FINANCIAL MANAGEMENT FOR SMALL BUSINESSES - FINANCIAL STATEMENTS & PRESENT VALUE MODELS



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FINANCIAL MANAGEMENT FOR SMALL BUSINESSES:
Financial Statements & Present Value Models
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List of Abbreviations

Table: List of Abbreviations

Value	Description	Chapter
A	constant loan payment	16
AE	annuity equivalent	8, 10, 12, 13
AIS	accrual income statement	4, 5, 6, 12, 13
AL	accrued liabilities	12, 13
AP	accounts payable	5, 12, 13
APR	annual percentage rate	16
AR	accounts receivable	12, 13
ATCF	after-tax cash flow	3, 12, 13
AVG	attachment value goods	21
CA	current assets	5
CE	cash expenses	5, 12
CFS	coordinated financial statements	4, 5, 6
CL	current liabilities	5
COGS	cost of goods sold	4, 5, 6, 12
CPI	consumer price index	14
CR	cash receipts	12
d	geometric decay	11
D	debt	5, 6, 7, 8, 15
Dep	depreciation	3, 12
E	equity	5, 6, 7, 8, 15
EBIT	earnings before interest and taxes	4, 5, 6, 12
EBT	earnings before taxes	4, 5, 6, 12
EV	expected value-variance	15
FTC	Federal Trade Commission	5
FV	future values	19
g	nominal growth rate	17
g^*	real growth rate	17
GAAP	generally accepted accounting practices	4
i	average interest rate	8, 9, 10, 14, 15, 16
i (superscript)	investment identifier	9, 10, 15
INV	inventories	5, 6
IRR	internal rate of return	9, 10, Appendix
IRRA	internal rate of return on Assets	12, 13

Value	Description	Chapter
IRRE	internal rate of return on Equity	12, 13
IRS	Internal Revenue Service	2, 3, 12
L	loan amount	16
LLC	Limited Liability Company	2
m	profit margin	5, 6, 10, 15, 16, 19, Appendix
MACRS	modified accelerated cost recovery system	12
MIRR	modified IRR	9, 10
mn	term	16
MNPV	modified NPV	9, 10
NIAT	net income after taxes	4, 5, 6, 12
NPC	net present cost	18
NPV	net present value	7–21
NWC	net working capital	5
OE	overhead expense	4, 5, 6
p	percent of loan to be paid as a refinance cost	16
PV	present value	1, 2, 7–20, Appendix
QTM	quantity theory of money	14
r	nominal discount rate	7–20, Appendix
r^f	APR	14, 16
rm	market rate of interest	7
r^*	real discount rate	17
R	constant value of net cash flow	8, 10, 11, 14, Appendix
	constant value of cash deposits	19
R_t^i	cash flow earned by investment i in the tth period	9
R (bold)	vector of cash flow values R_t^i earned by V_0^i	9
ROA	return on assets	5–8, 12, 15, Appendix
ROE	return on equity	5–8, 12, 15, Appendix
ROI	return on investment	7, 12
S	units sold	6
	liquidation value	7, 8
	sum of compounded periodic cash flows	9, 10,17
	standard deviation from a sample distribution	15
SCF	statement of cash flow	4, 12
SEC	Securities and Exchange Commission	5
SEG	socio-emotional goods	21
T	tax	3, 5, 11, 12

Value	Description	Chapter
t (subscript)	time period	7, 9–18
TI	total interest	16
V0	initial value	7–15, 17–20, Appendix
W	firm's wealth value	15
w	outcome variable	15
y	outcome variable	15
Excel Functions		
FV	future value	19, Appendix
IPMT	interest payment	16
IRR	internal rate of return	Appendix
NPER	number of periods	Appendix
NPV	net present value	Appendix
PMT	period payment	Appendix
PV	present value	Appendix
RATE	rate	Appendix
Functions		
Cov()	covariance	15
E()	expected value operator	15
Pr()	probability	15
$S(r, R_t^i)$	Sum of periodic cash flows	9, 10
$US0(r^f/m, mn)$	uniform series of $1 payments discounted at the actuarial rate for (mn) periods	16
Greek Characters		
Δ (Delta)	difference / change	4, 12, 13, 16
α (alpha)	endogenous projection constant	13
	tax adjustment rate for capital gains (losses)	17
β (beta)	coefficient multiplying an exogenous variable	13, 15
γ (gamma)	scaling factor	9
	rate of technological change	10
	financed proportion of the purchase price	14
	percentage between 0 and 100	15
	change in L_0	16
	% of the investment's value allowed to be deducted in a given year	18
δ (delta)	compounding / discounting factor	9, 10
	percent compensation for lost revenues	15
ϵt (epsilon)	average error in the tth period	13
ϵ (epsilon)	random variable component of a risky event	15

