

STATICS



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Statics

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Licensing

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Topic 01. Introduction to Statics

Topic 01 covers: 1) Describing Statics; 2) Explaining idealizations in Statics; 3) Outlining the fundamental approach in Statics.

Extra sheets on the last page are provided to: 1) review critical unit systems and prefixes, and 2) clarify the distinction between slugs and pounds, relevant to Statics

Statics

Topic 01: Introduction to Statics

Prof. J Lee

Objectives

- Describe Statics.
- Describe idealizations in Statics.
- Describe the fundamental approach in Statics.

Applications



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• Statics

: study of structural analysis of _____ in _____ under applied _____

• Equilibrium

: the system is either at _____ or moves at constant velocity.

• Idealization in Statics

: to simply complicated structures in practical environment for system analysis

• Particle

: has a _____, but a size and geometry is neglected.

: _____ of the body will _____ be involved in the analysis of the problem.

• Rigid Body

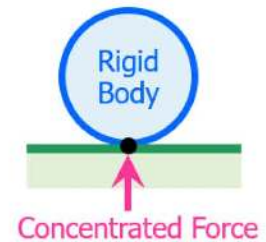
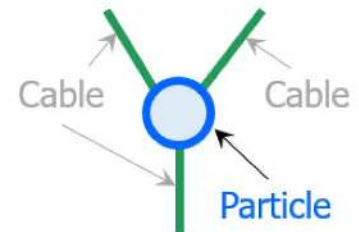
: a combination of a large number of particles in which all the particles remain at a fixed distance from one another before or after applying forces.

: _____ does _____ change when a force is applied.

: does NOT consider the material properties of the body.

• Concentrated Force

: assumes that the effect of loading acts at _____ on a body.



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• Applying Newton's Laws of Motion

• 1st Law (Equilibrium)

: A particle at rest stays at rest, and a particle in _____ moving with _____ velocity stays at constant velocity unless the particle is subjected to an unbalanced force.



• 3rd Law (Action - Reaction)

: The mutual forces of action and reaction between two bodies are _____, _____, and _____.



Extra Sheets: 1) System of Unit; 2) Slugs vs Pounds

System of Unit

	Length	Time	Mass	Force
International System of Units (SI)	m	s	kg	N ($\text{kg}\cdot\text{m}/\text{s}^2$)
US Customary (FPS)	ft	s	slug ($\text{lb}\cdot\text{s}^2/\text{ft}$)	lb

Gravity

- In SI Units: 9.8 m/s^2
- In US Customary: 32.2 ft/s^2

Prefix

Prefix	Symbol	Scientific Notation
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10
deci	d	10^{-1}
centi	c	10^{-2}
mili	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Slugs vs Pounds

Mass vs. Force

- Mass is a measure of how much matter an object contains. In the US customary system, the unit of mass is the slug [slug].
- Force is the interaction that causes an object to accelerate. In the US customary system, the unit of force is the pound-force [lb].

In the US customary system, a mass of 1 slug will accelerate at 1 ft/s^2 when subjected to a force of 1 lb. This definition aligns with $F = ma$: $1 \text{ lb} = 1 \text{ slug} \times 1 \text{ ft/s}^2$

- The relationship between mass and force is given by Newton's Second Law:

$$F = ma$$

, where

- F : the force [lb]
- m : the mass [slug]
- a : the acceleration [ft/s^2]

Weight

- Weight is a force caused by gravity acting on a mass. In the US system, the weight of an object is measured in pounds [lb].
- To connect weight (W) and mass (m), we use the acceleration due to gravity (g). Near the Earth's surface, $g \approx 32.2 \text{ ft/s}^2$. This means:

$$W [\text{lb}] = m [\text{slug}] \times g [\text{ft/s}^2]$$

Summary

- Use [slug] for mass in Statics problems.
- Use [lb] for force, including weight.
- Weight is a specific type of force: $W = mg$.

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Topic 02. Forces in 2D

Topic 2 covers: 1) Describing a force in 2D using Cartesian coordinates; 2) Determining the magnitude and direction of the resultant force in 2D.

Topic 2 is crucial for ensuring equilibrium of a body when forces are applied in a two-dimensional space (2D).

Extra Sheets at the end are provided to review vectors and essential theorems for right triangle force resolution, including the Pythagorean theorem and trigonometry, to help resolve forces using given proportions or angle information.

Statics

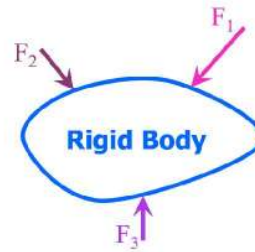
Topic 02: Forces in 2D using Cartesian Vectors

Prof. J Lee

Objectives

- Describe a force in 2D as Cartesian.
- Determine the **magnitude** and **direction** of the **resultant force in 2D**.

Applications



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How to make the Body in Equilibrium?

- Can we express **the three forces** as a **resultant force**?

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• What is Force

: an influence that can change the _____ of an object

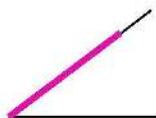
: a _____ quantity and therefore a force has both _____ and _____.

• How to Express the Direction of Force

(1) Using _____



(2) Using _____



Line of Action

()

: _____ in which

a vector _____

(3) Using _____ Vector

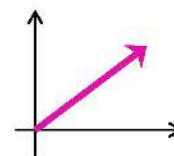


Unit Vector

: a vector with a _____ of _____



(4) Using _____ Vector



• What

: _____ vectors that define the _____ coordinate system

- _____ along the ____-axis
- _____ along the ____-axis
- _____ along the ____-axis

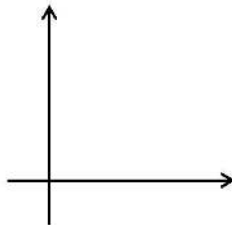
• How To Use

: To indicate the _____ of a _____

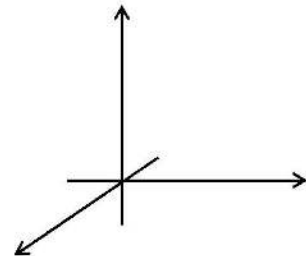
1D



2D



3D



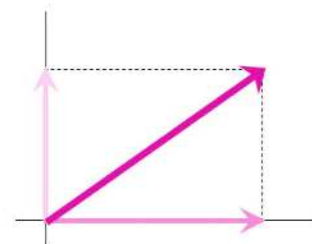
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• How to Express a Force in 2D as Cartesian Vectors

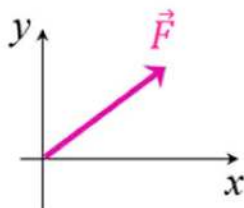
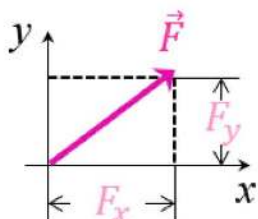
If the **magnitude** of $\vec{F}_x$ and $\vec{F}_y$ are **given**,

If the **magnitude** of $\vec{F}$ and the **angle** θ are **given**,

If the **magnitude** of $\vec{F}$ and the **"small slope triangle"** are **given**,



• How to Determine the Magnitude (F) and Direction (θ) of *when F_x and F_y are Given*



Magnitude of $\vec{F}$:

Direction of $\vec{F}$:

Note: θ is the angle between the force components $\vec{F}_x$ and $\vec{F}_y$ that form the resultant force $\vec{F}$.

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• What

: _____ force obtained by combining a system of forces acting on a rigid body via **vector addition**

• How To Find

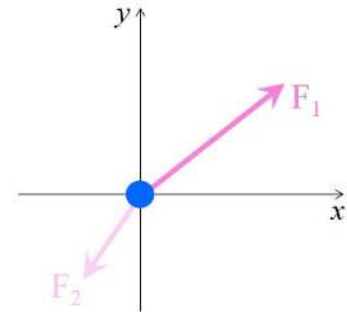
: by **adding** the forces via **vector addition**

: by **resolving** as Cartesian Vectors and **using scalar algebra**

Step 1: _____ the coordinates;

Step 2: Resolve each force into ____ and ____ components if it is ____D;

Step 3: _____ the components **using Scalar Algebra**.

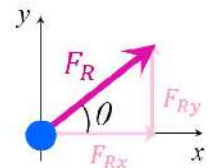


$$\begin{aligned}\vec{F}_1 &= \\ \vec{F}_2 &= \\ \vec{F}_R = \vec{F}_1 + \vec{F}_2 &= \underbrace{(F_{1x} - F_{2x})}_{F_{Rx}} \hat{i} + \underbrace{(F_{1y} - F_{2y})}_{F_{Ry}} \hat{j} \\ &= F_{Rx} \hat{i} + F_{Ry} \hat{j}\end{aligned}$$

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Magnitude of $\vec{F}_R$:

Direction of $\vec{F}_R$:

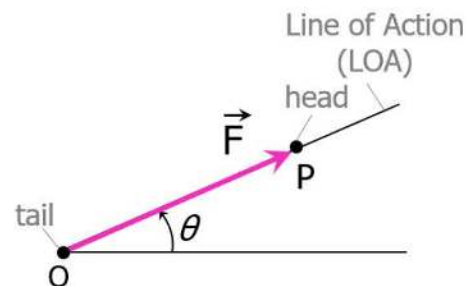


Extra Sheet for Vectors

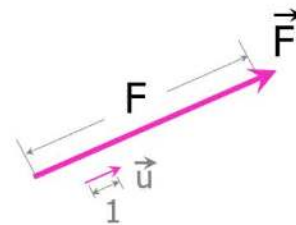
Vectors

- **Scalar:** Physical quantity that has **only** magnitude, having **no** direction.
- **Vector:** Physical quantity that has **both** magnitude and direction.
- **Line of Action (LOA):** Direction in which a vector points.

NOTE: θ is usually in CCW from x-axis.



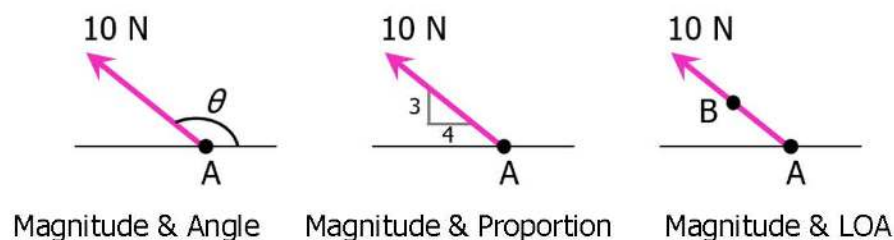
- **Unit Vector:** A vector with a magnitude of 1.



Vector Notation

	Typed	Hand-written
Vector	$\mathbf{F}$	$\vec{F}$
Scalar/Magnitude	F	F or $ \vec{F} $
Unit Vector	$\mathbf{u}$, or $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	$\hat{u}$ or $\hat{i}$, $\hat{j}$, $\hat{k}$ or $\underline{u}$, or $\underline{i}$, $\underline{j}$, $\underline{k}$

Depiction of Vectors

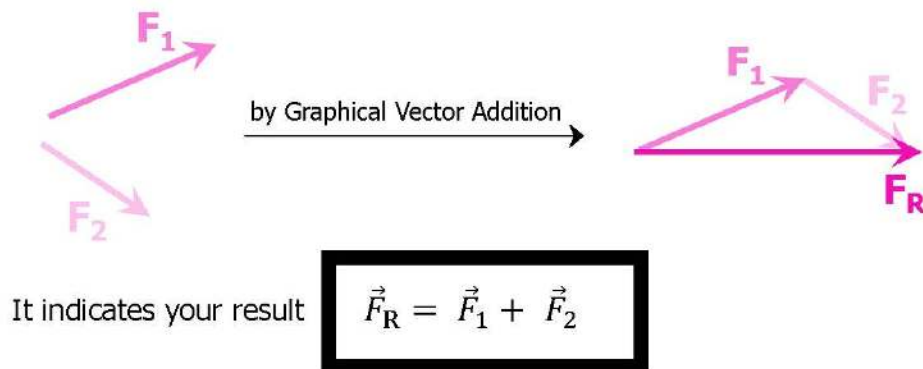


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• Vector Addition

by Graphical Vector Addition

- (1) **Draw the first** vector;
- (2) Place **the tail of the second** vector
at the head of the first vector;
- (3) Continue adding vectors *if more than two*, placing the tail of each subsequent vector at the head of the previous one;
- (4) Draw the resultant vector (as $\vec{F}_R$ in Figure below) as a straight arrow **from the tail of the first vector to the head of the last** vector in the sequence.



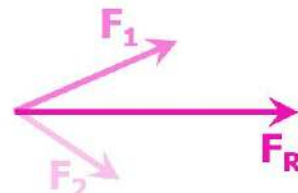
by Parallelogram Law

- (1) Make the tails concurrent;
- (2) Draw lines parallel to the vectors;
- (3) Draw the resultant force (as $\vec{F}_R$ in Figure).

It indicates your result

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

By Parallelogram Law



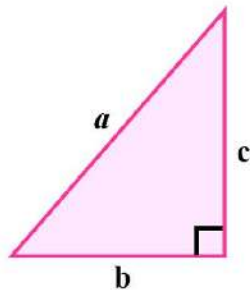
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Extra Sheet for Essential Theorems for Right Triangle Force Resolution

Essential Theorems for Right Triangle Force Resolution

Pythagorean Theorem

Note that the Pythagorean theorem applies only to Right Triangles.

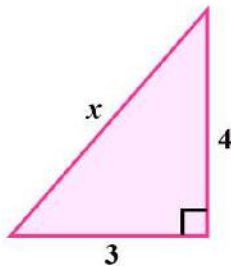


$$a^2 = b^2 + c^2$$

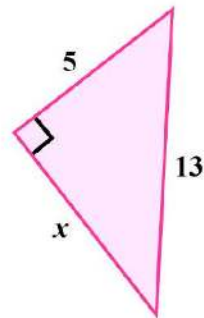
Hence, the length of the hypotenuse "a" can be determined by

$$\therefore a = \sqrt{b^2 + c^2}$$

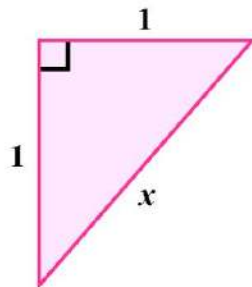
Examples



$$x = \sqrt{4^2 + 3^2} = 5$$



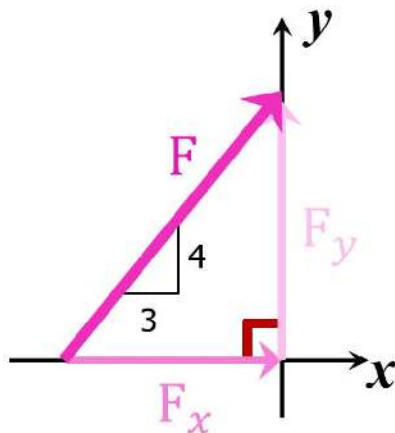
$$\begin{aligned} 13^2 &= 5^2 + x^2 \\ \rightarrow x^2 &= 13^2 - 5^2 \\ \therefore x &= \sqrt{13^2 - 5^2} = 12 \end{aligned}$$



$$x = \sqrt{1^2 + 1^2} = \sqrt{2}$$

How To Apply "Pythagorean Theorem" To Resolve Forces

If the magnitude of $\vec{F}$ is given as F , and **the slope of $\vec{F}$ is provided in proportional form** as described in Fig., determine the magnitude of x and y components of $\vec{F}$.



Using the Pythagorean theorem, the slope proportion of $\vec{F}$ is $\sqrt{3^2 + 4^2} = 5$.

Also, the ratio of $\vec{F}$ to $\vec{F}_x$ is given as 5 : 3.

This is expressed as the ratio

$$F : F_x = 5 : 3$$

$$\rightarrow 5 F_x = 3 F$$

$$\therefore F_x = \frac{3}{5} F$$

Similarly, the ratio of $\vec{F}$ to $\vec{F}_y$ is given as 5 : 4.

This is expressed as the ratio

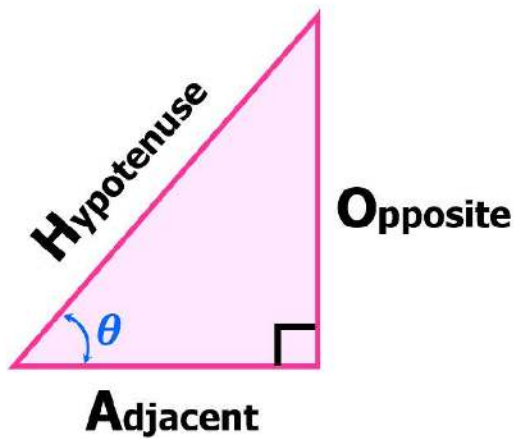
$$F : F_y = 5 : 4$$

$$\rightarrow 5 F_y = 4 F$$

$$\therefore F_y = \frac{4}{5} F$$

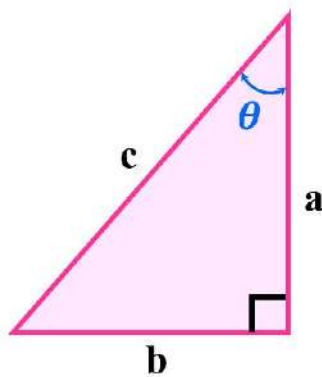
Trigonometry

Note that the trigonometry applies only to Right Triangles.



$$\begin{aligned}\sin \theta &= \frac{O}{H} \\ \cos \theta &= \frac{A}{H} \\ \tan \theta &= \frac{O}{A}\end{aligned}$$

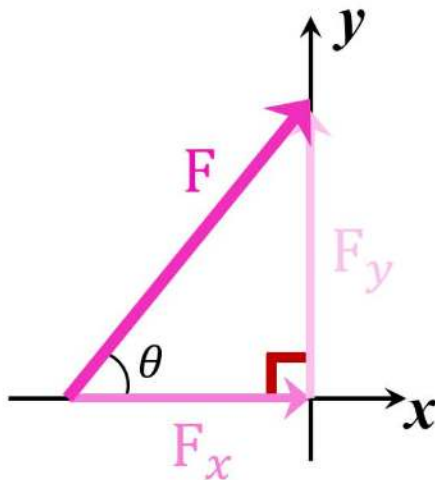
Examples



$$\sin \theta = \frac{b}{c} \quad \cos \theta = \frac{a}{c} \quad \tan \theta = \frac{b}{a}$$

How To Apply "Trigonometry" To Resolve Forces

If the magnitude of $\vec{F}$ is given as F , and **the slope of $\vec{F}$ is provided in an angle** as described in Fig., determine the magnitude of x and y components of $\vec{F}$.



Using the trigonometry,

$$\cos \theta = \frac{F_x}{F}$$

$$\therefore F_x = F \cos \theta$$

$$\sin \theta = \frac{F_y}{F}$$

$$\therefore F_y = F \sin \theta$$

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Topic 03. Forces in 3D using Coordinate Direction Angles and Projected Angles

Topic 03 covers: 1) Describing a force in 3D using Cartesian coordinates; 2) Describing a force in 3D with coordinate direction angles; 3) Describing a force in 3D using projected angles; 4) Determining the magnitude and coordinate direction angles of the resultant force in 3D.

This topic is essential for analyzing force components in three-dimensional space using the given or measured angle information.

At the end of this topic, an animation is provided to visually demonstrate how to express a force in 3D using the given information: 1) with the provided projected angle; 2) with the given coordinate direction angles.

