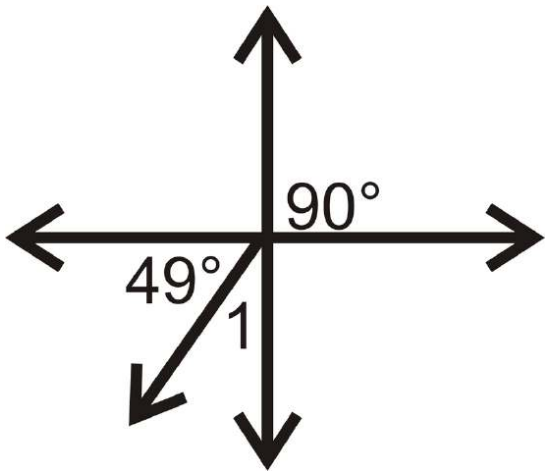

Figure 3.8.18

11.

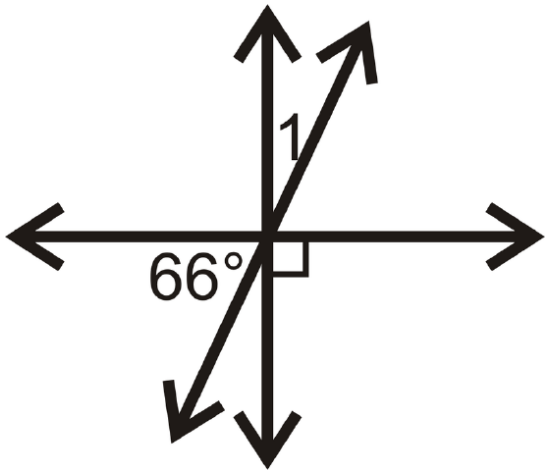

Figure 3.8.19

12.

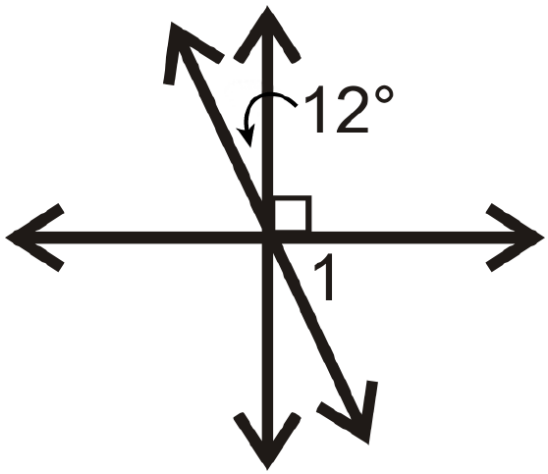

Figure 3.8.20

In questions 13-16, determine if $l \perp m$.

13.

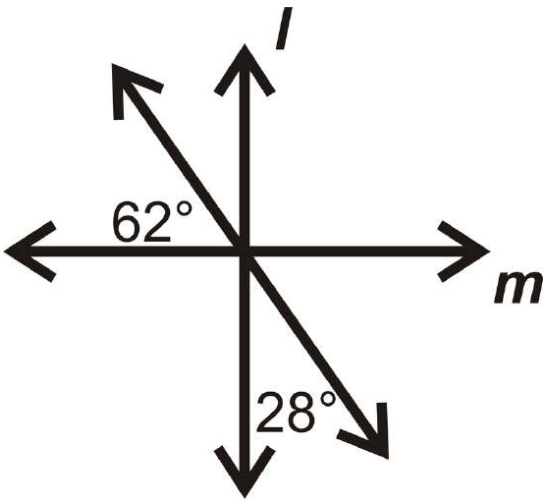

Figure 3.8.21

14.

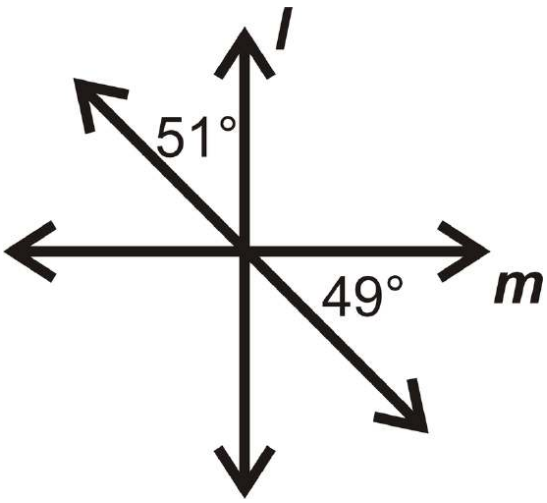

Figure 3.8.22

15.

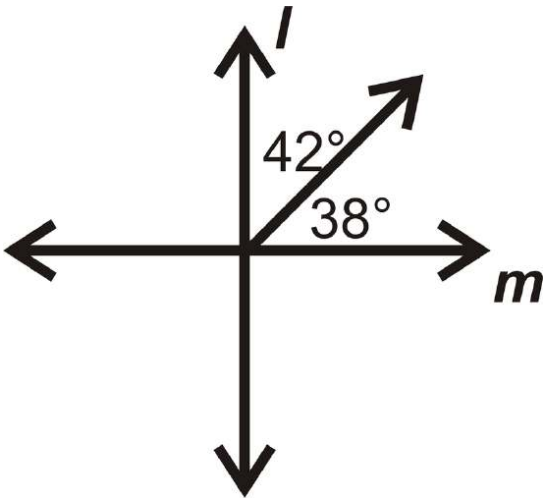

Figure 3.8.23

16.

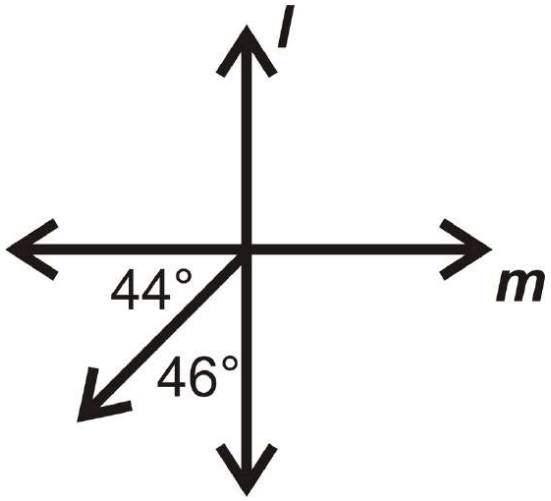

Figure 3.8.24

Fill in the blanks in the proof below.

17. Given: $l \perp m$, $l \perp n$ Prove: $m \parallel n$

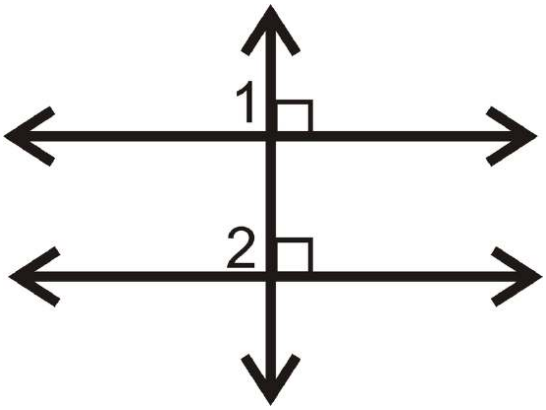

Figure 3.8.25

Statement	***Reason***
1.	1.
2. $\angle 1$ and $\angle 2$ are right angles	2.
3.	3. Definition of right angles
4.	4. Transitive ***PoE***
5. $m \parallel n$	5.

Resources

Vocabulary

Term	Definition
perpendicular	Two lines are ***perpendicular*** when they intersect to form a $90°$ angle.
Angle	A geometric figure formed by two rays that connect at a single point or vertex.
Perpendicular lines	Perpendicular lines are lines that intersect at a $90°$ angle.

Additional Resources

Video: Perpendicular Lines Principles - Basic

Activities: Perpendicular Lines Discussion Questions

Study Aids: Lines and Angles Study Guide

Practice: Angles and Perpendicular Lines

Real World: Turning The Tables

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3.9: Parallel Lines in the Coordinate Plane

Lines with the same slope that never intersect.

Parallel lines are two lines that never intersect. In the coordinate plane, that would look like this:

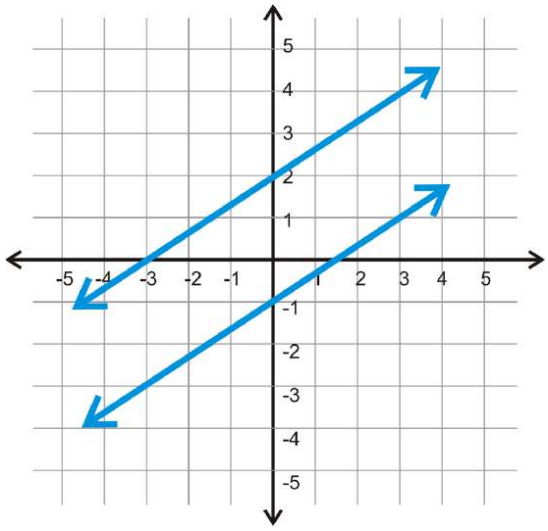

Figure 3.9.1

If we take a closer look at these two lines, the slopes are both $\frac{2}{3}$.

This can be generalized to any pair of parallel lines. Parallel lines always have the same slope and different y–intercepts.

What if you were given two parallel lines in the coordinate plane? What could you say about their slopes?

Video

Example 3.9.1

Find the equation of the line that is parallel to $y = \frac{1}{4}x + 3$ and passes through $(8, -7)$.

Solution

We know that parallel lines have the same slope, so the line will have a slope of $dfrac14$. Now, we need to find the y–intercept. Plug in 8 for x and -7 for y to solve for the *new* y–intercept (b).

$$\begin{aligned} -7 &= \frac{1}{4}(8) + b \\ -7 &= 2 + b \\ -9 &= b \end{aligned}$$

The equation of the parallel line is $y = \frac{1}{4}x - 9$.

Example 3.9.2

Are the lines $3x + 4y = 7$ and $y = \frac{3}{4}x + 1$ parallel?

Solution

First we need to rewrite the first equation in slope-intercept form.

$$\begin{aligned} 3x + 4y &= 7 \\ 4y &= -3x + 7 \\ y &= -\frac{3}{4}x + \frac{7}{4} \end{aligned}$$

The slope of this line is $-\frac{3}{4}$ while the slope of the other line is $\frac{3}{4}$. Because the slopes are different the lines are not parallel.

Example 3.9.3

Find the equation of the line that is parallel to $y = -\frac{1}{3}x + 4$ and passes through $(9, -5)$.

Solution

Recall that the equation of a line is y=mx+b, where m is the slope and b is the y–intercept. We know that parallel lines have the same slope, so the line will have a slope of −13. Now, we need to find the y–intercept. Plug in 9 for x and -5 for y to solve for the *new* y–intercept (b).

$$-5 = -\frac{1}{3}(9) + b$$
$$-5 = -3 + b$$
$$-2 = b$$

The equation of parallel line is $y = -\frac{1}{3}x - 2$.

Example 3.9.4

Find the equation of the lines below and determine if they are parallel.

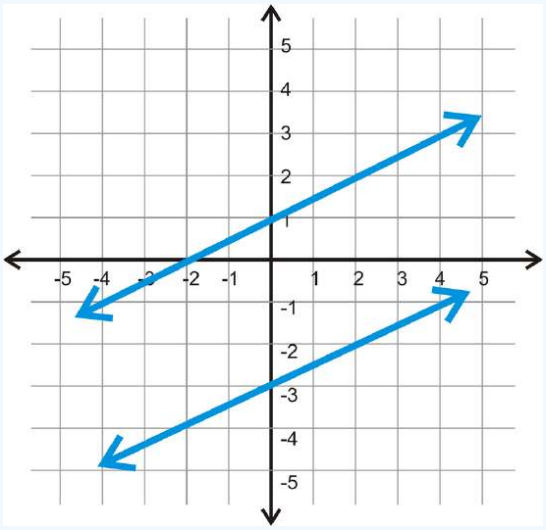

Figure 3.9.2

Solution

The top line has a y–intercept of 1. From there, use “rise over run” to find the slope. From the y–intercept, if you go up 1 and over 2, you hit the line again, $m = \frac{1}{2}$. The equation is $y = \frac{1}{2}x + 1$.

For the second line, the y–intercept is -3. The “rise” is 1 and the “run” is 2 making the slope $\frac{1}{2}$. The equation of this line is $y = \frac{1}{2}x - 3$.

The lines are **parallel** because they have the same slope.

Example 3.9.5

Find the equation of the line that is parallel to the line through the point marked with a blue dot.

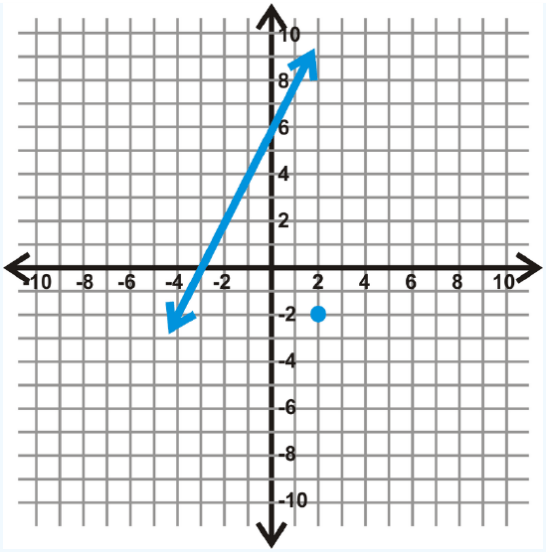

Figure 3.9.3

Solution

First, notice that the equation of the line is $y = 2x + 6$ and the point is $(2, -2)$. The parallel would have the same slope and pass through $(2, -2)$.

$$\begin{aligned} y &= 2x + b \\ -2 &= 2(2) + b \\ -2 &= 4 + b \\ -6 &= b \end{aligned}$$

The equation of the parallel line is $y = 2x + -6$.

Review

Determine if each pair of lines are parallel. Then, graph each pair on the same set of axes.

1. $y = 4x - 2$ and $y = 4x + 5$
2. $y = -x + 5$ and $y = x + 1$
3. $5x + 2y = -4$ and $5x + 2y = 8$
4. $x + y = 6$ and $4x + 4y = -16$

Determine the equation of the line that is ***parallel*** to the given line, through the given point.

5. $y = -5x + 1; (-2, 3)$
6. $y = \frac{2}{3}x - 2; (9, 1)$
7. $x - 4y = 12; (-16, -2)$
8. $3x + 2y = 10; (8, -11)$

Find the equation of the two lines in each graph below. Then, determine if the two lines are parallel.

9.

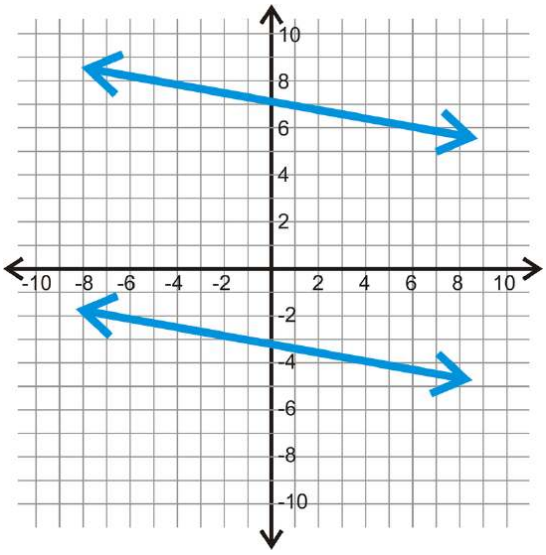

Figure **3.9.4**

For the line and point below, find a parallel line, through the given point.

10.

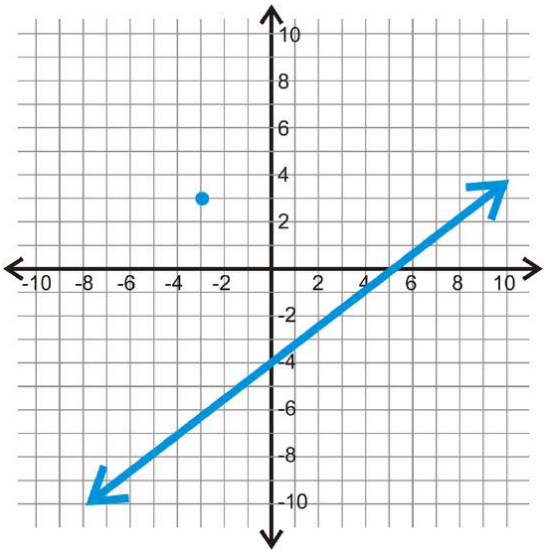

Figure **3.9.5**

11.

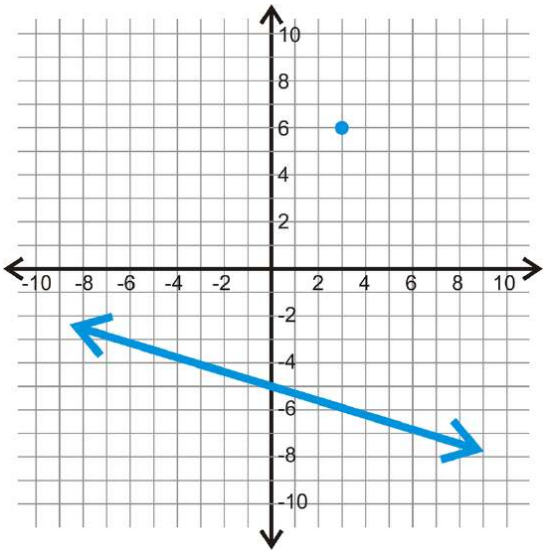

Figure **3.9.6**

12.

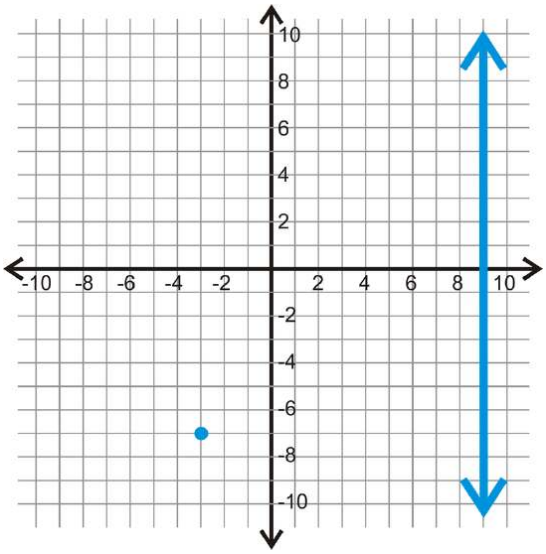

Figure **3.9.7**

13.

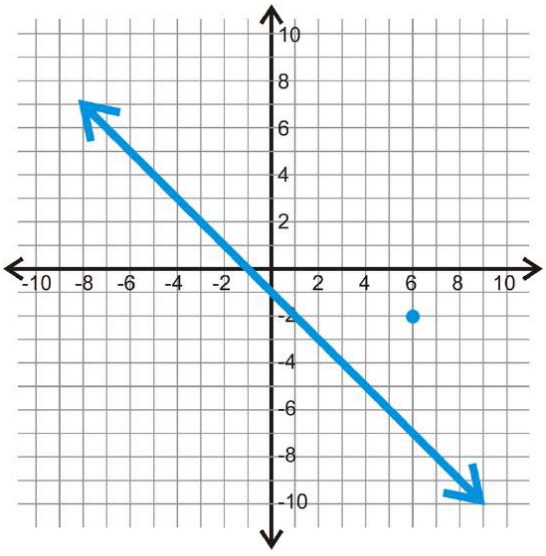

Figure **3.9.8**

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.8.

Vocabulary

Term	Definition
Parallel	Two or more lines are parallel when they lie in the same plane and never intersect. These lines will always have the same slope.

Additional Resource

Video: Equations of Parallel and Perpendicular Lines

Activities: Parallel Lines in the Coordinate Plane Discussion Questions

Study Aids: Lines in the Coordinate Plane

Practice: Parallel Lines in the Coordinate Plane

Real World: Parallel Lines In The Coordiante Plane

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3.10: Perpendicular Lines in the Coordinate Plane

Lines with opposite sign, reciprocal slopes that intersect at right angles.

Here you'll learn that the slopes of **perpendicular** lines are negative reciprocals of each other. You'll then apply this fact to determine if two lines are perpendicular and to find what their equations are.

Perpendicular lines are two lines that intersect at a , or right, angle. In the coordinate plane, that would look like this:

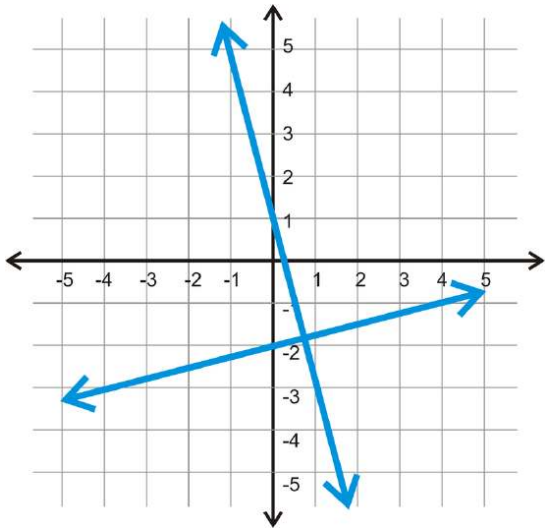

Figure **3.10.1**

If we take a closer look at these two lines, the slope of one is -4 and the other is .

This can be generalized to any pair of perpendicular lines in the coordinate plane. The slopes of perpendicular lines are opposite reciprocals of each other.

What if you were given two perpendicular lines in the coordinate plane? What could you say about their slopes?

Vocabulary

Term	Definition
perpendicular	Two lines that intersect at a **90°**, or right, angle. The slopes of perpendicular lines are opposite reciprocals of each other.

Additional Resource

Video: Equations of Parallel and Perpendicular Lines

Activities: Perpendicular Lines in the Coordinate Plane Discussion Questions

Study Aids: Lines in the Coordinate Plane

Practice: Perpendicular Lines in the Coordinate Plane

Real World: Perpendicular Lines In The Coordiante Plane

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3.11: Line Construction

Create lines that do not intersect or intersect at a 90 degree angle using a compass and straightedge.

Parallel and **Perpendicular Lines**

Constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

To construct **parallel lines**, remember that *if **corresponding angles** are congruent then lines are parallel*. This means that if you can copy an angle to create congruent corresponding angles, you can create parallel lines. This will be explored in Example A.

To construct **perpendicular lines**, remember that you already know how to construct a perpendicular bisector. You can use this method to construct a line perpendicular to another line through any given point.

Let's take a look at an exa

Use your straightedge to c oint like the one below. Then, construct a line through the point that is parallel to the original line.

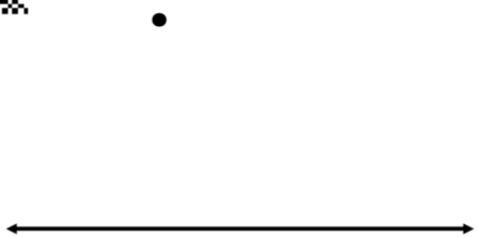

Figure **3.11.1**

Start by using your straightedge to draw a line through the point that intersects the original line. This will become the transversal after you have constructed the parallel line.

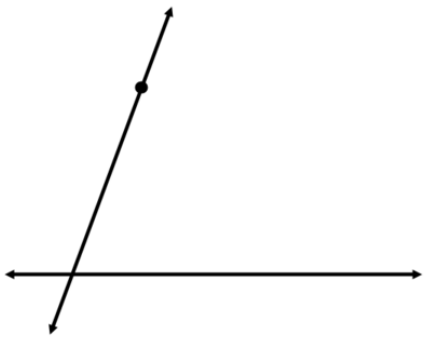

Figure **3.11.2**

Now, your goal is to copy one of the four angles created at the intersection of the two lines. Draw an arc through the angle you will copy:

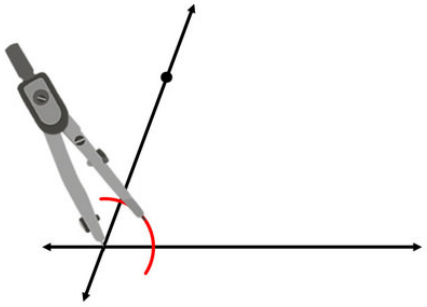

Figure **3.11.3**

Draw an arc with the same radius in the corresponding location, with the original point as the **vertex**.

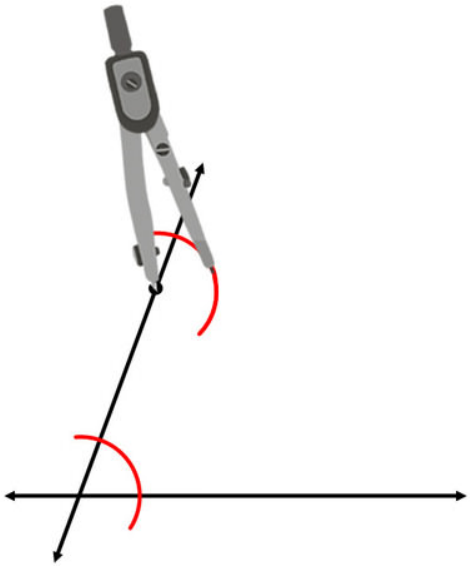

Figure **3.11.4**

Continue to copy the angle by measuring its width and marking off the correct width for the new angle.

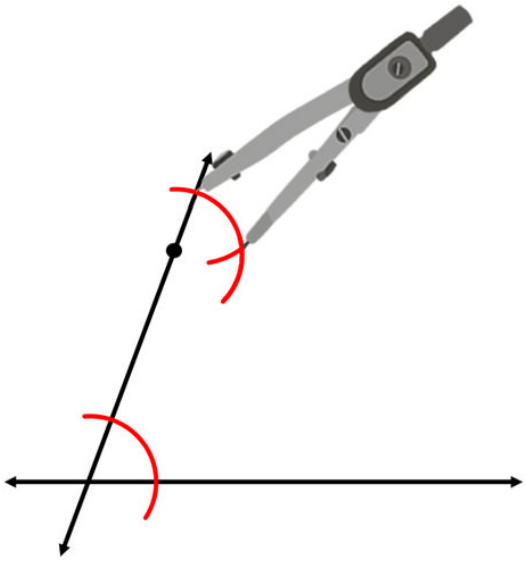

Figure **3.11.5**

Use your straightedge to construct the parallel line.

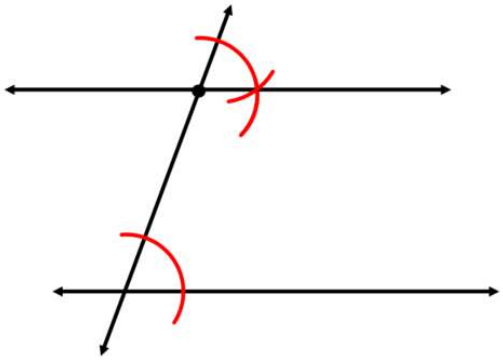

Figure **3.11.6**

Now, let's explain why the two lines in the second example above are parallel.

If corresponding angles are congruent, then lines are parallel. In the second example above, an angle was copied. This created congruent angles. Because the copied angle was put in the location of the corresponding angle, the construction created congruent corresponding angles. Therefore, the two lines must be parallel.

Use your straightedge to draw a line and a point like the one below. Then, construct a line through the point that is perpendicular to the original line.

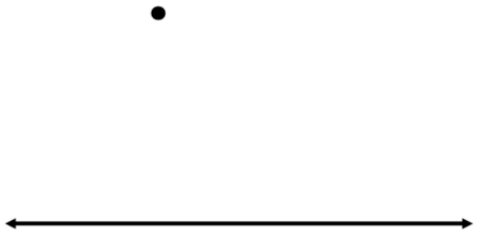

Figure **3.11.7**

You already know how to construct the perpendicular bisector of a segment. First, find a segment whose perpendicular bisector will pass through the given point. Draw a partial circle centered at the point that passes through the given line two times.

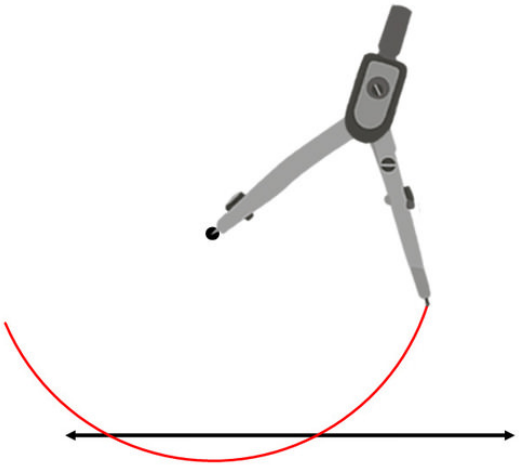

Figure **3.11.8**

The segment that connects the two points of intersection is the segment you will construct a perpendicular bisector for:

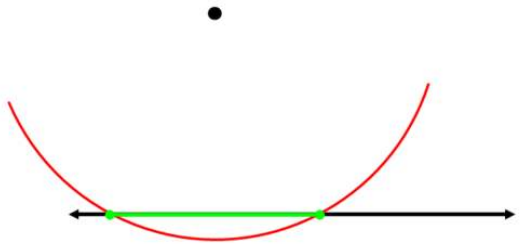

Figure **3.11.8**

Construct the perpendicular bisector of the green segment:

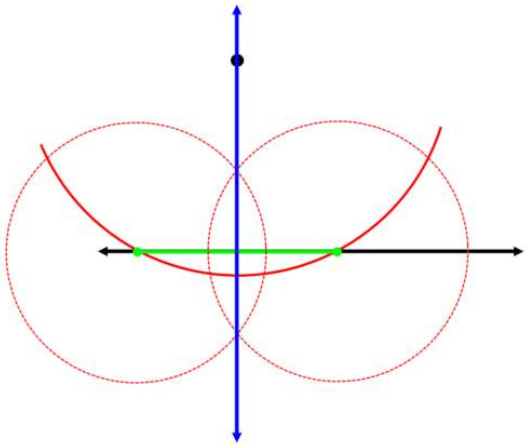

Figure **3.11.8**

The blue line is perpendicular to the original line and passes through the original point.

Example 3.11.1

Construct a triangle using a straightedge. Now construct the three altitudes of the triangle in order to find the **orthocenter** of the triangle.

Solution

Start with a generic triangle:

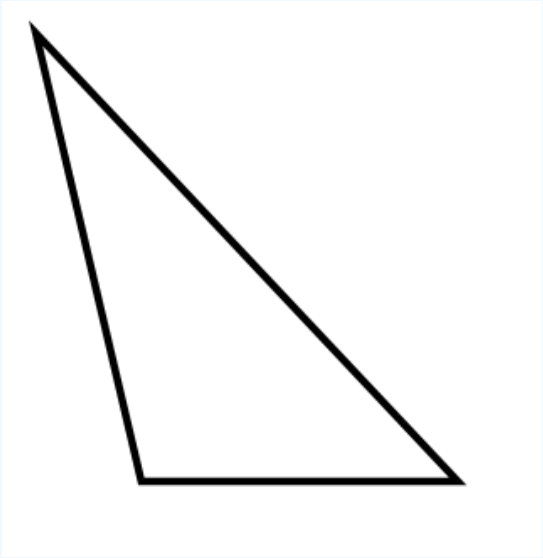

Figure **3.11.9**

Construct a perpendicular line through each side that passes through the opposite vertex. It might be necessary to extend the length of some of the sides. The blue line below is the first **altitude**:

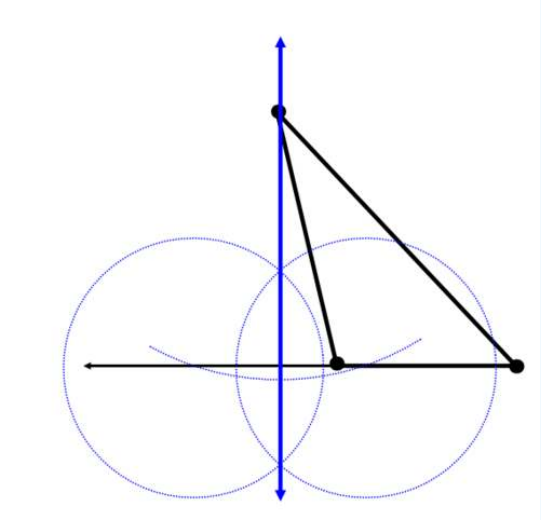

Figure **3.11.10**

The red line is the second altitude:

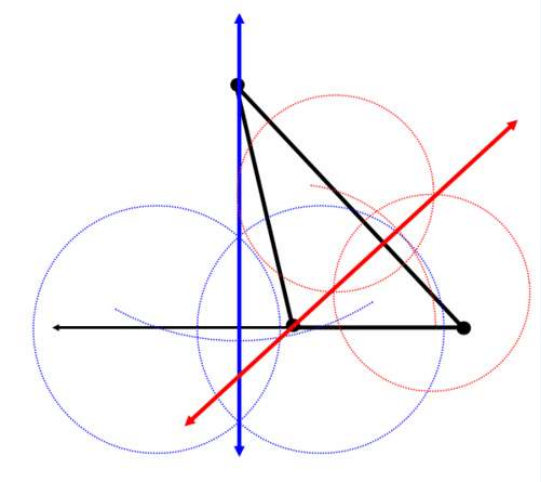

Figure **3.11.9**

The green line is the third altitude:

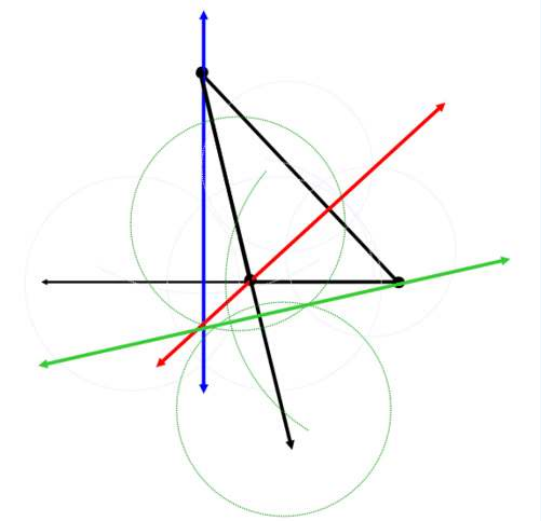

Figure **3.11.10**

The blue, red, and green lines intersect in a point. This point is the orthocenter.

Example 3.11.2

Draw a line segment. Construct a line parallel to the line segment.

Solution

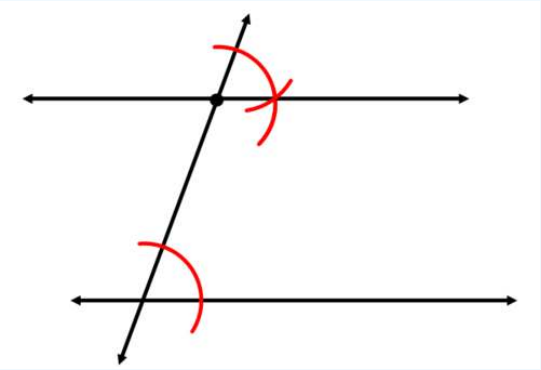

Figure **3.11.11**

Example 3.11.3

Extend your construction from Example 2 to construct a parallelogram.

Solution

Extend your construction from Example 2 to construct a parallelogram.

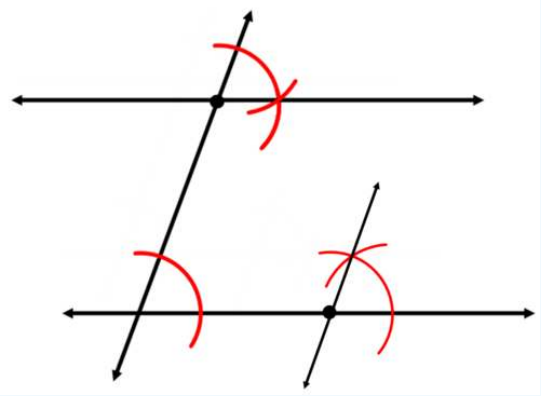

Figure **3.11.12**

Extend the new parallel line. The quadrilateral formed is a parallelogram.

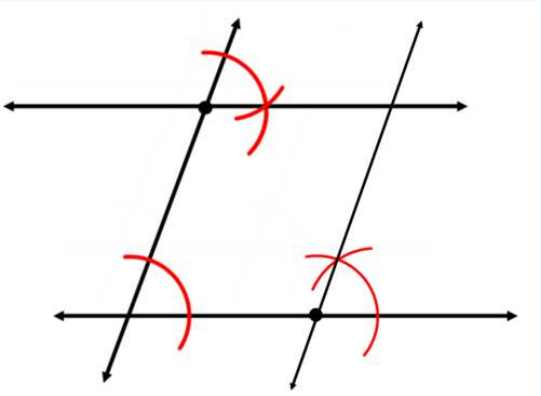

Figure **3.11.13**

Example 3.11.4

How can you be sure that your quadrilateral from Example 3 is a parallelogram?

Solution

A parallelogram is a quadrilateral with two pairs of parallel sides. The construction created two pairs of parallel lines, so the quadrilateral must be a parallelogram.

Review

Draw a line segment and a point.

1. Construct a line perpendicular to the line segment that passes through the point.
2. Construct a line parallel to the line segment that passes through the point.
3. Extend your construction to construct a rectangle. Explain what you did to construct the rectangle.

Draw another line segment and point.

4. Construct a line parallel to the line segment that passes through the point.
5. Extend your construction to construct a parallelogram. Explain what you did to construct the parallelogram.

Use a straightedge to draw a triangle.

6. Construct the altitudes of the triangle.
7. Find the point of intersection of the altitudes. What point is this?

8. Was your point of intersection inside or outside the triangle? If it was inside, create a new triangle that will cause the point of intersection to be **outside** the triangle. If it was outside, create a new triangle that will cause the point of intersection to be **inside** the triangle.

Draw another line segment and point.

9. Construct a line parallel to the line segment that passes through the point.

10. Extend your construction to construct a trapezoid. Explain what you did to construct the trapezoid.

Draw another line segment and point.

11. Construct a line perpendicular to the line segment that passes through the point.

12. Extend your construction to construct a square. Explain what you did to construct the square.

13. Justify why the shape you created in #12 must be a square.

14. Explain why the method for constructing parallel lines involves copying an angle.

15. Explain why the method for constructing perpendicular lines relies on the method for constructing a perpendicular bisector.

Vocabulary

Term	Definition
altitude	An altitude of a triangle is a line segment from a vertex and is perpendicular to the opposite side. It is also called the height of a triangle.
Corresponding Angles	Corresponding angles are two angles that are in the same position with respect to the transversal, but on different lines.
Perpendicular lines	Perpendicular lines are lines that intersect at a 90° angle.
Vertex	A vertex is a point of intersection of the lines or rays that form an angle.
Orthocenter	The orthocenter is the point at which the three altitudes of a triangle meet.

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CHAPTER OVERVIEW

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Thumbnail: Diagram of two congruent triangles. $\triangle ABC \cong \triangle DEF$ (CC BY-SA; Merovingian via Wikipedia)

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4.1: Classify Triangles

Classify triangles as acute, right, obtuse or equiangular and as scalene, isosceles or equilateral. Use classifications to find missing information.

Triangle Classification

A **triangle** is any closed figure made by three line segments intersecting at their endpoints. Every triangle has three **vertices** (the points where the segments meet), three **sides** (the segments), and three **interior angles** (formed at each vertex). All of the following shapes are triangles.

Figure 4.1.1

The sum of the interior angles in a triangle is 180°. This is called the **Triangle Sum Theorem** and is discussed further in the "Triangle Sum Theorem" concept.

Angles can be classified by their size as acute, obtuse or right. In any triangle, two of the angles will always be acute. The third angle can be acute, obtuse, or right. ***We classify each triangle by this angle***.

Right Triangle: A triangle with one **right angle**.

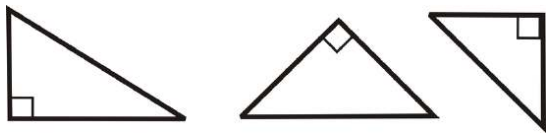


Figure 4.1.2

Obtuse Triangle: A triangle with one obtuse angle.

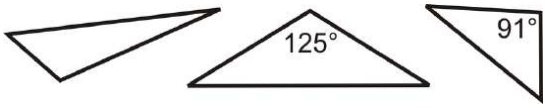


Figure 4.1.3

Acute Triangle: A triangle where all three angles are acute.

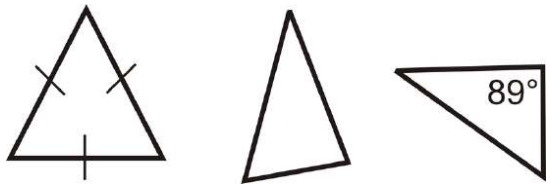


Figure 4.1.4

Equiangular Triangle: A triangle where all the angles are congruent.

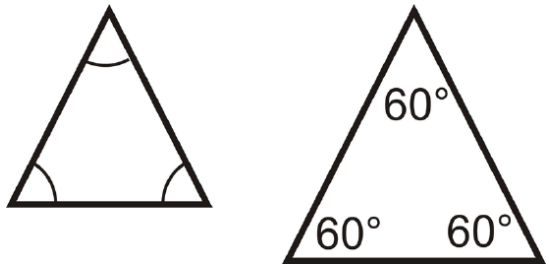


Figure 4.1.5

You can also classify a triangle by its sides.

Scalene Triangle: A triangle where all three sides are different lengths.

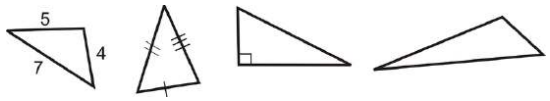


Figure **4.1.6**

Isosceles Triangle: A triangle with *at least* two congruent sides.

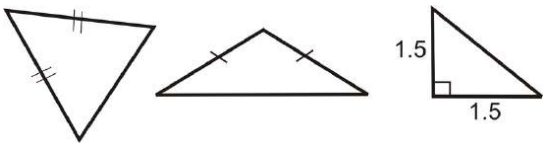


Figure **4.1.7**

Equilateral Triangle: A triangle with three congruent sides.

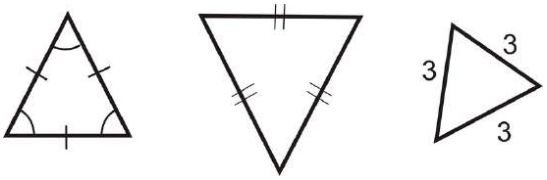


Figure **4.1.8**

Note that from the definitions, an **equilateral** triangle is also an isosceles triangle.

What if you were given the angle measures and/or side lengths of a triangle? How would you describe the triangle based on that information?

Example 4.1.1

Which of the figures bel s?

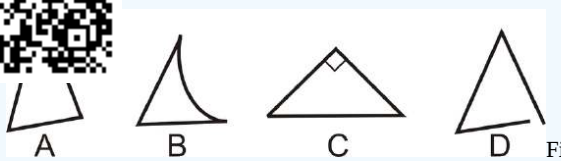


Figure **4.1.9**

Solution

B is not a triangle because it has one curved side. D is not closed, so it is not a triangle either.

Example 4.1.2

How many triangles are in the diagram below?

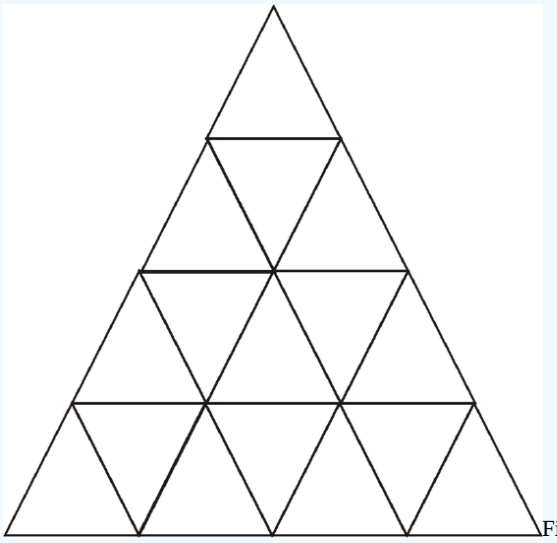


Figure 4.1.10

Solution

Start by counting the smallest triangles, 16.

Now count the triangles that are formed by 4 of the smaller triangles, 7.

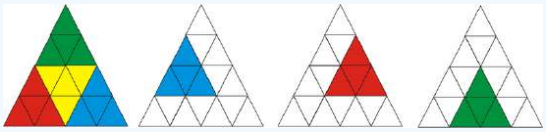


Figure \(\
PageIndex
{11}\)

Next, count the triangles that are formed by 9 of the smaller triangles, 3.

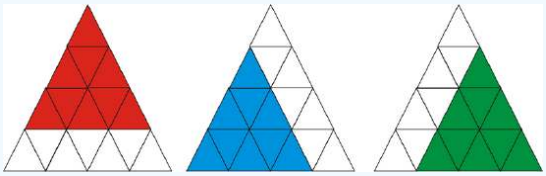


Figure 4.1.12

Finally, there is the one triangle formed by all 16 smaller triangles. Adding these numbers together, we get $16+7+3+1=27$.

Example 4.1.3

True or false: An equilateral triangle is **equiangular**.

Solution

True. Equilateral triangles have interior angles that are all congruent so they are equiangular.

Example 4.1.4

Which term best describes ΔRST below?

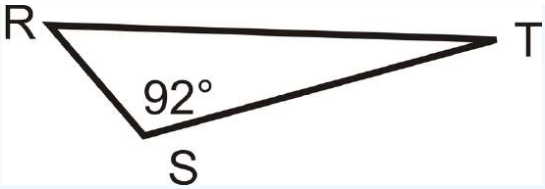


Figure **4.1.13**

Solution

This triangle has one labeled obtuse angle of $92°$. Triangles can have only one obtuse angle, so it is an obtuse triangle.

Example 4.1.5

Classify the triangle by its sides and angles.

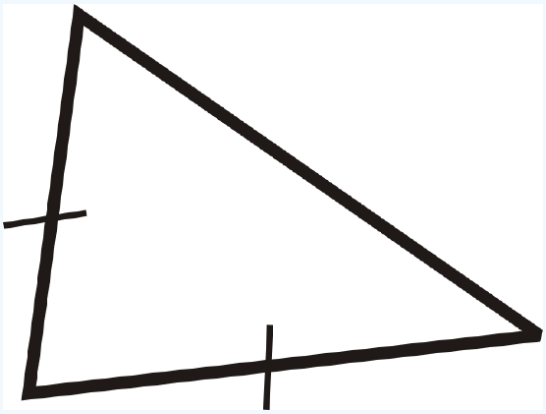


Figure **4.1.14**

Solution

We see that there are two congruent sides, so it is isosceles. By the angles, they all look acute. We say this is an acute isosceles triangle.

Review

For questions 1-6, classify each triangle by its sides and by its angles.

1.

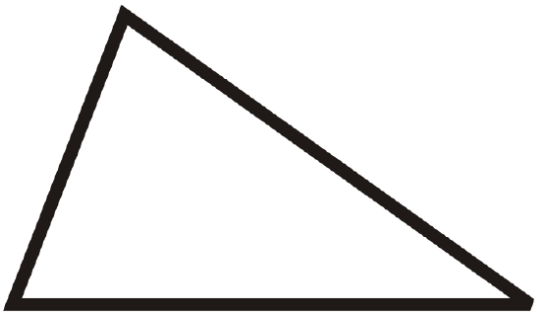


Figure **4.1.15**

2.

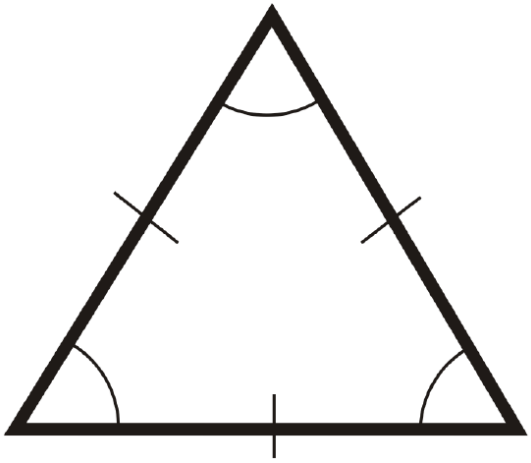


Figure **4.1.16**

3.

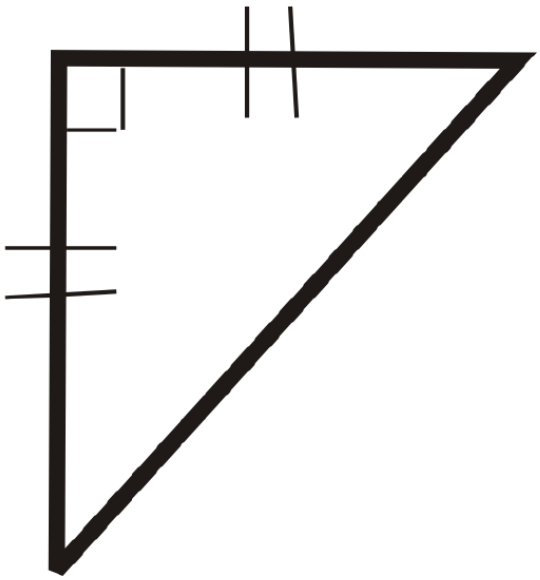


Figure **4.1.17**

4.

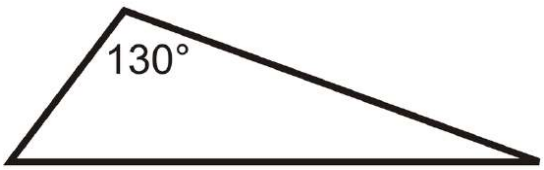


Figure **4.1.18**

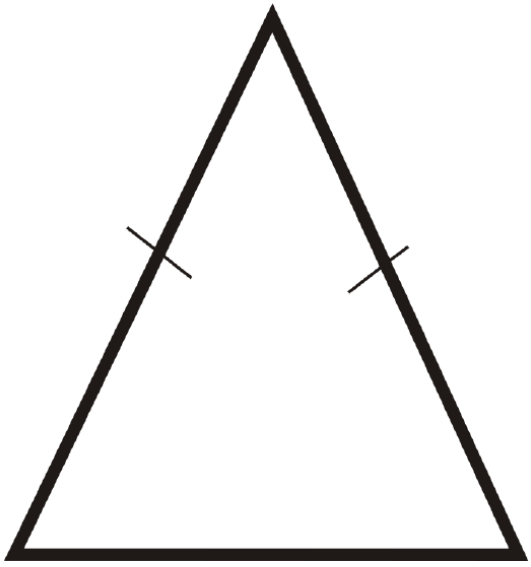
5. 

Figure **4.1.19**

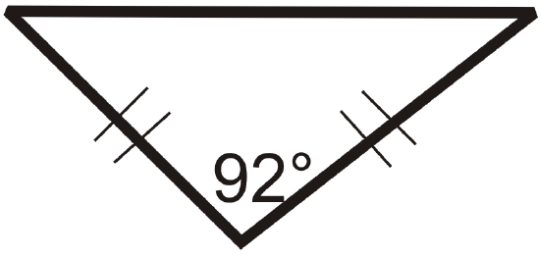
6. 

Figure **4.1.20**

7. Can you draw a triangle with a right angle and an obtuse angle? Why or why not?
8. In an isosceles triangle, can the angles opposite the congruent sides be obtuse?

For 9-10, determine if the statement is true or false.

9. Obtuse triangles can be isosceles.
10. A right triangle is acute.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.11.

Resources

Vocabulary

Term	Definition
Acute Triangle	An acute triangle has three angles that each measure less than 90 degrees.
equiangular triangle	A triangle with all congruent angles.
equilateral triangle	A triangle with three congruent sides.
Isosceles Triangle	An isosceles triangle is a triangle in which exactly two sides are the same length.
Obtuse Triangle	An obtuse triangle is a triangle with one angle that is greater than 90 degrees.
Scalene Triangle	A scalene triangle is a triangle in which all three sides are different lengths.
Triangle	A triangle is a polygon with three sides and three angles.
Interior angles	Interior angles are the angles inside a figure.
Right Angle	A right angle is an angle equal to 90 degrees.
Equilateral	A polygon is equilateral if all of its sides are the same length.
Equiangular	A polygon is equiangular if all angles are the same measure.

Additional Resources

Video: Angle Relationships and Types of Triangles

Activities: Triangle Classification Discussion Questions

Study Aids: Polygons Study Guide

Practice: Classify Triangles

Real World: Bridges Over Troubled Water

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4.2: Classify Triangles by Angle Measurement

Identify triangles as acute, right, obtuse or equiangular.

Triangle Classification by Angles

Figure **4.2.1**

Michael's mother bought a caddy that fits in a corner. She uses the caddy to store their brooms and mops in the garage. His mother's caddy had the appearance of a **right triangle** when viewed from above and fit perfectly in the corner. Michael liked the caddy, but he couldn't afford to buy one like hers, so he decided to make his own. He bought wood and nails and put it together, but when he tried to put it in the corner in his room, it wouldn't fit. He measures the angles of his caddy and realizes that the top is the **triangle** below:

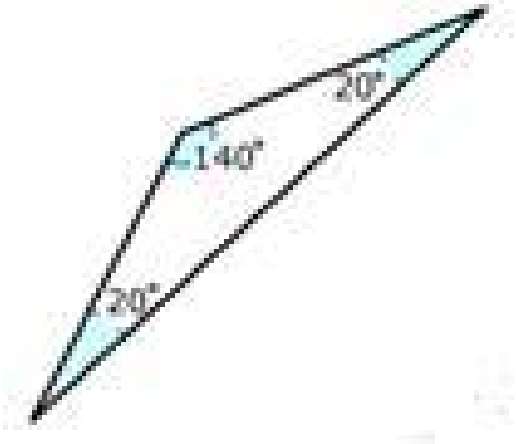


Figure **4.2.2**

What is the classification of Michael's triangle?

In this concept, you will learn how to use angles to classify triangles.

Classifying Triangles by Angles

The prefix “tri” means three. **Triangle** means three angles.

To classify a triangle according to its angles, you must look at the angles inside the triangle. Use the number of degrees in these angles to classify the triangle. Let’s look at a picture of a triangle to explain.

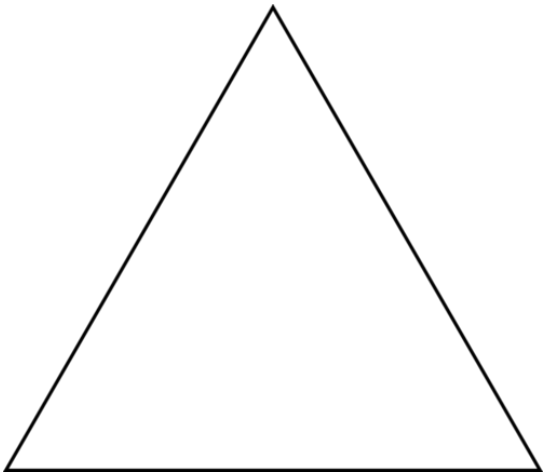


Figure **4.2.3**

Look at the measure of each angle inside the triangle to figure out what kind of triangle it is. There are four types of triangles based on angle measures.

A **right triangle** is a triangle that has one right angle and two acute angles. One of the angles in the triangle measures **90°** and the other two angles are less than 90. Here is a picture of a right triangle.

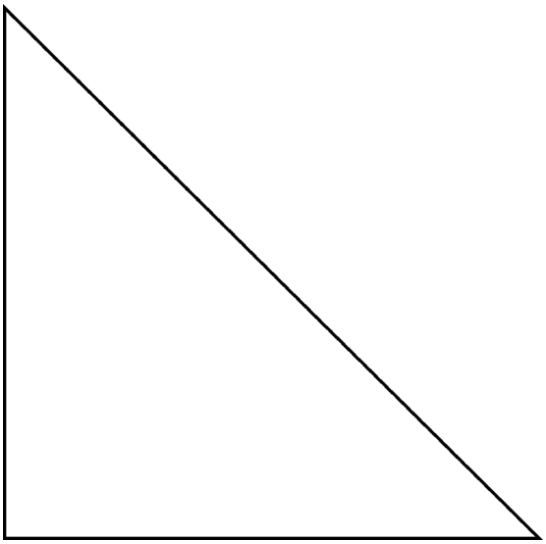


Figure **4.2.4**

You can see that the 90 degree angle is the one in the bottom left corner. You can even draw in the small box to identify it as a 90 degree angle. If you look at the other two angles you cans see that those angles are less than 90 degrees and are acute.

Let's look at an example of a right angle.

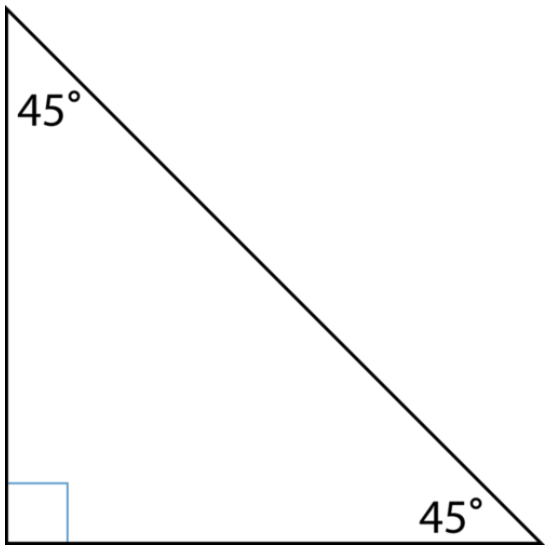


Figure 4.2.5

his triangle has one $90°$ angle and two $45°$ angles. Find the sum of the three angles.

$90 + 45 + 45 = 180°$

The sum of the three angles of a triangle is always equal to $180°$.

In an **equiangular triangle**, all three of the angles are equal.

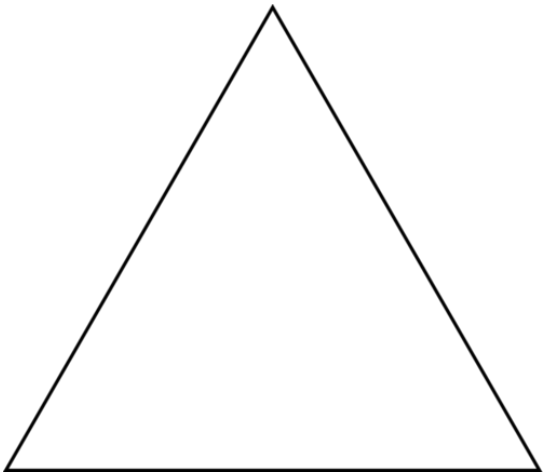


Figure 4.2.6

The three angles of this triangle are equal. This is an equiangular triangle.

You know that the sum of the three angles is equal to $180°$, therefore, for all three angles to be equal, each angle must be equal to $60°$.

$60 + 60 + 60 = 180°$

The sum of the angles is equal to $180°$.

In an **acute triangle**, all three angles of the triangle are less than 90 degrees. Here is an example of an acute triangle.

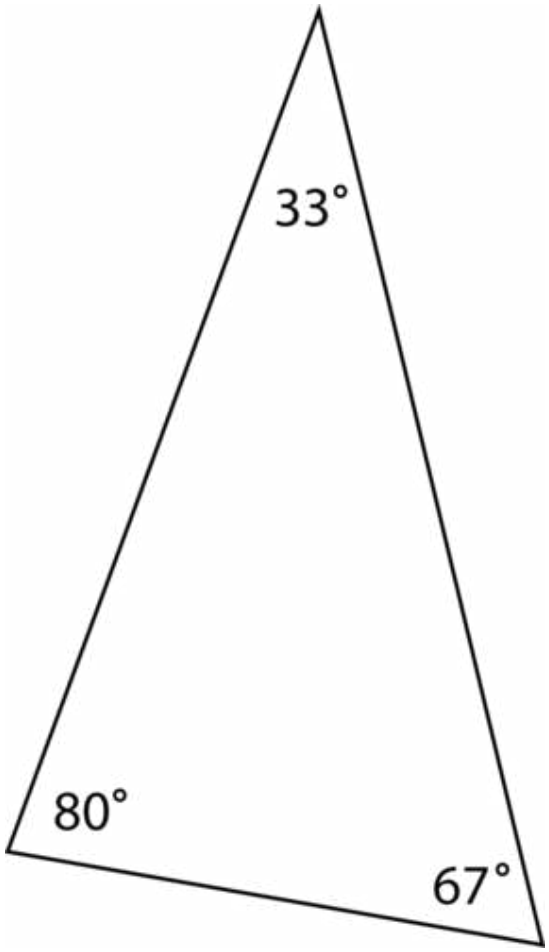


Figure 4.2.7

All three of these angles measure less than 90 degrees.

$33 + 80 + 67 = 180°$

The sum of the angles is equal to $180°$.

An **obtuse triangle** has one angle that is greater than 90 and two angles that are less than 90.

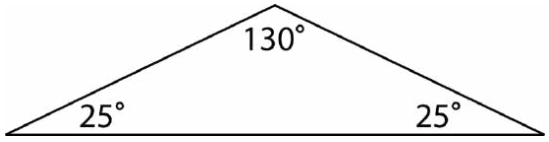


Figure 4.2.8

$130 + 25 + 25 = 180°$

The sum of the angles is equal to $180°$.

Example 4.2.1

Earlier, you were given a problem about Michael and his caddy.

He tried to build a caddy like his mother's caddy. Her caddy looked like a right triangle from above but he ended up building one that had the following measures and appearance from above:

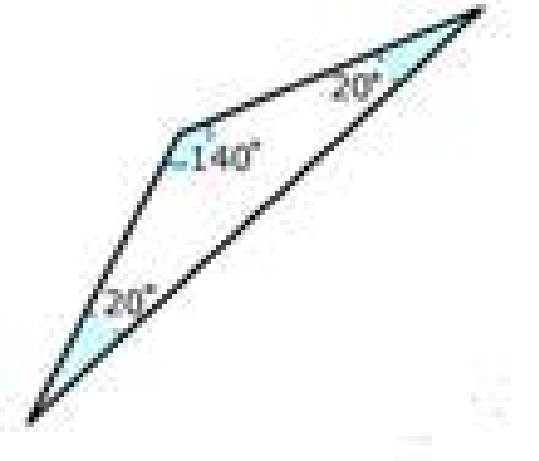


Figure **4.2.9**

Solution

What is the classification of Michael's triangle?

First, list the angle measures.

20, 20, 140

Next, determine if any of the angles are equal to 90 degrees or larger than 90 degrees.

Yes, one angle is larger than 90 degrees

Then, classify the triangle.

Obtuse

The answer is an obtuse triangle. Michael created an obtuse triangle instead of a right triangle.

Example **4.2.2**

Identify the type of triangle according to its angles.

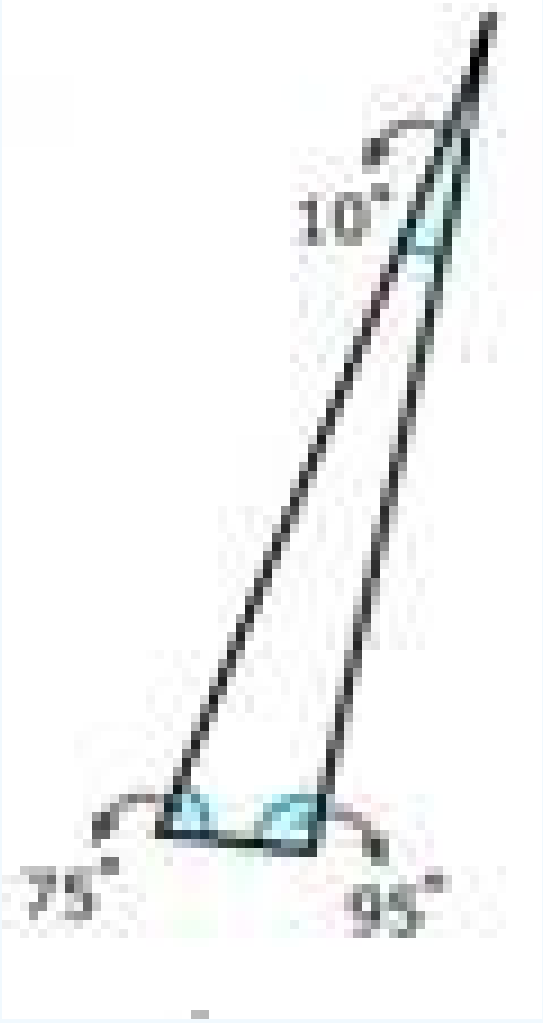


Figure **4.2.10**

Solution

First, list the angle measures.

10, 75, 95

Next, determine if any of the angles are equal to 90 degrees or larger than 90 degrees.

Yes, one angle is larger than 90 degrees

Then, classify the triangle.

Obtuse

The answer is an obtuse triangle.

Example **4.2.3**

Identify the type of triangle according to its angles.

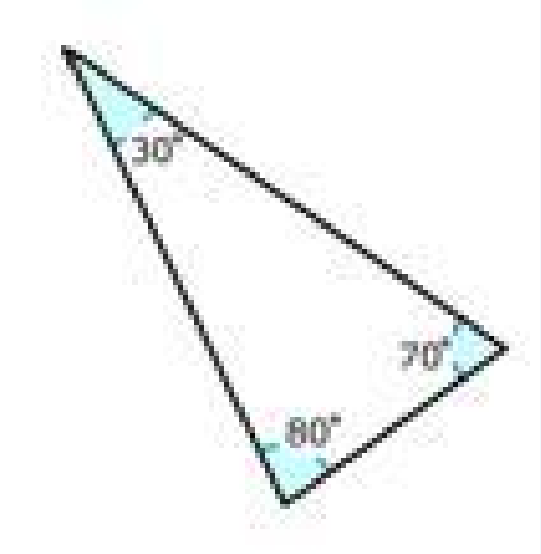


Figure **4.2.11**

Solution

First, list the angle measures.

30, 70 and 80

Next, determine if any of the angles are equal to 90 degrees or larger than 90 degrees.

No

Then, classify the angle.

Acute

The answer is an acute triangle.

Example Runaway argument for \eatSpaces?

Identify the type of triangle according to its angles.

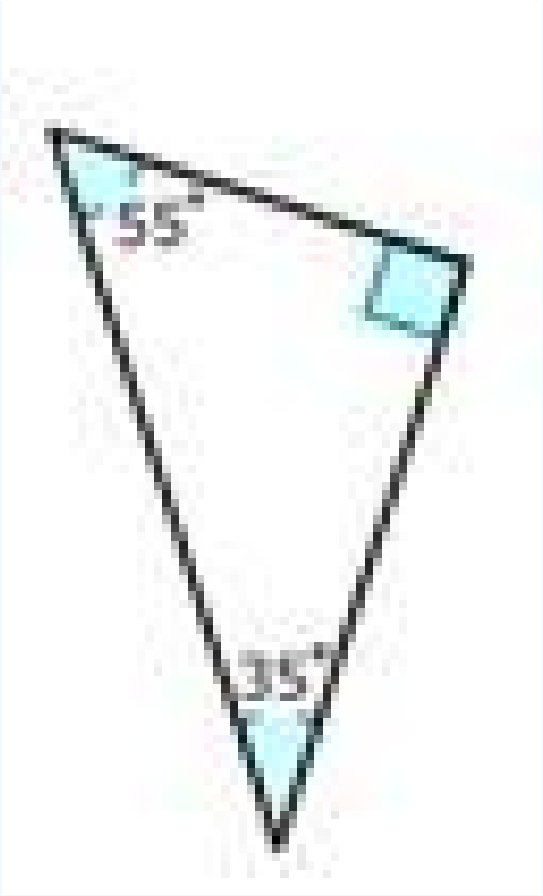


Figure 4.2.12

Solution

First, list the angle measures.

35, 55 and 90

Next, determine if any of the angles are equal to 90 degrees or larger than 90 degrees.

Yes, one of the angles is equal to 90 degrees

Then, classify the angle.

Right

The answer is a right triangle

Example 4.2.5

Classify the triangle by looking at the sum of its angles.

$40° + 60° + 80° = 180°$

Solution

First, list the angle measures.

40, 60 and 80

Next, determine if any of the angles are equal to 90 degrees or larger than 90 degrees.

No

Then, classify the angle.

Acute

The answer is an acute triangle.

Review

Classify each triangle according to its angles.

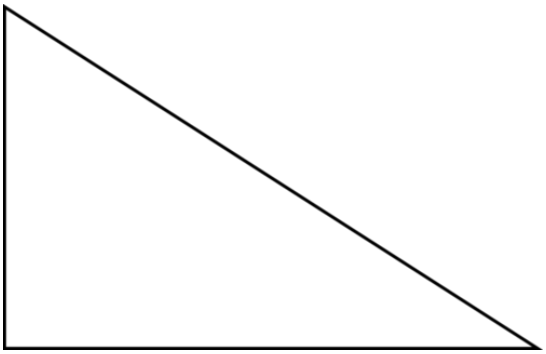
1. 

Figure **4.2.13**

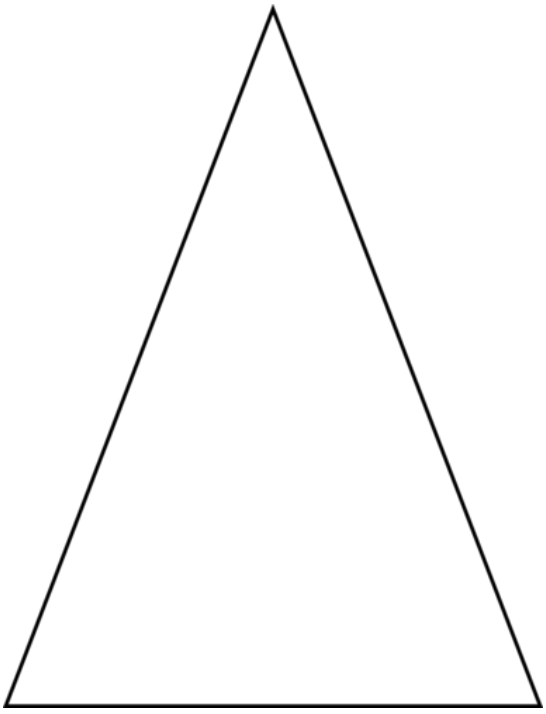
2. 

Figure **4.2.14**

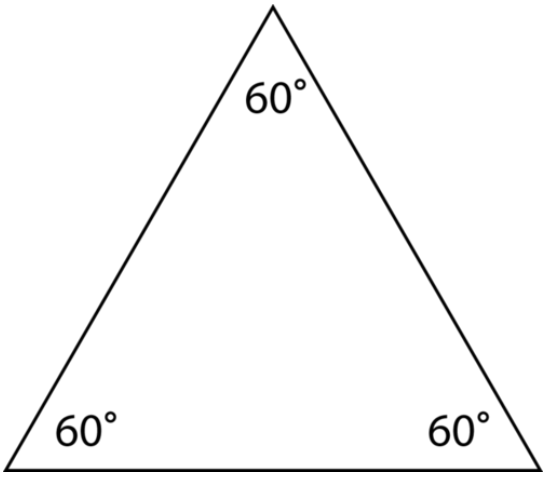
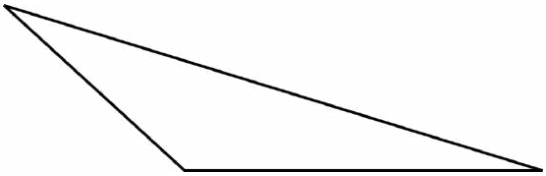
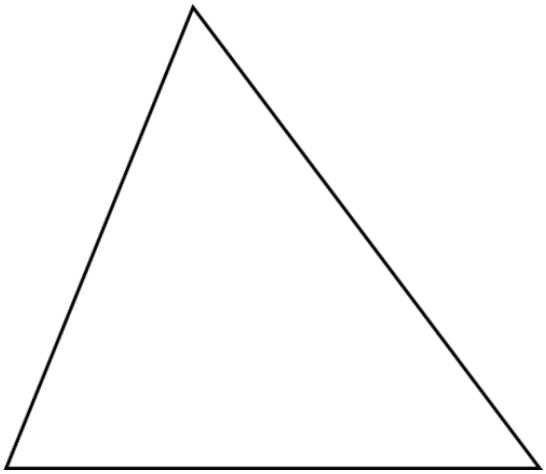
3. 

Figure 4.2.15

4. 
Figure 4.2.16

5. 
Figure 4.2.17

Classify the following triangles by looking at the sum of the angle measures.

6. $40 + 55 + 45 = 180°$
7. $20 + 135 + 25 = 180°$
8. $30 + 90 + 60 = 180°$
9. $60 + 60 + 60 = 180°$
10. $110 + 15 + 55 = 180°$
11. $105 + 65 + 10 = 180°$
12. $80 + 55 + 45 = 180°$
13. $70 + 45 + 65 = 180°$
14. $145 + 20 + 15 = 180°$
15. $60 + 80 + 40 = 180°$

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.7.

Resources

Vocabulary

Term	Definition
Acute Triangle	An acute triangle has three angles that each measure less than 90 degrees.
Equilateral Triangle	An equilateral triangle is a triangle in which all three sides are the same length.
Obtuse Triangle	An obtuse triangle is a triangle with one angle that is greater than 90 degrees.
Right Triangle	A right triangle is a triangle with one 90 degree angle.
Triangle	A triangle is a polygon with three sides and three angles.

Additional Resource

Video: Angle Relationships and Types of Triangles

Practice: Classify Triangles by Angle Measurement

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4.3: Classify Triangles by Side Measurement

Classify triangles as scalene, isosoles or equalateral by identifying the number of equal sides.

Triangle Classification by Side Lengths

Figure 4.3.1

Tasha goes sailing with her father on her father's old boat. The sail looks like a **triangle**. All of the sides of the sail are different lengths. Tasha wants to classify the triangle but she isn't sure how to name it. Given the side lengths of the triangle, how can Tasha classify the triangle?

In this concept, you will learn how to classify triangles by their side lengths.

Classifying Triangles by Side Lengths

You can use the lengths of the sides to help you classify triangles.

Let's look at how to classify triangles according to side length.

An **equilateral triangle** has side lengths that are the same. Here is an example.

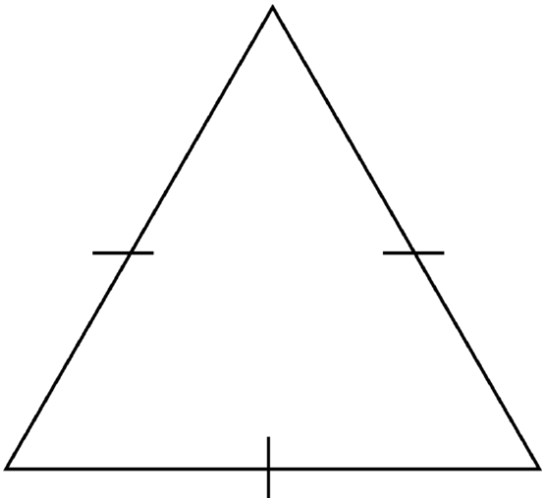

Figure 4.3.2

These little lines let you know that the side lengths are the same. Sometimes you will see these and sometimes you won't. You may have to figure it out on your own or by measuring with a ruler.

A **scalene triangle** is a triangle where the lengths of all three sides are different. Here is an example of a scalene triangle.

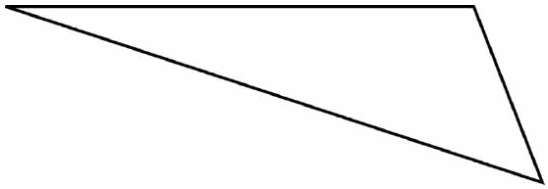

Figure **4.3.3**

You can see that all three sides of the triangle are different lengths.

An **isosceles triangle** has two side lengths that are the same and one side length that is different. Here is an example of an isosceles triangle.

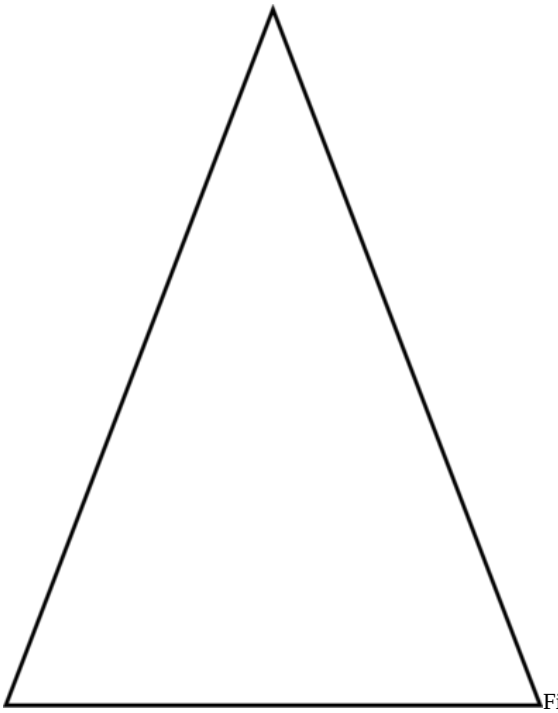

Figure **4.3.4**

Example 4.3.1

Earlier, you were given a problem about Tasha and her sail.

Her sail is shaped like a triangle with different side lengths. What classification should Tasha give the triangle?

Solution

First, determine if any of the side lengths are the same.

No

Then, classify the triangle.

Scalene

The answer is a scalene triangle.

Example 4.3.2

Classify this triangle as scalene, isosceles or equilateral according to its side lengths.

Side lengths, 6 cm, 4 cm, 6 cm

Angles 70, 70, 40 degrees

Solution

First, determine if any of the side lengths are the same.

Yes

Next, determine how many side lengths are the same.

Two

Then, classify the triangle.

Isosceles

The answer is an isosceles triangle.

Example 4.3.3

Classify this triangle according to its side lengths.

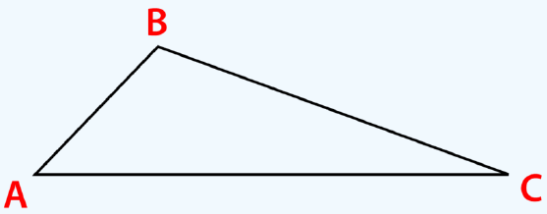

Figure 4.3.4

Solution

First, determine if any of the side lengths are the same.

No,

Then, classify the triangle.

Scalene

The answer is a scalene triangle.

Example 4.3.4

Classify the triangle according to its side lengths.

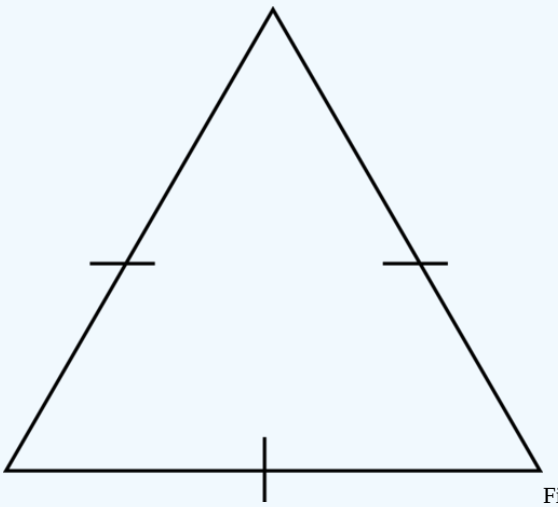

Figure 4.3.5

Solution

First, determine if any of the side lengths are the same.

Yes

Next, determine how many side lengths are the same.

All of the side lengths are the same

Then, classify the triangle.

Equilateral

The answer is an equilateral triangle.

Example 4.3.5

Classify the triangle according to its side lengths.

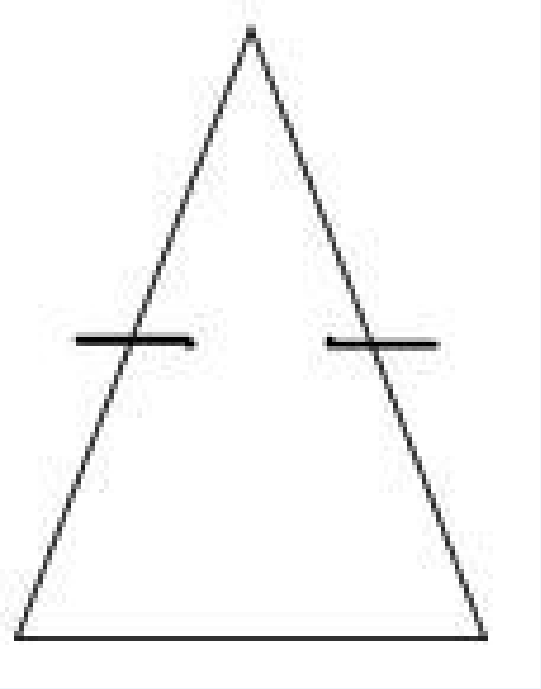

Figure 4.3.6

Solution

First, determine if any of the side lengths are the same.

Yes

Next, determine how many side lengths are the same.

Two

Then, classify the triangle.

Isosceles

The answer is an isosceles triangle.

Review

Answer the following questions using what you have learned about triangles their angles and side lengths.

1. If a triangle is a right triangle, then how many angles are acute?
2. How many angles in a right triangle are right angles?

3. How many degrees are there in a right triangle?
4. What is an obtuse angle?
5. How many obtuse angles are in an obtuse triangle?
6. If there is one obtuse angle, how many angles are acute?
7. If a triangle is equiangular, what is the measure of all three angles?
8. What does the word “interior angle” mean?
9. True or false. The side lengths of a scalene triangle are all equal.
10. True or false. The side lengths of a scalene triangle are all different.
11. True or false. The side lengths of an equilateral triangle are all equal.
12. True or false. An isosceles triangle has two side lengths the same and one different.
13. True or false. A scalene triangle can also be an isosceles triangle.
14. True or false. An equilateral triangle is also equiangular.
15. True or false. A scalene triangle can not be an acute triangle.

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.8.

Vocabulary

Term	Definition
Isosceles Triangle	An isosceles triangle is a triangle in which exactly two sides are the same length.
Scalene Triangle	A scalene triangle is a triangle in which all three sides are different lengths.
Triangle	A triangle is a polygon with three sides and three angles.

Additional Resources

Video: Angle Relationships and Types of Triangles

Practice: Classify Triangles by Side Measurement

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4.4: Isosceles Triangles

Properties of triangles with two equal sides/angles.

An isosceles triangle is a triangle that has ***at least*** two congruent sides. The congruent sides of the isosceles triangle are called the **legs**. The other side is called the **base**. The angles between the base and the legs are called **base angles**. The angle made by the two legs is called the **vertex angle**. One of the important properties of isosceles triangles is that their base angles are always congruent. This is called the **Base Angles Theorem.**

For $\DeltaDEF, if \overline{DE} \cong \overline{EF}$, then $\angle D \cong \angle F$.

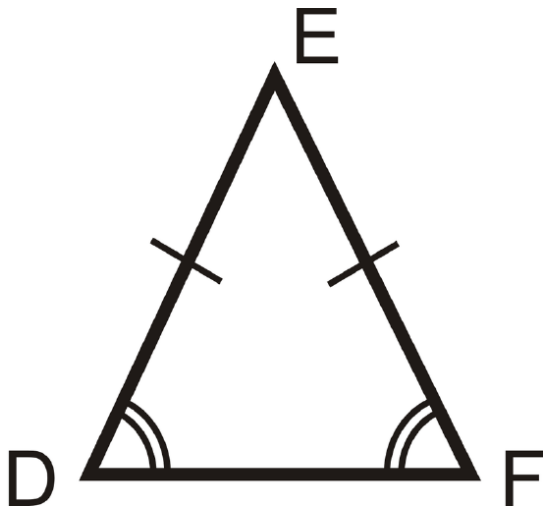


Figure 4.4.1

Another important property of isosceles triangles is that the angle bisector of the vertex angle is also the perpendicular bisector of the base. This is called the **Isosceles Triangle Theorem**. (*Note this is ONLY true of the vertex angle.*) The converses of the Base Angles Theorem and the Isosceles Triangle Theorem are both true as well.

Base Angles Theorem Converse: If two angles in a triangle are congruent, then the sides opposite those angles are also congruent. So for ΔDEF, if $\angle D \cong \angle F$, then $\overline{DE} \cong \overline{EF}$.

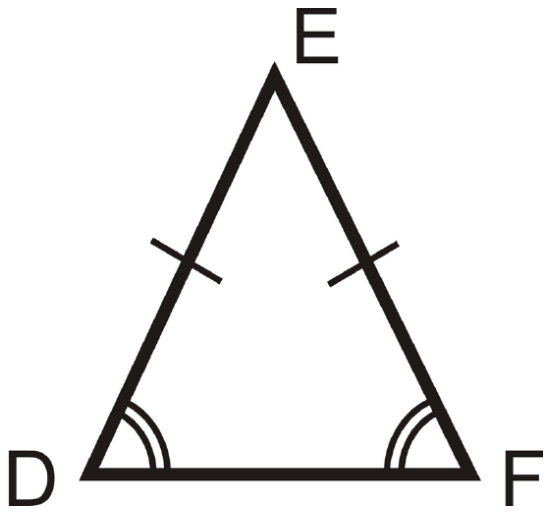


Figure 4.4.2

Isosceles Triangle Theorem Converse: The perpendicular bisector of the base of an isosceles triangle is also the angle bisector of the vertex angle. So for isosceles ΔDEF, if $\overline{EG} \perp \overline{DF}$ and $\overline{DG} \cong \overline{GF}$, then $\angle DEG \cong \angle FEG$.

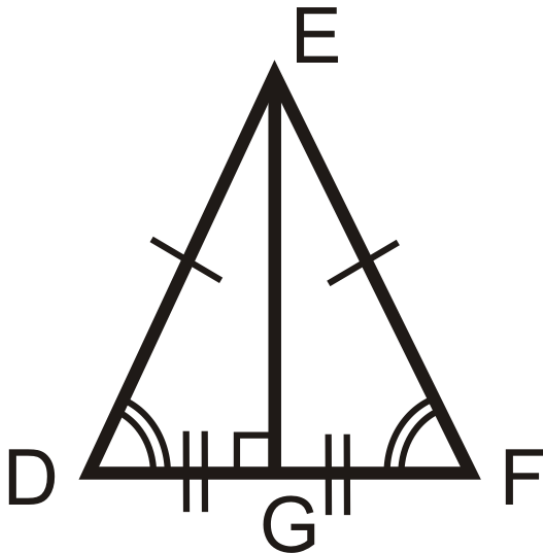


Figure 4.4.3

What if you were presented with an isosceles triangle and told that its base angles measure $x°$ and $y°$? What could you conclude about $x$and y?

Example 4.4.1

Find the value of x and angle.

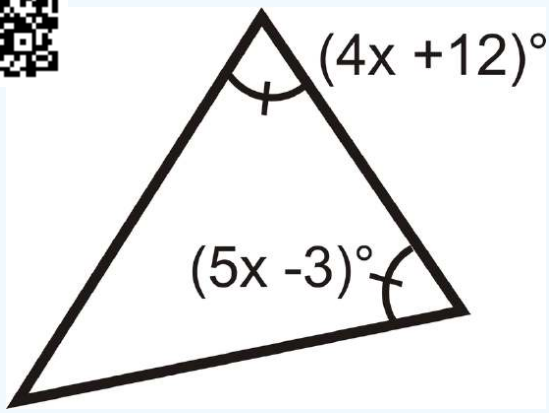


Figure 4.4.4

Solution

The two angles are equal, so set them equal to each other and solve for x.

$$(4x + 12)° = (5x - 3)°$$
$$15 = x$$

Substitute x=15; the base angles are $[4(15) + 12]°$, or 72^{\circ}\). The vertex angle is $180° - 72° - 72° = 36°$.

Example 4.4.2

True or false: Base angles of an isosceles triangle can be right angles.

Solution

This statement is false. Because the base angles of an isosceles triangle are congruent, if one base angle is a right angle then both base angles must be right angles. It is impossible to have a triangle with two right ($90°$) angles. The **Triangle Sum Theorem** states that the sum of the three angles in a triangle is $180°$. If two of the angles in a triangle are right angles, then the third angle must be $0°$ and the shape is no longer a triangle.

Example 4.4.3

Which two angles are congruent?

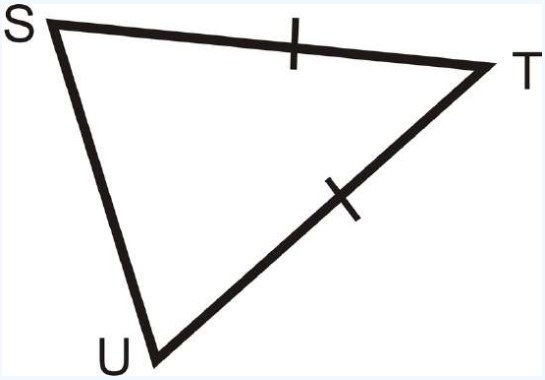


Figure 4.4.5

Solution

This is an isosceles triangle. The congruent angles are opposite the congruent sides. From the arrows we see that $\angle S \cong \angle U$.

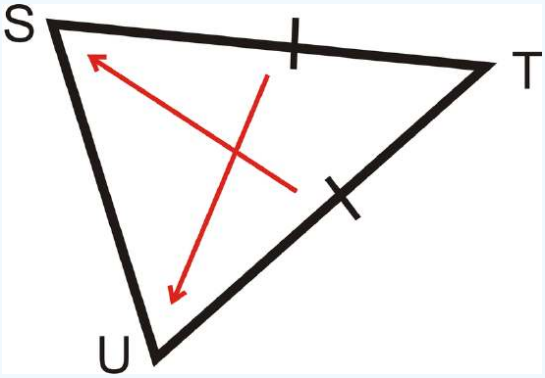


Figure 4.4.6

Example 4.4.4

If an isosceles triangle has base angles with measures of 47°, what is the measure of the vertex angle?

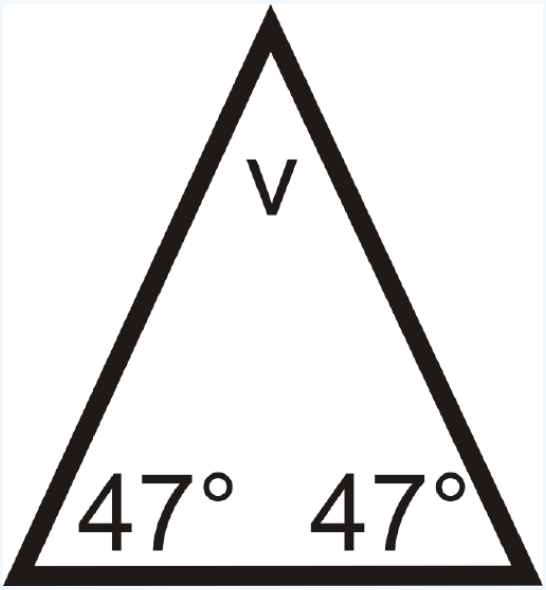


Figure 4.4.7

Solution

Draw a picture and set up an equation to solve for the vertex angle, v. Remember that the three angles in a triangle always add up to $180°$.

\begin{align*} ended with \end{align}

Example 4.4.5

If an isosceles triangle has a vertex angle with a measure of $116°$, what is the measure of each base angle?

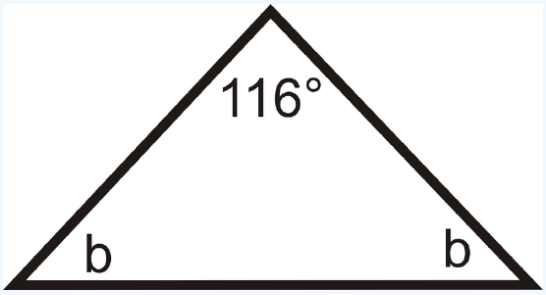


Figure 4.4.8

Solution

Draw a picture and set up and equation to solve for the base angles, b.

$$\begin{aligned} 116° + b + b &= 180° \\ 2b &= 64° \\ b &= 32° \end{aligned}$$

Review

Find the measures of $x$and/or y.

1.

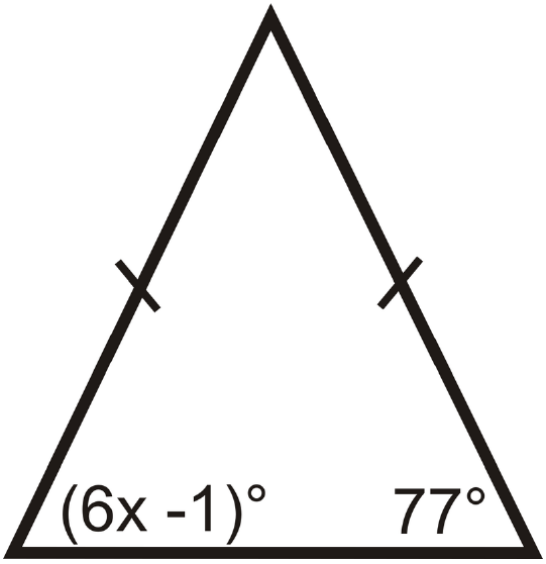


Figure 4.4.9

2.

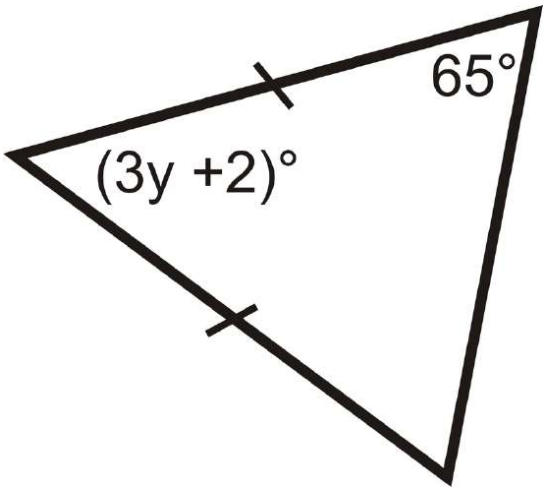


Figure 4.4.10

3.

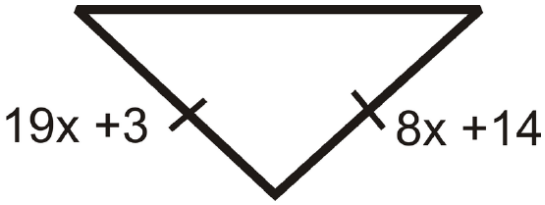


Figure 4.4.11

4.

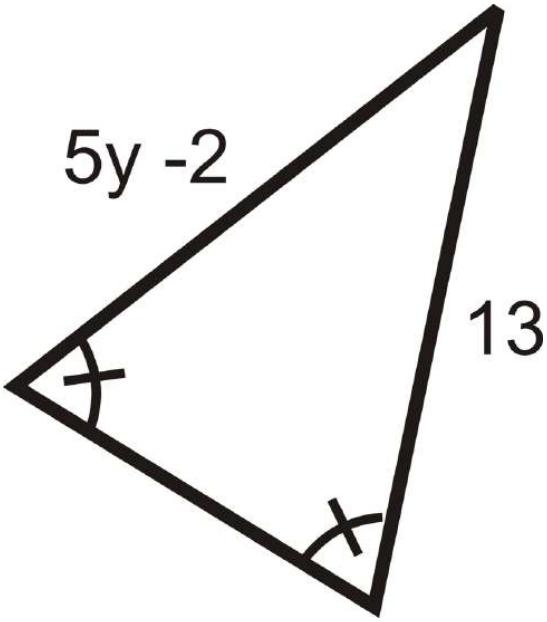
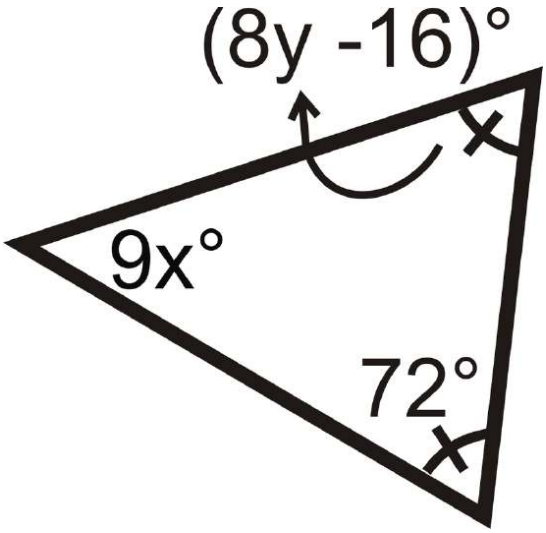


Figure 4.4.12



5.

Figure **4.4.13**

Determine if the following statements are true or false.

6. Base angles of an isosceles triangle are congruent.
7. Base angles of an isosceles triangle are complementary.
8. Base angles of an isosceles triangle can be equal to the vertex angle.
9. Base angles of an isosceles triangle are acute.

Fill in the proofs below.

10. Given: Isosceles \DeltaCIS, with base angles $\angle C$ and $\angle S$. \overline{IO}\) is the angle bisector of $\angle CIS$ Prove: $\overline{IO}$ is the perpendicular bisector of $\overline{CS}$

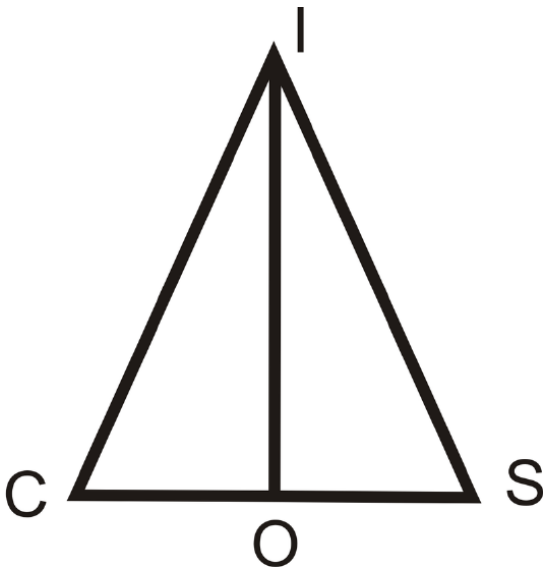


Figure **4.4.14**

Statement	*Reason*
1.	1. Given
2.	2. Base Angles Theorem
3. $\angle CIO \cong \angle SIO$	3.
4.	4. Reflexive *PoC*
5. \DeltaCIO ≅ \DeltaSIO	5.
6. $\overline{CO} \cong \overline{OS}$	6.
7.	7. *CPCTC*
8. $\angle IOC$ and $\angle IOS$ are supplementary	8.
9.	9. Congruent Supplements Theorem
10. $\overline{IO}$ is the perpendicular bisector of $\overline{CS}$	10.

11. Given: Isosceles \DeltaICS with $\angle C$ and $\angle S$. $\overline{IO}$ is the perpendicular bisector of $\overline{CS}$ Prove: $\overline{IO}$ is the angle bisector of $\angle CIS$

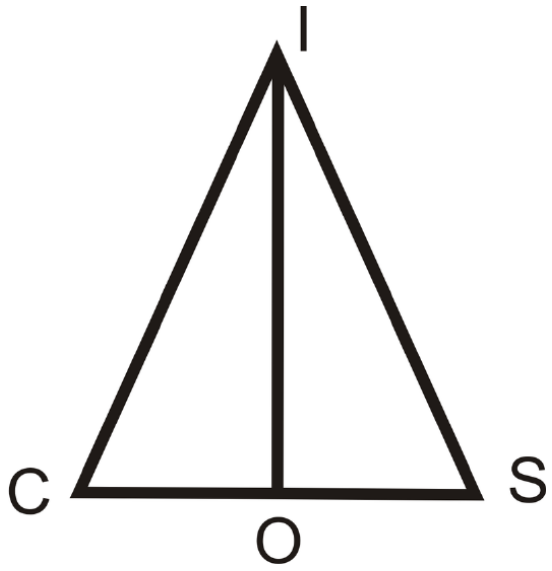


Figure **4.4.15**

Statement	*Reason*
1.	1.
2. $\angle C \cong \angle S$	2.
3. $CO\overline{AB} \cong OS\overline{AB}$	3.
4. $m\angle IOC = m\angle IOS = 90°$	4.
5.	5.
6.	6. *CPCTC*
7. $\overline{IO}$ is the angle bisector of $\angle CIS$	7.

On the x−y plane, plot the coordinates and determine if the given three points make a scalene or isosceles triangle.

12. $(-2, 1), (1, -2), (-5, -2)$
13. $(-2, 5), (2, 4), (0, -1)$
14. $(6, 9), (12, 3), (3, -6)$
15. $(-10, -5), (-8, 5), (2, 3)$
16. $(-1, 2), (7, 2), (3, 9)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.10.

Resources

Vocabulary

Term	Definition
Base	The side of a triangle parallel with the bottom edge of the paper or screen is commonly called the base. The base of an isosceles triangle is the non-congruent side in the triangle.
Base Angles	The base angles of an isosceles triangle are the angles formed by the base and one leg of the triangle.
Base Angles Theorem Converse	The base angles theorem converse states if two angles in a triangle are congruent, then the sides opposite those angles are also congruent.
Isosceles Triangle Theorem	The Isosceles Triangle Theorem states that the perpendicular bisector of the base of an isosceles triangle is also the angle bisector of the vertex angle.
Triangle Sum Theorem	The Triangle Sum Theorem states that the three interior angles of any triangle add up to 180 degrees.
Vertical Angles	Vertical angles are a pair of opposite angles created by intersecting lines.

Additional Resources

Video: Isosceles Triangles Principles - Basic

Activities: Isosceles Triangles Discussion Questions

Study Aids: Triangle Relationships Study Guide

Practice: Isosceles Triangles

Real World: Triangles in the Sky

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4.5: Equilateral Triangles

Properties of triangles with three equal sides.

Equilateral Triangle Theorem: All equilateral triangles are also equiangular. Furthermore, all equiangular triangles are also equilateral.

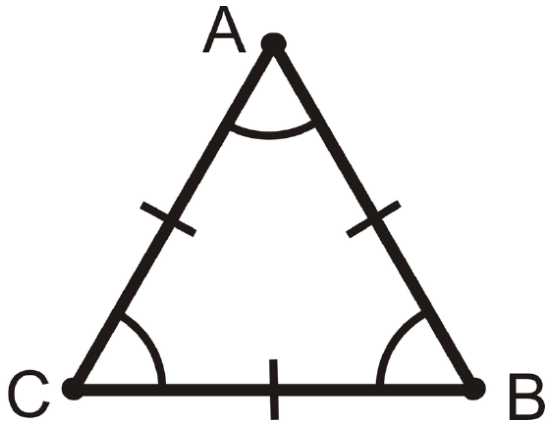


Figure **4.5.1**

If $\overline{AB} \cong \overline{BC} \cong \overline{AC}$, then $\angle A \cong \angle B \cong \angle C$. Conversely, if $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{AC}$.

What if you were presented with an equilateral triangle and told that its sides measure x, y, and 8? What could you conclude about x and y?

Example 4.5.1

Fill in the proof:

Given: Equilateral ΔRST

$\overline{RT} \cong \overline{ST} \cong \overline{RS}$

Prove: ΔRST is equiangular

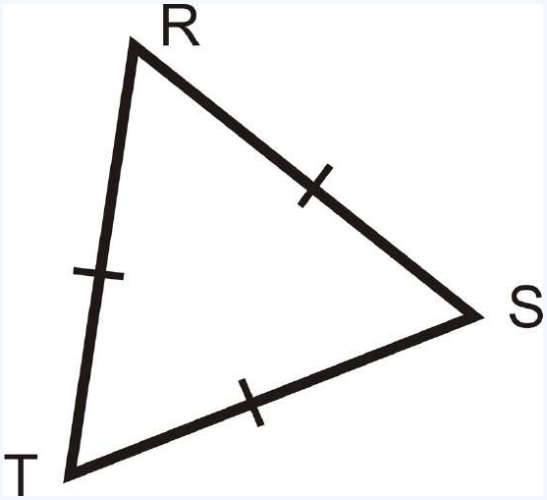


Figure **4.5.2**

Solution

Statement	*Reason*
1.	1. Given
2.	2. Base Angles Theorem
3.	3. Base Angles Theorem
4.	4. Transitive *PoC*
5. ΔRST is equiangular	5.

Statement	*Reason*
1. $RT\overline{AB} \cong ST\overline{AB} \cong RS\overline{AB}$	1. Given
2. $\angle R \cong \angle S$	2. Base Angles Theorem
3.$\angle T \cong \angle R$	3. Base Angles Theorem
4. $\angle T \cong \angle S$	4. Transitive *PoC*
5. ΔRST is equiangular	5. Definition of equiangular.

Example 4.5.2

True or false: All equilateral triangles are isosceles triangles.

Solution

This statement is true. The definition of an isosceles triangle is a triangle with at least two congruent sides. Since all equilateral triangles have three congruent sides, they fit the definition of an isosceles triangle.

Example 4.5.3

Find the value of x.

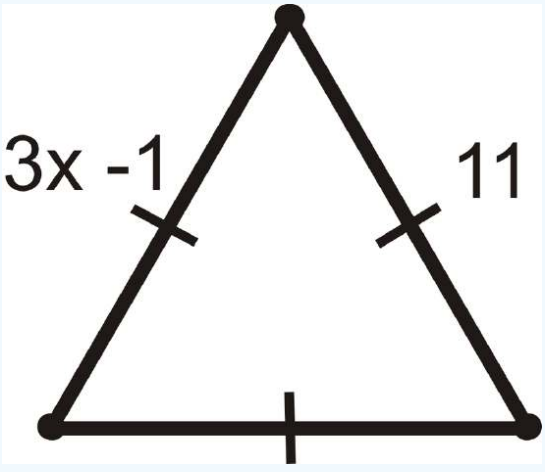


Figure 4.5.3

Solution

Because this is an equilateral triangle $3x - 1 = 11$. Solve for x.

Misplaced &

Example 4.5.4

Find the values of x and y.

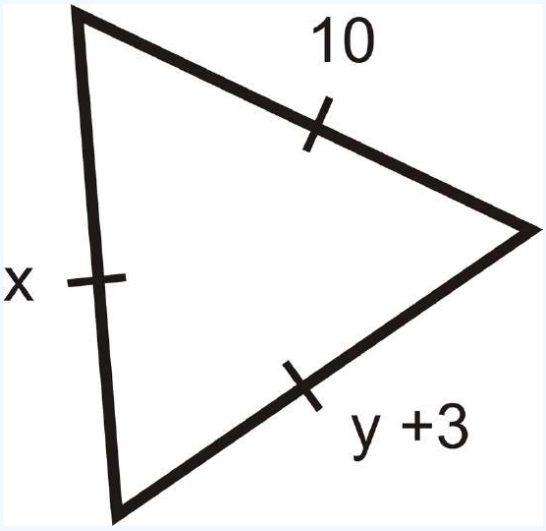


Figure 4.5.3

Solution

The markings show that this is an equilateral triangle since all sides are congruent. This means all sides must equal 10. We have $x = 10$ and $y + 3 = 10$ which means that $y = 7$.

Example 4.5.5

Two sides of an equilateral triangle are $2x + 5$ units and $x + 13$ units. How long is each side of this triangle?

Solution

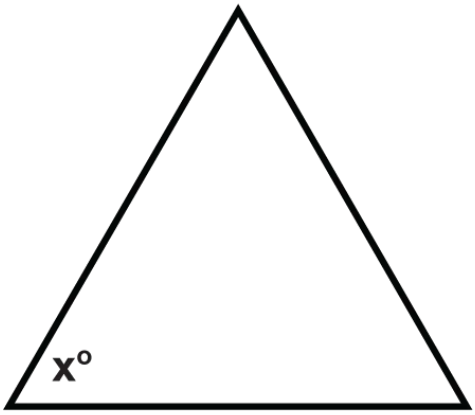
The two given sides must be equal because this is an equilateral triangle. Write and solve the equation for x.

Misplaced &

To figure out how long each side is, plug in 8 for x in either of the original expressions. $2(8) + 5 = 21$. Each side is 21 units.

Review

The following triangles are equilateral triangles. Solve for the unknown variables.



1.

Figure 4.5.4

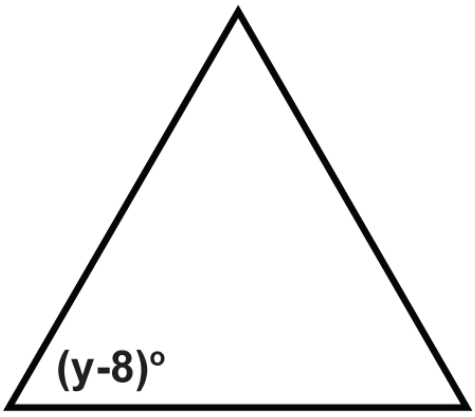


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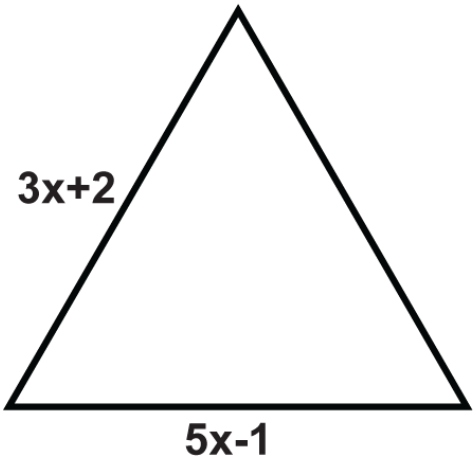
Figure 4.5.5



3.

Figure 4.5.6

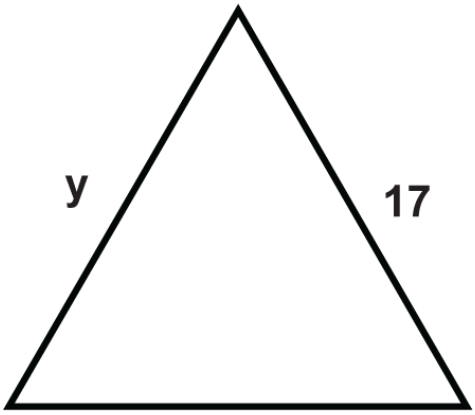


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Figure 4.5.7

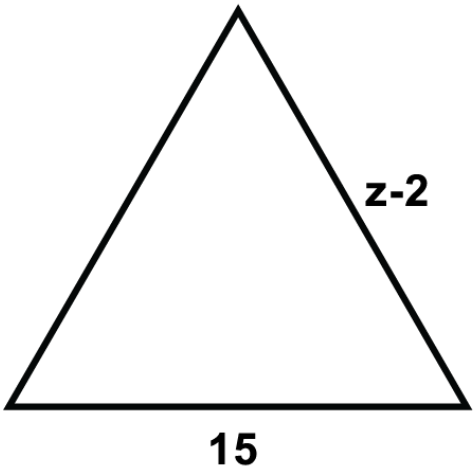

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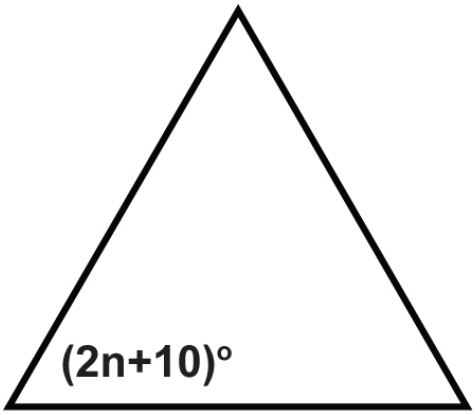
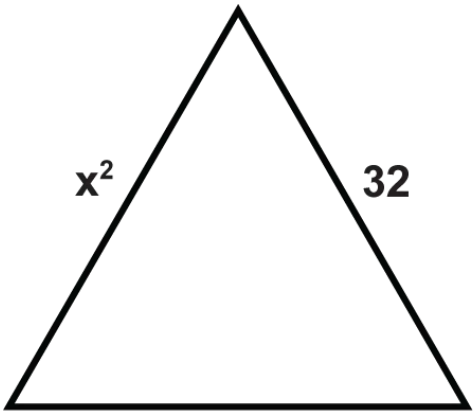
Figure 4.5.8



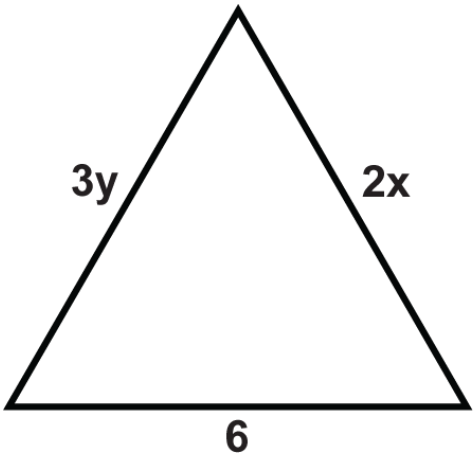
6.

Figure 4.5.9



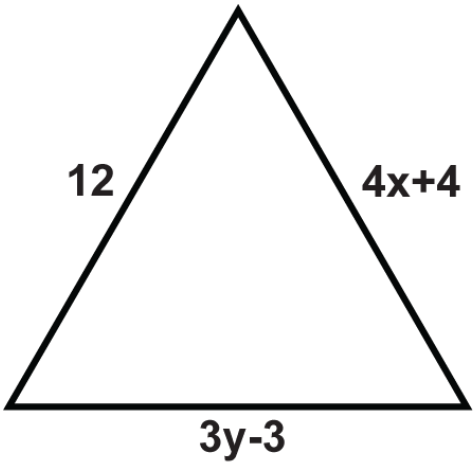
7.

Figure 4.5.10



8.

Figure 4.5.11



9.

Figure 4.5.12

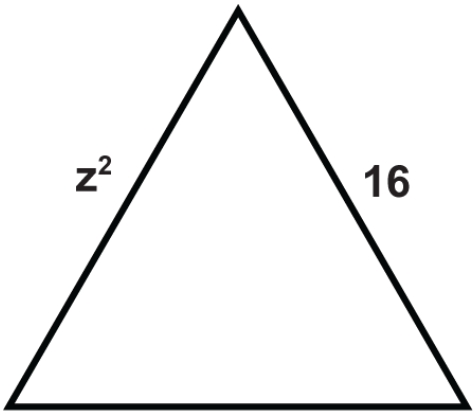


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Figure 4.5.13

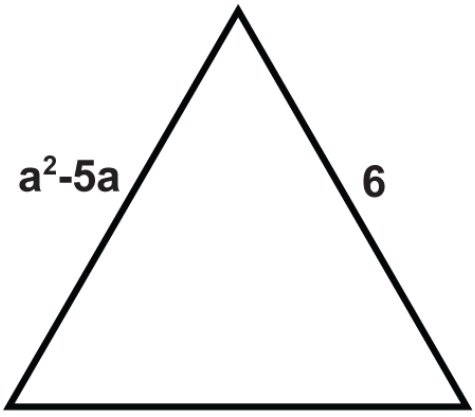

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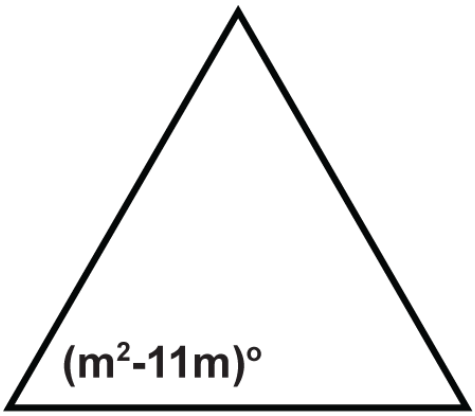
Figure 4.5.14



12.

Figure 4.5.15

13.

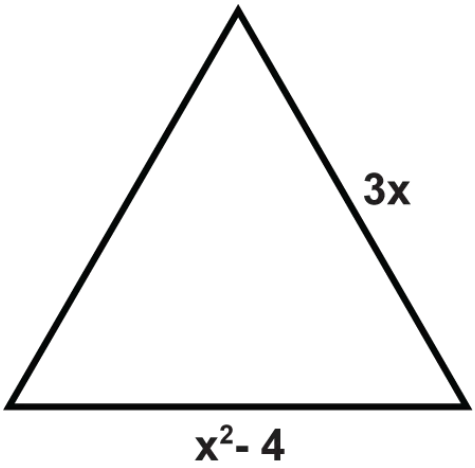


Figure 4.5.16

14.

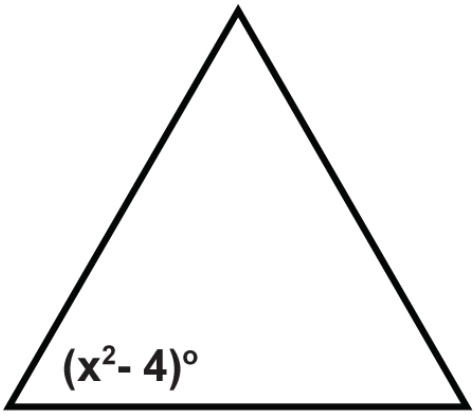


Figure 4.5.17

15. Find the measures of x and y.

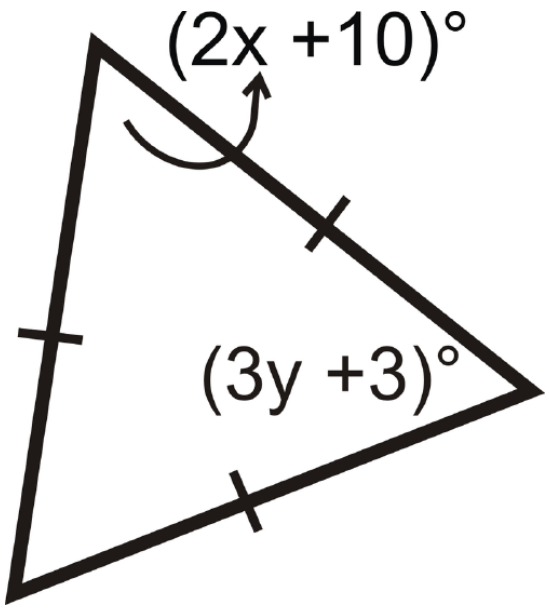


Figure 4.5.18

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.11.

Additional Resource

Video: Equilateral Triangl

Activities: Equilateral Tria uestions

Study Aids: Equilateral Tr Questions

Practice: Equilateral Triangles

Real World: Equilateral Triangles

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4.6: Area and Perimeter of Triangles

Area is half the base times the height while the perimeter is the sum of the sides.

The formula for the **area of a triangle** is half the **area of a parallelogram**.

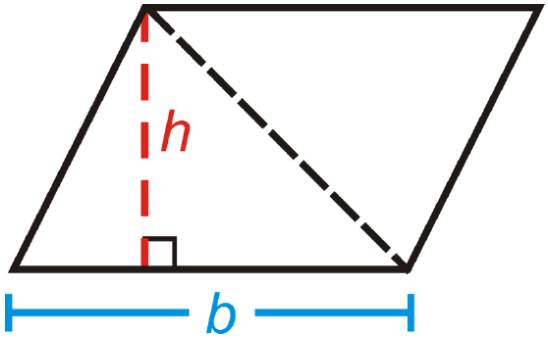

Figure 4.6.1

Area of a Triangle: $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$.

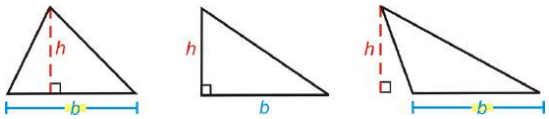

Figure 4.6.2

What if you were given a triangle and the size of its base and height? How could you find the total distance around the triangle and the amount of space it takes up?

For Examples 1 and 2, use gle.

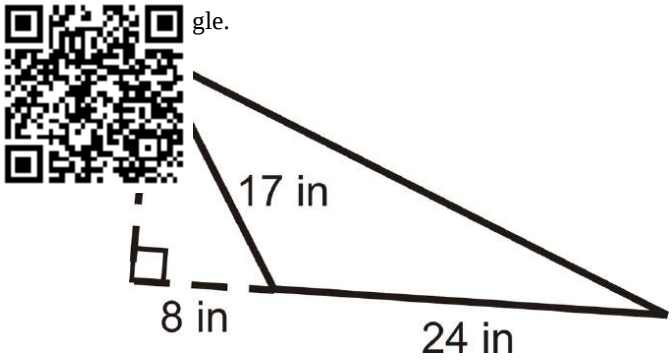

Figure 4.6.3

Example 4.6.1

Find the height of the triangle.

Solution

Use the Pythagorean Theorem to find the height.

$$8^2 + h^2 = 17^2$$
$$h^2 = 225$$
$$h = 15in$$

Example 4.6.2

Find the perimeter.

Solution

We need to find the hypotenuse. Use the Pythagorean Theorem again.

$$(8+24)^2+15^2=h^2$$
$$h^2=1249$$
$$h\approx 35.3in$$

The perimeter is $24+35.3+17\approx 76.3\ in$.

Example 4.6.3

Find the area of the triangle.

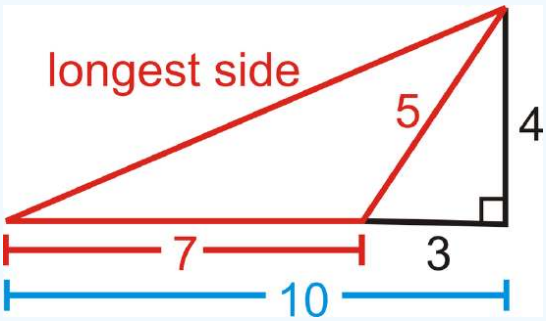

Figure 4.6.4

Solution

To find the area, we need to find the height of the triangle. We are given two sides of the small **right triangle**, where the hypotenuse is also the short side of the obtuse triangle.

$$3^2+h^2=5^2$$
$$9+h^2=25$$
$$h^2=16$$
$$h=4$$
$$A=\frac{1}{2}(4)(7)=14\ units^2$$

Example 4.6.4

Find the perimeter of the triangle in Example 3.

Solution

To find the perimeter, we need to find the longest side of the obtuse triangle. If we used the black lines in the picture, we would see that the longest side is also the hypotenuse of the right triangle with legs 4 and 10.

$$4^2+10^2=c^2$$
$$16+100=c^2$$
$$c=\sqrt{116}\approx 10.77$$

The perimeter is $7+5+10.77\approx 22.77$ units

Example 4.6.5

Find the area of a triangle with base of length $28\ cm$ and height of $15\ cm$.

Solution

The area is $dfrac12(28)(15)=210\ cm^2$.

Review

Use the triangle to answer the following questions.

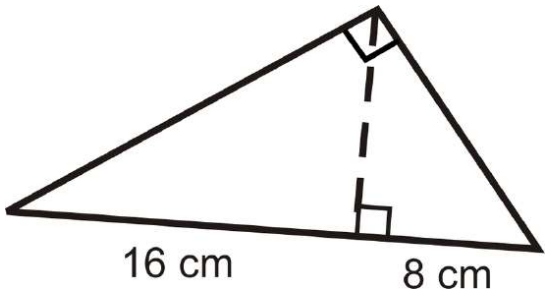

Figure 4.6.5

1. Find the height of the triangle by using the geometric mean.
2. Find the perimeter.
3. Find the area.

Find the area of the following shape.

4.

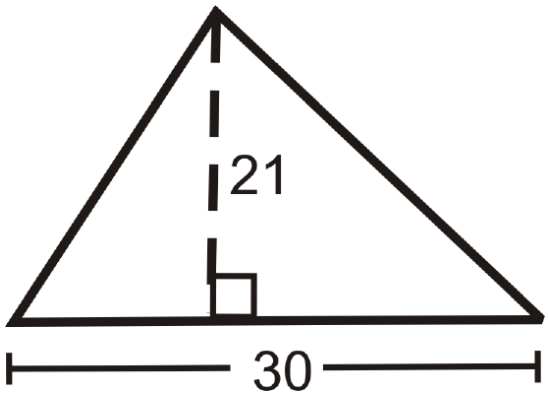

Figure 4.6.6

5. What is the height of a triangle with area $144\ m^2$ and a base of $24\ m$?

In questions 6-11 we are going to derive a formula for the area of an equilateral triangle.

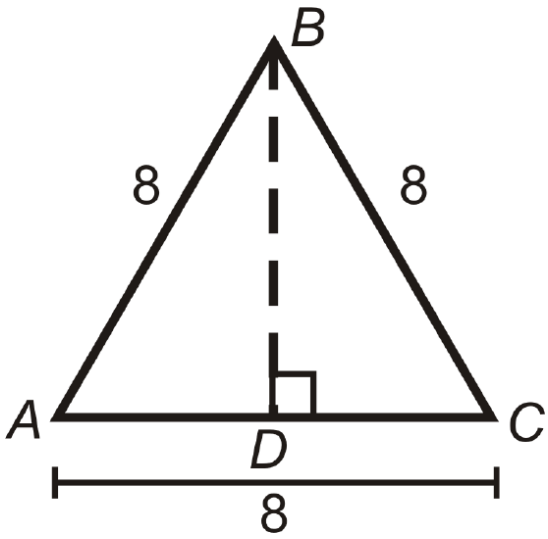

Figure 4.6.7

6. What kind of triangle is ΔABD? Find AD and BD.
7. Find the area of ΔABC.
8. If each side is x, what is AD and BD?
9. If each side is x, find the area of ΔABC.
10. Using your formula from #9, find the area of an equilateral triangle with 12 inch sides.

11. Using your formula from #9, find the area of an equilateral triangle with 5 inch sides.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.3.

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Perimeter	Perimeter is the distance around a two-dimensional figure.
Perpendicular	Perpendicular lines are lines that intersect at a 90° angle. The product of the slopes of two perpendicular lines is -1.
Right Angle	A right angle is an angle equal to 90 degrees.
Right Triangle	A right triangle is a triangle with one 90 degree angle.
Area of a Parallelogram	The area of a parallelogram is equal to the base multiplied by the height: $A = bh$. The height of a parallelogram is always perpendicular to the base (the sides are not the height).
Area of a Triangle	The area of a triangle is half the area of a parallelogram. Hence the formula: $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$.

Additional Resources

Video: **Area of a Triangle (Whole Numbers)**

Activities: **Area and Perimeter of Triangles Discussion Questions**

Study Aids: **Triangles and Quadrilaterals Study Guide**

Practice: **Area and Perimeter of Triangles**

Real World: **Perimeter**

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4.7: Triangle Area

Figure 4.7.1

Terry has two tickets to his favorite college football team's next home game, so he is planning to take his younger brother with him to the game. Terry wants to make two pennants for the game, so he buys a piece of material shaped like a **rectangle** with a base of 18 inches and a height of 12 inches. Terry plans to divide the material into two triangles. How can Terry figure out the area of each of the triangles that will be cut from the material for his pennants?

In this concept, you will learn to find the areas of triangles given a base and a height.

Finding the Area of a Triangle

Triangles are not parallelograms because they only have three sides, but they are related to parallelograms. Take a look the **parallelogram** below and see if you can figure out the connection.

Figure 4.7.2

If you look carefully, you will notice that the parallelogram can be divided into two triangles.

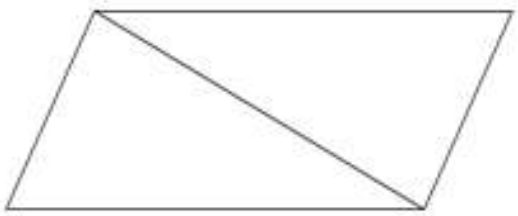

Figure 4.7.3

A **rectangle** is a type of parallelogram with four right angles. You can divide a rectangle into two triangles also.

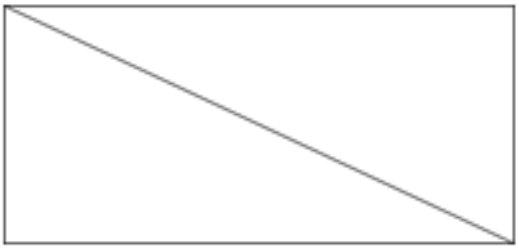

Figure 4.7.4

Notice that a rectangle is divided into two **right triangles**, triangles containing a right angle.

A **square** is a type of rectangle with four equal sides and four right angles. A square can also be divided into two right triangles.

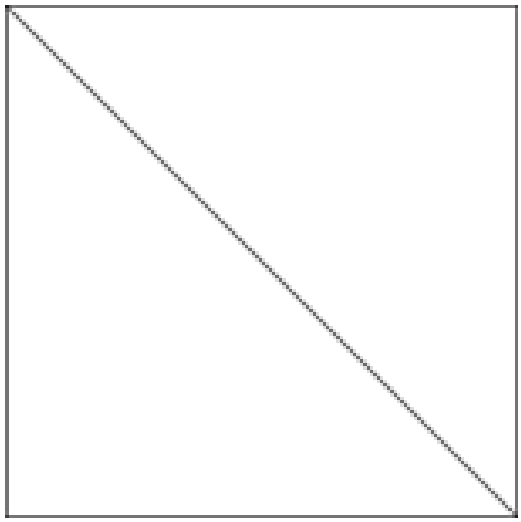

Figure 4.7.5

If a parallelogram can be divided into two triangles, then the area of a triangle is one-half the area of a parallelogram.

Let's look at how this works.

What is the area of this parallelogram?

Figure 4.7.6

To find the area of the parallelogram, multiply the base by the height.

$$A = bh$$
$$A = 2(5)$$
$$A = 10 sq.\ inches.$$

A parallelogram can be divided into two triangles.

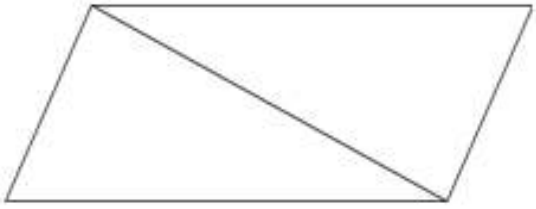

Figure 4.7.7

If you divide the area of the parallelogram in half, that will give you the area of one of the triangles.

$10 \div 2 = 5 sq.\ inches$

Based on this information, you can write the following formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

A triangle is one-half of a parallelogram, so the formula for the parallelogram multiplied by one-half is the formula for finding the area of a triangle.

Example 4.7.1

Earlier, you were given a problem about Terry and his sports pennants.

Terry has always been a Gator football fan, so he is excited about attending one of the team's home games. He has purchased material to make two pennants for the game. The material is shaped like a rectangle, and Terry plans to divide it into two triangles for the pennants. The material has a 38-inch base and a height of 30 inches. Terry wants to know the size of the two triangles after he cuts the material. How can Terry figure out the area of each of the triangles that will form his pennants?

Solution

Here is a look at the piece of material.

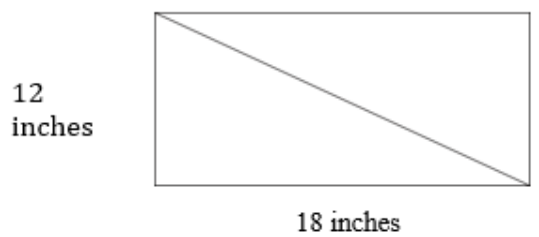

Figure 4.7.5

First, recall that the area of a rectangle is base times height and a triangle is one half the area of a rectangle.

$$\begin{aligned} Base \times Height &= Area\ of\ a\ Rectangle \\ 18 \times 12 &= 216 \\ &= 216\ square\ inches \end{aligned}$$

Next, substitute the area value for the rectangle (or the base\times height) in the triangle area formula.

$$\begin{aligned} A &= \frac{1}{2}bh \\ A &= \frac{1}{2}(216) \end{aligned}$$

Then, solve the equation.

$$\begin{aligned} A &= \frac{1}{2}(216) \\ A &= 108\ square\ inches \end{aligned}$$

The answer is each of the triangles will have an area of 108 square inches.

Example 4.7.2

Find the area of the triangle with a base of 6 feet and a height of 5 feet.

Solution

First, substitute the values for the base and height into the triangle area formula.

$$\begin{aligned} A &= \frac{1}{2}bh \\ A &= 12(6)(5) \end{aligned}$$

Next, solve the equation.

$$\begin{aligned} A &= 12(30) \\ A &= 15\ square feet \end{aligned}$$

The answer is 15 square feet.

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Example 4.7.3

Using the area of the following parallelogram, find the area of one of the triangles inside the parallelogram.

Solution

Area of a rectangle is 12 sq. inches

First, recall that the area of a rectangle is base times height and a triangle is one half the area of a rectangle.

Next, substitute the area value for the base and height in the triangle area formula.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(12)$$

Then, solve the equation.

$$A = \frac{1}{2}(12)$$
$$A = 6 \; square \; inches$$

The answer is 6 square inches.

Example 4.7.4

Using the area of the following parallelogram, find the area of one of the triangles inside the parallelogram.

Solution

Area of a parallelogram is 24 sq. feet

First, recall that the area of a parallelogram is base times height and a triangle is one half the area of a parallelogram.

Next, substitute the area value for the base and height in the triangle area formula.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(24)$$

Then, solve the equation.

$$A = \frac{1}{2}(24)$$
$$A = 12 \; square \; feet$$

The answer is 12 square feet.

Example 4.7.5

Using the area of the following parallelogram, find the area of one of the triangles inside the parallelogram.

Solution

Area of a parallelogram is 18 sq. feet

First, substitute the area value for the base and height in the triangle area formula.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(18)$$

Then, solve the equation.

$$A = \frac{1}{2}(18)$$
$$A = 9 \; square \; feet$$

The answer is 9 square feet.

Review

Find the area of each triangle given the following measurements.

1.

Figure **4.7.6**

Base = 10 in, Height = 4 in

2.

Figure **4.7.7**

Base = 16 meters, Height = 10 meters

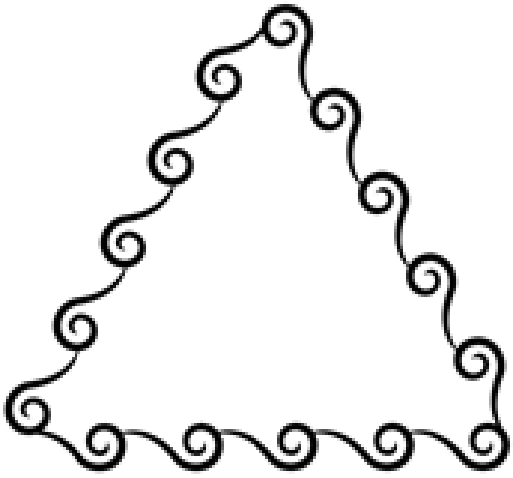
3. 

Figure **4.7.8**

Base = 8 in, Height = 6.5 in

2.

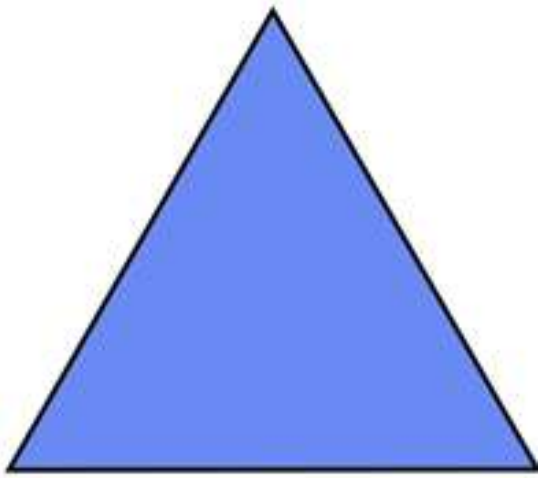


Figure **4.7.9**

Base = 10 cm, Height = 7 cm

2.

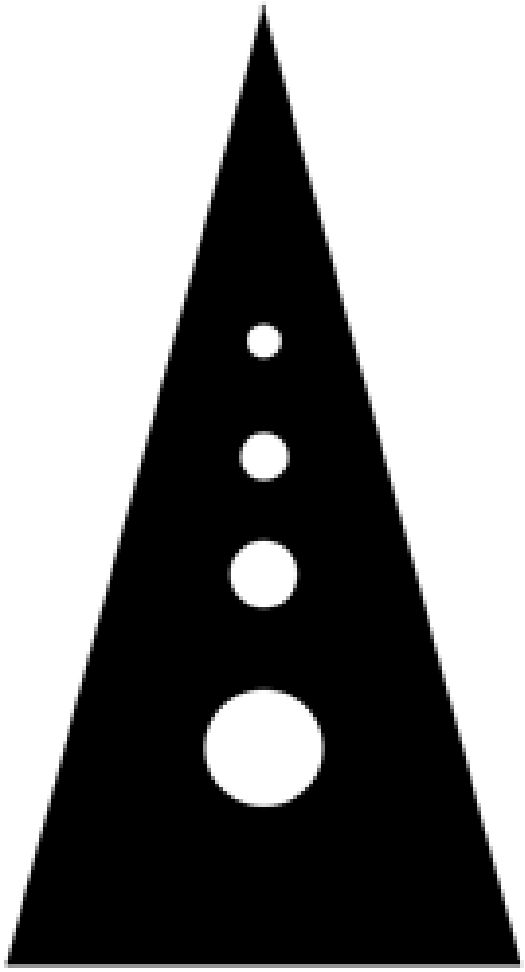


Figure **4.7.10**

Base = 5 ft, Height = 8.5 feet

Find the area of each triangle given the base and height.

6. Base = 4 in, Height = 5 in

7. Base = 6 in, Height = 4 in
8. Base = 8 ft, Height = 7 ft
9. Base = 10 meters, Height = 8 meters
10. Base = 10 meters, Height = 5 meters
11. Base = 12 feet, Height = 14 feet
12. Base = 11 feet, Height = 6 feet
13. Base = 14 inches, Height = 8 inches
14. Base = 22 feet, Height = 19 feet
15. Base = 30 cm, Height = 28 cm
16. Base = 18 inches, Height = 16 inches
17. Base = 13 meters, Height = 10 meters
18. Base = 18 meters, Height = 5.5 meters
19. Base = 12.5 feet, Height = 2.5 feet
20. Base = 13.75 inches, Height = 1.5 inches

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.3.

Resources

Vocabulary

Term	Definition
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Rectangle	A rectangle is a quadrilateral with four right angles.
Square	A square is a polygon with four congruent sides and four right angles.

Additional Resources

Video: Area of a Triangle (Whole Numbers)

Practice: Triangle Area

Real World: Building Triangles

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4.8: Unknown Dimensions of Triangles

Figure 4.8.1

Marie has a collection of photos of buildings shaped like pyramids. As she looked through the photos, she wondered how tall one of the buildings was. She knows that the area of the triangle shaped surface is 900 **square** feet. She also knows that the length of each side of the base of the pyramid (which is also the length of the base of each triangular side) is 60 feet. Given the base and area of the triangles forming the pyramid, how can Marie determine the height of the pyramid?

In this concept, you will learn how to find the unknown dimensions of triangles given the area and another dimension.

Finding Unknown Dimensions of Triangles

There may be a situation when you will be given the area of a triangle and one other dimension and asked to find the missing dimension. When this happens, you can figure out the missing dimension by using the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$

If you have been given the area and the base of a triangle, then you can use the formula to figure out the height. If you have been given the area and the height then you can figure out the base.

Now let's use the formula to find the base of a triangle when the area and height are given.

A triangle has an area of 36 square inches and a height of 6 inches. What is the measure of the base?

Start by looking at the formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

First, substitute the given information into the formula.

$$36 = \frac{1}{2}(6)b$$

Next, multiply one-half by six.

$$36 = 3b$$

Then, solve for the base by dividing both sides by 3.

$$\frac{36}{3} = \frac{3b}{3}$$
$$12 = b$$

The answer is 12. The base of this triangle is 12 inches.

Now, let's find the height of a triangle with a given area and base.

A triangle has an area of 64 square inches and a base of 8 inches. What is the measure of the base?

To figure this out, start by looking at the formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

First, substitute the given information into the formula.

$$64 = \frac{1}{2}(8)h$$

Next, multiply one-half by 8.

$$64 = 4h$$

Then, solve for the base by dividing both sides by 4.

$$\frac{64}{4} = \frac{4h}{4}$$
$$16 = h$$

The answer is 16. The height of this triangle is 16 inches.

Example 4.8.1

Earlier, you were given a problem about Marie and her photos of pyramid buildings.

Marie found out the area of each triangle shaped surface is 900 square feet. She also knows that the length of each side of the base of the pyramid (which is also the length of the base of each triangular side) is 60 feet. Marie found the formula for computing the area of a triangle, and decided to use it to figure out the triangle's height.

$$A = \frac{1}{2}bh$$

Solution

First, she should substitute the given information into the formula.

$$900 = \frac{1}{2}(60)h$$

Next, multiply one-half by 60.

$$900 = 30h$$

Then, solve for the base by dividing both sides by 30.

$$\frac{900}{30} = \frac{30h}{30}$$
$$30 = h$$

The answer is 30. The height of this pyramid is 30 feet.

Example 4.8.2

Find the missing dimension of the following triangle.

The area of the triangle is 48 square feet. The base of the triangle is 12 feet. What is the height?

Solution

Start by looking at the formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

First, fill in the given information.

$$48 = \frac{1}{2}(12)h$$

Next, multiply one-half by 12.

$$48 = 6h$$

Then, solve for the base by dividing 48 by 6.

$$\frac{48}{6} = \frac{6h}{6}$$
$$8 = h$$

The answer is 8. The height of this triangle is 8 feet.

Example 4.8.3

A triangle has an area of 42 sq. ft. If the base is 12 feet, what is the measure of the height?

Solution

To figure this out, start by looking at the formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

First, substitute the given information into the formula.

$$42 = \frac{1}{2}(12)h$$

Next, multiply one-half by 12.

$$42 = 6h$$

Then, solve for the base by dividing both sides by 6.

$$\frac{42}{6} = 6h$$
$$7 = 6h$$

The answer is 7. The height of this triangle is 7 feet.

Example 4.8.4

A triangle has an area of 16 sq. cm. If the height of the triangle is 4 cm, what is the measure of the base?

Solution

To figure this out, start by looking at the formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

First, substitute the given information into the formula.

$$16 = \frac{1}{2}b(4)$$

Next, multiply one-half by 4.

$$16 = 2b$$

Then, solve for the base by dividing both sides by 2.

$$\frac{16}{2} = \frac{2b}{2}$$
$$8 = b$$

The answer is 8. The height of this triangle is 8 cm.

Example 4.8.5

A triangle has an area of 40 sq. meters and a base of 10. What is the height?

Solution

Start by looking at the formula for finding the area of a triangle.

$$A = \frac{1}{2}bh$$

First, substitute the given information into the formula.

$$40 = \frac{1}{2}(10)h$$

Next, multiply one-half by 10.

$$40 = 5h$$

Then, solve for the base by dividing both sides by 5.

$$\frac{40}{5} = \frac{5h}{5}$$

$$8 = h$$

The answer is 8. The height of this triangle is 8 meters.

Review

Find the missing dimension given the area and either the base or height of a triangle.

1. Area = 15 sq. in, Base = 10 in, what is the height?
2. Area = 40 sq. in, Base = 20 in, what is the height?
3. Area = 24 sq. ft, Base = 8 ft, what is the height?
4. Area = 16 sq. m, Base = 8 m, what is the height?
5. Area = 25 sq. in, height = 5 in, what is the base?
6. Area = 36 sq. ft, Height = 6 ft, what is the base?
7. Area = 54 sq. cm, Height = 9 cm, what is the base?
8. Area = 80 sq. meters, Base = 16 meters, what is the height?
9. Area = 75 sq. meters, Base = 10 meters, what is the height?
10. Area = 90 sq. meters, Base = 30 meters, what is the height?
11. Area = 180 sq. meters, Base = 10 meters, what is the height?
12. Area = 90 sq. meters, Base = 15 meters, what is the height?
13. Area = 120 sq. meters, Base = 60 meters, what is the height?
14. Area = 150 sq. meters, Base = 50 meters, what is the height?
15. Area = 280 sq. meters, Base = 140 meters, what is the height?

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.4.

Resources

Vocabulary

Term	Definition
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Rectangle	A rectangle is a quadrilateral with four right angles.
Square	A square is a polygon with four congruent sides and four right angles.

Additional Resources

Video: Area of a Triangle (Whole Numbers)

Practice: Unknown Dimensions of Triangles

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4.9: CPCTC

Sides and angles of congruent triangles have the same measure.

Congruent Triangles

Two figures are congruent if they have exactly the same size and shape. If two triangles are congruent, they will have exactly the same three sides and exactly the same three angles. In other words, two triangles are congruent if you can turn, flip, and/or slide one so it fits exactly on the other.

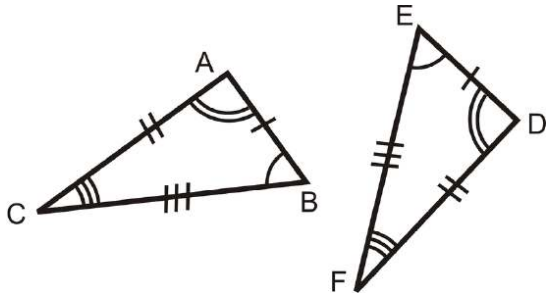


Figure 4.9.1

ΔABC and ΔDEF are congruent because

$$\begin{array}{ll} \overline{AB} \cong DE & \angle A \cong \angle D \\ \overline{BC} \cong EF\overline{AB}\ and & \angle B \cong \angle E \\ \overline{AC} \cong \overline{DF} & \angle C \cong \angle F \end{array}$$

Notice that when two triangles are congruent their three pairs of corresponding angles and their three pairs of corresponding sides are congruent.

When referring to corresponding congruent parts of congruent triangles, you can use the phrase **C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent, or its abbreviation **CPCTC**.

What if you were given two triangles with all the angle measures and all the side lengths marked? How could you tell if the two triangles were congruent?

Example 4.9.1

Determine if the triangl ng the definition of congruent triangles.

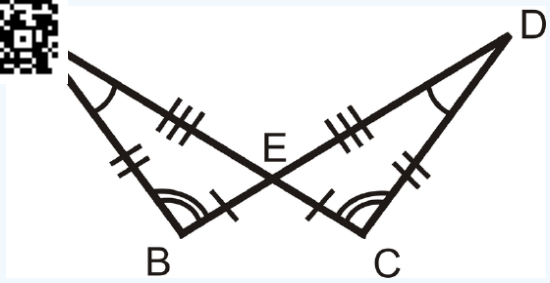


Figure 4.9.2

Solution

We can see from the markings that $\angle B \cong \angle C$, $\angle A \cong \angle D$, and $\angle AEB \cong \angle DEC$ because they are vertical angles. Also, we know that $\overline{BA} \cong \overline{CD}$, $\overline{EA} \cong \overline{ED}$, and $\overline{BE} \cong \overline{CE}$. Because three pairs of sides and three pairs of angles are all congruent and they are corresponding parts, this means that the two triangles are congruent.

Example 4.9.2

Determine if the triangles are congruent using the definition of congruent triangles.

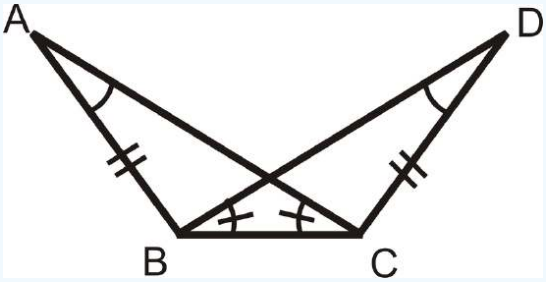


Figure 4.9.3

Solution

While there are congruent corresponding parts, there are only two pairs of congruent sides, the marked ones and the shared side. Without knowing whether or not the third pair of sides is congruent we cannot say if the triangles are congruent using the definition of congruent triangles. Note, this does not mean that the triangles are **not congruent**, it just means that we need more information in order to say they are congruent using the ***definition of congruent triangles*** (congruent triangles have three pairs of congruent angles and three pairs of congruent sides).

Example 4.9.3

Are the two triangles below congruent?

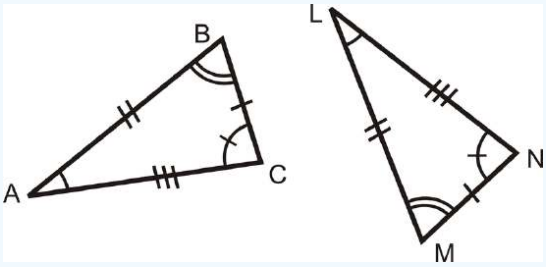


Figure 4.9.4

Solution

To determine if the triangles are congruent, match up sides with the same number of tic marks: $\overline{BC} \cong \overline{MN}$, $\overline{AB} \cong \overline{LM}$, $\overline{AC} \cong \overline{LN}$.

Next match up the angles with the same markings:

$\angle A \cong \angle L$, $\angle B \cong \angle M$, and $\angle C \cong \angle N$.

Lastly, we need to make sure these are *corresponding* parts. To do this, check to see if the congruent angles are opposite congruent sides. Here, $\angle A$ is opposite $\overline{BC}$ and $\angle L$ is opposite $\overline{MN}$. Because $\angle A \cong \angle L$ and $\overline{BC} \cong \overline{MN}$, they are corresponding. Doing this check for the other sides and angles, we see that everything matches up and the two triangles are congruent.

Example 4.9.4

If all three pairs of angles for two given triangles are congruent does that mean that the triangles are congruent?

Solution

Without knowing anything about the lengths of the sides you cannot tell whether or not two triangles are congruent. The two triangles described above *might* be congruent, but we would need more information to know for sure.

Example 4.9.5

Determine if the triangles are congruent using the definition of congruent triangles.

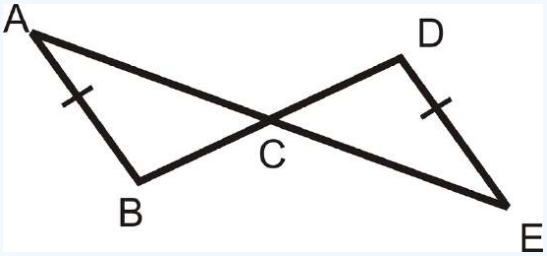


Figure 4.9.5

Solution

From the tic marks we can see that $\overline{AB} \cong \overline{DE}$. We also know that $\angle ACB \cong \angle ECD$ because they are vertical angles. However, this is not enough information to know whether or not the triangles are congruent.

Review

The following illustrations show two parallel lines cut by a transversal (You may assume that even though the lines are not marked parallel, the two lines that look parallel, actually are). Are the triangles formed by them definitively congruent?

1.

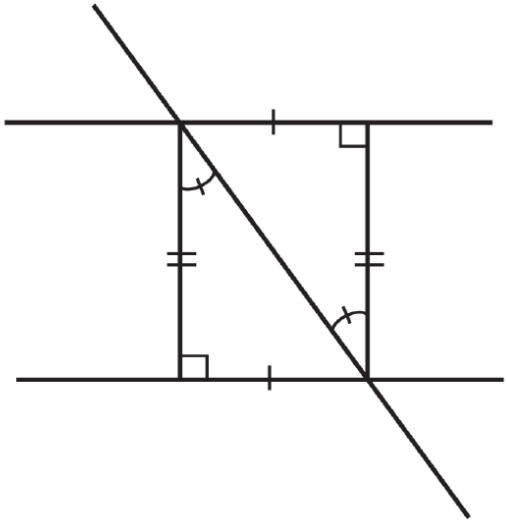


Figure 4.9.6

2.

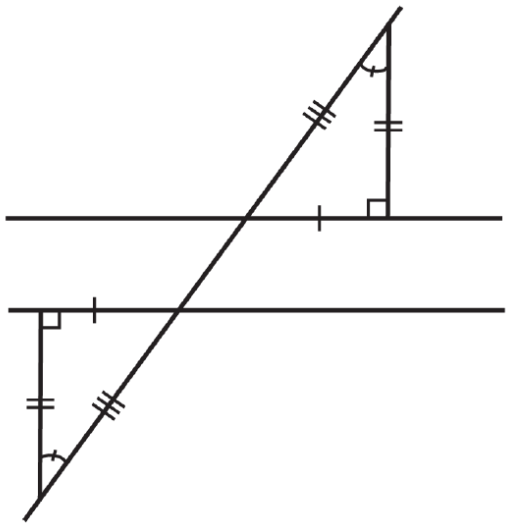


Figure 4.9.7

3.

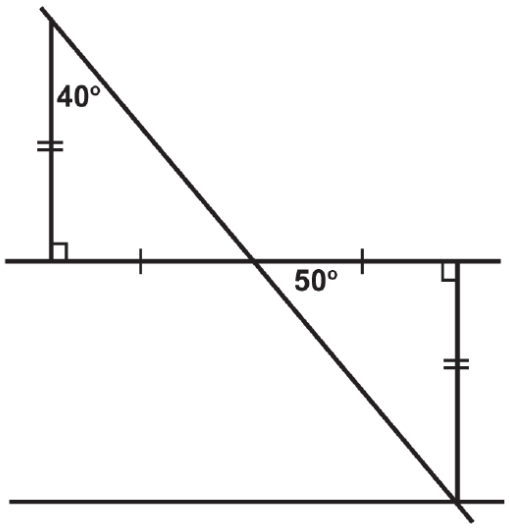


Figure 4.9.8

4.

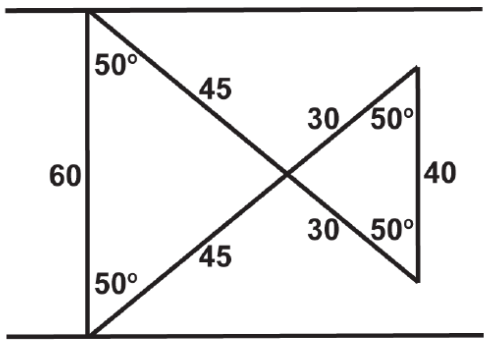
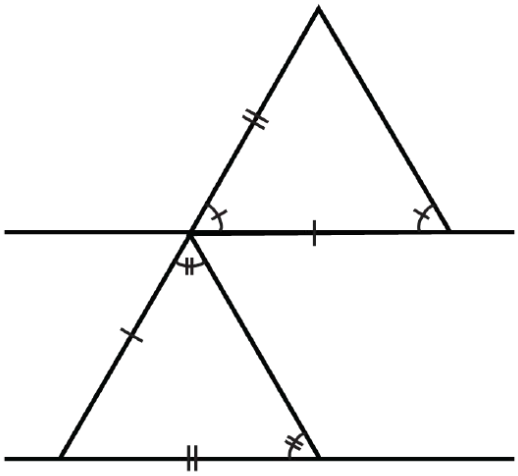


Figure 4.9.9



5.

Figure 4.9.10

Based on the following details, are the triangles definitively congruent?

6. Both triangles are right triangles in which one angle measures $55°$. All of their corresponding sides are congruent.
7. Both triangles are equiangular triangles.
8. Both triangles are equilateral triangles. All sides are 5 inches in length.
9. Both triangles are obtuse triangles in which one angle measures $35°$. Two of their corresponding sides are congruent.
10. Both triangles are obtuse triangles in which two of their angles measure $40°$ and $20°$. All of their corresponding sides are congruent.
11. Both triangles are isosceles triangles in which one angle measures $15°$.
12. Both triangles are isosceles triangles with two equal angles of $55°$. All corresponding sides are congruent.
13. Both triangles are acute triangles in which two of their angles measure $40°$ and $80°$. All of their corresponding sides are congruent.
14. Both triangles are acute triangles in which one angle measures $60°$. Two of their corresponding sides are congruent.
15. Both triangles are equilateral triangles.

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.3.

Resources

Vocabulary

Term	Definition
Third Angle Theorem	If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles is also congruent.

Additional Resources

Video: Congruent and Similar Triangles

Activities: Congruent Triangles Discussion Questions

Study Aids: Triangle Congruence Study Guide

Practice: CPCTC

Real World: Congruent Triangles

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4.10: Congruence Statements

Corresponding angles and sides of congruent triangles are congruent.

When stating that two triangles are congruent, the corresponding parts must be written in the same order. For example, if we know that ΔABC and ΔLMN are congruent then we know that:

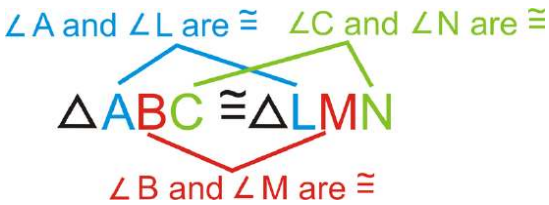


Figure **4.10.1**

Notice that the congruent sides also line up within the congruence statement.

$\overline{AB} \cong \overline{LM}$, $\overline{BC} \cong \overline{MN}$, $\overline{AC} \cong \overline{LN}$

We can also write this congruence statement five other ways, as long as the congruent angles match up. For example, we can also write $\Delta ABC \cong \Delta LMN$ as:

$\Delta ACB \cong \Delta LNM$ $\Delta BCA \cong \Delta MNL$ $\Delta BAC \cong \Delta MLN$
$\Delta CBA \cong \Delta NML$ $\Delta CAB \cong \Delta NLM$

What if you were told that $\Delta FGH \cong \Delta XYZ$? How could you determine which side in ΔXYZ is congruent to $\overline{GH}$ and which angle is congruent to $\angle F$?

Example 4.10.1

If $\Delta ABC \cong \Delta DEF$, wha ?

Solution

From this congruence st hree pairs of angles and three pairs of sides are congruent.

$\angle A \cong \angle D, \angle B \cong \angle E,$\angle C\cong \angle F\),

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$.

Example 4.10.2

If$\Delta KBP \cong \Delta MRS$, what else do you know?

Solution

From this congruence statement, we know three pairs of angles and three pairs of sides are congruent.

$\angle K \cong \angle M$, $\angle B \cong \angle R$, $\angle P \cong \angle S$,

$\overline{KB} \cong \overline{MR}$, $\overline{BP} \cong \overline{RS}$, $\overline{KP} \cong \overline{MS}$.

Example 4.10.3

Write a congruence statement for the two triangles below.

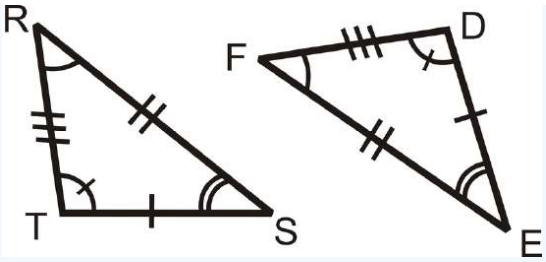


Figure **4.10.2**

Solution

Line up the corresponding angles in the triangles:

$\angle R \cong \angle F$, $\angle S \cong \angle E$ and $\angle T \cong \angle D$.

Therefore, one possible congruence statement is $\Delta RST \cong \angle FED$

Example 4.10.4

If $\Delta CAT \cong \Delta DOG$, what else do you know?

Solution

From this congruence statement, we know three pairs of angles and three pairs of sides are congruent.

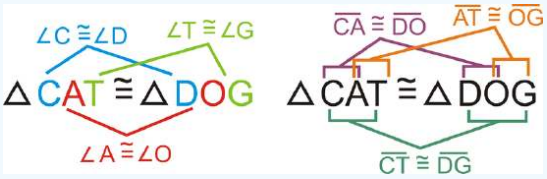


Figure **4.10.3**

Example 4.10.5

If $\Delta BUG \cong \Delta ANT$, what angle is congruent to $\angle N$?

Solution

Since the order of the letters in the congruence statement tells us which angles are congruent, $\angle N \cong \angle U$ because they are each the second of the three letters.

Review

For questions 1-4, determine if the triangles are congruent using the definition of congruent triangles. If they are, write the congruence statement.

1.

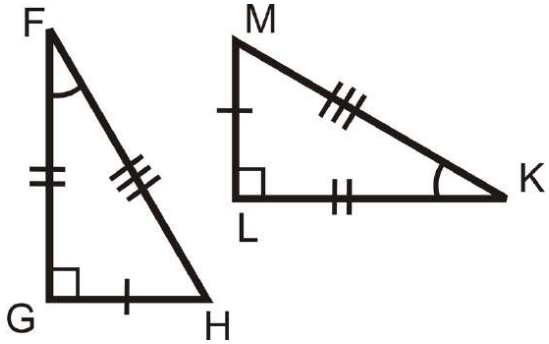


Figure **4.10.4**

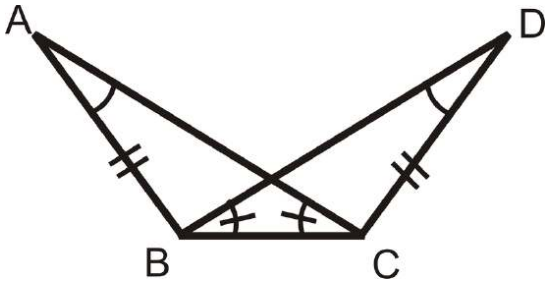
2. 

Figure 4.10.5

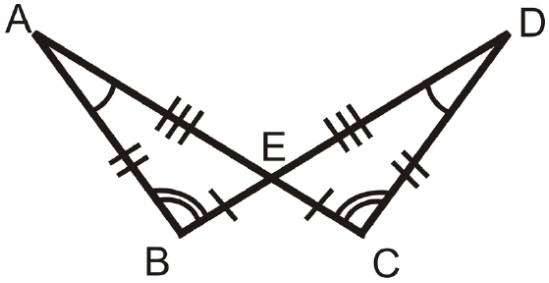
3. 

Figure 4.10.6

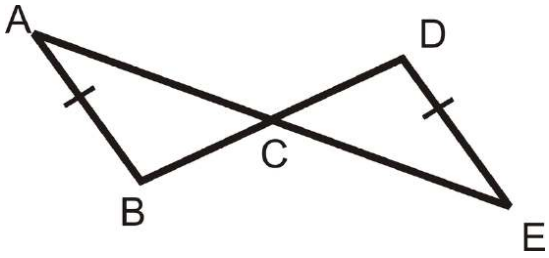
4. 

Figure 4.10.7

5. Suppose the two triangles below are congruent. Write a congruence statement for these triangles.

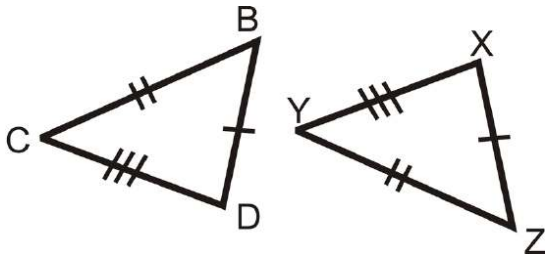


Figure 4.10.8

6. Explain how we know that if the two triangles are congruent, then $\angle B \cong \angle Z$.
7. If $\Delta TBS \cong \Delta FAM$, what else do you know?
8. If $\Delta PAM \cong \Delta STE$, what else do you know?
9. If $\Delta INT \cong \Delta WEB$, what else do you know?
10. If $\Delta ADG \cong \Delta BCE$, what angle is congruent to $\angle G$?

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.4.

Resources

Additional Resource

Video: Introduction to Cor

Activities: Congruence Sta ı Questions

Study Aids: Triangle Cong[redacted]

Practice: Congruence Statements

Real World: Congruent Statements

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4.11: Third Angle Theorem

Third angles are equal if the other two sets are each congruent.

If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles must also congruent. This is called the **Third Angle Theorem**.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

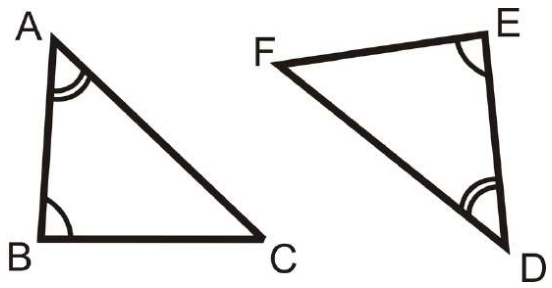

Figure 4.11.1

What if you were given ΔFGH and ΔXYZ and you were told that $\angle F \cong \angle X$ and $\angle G \cong \angle Y$? What conclusion could you draw about $\angle H$ and $\angle Z$?

Example 4.11.1

Determine the measure of each triangle.

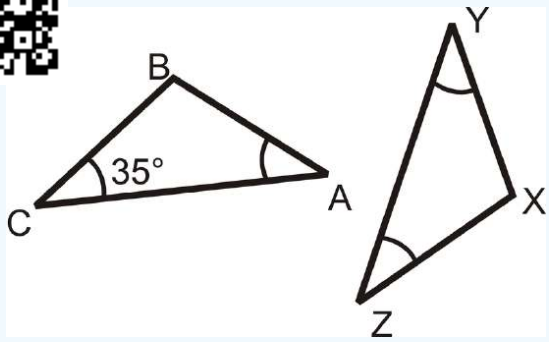

Figure 4.11.2

Solution

$m\angle C = m\angle A = m\angle Y = m\angle Z = 35$. By the **Triangle Sum Theorem** $m\angle B = m\angle X = 110$.

Example 4.11.2

Determine the measure of all the angles in each triangle.

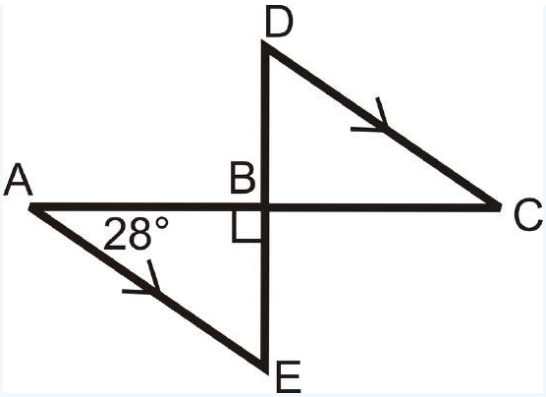

Figure 4.11.3

Solution

$m\angle A = 28$, $m\angle ABE = 90$ and by the Triangle Sum Theorem, $m\angle E = 62$. $m\angle D = m\angle E = 62$ because they are alternate interior angles and the lines are parallel. $m\angle C = m\angle A = 28$ because they are alternate interior angles and the lines are parallel. $m\angle DBC = m\angle ABE = 90$ because they are vertical angles.

Example 4.11.3

Determine the measure of the missing angles.

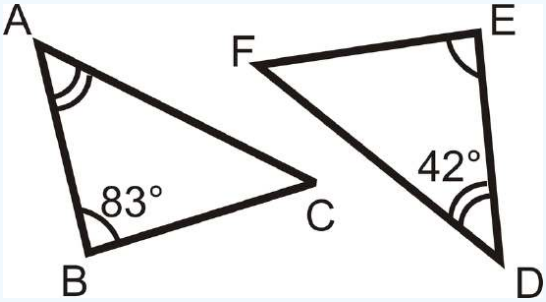

Figure 4.11.4

Solution

From the Third Angle Theorem, we know $\angle C \cong \angle F$. From the Triangle Sum Theorem, we know that the sum of the interior angles in each triangle is $180°$.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180° \\ m\angle D + m\angle B + m\angle C &= 180° \\ 42° + 83° + m\angle C &= 180° \\ m\angle C &= 55° = m\angle F \end{aligned}$$

Example 4.11.4

Explain why the Third Angle Theorem works.

Solution

The Third Angle Theorem is really like an extension of the Triangle Sum Theorem. Once you know two angles in a triangle, you automatically know the third because of the Triangle Sum Theorem. This means that if you have two triangles with two pairs of angles congruent between them, when you use the Triangle Sum Theorem on each triangle to come up with the third angle you will get the same answer both times. Therefore, the third pair of angles must also be congruent.

Example 4.11.5

Determine the measure of all the angles in the triangle:

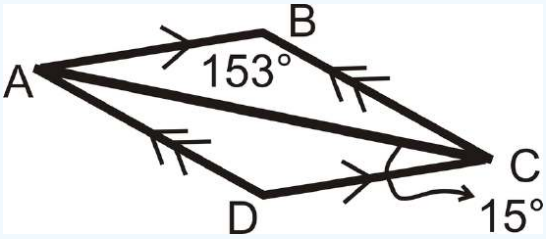

Figure 4.11.5

Solution

First we can see that $m\angle DCA = 15°$. This means that $m\angle BAC = 15°$ also because they are alternate interior angles. $m\angle ABC = 153°$ was given. This means by the Triangle Sum Theorem that $m\angle BCA = 12°$. This means that $m\angle CAD = 12°$ also because they are alternate interior angles. Finally, $m\angle ADC = 153°$ by the Triangle Sum Theorem.

Review

Determine the measures of the unknown angles.

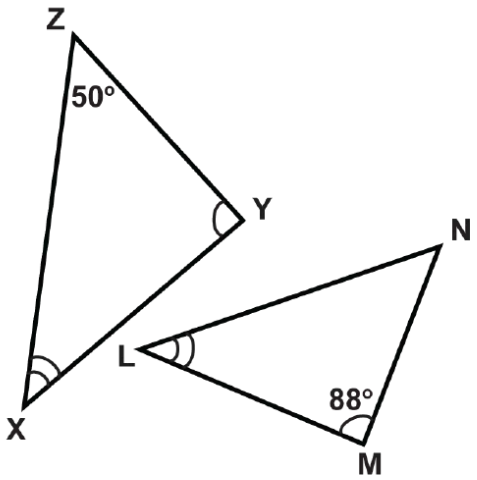

Figure 4.11.6

1. $\angle Y$
2. $\angle x$
3. $\angle N$
4. $\angle L$

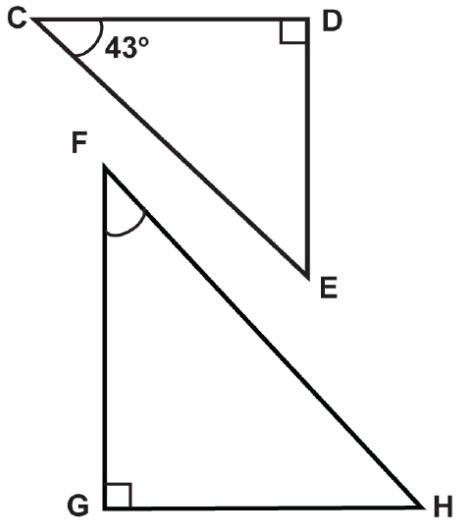

Figure 4.11.7

5. $\angle E$
6. $\angle F$
7. $\angle H$

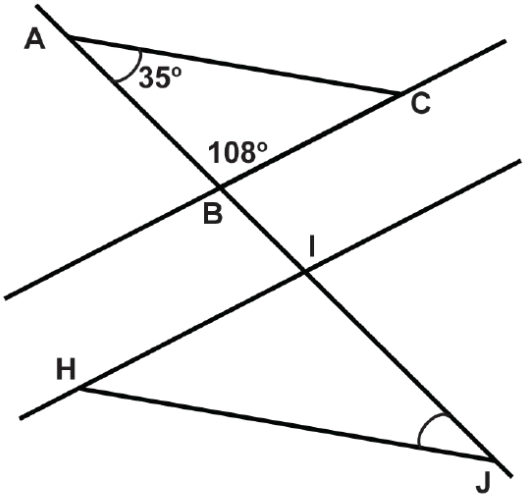

Figure 4.11.8

You may assume that $\overleftrightarrow{BC} \parallel \overleftrightarrow{HI}$.

8. $\angle ACB$
9. $\angle HIJ$
10. $\angle HJI$
11. $\angle IHJ$

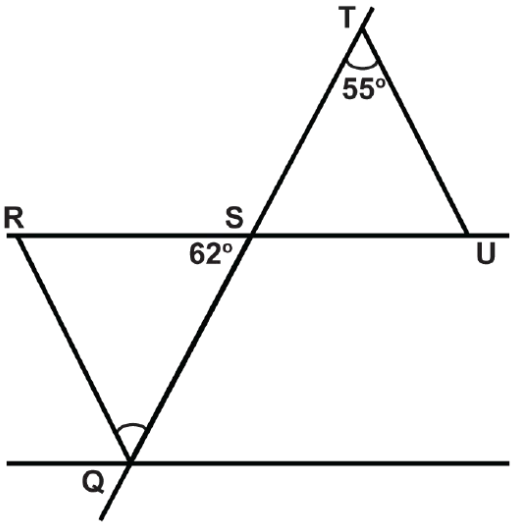

Figure 4.11.9

12. $\angle RQS$
13. $\angle SRQ$
14. $\angle TSU$
15. $\angle TUS$

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.5.

Vocabulary

Term	Definition
Triangle Sum Theorem	The ***Triangle Sum Theorem*** states that the measure of the three interior angles of any triangle will add up to $180°$.
Third Angle Theorem	If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles is also congruent.

Additional Resources

Video: The Third Angle Theorem Principles - Basic

Activities: Third Angle Theorem Discussion Questions

Study Aids: Triangle Congruence Study Guide

Practice: Third Angle Theorem

Real World: Third Angle Theorem

12. $\angle RQS$
13. $\angle SRQ$
14. $\angle TSU$
15. $\angle TUS$

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4.12: Congruent Triangles

Verify congruency with SSS, SAS, RHS, and ASA

Applications of Congruent Triangles

Two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs are congruent.

The following list summarizes the different criteria that can be used to show triangle congruence:

- AAS (Angle-Angle-Side): If two triangles have two pairs of congruent angles, and a non-common side of the angles in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.
- ASA (Angle-Side-Angle): If two triangles have two pairs of congruent angles and the common side of the angles (the side between the congruent angles) in one triangle is congruent to the corresponding side in the other triangle, then the triangles are congruent.
- SAS (Side-Angle-Side): If two triangles have two pairs of congruent sides and the included angle in one triangle is congruent to the included angle in the other triangle, then the triangles are congruent.
- SSS (Side-Side-Side): If two triangles have three pairs of congruent sides, then the triangles are congruent.
- **Right triangles only:** HL (Hypotenuse-Leg): If two **right** triangles have one pair of legs congruent and hypotenuses congruent, then the triangles are congruent.

If two triangles don't satisfy at least one of the criteria above, you cannot be confident that they are congruent.

Recognizing Perpendic

In the triangle below, ' the perpendicular bisector of AD\overline{AB}. Therefore \overline{AC}\cong \overline{CD}. Also, m\a c}and m\angle DCB=90^{\circ}, so \angle ACB \cong \angle DCB. You also know that \overline{BC} is a sid and is clearly congruent to itself (this is called the reflexive property).

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Image here.
Delete this
placeholder image

Figure 4.12.1

The triangles are congruent by SAS. *Note that even though these are right triangles, you would not use HL to show triangle congruence in this case since you are not given that the hypotenuses are congruent.*

Measuring Angles

Using the information from the previous problem, if $m\angle A = 50°$, what is $m\angle D$?

$m\angle D = 50°$

Since the triangles are congruent, all of their corresponding angles and sides must be congruent. \angle A\) and \angle D\) are corresponding angles, so $\angle A \cong \angle D$.

Congruent Triangles

Does one diagonal of a rectangle divide the rectangle into congruent triangles?

- Recall that a rectangle is a quadrilateral with four right angles.
- The opposite sides of a rectangle are congruent.

Figure \(\PageIndex{2}\)

There is more than enough information to show that \(\Delta EFG\cong \Delta GHE\).

- Method #1: The triangles have three pairs of congruent sides, so they are congruent by SSS.
- Method #2: The triangles have two pairs of congruent sides and congruent included angles, so they are congruent by SAS.
- Method #3: The triangles are right triangles with congruent hypotenuses and a pair of congruent legs, so they are congruent by HL.

Example 4.12.1

Max constructs a triangle using an online tool. He tells Alicia that his triangle has a 42^{\circ} angle, a side of length 12 and a side of length 8. With only this information, will Alicia be able to construct a triangle that must be congruent to Max's triangle?

Solution

If Max also told Alicia that the angle was in between the two sides, then she would be able to construct a triangle that must be congruent due to SAS. If the angle is not between the two sides, she cannot be confident that her triangle is congruent because SSA is not a criterion for triangle congruence. Because Max did not state where the angle was in relation to the sides, Alicia cannot create a triangle that must be congruent to Max's triangle.

Example 4.12.2

Are the following triangles congruent? Explain.

Figure **4.12.3**

Solution

Notice that besides the one pair of congruent sides and the one pair of congruent angles, $\overline{AC} \cong \overline{CA}$.

$\Delta ACB \cong \Delta CAD$ by SAS.

Example 4.12.3

Are the following triangles congruent? Explain.

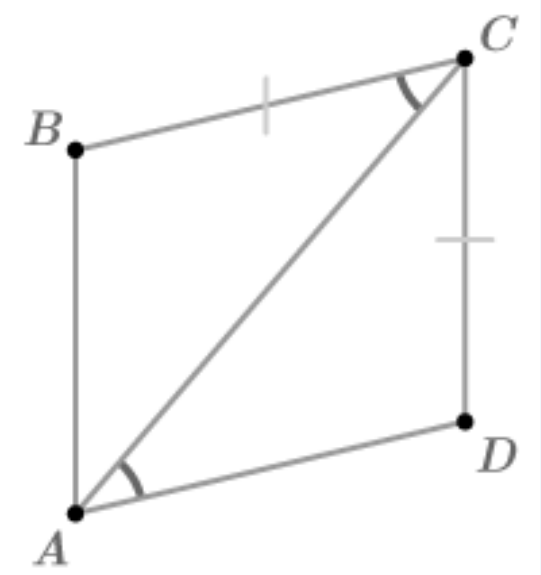

Figure 4.12.4

Solution

The congruent sides are not corresponding in the same way that the congruent angles are corresponding. The given information for ΔACB is SAS while the given information for ΔCAD is SSA. The triangles are not necessarily congruent.

Example 4.12.4

G is the midpoint of $\overline{EH}$. Are the following triangles congruent? Explain.

Figure 4.12.5

Solution

Because G\) is the midpoint of $\overline{EH}$, $\overline{EG} \cong \overline{GH}$. You also know that $\angle EGF \cong \angle HGI$ because they are vertical angles. $\Delta EGF \cong \Delta HGI$ by ASA.

Review

1. List the five criteria for triangle congruence and draw a picture that demonstrates each.

2. Given two triangles, do you always need at least three pieces of information about each triangle in order to be able to state that the triangles are congruent?

For each pair of triangles, tell whether the given information is enough to show that the triangles are congruent. If the triangles are congruent, state the criterion that you used to determine the congruence and write a congruency statement.

Note that the images are not necessarily drawn to scale.

3.

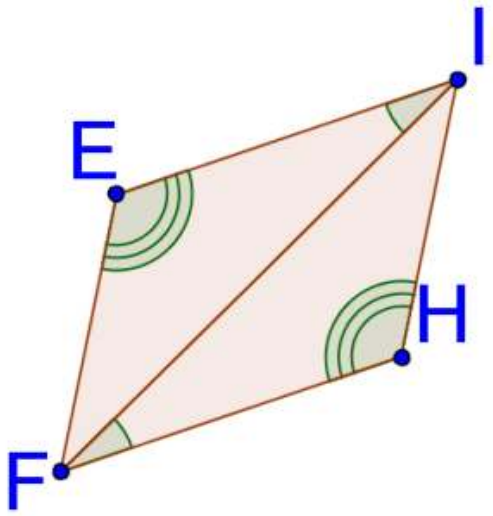

Figure **4.12.6**

4.

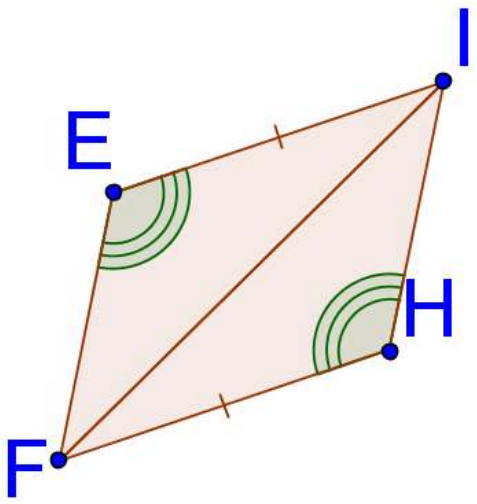

Figure **4.12.7**

5.

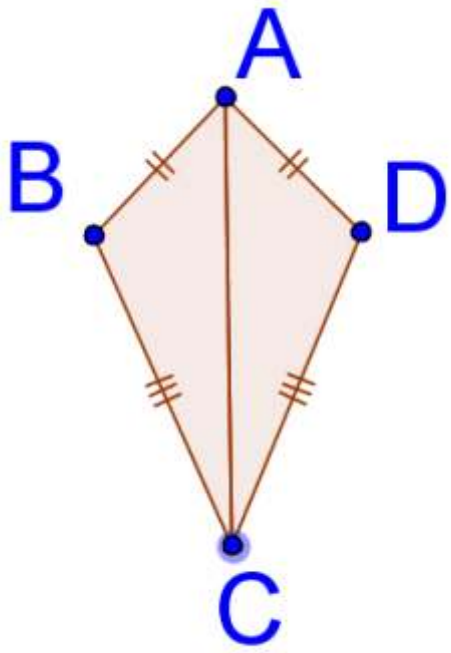

Figure **4.12.8**

6.

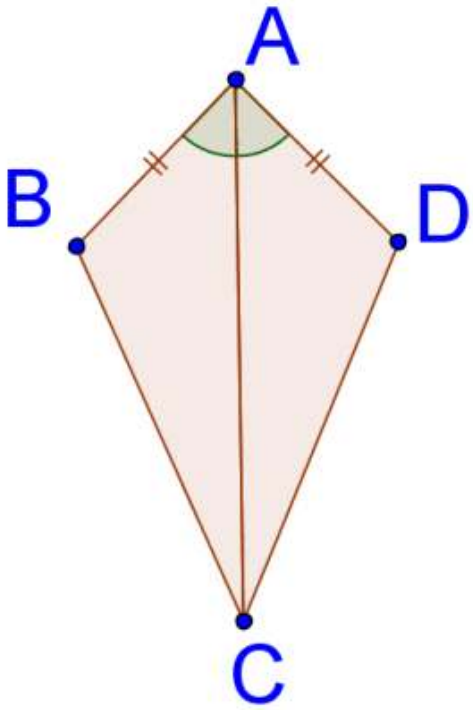

Figure **4.12.9**

7.

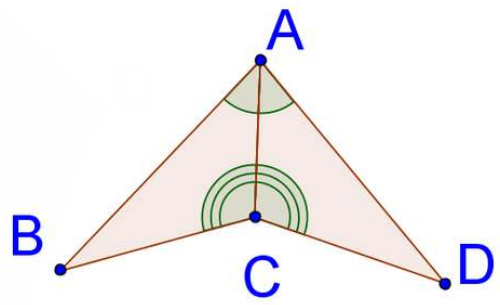

Figure **4.12.10**

8.

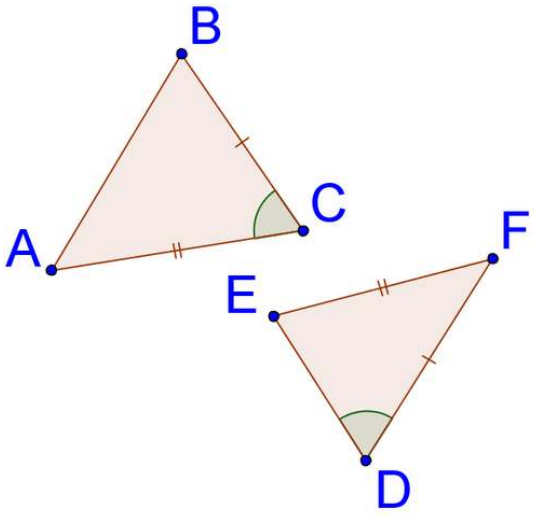

Figure **4.12.11**

For 9-11, state whether the given information about a hidden triangle would be enough for you to construct a triangle that must be congruent to the hidden triangle. Explain your answer.

9. ΔABC with $m\angle A = 72°,\ AB = 6\ cm,\ BC = 8\ cm.$

10. ΔABC with $m\angle A = 90°,\ AB = 4\ cm,\ BC = 5\ cm.$

11. ΔABC with $m\angle A = 72°,\ AB = 6\ cm,\ AC = 8\ cm.$

12. Recall that a square is a quadrilateral with four right angles and four congruent sides. Show and explain why a diagonal of a square divides the square into two congruent triangles.

13. Show and explain using a different criterion for triangle congruence why a diagonal of a square divides the square into two congruent triangles.

14. Recall that a kite is a quadrilateral with two pairs of adjacent, congruent sides. Will one of the diagonals of a kite divide the kite into two congruent triangles? Show and explain your answer.

15. In the picture below, G is the midpoint of both $\overline{EH}$ and $\overline{FI}$. Explain why $\overline{FH} \cong \overline{IE}$ and $\overline{FE} \cong \overline{HI}$.

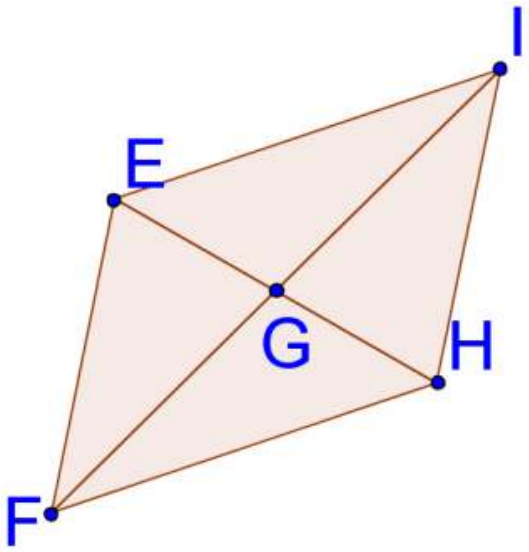

Figure 4.12.12

16. Explain why AAA is not a criterion for triangle congruence.

Review (Answers)

To see the Review answers, click here.

Additional Resources

Video: Congruent and Similar Triangles - KA

Practice: Congruent Triangles

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4.13: SSS

Three sets of equal side lengths determine congruence.

Side-Side-Side Postulate

If 3 sides in one triangle are **congruent** to 3 sides in another triangle, then the triangles are congruent.

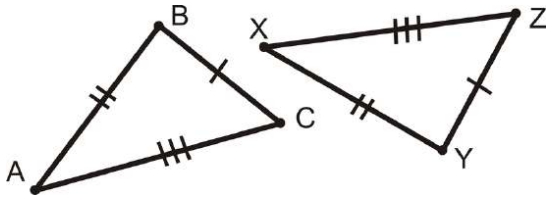


Figure 4.13.1

$\overline{BC} \cong \overline{YZ}$, $\overline{AB} \cong \overline{XY}$, and $\overline{AB} \cong \overline{XZ}$ then $\Delta ABC \cong \Delta XYZ$.

This is called the Side-Side-Side (**SSS**) Postulate and it is a shortcut for proving that two triangles are congruent. Before, you had to show **3 sides and 3 angles** in one triangle were congruent to **3 sides and 3 angles** in another triangle. Now you only have to show **3 sides** in one triangle are congruent to **3 sides** in another.

What if you were given two triangles and provided with information only about their side lengths? How could you determine if the two triangles were congruent?

Example 4.13.1

Determine if the two tri t.

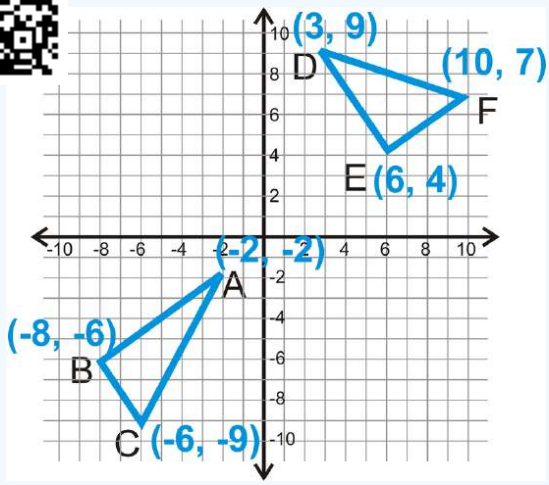


Figure 4.13.2

Solution

Start with ΔABC.

$$
\begin{aligned}
AB &= \sqrt{(-2-(-8))^2+(-2-(-6))^2} \\
&= \sqrt{(6)^2+(4)^2} \\
&= \sqrt{36+16} \\
&= \sqrt{52} \\
&= \sqrt{213} \\
BC &= \sqrt{(-8-(-6))^2+(-6-(-9))^2} \\
&= \sqrt{(-2)^2+(3)^2} \\
&= \sqrt{4+9} \\
&= \sqrt{13} \\
AC &= \sqrt{(-2-(-6))^2+(-2-(-9))^2} \\
&= \sqrt{(4)^2+(7)^2} \\
&= \sqrt{16+49} \\
&= \sqrt{65}
\end{aligned}
$$

Now find the sides of ΔDEF.

$$
\begin{aligned}
DE &= \sqrt{(3-6)^2+(9-4)^2} \\
&= \sqrt{(-3)^2+(5)^2} \\
&= \sqrt{9+25} \\
&= \sqrt{34} \\
EF &= \sqrt{(6-10)^2+(4-7)^2} \\
&= \sqrt{(-4)^2+(-3)^2} \\
&= \sqrt{16+9} \\
&= \sqrt{25} \\
&= 5 \\
DF &= \sqrt{(3-10)^2+(9-7)^2} \\
&= \sqrt{(-7)^2+(2)^2} \\
&= \sqrt{49+4} \\
&= \sqrt{53}
\end{aligned}
$$

No sides have equal measures, so the triangles are not congruent.

Example 4.13.2

Fill in the blanks in the proof below.

Given: $\overline{AB} \cong \overline{DC}$, $\overline{AC} \cong \overline{DB}$

Prove: $\Delta ABC \cong \Delta DCB$

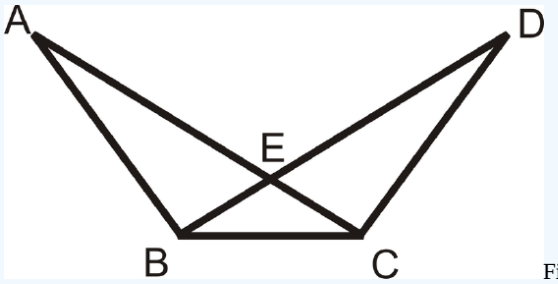


Figure 4.13.4

Solution

Statement	*Reason*
1.	1.
2.	2. Reflexive PoC
3. $\Delta ABC \cong \Delta DCB$	3.

Statement	*Reason*
1. $\overline{AB} \cong \overline{DC}, \overline{AC} \cong \overline{DB}$	1. Given
2. $\overline{BC} \cong \overline{CB}$	2. Reflexive PoC
3. $\Delta ABC \cong \Delta DCB$	3. SSS Postulate

Example 4.13.3

Write a **triangle congruence** statement based on the picture below:

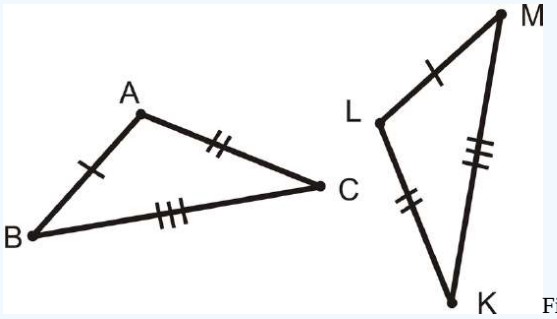


Figure 4.13.5

Solution

From the tic marks, we know AB\overline{AB}\cong LM\overline{AB}\), AC\overline{AB}\cong LK\overline{AB}\), \overline{BC}\cong MK\overline{AB}\). From the SSS Postulate, the triangles are congruent. Lining up the corresponding sides, we have \Delta ABC\cong \Delta LMK\).

Don’t forget ORDER MATTERS when writing congruence statements. Line up the sides with the same number of tic marks.

Example 4.13.4

Write a two-column proof to show that the two triangles are congruent.

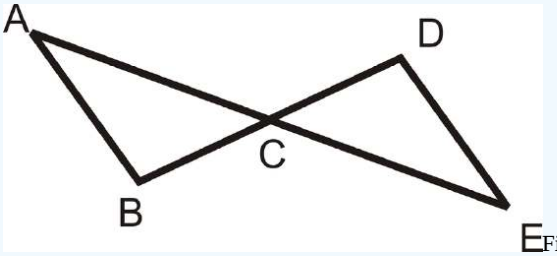


Figure 4.13.6

Given: $\overline{AB} \cong \overline{DE}$

C is the midpoint of $\overline{AE}$ and $\overline{DB}$.

Prove: $\Delta ACB \cong \Delta ECD$

Solution

Statement	*Reason*
1. $\overline{AB} \cong \overline{DE}$ C is the midpoint of $\overline{AE} and \backslash (\overline{DB}$	1.Given
2. $\overline{AC} \cong \overline{CE}, \overline{BC} \cong \overline{CD}$	2.Definition of a midpoint
3. $\Delta ACB \cong \Delta ECD$	3.SSS Postulate

Note that you must *clearly state* the three sets of sides are congruent BEFORE stating the triangles are congruent.

Example 4.13.5

The only way we will show two triangles are congruent in an $x-y$ plane is using SSS.

Find the lengths of all the line segments from both triangles to see if the two triangles are congruent.

Solution

To do this, you need to use the **distance formula**.

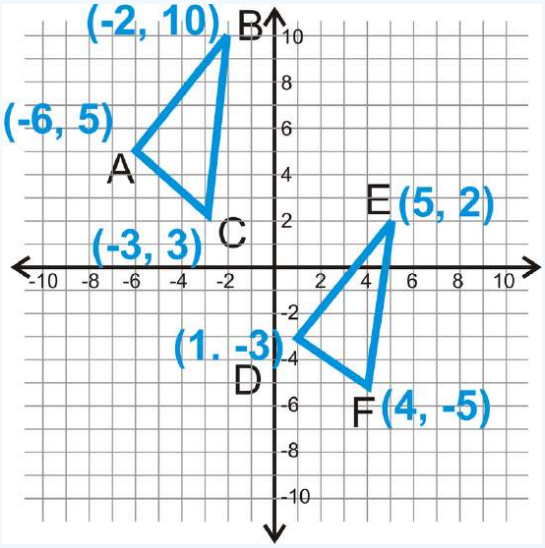


Figure 4.13.7

Begin with ΔABC and its sides.

$$\begin{aligned} AB &= \sqrt{(-6-(-2))^2+(5-10)^2} \\ &= (-4)^2+(-5)^2 \\ &= \sqrt{16+25} \\ &= \sqrt{41} \\ BC &= \sqrt{(-2-(-3))^2+(10-3)^2} \\ &= (1)^2+(7)^2 \\ &= \sqrt{1+49} \\ &= \sqrt{50} \\ &= \sqrt{52} \\ AC &= \sqrt{(-6-(-3))^2+(5-3)^2} \\ &= \sqrt{(-3)^2+(2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \end{aligned}$$

Now, find the lengths of all the sides in ΔDEF.

$$DE = \sqrt{(1-5)^2 + (-3-2)^2}$$
$$= \sqrt{(-4)^2 + (-5)^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$
$$EF = \sqrt{(5-4)^2 + (2-(-5))^2}$$
$$= \sqrt{(1)^2 + (7)^2}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$
$$DF = \sqrt{(1-4)^2 + (-3-(-5))^2}$$
$$= \sqrt{(-3)^2 + (2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

$AB = DE$, $BC = EF$, and $AC = DF$, so the two triangles are congruent by SSS.

Review

Are the pairs of triangles congruent? If so, write the congruence statement and why.

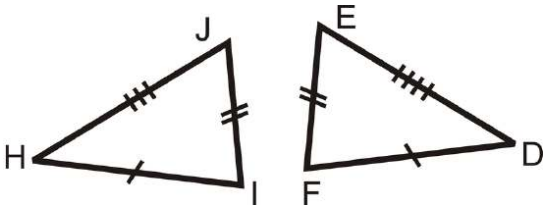
1. 

Figure 4.13.8

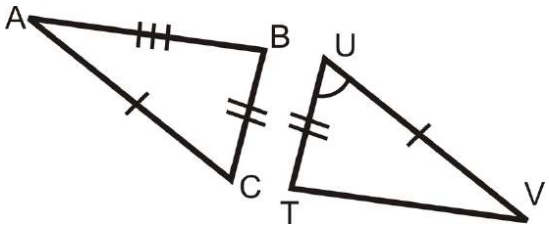
2. 

Figure 4.13.9

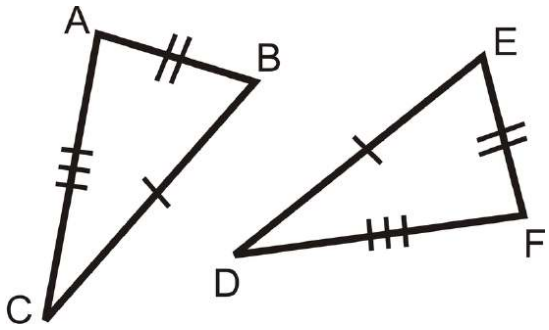
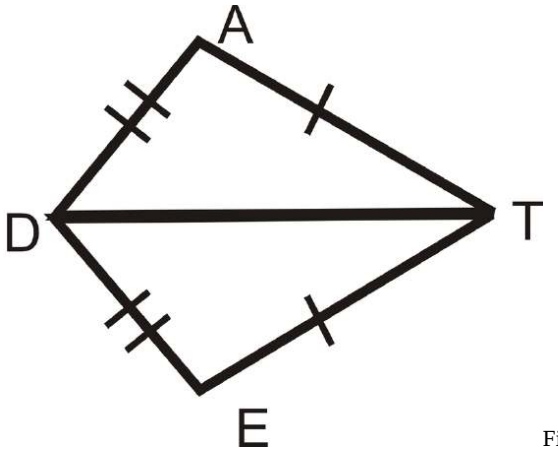
3. 

Figure 4.13.10



4. Figure **4.13.11**

State the additional piece of information needed to show that each pair of triangles is congruent.

5. Use SSS

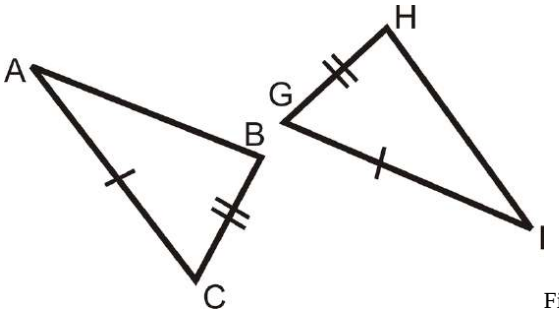


Figure **4.13.12**

6. Use SSS

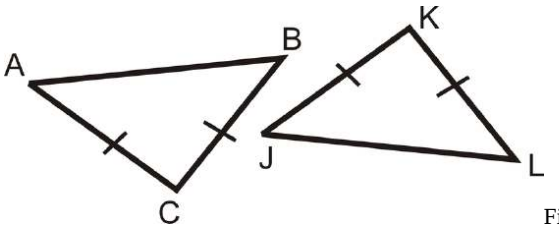


Figure **4.13.13**

Fill in the blanks in the proofs below.

7. Given: B is the midpoint of $\overline{DCAD} \cong \overline{AC}$ Prove: $\Delta ABD \cong \Delta ABC$

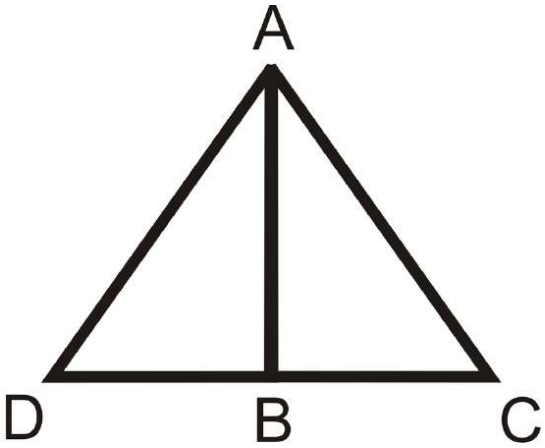


Figure **4.13.14**

Statement	*Reason*
1.	1.
2.	2. Definition of a Midpoint
3.	3. Reflexive *PoC*
4. $\Delta ABD \cong \Delta ABC$	4.

Find the lengths of the sides of each triangle to see if the two triangles are congruent. Leave your answers under the radical.

8.

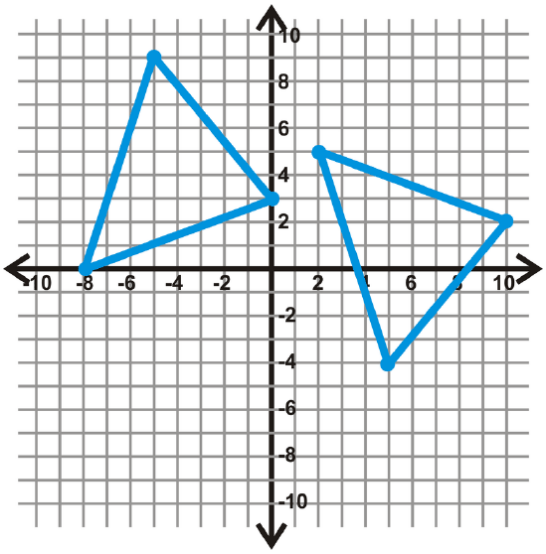


Figure 4.13.15

9.

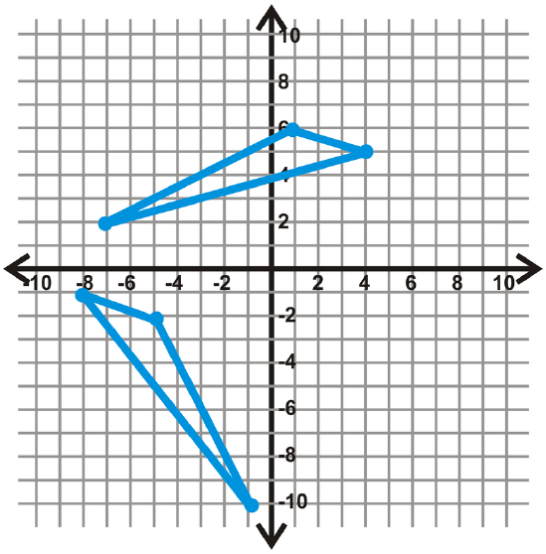


Figure 4.13.16

10. $\Delta ABC: A(-1,5), B(-4,2), C(2,-2)$ and $\Delta DEF: D(7,-5), E(4,2), F(8,-9)$
11. $\Delta ABC: A(-8,-3), B(-2,-4), C(-5,-9)$ and $\Delta DEF: D(-7,2), E(-1,3), F(-4,8)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.6.

Resources

Vocabulary

Term	Definition
Congruent	Congruent figures are identical in size, shape and measure.
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = (x_2 - x_1)^2 + (y_2 - y_1)^2$.
H-L (Hypotenuse-Leg) Congruence Theorem	If the hypotenuse and leg in one right triangle are congruent to the hypotenuse and leg in another right triangle, then the two triangles are congruent.
Side Side Side Triangle	A side side side triangle is a triangle where the lengths of all three sides are known quantities.
SSS	SSS means side, side, side and refers to the fact that all three sides of a triangle are known in a problem.
Triangle Congruence	Triangle congruence occurs if 3 sides in one triangle are congruent to 3 sides in another triangle.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Introduction to Congruent Triangles

Activities: SSS Triangle Congruence Discussion Questions

Study Aids: Triangle Congruence Study Guide

Practice: SSS

Real World: SSS Triangle Congruence

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4.14: SAS

Two sets of corresponding sides and included angles prove congruent triangles.

Side-Angle-Side Postulate

If two sides and the **included angle** in one triangle are **congruent** to two sides and the included angle in another triangle, then the two triangles are congruent. (When an angle is between two given sides of a polygon it is called an **included angle**.)

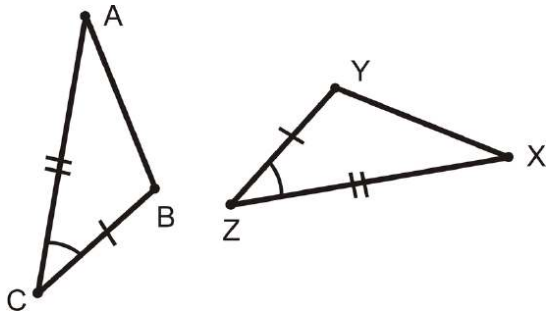


Figure **4.14.1**

$\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong YZ$, and $\angle C \cong \angle Z$, then $\Delta ABC \cong \Delta XYZ$.

This is called the Side-Angle-Side (**SAS**) Postulate and it is a shortcut for proving that two triangles are congruent. The placement of the word Angle is important because it indicates that the angle you are given is *between* the two sides.

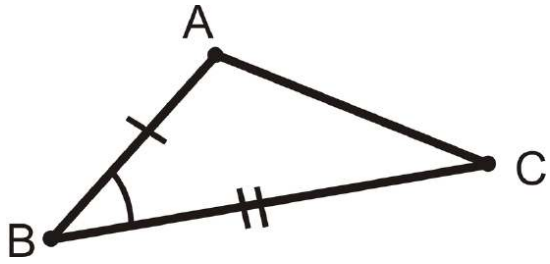


Figure **4.14.2**

$\angle B$ would be the included angle for sides $\overline{AB}$ and $\overline{BC}$.

What if you were given two triangles and provided with only two of their side lengths and the measure of the angle between those two sides? How could you determine if the two triangles were congruent?

Example 4.14.1

Is the pair of triangles c te the congruence statement and why.

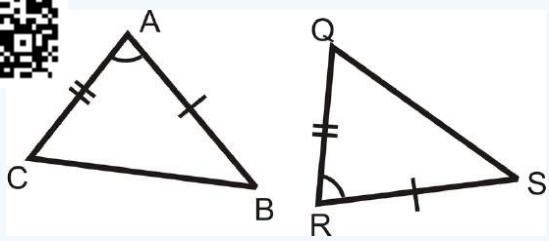


Figure **4.14.3**

Solution

The pair of triangles is congruent by the SAS postulate. $\Delta CAB \cong \Delta QRS$.

Example 4.14.2

State the additional piece of information needed to show that each pair of triangles is congruent.

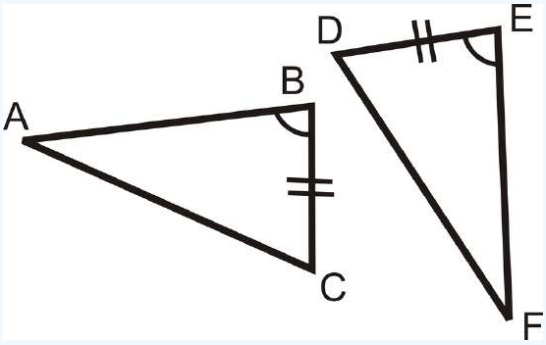


Figure 4.14.4

Solution

We know that one pair of sides and one pair of angles are congruent from the diagram. In order to know that the triangles are congruent by SAS we need to know that the pair of sides on the other side of the angle are congruent. So, we need to know that $\overline{EF} \cong \overline{BA}$.

Example 4.14.3

Fill in the blanks in the proof below.

Given:

$\overline{AB} \cong \overline{DC}$, $\overline{BE} \cong \overline{CE}$

Prove: $\Delta ABE \cong \Delta ACE$

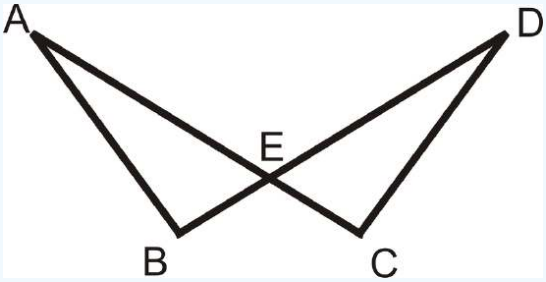


Figure 4.14.5

Solution

Statement	*Reason*
1.	1.
2. $\angle AEB \cong \angle DEC$	2.
3. $\Delta ABE \cong \Delta ACE$	3.

Statement	*Reason*
1. $\overline{AB} \cong \overline{DC}$, $\overline{BE} \cong \overline{CE}$	1. Given
2. $\angle AEB \cong \angle DEC$	2. Vertical Angle Theorem
3. $\Delta ABE \cong \Delta ACE$	3. SAS postulate

Example 4.14.4

What additional piece of information do you need to show that these two triangles are congruent using the SAS Postulate, $\angle ABC \cong \angle LKM$, $\overline{AB} \cong \overline{LK}$, $\overline{BC} \cong \overline{KM}$, or $\angle BAC \cong \angle KLM$?

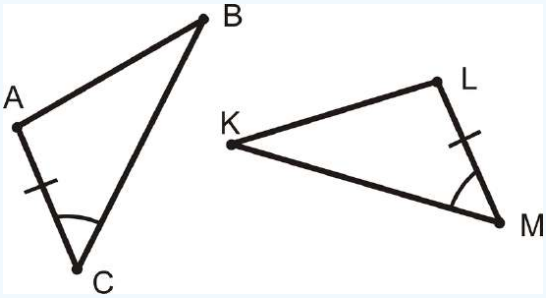


Figure 4.14.6

Solution

For the SAS Postulate, you need the side on the other side of the angle. In ΔABC, that is $\overline{BC}$ and in ΔLKM that is $\overline{KM}$. The answer is $\overline{BC} \cong \overline{KM}$.

Example 4.14.5

Is the pair of triangles congruent? If so, write the congruence statement and why.

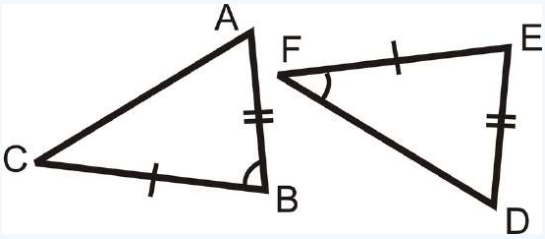


Figure 4.14.6

Solution

While the triangles have two pairs of sides and one pair of angles that are congruent, the angle is not in the same place in both triangles. The first triangle fits with SAS, but the second triangle is SSA. There is not enough information for us to know whether or not these triangles are congruent.

Review

Are the pairs of triangles congruent? If so, write the congruence statement and why.

1.

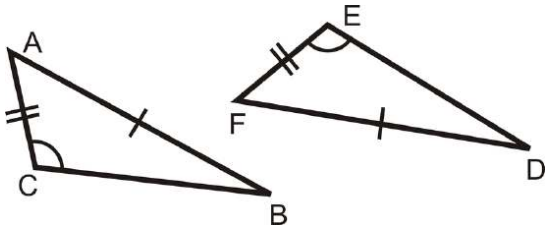


Figure 4.14.7

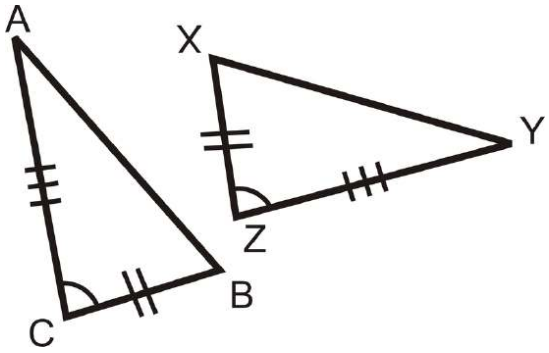
2. 

Figure 4.14.7

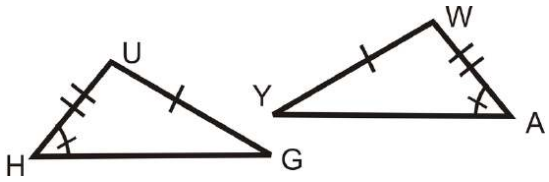
3. 

Figure 4.14.7

State the additional piece of information needed to show that each pair of triangles is congruent by SAS.

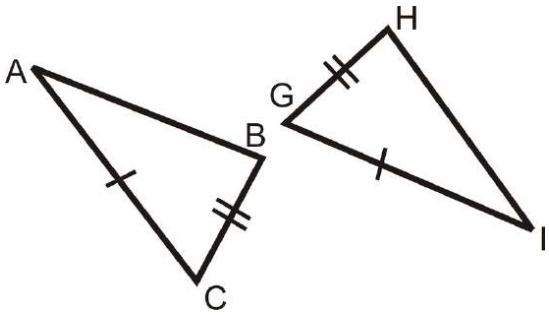
4. 

Figure 4.14.7

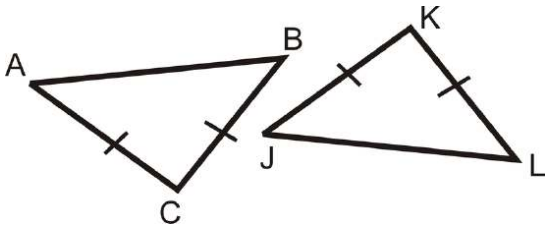
5. 

Figure 4.14.7

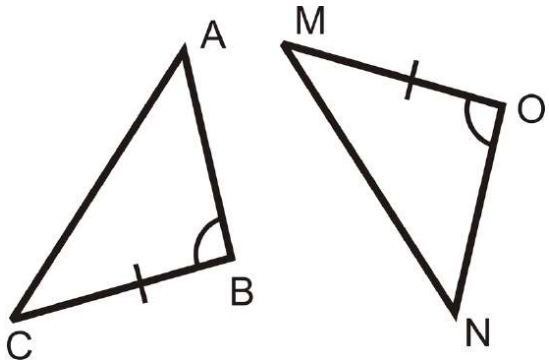
6. 

Figure 4.14.7

Fill in the blanks in the proofs below.

7. Given:

- B is a midpoint of $\overline{DC}$
- $\overline{AB} \perp \overline{DC}$

Prove: $\Delta ABD \cong \Delta ABC$

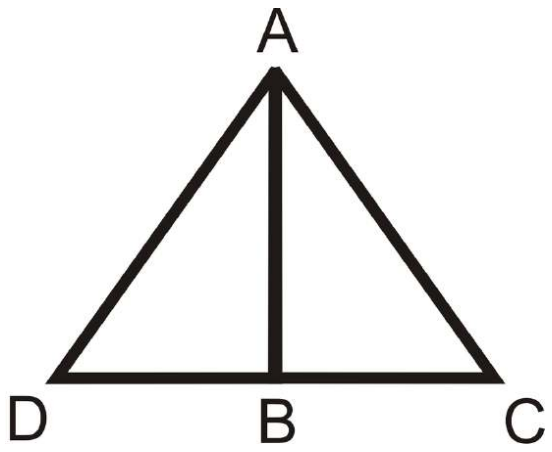


Figure 4.14.8

Statement	*Reason*
1. B is a midpoint of $\overline{DC}$, $\overline{AB} \perp \overline{DC}$	1.
2.	2. Definition of a midpoint
3. $\angle ABD$ and $\angle ABC$ are right angles	3.
4.	4. All right angles are \cong\)
5.	5.
6. $\Delta ABD \cong \Delta ABC$	6.

8. Given:
   - $\overline{AB}$ is an angle bisector of $\angle DAC$
   - $\overline{AD} \cong \overline{AC}$

   Prove: $\Delta ABD \cong \Delta ABC$

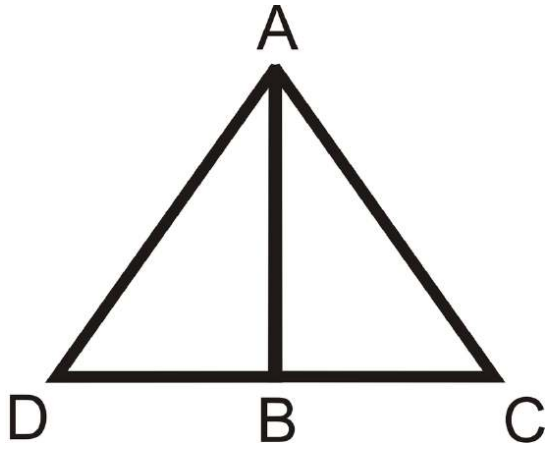


Figure 4.14.9

Statement	*Reason*
1.	1.
2. $\angle DAB \cong \angle BAC$	2.
3.	3. Reflexive *PoC*
4. $\Delta ABD \cong \Delta ABC$	4.

9. Given:
   - B is the midpoint of $\overline{DE}$ and $\overline{AC}$
   - $\angle ABE$ is a right angle

   Prove: $\Delta ABE \cong \Delta CBD$

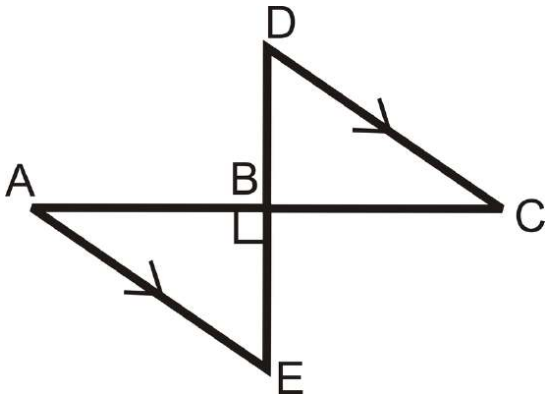


Figure 4.14.10

Statement	*Reason*
1.	1. Given
2. $\overline{DB} \cong \overline{BE}$, $\overline{AB} \cong \overline{BC}$	2.
3.	3. Definition of a Right Angle
4.	4. Vertical Angle Theorem
5. $\Delta ABE \cong \Delta CBD$	5.

10. Given:
    - $\overline{DB}$ is the angle bisector of $\angle ADC$
    - $\overline{AD} \cong \overline{DC}$

    Prove: $\Delta ABD \cong \Delta CBD$

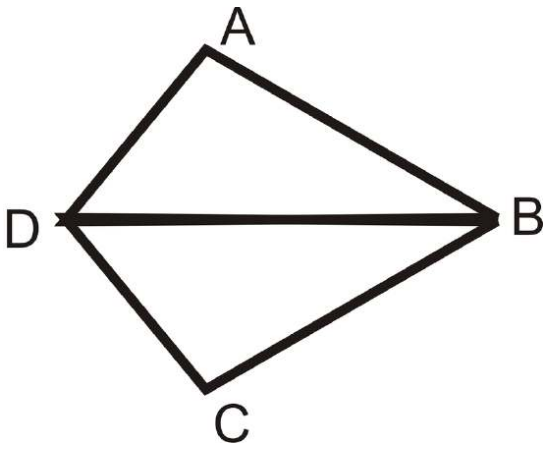


Figure **4.14.11**

Statement	*Reason*
1.	1.
2. $\angle ADB \cong \angle BDC$	2.
3.	3.
4. $\Delta ABD \cong \Delta CBD$	4.

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.7.

Resources

Vocabulary

Term	Definition
Base Angles	The base angles of an isosceles triangle are the angles formed by the base and one leg of the triangle.
Congruent	Congruent figures are identical in size, shape and measure.
Equilateral Triangle	An equilateral triangle is a triangle in which all three sides are the same length.
Included Angle	The included angle in a triangle is the angle between two known sides.
SAS	SAS means side, angle, side, and refers to the fact that two sides and the included angle of a triangle are known.
Side Angle Side Triangle	A side angle side triangle is a triangle where two of the sides and the angle between them are known quantities.
Triangle Congruence	Triangle congruence occurs if 3 sides in one triangle are congruent to 3 sides in another triangle.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Introduction to Congruent Triangles

Activities: SAS Triangle Congruence Discussion Questions

Study Aids: Triangle Congruence Study Guide

Practice: SAS

Real World: SSS Triangle Congruence

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4.15: ASA and AAS

Two sets of corresponding angles and any corresponding set of sides prove congruent triangles.

Angle-Side-Angle Postulate and Angle-Angle-Side Theorem

If two angles and one side in one triangle are **congruent** to the corresponding two angles and one side in another triangle, then the two triangles are congruent. This idea encompasses two **triangle congruence** shortcuts: Angle-Side-Angle and Angle-Angle-Side.

Angle-Side-Angle (ASA) Congruence Postulate: If two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the two triangles are congruent.

Angle-Angle-Side (AAS) Congruence Theorem: If two angles and a non-included side in one triangle are congruent to two angles and the corresponding non-included side in another triangle, then the triangles are congruent.

The placement of the word Side is important because it indicates where the side that you are given is in relation to the angles. The pictures below help to show the difference between the two shortcuts.

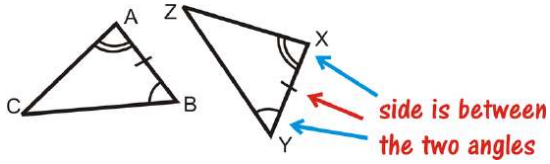


Figure 4.15.1

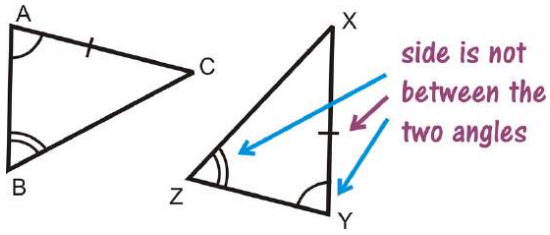


Figure 4.15.2

What if you were given two triangles and provided with only the measure of two of their angles and one of their side lengths? How could you determine if the two triangles were congruent?

Example 4.15.1

Can you prove that the f ıre congruent? Why or why not?

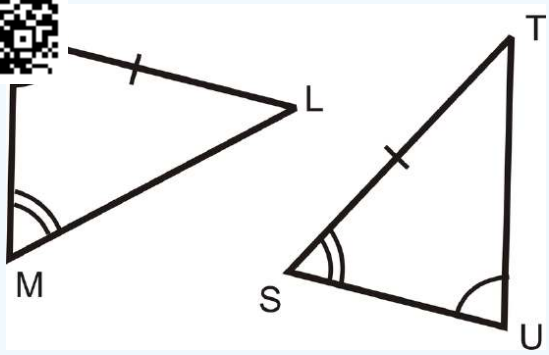


Figure 4.15.3

Solution

We cannot show the triangles are congruent because $\overline{KL}$ and $\overline{ST}$ are *not corresponding*, even though they are congruent. To determine if \($\overline{KL}$ and $\overline{ST}$ are corresponding, look at the angles around them, \($\angle K$ and $\angle L$ and \angle S\) and $\angle T$. $\angle K$ has **one** arc and \angle L is unmarked. $\angle S$ has **two** arcs and $\angle T$ is unmarked. In order to use AAS, $\angle S$ needs to be congruent to $\angle K$.

Example 4.15.2

Write a 2-column proof.

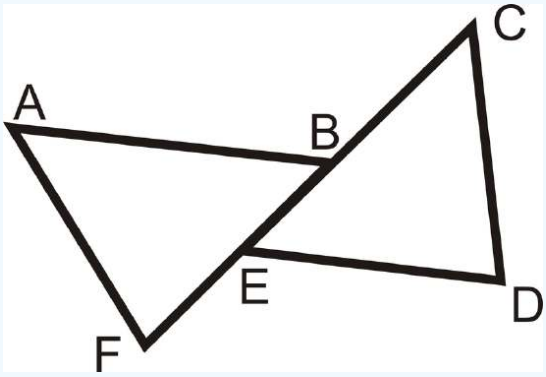


Figure 4.15.4

Given: $\overline{AB} \parallel \overline{ED}$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{ED}$

Prove: $\overline{AF} \cong \overline{CD}$

Solution

Statement	*Reason*
1. $\overline{AB} \parallel \overline{ED}$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{ED}$	1. Given
2. $\angle ABE \cong \angle DEB$	2. Alternate Interior Angles Theorem
3. $\triangle ABF \cong \triangle DEC$	3. ASA
4. $\overline{AF} \cong \overline{CD}$	4. **CPCTC** (Corresponding Parts of Congruent Triangles are Congruent)

Example 4.15.3

What information do you need to prove that these two triangles are congruent using the ASA Postulate, $\overline{AB} \cong UT\overline{AB}$, $\overline{AC} \cong \overline{UV}$, $\overline{BC} \cong \overline{TV}$, or $\angle B \cong \angle T$?

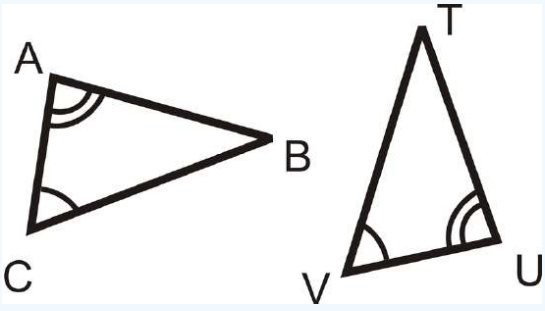


Figure 4.15.5

Solution

For ASA, we need the side between the two given angles, which is $\overline{AC}$ and $\overline{UV}$. The answer is $\overline{AC} \cong \overline{UV}$.

Example 4.15.4

Write a 2-column proof.

Given: $\angle C \cong \angle E$, $\overline{AC} \cong \overline{AE}$

Prove: $\Delta ACF \cong \Delta AEB$

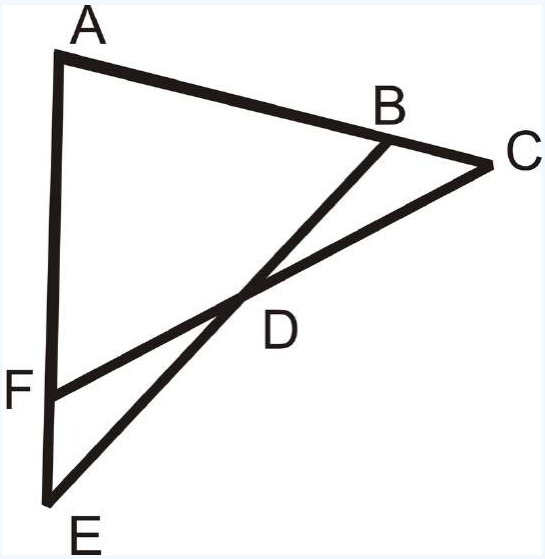


Figure **4.15.6**

Solution

Statement	*Reason*
1. $\angle C \cong \angle E$, $\overline{AC} \cong \overline{AE}$	1. Given
2. $\angle A \cong \angle A$	2. Reflexive *PoC*
3. $\Delta ACF \cong \Delta AEB$	3. ASA

Example 4.15.5

What information do you need to prove that these two triangles are congruent using ASA? AAS?

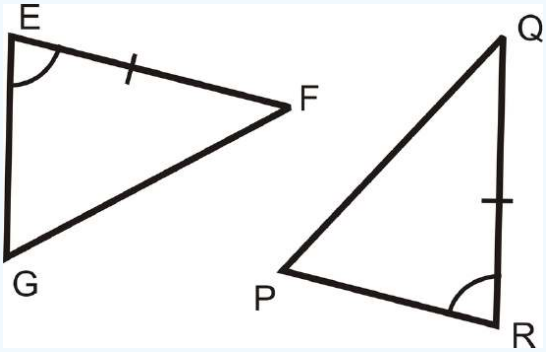


Figure **4.15.7**

Solution

For ASA, we need the angles on the other side of $\overline{EF}$ and $\overline{QR}$. $\angle F \cong \angle Q$

For AAS, we would need the other angle. $\angle G \cong \angle P$

Review

For questions 1-3, determine if the triangles are congruent. If they are, write the congruence statement and which congruence postulate or theorem you used.

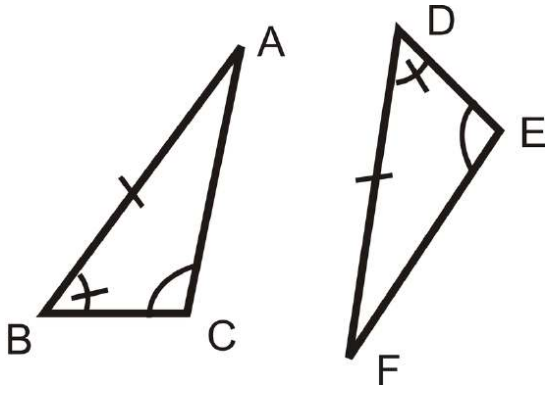
1. 

Figure 4.15.8

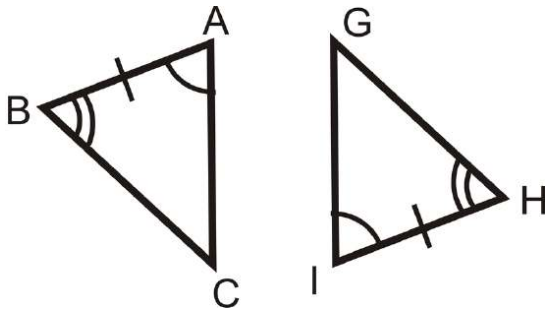
2. 

Figure 4.15.9

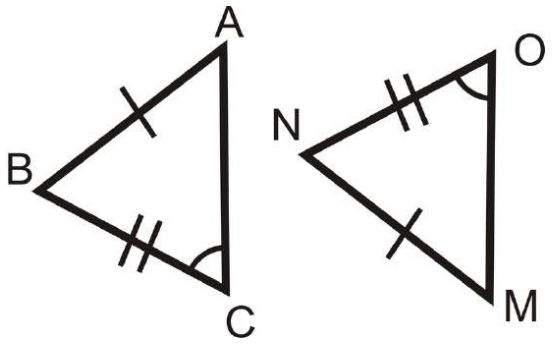
3. 

Figure 4.15.10

For questions 4-8, use the picture and the given information below.

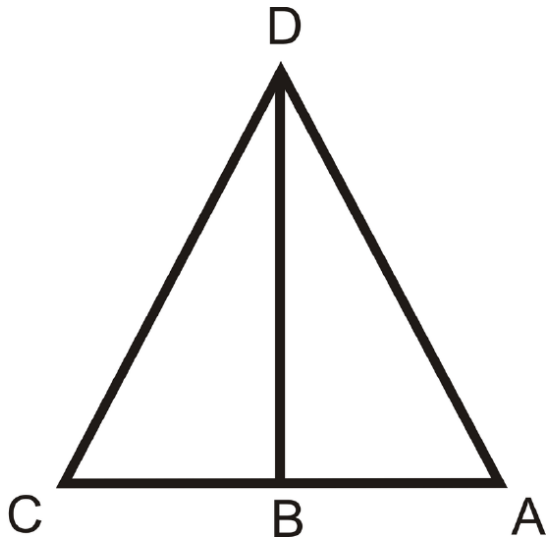


Figure 4.15.11

Given: $\overline{DB} \perp \overline{AC}$, $\overline{DB}$ is the angle bisector of $\angle CDA$

4. From $\overline{DB} \perp \overline{AC}$, which angles are congruent and why?
5. Because $\overline{DB}$ is the angle bisector of $\angle CDA$, what two angles are congruent?
6. From looking at the picture, what additional piece of information are you given? Is this enough to prove the two triangles are congruent?
7. Write a 2-column proof to prove $\Delta CDB \cong \Delta ADB$, using #4-6.
8. What would be your reason for $\angle C \cong \angle A$?

For questions 9-13, use the picture and the given information.

Given: $\overline{LP} \parallel \overline{NO}$, $\overline{LP} \cong \overline{NO}$

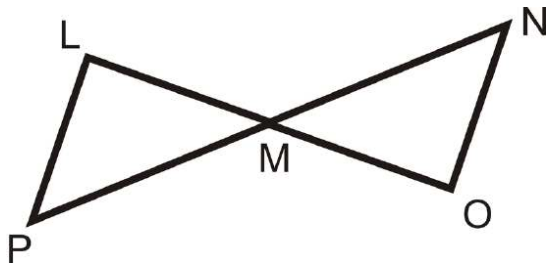


Figure **4.15.12**

9. From $\overline{LP} \parallel \overline{NO}$, which angles are congruent and why?
10. From looking at the picture, what additional piece of information can you conclude?
11. Write a 2-column proof to prove $\Delta LMP \cong \Delta OMN$.
12. What would be your reason for $\overline{LM} \cong \overline{MO}$?
13. Fill in the blanks for the proof below. Use the given from above. Prove: M is the midpoint of $\overline{PN}$.

Statement	***Reason***
1. $\overline{LP} \parallel \overline{NO}$, $\overline{LP} \cong \overline{NO}$	1. Given
2.	2. Alternate Interior Angles
3.	3. ASA
4. $\overline{LM} \cong \overline{MO}$	4.
5. M is the midpoint of $\overline{PN}$.	5.

Determine the additional piece of information needed to show the two triangles are congruent by the given postulate.

14. AAS

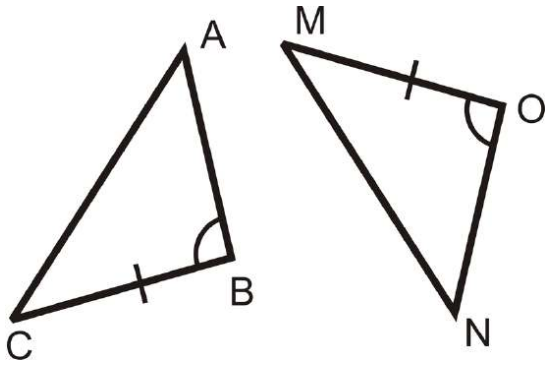


Figure **4.15.13**

15. ASA

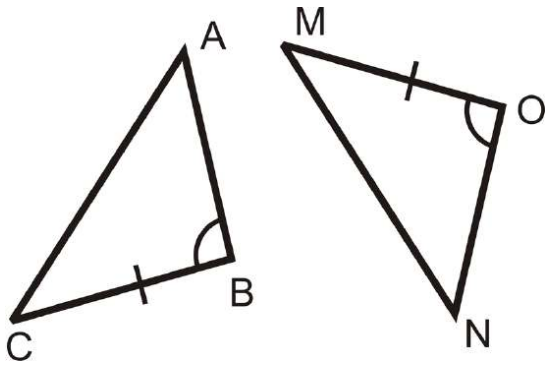


Figure **4.15.14**

16. ASA

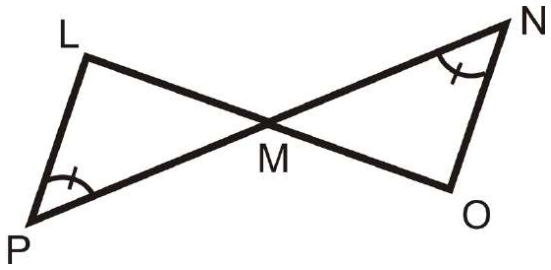


Figure **4.15.15**

17. AAS

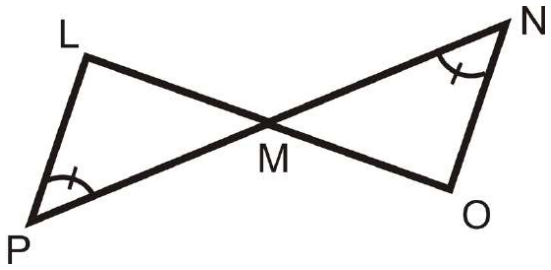


Figure **4.15.16**

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.8.

Resources

Vocabulary

Term	Definition
CPCTC	***Corresponding Parts of Congruent Triangles are Congruent***. It is used to show two sides or two angles in triangles are congruent after having proved that the triangles are congruent.
AAS (Angle-Angle-Side)	If two angles and a non-included side in one triangle are congruent to two angles and the corresponding non-included side in another triangle, then the triangles are congruent.
Angle Side Angle Triangle	The term 'angle-side-angle triangle' refers to a triangle with known measures of two angles and the length of the side between them.
ASA	ASA, angle-side-angle, refers to two known angles in a triangle with one known side between the known angles.
Congruent	Congruent figures are identical in size, shape and measure.
Triangle Congruence	Triangle congruence occurs if 3 sides in one triangle are congruent to 3 sides in another triangle.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Introduction to Congruent Triangles

Activities: ASA and AAS Triangle Congruence Discussion Questions

Study Aids: Triangle Congruence Study Guide

Practice: ASA and AAS

Real World: SSS Triangle Congruence

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4.16: HL

HL Triangle Congruence

Hypotenuse-Leg Congruence Theorem

If the hypotenuse and leg in one ***right*** triangle are congruent to the hypotenuse and leg in another ***right*** triangle, then the two triangles are congruent. This is called the **Hypotenuse-Leg (HL) Congruence Theorem**. Note that it will only work for **right** triangles.

If ΔABC and ΔXYZ are both right triangles and $\overline{AB} \cong \overline{XY}$ and $\overline{BC} \cong \overline{YZ}$ then $\Delta ABC \cong \Delta XYZ$.

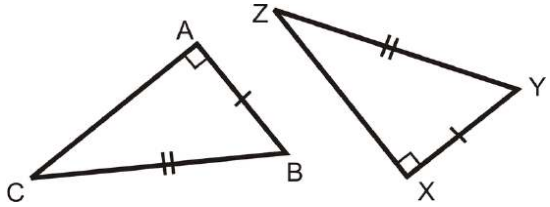

Figure **4.16.1**

What if you were given two right triangles and provided with only the measure of their hypotenuses and one of their **legs**? How could you determine if the two right triangles were congruent?

Example 4.16.1

Fill in the blanks in the

Given:

$\overline{SV} \perp \overline{WU}$

T is the midpoint of $\overline{SV}$ and $\overline{WU}$

Prove: $\overline{WS} \cong \overline{UV}$

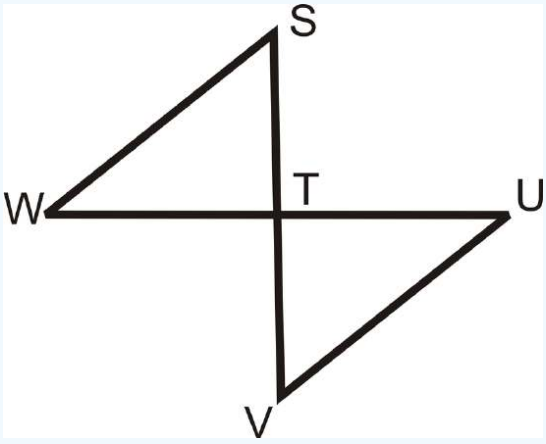

Figure **4.16.2**

Solution

Statement	Reason
1.	1.
2. $\angle STW$ and $\angle UTV$ are right angles	2.
3.	3.
4. $\overline{ST} \cong \overline{TV}$, $\overline{WT} \cong \overline{TU}$	4.
5. $\Delta STW \cong \Delta UTV$	5.
6. $\overline{WS} \cong \overline{UV}$	6.

Statement	Reason
1. $\overline{SV} \perp \overline{WU}$	1. Given
2. $\angle STW$ and $\angle UTV$ are right angles	2. Definition of perpendicular lines.
3. T is the midpoint of $\overline{SV}$ and $\overline{WU}$	3. Given
4. $\overline{ST} \cong \overline{TV}$, $\overline{WT} \cong \overline{TU}$	4. Definition of midpoint
5. $\Delta STW \cong \Delta UTV$	5. SAS
6. $\overline{WS} \cong \overline{UV}$	6. CPCTC

Note that even though these were right triangles, we did not use the HL congruence shortcut because we were not originally given that the two hypotenuses were congruent. The SAS congruence shortcut was quicker in this case.

Example 4.16.2

Explain why the HL Congruence shortcut works.

The Pythagorean Theorem, which says, for any ***right*** triangle, this equation is true:

$(leg)^2 + (leg)^2 = (hypotenuse)^2$

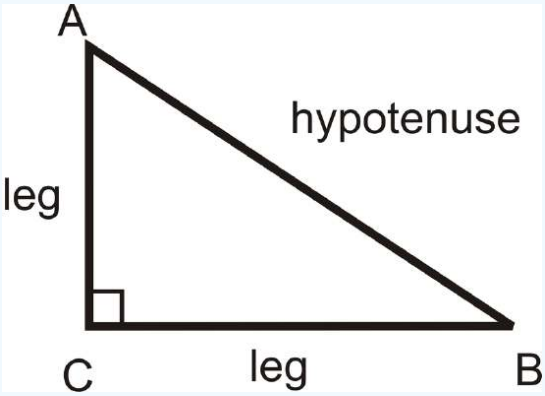

Figure **4.16.3**

Solution

What this means is that if you are given two sides of a **right triangle**, you can always find the third. Therefore, if you know two sides of a right triangle are congruent to two sides of another right triangle, then you can conclude that the third sides are also congruent. If three pairs of sides are congruent, then we know the triangles are congruent by SSS.

Example 4.16.3

What additional information would you need to prove that these two triangles were congruent using the HL Theorem?

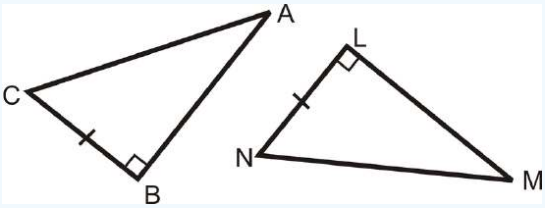

Figure 4.16.4

Solution

For HL, you need the hypotenuses to be congruent. $\overline{AC} \cong MN\overline{AB}$.

Example 4.16.4

Determine if the triangles are congruent. If they are, write the congruence statement and which congruence postulate or theorem you used.

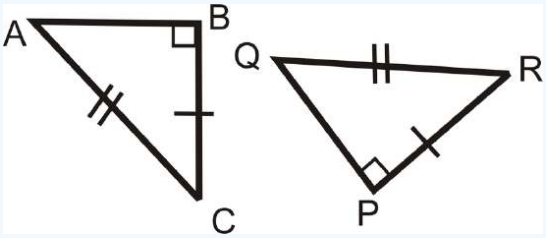

Figure 4.16.5

Solution

We know the two triangles are right triangles. The have one pair of legs that is congruent and their hypotenuses are congruent. This means that $\Delta ABC \cong \Delta RQP$ by HL.

Example 4.16.5

Determine the additional piece of information needed to show the two triangles are congruent by HL.

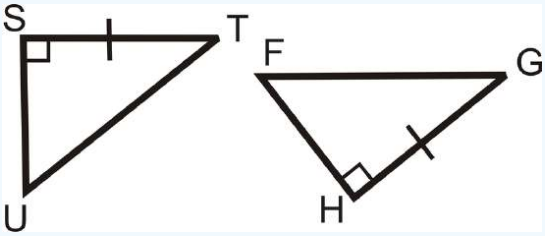

Figure 4.16.6

Solution

We already know one pair of legs is congruent and that they are right triangles. The additional piece of information we need is that the two hypotenuses are congruent, $\overline{UT} \cong \overline{FG}$.

Review

Using the HL Theorem, what additional information do you need to prove the two triangles are congruent?

1.

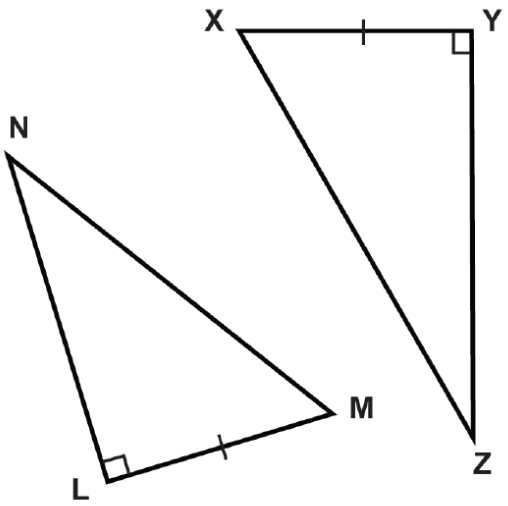

Figure 4.16.7

2.

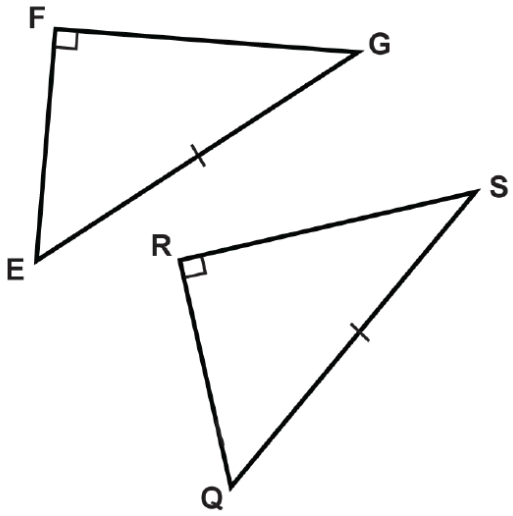

Figure 4.16.8

3.

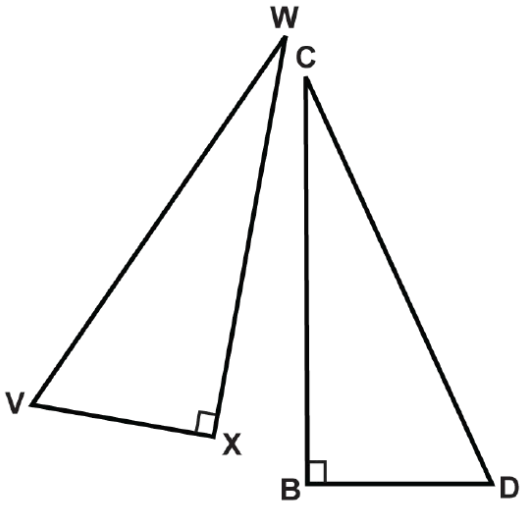

Figure 4.16.9

The triangles are formed by two parallel lines cut by a perpendicular transversal. C is the midpoint of $\overline{AD}$. Complete the proof to show the two triangles are congruent. Questions 4-7 are within the proof.

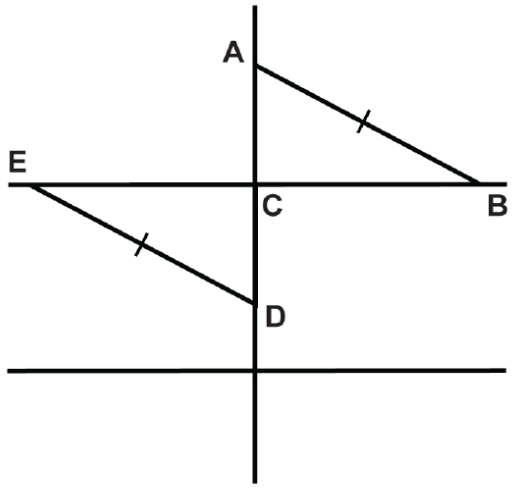

Figure **4.16.10**

Statement	Reason
1. $\angle ACB$ and $\angle DCE$ are right angles.	(4.)
2. (5.)	Definition of midpoint
3. (6.)	Given
4. $\Delta ACD \cong \Delta DCE$	(7.)

Based on the following details, are the two right triangles definitively congruent? The statements do not build off of each other.

8. The hypotenuses of two right triangles are congruent.
9. Both sets of legs in the two right triangles are congruent.
10. One set of legs are congruent in the two right triangles.
11. The hypotenuses and one pair of legs are congruent in the two right triangles.
12. One of the non right angles of the two right triangles is congruent.
13. All of the angles of the two right triangles are congruent.
14. All of the sides of the two right triangles are congruent.
15. Both triangles have one leg that is twice the length of the other.

Resources

Vocabulary

Term	Definition
hypotenuse	The side opposite the right angle in a right triangle.
legs	The two sides adjacent to the right angle.
right triangle	A triangle with exactly one right (90°) angle. The two sides adjacent to the right angle are called ***legs*** and the side opposite the right angle is called the ***hypotenuse***.
H-L (Hypotenuse-Leg) Congruence Theorem	If the hypotenuse and leg in one right triangle are congruent to the hypotenuse and leg in another right triangle, then the two triangles are congruent.

Additional Resources

Video: HL Triangle Congruence Principles - Basic

Activities: HL Triangle Congruence Discussion Questions

Study Aids: Triangle Congruence Study Guide

Practice: HL

Real World: Final Drafts

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4.17: Triangle Angle Sum Theorem

The interior angles of a triangle add to 180 degrees Use equations to find missing angle measures given the sum of 180 degrees.

Triangle Sum Theorem

The **Triangle Sum Theorem** says that the three interior angles of any triangle add up to $180°$.

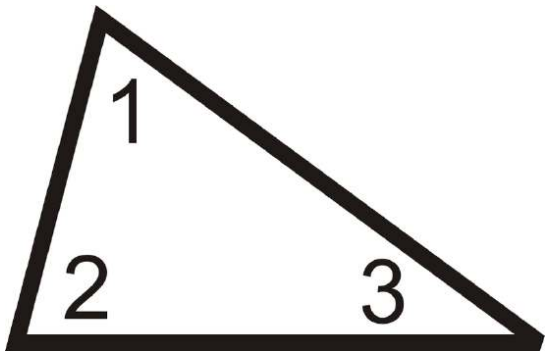

Figure 4.17.1

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Here is one proof of the Triangle Sum Theorem.

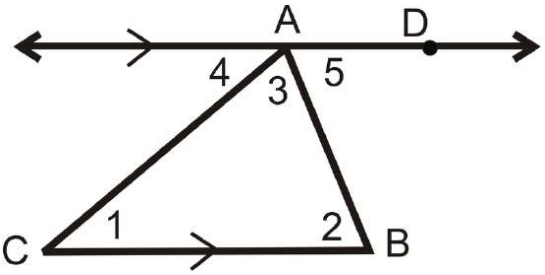

Figure 4.17.2

Given: ΔABC with $\overleftrightarrow{AD} \parallel \overline{BC}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Statement	*Reason*
1. $\Delta ABC with \overleftrightarrow{AD} \parallel \overline{BC}$	Given
2. \(angle 1\cong \angle 4,\: \angle 2\cong \angle 5\)	Alternate Interior Angles Theorem
3. $m\angle 1 = m\angle 4,\ m\angle 2 = m\angle 5$	\cong angles have = measures
4. $m\angle 4 + m\angle CAD = 180°$	Linear Pair Postulate
5. $m\angle 3 + m\angle 5 = m\angle CAD$	Angle Addition Postulate
6. $m\angle 4 + m\angle 3 + m\angle 5 = 180°$	Substitution PoE
7. $m\angle 1 + m\angle 3 + m\angle 2 = 180°$	Substitution PoE

You can use the Triangle S d missing angles in triangles.

What if you knew that two riangle measured $55°$? How could you find the measure of the third angle?

Example 4.17.1

Two interior angles of a $50°$ and $70°$. What is the third interior angle of the triangle?

Solution

$50° + 70° + x = 180°$.

Solve this equation and you find that the third angle is $60°$.

Example 4.17.2

Find the value of x and the measure of each angle.

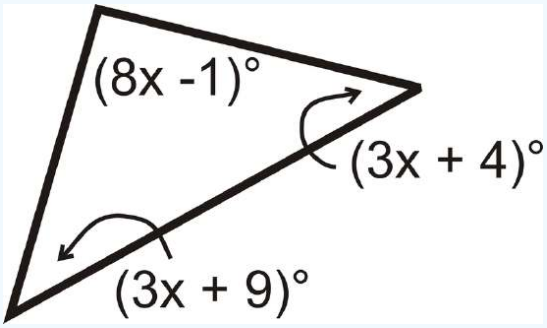

Figure 4.17.3

Solution

All the angles add up to $180°$.

$$\begin{aligned}(8x-1)° + (3x+9)° + (3x+4)° &= 180° \\ (14x+12)° &= 180° \\ 14x &= 168 \\ x &= 12\end{aligned}$$

Substitute in 12 for x to find each angle.

$[3(12)+9]° = 45°$ $[3(12)+4]° = 40°$ $[8(12)-1]° = 95°$

Example 4.17.3

What is m\angle T?

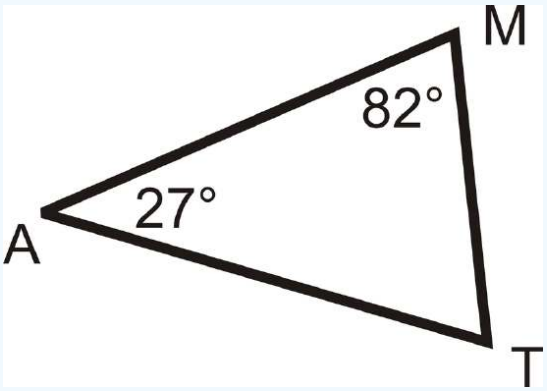

Figure 4.17.4

Solution

We know that the three angles in the triangle must add up to $180°$. To solve this problem, set up an equation and substitute in the information you know.

$$
\begin{aligned}
m\angle M + m\angle A + m\angle T &= 180^\circ \\
82^\circ + 27^\circ + m\angle T &= 180^\circ \\
109^\circ + m\angle T &= 180^\circ \\
m\angle T &= 71^\circ
\end{aligned}
$$

Example 4.17.4

What is the measure of each angle in an equiangular triangle?

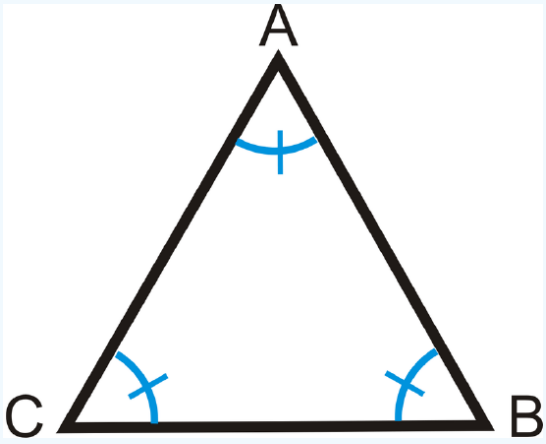

Figure 4.17.5

Solution

To solve, remember that ΔABC is an equiangular triangle, so all three angles are equal. Write an equation.

$$
\begin{aligned}
m\angle A + m\angle B + m\angle C &= 180^\circ \\
m\angle A + m\angle A + m\angle A &= 180^\circ && \textit{Substitute, all angles are equal.} \\
3m\angle A &= 180^\circ && \textit{Combine like terms.} \\
m\angle A &= 60^\circ
\end{aligned}
$$

If $m\angle A = 60^\circ$, then $m\angle B = 60^\circ$ and $m\angle C = 60^\circ$.

Each angle in an equiangular triangle is 60°.

Example 4.17.5

Find the measure of the missing angle.

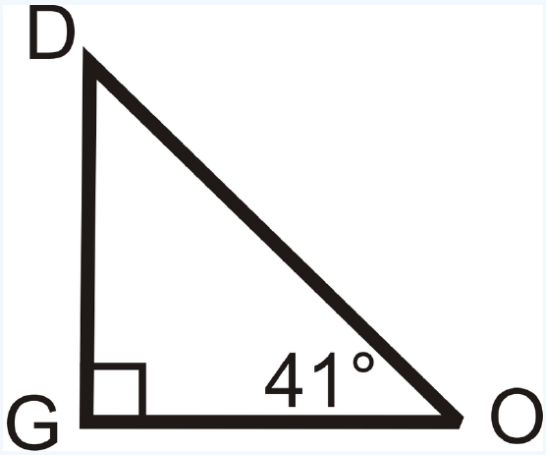

Figure 4.17.6

Solution

We know that $m\angle O = 41^\circ$ and $m\angle G = 90^\circ$ because it is a right angle. Set up an equation like in Example 3.

$$m\angle D + m\angle O + m\angle G = 180^\circ$$
$$m\angle D + 41^\circ + 90^\circ = 180^\circ$$
$$m\angle D + 41^\circ = 90^\circ$$
$$m\angle D = 49^\circ$$

Review

Determine $m\angle 1$ in each triangle.

1.

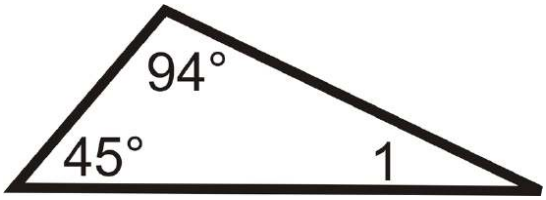

Figure 4.17.7

2.

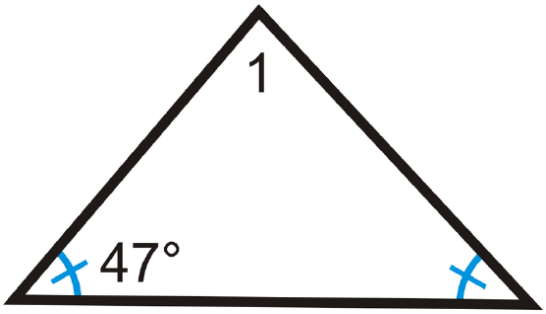

Figure 4.17.8

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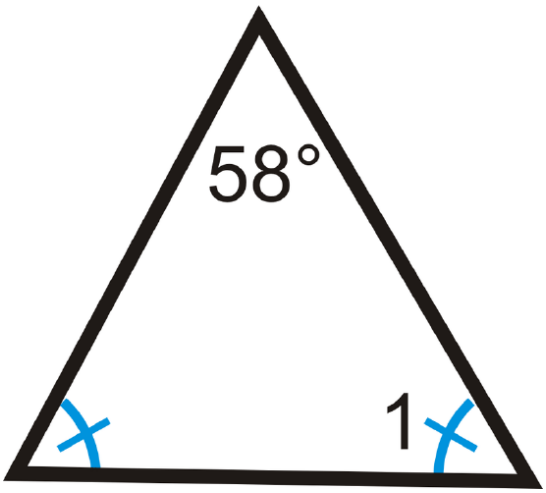

Figure 4.17.9

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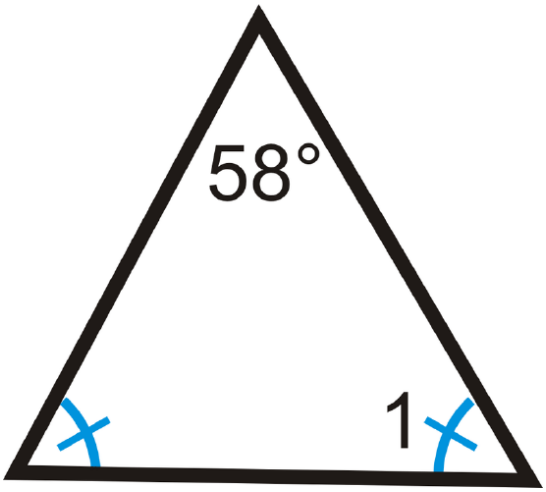

Figure 4.17.10

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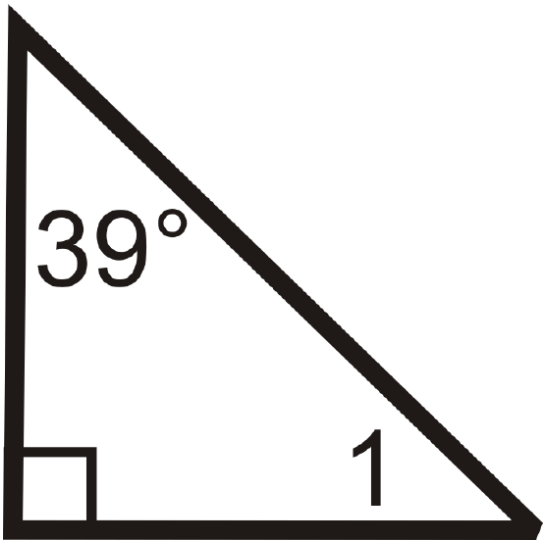

Figure 4.17.11

6.

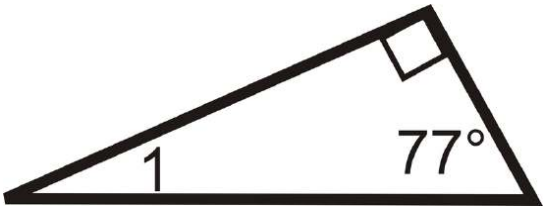

Figure 4.17.12

7.

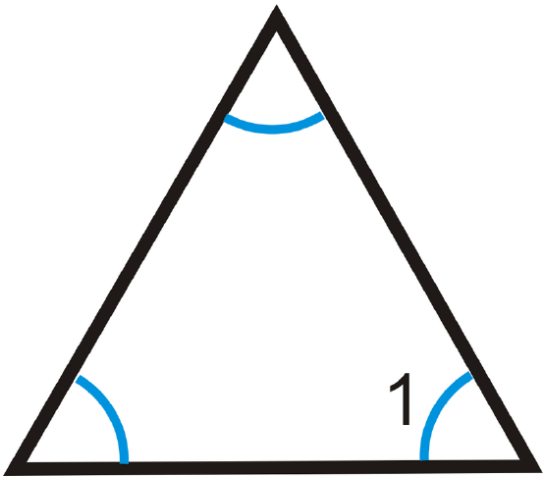

Figure 4.17.13

8. Two interior angles of a triangle measure $32°$ and $64°$. What is the third interior angle of the triangle?

9. Two interior angles of a triangle measure $111°$ and $12°$. What is the third interior angle of the triangle?

10. Two interior angles of a triangle measure $2°$ and $157°$. What is the third interior angle of the triangle?

Find the value of x and the measure of each angle.

11.

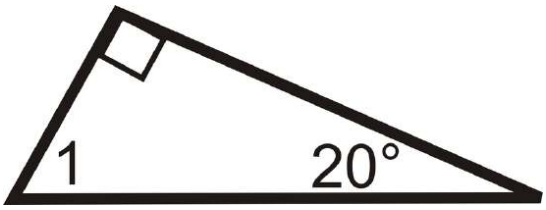

Figure 4.17.14

12.

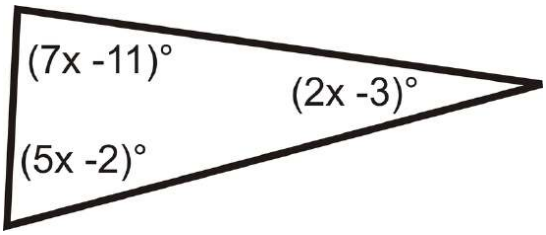

Figure 4.17.15

13.

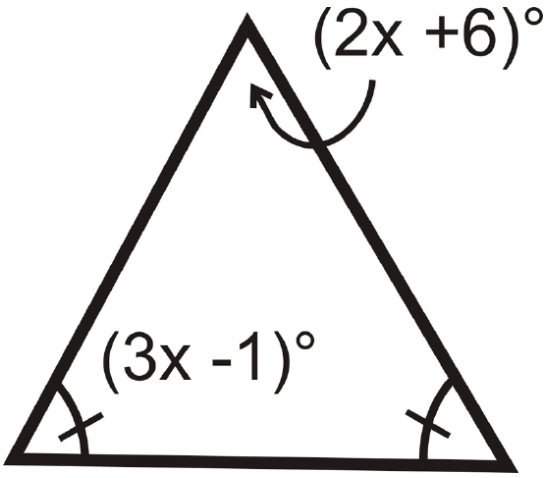

Figure 4.17.16

14.

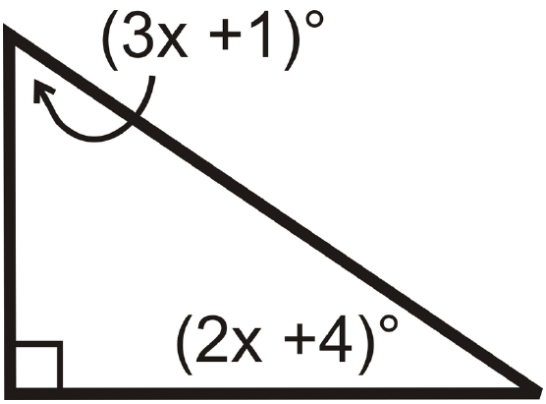

Figure 4.17.17

15.

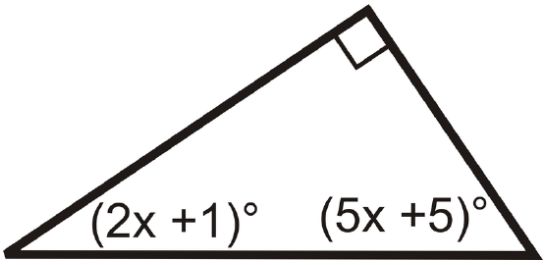

Figure 4.17.18

Review (Answers)

To see the Review answers, open this PDF file and look for section 4.1.

Resources

Vocabulary

Term	Definition
Triangle Sum Theorem	The Triangle Sum Theorem states that the three interior angles of any triangle add up to 180 degrees.

Additional Resources

Video: Triangle Sum Theorem Principles - Basic

Activities: Triangle Sum Theorem Discussion Questions

Study Aids: Triangle Relationships Study Guide

Practice: Triangle Angle Sum Theorem

Real World: Triangle Sum Theorem

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4.18: Exterior Angles and Theorems

Exterior angles equal the sum of the remote interiors.

Exterior Angles

An **Exterior Angle** is the angle formed by one side of a polygon and the extension of the adjacent side.

In all polygons, there are **two** sets of exterior angles, one that goes around clockwise and the other goes around counterclockwise.

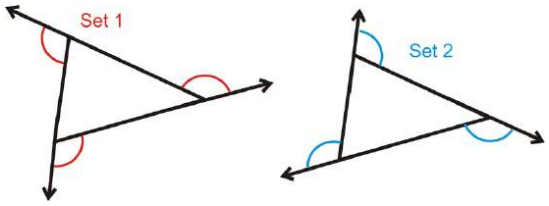


Figure 4.18.1

Notice that the interior angle and its adjacent exterior angle form a linear pair and add up to $180°$.

$m\angle 1 + m\angle 2 = 180°$

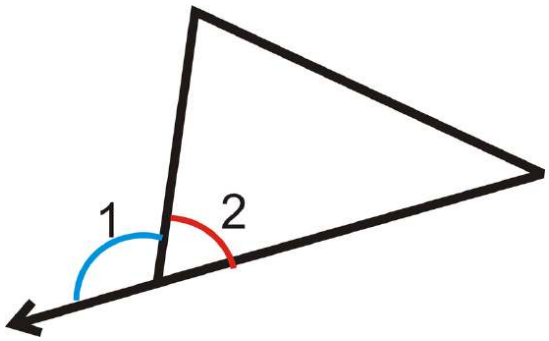


Figure 4.18.2

There are two important theorems to know involving exterior angles: the **Exterior Angle Sum Theorem** and the Exterior Angle Theorem.

The **Exterior Angle Sum Theorem** states that the exterior angles of any polygon will always add up to $360°$.

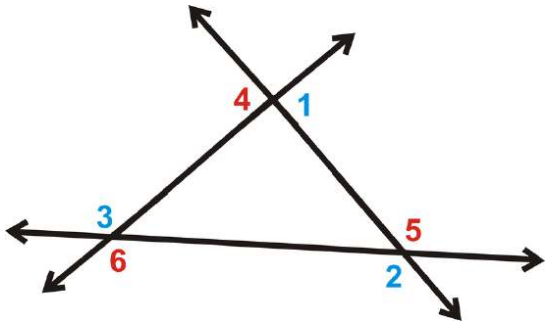


Figure 4.18.3

$$m\angle 1 + m\angle 2 + m\angle 3 = 360°$$

$$m\angle 4 + m\angle 5 + m\angle 6 = 360°.$$

The **Exterior Angle Theorem** states that an exterior angle of a triangle is equal to the sum of its **remote interior angles**. (**Remote Interior Angles** are the two interior angles in a triangle that are not adjacent to the indicated exterior angle.)

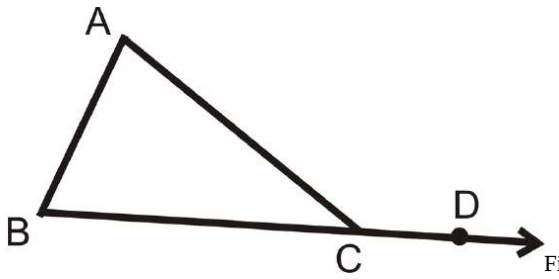


Figure 4.18.4

$m\angle A + m\angle B = m\angle ACD$

What if you knew that two of the exterior angles of a triangle measured $130°$? How could you find the measure of the third exterior angle?

Example 4.18.1

Two interior angles of a [redacted] $73°$. What are the measures of the three exterior angles of the triangle?

Solution

Remember that every in [redacted] a linear pair (adds up to $180°$) with an exterior angle. So, since one of the interior angles is $40°$ that means that one of the exterior angles is $140°$ (because $40 + 140 = 180$). Similarly, since another one of the interior angles is $73°$, one of the exterior angles must be $107°$. The third interior angle is not given to us, but we could figure it out using the **Triangle Sum Theorem**. We can also use the Exterior Angle Sum Theorem. If two of the exterior angles are $140°$ and $107°$, then the third Exterior Angle must be $113°$ since $140 + 107 + 113 = 360$.

So, the measures of the three exterior angles are 140, 107 and 113.

Example 4.18.2

Find the value of x and the measure of each angle.