Value	Description	Chapter
η (eta)	capital gains (loss) rate	14
θ (theta)	average tax adjustment coefficient	11, 17, 18, 19
λ (lambda)	average risk aversion coefficient	15
μ (mu)	the expected value of a probability density function	15
π (pi)	insurance premium paid to exchange a risky distribution with a sure value	12, 15
ρ (rho)	correlation coefficient between two random variables	15
σ (sigma)	variance of a probability density function	15
Ratios		
ATO	asset turnover ratio	6
ATOT	asset turnover time ratio	6
C	coverage ratio	14
CT	current ratio	6, 16
CTR	current-to-total returns ratio	14
DE	debt-to-equity ratio	6
DS	debt-to-service ratio	6
EM	equity multiplier ratio	6
ITO	inventory turnover ratio	6
ITOT	inventory turnover time ratio	6
PE	price-to-earnings ratio	6, 14
PTO	payable turnover ratio	6
PTOT	payable turnover time ratio	6
QK	quick ratio	6
RTO	receivable turnover ratio	6
RTOT	receivable turnover time ratio	6
SPELL	solvency, profitability, efficiency, liquidity, leverage	6
TIE	times interest earned ratio	6, 16

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Preface

LINDON ROBISON

This book intends to help students and others learn how to successfully manage the finances of small to medium size firms. The underlying assumption of this book is that successful financial managers need to master the construction and analysis of financial statements and present value models. Learning how to construct and analyze financial statements and present value models is the focus of this book that is divided into five parts.

Part I provides an introduction to management. Chapter 1 describes the firm management process—a process that includes identifying the firm's mission statement and strategic (long term) goals and tactical (short term) objectives; identifying the firm's strengths, weaknesses, opportunities, and threats; identifying and evaluating alternative strategies; and finally implementing and evaluating the preferred strategy. Chapter 1 notes that the management process has wide application including managing the financial resources of the small to medium size firm.

The firm's opportunities and threats are most often nested in factors outside the firm's control. For example, different ways to organize the firm (Chapter 2) and the tax environment facing the firm (Chapter 3) are factors external to the firm. They are important to discuss, though, because the firm can adopt different responses to the legal and tax environment in which it operates.

Part II focuses on the internal financial strengths and weaknesses of the firm and its ability to respond to external opportunities and threats. Chapter 4 focuses on the construction, analysis, and interpretation of coordinated financial statements (CFS). CFS are the primary tools for answering the question: what are the financial strengths and weaknesses of the firm?

An important consideration, especially when the focus is on small to medium-sized firms, is how to construct financial statements when the firm has incomplete records. While the data used in the financial management process and the construction of financial statements are most often assumed to be obtainable and accurate, the reality may be quite different. Small to medium-sized firms often lack the financial records required to conduct the analyses described in Part II of this book. Acquiring and sometimes guesstimating the missing data is almost an art form—a process that forms the nexus between theory and practice.

An important lesson to be learned about financial statements—even when we can construct them accurately—is that financial statements alone do not reveal completely the financial strengths and weaknesses of the firm. This more complete view of the firm's strengths and weaknesses requires ratios be constructed using data included in the firm's CFS (Chapter 5). Ratios constructed using the firm's CFS can be compared with similar firms, and significant deviations from the norm can be noted and given further attention. Ratios describing the firm's financial well-being can be described by the acronym SPELL: (S)olvency, (P)rofitability, (E)fficiency, (L)iquidity, and (L)everage.

Chapter 6 notes that the firm's CFS is a system. An important characteristic of an open system is that its parts are connected internally with endogenous variables and externally to factors outside of the firm with exogenous variables. Therefore, an activity outside of the firm described by an exogenous variable can change conditions inside of the firm described by endogenous variables. Because CFS are a system, we can analyze the firm's opportunities and threats presented by forces outside the firm. For example, we can ask: what if an exogenous variable changes, then how will the financial condition of the firm be affected? Or we may ask: if the firm has a financial goal, then how much must an exogenous variable change for the firm to reach its goal. Finally, we may ask: if one part of the system changes, what corresponding changes will result? One way to think of the CFS system is to compare it to a balloon: a squeeze somewhere in the balloon will produce a bulge somewhere else.

Throughout this book, we use data from a hypothetical (but not atypical) firm, HiQuality Nursery (HQN) to help make the analysis realistic. However, the financial analysis experience becomes authentic when students and others construct financial statements for actual firms, and analyze them to discover their strengths, weaknesses, opportunities, and threats. Often, these firms are ones in which the analyst has a personal interest.