Statics

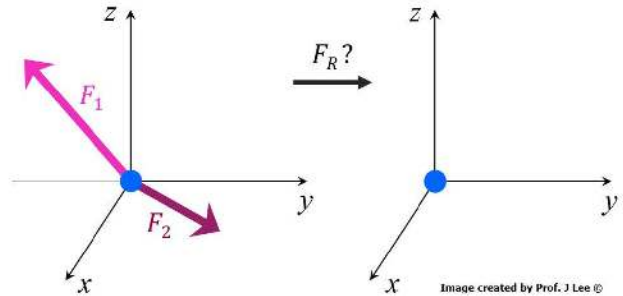
Topic 03: Forces in 3D using Coordinate Direction Angles & Projected Angles

Prof. J Lee

Objectives

- Describe a force in 3D **as Cartesian**.
- Describe a force in 3D **with the coordinate direction angles**.
- Describe a force in 3D **using the projected angles**.
- Determine the **magnitude** and **coordinate direction angles** of the **resultant force** in 3D.

Applications

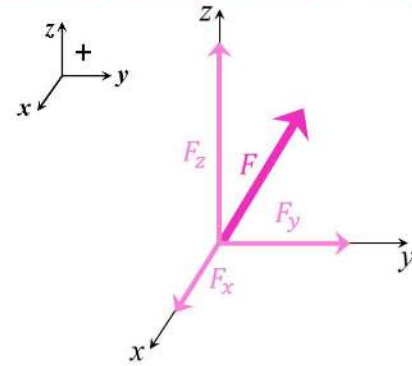
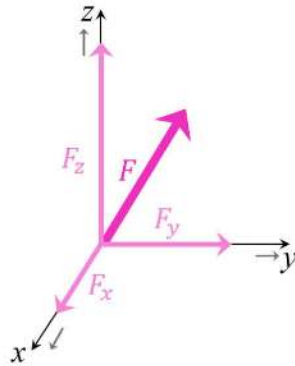
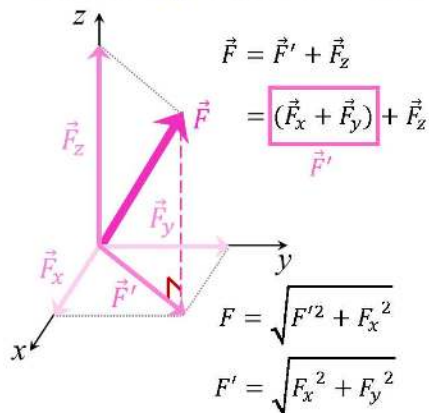


How to make the Particle in Equilibrium?

- Can we express the two forces in 3D as a resultant force?

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• How to Express a Force in 3D as Cartesian Vectors using the Coordinate Direction Angles



Angles from each _____ axis to the _____

Regardless of when $\vec{F}$ is directed, each of the angles will be $\leq \alpha, \beta, \gamma \leq$

$$\begin{aligned} \vec{F} &= \vec{F}_x + \vec{F}_y + \vec{F}_z \\ &= \text{using the Unit Vector} \\ &= \text{as Cartesian} \\ &= (\quad)\hat{i} + (\quad)\hat{j} + (\quad)\hat{k} \\ &\quad \text{using the Coordinate Direction Angles} \end{aligned}$$

Magnitude of $\vec{F}$:

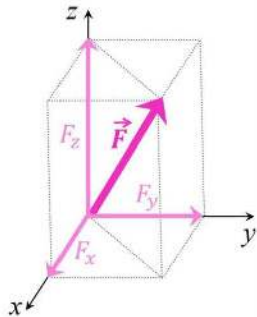
$$\alpha = (\quad)$$

$$\beta = (\quad)$$

$$\gamma = (\quad)$$

• How to Determine the Direction of a Force in 3D using the Unit Vector

If F_x , F_y , and F_z are given,



$$\begin{aligned}\vec{u} &= \frac{\vec{F}_x + \vec{F}_y + \vec{F}_z}{F} = \frac{F_x}{F} \hat{i} + \frac{F_y}{F} \hat{j} + \frac{F_z}{F} \hat{k} \\ &= \frac{F_x}{F} \hat{i} + \frac{F_y}{F} \hat{j} + \frac{F_z}{F} \hat{k} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}\end{aligned}$$

Magnitude of $\vec{u}$: $|\vec{u}| = 1$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

How to use the equation:

- If two angles are known, the **third** angle can be found from this equation.
- We can **check the Answer** using this equation.

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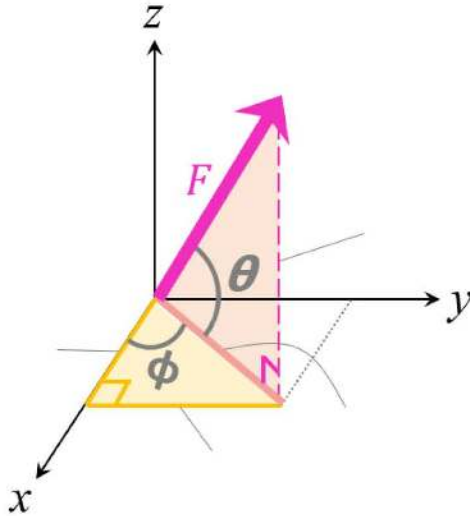
- How to Express a Force in 3D as Cartesian Vectors using **the Projected Angles**

$$\vec{F} =$$

$$= F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

If the Projected Angles, θ and ϕ , are given

$$=$$



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• How to Determine the Resultant Force, $\vec{F}_R$

Step 1: Define the Coordinates;

Step 2: Resolve each force in Cartesian;

Step 3: Add the corresponding vector components;

Step 4: Determine the **Magnitude** of the Resultant Force $\vec{F}_R$ using

Step 5: Determine the **Coordinate Direction Angles** of the Resultant Force $\vec{F}_R$ using

NOTE: *Coordinate direction angles* are measured **FROM** each _____ axis **TO** the _____.

According to its definition,
regardless of when $\vec{F}$ is directed, each of the angles will be $\leq \alpha, \beta, \gamma \leq$

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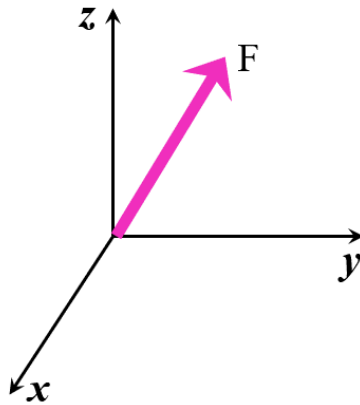
How to Express a Force in 3D using 1) the given Projected Angle; 2) the given Coordinate Direction Angles

Topic 03: Forces in 3D

Express a Force in 3D using the Projected Angles

$$\vec{F} =$$

using Projected Angle



using Coordinate Direction Angles

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Topic 04. Forces in 3D using Position Vectors

Topic 04 covers: 1) Describing a position vector; 2) Determining position vectors from given information; 3) Representing a force directed along a line using a position vector; 4) Calculating the magnitude and coordinate direction angles of the resultant force in 3D using position vectors.

This topic is essential for accurately representing forces in three-dimensional space using the given or measured geometric information.

At the end of this topic, an animation is provided to demonstrate how to find the point coordinates using the given information. Additionally, an Extra Sheet is provided for "Fundamental Concepts of Space."

Statics

Topic 04: Forces in 3D using Position Vectors

Prof. J Lee

Objectives

- Describe a **position vector**.
- Determine position vectors using the given information.
- Describe a force directed along a line using a position vector.
- Determine the **magnitude** and **coordinate direction angles** of the **resultant force in 3D** using the position vector.

Applications



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Position Vector ()

Topic 04 – 2

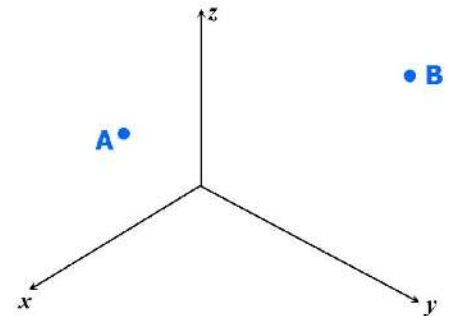
• What

A fixed vector which locates a point in space relative to another point

• How to Find ()

(1) Find the coordinates of _____ and _____ of the vector;

(2) Calculate



Exercise

Find the position vector $\vec{r}_{AB}$ from points A and B .

Solution:

Answer:

$$\vec{r}_{AB} = (\quad - \quad) \hat{i} + (\quad - \quad) \hat{j} + (\quad - \quad) \hat{k}$$

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Force directed along a Line

Topic 04 – 3

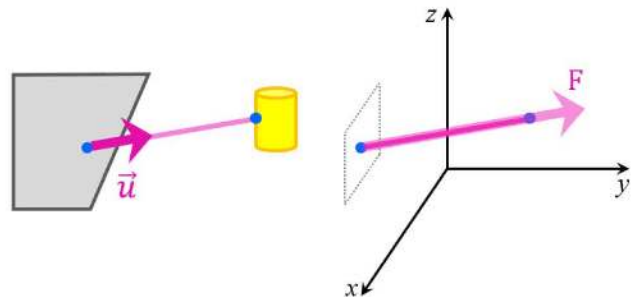
• How To Express a Force along a Line using the Position Vector

$\vec{F} =$
using the Unit Vector

 $=$
using the Position Vector



Photo taken by Prof. J Lee ©



• How to Determine

Step 1: Define the Coordinates;

Step 2: Express the coordinates of the given points;

Step 3: Find the position vector $\vec{r}$ using

Step 4: Find the unit vector $\vec{u} =$

Step 5: Present the force vector $\vec{F}$ using the unit vector $\vec{u}$ from (3)

Step 6: Find the magnitude and direction of $\vec{F}$

Magnitude of $\vec{F}$: $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ | Direction of $\vec{F}$: $\alpha = \cos^{-1}(\frac{F_x}{F})$, $\beta = \cos^{-1}(\frac{F_y}{F})$, and $\gamma = \cos^{-1}(\frac{F_z}{F})$.

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Practice Problem

Topic 04 – 4

Determine the position vector $\vec{r}_{AB}$ which extends from A to B.

Given: $F = 750 \text{ N}$; Geometry Info.

Find:

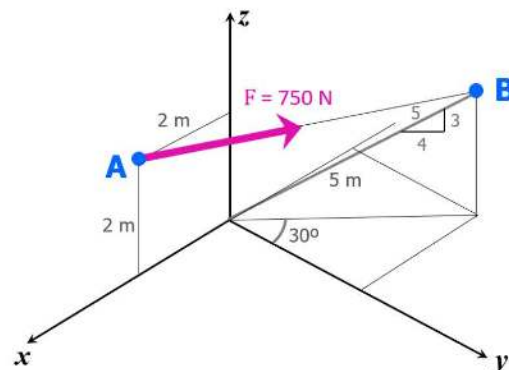
Solution: Step 1: Define the Coordinates;

Step 2: Express the coordinates of the given points;

A (, ,) m

B ($-[5(\frac{4}{5})] \sin 30^\circ$, $[5(\frac{4}{5})] \cos 30^\circ$, $5(\frac{3}{5})$) m = (-2, 3.464, 3) m

Step 3: Find the position vector $\vec{r}$ using



Answer:

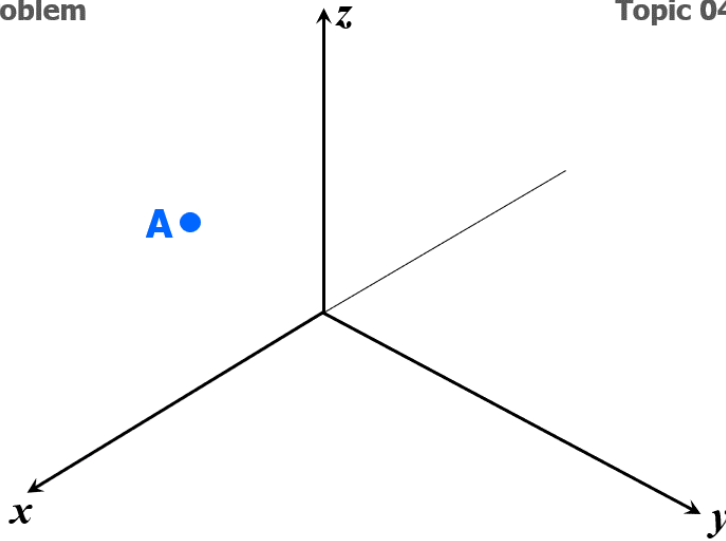
$\vec{r}_{AB} =$

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How to find the point coordinates using the given information

Practice Problem

Topic 04: Forces in 3D – Position Vector



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Extra Sheet for Fundamental Concepts of Space

Fundamental Concepts of Space

Point

a **geometric concept**, **not a physical quantity**, that represents a specific location in space. It serves as a reference for defining other physical and geometric properties.

Location

refers to the placement of a point in a given coordinate system.

Position

a **vector quantity** that describes a point's location relative to a reference.

Line

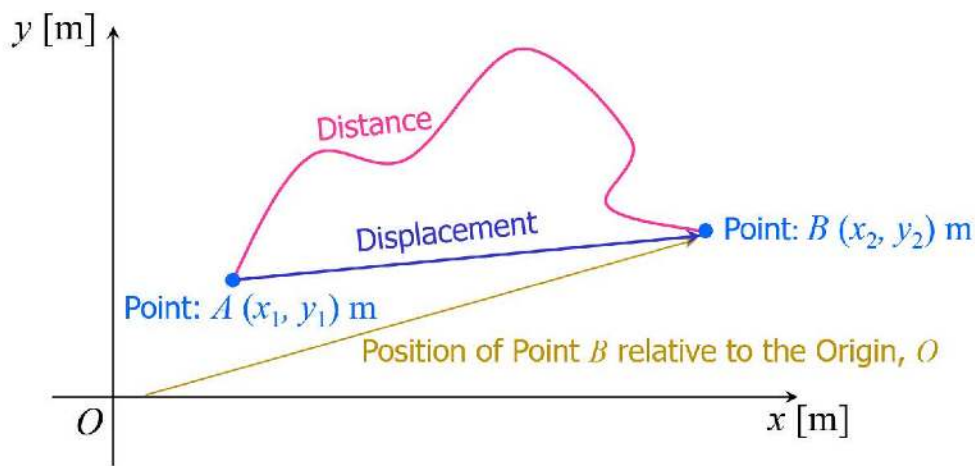
a **one-dimensional geometric object** that extends infinitely in both directions. It is defined by two distinct points and describes direction and connectivity in space.

Distance

a **scalar quantity** that represents **the total length traveled between two points**, regardless of direction. It is **always positive** and does not include directional information.

Displacement

a **vector quantity** that represents **the shortest straight-line path between two points, including direction**.



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Topic 05. Moment

Topic 05 covers: 1) Describing a moment; 2) Determining the moment about a point in 2D and 3D; 3) Calculating the magnitude and direction of the resultant moment about a point in 2D and 3D.

Understanding moments is crucial for ensuring the equilibrium of a body in Statics analysis, as it allows for the inclusion of rotational effects alongside forces.

At the end of this chapter, an animation is provided to demonstrate how to determine the direction of a moment using the right-hand rule. Additionally, extra sheets are provided to review the fundamental concepts required for this topic: 1) Vector Cross Product; 2) Determinant of Matrix.

Statics

Topic 05: Moment

Prof. J Lee

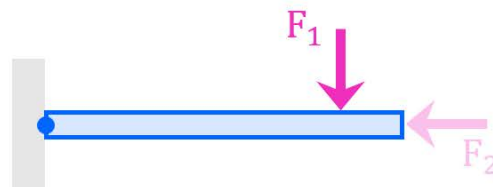
Objectives

- Describe a **Moment**.
- Determine the moment about a point.
- Determine the magnitude and direction of the resultant moment about a point.

Applications



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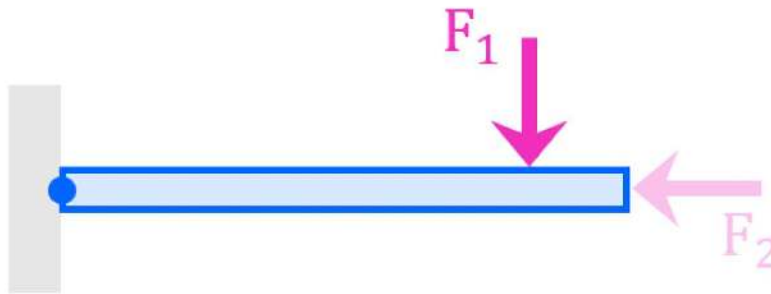


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• What

: A measure of _____ of the force to cause _____

: When a _____ is applied to a body, **the force** will produce a **tendency** for the body to
 _____ **ABOUT** a _____ or an _____, which is _____ on the _____ of the force.



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Magnitude

Moment about a Point from a Single Force:

Resultant Moment about a Point from Multiple Forces:



where

F : magnitude of the force; _____ value of the force

d :

_____ : _____ distance **FROM** the _____ **TO** the _____ of the force

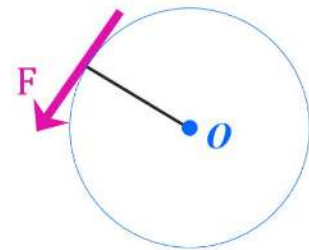
M_R : resultant moment

Unit: [_____] or [_____]

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Direction

- : using the **Right-Hand Rule**
- : **Right Thumb** indicates the Directional Sense of the Moment.
- : **CCW** is generally considered as the _____ direction.



• How To Determine the Moment about a Point

Step 1: Define the coordinates.

Step 2: Resolve $\vec{F}$ into $x - y$ components if required.

Step 3: Draw the _____ of $\vec{F}$.

Step 4: Determine the moment arm, d _____ of $\vec{F}$ where

Step 5: Calculate the **magnitude** of $\vec{M}_O$ using where F : _____ (_____ value) of $\vec{F}$

Step 6: Determine the **direction** of using the **Right-Hand Rule**.

- (1) **Put** right fingers (except the thumb) **from the** _____ **to** _____.
- (2) **Curl** them towards the direction of $\vec{F}$. Note that **only curl to the** _____ **direction**.
- (3) **Watch** the RIGHT thumb: generally upward _____ downward

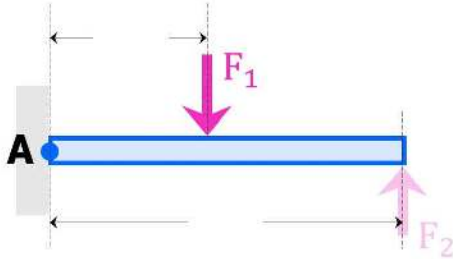
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Practice Problems

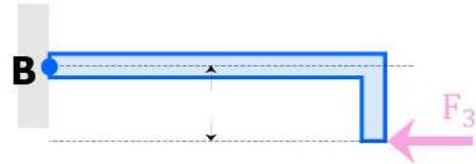
Topic 05 – 5

Determine the resultant moment.

(1) Moment about point A



(2) Moment about point B



Recall: **Moment Arm** ()

_____ distance **FROM** the _____

TO the _____ of the force

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How To Determine the Direction of a Moment using the Right-Hand Rule

How To Determine the Direction of **M**oment about Point **O**



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Topic 06. Couple Moment

Topic 06 covers: 1) Describing a couple and a couple moment; 2) Determining the couple moment.

Understanding couple moments is crucial in Statics because they represent a pair of equal and opposite forces whose effects do not cancel each other out but create a rotational effect about a point. Recognizing and calculating couple moments is essential for analyzing and ensuring the equilibrium of a system, as they contribute to the overall rotational balance alongside other forces and moments. This comprehensive understanding allows for accurate assessment of systems under various loading conditions.

At the end of this chapter, animations are provided to demonstrate how to determine a couple and to offer solutions for the practice problem.

Statics

Topic 06: Couple Moment

Prof. J Lee

Objectives

- Describe a Couple and a **Couple Moment**.
- Determine the Couple Moment.