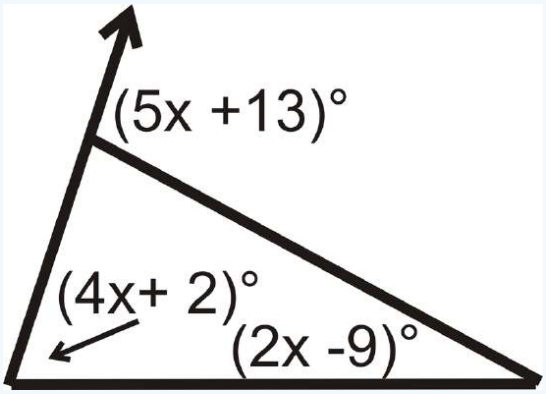


Figure 4.18.4

Solution

Set up an equation using the Exterior Angle Theorem.

$$\underbrace{(4x+2)^\circ + (2x-9)^\circ}_{\text{remote interior angles}} = \underbrace{(5x+13)^\circ}_{\text{exterior angle}}$$
$$(6x-7)^\circ = (5x+13)^\circ$$
$$x = 20$$

Substitute in 20 for x to find each angle.

$$[4(20)+2]^\circ = 82^\circ \quad [2(20)-9]^\circ = 31^\circ \quad \textit{Exterior angle}: \ [5(20)+13]^\circ = 113^\circ$$

Example 4.18.3

Find the measure of $\angle RQS$.

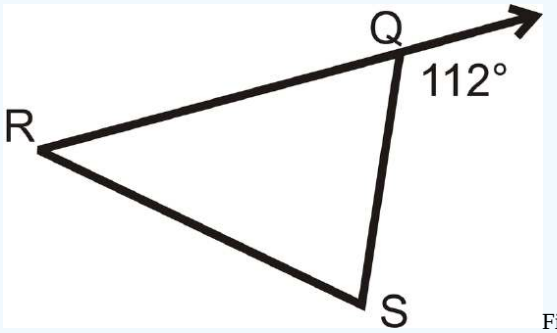


Figure 4.18.5

Solution

Notice that $112°$ is an exterior angle of ΔRQS and is supplementary to $\angle RQS$.

Set up an equation to solve for the missing angle.

$$112° + m\angle RQS = 180°$$
$$m\angle RQS = 68°$$

Example 4.18.4

Find the measures of the numbered interior and exterior angles in the triangle.

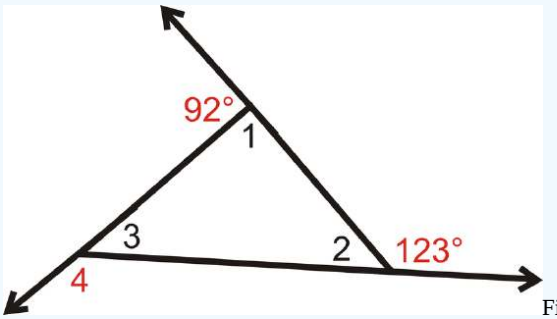


Figure 4.18.6

Solution

We know that $m\angle 1 + 92° = 180°$ because they form a linear pair. So, m\angle 1=88^{\circ}\).

Similarly, $m\angle 2 + 123° = 180°$ because they form a linear pair. So, m\angle 2=57^{\circ}\).

We also know that the three interior angles must add up to 180^{\circ}\) by the Triangle Sum Theorem.

$$m\angle 1 + m\angle 2 + m\angle 3 = 180° \quad \textit{by the Triangle Sum Theorem.}$$
$$88° + 57° + m\angle 3 = 180$$
$$m\angle 3 = 35°$$

Lastly, $m\angle 3 + m\angle 4 = 180°$ *because they form a linear pair.*

$$35° + m\angle 4 = 180°$$
$$m\angle 4 = 145°$$

Example 4.18.5

What is the value of p in the triangle below?

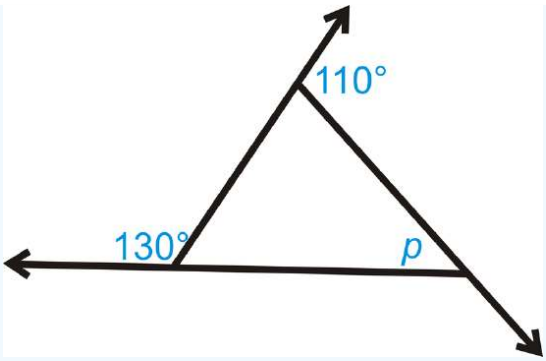


Figure 4.18.7

Solution

First, we need to find the missing exterior angle, which we will call x. Set up an equation using the Exterior Angle Sum Theorem.

$$\begin{aligned} 130^\circ + 110^\circ + x &= 360^\circ \\ x &= 360^\circ - 130^\circ - 110^\circ \\ x &= 120^\circ \end{aligned}$$

x and p add up to 180° because they are a linear pair.

$$\begin{aligned} x + p &= 180^\circ \\ 120^\circ + p &= 180^\circ \\ p &= 60^\circ \end{aligned}$$

Review

Determine $m\angle 1$.

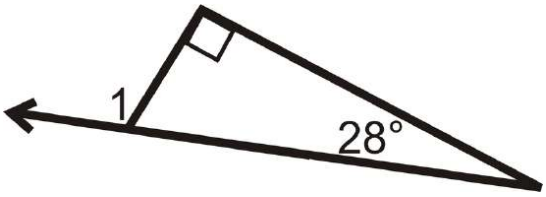
1. 

Figure 4.18.8

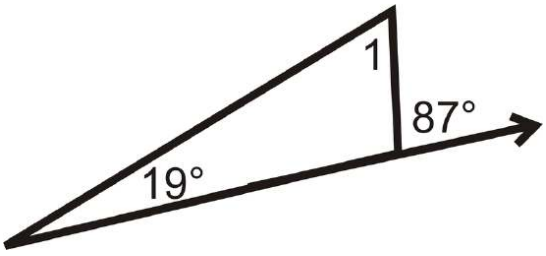
2. 

Figure 4.18.9

3.

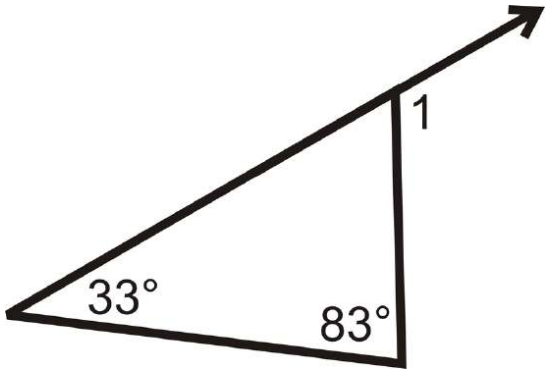


Figure 4.18.10

4.

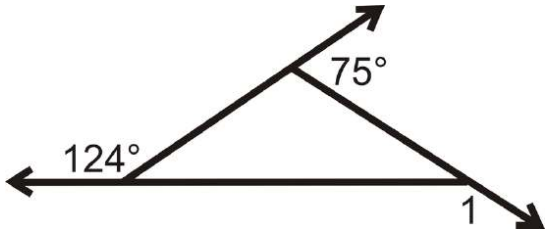


Figure 4.18.11

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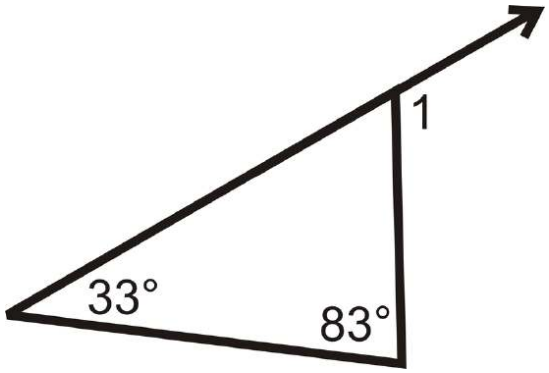

Figure 4.18.12

6.

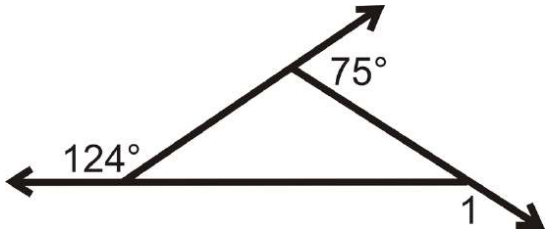

Figure 4.18.13

Use the following picture for the next three problems:

7. What is $m\angle 1 + m\angle 2 + m\angle 3$?
8. What is $m\angle 4 + m\angle 5 + m\angle 6$?
9. What is $m\angle 7 + m\angle 8 + m\angle 9$?

Solve for x.

10.

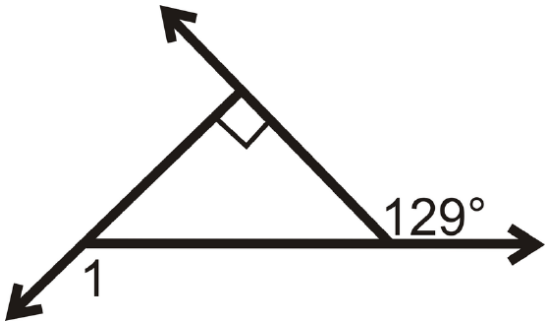

Figure 4.18.14

11.

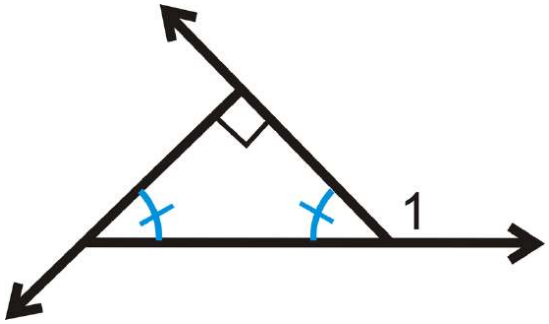

Figure 4.18.15

12.

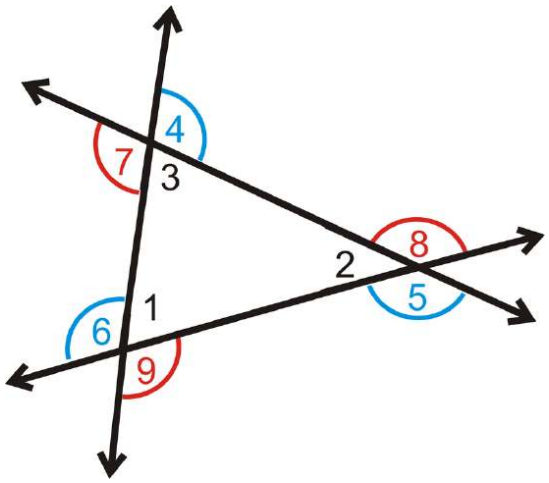

Figure 4.18.16

Resources

Vocabulary

Term	Definition
Exterior angles	An exterior angle is the angle formed by one side of a polygon and the extension of the adjacent side.
interior angles	The angles on the inside of a polygon.
remote interior angles	The remote interior angles (of a triangle) are the two interior angles that are not adjacent to the indicated exterior angle.
Triangle Sum Theorem	The ***Triangle Sum Theorem*** states that the three interior angles of any triangle will always add up to $180°$.
Exterior Angle Sum Theorem	Exterior Angle Sum Theorem states that the exterior angles of any polygon will always add up to 360 degrees.

Additional Resources

Video: Exterior Angles Theorems Examples - Basic

Activities: Exterior Angles Theorems Discussion Questions

Study Aids: Triangle Relationships Study Guide

Practice: Exterior Angles and Theorems

Real World: Exterior Angles Theorem

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4.19: Midsegment Theorem

Midsegment of a triangle joins the midpoints of two sides and is half the length of the side it is parallel to.

A line segment that connects two midpoints of the sides of a triangle is called a **midsegment**. $\overline{DF}$ is the midsegment between $\overline{AB}$ and $\overline{BC}$.

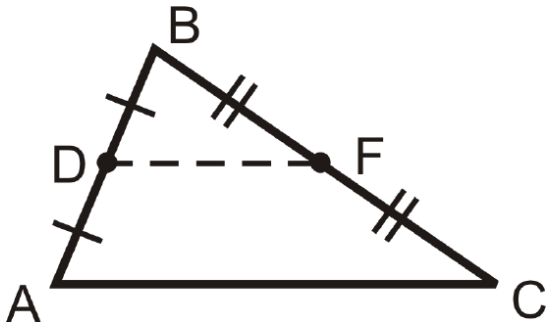


Figure 4.19.1

The tic marks show that D and F are midpoints. $\overline{AD} \cong \overline{DB}$ and $\overline{BF} \cong \overline{FC}$. For every triangle there are three midsegments.

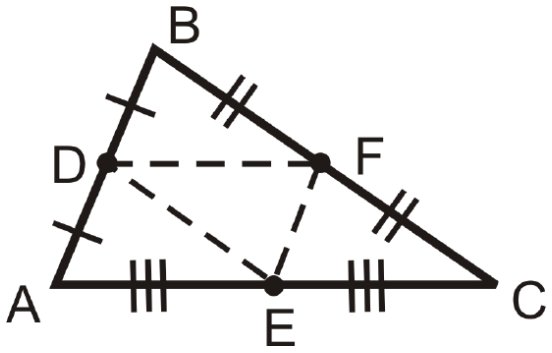


Figure 4.19.2

There are two important properties of midsegments that combine to make the **Midsegment Theorem**. The **Midsegment Theorem** states that the midsegment connecting the midpoints of two sides of a triangle is parallel to the third side of the triangle, and the length of this midsegment is half the length of the third side. So, if $\overline{DF}$ is a midsegment of ΔABC, then $DF = \frac{1}{2}AC = AE = EC$ and $\overline{DF} \parallel \overline{AC}$.

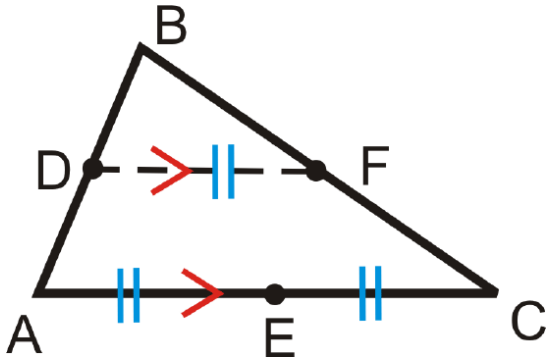


Figure 4.19.3

Note that there are two important ideas here. One is that the midsegment is parallel to a side of the triangle. The other is that the midsegment is always half the length of this side.

What if you were given ΔFGH and told that $\overline{JK}$ was its midsegment? How could you find the length of JK given the length of the triangle's third side, FH?

Example 4.19.1

Find the value of x and [illegible] dpoints.

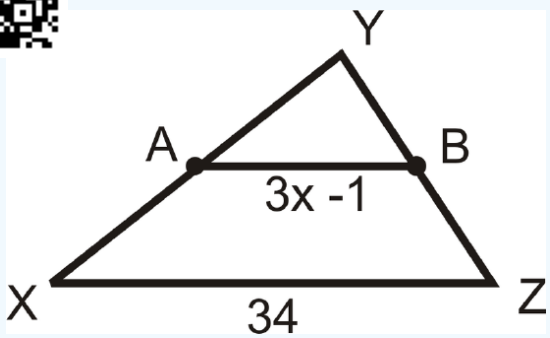


Figure 4.19.4

Solution

$AB = 34 \div 2 = 17$. To find x, set $3x - 1$ equal to 17.

$$\begin{aligned} 3x - 1 &= 17 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

Example 4.19.2

True or false: If a line passes through two sides of a triangle and is parallel to the third side, then it is a midsegment.

Solution

This statement is false. A line that passes through two sides of a triangle is only a midsegment if it passes through the **midpoints** of the two sides of the triangle.

Example 4.19.3

The vertices of ΔLMN are $L(4,5),\ M(-2,-7)\ and\ N(-8,3)$. Find the midpoints of all three sides, label them O, P and Q. Then, graph the triangle, plot the midpoints and draw the midsegments.

Solution

To solve this problem, use the **midpoint formula** 3 times to find all the midpoints. Recall that the midpoint formula is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

L and $M = \left(\frac{4 + (-2)}{2}, \frac{5 + (-7)}{2}\right) = (1, -1),\ point\ O$

M and $N = \left(\frac{-2 + (-8)}{2}, \frac{-7 + 3}{2}\right) = (-5, -2),\ point\ P$

L and $N = \left(\frac{4 + (-8)}{2}, \frac{5 + 3}{2}\right) = (-2, 4),\ point\ Q$

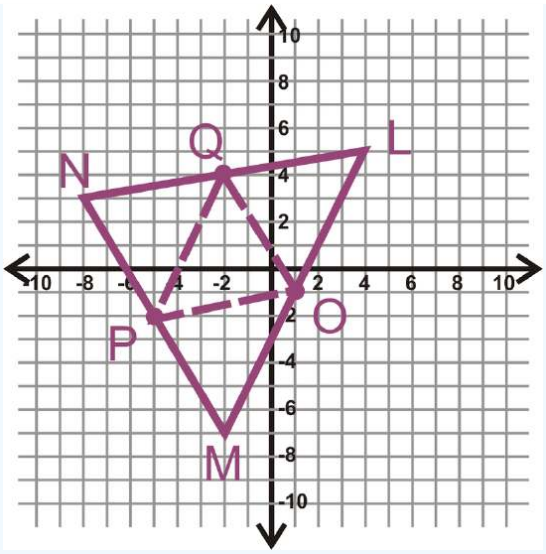


Figure 4.19.5

Example 4.19.4

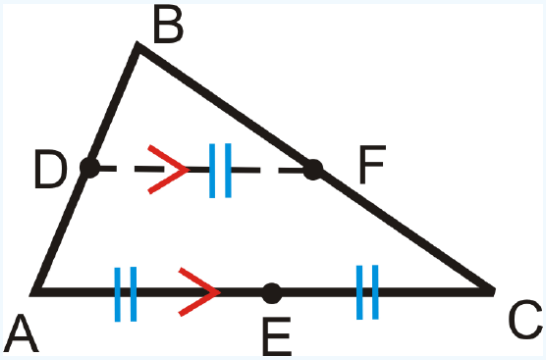


Figure 4.19.6

Mark all the **congruent** segments on ΔABC with midpoints D, E, and F.

Solution

Drawing in all three midsegments, we have:

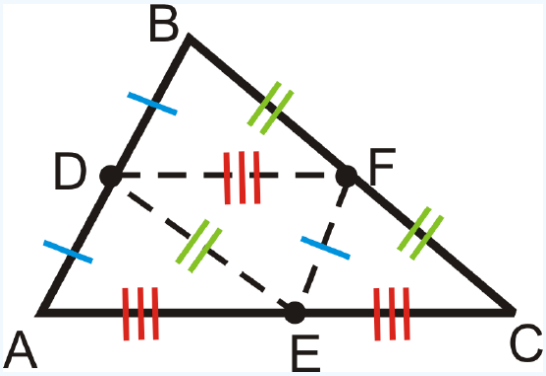


Figure 4.19.7

Also, this means the four smaller triangles are congruent by SSS.

Now, mark all the parallel lines on ΔABC, with midpoints D, E, and F.

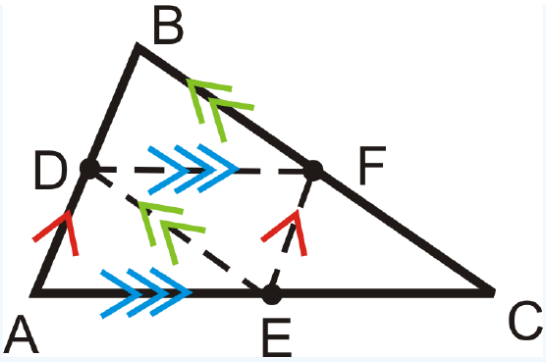


Figure 4.19.8

Example 4.19.5

M, N, and O are the midpoints of the sides of Δ\(xYZ\).

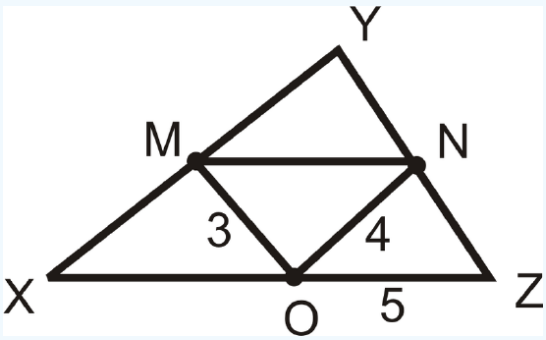


Figure 4.19.9

Solution

Find MN, XY, and the perimeter of Δ\(xYZ\).

Use the Midsegment Theorem:

$MN = OZ = 5$

$XY = 2(ON) = 2 \cdot 4 = 8$

Add up the three sides of ΔXYZ to find the perimeter.

$XY + YZ + XZ = 2 \cdot 4 + 2 \cdot 3 + 2 \cdot 5 = 8 + 6 + 10 = 24$

Remember: No line segment over MN means length or distance.

Review

Determine whether each statement is true or false.

1. The endpoints of a midsegment are midpoints.
2. A midsegment is parallel to the side of the triangle that it does not intersect.
3. There are three congruent triangles formed by the midsegments and sides of a triangle.
4. There are three midsegments in every triangle.

R, S, T, and U are midpoints of the sides of ΔXPO and ΔYPO

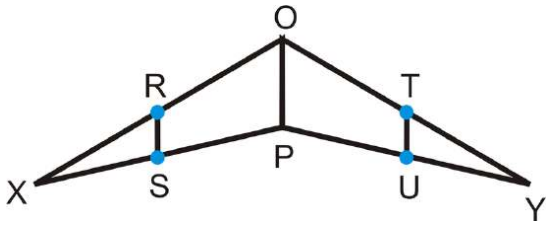


Figure 4.19.10

5. If $OP = 12$, find RS and TU.
6. If $RS = 8$, find TU.
7. If $RS = 2x$, and $OP = 20$, find x and TU.
8. If $OP = 4x$ and $RS = 6x - 8$, find x.

For questions 9-15, find the indicated variable(s). You may assume that all line segments within a triangle are midsegments.

9.

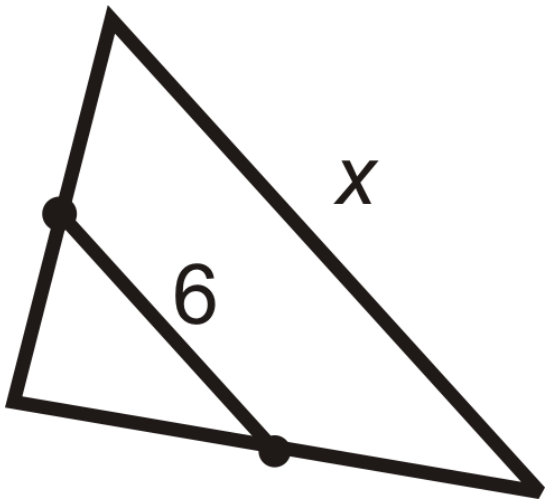


Figure 4.19.11

10.

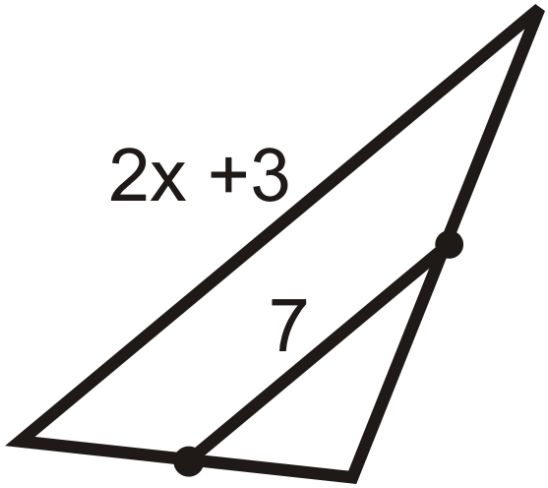


Figure 4.19.12

11.

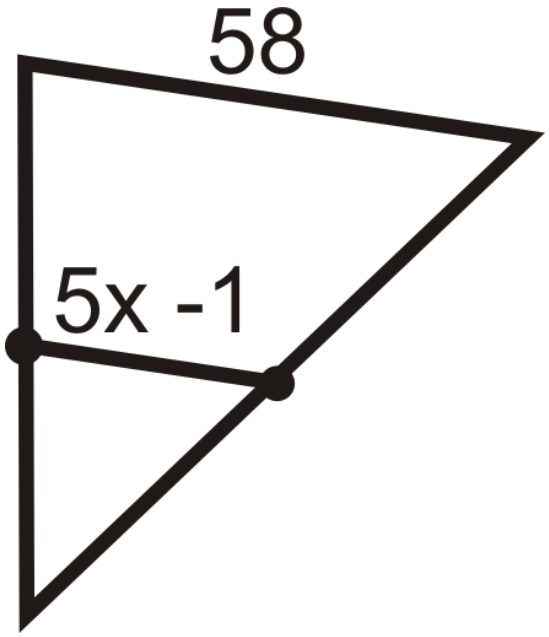


Figure 4.19.13

12.

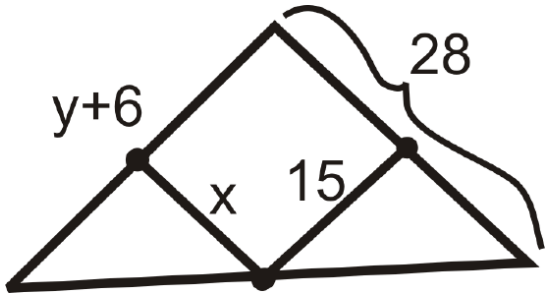


Figure 4.19.14

13.

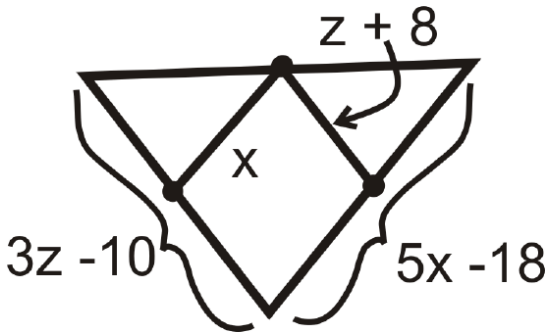


Figure 4.19.15

14.

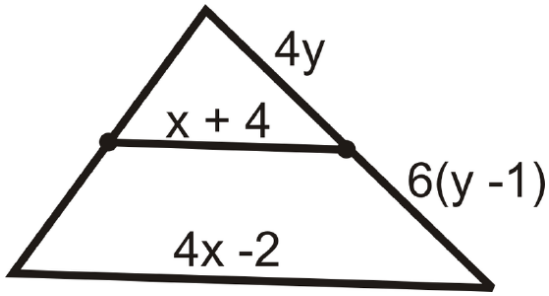


Figure 4.19.16

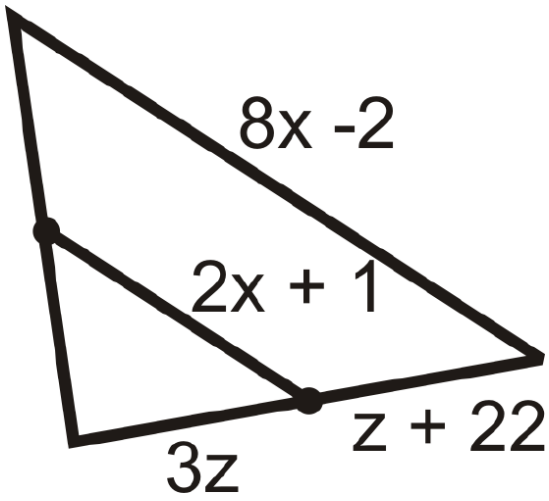
15. 

Figure **4.19.17**

16. The sides of ΔXYZ are 26, 38, and 42. ΔABC is formed by joining the midpoints of ΔXYZ.
    1. What are the lengths of the sides of ΔABC?
    2. Find the perimeter of ΔABC.
    3. Find the perimeter of ΔXYZ.
    4. What is the relationship between the perimeter of a triangle and the perimeter of the triangle formed by connecting its midpoints?

Coordinate Geometry Given the vertices of ΔABC below find the midpoints of each side.

17. $A(5,-2),\ B(9,4)\ and\ C(-3,8)$
18. $A(-10,1),\ B(4,11)\ and\ C(0,-7)$
19. $A(-1,3),\ B(5,7)\ and\ C(9,-5)$
20. $A(-4,-15),\ B(2,-1)\ and\ C(-20,11)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.1.

Resources

Vocabulary

Term	Definition
midsegment	A midsegment connects the midpoints of two sides of a triangle or the non-parallel sides of a trapezoid.
Congruent	Congruent figures are identical in size, shape and measure.
Midpoint Formula	The midpoint formula says that for endpoints (x_1, y_1) and (x_2, y_2), the midpoint is (\dfrac{x_1+x_2}{2}, \frac{y_1+y_2}{2})\).

Additional Resources

Video: Determining Unknown Values Using Properties of the Midsegments of a Triangle

Activities: Midsegment Theorem Discussion Questions

Study Aids: Bisectors, Medians, Altitudes Study Guide

Practice: Midsegment Theorem

Real World: Midsegment Theorem

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4.20: Perpendicular Bisectors

Intersect line segments at their midpoints and form 90 degree angles with them.

Perpendicular Bisector Theorem

A **perpendicular bisector** is a line that intersects a line segment at its midpoint and is perpendicular to that line segment, as shown in the construction below.

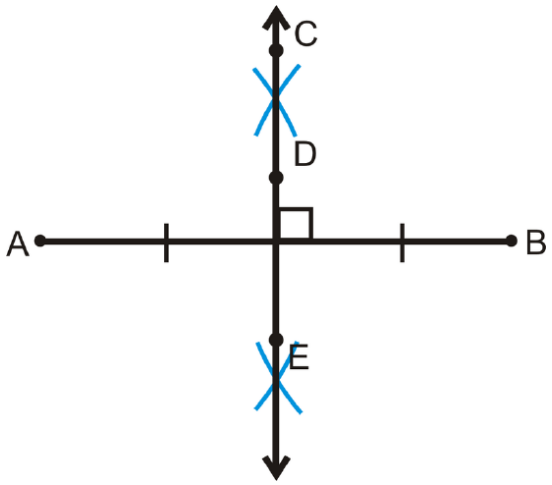


Figure **4.20.1**

One important property related to perpendicular bisectors is that if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. This is called the **Perpendicular Bisector Theorem.**

If $\overleftrightarrow{CD} \perp \overline{AB}$ and $AD = DB$, then $AC = CB$.

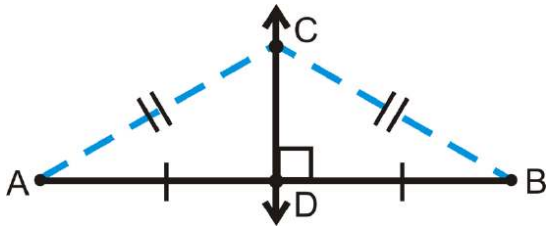


Figure **4.20.2**

In addition to the Perpendicular Bisector Theorem, the converse is also true.

Perpendicular Bisector Theorem Converse: If a point is equidistant from the endpoints of a segment, then the point is on the perpendicular bisector of the segment.

Using the picture above: If $AC = CB$, then $\overleftrightarrow{CD} \perp \overline{AB}$ and $AD = DB$.

When we construct perpendicular bisectors for the sides of a triangle, they meet in one point. This point is called the **circumcenter** of the triangle.

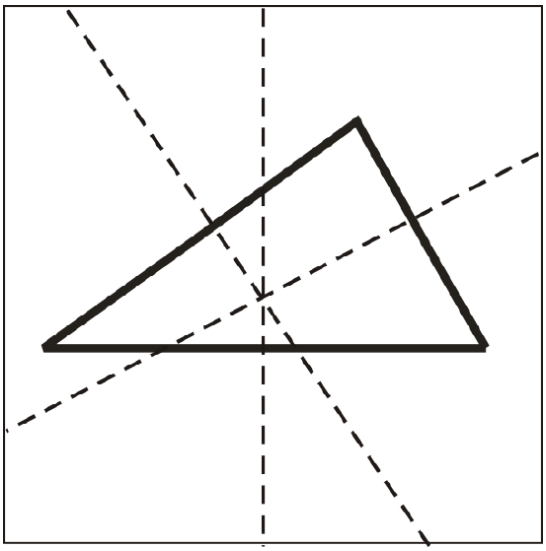


Figure 4.20.3

What if you were given ΔFGH and told that $\overleftrightarrow{GJ}$ was the perpendicular bisector of $\overline{FH}$? How could you find the length of FG given the length of GH\)?

Example 4.20.1

$\overleftrightarrow{OQ}$ is the perpendicular

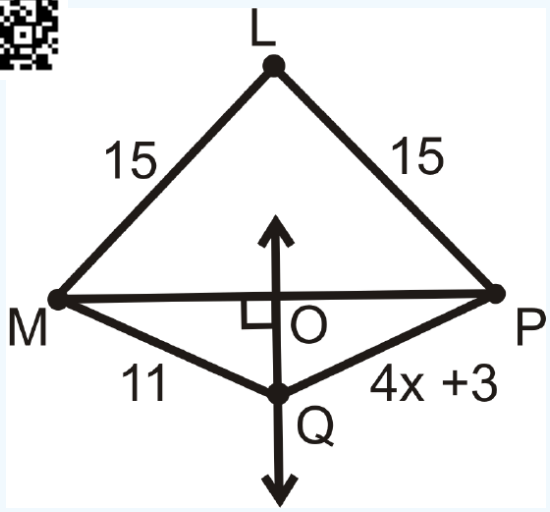


Figure 4.20.4

Which line segments are equal? Find x. Is L on $\overleftrightarrow{OQ}$? How do you know?

Solution

$ML = LP$, $MO = OP$, and $MQ = QP$.

$$\begin{aligned} 4x + 3 &= 11 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

Yes, L is on $\overleftrightarrow{OQ}$ because $ML = LP$ (the Perpendicular Bisector Theorem Converse).

Example 4.20.2

Determine if $\overleftrightarrow{ST}$ is the perpendicular bisector of $\overline{XY}$. Explain why or why not.

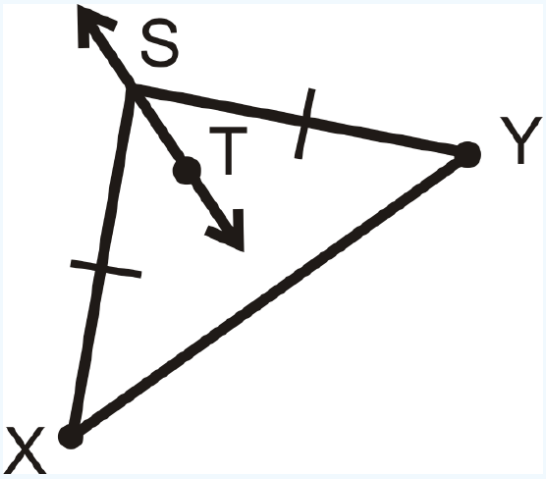


Figure 4.20.5

Solution

$\overleftrightarrow{ST}$ is not necessarily the perpendicular bisector of $\overline{XY}$ because not enough information is given in the diagram. There is no way to know from the diagram if $\overleftrightarrow{ST}$ will extend to make a right angle with $\overline{XY}$.

Example 4.20.3

If $\overleftrightarrow{MO}$– is the perpendicular bisector of $\overline{LN}$ and $LO = 8$, what is ON?

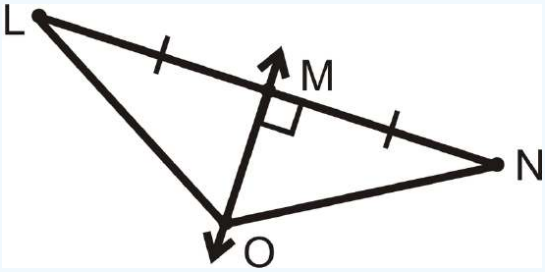


Figure 4.20.6

Solution

By the Perpendicular Bisector Theorem, $LO = ON$. So, $ON = 8$.

Example 4.20.4

Find x and the length of each segment.

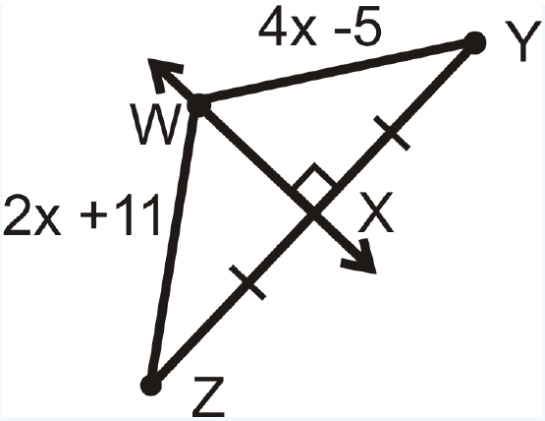


Figure 4.20.7

Solution

$\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{XZ}$ and from the Perpendicular Bisector Theorem $WZ = WY$.

$$2x + 11 = 4x - 5$$
$$16 = 2x$$
$$8 = x$$

$WZ = WY = 2(8) + 11 = 16 + 11 = 27$.

Example 4.20.5

Find the value of x. m is the perpendicular bisector of AB.

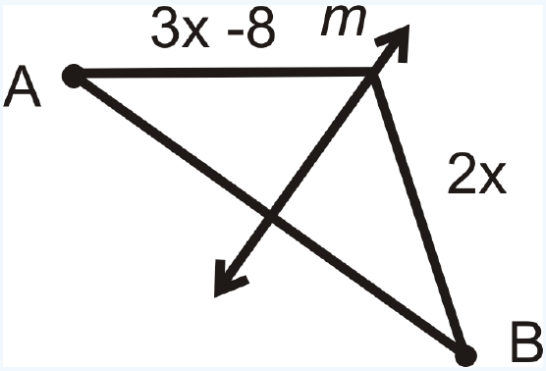


Figure 4.20.8

Solution

By the Perpendicular Bisector Theorem, both segments are equal. Set up and solve an equation.

$$3x - 8 = 2x$$
$$x = 8$$

Review

For questions 1-4, find the value of x. m\) is the perpendicular bisector ofAB.

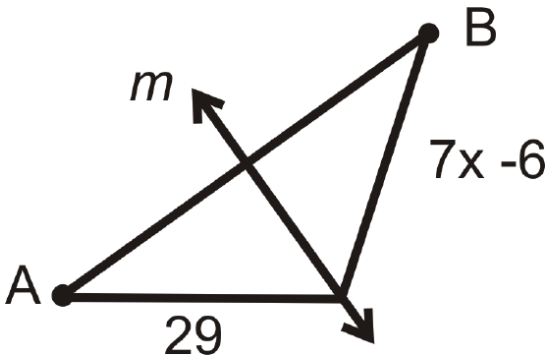
1. 

Figure 4.20.9

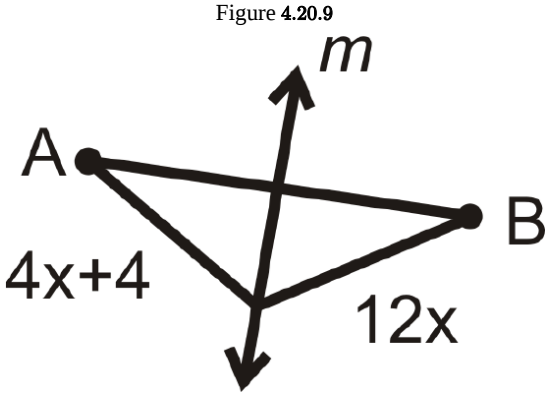
2. 

Figure 4.20.10

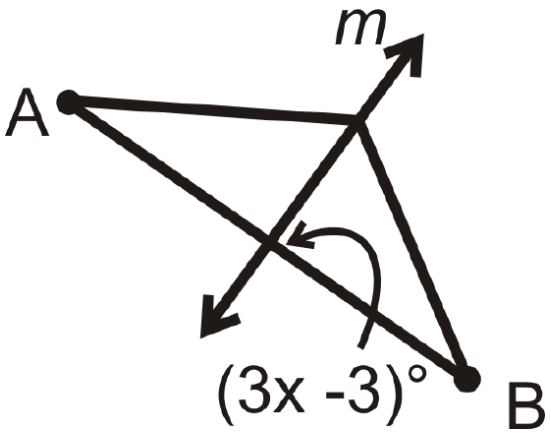
3. 

Figure 4.20.11

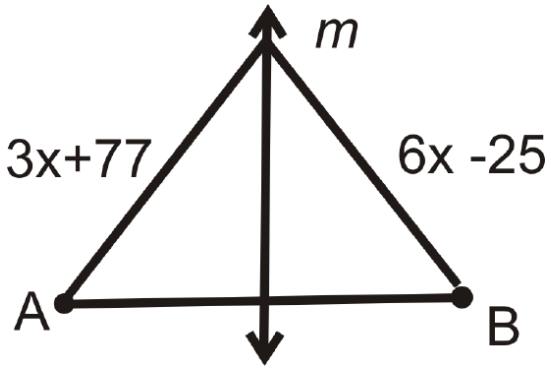
4. 

Figure 4.20.12

m is the perpendicular bisector of $\overline{AB}$.

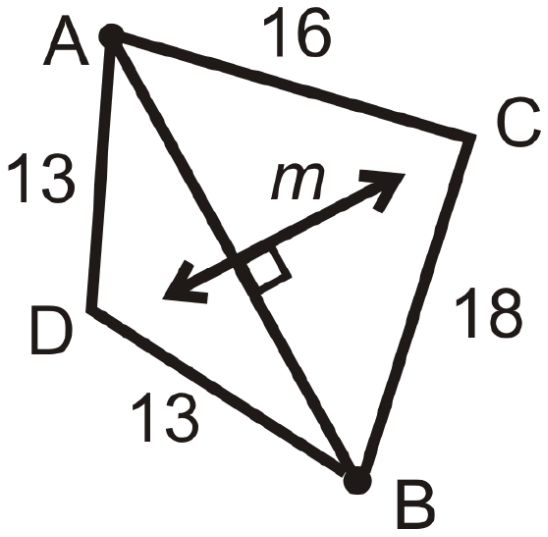


Figure **4.20.13**

5. List all the congruent segments.
6. Is C on m? Why or why not?
7. Is D on m? Why or why not?

For Question 8, determine if $\overleftrightarrow{ST}$ is the perpendicular bisector of \overline{XY}\). Explain why or why not.

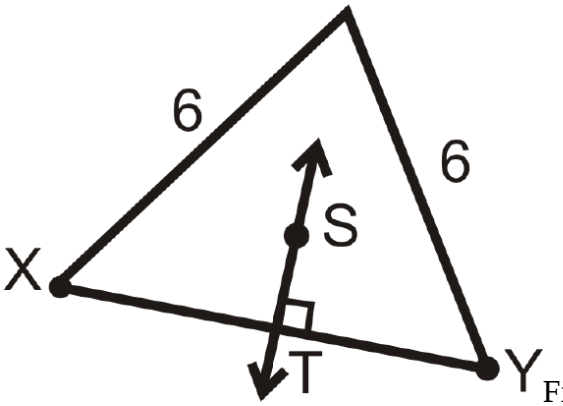
8. 

Figure \(\

PageIndex

{14}\)

9. In what type of triangle will all perpendicular bisectors pass through vertices of the triangle?
10. Fill in the blanks of the proof of the Perpendicular Bisector Theorem.

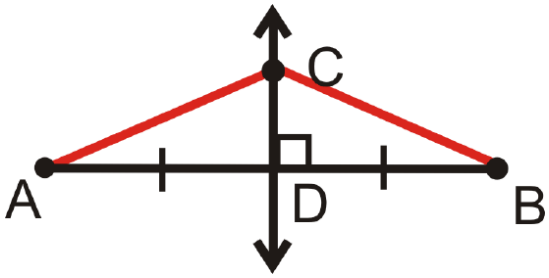


Figure **4.20.15**

Given: $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$

Prove: $\overline{AC} \cong \overline{CB}$

Statement	*Reason*
1.	1.
2. D is the midpoint of $\overline{AB}$	2.
3.	3. Definition of a midpoint
4. $\angle CDA$ and $\angle CDB$ are right angles	4.
5. $\angle CDA \cong \angle CDB$	5.
6.	6. Reflexive PoC
7. $\Delta CDA \cong \Delta CDB$	7.
8. $\overline{AC} \cong \overline{CB}$	8.

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.2.

Vocabulary

Term	Definition
circumcenter	The circumcenter is the point of intersection of the perpendicular bisectors of the sides in a triangle.
perpendicular bisector	A perpendicular bisector of a line segment passes through the midpoint of the line segment and intersects the line segment at $90°$.
Perpendicular Bisector Theorem Converse	If a point is equidistant from the endpoints of a segment, then the point is on the perpendicular bisector of the segment.

Additional Resources

Video: Perpendicular Bisectors Principles - Basic

Activites: Perpendicular Bisectors Discussion Questions

Study Aid: Bisectors, Medians, Altitudes Study Guide

Practice: Perpendicular Bisectors

Real World: Perpendicular Bisectors

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4.21: Angle Bisectors in Triangles

Construction and properties of bisectors, which cut angles in half.

Angle Bisector Theorem

An **angle bisector** cuts an angle exactly in half. One important property of angle bisectors is that if a point is on the bisector of an angle, then the point is equidistant from the sides of the angle. This is called the **Angle Bisector Theorem.**

In other words, if $\overrightarrow{BD}$ bisects $\angle ABC$, $\overrightarrow{BA} \perp FD\overline{AB}$, and, $\overrightarrow{BC} \perp \overline{DG}$ then $FD = DG$.

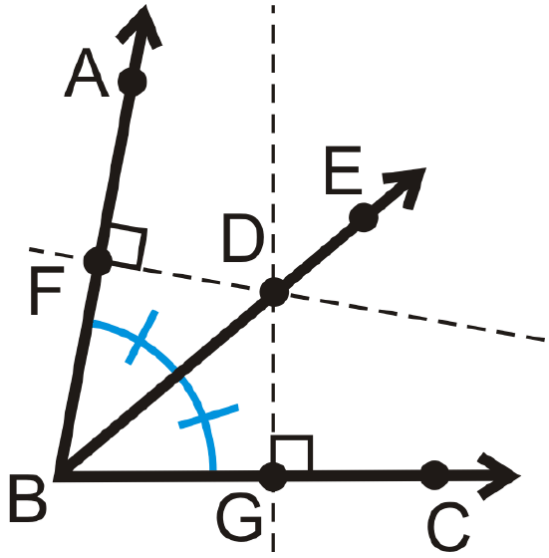


Figure 4.21.1

The converse of this theorem is also true.

Angle Bisector Theorem Converse: If a point is in the interior of an angle and equidistant from the sides, then it lies on the bisector of that angle.

When we construct angle bisectors for the angles of a triangle, they meet in one point. This point is called the **incenter** of the triangle.

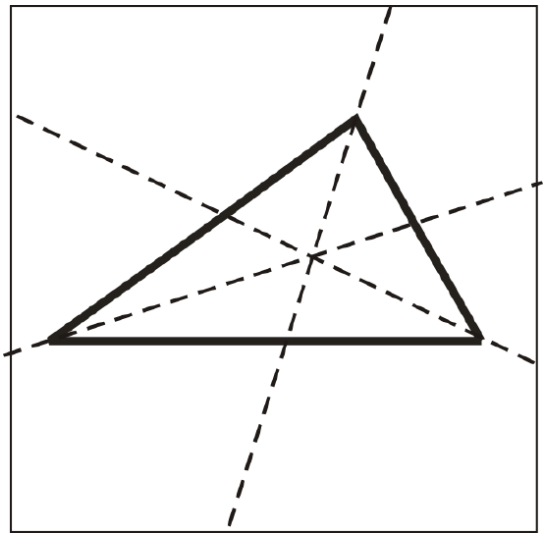


Figure 4.21.2

What if you were told that $\overrightarrow{GJ}$ is the angle bisector of $\angle FGH$? How would you find the length of FJ given the length of HJ?

Example 4.21.1

Is there enough informa $\overrightarrow{AB}$ is the angle bisector of $\angle CAD$? Why or why not?

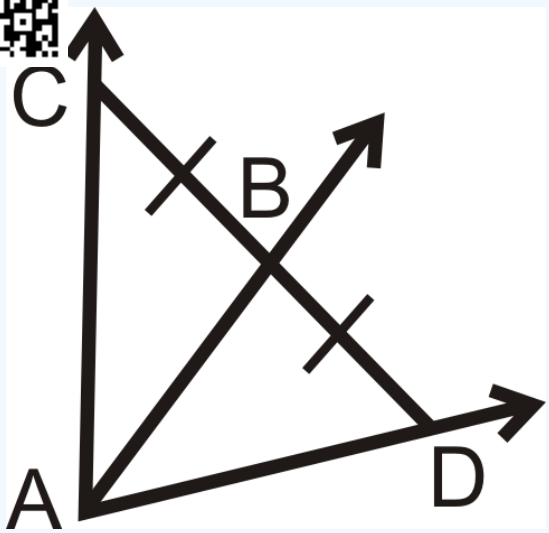


Figure 4.21.3

Solution

No because B is not necessarily equidistant from $\overline{AC}$ and $\overline{AD}$. We do not know if the angles in the diagram are right angles.

Example 4.21.2

A 108° angle is bisected. What are the measures of the resulting angles?

Solution

We know that to bisect means to cut in half, so each of the resulting angles will be half of 108. The measure of each resulting angle is 54°.

Example 4.21.3

Is Y on the angle bisector of $\angle XWZ$?

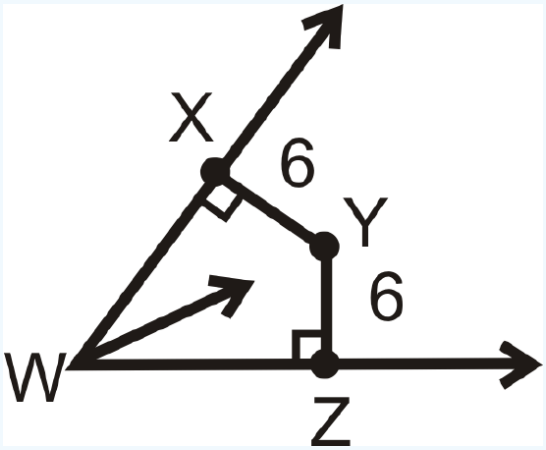


Figure 4.21.4

Solution

If Y is on the angle bisector, then $XY = YZ$ and both segments need to be perpendicular to the sides of the angle. From the markings we know $\overline{XY} \perp \overrightarrow{WX}$ and $\overline{ZY} \perp \overrightarrow{WZ}$. Second, $XY = YZ = 6$. So, yes, Y is on the angle bisector of $\angle XWZ$.

Example 4.21.4

$\overrightarrow{MO}$ is the angle bisector of $\angle LMN$. Find the measure of x.

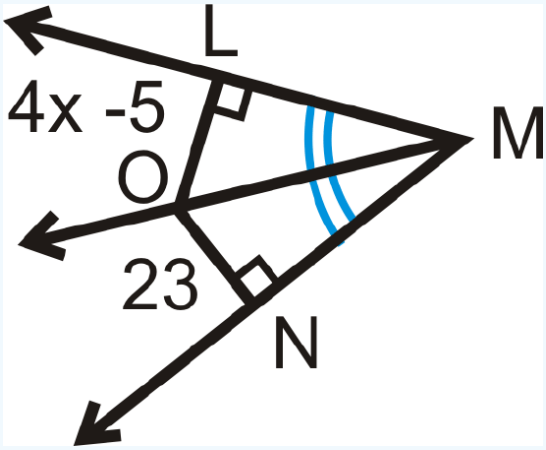


Figure 4.21.5

Solution

$LO = ON$ by the Angle Bisector Theorem.

$$\begin{aligned} 4x - 5 &= 23 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$

Example 4.21.5

$\overrightarrow{AB}$ is the angle bisector of $\angle CAD$. Solve for the missing variable.

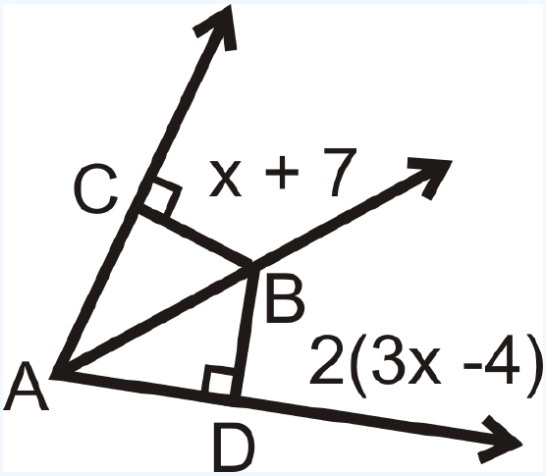


Figure 4.21.6

Solution

$CB = BD$ by the Angle Bisector Theorem, so we can set up and solve an equation for x.

$$\begin{aligned} x + 7 &= 2(3x - 4) \\ x + 7 &= 6x - 8 \\ 15x &= 5 \\ x &= 3 \end{aligned}$$

Review

For questions 1-4, $\overrightarrow{AB}$ is the angle bisector of $\angle CAD$. Solve for the missing variable.

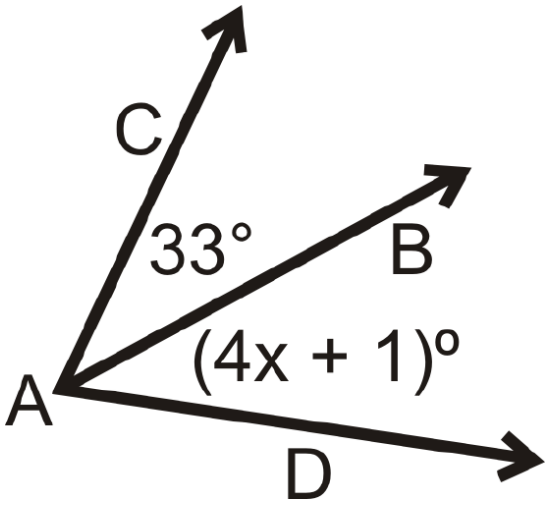
1. 

Figure 4.21.7

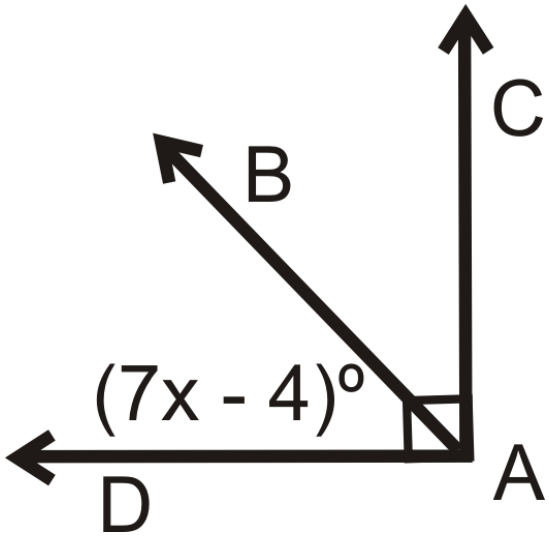
2. 

Figure 4.21.8

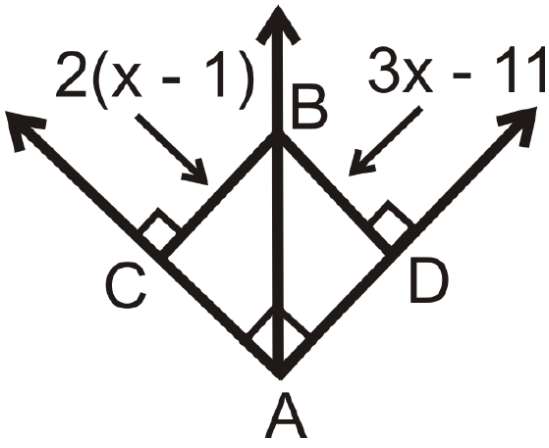
3. 

Figure 4.21.9

4.

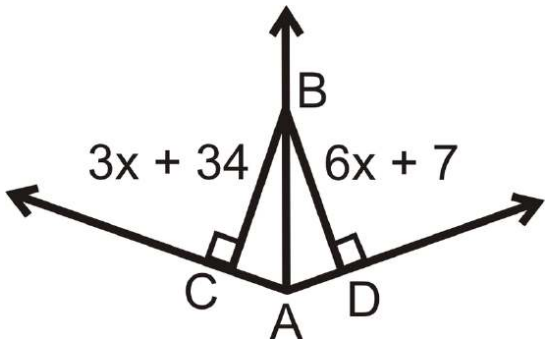


Figure 4.21.10

Is there enough information to determine if $\overrightarrow{AB}$ is the angle bisector of \angle CAD? Why or why not?

5.

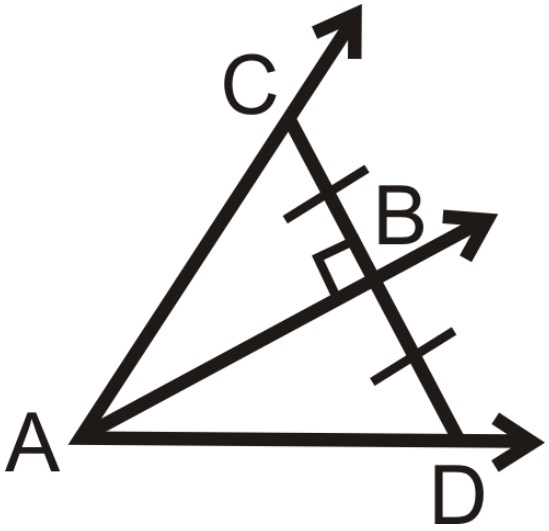


Figure 4.21.11

6.

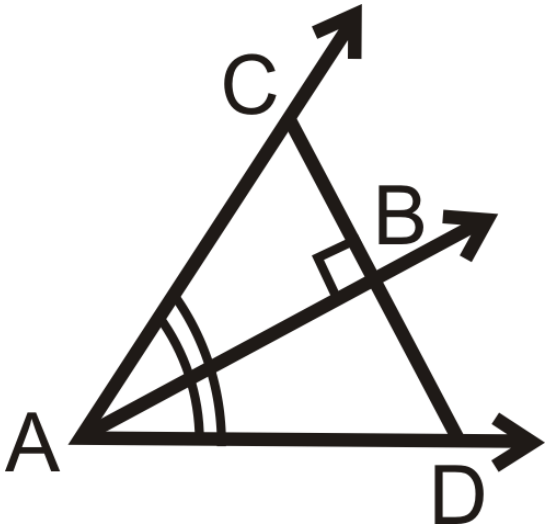


Figure 4.21.12

7. In what type of triangle will all angle bisectors pass through vertices of the triangle?
8. What is another name for the angle bisectors of the vertices of a square?
9. Draw in the angle bisectors of the vertices of a square. How many triangles do you have? What type of triangles are they?
10. Fill in the blanks in the Angle Bisector Theorem Converse.

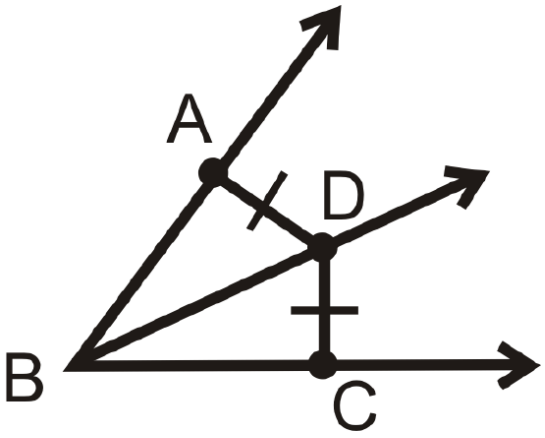


Figure **4.21.13**

Given: $\overline{AD} \cong \overline{DC}$, such that AD and DC are the shortest distances to $\overrightarrow{BA}$ and $\overrightarrow{BC}$

Prove: $\overrightarrow{BD} bisects \angle ABC$

Statement	*Reason*
1.	1.
2.	2. The shortest distance from a point to a line is perpendicular.
3. $\angle DAB$ and $\angle DCB$ are right angles	3.
4. $\angle DAB \cong \angle DCB$	4.
5. $\overline{BD} \cong \overline{BD}$	5.
6. $\Delta ABD \cong \Delta CBD$	6.
7.	7. CPCTC
8. $\overrightarrow{BD}$ bisects $\angle ABC$	8.

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.3.

Resources

Vocabulary

Term	Definition
angle bisector	An angle bisector is a ray that splits an angle into two congruent, smaller angles.
Angle Bisector Theorem	The angle bisector theorem states that if a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.
Angle Bisector Theorem Converse	The angle bisector theorem converse states that if a point is in the interior of an angle and equidistant from the sides, then it lies on the bisector of that angle.
incenter	The incenter is the point of intersection of the angle bisectors in a triangle.

Additional Resources

Video: Examples: Solving For Unknown Values Using Properties of Angle Bisectors

Activities: Angle Bisectors in Triangles Discussion Questions

Study Aids: Bisectors, Medians, Altitudes Study Guide

Practice: Angle Bisectors in Triangles

Real World: Perpendicular Bisectors

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4.22: Concurrence and Constructions

Constructions related to theorems involving circumcenters and other intersection points of three or more lines.

Given a triangle, what's the difference between the *inscribed circle* of the triangle and the *circumscribed circle* of the triangle?

Inscribed and Circumscribed Circles of Triangles

Given a triangle, an **inscribed circle** is the largest circle contained within the triangle. The inscribed circle will touch each of the three sides of the triangle in exactly one point. The center of the circle inscribed in a triangle is the **incenter** of the triangle, the point where the angle bisectors of the triangle meet.

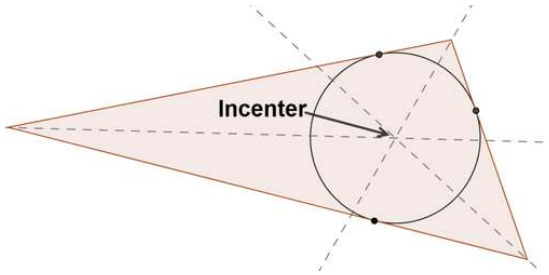

Figure **4.22.1**

To construct the **inscribed circle:**

1. Construct the incenter.
2. Construct a line perpendicular to one side of the triangle that passes through the incenter. The segment connecting the incenter with the point of intersection of the triangle and the perpendicular line is the radius of the circle.
3. Construct a circle centered at the incenter with the radius found in step 2.

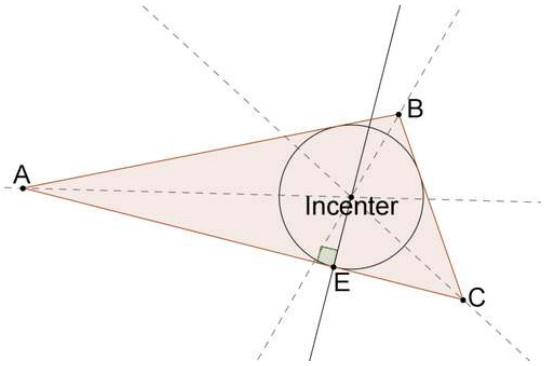

Figure **4.22.2**

The steps for constructing the inscribed circle for a given triangle will be explored in the problems below.

Given a triangle, the **circumscribed circle** is the circle that passes through all three vertices of the triangle. The center of the circumscribed circle is the **circumcenter** of the triangle, the point where the perpendicular bisectors of the sides meet.

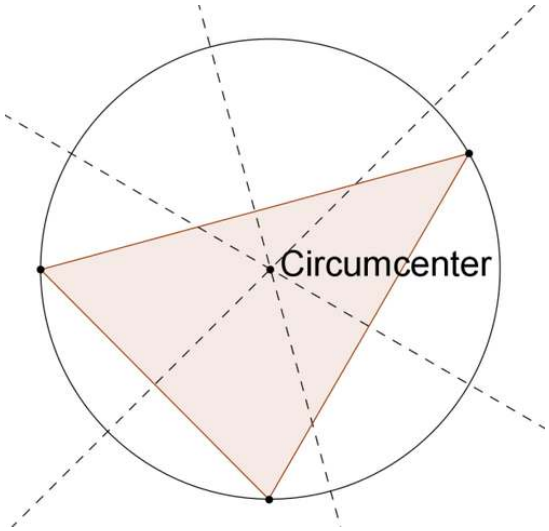

Figure **4.22.3**

To construct the **circumscribed circle:**

1. Construct the circumcenter.
2. Construct a circle centered at the circumcenter that passes through one of the vertices. This same circle should pass through *all three* vertices.

The steps for constructing the circumscribed circle for a given triangle will be explored in the Examples section.

Constructing Angle Bise

Draw a triangle. Construc s of two of its angles. Why is the point of intersection of the two angle bisectors the incenter of the circle?

Use your compass and stra ct the angle bisector of one of the angles.

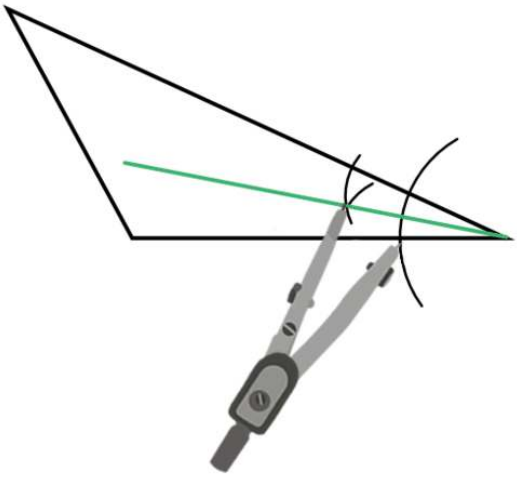

Figure **4.22.4**

Repeat with a second angle.

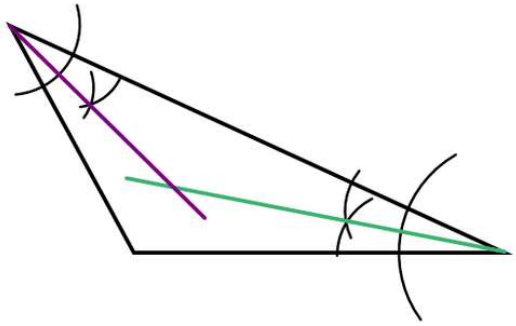

Figure **4.22.5**

The point of intersection of the angle bisectors is the incenter. It is not necessary to construct all three angle bisectors because they all meet in the same point. The third angle bisector does not provide any new information.

Constructing Perpendicular Lines

Construct a line perpendicular to one side of the triangle that passes through the incenter of the triangle.

Use your compass and straightedge to construct a line perpendicular to one side of the triangle that passes through the incenter.

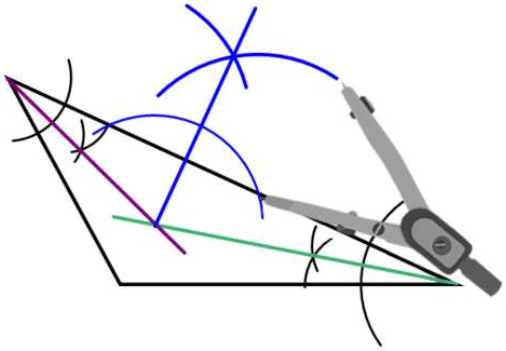

Figure **4.22.6**

Constructing Inscribed Circles

Construct a circle centered at the incenter that passes through the point of intersection of the side of the triangle and the perpendicular line from the problem above.

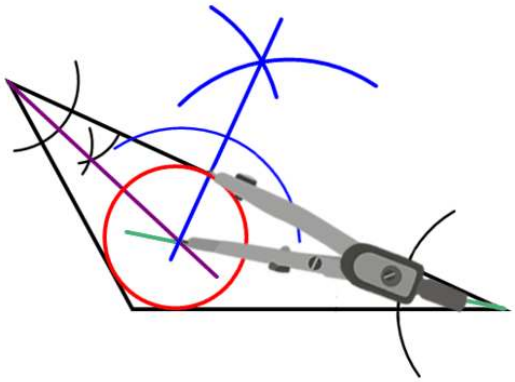

Figure **4.22.7**

Note that this circle touches each side of the triangle exactly once.

Example 4.22.1

Earlier, you were asked what is the difference between the inscribed circle of the triangle and the circumscribed circle of the triangle.

Solution

The **inscribed circle of a triangle** is inside the triangle. The circumscribed circle of a triangle is outside the triangle.

Example 4.22.2

Draw a triangle. Construct the perpendicular bisectors of two of its sides. Why is the point of intersection of the two perpendicular bisectors the circumcenter of the circle?

Solution

Use your compass and straightedge to construct the **perpendicular bisector** of one side.

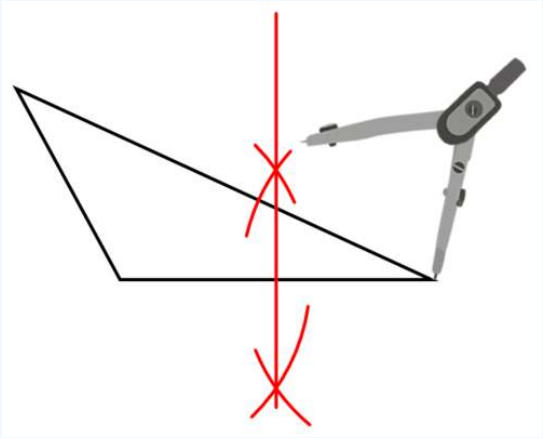

Figure 4.22.8

Repeat with a second side.

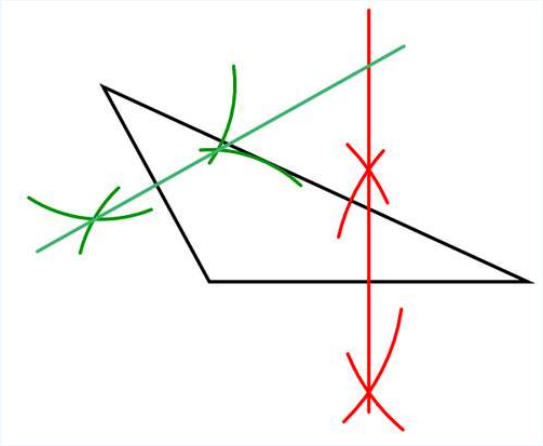

Figure 4.22.9

The point of intersection of the perpendicular bisectors is the circumcenter. It is not necessary to construct all three perpendicular bisectors because they all meet in the same point. The third perpendicular bisector does not provide any new information.

Example 4.22.3

Continue with your triangle from #2. Construct the circumscribed circle of the triangle.

Solution

Construct a circle centered at the circumcenter that passes through one of the vertices of the triangle. This circle should pass through *all three* vertices.

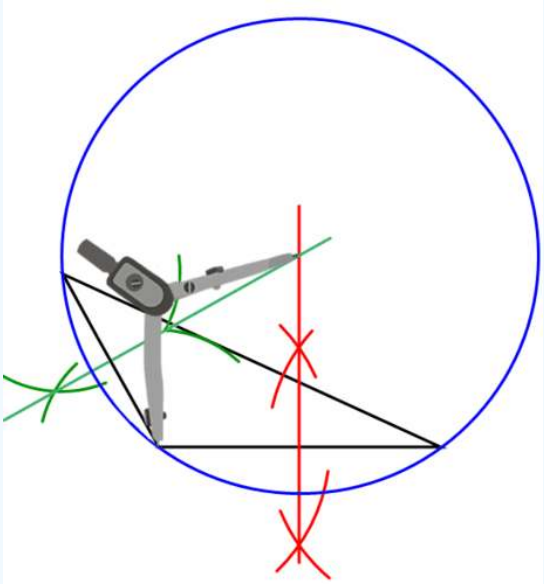

Figure 4.22.10

Example 4.22.4

Justify the statement: *The hypotenuse of a right triangle will be a diameter of the circumscribed circle of the triangle.*

Solution

Each of the angles that make up a triangle become inscribed angles of the circumscribed circle. A $90°$ angle will intercept an arc of $180°$, which is half a circle. Therefore, the side opposite the $90°$ angle of the triangle must be a diameter of the circle.

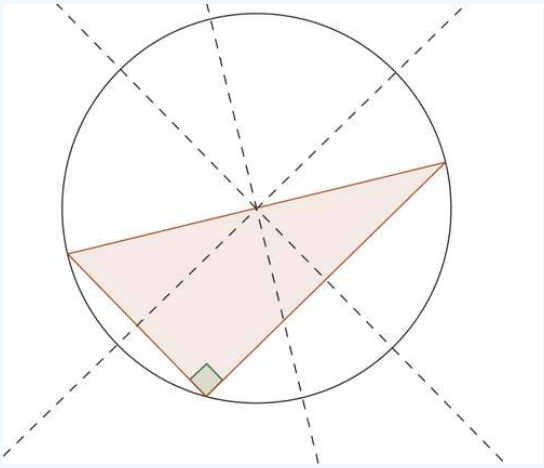

Figure 4.22.11

Review

1. Draw a triangle and construct the angle bisector of two of its angles.

2. Continue with your triangle from #1. Construct a line perpendicular to one side of the triangle that passes through the incenter of the triangle.

3. Continue with your triangle from #1 and #2. Construct the inscribed circle of the triangle.

4. Why was it not necessary to construct the angle bisector of *all three* of the angles of the triangle?

5. Explain why the incenter is equidistant from each of the sides of the triangle.

6. Draw a triangle and construct the perpendicular bisector of two of its sides.

7. Continue with your triangle from #5. Construct the circumscribed circle of the triangle.

8. Explain why the circumcenter is equidistant from each of the vertices of the triangle.

You work selling food from a food truck at a local park. You want to position your truck so that it is the same distance away from each of the three locations shown on the map below.

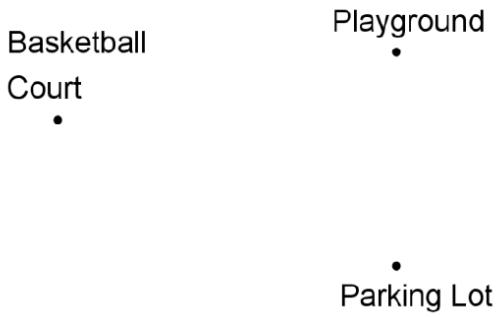

Figure **4.22.12**

9. Is the point of interest the incenter or the circumcenter?

10. Find the point on the map that is equidistant from each of the three locations.

11. How could you fold the map in two places to find the point equidistant from each of the three locations?

A new elementary school is to be constructed in your town. The plan is to build the school so that it is the same distance away from each of the three major roads shown in the map below.

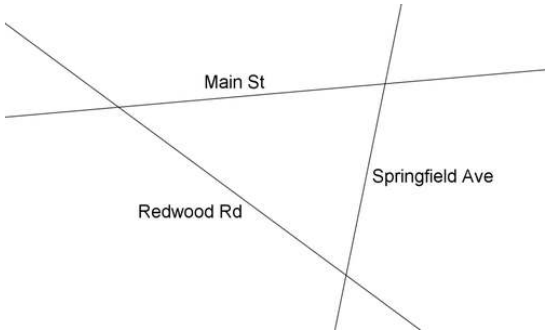

Figure **4.22.13**

12. Is the point of interest the incenter or the circumcenter?

13. Find the point on the map that is equidistant from each of the three roads.

14. How could you fold the map in two places to find the point equidistant from each of the three roads?

15. Justify the following statement: *Given any three non-collinear points, there exists exactly one circle that passes through the points.*

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.5.

Vocabulary

Term	Definition
perpendicular bisector	A perpendicular bisector of a line segment passes through the midpoint of the line segment and intersects the line segment at **90°**.
Inscribed Circle of a Triangle	The largest circle contained within the triangle. The inscribed circle will just touch each of the three sides of the triangle (it is tangent to each of the three sides of the triangle).

Additional Resources

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4.23: Medians

Line segment that joins a vertex and the midpoint of the opposite side of a triangle.

In a triangle, the line segment that joins a vertex and the midpoint of the opposite side is called a **median**.

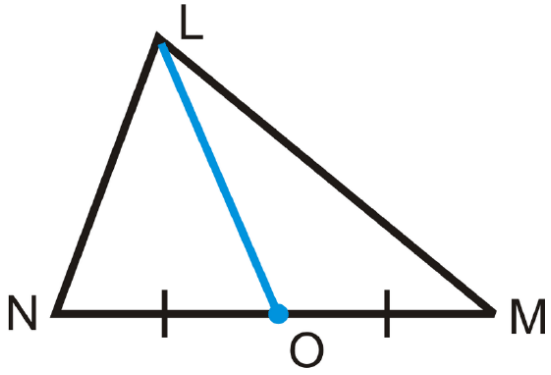

Figure 4.23.1

\overline{LO}\) is the median from L\) to the midpoint of \overline{NM}\).

If you draw all three medians they will intersect at one point called the **centroid**.

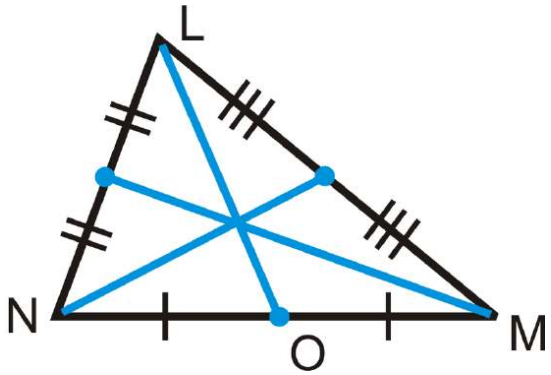

Figure 4.23.2

The centroid is the “balancing point” of a triangle. This means that if you were to cut out the triangle, the centroid is its center of gravity so you could balance it there.

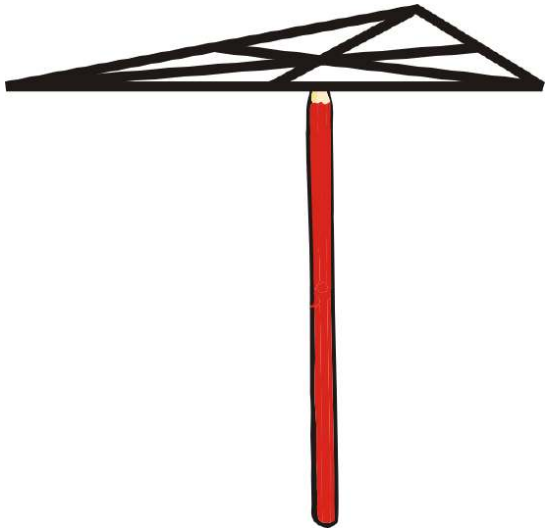

Figure 4.23.3

The **Median Theorem** states that the medians of a triangle intersect at a point called the centroid that is two-thirds of the distance from the vertices to the midpoint of the opposite sides.

So if G is the centroid, then:

$$AG = \frac{2}{3}AD, CG = \frac{2}{3}CF,\ EG = \frac{2}{3}BE$$

$$DG = \frac{1}{3}AD, FG = \frac{1}{3}CF,\ BG = \frac{1}{3}BE$$

$$\textit{And by substitution}: DG = \frac{1}{2}AG,\ FG = \frac{1}{2}CG,\ BG = \frac{1}{2}EG$$

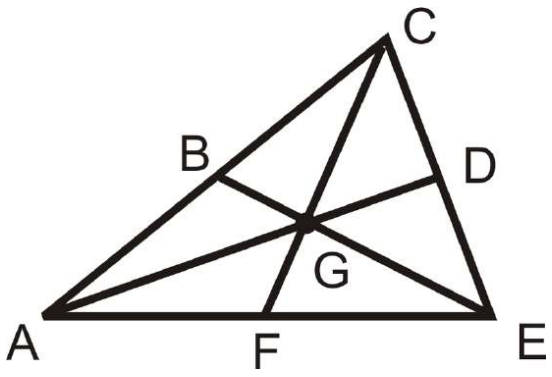

Figure 4.23.4

Example 4.23.1

B, D, and F are the mid[illegible] and G is the centroid. If $CG = 16$, find GF and CF.

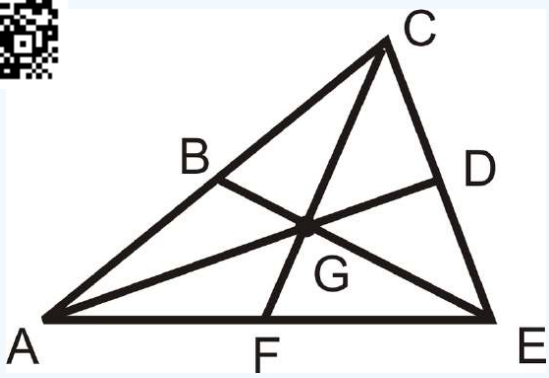

Figure 4.23.5

Solution

Use the Median Theorem.

$$CG = \frac{2}{3}CF$$
$$16 = \frac{2}{3}CF$$
$$CF = 24.$$

Therefore, $GF = 8$.

Example 4.23.2

True or false: The median bisects the side it intersects.

Solution

This statement is true. By definition, a median intersects a side of a triangle at its midpoint. Midpoints divide segments into two equal parts.

Example 4.23.3

I, K, and M are midpoints of the sides of ΔHJL.

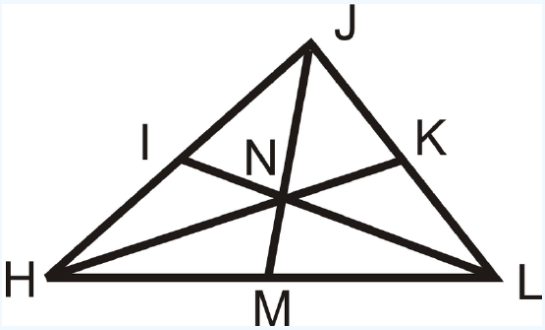

Figure 4.23.6

Solution

If $JM = 18$, find JN and NM. If $HN = 14$, find NK and HK.

Use the Median Theorem.

$JN = \frac{2}{3} \cdot 18 = 12. NM = JM - JN = 18 - 12.\ NM = 6.$

$14 = \frac{2}{3} \cdot HK$

$14 \cdot \frac{3}{2} = HK = 21.$ NK is a third of 21, $NK = 7$.

Example 4.23.4

H is the centroid of ΔABC and $DC = 5y - 16$. Find x and y.

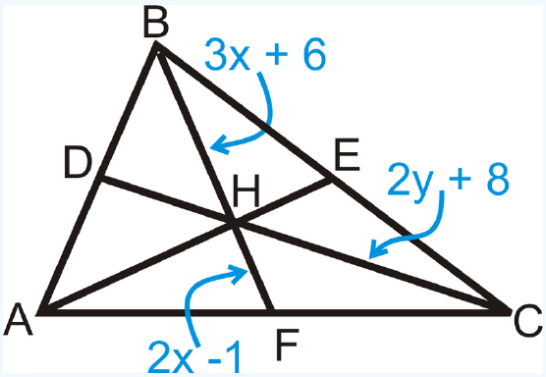

Figure 4.23.7

Solution

To solve, use the Median Theorem. Set up and solve equations.

$\frac{1}{2}BH = HF \rightarrow BH = 2HF$

$3x + 6 = 2(2x - 1)$

$3x + 6 = 4x - 2$

$8 = x$

$HC = \frac{2}{3}DC \rightarrow \frac{3}{2}HC = DC$

$\frac{3}{2}(2y + 8) = 5y - 16$

$3y + 12 = 5y - 16$

$28 = 2y \rightarrow 14 = y$

Example 4.23.5

B, D, and F are the midpoints of each side and G is the centroid. If $BG = 5$, find GE and BE

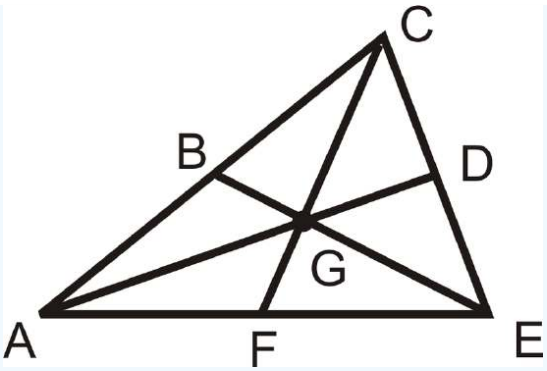

Figure 4.23.8

Solution

Use the Median Theorem.

$BG = \frac{1}{3}BE$

$5 = \frac{1}{3}BE$

$BE = 15.$

Therefore, $GE = 10$.

Review

For questions 1-4, B, D, and F are the midpoints of each side and G is the centroid. Find the following lengths.

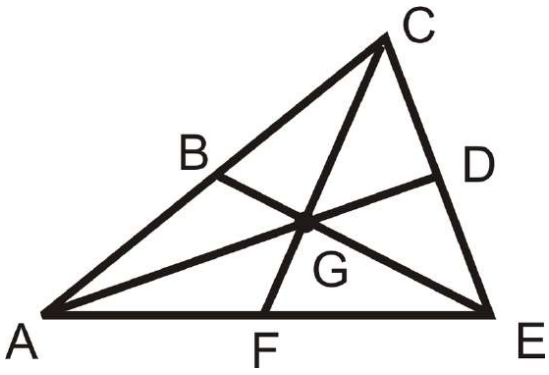

Figure 4.23.9

1. If $CG = 16$, find GF and CF
2. If $AD = 30$, find AG and GD
3. If $GF = x$, find GC and CF
4. If $AG = 9x$ and $GD = 5x - 1$, find x and AD.

Multi-step Problems Find the equation of a median in the x−y\) plane.

5. Plot $\Delta ABC:\ A(-6,4)$,\:B(−2,4)\)\:and\:C(6,−4)\)
6. Find the midpoint of $\overline{AC}$. Label it D.
7. Find the slope of $\overline{BD}$.
8. Find the equation of $\overline{BD}$.
9. Plot $\Delta DEF:\ D(-1,5),\ E(1,0),\ F(6,3)$
10. Find the midpoint of $\overline{EF}$. Label it G.
11. Find the slope of $\overline{DG}$.
12. Find the equation of \(overline{DG}\).

Determine whether the following statement is true or false.

13. The centroid is the balancing point of a triangle.

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.4.

Resources

Vocabulary

Term	Definition
centroid	The centroid is the point of intersection of the medians in a triangle.
Median	The median of a triangle is the line segment that connects a vertex to the opposite side's midpoint.

Additional Resources

Video: The Medians of a Triangle

Activities: Medians Discussion Questions

Study Aids: Bisectors, Medians, Altitudes Study Guide

Real World: Medians

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4.24: Altitudes

Height of a triangle or the line segment from a vertex and perpendicular to the opposite side.

In a triangle, a line segment from a vertex and perpendicular to the opposite side is called an **altitude**. It is also called the height of a triangle. The red lines below are all altitudes.

Figure 4.24.1

When a triangle is a right triangle, the altitude, or height, is the leg. If the triangle is obtuse, then the altitude will be outside of the triangle. If the triangle is acute, then the altitude will be inside the triangle.

What if you were given one or more of a triangle's angle measures? How would you determine where the triangle's altitude would be found?

Example 4.24.1

True or false: The altitu ngle are inside the triangle.

Solution

Every triangle has three otuse triangle, at least one of the altitudes will be outside of the triangle, as shown in the picture at the beginning of this section.

Example 4.24.2

Draw the altitude for the triangle shown.

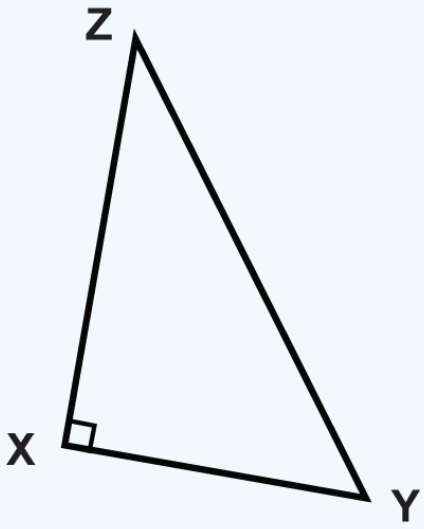


Figure 4.24.2

Solution

The triangle is a right triangle, so the altitude is already drawn. The altitude is $\overline{XZ}$.

Example 4.24.3

Which line segment is the altitude of ΔABC?

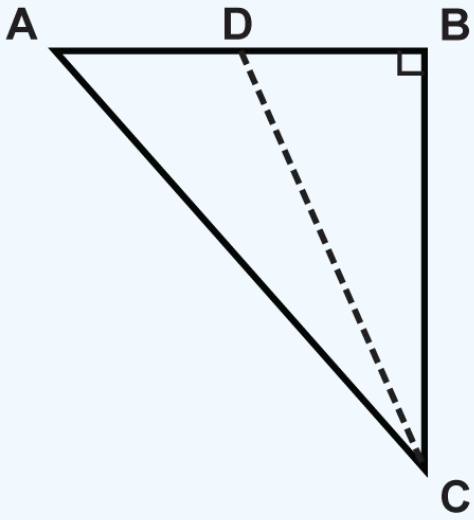


Figure 4.24.3

Solution

In a right triangle, the altitude, or the height, is the leg. If we rotate the triangle so that the right angle is in the lower left corner, we see that leg $\overline{BC}$ is the altitude.

Example 4.24.4

A triangle has angles that measure $55°$, $60°$, and $65°$. Where will the altitude be found?

Solution

Because all of the angle measures are less than $90°$, the triangle is an acute triangle. The altitude of any acute triangle is inside the triangle.

Example 4.24.5

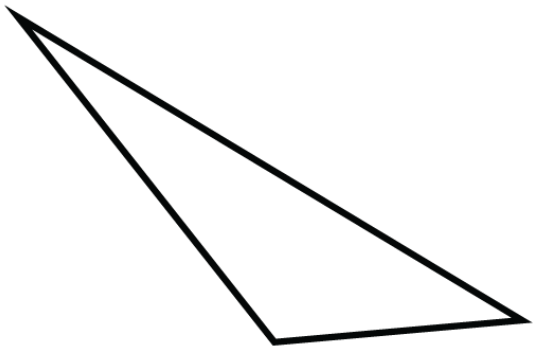
A triangle has an angle that measures $95°$. Where will an altitude be found?

Solution

Because $95° > 90°$, the triangle is obtuse and it will be outside the triangle.

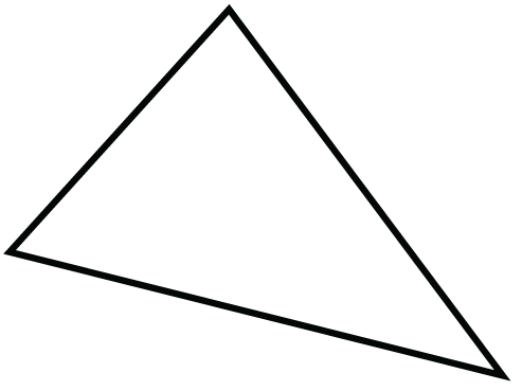
Review

Given the following triangles, tell whether the altitude is inside the triangle, outside the triangle, or at the leg of the triangle.



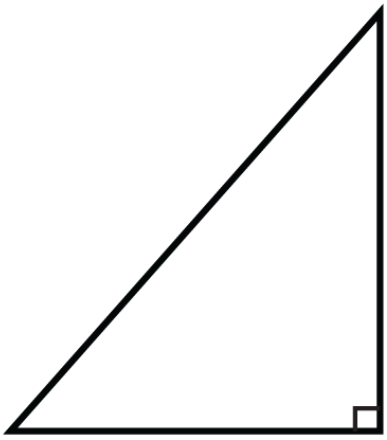
1.

Figure 4.24.4



2.

Figure 4.24.5



3.

Figure 4.24.6

4.

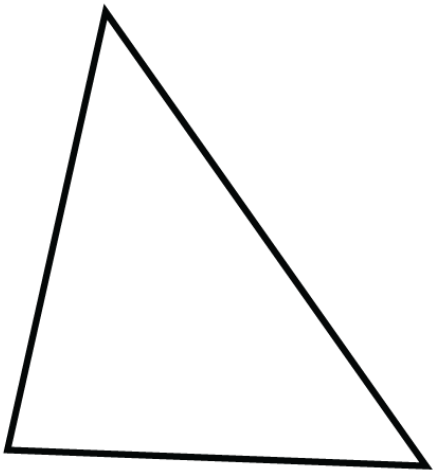


Figure 4.24.7

5.

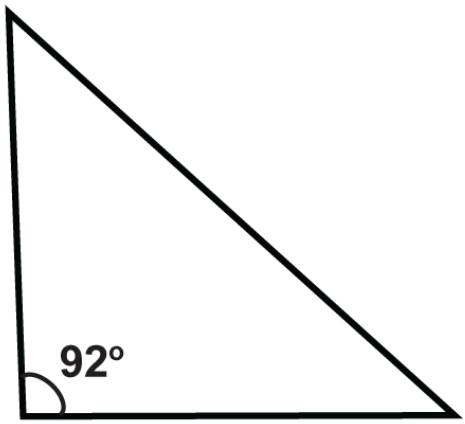


Figure 4.24.8

6. ΔJKL is an equiangular triangle.
7. ΔMNO is a triangle in which two the angles measure $30°$ and $60°$.
8. ΔPQR is an isosceles triangle in which two of the angles measure $25°$.
9. ΔSTU is an isosceles triangle in which two angles measures $45°$.

Given the following triangles, which line segment is the altitude?

10. ΔABC

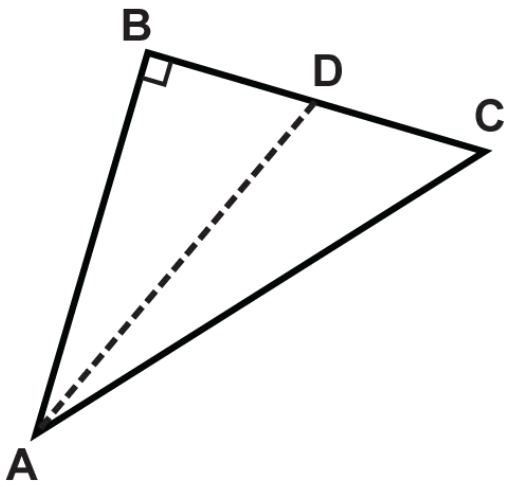


Figure 4.24.9

11. ΔEGH

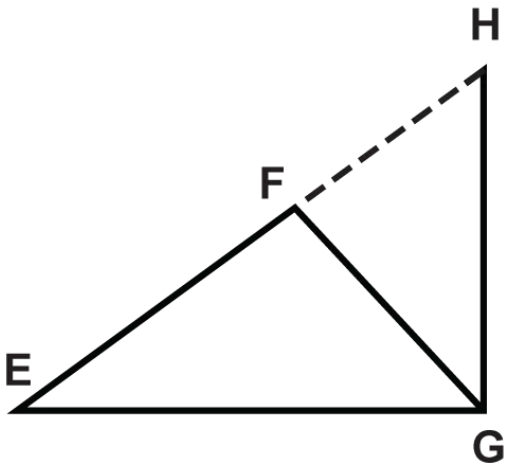


Figure 4.24.10

12. ΔIJK

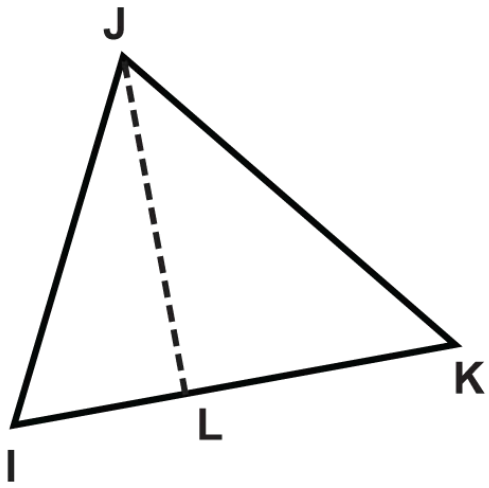


Figure 4.24.11

13. ΔMNO

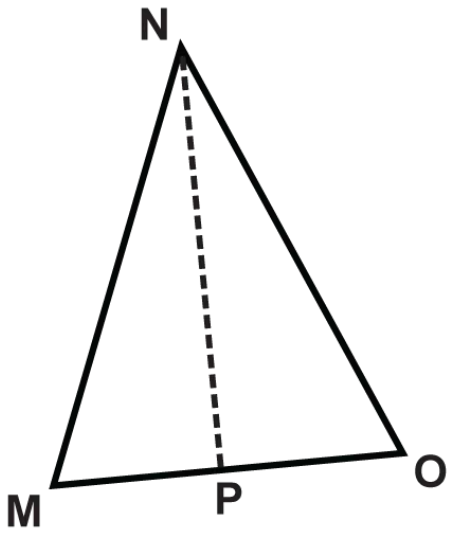


Figure 4.24.12

14. ΔRSQ

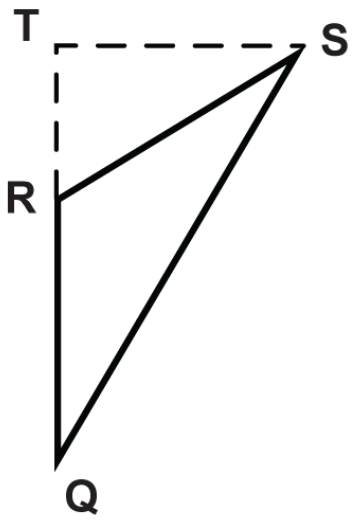


Figure 4.24.13

15. ΔUVW

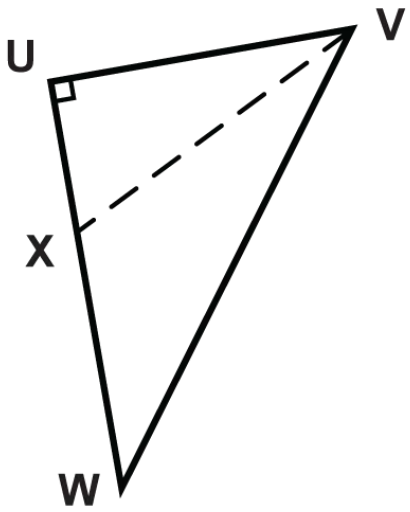


Figure 4.24.14

Resources

Review (Answers)

To see the Review answers and look for section 5.5.

Vocabulary

Term	Definition
altitude	An altitude of a triangle is a line segment from a vertex and is perpendicular to the opposite side. It is also called the height of a triangle.

Additional Resources

Video: Altitudes Principles - Basic

Activities: Altitudes Discussion Questions

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4.25: Comparing Angles and Sides in Triangles

Inequality theorems and ordering angles and sides of triangles.

SAS and SSS Inequality Theorems

Look at the triangle below. The sides of the triangle are given. Can you determine which angle is the largest? The largest angle will be opposite 18 because that is the longest side. Similarly, the smallest angle will be opposite 7, which is the shortest side.

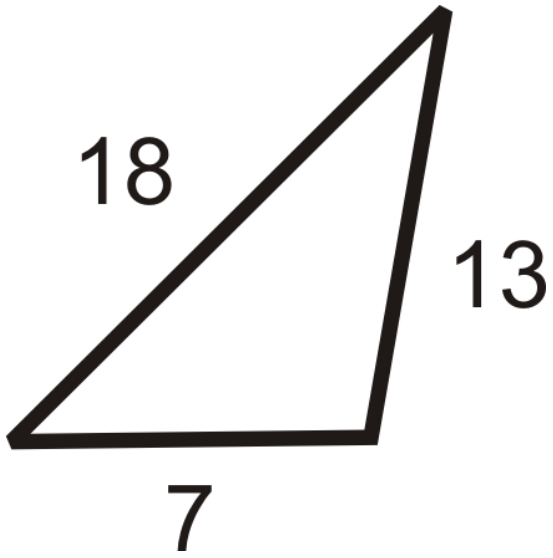


Figure **4.25.1**

This idea is actually a theorem: If one side of a triangle is longer than another side, then the angle opposite the longer side will be larger than the angle opposite the shorter side.

The converse is also true: If one angle in a triangle is larger than another angle in that triangle, then the side opposite the larger angle will be longer than the side opposite the smaller angle.

We can extend this idea into two theorems that help us compare sides and angles in **two** triangles If we have two congruent triangles ΔABC and ΔDEF, marked below:

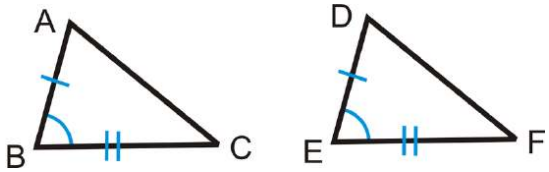


Figure **4.25.2**

Therefore, if $AB = DE$, $BC = EF$, and $m\angle B = m\angle E$, then $AC = DF$.

Now, let's make $m\angle B > m\angle E$. Would that make $AC > DF$? Yes. This idea is called the **SAS Inequality Theorem**.

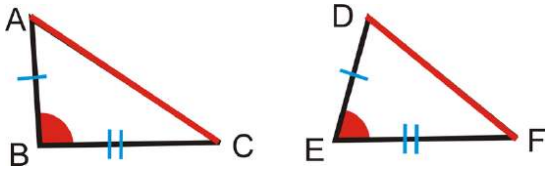


Figure **4.25.3**

The SAS Inequality Theorem: If two sides of a triangle are congruent to two sides of another triangle, but the included angle of one triangle has greater measure than the included angle of the other triangle, then the third side of the first triangle is longer than the third side of the second triangle.

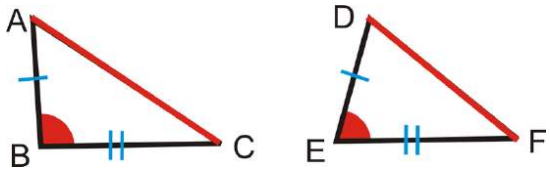


Figure 4.25.4

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $m\angle B > m\angle E$, then $\overline{AC} > \overline{DF}$.

If we know the third sides as opposed to the angles, the opposite idea is also true and is called the **SSS Inequality Theorem**.

SSS Inequality Theorem: If two sides of a triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle's two congruent sides is greater in measure than the included angle of the second triangle's two congruent sides.

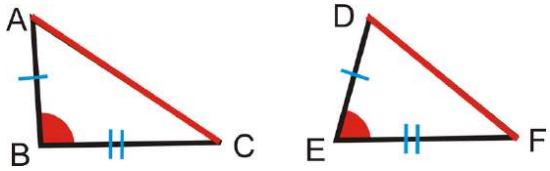


Figure 4.25.5

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{AC} > \overline{DF}$, then$m\angle B > m\angle E$.

What if you were told that a triangle has sides that measure 3, 4, and 5? How could you determine which of the triangle's angles is largest? Smallest?

Example 4.25.1

If $\overline{XM}$ is a median of ΔXYZ and $XY > XZ$, what can we say about $m\angle 1$ and $m\angle 2$?

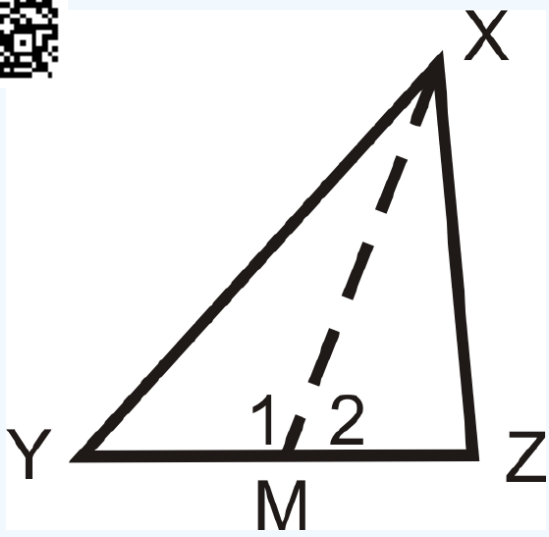


Figure 4.25.6

Solution

M is the midpoint of $\overline{YZ}$, so $YM = MZ$. $MX = MX$ by the Reflexive Property and we know $XY > XZ$.

We can use the SSS Inequality Theorem Converse to say $m\angle 1 > m\angle 2$.

Example 4.25.2

Below is isosceles triangle ΔABC. List everything you can about the sides and angles of the triangle and why.

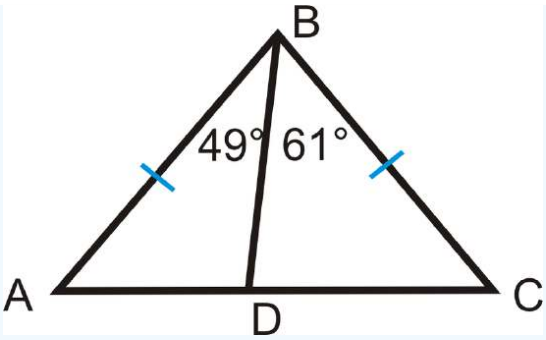


Figure 4.25.7

Solution

M is the midpoint of $\overline{YZ}$, so $YM = MZ$. $MX = MX$ by the Reflexive Property and we know $XY > XZ$.

$AB = BC$ because it is given.

$m\angle A = m\angle C$ because if sides are equal than their opposite angles must be equal..

$AD < DC$ because $m\angle ABD < m\angle CBD$ and because of the SAS Triangle Inequality Theorem.

Example 4.25.3

List the sides in order, from shortest to longest.

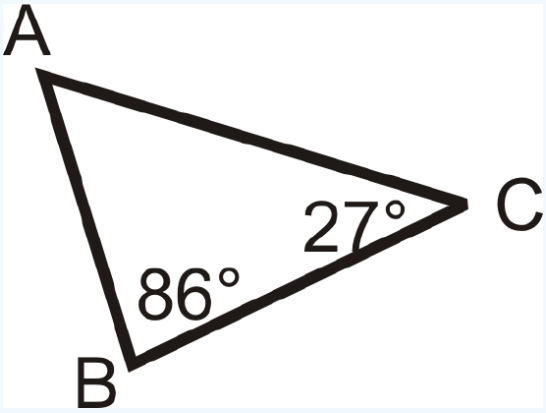


Figure 4.25.8

Solution

First, find m\angle A. From the **Triangle Sum Theorem**:

$$m\angle A + 86^\circ + 27^\circ = 180^\circ$$
$$m\angle A = 67^\circ$$

86° is the largest angle, so AC is the longest side. The next angle is 67°, so BC would be the next longest side. 27° is the smallest angle, so AB is the shortest side. In order, the answer is: AB, BC, AC.

Example 4.25.4

List the angles in order, from largest to smallest.

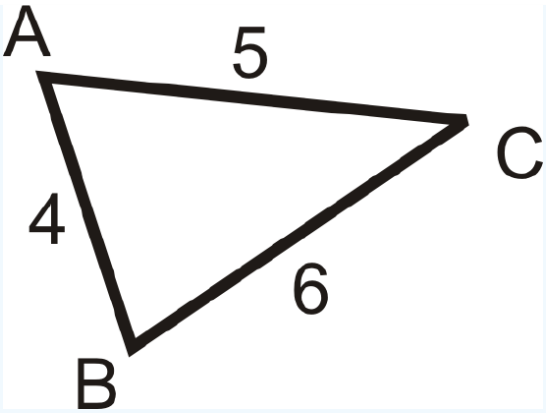


Figure 4.25.9

Solution

Just like with the sides, the largest angle is opposite the longest side. The longest side is BC, so the largest angle is $\angle A$. Next would be $\angle B$ and then $\angle C$.

Example 4.25.5

List the sides in order, from least to greatest.

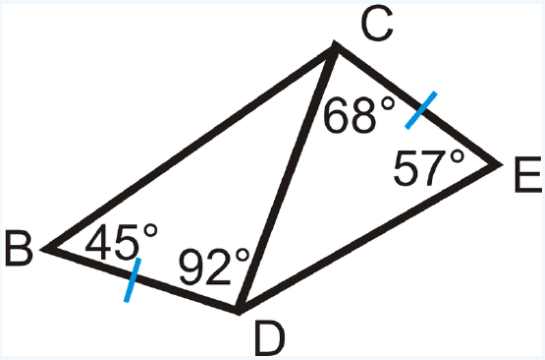


Figure 4.25.10

Solution

To solve, let's start with \Delta DCE\). The missing angle is 55°. By the theorem presented in this section, the sides, in order from least to greatest are CE, CD, and DE.

For ΔBCD, the missing angle is 43°. Again, by the theorem presented in this section, the order of the sides from least to greatest is BD, CD, and BC.

By the SAS Inequality Theorem, we know that $BC > DE$, so the order of all the sides would be: BD, CE, CD, DE, BC.

Review

For questions 1-3, list the sides in order from shortest to longest.

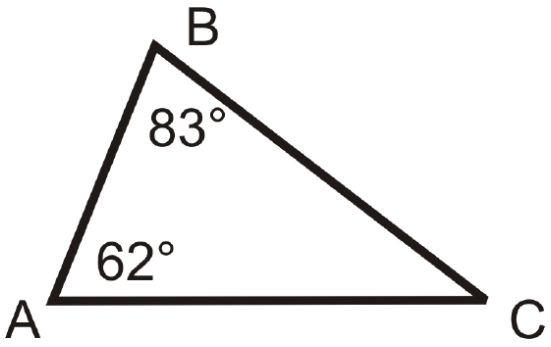
1. 

Figure **4.25.11**

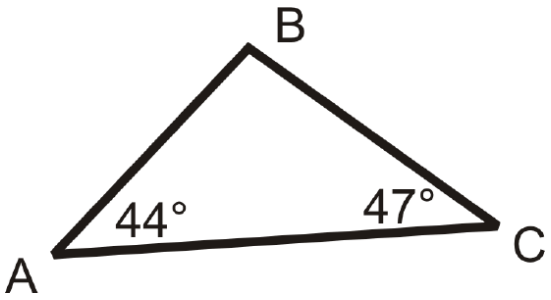
2. 

Figure **4.25.12**

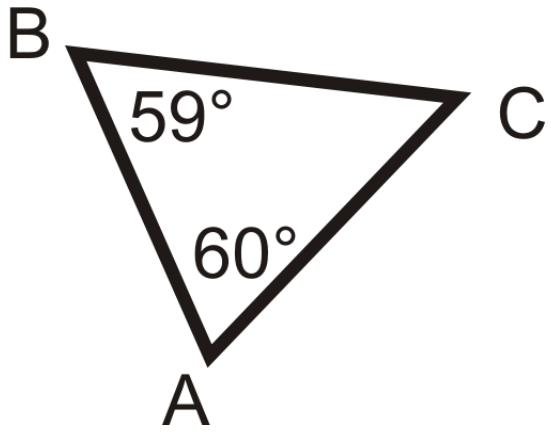
3. 

Figure **4.25.13**

For questions 4-6, list the angles from largest to smallest.

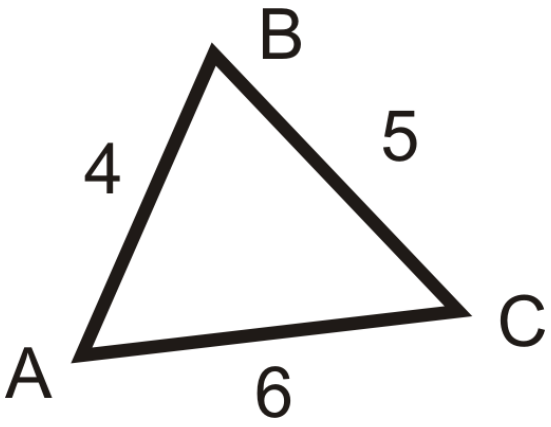
4. 

Figure **4.25.14**

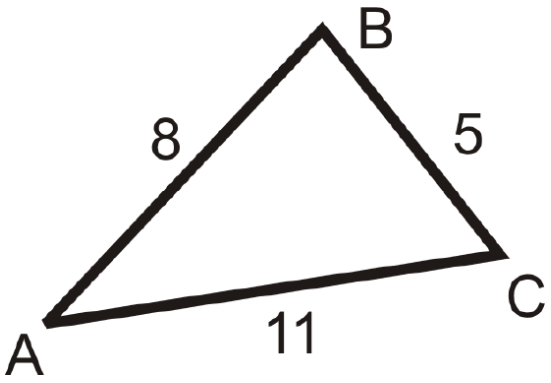
5. 

Figure 4.25.15

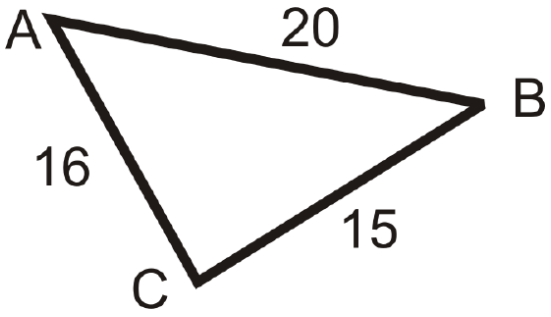
6. 

Figure 4.25.16

7. Draw a triangle with sides 3 cm, 4 cm, and 5 cm. The angle measures are $90°$, $53°$, and $37°$. Place the angle measures in the appropriate spots.
8. Draw a triangle with angle measures $56°$, $54°$ and the included side is 8 cm. What is the longest side of this triangle?
9. Draw a triangle with sides 6 cm, 7 cm, and 8 cm. The angle measures are $75.5°$, $58°$, and $46.5°$. Place the angle measures in the appropriate spots.
10. What conclusions can you draw about x?

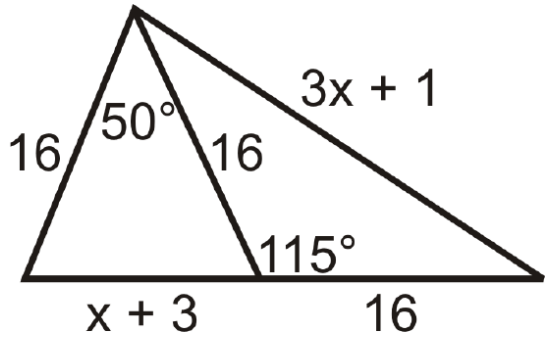


Figure 4.25.17

11. Compare $m\angle 1$ and $m\angle 2$.

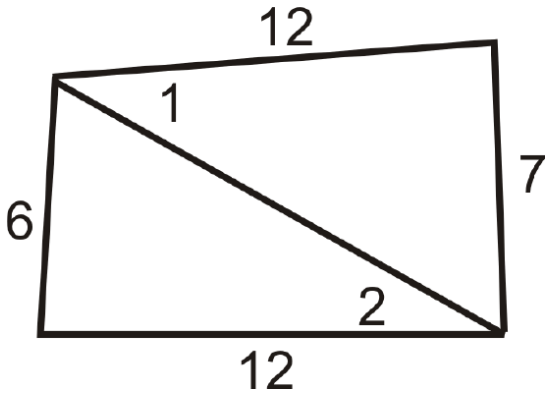


Figure 4.25.18

12. List the sides from shortest to longest.

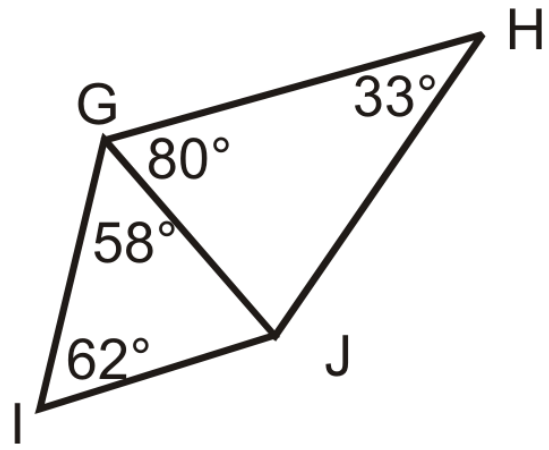


Figure 4.25.19

13. Compare $m\angle 1$ and $m\angle 2$. What can you say about $m\angle 3$ and $m\angle 4$?

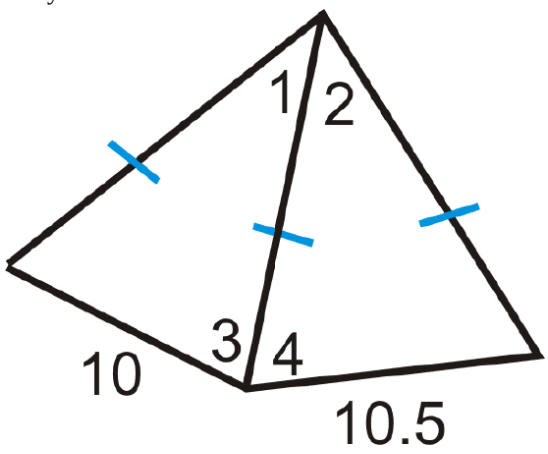


Figure 4.25.20

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.6.

Resources

Vocabulary

Term	Definition
SAS Inequality Theorem	The SAS Inequality Theorem states that if two sides of a triangle are congruent to two sides of another triangle, but the included angle of one triangle has greater measure than the included angle of the other triangle, then the third side of the first triangle is longer than the third side of the second triangle.
SSS Inequality Theorem	The SSS Inequality Theorem states that if two sides of a triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle's two congruent sides is greater in measure than the included angle of the second triangle's two congruent sides.
Triangle Sum Theorem	The Triangle Sum Theorem states that the three interior angles of any triangle add up to 180 degrees.

Additional Resources

Video: Comparing Angles & Sides in Triangles Principles - Basic

Activities: Comparing Angles and Sides in Triangles Discussion Questions

Study Aids: Inequalities in Triangles Study Guide

Practice: Comparing Angles and Sides in Triangles

Real World: Triangle Sum Theorem

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4.26: Triangle Inequality Theorem

Can any three lengths make a triangle? The answer is no. For example, the lengths 1, 2, 3 cannot make a triangle because $1 + 2 = 3$, so they would all lie on the same line. The lengths 4, 5, 10 also cannot make a triangle because $4 + 5 = 9 < 10$. Look at the pictures below:

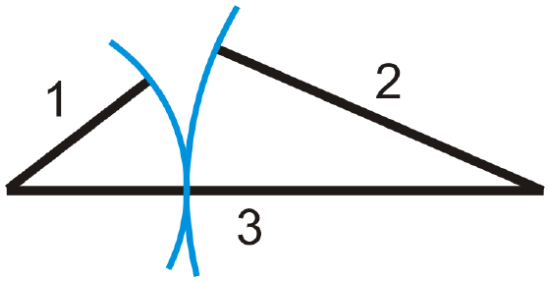

Figure 4.26.1

The arcs show that the two sides would never meet to form a triangle.

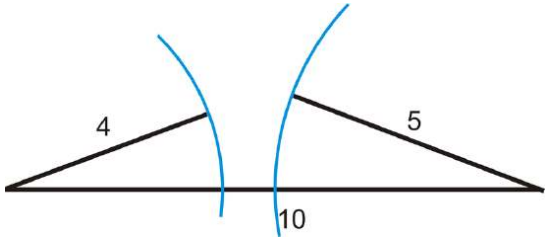

Figure 4.26.2

To make a triangle, two sides must add up to be **greater** than the third side. This is called the **Triangle Inequality Theorem.** This means that if you know two sides of a triangle, there are only certain lengths that the third side could be. If two sides have lengths a and b, then the length of the third side, s, has the range $a - b < s < a + b$.

What if you were given three lengths, like 5, 7 and 10? How could you determine if sides with these lengths form a triangle?

Example 4.26.1

Do the lengths 4.1, 3.5, ?

Solution

Use the Triangle Inequa ck to make sure that the smaller two numbers add up to be greater than the largest number.

$4.1 + 3.5 > 7.5$ and $7.6 > 7.5$ so **yes** these lengths make a triangle.

Example 4.26.2

Do the lengths 4, 4, 8 make a triangle?

Solution

Use the Triangle Inequality Theorem. Check to make sure that the smaller two numbers add up to be greater than the largest number.

$4 + 8 = 12$ and $12 > 11$ so **yes** these lengths make a triangle.

Example 4.26.3

Do the lengths 4, 11, 8 make a triangle?

Solution

Use the Triangle Inequality Theorem. Check to make sure that the smaller two numbers add up to be greater than the largest number.

$4 + 8 = 12$ and $12 > 11$ so **yes** these lengths make a triangle.

Example 4.26.4

Find the length of the third side of a triangle if the other two sides are 10 and 6.

Solution

The Triangle Inequality Theorem can also help you find the range of the third side. The two given sides are 6 and 10. The third side, s, must be between $10 - 6 = 4$ and $10 + 6 = 16$. In other words, the range of values for s is $4 < s < 16$.

Figure 4.26.3

Notice the range is no less than 4, and ***not equal*** to 4. The third side could be 4.1 because $4.1 + 6 > 10$. For the same reason, s cannot be greater than 16, but it could $15.9, 10 + 6 > 15.9$.

Example 4.26.5

The base of an isosceles triangle has length 24. What can you say about the length of each leg?

Solution

To solve this problem, remember that an isosceles triangle has two congruent sides (the legs). We have to make sure that the sum of the lengths of the legs is greater than 24. In other words, if x is the length of a leg:

$$x + x > 24$$
$$2x > 24$$
$$x > 12$$

Each leg must have a length greater than 12.

Review

Determine if the sets of lengths below can make a triangle. If not, state why.

1. 6, 6, 13
2. 1, 2, 3
3. 7, 8, 10
4. 5, 4, 3
5. 23, 56, 85
6. 30, 40, 50
7. 7, 8, 14
8. 7, 8, 15
9. 7, 8, 14.99

If two lengths of the sides of a triangle are given, determine the range of the length of the third side.

10. 8 and 9
11. 4 and 15
12. 20 and 32
13. 2 and 5

14. 10 and 8
15. x and $2x$
16. The legs of an isosceles triangle have a length of 12 each. What can you say about the length of the base?

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.7.

Resources

Vocabulary

Term	Definition
Triangle Inequality Theo	The Triangle Inequality Theorem states that in order to make a triangle, two sides must add up to be greater than the third side.

Additional Resources

Video: Triangle Inequality Theorem

Activities: Triangle Inequality Theorem Discussion Questions

Study Aids: Inequalities in Triangles Study Guide

Practice: Triangle Inequality Theorem

Real World: Triangle Inequality Theorem

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4.27: The Pythagorean Theorem

Square of the hypotenuse equals the sum of the squares of the legs in right triangles.

The two shorter sides of a **right triangle** (the sides that form the right angle) are the **legs** and the longer side (the side opposite the right angle) is the **hypotenuse**. For the Pythagorean Theorem, the legs are "a" and "b" and the hypotenuse is "c".

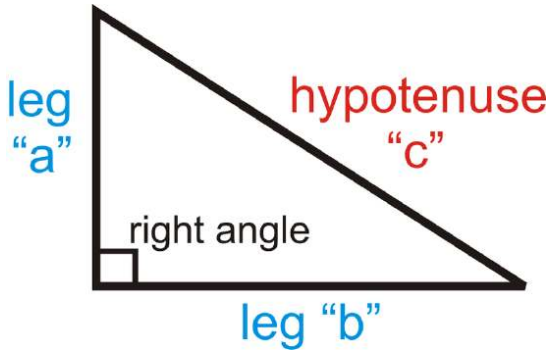

Figure 4.27.1

Pythagorean Theorem: Given a right triangle with legs of lengths a and b and a hypotenuse of length c, $a^2 + b^2 = c^2$.

The converse of the Pythagorean Theorem is also true. It allows you to prove that a triangle is a right triangle even if you do not know its angle measures.

Pythagorean Theorem Converse: If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

If $a^2 + b^2 = c^2$, then ΔABC is a right triangle.

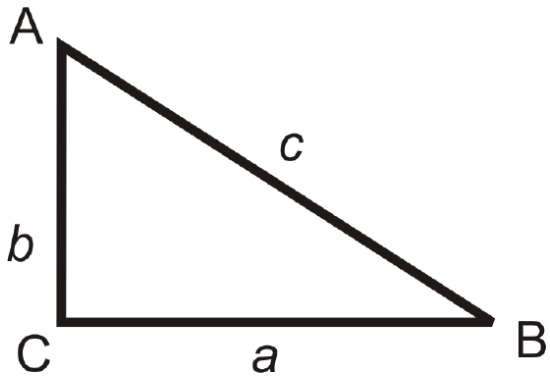

Figure 4.27.2

Pythagorean Triples

A combination of three numbers that makes the Pythagorean Theorem true is called a **Pythagorean triple**. Each set of numbers below is a Pythagorean triple.

$3, 4, 5$ $5, 12, 13$ $7, 24, 25$ $8, 15, 17$ $9, 12, 15$ $10, 24, 26$

Any multiple of a Pythagorean triple is also considered a Pythagorean triple. Multiplying 3, 4, 5 by 2 gives 6, 8, 10, which is another triple. To see if a set of numbers makes a Pythagorean triple, plug them into the Pythagorean Theorem.

What if you were told that a triangle had side lengths of 5, 12, and 13? How could you determine if the triangle were a right one?

Example 4.27.1

What is the diagonal of es 10 and 16?

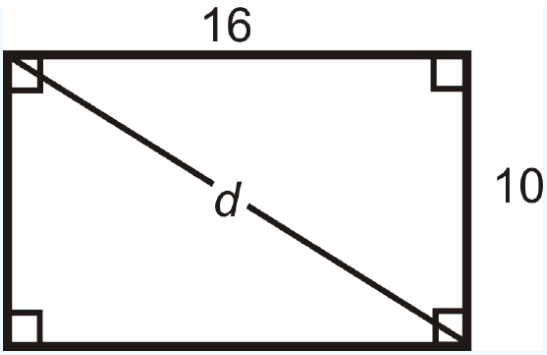

Figure 4.27.3

Solution

For any square and rectangle, you can use the Pythagorean Theorem to find the length of a diagonal. Plug in the sides to find d.

$$10^2 + 16^2 = d^2$$
$$100 + 256 = d^2$$
$$356 = d^2$$
$$d = \sqrt{356} = 2\sqrt{89} \approx 18.87$$

Example 4.27.2

Do 6, 7, and 8 make the sides of a right triangle?

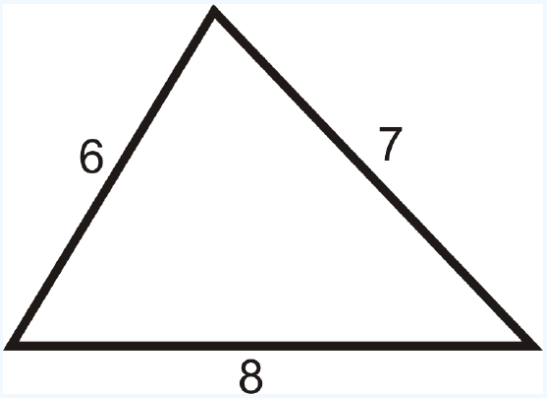

Figure 4.27.4

Solution

Plug the three numbers into the Pythagorean Theorem. Remember that the largest length will always be the hypotenuse, c. If $6^2 + 7^2 = 8^2$, then they are the sides of a right triangle.

$$6^2 + 7^2 = 36 + 49 = 85$$
$$8^2 = 64 \qquad 85 \neq 64, \textit{ so the lengths are not the sides of a right triangle.}$$

Example 4.27.3

Find the length of the hypotenuse.

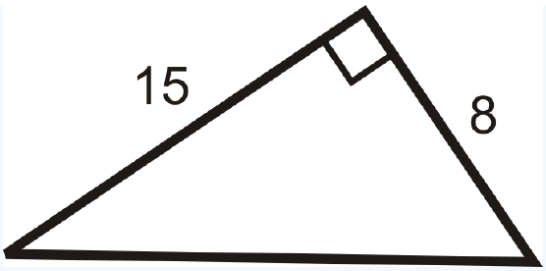

Figure 4.27.5

Solution

Use the Pythagorean Theorem. Set $a = 8$ and $b = 15$. Solve for c.

$$\begin{aligned} 8^2 + 152 &= c^2 \\ 64 + 225 &= c^2 \\ 289 &= c^2 \qquad \textit{Take the square root of both sides.} \\ 172 &= c \end{aligned}$$

Example 4.27.4

Is 20, 21, 29 a Pythagorean triple?

Solution

If $20^2 + 21^2 = 29^2$, then the set is a Pythagorean triple.

$$\begin{aligned} 20^2 + 21^2 &= 400 + 441 = 841 \\ 29^2 &= 841 \end{aligned}$$

Therefore, 20, 21, and 29 is a Pythagorean triple.

Example 4.27.5

Determine if the triangles below are right triangles.

Check to see if the three lengths satisfy the Pythagorean Theorem. Let the longest side represent c.

Solution

1.

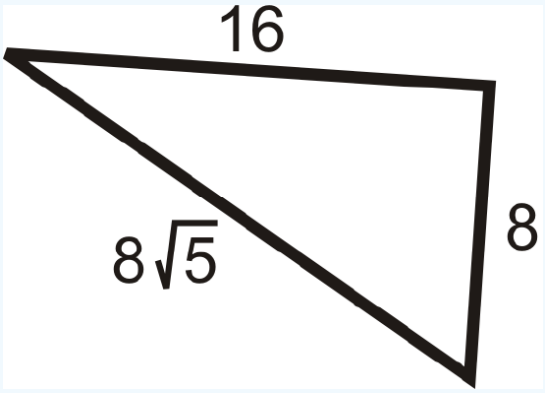

Figure 4.27.6

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 82 + 162 &\stackrel{?}{=} (8\sqrt{5})2 \\ 64 + 256 &\stackrel{?}{=} 64 \cdot 5 \\ 320 &= 320 \qquad \textit{Yes} \end{aligned}$$

2.

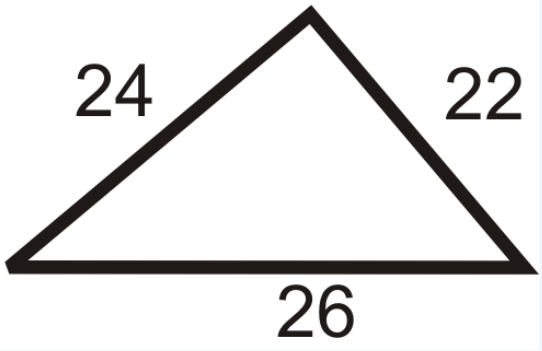

Figure 4.27.7

$$a^2 + b^2 = c^2$$
$$22^2 + 24^2 \stackrel{?}{=} 262$$
$$484 + 576 \stackrel{?}{=} 676$$
$$1060 \neq 676 \quad No$$

Review

Find the length of the missing side. Simplify all radicals.

1.

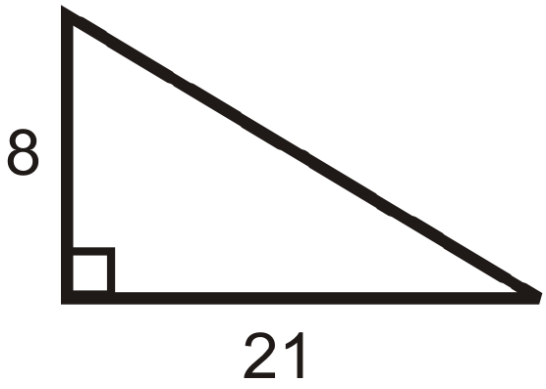

Figure 4.27.8

2.

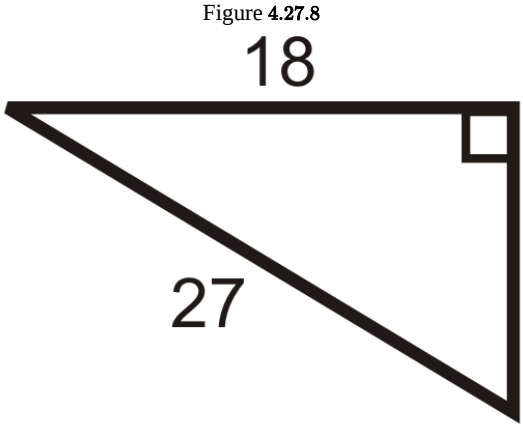

Figure 4.27.9

3.

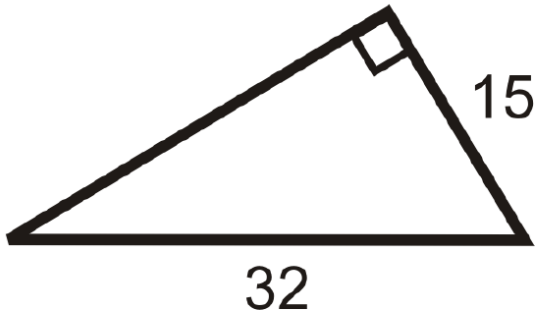

Figure 4.27.10

4.

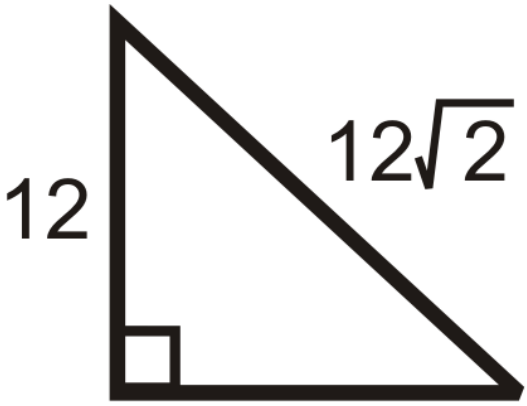

Figure 4.27.11

5.

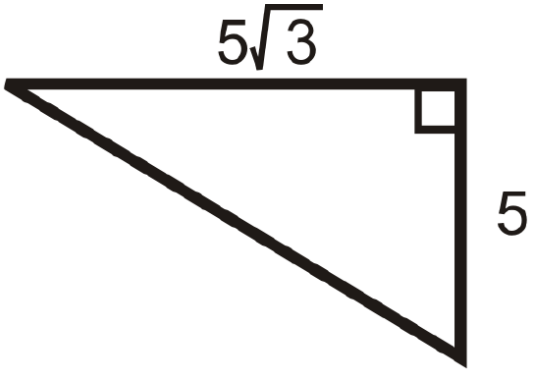

Figure 4.27.12

6.

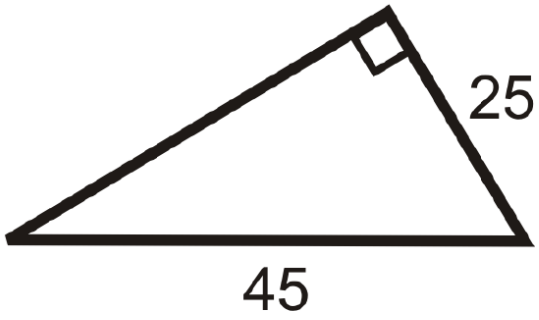

Figure 4.27.13

7. If the **legs of a right triangle** are 10 and 24, then the hypotenuse is _____.

8. If the sides of a rectangle are 12 and 15, then the diagonal is _____.
9. If the sides of a square are 16, then the diagonal is _____.
10. If the sides of a square are 9, then the diagonal is _____.

Determine if the following sets of numbers are Pythagorean Triples.

11. 12, 35, 37
12. 9, 17, 18
13. 10, 15, 21
14. 11, 60, 61
15. 15, 20, 25
16. 18, 73, 75

Determine if the following lengths make a right triangle.

17. 7, 24, 25
18. $\sqrt{5}, 2\sqrt{10}, 3\sqrt{5}$
19. $2\sqrt{3}, \sqrt{6}, 8$
20. 15, 20, 25
21. 20, 25, 30
22. $8\sqrt{3}, 6, 2\sqrt{39}$

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.2.

Vocabulary

Term	Definition
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2+b^2=c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Pythagorean triple	A combination of three numbers that makes the Pythagorean Theorem true.
Circle	A circle is the set of all points at a specific distance from a given point in two dimensions.
Conic	Conic sections are those curves that can be created by the intersection of a double cone and a plane. They include circles, ellipses, parabolas, and hyperbolas.
degenerate conic	A degenerate conic is a conic that does not have the usual properties of a conic section. Since some of the coefficients of the general conic equation are zero, the basic shape of the conic is merely a point, a line or a pair of intersecting lines.
Ellipse	Ellipses are conic sections that look like elongated circles. An ellipse represents all locations in two dimensions that are the same distance from two specified points called foci.
hyperbola	A hyperbola is a conic section formed when the cutting plane intersects both sides of the cone, resulting in two infinite “U”-shaped curves.
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Parabola	A parabola is the characteristic shape of a quadratic function graph, resembling a "U".
Pythagorean number triple	A Pythagorean number triple is a set of three whole numbers a,b and c that satisfy the Pythagorean Theorem, $a^2+b^2=c^2$.
Right Triangle	A right triangle is a triangle with one 90 degree angle.

Additional Resources

Video: Using The Pythagorean Theorem Principles - Basic

Activities: Pythagorean Theorem and Pythagorean Triples Discussion Questions

Study Aids: Pythagorean Theorem Study Guide

Practice: The Pythagorean Theorem

Real World: Pythagoras TV

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4.28: Basics of Pythagorean Theorem

Identify triples and calculate missing sides. Use indirect measurement applications

Derive and Use the Pythagorean Theorem

Figure 4.28.1

Tracy's kitten is stuck 2.8 yards above the ground on the window sill of her Grandma's house. Her grandma is not at home so she decides to rescue her terrified kitten with help from her friends. For safety reasons, the base of the ladder must be 0.7 yards away from the house and extend 1 yard above the window sill. The ladder she has borrowed is 4 yards in length.

As Tracy positions the ladder in place, she looks up and wonders if the ladder is going to be tall enough for her to rescue her kitten.

In this concept, you will learn to derive and use the **Pythagorean Theorem**.

Pythagorean Theorem

Triangles are often named according to the measure of the angles they contain. An **acute triangle** has three angles such that each of the three angles is less than $90°$. An **obtuse triangle** has two angles such that the measure of each of these angles is less than $90°$ and the measure of the third angle is greater than $90°$. A **right triangle** has one angle that has a measure of $90°$ and two acute angles such that the sum of their measures equals $90°$. The two acute angles of a right triangle are complementary – their sum equals $90°$.

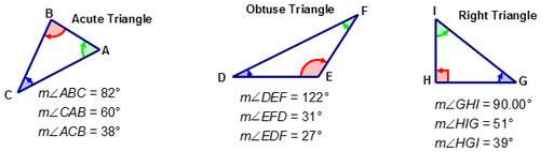

Figure 4.28.2

Of the three triangles shown in the above diagram, the right triangle is the one that is used most often in solving real world problems. This type of triangle has characteristics that are unique only to right triangles. Every triangle has three sides and three angles. The sides of a right triangle have names that apply only to a right triangle.

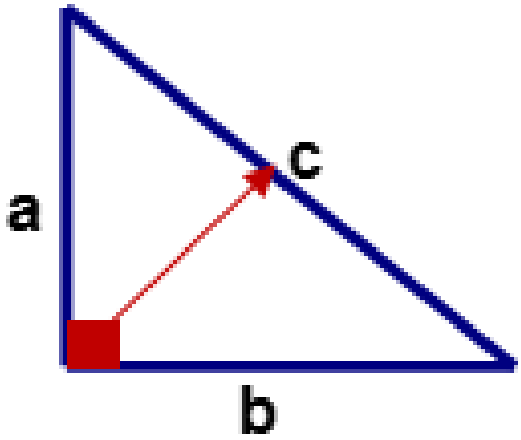

Figure 4.28.3

The two sides of the right triangle labeled ‘a’ and ‘b’ are called the **legs** of the triangle. The side of the triangle opposite the right angle (**90°**) labeled ‘c’ is called the **hypotenuse** and it is the longest side of the right triangle. The names of the legs ‘a’ and ‘b’ can be switched from one leg to the other but the hypotenuse ‘c’ must be the side opposite the right angle of the triangle.

The relationship between the hypotenuse and the **legs of a right triangle** is defined by the **Pythagorean Theorem** which states that the square of the length of the hypotenuse is equal to the sum of the length of leg ‘a’ squared and the length of leg ‘b’ squared. The Pythagorean Theorem is stated by the equation.

$$c^2 = a^2 + b^2$$

The equation representing the Pythagorean Theorem contains three pieces which are a,b and c. If you know two of these pieces, then the equation can be used to calculate the third piece. The Pythagorean Theorem can also be used to determine if a given triangle is indeed a right triangle.

Look at the triangle given below. There is no right angle marked to indicate that the triangle is a right triangle. Is the triangle a right triangle? The Pythagorean Theorem can be used to answer this question.

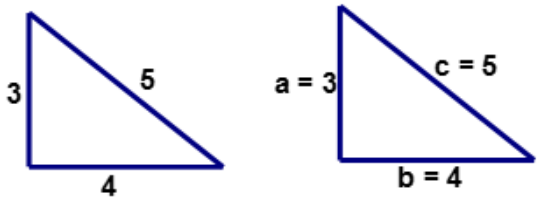

Figure 4.28.4

$$c^2 = a^2 + b^2$$

First, fill in the values for the letters a,b and c.

$$c^2 = a^2 + b^2$$
$$(5)^2 = (3)^2 + (4)^2$$

Next, perform the indicated operations on both sides of the equation.

$$(5)^2 = (3)^2 + (4)^2$$
$$(5 \times 5) = (3 \times 3) + (4 \times 4)$$
$$25 = 9 + 16$$

Then, simplify the equation by adding the values on the right side of the equation.

$$25 = 9 + 16$$
$$25 = 25$$

Both sides of the equation are equal. The values of (3, 4, and 5) resulted in \(c^2=a^2+b^2 being a true statement of equality. Therefore, the given triangle is a right triangle.

The values of (3, 4, and 5) for the positive integers (a,b,c) respectively, is a combination of numbers known as a **Pythagorean Triple** which are a set of three whole numbers that satisfy the Pythagorean Theorem $c^2 = a^2 + b^2$. Other Pythagorean Triples can be generated by multiplying the values of any known Pythagorean Triple by any positive integer. If the values (3, 4, and 5) are multiplied by 2 then another Pythagorean Triple (6, 8, and 10) is created. If the values (6, 8, and 10) are multiplied by 4 then another Pythagorean Triple (24, 32, and 40) is generated.

The newly generated values have been substituted into the Pythagorean Theorem, $c^2 = a^2 + b^2$, to ensure that they are Pythagorean Triples.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ (10)^2 &= (6)^2 + (8)^2 \\ (10 \times 10) &= (6 \times 6) + (8 \times 8) \\ 100 &= 36 + 64 \\ 100 &= 100 \end{aligned} \qquad \begin{aligned} c^2 &= a^2 + b^2 \\ (40)^2 &= (24)^2 + (32)^2 \\ (40 \times 40) &= (24 \times 24) + (32 \times 32) \\ 1600 &= 576 + 1024 \\ 1600 &= 1600 \end{aligned}$$

Each of the new Pythagorean Triples have resulted in $c^2 = a^2 + b^2$ being a true statement of equality.

If you know that a given triangle is a right triangle, then the Pythagorean Theorem can be used to calculate the length of an unknown side of the triangle when you know the lengths of two of the sides.

Let's look at an example.

For the following right triangle, calculate the length of the unknown side 'b?'

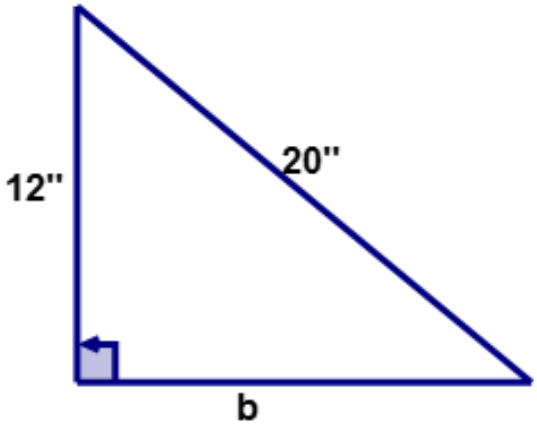

Figure 4.28.5

First, use the Pythagorean Theorem to calculate the length of side 'b.'

$$c^2 = a^2 + b^2$$

Next, fill in the values given for sides 'a' and side 'c.'

$$\begin{aligned} c^2 &= a^2 + b^2 \\ (20)^2 &= (12)^2 + b^2 \end{aligned}$$

Next, perform the indicated operations.

$$\begin{aligned} (20)^2 &= (12)^2 + b^2 \\ (20 \times 20) &= (12 \times 12) + b^2 \\ 400 &= 144 + b^2 \end{aligned}$$

Next, subtract 144 from both sides of the equation to isolate the variable and simplify both sides of the equation.

$$\begin{aligned} 400 &= 144 + b^2 \\ 400 - 144 &= 144 - 144 + b^2 \\ 256 &= b^2 \end{aligned}$$

Then, solve for 'b' by taking the square root of both sides of the equation. Remember taking the square root is the inverse operation of squaring.

$$256 = b^2$$
$$\sqrt{256} = \sqrt{b^2}$$
$$16 = b$$

The answer is 16.

The length of side 'b' is 16 inches.

Example 4.28.1

Earlier, you were given a problem about Tracy and her rescue mission. She needs to know if the ladder she is using is long enough to rescue her kitten.

Solution

First, draw and label a picture to model the problem.

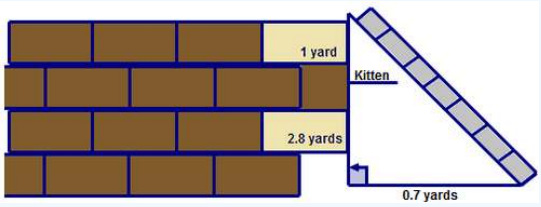

Figure 4.28.6

Next, look at the right triangle and determine the measurements of the two known sides.

$a = 3.8\ yards$

$b = 0.7\ yards$

Next, use the Pythagorean Theorem to calculate the length of the ladder which is the hypotenuse of the right triangle.

$$c^2 = a^2 + b^2$$

First, fill in the values for 'a' and 'b.'

$$c^2 = a^2 + b^2$$
$$c^2 = (3.8)^2 + (0.7)^2$$

Next, perform the indicated operations and simplify the equation.

$$c^2 = (3.8 \times 3.8) + (0.7 \times 0.7)$$
$$c^2 = 14.44 + 0.49$$
$$c^2 = 14.93\ yards$$

Then, solve for 'c' by taking the square root of each side of the equation.

$$c^2 = 14.93$$
$$\sqrt{c^2} = \sqrt{14.93}$$
$$c = 3.86$$

The answer is 3.86.

The ladder must be 3.86 yards in length to rescue the kitten. The ladder Tracy is using is long enough.

Example 4.28.2

Use the Pythagorean Theorem to calculate the length of the missing side of the following right triangle:

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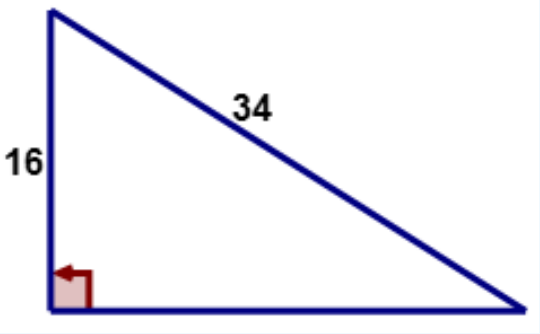

Figure 4.28.7

Solution

First, determine what the given values represent and what side is missing?

$$a = 16$$
$$b = ?$$
$$c = 34$$

Next, substitute the given values into equation that represents the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$(34)^2 = (16)^2 + b^2$$

Next, perform the indicated operations on both sides of the equation.

$$(34)^2 = (16)^2 + b^2$$
$$(34 \times 34) = (16 \times 16) + b^2$$
$$1156 = 256 + b^2$$

Next, subtract 256 from both sides of the equation to isolate the variable.

$$1156 = 256 + b^2$$
$$1156 - 256 = 256 - 256 + b^2$$

Next, simplify the equation.

$$1156 - 256 = 256 - 256 + b^2$$
$$900 = b^2$$

Then, take the square root of both sides of the equation to solve for the variable 'b.'

$$900 = b^2$$
$$\sqrt{900} = \sqrt{b^2}$$
$$30 = b$$

The answer is $b = 30$.

Example 4.28.3

Find the length of the hypotenuse of a right triangle given the lengths of legs 'a' and ''b' are 14 and 48 centimeters, respectively.

Solution

First, draw and label a right triangle to represent the given information.

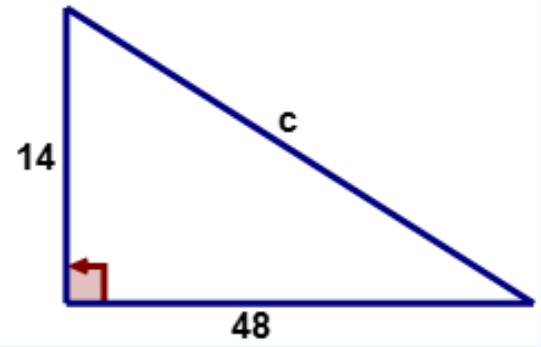

Figure 4.28.8

Next, substitute the given values into equation that represents the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = (14)^2 + (48)^2$$

Next, perform the indicated operations and simplify the equation.

$$c^2 = (14)^2 + (48)^2$$
$$c^2 = (14 \times 14) + (48 \times 48)$$
$$c^2 = 196 + 2304$$

Next, simplify the equation by adding the values on the right side of the equation.

$$c^2 = 196 + 2304$$
$$c^2 = 2500$$

Then, take the square root of both sides of the equation to solve for the variable 'c.'

$$c^2 = 2500$$
$$\sqrt{c^2} = 2500$$
$$c = 50$$

The answer is $c = 50$.

The length of the hypotenuse is 50 cm.

Example 4.28.4

If 'a' and 'b' are values such that $a,\ b \in N$ and $a > b$ then the sides of a right triangle are $a^2 + b^2$, $a^2 - b^2$, and $2ab$ as shown in the diagram below.

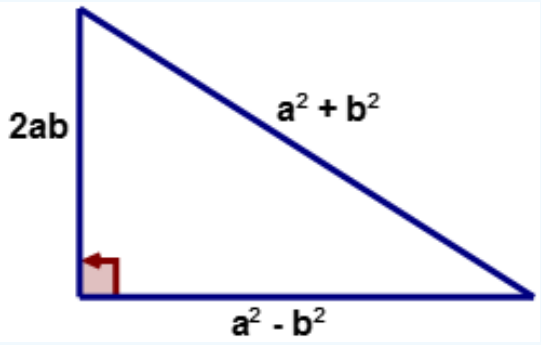

Figure 4.28.9

Solution

Complete the following table to generate Pythagorean Triples.

\(av	b	a^2-b^2	$2ab$	a^2+b^2
4	2	12	16	20
5	3	16	30	34
6	4	20	48	52

First, using $a=4 and \(b=2$ evaluate a^2-b^2 by substituting the given values into the expression.

$$a^2-b^2$$
$$(4)^2-(2)^2$$

Next, perform the indicated operations and simply the expression.

$$(4)^2-(2)^2$$
$$(4\times 4)-(2\times 2)$$
$$16-4$$
$$12$$

The answer is 12.

Then, place the answer in the table in the correct column.

Now, complete the a^2-b^2 column for the remaining given values of 'a' and 'b.'

First, using $a=5$ and $b=3$ evaluate a^2-b^2 by substituting the given values into the expression.

$$a^2-b^2$$
$$(5)^2-(3)^2$$

Next, perform the indicated operations and simply the expression.

$$(5)^2-(3)^2$$
$$(5\times 5)-(3\times 3)$$
$$25-9=16$$

The answer is 16.

Then, place the answer in the table in the correct column.

First, using $a=6$ and $b=4$ evaluate a^2-b^2 by substituting the given values into the expression.

$$a^2-b^2$$
$$(6)^2-(4)^2$$

Next, perform the indicated operations and simply the expression.

$$(6)^2-(4)^2$$
$$(6\times 6)-(4\times 4)$$
$$36-16$$
$$20$$

The answer is 20.

Then, place the answer in the table in the correct column.

First, using $a=4$ and $b=2$ evaluate $2ab$ by substituting the given values into the expression.

$$2ab$$

$$2(4)(2)$$

Next, perform the indicated operations.

$$2(4)(2)$$

$$8(2)$$

$$16$$

The answer is 16.

Then, place the answer in the table in the correct column.

Now, complete the $2ab$ column for the remaining given values of 'a' and 'b.'

First, using $a = 5$ and $b = 3$ evaluate $2ab$ by substituting the given values into the expression.

$$2ab$$
$$2(5)(3)$$

Next, perform the indicated operations.

$$2(5)(3)$$
$$10(3)$$
$$30$$

The answer is 30.

Then, place the answer in the table in the correct column.

First, using $a = 6$ and $b = 4$ evaluate $2ab$ by substituting the given values into the expression.

$$2ab$$
$$2(6)(4)$$

Next, perform the indicated operations.

$$2(6)(4)$$
$$12(4)$$
$$48$$

The answer is 48.

Then, place the answer in the table in the correct column.

First, using $a = 4$ and $b = 2$ evaluate $a^2 + b^2$ by substituting the given values into the expression.

$$a^2 + b^2$$
$$(4)^2(2)^2$$

Next, perform the indicated operations and simply the expression.

$$(4)^2 + (2)^2$$
$$(4 \times 4) + (2 \times 2)16 + 4$$
$$20$$

The answer is 20.

Then, place the answer in the table in the correct column.

Now, complete the $a^2 + b^2$ column for the remaining given values of 'a' and 'b.'

First, using $a = 5$ and $b = 3$ evaluate $a^2 + b^2$ by substituting the given values into the expression.

$$a^2 + b^2$$
$$(5)^2 + (3)^2$$

Next, perform the indicated operations and simply the expression.

$$(5)^2 + (3)^2$$
$$(5 \times 5) + (3 \times 3)$$
$$25 + 9$$
$$34$$

The answer is 34.

Then, place the answer in the table in the correct column.

First, using $a = 6$ and $b = 4$ evaluate $a^2 + b^2$ by substituting the given values into the expression.

$$a^2 + b^2$$

$$(6)^2 + (4)^2$$

Next, perform the indicated operations and simply the expression.

$$(6)^2 + (4)^2$$
$$(6 \times 6) + (4 \times 4)$$
$$36 + 16$$
$$52$$

The answer is 52.

Then, place the answer in the table in the correct column.

Example 4.28.5

Use the Pythagorean Theorem to find the length of the missing leg of the following right triangle.

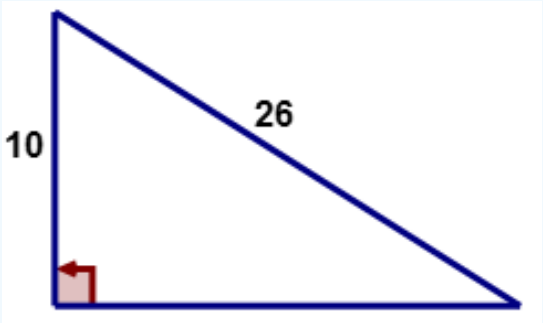

Figure 4.28.10

Solution

First, determine what the given values represent and what side is missing?

$$a = ?$$
$$b = 10$$
$$c = 26$$

Next, substitute the given values into equation that represents the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$(26)^2 = a^2 + (10)^2$$

Next, perform the indicated operations and simplify the equation.

$$(26)^2 = a^2 + (10)^2$$
$$(26 \times 26) = a^2 + (10 \times 10)$$
$$676 = a^2 + 100$$

Next, subtract 100 from both sides of the equation to isolate the variable.

$$676 = a^2 + 100$$
$$676 - 100 = a^2 + 100 - 100$$

Next, simplify the equation.

$$676 - 100 = a^2 + 100 - 100$$
$$576 = a^2$$

Then, take the square root of both sides of the equation to solve for the variable 'a.'

$$576 = a^2$$
$$\sqrt{576} = \sqrt{a^2}$$
$$24a$$

The answer is $a = 24$.

Review

Use the Pythagorean Theorem to find the missing dimensions of each right triangle.

1. $a = 3,\ b = 4,\ c = ?$
2. $a = 6,\ b = 8,\ c = ?$
3. $a = 9,\ b = 12,\ c = ?$
4. $a = 27,\ b = 36,\ c = ?$
5. $a = 15,\ b = 20,\ c = ?$
6. $a = 18,\ b = 24,\ c = ?$
7. $a = ?,\ b = 16,\ c = 20$
8. $a = ?,\ b = 28,\ c = 35$
9. $a = 30,\ b = ?,\ c = 50$
10. $a = 33,\ b = ?,\ c = 55$
11. $a = 1.5,\ b = ?,\ c = 2.5$
12. $a = 36,\ b = ?,\ c = 60$

State whether the following statements are True or False.

13. The Pythagorean Theorem will work for any triangle.
14. The longest side of a right triangle is called the hypotenuse.
15. A Pythagorean Triple can only be found in a right triangle.

Vocabulary

Term	Definition
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Pythagorean Triple	A Pythagorean Triple is a set of three whole numbers a,b and c that satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$.
Right Triangle	A right triangle is a triangle with one 90 degree angle.

Additional Resources

Video: Pythagorean Theorem 2

Practice: Basics of Pythagorean Theorem

Real World: Measuring the World

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4.29: Pythagorean Theorem to Classify Triangles

Lengths of triangle sides using the Pythagorean Theorem to classify triangles as obtuse, acute or right.

While painting a wall in your home one day, you realize that the wall you are painting seems "tilted", as though it might fall over. You realize that if the wall is standing upright, the angle between the wall and the floor is ninety degrees. After a few careful measurements, you find that the distance from the bottom of the ladder to the wall is 3 feet, the top of the ladder is at a point 10 feet up on the wall, and the ladder is 12 feet long. Can you determine if the wall is still standing upright, or if it is starting to lean?

Classifying Triangles by Using the Pythagorean Theorem

We can use the Pythagorean Theorem to help determine if a triangle is a **right triangle**, if it is acute, or if it is obtuse.

To help you visualize this, eral triangle with sides of length 5. We know that this is an **acute triangle**. If you plug in 5 for each number in th rem we get $5^2 + 5^2 = 5^2$ and $50 > 25$. Therefore, if $a^2 + b^2 > c^2$, then lengths a, b, and c make up an acute triangl $a^2 + b^2 < c^2$, then lengths a, b, and c make up the sides of an **obtuse triangle**. *It is important to note that the s the longest.*

Using the Pythagorean Theorem

Determine if the following lengths make an acute, right or obtuse triangle.

1. 5, 6, 7

Plug in each set of lengths into the Pythagorean Theorem.

$$5^2 + 6^2 ? 7^2$$

$$25 + 36 ? 49$$

$$61 > 49$$

Because $61 > 49$, this is an acute triangle.

2. 5, 10, 14

Plug in each set of lengths into the Pythagorean Theorem.

$$5^2 + 10^2 ? 14^2$$

$$25 + 100 ? 196$$

$$125 < 196$$

Because $125 < 196$, this is an obtuse triangle.

3. 12, 35, 37

Plug in each set of lengths into the Pythagorean Theorem.

$$12^2 + 35^2 ? 37^2$$

$$144 + 1225 ? 1369$$

$$1369 = 1369$$

Because the two sides are equal, this is a right triangle.

NOTE: All of the lengths in the above problem represent the lengths of the sides of a triangle. Recall the Triangle Inequality Theorem from geometry which states: The length of a side in a triangle is less than the sum of the other two sides. For example, 4, 7 and 13 cannot be the sides of a triangle because $4 + 7$ is not greater than 13.

Example 4.29.1

Earlier, you were given a problem asking if the wall is still standing upright, or if it is starting to lean.

Solution

The ladder is making a triangle with the floor as one side, the wall as another, and the ladder itself serves as the hypotenuse. To see if the wall is leaning, you can determine the type of triangle that is made with these lengths (right, acute, or obtuse). If the triangle is a right triangle, then the wall is standing upright. Otherwise, it is leaning.

Plugging the lengths of the sides into the Pythagorean Theorem:

$$3^2 + 10^2 ? 12^2$$

$$9 + 100 ? 144$$

$$109 < 144$$

Yes, you were right. Because $109 < 144$, this is an obtuse triangle. The wall is leaning with an angle greater than ninety degrees.

Example 4.29.2

Determine if the following lengths make an acute, right or obtuse triangle.

8, 15, 20

Solution

Plug in each set of lengths into the Pythagorean Theorem.

$$8^2 + 15^2 ? 20^2$$

$$64 + 225 ? 400$$

$$289 < 400$$

Because $289 < 400$, this is an obtuse triangle.

Example 4.29.3

Determine if the following lengths make an acute, right or obtuse triangle.

15, 22, 25

Solution

Plug in each set of lengths into the Pythagorean Theorem.

$$15^2 + 22^2 ? 25^2$$

$$225 + 484 ? 625$$

$$709 > 625$$

Because $709 > 625$, this is an acute triangle.

Review

Determine if each of the following lengths make a right triangle.

1. $9, 40, 41$.
2. $12, 24, 26$.
3. $5, 10, 14$.
4. $3, 3\sqrt{3}, 6$.

Determine if the following lengths make an acute, right or obtuse triangle.

5. $10, 15, 18$.

6. $4, 20, 21$.
7. $15, 16, 17$.
8. $15, 15, 15\sqrt{2}$.
9. $12, 17, 19$.
10. $3, 4, 5$.
11. $12, 12\sqrt{3}, 24$.
12. $2, 4, 5$.
13. $3, 5, 7$.
14. Explain why if $a^2 + b^2 < c^2$ then the triangle is obtuse.
15. Explain why if $a^2 + b^2 > c^2$ then the triangle is acute.

Vocabulary

Term	Definition
Acute Triangle	An acute triangle has three angles that each measure less than 90 degrees.
Obtuse Triangle	An obtuse triangle is a triangle with one angle that is greater than 90 degrees.
Right Triangle	A right triangle is a triangle with one 90 degree angle.

Additional Resources

Video: The Pythagorean Theorem and The Converse of the Pythagorean Theorem

Practice: Pythagorean Theorem to Classify Triangles

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4.30: Pythagorean Triples

Integer triples that make right triangles.

While working as an architect's assistant, you're asked to utilize your knowledge of the Pythagorean Theorem to determine if the lengths of a particular triangular brace support qualify as a **Pythagorean Triple**. You measure the sides of the brace and find them to be 7 inches, 24 inches, and 25 inches. Can you determine if the lengths of the sides of the triangular brace qualify as a Pythagorean Triple?

Pythagorean Triples

Pythagorean Triples are sets of whole numbers for which the Pythagorean Theorem holds true. The most well-known triple is 3, 4, 5. This means that 3 and 4 are the lengths of the legs and 5 is the hypotenuse. *The largest length is always the hypotenuse*. If we were to multiply any triple by a constant, this new triple would still represent sides of a right triangle. Therefore, 6, 8, 10 and 15, 20, 25, among countless others, would represent sides of a right triangle.

Identifying Pythagorean

Determine if the following orean Triples.

1. 7, 24, 25

Plug the given numbers in Theorem.

$$7^2 + 24^2 \stackrel{?}{=} 25^2$$

$$49 + 576 = 625$$

$$625 = 625$$

Yes, 7, 24, 25 is a Pythagorean Triple and sides of a right triangle.

2. 9, 40, 41

Plug the given numbers into the Pythagorean Theorem.

$$9^2 + 40^2 \stackrel{?}{=} 41^2$$

$$81 + 1600 = 1681$$

$$1681 = 1681$$

Yes, 9, 40, 41 is a Pythagorean Triple and sides of a right triangle.

3. 11, 56, 57

Plug the given numbers into the Pythagorean Theorem.

$$11^2 + 56^2 \stackrel{?}{=} 57^2$$

$$121 + 3136 = 3249$$

$$3257 \neq 3249$$

No, 11, 56, 57 do not represent the sides of a right triangle.

Example 4.30.1

Earlier, you were asked to utilize your knowledge of the Pythagorean Theorem to determine if the lengths of a particular triangular brace support qualify as a Pythagorean Triple.

Solution

Since you know that the sides of the brace have lengths of 7, 24, and 25 inches, you can substitute these values in the Pythagorean Theorem. If the Pythagorean Theorem is satisfied, then you know with certainty that these are indeed sides of a triangle with a right angle:

$$7^2 + 24^2 \stackrel{?}{=} 25^2$$

$$49 + 576 = 625$$

$$625 = 625$$

The Pythagorean Theorem is satisfied with these values as a lengths of sides of a right triangle. Since each of the sides is a whole number, this is indeed a set of Pythagorean Triples.

Example 4.30.2

Determine if the following lengths are Pythagorean Triples.

5, 10, 13

Solution

Plug the given numbers into the Pythagorean Theorem.

$$5^2 + 10^2 \stackrel{?}{=} 13^2$$

$$25 + 100 = 169$$

$$125 \neq 169$$

No, 5, 10, 13 is not a Pythagorean Triple and not the sides of a right triangle.

Example 4.30.3

Determine if the following lengths are Pythagorean Triples.

8, 15, 17

Solution

Plug the given numbers into the Pythagorean Theorem.

$$8^2 + 15^2 \stackrel{?}{=} 17^2$$

$$64 + 225 = 289$$

$$289 = 289$$

Yes, 8, 15, 17 is a Pythagorean Triple and sides of a right triangle.

Example 4.30.4

Determine if the following lengths are Pythagorean Triples.

11, 60, 61

Solution

Plug the given numbers into the Pythagorean Theorem.

$$11^2 + 60^2 \stackrel{?}{=} 61^2$$

$$121 + 3600 = 3721$$

$$3721 = 3721$$

Yes, 11, 60, 61 is a Pythagorean Triple and sides of a right triangle.

Review

1. Determine if the following lengths are Pythagorean Triples: 9, 12, 15.
2. Determine if the following lengths are Pythagorean Triples: 10, 24, 36.

3. Determine if the following lengths are Pythagorean Triples: 4, 6, 8.
4. Determine if the following lengths are Pythagorean Triples: 20, 99, 101.
5. Determine if the following lengths are Pythagorean Triples: 21, 99, 101.
6. Determine if the following lengths are Pythagorean Triples: 65, 72, 97.
7. Determine if the following lengths are Pythagorean Triples: 15, 30, 62.
8. Determine if the following lengths are Pythagorean Triples: 9, 39, 40.
9. Determine if the following lengths are Pythagorean Triples: 48, 55, 73.
10. Determine if the following lengths are Pythagorean Triples: 8, 15, 17.
11. Determine if the following lengths are Pythagorean Triples: 13, 84, 85.
12. Determine if the following lengths are Pythagorean Triples: 15, 16, 24.
13. Explain why it might be useful to know some of the basic Pythagorean Triples.
14. Prove that any multiple of 5, 12, 13 will be a Pythagorean Triple.
15. Prove that any multiple of 3, 4, 5 will be a Pythagorean Triple.

Vocabulary

Term	Definition
Pythagorean Triple	A Pythagorean Triple is a set of three whole numbers a,b and c that satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$.

Additional Resources

Video: **Pythagorean Triples**

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4.31: Converse of the Pythagorean Theorem

Identify right triangles using the theorem.

Derive and Use the Converse of the Pythagorean Theorem

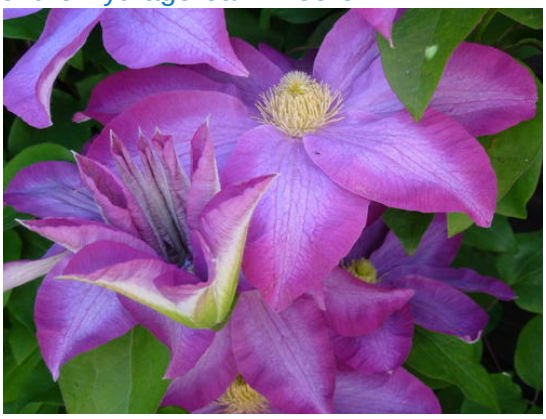

Figure **4.31.1**

Brandon has just purchased his first home and is very anxious to plant a flower garden in a triangular section of the backyard. Brandon has inherited a metal fence that his grandfather had made many years ago and he would like to use it in his garden. The fence consists of two pieces that are welded together to form a right angle and another long straight piece. He knows that the pieces of fence will fit in the garden if his triangular garden is a **right triangle**.

"If only I could use the garden measurements of 29 feet, 20 feet and 29 feet to figure out if my garden space forms a right triangle, then I could use Grandpa's fence." he thought to himself.

In this concept, you will learn to derive and use the converse of the **Pythagorean Theorem**.

The Converse of the Pythagorean Theorem

The **Pythagorean Theorem** states that for a right triangle the square of the length of the **hypotenuse** is equal to the sum of the squares of the lengths of the other two sides. This theorem can be modeled by the equation $c^2 = a^2 + b^2$ where 'c' represents the length of the hypotenuse, 'a' represents the length of one leg and 'b' represents the length of the other leg.

The Pythagorean Theorem can be used to calculate the length of an unknown side of a right triangle. The **Converse of the Pythagorean Theorem** states that if the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite the longest side is a right angle. A triangle that contains a right angle is a right triangle.

Let's look at the converse of the Pythagorean Theorem in action.

Determine whether or not the following triangles (which may not be drawn to scale) are right triangles. If the triangle is a right triangle, make any necessary changes to the triangle and draw it correctly.

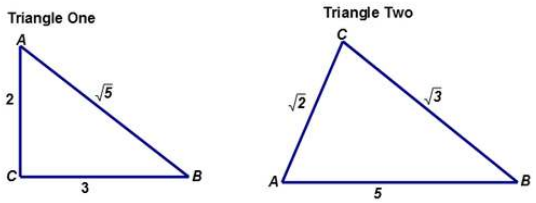

Figure **4.31.2**

Let's begin with Triangle One.

First, determine the longest side of the triangle.

The lengths of the sides are 2, 3 and $\sqrt{5}$. The length of side BC is $\sqrt{5} \approx 2.24$ *units*. The longest side of the triangle is AB which will be the hypotenuse of the triangle if ΔABC is a right triangle.

Next, substitute the values $(2, \sqrt{5}, 3)$ for (a, b, c) respectively into the equation $c^2 = a^2 + b^2$.

$$c^2 = a^2 + b^2$$
$$(3)^2 = (2)^2 + (\sqrt{5})^2$$

Next, square each of the terms indicated in the equation.

$$(3)^2 = (2)^2 + (\sqrt{5})^2$$
$$(3 \times 3) = (2 \times 2) + (\sqrt{5} \times \sqrt{5})$$

Next, evaluate each of the squares.

$$(3 \times 3) = (2 \times 2) + (\sqrt{5} \times \sqrt{5})$$
$$9 = 4 + 5$$

Then, simplify the right side of the equation by adding the numbers.

$$9 = 4 + 5$$
$$9 = 9$$

Both sides of the equation are equal. The square of the length of one side of the triangle is equal to the sum of the squares of the lengths of the other two sides. Triangle One is a right triangle. The length of BC should be 3 and the length of AB should be $\sqrt{5}$ since the right angle is located at C.

Let's look at Triangle Two.

First, determine the longest side of the triangle.

The lengths of the sides are $\sqrt{2}, \sqrt{3}$ and 5. The length of side AC is $\sqrt{2}, approximately 1.41 units in length. The length of side \(BC$ is $\sqrt{3}$, approximately 1.73 units in length. The longest side of the triangle is AB which will be the hypotenuse of the triangle if ΔABC is a right triangle.

Next, substitute the values $(\sqrt{2}, \sqrt{3}, 5)$ for (a, b, c) respectively into the equation $c^2 = a^2 + b^2$.

$$c^2 = a^2 + b^2$$
$$(5)^2 = (\sqrt{2})^2 + (\sqrt{3})^2$$

Next, square each of the terms indicated in the equation.

$$(5)^2 = (\sqrt{2})^2 + (\sqrt{3})^2$$
$$(5 \times 5) = (\sqrt{2} \times \sqrt{2}) + (\sqrt{3} \times \sqrt{3})$$

Next, evaluate each of the squares.

$$(5 \times 5) = (\sqrt{2} \times \sqrt{2}) + (\sqrt{3} \times \sqrt{3}$$
$$25 = 2 + 3$$

Then, simplify the right side of the equation by adding the numbers.

$$25 = 2 + 3$$
$$25 \neq 5$$

Both sides of the equation are not equal. The square of the length of one side of the triangle is NOT equal to the sum of the squares of the lengths of the other two sides. Triangle Two is not a right triangle.

Example 4.31.1

Earlier, you were given a problem about Brandon's flower garden and his metal fence. He needs to figure out if his garden space is a right triangle. How can Brandon figure this out using the measurements of the space?

Solution

First, determine the values for \((a,b,c) of a right triangle using the measurements of 29, 20 and 21 feet.

The longest side will represent 'c' the hypotenuse.

$a = 20$

$b = 21$

$c = 29$

Next, substitute the given values into the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$(29)^2 = (20)^2 + (21)^2$$

Next, square each of the terms indicated in the equation.

$$(29)^2 = (20)^2 + (21)^2$$
$$(29 \times 29) = (20 \times 20) + (21 \times 21)$$

Next, evaluate each of the squares and simplify the equation.

$$(29 \times 29) = (20 \times 20) + (21 \times 21)$$
$$841 = 400 + 441$$
$$841 = 841$$

The square of the length of one side of the triangle is equal to the sum of the squares of the lengths of the other two sides. The garden is a right triangle.

Brandon can use his Grandpa's fence for his flower garden.

Example 4.31.2

Are the set of numbers $(5, 8, 10)$ the lengths of the sides of a right triangle?

Solution

First, determine the values for each side of the right triangle. The length of the longest side will represent 'c,' the hypotenuse. The other two lengths will represent the lengths of the legs 'a' and 'b.'

$a = 5$

$b = 8$

$c = 10$

Next, substitute the given values into the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$(10)^2 = (5)^2 + (8)^2$$

Next, square each of the terms indicated in the equation.

$$(10)^2 = (5)^2 + (8)^2$$
$$(10 \times 10) = (5 \times 5) + (8 \times 8)$$

Next, evaluate each of the squares.

$$(10 \times 10) = (5 \times 5) + (8 \times 8)$$
$$100 = 25 + 64$$

Then, simplify the right side of the equation by adding the numbers.

$$100 = 25 + 64$$
$$100 \neq 89$$

Both sides of the equation are not equal. The square of the length of one side of the triangle is NOT equal to the sum of the squares of the lengths of the other two sides.

The set of numbers $(5, 8, 10)$ are not the lengths of the sides of a right triangle.

Example 4.31.3

Is the following triangle (which may or may not be drawn to scale) a right triangle?

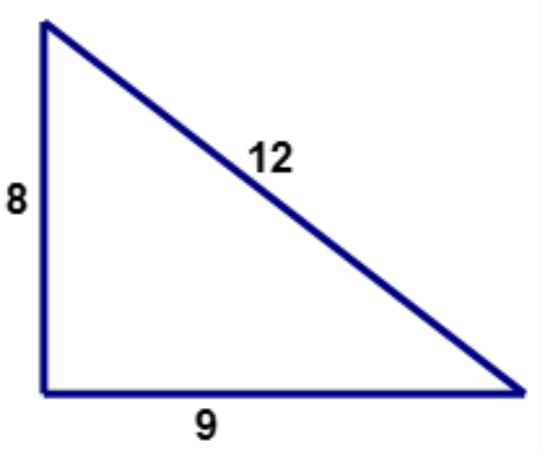

Figure 4.31.3

Solution

First, determine the values for (a, b, c) of a right triangle. The longest side will represent 'c' the hypotenuse.

$a = 8$

$b = 9$

$c = 12$

Next, substitute the given values into the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$(12)^2 = (8)^2 + (9)^2$$

Next, square each of the terms indicated in the equation.

$$(12)^2 = (8)^2 + (9)^2$$
$$(12 \times 12) = (8 \times 8) + (9 \times 9)$$

Next, evaluate each of the squares.

$$(12 \times 12) = (8 \times 8) + (9 \times 9)$$
$$144 = 64 + 81$$

Then, simplify the right side of the equation by adding the numbers.

$$144 = 64 + 81$$
$$144 \neq 145$$

Both sides of the equation are not equal. The square of the length of one side of the triangle is NOT equal to the sum of the squares of the lengths of the other two sides. The triangle is not a right triangle.

Example 4.31.4

Is the following triangle (which may or may not be drawn to scale) a right triangle?

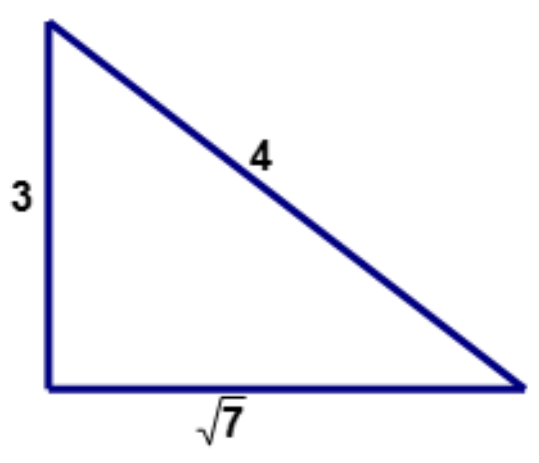

Figure 4.31.4

Solution

First, determine the values for (a, b, c) of a right triangle. The longest side will represent 'c' the hypotenuse.

$$\sqrt{7} \approx 2.65$$
$$a = 3$$
$$b = \sqrt{7}$$
$$c = 4$$

Next, substitute the given values into the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$(4)^2 = (3)^2 + (\sqrt{7})^2$$

Next, square each of the terms indicated in the equation.

$$(4)^2 = (3)^2 + (\sqrt{7})^2$$
$$(4 \times 4) = (3 \times 3) + (\sqrt{7} \times \sqrt{7})$$

Next, evaluate each of the squares.

$$(4 \times 4) = (3 \times 3) + (\sqrt{7} \times \sqrt{7}$$
$$16 = 9 + 7$$

Then, simplify the right side of the equation by adding the numbers.

Unknown environment 'ailgn*'

Both sides of the equation are equal. The square of the length of one side of the triangle is equal to the sum of the squares of the lengths of the other two sides. The triangle is a right triangle.

Review

Think about what you have learned about the Pythagorean Theorem and answer true or false for the following questions.

1. The Pythagorean Theorem will work for an acute triangle with all $60°$ angles.
2. The Pythagorean Theorem will work for a right triangle.
3. The Pythagorean Theorem will only work if the triangle is a right triangle.
4. The **legs of a right triangle** are considered the two shorter sides of the right triangle.
5. The hypotenuse is the longest side of a right triangle.
6. The converse of the Pythagorean Theorem is used to find the angle measures of an obtuse triangle.
7. A **Pythagorean Triple** is when you multiply all of the angle measures by three.
8. You can use the Pythagorean Theorem to figure out if the side lengths of a triangle make it a right triangle or not.

Identify whether or not each of the following values is a Pythagorean Triple. Write yes or no for your answer.

9. $(4, 5, 6)$

10. $(6, 8, 10)$

11. $(5, 6, 9)$

12. $(9, 12, 15)$

13. $(30, 40, 55)$

14. $(21, 28, 35)$

15. $(12, 16, 20)$.

Vocabulary

Term	Definition
directrix	The directrix of a parabola is the line that the parabola seems to curve away from. All points on a parabola are equidistant from the focus of the parabola and the directrix of the parabola.
focus	The focus of a parabola is the point that "anchors" a parabola. Any point on the parabola is exactly the same distance from the focus as from the directrix.
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Parabola	A parabola is the characteristic shape of a quadratic function graph, resembling a "U".
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Pythagorean Triple	A Pythagorean Triple is a set of three whole numbers a,b and c that satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$.
Right Triangle	A right triangle is a triangle with one 90 degree angle.

Additional Resources

Video: The Pythagorean Theorem and The Converse of the Pythagorean Theorem

Practice: Converse of the Pythagorean Theorem

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4.32: Pythagorean Theorem Applications

Discover, geometrically prove, and apply the Pythagorean Theorem.

Lengths of Triangle Sides Using the Pythagorean Theorem

You've just signed up to be an architect's assistant in a new office downtown. You're asked to draw a scale model of a sculpture for a business plaza. The sculpture has a large triangular piece where one of the angles between the sides is ninety degrees. This type of triangle is called a ‘‘right triangle’’. The architect you're working for comes into the room and tells you that the sides of the triangle that form the right angle are 9 feet and 12 feet. Can you tell how long the third side is?

Finding the Length of Triangle Sides Using Pythagorean Theorem

From Geometry, recall that the Pythagorean Theorem is $a^2 + b^2 = c^2$ where a and b are the legs of a right triangle and c is the hypotenuse. Also, the side opposite the angle is lower case and the angle is upper case. For example, angle A is opposite side a.

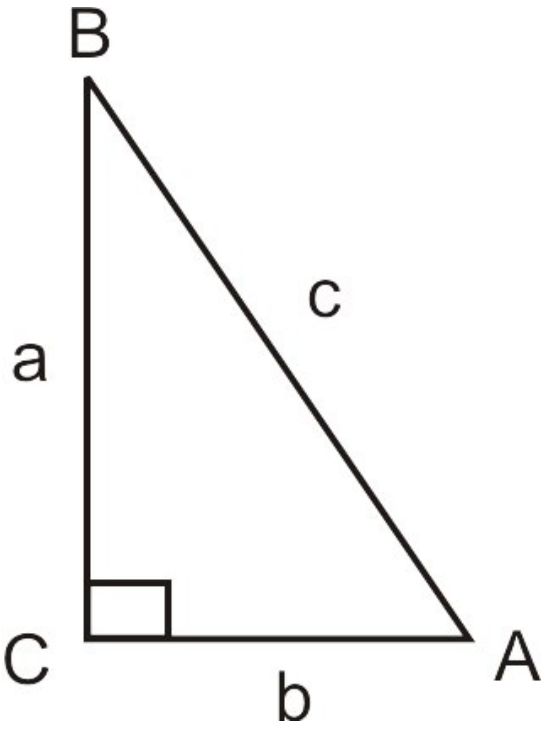

Figure 4.32.1

The Pythagorean Theorem is used to solve for the sides of a right triangle.

Using the Pythagore

1.

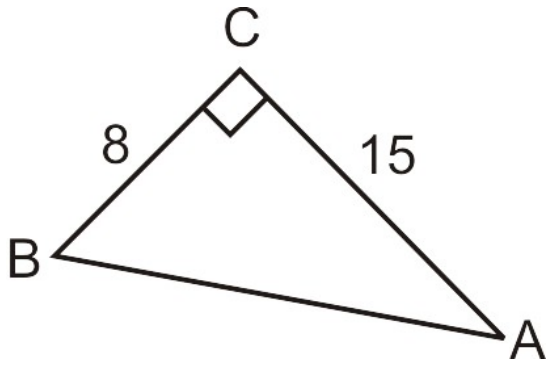

Figure 4.32.2

$a = 8, b = 15$, we need to find the hypotenuse.

$$\begin{aligned} 8^2 + 15^2 &= c^2 \\ 64 + 225 &= c^2 \\ 289 &= c^2 \\ 17 &= c \end{aligned}$$

Notice, we do not include -17 as a solution because a negative number cannot be a side of a triangle.

2.

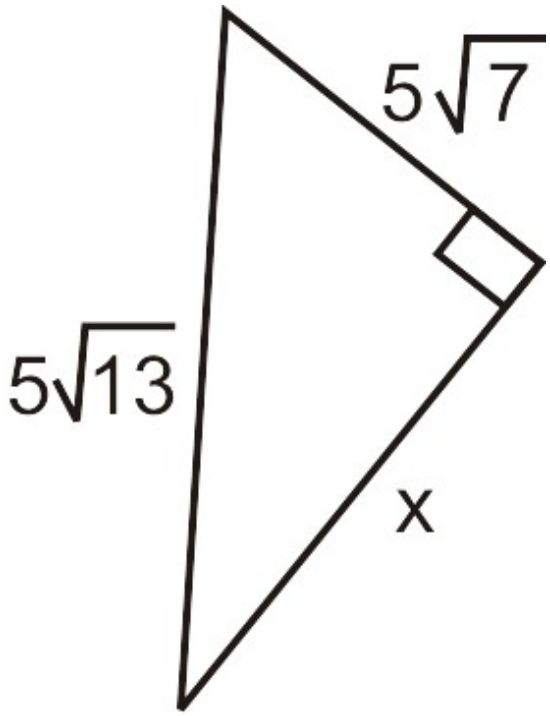

Figure 4.32.3

Use the Pythagorean Theorem to find the missing leg.

$$\begin{aligned} (5\sqrt{7})^2 + x^2 &= (5\sqrt{13})^2 \\ 25 \cdot 7 + x^2 &= 25 \cdot 13 \\ 175 + x^2 &= 325 \\ x^2 &= 150 \\ x &= 5\sqrt{6} \end{aligned}$$

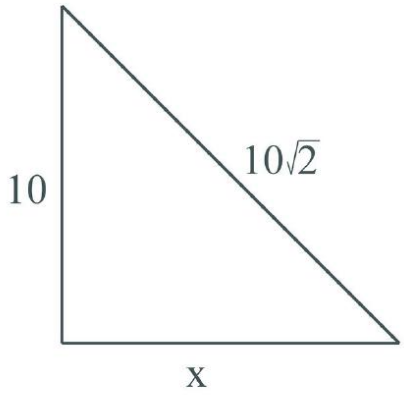

Figure 4.32.4

3. Use the Pythagorean Theorem to find the missing leg in the triangle above.

$$\begin{aligned} 10^2 + x^2 &= (10\sqrt{2})^2 \\ 100 + x^2 &= 100 \cdot 2 \\ 100 + x^2 &= 100 \\ x^2 &= 100 \\ x &= 10 \end{aligned}$$

Example 4.32.1

Earlier, you were given a problem asking you to draw a scale model of a sculpture for a business plaza.

Solution

With your knowledge of the Pythagorean Theorem, you can see that the triangle has sides with lengths 9 feet and 12 feet. You work to find the hypotenuse:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 9^2 + 12^2 &= c^2 \\ 81 + 144 &= c^2 \\ 225 &= c^2 \\ (\sqrt{225}) &= 15 = c \end{aligned}$$

With the knowledge that the length of the third side of the triangle is 15 feet, you are able to construct your scale model with ease.

Example 4.32.2

Use the Pythagorean Theorem to find the missing side of the following triangle:

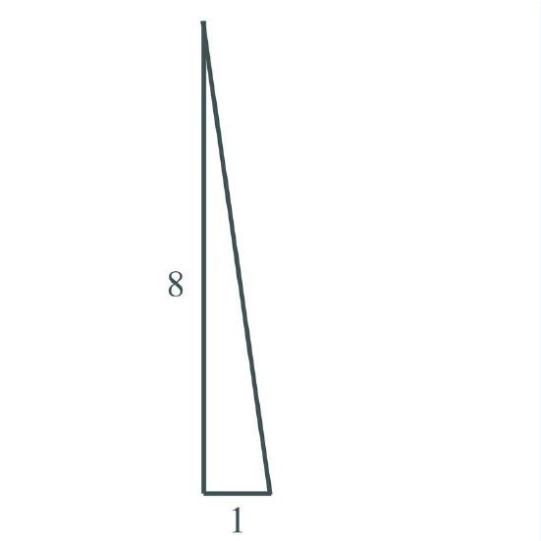

Figure 4.32.5

Solution

$a = 1$, $b = 8$, we need to find the hypotenuse.

$$\begin{aligned} 1^2 + 8^2 &= c^2 \\ 1 + 64 &= c^2 \\ 65 &= c^2 \\ \sqrt{65} &= c \end{aligned}$$

Example 4.32.3

Use the Pythagorean Theorem to find the missing side of the following triangle:

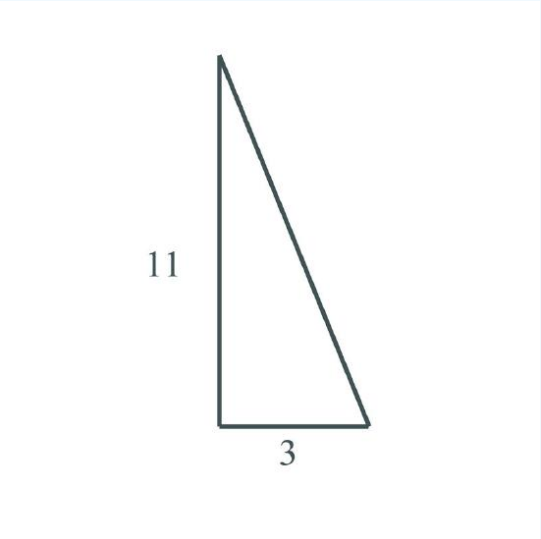

Figure 4.32.6

Solution

$a = 3$, $b = 11$, we need to find the length of side c, the hypotenuse.

$$\begin{aligned} 3^2 + 11^2 &= c^2 \\ 9 + 121 &= c^2 \\ 130 &= c^2 \\ \sqrt{130} &= c \end{aligned}$$

Example 4.32.4

Find the missing side of the right triangle below. Leave the answer in simplest radical form.

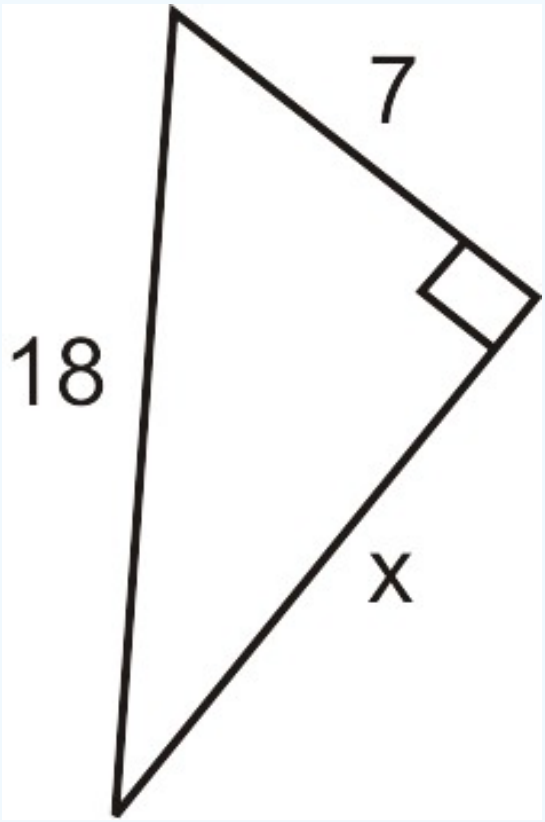

Figure 4.32.7

Solution

a=7, c=18, we need to find the length of side b.

$$\begin{aligned} 7^2 + b^2 &= 18^2 \\ 49 + b^2 &= 18^2 \\ 324 - 49 &= b^2 \\ 275 &= b^2 \\ \sqrt{275} &= b \end{aligned}$$

Review

Find the missing sides of the right triangles. Leave answers in simplest radical form.

1. If the legs of a right triangle are 3 and 4, then the hypotenuse is _____.
2. If the legs of a right triangle are 6 and 8, then the hypotenuse is _____.
3. If the legs of a right triangle are 5 and 12, then the hypotenuse is _____.
4. If the sides of a square are length 6, then the diagonal is _____.
5. If the sides of a square are 9, then the diagonal is _____.
6. If the sides of a square are x, then the diagonal is _____.
7. If the legs of a right triangle are 3 and 7, then the hypotenuse is _____.
8. If the legs of a right triangle are $2\sqrt{5}$ and 6, then the hypotenuse is _____.
9. If one leg of a right triangle is 4 and the hypotenuse is 8, then the other leg is _____.
10. If one leg of a right triangle is 10 and the hypotenuse is 15, then the other leg is _____.
11. If one leg of a right triangle is $4\sqrt{7}$ and the hypotenuse is $10\sqrt{6}$, then the other leg is _____.

12. If the legs of a right triangle are x and y, then the hypotenuse is _____.

Pythagorean Theorem Proof

Use the picture below to answer the following questions.

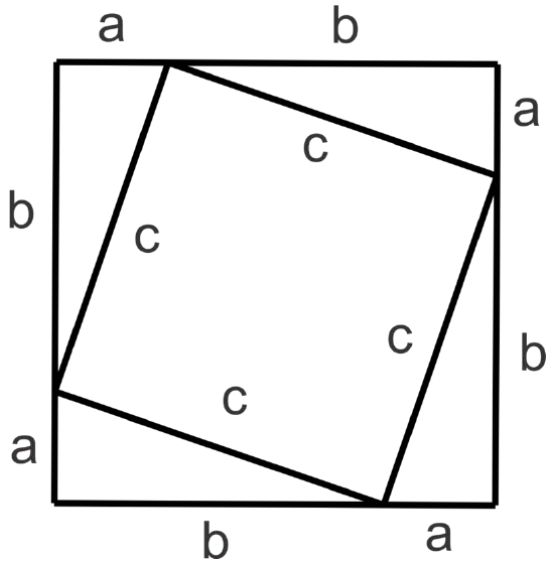

Figure 4.32.8

13. Find the area of the square in the picture with sides (a+b).
14. Find the sum of the areas of the square with sides c and the right triangles with legs a and b.
15. Explain why the areas found in the previous two problems should be the same value. Then, set the expressions equal to each other and simplify to get the Pythagorean Theorem.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.1.

Vocabulary

Term	Definition
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Practice: Pythagorean Theorem Applications

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4.33: Pythagorean Theorem and its Converse

Use a-squared + b-squared = c-squared to prove triangles are right triangles.

Suppose you're pitching a tent, and an 8-foot pole makes a right angle with the ground. If a rope goes from the top of the pole to a point on the ground 12 feet away from the base of the pole, how long is the rope? How would you calculate its length?

Pythagorean Theorem and its Converse

The Pythagorean Theorem

One of the most important theorems in mathematics and science is Pythagorean's Theorem. Simply put, it states, "The sum of the square of each leg of a right triangle is equal to the square of the **hypotenuse**."

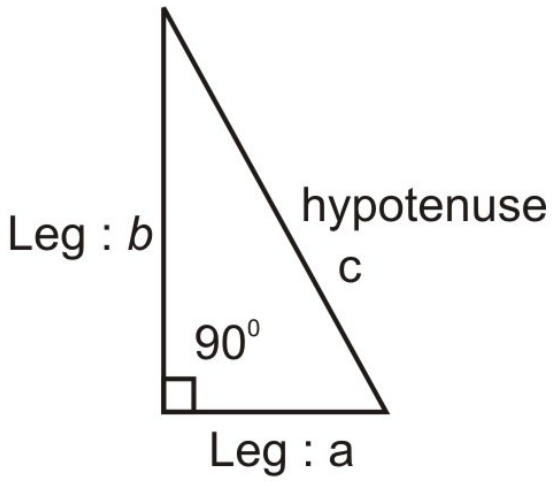

Figure **4.33.1**

A right triangle is a triangle with a right angle. Let's review basic right triangle anatomy.

The two segments forming the right angle ($90°$) are called the **legs** of the right triangle. The segment opposite the right angle is called the **hypotenuse.**

The Pythagorean Theorem states, $(leg_1)^2 + (leg_2)^2 = (hypotenuse)^2$:

$a^2 + b^2 = c^2$

Or, to find the hypotenuse, $c = \sqrt{a^2 + b^2}$.

Notice this relationship is only true for right triangles. In later courses, you will learn how to determine relationships with non-right triangles.

Although we usually refer to the Pythagorean Theorem when determining side lengths of a right triangle, the theorem originally made a statement about areas. If we build squares on each side of a right triangle, the Pythagorean Theorem says that the area of the square whose side is the hypotenuse is equal to the sum of the areas of the squares formed by the legs of the triangle.

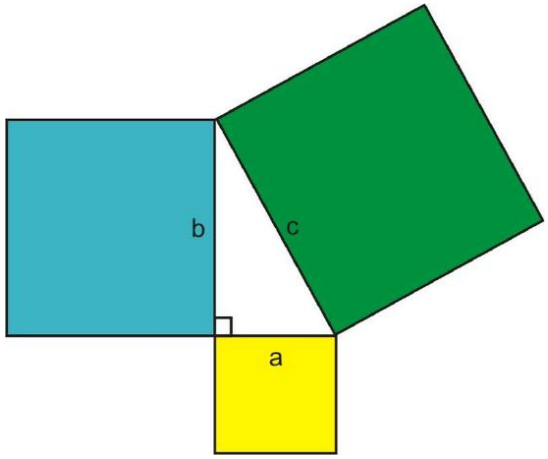

Figure **4.33.2**

The **Converse of the Pythagorean Theorem**

The **Converse of the Pythagorean Theorem** is also true. That is, if the lengths of three sides of a triangle make the equation $a^2 + b^2 = c^2$ true, then they represent the sides of a right triangle.

With this converse, you can use the Pythagorean Theorem to prove that a triangle is a right triangle, even if you do not know any of the triangle's angle measurements.

Let's complete the following problems involving right triangles and the Pythagorean Theorem:

1. Does the triangle below contain a right angle?

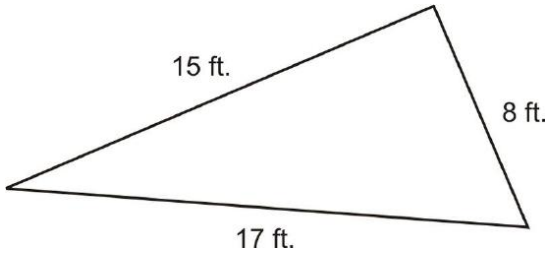

Figure **4.33.3**

This triangle does not have any right angle marks or measured angles, so you cannot assume you know whether the triangle is acute, right, or obtuse just by looking at it. Take a moment to analyze the side lengths and see how they are related. Two of the sides, 15 and 17, are relatively close in length. The third side, 8, is about half the length of the two longer sides.

To see if the triangle might be right, try substituting the side lengths into the Pythagorean Theorem to see if they make the equation true. The hypotenuse is always the longest side, so 17 should be substituted for c. The other two values can represent a and b and the order is not important.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 15^2 &= 17^2 \\ 64 + 225 &= 289 \\ 289 &= 289 \end{aligned}$$

Since both sides of the equation are equal, these values satisfy the Pythagorean Theorem. Therefore, the triangle described in the problem is a right triangle.

2. One leg of a right triangle is 5 more than the other leg. The hypotenuse is one more than twice the size of the short leg. Find the dimensions of the triangle.

Let x= length of the short leg. Then, $x + 5 = \textit{length of the long leg}$ and $2x + 1 = \textit{length of the hypotenuse}$.

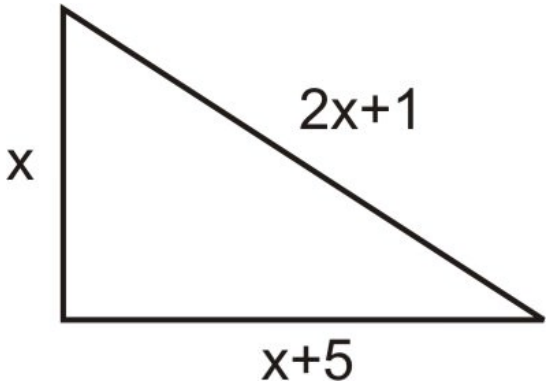

Figure 4.33.4

The sides of the triangle must satisfy the Pythagorean Theorem.

$$x^2 + (x+5)^2 = (2x+1)$$

Eliminate the parentheses. $x^2 + x^2 + 10x + 25 = 4x^2 + 4x + 1$

Move all terms to the right hand side of the equation. $0 = 2x^2 - 6x - 24$

Divide all terms by 2. $0 = x^2 - 3x - 12$

Solve using the quadratic formula. $x = \frac{3 \pm \sqrt{9+48}}{2} = \frac{3 \pm \sqrt{57}}{2}$

$x \neq 5.27$ *or* $x \neq -2.27$

The negative solution does not make sense in the context of this problem. So, use $x = 5.27$ and we get

$short\ -leg = 5.27$, $long\ -leg = 10.27$ and

$hypotenuse = 11.54$.

3. Find the area of the shaded region in the following diagram.

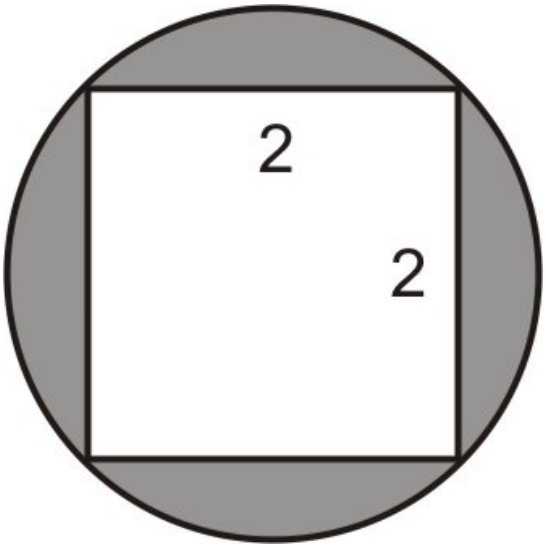

Figure 4.33.5

Draw the diagonal of the square on the figure.

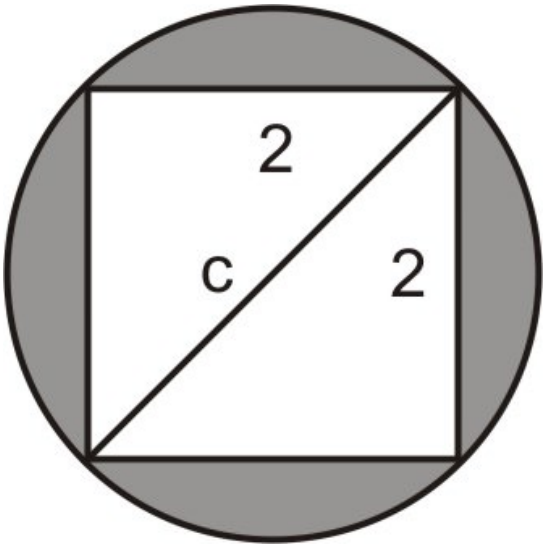

Figure 4.33.6

Notice that the diagonal of the square is also the diameter of the circle.

Define variables. Let $c = diameter\ of\ the\ circle$.

$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
Write the formula and solve. $$c^2 = 8 \Rightarrow c = \sqrt{8} \Rightarrow c = 2\sqrt{2}$$

The diameter of the circle is $2\sqrt{2}$. Therefore, the radius is $r = \sqrt{2}$.

The area of a circle is $A = \pi r^2 = \pi(\sqrt{2})^2 = 2\pi$.

The area of the shaded region is, therefore, 2\pi −4\neq 2.28\).

Example 4.33.1

Earlier, you were asked of a rope you used to pitch a tent. You know that the rope goes from the top of an 8 foot pole to a point on the ground 12 feet away from the base.

Solution

This situation can be represented by a right triangle.

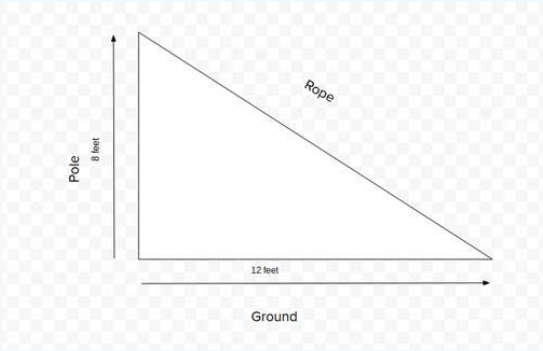

Figure 4.33.7

Assuming that the angle between the ground and the pole is 90 degrees, we can use the Pythagorean Theorem to solve for the length of the rope.

Let $a = 8\ feet (length\ of\ the\ pole)$, \(b = 12\ feet (the distance from the pole to the rope on the ground), and $c = the\ length\ of\ the\ rope$.

$$a^2 + b^2 = c^2$$
$$8^2 + 12^2 = c^2$$
$$208 = c^2$$
$$c = \sqrt{208}$$
$$c = \sqrt{16}\sqrt{13}$$
$$c = 4\sqrt{13} \text{ feet}$$

The rope is $4\sqrt{13}$ feet long.

Example 4.33.2

Determine whether or not a triangle with sides of lengths 5, 6 and 8 forms a right triangle.

Solution

Use the Converse of the Pythagorean Theorem:

Start with the Pythagorean equation.	$a^2 + b^2$	$= c^2$
Substitute in the values of the sides.	$5^2 + 6^2$	$\stackrel{?}{=} 8^2$
Simplify.	$25 + 36$	$\stackrel{?}{=} 64$
Check.	61	$\neq 64$

Since these lengths of sides do not satisfy the equation of the Pythagorean Theorem, the triangle is not a right triangle.

Review

Verify that each triangle is a right triangle.

1. $a = 12,\ b = 9,\ c = 15$
2. $a = 6,\ b = 6,\ c = 6\sqrt{2}$
3. $a = 8,\ b = 8\sqrt{3},\ c = 16$

Find the missing length of each right triangle.

4. $a = 12,\ b = 16,\ c = ?$
5. $a = ?,\ b = 20,\ c = 30$
6. $a = 4,\ b = ?,\ c = 11$

7.

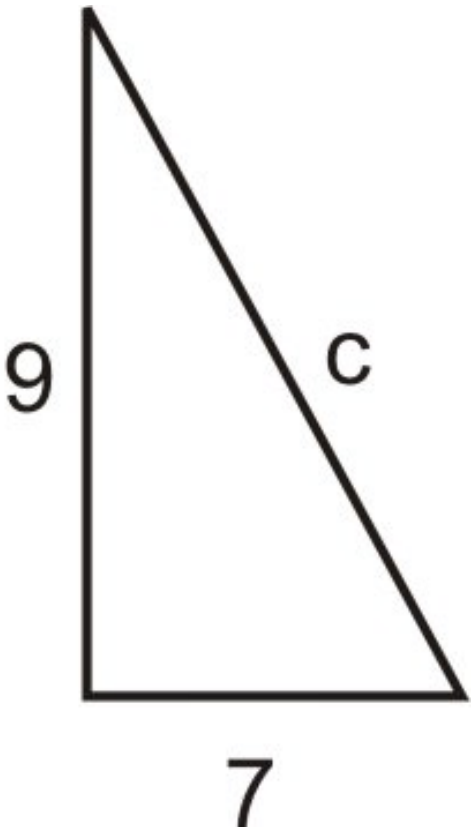

Figure **4.33.8**

8.

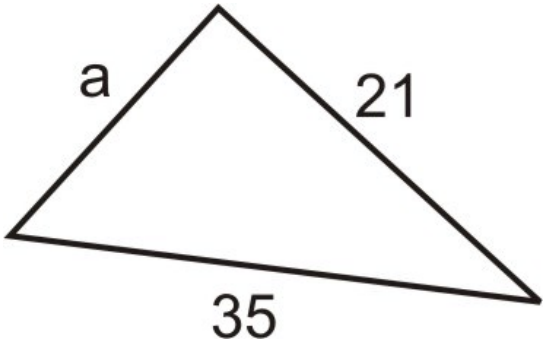

Figure **4.33.9**

9.

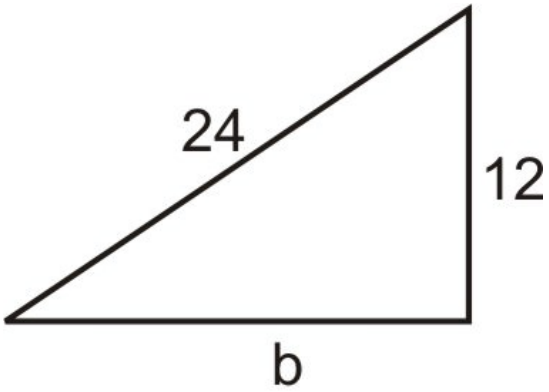

Figure **4.33.10**

10. One leg of a right triangle is 4 feet less than the hypotenuse. The other leg is 12 feet. Find the lengths of the three sides of the triangle.

11. One leg of a right triangle is 3 more than twice the length of the other. The hypotenuse is 3 times the length of the short leg. Find the lengths of the three legs of the triangle.
12. A regulation baseball diamond is a square with 90 feet between bases. How far is second base from home plate?
13. Emanuel has a cardboard box that measures $20\ cm \times 10\ cm \times 8\ cm (length \times width \times height)$. What is the length of the diagonal from a bottom corner to the opposite top corner?
14. Samuel places a ladder against his house. The base of the ladder is 6 feet from the house and the ladder is 10 feet long. How high above the ground does the ladder touch the wall of the house?
15. Find the area of the triangle by using the formula $A = \frac{1}{2} base \times height$.

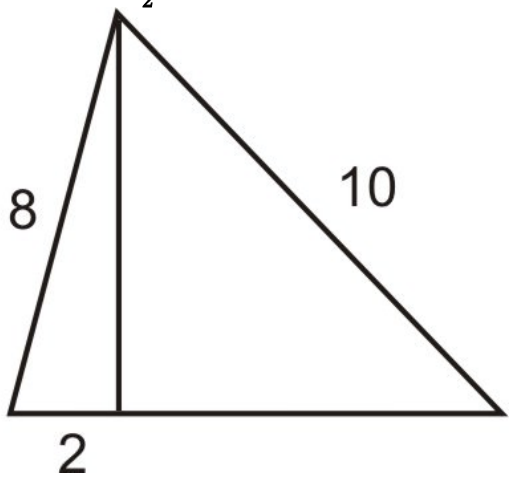

Figure **4.33.11**

16. Instead of walking along the two sides of a rectangular field, Mario decided to cut across the diagonal. He saves a distance that is half of the long side of the field. Find the length of the long side of the field given that the short side is 123 feet.
17. Marcus sails due north and Sandra sails due east from the same starting point. In two hours, Marcus's boat is 35 miles from the starting point and Sandra's boat is 28 miles from the starting point. How far are the boats from each other?
18. Determine the area of the circle.

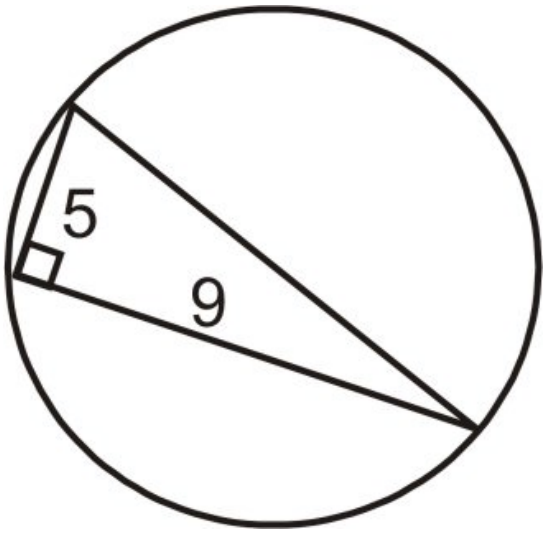

Figure **4.33.12**

19. In a right triangle, one leg is twice as long as the other and the perimeter is 28. What are the measures of the sides of the triangle?
20. Maria has a rectangular cookie sheet that measures $10\ inches \times 14\ inches$. Find the length of the diagonal of the cookie sheet.
21. Mike is loading a moving van by walking up a ramp. The ramp is 10 feet long and the bed of the van is 2.5 feet above the ground. How far does the ramp extend past the back of the van?

Quick Quiz

1. Identify the origin of $h(x) = \sqrt{x-2}+5$, and then graph the function.
2. Simplify $6\sqrt[3]{2}$ by rationalizing the denominator.
3. Simplify: $\sqrt[4]{-32}$. If the answer is not possible, explain why.
4. What is an extraneous solution? In what situations do such solutions occur?
5. Can sides with lengths of 3, 4, and 6 form a right triangle?
6. Solve $5 = y + \sqrt[3]{y+6}$.

Vocabulary

Term	Definition
converse of the Pythagorean Theorem	If the lengths of the three sides of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle.
converse	If a conditional statement is $p \rightarrow q$ (if p, then q), then the converse is $q \rightarrow p$ (if q, then p. Note that the converse of a statement is not true just because the original statement is true.
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Video: Pythagorean Theorem - Overview

Activities: Pythagorean Theorem and its Converse Discussion Questions

Practice: Pythagorean Theorem and its Converse

Real World: SF Map - PythThm

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4.34: Solving Equations Using the Pythagorean Theorem

Use a-squared + b-squared = c-squared to find missing side lengths of right triangles.

Figure 4.34.1

Gary wants to build a skateboard ramp, but it can't be too steep. If he has a platform 3 m high and a board 5 m long, how far out should the board extend from the platform?

In this concept, you will learn how to solve equations using the **Pythagorean Theorem**.

The **Pythagorean Theorem** states that the sum of the squares of the two **legs of a right triangle** is equal to the square of the **hypotenuse**. In a math sentence, where a and b are the legs and c is the hypotenuse, it looks like this:

$$c^2 = a^2 + b^2$$

Mathematically, you can use this equation to solve for any of the variables, not just the hypotenuse.

For example, the right triangle below has one leg equal to 3 and a hypotenuse of 5.

Solve for the other leg.

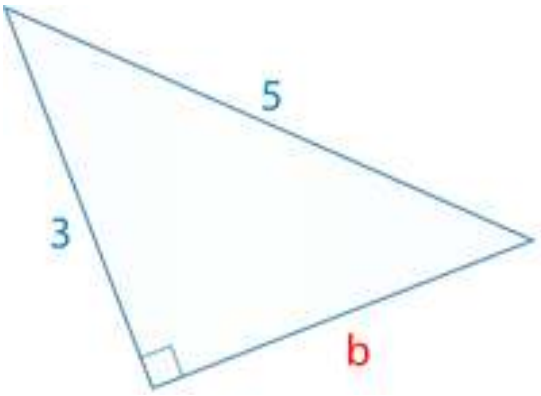

Figure 4.34.2

First, you can label either leg a or b. Remember that the legs are those sides adjacent to the right angle.

Next, fill into the Pythagorean Theorem the values that you know.

$$c^2 = a^2 + b^2$$
$$5^2 = 3^2 + b^2$$

Then, perform the calculations you are able to.

$$25 = 9 + b^2$$

Remember that your goal is to isolate the unknown variable on one side of the equation. In this case it is b and it is attached to a square and a+9. Perform the necessary operations to isolate b.

$$25 - 9 = 9 + b^2 - 9$$
$$16 = b^2$$
$$4 = b$$

The answer is 4.

Example 4.34.1

Earlier, you were given a problem about Gary and his skate board ramp.

Solution

One side, the base, was 4 m and the board, the hypotenuse, was 5 m. How high would the ramp be?

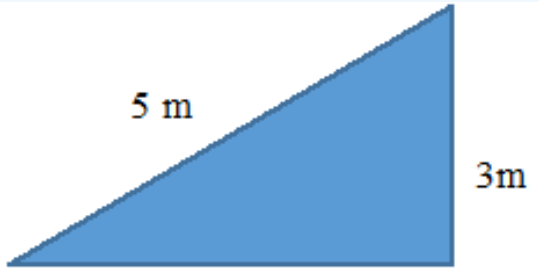

Figure 4.34.3

First, substitute.

$$5^2 = 3^2 + b^2$$

Next, perform the calculations.

$$25 = 9 + b^2$$
$$25 - 9 = 9 + b^2 - 9$$

Then, determine the square roots.

$$16 = b^2$$
$$4 = b$$

The answer is 4 m. Gary's board should extend 4 m from the base of the platform.

Example 4.34.2

Solve for b to the nearest tenth.

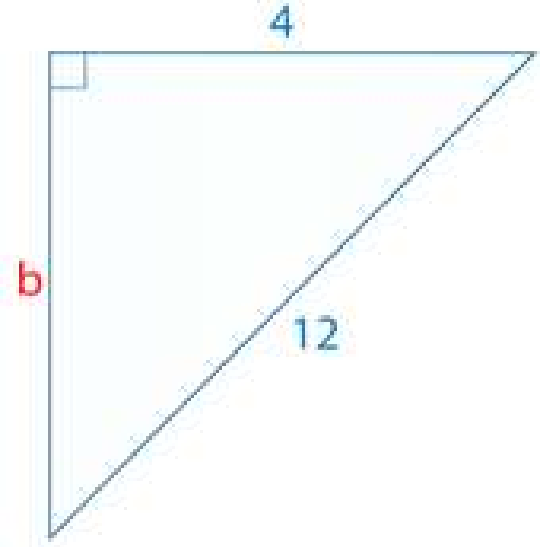

Figure 4.34.4

Solution

First, take the given lengths and substitute them into the formula.

$$4^2 + b^2 = 122$$
$$16 + b^2 = 144$$

Next, subtract 16 from both sides of the equation.

$$16 - 16 + b^2 = 144 - 16$$
$$b^2 = 128$$

Then take the square root of both sides of the equation.

$$b = 11.3137085 \ldots$$

Round to the tenths place

$$b \neq 11.3$$

The answer is 11.3

Example 4.34.3

A right triangle includes the dimensions of a, $b = 6$ and $c = 13$. Solve for a\).

Solution

First, substitute.

$$c^2 = a^2 + b^2$$
$$13^2 = a^2 + 6^2$$

Next, perform the calculations you are able to.

$$169 = a^2 + 36$$
$$169 - 36 = a^2 + 36 - 36$$
$$133 = a^2$$
$$11.532582594 \ldots = a$$
$$11.5 \approx a$$

The answer is $a = 11.5$.

Example 4.34.4

A right triangle with $a = 8$,b, and $c = 12$

Solution

First, substitute.

$$c^2 = a^2 + b^2$$
$$12^2 = 8^2 + b^2$$

Next, perform the calculations.

$$144 = 64 + b^2$$
$$144 - 64 = 64 + b^2 - 64$$

Then, determine the square roots.

$$80 = b^2$$
$$8.9 \neq b$$

The answer is 8.9

Example 4.34.5

A right triangle with $a = 6$, b, and $c = 10$

Solution

First, substitute.

$$c^2 = a^2 + b^2$$
$$10^2 = 6^2 + b^2$$

Next, perform the calculations.

$$100 = 36 + b^2$$
$$100 - 36 = 36 + b^2 - 36$$

Then, determine the square roots.

$$64 = b^2$$
$$8 = b$$

The answer is 8.

Review

Use the Pythagorean Theorem to find the length of each missing leg. You may round to the nearest tenth when necessary.

1. $a = 6,\ b = ?,\ c = 12$
2. $a = 9,\ b = ?,\ c = 15$
3. $a = 4,\ b = ?,\ c = 5$
4. $a = 9,\ b = ?,\ c = 18$
5. $a = 15,\ b = ?,\ c = 25$
6. $a = ?,\ b = 10,\ c = 12$
7. $a = ?,\ b = 11,\ c = 14$
8. $a = ?,\ b = 13,\ c = 15$

Write an equation using the Pythagorean Theorem and solve each problem.

Joanna laid a plank of wood down to make a ramp so that she could roll a wheelbarrow over a low wall in her garden. The wall is 1.5 meters tall, and the plank of wood touches the ground 2 meters from the wall. How long is the wooden plank?

9. Write the equation.
10. Solve for the answer.

Chris rode his bike 4 miles west and then 3 miles south. What is the shortest distance he can ride back to the point where he started?

11. Write the equation.
12. Solve the problem.

Naomi is cutting triangular patches to make a quilt. Each has a diagonal side of 14.5 inches and a short side of 5.5 inches. What is the length of the third side of each triangular patch?

13. Write the equation.
14. Solve the problem.

Resources

Vocabulary

Term	Definition
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Video: The Pythagorean Theorem with Variables: A Sample Application

Practice: Solving Equations Using the Pythagorean Theorem

Real World: Cryptic Notes

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4.35: Applications Using the Pythagorean Theorem

Word problems using the Pythagorean Theorem.

The Pythagorean Theorem and its **converse** have many applications for finding lengths and distances.

Real-World Application: Diagonal Length

Maria has a rectangular cookie sheet that measures $10\ inches \times 14\ inches$. Find the length of the diagonal of the cookie sheet.

Draw a sketch:

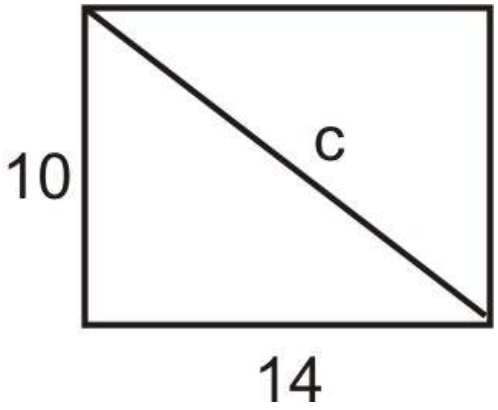

Figure 4.35.1

Define variables: Let $c = length\ of\ the\ diagonal$.

Write a formula: Use the Pythagorean Theorem: $a^2 + b^2 = c^2$

Solve the equation:

$$10^2 + 14^2 = c^2$$
$$100 + 196 = c^2$$
$$c^2 = 296 \Rightarrow c = \sqrt{296} \Rightarrow c = 2\sqrt{74}\ or\ c = 17.2\ inches$$

Check: $10^2 + 14^2 = 100 + 196 = 296$ and $c^2 = 17.2^2 = 296$. The solution checks out.

Real-World Application: Area of Shaded Regions

Find the area of the shaded region in the following diagram:

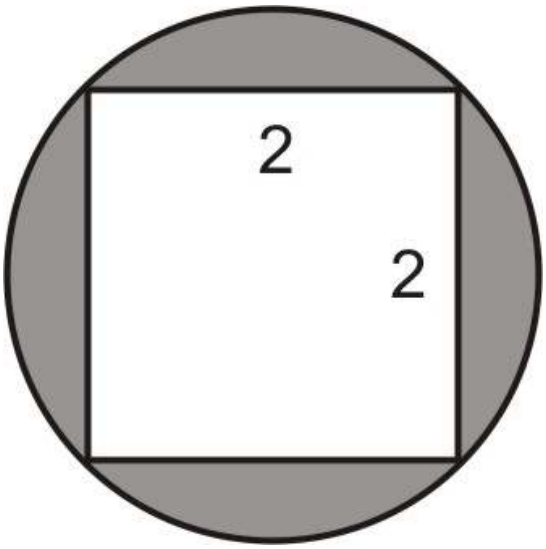

Figure 4.35.2

Draw the diagonal of the square in the figure:

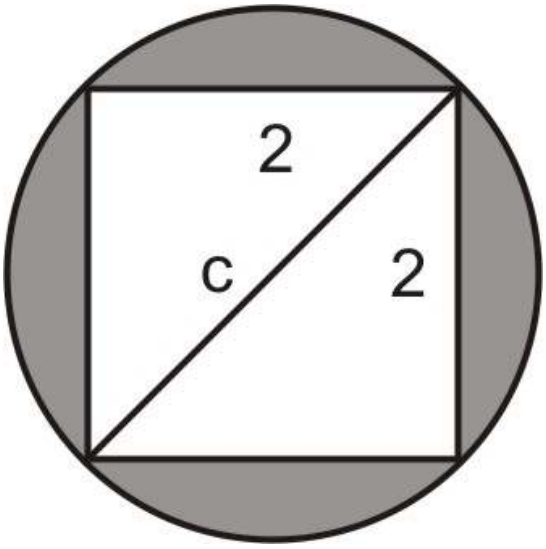

Figure 4.35.3

Notice that the diagonal of the square is also the diameter of the circle.

Define variables: Let $c = \textit{diameter of the circle}$.

Write the formula: Use the Pythagorean Theorem: $a^2 + b^2 = c^2$.

Solve the equation:

$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$c^2 = 8 \Rightarrow c = \sqrt{8} \Rightarrow c = 2\sqrt{2}$$

The diameter of the circle is $2\sqrt{2}$, therefore the radius $R = \sqrt{2}$.

Area of a circle formula: $A = \pi \cdot R^2 = \pi(\sqrt{2})^2 = 2\pi$.

The area of the shaded region is therefore $2\pi - 4 = 2.28$.

Real-World Application: Measuring the Sides of a Triangle

In a right triangle, one leg is twice as long as the other and the perimeter is 28. What are the measures of the sides of the triangle?

Make a sketch and define variables:

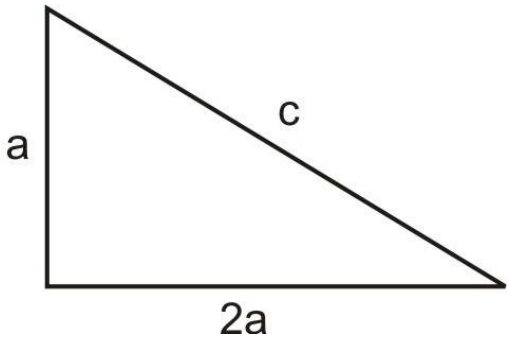

Figure 4.35.4

Let: $a = \textit{length of the short leg}$

$2a = \textit{length of the long leg}$

$c = \textit{length of the hypotenuse}$

Write formulas:

The sides of the triangle are related in two different ways.

The perimeter is 28, so $a + 2a + c = 28 \Rightarrow 3a + c = 28$

The triangle is a right triangle, so the measures of the sides must satisfy the Pythagorean Theorem:

$$a^2 + (2a)^2 = c^2 \Rightarrow a^2 + 4a^2 = c^2 \Rightarrow 5a^2 = c^2$$

$$or \qquad c = a\sqrt{5} = 2.236a$$

Solve the equation:

Plug the value of c we just obtained into the perimeter equation: $3a + c = 28$

$$3a + 2.236a = 28 \Rightarrow 5.236a = 28 \Rightarrow a = 5.35$$

The short leg is: $a = 5.35$

The long leg is: $2a = 10.70$

The hypotenuse is: $c = 11.95$

Check: The legs of the triangle should satisfy the Pythagorean Theorem:

$a^2 + b^2 = 5.35^2 + 10.70^2 = 143.1$, $c^2 = 11.95^2 = 142.80$. The results are approximately the same.

The perimeter of the triangle should be 28:

$a + b + c = 5.35 + 10.70 + 11.95 = 28$. **The answer checks out.**

Example 4.35.1

Mike is loading a movi up a ramp. The ramp is 10 feet long and the bed of the van is 2.5 feet above the ground. How far does th the back of the van?

Solution

Make a sketch:

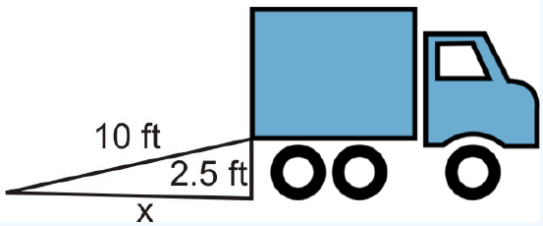

Figure 4.35.5

Define variables: Let $x = how\ far\ the\ ramp\ extends\ past\ the\ back\ of\ the\ van.$

Write a formula: Use the Pythagorean Theorem: $x^2 + 2.5^2 = 10^2$

Solve the equation:

$$x^2 + 6.25 = 100$$
$$x^2 = 93.5$$
$$x = \sqrt{93.5} = 9.7ft$$

Check by plugging the result in the Pythagorean Theorem:

$9.7^2 + 2.5^2 = 94.09 + 6.25 = 100.34 \neq 100$. So the ramp is 10 feet long. **The answer checks out.**

Review

1. In order to make a ramp that is $3\ fthighandcovers\(4\ ft$ of ground, how long must the ramp be?
2. A regulation baseball diamond is a square with 90 feet between bases. How far is second base from home plate?
3. Emanuel has a cardboard box that measures $20\ cm\ long \times 10\ cm\ wide \times 8\ cm\ deep$.
   a. What is the length of the diagonal across the bottom of the box?
   b. What is the length of the diagonal from a bottom corner to the opposite top corner?

4. Samuel places a ladder against his house. The base of the ladder is 6 feet from the house and the ladder is 10 feet long.
   a. How high above the ground does the ladder touch the wall of the house?
   b. If the edge of the roof is 10 feet off the ground and sticks out 1.5 feet beyond the wall, how far is it from the edge of the roof to the top of the ladder?
5. Find the area of the triangle below if the area of a triangle is defined as A=12 base\times height:

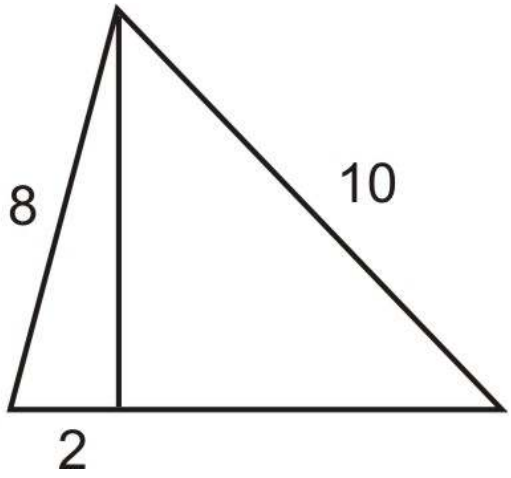

Figure **4.35.6**

6. Instead of walking along the two sides of a rectangular field, Mario decided to cut across the diagonal. He thus saves a distance that is half of the long side of the field.
   a. Find the length of the long side of the field given that the short side is 123 feet.
   b. Find the length of the diagonal.
7. Marcus sails due north and Sandra sails due east from the same starting point. In two hours Marcus' boat is 35 miles from the starting point and Sandra's boat is 28 miles from the starting point.
   a. How far are the boats from each other?
   b. Sandra then sails 21 miles due north while Marcus stays put. How far is Sandra from the original starting point?
   c. How far is Sandra from Marcus now?
8. Determine the area of the circle below. (Hint: the hypotenuse of the triangle is the diameter of the circle.)

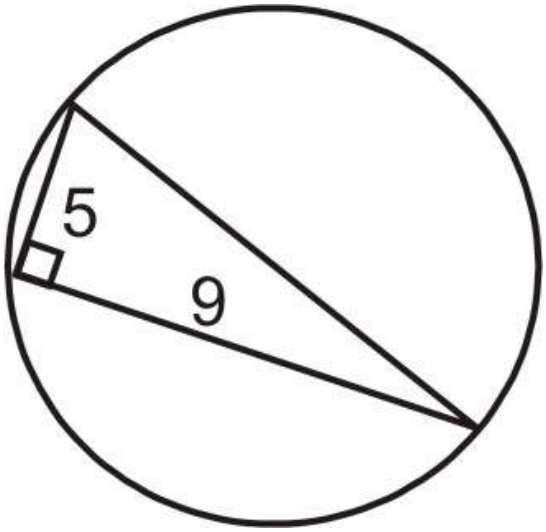

Figure **4.35.7**

9. A rectangle's length is $1\ in$ longer than its width and the diagonal has a length of $29\ in$. What are the lengths of the sides of the rectangle?
10. For an isosceles triangle with sides of the length given, find the length of each hypotenuse:

a. 1
b. 2
c. 3
d. n

Vocabulary

Term	Definition
converse	If a conditional statement is $p \rightarrow q$ (if p, then q), then the converse is $q \rightarrow p$ (if q, then p). Note that the converse of a statement is not true just because the original statement is true.

Additional Resources

Video: Solving Word Problems Involving Square Roots - Overview

Practice: Applications Using the Pythagorean Theorem

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4.36: Distance and Triangle Classification Using the Pythagorean Theorem

Find missing sides to calculate the area.

Applications of the Pythagorean Theorem

Find the Height of an Isosceles Triangle

One way to use The Pythagorean Theorem is to find the height of an isosceles triangle (see Example 1).

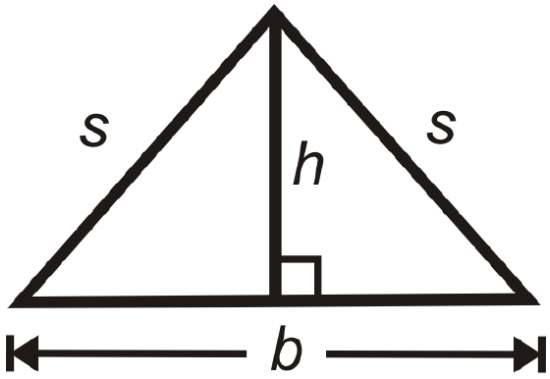

Figure **4.36.1**

Prove the **Distance Formula**

Another application of the Pythagorean Theorem is the Distance Formula. We will prove it here.

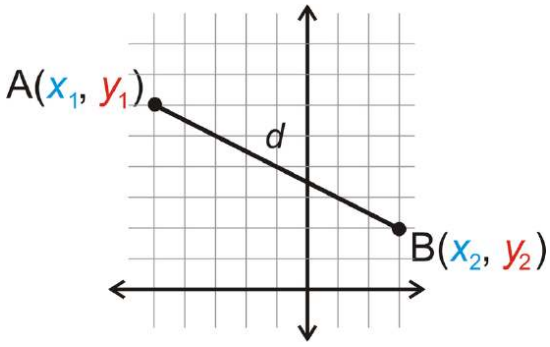

Figure **4.36.2**

Let's start with point $A(x_1, y_1)$ and point $B(x_2, y_2)$. We will call the distance between A and B, d.

Draw the vertical and horizontal lengths to make a right triangle.

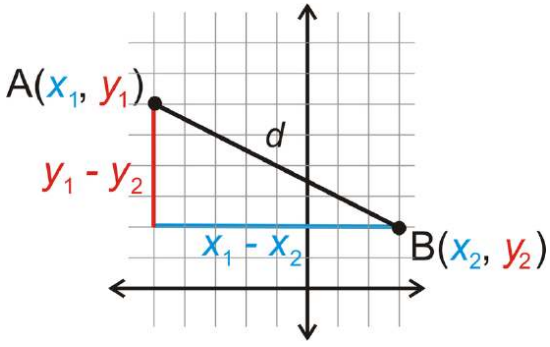

Figure **4.36.3**

Now that we have a right triangle, we can use the Pythagorean Theorem to find the hypotenuse, d.

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance Formula: The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Classify a Triangle as Acute, Right, or Obtuse

We can extend the converse of the Pythagorean Theorem to determine if a triangle is an obtuse or **acute triangle**.

Acute Triangles: If the sum of the squares of the two shorter sides in a right triangle is ***greater*** than the square of the longest side, then the triangle is ***acute.***

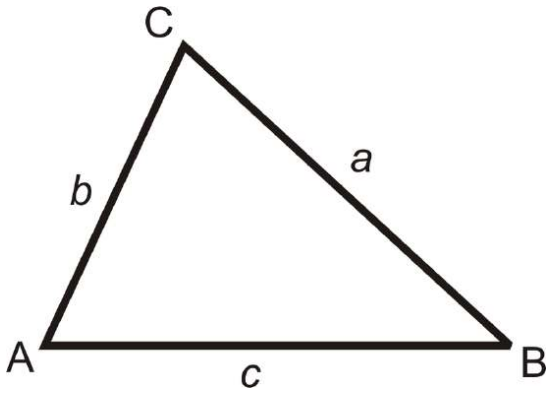

Figure 4.36.4

For $b < c$ and $a < c$, if $a^2 + b^2 > c^2$, then the triangle is acute.

Obtuse Triangles: If the sum of the squares of the two shorter sides in a right triangle is ***less*** than the square of the longest side, then the triangle is ***obtuse.***

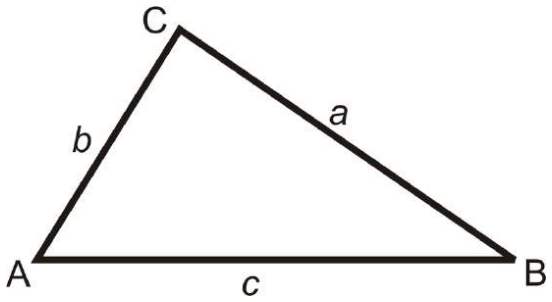

Figure 4.36.5

For $b < c$ and $a < c$, if $a^2 + b^2 < c^2$, then the triangle is obtuse.

What if you were given an equilateral triangle in which all the sides measured 4 inches? How could you use the Pythagorean Theorem to find the triangle's altitude?

Example 4.36.1

What is the height of the

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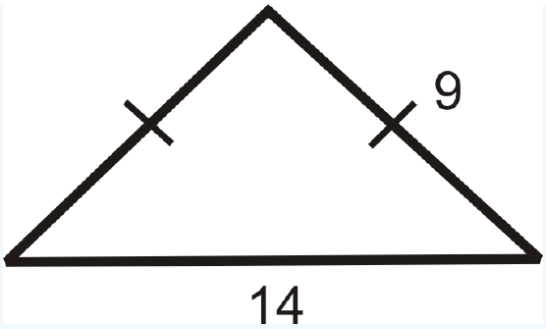


Figure 4.36.6

Solution

Draw the altitude from the **vertex** between the congruent sides, which will bisect the base.

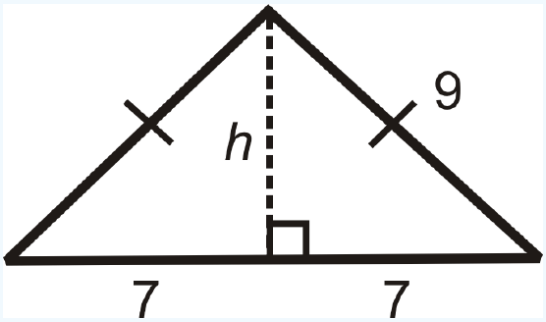


Figure 4.36.7

$$\begin{aligned} 7^2 + h^2 &= 9^2 \\ 49 + h^2 &= 81 \\ h^2 &= 32 \\ h &= \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2} \end{aligned}$$

Example 4.36.2

Find the distance between $(1, 5)$ and $(5, 2)$.

Solution

Make $A(1, 5)$ and $B(5, 2)$. Plug into the distance formula.

$$\begin{aligned} d &= \sqrt{(1-5)^2 + (5-2)^2} \\ &= \sqrt{(-4)^2 + (3)^2} \\ &= \sqrt{16+9} = \sqrt{25} = 5 \end{aligned}$$

Just like the lengths of the sides of a triangle, distances are always positive.

Example 4.36.3

Graph $A(-4, 1)$, $B(3, 8)$, and $C(9, 6)$. Determine if ΔABC is acute, obtuse, or right.

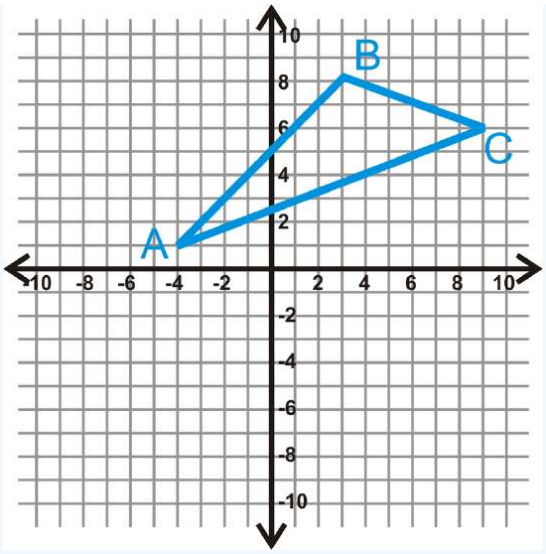


Figure 4.36.8

Solution

Use the distance formula to find the length of each side.

$$AB = \sqrt{(-4-3)^2 + (1-8)^2} = \sqrt{49+49} = \sqrt{98}$$
$$BC = \sqrt{(3-9)^2 + (8-6)^2} = \sqrt{36+4} = \sqrt{40}$$
$$AC = \sqrt{(-4-9)^2 + (1-6)^2} = \sqrt{169+25} = \sqrt{194}$$

Plug these lengths into the Pythagorean Theorem.

$$\sqrt{98})^2 + (\sqrt{40})^2 ? (\sqrt{194})^2$$
$$98 + 40 ? 194$$
$$138 < 194$$

ΔABC is an **obtuse triangle**.

For Examples 4 and 5, determine if the triangles are acute, right or obtuse.

Example 4.36.4

Set the longest side to c.

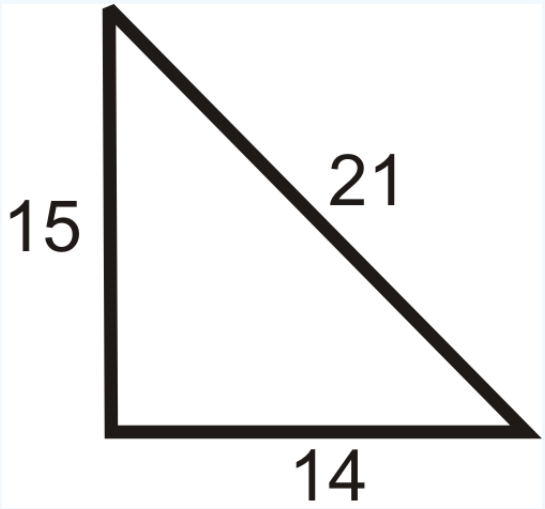


Figure 4.36.9

Solution

$$15^2 + 14^2 ? 21^2$$
$$225 + 196 ? 441$$
$$421 < 441$$

The triangle is obtuse.

Example 4.36.5

Set the longest side to c.

Solution

A triangle with side lengths 5, 12, 13.

$5^2 + 12^2 = 13^2$ so this triangle is right.

Review

Find the height of each isosceles triangle below. Simplify all radicals.

1.

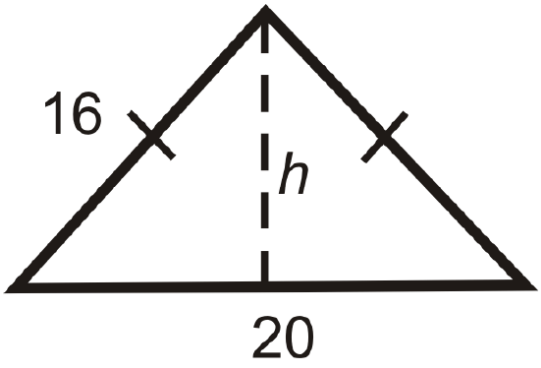


Figure 4.36.10

2.

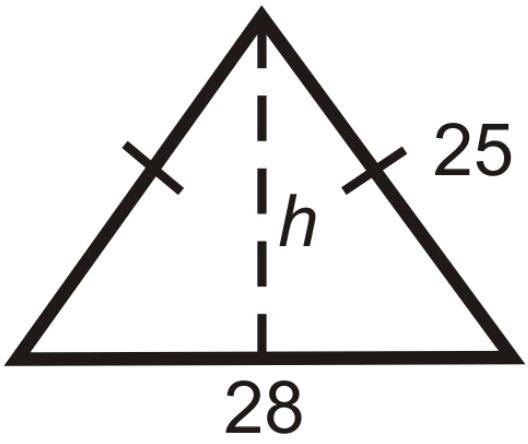


Figure 4.36.11

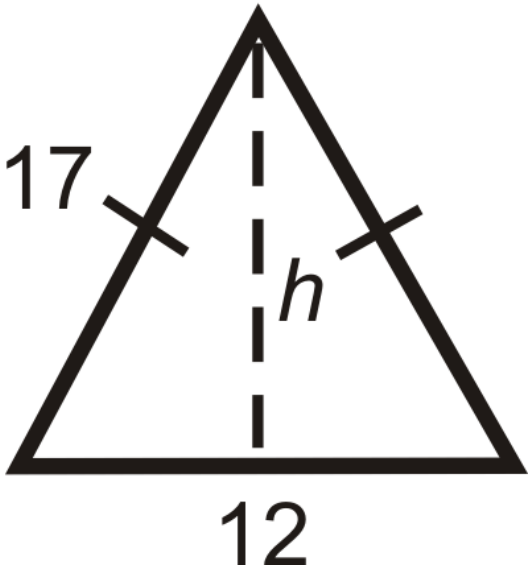
3. 

Figure **4.36.12**

Find the length between each pair of points.

4. $(-1, 6)$ and $(7, 2)$
5. $(10, -3)$ and $(-12, -6)$
6. $(1, 3)$ and $(-8, 16)$
7. What are the length and width of a 42” HDTV? Round your answer to the nearest tenth.
8. Standard definition TVs have a length and width ratio of 4:3. What are the length and width of a 42” Standard definition TV? Round your answer to the nearest tenth.

Determine whether the following triangles are acute, right or obtuse.

9. 7, 8, 9
10. 14, 48, 50
11. 5, 12, 15
12. 13, 84, 85
13. 20, 20, 24
14. 35, 40, 51
15. 39, 80, 89
16. 20, 21, 38
17. 48, 55, 76

Graph each set of points and determine whether \Delta{ABC} is acute, right, or obtuse, using the distance formula.

18. $A(3, -5), B(-5, -8), C(-2, 7)$
19. $A(5, 3), B(2, -7), C(-1, 5)$
20. $A(1, 6), B(5, 2), C(-2, 3)$
21. $A(-6, 1), B(-4, -5), C(5, -2)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.3.

Resources

Vocabulary

Term	Definition
acute triangle	A triangle where all angles are less than 90°.
Obtuse Triangle	An obtuse triangle is a triangle with one angle that is greater than 90 degrees.
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Vertex	A vertex is a point of intersection of the lines or rays that form an angle.

Additional Resources

Video: The Pythagorean Theorem and The Converse of the Pythagorean Theorem

Activities: Applications of the Pythagorean Theorem Discussion Questions

Practice: Distance and Triangle Classification Using the Pythagorean Theorem

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4.37: Distance Formula and the Pythagorean Theorem

Discover lengths of triangle sides using the Pythagorean Theorem. Identify distance as the hypotenuse of a right triangle. Determine distance between ordered pairs.

While walking to school one day, you decide to use your knowledge of the Pythagorean Theorem to determine how far it is between your home and school. You know that you walk 3 blocks east, and then turn and walk 7 blocks north to get to school. Is it possible to use the Pythagorean Theorem to help you determine the "straight line" distance between your home and school?

Determining the Distance Using the Pythagorean Theorem

You can use the Pythagorean Theorem is to find the distance between two points.

Consider the points $(-1, 6)$ and $(5, -3)$. If we plot these points on a grid and connect them, they make a diagonal line. Draw a vertical line down from $(-1, 6)$ and a horizontal line to the left of $(5, -3)$ to make a right triangle.

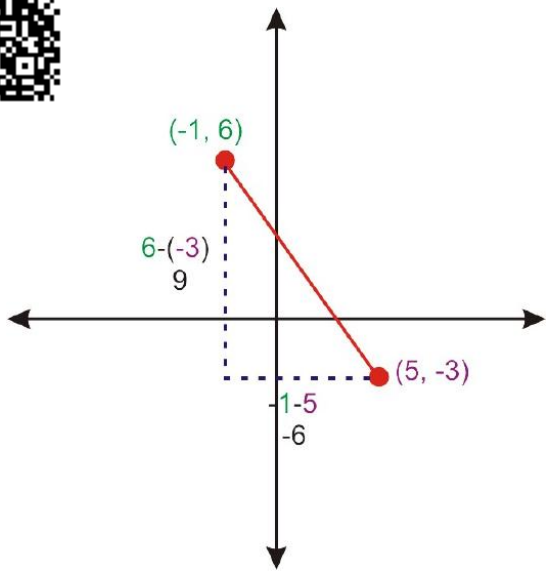

Figure 4.37.1

Now we can find the distance between these two points by using the vertical and horizontal distances that we determined from the graph.

$$\begin{aligned} 9^2 + (-6)^2 &= d^2 \\ 81 + 36 &= d^2 \\ 117 &= d^2 \\ \sqrt{117} &= d \\ 3\sqrt{13} &= d \end{aligned}$$

Notice, that the x−values were subtracted from each other to find the horizontal distance and the y−values were subtracted from each other to find the vertical distance. If this process is generalized for two points \((x_1, y_1) and \((x_2, y_2), the **Distance Formula** is derived.

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = d^2$$

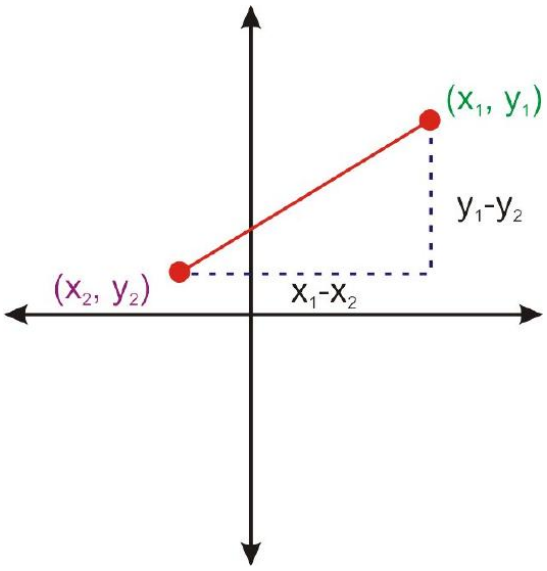

Figure 4.37.2

This is the Pythagorean Theorem with the vertical and horizontal differences between (x_1, y_1) and (x_2, y_2). Taking the square root of both sides will solve the right hand side for d, the distance.

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = d$$

This is the Distance Formula. The following problems show how to apply the distance formula.

Applying the Distance Formula

1. Find the distance between the two points.

$(4, 2)$ and $(-9, 5)$

Plug each pair of points into the distance formula.

$$\begin{aligned} d &= \sqrt{(4 - (-9))^2 + (2 - 5)^2} \\ &= \sqrt{13^2 + (-3)^2} \\ &= \sqrt{169 + 9} \\ &= \sqrt{178} \end{aligned}$$

2. Find the distance between the two points.

$(-10, 3)$ and $(0, -15)$

Plug each pair of points into the distance formula.

$$\begin{aligned} d &= \sqrt{(-10 - 0)^2 + (3 - (-15))^2} \\ &= \sqrt{(-10)^2 + (18)^2} \\ &= \sqrt{100 + 324} \\ &= \sqrt{424} = 2\sqrt{106} \end{aligned}$$

3. Find the distance between the two points.

$(3, 1)$ and $(2, -7)$

Plug each pair of points into the distance formula.

$$\begin{aligned} d &= \sqrt{(3 - 2)^2 + (1 - (-7))^2} \\ &= \sqrt{(1)^2 + (8)^2} \\ &= \sqrt{1 + 64} \\ &= \sqrt{65} \end{aligned}$$

Example 4.37.1

Earlier, you were asked to use your knowledge of the Pythagorean Theorem to determine how far it is between your home and school.

Solution

Since you know that the trip to school involves walking 3 blocks east followed by 7 blocks north, you can construct a triangle on a coordinate system out of these lengths, like this:

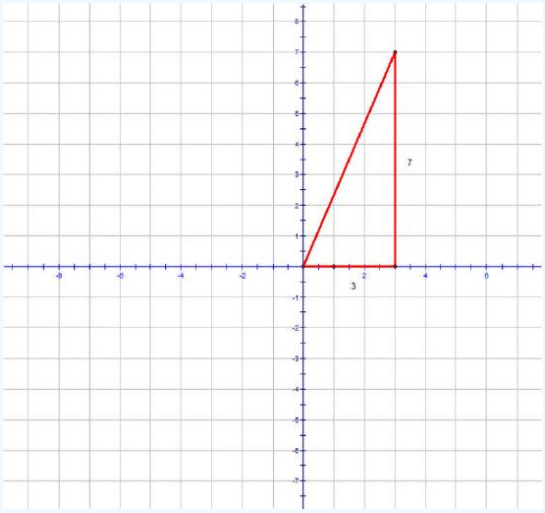

Figure 4.37.3

Since you went three blocks to the east, the school has an "x" coordinate of 3. Likewise, since you went 7 blocks north, the school has a "y" coordinate of 7. To find the straight-line distance to school, you can use the Distance Formula:

$$\begin{aligned} d &= \sqrt{(3-0)^2+(7-0)^2} \\ &= \sqrt{(3)^2+(7)^2} \\ &= \sqrt{58} \end{aligned}$$

This is a straight line distance of approximately 7.6 blocks.

Example 4.37.2

Find the distance between the two points.

\((3, 1) and \((2, -7)

Solution

Plug each pair of points into the distance formula.

\(\begin{align*}d&=\sqrt{(3−2)^2+(1−(−7))^2 \ &=\sqrt{(1)^2+(8)^2} \ &=\sqrt{1+64} \ &=\sqrt{65}\end{align*}\)

Example 4.37.3

Find the distance between the two points.

\((5, -8) and \((0, 3)

Solution

Plug each pair of points into the distance formula.

$$d = \sqrt{(5-0)^2 + (-8-(3))^2}$$
$$= \sqrt{(5)^2 + (-11)^2}$$
$$= \sqrt{25+121}$$
$$= \sqrt{146}$$

Example 4.37.4

Find the distance between the two points.

$(2, 6)$and $(2, 9)$

Solution

Plug each pair of points into the distance formula.

\(\begin{align*}d &=\sqrt{(2−2)^2+(6−9)^2} \ &=\sqrt{(0)^2+(−3)^2 \ &=\sqrt{9} \ =3\end{align*}\)

Review

Find the distance between each pair of points. Round each answer to the nearest tenth.

1. $(2, 4)$ and $(5, 10)$
2. $(1, 5)$ and $(8, 9)$
3. $(-2, 3)$ and $(6, 4)$
4. $(5, 7)$ and $(5, 10)$
5. $(8, 12)$ and $(15, 12)$
6. $(1, -4)$ and $(25, -2)$
7. $(5, -6)$ and $(3, 7)$
8. $(12, -9)$ and $(-1, 5)$
9. $(-3, 14)$ and $(8, 10)$
10. $(-11, 3)$ and $(-5, 1)$
11. $(5, 2)$ and $(11, 13)$
12. $(8, 10)$ and $(9, -6)$

Find the perimeter of each triangle. Round each answer to the nearest tenth.

13. $A(3, -5)$, $B(-5, -8)$, $C(-2, 7)$
14. $A(5, 3)$, $B(2, -7)$, $C(-1, 5)$
15. $A(1, 2)$, $B(1, 5)$, $C(4, 5)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.4.

Vocabulary

Term	Definition
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Additional Resources

Video: The Distance Formula

Practice: Distance Formula and the Pythagorean Theorem

Real World: It All Depends on Distance

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4.38: Distance Between Parallel Lines

Length of a perpendicular segment between parallel lines.

All vertical lines are in the form $x = a$, where a is the x-intercept. To find the distance between two vertical lines, count the squares between the two lines. You can use this method for horizontal lines as well. All horizontal lines are in the form $y = b$, where b is the y-intercept.

In general, the shortest distance between two parallel lines is the length of a **perpendicular** segment between them. There are infinitely many perpendicular segments between two parallel lines, but they will all be the same length.

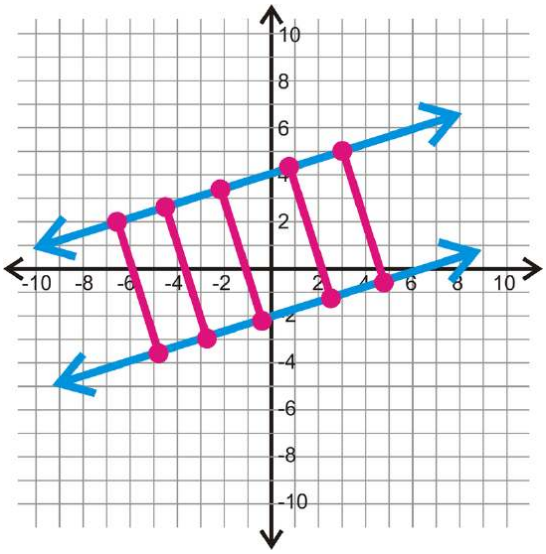

Figure 4.38.1

Remember that distances are always positive!

Example 4.38.1

Find the distance betwe $x = 3$ $x = -5$.

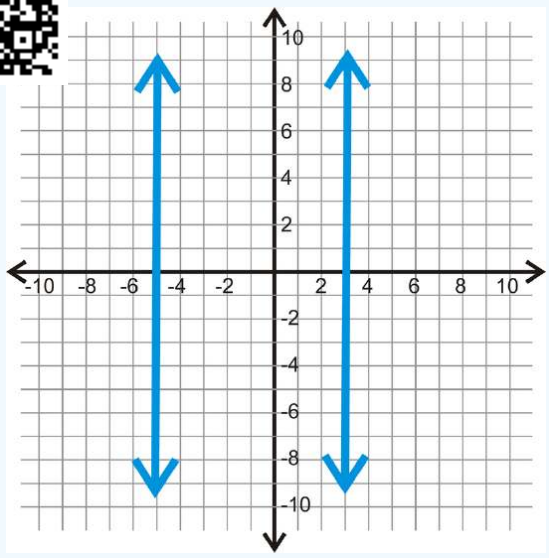

Figure 4.38.2

Solution

The two lines are $3–(–5)$ units apart, or 8 units apart.

Example 4.38.2

Find the distance between $x = -5$ and $x = -10$.

Solution

The two lines are $-5-(-10)$ units apart, or 5 units apart.

Example 4.38.3

Find the distance between $y = 5$ and $y = -8$.

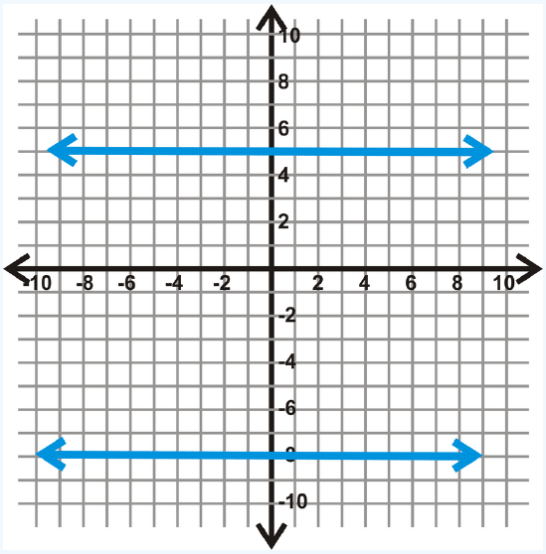

Figure 4.38.3

Solution

The two lines are $5-(-8)$ units apart, or 13 units apart.

Example 4.38.4

Find the distance between $y = x + 6$ and $y = x - 2$.

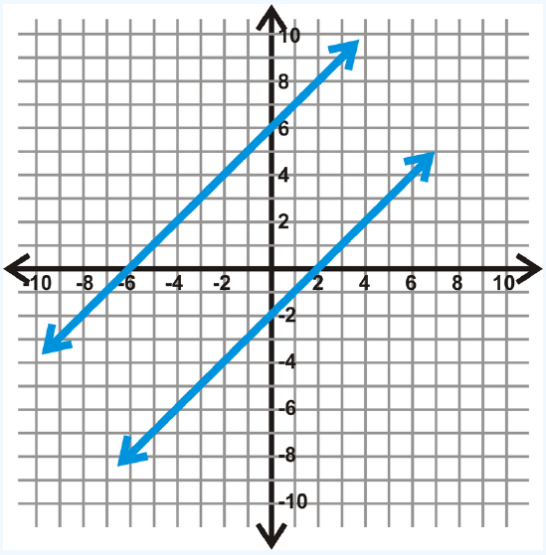

Figure 4.38.4

Solution

Step 1: Find the perpendicular slope.

$m = 1$, so $m_{\perp} = -1$.

Step 2: Find the y-intercept of the top line, $y = x + 6$.

The intercept is $(0, 6)$.

Step 3: Use the slope and count down 1 and to the right 1 until you hit $y = x - 2$.

Always rise/run the same amount for $m = 1$ or $m = -1$.

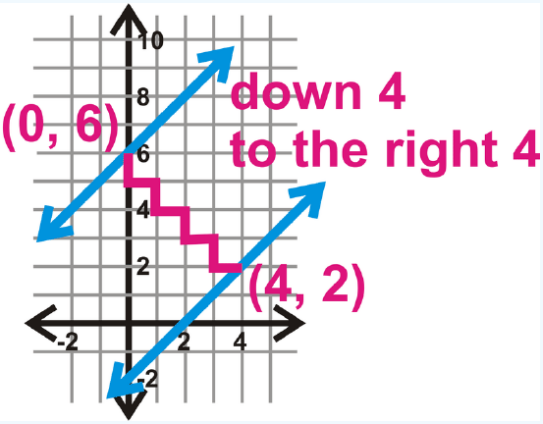

Figure 4.38.5

Step 4: Use these two points in the **distance formula** to determine how far apart the lines are.

$$\begin{aligned} d &= \sqrt{(0-4)^2 + (6-2)^2} \\ &= \sqrt{(-4)^2 + (4)^2} \\ &= \sqrt{16+16} \\ &= \sqrt{32} = 5.66 \; units \end{aligned}$$

Example 4.38.5

Find the distance between $y = -x - 1$, and $y = -x - 3$.

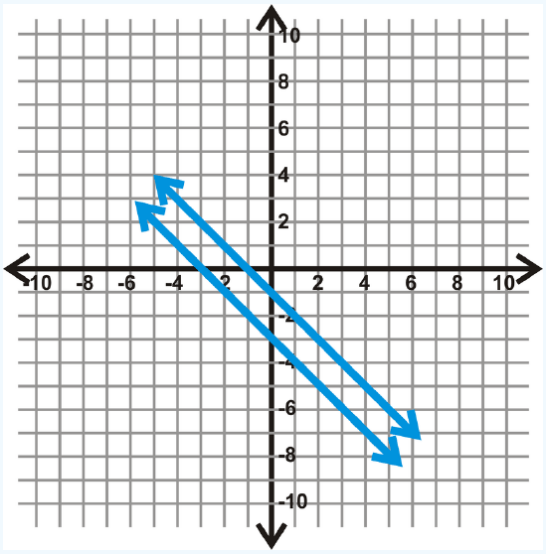

Figure 4.38.6

Solution

Step 1: Find the perpendicular slope.

$m = -1$, so $m_{\perp} = 1$.

Step 2: Find the y-intercept of the top line, $y = -x - 1$.

The intercept is $(0, -1)$.

Step 3: Use the slope and count down 1 and to the *left* 1 until you hit $y = x - 3$.

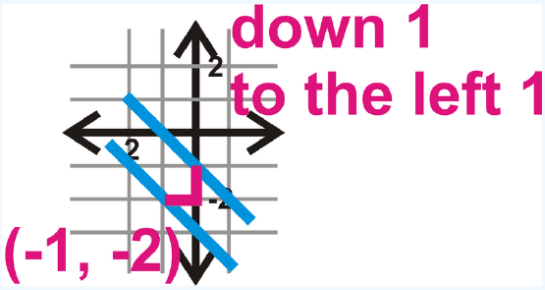

Figure 4.38.7

Step 4: Use these two points in the distance formula to determine how far apart the lines are.

$$\begin{aligned} d &= \sqrt{(0-(-1))^2 + (-1-(-2))^2} \\ &= \sqrt{(1)^2 + (1)^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2} = 1.41 \ units \end{aligned}$$

Review

Use each graph below to determine how far apart each pair of parallel lines is.

1.

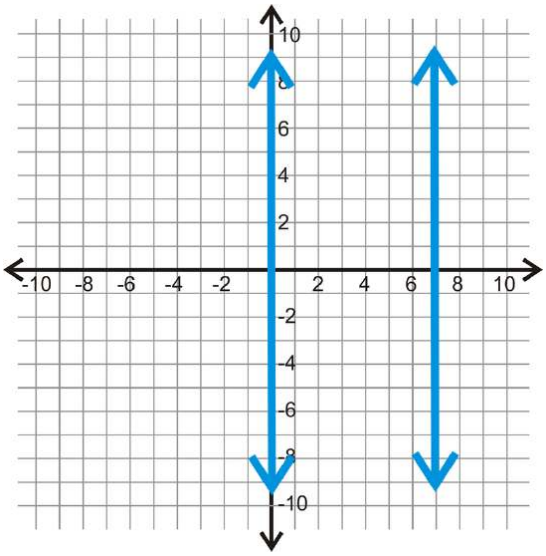

Figure 4.38.8

2.

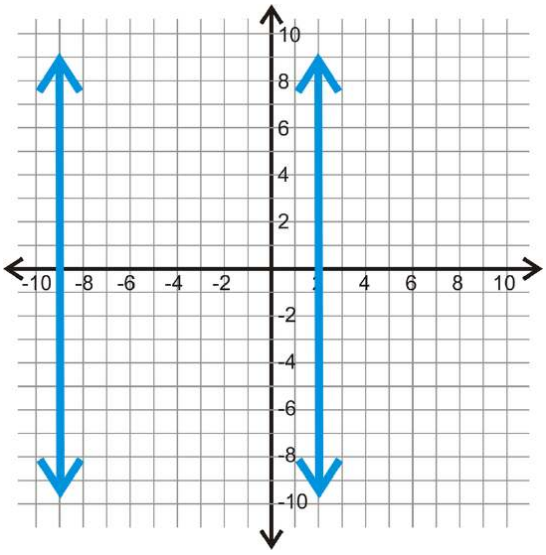

Figure 4.38.9

3.

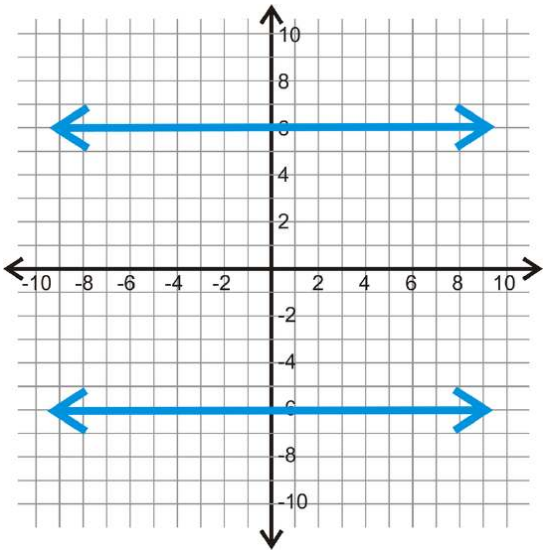

Figure 4.38.10

4.

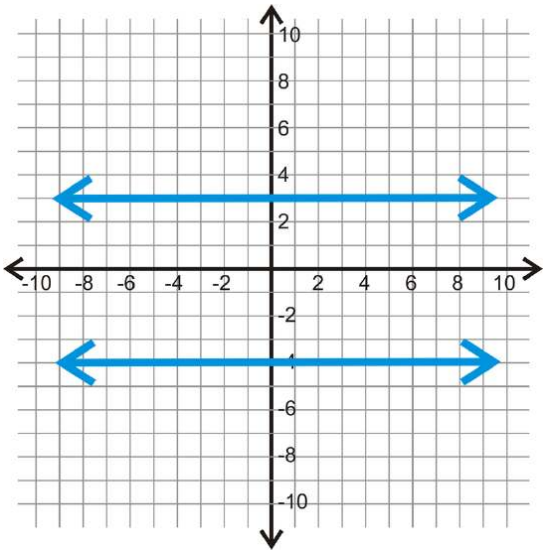

Figure 4.38.11

Determine the shortest distance between the each pair of parallel lines. Round your answer to the nearest hundredth.

5. $x = 5, x = 1$
6. $y = -6, y = 4$
7. $y = 3, y = 15$
8. $x = -10, x = -1$
9. $x = 8, x = 0$
10. $y = 7, y = -12$

Find the distance between the given parallel lines.

11. $y = x - 3, y = x + 11$
12. $y = -x + 4$, $y = -x$
13. $y = -x - 5$, $y = -x + 1$
14. $y = x + 12$, $y = x - 6$

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.11.

Resources

Vocabulary

Term	Definition
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Perpendicular	Perpendicular lines are lines that intersect at a $90°$ angle. The product of the slopes of two perpendicular lines is -1.

Additional Resources

Video: Finding The Distance Between Parallel Lines Principles - Basic

Activities: Distance Between Parallel Lines Discussion Questions

Study Aids: Lines in the Coordinate Plane

Practice: Distance Between Parallel Lines

Real World: Flipping Out

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4.39: The Distance Formula and Algebra

Using the Pythagorean Theorem to determine distances

Suppose you and your friend were on a scavenger hunt. Starting out from the same place, you walked 5 blocks east and 3 blocks north. Your friend walked 7 blocks west and 2 blocks south. If each block were a tenth of a mile long, could you calculate how far apart you and your friend were? How would you do it?

The Distance Formula

To understand the distance formula, we will first look at the following problem:

Find the length of the segment connecting $(1, 5)$ and $(5, 2)$.

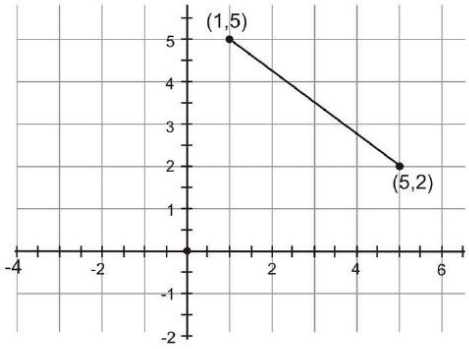

Figure 4.39.1

The question asks you to identify the length of the segment. Because the segment is not parallel to either axis, it is difficult to measure given the coordinate grid.

However, it is possible to think of this segment as the hypotenuse of a right triangle. Draw a vertical line and a horizontal line. Find the point of intersection. This point represents the third vertex in the right triangle.

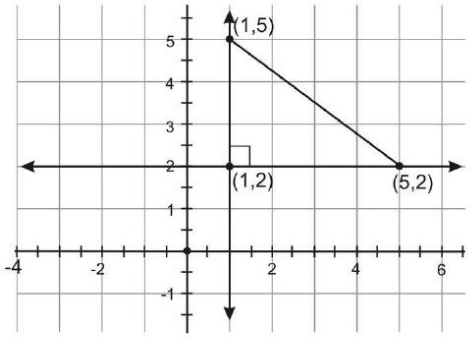

Figure 4.39.2

You can easily count the lengths of the legs of this triangle on the grid. The vertical leg extends from $(1, 2)$ to $(1, 5)$, so it is $|5 - 2| = |3| = 3$ *units* long. The horizontal leg extends from $(1, 2)$ to $(5, 2)$, so it is $|5 - 1| = |4| = 4$ *units* long. Use the Pythagorean Theorem with these values for the lengths of each leg to find the length of the hypotenuse.