Part III of this book introduces present value (PV) models. PV models compare a challenging investment(s) to a defending investment(s) by converting future dollars to their value in the present. A key principle that should guide the construction of PV models is to compare challenging and defending investments using homogeneous measures—the subject of Chapter 7. Chapter 8 describes the different kinds of PV models and the questions each model is designed to answer. Chapters 9, 10, and 11 remind those solving PV problems that comparisons between challenger(s) and defenders(s) must use homogeneous measures. Chapter

9 describes how to compare investments with different sizes. Chapter 10 describes how to compare investments with different terms. Chapter 11 describes how to introduce taxes into PV models to produce consistent comparisons.

Part IV describes more details about the properties of PV models. Chapter 12 provides the theoretical basis for PV models by comparing their construction to the process of producing earnings measures derived from accrual income statements. Chapter 13 builds PV model templates that can be used to solve practical PV problems. Besides illustrating how to solve practical PV problems, it describes how to forecast exogenous variables. Chapter 14 provides PV model properties to describe an investment's liquidity.

We live in and make capital budgeting decisions in a world of risk and uncertainty. So how do we make choices when we can't be sure what the outcomes will be? Formal insurance programs are one way of addressing the risk firms face. However, there are other risk responses the firm can employ. Learning about risk and applying this knowledge to the purchase of insurance and adopting other risk management alternatives is the focus of Chapter 15. Some of the concepts covered in Chapter 14 are essential statistical tools needed to plan and make important decisions in a risky and uncertain world.

Armed with a knowledge of PV model building principles and tools, the analyst is prepared to construct PV models for specific investments. Specific investments, while similar, may have some distinct characteristics, depending on the type of investment activity under examination.

Part V considers six different investment strategies. Chapter 16 considers taking out and repaying loans, emphasizing that loan analysis is essentially a present value problem. Chapter 17 considers purchasing, using, or selling land. An important feature of land purchases and sales is transaction costs, which are included in the land models. Chapter 18 recognizes that the control and use of investments can be acquired through leases as well as through purchases. So, in this chapter, models are constructed that can be used to find the present values of leases. Chapter 19 reviews financial investments, a separate class of investments, especially relevant for personal financial management decisions.

Chapter 20 prepares students to observe financial opportunities and threats using the term structure of interest rates, an important financial tool. Finally, Chapter 21 reminds readers that "money can't buy love" nor most other relational goods. Thus, Chapter 22 enlarges the management process to include relational goods as well as commodities.

To solve many of the problems in this text we employ Excel spreadsheets that are now generally available and which have become the "industry standard" for solving financial management problems. Excel spreadsheets enable the analyst to solve complicated financial problems and examine the robustness of a solution under different possible scenarios. Although students are expected to come to this class with Excel experience, we include in this book an Appendix that provides a brief review of Excel methods employed in this text.

Finally, throughout this text we identify key paragraphs for understanding the material in particular chapters. These key paragraphs are highlighted and identified with a magnifying glass icon to suggest that the reader pay them particular attention.

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Dedication

LINDON ROBISON

This book is dedicated to our friend and colleague J. Roy Black.

We also dedicate this book to all the students who have passed Agribusiness Management ABM 435 at Michigan State University.

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1: Financial Management and the Firm

Learning Objectives

After completing this chapter, you should be able to: (1) recognize the six steps included in the management process; (2) apply the management process to better manage the financial resources of the small to medium-size firm; and (3) apply the management process to other activities such as being a successful student.

To achieve your learning goals, you should complete the following objectives:

- Understand the need for a concise firm mission statement.
- Learn how to distinguish between the firm's strategic (long-term) goals and its tactical (short-term) objectives.
- Learn how to choose goals and objectives that will successfully guide financial managers in the financial management process.
- Learn how to identify a firm's (internal) strengths and weaknesses.
- Learn how to identify a firm's (external) opportunities and threats.
- Learn how to develop a strategy to reach the firm's goals and objectives that take advantage of the firm's strengths and opportunities and minimize the limitations of its weaknesses and threats.
- Learn how to implement and evaluate outcomes of the firm's strategy for achieving its goals and objectives.
- Learn how to apply the management process to one's own efforts to succeed in this class.

Introduction to Management

Financial management is about, above all else, management. The verb manage comes from an Italian verb meaning "to handle" as in how a rider handles a horse. The management process can be applied to a wide variety of organizations and resources. In this book, we apply the management process to managing the financial resources of the small business as opposed to larger corporations.

The Management Process

The management process includes six steps: 1) develop the firm's mission statement; 2) choose the firm's strategic (long-term) goals and tactical (short-term) objectives; 3) identify the firm's strengths, weaknesses, opportunities, and threats; 4) develop the firm's strategy for accomplishing its strategic goals and tactical objectives; 5) implement the firm's strategy; and 6) evaluate the firm's performance.