Applications



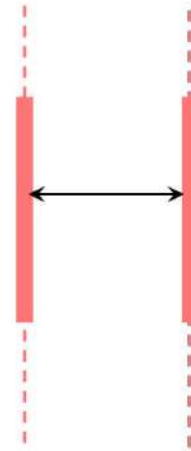
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• What

_____ Forces:

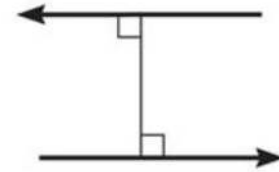
- (1)
- (2) Magnitude
- (3) Direction
- (4) Separated by a Distance,



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• What

: a moment produced by _____



• How To Determine

Magnitude



d : _____ distance _____ the _____

Direction

Using the Right-Hand rule

- Select one member of the couple for the couple moment.
- Position your right hand with four fingers along the moment arm originating from the other force.
- Curl your fingers towards the chosen force representing the couple moment.
- See the direction indicated by your right thumb for the direction of the couple moment.

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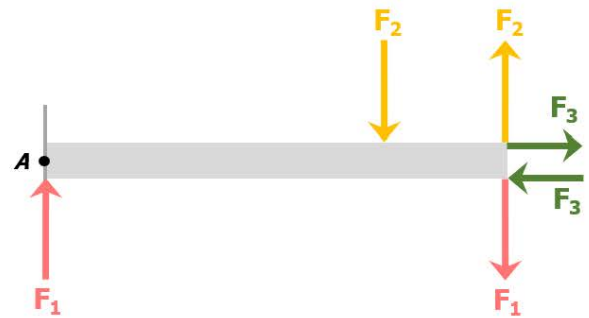
• Characteristics

- The couple moment depends on the _____ of the force and _____ between the forces.
- The couple moment is a _____ vector: can be moved anywhere on the body and have the _____ external effect on the body.
- Moments due to couples can be _____ using the same as adding any vectors.

Practice Problem

Topic 06 – 5

Determine the resultant couple moment acting on the beam.



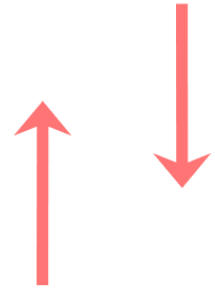
Answer:

$$\vec{M}_R = -d_1 F_1 + d_2 F_2 - d_3 F_3$$

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How To Determine a Couple

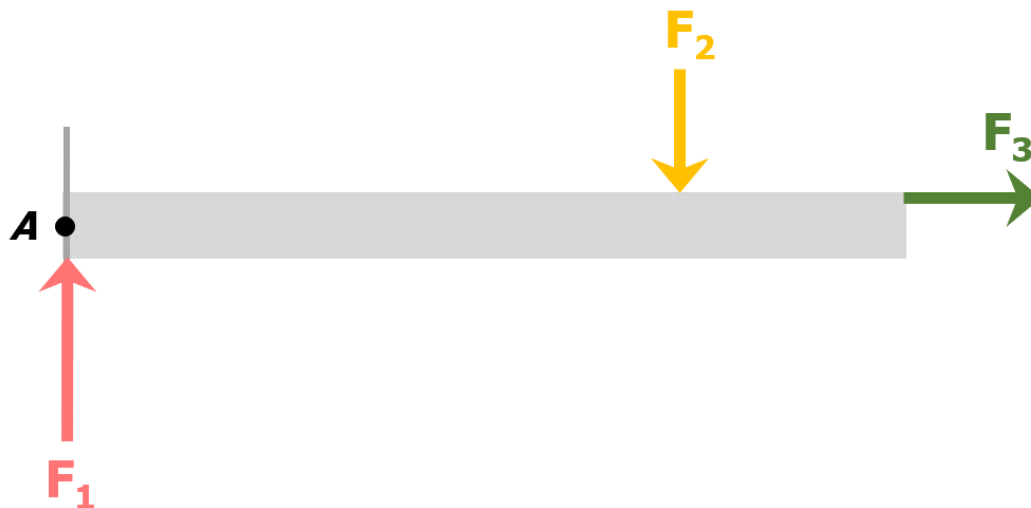
Two Forces:



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Topic 06

Practice Problem Determine the resultant couple moment.



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Topic 07. Equilibrium in 2D

Topic 07 covers: 1) Describing a free body diagram; 2) Explaining the characteristics of support reactions; 3) Drawing free body diagrams; 4) Applying equations of equilibrium to solve 2D problems; 5) Determining support reactions.

Understanding equilibrium in 2D is essential in Statics, as it enables accurate analysis of systems where forces and moments are in balance. It also involves understanding the characteristics of the supports applied to the system and how they influence the overall stability. Mastering these concepts is crucial for ensuring that structures remain stable and properly supported under various loads.

At the end of this chapter, animations are provided to demonstrate how to resolve a normal force on an inclined surface with an angle and to present the solution for the Practice Problem on Page 8. Additionally, extra sheets are provided to review the fundamental concepts required for this topic: 1) Parallelogram Properties; 2) Trigonometric Identities.

Statics

Topic 07: Equilibrium in 2D

Prof. J Lee

Objectives

- Describe a free body diagram.
- Explain the characteristics of support reactions.
- Draw free body diagrams.
- Apply equations of equilibrium to solve 2D problems.
- Determine the support reactions.

Applications



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Free Body Diagram ()

Topic 07 – 2

- **What**

: a drawing that shows _____ external _____ acting on a body

- **Why**

: To apply Newton's 1st law of motion (Equation of Equilibrium) for system analysis



- **How**

Step 1: Choose a body which you need to analyze.

Step 2: Draw an _____ shape of the body.

Step 3: _____ the body from its surroundings.

Step 4: Show all external (active and reactive) forces and couple moments which are acting on the body.

Step 5: Label all known and unknown forces, couples, and dimensions.

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- **What**

: a force at the support to _____ motion of a body acting

in an _____ **direction** to the _____ force

- **General Characteristics**

(1) **Prevent** the _____ of a Body: create a _____ on the body in the **opposite** directions.

(2) **Prevent** the _____ of a Body: create a couple _____ on the body in the **opposite** directions.

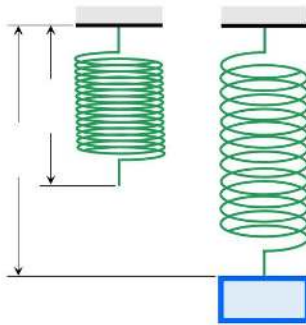
Note: Reaction forces at support depend on the _____ of supports.

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Types of Support

Topic 07 – 4

• Spring



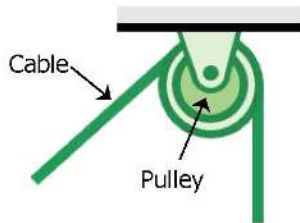
Spring Force by Hooke's Law

where

k : spring constant (stiffness)

Deformation = _____ length – _____ stretched length

• Cables and Pulleys



Assumptions:

- (1) _____ of cables are negligible;
- (2) Cables can _____ stretch (_____ Deformation);
- (3) Cables can support only a _____;
- (4) _____ of tension which passes over a pulley must be _____.

Forces by a Cable which passes through a Pulley

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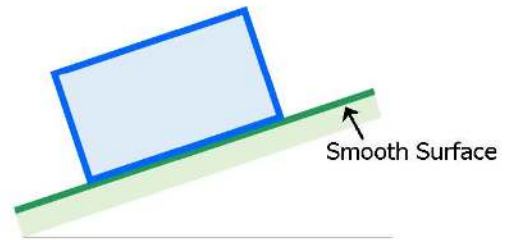
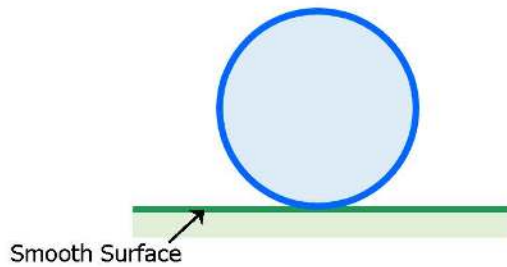
• Smooth Surface Contact

Normal Force

N the surface of contact

: **N** is the force component _____ **to the surface** of contact **to prevent the object to fall**.

: **N** is applied to an object from the surface to **the** _____ **of the object**.



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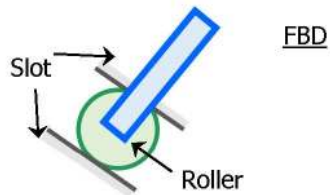
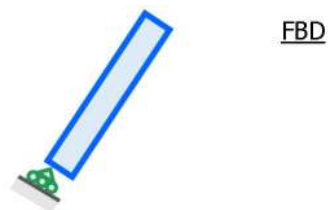
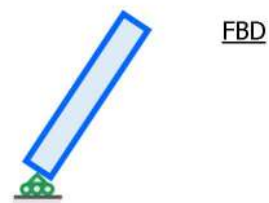
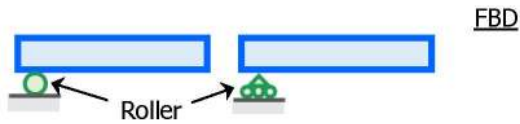
Types of Support

Topic 07 – 6

• Roller and Rocker

: can only resist a force _____ to the supporting surface.

: can _____ resist a _____.



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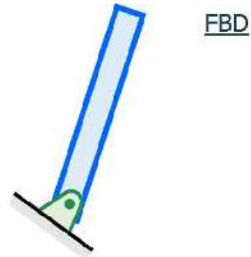
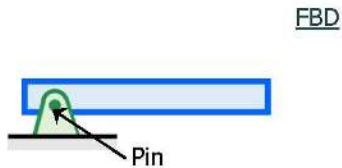
Types of Support

Topic 07 – 7

• Pin

: can only resist forces in _____ and _____ directions.

: can _____ resist a _____.



• Fixed Support

: can only resist forces in _____ and _____ directions.

: _____ resist a _____.

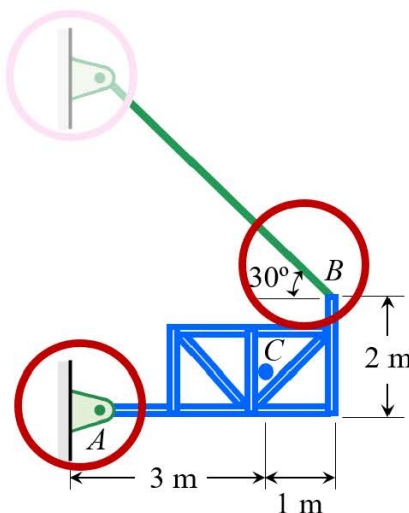
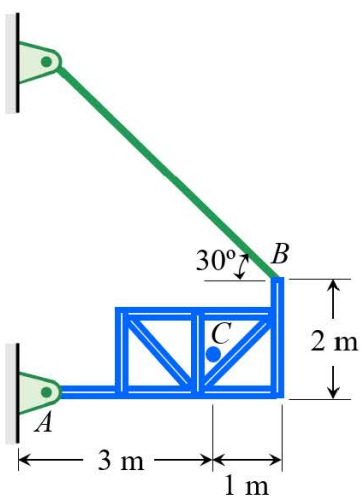


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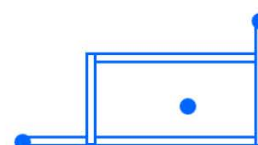
Free Body Diagram: Practice Problem

Topic 07 – 8

Idealized Model



Free Body Diagram

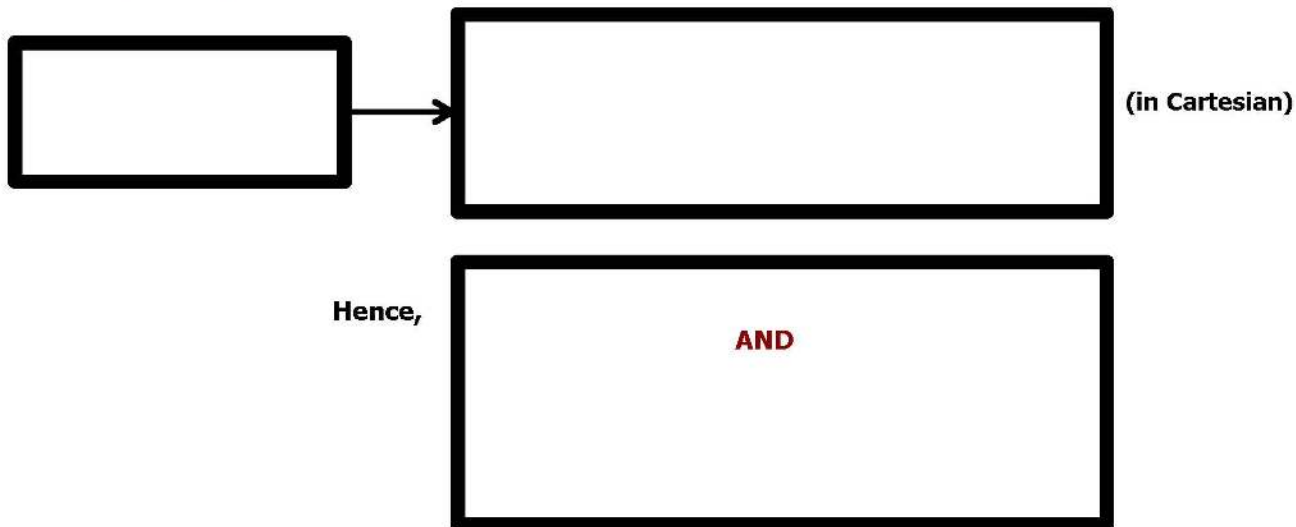


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•What

: Coplanar forces in a ____D system can be resolved into their ____ - and ____ - components.

: **If a body is in Equilibrium**, then _____ parts of the body must be in Equilibrium.



Note: Count the _____ of _____ knowns from given problems. Then, decide how many equations will be needed

from Moment about _____ to solve the unknowns.

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• How TO

Step 1. Define the coordinate system.

Step 2. Draw a _____.

Step 3. Present each force as cartesian.

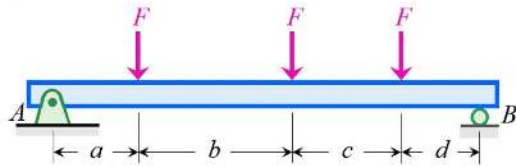
Step 4. Apply Equations of Equilibrium.

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Practice Problem

Topic 07 – 11

(1) Idealized Model



(2) Free Body Diagram

(3) Equilibrium Equations

Check # of Unknowns:

Then, how many equations you need? _____

Recall:

In 2D Equilibrium, always Moment about a _____!

Mag of Moment:

where

d : _____ distance **FROM** the _____

TO the _____ of the force

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Animation on Resolving Normal Force on an Inclined Surface with an Angle

Normal Force N on a Titled Surface : How To determine the x and y components using the Given Angle



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Animation on Resolving Normal Force on an Inclined Surface with an Angle

Free Body Diagram: Practice Problem

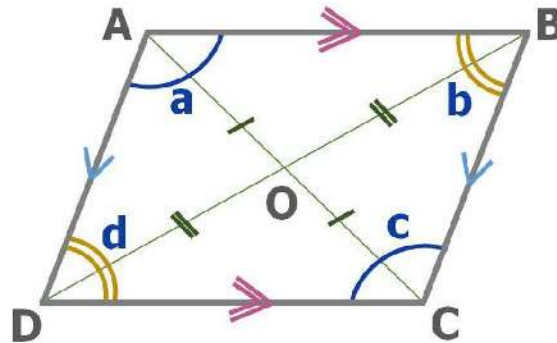
Topic 07 – 8

Idealized Model	Free Body Diagram

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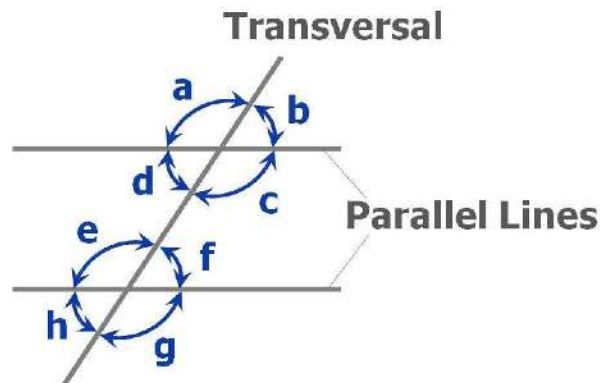
Extra Sheets for (1) Parallelogram Properties; (2) Trigonometric Identities

Parallelogram Properties



- Two pairs of Parallel Lines: $\overline{AB} // \overline{CD}$ $\overline{AD} // \overline{BC}$
- Congruent Opposite Sides: $\overline{AB} = \overline{CD}$ $\overline{AD} = \overline{BC}$
- Congruent Opposite Angles: $a = c$ $b = d$
- Bisected Diagonals: $\overline{AO} = \overline{OC}$ $\overline{BO} = \overline{OD}$

Angles in Parallel Lines



- Corresponding Angles: $a = e$ $b = f$ $c = g$ $d = h$
- Vertical Angles: $a = c$ $b = d$ $e = g$ $f = h$
- Alternate Interior Angles: $d = f$ $c = e$
- Alternate Exterior Angles: $a = g$ $b = h$

Trigonometric Identities

Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Opposite Angle Identities

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Sum & Difference Identities

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Double-Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Product-Sum Identities

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

Cofunction Identities

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\sec(90^\circ - \theta) = \csc \theta$$

$$\sin(90^\circ + \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\csc(90^\circ - \theta) = \sec \theta$$

$$\cos(90^\circ + \theta) = -\sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

$$\cot(90^\circ - \theta) = \tan \theta$$

$$\cot(90^\circ + \theta) = -\cot \theta$$

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Topic 08. Equilibrium in 3D using Scalar Analysis

Topic 08 covers: 1) Describing the characteristics of support reactions in 3D; 2) Drawing free body diagrams for 3D systems; 3) Applying equations of equilibrium to solve 3D problems using scalar analysis; 4) Determining support reactions.

Understanding equilibrium in 3D is important in Statics, as it allows for accurate analysis of systems where forces and moments are balanced in three-dimensional space. It also requires understanding the characteristics of the supports applied to the system and how they influence the overall stability. Mastering these concepts is critical for ensuring that structures remain stable and properly supported under complex loading conditions in multiple directions.

At the end of this chapter, an animation is provided to demonstrate how to determine the moment arm for a moment about an axis in 3D.

Statics

Topic 08: Equilibrium in 3D using Scalar Analysis

Prof. J Lee

Objectives

- Describe the characteristics of support reactions.
- Draw free body diagram for 3D systems.
- Apply equations of equilibrium to solve 3D problems using scalar analysis.
- Determine the support reactions.

Applications



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Types of Support in 3D

Topic 08 – 2

• General Characteristics

(1) **Prevent** the _____ of a Body

(2) **Prevent** the _____ of a Body

Note: Reaction forces at support depend on the _____ of supports.

Types of Support	Reaction(s) & <u>FBD</u>	# of Unknowns
Smooth Surface Support 	 i) a point contact ii) the surface	
Roller 	 i) a point contact ii) the surface	
Single Smooth Pin 		
Ball and Socket 		

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Types of Support in 3D

Topic 08 – 3

Types of Support	Reaction(s) & <u>FBD</u>	# of Unknowns
Single Journal Bearing 		
Single Hinge 		
Fixed Support 		

Note: For _____, _____, and _____, if the body is supported elsewhere,

the couple moments are generally _____ applied.

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• What

If a body is in Equilibrium, then _____ parts of the body must be in Equilibrium.



• When

(1) when the geometry is _____.

(2) when all forces are either _____ or _____.