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = \sqrt{c^2}$$
$$5 = c$$

The segment connecting $(1, 5)$ and $(5, 2)$ is 5 units long.

Mathematicians have simplified this process and created a formula that uses these steps to find the distance between any two points in the coordinate plane. If you use the distance formula, you don't have to draw the extra lines.

The **distance formula** states: Given points (x_1, y_1) and (x_2, y_2), the length of the segment connecting those two points is $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$.

Let's use the distance formula to complete the following problems:

1. Find the distance between $(-3, 5)$ and $(4, -2)$.

Use the distance formula. Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (4, -2)$.

$$d = \sqrt{(-2 - 5)^2 + (4 - (-3))^2} \rightarrow \sqrt{(-7)^2 + 7^2}$$
$$d = \sqrt{98} = 7\sqrt{2} \text{ units}$$

2. At 8 a.m. one day, Amir decides to walk in a straight line on the beach. After two hours of making no turns and traveling at a steady rate, Amir was two miles east and four miles north of his starting point. How far did Amir walk and what was his walking speed?

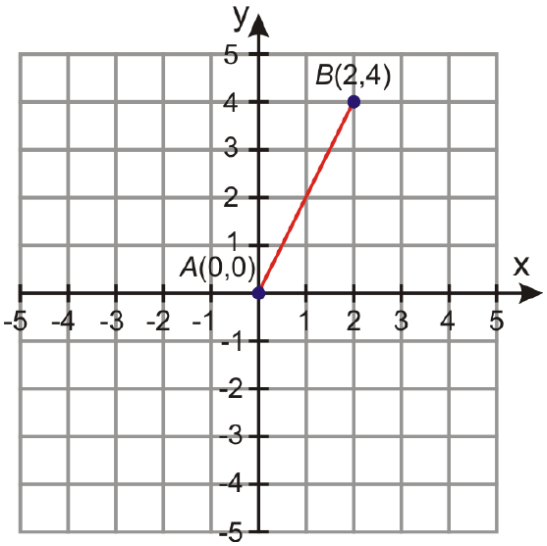

Figure **4.39.3**

Plot Amir's route on a coordinate graph. We can place his starting point at the origin, $A = (0, 0)$. Then, his ending point will be at the point $B = (2, 4)$. The distance can be found with the distance formula.

$$d = \sqrt{(4 - 0)^2 + (2 - 0)^2} = \sqrt{(4)^2 + (2)^2} = \sqrt{16 + 4} = \sqrt{20}.$$
$$d = 4.47 \text{ miles}$$

Since Amir walked 4.47 miles in 2 hours, his speed is:

$$Speed = \frac{4.47 \text{ miles}}{2 \text{ hours}} = 2.24\, mi/h$$

Example 4.39.1

Earlier, you were told that on a scavenger hunt, starting out from the same place, you walked 5 blocks east and 3 blocks north and your friend walked 7 blocks west and 2 blocks south. Each block is a tenth of a mile long. How far apart were you and your friend?

Solution

It may be helpful to draw a graph to visualize the situation. Assuming that the starting point was at (0,0) and the distance between each point is equal to one block, if you walked 5 blocks east and 3 blocks north, your coordinates would be $(-5, 3)$. Your friend walked 7 blocks west and 2 blocks south, so their coordinates would be $(7, -2)$

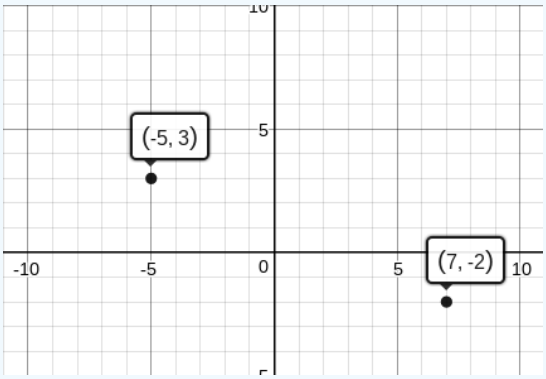

Figure 4.39.4

Now, you can calculate the distance between these two points using the distance formula. All you need to do is plug the points into the formula and solve.

\(\begin{aligned} d&=\sqrt{\left(y_{2}-y_{1}\right)^{2}+\left(x_{2}-x_{1}\right)^{2}} \

d&=\sqrt{(−2−3)^2+(7−(−5))^2}\rightarrow \sqrt{(−5)^2+12^2} \

d&=\sqrt{169}=13\end{aligned}\)

You and your friend are 13 units away from each other. Since each block is a tenth of a mile long, you can multiply 13 by one tenth to get the actual distance.

$$13 \times \frac{1}{10} = \frac{13}{10} = 1.3$$

You and your friend are 1.3 miles away from each other.

Example 4.39.2

Point $A = (6, -4)$ and point $B = (2, k)$. What is the value of k such that the distance between the two points is 5?

Solution

Use the distance formula.

$$d = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2} \Rightarrow 5 = \sqrt{(4 - k)^2 + (6 - 2)^2}$$

\(\begin{aligned} Square\: both \:sides \:of \:the \:equation. &\qquad& 5^2&=[\sqrt{(4−k)^2+(6−2)^2}]^2 \

Simplify.&\qquad& 25&=(−4−k)^2+16 \ Eliminate\: the \:parentheses. &\qquad& 0&=k^2+8k+16−9 \

Simplify.&\qquad& 0&=k^2+8k+7 \

Find \:k \:using \:the \:quadratic \:formula. & \qquad& k&=\dfrac{−8\pm\sqrt{64−28}}{2}=\dfrac{−8\pm\sqrt{36}}{2}=\dfrac{−8\pm6}{2}\end{aligned}\)

$k = -7$ or $k = -1$. There are two possibilities for the value of k. Let's graph the points to get a visual representation of our results.

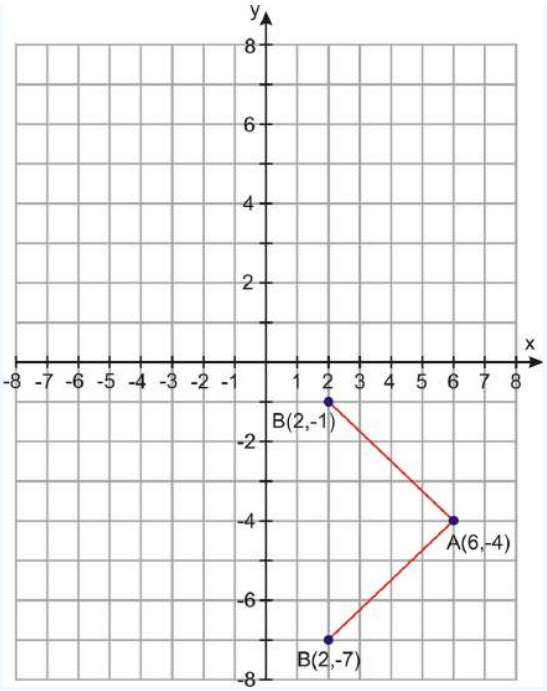

Figure 4.39.5

Review

In 1–10, find the distance between the two points.

1. (x_1, y_1) and (x_2, y_2)
2. $(7, 7)$ and $(-7, 7)$
3. $(-3, 6)$ and $(3, -6)$
4. $(-3, -1)$ and $(-5, -8)$
5. $(3, -4)$ and $(6, 0)$
6. $(-1, 0)$ and $(4, 2)$
7. $(-3, 2)$ and $(6, 2)$
8. $(0.5, -2.5)$ and $(4, -4)$
9. $(12, -10)$ and $(0, -6)$
10. $(2.3, 4.5)$ and $(-3.4, -5.2)$
11. Find all points having an x-coordinate of –4 and whose distance from point $(4, 2)$ is 10.
12. Find all points having a y-coordinate of 3 and whose distance from point $(-2, 5)$ is 8.
13. Michelle decides to ride her bike one day. First she rides her bike due south for 12 miles, and then the direction of the bike trail changes and she rides in the new direction for a while longer. When she stops, Michelle is 2 miles south and 10 miles west of her starting point. Find the total distance that Michelle covered from her starting point.

Mixed Review

14. Solve $(x - 4)^2 = 121$.
15. What is the GCF of $21ab^4$ and $15a^7b^2$?
16. Evaluate ${}_{10}C_7$ and explain its meaning.
17. Factor $6x^2 + 17x + 5$.
18. Find the area of a rectangle with a length of $(16 + 2m)$ and a width of $(12 + 2m)$.
19. Factor $x^2 - 81$.

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.7.

Vocabulary

Term	Definition
Distance Formula	The distance between two points \)(x_1,y_1)\) and \)(x_2,y_2)\) can be defined as \)d=\sqrt{(x_2−x_1)^2+(y_2−y_1)^2}\).
Midpoint Formula	The midpoint formula says that for endpoints (x_1, y_1) and (x_2, y_2), the midpoint is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

Additional Resources

Video: Distance Formula

Activities: Distance Formula Discussion Questions

Practice: The Distance Formula and Algebra

Real World: SF Map - PythThm

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4.40: Applications of the Distance Formula

Length between two points based on a right triangle.

Distance Formula in the Coordinate Plane

The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This is called the **distance formula**. Remember that distances are always positive!

What if you were given the coordinates of two points? How could you find how far apart these two points are?

Example 4.40.1

Find the distance betwe $(-2, -3)$ $(3, 9)$.

Solution

Use the distance formul , and simplify.

$$\begin{aligned} d &= \sqrt{(3-(-2))^2 + (9-(-3))^2} \\ &= \sqrt{(5)^2 + (12)^2} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} = 13 \text{ units} \end{aligned}$$

Example 4.40.2

Find the distance between $(12, 26)$ and $(8, 7)$.

Solution

Use the distance formula, plug in the points, and simplify.

$$\begin{aligned} d &= \sqrt{(8-12)^2 + (7-26)^2} \\ &= \sqrt{(-4)^2 + (-19)^2} \\ &= \sqrt{16 + 361} \\ &= \sqrt{377} \approx 19.42 \text{ units} \end{aligned}$$

Example 4.40.3

Find the distance between $(4, -2)$ and $(-10, 3)$.

Solution

Plug in $(4, -2)$ for (x_1, y_1) and $(-10, 3)$ for (x_2, y_2) and simplify.

$$\begin{aligned} d &= \sqrt{(-10-4)^2 + (3+2)^2} \\ &= \sqrt{(-14)^2 + (5)^2} \\ &= \sqrt{196 + 25} \\ &= \sqrt{221} \approx 14.87 \text{ units} \end{aligned}$$

Example 4.40.4

Find the distance between $(3, 4)$ and $(-1, 3)$.

Solution

Plug in (3, 4)\) for (x_1, y_1) and $(-1, 3)$ for (x_2, y_2) and simplify.

$$\begin{aligned} d &= \sqrt{(-1-3)^2+(3-4)^2} \\ &= \sqrt{(-4)^2+(-1)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \approx 4.12 \text{ units} \end{aligned}$$

Example 4.40.5

Find the distance between $(4, 23)$ and $(8, 14)$.

Solution

Plug in $(4, 23)$ for (x_1, y_1) and $(8, 14)$ for (x_2, y_2) and simplify.

$$\begin{aligned} d &= \sqrt{(8-4)^2+(14-23)^2} \\ &= \sqrt{(4)^2+(-9)^2} \\ &= \sqrt{16+81} \\ &= \sqrt{97} \approx 9.85 \text{ units} \end{aligned}$$

Review

Find the distance between each pair of points. Round your answer to the nearest hundredth.

1. $(4, 15)$ and $(-2, -1)$
2. $(-6, 1)$ and $(9, -11)$
3. $(0, 12)$ and $(-3, 8)$
4. $(-8, 19)$ and $(3, 5)$
5. $(3, -25)$ and $(-10, -7)$
6. $(-1, 2)$ and$(8, -9)$
7. $(5, -2)$ and $(1, 3)$
8. $(-30, 6)$ and $(-23, 0)$
9. $(2, -2)$ and $(2, 5)$
10. $(-9, -4)$ and $(1, -1)$

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.10.

Resource

Vocabulary

Term	Definition
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2+b^2=c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Video: The Distance Formula

Activities: Distance Formula in the Coordinate Plane Discussion Questions

Study Aids: Segments Study Guide

Practice: Applications of the Distance Formula

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4.41: Special Right Triangles and Ratios

Properties of 30-60-90 and 45-45-90 triangles.

The **Pythagorean Theorem** is great for finding the third side of a right triangle when you already know two other sides. There are some triangles like 30-60-90 and 45-45-90 triangles that are so common that it is useful to know the side ratios without doing the Pythagorean Theorem each time. Using these patterns also allows you to totally solve for the missing sides of these special triangles when you only know one side length.

Given a 45-45-90 right triangle with sides 6 inches, 6 inches and x inches, what is the value of x?

Special Right Triangles

There are three types of special right triangles, 30-60-90 triangles, 45-45-90 triangles, and Pythagorean triple triangles.

30-60-90 Triangles

A 30-60-90 right triangle has side ratios x, $x\sqrt{3}$, $2x$.

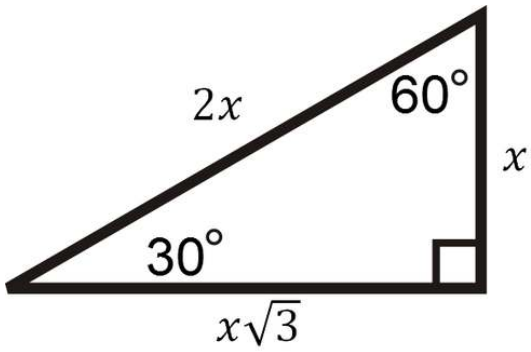

Figure 4.41.1

Confirm with Pythagorean Theorem:

$$\begin{aligned} x^2 + (x\sqrt{3})^2 &= (2x)^2 \\ x^2 + 3x^2 &= 4x^2 \\ 4x^2 &= 4x^2 \end{aligned}$$

45-45-90 Triangles

A 45-45-90 right triangle has side ratios x, x, $x\sqrt{2}$.

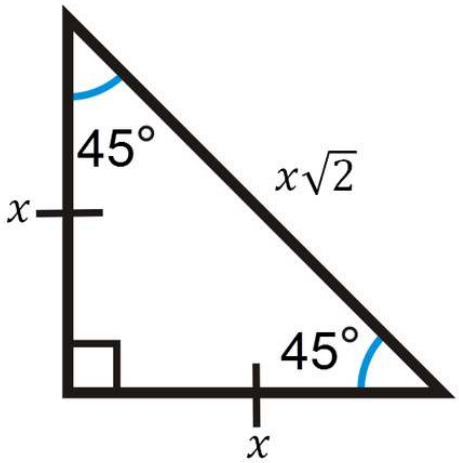

Figure 4.41.2

Confirm with Pythagorean Theorem:

$$x^2 + x^2 = (x\sqrt{2})^2$$
$$2x^2 = 2x^2$$

Note that the order of the side ratios $x, x\sqrt{3}, 2x$ and $x, x, x\sqrt{2}$ is important because each side ratio has a corresponding angle. In all triangles, the smallest sides correspond to smallest angles and largest sides always correspond to the largest angles.

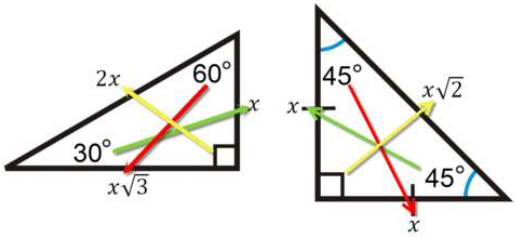

Figure **4.41.3**

Pythagorean Triple Triangles

Pythagorean number triples are special right triangles with integer sides. While the angles are not integers, the side ratios are very useful to know because they show up everywhere. Knowing these number triples also saves a lot of time from doing the Pythagorean Theorem repeatedly. Here are some examples of Pythagorean number triples:

- 3, 4, 5
- 5, 12, 13
- 7, 24, 25
- 8, 15, 17
- 9, 40, 41

More Pythagorean number triples can be found by scaling any other **Pythagorean number triple**. For example:

$3, 4, 5 \rightarrow 6, 8, 10$ (scaled by a factor of 2)

Even more Pythagorean number triples can be found by taking any odd integer like 11, squaring it to get 121, halving the result to get 60.5. The original number 11 and the two numbers that are 0.5 above and below (60 and 61) will always be a Pythagorean number triple.

$11^2 + 60^2 = 61^2$

Example 4.41.1

Earlier you were asked about a 45-45-90 right triangle with sides 6 inches, 6 inches and x inches.

Solution

If you can recognize the pattern for 45-45-90 right triangles, a right triangle with legs 6 inches and 6 inches has a hypotenuse that is $6\sqrt{2}$ inches. $x = 6\sqrt{2}$.

Example 4.41.2

A 30-60-90 right triangle has hypotenuse of length 10. What are the lengths of the other two sides?

Solution

The hypotenuse is the side opposite 90. Sometimes it is helpful to draw a picture or make a table.

30	60	90
x	$x\sqrt{3}$	$2x$
		10

From the table you can write very small subsequent equations to solve for the missing sides.

$$10 = 2x$$
$$x = 5$$
$$x\sqrt{3} = 5\sqrt{3}$$

The other sides are 5 and $5\sqrt{3}$.

Example 4.41.3

A 30-60-90 right triangle has a side length of 18 inches corresponding to 60 degrees. What are the lengths of the other two sides?

Solution

Make a table with the side ratios and the information given, then write equations and solve for the missing side lengths.

30	60	90
x	$x\sqrt{3}$	$2x$
	18	

$$18 = x\sqrt{3}$$
$$\frac{18}{\sqrt{3}} = x$$
$$x = 18\sqrt{3} = 18\sqrt{3} \cdot \sqrt{3}\sqrt{3} = 18\sqrt{33} = 6\sqrt{3}$$
$$x = 6\sqrt{3}$$

Note that you need to rationalize denominators.

Now use the calculated x value to solve for $2x$.

$$2x = 2(6\sqrt{3})$$
$$2x = 12\sqrt{3}$$

The other sides are $6\sqrt{3}$ and $12\sqrt{3}$.

Example 4.41.4

Using your knowledge of special right triangle ratios, solve for the missing sides of the right triangle.

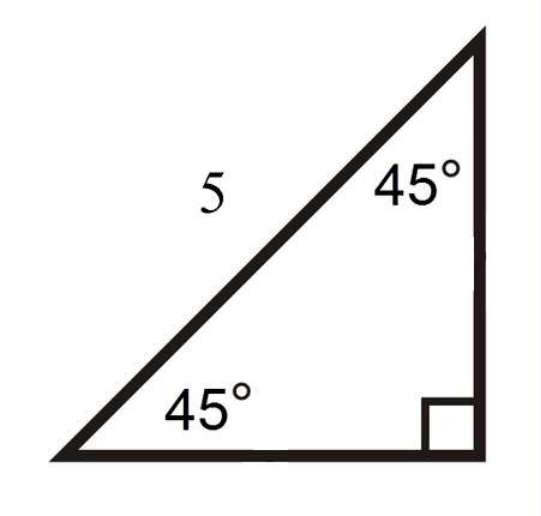

Figure 4.41.4

Solution

The other sides are each $\frac{5\sqrt{2}}{2}$.

45	45	90
x	x	$x\sqrt{2}$
		5

$$x\sqrt{2} = 5$$
$$x = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

The other sides are each $\frac{5\sqrt{2}}{2}$.

Example 4.41.5

Using your knowledge of special right triangle ratios, solve for the missing sides of the right triangle.

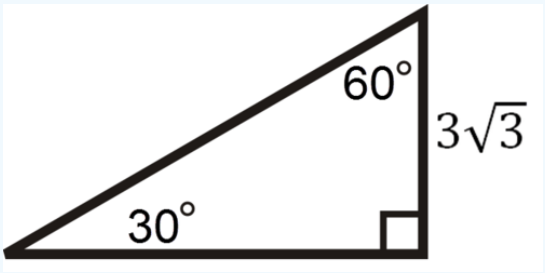

Figure 4.41.5

Solution

The other sides are 9 and $6\sqrt{3}$.

30	60	90
x	$x\sqrt{3}$	$2x$
\(3\sqrt{3}		

$$x = 3\sqrt{3}$$
$$2x = 6\sqrt{3}$$
$$x\sqrt{3} = 3\sqrt{3} \cdot \sqrt{3} = 9$$

The other sides are 9 and $6\sqrt{3}$.

Review

For 1-4, find the missing sides of the **45-45-90 triangle** based on the information given in each row.

Problem Number	Side Opposite 45°	Side Opposite 45°	Side Opposite 90°
1.	3		
2.		7.2	
3.			16
4.	$5\sqrt{2}$		

For 5-8, find the missing sides of the **30-60-90 triangle** based on the information given in each row.

Problem Number	Side Opposite $30°$	Side Opposite $60°$	Side Opposite $90°$
5.	$3\sqrt{2}$		
6.		4	
7.			15
8.			$12\sqrt{3}$

Use the picture below for 9-11.

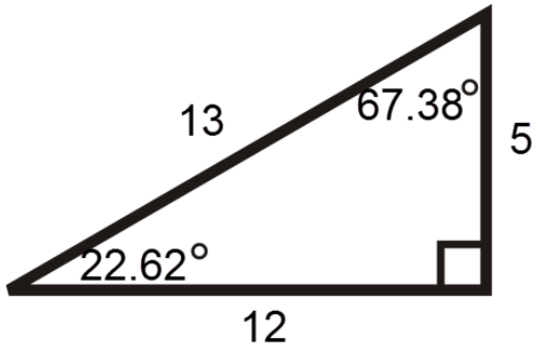

Figure $4.41.6$

9. Which angle corresponds to the side that is 12 units?

10. Which side corresponds to the right angle?

11. Which angle corresponds to the side that is 5 units?

12. A right triangle has an angle of $\frac{\pi}{6}$ radians and a hypotenuse of 20 inches. What are the lengths of the other two sides of the triangle?

13. A triangle has two angles that measure $\frac{\pi}{4}$ radians. The longest side is 3 inches long. What are the lengths of the other two sides?

For 14-19, verify the Pythagorean Number Triple using the Pythagorean Theorem.

14. 3, 4, 5

15. 5, 12, 13

16. 7, 24, 25

17. 8, 15, 17

18. 9, 40, 41

19. 6, 8, 10

20. Find another Pythagorean Number Triple using the method explained for finding “11, 60, 61”.

Vocabulary

Term	Definition
30-60-90 Triangle	A 30-60-90 triangle is a special right triangle with angles of $30°$, $60°$, and $90°$.
45-45-90 Triangle	A 45-45-90 triangle is a special right triangle with angles of $45°$, $45°$, and $90°$.
Pythagorean number triple	A Pythagorean number triple is a set of three whole numbers a,b and c that satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Video: Solving Special Right Triangles

Practice: Special Right Triangles and Ratios

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4.42: 45-45-90 Right Triangles

Leg times $\sqrt{2}$ equals hypotenuse.

A right triangle with congruent legs and acute angles is an **Isosceles Right Triangle**. This triangle is also called a **45-45-90 triangle** (named after the angle measures).

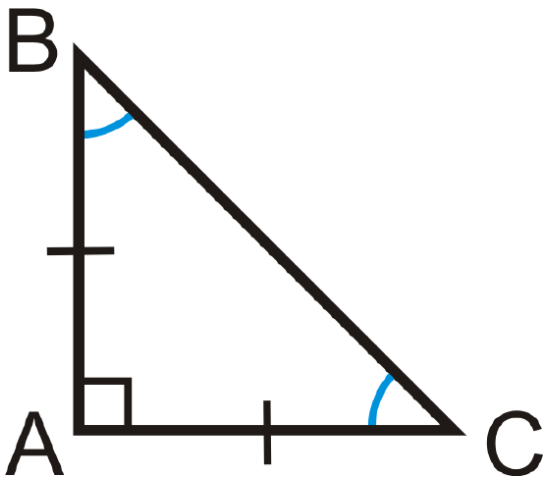

Figure 4.42.1

ΔABC is a right triangle with $m\angle A = 90°$, $\overline{AB} \cong \overline{AC}$ and $m\angle B = m\angle C = 45°$.

45-45-90 Theorem: If a right triangle is isosceles, then its sides are in the ratio $x : x : x\sqrt{2}$. For any isosceles right triangle, the legs are x and the **hypotenuse** is always $x\sqrt{2}$.

What if you were given an isosceles right triangle and the length of one of its sides? How could you figure out the lengths of its other sides?

Example 4.42.1

Find the length of x.

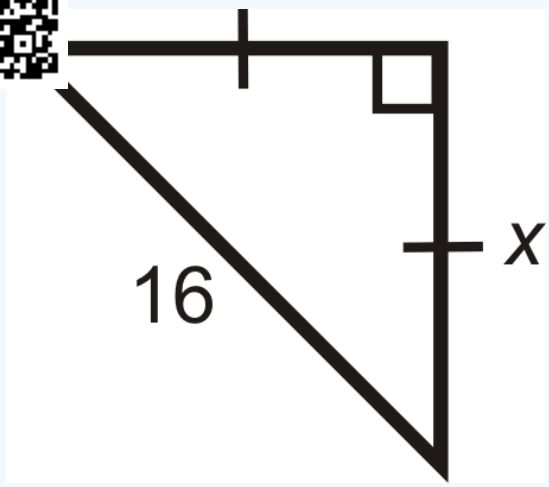

Figure 4.42.2

Solution

Use the $x : x : x\sqrt{2}$ ratio.

Here, we are given the hypotenuse. Solve for x in the ratio.

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$x\sqrt{2} = 16$

$x = \frac{16}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$

Example 4.42.2

Find the length of x, where x is the hypotenuse of a 45-45-90 triangle with leg lengths of $5\sqrt{3}$.

Solution

Use the $x : x : x\sqrt{2}$ ratio.

$x = 5\sqrt{3} \cdot \sqrt{2} = 5\sqrt{6}$

Example 4.42.3

Find the length of the missing side.

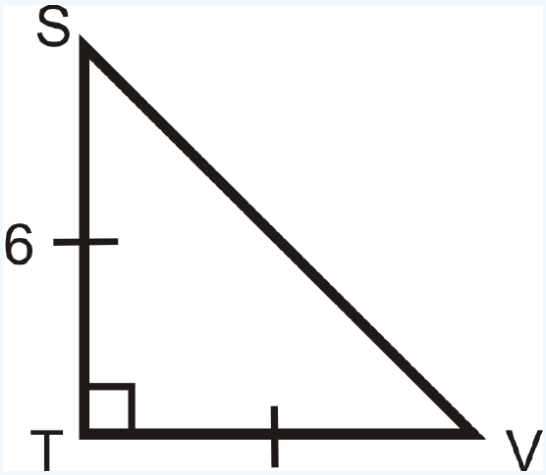

Figure 4.42.3

Solution

Use the $x : x : x\sqrt{2}$ ratio. $TV = 6$ because it is equal to ST. So, $SV = 6 \cdot \sqrt{2} = 6\sqrt{2}$.

Example 4.42.4

Find the length of the missing side.

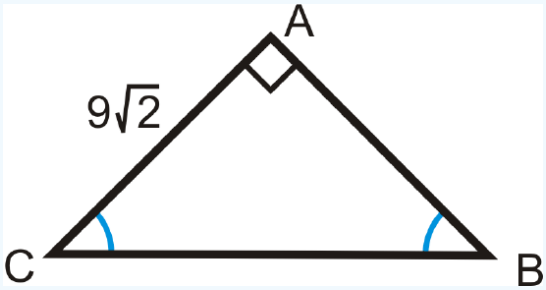

Figure 4.42.4

Solution

Use the $x : x : x\sqrt{2}$ ratio. $AB = 9\sqrt{2}$ because it is equal to AC. So, $BC = 9\sqrt{2} \cdot \sqrt{2} = 9 \cdot 2 = 18$.

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Example 4.42.5

A square has a diagonal with length 10, what are the lengths of the sides?

Solution

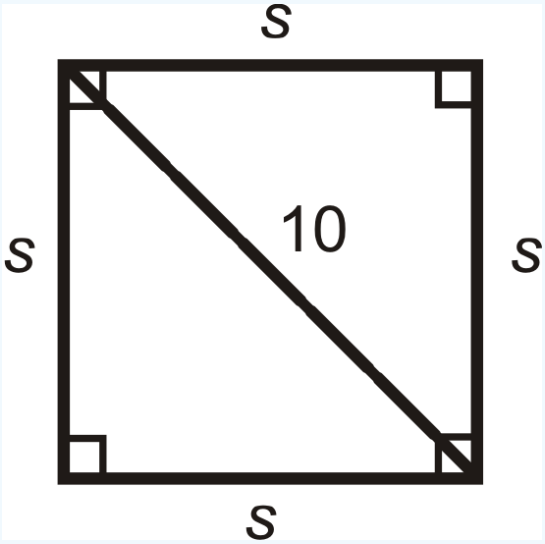

Figure 4.42.5

We know half of a square is a 45-45-90 triangle, so $10 = s\sqrt{2}$.

$s\sqrt{2} = 10$

$s = 10\sqrt{2} \cdot \sqrt{2}\sqrt{2} = 10\sqrt{22} = 5\sqrt{2}$

Review

1. In an isosceles right triangle, if a leg is 4, then the hypotenuse is _____.
2. In an isosceles right triangle, if a leg is x, then the hypotenuse is _____.
3. A square has sides of length 15. What is the length of the diagonal?
4. A square's diagonal is 22. What is the length of each side?

For questions 5-11, find the lengths of the missing sides. Simplify all radicals.

5.

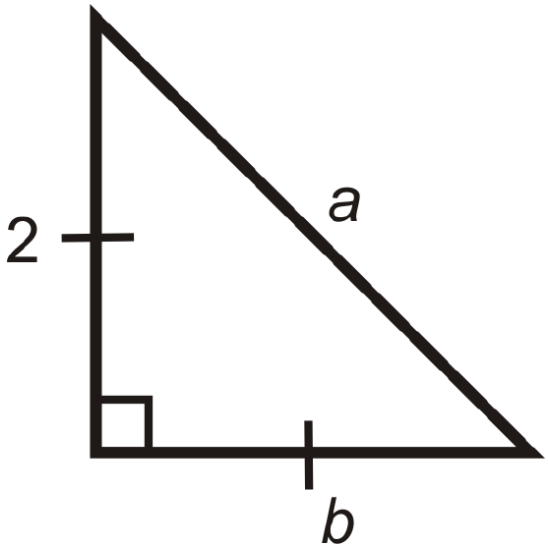

Figure 4.42.6

6.

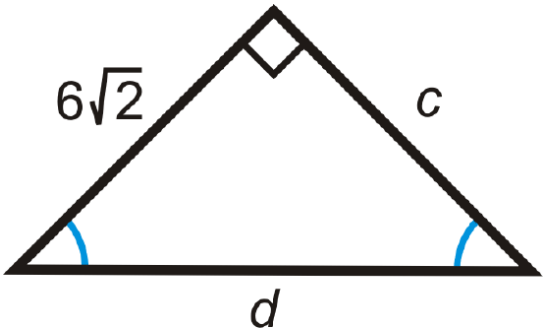

Figure 4.42.7

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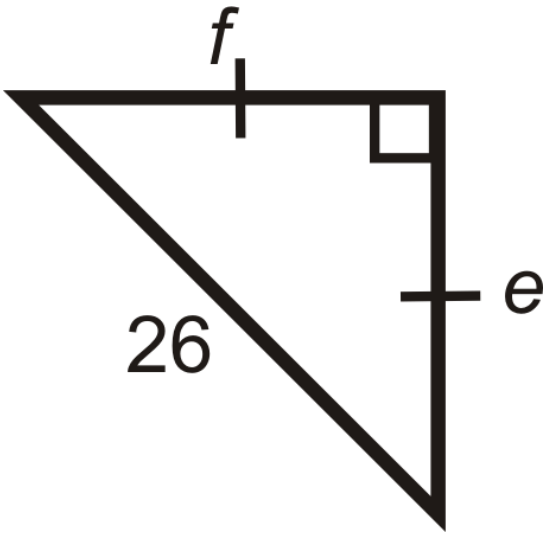

Figure 4.42.8

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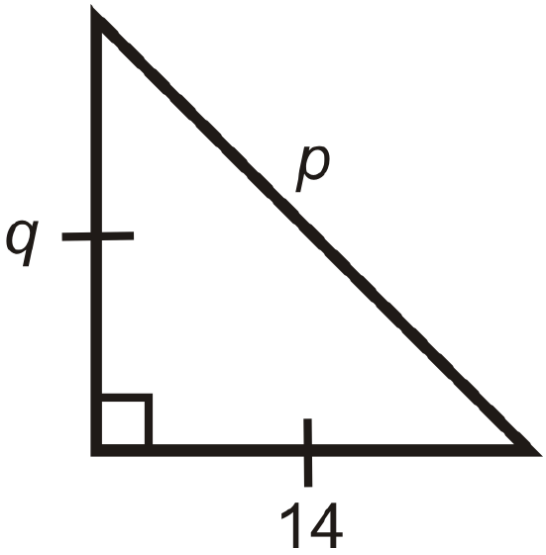

Figure 4.42.9

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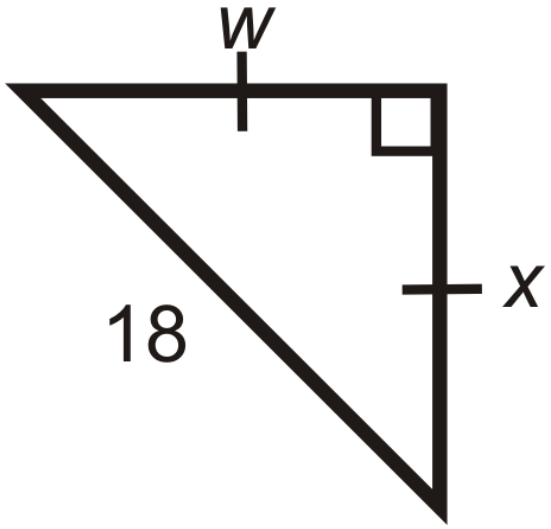

Figure 4.42.10

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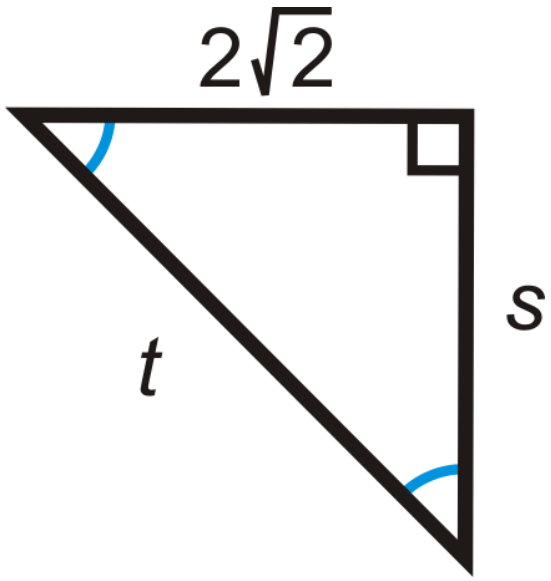

Figure 4.42.11

11.

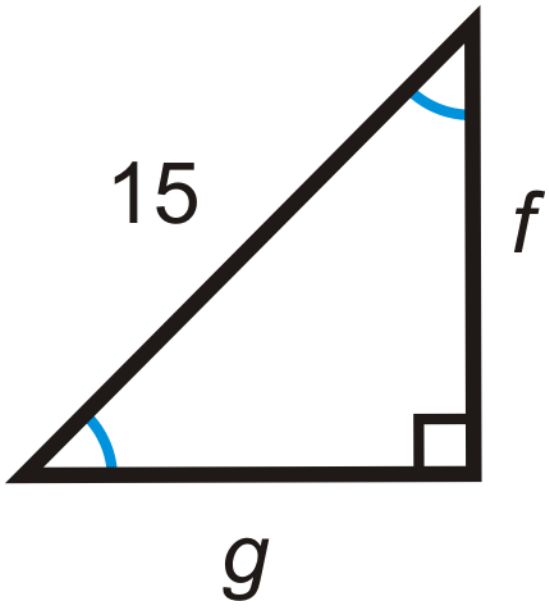

Figure 4.42.12

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.5.

Resources

Vocabulary

Term	Definition
45-45-90 Theorem	For any isosceles right triangle, if the legs are x units long, the hypotenuse is always $x\sqrt{2}$.
45-45-90 Triangle	A 45-45-90 triangle is a special right triangle with angles of **45°**, **45°**, and **90°**.
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Radical	The Missing argument for \sqrt, or square root, sign.

Additional Resources

Video: Solving Special Right Triangles

Activities: 45-45-90 Right Triangles Discussion Questions

Study Aids: Special Right Triangles Study Guide

Practice: 45-45-90 Right Triangles

Real World: Fighting the War on Drugs Using Geometry and Special Triangles

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4.43: 30-60-90 Right Triangles

Hypotenuse equals twice the smallest leg, while the larger leg is $\sqrt{3}$ times the smallest.

One of the two special right triangles is called a **30-60-90 triangle**, after its three angles.

30-60-90 Theorem: If a triangle has angle measures **30°**, **60°** and **90°**, then the sides are in the ratio $x : x\sqrt{3} : 2x$.

The shorter leg is always x, the longer leg is always $x\sqrt{3}$, and the **hypotenuse** is always $2x$. If you ever forget these theorems, you can still use the **Pythagorean Theorem**.

What if you were given a 30-60-90 right triangle and the length of one of its side? How could you figure out the lengths of its other sides?

Example 4.43.1

Find the value of x and y.

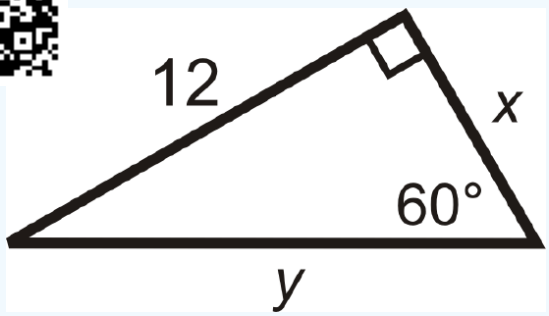

Figure 4.43.1

Solution

We are given the longer leg.

$$x\sqrt{3} = 12$$
$$x = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

The hypotenuse is

$$y = 2(4\sqrt{3}) = 8\sqrt{3}$$

Example 4.43.2

Find the value of x and y.

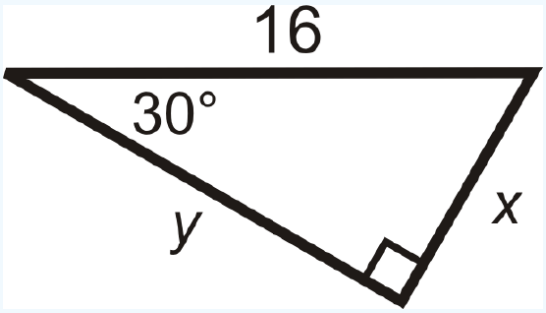

Figure 4.43.2

Solution

We are given the hypotenuse.

$2x = 16$
$x = 8$
The longer leg is
$y = 8 \cdot \sqrt{3} = 8\sqrt{3}$

Example 4.43.3

Find the length of the missing sides.

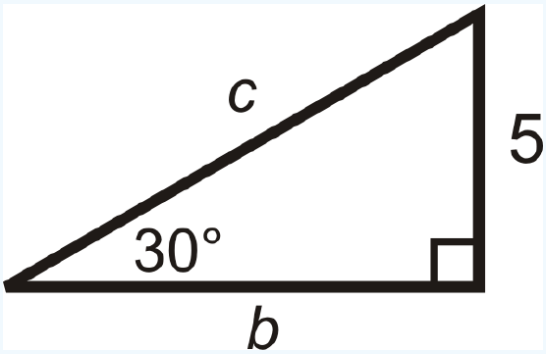

Figure 4.43.3

Solution

We are given the shorter leg. If $x = 5$, then the longer leg, $b = 5\sqrt{3}$, and the hypotenuse, $c = 2(5) = 10$.

Example 4.43.4

Find the length of the missing sides.

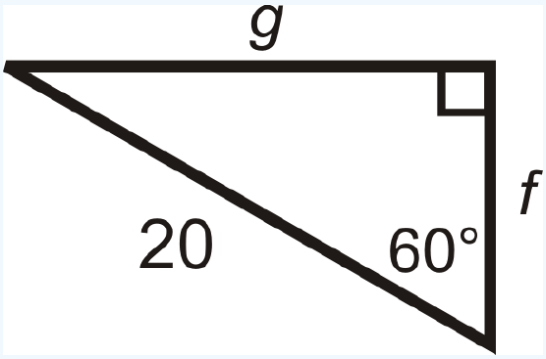

Figure 4.43.4

Solution

We are given the hypotenuse. $2x = 20$, so the shorter leg, $f = \frac{20}{2} = 10$, and the longer leg, $g = 10\sqrt{3}$.

Example 4.43.5

A rectangle has sides 4 and $4\sqrt{3}$. What is the length of the diagonal?

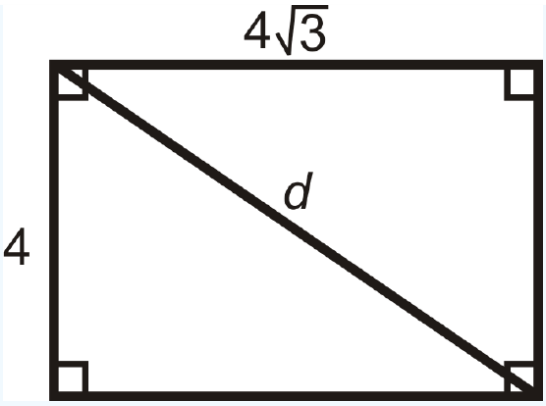

Figure 4.43.5

Solution

The two lengths are x, $x\sqrt{3}$, so the diagonal would be $2x$, or $2(4) = 8$.

If you did not recognize this is a 30-60-90 triangle, you can use the Pythagorean Theorem too.

$$4^2 + (4\sqrt{3})^2 = d^2$$
$$16 + 48 = d^2$$
$$d = \sqrt{64} = 8$$

Review

1. In a 30-60-90 triangle, if the shorter leg is 5, then the longer leg is _____ and the hypotenuse is _____.
2. In a 30-60-90 triangle, if the shorter leg is x, then the longer leg is _____ and the hypotenuse is _____.
3. A rectangle has sides of length 6 and $6\sqrt{3}$. What is the length of the diagonal?
4. Two (opposite) sides of a rectangle are 10 and the diagonal is 20. What is the length of the other two sides?

For questions 5-12, find the lengths of the missing sides. Simplify all radicals.

5.

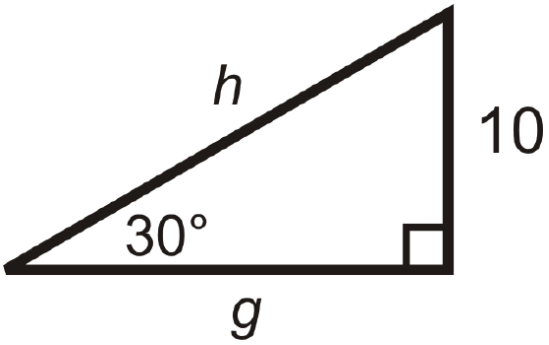

Figure 4.43.6

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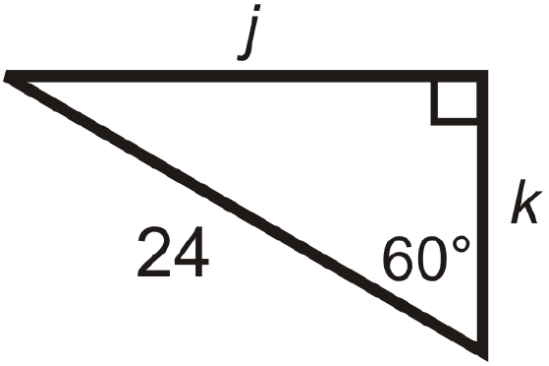

Figure 4.43.7

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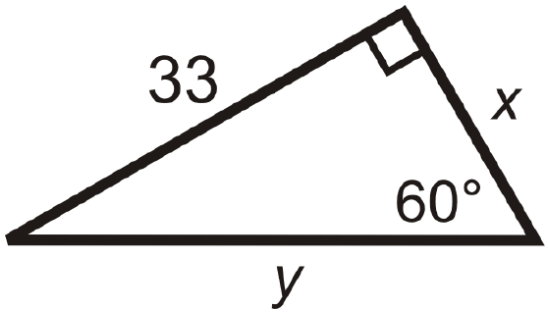

Figure 4.43.8

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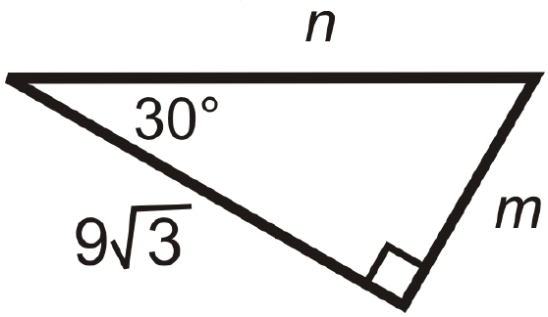

Figure 4.43.9

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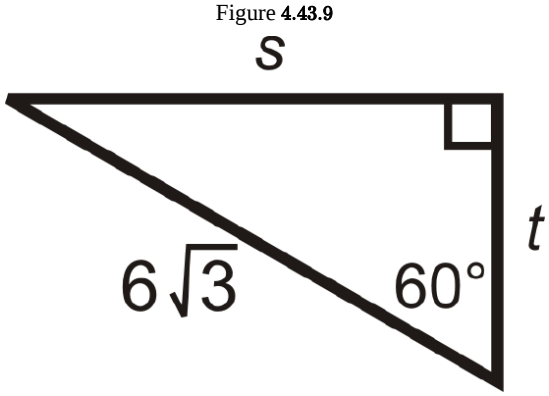

Figure 4.43.10

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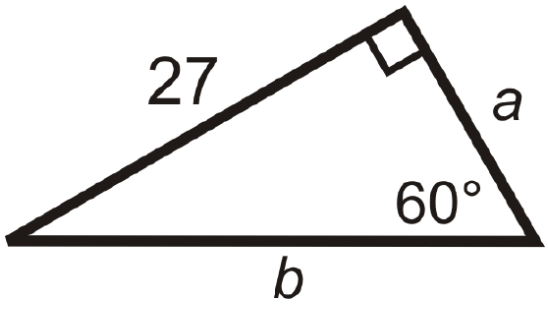

Figure 4.43.11

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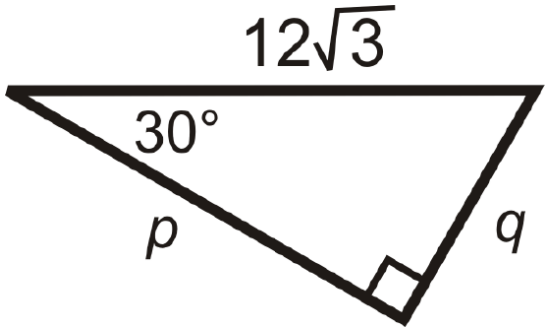

Figure **4.43.12**

12.

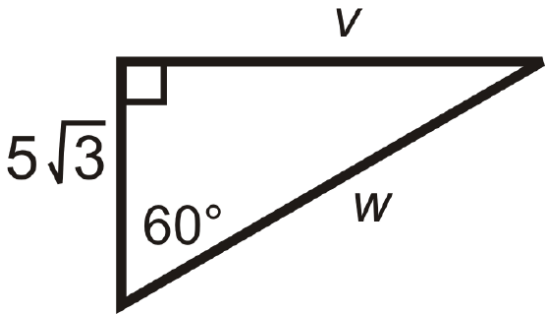

Figure **4.43.13**

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.6.

Resources

Vocabulary

Term	Definition
30-60-90 Theorem	If a triangle has angle measures of 30, 60, and 90 degrees, then the sides are in the ratio $x : x\sqrt{3} : 2x$
30-60-90 Triangle	A 30-60-90 triangle is a special right triangle with angles of **30°**, **60°**, and **90°**.
Hypotenuse	The hypotenuse of a right triangle is the longest side of the right triangle. It is across from the right angle.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Radical	The Missing argument for \sqrt, or square root, sign.

Additional Resources

Video: Solving Special Right Triangles

Activities: 30-60-90 Right Triangles Discussion Questions

Study Aids: Special Right Triangles Study Guide

Practice: 30-60-90 Right Triangles

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CHAPTER OVERVIEW

5: Quadrilaterals and Polygons

5.1: Squares
5.2: Rectangles
5.3: Square and Rectangle Area and Perimeter
5.4: Perimeter of Squares and Rectangles
5.5: Area of Squares and Rectangles
5.6: Unknown Dimensions of Squares and Rectangles
5.7: Quadrilateral Classification
5.8: Parallelogram Classification
5.9: Parallelograms
5.10: Area of a Parallelogram
5.11: Unknown Dimensions of Parallelograms
5.12: Estimation of Parallelogram Area in Scale Drawings
5.13: Trapezoids
5.14: Area and Perimeter of Trapezoids
5.15: Area of Parallelograms- Squares, Rectangles and Trapezoids
5.16: Kites
5.17: Area and Perimeter of Rhombuses and Kites
5.18: Area and Perimeter of Composite Shapes
5.19: Quadrilateral Classification in the Coordinate Plane
5.20: Regular and Irregular Polygons
5.21: Area of Regular and Irregular Polygons
5.22: Area and Perimeter of Similar Polygons
5.23: Construct Regular Polygons
5.24: Congruent Polygons
5.25: Corresponding Parts of Congruent Figures
5.26: Determine Missing Angle Measures
5.27: Interior Angles in Convex Polygons
5.28: Exterior Angles in Convex Polygons

Thumbnail: Four quadrilaterals of different types. (Public Domain; Jim.belk via Wikipedia)

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Identify the properties of squares.

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If you provide content to customers through CloudFront, you can find steps to troubleshoot and help prevent this error by reviewing the CloudFront documentation.

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Video: Overview of Squares and Rectangles

Practice: Squares

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5.2: Rectangles

Identify the properties of rectangles. Determine lengths of missing sides.

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Video: Overview of Squares and Rectangles

Practice: Rectangles

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5.3: Square and Rectangle Area and Perimeter

Compute edge and coverage measures of rectilinear quadrilaterals, given linear measures.

Area and **Perimeter** of Rectangles

To find the **area of a rectangle**, calculate $A = bh$, where b is the base (width) and h is the height (length). The **perimeter of a rectangle** will always be $P = 2b + 2h$.

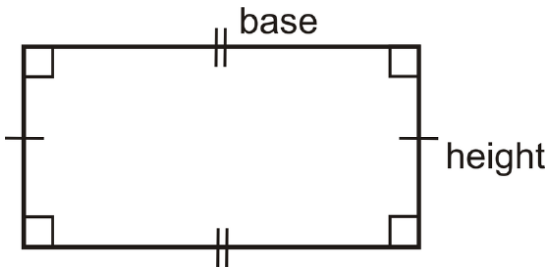

Figure 5.3.1

If a rectangle is a square, with sides of length s, then perimeter is $P_{square} = 2s + 2s = 4s$ and area is $A_{sqaure} = s \cdot s = s^2$.

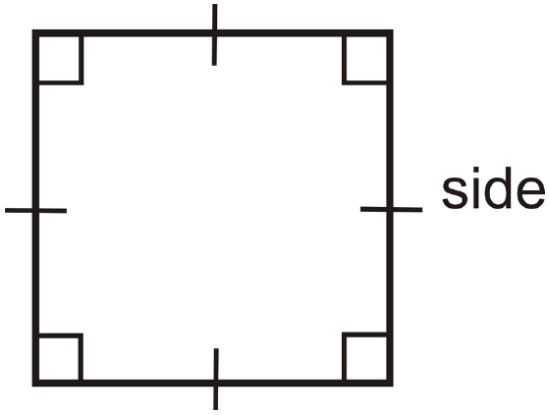

Figure 5.3.2

What if you were given a rectangle and the size of its base and height? How could you find the total distance around the rectangle and the amount of space it takes up?

Example 5.3.1

The area of a square is $75\ \text{in}^2$. [illegible]neter.

Solution

To find the perimeter, w[illegible]ngth of the sides.

$$A = s^2 = 75\ \text{in}^2$$
$$s = \sqrt{75} = 5\sqrt{3}\ \text{in}$$

From this, $P = 4(5\sqrt{3}) = 20\sqrt{3}\ \text{in}$

Example 5.3.2

Draw two different rectangles with an area of '^' allowed only in math mode.

Solution

Think of all the different factors of 36. These can all be dimensions of the different rectangles.

Figure 5.3.3

Other possibilities could be 6×6, 2×18, and 1×36.

Example 5.3.3

Find the area and perimeter of a rectangle with sides 4 cm by 9 cm.

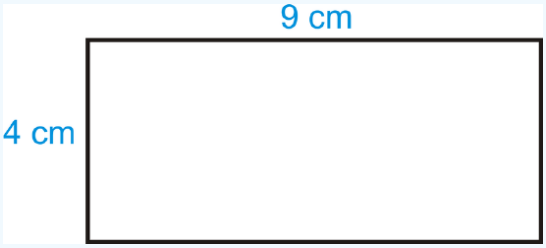

Figure 5.3.4

Solution

The perimeter is $4 + 9 + 4 + 9 = 26$ cm. The area is $A = 9 \cdot 4 = 36$ cm^2.

Example 5.3.4

Find the area and perimeter of a square with side 5 in.

Solution

The perimeter is $4(5) = 20$ *in* and the area is $5^2 = 25$ in^2.

Example 5.3.5

Find the area and perimeter of a rectangle with sides 13 m and 12 m^2.

Solution

The perimeter is $2(13) + 2(12) = 50$ m. The area is $13(12) = 156$ m^2.

Review

1. Find the area and perimeter of a square with sides of length 12 in.
2. Find the area and perimeter of a rectangle with height of 9 cm and base of 16 cm.
3. Find the area and perimeter of a rectangle if the height is 8 and the base is 14.
4. Find the area and perimeter of a square if the sides are 18 ft.
5. If the area of a square is 81 ft^2, find the perimeter.
6. If the perimeter of a square is 24 in, find the area.
7. The perimeter of a rectangle is 32. Find two different dimensions that the rectangle could be.
8. Draw two different rectangles that haven an area of 90 mm^2.
9. True or false: For a rectangle, the bigger the perimeter, the bigger the area.
10. Find the perimeter and area of a rectangle with sides 17 in and 21 in.

Vocabulary

Term	Definition
area	The amount of space inside a figure. Area is measured in square units.
perimeter	The distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write ***units.***
Area of a Rectangle	To find the area 'A' of a rectangle, calculate $A = bh$, where b is the base (width) and h is the height (length).
Perimeter of a Rectangle	The perimeter 'P' of a rectangle is equal to twice the base added to twice the height: $P = 2b + 2h$.

Additional Resources

Video: Determine the Area of a Rectangle Involving Whole Numbers

Activities: Area and Perimeter of Rectangles Discussion Questions

Study Aids: Triangles and Quadrilaterals Study Guide

Practice: Square and Rectangle Area and Perimeter

Real World: Perimeter

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5.4: Perimeter of Squares and Rectangles

Use basic quadrilateral formulas to find total edge measures of rectilinear quadrilaterals.

Find Perimeter and Area of Squares and Rectangles Using Formulas

Figure 5.4.1

Raj has a small patch of land that he wants to turn into a vegetable garden. However, he wants to build a small fence around it before he plants anything. He knows that the **area** of the land is 240 square feet and that the length of one side is 15 feet. In order to build a fence, he needs to know the width of the land too. How can Raj write and solve an equation to determine the width of his future garden?

In this concept, you will learn to find the **perimeter** and area of squares and rectangles using formulas.

Perimeter and Area of Squares and Rectangles

A **formula** is a method that has been proven to work when solving specific types of problems.

Let's explore some of those familiar formulas by looking at rectangles, squares, area and perimeter.

The **perimeter** of a figure is the distance around the figure. Perimeter is the sum of all of the sides in a square or rectangle. Since a rectangle has two sets of parallel sides, the formula for determining perimeter (P) of a rectangle is:

$$P = 2L + 2W$$

$L = length\ and\ W = width$

Let's look at an example.

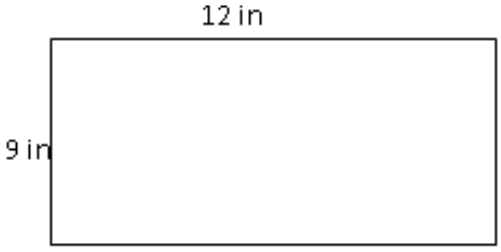

Figure 5.4.2

The rectangle above shows its dimensions. Find the perimeter.

First, substitute the values for the width (W) and the length (L) into the perimeter formula.

$$P = 2(12) + 2(9)$$

Next, complete the multiplication and addition to find the perimeter.

$$P = 2(12) + 2(9)$$
$$P = 24 + 18$$
$$P = 42$$

The answer is 42.

The perimeter of the rectangle is 42 inches.

Area is the amount of square units inside the figure. Area is found by multiplying the length\times width. The formula for finding the area of a rectangle is:

$$A = L \times W$$

You can use the dimensions from the rectangle above to find the area of this rectangle.

First, fill the values for L and W into the formula for area.

$$A = L \times W$$
$$A = 12 \times 9$$

Next, solve for the area by multiplying.

$$A = 12 \times 9$$
$$A = 108$$

The answer is 108.

The area of the rectangle is $108\ in^2$.

Notice that the unit of measurement for area is squared. That is because you multiplied a unit measure times itself: $in \times in = in^2$. Area is always written in square units.

You can also find the perimeter and area of a square. Remember that a square has four equal sides given the symbol s. You can use the following formula for finding the perimeter of a square:

$$P = 4s$$

Let's look at an example.

A rectangle has a length of 12 feet and a perimeter of 72 feet. Write and solve an equation to determine the width of the rectangle.

First, substitute the values for the perimeter (P) and the length (L) into the perimeter formula.

$$72 = 2(12) + 2W$$

Next, complete the multiplication.

$$72 = 24 + 2W$$

Then, subtract 24 from both sides to get your variable alone on the right side.

$$72 - 24 = 24 - 24 + 2W$$
$$48 = 2W$$

Then, multiply both sides by the reciprocal of 2 to isolate your variable.

$$\frac{1}{2} \times \frac{48}{1} = \frac{1}{2} \times \frac{2W}{1}$$
$$\frac{48}{2} = W$$
$$W = 24$$

The answer is 24.

The width of the rectangle is 24 feet.

Example 5.4.1

Earlier, you were given a problem about Raj and his garden fence.

He knows that the area of the land is 240 square feet, the length of one side is 15 feet, and needs to know the width of the land.

Solution

First, substitute the values for the area and the side length into the area formula.

$$A = L \times W$$
$$240 \text{ ft}^2 = 15 \text{ ft} \times W$$

Next, multiply both sides by the reciprocal of 15 in order to isolate the variable W.

$$\frac{1}{15} \times \frac{240}{1} = \frac{1}{15} \times \frac{15}{1} \times W$$
$$W = 16$$

The answer is 16.

The width of Raj's garden is 16 ft. Therefore the dimensions of the garden are 16 ft by 15 ft.

Example 5.4.2

A square has a perimeter of 196 inches. Determine the length of one side of the square.

Solution

First, substitute the value for the perimeter (P) into the perimeter formula.

$$196 = 4s$$

Next, multiply both sides by the reciprocal of 4 to isolate your variable.

$$\frac{1}{4} \times \frac{196}{1} = \frac{1}{4} \times \frac{4s}{1}$$
$$\frac{196}{4} = s$$
$$s = 49$$

The answer is 49.

Example 5.4.3

Find the perimeter of the following square if the side length is 4.5 *inches*.

Solution

First, substitute the value for the side length into the perimeter formula.

$$P = 4s$$
$$P = 4(4.5)$$

Next, multiply by 4 to solve for the perimeter.

$$P = 4(4.5)$$
$$P = 18$$

The answer is 18.

Example 5.4.4

Can you find the area of the square in Example 1?

Solution

First, substitute the value for the side length into the area formula.

$$A = s \times s$$
$$A = 4.5 \times 4.5$$

Next, multiply to solve for the area.

$$A = 4.5 \times 4.5$$
$$A = 20.25$$

The answer is 20.25.

The area of the square is 20.25 in^2.

Example 5.4.5

A square has an area of 144 sq. meters. What is the side length?

Solution

First, substitute the value for the area into the area formula.

$$A = s \times s$$
$$144 \text{ m}^2 = s \times s$$

Next, take the square root of the area to isolate s. Remember that the opposite of square is square root.

$$144 = s \times s$$
$$144 = s^2$$
$$s = \sqrt{144}$$
$$s = 12$$

The answer is 12.

Review

Find the area and perimeter of each square or rectangle using formulas and the given dimensions. Each problem will have two answers.

1. A square with a side length of 5 inches.
2. A rectangle with a length of 5 inches and a width of 3 inches.
3. A rectangle with a length of 8 cm and a width of 6 cm.
4. A square with a side length of 11 feet.
5. A rectangle with a length of 9 inches and a width of 4.5 inches.
6. A square with a side length of 7 feet.
7. A rectangle with a length of 12 meters and a width of 11 meters.
8. A square with a side length of 13 meters.
9. A rectangle with a length of 15 feet and a width of 8 feet.
10. A square with a side length of 12.5 feet.

Find the missing side length given the area of each square.

11. $A = 64 \text{ in}^2$
12. $A = 36 \text{ in}^2$
13. $A = 81 \text{ m}^2$
14. $A = 100 \text{ in}^2$
15. $A = 144 \text{ ft}^2$
16. $A = 121 \text{ cm}^2$
17. $A = 4 \text{ mm}^2$

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.13.

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Formula	A formula is a type of equation that shows the relationship between different variables.
Perimeter	Perimeter is the distance around a two-dimensional figure.
Square Unit	A Square Unit is a square with four sides that each measure 1 unit.

Additional Resources

Video: Determine the Area of a Rectangle Involving Whole Numbers

Practice: Perimeter of Squares and Rectangles

Real World: A Stitch in Time

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5.5: Area of Squares and Rectangles

Find the area of squares and rectangles using formulas: base times height and side squared.

Figure **5.5.1**

Allen bought a new house and is planning to replace the carpet in two of the bedrooms. One bedroom is a shaped like a square and the other is rectangular. He needs to figure out how much carpet is needed to cover both bedroom floors.

The square bedroom has side lengths of 9 feet.

Figure **5.5.2**

The rectangular bedroom has a length of 12 feet and a width of 8 feet.

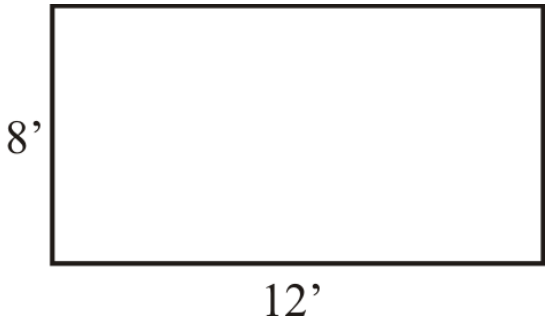

Figure **5.5.3**

In order for Allen to figure out how much carpet he needs to buy for these two bedrooms, he needs to find the area of each bedroom.

In this concept, you will learn how to find the area of squares and rectangles.

Area

The inside space of a figure is known as the **area** of the figure. Carpeting a floor, grass on the ground, or anything else covering the space inside of a figure are all examples of area.

There is a formula for finding the area of different shapes. In this concept, this concept will show you how to find the area of both squares and rectangles.

This is the formula used to find the area of a square:

$$A = s \cdot s$$

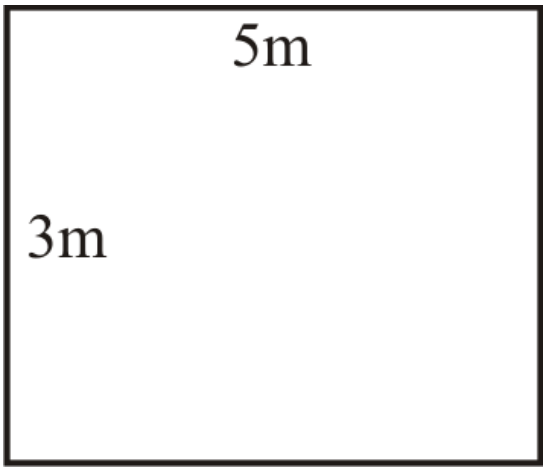

Figure 5.5.4

Remember, the dot is another symbol for multiplication. To figure out the area of the square, multiply one side times the other side. Since all sides in a square are equal, the two numbers will be the same. The unit of measurement is kept in the equation to help you remember what the correct label should be.

Here is how to use the formula for finding the area of a square:

$$A = 6 \text{ ft} \cdot 6 \text{ ft}$$

Now, think of the equation as multiplying two different things; the numbers and the units of measurement.

$$A = 6 \cdot 6$$

$$A = ft \cdot \text{ ft}$$

Solve both equations.

$$A = 6 \times 6 = 36$$

$$A = ft \times \text{ ft} = ft^2$$

Think back to what you know about exponents. When you multiply two of the same thing together, you can write it in exponential form. Another way to label area is to write square feet (sq. ft.).

The answer is $36\ sq.\ \text{ft}$. or $36\ \text{ft}^2$

This is the formula used to find the area of a rectangle:

$$A = lw$$

To find the area of a rectangle, use the measurements for length and width instead of side length. These numbers may be different from each other because not all rectangles have the same side lengths like squares do.

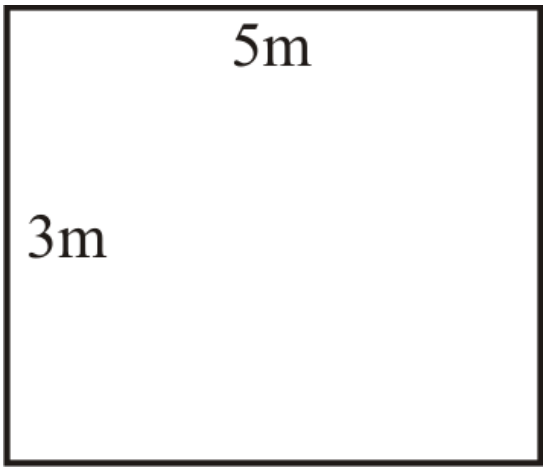

Figure 5.5.5

This rectangle measures a length of 5 meters and a width of 3 meters. Just like the area formula for a square, multiply those two numbers to find the area of the rectangle. Remember, two variables next to each other in an equation indicate multiplication, just like the dot symbol or traditional multiplication symbol. The symbols will be used interchangeably.

To find the area of a rectangle, multiply length times width.

$$A = 5 \text{ m} \cdot 3 \text{ m}$$

$$A = 5 \times 3$$

$$A = meters \times meters$$

The formula shows 5 meters times 3 meters. Multiply the measurement part (5×3), then multiply the units of measurement.

The answer is 15 sq. m or 15 m^2.

The label for area of a rectangle is similar to the label for area of a square from above. The label can be written in exponential form or written in abbreviation.

Example 5.5.1

Earlier, you were given a problem about Allen and his new house that he wants to re-carpet.

The dimensions of Allen's two bedrooms are given in the beginning of the concept. His square bedroom has side lengths of 9 feet. The other room is a rectangle with dimensions of 12 feet by 8 feet. To begin solving this problem, find the area of each of the bedrooms.

Solution

The square bedroom has a side length of 9 feet.

$$A = s \cdot s$$

$$A = 9 \times 9 = 81 \text{ sq. feet}$$

The square bedroom has an area of 81 square feet.

The rectangular bedroom has a length of 12 feet and width of 8 feet.

$$A = 12 \text{ ft} \cdot 8 \text{ ft}$$

$$A = 12 \times 8$$

$$A = feet \times feet$$

The rectangular bedroom has an area of 96 square feet.

Allen wants the total area of his two bedrooms to know how much carpet to buy, so add the two areas together.

$$81 + 96 = 177 \text{ square feet}$$

The answer is 177 square feet.

Example 5.5.2

Find the area of a rectangle with a length of 7\text{ ft }. and a width of 4\text{ ft }.

Solution

First, write out the equation

$$A = lw$$

$$A = 7 \text{ ft } \cdot 4 \text{ ft}$$

Next, multiply the two numbers.

$$A = 7 \cdot 4 = 28$$

Then, multiply the units of measurement.

$$A = \text{ ft } \cdot \text{ ft } = \text{ft}^2$$

Now, put your answer together to show the final solution.

$$A = 28 \text{ ft }^2$$

The answer is 28 sq. ft .

Once you do more examples you may find yourself skipping the step of multiplying the units of measurement. This may become natural to square the unit of measurement for any area problem.

Example 5.5.3

Find the area of a square with side lengths of 10 inches.

Solution

First, write your equation.

$$A = s \cdot s$$

$$A = 10 \text{ in } \cdot 10 \text{ in}$$

Next, solve the equation by multiplying the two side lengths.

$$A = 10 \cdot 10 = 100$$

Then, write your final solution. Remember to write the squared unit of measurement after the numerical answer.

$$A = 100 \text{ in}^2$$

The answer is 100 sq. in.

Example 5.5.4

Find the area of a rectangle with a length of 6 feet and a width of 5 feet.

Solution

First, write your equation.

$$A = lw$$

$$A = 6 \text{ ft } \cdot 5 \text{ ft}$$

Next, solve the equation by multiplying the length times the width.

$$A = 6 \cdot 5 = 30$$

Then, write your final solution. Remember to write the squared unit of measurement after the numerical answer.

$$A = 30 \text{ ft}^2$$

The answer is 30 sq.\text{ ft }.

Example 5.5.5

Find the area of a rectangle with a length of 9 meters and a width of 8 meters.

Solution

First, write your equation.

$$A = lw$$

$$A = 9 \text{ m} \cdot 8 \text{ m}$$

Next, solve the equation by multiplying the length times the width.

$$A = 9 \cdot 8 = 72$$

Then, write your final solution. Remember to write the squared unit of measurement after the numerical answer.

$$A = 72 \text{ m}^2$$

The answer is 72 sq. m

Review

Find the area of each of the following figures. Make sure you label your answer correctly.

1. A square with a side length of 6 inches.
2. A square with a side length of 4 inches.
3. A square with a side length of 8 centimeters.
4. A square with a side length of 12 centimeters.
5. A square with a side length of 9 meters.
6. A rectangle with a length of 6 inches and a width of 4 inches.
7. A rectangle with a length of 9 meters and a width of 3 meters.
8. A rectangle with a length of 4 meters and a width of 2 meters.
9. A rectangle with a length of 17 feet and a width of 12 feet.
10. A rectangle with a length of 22 feet and a width of 18 feet.
11. A square with a side length of 13 feet.
12. A square with a side length of 18 feet.
13. A square with a side length of 21 feet.
14. A rectangle with a length of 18 feet and a width of 13 feet.
15. A rectangle with a length of 60 feet and a width of 27 feet.
16. A rectangle with a length of 57 feet and a width of 22 feet.

Resources

Additional Resource

Video: Determine the Area olving Whole Numbers

Practice: Area of Squares a

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5.6: Unknown Dimensions of Squares and Rectangles

Compute linear measures, given edge or surface measures of rectilinear quadrilaterals.

Figure 5.6.1

Andrea is out Christmas shopping with her mom and spots a sale on Christmas lights. She asks her mom if she can get some to string around the perimeter of her bedroom. Her mom agrees, so they look at the packages to try to figure out how many they will need. Andrea's mom can't remember the exact dimensions of Andrea's room, but she remembers that it is 81 square feet and is a square. How can Andrea figure out how many feet of Christmas lights she needs?

In this concept, you will learn how to use formulas to solve for unknown dimensions when given perimeter or **area**.

Finding Unknown Dimensions Using Formulas

Perimeter is the distance around a figure. **Area** is the number of square units contained inside a figure. The perimeters and areas of rectangles and squares can be found using the formulas below.

- Rectangle Perimeter: $P = 2l + 2w$
- Square Perimeter: $P = 4s$
- Rectangle Area: $A = lw$
- Square Area: $A = s^2$

An **equation** is a mathematical sentence that says that two mathematical expressions are equal. The perimeter and area formulas above are examples of equations. The variables in these equations are P, A, l, w, and s.

Sometimes you will know the area or perimeter of a rectangle or a square, but not know information about all of its sides. In this case, you can substitute the information you know into the relevant formula and **solve** for the unknown value. Remember that when you **solve** an equation you are looking for the value of the variable that makes both sides of the equation equal.

Here is an example.

The width of a rectangle is 10 feet and the perimeter is 50 feet. What is the length of the rectangle?

First, figure out which formula is relevant. This question is about the perimeter of a rectangle, so you will be using the rectangle perimeter formula.

$P = 2l + 2w$

Next, figure out which variables you have values for and which variable you are solving for. The problem says that the width is 10 feet, so w=10. The problem also says that the perimeter is 50 feet, so $P = 50$. You are solving for the length.

$P = 50$
$l = ?$
$w = 10$

Now, substitute the values for P and w into the formula and simplify.

Next, solve the equation. Remember you are looking for the value of l that will make both sides of the equation equal. First, figure out the value of the $2l$ in the equation. Ask yourself a question.

"50 is equal to what number plus 20?"

Then, answer your question. 30 plus 20 is equal to 50, so $2l$ must be equal to 30.

Now, you have a simpler equation to solve.

$2l = 30$

Next, ask yourself another question.

"2 times what number is equal to 30?"

Finally, answer your question. 2 times 15 is equal to 30, so l must be equal to 15.

The answer is that the length of the rectangle is 15 feet.

You can check your answer by substituting all three values into the rectangle perimeter formula, evaluating, and making sure both sides of the equation end up equal to each other.

The answer is correct.

Here is another example.

If the area of a square is 144 square miles, what is the length of one of the sides of the square?

First, figure out which formula is relevant. This question is about the area of a square, so you will be using the square area formula.

$A = s^2$

Next, figure out which variable you have a value for and which variable you are solving for. The problem says that the area is 144 square miles, so A=144. You are solving for the length of the side.

Now, substitute the value for A into the formula.

Next, solve the equation. Remember you are looking for the value of s that will make both sides of the equation equal. Ask yourself a question.

"144 is equal to what number times itself?"

Then, answer your question. 12 times 12 is equal to 144, so s must be equal to 12.

The answer is that the side length of the square is 12 miles.

Now, check your answer by substituting both values into the square area formula, evaluating, and making sure both sides of the equation end up equal to each other.

The answer is correct.

Example 5.6.1

Earlier, you were given a problem about Andrea and the Christmas lights.

She's buying some to string around the perimeter of her room and needs to know how many feet of lights to buy. She knows her square room has an area of 81 square feet.

Solution

First, figure out the length of each side of her room. Her room is a square and you know its area, so you will be using the square area formula.

$A = s^2$

Now, you know that the area of the square is 81 square feet, so $A = 81$. Substitute this value for A into the formula.

Next, ask yourself a question to help solve the equation.

"81 is equal to what number times itself?"

Then, answer your question. 9 times 9 is equal to 81, so s must be equal to 9. Each side of Andrea's room is 9 feet.

Now, you need to figure out the perimeter of Andrea's room so Andrea will know how many feet of lights she needs. You know the room is a square with a side length of 9 feet. Substitute s=9 into the square perimeter formula.

The answer is that the perimeter of her room is 36 feet, so she needs 36 feet of lights.

Example 5.6.2

The Hegazzi family is designing a summer garden based on the model shown below. The bottom of the plot is a square and the top of the plot is a rectangle. If the total area of both plots in the garden is 139 square feet, what is the length of one side of the square plot (indicated with a question mark below)?

Figure 5.6.2

Solution

First, use the given information to find the area of the rectangle plot. You will then use this information to help you to find the area of the square plot, which will allow you to find the side length of the square plot.

The rectangle has a length of 15 feet and a width of 6 feet. The formula to find the area of a rectangle is $A = lw$. Substitute the values for the length and the width of the rectangle into the formula and evaluate to find the area.

Next, find the area of the square plot. You know that the total area is 139 square feet and the area of the rectangle is 90 square feet. You also know that the total area is equal to the rectangle area plus the square area. Substitute the values that you know into this relationship.

Now, solve the equation by asking yourself a question.

"139 is equal to 90 plus what number?"

Next, answer the question. 90 plus 10 is equal to 100. 100 plus 39 is equal to 139. So 90 plus 49 must equal 139. The area of the square plot is 49 square feet.

Now, use the area of the square plot to help you to find the side length of the square plot. Substitute the value for A into the square area formula.

Next, solve the equation.

"49 is equal to what number times itself?"

Then, answer your question. 7 times 7 is equal to 49, so s must be equal to 7.

The answer is that the side length of the square plot is 7 feet.

Example 5.6.3

If the perimeter of a rectangle is 30 feet, with a length of 9 feet, what is the width of the rectangle?

Solution

First, figure out which formula is relevant. This question is about the perimeter of a rectangle, so you will be using the rectangle perimeter formula.

$P = 2l + 2w$

Next, figure out which variables you have values for and which variable you are solving for. The problem says that the perimeter is 30 feet, so P=30\). The problem also says the length is 9 feet, so l=9\). You are solving for the width.

$P = 30$
$l = 9$
$w = ?$

Now, substitute the values for P and l into the formula and simplify.

Next, solve the equation. First, figure out the value of the $2w$ in the equation. Ask yourself a question.

"30 is equal to 18 plus what number?"

Then, answer your question. 18 plus 12 is equal to 30, so $2w$ must be equal to 12.

Now, you have a simpler equation to solve.

$2w = 12$

Next, ask yourself another question.

"2 times what number is equal to 12?"

Finally, answer your question. 2 times 6 is equal to 12, so w must be equal to 6.

The answer is that the width of the rectangle is 6 feet.

Now, check your answer by substituting all three values into the rectangle perimeter formula, evaluating, and making sure both sides of the equation end up equal to each other.

The answer is correct.

Example 5.6.4

If the area of a square is 225 square feet, what is the length of one of its sides?

Solution

First, figure out which formula is relevant. This question is about the area of a square, so you will be using the square area formula.

$A = s^2$

Next, figure out which variable you have a value for and which variable you are solving for. The problem says that the area is 225 square feet, so $A = 225$. You are solving for the length of the side.

Now, substitute the value for A into the formula.

Next, solve the equation by asking yourself a question.

"225 is equal to what number times itself?"

Then, answer your question. 15 times 15 is equal to 225, so s must be equal to 15.

The answer is that the side length of the square is 15 feet.

Now, check your answer by substituting both values into the square area formula, evaluating, and making sure both sides of the equation end up equal to each other.

The answer is correct.

Example 5.6.5

If the perimeter of a square is 24 meters, what is the length of one of the sides?

Solution

First, figure out which formula is relevant. This question is about the perimeter of a square, so you will be using the square perimeter formula.

$P = 4s$

Next, figure out which variable you have a value for and which variable you are solving for. The problem says that the perimeter is 24 meters, so $P = 24$. You are solving for the length of the side.

Now, substitute the value for P into the formula.

Next, solve the equation by asking yourself a question.

"24 is equal to 4 times what number?"

Then, answer your question. 4 times 6 is equal to 24, so s must be equal to 6.

The answer is that the side length of the square is 6 meters.

Now, check your answer by substituting both values into the square perimeter formula, evaluating, and making sure both sides of the equation end up equal to each other.

The answer is correct.

Review

Given the area, find the side length for each square.

1. $A = 16$ sq. ft.
2. $A = 64$ sq. m.
3. $A = 100$ sq. miles
4. $A = 121$ sq. inches
5. $A = 144$ sq. ft.

Given the area and length, find the width of each rectangle.

6. $A = 24$ sq. feet , length $= 8$ feet
7. $A = 48$ sq. feet , length $= 12$ feet
8. $A = 64$ sq. feet , length $= 10$ feet
9. $A = 120$ sq. meters , length $= 40$ meters
10. $A = 130$ sq. feet , length $= 13$ feet
11. $A = 90$ sq. inches , length $= 45$ inches

Given the perimeter, find the side length of each square.

12. $P = 48$ inches
13. $P = 64$ inches
14. $P = 90$ inches
15. $P = 35$ feet

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.15.

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Measurement	A measurement is the weight, height, length or size of something.

Additional Resources

Video: Determine the Area of a Rectangle Involving Whole Numbers

Practice: Unknown Dimensions of Squares and Rectangles

Real World: Yard Waste

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5.7: Quadrilateral Classification

Categorize quadrilaterals based on sides and angles. Identify them visually, and by description. Find lengths and angles, determine shapes, and prove congruence and if lines are parallel.

Figure 5.7.1

Rasha is helping her father paint the door frame of their home. She calls out the names of each shape that she paints. She reaches a shape that she cannot name. She sees that it has four sides and one set of parallel sides. What shape is she painting?

In this concept, you will learn how to identify and classify quadrilaterals.

Classifying Quadrilaterals

Polygons are closed shapes with sides made up of lines. Any closed shape with straight edges, such as a triangle or **rectangle**, is a **polygon**.

Polygons with four sides, such as rectangles and squares, are called **quadrilaterals.** The prefix of this word "quad" means four, therefore, a **quadrilateral** is a polygon that has four sides. Quadrilaterals also have four angles, and these four angles have a sum of 360°. This is true for all quadrilaterals

Another important feature of quadrilaterals is that many have one or even two pairs of parallel sides. Look at the quadrilaterals below.

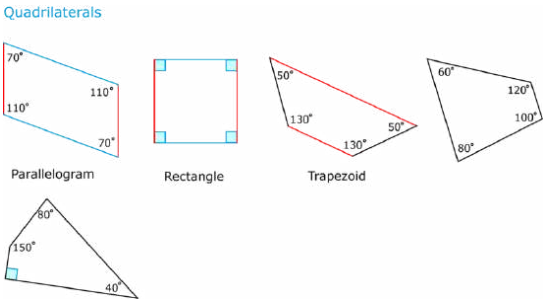


Figure 5.7.2

Let's look at the different types of quadrilaterals in detail.

A **parallelogram** is a quadrilateral with opposite sides parallel. The first figure above is a parallelogram. There are a few special features of parallelograms. One is that each pair of parallel sides is congruent. In the first figure above, the two short sides are the same length and the two long sides are the same length. This special relationship between the pairs of sides affects the angles of a parallelogram: the angles opposite each other are congruent. The 70° angles are opposite each other, and the 110° angles are opposite each other. This relationship exists in any parallelogram, no matter the length of the sides or the sizes of the angle pairs.

There are three special kinds of parallelograms.

A **rectangle** is a quadrilateral because it has four sides, and it is a parallelogram because it has two pairs of parallel, congruent sides. All four angles are right angles.

A **square** has two pairs of parallel sides and four right angles. All four of its sides are congruent.

The third special parallelogram is called a **rhombus**. Like a square, a rhombus has four congruent sides and pairs of congruent angles opposite each other. However, right angles are not required for a rhombus. So a rhombus canbe a square, but a square isn't necessarily a rhombus because of the right angles necessary in a square.

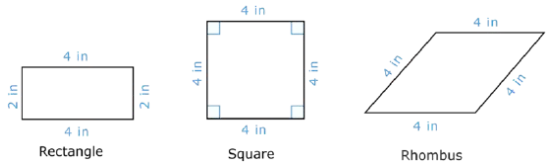


Figure **5.7.3**

Let's look at an example.

Identify the shape below as a rectangle, square, rhombus, or just a parallelogram.

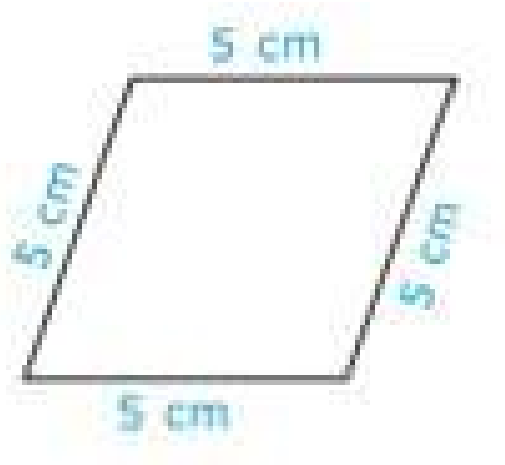


Figure **5.7.4**

First, check the lengths of the sides.

The shape has four sides of equal lengths.

Next, check to see if the shape has four right angles.

The shape does not have four right angles.

Then, identify the shape.

Rhombus.

The answer is a rhombus.

Now let's look at one other special quadrilateral.

A **trapezoid** is a quadrilateral that has only one pair of parallel sides.

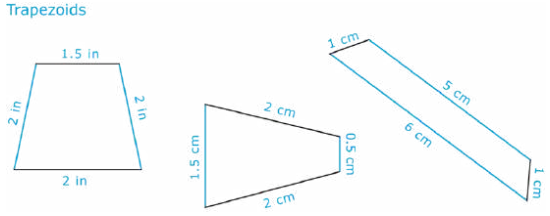


Figure **5.7.5**

Example 5.7.1

Earlier, you were given a problem about Rasha and the door frame of her home.

She paints a shape that has four sides and one set of parallel sides. What shape is she painting?

Solution

First, check the number of sides.

The shape has four sides.

Next, check to see if the shape has parallel sides.

The shape has one set of parallel sides.

Then, identify the shape.

Trapezoid.

The answer is that Rasha is painting a trapezoid.

Example 5.7.2

Identify the shape below as a rectangle, square, rhombus, or parallelogram.

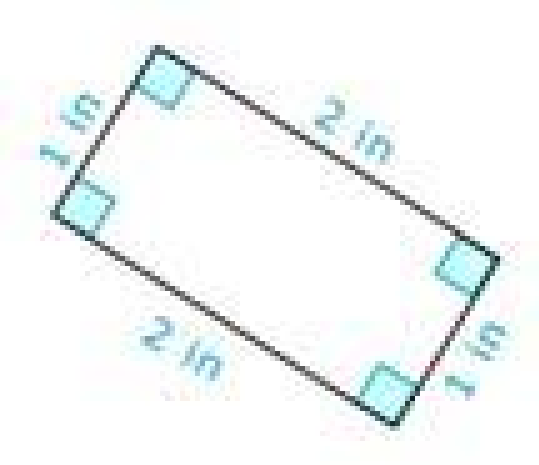


Figure 5.7.6

Solution

First, check the lengths of the sides.

The opposite sides are congruent.

Next, check to see if the shape has four right angles.

The shape has four right angles.

Then, identify the shape.

Rectangle.

The answer is rectangle.

Example 5.7.3

Identify each type of quadrilateral based on its description.

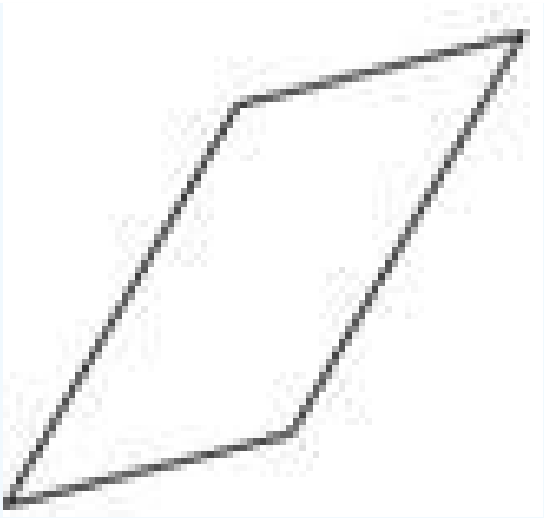


Figure 5.7.7

Solution

First, check the lengths of the sides.

The length of the sides is unknown, but the opposite sides appear to be congruent.

Next, check to see if the shape has four right angles.

The shape does not have four right angles.

Then, identify the shape.

Parallelogram.

The answer is parallelogram.

Example 5.7.4

A four sided figure with opposite sides parallel and congruent with four right angles.

Solution

First, check the number of sides.

The shape has four sides.

Next, note the relationship between the opposite sides.

The opposite sides are parallel and congruent.

Then, classify the shape.

Square.

The answer is a square.

Example 5.7.5

A four sided figure with two pairs of parallel sides. Opposite sides are congruent.

Solution

First, check the number of sides.

The shape has four sides.

Next, note the relationship between the opposite sides.

The opposite sides are parallel and the parallel sides are congruent.

Then, classify the shape.

Parallelogram.

The answer is a parallelogram.

Review

Identify each quadrilateral as a parallelogram, rectangle, square, rhombus, or trapezoid.

1.

Figure **5.7.8**

2.

Figure **5.7.9**

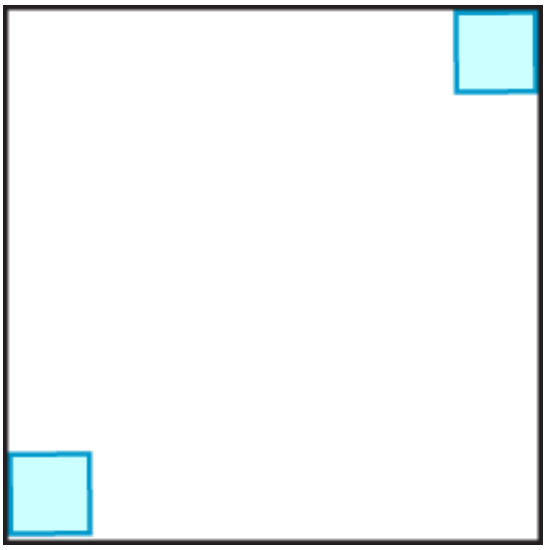
3. 

Figure **5.7.10**

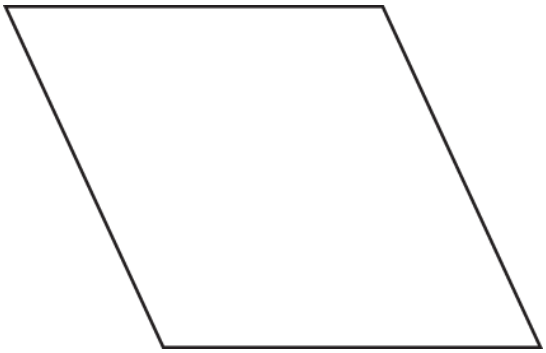
4. 

Figure **5.7.11**

Look at each image and name the quadrilateral pictured.

5.

Figure **5.7.12**

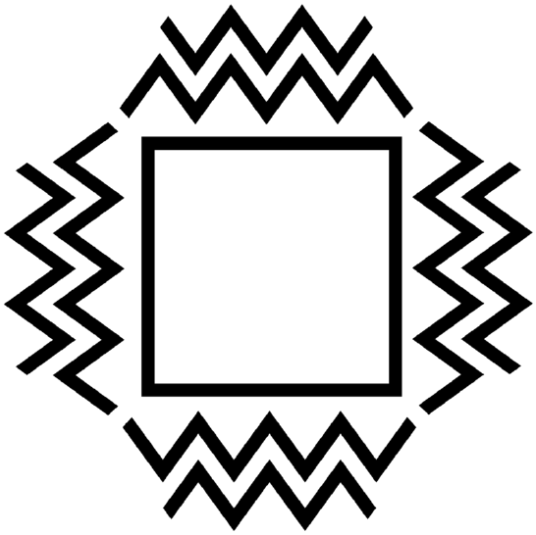
6. 

Figure **5.7.13**

7.

Figure **5.7.14**

8.

Figure 5.7.15

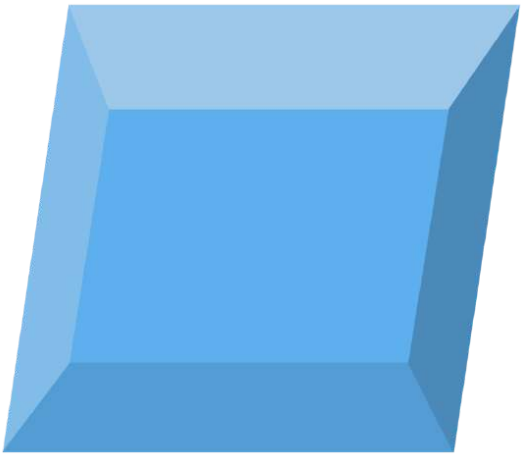
9. 

Figure 5.7.16

Name the geometric figure described below or answer each question.

10. Has four sides and four angles
11. Has one pair of opposite sides that are parallel
12. Has four right angles and four congruent sides
13. A parallelogram with four right angles.
14. A parallelogram with four congruent sides
15. Has four right angles and opposite sides congruent

Resources

Vocabulary

Term	Definition
Kite	A kite is a quadrilateral with distinct adjacent congruent sides.
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Polygon	A polygon is a simple closed figure with at least three straight sides.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.
Rectangle	A rectangle is a quadrilateral with four right angles.
Rhombus	A rhombus is a quadrilateral with four congruent sides.
Trapezoid	A trapezoid is a quadrilateral with exactly one pair of parallel opposite sides.

Additional Resources

Video: Quadrilateral Properties

Activities: Quadrilateral Classification Discussion Questions

Study Aids: Parallelograms Study Guide

Practice: Quadrilateral Classification

Real World: Quadrilateral Classification

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5.8: Parallelogram Classification

Rectangles, rhombuses, and squares are parallelograms defined by their diagonals, angles, and sides.

Classifying Parallelograms

Rectangles, rhombuses (also called rhombi) and squares are all more specific versions of parallelograms, also called special parallelograms.

- A quadrilateral is a **rectangle** if and only if it has four right (congruent) angles.

Figure 5.8.1

$ABCD$ is a rectangle if and only if $\angle A \cong \angle B \cong \angle C \cong \angle D$.

- A quadrilateral is a **rhombus** if and only if it has four congruent sides.

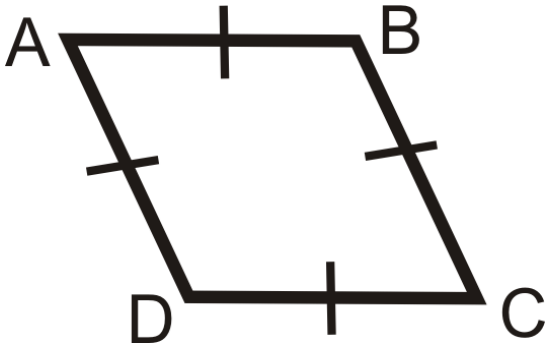


Figure 5.8.2

$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

- A quadrilateral is a **square** if and only if it has four right angles and four congruent sides. By definition, ***a square is a rectangle and a rhombus.***

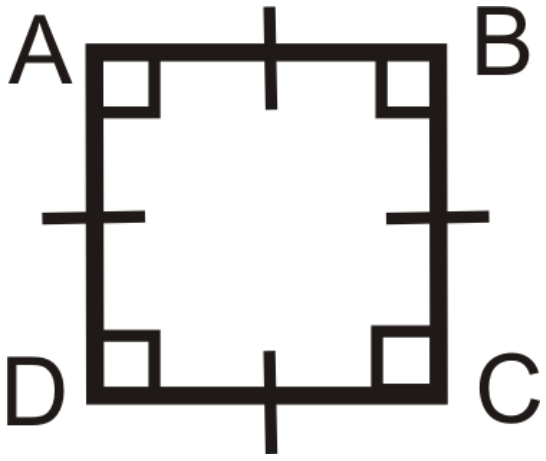


Figure 5.8.3

$ABCD$ is a square if and only if $\angle A \cong \angle B \cong \angle C \cong \angle D$ *and* $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

You can always show that a **parallelogram** is a rectangle, rhombus, or square by using the definitions of these shapes. There are some additional ways to prove parallelograms are rectangles and rhombuses, shown below:

1. A parallelogram is a **rectangle** if the diagonals are congruent.

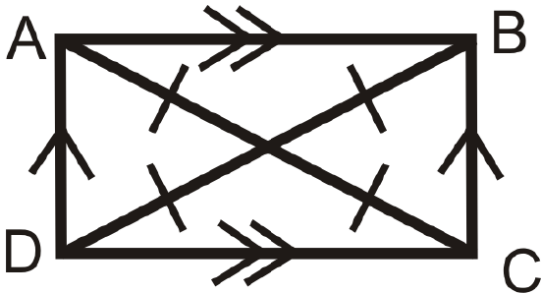


Figure 5.8.4

$ABCD$ is parallelogram. If $\overline{AC} \cong \overline{BD}$, then $ABCD$ is also a rectangle.

2. A parallelogram is a **rhombus** if the diagonals are perpendicular.

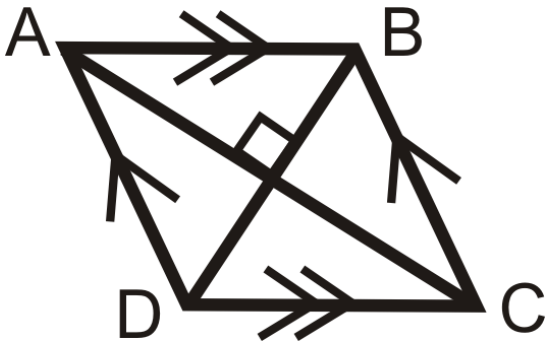


Figure 5.8.5

$ABCD$ is a parallelogram. If $\overline{AC} \perp \overline{BD}$, then $ABCD$ is also a rhombus.

3. A parallelogram is a **rhombus** if the diagonals bisect each angle.

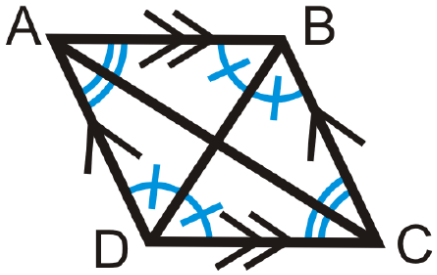


Figure 5.8.6

$ABCD$ is a parallelogram. If $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$ **and** $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$, then $ABCD$ is also a rhombus.

What if you were given a parallelogram and information about its diagonals? How could you use that information to classify the parallelogram as a rectangle, rhombus, and/or square?

Example 5.8.1

Is a rectangle SOMETIMES, ALWAYS, or NEVER a parallelogram? Explain why.

Solution

A rectangle has two sets of parallel sides, so it is ALWAYS a parallelogram.

Example 5.8.2

Is a quadrilateral SOMETIMES, ALWAYS, or NEVER a pentagon? Explain why.

Solution

A quadrilateral has four sides, so it will NEVER be a pentagon with five sides.

Example 5.8.3

What typed of parallelogram are the figures below?

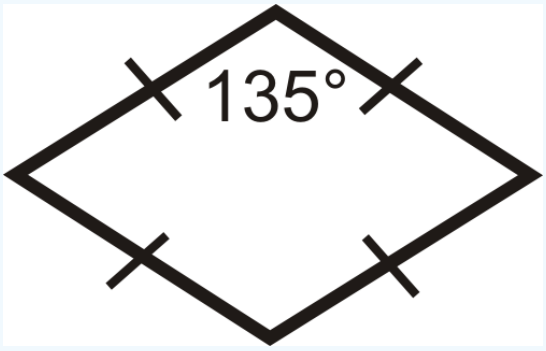


Figure 5.8.7

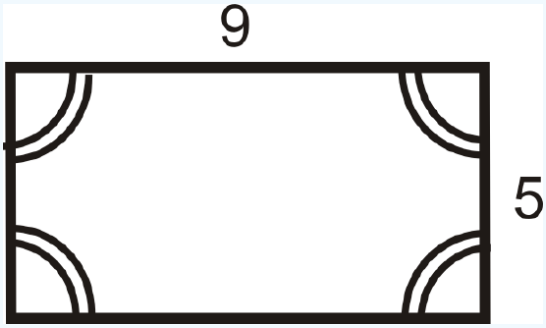


Figure 5.8.8

Solution

For the first figure, all sides are congruent and one angle is $135°$, so the angles are not congruent. This is a rhombus.

For the second figure, all four angles are congruent but the sides are not. This is a rectangle.

Example 5.8.4

Is a rhombus SOMETIMES, ALWAYS, or NEVER a square? Explain why.

Solution

A rhombus has four congruent sides and a square has four congruent sides ***and*** angles. Therefore, a rhombus is a square when it has congruent angles. This means a rhombus is SOMETIMES a square.

Example 5.8.5

List ***everything*** you know about the square $SQRE$.

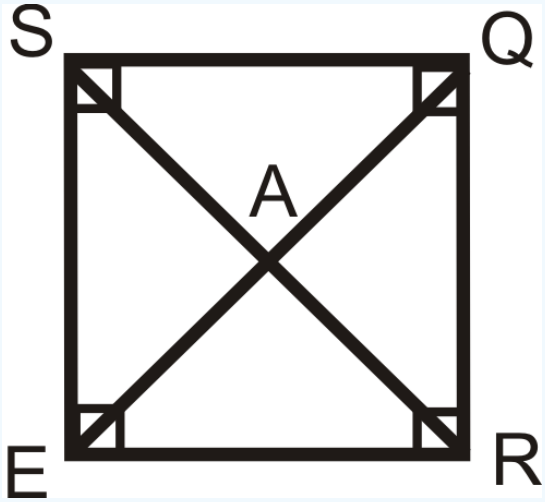


Figure 5.8.9

Solution

A square has all the properties of a parallelogram, rectangle and rhombus.

Properties of a Parallelogram	*Properties of a Rhombus*	*Properties of a Rectangle*
• $\overline{SQ} \parallel \overline{ER}$	• $\overline{SQ} \cong \overline{ER} \cong \overline{SE} \cong \overline{QR}$	• $m\angle SER = m\angle SQR = m\angle QSE = m\angle QRE = 90^\circ$
• $\overline{SE} \parallel \overline{QR}$	• $\overline{SR} \perp \overline{QE}$	
	• $\angle SEQ \cong \angle QER \cong \angle SQE \cong \angle EQR$	• $\overline{SR} \cong \overline{QE}$
	• $\angle QSR \cong \angle RSE \cong \angle QRS \cong \angle SRE$	• $\overline{SA} \cong \overline{AR} \cong \overline{QA} \cong \overline{AE}$

All the bisected angles are 45°.

Review

1. $RACE$ is a rectangle. Find:
    1. RG
    2. AE
    3. AC
    4. EC
    5. $m\angle RAC$

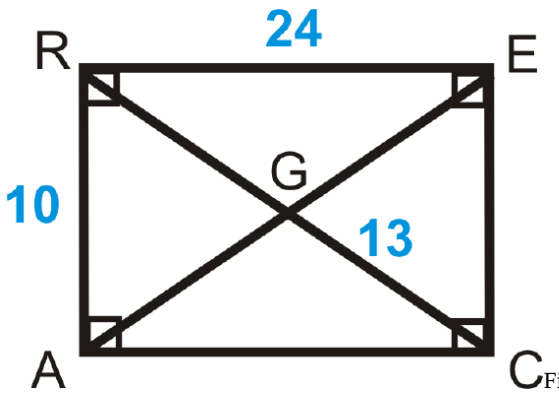


Figure 5.8.10

2. $DIAM$ is a rhombus. Find:
    1. MA
    2. MI
    3. DA
    4. $m\angle DIA$
    5. $m\angle MOA$

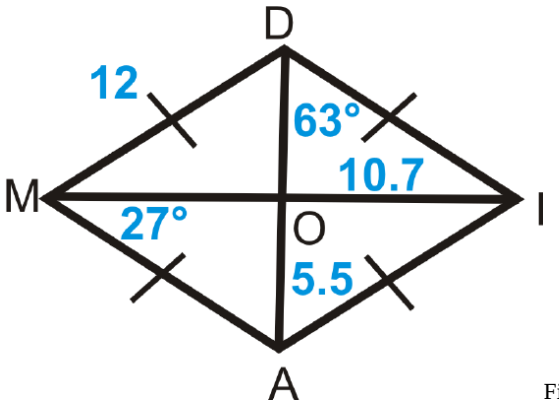


Figure 5.8.11

3. $CUBE$ is a square. Find:
    1. $m\angle UCE$
    2. $m\angle EYB$
    3. $m\angle UBY$
    4. $m\angle UEB$

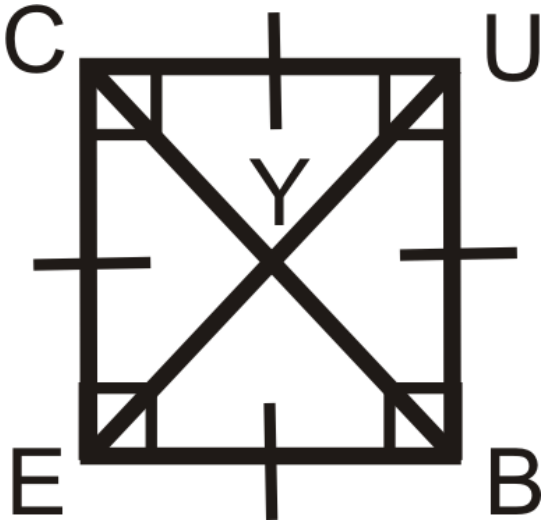


Figure 5.8.12

For questions 4-15, determine if the quadrilateral is a parallelogram, rectangle, rhombus, square or none.

4.

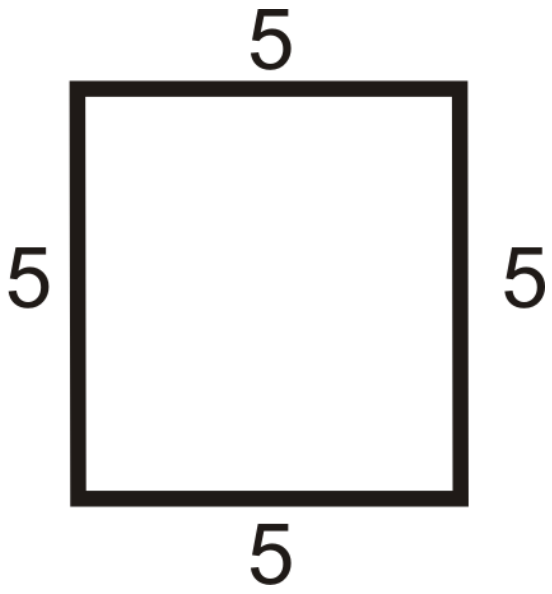


Figure 5.8.13

5.

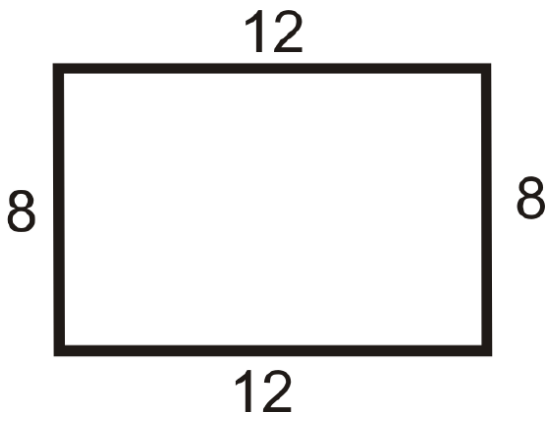


Figure 5.8.14

6.

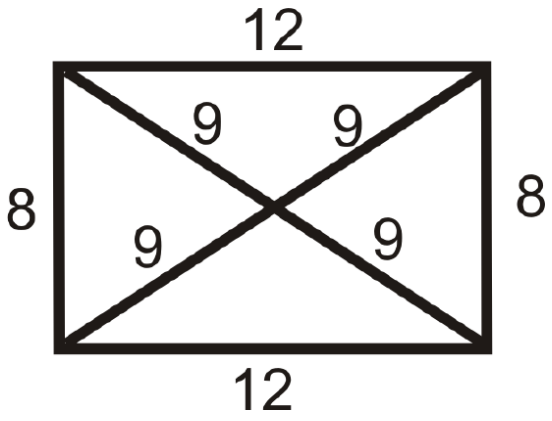


Figure 5.8.15

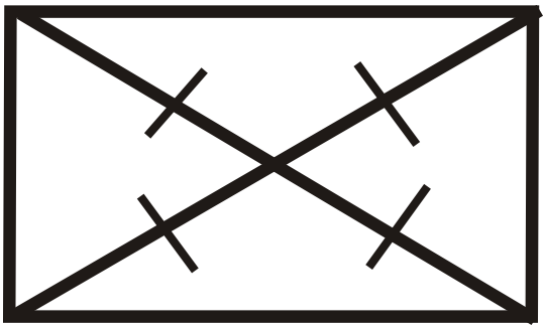
7. 

Figure 5.8.16

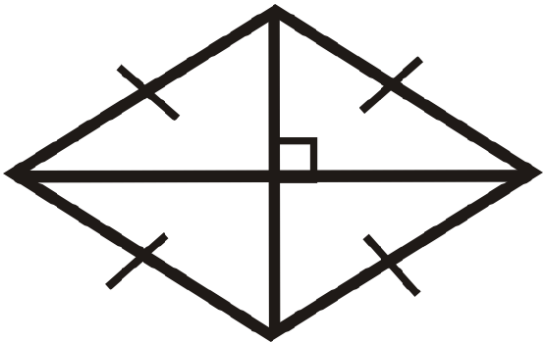
8. 

Figure 5.8.17

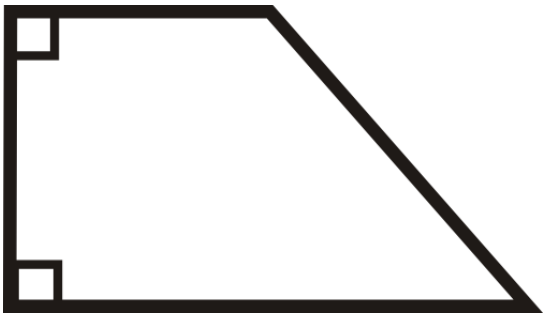
9. 

Figure 5.8.18

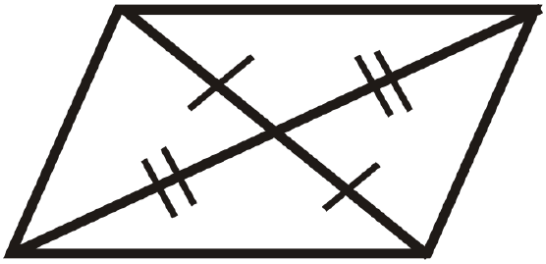
10. 

Figure 5.8.19

11.

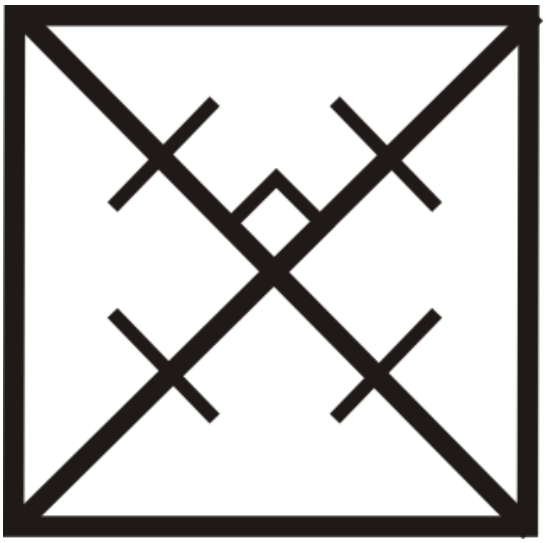


Figure **5.8.20**

12.

Figure **5.8.21**

13.

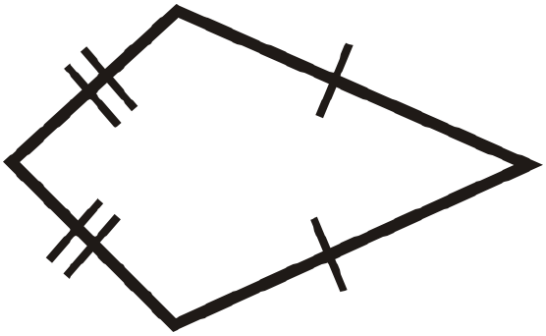


Figure **5.8.22**

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14.

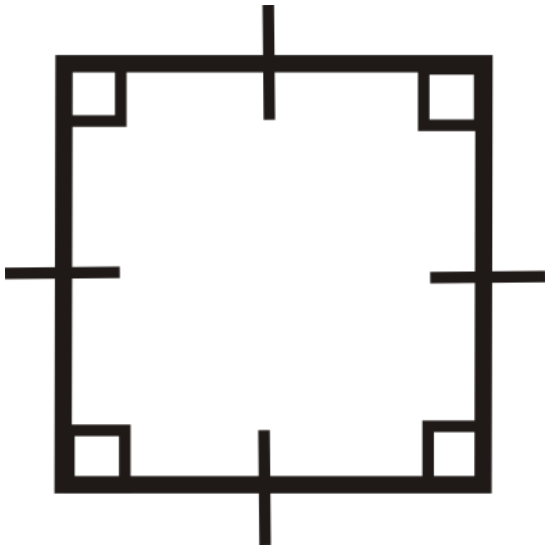

Figure 5.8.23

15.

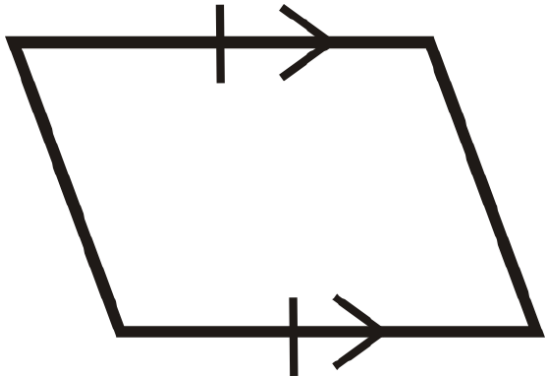

Figure 5.8.24

For questions 16-19 determine if the following are ALWAYS, SOMETIME, or NEVER true. Explain your reasoning.

16. A rectangle is a rhombus.
17. A square is a parallelogram.
18. A parallelogram is regular.
19. A square is a rectangle.

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.5.

Vocabulary

Term	Definition
rectangle	A parallelogram is a ***rectangle*** if and only if it has four right (congruent) angles
rhombus	A parallelogram is a ***rhombus*** if and only if it has four congruent sides
square	A parallelogram is a ***square*** if and only if it has four right angles and four congruent sides.
converse	If a conditional statement is $p \rightarrow q$ (if p, then q), then the converse is $q \rightarrow p$ (if q, then p. Note that the converse of a statement is not true just because the original statement is true.
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$ or $\angle B \cong \angle B$

Additional Resources

Video: Classifying Parallelograms Principles - Basic

Activities: Parallelogram Classification Discussion Questions

Study Aids: Parallelograms Study Guide

Practice: Parallelogram Classification

Real World: Parallelograms

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5.9: Parallelograms

Find unknown angle measurements of quadrilaterals with two pairs of parallel sides.

A **parallelogram** is a quadrilateral with two pairs of parallel sides.

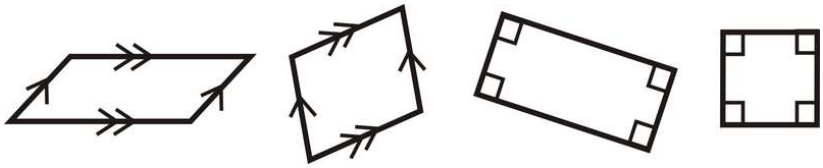

Figure 5.9.1

Notice that each pair of sides is marked parallel (for the last two shapes, remember that when two lines are perpendicular to the same line then they are parallel). Parallelograms have a lot of interesting properties.

Facts about Parallelograms

1. **Opposite Sides Theorem:** If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.

If

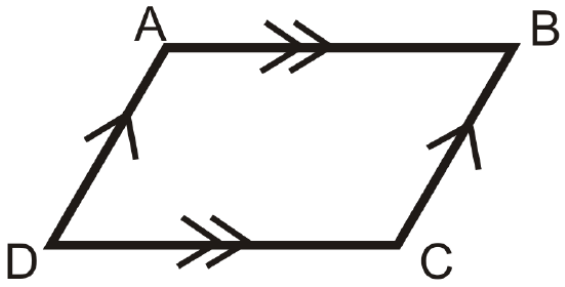

Figure 5.9.2

then

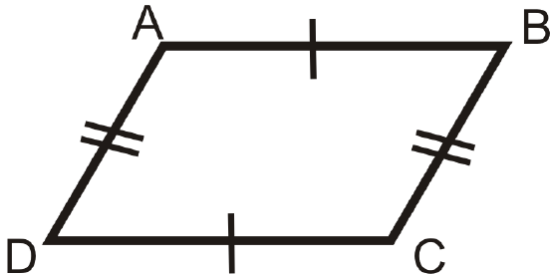

Figure 5.9.3

2. **Opposite Angles Theorem:** If a quadrilateral is a parallelogram, then both pairs of opposite angles are congruent.

If

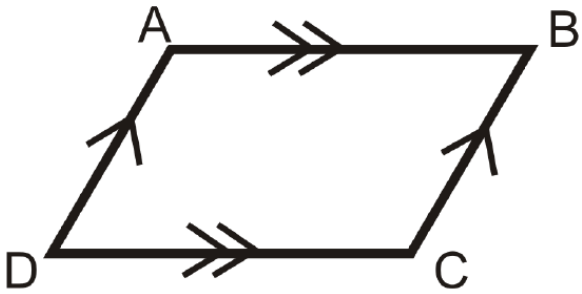

Figure 5.9.4

then

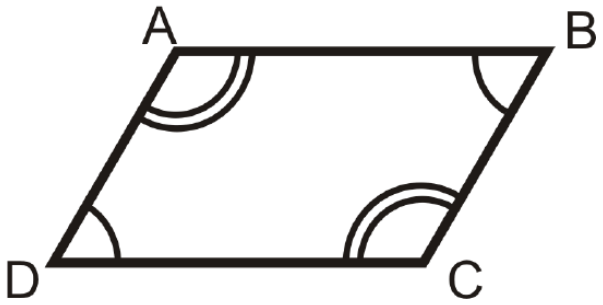

Figure 5.9.5

3. **Consecutive Angles Theorem:** If a quadrilateral is a parallelogram, then all pairs of consecutive angles are supplementary.

If

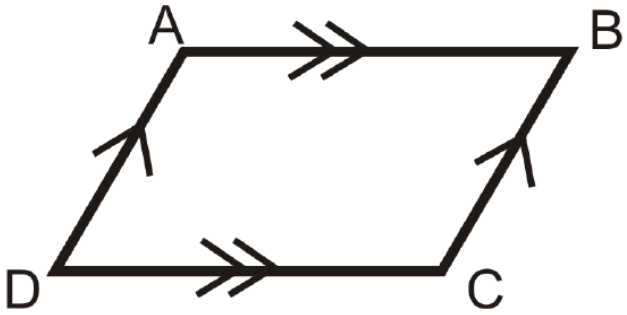

Figure 5.9.6

then

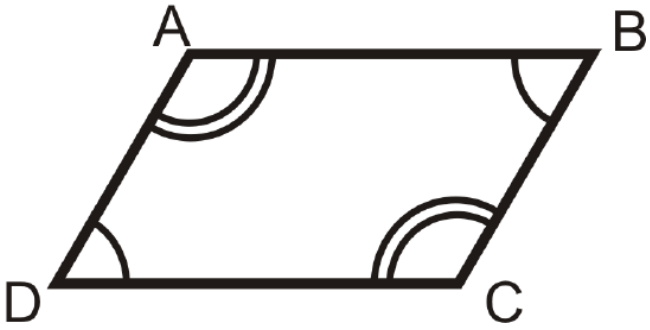

Figure 5.9.7

$$m\angle A + m\angle D = 180^\circ$$
$$m\angle A + m\angle B = 180^\circ$$
$$m\angle B + m\angle C = 180^\circ$$
$$m\angle C + m\angle D = 180^\circ$$

4. **Parallelogram Diagonals Theorem:** If a quadrilateral is a parallelogram, then the diagonals bisect each other.

If

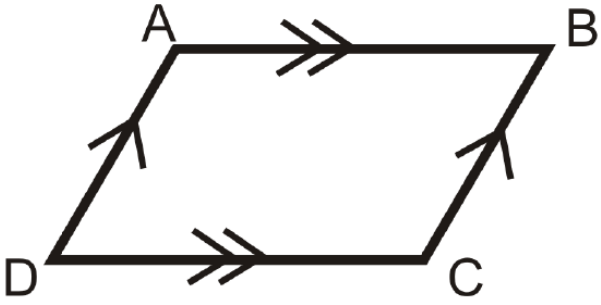

Figure 5.9.8

then

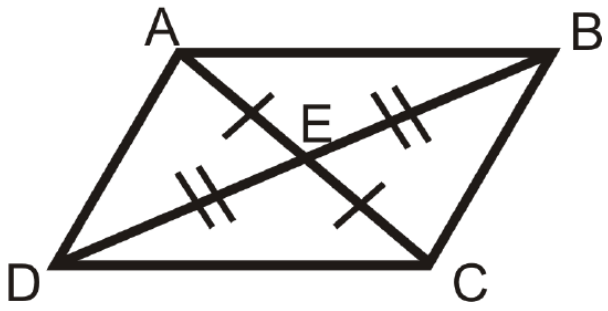

Figure 5.9.9

What if you were told that $FGHI$ is a parallelogram and you are given the length of FG\) and the measure of $\angle F$? What can you determine about HI, $\angle H$, $\angle G$, and $\angle I$?

Example 5.9.1

Show that the diagonals $\overline{FH}$ and $\overline{JG}$ bisect each other.

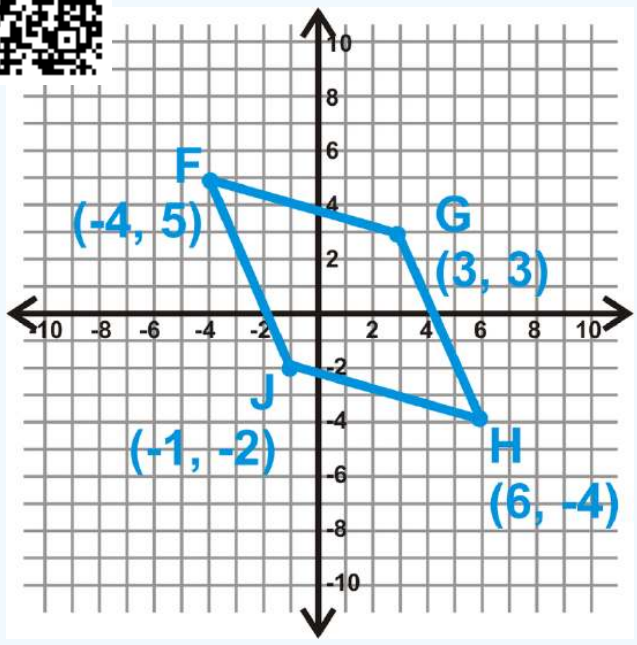

Figure 5.9.10

Solution

Find the midpoint of each diagonal.

Missing \begin{aligned} or extra \end{aligned}

Because they are the same point, the diagonals intersect at each other's midpoint. This means they bisect each other.

Example 5.9.2

Find the measures of a and b in the parallelogram below:

Figure 5.9.11

Solution

Consecutive angles are supplementary so $127^\circ + m\angle b = 180^\circ$ which means that $m\angle b = 53^\circ$. a and b are alternate interior angles and since the lines are parallel (since its a parallelogram), that means that $m\angle a = m\angle b = 53^\circ$.

Example 5.9.3

$ABCD$ is a parallelogram. If $m\angle A = 56^\circ$, find the measure of the other angles.

Solution

First draw a picture. When labeling the vertices, the letters are listed, in order.

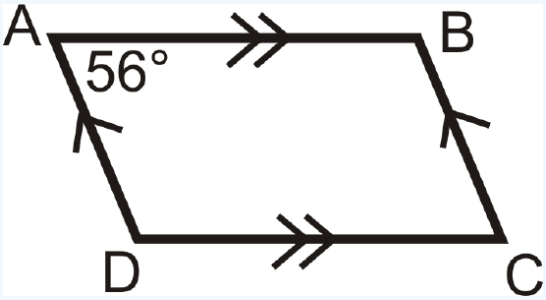

Figure 5.9.12

If $m\angle A = 56^\circ$, then $m\angle C = 56^\circ$ by the Opposite Angles Theorem.

$$m\angle A + m\angle B = 180^\circ \quad \text{by the Consecutive Angles Theorem.}$$
$$56^\circ + m\angle B = 180^\circ$$
$$m\angle B = 124^\circ \quad m\angle D = 124^\circ \quad \text{because it is an opposite angle to } \angle B$$

Example 5.9.4

Find the values of x and y.

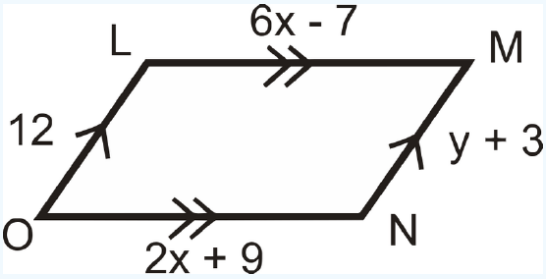

Figure 5.9.13

Solution

Remember that opposite sides of a parallelogram are congruent. Set up equations and solve.

$$6x - 7 = 2x + 9$$
$$4x = 16$$
$$x = 4$$
$$y + 3 = 12$$
$$y = 9$$

Example 5.9.5

Prove the Opposite Sides Theorem.

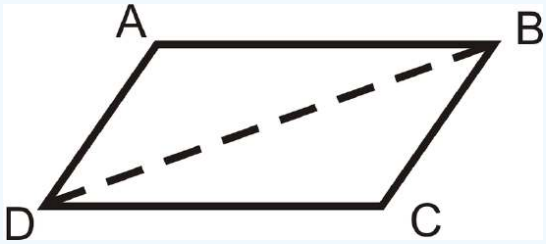

Figure 5.9.14

Solution

Given: $ABCD$ is a parallelogram with diagonal $\overline{BD}$

Prove: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

Statement	*Reason*
1. $ABCD$ is a parallelogram with diagonal $\overline{BD}$	1. Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	2. Definition of a parallelogram
3. $\angle ABD \cong \angle BDC$, $\angle ADB \cong \angle DBC$	3. Alternate Interior Angles Theorem
4. $\overline{DB} \cong \overline{DB}$	4. Reflexive PoC
5. $\Delta ABD \cong \Delta CDB$	5. ASA
6. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$	6. CPCTC

The proof of the Opposite Angles Theorem is almost identical.

Review

$ABCD$ is a parallelogram. Fill in the blanks below.

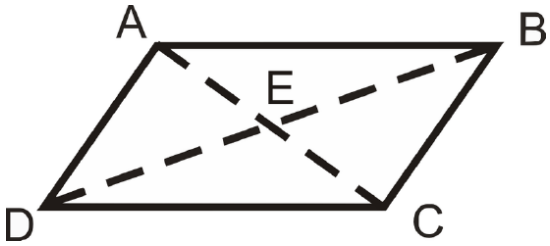

Figure 5.9.15

1. If $AB = 6$, then '_' allowed only in math mode.
2. If $AE = 4$, then '_' allowed only in math mode.
3. If $m\angle ADC = 80°$, '_' allowed only in math mode.
4. If $m\angle BAC = 45°$, '_' allowed only in math mode.
5. If $m\angle CBD = 62°$, '_' allowed only in math mode.
6. If $DB = 16$, then '_' allowed only in math mode.
7. If$m\angle B = 72°$ in parallelogram $ABCD$, find the other three angles.
8. If $m\angle S = 143°$ in parallelogram $PQRS$, find the other three angles.
9. If $\overline{AB} \perp \overline{BC}$ in parallelogram $ABCD$, find the measure of all four angles.
10. If $m\angle F = x°$ in parallelogram $EFGH$, find the other three angles.

For questions 11-18, find the values of the variable(s). All the figures below are parallelograms.

11.

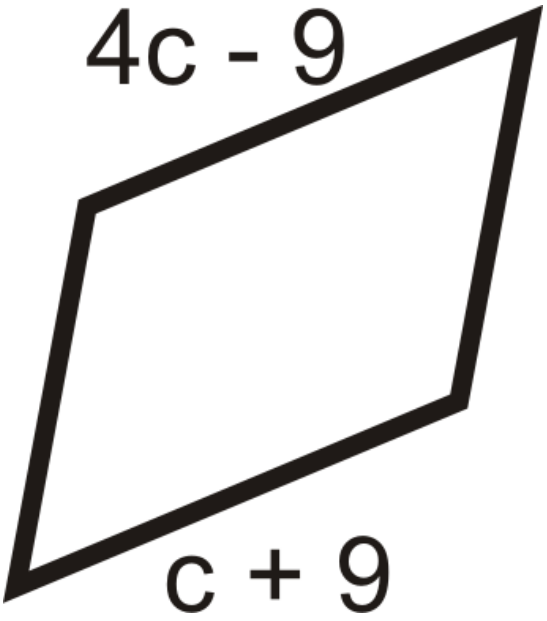

Figure 5.9.16

12.

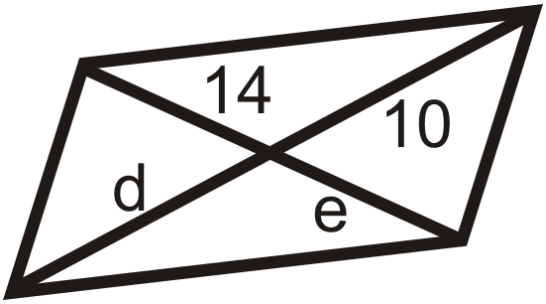

Figure 5.9.17

13.

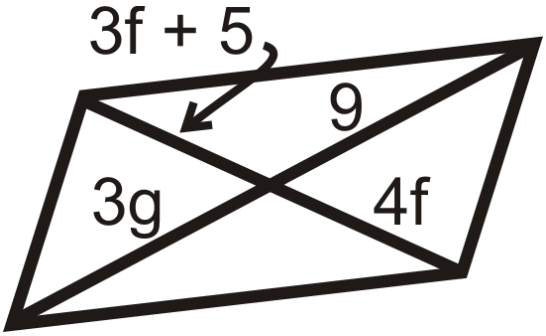

Figure 5.9.18

14.

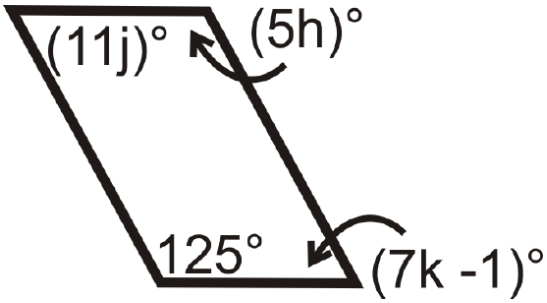

Figure 5.9.19

15.

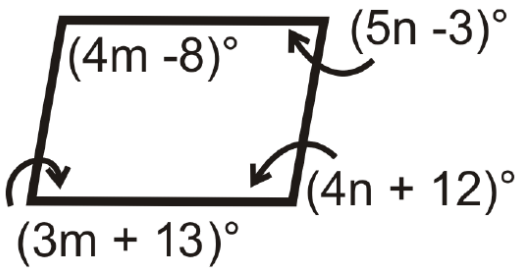

Figure 5.9.20

16.

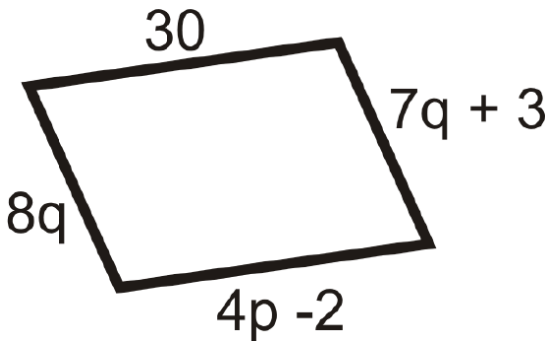

Figure 5.9.21

17.

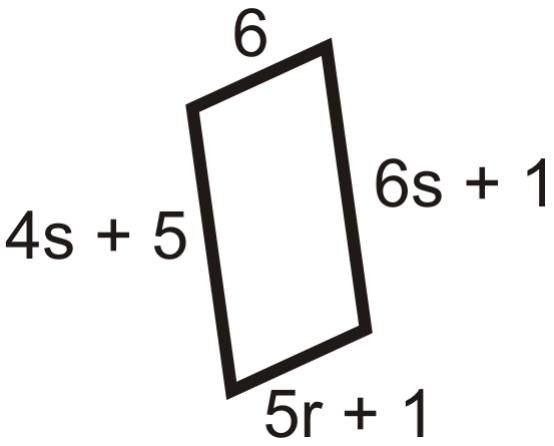

Figure 5.9.22

18.

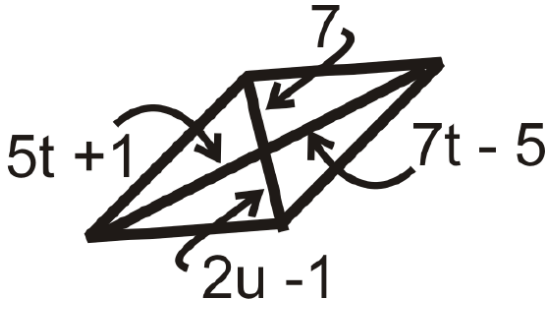

Figure 5.9.23

Use the parallelogram $WAVE$ to find:

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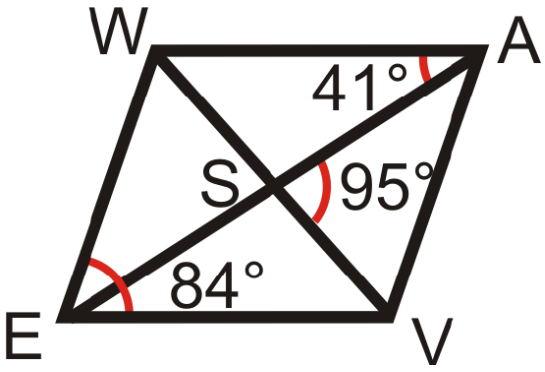

Figure 5.9.24

19. $m\angle AWE$
20. $m\angle ESV$
21. $m\angle WEA$
22. $m\angle AVW$

Find the point of intersection of the diagonals to see if $EFGH$ is a parallelogram.

23. $E(-1,3), F(3,4), G(5,-1), H(1,-2)$
24. $E(3,-2), F(7,0), G(9,-4), H(5,-4)$
25. $E(-6,3), F(2,5), G(6,-3), H(-4,-5)$
26. $E(-2,-2), F(-4,-6), G(-6,-4), H(-4,0)$

Fill in the blanks in the proofs below.

27. ***Opposite Angles Theorem***

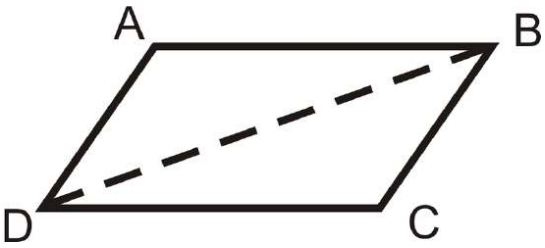

Figure 5.9.25

Given: $ABCD$ is a parallelogram with diagonal $\overline{BD}$

Prove: $\angle A \cong \angle C$

Statement	*Reason*
1.	1. Given
2. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	2.
3.	3. Alternate Interior Angles Theorem
4.	4. Reflexive PoC
5. $\Delta ABD \cong \Delta CDB$	5.
6. $\angle A \cong \angle C$	6.

28. ***Parallelogram Diagonals Theorem***

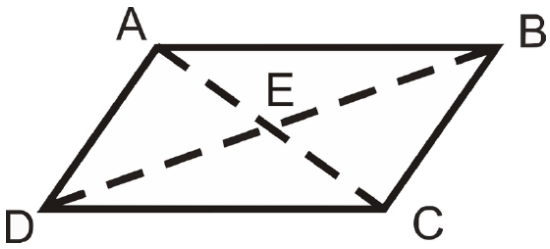

Figure **5.9.26**

Given: $ABCD$ is a parallelogram with diagonals $\overline{BD}$ and $\overline{AC}$

Prove: $\overline{AE} \cong \overline{EC}$, $\overline{DE} \cong \overline{EB}$

Statement	***Reason***
1.	1.
2.	2. Definition of a parallelogram
3.	3. Alternate Interior Angles Theorem
4. $\overline{AB} \cong \overline{DC}$	4.
5.	5.
6. $\overline{AE} \cong \overline{EC}, \overline{DE} \cong \overline{EB}$	6.

29. Find x, $y°$, and $z°$. (The two quadrilaterals with the same side are parallelograms.)

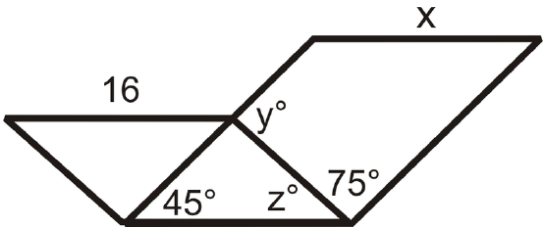

Figure **5.9.27**

Vocabulary

Term	Definition
parallelogram	A quadrilateral with two pairs of parallel sides. A parallelogram may be a **rectangle**, a **rhombus**, or a **square**, but need not be any of the three.

Additional Resources

Video: Parallelograms Principles - Basic

Activities: Parallelograms Discussion Questions

Study Aids: Parallelograms Study Guide

Practice: Parallelograms

Real World: Parallelograms

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5.10: Area of a Parallelogram

Use $A = bh$ to find area.

A **parallelogram** is a quadrilateral whose opposite sides are parallel.

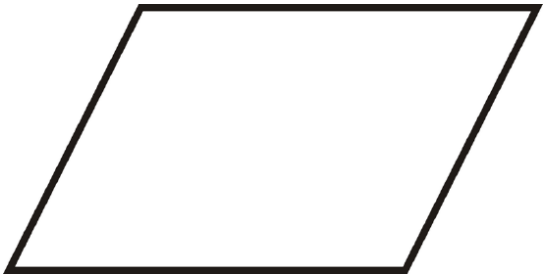

Figure 5.10.1

To find the **area** of a parallelogram, make it into a rectangle.

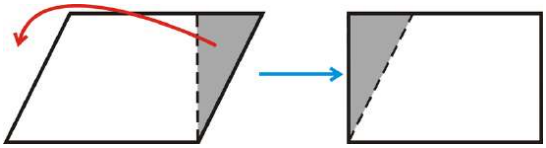

Figure 5.10.2

From this, we see that the area of a parallelogram is the same as the area of a rectangle. The area of a parallelogram is $A = bh$. The height of a parallelogram is always perpendicular to the base. This means that the sides are not the height.

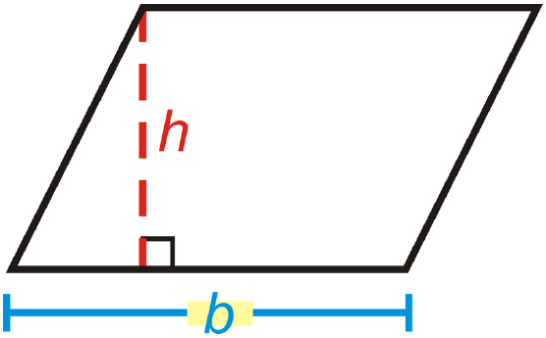

Figure 5.10.3

What if you were given a parallelogram and the size of its base and height? How could you find the amount of space the parallelogram takes up?

Example 5.10.1

Find the area of the para

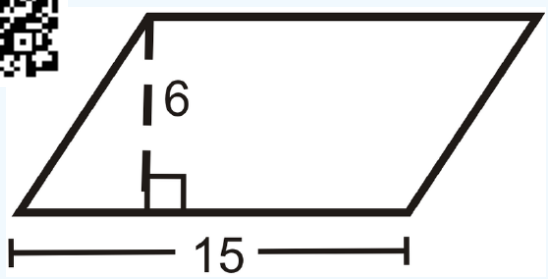

Figure 5.10.4

Solution

Area is $15(6) = 90 \text{ un}^2$.

Example 5.10.2

Find the area of the parallelogram with a base of 10 m and a height of 12 m.

Solution

Area is 10(12)=120\text{ m}^2\).

Example 5.10.3

Find the area of the parallelogram.

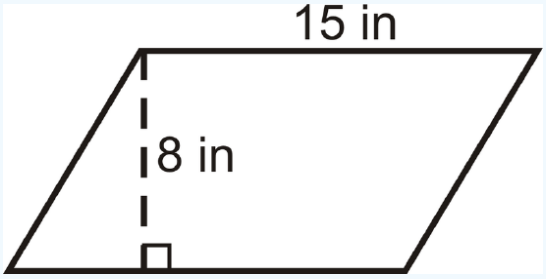

Figure 5.10.5

Solution

$A = 15 \cdot 8 = 120 \text{ in}^2$

Example 5.10.4

If the area of a parallelogram is 56 units^2 and the base is 4 units, what is the height?

Solution

Solve for the height in $A = bh$.

$$56 \text{ units} = 4h$$

$$14 \text{ units} = h$$

Example 5.10.5

If the height of a parallelogram is 12 m and the area is $60m^2$, how wide is the base?

Solution

Solve for the base in $A = bh$.

$$60 \text{ units} = 12b$$

$$5 \text{ units} = b$$

Review

1. Find the area of a parallelogram with height of 20 m and base of 18 m.
2. Find the area of a parallelogram with height of 12 m and base of 15 m.
3. Find the area of a parallelogram with height of 40 m and base of 33 m.
4. Find the area of a parallelogram with height of 32 m and base of 21 m.
5. Find the area of a parallelogram with height of 25 m and base of 10 m.

Find the area of the parallelogram.

6.

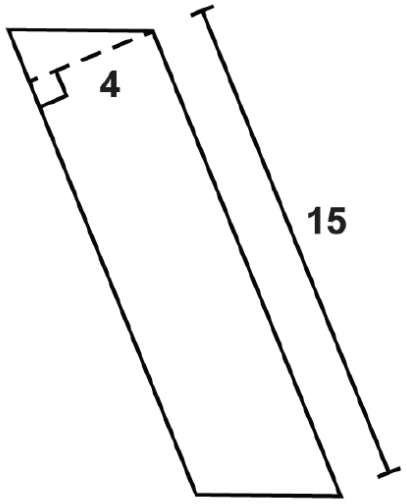

Figure 5.10.6

7.

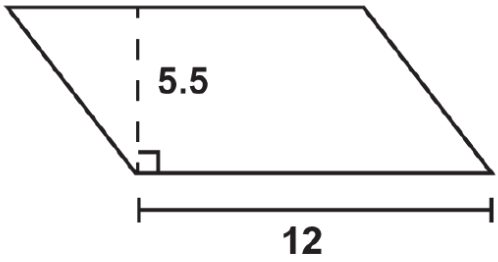

Figure 5.10.7

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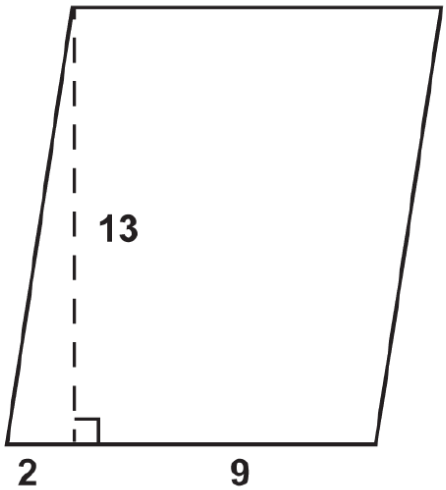
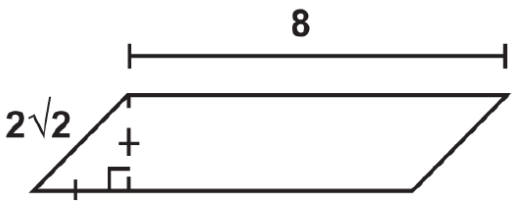
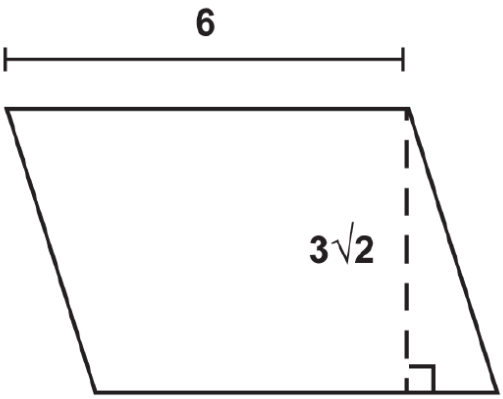


Figure 5.10.8



9.

Figure 5.10.9

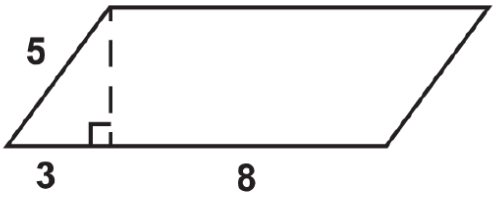


10.

Figure 5.10.10

11.

Figure 5.10.11



12.

Figure 5.10.12

13. If the area of a parallelogram is 42 units^2 and the base is 6 units, what is the height?
14. If the area of a parallelogram is 48 units^2 and the height is 6 units, what is the base?
15. If the base of a parallelogram is 9 units and the area is 108 units^2, what is the height?
16. If the height of a parallelogram is 11 units and the area is 27.5 units^2, what is the base?

Vocabulary

Term	Definition
area	The amount of space inside a figure. Area is measured in square units.
perimeter	The distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write ***units.***
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Area of a Parallelogram	The area of a parallelogram is equal to the base multiplied by the height: $A = bh$. The height of a parallelogram is always perpendicular to the base (the sides are not the height).

Additional Resources

Video: Area of a Parallelogram (Whole Numbers)

Activities: Area of a Parallelogram Discussion Questions

Study Aids: Triangles and Quadrilaterals Study Guide

Practice: Area of a Parallelogram

Real World: Perimeter

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5.11: Unknown Dimensions of Parallelograms

Use formula $A = bh$ to solve for unknown variable

Figure 5.11.1

Michael is in charge of designing a banner for the school's Homecoming activities. The banner will be divided into 4 different colored parallelograms each representing a different grade. The banner's quilt-style design requires each of the parallelograms to have an **area** of at least 720 square inches and a base of 36 inches. How can Michael use this information to determine the minimal height of each parallelogram?

In this concept, you will learn how to find unknown dimensions of parallelograms given the area and another dimension.

Finding Unknown Dimensions of Parallelograms

Finding the missing dimensions in parallelograms requires you to work backwards. You can figure out a missing dimension if you have been given the area and another measurement, such as the area and the height or the area and the base.

The area of a parallelogram is determined by multiplying the base and the height.

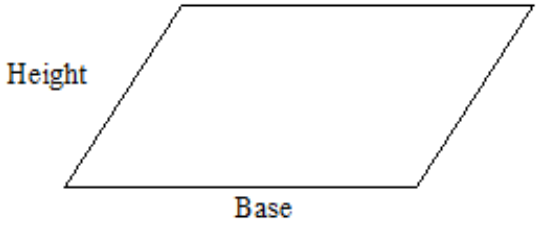

Figure 5.11.2

$$\text{Base} \times \text{Height} = \text{Area of a parallelogram}$$

If you know the area and the base or the area and height, you can use this formula and division to find the missing dimension.

Let's begin with a parallelogram where the base is the missing dimension.

A parallelogram has an area of 48 square inches and a height of 6 inches. What is the measurement of the base?

First, substitute the known information into the formula for computing the area of a parallelogram.

$$\begin{aligned} \text{Base} \times \text{Height} &= \text{Area of a parallelogram} \\ \text{Base} \times 6 \text{ inches} &= 48 \text{ square inches} \end{aligned}$$

Next, to get the base by itself on one side of the equal sign, divide both sides by the known height, 6 inches.

$$\frac{\text{Base (6 inches)}}{6 \text{ inches}} = \frac{48 \text{ square inches}}{6 \text{ inches}}$$

On the left side of the equation, the 6 inches will cancel leaving the base. On the right side of the equation 48 square inches are divided by 6 inches, leaving 8 inches.

Base = 8 inches

The answer is the base is 8 inches.

To find the missing base, you simply divided the area by the given height. This will work the same way if you are looking for the height. To find the missing height, divide the area by the given base.

Consider this parallelogram where height is the missing dimension.

A parallelogram has an area of 54 square feet and a base of 9 feet. What is the height of the parallelogram?

Start by working backwards.

To find the missing height, divide the given area by the given base measurement.

$54 \div 9 = 6$

The height of the parallelogram is 6 feet.

Example 5.11.1

Earlier, you were given a problem about Michael and the Homecoming banner.

The banner will hang at the entrance of the school during the week of Homecoming, so Michael wants to make sure each class is represented equally on the banner. Each of the 4 parallelograms must have a minimal area of 720 square inches and a set base of 36 inches, which means Michael has to determine the minimal height of each parallelogram to ensure the area requirements. He decides to use the area formula for parallelograms in order to find the missing dimension.

Solution

First, he substitutes the known information into the formula for computing the area of a parallelogram.

$$\begin{aligned} \text{Base} \times \text{Height} &= \text{Area of a parallelogram} \\ 36 \text{ inches} \times \text{Height} &= 720 \text{ square inches} \end{aligned}$$

Next, to get the base by itself on one side of the equal sign, divide both sides by the known height, 6 inches.

$$\frac{(36 \text{ inches}) \text{ Height}}{36 \text{ inches}} = \frac{720 \text{ square inches}}{36 \text{ inches}}$$

Notice, that Michael divided the given area by the given base to find the missing height.

$720 \div 36 = 20$

$\text{Height} = 20 \text{ inches}$

The answer is the minimal height of the parallelograms must be 20 inches.

Example 5.11.2

Find the parallelogram's missing dimension.

The area of the parallelogram is 169 square inches. The length of the base is 13 inches. What is the height?

Solution

First, substitute the known information into the formula for computing the area of a parallelogram.

$$\begin{aligned} \text{Base} \times \text{Height} &= \text{Area of a parallelogram} \\ 13 \text{ inches} \times \text{Height} &= 169 \text{ square inches} \end{aligned}$$

Next, to get the base by itself on one side of the equal sign, divide both sides by the known height, 6 inches.

$$\frac{(13 \text{ inches}) \text{ Height}}{13 \text{ inches}} = \frac{169 \text{ square inches}}{13 \text{ inches}}$$

Notice, that you simply divided the given area by the given base to find the missing height.

$169 \div 13 = 13$

$\text{Height} = 13 \text{ inches}$

The answer is the height is 13 inches.

Example 5.11.3

Find the parallelogram's missing dimension using the given measurements.

Area = 25 square meters

Base = 5 meters

Solution

First, divide the given area by the given base to find the missing height.

$25 \div 5 = 5$

$\text{Height} = 5 \text{ meters}$

The answer is the height is 5 meters.

Example 5.11.4

Find the parallelogram's missing dimension using the given measurements.

Area = 81 square feet

Base = 27 feet

Solution

First, divide the given area by the given base to find the missing height.

$81 \div 27 = 3$

$\text{Height} = 3 \text{ feet}$

The answer is the height is 3 feet.

Example 5.11.5

Find the parallelogram's missing dimension using the given measurements.

Area = 36 square inches

Height = 2 inches

Solution

First, divide the given area by the given height to find the missing base.

$36 \div 2 = 18$

$\text{Height} = 18 \text{ inches}$

The answer is the height is 18 inches.

Review

Use the given area and other dimension to find the missing base or height.

1. Area = 22 sq. inches, Base = 11 inches
2. Area = 50 sq. miles, Base = 10 miles
3. Area = 48 sq. inches, Base = 8 inches
4. Area = 30 sq. meters, Base = 15 meters
5. Area = 45 sq. feet, Height = 3 feet
6. Area = 88 sq. feet, Height = 8 feet
7. Area = 121 sq. feet, Height = 11 feet
8. Area = 160 sq. miles, Height = 20 miles

9. Area = 90 sq. meters, Height = 30 meters
10. Area = 100 sq. feet, Base = 25 feet
11. Area = 120 sq. feet, Base = 20 feet
12. Area = 144 sq. feet, Base = 12 feet
13. Area = 200 sq. feet, Base = 20 feet
14. Area = 400 sq. feet, Base = 200 feet
15. Area = 360 sq. feet, Base = 100 feet

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.2.

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Rectangle	A rectangle is a quadrilateral with four right angles.

Additional Resources

Video: Area of a Parallelogram (Whole Numbers)

Practice: Unknown Dimensions of Parallelograms

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5.12: Estimation of Parallelogram Area in Scale Drawings

Use a formula and a scale to determine actual dimensions from scale dimensions.

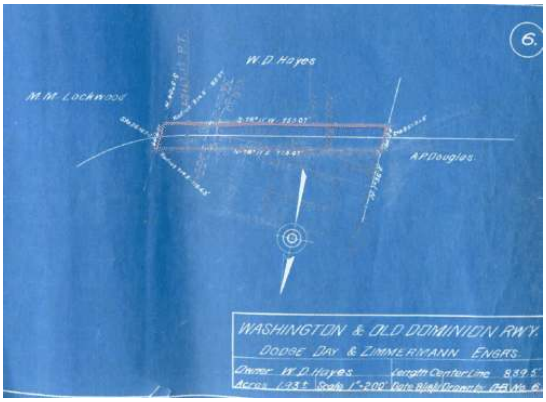

Figure 5.12.1

The town planned to put in a row of shuffleboard courts along a **parallelogram**-shaped piece of property near Jaxon's home. If the drawing shows a parallelogram with a $15''$ base and a $1''$ height, and the scale is $1 \text{ inch } = 12 \text{ feet}$, how big is the entire **area**?
In this concept, you will learn to estimate the area of a parallelogram on a **scale drawing**.

Estimating the Area of a Parallelogram

A **scale drawing** is a drawing that uses a small measurement to represent the actual measurement.

If you are given the scale, you can use it to estimate the actual areas of a parallelogram.

For example:

Estimate the area of this garden.

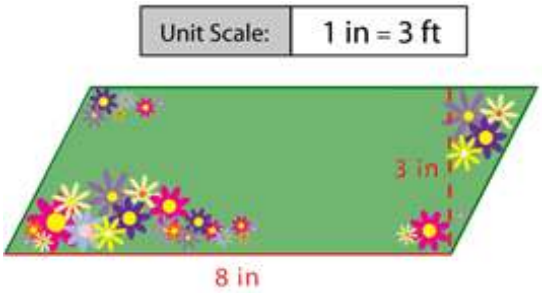

Figure 5.12.2

The formula for the area of a parallelogram is $A = bh$.

If you multiply the dimensions given for the base and height on the scale drawing, you will get the square inches of the drawing, not the area of the actual garden. You will need to use the scale to convert the dimensions to their actual sizes.

First, the scale reads $1 \text{ in } = 3 \text{ ft}$. This means that each inch on the drawing represents 3 feet of actual ground. There are 3 feet per inch, so multiply the given base and height times 3.

$$8 \text{ in } \times \frac{3 \text{ ft}}{\text{in}}$$

Writing the equation with two fractions, you can see that the inches cancel, and you are left with feet.

$$\frac{8 \text{ in}}{1} \times \frac{3 \text{ ft}}{\text{in}} = 24 \text{ ft}$$

This means that the actual base is 24\text{ ft }.

$$\frac{3 \text{ in}}{1} \times \frac{3 \text{ ft}}{\text{in}} = 9 \text{ ft}$$

This means that the actual height is 9 ft.

Next, round those values to the nearest 10.

$24 \approx 20$

The approximate base is 20.

$9 \approx 10$

The approximate height is 10.

Then, insert your approximations into the formula.

$$A = bh$$
$$A = 20' \times 10'$$
$$A = 200 \text{ sq ft}$$

The answer is that the estimate is 200 square feet.

You can check your estimate by inserting the actual dimensions into the formula.

$$A = bh$$
$$A = 24' \times 9'$$
$$A = 216 \text{ sq ft}$$

The actual area is 216 square feet.

$216 \approx 200$

The estimate is reasonable.

Estimating can help you visualize the actual size of something and determine whether or not your answer is correct.

Remember that when rounding 4, 5, or 6, you may get a significantly higher or lower area than actual.

Example 5.12.1

Earlier, you were give a problem about Jaxon and the blueprint of the shuffleboard court area.

Jaxon wants to estimate the size of the actual area and knows that the scale is 1 inch=12 feet. How many square feet will there be for the courts if the drawing has a base of 15 inches and a height of 1 inch?

Solution

First, determine the actual dimensions.

$$15 \text{ in} \times \frac{12 \text{ ft}}{\text{in}} = 180 \text{ ft}$$
$$b = 180'$$
$$1 \text{ in} \times \frac{12 \text{ ft}}{\text{in}} = 12 \text{ ft}$$
$$h = 12'$$

Next, round the values, and substitute into the formula.

$$A = bh$$
$$A = 180' \times 10'$$
$$A = 1,800 \text{ sq ft}$$

Jaxon's estimate is 1,800 square feet.

Then, find the actual square footage by using the actual dimensions.

$$A = 180' \times 12'$$
$$A = 2,160 \text{ sq ft}$$

The actual area is 2,160 square feet.

Example 5.12.2

Looking at the same drawing, use a scale of $1'' = 2$ feet .

Solution

First, the scale reads 1 in $= 2$ ft , or 2 feet per inch, so multiply the given base and height times 2.

$$8 \text{ in } \times \frac{2 \text{ ft}}{\text{in}} = 16 \text{ ft}$$

The actual base is 16\text{ ft }.

$$3 \text{ in } \times \frac{2 \text{ ft}}{\text{in}} = 6 \text{ ft}$$

This means that the actual height is 6 ft.

Next, round those values to the nearest 10.

16 rounds to 20

The approximate base is 20.

6 rounds to 10

The approximate height is 10.

Then, insert your approximations into the formula.

$$A = bh$$
$$A = 20' \times 10'$$
$$A = 200 \text{ sq ft}$$

The answer is that the estimate is 200 square feet.

You can check your estimate by inserting the actual dimensions into the formula.

$$A = bh$$
$$A = 16' \times 6'$$
$$A = 96 \text{ sq ft}$$

The actual area is 96 square feet.

The estimate is high.

Example 5.12.3

Use a scale of 1''=4\text{ ft } to estimate the area that has a base of 6 inches and a height of 4 inches.

Solution

First, determine the actual dimensions.

$$6 \text{ in } \times \frac{4 \text{ ft}}{\text{in}} = 24 \text{ ft}$$
$$b = 24'$$
$$4 \text{ in } \times \frac{4 \text{ ft}}{\text{in}} = 16 \text{ ft}$$
$$h = 16'$$

Next, round the values, and substitute into the formula.

$$A = 20' \times 20'$$
$$A = 400 \text{ sq ft}$$

The answer is that the estimate is 400 square feet.

Then, find the actual square footage by using the actual dimensions.

$$A = 24' \times 16'$$
$$A = 384 \text{ sq ft}$$

The actual area is 384 square feet.

The estimate is reasonable.

Example 5.12.4

Estimate the area of a parallelogram with a base of 59 inches and a height of 67 inches. Then solve for the actual area.

Solution

First, round the values, and substitute them into the formula.

$$A = 60'' \times 70''$$
$$A = 420 \text{ sq in}$$

Next, use the actual dimensions in the formula.

$$A = 59'' \times 67''$$
$$A = 3,953 \text{ sq in}$$

The answer is that the area is 3,953 square inches.

Example 5.12.5

A parallelogram has a base of 12 cm and height of 9 cm. What is its area?

Solution

First, write the formula.

$$A = bh$$

Next, substitute.

$$A = 12 \text{ cm} \times 9 \text{ cm}$$

Then, solve.

$$A = 108 \text{ sq cm}$$

The answer is 108 square centimeters.

If you rounded and multiplied in your head, your estimate should have been 100 square centimeters.

Review

For each parallelogram, find each new area using the scale $1'' = 2$ feet .

1. Base of 6 inches, height of 4 inches
2. Base of 8 inches, height of 6 inches
3. Base of 4 inches, height of 4 inches
4. Base of 5 inches, height of 4 inches
5. Base of 6 inches, height of 6 inches
6. Base of 10 inches, height of 8 inches
7. Base of 11 inches, height of 12 inches
8. Base of 15 inches, height of 9 inches
9. Base of 15 inches, height of 12 inches

Solve each problem.

10. A parallelogram has an area of 390 square centimeters. If its height is 15 cm, what is its base?
11. What is the height of a parallelogram whose base is 28 inches and area is 1,176 square inches?
12. Donna wants to cover her parallelogram-shaped crafts box in fabric. The base of the lid is 32.7 cm and the height is 12.2 cm. What is the area of the lid?
13. John is planting grass in a patch of lawn that is shaped like a parallelogram. The height of the parallelogram is 34 feet. The other border is 65 feet. How many square feet of grass will John plant?

14. Kara and Sharice are in a quilting competition. Both are stitching parallelogram-shaped quilts. So far Kara's has an area of 2,278 square inches and a height of 44 inches. Sharice's quilt has an area of 2,276 square inches and a height of 47 inches. Whose quilt is longer? By how many inches is it longer?
15. Denise bought a picture frame in the shape of a parallelogram. The area of the picture frame is 36,795 square centimeters. If its height is 165 centimeters, what is its base?

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.11.

Resources

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Perimeter	Perimeter is the distance around a two-dimensional figure.
Scale Drawing	A scale drawing is a drawing that is done with a scale so that specific small units of measure represent larger units of measure.

Additional Resources

Video: Area of a Parallelogram (Whole Numbers)

Practice: Estimation of Parallelogram Area in Scale Drawings

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5.13: Trapezoids

Determine unknown angle measurements of quadrilaterals with exactly one pair of parallel sides.

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

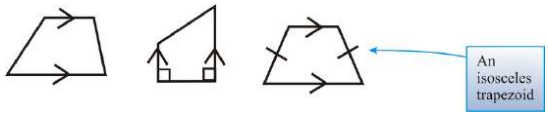

Figure **5.13.1**

An **isosceles trapezoid** is a trapezoid where the non-parallel sides are congruent.

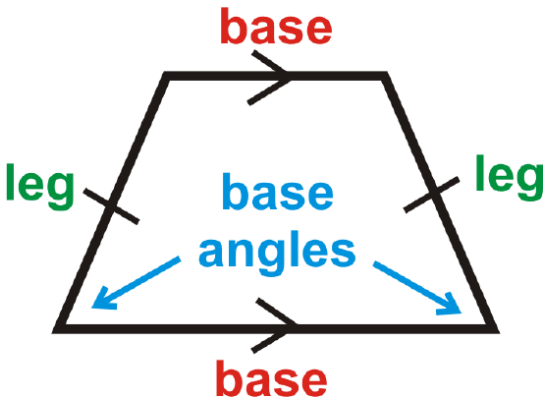

Figure **5.13.2**

The base angles of an isosceles trapezoid are congruent. If \(*ABCD*\) is an isosceles trapezoid, then $\angle A \cong \angle B$ and $\angle C \cong \angle D$.

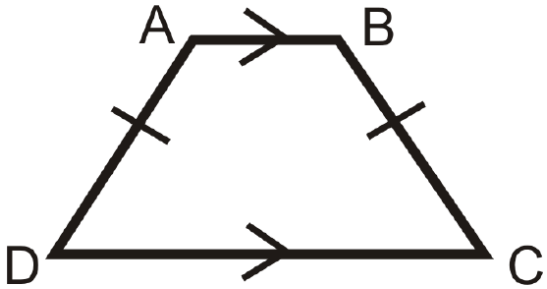

Figure **5.13.3**

The converse is also true. If a trapezoid has congruent base angles, then it is an isosceles trapezoid. The **diagonals** of an isosceles trapezoid are also congruent. The **midsegment** (of a trapezoid) is a line segment that connects the midpoints of the non-parallel sides:

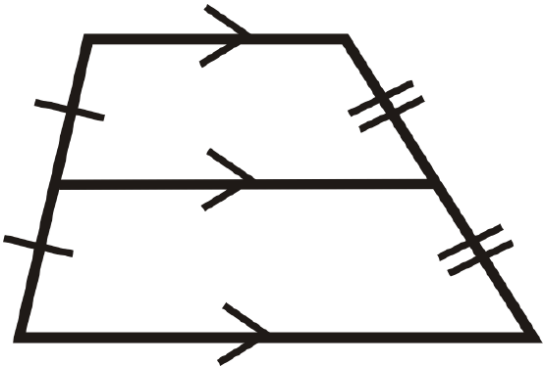

Figure **5.13.4**

There is only one midsegment in a trapezoid. It will be parallel to the bases because it is located halfway between them.

Midsegment Theorem: The length of the **midsegment of a trapezoid** is the average of the lengths of the bases.

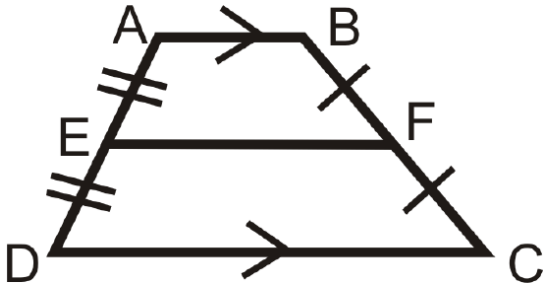

Figure 5.13.5

If $\overline{EF}$ is the midsegment, then $EF = \frac{AB + CD}{2}$.

What if you were told that the polygon $ABCD$ is an isosceles trapezoid and that one of its base angles measures $38°$? What can you conclude about its other base angle?

For Examples 1 and 2, use mation:

\($TRAP$\) is an isosceles t

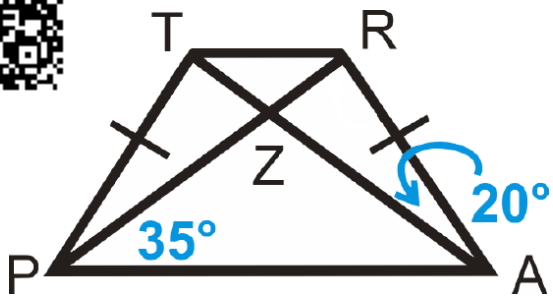

Figure 5.13.6

Example 5.13.1

Find $m\angle TPA$.

Solution

$\angle TPZ \cong \angle RAZ$ so $m\angle TPA = 20° + 35° = 55°$.

Example 5.13.2

Find $m\angle ZRA$.

Solution

Since $m\angle PZA = 110°$, $m\angle RZA = 70°$ because they form a linear pair. By the Triangle Sum Theorem, $m\angle ZRA = 90°$.

Example 5.13.3

Look at trapezoid $TRAP$ below. What is $m\angle A$?

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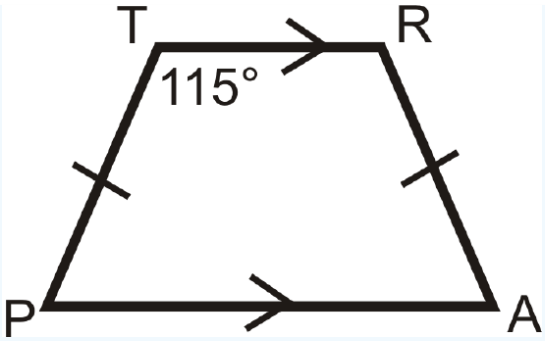


Figure 5.13.7

Solution

$TRAP$ is an isosceles trapezoid. $m\angle R = 115°$ also.

To find \(m\angle , set up an equation.

$$\begin{aligned} 115° + 115° + m\angle A + m\angle P &= 360° \\ 230° + 2m\angle A &= 360° \quad \rightarrow m\angle A = m\angle P \\ 2m\angle A &= 130° \\ m\angle A &= 65° \end{aligned}$$

Notice that $m\angle R + m\angle A = 115° + 65° = 180°$. These angles will always be supplementary because of the Consecutive Interior Angles Theorem.

Example 5.13.4

Is $ZOID$ an isosceles trapezoid? How do you know?

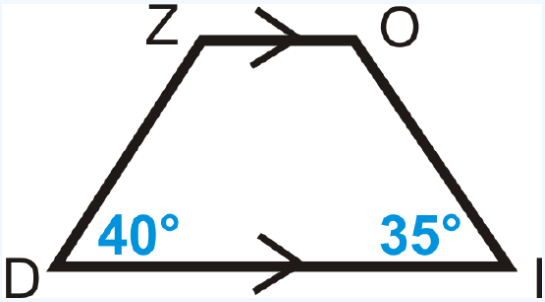


Figure 5.13.8

Solution

$40° \neq 35°$, $ZOID$ is not an isosceles trapezoid.

Example 5.13.5

Find x. All figures are trapezoids with the midsegment marked as indicated.

a.

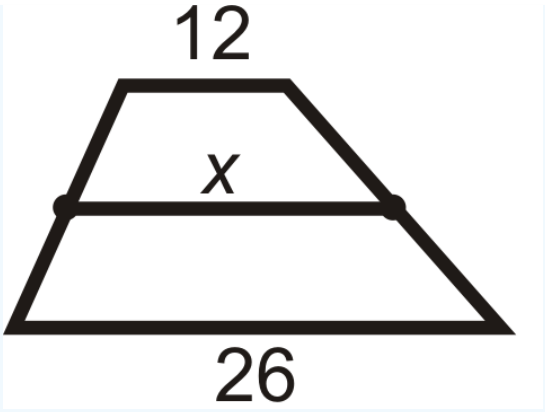


Figure 5.13.10

b.

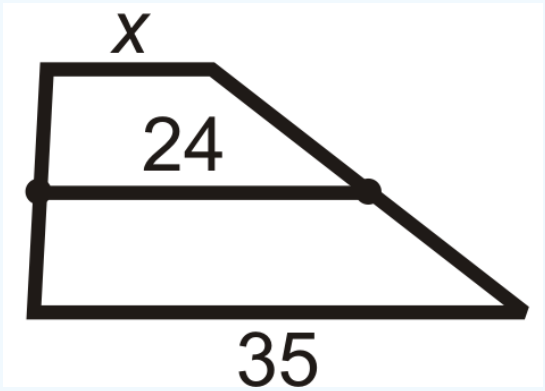


Figure 5.13.10

c.

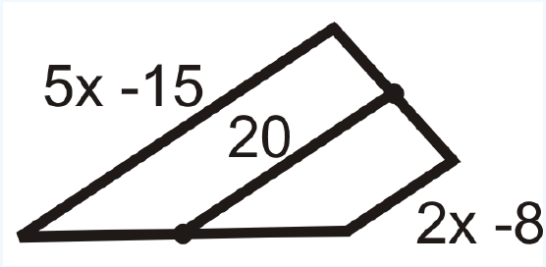


Figure 5.13.11

Solution

a. x is the average of 12 and 26. $\frac{12+26}{2}=\frac{38}{2}=19$

b. 24 is the average of x and 35.

$$\begin{aligned}\frac{x+35}{2}&=24\\ x+35&=48\\ x&=13\end{aligned}$$

c. 20 is the average of $5x-15$ and $2x-8$.

$$\begin{aligned}\frac{5x-15+2x-8}{2}&=20\\ 7x-23&=40\\ 7x&=63\\ x&=9\end{aligned}$$

Review

1. Can the parallel sides of a trapezoid be congruent? Why or why not?

For questions 2-8, find the length of the midsegment or missing side.

2.

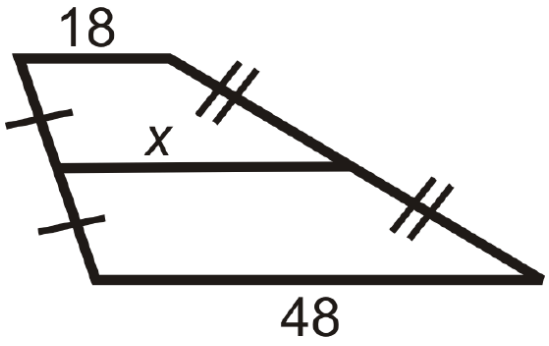


Figure 5.13.12

3.

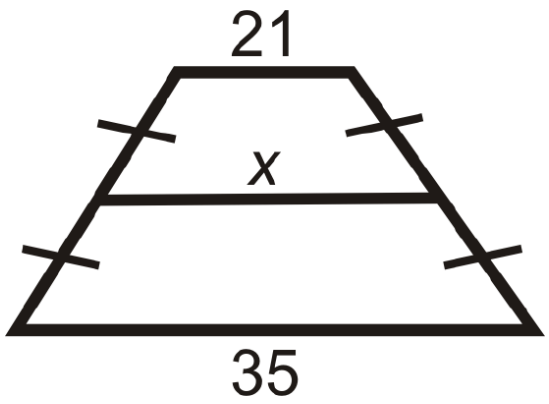


Figure 5.13.13

4.

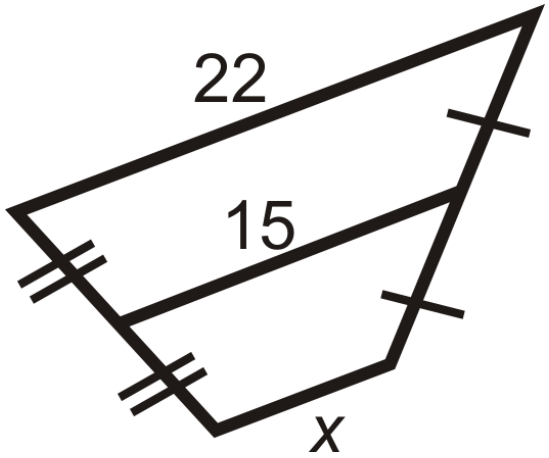


Figure 5.13.14

5.

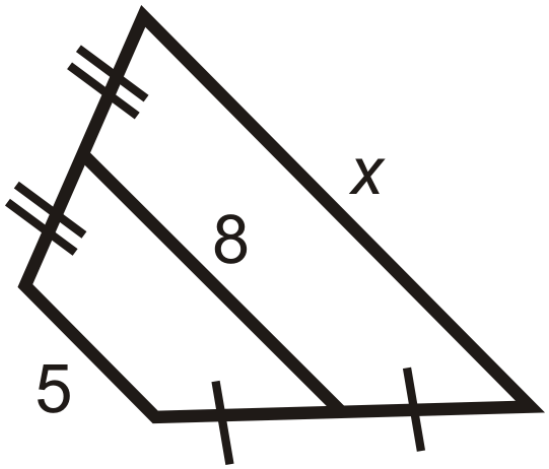


Figure 5.13.15

6.

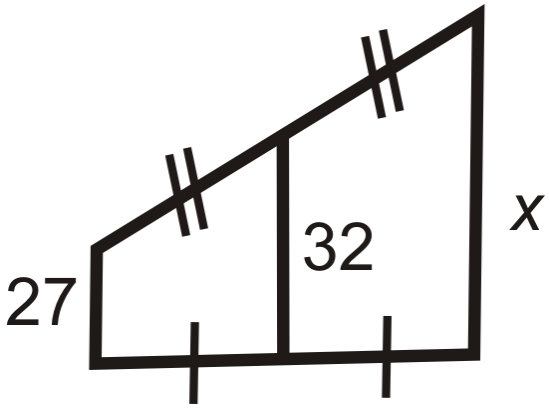


Figure 5.13.16

7.

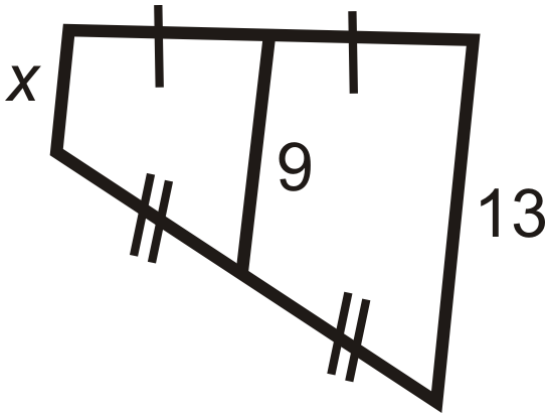


Figure 5.13.17

Find the value of the missing variable(s).

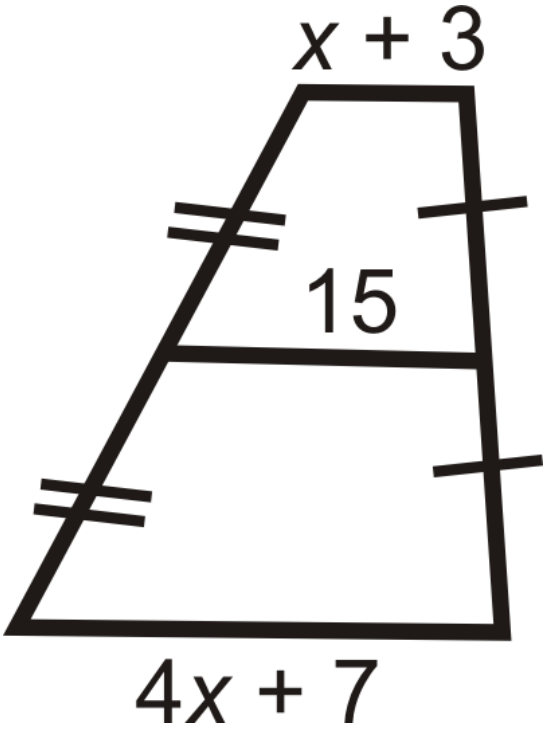
8. 

Figure 5.13.18

Find the lengths of the diagonals of the trapezoids below to determine if it is isosceles.

9. A(−3,2), B(1,3), C(3,−1), D(−4,−2)
10. A(−3,3), B(2,−2), C(−6,−6), D(−7,1)

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.6.

Vocabulary

Term	Definition
isosceles trapezoid	An ***isosceles trapezoid*** is a trapezoid where the non-parallel sides are congruent.
midsegment (of a trapezoid)	A line segment that connects the midpoints of the non-parallel sides.
trapezoid	A quadrilateral with exactly one pair of parallel sides.
Diagonal	A diagonal is a line segment in a polygon that connects nonconsecutive vertices
midsegment	A midsegment connects the midpoints of two sides of a triangle or the non-parallel sides of a trapezoid.

Additional Resources

Video: Trapezoids Examples - Basic

Activities: Trapezoids Discussion Questions

Study Aids: Trapezoids and Kites Study Guide

Practice: Trapezoids

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5.14: Area and Perimeter of Trapezoids

Area is the height times the average of the bases; perimeter is the sum of the sides.

A **trapezoid** is a quadrilateral with one pair of parallel sides. The parallel sides are called the bases and we will refer to the lengths of the bases as b_1 and b_2. The perpendicular distance between the parallel sides is the height of the trapezoid. The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$ where h is always perpendicular to the bases.

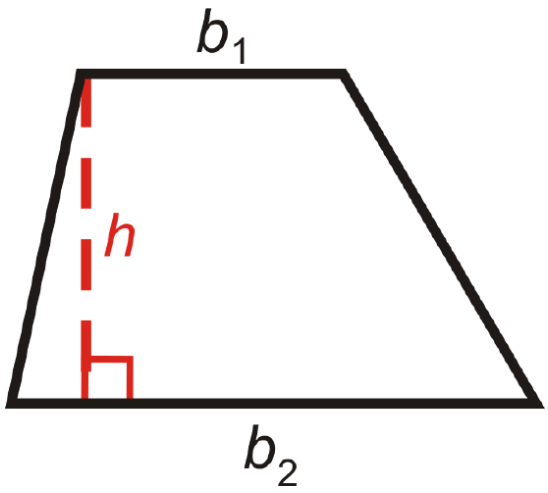

Figure 5.14.1

What if you were given a trapezoid and the size of its two bases as well as its height? How could you find the total distance around the trapezoid and the amount of space it takes up?

Example 5.14.1

Find the area of the trap

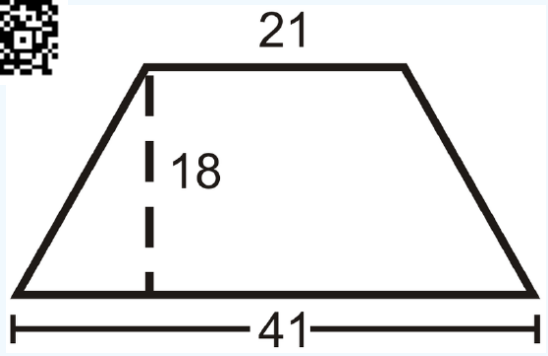

Figure 5.14.2

Solution

Use the formula for the area of a trapezoid.

$\frac{1}{2}(18)(41 + 21) = 558 \text{ units}^2$

Example 5.14.2

Find the area of the trapezoid. *Round your answers to the nearest hundredth.*

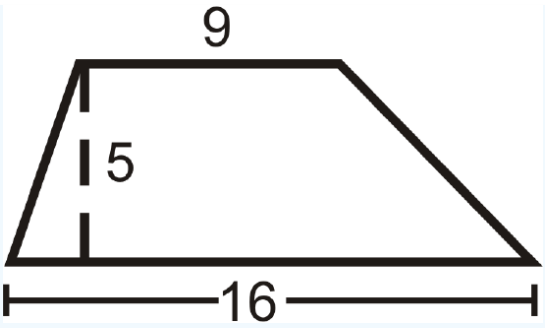

Figure 5.14.3

Solution

Use the formula for the area of a trapezoid.

$\frac{1}{2}(5)(16+9) = 62.5 \text{ units}^2$

Example 5.14.3

Find the area of the trapezoid.

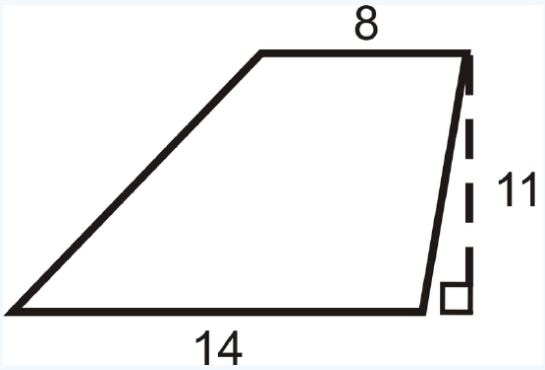

Figure 5.14.4

Solution

$A = \frac{1}{2}(11)(14+8)$

$A = \frac{1}{2}(11)(22)$

$A = 121 \text{ units}^2$

Example 5.14.4

Find the area of the trapezoid.

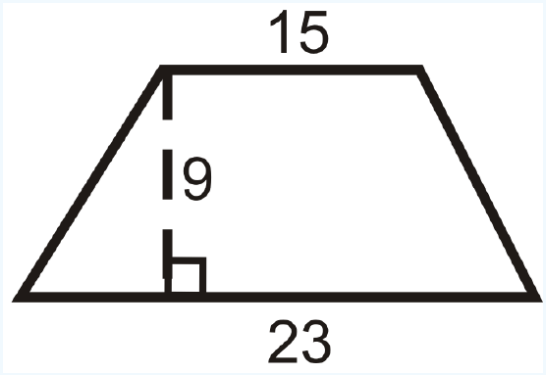

Figure 5.14.5

Solution

$A = \frac{1}{2}(9)(15 + 23)$

$A = \frac{1}{2}(9)(38)$

$A = 171 \text{ units}^2$

Example 5.14.5

Find the perimeter and area of the trapezoid.

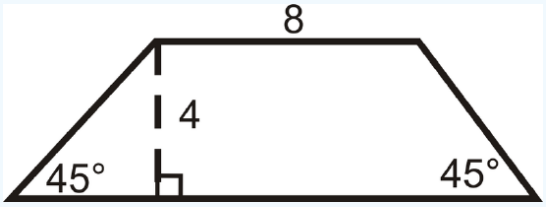

Figure 5.14.6

Solution

Even though we are not told the length of the second base, we can find it using special right triangles. Both triangles at the ends of this trapezoid are isosceles right triangles, so the hypotenuses are $4\sqrt{2}$ and the other legs are of length 4.

$P = 8 + 4\sqrt{2} + 16 + 4\sqrt{2}$ $\qquad$ $A = \frac{1}{2}(4)(8 + 16)$

$P = 24 + 8\sqrt{2} \approx 35.3 \text{ units}$ $\qquad$ $A = 48 \text{ units}^2$

Review

Find the area and perimeter of the following shapes. *Round your answers to the nearest hundredth.*

1.

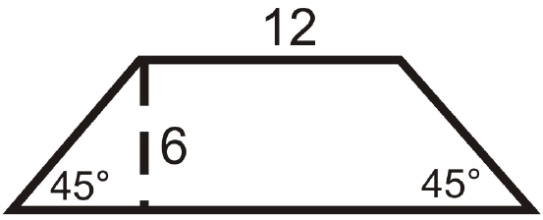

Figure 5.14.7

2.

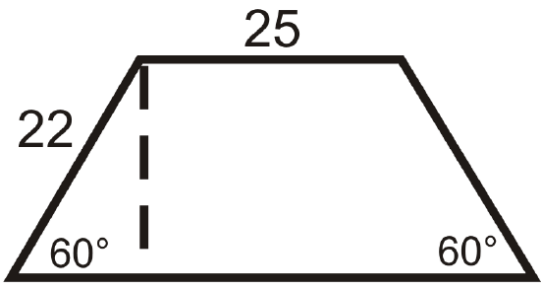

Figure 5.14.8

Find the area of the following trapezoids.

3. Trapezoid with bases 3 in and 7 in and height of 3 in.
4. Trapezoid with bases 6 in and 8 in and height of 5 in.
5. Trapezoid with bases 10 in and 26 in and height of 2 in.
6. Trapezoid with bases 15 in and 12 in and height of 10 in.
7. Trapezoid with bases 4 in and 23 in and height of 21 in.
8. Trapezoid with bases 9 in and 4 in and height of 1 in.

9. Trapezoid with bases 12 in and 8 in and height of 16 in.
10. Trapezoid with bases 26 in and 14 in and height of 19 in.

Use the given figures to answer the questions.

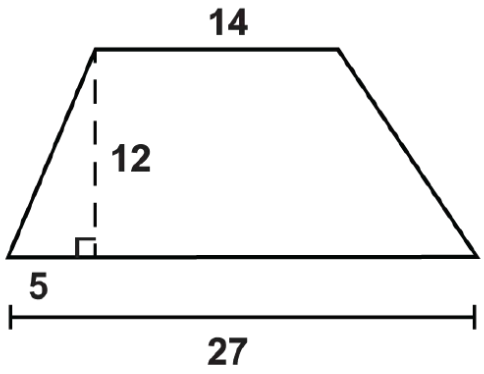

Figure **5.14.9**

11. What is the perimeter of the trapezoid?
12. What is the area of the trapezoid?

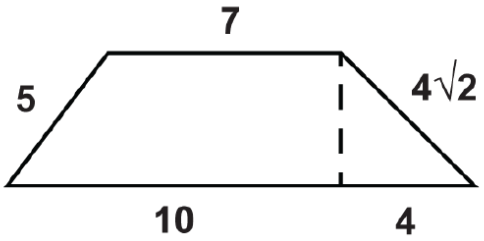

Figure **5.14.10**

13. What is the perimeter of the trapezoid?
14. What is the area of the trapezoid?

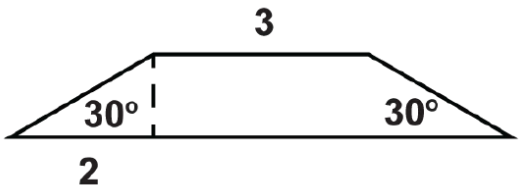

Figure 5.14.11

15. What is the perimeter of the trapezoid?
16. What is the area of the trapezoid?

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.5.

Vocabulary

Term	Definition
area	The amount of space inside a figure. Area is measured in square units.
isosceles trapezoid	An ***isosceles trapezoid*** is a trapezoid where the non-parallel sides are congruent.
midsegment (of a trapezoid)	A line segment that connects the midpoints of the non-parallel sides.
perimeter	The distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write ***units.***
trapezoid	A quadrilateral with exactly one pair of parallel sides.

Additional Resources

Video: Area and Perimeter of Trapezoids Examples - Basic

Activities: Area and Perimeter of Trapezoids Discussion Questions

Study Aids: Triangles and Quadrilaterals Study Guide

Practice: Area and Perimeter of Trapezoids

Real World: Perimeter

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5.15: Area of Parallelograms- Squares, Rectangles and Trapezoids

Find the area of rectangles, parallelograms, trapezoids.

Find the Dimensions and Area of Quadrilaterals

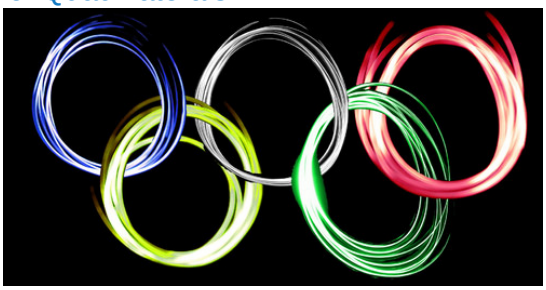

Figure 5.15.1

Montgomery Middle School is going to host a school wide Olympics for the first time. The initial preparations have already started and a fantastic two day Olympic event will take place in six weeks. Mrs. Meery's class will be building the platform for the award's ceremony. The platform is in the shape of a trapezoid with bases 35 feet and 41 feet and a height of 7.5 feet. If one bucket of cement covers 25 square feet, how many buckets will the students need to build the platform?

In this concept, you will learn to find the dimensions and areas of different quadrilaterals.

Quadrilaterals

Quadrilaterals are four-sided polygons. They are classified by the characteristics of their sides and angles. Rectangles, squares, parallelograms, rhombi, and trapezoids are all quadrilaterals. The images below show some of the quadrilaterals and their characteristics.

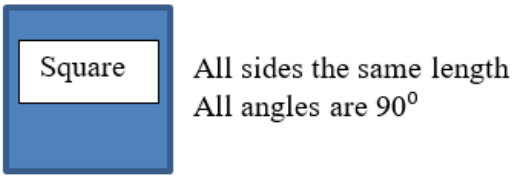

Figure 5.15.2

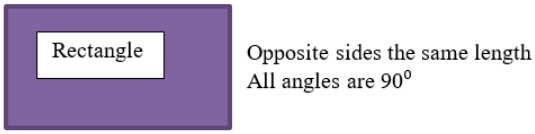

Figure 5.15.3

Figure 5.15.4

Figure 5.15.5

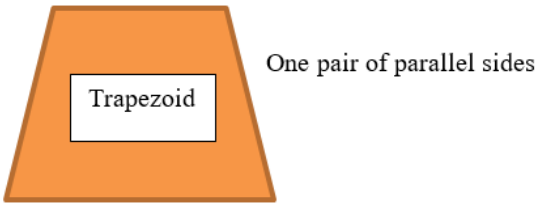

Figure 5.15.6

Each of these quadrilaterals has its own **area** formula.

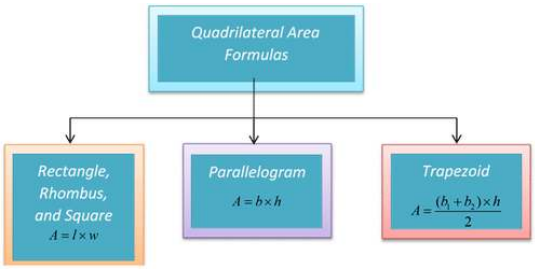

Figure 5.15.7

Let's look at an example.

What is the area of a rectangle with a length of 6 inches and a width of 4 inches?

First, fill in what you know into the formula for the area of a rectangle.

$$A = l \times w$$
$$A = 6 \times 4$$

Next, solve for A.

$$A = 6 \times 4$$
$$A = 24$$

The answer is 24.

The area of the rectangle is 24 in^2, or 24 square inches.

Let's look at another example.

What is the area of the parallelogram below?

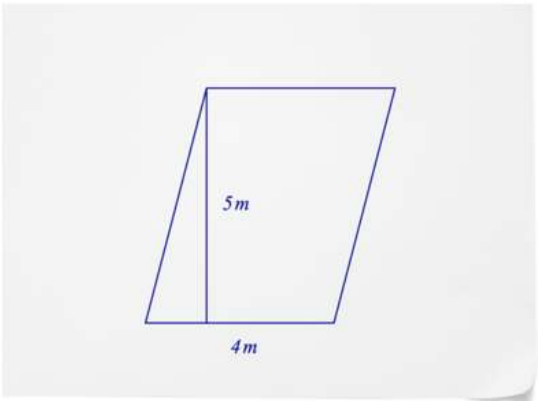

Figure 5.15.8

First, substitute what you know into the formula for the area of a parallelogram.

$$A = b \times h$$
$$A = 4 \times 5$$

Next, solve for A.

$$A = 4 \times 5$$
$$A = 20$$

The answer is 20.

The area of the parallelogram is $20\ \text{m}^2$, or 20 square meters.

Let's look at one more example.

Find the area of the trapezoid below.

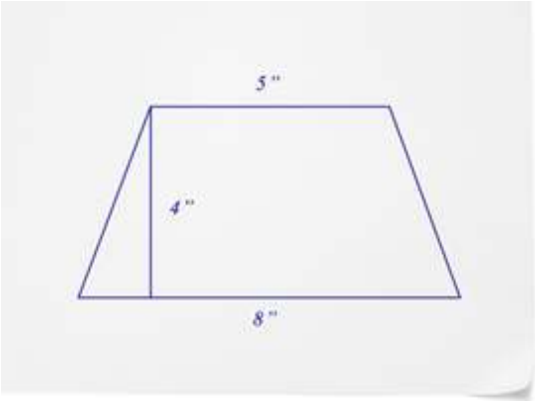

Figure 5.15.9

First, substitute what you know into the formula for the area of a trapezoid.

$$A = \frac{(b_1 + b_2) \times h}{2}$$
$$A = \frac{(5 + 8) \times 4}{2}$$

Next, solve for A.

$$A = \frac{(5 + 8) \times 4}{2}$$
$$A = 26$$

The answer is 26.

The area of the trapezoid is $26\ \text{in}^2$, or 26 square inches.

Example 5.15.1

Earlier, you were given a problem about the school's podium.

The podium is in the shape of a trapezoid that has base measures of 35 feet and 41 feet and a height of 7.5 feet. The podium is being made of cement where one bucket of cement covers 25 square feet. You need to find the area of the podium and then the number of buckets of cement to make it.

Solution

First, substitute what you know into the formula for the area of a trapezoid.

$$A = \frac{(b_1 + b_2) \times h}{2}$$
$$A = \frac{(35 + 41) \times 7.5}{2}$$

Next, solve for A.

$$A = \frac{(35 + 41) \times 7.5}{2}$$
$$A = 285$$

Then, divide the area by 25 in order to determine the number of buckets of cement to use.

Misplaced '#'

Misplaced '#'

The answer is 11.4.

The students will have to buy 12 buckets of cement for their project.

Example 5.15.2

A parallelogram has an area of $105\ m^2$. The height of the parallelogram is 7 m. What is its base?

Solution

First, substitute what you know into the formula for the area of a parallelogram.

$$A = b \times h$$
$$105 = b \times 7$$

Next, divide both sides by 7 to solve for b.

$$105 = b \times 7$$
$$\frac{105}{7} = \frac{7b}{7}$$
$$b = 15$$

The answer is 15.

The base of the parallelogram has a length of 15 m.

Example 5.15.3

What is the area of a square with a side length of 4.5 inches?

Solution

First, substitute what you know into the formula for the area of a square.

$$A = l \times w$$
$$A = 4.5 \times 4.5$$

Next, solve for A.

$$A = 4.5 \times 4.5$$
$$A = 20.25$$

The answer is 20.25. Therefore the area of the square is $20.25\ in^2$, or 20.25 square inches.

Example 5.15.4

What is the area of a rectangle with a length of 8 feet and a width of 6.25 feet?

Solution

First, substitute what you know into the formula for the area of a rectangle.

$$A = l \times w$$
$$A = 8 \times 6.25$$

Next, solve for A.

$$A = 8 \times 6.25$$
$$A = 50$$

The answer is 50.

The area of the rectangle is 50 ft^2\), or 50 square feet.

Example 5.15.5

What is the area of a parallelogram with a base of 10 meters and a height of 7.5 meters?

Solution

First, substitute what you know into the formula for the area of a parallelogram.

$$A = b \times h$$
$$A = 10 \times 7.5$$

Next, solve for A.

$$A = 10 \times 7.5$$
$$A = 75$$

The answer is 75.

The area of the parallelogram is 75 m2, or 75 square meters.

Review

1. $l = 10$ in , $w = 7.5$ in

2. $l = 12$ ft , $w = 9$ ft

3. $l = 14$ ft , $w = 11$ ft

4. $l = 21$ ft , $w = 19$ ft

Find the area of each parallelogram.

5. $b = 11$ ft , $h = 9$ ft

6. $b = 13$ in , $h = 11$ in

7. $b = 22$ ft , $h = 19$ ft

8. $b = 31$ meters , $h = 27$ meters

Find the area of each trapezoid.

9. $Bases = 5$ in $and\, 8in$, $height = 4$ inches

10. $Bases = 6\, in\, and\, 8$ in , $height = 5$ inches

11. $Bases = 10$ feet $and\, 12$ feet , $height = 9$ feet

Find the area of each square.

12. side length of 8 inches

13. side length of 15 feet

14. side length of 22.5 mm

15. side length of 18.25 cm

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.2.

Resources

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Perpendicular	Perpendicular lines are lines that intersect at a **90°** angle. The product of the slopes of two perpendicular lines is -1.
Polygon	A polygon is a simple closed figure with at least three straight sides.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.

Additional Resources

Video: Determine the Area of a Rectangle Involving Whole Numbers

Practice: Area of Parallelograms: Squares, Rectangles and Trapezoids

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5.16: Kites

Quadrilaterals with two distinct sets of adjacent, congruent sides.

A **kite** is a quadrilateral with two distinct sets of adjacent congruent sides. It looks like a kite that flies in the air.

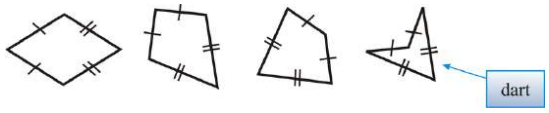

Figure **5.16.1**

From the definition, a kite could be concave. If a kite is concave, it is called a **dart.** The word **distinct** in the definition means that the two pairs of congruent sides have to be different. This means that a square or a rhombus is not a kite.

The angles between the congruent sides are called **vertex angles.** The other angles are called **non-vertex angles.** If we draw the diagonal through the vertex angles, we would have two congruent triangles.

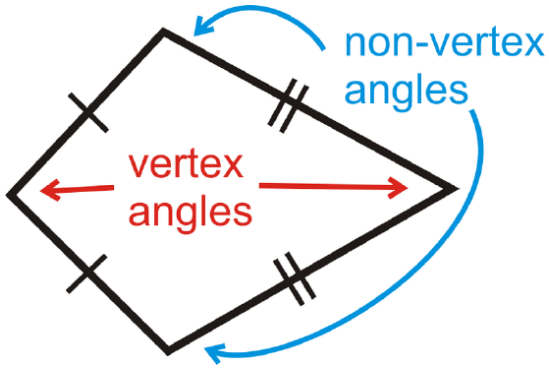

Figure **5.16.2**

Facts about Kites

1. The non-vertex angles of a kite are congruent.

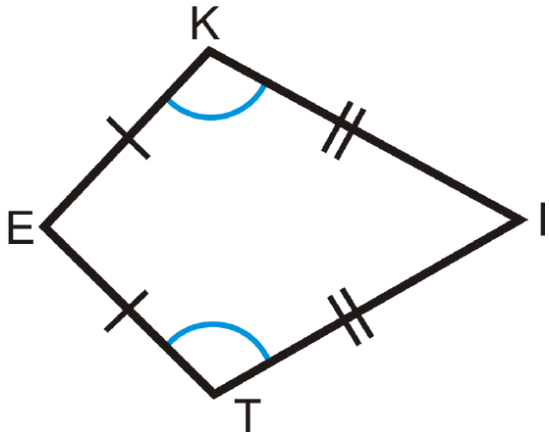

Figure **5.16.3**

If $KITE$ is a kite, then $\angle K \cong \angle T$.

2. The diagonal through the vertex angles is the angle bisector for both angles.

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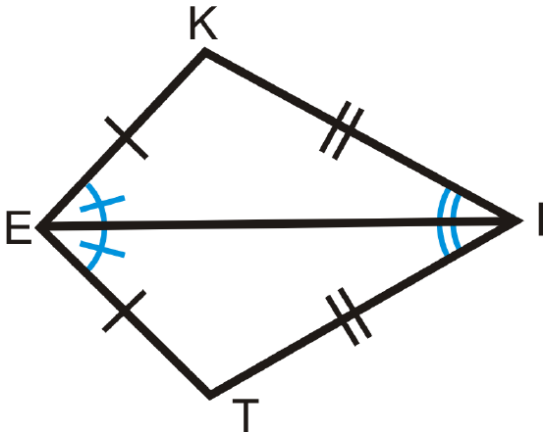

Figure \(\
PageIndex
{4}\)

If $KITE$ is a kite, then $\angle KEI \cong \angle IET$ and $\angle KIE \cong \angle EIT$.

3. **Kite Diagonals Theorem:** The diagonals of a kite are perpendicular.

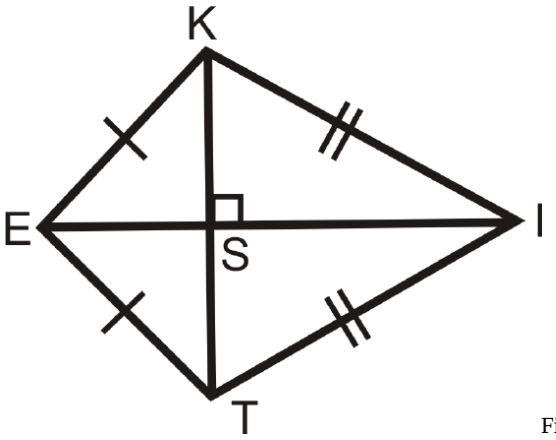

Figure **5.16.5**

ΔKET and ΔKIT are isosceles triangles, so $\overline{EI}$ is the perpendicular bisector of $\overline{KT}$ (Isosceles Triangle Theorem).

What if you were told that $WIND$ is a kite and you are given information about some of its angles or its diagonals? How would you find the measure of its other angles or its sides?

For Examples 1 and 2, use mation:

$KITE$ is a kite.

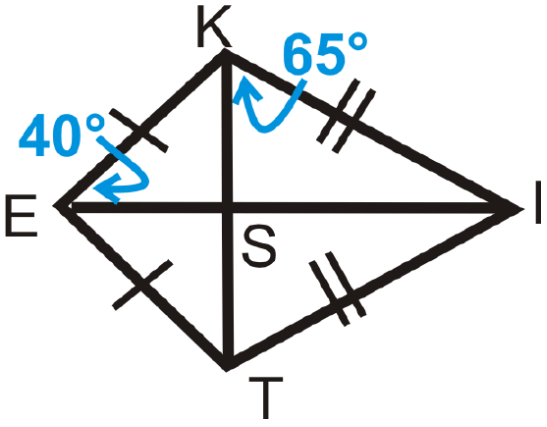

Figure 5.16.6

Example 5.16.1

Find $m\angle KIS$.

Solution

$m\angle KIS = 25°$ by the **Triangle Sum Theorem** (remember that \angle KSI is a right angle because the diagonals are perpendicular.)

Example 5.16.2

Find $m\angle IST$.

Solution

$m\angle IST = 90°$ because the diagonals are perpendicular.

Example 5.16.3

Find the missing measures in the kites below.

a.

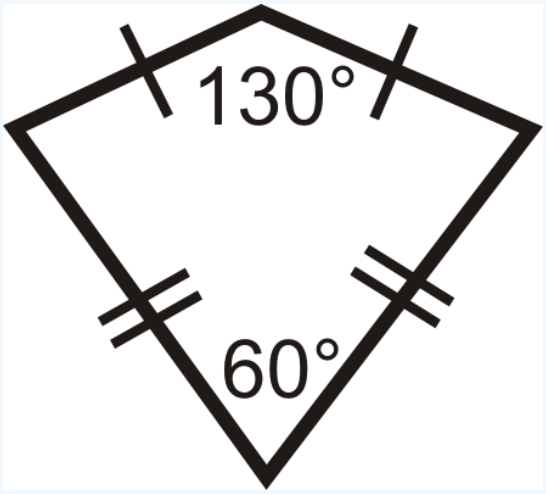

Figure 5.16.7

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b.

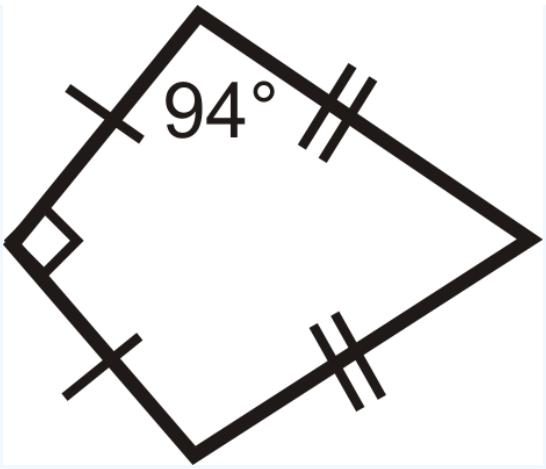

Figure 5.16.8

Solution

a. The two angles left are the non-vertex angles, which are congruent.

$$\begin{aligned} 130^\circ + 60^\circ + x + x &= 360^\circ \\ 2x &= 170^\circ \\ x &= 85^\circ \qquad Both angles are 85^\circ \end{aligned}$$

b. The other non-vertex angle is also 94°. To find the fourth angle, subtract the other three angles from 360°.

$$\begin{aligned} 90^\circ + 94^\circ + 94^\circ + x &= 360^\circ \\ x &= 82^\circ \end{aligned}$$

Example 5.16.4

Use the Pythagorean Theorem to find the lengths of the sides of the kite.

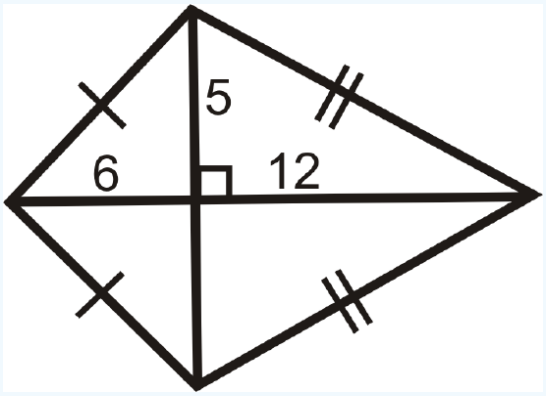

Figure 5.16.9

Solution

Recall that the Pythagorean Theorem says $a^2 + b^2 = c^2$, where c is the hypotenuse. In this kite, the sides are the hypotenuses.

$$\begin{aligned} 6^2 + 5^2 &= h^2 \\ 36 + 25 &= h^2 \\ 61 &= h^2 \\ \sqrt{61} &= h \end{aligned} \qquad \begin{aligned} 12^2 + 5^2 &= j^2 \\ 144 + 25 &= j^2 \\ 169 &= j^2 \\ 13 &= j \end{aligned}$$

Example 5.16.5

Prove that the non-vertex angles of a kite are congruent.

Given: $KITE$ with $\overline{KE} \cong \overline{TE}$ and $\overline{KI} \cong \overline{TI}$

Prove: $\angle K \cong \angle T$

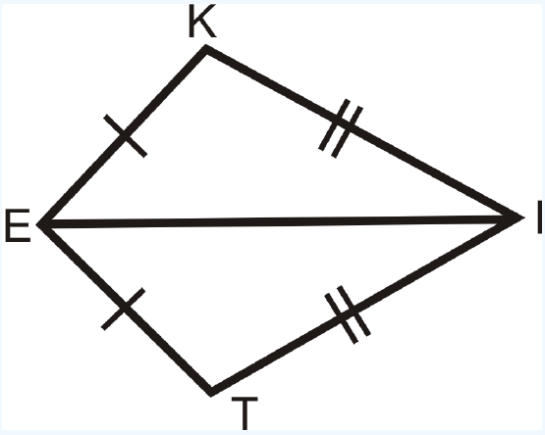

Figure 5.16.10

Solution

Statement	*Reason*
1. $\overline{KE} \cong \overline{TE}$ and $\overline{KI} \cong \overline{TI}$	1. Given
2.$\overline{EI} \cong \overline{EI}$	2. Reflexive PoC
3. $\Delta EKI \cong \Delta ETI$	3. SSS
4. $\angle K \cong \angle T$	4. CPCTC

Review

For questions 1-6, find the value of the missing variable(s). All figures are kites.

1.

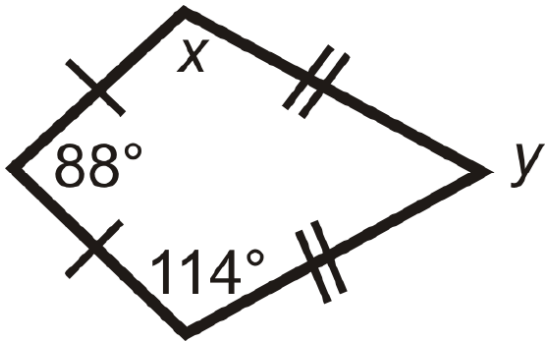

Figure 5.16.11

2.

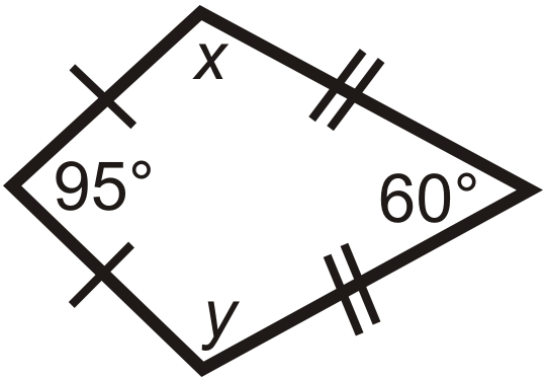

Figure 5.16.12

3.

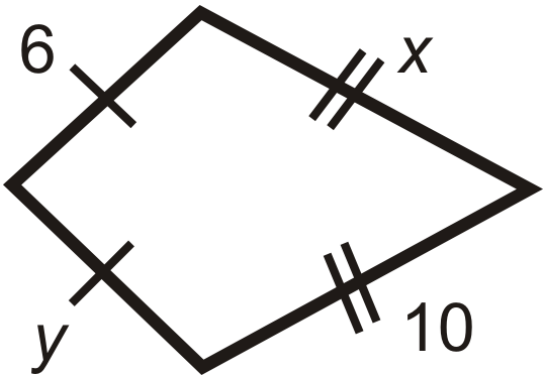

Figure 5.16.13

4.

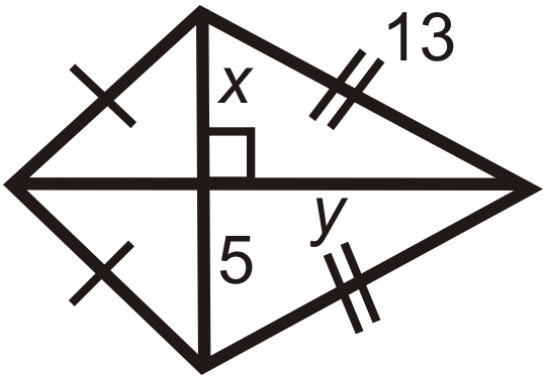

Figure 5.16.14

5.

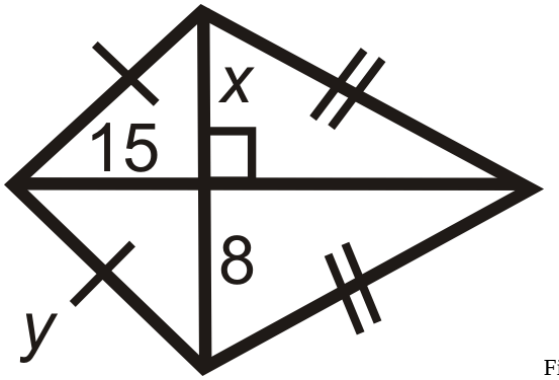

Figure 5.16.15

6.

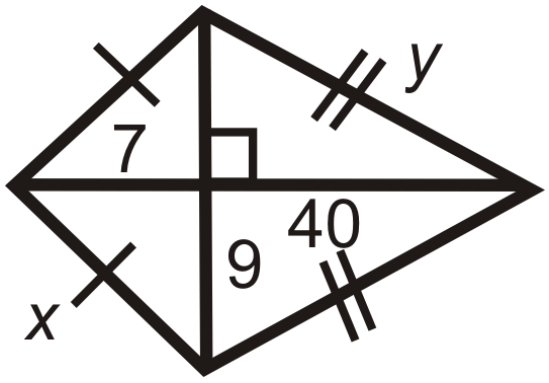

Figure 5.16.16

For questions 7-11, find the value of the missing variable(s).

7.

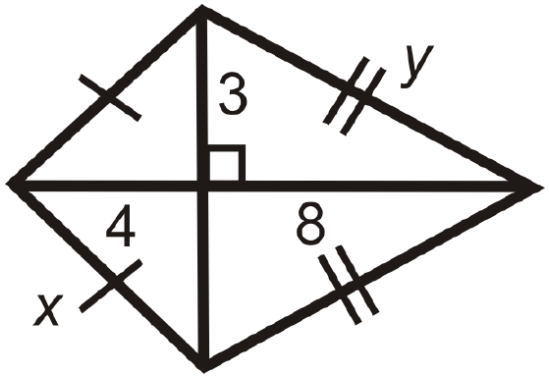

Figure 5.16.17

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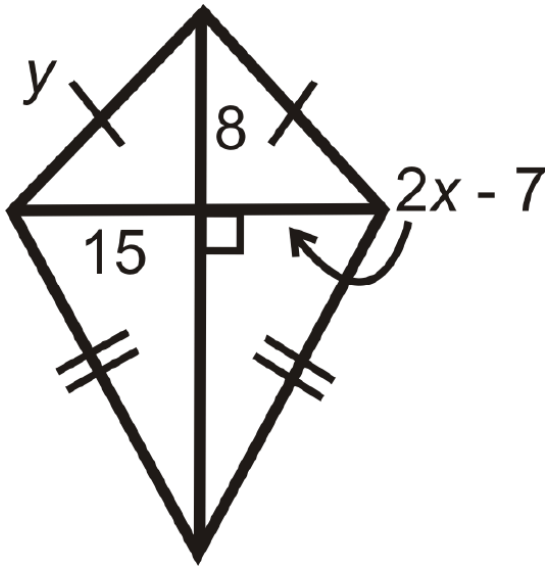

Figure 5.16.18

9.

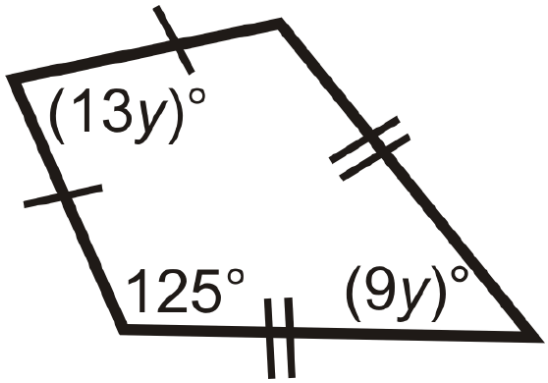

Figure 5.16.19

10.

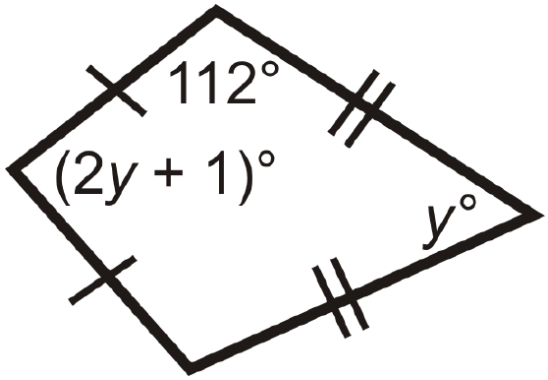

Figure 5.16.20

11.

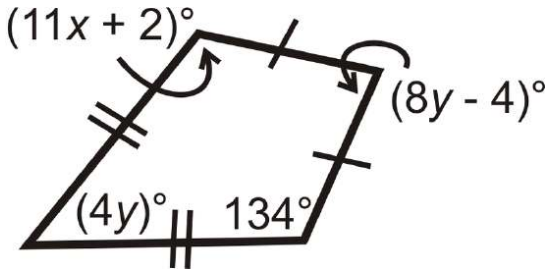

Figure 5.16.21

12. Fill in the blanks to the proof below.

Given: $\overline{KE} \cong \overline{TE}$ and $\overline{KI} \cong \overline{TI}$

Prove: $\overline{EI}$ is the angle bisector of $\angle KET$ and $\angle KIT$

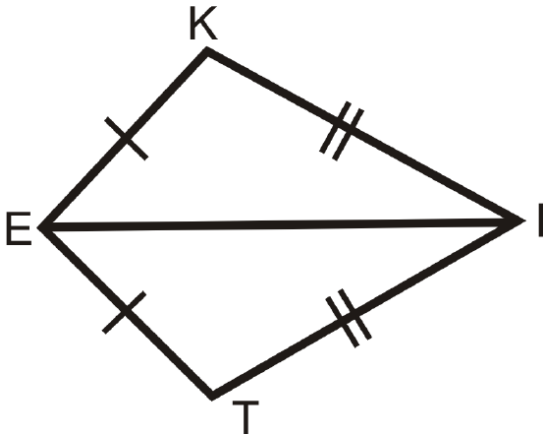

Figure 5.16.22

Statement	Reason
1. $\overline{KE} \cong \overline{TE}$ and $\overline{KI} \cong \overline{TI}$	1.
2. $\overline{EI} \cong \overline{EI}$	2.
3. $\Delta EKI \cong \Delta ETI$	3.
4.	4. CPCTC
5. $\overline{EI}$ is the angle bisector of $\angle KET$ and \angle KIT\)	5.

13. Fill in the blanks to the proof below.

Given: $\overline{EK} \cong \overline{ET}$, $\overline{KI} \cong \overline{IT}$

Prove: $\overline{KT} \perp \overline{EI}$

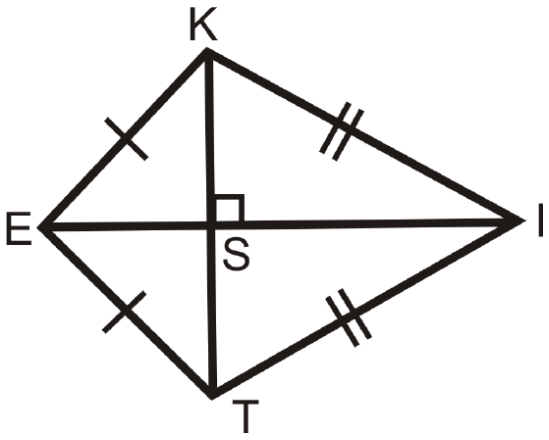

Figure **5.16.23**

Statement	Reason
1. $\overline{KE} \cong \overline{TE}$ and $\overline{KI} \cong \overline{TI}$	1.
2.	2. Definition of isosceles triangles
3. $\overline{EI}$ is the angle bisector of $\angle KET$ and $\angle KIT$	3.
4.	4. Isosceles Triangle Theorem
5. $\overline{KT} \perp \overline{EI}$	5.

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.7.

Vocabulary

Term	Definition
kite	A quadrilateral with distinct adjacent congruent sides.
Triangle Sum Theorem	The Triangle Sum Theorem states that the three interior angles of any triangle add up to 180 degrees.
Vertical Angles	Vertical angles are a pair of opposite angles created by intersecting lines.

Additional Resources

Video: Kites Principles - Basic

Activities: Kites Discussion Questions

Study Aids: Trapezoids and Kites Study Guide

Practice: Kites

Real World: Go Fly a Kite

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5.17: Area and Perimeter of Rhombuses and Kites

Area is half the product of the diagonals; perimeter is the sum of the sides.

Recall that a **rhombus** is a quadrilateral with four **congruent** sides and a **kite** is a quadrilateral with distinct adjacent congruent sides. Both of these quadrilaterals have perpendicular diagonals, which is how we are going to find their areas.

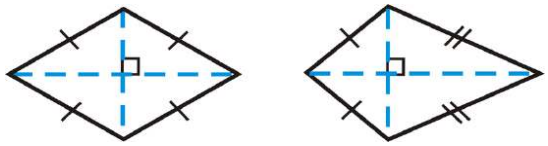

Figure **5.17.1**

Notice that the diagonals divide each quadrilateral into 4 triangles. If we move the two triangles on the bottom of each quadrilateral so that they match up with the triangles above the horizontal diagonal, we would have two rectangles.

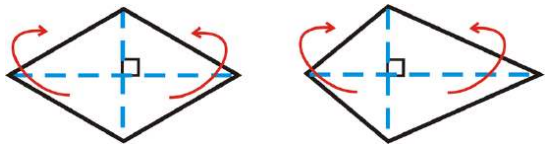

Figure **5.17.2**

So, the height of these rectangles is half of one of the diagonals and the base is the length of the other diagonal.

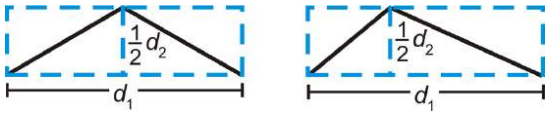

Figure **5.17.3**

The **area of a rhombus or a kite** is $A = \frac{1}{2}d_1d_2$

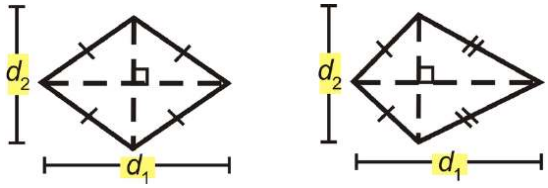

Figure **5.17.4**

What if you were given a kite or a rhombus and the size of its two diagonals? How could you find the total distance around the kite or rhombus and the amount of space it takes up?

Example 5.17.1

Find the perimeter and a w.

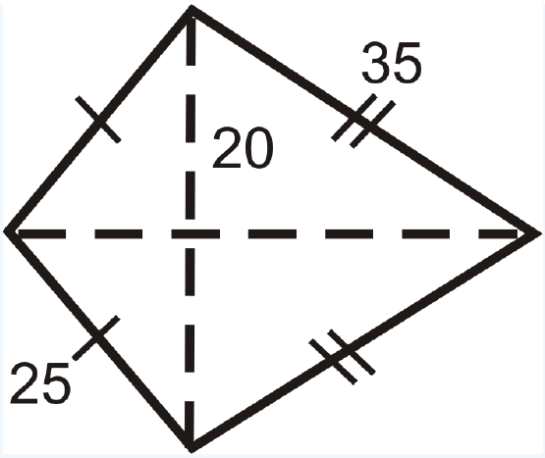

Figure 5.17.5

Solution

In a kite, there are two pairs of congruent triangles. Use the Pythagorean Theorem to find the lengths of sides or diagonals.

$$\textit{Smaller diagonal portion}$$

$$20^2 + d_s^2 = 25^2$$

$$d_s^2 = 225$$

$$d_s = 15\ units$$

$$\textit{Larger diagonal portion}$$

$$20^2 + d_l^2 = 352$$

$$d_l^2 = 825$$

$$d_l = 5units$$

$$A = \frac{1}{2}(15+5)(40) \cong 874.5units^2$$

$$P = 2(25) + 2(35) = 120\ units$$

Example 5.17.2

Find the area of a rhombus with diagonals of 6 in and 8 in.

Solution

The area is $\frac{1}{2}(8)(6) = 24in^2$.

Example 5.17.3

Find the perimeter and area of the rhombus below.

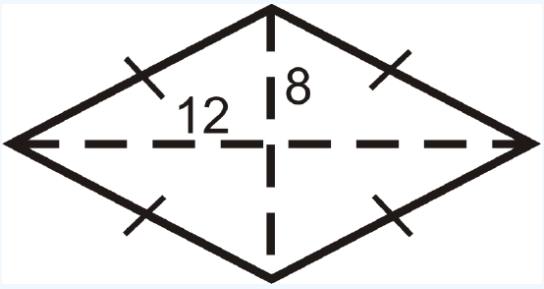

Figure 5.17.6

Solution

In a rhombus, all four triangles created by the diagonals are congruent.

To find the perimeter, you must find the length of each side, which would be the hypotenuse of one of the four triangles. Use the Pythagorean Theorem.

$$12^2 + 8^2 = side^2 \qquad A = 12 \cdot 16 \cdot 24$$

$$144 + 64 = side^2 \qquad A = 192 units^2$$

$$side = \sqrt{208}$$

$$P = 4\sqrt{13} = 4(4\sqrt{13}) = 16\sqrt{13} units$$

Example 5.17.4

Find the perimeter and area of the rhombus below.

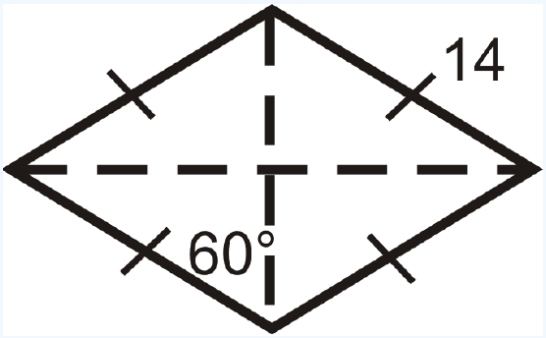

Figure 5.17.7

Solution

In a rhombus, all four triangles created by the diagonals are congruent.

Here, each triangle is a 30-60-90 triangle with a hypotenuse of 14. From the special right triangle ratios the short leg is 7 and the long leg is $7\sqrt{3}$.

$$P = 4 \cdot 14 = 56 units \qquad A = 12 \cdot 14 \cdot 14\sqrt{3} = 98\sqrt{3} units^2$$

Example 5.17.5

The vertices of a quadrilateral are $A(2,8), B(7,9), C(11,2),$ *and* $D(3,3)$. Show $ABCD$ is a kite and find its area.

Solution

After plotting the points, it looks like a kite. $AB = AD$ and $BC = DC$. The diagonals are perpendicular if the slopes are negative reciprocals of each other.

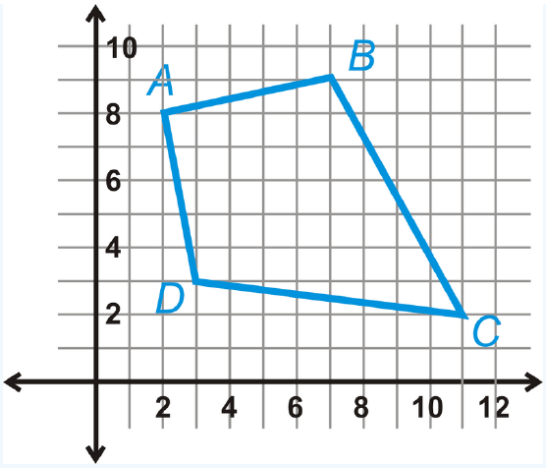

Figure 5.17.8

$m_{AC} = \frac{2-8}{11-2} = -\frac{6}{9} = -\frac{2}{3}$

$m_{BD} = 9 - 37 - 3 = 64 = 32$

The diagonals are perpendicular, so $ABCD$ is a kite. To find the area, we need to find the length of the diagonals, AC and BD.

$d_1 = \sqrt{(2-11)^2 + (8-2)^2}$

$= \sqrt{(-9)^2 + 6^2}$

\(=\sqrt{81+36=\sqrt{117}=3\sqrt{13}\)

$d_2 = \sqrt{(7-3)^2 + (9-3)^2}$

$= \sqrt{4^2 + 6^2}$

$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

Plug these lengths into the area formula for a kite. $A = 12(3\sqrt{13})(2\sqrt{13}) = 39 units^2$

Review

1. Do you think all rhombi and kites with the same diagonal lengths have the same area? *Explain* your answer.

Find the area of the following shapes. *Round your answers to the nearest hundredth.*

2.

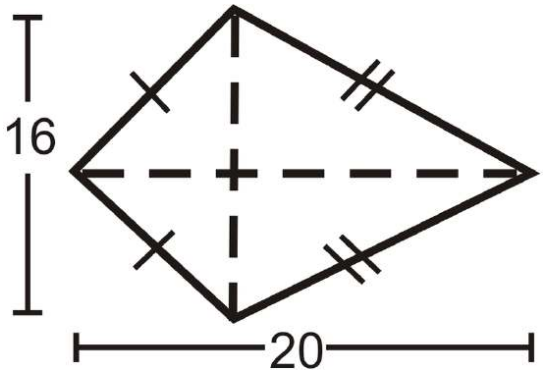

Figure 5.17.9

3.

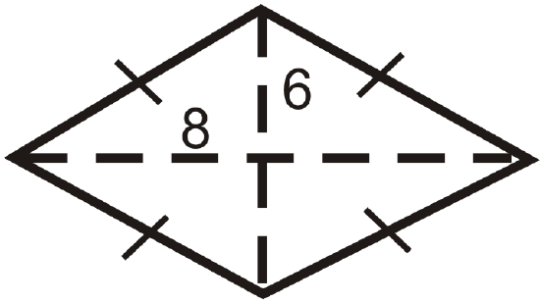

Figure **5.17.10**

4.

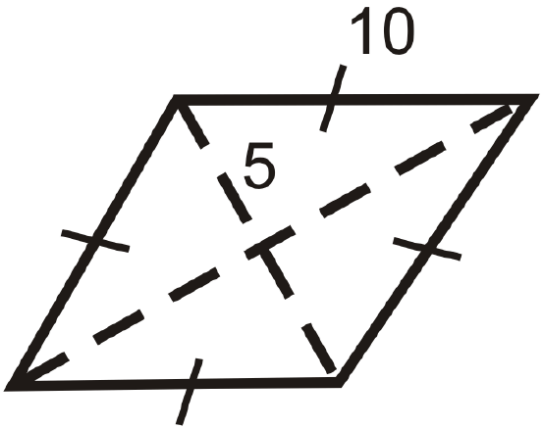

Figure **5.17.11**

5.

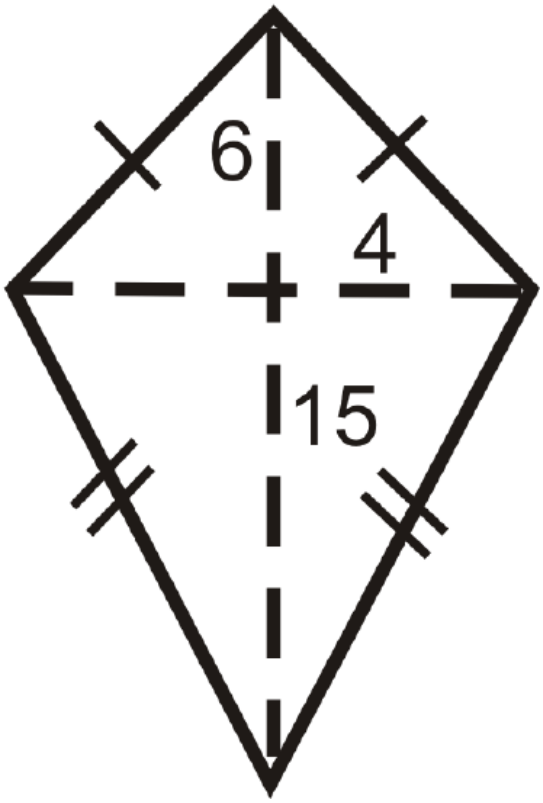

Figure **5.17.12**

6.

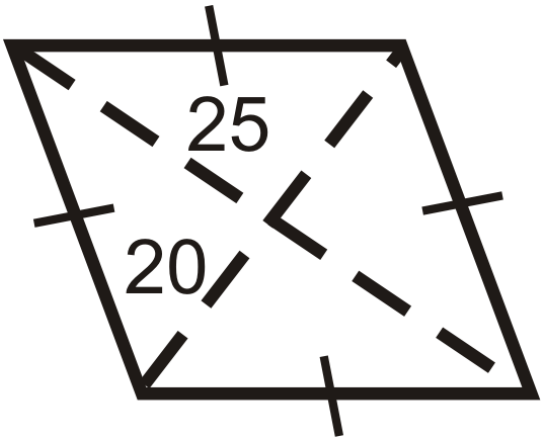

Figure **5.17.13**

7.

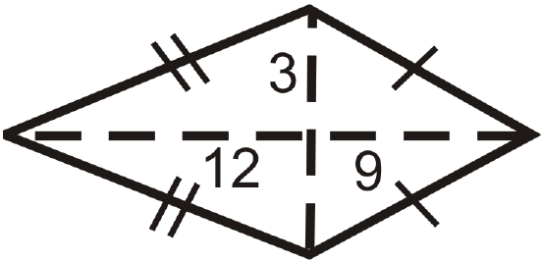

Figure **5.17.14**

Find the area and perimeter of the following shapes. *Round your answers to the nearest hundredth.*

8.

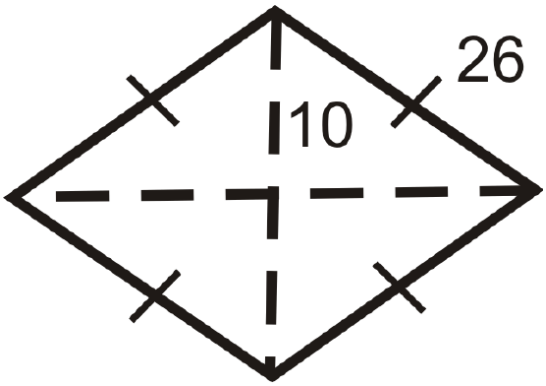

Figure **5.17.15**

9.

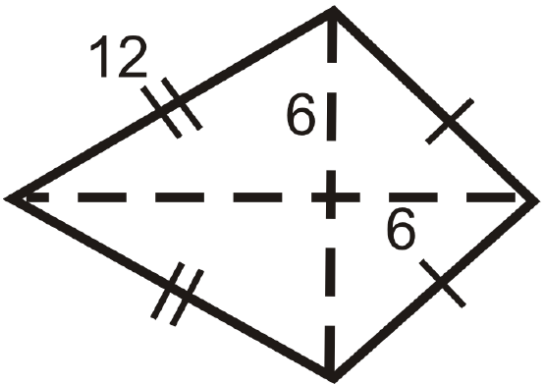

Figure **5.17.16**

10.

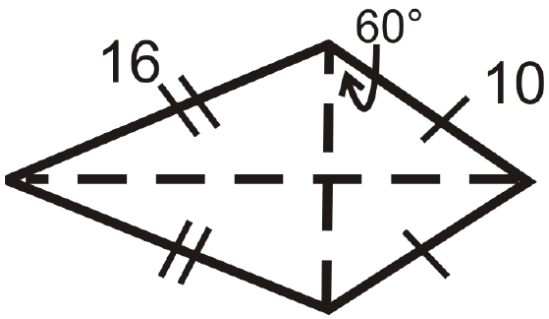

Figure 5.17.17

11.

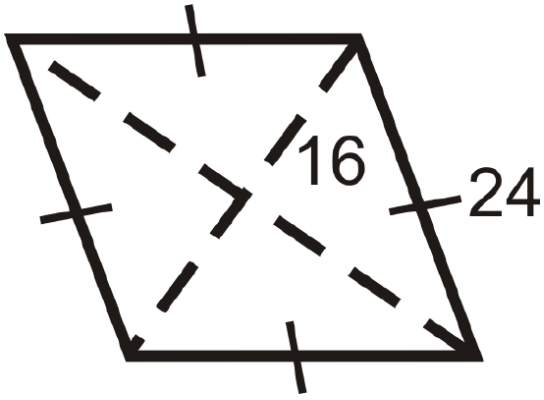

Figure 5.17.18

For Questions 12 and 13, the area of a rhombus is $32\ units^2$.

12. What would the product of the diagonals have to be for the area to be $32 units^2$?
13. List two possibilities for the length of the diagonals, based on your answer from #12.

For Questions 14 and 15, the area of a kite is $54\ units^2$.

14. What would the product of the diagonals have to be for the area to be $54\ units^2$?
15. List two possibilities for the length of the diagonals, based on your answer from #14.

Sherry designed the logo for a new company, made up of 3 congruent kites.

16. What are the lengths of the diagonals for one kite?
17. Find the area of one kite.
18. Find the area of the entire logo.

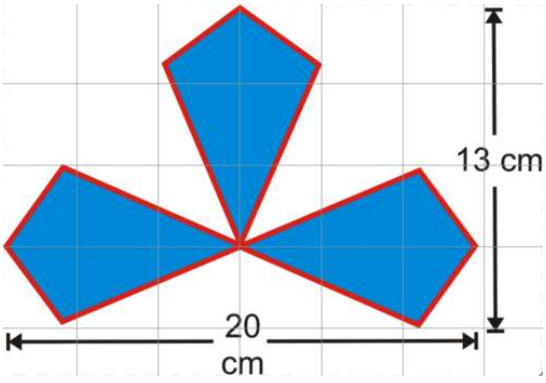

Figure 5.17.19

Vocabulary

Term	Definition
area	The amount of space inside a figure. Area is measured in square units.
kite	A quadrilateral with distinct adjacent congruent sides.
perimeter	The distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write ***units.***
Congruent	Congruent figures are identical in size, shape and measure.
Legs of a Right Triangle	The legs of a right triangle are the two shorter sides of the right triangle. Legs are adjacent to the right angle.

Additional Resources

Video: Area and Perimeter of Rhombuses and Kites Principles - Basic

Activities: Area and Perimeter of Rhombuses and Kites Discussion Questions

Study Aids: Triangles and Quadrilaterals Study Guide

Practice: Area and Perimeter of Rhombuses and Kites

Real World: Perimeter

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5.18: Area and Perimeter of Composite Shapes

Find area and perimeter of figures made up of two or more common shapes.

Area of Composite Shapes

Perimeter is the distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc.), write "units."

Area is the amount of space inside a figure. If two figures are congruent, they have the same area (**Congruent Areas Postulate**).

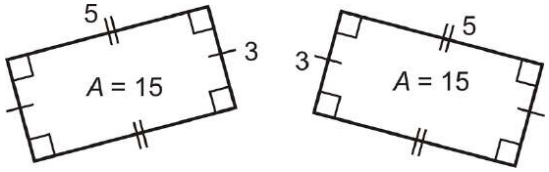


Figure 5.18.1

A **composite shape** is a shape made up of other shapes. To find the area of such a shape, simply find the area of each part and add them up.

Area Addition Postulate: If a figure is composed of two or more parts that do not overlap each other, then the area of the figure is the sum of the areas of the parts.

Consider a basic house drawn as a triangle on top of a square. How could you find the area of this composite shape?

Example 5.18.1

Find the area of the figu s that look like right angles are right angles.

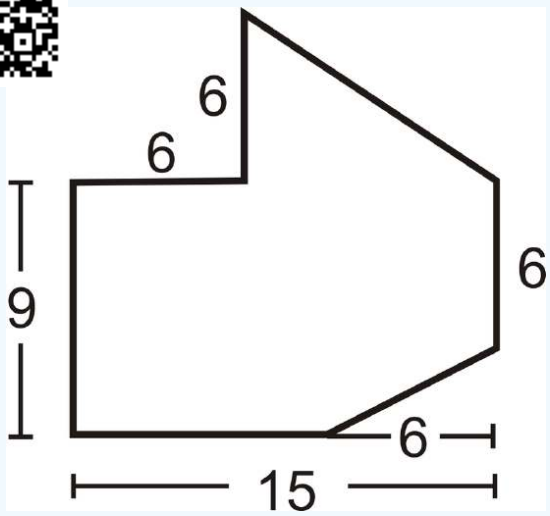


Figure 5.18.2

Solution

Divide the figure into a triangle and a rectangle with a small rectangle cut out of the lower right-hand corner.

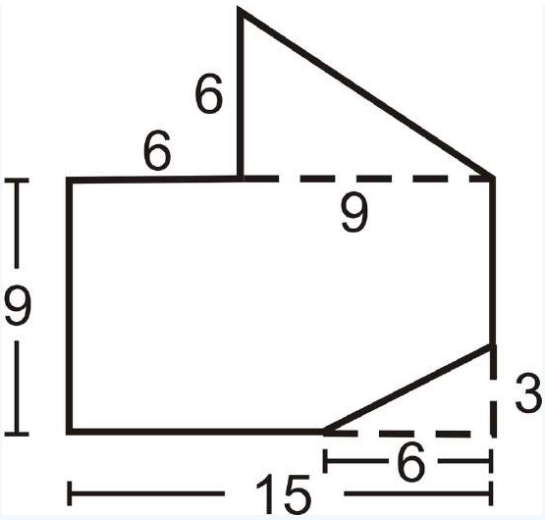


Figure 5.18.3

Misplaced &

Example 5.18.2

Divide the shape into two rectangles and one triangle. Find the area of the two rectangles and triangle. All angles that look like right angles are right angles.

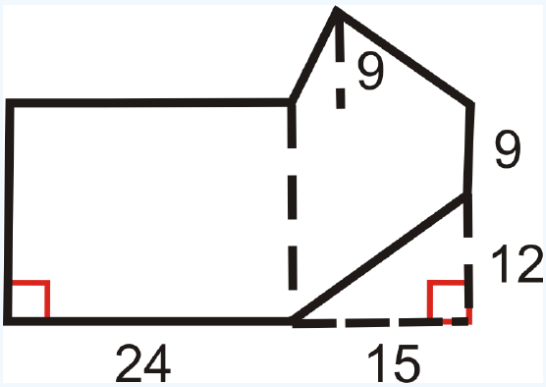


Figure 5.18.4

Solution

Rectangle #1: $\textbf{Area} = 24(9+12) = 504 \text{ units}^2$

Rectangle #2: $\textbf{Area} = 15(9+12) = 315 \text{ units}^2$

Triangle: $\textbf{Area} = 15(9)2 = 67.5 \text{ units}^2$

Example 5.18.3

Find the area of the entire shape from Example 2 (you will need to subtract the area of the small triangle in the lower right-hand corner).

Solution

$$\textbf{Total Area} = 504 + 315 + 67.5 - \frac{15(12)}{2} = 796.5 \text{ units}^2$$

Example 5.18.4

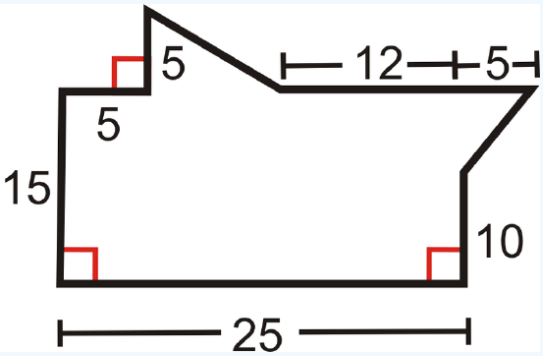


Figure 5.18.5

All angles that look like right angles are right angles.

a. Divide the shape into two triangles and one rectangle.
b. Find the area of the two triangles and rectangle.
c. Find the area of the entire shape.

Solution

a. One triangle on the top and one on the right, the rest of the shape is a rectangle.
b. The area of the triangle on top is $\frac{8(5)}{2} = 20$ units^2. The area of the triangle on the right is $\frac{5(5)}{2} = 12.5$ units^2. The area of the rectangle is 375 units^2.
c. The total area is 407.5 units^2.

Review

Use the picture below for questions 1-4. The composite shape is formed of a square within a square.

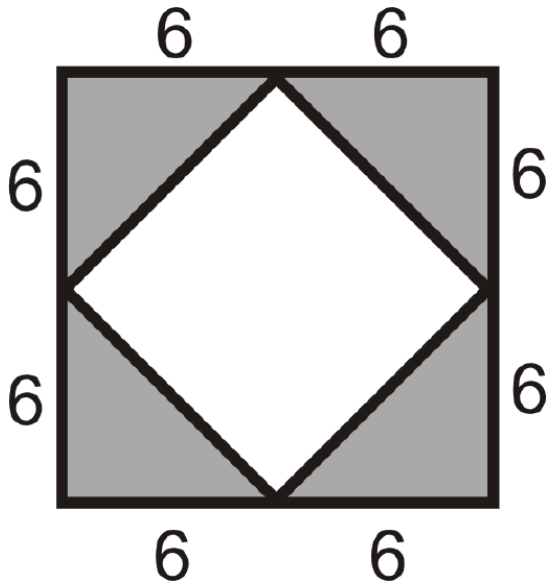


Figure 5.18.6

1. Find the area of the outer square.
2. Find the area of one grey triangle.
3. Find the area of all four grey triangles.
4. Find the area of the inner square.

Find the areas of the figures below. You may assume all sides are perpendicular.

5.

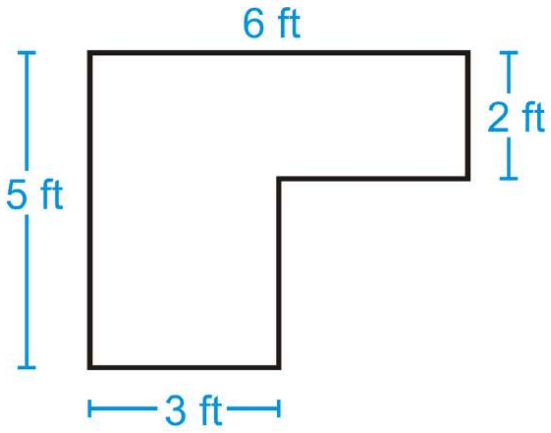


Figure **5.18.7**

6.

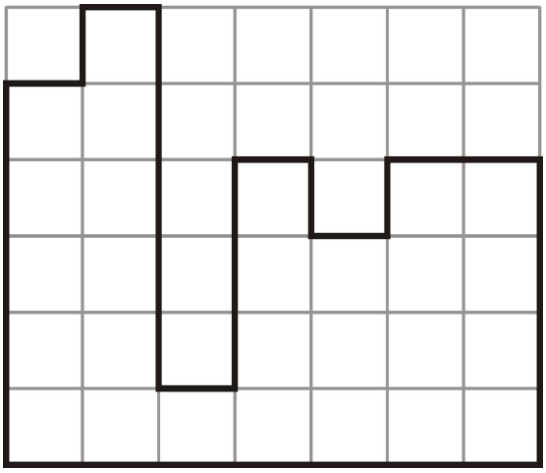


Figure **5.18.8**

Find the areas of the composite figures.

7.

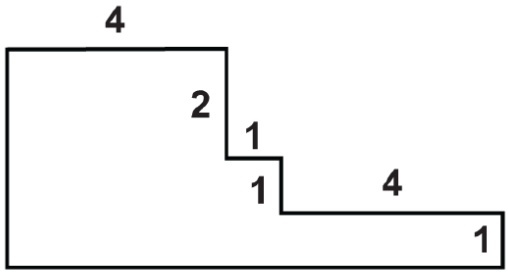


Figure **5.18.9**

8.

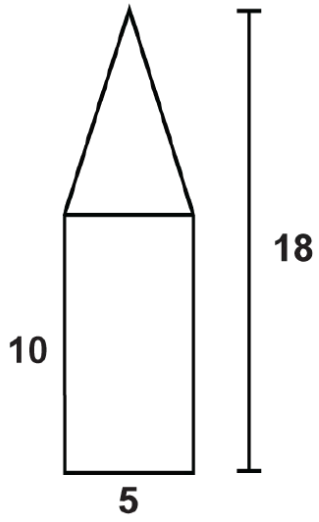


Figure 5.18.10

9.

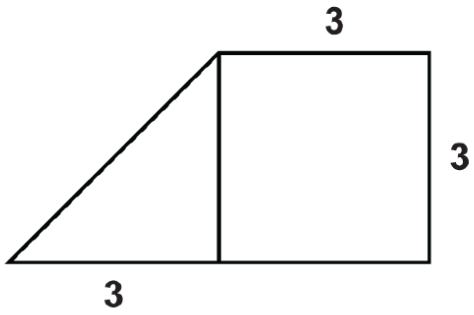


Figure 5.18.11

10.

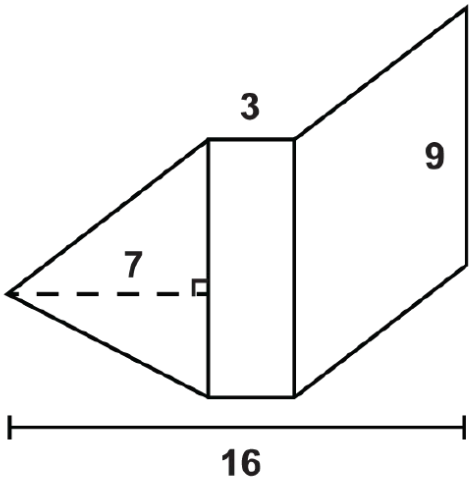
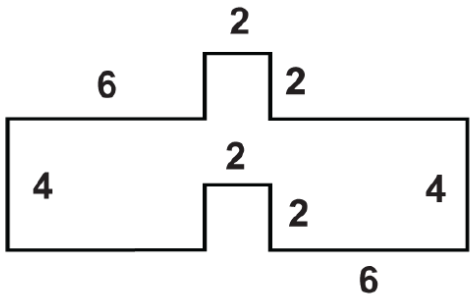
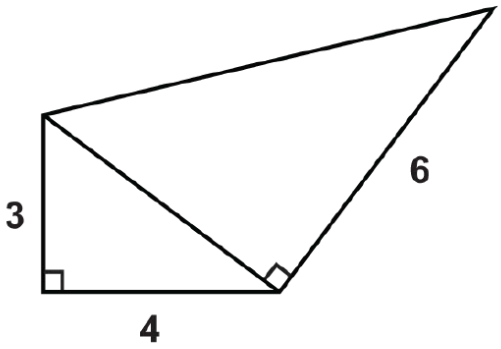


Figure 5.18.12



11.

Figure 5.18.13



12.

Figure 5.18.14

Use the figure to answer the questions.

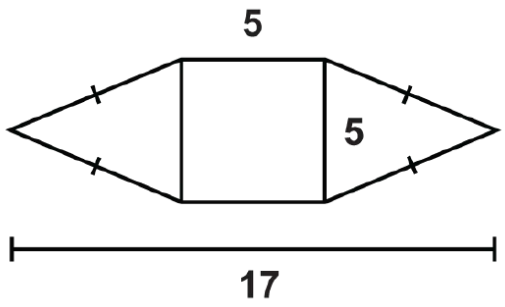


Figure 5.18.15

13. What is the area of the square?
14. What is the area of the triangle on the left?
15. What is the area of the composite figure?

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.6.

Vocabulary

Term	Definition
area	The amount of space inside a figure. Area is measured in square units.
composite shape	A shape made up of other shapes.
perimeter	The distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write ***units.***
Composite	A number that has more than two factors.

Additional Resources

Video: Area of a Triangle (Whole Numbers)

Activites: Area of Composite Shapes Discussion Questions

Study Aids: Perimeter and Area Study Guide

Practice: Area and Perimeter of Composite Shapes

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5.19: Quadrilateral Classification in the Coordinate Plane

Identify and calculate area of shapes based on coordinates on a plane.

Quadrilateral Classification

What if you were given a **quadrilateral** in the coordinate plane? How could you determine if that quadrilateral qualifies as one of the special quadrilaterals: parallelograms, squares, rectangles, rhombuses, kites, or trapezoids?

When working in the coordinate plane, you will sometimes want to know what type of shape a given shape is. You should easily be able to tell that it is a quadrilateral if it has four sides. But how can you classify it beyond that?

First you should graph the shape if it has not already been graphed. Look at it and see if it looks like any special quadrilateral. Do the sides appear to be congruent? Do they meet at right angles? This will give you a place to start.

Once you have a guess for what type of quadrilateral it is, your job is to prove your guess. To prove that a quadrilateral is a **parallelogram**, **rectangle**, **rhombus**, square, **kite** or **trapezoid**, you must show that it meets the definition of that shape OR that it has properties that only that shape has.

If it turns out that your guess was wrong because the shape does not fulfill the necessary properties, you can guess again. If it appears to be no type of special quadrilateral then it is simply a *quadrilateral*.

The examples below will help you to see what this process might look like.

Classifying Parallelogra

Determine what type of pa is: $T(0, 10)$, $U(4, 2)$, $N(-2, -1)$, *and* $E(-6, 7)$.

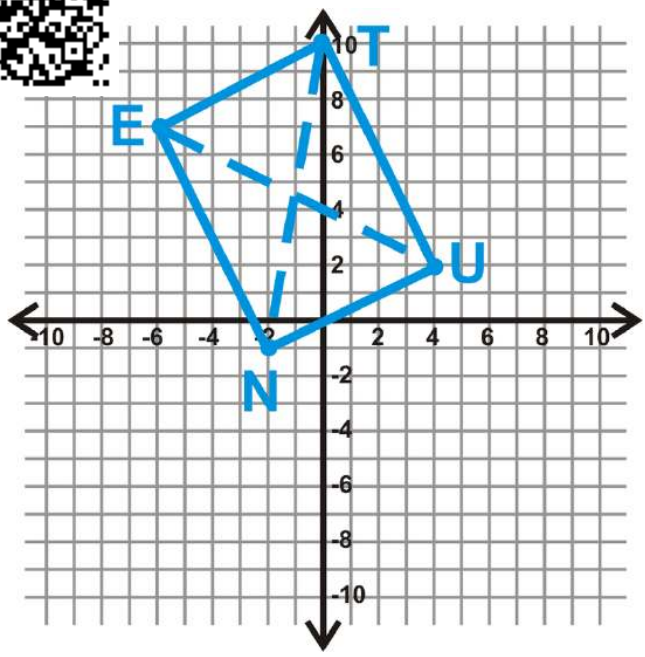

Figure 5.19.1

This *looks* like a rectangle. Let's see if the diagonals are equal. If they are, then TUNE is a rectangle.

$$\begin{aligned} EU &= \sqrt{(-6-4)^2+(7-2)^2} & TN &= \sqrt{(0+2)^2+(10+1)^2} \\ &= \sqrt{(-10)^2+5^2} & &= \sqrt{2^2+11^2} \\ &= \sqrt{100+25} & &= \sqrt{4+121} \\ &= \sqrt{125} & &= \sqrt{125} \end{aligned}$$

If the diagonals are also perpendicular, then $TUNE$ is a square.

$$\text{Slope of } EU = \frac{7-2}{-6-4} = -\frac{5}{10} = -\frac{1}{2} \qquad \text{Slope of } TN = \frac{10-(-1)}{0-(-2)} = \frac{11}{2}$$

The slope of $EU \neq$ slope of TN, so $TUNE$ is a rectangle.

Determining if a Quadri elogram

A quadrilateral is defined l $= 2x + 1$, $y = -x + 5$, $y = 2x - 4$, and $y = -x - 5$. Is this quadrilateral a parallelogram?

To check if its a parallelc heck that it has two pairs of parallel sides. From the equations we can see that the slopes of the lines are 2, ause two pairs of slopes match, this shape has two pairs of parallel sides and is a parallelogram.

Determining Types of Quadrilaterals

Determine what type of quadrilateral $RSTV$ is. Simplify all radicals.

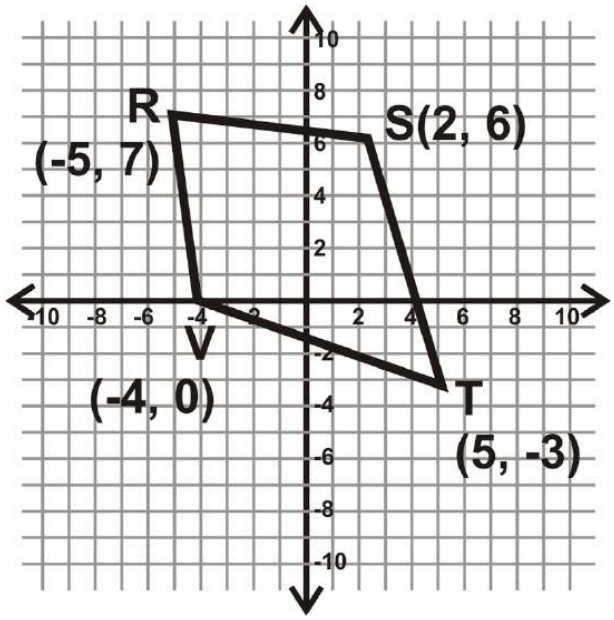

Figure 5.19.2

There are two directions you could take here. First, you could determine if the diagonals bisect each other. If they do, then it is a parallelogram. Or, you could find the lengths of all the sides. Let's do this option.

$$\begin{aligned} RS &= \sqrt{(-5-2)^2 + (7-6)^2} & ST &= \sqrt{(2-5)^2 + (6-(-3))^2} \\ &= \sqrt{(-7)^2 + 1^2} & &= \sqrt{(-3)^2 + 9^2} \\ &= \sqrt{50} = 5\sqrt{2} & &= \sqrt{90} = 3\sqrt{10} \\ RV &= \sqrt{(-5-(-4))^2 + (7-0)^2} & VT &= \sqrt{(-4-5)^2 + (0-(-3))^2} \\ &= \sqrt{(-1)^2 + 7^2} & &= \sqrt{(-9)^2 + 3^2} \\ &= \sqrt{50} = 5\sqrt{2} & &= \sqrt{90} = 3\sqrt{10} \end{aligned}$$

From this we see that the adjacent sides are congruent. Therefore, $RSTV$ is a kite.

Algebra Review: When asked to "simplify the radical," pull all square numbers (1, 4, 9, 16, 25, ...) out of the radical. Above $\sqrt{50} = \sqrt{25 \cdot 2}$. We know $\sqrt{25} = 5$, so $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$.

Recognizing Parallelogi

Is the quadrilateral $ABCD$

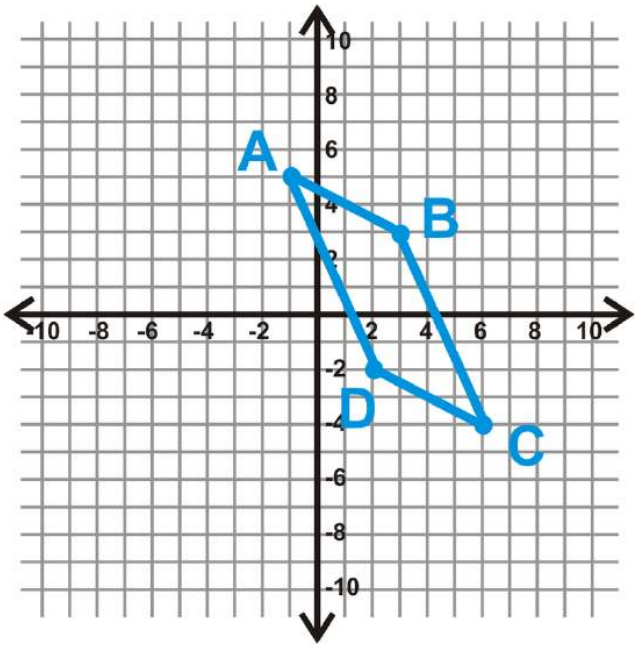

Figure 5.19.2

We have determined there are four different ways to show a quadrilateral is a parallelogram in the $x-y$ plane. Let's check if a pair of opposite sides are congruent and parallel. First, find the length of AB and CD.

$$\begin{aligned} AB &= \sqrt{(-1-3)^2+(5-3)^2} \\ &= \sqrt{(-4)^2+2^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \end{aligned} \qquad \begin{aligned} CD &= \sqrt{(2-6)^2+(-2+4)^2} \\ &= \sqrt{(-4)^2+2^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \end{aligned}$$

$AB = CD$, so if the two lines have the same slope, $ABCD$ is a parallelogram.

$$\text{Slope } AB = \frac{5-3}{-1-3} = \frac{2}{-4} = -\frac{1}{2} \quad \text{Slope } CD = \frac{-2+4}{2-6} = \frac{2}{-4} = -\frac{1}{2}$$

Therefore, $ABCD$ is a parallelogram.

Example 5.19.1

A quadrilateral is defined by the four lines $y = 2x + 1$, $y = -2x + 5$, $y = 2x - 4$, and $y = -2x - 5$. Is this quadrilateral a rectangle?

Solution

To be a rectangle a shape must have four right angles. This means that the sides must be perpendicular to each other. From the given equations we see that the slopes are 2,−2, 2, and −2. Because the slopes are not opposite reciprocals of each other, the sides are not perpendicular, and the shape is not a rectangle.

Example 5.19.2

Determine what type of quadrilateral $ABCD$ is. $A(-3,3)$, $B(1,5)$, $C(4,-1)$, $D(1,-5)$. Simplify all radicals.

Solution

First, graph $ABCD$. This will make it easier to figure out what type of quadrilateral it is. From the graph, we can tell this is not a parallelogram. Find the slope of \overline{BC} and \overline{AD} to see if they are parallel.

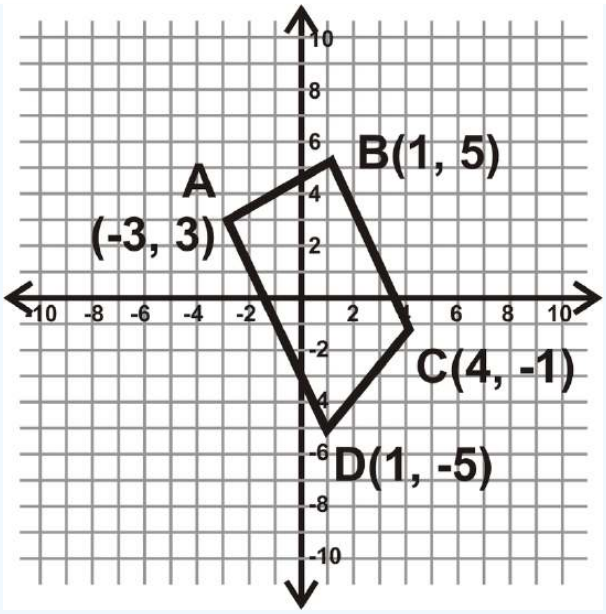

Figure 5.19.4

$$\text{Slope of } \overline{BC} = \frac{5-(-1)}{1-4} = \frac{6}{-3} = -2$$
$$\text{Slope of } \overline{AD} = \frac{3-(-5)}{-3-1} = \frac{8}{-4} = -2$$

We now know $\overline{BC} \parallel \overline{AD}$. This is a trapezoid. To determine if it is an isosceles trapezoid, find AB and CD.

$$\begin{aligned} AB &= \sqrt{(-3-1)^2+(3-5)^2} \\ &= \sqrt{(-4)^2+(-2)^2} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned} \qquad \begin{aligned} ST &= \sqrt{(4-1)^2+(-1-(-5))^2} \\ &= \sqrt{3^2+4^2} \\ &= \sqrt{25} = 5 \end{aligned}$$

$AB \neq CD$, therefore this is only a trapezoid.

Example 5.19.3

Determine what type of quadrilateral $EFGH$ is. $E(5,-1)$, $F(11,-3)$, $G(5,-5)$, $H(-1,-3)$

Solution

We will not graph this example. Let's find the length of all four sides.

$$\begin{aligned} EF &= \sqrt{(5-11)^2+(-1-(-3))^2} \\ &= \sqrt{(-6)^2+2^2} \\ &= \sqrt{40} = 2\sqrt{10} \end{aligned} \qquad \begin{aligned} FG &= \sqrt{(11-5)^2+(-3-(-5))^2} \\ &= \sqrt{6^2+2^2} \\ &= \sqrt{40} = 2\sqrt{10} \end{aligned}$$

$$\begin{aligned} GH &= \sqrt{(5-(-1))^2+(-5-(-3))^2} \\ &= \sqrt{6^2+(-2)^2} \\ &= \sqrt{40} = 2\sqrt{10} \end{aligned} \qquad \begin{aligned} H &= \sqrt{(-1-5)^2+(-3-(-1))^2} \\ &= \sqrt{(-6)^2+(-2)^2} \\ &= \sqrt{40} = 2\sqrt{10} \end{aligned}$$

All four sides are equal. That means, this quadrilateral is either a rhombus or a square. The difference between the two is that a square has four 90° angles and congruent diagonals. Let's find the length of the diagonals.

$$\begin{aligned} EG &= \sqrt{(5-5)^2+(-1-(-5))^2} \\ &= \sqrt{0^2+4^2} \\ &= \sqrt{16} = 4 \end{aligned} \qquad \begin{aligned} FH &= \sqrt{(11-(-1))^2+(-3-(-3))^2} \\ &= \sqrt{12^2+0^2} \\ &= \sqrt{144} = 12 \end{aligned}$$

The diagonals are not congruent, so $EFGH$ is a rhombus.

Review

1. If a quadrilateral has exactly one pair of parallel sides, what type of quadrilateral is it?
2. If a quadrilateral has two pairs of parallel sides and one right angle, what type of quadrilateral is it?
3. If a quadrilateral has perpendicular diagonals, what type of quadrilateral is it?
4. If a quadrilateral has diagonals that are perpendicular and congruent, what type of quadrilateral is it?
5. If a quadrilateral has four congruent sides and one right angle, what type of quadrilateral is it?

Determine what type of quadrilateral $ABCD$ is.

6. $A(-2,4)$, $B(-1,2)$, $C(-3,1)$, $D(-4,3)$
7. $A(-2,3)$, $B(3,4)$, $C(2,-1)$, $D(-3,-2)$
8. $A(1,-1)$, $B(7,1)$, $C(8,-2)$, $D(2,-4)$
9. $A(10,4)$, $B(8,-2)$, $C(2,2)$, $D(4,8)$
10. $A(0,0)$, $B(5,0)$, $C(0,4)$, $D(5,4)$
11. $A(-1,0)$, $B(0,1)$, $C(1,0)$, $D(0,-1)$
12. $A(2,0)$, $B(3,5)$, $C(5,0)$, $D(6,5)$

SRUE is a rectangle and PRUC is a square.

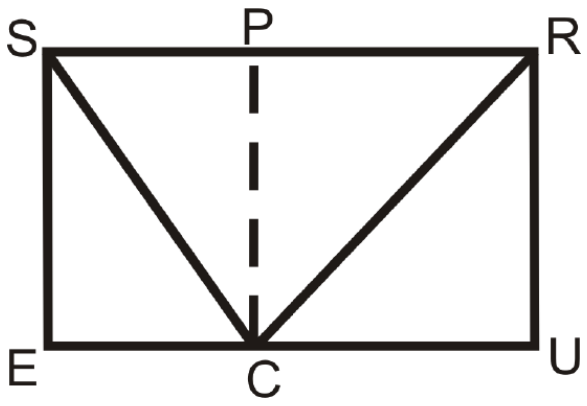

Figure 5.19.5

13. What type of quadrilateral is $SPCE$?
14. If $SR = 20$ and $RU = 12$, find CE.
15. Find SC and RC based on the information from part b. Round your answers to the nearest hundredth.

Review (Answers)

To view the Review answers, open this PDF file and look for section 6.8.

Vocabulary

Term	Definition
Kite	A kite is a quadrilateral with distinct adjacent congruent sides.
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.
Rectangle	A rectangle is a quadrilateral with four right angles.
Rhombus	A rhombus is a quadrilateral with four congruent sides.
Trapezoid	A trapezoid is a quadrilateral with exactly one pair of parallel opposite sides.

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5.20: Regular and Irregular Polygons

Understand and identify regular and irregular polygons.

Figure 5.20.1

Logan has been tracing street signs around his city. He is preparing to trace the above street sign. He outlines the shape and he draws the round edges as straight lines. Will the resulting shape be a regular or irregular **polygon**, and what type of polygon will it be?

In this concept, you will learn how to determine if a polygon is regular or irregular.

Determining if Polygons are Regular or Irregular

A polygon can have a certain number of sides, but the sides do not necessarily have to be the same length. Both of the polygons below are pentagons because they both have five angles and sides, but look at the differences.

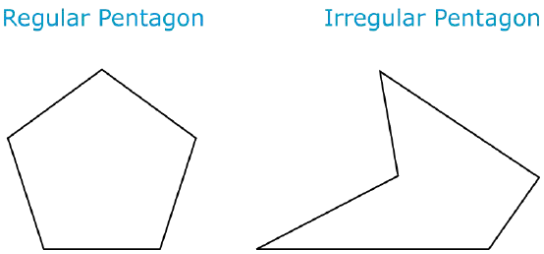

Figure 5.20.2

In the first pentagon, all of the angles are congruent and all of the sides are congruent. In the second pentagon, the angles have different measures and the sides have different lengths.

A **regular polygon** has congruent angles and congruent sides. Any polygon can be a regular polygon.

Look at the regular polygons below.

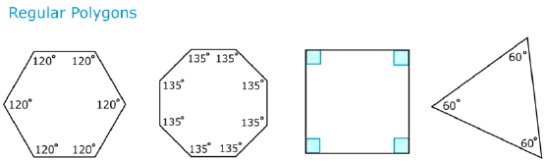

Figure 5.20.3

As you can see, a regular hexagon has six congruent sides and six congruent angles. A regular hexagon, no matter how long its sides, will always have angles that measure $120°$.

A regular octagon has eight congruent angles and eight congruent sides. Every regular octagon has the same angle measures.

The term "regular **rectangle**" is not usually used because a rectangle with congruent sides is actually a **square**. The four angles in a square are always $90°$, and their sum is always $360°$. However, there is a special name for a "regular triangle." A triangle with three

congruent sides is called an **equilateral triangle**. Its three angles are always $60°$, and their sum is always $180°$.

Any polygon that does not have all congruent sides is an **irregular polygon.** Irregular polygons can still be pentagons, hexagons and nonagons, but they do not have congruent angles or equal sides. Here are some examples of irregular polygons.

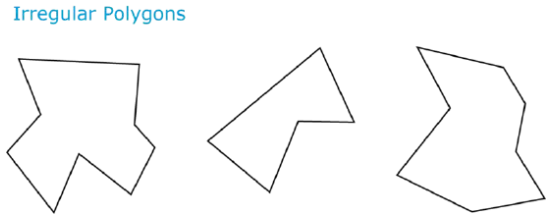

Figure 5.20.4

In these polygons, some sides are clearly longer than others and some angles are wider than others, so they must be irregular. Notice that irregular polygons tend to look uneven or lopsided, while regular polygons look more orderly and symmetrical.

Let's look at an example of identifying irregular and regular polygons.

Identify the polygon below and tell whether it is regular or irregular.

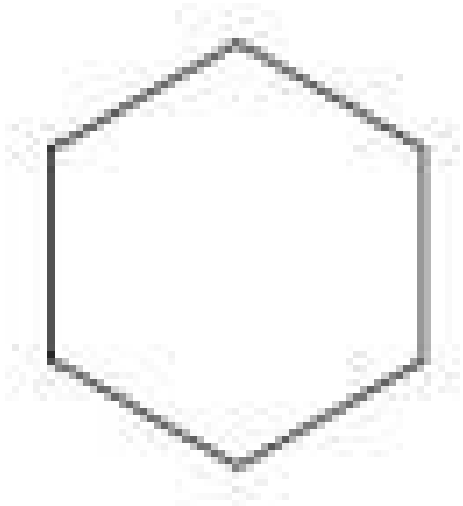

Figure 5.20.5

First, count the number of sides.

The shape has 6 sides.

Next, determine if the sides are congruent.

The sides are congruent.

Then, label the polygon as regular or irregular.

Regular.

The answer is that the polygon is a regular hexagon.

When lines intersect in the geometric plane, they form polygons. Apply what you know in order to classify these polygons. Find side lengths, or solve for unknown angle measures.

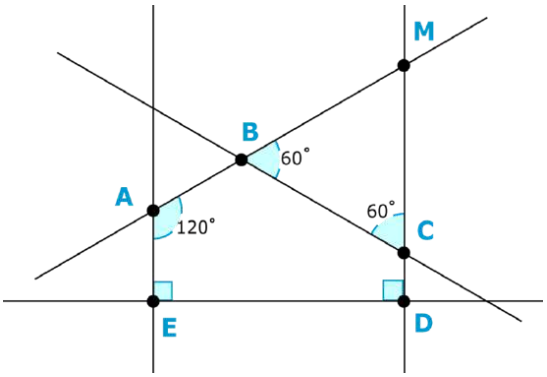

Figure 5.20.6

The intersection of these five lines has created several different polygons. First, let's identify them.

The largest is $AMDE$. Within this figure there are two figures.

One is figure $ABCDE$. $ABCDE$ has five angles and five sides, so it is a pentagon. Two of its angles measure $90°$, and one measures $120°$ Which means that the polygon is an irregular pentagon. In order to find the measures of the two remaining angles, you need to take a look at the third figure.

The third figure is triangle MBC. It has two $60°$ angles. Angles ABC and MBC are supplementary. In other words, together they form a straight line. A straight line measures $180°$. Therefore the sum of these two angles is 180^{\circ}\). Simply subtract to find the measure of ABC.

$180 - 60 = 120°$

Angle ABC is $120°$.

Now let's find the measure of angle BCD. You now know four of the five angles in figure $ABCDE$. Because this is a pentagon, you also know that its interior angles must have a sum of $540°$. You can set up an equation to find the measure of the unknown angle.

$$\begin{aligned} 90 + 90 + 120 + 120 + \angle BCD &= 540° \\ 420 + \angle BCD &= 540° \\ \angle BCD &= 540 - 420 \\ \angle BCD &= 120° \end{aligned}$$

The fifth and final angle must measure $120°$. Let's add up all of the angles in the pentagon to be sure they total $540°$.

$90 + 90 + 120 + 120 + 120 = 540°$

There is still one unknown angle, angle AMD\). There are two different ways to find the measure of the third angle in triangle BMD. You know that the three angles in a triangle have a sum of $180°$. It is also the fourth angle in **quadrilateral** $AMDE$, and you know that the four angles in a quadrilateral have a sum of $360°$. You can set up an equation to find AMD.

$$\begin{aligned} 60 + 60 + \angle AMD &= 180° \\ 120 + \angle AMD &= 180° \\ \angle AMD &= 180 - 120 \\ \angle AMD &= 60° \end{aligned}$$

Example 5.20.1

Figure **5.20.7**

Earlier, you were given a problem about Logan and the above street sign.

If he traces the shape of the sign, will the resulting shape be a regular or irregular polygon, and what type of polygon will it be?

Solution

First, count the number of sides.

The shape has 6 sides.

Next, determine if the sides are congruent.

The sides are not congruent.

Then, label the polygon as regular or irregular.

Irregular.

The answer is that the polygon is an irregular hexagon.

Example 5.20.2

Name this figure.

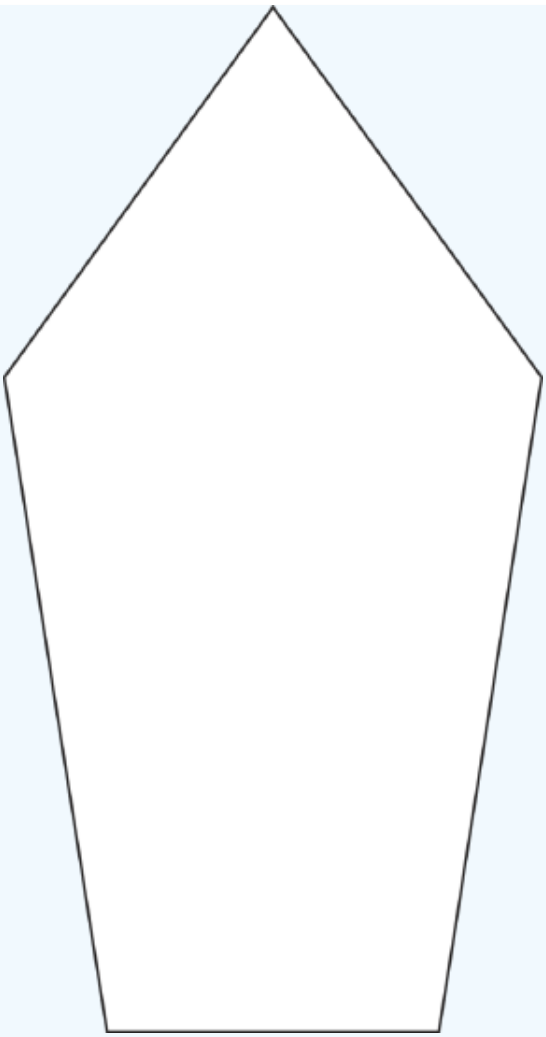

Figure 5.20.8

Solution

First, count the number of sides.

The shape has 5 sides.

Next, determine if the sides are congruent.

The sides are not congruent.

Then, label the polygon as regular or irregular.

Irregular.

The answer is the polygon is an irregular pentagon.

Example 5.20.3

Name this figure.

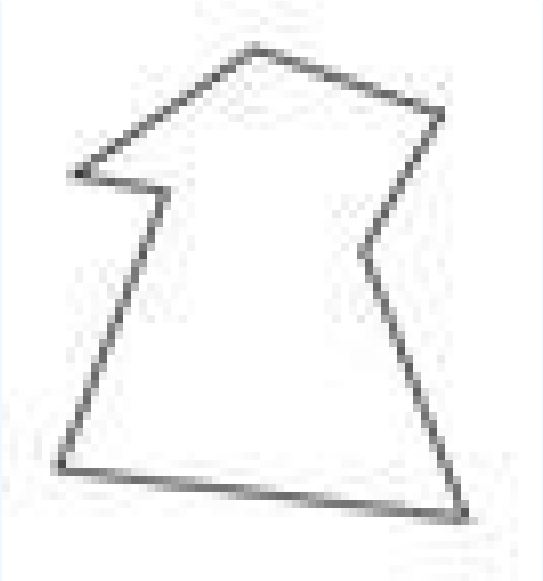

Figure **5.20.9**

Solution

First, count the number of sides.

The shape has 7 sides.

Next, determine if the sides are congruent.

The sides are not congruent.

Then, label the polygon as regular or irregular.

Irregular.

The answer is the polygon is an irregular heptagon.

Example 5.20.4

Classify the figure as irregular or regular and identify the polygon.

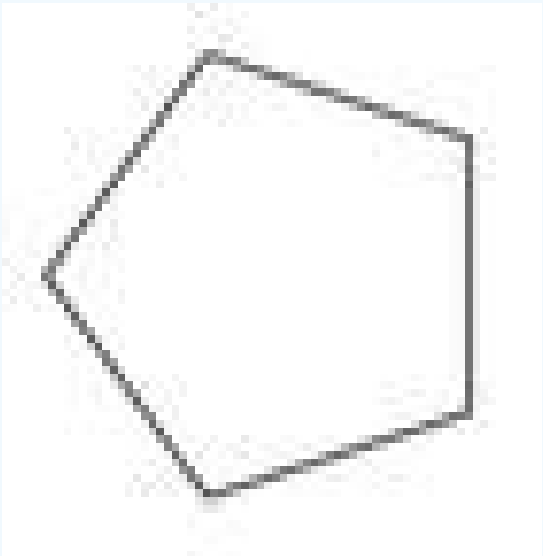

Figure **5.20.10**

Solution

First, count the number of sides.

The shape has 5 sides.

Next, determine if the sides are congruent.

The sides appear to be congruent.

Then, label the polygon as regular or irregular.

Regular.

The answer is that the polygon is a regular pentagon.

Example 5.20.5

Identify the polygon.

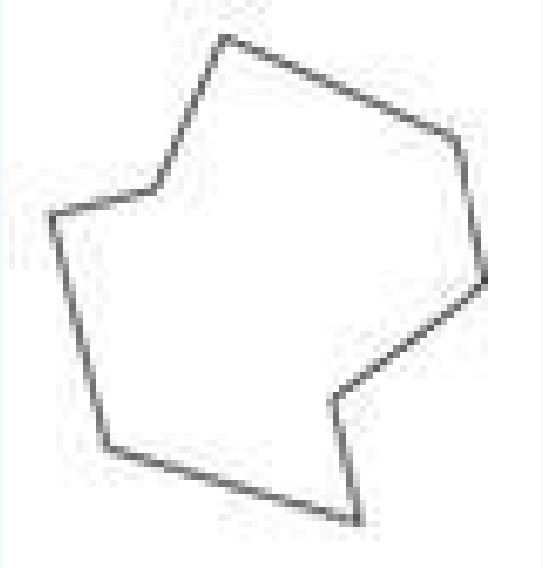

Figure 5.20.11

Solution

First, count the number of sides.

The shape has 8 sides.

Next, determine if the sides are congruent.

The sides are not congruent.

Then, label the polygon as regular or irregular.

Irregular.

The answer is that the polygon is an irregular octagon.

Review

Identify each quadrilateral by its description.

1. A quadrilateral with one pair of parallel sides.
2. A quadrilateral with opposites sides congruent and parallel, with four right angles.
3. A quadrilateral with four congruent parallel sides and four right angles.
4. A rectangle with four congruent parallel sides.

Identify the polygons in the diagram. Then find the measures of the unknown angles.

5.

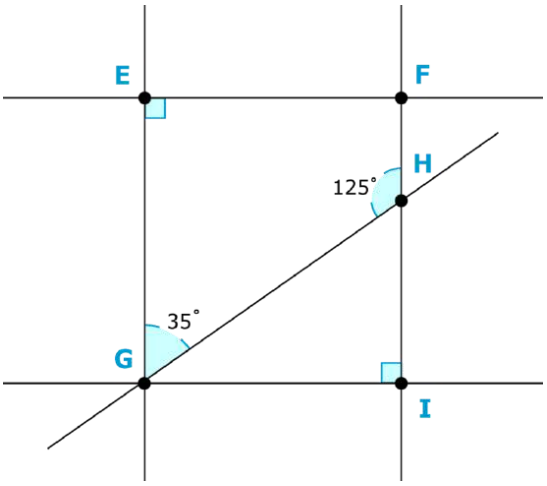

Figure 5.20.12

Answer true or false for each of the following questions.

6. A **rhombus** is always a square.
7. A **parallelogram** has opposite sides that are parallel.
8. A rectangle is a type of parallelogram.
9. Squares, rectangles and rhombi are parallelograms with four right angles.
10. A trapezoid has four right angles.
11. A trapezoid has one pair of parallel sides.
12. A regular polygon has congruent sides and angles.
13. A pentagon can not be an irregular polygon.

Identify each polygon below.

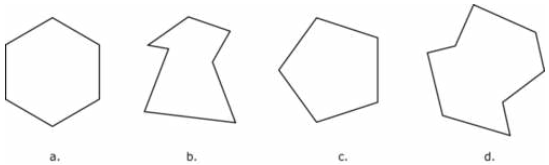

Figure 5.20.13

14. a
15. b
16. c
17. d

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.11.

Resources

Vocabulary

Term	Definition
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Polygon	A polygon is a simple closed figure with at least three straight sides.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.
Rectangle	A rectangle is a quadrilateral with four right angles.
Rhombus	A rhombus is a quadrilateral with four congruent sides.
Square	A square is a polygon with four congruent sides and four right angles.

Additional Resources

Video: Introduction to Polygons

Practice: Regular and Irregular Polygons

Real World: A Perfect Pentagon

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5.21: Area of Regular and Irregular Polygons

Half the apothem times the perimeter.

Area of Regular Polygons

What if you were asked to find the distance across The Pentagon in Arlington, VA? The Pentagon, which also houses the Department of Defense, is composed of two regular pentagons with the same center. The entire area of the building is 29 acres (40,000 square feet in an acre), with an additional 5 acre courtyard in the center. The length of each outer wall is 921 feet. What is the total distance across the pentagon? Round your answer to the nearest hundredth.

Figure 5.21.1

A regular polygon is a polygon with congruent sides and angles. Recall that the perimeter of a square is 4 times the length of a side because each side is congruent. We can extend this concept to any regular polygon.

Perimeter of a Regular Polygon: If the length of a side is s and there are n sides in a regular polygon, then the perimeter is $P = ns$.

In order to find the area of a regular polygon, we need to define some new terminology. First, all regular polygons can be inscribed in a circle. So, regular polygons have a **center** and **radius**, which are the center and radius of the circumscribed circle. Also like a circle, a regular polygon will have a central angle formed. In a regular polygon, however, the central angle is the angle formed by two radii drawn to consecutive vertices of the polygon. In the picture below, the central angle is $\angle BAD$. Also, notice that ΔBAD is an isosceles triangle. Every regular polygon with n sides is formed by n isosceles triangles. The height of these isosceles triangles is called the **apothem**.

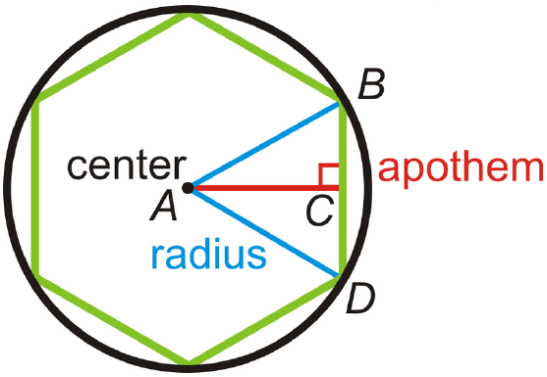

Figure 5.21.2

The area of each triangle is $A_{\Delta} = \frac{1}{2}bh = \frac{1}{2}sa$, where s is the length of a side and a is the apothem. If there are n sides in the regular polygon, then it is made up of n congruent triangles.

Area of a Regular Polygon: If there are n sides with length s in a regular polygon and a is the apothem, then $A = \frac{1}{2}asn$ or $A = \frac{1}{2}aP$, where P is the perimeter.

Calculating the Perimet

What is the perimeter of a h 4 inch sides?

If each side is 4 inches and there are 8 sides, that means the perimeter is $8(4 \text{ in }) = 32 \text{ inches}$.

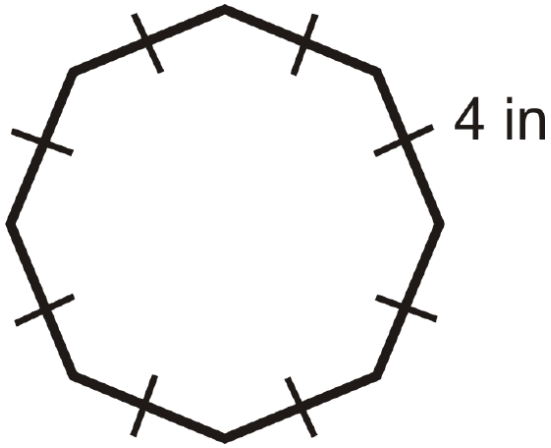

Figure 5.21.3

Finding Side Lengths

The perimeter of a regular heptagon is 35 cm. What is the length of each side?

If $P = ns$, then $35 \text{ cm } = 7s$. Therefore, $s = 5 \text{ cm}$.

Finding the Length of the Apothem

Find the length of the apothem in the regular octagon. Round your answer to the nearest hundredth.

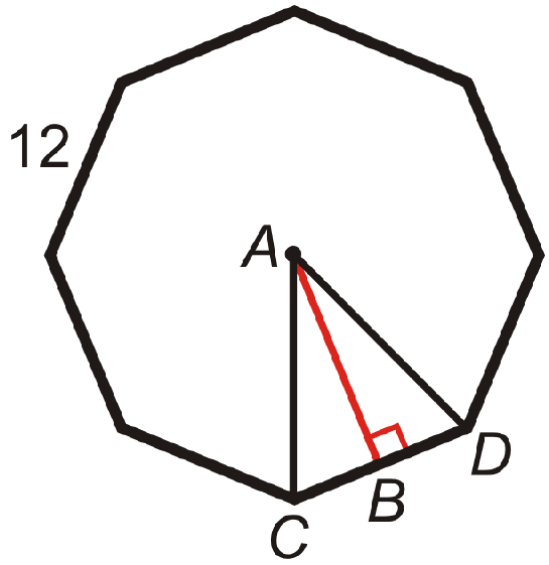

Figure 5.21.4

To find the length of the apothem, AB, you will need to use the trig ratios. First, find $m\angle CAD$. There are 360° around a point, so $m\angle CAD = \frac{360^\circ}{8} = 45^\circ$. Now, we can use this to find the other two angles in ΔCAD. m\angle ACB\) and m\angle ADC\) are equal because \Delta CAD\) is a right triangle.

$$\begin{aligned} m\angle CAD + m\angle ACB + m\angle ADC &= 180^\circ \\ 45^\circ + 2m\angle ACB &= 180^\circ \\ 2m\angle ACB &= 135^\circ \\ m\angle ACB &= 67.5^\circ \end{aligned}$$

To find AB, we must use the tangent ratio. You can use either acute angle.

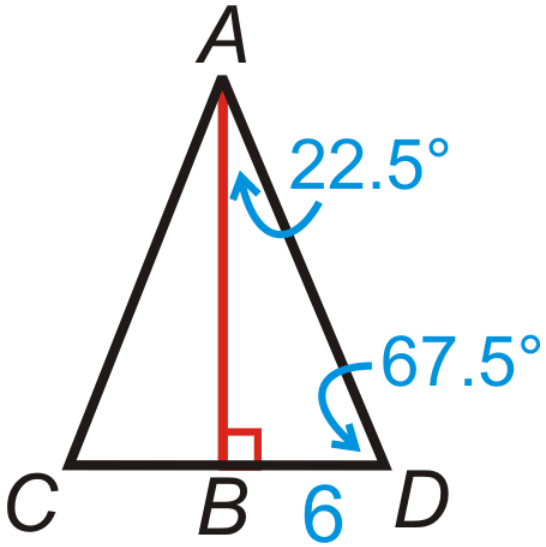

Figure 5.21.5

$$tan67.5^\circ = \frac{AB}{6}$$
$$AB = 6 \cdot tan67.5^\circ \approx 14.49$$

Pentagon Problem Rev

From the picture below, w otal distance across the Pentagon is the length of the apothem plus the length of the radius. If the total area 34 acres, that is 2,720,000 square feet. Therefore, the area equation is $2720000 = \frac{1}{2}a(921)(5)$ and the apot o find the radius, we can either use the Pythagorean Theorem, with the apothem and half the length of a side or the sine ratio. Recall from Example 5, that each central angle in a pentagon is 72°, so we would use half of that for the right triangle.

$$sin36^\circ = \frac{460.5}{r} \rightarrow r = \frac{460.5}{sin36^\circ} \approx 783.45 \text{ ft}$$

Therefore, the total distance across is $590.66 + 783.45 = 1374.11\ ft$.

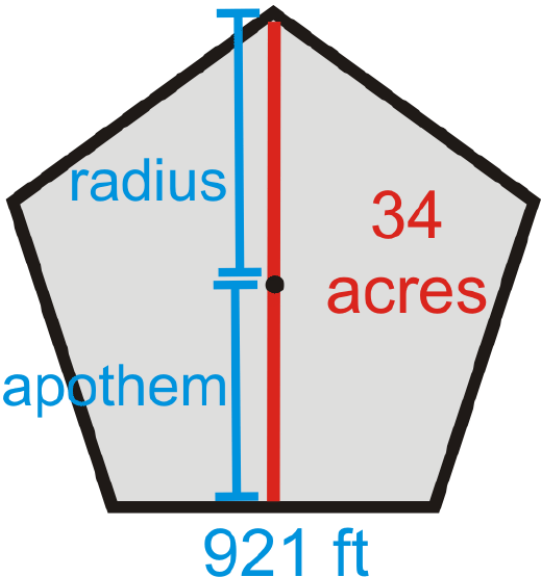

Figure 5.21.6

Example 5.21.1

Find the area of the regular octagon in Example C.

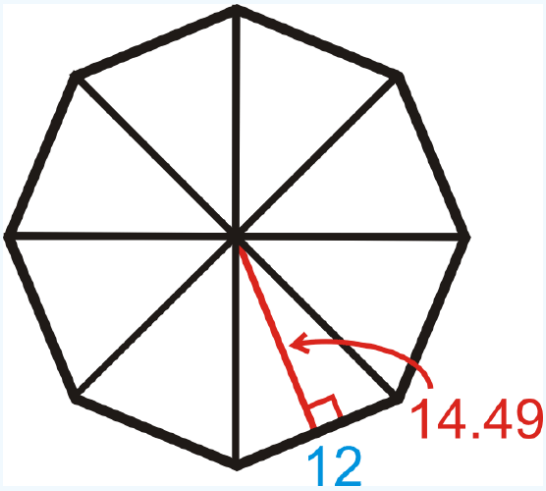

Figure 5.21.7

Solution

The octagon can be split into 8 congruent triangles. So, if we find the area of one triangle and multiply it by 8, we will have the area of the entire octagon.

$$A_{octagon} = 8(12 \cdot 12 \cdot 14.49) = 695.52 units^2$$

Example 5.21.2

Find the area of the regular polygon with radius 4.

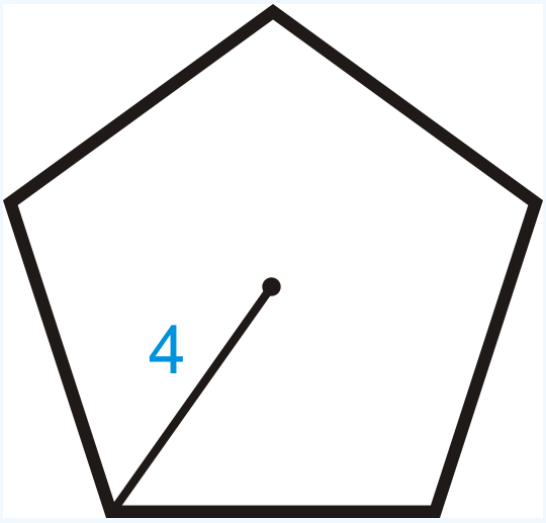

Figure 5.21.8

Solution

INSERTB In this problem we need to find the apothem and the length of the side before we can find the area of the entire polygon. Each central angle for a regular pentagon is $\frac{360^\circ}{5} = 72^\circ$. So, half of that, to make a right triangle with the apothem, is 36°. We need to use sine and cosine.

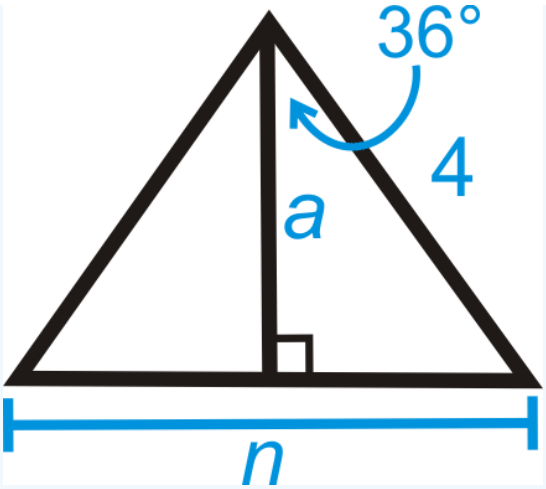

Figure 5.21.9

$$\begin{aligned} \sin 36^\circ &= \tfrac{.5n}{4} & \cos 36^\circ &= \tfrac{a}{4} \\ 4\sin 36^\circ &= \tfrac{1}{2}n & 4\cos 36^\circ &= a \\ 8\sin 36^\circ &= n & a &\approx 3.24 \\ n &\approx 4.7 \end{aligned}$$

Using these two pieces of information, we can now find the area. $A = 12(3.24)(5)(4.7) \approx 38.07 units^2$.

Example 5.21.3

The area of a regular hexagon is $54\sqrt{3}$ and the perimeter is 36. Find the length of the sides and the apothem.

Solution

Plug in what you know into both the area and the perimeter formulas to solve for the length of a side and the apothem.

$$\begin{aligned} P &= sn & A &= \tfrac{1}{2}aP \\ 36 &= 6s & 54\sqrt{3} &= \tfrac{1}{2}a(36) \\ s &= 6 & 54\sqrt{3} &= 18a \\ 3\sqrt{3} &= a \end{aligned}$$

Review

Use the regular hexagon below to answer the following questions. Each side is 10 cm long.

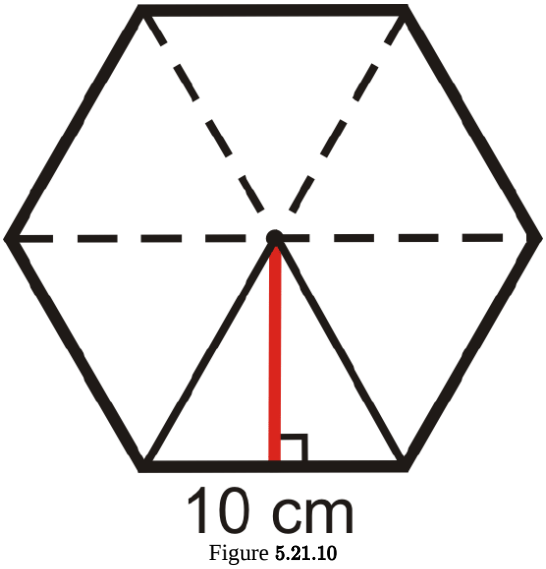

Figure 5.21.10

1. Each dashed line segment is a(n) _____.
2. The red line segment is a(n) _____.
3. There are _____ congruent triangles in a regular hexagon.
4. In a regular hexagon, all the triangles are _____.
5. Find the radius of this hexagon.
6. Find the apothem.
7. Find the perimeter.
8. Find the area.

Find the area and perimeter of each of the following regular polygons. Round your answer to the nearest hundredth.

9.

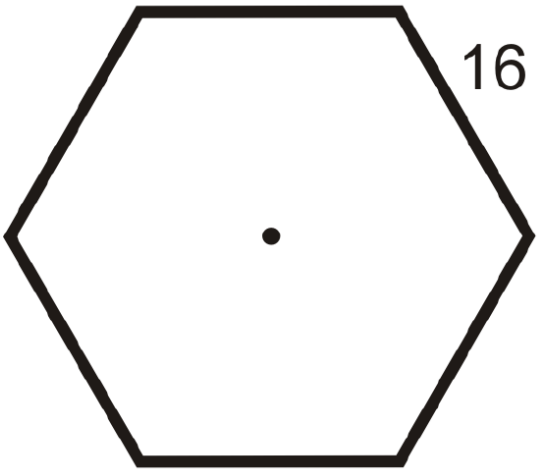

Figure **5.21.11**

10.

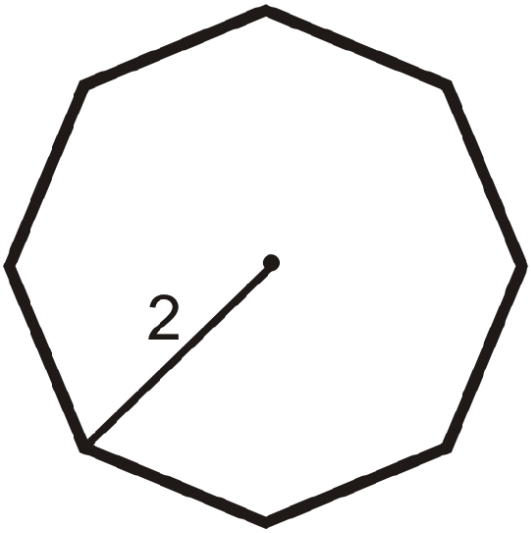

Figure **5.21.12**

11.

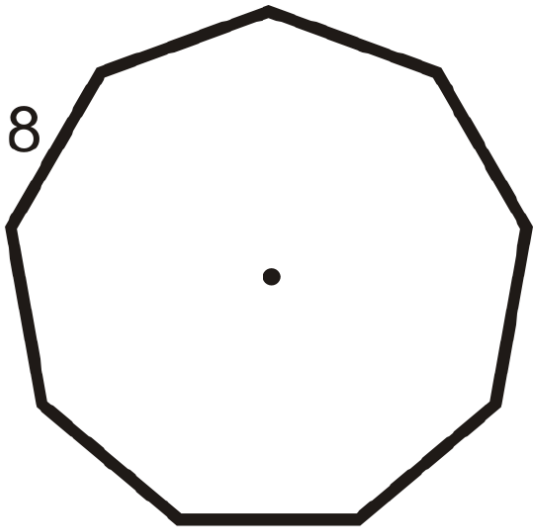

Figure 5.21.13

12.

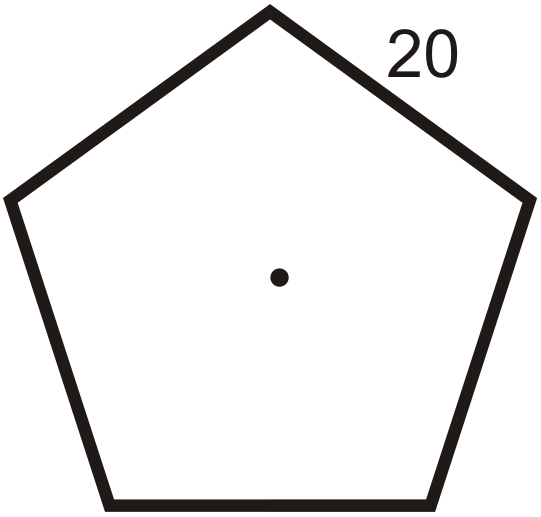

Figure 5.21.14

13. If the perimeter of a regular decagon is 65, what is the length of each side?
14. A regular polygon has a perimeter of 132 and the sides are 11 units long. How many sides does the polygon have?
15. The area of a regular pentagon is 440.44 in^2 and the perimeter is 80 in. Find the length of the apothem of the pentagon.
16. The area of a regular octagon is 695.3 cm^2 and the sides are 12 cm. What is the length of the apothem?

A regular 20-gon and a regular 40-gon are inscribed in a circle with a radius of 15 units.

17. ***Challenge*** Derive a formula for the area of a regular hexagon with sides of length s. Your only variable will be s. HINT: Use 30-60-90 triangle ratios.
18. ***Challenge*** in the following steps you will derive an alternate formula for finding the area of a regular polygon with n sides.

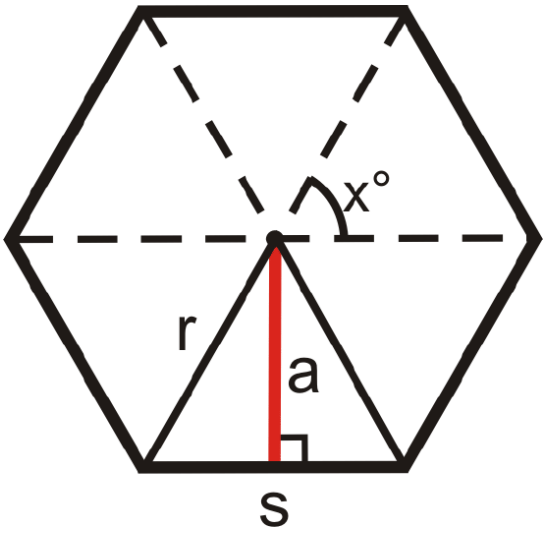

Figure 5.21.15

We are going to start by thinking of a polygon with n sides as n congruent isosceles triangles. We will find the sum of the areas of these triangles using trigonometry. First, the area of a triangle is $\frac{1}{2}bh$. In the diagram to the right, this area formula would be $\frac{1}{2}sa$, where s is the length of a side and a is the length of the apothem. In the diagram, x represents the measure of the vertex angle of each isosceles triangle.

a. The apothem, a, divides the triangle into two congruent right triangles. The top angle in each is $\frac{x^\circ}{2}$. Find $sin(\frac{x^\circ}{2})$ and $cos(\frac{x^\circ}{2})$.
b. Solve your sin equation to find an expression for s in terms of r and x.
c. Solve your cos equation to find an expression for a in terms of r and x.
d. Substitute these expressions into the equation for the area of one of the triangles, \dfrac{1}{2}sa\).
e. Since there will be n triangles in an n-gon, you need to multiply your expression from part d by n to get the total area.
f. How would you tell someone to find the value of x for a regular n-gon?

Use the formula you derived in problem 18 to find the area of the regular polygons described in problems 19-22. Round your answers to the nearest hundredth.

19. Decagon with radius 12 cm.
20. 20-gon with radius 5 in.
21. 15-gon with radius length 8 cm.
22. 45-gon with radius length 7 in.

Review (Answers)

To view the Review answers, open this PDF file and look for section 10.12.

Additional Resources

Video: Area of Regular Polygons Principles

Practice: Area of Regular and Irregular Polygons

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5.22: Area and Perimeter of Similar Polygons

Ratio of the areas is the square of the scale factor; ratio of perimeters is the scale factor.

Polygons are **similar** when their corresponding angles are equal and their corresponding sides are in the same **proportion**. Just as their corresponding sides are in the same proportion, perimeters and areas of similar polygons have a special relationship.

Perimeters: The **ratio** of the perimeters is the same as the **scale factor**. In fact, the ratio of any part of two similar shapes (diagonals, medians, midsegments, altitudes, etc.) is the same as the scale factor.

Areas: If the scale factor of the sides of two similar polygons is $\frac{m}{n}$, then the ratio of the areas is $\left(\frac{m}{n}\right)^2$ (**Area of Similar Polygons Theorem**). You square the ratio because area is a two-dimensional measurement.

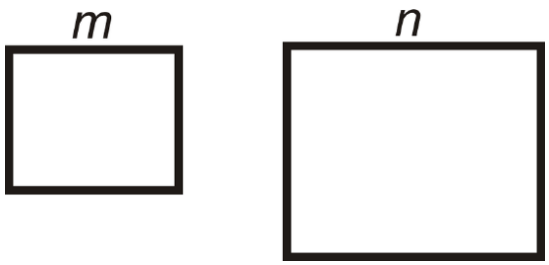

Figure 5.22.1

What if you were given two similar triangles and told what the scale factor of their sides was? How could you find the ratio of their perimeters and the ratio of their areas?

Example 5.22.1

The two rectangles belo the scale factor and the ratio of the perimeters and verify that the two results are the same.

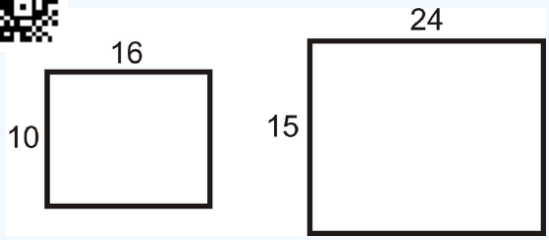

Figure 5.22.2

Solution

The scale factor is $\frac{16}{24} = \frac{2}{3}$.

$$P_{\text{small}} = 2(10) + 2(16) = 52 \text{ units}$$
$$P_{\text{large}} = 2(15) + 2(24) = 78 \text{ units}$$

The ratio of the perimeters is $\frac{52}{78} = \frac{2}{3}$.

Example 5.22.2

Find the area of each rectangle from Example 1. Then, find the ratio of the areas and verify that it fits the Area of Similar Polygons Theorem.

Solution

$A_{\text{small}} = 10 \cdot 16 = 160 \text{ units}^2$
$A_{\text{large}} = 15 \cdot 24 = 360 \text{ units}^2$

The ratio of the areas would be $\frac{160}{360} = \frac{4}{9}$.

The ratio of the sides, or scale factor is $\frac{2}{3}$ and the ratio of the areas is $\frac{4}{9}$. Notice that the ratio of the areas is the square of the scale factor.

Example 5.22.3

Find the ratio of the areas of the rhombi below. The rhombi are similar.

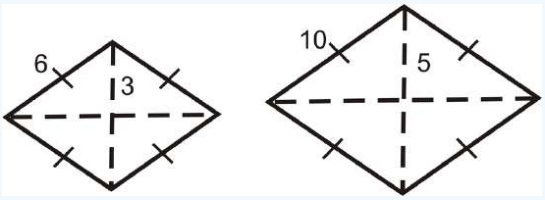

Figure 5.22.3

Solution

Find the ratio of the sides and square it.

$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$

Example 5.22.4

Two trapezoids are similar. If the scale factor is 34 and the area of the smaller **trapezoid** is 81 cm^2, what is the area of the larger trapezoid?

Solution

First, the ratio of the areas is $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$. Now, we need the area of the larger trapezoid. To find this, set up a proportion using the area ratio.

$\frac{9}{16} = \frac{81}{A} \rightarrow 9A = 1296$

$A = 144 \text{ cm}^2$

Example 5.22.5

Two triangles are similar. The ratio of the areas is $\frac{25}{64}$. What is the scale factor?

Solution

The scale factor is $\sqrt{\frac{25}{64}} = \frac{5}{8}$.

Example 5.22.6

Using the ratios from Example 5, find the length of the base of the smaller triangle if the length of the base of the larger triangle is 24 units.

Solution

Set up a proportion using the scale factor.

$\frac{5}{8} = \frac{b}{24} \rightarrow 8b = 120$

$b = 15 \text{ units}$

Review

Determine the ratio of the areas, given the ratio of the sides of a polygon.

1. $\frac{3}{5}$

2. $\frac{1}{4}$
3. $\frac{7}{2}$
4. $\frac{6}{11}$

Determine the ratio of the sides of a polygon, given the ratio of the areas.

5. $\frac{1}{36}$
6. $\frac{4}{81}$
7. $\frac{49}{9}$
8. $\frac{25}{144}$

This is an equilateral triangle made up of 4 congruent equilateral triangles.

9. What is the ratio of the areas of the large triangle to one of the small triangles?

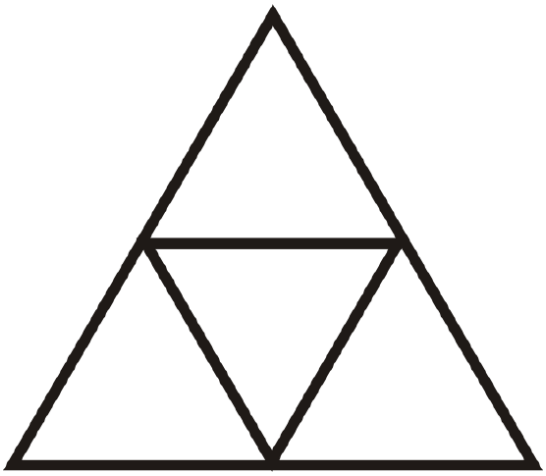

Figure 5.22.4

10. What is the scale factor of large to small triangle?
11. If the area of the large triangle is 20 units^2, what is the area of a small triangle?
12. If the length of the altitude of a small triangle is $2\sqrt{3}$, find the perimeter of the large triangle.

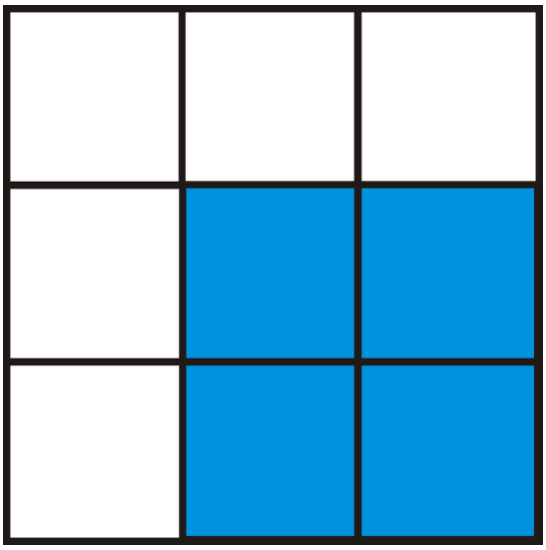

Figure 5.22.5

13. Find the perimeter of the large square and the blue square.
14. Find the scale factor of the blue square and large square.

15. Find the ratio of their perimeters.
16. Find the area of the blue and large squares.
17. Find the ratio of their areas.
18. Find the length of the diagonals of the blue and large squares. Put them into a ratio. Which ratio is this the same as?
19. Two rectangles are similar with a scale factor of $\frac{4}{7}$. If the area of the larger rectangle is 294 in^2, find the area of the smaller rectangle.
20. Two triangles are similar with a scale factor of $\frac{1}{3}$. If the area of the smaller triangle is 22 ft^2, find the area of the larger triangle.
21. The ratio of the areas of two similar squares is $\frac{16}{81}$. If the length of a side of the smaller square is 24 units, find the length of a side in the larger square.
22. The ratio of the areas of two right triangles is $\frac{4}{9}$. If the length of the hypotenuse of the larger triangle is 48 units, find the length of the smaller triangle's hypotenuse.

Questions 23-26 build off of each other. You may assume the problems are connected.

23. Two similar rhombi have areas of 72 units^2 and 162 units^2. Find the ratio of the areas.
24. Find the scale factor.
25. The diagonals in these rhombi are congruent. Find the length of the diagonals and the sides.
26. What type of rhombi are these quadrilaterals?

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.7.

Vocabulary

Term	Definition
area	The amount of space inside a figure. Area is measured in square units.
Perimeter	Perimeter is the distance around a two-dimensional figure.
Proportion	A proportion is an equation that shows two equivalent ratios.
Ratio	A ratio is a comparison of two quantities that can be written in fraction form, with a colon or with the word "to".
Scale Factor	A scale factor is a ratio of the scale to the original or actual dimension written in simplest form.
Trapezoid	A trapezoid is a quadrilateral with exactly one pair of parallel opposite sides.

Additional Resources

Video: Area and Perimeter of Similar Polygons Principles - Basic

Activities: Area and Perimeter of Similar Polygons Discussion Questions

Study Aids: Perimeter and Area Study Guide

Real World: In the Doll's House

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5.23: Construct Regular Polygons

Construct drawings of equilateral triangles, squares, and regular polygons using a compass and straightedge. Create polygons using Geogebra.

Constructions of Regular Polygons

Use your compass to construct a circle like the one shown below on a piece of paper. Describe how to fold the paper two times in order to help you construct a square.

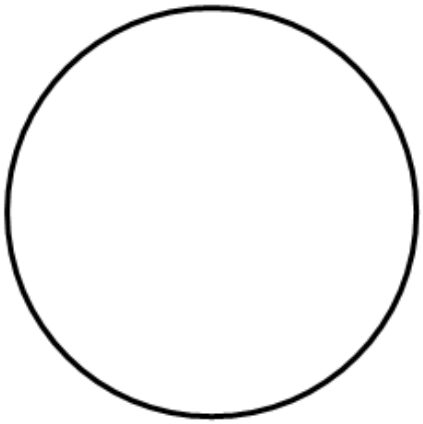

Figure 5.23.1

Regular Polygons

A **regular polygon** is a polygon that is **equiangular** and **equilateral**. This means that all its angles are the same measure and all its sides are the same length.

The most basic example of a regular polygon is an **equilateral triangle**, a triangle with three congruent sides and three congruent angles. **Squares** are also regular polygons, because all their angles are the same ($90°$) and all their sides are the same length. Regular polygons with five or more sides do not have special names. Instead, the word *regular* is used to describe them. For example, a **regular hexagon** is a hexagon (6 sided polygon) whose angles are all the same measure and sides are all the same length.

All regular polygons have **rotation symmetry**. This means that a rotation of less than $360°$ will carry the regular polygon onto itself. In fact, a regular n-sided polygon has rotation symmetry for any multiple of $\frac{360°}{n}$.

Constructions are step-by-step processes used to create accurate geometric figures. To create a construction by hand, there are a few tools that you can use:

1. **Compass:** A device that allows you to create a circle with a given radius. Not only can compasses help you to create circles, but also they can help you to copy distances.
2. **Straightedge:** Anything that allows you to produce a straight line. A straightedge should not be able to measure distances. An index card works well as a straightedge. You can also use a ruler as a straightedge, as long as you only use it to draw straight lines and not to measure.
3. **Paper:** When a geometric figure is on a piece of paper, the paper itself can be folded in order to construct new lines.

You can construct ***some*** regular polygons by hand if you remember the definitions and properties of these regular polygons. With the additional help of geometry software or a protractor, you can construct ***any*** regular polygon.

Let's take a look at an exa

$\overline{AB}$ is one side of what w[illegible]al triangle ΔABC. You need to put point C in the correct place in order to make the equilateral triangle. Where [illegible] placed with respect to points A and B?

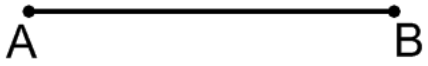

Figure 5.23.2

Let the distance between A and B be d. Point C needs to be d away from point A and also d away from point B.

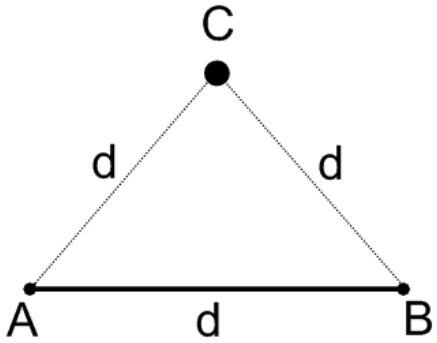

Figure 5.23.3

Use a straightedge to draw line segment $\overline{AB}$. Use the ideas from Example A to construct equilateral triangle ΔABC.

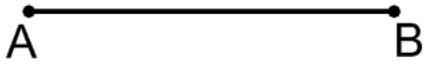

Figure 5.23.4

Use a compass to measure the length of $\overline{AB}$.

Figure 5.23.5

Make a partial circle of points that are the length of $\overline{AB}$ away from point A.

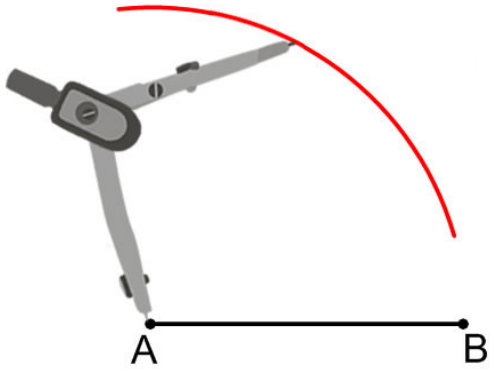

Figure 5.23.6

Make another partial circle of points that are the length of $\overline{AB}$ away from point B.

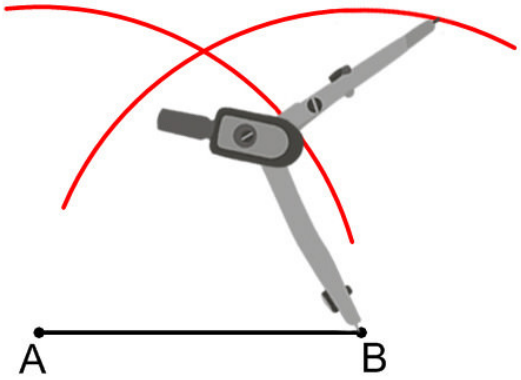

Figure 5.23.7

The point of intersection of these two partial circles is point C.

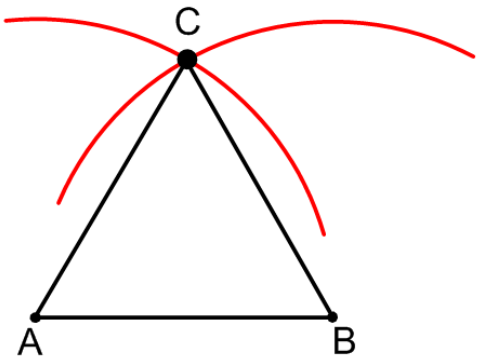

Figure 5.23.8

Points A, B, C, and D are on a circle centered at point O. Prove that $ABCD$ is a square.

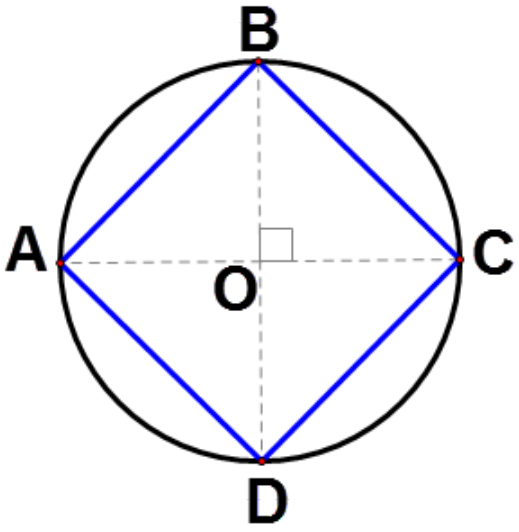

Figure 5.23.9

$\overline{AO} \cong \overline{BO} \cong \overline{CO} \cong \overline{DO}$ because they are all radii of the same circle. Since$\angle BOC$ is a right angle, $\angle BOA$,\angle AOD\) and $\angle COD$ must also be right angles. Therefore, $\angle BOC \cong \angle BOA \cong \angle AOD \cong \angle COD$. This means that $\Delta BOC \cong \Delta AOB \cong \Delta DOA \cong \Delta COD$ by SAS\cong\). $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ because they are corresponding parts of congruent triangles.

All four triangles are isosceles because they each have two congruent sides. This means that their base angles are congruent. Because the vertex angle of each triangle is $90°$, the base angles of each triangle must be $45°(90 + 45 + 45 = 180)$. The four angles that make up the quadrilateral are each made of two of these $45°$ angles, and are each therefore $90°$.

Because the quadrilateral has four congruent sides and four $90°$ angles, it is a square.

Example 5.23.1

Earlier, you were asked to describe how to fold the paper two times in order to help you construct a square.

Solution

Fold the circle so that the two halves overlap to create a crease that is the diameter.

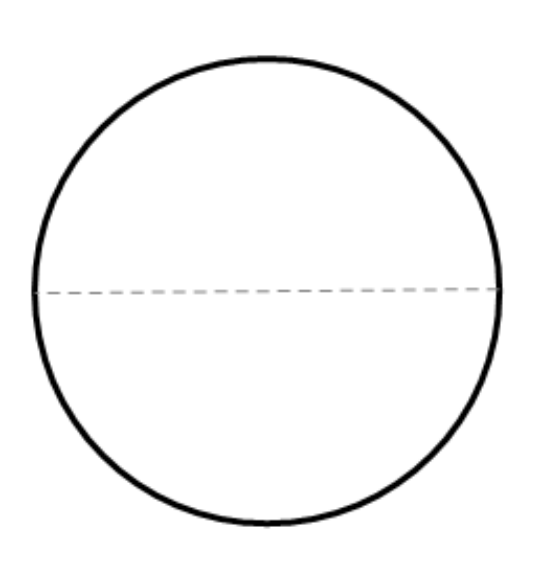

Figure 5.23.10

Fold the circle in half again to create the perpendicular bisector of the diameter. To do this, fold so that the two endpoints of the diameter meet. The second crease will also be a diameter.

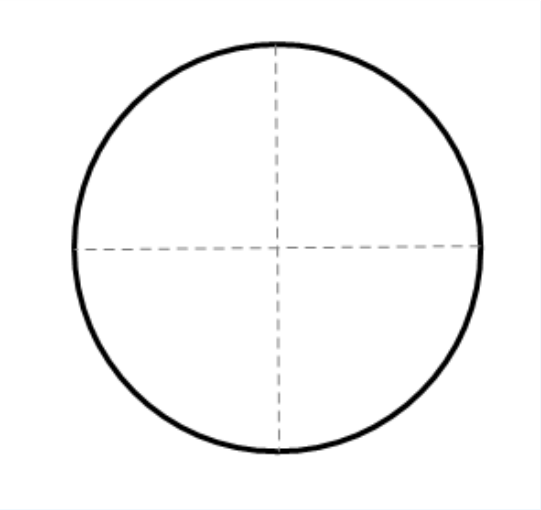

Figure **5.23.11**

Note that the two diameters are perpendicular to one another. Connect the four points of intersection on the circle to construct the square.

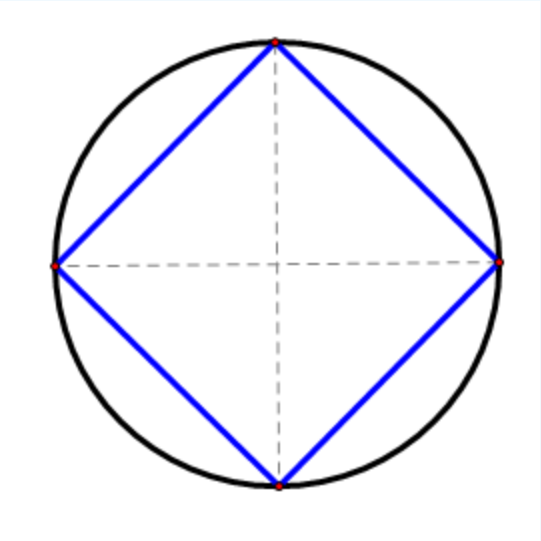

Figure **5.23.12**

You can be certain that this is a square due to the proof in Example C.

Example 5.23.2

The regular hexagon below has been divided into six congruent triangles. What type of triangles are they? Explain.

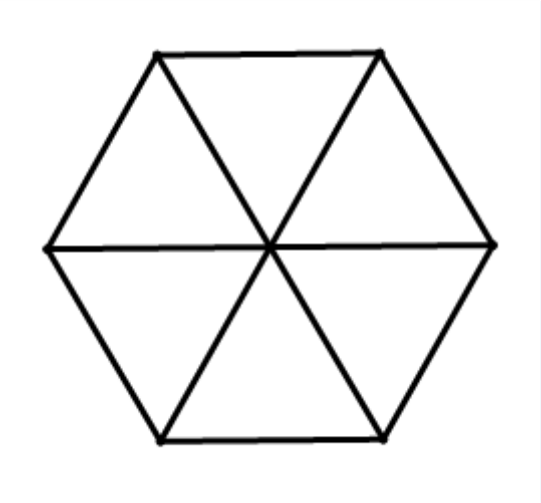

Figure 5.23.13

Solution

They must be equilateral triangles.

- A full circle is $360°$, so each angle at the center of the hexagon must be $\frac{360°}{6} = 60°$. **This is also why regular hexagons demonstrate rotation symmetry at multiples of* $60°$*.**
- The six triangles are congruent, so the six segments connecting the center of the hexagon to the vertices must be congruent. This means the six triangles are all isosceles.
- The base angles of each of the isosceles triangles must be $\frac{180 - 60}{2} = 60°$.
- The measure of each angle of all of the triangles is $60°$, so all triangles are equilateral.

Example 5.23.3

Six points have been evenly spaced around the circle below. Explain why a regular hexagon is created when these points are connected.

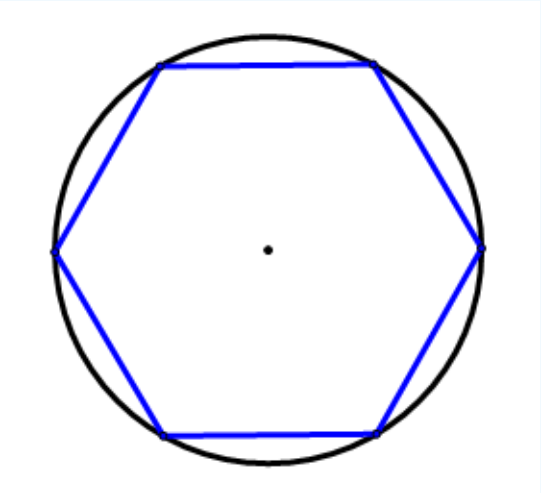

Figure 5.23.14

Solution

Because the six points are evenly spaced, each of the segments connecting the six points must be the same length. Therefore, the polygon must be regular. Because there are six sides, it must be a regular hexagon.

Example 5.23.4

Construct a regular hexagon inscribed in a circle.

Solution

“Inscribed in a circle” means all six vertices of the hexagon are on the same circle. Start by constructing a circle and a point on the circle.

Figure 5.23.15

You know that the radius of the circle is the same as the length of each side of the circle (see guided practice #1). Therefore, your goal is to place six points around the circle that are the same distance apart from one another as the radius of the circle. Keep your compass open to the same width as the radius of the circle and make one new mark on the circle.

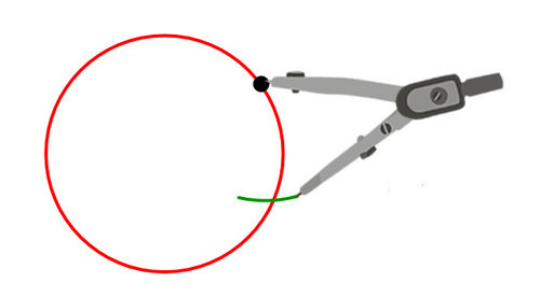

Figure 5.23.16

Continue to make new marks around the circle that are the same distance apart from one another.

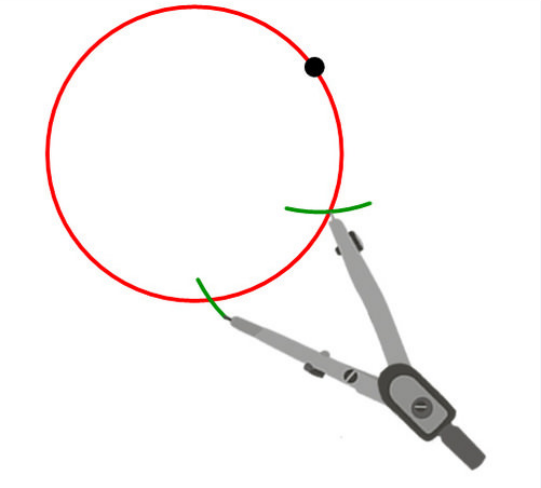

Figure **5.23.17**

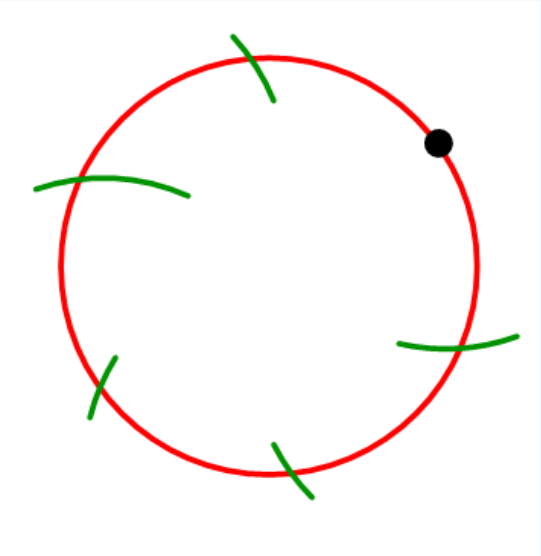

Figure **5.23.18**

Connect the intersection points to form the regular hexagon.

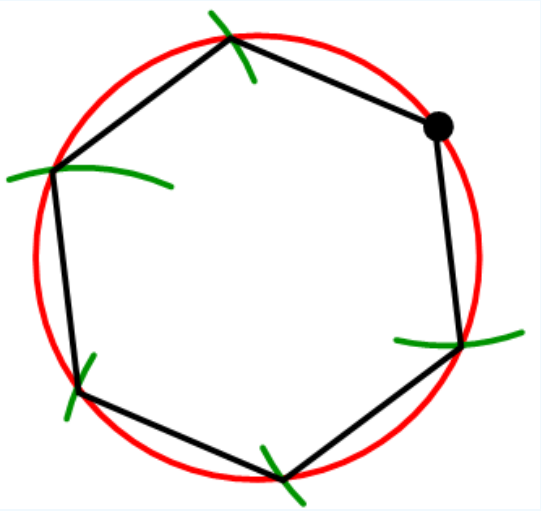

Figure **5.23.19**

Review

1. Construct an equilateral triangle.

2. Construct another equilateral triangle.

3. Explain why your process for constructing equilateral triangles works.

4. Construct a square inscribed in a circle by making two folds.

5. Justify why the polygon you've created is actually a square.

Use your straightedge to construct $\overline{AB}$.

6. Construct the perpendicular bisector of $\overline{AB}$.

7. Construct a circle with diameter $\overline{AB}$.

8. Construct a square inscribed in the circle by connecting the four endpoints of the diameters.

9. Extend your construction to a regular octagon by bisecting each of the right angles at the center of the circle.

10. Construct a regular hexagon inscribed in a circle.

11. Explain why the method for constructing a regular hexagon relies on a circle.

12. Explain how you could extend your construction of the regular hexagon to a construction of a regular 12-gon.

13. Construct an equilateral triangle. Explain how you could construct the circle that passes through the three points of the equilateral triangle.

14. Given an equilateral triangle inscribed in a circle, how could you extend the construction to construct a regular hexagon?

15. Given a circle and a protractor, explain how you could create a regular pentagon.

Review (Answers)

To see the Review answers, open this PDF file and look for section 5.4.

Vocabulary

Term	Definition
Regular Polygon	A regular polygon is a polygon with all sides the same length and all angles the same measure.
Equilateral	A polygon is equilateral if all of its sides are the same length.
Equiangular	A polygon is equiangular if all angles are the same measure.

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5.24: Congruent Polygons

Recognize and understand congruent polygons.

Congruent Figures

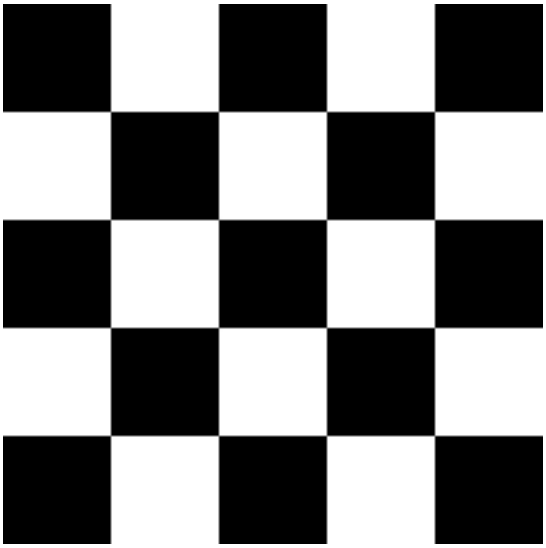


Figure 5.24.1

Chelsie wants to put a checkerboard pattern on her locker for her birthday. She has to make the pattern on poster board then tape it to her locker so that she does not permanently damage her locker. She decides that the easiest technique is to get construction paper and cut out enough squares to make her pattern. How can she tell if the squares that she cuts out are **congruent**?

In this concept, you will learn about congruent figures.

Congruent Figures

Congruent figures have exactly the same size and shape. They have congruent sides and congruent angles.

Here are some pairs of congruent figures.

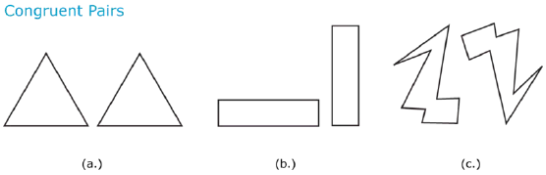


Figure 5.24.2

Compare the figures in each pair. They are exactly the same. If you're not sure, imagine that you could cut out one figure and place it on top of the other. If they match exactly, they are congruent.

Test for congruency by comparing each side and **angle** of two figures. Each side and angle of one figure corresponds to a side or angle in the other. These are called **corresponding parts**. For instance, the top point of one triangle corresponds to the top point of the other triangle in a congruent pair.

It is not always easy to see the corresponding parts of two figures. One figure may be rotated differently so that the corresponding parts appear to be in different places. If you're not sure, trace one figure and place it on top of the other to see if you can make them match.

Is the pair of figures below congruent?

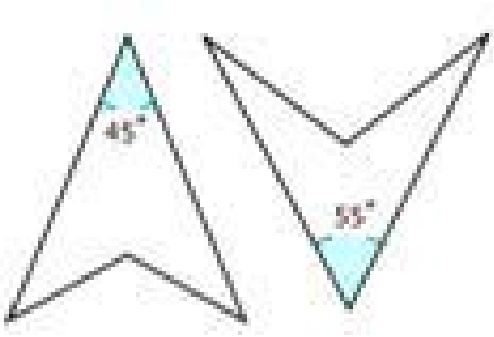


Figure **5.24.3**

First, look at the shape of each figure.

The shapes look similar.

Next, determine if there are corresponding parts.

The given angles are not corresponding because they have different values.

Then, determine whether the shapes are corresponding.

No.

The answer is that the pair of figures are not congruent.

You can use information about one figure in a pair of congruent figures to find the measure of a corresponding angle or side in the other figure. Let's see how this works. Take a look at the congruent figures below.

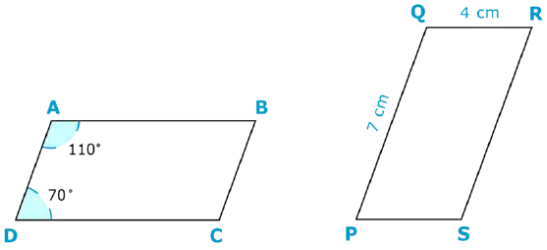


Figure **5.24.4**

These two parallelograms are congruent. Find the corresponding parts.

First, trace one parallelogram and place it on top of the other. Rotate it until the parts correspond.

You can see that side AB corresponds to side PQ because they are congruent. Therefore,

$AB \cong PQ$.

List all of the congruent sides.

$$AB \cong PQ$$
$$BC \cong QR$$
$$AD \cong PS$$
$$DC \cong SR$$

List the congruent angles.

$$\angle A \cong \angle P \qquad \angle D \cong \angle S$$
$$\angle B \cong \angle Q \qquad \angle C \cong \angle R$$

Now let's find the length of side AB.

AB is congruent to PQ, so if you find the length of PQ then it will be the same for AB. PQ is 7 centimeters. Therefore AB must also be 7 centimeters long.

Now let's look at the angles. Let's find the measure of $\angle C$. The angle corresponds to \angle R, but you do not know the measure of $\angle R$ either. However, you do know the measures of two of the angles in the first parallelogram: $70°$ and $110°$. You do not know the

measure of \angle B, but this time you do know the measure of its corresponding angle, $\angle Q$. These two angles are congruent, so you know that \angle B must measure 70°. Now you know three of the angles in the first figure, so you can subtract to find the measure of $\angle C$.

$$360 - (70 + 110 + 70) = \angle C$$
$$360 - 250 = \angle C$$
$$110^\circ = \angle C$$

Example 5.24.1

Earlier, you were given a problem about Chelsie and her checkerboard themed locker.

She has cut out the shapes, how can she tell if the squares that she cut out are congruent?

Solution

First, measure the lengths of the sides.

All of the sides are equal.

Next, measure the angles.

All of the angles should be equal.

Then, draw a conclusion.

The shapes are congruent.

The answer is that the shapes are congruent.

Example 5.24.2

Is the pair below congruent?

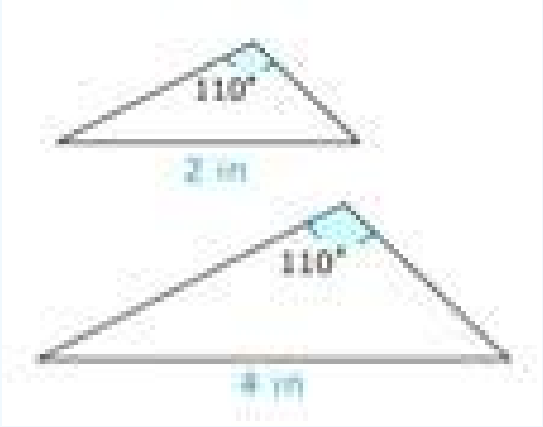


Figure 5.24.5

Solution

First, look at the shape of each figure.

The shapes look similar but one shape is smaller than the other.

Next, determine if there are corresponding parts.

There appears to be corresponding angles, but the sides that are in the same position are not congruent.

Then, determine if the shapes are corresponding.

No.

The answer is that the pair of figures are not congruent.

Example 5.24.3

Is the pair of figures congruent?

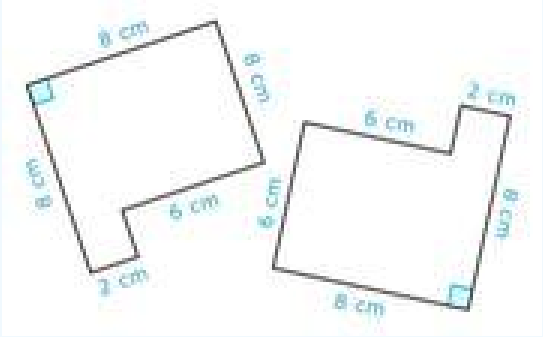


Figure 5.24.6

Solution

First, look at the shape of each figure.

The shapes look similar.

Next, determine if there are corresponding parts.

There appears to be corresponding angles, but not all of the sides that are in the same position are congruent.

Then, determine if the shapes are corresponding.

No.

The answer is that the pair of figures are not congruent.

Example 5.24.4

What is the measure of $\angle M$?

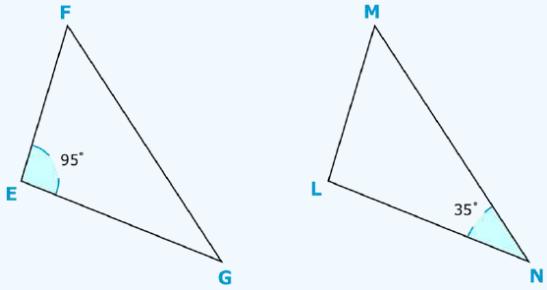


Figure \(\
PageIndex
{7}\)

Solution

First, identify the angle that corresponds with angle L.

$Angle L = Angle E = 95°$

Next, understand that the sum of three interior angles is 180o and write an equation.

$95 + 35 + x = 180$

Then, solve for the missing angle measure.

$130 + x = 180$
$x = 50$

The answer is that the measure of angle M is $50°$.

Example 5.24.5

Triangle DEF is congruent with triangle KLM. If $angle D = 42°$, what is the measure of angle K?

Solution

First, determine if angle K is congruent to angle D.

Yes, it is.

Next, remember the relationship between congruent parts.

Congruent parts are equal.

Then, write the measure of angle K.

$42°$

The answer is that angle $K = 42°$.

Review

Name the corresponding parts to those given below.

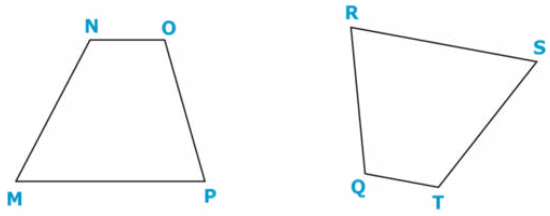


Figure 5.24.8

1. $\angle R$
2. MN
3. $\angle O$

Use the relationships between congruent figures to find the measure of g. Show your work.

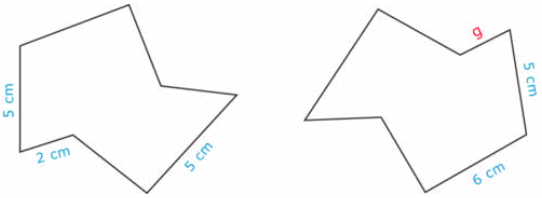
4. 

Figure 5.24.9

Use the relationships between congruent figures to find the measure of $\angle T$. Show your work.

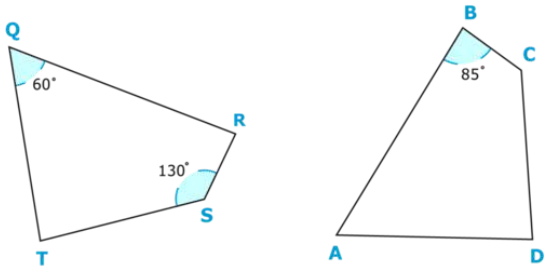
5. 

Figure 5.24.10

Answer each of the following questions.

6. Triangles ABC and DEF are congruent. If the measure of angle A is $58°$, what is the measure of angle D if it corresponds to angle A?
7. True or false. Congruent figures are exactly the same in every way.

Identify the given triangles as visually congruent or not.

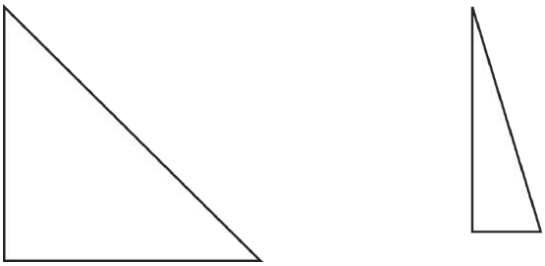
8. 

Figure 5.24.11

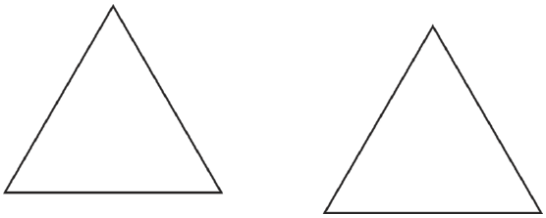
9. 

Figure 5.24.12

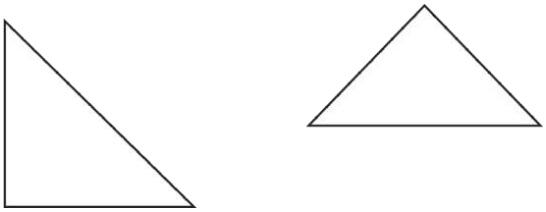
10. 

Figure 5.24.13

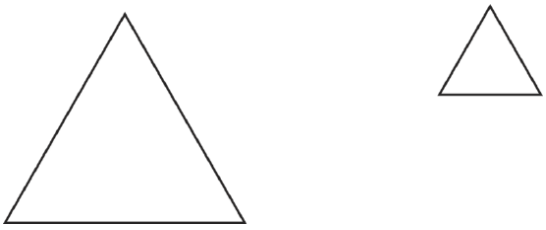
11. 

Figure 5.24.14

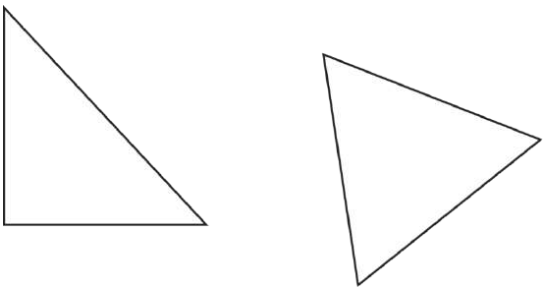
12. 

Figure 5.24.15

Answer each of the following questions.

13. Triangles ABC and DEF are congruent. Does this mean that their angle measures are the same? Why?
14. Define Congruent.
15. True or false. If two figures are congruent, then they have the same length sides but not the same angle measures.

16. Triangles ABC and DEF are congruent. If the measure of angle A is 58°, what is the measure of angle D if it corresponds to angle A?

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.12.

Resources

Vocabulary

Term	Definition
Angle	A geometric figure formed by two rays that connect at a single point or vertex.
Congruent	Congruent figures are identical in size, shape and measure.
Trapezoid	A trapezoid is a quadrilateral with exactly one pair of parallel opposite sides.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Congruent and Similar Triangles

Practice: Congruent Polygons

Real World: Copycats

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5.25: Corresponding Parts of Congruent Figures

Identify matching side lengths and angle measures in congruent figures.

Figure 5.25.1

Melvin lives in a house that has a lot of mirrors. He enjoys looking in the mirrors and playing with the mirrors. One day, his mother tells him that the parallelogram shaped mirror by the front door is **congruent** to the parallelogram shaped mirror next to the pictures in the hallway. Melvin draws the to shapes as best as he can and labels the parts that he knows for each shape.

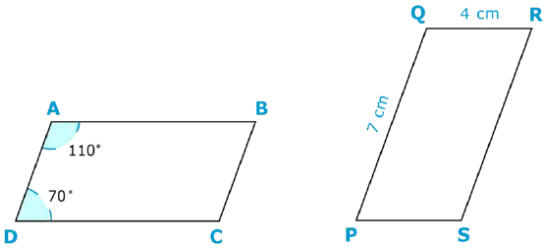

Figure 5.25.2

What is the length of $\overline{AB}$?

In this concept, you will learn about **corresponding** parts of congruent figures.

Corresponding Parts of Congruent Figures

The word **corresponding** refers to parts that match between two congruent triangles. You can identify corresponding angles and corresponding sides.

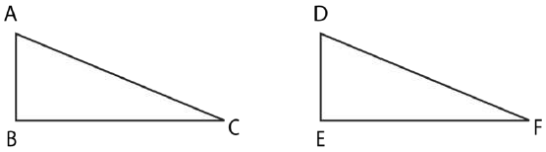

Figure 5.25.3

First, name the corresponding angles. Corresponding angles are matching angles between the two triangles. Corresponding angles will have the same measure in congruent triangles.

$\angle A \cong \angle D \angle B \cong \angle E \angle C \cong \angle F$

Here the angles are connected with the symbol for congruent. When you see the equals sign with a squiggly line on top, you know that the items on each side of the equation are congruent.

Next, name the corresponding sides. Corresponding sides are matching sides between two triangles. They will have the same length in congruent triangles.

$\overline{AB} \cong \overline{DE}$
$\overline{AC} \cong \overline{DF}$
$\overline{BC} \cong \overline{EF}$

Example 5.25.1

Earlier, you were given a problem about Melvin and his house of mirrors.

He learns that two of the mirrors are congruent, so he draws the shapes as best as he can and fills in what he knows about each shape.

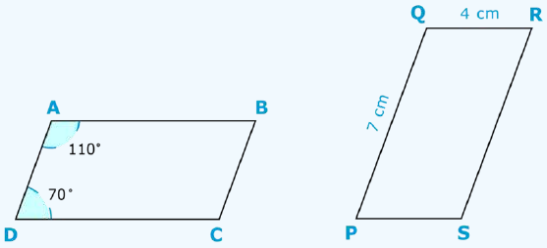

Figure 5.25.4

What is the length of $\overline{AB}$?

Solution

First, rotate the second shape so that it looks the same as the first shape.

Next, identify the side on the second triangle that corresponds to $\overline{AB}$.

$\overline{PQ}$

Then, write the length of the corresponding side.

7 cm

The answer is 7 cm. The length of $\overline{AB}$ is 7 cm.

Example 5.25.2

These two figures are congruent. Which angle is congruent to angle A?

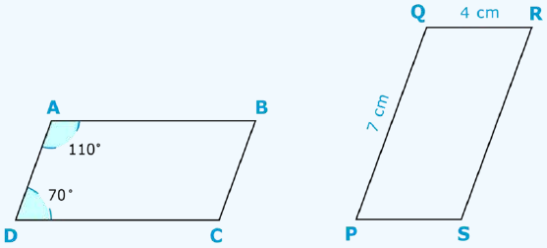

Figure 5.25.5

Solution

First, turn the second figure so that it is arranged like the first shape.

Then, identify the angle on the second figure that is in the same position as angle A in the first triangle.

P

The answer is angle P. $\angle A \cong \angle P$

Use the following diagram of two congruent triangles to answer the following questions.

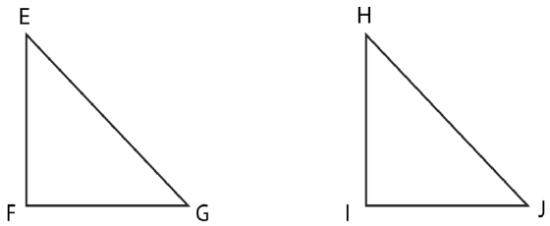

Figure 5.25.6

Example 5.25.3

Angle E is congruent to angle _____

Solution

First, make sure that the triangles are arranged so that they look the same.

Yes

Then, identify the angle on the second triangle that is in the position that angle E is in the first triangle.

H

The answer is angle H. Angle E is congruent to angle H.

Example 5.25.4

Missing open brace for subscript

Solution

First, make sure that the triangles are arranged so that they look the same.

Yes

Then, identify the line segment on the second triangle that is in the position that $\overline{FG}$ is in the first triangle.

$\overline{IJ}$

The answer is $\overline{IJ}$. $\overline{FG} \cong \overline{IJ}$.

Example 5.25.5

Angle J is congruent to angle _____

Solution

First, make sure that the triangles are arranged so that they look the same.

Yes

Then, identify the angle on the second triangle that is in the position that angle J is in the first triangle.

G

The answer is angle G. Angle J is congruent to angle G.

Review

Use the following triangles to answer the questions.

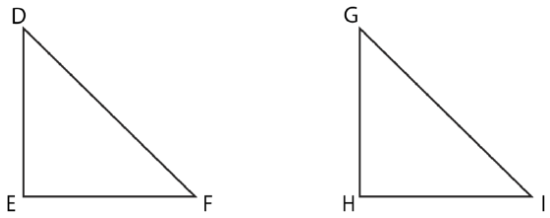

Figure **5.25.7**

1. Are these two triangles **similar** or congruent?
2. How do you know?
3. Side DE is congruent to which other side?
4. Side DF is congruent to which other side?
5. Side EF is congruent to which other side?
6. If the side length of DE is 10, what is the side length of GH?
7. If the side length of HI is 8, which other side is also 8?

Use the following figures to answer the questions.

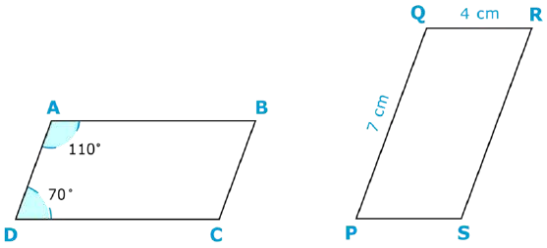

Figure **5.25.8**

8. These two figures are congruent. Explain two ways that you can determine whether or not figures are congruent.
9. Angle Q is congruent to which other angle?
10. Angle R is congruent to which other angle?
11. Angle S is congruent to which other angle?
12. If PQ is 7 cm, which other sides are 7 cm?
13. If QR is 4 cm, which other sides are 4 cm?
14. If Angle D is 70 degrees, name another 70 degree angle.
15. If Angle A is 110 degrees, name another 110 degree angle.

Resources

Vocabulary

Term	Definition
Congruent	Congruent figures are identical in size, shape and measure.
Corresponding	The corresponding sides between two triangles are sides in the same relative position.
Similar	Two figures are similar if they have the same shape, but not necessarily the same size.

Additional Resources

Video: Congruent and Similar Triangles

Practice: Corresponding Parts of Congruent Figures

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5.26: Determine Missing Angle Measures

Find missing angle measures of quadrilaterals given a sum of 360 degrees.

Understanding the Angle Measures of Quadrilaterals

Figure **5.26.1**

Stephanie, a Math 12 student, is helping to raise money to participate in this year's Math Olympiad in Dallas, Texas. Since Stephanie is very artistic in making jewelry, she has decided to make and sell necklaces to raise money. Several customers have asked her what shape she has used for the blue bead of a particularly geometric looking necklace. She knows what shape it is but is not sure of how to explain this shape to her customers.

What can Stephanie tell her customers about this shape?

In this concept, you will learn to understand the angle measures of quadrilaterals.

Quadrilaterals

A **quadrilateral** is a two-dimensional closed shape that has four straight sides.

Three special quadrilaterals are a **parallelogram** which is a quadrilateral with opposite sides parallel and **congruent**, a **trapezoid** which is a quadrilateral that has one pair of opposite sides parallel and a **kite** which is a quadrilateral that has two pairs of congruent adjacent sides.

Three special parallelograms are a **rectangle** which has every angle a right angle and opposite sides both congruent and parallel, a **square** which has every angle a right angle, opposite sides parallel and all sides congruent and a **rhombus** which has every side equal in length, opposite sides parallel, but not every angle is a right angle.

Remember a quadrilateral is a two-dimensional closed shape that has four straight sides and four angles.

The sum of the interior angles of a quadrilateral is **360°** regardless of the type of quadrilateral. The following diagram will show various quadrilaterals and their interior angles.

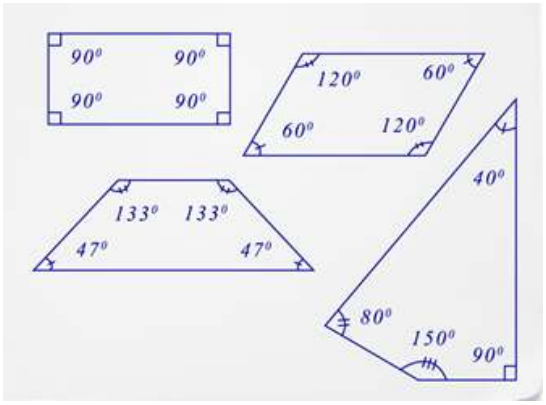

Figure **5.26.2**

If you add the measure of the four interior angles of each quadrilateral shown above, the sum will equal $360°$. Each of these quadrilaterals can be divided into two triangles. Remember the sum of the interior angles of a triangle equals $180°$. If the closed figure contains two triangles then the sum of the interior angles equals $2(180°) = 360°$.

Let's apply these facts to an example.

For the following quadrilateral, determine the measure of \angle M\).

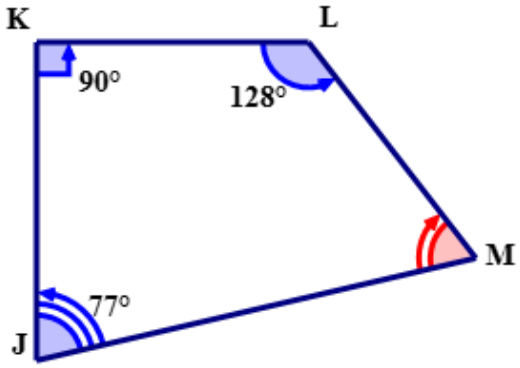

Figure 5.26.3

First, write an equation to represent the sum of the interior angles of quadrilateral $JKLM$.

$$\angle J + \angle K + \angle L + \angle M = 360°$$

Next, substitute into the equation, the given measures of angles J\),K\) and L\).

$$\angle J + \angle K + \angle L + \angle M = 360°$$
$$77° + 90° + 128° + \angle M = 360°$$

Next, simplify the left side of the equation.

$$77° + 90° + 128° + \angle M = 360°$$
$$295° + \angle M = 360°$$

Then, subtract $295°$ from both sides of the equation to solve for the measure of $\angle M$.

$$295° + \angle M = 360°$$
$$295° - 295° + \angle M = 360° - 295°$$
$$\angle M = 65°$$

The answer is 65.

The measure of $\angle M$ is $65°$.

This method can be used to calculate the measure of the missing angle of any quadrilateral when the measures of three of the angles are known.

Example 5.26.1

Earlier, you were given a problem about Stephanie and her necklaces. She needs to figure out what to tell her customers about the shape of the blue bead. What can she tell them?

Solution

First, draw the shape and include all important markings.

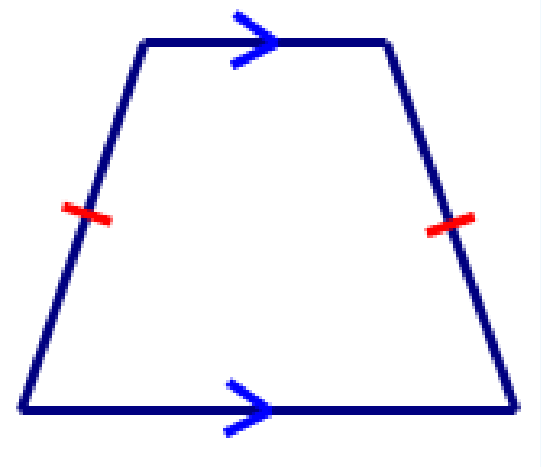

Figure 5.26.4

Next, explain the markings on the sides of the shape.

The arrows on the top and bottom sides of the figure indicate that this pair of opposite sides are parallel. The tic mark on the other two sides indicate that these sides are congruent (equal in length).

Then, name the shape.

The shape of the blue bead is an isosceles trapezoid.

Example 5.26.2

A quadrilateral $ABCD$ having one pair of opposite sides parallel has angle measures such that $\angle A = 44°$. Find the measure of $\angle C$ in the following quadrilateral.

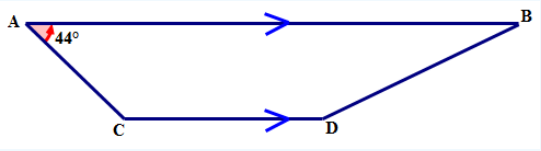

Figure 5.26.5

Solution

First, name the type of quadrilateral and write down what you know.

A quadrilateral with one pair of opposite sides parallel is called a trapezoid. $\angle A$ and $\angle C$ are on the same side of leg $\angle AC$ and are supplementary.

Next, write an equation to model what you know about the two angles.

$$\angle A + \angle C = 180°$$

Next, fill into the equation the measure of $\angle A$.

$$\angle A + \angle C = 180°$$
$$44° + \angle C = 180°$$

Then, subtract $44°$ from both sides of the equation to solve for \angle C\).

$$44° + \angle C = 180°$$
$$44° - 44° + \angle C = 180° - 44°$$
$$\angle C = 136°$$

The answer is 136.

The measure of $\angle C$ is $136°$.

Example 5.26.3

For the following quadrilateral, find the measure of the unknown angle.

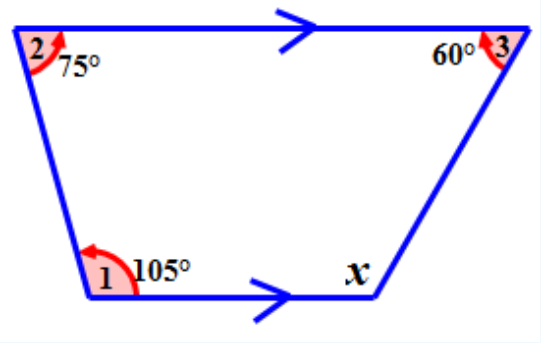

Figure 5.26.7

Solution

First, write an equation to model the sum of the interior angles of a quadrilateral.

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$$

Next, fill into the equation the measures of any known angles.

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$$
$$105^\circ + 75^\circ + 60^\circ + \angle 4 = 360^\circ$$

Next, let $\angle 4 = x$

$$105^\circ + 75^\circ + 60^\circ + \angle 4 = 360^\circ$$
$$105^\circ + 75^\circ + 60^\circ + x = 360^\circ$$

Next, simplify the left side of the equation.

$$105^\circ + 75^\circ + 60^\circ + x = 360^\circ$$
$$240^\circ + x = 360^\circ$$

Next, subtract 240° from both sides of the equation to solve for 'x'.

$$240^\circ + x = 360^\circ$$
$$240^\circ - 240^\circ + x = 360^\circ$$
$$x = 360^\circ - 240^\circ = 120^\circ$$

The answer is 120.

The measure of $\angle 4$ is 120°.

Example 5.26.4

What is the measure of the fourth angle of a quadrilateral which has three angles that measure 110°, 80° and 95°?

Solution

First, write an equation to model the sum of the interior angles of the quadrilateral.

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$$

Next, fill into the equation the measures of the three known angles.

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$$
$$110^\circ + 95^\circ + 80^\circ + \angle 4 = 360^\circ$$

Next, simplify the left side of the equation.

$$110^\circ + 95^\circ + 80^\circ + \angle 4 = 360^\circ$$
$$285^\circ + \angle 4 = 360^\circ$$

Then, subtract 285° from both sides of the equation to solve for the measure of $\angle 4$.

$$285^\circ + \angle 4 = 360^\circ$$
$$285^\circ - 285^\circ + \angle 4 = 360^\circ - 285^\circ$$
$$\angle 4 = 75^\circ$$

Example 5.26.5

What is the name of the quadrilateral that has all sides equal in length, opposite sides parallel and every angle a right angle?

Solution

First, draw the quadrilateral to represent the information given in the problem.

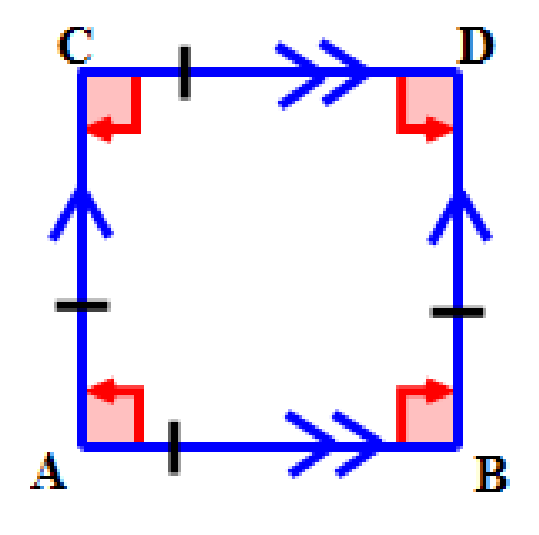

Figure 5.26.8

Then, name the quadrilateral.

This is a special parallelogram called a square.

Review

Use what you have learned about quadrilaterals to figure out the missing angle measure of each quadrilateral based on three given angles.

1. $120^\circ, 120^\circ, 60^\circ, ?$
2. $50^\circ, 70^\circ, 130^\circ, ?$
3. $52^\circ, 128^\circ, 52^\circ, ?$
4. $47^\circ, 55^\circ, 120^\circ, ?$
5. $80^\circ, 80^\circ, 100^\circ, ?$
6. $105^\circ, 105^\circ, 85^\circ, ?$
7. $97^\circ, 97^\circ, 35^\circ, ?$
8. $120^\circ, 120^\circ, 40^\circ, ?$
9. $88^\circ, 90^\circ, 60^\circ, ?$
10. $25^\circ, 85^\circ, 85^\circ, ?$
11. $90^\circ, 90^\circ, 90^\circ, ?$
12. $140^\circ, 150^\circ, 45^\circ, ?$
13. $80^\circ, 80^\circ, 120^\circ, ?$
14. $75^\circ, 95^\circ, 110^\circ, ?$
15. $80^\circ, 50^\circ, 95^\circ, ?$

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.8.

Vocabulary

Term	Definition
Congruent	Congruent figures are identical in size, shape and measure.
Parallelogram	A parallelogram is a quadrilateral with two pairs of parallel sides.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.
Rectangle	A rectangle is a quadrilateral with four right angles.
Rhombus	A rhombus is a quadrilateral with four congruent sides.
Square	A square is a polygon with four congruent sides and four right angles.
Trapezoid	A trapezoid is a quadrilateral with exactly one pair of parallel opposite sides.

Additional Resources

Video: Quadrilateral Properties

Practice: Determine Missing Angle Measures

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5.27: Interior Angles in Convex Polygons

Use the formula $(x-2)180$ to find the sum of the interior angles of any polygon.

The **interior angle** of a polygon is one of the angles on the inside, as shown in the picture below. A polygon has the same number of interior angles as it does sides.

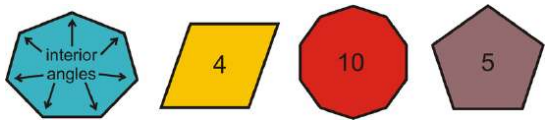

Figure 5.27.1

The sum of the interior angles in a polygon depends on the number of sides it has. The **Polygon Sum Formula** states that for any n−gon, the interior angles add up to $(n-2) \times 180°$.

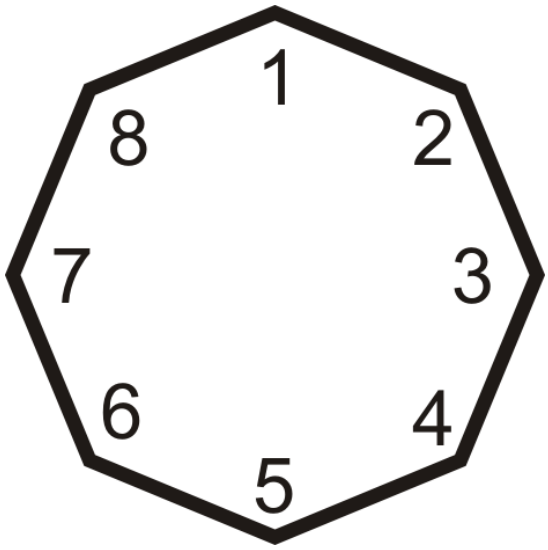

Figure 5.27.2

$\rightarrow n = 8$

$(8-2) \times 180°$

$6 \times 180°$

$1080°$

Once you know the sum of the interior angles in a polygon it is easy to find the measure of ONE interior angle if the polygon is **regular**: all sides are congruent and all angles are congruent. Just divide the sum of the angles by the number of sides.

Regular Polygon Interior Angle Formula: For any equiangular n−gon, the measure of each angle is $\frac{(n-2) \times 180°}{n}$.

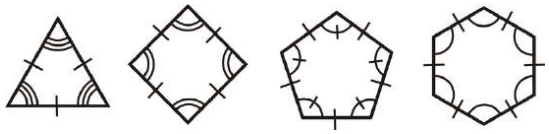

Figure 5.27.3

In the picture below, if all eight angles are congruent then each angle is $\frac{(8-2) \times 180°}{8} = \frac{6 \times 180°}{8} = \frac{1080°}{8} = 135°$.

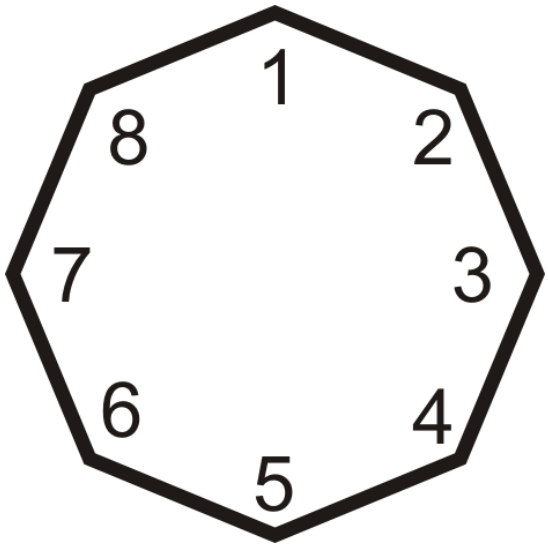

Figure 5.27.4

What if you were given an equiangular seven-sided convex polygon? How could you determine the measure of its interior angles?

Example 5.27.1

The interior angles of a $2x°$, $2x°$, and $2x°$. What is x?

Solution

From the Polygon Sum hat a pentagon has interior angles that sum to $(5-2) \times 180° = 540°$.

Write an equation and solve for x.

$$\begin{aligned} x° + x° + 2x° + 2x° + 2x° &= 540° \\ 8x &= 540 \\ x &= 67.5 \end{aligned}$$

Example 5.27.2

What is the sum of the interior angles in a 100-gon?

Solution

Use the Polygon Sum Formula. $(100-2) \times 180° = 17,640°$.

Example 5.27.3

The interior angles of a polygon add up to $1980°$. How many sides does it have?

Solution

Use the Polygon Sum Formula and solve for n\).

$$\begin{aligned} (n-2) \times 180° &= 1980° \\ 180°n - 360° &= 1980° \\ 180°n &= 2340° \\ n &= 13 \end{aligned}$$

The polygon has 13 sides.

Example 5.27.4

How many degrees does ***each angle*** in an equiangular nonagon have?

Solution

First we need to find the sum of the interior angles; set $n = 9$.

$$(9-2) \times 180^\circ = 7 \times 180^\circ = 1260^\circ$$

"Equiangular" tells us every angle is equal. So, each angle is $\frac{1260^\circ}{9} = 140^\circ$.

Example 5.27.5

An interior angle in a regular polygon is 135°. How many sides does this polygon have?

Solution

Here, we will set the Regular Polygon Interior Angle Formula equal to 135° and solve for n.

$$\frac{(n-2) \times 180^\circ}{n} = 135^\circ$$
$$180^\circ n - 360^\circ - 360^\circ = 135^\circ n$$
$$n = -45^\circ$$
$$n = 8 \quad \text{The polygon is an octagon.}$$

Review

1. Fill in the table.

# of sides	Sum of the Interior Angles	Measure of Each Interior Angle in a Regular n−gon
3		60°
4	360°	
5	540°	108°
6		120°
7		
8		
9		
10		
11		
12		

2. What is the sum of the angles in a 15-gon?
3. What is the sum of the angles in a 23-gon?
4. The sum of the interior angles of a polygon is 4320°. How many sides does the polygon have?
5. The sum of the interior angles of a polygon is 3240°. How many sides does the polygon have?
6. What is the measure of each angle in a regular 16-gon?
7. What is the measure of each angle in an equiangular 24-gon?
8. Each interior angle in a regular polygon is 156°. How many sides does it have?
9. Each interior angle in an equiangular polygon is 90°. How many sides does it have?

For questions 10-18, find the value of the missing variable(s).

10.

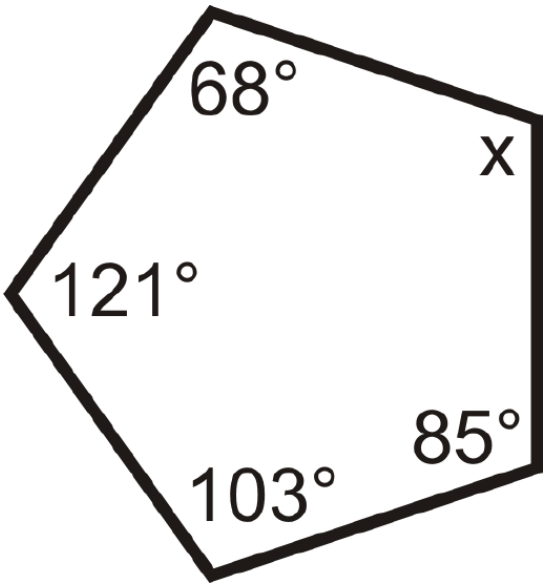

Figure **5.27.5**

11.

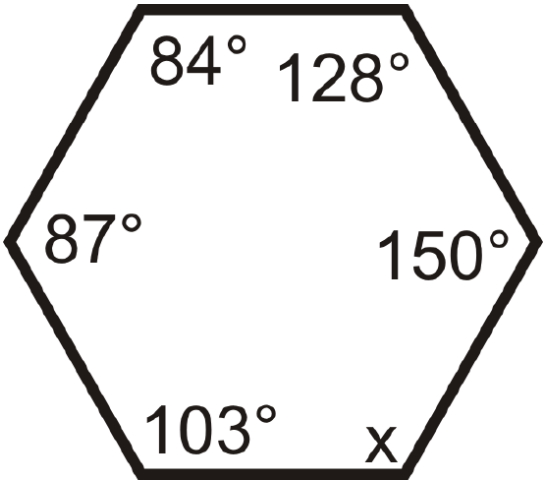

Figure **5.27.6**

12.

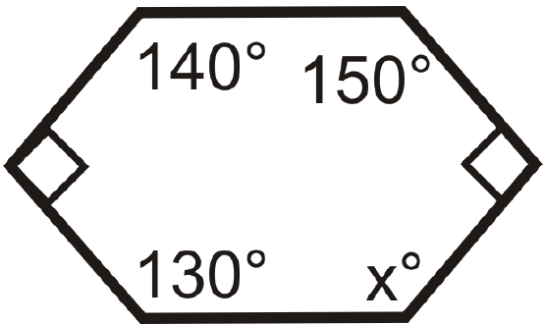

Figure **5.27.7**

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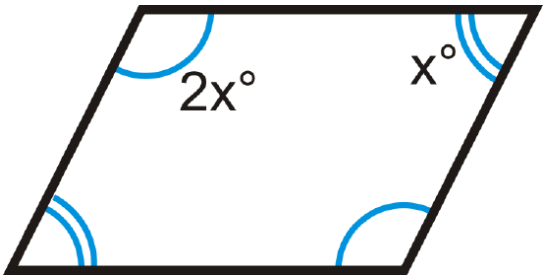

Figure **5.27.8**

14.

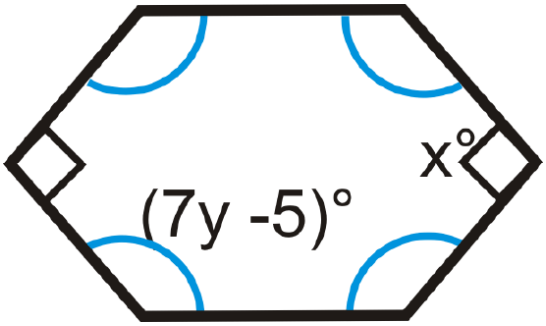

Figure **5.27.9**

15.

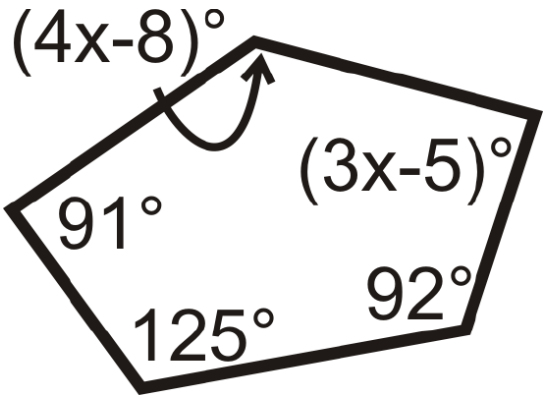

Figure **5.27.10**

16.

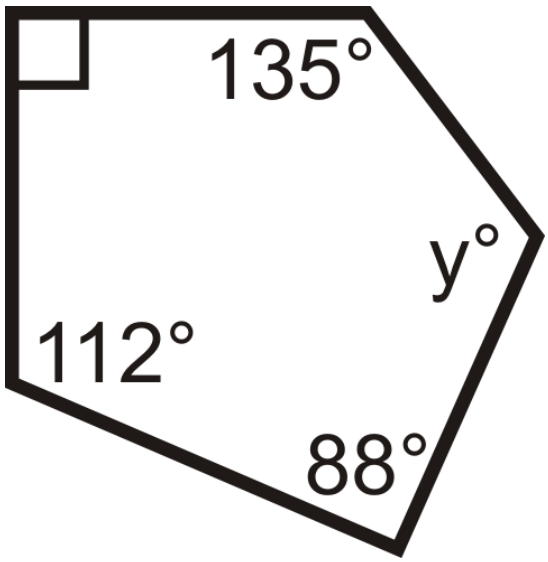

Figure **5.27.11**

17.

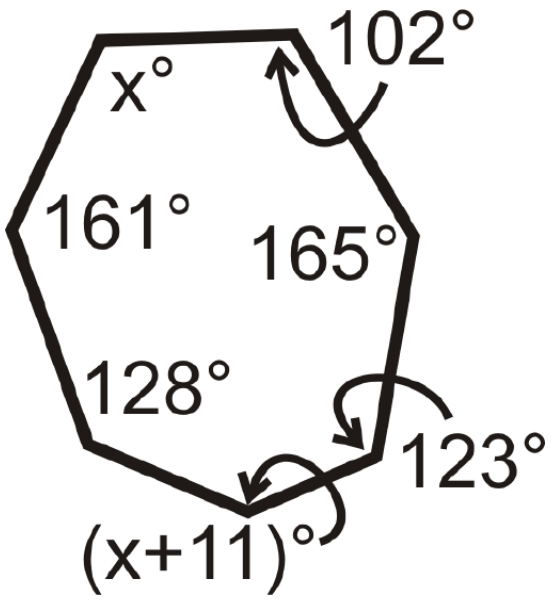

Figure **5.27.12**

18.

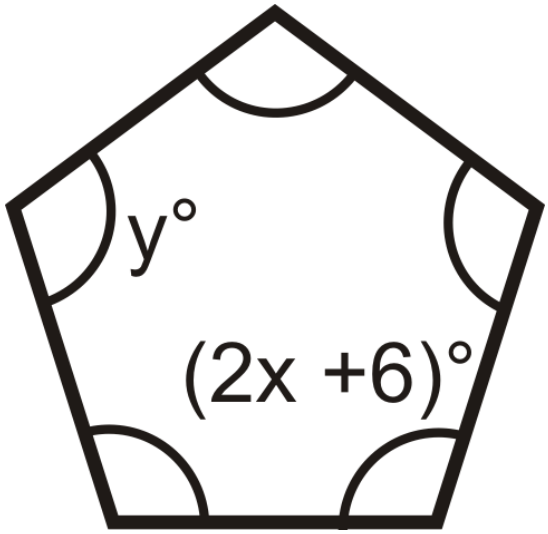

Figure **5.27.13**

19. The interior angles of a hexagon are $x°$, $(x+1)°$, $(x+2)°$, $(x+3)°$,$(x+4)°$, and $(x+5)°$. What is x?

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.1.

Resources

Vocabulary

Term	Definition
Interior angles	Interior angles are the angles inside a figure.
Polygon Sum Formula	The Polygon Sum Formula states that for any polygon with n sides, the interior angles add up to $(n-2) \times 180$ degrees.

Additional Resources

Video: Interior and Exterior Angles of a Polygon

Activities: Interior Angles in Convex Polygons Discussion Questions

Study Aids: Polygons Study Guide

Practice: Interior Angles in Convex Polygons

Real World: Interior Angles In Convex Polygons

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5.28: Exterior Angles in Convex Polygons

Measure of angles on the outside of a polygon formed by extending a side.

Exterior Angle Sum Theorem

An **exterior angle** is an angle that is formed by extending a side of the polygon.

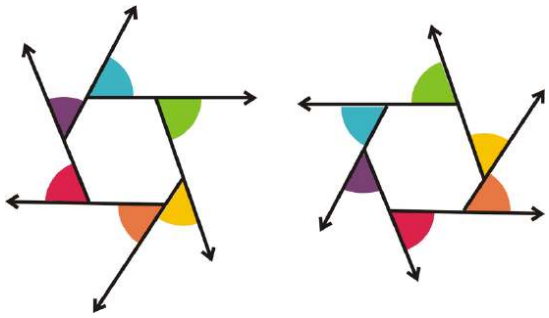

Figure 5.28.1

As you can see, there are two sets of exterior angles for any vertex on a polygon, one going around clockwise (1st hexagon), and the other going around counter-clockwise (2nd hexagon). The angles with the same colors are vertical and congruent.

The **Exterior Angle Sum Theorem** states that the sum of the exterior angles of ANY convex polygon is $360°$. If the polygon is regular with n sides, this means that each exterior angle is $\frac{360°}{n}$.

What if you were given a seven-sided **regular polygon**? How could you determine the measure of each of its exterior angles?

Example 5.28.1

What is the measure of of a regular 12-gon?

Solution

Divide $360°$ by the giver

$30°$

Example 5.28.2

What is the measure of each exterior angle of a regular 100-gon?

Solution

Divide $360°$ by the given number of sides.

$3.6°$

Example 5.28.3

What is y?

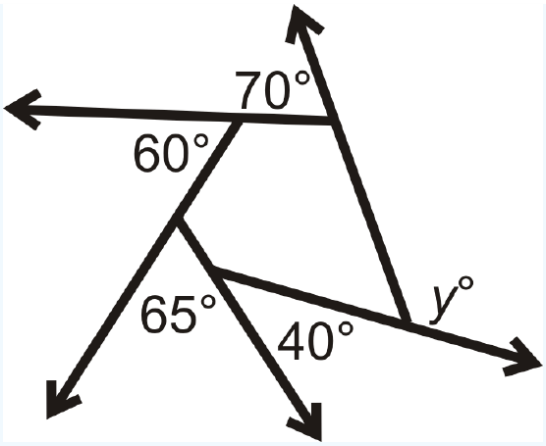

Figure 5.28.2

Solution

y is an exterior angle and all the given angles add up to $360°$. Set up an equation.

$$70° + 60° + 65° + 40° + y = 360°$$
$$y = 125°$$

Example 5.28.4

What is the measure of each exterior angle of a regular heptagon?

Solution

Because the polygon is regular, the interior angles are equal. It also means the exterior angles are equal. $\frac{360°}{7} \approx 51.43°$

Example 5.28.5

What is the sum of the exterior angles in a regular 15-gon?

Solution

The sum of the exterior angles in any convex polygon, including a regular 15-gon, is $360°$.

Review

1. What is the measure of each exterior angle of a regular decagon?
2. What is the measure of each exterior angle of a regular 30-gon?
3. What is the sum of the exterior angles of a regular 27-gon?

Find the measure of the missing variables:

4.

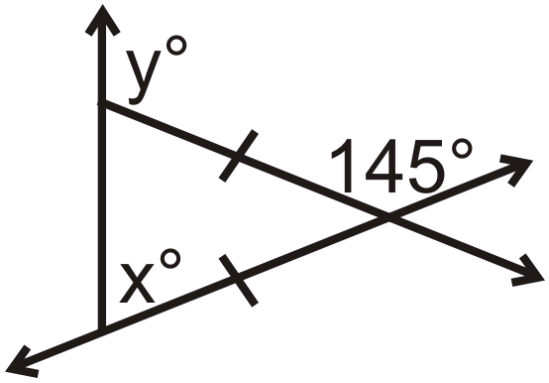

Figure 5.28.3

5.

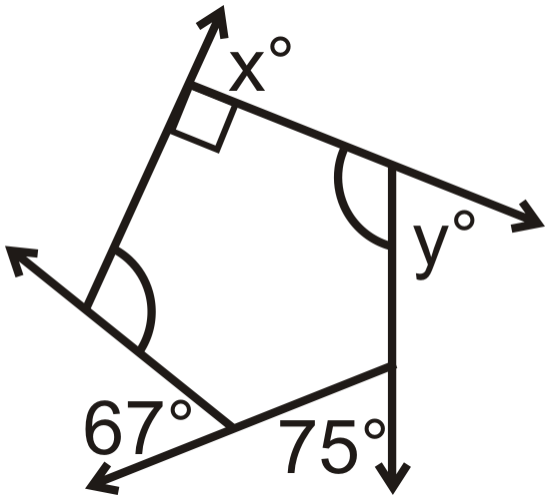

Figure 5.28.4

6. The exterior angles of a quadrilateral are $x°$, $2x°$, $3x°$, and $4x°$. What is x?

Find the measure of each exterior angle for each regular polygon below:

7. octagon
8. nonagon
9. triangle
10. pentagon

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.2.

Resources

Vocabulary

Term	Definition
exterior angle	An angle that is formed by extending a side of the polygon.
regular polygon	A polygon in which all of its sides and all of its angles are congruent.
Exterior Angle Sum Theorem	Exterior Angle Sum Theorem states that the exterior angles of any polygon will always add up to 360 degrees.

Additional Resources

Video: Interior and Exterior Angles of a Polygon

Activities: Exterior Angles in Convex Polygons Discussion Questions

Study Aids: Polygons Study Guide

Real World: Exterior Angles Theorem

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CHAPTER OVERVIEW

6: Circles

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6.2: Identify Circle Components
6.3: Use Diameter, Radius and Pi
6.4: Circumference
6.5: Diameter or Radius of a Circle Given Circumference
6.6: Circle Area
6.7: Areas of Combined Figures Involving Semicircles
6.8: Radius or Diameter of a Circle Given Area
6.9: Arcs in Circles
6.10: Area of Sectors and Segments
6.11: Arc Length
6.12: Chords and Central Angle Arcs
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6.14: Inscribed Angles in Circles
6.15: Inscribed Quadrilaterals in Circles
6.16: Angles On and Inside a Circle
6.17: Angles Outside a Circle
6.18: Tangent Lines
6.19: Intersecting Secants Theorem
6.20: Tangent Secant Theorem
6.21: Circles in the Coordinate Plane

Thumbnail: A circular sector is shaded in green. Its curved boundary of length L is a circular arc. (CC BY-SA 3.0; Cburnett via Wikipedia)

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6.1: Semicircles and Quarter Circles

Visually identify semicircles and quarter circles.

Video: Identifying Semi-circles and Quarter-circles

Practice: Semicircles and Quarter Circles

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6.2: Identify Circle Components

Components, circumference, and area of circles.

Parts of Circles

Circles

A **circle** is the set of all points in the plane that are the same distance away from a specific point, called the **center**. The center of the circle below is point A. We call this circle "circle A," and it is labeled $\odot A$.

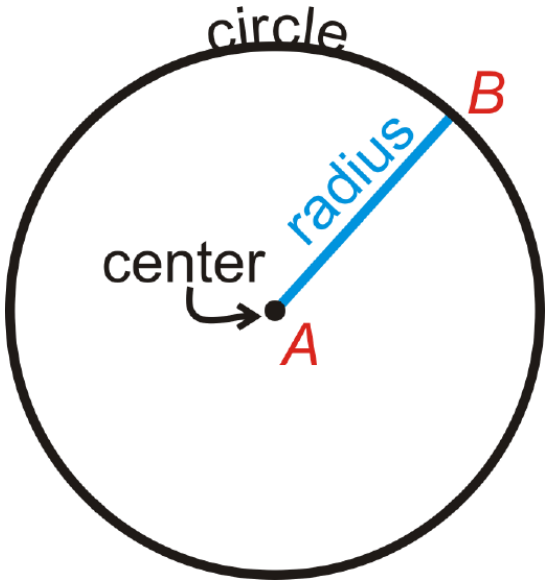

Figure **6.2.1**

Important Circle Parts

Radius: The distance from the center of the circle to its outer rim.

Chord: A line segment whose endpoints are on a circle.

Diameter: A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.

Secant: A line that intersects a circle in two points.

Tangent: A line that intersects a circle in exactly one point.

Point of Tangency: The point where a tangent line touches the circle.

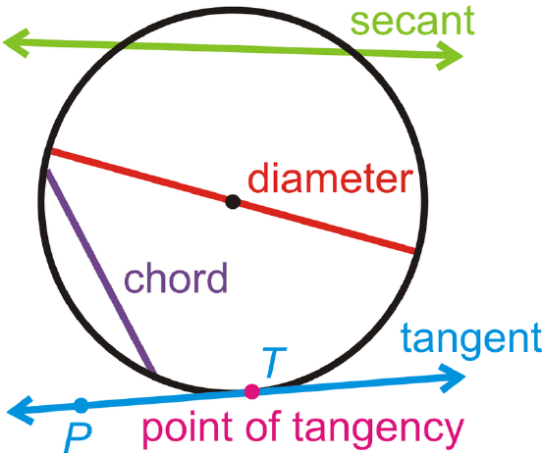

Figure **6.2.2**

The tangent ray $\overrightarrow{TP}$ and tangent segment $\overline{TP}$ are also called tangents.

Tangent Circles: Two or more circles that intersect at one point.

Concentric Circles: Two or more circles that have the same center, but different radii.

Congruent Circles: Two or more circles with the same radius, but different center

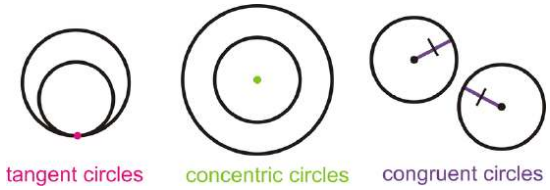

Figure **6.2.3**

What if you drew a line through a circle from one side to the other that does not pass through the center? What if you drew a line outside a circle that touched the circle at one point? What would you call these lines you drew?

Example 6.2.1

Find the parts of $\odot A$ th [redacted] ription.

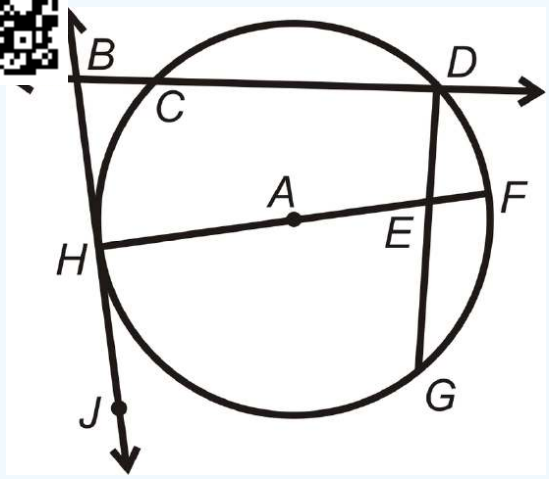

Figure **6.2.4**

a. A radius
b. A chord
c. A tangent line
d. A point of tangency
e. A diameter
f. A secant

Solution

a. \overline{HA}\) or \overline{AF}\)
b. $\overline{CD}$, $\overline{HF}$, or \overline{DG}\)
c. $\overleftrightarrow{BJ}$
d. $Point H$
e. $\overline{HF}$
f. $\overleftrightarrow{BD}$

Example 6.2.2

Draw an example of how two circles can intersect with no, one and two points of intersection. You will make three separate drawings.

Solution

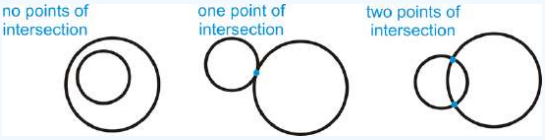

Figure 6.2.5

Example 6.2.3

Determine if any of the following circles are congruent.

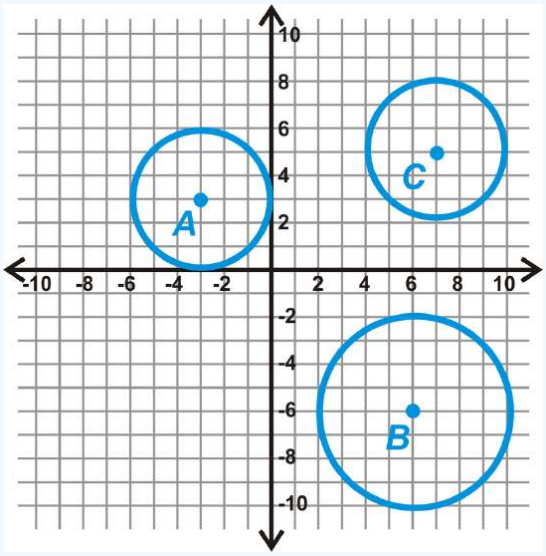

Figure 6.2.6

Solution

From each center, count the units to the outer rim of the circle. It is easiest to count vertically or horizontally. Doing this, we have:

$$\text{Radius of } \odot A = 3 \text{ unit}$$
$$\text{Radius of } \odot B = 4 \text{ unit}$$
$$\text{Radius of } \odot C = 3 \text{ unit}$$

From these measurements, we see that $\odot A \cong \odot C$.

Notice the circles are congruent. The lengths of the radii are equal.

Example 6.2.4

Is it possible to have a line that intersects a circle three times? If so, draw one. If not, explain.

Solution

It is not possible. By definition, all lines are *straight*. The maximum number of times a line can intersect a circle is twice.

Example 6.2.5

Are all circles similar?

Solution

Yes. All circles are the same shape, but not necessarily the same size, so they are similar.

Review

Determine which term best describes each of the following parts of $\odot P$.

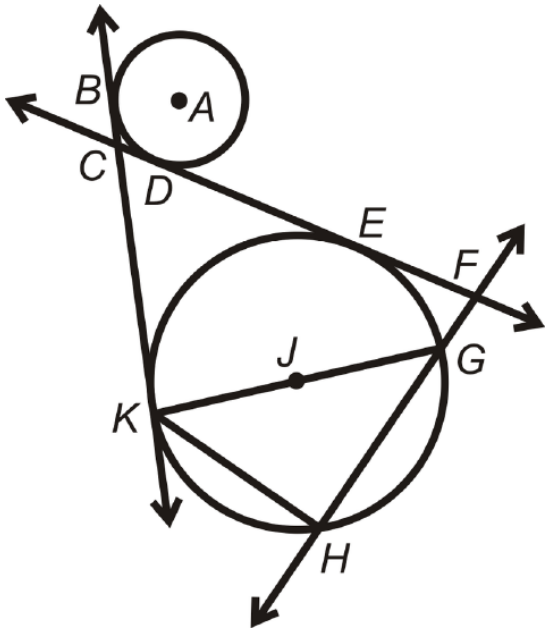

Figure 6.2.7

1. $\overline{KG}$
2. $\overleftrightarrow{FH}$
3. $\overline{FH}$
4. E
5. $\overleftrightarrow{BK}$
6. $\overleftrightarrow{CF}$
7. A
8. $\overline{JG}$
9. What is the longest chord in any circle?

Use the graph below to answer the following questions.

10. Find the radius of each circle.
11. Are any circles congruent? How do you know?
12. Find all the common tangents for $\odot B$ and $\odot C$.
13. $\odot C$ and $\odot E$ are externally tangent. What is CE?
14. Find the equation of $\overline{CE}$.

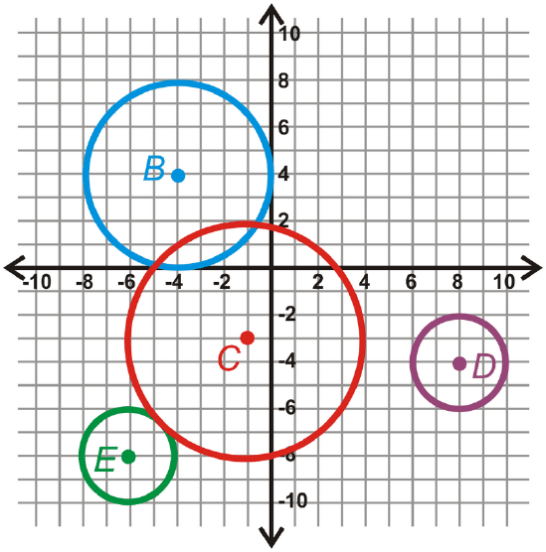

Figure 6.2.8

Resources

Vocabulary

Term	Definition
chord	A line segment whose endpoints are on a circle.
Circle	A circle is the set of all points at a specific distance from a given point in two dimensions.
concentric circles	Two or more circles that have the same center, but different radii.
congruent circles	Two or more circles with the same radius, but different centers.
Diameter	Diameter is the measure of the distance across the center of a circle. The diameter is equal to twice the measure of the radius.
point of tangency	The point where the tangent line touches the circle.
Radius	The radius of a circle is the distance from the center of the circle to the edge of the circle.
Circumference	The circumference of a circle is the measure of the distance around the outside edge of a circle.
Secant	The secant of an angle in a right triangle is the value found by dividing length of the hypotenuse by the length of the side adjacent the given angle. The secant ratio is the reciprocal of the cosine ratio.
Tangent	The tangent of an angle in a right triangle is a value found by dividing the length of the side opposite the given angle by the length of the side adjacent to the given angle.
Tangent Circles	Tangent Circles are two or more circles that intersect at one point.

Additional Resources

Video: Introduction to Circles

Activities: Parts of Circles Discussion Questions

Study Aids: Properties of a Circle Study Guide

Practice: Identify Circle Components

Real World: A Roundabout Way Home

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6.3: Use Diameter, Radius and Pi

Identify the constant ratio $\pi = \frac{C}{d}$.

Pi

Figure **6.3.1**

Carla has to go to the store and get a tablecloth for the round table in the dining room. In order to get the right size tablecloth, she needs to know the distance from one side of the table to the other side. The 3-foot long yardstick she has can only measure up to the center of the table, or halfway across. How can Carla figure out the distance across the circular table with only the yardstick and then use that information to find out the **circumference** of the table?

In this concept, you will learn the relationship between **diameter** and **radius** and identify **pi** as the ratio of diameter to circumference in a circle.

Circle Relationships

To work with circles, you need to know the different parts of a circle and how these key parts are measured.

You can measure the distance across the center of the circle. This distance is called the **diameter** of the circle.

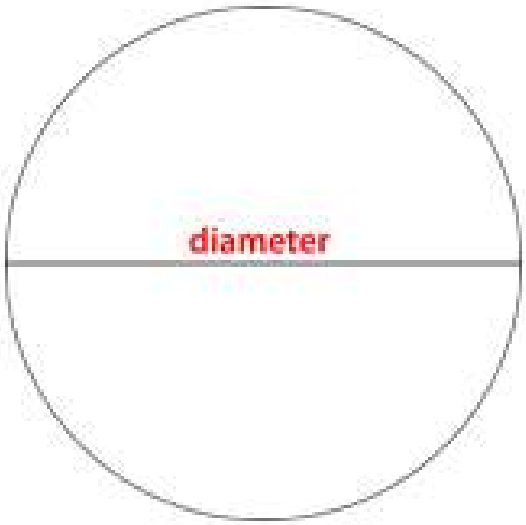

Figure **6.3.2**

You can measure the distance from the center of the circle to the outer edge. This distance is called the **radius**.

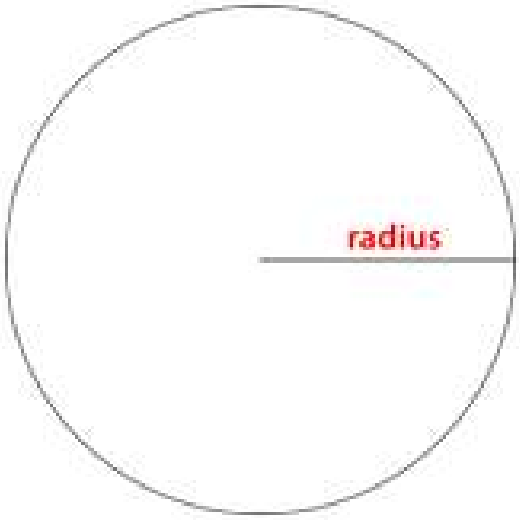

Figure **6.3.3**

Notice that the radius is one-half of the measure of the diameter.

You can measure the perimeter of the circle too. This distance is called the **circumference** of the circle.

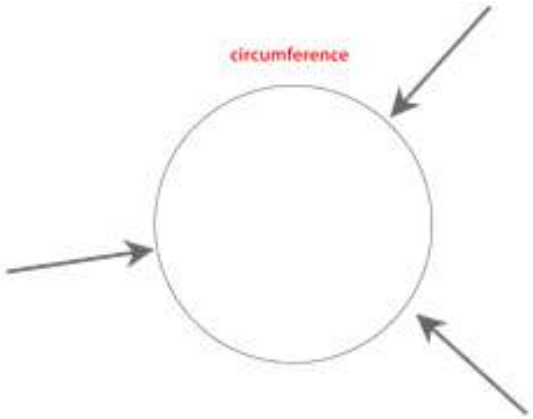

Figure **6.3.4**

Archimedes, a mathematician in ancient Greece, is credited with figuring out the relationship between the diameter and circumference of a circle. Archimedes discovered that if you take the distance across the circle (diameter) and stretch it around the circumference, it would take a little over 3 lengths of the diameter to go around the circumference of the circle.

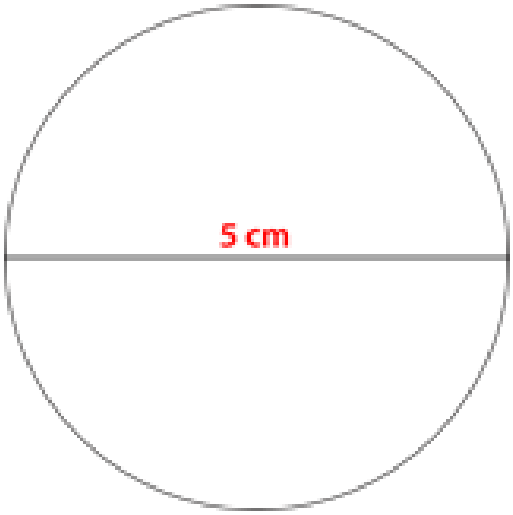

Figure 6.3.5

Let's say that the diameter of this circle is 5 cm, in that case the circumference of the circle is three and a little more times the 5 cm, or a little less than 16 cm.

The ratio of the diameter to the circumference is called pi, π. The numerical value of pi is a non-terminating decimal, which means it cannot be written precisely as a numeral since the decimal places never end or form a pattern. So, the number 3.14 is used to represent pi and is used for estimating the circumference of a circle.

Example 6.3.1

Earlier, you were given a problem about Carla and her trouble measuring the diameter of the round table.

Carla was still debating how to find the diameter of the round table when her mother walked in the dining room. When Carla explained the problem to her mom, her mom told her if the 3-foot long yardstick measured the distance from the edge of the table to the center point, then that was the radius of the table. Knowing the radius of the table was 3 feet, Carla knew she could use the relationship formula for radius and diameter to figure out the distance from one side of the table to the other.

Solution

First, Carla substituted the given measurement in the formula for diameter and radius.

$$\begin{aligned} 2(\text{ radius }) &= \text{ diameter} \\ 2(3 \text{ feet }) &= \text{ diameter} \end{aligned}$$

Next, she multiplied 2 times 3.

$$6 \text{ feet } = \text{ diameter}$$

The table's diameter is 6 feet.

Carla could use this information to figure out the circumferences of the table by multiplying the diameter and pi (3.14).

Example 6.3.2

Use the given value for the radius to find the diameter.

If the radius of a circle is 3, what is the diameter?

Solution

First, note the definition of radius and diameter. The radius is one-half the diameter or the diameter is twice the length of the radius.

$$2(\text{ radius }) = \text{ diameter}$$

Next, multiply the radius by 2 to get 6.

$$2(3) = 6$$

The answer is 6.

Example 6.3.3

If a circle has a diameter of 14 inches, what is its radius?

Solution

First, substitute the given measurement in the formula for diameter and radius.

$$\begin{aligned} 2(\text{ radius }) &= \text{ diameter} \\ 2r &= 14 \text{ inches} \end{aligned}$$

Next, divide both sides by 2.

$$\begin{aligned} \frac{2r}{2} &= \frac{14}{2} \\ r &= 7 \end{aligned}$$

The answer is 7.

Example 6.3.4

If a circle has a radius of 11 inches, what is its diameter?

Solution

First, substitute the given measurement in the formula for diameter and radius.

$$\begin{aligned} 2(\text{ radius }) &= \text{ diameter} \\ 2(11 \text{ inches }) &= \text{ diameter} \end{aligned}$$

Next, multiply 2 and 11.

$$22 = \text{ diameter}$$

The answer is 22.

Example 6.3.5

If a circle has a diameter of 18 inches, what is its radius?

Solution

First, substitute the known values in the equation.

$$2(r) = 18$$

Next, do the math and divide both sides by 2.

$$\begin{aligned} \frac{2r}{2} &= \frac{18}{2} \\ r &= 9 \end{aligned}$$

The answer is 9.

Review

Given each radius, determine the diameter.

1. $r = 4$ in.
2. $r = 6$ in.
3. $r = 5$ in.
4. $r = 12$ in.
5. $r = 16$ ft.
6. $r = 28$ mm.

7. $r = 12.5$ ft.
8. $r = 1.25$ m.

Given each diameter, determine the radius.

9. $d = 12$ m.
10. $d = 18$ m.
11. $d = 12.5$ in.
12. $d = 18.5$ ft.
13. $d = 9.8$ in.
14. $d = 1.45$ mm.
15. $d = 1.75$ ft.
16. $d = 2.5$ ft.
17. $d = 221.25$ m.

Vocabulary

Term	Definition
\pi	π (Pi) is the ratio of the circumference of a circle to its diameter. It is an irrational number that is approximately equal to 3.14.
Archimedes	Archimedes was a Greek mathematician and philosopher. Among many other things, he identified 3.14 as pi.
Circumference	The circumference of a circle is the measure of the distance around the outside edge of a circle.
Diameter	Diameter is the measure of the distance across the center of a circle. The diameter is equal to twice the measure of the radius.
Pi	π (Pi) is the ratio of the circumference of a circle to its diameter. It is an irrational number that is approximately equal to 3.14.
Radius	The radius of a circle is the distance from the center of the circle to the edge of the circle.

Video: Circles: Radius, Diameter and Circumference

Practice: Use Diameter, Radius and Pi

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6.4: Circumference

Find the circumference of a circle using the formulas: $C = \pi d$; $C = 2\pi r$.

Circumference is the distance around a **circle**. The circumference can also be called the perimeter of a circle. However, we use the term circumference for circles because they are round.

Circumference Formula: $C = \pi d$ where the **diameter** $d = 2r$, or twice the **radius**. So $C = 2\pi r$ as well.

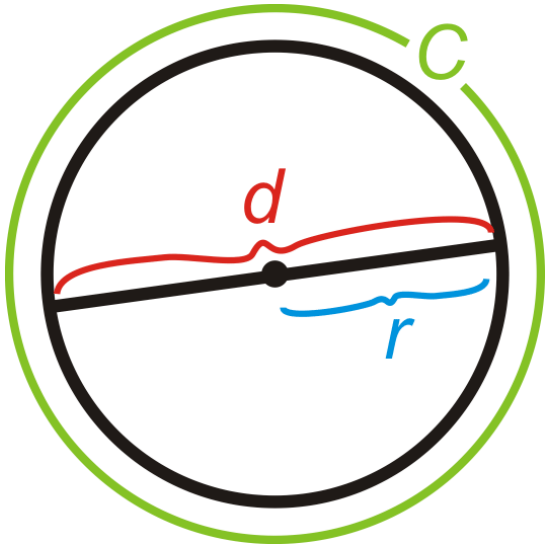

Figure 6.4.1

π, or **"pi"** is the ratio of the circumference of a circle to its diameter. It is approximately equal to 3.14159265358979323846...

You should have a \pi button on your calculator. If you don't, you can use 3.14 as an approximation for π. You can also leave your answers in terms of π for many problems.

What if you were given the radius or diameter of a circle? How could you find the distance around that circle?

Example 6.4.1

Find the circumference dius of 7 cm.

Solution

Plug the radius into the

$C = 2\pi(7) = 14\pi \approx 44$ cm

Example 6.4.2

The circumference of a circle is 64π units. Find the diameter.

Solution

Again, you can plug in what you know into the circumference formula and solve for d.

$$64\pi = \pi d$$
$$64 \text{ units} = d$$

Example 6.4.3

A circle is inscribed in a square with 10 in. sides. What is the circumference of the circle? Leave your answer in terms of π.

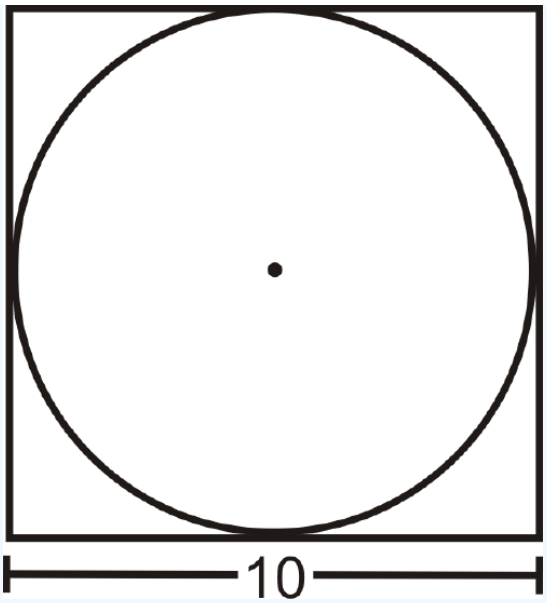

Figure 6.4.2

Solution

From the picture, we can see that the diameter of the circle is equal to the length of a side. $C = 10\pi$ in.

Example 6.4.4

Find the perimeter of the square in Example 3. Is it more or less than the circumference of the circle? Why?

Solution

The perimeter is $P = 4(10) = 40$ in. In order to compare the perimeter with the circumference we should change the circumference into a decimal.

$C = 10\pi \approx 31.42$ in. This is less than the perimeter of the square, which makes sense because the circle is inside the square.

Example 6.4.5

The tires on a compact car are 18 inches in diameter. How far does the car travel after the tires turn once? How far does the car travel after 2500 rotations of the tires?

Figure 6.4.3

Solution

One turn of the tire is the circumference. This would be $C = 18\pi \approx 56.55$ in. 2500 rotations would be $2500 \cdot 56.55$ in approx 141,375 in, 11,781 ft, or 2.23 miles.

Review

Fill in the following table. Leave all answers in terms of π.

	diameter	*radius*	*circumference*
1.	15		
2.		4	
3.	6		
4.			84π
5.		9	
6.			25π
7.			2π
8.	36		

9. Find the circumference of a circle with $d = \frac{20}{\pi}$ cm .

Square $PQSR$ is inscribed in $\odot T$. $RS = 8\sqrt{2}$.

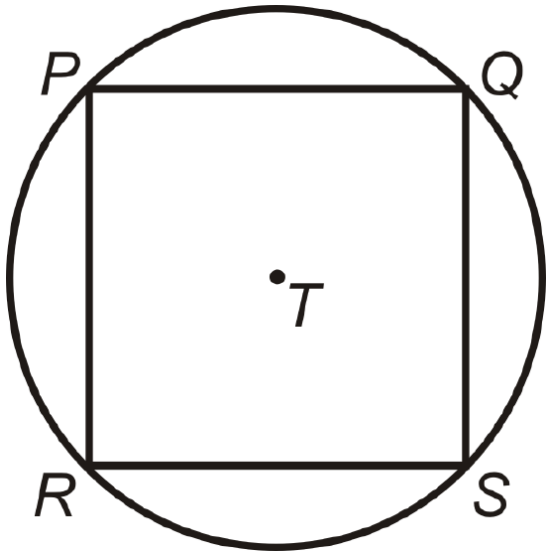

Figure 6.4.4

10. Find the length of the diameter of $\odot T$.
11. How does the diameter relate to $PQSR$?
12. Find the perimeter of $PQSR$.
13. Find the circumference of $\odot T$.

For questions 14-17, a truck has tires with a 26 in diameter.

14. How far does the truck travel every time a tire turns exactly once? What is this the same as?
15. How many times will the tire turn after the truck travels 1 mile? (1 mile = 5280 feet)
16. The truck has travelled 4072 tire rotations. How many miles is this?
17. The average recommendation for the life of a tire is 30,000 miles. How many rotations is this?

Vocabulary

Term	Definition
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
Circumference	The circumference of a circle is the measure of the distance around the outside edge of a circle.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
pi	(or π) The ratio of the circumference of a circle to its diameter.
Radius	The radius of a circle is the distance from the center of the circle to the edge of the circle.
Congruent	Congruent figures are identical in size, shape and measure.
Regular Polygon	A regular polygon is a polygon with all sides the same length and all angles the same measure.

Additional Resources

Video: Circumference of a Circle

Activities: Circumference Discussion Questions

study Aids: Circumference and Arc Length Study Guide

Practice: Circumference

Real World: A Roundabout Way Home

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6.5: Diameter or Radius of a Circle Given Circumference

Find the diameter or radius of a circle using the formulas: $C = \pi d$; $C = 2\pi r$.

Diameter or Radius of a Circle Given Circumference

Figure 6.5.1

Marcus is helping his coach design an obstacle course for the weekend's fitness tournament. They dug a circular water pit with a circumference of 18 feet. Marcus has to cut a balance beam that the students will use to cross the pit. The beam is going across the center of the pit. How can Marcus determine how long the balance beam should be to cover the diameter of the pit?

In this concept, you will learn to find the diameter or radius of a circle given the circumference.

Finding the Diameter or Radius

The circumference of a circle is equal to pi times the diameter.

$$C = \pi d$$

The diameter is two times the radius, so the equation for the circumference of a circle using the radius is two times pi times the radius.

$$C = 2\pi r$$

If you are given the circumference but not the radius or the diameter, you can still solve for one or the other using these formulas.

Let's look at an example.

Find the diameter of a circle with a circumference of 21.98 feet.

To work on this problem, you will need the formula for finding the circumference of a circle using the diameter.

$$C = \pi d$$

First, fill in the given information.

$$21.98 = (3.14)d$$

Next, solve the problem by dividing both sides by 3.14. The 3.14 will cancel out on the right side of the equation, leaving d. Then divide 21.98 by 3.14.

$$3.14)\overline{21.98}$$

When dividing decimals, move the decimal point two places to make the divisor a whole number then divide as usual.

$$7$$

$$314)\overline{2198}$$

The answer is, the diameter is 7 feet.

You can use the same approach to solving for the radius as you did the diameter. Just remember to divide the diameter by two to get the radius.

If you were asked to find the radius instead of the diameter, you would simply divide 7 feet by 2 because the radius is one-half the measure of the diameter.

$$7 \div 2 = 3.5$$

The radius of the circle is 3.5 feet.

You can also use the circumference and radius equation.

Practice Long Division

You may use this tool to practice long division if you need a refresh on the process:

Example 6.5.1

Earlier, you were given a problem about Marcus and the obstacle course.

The circumference of the circular water pit is 18 feet. How can Marcus use this information to determine how long the beam needs to be to go across the center of the pit? Marcus needs to figure out the diameter of the pit.

Solution

He will need to use the formula for finding the circumference of a circle using the diameter.

$$C = \pi d$$

First, fill in the given information.

$$18 = (3.14)d$$

Next, solve the problem by dividing both sides by 3.14. The 3.14 will cancel out on the right side of the equation, leaving d. Then divide 18 by 3.14.

$$3.14)\overline{18.00}$$

When dividing decimals, move the decimal point two places to make the divisor a whole number then divide as usual.

$$5.72$$

$$314)\overline{1800.00}$$

The diameter is 5.72 feet, so the beam needs to be at least that long.

Example 6.5.2

Use the equation for the circumference of a circle to answer the following question.

What is the radius of a circle with a circumference of 34.54 feet?

Solution

To work on this problem, you will need the formula for finding the circumference of a circle using the diameter.

$$C = \pi d$$

First, fill in the given information.

$$34.54 = (3.14)d$$

Next, solve the problem by dividing both sides by 3.14. The 3.14 will cancel out on the right side of the equation, leaving d. Then divide 34.54 by 3.14.

$$3.14\overline{)34.54}$$

When dividing decimals, move the decimal point two places to make the divisor a whole number then divide as usual.

$$11$$

$$314\overline{)3454}$$

The diameter of this circle is 11 feet.

Then, because the radius is half of the diameter, divide the diameter by 2.

$$\frac{11}{2} = 5.5$$

The radius of this circle is 5.5 feet.

Example 6.5.3

Find the diameter given the circumference.

$$C = 31.4m$$

Solution

To work on this problem, you will need the formula for finding the circumference of a circle using the diameter.

$$C = \pi d$$

First, fill in the given information.

$$31.4 = (3.14)d$$

Next, solve the problem by dividing both sides by 3.14.

$$10$$

$$314\overline{)3140}$$

The diameter of this circle is 10 m.

Example 6.5.4

Find the diameter given the circumference.

$$C = 28.26in$$

To work on this problem, you will need the formula for finding the circumference of a circle using the diameter.

$$C = \pi d$$

Solution

First, fill in the given information.

$$28.26 = (3.14)d$$

Next, solve the problem by dividing both sides by 3.14.

$$9$$

$$314\overline{)2826}$$

The diameter of this circle is 9 inches.

Example 6.5.5

Find the diameter given the circumference.

$$C = 23.55in$$

Solution

To work on this problem, you will need the formula for finding the circumference of a circle using the diameter.

$$C = \pi d$$

First, fill in the given information.

$$23.55 = (3.14)d$$

Next, solve the problem by dividing both sides by 3.14.

$$\begin{array}{r} 11 \\ 314\overline{)3454} \end{array}$$

The diameter is 11 inches.

Review

Find the diameter of each circle given the circumference.

1. $C = 37.68$ in.
2. $C = 40.82$ in.
3. $C = 18.84$ in.
4. $C = 28.26$ ft.
5. $C = 56.52$ m.
6. $C = 17.27$ m.
7. $C = 19.468$ ft.
8. $C = 30.772$ ft.

Find the radius of each circle given the circumference.

9. $C = 25.2$ in.
10. $C = 37.68$ in.
11. $C = 12.56$ in.
12. $C = 15.7$ ft.
13. $C = 7.85$ m.
14. $C = 50.24$ m.
15. $C = 21.98$ m.
16. $C = 14.13$ ft.

Resources

Additional Resource

Video: Circles: Radius, Di ference

Practice: Diameter or Radi n Circumference

Real World: Put the Pedal

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6.6: Circle Area

Area of a Circle

To find the area of a circle, all you need to know is its **radius**. If r is the radius of a circle, then its **area** is $A = \pi r^2$.

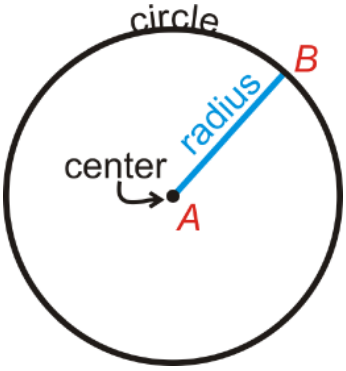

Figure **6.6.1**

We will leave our answers in terms of π, unless otherwise specified.

What if you were given the radius or **diameter** of a circle? How could you find the amount of space the circle takes up?

Example 6.6.1

Find the area of a circle 12 cm.

Solution

If $d = 12$ cm , then $r = 6$ cm $A = \pi(6^2) = 36\pi$ cm ^{2}.

Example 6.6.2

If the area of a circle is 20π units , what is the radius?

Solution

Plug in the area and solve for the radius.

$$20\pi = \pi r^2$$
$$20 = r^2$$
$$r = \sqrt{20} = 2\sqrt{5} \text{ units}$$

Example 6.6.3

A circle is inscribed in a square. Each side of the square is 10 cm long. What is the area of the circle?

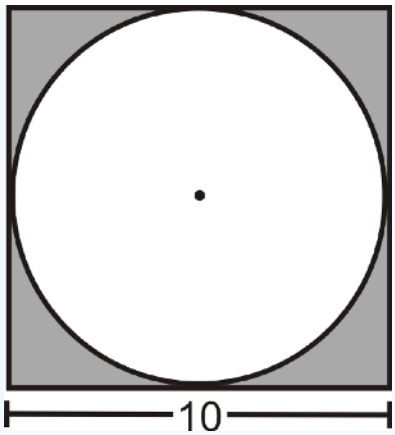

Figure 6.6.2

Solution

The diameter of the circle is the same as the length of a side of the square. Therefore, the radius is 5 cm.

$$A = \pi 5^2 = 25\pi \text{ cm}^2$$

Example 6.6.4

Find the area of the shaded region from Example 3.

Solution

The area of the shaded region would be the area of the square minus the area of the circle.

$A = 102 - 25\pi = 100 - 25\pi \approx 21.46 \text{ cm}^2$

Example 6.6.5

Find the diameter of a circle with area 36π.

Solution

First, use the formula for the area of a circle to solve for the radius of the circle.

$$A = \pi r^2$$
$$36\pi = \pi r^2$$
$$36 = r^2$$
$$r = 6$$

If the radius is 6 units, then the diameter is 12 units.

Review

Fill in the following table. Leave all answers in terms of π.

	radius	*Area*	*circumference*
1.	2		
2.		16π	
3.			10π
4.			24π
5.	9		
6.		90π	
7.			35π
8.	7π		
9.			60
10.		36	

Find the area of the shaded region. Round your answer to the nearest hundredth.

11.

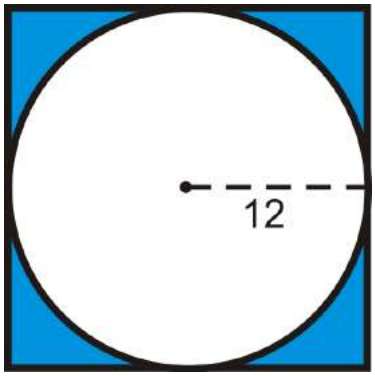

Figure 6.6.3

12.

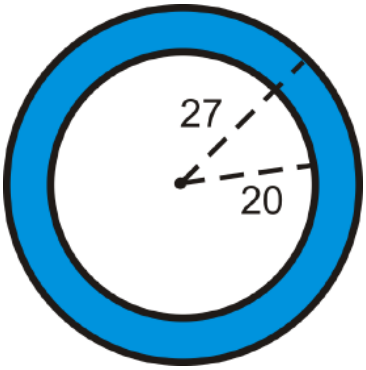

Figure 6.6.4

13.

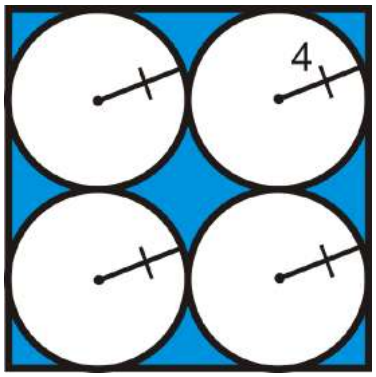

Figure 6.6.5

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.10.

Vocabulary

Term	Definition
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
circumference	The distance around a circle.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
pi	(or π) The ratio of the circumference of a circle to its diameter.
radius	The distance from the center to the outer rim of a circle.

Additional Resources

Video: Determine the Area of a Circle

Activities: Area of a Circle Discussion Questions

Study Aids: Circumference and Arc Length Study Guide

Practice: Circle Area

Real World: Ringside Seats

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6.7: Areas of Combined Figures Involving Semicircles

Calculate areas of irregular shapes.

Areas of Combined Figures Involving Circles

Figure **6.7.1**

Most of the outdoor basketball court was already repainted green. The only part left to redo was the brown key. The painters need to know the total **area** of the space remaining in order to buy enough paint. Given the dimensions shown below, what area should the workers use to calculate their supplies?

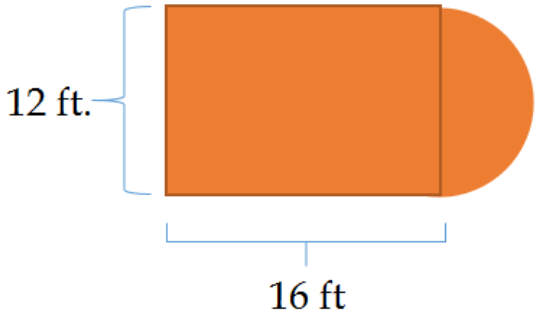

Figure **6.7.2**

In this concept, you will learn how to find the area of composite figures containing circles.

Finding the Area of Composite Figures

The formula to find the area of a rectangle is $A = lw$. The formula to find the area of a **circle** is $A = \pi r^2$. To find the area of a composite shape, simply find the area of each individual shape and add them together. The order in which you calculate the areas does not matter, and the **commutative property** states that it does not matter which order you add them in.

Let's look at an example.

Find the area of the figure below.

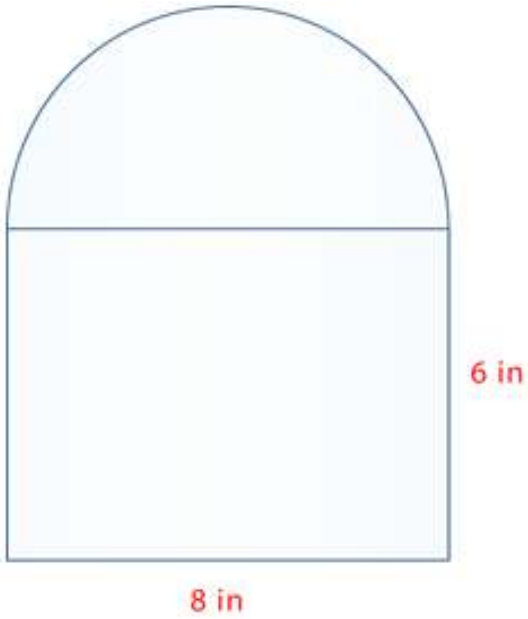

Figure 6.7.3

First, find the area of the rectangle.

$$A = lw$$
$$A = 6(8)$$
$$A = 48$$

The area of the rectangle is 48 square inches.

Next, recognize that you have been given a **diameter** and need to divide that by 2 to get the radius.

$$A = \pi r^2$$
$$A = 3.14(4)^2$$
$$A = 3.14(16)$$
$$A = 50.24 \text{ sq. in.}$$

The area of a full circle is 50.24 square inches.

Then, find the area of one half the circle, a semi-circle, by dividing your final answer by 2.

$$A = 25.12$$

The area of the semi-circle is 25.12 square inches.

Finally, add the two areas together.

$$A = A_r + A_{sc}$$
$$A = 48 + 25.12$$
$$A = 73.12 \text{ sq. in.}$$

The answer is the composite figure has an area of 73.12 square inches.

Example 6.7.1

Earlier, you were given a problem about the outdoor basketball court, which needs some paint.

The workers had the following court dimensions and need to know the total area to figure out their supplies.

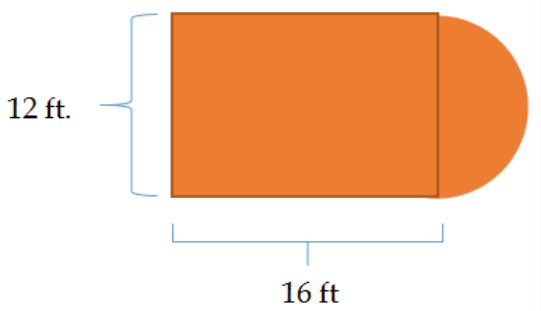

Figure 6.7.4

Solution

First, find the area of the rectangle.

$$A = lw$$
$$A = 16(12)$$
$$A = 192$$

The area of the rectangle is 192 square feet.

Next, recognize that you have been given a diameter and need to divide that by 2 to get the radius. The problem states that the diameter of the circle is the same as the width of the rectangle, 3 feet.

$$A = \pi r^2$$
$$A = 3.14(6)^2$$
$$A = 3.14(36)$$
$$A = 113.04$$

The area for a full circle is approximately 113 square feet.

Then, remember that you have a semi-circle and divide this area by 2.

$$A_{sc} = 56.5$$

The area of the semi-circle is 56.5 square feet.

Add the two areas together.

$$A = A_r + A_{sc}$$
$$A = 192 + 56.5$$
$$A = 248.5 \text{ sq. ft.}$$

The answer is the composite figure has an area $A = 248.5$ square feet . The workers will need to by enough paint to cover 248.5 sq. ft.

Example 6.7.2

The figure below is two semi-circles with a square in the middle. The square has a side length of 6 inches.

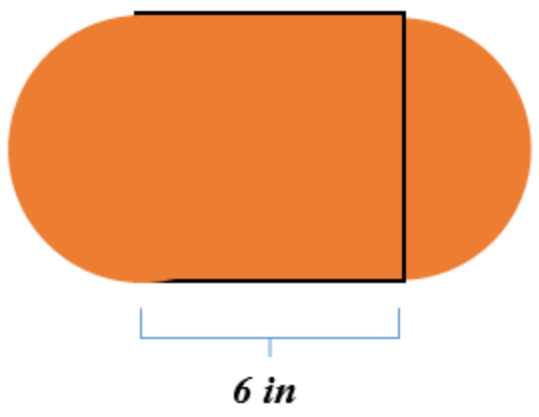

Figure 6.7.5

Solution

Next, determine the area of the square.

$$A = s^2$$
$$A = 6^2$$
$$A = 36$$

The area of the square is 36 square inches.

Then, recognize that since the square has four equal sides, the diameter of the circle is also 6 inches.

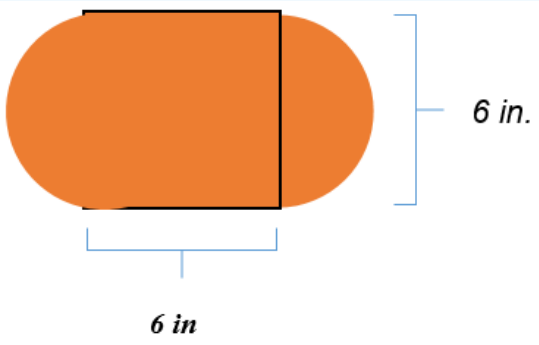

Figure 6.7.6

The formula for the area of one semi-circle is the formula for the area of a circle divided by 2.

$$A_{sc} = \frac{\pi r^2}{2}$$

Since there are two semi-circles the same size, you may simply add them together to end up with the area for the complete circle.

Remember that you have been given a diameter, not a radius, so divide the diameter by 2.

Then, use the formula for the area of a circle.

$$A = \pi r^2$$
$$A = 3.14(3)^2$$
$$A = 3.14(9)$$
$$A = 28.26$$

The area of the two semi-circles that form one full circle is 28.26 square inches.

Finally, add the two areas together.

$$A = A_r + A_{sc}$$
$$A = 36 + 28.26$$
$$A = 64.26$$

The answer is the composite figure has an area $A = 64.26$ square inches

Example 6.7.3

Find the composite area of a rectangle with a length of 5 feet and a width of 3 feet connected to a semi-circle with the same diameter as the width.

Solution

First, find the area of the rectangle.

$$A = lw$$
$$A = 5(3)$$
$$A = 15$$

The area of the rectangle is 15 square feet.

Next, recognize that you have been given a diameter and need to divide that by 2 to get the radius. The problem states that the diameter of the circle is the same as the width of the rectangle, 3 feet.

$$A = \pi r^2$$
$$A = 3.14(1.5)^2$$
$$A = 3.14(2.25)$$
$$A = 7.065$$

The area for a full circle is 7.065 square feet.

Then, remember that you have a semi-circle and divide this area by 2.

$$A_{sc} = 3.53$$

The area of the semi-circle is 3.53 square feet.

Add the two areas together.

$$A = A_r + A_{sc}$$
$$A = 15 + 3.53$$
$$A = 18.53 \text{ sq. ft.}.$$

The answer is the composite figure has an area $A = 18.53$ square feet .

Example 6.7.4

Find the composite area of a square with a side length of 4 mm and a semi-circle with a diameter of the same side length as the square.

Solution

First, find the area of the square.

$$A = s^2$$
$$A = 4^2$$
$$A = 16$$

The area of the square is 16 square millimeters.

Next, recognize that you have been given a diameter and need to divide that by 2 to get the radius. The problem states that the diameter of the circle is the same as the side length of the square, 4 mm.

$$A = \pi r^2$$
$$A = 3.14(2)^2$$
$$A = 3.14(4)$$
$$A = 12.56$$

The area for a full circle is 12.56 square millimeters.

Then, remember that you have a semi-circle and divide this area by 2.

$$A_{sc} = 6.28$$

The area of the semi-circle is 6.28 square millimeters.

Add the two areas together.

$$A = AS + A_{sc}$$
$$A = 16 + 6.28$$
$$A = 22.28 \text{ sq. mm.}$$

The answer is the composite figure has an area $A = 22.28$ sq. mm.

Example 6.7.5

Find the area of a figure that is made up of a square and a semi-circle. The square has a side length of 8 inches. The circle's diameter is the same as the side length of the square.

Solution

First, find the area of the square.

$$A = s^2$$
$$A = 8^2$$
$$A = 64$$

The area of the square is 64 square inches.

Next, recognize that you have been given a diameter and need to divide that by 2 to get the radius. The problem states that the diameter of the circle is the same as the side length of the square, 8 inches.

$$A = \pi r^2$$
$$A = 3.14(4)^2$$
$$A = 3.14(16)$$
$$A = 50.24$$

The area for a full circle is 50.24 square inches.

Then, remember that you have a semi-circle and divide this area by 2.

$$A_{sc} = 25.12$$

The area of the semi-circle is 25.12 square inches.

Add the two areas together.

$$A = A_r + A_{sc}$$
$$A = 64 + 25.12$$
$$A = 79.12 \text{ square inches}$$

The answer is the composite figure has an area $A = 79.12$ sq. in.

Review

Find the area of each combined figure.

1. A square and a semi-circle. The square has a side length of 11 mm. The diameter of the circle matches the square's side.
2. A square and a semi-circle. The square has a side length of 8.5 inches. The diameter of the circle matches the square's side.
3. A square and a semi-circle. The square has a side length of 7.25 inches. The diameter of the circle matches the square's side.
4. A square and a semi-circle. The square has a side length of 13 feet. The diameter of the circle matches the square's side.
5. A square and a semi-circle. The square has a side length of 15.5 feet. The diameter of the circle matches the square's side.
6. A rectangle and a semi-circle. The rectangle has a length of 8 feet and a width of 5 feet. The diameter of the circle matches the width.
7. A rectangle and a semi-circle. The rectangle has a length of 8.5 feet and a width of 6 feet. The diameter of the circle matches the width.

8. A rectangle and a semi-circle. The rectangle has a length of 9 inches and a width of 4.5 inches. The diameter of the circle matches the length.
9. A rectangle and a semi-circle. The rectangle has a length of 7 feet and a width of 4 feet. The diameter of the circle matches the length.
10. A rectangle and a semi-circle. The rectangle has a length of 5.5 feet and a width of 3.5 feet. The diameter of the circle matches the width.
11. A triangle and a semi-circle. The triangle has a base of 5 inches and a height of 4 inches. The diameter of the circle matches the base of the triangle.
12. A triangle and a semi-circle. The triangle has a base of 7 inches and a height of 6 inches. The diameter of the circle matches the base of the triangle.
13. A triangle and a semi-circle. The triangle has a base of 5.5 inches and a height of 4 inches. The diameter of the circle matches the base of the triangle.

Solve each problem.

14. Rob is painting large polka dots on a sheet for the backdrop of the school musical. He painted 16 polka dots, each with a radius of 3 feet. What is the total area that the polka dots cover?
15. The librarian is having the library at her school carpeted. The library is a circular room with a diameter of 420 feet. How many square feet of carpet will she need to order?

Resources

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Circle	A circle is the set of all points at a specific distance from a given point in two dimensions.
Diameter	Diameter is the measure of the distance across the center of a circle. The diameter is equal to twice the measure of the radius.

Additional Resources

Video: Area and Perimeter

Practice: Areas of Combined Figures Involving Semicircles

Real World: Shooting Circles

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6.8: Radius or Diameter of a Circle Given Area

To find the radius, divide the area by pi, then take the square root.

Figure 6.8.1

Clara took her little sister, Grace, to the fish pond at the local park. Grace saw a penny in the center of the pond and wanted Clara to reach it for her. A sign stated that the **area** of the pond was 113.04 sq. ft. Can Clara reach the penny without falling in?

In this concept, you will learn how to find the radius (and **diameter**) of a **circle** if you know its area.

Finding the Radius or Diameter of a Circle Given Area

The formula for the area of a circle, $A = \pi r^2$, can also be used to solve for the radius and diameter.

Let's look an example.

The area of a circle is 113.04 square inches. What is its radius?

First, write the formula.

$$A = \pi r^2$$

Next, substitute in what you know.

$$113.04 = (3.14)r^2$$

Next, begin isolating the r by dividing both sides of the equation by 3.14.

$$36 = r^2$$

Then, take the square root of both sides.

$$6 = r$$

The answer is r=6. The radius of the circle is 6 inches.

Example 6.8.1

Earlier, you were given a problem about Clara and Grace, who were at the circular 113.04 sq. ft. fish pond.

Clara wondered if she could reach a penny in the middle without falling in.

Solution

First, write the formula.

$$A = \pi r^2$$

Next, substitute in what you know.

$$113.04 = (3.14)r^2$$

Then, begin isolating the r by dividing both sides of the equation by 3.14.

$$36 = r^2$$

Take the square root of both sides.

$$6 = r$$

The answer is r=6. The radius of the circle is 6 feet. Unless Clara wants to swim with the goldfish, she'd best leave the penny where it is!

Example 6.8.2

What is the diameter of a circle if its area is $379.94 cm^2$?

Solution

First, write the formula.

$$A = \pi r^2$$

Next, substitute in what you know.

$$379.94 = (3.14)r^2$$

Next, begin isolating the r by dividing both sides of the equation by 3.14.

$$121 = r^2$$

Then, take the square root of both sides.

$$11 = r$$

Remember that you are solving for the diameter.

$$d = 2r$$
$$d = 2 \times 11$$
$$d = 22$$

The answer is the diameter, d=22 cm.

Example 6.8.3

Solve for the radius of a circle if area=153.86 sq. in.

Solution

First, write the formula.

$$A = \pi r^2$$

Next, substitute in what you know.

$$153.86 = (3.14)r^2$$

Next, begin isolating the r by dividing both sides of the equation by 3.14.

$$49 = r^2$$

Then, take the square root of both sides.

$$7 = r$$

The answer is $r = 7$. The radius of the circle is 7 inches.

Example 6.8.4

Find the radius of a circle with an area of 379.94 sq. ft.

Solution

First, write the formula.

$$A = \pi r^2$$

Next, substitute in what you know.

$$379.94 = (3.14)r^2$$

Then, begin isolating the r by dividing both sides of the equation by 3.14.

$$121 = r^2$$

Take the square root of both sides.

$$11 = r$$

The answer is r=11. The radius of the circle is 11 feet.

Example 6.8.5

The area of a circle is 452.16 sq. m. Find its radius.

Solution

First, write the formula.

$$A = \pi r^2$$

Next, substitute in what you know.

$$452.16 = (3.14)r^2$$

Then, begin isolating the r by dividing both sides of the equation by 3.14.

$$144 = r^2$$

Take the square root of both sides.

$$12 = r$$

The answer is r=12. The radius of the circle is 12 meters.

Review

Find each radius given the area of the circle.

1. 12.56 sq. in.
2. 78.5 sq. m
3. 200.96 sq. cm
4. 254.34 sq. in.
5. 7.07 sq. ft.
6. 28.26 sq. m

Find each diameter given the area of the circle.

7. 12.56 sq. in.
8. 78.5 sq. m
9. 200.96 sq. cm
10. 254.34 sq. in.
11. 7.07 sq. ft.
12. 28.26 sq. m
13. 615.44 sq. ft.
14. 176.625 sq. m
15. 113.04 sq. ft.

Resources

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Circle	A circle is the set of all points at a specific distance from a given point in two dimensions.
Diameter	Diameter is the measure of the distance across the center of a circle. The diameter is equal to twice the measure of the radius.

Additional Resources

Video: Determine the Area of a Circle

Practice: Radius or Diameter of a Circle Given Area

Real World: Circles in the Cereal

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6.9: Arcs in Circles

Sections of a circle and central angles.

A **circle** has 360°. An **arc** is a section of the circle. A **semicircle** is an arc that measures 180°.

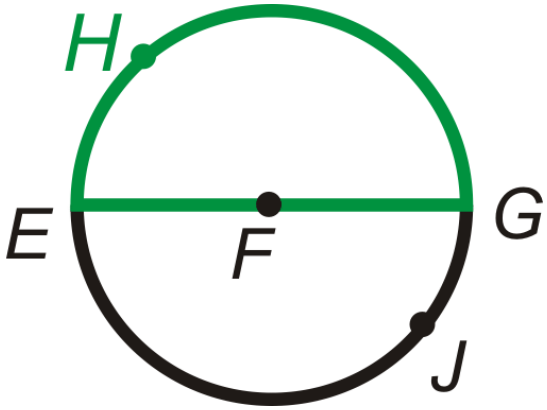

Figure 6.9.1

$\overset{\frown}{EHG}$ and $\overset{\frown}{EJG}$ are semicircles

A **central angle** is the angle formed by two radii with its vertex at the center of the circle. A **minor arc** is an arc that is less than 180°. A **major arc** is an arc that is greater than 180°. Always use 3 letters to label a major arc.

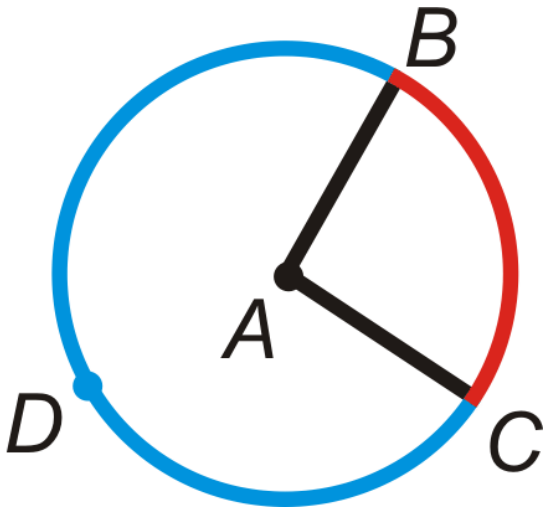

Figure 6.9.2

The central angle is $\angle BAC$. The minor arc is $\overset{\frown}{BC}$. The major arc is $\overset{\frown}{BDC}$.

An arc can be measured in degrees or in a linear measure (cm, ft, etc.). In this concept we will use degree measure. **The measure of a minor arc is the same as the measure of the central angle that corresponds to it**. The measure of a major arc is 360° minus the measure of the corresponding minor arc. The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs (**Arc Addition Postulate**).

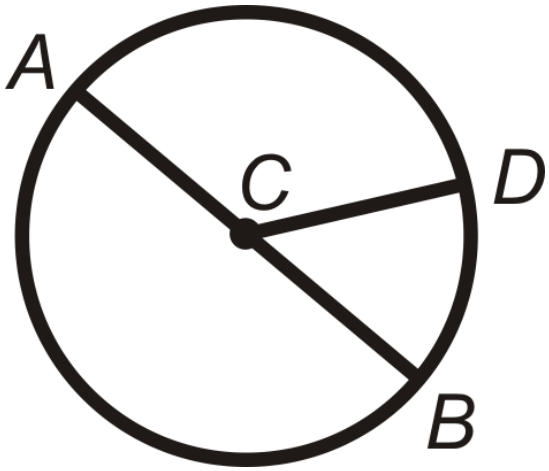

Figure **6.9.3**

$m\widehat{AD} + m\widehat{DB} = m\widehat{ADB}$

What if a circle were divided into pieces by various radii? How could you find the measures of the arcs formed by these radii?

Example 6.9.1

Find $m\widehat{AB}$ and $m\widehat{ADB}$ i

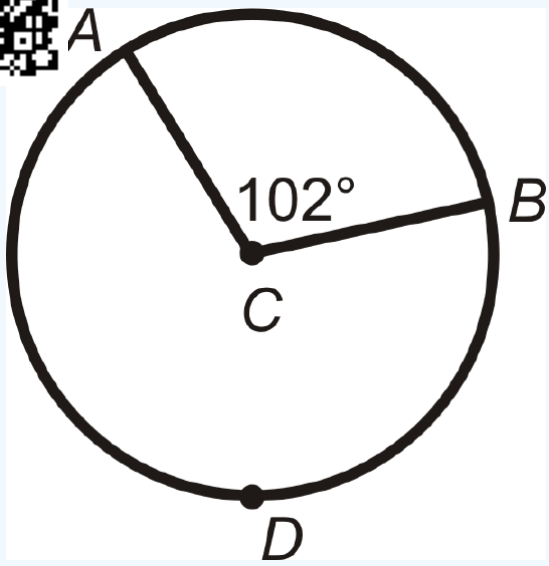

Figure **6.9.4**

Solution

$m\widehat{AB} = m\angle ACB$. So, $m\widehat{AB} = 102°$.

$$m\widehat{ADB} = 360° - m\widehat{AB} = 360° - 102° = 258°$$

Example 6.9.2

Find the measures of the minor arcs in $\odot A$. $\overline{EB}$ is a **diameter**.

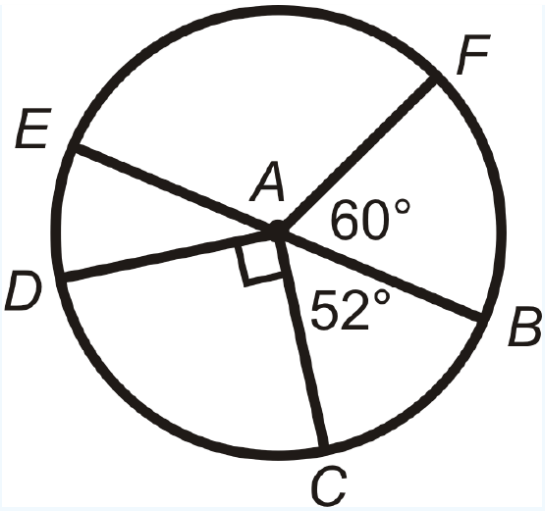

Figure 6.9.5

Solution

Because $\overline{EB}$ is a diameter, $m\angle EAB = 180°$. Each arc has the same measure as its corresponding central angle.

$$m\widehat{BF} = m\angle FAB = 60°$$
$$m\widehat{EF} = m\angle EAF = 120° \rightarrow 180° - 60°$$
$$m\widehat{ED} = m\angle EAD = 38° \rightarrow 180° - 90° - 52°$$
$$m\widehat{DC} = m\angle DAC = 90°$$
$$m\widehat{BC} = m\angle BAC = 52°$$

Example 6.9.3

Find the measures of the indicated arcs in $\odot A$. $\overline{EB}$ is a diameter.

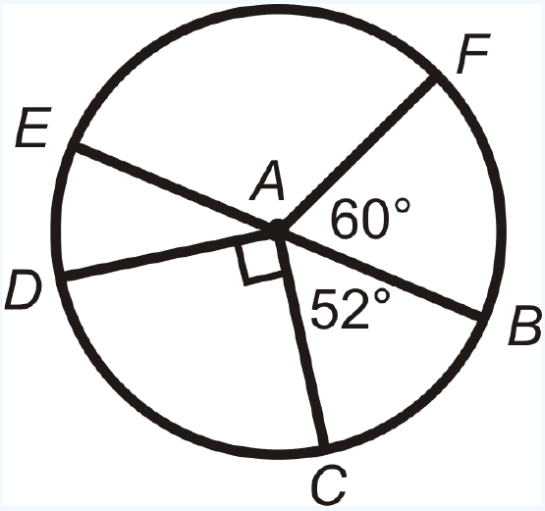

Figure 6.9.6

Use the Arc Addition Postulate.

a. $m\widehat{FED}$
b. $m\widehat{CDF}$
c. $m\widehat{DFC}$

Solution

a. $m\widehat{FED} = m\widehat{FE} + m\widehat{ED} = 120° + 38° = 158°$

b. $m\overset{\frown}{CDF} = m\overset{\frown}{CD} + m\overset{\frown}{DE} + m\overset{\frown}{EF} = 90° + 38° + 120° = 248°$

c. $m\overset{\frown}{DFC} = m\overset{\frown}{ED} + m\overset{\frown}{EF} + m\overset{\frown}{FB} + m\overset{\frown}{BC} = 38° + 120° + 60° + 52° = 270°$

Example 6.9.4

List the congruent arcs in $\odot C$ below. $\overline{AB}$ and $\overline{DE}$ are diameters.

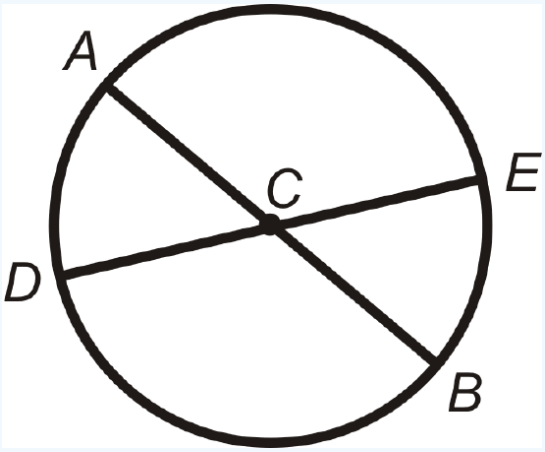

Figure 6.9.7

Solution

$\angle ACD \cong \angle ECB$ because they are vertical angles. $\angle DCB \cong \angle ACE$ because they are also vertical angles.

$\overset{\frown}{AD} \cong \overset{\frown}{EB}$ and $\overset{\frown}{AE} \cong \overset{\frown}{DB}$

Example 6.9.5

For each of the circles below, are the blue arcs congruent? Explain why or why not.

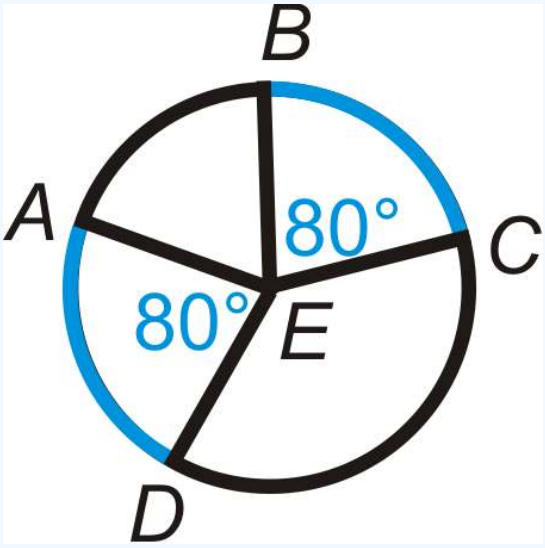

Figure 6.9.8

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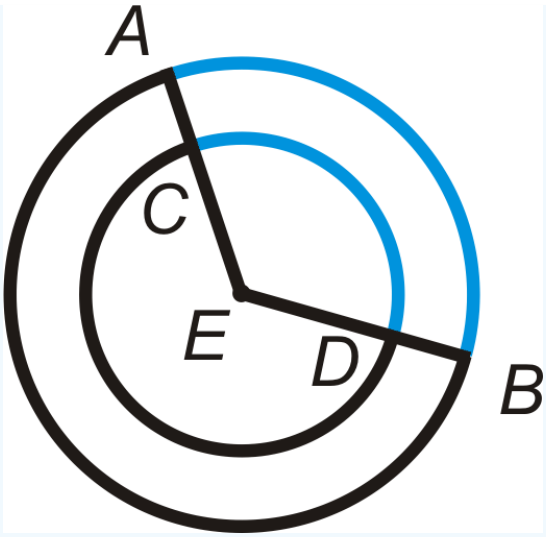

Figure **6.9.9**

Solution

For the first circle, $\overset{\frown}{AD} \cong \overset{\frown}{BC}$ because they have the same central angle measure and are in the same circle.

For the second circle, the two arcs have the same measure, but are not congruent because the circles have different radii.

Review

Determine whether the arcs below are a minor arc, major arc, or semicircle of $\odot G$. $\overline{EB}$ is a diameter.

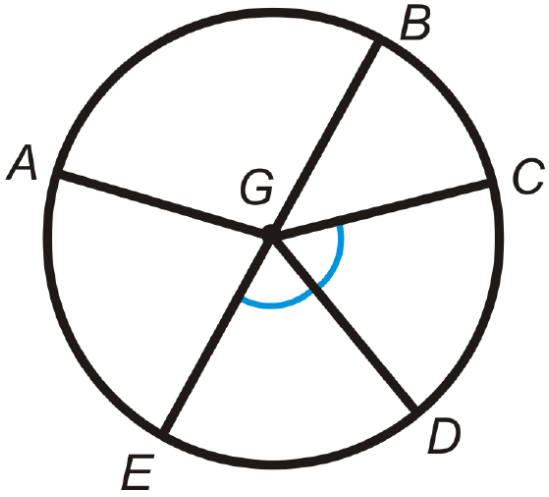

Figure **6.9.10**

1. $\overset{\frown}{AB}$
2. $\overset{\frown}{ABD}$
3. $\overset{\frown}{BCE}$
4. $\overset{\frown}{CAE}$
5. $\overset{\frown}{ABC}$
6. $\overset{\frown}{EAB}$
7. Are there any congruent arcs? If so, list them.
8. If $m\overset{\frown}{BC} = 48°$, find m\widehat{CD}\).
9. Using #8, find m\widehat{CAE}\).

Find the measure of the minor arc and the major arc in each circle below.

10.

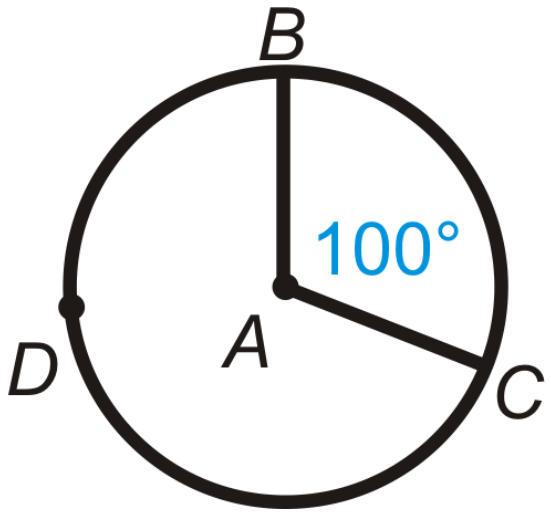

Figure **6.9.11**

11.

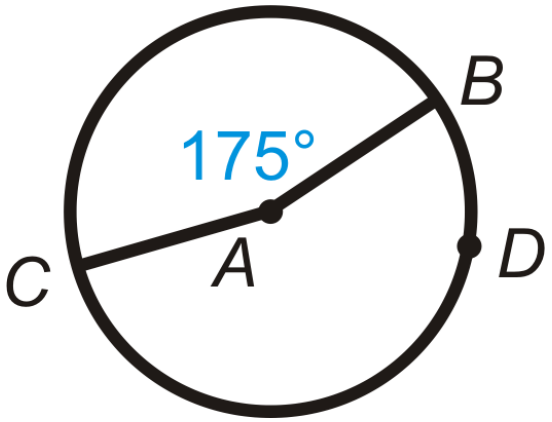

Figure **6.9.12**

12.

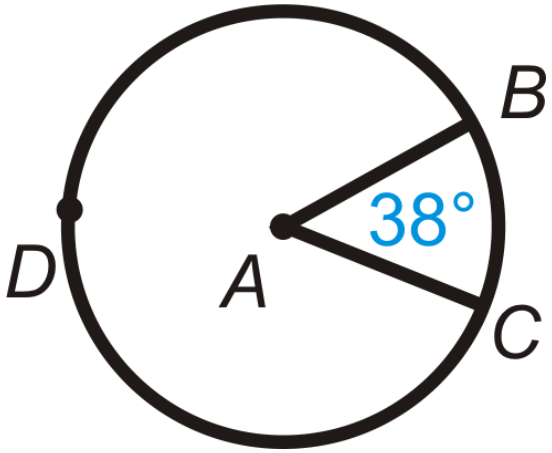

Figure **6.9.13**

13.

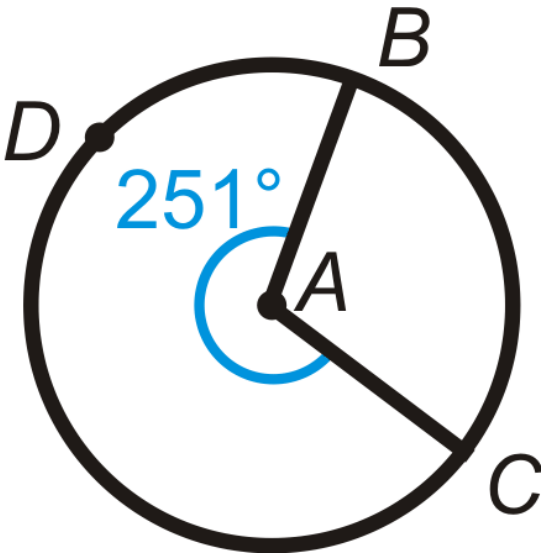

Figure **6.9.14**

14.

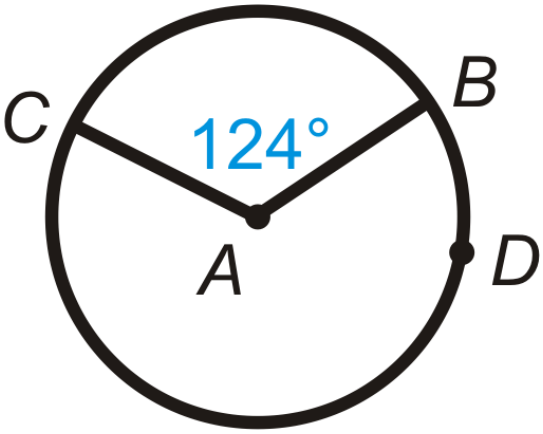

Figure **6.9.15**

15.

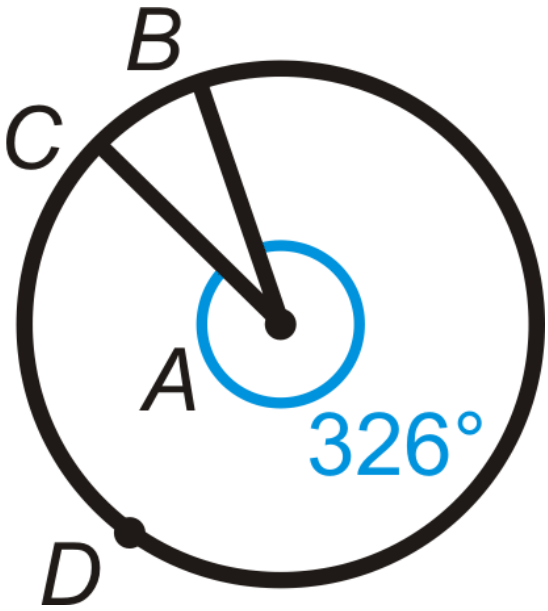

Figure **6.9.16**

Determine if the blue arcs are congruent. If so, state why.

16.

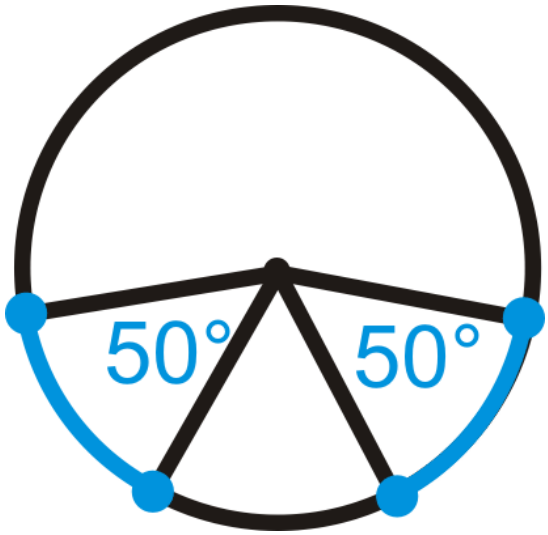

Figure **6.9.17**

17.

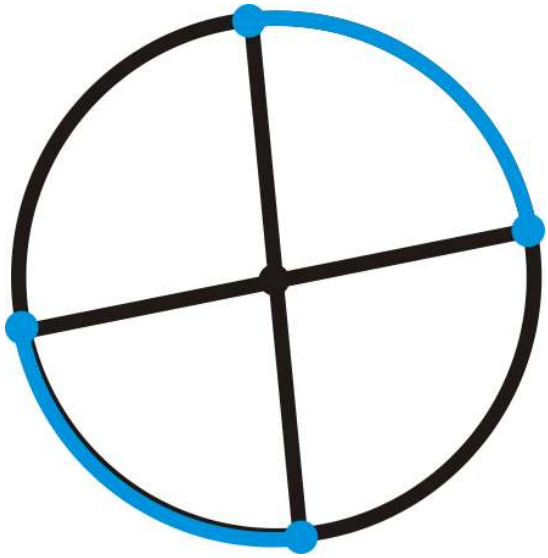

Figure **6.9.17**

18.

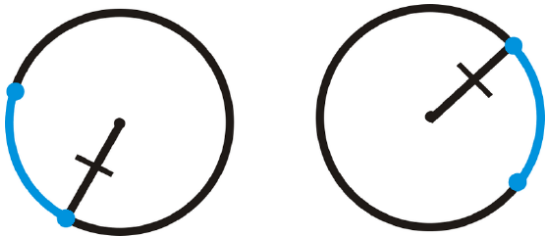

Figure **6.9.17**

Find the measure of the indicated arcs or central angles in \bigodot A\).\overline{DG}\) is a diameter.

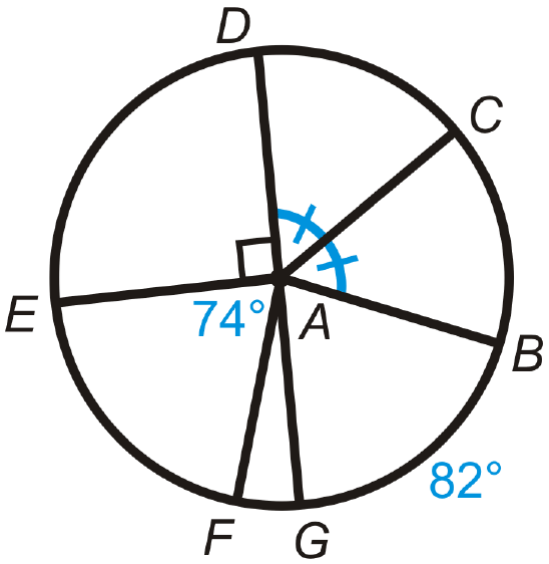

Figure **6.9.18**

19. \widehat{DE}\)
20. \widehat{DC}\)
21. \widehat{GAB}\)
22. \widehat{FG}\)
23. \widehat{EDB}
24. \widehat{EAB}\)
25. \widehat{DCF}\)
26. \widehat{DBE}\)

Find the measure of x\) in \bigodot P\).

27.

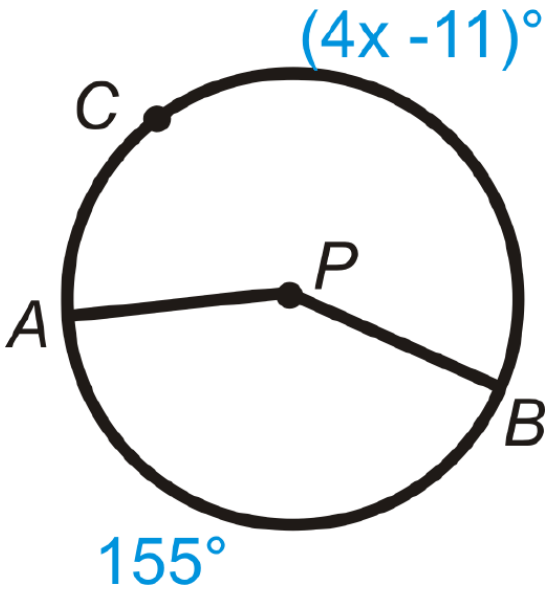

Figure **6.9.19**

28.

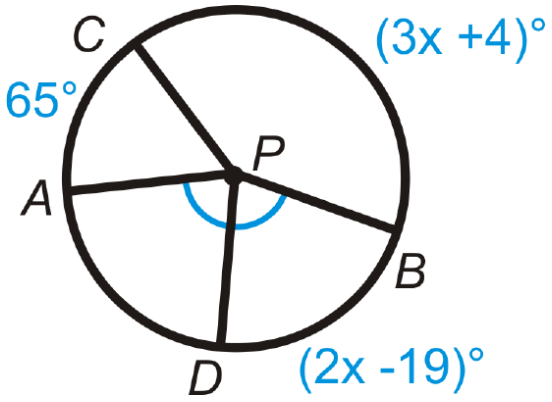

Figure 6.9.20

29.

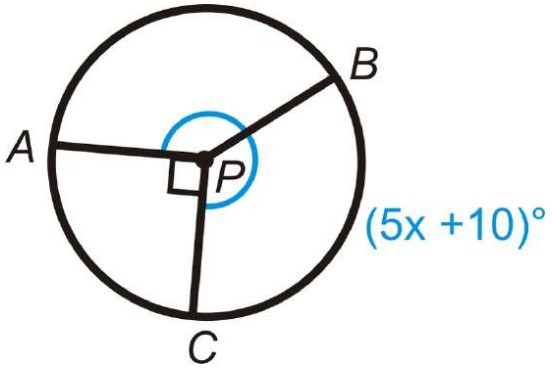

Figure 6.9.21

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.3.

Vocabulary

Term	Definition
arc	A single section of the circle, that describes a particular angle.
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
major arc	An arc that is greater than $180°$.
minor arc	An arc that is less than $180°$.
radius	The distance from the center to the outer rim of a circle.
semicircle	An arc that measures $180°$.
Arc Addition Postulate	Arc addition postulate states that the measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.
Diameter	Diameter is the measure of the distance across the center of a circle. The diameter is equal to twice the measure of the radius.

Additional Resources

Video: Arcs in Circles Principles - Basic

Activities: Arcs in Circles Discussion Questions

Study Aids: Properties of a Circle Study Guide

Practice: Arcs in Circles

Real World: Farming the Arc

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6.10: Area of Sectors and Segments

Area of parts of a circle.

Area of Sectors and Segments

A **sector of a circle** is the area bounded by two radii and the **arc** between the endpoints of the radii. If r is the **radius** and \widehat{AB} is the arc bounding a **sector**, then the **area** of the sector is $A = \frac{m\widehat{AB}}{360^\circ} \cdot \pi r^2$.

Figure **6.10.1**

A **segment of a circle** is the area of a circle that is bounded by a **chord** and the arc with the same endpoints as the chord. The **area** of a segment is $A_{segment} = A_sector - A_{\Delta ABC}$

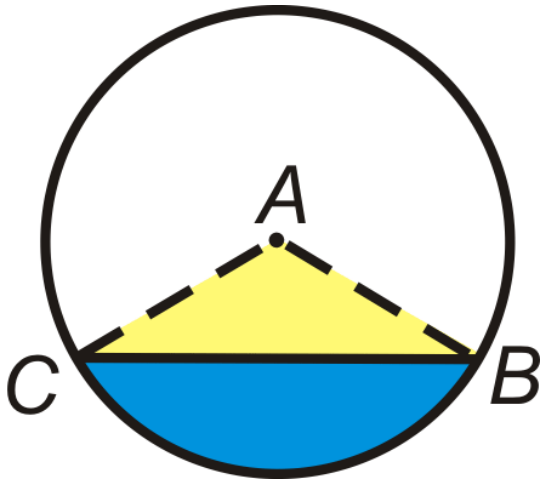

Figure **6.10.2**

What if you were given a circle with two radii in which the region between those two radii was shaded? How could you find the area of that shaded region of the circle?

Example 6.10.1

The area of a sector is 135π sure is 216^{\circ}. What is the radius of the circle?

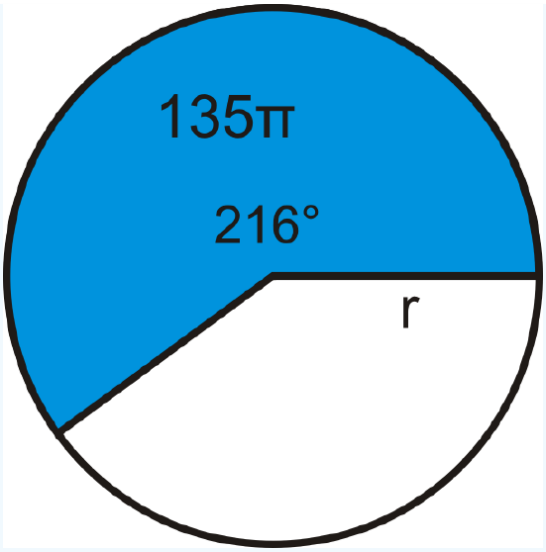

Figure 6.10.3

Solution

Plug in what you know to the sector area formula and solve for r.

$$135\pi = \frac{216^\circ}{360^\circ} \cdot \pi r^2$$
$$135 = 35 \cdot r^2$$
$$\frac{5}{3} \cdot 135 = r^2$$
$$225 = r^2 \rightarrow r = \sqrt{225} = 15$$

Example 6.10.2

Find the area of the shaded region. The quadrilateral is a square.

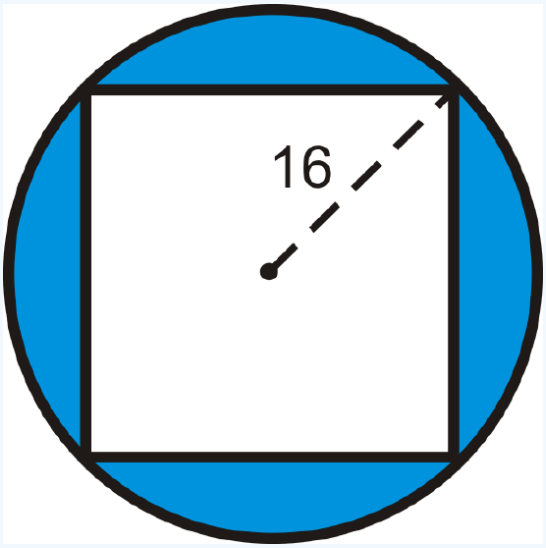

Figure 6.10.4

Solution

The radius of the circle is 16, which is also half of the diagonal of the square. So, the diagonal is 32 and the sides would be \dfrac{32}{\sqrt{2}}\cdot \dfrac{\sqrt{2}}{\sqrt{2}}=16\sqrt{2} because each half of a square is a 45-45-90 triangle.

$$A_{\text{circle}} = 16^2\pi = 256\pi$$
$$A_{\text{square}} = (16\sqrt{2})^2 = 256 \cdot 2 = 512$$

The area of the shaded region is $256\pi - 512 \approx 292.25$

Example 6.10.3

Find the area of the blue sector. Leave your answer in terms of π.

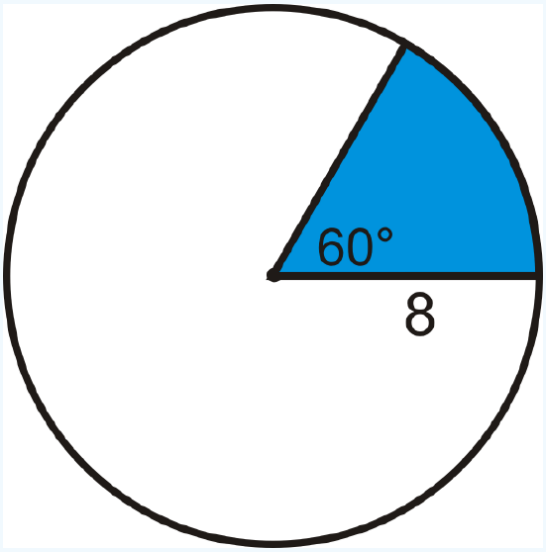

Figure 6.10.5

Solution

In the picture, the central angle that corresponds with the sector is $60°$. $60°$ is 16 of $360°$, so this sector is 16 of the total area.

$\text{area of blue sector } = 16 \cdot \pi 8^2 = \frac{32}{3}\pi$

Example 6.10.1

The area of a sector is 8π and the radius of the circle is 12. What is the central angle?

Solution

Plug in what you know to the sector area formula and then solve for the central angle, which we will call x.

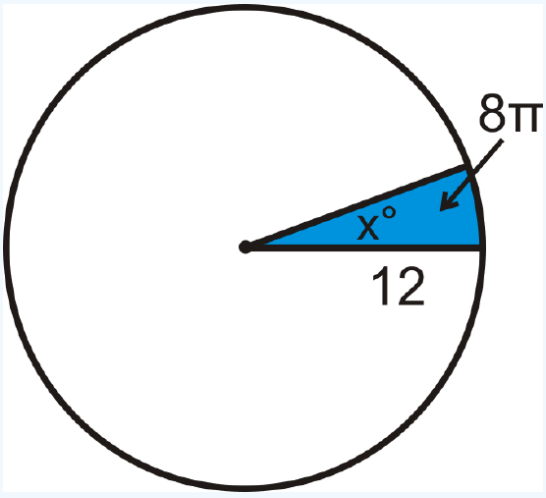

Figure 6.10.6

$$8\pi = \frac{x}{360^\circ} \cdot \pi 12^2$$
$$8\pi = \frac{x}{360^\circ} \cdot 144\pi$$
$$8 = \frac{2x}{5^\circ}$$
$$x = 8 \cdot \frac{5^\circ}{2} = 20^\circ$$

Example 6.10.1

Find the area of the blue segment below.

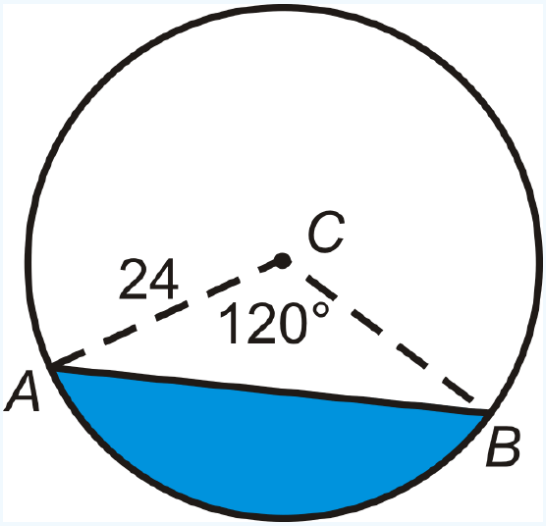

Figure 6.10.7

Solution

The area of the segment is the area of the sector minus the area of the isosceles triangle made by the radii. If we split the isosceles triangle in half, each half is a 30-60-90 triangle, where the radius is the hypotenuse. The height of \DeltaABC is 12 and the base is 2(123–√)=243–√.

$$A_{\text{sector}} = \tfrac{120}{360}\pi \cdot 24^2 = 192\pi \qquad A_{\triangle} = \tfrac{1}{2}(24\sqrt{3})(12) = 144\sqrt{3}$$

The area of the segment is $A = 192\pi - 144\sqrt{3} \approx 353.8$ units.

Review

Find the area of the blue sector or segment in ⊙A. Leave your answers in terms of \pi . Round any decimal answers to the nearest hundredth.

1.

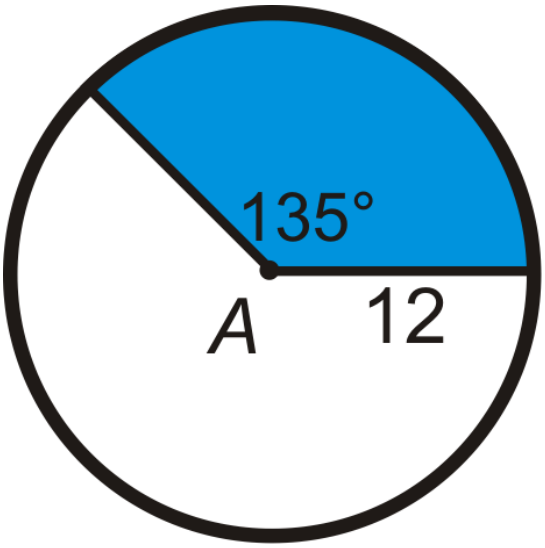

Figure **6.10.8**

2.

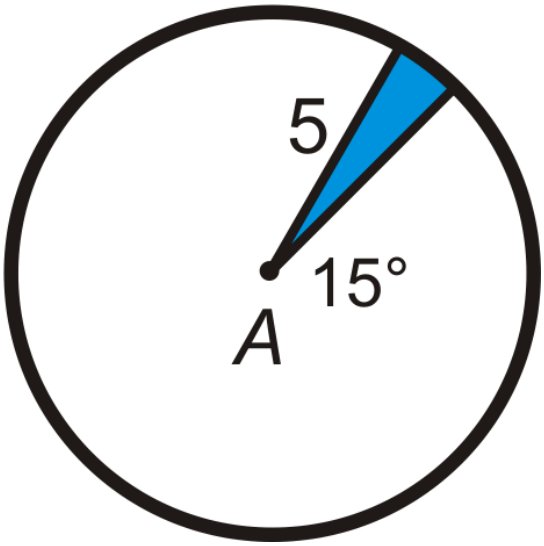

Figure **6.10.9**

3.

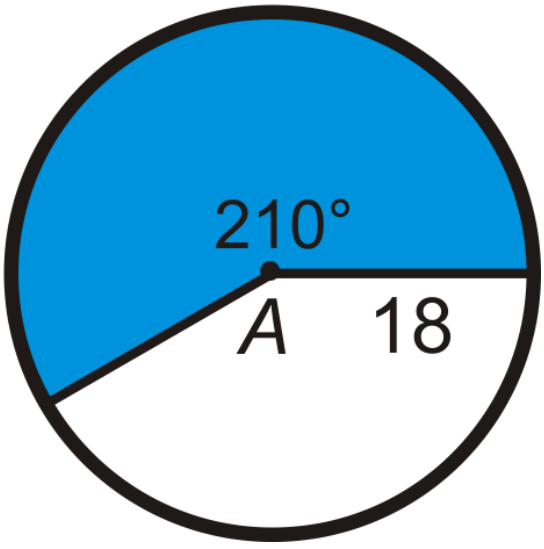

Figure **6.10.10**

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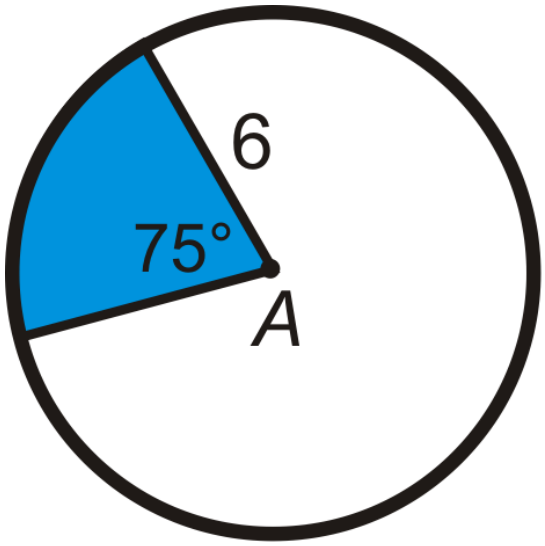

Figure 6.10.11

5.

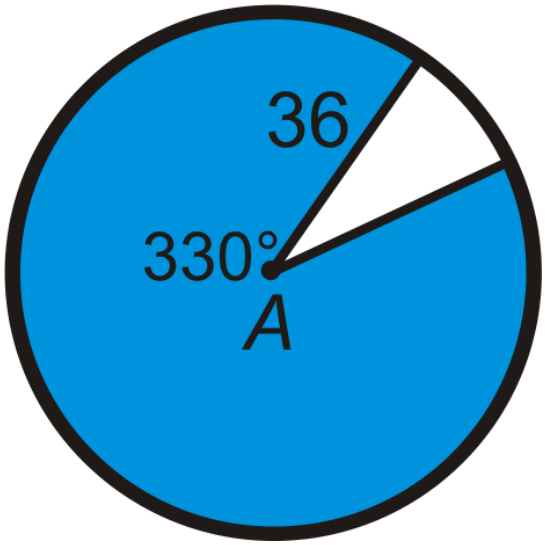

Figure 6.10.12

Find the radius of the circle. Leave your answer in terms of \pi .

6.

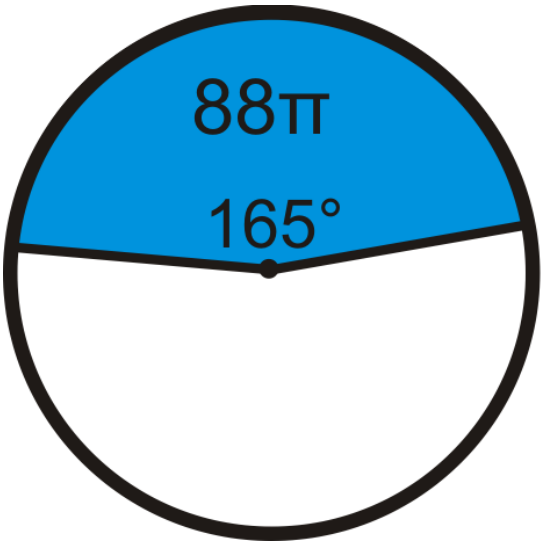

Figure **6.10.13**

7.

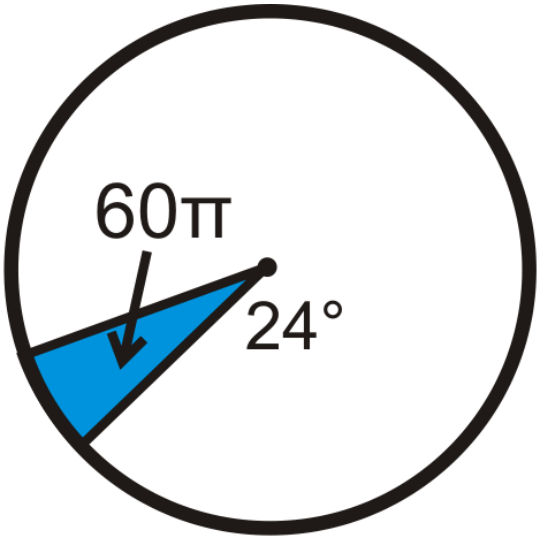

Figure **6.10.14**

8.

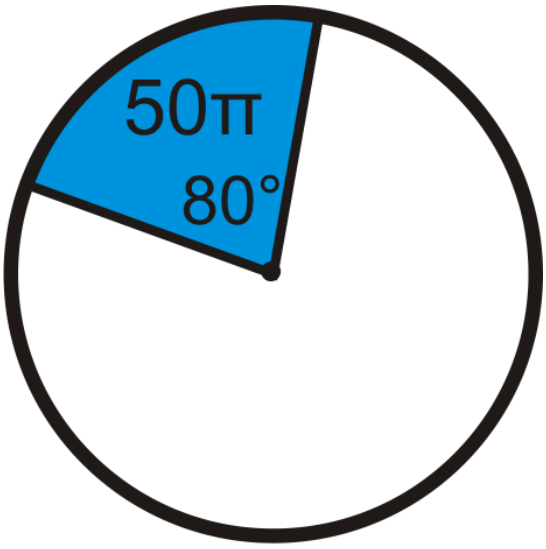

Figure **6.10.15**

Find the central angle of each blue sector. Round any decimal answers to the nearest tenth.

9.

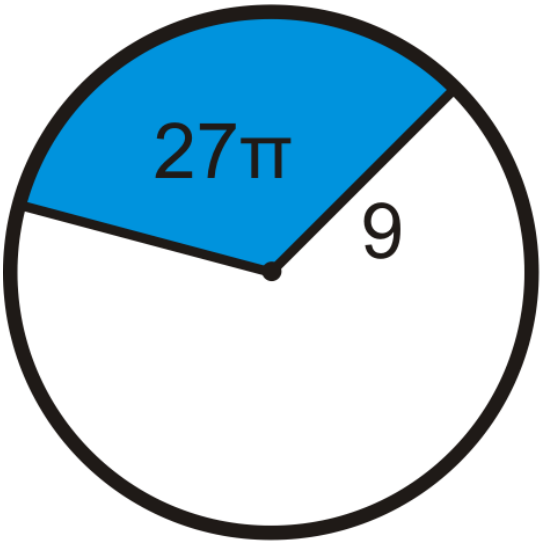

Figure **6.10.16**

10.

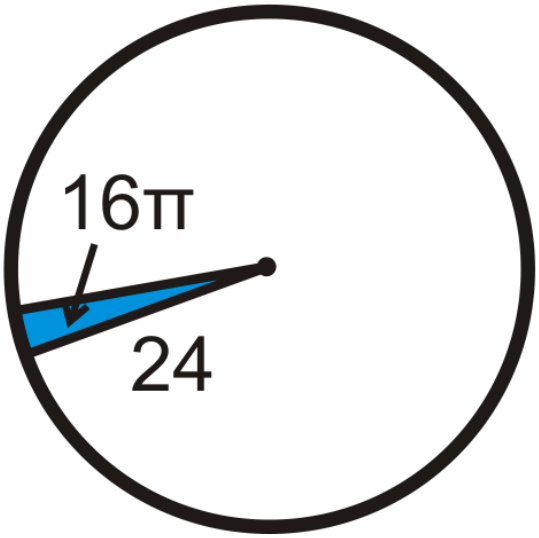

Figure **6.10.17**

11.

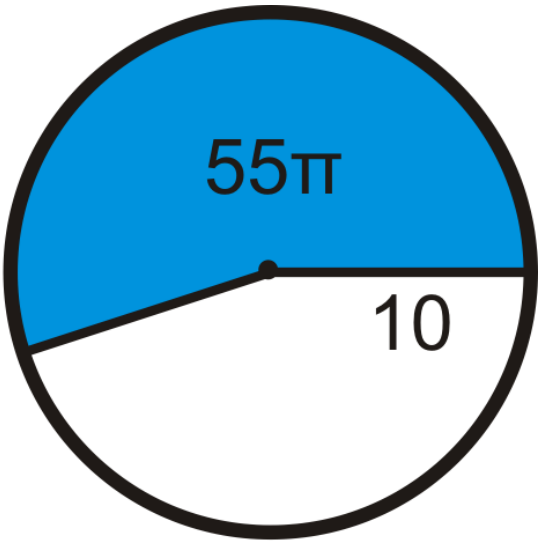

Figure 6.10.18

12. Find the area of the sector in ⊙A. Leave your answer in terms of \pi .

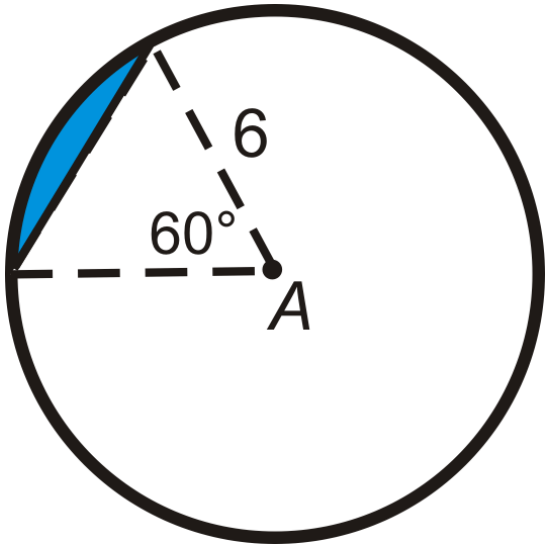

Figure 6.10.19

13. Find the area of the equilateral triangle.
14. Find the area of the segment. Round your answer to the nearest hundredth.
15. Find the area of the sector in ⊙A. Leave your answer in terms of \pi .
16. Find the area of the right triangle.

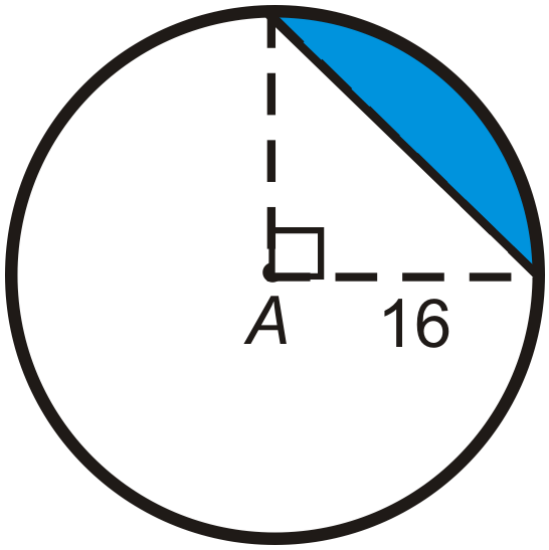

Figure 6.10.20

17. Find the area of the segment. Round your answer to the nearest hundredth.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.11.

Vocabulary

Term	Definition
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
pi	(or \pi) The ratio of the circumference of a circle to its diameter.
radius	The distance from the center to the outer rim of a circle.
Arc	An arc is a section of the circumference of a circle.
arc length	In calculus, arc length is the length of a plane function curve over an interval.
radian	A radian is a unit of angle that is equal to the angle created at the center of a circle whose arc is equal in length to the radius.
Scale Factor	A scale factor is a ratio of the scale to the original or actual dimension written in simplest form.
Sector	A sector of a circle is a portion of a circle contained between two radii of the circle. Sectors can be measured in degrees.
Sector of a Circle	A sector of a circle is the area bounded by two radii and the arc between the endpoints of the radii.

Additional Resources

Video: Area of Sectors and Segments Principles - Basic

Activities: Area of Sectors and Segments Discussion Questions

Study Aids: Circumference and Arc Length Study Guide

Practice: Area of Sectors and Segments

Real World: Watch Out for the Weather!

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6.11: Arc Length

Portion of a circle's circumference.

One way to measure arcs is in degrees. This is called the "**arc** measure" or "degree measure" (see Arcs in Circles). Arcs can also be measured in length, as a portion of the **circumference**. **Arc length** is the length of an arc or a portion of a **circle**'s circumference. The arc length is directly related to the degree arc measure.

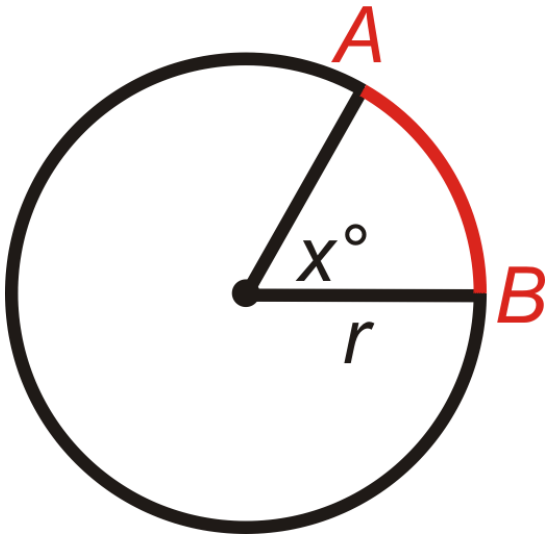

Figure **6.11.1**

Arc Length Formula: The length of $\widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot \pi d$ or $\frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$.

What if you were given the angle measure of a circle's arc? How could you find the length of that arc?

Example 6.11.1

Find the arc length of $\widehat{PQ}$. Leave your answers in terms of π.

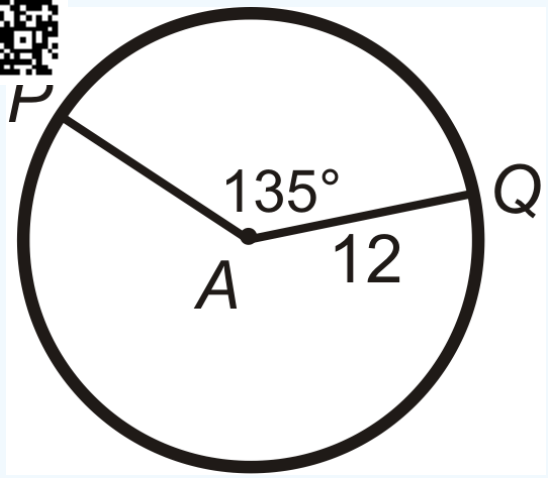

Figure **6.11.2**

Solution

Use the Arc Length formula.

$$\widehat{PQ} = \frac{135}{360} \cdot 2\pi(12)$$
$$\widehat{PQ} = \frac{3}{8} \cdot 24\pi$$
$$\widehat{PQ} = 9\pi$$

Example 6.11.2

A typical large pizza has a **diameter** of 14 inches and is cut into 8 pieces. Think of the crust as the circumference of the pizza. Find the *length* of the crust for the entire pizza. Then, find the length of the crust for one piece of pizza if the entire pizza is cut into 8 pieces.

Figure **6.11.3**

Solution

The entire length of the crust, or the circumference of the pizza, is $14\pi \approx 44in$. In $\frac{1}{8}$ of the pizza, one piece would have $\frac{44}{8} \approx 5.5inches$ of crust.

Example 6.11.3

Find the length of $\widehat{PQ}$. Leave your answer in terms of π.

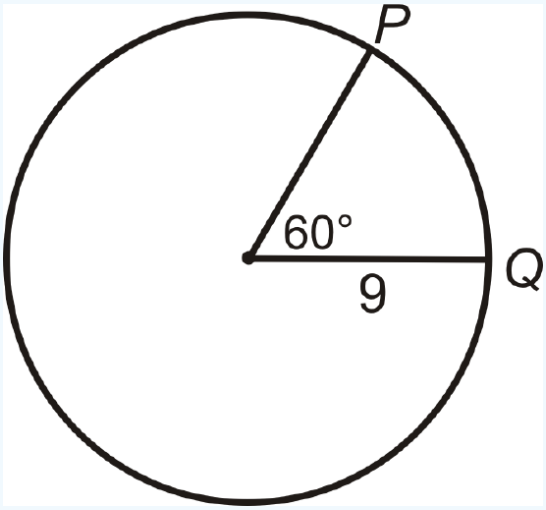

Figure **6.11.4**

Solution

In the picture, the central angle that corresponds with $\overset{\frown}{PQ}$ is $60°$. This means that $m\overset{\frown}{PQ} = 60°$. Think of the arc length as a portion of the circumference. There are $360°$ in a circle, so 60^{\circ}\) would be 16 of that ($60°360° = 16$). Therefore, the length of $\overset{\frown}{PQ}$ is 16 of the circumference. \text{length of } \widehat{PQ}=16\cdot 2\pi (9)=3\pi \text{ units}\).

Example 6.11.4

The arc length of a circle is $\overset{\frown}{AB} = 6\pi$ and is 14 the circumference. Find the **radius** of the circle.

Solution

If 6\pi is 14 the circumference, then the total circumference is $4(6\pi) = 24\pi$. To find the radius, plug this into the circumference formula and solve for r.

$$24\pi = 2\pi r$$
$$12 \text{ units } = r$$

Example 6.11.5

Find the measure of the central angle or $\overset{\frown}{PQ}$.

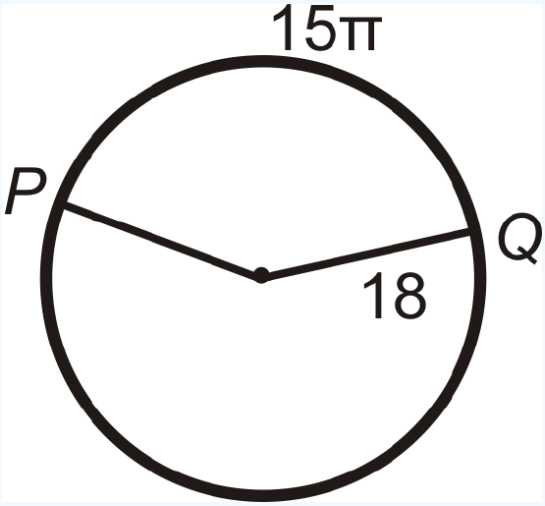

Figure 6.11.5

Solution

Let's plug in what we know to the Arc Length Formula.

$$15\pi = \frac{m\overset{\frown}{PQ}}{360°} \cdot 2\pi(18)$$
$$15 = \frac{m\overset{\frown}{PQ}}{10°}$$
$$150° = m\overset{\frown}{PQ}$$

Review

Find the arc length of $\overset{\frown}{PQ}$ in $\odot A$. Leave your answers in terms of π.

1.

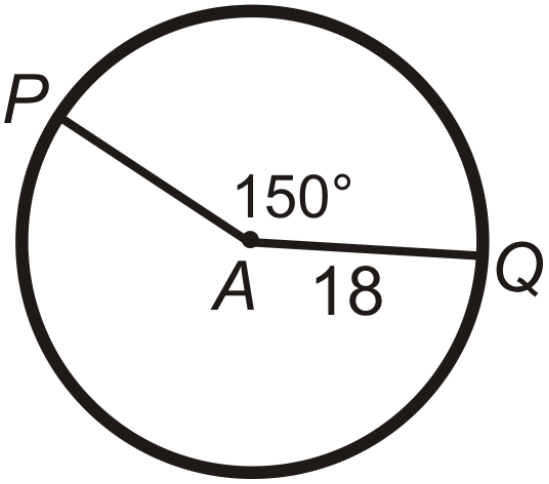

Figure **6.11.6**

2.

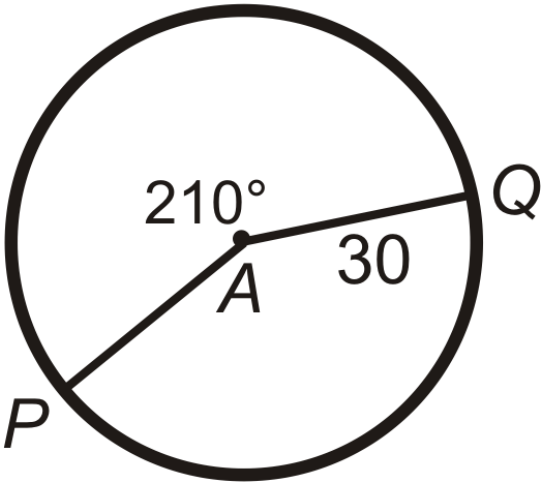

Figure **6.11.7**

3.

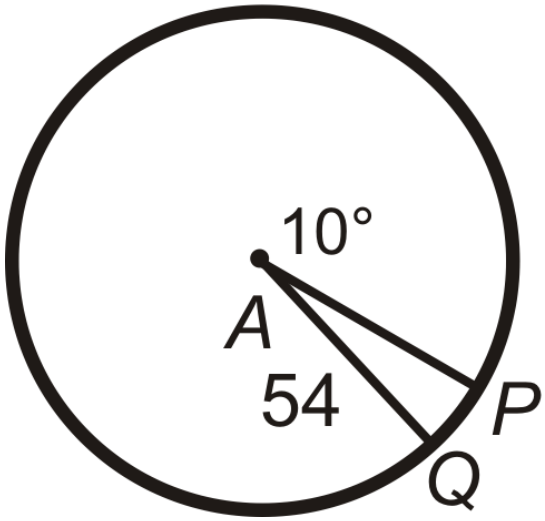

Figure **6.11.8**

4.

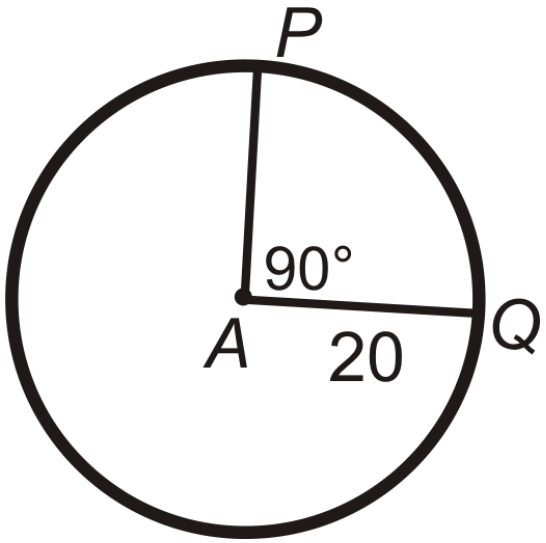

Figure 6.11.9

Find PA (the radius) in $\odot A$. Leave your answer in terms of π.

5.

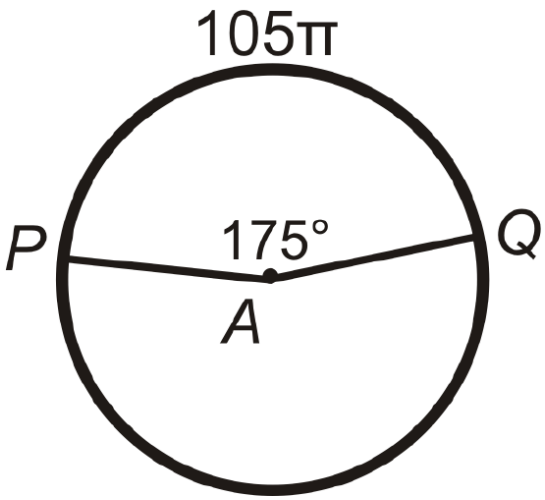

Figure 6.11.11

6.

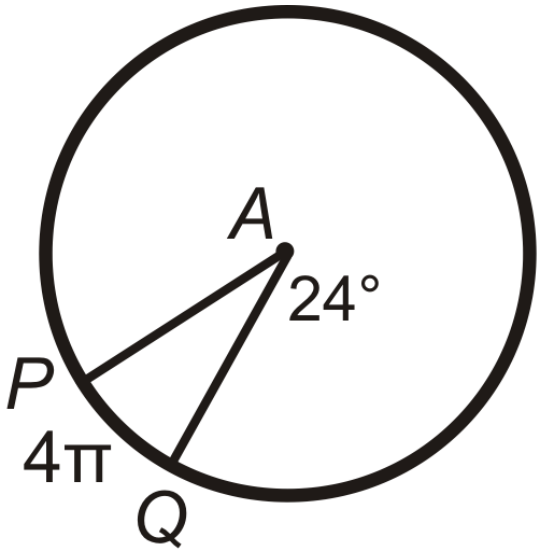

Figure 6.11.12

7.

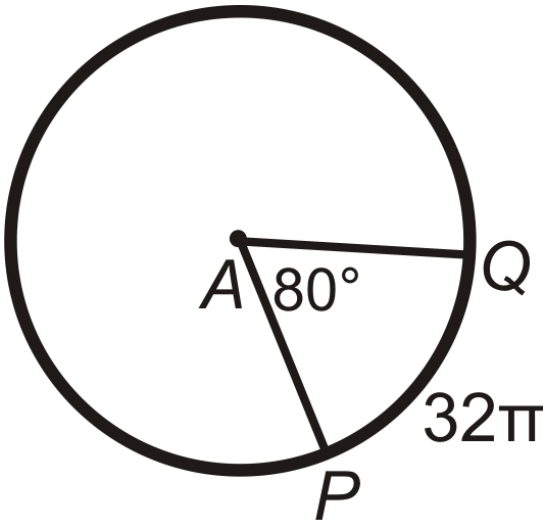

Figure **6.11.13**

Find the central angle or $m\widehat{PQ}$ in $\odot A$. Round any decimal answers to the nearest tenth.

8.

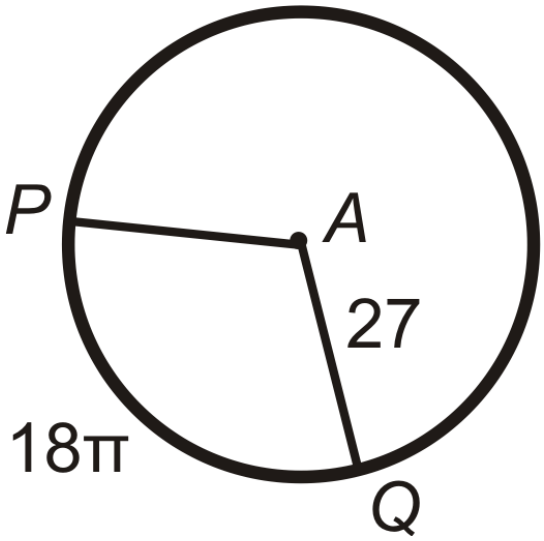

Figure **6.11.14**

9.

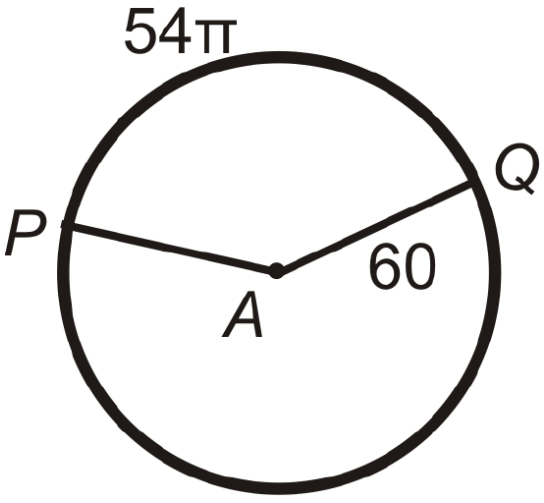

Figure **6.11.15**

10.

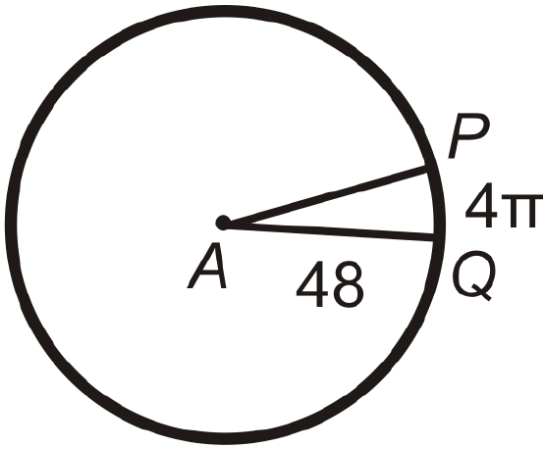

Figure **6.11.16**

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.9.

Vocabulary

Term	Definition
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
pi	(or π) The ratio of the circumference of a circle to its diameter.
radius	The distance from the center to the outer rim of a circle.
Arc	An arc is a section of the circumference of a circle.
arc length	In calculus, arc length is the length of a plane function curve over an interval.
Circumference	The circumference of a circle is the measure of the distance around the outside edge of a circle.
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
radian	A radian is a unit of angle that is equal to the angle created at the center of a circle whose arc is equal in length to the radius.
Sector	A sector of a circle is a portion of a circle contained between two radii of the circle. Sectors can be measured in degrees.

Additional Resources

Video: Arc Length Principles - Basic

Activities: Arc Length Discussion Questions

Study Aids: Circumference and Arc Length Study Guide

Practice: Arc Length

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6.12: Chords and Central Angle Arcs

Arcs determined by angles whose vertex is the center of a circle and chords (segments that connect two points on a circle).

Chords in Circles

Chord Theorems

There are several important theorems about chords that will help you to analyze circles better.

1. **Chord Theorem #1:** In the same **circle** or congruent circles, minor arcs are congruent if and only if their corresponding chords are congruent.

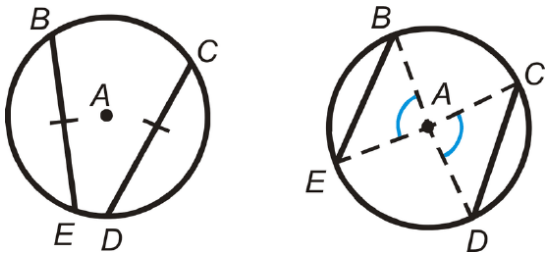

Figure **6.12.1**

In both of these pictures, $\overline{BE} \cong \overline{CD}$ and $\overset{\frown}{BE} \cong \overset{\frown}{CD}$.

2. **Chord Theorem #2:** The perpendicular bisector of a chord is also a **diameter**.

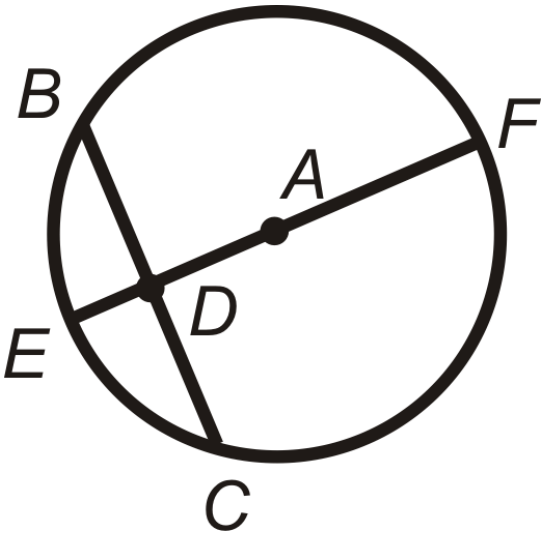

Figure **6.12.2**

If $\overline{AD} \perp \overline{BC}$ and $\overline{BD} \cong \overline{DC}$ then $\overline{EF}$ is a diameter.

3. **Chord Theorem #3:** If a diameter is perpendicular to a chord, then the diameter bisects the chord and its corresponding arc.

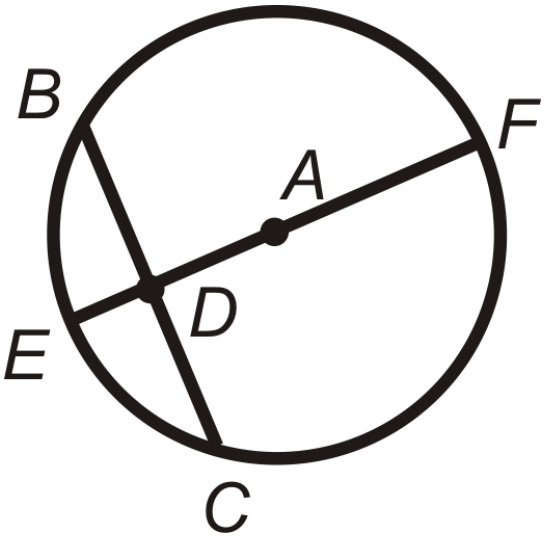

Figure **6.12.3**

If $\overline{EF} \perp \overline{BC}$, then $\overline{BD} \cong \overline{DC}$

4. **Chord Theorem #4:** In the same circle or congruent circles, two chords are congruent if and only if they are equidistant from the center.

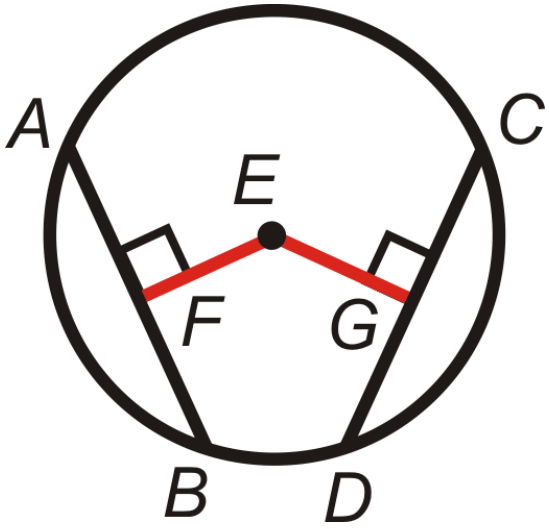

Figure **6.12.4**

The shortest distance from any point to a line is the perpendicular line between them. If $FE = EG$ and $\overline{EF} \perp \overline{EG}$, then $\overline{AB}$ and $\overline{CD}$ are equidistant to the center and $\overline{AB} \cong \overline{CD}$.

What if you were given a circle with two chords drawn through it? How could you determine if these two chords were congruent?

Example 6.12.1

Find the value of x and y

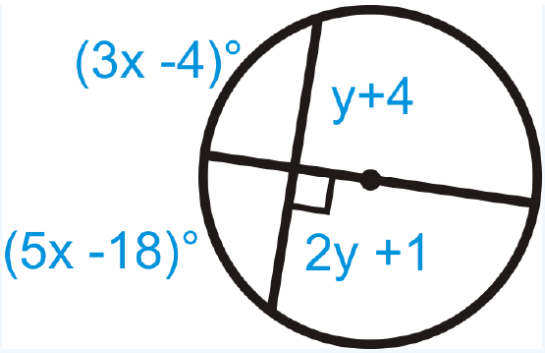

Figure 6.12.5

Solution

The diameter is perpendicular to the chord, which means it bisects the chord and the arc. Set up equations for x and y.

$$\begin{aligned} (3x-4)^\circ &= (5x-18)^\circ \quad y+4 = 2y+1 \\ 14 &= 2x \qquad\qquad\quad 3 = y \\ 7 &= x \end{aligned}$$

Example 6.12.2

$BD = 12$ and $AC = 3$ in $\odot A$. Find the **radius**.

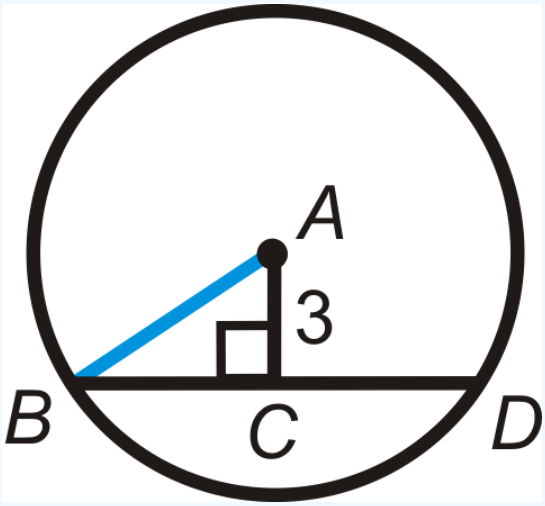

Figure 6.12.6

Solution

First find the radius. $\overline{AB}$ is a radius, so we can use the right triangle \Delta ABC\) with hypotenuse $\overline{AB}$. From Chord Theorem #3, $BC = 6$.

$$\begin{aligned} 3^2 + 6^2 &= AB^2 \\ 9 + 36 &= AB^2 \\ AB &= \sqrt{45} = 3\sqrt{5} \end{aligned}$$

Example 6.12.3

Use $\odot A$ to answer the following.

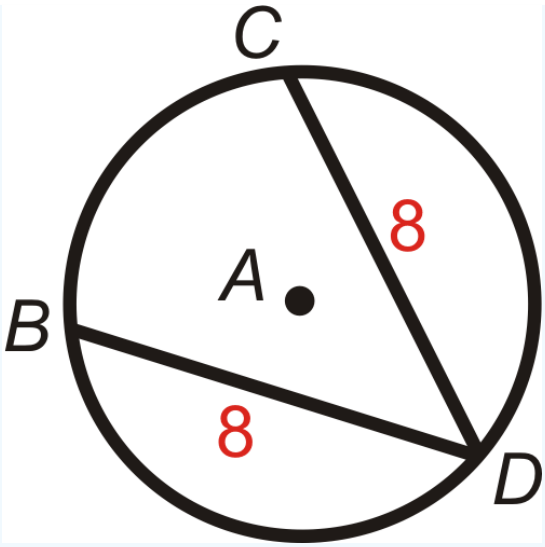

Figure 6.12.7

a. If $m\widehat{BD} = 125°$, find $m\widehat{CD}$.
b. If $m\widehat{BC} = 80°$, find $m\widehat{CD}$.

Solution

1. $BD = CD$, which means the arcs are congruent too. $m\widehat{CD} = 125°$.
2. $m\widehat{CD} \cong m\widehat{BD}$ because $BD = CD$.

$$\begin{aligned} m\widehat{BC} + m\widehat{CD} + m\widehat{BD} &= 360° \\ 80° + 2m\widehat{CD} &= 360° \\ 2m\widehat{CD} &= 280° \\ m\widehat{CD} &= 140° \end{aligned}$$

Example 6.12.4

Find the values of x and y.

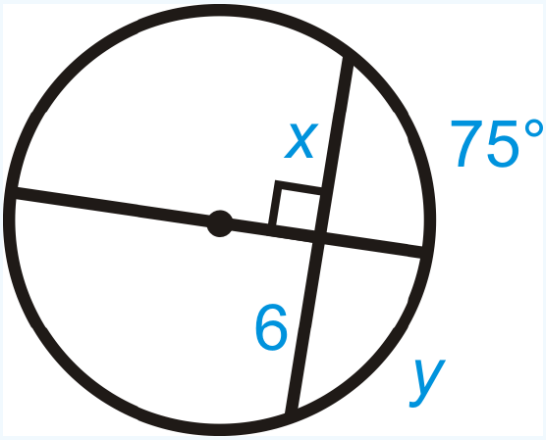

Figure 6.12.8

Solution

The diameter is perpendicular to the chord. From Chord Theorem #3, $x = 6$ and $y = 75°$.

Example 6.12.5

Find the value of x.

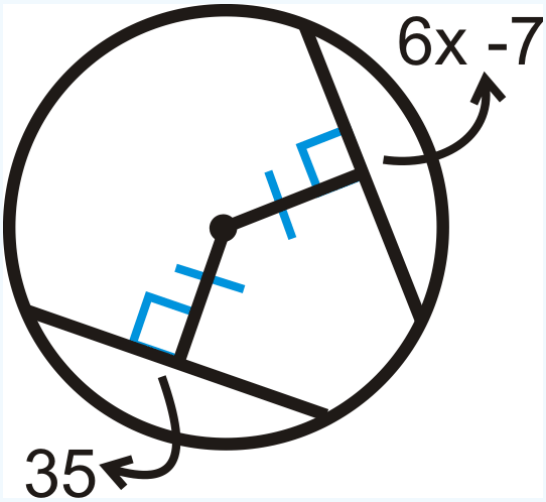

Figure 6.12.9

Solution

Because the distance from the center to the chords is equal, the chords are congruent.

$$6x - 7 = 35$$
$$6x = 42$$
$$x = 7$$

Review

Fill in the blanks.

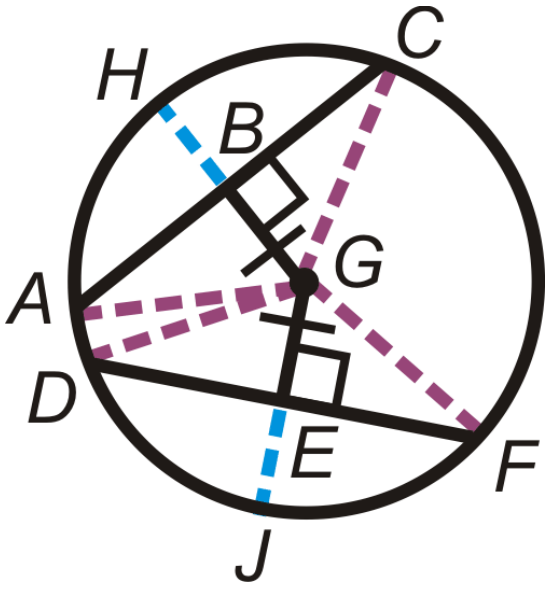

Figure 6.12.10

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2. '_' allowed only in math mode
3. '_' allowed only in math mode
4. '_' allowed only in math mode
5. '_' allowed only in math mode
6. '_' allowed only in math mode

7. List all the congruent radii in $\odot G$.

Find the value of the indicated arc in $\odot A$.

8. $m\widehat{BC}$

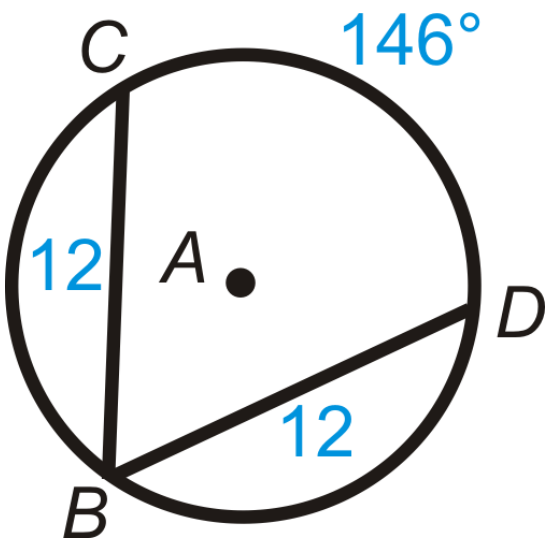

Figure **6.12.11**

9. $m\widehat{BD}$

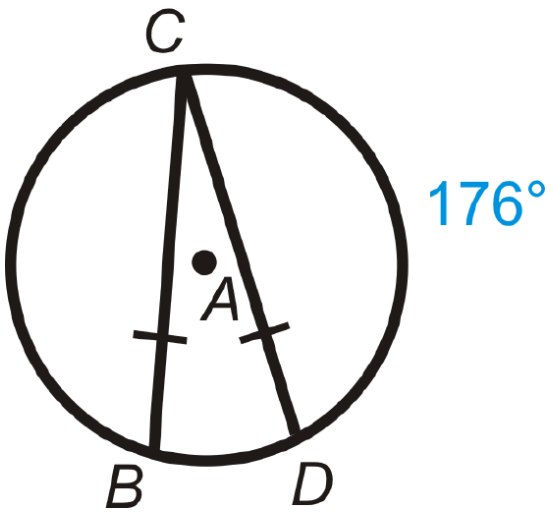

Figure **6.12.12**

10. $m\widehat{BC}$

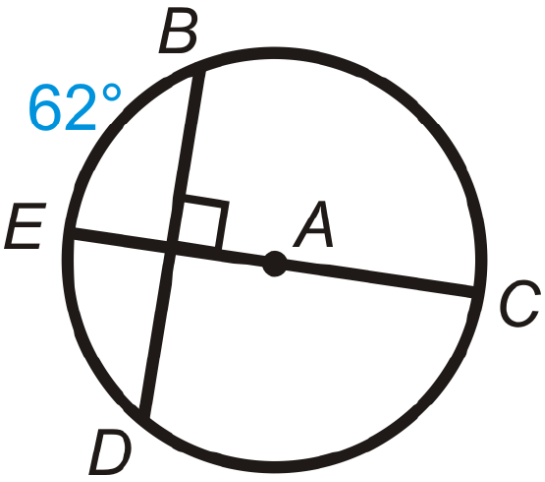

Figure **6.12.13**

11. $m\widehat{BD}$

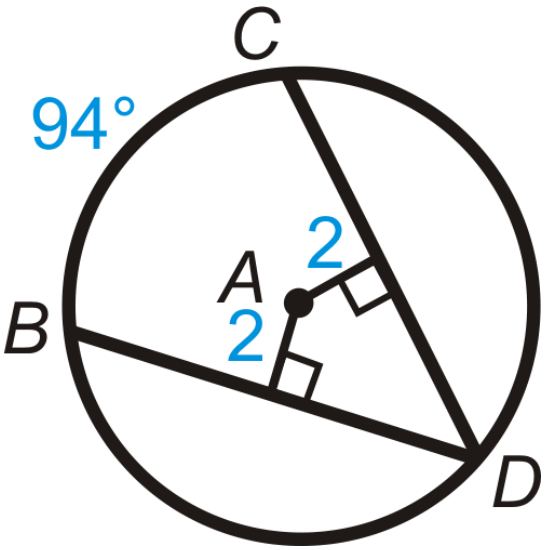

Figure **6.12.14**

12. $m\widehat{BD}$

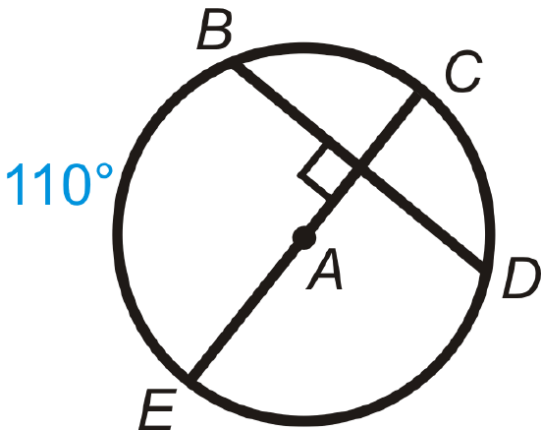

Figure **6.12.15**

13. $m\widehat{BD}$

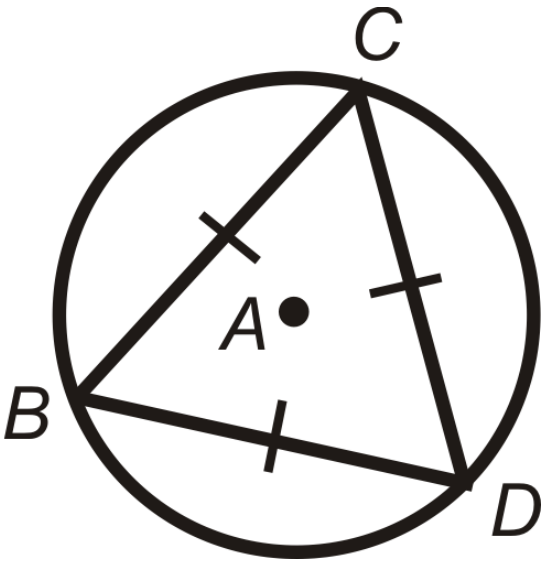

Figure **6.12.16**

Find the value of x and/or y.

14.

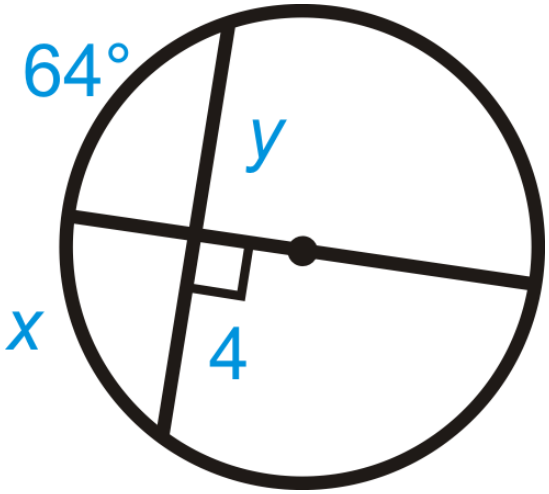

Figure **6.12.17**

15.

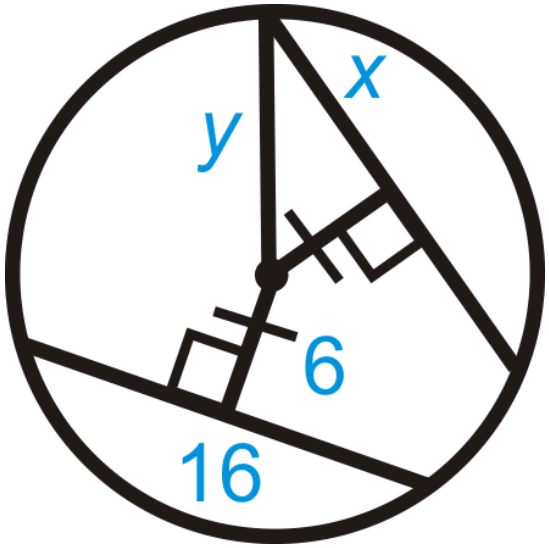

Figure **6.12.18**

16.

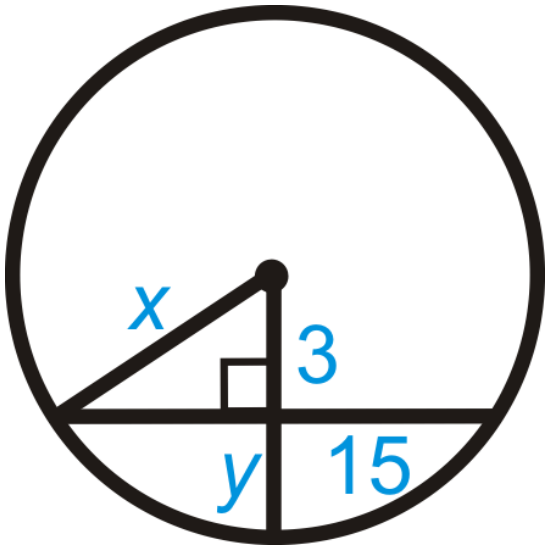

Figure 6.12.19

17. $AB = 32$

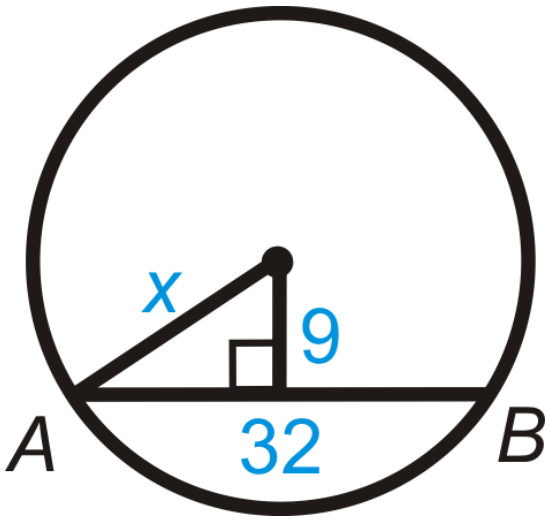

Figure 6.12.20

18.

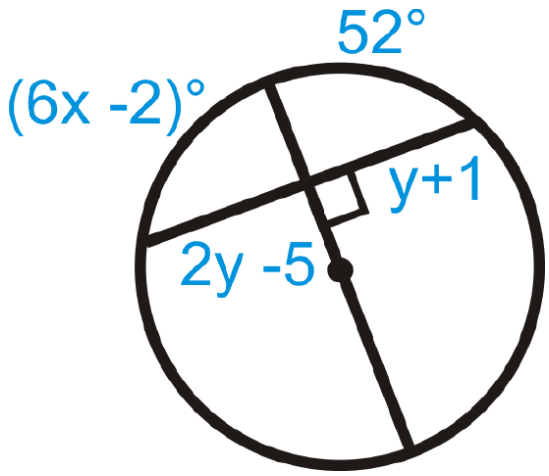

Figure 6.12.21

19.

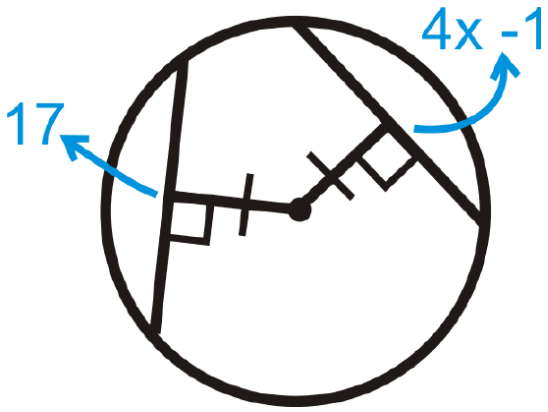

Figure 6.12.22

20.

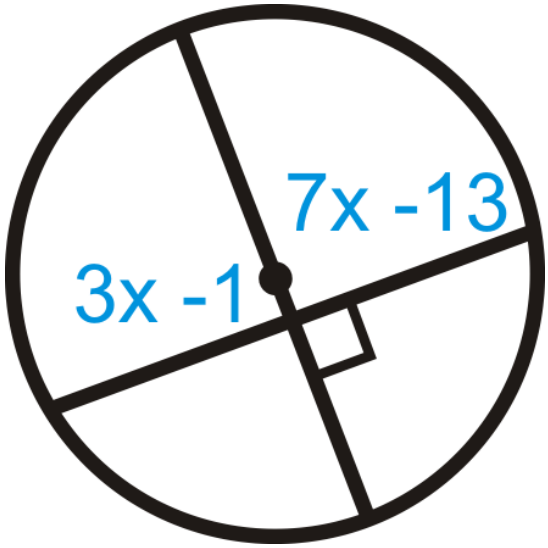

Figure 6.12.23

21.

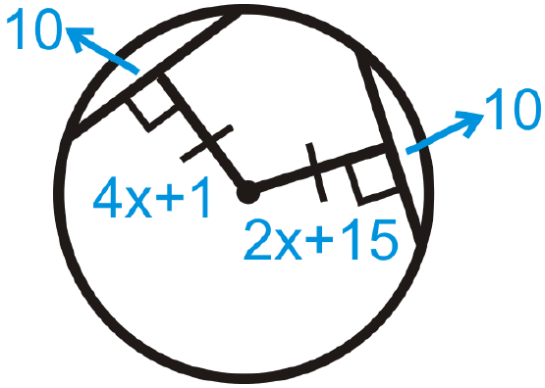

Figure 6.12.24

22. $AB = 20$

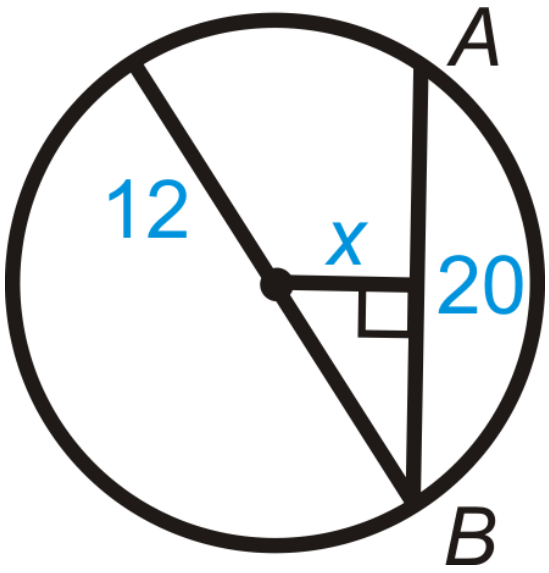

Figure **6.12.25**

23. Find $\widehat{mAB}$ in Question 17. Round your answer to the nearest tenth of a degree.
24. Find $\widehat{mAB}$ in Question 22. Round your answer to the nearest tenth of a degree.

In problems 25-27, what can you conclude about the picture? State a theorem that justifies your answer. You may assume that A is the center of the circle.

25.

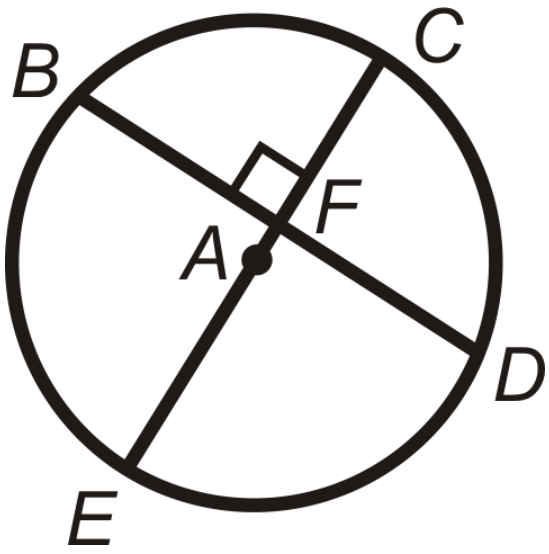

Figure **6.12.26**

26.

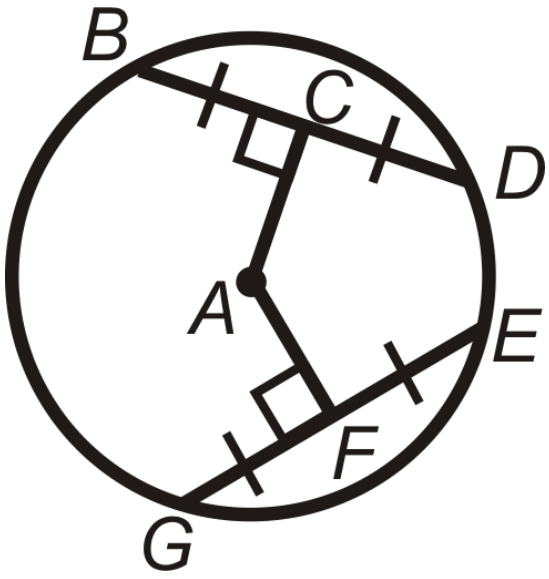

Figure **6.12.27**

27.

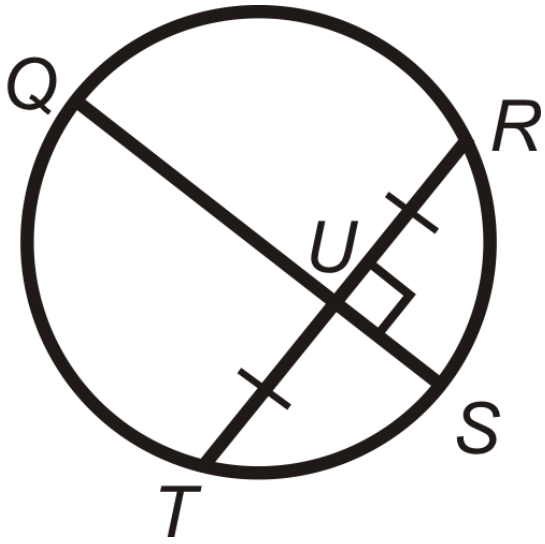

Figure **6.12.28**

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.4.

Vocabulary

Term	Definition
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
radius	The distance from the center to the outer rim of a circle.

Additional Resources

Video: Chords in Circles Principles - Basic

Activities: Chords in Circles Discussion Questions

Study Aids: Circles: Segments and Lengths Study Guide

Practice: Chords and Central Angle Arcs

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6.13: Segments from Chords

Apply the Intersecting Chords Theorem.

When we have two chords that intersect inside a **circle**, as shown below, the two triangles that result are similar.

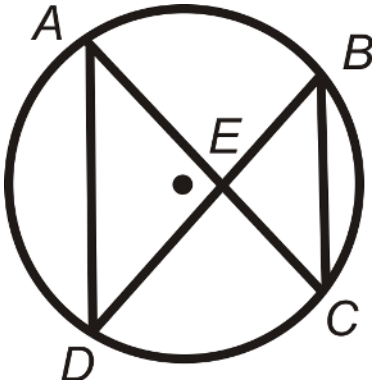

Figure **6.13.1**

This makes the corresponding sides in each triangle proportional and leads to a relationship between the segments of the chords, as stated in the **Intersecting Chords Theorem**.

Intersecting Chords Theorem: If two chords intersect inside a circle so that one is divided into segments of length a and b and the other into segments of length c and d then $ab = cd$.

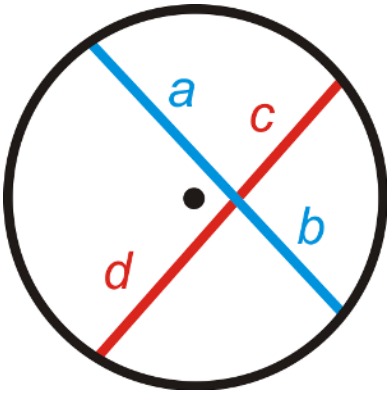

Figure **6.13.2**

What if you were given a circle with two chords that intersect each other? How could you use the length of some of the segments formed by their intersection to determine the lengths of the unknown segments?