• How

d : _____ distance from the _____ to the _____ of $\vec{F}$



Note: If the forces are (1) _____ to the axis,
or (2) _____ through the axis



about the axis.

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Practice Problem

Topic 08 – 5

1. Draw a free body diagram for the system. The mass of the plate is 100 kg.

Given:

Find:

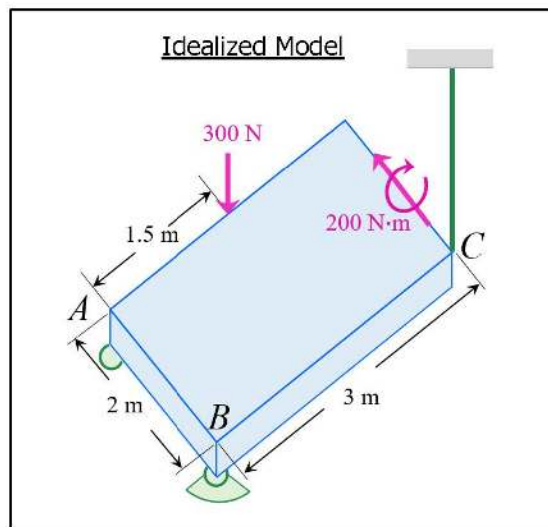
Solutions:

Types of the supports

A:

B:

C:



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Practice Problem

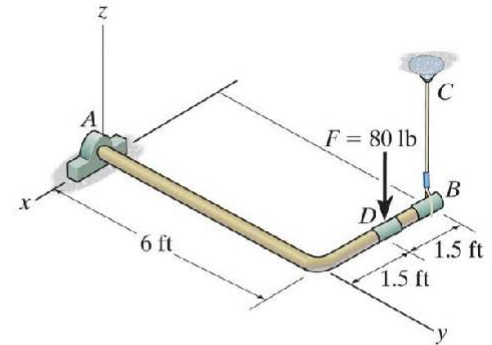
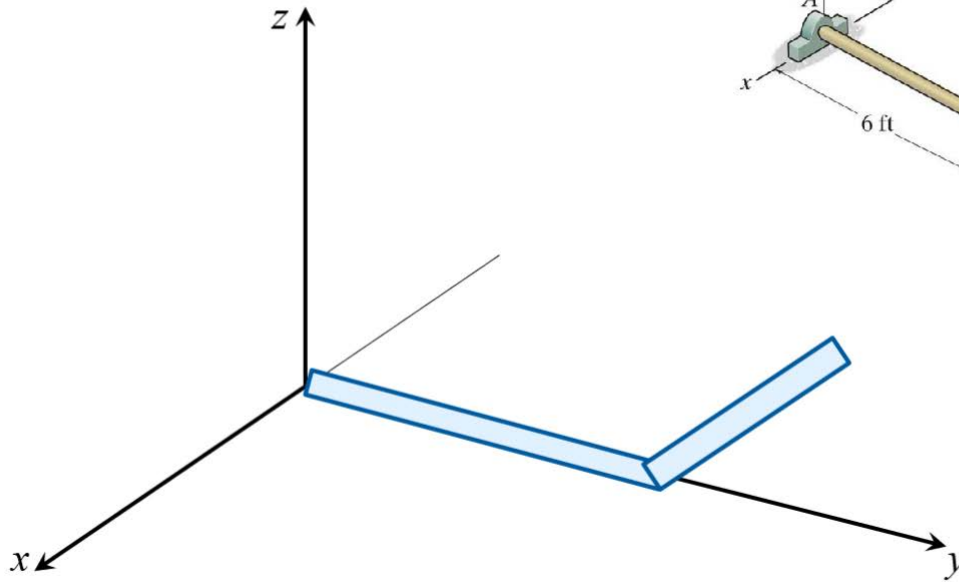
Topic 08 – 6

2. Determine the components of reaction that the thrust bearing A and cable BC exert on the bar.

Given: A : Thrust Bearing ; B : Cable ; $F = 80 \text{ lb}$

Find:

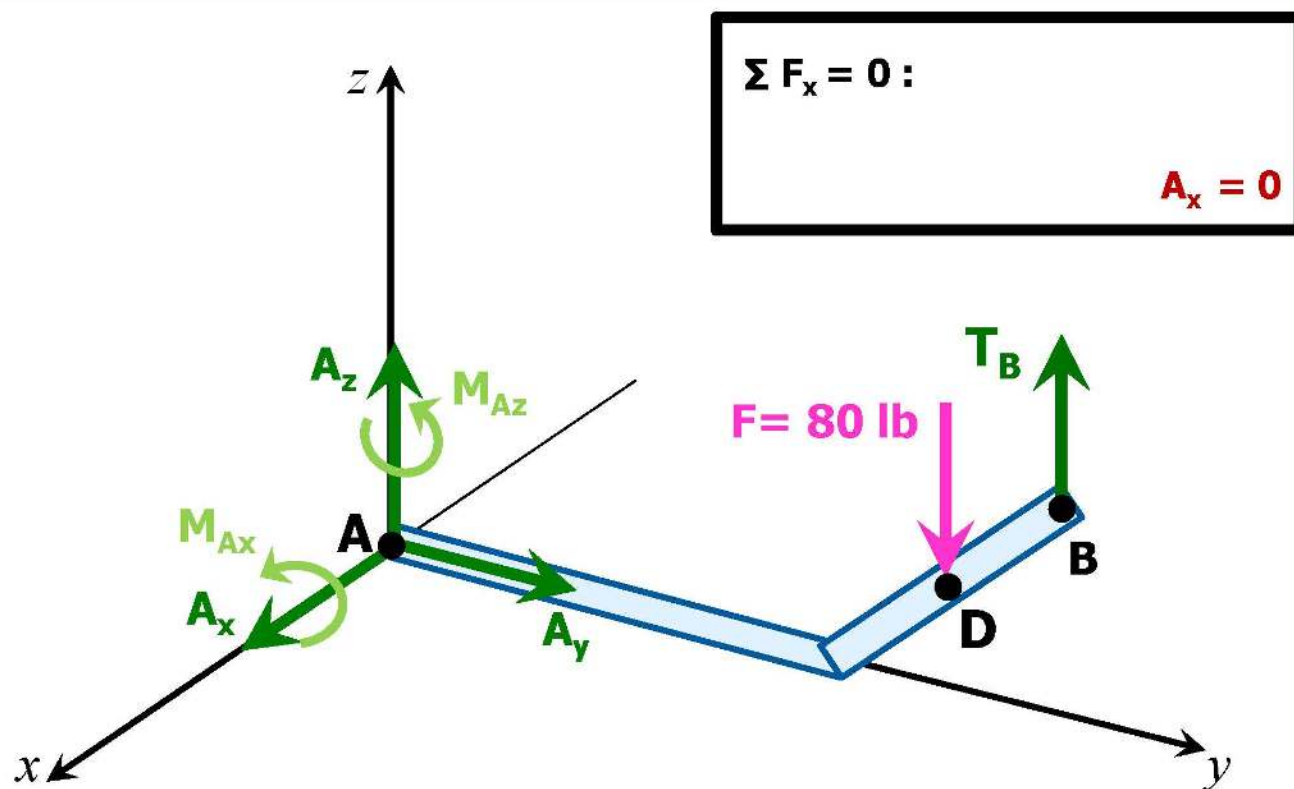
Solution:



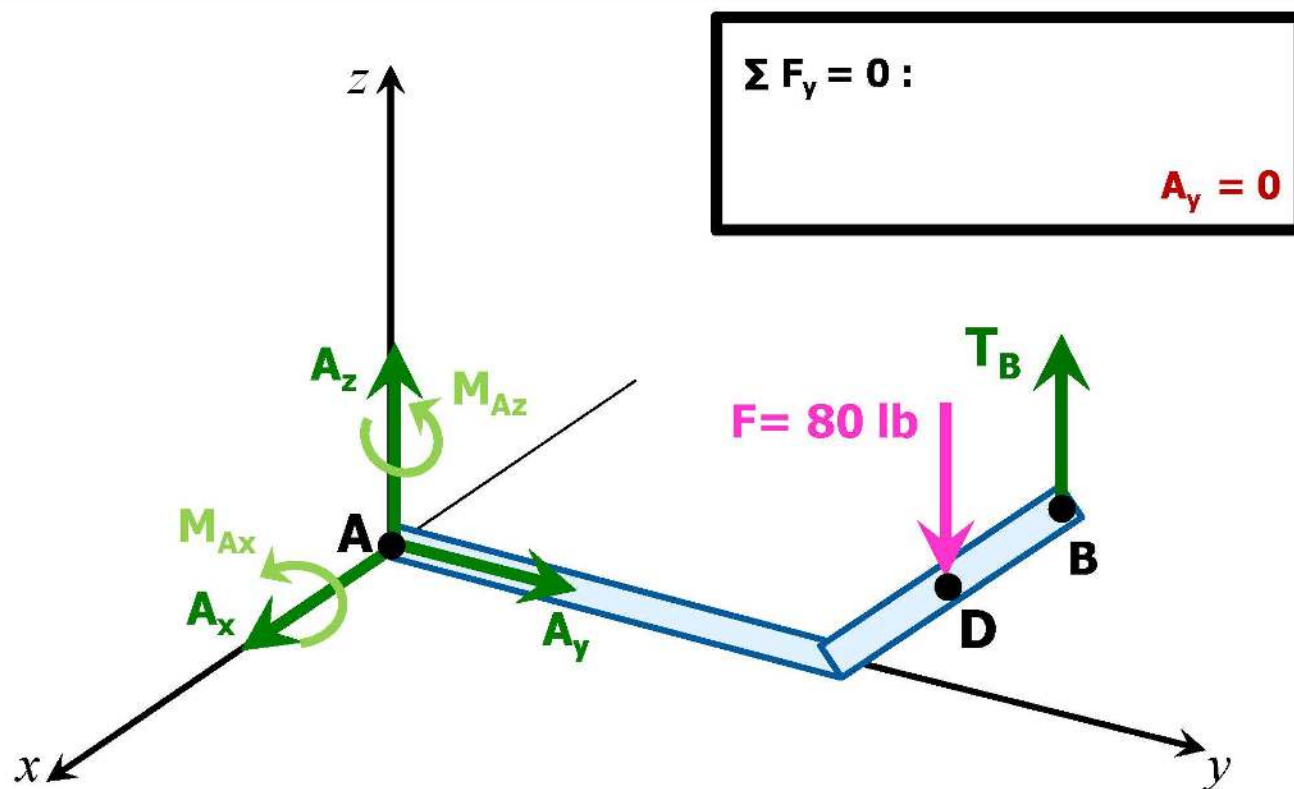
Answer:

$$A_x = 0 ; A_y = 0 ; A_z = 40.0 \text{ lb} ; M_{Ax} = 240 \text{ lb} \cdot \text{ft} ; M_{Az} = 0$$

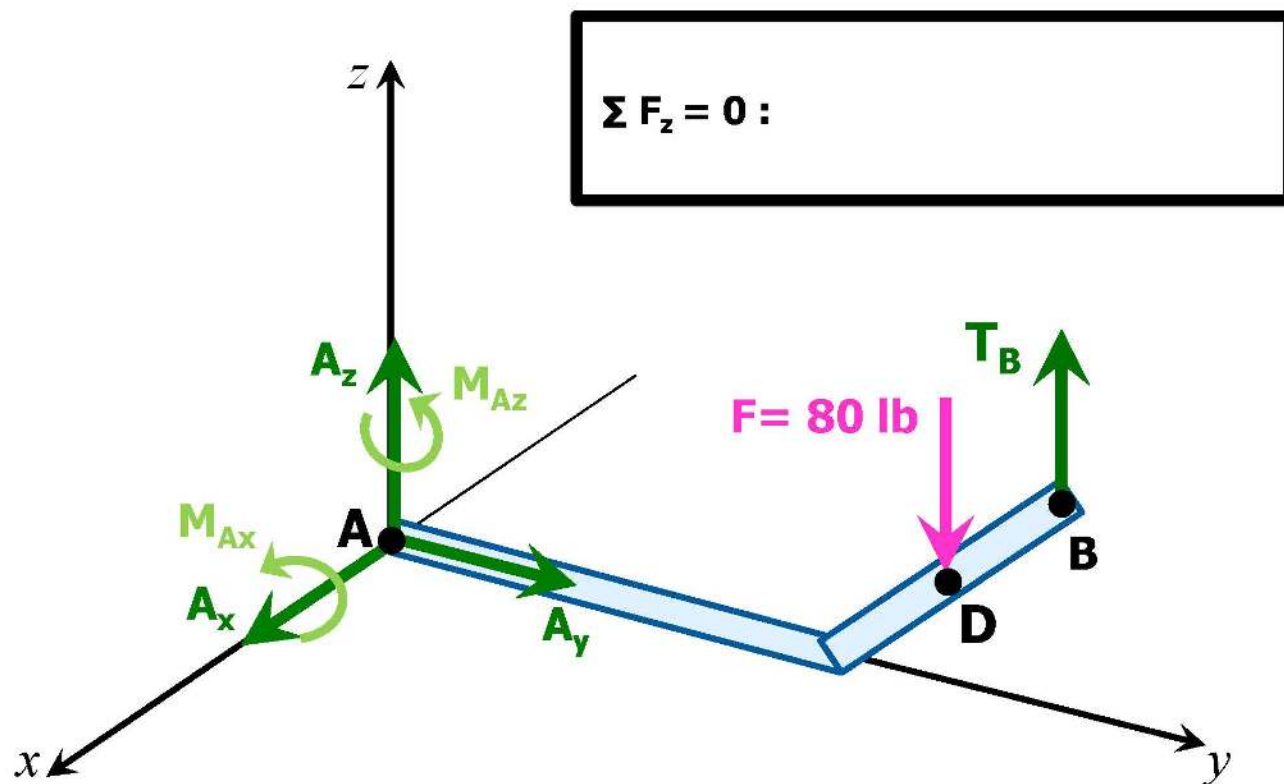
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In 3D, $\Sigma \mathbf{M} = 0$