Example 6.13.1

Find x. Simplify any rac

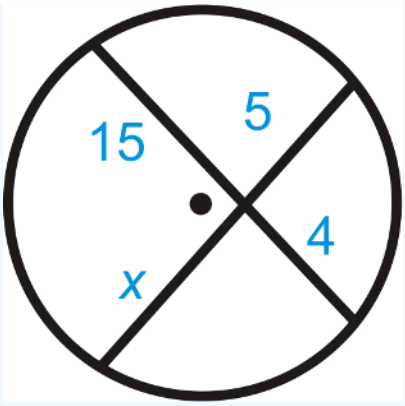

Figure 6.13.3

Solution

Use the Intersecting Chords Theorem.

$$15 \cdot 4 = 5 \cdot x$$

$$60 = 5x$$

$$x = 12$$

Example 6.13.2

Find x. Simplify any radicals.

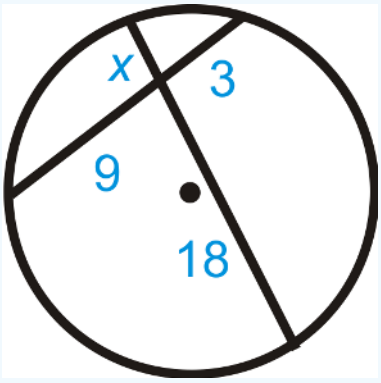

Figure 6.13.4

Solution

Use the Intersecting Chords Theorem.

$$18 \cdot x = 9 \cdot 3$$
$$18x = 27$$
$$x = 1.5$$

Example 6.13.3

Find x in each diagram below.

a.

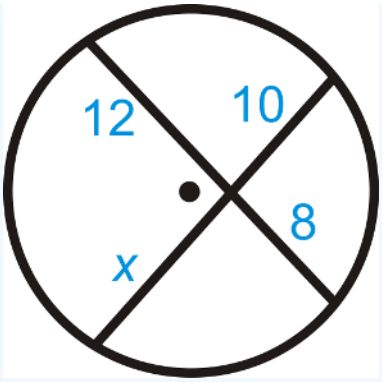

Figure 6.13.5

b.

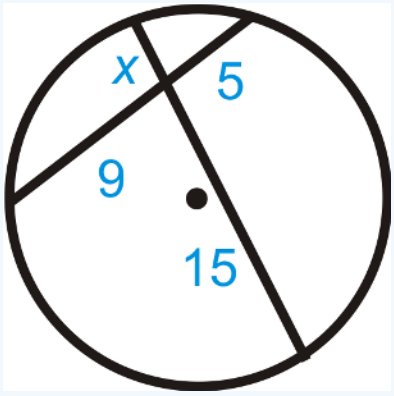

Figure \(\PageIndex{6}\

Solution

a. Use the formula from the Intersecting Chords Theorem.

$$12 \cdot 8 = 10 \cdot x$$
$$96 = 10x$$
$$9.6 = x$$

a. Use the formula from the Intersecting Chords Theorem.

$$x \cdot 15 = 5 \cdot 9$$
$$15x = 45$$
$$x = 3$$

Example 6.13.4

Solve for x in each diagram below.

a.

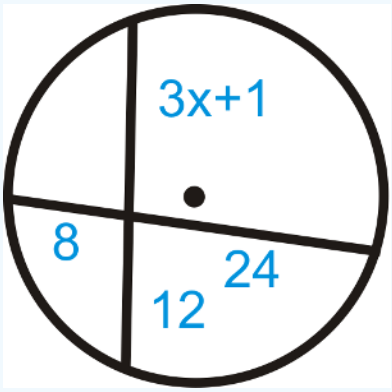

Figure 6.13.7

b.

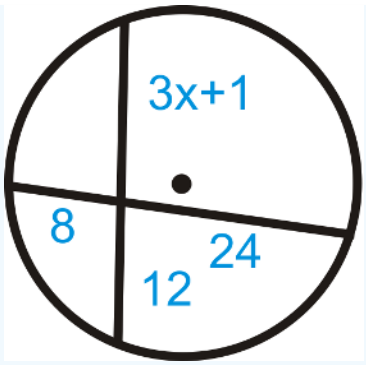

Figure **6.13.8**

Solution

a. Use the Intersecting Chords Theorem.

$$\begin{aligned} 8 \cdot 24 &= (3x+1) \cdot 12 \\ 192 &= 36x + 12 \\ 180 &= 36x \\ 5 &= x \end{aligned}$$

b. Use the Intersecting Chords Theorem.

$$\begin{aligned} (x-5)21 &= (x-9)24 \\ 21x - 105 &= 24x \\ 111 &= 3x \\ 37 - 216 &= x \end{aligned}$$

Example 6.13.5

Ishmael found a broken piece of a CD in his car. He places a ruler across two points on the rim, and the length of the **chord** is 9.5 cm. The distance from the midpoint of this chord to the nearest point on the rim is 1.75 cm. Find the **diameter** of the CD.

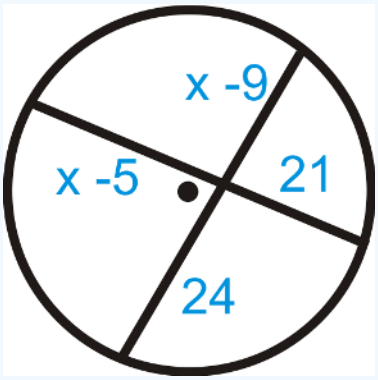

Figure **6.13.9**

Solution

Think of this as two chords intersecting each other. If we were to extend the 1.75 cm segment, it would be a diameter. So, if we find x in the diagram below and add it to 1.75 cm, we would find the diameter.

$$\begin{aligned} 4.25 \cdot 4.25 &= 1.75 \cdot x \\ 18.0625 &= 1.75x \end{aligned}$$

$$x \approx 10.3cm, making the diameter 10.3 + 1.75 \approx 12cm, which\ is\ the\ actual\ diameter\ of\ a\ CD.$$

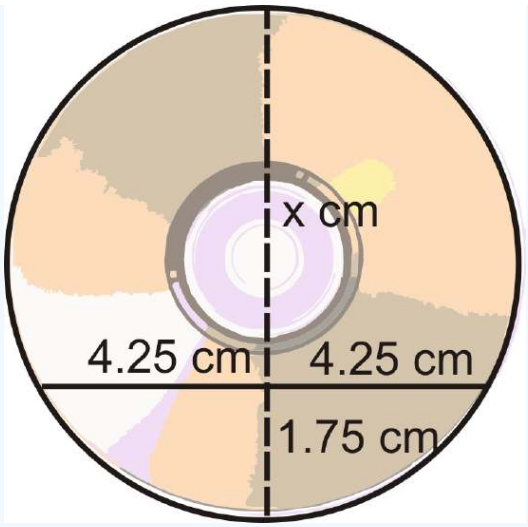

Figure **6.13.10**

Review

Fill in the blanks for each problem below and then solve for the missing segment.

1.

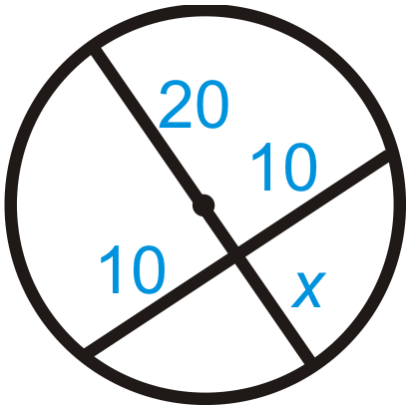

Figure **6.13.11**

Missing open brace for subscript

2.

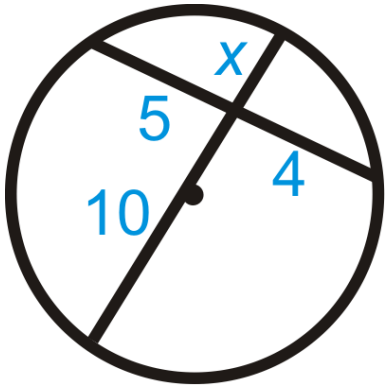

Figure **6.13.12**

'_' allowed only in math mode

Find x in each diagram below. Simplify any radicals.

3.

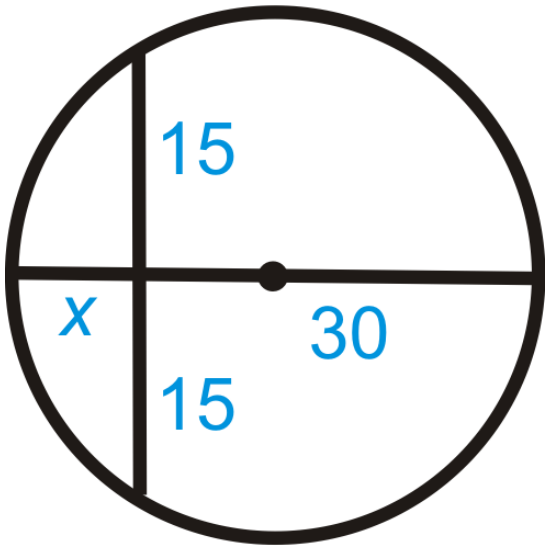

Figure **6.13.13**

4.

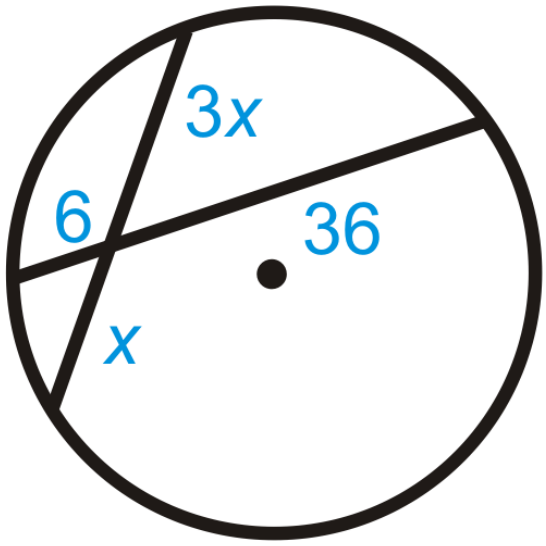

Figure **6.13.14**

5.

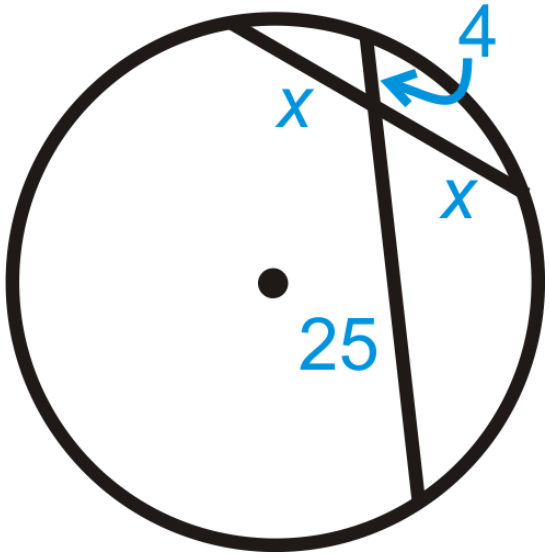

Figure **6.13.15**

Find the value of x.

6.

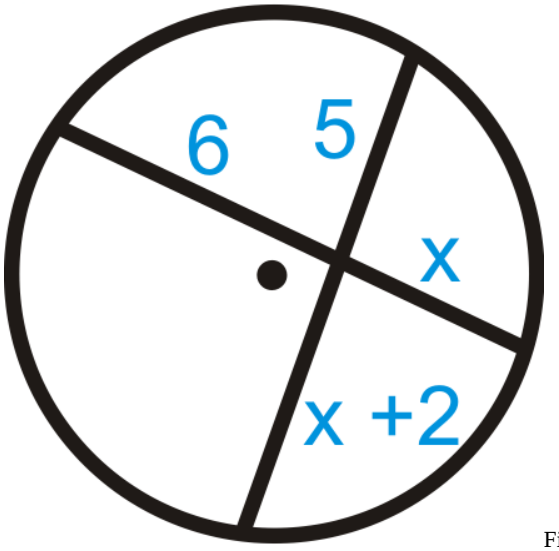

Figure **6.13.16**

7.

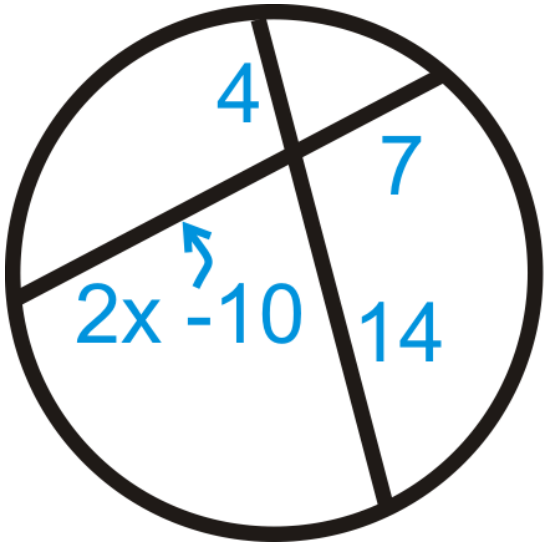

Figure **6.13.17**

8.

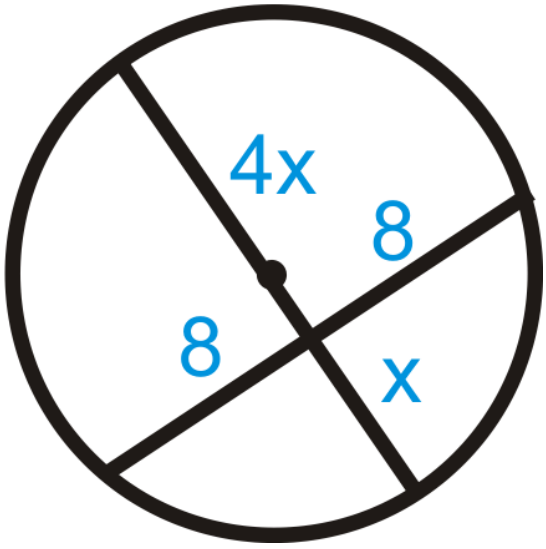

Figure **6.13.18**

9. Suzie found a piece of a broken plate. She places a ruler across two points on the rim, and the length of the chord is 6 inches. The distance from the midpoint of this chord to the nearest point on the rim is 1 inch. Find the diameter of the plate.
10. Fill in the blanks of the proof of the Intersecting Chords Theorem.

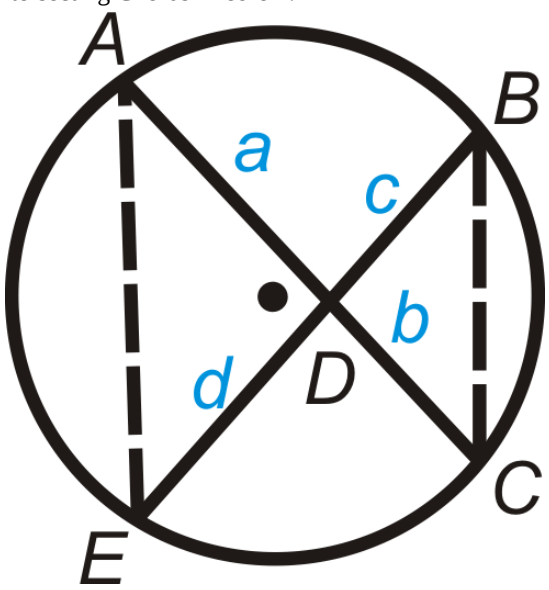

Figure **6.13.19**

Given: Intersecting chords $\overline{AC}$ and $\overline{BE}$.

Prove: $ab = cd$

Statement	***Reason***
1. Intersecting chords $\overline{AC}$ and $\overline{BE}$ with segments a, b, c, and d.	1.
2.	2. Congruent Inscribed Angles Theorem
3. $\Delta ADE \sim \Delta BDC$	3.
4.	4. Corresponding parts of similar triangles are proportional
5. $ab = cd$	5.

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
inscribed angle	An angle with its vertex on the circle and whose sides are chords.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
radius	The distance from the center to the outer rim of a circle.
Intersecting Chords Theorem	According to the Intersecting Chords Theorem, if two chords intersect inside a circle so that one is divided into segments of length a and b and the other into segments of length c and d, then $ab = cd$.

Additional Resources

Video: Segments from Chords Principles - Basic

Activities: Segments from Chords Discussion Questions

Study Aids: Circles: Segments and Lengths Study Guide

Practice: Segments from Chords

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6.14: Inscribed Angles in Circles

Vertex on a circle and chords as sides, and whose measure equals half the intercepted arc.

An **inscribed angle** is an angle with its vertex on the **circle** and whose sides are chords. The **intercepted arc** is the **arc** that is inside the inscribed angle and whose endpoints are on the angle. The vertex of an inscribed angle can be anywhere on the circle as long as its sides intersect the circle to form an intercepted arc.

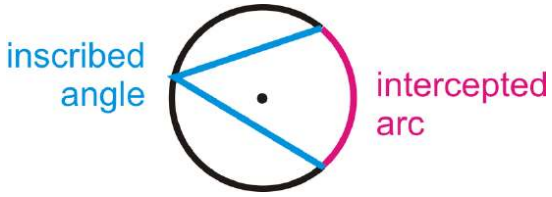


Figure **6.14.1**

The **Inscribed Angle Theorem** states that the measure of an inscribed angle is half the measure of its intercepted arc.

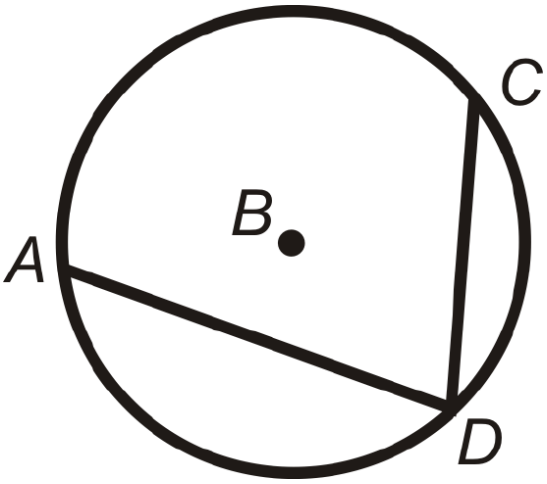


Figure **6.14.2**

$m\angle ADC = \frac{1}{2}m\overset{\frown}{AC}$ and $m\overset{\frown}{AC} = 2m\angle ADC$

Inscribed angles that intercept the same arc are congruent. This is called the **Congruent Inscribed Angles Theorem** and is shown below.

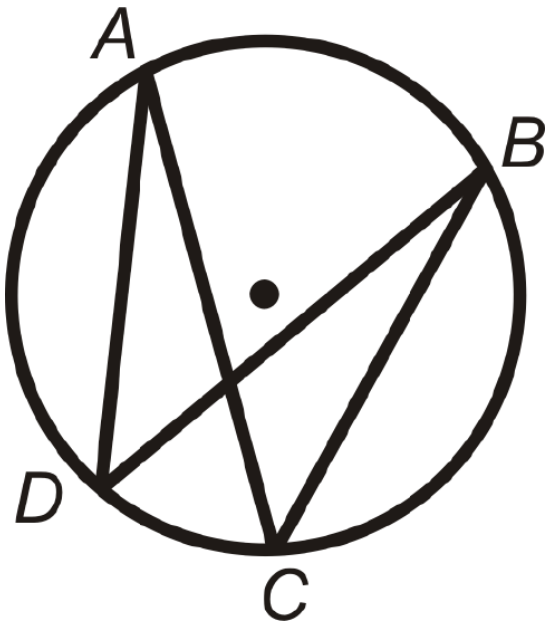


Figure **6.14.3**

$\angle ADB$ and $\angle ACB$ intercept $\overset{\frown}{AB}$, so $m\angle ADB = m\angle ACB$. Similarly, $\angle DAC$ and $\angle DBC$ intercept $\overset{\frown}{DC}$, so $m\angle DAC = m\angle DBC$.

An angle **intercepts** a semicircle if and only if it is a right angle (**Semicircle Theorem**). Anytime a right angle is inscribed in a circle, the endpoints of the angle are the endpoints of a **diameter** and the diameter is the hypotenuse.

What if you had a circle with two chords that share a common endpoint? How could you use the arc formed by those chords to determine the measure of the angle those chords make inside the circle?

Example 6.14.1

Find $m\overset{\frown}{DC}$ and $m\angle ADB$

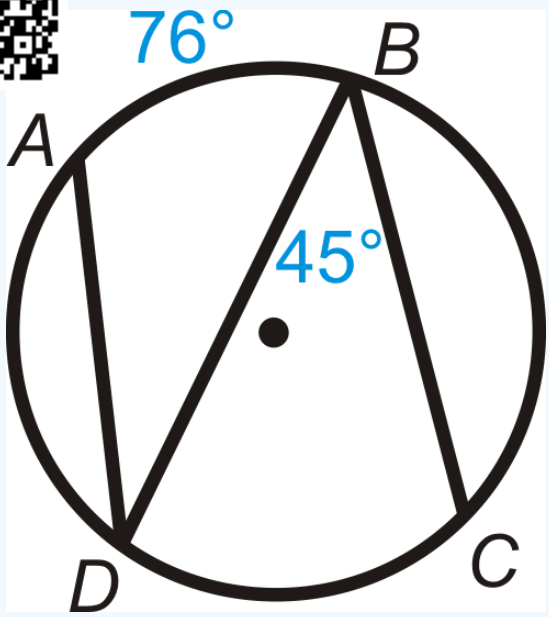


Figure **6.14.4**

Solution

From the Inscribed Angle Theorem:

$$m\overset{\frown}{DC} = 2 \cdot 45^\circ = 90^\circ$$
$$m\angle ADB = 12 \cdot 76^\circ = 38^\circ$$

Example 6.14.2

Find $m\angle ADB$ and $m\angle ACB$.

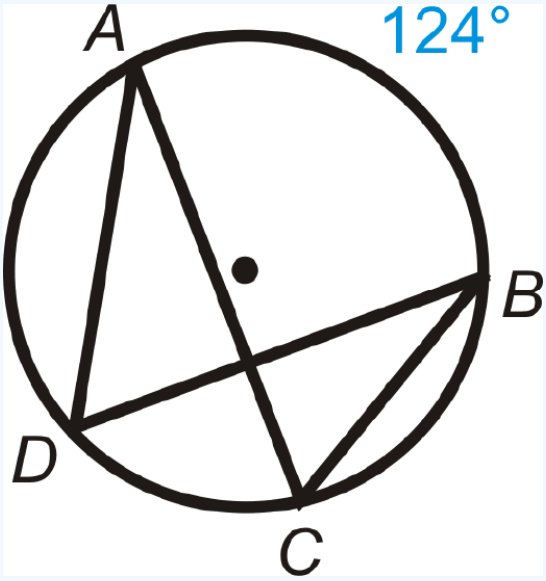


Figure **6.14.5**

Solution

The intercepted arc for both angles is $\overset{\frown}{AB}$. Therefore,

$$m\angle ADB = 12 \cdot 124^\circ = 62^\circ$$
$$m\angle ACB = 12 \cdot 124^\circ = 62^\circ$$

Example 6.14.3

Find $m\angle DAB$ in $\odot C$.

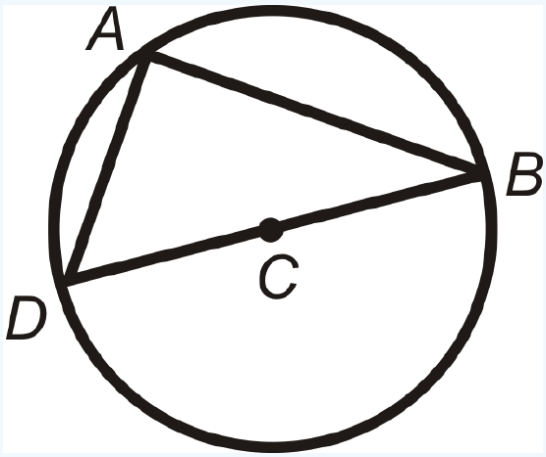


Figure **6.14.6**

Solution

C is the center, so $\overline{DB}$ is a diameter. $\angle DAB$'s endpoints are on the diameter, so the **central angle** is 180°.

$$m\angle DAB = \frac{1}{2} \cdot 180^\circ = 90^\circ.$$

Example 6.14.4

Fill in the blank: An inscribed angle is _____ the measure of the intercepted arc.

Solution

half

Example 6.14.5

Fill in the blank: A central angle is _____ the measure of the intercepted arc.

Solution

equal to

Review

Fill in the blanks.

1. An angle inscribed in a _____ is $90°$.
2. Two inscribed angles that intercept the same arc are _____.
3. The sides of an inscribed angle are _____.
4. Draw inscribed angle $\angle JKL$ in $\odot M$. Then draw central angle $\angle JML$. How do the two angles relate?

Find the value of x and/or y in $\odot A$.

5.

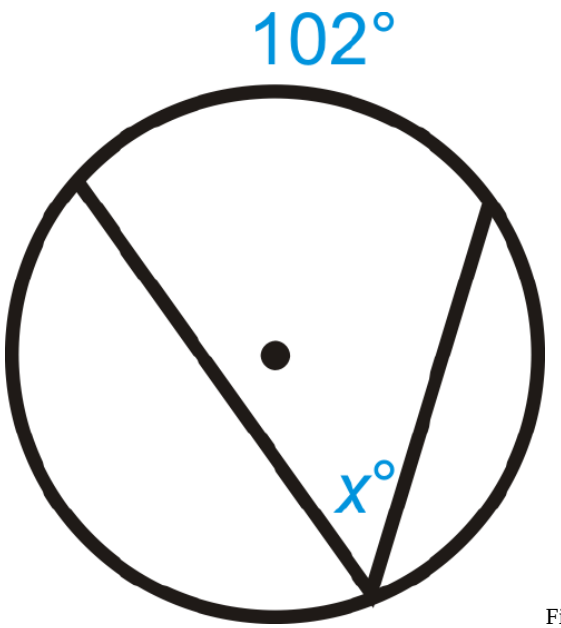


Figure 6.14.7

6.

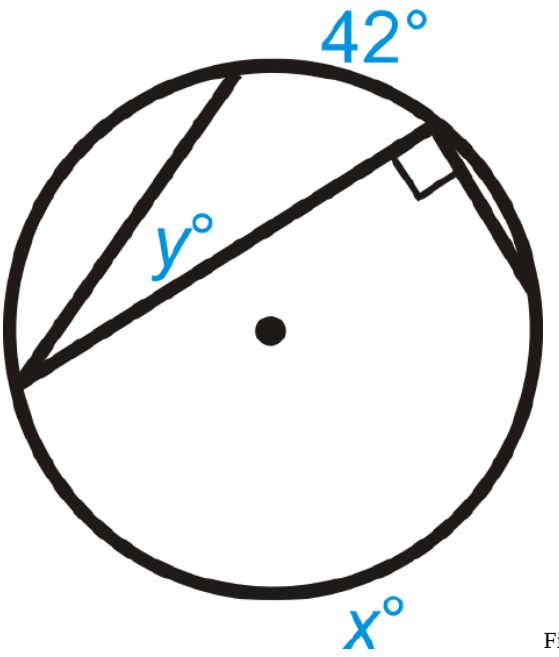


Figure **6.14.8**

7.

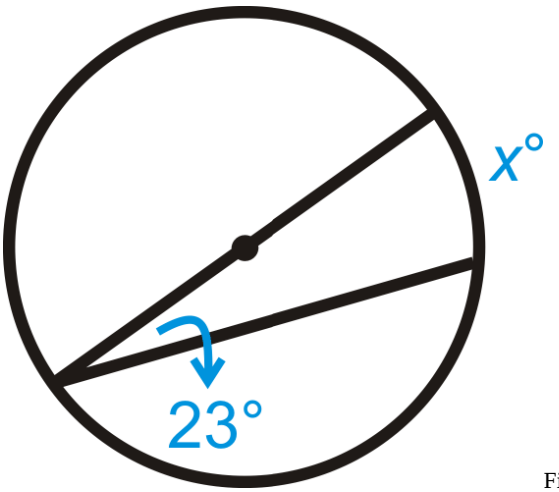


Figure **6.14.9**

8.

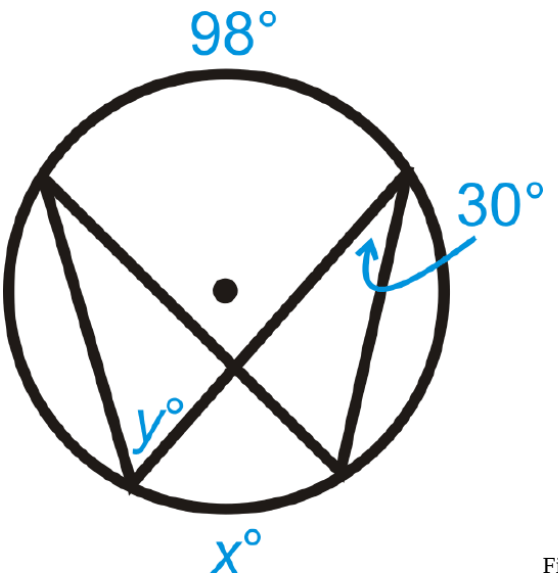
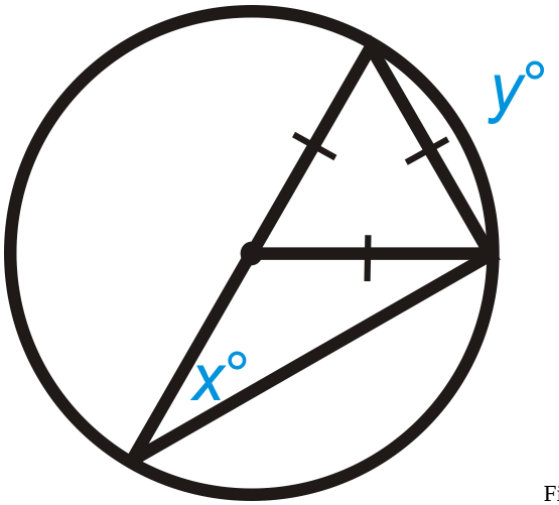
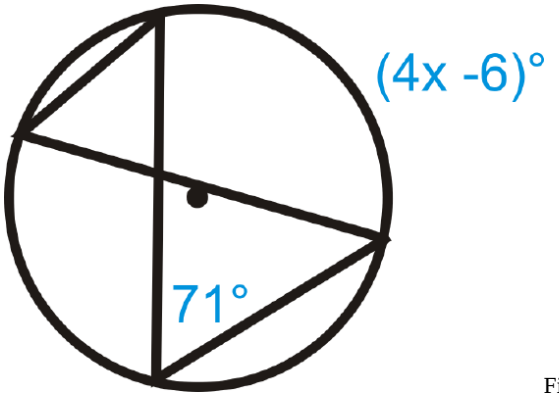


Figure **6.14.10**

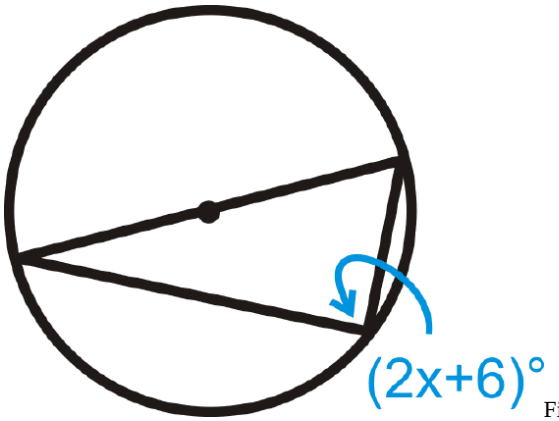


9. Figure 6.14.11

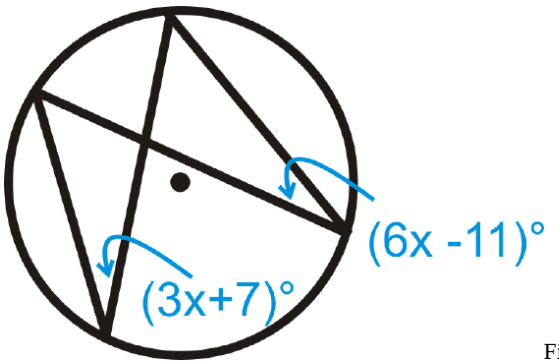
Solve for x.



10. Figure 6.14.12



11. Figure 6.14.13



12. Figure 6.14.14

13.

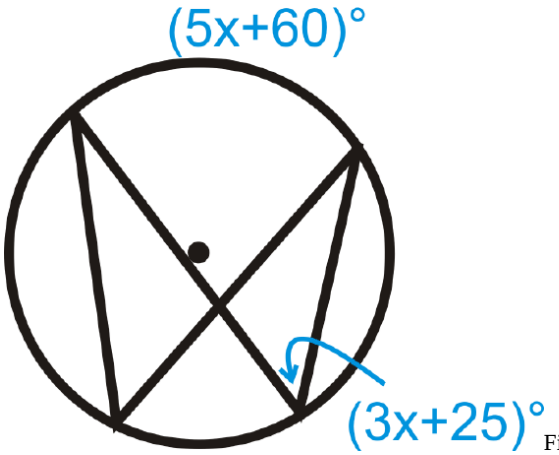


Figure **6.14.15**

14. Fill in the blanks of the Inscribed Angle Theorem proof.

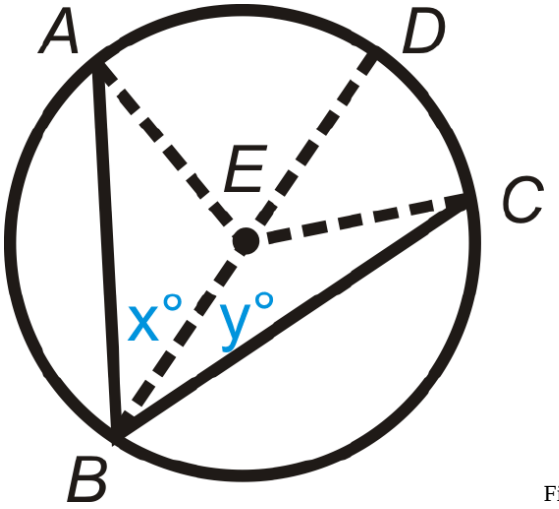


Figure **6.14.16**

Given: Inscribed $\angle ABC$ and diameter $\overline{BD}$

Prove: \(m\angle ABC=12m\widehat{AC}

Statement	*Reason*
1. Inscribed $\angle ABC$ and diameter $\overline{BD}$ $m\angle ABE = x°$ and $m\angle CBE = y°$	1.
2. $x° + y° = m\angle ABC$	2.
3.	3. All radii are congruent
4.	4. Definition of an isosceles triangle
5. $m\angle EAB = x°$ and $m\angle ECB = y°$	5.
6. $m\angle AED = 2x°$ and $m\angle CED = 2y°$	6.
7. $m\widehat{AD} = 2x°$ and $m\widehat{DC} = 2y°$	7.
8.	8. Arc Addition Postulate
9. $m\widehat{AC} = 2x° + 2y°$	9.
10.	10. Distributive PoE
11. $m\widehat{AC} = 2m\angle ABC$	11.
12. $m\angle ABC = \frac{1}{2}m\widehat{AC}$	12.

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
Inscribed Angle	An inscribed angle is an angle with its vertex on the circle. The measure of an inscribed angle is half the measure of its intercepted arc.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
radius	The distance from the center to the outer rim of a circle.
Arc	An arc is a section of the circumference of a circle.
Intercepts	The intercepts of a curve are the locations where the curve intersects the x and y axes. An x intercept is a point at which the curve intersects the x-axis. A y intercept is a point at which the curve intersects the y-axis.
Inscribed Angle Theorem	The Inscribed Angle Theorem states that the measure of an inscribed angle is half the measure of its intercepted arc.
Semicircle Theorem	The Semicircle Theorem states that any time a right angle is inscribed in a circle, the endpoints of the angle are the endpoints of a diameter and the diameter is the hypotenuse.

Additional Resources

Video: Inscribed Angles in Circles Principles - Basic

Activities: Inscribed Angles in Circles Discussion Questions

Study Aids: Inscribed in Circles Study Guide

Practice: Inscribed Angles in Circles

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6.15: Inscribed Quadrilaterals in Circles

Quadrilaterals with every vertex on a circle and opposite angles that are supplementary.

An **inscribed polygon** is a polygon where every vertex is on the **circle**, as shown below.

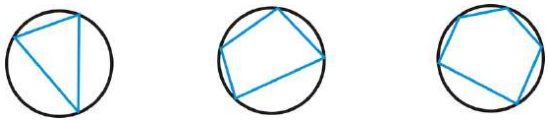

Figure 6.15.1

For inscribed quadrilaterals in particular, the opposite angles will always be supplementary.

Inscribed Quadrilateral Theorem: A quadrilateral can be inscribed in a circle if and only if the opposite angles are supplementary.

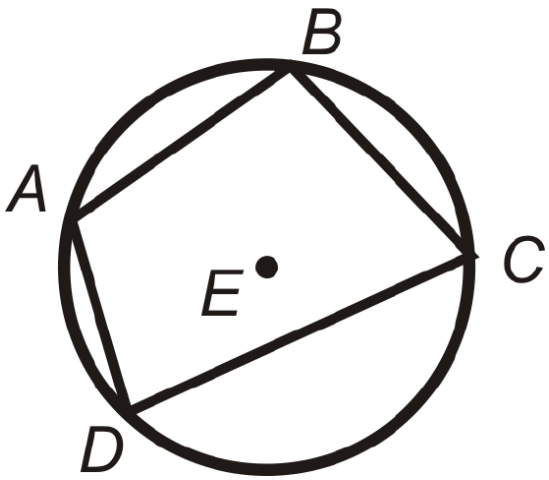

Figure 6.15.2

If $ABCD$ is inscribed in $\odot E$, then $m\angle A + m\angle C = 180°$ and $m\angle B + m\angle D = 180°$. Conversely, If $m\angle A + m\angle C = 180°$ and $m\angle B + m\angle D = 180°$, then $ABCD$ is inscribed in $\odot E$.

What if you were given a circle with a quadrilateral inscribed in it? How could you use information about the arcs formed by the quadrilateral and/or the quadrilateral's angle measures to find the measure of the unknown quadrilateral angles?

Example 6.15.1

a.

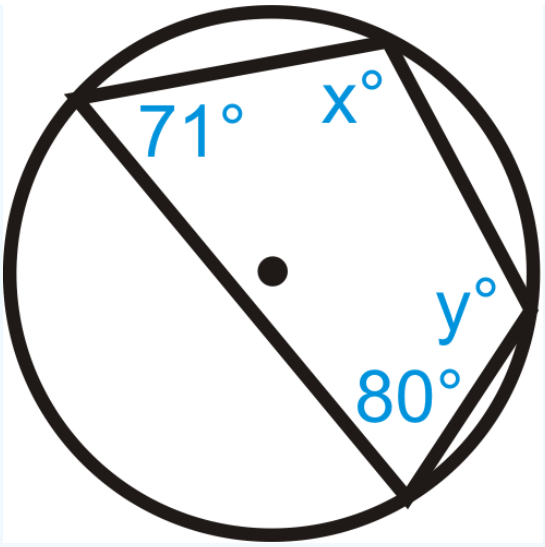

Figure 6.15.3

b.

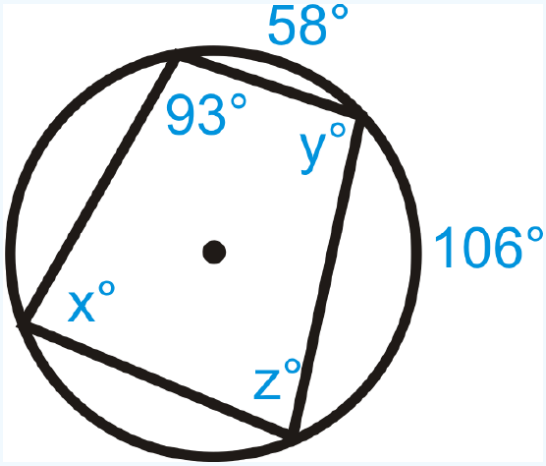

Figure 6.15.4

Solution

a. $x + 80^\circ = 180^\circ$ $\quad$ $y + 71^\circ = 180^\circ$

$x = 100^\circ$ $\quad$ $y = 109^\circ$

b. $z + 93^\circ = 180^\circ$ $\quad$ $x = \frac{1}{2}(58^\circ + 106^\circ)$ $\quad$ $y + 82^\circ = 180^\circ$

$z = 87^\circ$ $\quad$ $x = 82^\circ$ $\quad$ $y = 98^\circ$

Example 6.15.2

Find x and y in the picture below.

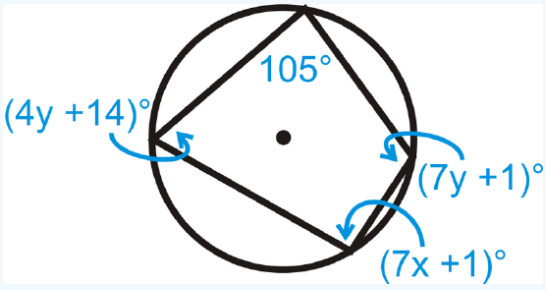

Figure 6.15.5

Solution

$$\begin{aligned} (7x+1)^\circ + 105^\circ &= 180^\circ \\ 7x + 106^\circ &= 180^\circ \\ 7x &= 74 \\ x &= 10.57 \end{aligned} \qquad \begin{aligned} (4y+14)^\circ + (7y+1)^\circ &= 180^\circ \\ 11y + 15^\circ &= 180^\circ \\ 11y &= 165 \\ y &= 15 \end{aligned}$$

Example 6.15.3

Find the values of x and y in $\odot A$.

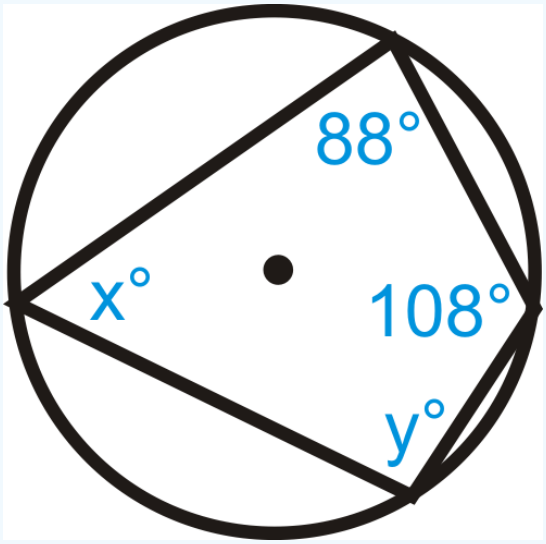

Figure 6.15.6

Solution

Use the Inscribed Quadrilateral Theorem. $x^\circ + 108^\circ = 180^\circ$ so $x = 72^\circ$. Similarly, $y^\circ + 88^\circ = 180^\circ$ so $y = 92^\circ$.

Example 6.15.4

Quadrilateral $ABCD$ is inscribed in $\odot E$. Find $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$.

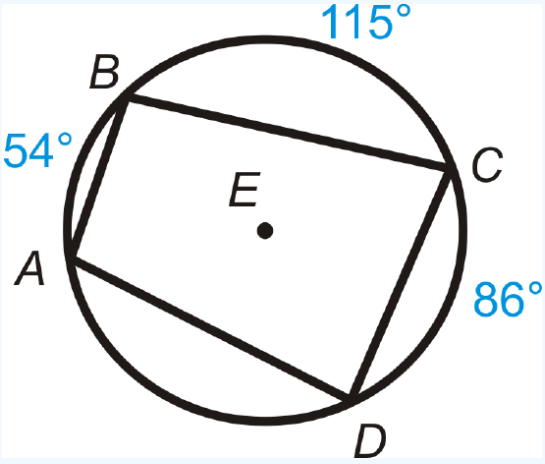

Figure 6.15.7

Solution

First, note that $m\widehat{AD} = 105^\circ$ because the complete circle must add up to 360°.

$$m\angle A = \frac{1}{2}m\overset{\frown}{BD} = 12(115 + 86) = 100.5^\circ$$
$$m\angle B = \frac{1}{2}m\overset{\frown}{AC} = 12(86 + 105) = 95.5^\circ$$
$$m\angle C = 180^\circ - m\angle A = 180^\circ - 100.5^\circ = 79.5^\circ$$
$$m\angle D = 180^\circ - m\angle B = 180^\circ - 95.5^\circ = 84.5^\circ$$

Review

Fill in the blanks.

1. A(n) _____ polygon has all its vertices on a circle.
2. The _____ angles of an inscribed quadrilateral are _____.

Quadrilateral $ABCD$ is inscribed in $\odot E$. Find:

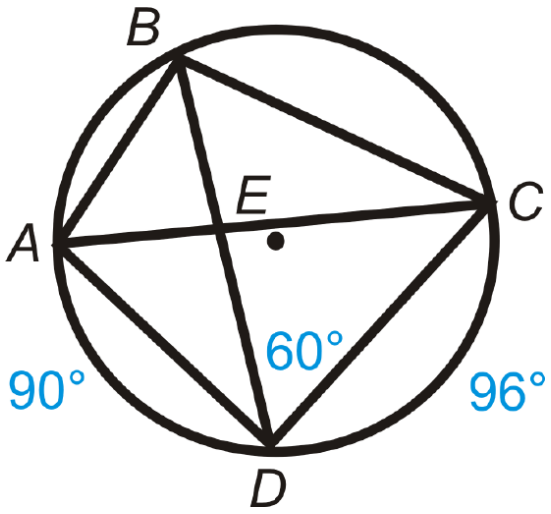

Figure 6.15.8

3. $m\angle DBC$
4. $m\overset{\frown}{BC}$
5. $m\overset{\frown}{AB}$
6. $m\angle ACD$
7. $m\angle ADC$
8. $m\angle ACB$

Find the value of x and/or y in $\odot A$.

9.

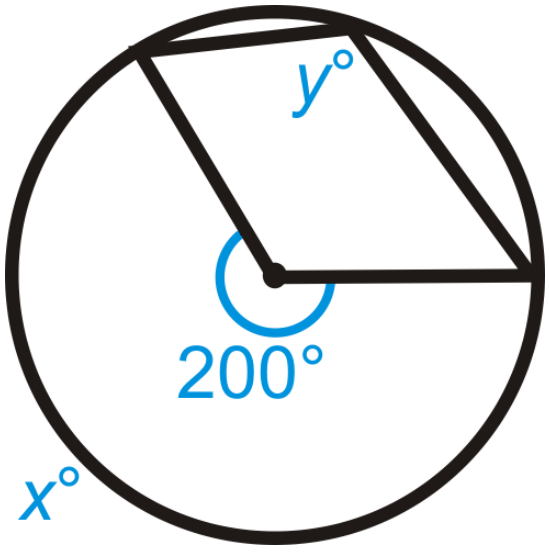

Figure **6.15.9**

10.

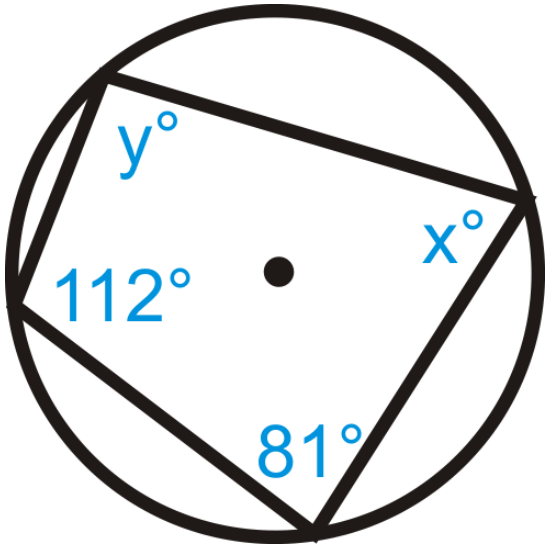

Figure **6.15.10**

11.

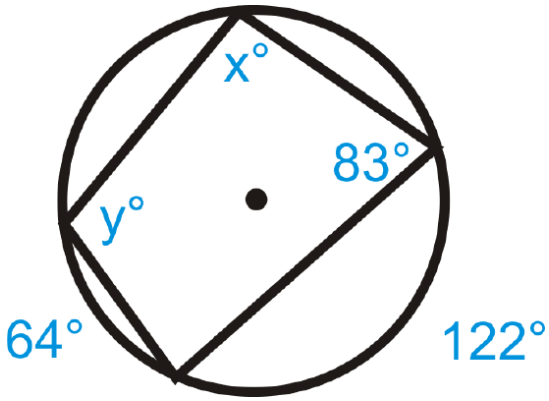

Figure **6.15.11**

Solve for x.

12.

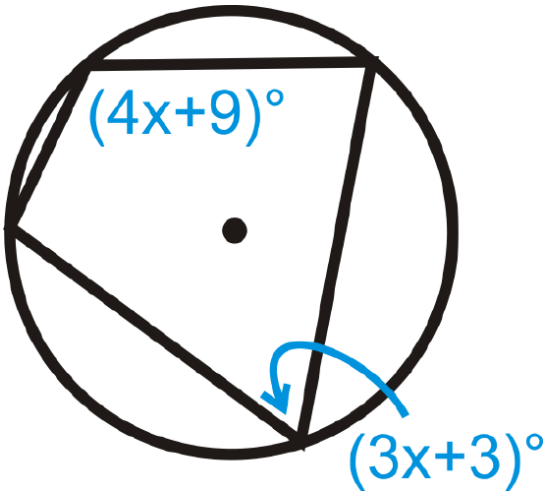

Figure **6.15.12**

13.

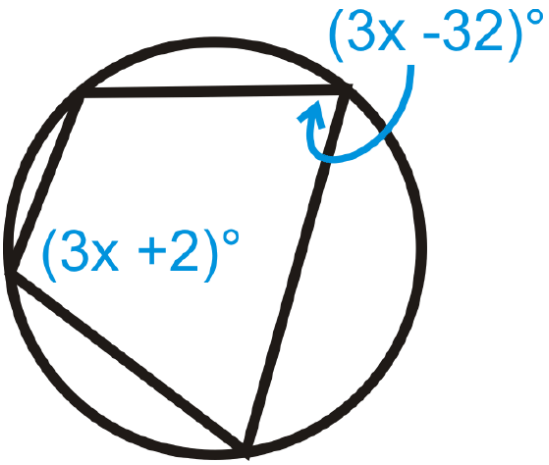

Figure **6.15.13**

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
inscribed angle	An angle with its vertex on the circle and whose sides are chords.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
radius	The distance from the center to the outer rim of a circle.
Inscribed Polygon	An inscribed polygon is a polygon with every vertex on a given circle.
Inscribed Quadrilateral Theorem	The Inscribed Quadrilateral Theorem states that a quadrilateral can be inscribed in a circle if and only if the opposite angles of the quadrilateral are supplementary.
Cyclic Quadrilaterals	A cyclic quadrilateral is a quadrilateral that can be inscribed in a circle.

Additional Resources

Video: Inscribed Quadrilaterals in Circles Principles - Basic

Activities: Inscribed Quadrilaterals in Circles Discussion Questions

Study Aids: Inscribed in Circles Study Guide

Practice: Inscribed Quadrilaterals in Circles

Real World: Sunrise at Stonehenge

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6.16: Angles On and Inside a Circle

Angles whose vertices are on the circumference of a circle or formed by tangent lines and chords.

When we say an angle is ***on*** a circle, we mean the vertex is on the edge of the circle. One type of angle *on* a circle is the **inscribed angle** (see Inscribed Angles in Circles). Another type of angle *on* a circle is one formed by a tangent and a **chord**.

Chord/Tangent Angle Theorem: The measure of an angle formed by a chord and a tangent that intersect on a circle is half the measure of the **intercepted arc**.

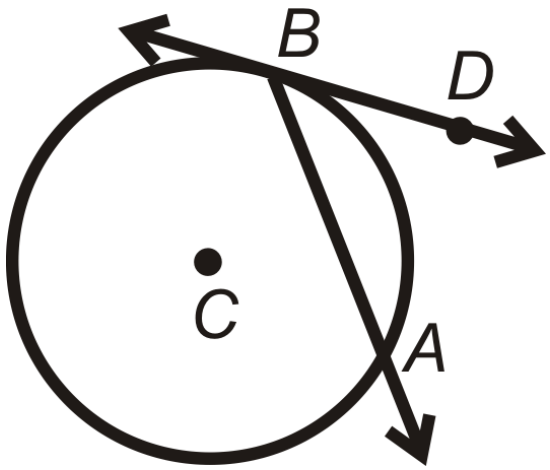


Figure **6.16.1**

$$m\angle DBA = \frac{1}{2}m\widehat{AB}$$

If two angles, with their vertices on the circle, intercept the same arc then the angles are congruent.

An angle is *inside* a circle when the vertex lies anywhere inside the circle.

Intersecting Chords Angle Theorem: The measure of the angle formed by two chords that intersect *inside* a circle is the average of the measures of the intercepted arcs.

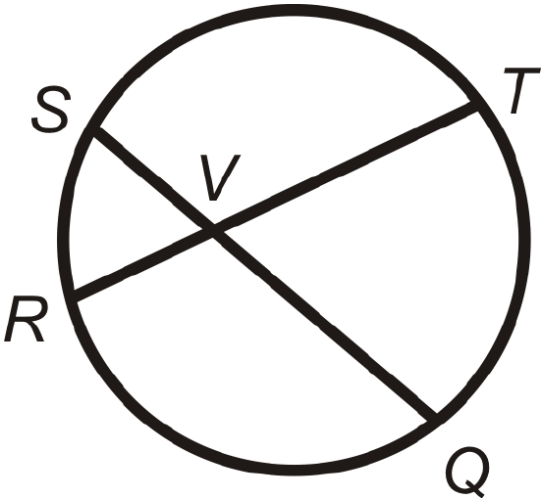


Figure **6.16.2**

$$m\angle SVR = \frac{1}{2}(m\widehat{SR} + m\widehat{TQ}) = m\widehat{SR} + m\widehat{TQ}2 = m\angle TVQ$$

$$m\angle SVT = \frac{1}{2}(m\widehat{ST} + m\widehat{RQ}) = m\widehat{ST} + m\widehat{RQ}2 = m\angle RVQ$$

What if you were given a circle with either a chord and a tangent or two chords that meet at a common point? How could you use the measure of the arc(s) formed by those circle parts to find the measure of the angles they make on or inside the circle?

Example 6.16.1

Find x.

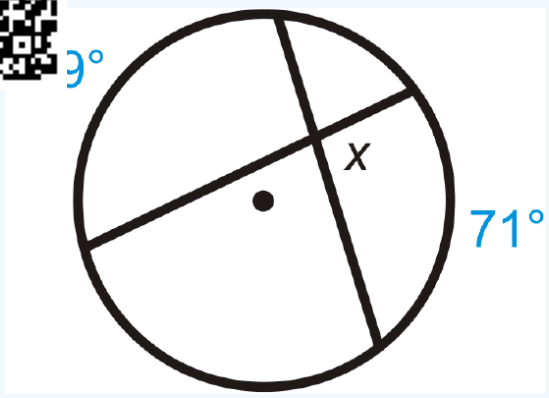


Figure 6.16.3

Solution

Use the Intersecting Chords Angle Theorem to write an equation.

$$x = \frac{129^\circ + 71^\circ}{2} = \frac{200^\circ}{2} = 100^\circ$$

Example 6.16.2

Find x.

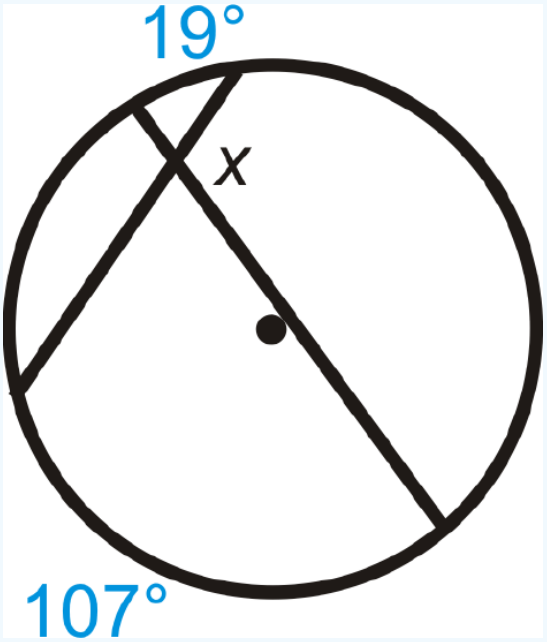


Figure 6.16.4

Solution

Use the Intersecting Chords Angle Theorem to write an equation.

x is supplementary to the angle that is the average of the given intercepted arcs. We call this supplementary angle y.

$$y = \frac{19^\circ + 107^\circ}{2} = \frac{126^\circ}{2} = 63^\circ \qquad x + 63^\circ = 180^\circ;\ x = 117^\circ$$

Example 6.16.3

Find $m\angle BAD$.

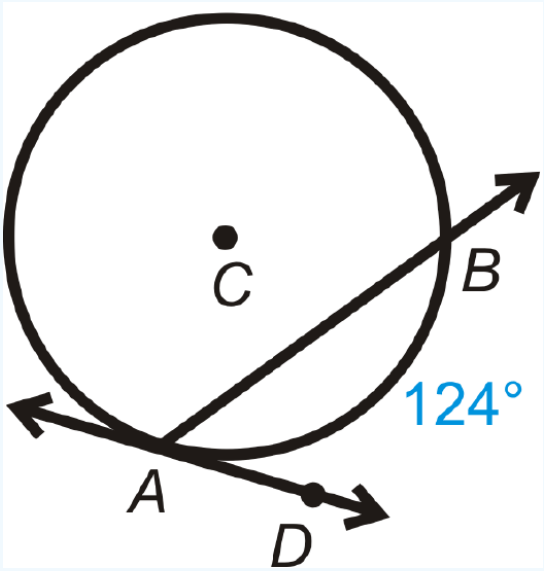


Figure 6.16.5

Solution

Use the Chord/Tangent Angle Theorem. $m\angle BAD = \frac{1}{2}m\overset{\frown}{AB} = \frac{1}{2} \cdot 124^\circ = 62^\circ$.

Example 6.16.4

Find a, b, and c.

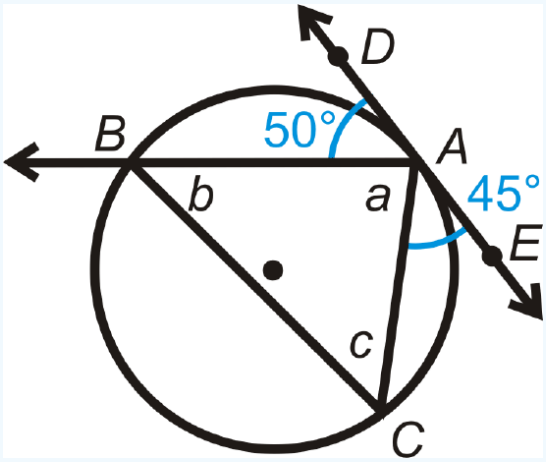


Figure 6.16.6

Solution

$$50^\circ + 45^\circ + m\angle a = 180^\circ \quad \text{straight angle}$$
$$m\angle a = 85^\circ$$
$$m\angle b = \tfrac{1}{2} \cdot m\overset{\frown}{AC}$$
$$\overset{\frown}{AC} = 2 \cdot m\angle EAC = 2 \cdot 45^\circ = 90^\circ$$
$$m\angle b = \tfrac{1}{2} \cdot 90^\circ = 45^\circ$$
$$85^\circ + 45^\circ + m\angle c = 180^\circ \quad \text{Triangle Sum Theorem}$$
$$m\angle c = 50^\circ$$

Example 6.16.5

Find $m\widehat{AEB}$.

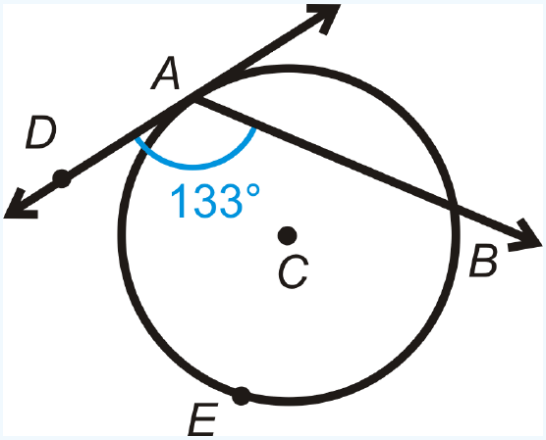


Figure 6.16.7

Solution

Use the Chord/Tangent Angle Theorem. $m\widehat{AEB} = 2 \cdot m\angle DAB = 2 \cdot 133° = 266°$.

Review

Find the value of the missing variable(s).

1.

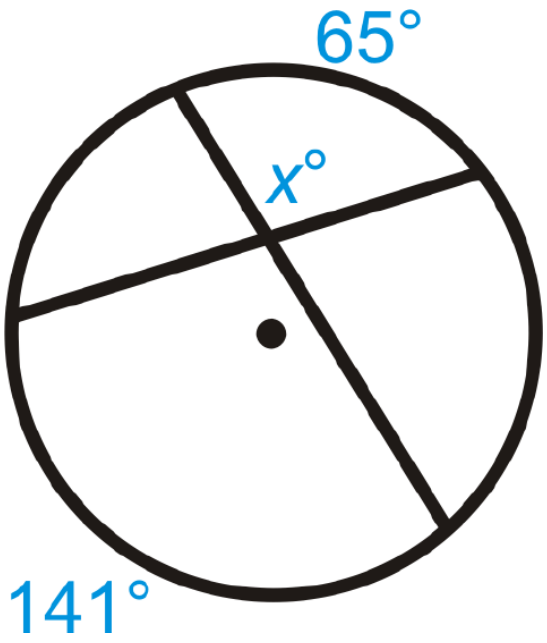


Figure 6.16.8

2.

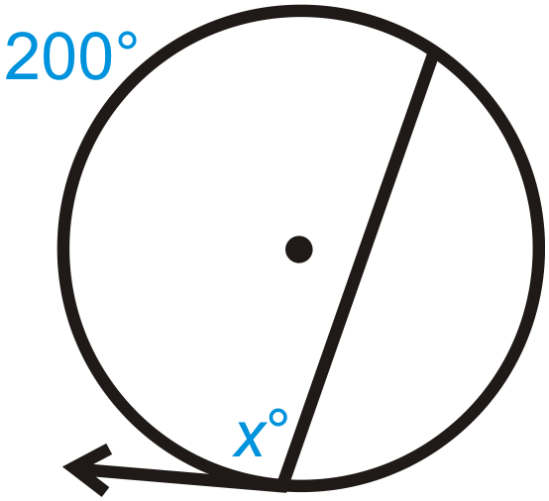


Figure **6.16.9**

3.

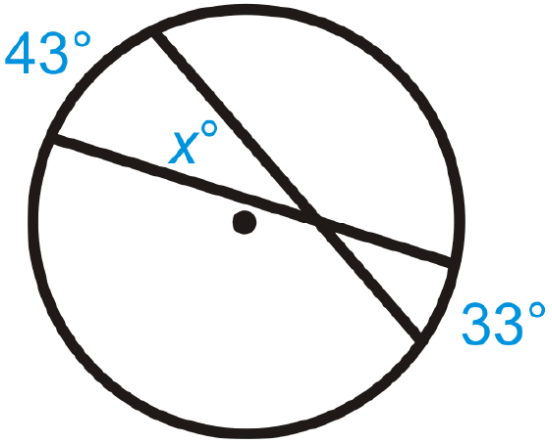


Figure **6.16.10**

4.

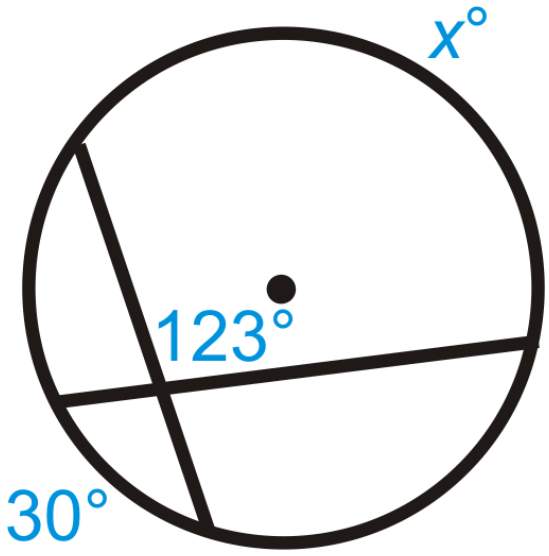


Figure **6.16.11**

5.

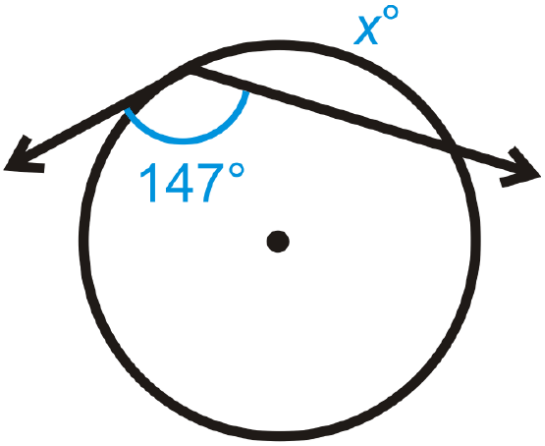


Figure **6.16.112**

6.

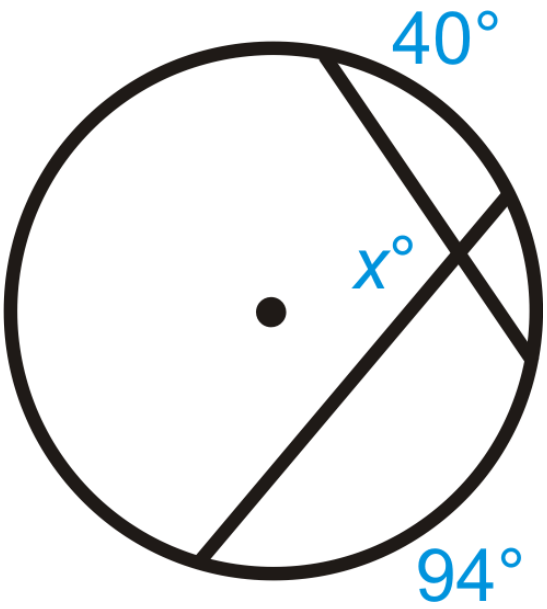


Figure **6.16.13**

7. $y \neq 60°$

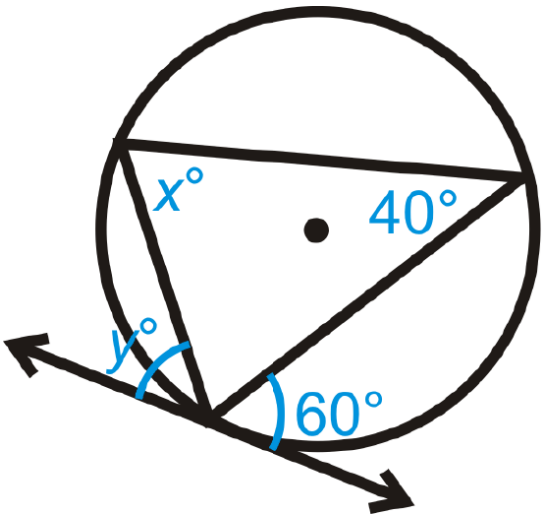


Figure **6.16.14**

Solve for x.

8. Fill in the blanks of the proof for the Intersecting Chords Angle Theorem

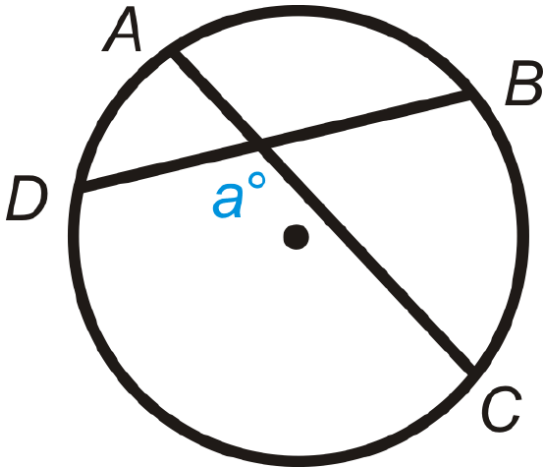
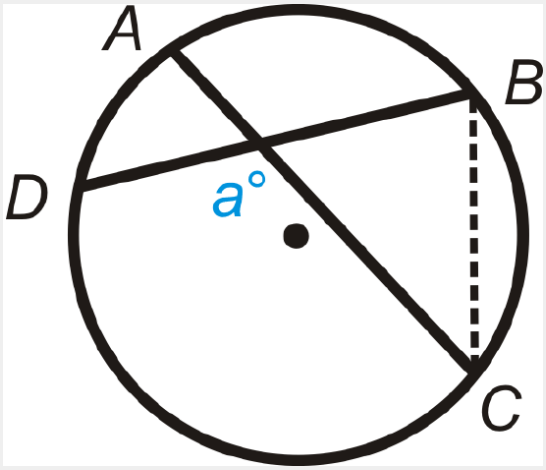


Figure **6.16.15**

Given: Intersecting chords $\overline{AC}$ and $\overline{BD}$.

Prove: $m\angle a = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$

Statement	***Reason***
1. Intersecting chords $\overline{AC}$ and $\overline{BD}$.	1.
2. Draw $\overline{BC}$  Figure **6.16.16**	2. Construction
3. $m\angle DBC = \frac{1}{2}m\widehat{DC}$ $m\angle ACB = \frac{1}{2}m\widehat{AB}$	3.
4. $m\angle a = m\angle DBC + m\angle ACB$	4.
5. $m\angle a = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{AB}$	5.

Fill in the blanks.

9. If the vertex of an angle is _____ a circle, then its measure is the average of the _____ arcs.
10. If the vertex of an angle is _____ a circle, then its measure is _____ the intercepted arc.

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
inscribed angle	An angle with its vertex on the circle and whose sides are chords.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
point of tangency	The point where the tangent line touches the circle.
radius	The distance from the center to the outer rim of a circle.
Chord/Tangent Angle Theorem	The Chord/Tangent Angle Theorem states that the measure of an angle formed by a chord and a tangent that intersect on a circle is half the measure of the intercepted arc.
Intersecting Chords Angle Theorem	The Intersecting Chords Angle Theorem states that the measure of the angle formed by two chords that intersect inside a circle is the average of the measures of the intercepted arcs.

Additional Resources

Video: Angles On and Inside a Circle Principles - Basic

Activities: Angles On and Inside a Circle Discussion Questions

Study Aids: Arcs and Angles Study Guide

Practice: Angles On and Inside a Circle

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6.17: Angles Outside a Circle

Angles formed by tangents and/or secants.

An angle is *outside* a circle if its vertex is outside the circle and its sides are tangents or secants. The possibilities are: an angle formed by two tangents, an angle formed by a **tangent** and a **secant**, and an angle formed by two secants.

Outside Angle Theorem: The measure of an angle formed by two secants, two tangents, or a secant and a tangent from a point outside the circle is half the difference of the measures of the intercepted arcs.

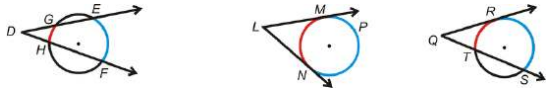

Figure **6.17.1**

$$m\angle D = \frac{m\widehat{EF} - m\widehat{GH}}{2},\ m\angle L = \frac{m\widehat{MPN} - m\widehat{MN}}{2},\ m\angle Q = \frac{m\widehat{RS} - m\widehat{RT}}{2}$$

What if you were given a circle with either two secants, two tangents, or one of each that share a common point outside the circle? How could you use the measure of the arcs formed by those circle parts to find the measure of the angle they make outside the circle?

Example 6.17.1

Find the measure of x.

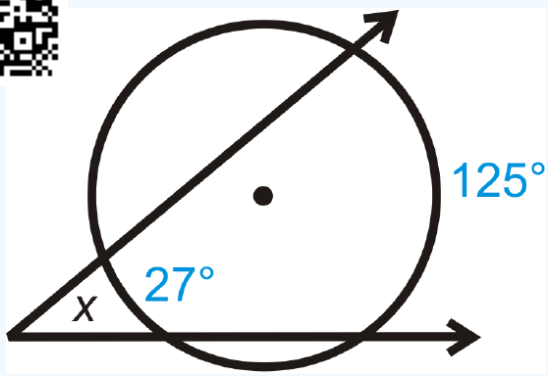

Figure **6.17.2**

Solution

Use the Outside Angle Theorem.

$$x = \frac{125^\circ - 27^\circ}{2} = \frac{98^\circ}{2} = 49^\circ$$

Example 6.17.2

Find the measure of x.

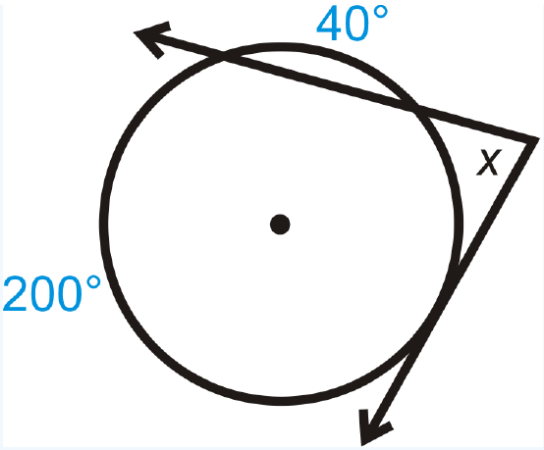

Figure 6.17.3

Solution

Use the Outside Angle Theorem.

$40°$ is not the **intercepted arc**. The intercepted arc is $120°$, $(360° - 200° - 40°)$. $x = \frac{200° - 120°}{2} = \frac{80°}{2} = 40°$

Example 6.17.3

Find the value of x.

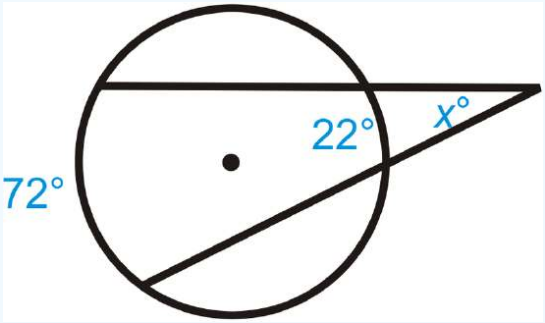

Figure 6.17.4

Solution

$x = \frac{72° - 22°}{2} = \frac{50°}{2} = 25°$.

Example 6.17.4

Find the value of x.

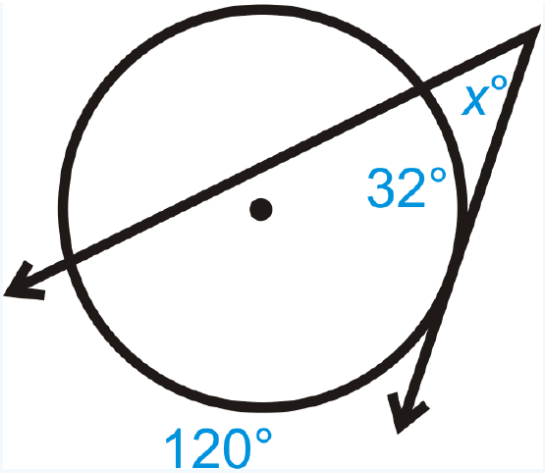

Figure 6.17.5

Solution

$x = \frac{120^\circ - 32^\circ}{2} = \frac{88^\circ}{2} = 44^\circ$.

Example 6.17.5

Find the value of x.

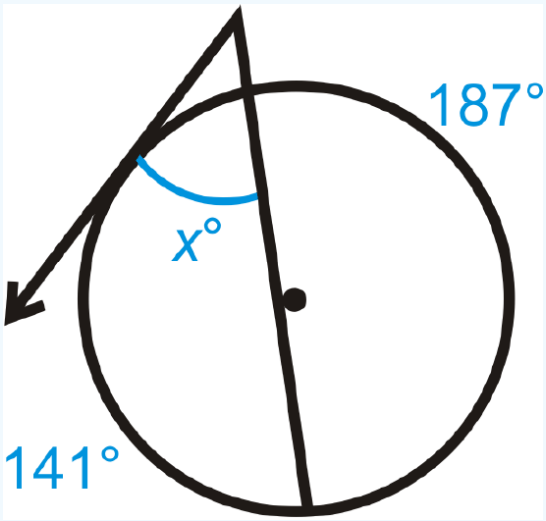

Figure 6.17.6

Solution

First note that the missing arc by angle x measures 32° because the complete circle must make 360°. Then, $x = \frac{141^\circ - 32^\circ}{2} = \frac{109^\circ}{2} = 54.5^\circ$.

Review

Find the value of the missing variable(s).

1.

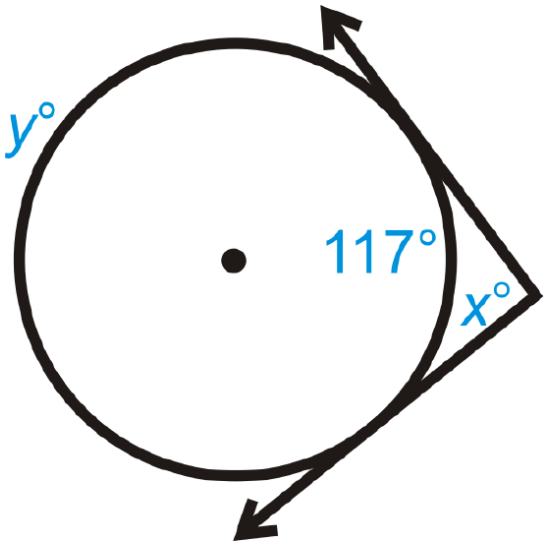

Figure **6.17.7**

2.

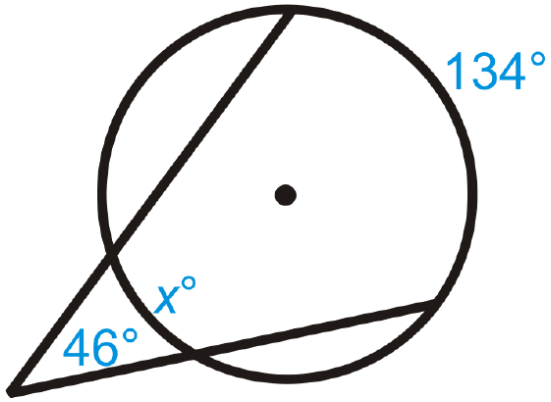

Figure **6.17.8**

3.

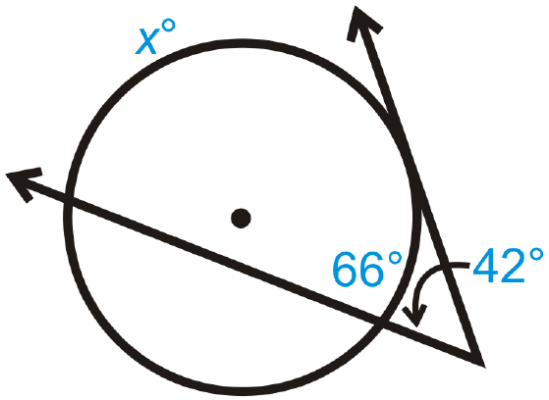

Figure **6.17.9**

4.

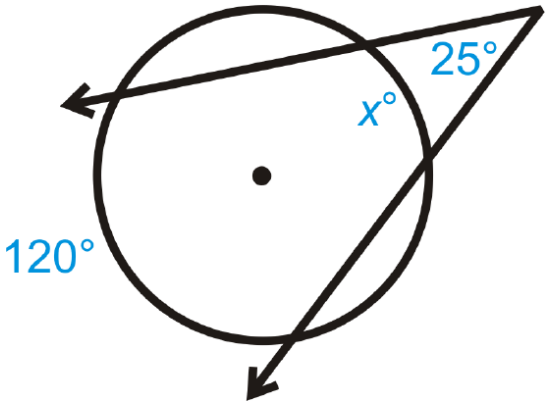

Figure **6.17.10**

5.

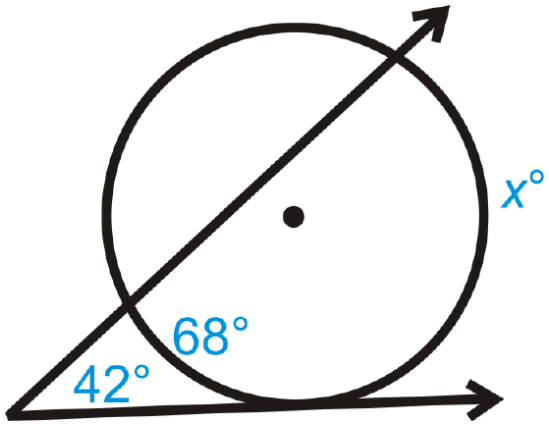

Figure **6.17.11**

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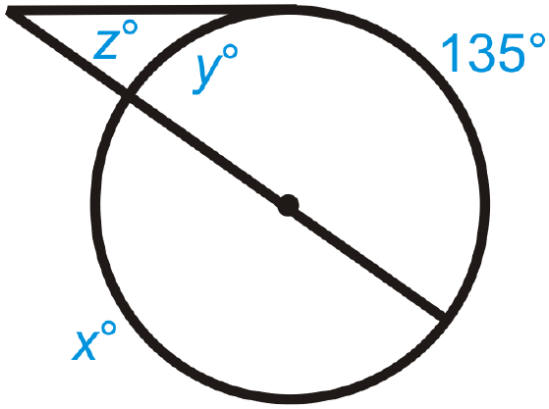


Figure **6.17.12**

7.

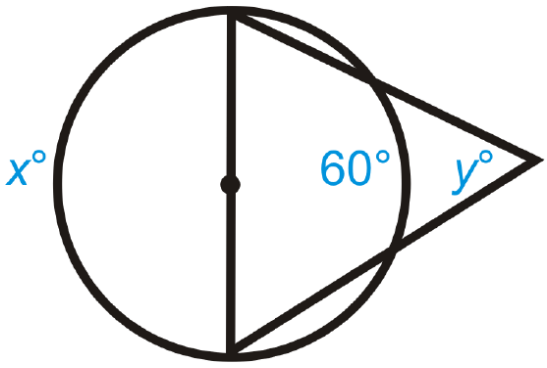


Figure **6.17.13**

Solve for x.

8.

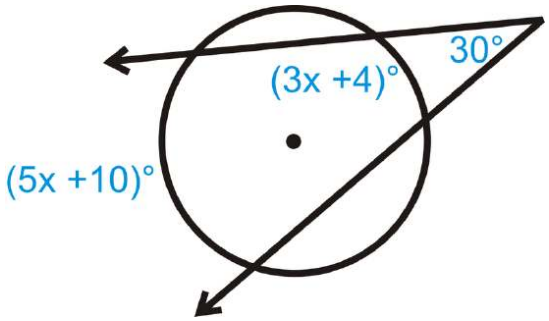


Figure **6.17.14**

9.

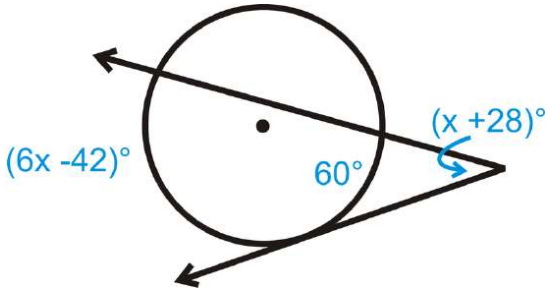


Figure **6.17.15**

10. Fill in the blanks of the proof for the Outside Angle Theorem

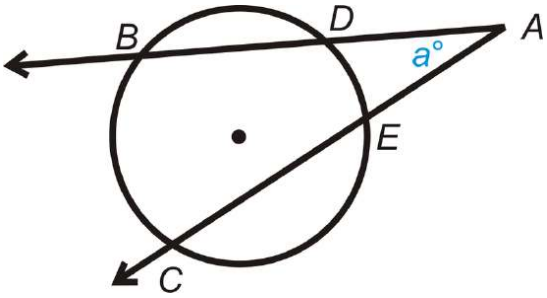
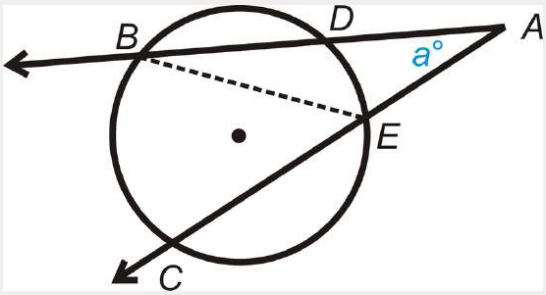


Figure **6.17.16**

Given: Secant rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$

Prove: $m\angle a = 12(m\widehat{BC} - m\widehat{DE})$

Statement	*Reason*
1. Intersecting secants $\overrightarrow{AB}$ and $\overrightarrow{AC}$.	1.
2. Draw $\overline{BE}$.  Figure 6.17.17	2. Construction
3. $m\angle BEC = \frac{1}{2}m\overset{\frown}{BC}$ $m\angle DBE = \frac{1}{2}m\overset{\frown}{DE}$	3.
4. $m\angle a + m\angle DBE = m\angle BEC$	4.
5.	5. Subtraction PoE
6.	6. Substitution
7. $m\angle a = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{DE})$	7.

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.8.

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
inscribed angle	An angle with its vertex on the circle and whose sides are chords.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
point of tangency	The point where the tangent line touches the circle.
radius	The distance from the center to the outer rim of a circle.
Secant	The secant of an angle in a right triangle is the value found by dividing length of the hypotenuse by the length of the side adjacent the given angle. The secant ratio is the reciprocal of the cosine ratio.
Tangent	The tangent of an angle in a right triangle is a value found by dividing the length of the side opposite the given angle by the length of the side adjacent to the given angle.
Outside Angles Theorem	The Outside Angles Theorem states that the measure of an angle formed by two secants, two tangents, or a secant and a tangent from a point outside the circle is half the difference of the measures of the intercepted arcs.

Additional Resources

Video: Angles Outside a Circle Principles - Basic

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Practice: Angles Outside a Circle

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6.18: Tangent Lines

Lines perpendicular to the radius drawn to the point of tangency.

Tangent Line Theorems

There are two important theorems about tangent lines.

1. **Tangent to a Circle Theorem**: A line is tangent to a circle if and only if the line is perpendicular to the **radius** drawn to the **point of tangency**.

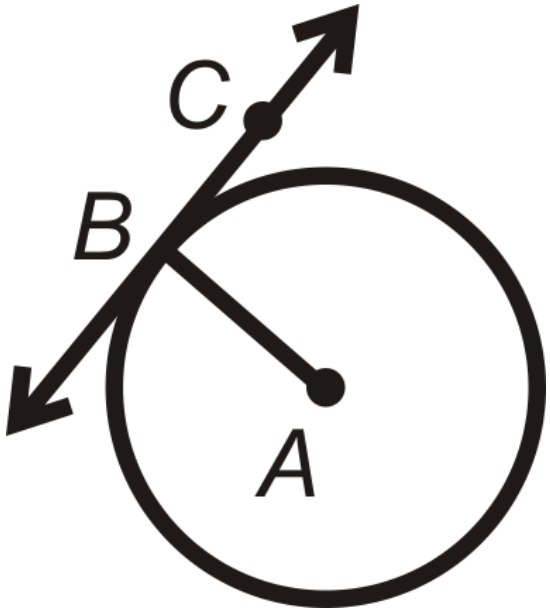

Figure **6.18.1**

$\overleftrightarrow{BC}$ is tangent at point B if and only if $\overleftrightarrow{BC} \perp \overline{AB}$.

This theorem uses the words “if and only if,” making it a biconditional statement, which means the converse of this theorem is also true.

2. **Two Tangents Theorem:** If two tangent segments are drawn to one circle from the same external point, then they are congruent.

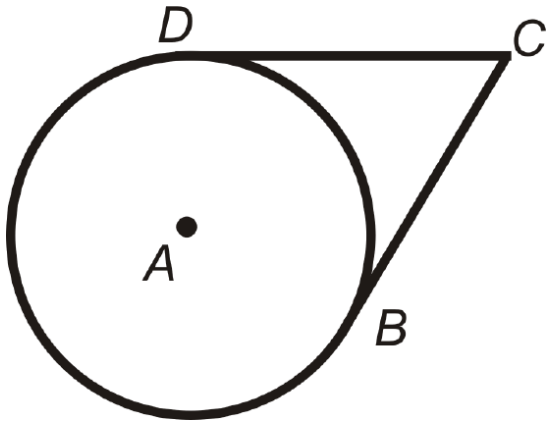

Figure **6.18.2**

$\overline{BC}$ and $\overline{DC}$ have C as an endpoint and are tangent; $\overline{BC} \cong \overline{DC}$.

What if a line were drawn outside a circle that appeared to touch the circle at only one point? How could you determine if that line were actually a tangent?

Example 6.18.1

Determine if the triangle angle.

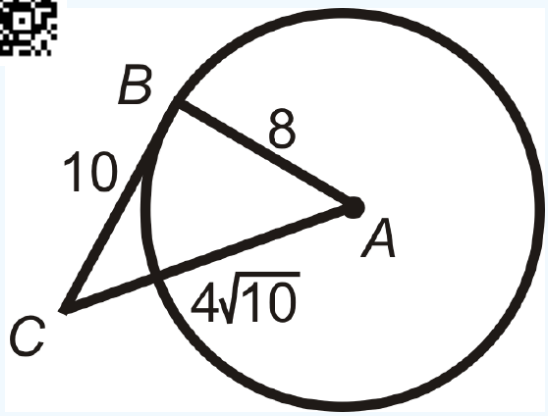

Figure **6.18.3**

Solution

Use the Pythagorean Theorem. $4\sqrt{10}$ is the longest side, so it will be c.

Does

$$8^2 + 10^2 = (4\sqrt{10})^2?$$
$$64 + 100 \neq 160$$

ΔABC is not a right triangle. From this, we also find that $\overline{CB}$ is not tangent to $\odot A$.

Example 6.18.2

If D and C are the centers and AE is tangent to both circles, find DC.

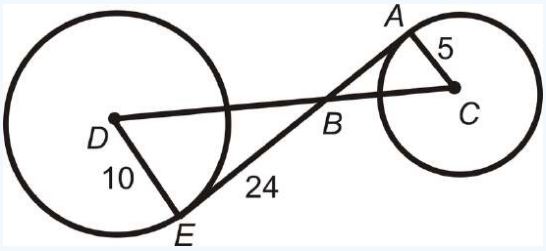

Figure **6.18.4**

Solution

$\overline{AE} \perp \overline{DE}$ and $\overline{AE} \perp \overline{AC}$ and $\Delta ABC \sim \Delta DBE$ by AA Similarity.

To find DB, use the Pythagorean Theorem.

$$10^2 + 24^2 = DB^2$$
$$100 + 576 = 676$$
$$DB = \sqrt{676} = 26$$

To find BC, use similar triangles.

$$\frac{5}{10} = \frac{BC}{26} \rightarrow BC = 13.\ DC = DB + BC = 26 + 13 = 39$$

Example 6.18.3

$\overline{CB}$ is tangent to $\odot A$ at point B. Find AC. Reduce any radicals.

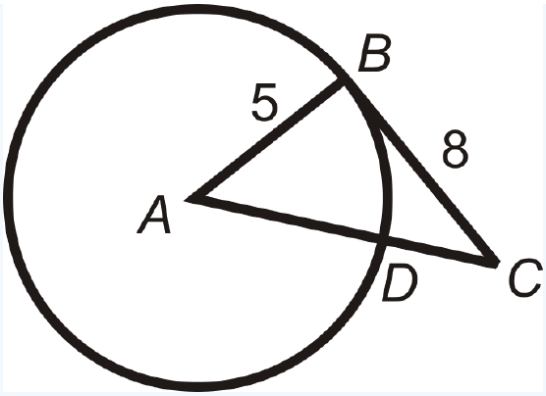

Figure 6.18.5

Solution

$\overline{CB}$ is tangent, so $\overline{AB} \perp \overline{CB}$ and ΔABC a right triangle. Use the Pythagorean Theorem to find AC.

$$5^2 + 8^2 = AC^2$$
$$25 + 64 = AC^2$$
$$89 = AC^2$$
$$AC = \sqrt{89}$$

Example 6.18.4

Using the answer from Example A above, find DC in $\odot A$. Round your answer to the nearest hundredth.

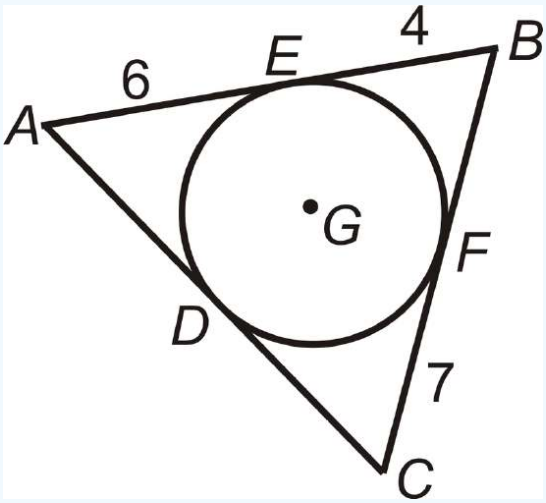

Figure 6.18.6

Solution

$DC = AC - AD$

$DC = \sqrt{89} - 5 \approx 4.43$

Example 6.18.5

Find the perimeter of ΔABC.

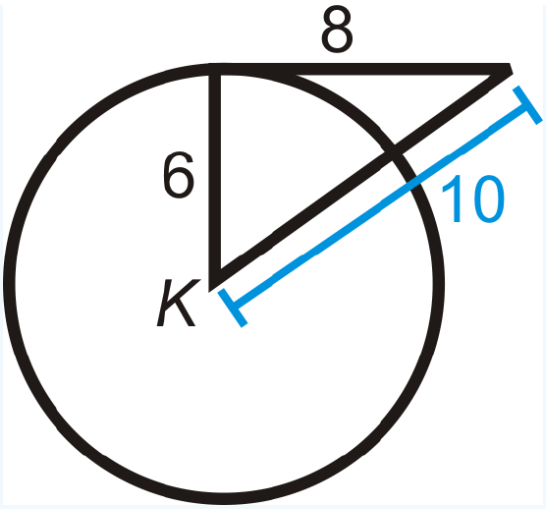

Figure 6.18.7

Solution

$AE = AD$, $EB = BF$, and $CF = CD$. Therefore, the perimeter of

$\Delta ABC = 6 + 6 + 4 + 4 + 7 + 7 = 34$.

$\odot G$ is ***inscribed*** in ΔABC. A circle is inscribed in a polygon if every side of the polygon is tangent to the circle.

Review

Determine whether the given segment is tangent to $\odot K$.

1.

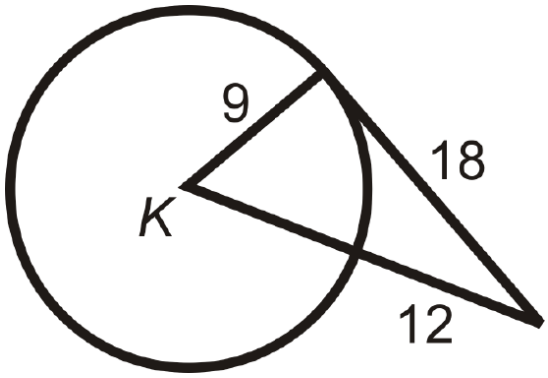

Figure 6.18.8

2.

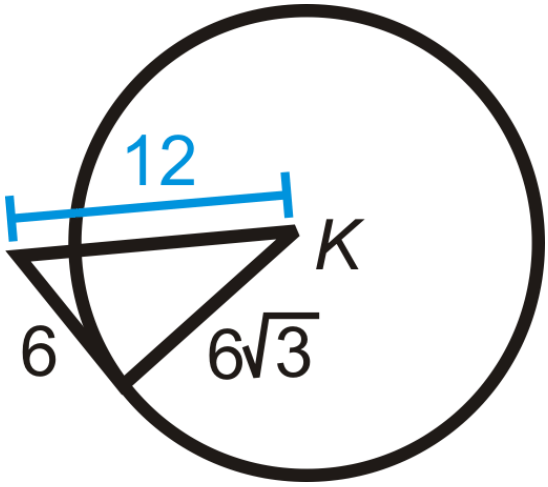

Figure 6.18.9

3.

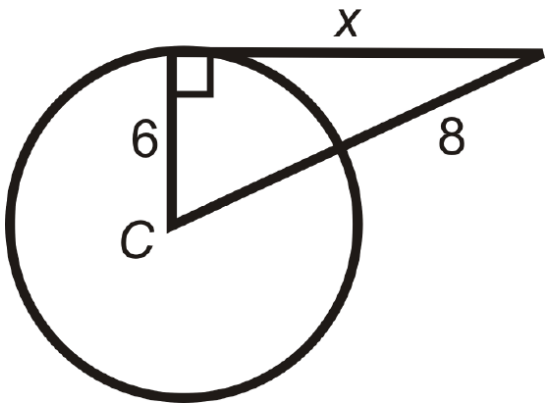

Figure 6.18.10

Find the value of the indicated length(s) in $\odot C$. A and B are points of tangency. Simplify all radicals.

4.

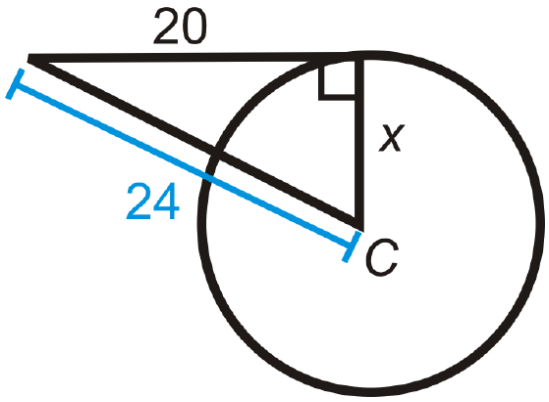

Figure 6.18.11

5.

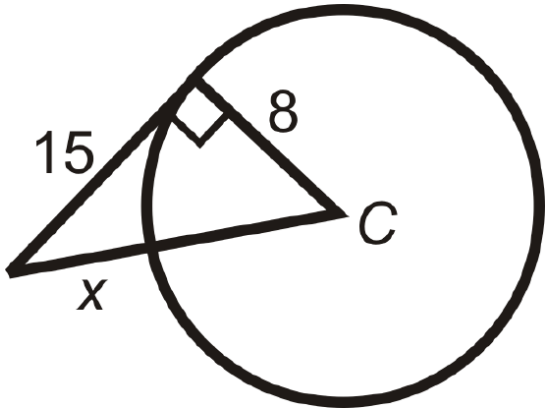

Figure 6.18.12

6.

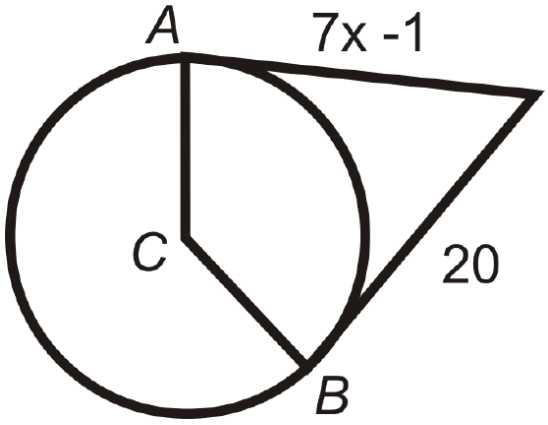

Figure 6.18.13

7.

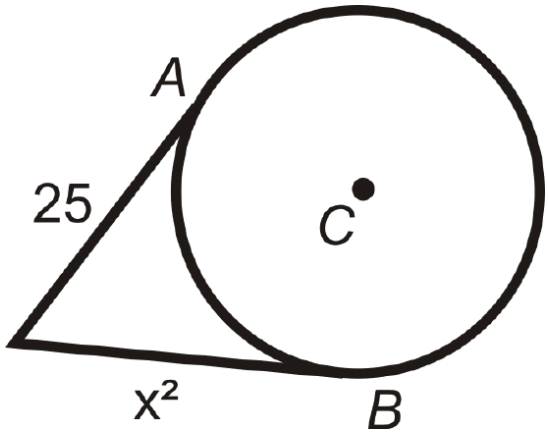

Figure 6.18.14

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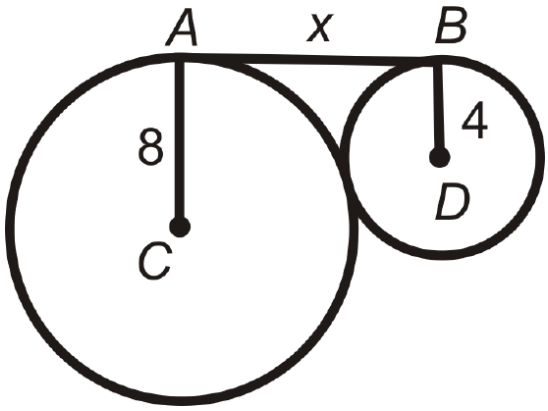

Figure 6.18.15

9.

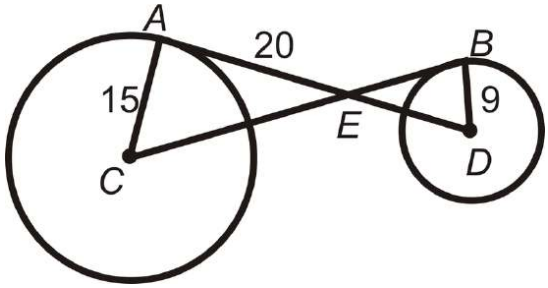

Figure 6.18.16

A and B are points of tangency for $\odot C$ and $\odot D$.

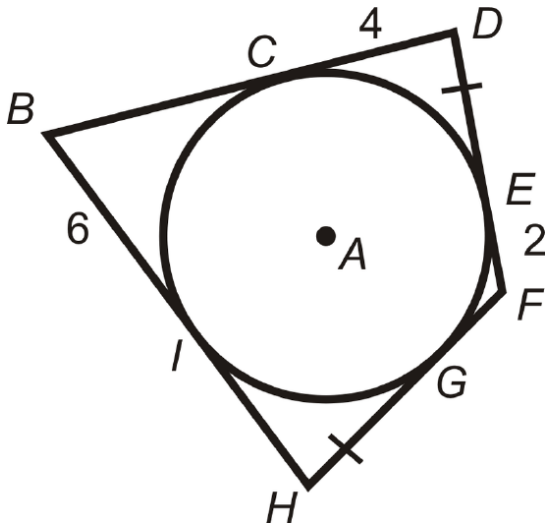

Figure 6.18.17

10. Is $\Delta AEC \sim \Delta BED$? Why?
11. Find CE.
12. Find BE.
13. Find ED.
14. Find BC and AD.

$\odot A$ is inscribed in $BDFH$.

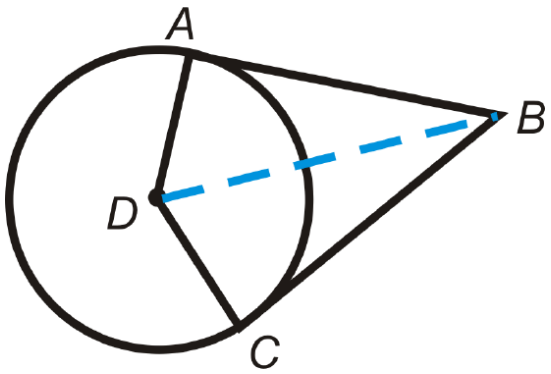

Figure 6.18.18

15. Find the perimeter of $BDFH$.
16. What type of quadrilateral is $BDFH$? How do you know?
17. Draw a circle inscribed in a square. If the radius of the circle is 5, what is the perimeter of the square?
18. Can a circle be inscribed in a rectangle? If so, draw it. If not, explain.
19. Draw a triangle with two sides tangent to a circle, but the third side is not.
20. Can a circle be inscribed in an obtuse triangle? If so, draw it. If not, explain.
21. Fill in the blanks in the proof of the Two Tangents Theorem.

Given: $\overline{AB}$ and $\overline{CB}$ with points of tangency at A and C. $\overline{AD}$ and $\overline{DC}$ are radii.

Prove: $\overline{AB} \cong \overline{CB}$

Statement	*Reason*
1.	1.
2. $\overline{AD} \cong \overline{DC}$	2.
3. $\overline{DA} \perp \overline{AB}$ and $\overline{DC} \perp \overline{CB}$	3.
4.	4. Definition of perpendicular lines
5.	5. Connecting two existing points
6. ΔADB and ΔDCB are right triangles	6.
7.$\overline{DB} \cong \overline{DB}$	7.
8.$\Delta ABD \cong \Delta CBD$	8.
9.$\overline{AB} \cong \overline{CB}$	9.

22. Fill in the blanks, using the proof from #21.
    1. $ABCD$ is a _____ (type of quadrilateral).
    2. The line that connects the _____ and the external point B _____ $\angle ABC$.
23. Points A, B, and C are points of tangency for the three tangent circles. *Explain* why $\overline{AT} \cong \overline{BT} \cong \overline{CT}$.

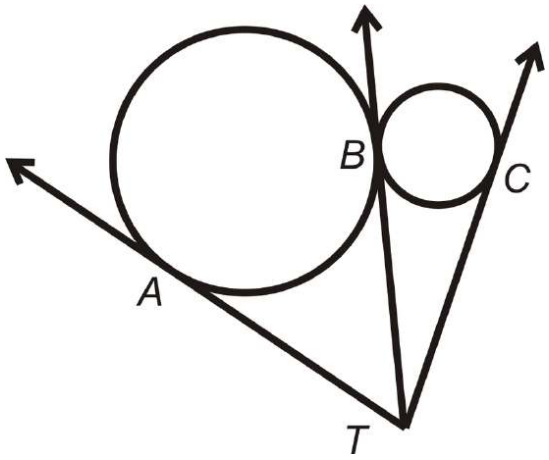

Figure **6.18.19**

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.2.

Resources

Vocabulary

Term	Definition
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
point of tangency	The point where the tangent line touches the circle.
radius	The distance from the center to the outer rim of a circle.
Tangent	The tangent of an angle in a right triangle is a value found by dividing the length of the side opposite the given angle by the length of the side adjacent to the given angle.
Tangent to a Circle Theorem	A line is tangent to a circle if and only if the line is perpendicular to the radius drawn to the point of tangency.
Two Tangent Theorem	The Two-Tangent Theorem states that if two tangent segments are drawn to one circle from the same external point, then they are congruent.

Additional Resources

Video: Tangent Lines Principles - Basic

Activities: Tangent Lines Discussion Questions

Study Aids: Properties of a Circle Study Guide

Practice: Tangent Lines

Real World: Swing Rides

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6.19: Intersecting Secants Theorem

Prove and use theorems involving lines that intersect a circle at two points.

Segments from Secants

When two secants intersect outside a **circle**, the circle divides the secants into segments that are proportional with each other.

Two Secants Segments Theorem: If two secants are drawn from a common point outside a circle and the segments are labeled as below, then $a(a+b) = c(c+d)$.

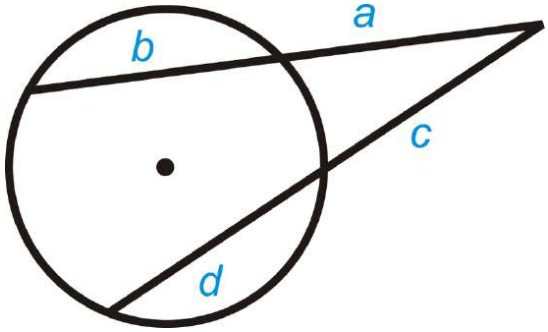


Figure 6.19.1

What if you were given a circle with two secants that intersect outside the circle? How could you use the length of some of the segments formed by their intersection to determine the lengths of the unknown segments?

Example 6.19.1

Find x. Simplify any rac

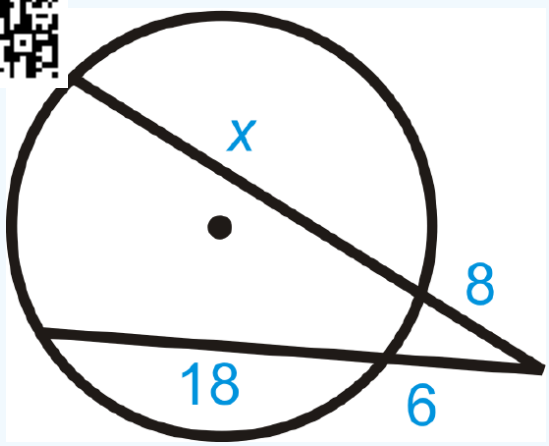


Figure 6.19.2

Solution

Use the Two Secants Segments Theorem.

$$\begin{aligned} 8(8+x) &= 6(6+18) \\ 64+8x &= 144 \\ 8x &= 80 \\ x &= 10 \end{aligned}$$

Example 6.19.2

Find x. Simplify any radicals.

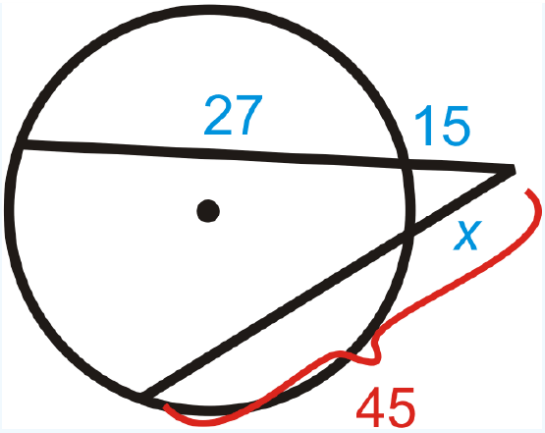


Figure 6.19.3

Solution

Use the Two Secants Segments Theorem.

$$\begin{aligned} 15(15+27) &= x \cdot 45 \\ 630 &= 45x \\ x &= 14 \end{aligned}$$

Example 6.19.3

Find the value of x.

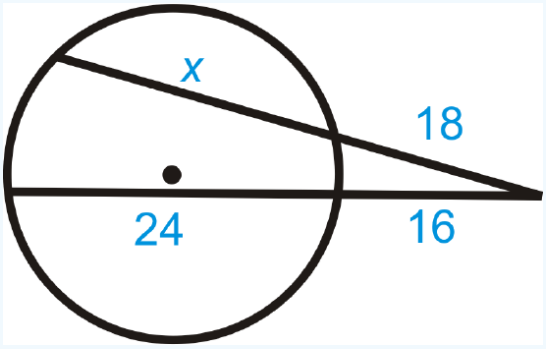


Figure 6.19.4

Solution

Use the Two Secants Segments Theorem.

$$\begin{aligned} 18 \cdot (18+x) &= 16 \cdot (16+24) \\ 324+18x &= 256+384 \\ 18x &= 316 \\ x &= 17\frac{5}{9} \end{aligned}$$

Example 6.19.4

Find the value of x.

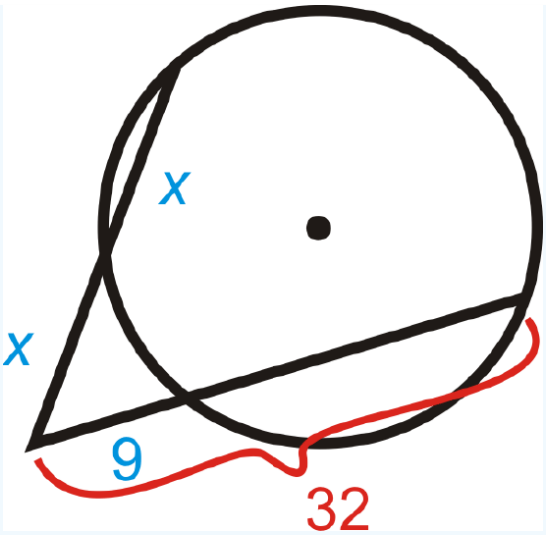


Figure **6.19.5**

Solution

Use the Two Secants Segments Theorem.

$$\begin{aligned} x \cdot (x + x) &= 9 \cdot 32 \\ 2x^2 &= 288 \\ x^2 &= 144 \\ x &= 12,\ x \neq -12 \text{(length is not negative)} \end{aligned}$$

Example 6.19.5

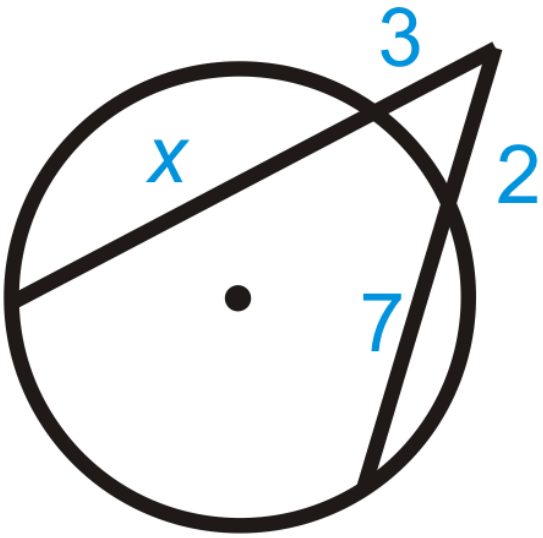
True or False: Two secants will always intersect outside of a circle.

Solution

False. If the two secants are parallel, they will never intersect. It's also possible for two secants to intersect inside a circle.

Review

Fill in the blanks for each problem below. Then, solve for the missing segment.



1.

Figure **6.19.6**

'_' allowed only in math mode

2.

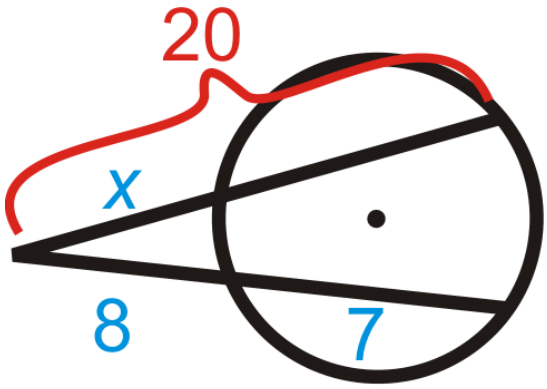


Figure 6.19.7

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Find x in each diagram below. Simplify any radicals.

3.

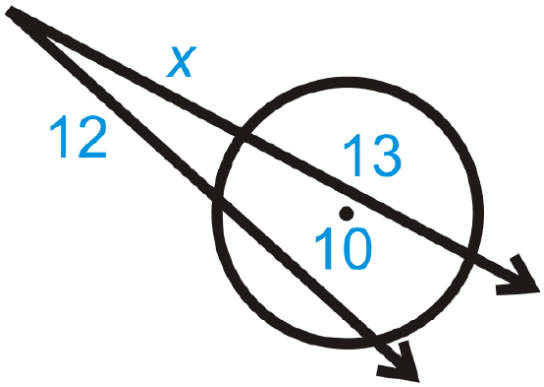

Figure 6.19.8

4.

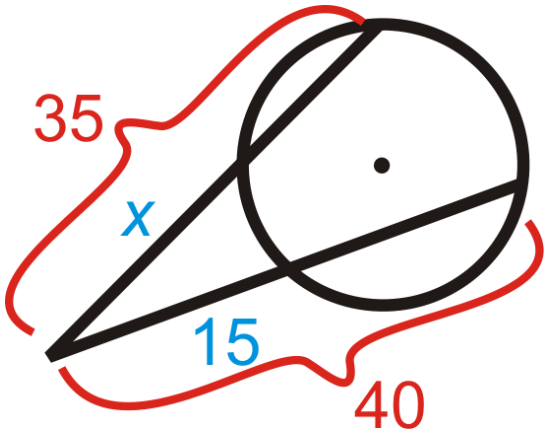

Figure 6.19.9

5.

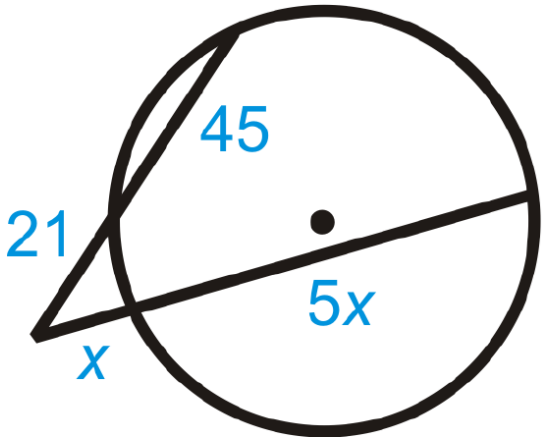

Figure **6.19.10**

6. Fill in the blanks of the proof of the Two Secants Segments Theorem.

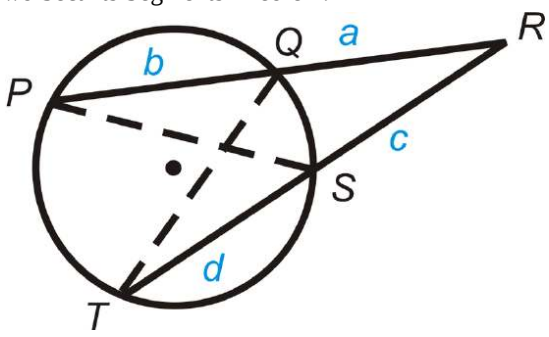

Figure **6.19.11**

Given: Secants $\overline{PR}$ and $\overline{RT}$

Prove: $a(a+b)=c(c+d)$

Statement	***Reason***
1. Secants $\overline{PR}$ and $\overline{RT}$ with segments a, b, c, and d.	1. Given
2. $\angle R \cong \angle R$	2. Reflexive PoC
3. $\angle QPS \cong \angle STQ$	3. **Congruent** Inscribed Angles Theorem
4. $\Delta RPS \sim \Delta RTQ$	4. AA Similarity Postulate
5. $ac+d=ca+b$	5. Corresponding parts of similar triangles are proportional
6. $a(a+b)=c(c+d)$	6. Cross multiplication

Solve for the unknown variable.

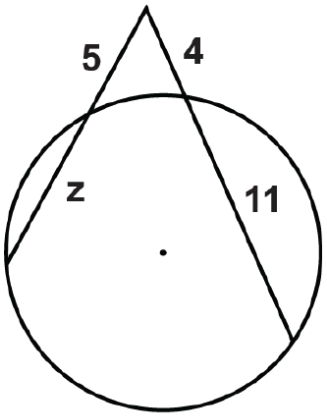

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Figure **6.19.12**

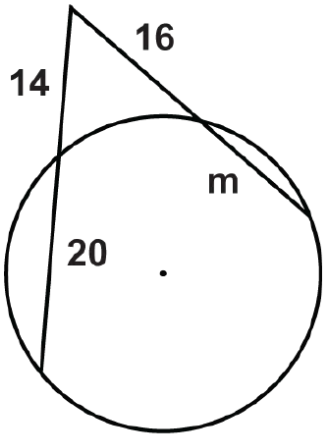

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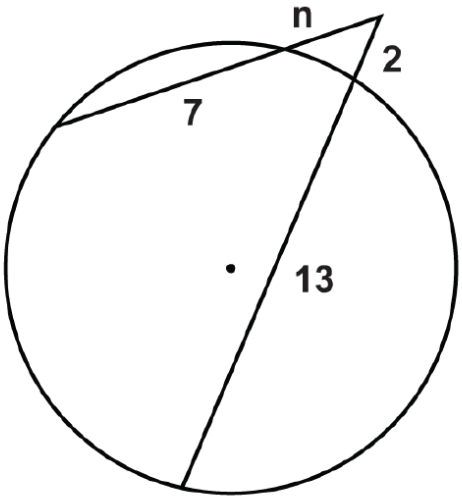
Figure **6.19.13**



9.

Figure **6.19.14**

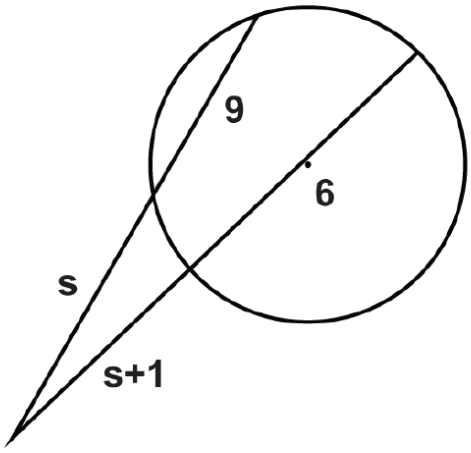


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Figure **6.19.15**

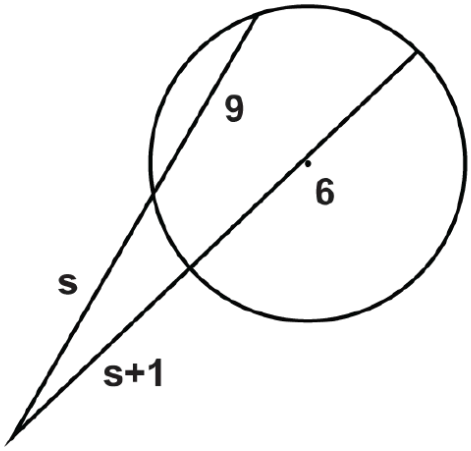

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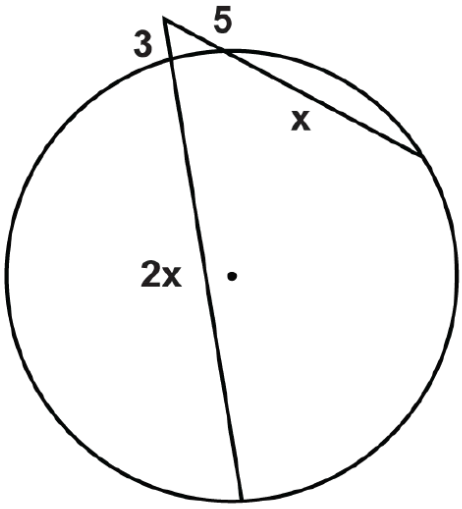
Figure **6.19.16**



12.

Figure **6.19.17**

13.

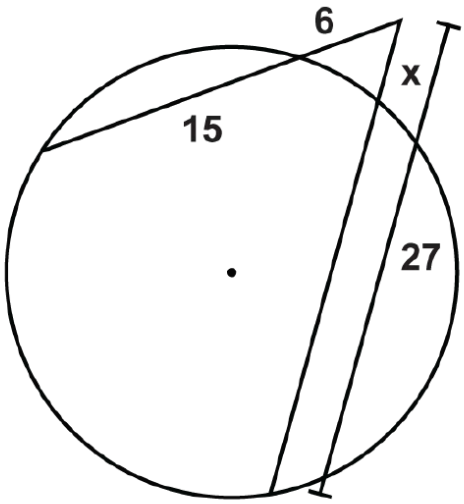


Figure 6.19.18

14.

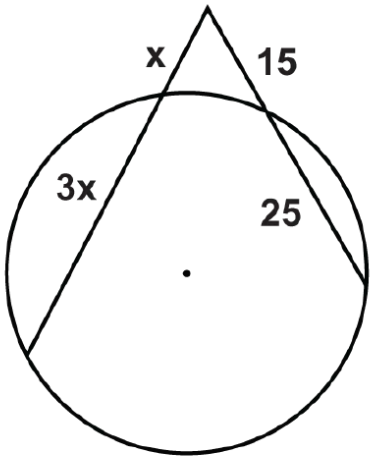


Figure 6.19.19

15.

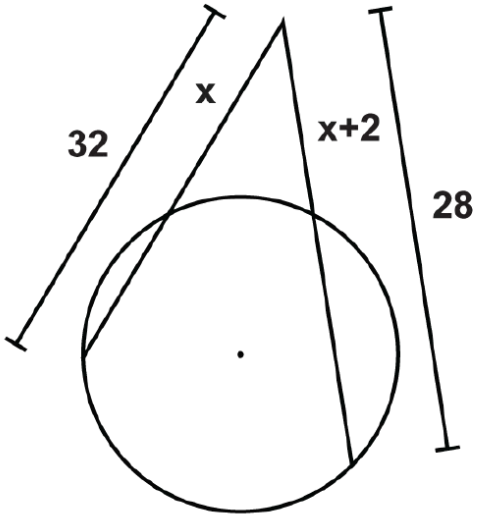


Figure 6.19.12

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.10.

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
Inscribed Angle	An inscribed angle is an angle with its vertex on the circle. The measure of an inscribed angle is half the measure of its intercepted arc.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
point of tangency	The point where the tangent line touches the circle.
radius	The distance from the center to the outer rim of a circle.
AA Similarity Postulate	If two angles in one triangle are congruent to two angles in another triangle, then the two triangles are similar.
Congruent	Congruent figures are identical in size, shape and measure.
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$ or $\angle B \cong \angle B$
Secant	The secant of an angle in a right triangle is the value found by dividing length of the hypotenuse by the length of the side adjacent the given angle. The secant ratio is the reciprocal of the cosine ratio.
secant line	A secant line is a line that joins two points on a curve.
Tangent line	A tangent line is a line that "just touches" a curve at a single point and no others.
Two Secants Segments Theorem	Two secants segments theorem states that if you have a point outside a circle and draw two secant lines from it, there is a relationship between the line segments formed.

Additional Resources

Video: Segments from Secants Principles - Basic

Activities: Segments from Secants Discussion Questions

Study Aids: Circles: Segments and Lengths Study Guide

Practice: Intersecting Secants Theorem

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6.20: Tangent Secant Theorem

Product of the outside segment and whole secant equals the square of the tangent to the same point.

Segments from Secants and Tangents

If a tangent and **secant** meet at a common point outside a **circle**, the segments created have a similar relationship to that of two secant rays.

Tangent Secant Segment Theorem: If a tangent and a secant are drawn from a common point outside the circle (and the segments are labeled like the picture below), then $a^2 = b(b+c)$.

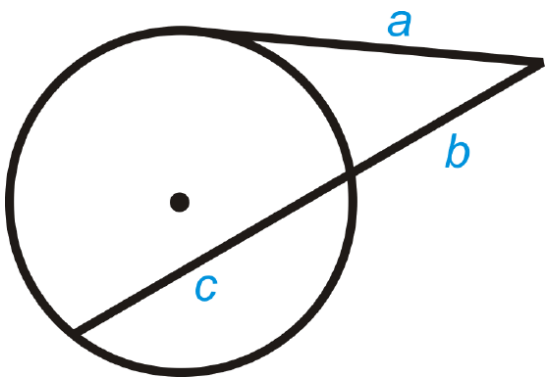

Figure 6.20.1

What if you were given a circle with a tangent and a secant that intersect outside the circle? How could you use the length of some of the segments formed by their intersection to determine the lengths of the unknown segments?

Example 6.20.1

Find x. Simplify any rac

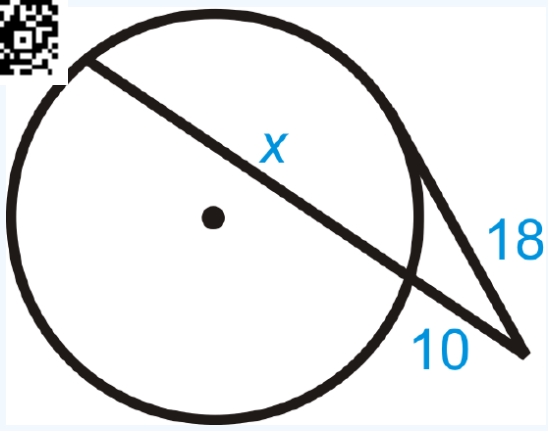

Figure 6.20.2

Solution

Use the Tangent Secant Segment Theorem.

$$\begin{aligned} 18^2 &= 10(10+x) \\ 324 &= 100 + 10x \\ 224 &= 10x \\ x &= 22.4 \end{aligned}$$

Example 6.20.2

Find x. Simplify any radicals.

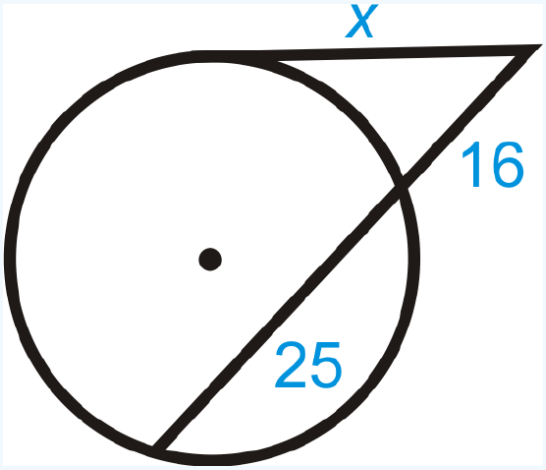

Figure 6.20.3

Solution

Use the Tangent Secant Segment Theorem.

$$x^2 = 16(16 + 25)$$
$$x^2 = 656$$
$$x = 4\sqrt{41}$$

Example 6.20.3

Find the length of the missing segment.

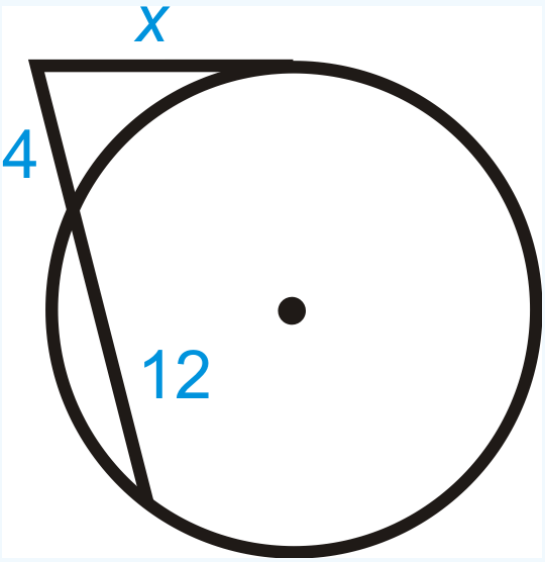

Figure 6.20.4

Solution

Use the Tangent Secant Segment Theorem.

$$x^2 = 4(4 + 12)$$
$$x^2 = 4 \cdot 16 = 64$$
$$x = 8$$

Example 6.20.4

Fill in the blank and then solve for the missing segment.

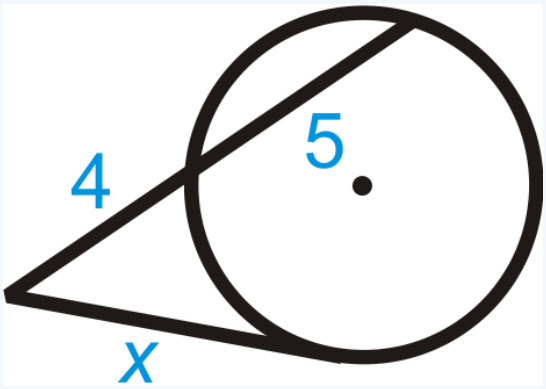

Figure 6.20.5

Solution

'_' allowed only in math mode

$$x^2 = 4(4+5)$$
$$x^2 = 36$$
$$x = 6$$

Example 6.20.5

Find the value of the missing segment.

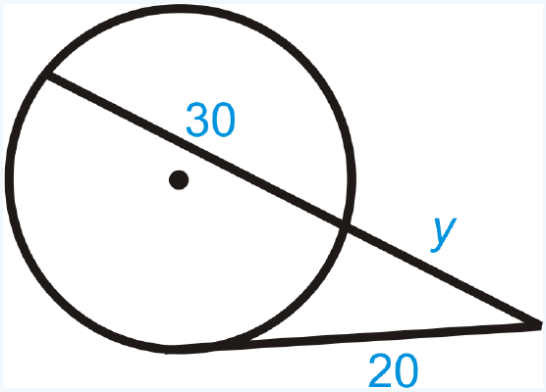

Figure 6.20.6

Solution

Use the Tangent Secant Segment Theorem.

$$20^2 = y(y+30)$$
$$400 = y^2 + 30y$$
$$0 = y^2 + 30y - 400$$
$$0 = (y+40)(y-10)$$
$$y = \cancel{-40}, 10$$

Review

Fill in the blanks for each problem below and then solve for the missing segment.

1.

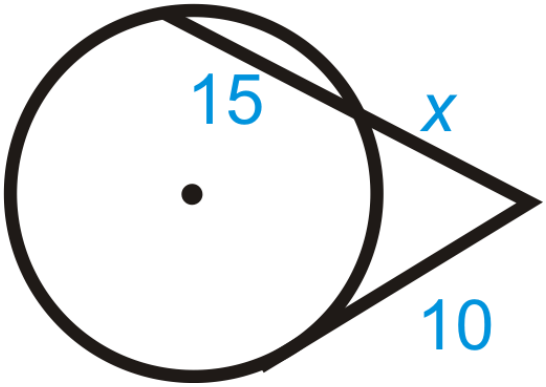

Figure **6.20.7**

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Find x in each diagram below. Simplify any radicals.

2.

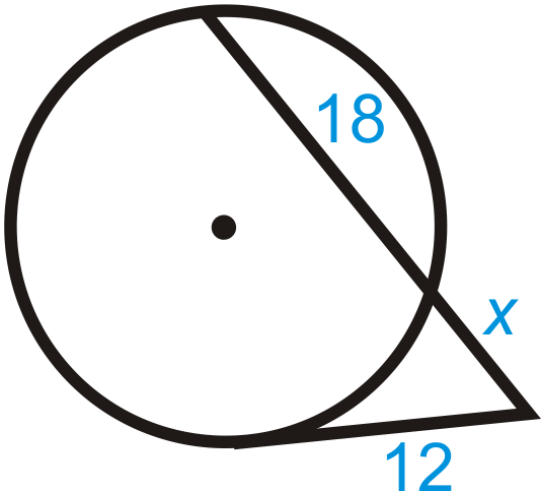


Figure **6.20.8**

3.

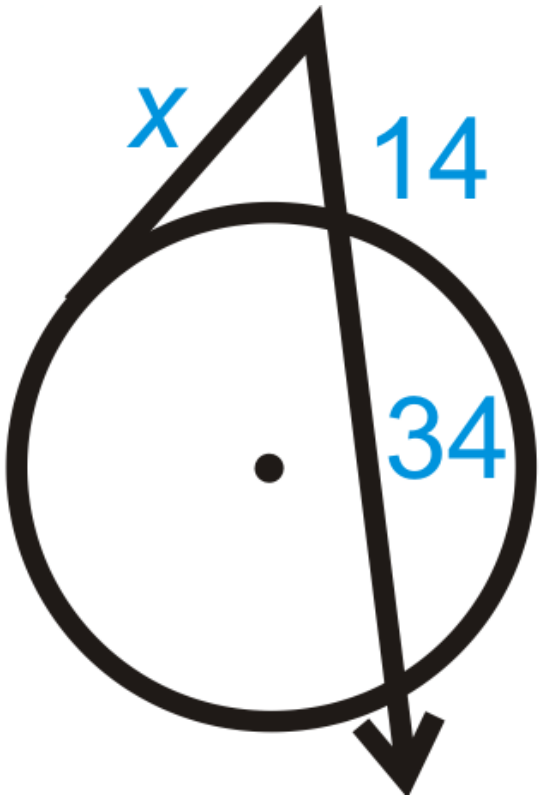


Figure 6.20.9

4.

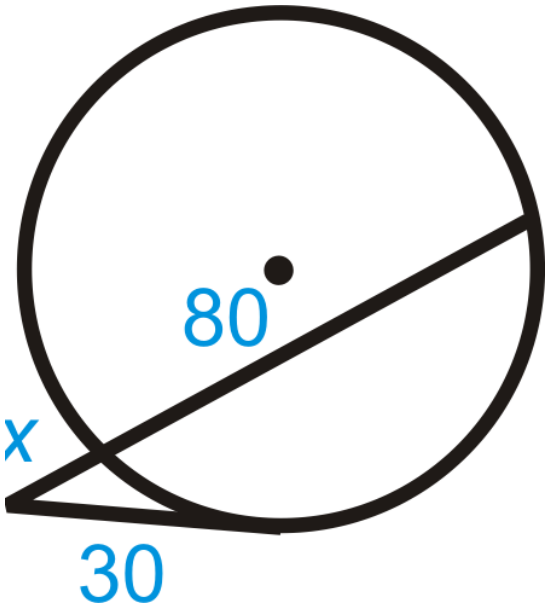


Figure 6.20.10

5. Describe and correct the error in finding y.

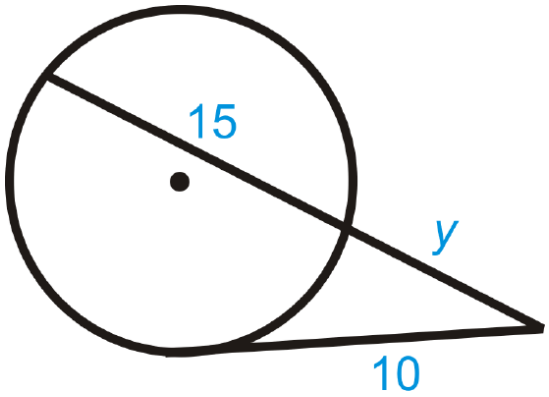
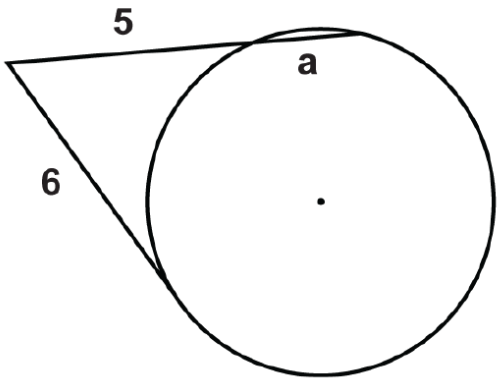


Figure 6.20.11

$$10 \cdot 10 = y \cdot 15y$$
$$100 = 15y^2$$
$$\frac{20}{3} = y^2$$
$$\frac{2\sqrt{15}}{3} = y \leftarrow \text{y is not correct}$$

Solve for the unknown variable.



6.

Figure 6.20.12

7.

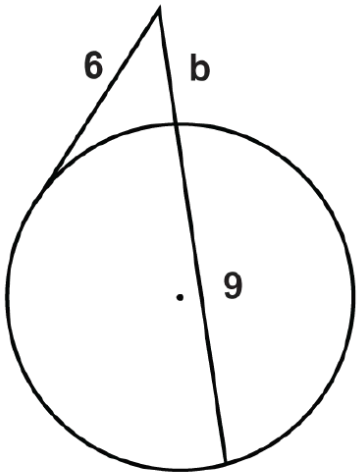


Figure 6.20.13

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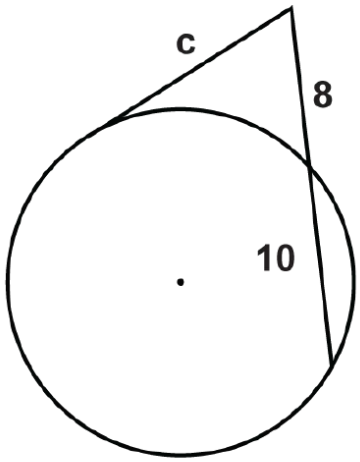

Figure 6.20.14

9.

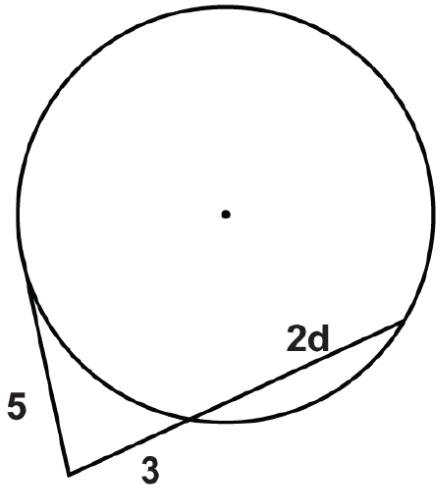

Figure 6.20.15

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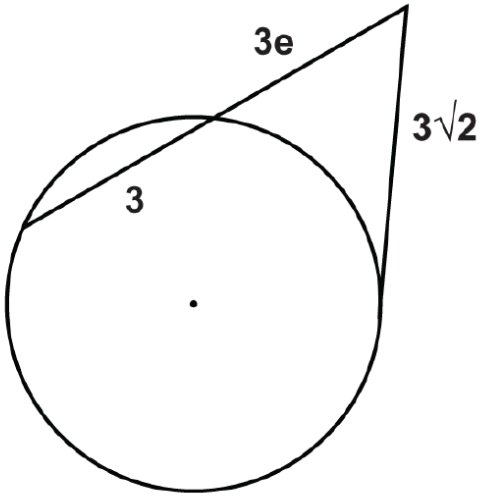

Figure 6.20.16

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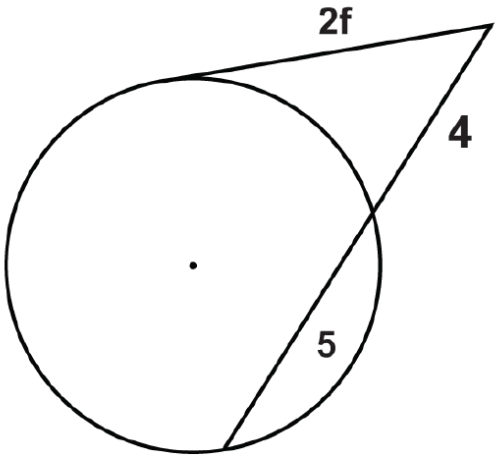

Figure 6.20.17

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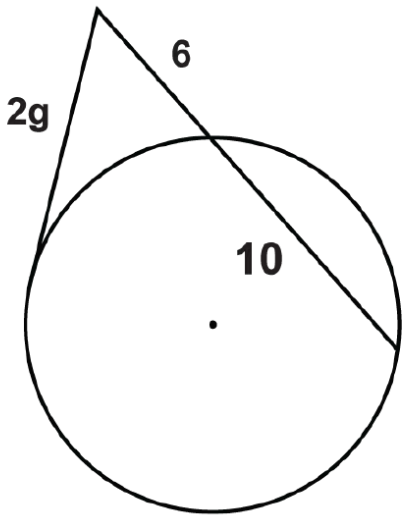

Figure 6.20.18

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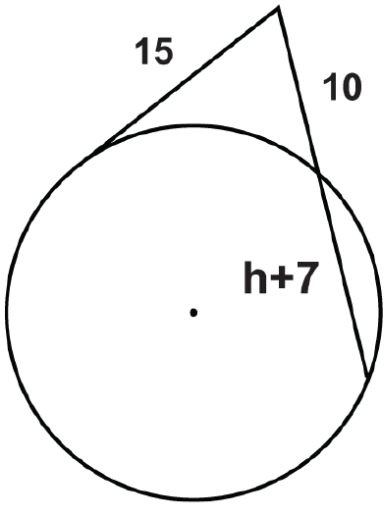

Figure 6.20.19

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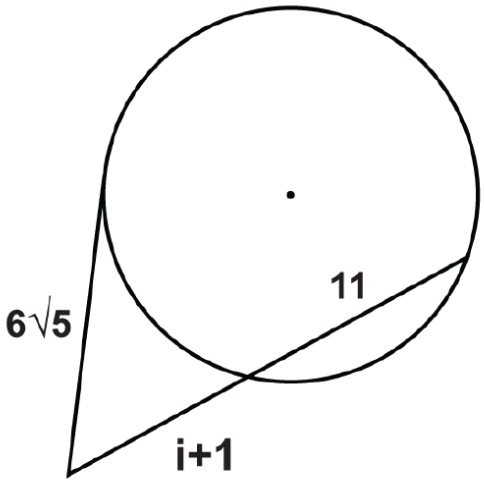

Figure 6.20.20

15.

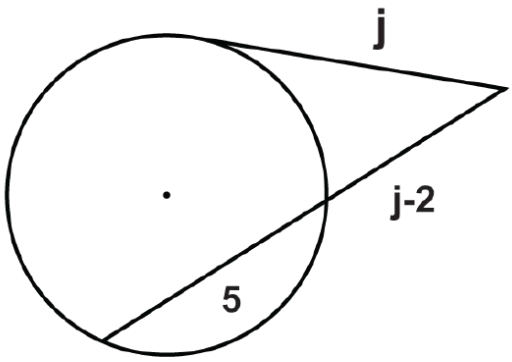

Figure 6.20.21

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.11.

Vocabulary

Term	Definition
central angle	An angle formed by two radii and whose vertex is at the center of the circle.
chord	A line segment whose endpoints are on a circle.
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
inscribed angle	An angle with its vertex on the circle and whose sides are chords.
intercepted arc	The arc that is inside an inscribed angle and whose endpoints are on the angle.
point of tangency	The point where the tangent line touches the circle.
radius	The distance from the center to the outer rim of a circle.
Secant	The secant of an angle in a right triangle is the value found by dividing length of the hypotenuse by the length of the side adjacent the given angle. The secant ratio is the reciprocal of the cosine ratio.
Tangents Secant Segments Theorem	If a tangent and a secant are drawn from a common point outside the circle (and the segments are labeled like the picture below), then $a^2 = b(b + c)$.

Additional Resources

Activities: Segments from Secants and Tangents Discussion Questions

Study Aids: Circles: Segments and Lengths Study Guide

Practice: Tangent Secant Theorem

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6.21: Circles in the Coordinate Plane

Graph a circle. Use (h, k) as the center and a point on the circle. Formula: $(x-h)^2+(y-k)^2=r^2$ where (h, k) is the center and r is the radius.

Recall that a **circle** is the set of all points in a plane that are the same distance from the center. This definition can be used to find an equation of a circle in the coordinate plane.

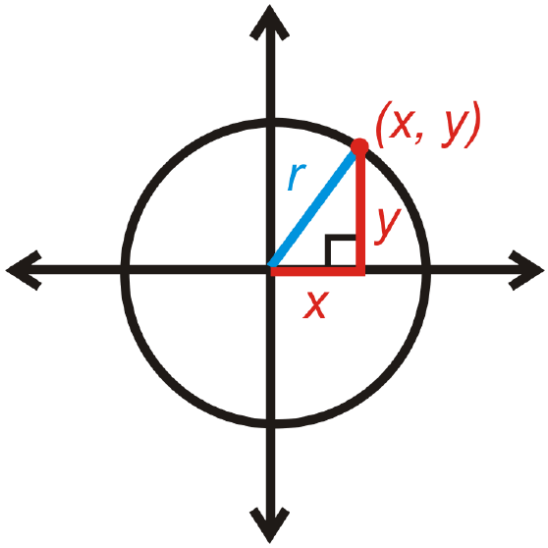

Figure **6.21.1**

Let's start with the circle centered at $(0, 0)$. If (x, y) is a point on the circle, then the distance from the center to this point would be the **radius**, r. x is the horizontal distance and y is the vertical distance. This forms a right triangle. From the Pythagorean Theorem, the equation of a circle *centered at the* ***origin*** is $x^2+y^2=r^2$.

The center does not always have to be on $(0, 0)$. If it is not, then we label the center (h, k). We would then use the **Distance Formula** to find the length of the radius.

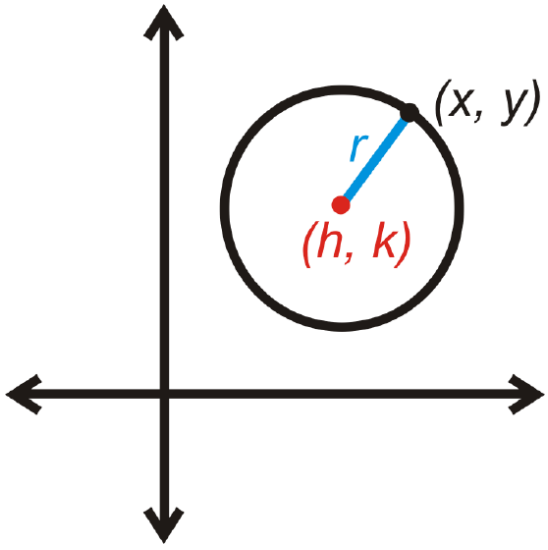

Figure **6.21.2**

$$r=\sqrt{(x-h)^2+(y-k)^2}$$

If you square both sides of this equation, then you would have the standard equation of a circle. The standard equation of a circle with center (h, k) and radius r is $r^2=(x-h)^2+(y-k)^2$.

What if you were given the length of the radius of a circle and the coordinates of its center? How could you write the equation of the circle in the coordinate plane?

Example 6.21.1

Find the center and radi circle.

$(x+2)^2+(y-5)^2=49$

Solution

Rewrite the equation as $(x-(-2))^2+(y-5)^2=7^2$. The center is $(-2,5)$ and $r=7$.

Keep in mind that, due to the minus signs in the formula, the coordinates of the center have the ***opposite signs*** of what they may initially appear to be.

Example 6.21.2

Find the center and radius of the following circle.

Find the equation of the circle with center $(4,-1)$ and which passes through $(-1,2)$.

Solution

First plug in the center to the standard equation.

$$(x-4)^2+(y-(-1))^2=r^2$$
$$(x-4)^2+(y+1)^2=r^2$$

Now, plug in (-1, 2) for x and y and solve for r.

$$(-1-4)^2+(2+1)^2=r^2$$
$$(-5)^2+(3)^2=r^2$$
$$25+9=r^2$$
$$34=r^2$$

Substituting in 34 for r^2, the equation is $(x-4)^2+(y+1)^2=34$.

Example 6.21.3

Graph $x^2+y^2=9$.

Solution

The center is $(0,0)$. Its radius is the square root of 9, or 3. Plot the center, plot the points that are 3 units to the right, left, up, and down from the center and then connect these four points to form a circle.

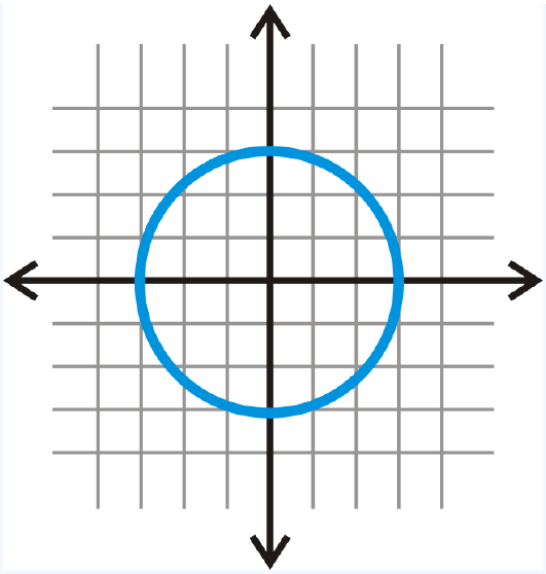

Figure 6.21.3

Example \(\PageIndex{4}\)

Find the equation of the circle below.

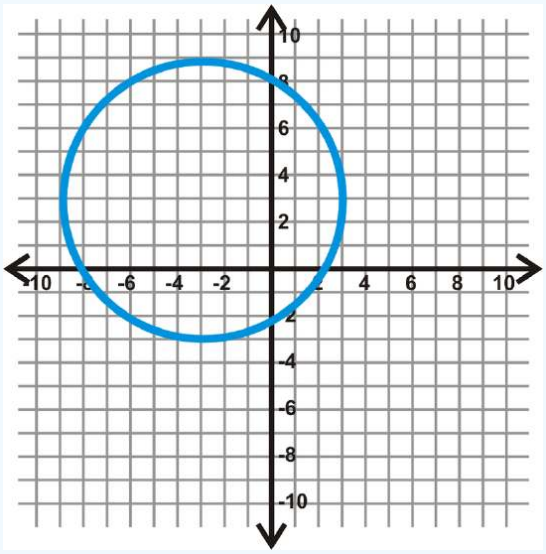

Figure 6.21.4

Solution

First locate the center. Draw in the horizontal and vertical diameters to see where they intersect.

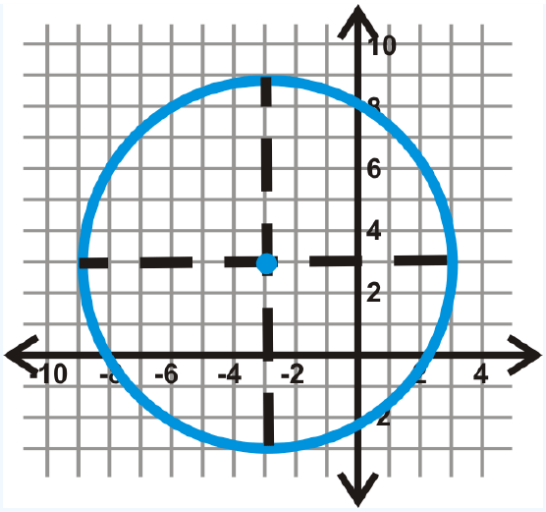

Figure 6.21.5

From this, we see that the center is $(-3, 3)$. If we count the units from the center to the circle on either of these diameters, we find $r = 6$. Plugging this into the equation of a circle, we get: $(x-(-3))^2+(y-3)^2=6^2$ or $(x+3)^2+(y-3)^2=36$.

Example 6.21.5

Determine if the following points are on $(x+1)^2+(y-5)^2=50$.

Solution

Plug in the points for x and y in $(x+1)^2+(y-5)^2=50$.

1. $(8,-3)$

$$(8+1)^2+(-3-5)^2=50$$
$$9^2+(-8)^2=50$$
$$81+64\neq 50$$

$(8,-3)$ is *not* on the circle

2. $(-2,-2)$

$$(-2+1)^2+(-2-5)^2=50$$
$$(-1)^2+(-7)^2=50$$
$$1+49=50$$

$(-2,-2)$ is on the circle

Review

Find the center and radius of each circle. Then, graph each circle.

1. $(x+5)^2+(y-3)^2=16$
2. \(x^2+(y+8)^2=4
3. \((x−7)^2+(y−10)^2=20
4. \((x+2)^2+y^2=8

Find the equation of the circles below.

5.

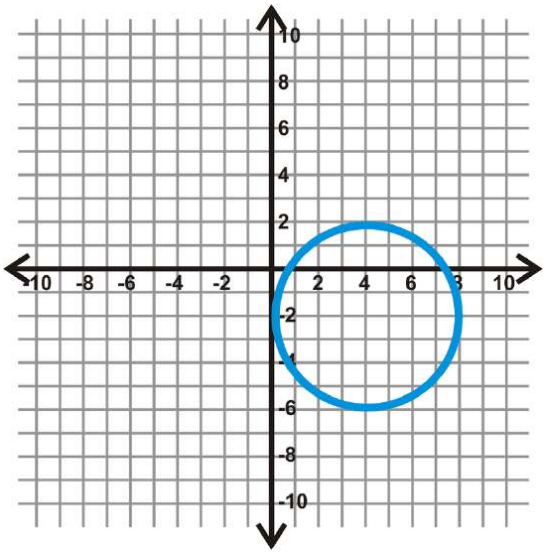

Figure 6.21.6

6.

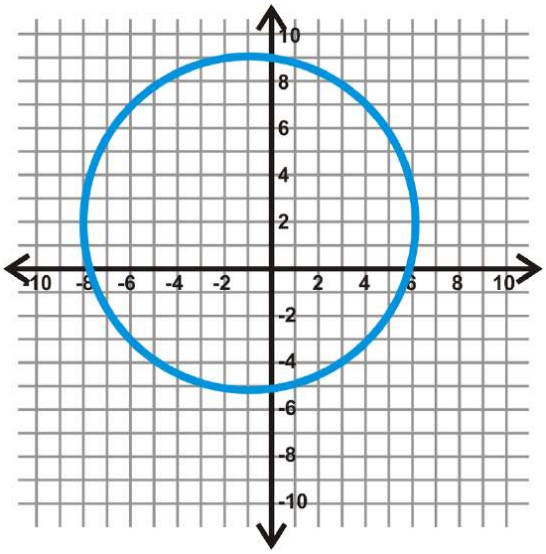

Figure 6.21.7

7.

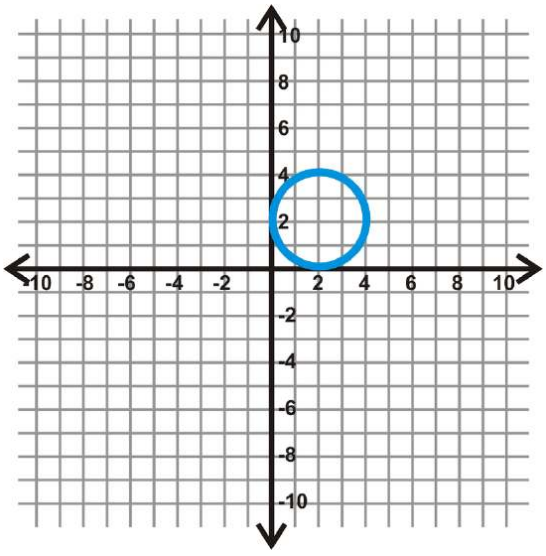

Figure 6.21.8

8.

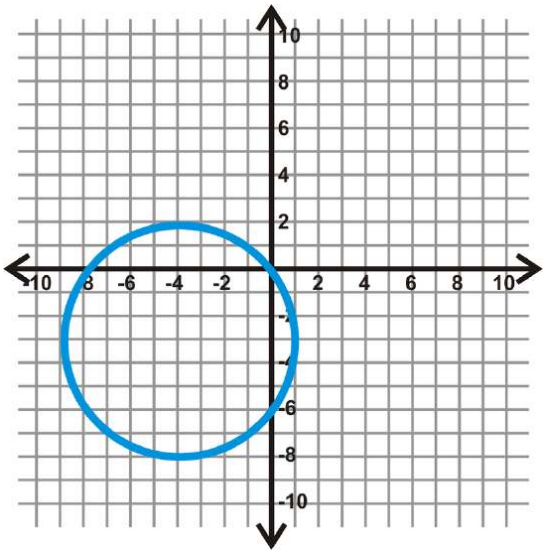

Figure 6.21.9

9. Is (-7, 3) on $(x+1)^2+(y-6)^2=45$?
10. Is (9, -1) on $(x-2)^2+(y-2)^2=60$?
11. Is (-4, -3) on $(x+3)^2+(y-3)^2=37$?
12. Is (5, -3) on $(x+1)^2+(y-6)^2=45$?

Find the equation of the circle with the given center and point on the circle.

13. center: (2, 3), point: (-4, -1)
14. center: (10, 0), point: (5, 2)
15. center: (-3, 8), point: (7, -2)
16. center: (6, -6), point: (-9, 4)

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.12.

Vocabulary

Term	Definition
circle	The set of all points that are the same distance away from a specific point, called the ***center***.
radius	The distance from the center to the outer rim of a circle.
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.
Origin	The origin is the point of intersection of the x and y axes on the Cartesian plane. The coordinates of the origin are (0, 0).

Additional Resources

Video: Graphing Circles Principles - Basic

Activities: Circles in the Coordinate Plane Discussion Questions

Study Aids: Properties of a Circle Study Guide

Practice: Circles in the Coordinate Plane

Real World: GPS Geometry

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CHAPTER OVERVIEW

7: Similarity

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7.4: Corresponding Parts of Similar Figures
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Thumbnail: Similar Triangles. (CC BY-SA 3.0; Nguyenthephuc via Wikipedia).

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7.1: Similar Figures

Figures that have the same shape but not the same size.

Triangle Identification as Similar, Congruent, or Neither

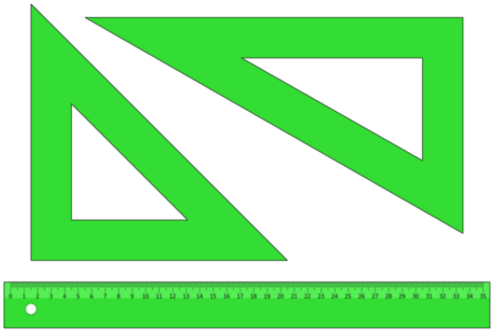


Figure **7.1.1**

Gabriel is in geometry. He uses different tools to measure different items. One day he sits a couple of his tools together and he notices that two of his tools are triangles. His class has recently compared triangles and identified them as **similar**, **congruent** or neither. The class is supposed to be using their measuring tools to measure items around the classroom and determine if they are similar, congruent or neither. Gabriel takes a good look at the two shapes then decides to identify them so that he can add them to his list. Are Gabriel's triangles similar, congruent or neither?

In this concept, you will learn to distinguish between triangles.

Identifying Triangles as Similar, Congruent, or Neither

Congruent means being exactly the same. When two line segments have the same length, they are congruent. When two figures have the same shape and size, they are congruent.

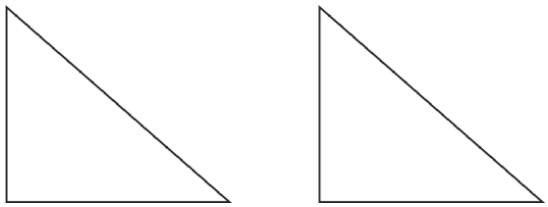


Figure **7.1.2**

These two triangles are congruent. They are exactly the same in every way. They are the same size and the same shape. Their side lengths are the same and that their angle measures are the same.

Sometimes, two figures will be **similar.** Similar means that the figures have the same shape, but not the same size. Similar figures are not congruent.

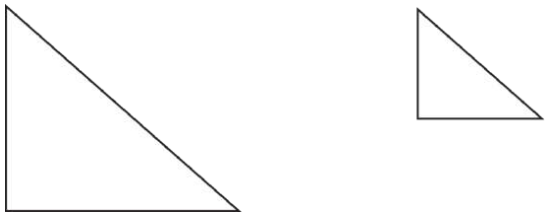


Figure **7.1.3**

These two triangles are similar. They are the same shape, but they are not the same size.

Example 7.1.1

Earlier, you were given a problem about Gabriel and his measuring tools.

He notices that two of his tools are triangles. Are his triangles similar, congruent or neither?

Solution

First, check to see if the triangles are the same size.

No

Next, check to see if the triangles are the same shape.

No

Then, identify the triangles.

The answer is that the triangles are neither similar, nor congruent. Gabriel will list the triangles as neither.

Example 7.1.2

Identify the following triangles as similar, congruent or neither.

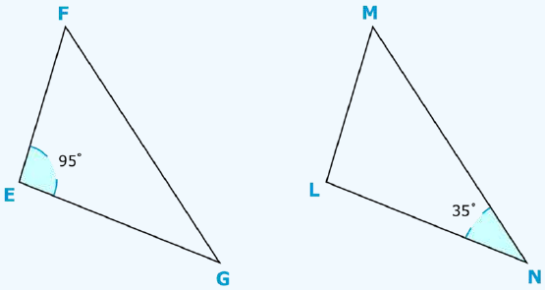


Figure **7.1.4**

Solution

First, check to see if the triangles have the same size.

Yes

Next, check to see if the triangles have the same shape.

Yes

Then, identify the triangles.

Congruent

The answer is that the triangles are congruent.

Example 7.1.3

Identify the following triangles as similar, congruent, or neither.

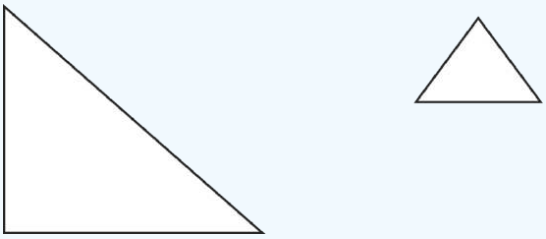


Figure **7.1.5**

Solution

First, check to see if the triangles are the same size.

No

Next, check to see if the triangles are the same shape.

No

Then, identify the triangles.

Neither congruent, nor similar

The answer is that the triangles are neither congruent, nor similar.

Example 7.1.4

Identify the following triangles as similar, congruent, or neither.

Figure **7.1.6**

Solution

First, check to see if the triangles are the same size.

No

Next, check to see if the triangles are the same shape.

Yes

Then, identify the triangles.

Similar

The answer is that the triangles are similar.

Example 7.1.5

Identify the following triangles as similar, congruent, or neither.

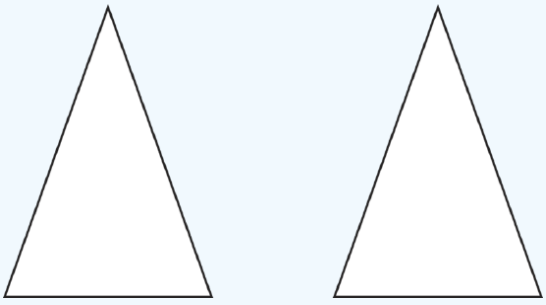


Figure **7.1.7**

Solution

First, check to see if the triangles are the same size.

Yes

Next, check to see if the triangles are the same shape.

Yes

Then, identify the triangles.

Congruent

The answer is that the triangles are congruent.

Review

Identify the given triangles as visually similar, congruent or neither.

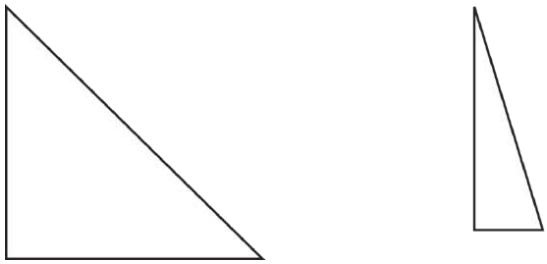
1. 

Figure 7.1.8

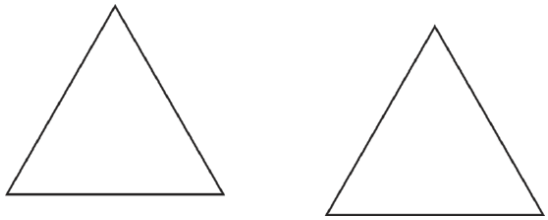
2. 

Figure 7.1.9

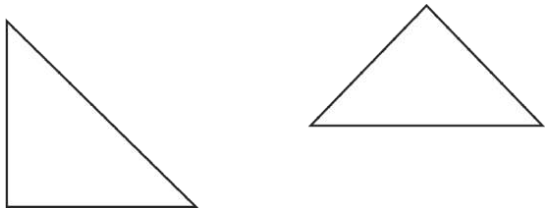
3. 

Figure 7.1.10

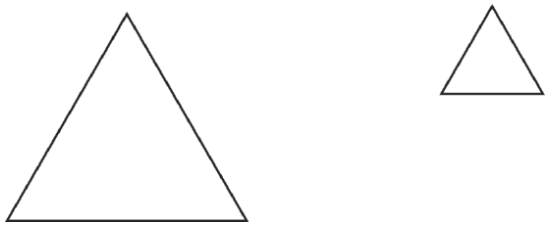
4. 

Figure 7.1.11

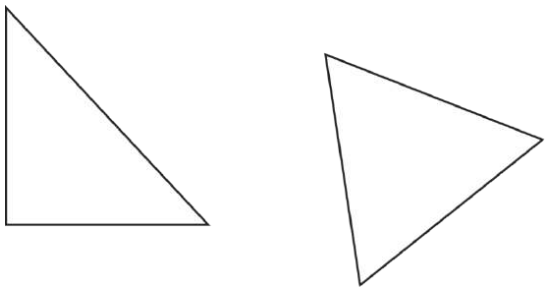
5. 

Figure 7.1.12

Answer each of the following questions.

6. Triangles ABC and DEF are congruent. Does this mean that their angle measures are the same? Why?
7. True or false. If triangles DEF and GHI are similar, then the side lengths are different but the angle measures are the same.
8. True or false. Similar figures have exactly the same size and shape.
9. True or false. Congruent figures are exactly the same in every way.
10. Triangles LMN and HIJ are similar. If this is true, then the side lengths are the same, true or false.
11. True or false. To figure out if two figures are similar, then their side lengths form a proportion.
12. Define similar figures
13. Define congruent figures.
14. Use a ruler to draw a congruent pair of triangles.
15. Use a ruler to draw a pair of triangles that is similar.

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.15.

Resources

Vocabulary

Term	Definition
Congruent	Congruent figures are identical in size, shape and measure.
Similar	Two figures are similar if they have the same shape, but not necessarily the same size.

Additional Resources

Video: Congruent and Similar Triangles - KA

Practice: Similar Figures

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7.2: Ratio and Proportion in Similar Figures

Simplify ways to compare two numbers.

Proportions

A **proportion** is two ratios that are set equal to each other. Usually the ratios in proportions are written in fraction form. An example of a proportion is $\frac{2}{x} = 510$. To solve a proportion, you need to **cross-multiply.** The **Cross-Multiplication Theorem**, which allows us to solve proportions using this method, states that if a, b, c, and d are real numbers, with $b \neq 0$ and $d \neq 0$ and if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{d} = \frac{b}{c}$. Cross-multiplying allows us to get rid of the fractions in our equation. The Cross-Multiplication Theorem has several sub-theorems, called **corollaries.**

Corollary #1: If a, b, c, and d are nonzero and $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

Switch b and c.

Corollary #2: If a, b, c, and d are nonzero and $\frac{a}{b} = \frac{c}{d}$, then $\frac{d}{b} = \frac{c}{a}$.

Switch a and d.

Corollary #3: If a, b, c, and d are nonzero and $\frac{a}{b} = \frac{c}{d}$\), then $\frac{b}{a} = \frac{c}{d}$.

*Flip each **ratio** upside down.*

Corollary #4: If a, b, c, and d are nonzero and \dfrac{a}{b}=\dfrac{c}{d}\), then $\frac{a+b}{b} = \frac{c+d}{d}$.

Corollary #5: If a, b, c, and d are nonzero and $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$.

What if you were told that a scale model of a python is in the ratio of 1:24? If the model measures 0.75 feet long, how long is the real python?

Example 7.2.1

In the picture, $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$.

Find the measures of AC

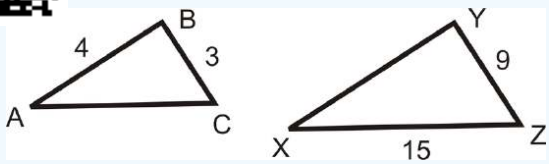

Figure **7.2.1**

Solution

Plug in the lengths of the sides we know.

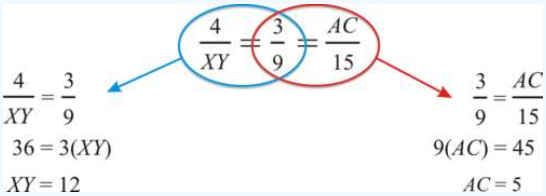

Figure **7.2.2**

Example 7.2.2

In the picture, $\frac{AB}{BE} = \frac{AC}{CD}$. Find BE.

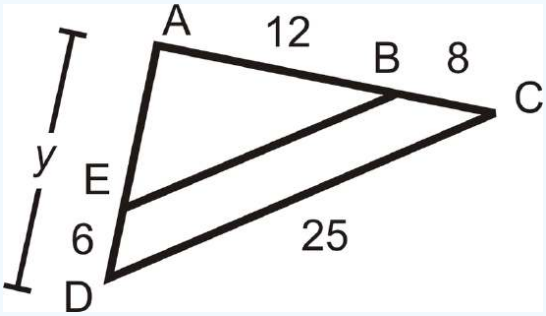

Figure 7.2.3

Solution

Substitute in the lengths of the sides we know.

$$\frac{12}{BE} = \frac{20}{25} \rightarrow 20(BE) = 12(25)$$
$$BE = 15$$

Example 7.2.3

Solve the proportions. Remember, to solve a proportion, you need to **cross-multiply.**

a. $\frac{4}{5} = \frac{x}{30}$
b. $\frac{y+1}{8} = \frac{5}{20}$
c. $\frac{6}{5} = \frac{2x+4}{x-2}$

Solution

1.

$$\frac{4}{5} = \frac{x}{30}$$
$$4 \cdot 30 = 5 \cdot x$$
$$120 = 5x$$
$$24 = x$$

Figure 7.2.4

2.

$$\frac{y+1}{8} = \frac{5}{20}$$
$$(y+1) \cdot 20 = 5 \cdot 8$$
$$20y + 20 = 40$$
$$20y = 20$$
$$y = 1$$

Figure 7.2.5

3.

$$\frac{6}{5} = \frac{2x+4}{x-2}$$
$$6 \cdot (x-2) = 5 \cdot (2x+4)$$
$$6x-12 = 10x+20$$
$$-32 = 4x$$
$$-8 = x$$

Figure 7.2.6

Example 7.2.4

Your parents have an architect's drawing of their home. On the paper, the house's dimensions are 36 in by 30 in. If the shorter length of the house is actually 50 feet, what is the longer length?

Solution

To solve, first set up a proportion. If the shorter length is 50 feet, then it lines up with 30 in, the shorter length of the paper dimensions.

$$\frac{30}{3}6 = \frac{50}{x} \rightarrow 30x = 1800$$
$$x = 60 \quad \text{The longer length is 60 feet.}$$

Example 7.2.5

Suppose we have the proportion $\frac{2}{5} = \frac{14}{35}$. Write three true proportions that follow.

Solution

First of all, we know this is a true proportion because you would multiply $\frac{2}{5}$ by $\frac{7}{7}$ to get $\frac{14}{35}$. Using the first three corollaries:

1. $\frac{2}{14} = \frac{5}{35}$
2. $\frac{35}{5} = \frac{14}{2}$

3. $\frac{5}{2} = \frac{35}{14}$

Review

Solve each proportion.

1. $\frac{x}{10} = \frac{42}{35}$
2. $\frac{x}{x-2} = \frac{5}{7}$
3. $\frac{6}{9} = \frac{y}{24}$
4. $\frac{x}{9} = \frac{16}{x}$
5. $\frac{y-3}{8} = \frac{y+6}{5}$
6. $\frac{20}{z+5} = \frac{16}{7}$
7. Shawna drove 245 miles and used 8.2 gallons of gas. At the same rate, if she drove 416 miles, how many gallons of gas will she need? Round to the nearest tenth.
8. The president, vice-president, and financial officer of a company divide the profits is a 4:3:2 ratio. If the company made $1,800,000 last year, how much did each person receive?

Given the true proportion, $\frac{10}{6} = \frac{15}{d} = \frac{x}{y}$ and d, x, and y are nonzero, determine if the following proportions are also true.

9. $\frac{10}{y} = \frac{x}{6}$
10. $\frac{15}{10} = \frac{d}{6}$
11. $\frac{6+10}{10} = \frac{y+x}{x}$
12. $\frac{15}{x} = \frac{y}{d}$

For questions 13-16, $\frac{AE}{ED} = \frac{BC}{CD}$ and $\frac{ED}{AD} = \frac{CD}{DB} = \frac{EC}{AB}$.

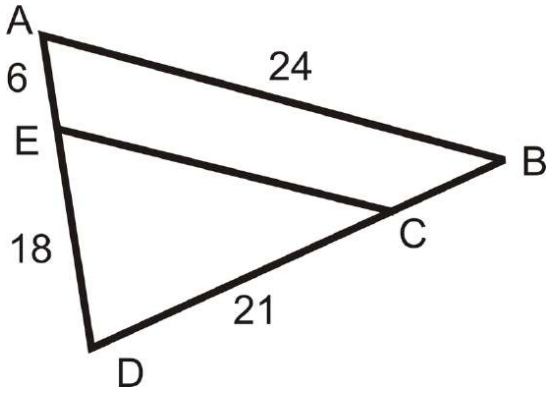

Figure **7.2.7**

13. Find DB.
14. Find EC.
15. Find CB.
16. Find AD.

Resources

Review (Answers)

To see the Review answers and look for section 7.2.

Vocabulary

Term	Definition
corollary	A theorem that follows directly from another theorem.
proportion	Two ratios that are set equal to each other.
ratio	A way to compare two numbers. Ratios can be written in three ways: \dfrac{a}{b}, a:b, and a to b.
Cross Products	To simplify a proportion using cross products, multiply the diagonals of each ratio.
Cross-Multiplication Theorem	The Cross-Multiplication theorem states that if a, b, c and d are real numbers, with $b \neq 0$ and $d \neq 0$ and if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{d} = \frac{b}{c}$.

Additional Resources

Video: Ratios

Activities: Forms of Ratios Discussion Questions

Study Aids: Ratios and Proportions Study Guide

Practice: Ratio and Proportion in Similar Figures

Real World: In the Doll's House

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7.3: Similar Polygons and Scale Factors

Similar Polygons

Similar polygons are two polygons with the same shape, but not the same size. Similar polygons have corresponding angles that are **congruent**, and corresponding sides that are **proportional.**

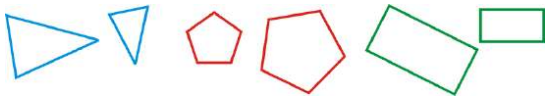


Figure **7.3.1**

These polygons are not similar:

Figure **7.3.2**

Scale Factors

Think about similar polygons as enlarging or shrinking the same shape. The symbol \sim\) is used to represent similarity. Specific types of triangles, quadrilaterals, and polygons will always be similar. For example, all equilateral triangles are similar and all squares are similar**.** If two polygons are similar, we know the lengths of corresponding sides are proportional. In similar polygons, the ratio of one side of a polygon to the corresponding side of the other is called the **scale factor**. The ratio of all parts of a polygon (including the perimeters, diagonals, medians, midsegments, altitudes) is the same as the ratio of the sides.

What if you were told that two pentagons were similar and you were given the lengths of each pentagon's sides. How could you determine the scale factor of pentagon #1 to pentagon #2?

Example 7.3.1

$ABCD$ and $UVWX$ are l o rectangles similar?

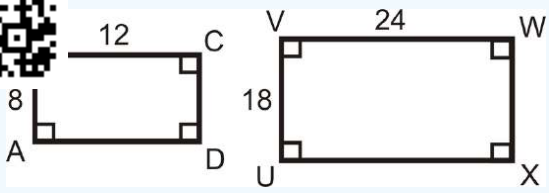


Figure **7.3.3**

Solution

All the corresponding angles are congruent because the shapes are rectangles.

Let's see if the sides are proportional. $\frac{8}{12}=\frac{2}{3}$ and $\frac{18}{24}=\frac{3}{4}$. $\frac{2}{3}\neq\frac{3}{4}$, so the sides are ***not*** in the same **proportion**, and the rectangles are ***not*** similar.

Example 7.3.2

$\Delta ABC \sim \Delta MNP$. The perimeter of ΔABC is 150, $AB=32$ and $MN=48$. Find the perimeter of ΔMNP.

Solution

From the similarity statement, AB and MN are corresponding sides. The scale factor is $\frac{32}{48}=\frac{2}{3}$ or $\frac{3}{2}$. \Delta ABC\) is the smaller triangle, so the perimeter of ΔMNP is $\frac{3}{2}(150)=225$.

Example 7.3.3

Suppose $\Delta ABC \sim \Delta JKL$. Based on the similarity statement, which angles are congruent and which sides are proportional?

Solution

Just like in a congruence statement, the congruent angles line up within the similarity statement. So, $\angle A \cong \angle J$, $\angle B \cong \angle K$, and \angle C\cong \angle L\). Write the sides in a proportion: $\frac{AB}{JK} = \frac{BC}{KL} = \frac{AC}{JL}$. Note that the proportion could be written in different ways. For example, $\frac{AB}{BC} = \frac{JK}{KL}$ is also true.

Example 7.3.4

$MNPQ \sim RSTU$. What are the values of x, y and z?

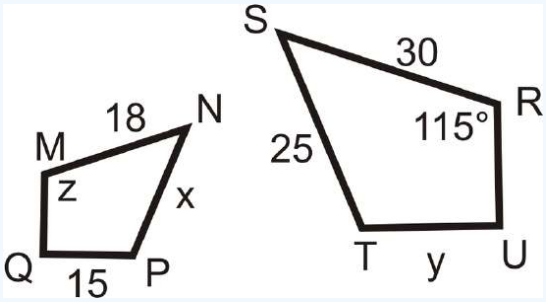


Figure 7.3.4

Solution

In the similarity statement, $\angle M \cong \angle R$, so $z = 115°$. For x and y, set up proportions.

$$\frac{18}{30} = \frac{x}{25} \qquad \frac{18}{30} = \frac{15}{y}$$

$$450 = 30x \qquad 18y = 450$$

$$x = 15 \qquad y = 25$$

Example 7.3.5

$ABCD \sim AMNP$. Find the scale factor and the length of BC.

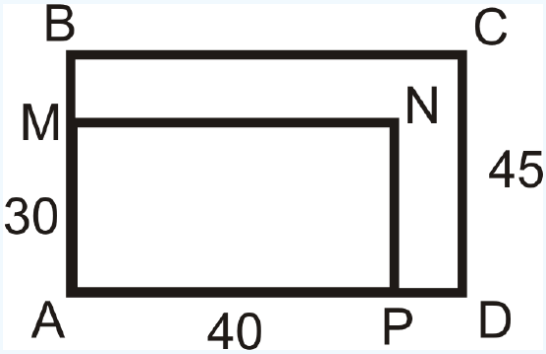


Figure 7.3.5

Solution

Line up the corresponding sides, AB and $AM = CD$, so the scale factor is $\frac{30}{45} = \frac{2}{3}$ or $\frac{3}{2}$. Because BC is in the bigger rectangle, we will multiply 40 by $\frac{3}{2}$ because $\frac{3}{2}$ is greater than 1. $BC = \frac{3}{2}(40) = 60$.

Review

For questions 1-8, determine whether the following statements are true or false.

1. All equilateral triangles are similar.
2. All isosceles triangles are similar.
3. All rectangles are similar.
4. All rhombuses are similar.
5. All squares are similar.
6. All congruent polygons are similar.
7. All similar polygons are congruent.
8. All regular pentagons are similar.
9. $\Delta BIG \sim \Delta HAT$. List the congruent angles and proportions for the sides.
10. If $BI = 9$ and $HA = 15$, find the scale factor.
11. If $BG = 21$, find HT.
12. If $AT = 45$, find IG.
13. Find the perimeter of ΔBIG and ΔHAT. What is the ratio of the perimeters?
14. An NBA basketball court is a rectangle that is 94 feet by 50 feet. A high school basketball court is a rectangle that is 84 feet by 50 feet. Are the two rectangles similar?
15. HD TVs have sides in a ratio of 16:9. Non-HD TVs have sides in a ratio of 4:3. Are these two ratios equivalent?

Use the picture to the right to answer questions 16-20.

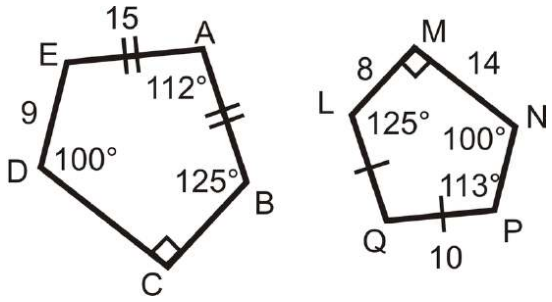


Figure 7.3.6

16. Find $m\angle E$ and $m\angle Q$.
17. $ABCDE \sim QLMNP$, find the scale factor.
18. Find BC.
19. Find CD.
20. Find NP.

Determine if the following triangles and quadrilaterals are similar. If they are, write the similarity statement.

21.

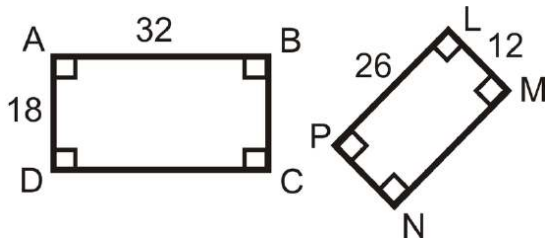


Figure 7.3.7

22.

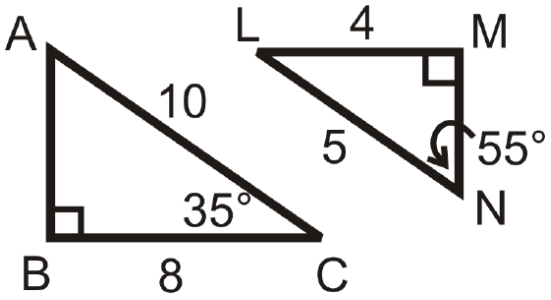


Figure 7.3.8

23.

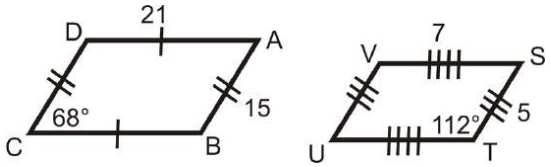


Figure 7.3.9

24.

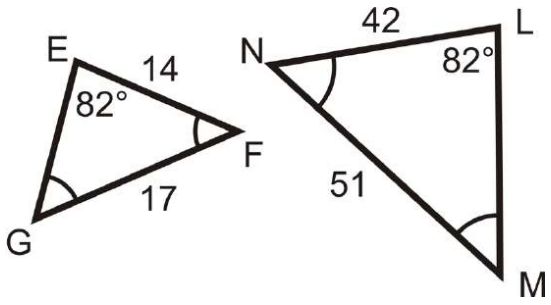


Figure 7.3.10

25.

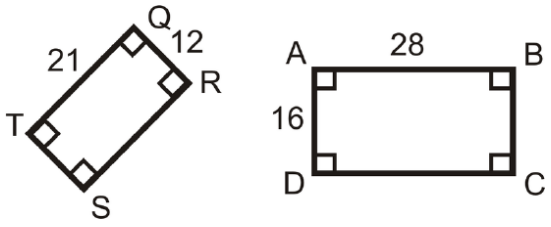


Figure 7.3.11

26.

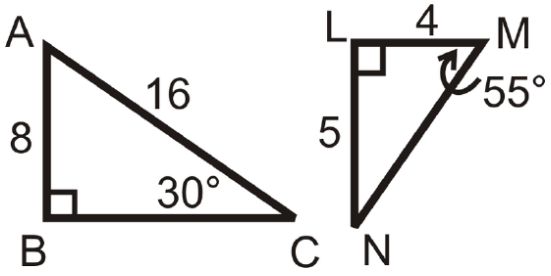


Figure 7.3.12

27.

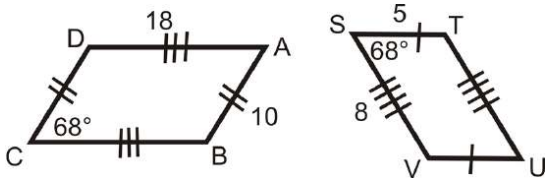


Figure 7.3.13

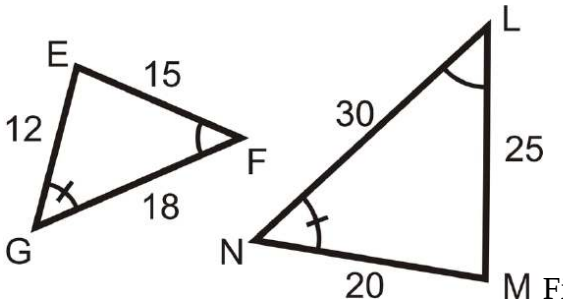
28. 

Figure \(\

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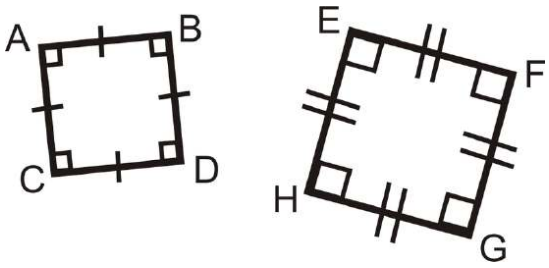
29. 

Figure **7.3.15**

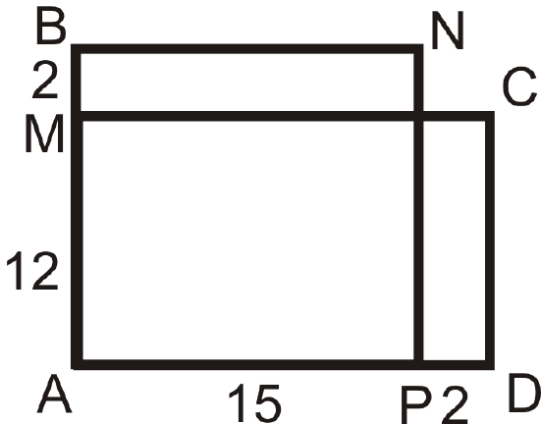
30. 

Figure **7.3.16**

Resources

Vocabulary

Term	Definition
similar polygons	Two polygons with the same shape, but not the same size. The corresponding angles of similar polygons are ***congruent*** (exactly the same) and the corresponding sides are ***proportional*** (in the same ratio). In similar polygons, the ratio of one side of a polygon to the corresponding side of the other is called the ***scale factor***.
Congruent	Congruent figures are identical in size, shape and measure.
Proportion	A proportion is an equation that shows two equivalent ratios.
Scale Factor	A scale factor is a ratio of the scale to the original or actual dimension written in simplest form.
sine	The sine of an angle in a right triangle is a value found by dividing the length of the side opposite the given angle by the length of the hypotenuse.
Trigonometric Ratios	Ratios that help us to understand the relationships between sides and angles of right triangles.

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7.4: Corresponding Parts of Similar Figures

Identify matching side lengths and angle measures in similar figures.

Figure 7.4.1

Maria has an art project due for school. She decides to make a small replica of her bedroom. She begins by measuring the angles of her bedroom, then she draws **similar** figures. The larger figure represents her bedroom and the smaller figure represents her replica.

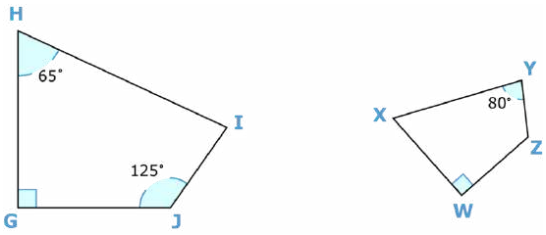

Figure 7.4.2

Her bedroom door is located in the middle of $\overline{IJ}$. For her replica, the door must be placed in the middle of the **corresponding** side. What side of the replica corresponds to $\overline{IJ}$?

In this concept, you will learn about corresponding sides of similar figures.

Corresponding Parts of Similar Figures

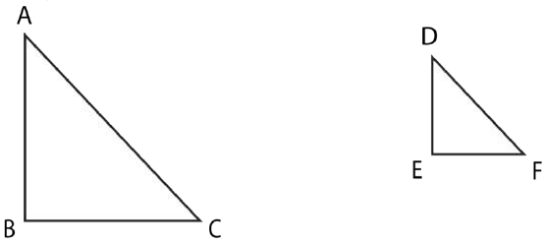

Figure 7.4.3

Triangle ABC is **similar** to triangle DEF. This means that while they are the same shape, they aren't the same size. In fact, there is a relationship between the corresponding parts of the triangle.

The side lengths are corresponding even though they aren't **congruent**. Use the symbol for similar ("~") to show the relationship between the corresponding sides of the two triangles.

$$\overline{AB} \sim \overline{DE}$$

$$\overline{BC} \sim \overline{EF}$$

$$\overline{AC} \sim \overline{DF}$$

Example 7.4.1

Earlier, you were given a problem about Maria and her bedroom.

She is making a replica of her bedroom for her art class. She draws two similar figures to make sure that she is placing everything in the right position.

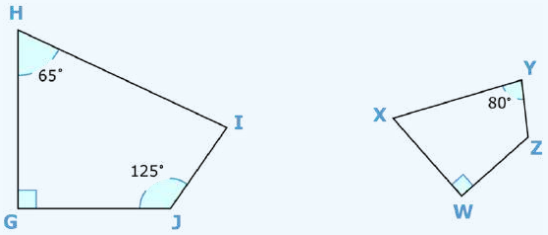

Figure 7.4.4

Solution

She needs to place her bedroom door in the middle of the side that corresponds to $\overline{IJ}$. What side corresponds to $\overline{IJ}$?

First, rotate the second figure so that it looks like the first figure.

Then, locate the side on the second figure that is in the position that $\overline{IJ}$ is in the first figure.

$\overline{YZ}$

The answer is that $\overline{YZ}$ corresponds to $\overline{IJ}$. Maria needs to place her bedroom door in the middle of $\overline{YZ}$ on the replica.

Example 7.4.2

The figures below are similar. What side corresponds $\overline{NO}$?

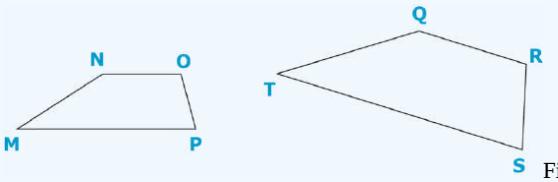

Figure 7.4.5

Solution

First, position the second figure so that it looks like the first figure.

Then, identify the side in the second figure that is in the same position as $\overline{NO}$ is in the first figure.

$\overline{QR}$

The answer is that $\overline{QR}$ corresponds to $\overline{NO}$.

Example 7.4.3

Missing open brace for subscript

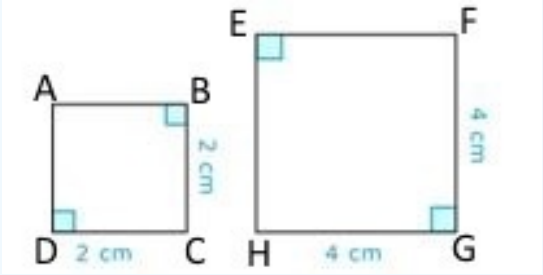

Figure 7.4.6

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Solution

First, look to see which side in the second triangle corresponds to the given segment.

$\overline{EF}$

Then, complete the problem.

$\overline{AB} \sim \overline{EF}$

The answer is that $\overline{AB} \sim \overline{EF}$.

Example 7.4.4

What makes the two squares similar if they both have four right angles?

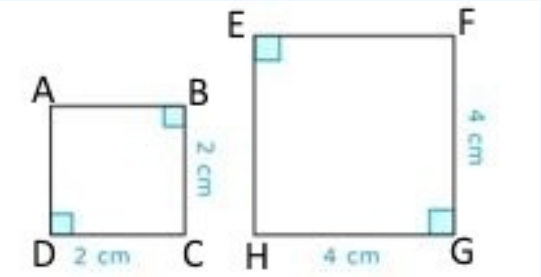

Figure 7.4.7

Solution

First, check to see if the side lengths are the same.

No

Next, check to see if the shape is the same.

Yes

Then, state why the two squares are similar.

Same shape, different sizes

The answer is that the two squares are similar because they have the same shape, but different sizes.

Example 7.4.5

In the triangle pair, are the two triangles similar or congruent?

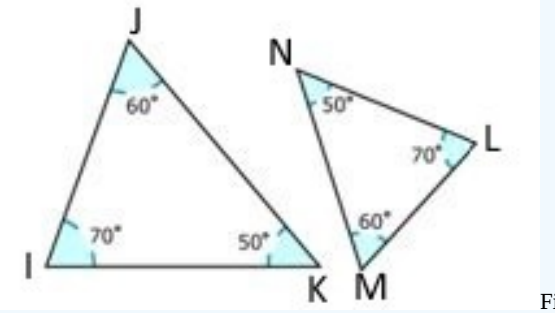

Figure 7.4.8

Solution

First, check to see if the two triangles are the same size.

No

Next, check to see if the two triangles are the same shape.

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Yes

Then, determine if the two triangles are similar or congruent.

Similar

The two triangles are similar.

Review

Use the following figures to answer each question.

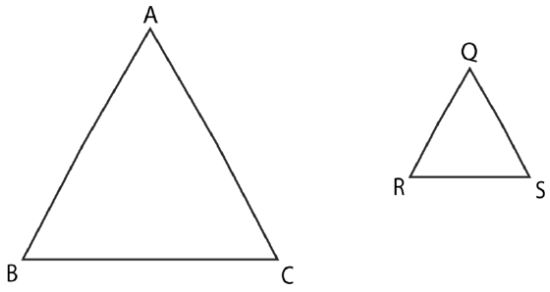

Figure **7.4.9**

1. Are these two triangles similar or congruent?
2. How do you know?
3. Which side is congruent to $\overline{AB}$?
4. Which side is congruent to $\overline{AC}$?
5. Which side is congruent to $\overline{RS}$?
6. Which angle is congruent to angle A?
7. Which angle is congruent to angle B?
8. Which angle is congruent to angle C?

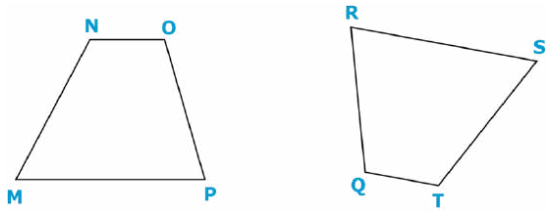

Figure **7.4.10**

9. Are the two figures similar or congruent?
10. Why?
11. Which side is congruent to $\overline{NO}$?
12. Which side is congruent to $\overline{MN}$?
13. Which side is congruent to $\overline{ST}$?
14. Which side is congruent to $\overline{QT}$?
15. Which side is congruent to $\overline{OP}$?

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.17.

Resources

Vocabulary

Term	Definition
Congruent	Congruent figures are identical in size, shape and measure.
Corresponding	The corresponding sides between two triangles are sides in the same relative position.
Similar	Two figures are similar if they have the same shape, but not necessarily the same size.

Additional Resources

Video: Congruent and Similar Triangles

Practice: Corresponding Parts of Similar Figures

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7.5: Indirect Measurement

Find lengths using proportions of similar triangles.

Unknown Measures of Similar Figures

Figure **7.5.1**

Shelby's father is a former Olympic swimmer. He loves swimming, but he cannot build an Olympic sized swimming pool in his backyard because he does not have the space. He decides to build a smaller pool that is **similar** to the dimensions of an Olympic pool. An Olympic pool has a length of 50 meters and a width of 25 meters. His pool has a length of 25 meters. What is the width of his pool?

In this concept, you will learn how to find the unknown measures of similar figures.

Finding Unknown Measures of Similar Figures

If you know the length of a side in one figure, you can use the **scale factor** to find the measure of the corresponding side in a similar figure. Let's see how this works.

Side a in triangle ABC corresponds to side x in the smaller triangle XYZ. Side x is 4 meters long and the scale factor is 6. What is the measure of side a?

You have been told that two sides, a and x, correspond in a small triangle and a large one. Side x is 4 meters long, and the scale factor tells you that side a will be six times longer. Let's write this out and solve.

$$\text{side } x \times \text{ scale factor } = \text{ side } a$$

$$4 \times 6 = \text{ side a}$$

$$24 \text{ m } = \text{ side a}$$

Side a must have a length of 24 meters.

You can check by setting up a ratio that compares the lengths of the two sides. If the scale factor is 6, then your work is accurate.

$$\frac{a}{x} = \frac{24}{4} = 6$$

Sometimes, you can figure out missing side lengths by looking at the given measures. Always look at the diagram of the figures and see if you can determine the missing length without measuring.

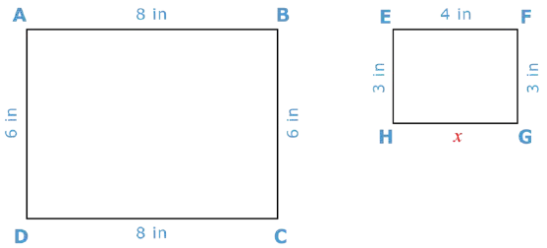

Figure **7.5.2**

Look at these two rectangles. First, look and see if you can figure out the relationship between the two figures. To do this, you compare the side lengths of each part of the two figures.

You need to figure out the measurement of side GH in the second rectangle.

You can see that the measurements in the second rectangle are half as big as the measurements in the first. Also, you know that the opposite sides of a rectangle are congruent. Therefore, the missing side length is 4.

Similar figures that are related by a scale factor are often seen in maps, architectural blueprints and diagrams. In most of these cases, the scale factor is given so that you know how to enlarge the items in the drawing to their real sizes. Take a look at the floor plan below. It shows where the furniture is located in a living room.

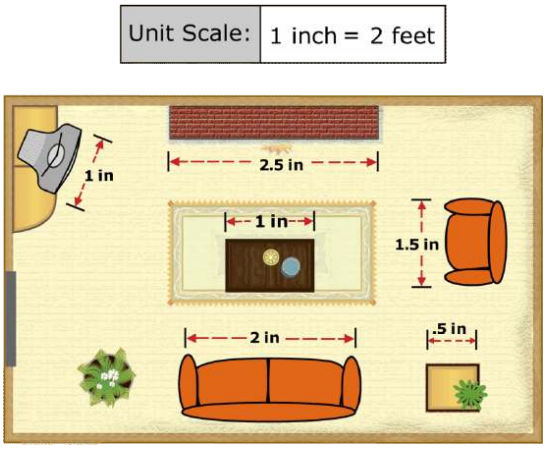

Figure **7.5.3**

It tells you that one inch in the drawing is equal to two feet in actual size. Therefore, if you know the size in inches of any object in the floor plan, you can find its actual size in feet. Let's give it a try.

Find the sofa on the floor plan then find its length in inches. The sofa in the floor plan is 2 inches long. This is like knowing the length of one side in a similar figure. Now use the scale factor as you would to find the length of the corresponding side in a similar figure (in this case the "corresponding side" is the actual sofa). Simply multiply the length you know by the scale factor:

$$\text{sofa drawing} \times \text{scale factor} = \text{actual sofa size}$$

$$2 \text{ inches} \times 2 = 4 \text{ feet}$$

The sofa is 4 feet long.

Next, let's calculate the real length of the fireplace.Use a ruler to measure the fireplace in the drawing. It is 2.5 inches long. Multiply this by the scale factor to find the length in feet.

$$\text{fireplace drawing} \times \text{scale factor} = \text{actual fireplace length}$$

$$2.5 \text{ inches} \times 2 = 5 \text{ feet}$$

The real length of the fireplace is 5 feet.

You can also reverse the process to take an actual size and reduce it.

Example 7.5.1

Earlier, you were given a problem about Shelby's father and his swimming pool.

He builds a pool that is similar to an Olympic pool. An Olympic pool has a length of 50 meters and a width of 25 meters. If his pool has a length of 25 meters, what is the width of his pool?

Solution

First, set up an equation that can be used to solve for the width of his pool.

$$\frac{50}{25} = \frac{25}{x}$$

Next, cross multiply.

$50x = 625$

Then, divide both sides of the equation by 50.

$x = 12.5$

The answer is that his pool has a width of 12.5 meters.

Example 7.5.2

Use the scale factor of the similar figures below to find the measure of KJ.

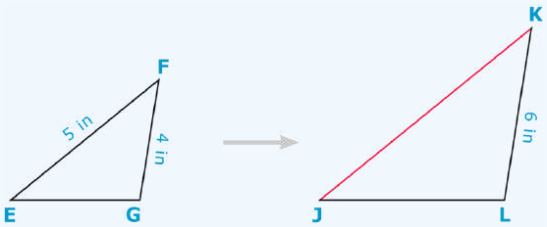

Figure 7.5.4

Solution

First, set up a proportion to solve for the missing side.

$$\frac{KJ}{5} = \frac{6}{4}$$

This proportion is written so that the corresponding sides form the two ratios of the proportion. You can say that KJ is the unknown in this proportion.

Next, cross multiply.

$$KJ \times 4 = 4KJ$$
$$5 \times 6 = 30$$
$$4KJ = 30$$

Then solve the equation for KJ by dividing both sides of the equation by 4.

$$30 \backslash \text{divide} 4 = 7.5$$
$$KJ = 7.5$$

The answer is that the side length of KJ is 7.5.

Example 7.5.3

Chris is making a drawing of his school and the grounds around it. The basketball court is 75 feet long and 40 feet wide. If Chris uses a scale factor in which 1 inch equals 10 feet, what should the dimensions of the basketball court be in his drawing?

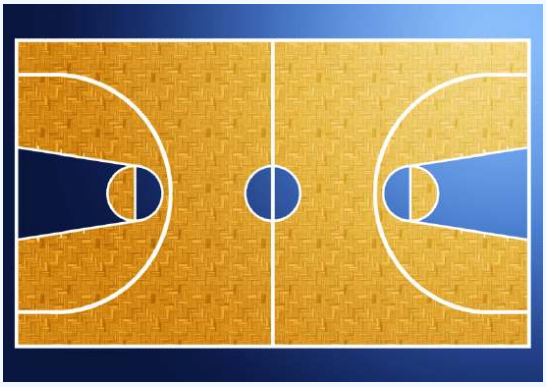

Figure 7.5.5

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Solution

First, write the information that you know.

The actual size of the basketball court and the scale factor Chris is using for his drawing.

Next, set up an equation to find the length Chris should draw.

$$\begin{aligned} \text{drawing length} \times \text{scale factor} &= \text{actual basketball court length} \\ \text{drawing length} \times 10 &= 75 feet \\ \text{drawing length} &= 75 \backslash \text{divide} 10 \\ \text{drawing length} &= 7.5 \text{ inches} \end{aligned}$$

The length of the basketball court in Chris's drawing should be 7.5 inches.

Then, use the same process to find the width Chris should draw.

$$\begin{aligned} \text{drawing width} \times \text{scale factor} &= \text{actual basketball court width} \\ \text{drawing width} \times 10 &= 40 \text{ feet} \\ \text{drawing width} &= 40 \backslash \text{divide} 10 \\ \text{drawing width} &= 4 \text{ inches} \end{aligned}$$

The answer is that Chris should represent the basketball court as a 7.5 by 4 inch rectangle on his drawing.

Example 7.5.4

Solve for x.

$\frac{3}{4} = \frac{x}{12}$

Solution

First, cross multiply.

$4x = 36$

Next, divide both sides of the equation by 4.

$x = 9$

Then, state the solution.

9

The answer is that x equals 9.

Example 7.5.5

Solve for x.

$\frac{3}{6} = \frac{1}{x}$

Solution

First, cross multiply.

$3x = 6$

Next, divide both sides of the equation by 3.

$x = 2$

Then, state the solution.

2

The answer is that x equals 2.

Review

Solve each problem.

1. Side m in triangle LMN corresponds to side c in the smaller triangle BCD. Side m is 12 cm long and the scale factor is 4. What is the measure of side c?
2. Side q in triangle PQR corresponds to side y in the smaller triangle XYZ. Side y is 8 inches long and the scale factor is 7. What is the measure of side q?

Solve each proportion for the missing side length.

3. $\frac{7}{10} = \frac{x}{30}$
4. $\frac{1.5}{3} = \frac{x}{6}$

Now use the scale factor to create a new ratio.

5. $\frac{1}{3}$, scale factor 4
6. $\frac{8}{5}$, scale factor 5
7. $\frac{9}{3}$, scale factor 3
8. Find the scale factor of the similar figures below and then use it to find the measure of LO\).

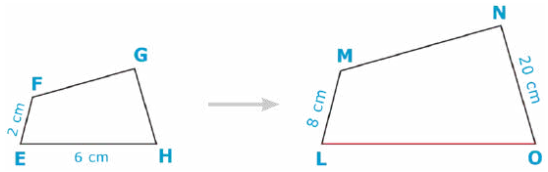

Figure 7.5.6

9. Use the scale factor of the similar figures below to find the measure of JK.

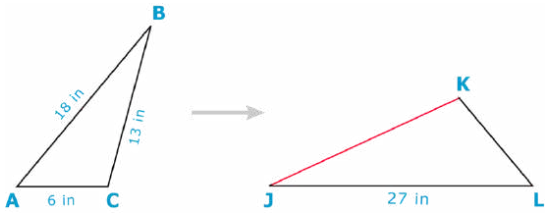

Figure 7.5.7

Use the map below and a ruler to answer the questions that follow.

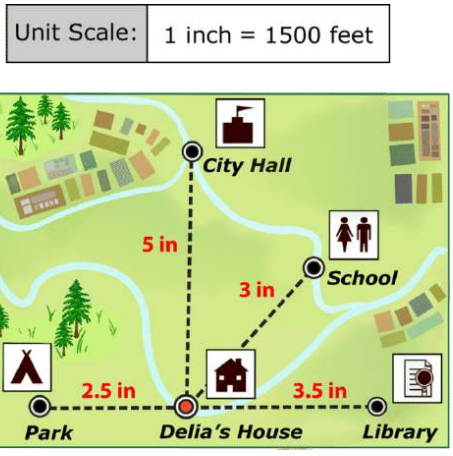

Figure 7.5.8

10. How far does Delia live from her school?
11. How far is it from the library to the park?

12. How far does Delia live from City Hall?
13. Delia drew another point to show the police station on her map. She drew it 1.5 inches away from the City Hall. What is the actual distance between the police station and City Hall?
14. How far does Delia live from the park?
15. How far does she live from the library?
16. What is the farthest that Delia will travel to any one item on her map?

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.15.

Resources

Vocabulary

Term	Definition
Indirect Measurement	Indirect measurement is the process of using the characteristics of similar triangles to measure distances.
Scale Factor	A scale factor is a ratio of the scale to the original or actual dimension written in simplest form.
Similar	Two figures are similar if they have the same shape, but not necessarily the same size.

Additional Resources

Video: Congruent and Similar Triangles

Practice: Indirect Measurement

Real World: Mighty Measurements

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7.6: Indirect Measurement Applications

Use ratios and proportions to solve for missing lengths in similar figures in real world situations.

Indirect Measurement

An application of similar triangles is to measure lengths **indirectly.** You can use this method to measure the width of a river or canyon or the height of a tall object. The idea is that you model a situation with similar triangles and then use proportions to find the missing measurement indirectly.

What if you were standing next to a building and wanted to know how tall the building was? How could you use your own height and the length of the shadows cast by you and the building to determine the building's height?

For Examples 1, 2, and 3, formation:

In order to estimate the wi llowing technique can be used. Use the diagram.

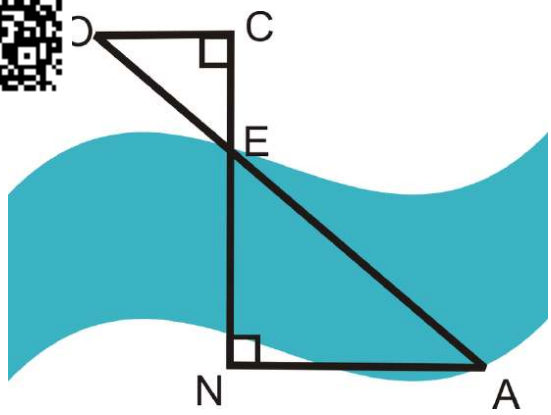

Figure **7.6.1**

Place three markers, O, C, and E on the upper bank of the river. E is on the edge of the river and $\overline{OC} \perp \overline{CE}$. Go across the river and place a marker, N so that it is collinear with C and E. Then, walk along the lower bank of the river and place marker A\), so that $\overline{CN} \perp \overline{NA}$. $OC = 50 feet$, $CE = 30\ feet$, $NA = 80\ feet$.

Example 7.6.1

Is $\Delta OCE \sim \Delta ANE$? How do you know?

Solution

Yes. $\angle C \cong \angle N$ because they are both right angles. $\angle OEC \cong \angle AEN$ because they are vertical angles. This means $\Delta OCE \sim \Delta ANE$ by the AA Similarity Postulate.

Example 7.6.2

Is $\overline{OC} \parallel \overline{NA}$? How do you know?

Solution

Since the two triangles are similar, we must have $\angle EOC \cong \angle EAN$. These are alternate interior angles. When alternate interior angles are congruent then lines are parallel, so $\overline{OC} \parallel \overline{NA}$.

Example 7.6.3

What is the width of the river? Find EN.

Solution

Set up a **proportion** and solve by cross-multiplying.

$$\frac{30\ ft}{EN} = \frac{50\ ft}{80\ ft}$$

$$50(EN) = 2400$$

$$EN = 48$$

The river is 48 feet wide.

Example 7.6.4

A tree outside Ellie's building casts a 125 foot shadow. At the same time of day, Ellie casts a 5.5 foot shadow. If Ellie is 4 feet 10 inches tall, how tall is the tree?

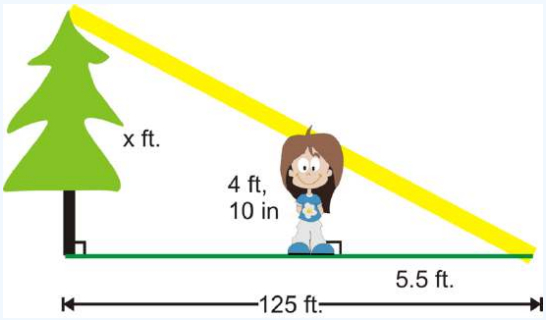

Figure 7.6.2

Solution

To solve, start by drawing a picture. We see that the tree and Ellie are parallel, so the two triangles are similar.

$$\frac{4\ ft, 10\ in}{x} = \frac{5.5\ ft}{125\ ft}$$

The measurements need to be in the same units. Change everything into inches and then we can cross multiply.

$$\frac{58\ in}{x} = \frac{66\ in}{1500\ in}$$

$$87000 = 66x$$

$$x \approx 1318.\overline{18} in or 109.85\ ft$$

Example 7.6.5

Cameron is 5 ft tall and casts a 12 ft shadow. At the same time of day, a nearby building casts a 78 ft shadow. How tall is the building?

Solution

To solve, set up a proportion that compares height to shadow length for Cameron and the building. Then solve the equation to find the height of the building. Let x represent the height of the building.

\dfrac{5ft}{12\: ft}=\dfrac{x}{78\: ft}\)

12x=390\)

x=32.5\: ft\)

The building is 32.5 feet tall.

Review

The technique from the guided practice section was used to measure the distance across the Grand Canyon. Use the picture below and $OC = 72\ ft$, $CE = 65\ ft$, and $NA = 14,400\ ft$ for problems 1 - 3.

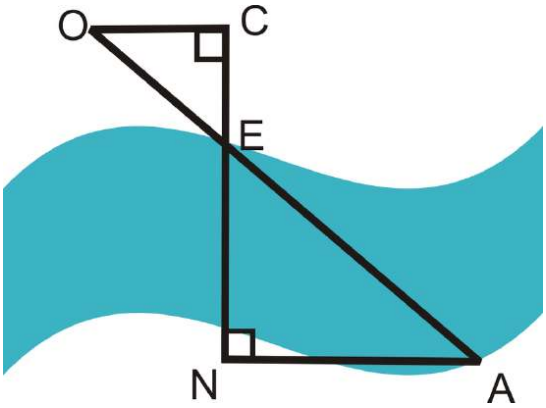

Figure **7.6.3**

1. Find EN (the distance across the Grand Canyon).

2. Find OE.

3. Find EA.

4. Mark is 6 ft tall and casts a 15 ft shadow. A the same time of day, a nearby building casts a 30 ft shadow. How tall is the building?

5. Karen and Jeff are standing next to each other. Karen casts a 10 ft shadow and Jeff casts an 8 ft shadow. Who is taller? How do you know?

6. Billy is 5 ft 9 inches tall and Bobby is 6 ft tall. Bobby's shadow is 13 feet long. How long is Billy's shadow?

7. Sally and her little brother are walking to school. Sally is 4 ft tall and has a shadow that is 3 ft long. Her little brother's shadow is 2 ft long. How tall is her little brother?

8. Ryan is outside playing basketball. He is 5 ft tall and at this time of day is casting a 12 ft shadow. The basketball hoop is 10 ft tall. How long is the basketball hoop's shadow?

9. Jack is standing next to a very tall tree and wonders just how tall it is. He knows that he is 6 ft tall and at this moment his shadow is 8 ft long. He measures the shadow of the tree and finds it is 90 ft. How tall is the tree?

10. Thomas, who is 4 ft 9 inches tall is casting a 6 ft shadow. A nearby building is casting a 42 ft shadow. How tall is the building?

Resources

Vocabulary

Term	Definition
AA Similarity Postulate	If two angles in one triangle are congruent to two angles in another triangle, then the two triangles are similar.
Proportion	A proportion is an equation that shows two equivalent ratios.

Additional Resources

Video: Scale and Indirect Measurements

Activities: Indirect Measurement Discussion Questions

Study Aids: Polygon Similarity Study Guide

Practice: Indirect Measurement Applications

Real World: Mighty Measurements

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7.7: AA Similarity

Two triangles are similar if two pairs of angles are congruent.

AA Similarity Postulate

By definition, two triangles are similar if all their corresponding angles are congruent and their corresponding sides are proportional. It is not necessary to check all angles and sides in order to tell if two triangles are similar. In fact, if you only know that two pairs of corresponding angles are congruent that is enough information to know that the triangles are similar. This is called the **AA Similarity Postulate.**

AA Similarity Postulate: If two angles in one triangle are congruent to two angles in another triangle, then the two triangles are similar.

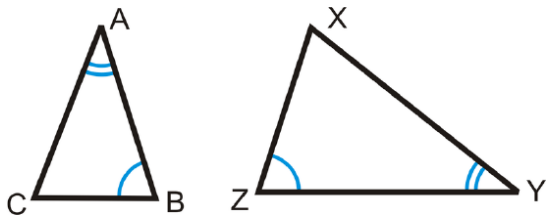

Figure **7.7.1**

If $\angle A \cong \angle Y$ and $\angle B \cong \angle Z$, then $\Delta ABC \sim \Delta YZX$.

What if you were given a pair of triangles and the angle measures for two of their angles? How could you use this information to determine if the two triangles are similar?

Example 7.7.1

Are the triangles similar? Write a similarity statement.

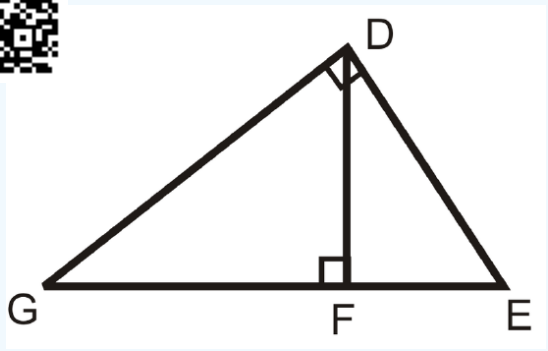

Figure **7.7.2**

Solution

Yes, there are three **similar triangles** that each have a right angle. $DGE \sim FGD \sim FDE$.

Example 7.7.2

Are the triangles similar? If so, write a similarity statement.

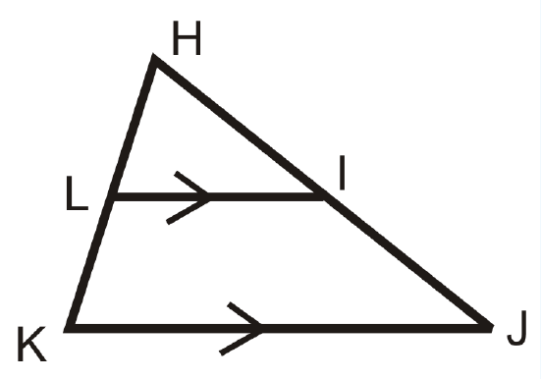

Figure 7.7.3

Solution

By the reflexive property, $\angle H \cong \angle H$. Because the horizontal lines are parallel, $\angle L \cong \angle K$ (corresponding angles). So yes, there is a pair of similar triangles. $HLI \sim HKJ$.

Example 7.7.3

Determine if the following two triangles are similar. If so, write the similarity statement.

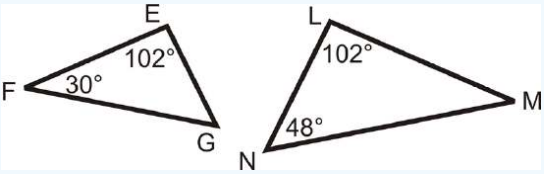

Figure 7.7.4

Solution

Compare the angles to see if we can use the AA Similarity Postulate. Using the **Triangle Sum Theorem**, $m\angle G = 48°$ and $m\angle M = 30°$. So, $\angle F \cong \angle M$, $\angle E \cong \angle L$ and $\angle G \cong \angle N$ and the triangles are similar. $\Delta FEG \sim \Delta MLN$.

Example 7.7.4

Determine if the following two triangles are similar. If so, write the similarity statement.

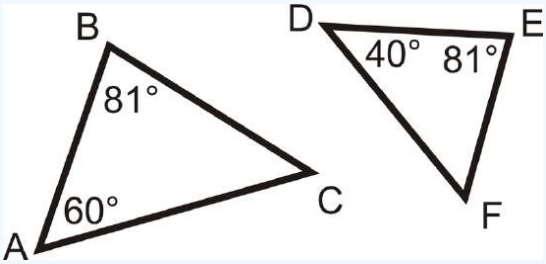

Figure 7.7.5

Solution

Compare the angles to see if we can use the AA Similarity Postulate. Using the Triangle Sum Theorem, $m\angle C = 39°$ and $m\angle F = 59°$. $m\angle C \neq m\angle F$, So ΔABC and ΔDEF are not similar.

Example 7.7.5

$\Delta LEG \sim \Delta MAR$ by AA. Find GE and MR.

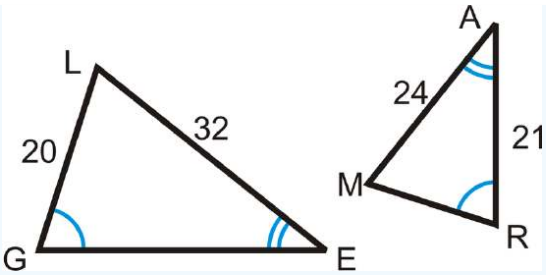

Figure 7.7.6

Solution

Set up a proportion to find the missing sides.

$$\begin{aligned} \frac{24}{32} &= \frac{MR}{20} & \frac{24}{32} &= \frac{21}{GE} \\ 480 &= 32MR & 24GE &= 672 \\ 15 &= MR & GE &= 28 \end{aligned}$$

When two triangles are similar, the corresponding sides are proportional. But, what are the corresponding sides? Using the triangles from this example, we see how the sides line up in the diagram to the right.

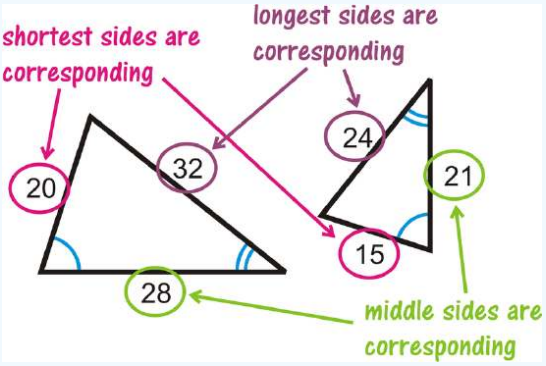

Figure 7.7.7

Review

Use the diagram to complete each statement.

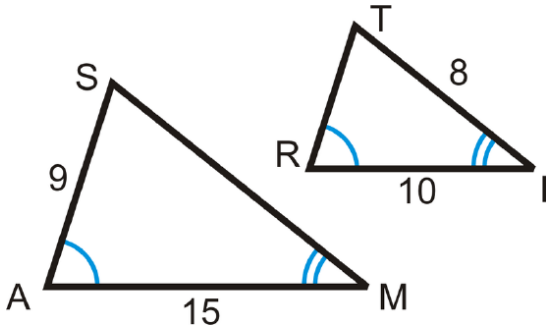

Figure 7.7.8

1. Missing open brace for subscript
2. $\frac{SA}{?} = \frac{SM}{?} = \frac{?}{RI}$
3. Missing open brace for subscript
4. Missing open brace for subscript
5. $\frac{9}{?} = \frac{?}{8}$

Answer questions 6-9 about trapezoid $ABCD$.

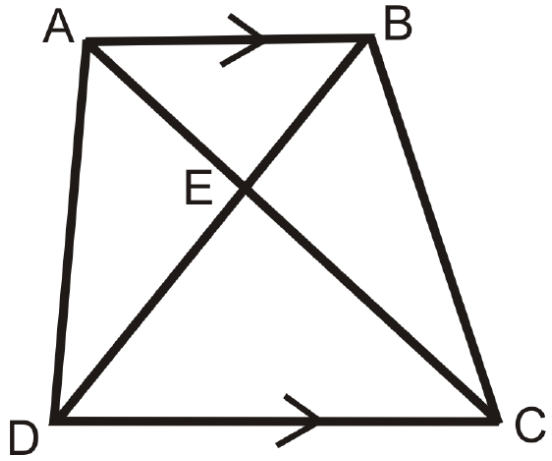

Figure 7.7.9

6. Name two similar triangles. How do you know they are similar?
7. Write a true proportion.
8. Name two other triangles that might *not* be similar.
9. If $AB = 10$, $AE = 7$, and $DC = 22$, find AC. Be careful!

Use the triangles to the left for questions 10-14.

$AB = 20$, $DE = 15$, and $BC = k$.

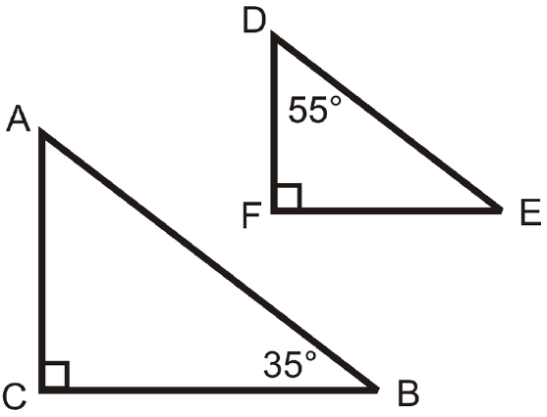

Figure 7.7.10

10. Are the two triangles similar? How do you know?
11. Write an expression for FE in terms of k.
12. If $FE = 12$, what is k?
13. Fill in the blanks: If an acute angle of a _____ triangle is congruent to an acute angle in another _____ triangle, then the two triangles are _____.
14. ***Writing*** How do congruent triangles and similar triangles differ? How are they the same?

Are the following triangles similar? If so, write a similarity statement.

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15.

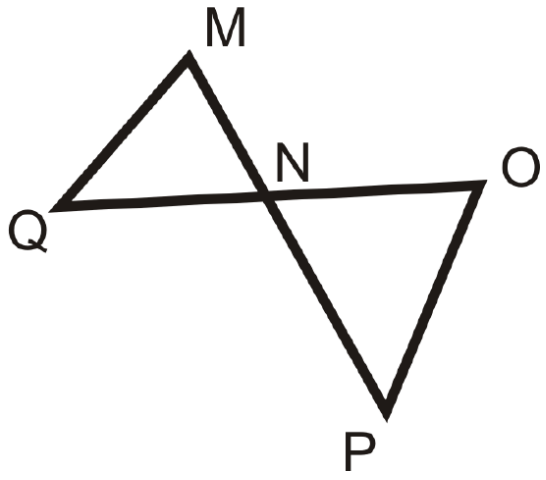

Figure **7.7.11**

16.

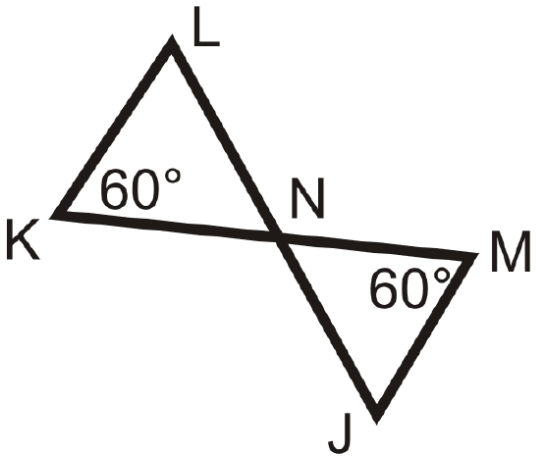

Figure **7.7.12**

17.

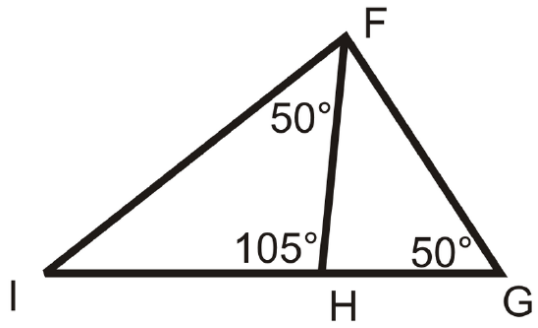

Figure **7.7.13**

18.

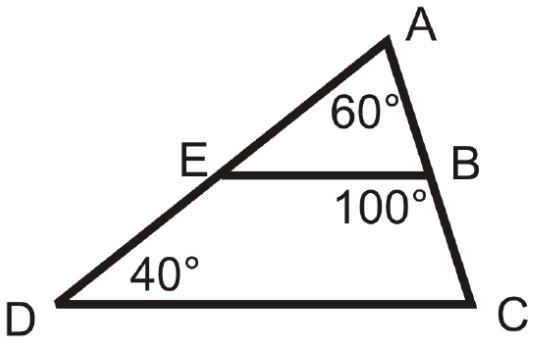

Figure **7.7.14**

19.

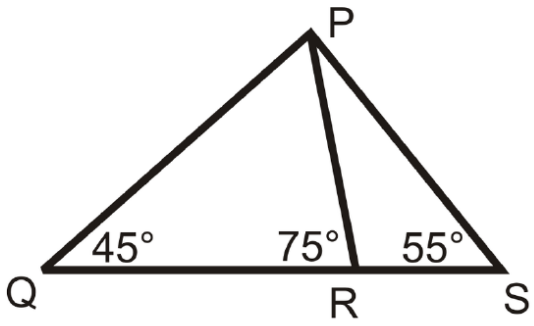

Figure **7.7.15**

20.

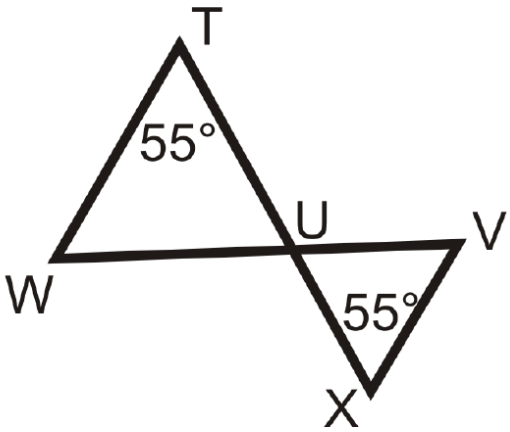

Figure **7.7.16**

21.

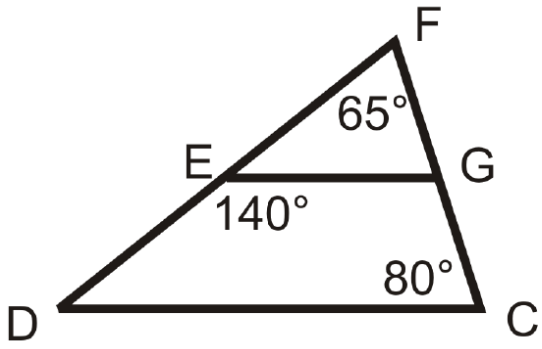

Figure **7.7.17**

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.4.

Resources

Vocabulary

Term	Definition
similar triangles	Two triangles where all their corresponding angles are ***congruent*** (exactly the same) and their corresponding sides are ***proportional*** (in the same ratio).
AA Similarity Postulate	If two angles in one triangle are congruent to two angles in another triangle, then the two triangles are similar.
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
Triangle Sum Theorem	The Triangle Sum Theorem states that the three interior angles of any triangle add up to 180 degrees.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Congruent and Similar Triangles

Activities: AA Similarity Discussion Questions

Study Aids: Polygon Similarity Study Guide

Practice: AA Similarity

Real World: Crazy Quilt

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7.8: SSS Similarity

Triangles are similar if their corresponding sides are proportional.

SSS Similarity Theorem

By definition, two triangles are similar if all their corresponding angles are **congruent** and their corresponding sides are proportional. It is not necessary to check all angles and sides in order to tell if two triangles are similar. In fact, if you know only that all sides are proportional, that is enough information to know that the triangles are similar. This is called the **SSS Similarity Theorem**.

SSS Similarity Theorem: If all three pairs of corresponding sides of two triangles are proportional, then the two triangles are similar.

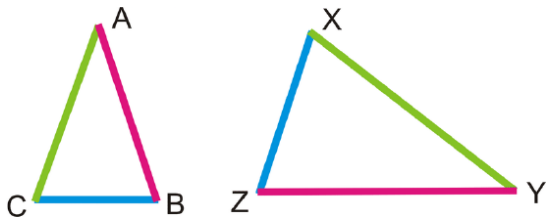

Figure **7.8.1**

If $\frac{AB}{YZ} = \frac{BC}{ZX} = \frac{AC}{XY}$, then $\Delta ABC \sim \Delta YZX$.

What if you were given a pair of triangles and the side lengths for all three of their sides? How could you use this information to determine if the two triangles are similar?

For Examples 1 and 2, use ram:

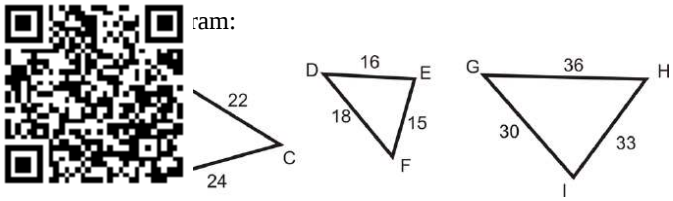

Figure **7.8.2**

Example 7.8.1

Is $\Delta DEF \sim \Delta GHI$?

Is $\frac{15}{30} = \frac{16}{33} = \frac{18}{36}$?

Solution

$\frac{15}{30} = \frac{1}{2}$, $\frac{16}{33} = \frac{16}{33}$, and $\frac{18}{36} = \frac{1}{2}$. $\frac{1}{2} \neq \frac{16}{33}$, ΔDEF is **not** similar to ΔGHI.

Example 7.8.2

Is $\Delta ABC \sim \Delta GHI$?

Is $\frac{20}{30} = \frac{22}{33} = \frac{24}{36}$?

Solution

$\frac{20}{30} = dfrac23$, $\frac{22}{33} = \frac{2}{3}$, and $\frac{24}{36} = \frac{2}{3}$. All three ratios reduce to $\frac{2}{3}$, $\Delta ABC \sim \Delta GHI$.

Example 7.8.3

Determine if the following triangles are similar. If so, explain why and write the similarity statement.

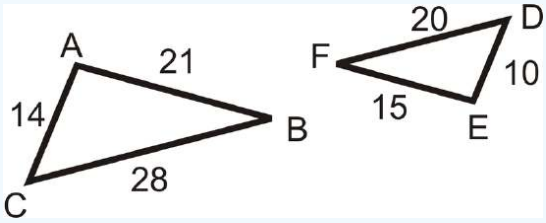

Figure 7.8.3

Solution

We will need to find the ratios for the corresponding sides of the triangles and see if they are all the same. Start with the longest sides and work down to the shortest sides.

$$\frac{BC}{FD} = \frac{28}{20} = \frac{7}{5}$$
$$\frac{BA}{FE} = \frac{21}{15} = \frac{7}{5}$$
$$\frac{AC}{ED} = \frac{14}{10} = \frac{7}{5}$$

Since all the ratios are the same, $\Delta ABC \sim \Delta EFD$ by the SSS Similarity Theorem.

Example 7.8.4

Find $x and \(y, such that \(\Delta ABC \sim \Delta DEF$.

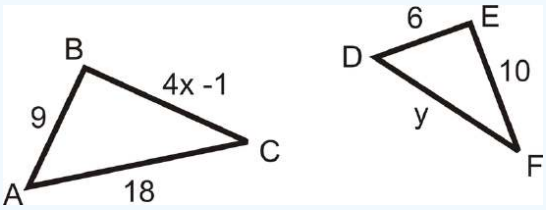

Figure 7.8.4

Solution

According to the similarity statement, the corresponding sides are: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. Substituting in what we know, we have $\frac{9}{6} = \frac{4x-1}{10} = \frac{18}{y}$.

$$\frac{9}{6} = \frac{4x-1}{10} \qquad \frac{9}{6} = \frac{18}{y}$$
$$9(10) = 6(4x-1) \qquad 9y = 18(6)$$
$$90 = 24x - 6 \qquad 9y = 108$$
$$96 = 24x \qquad y = 12$$
$$x = 4$$

Example 7.8.5

Determine if the following triangles are similar. If so, explain why and write the similarity statement.

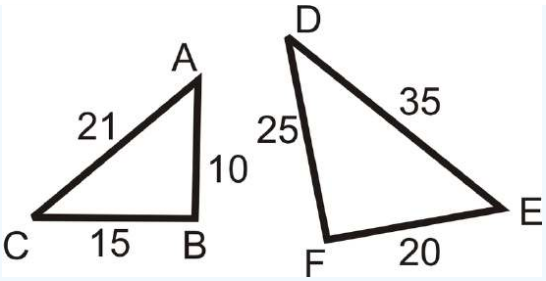

Figure 7.8.5

Solution

We will need to find the ratios for the corresponding sides of the triangles and see if they are all the same. Start with the longest sides and work down to the shortest sides.

$$\frac{AC}{ED} = \frac{21}{35} = \frac{3}{5}$$
$$\frac{BC}{FD} = \frac{15}{25} = \frac{3}{5}$$
$$\frac{AB}{EF} = \frac{10}{20} = \frac{1}{2}$$

Since the ratios are not all the same, the triangles are not similar.

Review

Fill in the blanks.

1. If all three sides in one triangle are _____ to the three sides in another, then the two triangles are similar.
2. Two triangles are similar if the corresponding sides are _____.

Use the following diagram for questions 3-5. *The diagram is to scale.*

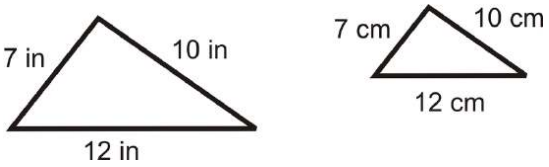

Figure 7.8.6

3. Are the two triangles similar? Explain your answer.
4. Are the two triangles congruent? Explain your answer.
5. What is the scale factor for the two triangles?

Fill in the blanks in the statements below. Use the diagram to the left.

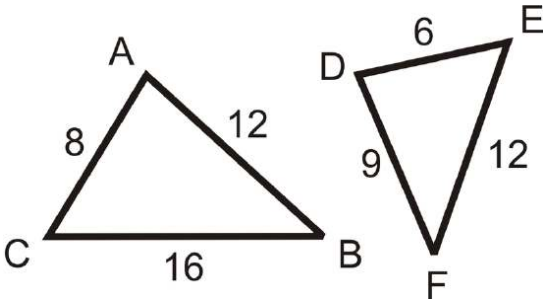

Figure 7.8.7

6. Missing open brace for subscript
7. $\frac{AB}{?} = \frac{BC}{?} = \frac{AC}{?}$
8. If ΔABC had an altitude, $AG = 10$, what would be the length of altitude $\overline{DH}$?
9. Find the perimeter of ΔABC and ΔDEF. Find the **ratio** of the perimeters.

Use the diagram to the right for questions 10-15.

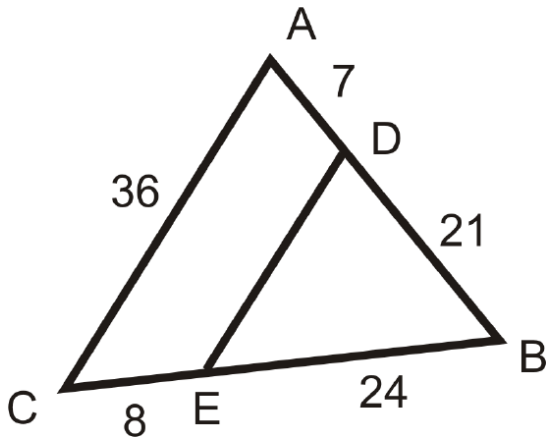

Figure \(\
PageIndex
{8}\)

10. Missing open brace for subscript
11. Why are the two triangles similar?
12. Find ED.
13. $\frac{BD}{?}=\frac{?}{BC}=\frac{DE}{?}$
14. Is $\frac{AD}{DB}=\frac{CE}{EB}$ true?
15. Is $\frac{AD}{DB}=\frac{AC}{DE}$ true?

Find the value of the missing variable(s) that makes the two triangles similar.

16.

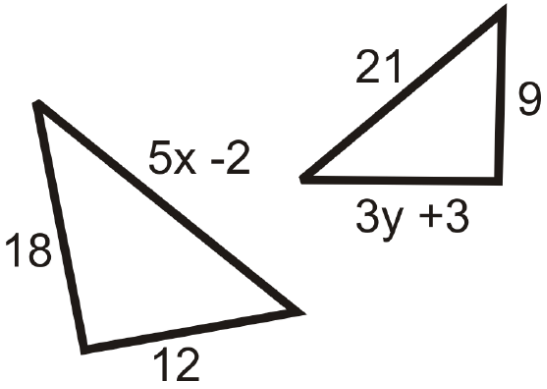

Figure **7.8.9**

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.6.

Resources

Vocabulary

Term	Definition
AAA Similarity Theorem	AAA Similarity Theorem states that if all three pairs of corresponding sides of two triangles are proportional, then the two triangles are similar.
Congruent	Congruent figures are identical in size, shape and measure.
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
Ratio	A ratio is a comparison of two quantities that can be written in fraction form, with a colon or with the word “to”.
SSS	SSS means side, side, side and refers to the fact that all three sides of a triangle are known in a problem.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Congruent and Similar Triangles

Activities: SSS Similarity Discussion Questions

Study Aids: Polygon Similarity Study Guide

Practice: SSS Similarity

Real World: Crazy Quilt

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7.9: SAS Similarity

Triangles are similar if two pairs of sides are proportional and the included angles are congruent.

SAS Similarity Theorem

By definition, two triangles are similar if all their corresponding angles are **congruent** and their corresponding sides are proportional. It is not necessary to check all angles and sides in order to tell if two triangles are similar. In fact, if you know only that two pairs of sides are proportional and their included angles are congruent, that is enough information to know that the triangles are similar. This is called the **SAS** Similarity Theorem.

SAS Similarity Theorem: If two sides in one triangle are proportional to two sides in another triangle and the included angle in both are congruent, then the two triangles are similar.

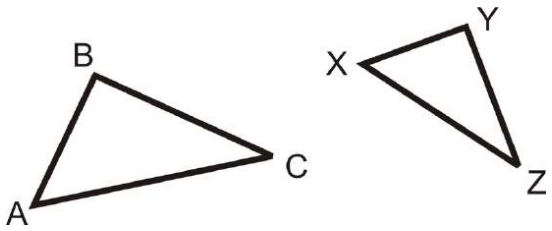

Figure **7.9.1**

If $\frac{AB}{XY} = \frac{AC}{XZ}$ and $\angle A \cong \angle X$, then $\Delta ABC \sim \Delta XYZ$.

What if you were given a pair of triangles, the lengths of two of their sides, and the measure of the angle between those two sides? How could you use this information to determine if the two triangles are similar?

Example 7.9.1

Determine if the followi ilar. If so, write the similarity theorem and statement.

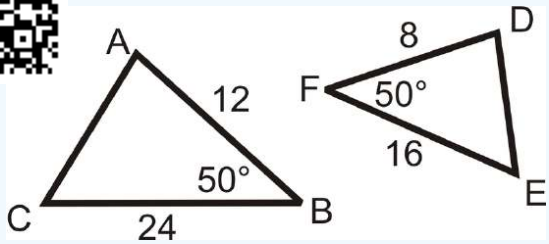

Figure **7.9.2**

Solution

We can see that $\angle B \cong \angle F$ and these are both included angles. We just have to check that the sides around the angles are proportional.

$$\frac{AB}{DF} = \frac{12}{8} = \frac{3}{2}$$
$$\frac{BC}{FE} = \frac{24}{16} = \frac{3}{2}$$

Since the ratios are the same $\Delta ABC \sim \Delta DFE$ by the SAS Similarity Theorem.

Example 7.9.2

Determine if the following triangles are similar. If so, write the similarity theorem and statement.

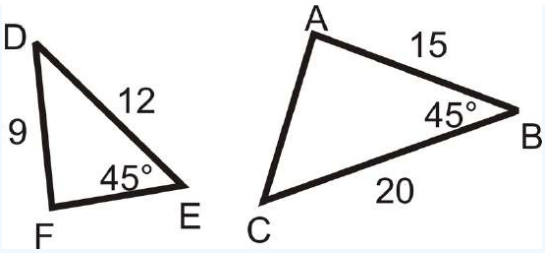

Figure 7.9.3

Solution

The triangles are not similar because the angle is not the included angle for both triangles.

Example 7.9.3

Are the two triangles similar? How do you know?

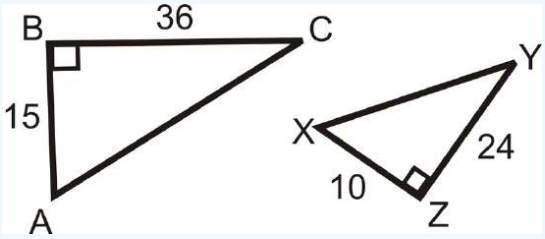

Figure 7.9.4

Solution

We know that $\angle B \cong \angle Z$ because they are both right angles and $\frac{10}{15} = \frac{24}{36}$. So, $\frac{AB}{XZ} = \frac{BC}{ZY}$ and $\Delta ABC \sim \Delta XZY$ by SAS.

Example 7.9.4

Are there any similar triangles in the figure? How do you know?

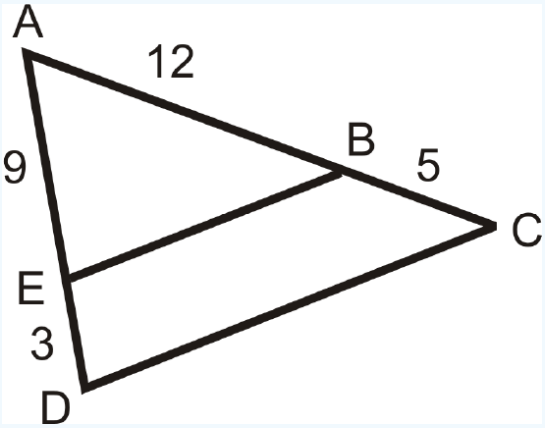

Figure 7.9.5

Solution

$\angle A$ is shared by ΔEAB and ΔDAC, so it is congruent to itself. Let's see if $\frac{AE}{AD} = \frac{AB}{AC}$.

$$\frac{9}{9+3} = \frac{12}{12+5}$$

$$\frac{9}{12} = \frac{3}{4} \neq \frac{12}{17}$$

The two triangles are not similar.

Example 7.9.5

From Example 4, what should BC equal for $\Delta EAB \sim \Delta DAC$?

Solution

The proportion we ended up with was $\frac{9}{12} = \frac{3}{4} \neq \frac{12}{17}$. AC needs to equal 16, so that $\frac{12}{16} = dfrac34$. $AC = AB + BC$ and $16 = 12 + BC$. BC should equal 4.

Review

Fill in the blanks.

1. If two sides in one triangle are _____ to two sides in another and the _____ angles are _____, then the triangles are _____.

Determine if the following triangles are similar. If so, write the similarity theorem and statement.

2.

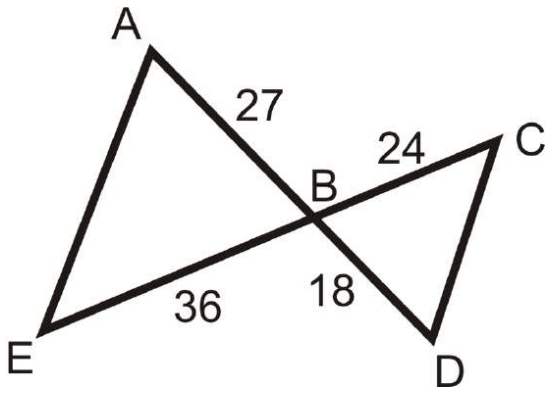

Figure 7.9.6

Find the value of the missing variable(s) that makes the two triangles similar.

3.

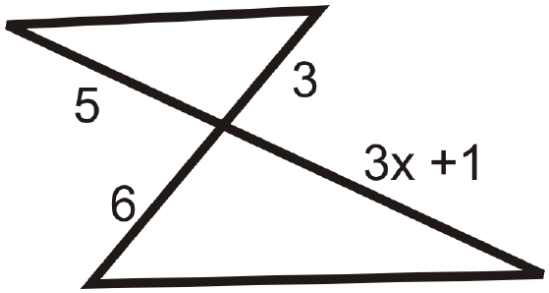

Figure 7.9.7

4.

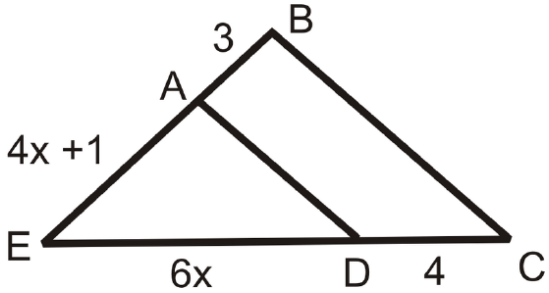

Figure 7.9.8

5.

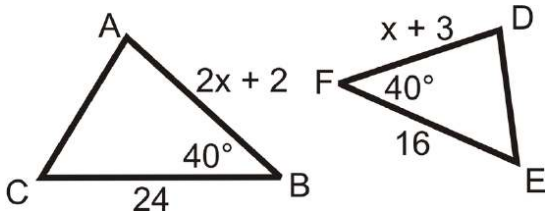

Figure 7.9.9

Determine if the triangles are similar. If so, write the similarity theorem and statement.

6. ΔABC is a right triangle with legs that measure 3 and 4. ΔDEF is a right triangle with legs that measure 6 and 8.
7. ΔGHI is a right triangle with a leg that measures 12 and a hypotenuse that measures 13. ΔJKL is a right triangle with legs that measure 1 and 2.

8.

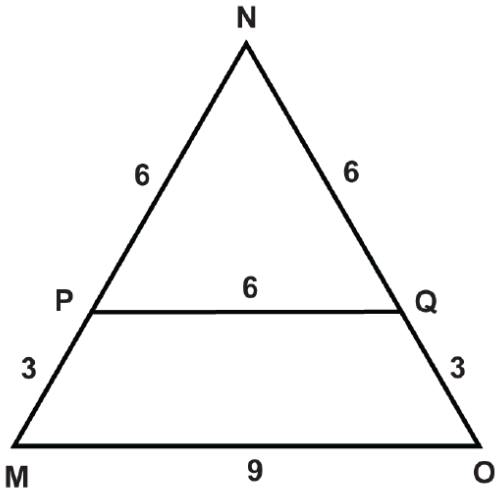

Figure 7.9.10

9.

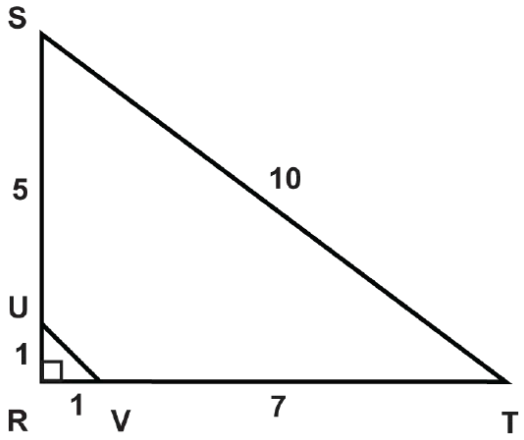

Figure 7.9.11

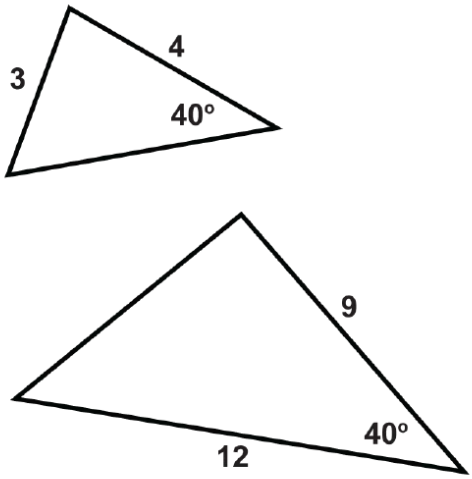

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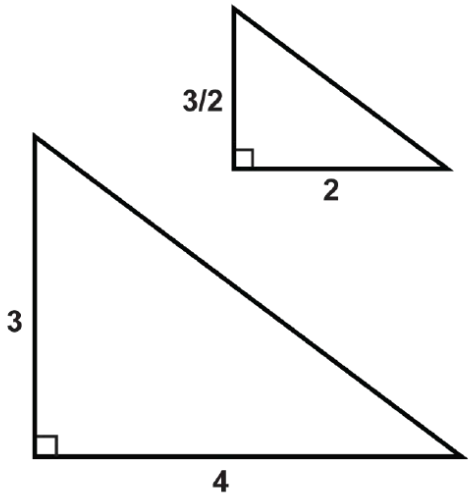
Figure 7.9.12



11.

Figure 7.9.13

12. $\overline{AC} = 3$

$\overline{DF} = 6$

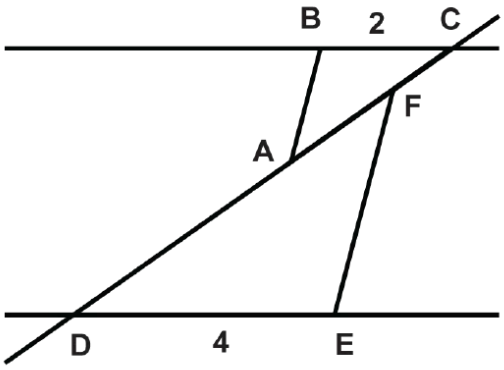


Figure 7.9.14

13.

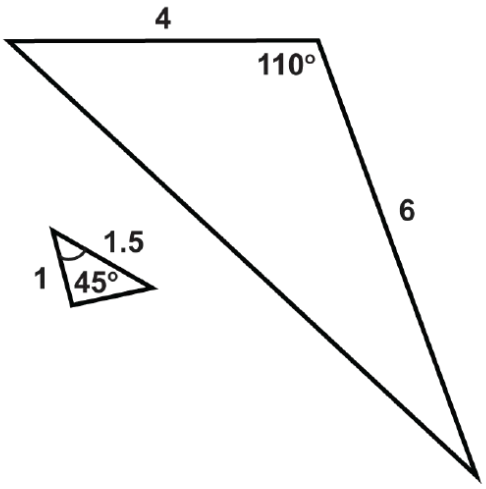


Figure 7.9.15

14.

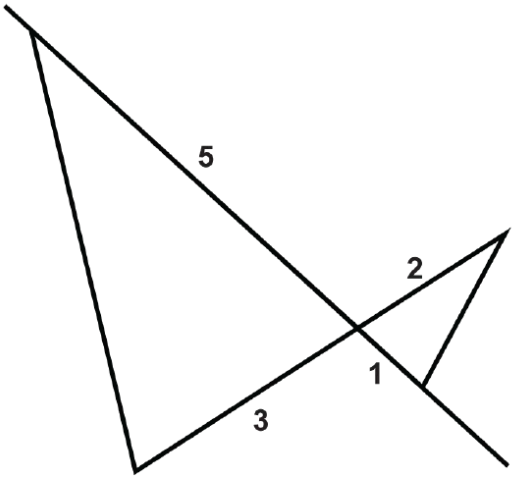
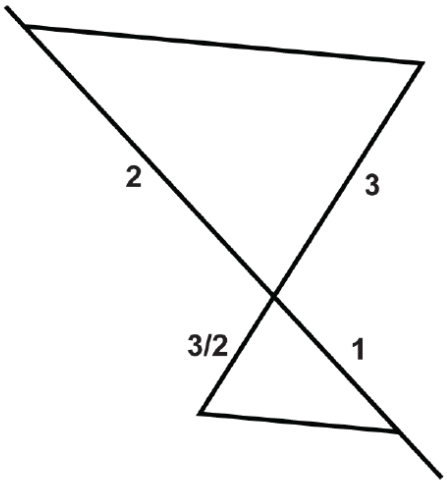


Figure 7.9.16



15.

Figure \(\PageIndex{17}\

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.7.

Resources

Vocabulary

Term	Definition
AA Similarity Postulate	If two angles in one triangle are congruent to two angles in another triangle, then the two triangles are similar.
Congruent	Congruent figures are identical in size, shape and measure.
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
SAS	SAS means side, angle, side, and refers to the fact that two sides and the included angle of a triangle are known.
SAS Similarity Theorem	The SAS Similarity Theorem states that if two sides in one triangle are proportional to two sides in another triangle and the included angle in both are congruent, then the two triangles are similar.
Similarity Transformation	A similarity transformation is one or more rigid transformations followed by a dilation.

Additional Resources

Video: Congruent and Similar Triangles

Activities: SAS Similarity Discussion Questions

Study Aids: Polygon Similarity Study Guide

Practice: SAS Similarity

Real World: Triangle Similarity

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7.10: Proportional Triangles

Sides divided by a line parallel to the third side of a triangle.

Triangle Proportionality Theorem

Think about a **midsegment** of a triangle. A midsegment is **parallel** to one side of a triangle and divides the other two sides into **congruent** halves. The midsegment divides those two sides **proportionally.** But what about another line that is parallel, but does not divide the other two sides into congruent halves? In fact, such a line will still divide the sides proportionally. This is called the **Triangle Proportionality Theorem.**

Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

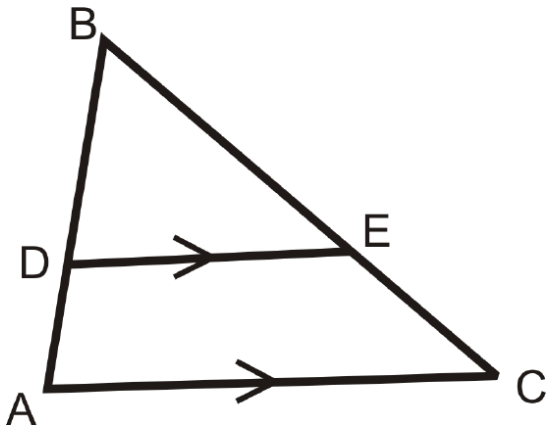

Figure **7.10.1**

If $\overline{DE} \parallel \overline{AC}$, then $\frac{BD}{DA} = \frac{BE}{EC}$. ($\frac{DA}{BD} = \frac{EC}{BE}$ *is also a true* ***proportion***.)

The converse of this theorem is also true.

Triangle Proportionality Theorem Converse: If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

If $\frac{BD}{DA} = \frac{BE}{EC}$, then $\overline{DE} \parallel \overline{AC}$.

What if you were given a triangle with a line segment drawn through it from one side to the other? How could you use information about the triangle's side lengths to determine if that line segment is parallel to the third side?

Use the diagram to answer : $\overline{DB} \parallel \overline{FE}$.

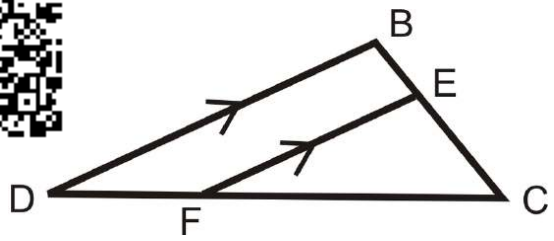

Figure **7.10.2**

Example 7.10.1

Name the similar triangles. Write the similarity statement.

Solution

$\Delta DBC \sim \Delta FEC$

Example 7.10.2

$\frac{FC+?}{FC} = \frac{?}{FE}$

Solution

DF; DB

Example 7.10.3

A triangle with its midsegment is drawn below. What is the ratio that the midsegment divides the sides into?

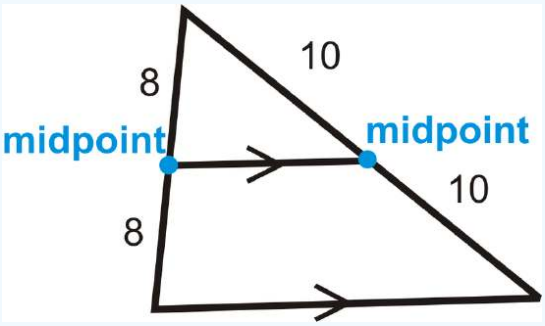

Figure 7.10.3

Solution

The midsegment splits the sides evenly. The ratio would be 8:8 or 10:10, which both reduce to 1:1.

Example 7.10.4

In the diagram below, $\overline{EB} \parallel \overline{CD}$. Find BC.

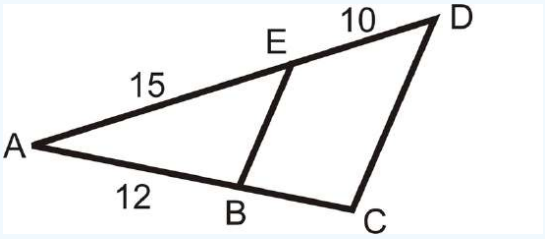

Figure 7.10.4

Solution

To solve, set up a proportion.

$$\frac{10}{15} = \frac{BC}{12} \rightarrow 15(BC) = 120$$
$$BC = 8$$

Example 7.10.5

Is $\overline{DE} \parallel \overline{CB}$?

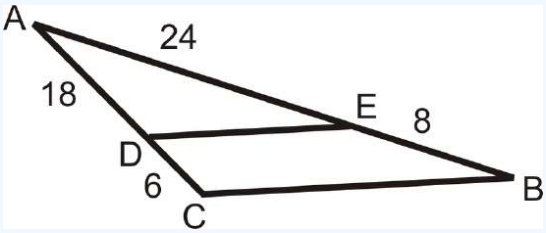

Figure 7.10.5

Solution

If the ratios are equal, then the lines are parallel.

$$\frac{6}{18} = \frac{8}{24} = \frac{1}{3}$$

Because the ratios are equal, $\overline{DE} \parallel \overline{CB}$.

Review

Use the diagram to answer questions 1-7. $\overline{AB} \parallel \overline{DE}$.

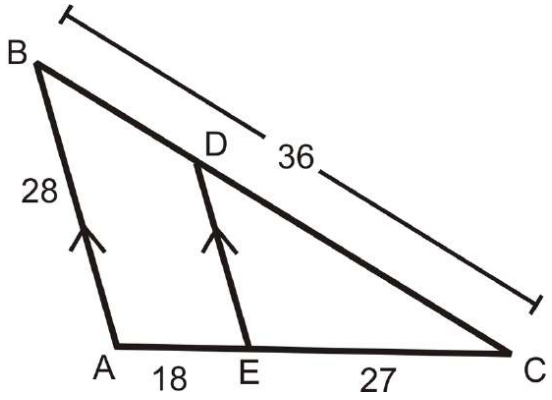

Figure 7.10.6

1. Find BD.
2. Find DC.
3. Find DE.
4. Find AC.
5. What is $BD : DC$?
6. What is $DC : BC$?
7. Why $BD : DC \neq DC : BC$?

Use the given lengths to determine if $\overline{AB} \parallel \overline{DE}$.

8.

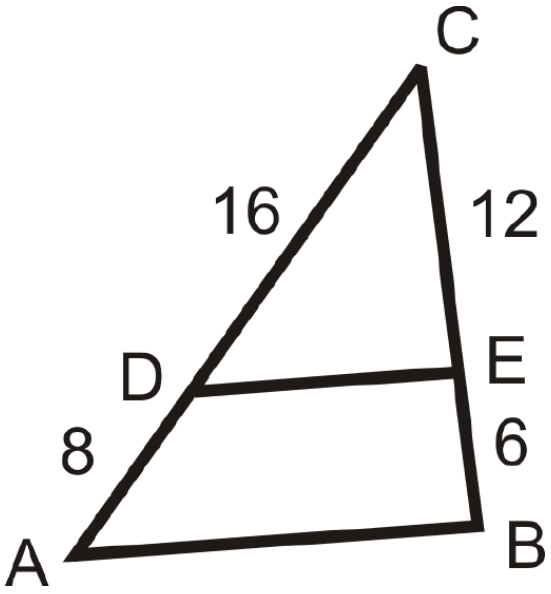

Figure 7.10.7

9.

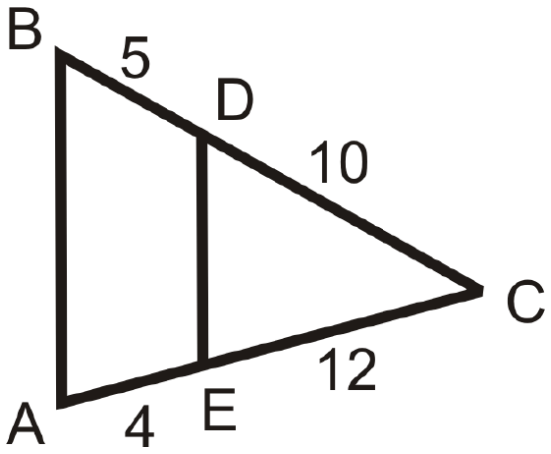

Figure 7.10.8

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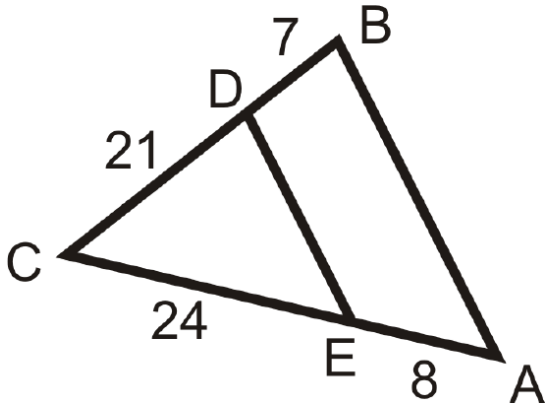

Figure 7.10.9

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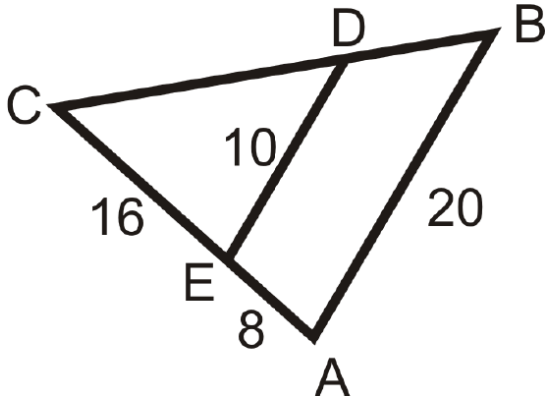

Figure 7.10.10

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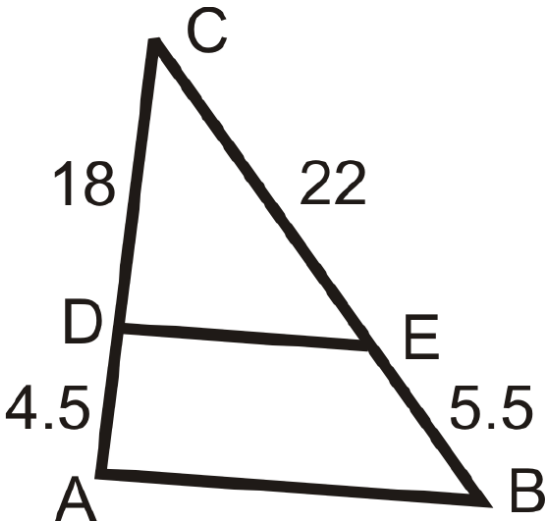

Figure **7.10.11**

13.

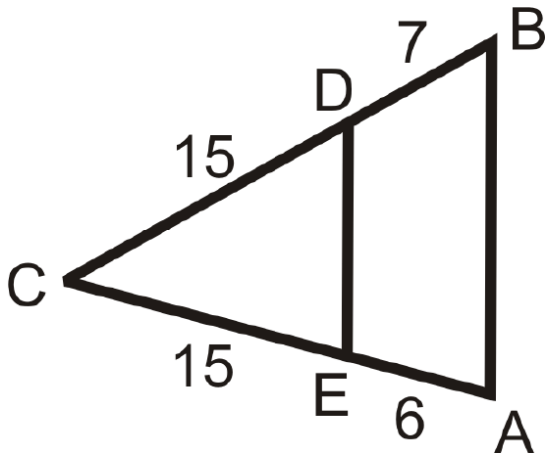

Figure **7.10.12**

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.8.

Resources

Vocabulary

Term	Definition
Congruent	Congruent figures are identical in size, shape and measure.
midsegment	A midsegment connects the midpoints of two sides of a triangle or the non-parallel sides of a trapezoid.
Parallel	Two or more lines are parallel when they lie in the same plane and never intersect. These lines will always have the same slope.
Proportion	A proportion is an equation that shows two equivalent ratios.
Triangle Proportionality Theorem	The Triangle Proportionality Theorem states that if a line is parallel to one side of a triangle and it intersects the other two sides, then it divides those sides proportionally.
Triangle Proportionality Theorem Converse	The Triangle Proportionality Theorem converse states that if a line divides two sides of a triangle proportionally, then it is parallel to the third side.

Additional Resources

Video: Using the Properties of the Triangle Proportionality Theorem to Solve for Unknown Values

Activities: Triangle Proportionality Discussion Questions

Study Aids: Proportionality Relationships Study Guide

Practice: Proportional Triangles

Real World: Triangle Proportionality

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7.11: Inscribed Similar Triangles

Division of a right triangle into similar triangles using an altitude.