In Scalar Analysis,

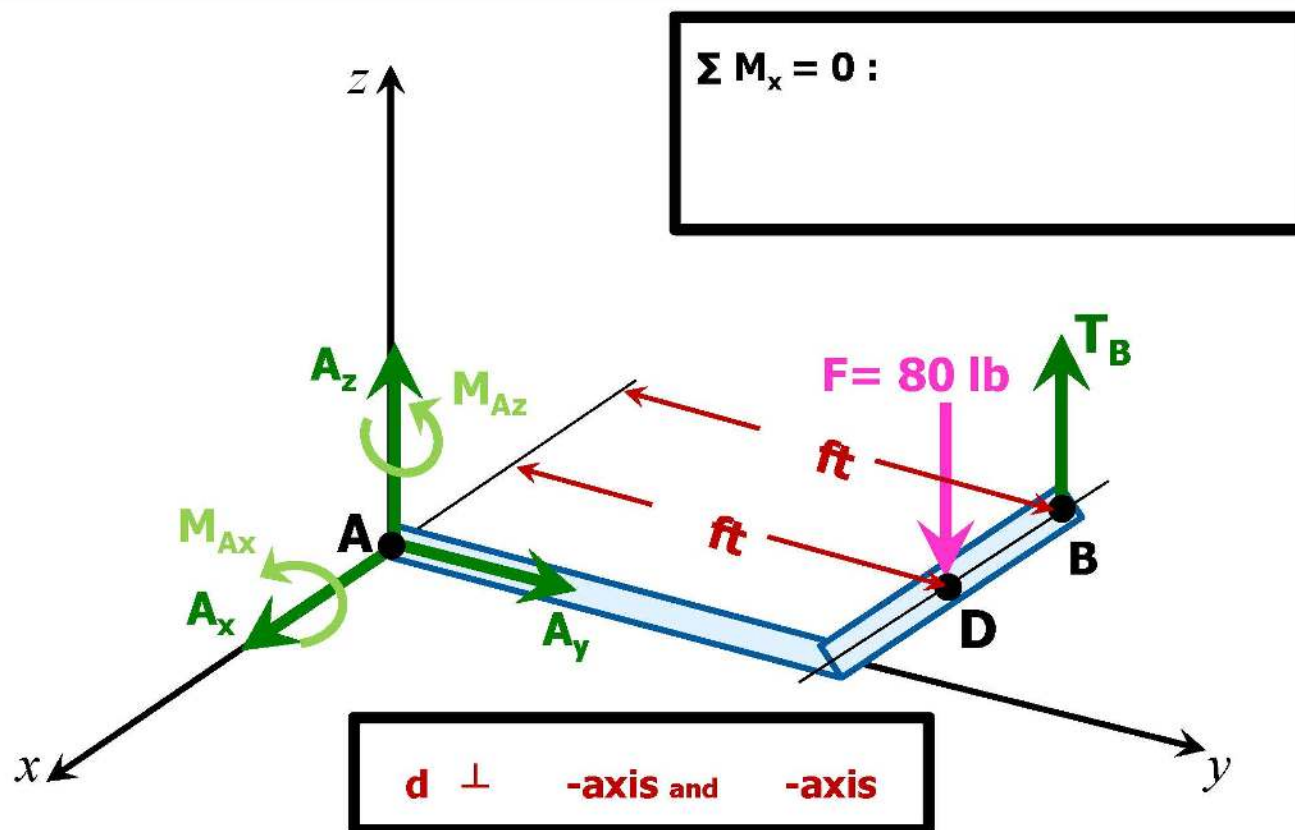
$M =$

where $d \perp$

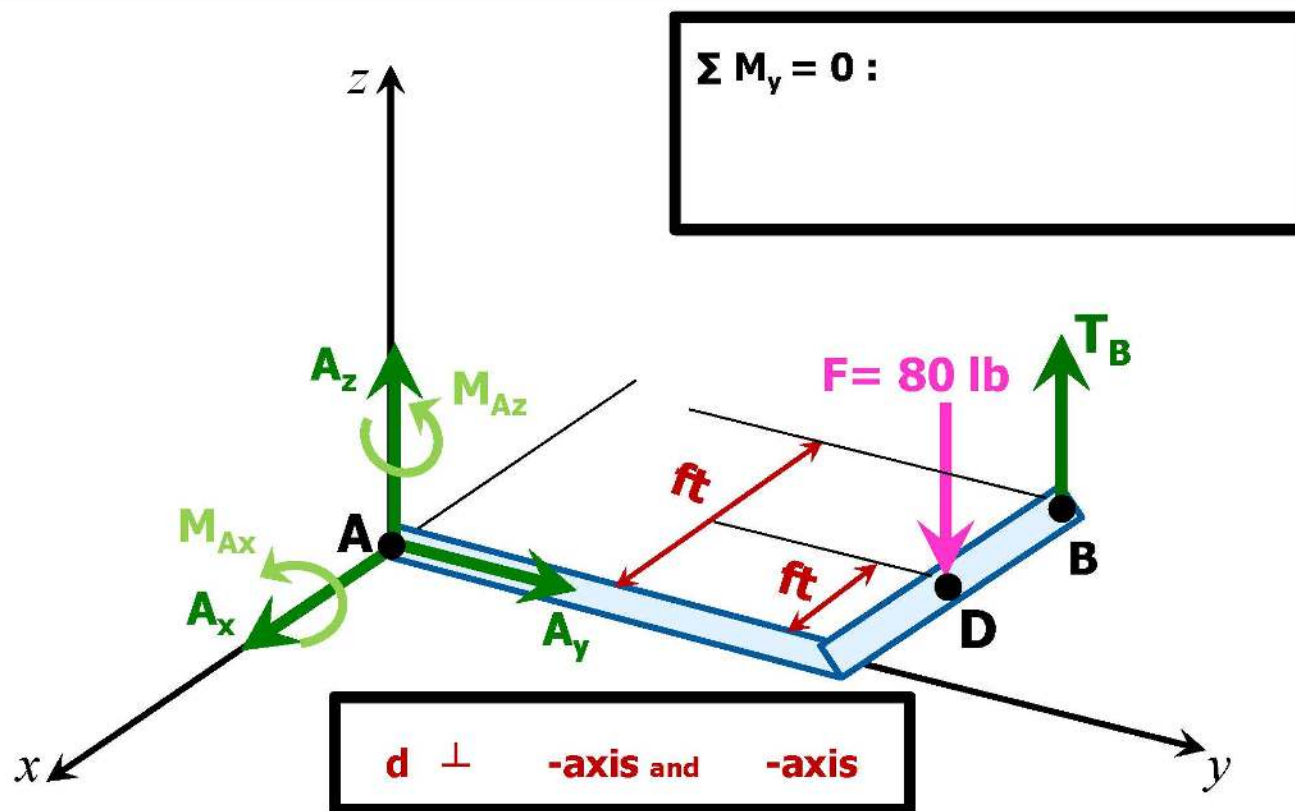
AND $\perp$

**If $\vec{F}$ is (1)
(2)**

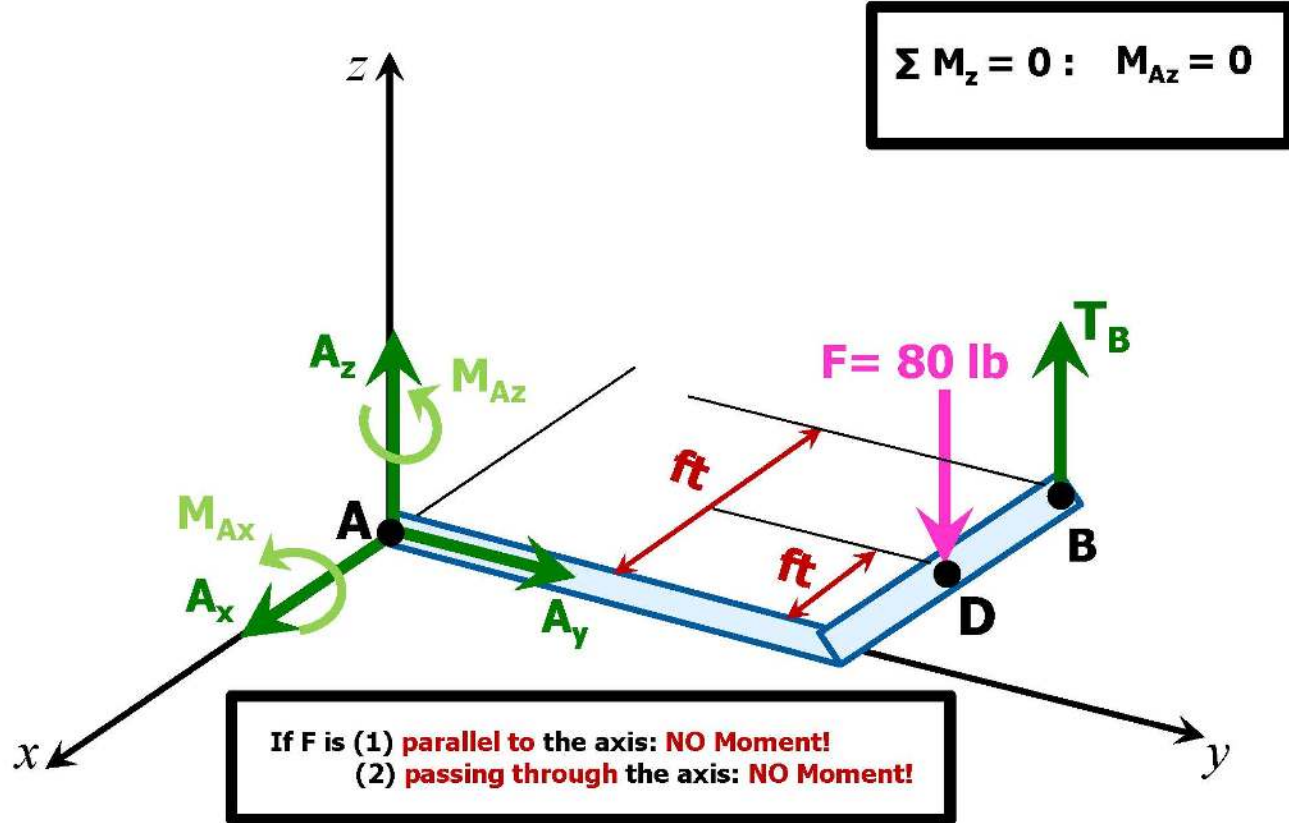
**to the Axis: NO Moment!
through the Axis: NO Moment!**



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How To Determine the Moment Arm for a Moment about an Axis in 3D

$$\text{In 3D, } \Sigma M_{\text{Axis}} = 0$$

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Topic 09. Equilibrium in 3D using Vector and Scalar Analysis

Topic 09 covers: 1) Identifying appropriate strategies to solve equilibrium problems in 3D; 2) Applying equations of equilibrium to solve 3D problems using both vector and scalar analysis; 3) Determining support reactions; 4) Describing statically indeterminate systems and improper constraints.

Understanding equilibrium in 3D requires not only the application of forces and moments but also the ability to select the appropriate methods to solve complex problems. This includes analyzing statically indeterminate systems, where the number of unknowns exceeds the available equations, and recognizing improper constraints that can destabilize the system. Mastering these concepts is essential for solving intricate 3D equilibrium problems and ensuring structural stability under various loading conditions.

At the end of this chapter, animations are provided to demonstrate 1) the process of solving both 2D and 3D equilibrium problems; 2) how to determine the moment arm for a moment about an axis in 3D.

Statics

Topic 09: Equilibrium in 3D using Vector & Scalar Analysis

Prof. J Lee

Objectives

- Identify appropriate strategies to solve equilibrium problems in 3D.
- Apply equations of equilibrium to solve 3D problems using vector + scalar analysis.
- Determine the reactions.
- Describe the statically indeterminate and improper constraints.

Applications



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Equilibrium in 3D using **Vector + Scalar Analysis**

Topic 09 – 2

• When

- (1) Forces are directed at _____.
- (2) Forces can be indicated with _____ vectors.

• How

Use “_____ Analysis” for resolving _____ as Cartesian and “_____ Analysis” for applying Equilibrium Equations.

Step 1: Define the coordinates.

Step 2: Apply the Vector Analysis to present each force as _____.

Recall: $\vec{F} = \text{_____}$ where $\vec{r}_{AB} = (\text{_____}) \hat{i} + (\text{_____}) \hat{j} + (\text{_____}) \hat{k}$

$$r_{AB} = (B_x - A_x) \quad (B_y - A_y) \quad (B_z - A_z)$$

Step 3: Apply the Scalar Analysis to apply Equilibrium Equations:

Note: For the moment equilibrium, apply _____ analysis for each force Cartesian component.

Recall: _____ where d : _____ distance from the _____ to the _____ of $\vec{F}$

AND d

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Practice Problem

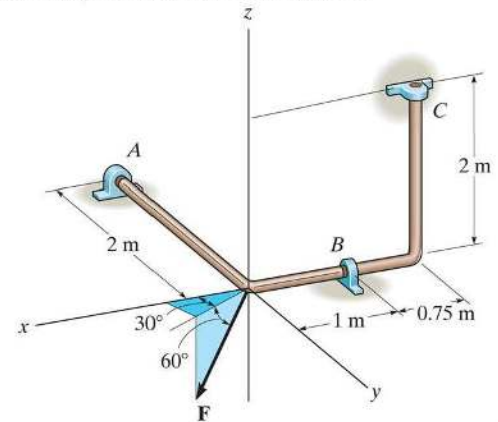
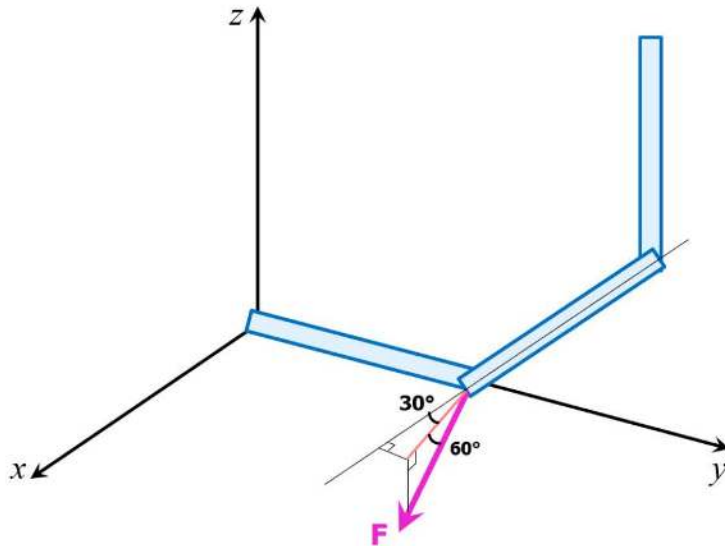
Topic 09 – 5

The bent rod is supported at A , B , and C by smooth journal bearings. Determine the components of reaction at the bearings if the rod is subjected to the force $F = 800$ N. The bearings are in proper alignment and exert only force reactions on the rod.

Given: $F = 800$ N; A, B, C : Smooth Journal Bearing

Find:

Solution:



Recall:

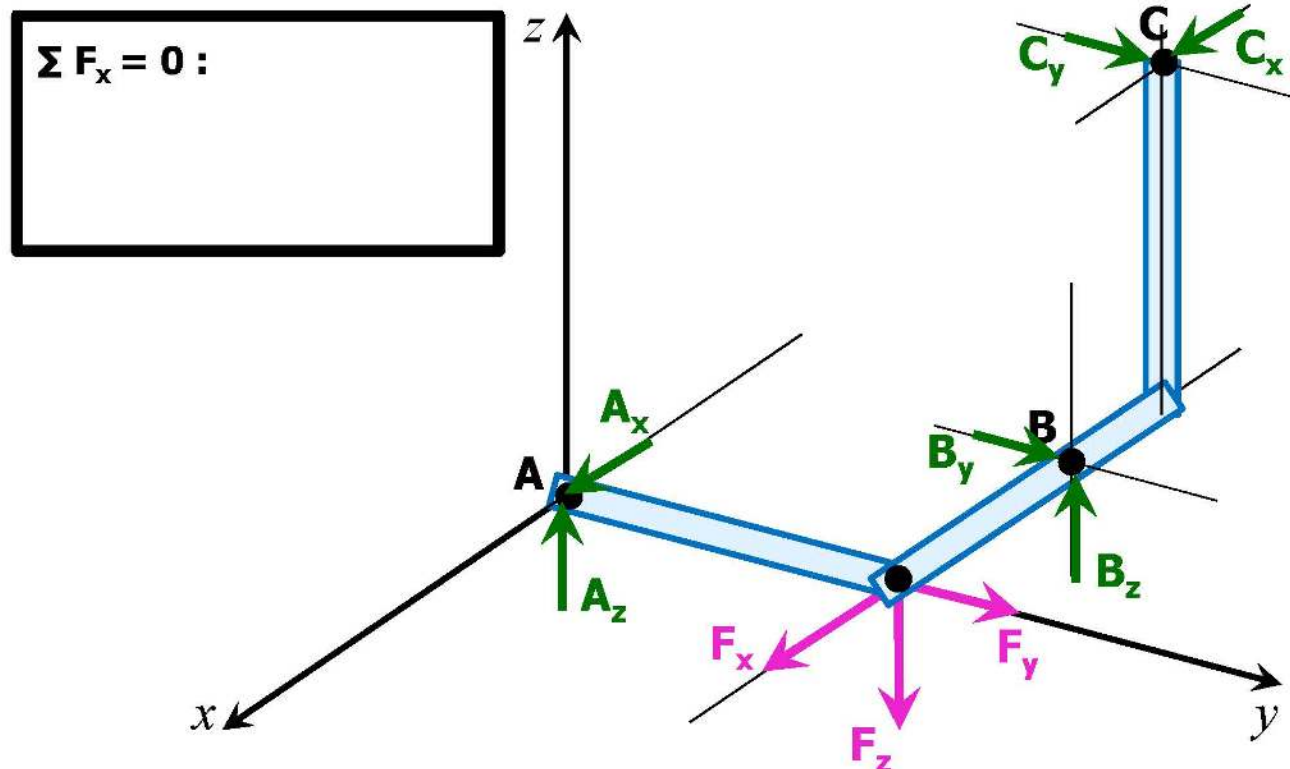
For Smooth Journal Bearing, the couple moments are generally _____ applied if the body supported _____

because the force reactions developed by bearings are sufficient for equilibrium.

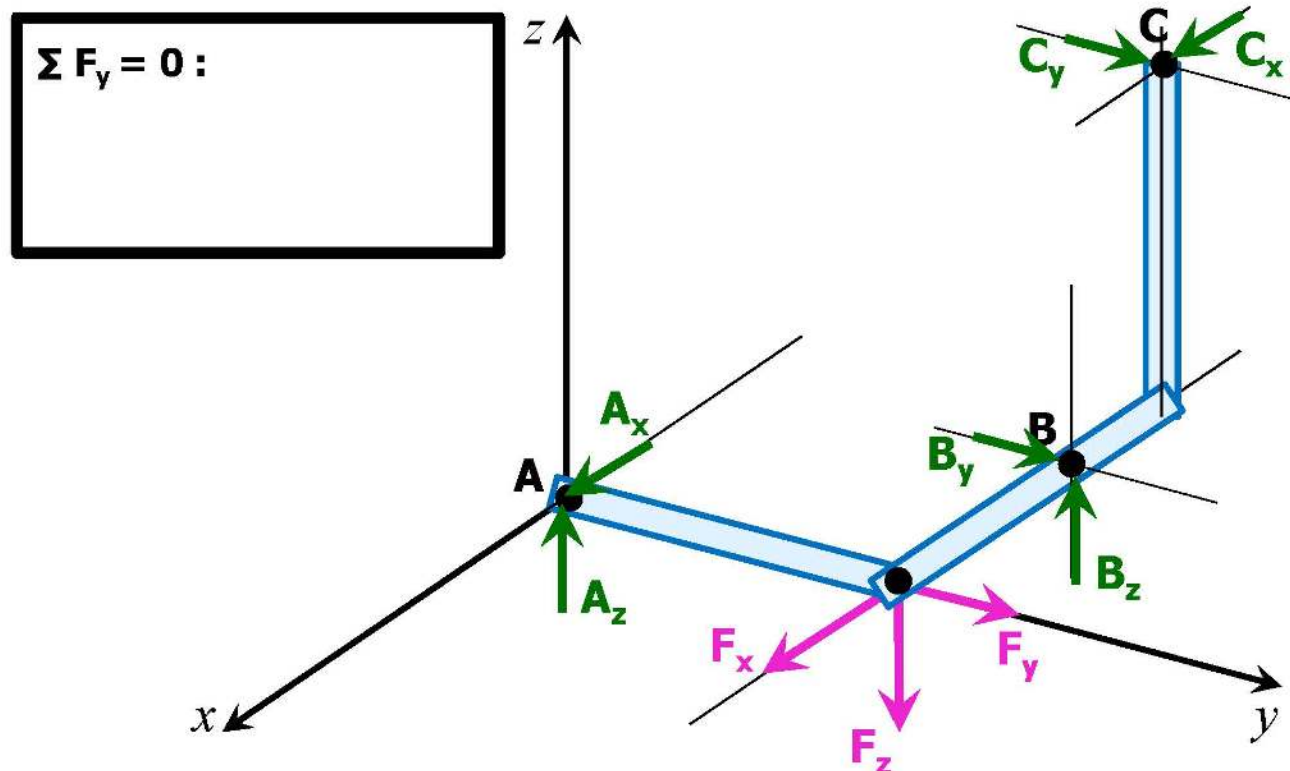
Answer:

$$\begin{aligned} A_x &= 400 \text{ N}, & B_y &= 600 \text{ N}, & C_x &= 53.6 \text{ N}, \\ A_z &= 800 \text{ N}, & B_z &= -107 \text{ N}, & C_y &= 80.0 \text{ N} \end{aligned}$$

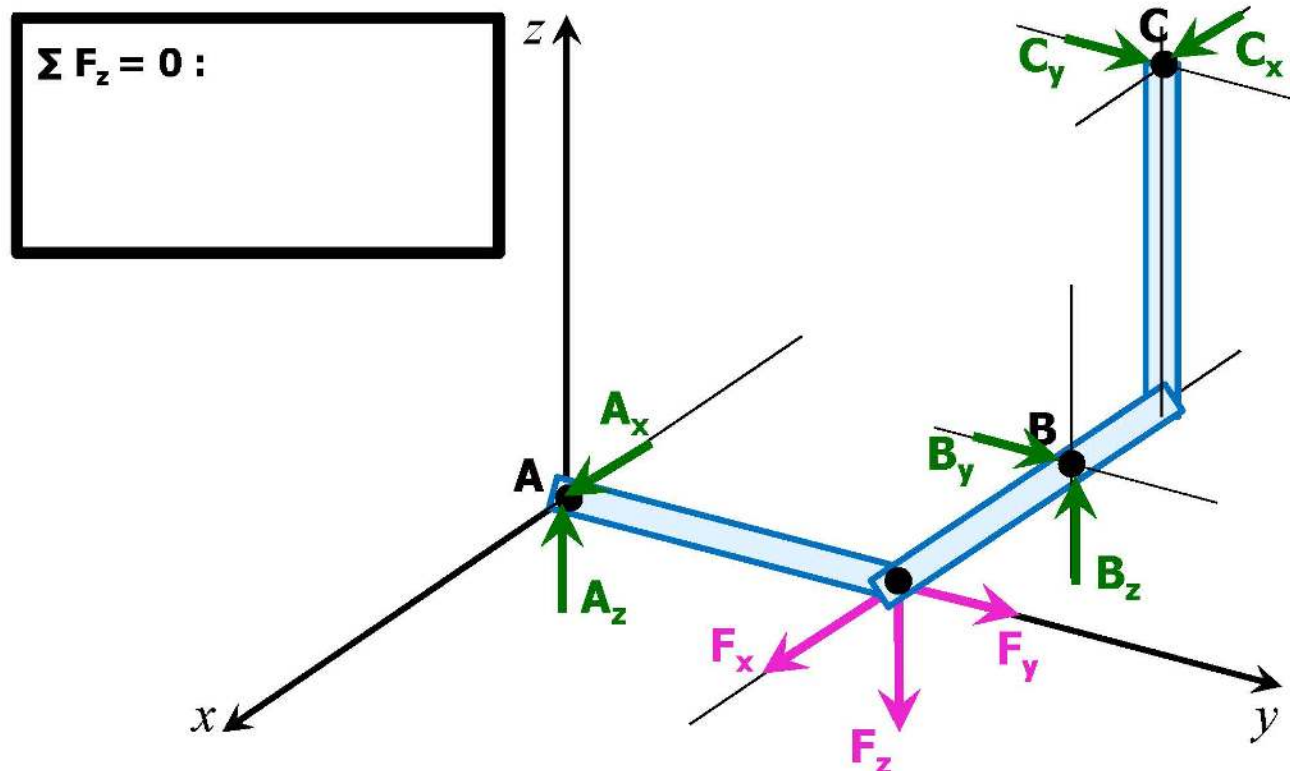
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In 3D, $\Sigma M = 0$

In Scalar Analysis,

$M =$

where $d \perp$

AND $\perp$

**If $\vec{F}$ is (1)
(2)**

**to the Axis: NO Moment!
through the Axis: NO Moment!**

$$\Sigma M_x = 0 :$$

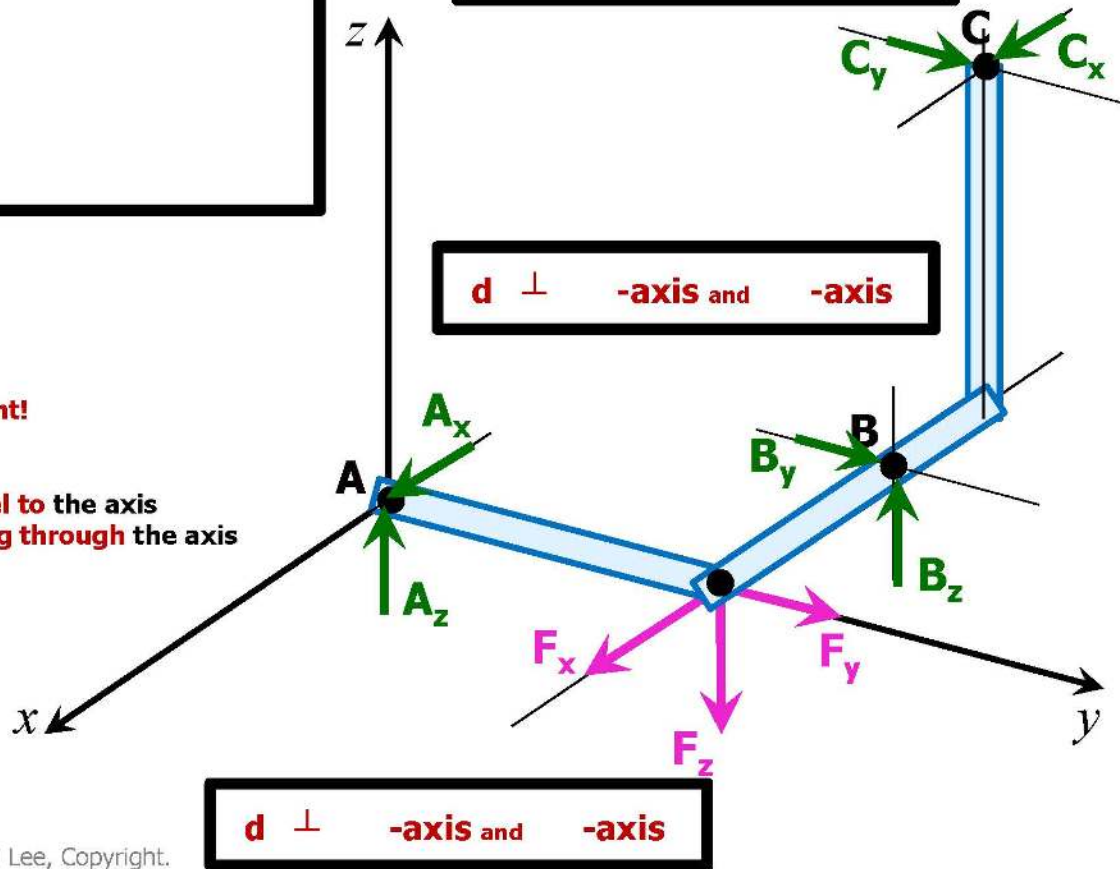
$d \perp$ -axis and -axis

$d \perp$ -axis and -axis

NO Moment!

If F is

- (1) **parallel to the axis**
- (2) **passing through the axis**

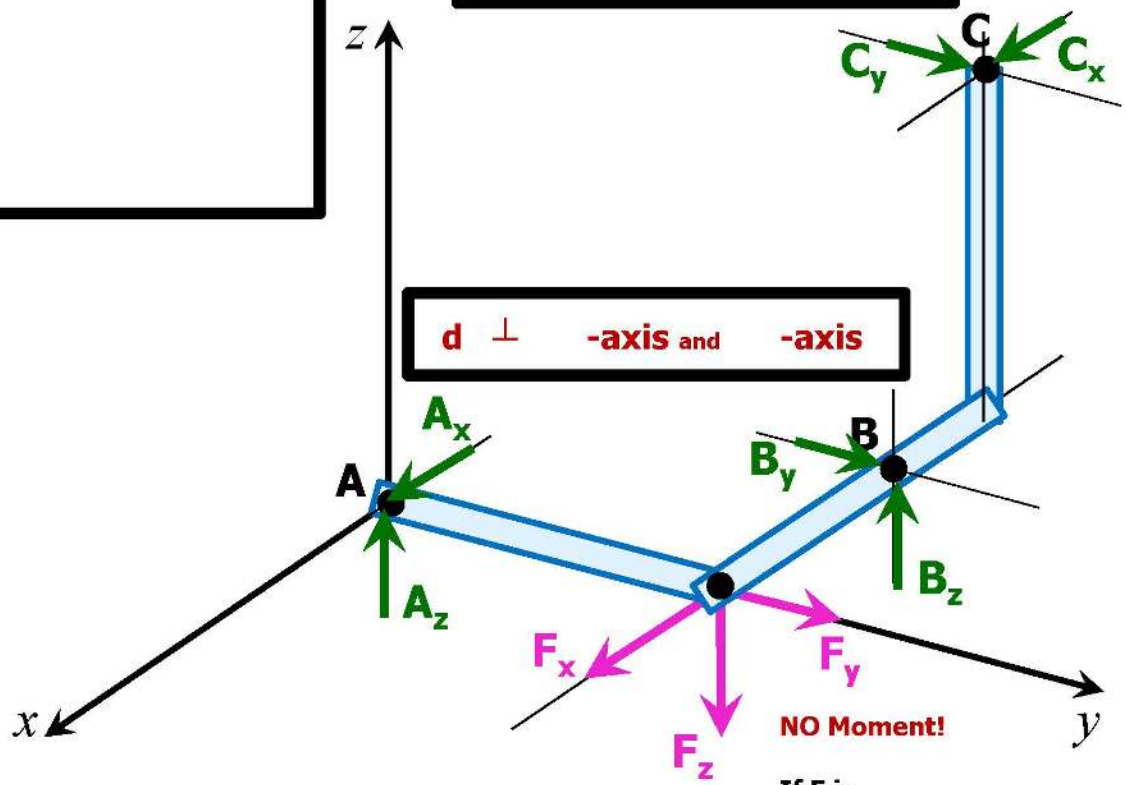


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$$\Sigma M_y = 0 :$$

$d \perp$ -axis and -axis

$d \perp$ -axis and -axis



If F is
 (1) **parallel to the axis**
 (2) **passing through the axis**

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$$\Sigma M_z = 0 :$$

$d \perp$ -axis and -axis

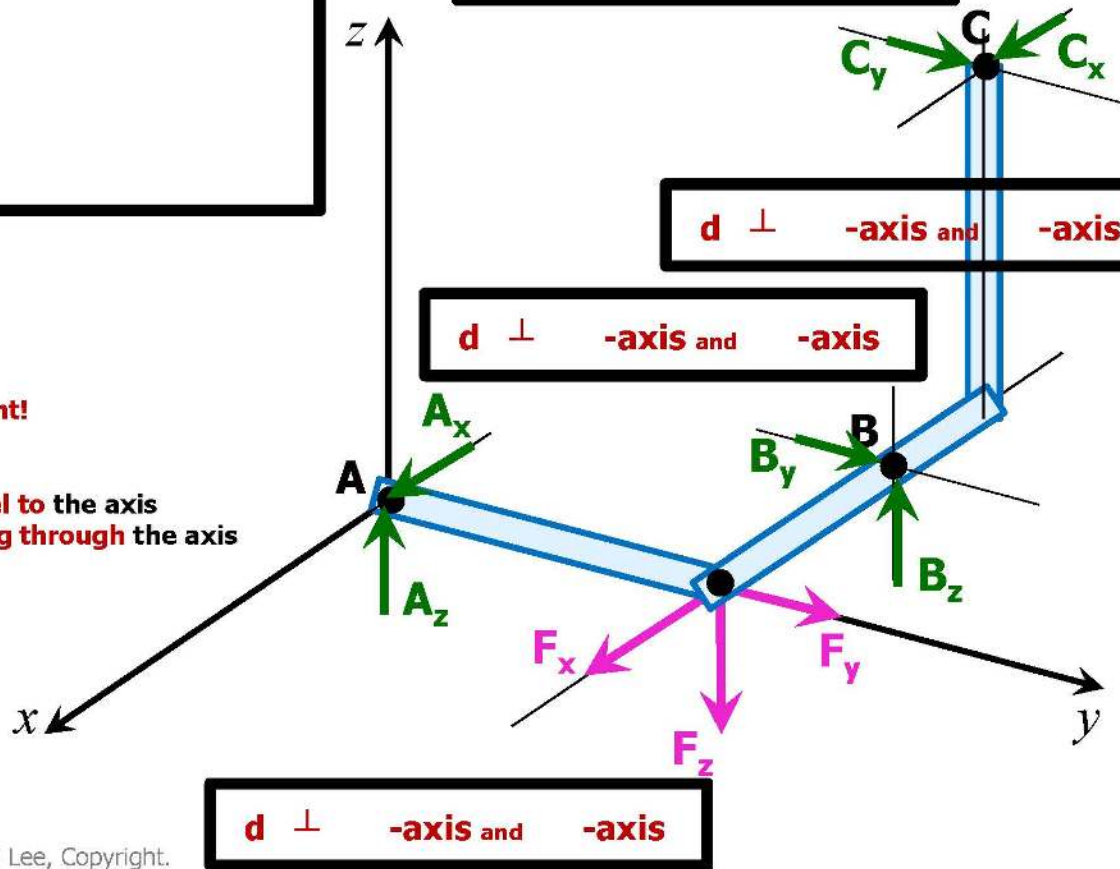
$d \perp$ -axis and -axis

$d \perp$ -axis and -axis

NO Moment!

If F is

- (1) **parallel to the axis**
- (2) **passing through the axis**



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Process for Equilibrium Problems in 2D and 3D

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How To Determine the Moment Arm for a Moment about an Axis in 3D

$$\text{In 3D, } \Sigma M_{\text{Axis}} = 0$$

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Topic 10. Moment about a Point in 3D

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Topic 12. Equivalent System: Distributed Loads

Topic 12 covers: 1) Explaining an equivalent system; 2) Describing a distributed load; 3) Determining an Equivalent Single Resultant Force and a Resultant Couple Moment from Distributed Loads; 4) Finding the Centroid for rectangular and right triangle-shaped bodies.

Converting distributed loads into an equivalent force and moment is crucial in Statics analysis, as many engineering systems can be effectively modeled as distributed load systems. Mastering these concepts is vital for analyzing complex equilibrium problems and maintaining structural stability under various loading conditions.

Animations are included to demonstrate: 1) how to convert a distributed load into an equivalent force; 2) how to find the centroid for a rectangle and a right triangle.

Extra sheets are provided at the end to review integrals and centroid calculations for simpler geometries, aiding in determining areas when irregular loading shapes are applied to a body.

Statics

Topic 12: Equivalent System: Distributed Loads

Prof. J Lee

Objectives & Applications

Topic 12 – 1

Objectives

- Explain an Equivalent System.
- Explain a Distributed Load.
- Determine an Equivalent Single Resultant Force and a Resultant Couple Moment from Distributed Loads.
- Determine the Centroid for Rectangle and Right Triangle Shaped Bodies.

Applications



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Equivalent System

Topic 12 – 2

• What

: a _____ resultant _____ and _____ that have the _____ external effect *with the original forces and moments*
in order to make **easier to understand the overall effects of the loadings**

• Characteristics

- Force is a _____ vector. 

: Force can move to any point on its _____ since it will have the _____ external effect by the principle of transmissibility.

- Couple moment is a _____ vector.

: Couple moment can be applied at any point on the body: _____ external effect

- If a point is _____ on the LOA of the forces, then the moving forces **require** _____ **couple moment** to make the same external effect.

Practice Problem

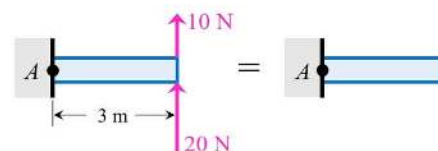
When A is on LoA of $\vec{F}$



$$\vec{F}_R =$$

$$(\vec{M}_R)_A =$$

When A is NOT on LoA of $\vec{F}$



$$\vec{F}_R =$$

$$(\vec{M}_R)_A =$$

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Distributed Load

Topic 12 – 3

• What

: When **forces are caused by winds, fluids, or the weight of items on a** _____'s _____,

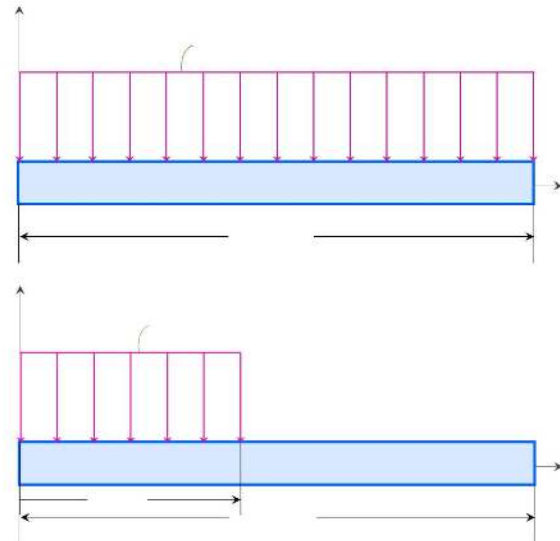
the surface area of the body is subjected to **a distributed load**.

: For analysis, assume that **a distributed load is a load along** _____ of a flat rectangular body.

: defined as a _____ **function**, _____ as a function of _____ :

:

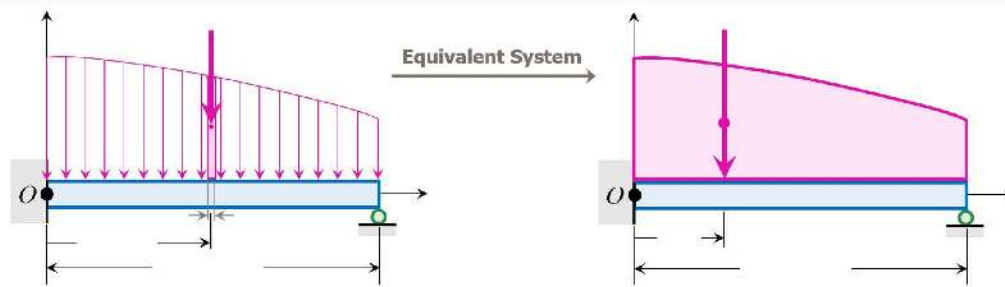
: Unit [_____]



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Resultant Force of a Distributed Load

Topic 12 – 4



Force per Unit Length, **Loading Function:**

$w(x)$

Force for an Element of Length dx

:

$=$

$[]$

Magnitude of Resultant Force, $F_R =$

$=$

$=$ the under the loading diagram

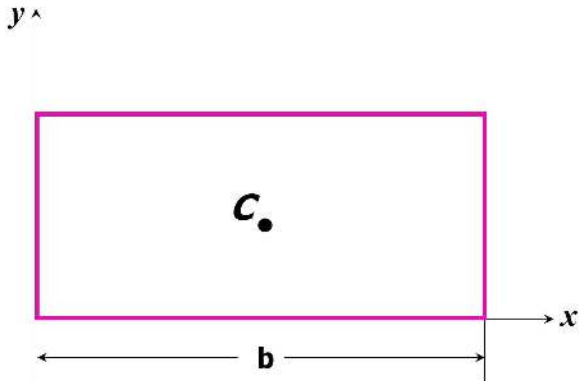
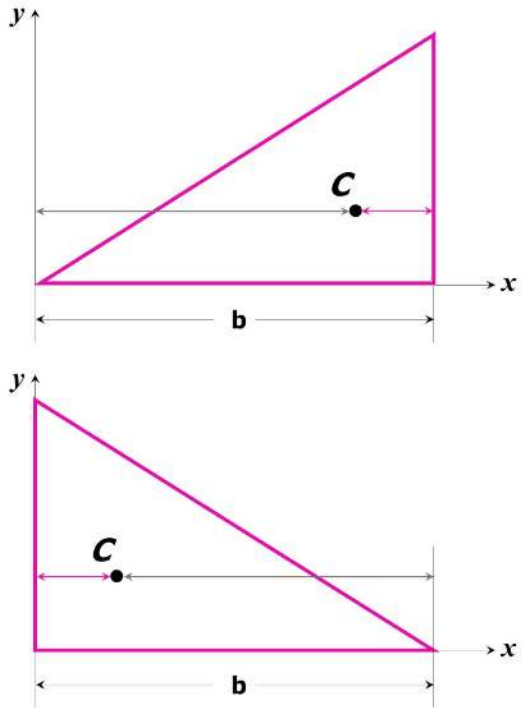
Location of Resultant Force

$(\vec{M}_R)$ $=$ $\rightarrow$ $= \int$ $=$

$\bar{x} =$ $= \frac{\int_L x w(x) dx}{F_R}$ $=$

$=$ the of the Area under the Distributed Loading

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Rectangle	Right Triangle
 A diagram of a rectangle with width b and height h. The centroid C is marked at the center. A coordinate system (x, y) is shown with the origin at the bottom-left corner.	 Two diagrams of right triangles. The top triangle has its right angle at the bottom-right corner, with base b and height h. The centroid C is marked. The bottom triangle has its right angle at the bottom-left corner, with base b and height h. The centroid C is marked. Both diagrams include a coordinate system (x, y) with the origin at the right-angle vertex.

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Animation to Demonstrate How to Find the Centroid for 1) a Rectangle; 2) a Right Triangle

Centroid: Rectangle

Topic 12

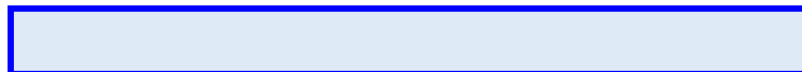


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Animation to Demonstrate How to Convert a Distributed Load into an Equivalent Force

Distributed Load

Topic 12



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Extra Sheets for 1) Integral; 2) Centroid for Simpler Geometry

Integral

$$\int a \, dx = a x$$

$$\int x \, dx = \frac{x^2}{2} \quad \int x^n \, dx = \frac{x^{n+1}}{n+1} \quad \int a^x \, dx = \frac{a^x}{\ln a}$$

$$\int \frac{1}{x} \, dx = \ln|x| \quad \int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln|ax+b| \quad \int \frac{1}{(x+a)^2} \, dx = -\frac{1}{x+a}$$

$$\int \ln(x) \, dx = x \ln(x) - x \quad \int \ln(ax) \, dx = x \ln(ax) - x \quad \int \ln(ax+b) \, dx = \left(x + \frac{b}{a}\right) \ln(ax+b) - x$$

$$\int e^x \, dx = e^x \quad \int e^{ax} \, dx = \frac{1}{a} e^{ax} \quad \int x e^x \, dx = (x-1) e^x \quad \int x e^{ax} \, dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

$$\int \sin x \, dx = -\cos x \quad \int \sin ax \, dx = -\frac{\cos ax}{a} \quad \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$\int \cos x \, dx = \sin x \quad \int \cos ax \, dx = \frac{\sin ax}{a} \quad \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$\int \tan x \, dx = -\ln|\cos x| = \ln|\sec x|$$

Integral Rules

• Multiplication by Constant

• Power Rule ($n \neq -1$)

• Sum & Difference Rules

$$\int a f(x) \, dx = a \int f(x) \, dx \quad \int x^n \, dx = \frac{x^{n+1}}{(n+1)} \quad \int (f \pm g) \, dx = \int f \, dx \pm \int g \, dx$$

(where a is the constant)

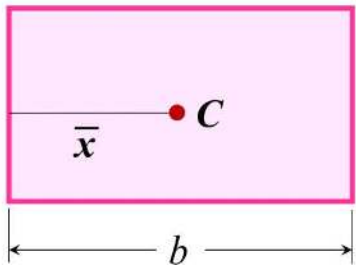
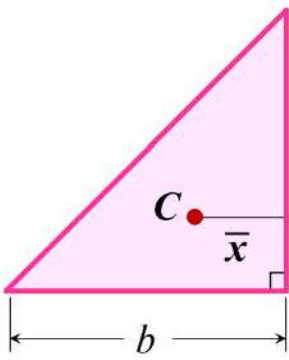
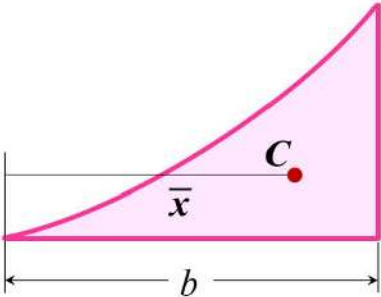
Recall

$$\sqrt{x} = (x)^{\frac{1}{2}}$$

$$\sqrt[n]{x} = (x)^{\frac{1}{n}}$$

Centroid

Center of mass; A point where all the medians of an object

Shape	Figure	$\bar{x}$
Rectangle		$\frac{b}{2}$
Right Triangle		$\frac{b}{3}$
Parabolic Spandrel		$\frac{3b}{4}$

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Topic 13. Internal Forces: Shear and Moment Diagram

Topic 13 covers: 1) Explaining internal forces; 2) Explaining and drawing shear and moment diagrams; 3) Determining internal forces and bending moments in 2D systems.