Inscribed Similar Triangles Theorem

Remember that if two objects are similar, their corresponding angles are congruent and their sides are proportional in length. The altitude of a right triangle creates similar triangles.

Inscribed Similar Triangles Theorem: If an altitude is drawn from the right angle of any right triangle, then the two triangles formed are similar to the original triangle and all three triangles are similar to each other.

In ΔADB, $m\angle A = 90°$ and $\overline{AC} \perp \overline{DB}$:

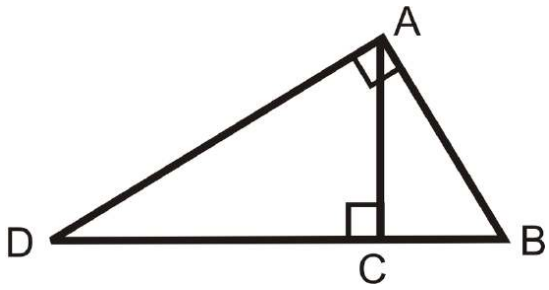


Figure **7.11.1**

So, $\Delta ADB \sim \Delta CDA \sim \Delta CAB$:

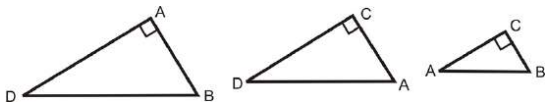


Figure **7.11.2**

This means that all of the corresponding sides are proportional. You can use this fact to find missing lengths in right triangles.

What if you drew a line from the right angle of a right triangle **perpendicular** to the side that is opposite that angle? How could you determine the length of that line?

Example 7.11.1

Find the value of x.

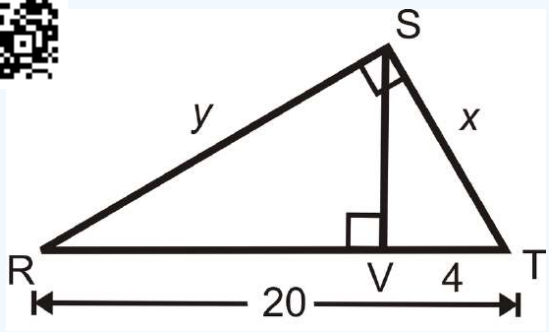


Figure **7.11.3**

Solution

Set up a **proportion**.

$$\frac{\text{shorter leg in } \Delta SVT}{\text{shorter leg in } \Delta RST} = \frac{\text{hypotenuse in } \Delta SVT}{\text{hypotenuse in } \Delta RST}$$
$$\frac{4}{x} = \frac{x}{20}$$
$$x^2 = 80$$
$$x = \sqrt{80} = 4\sqrt{5}$$

Example 7.11.2

Now find the value of y in ΔRST above.

Solution

Use **the Pythagorean Theorem**.

$$y^2 + (4\sqrt{5})^2 = 20^2$$
$$y^2 + 80 = 400$$
$$y^2 = 320$$
$$y = \sqrt{320} = 8\sqrt{5}$$

Example 7.11.3

Find the value of x.

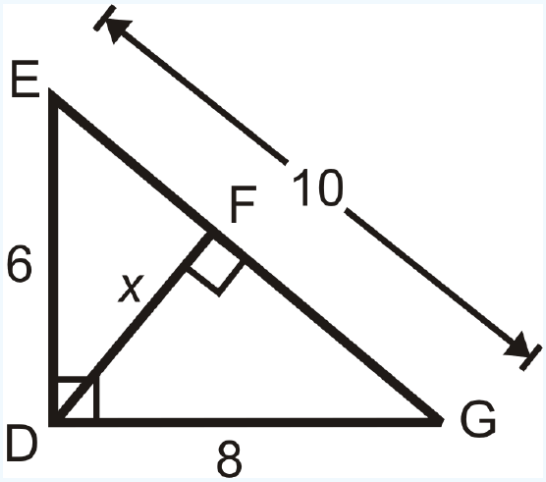


Figure 7.11.4

Solution

Separate the triangles to find the corresponding sides.

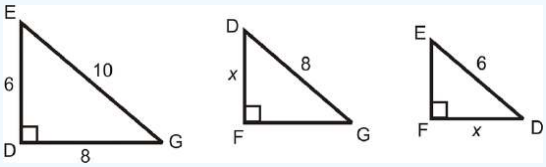


Figure 7.11.5

Set up a proportion.

$$\frac{\text{shorter leg in } \Delta EDG}{\text{shorter leg in } \Delta DFG} = \frac{\text{hypotenuse in } \Delta EDG}{\text{hypotenuse in } \Delta DFG}$$
$$\frac{6}{x} = \frac{10}{8}$$
$$48 = 10x$$
$$4.8 = x$$

Example 7.11.4

Find the value of x.

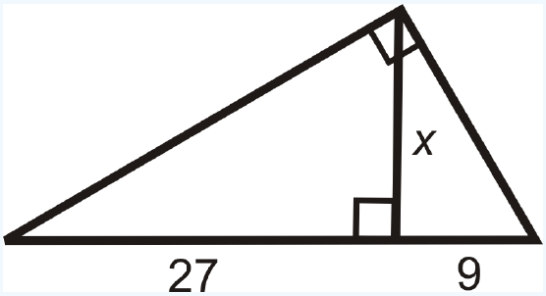


Figure 7.11.6

Solution

Set up a proportion.

$$\frac{\text{shorter leg of smallest } \Delta}{\text{shorter leg of middle } \Delta} = \frac{\text{longer leg of smallest } \Delta}{\text{longer leg of middle } \Delta}$$

$$\frac{9}{x} = \frac{x}{27}$$

$$x^2 = 243$$

$$x = \sqrt{243} = 9\sqrt{3}$$

Example 7.11.5

Find the values of x and y.

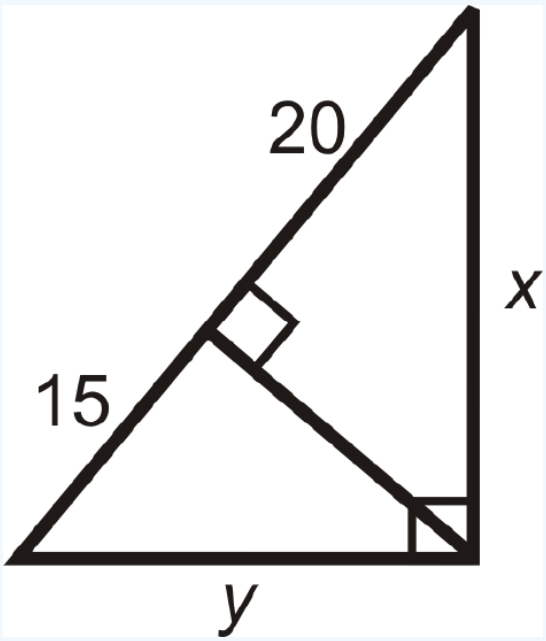


Figure 7.11.7

Separate the triangles. Write a proportion for x.

Solution

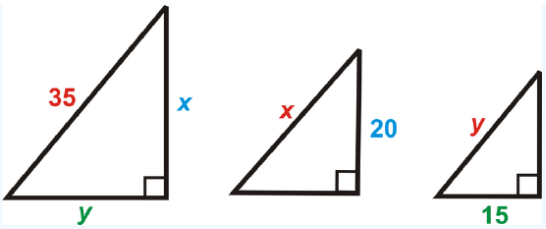


Figure 7.11.8

$$\frac{20}{x} = \frac{x}{35}$$
$$x^2 = 20 \cdot 35$$
$$x = \sqrt{20 \cdot 35}$$
$$x = 10\sqrt{7}$$

Set up a proportion for y. Or, now that you know the value of x\) you can use the Pythagorean Theorem to solve for y. Use the method you feel most comfortable with.

$$\frac{15}{y} = \frac{y}{35} \qquad (10\sqrt{7})^2 + y^2 = 35^2$$
$$y^2 = 15 \cdot 35 \qquad 700 + y^2 = 1225$$
$$y = \sqrt{15 \cdot 35} \qquad y = \sqrt{525} = 5\sqrt{21}$$
$$y = 5\sqrt{21}$$

Review

Fill in the blanks.

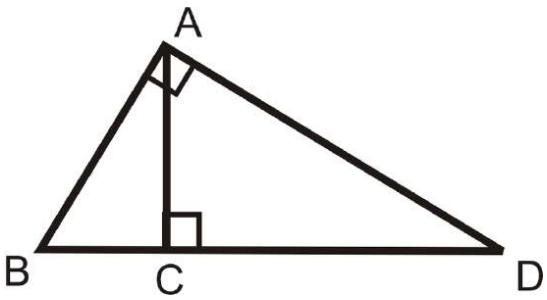
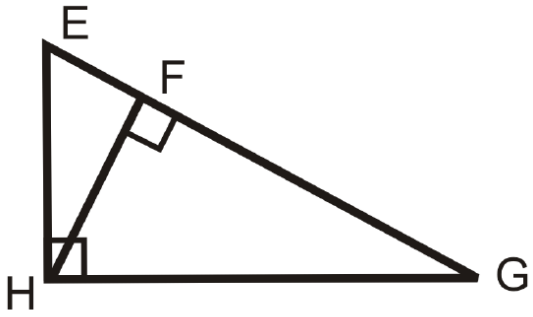


Figure 7.11.9

1. Missing open brace for subscript
2. $\frac{BC}{?} = \frac{?}{CD}$
3. $\frac{BC}{AB} = \frac{AB}{?}$
4. $\frac{?}{AD} = \frac{AD}{BD}$

Write the similarity statement for the right triangles in each diagram.



5.

Figure 7.11.10

6.

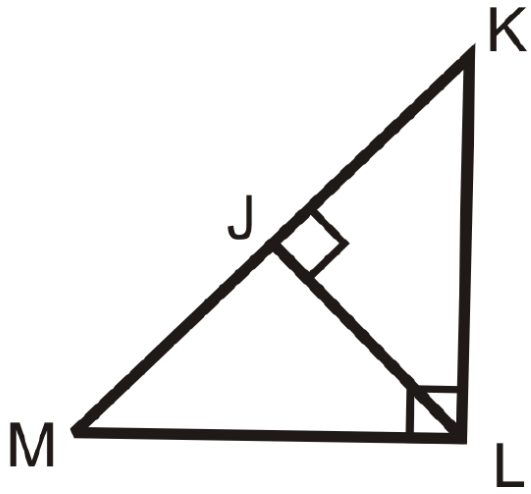


Figure **7.11.11**

Use the diagram to answer questions 7-10.

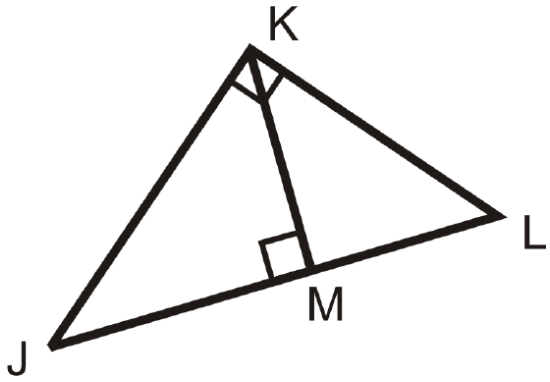


Figure **7.11.12**

7. Write the similarity statement for the three triangles in the diagram.
8. If $JM = 12$ and $ML = 9$, find KM.
9. Find JK.
10. Find KL.

Find the length of the missing variable(s). Simplify all radicals.

11.

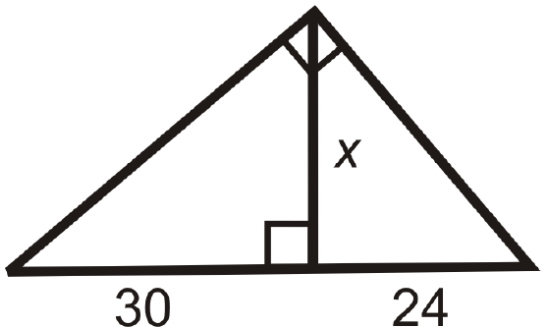


Figure **7.11.13**

12.

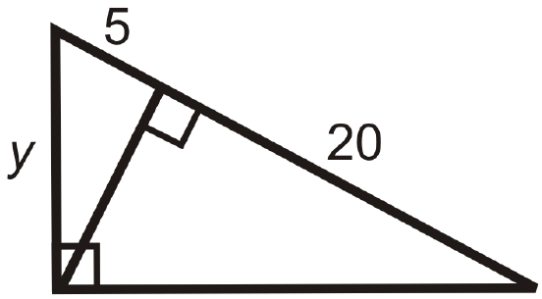


Figure 7.11.14

13.

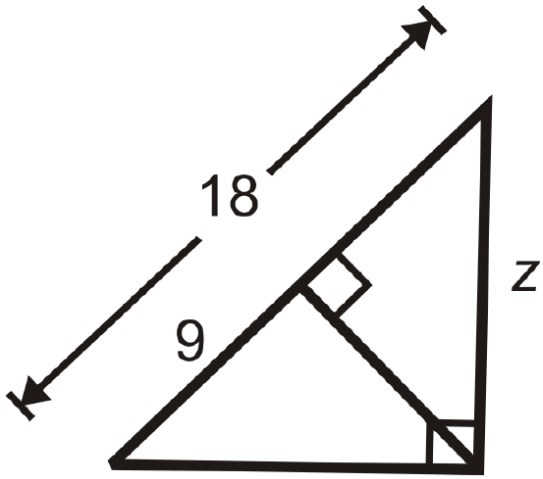


Figure 7.11.15

14.

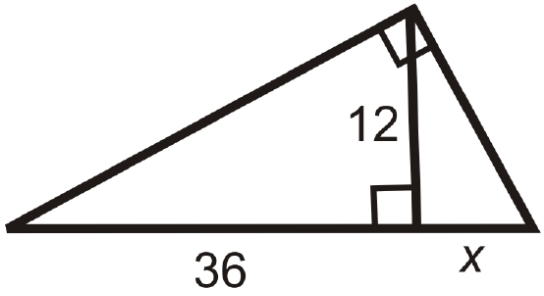


Figure 7.11.16

15.

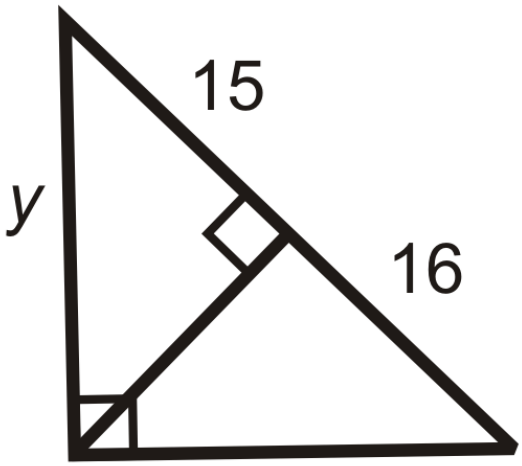


Figure 7.11.17

16.

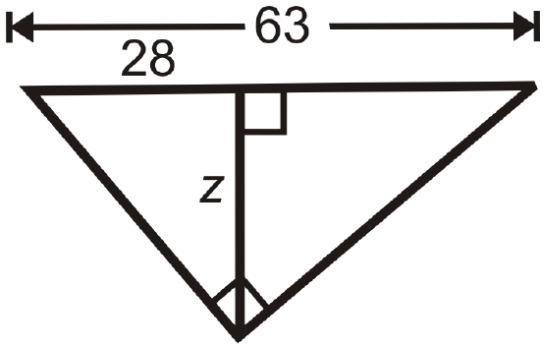


Figure 7.11.18

17.

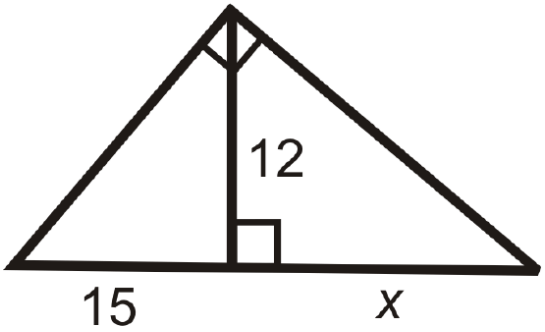


Figure 7.11.19

18.

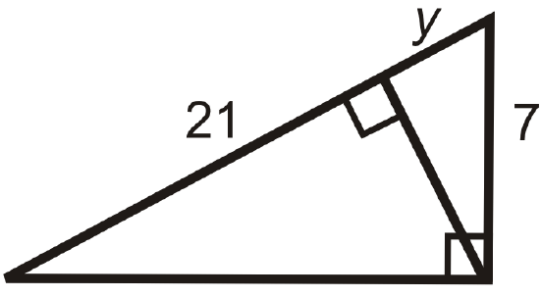


Figure 7.11.20

19.

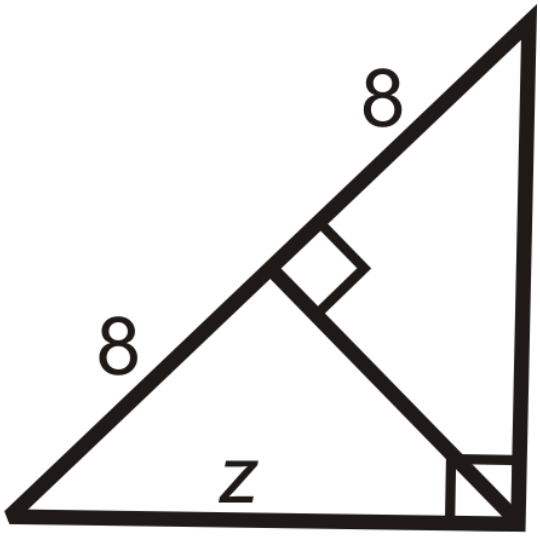


Figure 7.11.21

20.

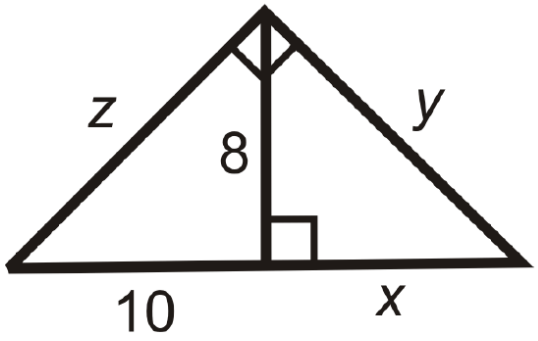


Figure **7.11.22**

21.

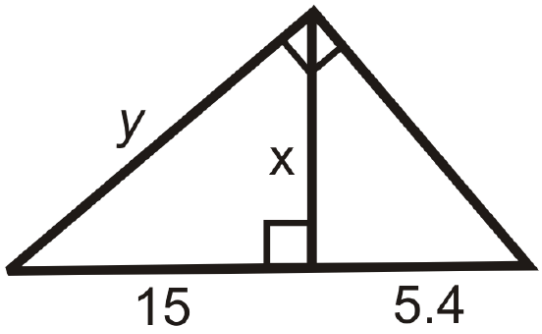


Figure **7.11.23**

22.

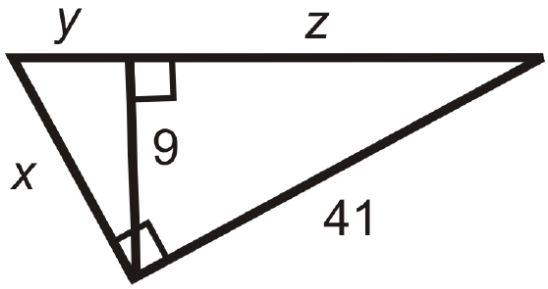


Figure **7.11.24**

23. Fill in the blanks of the proof for the Inscribed Similar Triangles Theorem.

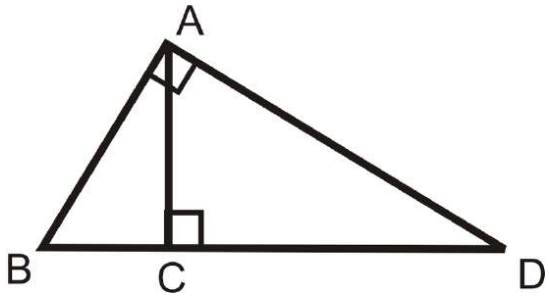


Figure **7.11.25**

Given: ΔABD with $\overline{AC} \perp \overline{DB}$ and $\angle DAB$ is a right angle.

Prove: $\Delta ABD \sim \Delta CBA \sim \Delta CAD$

Statement	*Reason*
1.	1. Given
2. $\angle DCA$ and $\angle ACB$ are right angles	2.
3. $\angle DAB \cong \angle DCA \cong \angle ACB$	3.
4.	4. Reflexive PoC
5.	5. AA Similarity Postulate
6. $B \cong \angle B$	6.
7. $\Delta CBA \cong \Delta ABD$	7.
8. $\Delta CAD \cong \Delta CBA$	8.

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.4.

Vocabulary

Term	Definition
Inscribed Similar Triangles Theorem	The Inscribed Similar Triangles Theorem states that if an altitude is drawn from the right angle of any right triangle, then the two triangles formed are similar to the original triangle and all three triangles are similar to each other.
Perpendicular	Perpendicular lines are lines that intersect at a 90° angle. The product of the slopes of two perpendicular lines is -1.
Proportion	A proportion is an equation that shows two equivalent ratios.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Video: Inscribed Similar Triangles Principles - Basic

Activities: Inscribed Similar Triangles Discussion Questions

Study Aids: Right Triangle Similarity Study Guide

Practice: Inscribed Similar Triangles

Real World: Inscribed Similar Triangles

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7.12: Parallel Lines, Transversals, and Proportionality

As demonstrated by the the Triangle Proportionality Theorem, three or more parallel lines cut by two transversals divide them proportionally.

Triangle Proportionality Theorem

The **Triangle Proportionality Theorem** states that if a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally. We can extend this theorem to a situation outside of triangles where we have multiple parallel lines cut by transversals.

Theorem: If two or more parallel lines are cut by two transversals, then they divide the transversals proportionally.

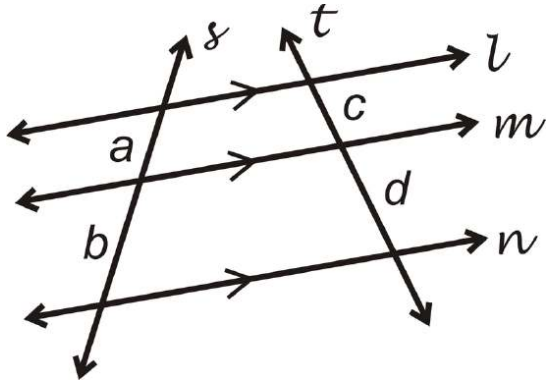

Figure **7.12.1**

If $l \parallel m \parallel n$, then $\frac{a}{b} = \frac{c}{d}$ or $\frac{a}{c} = \frac{b}{d}$.

Note that this theorem works for ***any*** number of parallel lines with ***any*** number of transversals. When this happens, all corresponding segments of the transversals are proportional.

What if you were looking at a map that showed four parallel streets (A, B, C, and D) cut by two avenues, or transversals, (1 and 2)? How could you determine the distance you would have to travel down Avenue 2 to reach Street C from Street B given the distance down Avenue 1 from Street A to Street B, the distance down Avenue 1 from Street B to C, and the distance down Avenue 2 from Street A to B?

Example 7.12.1

Find a, b, and c.

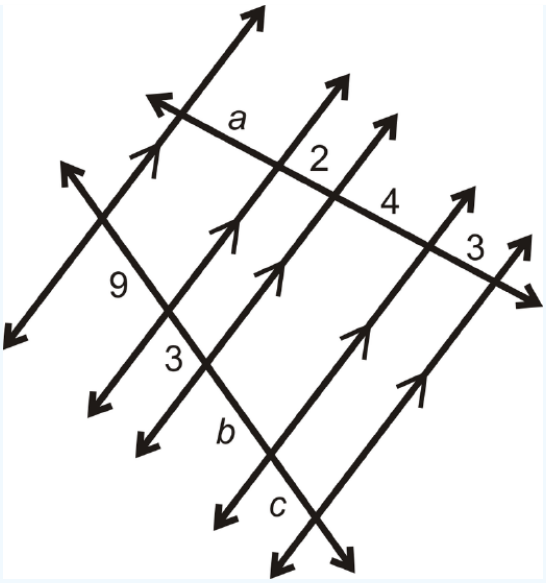

Figure **7.12.2**

Solution

Line up the segments that are opposite each other.

$$\begin{array}{lll} \frac{a}{9}=\frac{2}{3} & \frac{2}{3}=\frac{4}{b} & \frac{2}{3}=\frac{3}{c} \\ 3a=18 & 2b=12 & 2c=9 \\ a=6 & b=6 & c=4.5 \end{array}$$

Example 7.12.1

Below is a street map of part of Washington DC. R Street, Q Street, and O Street are parallel and 7th Street is **perpendicular** to all three. All the measurements are given on the map. What are x and y?

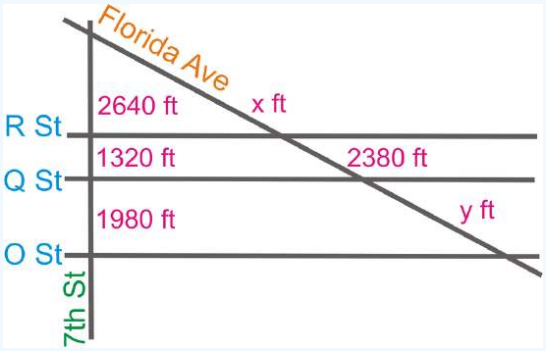

Figure **7.12.3**

Solution

To find x and y, you need to set up a **proportion** using the parallel lines.

$$\frac{2640}{x}=\frac{1320}{2380}=\frac{1980}{y}$$

From this, $x=4760$ ft and $y=3570$ ft .

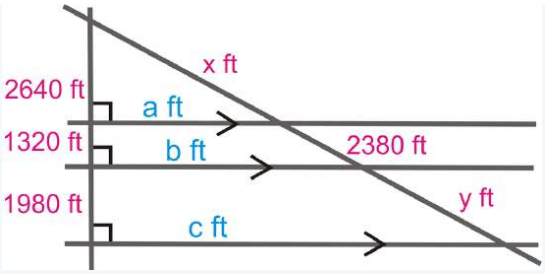

Figure 7.12.4

Example 7.12.1

Find a.

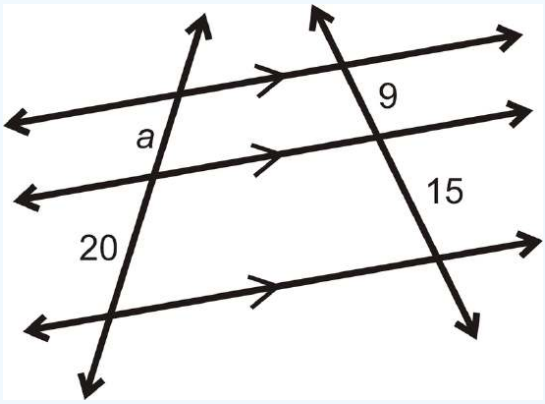

Figure 7.12.5

Solution

The three lines are marked parallel, so to solve, s et up a proportion.

$$\frac{a}{20} = \frac{9}{15}$$
$$180 = 15a$$
$$a = 12$$

Example 7.12.1

Find b.

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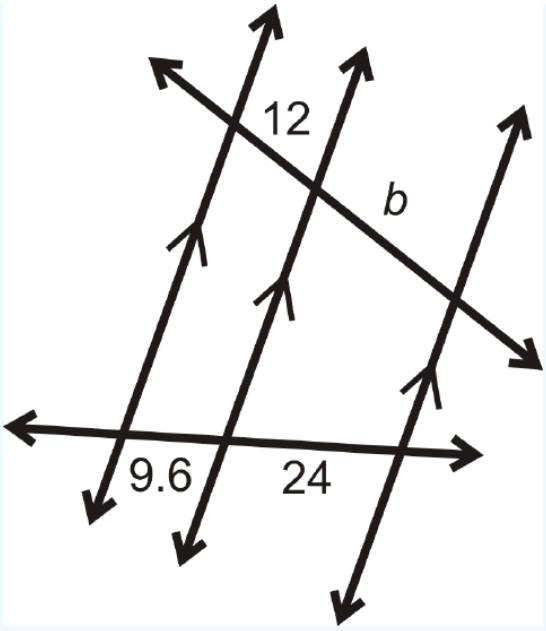

Figure 7.12.6

Solution

To solve, set up a proportion.

$$\frac{12}{9.6} = \frac{b}{24}$$
$$288 = 9.6b$$
$$b = 30$$

Example 7.12.1

Find the value of x\) that makes the lines parallel.

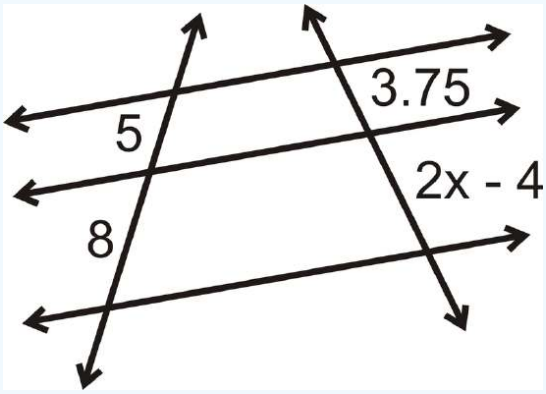

Figure 7.12.7

Solution

To solve, set up a proportion and solve for x.

$$\frac{5}{8} = \frac{3.75}{2x-4} \rightarrow 5(2x-4) = 8(3.75)$$
$$10x - 20 = 30$$
$$10x = 50$$
$$x = 5$$

Review

Find the value of each variable in the pictures below.

1.

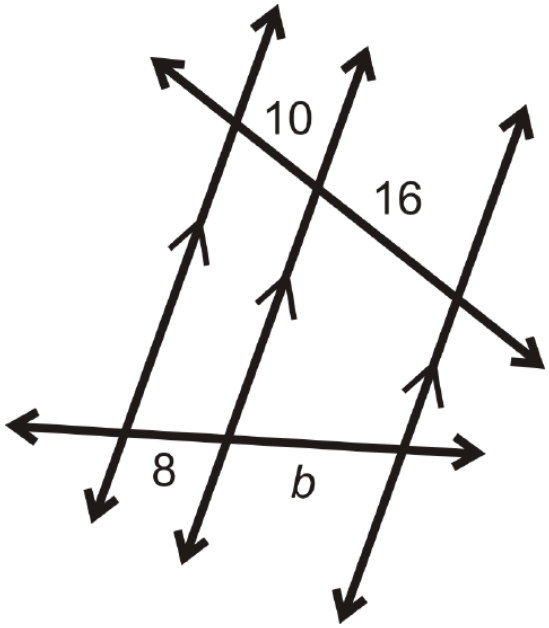

Figure **7.12.8**

2.

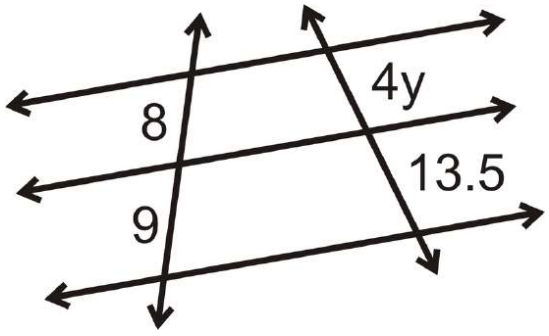

Figure **7.12.9**

3.

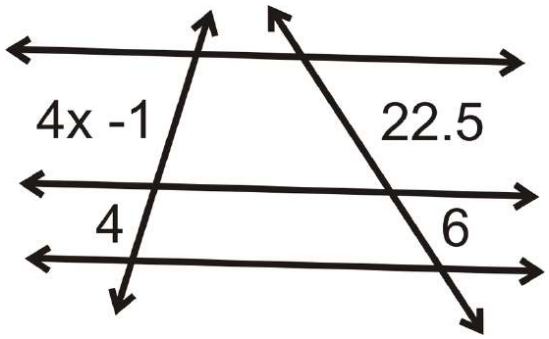

Figure **7.12.10**

4.

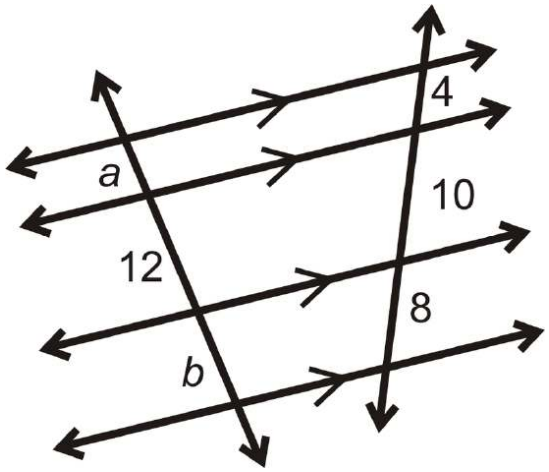

Figure 7.12.11

5.

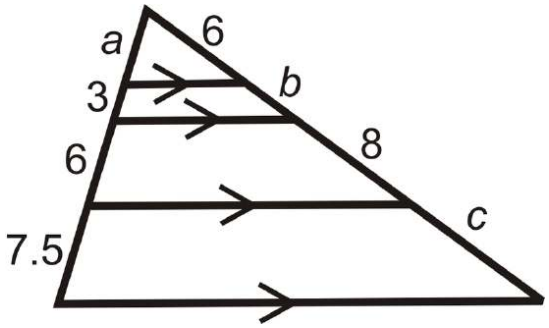

Figure 7.12.12

The street map shows part of New Orleans. Burgundy St., Dauphine St. and Royal St. are parallel to each other. If Spain St. is perpendicular to all three, find the indicated distances.

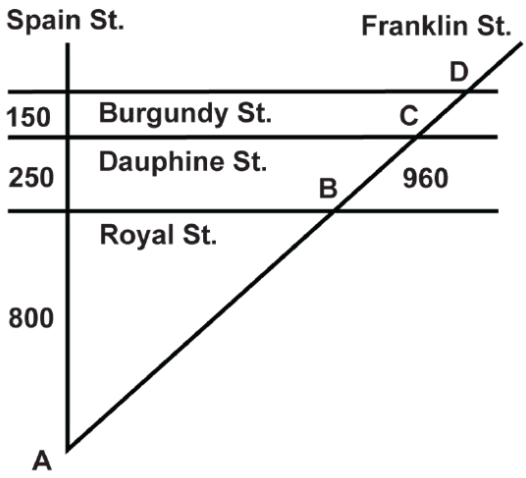

Figure 7.12.13

6. What is the distance between points A and B?
7. What is the distance between points C and D?
8. What is the distance between points A and D?

Using the diagram, answer the questions.

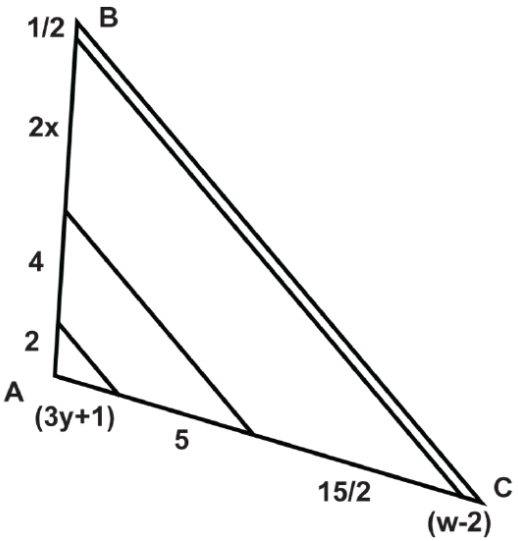

Figure 7.12.14

9. What is the value of w?
10. What is the value of x?
11. What is the value of y?
12. What is the length of $\overline{AB}$?
13. What is the length of $\overline{AC}$?

Using the diagram, fill in the blank.

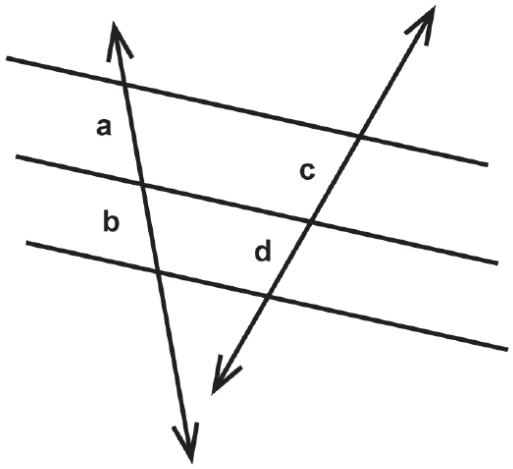

Figure 7.12.15

14. If b is one-third d, then a is _____.
15. If c is two times a, then b is _____.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.9.

Vocabulary

Term	Definition
Coordinate Plane	The coordinate plane is a grid formed by a horizontal number line and a vertical number line that cross at the (0, 0) point, called the origin. The coordinate plane is also called a Cartesian Plane.
Perpendicular	Perpendicular lines are lines that intersect at a 90∘ angle. The product of the slopes of two perpendicular lines is -1.
Proportion	A proportion is an equation that shows two equivalent ratios.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.
transversal	A transversal is a line that intersects two other lines.
Triangle Proportionality Theorem	The Triangle Proportionality Theorem states that if a line is parallel to one side of a triangle and it intersects the other two sides, then it divides those sides proportionally.

Additional Resources

Video: Parallel Lines and Transversals Examples - Basic

Activities: Parallel Lines and Transversals Discussion Questions

Study Aids: Proportionality Relationships Study Guide

Practice: Parallel Lines, Transversals, and Proportionality

Real World: Parallel Lines And Transversals

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7.13: Proportions and Angle Bisectors

Angle bisectors divide triangles proportionally.

Angle Bisector Theorem

When an angle within a triangle is bisected, the bisector divides the triangle proportionally. This idea is called the **Angle Bisector** Theorem.

Angle Bisector Theorem: If a ray bisects an angle of a triangle, then it divides the opposite side into segments that are proportional to the lengths of the other two sides.

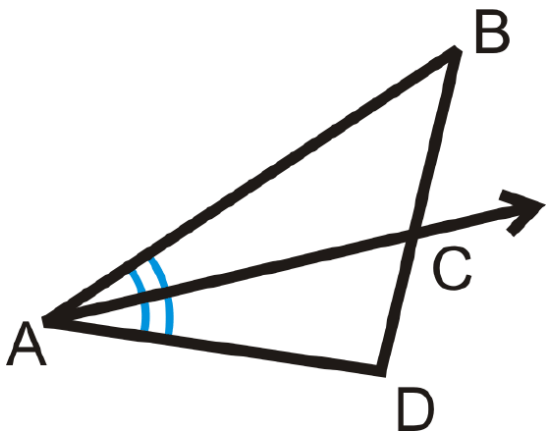


Figure **7.13.1**

If $\Delta BAC \cong \Delta CAD$, then $\frac{BC}{CD} = \frac{AB}{AD}$.

What if you were told that a ray was an angle bisector of a triangle? How would you use this fact to find unknown values regarding the triangle's side lengths?

Example 7.13.1

Fill in the missing varia

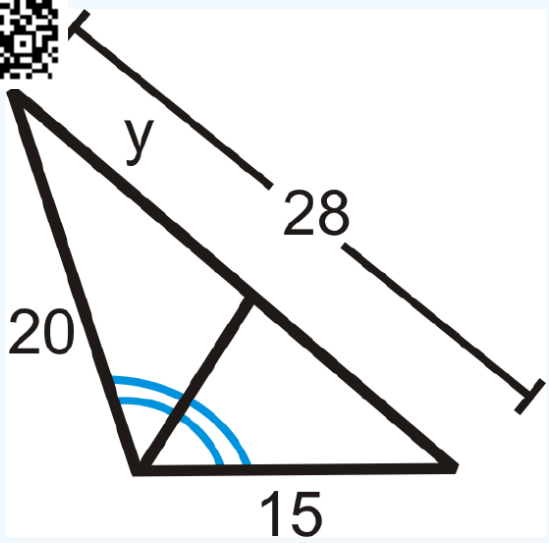


Figure **7.13.2**

Solution

Set up a **proportion** and solve.

$$\frac{20}{y} = \frac{15}{28 - y}$$
$$15y = 20(28 - y)$$
$$15y = 560 - 20y$$
$$35y = 560$$
$$y = 16$$

Example 7.13.2

Fill in the missing variable:

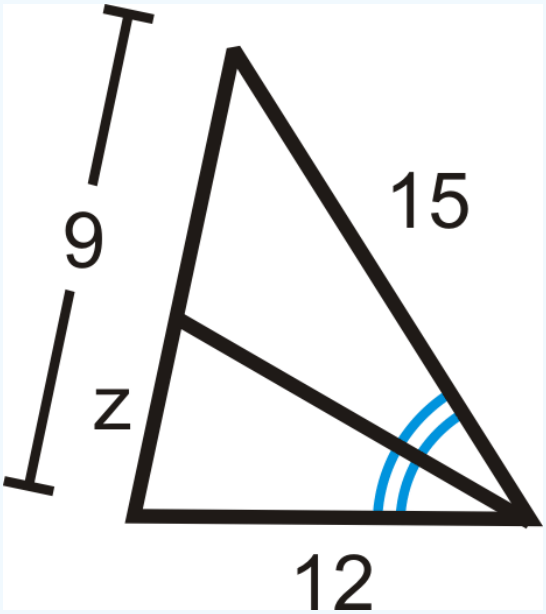


Figure 7.13.3

Solution

Set up a proportion and solve.

$$\frac{12}{z} = \frac{15}{9 - z}$$
$$15z = 12(9 - z)$$
$$15z = 108 - 12z$$
$$27z = 108$$
$$z = 4$$

Example 7.13.3

Find x.

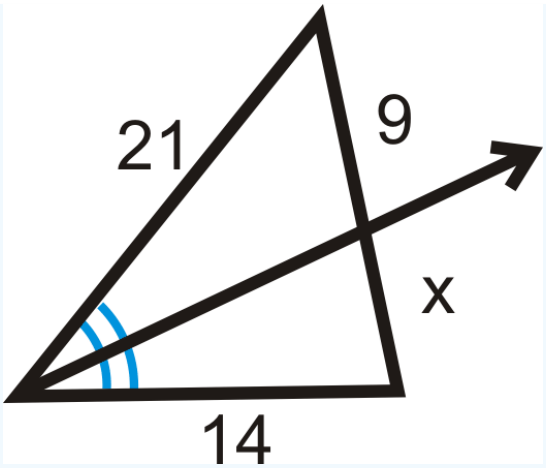


Figure 7.13.4

Solution

The ray is the angle bisector and it splits the opposite side in the same **ratio** as the other two sides. The proportion is:

$$\frac{9}{x} = \frac{21}{14}$$
$$21x = 126$$
$$x = 6$$

Example 7.13.4

Find the value of x that would make the proportion true.

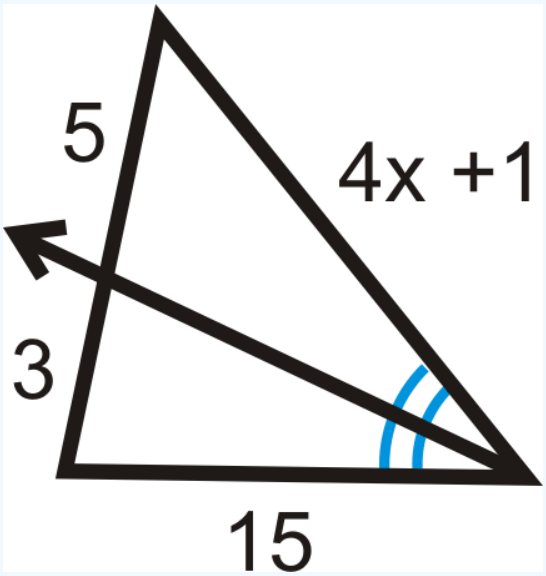


Figure 7.13.5

Solution

You can set up this proportion like the previous example.

$$\frac{5}{3} = \frac{4x+1}{15}$$
$$75 = 3(4x+1)$$
$$75 = 12x+3$$
$$72 = 12x$$
$$6 = x$$

Example 7.13.5

Find the missing variable:

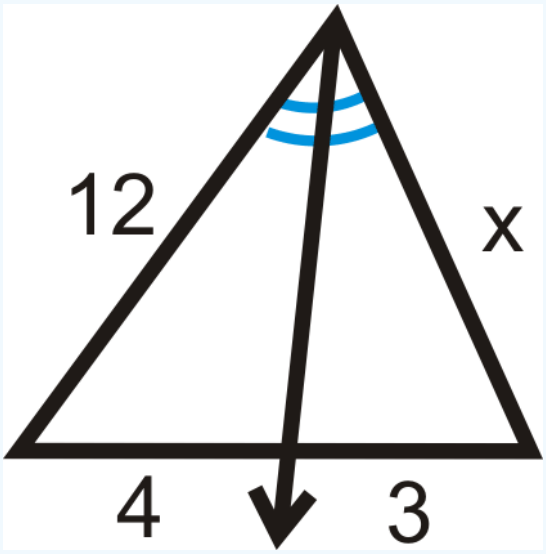


Figure 7.13.6

Solution

Set up a proportion and solve like in the previous examples.

$$\frac{12}{4} = \frac{x}{3}$$
$$36 = 4x$$
$$x = 9$$

Review

Find the value of the missing variable(s).

1.

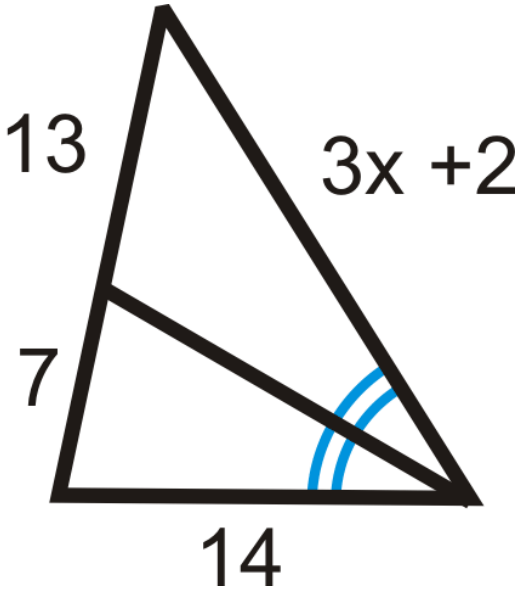


Figure 7.13.7

2.

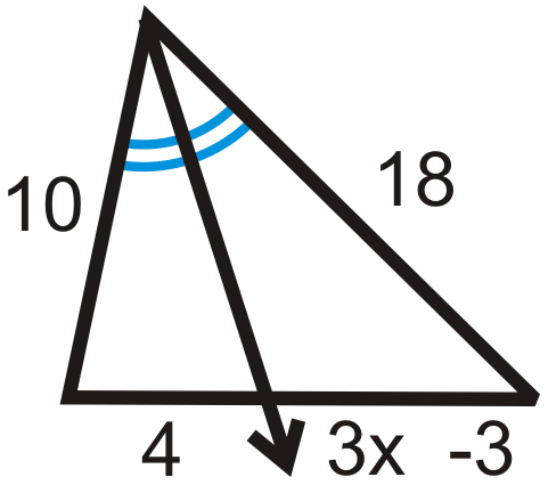


Figure **7.13.8**

Solve for the unknown variable.

3.

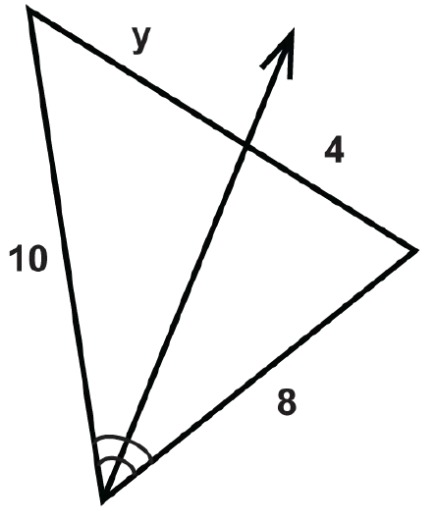


Figure **7.13.9**

4.

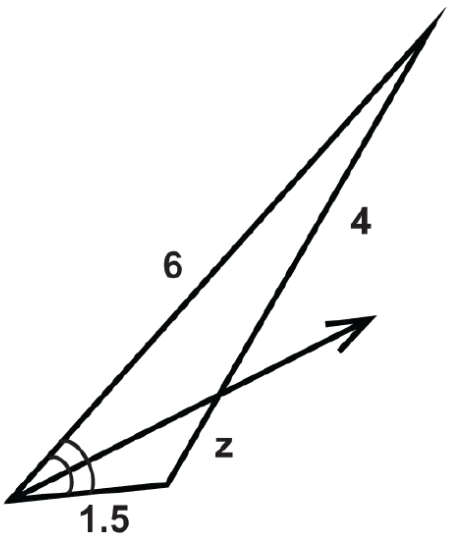


Figure **7.13.10**

5.

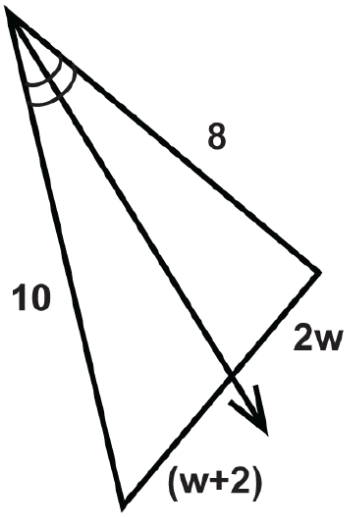


Figure 7.13.11

6.

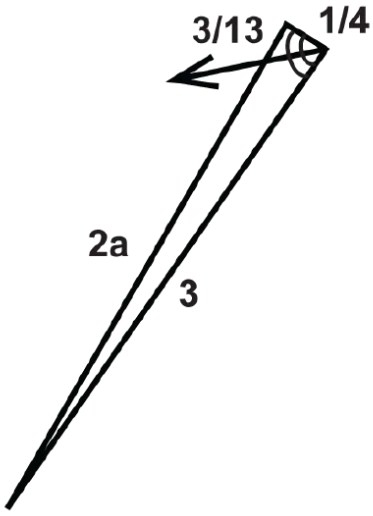


Figure 7.13.12

7.

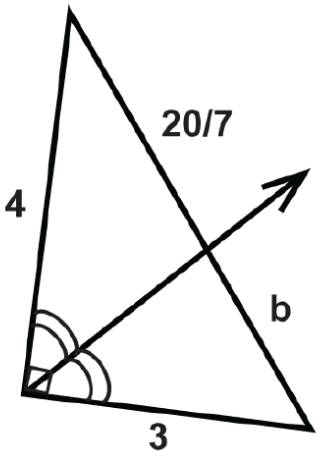


Figure 7.13.13

8.

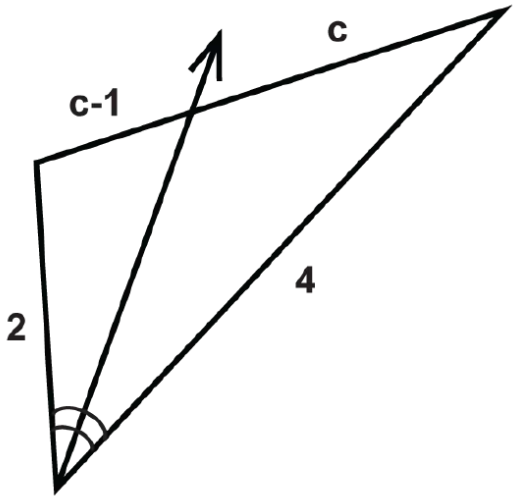


Figure 7.13.14

9.

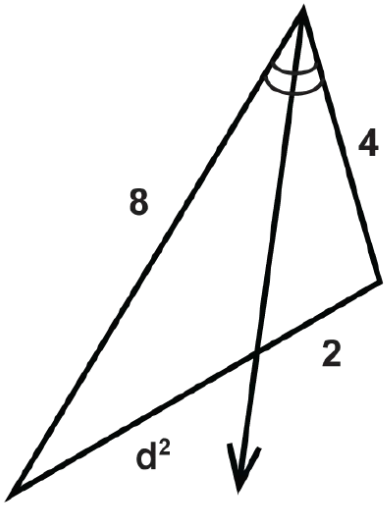


Figure 7.13.15

10.

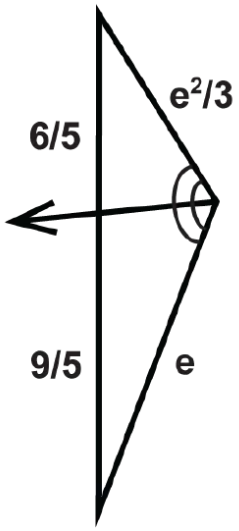


Figure 7.13.16

11.

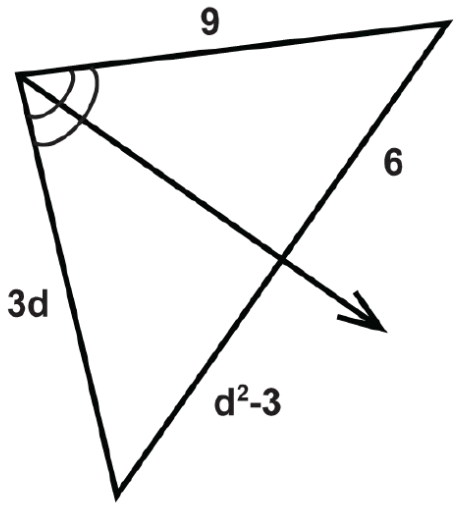


Figure 7.13.17

12.

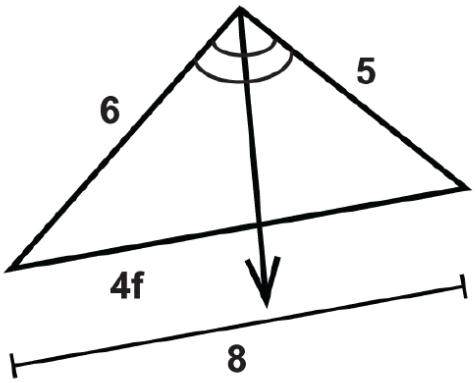


Figure 7.13.18

13.

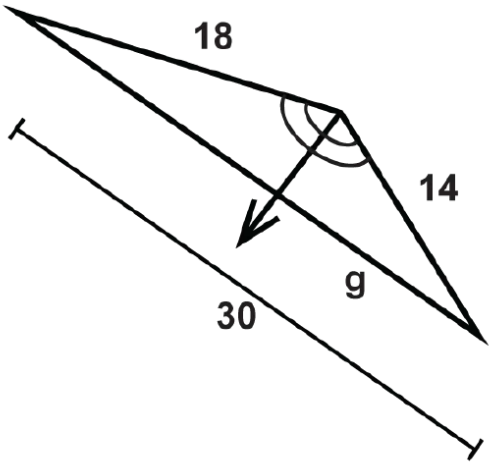
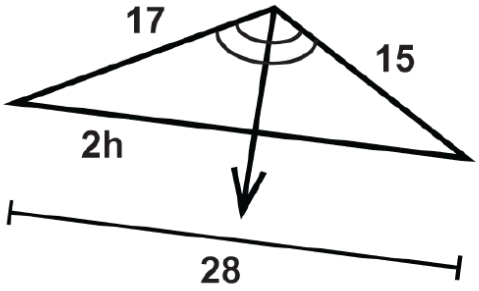
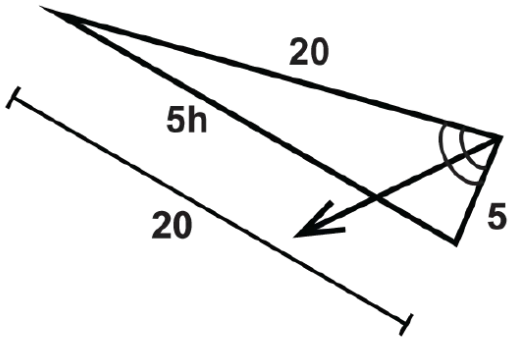


Figure 7.13.19



14.

Figure **7.13.20**



15.

Figure **7.13.21**

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.10.

Resources

Vocabulary

Term	Definition
angle bisector	A ray that divides an angle into two congruent angles.
Angle Bisector Theorem	The angle bisector theorem states that if a point is on the bisector of an angle, then the point is equidistant from the sides of the angle.
Proportion	A proportion is an equation that shows two equivalent ratios.
Ratio	A ratio is a comparison of two quantities that can be written in fraction form, with a colon or with the word “to”.

Additional Resources

Video: Using the Properties of the Triangle Angle Bisector Theorem to Determine Unknown Values

Activities: Proportions with Angle Bisectors Discussion Questions

Study Aids: Proportionality Relationships Study Guide

Real World: Triangle Proportionality

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7.14: Theorems Involving Similarity

Prove and use the Triangle Proportionality, Triangle Angle Bisector, and Pythagorean Theorems.

Can you find any similar triangles in the picture below?

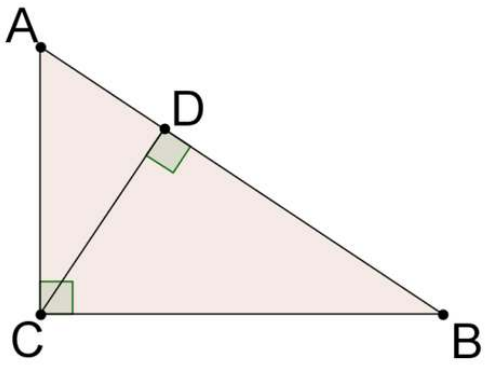


Figure **7.14.1**

Theorems on Similar Triangles

If two triangles are similar, then their corresponding angles are congruent and their corresponding sides are proportional. There are many theorems about triangles that you can prove using similar triangles.

1. **Triangle Proportionality Theorem**: A line parallel to one side of a triangle divides the other two sides of the triangle proportionally. *This theorem and its converse will be explored and proved in #1 and #2, and the Review exercises.*
2. **Triangle Angle Bisector Theorem**: The angle bisector of one angle of a triangle divides the opposite side of the triangle into segments proportional to the lengths of the other two sides of the triangle. *This theorem will be explored and proved in #3.*
3. **Pythagorean Theorem**: For a right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$. *This theorem will be explored and proved in the Examples problems.*

Let's take a look at some p ing triangle similarity.

1. Prove that $\Delta ADE \sim \Delta ABC$

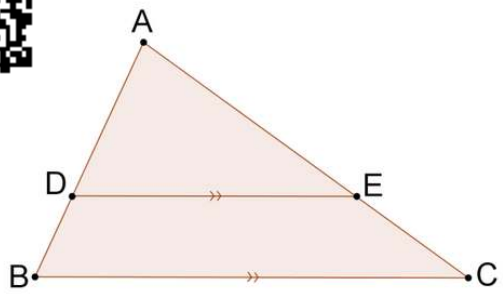


Figure **7.14.2**

The two triangles share $\angle A$. Because $\overline{DE} \parallel \overline{BC}$, corresponding angles are congruent. Therefore, $\angle ADE \cong \angle ABC$. The two triangles have two pairs of congruent angles. Therefore, $\Delta ADE \sim \Delta ABC$ by AA\sim\).

2. Use your result from #1 to prove that $\frac{AB}{AD} = \frac{AC}{AE}$. Then, use algebra to show that $\frac{DB}{AD} = \frac{EC}{AE}$.

\(**\Delta ADE\sim \Delta ABC\)** which means that corresponding sides are proportional. Therefore, $\frac{AB}{AD} = \frac{AC}{AE}$. Now, you can use algebra to show that the second proportion must be true. Remember that $AB = AD + DB$ and $AC = AE + EC$.

$$\frac{AB}{AD} = \frac{AC}{AE}$$
$$\rightarrow \frac{AD + DB}{AD} = AE + \frac{EC}{AE}$$
$$\rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$
$$\rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

You have now proved the triangle proportionality theorem: a line parallel to one side of a triangle divides the other two sides of the triangle proportionally.

3. Consider ΔABC with $\overline{AE}$ the angle bisector of $\angle BAC$ and point D constructed so that $\overline{DC} \parallel \overline{AE}$. Prove that $\frac{EB}{BA} = \frac{EC}{CA}$.

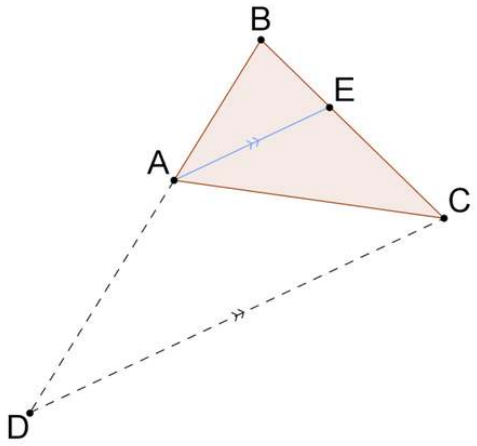


Figure **7.14.3**

By the triangle proportionality theorem, $\frac{EB}{EC} = \frac{BA}{AD}$. Multiply both sides of this proportion by $\frac{EC}{BA}$.

$$(\frac{EC}{BA}) \cdot \frac{EB}{EC}$$
$$\rightarrow \frac{EB}{BA} = \frac{BA}{AD} \cdot (\frac{EC}{BA}) = \frac{EC}{AD}$$

Now all you need to show is that $AD = CA$ in order to prove the desired result.

- Because $\overline{AE}$ is the angle bisector of $\angle BAC$, $\angle BAE \cong \angle EAC$.
- Because $\overline{DC} \parallel \overline{AE}$, $\angle BAE \cong \angle BDC$ (corresponding angles).
- Because $\overline{DC} \parallel \overline{AE}$, $\angle EAC \cong \angle DCA$ (alternate interior angles).
- Thus, $\angle BDC \cong \angle DCA$ by the transitive property.

Therefore, ΔADC is isosceles because its base angles are congruent and it must be true that $\overline{AD} \cong \overline{CA}$. This means that $AD = CA$. Therefore:

$$\frac{EB}{BA} = \frac{EC}{CA}$$

This proves the triangle angle bisector theorem: the angle bisector of one angle of a triangle divides the opposite side of the triangle into segments proportional to the lengths of the other two sides of the triangle.

Example 7.14.1

Earlier, you were asked can you find any similar triangles in the picture below.

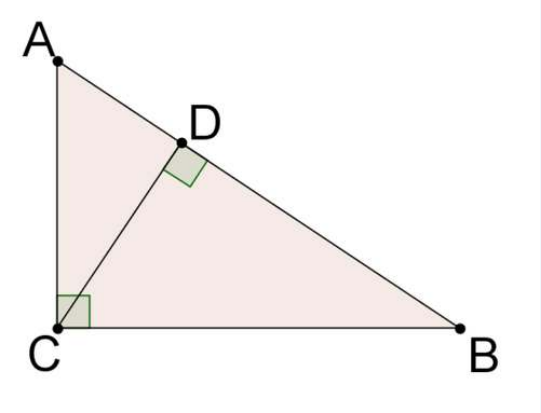


Figure 7.14.4

Solution

There are three triangles in this picture: ΔBAC, ΔBCD, ΔCAD. All three triangles are right triangles so they have one set of congruent angles (the right angle). ΔBAC and ΔBCD share $\angle B$, so $\Delta BAC \sim \Delta BCD$by\\($AA \sim$. Similarly, ΔBAC and ΔCAD share $\angle C$, so $\Delta BAC \sim \Delta CAD$ by $AA \sim$. By the transitive property, all three triangles must be similar to one another.

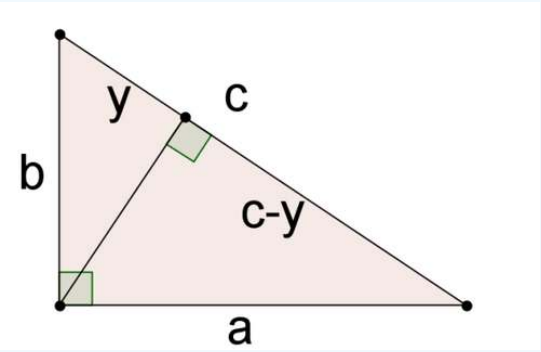


Figure 7.14.5

The large triangle above has sides a, b, and c. Side c has been divided into two parts: y and $c - y$. In the Concept Problem Revisited you showed that the three triangles in this picture are similar.

Example 7.14.2

Explain why $\frac{a}{c} = \frac{c-y}{a}$.

Solution

When triangles are similar, corresponding sides are proportional. Carefully match corresponding sides and you see that $\frac{a}{c} = \frac{c-y}{a}$.

Example 7.14.3

Explain why $\frac{b}{c} = \frac{y}{b}$.

Solution

When triangles are similar, corresponding sides are proportional. Carefully match corresponding sides and you see that $\frac{b}{c} = \frac{y}{b}$.

Example 7.14.4

Use the results from #2 and #3 to show that $a^2 + b^2 = c^2$.

Solution

Cross multiply to rewrite each equation. Then, add the two equations together.

$$\frac{a}{c} = \frac{c-y}{a} \rightarrow a^2 = c^2 - cy$$
$$\frac{b}{c} = \frac{y}{b} \rightarrow b^2 = cy$$
$$\rightarrow a^2 + b^2 = c^2 - cy + cy$$
$$\rightarrow a^2 + b^2 = c^2$$

You have just proved the Pythagorean Theorem using similar triangles.

Review

Solve for x in each problem.

1.

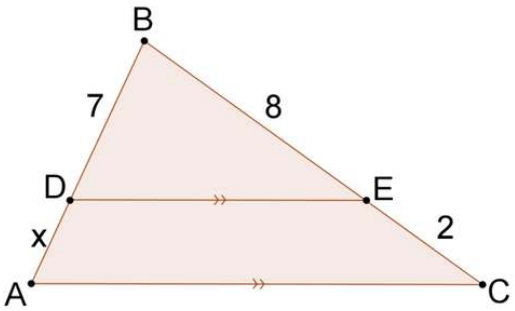


Figure 7.14.6

2.

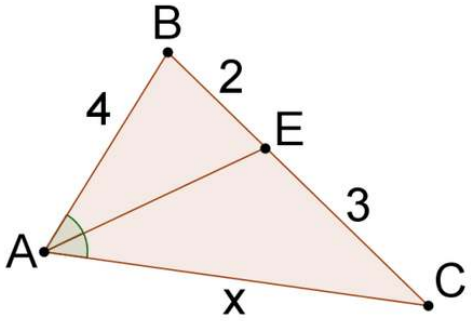


Figure 7.14.7

3.

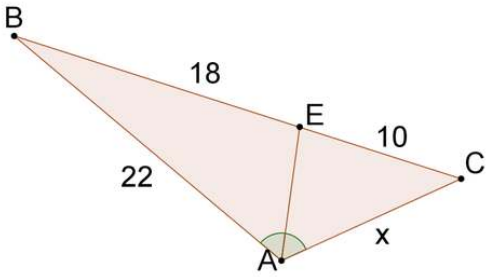


Figure 7.14.8

4.

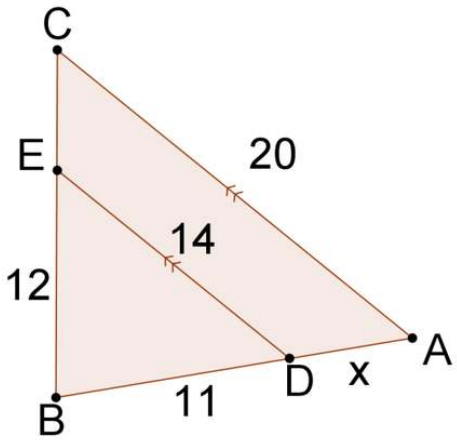


Figure **7.14.9**

5.

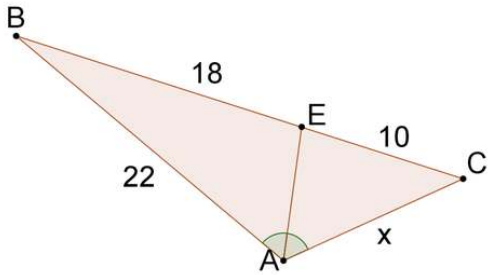


Figure **7.14.10**

6.

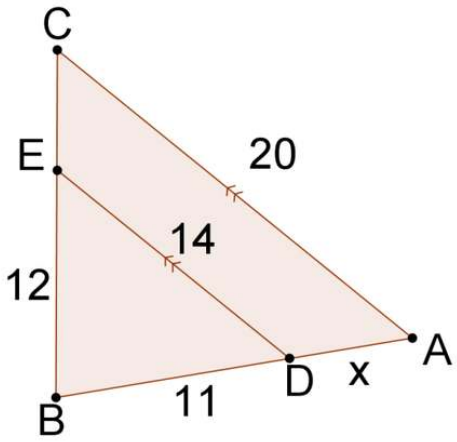
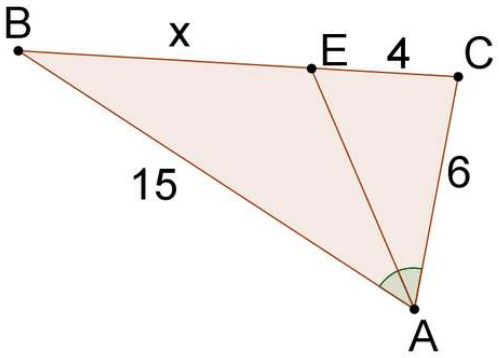


Figure **7.14.11**



7.

Figure **7.14.12**

Use the picture below for #8-#10.

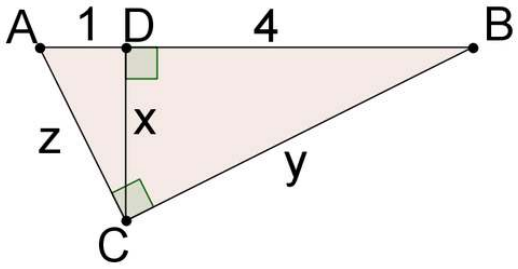


Figure **7.14.13**

8. Solve for x.

9. Solve for z.

10. Solve for y.

Use the picture below for #11-#13.

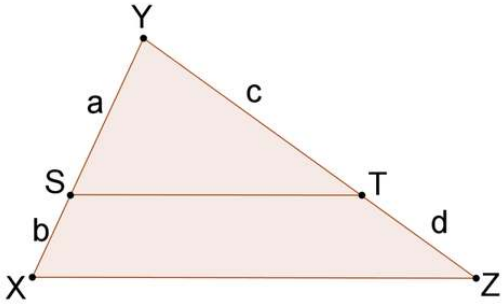


Figure **7.14.14**

11. Assume that $\frac{b}{a} = \frac{d}{c}$. Use algebra to show that $\frac{b+a}{a} = \frac{d+c}{c}$.

12. Prove that $\Delta YST \sim \Delta YXZ$

13. Prove that $\overline{ST} \parallel \overline{XZ}$

14. Prove that a segment that connects the midpoints of two sides of a triangle will be parallel to the third side of the triangle.

15. Prove the Pythagorean Theorem using the picture below.

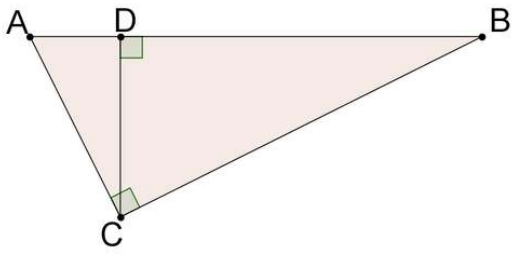


Figure **7.14.15**

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.6.

Vocabulary

Term	Definition
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.
Triangle Proportionality Theorem	The Triangle Proportionality Theorem states that if a line is parallel to one side of a triangle and it intersects the other two sides, then it divides those sides proportionally.
Triangle Angle Bisector Theorem	The angle bisector of one angle of a triangle divides the opposite side of the triangle into segments proportional to the lengths of the other two sides of the triangle.

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7.15: Dilation of a Shape

Larger or smaller version of a figure that preserves its shape.

Dilation

Two figures are similar if they are the same shape but not necessarily the same size. One way to create similar figures is by dilating. A dilation makes a figure larger or smaller but the new resulting figure has the same shape as the original.

Dilation: An enlargement or reduction of a figure that preserves shape but not size. All dilations are similar to the original figure.

Dilations have a **center** and a **scale factor**. The center is the point of reference for the dilation and the scale factor tells us how much the figure stretches or shrinks. A scale factor is labeled k. Only positive scale factors, k, will be considered in this text.

If the dilated image is smaller than the original, then $0 < k < 1$.

If the dilated image is larger than the original, then $k > 1$.

A dilation, or image, is always followed by a $'$.

Label It	*Say It*
$'$	"prime" (copy of the original)
A'	"a prime" (copy of point A)
A''	"a double prime" (second copy)

What if you enlarged or reduced a triangle without changing its shape? How could you find the scale factor by which the triangle was stretched or shrunk?

Example 7.15.1

Find the perimeters of $KLMN$ and $K'L'M'N'$. Compare this **ratio** to the scale factor.

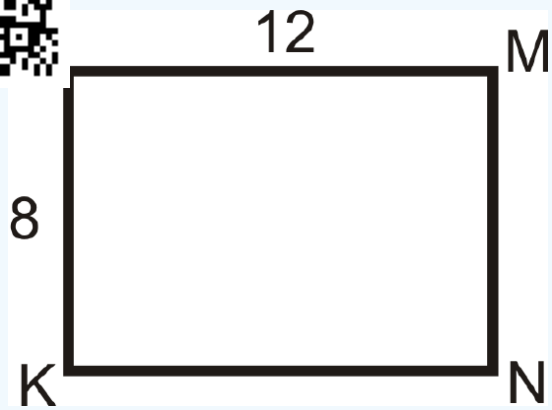

Figure 7.15.1

Solution

The perimeter of $KLMN = 12 + 8 + 12 + 8 = 40$. The perimeter of $K'L'M'N' = 24 + 16 + 24 + 16 = 80$. The ratio is 80:40, which reduces to 2:1, which is the same as the scale factor.

Example 7.15.2

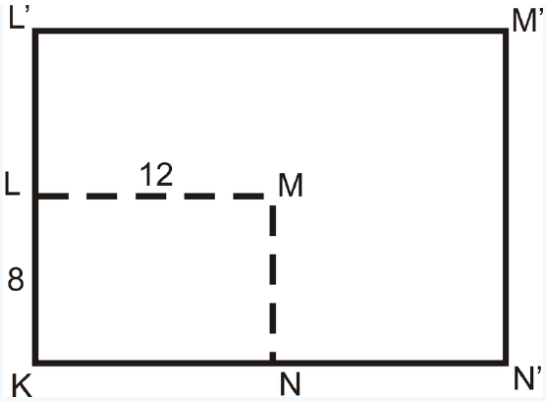

Figure 7.15.2

ΔABC is a dilation of ΔDEF. If P is the center of dilation, what is the scale factor?

Solution

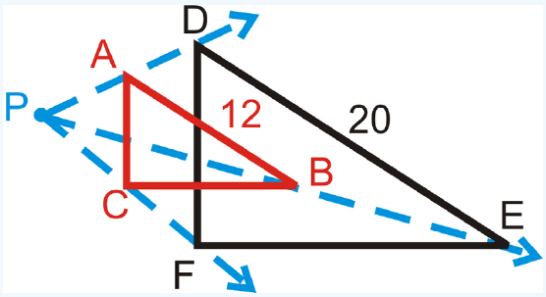

Figure 7.15.3

Because ΔABC is a dilation of ΔDEF, then $\Delta ABC \sim \Delta DEF$. The scale factor is the ratio of the sides. Since ΔABC is smaller than the original, ΔDEF, the scale factor is going to be less than one, $\frac{12}{20} = \frac{3}{5}$.

If ΔDEF was the dilated image, the scale factor would have been $\frac{5}{3}$.

Example 7.15.3

The center of dilation is P and the scale factor is 3.

Find Q'.

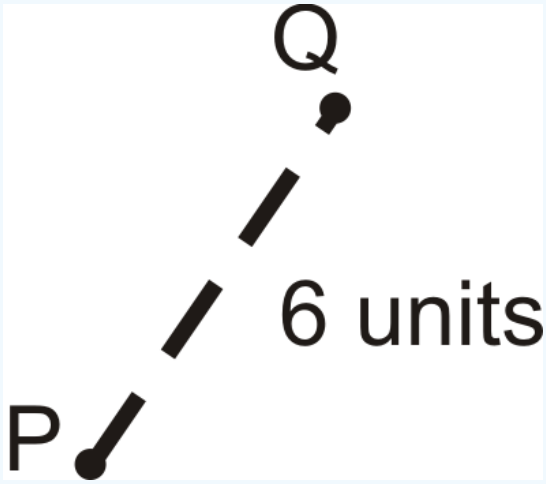

Figure 7.15.4

Solution

If the scale factor is 3 and Q is 6 units away from P, then Q' is going to be $6 \times 3 = 18$ units away from P. The dilated image will be on the same line as the original image and center.

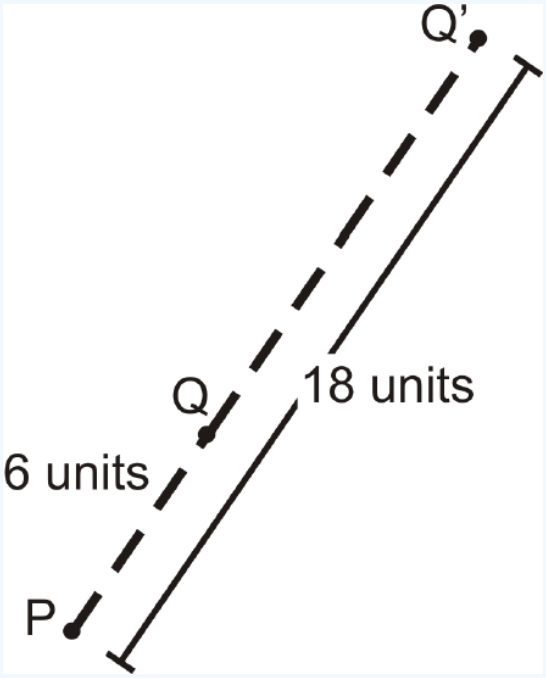

Figure 7.15.5

Example 7.15.4

Using the picture above, change the scale factor to 13.

Find Q'' using this new scale factor.

Solution

The scale factor is $\frac{1}{3}$, so Q'' is going to be $6 \times \frac{1}{3} = 2$ units away from P. Q'' will also be collinear with Q and center.

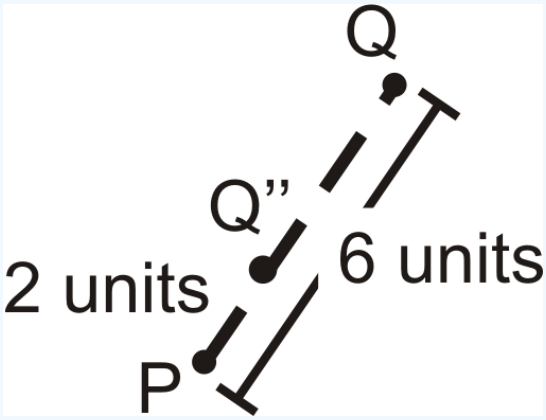

Figure 7.15.6

Example 7.15.5

$KLMN$ is a rectangle. If the center of dilation is K and $k = 2$, draw $K'L'M'N'$.

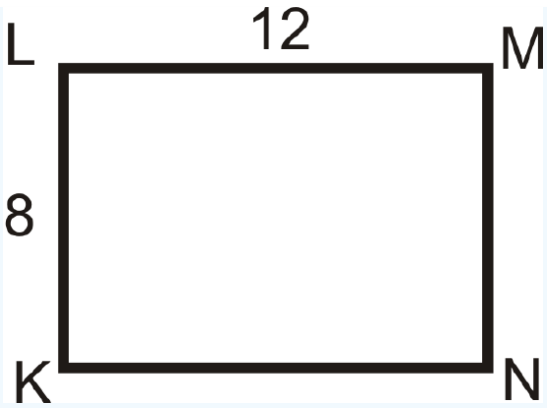

Figure 7.15.7

Solution

If K is the center of dilation, then K and K' will be the same point. From there, L' will be 8 units above L and N' will be 12 units to the right of N.

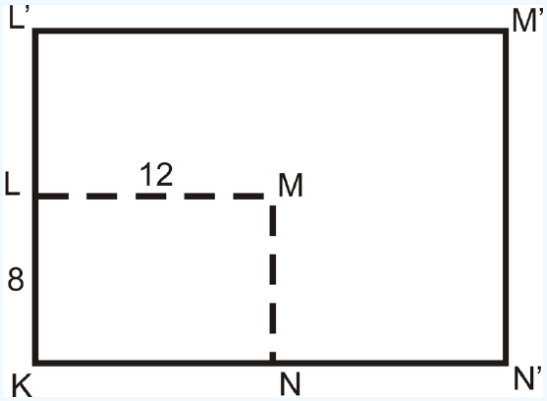

Figure 7.15.8

Review

For the given shapes, draw the dilation, given the scale factor and center.

1. $k = 3.5$, center is A

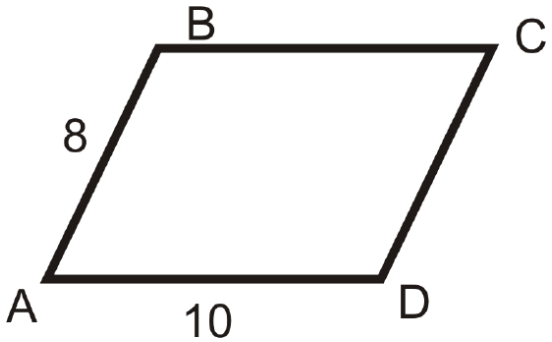

Figure 7.15.9

2. $k = 2$, center is D

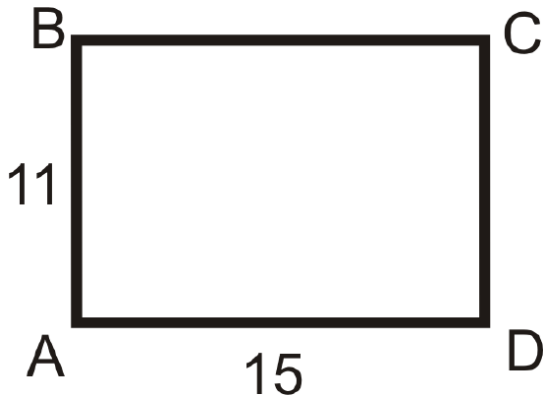

Figure 7.15.10

3. $k = \frac{3}{4}$, center is A

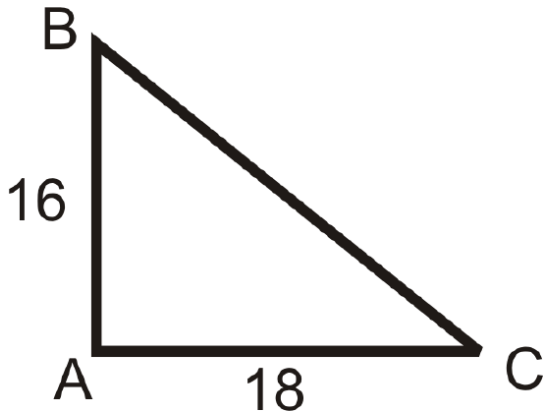

Figure 7.15.11

4. $k = \frac{2}{5}$, center is A\)

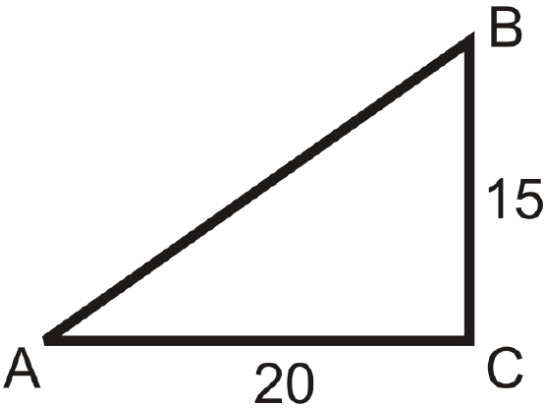

Figure 7.15.12

In the four questions below, you are told the scale factor. Determine the dimensions of the dilation. In each diagram, the **black** figure is the original and P is the center of dilation.

5. $k = 4$

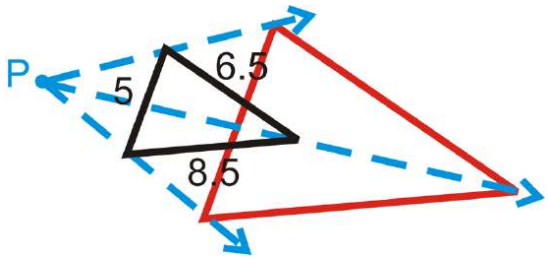

Figure **7.15.13**

6. $k = \frac{1}{3}$

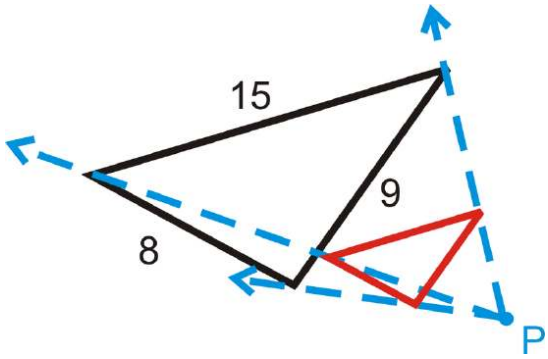

Figure **7.15.14**

7. $k = 2.5$

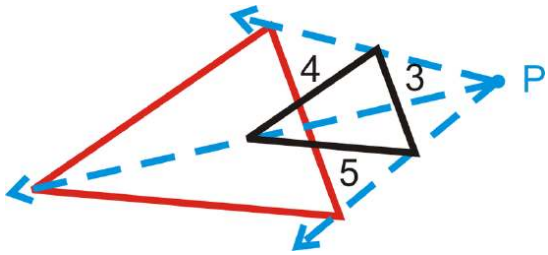

Figure **7.15.15**

8. $k = \frac{1}{4}$

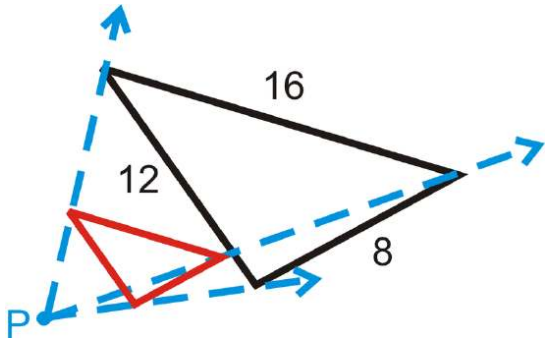

Figure **7.15.16**

In the three questions below, find the scale factor, given the corresponding sides. In each diagram, the **black** figure is the original and P is the center of dilation.

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9.

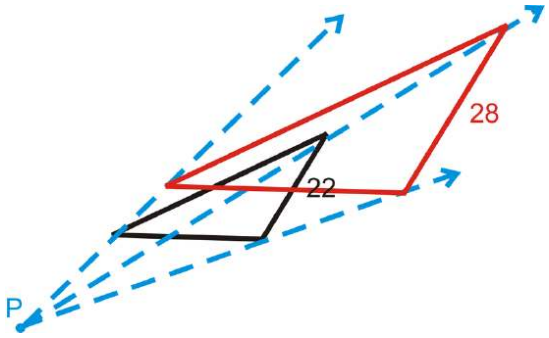

Figure 7.15.17

10.

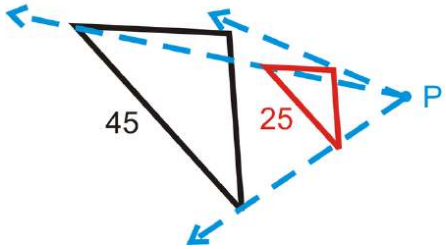

Figure 7.15.18

11.

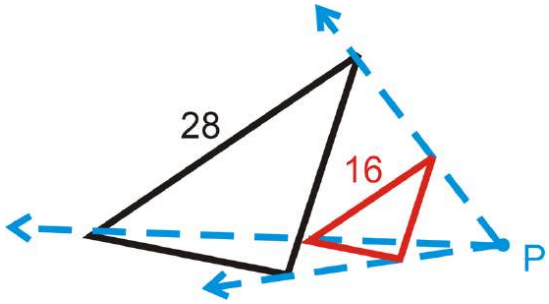

Figure 7.15.19

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.11.

Vocabulary

Term	Definition
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
Quadrilateral	A quadrilateral is a closed figure with four sides and four vertices.
Ratio	A ratio is a comparison of two quantities that can be written in fraction form, with a colon or with the word “to”.
Scale Factor	A scale factor is a ratio of the scale to the original or actual dimension written in simplest form.
Transformation	A transformation moves a figure in some way on the coordinate plane.
Vertex	A vertex is a point of intersection of the lines or rays that form an angle.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Dilation Principles - Basic

Activities: Dilation Discussion Questions

Study Aids: Types of Transformations Study Guide

Practice: Dilation of a Shape

Real World: The CSI Effect

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7.16: Dilation in the Coordinate Plane

Multiplication of coordinates by a scale factor given the origin as the center.

Dilation in the Coordinate Plane

Two figures are similar if they are the same shape but not necessarily the same size. One way to create similar figures is by dilating. A dilation makes a figure larger or smaller such that the new image has the same shape as the original.

Dilation: An enlargement or reduction of a figure that preserves shape but not size. All dilations are similar to the original figure.

Dilations have a **center** and a **scale factor**. The center is the point of reference for the dilation and the scale factor tells us how much the figure stretches or shrinks. A scale factor is labeled k. Only positive scale factors, k, will be considered in this text.

If the dilated image is smaller than the original, then $0 < k < 1$.

If the dilated image is larger than the original, then $k > 1$.

To dilate something in the coordinate plane, multiply each coordinate by the scale factor. This is called **mapping**. For any dilation the mapping will be $(x, y) \rightarrow (kx, ky)$. In this text, the center of dilation will always be the origin.

What if you were given the coordinates of a figure and were asked to dilate that figure by a scale factor of 2? How could you find the coordinates of the dilated figure?

For Examples 1 and 2, use uctions:

Given A and the scale fact oordinates of the dilated point, A'. You may assume the center of dilation is the origin. Remember that the mappi $(x, y) \rightarrow (kx, ky)$.

Example 7.16.1

$A(-4, 6), k = 2$

Solution

$A'(-8, 12)$

Example 7.16.2

$A(9, -13), k = \frac{1}{2}$

Solution

$A'(4.5, -6.5)$

Example 7.16.3

Quadrilateral EFGH\) has vertices $E(-4, -2)$, $F(1, 4)$, $G(6, 2)$ and $H(0, -4)$. Draw the dilation with a scale factor of 1.5.

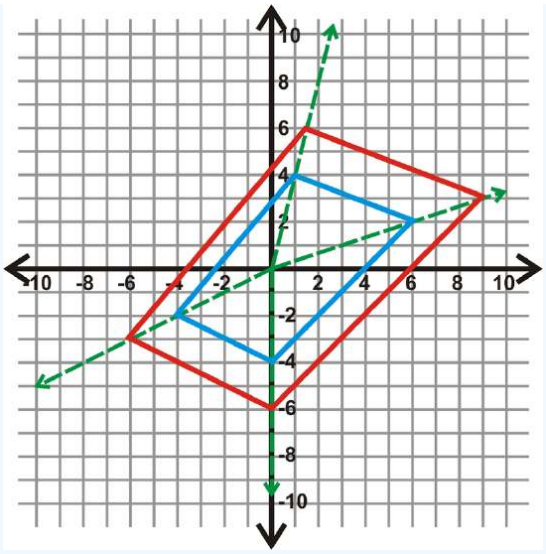

Figure 7.16.1

Solution

Remember that to dilate something in the coordinate plane, multiply each coordinate by the scale factor.

For this dilation, the mapping will be \((x,y)\rightarrow (1.5x, 1.5y) .

$$E(-4,-2) \rightarrow (1.5(-4), 1.5(-2)) \rightarrow E'(-6,-3)$$
$$F(1,4) \rightarrow (1.5(1), 1.5(4)) \rightarrow F'(1.5,6)$$
$$G(6,2) \rightarrow (1.5(6), 1.5(2)) \rightarrow G'(9,3)$$
$$H(0,-4) \rightarrow (1.5(0), 1.5(-4)) \rightarrow H'(0,-6)$$

In the graph above, the blue quadrilateral is the original and the red image is the dilation.

Example 7.16.4

Determine the coordinates of ΔABC and $\Delta A'B'C'$ and find the scale factor.

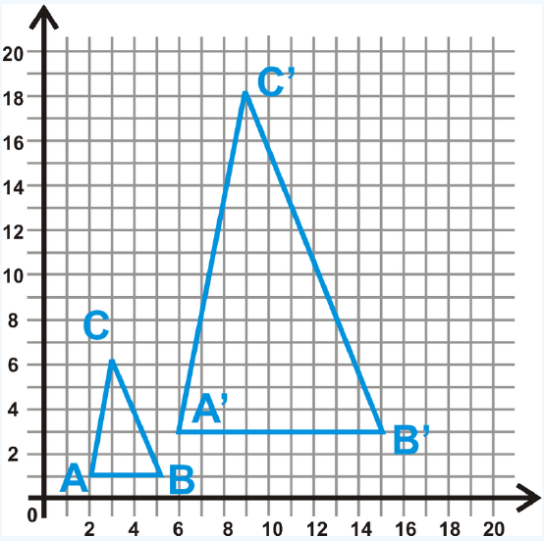

Figure 7.16.2

Solution

The coordinates of the vertices of ΔABC are $A(2,1)$, B(5,1)\) and C(3,6)\). The coordinates of the vertices of $\Delta A'B'C'$ are A′(6,3)\), $B'(15,3)$ and C′(9,18)\). Each of the corresponding coordinates are three times the original, so $k=3$.

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Example 7.16.5

Show that dilations preserve shape by using the **distance formula**. Find the lengths of the sides of both triangles in Example B.

Solution

ΔABC	$\Delta A'B'C'$
$AB = \sqrt{(2-5)^2 + (1-1)^2} = \sqrt{9} = 3$	$A'B' = \sqrt{(6-15)^2 + (3-3)^2} = \sqrt{81} = 9$
$AC = \sqrt{(2-3)^2 + (1-6)^2} = \sqrt{26}$	$\sqrt{(6-9)^2 + (3-18)^2} = 3\sqrt{26}$
$CB = \sqrt{(3-5)^2 + (6-1)^2} = \sqrt{29}$	$\sqrt{(9-15)^2 + (18-3)^2} = 3\sqrt{29}$

From this, we also see that all the sides of $\Delta A'B'C'$ are three times larger than ΔABC.

Review

Given A and A', find the scale factor. You may assume the center of dilation is the origin.

1. $A(8, 2), A'(12, 3)$
2. $A(-5, -9), A'(-45, -81)$
3. $A(22, -7), A(11, -3.5)$

The origin is the center of dilation. Draw the dilation of each figure, given the scale factor.

4. $A(2, 4), B(-3, 7), C(-1, -2); k = 3$
5. $A(12, 8), B(-4, -16), C(0, 10); k = 34$

Multi-Step Problem Questions 6-9 build upon each other.

6. Plot $A(1, 2), B(12, 4), C(10, 10)$. Connect to form a triangle.
7. Make the origin the center of dilation. Draw 4 rays from the origin to each point from #21. Then, plot $A'(2, 4), B'(24, 8), C'(20, 20)$. What is the scale factor?
8. Use $k = 4$, to find $A''B''C''$. Plot these points.
9. What is the scale factor from $A'B'C'$ to $A''B''C''$?

If O is the origin, find the following lengths (using 6-9 above). Round all answers to the nearest hundredth.

10. OA
11. AA'
12. AA''
13. OA'
14. OA''
15. AB
16. $A'B'$
17. $A''B''$
18. Compare the ratios $OA : OA'$ and $AB : A'B'$. What do you notice? Why do you think that is?
19. Compare the ratios $OA : OA''$ and $AB : A''B''$. What do you notice? Why do you think that is?

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.12.

Vocabulary

Term	Definition
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
Distance Formula	The distance between two points (x_1, y_1) and (x_2, y_2) can be defined as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Mapping	Mapping is a procedure involving the plotting of points on a coordinate grid to see the behavior of a function.
Scale Factor	A scale factor is a ratio of the scale to the original or actual dimension written in simplest form.

Additional Resources

Video: Dilation in the Coordinate Plane Principles - Basic

Activities: Dilation in the Coordinate Plane Discussion Questions

Study Aids: Types of Transformations Study Guide

Practice: Dilation in the Coordinate Plane

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7.17: Mapping Dilations

Figures that can be carried to each other using one or more rigid transformations followed by a dilation. Rules that describe given size changes of images

Rules for Dilations

The figure below shows a dilation of two trapezoids. Write the mapping rule for the dilation of Image A to Image B.

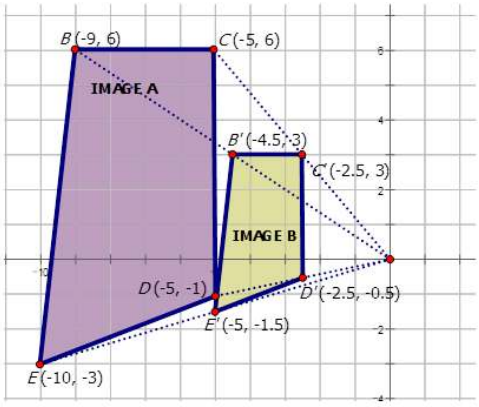

Figure **7.17.1**

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A dilation is a type of transformation that enlarges or reduces a figure (called the preimage) to create a new figure (called the image). The scale factor, r, determines how much bigger or smaller the dilation image will be compared to the preimage.

Look at the diagram below:

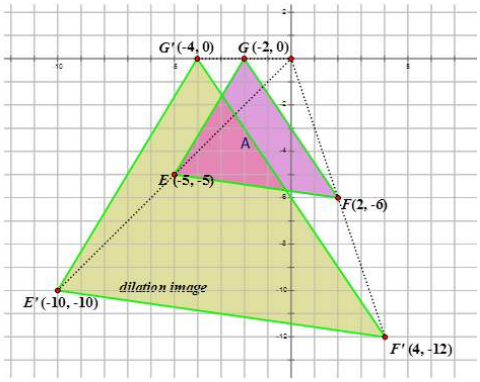

Figure **7.17.2**

The Image A has undergone a dilation about the origin with a scale factor of 2. Notice that the points in the dilation image are all double the coordinate points in the preimage. A dilation with a scale factor k about the origin can be described using the following notation:

$D_k(x, y) = (kx, ky)$

k will always be a value that is greater than 0.

Scale Factor, k	Size change for preimage
$k > 1$	Dilation image is larger than preimage
$0 < k < 1$	Dilation image is smaller than preimage
$k = 1$	Dilation image is the same size as the preimage

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Let's draw the dilation image for the following mappings:

1. The mapping rule for the dilation applied to the triangle below is $(x, y) \rightarrow (1.5x, 1.5y)$.

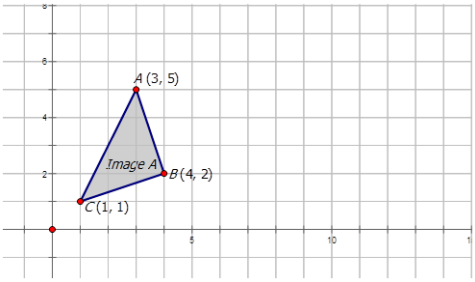

Figure **7.17.3**

With a scale factor of 1.5, each coordinate point will be multiplied by 1.5.

Image A	$A(3, 5)$	$B(4, 2)$	$C(1, 1)$
Dilation Image	$A'(4.5, 7.5)$	$B'(6, 3)$	$C'(1.5, 1.5)$

The dilation image looks like the following:

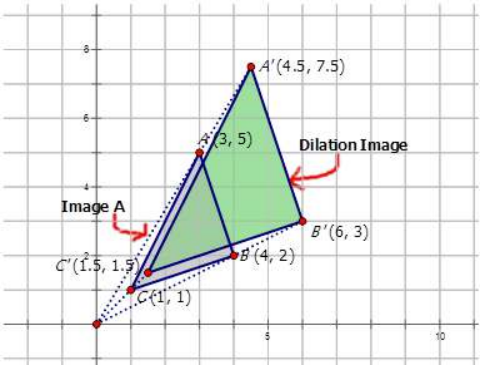

Figure **7.17.4**

2. The mapping rule for the dilation applied to the diagram below is $(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$.

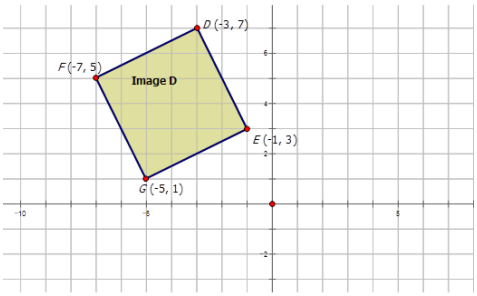

Figure **7.17.5**

With a scale factor of $\frac{1}{3}$, each coordinate point will be multiplied by $\frac{1}{3}$.

Image D	$D(-3,7)$	$E(-1,3)$	$F(-7,5)$	$G(-5,1)$
Dilation Image	$D'(-1,2.3)$	$E'(-0.3,1)$	$F'(-2.3,1.7)$	$G'(-1.7,0.3)$

The dilation image looks like the following:

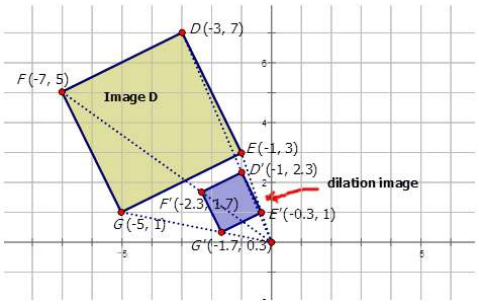

Figure **7.17.6**

Now, let's write the notation that represents the dilation of the preimage A to the dilation image J in the diagram below:

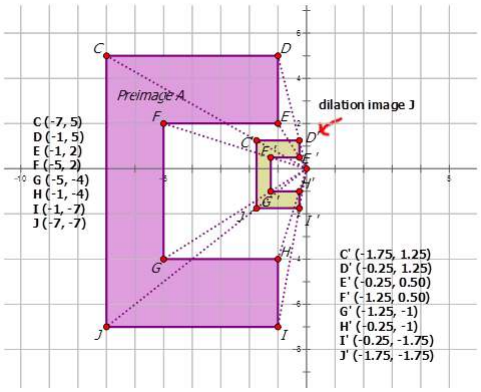

Figure **7.17.7**

First, pick a point in the diagram to use to see how it has been affected by the dilation.

$C: (-7, 5)$ $C': (-1.75, 1.25)$

Notice how both the x- and y-coordinates are multiplied by $\frac{1}{4}$. This indicates that the preimage A undergoes a dilation about the origin by a scale factor of $\frac{1}{4}$ to form the dilation image J. Therefore the mapping notation is $(x, y) \rightarrow (\frac{1}{4}x, \frac{1}{4}y)$.

Example 7.17.1

Earlier, you were asked to write the mapping rule for the dilation of Image A to Image B in the image below:

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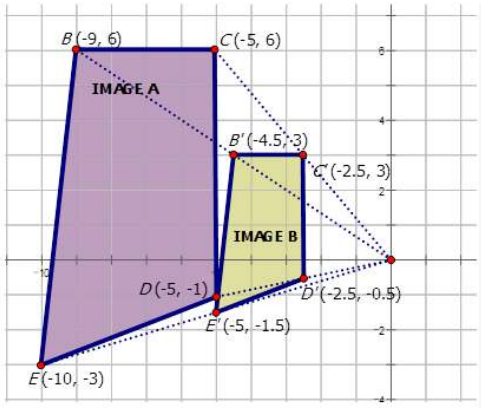

Figure **7.17.8**

Solution

Look at the points in each image:

Image *A*	$B(-9,6)$	$C(-5,6)$	$D(-5,-1)$	$E(-10,-3)$
Image *B*	$B'(-4.5,3)$	$C'(-2.5,3)$	$D'(-2.5,-0.5)$	$E'(-5,-1.5)$

Notice that the coordinate points in Image B (the dilation image) are \dfrac{1}{2}\) that found in Image A\). Therefore the Image A undergoes a dilation about the origin of scale factor \dfrac{1}{2}\). To write a mapping rule for this dilation you would write: (x,y)\rightarrow (\dfrac{1}{2}x,\dfrac{1}{2}y)\).

Example 7.17.2

Thomas describes a dilation of point JT with vertices $J(-2,6)$ to $T(6,2)$ to point $J'T'$ with vertices $J'(-4,12)$ and $T'(12,4)$. Write the notation to describe this dilation for Thomas.

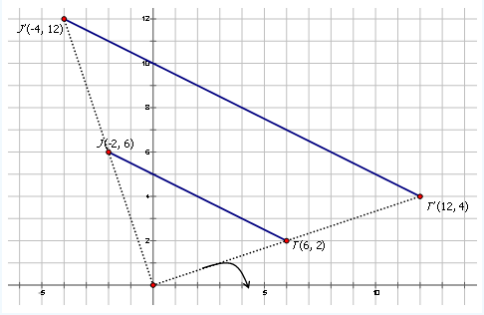

Figure **7.17.9**

Solution

Since the x- and y-coordinates are each multiplied by 2, the *scale factor* is 2. The mapping notation is: $(x,y)\rightarrow(2x,2y)$

Example 7.17.3

Given the points $A(12,8)$ and $B(8,4)$ on a line undergoing a dilation to produce $A'(6,4)$ and $B'(4,2)$, write the notation that represents the dilation.

Solution

In order to write the notation to describe the dilation, choose one point on the preimage and then the corresponding point on the dilation image to see how the point has moved. Notice that point EA\) is:

$A(12,8)\rightarrow A'(6,4)$

Since both x- and y-coordinates are multiplied by $\frac{1}{2}$, the dilation is about the origin has a scale factor of $\frac{1}{2}$. The notation for this dilation would be: $(x,y)\rightarrow(\frac{1}{2}x,\frac{1}{2}y)$.

Example 7.17.4

Janet was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.

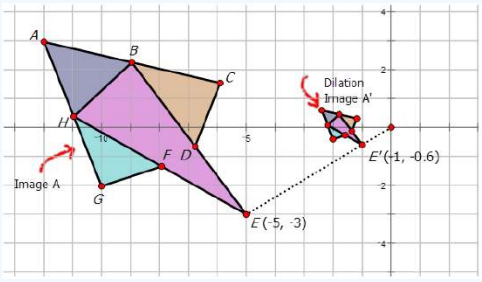

Figure 7.17.10

Solution

In order to write the notation to describe the dilation, choose one point on the preimage A and then the corresponding point on the dilation image A' to see how the point has changed. Notice that point E is shown in the diagram:

$E(-5,-3) \rightarrow E'(-1,-0.6)$

Since both x- and y-coordinates are multiplied by $\frac{1}{5}$, the dilation is about the origin has a scale factor of $\frac{1}{5}$. The notation for this dilation would be: $(x,y) \rightarrow (\frac{1}{5}x, \frac{1}{5}y)$.

Review

Complete the following table. Assume that the center of dilation is the origin.

Starting Point	D_2	D_5	$D_{\frac{1}{2}}$	$D_{\frac{3}{4}}$
1. (1, 4)				
2. (4, 2)				
3. (2, 0)				
4. (-1, 2)				
5. (-2, -3)				
6. (9, 4)				
7. (-1, 3)				
8. (-5, 2)				
9. (2, 6)				
10. (-5, 7)				

Write the notation that represents the dilation of the preimage to the image for each diagram below.

11.

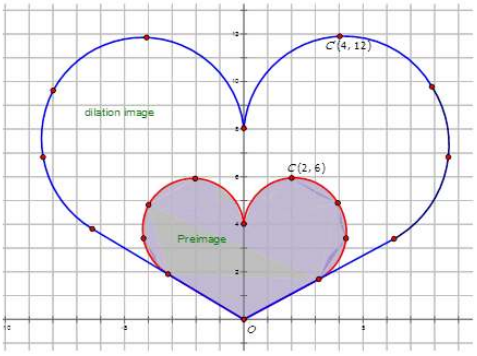

Figure 7.17.11

12.

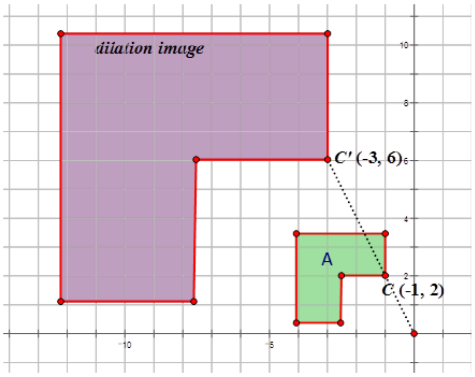

Figure 7.17.12

13.

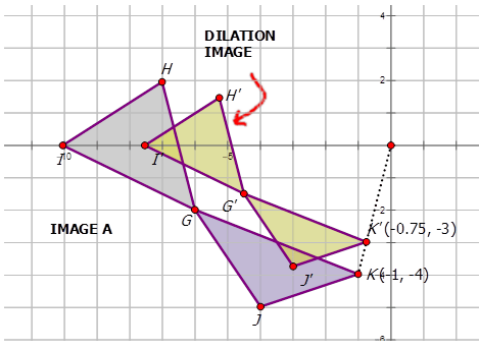

Figure 7.17.13

14.

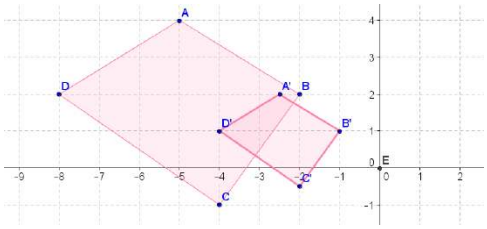

Figure 7.17.14

15.

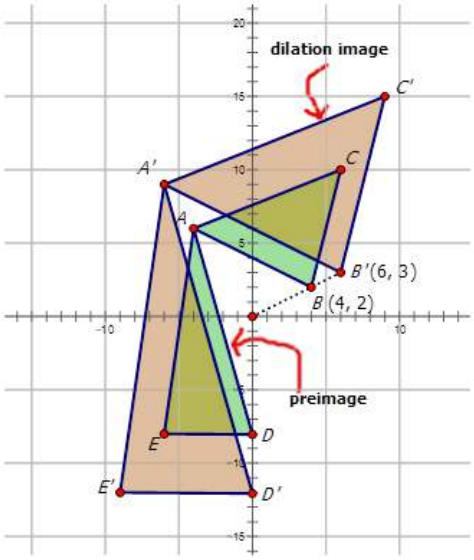

Figure **7.17.15**

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.12.

Additional Resources

Video: SAT Math (Geometry) - Practice 2.7

Practice: Mapping Dilations

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7.18: Self-Similarity and Fractals

One part of an object can be enlarged or shrunk to look like the whole object.

Self-Similarity

When one part of an object can be enlarged (or shrunk) to look like the whole object it is **self-similar**.

To explore self-similarity, we will go through some examples. Typically, each step of a process is called an **iteration**. The first level is called **Stage 0.**

What if you were given an object, like a triangle or a snowflake, in which a part of it could be enlarged (or shrunk) to look like the whole object? What would each successive iteration of that object look like?

Example 7.18.1

The Sierpinski triangle [illegible]y connecting the midpoints of the sides and shading the central triangle (Stage 1). Repeat this process for t[illegible]es in Stage 1 to get Stage 2.

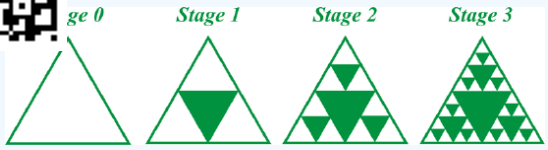

Figure 7.18.1

Example 7.18.2

Determine the number of shaded and unshaded triangles in each stage of the Sierpinkski triangle. Determine if there is a pattern.

	Stage 0	*Stage 1*	*Stage 2*	*Stage 3*
Unshaded	1	3	9	27
Shaded	0	1	4	13

Solution

The number of unshaded triangles seems to be powers of $3: 3^0, 3^1, 3^2, 3^3, \ldots$. The number of shaded triangles is the sum the number of shaded and unshaded triangles from the previous stage. For Example, the number of shaded triangles in Stage 4 would equal $27 + 13 = 40$.

Example 7.18.3

Like the Sierpinski triangle, a fractal is another self-similar object that is repeated at smaller scales. Below are the first three stages of the Koch snowflake.

Figure 7.18.2

Example 7.18.4

Determine the number of edges and the perimeter of each snowflake shown in Example 3. Assume that the length of one side of the original (stage 0) equilateral triangle is 1.

	Stage 0	*Stage 1*	*Stage 2*
Number of Edges	3	12	48
Edge Length	1	$\frac{1}{3}$	$\frac{1}{9}$
Perimeter	3	4	$\frac{48}{9} = \frac{15}{3}$

Example 7.18.5

The Cantor set is another example of a fractal. It consists of dividing a segment into thirds and then erasing the middle third.

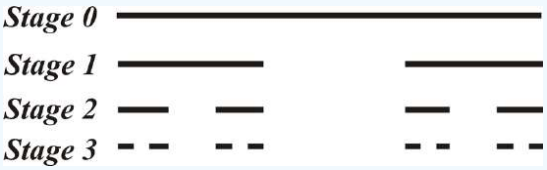

Figure 7.18.3

Review

1. Draw Stage 4 of the Cantor set.
2. Use the Cantor Set to fill in the table below.

	Number of Segments	*Length of each Segment*	*Total Length of the Segments*
Stage 0	1	1	1
Stage 1	2	$\frac{1}{3}$	$\frac{2}{3}$
Stage 2	4	$\frac{1}{9}$	$\frac{4}{9}$
Stage 3			
Stage 4			
Stage 5			

3. How many segments are in Stage n?
4. Draw Stage 3 of the Koch snowflake.
5. A variation on the Sierpinski triangle is the Sierpinski carpet, which splits a square into 9 equal squares, coloring the middle one only. Then, split the uncolored squares to get the next stage. Draw the first 3 stages of this fractal.
6. How many colored vs. uncolored square are in each stage?
7. Fractals are very common in nature. For example, a fern leaf is a fractal. As the leaves get closer to the end, they get smaller and smaller. Find three other examples of fractals in nature.

Figure **7.18.4**

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.13.

Vocabulary

Term	Definition
self-similar	When one part of an object can be enlarged (or shrunk) to look like the whole object.
Iteration	An iteration is a single step within a process.
midpoint	The midpoint of a line segment is the point on the line segment that splits the segment into two congruent parts.

Additional Resources

Video: Self-Similarity Examples - Basic

Activities: Self-Similarity Discussion Questions

Study Aids: Self-Similarity Study Guide

Practice: Self-Similarity and Fractals

Real World: Fractals Everywhere

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CHAPTER OVERVIEW

8: Rigid Transformations

8.1: Identify Transformation Types
8.2: Lines of Symmetry
8.3: Reflection Symmetry
8.4: Rotation Symmetry
8.5: Geometric Translation
8.6: Sliding Figures
8.7: Geometry Software for Translations
8.8: Translation Applications in Circle Similarity
8.9: Translation Notation
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8.12: Geometry Software and Graphing Rotations
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8.14: Rules for Reflections
8.15: Geometry Software for Reflections
8.16: Reflecting Figures
8.17: Composite Transformations
8.18: Notation for Composite Transformations
8.19: Tessellations

Thumbnail: Reflection symmetries in a hexagon (Public Domain; Jim.belk via Wikipedia)

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8.1: Identify Transformation Types

Identify transformations, translations, reflections and rotations.

Transformations in the Plane

Transformation is a process that changes the shape, size or position of a figure to create a new image. It is a function that takes points in the plane as inputs and gives other points as outputs. You can think of a transformation as a rule that tells you how to create new points.

Suppose you have a transformation F that applies a horizontal stretch factor of two to each point. Below, this transformation is applied to triangle S to create triangle S'.

- S' is considered the image of S by F.
- It is also correct to say that F maps S to S'.
- Each of the points in the image are labeled with a $'$ symbol, which is read as "prime."

This helps to show how points on S correspond to points on S'. For example, you could say that "point A maps to point A-prime."

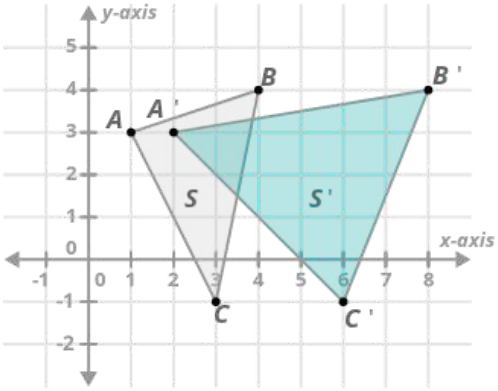

Figure 8.1.1

Some transformations preserve length and angles. Preserving length means that if a line segment is 3 units, its image will also be 3 units. Similarly, preserving angles means if an angle is $60°$, its image will also be $60°$.

- A transformation that preserves length and angles is called a rigid transformation.

Recognizing Rigid T

1. Is a horizontal stretch a d transformation?

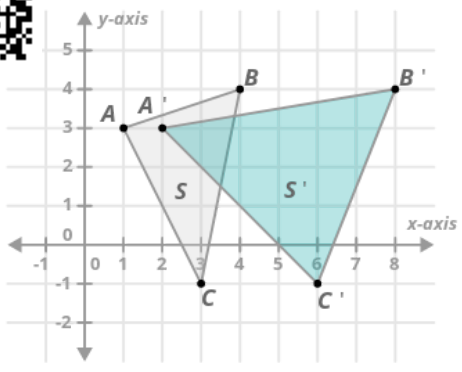

Figure 8.1.2

No. You can prove this using the picture above by showing that length is not preserved.

$$
\begin{aligned}
&\textit{The length of } \overline{AC} \\
&= \sqrt{(3-1)^2 + (-1-3)^2} \\
&= \sqrt{(2)^2 + (-4)^2} \\
&= \sqrt{4 + 16} \\
&= \sqrt{20} \\
&= 2\sqrt{5} \ \textit{units}
\end{aligned}
$$

Figure 8.1.3

$$
\begin{aligned}
&\textit{The length of } \overline{A'C'} \\
&= \sqrt{(6-2)^2 + (-1-3)^2} \\
&= \sqrt{(4)^2 + (-4)^2} \\
&= \sqrt{16 + 16} \\
&= \sqrt{32} \\
&= 4\sqrt{2} \ \textit{units}
\end{aligned}
$$

Figure 8.1.4

2. A transformation reflects points in shape K across $\overleftrightarrow{AB}$ to create shape K'. Is this reflection a rigid transformation?

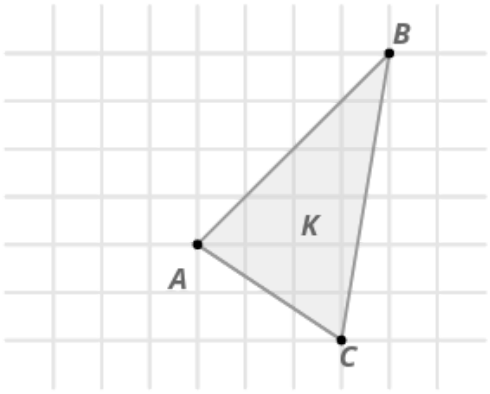

Figure 8.1.5

Yes, reflections are rigid transformations. You can verify that the distances between the points are preserved.

3. A transformation translates points in shape K along vector $\rightarrow v$ to create shape K'. Is this translation a rigid transformation?

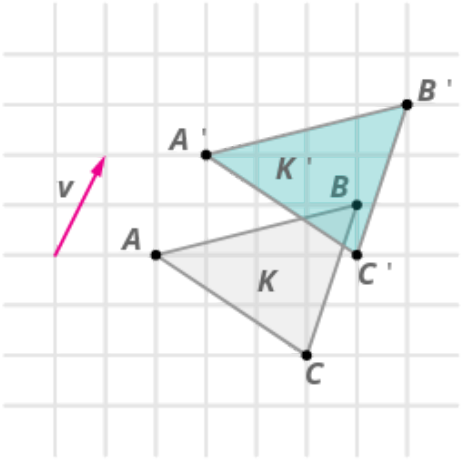

Figure **8.1.6**

Yes, translations are rigid transformations. You can verify that the distances between the points are preserved.

Example 8.1.1

You slide a book across your desk. You pour soda from a can into a big glass. Describe these actions as transformations.

Solution

Sliding a book across your desk is a rigid transformation because a book is a rigid object that does not change shape. The distances and angles that make up the book don't change once the book is in a new location. Pouring soda, on the other hand, is not a rigid transformation. Liquid is not a rigid object and it can change shape depending on its surroundings. The overall shape of the soda in the can will be different from the overall shape of the soda in the glass.

Example 8.1.2

A transformation rotates points in shape K around point D to create shape K'. Does this rotation look like a rigid transformation? Use algebra to prove your answer.

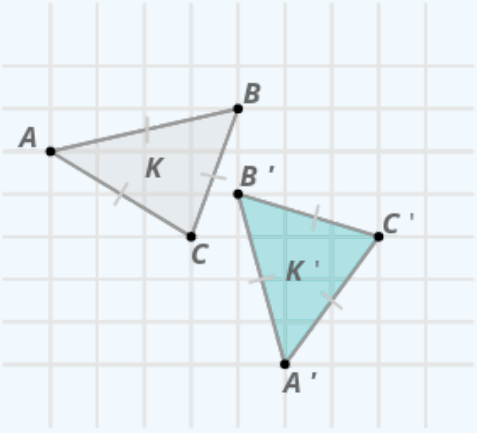

Figure **8.1.7**

Solution

This appears to be a rigid transformation.

With the Pythagorean Theorem, you can show that the corresponding sides are the same length. For example:

The length of $\overline{AC}$

$$= \sqrt{3^2 + 2^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{13} \text{ units}$$

The length of $\overline{A'C'}$

$$= \sqrt{3^2 + 2^2}$$
$$= \sqrt{9 + 4}$$
$$= \sqrt{13} \text{ units}$$

Thus it is indeed a rigid transformation.

Example 8.1.3

What makes a transformation a rigid transformation?

Solution

Rigid transformations preserve distance and angles. All corresponding sides will be the same length and all corresponding angles will be the same measure.

Review

For 1-7, define each statement as true or false and justify your answer.

1. Translations are rigid transformations.

2. Rotations are rigid transformations.

3. Horizontal stretches are rigid transformations.

4. Rigid transformations preserve location in the plane.

5. Corresponding sides in rigid transformations are the same length.

6. If it's not a rigid transformation, it's not a real transformation.

7. Reflections are rigid transformations.

Use the following image for 8-9.

Figure 8.1.8

8. Describe the transformation in your own words. Does it look like a rigid transformation?

9. Prove your answer to #8 by comparing the lengths of two sides.

Use the following image for 10-11.

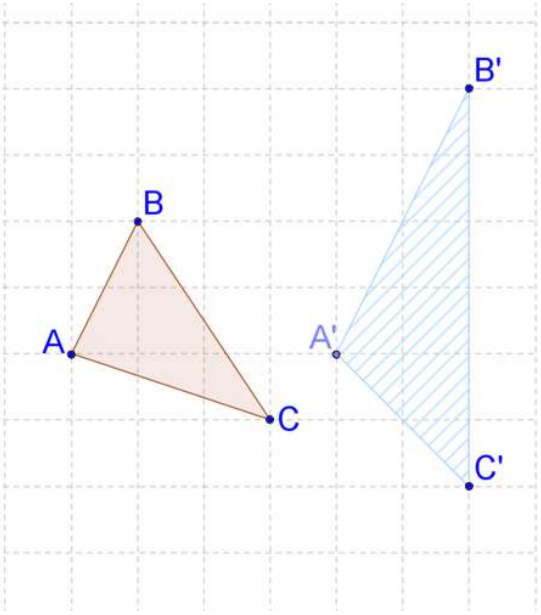

Figure **8.1.9**

10. Describe the transformation in your own words. Does it look like a rigid transformation?

11. Prove your answer to #10 by comparing the lengths of two sides.

Use the following image for 12-13.

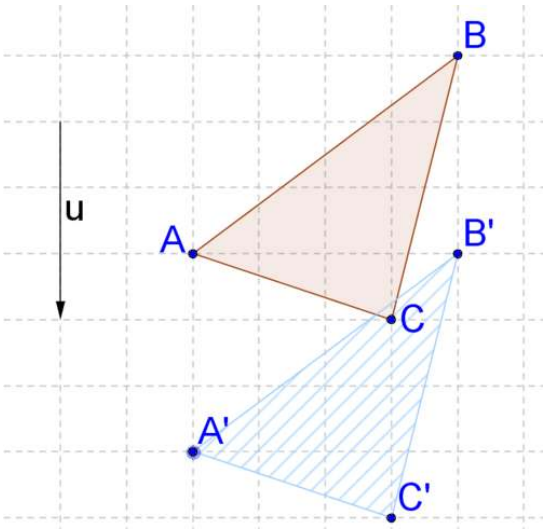

Figure **8.1.10**

12. Describe the transformation in your own words. Does it look like a rigid transformation?

13. Prove your answer to #12 by comparing the lengths of two sides.

Use the following image for 14-15.

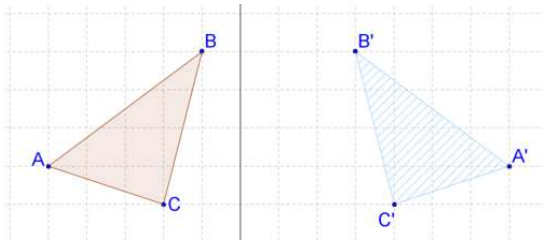

Figure **8.1.11**

14. Describe the transformation in your own words. Does it look like a rigid transformation?

15. Prove your answer to #14 by comparing the lengths of two sides.

16. A transformation can be thought of as a movement of an object in a plane such that angle measures and segment lengths are preserved. Imagine a triangle that you've cut out, lying flat on a piece of poster paper. How can the triangle be moved so that the measures of angles and the lengths of segments are preserved? How can the different types of movements be categorized and defined? Be creative. Draw, write, and discuss how to more specifically describe and define the types and methods of rigid motion transformations.

17. There is one type of rigid motion transformation which seems to require the triangle described above to be lifted from the poster board and flipped. (Mathematically, that's not what happens, but it looks that way from our 3-dimensional perspective.) How can this transformation be defined and described?

18. Sometimes we can describe transformations in the coordinate plane. For example, one can specifically alter the coordinates of the vertices of a polygon according to a rule. Think about how you can modify the coordinates of a point and what the consequences of each type of modification would be. Experiment. Draw a polygon on a coordinate plane, and alter the coordinates according to different rules, then draw the resulting images. Write about and discuss your conclusions.

19. Define and describe the transformations that map A to A' to A'' to A''' below.

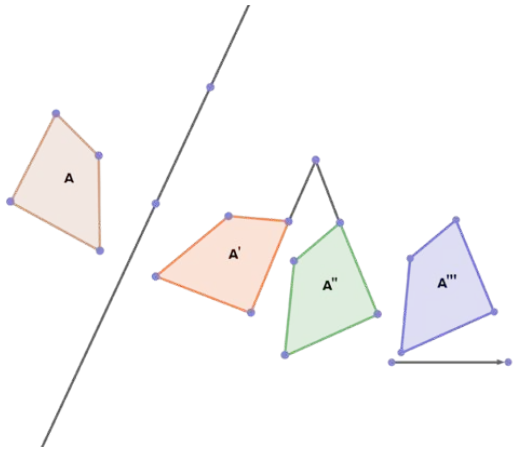

Figure **8.1.12**

20. Some of the images below represent rigid motion transformations of polygon B and some do not. Decide which do and which don't, and explain your choices. Define and describe the ones that do.

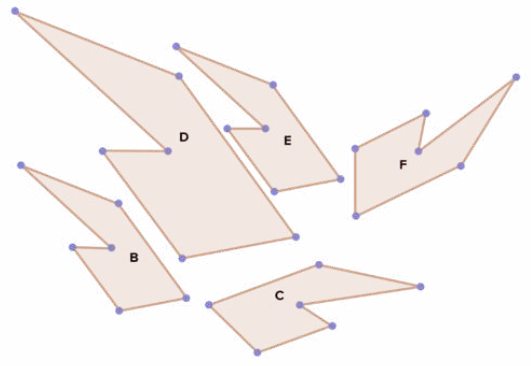

Figure **8.1.13**

Review (Answers)

To see the Review answers, click here.

Additional Resources

Practice: Identify Transformation Types

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8.2: Lines of Symmetry

Identify existence and number of lines of symmetry.

Identify Lines of Symmetry

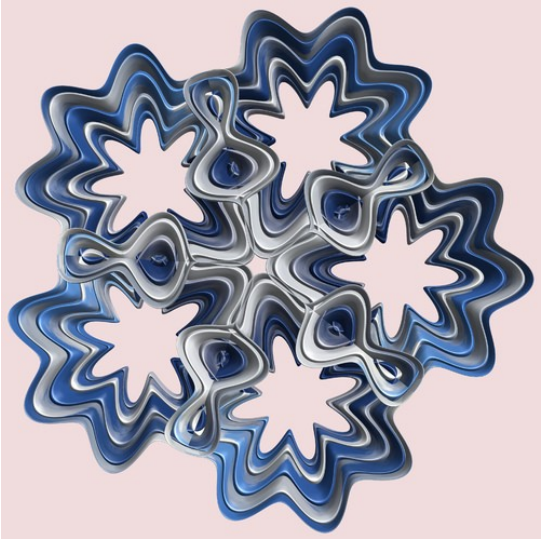


Figure **8.2.1**

Symmetry is the key to the balance of a design or structure.

What is symmetry? How does an image have symmetry? What would you have to do to be sure that an image or construction was symmetrical?

In this concept, you will learn to identify lines of symmetry.

Symmetry

Sometimes, a figure will have parts that mirror themselves within one object. In this case, parts of the object match other parts of the picture. This is called **symmetry**.

Let's look at an example.

Figure **8.2.2**

Look at this heart. It has two sides that match. The heart is symmetrical because there is symmetry in its design. This heart can be divided in half vertically where one half matches the other half.

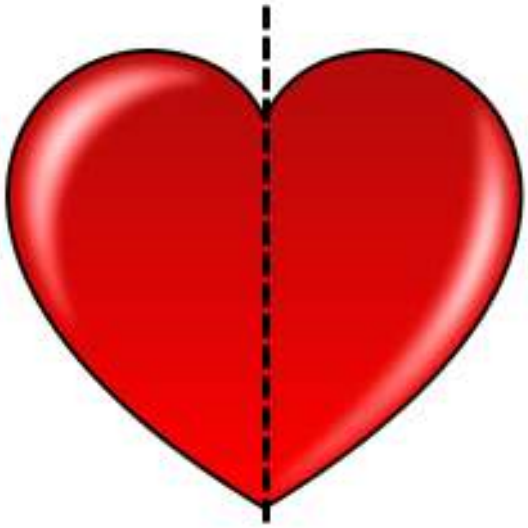


Figure 8.2.3

This line that divides the heart into matching parts is called the **line of symmetry**.

You can determine other **lines of symmetry** by looking at other objects.

Let's look at another example.

Figure 8.2.4

Look at this cross. It has two lines of symmetry. If you look, the cross can be divided in half perfectly vertically and in half horizontally.

Figure 8.2.5

This means that there are two lines of symmetry in the cross.

Example 8.2.1

Earlier, you were given a problem about questions on symmetry.

1. What is symmetry?
2. How does an image have symmetry?
3. What would you have to do to be sure that your image was symmetrical?

Solution

1. Symmetry is when two halves of an object match. In other words, you can divide the object into parts and the parts are congruent. A heart is a symmetrical object, so is the letter A.
2. An image has symmetry because it can be divided in half so that one half of the image matches the other half.
3. For an image to have a line of symmetry the image must be able to fold alone the line of symmetry so that the two parts are mirror images of each other. The folded line would be the line of symmetry. An image that is symmetrical would have a line of symmetry such as a square (four lines of symmetry), a circle (infinite lines of symmetry) or a butterfly (one line of symmetry).

Example 8.2.2

Does this figure have symmetry? Can it be a **reflection**?

Figure **8.2.6**

Solution

First, try to divide this butterfly evenly so that one side can reflect the other. Can you draw a line of symmetry?

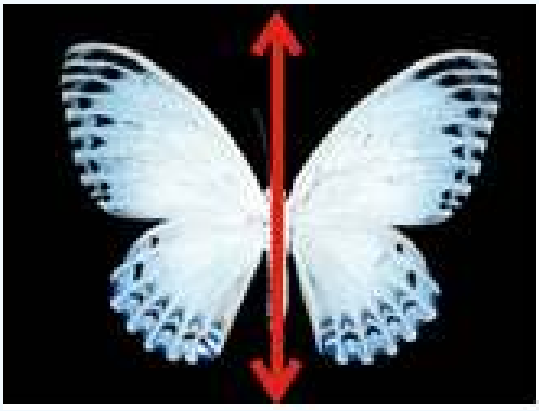


Figure **8.2.7**

The answer is yes.

Therefore, it has symmetry.

Example 8.2.3

Does the figure below have symmetry? Can it be a reflection?

Figure 8.2.8

Solution

First, try to divide this flower evenly so that one side can reflect the other. Can you draw a line of symmetry?

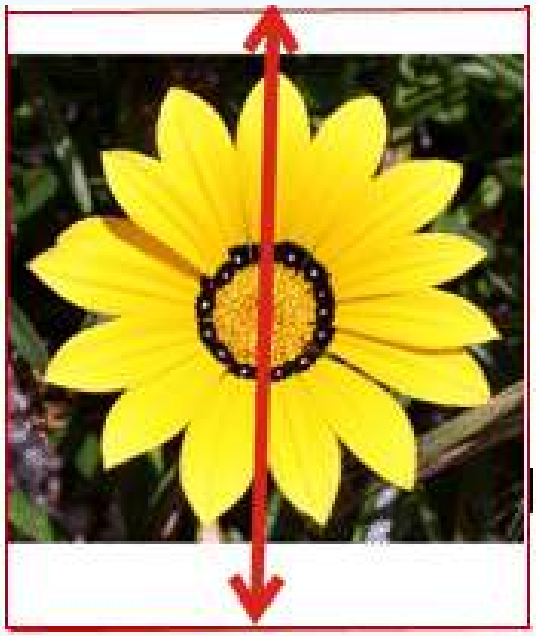


Figure 8.2.9

The answer is yes.

Therefore, it has symmetry.

Example 8.2.4

How many lines of symmetry does this figure have?

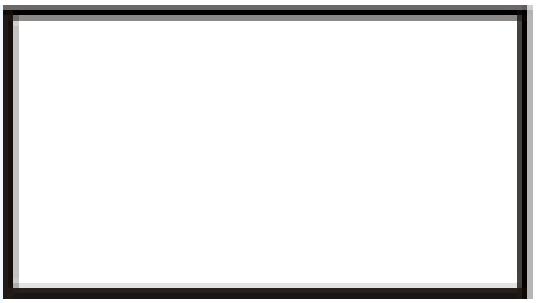


Figure 8.2.10

Solution

First, try to divide this rectangle evenly so that one side can reflect the other. Can you draw a line of symmetry?

Figure 8.2.11

The answer is yes. Therefore, a rectangle has two lines of symmetry.

Example 8.2.5

Do these figures have symmetry?

Figure 8.2.12

Solution

First, try to divide these images evenly so that one side can reflect the other. Can you draw a line of symmetry for any of these images?

The answer is no.

Therefore, none of these images have a line of symmetry.

Review

Use the illustration to answer each question.

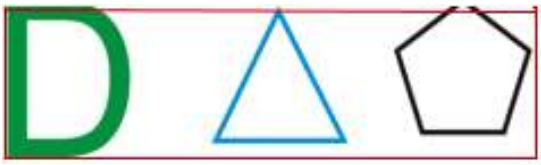


Figure 8.2.13

1. Do these figures have symmetry?
2. Can they be reflections?
3. How many lines of symmetry does each figure have?

Find all lines of symmetry for the shapes below.

4.

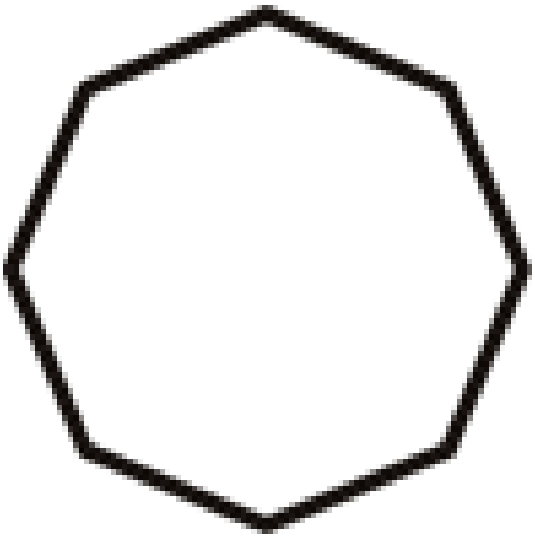


Figure **8.2.14**

5.

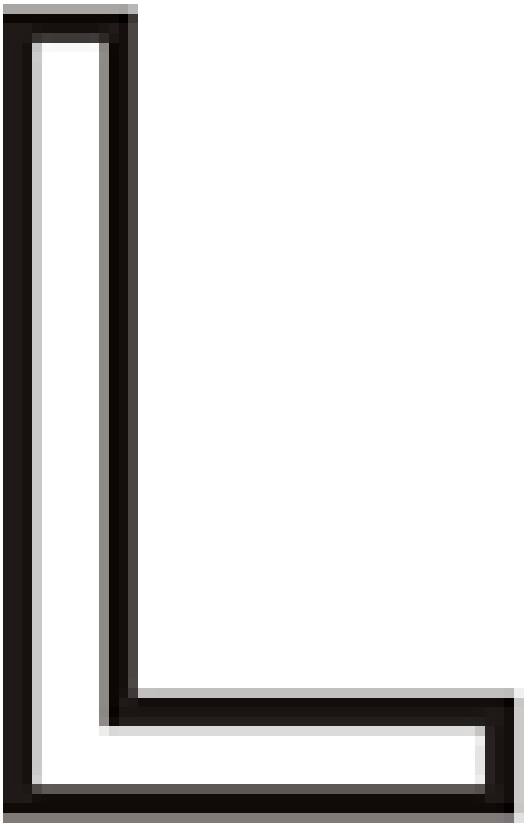
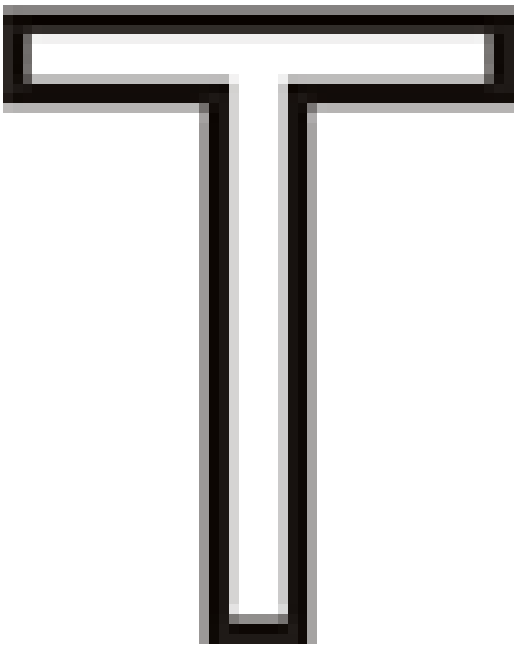


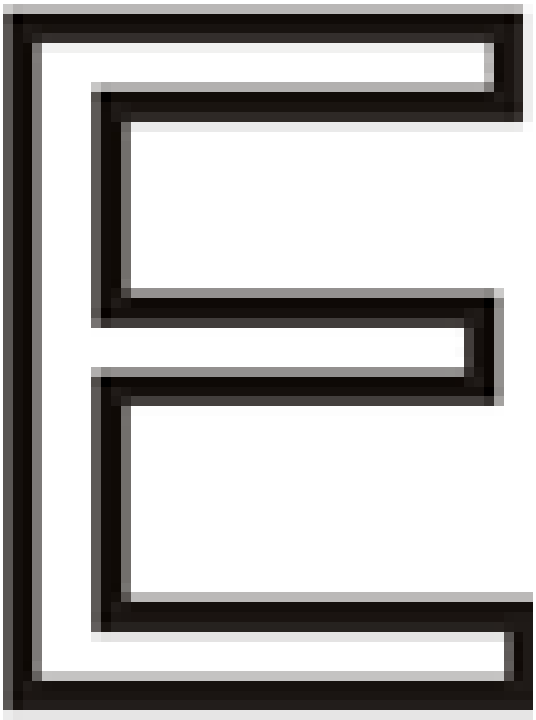
Figure **8.2.15**



6.

Figure **8.2.16**

Name the number of lines of symmetry for each letter.



7.

Figure **8.2.17**

8.

Figure **8.2.18**

9.

Figure **8.2.19**

10.

Figure 8.2.20

11.

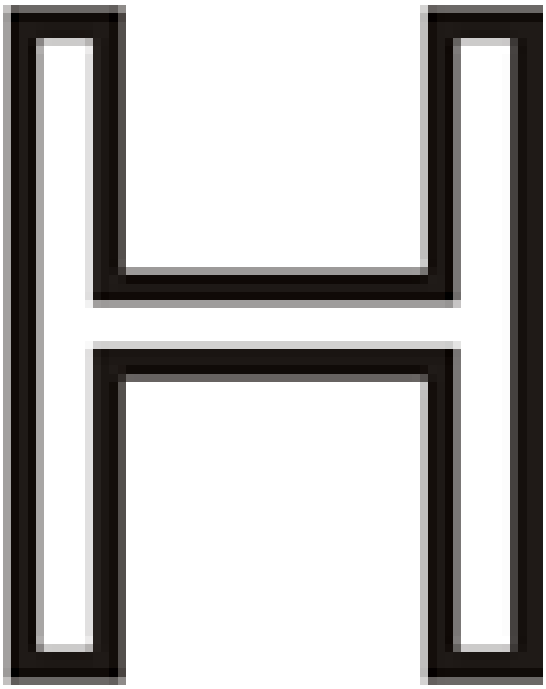


Figure 8.2.21

Answer each question true or false.

12. All triangles have symmetry.

13. All circles have symmetry.

14. The letter "x" has two lines of symmetry.

15. The letter "s" has two lines of symmetry.

Review (Answers)

To see the Review answers, open this PDF file and look for section 6.13.

Resources

Vocabulary

Term	Definition
Lines of Symmetry	Lines of symmetry are the lines that can be drawn to divide a figure into equal halves.
Reflection	A reflection is a transformation that flips a figure on the coordinate plane across a given line without changing the shape or size of the figure.
Symmetry	A figure has symmetry if it can be transformed and still look the same.

Additional Resources

Video: Rotational Symmetry Tutorial

Practice: Lines of Symmetry

Real World: The Balance of Nature

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8.3: Reflection Symmetry

Property of a shape with one or more lines of symmetry.

A **line of symmetry** is a line that passes through a figure such that it splits the figure into two **congruent** halves such that if one half were folded across the line of **symmetry** it would land directly on top of the other half.

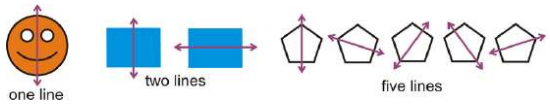


Figure **8.3.1**

Reflection symmetry is present when a figure has one or more lines of symmetry. These figures have reflection symmetry:

Figure **8.3.2**

These figures do not have reflection symmetry:

Figure **8.3.3**

What if you had a six-pointed star, you drew a line down it, and then you folded it along that line? If the two sides of the star lined up, what would that mean about the line?

Example 8.3.1

Find all lines of symmet ow.

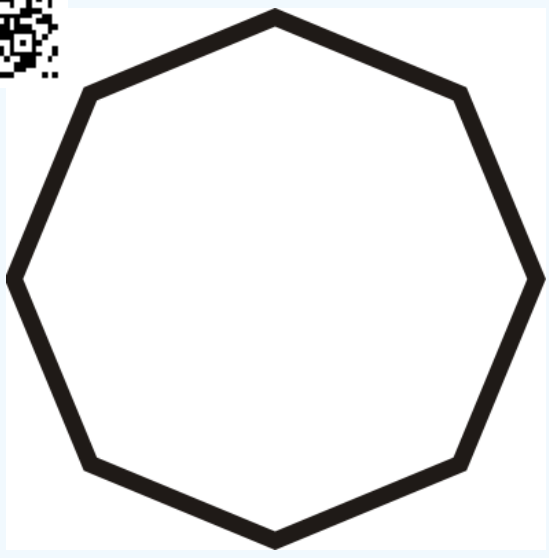


Figure **8.3.4**

Solution

Draw lines through the figure so that the lines perfectly cut the figure in half. This figure has eight lines of symmetry.

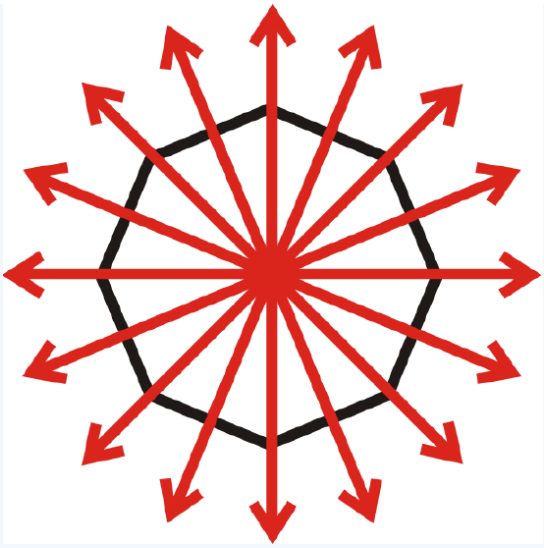


Figure 8.3.5

Example 8.3.2

Find all lines of symmetry for the shape below.

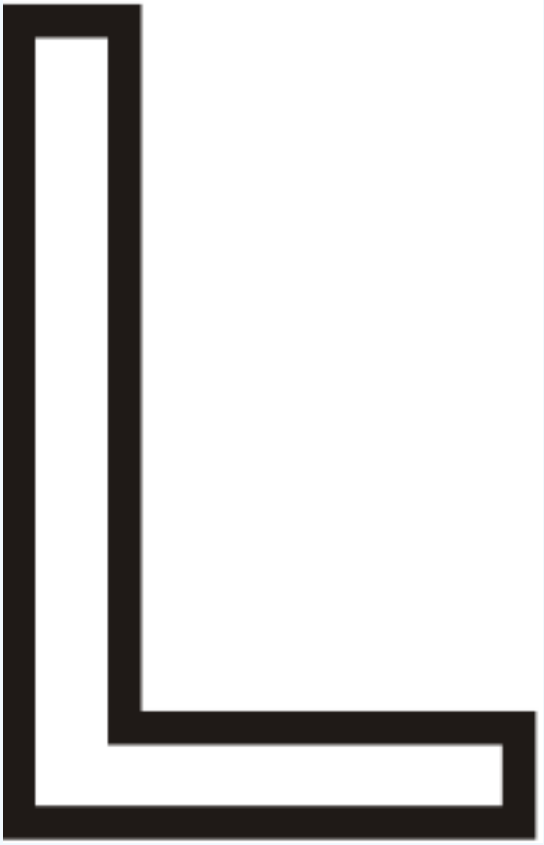


Figure 8.3.6

Solution

You cannot draw lines through the figure so that the lines perfectly cut the figure in half, so this figure has no lines of symmetry.

Example 8.3.3

Find all lines of symmetry for the shape below.

Figure 8.3.7

Solution

This figure has two lines of symmetry.

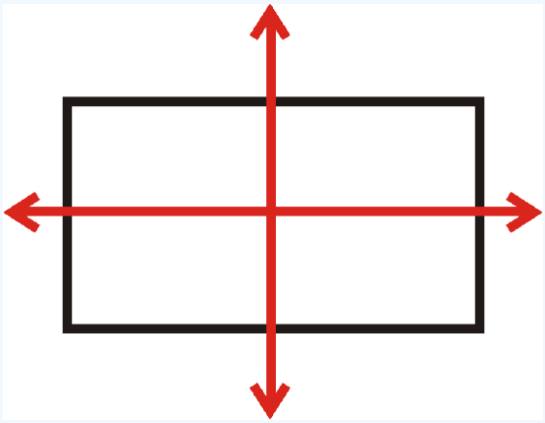


Figure 8.3.8

Example 8.3.4

Does the figure below have reflection symmetry?

Figure 8.3.9

Solution

Yes, this figure has reflection symmetry.

Figure 8.3.10

Example 8.3.5

Does the figure below have reflection symmetry?

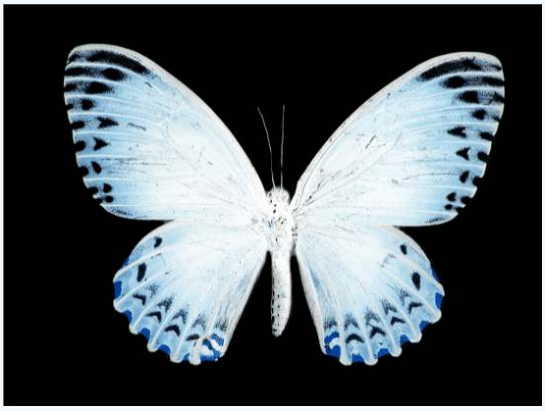


Figure 8.3.11

Solution

Yes, this figure has reflection symmetry.

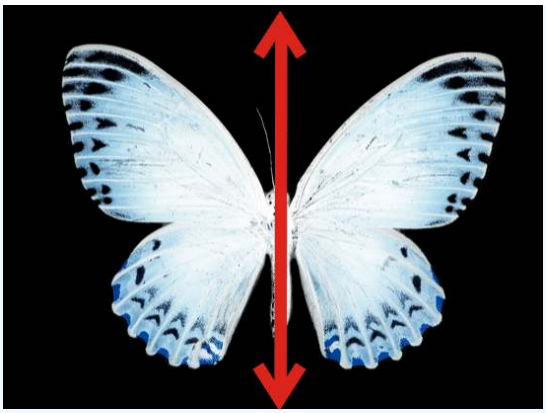


Figure 8.3.12

Review

Determine whether each statement is true or false.

1. All right triangles have **line symmetry**.
2. All isosceles triangles have line symmetry.
3. Every rectangle has line symmetry.
4. Every rectangle has exactly two lines of symmetry.
5. Every parallelogram has line symmetry.
6. Every square has exactly two lines of symmetry.
7. Every regular polygon has three lines of symmetry.
8. Every sector of a circle has a line of symmetry.

Draw the following figures.

9. A quadrilateral that has two pairs of congruent sides and exactly one line of symmetry.
10. A figure with infinitely many lines of symmetry.

Find all lines of symmetry for the letters below.

11.

Figure 8.3.13

12.

Figure 8.3.14

13.

Figure **8.3.15**

14.

Figure **8.3.16**

15.

Figure **8.3.17**

Determine if the words below have reflection symmetry.

16. **OHIO**
17. **MOW**
18. **WOW**
19. **KICK**
20. **pod**

Trace each figure and then draw in all lines of symmetry.

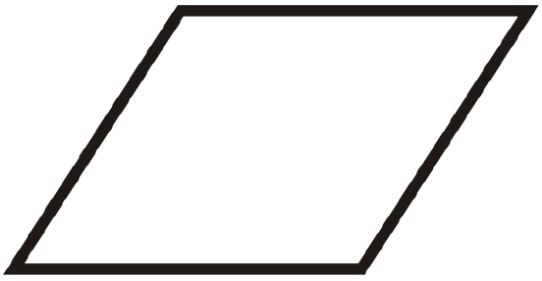
21. 

Figure **8.3.18**

22.

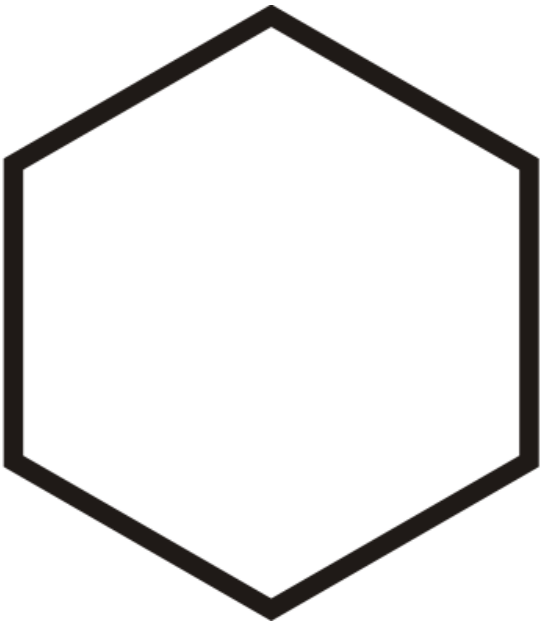


Figure 8.3.19

23.

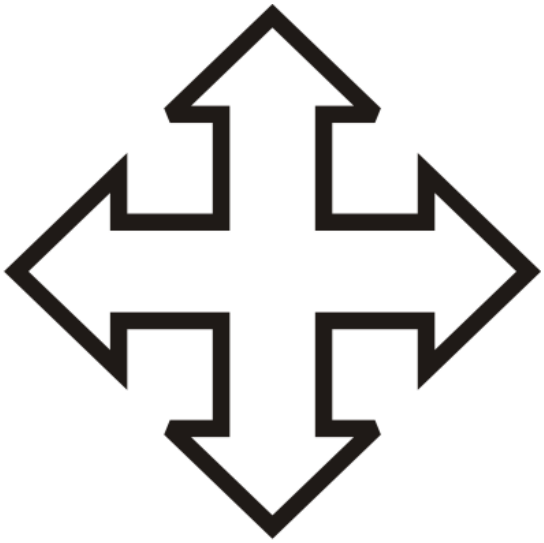


Figure 8.3.20

Determine if the figures below have reflection symmetry. Identify all lines of symmetry.

24.

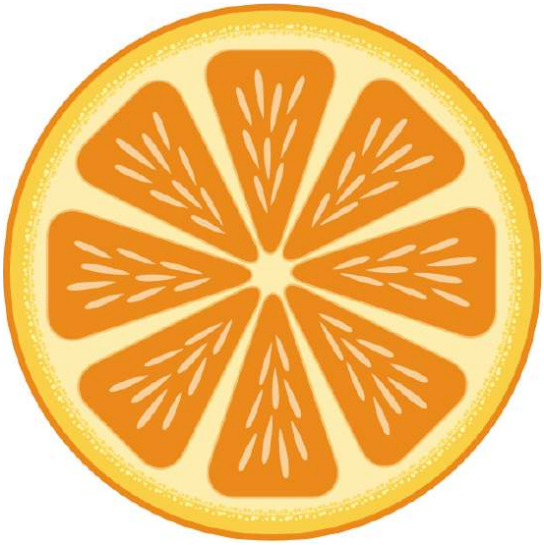


Figure 8.3.13

25.

Figure 8.3.13

26.

Figure 8.3.13

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.1.

Vocabulary

Term	Definition
Line of Symmetry	A line of symmetry is a line that can be drawn to divide a figure into equal halves.
Congruent	Congruent figures are identical in size, shape and measure.
Isosceles Triangle	An isosceles triangle is a triangle in which exactly two sides are the same length.
Line Symmetry	A figure has line symmetry or reflection symmetry when it can be divided into equal halves that match.
reflection symmetry	A figure has reflection symmetry if it can be reflected across a line and look exactly the same as it did before the reflection.
Symmetry	A figure has symmetry if it can be transformed and still look the same.

Additional Resources

Video: Reflection Symmetry Principles - Basic

Activities: Reflection Symmetry Discussion Questions

Study Aids: Symmetry and Tessellations Study Guide

Practice: Reflection Symmetry

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8.4: Rotation Symmetry

Rotation less than 360 degrees that carries a shape onto itself.

Rotational symmetry is present when a figure can be rotated (less than $360°$) such that it looks like it did before the **rotation**. The **center of rotation** is the point a figure is rotated around such that the rotational **symmetry** holds.

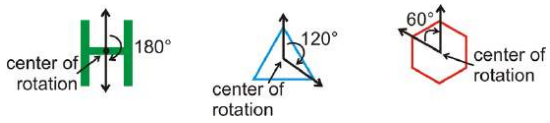

Figure 8.4.1

For the H, we can rotate it twice, the triangle can be rotated 3 times and still look the same and the hexagon can be rotated 6 times.

What if you had a six-pointed star and you rotated that star less than $360°$? If the rotated star looked exactly the same as the original star, what would that say about the star?

Example 8.4.1

Determine if the figure l l symmetry. Find the angle and how many times it can be rotated.

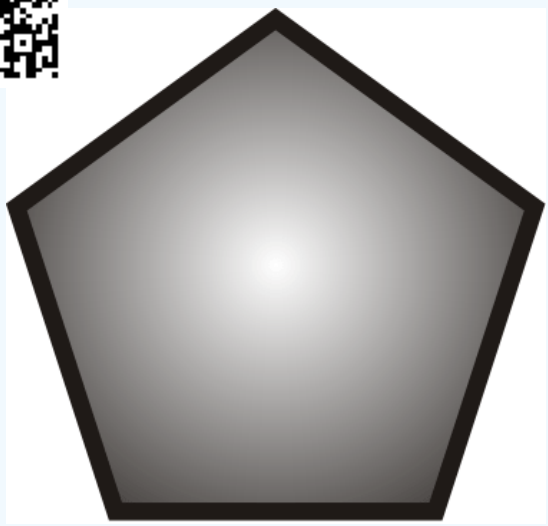

Figure 8.4.2

Solution

The pentagon can be rotated 5 times. Because there are 5 lines of rotational symmetry, the angle would be $\frac{360°}{5} = 72°$.

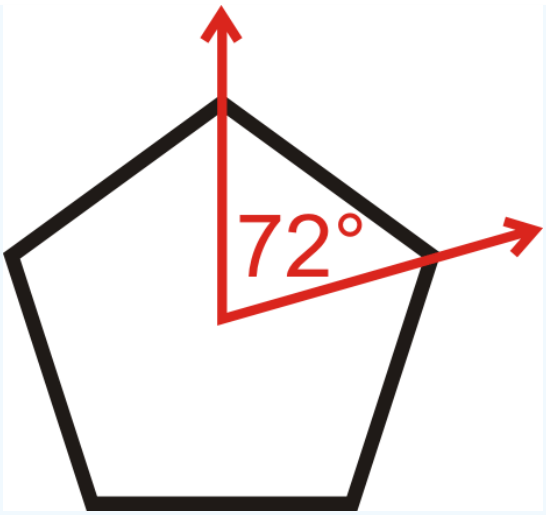

Figure 8.4.3

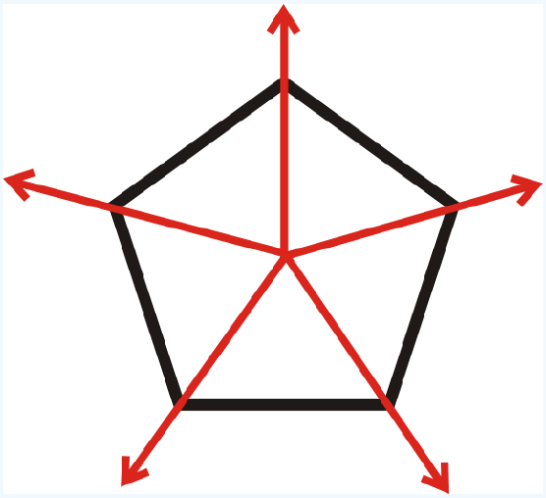

Figure 8.4.4

Example 8.4.2

Determine if the figure below has rotational symmetry. Find the angle and how many times it can be rotated.

Figure 8.4.5

Solution

The N can be rotated twice. This means the angle of rotation is 180°.

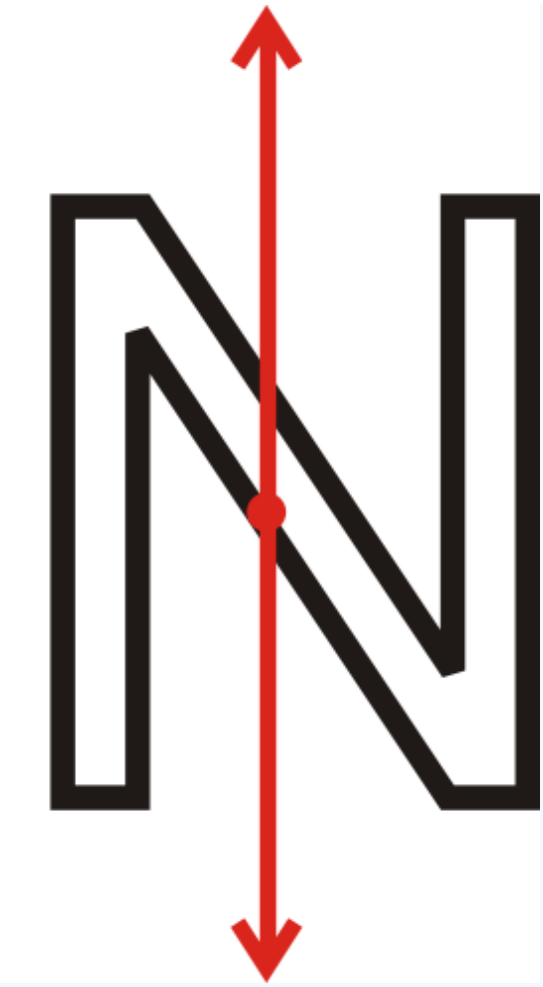

Figure 8.4.6

Example 8.4.3

Determine if the figure below has rotational symmetry. Find the angle and how many times it can be rotated.

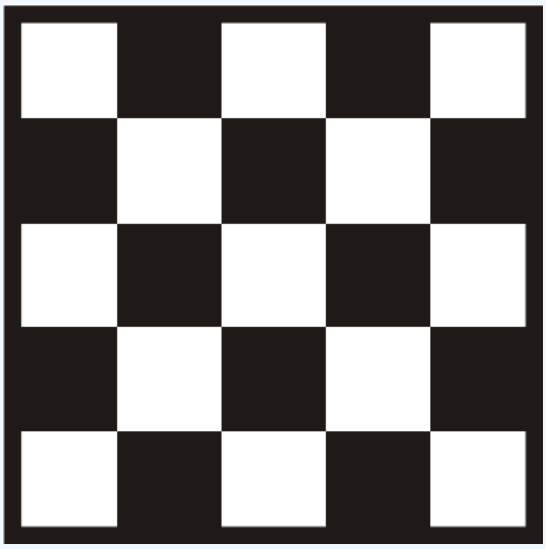

Figure 8.4.7

Solution

The checkerboard can be rotated 4 times. There are 4 lines of rotational symmetry, so the angle of rotation is $\frac{360^\circ}{4} = 90^\circ$.

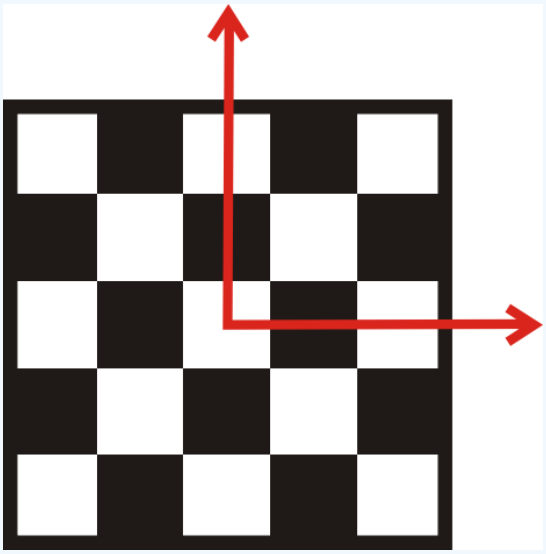

Figure 8.4.8

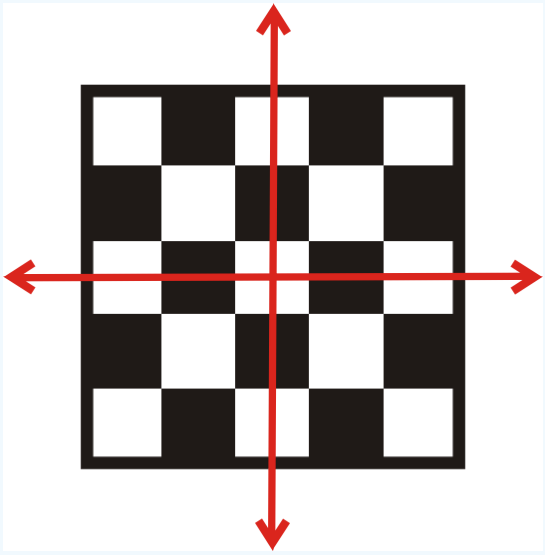

Figure 8.4.9

Example 8.4.4

Find the angle of rotation and the number of times each figure can rotate.

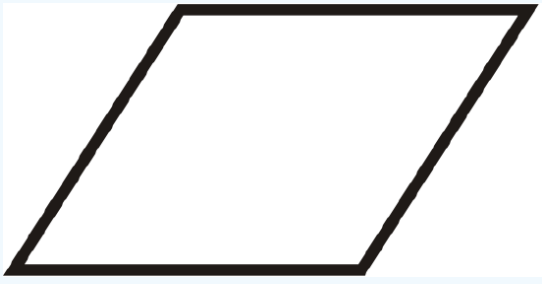

Figure 8.4.10

Solution

The parallelogram can be rotated twice. The angle of rotation is 180°.

Example 8.4.5

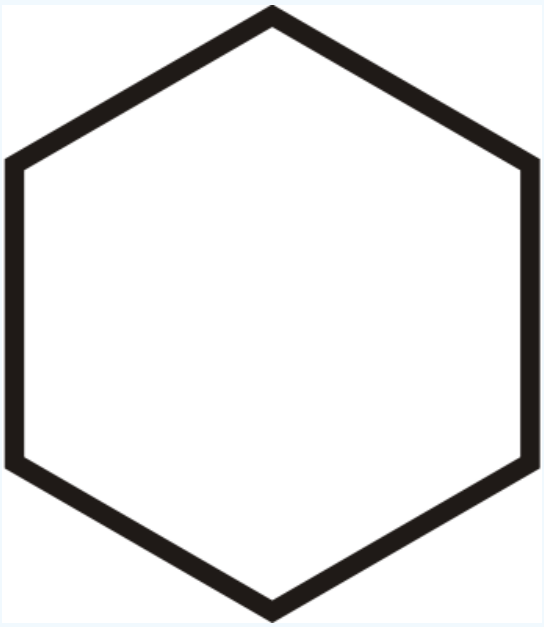

Figure 8.4.11

Solution

The hexagon can be rotated six times. The angle of rotation is $60°$.

Review

1. If a figure has 3 lines of rotational symmetry, it can be rotated _____ times.
2. If a figure can be rotated 6 times, it has _____ lines of rotational symmetry.
3. If a figure can be rotated n times, it has _____ lines of rotational symmetry.
4. To find the angle of rotation, divide $360°$ by the total number of _____.
5. Every square has an angle of rotation of _____.

Determine whether each statement is true or false.

6. Every parallelogram has rotational symmetry.
7. Every figure that has line symmetry also has rotational symmetry.

Determine whether the words below have rotation symmetry.

8. **OHIO**
9. **MOW**
10. **WOW**
11. **KICK**
12. **pod**

Find the angle of rotation and the number of times each figure can rotate.

13.

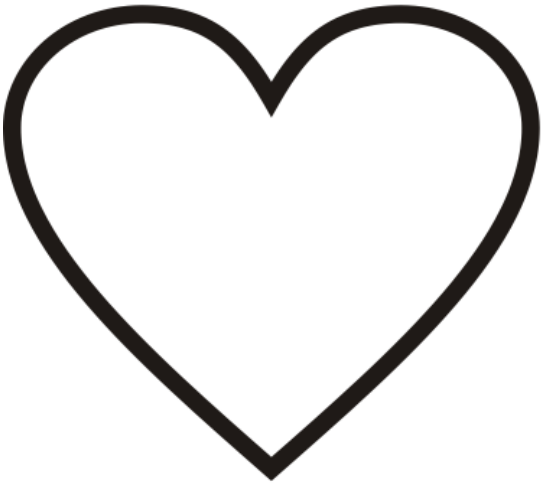

Figure 8.4.12

14.

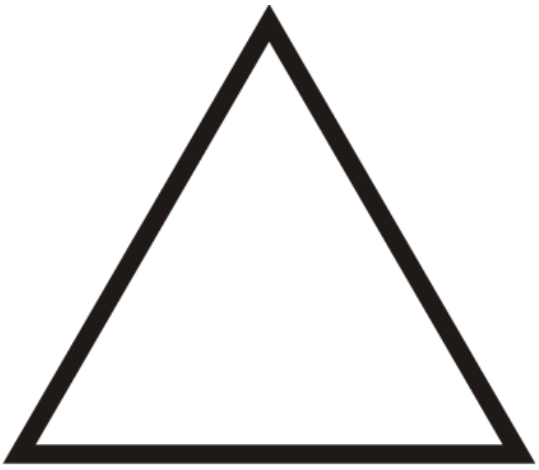

Figure 8.4.13

15.

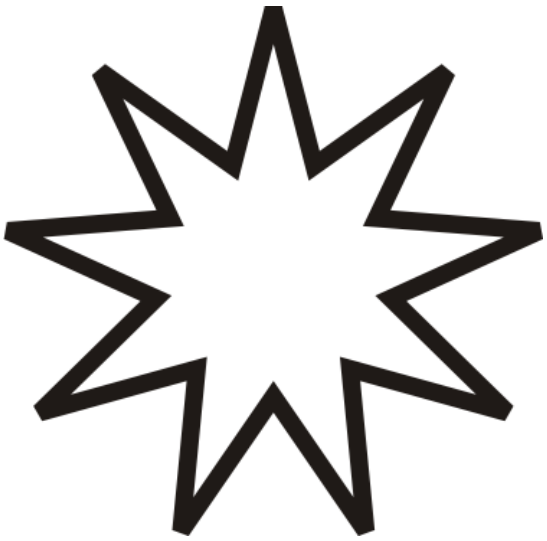

Figure 8.4.14

Determine if the figures below have rotation symmetry. Identify the angle of rotation.

16.

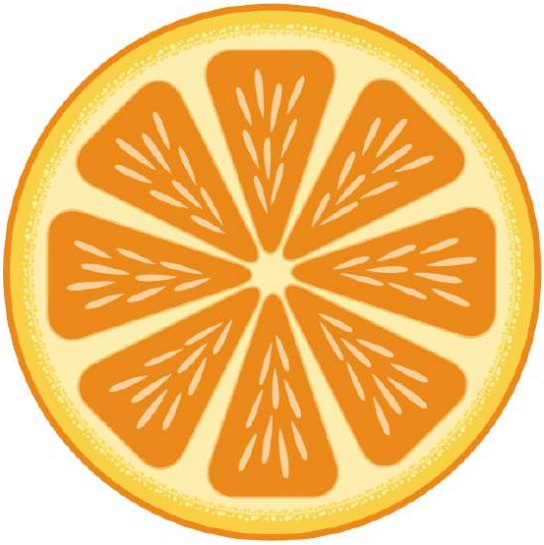

Figure 8.4.15

17.

Figure 8.4.16

18.

Figure 8.4.17

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.2.

Vocabulary

Term	Definition
rotational symmetry	When a figure can be rotated (less than 360^{\circ}\)) such that it looks like it did before the rotation. The ***center of rotation*** is the point a figure is rotated around such that the rotational symmetry holds.
Center of Rotation	In a rotation, the center of rotation is the point that does not move. The rest of the plane rotates around this fixed point.
Rotation	A rotation is a transformation that turns a figure on the coordinate plane a certain number of degrees about a given point without changing the shape or size of the figure.
Rotation Symmetry	A figure has rotational symmetry if it can be rotated less than **360°** around its center point and look exactly the same as it did before the rotation.
Symmetry	A figure has symmetry if it can be transformed and still look the same.

Additional Resources

Video: Rotation Symmetry Principles - Basic

Activities: Rotation Symmetry Discussion Questions

Study Aids: Symmetry and Tessellations Study Guide

Practice: Rotation Symmetry

Real World: This End Up

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8.5: Geometric Translation

Understand translations as movement of every point in a figure the same distance in the same direction. Graph images given preimage and translation.

Translations

A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change the size or shape of a figure.

The rigid transformations are **translations**, reflections, and rotations. The new figure created by a transformation is called the **image**. The original figure is called the **preimage**. If the preimage is A, then the image would be A', said "a prime." If there is an image of A', that would be labeled A'', said "a double prime."

A **translation** is a transformation that moves every point in a figure the same distance in the same direction. For example, this transformation moves the parallelogram to the right 5 units and up 3 units. It is written $(x, y) \rightarrow (x + 5, y + 3)$.

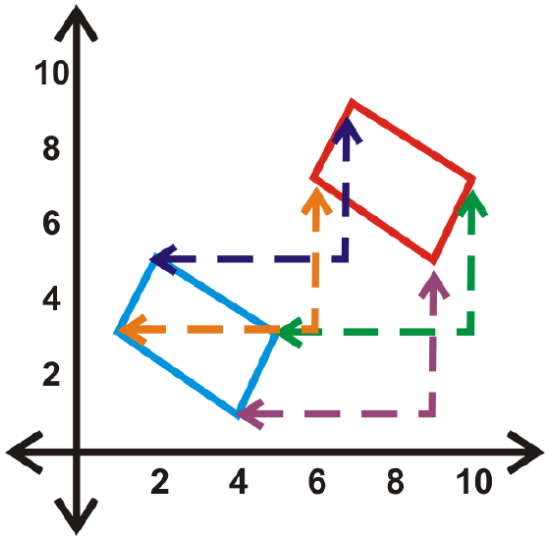

Figure 8.5.1

What if you were given the coordinates of a quadrilateral and you were asked to move that quadrilateral 3 units to the left and 2 units down? What would its new coordinates be?

Example 8.5.1

Triangle ΔABC has coo $A(-1, 4)$, $B(7, -5)$ and $C(-2, -2)$. Translate ΔABC to the left 4 units and up 5 units. Determine the coordinates of $\Delta A'B'C'$.

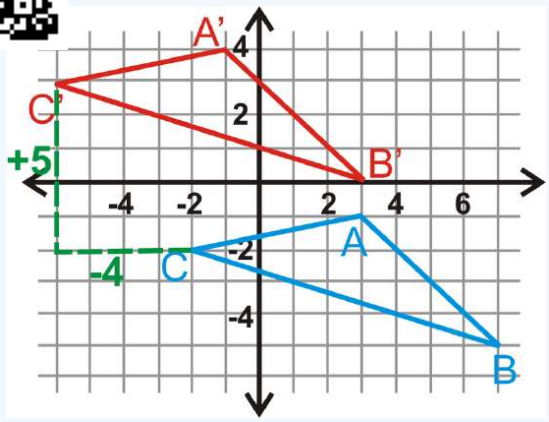

Figure 8.5.2

Solution

Graph ΔABC. To translate ΔABC, subtract 4 from each x value and add 5 to each y value of its coordinates.

$$A(3,-1) \rightarrow (3-4,-1+5) = A'(-1,4)$$
$$B(7,-5) \rightarrow (7-4,-5+5) = B'(3,0)$$
$$C(-2,-2) \rightarrow (-2-4,-2+5) = C'(-6,3)$$

The rule would be $(x,y) \rightarrow (x-4, y+5)$.

Example 8.5.2

Using the translation $(x,y) \rightarrow (x+2, y-5)$, what is the image of $A(-6,3)$?

Solution

$A'(-4,-2)$

Example 8.5.3

Graph square $S(1,2)$, $Q(4,1)$, $R(5,4)$ and $E(2,5)$. Find the image after the translation $(x,y) \rightarrow (x-2, y+3)$. Then, graph and label the image.

Solution

We are going to move the square to the left 2 and up 3.

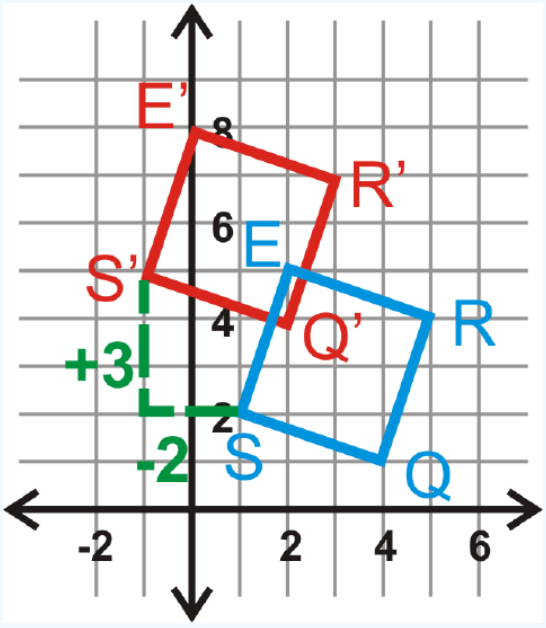

Figure 8.5.3

$$(x,y) \rightarrow (x-2, y+3)$$
$$S(1,2) \rightarrow S'(-1,5)$$
$$Q(4,1) \rightarrow Q'(2,4)$$
$$R(5,4) \rightarrow R'(3,7)$$
$$E(2,5) \rightarrow E'(0,8)$$

Example 8.5.4

Find the translation rule for ΔTRI to $\Delta T'R'I'$.

Solution

Look at the movement from T to T'. The translation rule is $(x,y) \rightarrow (x+6, y-4)$.

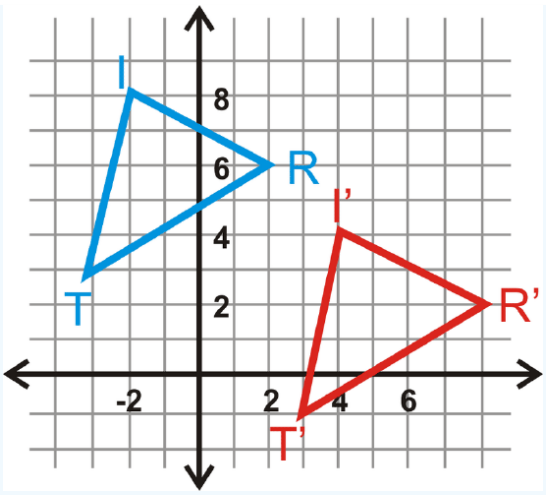

Figure 8.5.4

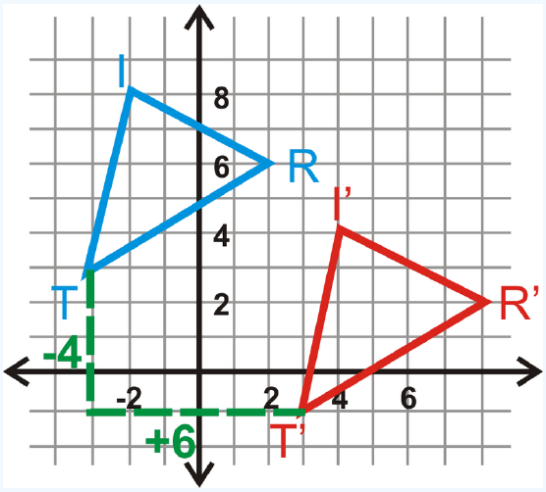

Figure 8.5.5

Review

Use the translation $(x, y) \rightarrow (x + 5, y - 9)$ for questions 1-7.

1. What is the image of $A(-1, 3)$?
2. What is the image of $B(2, 5)$?
3. What is the image of $C(4, -2)$?
4. What is the image of A'?
5. What is the preimage of $D'(12, 7)$?
6. What is the image of A''?
7. Plot A, A', A'', and A''' from the questions above. What do you notice?

The vertices of ΔABC are $A(-6, -7)$, $B(-3, -10)$ and $C(-5, 2)$. Find the vertices of $\Delta A'B'C'$, given the translation rules below.

8. $(x, y) \rightarrow (x - 2, y - 7)$
9. $(x, y) \rightarrow (x + 11, y + 4)$
10. $(x, y) \rightarrow (x, y - 3)$
11. $(x, y) \rightarrow (x - 5, y + 8)$
12. $(x, y) \rightarrow (x + 1, y)$
13. $(x, y) \rightarrow (x + 3, y + 10)$

In questions 14-17, $\Delta A'B'C'$ is the image of ΔABC. Write the translation rule.

14.

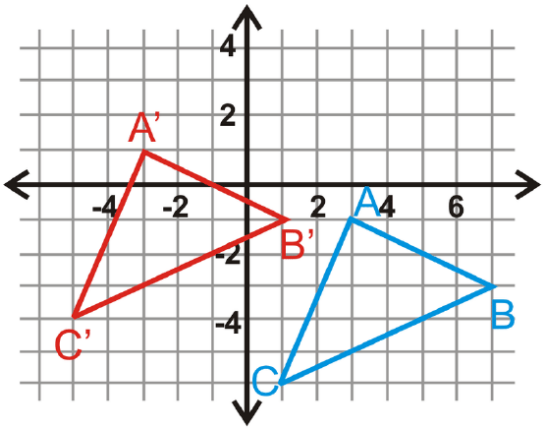

Figure 8.5.6

15.

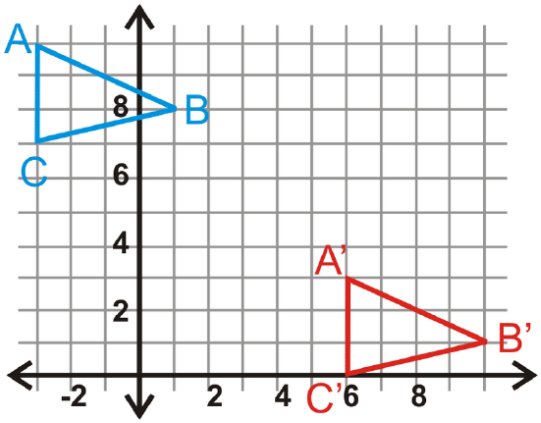

Figure 8.5.7

16.

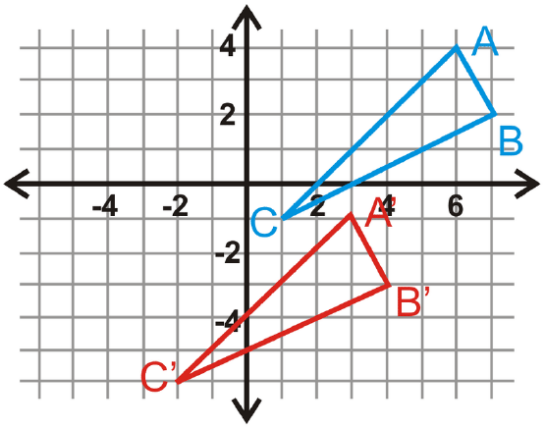

Figure 8.5.8

17.

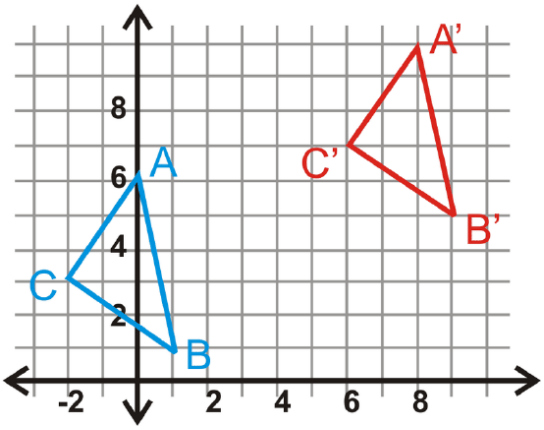

Figure **8.5.9**

Use the triangles from #17 to answer questions 18-20.

18. Find the lengths of all the sides of ΔABC.
19. Find the lengths of all the sides of $\Delta A'B'C'$.
20. What can you say about ΔABC and $\Delta A'B'C'$? Can you say this for *any* translation?
21. If $\Delta A'B'C'$ was the *preimage* and ΔABC was the image, write the translation rule for #14.
22. If $\Delta A'B'C'$ was the *preimage* and ΔABC was the image, write the translation rule for #15.
23. Find the translation rule that would move A to $A'(0, 0)$, for #16.
24. The coordinates of ΔDEF are $D(4, -2)$, $E(7, -4)$ and $F(5, 3)$. Translate ΔDEF to the right 5 units and up 11 units. Write the translation rule.
25. The coordinates of quadrilateral $QUAD$ are $Q(-6, 1)$, $U(-3, 7)$, $A(4, -2)$ and $D(1, -8)$. Translate $QUAD$ to the left 3 units and down 7 units. Write the translation rule.

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.3.

Additional Resources

Video: Transformation: Translation Principles - Basic

Activities: Translations Discussion Questions

Study Aids: Types of Transformations Study Guide

Practice: Geometric Translation

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8.6: Sliding Figures

Use coordinates to move figures on the coordinate plane.

Transformation Classification

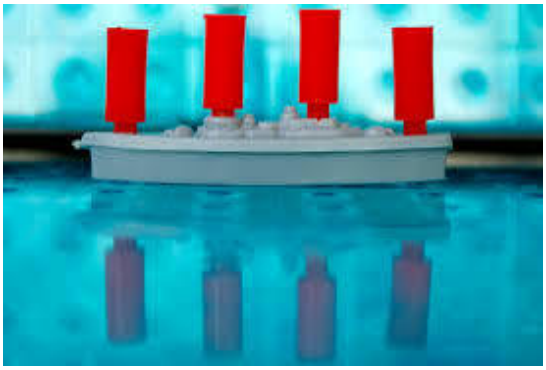

Figure **8.6.1**

Mrs. Gilcrest has designed a special version of the game Battleship. Her version requires students to use coordinates such as (1, 2) to guess the location of a ship. The ships are strategically placed so that each is a translation of the other. In order to keep the game easier for the students, she translated the ships the same distance up or down, right or left. If the first ship is at (3, 6), (3, 7), (3, 8) and the second ship is translated 4 units to the right and 4 units up, what are the coordinates of the second ship?

In this concept, you will learn how to classify transformations.

Classifying Transformations

The **coordinate plane** is a representation of two-dimensional space. It has a **horizontal axis**, called the x-axis, and a **vertical axis**, called the y-axis. You can graph and move geometric figures on the coordinate plane.

Remember the three types of transformations: translation, reflection and rotation.

Now let's look at performing each type of **transformation** in the coordinate plane.

When you perform translations, you slide a figure left or right, up or down. This means that on the coordinate plane, the coordinates for the vertices of the figure will change. Take a look at the example below.

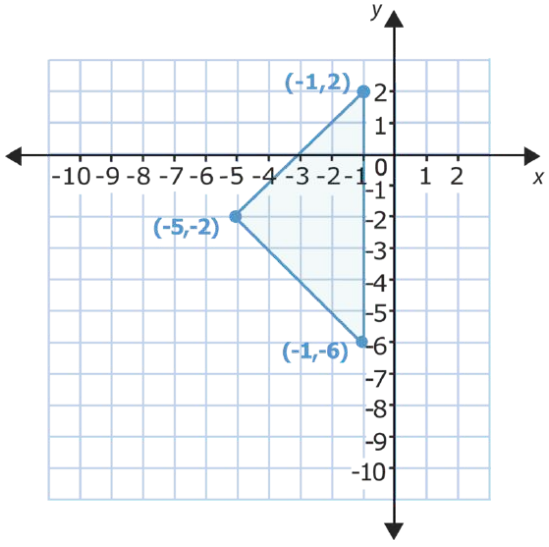

Figure **8.6.2**

Now let's look at performing a translation or slide of this figure.

You can choose the number of places that you want to move the triangle and the direction that you wish to move it in. If you slide this triangle 3 places down, all of its vertices will shift 3 places down the y-axis and the y-coordinate in each pair will decrease by 3.

Let's see why this happens.

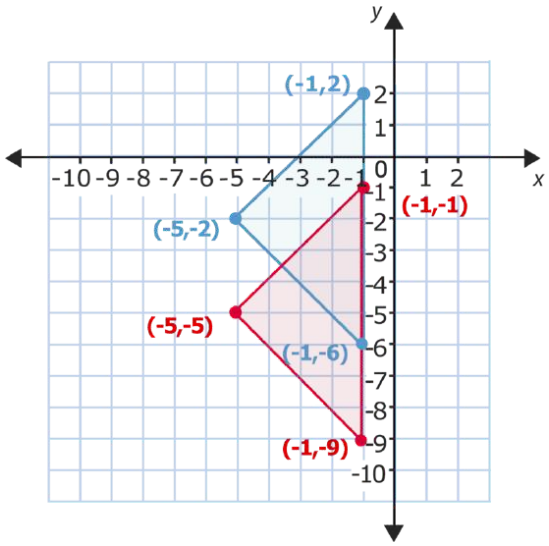

Figure **8.6.3**

You can see the change in all of the y-coordinates. Compare the top points. The y-coordinate on the left is 2. The y-coordinate for the corresponding point in the triangle after it moves is -1. The y-coordinate decreases by 3. Now compare the left-hand point of each triangle. The y-coordinate originally is -2, and the y-coordinate after the translation is -5. Again, the difference shows a change of -3 in the y-coordinate. For the last point, the y-coordinate starts out as -6, and shifts to -9 after the downward slide. For each point, the y-coordinate decreases by 3 while the x-coordinates stay the same.

You can translate figures in other ways, too. As you might guess, you move figures right or left on the coordinate grid by their x-coordinates. You can also move figures diagonally by changing both their x and y-coordinates. One way to recognize translations, then, is to compare their points. The x-coordinates will all change the same way, and the y-coordinates will all change the same way.

Here is an example of how to graph a translation.

Slide the following figure 5 places to the right.

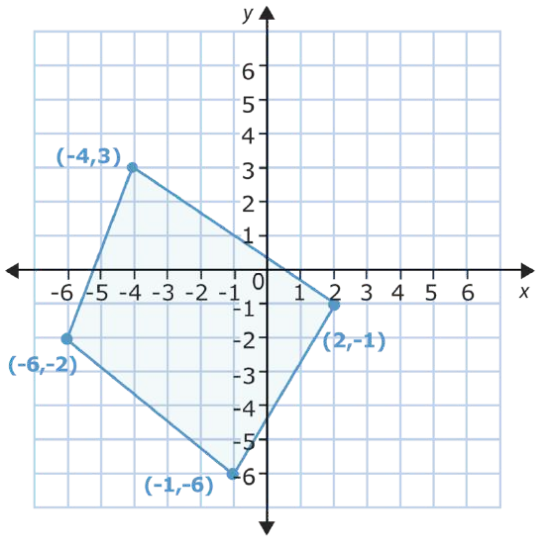

Figure **8.6.4**

In this translation, you will move the figure to the right. That means the x-coordinates for each point will change but the y-coordinates will not. You simply count 5 places to the right from each point and make a new point.

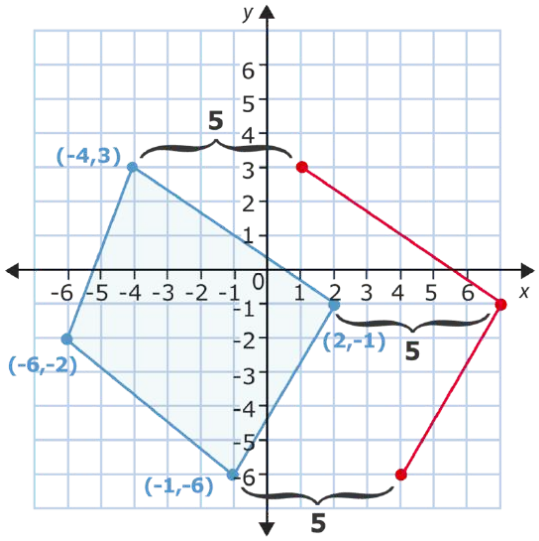

Figure 8.6.5

Once you relocate each point 5 places to the right, you can connect them to make the new figure that shows the translation.

You can check to see if you performed the translation correctly by adding 5 to each x-coordinate (because you moved to the right) and then checking these against the ordered pairs of the figure you drew. This is called **coordinate notation**. Notice that each point is represented by coordinates.

$$\begin{array}{cccc} (-4,3) & (-6,-2) & (-1,-6) & (2,-1) \\ +5 & +5 & +5 & +5 \\ (1,3) & (-1,-2) & (4,-6) & (7,-1) \end{array}$$

These are the points you graphed, so you have performed the translation correctly.

Example 8.6.1

Earlier, you were given a problem about Mrs. Gilcrest and her special version of the game Battleship.

She translated the ships the same distance up or down, right or left. If the first ship is at (3, 6), (3, 7), (3, 8) and the second ship is translated 4 units to the right and 4 units up, what are the coordinates of the second ship?

Solution

First, remember the signs associated with a move to the right and a move up.

Right is a move in the positive direction on the x-axis and up is a move in the positive direction on the y-axis.

Next, add the moves to the coordinates.

(3+4, 6+4), (3+4, 7+4), (3+4, 8+4)

Then write the new vertices

(7, 10), (7, 11), (7, 12)

The coordinates of the second ship are (7, 10), (7, 11), (7, 12).

Example 8.6.2

Solve this problem.

Slide the following figure 4 places to the left and 2 places up.

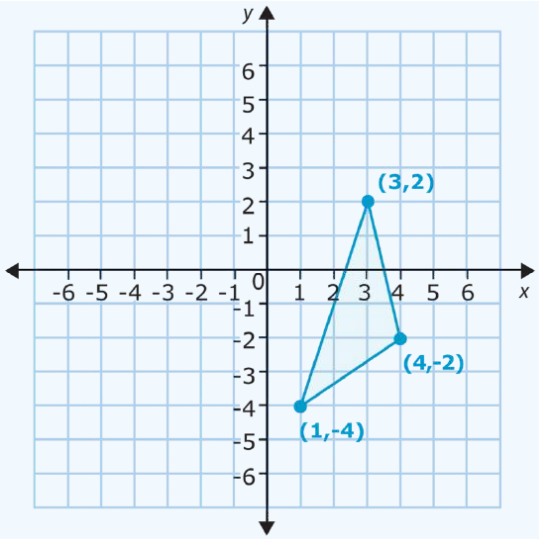

Figure 8.6.6

Solution

First, graph the new points.

Graph each point by counting 4 places to the left, and from there 2 places up.

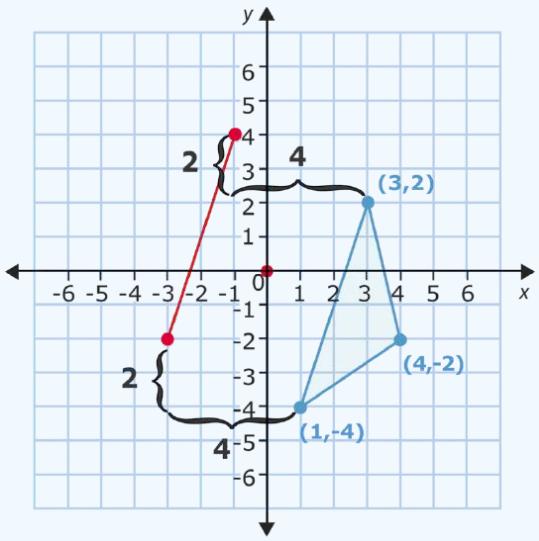

Figure 8.6.7

Then, form the new triangle.

Connect the new points.

You can check the translation by changing the x- and y-coordinates in the ordered pairs and then comparing these to the points you graphed. This time subtract 4 from each x-coordinate and add 2 to each y-coordinate. Let's see what happens.

$$\begin{array}{ccc} (3,2) & (4,-2) & (1,-4) \\ -4+2 & -4+2 & -4+2 \\ (-1,4) & (0,0) & (-3,-2) \end{array}$$

Example 8.6.3

Translate triangle \(ABC\) (0, 1), (1, 3), (4, 0) up 4.

Solution

First, remember whether up is a move on the x-axis or the y-axis.

Up is a move on the y-axis.

Next, add 4 to each of the y-values.

(0, 1+4), (1, 3+4), (4, 0+4)

Then, write the new vertices.

(0, 5), (1, 7), (4, 4)

The new triangle has coordinates (0, 5), (1, 7), (4, 4).

Example 8.6.4

Translate Triangle DEF (-3, 2),(1, 6),(2, 1) down 2.

Solution

First, remember whether down is a move on the x-axis or the y-axis.

Down is a move on the y-axis.

Next, subtract 2 from each of the y-values.

(-3, 2-2), (1, 6-2), (2, 1-2)

Then, write the new vertices.

(-3, 0), (1, 4), (2, -1)

The new triangle has coordinates (-3, 0), (1, 4), (2, -1)

Example 8.6.5

Translate triangle XYZ (-5, 4), (1, 8), (3, 5) to the right 3.

Solution

First, remember whether right is a move on the x-axis or the y-axis.

Right is a move on the x-axis.

Next, add 3 to each x-value.

(-5+3, 4), (1+3, 8), (3+3, 5)

Then write the new vertices

(-2, 4), (4, 8), (6, 5)

The new triangle has coordinates (-2, 4), (4, 8), (6, 5).

Review

Identify the transformations shown below as a translation, reflection, or rotation.

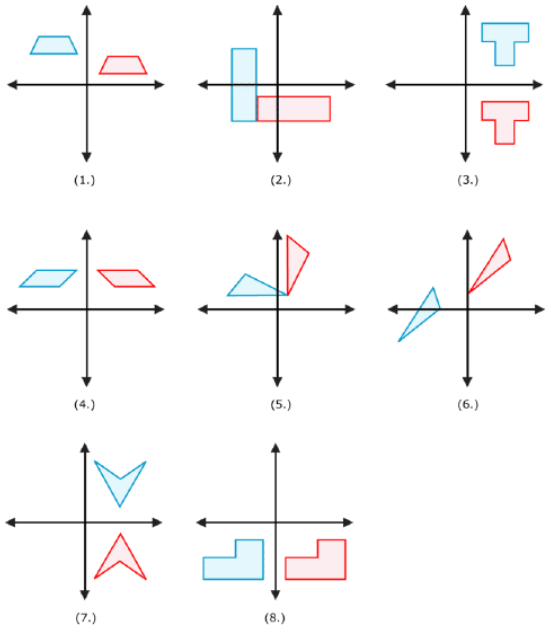

Figure 8.6.8

9. True or false. This figure has been translated 5 places to the right.

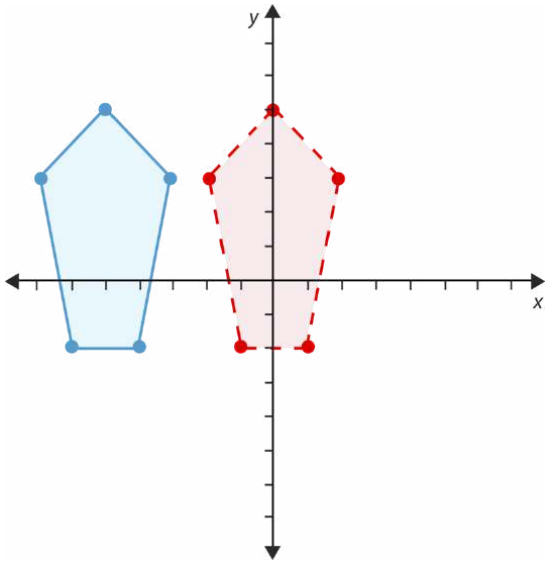

Figure 8.6.9

Translate each figure to the right 6 places and up 1. Then write the new coordinates for the figure.

10. Triangle ***DEF*** (-1, 2)(1, 6)(2, 1)
11. Triangle ***DEF*** (-3, 2)(1, 6)(2, 1)
12. Triangle ***DEF*** (0, 2)(1, 6)(2, 1)
13. Triangle ***DEF*** (4, -2)(1, 6)(2, 1)
14. Triangle ***DEF*** (5, 3)(1, 6)(2, 1)
15. Triangle ***DEF*** (4, 4)(1, 6)(2, 1)

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.18.

Resources

Vocabulary

Term	Definition
x-axis	The x-axis is the horizontal axis in the coordinate plane, commonly representing the value of the input or independent variable.
y-axis	The y-axis is the vertical number line of the Cartesian plane.
Coordinate Plane	The coordinate plane is a grid formed by a horizontal number line and a vertical number line that cross at the (0, 0) point, called the origin. The coordinate plane is also called a Cartesian Plane.
Transformation	A transformation moves a figure in some way on the coordinate plane.

Additional Resources

Video: Identify the Quadrant of a Point of the Coordinate System

Practice: Sliding Figures

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8.7: Geometry Software for Translations

Perform and analyze translations using Geogebra.

Geometry software has a **translation** function that makes it easy to translate an object along a vector. Can you perform a translation using geometry software without that function?

Translations

Recall that a **translation** is one example of a **rigid transformation**. A translation moves each point in a shape a specified distance in a specified direction as defined by a vector. Below, the parallelogram has been translated along vector \rightarrow{v}\) to create a new parallelogram (the image).

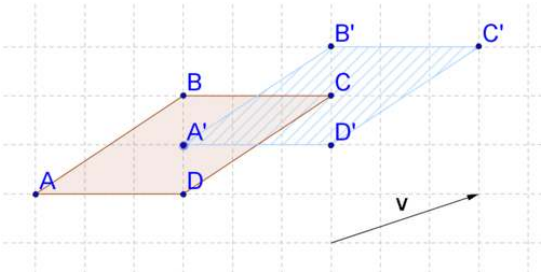

Figure **8.7.1**

When you are working on a grid (or graph paper), performing translations is relatively easy. In the above translation, you can see that the vector moves each point one unit up and three units to the right. To perform the translation, just move each point that defines the parallelogram one unit up and three units to the right. But what if the grid is not there? Then, it is not as easy to describe the translation because there are no grid lines as a guide.

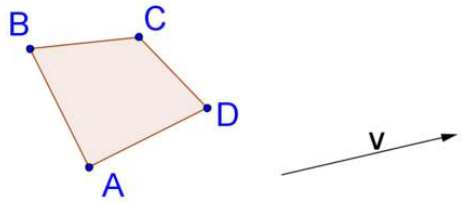

Figure **8.7.2**

To perform a translation without a grid, you need to:

1. Find lines parallel to the vector through each of the points that define the shape.
2. Move the length of the vector along each of those lines.

Doing this by hand requires careful construction of parallel lines and copying of line segments. Geometry software simplifies this process, because geometry software has a “translate” button. *Geogebra* is one example of geometry software that is free to download. To perform a translation in Geogebra, first create your polygon.

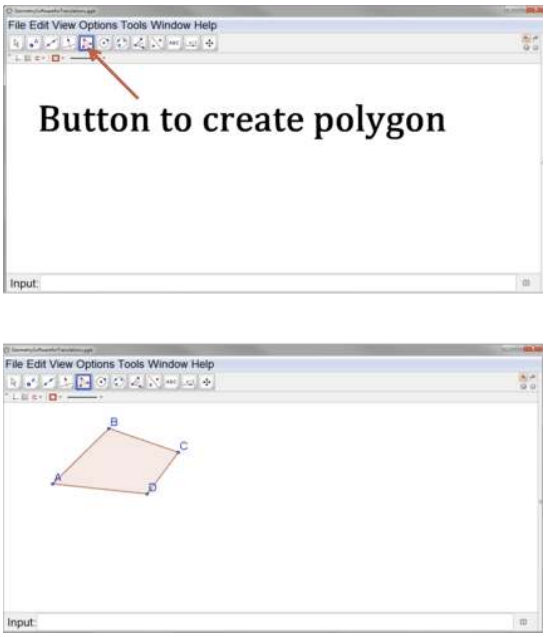

Figure 8.7.3

Next, create the vector that will define your translation. Below, a vector has been created from point G to point H.

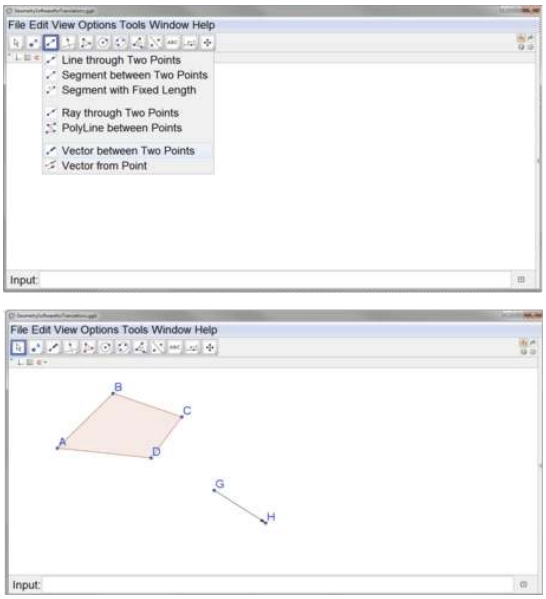

Figure 8.7.4

To perform the translation, select the translation button, then the polygon, and then the vector.

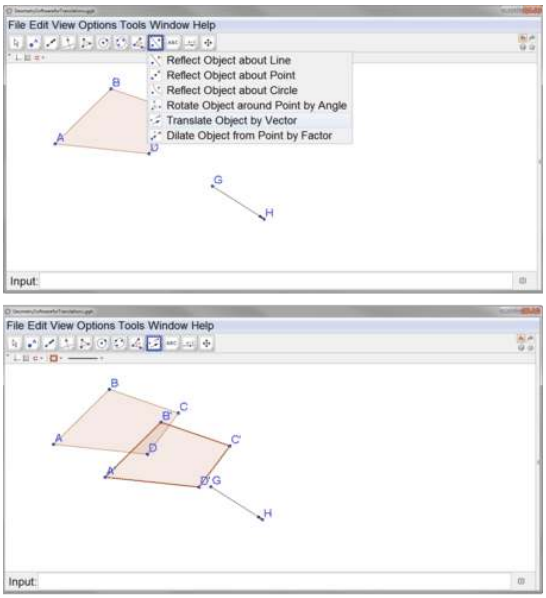

Figure 8.7.5

Note that the points defining the image are labeled with prime notation. Remember that the specific location of the vector does not matter. At this point you can drag the vector to another location and your translation will not change. You can also move points G or H to redefine the vector (or change your original polygon) and the location of your image will move accordingly.

Let's take a look at an exa

Show that the segment co AA' s parallel to the vector connecting points G and H.

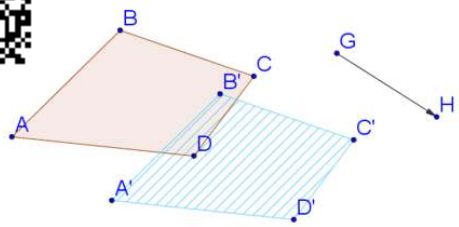

Figure 8.7.6

Use Geogebra to construct a line parallel to the vector through point A. You can see that it also passes through A'. This means the line segment connecting A with A' is parallel to the vector.

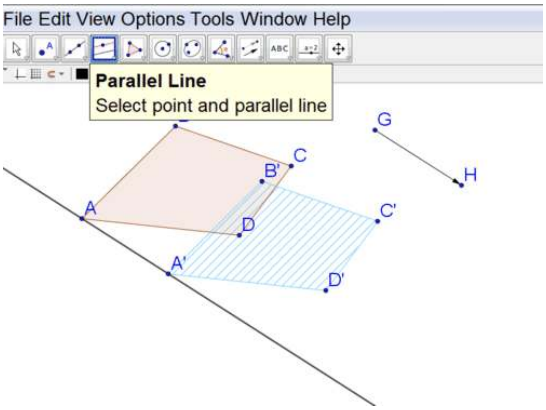

Figure 8.7.7

Translate the polygon without using the translate button.

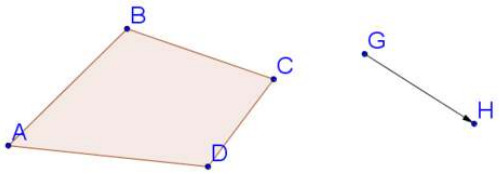

Figure 8.7.8

You are going to use Geogebra to do the same steps you would do if you were translating by hand. First, construct lines parallel to the vector through each of the four points defining the quadrilateral, using the method from Example A.

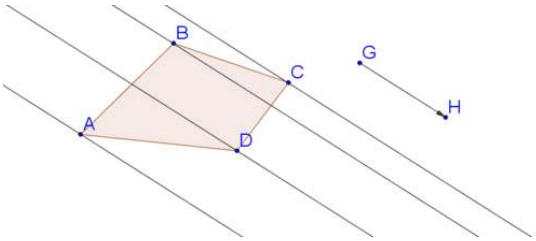

Figure 8.7.9

Next, copy the vector and paste it so that it starts at point A. Repeat for points B, C, and D. You might have to make sure that the copies of point H are not hidden.

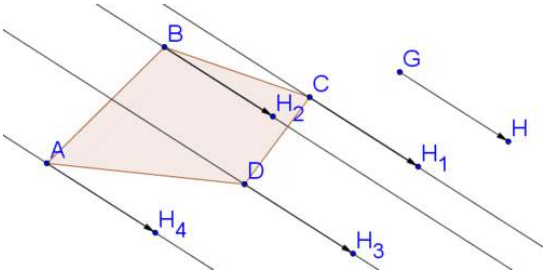

Figure 8.7.10

Form a polygon with the copies of point H.

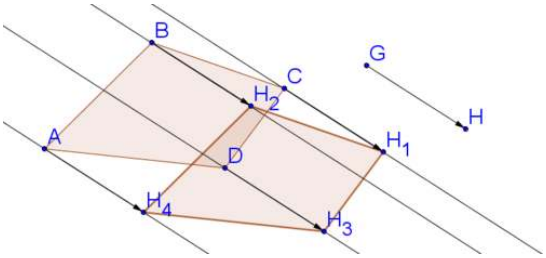

Figure 8.7.11

Now, let's compare and contrast the two methods for translating a polygon using Geogebra.

Using the translate button is faster, but performing the steps of constructing the parallel lines and copying the vectors also works and helps to show the connection between parallel lines, vectors, and translations. In both cases you can move the vector or the original shape after the translation has taken place and the image will change and move accordingly.

Example 8.7.1

Earlier, you were asked can you perform a translation using geometry software without the function.

Solution

You can perform a translation using geometry software without the “translate” button by constructing parallel lines and copying and pasting the vector onto those lines.

Example 8.7.2

Create your own polygon and vector in Geogebra.

Example 8.7.3

Translate the polygon along the vector using the translate button.

Example 8.7.4

Try translating the polygon along the vector without using the translate button by constructing parallel lines and copying and pasting the vector as shown in Example B. How can you verify that you have done this correctly?

Solution

Answers will vary depending on what polygon you construct. Look back at the guidance section for help using Geogebra. To check that you have performed the translation without the translate button correctly, just make sure that both images ended up in the same place.

Review

1. Create a polygon in Geogebra.
2. Create a vector that will move the polygon to the right.
3. Translate the polygon along the vector from #2 using the translate button.
4. Create another vector that will move the polygon up and to the left.
5. Translate the polygon along the vector from #4 using the translate button.
6. Create a third vector that will move the polygon straight down.
7. Translate the polygon along the vector from #6 by constructing parallel lines and copying and pasting the vector.
8. Check your work to #7 by using the translate button to translate the polygon along the vector from #6. How can you tell if you performed the translation correctly?
9. Translations are rigid transformations which means that distance is preserved. Verify that distance has been preserved by using Geogebra to measure sides of your original polygon and their images. *Select "distance or length" from one of the drop down menus. Then, click on each line segment that you want to measure to see its length.*

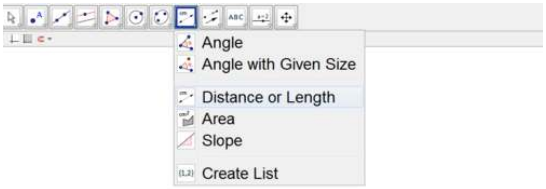

Figure **8.7.12**

10. Translations are rigid transformations which means that angles are preserved. Verify that angles have been preserved by using Geogebra to measure two corresponding angles. *Select "angle" from the same drop down menu as in #9. Then, tell Geogebra what angle you want to measure by clicking on the three points you would use to name the angle. You must click on the points in clockwise order for it to measure the correct angle.*
11. Construct a circle in Geogebra.
12. Create a vector that will move the circle to the right.
13. Translate the circle along the vector from #12 using the translate button.
14. Create another vector that will move the circle up and to the left.
15. Translate the circle along the vector from #14 using the translate button.
16. Explore how you might be able to translate the circle without using the translate button by creating parallel lines and copying and pasting the vector.

Review (Answers)

To see the Review answers, open this PDF file and look for section 2.3.

Vocabulary

Term	Definition
Translation	A translation is a transformation that slides a figure on the coordinate plane without changing its shape, size, or orientation.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

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8.8: Translation Applications in Circle Similarity

Solve problems involving ratios and circles and use transformations to show circles are similar.

Circles and Similarity

Sean has two circles, one with a **radius** of 1 inch and another with a radius of 3 inches.

1. What is the ratio between the radii of the circles?
2. What is the scale factor between the two circles?
3. What is the ratio between the circumferences of the circles?
4. What is the ratio between the areas of the circles?
5. What do area ratios and circumference ratios have to do with scale factor?

A **circle** is a set of points equidistant from a given point. The **radius** of a circle, r, is the distance from the center of the circle to the circle. A circle's size depends *only* on its radius.

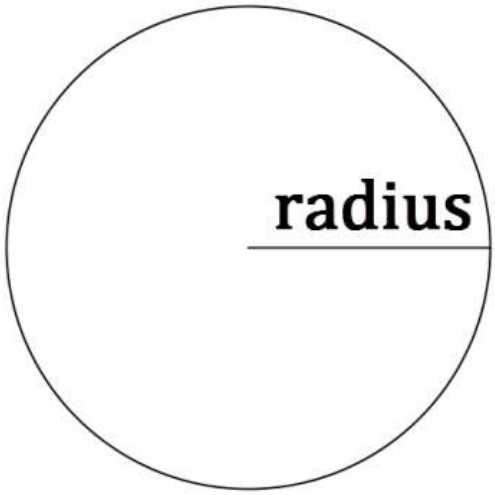

Figure 8.8.1

Two figures are **similar** if a **similarity transformation** will carry one figure to the other. A **similarity transformation** is one or more rigid transformations followed by a **dilation**. In the examples, you will show that a similarity transformation exists between any two circles and therefore, all circles are similar.

Recall two important formulas related to circles:

- Circumference (Perimeter) of a Circle: $C = 2\pi r$
- Area of a Circle: $A = \pi r^2$

Once you have shown that all circles are similar, you will explore how the circumferences and areas of circles are related.

Proving Circle Simila

1. Consider circle A, cent ı radius r_A, and circle D, centered at point D with radius rD. Perform a **rigid transformation** to brir D.

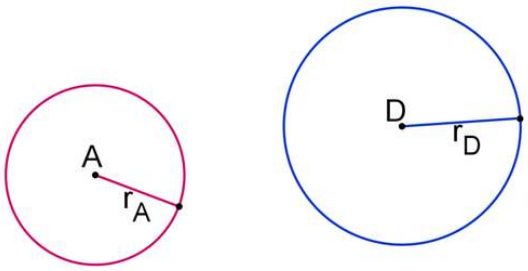

Figure 8.8.2

Draw a vector from point A to point D. Translate Circle A\) along the vector to create circle A'. Note that $r_A \cong r'_A$.

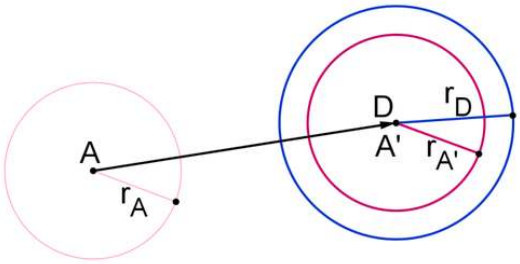

Figure 8.8.3

2. Dilate circle A to map it to circle D. Can you be confident that the circles are similar?

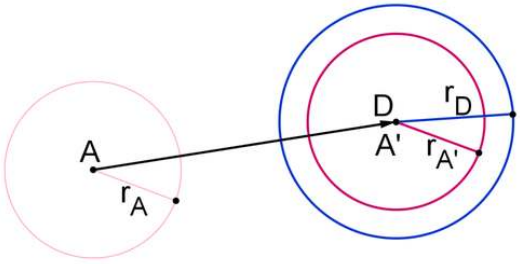

Figure 8.8.4

Because the size of a circle is completely determined by its radius, you can use the radii to find the correct scale factor. Dilate circle A' about point A' by a scale factor of $\frac{r_D}{r_{A'}}$.

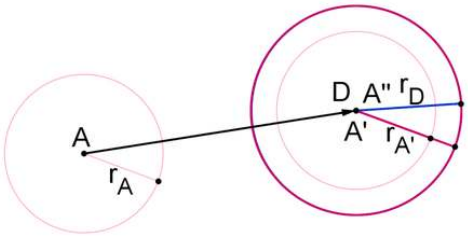

Figure 8.8.5

Circle A'' is the same Circle As circle D. You can be confident of this because $r_{A''} = \frac{r_D}{r_{A'}} \cdot r_{A'} = r_D$ and point A'' is the same as point D. Because a circle is defined by its center and radius, if two circles have the same center and radius then they are the same circle.

This means that circle A is similar to circle D, because a similarity transformation (translation then dilation) mapped circle A to circle D\).

Circle A and circle D were two random circles. This proves that in general, all circles are similar.

3. Show that circle A with center $(-3, 4)$ and radius 2 is similar to circle B with center $(3, 2)$ and radius 4.

Translate circle A along the vector from $(-3, 4)$ to $(3, 2)$. Then, dilate the image about its center by a scale factor of 2. You will have mapped circle A to circle B with a similarity transformation. This means that circle A is similar to circle B.

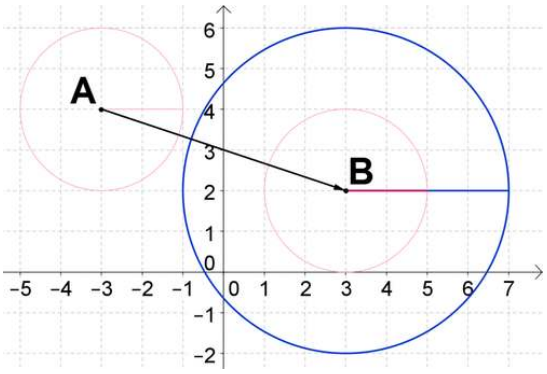

Figure 8.8.6

Example 8.8.1

Earlier, you were given a problem about Sean and his two circles.

Sean has two circles, one with a radius of 1 inch and another with a radius of 3 inches.

a. What is the ratio between the radii of the circles?
b. What is the scale factor between the two circles?
c. What is the ratio between the circumferences of the circles?
d. What is the ratio between the area of the circles?
e. What do area ratios and circumference ratios have to do with scale factor?

Solution

a. The ratio between the radii is 31.
b. A scale factor exists because any two circles are similar. You can use the radii to determine the scale factor. The ratio between the radii is $\frac{3}{1}$ so the scale factor is $\frac{3}{1} = 3$.
c. The circumference of the smaller circle is $C = 2\pi(1) = 2\pi$. The circumference of the larger circle is $C = 2\pi(3) = 6\pi$. The ratio between the circumferences is $\frac{6\pi}{2\pi} = \frac{3}{1}$.
d. The area of the smaller circle is $A = \pi(1)2 = \pi$. The area of the larger circle is $A = \pi(3)^2 = 9\pi$. The ratio between the areas is $\frac{9\pi}{\pi} = \frac{9}{1}$. Note that $\frac{9}{1} = (\frac{3}{1})^2$.
e. The area ratio is the scale factor squared, because area is a two dimensional measurement. The circumference ratio is equal to the scale factor, because circumference is a one dimensional measurement.

Example 8.8.2

Show that Circle A with center $(-1, 7)$ and radius 2 is similar to Circle B with center $(4, 6)$ and radius 3.

Solution

Translate Circle A along the vector from $(-1, 7)$ to $(4, 6)$. Then, dilate the image about its center with a scale factor of $\frac{3}{2}$. You will have mapped Circle A to Circle B with a similarity transformation. This means that Circle A is similar to Circle B.

Example 8.8.3

The ratio of the circumference of circle D to the circumference of circle C is $\frac{4}{3}$. What is the ratio of their areas?

Solution

The ratio of the circumferences is the same as the scale factor. Therefore, the scale factor is $\frac{4}{3}$. The ratio of the areas is the scale factor squared. Therefore, the ratio of the areas is $(\frac{4}{3})^2 = \frac{16}{9}$.

Example 8.8.4

The ratio of the area of circle F to the area of circle E is $\frac{9}{4}$. What is the ratio of their radii?

Solution

The ratio of the areas is the scale factor squared. Therefore, the scale factor is $\sqrt{\frac{9}{4}} = \frac{3}{2}$. The ratio of the radii is the same as the scale factor, so the ratio of the radii is $\frac{3}{2}$.

Review

For #1-#10, show that the circles are similar by describing the similarity transformation necessary to map one circle onto the other.

1. Circle A with center (2,7)\) and radius 4. Circle B with center $(1, -4)$ and radius 3.
2. Circle A with center (6,4)\) and radius 3. Circle B with center $(-5, 6)$ and radius 5.
3. Circle A with center (1,4)\) and radius 2. Circle B with center $(3, -2)$ and radius 7.
4. Circle A with center (8,1)\) and radius 6. Circle B with center $(6, -4)$ and radius 8.
5. Circle A with center (−2,10)\) and radius 3. Circle B with center $(-1, -4)$ and radius 6.
6. Circle A with center (−1,5) and radius 4. Circle B with center $(-1, 5)$ and radius 5.
7. Circle A with center (−4,−2) and radius 1. Circle B with center $(1, 8)$ and radius 4.
8. Circle A with center (10,3) and radius 5. Circle B with center $(4, 2)$ and radius 8.
9. Circle A with center (12,4) and radius 10. Circle B with center $(12, 4)$ and radius 12.
10. Circle A with center (−7,6) and radius 9. Circle B with center $(1, -4)$ and radius 9.
11. The ratio of the circumference of Circle A to the circumference of Circle B is $\frac{2}{3}$. What is the ratio of their radii?
12. The ratio of the area of Circle A to the area of Circle B is $\frac{6}{1}$. What is the ratio of their radii?
13. The ratio of the radius of Circle A to the radius of Circle B is $\frac{5}{9}$. What is the ratio of their areas?
14. The ratio of the area of Circle A to the area of Circle B is $\frac{12}{5}$. What is the ratio of their circumferences?
15. To show that any two circles are similar you need to perform a translation and/or a dilation. Why won't you ever need to use reflections or rotations?

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.1.

Vocabulary

Term	Definition
Circle	A circle is the set of all points at a specific distance from a given point in two dimensions.
Dilation	To reduce or enlarge a figure according to a scale factor is a dilation.
Radius	The radius of a circle is the distance from the center of the circle to the edge of the circle.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.
Similarity Transformation	A similarity transformation is one or more rigid transformations followed by a dilation.

Additional Resources

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8.9: Translation Notation

Graphical introduction to image translations. Learn to use notation to describe mapping rules,and graph images given preimage and translation

Rules for Translations

Jack describes a translation as point moving from $J(-2,6)$ to $J'(4,9)$. Write the mapping rule to describe this translation for Jack.

In geometry, a transformation is an operation that moves, flips, or changes a shape (called the preimage) to create a new shape (called the image). A translation is a type of transformation that moves each point in a figure the same distance in the same direction. Translations are often referred to as slides. You can describe a translation using words like "moved up 3 and over 5 to the left" or with notation. There are two types of notation to know.

1. One notation looks like $T_{\langle 3,5\rangle}$. This notation tells you to add 3 to the x values and add 5 to the y values.
2. The second notation is a mapping rule of the form $(x,y)\rightarrow(x-7,y+5)$. This notation tells you that the x and y coordinates are translated to $x-7$ and $y+5$.

The mapping rule notation is the most common.

Let's write a mapping rule for each of the following translations:

1. Sarah describes a translation as point P moving from $P(-2,2)$ to $P'(1,-1)$.

In general, $P(x,y)\rightarrow P'(x+a,y+b)$.

In this case, $P(-2,2)\rightarrow P'(-2+a,2+b)$ or $P(-2,2)\rightarrow P'(1,-1)$

Therefore:

$$\begin{aligned} -2+a&=1 \quad \text{and} \quad 2+b=-1\\ a&=3 \qquad\qquad\quad b=-3 \end{aligned}$$

The rule is: $(x,y)\rightarrow(x+3,y-3)$

2. Mikah describes a translation as point D in a diagram moving from $D(1,-5)$ to $D'(-3,1)$.

In general, $P(x,y)\rightarrow P'(x+a,y+b)$.

In this case, $D(1,-5)\rightarrow D'(1+a,-5+b)$ or $D(1,-5)\rightarrow D'(-3,1)$

Therefore:

$$\begin{aligned} 1+a&=-3 \quad \text{and} \quad -5+b=1\\ a&=-4 \qquad\qquad\quad b=6 \end{aligned}$$

The rule is: $(x,y)\rightarrow(x-4,y+6)$

3. The translation of the preimage A to the translated image J in the diagram below:

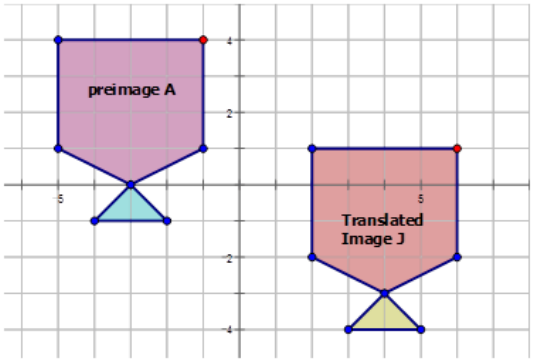

Figure 8.9.1

First, pick a point in the diagram to use to see how it is translated.

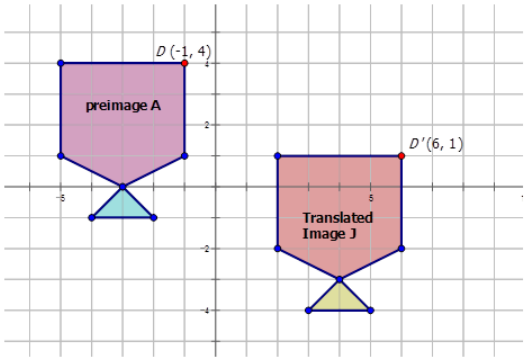

Figure 8.9.2

$D: (-1, 4) D'$: $(6, 1)$

$D(x, y) \rightarrow D'(x + a, y + b)$

So: $D(-1, 4) \rightarrow D'(-1 + a, 4 + b)$ or $D(-1, 4) \rightarrow D'(6, 1)$

Therefore:

$$\begin{aligned} -1 + a &= 6 \quad \text{and} \quad 4 + b = 1 \\ a &= 7 \qquad\qquad b = -3 \end{aligned}$$

The rule is: $(x, y) \rightarrow (x + 7, y - 3)$

Example 8.9.1

Earlier, you were told that Jack described a translation as point J moving from $J(-2, 6)$ to $J'(4, 9)$. What is the mapping rule that describes this translation?

Solution

$(x, y) \rightarrow (x + 6, y + 3)$

Example 8.9.2

Write the mapping rule that represents the translation of the red triangle to the translated green triangle in the diagram below.

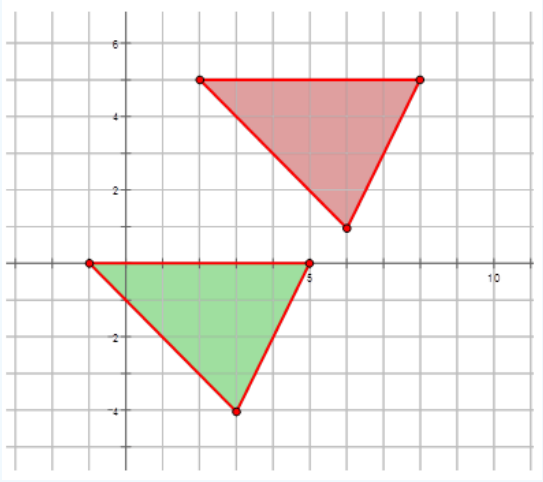

Figure 8.9.3

Solution

$(x, y) \rightarrow (x - 3, y - 5)$

Example 8.9.3

The following pattern is part of wallpaper found in a hotel lobby. Write the mapping rule that represents the translation of one blue trapezoid to a translated blue trapezoid shown in the diagram below.

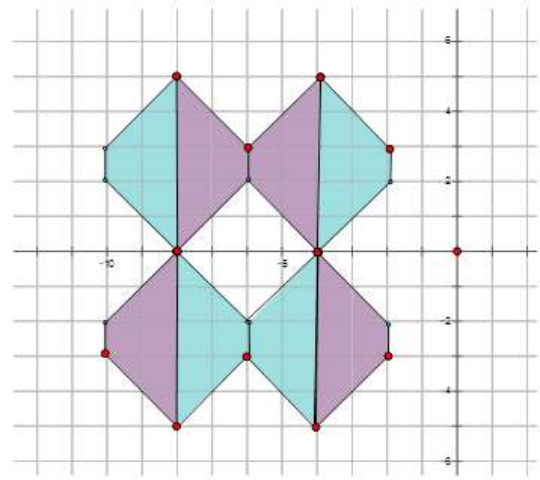

Figure 8.9.4

Solution

If you look closely at the diagram below, there two pairs of trapezoids that are translations of each other. Therefore you can choose one blue trapezoid that is a translation of the other and pick a point to find out how much the shape has moved to get to the translated position.

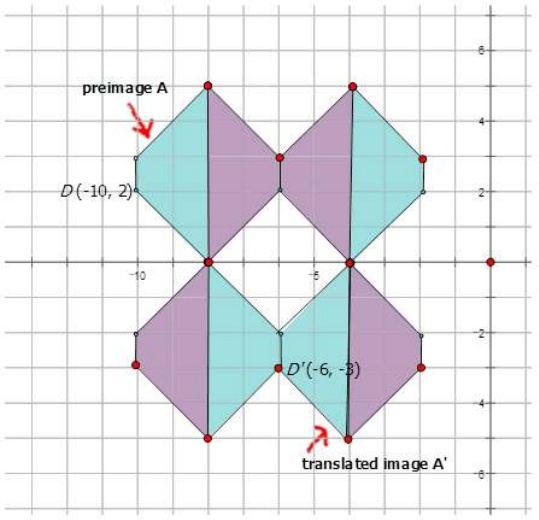

Figure 8.9.5

For those two trapezoids: $(x, y) \rightarrow (x + 4, y - 5)$

Review

Write the mapping rule to describe the movement of the points in each of the translations below.

1. $S(1, 5) \rightarrow S'(2, 7)$
2. $W(-5, -1) \rightarrow W'(-3, 1)$
3. $Q(2, -5) \rightarrow Q'(-6, 3)$
4. $M(4, 3) \rightarrow M'(-2, 9)$
5. $B(-4, -2) \rightarrow B'(2, -2)$
6. $A(2, 4) \rightarrow A'(2, 6)$

7. $C(-5,-3) \rightarrow C'(-3,4)$
8. $D(4,-1) \rightarrow D'(-4,2)$
9. $Z(7,2) \rightarrow Z'(-3,6)$
10. $L(-3,-2) \rightarrow L'(4,-1)$

Write the mapping rule that represents the translation of the preimage to the image for each diagram below.

11.

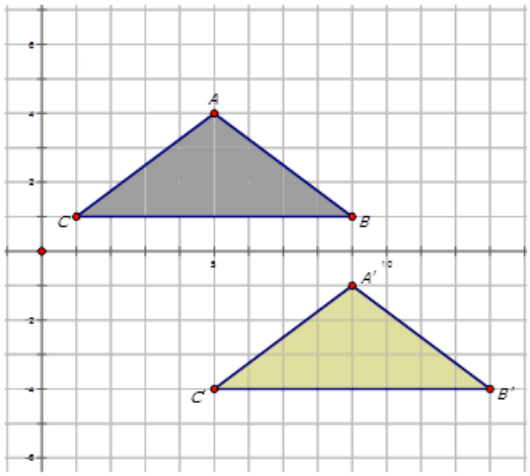

Figure **8.9.6**

12.

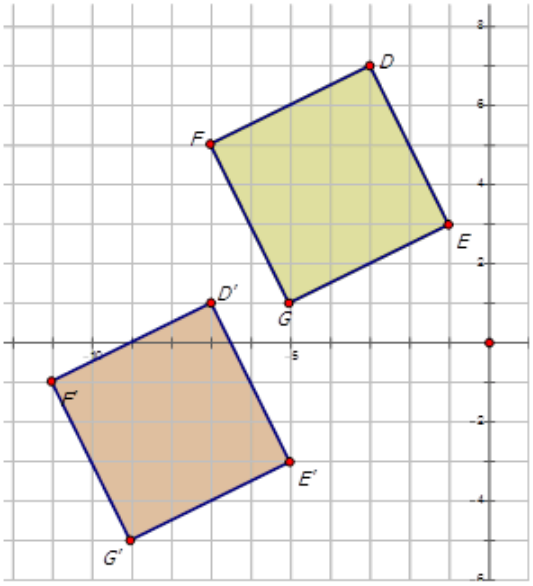

Figure **8.9.7**

13.

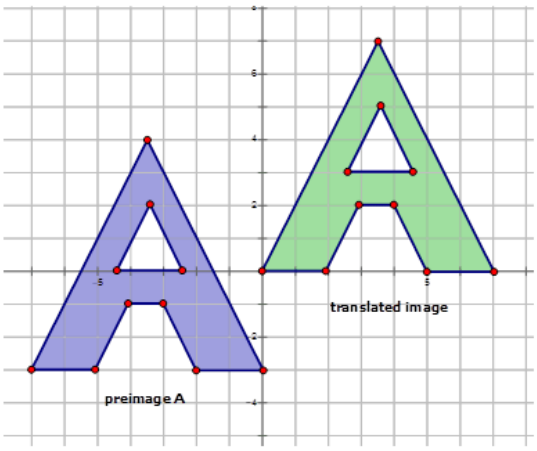

Figure 8.9.8

14.

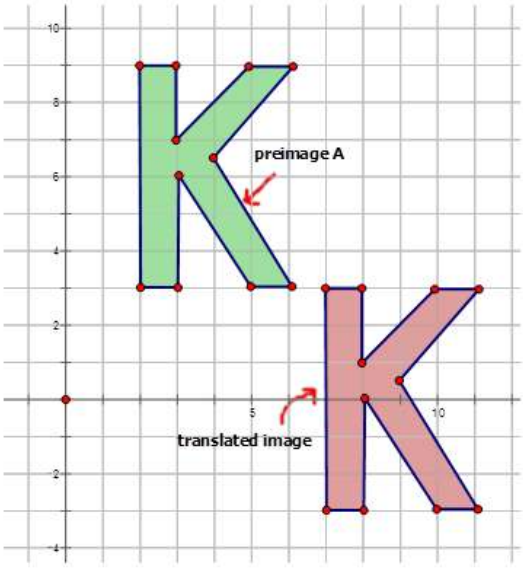

Figure 8.9.9

15.

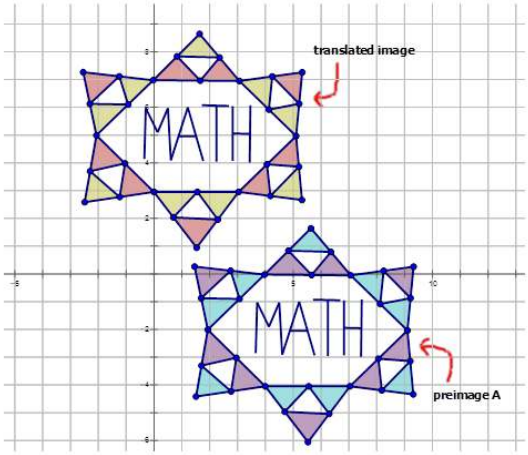

Figure 8.9.10

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.3.

Additional Resources

Video: Quadrants of Coordinate Plane

Practice: Translation Notation

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8.10: Defining Rotation

Transformations by which a figure is turned around a fixed point to create an image.

Rotations

A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change the size or shape of a figure.

The rigid transformations are translations, reflections, and **rotations**. The new figure created by a transformation is called the **image**. The original figure is called the **preimage**. If the preimage is A, then the image would be A', said "a prime." If there is an image of A', that would be labeled A'', said "a double prime."

A **rotation** is a transformation where a figure is turned around a fixed point to create an image. The lines drawn from the preimage to the **center of rotation** and from the center of rotation to the image form the **angle of rotation**.

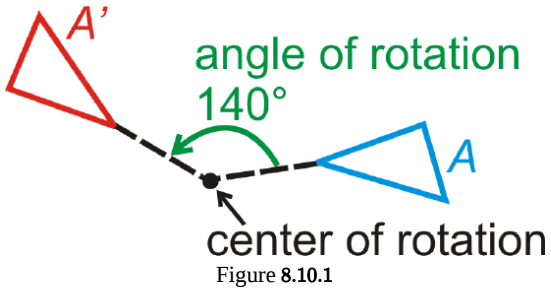


Figure 8.10.1

While we can rotate any image any amount of degrees, $90°$, $180°$ and $270°$ rotations are common and have rules worth memorizing.

Rotation of $180°$: $(x, y) \rightarrow (-x, -y)$

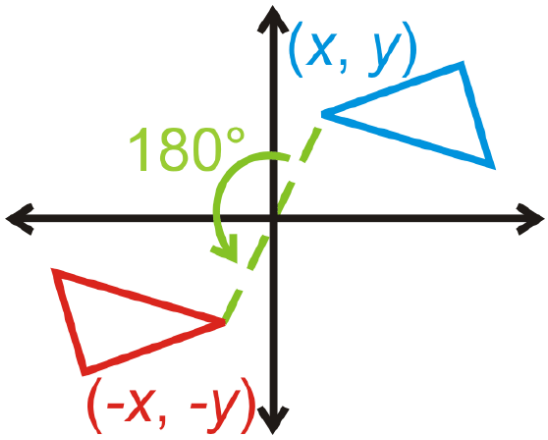


Figure 8.10.2

Rotation of $90°$: $(x, y) \rightarrow (-y, x)$

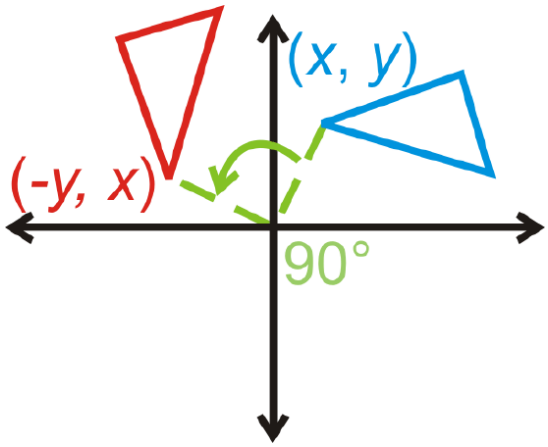


Figure 8.10.3

Rotation of 270°: $(x, y) \rightarrow (y, -x)$

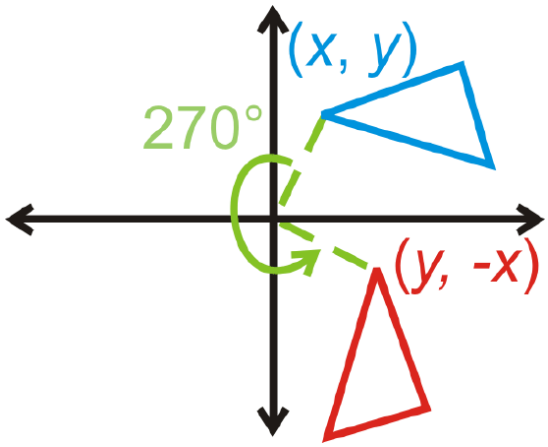


Figure 8.10.4

What if you were given the coordinates of a quadrilateral and you were asked to rotate that quadrilateral $270°$ about the **origin**? What would its new coordinates be?

Example 8.10.1

A rotation of $80°$ clockw hat counterclockwise rotation?

Solution

There are $360°$ around a tation clockwise is the same as a $360° - 80° = 280°$ rotation counterclockwise.

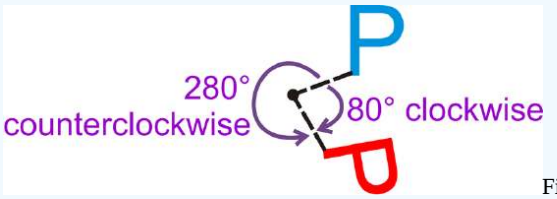


Figure 8.10.6

Example 8.10.2

A rotation of \(160^{\circ} counterclockwise is the same as what clockwise rotation?

Solution

$360° - 160° = 200°$ clockwise rotation.

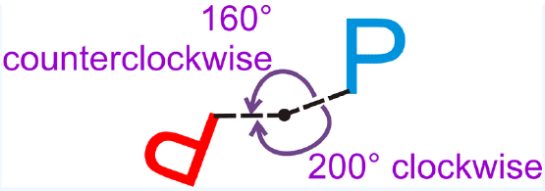


Figure 8.10.7

Example 8.10.3

Rotate $\overline{ST}$ 90°.

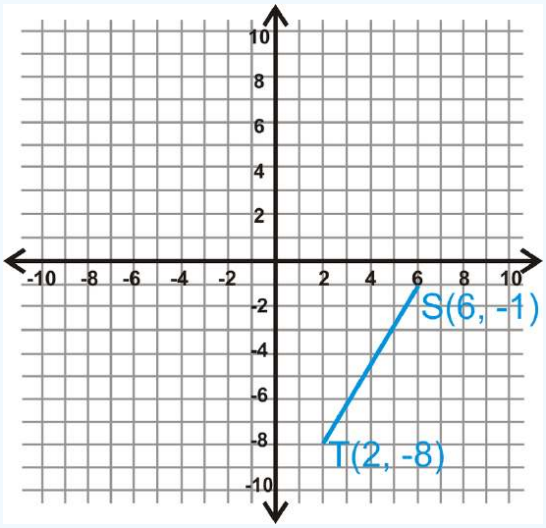


Figure 8.10.8

Solution

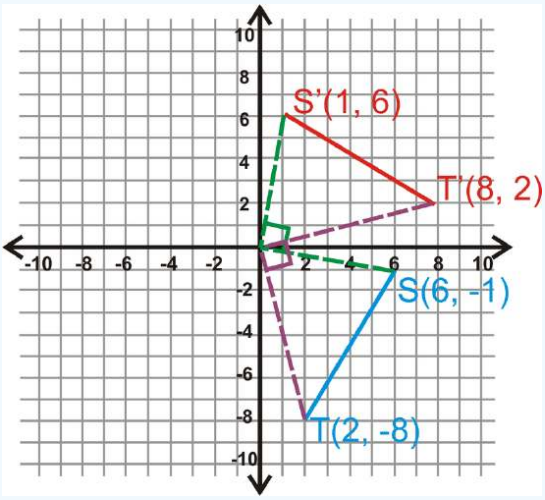


Figure 8.10.9

Example 8.10.4

The rotation of a quadrilateral is shown below. What is the measure of x and y?

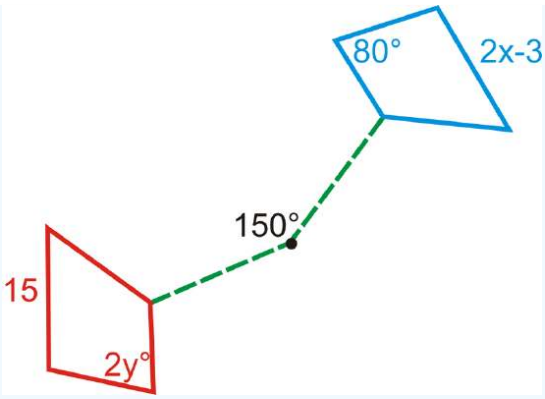


Figure 8.10.10

Solution

Because a rotation produces congruent figures, we can set up two equations to solve for x and y.

$$\begin{aligned} 2y &= 80^\circ \quad 2x - 3 = 15 \\ y &= 4 \quad y = 40^\circ \quad\quad 2x = 18 \\ & \qquad\qquad\quad x = 9 \end{aligned}$$

Example 8.10.5

Rotate ΔABC, with vertices $A(7,4)$, $B(6,1)$, and $C(3,1)$, 180° about the origin. Find the coordinates of $\Delta A'B'C'$.

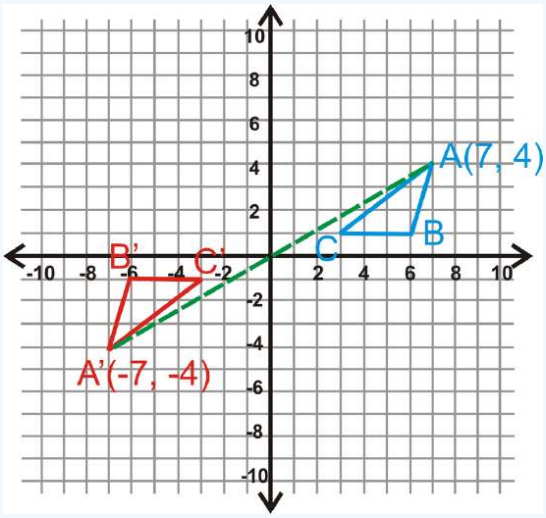


Figure 8.10.11

Solution

Use the rule above to find $\Delta A'B'C'$.

$$\begin{aligned} A(7,4) &\rightarrow A'(-7,-4) \\ B(6,1) &\rightarrow B'(-6,-1) \\ C(3,1) &\rightarrow C'(-3,-1) \end{aligned}$$

Review

In the questions below, every rotation is *counterclockwise*, unless otherwise stated.

1. If you rotated the letter p 180^{\circ}\) counterclockwise, what letter would you have?
2. If you rotated the letter p 180° *c*\(*ockwise*, what letter would you have?
3. A 90° clockwise rotation is the same as what counterclockwise rotation?
4. A 270° clockwise rotation is the same as what counterclockwise rotation?

5. A 210° counterclockwise rotation is the same as what clockwise rotation?
6. A 120° counterclockwise rotation is the same as what clockwise rotation?
7. A 340° counterclockwise rotation is the same as what clockwise rotation?
8. Rotating a figure 360° is the same as what other rotation?
9. Does it matter if you rotate a figure 180° clockwise or counterclockwise? Why or why not?
10. When drawing a rotated figure and using your protractor, would it be easier to rotate the figure 300° counterclockwise or 60° clockwise? Explain your reasoning.

Rotate each figure in the coordinate plane the given angle measure. The center of rotation is the origin.

11. 180°

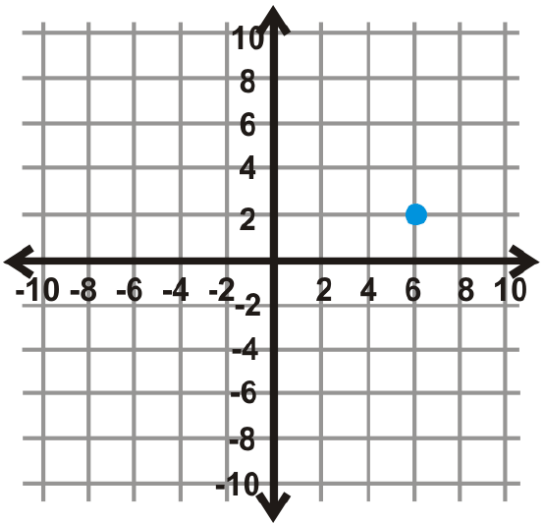


Figure 8.10.12

12. 90°

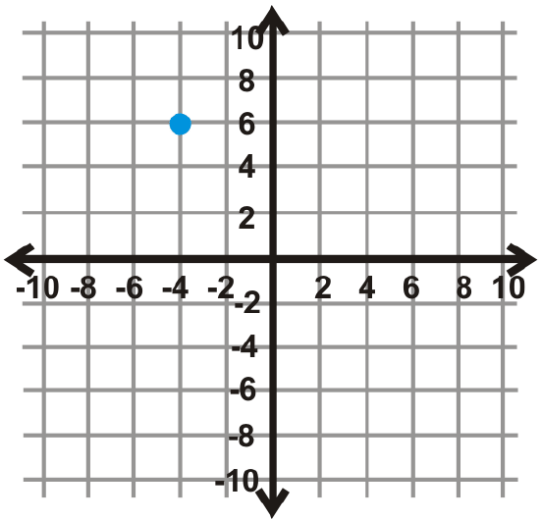


Figure \(\
PageIndex
{13}\)

13. 180°

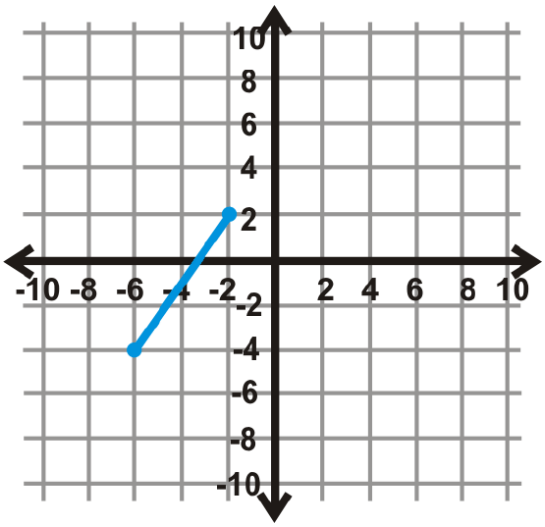


Figure 8.10.14

14. 270°

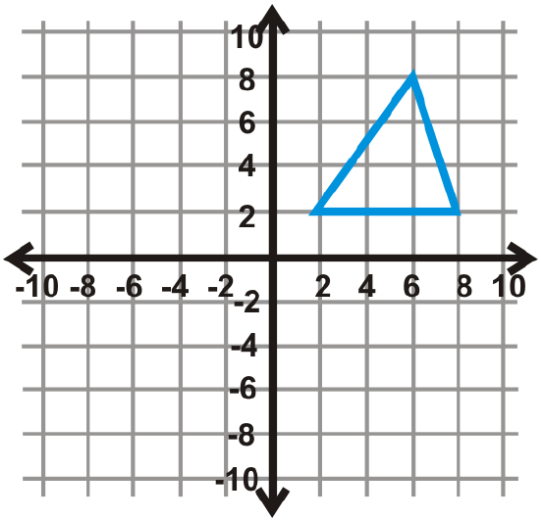


Figure 8.10.15

15. 90°

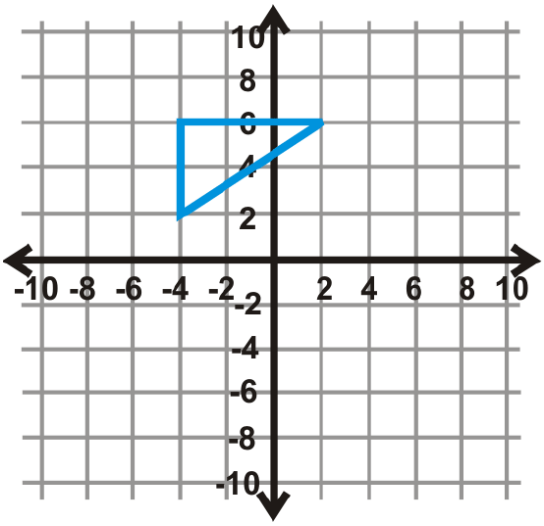


Figure 8.10.16

16. 270°

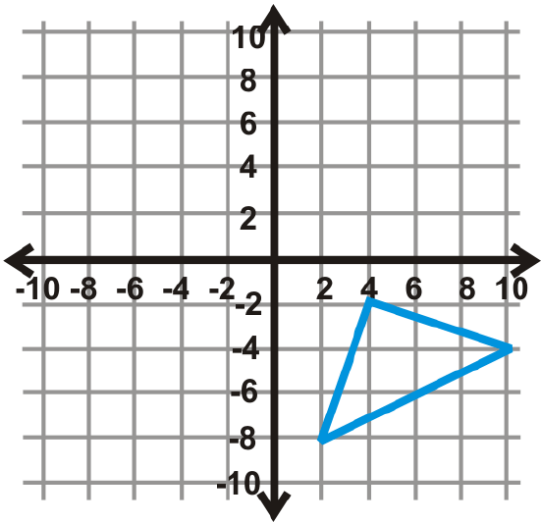


Figure 8.10.17

17. 180°

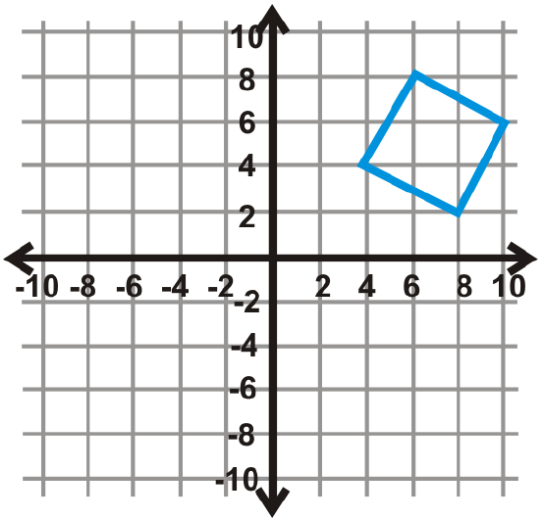


Figure 8.10.18

18. 270°

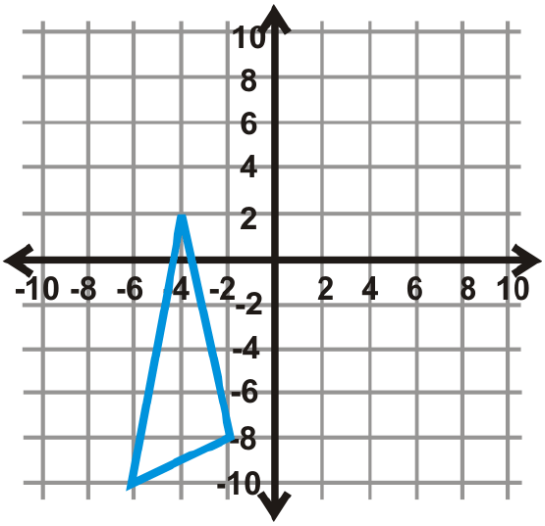


Figure 8.10.19

19. 90°

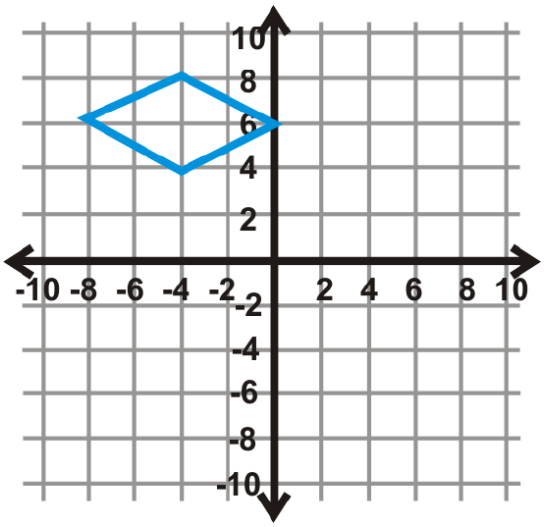


Figure 8.10.20

Find the measure of x in the rotations below. The blue figure is the preimage.

20.

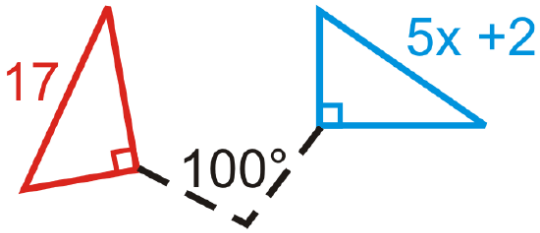


Figure 8.10.21

21.

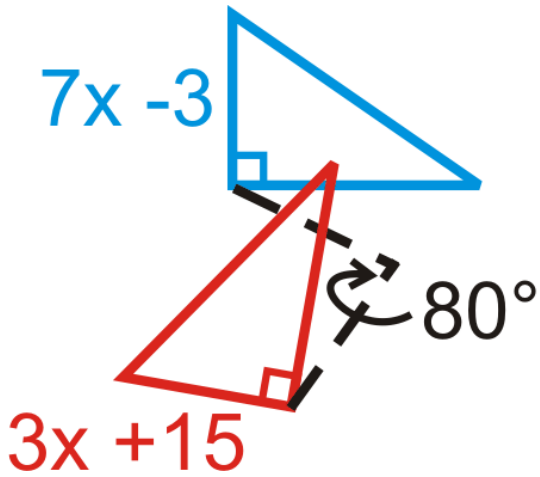
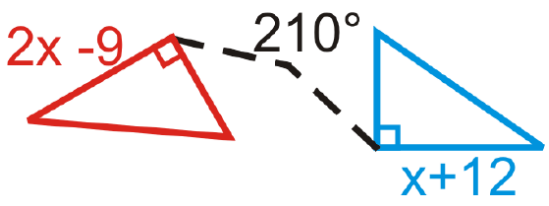


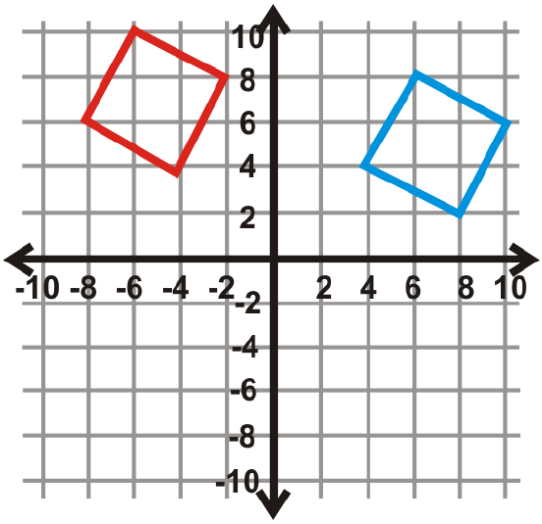
Figure 8.10.22



22.

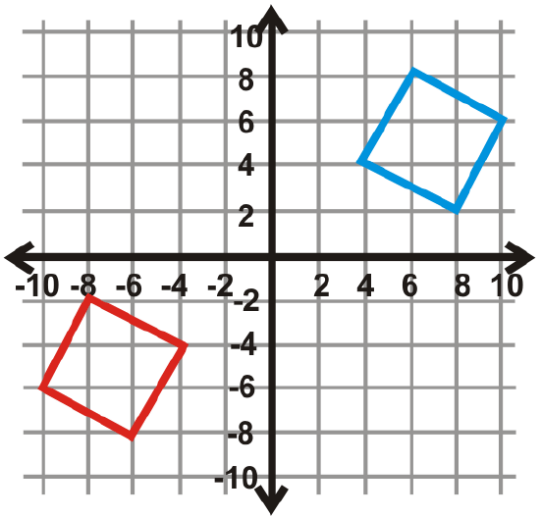
Figure 8.10.23

Find the angle of rotation for the graphs below. The center of rotation is the origin and the blue figure is the preimage. Your answer will be $90°$, $270°$, or $180°$.



23.

Figure 8.10.24



24.

Figure 8.10.25

25.

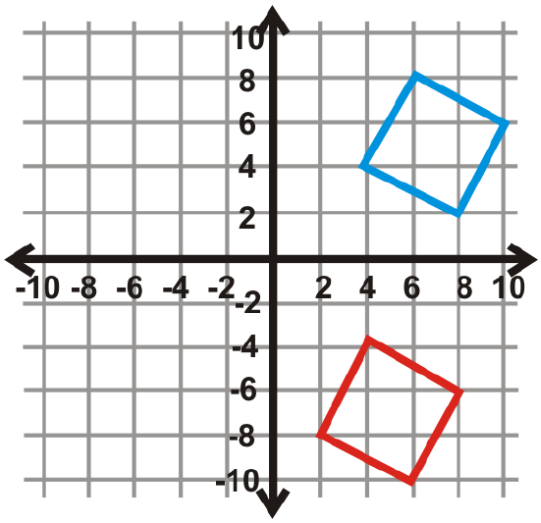


Figure 8.10.26

26.

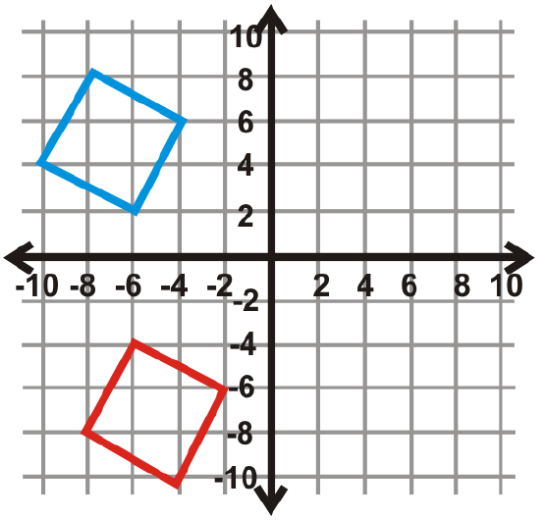


Figure 8.10.27

27.

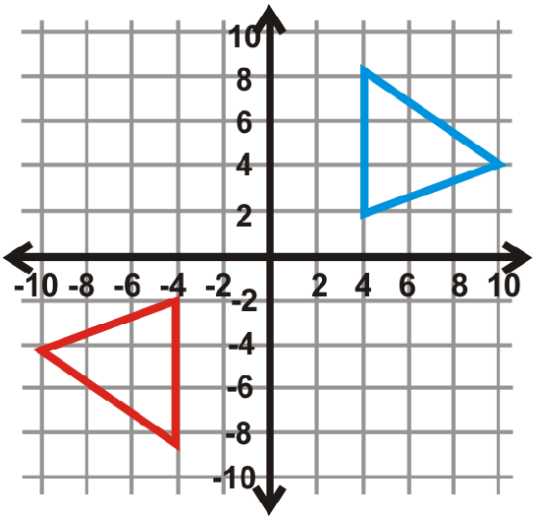


Figure 8.10.28

28.

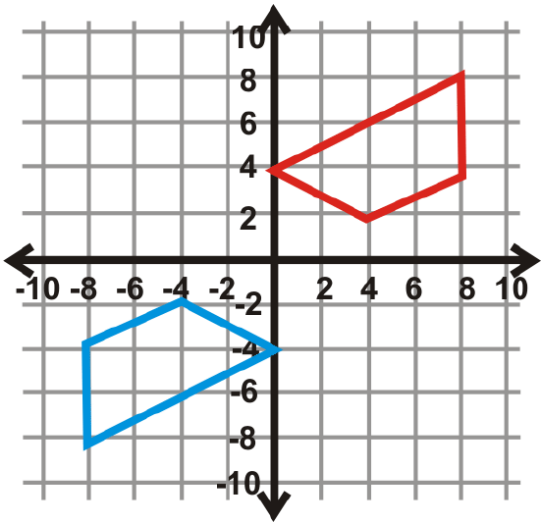


Figure 8.10.29

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.4.

Vocabulary

Term	Definition
Center of Rotation	In a rotation, the center of rotation is the point that does not move. The rest of the plane rotates around this fixed point.
Image	The image is the final appearance of a figure after a transformation operation.
Origin	The origin is the point of intersection of the x and y axes on the Cartesian plane. The coordinates of the origin are (0, 0).
Preimage	The pre-image is the original appearance of a figure in a transformation operation.
Rotation	A rotation is a transformation that turns a figure on the coordinate plane a certain number of degrees about a given point without changing the shape or size of the figure.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Additional Resources

Video: Transformation: Rotation Principles - Basic

Activities: Rotations Discussion Questions

Study Aids: Types of Transformations Study Guide

Practice: Defining Rotation

Real World: Radical Rotations

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8.11: Rotation Rules

State rules that describe given rotations.

Rules for Rotations

The figure below shows a pattern of two fish. Write the mapping rule for the rotation of Image A to Image B.

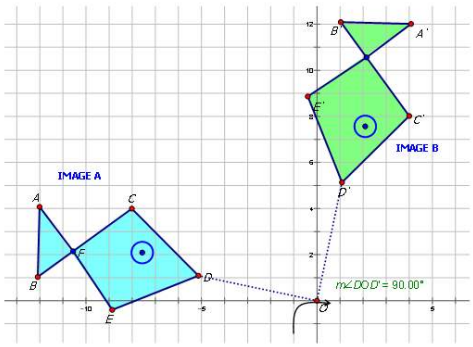

Figure **8.11.1**

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A rotation is an example of a transformation where a figure is rotated about a specific point (called the center of rotation), a certain number of degrees. Common rotations about the origin are shown below:

Center of Rotation	Angle of Rotation	Preimage (Point P)	Rotated Image (Point P′)	Notation (Point P′)
(0, 0)	**90°**(or **−270°**)	(x, y)	$(-y, x)$	$(x, y) \rightarrow (-y, x)$
(0, 0)	**180°**(or **−180°**)	(x, y)	$(-x, -y)$	$(x, y) \rightarrow (-x, -y)$
(0, 0)	**270°**(or **−90°**)	(x, y)	$(y, -x)$	$(x, y) \rightarrow (y, -x)$

You can describe rotations in words, or with notation. Consider the image below:

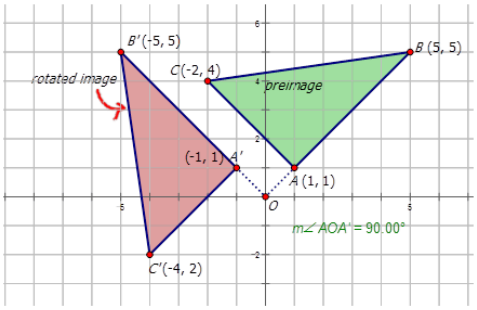

Figure **8.11.2**

Notice that the preimage is rotated about the origin **90°**CCW. If you were to describe the rotated image using notation, you would write the following:

$R_{90°}(x, y) = (-y, x)$

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Access to this content has been restricted. Contact your internet service provider for help.

Let's write the notation to describe the following CCW rotations on the point (3, 2)\) and draw the image:

1. about the origin at $90°$

Rotation about the origin at $90°$: \($R_{90°}(x, y) = (-y, x)$

2. about the origin at $180°$

Rotation about the origin at $180°$: $R_{180°}(x, y) = (-x, -y)$

3. about the origin at $270°$

Rotation about the origin at 270^{\circ}: $R_270°(x, y) = (y, -x)$

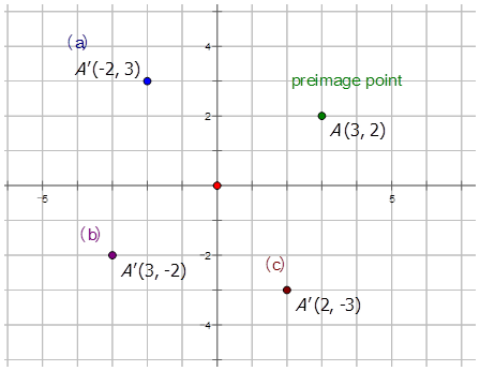

Figure **8.11.3**

Now let's perform the following rotations on Image *A* shown below in the diagram below and describe the rotations:

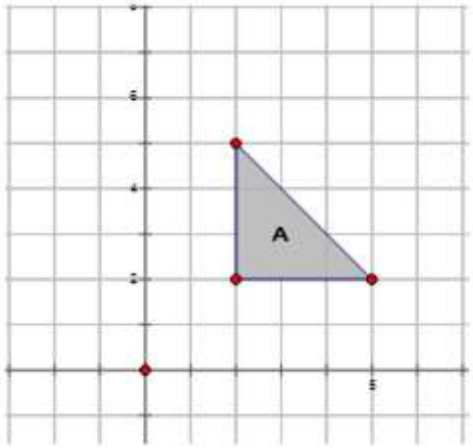

Figure **8.11.4**

1. about the origin at $90°$, and label it B.

Rotation about the origin at $90°$: $R_{90°}A \rightarrow B = R_{90°}(x, y) \rightarrow (-y, x)$

2. about the origin at $180°$, and label it O.

Rotation about the origin at $180°$: $R_{180°}A \rightarrow O = R_{180°}(x, y) \rightarrow (-x, -y)$

3. about the origin at $270°$, and label it Z.