Understanding internal forces is essential in Statics, as these forces are responsible for the internal stability and structural integrity of beams and other elements under load. Shear and moment diagrams play a crucial role in visualizing how internal forces vary across a structure, helping engineers identify critical points of stress and design appropriate reinforcements. Mastering these concepts ensures accurate analysis and safe design of engineering structures, particularly when subject to complex loading conditions.

Statics

Topic 13: Internal Forces: Shear and Moment Diagrams

Prof. J Lee

Objectives

- Explain the internal forces.
- Explain and draw shear and moment diagrams.
- Determine internal forces and bending moments in 2D systems.

Applications



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• What

: In 2D cases, typical internal forces are

(1) _____ force (): an axial force acting _____ to the cut section;

(2) _____ force (): a force acting _____ the surface;

(3) _____ moment ()

• Sign Convention



The forces on the left/right sides of the imaginary cut section at A are _____ **in Magnitude**, but _____ **in Direction** as when the two sides are reconnected, **the net forces are zero at the section**.

• Shear and Moment Diagrams

: graphical descriptions of how the internal _____ () and _____ () vary throughout the length of the member

: used - to _____ a structural member;

- to find distribution of forces acting within the member to check the member can support the applied loads _____ failure

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Shear and Moment Diagrams

Topic 13 – 3

• Steps

Given: Bearing supports at A and C.

1. Define the coordinates:

2. Draw a Global FBD and determine the reactions from a Global FBD:

$$\sum M_A = 0: -()5 + ()C_y = 0 \quad \therefore C_y = 2.50$$

$$\sum F_y = 0: A_y - 5 + = 0 \quad \therefore A_y = 2.50$$

3. Starting with the _____ end, _____ the beam into sections after each concentrated force or moment.

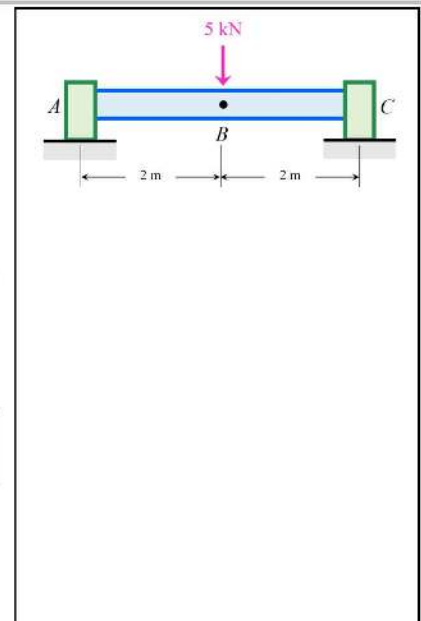
4. Draw a FBD of the _____ section, and write expressions for _____ and _____ in each section. Solve equilibrium equations to determine $V(x)$ and $M(x)$ within the ranges of _____.

_____ : the distance **FROM** the _____ **TO** the _____

5. Draw axes for shear () and moment () as a function of length of the beam, _____.

6. Draw a dashed line where each force or moment is applied or changed.

7. Plot $V(x)$ and $M(x)$ for the section.



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Practice Problem

Topic 13 – 4

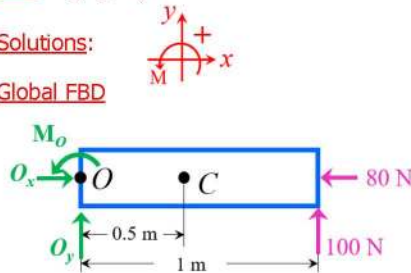
Determine the internal normal, shear, and bending moment at C .

Given: two active forces; geometric information; Fixed support at O

Find: N_C, V_C, M_C

Solutions:

Global FBD



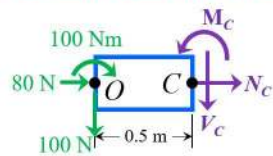
Applying the Equilibrium Equations to Determine the Reactions at O ,

$$\sum F_x = 0: O_x - 80 = 0 \quad \therefore O_x = 80 \text{ N}$$

$$\sum F_y = 0: O_y + 100 = 0 \quad \therefore O_y = -100 \text{ N}$$

$$\sum M_O = 0: M_o + 100 = 0 \quad \therefore M_o = -100 \text{ Nm}$$

FBD for the Left Cut Part at Cut C



$$\sum F_x = 0: 80 + N_C = 0$$

$$\therefore N_C = -80 \text{ N}$$

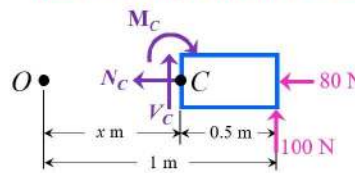
$$\sum F_y = 0: -100 - V_C = 0$$

$$\therefore V_C = -100 \text{ N}$$

$$\sum M_C = 0: -100 + (0.5) 100 + M_C = 0$$

$$\therefore M_C = 50 \text{ Nm}$$

FBD for the Right Cut Part at Cut C



$$\sum F_x = 0: -N_C - 80 = 0$$

$$\therefore N_C = -80 \text{ N}$$

$$\sum F_y = 0: V_C + 100 = 0$$

$$\therefore V_C = -100 \text{ N}$$

$$\sum M_C = 0: -M_C + (0.5) 100 = 0$$

$$\therefore M_C = 50 \text{ Nm}$$

Answer:

$$N_C = -80.0 \text{ N}; V_C = 100 \text{ N}; M_C = 50.0 \text{ Nm}$$

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Topic 14. Truss: Method of Joints

Topic 14 covers: 1) Explaining a simple truss; 2) Explaining the assumptions used in force analysis on a truss; 3) Identifying zero force members; 4) Determining the forces in members of a simple truss using the method of joints.

Understanding trusses is essential in Statics, as truss structures are widely used in engineering applications due to their efficiency in carrying loads. The method of joints provides a systematic approach to analyzing the forces within individual members of a truss, ensuring the stability and integrity of the structure. Identifying zero force members is also critical for simplifying the analysis and optimizing the design. Mastering these concepts ensures accurate force analysis in truss systems and contributes to the safe and efficient design of engineering structures.

Statics

Topic 14: Truss: Method of Joints

Prof. J Lee

Objectives & Applications

Topic 14 – 1

Objectives

- Explain a simple truss.
- Explain the assumptions used in force analysis on a truss.
- Identify zero force members.
- Determine the forces in members of a simple truss using the method of joints.

Applications



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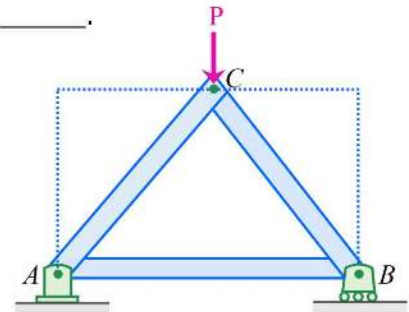
• What

: a structure composed of slender _____ joined together at their _____ points.

- Planar Trusses: the loading acts in the _____ plane as the truss.
- Simple Truss: A planar truss which begins **with a triangular element** and **can be expanded** by adding _____ members and _____.

• Assumptions for Truss Analysis

- (1) All loads are applied at the _____; _____ of the members is neglected.
- (2) The members are joined by smooth _____:
 - Only _____ reaction **at joints**
 - _____ moments **at joints**
- (3) The members are loaded in either "**Tension**" () or "**Compression**" ().



• Truss Analysis Methods

- (1) Method of _____: to determine forces in _____ member – Topic
- (2) Method of _____: to determine forces for only few specific members – Topic

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Truss Analysis – Method of Joints

Topic 14 – 3

• Steps

1. Define the coordinates:
2. Determine the reactions from the global FBD.

$$\Sigma M = 0:$$

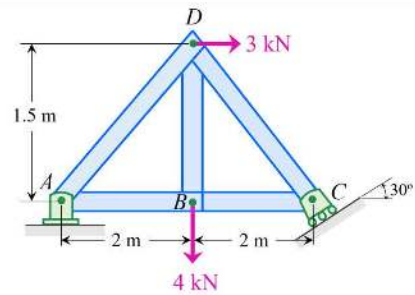
$$\Sigma F_x = 0: A_x - 3 + R_C \sin 30^\circ = 0$$

$$\Sigma F_y = 0: A_y - 4 + R_C \cos 30^\circ = 0$$

$$\therefore R_C = 3.608 \text{ kN}$$

$$\therefore A_x = -1.196 \text{ kN}$$

$$\therefore A_y = 0.875 \text{ kN}$$



3. Determine the member forces at each joint.

- (1) Draw a FBD for each joint and apply the equilibrium equations to the joint.

Note: Important Assumptions

All unknown member forces act in **Tension** (). If the answer is **positive**, the assumed direction () is correct, **otherwise** the member is in ().

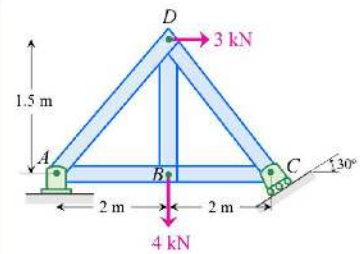
Recall: Only _____ reactions at Joints.
:

$$\therefore F_{AB} = 2.362 \text{ kN}$$

$$F_{AD} = -1.458 \text{ kN}$$

- (2) Repeat Step 3(1) with each successive joint until the forces in each member are known.

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$$\therefore F_{BC} = 2.362 \text{ kN}$$

$$F_{BD} = 4 \text{ kN}$$

$$\therefore F_{CD} = -5.208 \text{ kN}$$

$$F_{CB} = 2.362 \text{ kN}$$

Answer:

$$F_{AB} = 2.36 \text{ kN ()}$$

$$F_{BD} = 4.00 \text{ kN ()}$$

$$F_{CB} = 2.36 \text{ kN ()}$$

$$F_{AD} = 1.46 \text{ kN ()}$$

$$F_{CD} = 5.21 \text{ kN ()}$$

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• What

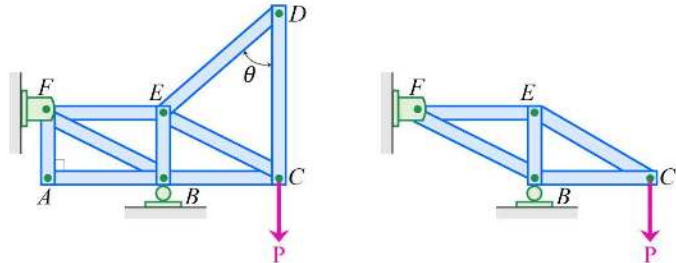
: A member that carries _____

(1) _____ members **at a joint** & _____ **load and reaction at the joint**

→ Then, **the two members** are zero-force members.

Steps:

1. Select a joint.
2. Two members at the joint?
3. Non-collinear at the joint?
4. No load and reactions at the joint?

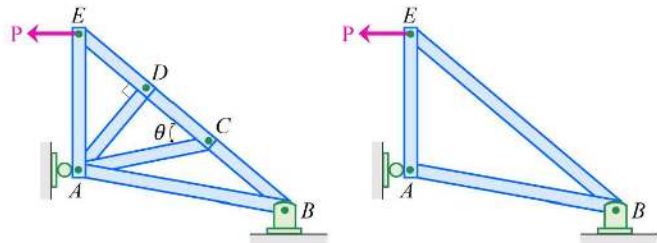


(2) _____ members **at a joint** & _____ members **with no load or reaction at the joint**

→ Then, **the _____ member** is a zero-force member.

Steps:

1. Select a joint.
2. Three members at the joint?
3. Two-collinear at the joint?
4. No load and reactions at the joint?



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Topic 15. Truss: Method of Sections

Topic 15 covers: 1) Explaining the method of sections for truss analysis; 2) Determining the forces in members of a simple truss using the method of sections.

Understanding the method of sections is crucial in Statics, as it provides an efficient approach for analyzing specific members within a truss without evaluating the entire structure. This method allows engineers to cut through the truss and focus on a section to directly solve for internal forces in targeted members. Mastering the method of sections is essential for accurate and efficient force analysis, contributing to the stability and safety of truss structures in engineering applications. This approach complements the method of joints and enhances the ability to design and analyze complex truss systems effectively.

Statics

Topic 15: Truss: Method of Sections

Prof. J Lee

Objectives

- Explain a method of sections for truss analysis.
- Determine the forces in members of a simple truss using the method of sections.

Applications



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Truss Analysis – Method of Sections

Topic 15 – 2

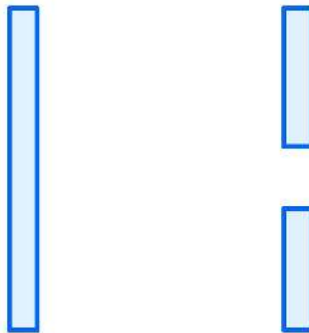
• What

: to find forces in only a few _____ members

: a truss is divided into _____ parts by taking an *imaginary* _____ through the truss.

: _____ forces at **the cut member** will also be either _____ or _____

with the _____ magnitude (by the Newton's _____ Law).



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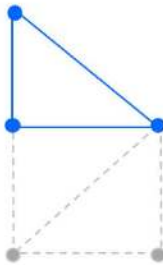
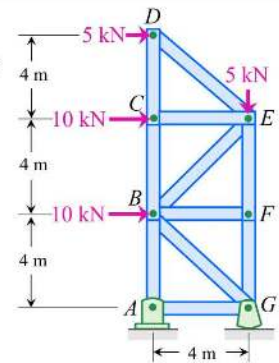
Truss Analysis – Method of Sections

Topic 15 – 3

• Steps

1. "_____ " the truss based on where you need to determine forces and where the total number of unknowns does not exceed _____ in general: i.e., **To find the forces in member BE , EF , and CB**
2. Draw a _____ for the _____ truss.
3. Decide **a** _____ **of the cut** to identify the force in members: Only select _____ section!
4. If necessary, determine the external _____ using the global FBD of the entire truss.
5. Draw another _____ of the selected section indicating _____ forces at the cut members.

Note: Assume the internal forces in members are in _____. Then, apply the equilibrium equations to the selected section of the truss to solve for the unknown force members.



Answer: $F_{BE} =$ () ; $F_{EF} =$ () ; $F_{CB} =$ ()

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Topic 16. Frames and Machines

Topic 16 covers: 1) Explaining frames and machines; 2) Describing two-force and three-force members and their characteristics; 3) Identifying whether frames or machines contain two-force and/or three-force members and applying these characteristics for static analysis; 4) Drawing free body diagrams of a frame or machine and its members; 5) Determining the forces acting at the joints and supports of a frame or machine.

Understanding frames and machines is essential in Statics, as these structures are commonly used in engineering to support loads and perform mechanical tasks. Frames and machines often consist of interconnected members that transfer forces through joints and supports. Analyzing these systems requires identifying two-force and three-force members and applying their unique characteristics to simplify and accurately model forces within the structure. Mastering these concepts enables engineers to conduct precise static analysis, ensuring the stability and functionality of frames and machines under various loading conditions.

Statics

Topic 16: Frames and Machines

Prof. J Lee

Objectives & Applications

Topic 16 – 1

Objectives

- Explain frames and machines.
- Explain two-force and three-force members and their characteristics.
- Identify whether the given frames or machines contain two-force and/or three-force members and apply their characteristics for static analysis.
- Draw free body diagrams of a frame or machine and its members.
- Determine the forces acting at the joints and supports of a frame or machine.

Applications



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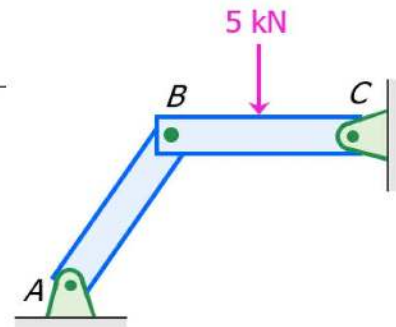
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• What

: Two common types of structures which are composed of _____ force _____

• Frames

- used to _____ loads
- generally _____



• Machines

- used to _____ or _____ the effect of forces
- contain _____ parts



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Two-Force Members

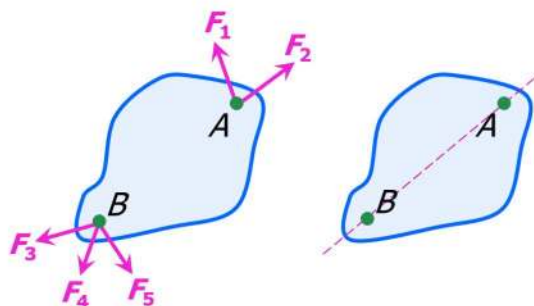
Topic 16 – 3

• What

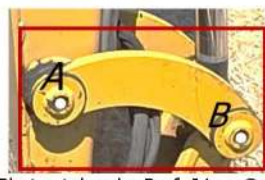
: Forces applied at only _____ points on the member.

• Characteristics

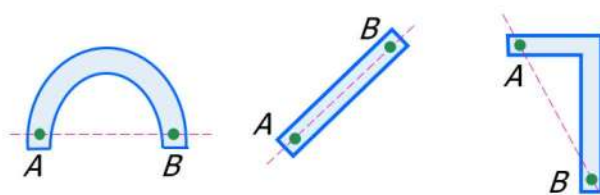
- (1) Have the _____ directed along the member
- (2) Have the _____ magnitude & _____ direction
- (3) Assumes that the _____ of the member is negligible.



• Examples



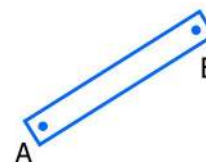
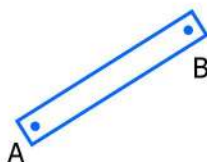
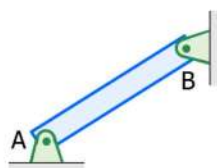
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• Exercise

FBD

FBD using the Two-Force Member



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Three-Force Members

Topic 16 – 4

• What

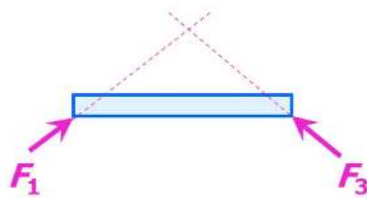
: A member is subjected to only _____.