Rotation about the origin at $270°$: $R_{270°}A \rightarrow Z = R270°(x, y) \rightarrow (y, -x)$

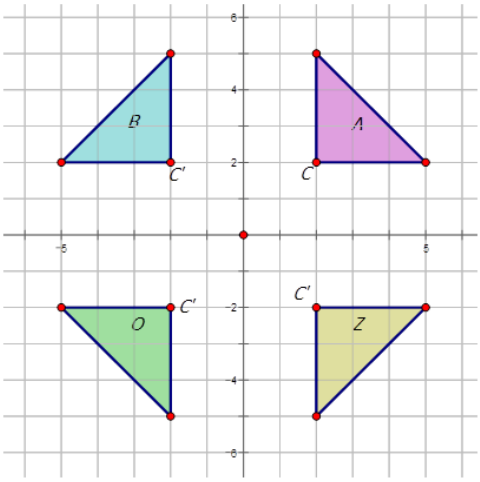

Figure **8.11.5**

Finally, let's write the notation that represents the rotation of the preimage A to the rotated image J in the diagram below:

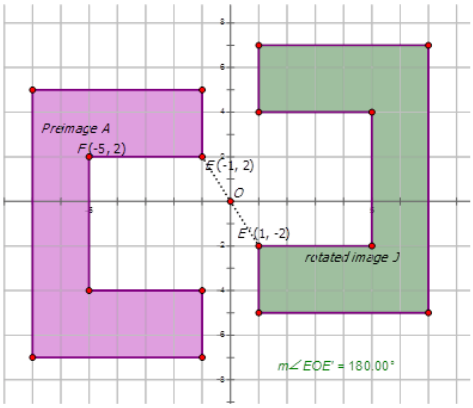

Figure **8.11.6**

First, pick a point in the diagram to use to see how it is rotated.

$E : (-1, 2)$ $E' : (1, -2)$

Notice how both the x- and y-coordinates are multiplied by -1. This indicates that the preimage A is reflected about the origin by $180°$CCW to form the rotated image J. Therefore the notation is $R_{180°}A \rightarrow J = R_{180°}(x, y) \rightarrow (-x, -y)$.

Example 8.11.1

Earlier, you were given the figure below which shows a pattern of two fish. Write the mapping rule for the rotation of Image A to Image B.

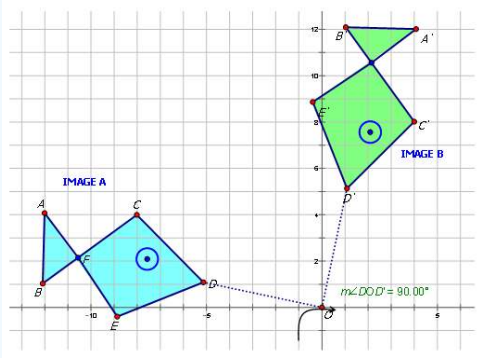

Figure 8.11.7

Solution

Notice that the angle measure is $90°$ and the direction is clockwise. Therefore the Image A has been rotated $-90°$ to form Image B. To write a rule for this rotation you would write: $R_{270°}(x, y) = (-y, x)$.

Example Runaway argument for \eatSpaces?

Thomas describes a rotation as point J moving from $J(-2, 6)$ to $J'(6, 2)$. Write the notation to describe this rotation for Thomas.

$J : (-2, 6)$ $J' : (6, 2)$

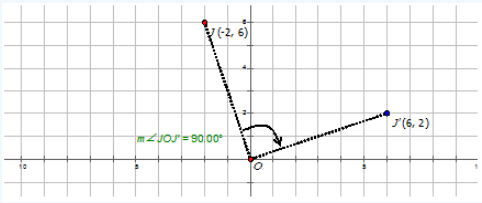

Figure 8.11.8

Solution

Since the x-coordinate is multiplied by -1, the y-coordinate remains the same, and finally the x- and y-coordinates change places, this is a rotation about the origin by $270°$ or $-90°$. The notation is: $R_{270°}J \rightarrow J' = R_270°(x, y) \rightarrow (y, -x)$

Example 8.11.1

Write the notation that represents the rotation of the yellow diamond to the rotated green diamond in the diagram below.

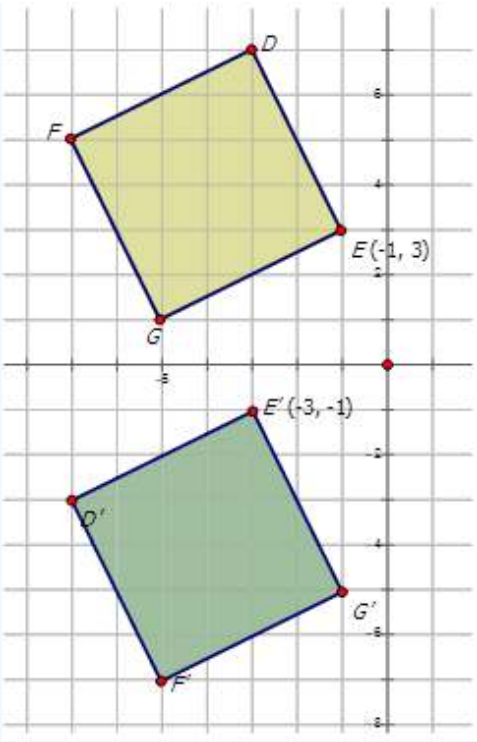

Figure **8.11.9**

Solution

In order to write the notation to describe the rotation, choose one point on the preimage (the yellow diamond) and then the rotated point on the green diamond to see how the point has moved. Notice that point E\) is shown in the diagram:

$E(-1, 3) \rightarrow E'(-3, -1)$

Since both x- and y-coordinates are reversed places and the y-coordinate has been multiplied by -1, the rotation is about the origin **90°**. The notation for this rotation would be: $R_{90°}(x, y) \rightarrow (-y, x)$.

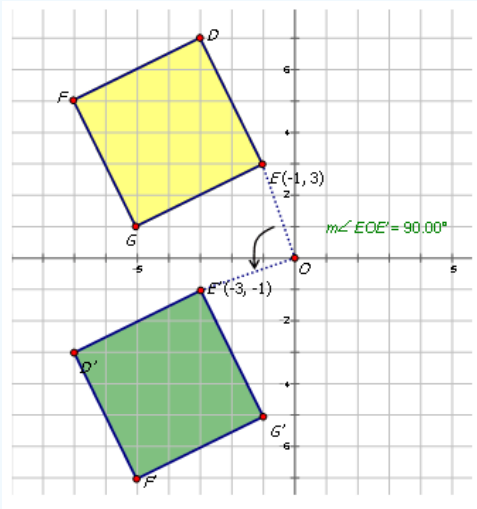

Figure **8.11.10**

Example 8.11.1

Karen was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.

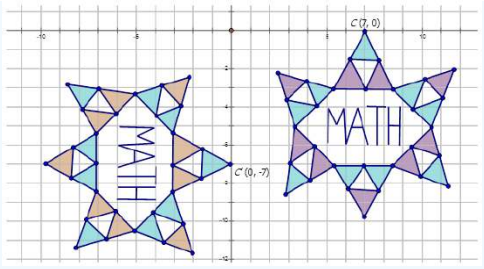

Figure **8.11.11**

Solution

In order to write the notation to describe the transformation, choose one point on the preimage (purple and blue diagram) and then the transformed point on the orange and blue diagram to see how the point has moved. Notice that point C is shown in the diagram:

$C(7,0) \rightarrow C'(0,-7)$

Since the x-coordinates only are multiplied by -1, and then x- and y-coordinates change places, the transformation is a rotation is about the origin by $270°$. The notation for this rotation would be: $R_{270°}(x,y) \rightarrow (y,-x)$.

Review

Complete the following table:

Starting Point	**90° Rotation**	**180° Rotation**	**270° Rotation**	**360° Rotation**
1. $(1,4)$				
2. $(4,2)$				
3. $(2,0)$				
4. $(-1,2)$				
5. $(-2,-3)$				

Write the notation that represents the rotation of the preimage to the image for each diagram below.

6.

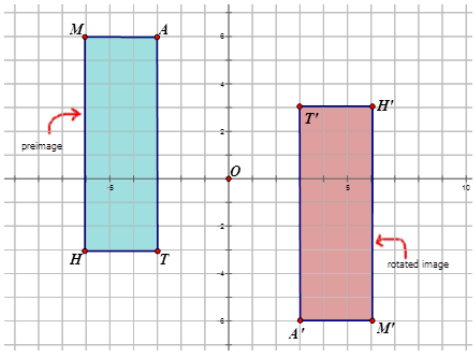

Figure **8.11.12**

7.

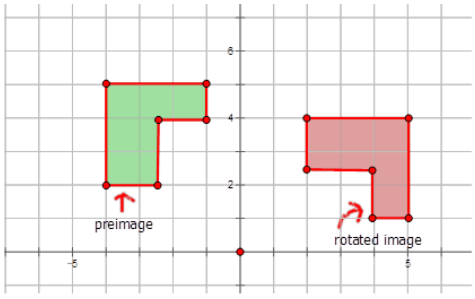

Figure **8.11.13**

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8.

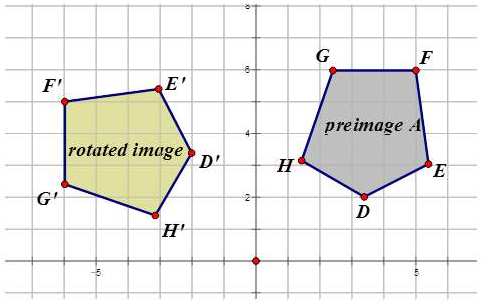

Figure 8.11.14

9.

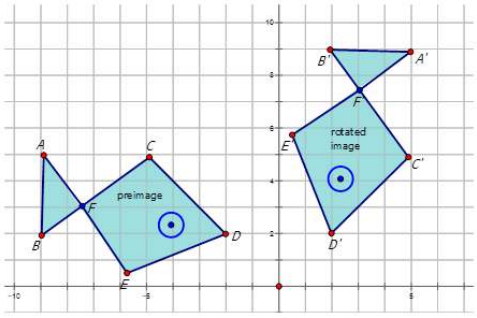

Figure 8.11.15

10.

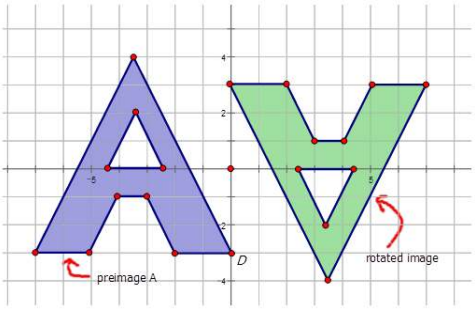

Figure 8.11.16

Write the notation that represents the rotation of the preimage to the image for each diagram below.

11.

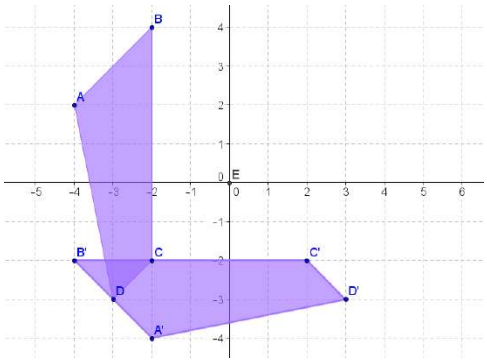

Figure 8.11.17

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12.

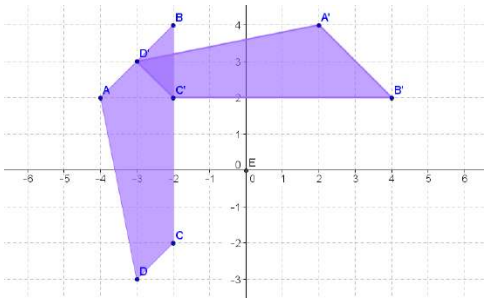

Figure **8.11.18**

13.

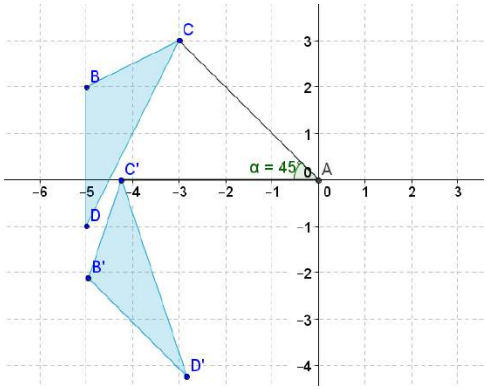

Figure **8.11.19**

14.

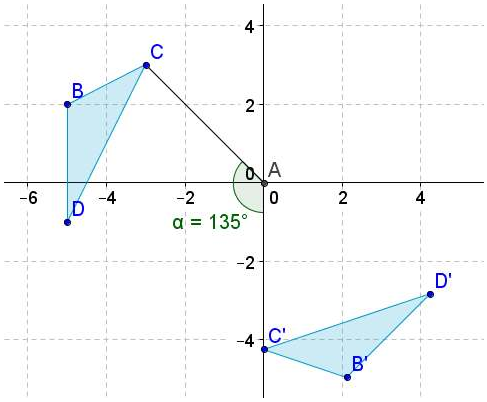

Figure **8.11.20**

15.

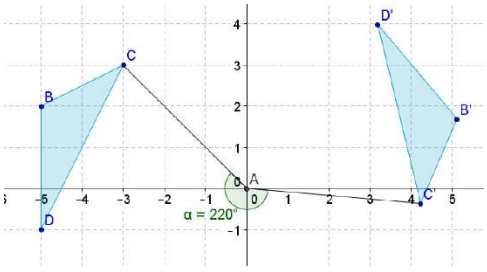

Figure **8.11.21**

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.9.

Additional Resources

Video: Rules for Rotations

Practice: Rotation Rules

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8.12: Geometry Software and Graphing Rotations

Graph rotated images given preimage and number of degrees. Perform rotations using Geogebra.

Quadrilateral $WXYZ$ has coordinates $W(-5,-5)$, $X(-2,0)$, \(Y(2,3)$ and $Z(-1,3)$. Draw the quadrilateral on the Cartesian plane. Rotate the image $110°$ counterclockwise about the point X. Show the resulting image.

Graphs of Rotations

In geometry, a **transformation** is an operation that moves, flips, or changes a shape to create a new shape. A **rotation** is an example of a transformation where a figure is rotated about a specific point (called the center of rotation), a certain number of degrees.

For now, in order to graph a rotation in general you will use geometry software. This will allow you to rotate any figure any number of degrees about any point. There are a few common rotations that are good to know how to do without geometry software, shown in the table below.

Center of Rotation	Angle of Rotation	Preimage (Point P)	Rotated Image (Point P')
$(0,0)$	$90°$ (or \(−270^{\circ})	(x,y)	$(-y,x)$
$(0,0)$	$180°$ (or $-180°$)	(x,y)	$(-x,-y)$
$(0,0)$	$270°$ (or $-90°$)	(x,y)	$(y,-x)$

We couldn't verify the security of your connection.

Access to this content has been restricted. Contact your internet service provider for help.

Let's draw the preimage and image and properly label each for the following transformation:

Line $\overline{AB}$ drawn from $(-4,2)$ to $(3,2)$ has been rotated about the origin at an angle of $90°$ CW.

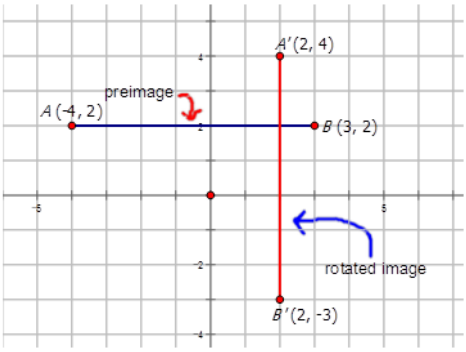

Figure **8.12.1**

Now, let's draw and label the rotated image for the following rotations:

1. The diamond $ABCD$ is rotated $145°$ CCW about the origin to form the image $A'B'C'D'$.

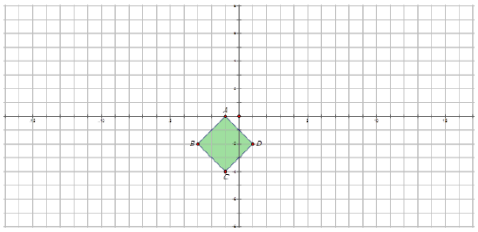

Figure 8.12.2

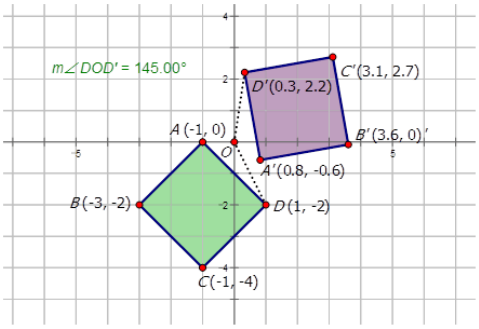

Figure 8.12.3

Notice the direction is counter-clockwise.

2. The following figure is rotated about the origin $200°$ CW to make a rotated image.

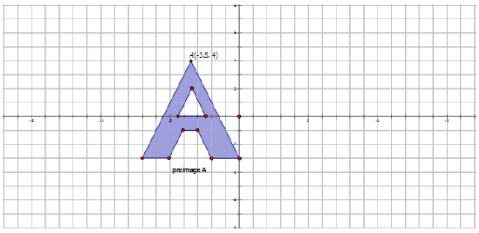

Figure 8.12.4

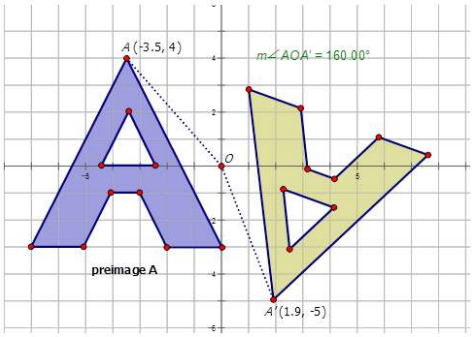

Figure 8.12.5

Notice the direction of the rotation is counter-clockwise, therefore the angle of rotation is $160°$.

Example 8.12.1

Earlier, you were asked about the quadrilateral $WXYZ$ has coordinates $W(-5,-5)$, $X(-2,0)$, Y(2,3)\) and $Z(-1,3)$. Draw the quadrilateral on the Cartesian plane. Rotate the image $110°$ counterclockwise about the point X\). Show the resulting image.

Solution

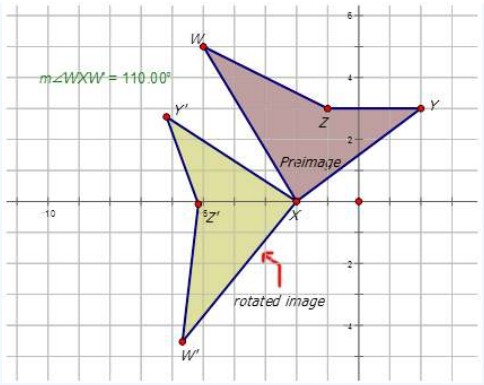

Figure **8.12.6**

Example 8.12.2

Line $\overline{ST}$ drawn from $(-3, 4)$ to $(-3, 8)$ has been rotated $60°$ CW about the point S. Draw the preimage and image and properly label each.

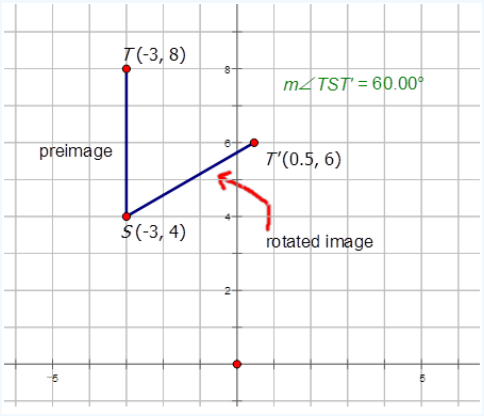

Figure **8.12.7**

Solution

Notice the direction of the angle is clockwise, therefore the angle measure is $60°$ CW or $-60°$.

Example 8.12.3

The polygon below has been rotated $155°$ CCW about the origin. Draw the rotated image and properly label each.

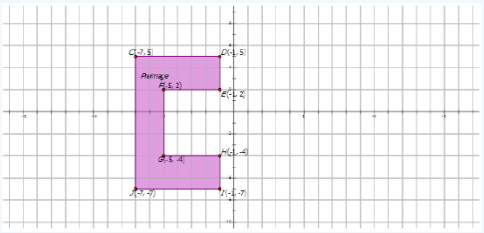

Figure **8.12.8**

Solution

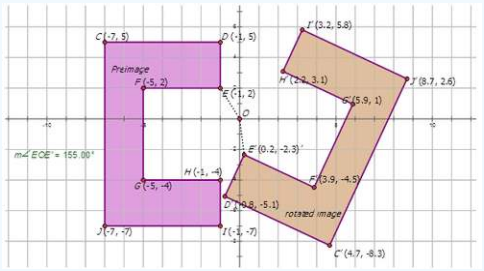

Figure **8.12.9**

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Notice the direction of the angle is counter-clockwise, therefore the angle measure is $155°$ CCW or $155°$.

Example 8.12.4

The purple pentagon is rotated about the point A $225°$. Find the coordinates of the purple pentagon. On the diagram, draw and label the rotated pentagon.

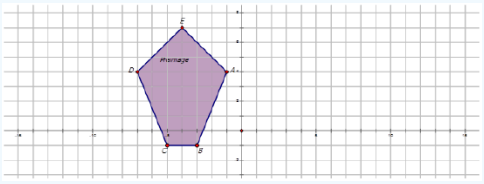

Figure 8.12.10

Solution

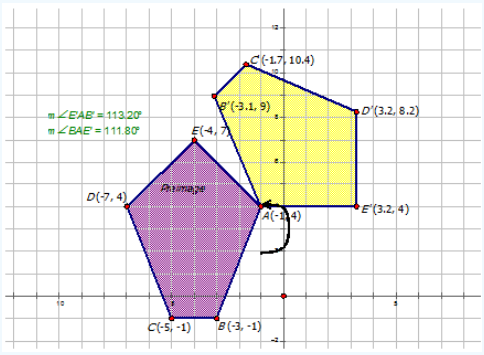

Figure 8.12.11

The measure of $\angle BAB' = m\angle BAE' + m\angle E'AB'$. Therefore $\angle BAB' = 111.80° + 113.20°$ or $225°$. Notice the direction of the angle is counter-clockwise, therefore the angle measure is $225°$ CCW or $225°$.

Review

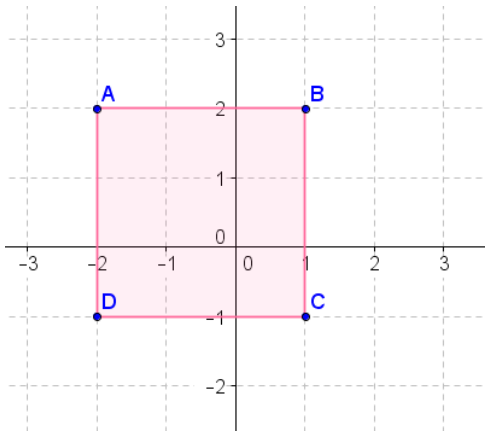

Figure 8.12.12

1. Rotate the above figure $90°$ clockwise about the origin.
2. Rotate the above figure $270°$ clockwise about the origin.
3. Rotate the above figure $180°$ about the origin.

https://k12.libretexts.org/@go/page/6056

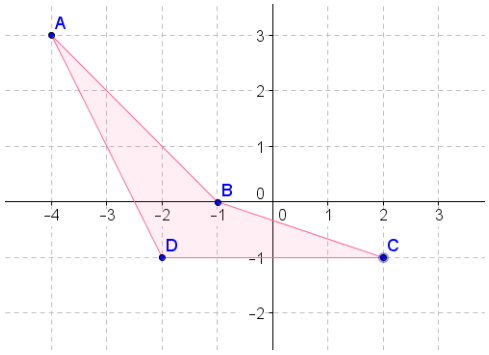

Figure **8.12.13**

4. Rotate the above figure **90°** counterclockwise about the origin.
5. Rotate the above figure **270°** counterclockwise about the origin.
6. Rotate the above figure **180°** about the origin.

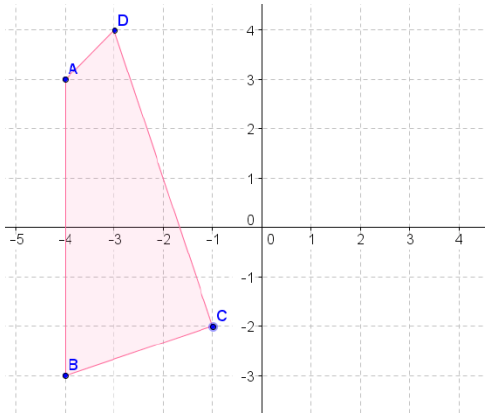

Figure **8.12.14**

7. Rotate the above figure **90°** clockwise about the origin.
8. Rotate the above figure **270°** clockwise about the origin.
9. Rotate the above figure **180°** about the origin.

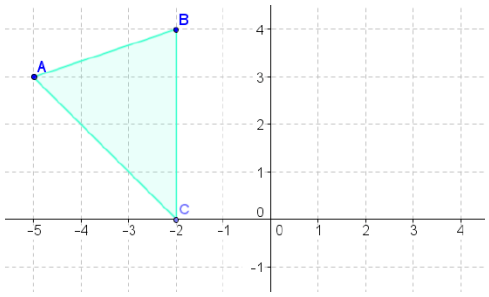

Figure **8.12.15**

10. Rotate the above figure **90°** counterclockwise about the origin.
11. Rotate the above figure **270°** counterclockwise about the origin.
12. Rotate the above figure **180°** about the origin.

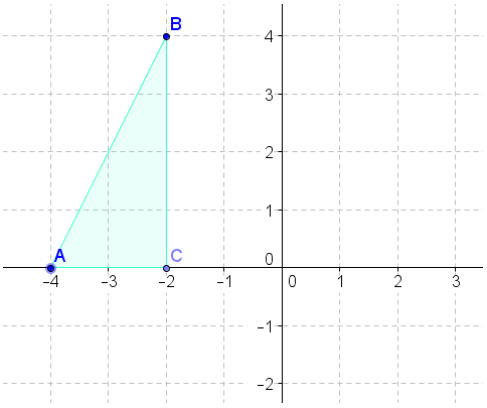

Figure **8.12.16**

13. Rotate the above figure **90°** clockwise about the origin.
14. Rotate the above figure **270°** clockwise about the origin.
15. Rotate the above figure **180°** about the origin.

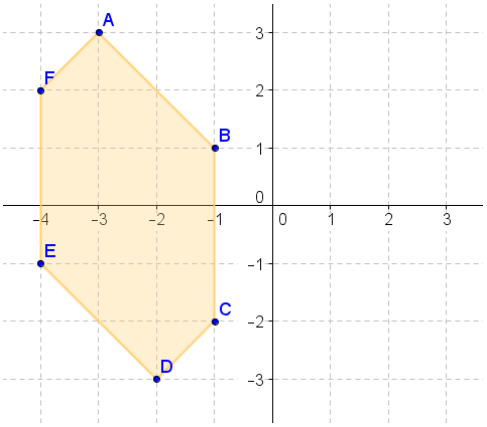

Figure **8.12.17**

16. Rotate the above figure **90°** counterclockwise about the origin.
17. Rotate the above figure **270°** counterclockwise about the origin.
18. Rotate the above figure **180°** about the origin.

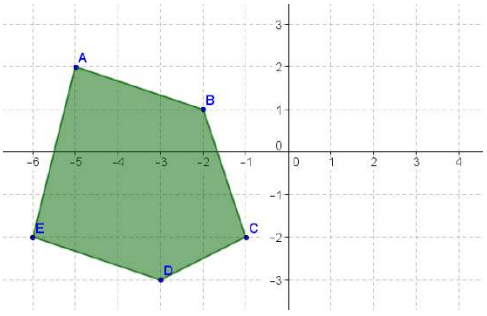

Figure **8.12.18**

19. Rotate the above figure **90°** clockwise about the origin.
20. Rotate the above figure **270°** clockwise about the origin.
21. Rotate the above figure **180°** about the origin.

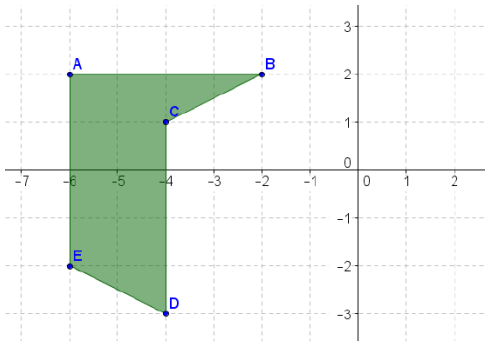

Figure **8.12.19**

22. Rotate the above figure **90°** counterclockwise about the origin.
23. Rotate the above figure **270°** counterclockwise about the origin.
24. Rotate the above figure **180°** about the origin.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.8.

Vocabulary

Term	Definition
Rotation	A rotation is a transformation that turns a figure on the coordinate plane a certain number of degrees about a given point without changing the shape or size of the figure.

Additional Resources

Practice: Geometry Software and Graphing Rotations

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8.13: Defining Reflection

Transformations that turn a figure into its mirror image by flipping it over a line.

Reflections

A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change the size or shape of a figure.

The rigid transformations are translations, **reflections**, and rotations. The new figure created by a transformation is called the **image**. The original figure is called the **preimage**. If the preimage is A, then the image would be A', said "a prime." If there is an image of A', that would be labeled A'', said "a double prime."

A **reflection** is a transformation that turns a figure into its mirror image by flipping it over a line. The **line of reflection** is the line that a figure is reflected over. If a point is on the line of reflection then the image is the same as the preimage. Images are always congruent to preimages.

Figure 8.13.1

While you can reflect over any line, some common lines of reflection have rules that are worth memorizing:

Reflection over the y-axis: $(x, y) \rightarrow (-x, y)$

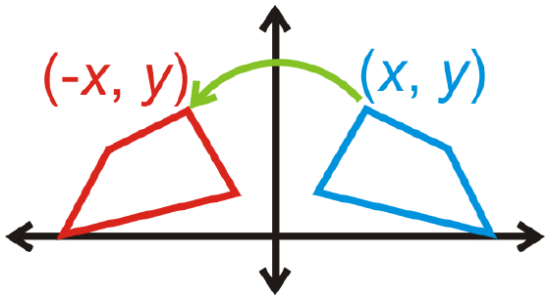

Figure 8.13.2

Reflection over the x-axis: $(x, y) \rightarrow (x, -y)$

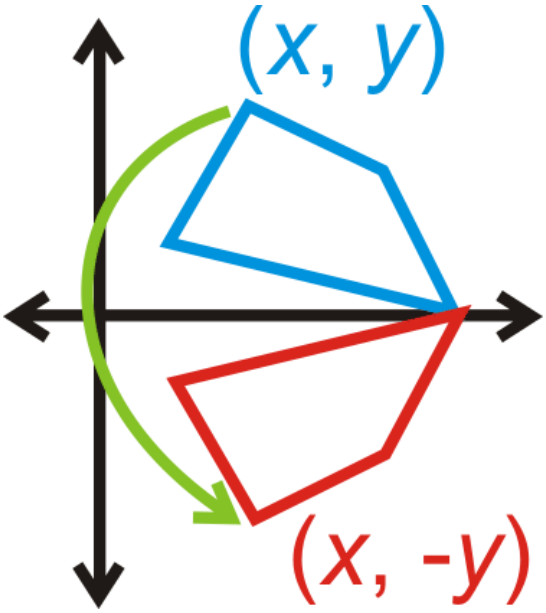

Figure **8.13.3**

Reflection over $y = x$: $(x, y) \rightarrow (y, x)$

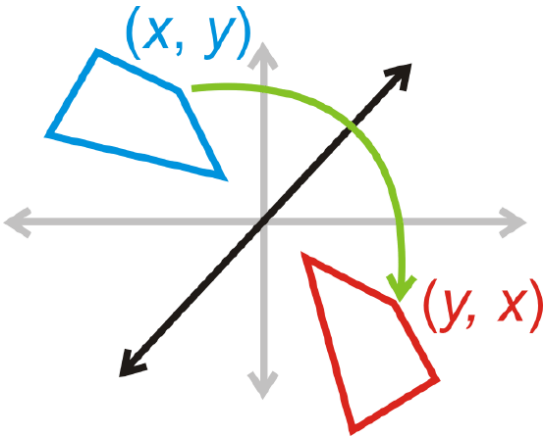

Figure **8.13.4**

Reflection over $y = -x$: $(x, y) \rightarrow (-y, -x)$

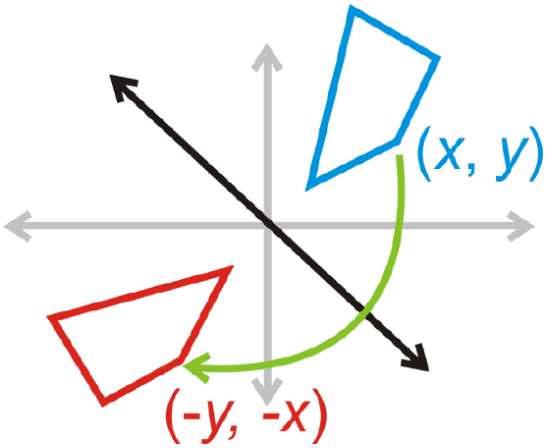

Figure **8.13.5**

What if you were given the coordinates of a quadrilateral and you were asked to reflect that quadrilateral over the y-axis? What would its new coordinates be?

Example 8.13.1

Reflect ΔABC over the y ordinates of the image.

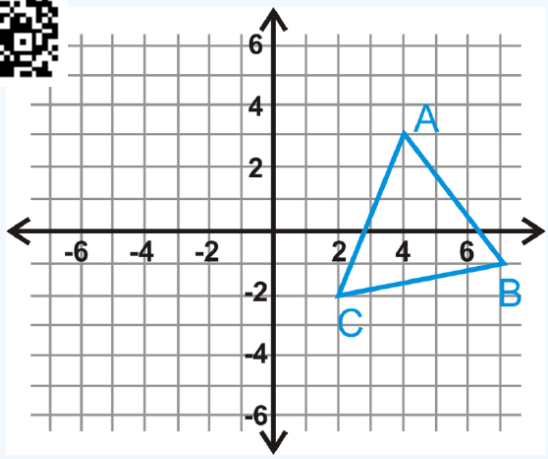

Figure 8.13.6

Solution

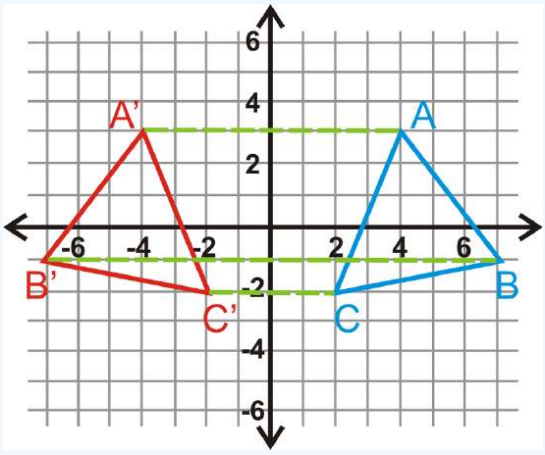

Figure 8.13.7

$\Delta A'B'C'$ will be the same distance away from the y-axis as ΔABC, but on the other side. Hence, their x-coordinates will be opposite.

$A(4,3) \rightarrow A'(-4,3)$
$B(7,-1) \rightarrow B'(-7,-1)$
$C(2,-2) \rightarrow C'(-2,-2)$

Example 8.13.2

Reflect the letter "F" over the x-axis.

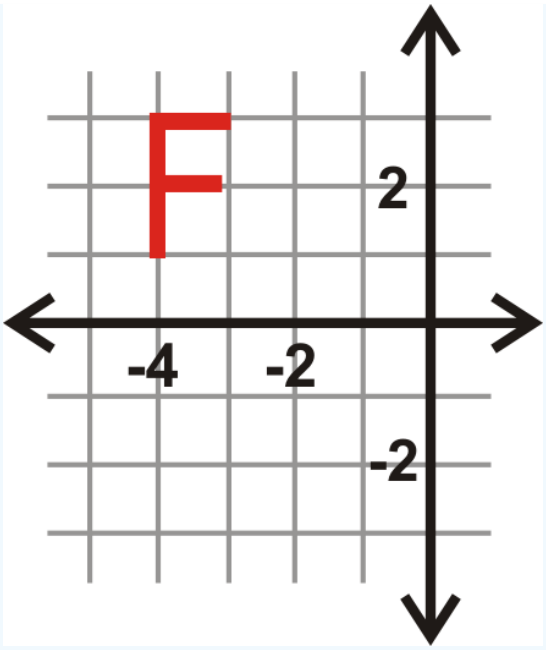

Figure 8.13.8

Solution

When reflecting the letter F over the x-axis, the y–coordinates will be the same distance away from the x-axis, but on the other side of the x-axis. Hence, their y-coordinates will be opposite.

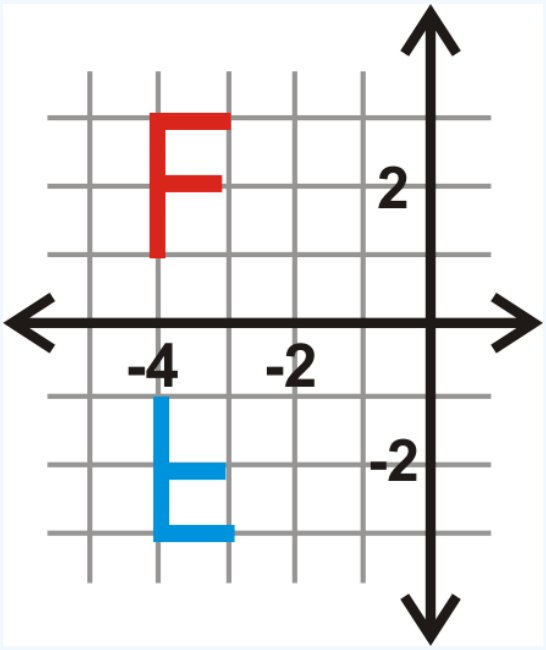

Figure 8.13.9

Example 8.13.3

Reflect the triangle ΔABC with vertices $A(4,5)$, $B(7,1)$ and $C(9,6)$ over the line $x=5$. Find the coordinates of A', B', and C'.

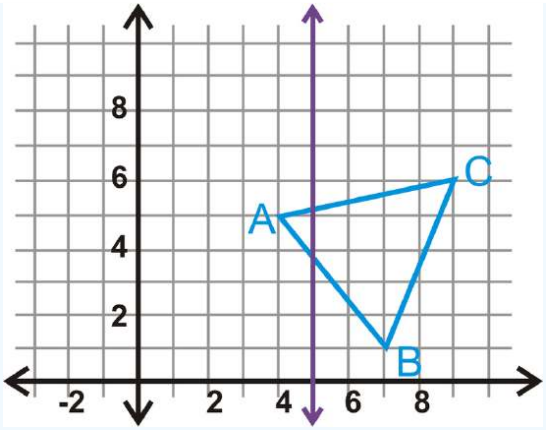

Figure 8.13.10

Solution

The image's vertices are the same distance away from $x = 5$ as those of the preimage.

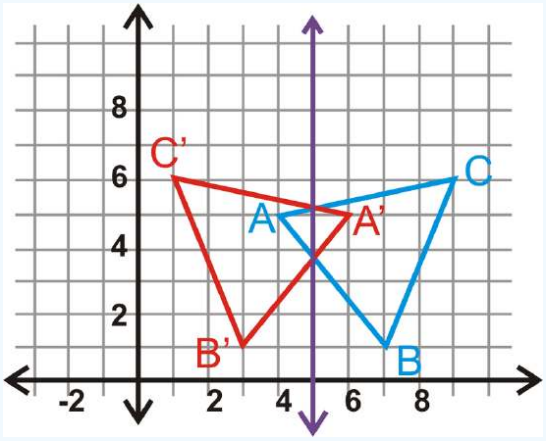

Figure 8.13.11

$A(4,5) \rightarrow A'(6,5)$
$B(7,1) \rightarrow B'(3,1)$
$C(9,6) \rightarrow C'(1,6)$

Example 8.13.4

Reflect the line segment $\overline{PQ}$ with endpoints $P(-1,5)$ and $Q(7,8)$ over the line $y = 5$.

Solution

P is on the line of reflection, which means P' has the same coordinates. Q' is the same distance away from \(y=5, but on the other side.

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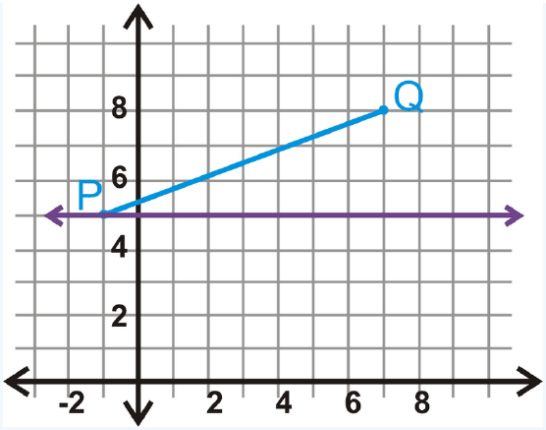

Figure 8.13.12

$P(-1,5) \rightarrow P'(-1,5)$
$Q(7,8) \rightarrow Q'(7,2)$

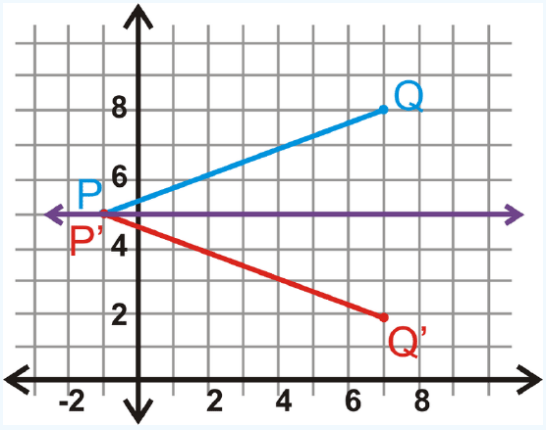

Figure 8.13.13

Example 8.13.5

A triangle ΔLMN and its reflection, $\Delta L'M'N'$ are below. What is the line of reflection?

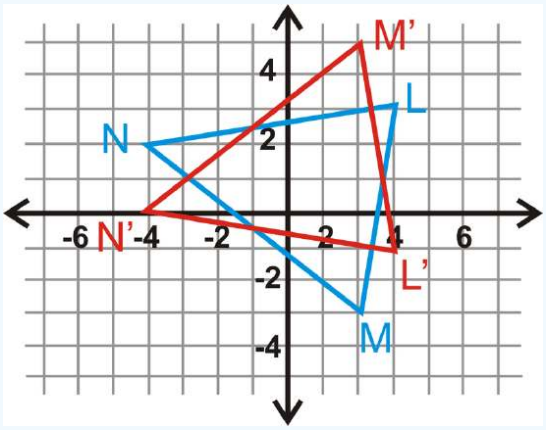

Figure \(\PageIndex{14}\)

Solution

Looking at the graph, we see that the corresponding parts of the preimage and image intersect when $y = 1$. Therefore, this is the line of reflection.

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If the image does not intersect the preimage, find the midpoint between the preimage point and its image. This point is on the line of reflection.

Example 8.13.6

Reflect the trapezoid $TRAP$ over the line $y = -x$.

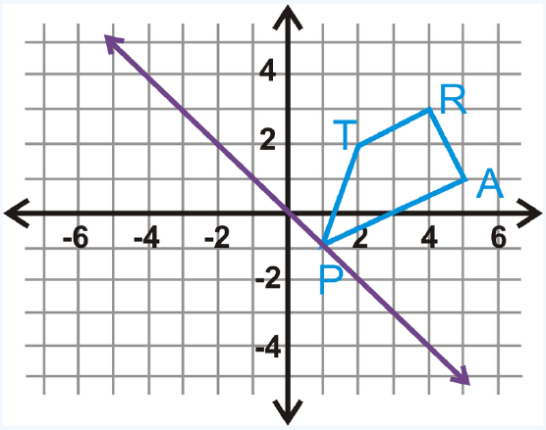

Figure 8.13.15

Solution

The purple line is $y = -x$. You can reflect the trapezoid over this line.

$T(2, 2) \rightarrow T'(-2, -2)$
$R(4, 3) \rightarrow R'(-3, -4)$
$A(5, 1) \rightarrow A'(-1, -5)$
$P(1, -1) \rightarrow P'(1, -1)$

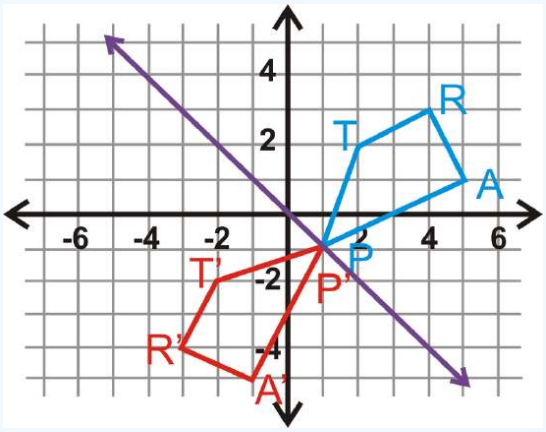

Figure 8.13.16

Review

1. If $(5, 3)$ is reflected over the y-axis, what is the image?
2. If $(5, 3)$ is reflected over the x-axis, what is the image?
3. If $(5, 3)$ is reflected over $y = x$, what is the image?
4. If $(5, 3)$ is reflected over $y = -x$, what is the image?
5. Plot the four images. What shape do they make? Be specific.
6. Which letter is a reflection over a vertical line of the letter "b"?
7. Which letter is a reflection over a horizontal line of the letter "b"?

Reflect each shape over the given line.

8. y-axis

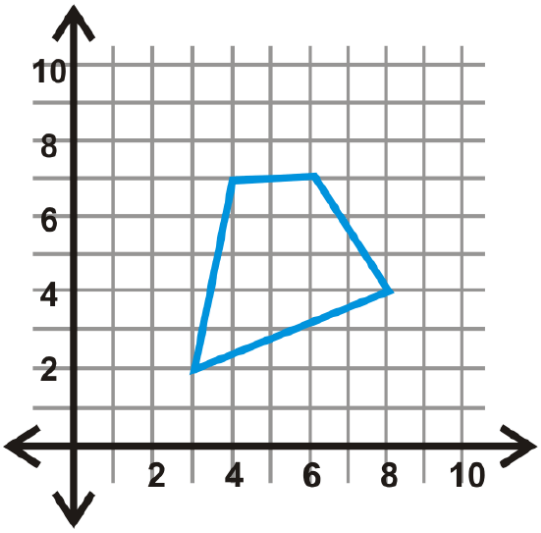

Figure 8.13.17

9. x-axis

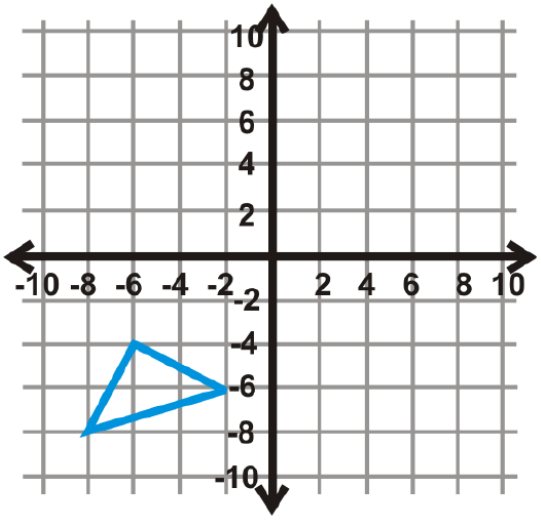

Figure 8.13.18

10. $y = 3$

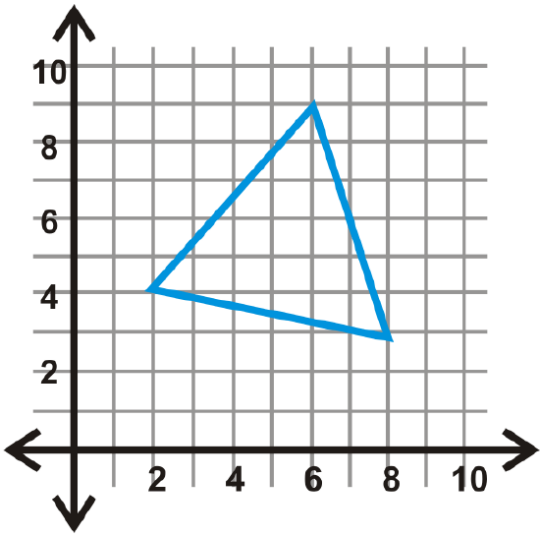

Figure 8.13.19

11. $x = -1$

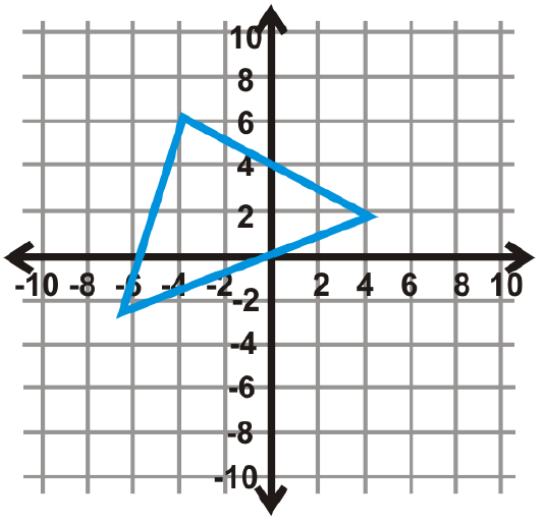

Figure 8.13.20

12. x-axis

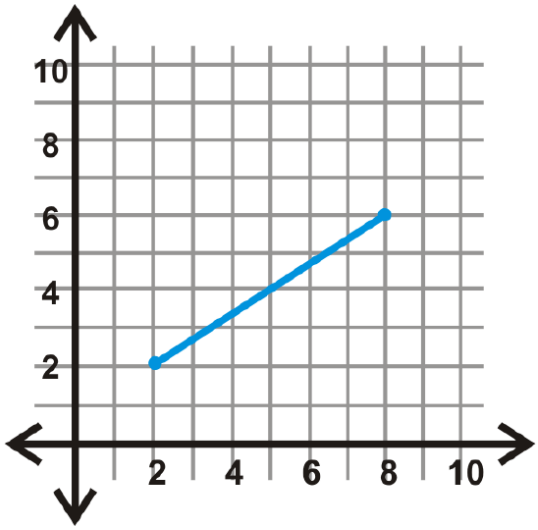

Figure 8.13.21

13. y-axis

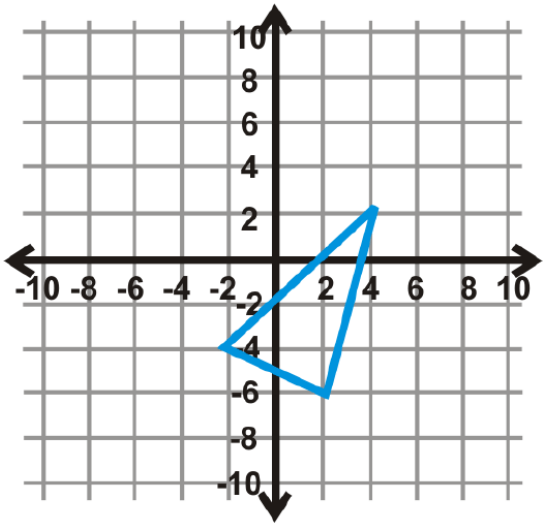

Figure 8.13.22

14. $y = x$

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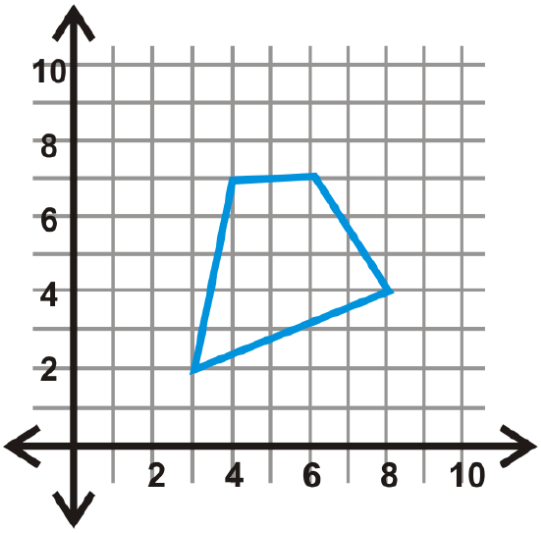

Figure **8.13.23**

15. $y = -x$

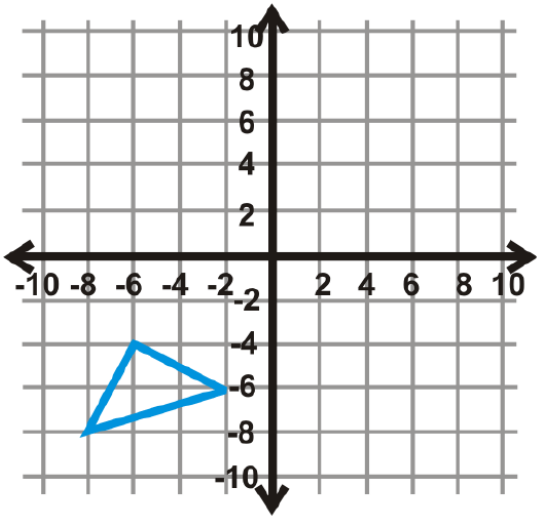

Figure \(\PageIndex{24\)

16. $x = 2$

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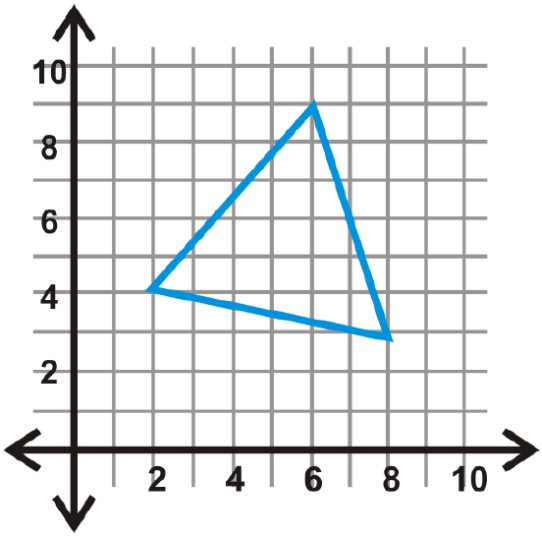

Figure 8.13.25

17. $y = -4$

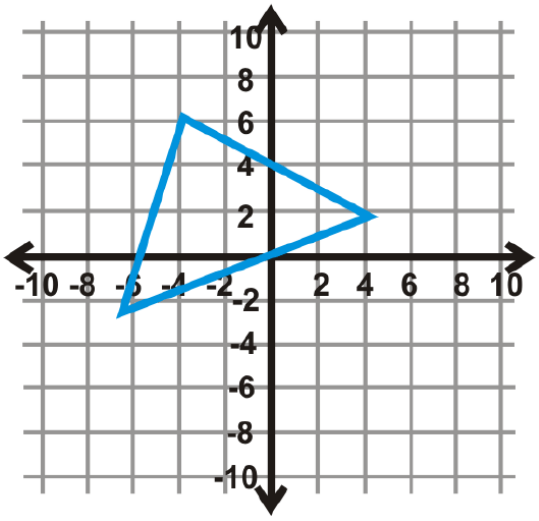

Figure 8.13.26

18. $y = -x$

https://k12.libretexts.org/@go/page/6087

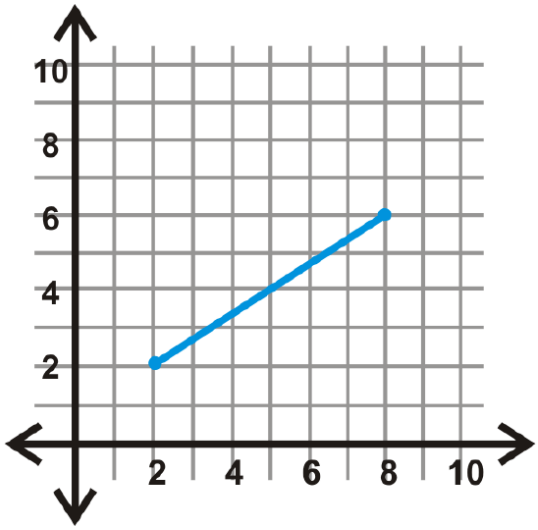

Figure 8.13.27

19. $y = x$

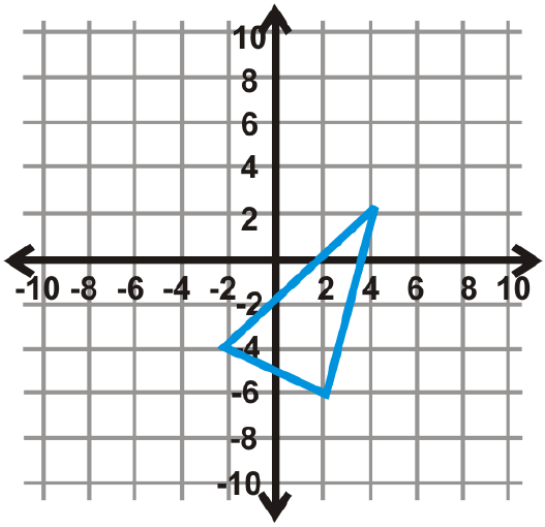

Figure 8.13.28

Find the line of reflection the blue triangle (preimage) and the red triangle (image).

20.

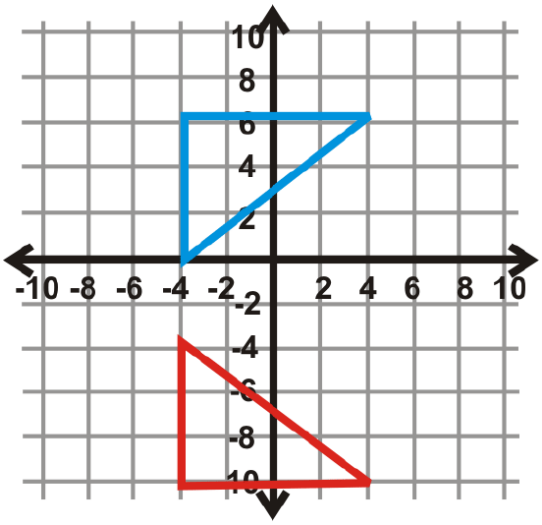

Figure 8.13.29

21.

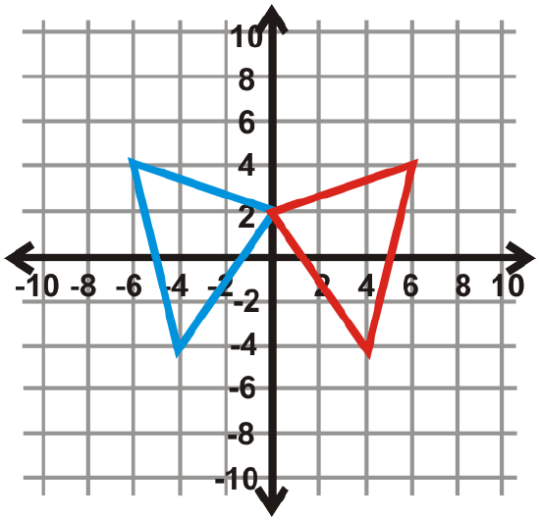

Figure 8.13.30

22.

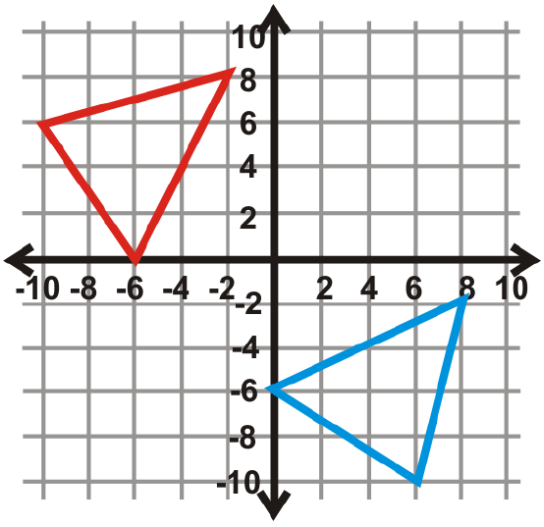

Figure 8.13.31

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Vocabulary

Term	Definition
Coordinate Plane	The coordinate plane is a grid formed by a horizontal number line and a vertical number line that cross at the $(0, 0)$ point, called the origin. The coordinate plane is also called a Cartesian Plane.
Geometric Patterns	Geometric patterns are visual patterns of geometric figures that follow a rule.
Image	The image is the final appearance of a figure after a transformation operation.
perpendicular bisector	A perpendicular bisector of a line segment passes through the midpoint of the line segment and intersects the line segment at $90°$.
Perpendicular lines	Perpendicular lines are lines that intersect at a $90°$ angle.
Preimage	The pre-image is the original appearance of a figure in a transformation operation.
Reflection	A reflection is a transformation that flips a figure on the coordinate plane across a given line without changing the shape or size of the figure.
Transformation	A transformation moves a figure in some way on the coordinate plane.
Rigid Transformation	A rigid transformation is a transformation that preserves distance and angles, it does not change the size or shape of the figure.

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.5.

Additional Resources

Video: Transformation: Reflection Principles - Basic

Activities: Reflections Discussion Questions

Study Aids: Types of Transformations Study Guide

Practice: Defining Reflection

Real World: Reflecting Reality

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8.14: Rules for Reflections

Identify and state rules describing reflections using notation

The figure below shows a pattern of two fish. Write the mapping rule for the reflection of Image A to Image B.

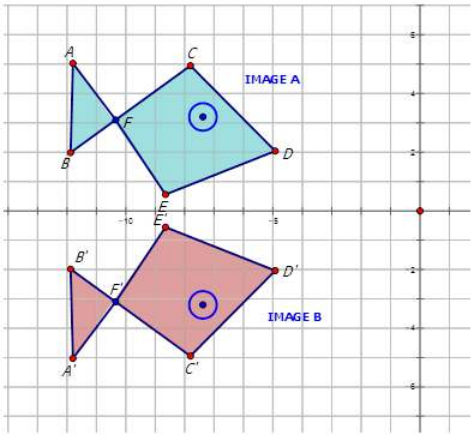

Figure 8.14.1

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A reflection is an example of a transformation that takes a shape (called the preimage) and flips it across a line (called the line of reflection) to create a new shape (called the image). By examining the coordinates of the reflected image, you can determine the line of reflection. The most common lines of reflection are the x-axis, the y-axis, or the lines $y = x$ or $y = -x$.

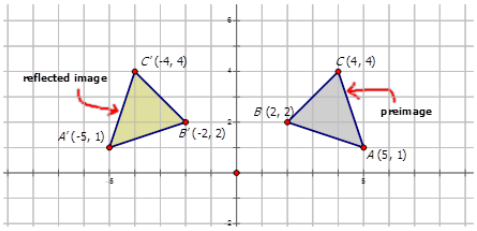

Figure 8.14.2

The preimage above has been reflected across the y-axis. This means, all of the x-coordinates have been multiplied by -1. You can describe the reflection in words, or with the following notation:

$r_{y-axis}(x, y) \rightarrow (-x, y)$

Notice that the notation tells you exactly how each (x, y) point changes as a result of the transformation.

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Let's find the image of the point $(3, 2)$ that has undergone a reflection across the following lines:

1. The x-axis,

Reflection across the x-axis: $r_{x-axis}(3, 2) \rightarrow (3, -2)$

2. The y-axis

Reflection across the y-axis: $r_{y-axis}(3, 2) \rightarrow (3, -2)$

3. The line $y = x$

Reflection across the line $y = x$: $r_{y=x}(3, 2) \rightarrow (2, 3)$

4. The line$y = -x$.

Reflection across the line$y = -x$: $r_{y=-x}(3, 2) \rightarrow (-2, -3)$

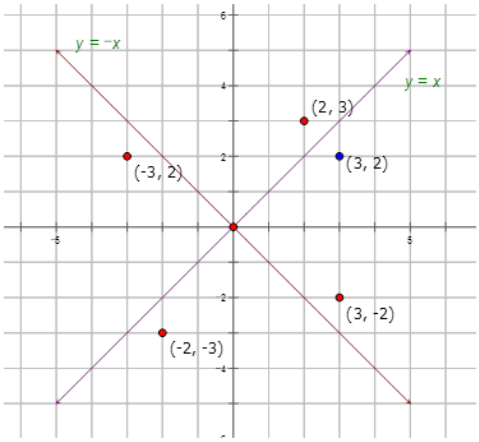

Figure 8.14.4

Now, let's reflect Image A in the diagram below across the following lines and write the notation for each reflection:

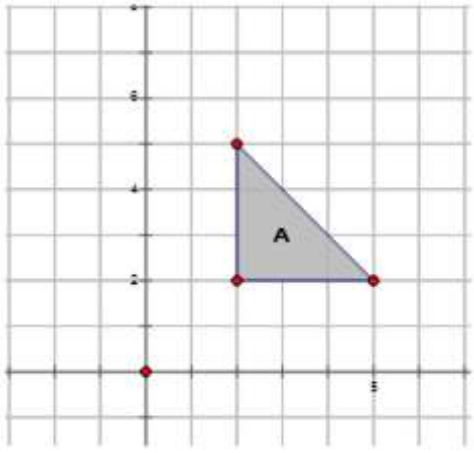

Figure 8.14.5

1. Across the y-axis and label it B.

Reflection across the y-axis: $r_{y-axis}A \rightarrow B = r_{y-axis}(x, y) \rightarrow (-x, y)$

2. Across the x-axis and label it O.

Reflection across the x-axis: $r_{x-axis}A \rightarrow O = r_{x-axis}(x, y) \rightarrow (x, -y)$

3. Across the line $y = -x$ and label it Z.

Reflection across the $y = -x$: $r_{y=-x}A \rightarrow Z = r_{y=-x}(x, y) \rightarrow (-y, -x)$

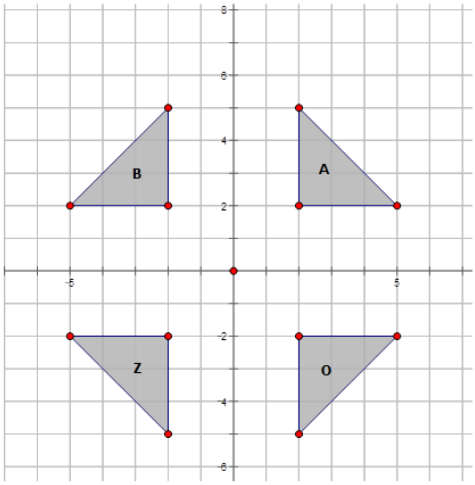

Figure 8.14.6

Finally, let's write the notation that represents the reflection of the preimage to the image in the diagram below:

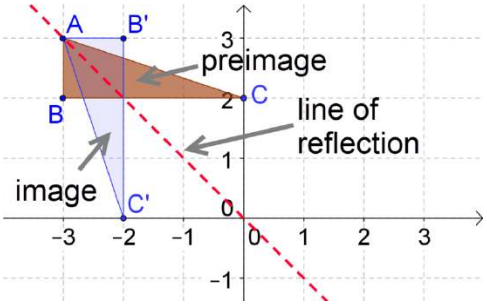

Figure 8.14.7

This is a reflection across the line $y = -x$. The notation is $r_{y=-x}(x, y) \rightarrow (-y, -x)$.

Example 8.14.1

Earlier, you were given a problem about the figure below that shows a pattern of two fish. Write the mapping rule for the reflection of Image A\) to Image B\).

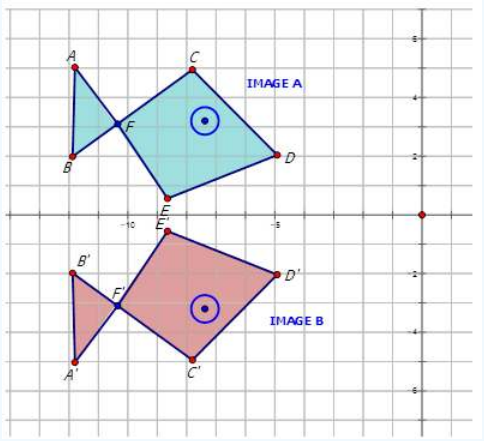

Figure 8.14.8

Solution

To answer this question, look at the coordinate points for Image A and Image B.

Image A	$A(-11.8, 5)$	$B(-11.8, 2)$	$C(-7.8, 5)$	$D(-4.9, 2)$	$E(-8.7, 0.5)$	$F(-10.4, 3.1)$
Image B	$A'(-11.8, -5)$	$B'(-11.8, -2)$	$C'(-7.8, -5)$	$D'(-4.9, -2)$	$E'(-8.7, -0.5)$	$F'(-10.4, -3.1)$

Notice that all of the $y-coordinateshavechangedsign.ThereforeImageAhasreflectedacrossthe\(x$-axis. To write a rule for this reflection you would write: $r_{x-axis}(x, y) \rightarrow (x, -y)$.

Example 8.14.2

Thomas describes a reflection as point $Jmovingfrom\(J(-2, 6)$ to $J'(-2, -6)$. Write the notation to describe this reflection for Thomas.

$$J : (-2, 6) \qquad J' : (-2, -6)$$

Solution

Since the y-coordinate is multiplied by -1 and the $x-coordinateremainsthesame, thisisareflectioninthe\(x$-axis. The notation is: $r_{x-axis}J \rightarrow J' = r_{x-axis}(-2, 6) \rightarrow (-2, 6)$

Example 8.14.3

Write the notation that represents the reflection of the yellow diamond to the reflected green diamond in the diagram below.

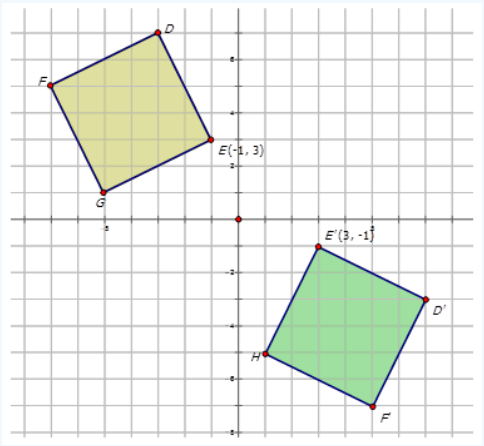

Figure 8.14.9

Solution

In order to write the notation to describe the reflection, choose one point on the preimage (the yellow diamond) and then the reflected point on the green diamond to see how the point has moved. Notice that point E is shown in the diagram:

$$E(-1,3) \rightarrow E'(3,-1)$$

Since both x- and y-coordinates are reversed numbers, the reflection is in the line $y = x$. The notation for this reflection would be: $r_{y=x}(x,y) \rightarrow (y,x)$.

Example 8.14.4

Karen was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.

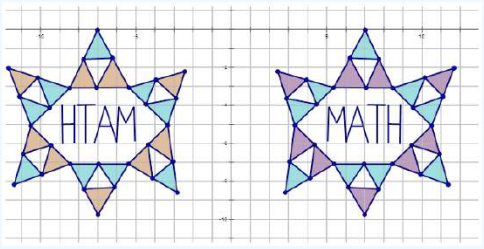

Figure **8.14.10**

Solution

In order to write the notation to describe the transformation, choose one point on the preimage (purple and blue diagram) and then the transformed point on the orange and blue diagram to see how the point has moved. Notice that point A is shown in the diagram:

$$C(7,0) \rightarrow C'(-7,0)$$

Since both x-coordinates only are multiplied by -1, the transformation is a reflection is in y-axis. The notation for this reflection would be: $r_{y-axis}(x,y) \rightarrow (-x,y)$.

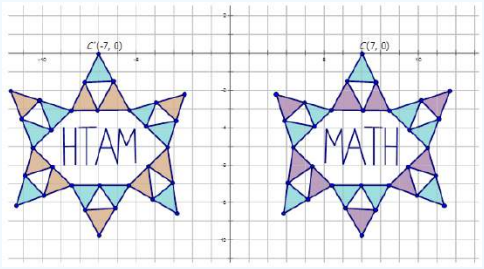

Figure **8.14.11**

Review

Write the notation to describe the movement of the points in each of the reflections below.

1. $S(1,5) \rightarrow S'(-1,5)$
2. $W(-5,-1) \rightarrow W'(5,-1)$
3. $Q(2,-5) \rightarrow Q'(2,5)$
4. $M(4,3) \rightarrow M'(-3,-4)$
5. $B(-4,-2) \rightarrow B'(-2,-4)$
6. $A(3,5) \rightarrow A'(-3,5)$
7. $C(1,2) \rightarrow C'(2,1)$
8. $D(2,-5) \rightarrow D'(5,-2)$
9. $E(3,1) \rightarrow E'(-3,1)$
10. $F(-4,2) \rightarrow F'(-4,-2)$
11. $G(1,3) \rightarrow G'(1,-3)$

Write the notation that represents the reflection of the preimage image for each diagram below.

12.

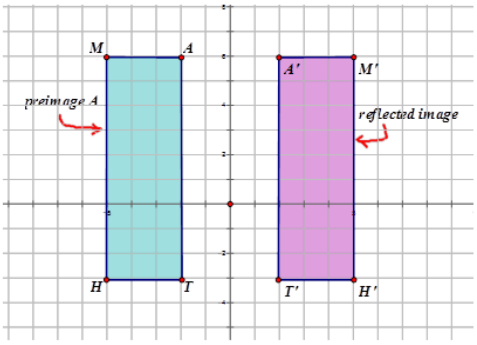

Figure 8.14.12

13.

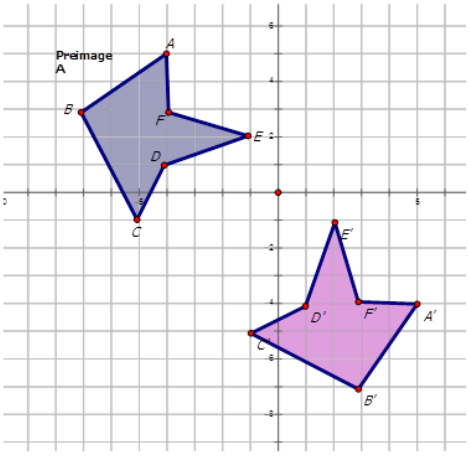

Figure 8.14.13

14.

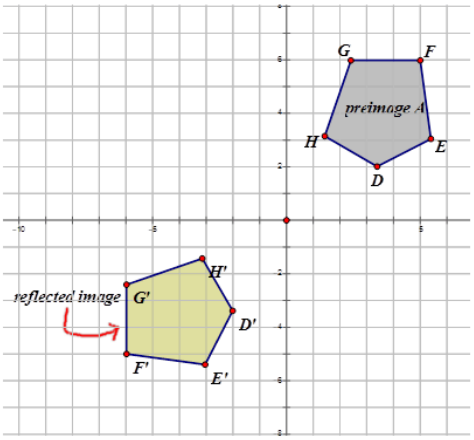

Figure 8.14.14

15.

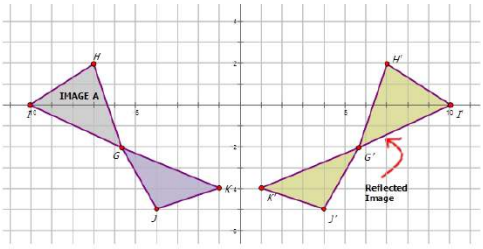

Figure 8.14.15

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.6.

Additional Resources

Practice: Rules for Reflections

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8.15: Geometry Software for Reflections

Graph images given preimage and line of reflection. Perform reflections using Geogebra.

Graphs of Reflections

Triangle A has coordinates $E(-5,-5)$, $F(2,-6)$ and $G(-2,0)$. Draw the triangle on the Cartesian plane. Reflect the image across the y-axis. State the coordinates of the resulting image.

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A reflection is an example of a transformation that takes a shape (called the preimage) and flips it across a line (called the line of reflection) to create a new shape (called the image).

To graph a reflection, you can visualize what would happen if you flipped the shape across the line.

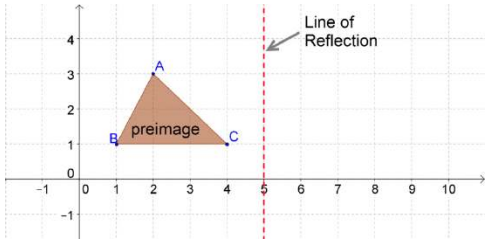

Figure 8.15.1

Each point on the preimage will be the same distance from the line of reflection as it's corresponding point in the image. For example, for the pair of triangles below, both A and A' are 3 units away from the line of reflection.

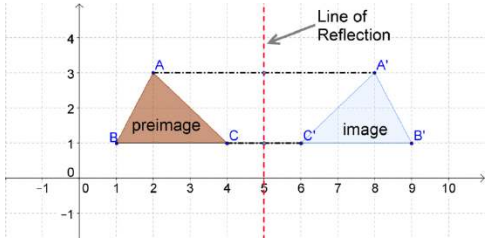

Figure 8.15.2

For common reflections, you can also remember what happens to their coordinates:

- reflections across the x-axis: y values are multiplied by -1.
- reflections across the y-axis: x values are multiplied by -1.
- reflections across the line $y = x$: x and y values switch places.
- reflections across the line $y = -x$. x and y values switch places and are multiplied by -1.

Knowing the rules above will allow you to recognize reflections even when a graph is not available.

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Let's draw the reflections described in the following problems. Include the preimage if not given.

1. Line $\overline{AB}$ drawn from $(-4, 2)$ to $(3, 2)$ has been reflected across the x-axis.

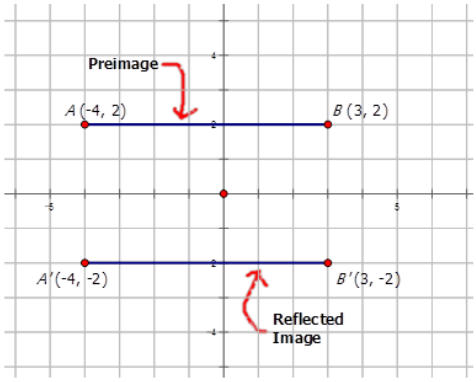

Figure 8.15.3

2. Triangle ABC is reflected across the line $y = -x$ to form the image $A'B'C'$. Draw and label the reflected image.

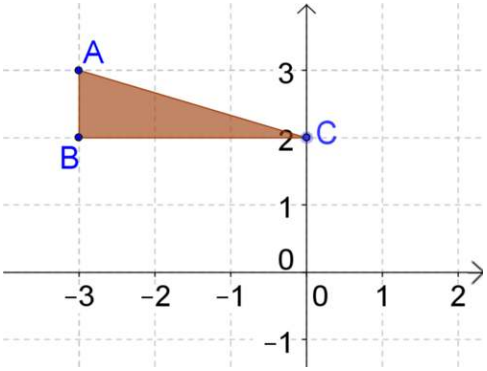

Figure 8.15.4

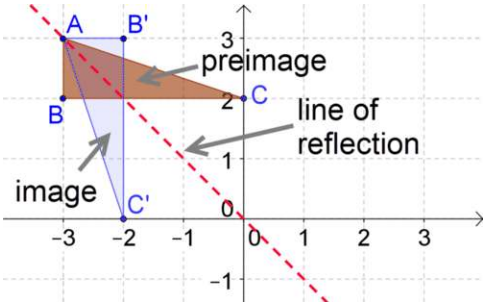

Figure 8.15.5

Now, let's find find the coordinates of the following reflected image and draw the image:

The diamond $ABCD$ is reflected across the line $y = x$ to form the image $A'B'C'D'$.

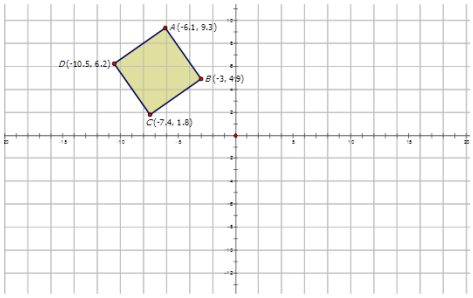

Figure 8.15.6

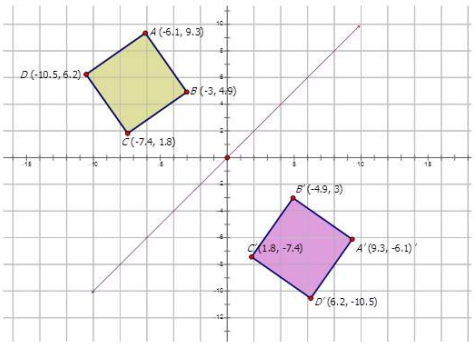

Figure 8.15.7

Example 8.15.1

Earlier, you were given the coordinates of Triangle A, $E(-5, -5)$, $F(2, -6)$, and $G(-2, 0)$, and asked to reflect the image across the y-axis. What are the coordinates of the reflected triangle?

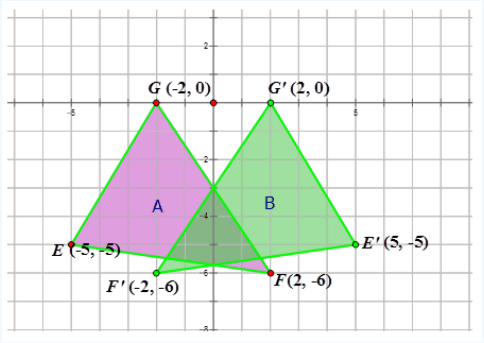

Figure 8.15.8

Solution

The coordinates of the new image (B) are $E'(5, -5)$, $F'(2, -6)$ and $G'(2, 0)$.

Example 8.15.2

Line $\overline{ST}$ drawn from $(-3, 4)$ to $(-3, 8)$ has been reflected across the line $y = -x$. Draw the preimage and image and properly label each.

Solution

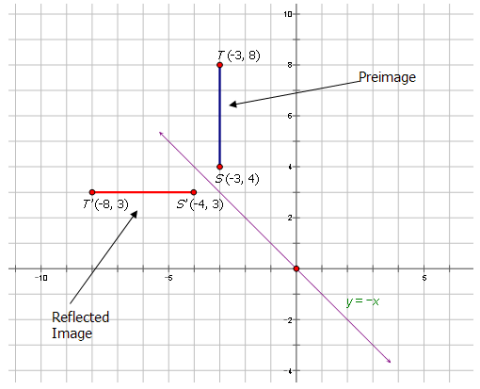

Figure 8.15.9

Example 8.15.3

The polygon below has been reflected across the y-axis. Draw the reflected image and properly label each.

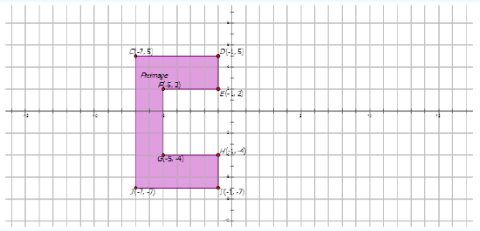

Figure 8.15.10

Solution

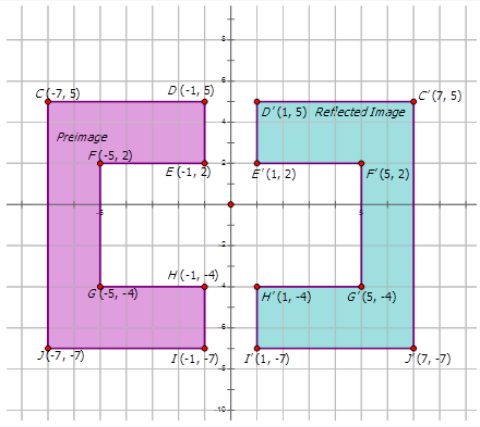

Figure 8.15.11

Example 8.15.4

The purple pentagon is reflected across the y-axis to make the new image. Find the coordinates of the purple pentagon. On the diagram, draw and label the reflected pentagon.

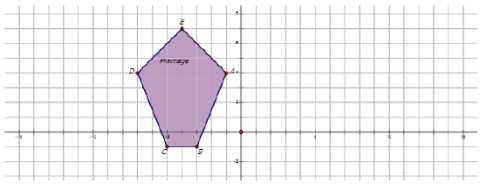

Figure 8.15.12

https://k12.libretexts.org/@go/page/6125

Solution

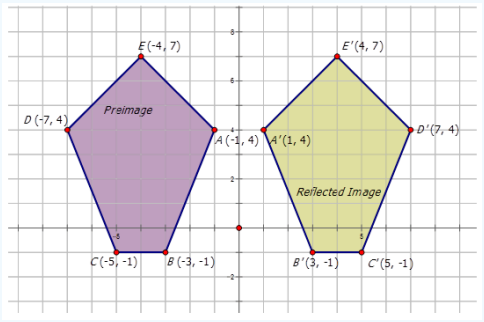

Figure **8.15.13**

Reviews

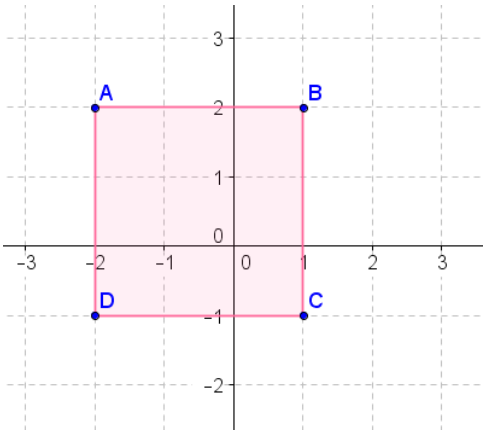

Figure **8.15.14**

1. Reflect the above figure across the x-axis.
2. Reflect the above figure across the y-axis.
3. Reflect the above figure across the line $y = x$.

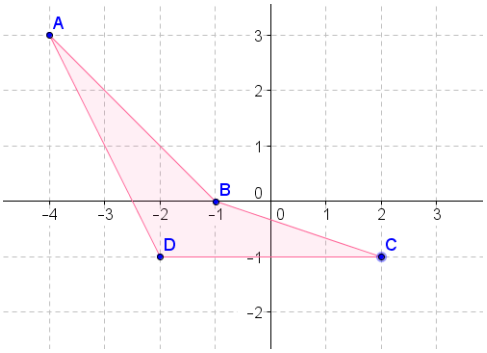

Figure **8.15.15**

4. Reflect the above figure across the x-axis.
5. Reflect the above figure across the y-axis.
6. Reflect the above figure across the line $y = x$.

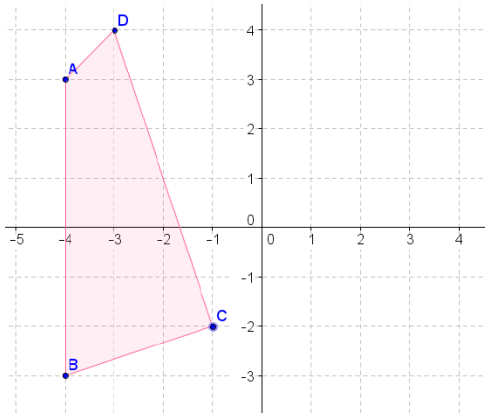

Figure **8.15.16**

7. Reflect the above figure across the x-axis.
8. Reflect the above figure across the y-axis.
9. Reflect the above figure across the line $y = x$.

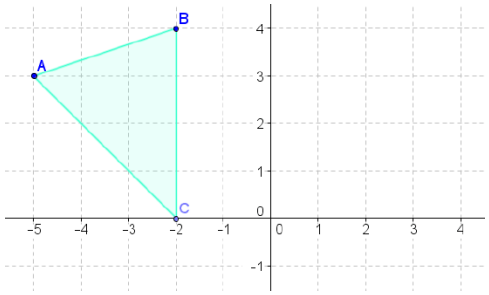

Figure **8.15.17**

10. Reflect the above figure across the x-axis.
11. Reflect the above figure across the y-axis.
12. Reflect the above figure across the line $y = x$.

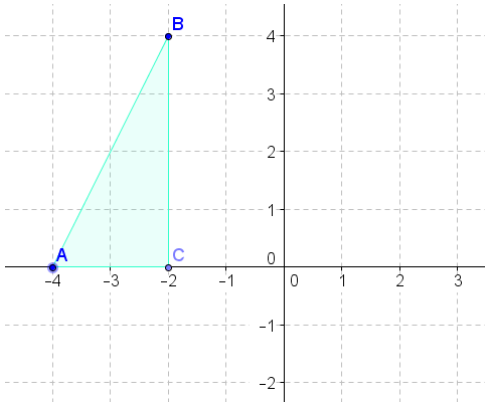

Figure **8.15.18**

13. Reflect the above figure across the x-axis.
14. Reflect the above figure across the y-axis.
15. Reflect the above figure across the line $y = x$.

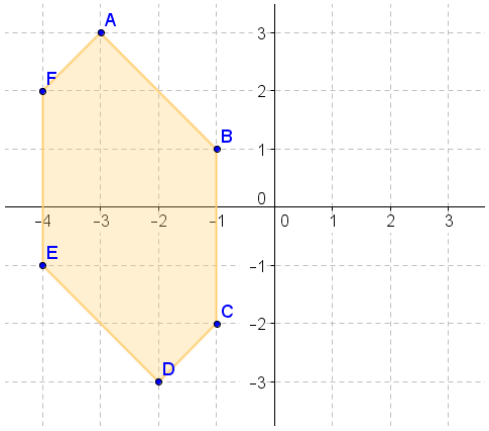

Figure **8.15.19**

16. Reflect the above figure across the x-axis.
17. Reflect the above figure across the y-axis.
18. Reflect the above figure across the line $y = x$.

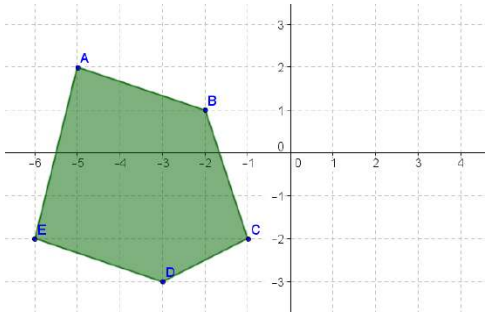

Figure **8.15.20**

19. Reflect the above figure across the x-axis.
20. Reflect the above figure across the y-axis.
21. Reflect the above figure across the line $y = x$.

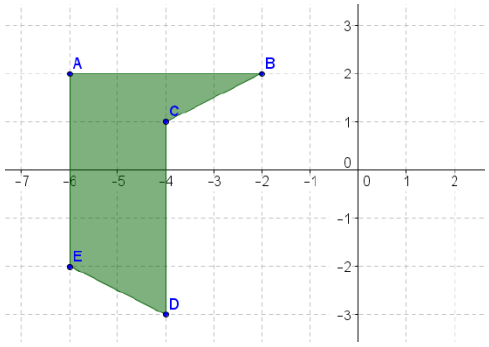

Figure **8.15.21**

22. Reflect the above figure across the x-axis.
23. Reflect the above figure across the y-axis.
24. Reflect the above figure across the line $y = x$.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.5.

Additional Resources

Practice: Geometry Software for Reflections

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8.16: Reflecting Figures

Move figures on the coordinate plane using patterns.

Translations, Rotations, and Reflections

Tyler takes a picture of an item and its **reflection**. He places a **coordinate plane** over the picture. The coordinate plane is positioned so that the x−axis separates the image from the reflection. He then makes the grid according to the key features of the picture, so that a point at $(2, 0)$ is reflected at the point $(-2, 0)$. If the original coordinates of the image are $(3, 0)$, $(4, 6)$ and $(5, 1)$, what are the coordinates of the reflection?

Figure 8.16.1

In this concept, you will learn how to find the coordinates for translations, rotations and reflections.

Finding Coordinates for Translations, Rotations, and Reflections

When you perform translations, you slide a figure left or right, up or down. This means that on the coordinate plane, the coordinates for the vertices of the figure will change.

To graph a **translation**, perform the same change for each point.

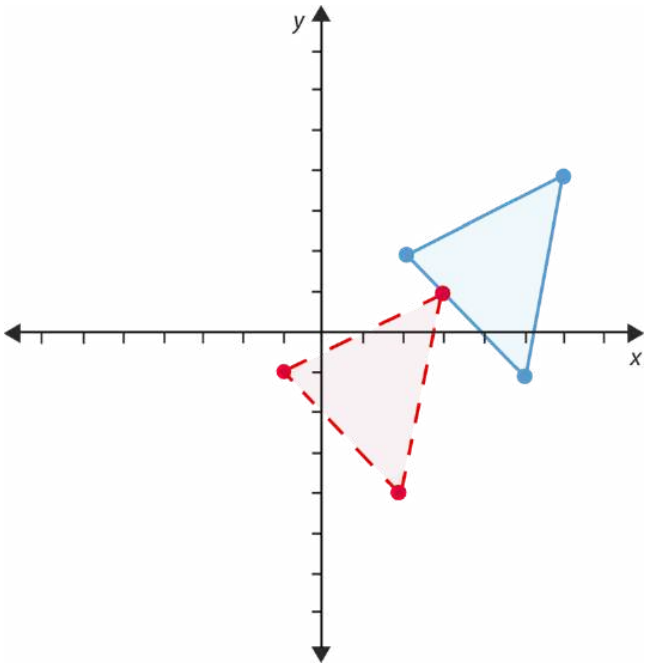

Figure 8.16.2

You can identify a reflection by the changes in its coordinates. In a reflection, the figure flips across a line to make a mirror image of itself. Take a look at the reflection below.

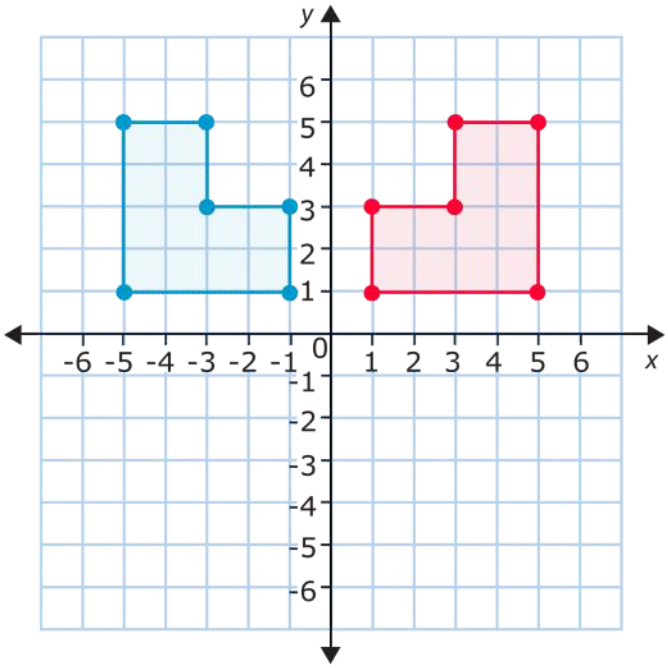

Figure **8.16.3**

Figures are usually reflected across either the x− or the y-axis. In this case, the figure is reflected across the y-axis**.** If you compare the figures in the first example vertex by vertex, you see that the x−coordinates change but the y−coordinates stay the same. This is because the reflection happens from left to right across the y-axis. When you reflect across the x−axis, the y−coordinates change and the x−coordinates stay the same. Take a look at this example.

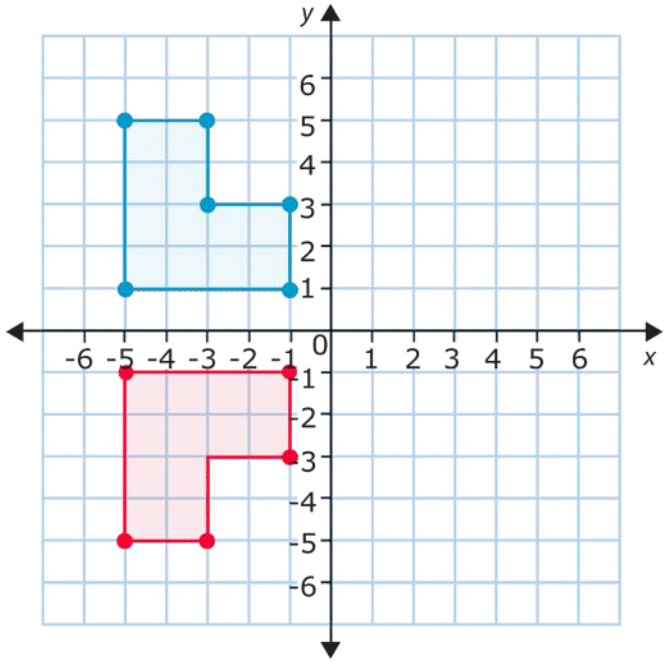

Figure **8.16.4**

In the figure above the coordinates for the upper-left vertex of the original figure are $(-5, 5)$. After you reflect it across the x−axis, the coordinates for the corresponding vertex are $(-5, -5)$. How about the lower-right vertex? It starts out at $(-1, 1)$, and after the flip it is at $(-1, -1)$. As you can see, the x−coordinates stay the same while the y−coordinates change. In fact, the y−coordinates all become the opposite integers of the original y−coordinates. This indicates that this is a vertical (up/down) reflection or a reflection over the x−axis.

In a horizontal (left/right) reflection or a reflection over the y-axis, the x−coordinates would become integer opposites. Let's look at an example.

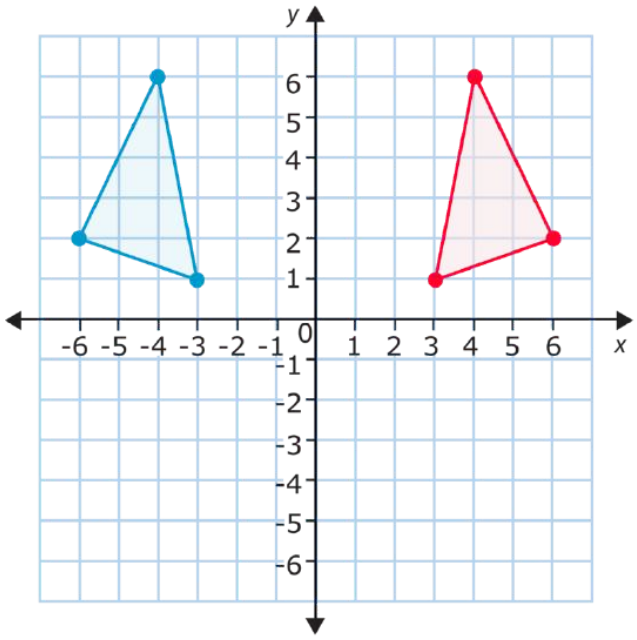

Figure 8.16.5

This is a reflection across the y-axis. Compare the points. Notice that the y−coordinates stay the same. The x−coordinates become the integer opposites of the original x−coordinates. Look at the top point of the triangle, for example. The coordinates of the original point are $(-4, 6)$, and the coordinates of the new point are $(4, 6)$. The x−coordinate has switched from -4 to 4.

You can recognize reflections by these changes to the x− and y−coordinates. If you reflect across the x−axis, the y−coordinates will become opposite. If you reflect across the y-axis, the x−coordinates will become opposite.

You can also use this information to graph reflections. To graph a reflection, you need to decide whether the reflection will be across the x−axis or the y-axis, and then change either the x− or y−coordinates.

Now let's look at the third kind of **transformation**: rotations. A **rotation** is a transformation that turns the figure in either a clockwise or counterclockwise direction. The figure below has been rotated. What are its new coordinates?

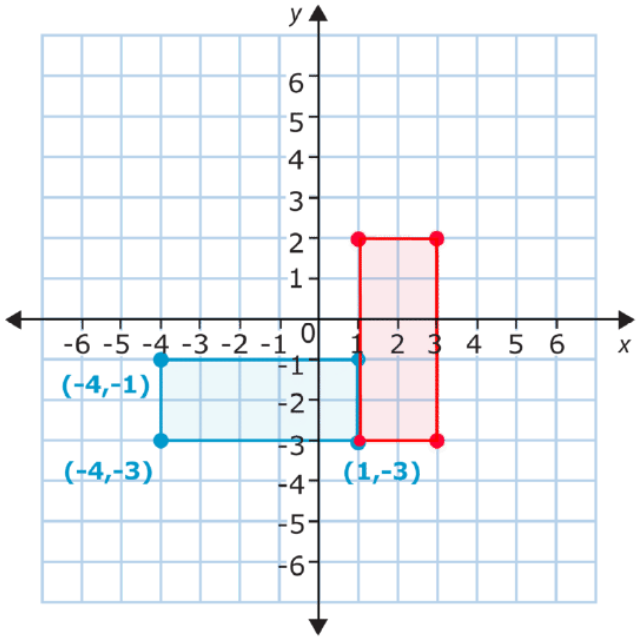

Figure 8.16.6

The new coordinates of the rectangle's vertices are $(1, -3)$, $(1, 2)$, $(3, 2)$, and $(3, -3)$. As you can see, both the x− and y−coordinates changed. One of the points remained exactly the same. That means the figure was rotated about this point. Imagine you put your finger on this corner of the rectangle and spun it. That's what happened in the rotation. The rectangle has been rotated $90°$ clockwise.

When you graph a rotation, you first need to know how much the figure will be rotated. Rotating the rectangle above $90°$ stands it up on end. Rotating it $180°$ would make it flat again. You also need to know which point you will rotate it around. This is the point that stays the same.

Next, you need to count how many units long and wide the figure is. The figure above stretches from 1 on the x−axis to -4 on the x −axis. This is a total of 5 units along the x−axis. When you rotate a figure $90°$, the distance on the x−axis becomes the distance on the y-axis. Look at the rectangle. The long sides are horizontal at first, but after you rotate it, they become the vertical sides. This means that the x−distance of 5 will become a y−distance of 5.

Now, remember the point $(1, -3)$ stays the same, so it is one corner of the rotated figure. You add 5 to the y−coordinate to find the next vertex of the rectangle. $-3 + 5 = 2$. This puts a vertex at (1, 2)\).

To find the other points of the rotated rectangle, you need to think about its width. Find the width, or short side, of the original rectangle by counting the units between vertices along the y-axis. The rectangle covers 2 units on the y-axis. As you might guess, this becomes the x−distance in the rotated figure. In other words, you add 2 to the x−coordinate of the point that stays the same. $1 + 2 = 3$, so another vertex of the rectangle will be $(3, -3)$. To find the fourth and final vertex, add 2 to the x−coordinate of the other ordered pair, $(1, 2)$. This puts the last vertex at $(3, 2)$.

Write each set of coordinates to show a reflection in the y-axis.

Example 8.16.1

Earlier, you were given a problem about Tyler and his picture.

If the original coordinates of the image are $(3, 0)$, $(4, 6)$ and $(5, 1)$, what are the coordinates of the reflection?

Solution

First, remember how to reflect across the x−axis.

Flip the shape across the x−axis.

Next, remember the rules.

The y–coordinates will change to their integer opposites.

Then, write the new points.

$(3, 0)$, $(4, -6)$, $(5, -1)$

The answer is that the coordinates of the reflection are $(3, 0)$, $(4, -6)$, $(5, -1)$.

Example 8.16.2

The figure below is reflected across the x–axis. What are the coordinates of the reflection?

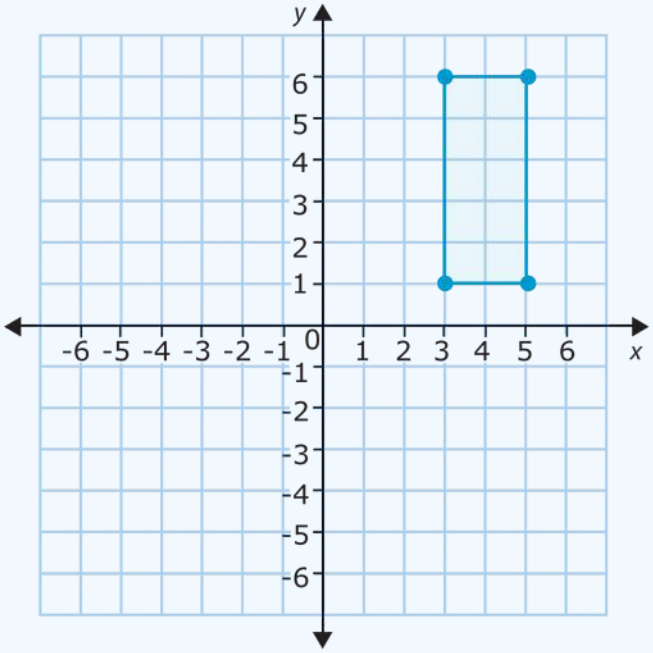

Figure 8.16.7

Solution

First, remember how to reflect across the x–axis.

Flip the shape across the x–axis.

Next, remember the rules.

The y–coordinates will change to their integer opposites.

Then, write the new points.

$(3, -6)$, $(3, -1)$, $(5, -6)$, $(5, -1)$

The answer is that the coordinates of the reflection are $(3, -6)$, $(3, -1)$, $(5, -6)$, $(5, -1)$.

Example 8.16.3

Determine if the change is the result of a translation, reflection or rotation.

After a transformation, the vertices $(3, 1)$, $(0, 3)$, $(1, 2)$ become $(3, -1)$, $(0, -3)$, $(1, -2)$.

Solution

First, look at the coordinates to see which coordinate changes.

The x–coordinates stay the same and the y–coordinates change for each of the points.

Next, determine the type of change for the coordinates.

The new y–coordinates are opposites of the original coordinates.

Then, state the type of transformation.

Reflection

The answer is that the change is the result of a reflection.

Example 8.16.4

Determine if the change is the result of a translation, reflection or rotation.

After a transformation, the vertices $(-3, 6)$, $(-2, 3)$, $(2, 3)$, $(3, 6)$ become $(-3, -1)$, $(-2, 8)$, $(2, 8)$, $(3, 11)$.

Solution

First, look at the coordinates to see which coordinate changes.

The x–coordinates stay the same and the y–coordinates change for each of the points.

Next, determine the type of change for the coordinates.

The new y–coordinates are each 5 more than the original coordinates.

Then, state the type of transformation.

Translation

The answer is that the change is the result of a translation.

Example 8.16.5

The points $(1, 2)$, $(3, 7)$, $(3, 4)$, $(5, 6)$ are reflected over the y-axis. What are the new coordinates?

Solution

First, remember the changes that occur with a reflection over the y-axis.

The x–coordinates become the opposite and the y–coordinates stay the same.

Then, write the new coordinates.

$(-1, 2)$, $(-3, 7)$, $(-3, 4)$, $(-5, 6)$

The answer is that the new coordinates are $(-1, 2)$, $(-3, 7)$, $(-3, 4)$, $(-5, 6)$.

True or false. When there is a reflection in the y-axis, both coordinates change to opposites.

Review

Use this figure to answer each question. Be sure to write everything in **coordinate notation** when possible.

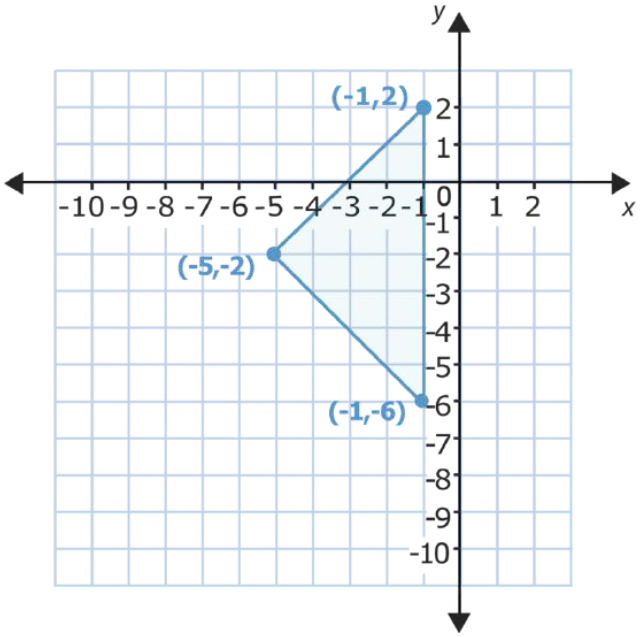

Figure 8.16.8

1. Translate this figure three units up.
2. Translate this figure four units to the right.
3. Translate this figure five units down.
4. Translate this figure six units to the left.
5. Translate this figure one unit down and two units to the right.
6. Translate this figure two units up and one unit to the left.
7. Translate this figure three units up and one unit to the right.
8. Rotate this figure 180 degrees.
9. Rotate this figure 90 degrees.
10. Reflect this figure over the x axis.
11. Reflect this figure over the y axis.
12. Translate this figure five units up and three units to the right.
13. Translate this figure six units down and four units to the left.
14. True or false. The figure below is an image of a reflection.

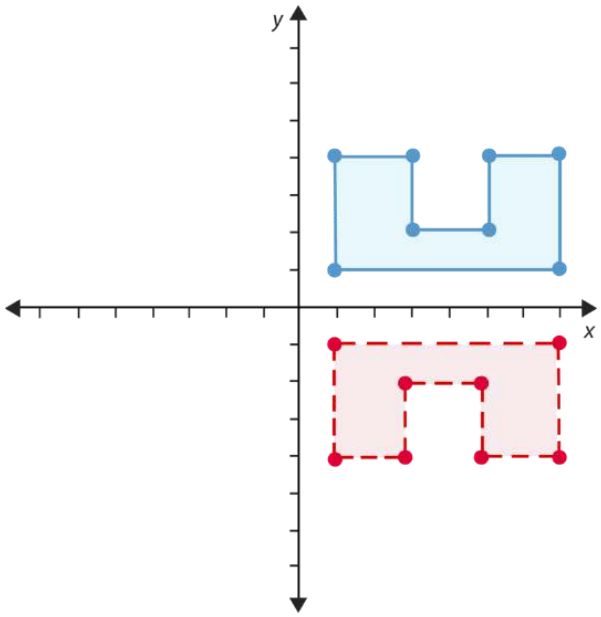

Figure **8.16.9**

15. True or false. This figure has been rotated **180°**.

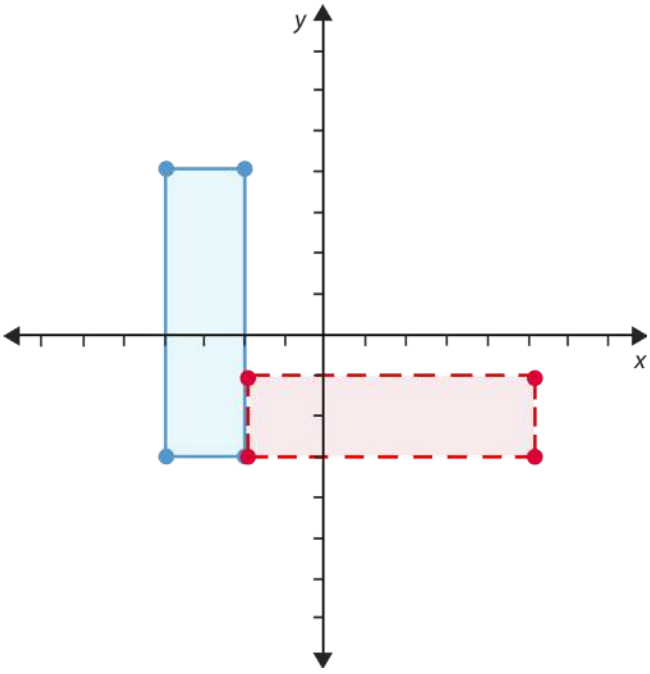

Figure **8.16.10**

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.19.

Resources

Vocabulary

Term	Definition
x−axis	The x−axis is the horizontal axis in the coordinate plane, commonly representing the value of the input or independent variable.
y-axis	The y-axis is the vertical number line of the Cartesian plane.
Coordinate Notation	A coordinate point is the description of a location on the coordinate plane. Coordinate points are commonly written in the form (x, y) where x is the horizontal distance from the origin, and y is the vertical distance from the origin.
Coordinate Plane	The coordinate plane is a grid formed by a horizontal number line and a vertical number line that cross at the $(0, 0)$ point, called the origin. The coordinate plane is also called a Cartesian Plane.
Reflection	A reflection is a transformation that flips a figure on the coordinate plane across a given line without changing the shape or size of the figure.
Rotation	A rotation is a transformation that turns a figure on the coordinate plane a certain number of degrees about a given point without changing the shape or size of the figure.
Transformation	A transformation moves a figure in some way on the coordinate plane.
Translation	A translation is a transformation that slides a figure on the coordinate plane without changing its shape, size, or orientation.

Additional Resources

Practice: Reflecting Figures

Real World: Mirror, Mirror

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8.17: Composite Transformations

Learn how to compose transformations of a figure on a coordinate plane, and understand the order in which to apply them.

Transformations Summary

A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the **image**. The original figure is called the **preimage**.

There are three rigid transformations: translations, rotations and reflections. A **translation** is a transformation that moves every point in a figure the same distance in the same direction. A **rotation** is a transformation where a figure is turned around a fixed point to create an image. A **reflection** is a transformation that turns a figure into its mirror image by flipping it over a line.

Composition of Transformations

A **composition (of transformations)** is when more than one transformation is performed on a figure. Compositions can always be written as one rule. You can compose any transformations, but here are some of the most common compositions:

1. A **glide reflection** is a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.

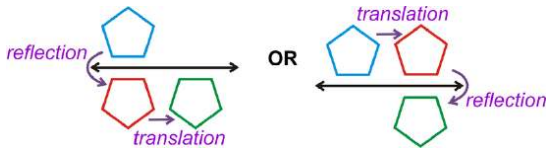

Figure 8.17.1

2. The composition of two reflections over parallel lines that are h units apart is the same as a translation of $2h$ units (**Reflections over Parallel Lines Theorem**).

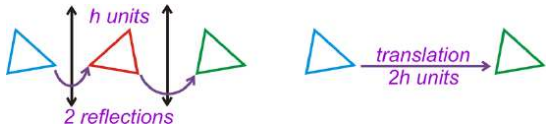

Figure 8.17.2

If you compose two reflections over each axis, then the final image is a rotation of $180°$ around the origin of the original (**Reflection over the Axes Theorem**).

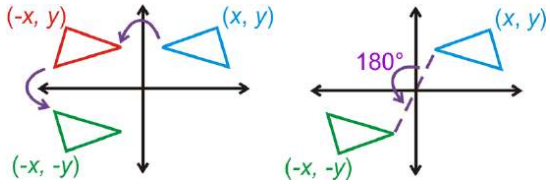

Figure 8.17.3

A composition of two reflections over lines that intersect at $x°$ is the same as a rotation of $2x°$. The center of rotation is the point of intersection of the two lines of reflection (**Reflection over Intersecting Lines Theorem**).

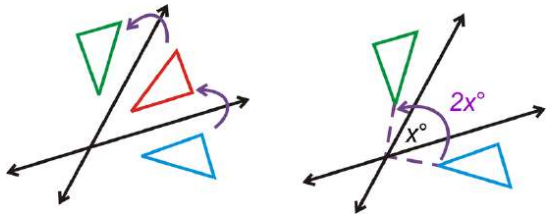

Figure 8.17.4

What if you were given the coordinates of a quadrilateral and you were asked to reflect the quadrilateral and then translate it? What would its new coordinates be?

Example 8.17.1

Reflect ΔABC over the y slate the image 8 units down.

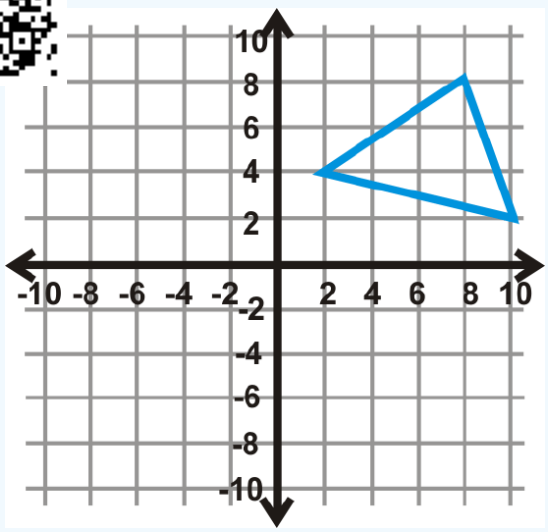

Figure 8.17.5

Solution

The green image to the left is the final answer.

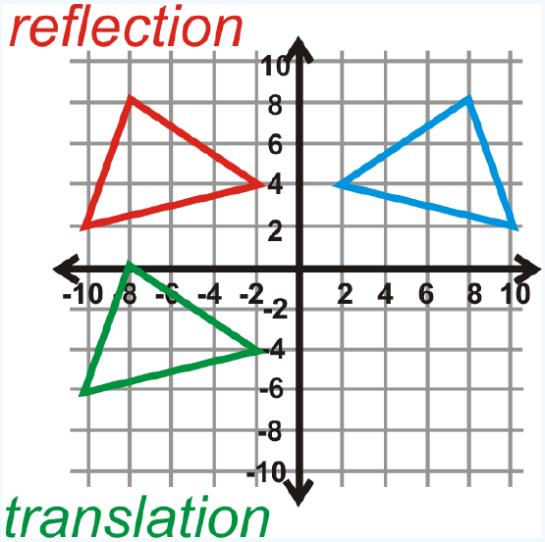

Figure 8.17.6

$$A(8,8) \rightarrow A''(-8,0)$$
$$B(2,4) \rightarrow B''(-2,-4)$$
$$C(10,2) \rightarrow C''(-10,-6)$$

Example 8.17.2

Write a single rule for ΔABC to $\Delta A''B''C''$ from Example 1.

Solution

Looking at the coordinates of A to A'', the x−value is the opposite sign and the y−value is $y-8$. Therefore the rule would be $(x, y) \rightarrow (-x, y-8)$.

Example 8.17.3

Reflect ΔABC over $y=3$ and then reflect the image over $y=-5$.

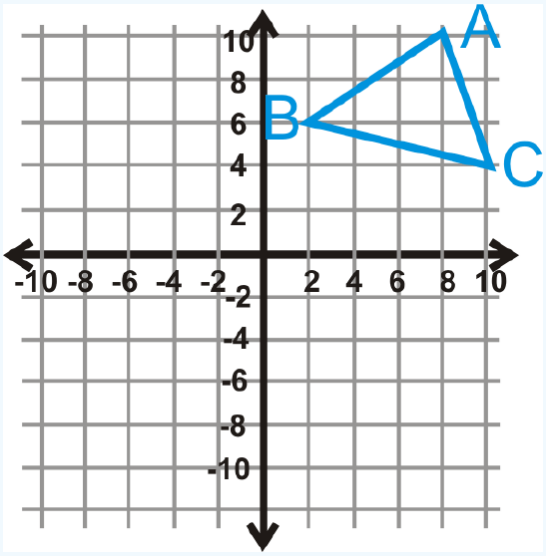

Figure 8.17.7

Solution

Order matters, so you would reflect over $y=3$ first, (red triangle) then reflect it over $y=-5$ (green triangle).

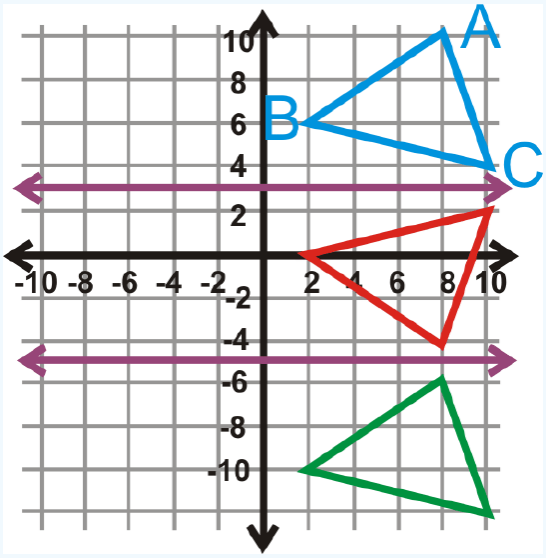

Figure 8.17.8

Example 8.17.4

A square is reflected over two lines that intersect at a $79°$ angle. What one transformation will this be the same as?

Solution

From the Reflection over Intersecting Lines Theorem, this is the same as a rotation of $2 \cdot 79°$=178^{\circ}\).

Example 8.17.5

ΔDEF has vertices $D(3,-1)$, $E(8,-3)$, and $F(6,4)$. Reflect ΔDEF over $x=-5$ and then $x=1$. Determine which one translation this double reflection would be the same as.

Solution

From the Reflections over Parallel Lines Theorem, we know that this double reflection is going to be the same as a single translation of $2(1-(-5))$ or 12 units.

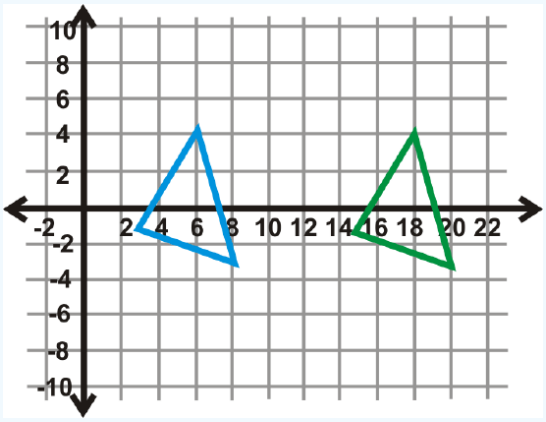

Figure 8.17.9

Example 8.17.6

Reflect ΔDEF from Question 2 over the x-axis, followed by the y-axis. Find the coordinates of $\Delta D''E''F''$ and the one transformation this double reflection is the same as.

Solution

$\Delta D''E''F''$ is the green triangle in the graph to the left. If we compare the coordinates of it to ΔDEF, we have:

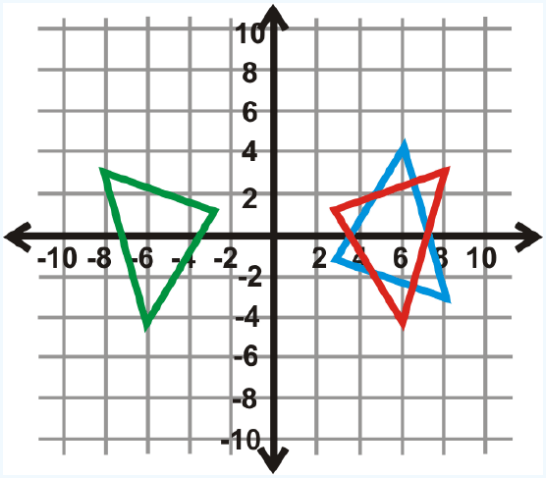

Figure 8.17.10

$$D(3,-1) \rightarrow D''(-3,1)$$
$$E(8,-3) \rightarrow E''(-8,3)$$
$$F(6,4) \rightarrow F''(-6,-4)$$

Review

1. *Explain* why the composition of two or more isometries must also be an isometry.
2. What one transformation is the same as a reflection over two parallel lines?
3. What one transformation is the same as a reflection over two intersecting lines?

Use the graph of the square to the left to answer questions 4-6.

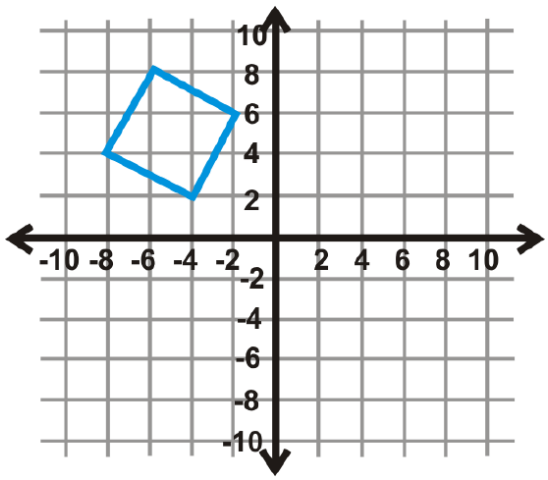

Figure 8.17.11

4. Perform a glide reflection over the x-axis and to the right 6 units. Write the new coordinates.
5. What is the rule for this glide reflection?
6. What glide reflection would move the image back to the preimage?

Use the graph of the square to the left to answer questions 7-9.

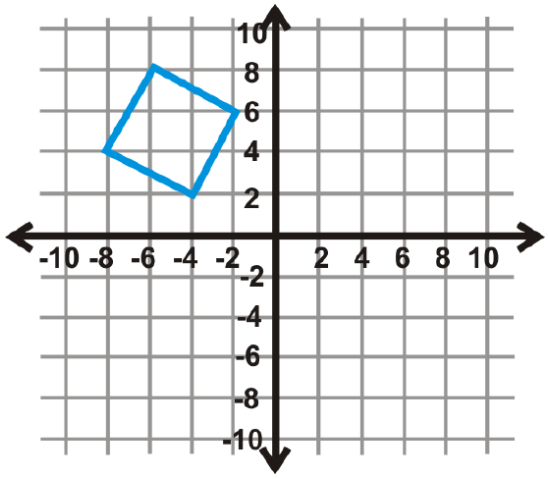

Figure 8.17.12

7. Perform a glide reflection to the right 6 units, then over the x-axis. Write the new coordinates.
8. What is the rule for this glide reflection?
9. Is the rule in #8 different than the rule in #5? Why or why not?

Use the graph of the triangle to the left to answer questions 10-12.

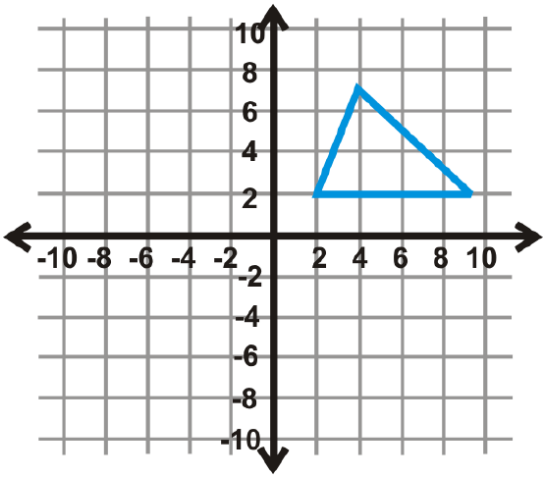

Figure 8.17.13

10. Perform a glide reflection over the y-axis and down 5 units. Write the new coordinates.
11. What is the rule for this glide reflection?
12. What glide reflection would move the image back to the preimage?

Use the graph of the triangle to the left to answer questions 13-15.

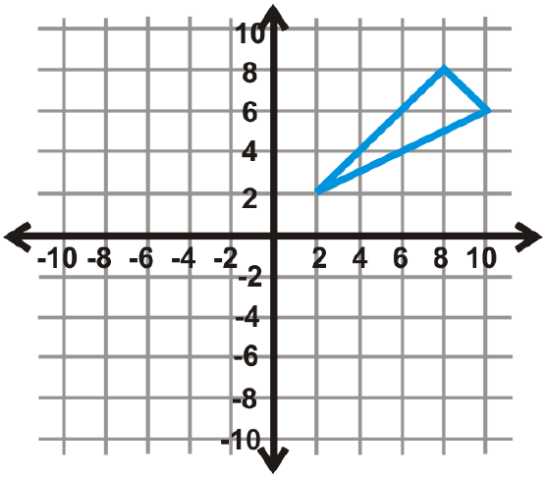

Figure 8.17.14

13. Reflect the preimage over $y = -1$ followed by $y = -7$. Draw the new triangle.
14. What one transformation is this double reflection the same as?
15. Write the rule.

Use the graph of the triangle to the left to answer questions 16-18.

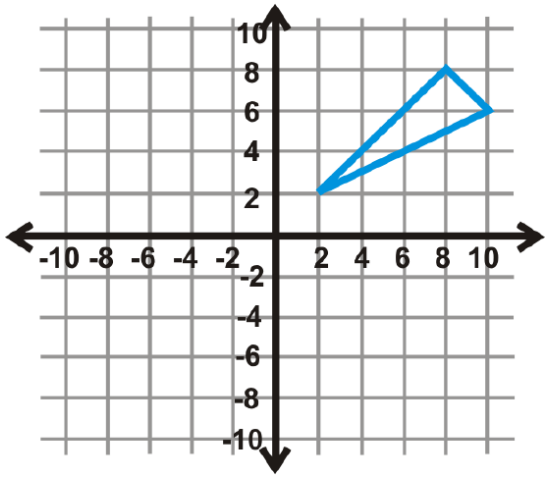

Figure 8.17.15

16. Reflect the preimage over $y = -7$ *followed* by $y = -1$. Draw the new triangle.
17. What one transformation is this double reflection the same as?
18. Write the rule.
19. How do the final triangles in #13 and #16 differ?

Use the trapezoid in the graph to the left to answer questions 20-22.

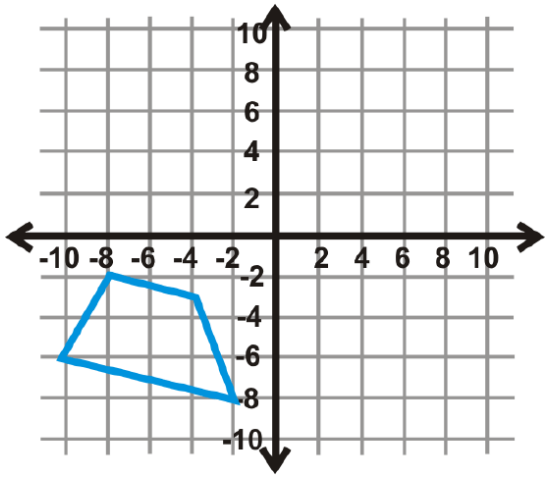

Figure 8.17.16

20. Reflect the preimage over the x-axis then the y-axis. Draw the new trapezoid.
21. Now, start over. Reflect the trapezoid over the y-axis then the x-axis. Draw this trapezoid.
22. Are the final trapezoids from #20 and #21 different? Why do you think that is?

Answer the questions below. Be as specific as you can.

23. Two parallel lines are 7 units apart. If you reflect a figure over both how far apart with the preimage and final image be?
24. After a double reflection over parallel lines, a preimage and its image are 28 units apart. How far apart are the parallel lines?
25. Two lines intersect at a $165°$ angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
26. What is the center of rotation for #25?
27. Two lines intersect at an $83°$ angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
28. A preimage and its image are $244°$ apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
29. A preimage and its image are $98°$ apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
30. After a double reflection over parallel lines, a preimage and its image are 62 units apart. How far apart are the parallel lines?

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.6.

Vocabulary

Term	Definition
composition (of transformations)	When more than one transformation is performed on a figure.
Rotation	A rotation is a transformation that turns a figure on the coordinate plane a certain number of degrees about a given point without changing the shape or size of the figure.
Reflection	A reflection is a transformation that flips a figure on the coordinate plane across a given line without changing the shape or size of the figure.
Glide Reflection	A reflection followed by a translation where the line of reflection is parallel to the direction of translation is called a glide reflection or a walk.
Composite Transformation	A composite transformation, also known as composition of transformation, is a series of multiple transformations performed one after the other.

Additional Resources

Video: Composing Transformations Principles - Basic

Activities: Composition of Transformations Discussion Questions

Study Aids: Composition of Transformations Study Guide

Practice: Composite Transformations

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8.18: Notation for Composite Transformations

Interpret and use notation for combined transformations

The figure below shows a composite **transformation** of a trapezoid. Write the mapping rule for the composite transformation.

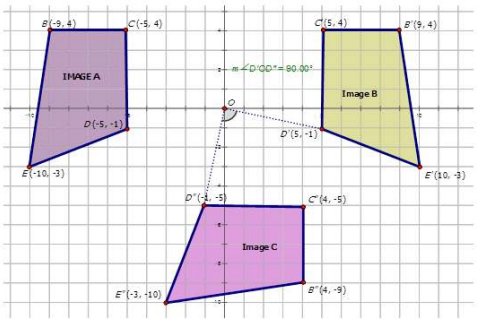

Figure 8.18.1

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image). The order of transformations performed in a composite transformation matters.

To describe a composite transformation using notation, state each of the transformations that make up the composite transformation and link them with the symbol $\circ$. The transformations are performed in order from right to left. Recall the following notation for translations, **reflections**, and rotations:

- Translation: $T_{a,b}: (x, y) \rightarrow (x + a, y + b)$ is a translation of a units to the right and b units up.
- Reflection: $r_{y-axis}(x, y) \rightarrow (-x, y)$.
- **Rotation**: $R_{90^\circ}(x, y) = (-y, x)$

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Let's graph the line described below and the composite image defined by $r_{y-axis} \circ R_{90^\circ}$:

The first translation is a 90° CCW turn about the origin to produce $X'Y'$. The second translation is a reflection about the y-axis to produce $X''Y''$.

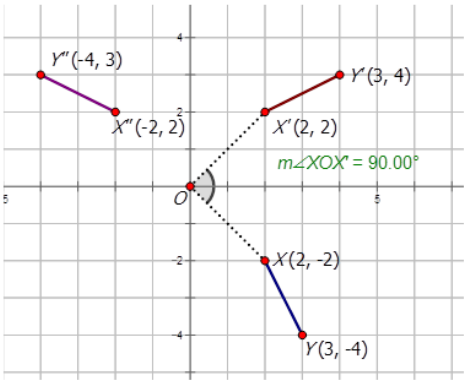

Figure 8.18.2

Now, let's graph the composite images described in the following problems:

1. Image A with vertices $A(3,5)$, $B(4,2)$ and $C(1,1)$ undergoes a composite transformation with mapping rule $r_{x-axis} \circ r_{y-axis}$.

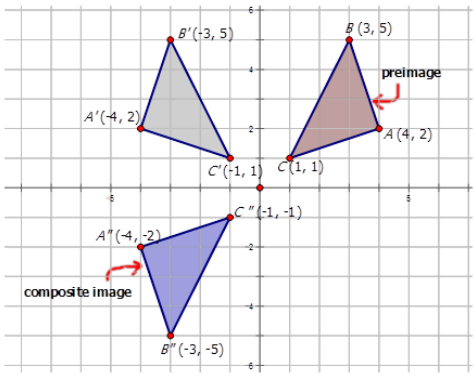

Figure 8.18.3

2. Image D with vertices $D(-3,7)$, $E(-1,3)$, $F(-7,5)$ and $G(-5,1)$ undergoes a composite transformation with mapping rule $T_{3,4} \circ r_{x-axis}$.

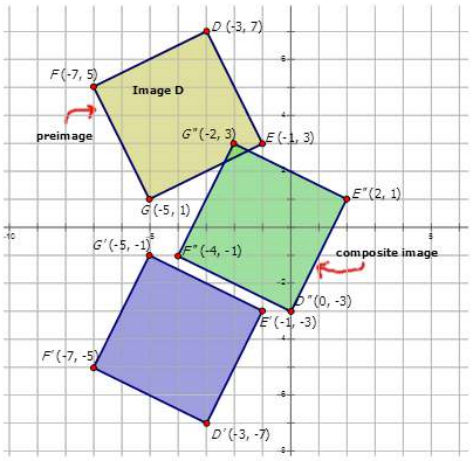

Figure 8.18.4

Example 8.18.1

Earlier, you were asked to write the mapping rule for the following composite transformation:

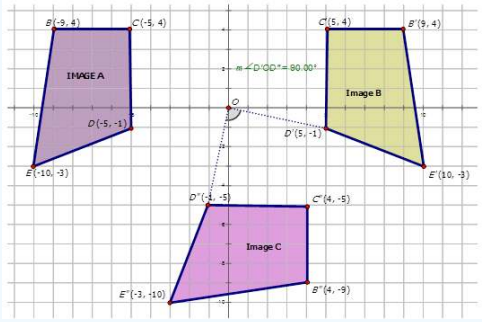

Figure 8.18.5

Solution

The transformation from Image A to Image B is a reflection across the y-axis. The notation for this is r_{y-axis}. The transformation for image B to form image C is a rotation about the origin of $90°CW$. The notation for this transformation is $R_{270°}$. Therefore, the notation to describe the transformation of Image A to Image C is $R_{270°} \circ r_{y-axis}$

Example 8.18.2

Graph the line XY given that $X(2, -2)$ and $Y(3, -4)$. Also graph the composite image that satisfies the rule $R_{90°} \circ r_{y-axis}$

Solution

The first transformation is a reflection about the y-axis to produce $X'Y'$. The second transformation is a $90°CCW$ turn about the origin to produce $X''Y''$.

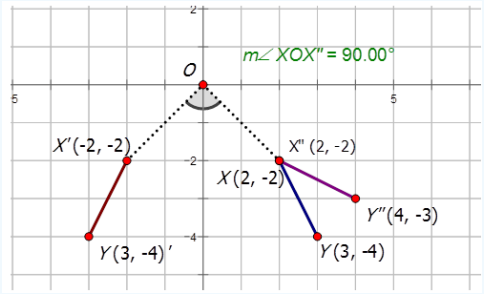

Figure 8.18.6

Example 8.18.3

Describe the composite transformations in the diagram below and write the notation to represent the transformation of figure $ABCD$ to $A''B''C''D''$.

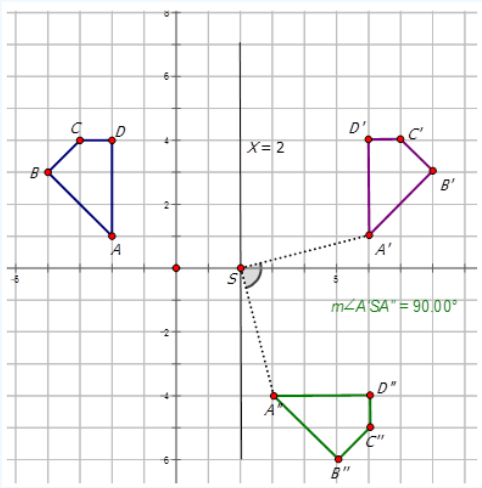

Figure 8.18.7

Solution

There are two transformations shown in the diagram. The first transformation is a reflection about the line X=2\) to produce $A'B'C'D'$. The second transformation is a $90°$CW (or $270°CCW$) rotation about the point $(2,0)$ to produce the figure $A''B''C''D''$. Notation for this composite transformation is:

$R_{270°} \circ r_{x=2}$

Example 8.18.4

Describe the composite transformations in the diagram below and write the notation to represent the transformation of figure ABC to $A''B''C''$.

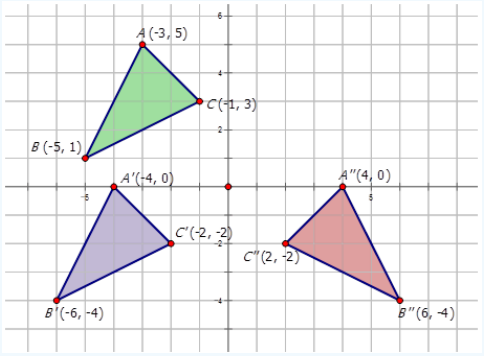

Figure **8.18.8**

Solution

There are two transformations shown in the diagram. The first transformation is a translation of 1 unit to the left and 5 units down to produce $A'B'C'$. The second reflection in the y-axis to produce the figure $A''B''C''$. Notation for this composite transformation is:

$r_{y-axis} \circ T_{-1,-5}$

Review

Complete the following table:

Starting Point	$T_{3,-4} \circ R_{90°}$	$r_{x-axis} \circ r_{y-axis}$	$T_{1,6} \circ r_{x-axis}$	$r_{y-axis} \circ R_{180°}$
1. $(1,4)$				
2. $(4,2)$				
3. $(2,0)$				
4. $(-1,2)$				
5. $(-2,-3)$				
6. $(4,-1)$				
7. $(3,-2)$				
8. $(5,4)$				
9. $(-3,7)$				
10. $(0,0)$				

Write the notation that represents the composite transformation of the preimage A to the composite images in the diagrams below.

11.

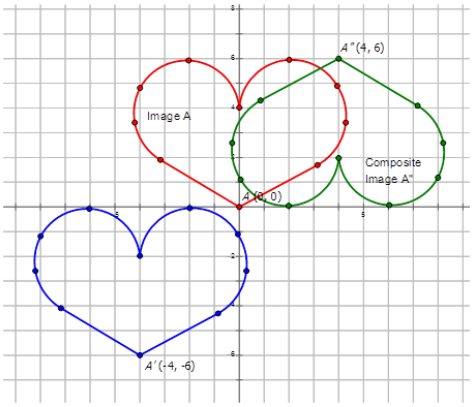

Figure 8.18.9

12.

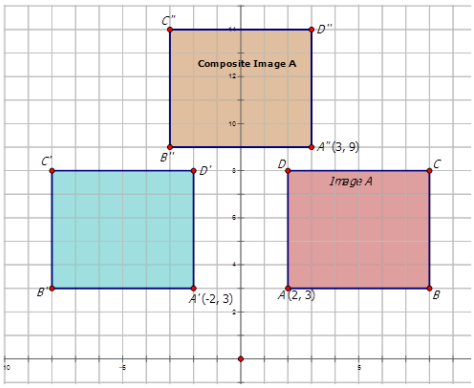

Figure 8.18.10

13.

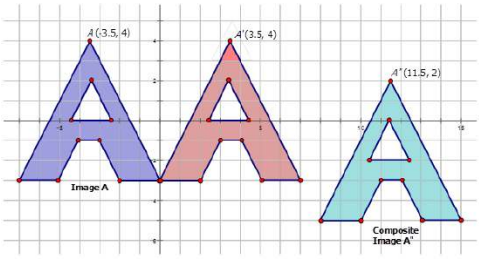

Figure 8.18.11

14.

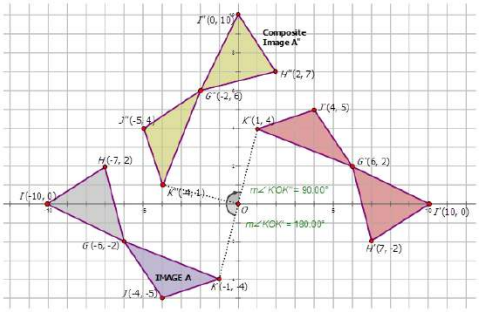

Figure 8.18.12

15.

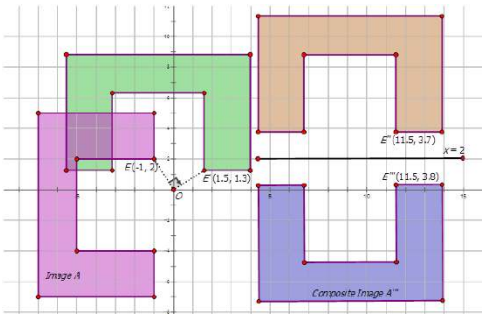

Figure **8.18.13**

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.15.

Vocabulary

Term	Definition
Reflections	Reflections are transformations that result in a "mirror image" of a parent function. They are caused by differing signs between parent and child functions.
Rotation	A rotation is a transformation that turns a figure on the coordinate plane a certain number of degrees about a given point without changing the shape or size of the figure.
Transformation	A transformation moves a figure in some way on the coordinate plane.

Additional Resources

Practice: Notation for Composite Transformations

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8.19: Tessellations

Tiling over a plane such that the figures fill the plane with no overlaps or gaps.

A **tessellation** is a tiling over a plane with one or more figures such that the figures fill the plane with no overlaps and no gaps. You have probably seen tessellations before. Examples of a tessellation are: a tile floor, a brick or block wall, a checker or chess board, and a fabric pattern. The following pictures are also examples of tessellations.

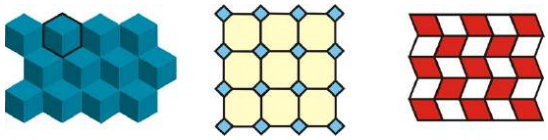

Figure 8.19.1

Notice the hexagon (cubes, first tessellation) and the quadrilaterals fit together perfectly. If we keep adding more, they will entirely cover the plane with no gaps or overlaps.

We are only going to worry about tessellating regular polygons. To tessellate a shape, it must be able to exactly surround a point, or the sum of the angles around each point in a tessellation must be $360°$. The only regular polygons with this feature are equilateral triangles, squares, and regular hexagons.

What if you were given a hexagon and asked to tile it over a plane such that it would fill the plane with no overlaps and no gaps?

Example 8.19.1

How many regular hexa one point?

Solution

First, recall how many cle, and then figure out how many degrees are in each angle of a regular hexagon. There are $360°$ in a circle and $120°$ in each interior angle of a hexagon, so $\frac{360}{120} = 3$ hexagons will fit around one point.

Example 8.19.2

Does a regular octagon tessellate?

Solution

First, recall that there are $1080°$ in a pentagon. Each angle in a regular pentagon is $1080° \backslash \text{divide} 8 = 135°$. From this, we know that a regular octagon will not tessellate by itself because $135°$ does not go evenly into $360°$.

Example 8.19.3

Draw a tessellation of equilateral triangles.

Solution

In an equilateral triangle each angle is $60°$. Therefore, six triangles will perfectly fit around each point.

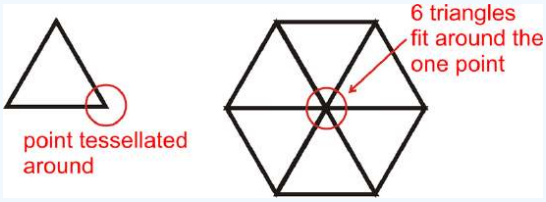

Figure 8.19.2

Extending the pattern, we have:

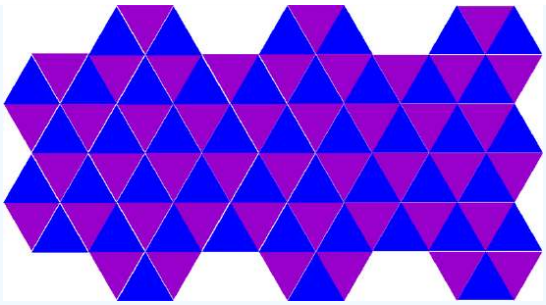

Figure 8.19.3

Example 8.19.4

Does a regular pentagon tessellate?

Solution

First, recall that there are 540° in a pentagon. Each angle in a regular pentagon is $540^\circ \backslash \text{divide} 5 = 108^\circ$. From this, we know that a regular pentagon will not tessellate by itself because 108° times 2 or 3 does not equal 360°.

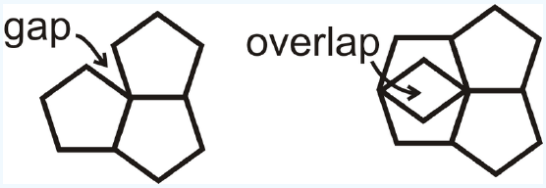

Figure 8.19.4

Example 8.19.5

How many squares will fit around one point?

Solution

First, recall how many degrees are in a circle, and then figure out how many degrees are in each angle of a square. There are 360° in a circle and 90° in each interior angle of a square, so $\frac{360}{90} = 4$ squares will fit around one point.

Review

1. Tessellate a square. Add color to your design.
2. What is an example of a tessellated square in real life?
3. Tessellate a regular hexagon. Add color to your design.
4. You can also tessellate two regular polygons together. Try tessellating a regular hexagon and an equilateral triangle. First, determine how many of each fit around a point and then repeat the pattern. Add color to your design.
5. Does a regular dodecagon (12-sided shape) tessellate? Why of why not?
6. Does a kite tessellate? Why or why not?

Do the following figures tessellate?

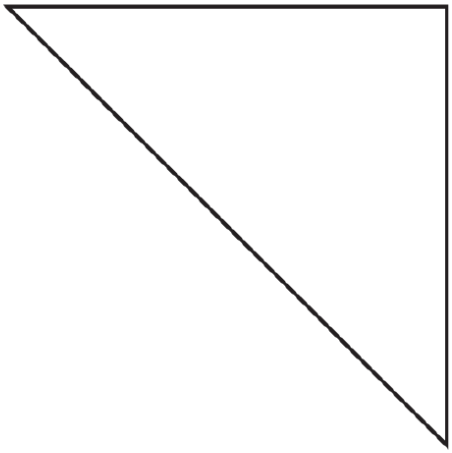

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Figure 8.19.5

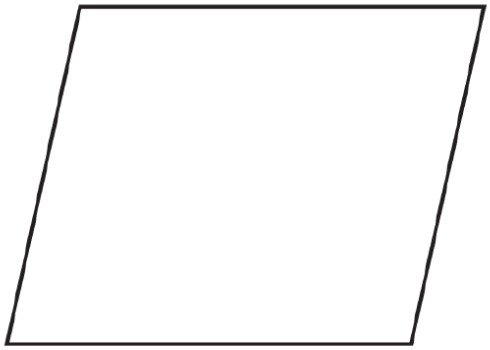

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Figure 8.19.6

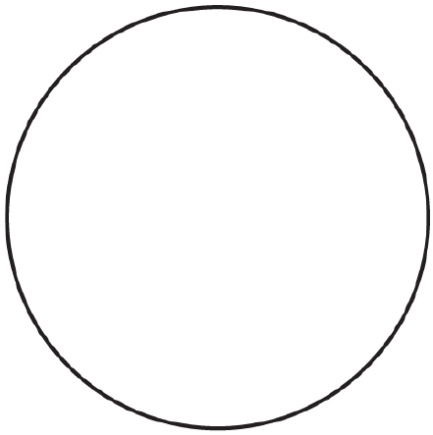

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Figure 8.19.7

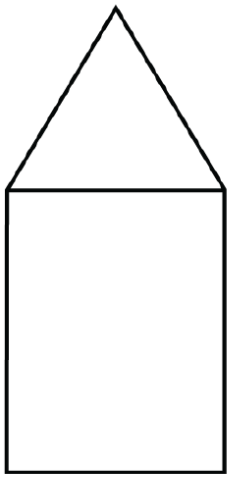

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Figure 8.19.8

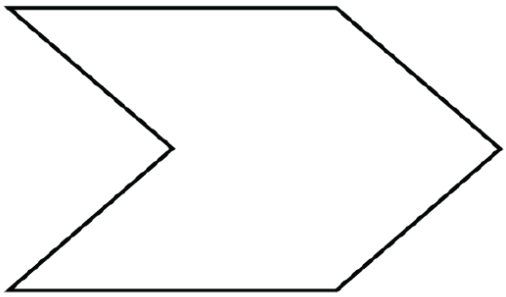

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Figure 8.19.9\

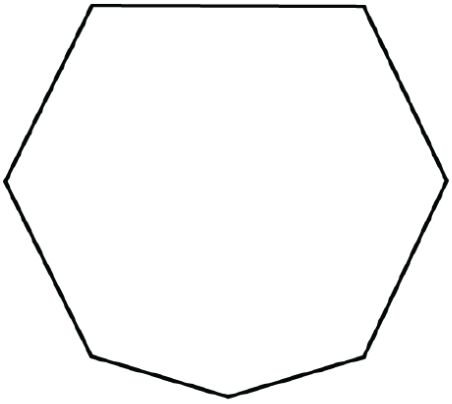

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Figure 8.19.10

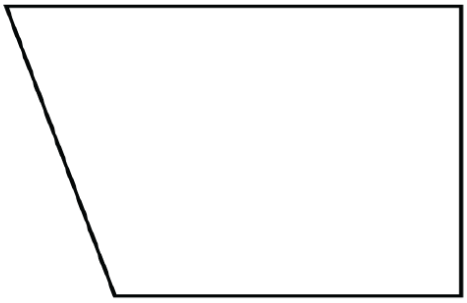

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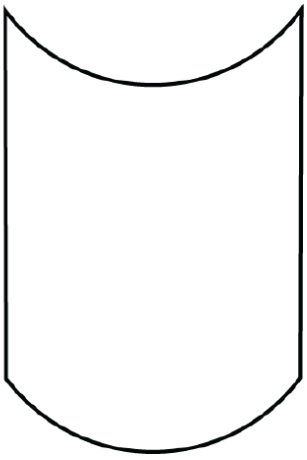
Figure 8.19.11



14.

Figure 8.19.12

Review (Answers)

To see the Review answers, open this PDF file and look for section 12.7.

Vocabulary

Term	Definition
tessellation	A tiling over a plane with one or more figures such that the figures fill the plane with no overlaps and no gaps.
line segment	A line segment is a part of a line that has two endpoints.

Additional Resources

Video: Tessellations Principles - Basic

Activities: Symmetry and Tessellations Study Guide

Practice: Tessellations

Real World: The Master of Tessellations

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CHAPTER OVERVIEW

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Thumbnail: Rotating cube (CC BY-SA 3.0 Unported; Aldoaldoz/modified by Delmar Larsen via Wikipedia)

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9.1: Polyhedrons

3-D figures formed by polygons enclosing regions in space.

A **polyhedron** is a 3-dimensional figure that is formed by polygons that enclose a region in space. Each **polygon** in a polyhedron is a **face.** The line segment where two faces intersect is an **edge.** The point of intersection of two edges is a **vertex.**

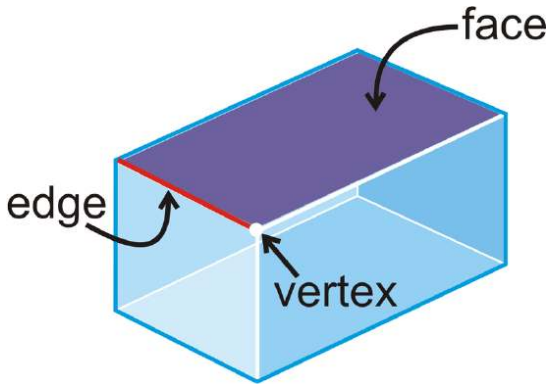

Figure 9.1.1

Examples of polyhedrons include a cube, prism, or pyramid. Cones, spheres, and cylinders are not polyhedrons because they have surfaces that are not polygons. The following are more examples of polyhedrons:

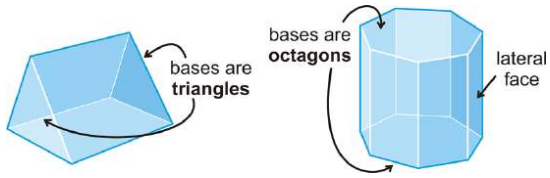

Figure 9.1.2

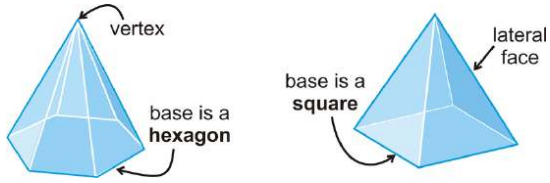

Figure 9.1.3

The number of faces (F), vertices (V) and edges (E) are related in the same way for any polyhedron. Their relationship was discovered by the Swiss mathematician Leonhard Euler, and is called Euler's Theorem.

Euler's Theorem: $F + V = E + 2$.

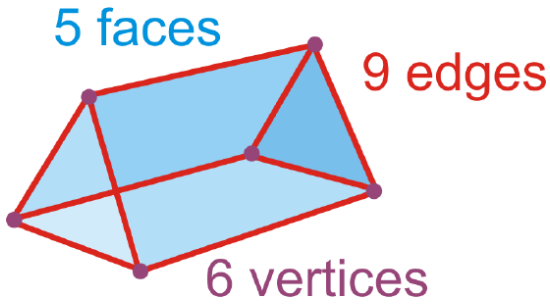

Figure 9.1.4

$Faces + Vertices = Edges + 2$

$5 + 6 = 9 + 2$

A **regular polyhedron** is a polyhedron where all the faces are congruent regular polygons. There are only **five regular polyhedra, called the Platonic solids.**

1. **Regular Tetrahedron:** A 4-faced polyhedron and all the faces are equilateral triangles.
2. **Cube:** A 6-faced polyhedron and all the faces are squares.
3. **Regular Octahedron:** An 8-faced polyhedron and all the faces are equilateral triangles.
4. **Regular Dodecahedron:** A 12-faced polyhedron and all the faces are regular pentagons.
5. **Regular Icosahedron:** A 20-faced polyhedron and all the faces are equilateral triangles.

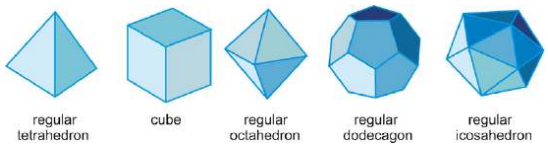

Figure **9.1.5**

What if you were given a solid three-dimensional figure, like a carton of ice cream? How could you determine how the faces, vertices, and edges of that figure are related?

Example 9.1.1

a.

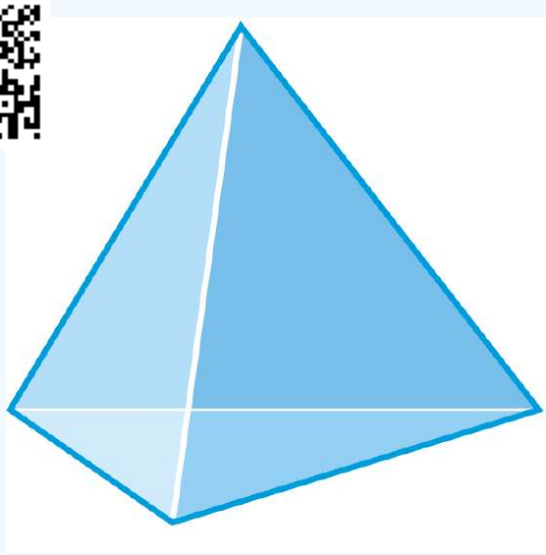

Figure **9.1.6**

b.

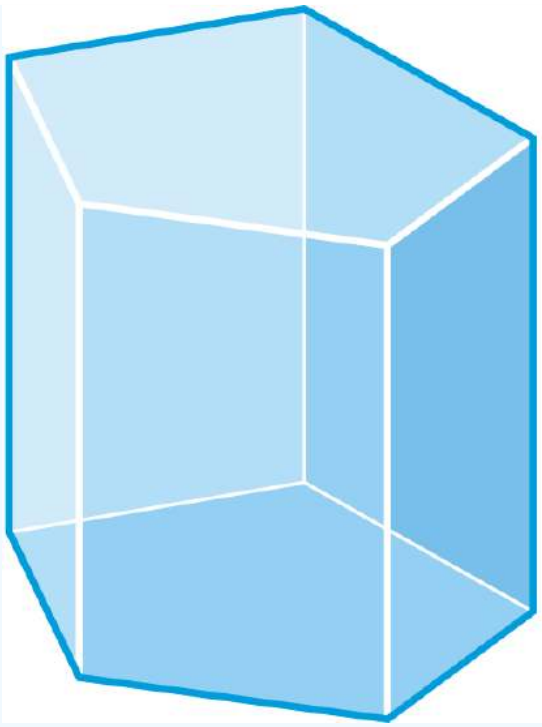

Figure **9.1.7**

c.

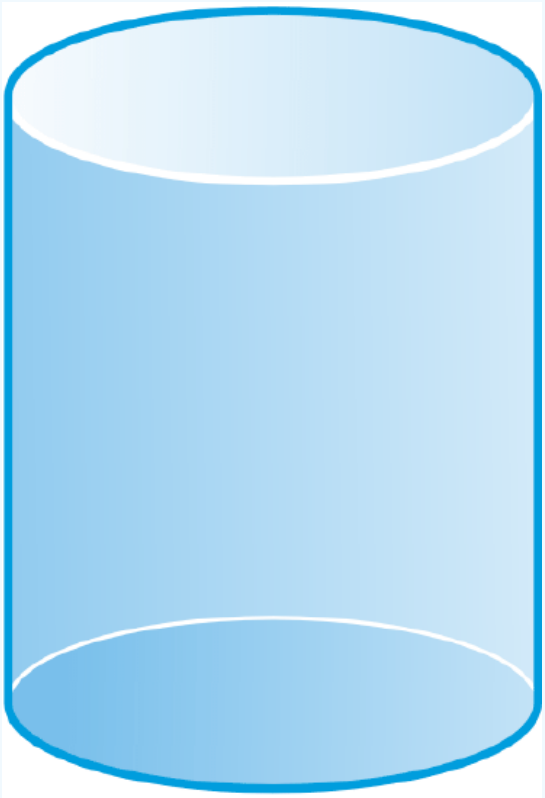

Figure **9.1.8**

Solution

The base is a triangle and all the sides are triangles, so this is a triangular pyramid, which is also known as a **tetrahedron**. There are 4 faces, 6 edges and 4 vertices.

Example 9.1.2

In a six-faced polyhedron, there are 10 edges. How many vertices does the polyhedron have?

Solution

Solve for V in Euler's Theorem.

$$F + V = E + 2$$
$$6 + V = 10 + 2$$
$$V = 6$$

Therefore, there are 6 vertices.

Example 9.1.3

Markus counts the edges, faces, and vertices of a polyhedron. He comes up with 10 vertices, 5 faces, and 12 edges. Did he make a mistake?

Solution

Plug all three numbers into Euler's Theorem.

$$F + V = E + 2$$
$$5 + 10 = 12 + 2$$
$$15 \neq 14$$

Because the two sides are not equal, Markus made a mistake.

Example 9.1.4

Find the number of faces, vertices, and edges in an octagonal prism.

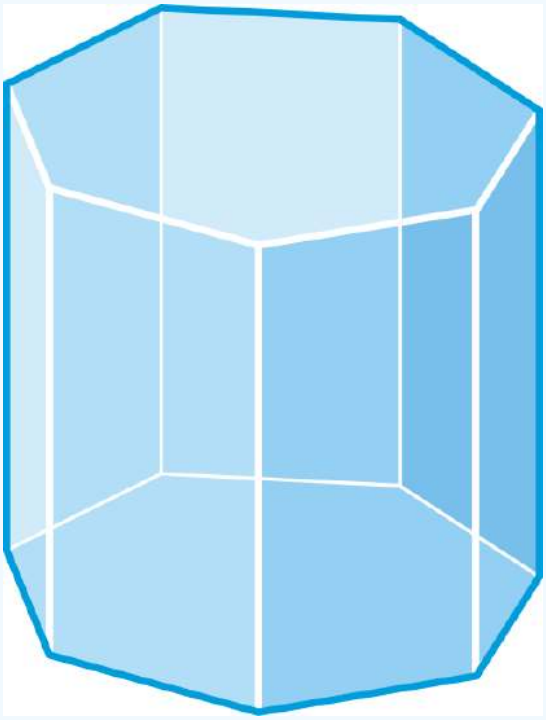

Figure **9.1.9**

Solution

There are 10 faces and 16 vertices. Use Euler's Theorem, to solve for E.

$$F + V = E + 2$$
$$10 + 16 = E + 2$$
$$24 = E$$

Therefore, there are 24 edges.

Example 9.1.5

A **truncated icosahedron** is a polyhedron with 12 regular pentagonal faces, 20 regular hexagonal faces, and 90 edges. This icosahedron closely resembles a soccer ball. How many vertices does it have? Explain your reasoning.

Figure 9.1.10

Solution

We can use Euler's Theorem to solve for the number of vertices.

$$F + V = E + 2$$
$$32 + V = 90 + 2$$
$$V = 60$$

Therefore, it has 60 vertices.

Review

Complete the table using Euler's Theorem.

	Name	*Faces*	*Edges*	*Vertices*
1.	Rectangular Prism	6	12	
2.	Octagonal Pyramid		16	9
3.	Regular Icosahedron	20		12
4.	Cube		12	8
5.	Triangular Pyramid	4		4
6.	Octahedron	8	12	
7.	Heptagonal Prism		21	14
8.	Triangular Prism	5	9	

Determine if the following figures are polyhedra. If so, name the figure and find the number of faces, edges, and vertices.

9.

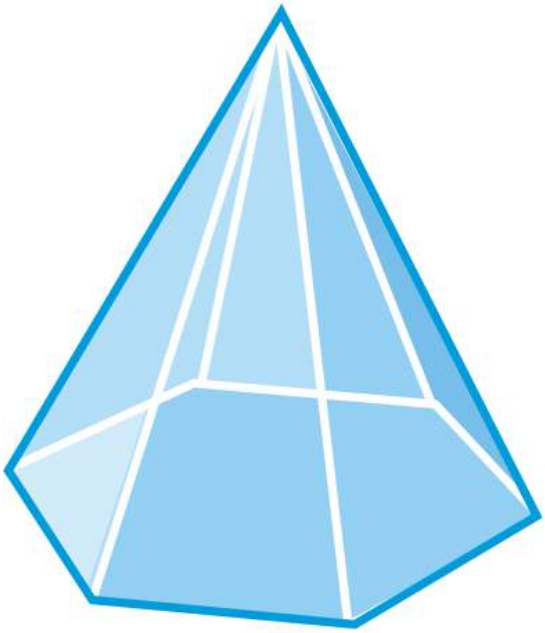

Figure **9.1.11**

10.

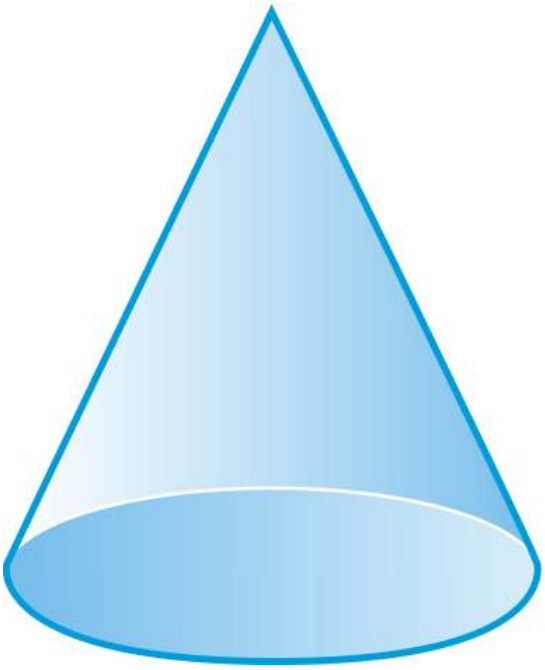

Figure **9.1.12**

11.

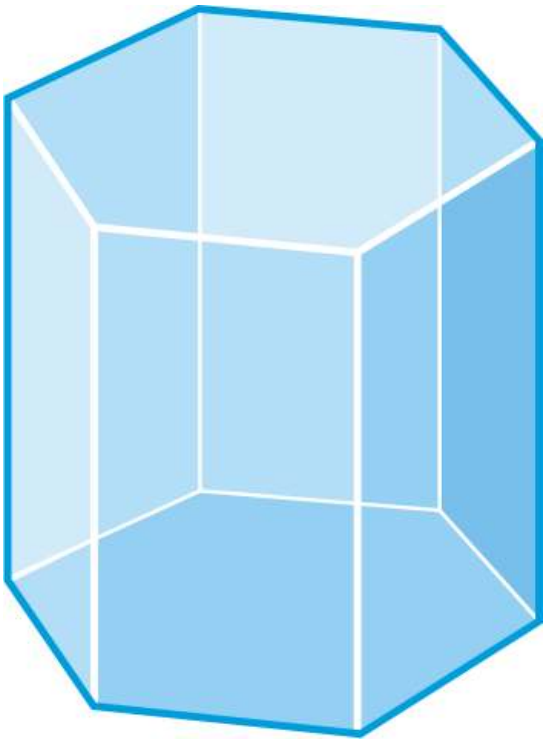

Figure **9.1.13**

12.

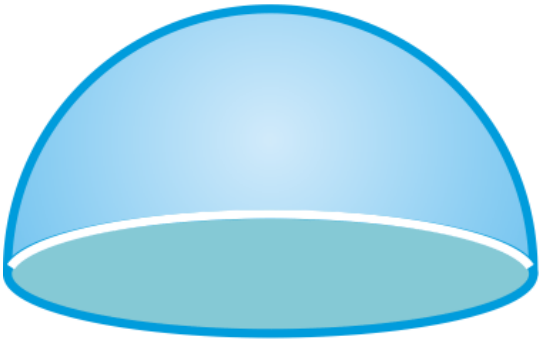

Figure **9.1.14**

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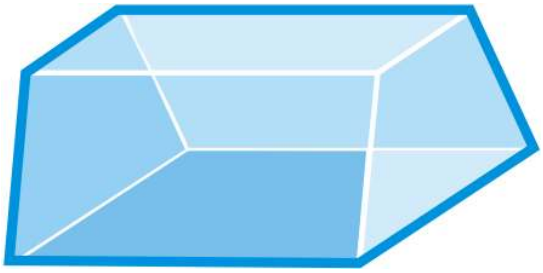

Figure **9.1.15**

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.1.

Additional Resources

Video: Polyhedrons Principles - Basic

Activities: Polyhedrons Discussion Questions

Study Aids: Polyhedra Study Guide

Practice: Polyhedrons

Real World: Roly Poly Polyhedron!

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9.2: Faces, Edges, and Vertices of Solids

Leaning on what makes a solid, identify and count the elements, including faces, edges, and vertices of prisms, cylinders, cones

Figure **9.2.1**

Gayle chose migratory birds as the topic of her biology class project. She has chosen 12 pictures of different birds to display as part of her photo essay. She plans to purchase cube-shaped photo frames that allow her to place a photo on each side of the cube. How many of the photo frames does Gayle need to frame the 12 pictures?

In this concept, you will learn to identify the faces, edges, and vertices of solid figures.

Identifying Faces, Edges, and Vertices of Solid Figures

A **solid figure** can be defined by the number and combination of certain parts. These parts are:

- Faces
- Edges
- Vertices

These three parts are relevant to any solid that has flat surfaces, places where flat surfaces intersect and places where planes intersect.

Let's define each part.

A **face** is any flat surface. The flat surface that makes the front of this cube is called a face. Many solid figures have more than one face.

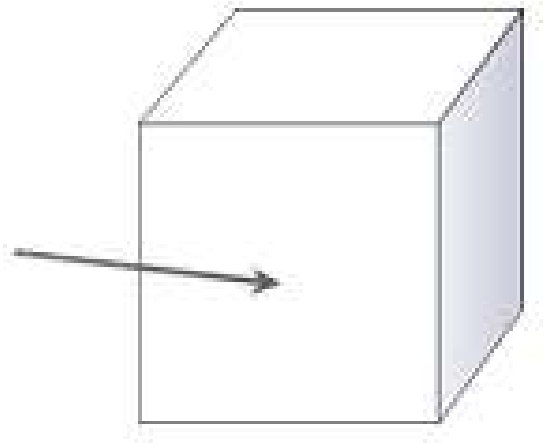

Figure **9.2.2**

An **edge** is the line segment where two faces meet. You can see by looking at this cube that the faces intersect in a line. Many solid figures have more than one edge.

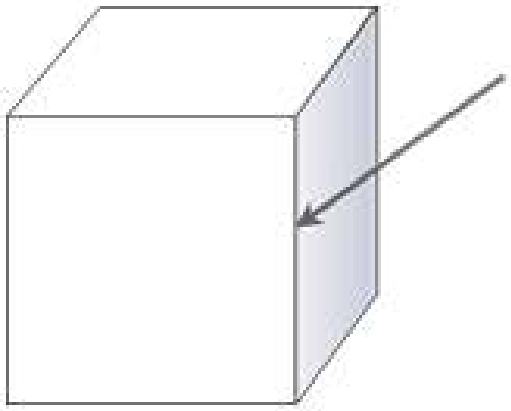

Figure 9.2.3

A **vertex** is a point where several planes meet in a point. The arrow here is pointing to a vertex of this cube. Many solids have more than one vertex, or many **vertices**.

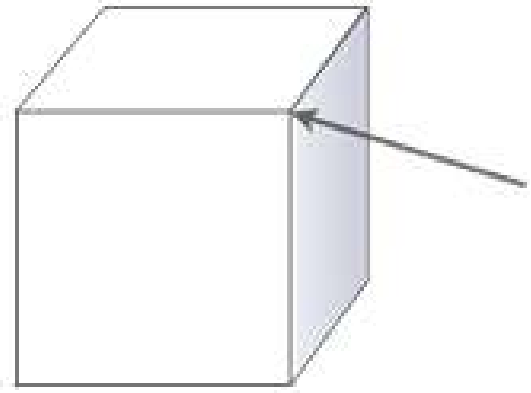

Figure 9.2.4

Let's look at identifying the number of faces, edges and vertices of a solid figure.

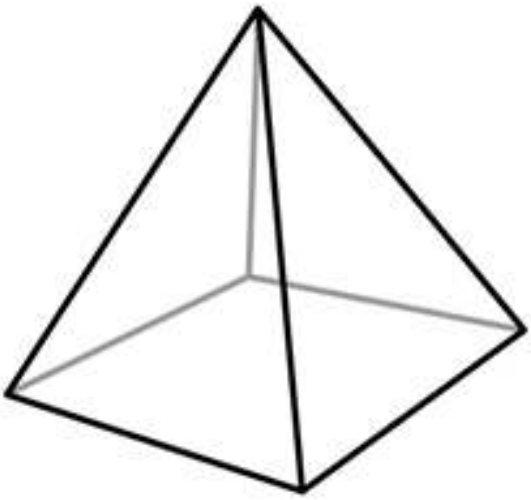

Figure 9.2.5

This is a square **pyramid**.

The faces are the triangle sides and the square bottom. Given this, there are 5 faces in this pyramid.

The edges are where two flat surfaces meet in a line segment. Given this, there are 8 edges in this pyramid. Edges come together at a vertex. There are five vertices in this pyramid.

Example 9.2.1

Earlier, you were given a problem about Gayle and the pictures for her photo essay.

Gayle has already cropped and printed the 12 photos that will be part of biology class project. The 12 photos will be framed in cube-shaped frames. Gayle has to figure out how many frames she needs for the 12 photos. To do this, she must determine how many faces are on a cube.

Solution

To do this, Gayle needs to first visualize a cube.

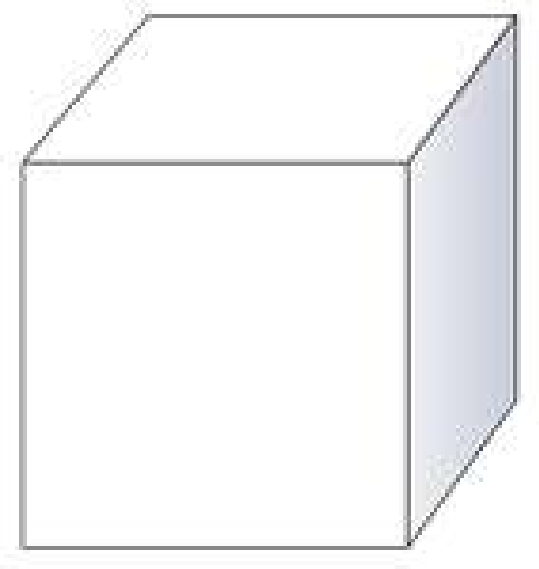

Figure 9.2.6

Next, Gayle can use the definition of a cube to determine how many faces or flat planes there are or she can count the planes herself.

A cube is a special case of a rectangular **prism**, which means it has two congruent flat bases and four flat planes or faces. This means there are a total of 6 flat planes in a cube.

12 divided by 6 is 2.

The answer is Gayle needs 2 photo cubes to display the 12 photos.

Example 9.2.2

Answer the following question about the solid figure below.

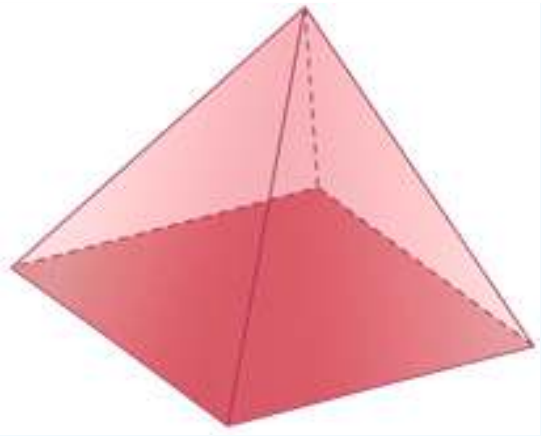

Figure **9.2.7**

How many edges, faces and vertices are there in this figure?

Solution

First, count the edges, which will be line segments.

There are 8 line segments, so there are 8 edges.

Next, count the faces, which will be flat surfaces or flat planes.

There are 5 flat planes, so there are 5 faces.

Then, count the vertices, which are points or corners where several edges or faces meet.

There 5 points, so there are 5 vertices.

The answer is, there are five vertices, five faces and eight edges.

Example 9.2.3

What are the points on a solid figure called?

Solution

First, recall that points are the places where more than two faces meet.

The answer is vertices.

Example 9.2.4

What is the place where two faces meet called?

Solution

First, recall that when two faces meet, they form a line segment.

The answer is an edge.

Example 9.2.5

What are the flat sides of a figure called?

Solution

First, recall that the flat sides are also called flat planes.

The answer is faces.

Review

Identify the part of each solid indicated by the arrow.

1.

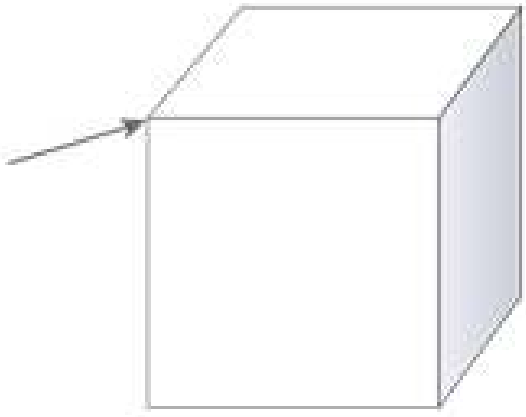

Figure **9.2.8**

2.

Figure **9.2.9**

3.

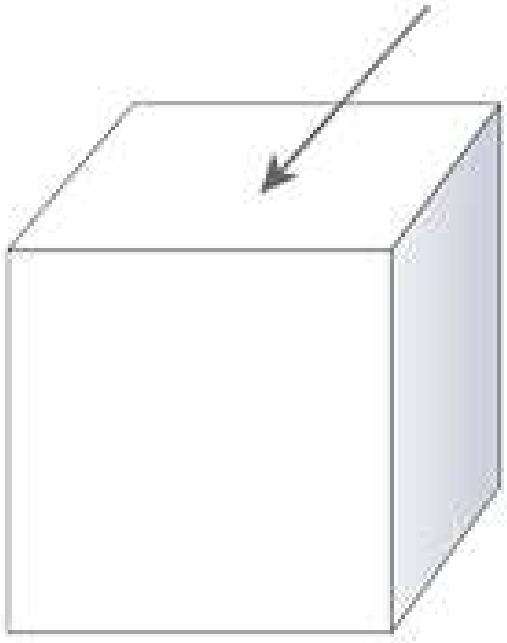

Figure **9.2.10**

Answer each of the following questions about the parts of a solid figure.

4. Define face.

5. Define edge.
6. Define vertex.
7. How many edges in a cube?
8. Which plane figures make up the faces of a square pyramid?
9. Which plane figures make up the faces of a rectangular prism?
10. How many vertices are there in a triangular pyramid?

Identify each figure.

11.

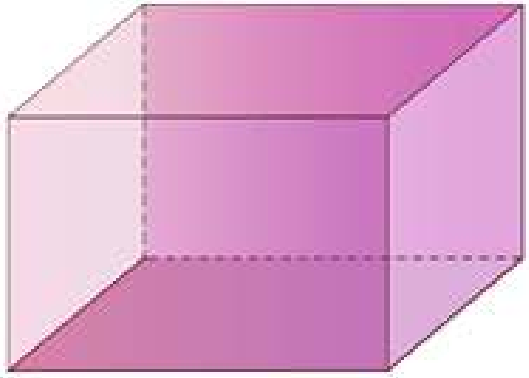

Figure 9.2.11

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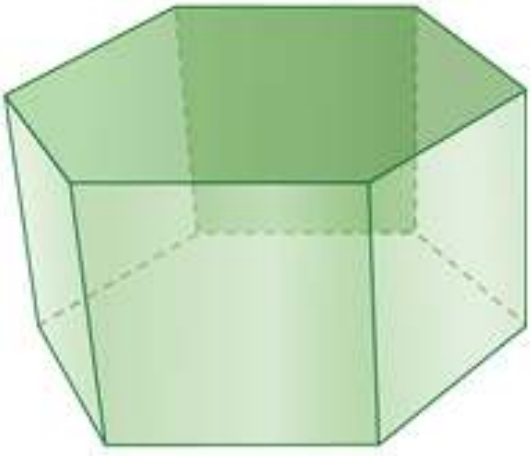

Figure 9.2.12

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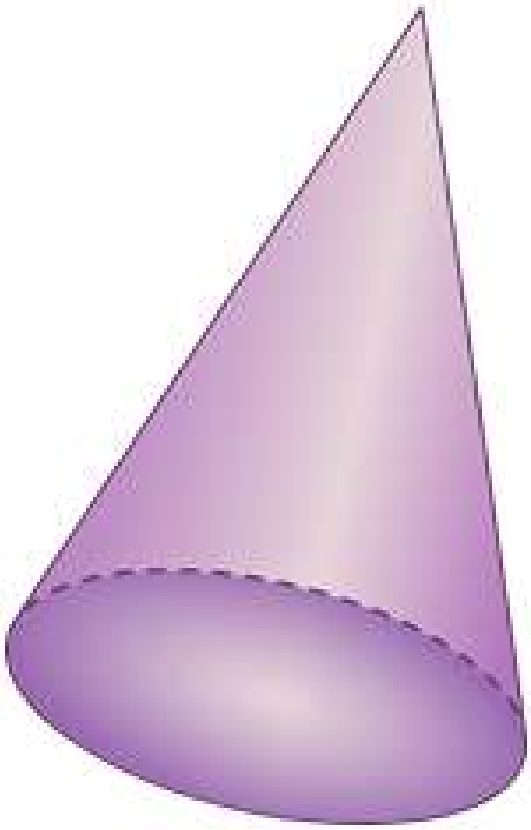

Figure **9.2.13**

14.

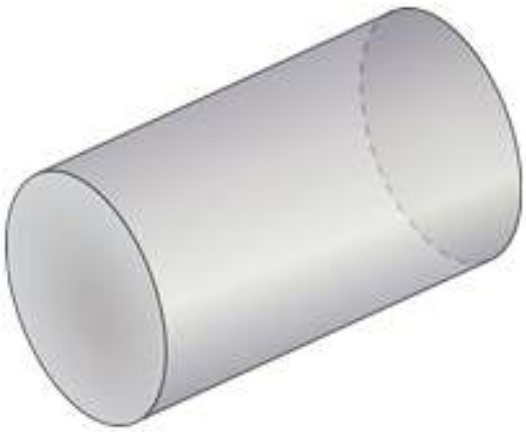

Figure **9.2.14**

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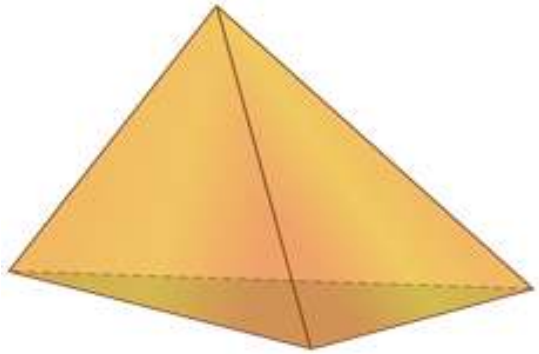

Figure **9.2.15**

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.13.

Vocabulary

Term	Definition
Cone	A cone is a solid three-dimensional figure with a circular base and one vertex.
Cylinder	A cylinder is a solid figure with two parallel congruent circular bases.
Edge	An edge is the intersection between two faces of a figure. An edge is a line segment.
Face	A face is one of the flat surfaces on a solid figure.
Plane Figure	A plane figure is a flat, two-dimensional figure.
Prism	A prism is a three-dimensional object with two congruent parallel bases that are polygons.
Pyramid	A pyramid is a three-dimensional object with a base that is a polygon and triangular faces that meet at one vertex.
Solid Figure	A solid figure is a three-dimensional figure with height, width and depth.
Sphere	A sphere is a round, three-dimensional solid. All points on the surface of a sphere are equidistant from the center of the sphere.
Vertex	A vertex is a corner of a three-dimensional object. It is the point where three or more faces meet.

Additional Resources

Video: Polyhedrons - Faces, Vertices, Edges

Practice: Faces, Edges, and Vertices of Solids

Real World: Erupting Cones

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9.3: Cross-Sections and Nets

Cross-sections are intersections of a plane with a solid. Nets are unfolded, flat representations of the sides of a 3-D shape that allow you to find the sum of the area of the faces by showing the edges and faces in 2-D.

While our world is three dimensional, we are used to modeling and thinking about three dimensional objects on paper (in two dimensions). There are a few common ways to help think about three dimensions in two dimensions. One way to "view" a three-dimensional figure in a two-dimensional plane (like on a piece of paper or a computer screen) is to use cross-sections. Another way to "view" a three-dimensional figure in a two-dimensional plane is to use a **net**.

Cross-Section: The intersection of a plane with a solid.

The cross-section of the peach plane and the tetrahedron is a *triangle*.

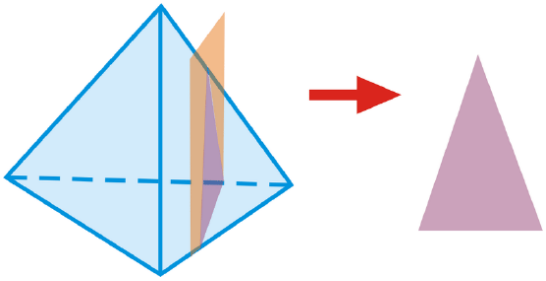

Figure **9.3.1**

Net: An unfolded, flat representation of the sides of a three-dimensional shape.

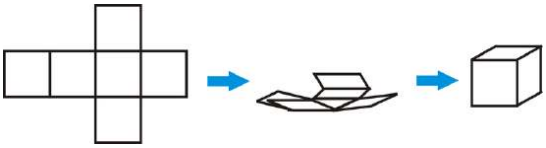

Figure **9.3.2**

It is good to be able to visualize cross sections and nets as the three dimensional objects they represent.

What if you were given a three-dimensional figure like a pyramid and you wanted to know what that figure would look like in two dimensions? What would a flat slice or an unfolded flat representation of that solid look like?

Example 9.3.1

Describe the **cross secti** tersection of the plane and the solid.

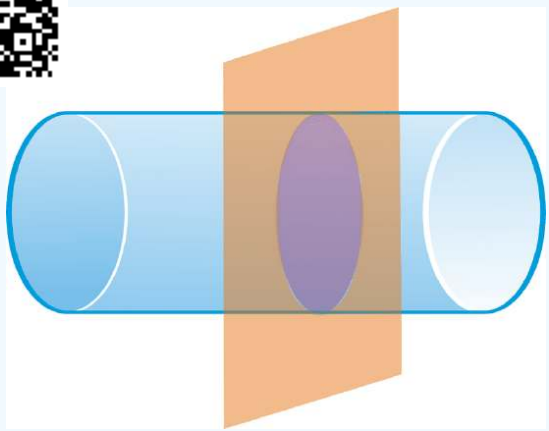

Figure **9.3.3**

Solution

Circle

Example 9.3.2

Determine what shape is formed by the following net.

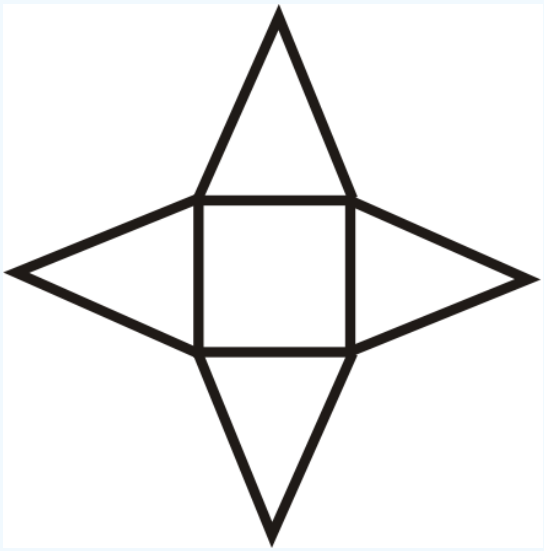

Figure **9.3.4**

Solution

Square-based pyramid

Example 9.3.3

What is the shape formed by the intersection of the plane and the regular octahedron?

a.

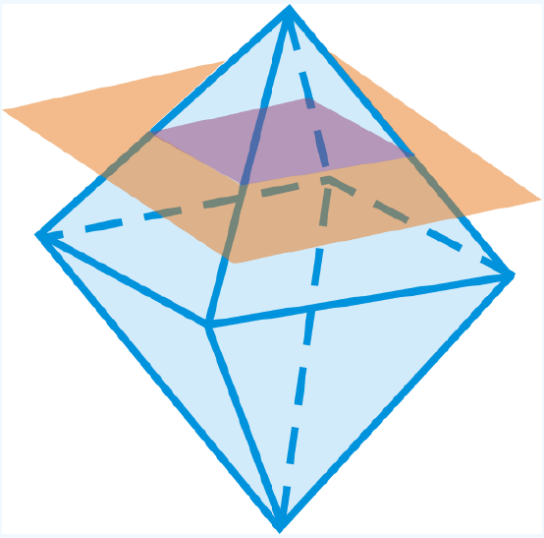

Figure **9.3.5**

b.

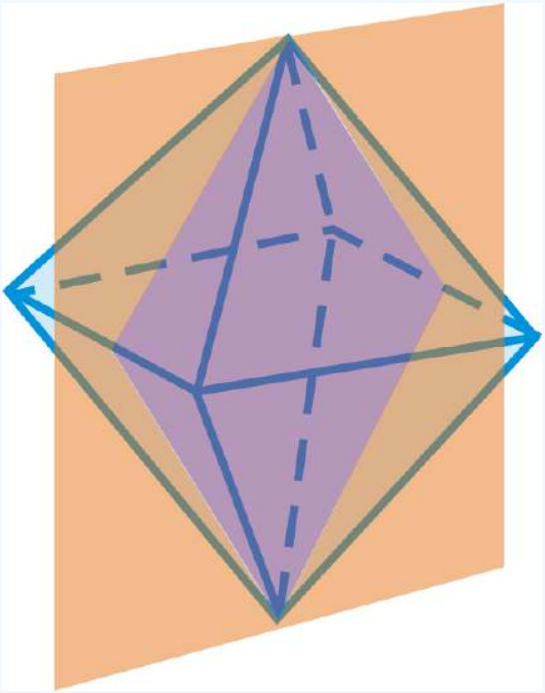

Figure **9.3.6**

c.

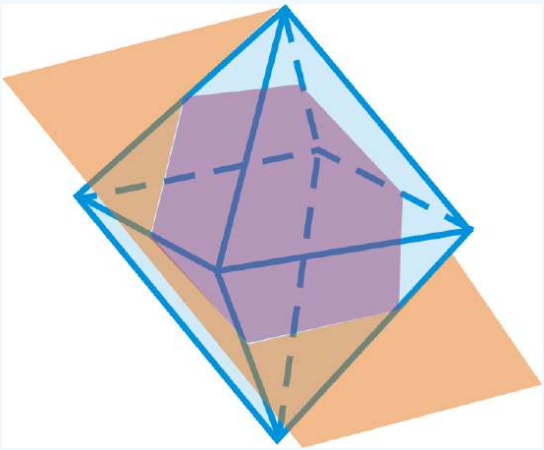

Figure \(\PageIndex{7}\

Solution

a. Square
b. Rhombus
c. Hexagon

Example 9.3.4

What kind of figure does this net create?

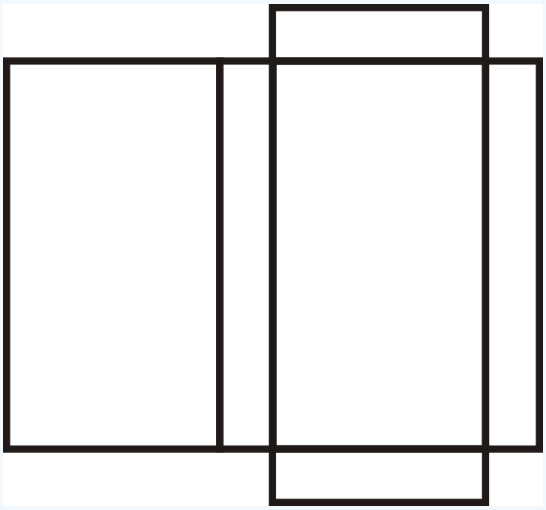

Figure **9.3.8**

Solution

The net creates a rectangular prism.

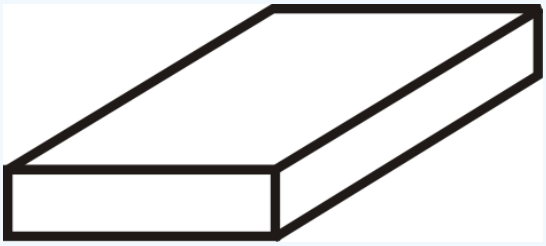

Figure **9.3.9**

Example **9.3.5**

Draw a net of the right triangular prism below.

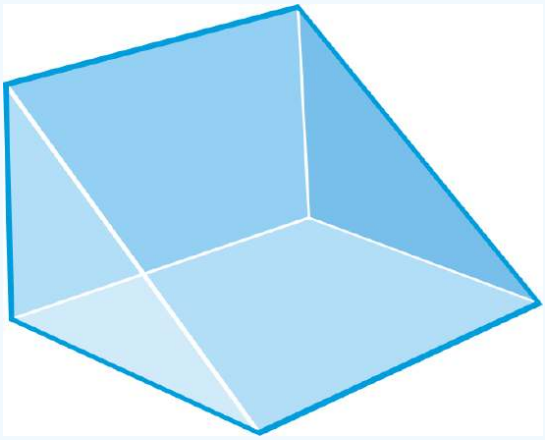

Figure **9.3.10**

Solution

The net will have two triangles and three rectangles. The rectangles are different sizes and the two triangles are the same.

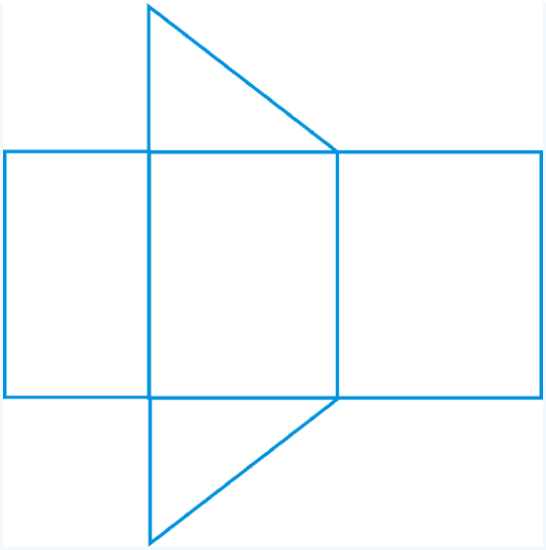

Figure **9.3.11**

There are several different nets of any **polyhedron**. For example, this net could have the triangles anywhere along the top or bottom of the three rectangles.

Review

Describe the cross section formed by the intersection of the plane and the solid.

1.

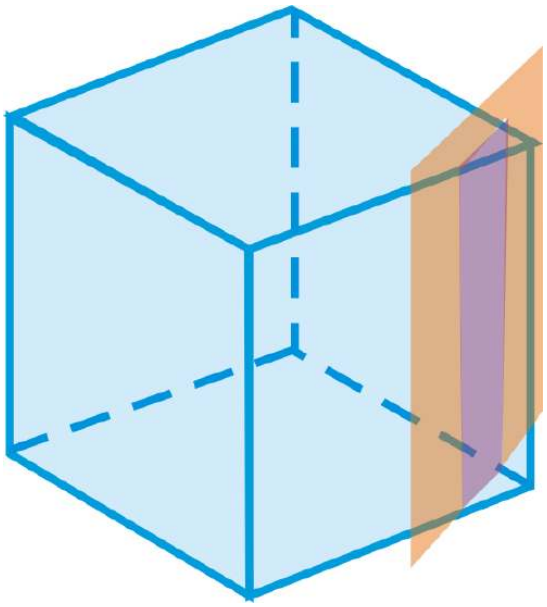

Figure **9.3.12**

2.

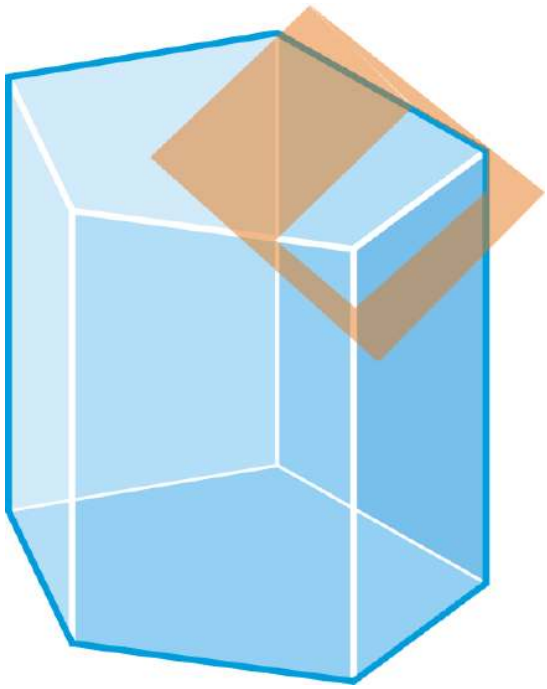

Figure **9.3.13**

Draw the net for the following solids.

3.

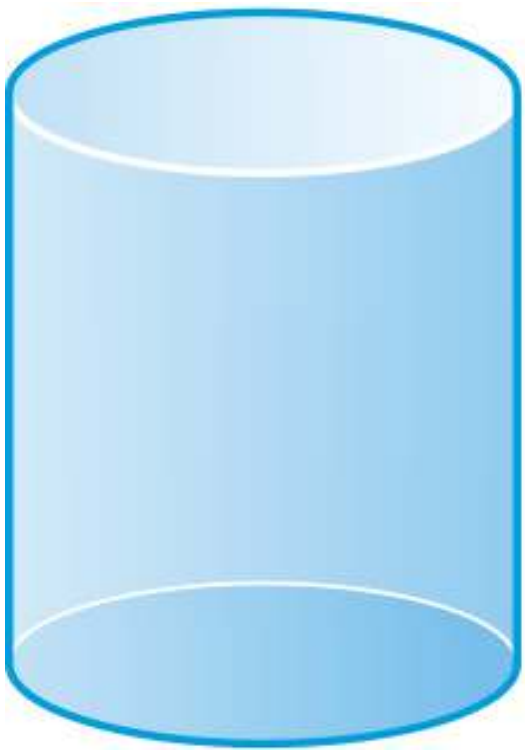

Figure **9.3.14**

4.

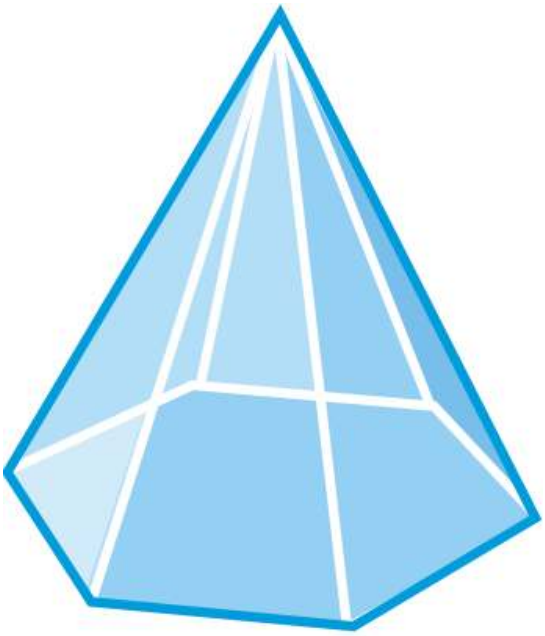

Figure 9.3.15

5.

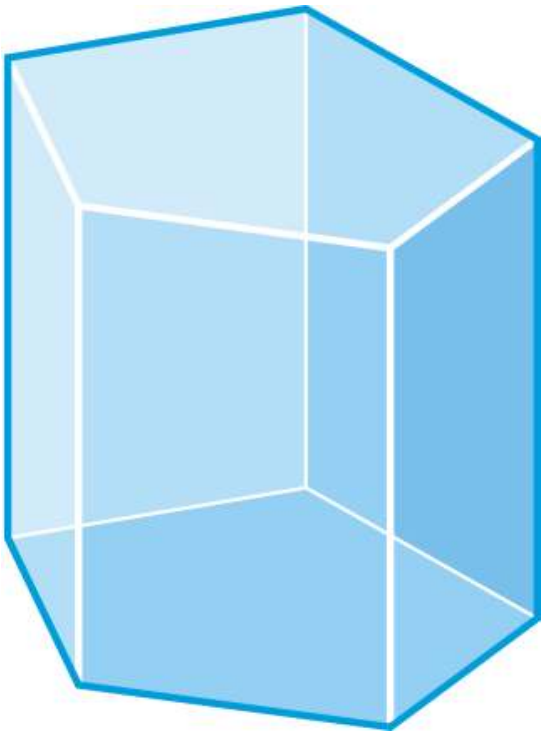

Figure 9.3.16

6.

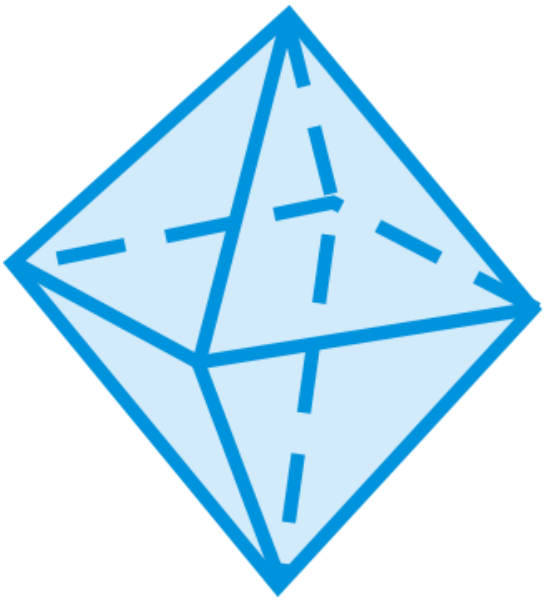

Figure **9.3.17**

7.

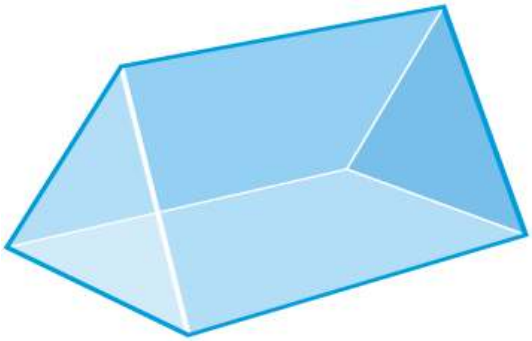

Figure **9.3.18**

8.

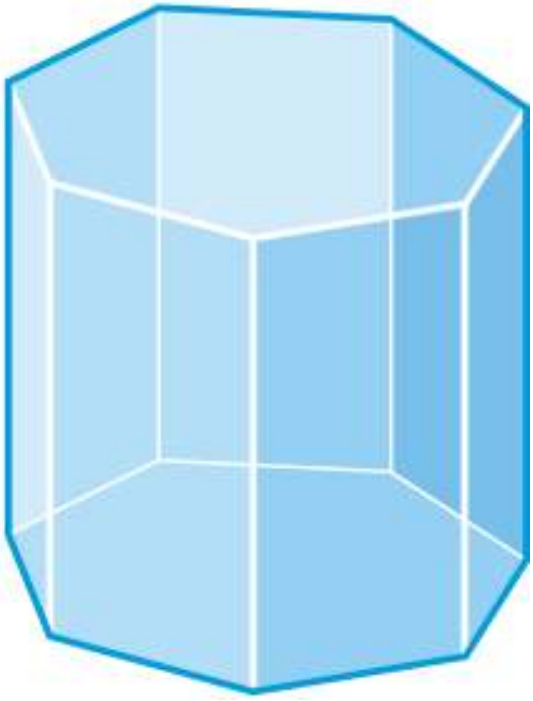

Figure **9.3.19**

Determine what shape is formed by the following nets.

9.

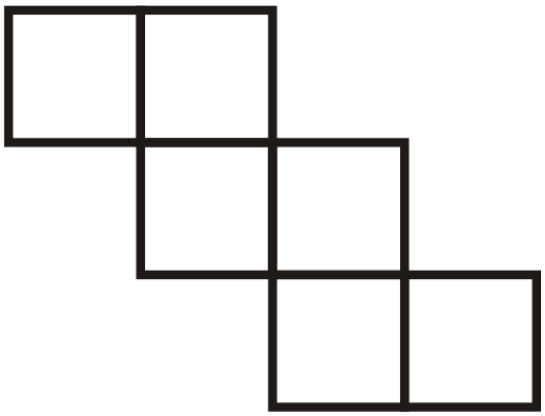

Figure 9.3.20

10.

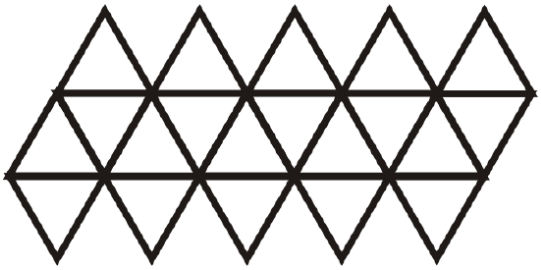

Figure 9.3.21

11.

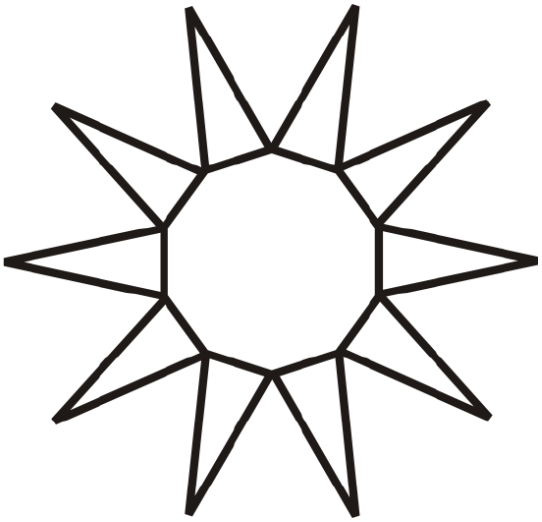

Figure 9.3.22

12.

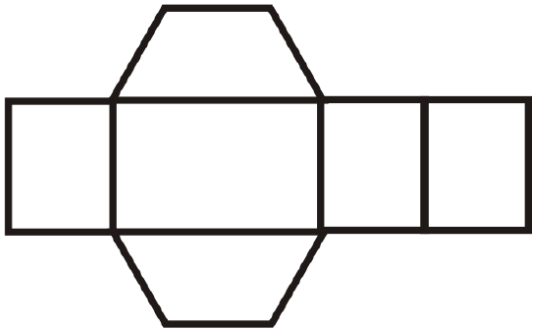

Figure \(\PageIndex{23}\

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.2.

Vocabulary

Term	Definition
cross-section	The intersection of a plane with a solid.
net	An unfolded, flat representation of the sides of a three-dimensional shape.
cross section	A cross section is the intersection of a three-dimensional solid with a plane.
Polyhedron	A polyhedron is a solid with no curves surfaces or edges. All faces are polygons and all edges are line segments.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

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9.4: Surface Area

Surface Area and Nets

In geometry, a net is a 2-dimensional shape that can be folded to form a 3-dimensional shape or a solid. Or a net is a drawing made when the surface of a 3-dimensional figure is laid out flat showing each face and edge of the figure in 2-dimension.

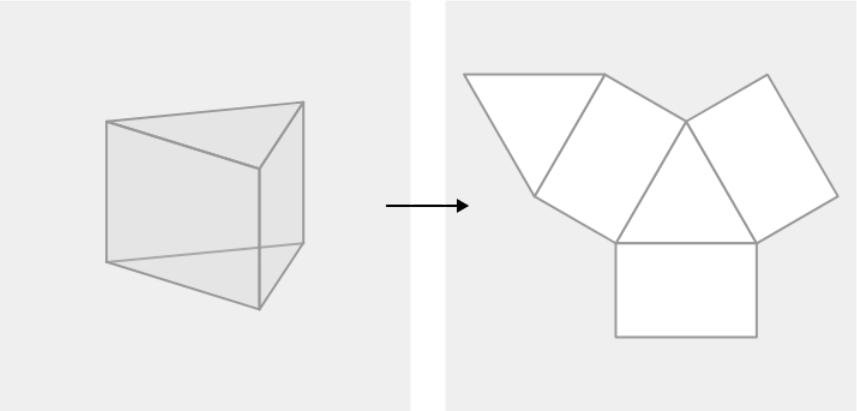

Figure **9.4.1**

Here are some steps to determine whether a net forms a solid:

1. Make sure that the solid and the net have the same number of faces and that the shapes of the faces of the solid match the shapes of the corresponding faces in the net.

2. Visualize how the net is to be folded to form the solid and make sure that all the sides fit together properly.

Nets are helpful when we need to find the surface area of the solids. In the image above, you can see a triangular prism when unfolded consists of two triangles and three rectangles. The triangles are the bases of the prism and the rectangles are the lateral faces.

Surface Area of a Solid

The surface area of a 3-dimensional object is the measure of the total area of all its faces. This means that one way to find the surface area of a solid is to find the area of its net.

$$Surface\ area\ of\ rectangular\ prism = 2ab + 2ac + 2bc = 2(ab + ac + bc)$$

The surface area of a rectangular prism is the area of the six rectangles that cover it. But we don't have to figure out all six because we know that the top and bottom are the same, the front and back are the same, and the left and right sides are the same.

Nets of Solids

Explore the nets and see how the 2d shape transforms into a 3d shape.

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Delete this
placeholder image

Figure **9.4.2**

Note that there could be a couple different interpretations of any net. Most prisms have multiple nets.

Find the approximate surf gular prism below. The base is an equilateral triangle.

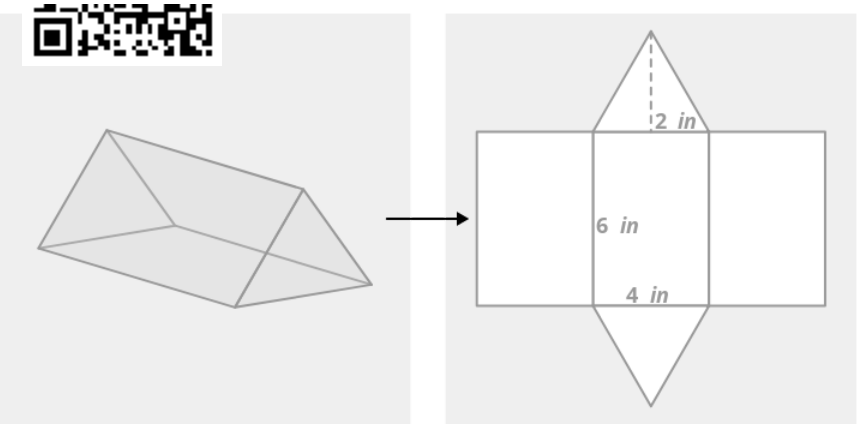

Figure 9.4.3

The net is made of three congruent rectangles and two congruent equilateral triangles.

The surface area is the sum of the areas of the five shapes. To find the area of the triangle, you need to know the height of the triangle. Using the Pythagorean Theorem, you can determine that the height is approximately 3.46 inches.

$$H^2 = P^2 + B^2$$

$$4^2 = P^2 + 2^2$$

$$P^2 = 16 - 4 = 12$$

$$P = 3.46 inches$$

$$\begin{aligned}
Area_{triangle} &= \frac{1}{2}\, bh \\
Area_{triangle} &= \frac{1}{\cancel{2}}\,(\cancel{4}^{2})\,(3.46) \\
Area_{triangle} &= 6.93\ in^2 \\
\hline
Area_{rectangle} &= bh \\
Area_{rectangle} &= (4)\,(6) \\
Area_{rectangle} &= 24\ in^2 \\
\hline
&Total\ surface\ area \\
&= 2\,Area_{triangle} + 3\,Area_{rectangle} \\
&= 2\,(6.93) + 3(24) \\
&= 13.86 + 72 \\
&= 85.86\ in^2
\end{aligned}$$

Figure 9.4.4

Draw the net of a square pyramid.

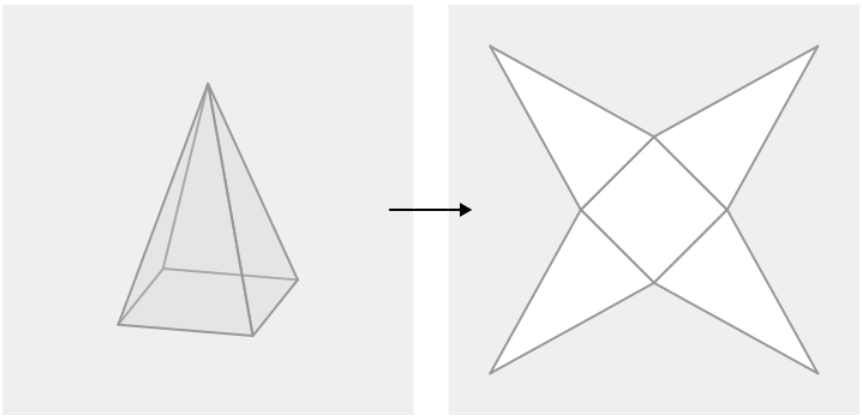

Figure 9.4.5

A square pyramid has a height of 20 inches. Each side of the square base is 12 inches. What is the surface area of the pyramid?

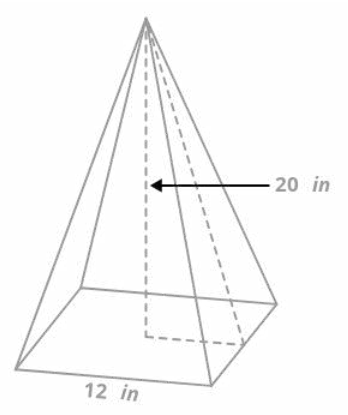

Figure 9.4.6

In order to find the surface area, you will need to determine the area of the triangle faces. In order to find the area of each triangle face, you will need the base and height of the triangle. In order to do this, imagine a right triangle standing upright in the pyramid.

The height of this triangle is 20 inches and the base of the triangle is 6 inches. It's hypotenuse, which is the height of the triangle face, can be determined with the Pythagorean Theorem.

Now you can find the area of each of the five shapes that make up the net in order to find the surface area.

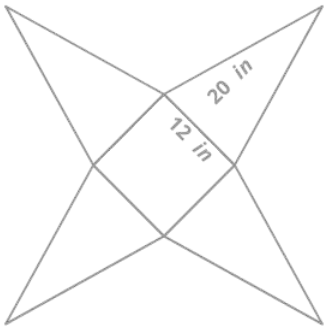

Figure 9.4.7

$$H^2 = P^2 + B^2$$

$$H^2 = 20^2 + 6^2$$

$$H^2 = 400 + 36$$

\(H=\sqrt{436}=20.88 inches

$$
\begin{aligned}
Area_{triangle} &= \frac{1}{2}\, bh \\
Area_{triangle} &= \frac{1}{2}\,(12)\,(20.88) \\
Area_{triangle} &= 125.28\ in^2 \\
\hline
Area_{square} &= sides^{\,2} \\
Area_{square} &= (12)^2 \\
Area_{square} &= 144\ in^2 \\
\hline
&Total\ surface\ area \\
&= 4\,Area_{triangle} + Area_{square} \\
&= 4\,(125.28) + (144) \\
&= 645.12\ in^2
\end{aligned}
$$

Figure 9.4.8

Identify the solid whose net is given below, based on the base shape.

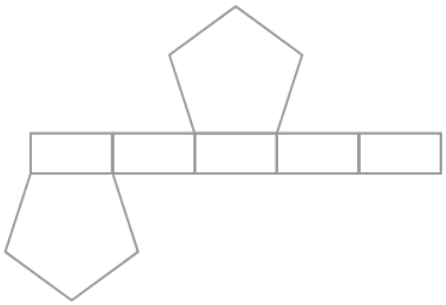

Figure 9.4.9

The solid is _____.

CK-12 PLIX: Cross-Sections and Nets

Example 9.4.1

A cup of paint covers about 22 square feet. You need to paint all faces of a cube to use as a prop in a play. Each edge of the cube is 2.5 feet long. How much paint will you need to buy?

Solution

One cup of paint covers about 22 square feet. Each edge of the cube is 2.5 feet long. In order to figure out how much paint you will need, you should find the surface area of the cube.

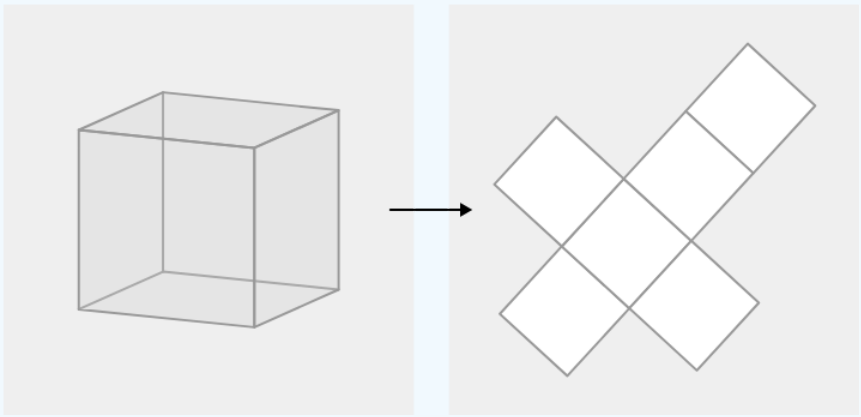

Figure 9.4.10

The cube has six congruent square faces. The area of each square is $(2.5)^2 = 6.25 ft^2$. The total surface area of the cube is $37.5 ft^2$. You will need $37.522 \approx 1.7$ cups of paint.

Example 9.4.2

Draw the net for a pentagonal prism.

Solution

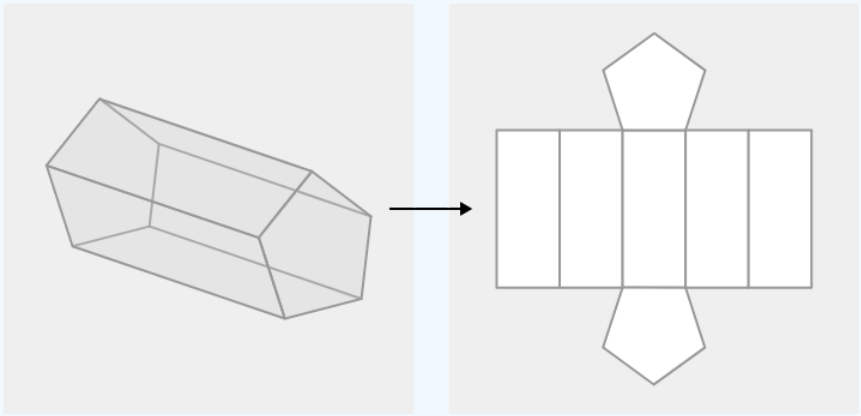

Figure 9.4.11

Example 9.4.3

The base of a pentagonal prism is the following pentagon:

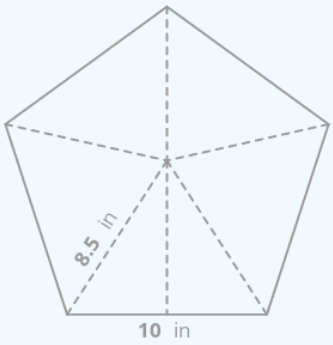

Figure 9.4.12

Find the area of the pentagon by dissecting the pentagon into five triangles and finding the area of each triangle.

Solution

Use the Pythagorean Theorem to find the height of each triangle.

$$H^2 = P^2 + B^2$$

$$8.5^2 = P^2 + 5^2$$

$$P^2 = 72.25 - 25 = 47.25$$

$$P = \sqrt{47.25} = 6.87 inches$$

The area of each triangle is approximately

$$Area_{triangle} = \frac{1}{2} bh$$

$$Area_{triangle} = \frac{1}{2} (10) (6.87)$$

$$Area_{triangle} = 34.37 \ in^2$$

Therefore, the area of the pentagon is:

$$= 5 \, Area_{triangle}$$

$$= 5 (34.37)$$

$$= 171.85 \ in^2$$

Figure **9.4.13**

Example 9.4.4

Find the surface area for a pentagonal prism with a height of 25 inches and a base given in Example 3.

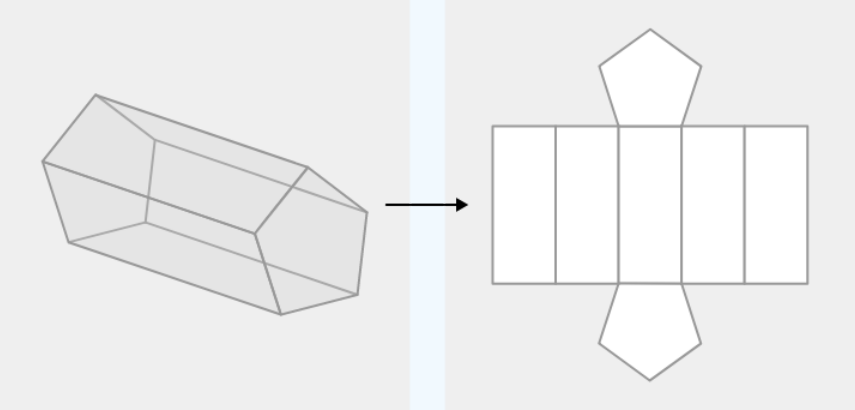

Figure **9.4.13**

Solution

The pentagonal prism is made of five rectangular faces and two pentagonal faces. Each rectangle is 10 inches by 25 inches.

$$\begin{aligned} Area_{rectangle} &= lb \\ Area_{rectangle} &= (10)(25) \\ Area_{rectangle} &= 250\ in^2 \end{aligned}$$

$$\begin{aligned} &Total\ surface\ area \\ &= 5\,Area_{rectangle} + 2\,Area_{pentagon} \\ &= 5\,(250) + 2\,(171.85) \\ &= 1250 + 343.7 \\ &= 1593.7\ in^2 \end{aligned}$$

Figure 9.4.14

Review

1. Explain the connection between the surface area of a solid and the net of a solid.

2. When stating a surface area, why do you use square units such as "in^2"?

A triangular pyramid has four congruent equilateral triangle faces. Each edge of the pyramid is 6 inches.

3. Draw a net for the pyramid.

4. Find the area of one triangle face.

5. Find the surface area of the pyramid.

A 20 inch tall hexagonal pyramid has a regular hexagon base that can be divided into six equilateral triangles with side lengths of 12 inches, as shown below.

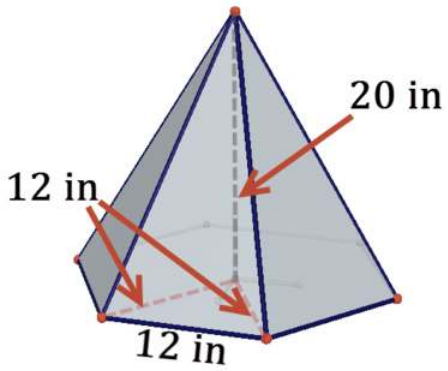

Figure 9.4.15

6. Draw a net for the pyramid.

7. Find the area of the hexagon base.

8. The pyramid has 6 triangular faces. Use the Pythagorean Theorem to help you to find the height of each of these triangles.

9. Find the total surface area of the pyramid.

A square prism is topped with a square pyramid to create the composite solid below.

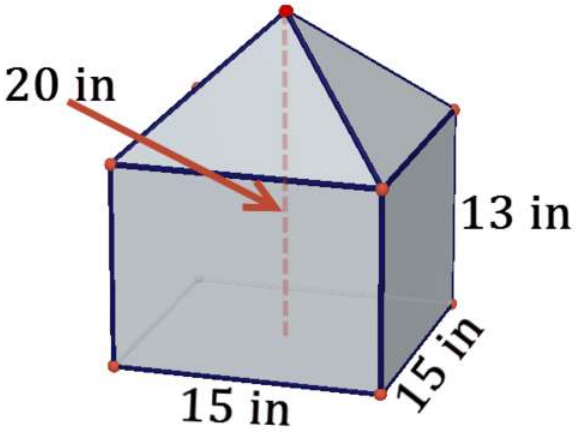

Figure 9.4.16

10. Draw a net for the solid.

11. There are four triangular faces. Use the Pythagorean Theorem to help you find the height of each of these triangles.

12. Find the total surface area of the solid.

A solid has the following net.

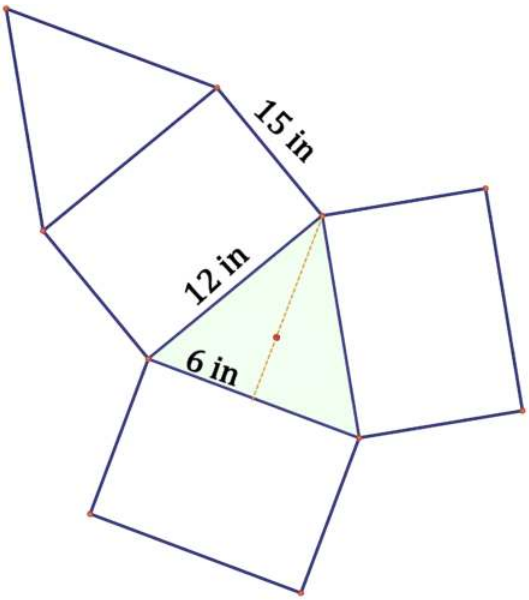

Figure 9.4.17

13. What type of solid is this?

14. Find the surface area of the solid.

15. What would the net of a cylinder look like? Try to make a sketch.

16. What would the net of a cone look like? Try to make a sketch.

16. Graph the points $(2, 3)$ and $(12, 3)$. Label them A and B and connect them to form a segment. Draw vertical segments connecting them to the x-axis. Rotate this shape around the x-axis to form a solid. What figure have you made? Explain. Find the surface area and volume of this figure.

17. Graph the points $(2, 0)$ and $(10, 6)$. Label them A and B and connect them to form a segment. Draw a vertical segment from B to the x-axis. Rotate this shape around the x-axis to form a solid. What figure have you made? Explain. Find the surface area and volume of this figure.

18. Graph the points $(24, 10)$ and $(12, 5)$. Form a solid of revolution as described above. Find its surface area and volume.

Review (Answers)

To see the Review answers, open this PDF file and look for section 1.11.

Additional Resources

Practice: Surface Area

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9.5: Volume

Number of unit cubes to fill prisms, spheres, pyramids, cylinders, cones, and composite solids.

Volume of Solids

The volume of a solid is the measure of how much space an object takes up. It is measured by the number of unit cubes it takes to fill up the solid.

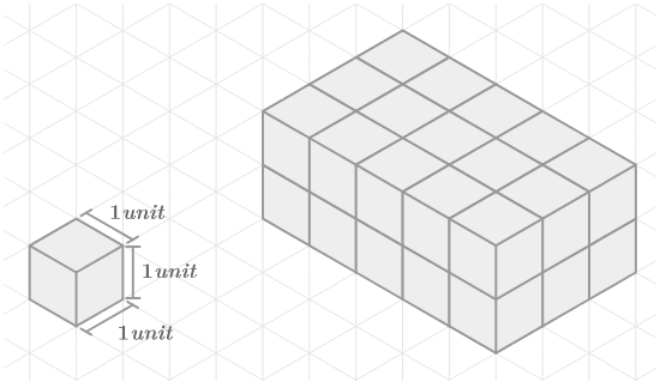


Figure 9.5.1

Volume of a Prism

A prism is a solid with two congruent polygon bases that are parallel and connected by rectangles. Prisms are named by their base shape.

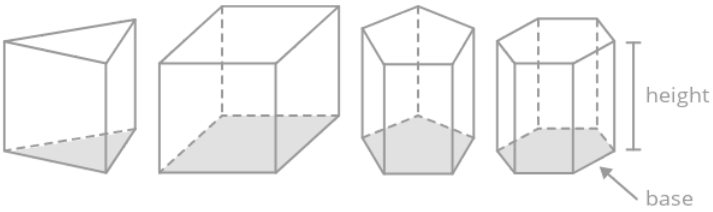


Figure 9.5.2

To find the volume of a prism, find the area of its base and multiply by its height.

$V_{prism} = A_{base} \cdot h$

Volume of a Cylinder

A cylinder is a three-dimensional solid consisting of two congruent, parallel, circular sides (the bases), joined by a curved surface.

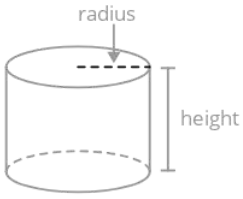


Figure 9.5.3

To find the volume of a cylinder, find the area of its circular base and multiply by its height.

$V_{cylinder} = \pir^2h$

CK-12 PLIX: Surface Area and Volume of Cylinders

Volume of a Pyramid

A pyramid is a three dimensional solid with a polygonal base. Each corner of a polygon is attached to a singular vertex, which gives the pyramid its distinctive shape. Each base edge and the vertex form a triangle. Pyramids are named by their base shape.

To find the volume of a pyramid, find the volume of the prism with the same base and divide by three.

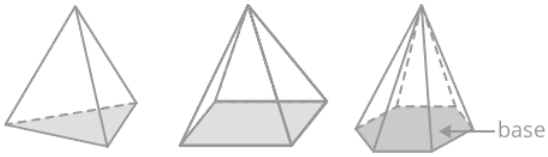


Figure 9.5.4

$$V_{pyramid} = \frac{A_{base} \cdot h}{3}$$

Volume of a Cone

A cone is a three-dimensional solid with a circular base whose lateral surface meets at a point called the vertex.

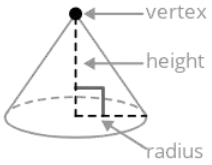


Figure 9.5.5

To find the volume of a cone, find the volume of the cylinder with the same base and divide by three.

$$V_{cone} = \frac{\pi r^2 h}{3}$$

Consider:

What is the ratio between the volume of a cylinder and of a cone, having the same radius and height?

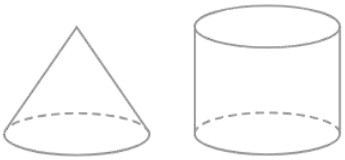


Figure 9.5.6

The ratio between the volume of a cylinder and a cone with the same radius and height is _____.

Volume of a Sphere

A sphere is the set of all points in space equidistant from a center point. The distance from the center point to the sphere is called the radius.

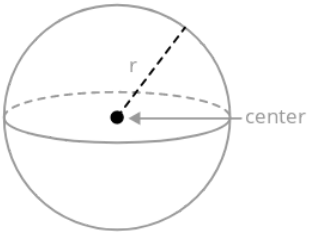


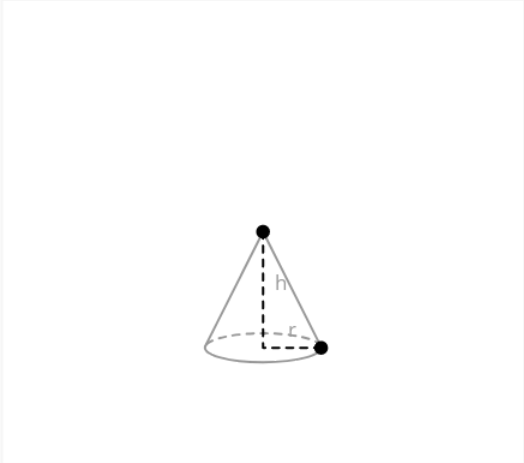
Figure 9.5.7

The volume of a sphere relies on its radius.

$$V_{sphere} = \frac{4}{3}\pi r^3$$

Calculating Volume

Click the small blue arrow next to the image below and drag the orange dots to adjust the dimensions of the solid given below and observe how the volume changes.



$$V_{cone} = \frac{\pi r^2 h}{3}$$

Figure \(\PageIndex{8}\)

Volume of a Composite Solid

A composite solid is a solid made up of common geometric solids. The solids that it is made up of are generally prisms, pyramids, cones, cylinders and spheres.

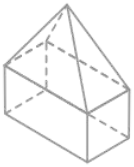


Figure **9.5.8**

The volume of a composite solid is the sum of the volumes of the individual solids that make up the composite.

Let's look at some problen e volume.

1. Find the volume of the ı elow.

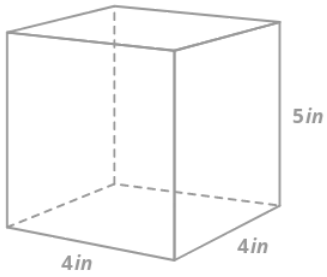


Figure 9.5.9

To find the volume of the prism, you need to find the area of the base and multiply by the height. Note that for a rectangular prism, any face can be the “base”, not just the face that appears to be on the bottom.

$$\begin{aligned} Volume &= Area_{base} \cdot height \\ Volume &= (length \cdot width) \cdot height \\ Volume &= (4)\ \ (4)\ \ (5) \\ Volume &= 80\ in^3 \end{aligned}$$

Figure 9.5.10

2. Find the volume of the cone below.

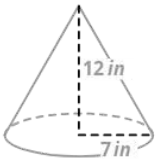


Figure 9.5.11

To find the volume of the cone, you need to find the area of the circular base, multiply by the height, and divide by three.

$$\begin{aligned} Volume &= \frac{1}{3}\ Area_{base} \cdot height \\ Volume &= \frac{1}{3}\,\pi\, r^2\, h \\ Volume &= \frac{1}{3}\,\pi\,(7)^2\ \ (12) \\ Volume &= 196\,\pi\ cm^3 \end{aligned}$$

Figure 9.5.12

3. Find the volume of a sphere with radius 4 cm.

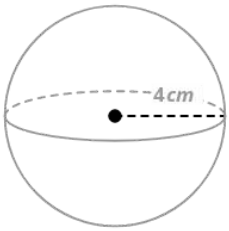


Figure 9.5.11

$$\begin{aligned}\text{Volume of Sphere} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(4)^3 \\ &= \frac{256\pi}{3} cm^3\end{aligned}$$

Example 9.5.1

The composite solid below is made of a cube and a square pyramid. The length of each edge of the cube is 12 feet and the overall height of the solid is 22 feet. What is the volume of the solid? Why might you want to know the volume of the solid?

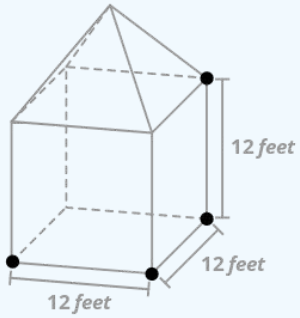


Figure 9.5.1

Solution

To find the volume of the solid, find the sum of the volumes of the prism (the cube) and the pyramid. Note that since the overall height is 22 feet and the height of the cube is 12 feet, the height of the pyramid must be 10 feet.

$$\begin{aligned}\text{Volume of Prism (Cube)} &= A_{\text{base}} \cdot h \\ &= (12 \cdot 12)(12) \\ &= 1728 \text{ft}^3 \\ \text{Volume of Pyramid} &= \frac{A_{\text{base}} \cdot h}{3} \\ &= \frac{(12 \cdot 12)(10)}{3} \\ &= 480 \text{ft}^3 \\ &= \text{Volume of Prism (Cube)} + \text{Volume of Pyramid} \\ &= 1728 + 480 \\ &= 2208 \text{ft}^3\end{aligned}$$

The volume helps you to know how much the solid will hold. One cubic foot holds about 7.48 gallons of liquid, so

$$\begin{aligned}\text{Gallons of liquid the solid can hold} &= \text{Volume of solid . Number of gallons/cubic foot} \\ &= (2208)(7.48) \\ &= 16,515.84 \text{ gallons}\end{aligned}$$

Example 9.5.2

The area of the base of the pyramid below is $100 cm^2$. The height is 5 cm. What is the volume of the pyramid?

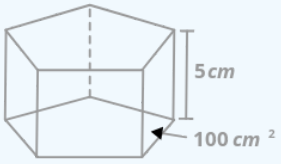


Figure 9.5.12

Solution

$V_{prism} = A_{base} \cdot h = (100 \text{ cm}^2) \cdot (5cm) = 500 \text{ cm}^3$

Example 9.5.3

The volume of a sphere is $\frac{500\pi}{3} in^3$. What is the radius of the sphere?

Solution

To find the radius, use the formula: $Volume\ of\ Sphere = \frac{4}{3}\pir^3$

$$\frac{500\pi}{3} = \frac{4}{3}\pi r^3$$
$$4r^3 = 500$$
$$r^3 = 125$$
$$r = 5 \text{ in}$$

Example 9.5.4

The volume of a square pyramid is $64in^3$. The height of the pyramid is three times the length of a side of the base. What is the height of the pyramid?

Solution

$$V = \frac{A_{base} \cdot h}{3}$$
$$64 = \frac{(s^2)\ (3s)}{3}$$
$$64 = s^3$$
$$\sqrt[3]{64} = s$$
$$4 = s$$

Figure 9.5.13

Side $s = 4$ in and height $h = 3(4) = 12in$.

Review

Find the volume of each solid or composite solid.

1.

Figure 9.5.14

2.

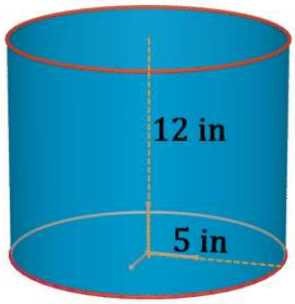


Figure 9.5.15

3.

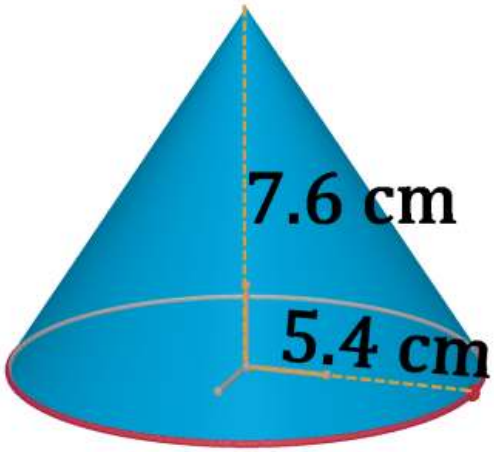


Figure 9.5.16

4.

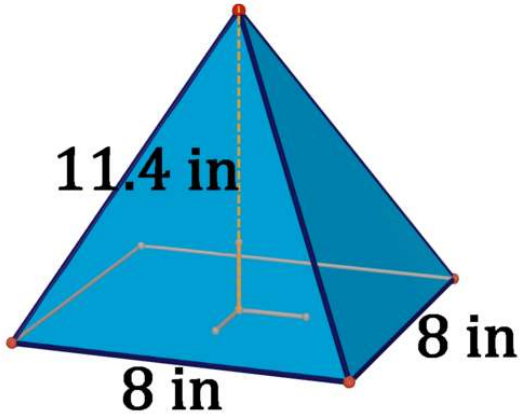


Figure 9.5.17

5. The base is an equilateral triangle.

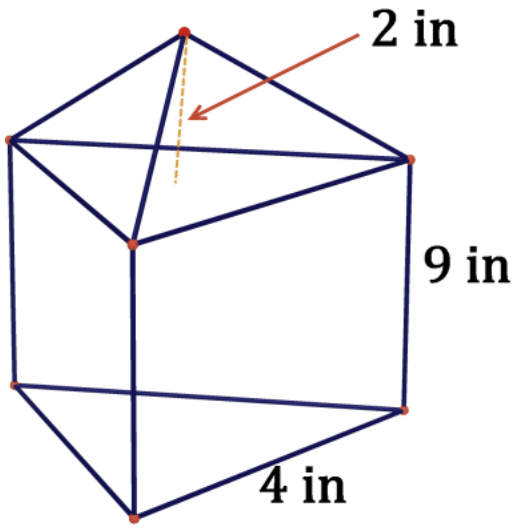


Figure 9.5.18

6.

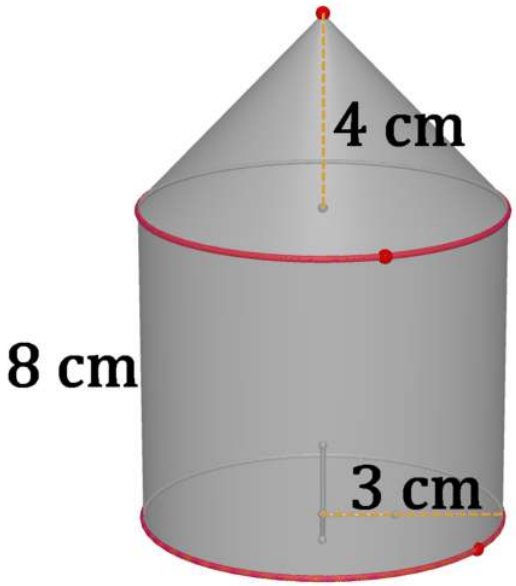


Figure 9.5.19

7. Explain why the formula for the volume of a prism involves the area of the base.

8. How is a cylinder related to a prism?

9. How is a pyramid related to a cone?

10. How is a sphere related to a circle?

11. If one cubic centimeter will hold 1 milliliter of water, approximately how many liters of water will the solid in #1 hold? (One liter is 1000 milliliters).

12. If one cubic centimeter will hold 1 milliliter of water, approximately how many liters of water will the solid in #3 hold? (One liter is 1000 milliliters).

13. If 231 cubic inches will hold one gallon of water, approximately how many gallons of water will the solid in #5 hold?

14. The volume of a cone is $125\pi in^3$. The height is three times the length of the radius. What is the height of the cone?

15. The volume of a pentagonal prism is $360in^3$. The height of the prism is 3 *in*. What is the area of the pentagon base?

16. The figure below features a cylinder whose height is 3 units. Inside the cylinder is a hemisphere (half of a sphere). And inside the hemisphere is a cone. Find the volume of the cylinder and the cone. Find the volume of the sphere and then the hemisphere. Do you see a relationship between the volume of the cylinder and cone, and that of the hemisphere? Explain. Use this to derive the formula for volume of a sphere from the formulas for a cylinder and cone.

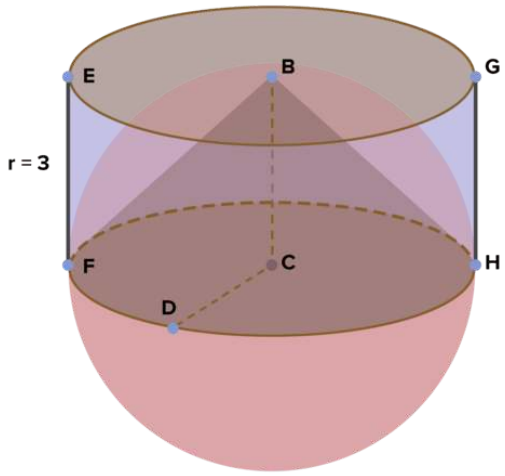


Figure 9.5.20

17. The isometric grid below is composed of equilateral triangles whose side are of unit length 1. Find the volume for each of the pyramids. What do you observe? Explain.

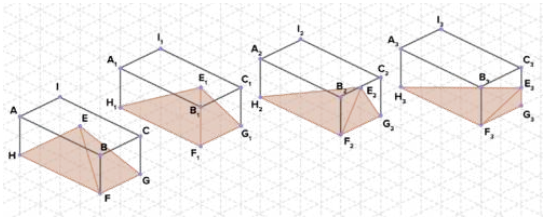


Figure 9.5.21

18. Find the surface area and volume of the following pyramid. Each line segment in the background grid is length 1.

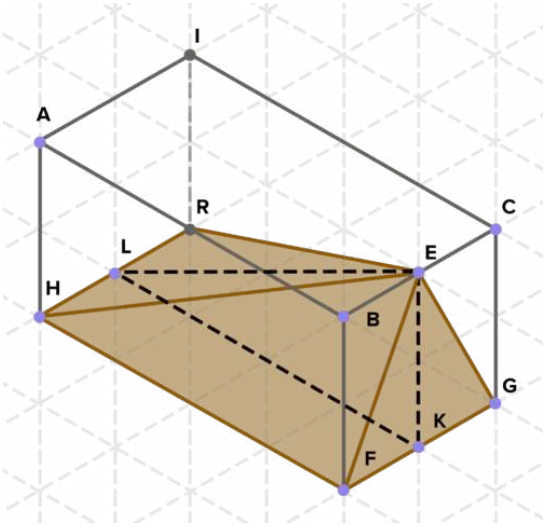


Figure 9.5.22

Review (Answers)

To see the Review answers, click here.

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9.6: Cross Sections and Basic Solids of Revolution

Cross sections of solids and solids created by rotating shapes around lines.

The shaded figure below is rotated around the line. What is the volume of the solid that is created?

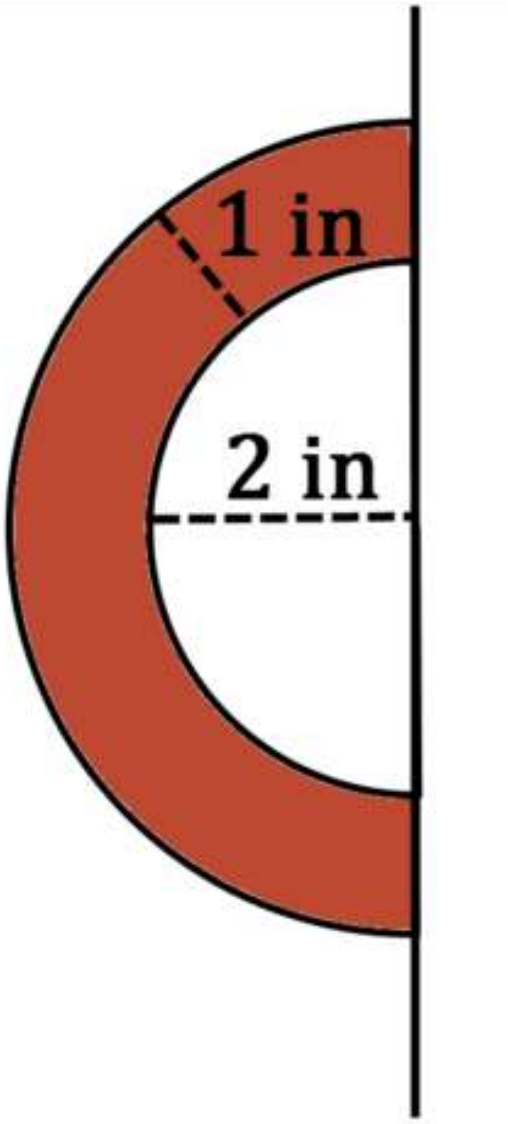

Figure **9.6.1**

Cross Sections

Recall that a **cross section** is the shape you see when you make one slice through a solid. A solid can have many different cross sections depending on where you make the slice. Consider a hexagonal pyramid. Cross sections *perpendicular* to the base will be triangles. Cross sections *parallel* to the base will be hexagons. It is also possible to take cross sections using planes that are neither parallel nor perpendicular to the base. Below, a hexagonal pyramid has been sliced at a slant. The cross section is a pentagon.

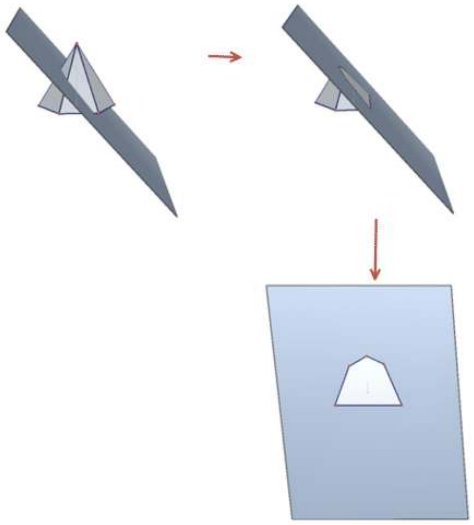

Figure **9.6.2**

Cross sections are one way that two dimensional objects are connected to three dimensional objects.

A second connection between two and three dimensions comes from the fact that three dimensional solids can be created by rotating two dimensional objects around a line.

Let's look at some problems about cross sections.

1. Identify the solid that is created when the following shape is rotated around the line.

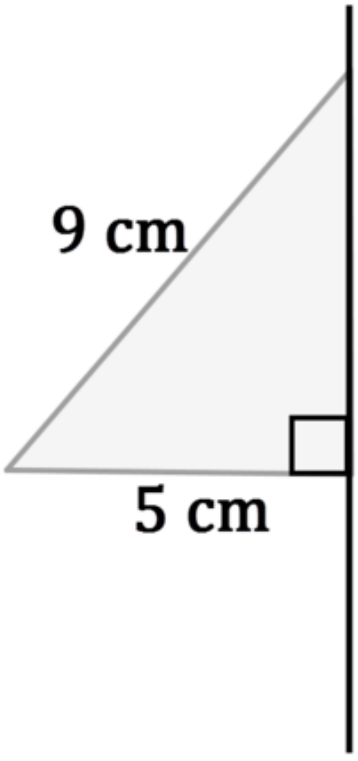

Figure **9.6.3**

The solid is a cone with radius 5 cm.

2. Find the volume of the solid from #1.

The volume of a cone is $V = \frac{\pi r^2 h}{3}$. The radius is 5 *cm*. To find the height of the cone, use the Pythagorean Theorem:

$$5^2 + h^2 = 9^2$$

$$h = \sqrt{56} \approx 7.48\ cm$$

The volume of the cone is:

$$V = \frac{\pi(5^2)(7.48)}{3} \approx 195.83 cm^3$$

3. Identify at least three two dimensional shapes created by cross sections of the solid from first problem.

Cross sections taken parallel to the base will be circles. Cross sections taken perpendicular to the base will be triangles. When the cross section is taken at a slant, there are many other possibilities. Two additional cross sections are an ellipse or a filled in parabola.

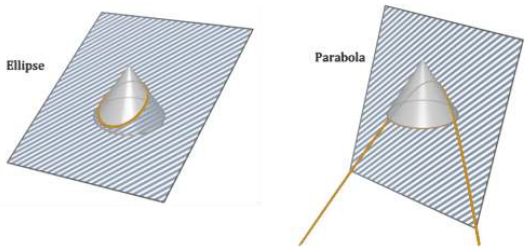

Figure 9.6.4

Example 9.6.1

Earlier, you were asked what is the volume of the solid that was created.

The shaded figure below is rotated around the line.

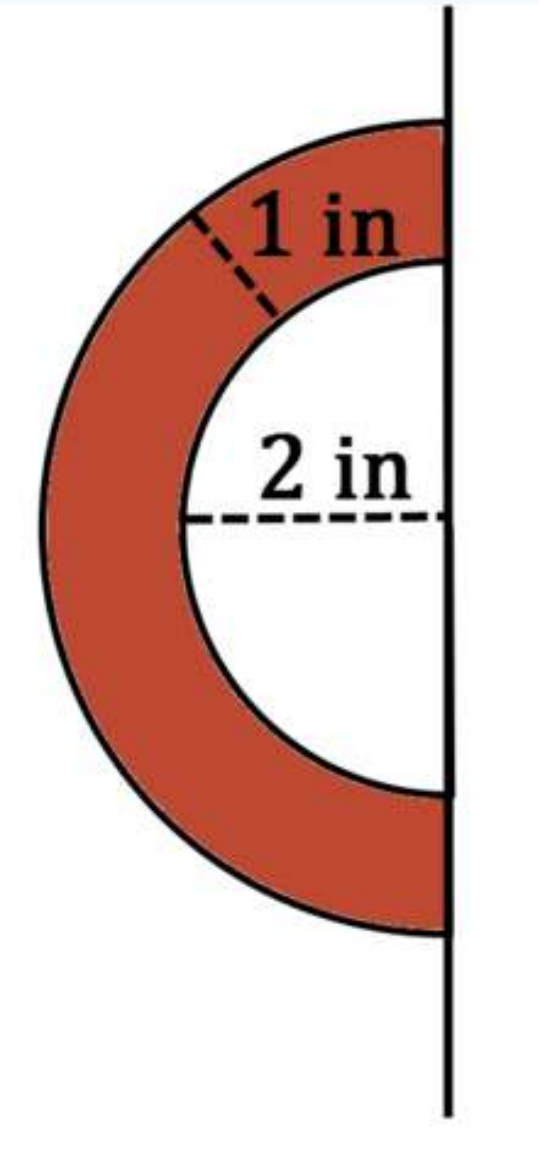

Figure 9.6.5

Solution

The resulting solid is a sphere with a sphere removed from the center. The volume of the large sphere is $\frac{4\pi(3^3)}{3} = 36\pi in^3$. The volume of the small sphere is $\frac{4\pi(2^3)}{3} = 32\pi 3in^3$. The volume of the resulting solid is:

$$V = 36\pi - \frac{32\pi}{3} = \frac{76\pi}{3} \text{ in}^3$$

Use the picture below for #2-#4.

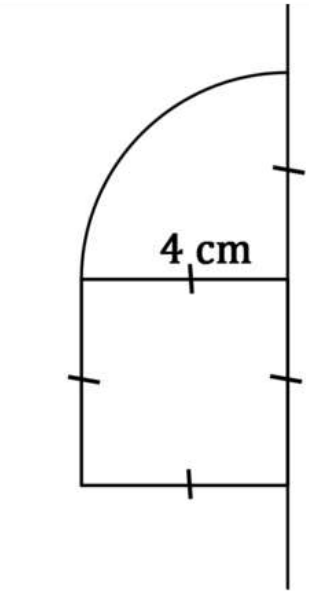

Figure 9.6.6

Example 9.6.2

Identify the solid that is created when the figure above is rotated around the line.

Solution

The solid is a cylinder with a hemisphere on top. The radius of each is 4 cm.

Example 9.6.3

Find the volume of the solid from #2.

Solution

The volume of the cylinder is $\pi(4^2)(4) = 64\pi cm^3$. The volume of the hemisphere is $\frac{4\pi(4^3)}{6} = \frac{128\pi}{3} cm^3$. The volume of the whole solid is $\frac{320\pi}{3} cm^3$.

Example 9.6.4

Identify at least three two dimensional shapes created by cross sections of the solid from #2.

Solution

Possible answers: Cross sections taken parallel to the base will be circles. Cross sections taken perpendicular to the base will be rectangles with half circles on top. Some cross sections taken at a slant will be ellipses.

Review

Use the picture below for #1-#3.

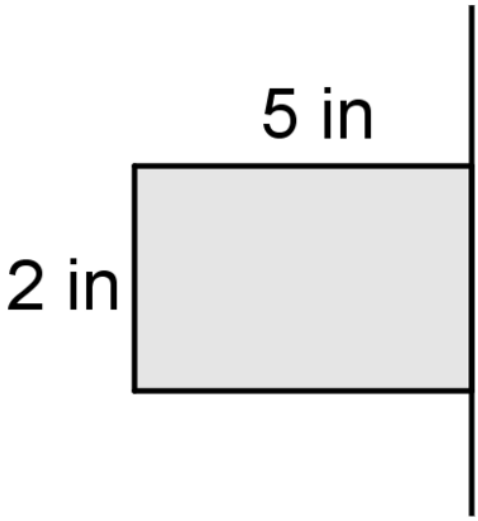

Figure **9.6.7**

1. Describe the solid that is created when the figure above is rotated around the line.

2. Find the volume of the solid.

3. Identify at least 3 two dimensional shapes created by cross sections of the solid.

Use the picture below for #4-#6.

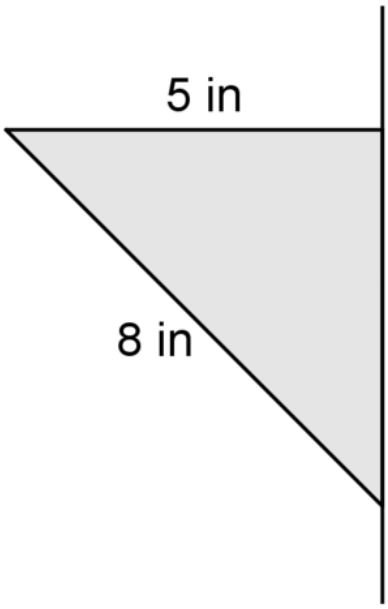

Figure **9.6.8**

4. Describe the solid that is created when the figure above is rotated around the line.

5. Find the volume of the solid.

6. Identify at least 3 two dimensional shapes created by cross sections of the solid.

Use the picture below for #7-#9.

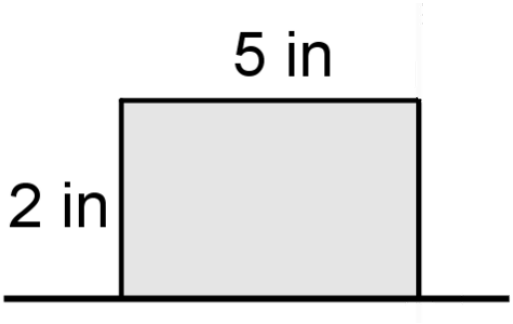

Figure **9.6.9**

7. Describe the solid that is created when the figure above is rotated around the line.

8. Find the volume of the solid.

9. Identify at least 2 two dimensional shapes created by cross sections of the solid.

Use the picture below for #10-#12.

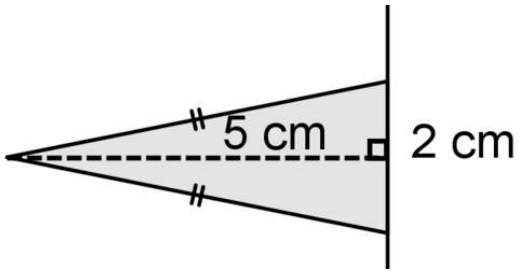

Figure **9.6.10**

10. Describe the solid that is created when the figure above is rotated around the line.

11. Find the volume of the solid.

12. Identify at least 2 two dimensional shapes created by cross sections of the solid.

Use the picture below for #13-#15.

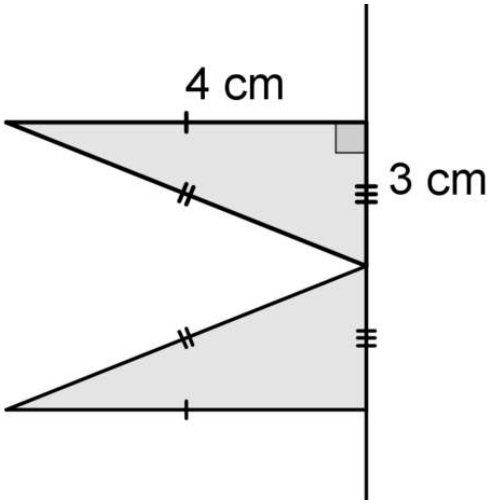

Figure **9.6.11**

13. Describe the solid that is created when the figure above is rotated around the line.

14. Find the volume of the solid.

15. Identify at least 2 two dimensional shapes created by cross sections of the solid.

16. Below are cross-sections of various solids. Find the area of each cross-section. (On the isometric grid paper shown, a side of an equilateral triangle represents 1 unit of length in any direction.)

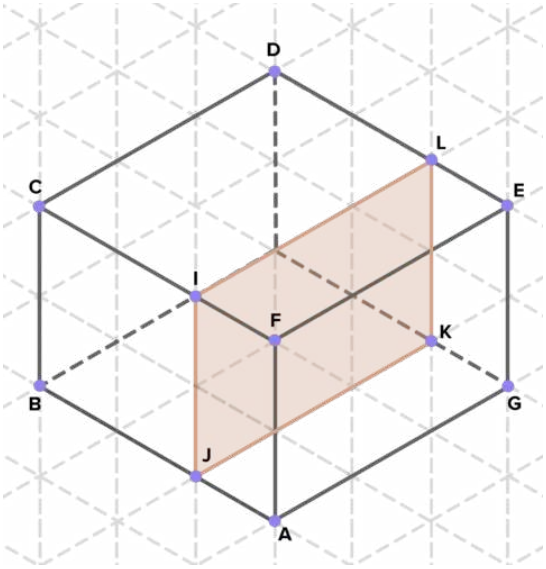

Figure 9.6.12

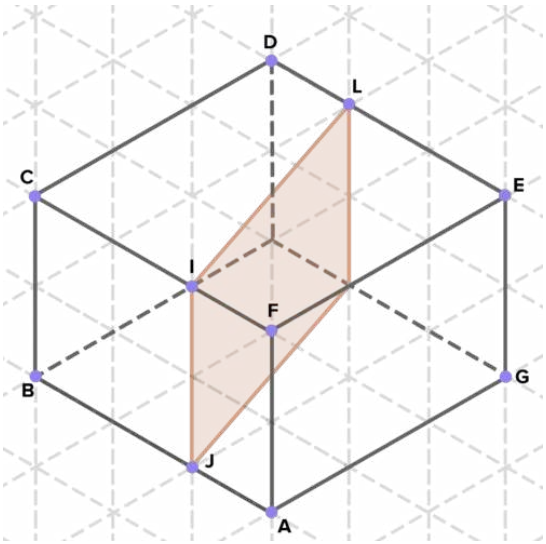

Figure 9.6.13

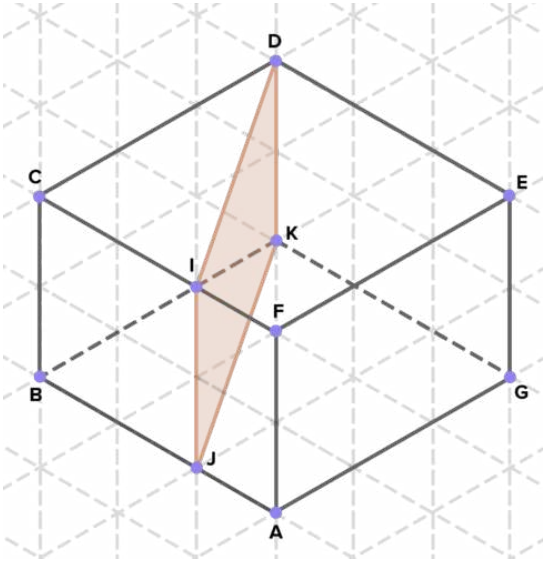

Figure **9.6.14**

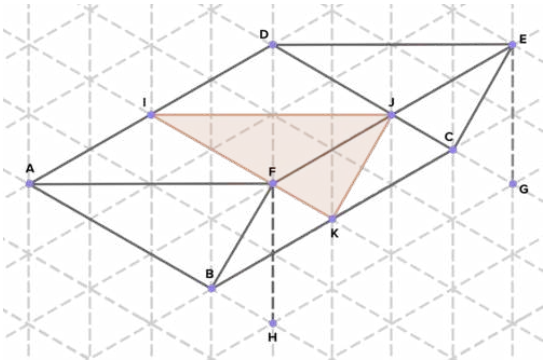

Figure **9.6.15**

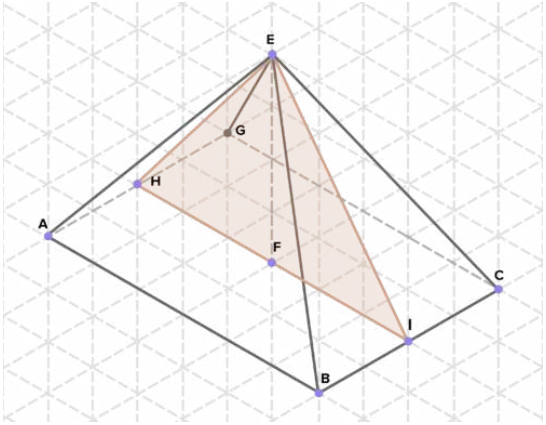

Figure **9.6.16**

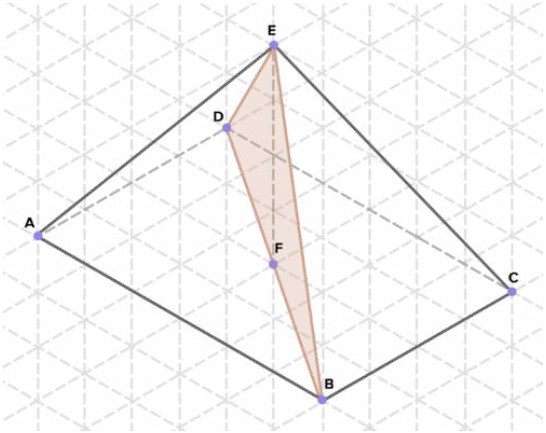

Figure **9.6.17**

Review (Answers)

To see the Review answers, click here.

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9.7: Composite Solids

Solids made up of two or more solids.

A **composite solid** is a solid that is composed, or made up of, two or more solids. The solids that it is made up of are generally prisms, pyramids, cones, cylinders, and spheres. In order to find the surface area and **volume** of a composite solid, you need to know how to find the surface area and volume of prisms, pyramids, cones, cylinders, and spheres. For more information on any of those specific solids, consult the concept that focuses on them. This concept will assume knowledge of those five solids.

Most composite solids problems that you will see will be about volume, so most of the examples and practice problems below are about volume. There is one surface area example as well.

What if you built a solid three-dimensional house model consisting of a pyramid on top of a square prism? How could you determine how much two-dimensional and three-dimensional space that model occupies?

Example 9.7.1

Find the volume of the f

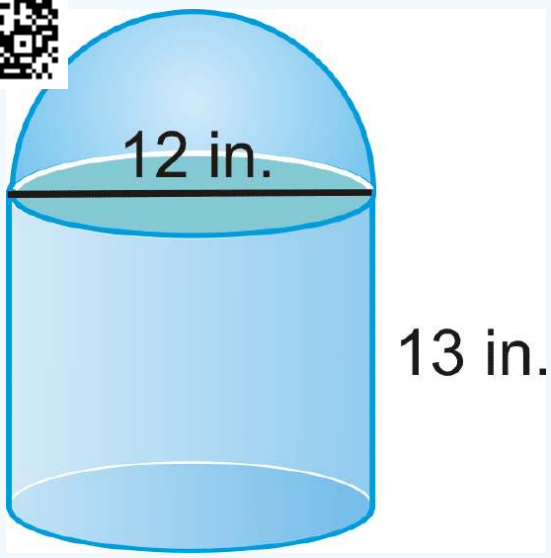

Figure 9.7.1

Solution

Use what you know about cylinders and spheres. The top of the solid is a hemisphere.

$$V_{cylinder} = \pi 6^2(13) = 468\pi$$
$$V_{hemisphere} = 12(\frac{4}{3}\pi 6^3) = 144\pi$$
$$V_{total} = 468\pi + 144\pi = 612\pi \text{ in}^3$$

Example 9.7.2

Find the volume of the base prism. Round your answer to the nearest hundredth.

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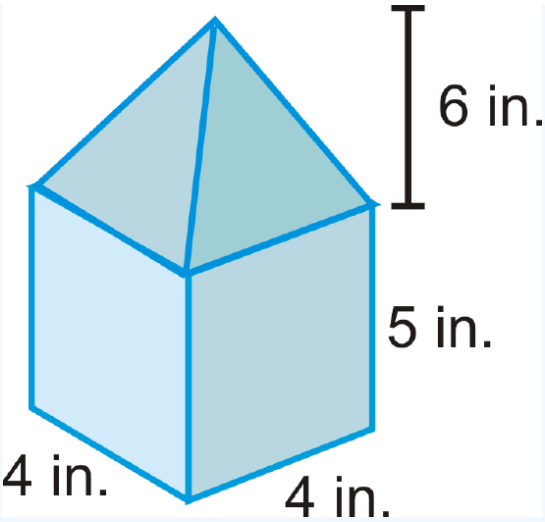

Figure 9.7.2

Solution

Use what you know about prisms.

$$V_{prism} = B \cdot h$$
$$V_{prism} = (4 \cdot 4) \cdot 5$$
$$V_{prism} = 80 \text{ in}^3$$

Example 9.7.3

Find the volume of the solid below.

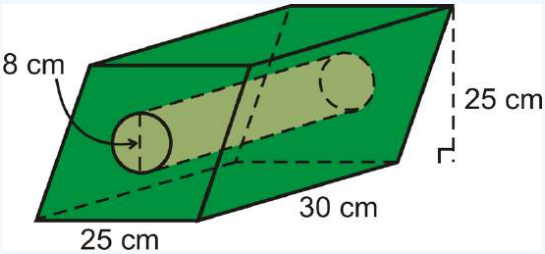

Figure 9.7.3

Solution

This solid is a parallelogram-based prism with a cylinder cut out of the middle.

$$V_{prism} = (25 \cdot 25)30 = 18,750 \text{ cm}^3$$
$$V_{cylinder} = \pi(4)^2(30) = 480\pi \text{ cm}^3$$

The total volume is $18750 - 480\pi \approx 17,242.04 cm^3$.

Example 9.7.4

Find the volume of the composite solid. All bases are squares.

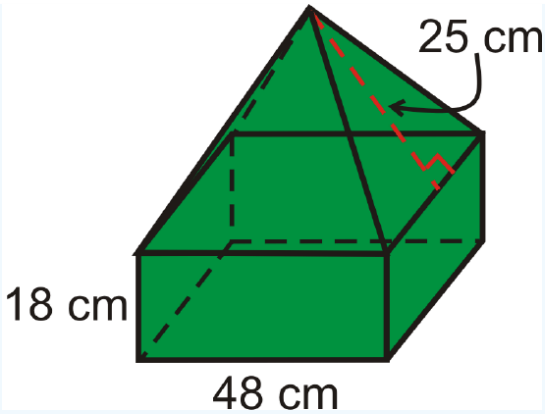

Figure 9.7.4

Solution

This is a square prism with a square pyramid on top. First, we need the height of the pyramid portion. Using the **Pythagorean Theorem**, we have, $h = \sqrt{25^2 - 24^2} = 7$.

$$V_{prism} = (48)(48)(18) = 41,472 \text{ cm}^3$$
$$V_{pyramid} = \frac{1}{3}(48^2)(7) = 5376 \text{ cm}^3$$

The total volume is $41,472 + 5376 = 46,848 cm^3$.

Example 9.7.5

Find the surface area of the following solid.

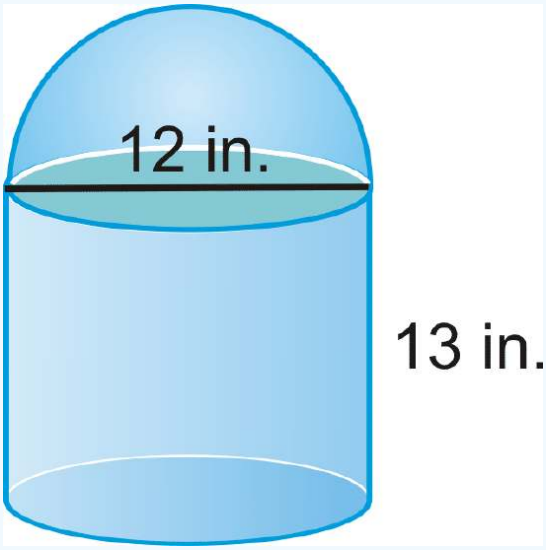

Figure 9.7.5

Solution

This solid is a cylinder with a hemisphere on top. It is one solid, so do not include the bottom of the hemisphere or the top of the cylinder.

$$\begin{aligned} SA &= LA_{cylinder} + LA_{hemisphere} + A_{base\ circle} \\ &= 2\pi rh + \frac{1}{2}4\pi r^2 + \pi r^2 \\ &= 2\pi(6)(13) + 2\pi 6^2 + \pi 6^2 \\ &= 156\pi + 72\pi + 36\pi \\ &= 264\pi in^2 \end{aligned}$$

> "LA" stands for lateral area.

Review

Round your answers to the nearest hundredth. The solid below is a cube with a cone cut out.

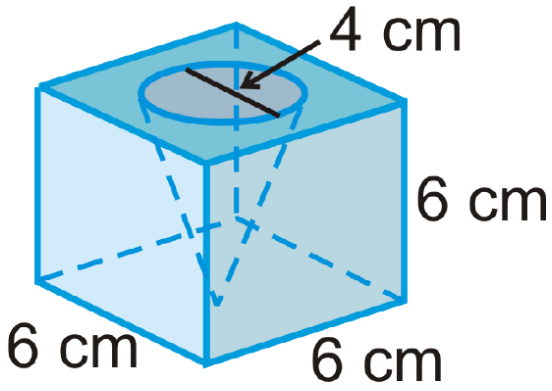

Figure **9.7.6**

1. Find the volume of the cube.
2. Find the volume of the cone.
3. Find the volume of the entire solid.

The solid below is a cylinder with a cone on top.

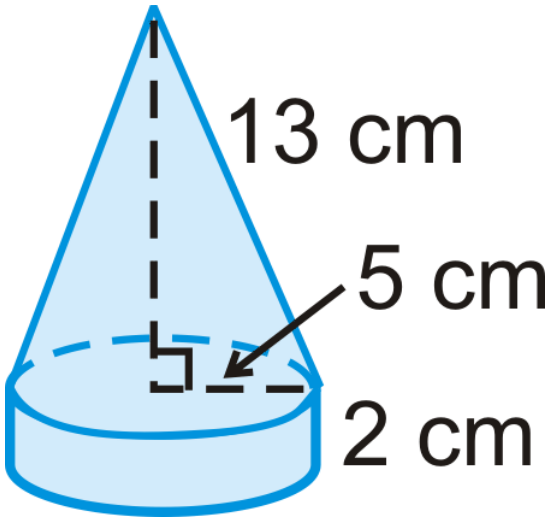

Figure **9.7.7**

4. Find the volume of the cylinder.
5. Find the volume of the cone.
6. Find the volume of the entire solid.

7.

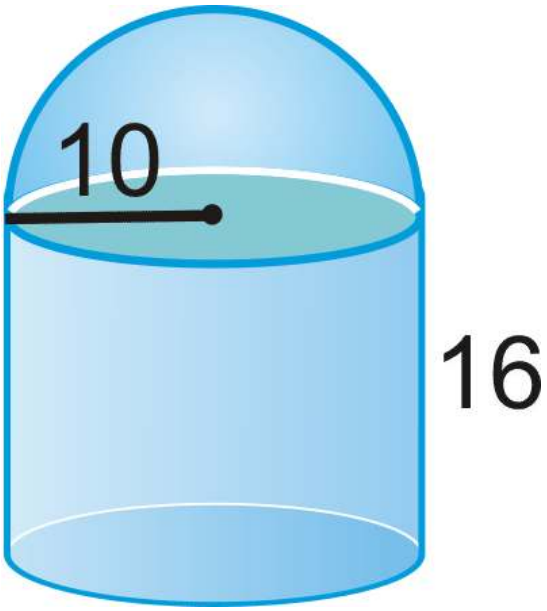

Figure **9.7.8**

8.

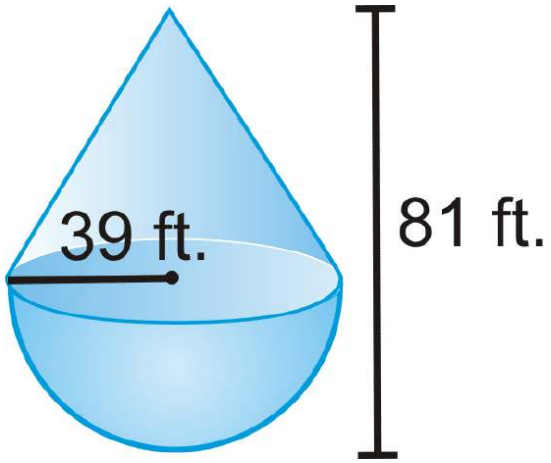

Figure **9.7.9**

9. You may assume the bottom is *open*.

Find the volume of the following shapes. Round your answers to the nearest hundredth.

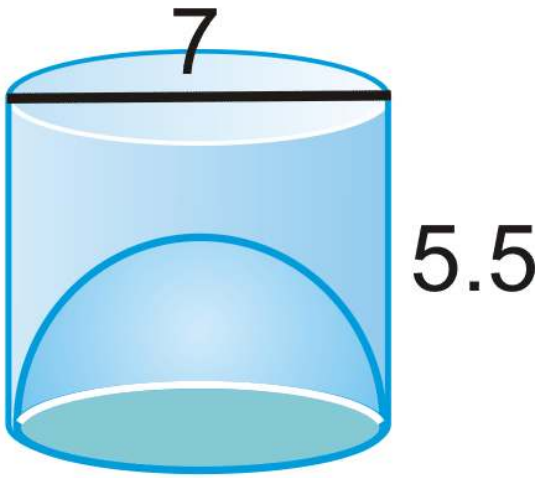

Figure **9.7.10**

10.

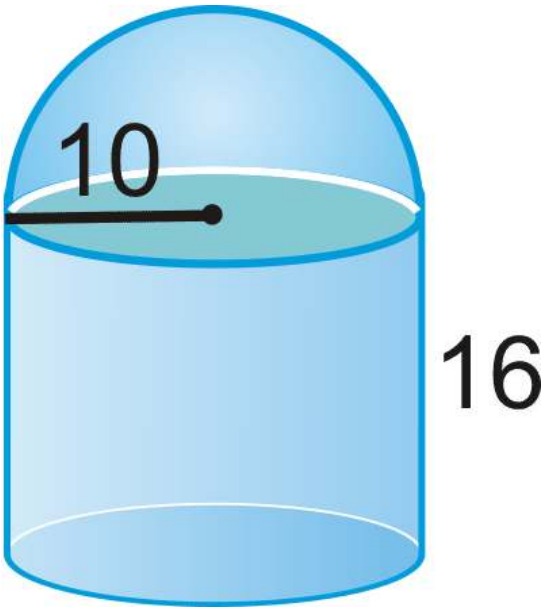

Figure 9.7.11

11.

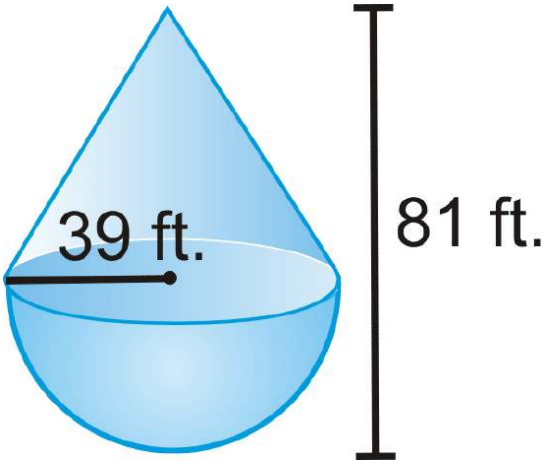

Figure 9.7.12

12.

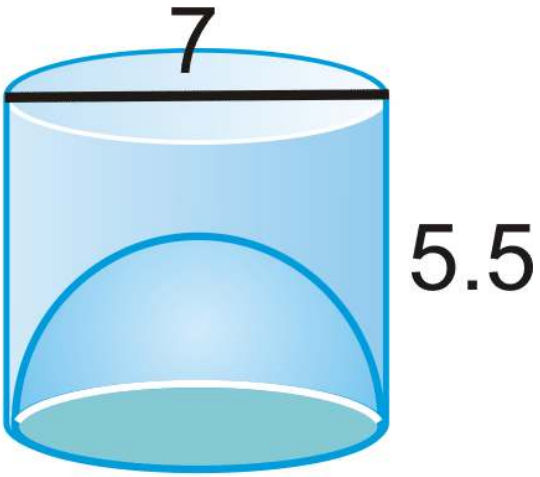

Figure 9.7.1: Copy and Paste Caption here. (Copyright; author via source)

13. A sphere has a radius of 5 cm. A right cylinder has the same radius and volume. Find the height of the cylinder.

The bases of the prism are squares and a cylinder is cut out of the center.

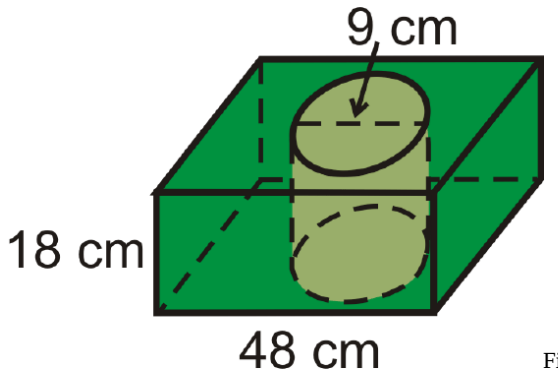

Figure **9.7.13**

14. Find the volume of the prism.
15. Find the volume of the cylinder in the center.
16. Find the volume of the figure.

This is a prism with half a cylinder on the top.

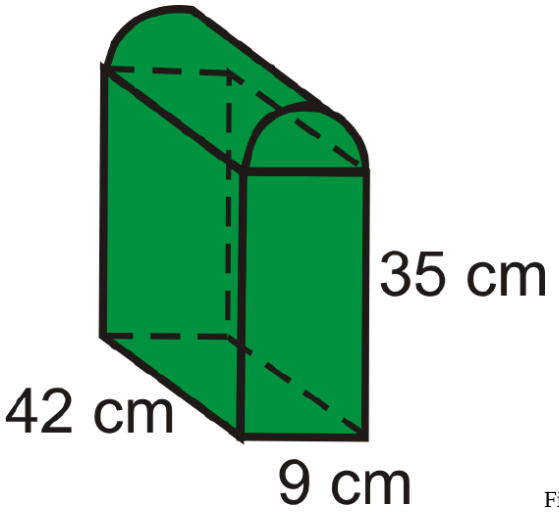

Figure **9.7.14**

17. Find the volume of the prism.
18. Find the volume of the half-cylinder.
19. Find the volume of the entire figure.

Tennis balls with a 3 inch diameter are sold in cans of three. The can is a cylinder. Round your answers to the nearest hundredth.

Figure 9.7.15

20. What is the volume of one tennis ball?
21. What is the volume of the cylinder?
22. Assume the balls touch the can on the sides, top and bottom. What is the volume of the space *not* occupied by the tennis balls?

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.8.

Vocabulary

Term	Definition
composite solid	A solid that is composed, or made up of, two or more solids.
volume	A three-dimensional measurement that is a measure of how much three-dimensional space a solid occupies.
Pythagorean Theorem	The Pythagorean Theorem is a mathematical relationship between the sides of a right triangle, given by $a^2 + b^2 = c^2$, where a and b are legs of the triangle and c is the hypotenuse of the triangle.

Additional Resources

Video: Composite Solids Principles - Basic

Activities: Composite Solids Discussion Questions

Study Aids: Surface Area and Volume Study Guide

Practice: Composite Solids

Real World: Nature's Arches

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9.8: Area and Volume of Similar Solids

Solve problems using ratios between similar solids.

Understand Scale Relationships

Figure **9.8.1**

Tim has a cube with a side length of 4 inches. He has a similar cube with dimensions that are twice the first cube. How does the **volume** of the larger cube compare to the volume of the smaller cube?

In this concept, you will learn to understand scale relationships of **area** and volume.

Scale Relationships

You can compare the scale relationships of distance, area and volume when looking at three–dimensional figures. Some examples of three–dimensional figures include a prism or a pyramid. When you compare different measurements, you will see the proportional relationships between them.

Let's look at a situation involving volume.

Brooke has a **scale model** of a warehouse. A storage unit is shaped like a rectangular prism and has the dimensions 4 in. by 3 in. by 6 in. If the scale of the model is $\mathbf{0.5\ in = 2\ ft}$, what are the actual dimensions of the storage unit? What is the volume?

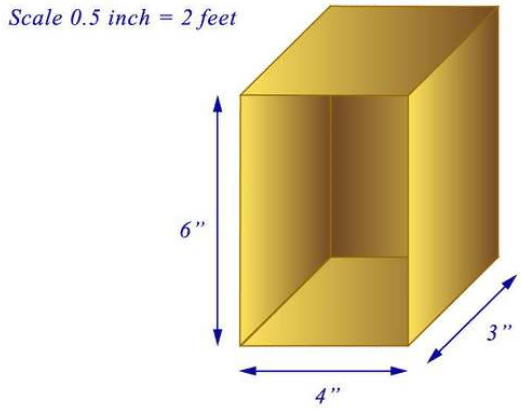

Figure **9.8.2**

First, notice that there are two parts to this problem. The first part is figuring out the actual dimensions given that Brooke has a scale model. The second part is figuring out the volume. Start by using the scale to write a proportion for the length, width, and height.

$$\begin{array}{lll} \text{length} & \text{width} & \text{height} \\ \frac{0.5\text{in}}{2\text{ft}} = \frac{4\text{in}}{x\text{ft}} & \frac{0.5\text{in}}{2\text{ft}} = \frac{3\text{in}}{x\text{ft}} & \frac{0.5\text{in}}{2\text{ft}} = \frac{6\text{in}}{x\text{ft}} \end{array}$$

Next, cross multiply for each dimension.

$$\begin{array}{lll} \text{length} & \text{width} & \text{height} \\ \frac{0.5}{2} = \frac{4}{x} & \frac{0.5}{2} = \frac{3}{x} & \frac{0.5}{2} = \frac{6}{x} \\ 0.5x = 2 \times 4 & 0.5x = 2 \times 3 & 0.5x = 2 \times 6 \\ 0.5x = 8 & 0.5x = 6 & 0.5x = 12 \end{array}$$

Then, divide both sides by 0.5 to solve for x.

$$\begin{array}{lll} \text{length} & \text{width} & \text{height} \\ 0.5x = 8 & 0.5x = 6 & 0.5x = 12 \\ \frac{0.5x}{0.5} = \frac{8}{0.5} & \frac{0.5x}{0.5} = \frac{6}{0.5} & \frac{0.5x}{0.5} = \frac{12}{0.5} \\ x = 16 & x = 12 & x = 24 \end{array}$$

The answers are 16, 12, and 24.

The length of the storage unit is 16 ft, the width is 12 ft, and the height is 24 ft.

Then, you need to calculate the volume of the storage unit.

$$\begin{aligned} V &= l \times w \times h \\ V &= 16 \times 12 \times 24 \\ V &= 4608 \end{aligned}$$

The answer is 4608.

The volume of the storage unit is $4608\ ft^3$ or 4608 cubic feet.

There is a relationship between the area of the base of the prism and the volume of the prism. Let's take a look at how the area of the base of the prism relates to the volume of the prism using the storage unit problem.

$$\begin{aligned} A &= l \times w \\ A &= 16 \times 12 \\ A &= 192 \end{aligned}$$

The area of the prism is 192 ft^{2} or 192 square feet.

Now, to see the relationship between the volume of a prism and the area of a prism, divide the volume by the area.

$$\begin{aligned} \frac{V}{A} &= 4608192 \\ \frac{V}{A} &= 24 \end{aligned}$$

The answer is 24.

Notice that this is the height of the prism.

Example 9.8.1

Earlier, you were given a problem about Tim's cubes.

Tim has two cubes where the larger one is twice the size of the smaller one. This means that there is a scale factor of 2.

Solution

First, find the dimensions of the larger cube.

The smaller cube has a side measure of 4 inches. Since this a cube, the length=width = height =4 inches.

The larger cube will have a side measure of 4 in\times 2=8 in. Therefore the length=width = height =8 inches.

Next find the volume of both cubes and compare.

Volume of smaller cube	Volume of larger cube
$V = l \times w \times h$	$V = l \times w \times h$
$V = 4\text{in} \times 4\text{in} \times 4\text{in}$	$V = 8\text{in} \times 8\text{in} \times 8\text{in}$
$V = 64\text{in}^3$	$V = 512\text{in}^3$

Then, compare the volume of the larger cube to the smaller cube.

$$\frac{\text{Volume of larger cube}}{\text{Volume of smaller cube}} = \frac{512}{64}$$
$$\frac{\text{Volume of larger cube}}{\text{Volume of smaller cube}} = 8$$

The answer is 8.

The volume of the larger cube is 8 times the volume of the smaller cube.

Example 9.8.2

Prove that the height of the following prism can be found by using a ratio of volume to area for a prism with a length of 6 inches, a width of 5 inches and a height of 9 inches.

Solution

First, calculate the volume of the prism.

$$V = l \times w \times h$$
$$V = 6 \times 5 \times 9$$
$$V = 270$$

Next, calculate the area of the prism.

$$A = l \times w$$
$$A = 6 \times 5$$
$$A = 30$$

Then, divide the volume by the area.

$$\frac{V}{A} = \frac{270}{30}$$
$$\frac{V}{A} = 9$$

The answer is 9.

Notice that this is the height of the prism.

Example 9.8.3

A prism has a length of 16 feet, a width of 12 feet, and a height of 18 feet. Find the volume of the prism.

Solution

$$V = l \times w \times h$$
$$V = 16 \times 12 \times 18$$
$$V = 3456$$

The answer is 3456.

The volume of the prism is $3456 ft^3$ or 3456 cubic feet.

Example 9.8.4

A prism has a length of 16 feet, a width of 12 feet, and a height of 18 feet. Find the area of the base.

Solution

$$A = l \times w$$
$$A = 16 \times 12$$
$$A = 192$$

The answer is 192.

The area of the prism is $192 ft^2$ or 192 square feet.

Example 9.8.5

Write a ratio comparing the volume to the area of the prism and simplify.

Solution

Misplaced &

The answer is 18.

The height of the prism is 18 feet.

Review

Solve each problem.

1. A cube measures 8 feet on each side. A similar cube has dimensions that are twice as large. How does the volume of the larger cube compare to the volume of the smaller cube? Write a ratio to show the comparison.

2. A cube measures 3 inches on each side. A similar cube has dimensions that are half that of the other cube. How does the volume of the larger cube compare to the volume of the smaller cube? Write a ratio to show the comparison

3. A scale model of a sandbox has dimensions 0.5 inch by 3 inches by 4 inches. If the scale of the model is 1 inch=1 foot, what is the volume of the actual sandbox?

4. A cube measures 5 inches on each side. A similar cube has dimensions that are 3 times as large. How does the volume of the larger cube compare to the volume of the smaller cube? Write a ratio to show the comparison.

5. A shipping box measures 16 inches by 12 inches by 8 inches. A second box has a similar size but each dimension is 14 as long. How does the volume of the second box compare to the volume of the first box?

6. Rina's fish tank has a volume of 8,000 cubic inches. The dimensions of AVA's fish tank are all 12 the size of Rina's. What is the volume of AVA's fish tank?

7. A prism has a width of 6 feet, a length of 8 feet and a height of 12 feet. What is the volume of the prism?

8. What is the area of the base of this prism?

9. What would the volume be of a prism 14 the size of the one described above?

10. What would the volume be of a prism 12 the size of the one described above?

11. What would the volume be of a prism twice the size of the one described above?

12. What ratio can you use to discover the relationship between volume and area?

13. Which measurement will you find when you simplify this ratio?

14. True or false. You can use scale measurement to find the height of a prism.

15. True or false. You can use scale measurement to find the dimensions of a prism.

Resources

Review (Answers)

To see the Review answers e and look for section 4.11.

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Scale Model	A scale model is a model that represents a three-dimensional space.
Three – Dimensional	A figure drawn in three dimensions is drawn using length, width and height or depth.
Two – Dimensional	A figure drawn in two dimensions is only drawn using length and width.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Video: Similar Solids Principles - Basic

Activities: Area and Volume of Similar Solids Discussion Questions

Study Aids: Surface Area and Volume Study Guide

Practice: Area and Volume of Similar Solids

Real world: Is There Life Out There?

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9.9: Surface Area and Volume Applications

Use geometric solids to model real world objects and solve problems. Find density by the ratio of mass, people, or things to volume or area.

Modeling in Three Dimensions

Mark wants to make an open box from an 8.5 inch by 11 inch piece of paper by cutting squares out of each corner, folding up the sides, and securing with tape. How does the volume of his box relate to the size of the squares he cuts out?

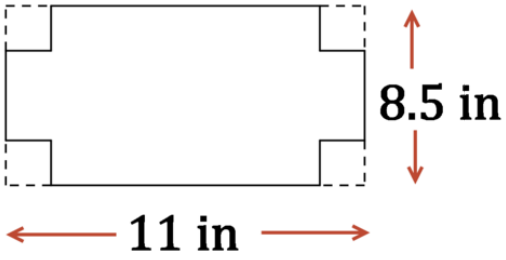

Figure **9.9.1**

Modeling in Three Dimensions

You live in a three dimensional world. Look around and observe what you see over the course of your day. Can you find examples of prisms? Cylinders? Pyramids? Cones? Spheres?

While most objects in your daily life are not ***perfect*** prisms, pyramids, cylinders, cones, or spheres, most are close to one of these five solids or a combination of these solids. Modeling in three dimensions is about being able to choose the best solid to help analyze a real world three dimensional situation, and then using your geometry knowledge to make decisions about the real life situation. You should ask yourself:

- What solid or solids are the best model of this real life object?
- What problems am I trying to solve or decisions am I trying to make about the real life object?
- What information about the real life object am I given and where does it fit in my model?

Real-World Applicati rees

A big storm causes a large ' yard. The main portion of the tree trunk measures about 9 feet around and is 40 feet long. You plan to chop up sell as fire wood. Approximately what volume of wood will you get from the tree?

A tree trunk is best mod Here, you are looking for the approximate volume of the cylinder. Two pieces of information are given.

1. *"The main portion of the tree trunk measures about 9 feet around"* --- This is the **circumference** of the base of the cylinder. You can use this measurement to find the radius of the cylinder.

$$2\pi r = 9 \rightarrow r = \frac{9}{2\pi} \rightarrow r \approx 1.43 \text{ ft}$$

2. "40 feet long" --- This is the height of the cylinder.

$$h = 40 \text{ ft}$$

The volume of the wood from the tree is approximately:

$$V = \pi r^2 h = \pi(1.43^2)(40) \approx 257 \text{ ft}^3$$

Real-World Application: Buying Wood

Wood is commonly sold in cords. A cord of wood is a stack of tightly packed wood that measures 4 feet by 4 feet by 8 feet. Approximately how many cords of wood will the chopped tree from the first problem produce?

A cord of wood is best modeled by a rectangular prism. $V = A_{Base} \cdot h$, so $V = (4 \cdot 4) \cdot 8 = 128 ft^3$. Each cord of wood is approximately 128 cubic feet of wood. Since the tree from the first problem produced 257 cubic feet of wood, this is $\frac{257}{128} \approx 2$ cords of wood.

Now, let's come up with an equation that relates the length around a tree in feet, the height of a tree in feet, and the approximate number of cords of wood that a tree will produce.

You want to come up with an equation that takes an input of circumference and height and produces an output of cords of wood. Think back to the steps taken in Examples A and B and repeat these steps with variables for circumference and height instead of specific values.

- Let $C =$ **distance around the tree**
- Let $h =$ **height of tree**

Use the distance around the tree to find the radius:

$$C = 2\pi r \rightarrow r = C^2\pi$$

The volume of the wood from the tree is:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(\frac{C}{2\pi})^2 h \\ &= \frac{C^2 h}{4\pi} \end{aligned}$$

Once you have the volume of the wood from a given tree, to find the number of cords of wood divide the volume by 128 ft3, which is the number of cubic feet in a cord of wood.

$$\textbf{Number of Cords} = \frac{V}{128} = \frac{\frac{C^2 h}{4\pi}}{128} = \frac{C^2 h}{512\pi}$$

Test this formula using the original information from the first problem about chopping trees to see if you get the correct answer to the second problem. In this first problem, C=9 ft and h=40 ft.

$$\textbf{Number of Cords} = \frac{C^2 h}{512\pi} = \frac{(9^2)(40)}{512\pi} \approx 2.01$$

This matches the answer to , so you can feel confident that your equation is correct.

Example 9.9.1

Earlier, you were asked how the volume of Mark's box relate to the size of the squares he cut out.

Solution

Let the length of the side of the square that Mark cuts out of each corner be x. The portion of the paper that will become the base of the box once it is made is shaded below in red.

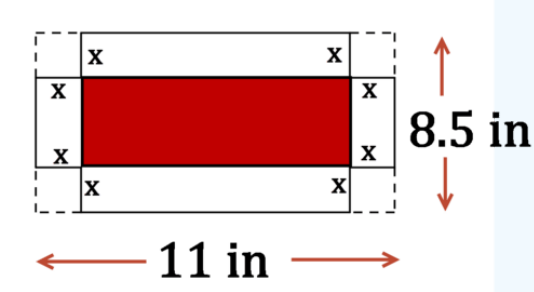

Figure 9.9.2

The box is a rectangular prism. The volume of the box is therefore \(V=A_{Base}\cdot h.

- $A_{Base} = (11 - 2x)(8.5 - 2x) = 4x^2 - 39x + 93.5$
- $h = x$

Therefore, the volume of the box in terms of the size of the square is:

$$V = x(4x2 - 39x + 93.5) = 4x^3 - 39x^2 + 93.5x.$$

Mark can use this formula to determine the volume of the box given the length of the side of the squares he cuts out. For example, if he cuts out squares that are 2 inch by 2 inch, then x=2. The volume of the box would be:

$$V = 4(2^3) - 39(2^2) + 93.5(2) = 63in^3$$

Example 9.9.2

Graph the equation $y = 4x^3 - 39x^2 + 93.5x$ with a graphing calculator. What do the points on this graph represent? What portion of this graph is relevant to this problem?

Solution

The points on the graph represent the volume of the box given the length of the side of each square cut out.

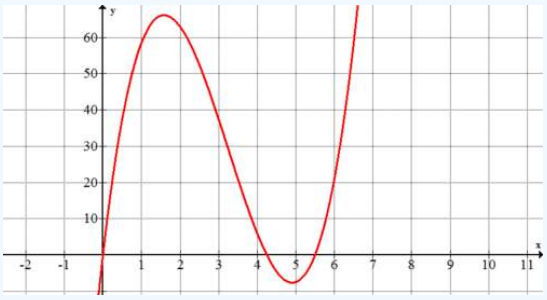

Figure 9.9.3

Because Mark can't cut out a square with a negative side length or a square with a side length greater than 4.25 *in* (because the paper is only 8.5 inches wide), the portion of the graph that is relevant is the portion with x values between 0 and 4.25.

Figure 9.9.4

Example 9.9.3

Approximately what size squares will maximize the volume of the box (cause the box to have the greatest possible volume)? How does the graph from #1 help you to answer this question?

Solution

The maximum volume looks to occur with squares that are approximately 1.6 inches by 1.6 inches. The volume at that point looks to be around $66in^3$. The graph helps to answer this question because the peak on the graph is where the maximum volume occurs.

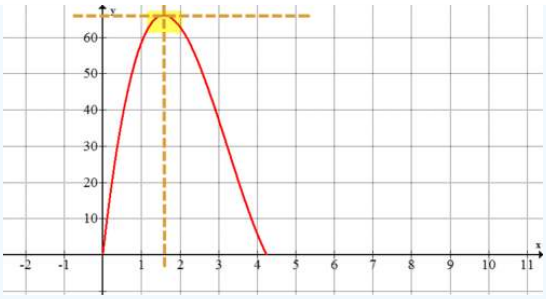

Figure **9.9.5**

Example 9.9.4

Does the size of square that maximizes volume also maximize surface area of the box? Explain.

Solution

The surface area of the open box will be the area of the unfolded box (the net). The more you cut out of the paper, the smaller the surface area. Therefore, the size of square that maximizes volume does not also maximize surface area of the box.

Review

An 11 inch tall roll of paper towels has an inner cardboard tube with a diameter of 1.5 inches. The width of the paper towel on the roll is 2 inches and each paper towel is 0.015 inches thick.

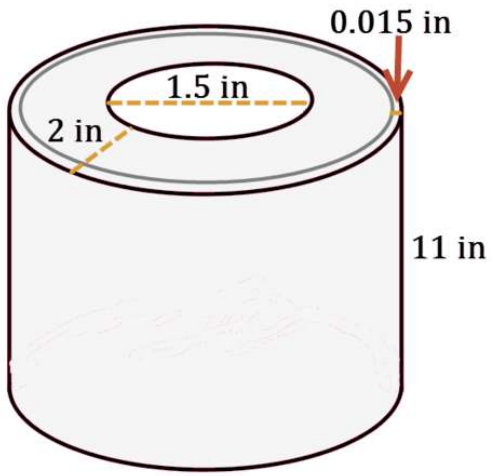

Figure **9.9.6**

1. What is the volume of paper towels? Will the volume change if the roll of paper towels is unrolled?

2. If the whole roll of paper towels is unrolled, how long will the chain of unrolled paper towels be? [Hint: Use your answer to #1 to help]

3. Come up with an equation that generalizes the relationship between the variables: diameter of tube, width of paper towel on roll, thickness of paper towel, and length of unrolled paper towels. Why does the height of the paper towel roll not matter in this relationship?

In order to decorate a cake with purple frosting, Sam plans to fill a zipper sandwich bag with frosting, cut off one of the tips of the bag, and squeeze the frosting out of the corner (see picture below).

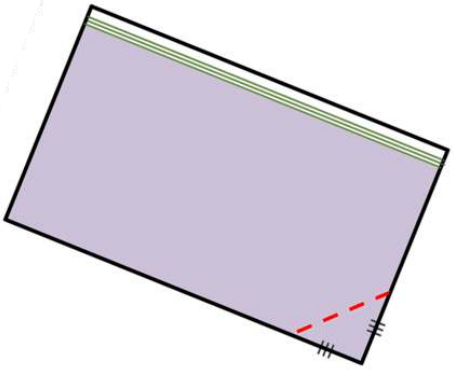

Figure 9.9.7

4. Assuming he cuts an isosceles triangle out of the corner, what length of cut should he make to pipe frosting with a 1 centimeter diameter. In other words, what should the length of the red dotted line be?

5. Give an equation that shows the relationship between the length of the cut made and the diameter of the frosting as it comes out of the bag.

6. You are packing up yearbooks that measure 11 inches by 14 inches by 1.5 inches. You have boxes that measure 12 inches by 30 inches by 10 inches. How many books can you fit in each box?

A certain burning candle loses \(7 in^{3} of volume each hour.

Figure 9.9.8

7. If the original candle was 10 inches tall with a diameter of 3 inches, what is the volume of the candle after 3 hours?

8. Create an equation that relates the original height of the candle, the diameter of the candle, the time burned, and the current volume of the candle for all candles that lose $7in^3$ of volume per hour.

9. Mike and his friends are having a water balloon fight. Each approximately spherical balloon can hold one cup of water. If one cup of water has a volume of approximately $14.44in^3$, what is the radius of each filled balloon?

Marissa works at an ice cream shop. The sugar cones have a diameter of 2 inches and a height of 4 inches. For a single scoop cone, she packs the cone with ice cream and then puts a scoop on top. For a double scoop cone, she packs the cone with ice cream and then puts two scoops on top (see sketch below). For a triple scoop cone, she packs the cone with ice cream and then puts three scoops on top. One cup of ice cream has a volume of $14.44 in^3$.

Figure **9.9.9**

10. Approximately how much ice cream (in cups) does a single scoop cone get?

11. Approximately how much ice cream (in cups) does a double scoop cone get?

12. Approximately how much ice cream (in cups) does a triple scoop cone get?

13. Describe at least two reasons why your answers to the previous three questions are only approximate.

14. If a single scoop cone costs $2.50 and a double scoop cone costs $3.50. Which type of cone is a better deal considering cost per cup of ice cream?

15. If a single scoop cone costs $2.50, what should a triple scoop cone cost if it is fairly priced?

Review (Answers)

To see the Review answers, open this PDF file and look for section 9.5.

Vocabulary

Term	Definition
Circumference	The circumference of a circle is the measure of the distance around the outside edge of a circle.

Additional Resources

Practice: Surface Area and Volume Applications

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9.10: Surface Area and Volume of Prisms

3-D figures with 2 congruent bases in parallel planes and rectangles for their other faces.

Prisms

A **prism** is a 3-dimensional figure with 2 congruent bases, in parallel planes, in which the other faces are rectangles.

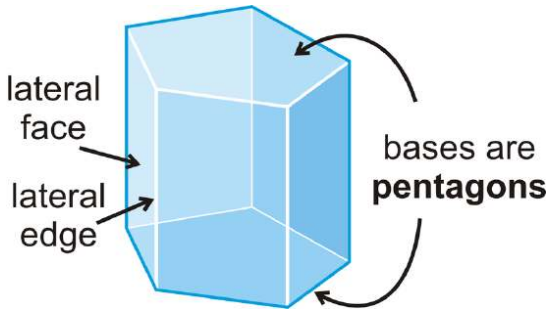

Figure **9.10.1**

The non-base faces are **lateral faces.** The edges between the **lateral faces** are **lateral edges**.

This particular example is a ***pentagonal prism*** because its base is a pentagon. Prisms are named by the shape of their base. Prisms are classified as either **right prisms** (prisms where all the lateral faces are perpendicular to the bases) or **oblique prisms** (prisms that lean to one side, whose base is a parallelogram rather than a rectangle, and whose height is perpendicular to the base's plane), as shown below.

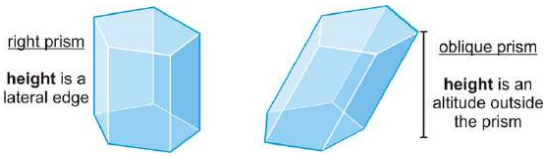

Figure **9.10.2**

Surface Area

To find the **surface area** of a prism, find the sum of the areas of its faces. The **lateral area** is the sum of the areas of the lateral faces. The basic unit of area is the square unit.

$$SurfaceArea = B_1 + B_2 + L_1 + L_2 + L_3$$

$$LateralArea = L_1 + L_2 + L_3$$

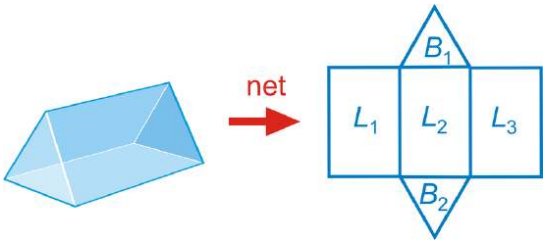

Figure **9.10.3**

Volume

To find the **volume** of any solid you must figure out how much space it occupies. The basic unit of volume is the cubic unit.

For prisms in particular, to find the volume you must find the area of the base and multiply it by the height.

Volume of a Prism: $V = B \cdot h$, where $B = area\ of\ base$.

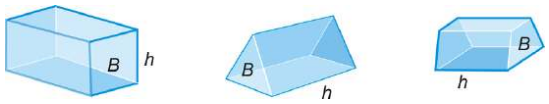

Figure 9.10.4

If an **oblique prism** and a **right prism** have the same base area and height, then they will have the same volume. This is due to **Cavalieri's Principle**, which states that if two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.

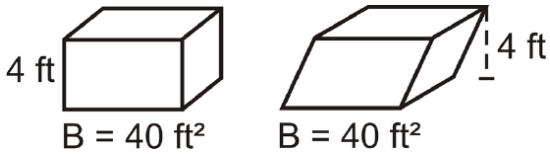

Figure 9.10.5

What if you were given a solid three-dimensional figure with two congruent bases in which the other faces were rectangles? How could you determine how much two-dimensional and three-dimensional space that figure occupies?

Example 9.10.1

The total surface area of n is 540 units^2. What is x?

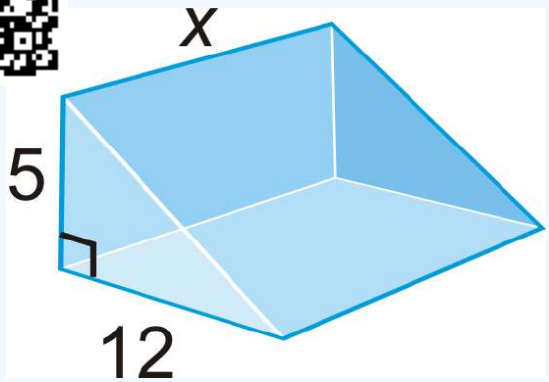

Figure 9.10.6

Solution

The total surface area is equal to:

$$A_{2\ triangles} + A_{3\ rectangles} = 540$$

The hypotenuse of the triangle bases is 13, $5^2 + 12^2$. Let's fill in what we know.

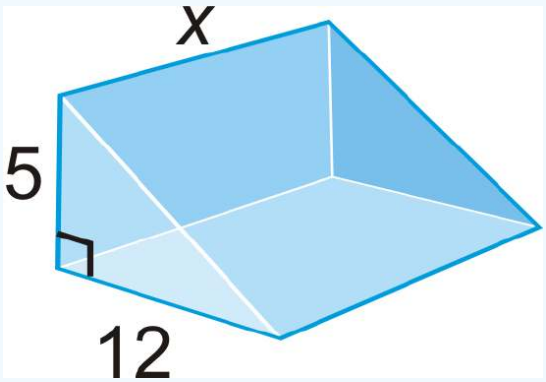

Figure 9.10.7

$$A_{2\ triangles} = 2(\frac{1}{2} \cdot 5 \cdot 12) = 60$$

$$A_{3\ rectangles} = 5x + 12x + 13x = 30x$$
$$60 + 30x = 540$$
$$30x = 480$$
$$x = 16 \text{ units} \qquad \text{The height is 16 units.}$$

Example 9.10.2

Find the volume of the right rectangular prism below.

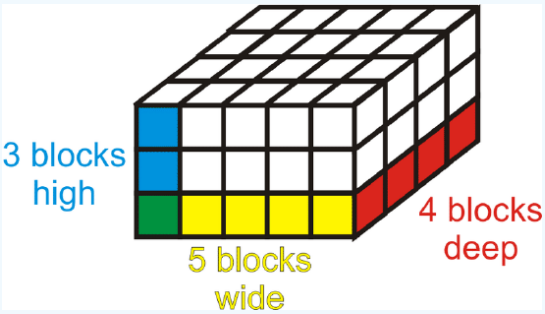

Figure 9.10.8

Solution

The area of the base is $(5)(4) = 20$ and the height is 3. So the total volume is $(20)(3) = 60\ \text{unit}^3$

Example 9.10.3

Find the surface area of the prism below.

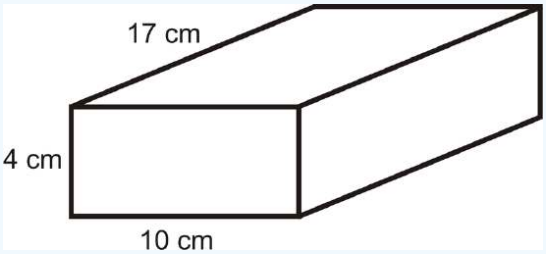

Figure 9.10.9

Solution

To solve, draw the net of the prism so that we can make sure we find the area of ALL faces.

Using the net, we have:

$$SA_{prism} = 2(4)(10) + 2(10)(17) + 2(17)(4)$$
$$= 80 + 340 + 136$$
$$= 556\ \text{cm}^2$$

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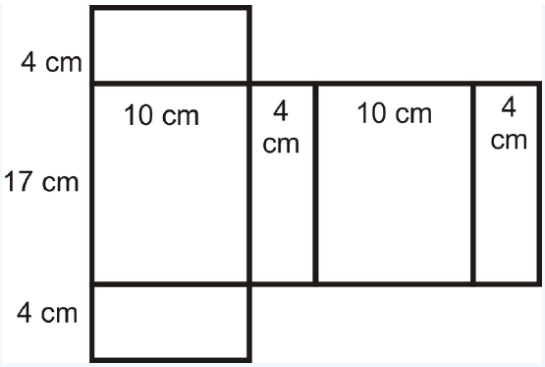

Figure 9.10.10

Example 9.10.4

Find the surface area of the prism below.

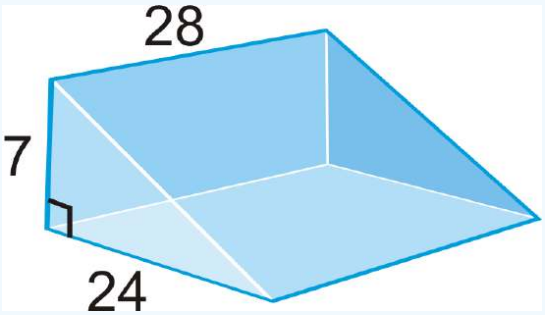

Figure 9.10.11

Solution

This is a right triangular prism. To find the surface area, we need to find the length of the hypotenuse of the base because it is the width of one of the lateral faces. We can use the Pythagorean Theorem to find this length.

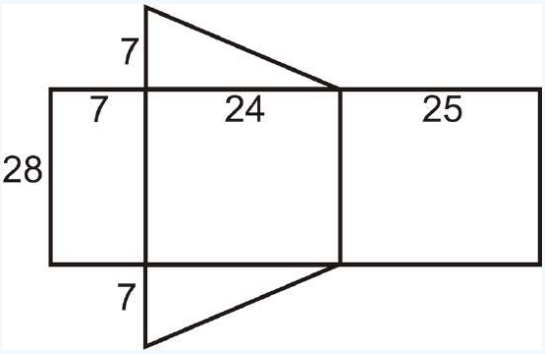

Figure 9.10.12

$$7^2 + 24^2 = c^2$$
$$49 + 576 = c^2$$
$$625 = c^2 \qquad c = 25$$

Looking at the net, the surface area is:

$$SA = 28(7) + 28(24) + 28(25) + 2(\frac{1}{2} \cdot 7 \cdot 24)$$
$$SA = 196 + 672 + 700 + 168 = 1736 units^2$$

Example 9.10.5

You have a small, triangular prism-shaped tent. How much volume does it have once it is set up?

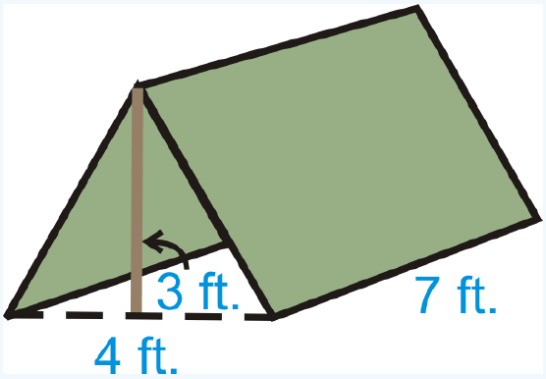

Figure 9.10.13

Solution

First, we need to find the area of the base.

$$B = \frac{1}{2}(3)(4) = 6 \text{ ft}^2$$
$$V = Bh = 6(7) = 42 \text{ ft}^3$$

Even though the height in this problem does not look like a "height," it is because it is the perpendicular segment connecting the two bases.

Review

1. What type of prism is this?

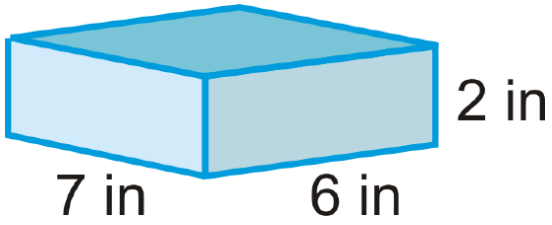

Figure 9.10.14

2. Draw the net of this prism.
3. Find the area of the bases.
4. Find the area of lateral faces, or the lateral surface area.
5. Find the total surface area of the prism.
6. How many one-inch cubes can fit into a box that is 8 inches wide, 10 inches long, and 12 inches tall? Is this the same as the volume of the box?
7. A cereal box in 2 inches wide, 10 inches long and 14 inches tall. How much cereal does the box hold?
8. A can of soda is 4 inches tall and has a diameter of 2 inches. How much soda does the can hold? Round your answer to the nearest hundredth.
9. A cube holds 216 in^3. What is the length of each edge?
10. A cube has sides that are 8 inches. What is the volume?

Use the right triangular prism to answer questions 11-15.

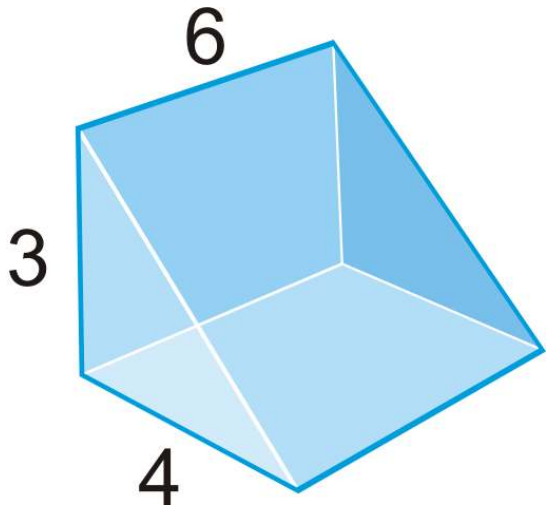

Figure **9.10.15**

11. Find the volume of the prism.
12. What shape are the bases of this prism? What are their areas?
13. What are the dimensions of each of the lateral faces? What are their areas?
14. Find the lateral surface area of the prism.
15. Find the total surface area of the prism.
16. Describe the difference between *lateral* surface area and *total* surface area.
17. Fuzzy dice are cubes with 4 inch sides.

Figure **9.10.16**

1. What is the volume and surface area of one die?
2. What is the volume and surface area of both dice?

Find the volume of the following solids. Round your answers to the nearest hundredth.

18. bases are isosceles trapezoids

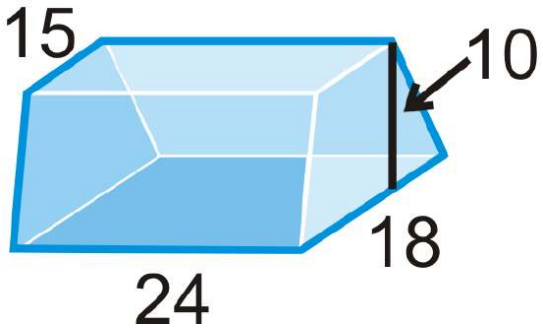

Figure **9.10.17**

19.

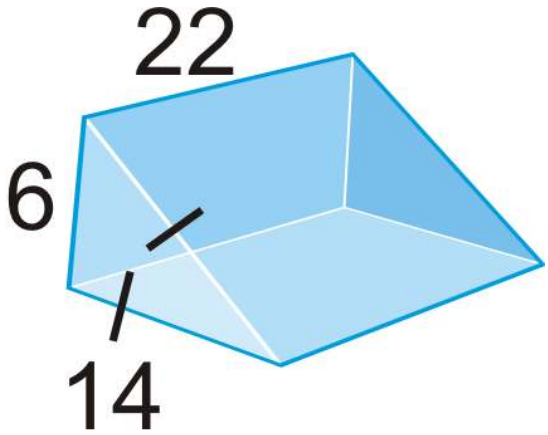

Figure **9.10.18**

20.

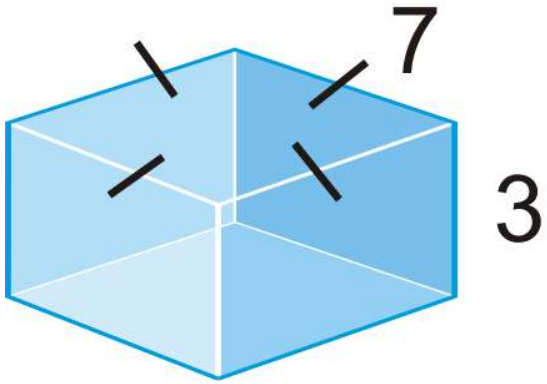

Figure **9.10.19**

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21.

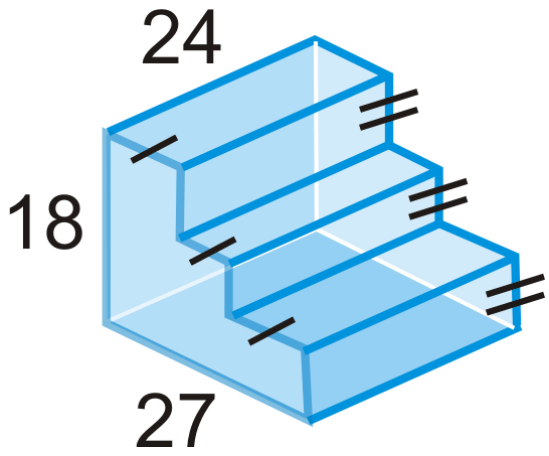

Figure 9.10.20

Find the value of x, given the surface area.

22. $V = 504 \text{ unit}^3$

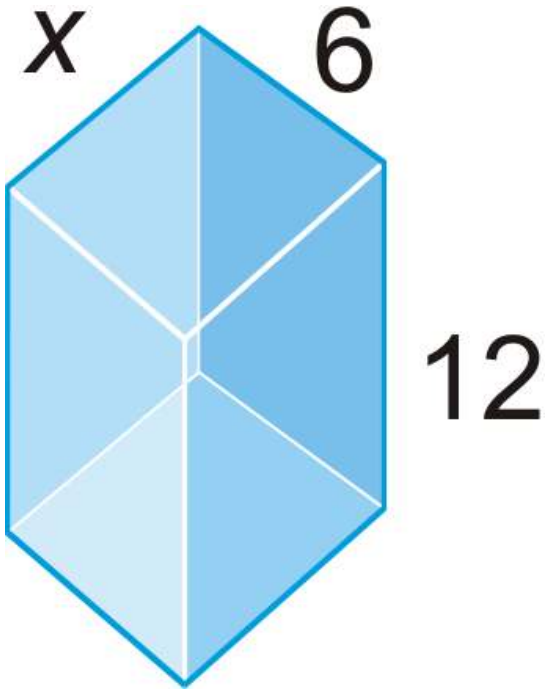

Figure 9.10.21

23. $V = 2688 \text{ unit}^3$

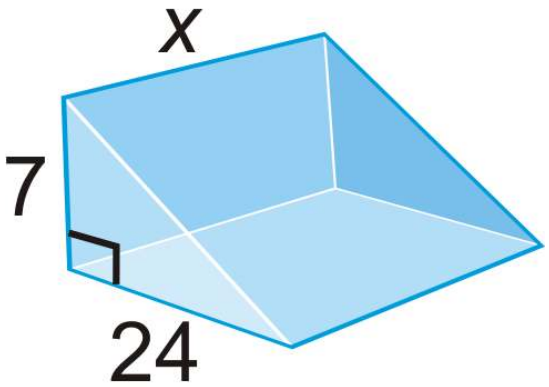

Figure 9.10.22

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.3.

Vocabulary

Term	Definition
lateral edges	Edges between the lateral faces of a prism.
oblique prism	A prism that leans to one side and whose height is perpendicular to the base's plane.
prism	is a 3-dimensional figure with 2 congruent bases, in parallel planes, and in which the other faces are rectangles.
right prism	A prism where all the lateral faces are perpendicular to the bases.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.
Lateral Faces	All faces of a prism aside from the base are known as lateral faces.

Additional Resources

Video: Prisms Principles - Basic

Activities: Prisms Discussion Questions

Study Aids: Prisms and Cylinders Study Guide

Practice: Surface Area and Volume of Prism

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9.11: Surface Area of Prisms

Use formulas to find the surface area of a rectangular prisms and triangular prisms.

Surface Area of Triangular Prisms

Figure **9.11.1**

Max bought a light **prism** for his mom for Mother's Day. He wants to figure out how much wrapping paper is needed to wrap it. To do this, he needs to figure out the **surface area** of the gift. The triangular end has a base of 3 cm and height of 4 cm. The length of each side is 6 cm and the height of the prism (length of the rectangular side) is 8 cm. What is the surface **area** of the gift that Max bought for his mom?

In this concept, you will learn how to calculate the surface area of a **triangular prism**.

Finding the Surface Area of a Triangular Prism

Area is the space that is contained in a two-dimensional figure**. Surface area** is the total area of all of the sides and faces of a three-dimensional figure. To find the surface area, the area of each face is calculated and then add these areas together.

One way to do this is to use a **net**, since a net is a two-dimensional representation of a three-dimensional solid**,** or an unfolded picture of a solid.

What is the surface area of the figure below?

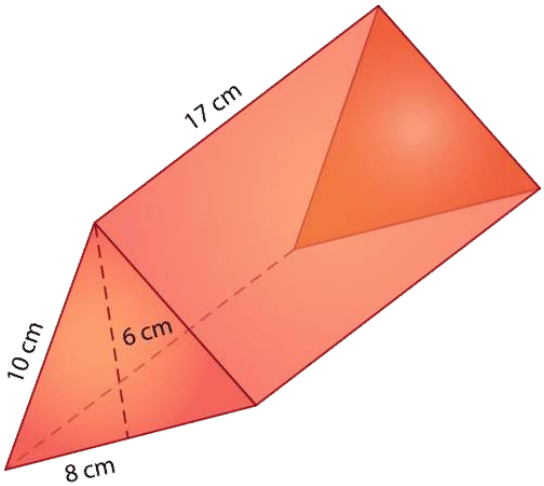

Figure **9.11.2**

The net for this triangular prism is as follows:

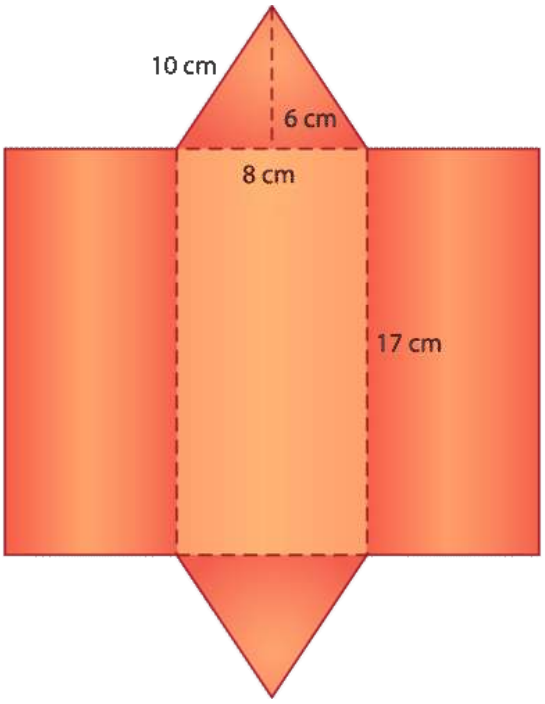

Figure **9.11.3**

Now, let's fill in the measurements for the sides of each face in order to calculate their area. Triangular prisms have their own formula for finding surface area because they have two triangular faces opposite each other.

The formula $A = \frac{1}{2}bh$ is used to find the area of the top and bases triangular faces, where A = area, b = base, and h = height. The formula $A = lw$ is used to find the area of the three rectangular side faces, where A = area, l = length, and w = width.

Plugging in the measurements that are given in the net, calculate the area of each face. Remember to use the correct area formula for the triangles and rectangles.

Bottom face	Top face	side	side	side				
$A = \frac{1}{2}bh$	$A = \frac{1}{2}bh$	$A = lw$	$A = lw$	$A = lw$				
$\frac{1}{2}(8)(6)+$	$\frac{1}{2}(8)(6)+$	$17 \times 10+$	$17 \times 10+$	17×8				
24	+	24	+	170	+	170	+	136
$= 524\ \text{cm}^2$								

When you add these values together, you get a surface area of 524 square centimeters for this triangular prism.

$$SA = bh + (s_1 + s_2 + s_3)H$$

You can also use one formula to calculate the surface area of a triangular prism which can save time over the process of using a net to derive the areas:

where b = base; h = height of the triangle; s_{1}\), s_{2}\), and s_{3}\) = the length of each side of the triangle base, and H = the height of the prism (which is the length of the rectangles).

First, find the area of the two triangular faces. Each face will have an area of 12bh. Remember, you can use a formula to calculate the area of a pair of faces. Therefore, you can double this formula to find the area of both triangular faces at once which results in the formula 2(12bh). The 2 multiplied by the 12 equals 1, or cancels each other out, and you're left with bh.

Next, you need to calculate the area of each of the rectangular side faces. The length of each rectangle is the same as the height of the prism, so call this H. The width of each rectangle is actually the same as the sides of the triangular base. Instead of multiplying the length and width for each rectangle, you can combine this information. Since there are 3 rectangular widths (all equal to the sides of the triangles), multiply the perimeter of the triangular base by the height of the rectangles, H, which will give the area of all three rectangles.

If you put these pieces together—the area of the bases and the area of the side faces—you get this formula:

$SA = bh + (s_1 + s_2 + s_3)H$

where bh = the area of the triangle top and base, and $(s_1 + s_2 + s_3)H$ = the area of the rectangular side faces.

Remember that the height of the triangular base (h) is not necessarily the same as the height of the prism (H).

Example 9.11.1

Earlier, you were given a problem about the gift Max bought for his mom.

The triangular end has a base of 3 cm and height of 4 cm. The length of each side is 6 cm and the height of the prism (length of the rectangle) is 8 cm. What is the surface area of this triangular prism?

Solution

First, substitute the given values into the formula.

$$SA = bh + (s_1 + s_2 + s_3)H$$
$$SA = 3(4) + (3 + 6 + 6)(8)$$

Next, multiply the base times height for the area of the triangles (bh), and add their three sides ($s_1 + s_2 + s_3$).

$$SA = 12 + 15(8)$$

Then, multiply the sum of the triangle sides by the height of the prism (H\)) and add the values together for the answer, remembering to include the appropriate unit of measurement.

$$SA = 12 + 120$$
$$SA = 132 \text{ cm}^2$$

The answer is the surface area of the gift Max bought for his mom is 132 square centimeters.

Example 9.11.1

Calculate the surface area of the following triangular prism:

Solution

First, substitute the given values into the formula.

$$SA = bh + (s_1 + s_2 + s_3)H$$
$$SA = 4(6) + (4 + 7 + 7)(12)$$

Next, multiply the base times height for the area of the triangles (bh) and add the three sides of the triangle ($s_1 + s_2 + s_3$).

$$SA = 24 + 18(12)$$

Then, multiply the sum of the triangle sides by the height of the prism (H) and add the values together for the answer, making sure to include the appropriate unit of measurement.

$$SA = 24 + 216$$
$$SA = 240cm^2$$

The answer is the surface area of the above triangular prism is 240 square centimeters.

Example 9.11.1

A triangular prism has a triangular end with a base of 5 inches and a height of 4 inches. The length of each side is 8 inches and the width of each side is 6 inches. What is the surface area of the prism?

Solution

First, substitute the given values into the formula.

$$SA = bh + (s_1 + s_2 + s_3)H$$
$$SA = 5(4) + (5 + 6 + 6)(8)$$

Next, multiply the base times height for the area of the triangles (bh), and add their three sides (s_{1} + s_{2} + s_{3}\)).

$$SA = 20 + 17(8)$$

Then, multiply the sum of the triangle sides by the height of the prism (H) and add the values together for the answer, remembering to include the appropriate unit of measurement.

$$SA = 20 + 136$$
$$SA = 156 \text{ in}^2$$

The answer is the surface area of the above triangular prism is 156 square inches.

Example 9.11.1

A triangular prism has a triangular end with a base of 8 feet and a height of 6 feet. The length of each side is 10 feet and the width of each side is 6 feet. What is the surface area of the prism?

Solution

First, substitute the given values into the formula.

$$SA = bh + (s_1 + s_2 + s_3)H$$
$$SA = 8(6) + (8 + 6 + 6)(10)$$

Next, multiple the base times height for the area of the triangles (bh), and add the three sides of the triangle (s_{1} + s_{2} + s_{3}\)).

$$SA = 48 + 20(10)$$

Then, multiply the sum of the triangle sides by the height of the prism (H) and add the values together for the answer, making sure to include the appropriate unit of measurement.

$$SA = 48 + 200$$
$$SA = 248 \text{ ft}^2$$

The answer is the surface area of the above triangular prism is 248 square feet.

Example 9.11.1

A triangular prism has a triangular end with a base of 12 inches and a height of 9 inches. The length of each side is 14 inches and the width of each side is 9 inches. What is the surface area of the prism?

Solution

First, substitute the given values into the formula.

$$SA = bh + (s_1 + s_2 + s_3)H$$
$$SA = 12(9) + (9 + 9 + 9)(14)$$

Next, multiply the base times height for the area of the triangles (bh\)), and add the three sides of the triangle ($s_1 + s_2 + s_3$).

$$SA = 108 + 27(14)$$

Then, multiply the sum of the triangle sides by the height of the prism (H\)) and add the values together for the answer, making sure to include the appropriate unit of measurement.

$$SA = 108 + 378$$
$$SA = 486 \text{ in}^2$$

The answer is the surface area of the above triangular prism is 486 square inches.

Review

Use the following figure to answer each question.

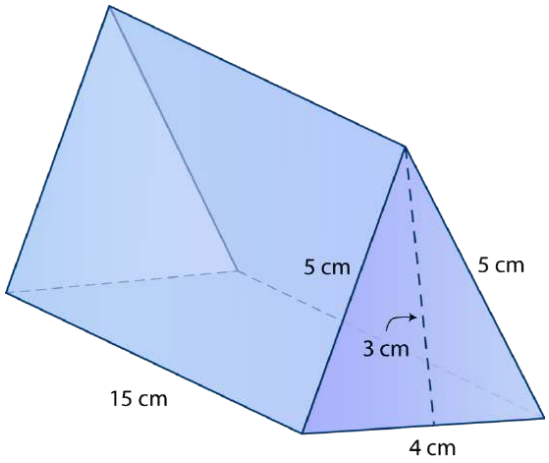

Figure **9.11.4**

1. What unit are the measurements of this figure?
2. What is the length of the base?
3. What is the width of the base?
4. What is the shape of the sides?
5. What is the measure of the triangular side?
6. What is the formula for finding the surface area of a triangular prism?
7. What is the surface area of this figure?

Find the surface area of each triangular prism.

8. A triangular prism has a triangular end with a base of 4 inches and a height of 3 inches. The length of each side is 5 inches and the width of each side is 3 inches. What is the surface area of the prism?
9. A triangular prism has a triangular end with a base of 4 feet and a height of 5 feet. The length of each side is 6 feet and the width of each side is 5 feet. What is the surface area of the prism?
10. A triangular prism has a triangular end with a base of 3 inches and a height of 3 inches. The length of each side is 4 inches and the width of each side is 3 inches. What is the surface area of the prism?
11. A triangular prism has a triangular end with a base of 6 meters and a height of 5 meters. The length of each side is 8 meters and the width of each side is 5 meters. What is the surface area of the prism?
12. A triangular prism has a triangular end with a base of 15 feet and a height of 10 feet. The length of each side is 12 feet and the width of each side is 10 feet. What is the surface area of the prism?
13. A triangular prism has a triangular end with a base of 9 inches and a height of 7 inches. The length of each side is 9 inches and the width of each side is 7 inches. What is the surface area of the prism?
14. A triangular prism has a triangular end with a base of 8 inches and a height of 6 inches. The length of each side is 15 inches and the width of each side is 6 inches. What is the surface area of the prism?
15. A triangular prism has a triangular end with a base of 5.5 feet and a height of 4 feet. The length of each side is 9 feet and the width of each side is 5 feet. What is the surface area of the prism?

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.5.

Resources

Vocabulary

Term	Definition
Area	Area is the space within the perimeter of a two-dimensional figure.
Prism	A prism is a three-dimensional object with two congruent parallel bases that are polygons.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Triangular Prism	A triangular prism is a prism made up of two triangular bases and three rectangular faces.

Additional Resources

Video: Find the Surface Area of an Open Top Box

Practice: Surface Area of Prisms

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9.12: Volume of Prisms

Use the formula $V = Bh$

Figure **9.12.1**

Karen and Lance are building a backyard playground for their two children. They have a swing set, a merry-go-round, and a sandbox already set up. Lance is going to the store to buy sand for the sandbox and needs to know how much sand it can hold. The sandbox is 6 feet wide, 8 feet long, and 1 feet deep. How can Lance use a formula to calculate the total volume of this sandbox?

In this concept, you will learn to find volumes of rectangular and triangular prisms using formulas.

Finding Volume of Prisms

Volume is the amount of space inside a solid figure.

Filling solid figures with cubes is a simple, easy way to understand volume. If you can count the cubes, you can figure out the volume. However, sometimes you will have to figure out the volume of a prism when there aren't any cubes drawn inside it.

Take a look at the prism below.

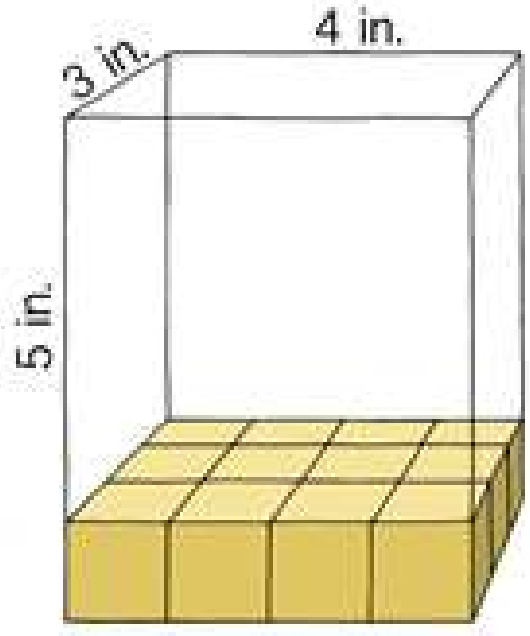

Figure **9.12.2**

This **rectangular prism** has a height of 5 inches, a width of 3 inches, and a length of 4 inches.

Here is a formula for finding the volume of this type prism.

$$V = Bh$$

B means the area of the base, which is the length times the width, and h means the height.

So, first figure out the area of the base.

$$A = 3 \times 4 = 12$$

Next, multiply B by h.

$$h = 5$$
$$V = 12 \times 5 = 60$$

The volume is 60 cubic inches or in^3. Remember, volume is in cubic units.

Let's look at another example. Find the volume using the volume formula.

Figure **9.12.3**

$$V = Bh$$

First, figure the area of the base.

The area of the base is $2 \times 8 = 16$

Next, multiply the B by h. The height is 3 inches.

$$V = 16 \times 3$$
$$V = 48 \text{ in}^3$$

The volume of this rectangular prism is 48 in^3.

You can use the same formula for finding the volume of the **triangular prism**. Except this time, the area of the base is a triangle and not a rectangle.

Take a look at the triangular prism below.

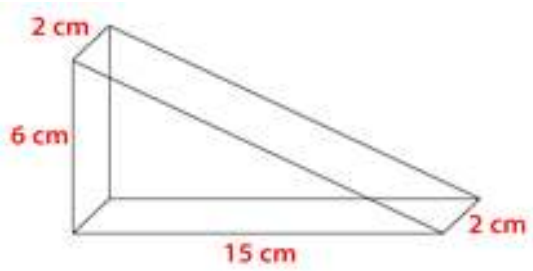

Figure **9.12.4**

To find the volume of a triangular prism, multiply the area of the base (B) with the height of the prism.

$$V = Bh$$

First, find the area of the triangular base using the formula for area of a triangle.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(15 \times 6)$$
$$A = \frac{1}{2}(90)$$
$$A = 45 \text{ sq. units}$$

Next, multiply this by the height.

$$V = Bh$$
$$V = (45)h$$
$$V = 45(2)$$
$$V = 90 \text{ cubic centimeters} or \text{ cm}^3$$

The volume of the prism is 90 cm^3.

Example 9.12.1

Earlier, you were given a problem about Lance and Karen's sandbox.

The sandbox is 6 feet wide, 8 feet long, and 1 feet deep. Lance needs to know the volume.

Solution

To find the volume of the sandbox, which is a prism with one of its bases removed, use the following formula.

$$V = Bh$$

First, substitute in the given values. Remember, B is length times width.

$$V = (8 \times 6)(1)$$
$$V = 48 \text{ ft}^3$$

This is the total volume.

Next, to find out how much sand he needs to fill the sandbox halfway, divide the total volume by 2.

$$48 \div 2 = 24$$

Lance needs 24 cubic feet of sand to fill the sandbox halfway.

Example 9.12.2

Find the volume of the prism.

Figure 9.12.5

Solution

To find the volume of a prism, use the following formula.

$$V = Bh$$

First, substitute in the given values. Remember B is length times width.

$$V = (16 \times 9)(4)$$
$$V = 576 \text{ cm}^3$$

The answer is 576 cm^3.

Example 9.12.3

Find the volume of the prism.

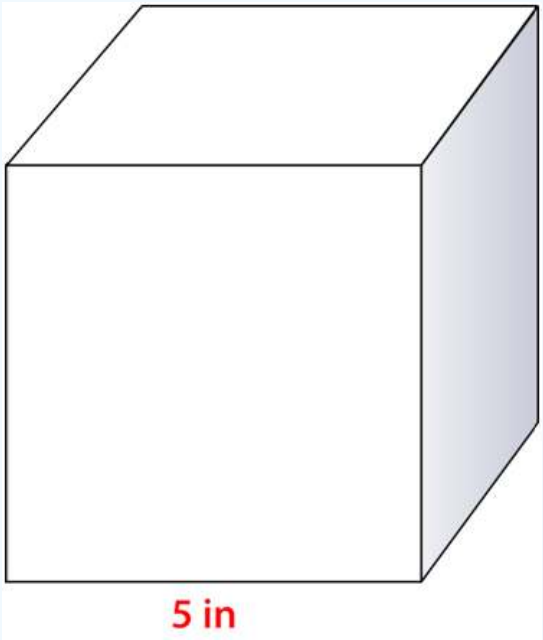

Figure 9.12.6

Solution

First, to find the volume of a prism, use the following formula.

$$V = Bh$$

Next, substitute in the given values. Remember B is length times width. This is a square cube, so the length, width, and height are the same.

$$V = (5 \times 5)(5)$$
$$V = 125 \text{ in}^3$$

The answer is $125\ \text{in}^3$.

Example 9.12.4

Find the volume of the prism.

Figure 9.12.7

Solution

First, to find the volume of a prism, use the following formula.

$$V = Bh$$

First, substitute in the given values. Remember, B is length times width.

$$V = (30 \times 5)(3)$$
$$V = 450\ \text{in}^3$$

The answer is $450\ \text{in}^3$.

Example 9.12.5

Find the volume of the prism.

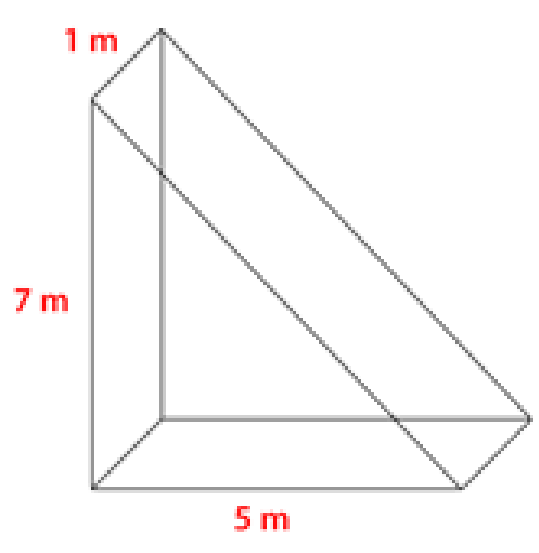

Figure 9.12.8

Solution

To find the volume of a triangular prism, multiply the area of the base (B) with the height of the prism.

$$V = Bh$$

First, find the area of the triangular base using the formula for area of a triangle.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(5 \times 7)$$
$$A = \frac{1}{2}(35)$$
$$A = 17.5 \text{ sq. m}$$

Next, multiply this by the height.

$$V = Bh$$
$$V = (17.5)1$$
$$V = 17.5 \text{ cm}^3$$

The volume of the prism is 17.5 cm^3.

Review

Find the volume of each rectangular prism. Remember to label your answer in cubic units.

1. Length = 5 in, width = 3 in, height = 4 in
2. Length = 7 m, width = 6 m, height = 5 m
3. Length = 8 cm, width = 4 cm, height = 9 cm
4. Length = 8 cm, width = 4 cm, height = 12 cm
5. Length = 10 ft, width = 5 ft, height = 6 ft
6. Length = 9 m, width = 8 m, height = 11 m
7. Length = 5.5 in, width = 3 in, height = 5 in
8. Length = 6.6 cm, width = 5 cm, height = 7 cm
9. Length = 7 ft, width = 4 ft, height = 6 ft
10. Length = 15 m, width = 8 m, height = 10 m

Find the volume of each triangular prism. Remember that h\) means the height of the triangular base and H\) means the height of the whole prism.

11. $b = 6 \text{ in}, h = 4 \text{ in}, H = 5 \text{ in}$
12. $b = 7 \text{ in}, h = 5 \text{ in}, H = 9 \text{ in}$
13. $b = 10 \text{ m}, h = 8 \text{ m}, H = 9 \text{ m}$
14. $b = 12 \text{ m}, h = 10 \text{ m}, H = 13 \text{ m}$
15. $b = 8 \text{ cm}, h = 6 \text{ cm}, H = 9 \text{ cm}$

Answer true or false for each of the following questions.

16. Volume is the amount of space that a figure can hold inside it.
17. The volume of a rectangular prism is always greater than the volume of a cube.
18. The volume of a triangular prism is less than a rectangular prism with the same size base.
19. A painter would need to know the **surface area** of a house to do his/her job correctly.
20. If Marcus is covering his book with a book cover, Marcus is covering the surface area of the book.

Vocabulary

Term	Definition
Net	A net is a diagram that shows a “flattened” view of a solid. In a net, each face and base is shown with all of its dimensions. A net can also serve as a pattern to build a three-dimensional solid.
Rectangular Prism	A rectangular prism is a prism made up of two rectangular bases and four rectangular faces.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Triangular Prism	A triangular prism is a prism made up of two triangular bases and three rectangular faces.

Additional Resources

Video: Solid Geometry Volume

Practice: Volume of Prisms

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9.13: Volume of Prisms Using Unit Cubes

Use cubes to find the volume of a prism.

Figure **9.13.1**

Jerry's assignment in his carpentry class is to build a wooden storage container that will hold at least 48 cubic units of material. Jerry has already started on the rectangular container, which is 4 units wide and 2 units long. Jerry isn't sure what height the container should be in order to hold a **volume** of 48 cubic units. How can Jerry use this information to determine the height of the container?

In this concept, you will learn to identify the volume of prisms using unit cubes.

Finding Volume of Prisms Using Unit Cubes

Volume is the amount of space inside a solid figure.

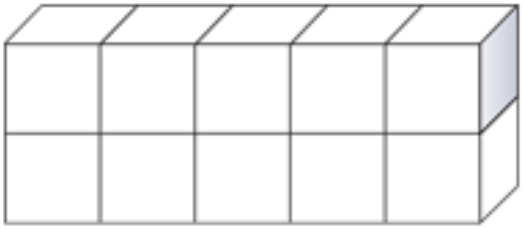


Figure **9.13.2**

These cubes make up a **rectangular prism**. The cubes represent the volume of the prism. This prism is five cubes by two cubes by one cube. In other words, it is five cubes long, by two cubes high by one cube wide. You can multiply each of these values together to get the volume of the rectangular prism.

$5 \times 2 \times 1 = 10$ cubic units

The volume of the rectangular prism is 10 cubic units or units^3. The units are cubic units because you multiplied the units 3 times when you multiplied the height, length, and width.

Here is another prism.

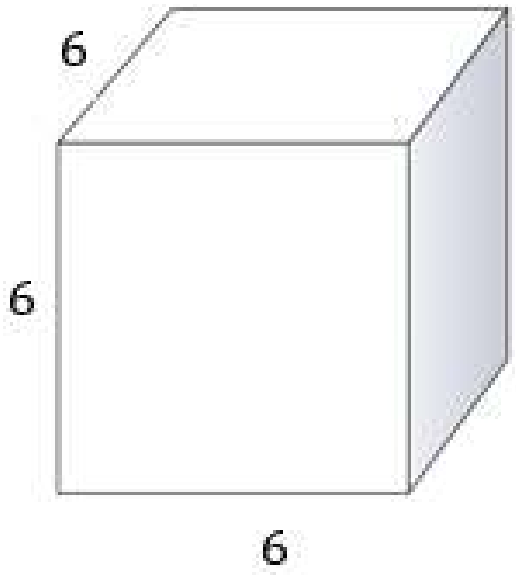


Figure 9.13.3

First, to figure out the volume of this prism, identify the measurements.

Length - 6

Width - 6

Height - 6

Next, substitute these values in the formula for volume then solve.

$$\text{Length} \times \text{Width} \times \text{Height} = \text{Volume in cubic units}$$

$$6 \times 6 \times 6 = 216 \text{ cubic units}$$

The volume of the cube is **216 cubic units**.

Example 9.13.1

Earlier, you were given a problem about Jerry and the storage container he is building.

The container is 4 units wide and 2 units long when he learned about the volume requirement. The container needs to have a volume of 48 cubic units. How can Jerry use this information to determine the height the container should be?

Solution

First, to figure the height the container should be, identify the given measurements.

Length - 2 units

Width - 4 units

Height - ?

Volume - 48 cubic units

Next, substitute these values into the formula for volume.

$$\text{Length} \times \text{Width} \times \text{Height} = Volume in cubic units$$
$$2 \times 4 \times H = 48 \text{ cubic units}$$

Then, multiply.

$$8(H) = 48$$

Then, divide both sides by 8.

$$H = 6 \text{ units}$$

The container should have a height of 6 units.

Example 9.13.2

What is the volume of this figure?

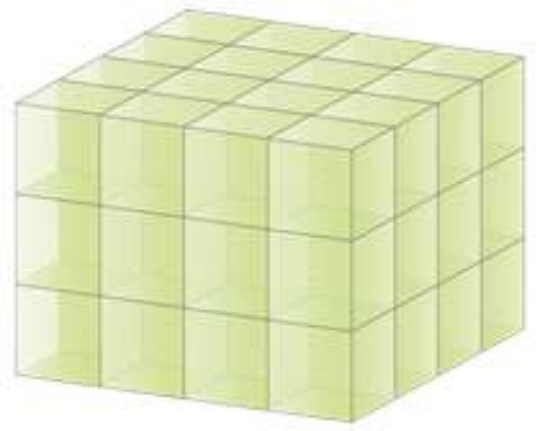


Figure 9.13.4

Solution

First, to figure out the volume of this prism, identify the measurements.

Length - 4 units

Width - 4 units

Height - 3 units

Next, substitute these values into the formula for volume then solve.

$$\text{Length} \times \text{Width} \times \text{Height} = \text{Volume in cubic units}$$
$$4 \times 4 \times 3 = 216 \text{ cubic units}$$

The volume of the prism is **48 cubic units**, or **48 units^3**.

Example 9.13.3

Find the volume of the prism.

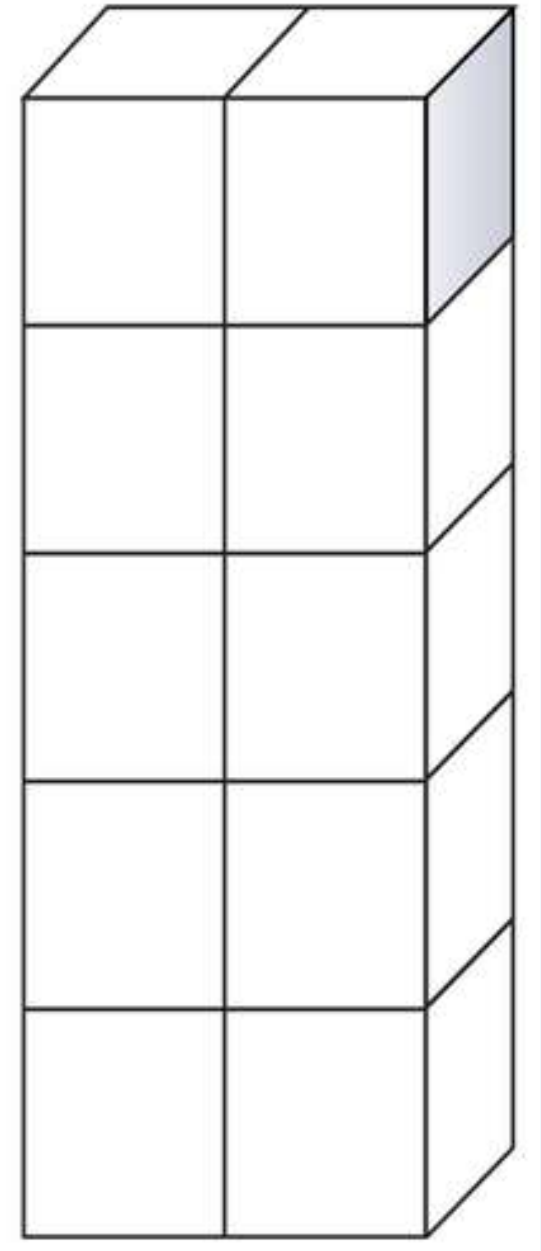


Figure **9.13.5**

Solution

First, to figure out the volume of this prism, identify the measurements.

Length - 1

Width - 2

Height - 5

Next, substitute these values into the formula for volume then solve.

$$\text{Length} \times \text{Width} \times \text{Height} = \text{Volume in cubic units}$$
$$1 \times 2 \times 5 = 10 \text{ cubic units}$$

The volume of the prism is 10 cubic units.

Example 9.13.4

Find the volume of the prism.

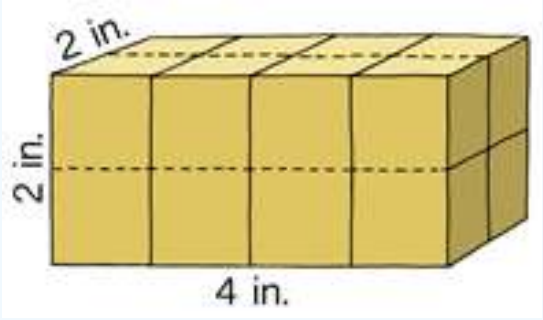


Figure 9.13.6

Solution

First, to figure out the volume of this prism, identify the measurements.

Length - 2 inches

Width - 4 inches

Height - 2 inches

Next, substitute these values into the formula for volume then solve.

$$\text{Length} \times \text{Width} \times \text{Height} = \text{Volume in cubic units}$$
$$2 \times 4 \times 2 = 16 \text{ cubic inches}$$

The volume of the cube is 16 cubic inches.

Example 9.13.5

Find the volume of the prism.

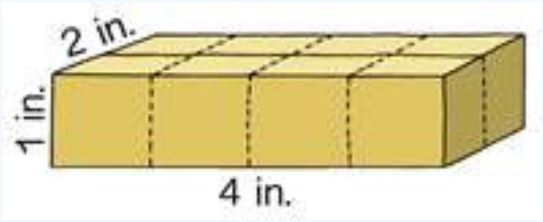


Figure 9.13.7

Solution

First, to figure out the volume of this prism, identify the measurements.

Length - 2 inches

Width - 4 inches

Height - 1 inch

Next, substitute these values into the formula for volume then solve.

$$\text{Length} \times \text{Width} \times \text{Height} = \text{Volume in cubic units}$$
$$2 \times 4 \times 1 = 8 \text{ cubic units}$$

The volume of the prism is 8 cubic inches.

Review

Find the volume of each prism.

1.

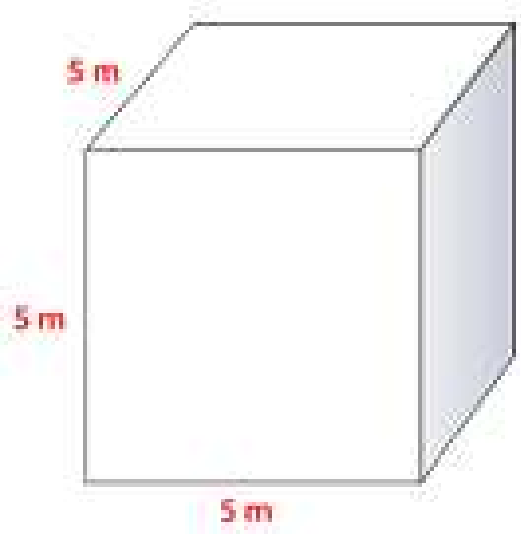


Figure **9.13.8**

2.

Figure **9.13.9**

3.

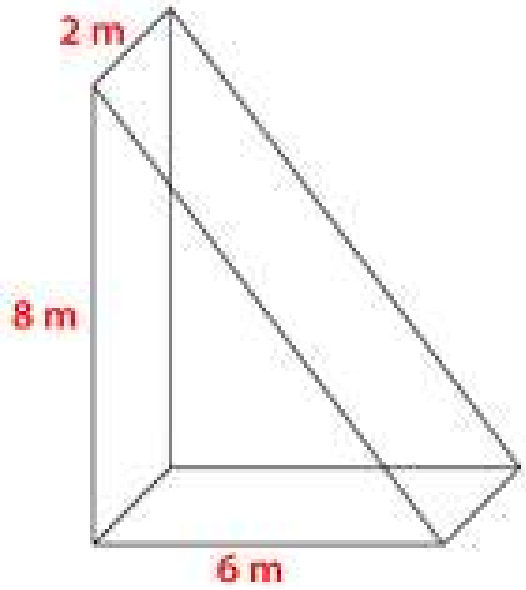


Figure **9.13.10**

4.

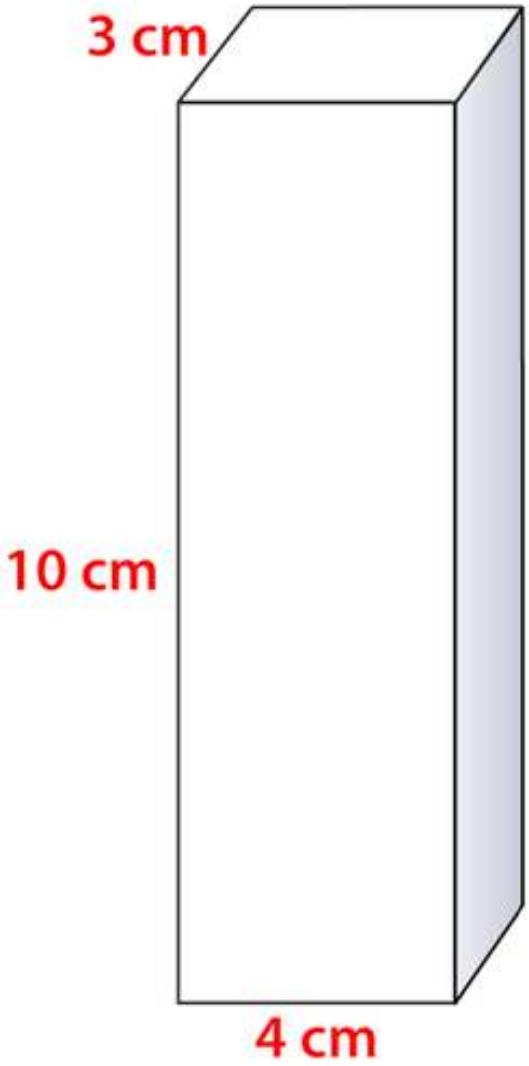


Figure **9.13.11**

5.

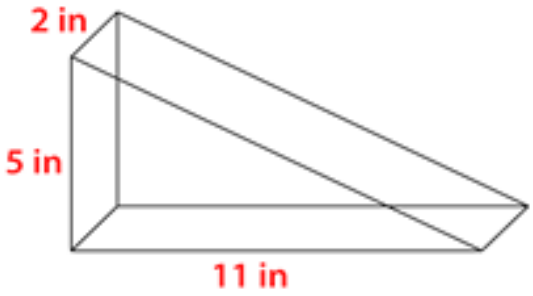


Figure **9.13.12**

6.

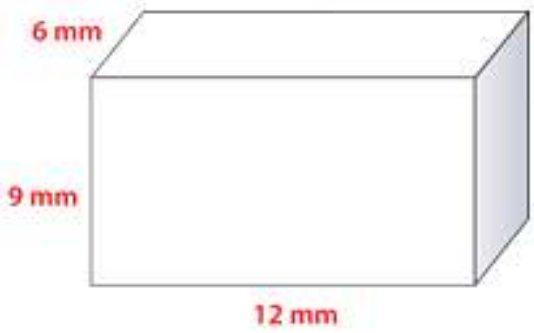


Figure **9.13.13**

7.

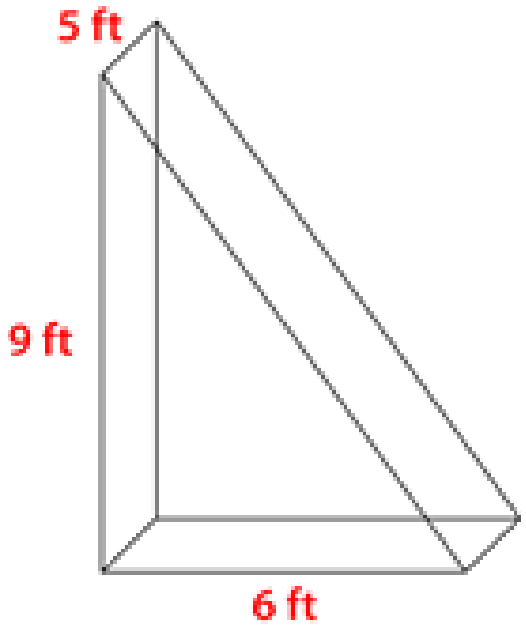


Figure **9.13.14**

8.

Figure **9.13.15**

9.

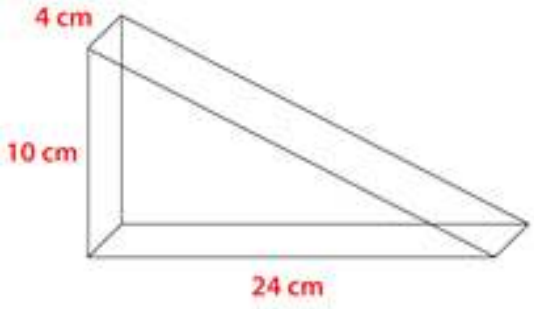
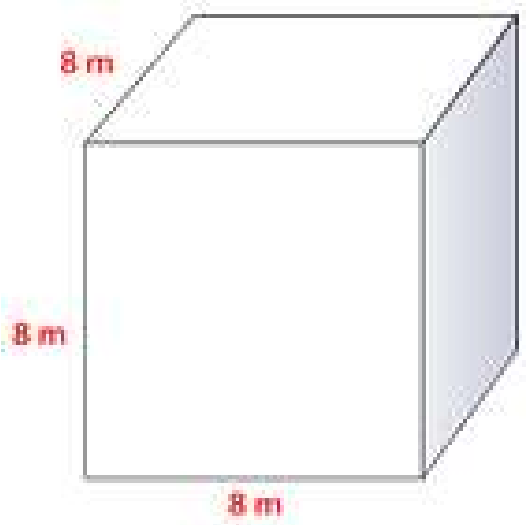


Figure **9.13.16**



10.

Figure **9.13.17**

Identify each type of prism.

11.

Figure **9.13.18**

12.

Figure **9.13.19**

13.

Figure 9.13.20

14.

Figure 9.13.21

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.15.

Vocabulary

Term	Definition
Net	A net is a diagram that shows a “flattened” view of a solid. In a net, each face and base is shown with all of its dimensions. A net can also serve as a pattern to build a three-dimensional solid.
Rectangular Prism	A rectangular prism is a prism made up of two rectangular bases and four rectangular faces.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Triangular Prism	A triangular prism is a prism made up of two triangular bases and three rectangular faces.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Solid Geometry Volume

Practice: Volume of Prisms Using Unit Cubes

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9.14: Volume of Rectangular Prisms

Volume equals length times width times height

Figure **9.14.1**

Ben's mom has grown frustrated with Ben's extensive Lego collection thrown all over his bedroom floor. She gives him a choice of three different boxes in which to store them. Ben wants to choose the box that will hold the most Legos. While the boxes are different in dimensions, Ben cannot figure out which will hold the most. The box measurements are as follows:

Box A: height = 5 inches, length = 18 inches, width = 10

Box B: height = 8 inches, length = 10 inches, width = 12 inches

Box C: height = 6 inches, length = 14 inches, width = 10 inches.

Which box has the greatest **volume**?

In this concept, you will learn how to figure out the volume of rectanglar prisms.

Finding the Volume of a Rectangular **Prism**

Volume is the measure of how much three-dimensional space an object takes up or holds.

Imagine a fish aquarium. Its length, width, and height determine how much water the tank will hold. If you fill it with water, the amount of water is the volume that the tank will hold. You measure volume in **cubic units**, because you are multiplying three dimensions: length, width, and height.

One way to find the volume of a prism is to consider how many unit cubes it can contain. A unit cube is simply a cube measuring one inch, one centimeter, one foot, or whatever unit of measurement you are using, on all sides. Here are some unit cubes.

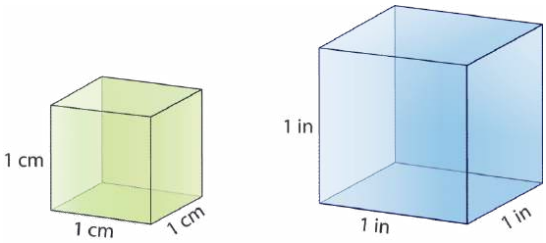

Figure **9.14.2**

To use unit cubes to calculate volume, simply count the number of unit cubes that fit into the prism. Begin by counting the number of cubes that cover the bottom of the prism, and then count each layer. Let’s see how this works.

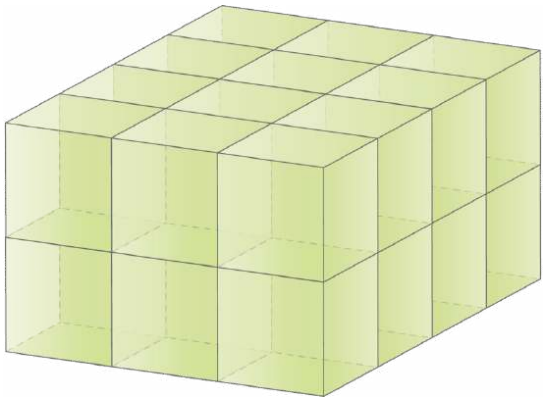

Figure **9.14.3**

How many cubes do you see here? If you count all of the cubes, you will see that there are 24 cubes in this prism.

The volume of this prism is **24 units^3** or cubic units.

Find the volume of the following figure using unit cubes.

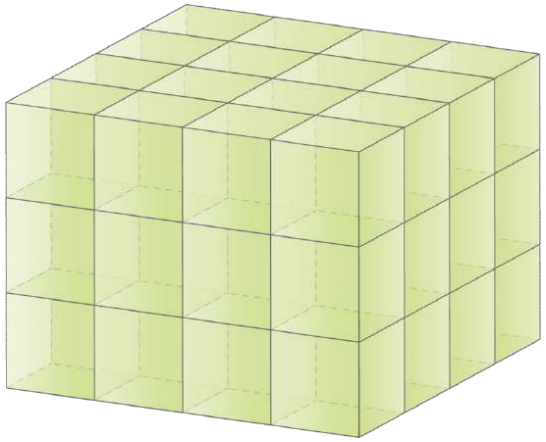

Figure **9.14.4**

How many cubes are in this figure? If you count the cubes, you will get a total of 48 cubes. The volume of this prism is 48 cubic units or **units^3**.

If you look carefully, you will see that the volume of the rectangular prism is a function of multiplying the length \times the width \times the height. You just discovered the formula for finding the volume of a rectangular prism. Now let's refine that formula a little further. Here is the formula.

$$V = Bh$$

The volume is equal to the B, base area of the prism, times the height of the prism.

Let's look at an example.

Find the volume of the prism below.

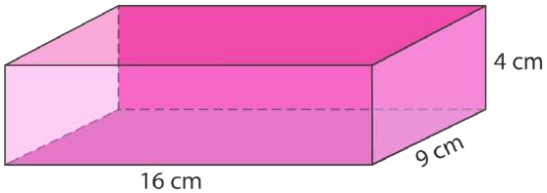

Figure **9.14.5**

Simply put the values for the length, width, and height in for the appropriate variables in the formula, then solve for V, volume.

First find the area of the base. This is the rectangular side on the bottom. Remember, to find the area of a rectangle, multiply the length times the width.

$$B = lw$$
$$B = 16 \times 9$$
$$B = 144 \text{ cm}^2$$

The base area is 144 square centimeters. Now multiply this by the height.

$$V = Bh$$
$$V = 144 \times 4$$
$$V = 576 \text{ cm}^3$$

You can use the following formula for volume of a rectangular prism. This combines the two steps that you completed above:

$$V = lwh$$
$$V = (16)(9)(4)$$
$$V = 576 \text{ cm}^3$$

The volume of this rectangular prism is 576 cubic centimeters.

You can work with the same rectangular prism, but fill it with unit cubes.

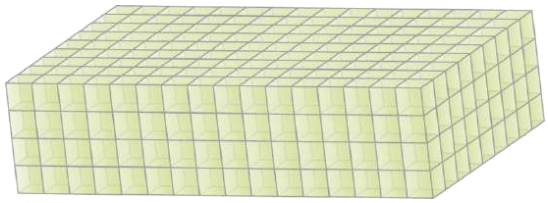

Figure **9.14.6**

You can count the unit cubes here to find the volume of the rectangular prism. However, you save time by using the formula for volume.

Let's look at another example.

Find the volume of a container with a length of 15 ft, width of 12 ft, and height of 11 ft.

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (15)(12)(11)$$
$$V = (180)(11)$$

Next, multiply the results by the value for the height:

$$V = (180)(11)$$
$$V = 1,980$$

Then, record the answer including the appropriate unit of measurement:

$$V = 1,980 \text{ ft}^3$$

The answer is the container has a volume of 1,980 cubic feet.

Example 9.14.1

Earlier, you were given a problem about Ben, who is searching for the box that will hold the most Legos.

Ben needs to figure out which of the following boxes has the greatest volume.

Box A: height = 5 inches, length = 18 inches, width = 10 inches

Box B: height = 8 inches, length = 10 inches, width = 12 inches

Box C: height = 6 inches, length = 14 inches, width = 10 inches

Solution

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$\text{Box A: } V = (5)(18)(10)$$

$$V = (90)(10)$$

$$\text{Box B: } V = (8)(10)(12)$$

$$V = (80)(12)$$

$$\text{Box C: } V = (6)(14)(10)$$

$$V = (84)(10)$$

Next, multiply the results by the value for the height:

$$\text{Box A: } V = (90)(10)$$

$$V = 900$$

$$\text{Box B: } V = (80)(12)$$

$$V = 960$$

$$\text{Box C: } V = (84)(10)$$

$$V = 840$$

Then, record the answer including the appropriate unit of measurement:

$$\text{Box A: } V = 900 \text{ in}^2$$

$$\text{Box B: } V = 960 \text{ in}^2$$

$$\text{Box C: } V = 840 \text{ in}^2$$

The answer is Box B has the greatest volume and therefore can hold the most Legos.

Example 9.14.2

Carla is cleaning out her fish tank, so she filled the bathtub to the rim with water for her fish to swim in while she empties their tank. If the bathtub is 5.5 feet long, 3.3 feet wide, and 2.2 feet deep, how much water can it hold?

Solution

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (5.5)(3.3)(2.2)$$
$$V = (18.15)(2.2)$$

Next, multiply the results by the value for the height:

$$V = (18.15)(2.2)$$
$$V = 39.93$$

Then, record the answer including the appropriate unit of measurement:

$$V = 39.93 \text{ ft}^3$$

The answer is Carla's bathtub can hold 39.93 cubic feet of water.

Example 9.14.3

Find the volume of a container with a length of 10 inches, width of 8 inches, and height of 6 inches.

Solution

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (10)(8)(6)$$
$$V = (80)(6)$$

Next, multiply the results by the value for the height:

$$V = (80)(6)$$
$$V = 480$$

Then, record the answer including the appropriate unit of measurement:

$$V = 480 \text{ in}^3$$

The answer is the container has a volume of 480 cubic inches.

Example 9.14.4

Find the volume of a container with a length of 8 meters, width of 7 meters, and height of 3 meters.

Solution

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (8)(7)(3)$$
$$V = (56)(3)$$

Next, multiply the results by the value for the height:

$$V = (56)(3)$$
$$V = 168$$

Then, record the answer including the appropriate unit of measurement:

$$V = 168 \text{ m}^3$$

The answer is the container has a volume of 168 cubic meters.

Review

Find the volume of each rectangular prism. Remember to label your answer in cubic units.

1. Length = 5 in, width = 3 in, height = 4 in
2. Length = 7 m, width = 6 m, height = 5 m
3. Length = 8 cm, width = 4 cm, height = 9 cm
4. Length = 8 cm, width = 4 cm, height = 12 cm
5. Length = 10 ft, width = 5 ft, height = 6 ft
6. Length = 9 m, width = 8 m, height = 11 m
7. Length = 5.5 in, width = 3 in, height = 5 in
8. Length = 6.6 cm, width = 5 cm, height = 7 cm
9. Length = 7 ft, width = 4 ft, height = 6 ft
10. Length = 15 m, width = 8 m, height = 10 m
11. Length = 10.5 m, width = 11 m, height = 4 m
12. Length = 12 ft, width = 12 ft, height = 8 ft
13. Length = 16 in, width = 8 in, height = 8 in
14. Length = 12 m, width = 12 m, height = 12 m
15. Length = 24 in, width = 6 in, height = 6 in

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.10.

Resources

Vocabulary

Term	Definition
Cubic Units	Cubic units are three-dimensional units of measure, as in the volume of a solid figure.
Prism	A prism is a three-dimensional object with two congruent parallel bases that are polygons.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Video: Solid Geometry Volume

Practice: Volume of Rectangular Prisms

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9.15: Volume of Triangular Prisms

Volume equals base times height times half of length.

Figure **9.15.1**

Maggie is enjoying her favorite chocolate. She wonders how much chocolate can fit into the packaging, which is in the shape of a triangular **prism**. She measures the cardboard prism and finds that the triangle end, or base, is 1.4 inches, and the height of the triangle is 1.25 inches. The height of the bar when stood on its end is 8.25 inches. What is the **volume** of the chocolate package?

In this concept, you will learn how to calculate the volume of a triangular prism.

Finding the Volume of a Triangular Prism

With a triangular prism, the two parallel faces are triangles and the other faces are rectangles.

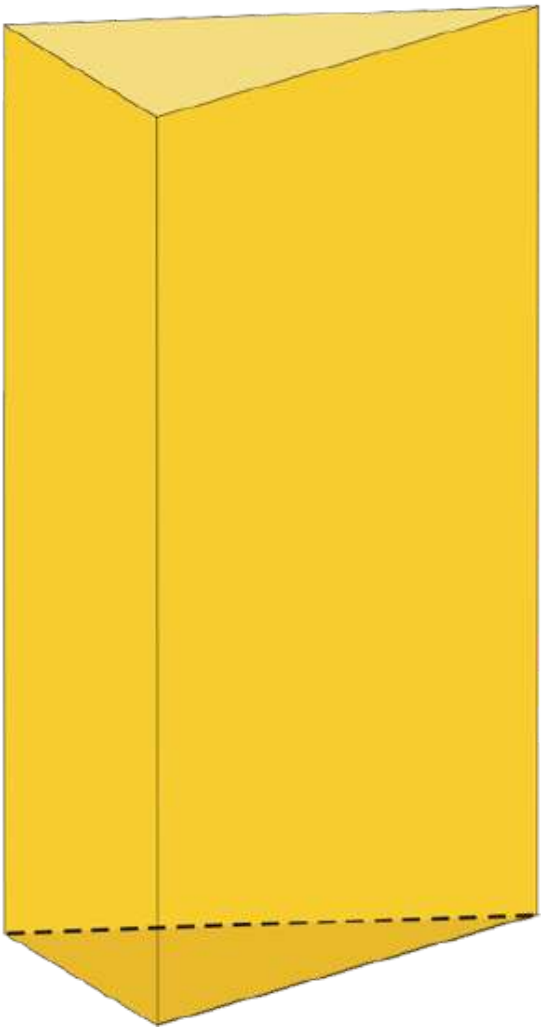

Figure 9.15.2

You calculate the volume of triangular prisms almost the same way that you find the volume of rectangular prisms. You still use the formula $V = Bh$. However, this time the base of the prism is a triangle, not a rectangle. Therefore, you need to use the area formula for a triangle to find the area of the base, B. Then you can multiply this amount by the height of the rectangle to find the volume of the triangular prism.

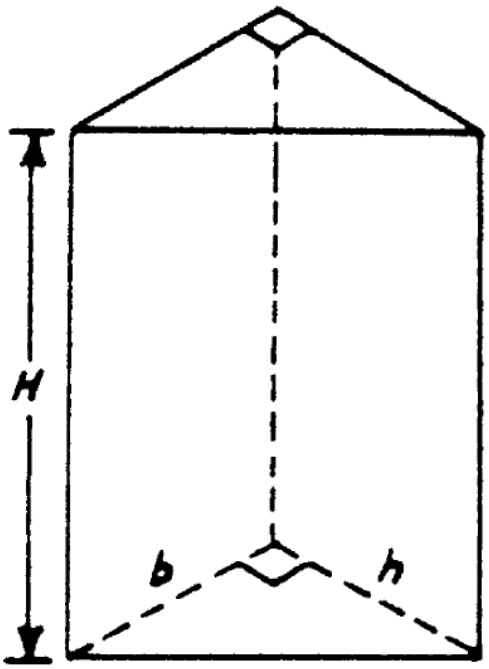

Figure 9.15.3

Let's look at an example.

What is the volume of this triangular prism where the height of the triangular base is 7 cm and the width of the base of the triangle is 5 cm?

First ,you need to find the area of the triangular base. You use the formula for the area of a triangle which is $\frac{1}{2}bh$. Remember, you use the height and base measurements for the triangular face, not the height, H, measurement for the whole prism which is the length of the rectangle.

So there are two things that you need to accomplish: You need to find the area of one of the triangular bases, and then you can take that measurement and multiply it with the height of the entire prism.

$$\begin{aligned} V &= BH \\ b &= \frac{1}{2}bh \\ b &= \frac{1}{2}(5)(7) \\ b &= 35 \\ V &= (35)H \\ V &= (35)(12) \\ V &= 420 \text{ cm}^3 \end{aligned}$$

The answer is the volume of this triangular prism is 420centimeters^3.

Example 9.15.1

Earlier, you were given a problem about how Maggie wants to find the volume of chocolate that will fit inside the packaging.

The triangular base is 1.4 inches; the height of the triangle is 1.25 inches, and the height of the bar when stood on its end is 8.25 inches.

Solution

First, plug in the values for the area formula of a triangle to find the base area, B.

$$b = \frac{1}{2}bh$$
$$b = \frac{1}{2}(1.4)(1.25)$$
$$b = 0.7(1.25)$$
$$b = 0.88 \text{ in}^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 0.88 \times 8.25$$
$$V = 0.88 \times 8.25$$
$$V = 7.26$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 7.26 \text{ in}^3$$

The answer is the package has a volume of 7.26 cubic inches.

Example 9.15.2

Every year Jeanie gets a bottle of her favorite perfume for her birthday. The perfume comes in a bottle shaped like a triangular prism. How much perfume does the bottle hold when it is full?

Figure 9.15.4

Solution

First, plug in the values for the area formula of a triangle to find the base area, B.

$$b = \frac{1}{2}bh$$
$$b = \frac{1}{2}(6)(4)$$
$$b = 3(4)$$
$$b = 12 \text{ cm}^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 12 \times 9$$
$$V = 12 \times 9$$
$$V = 108$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 108 \text{ cm}^3$$

The answer is the perfume bottle holds 108 cubic centimeters of perfume when full.

Example 9.15.3

What is the volume of a triangular prism with the following dimensions: $b = 12$ in, $h = 10$ in, $h = 15$ in

Solution

First, plug in the values for the area formula of a triangle to find the base area, B\).

$$b = \frac{1}{2}bh$$
$$b = \frac{1}{2}(12)(10)$$
$$b = 6(10)$$
$$b = 60 \text{ in}^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 60 \times 15$$
$$V = 60 \times 15$$
$$V = 900$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 900 \text{ in}^3$$

The answer is the volume of this triangular prism is 900 in^3.

Example 9.15.4

What is the volume of a triangular prism with the following dimensions: $b = 7$ cm, $h = 5$ cm, $h = 9$ cm

Solution

First, plug in the values for the area formula of a triangle to find the base area, B.

$$b = \frac{1}{2}bh$$
$$b = \frac{1}{2}(7)(5)$$
$$b = 3.5(5)$$
$$b = 17.5cm^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together. V\(V=B\(h=17.5\times 9

$$V = 17.5 \times 9$$
$$V = 157.5$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 157.5 \text{ cm}^3$$

The answer is the volume of this triangular prism is 157.5 cm^3.

Example 9.15.5

What is the volume of a triangular prism with the following dimensions: $b = 4$ mm, $h = 3$ mm, $h = 5$ mm

Solution

First, plug in the values for the area formula of a triangle to find the base area, B

$$b = \frac{1}{2}bh$$
$$b = \frac{1}{2}(4)(3)$$
$$b = 2(3)$$
$$b = 6 \text{ mm}^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 6 \times 5$$
$$V = 6 \times 5$$
$$V = 30$$

Last, record the answer, making sure to include the appropriate unit of measurement.

$$V = 30 \text{ mm}^3$$

The answer is the volume of this triangular prism is 30 mm^3.

Review

Find the volume of each triangular prism. Remember that h means the height of the triangular base and H means the height of the whole prism.

1. $b = 6$ in, $h = 4$ in, $h = 5$ in
2. $b = 7$ in, $h = 5$ in, $h = 9$ in
3. $b = 10$ m, $h = 8$ m, $h = 9$ m
4. $b = 12$ m, $h = 10$ m, $h = 13$ m
5. $b = 8$ cm, $h = 6$ cm, $h = 9$ cm
6. $b = 9$ cm, $h = 7$ cm, $h = 8$ cm
7. $b = 5.5$ m, $h = 4$ m, $h = 4$ m
8. $b = 11$ cm, $h = 9$ cm, $h = 8$ cm
9. $b = 20$ ft, $h = 17$ ft, $h = 19$ ft
10. $b = 20$ ft, $h = 18$ ft, $h = 15$ ft.
11. $b = 18$ ft, $h = 16$ ft, $h = 17$ ft.
12. $b = 24$ ft, $h = 21$ ft, $h = 19$ ft.
13. $b = 24.5$ ft, $h = 18$ ft, $h = 16$ ft.
14. $b = 99$ ft, $h = 80$ ft, $h = 75$ ft.
15. $b = 100$ ft, $h = 80$ ft, $h = 110$ ft.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.11.

Resources

Vocabulary

Term	Definition
Cubic Units	Cubic units are three-dimensional units of measure, as in the volume of a solid figure.
Prism	A prism is a three-dimensional object with two congruent parallel bases that are polygons.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Solid Geometry Volume

Practice: Volume of Triangular Prisms

Real World: Rainbow Connection

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9.16: Surface Area and Volume of Pyramids

Surface area and volume of solids with a base and lateral faces that meet at a common vertex.

Pyramids

A **pyramid** is a solid with one **base** and **lateral faces** that meet at a common **vertex**. The edges between the lateral faces are **lateral edges**. The edges between the base and the lateral faces are **base edges.**

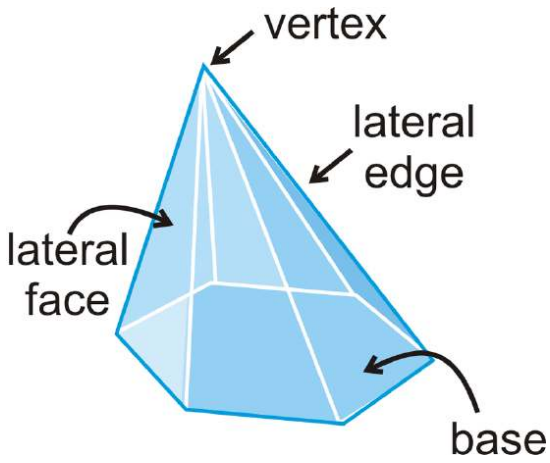

Figure **9.16.1**

A **regular pyramid** is a pyramid where the base is a regular polygon. All regular pyramids also have a **slant height**, which is the height of a lateral face. A non-regular pyramid does not have a slant height.

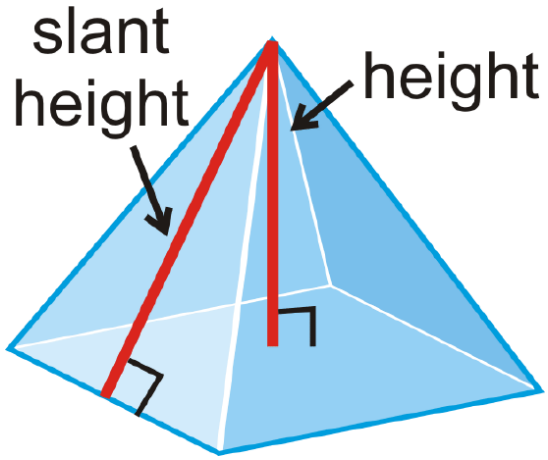

Figure **9.16.2**

Surface Area

Surface area is a two-dimensional measurement that is the total area of all surfaces that bound a solid. The basic unit of area is the square unit. For pyramids, we will need to use the slant height, which is labeled l, to find the area of each triangular face.

Surface Area of a Regular Pyramid: If B is the area of the base, and n is the number of triangles, then $SA = B + \frac{1}{2}nbl$.

The net shows the surface area of a pyramid. If you ever forget the formula, use the net.

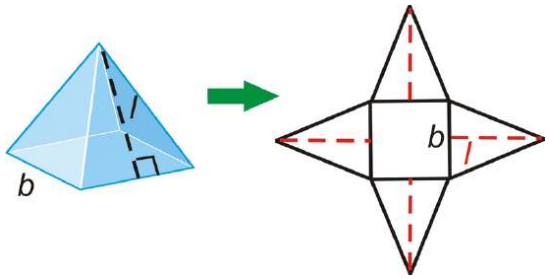

Figure **9.16.3**

Volume

To find the **volume** of any solid you must figure out how much space it occupies. The basic unit of volume is the cubic unit.

Volume of a Pyramid: $V = \frac{1}{3}Bh$ where B is the area of the base.

Figure **9.16.4**

What if you were given a solid three-dimensional figure with one base and lateral faces that meet at a common vertex? How could you determine how much two-dimensional and three-dimensional space that figure occupies?

Example 9.16.1

Find the slant height of the square pyramid.

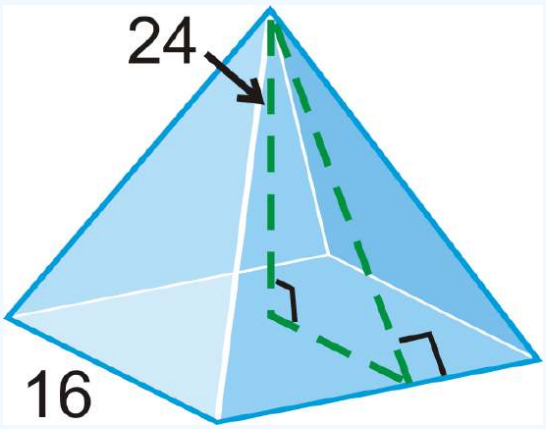

Figure **9.16.5**

Solution

The slant height is the hypotenuse of the right triangle formed by the height and half the base length. Use the Pythagorean Theorem.

$$8^2 + 24^2 = l^2$$
$$640 = l^2$$
$$l = \sqrt{640} = 8\sqrt{10}$$

Example 9.16.2

Find the surface area of the **regular** triangular pyramid.

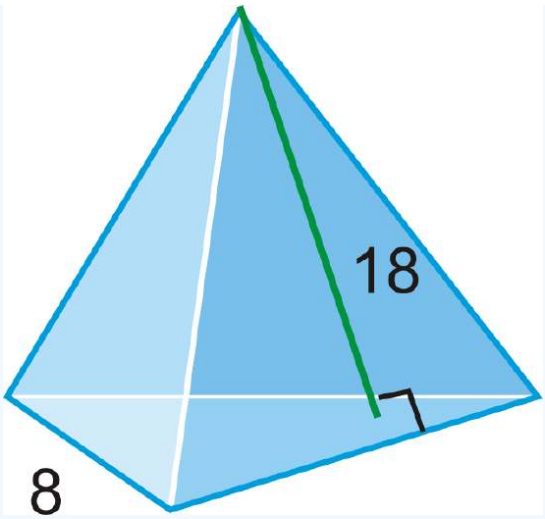

Figure 9.16.6

Solution

"Regular" tells us the base is an equilateral triangle. Let's draw it and find its area.

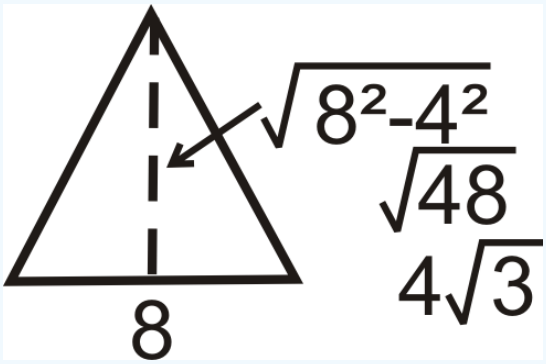

Figure 9.16.7

$B = \frac{1}{2} \cdot 8 \cdot 4\sqrt{3} = 16\sqrt{3}$

The surface area is:

$SA = 16\sqrt{3} + \frac{1}{2} \cdot 3 \cdot 8 \cdot 18 = 16\sqrt{3} + 216 \approx 243.71$

Example 9.16.3

If the lateral surface area of a regular square pyramid is $72\ ft^2$ and the **base edge** is equal to the slant height. What is the length of the base edge?

Solution

In the formula for surface area, the lateral surface area is $\frac{1}{2}nbl$. We know that $n = 4$ and $b = l$. Let's solve for b.

$$\frac{1}{2}nbl = 72 \text{ ft}^2$$
$$\frac{1}{2}(4)b^2 = 7^2$$
$$2b^2 = 7^2$$
$$b^2 = 36$$
$$b = 6\text{feet}$$

Example 9.16.4

Find the height and then volume of the pyramid.

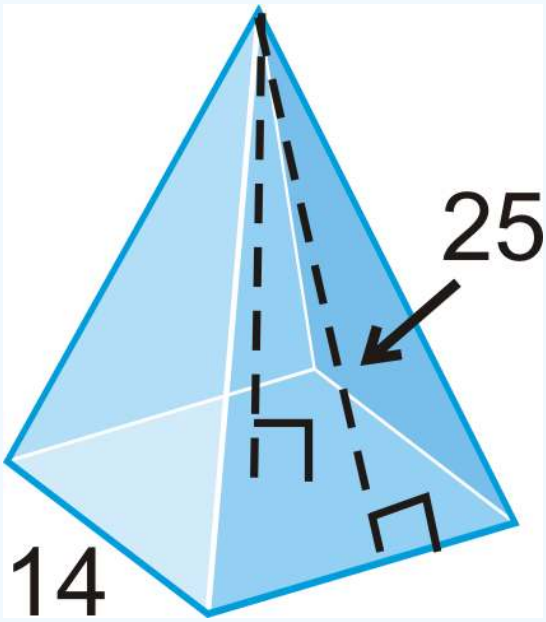

Figure 9.16.8

Solution

In this example, we are given the *slant* height. Use the Pythagorean Theorem.

$$\begin{aligned} 7^2 + h^2 &= 25^2 \\ h^2 &= 576 \\ h &= 24 \end{aligned}$$

$V = 13(142)(24) = 1568 \text{ units}^3$

Example 9.16.5

Find the volume of the pyramid with a right triangle as its base.

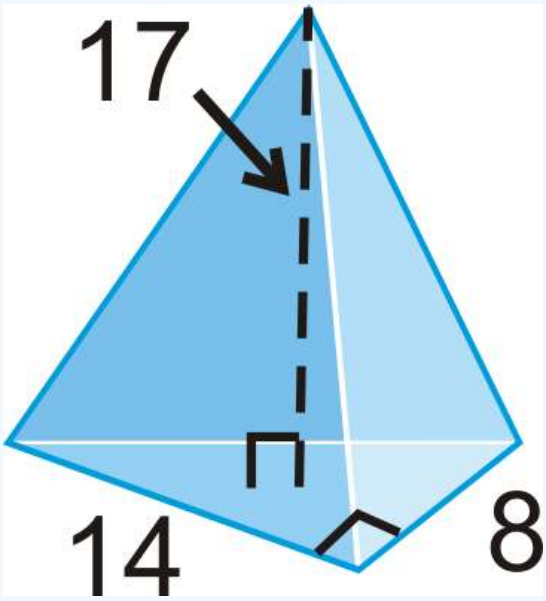

Figure 9.16.9

Solution

The base of the pyramid is a right triangle. The area of the base is $\frac{1}{2}(14)(8) = 56 \text{ units}^2$.

$V = \frac{1}{3}(56)(17) \approx 317.33 \text{ units}^3$

Example 9.16.6

A rectangular pyramid has a base area of $56\ cm^2$ and a volume of $224\ cm^3$. What is the height of the pyramid?

Solution

Use the formula for volume and plug in the information we were given. Then solve for the height.

$$V = \frac{1}{3}Bh$$
$$224 = \frac{1}{3} \cdot 56h$$
$$12 = h$$

Review

Fill in the blanks about the diagram to the left.

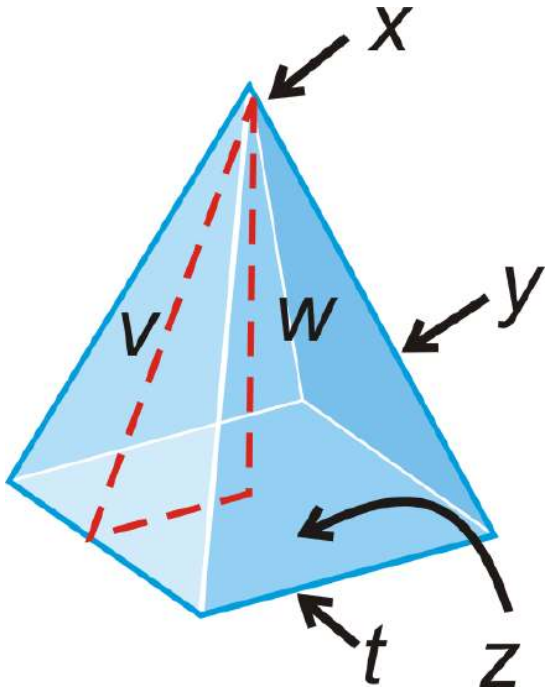

Figure 9.16.10

1. x is the _____.
2. The slant height is _____.
3. y is the _____.
4. The height is _____.
5. The base is _____.
6. The base edge is _____.

For questions 7-8, sketch each of the following solids and answer the question. Your drawings should be to scale, but not one-to-one. Leave your answer in simplest radical form.

7. Draw a square pyramid with an edge length of 9 in and a 12 in height. Find the slant height.
8. Draw an equilateral triangle pyramid with an edge length of 6 cm and a height of 6 cm. What is the height of the base?

Find the slant height, l, of one lateral face in each pyramid. Round your answer to the nearest hundredth.

9.

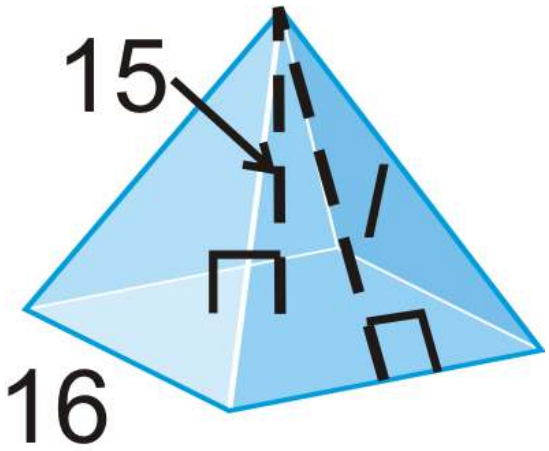

Figure **9.16.11**

10.

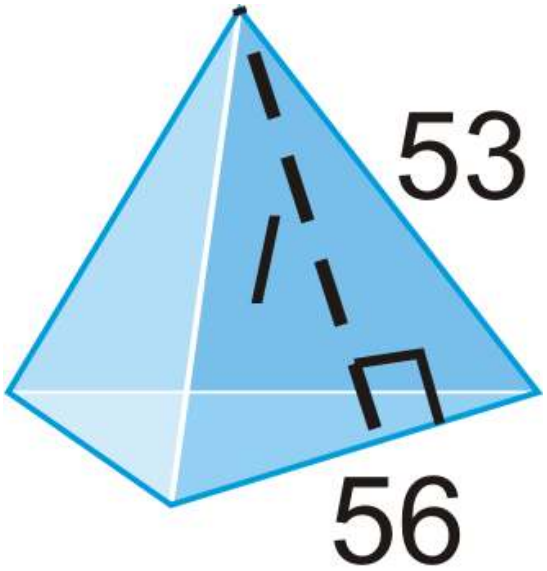

Figure **9.16.12**

Find the surface area and volume of the regular pyramid. Round your answers to the nearest hundredth.

11.

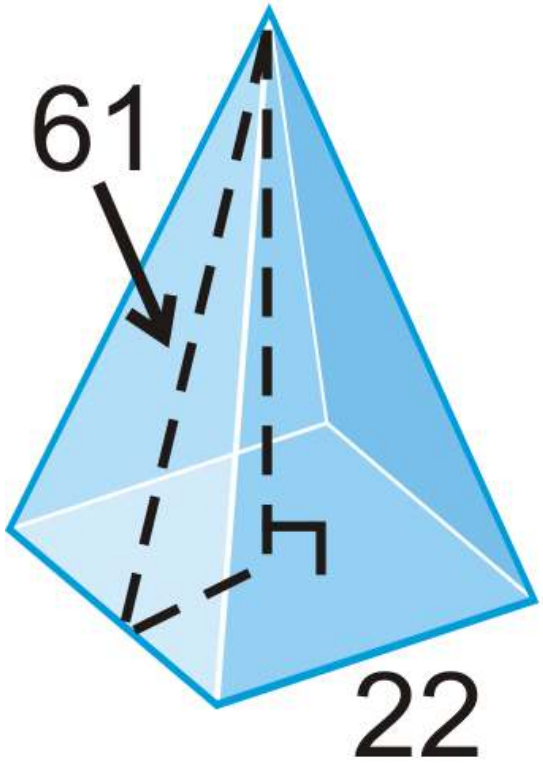

Figure **9.16.13**

12.

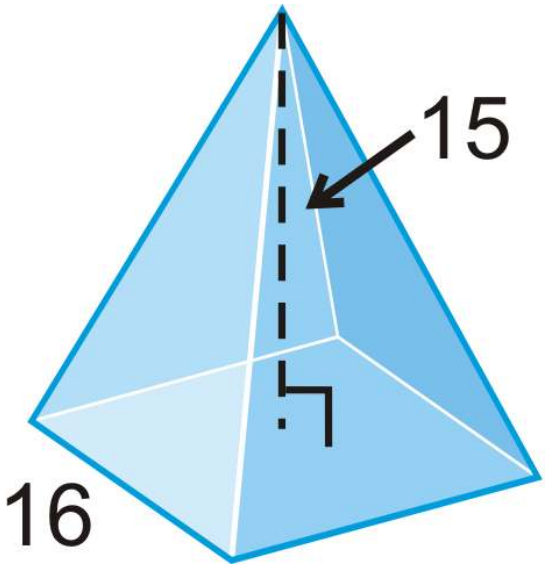

Figure **9.16.14**

13.

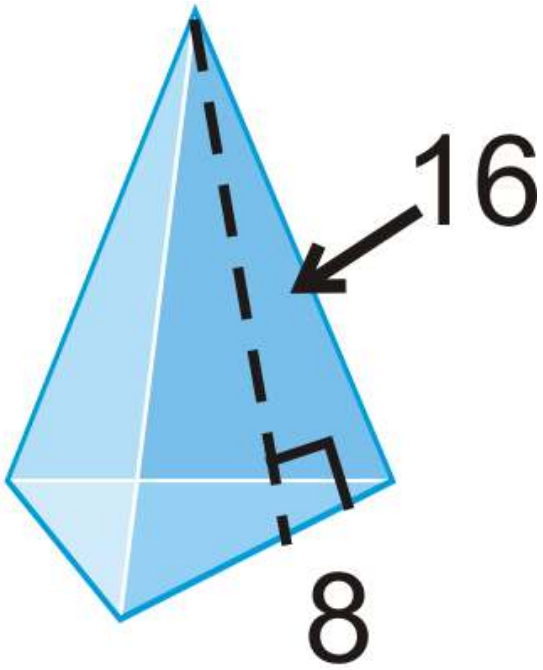

Figure **9.16.15**

14.

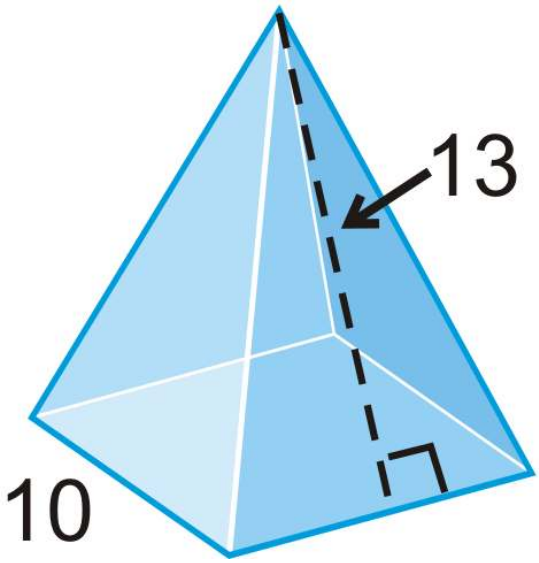

Figure **9.16.16**

15.

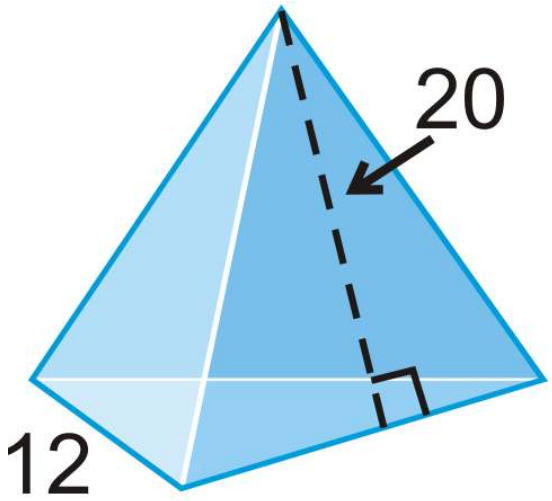

Figure 9.16.17

16.

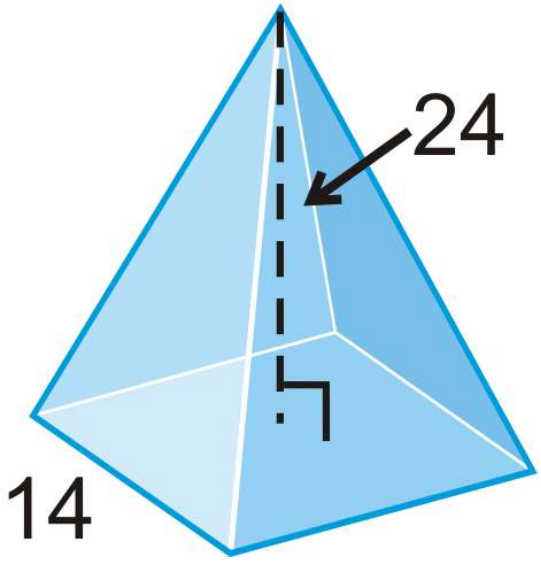

Figure 9.16.18

17. A ***regular tetrahedron*** has four equilateral triangles as its faces.
    1. Find the height of one of the faces if the edge length is 6 units.
    2. Find the area of one face.
    3. Find the total surface area of the regular tetrahedron.
18. If the surface area of a square pyramid is $40\ ft^2$ and the base edge is 4 ft, what is the slant height?
19. If the lateral area of a square pyramid is $800\ in^2$ and the slant height is 16 in, what is the length of the base edge?
20. If the lateral area of a regular triangle pyramid is $252\ in^2$ and the base edge is 8 in, what is the slant height?
21. The volume of a square pyramid is 72 square inches and the base edge is 4 inches. What is the height?
22. The volume of a triangle pyramid is $170\ in^3$ and the base area is $34\ in^2$. What is the height of the pyramid?

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.5.

Vocabulary

Term	Definition
lateral edges	Edges between the lateral faces of a prism.
lateral faces	The non-base faces of a prism.
Pyramid	A pyramid is a three-dimensional object with a base that is a polygon and triangular faces that meet at one vertex.
regular pyramid	is a pyramid where the base is a regular polygon. All regular pyramids also have a ***slant height***, which is the height of a lateral face.
Cone	A cone is a solid three-dimensional figure with a circular base and one vertex.
Vertex	A vertex is a point of intersection of the lines or rays that form an angle.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.
Cavalieri's Principle	States that if two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.
Base Edge	The base edge is the edge between the base and the lateral faces of a prism.
Slant Height	The slant height is the height of a lateral face of a pyramid.
Apothem	The apothem of a regular polygon is a perpendicular segment from the center point of the polygon to the midpoint of one of its sides.

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9.17: Volume of Pyramids

Volume equals area of the base times $\frac{1}{3}$ of height.

Figure **9.17.1**

Michael and Juanita are interested in the amount of sand required to build a sand **pyramid** of the size that they see while at a sand sculpture exhibit. They carefully measure the width of the square base of the sand pyramid which is 4 feet and the height of the pyramid which is 5 feet. How much sand is required to build this sand pyramid?

In this concept, you will learn how to calculate the **volume** of a pyramid.

Finding the Volume of a Pyramid

Pyramids are solid shapes that exist in three-dimensional space. A pyramid has sides that are triangular faces and a base that can be any shape.

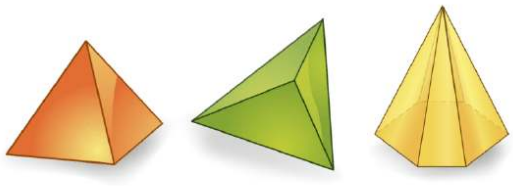

Figure **9.17.2**

Volume is the measure of how much space a three-dimensional figure takes up or holds. Volume is measured in cubic units, because you are dealing with three dimensions: length, width, and height.

You can start to understand the volume of a pyramid by comparing a pyramid with a figure that is similar to it. You know that a prism has length, width and height. The parallel bases of a prism can be any polygon. This one has a square base; so it is called a cube.

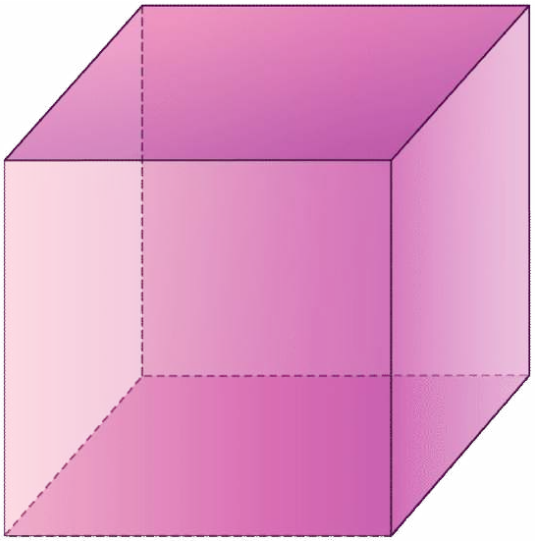

Figure **9.17.3**

Now think about a pyramid. It has a base that can be any polygon. The base of this pyramid is a square.

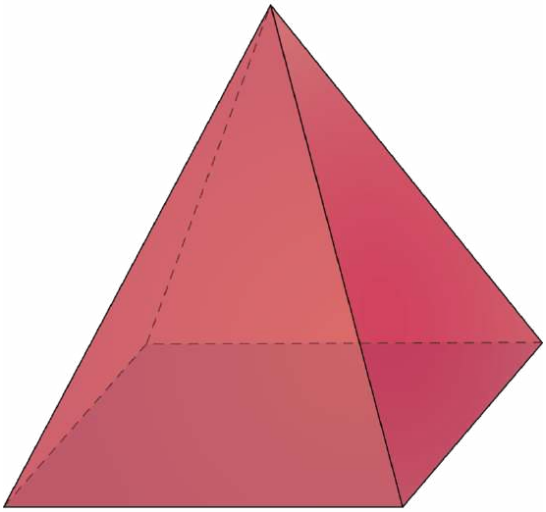

Figure **9.17.4**

These are both similar given that they have square bases. Think about the volume of a pyramid by looking at the volume of the cube or prism. Look at this picture that demonstrates the relationship of a cube to a square pyramid:

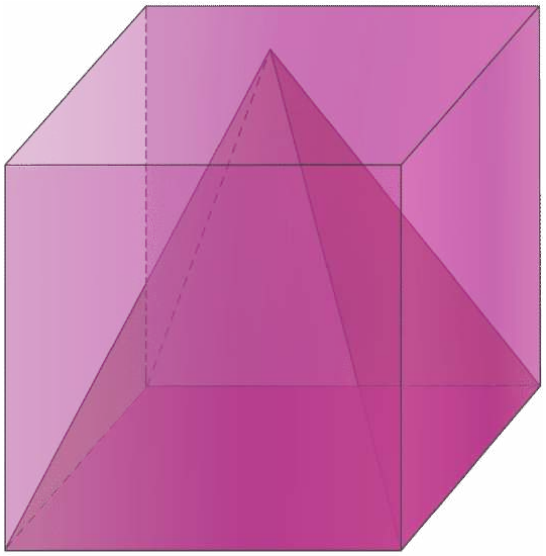

Figure **9.17.5**

To find the volume of this cube, multiply the base times the width times the height. To find the volume of the pyramid, take the area of the base, B and multiply it times the height and then multiply it by $\frac{1}{3}$. Here is the formula for finding the volume of a pyramid.

\($V = \frac{1}{3}Bh$

One thing to keep in mind is that pyramids can be tricky because they can have many different bases. B in the formula means that you need to find the area of the base. If the base is a square, you will need to use the formula for area of a square, $B = s^2$. If the base is a rectangle, the area of the base is $B = lw$. If the base is a triangle, use the formula $B = \frac{bh}{2}$. Making sure that you have the correct formula is essential in your work with pyramids.

Let's look at an example. What is the volume of the pyramid below?

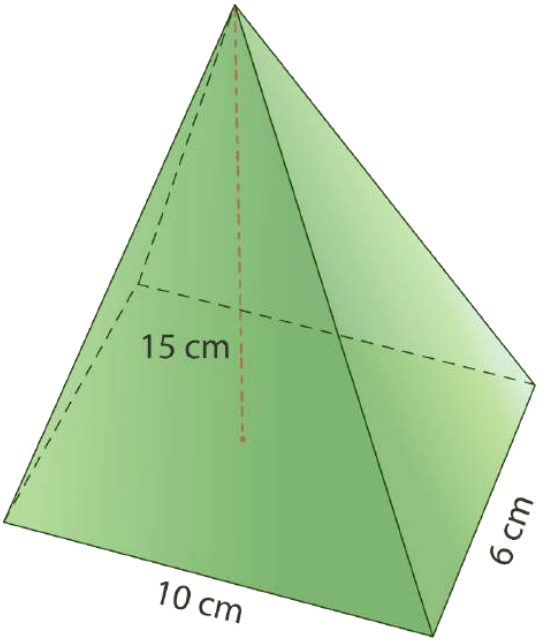

Figure **9.17.6**

First, decide what shape the base of the pyramid is. One side is 10 centimeters and the other is 6 centimeters, so it must be a rectangle. Use the area formula for rectangles to find B, the base area.

$$B = lw$$
$$B = 10(6)$$
$$B = 60 \text{ cm}^2$$

The area of this pyramid's base is 60 square centimeters. Now multiply this by the height and 13 according to the formula.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(60)(15)$$
$$V = \frac{900}{3}$$
$$V = 300 \text{ cm}^3$$

The answer is the volume of the pyramid is 300 cubic centimeters.

Remember, you measure volume in three dimensions, so write the answer in cubic units.

Let's look at another example.

Find the volume of the figure below.

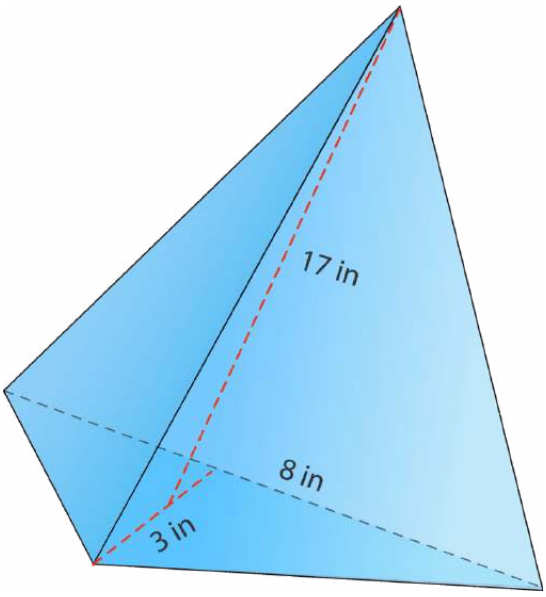

Figure **9.17.7**

This time the base is a triangle, so you'll need to use the area formula for triangles to find the base area. Be careful not to confuse the height of the base triangle with the height of the whole pyramid.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(8)(3)$$
$$B = \frac{24}{2}$$
$$B = 12in^2$$

The base area for this triangular pyramid is 12 square inches. Put this into the formula and solve for V, volume.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(12)(17)$$
$$V = \frac{204}{3}$$
$$V = 68 \text{ in}^3$$

The answer is the volume of this pyramid is 68 cubic inches.

Example 9.17.1

Earlier, you were given a problem about Michael and Juanita's interest in the sand sculpture pyramid.

They want to figure out the volume of sand required to create a sand sculpture pyramid with base 4 feet and height 5 feet:

Solution

First, determine the shape of the base and use the correct formula to find the base area. Since the base is a square, use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (4)^2$$
$$B = 16 \text{ ft}^2$$

The base area of the pyramid is 16 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(16)(5)$$
$$V = 803$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = 803$$
$$V = 26.67 \text{ ft}^3$$

The answer is the volume of sand required to create the sand sculpture pyramid is 26.67 cubic feet.

Example 9.17.2

Felice bought the candle below for her friend's birthday. The package says that the candle burns one hour for every 20 cubic centimeters of wax. What is the volume of the candle so that Felice can figure out how long it will burn?

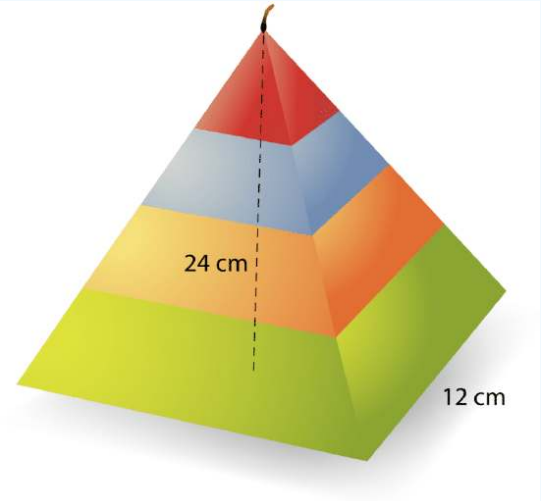

Figure 9.17.8

Solution

First, determine the shape of the base and use the correct formula to find the base area. Since the base is a square, you use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (12)^2$$
$$B = 144 \text{ cm}^2$$

The base area of the pyramid is 144 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(144)(24)$$
$$V = \frac{3456}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{3456}{3}$$
$$V = 1,152 \text{ cm}^3$$

The answer is the volume of the candle is 1,152 cubic cm.

Example 9.17.3

Find the volume of a square pyramid with a base of 8 cm and a height of 6 cm.

Solution

First, determine the shape of the base and use the correct formula to find the base area. The base is a square, so use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (8)^2$$
$$B = 64 \text{ cm}^2$$

The base area of the pyramid is 64 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(64)(6)$$
$$V = \frac{384}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{384}{3}$$
$$V = 128 \text{ cm}^3$$

The answer is the volume of the pyramid is 128 cubic cm.

Example 9.17.4

Find the volume of a rectangular pyramid with a length of 10 cm, a width of 8 cm and a height of 9 cm.

Solution

First, determine the shape of the base and use the correct formula to find the base area. Since the base is a rectangle, use the area formula for squares to find the base area.

$$B = lw$$
$$B = (10)(8)$$
$$B = 80 \text{ cm}^2$$

The base area of the pyramid is 80 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(80)(9)$$
$$V = \frac{720}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{720}{3}$$
$$V = 240 \text{ cm}^3$$

The answer is the volume of the pyramid is 240 cubic cm.

Example 9.17.5

Find the volume of a square pyramid with a base of 5.5 in and a height of 4 in.

Solution

First, determine the shape of the base and use the correct formula to find the base area. The base is a square, so use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (5.5)^2$$
$$B = 30.25in^2$$

The base area of the pyramid is 30.25 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(30.25)(4)$$
$$V = \frac{121}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{121}{3}$$
$$V = 40.33 \text{ in}^3$$

The answer is the volume of the pyramid is 40.33 cubic inches.

Review

Find the volume of each of the following pyramids.

1. A square pyramid with a base of 6 ft and a height of 9 ft.
2. A square pyramid with a base of 8 m and a height of 10 m.
3. A square pyramid with a base of 11 in and a height of 13 in.
4. A square pyramid with a base of 9 ft and a height of 14 ft.
5. A square pyramid with a base of 4.5 in and a height of 5 inches.
6. A rectangular pyramid with a base length of 4 in, a base width of 3 in and a height of 5 in.
7. A rectangular pyramid with a base length of 5 ft, a base width of 4 ft and a height of 6 ft.
8. A rectangular pyramid with a base length of 7 m, a base width of 4 m and a height of 9 m.
9. A triangular pyramid with a base length of 5 in and a base height of 4 inches with a pyramid height of 6 inches.
10. A triangular pyramid with a base length of 8 ft and a base height of 7 ft with a pyramid height of 9 ft.

11. A square pyramid with a base of 8 feet and height of 4 feet.
12. A rectangular pyramid with a length of 5 inches, a width of 4 inches and a height of 6 inches.
13. A square pyramid with a base of 3.5 feet and a height of 6.5 feet.
14. A square pyramid with a base of 6.5 feet and a height of 8.5 feet.
15. A rectangular pyramid with a width of 4 feet, a length of 6 feet and a height of 7.5 feet.

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.14.

Vocabulary

Term	Definition
Cone	A cone is a solid three-dimensional figure with a circular base and one vertex.
Pyramid	A pyramid is a three-dimensional object with a base that is a polygon and triangular faces that meet at one vertex.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Volume of a Pyramid - Overview

Practice: Volume of Pyramids

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9.18: Surface Area and Volume of Cylinders

Surface area and volume of solids with congruent circular bases in parallel planes.

Cylinders

A **cylinder** is a solid with congruent circular bases that are in parallel planes. The space between the circles is enclosed.

A cylinder has a **radius** and a **height.**

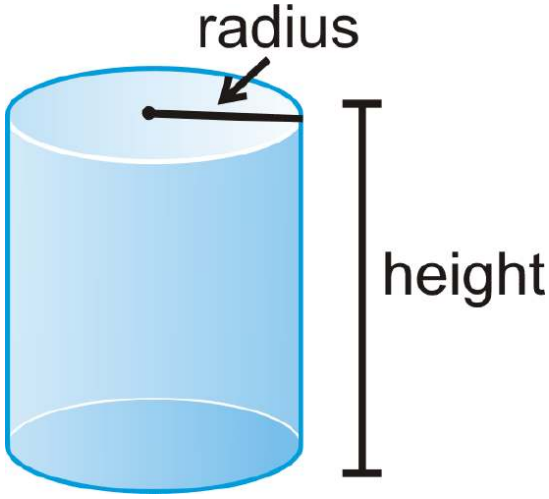

Figure 9.18.1

A cylinder can also be **oblique** (slanted) like the one below.

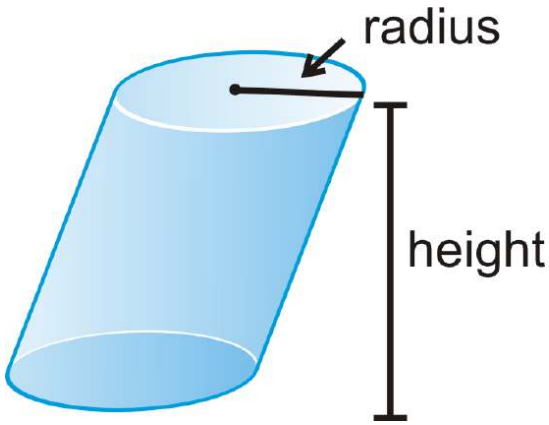

Figure 9.18.2

Surface Area

Surface area is the sum of the area of the faces of a solid. The basic unit of area is the square unit.

Surface Area of a Right Cylinder: $SA = 2\pi r^2 + 2\pi rh$.

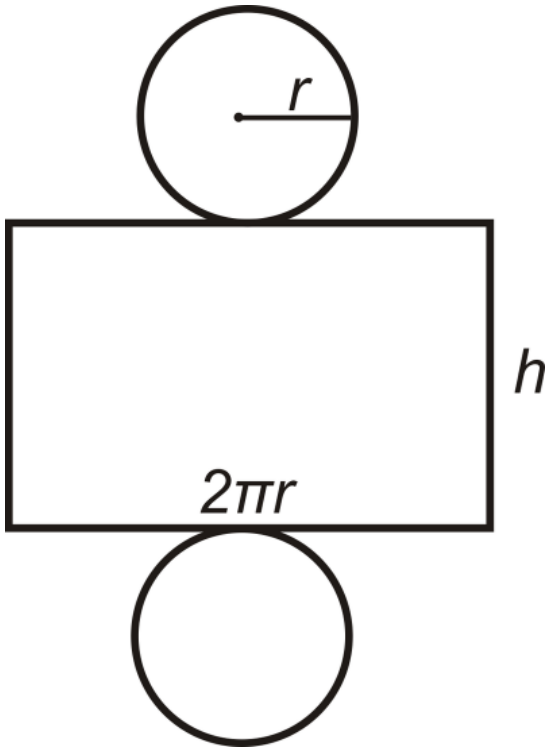

Figure 9.18.3

$$\underbrace{2\pi r^2}_{\text{area of both circles}} + \underbrace{2\pi rh}_{\text{length of rectangle}}$$

Volume

To find the **volume** of any solid you must figure out how much space it occupies. The basic unit of volume is the cubic unit. For cylinders, volume is the area of the circular base times the height.

Volume of a Cylinder: $V = \pi r^2 h$

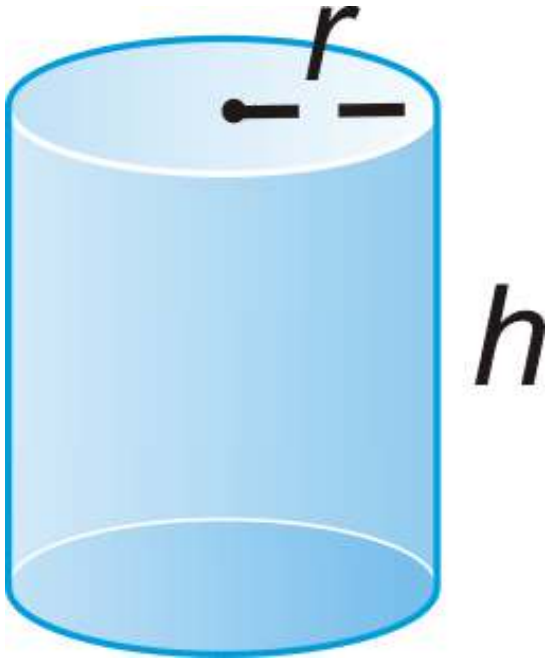

Figure 9.18.4

If an **oblique cylinder** has the same base area and height as another cylinder, then it will have the same volume. This is due to **Cavalieri's Principle**, which states that if two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.

What if you were given a solid three-dimensional figure with congruent enclosed circular bases that are in parallel planes? How could you determine how much two-dimensional and three-dimensional space that figure occupies?

Example 9.18.1

If the volume of a cylin[illegible] height is 4 in, what is the radius?

Solution

Solve for r.

$$\begin{aligned} 484\pi &= \pi r^2(4) \\ 121 &= r^2 \\ 11 \text{ in} &= r \end{aligned}$$

Example 9.18.2

The circumference of the base of a cylinder is 80π cm and the height is 36 cm. Find the total surface area.

Solution

We need to solve for the radius, using the circumference.

$$\begin{aligned} 2\pi r &= 80\pi \\ r &= 40 \end{aligned}$$

Now, we can find the surface area.

$$\begin{aligned} SA &= 2\pi(40)^2 + (80\pi)(36) \\ &= 3200\pi + 2880\pi \\ &= 6080\pi \text{ units}^2 \end{aligned}$$

Example 9.18.3

Find the surface area of the cylinder.

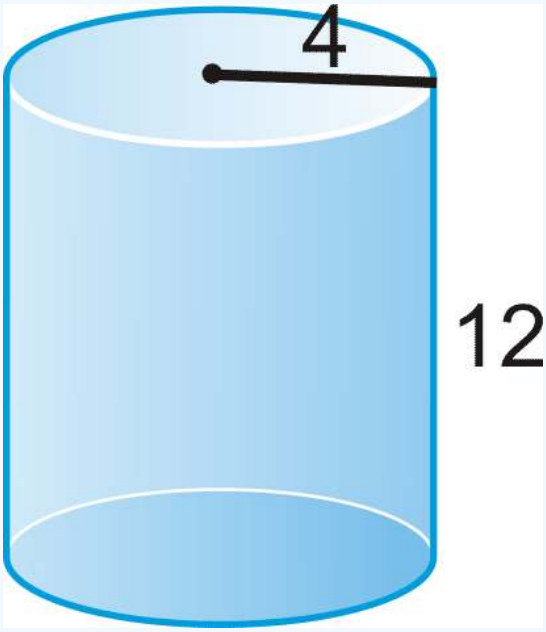

Figure 9.18.5

Solution

$r = 4$ and $h = 12$.

$$\begin{aligned} SA &= 2\pi(4)^2 + 2\pi(4)(12) \\ &= 32\pi + 96\pi \\ &= 128\pi \text{ units}^2 \end{aligned}$$

Example 9.18.4

The circumference of the base of a cylinder is 16π and the height is 21. Find the surface area of the cylinder.

Solution

We need to solve for the radius, using the circumference.

$$\begin{aligned} 2\pi r &= 16\pi \\ r &= 8 \end{aligned}$$

Now, we can find the surface area.

$$\begin{aligned} SA &= 2\pi(8)2 + (16\pi)(21) \\ &= 128\pi + 336\pi \\ &= 464\pi \text{ units}^2 \end{aligned}$$

Example 9.18.5

Find the volume of the cylinder.

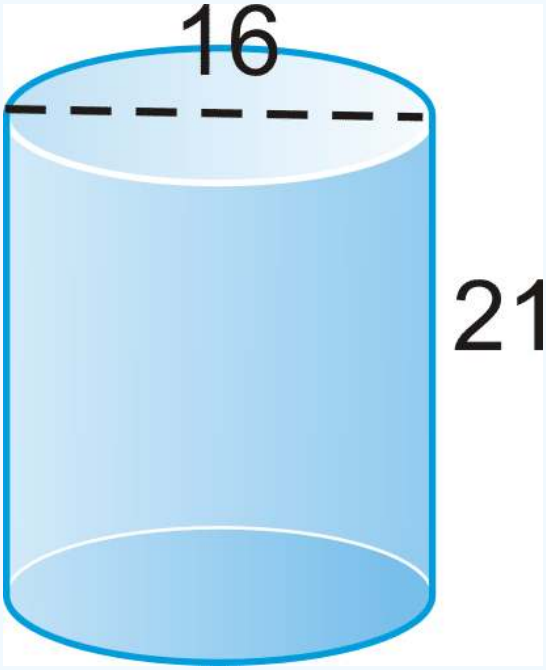

Figure 9.18.6

Solution

If the diameter is 16, then the radius is 8.

$$\begin{aligned} V &= \pi 8^2(21) \\ &= 1344\pi \text{ units}^3 \end{aligned}$$

Review

1. Two cylinders have the same surface area. Do they have the same volume? How do you know?
2. A cylinder has $r = h$ and the radius is 4 cm. What is the volume?
3. A cylinder has a volume of 486π ft^3. If the height is 6 ft., what is the diameter?

4. A right cylinder has a 7 cm radius and a height of 18 cm. Find the volume.

Find the volume of the following solids. Round your answers to the nearest hundredth.

5.

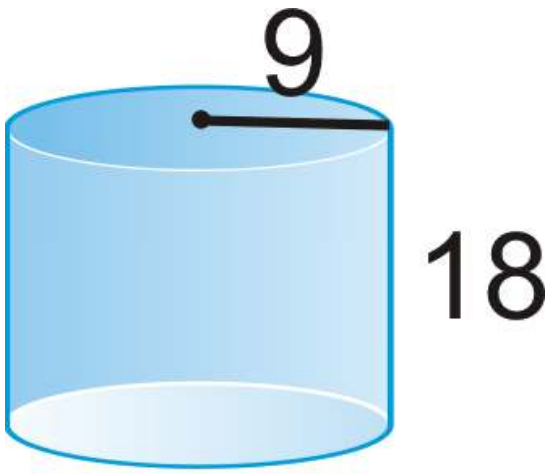

Figure 9.18.7

6.

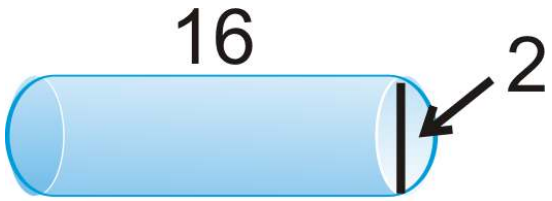

Figure 9.18.8

Find the value of x, given the volume.

7. $V = 6144\pi$ units^3

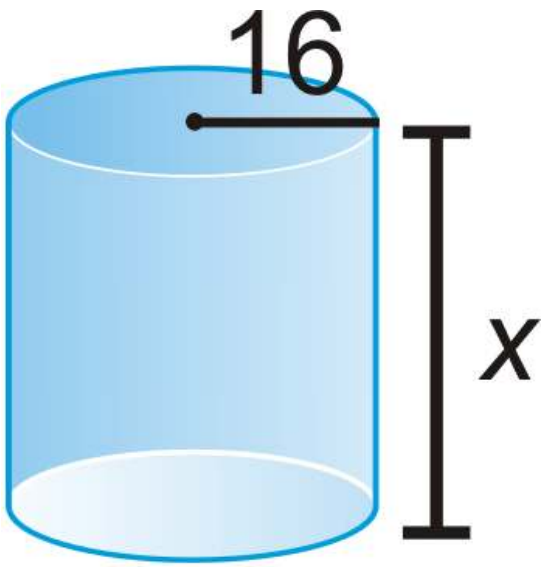

Figure 9.18.9

8. The area of the base of a cylinder is 49π in^2 and the height is 6 in. Find the volume.
9. The circumference of the base of a cylinder is 34π cm and the height is 20 cm. Find the total surface area.
10. The lateral surface area of a cylinder is 30π m^2 and the height is 5 m. What is the radius?

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.4.

Vocabulary

Term	Definition
cylinder	A solid with congruent circular bases that are in parallel planes. The space between the circles is enclosed. A cylinder has a ***radius*** and a ***height*** and can also be ***oblique*** (slanted).
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.
Cavalieri's Principle	States that if two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.
Oblique Cylinder	An oblique cylinder is a cylinder with bases that are not directly above one another.

Additional Resources

Video: Cylinders Principles - Basic

Activities: Cylinders Discussion Questions

Study Aids: Prisms and Cylinders Study Guide

Practice: Surface Area and Volume of Cylinders

Real World: Drill Teams

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9.19: Surface Area of Cylinders

$SA = 2\pi r(h + r)$

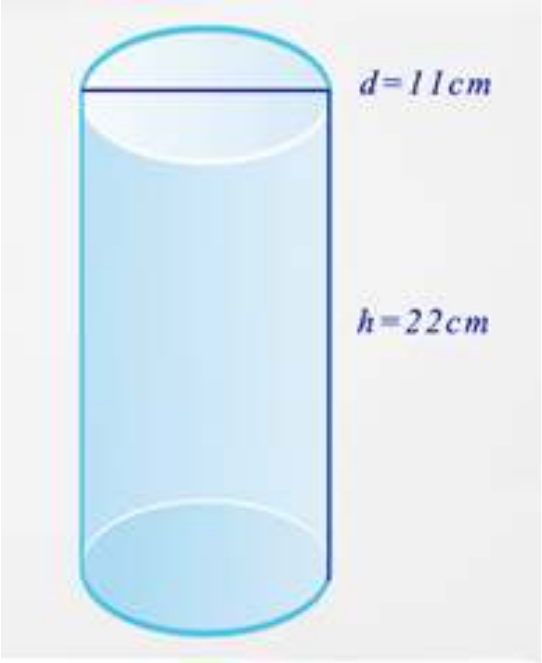

Figure **9.19.1**

Mrs. Johnson is wrapping a cylindrical package in brown paper so that she can mail it to her son. Based on the dimensions shown, how much paper will she need to cover the package?

In this concept, you will learn to find the **surface area** of cylinders.

Surface Area

A **cylinder** has two parallel congruent circular bases with a curved rectangle as its side. One way is to use a **net**.

A **net** is a two-dimensional diagram of a three-dimensional figure. If you could unfold a cylinder (like a can) so that it is completely flat, you would have something that looks like this.

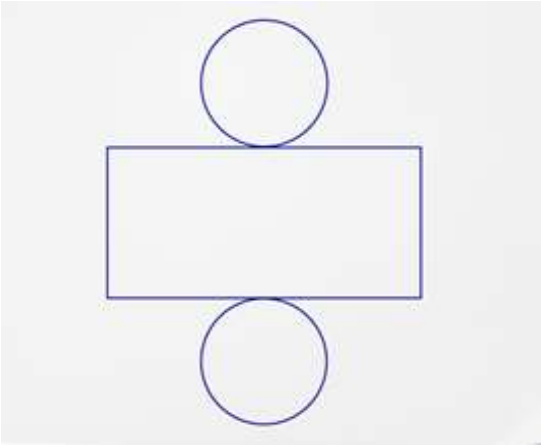

Figure **9.19.2**

With the net of a cylinder, you would need to calculate the area of each circle and the area of the curved side of the cylinder. Then you could add these values together to find the k.

The formula Misplaced & can be used to find the area of a circle bases. Look closely again at the cylinder above. The two circular faces are congruent, so they must have the same radius and diameter.

Let's look at an example.

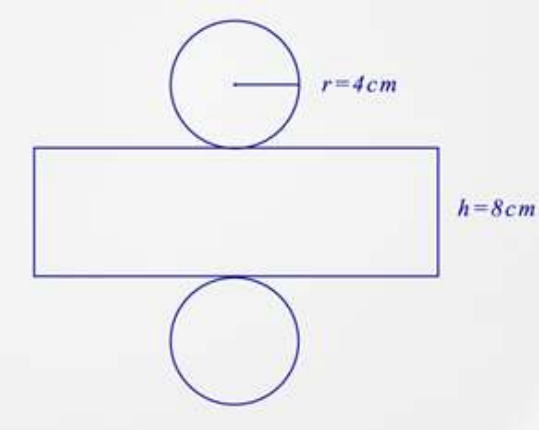

Figure 9.19.3

First, calculate the area of the circular bases.

$$A = \pi r^2$$
$$A = \pi(4)^2$$
$$A = \pi(16)$$
$$A = 50.3$$

Next, find the length of the cylinder's side. You know the width is 8 cm but the length is not shown. The cylinder was unwrapped to form the net. Therefore, the circumference of the circle would be the length of the side.

$$C = 2\pi r$$
$$C = 2\pi \times 4$$
$$C = 25.1$$

Then, calculate the area of the side.

$$A = l \times w$$
$$A = 25.1 \times 8$$
$$A = 200.8$$

Then, find the surface area of the cylinder by adding the side area to the top and bottom area.

$$SA = bottom + top + side$$
$$SA = 50.3 + 50.3 + 200.8$$
$$SA = 301.4$$

The answer is 301.4.

The surface area of the cylinder is $301.4\ \text{cm}^2$.

Putting this all together, you can use the following formula to find the surface area of the cylinder:

$$SA = 2\pi r^2 + 2\pi rh$$

The formula $2\pi r^2$ represents the area of the top and bottom circles of the cylinder. The $2\pi rh$ represents the perimeter ($2\pi r$) multiplied by the height, h.

Let's look at an example.

What is the surface area of the figure below?

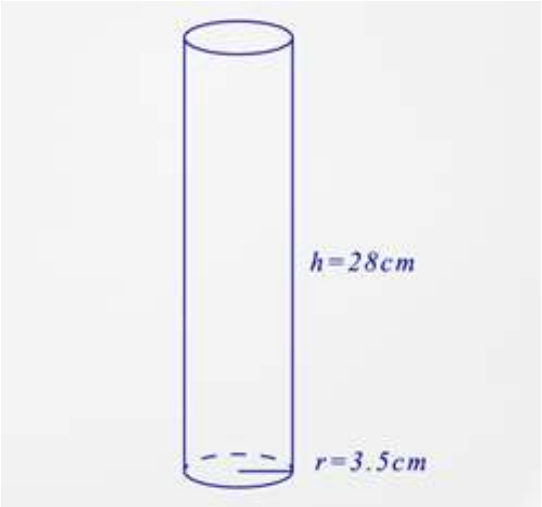

Figure 9.19.4

First, substitute what you know into the surface area formula.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi(3.5)^2 + 2\pi(3.5)(28)$$

Next, calculate the surface area.

$$SA = 2\pi(3.5)^2 + 2\pi(3.5)(28)$$
$$SA = 2\pi(12.25) + 2\pi(98)$$
$$SA = 76.97 + 615.75$$
$$SA = 692.72$$

The answer is 692.72.

The surface area of the cylinder is $692.7\ \text{cm}^2$.

Sometimes, you can have a cylinder that has been cut. This is called a **truncated cylinder**. This is where you only see a section of the cylinder and will need to figure out the surface area of what you see.

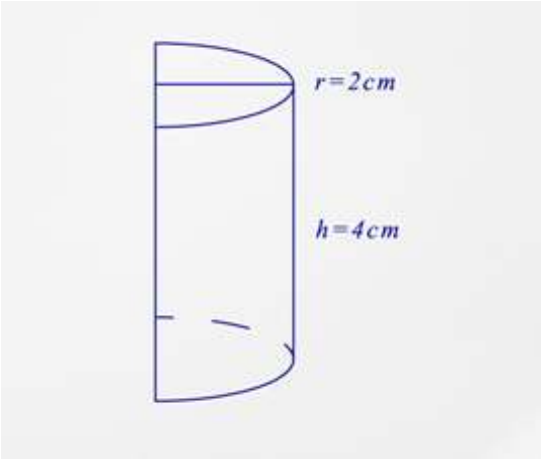

Figure 9.19.5

Example 9.19.1

Earlier, you were given a problem about Mrs. Johnson's cylindrical wrapping.

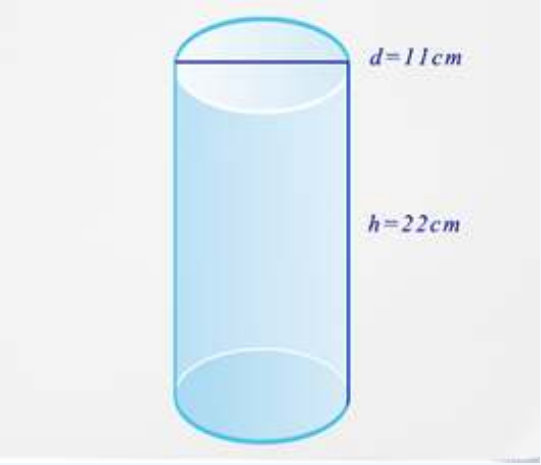

Figure 9.19.6

Solution

First, you need to find the radius. Remember, the radius is half the measure of the diameter.

$$r = \frac{d}{2}$$
$$r = \frac{11}{2}$$
$$r = 5.5v$$

Next, substitute what you know into the surface area formula.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi(5.5)^2 + 2\pi(5.5)(22)$$

Then, calculate the surface area.

$$SA = 2\pi(5.5)^2 + 2\pi(5.5)(22)$$
$$SA = 2\pi(30.25) + 2\pi(121)$$
$$SA = 190.1 + 760.3$$
$$SA = 950.4$$

The answer is 950.4.

Mrs. Johnson needs $950.4\ cm^2$ of brown paper to wrap her parcel.

Example 9.19.2

What is the surface area of the figure below?

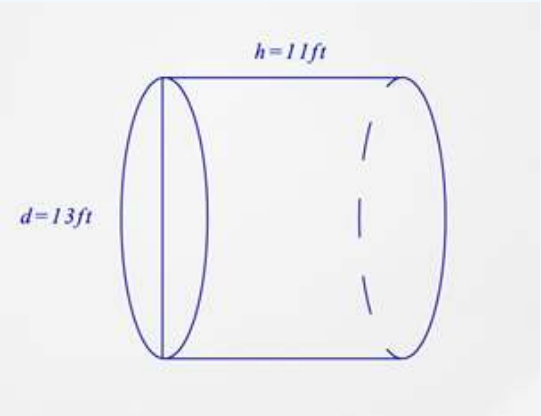

Figure 9.19.7

Solution

First, you need to find the radius. Remember that the radius is half the measure of the diameter.

$$r = \frac{d}{2}$$
$$r = \frac{13}{2}$$
$$r = 6.5$$

Next, substitute what you know into the surface area formula.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi(6.5)^2 + 2\pi(6.5)(11)$$

Then, calculate the surface area.

$$SA = 2\pi(6.5)^2 + 2\pi(6.5)(11)$$
$$SA = 2\pi(42.25) + 2\pi(71.5)$$
$$SA = 265.5 + 449.2$$
$$SA = 714.7$$

The answer is 714.7.

The surface area of the cylinder is \(714.7 \: ft^{2}.

Example 9.19.3

Find the surface area of a cylinder with a radius of 6 inches and a height of 5 inches.

Solution

First, substitute what you know into the surface area formula.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi(6)^2 + 2\pi(6)(5)$$

Next, calculate the surface area.

$$SA = 2\pi(6)^2 + 2\pi(6)(5) \; SA \quad = 2\pi(36) + 2\pi(30)$$
$$SA = 226.2 + 188.5$$
$$SA = 414.7$$

The answer is 414.7.

The surface area of the cylinder is $414.7 \; in^2$.

Example 9.19.4

Find the surface area of a cylinder with a radius of 4 cm and a height of 12 cm.

Solution

First, substitute what you know into the surface area formula.

Misplaced &

Next, calculate the surface area.

$$SA = 2\pi(4)^2 + 2\pi(4)(12)$$
$$SA = 2\pi(16) + 2\pi(48)$$
$$SA = 100.5 + 301.6$$
$$SA = 402.1$$

The answer is 402.1.

The surface area of the cylinder is $402.1 \; cm^2$.

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Example 9.19.5

Find the surface area of a cylinder with a diameter of 10 meters and a height of 15 meters.

Solution

First, you need to find the radius. Remember that the radius is half the measure of the diameter.

$$r = \frac{d}{2}$$
$$r = \frac{10}{2}$$
$$r = 5$$

Next, substitute what you know into the surface area formula.

$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi(5)^2 + 2\pi(5)(15)$$

Then, calculate the surface area.

$$SA = 2\pi(5)^2 + 2\pi(5)(15)$$
$$SA = 2\pi(25) + 2\pi(75)$$
$$SA = 157.1 + 471.2$$
$$SA = 628.3$$

The answer is 628.3.

The surface area of the cylinder is $628.3\ m^2$.

Review

Use the diagrams to answer the questions below each one.

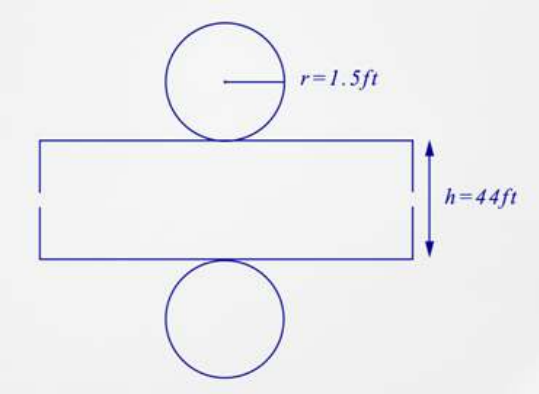

Figure 9.19.8

1. What is the name of this figure?
2. What is the shape of the base of this figure?
3. How many bases are there?
4. What is the surface area of this figure?
5. Which measurement is needed the radius or the diameter?

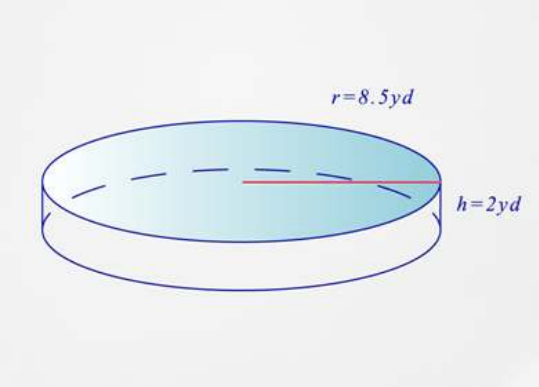

Figure **9.19.9**

6. What is the name of this figure?
7. Which measurement is given the radius or the diameter?
8. What is the surface area of the figure?

Use what you have learned to answer each question.

9. A cylindrical water tank is 35 long and 10 feet across. How much sheet metal is the tank made of?
10. Did you use area or surface area to solve this problem?
11. True or false. You can only find the surface area if you know the volume.
12. True or false. Surface area and volume measure the same thing.
13. True or false. Surface area measures the outside of a cylinder.
14. True or false. You need the radius to find the surface area of a cylinder.
15. True or false. The radius is one-half of the diameter.

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.7.

Vocabulary

Term	Definition
Cylinder	A cylinder is a solid figure with two parallel congruent circular bases.
Net	A net is a diagram that shows a “flattened” view of a solid. In a net, each face and base is shown with all of its dimensions. A net can also serve as a pattern to build a three-dimensional solid.
Prism	A prism is a three-dimensional object with two congruent parallel bases that are polygons.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Three – Dimensional	A figure drawn in three dimensions is drawn using length, width and height or depth.
Truncated Cylinder	A cylinder that is cut in part from a complete cylinder.

Additional Resources

Video: Cylinder Volume and Surface Area

Practice: Surface Area of Cylinders

Real World: Why Do Supertankers Float?

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9.20: Volume of Cylinders

Volume equals the area of top or bottom times height.

Figure 9.20.1

Ramona prepared a large pot of fruit preserves, but she ran out of canning jars before she had canned all of the preserves. She took two food storage containers out of the cupboard. The cylindrical containers have the dimensions below. There isn't enough of the extra preserve for both containers, so she only needs one of the containers and it needs to be the one with the greatest **volume**.

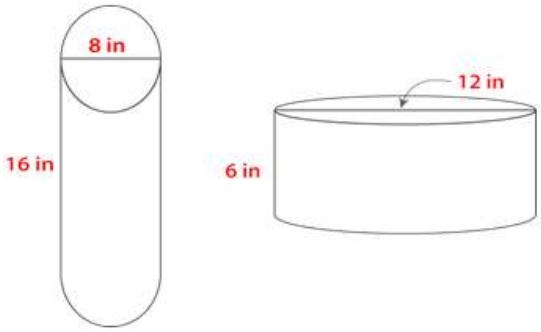

Figure 9.20.2

How can Ramona use these dimensions to figure out which container will hold the most?

In this concept, you will learn to use formulas to find the volume of cylinders.

Finding Volume of Cylinders

Volume is the amount of space contained within a solid figure. In the case of cylinders, you can think of volume as the capacity, or the amount of a liquid or substance, it can hold.

Unlike prisms, unit cubes do not fit nicely into a **cylinder**.

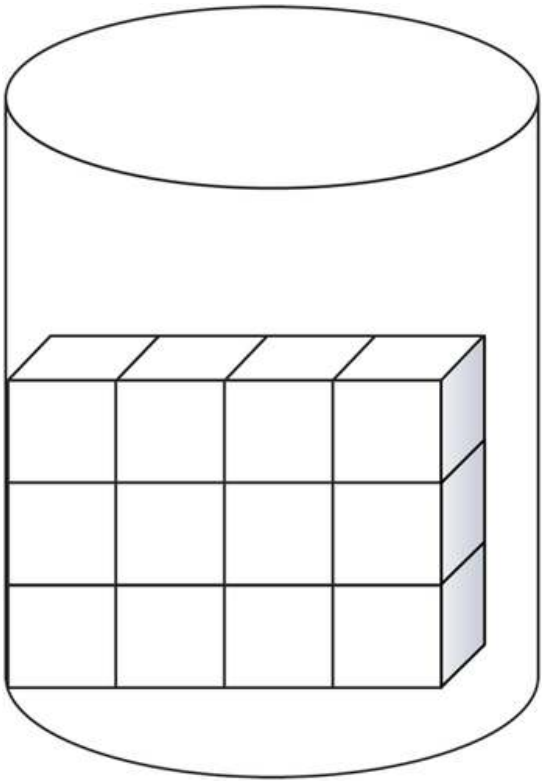

Figure **9.20.3**

So, to calculate the volume of a cylinder accurately, you need to use a formula instead of unit cubes.

To calculate the volume of a cylinder, start by calculating the area of the circular base. That will give you a measure for the number of unit cubes that can fit across the bottom of the cylinder. The height of the cylinder will show how high cubes can be stacked inside the cylinder.

Here is the formula for finding the volume of a cylinder.

Misplaced &

Let’s apply this formula.

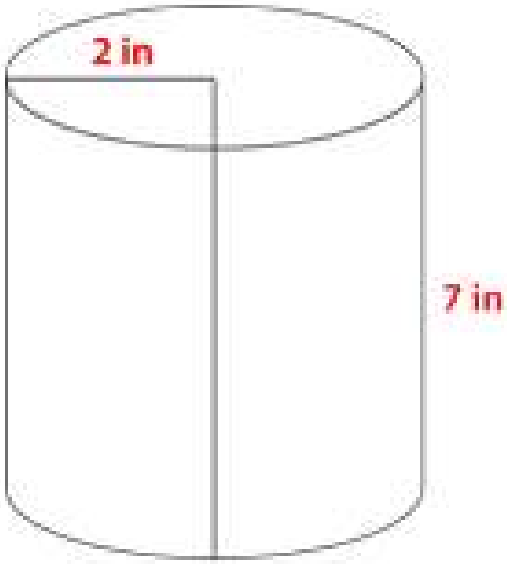

Figure **9.20.4**

The **radius** of the circular base is 2 inches. The height of the cylinder is 7 inches. If you take both of these given measures and substitute them into the formula, you can solve for the volume of the cylinder.

$$V = \pi r^2 h$$
$$V = (3.14)(2^2)(7)$$
$$V = (3.14)(4)(7)$$
$$V = (3.14)(28)$$
$$V = 87.92 \text{ in}^3$$

The volume of the cylinder is 87.92 in^3.

Example 9.20.1

Earlier, you were given a problem about Ramona and her extra fruit preserve.

Ramona has two food storage containers, but isn't sure which will hold the most. How can Raymond use these dimensions to figure out which will hold the most?

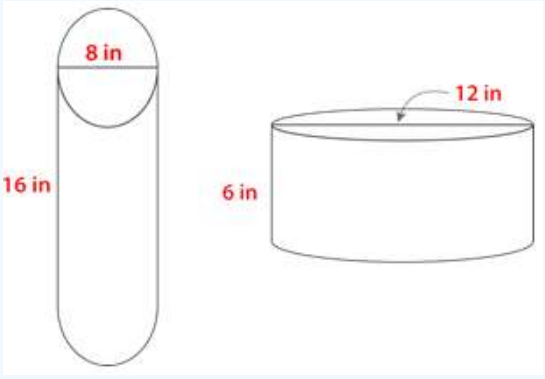

Figure 9.20.5

Solution

Start with the container on the left.

First, note that the diameter is twice the radius, so divide the diameter by 2 to get the radius.

8\divide 2=4

Radius is 4.

Next, substitute the given information into the formula then solve.

$$V = \pi r^2 h$$
$$V = \pi(4)^2(16)$$
$$V = \pi(16)(16)$$
$$V = 256\pi$$
$$V = 803.84 \text{ in}^3$$

The first container holds 803.84 cubic inches.

Next, consider the other container.

First, note that the diameter is twice the radius, so divide the diameter by 2 to get the radius.

12\divide 2=6

Radius is 6.

Next, substitute the given information into the formula then solve.

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$$V = \pi r^2 h$$
$$V = \pi(6)^2(6)$$
$$V = \pi(36)(6)$$
$$V = 216\pi$$
$$V = 678.24 \text{ in}^3$$

The second container holds 678.24 cubic inches.

The container on the left has the greatest volume.

Example 9.20.2

Use the formula for finding the volume of a cylinder to answer the following question.

A water tank has a radius of 50 feet and a height of 400 feet. How many cubic feet of water will the tank hold when it is full?

Solution

First, substitute the given information into the formula and solve.

$$V = \pi r^2 h$$
$$V = \pi(502)(400)$$
$$V = \pi(2,500)(400)$$
$$V = 1,000,000\pi$$
$$V = 3,140,000 \text{ in}^3$$

The water tank will hold more than 3 *million* cubic feet of water!

Example 9.20.3

Find the volume of the cylinder.

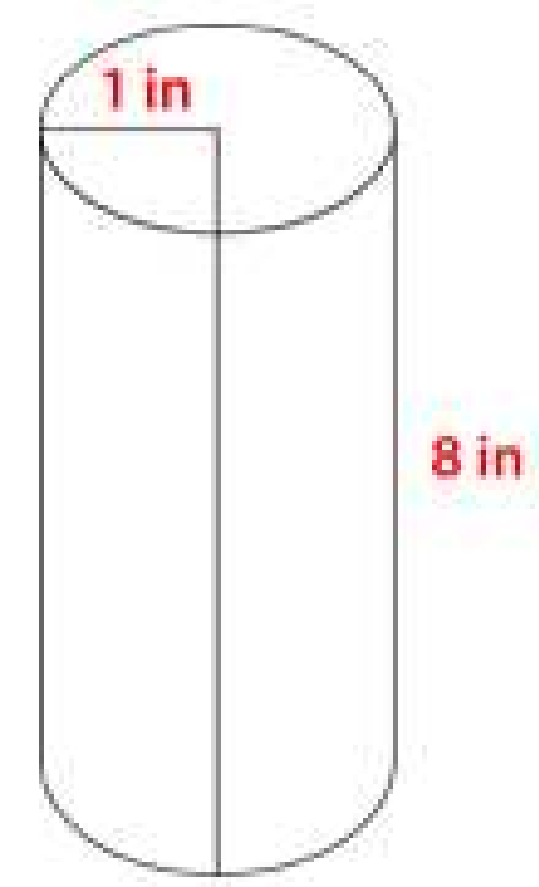

Figure 9.20.6

https://k12.libretexts.org/@go/page/6290

Solution

First, substitute the given information into the formula then solve.

$$V = \pi r^2 h$$
$$V = \pi(1^2)(8)$$
$$V = \pi(1)(8)$$
$$V = 8\pi$$
$$V = 25.12 \text{ in}^3$$

The answer is 25.12 cubic inches.

Example 9.20.4

Find the volume of the cylinder.

d=10 \text{ ft}\), $h = 12$ ft

Solution

First, note that the diameter is twice the radius, so divide the diameter by 2 to get the radius.

10\divide 2=5\)

Radius is 5.

Next, substitute the given information into the formula then solve.

$$V = \pi r^2 h$$
$$V = \pi(5)^2(12)$$
$$V = \pi(25)(12)$$
$$V = 300\pi$$
$$V = 942 \text{ ft}^3$$

The answer is 942 cubic feet.

Example 9.20.5

Find the volume of the cylinder.

$r = 6$ in, $h = 10$ in

Solution

First, substitute the given information into the formula then solve.

$$V = \pi r^2 h$$
$$V = \pi(6)^2(10)$$
$$V = \pi(36)(10)$$
$$V = 360\pi$$
$$V = 1130.4 \text{ in}^3$$

The answer is 1130.4 cubic inches.

Review

Find the volume of each of the following cylinders.

1. $r = 5$ in, $h = 8$ in
2. $r = 4$ in, $h = 7$ in
3. $r = 3$ ft, $h = 5$ ft
4. $r = 3$ ft, $h = 8$ ft
5. $r = 4$ cm, $h = 9$ cm

6.

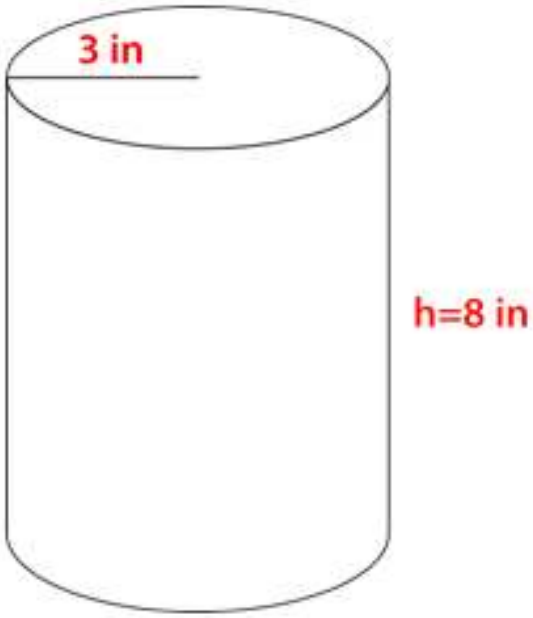

Figure **9.20.7**

7.

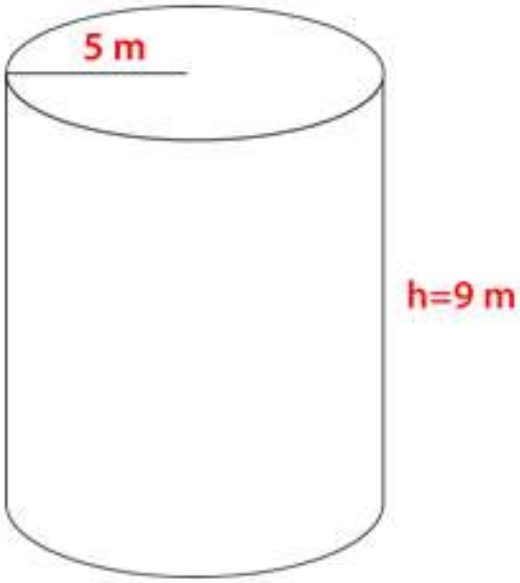

Figure **9.20.8**

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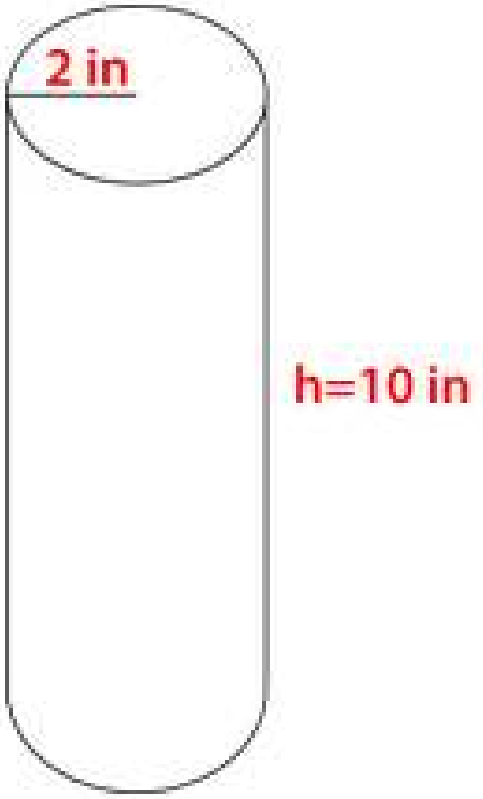

Figure **9.20.9**

9.

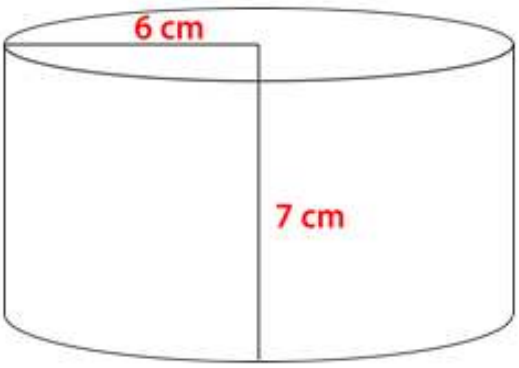

Figure **9.20.10**

10.

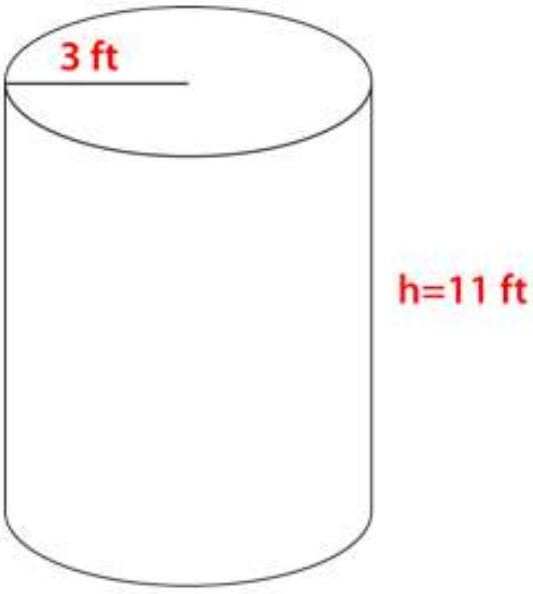

Figure 9.20.11

11.

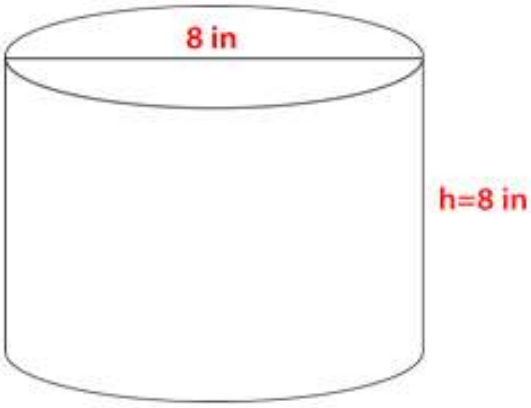

Figure 9.20.12

12.

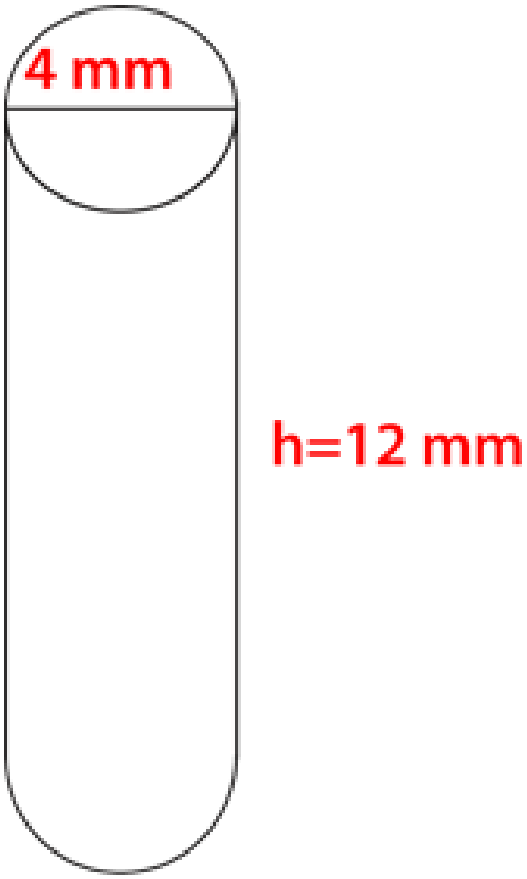

Figure **9.20.13**

13.

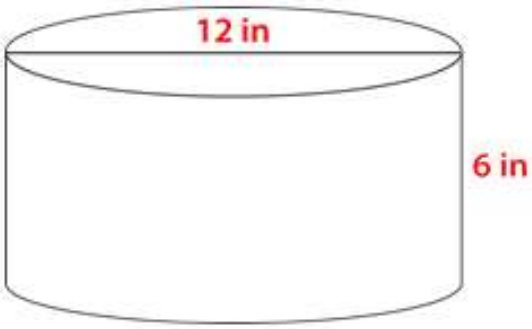

Figure **9.20.14**

14.

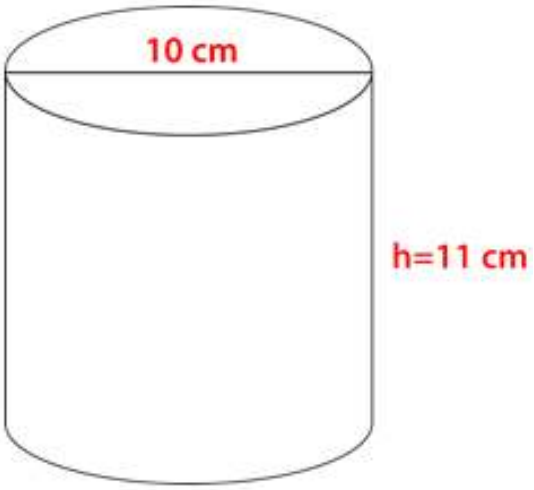

Figure 9.20.15

15.

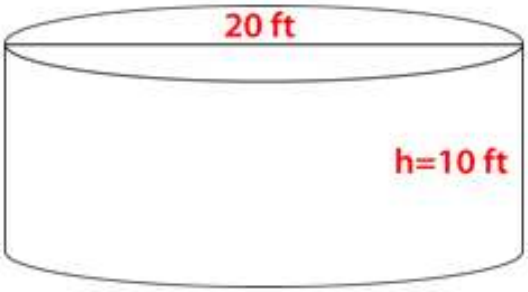

Figure 9.20.16

Vocabulary

Term	Definition
Cylinder	A cylinder is a solid figure with two parallel congruent circular bases.
Radius	The radius of a circle is the distance from the center of the circle to the edge of the circle.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Solid Geometry Volume

Practice: Volume of Cylinders

Real World: Why Do Supertankers Float?

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9.21: Heights of Cylinders Given Surface Area or Volume

Use formulas to find the height of a cylinder, given the volume or surface area.

Figure **9.21.1**

Gregory's family just bought a hot tub for their lake house. The hot tub company told his family that the tub holds 125 cubic feet of water. Gregory is interested to know how deep the hot tub is. He measures the diameter of the top and finds that the hot tub is 6 feet across. What is the height, or depth, of the hot tub?

In this concept, you will learn how to calculate the height of a cylinder when given the **volume** and radius or diameter.

Finding the Height of a Cylinder Given Volume

Sometimes you will know the volume and radius of a cylinder and you won't know the height of it. Think about a water tower that is cylindrical in shape. You might know how much volume the tank will hold and the radius of the tank, but not the height of it. When this happens, you can use the formula for the volume of a cylinder to find the missing height:

$$\text{Missing } \backslash\text{end}\{\text{aligned}\}$$

Let's look at an example.

A cylinder with a radius of 2 inches has a volume of 125.6 cubic inches. What is the height of the cylinder?

The volume and the radius are given, so substitute these into the formula and then solve for h, the height.

$$\begin{aligned} V &= \pi r^2 h \\ 125.6 &= (3.14)(22)h \\ 125.6 &= (3.14)(4)h \\ 125.6 &= 12.56h \\ 125.6 &\backslash \text{divide} 12.56 \\ 10 \text{ in} &= 12.56h \backslash \text{divide} 12.56 = h \end{aligned}$$

The height of the cylinder is 10 inches.

Check your work by substituting the answer in for the height. You should get a volume of 125.6 cubic inches.

$$V = \pi r^2 h$$
$$V = \pi(2)^2(10)$$
$$V = \pi(4)(10)$$
$$V = 40\pi$$
$$V = 125.6 \text{ in}^3$$

What is the height of a cylinder that has a radius of 6 cm and a volume of 904.32 cubic cm?

Again, you have been given the volume and the radius. Put this information into the formula along with the value of pi and solve for h, the height.

$$V = \pi r^2 h$$
$$904.32 = (3.14)(62)h$$
$$904.32 = (3.14)(36)h$$
$$904.32 = 113.04h$$
$$904.32 \div 113.04 = 113.04h \div 113.04$$
$$8 \text{ cm} = h$$

The height of this cylinder is 8 centimeters.

Example 9.21.1

Earlier, you were given a problem about Gregory and his family's hot tub.

To figure this out, use the formula for volume of a cylinder. He already knows the volume of the tub is 125 cubic feet and the diameter is 6 feet.

Solution

First, divide the diameter by 2 and plug the values for volume, pi, and radius into the formula for volume of a cylinder.

$$r = 6 \div 2$$
$$r = 3$$
$$V = \pi r^2 h$$
$$125 = (3.14)(3^2)h$$

Next, square the radius and multiply the values together.

$$125 = (3.14)(3^2)h$$
$$125 = (3.14)(9)h$$
$$125 = 28.26h$$

Last, divide both sides by 200.96 for the answer, remembering to include the appropriate unit of measurement.

$$125 = 28.26h$$
$$125 \div 28.26 = 28.26h \div 28.26$$
$$4.42 \text{ ft} = hv$$

The answer is Gregory's hot tub is 4.42 feet deep.

Example 9.21.2

Javier wants to construct a cylindrical container to hold enough water for his pet fish. He read that the fish needs to live in 2,110.08 cubic inches of water. If he constructs a tank with a diameter of 16 inches, how tall must he make it so that it holds the right amount of water?

Solution

First, divide the diameter by 2 and plug the values for volume, pi, and radius into the formula for volume of a cylinder.

$$r = 16 \div 2$$
$$r = 8$$
$$V = (3.14)(82)h$$
$$2,110.08 = \pi r^2 h$$

Next, square the radius and multiply the values together.

$$2,110.08 = (3.14)(8^2)h$$
$$2,110.08 = (3.14)(64)h$$
$$2,110.08 = 200.96h$$

Then, divide both sides by 200.96 for the answer, remembering to include the appropriate unit of measurement.

$$2,110.08 = 200.96h$$
$$2,110.08 \divide 200.96 = 200.96h \divide 200.96$$
$$10.5 \text{ in} = h$$

The answer is Javier must make his tank 10.5 inches tall for his tank to hold 2,110.08 cubic inches of water.

Example 9.21.3

Find the height of a cylinder with radius = 6 inches and volume = 904.32 cubic inches.

Solution

First, plug the values of the volume, pi, and radius into the formula for volume of a cylinder.

$$V = \pi r^2 h$$
$$904.32 = (3.14)(6^2)h$$

Next, square the radius and multiply the values together.

$$904.32 = (3.14)(62)h$$
$$904.32 = (3.14)(36)h$$
$$904.32 = 113.04h$$

Last, divide each side by 113.04 for the answer, remembering to include the appropriate unit of measurement.

$$904.32 = 113.04h$$
$$904.32 \divide 113.04 = 113.04h \divide 113.04$$
$$8 \text{ in} = h$$

The answer is the height of the cylinder is 8 inches.

Example 9.21.4

Find the height of a cylinder with radius = 3 meters and volume = 354.34 cubic meters.

Solution

First, plug the values of the volume, pi, and radius into the formula for volume of a cylinder.

$$V = \pi r^2 h$$
$$354.34 = (3.14)(3^2)h$$

Next, square the radius and multiply the values together.

$$354.34 = (3.14)(3^2)h$$
$$904.32 = (3.14)(9)h$$
$$354.34 = 28.26h$$

Last, divide each side by 28.26 for the answer, remembering to include the appropriate unit of measurement.

$$354.34v = 28.26h$$
$$354.34 \divide 28.26 = 28.26h \divide 28.26$$
$$9 \text{ m} = h$$

The answer is the height of the cylinder is 9 meters.

Example 9.21.5

Find the height of a cylinder with radius = 5 feet and volume = 785 cubic feet.

Solution

First, plug the values of the volume, pi, and radius into the formula for volume of a cylinder.

$$V = \pi r^2 h$$
$$785 = (3.14)(52)h$$

Next, square the radius and multiply the values together.

$$785 = (3.14)(52)h$$
$$785 = (3.14)(25)h$$
$$785 = 78.5h$$

Last, divide each side by 78.5 for the answer, remembering to include the appropriate unit of measurement.

$$785 = 78.5h$$
$$785 \backslash \text{divide} 78.5 = 78.5 \backslash \text{divide} 78.5$$
$$10 \text{ ft} = h$$

The answer is the height of the cylinder is 10 feet.

Review

Given the volume and the radius, find the height of each cylinder.

1. $r = 6$ in, Misplaced &
2. $r = 5$ in, Misplaced &
3. $r = 7$ ft, Misplaced &
4. $r = 8$ ft, Misplaced &
5. $r = 7$ ft, Misplaced &
6. $r = 12$ m, Misplaced &
7. $r = 9$ m, Misplaced &
8. $r = 10$ m, Misplaced &
9. $r = 12$ in, Misplaced &
10. $r = 11$ ft, Misplaced &
11. $r = 10$ in, Misplaced &
12. $r = 9.5$ in, Misplaced &
13. $r = 8$ m, Misplaced &
14. $r = 14$ ft, Misplaced &
15. $r = 13.5$ in, Misplaced &

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.13.

Resources

Vocabulary

Term	Definition
Cubic Units	Cubic units are three-dimensional units of measure, as in the volume of a solid figure.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Cylinder Volume and Surface Area

Practice: Heights of Cylinders Given Surface Area or Volume

Real World: We All Scream for Ice Cream

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9.22: Surface Area and Volume of Cones

Surface area equals the area of the circle plus the area of the outside of the cone

Cones

A **cone** is a solid with a circular base and sides that taper up towards a vertex. A cone is generated from rotating a right triangle, around one leg. A cone has a **slant height**.

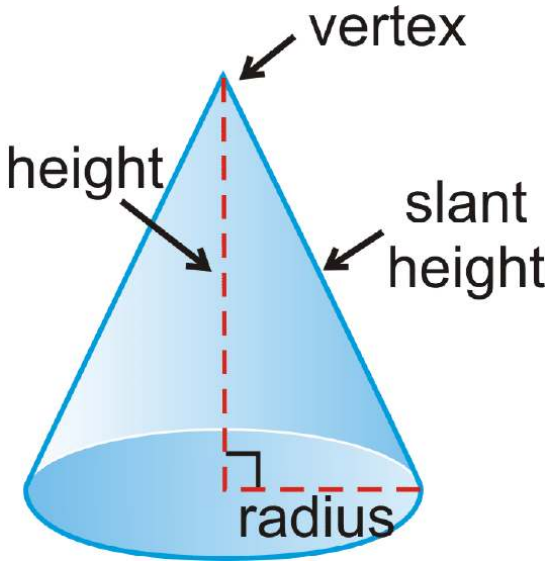


Figure **9.22.1**

Surface Area

Surface area is a two-dimensional measurement that is the total area of all surfaces that bound a solid. The basic unit of area is the square unit. For the surface area of a cone we need the sum of the area of the base and the area of the sides.

Surface Area of a Right Cone: $SA = \pi r^2 + \pi rl$.

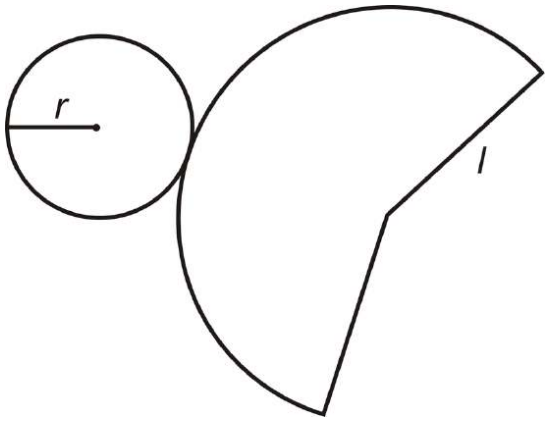


Figure **9.22.2**

Area of the base: πr^2

Area of the sides: πrl

Volume

To find the **volume** of any solid you must figure out how much space it occupies. The basic unit of volume is the cubic unit.

Volume of a Cone: $V = \frac{1}{3}\pi r^2 h$.

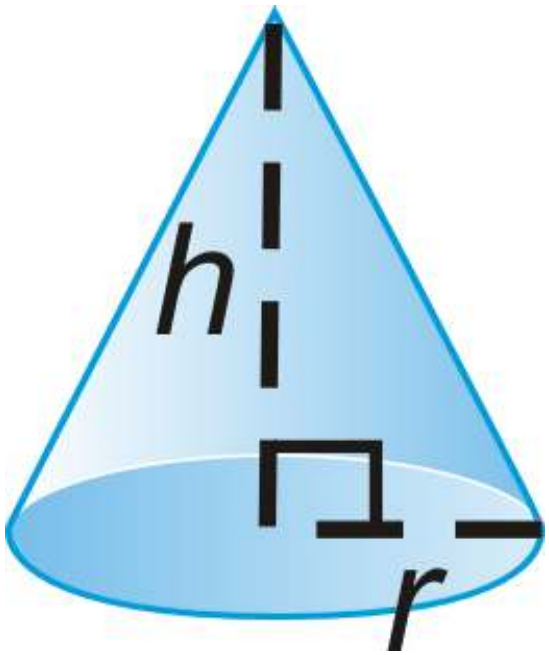


Figure **9.22.3**

What if you were given a three-dimensional solid figure with a circular base and sides that taper up towards a vertex? How could you determine how much two-dimensional and three-dimensional space that figure occupies?

Example 9.22.1

The surface area of a co 36π dius is 4 units. What is the slant height?

Solution

Plug what you know int e surface area of a cone and solve for l.

$$\begin{aligned} 36\pi &= \pi 4^2 + \pi 4l \\ 36 &= 16 + 4l \qquad \text{When each term has a } \pi\text{, they cancel out.} \\ 20 &= 4l \\ 5 &= l \end{aligned}$$

Example 9.22.2

The volume of a cone is $484\pi \text{ cm}^3$ and the height is 12 cm. What is the radius?

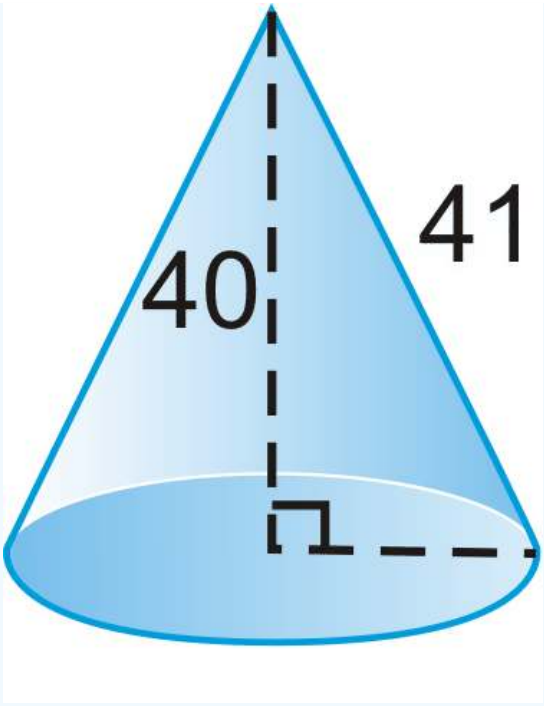


Figure 9.22.4

Solution

Plug what you know to the volume formula.

$$\begin{aligned} 484\pi &= \frac{1}{3}\pi r^2(12) \\ 121 &= r^2 \\ 11 \text{ cm} &= r \end{aligned}$$

Example 9.22.3

What is the surface area of the cone?

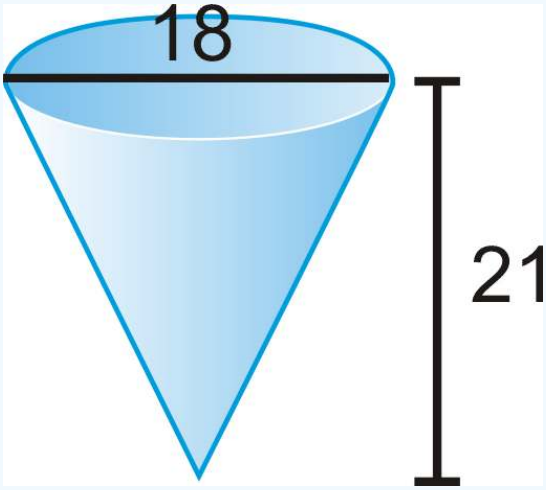


Figure 9.22.5

Solution

First, we need to find the slant height. Use the Pythagorean Theorem.

\(\begin{aligned} l^{2}&=9^{2}+21^{2} \ &=81+441 \ l&=\sqrt{522}\cong 22.85\end{aligned}]\)

The total surface area, then, is $SA = \pi 9^2 + \pi(9)(22.85) \cong 900.54 \text{ units}^2$.

Example 9.22.4

Find the volume of the cone.

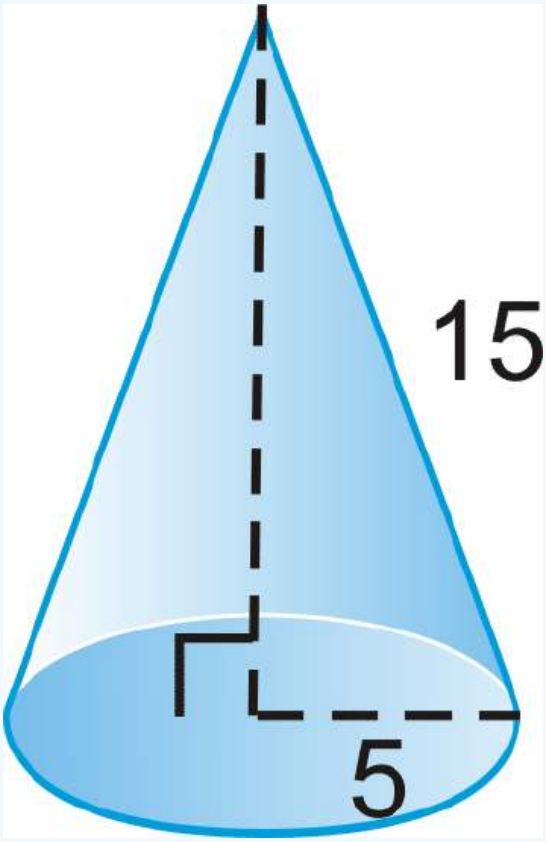


Figure 9.22.6

Solution

First, we need the height. Use the Pythagorean Theorem.

$$5^2 + h^2 = 15^2$$
$$h = \sqrt{200} = 10\sqrt{2}$$
$$V = 13(5^2)(10\sqrt{2})\pi \cong 370.24 \text{ units}^3$$

Example 9.22.5

Find the volume of the cone.

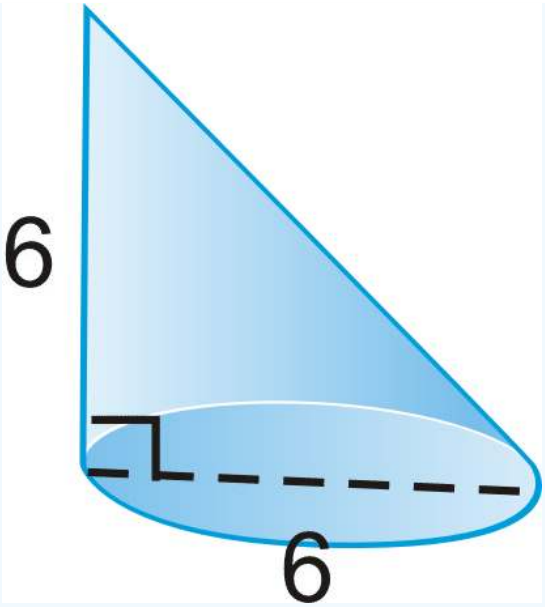


Figure 9.22.7

Solution

We can use the same volume formula. Find the *radius*.

$$V = \frac{1}{3}\pi(3^2)(6) = 18\pi \cong 56.55 \text{ units}^3$$

Review

Use the cone to fill in the blanks.

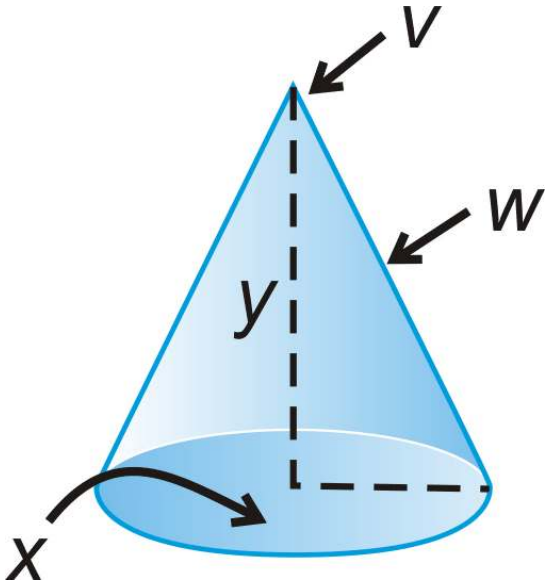


Figure 9.22.8

1. v is the _____.
2. The height of the cone is _____.
3. x is a _____ and it is the _____ of the cone.
4. w is the _____ _____.

Sketch the following solid and answer the question. Your drawing should be to scale, but not one-to-one. Leave your answer in simplest radical form.

5. Draw a right cone with a radius of 5 cm and a height of 15 cm. What is the slant height?

Find the slant height, l, of one lateral face in the cone. Round your answer to the nearest hundredth.

6.

Figure **9.22.9**

Find the surface area and volume of the right cones. Round your answers to 2 decimal places.

7.

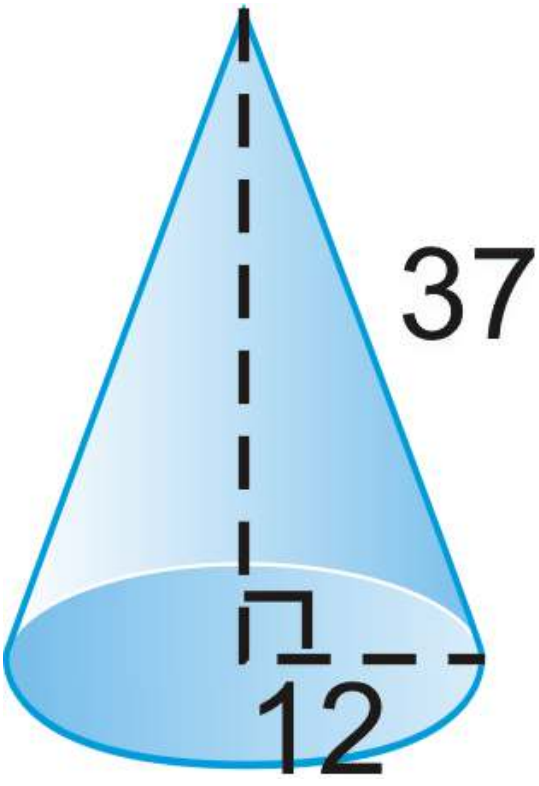


Figure **9.22.10**

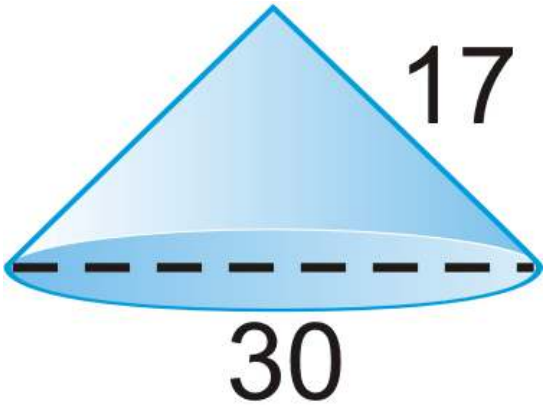
8. 

Figure **9.22.11**

9. If the lateral surface area of a cone is $30\pi \text{ cm}^2$ and the radius is 5 cm, what is the slant height?
10. If the surface area of a cone is $105\pi \text{ cm}^2$ and the slant height is 8 cm, what is the radius?
11. If the volume of a cone is$30\pi \text{ cm}^3$ and the radius is 5 cm, what is the height?
12. If the volume of a cone is $105\pi \text{ cm}^3$ and the height is 35 cm, what is the radius?

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.6.

Vocabulary

Term	Definition
cone	is a solid with a circular base and sides that taper up towards a vertex. A cone has a ***slant height***.
Slant Height	The slant height is the height of a lateral face of a pyramid.

Additional Resources

Video: Surface Area of Cone - Overview

Activities: Cones Discussion Questions

Study Aids: Pyramids and Cones Study Guide

Practice: Surface Area and Volume of Cones

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9.23: Surface Area of Cones

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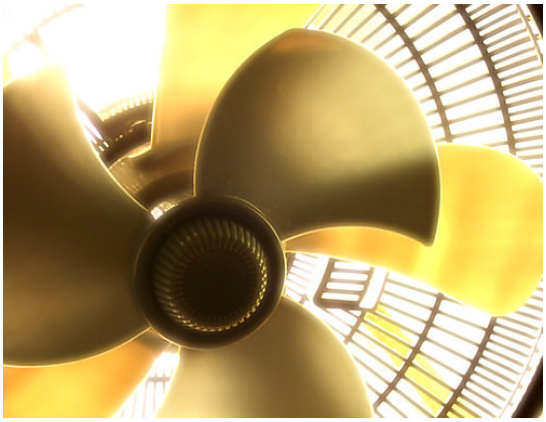

Figure **9.23.1**

Robbie is building a base for a cylindrical fan to put in his room. The truncated **cone** has a larger radius of 3 feet, a smaller radius of 2 feet, and a **slant height** of 3 ft. He is going to paint the stand and needs to find the **surface area** of the base to figure out how much paint to buy. How can Robbie figure out the surface area?

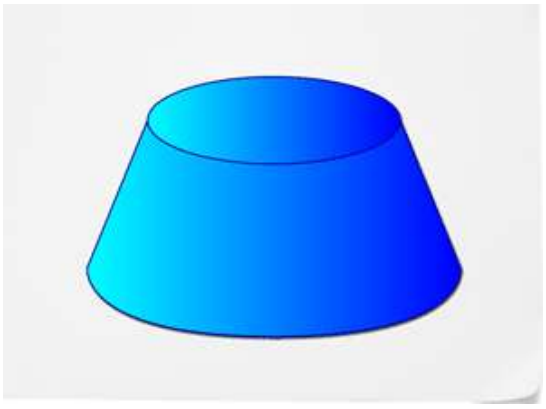

Figure **9.23.2**

In this concept, you will learn to find the surface area cones and truncated cones.

Surface Area

Cones are three-dimensional figures that have a circular base and a point at the top. You can calculate the surface area of a cone using a **net** or a formula. **Surface area** is the total of the areas of each face in a solid figure.

Here is what the net of a cone would look like.

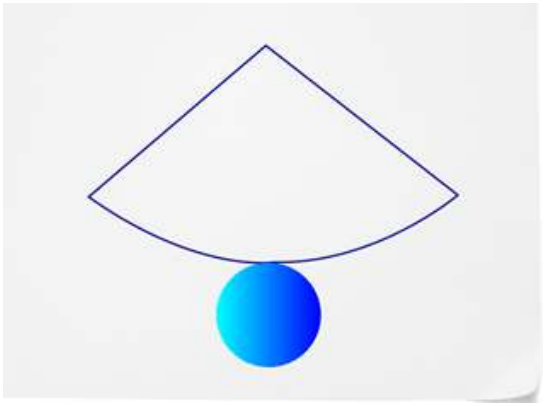

Figure **9.23.3**

The shaded circle is the base. Remember, cones always have circular bases. The un-shaded portion of the cone represents its side. Technically you don't call this a face because it has a round edge.

To find the surface area of a cone, you need to calculate the area of the circular base and the side and add them together. The formula for finding the area of a circle is Misplaced &, where r is the radius of the circle. You use this formula to find the area of the circular base.

To find the area of the cone's side, you multiply the radius (r), the slant height (s), and π.

$$A = \pi rs$$

Let's look at an example.

Find the surface area of the following cone.

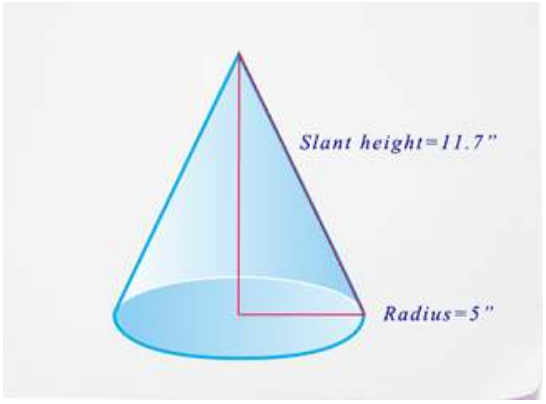

Figure \(\
PageIndex
{4}\)

First, calculate the area of the base.

$$A = \pi r^2$$
$$A = \pi(5)^2$$
$$A = 78.5$$

Next, calculate the area of the **sector**, also known as the **lateral surface area**.

$$A = \pi(5)(11.7)$$
$$A = 183.8$$

Then, add the area of the base to the lateral surface area.

$$A = \text{base} + \text{lateral Surface}$$
$$A = 78.5 + 183.8$$
$$A = 262.3$$

The answer is 262.3.

The surface area of the cone is $262.3\ \text{in}^2$.

A short cut would be to simply use the formula to find the surface area of a cone. Here is the formula for finding the surface area of a cone: Misplaced &.

The first part of the formula, πr^2, is simply the area formula for circles. This represents the base area. The second part, \pi rs, represents the lateral surface area of the cone's side. You can simply put the pieces together and solve for the area of both parts at once.

A **truncated cone** is one where the point of the cone is cut off leaving two circular bases and a side face.

Below is an image of what a truncated cone looks like.

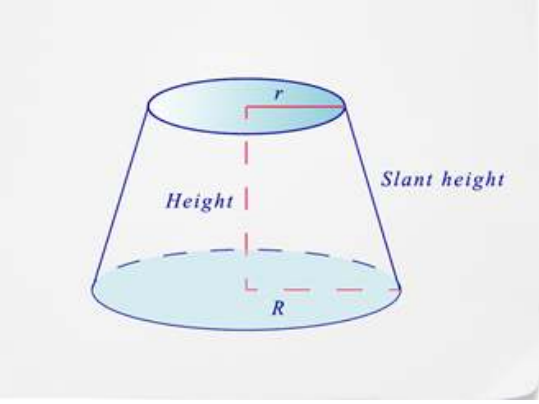

Figure **9.23.5**

Notice that with a truncated cone you will have two different circular bases - a top radius and a bottom one. You will have to find the area of both bases plus the area of the sector to find the surface area.

The formula for finding the surface area of a truncated cone is:

$SA = \pi[s(R + r) + r^2 + r^2]$

Notice that s stands for slant height, the capital R stands for the larger radius and the lowercase r stands for the smaller radius.

Let's look at an example.

What is the surface area of a truncated cone with a slant height of 6 cm, a radius of 8 cm and a radius of 6 cm?

First, fill in all you know into the surface area formula.

$$SA = \pi[s(R + r) + r^2 + r^2]$$
$$SA = \pi[6(8 + 6) + 8^2 + 6^2]$$

Next, use algebra to solve for the surface area.

$$SA = \pi[6(8 + 6) + 8^2 + 6^2]$$
$$SA = \pi[6(14) + 64 + 36]$$
$$SA = \pi[84 + 64 + 36]$$
$$SA = \pi[184]$$
$$SA = 578.1$$

The answer is 578.1.

The surface area of the cone is $578.1\ \text{cm}^2$.

Example 9.23.1

Earlier, you were given a problem about Robbie's fan stand.

Robbie is building a stand in the shape of a truncated cone. The bottom circular base has a radius of 3 ft. The circular top has a radius of 2 ft. The slant height for the stand is 3 ft. Robbie has to find the surface area so that he knows how much paint he

needs.

Solution

First, substitute what you know into the surface area formula.

$$SA = \pi[s(R + r) + r^2 + r^2]$$
$$SA = \pi[3(3 + 2) + 3^2 + 2^2]$$

Next, use algebra to solve for the surface area.

$$SA = \pi[3(3 + 2) + 3^2 + 2^2]$$
$$SA = \pi[3(5) + 9 + 4]$$
$$SA = \pi[15 + 9 + 4]$$
$$SA = \pi[28]$$
$$SA = 87.96$$

The answer is 87.96.

Robbie needs enough paint to cover $88\ \text{ft}^2$.

Example 9.23.2

Trey is decorating conical party hats for his party by wrapping them in colored tissue paper. Each hat has a radius of 4.2 centimeters and a slant height of 8.6 centimeters. If he wants to wrap six party hats, how much paper will he need?

Solution

First, substitute what you know into the surface area formula.

$$SA = \pi r^2 + \pi rs$$
$$SA = \pi(4.2)2 + \pi(4.2)(8.6)$$

Next, use algebra to solve for the surface area.

$$SA = \pi(4.2)^2 + \pi(4.2)(8.6)$$
$$SA = \pi(17.64) + \pi(36.12)$$
$$SA = 55.4 + 113.5$$
$$SA = 168.9$$

Then, find the total area of the 6 hats.

Misplaced &

The answer is 1013.4.

Trey would need 1013.4 cm^2 of paper to make the six hats.

Example 9.23.3

Find the surface area of a cone with a radius of 4 inches and a slant height of 6 inches.

Solution

First, substitute what you know into the surface area formula.

$$SA = \pi r^2 + \pi rs$$
$$SA = \pi(4)2 + \pi(4)(6)$$

Next, use algebra to solve for the surface area.

$$SA = \pi(4)^2 + \pi(4)(6)$$
$$SA = \pi(16) + \pi(24)$$
$$SA = 50.3 + 75.4$$
$$SA = 125.7$$

The answer is 125.7.

The surface area of the cone is $125.7\ \text{in}^2$.

Example 9.23.4

Find the surface area of a cone with a radius of 5 feet and a slant height of 8 feet.

Solution

First, substitute what you know into the surface area formula.

$$SA = \pi r^2 + \pi rs$$
$$SA = \pi(5)2 + \pi(5)(8)$$

Next, use algebra to solve for the surface area.

\(\begin{aligned}SA&=\pi (5)^{2}+\pi (5)(8) \\ SA&=\pi (25)+\pi (40) \\ SA&=78.5+125.7 \\ SA&=204.2\end{aligned}\)

The answer is 204.2.

The surface area of the cone is $204.2\ \text{ft}^2$.

Example 9.23.5

Find the surface area of a cone with a radius of 3 inches and a slant height of 4.5 inches.

Solution

First, substitute what you know into the surface area formula.

$$SA = \pi r^2 + \pi rs$$
$$SA = \pi(3)^2 + \pi(3)(4.5)$$

Next, use algebra to solve for the surface area.

$$SA = \pi(3)^2 + \pi(3)(4.5)$$
$$SA = \pi(9) + \pi(13.5)$$
$$SA = 28.3 + 42.4$$
$$SA = 70.7$$

The answer is 70.7.

The surface area of the cone is $70.7\ \text{in}^2$.

Review

Answer the questions below each of the following diagrams

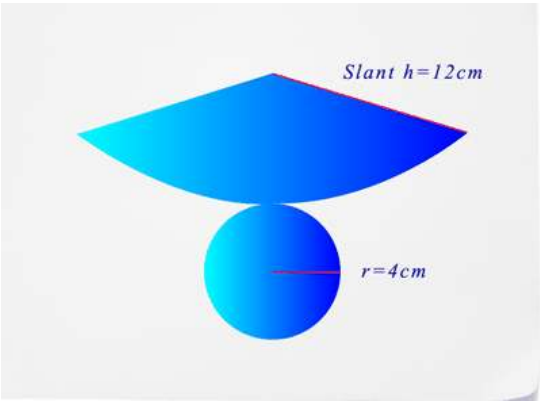

Figure 9.23.6

1. What is the name of the figure represented in this net?
2. What is the diameter of this figure?
3. What is the slant height of the figure?
4. What is the surface area of the figure?

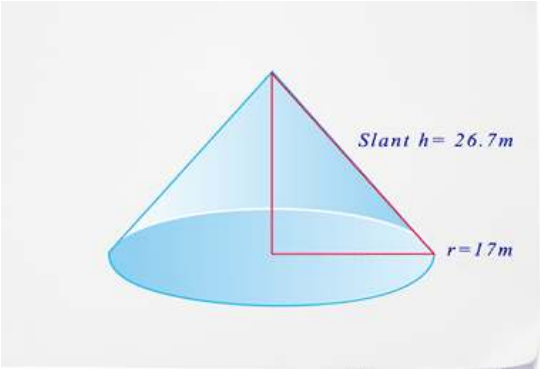

Figure **9.23.7**

5. What is the name of this figure?
6. What is the shape of the base?
7. What is the diameter of the base?
8. What is the surface area of this figure?

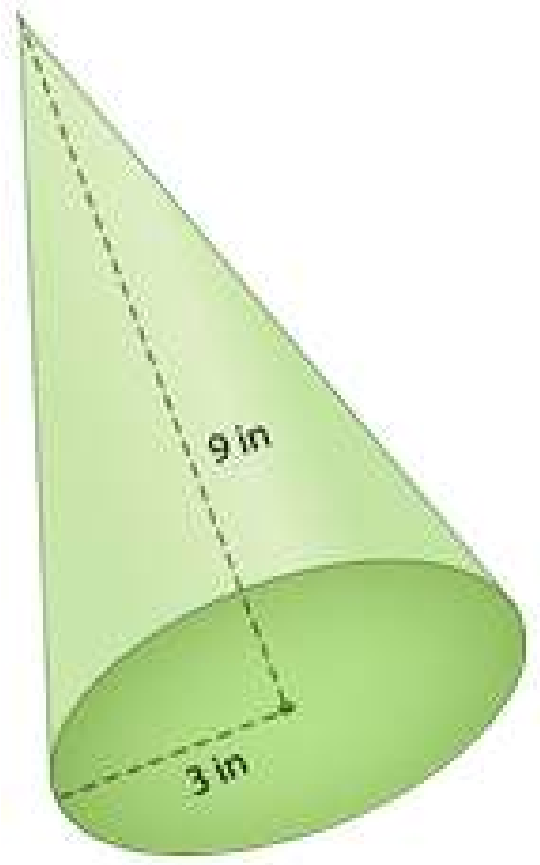

Figure **9.23.8**

9. What is the name of this figure?
10. What is the shape of the base?
11. What is the diameter of the base?
12. What is the surface area of this figure?

Find the surface area of each cone.

13. $r = 4$ in, $sh = 5$ in
14. $r = 5$ m, $sh = 7$ m
15. $r = 3$ cm, $sh = 6$ cm

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.9.

Resources

Vocabulary

Term	Definition
Cone	A cone is a solid three-dimensional figure with a circular base and one vertex.
Net	A net is a diagram that shows a “flattened” view of a solid. In a net, each face and base is shown with all of its dimensions. A net can also serve as a pattern to build a three-dimensional solid.
Sector	A sector of a circle is a portion of a circle contained between two radii of the circle. Sectors can be measured in degrees.
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.
Slant Height	The slant height is the height of a lateral face of a pyramid.

Additional Resources

Video: Surface Area of Cone - Overview

Practice: Surface Area of Cones

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9.24: Volume of Cones

Volume equals the area of the base times $\frac{1}{3}$ of the height.

Figure 9.24.1

Finding the Volume of a Cone

To figure out the volume of a cone, let's first look at how you can compare it with another solid figure. The closest solid figure to a cone is a cylinder. Both cones and cylinders have circular bases. The difference between the two is while a cylinder has two circular bases, a cone only has one.

Here is a cylinder and the formula for finding the volume of a cylinder.

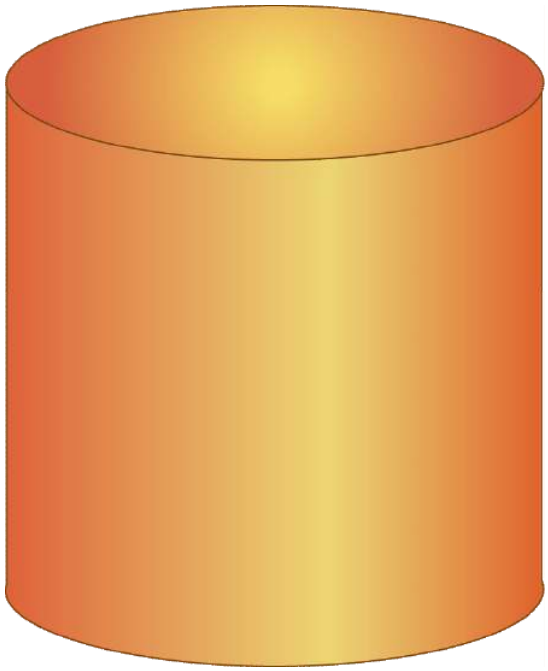

Figure **9.24.2**

Misplaced &

In the picture below, you can see how a cylinder and a cone are related:

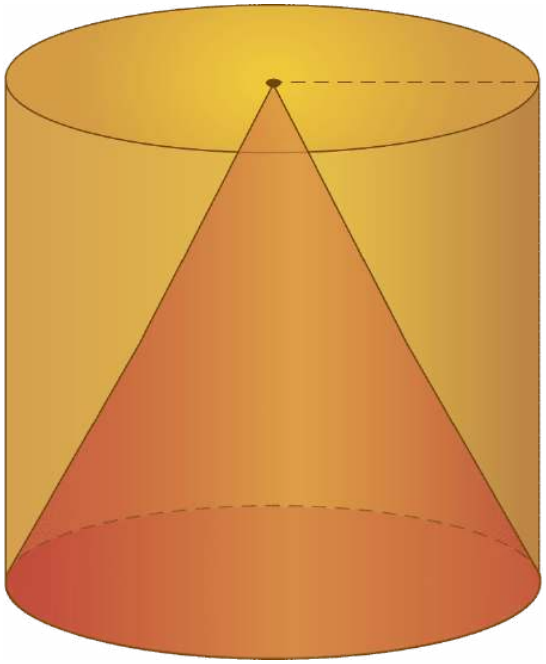

Figure **9.24.3**

To find the volume of the cone, you use a formula similar to that of a **pyramid**, except that you are going to take into consideration that the base of a cone is a circle. Therefore, you will need to find the area of the circle to find the volume of the cone. The formula for the area of a circle is πr^2. Take that measurement and multiply it by the height of the cone. Next, because the cone fills a portion of the cylinder, multiply the product by one-third.

Misplaced &

Let's look at an example. What is the volume of the cone below?

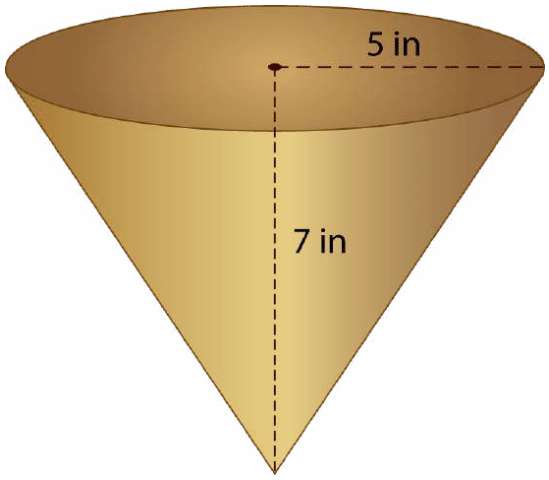

Figure 9.24.4

First, substitute the given values into the formula.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}(3.14)(52)(7)$$
$$V = \frac{1}{3}(3.14)(25)(7)$$
$$V = \frac{1}{3}(3.14)(175)$$
$$V = \frac{549.5}{3}$$
$$V = 183.16 \text{ in}^3$$

The answer is the volume of the cone is 183.16 cubic inches.

Let's look at another example. Find the volume of the following cone.

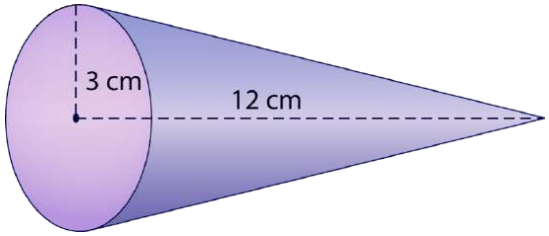

Figure 9.24.5

First, substitute the given values into the formula.

$$V = \frac{1}{3}(3.14)(3^2)(12)$$
$$V = \frac{1}{3}(3.14)(9)(12)$$
$$V = \frac{1}{3}(3.14)(108)$$
$$V = \frac{339.12}{3}$$
$$V = 113.04cm^3$$

The volume of the cone is 113.04 cm^3.

Example 9.24.1

Earlier, you were given a problem about Francine's new dessert cups.

She needs to figure out the volume of ice cream that will fit into one dessert cup. The diameter of the cup is 3 inches and the height is 6 inches.

Solution

First, find the value of the radius by dividing the diameter by 2, and substitute the values for pi, the radius, and the height of the cone into the formula for volume of a cone.

$$r = 1.5$$
$$r = 3\backslash\text{divide}2$$
$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}(3.14)(1.52)(6)$$

Next, square the radius and multiply the values together.

$$V = \frac{1}{3}(3.14)(1.52)(6)$$
$$V = \frac{1}{3}(3.14)(2.25)(6)$$
$$V = \frac{1}{3}(3.14)(13.5)$$
$$V = 42.393$$

Then, divide by 3 and record the answer, remembering to include the appropriate unit of measurement.

$$V = 42.393$$
$$V = 14.13 \text{ in}^3$$

The answer is the volume of ice cream that will fill Francine's cone-shaped desert cup is 16.75 cubic inches.

Example 9.24.2

Find the volume of a cone with a radius of 2 inches and a height of 4 inches.

Solution

First, substitute the values for pi, the radius, and the height of the cone into the formula for volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}(3.14)(2^2)(4)$$

Next, square the radius and multiply the values together.

$$V = \frac{1}{3}(3.14)(2^2)(4)$$
$$V = \frac{1}{3}(3.14)(4)(4)$$
$$V = \frac{1}{3}(3.14)(16)$$
$$V = 50.243$$

Then, divide by 3 and record the answer, remembering to include the appropriate unit of measurement.

$$V = \frac{50.24}{3}$$
$$V = 16.75 \text{ in}^3$$

The answer is the volume of the cone is 16.75 cubic inches.

Example 9.24.3

Find the volume of a cone with a radius of 5 ft and a height of 8 ft.

Solution

First, substitute the values for pi, the radius, and the height of the cone into the formula for volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}(3.14)(52)(8)$$

Next, square the radius and multiply the values together.

$$V = \frac{1}{3}(3.14)(52)(8)$$
$$V = \frac{1}{3}(3.14)(25)(8)$$
$$V = \frac{1}{3}(3.14)(200)$$
$$V = \frac{628}{3}$$

Then, divide by 3 and record the answer, remembering to include the appropriate unit of measurement.

$$V = \frac{628}{3}$$
$$V = 209.33 \text{ in}^3$$

The answer is the volume of the cone is 209.33 cubic inches.

Example 9.24.4

A cone with a radius of 4 m and a height of 9 m.

Solution

First, substitute the values for pi, the radius, and the height of the cone into the formula for volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}(3.14)(4^2)(9)$$

Next, square the radius and multiply the values together.

$$V = \frac{1}{3}(3.14)(4^2)(9)$$
$$V = \frac{1}{3}(3.14)(16)(9)$$
$$V = \frac{1}{3}(3.14)(144)$$
$$V = \frac{452.16}{3}$$

Then, divide by 3 and record the answer, remembering to include the appropriate unit of measurement.

$$V = \frac{452.16}{3}$$
$$V = 150.72 \text{ m}^3$$

The answer is the volume of the cone is 150.72 cubic meters.

Review

Find the volume of each cone.

1. A cone with a radius of 3 inches and a height of 7 inches.
2. A cone with a radius of 5 ft and a height of 9 ft.
3. A cone with a radius of 6 meters and a height of 10 meters.
4. A cone with a radius of 10 inches and a height of 12 inches.
5. A cone with a radius of 12 mm and a height of 14 mm.
6. A cone with a radius of 5 ft and a height of 12 feet.
7. A cone with a radius of 4.5 inches and a height of 7 inches.
8. A cone with a radius of 3.5 inches and a height of 5.5 inches.
9. A cone with a radius of 7 cm and a height of 13 cm.
10. A cone with a radius of 8 cm and a height of 11 cm.
11. A cone with a radius of 7.5 inches and a height of 11 inches.

12. A cone with a radius of 11.5 inches and a height of 15 inches.
13. A cone with a radius of 12.5 feet and a height of 15 feet.

Solve each word problem.

14. A cone has a radius of 6 meters and a height of 14 meters. What is its volume?
15. The containers of icing for Tina's cake decorator are cones. Each container has a radius of 2.4 inches and a height of 7 inches. If Tina buys containers of red, yellow, and blue icing, how much icing will she buy?

Review (Answers)

To see the Review answers, open this PDF file and look for section 10.15.

Vocabulary

Term	Definition
Cone	A cone is a solid three-dimensional figure with a circular base and one vertex.
Pyramid	A pyramid is a three-dimensional object with a base that is a polygon and triangular faces that meet at one vertex.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Determine the Volume of a Cone

Practice: Volume of Cones

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9.25: Surface Area and Volume of Spheres

Derive and use the formula: volume equals $\frac{4}{3}$ times pi times the radius cubed.

Spheres

Consider a solid figure consisting of the set of all points in three-dimensional space that are equidistant from a single point. How could you determine how much two-dimensional and three-dimensional space that figure occupies?

A **sphere** is the set of all points in three-dimensional space that are equidistant from a single point. The **radius** of a sphere has one endpoint on the sphere surface and the other endpoint at the center of that sphere. The **diameter** of a sphere must contain the center.

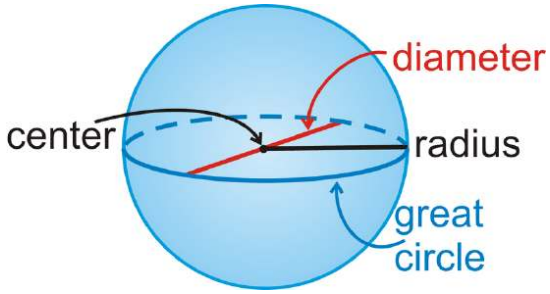

Figure **9.25.1**

A great circle is the largest circular cross-section in a sphere. **The circumference of a sphere is the circumference of a great circle.** Every great circle divides a sphere into two congruent **hemispheres.**

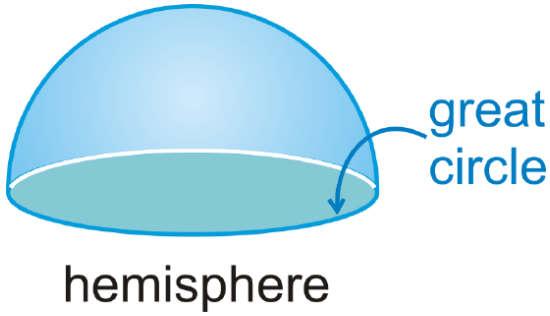

Figure **9.25.2**

Surface Area

Surface area is a two-dimensional measurement that is the total area of all surfaces that bound a solid. The basic unit of area is the square unit.

Surface Area of a Sphere: $SA = 4\pi r^2$

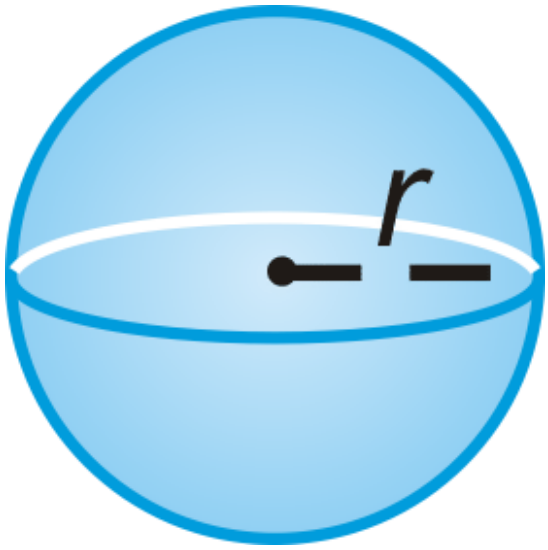

Figure 9.25.3

Volume

To find the **volume** of any solid you must figure out how much space it occupies. The basic unit of volume is the cubic unit.

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

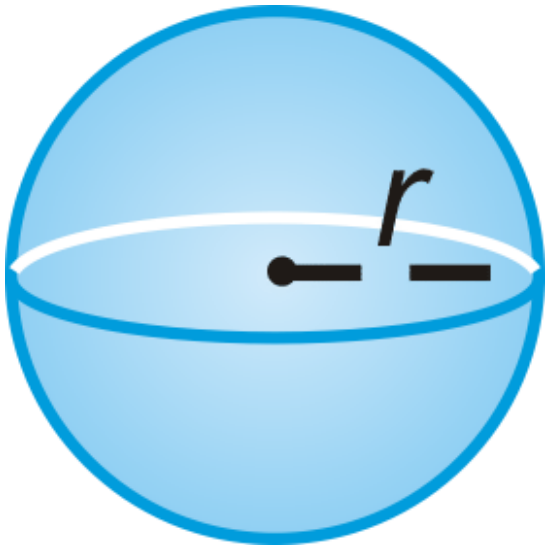

Figure 9.25.4

Example 9.25.1

Find the surface area of hemisphere with a circular base.

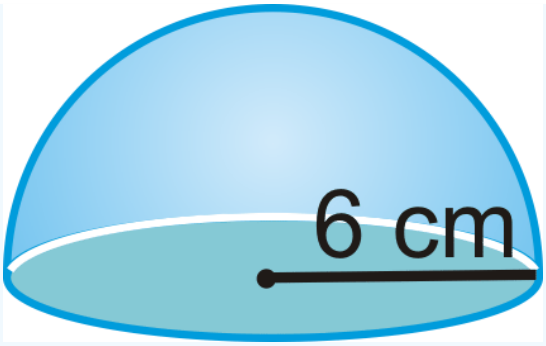

Figure 9.25.5

Solution

Use the formula for surface area:

$$\begin{aligned} SA &= \pi r^2 + 124\pi r^2 \\ &= \pi(6^2) + 2\pi(6^2) \\ &= 36\pi + 72\pi = 108\pi \text{ cm}^2 \end{aligned}$$

Example 9.25.2

A sphere has a volume of 14,137.167 ft3. What is the radius?

Solution

Use the formula for volume, plug in the given volume and solve for the radius, r:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 14,137.167 &= \frac{4}{3}\pi r^3 \\ \frac{3}{4\pi} \cdot 14,137.167 &= r^3 \\ 3375 &\approx r^3 \end{aligned}$$

At this point, you will need to take the cubed root of 3375. Your calculator might have a button that looks like \sqrt[3]{ }, or you can use $3375^{\frac{1}{3}}$.

$$\sqrt[3]{3375} = 15 \approx r$$

Example 9.25.3

The circumference of a sphere is 26\pi feet. What is the radius of the sphere?

Solution

The circumference is referring to the circumference of a great circle.

Use $C = 2\pi r$:

$$\begin{aligned} 2\pi r &= 26\pi \\ r &= 13 \text{ ft} \end{aligned}$$

Example 9.25.4

Find the surface area of a sphere with a radius of 14 feet.

Solution

Use the formula for surface area:

$SA = 4\pi(14)^2$

$= 784\pi \text{ ft}^2$\)

Example 9.25.5

Find the volume of a sphere with a radius of 6 m.

Solution

Use the formula for volume:

$$\begin{aligned} V &= \frac{4}{3}\pi 63 \\ &= \frac{4}{3}\pi(216) \\ &= 288\pi \text{ m}^3 \end{aligned}$$

Review

1. Are there any cross-sections of a sphere that are not a circle? Explain your answer.
2. List all the parts of a sphere that are the ***same*** as a circle.
3. List any parts of a sphere that a circle does not have.

For 4 - 11, find the surface area and volume of a sphere with the given dimension. Leave your answer in terms of π.

4. a radius of 8 in.
5. a diameter of 18 cm.
6. a radius of 20 ft.
7. a diameter of 4 m.
8. a radius of 15 ft.
9. a diameter of 32 in.
10. a circumference of 26π cm.
11. a circumference of 50π yds.
12. The surface area of a sphere is $121\pi \text{ in}^2$. What is the radius?
13. The volume of a sphere is $47916\pi \text{ m}^3$. What is the radius?
14. The surface area of a sphere is $4\pi \text{ ft}^2$. What is the volume?
15. The volume of a sphere is $36\pi \text{ mi}^3$. What is the surface area?
16. Find the radius of the sphere that has a volume of 335 cm^3. Round your answer to the nearest hundredth.
17. Find the radius of the sphere that has a surface area $225\pi \text{ ft}^2$.

Find the surface area and volume of the following shape. Leave your answers in terms of \pi .

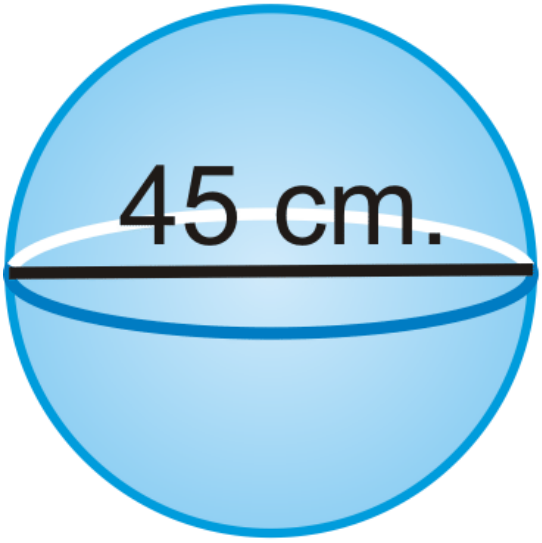

Figure 9.25.6

Review (Answers)

To see the Review answers, open this PDF file and look for section 11.7.

Vocabulary

Term	Definition
diameter	A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.
Sphere	A sphere is a round, three-dimensional solid. All points on the surface of a sphere are equidistant from the center of the sphere.
Radius	The radius of a circle is the distance from the center of the circle to the edge of the circle.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.
Cavalieri's Principle	States that if two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.

Additional Resources

Video: Spheres Principles - Basic

Activities: Spheres Discussion Questions

Study Aids: Spheres Study Guide

Practice: Surface Area and Volume of Spheres

Real World: Where We Live!

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9.26: Surface Area of Spheres

Surface area from radius

Surface Area of Spheres

Figure **9.26.1**

Erin is decorating a sphere shaped piñata with pink construction paper. Each sphere has a radius of 2.4 feet and Erin made four piñatas. How much construction paper will she need to cover each piñata completely? How much will she need in total?

In this concept, you will learn to calculate the **surface area** of spheres.

Surface Area

A **sphere** is a solid figure that exists in three-dimensional space. Spheres consist of all the points that are equidistant from a center point. Every point on the sphere is the distance of the radius from the center. Spheres are perfectly round.

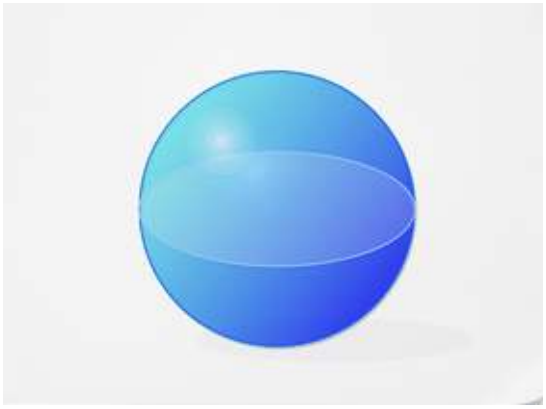

Figure **9.26.2**

Spheres do not have any faces because they are round. Still, you can think of its surface as a flat plane that you can unroll. Imagine you could wrap a sphere in wrapping paper, like a present. The amount of wrapping paper needed to cover the figure exactly represents its surface area.

If you could “unroll” the sphere and show it as a rectangle, the rectangle would have a width that is equivalent to the diameter of the sphere. Its length would be the same as the circumference of the sphere (circumference is the distance around a circle). Now this gives you something you can work with, because you can use the area formula for rectangles to find the area of the “unrolled” sphere.

The formula for the area of rectangles is: $A = lw$.

The length is the circumference, so the formula becomes:

$$l = C$$
$$C = 2\pi r$$
$$\therefore A = 2\pi r \times w$$

Now, the width is equal to the diameter of the “roll”. The diameter is twice the radius. Now your formula becomes:

$$w = d$$
$$d = 2r$$
$$\therefore A = 2\pi r \times 2r$$
$$A = 4\pi r^2$$

So, the formula you use for finding the surface area of a sphere is $SA = 4\pi r^2$.

All you need to do is substitute the measure of the radius for r in the formula and solve for SA, the surface area.

Let’s look at an example.

What is the surface area of the sphere below?

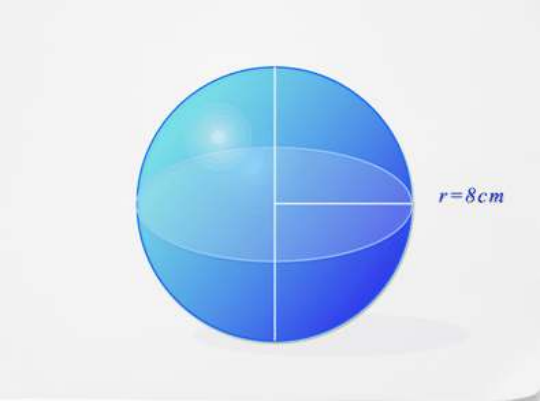

Figure **9.26.3**

First, substitute what you know into the surface area formula.

$$SA = 4\pi r^2$$
$$SA = 4\pi(8)^2$$

Next, use algebra to calculate the surface area.

$$SA = 4\pi(8)^2$$
$$SA = 4\pi(64)$$
$$SA = 256\pi$$
$$SA = 804.25$$

The answer is 804.25.

The surface area of the sphere is $804.25\ \text{cm}^2$.

If you wanted to make your measurement more accurate, you could say that the surface area is $256\pi\ \text{cm}^2$.

Example 9.26.1

Earlier, you were given a problem about Erin and her piñatas.

Solution

Erin wants to know how much pink construction paper she needs to decorate a sphere shaped piñata with a radius of 2.4 feet. She also wants to know the total amount to decorate four piñatas.

First, substitute what you know into the surface area formula.

$$SA = 4\pi r^2$$
$$SA = 4\pi(2.4)^2$$

Next, use algebra to calculate the surface area.

$$SA = 4\pi(2.4)^2$$
$$SA = 4\pi(5.76)$$
$$SA = 3.04\pi$$
$$SA = 72.38$$

Then multiply by 4 to find the total amount of construction paper.

$$SA_{Total} = 4 \times 72.38$$
$$SA_{Total} = 289.52$$

The answer is 289.52.

Erin would need $72.38\ \text{ft}^2$ of pink construction paper for one piñata and $289.52\ \text{ft}^2$ for all four.

Example 9.26.2

The sphere below has a radius of 8.5 inches. What is the surface area of the sphere?

Figure 9.26.4

Solution

First, substitute what you know into the surface area formula.

$$SA = 4\pi r^2$$
$$SA = 4\pi(8.5)^2$$

Next, use algebra to calculate the surface area.

$$SA = 4\pi(8.5)^2$$
$$SA = 4\pi(72.25)$$
$$SA = 89\pi$$
$$SA = 907.9$$

The answer is 907.9.

The surface area of the sphere is $907.9\ \text{in}^2$.

If you wanted to make your measurement more accurate, you could say that the surface area is $289\pi\ \text{in}^2$.

Example 9.26.3

Find the surface area of a sphere with a radius of 5 inches.

Solution

First, substitute what you know into the surface area formula.

$$SA = 4\pi r^2$$
$$SA = 4\pi(5)^2$$

Next, use algebra to calculate the surface area.

$$SA = 4\pi(5)^2$$
$$SA = 4\pi(25)$$
$$SA = 100\pi$$
$$SA = 314.16$$

The answer is 314.16.

The surface area of the sphere is $314.16\ \text{in}^2$ or $100\pi\ \text{in}^2$.

Example 9.26.4

Find the surface area of a sphere with a radius of 8 meters.

Solution

First, substitute what you know into the surface area formula.

$$SA = 4\pi r^2$$
$$SA = 4\pi(8)^2$$

Next, use algebra to calculate the surface area.

$$SA = 4\pi(8)^2$$
$$SA = 4\pi(64)$$
$$SA = 256\pi$$
$$SA = 804.25$$

The answer is 804.25.

The surface area of the sphere is $804.25\ \text{m}^2$ or $256\pi\ \text{m}^2$.

Example 9.26.5

Find the surface area of a sphere with a radius of 12 feet.

Solution

First, substitute what you know into the surface area formula.

$$SA = 4\pi r^2$$
$$SA = 4\pi(12)^2$$

Next, use algebra to calculate the surface area.

$$SA = 4\pi(12)^2$$
$$SA = 4\pi(144)$$
$$SA = 576\pi$$
$$SA = 1809.6$$

The answer is 1809.6.

The surface area of the sphere is $1809.6\ \text{ft}^2$ or $576\pi\ \text{ft}^2$.

Review

Find the surface area of each sphere. Use 3.14 to approximate pi.

1. A sphere with a radius of 4 in.
2. A sphere with a radius of 2 in.
3. A sphere with a radius of 3.5 ft.
4. A sphere with a radius of 6.7 in.
5. A sphere with a radius of 12 cm.
6. A sphere with a radius of 1.6 ft.
7. A sphere with a radius of 9 m.
8. A sphere with a diameter of 9 m.
9. A sphere with a diameter of 18 in.
10. A sphere with a diameter of 10 cm.
11. A sphere with a diameter of 12 m.
12. A sphere with a diameter of 13 ft.
13. A sphere with a diameter of 15 m.
14. What is the surface area of a sphere whose diameter is 22 centimeters?
15. Bruce is making a sculpture in his art class that is made of 3 spheres. Each sphere has a radius of 2.3 feet. He will paint them all with blue poster paint. If each bottle of paint covers 20 square feet, how many bottles will Bruce need to buy?

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.14.

Resources

Vocabulary

Term	Definition
Surface Area	Surface area is the total area of all of the surfaces of a three-dimensional object.

Additional Resources

Practice: Surface Area of Spheres

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9.27: Volume of Spheres

Figure 9.27.1

Maria has a paperweight that is a glass **sphere**. The sphere is full of liquid. If the diameter of the paperweight is 6 inches, how much liquid does it contain?

In this concept, you will learn to calculate the **volume** of spheres.

Volume

Volume is the measure of three-dimensional space a figure takes up. You can also think of it as how much space the figure "holds."

To find the volume of spheres, you can use pyramids. Imagine a pyramid with its base on the surface of the sphere and its point as the center of the sphere. The radius of the sphere would be the height of the pyramid.

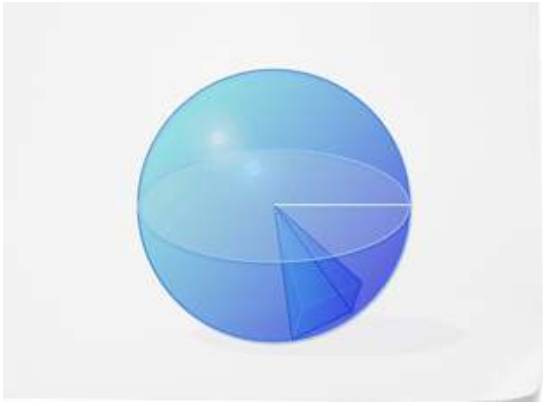

Figure 9.27.2

The pyramid makes up a portion of the sphere's volume. If you can fill the whole sphere with pyramids like this, you would know the volume of the sphere. It would be equal to the volumes of all the pyramids put together. How many pyramids would it take to fill a sphere? That depends on the surface area of the sphere.

You can combine the surface area of a sphere with the volume formula for a pyramid to calculate the volume of all the pyramids contained within the sphere.

The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$.

You know that the height of the pyramid is the radius (r) of the sphere and the base of the pyramid is the surface area of the sphere ($SA = 4\pi r^2$). If you substitute this information into the formula for the volume of a pyramid, you would get the following.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(4\pi r^2)r$$
$$V = \frac{4}{3}\pi r^3$$

The volume of a sphere can be found using the formula

$$V = \frac{4}{3}\pi r^3$$

Again, all you need to know is the radius of the sphere. You put the value in for r in the formula and solve for V, the volume.

Let's look at an example.

Find the volume of the sphere below.

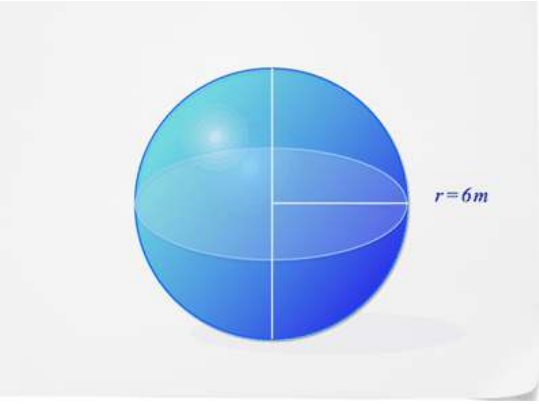

Figure 9.27.3

First, substitute what you know into the volume formula.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(6)^3$$

Next, use algebra to calculate the volume.

$$V = \frac{4}{3}\pi(6)^3$$
$$V = \frac{4}{3}\pi(216)$$
$$V = 288\pi$$
$$V = 904.78$$

The answer is 904.78.

The volume of the sphere is $904.78\ \mathrm{m}^3$.

If you wanted to make your measurement more accurate, you could say that the volume is $288\pi m^3$.

Example 9.27.1

Earlier, you were given a problem about Maria's liquid paperweight.

Maria has a paperweight with a diameter of 6 inches that is full of water, and she wants to find out how much liquid is inside.

Solution

First, find the radius of the sphere. Remember the radius is half the diameter.

$$r = \frac{d}{2}$$
$$r = \frac{6}{2}$$
$$r = 3$$

Next, substitute what you know into the volume formula.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(3)^3$$

Then, use algebra to calculate the volume.

$$V = \frac{4}{3}\pi(3)^3$$
$$V = \frac{4}{3}\pi$$
$$V = (27)^3 6\pi$$
$$V = 113.10v$$

The answer is 113.10.

The volume contained in Maria's paperweight is $113.10\ \text{in}^3$ or $36\pi\ \text{in}^3$.

Example 9.27.2

Find the volume of the following sphere.

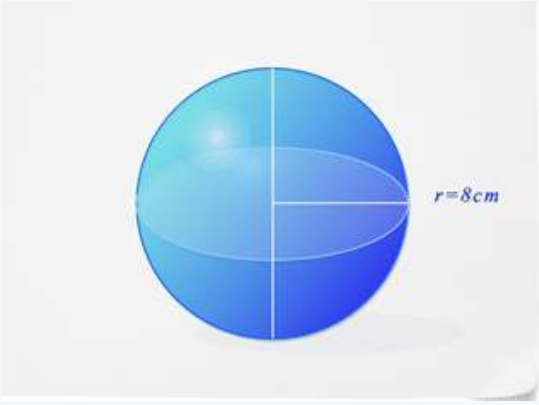

Figure 9.27.4

Solution

First, substitute what you know into the volume formula.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(8)^3$$

Next, use algebra to calculate the volume.

$$V = \frac{4}{3}\pi(8)^3$$
$$V = \frac{4}{3}\pi(512)$$
$$V = 682.67\pi$$
$$V = 2144.66$$

The answer is 2144.66. The volume of the sphere is $2144.66cm^3$.

If you wanted to make your measurement more accurate, you could say that the volume is $682.67\pi cm^3$.

Example 9.27.3

Find the volume of a sphere with a radius of 4 inches.

Solution

First, substitute what you know into the volume formula.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(4)^3$$

Next, use algebra to calculate the volume.

$$V = \frac{4}{3}\pi(4)^3$$
$$V = \frac{4}{3}\pi(64)$$
$$V = 85.33\pi$$
$$V = 268.08$$

The answer is 268.08.

The volume of the sphere is $\mathbf{268.08\ in^3}$.

Example 9.27.4

Find the volume of a sphere with a radius of 5 ft.

Solution

First, substitute what you know into the volume formula.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(5)^3$$

Next, use algebra to calculate the volume.

$$V = \frac{4}{3}\pi(5)^3$$
$$V = \frac{4}{3}\pi(125)$$
$$V = 166.67\pi$$
$$V = 523.60$$

The answer is 523.60.

The volume of the sphere is $\mathbf{523.60\ ft^3}$.

Example 9.27.5

Find the volume of a sphere with a radius of 3.5 inches.

Solution

First, substitute what you know into the volume formula.

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi(3.5)^3$$

Next, use algebra to calculate the volume.

$$V = \frac{4}{3}\pi(3.5)^3$$
$$V = \frac{4}{3}\pi(42.875)$$
$$V = 57.17\pi$$
$$V = 179.59$$

The answer is 179.59.

The volume of the sphere is $\mathbf{179.59\ in^3}$.

Review

Find the volume of each sphere. You may round to the nearest hundredth when necessary.

1. A sphere with a radius of 3 m.

2. A sphere with a radius of 2.5 m.

3. A sphere with a radius of 5 in.

4. A sphere with a radius of 6 in.

5. A sphere with a radius of 7 ft.

6. A sphere with a radius of 4.5 cm.

7. A sphere with a radius of 5.5 m.

8. A sphere with a radius of 13 mm.

9. A sphere with a diameter of 8 in.

10. A sphere with a diameter of 10 ft.

11. A sphere with a diameter of 3 m.

12. A sphere with a diameter of 13 m.

13. A sphere with a diameter of 22 ft.

Use what you have learned to solve each problem.

14. A sphere has a diameter of 12 feet. What is its volume?

15. Kelly has a perfume bottle in the shape of a sphere. The diameter of the bottle is 6 inches. How much perfume does Kelly have left if the bottle is only half full?

Review (Answers)

To see the Review answers, open this PDF file and look for section 8.15.

Vocabulary

Term	Definition
Sphere	A sphere is a round, three-dimensional solid. All points on the surface of a sphere are equidistant from the center of the sphere.
Volume	Volume is the amount of space inside the bounds of a three-dimensional object.

Additional Resources

Video: Volume of a Sphere

Practice: Volume of Spheres

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