• Characteristics

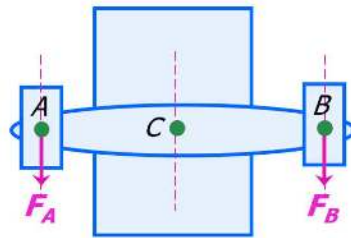
- (1) Have the _____ forces **OR**
- (2) Have the _____ forces
- (3) Assumes that the _____ of the member is negligible.

• Examples

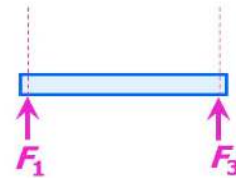
Force System



Force System



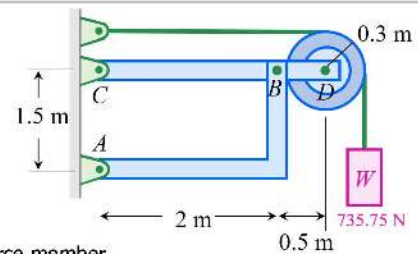
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• Steps

1. Define the coordinates:
2. Determine disassembled members: i.e., to determine the forces at B and D
3. Draw FBDs for the members:
 - (1) Place FBDs in order (members that interact).
 - (2) Identify any two-force or three-force members: Y | N if yes, why? member _____ is ____-force member
 - (3) Forces on contacting surfaces are _____ and _____ by Newton's _____ law of motion.



4. Apply Equilibrium equation to each FBD:
 - (1) Equilibrium equation for _____:
 - (2) Equilibrium equation for _____: $F_{AB} = -1532.8 \text{ N} = 1532.8 \text{ N}$

$$B_x = (\text{---}) = 1226.25 \text{ N} \quad B_y = (\text{---}) = 919.69 \text{ N}$$

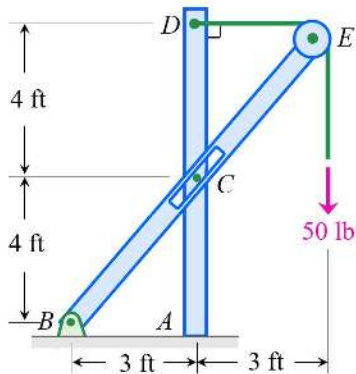
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Answer: $B_x =$ _____ ; $B_y = 920 \text{ N}$; $D_x = 736 \text{ N}$; $D_y = 736 \text{ N}$

Practice Problem

Topic 16 – 6

Disassemble members and draw FBDs for the members to find the forces at A , B and C .



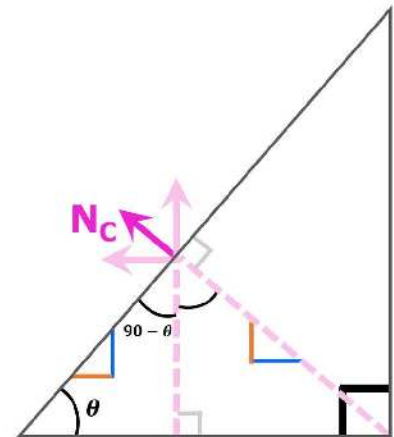
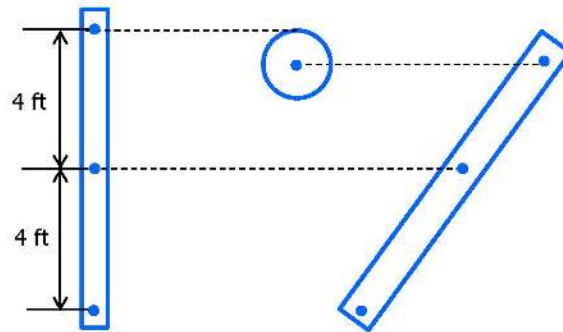
Given: A:
B:
C: smooth
D:
E: pulley and cable

Find: Dissembled Members:

; FBDs

Solution: Does the system include 2- or 3 -force members?

FBDs

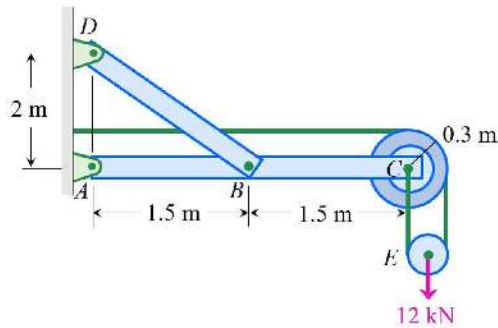


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Practice Problem

Topic 16 – 7

Disassemble members and draw FBDs for the members to find the forces at A and D .



Given: A ; B ; D :

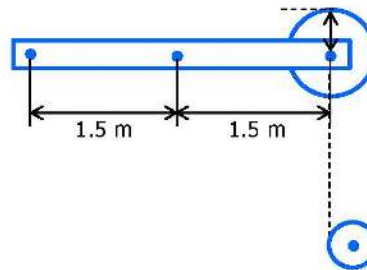
C ; E : Pulley & Cable

Find: Disassembled Members:

; FBDs

Solution: Does the system include 2- or 3 -force members?

FBDs



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Topic 18. Centroids

Topic 18 covers: 1) Explaining the concept of a centroid; 2) Determining the centroid of an area for a body or composite bodies.

Understanding centroids is essential in Statics, as the centroid represents the center of area, mass, or volume for a shape or structure, where its properties are evenly distributed. Calculating the centroid allows engineers to predict how a structure will behave under various loading conditions and is fundamental for further analysis in stability, support, and design. Mastering these concepts enables accurate determination of the center point for simple and composite bodies, which is critical for ensuring balanced and stable structures.

At the end of this chapter, extra sheets are provided to review centroid calculations for simpler geometries and integrals, assisting in determining areas for bodies with irregular geometries.

Statics

Topic 18: Centroids

Prof. J Lee

Objectives & Applications

Topic 18 – 1

Objectives

- Explain the centroid.
- Determine the centroid of an area for a body or composite bodies.

Applications



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Centroid ()

Topic 18 – 2

• What

: a point which defines the _____ of an object

- If the body is symmetric, the centroid will lie on the _____ of _____.

- The centroid is not necessarily located _____ the _____.

• How

: Assuming that a body is composed of an _____ number of particles with differential size,

and each particle has a centroid with an area of _____

In _____ :

$$= \frac{\sum x_i A_i}{\sum A_i}, \quad = \frac{\sum y_i A_i}{\sum A_i}$$

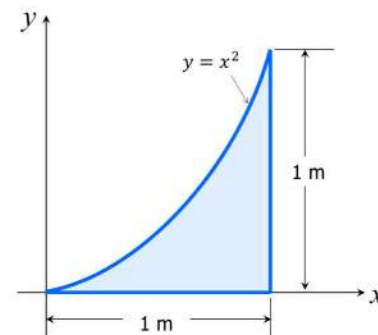
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How To Determine the Centroid of an Area

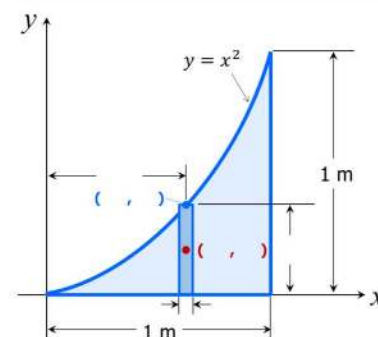
Topic 18 – 3

• Steps

1. Choose a coordinate _____ of integration (_____ - OR _____ - direction).
2. Choose a point on the boundary of the function and set the point coordinate as (_____ , _____).
3. Draw a rectangular differential _____ which includes the point.
4. Set the centroid of the element (_____ , _____) and express the centroid of the element _____ in terms of the geometry given, (_____ , _____) = (_____ , _____).
5. Indicate the area of the element (_____) in terms of the given geometry as _____
6. Apply the equations of (_____ , _____) and integrate with respect to the selected direction.



When _____ - direction is considered



$$\bar{x} = \text{_____} = \text{_____} = \text{_____} = \text{_____} = 0.75 \text{ m}$$

$$\bar{y} = \text{_____} = \text{_____} = \text{_____} = \text{_____} = 0.3 \text{ m}$$

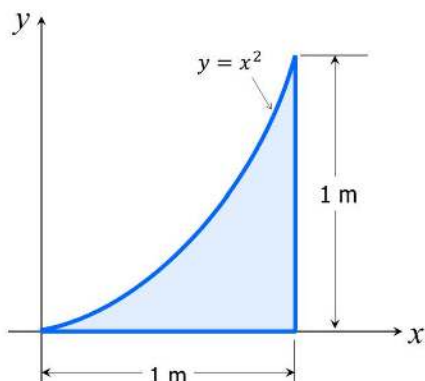
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Answer:

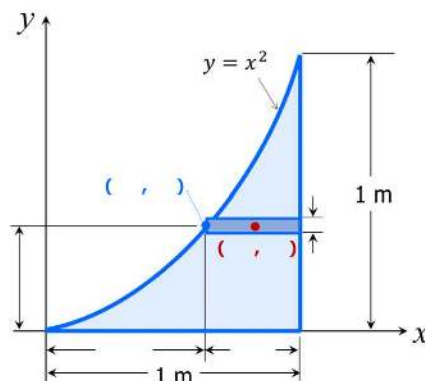
$$(\bar{x}, \bar{y}) = (\text{_____} , \text{_____})$$

How To Determine the Centroid of an Area

Topic 18 – 4



When - direction is considered



Step 4: $\tilde{x} =$; $\tilde{y} =$ $\rightarrow (\tilde{x}, \tilde{y}) = ($, $)$

Step 5: $dA =$

$\bar{x} =$ = = = = 0.75 m

Recall: $y =$
 $\therefore x =$

$\bar{y} =$ = = = = 0.3 m

Answer: $(\bar{x}, \bar{y}) = (0.75, 0.30)$

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Centroid of an Area for Composite Bodies

Topic 18 – 5

• Composite Body

: a series of connected _____ shaped bodies, e.g., rectangular, triangular, semicircular, etc.

• Steps

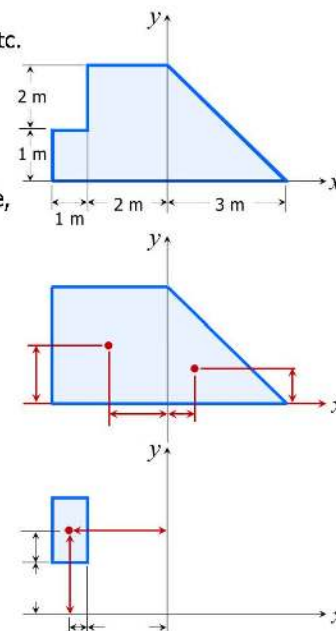
1. Divide the composite body into _____ shaped parts.
2. If a composite body has a _____ having no material, consider the composite body w/o hole, and then put ____ sign to the area or size of an additional composite part.

$$\bar{x} = \text{_____}, \quad \bar{y} = \text{_____}$$

Segment	A ()	$\bar{x}$ ()	$\bar{y}$ ()	$\bar{x}A$ ()	$\bar{y}A$ ()
1					
2					
3					
$\Sigma A =$		$\Sigma \bar{x}A =$		$\Sigma \bar{y}A =$	

$$\bar{x} = \text{_____} = \text{_____} = -0.348 \text{ m}, \quad \bar{y} = \text{_____} = \text{_____} = 1.22 \text{ m}$$

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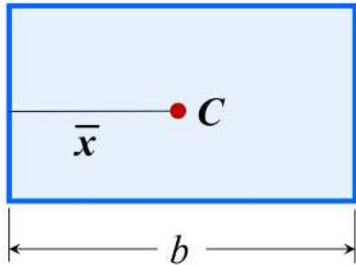
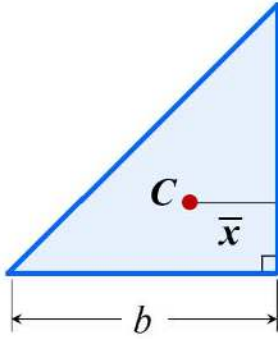
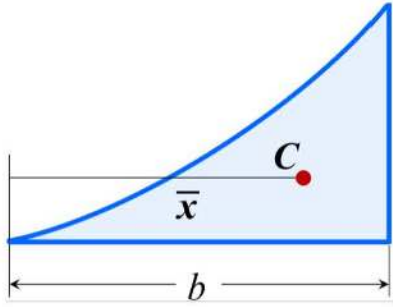
Answer:

$$(\bar{x}, \bar{y}) = (\text{_____}, \text{_____})$$

Extra Sheets for (1) Centroids for Simpler Body Shape; (2) Integral

Centroid

Center of mass; A point where all the medians of an object

Shape	Figure	$\bar{x}$
Rectangle		$\frac{b}{2}$
Right Triangle		$\frac{b}{3}$
Parabolic Spandrel		$\frac{3b}{4}$

Integral

$$\int a \, dx = a x$$

$$\int x \, dx = \frac{x^2}{2} \quad \int x^n \, dx = \frac{x^{n+1}}{n+1} \quad \int a^x \, dx = \frac{a^x}{\ln a}$$

$$\int \frac{1}{x} \, dx = \ln|x| \quad \int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln|ax+b| \quad \int \frac{1}{(x+a)^2} \, dx = -\frac{1}{x+a}$$

$$\int \ln(x) \, dx = x \ln(x) - x \quad \int \ln(ax) \, dx = x \ln(ax) - x \quad \int \ln(ax+b) \, dx = \left(x + \frac{b}{a}\right) \ln(ax+b) - x$$

$$\int e^x \, dx = e^x \quad \int e^{ax} \, dx = \frac{1}{a} e^{ax} \quad \int x e^x \, dx = (x-1) e^x \quad \int x e^{ax} \, dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

$$\int \sin x \, dx = -\cos x \quad \int \sin ax \, dx = -\frac{\cos ax}{a} \quad \int x \sin ax \, dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$\int \cos x \, dx = \sin x \quad \int \cos ax \, dx = \frac{\sin ax}{a} \quad \int x \cos ax \, dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$\int \tan x \, dx = -\ln|\cos x| = \ln|\sec x|$$

Integral Rules

• Multiplication by Constant

• Power Rule ($n \neq -1$)

• Sum & Difference Rules

$$\int a f(x) \, dx = a \int f(x) \, dx \quad \int x^n \, dx = \frac{x^{n+1}}{(n+1)} \quad \int (f \pm g) \, dx = \int f \, dx \pm \int g \, dx$$

(where a is the constant)

Recall

$$\sqrt{x} = (x)^{\frac{1}{2}} \quad \sqrt[n]{x} = (x)^{\frac{1}{n}}$$

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Topic 19. (Area) Moment of Inertia

Topic 19 covers: 1) Explaining the moment of inertia (MoI) for an area; 2) Describing the parallel-axis theorem; 3) Determining the moment of inertia about an axis using the parallel-axis theorem.

The moment of inertia (MoI) for an area is a crucial concept in Statics, as it quantifies the distribution of an area relative to an axis and its resistance to bending. Understanding MoI is essential for analyzing how forces and loads impact a structure's stability and deformation. The parallel-axis theorem further aids in calculating the MoI about different axes, which is especially valuable when working with composite bodies or shifted axes. Mastering these concepts is fundamental for the structural analysis and design of beams, frames, and other engineering structures.

Statics

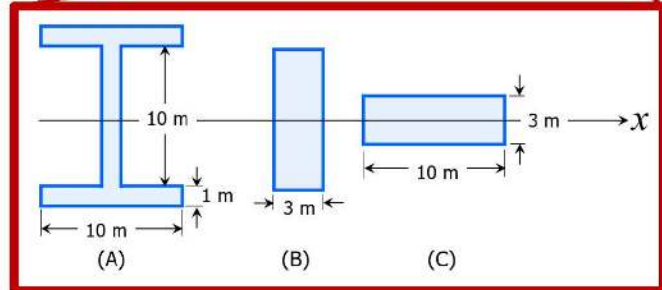
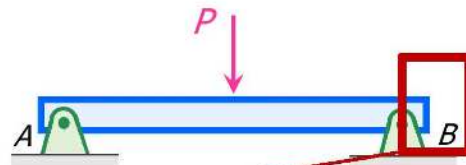
Topic 19: (Area) Moment of Inertia

Prof. J Lee

Objectives

- Explain the moment of inertia (MoI) for an area.
- Explain the parallel-axis theorem.
- Determine the MoI about an axis using the parallel-axis theorem.

Applications



To examine the strength and deflection of beams
(e.g. _____)

Bending Stress:

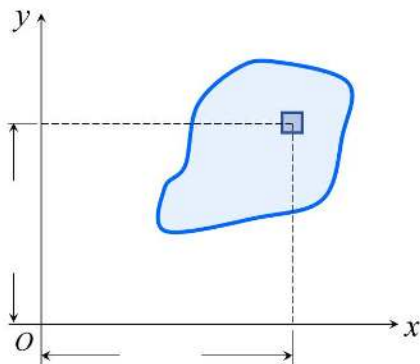
$$\sigma = \frac{M y}{I}$$

(Area) Moment of Inertia ()

Topic 19 – 2

• What

- : also called “_____ MoI”
- : _____ property of an _____ which reflects how its points are distributed with respect to an arbitrary _____
- : concerned with the _____ stresses developed in the body whereas _____ MoI is concerned with _____ body.
- : related not only to _____ but also to _____ of mass.
- : Mass far away from the _____ of _____ MoI more than the same mass _____ the axis of rotation.



Moment of Inertia for a Differential Area

$$dI_x = \quad \quad \quad dI_y = \quad \quad \quad$$

Moment of Inertia for the Entire Area

$$I_x = \quad = \quad , \quad I_y = \quad = \quad$$

Unit: [] or []

Note: MOI is always _____.

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• What

: _____ about **an arbitrary axis** which is _____ to the _____ axis

: Used to find _____ about an arbitrary axis which is _____ to the axes passing through the _____.

$$I_x =$$

where $I_{x'}$ and $I_{y'}$: MOI about the _____ oidal _____

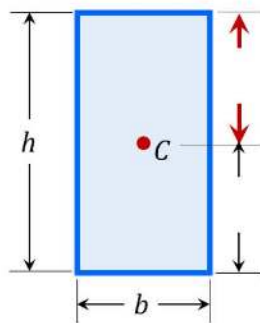
A:

$$I_y =$$

d: _____ distance **between** the specified axis **and** _____ axis

• How

Example: Determine MoI about an axis passing through the TOP :



$$I_{CEN} =$$

MoI about the axis passing through the TOP

$$I_{TOP} =$$

$$\therefore I_{CEN}$$

$$I_{TOP}$$

Note: I_{CEN} is always the _____.

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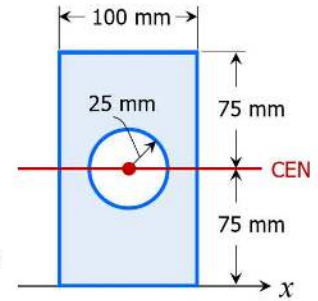
Moment of Inertia for Composite Area

Topic 19 – 4

$I =$

• Steps

1. _____ the body into simpler shaped parts (e.g. rectangle, triangle, and/or circle, etc.).
2. Locate the _____ of each part and indicate the _____ distance from the _____ axis to each _____ of the parts. i.e., to find I_x
3. Determine the _____ of each part about the reference axis using the parallel-axis theorem:
4. The MoI of the entire area about the reference axis is determined by the summation of the individual MoIs obtained in Step 3.



$$b_1 =$$

$$h_1 =$$

$$A_1 =$$

$$= 15000 \text{ mm}^2$$

$$d_1 =$$

$$r_2 =$$

$$A_2 =$$

$$d_2 =$$

Answer: $I_x = 101 \times 10^6$

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