These six steps are illustrated in Figure 1.1 followed by narrative that describes them in more detail. Note that as one moves from one management step to the other, one moves toward the center of the circle in Figure 1.1. And as one moves toward the center of the circle, each activity is constrained by previous management choices.

Figure 1.1. The Managerial Process in Six Steps



So what have we learned? We learned that the management process includes six steps: 1) develop a mission statement; 2) choose strategic goals and tactical objectives; 3) identify the firm's strengths, weaknesses, opportunities, and threats; 4) identify strategies, 5) implement strategies; and 6) evaluate performance.

Develop the Firm's Mission Statement

The management process begins with a mission statement. The firm's mission statement explains why the firm exists and what it values. The mission statement may also establish criteria for selecting activities in which the firm will participate. Mission statements might also describe the firm's customers, the markets in which it will participate, and the firm's social responsibilities. Mission statements are often succinct, short, easy to express and remember, clear, and flexible enough to describe the entire range of the firm's activities. For example, a mission for a vegetable farm may be: "our mission is to provide wholesome and safe vegetables grown in an environmentally healthy and worker-friendly environment."

Motives play a critical role in the formulation of the firm's mission statement. Motives spring from physical and socio-emotional needs and the relative importance of these motives will determine how the firm manages its resources to satisfy its needs. Therefore, the relative importance of the financial manager's motives should be reflected in the mission statement. To illustrate, motives may reflect the need to increase one's own consumption, to act in accordance with one's internalized set of values, to earn the good will of others, to increase one's sense of belonging in one's community or other organizations, and to improve the well-being of a disadvantaged group. The mission statement needs to guide the firm's efforts to manage its resources depending on the relative importance of these competing motives.

One word sometimes associated with management is the word "vision." Vision is the ability to imagine or picture, in one's mind, something that has not yet been created physically. Therefore, vision is a necessary condition for any kind of physical creation because we must create an event, outcome, or thing in our mind before we can create it physically. An architect must imagine a building and reflect that vision in plans which the builders will follow to create a building. A football coach imagines an offensive play and reflects it in a drawing before it can be executed in a game. A financial manager must imagine a project that will produce valued outcomes before committing time, energy, and other resources to the project. A vision of what the manager wants the firm to achieve is reflected in the firm's mission statement. A biblical expression highlights the importance of vision: "where there is no vision, the people perish..." (Proverbs 29:18).

So what have we learned? We learned that the management process begins with a vision that describes what the firm wants to do and become. Without a vision, the firm is unlikely to succeed.

Choose the Firm's Strategic Goals and Tactical Objectives

Strategic goals set the long-term direction of the firm. Strategic goals are consistent with the firm's mission statement—what it values, why it exists, and what its purpose is. Strategic goals direct the firm's efforts toward achieving its mission over the long run. Strategic goals also call for tactical objectives, specific actions needed to achieve strategic goals. For example, a strategic

goal may be to increase the firm's revenues by 10%. The objectives consistent with this goal may be to develop new products, to focus on a particular marketing strategy, or to increase the firm's sales force. The firm's objectives transform the firm's strategic goals into an action plan. Tactical objectives describe what short term actions are required if the firm is to reach its long-term goals.

The importance of goals and objectives

There are at least four reasons management requires the firm to choose goals and objectives:

1. Firm goals and objectives express what the firm believes is possible and desirable to achieve. By choosing the firm's goals and objectives, the manager is expressing confidence in what the firm can achieve. By declaring the firm's goals and objectives, the manager is also declaring its commitment of time, energy, and resources to achieve a desired end. Declaring the firm's confidence and commitments in the form of its goals and objectives is the first reason for setting strategic goals and tactical objectives.
2. As the firm works to achieve its goals and objectives, it faces a multitude of choices, including how best to allocate its limited resources. Goals and objectives guide the firm in its allocation decisions by asking: what choices and resource allocations will best enable the firm to reach its goals and objectives? Therefore, firm goals and objectives provide criteria for making choices, the second reason for setting goals and objectives.
3. Most firms include multiple actors with diverse assignments. Goals and objectives provide a means for rallying firm members to support a common cause. Goals and objectives provide a means for seeing one's individual efforts in the context of the overall goals and objectives of the firm and to identify opportunities for synergism within the firm. Ideally, the firm's goals and objectives represent a consensus of firm members' beliefs in what is possible and desirable and how to best achieve them so that all parts of the firm work cooperatively and enthusiastically together. Providing a framework for firm members to work synergistically is the third reason for setting goals and objectives.
4. Finally, goals and objectives provide a measure against which the firm can evaluate its performance. It allows the firm to ask and answer questions such as: Where are we? How are we doing? Can we get where we want to be from here? Are we closer to reaching our goals than before? Measuring one's efforts against a standard is the fourth reason for setting goals and objectives.

The characteristics of good goals and objectives.

Some goals are better than others. The four reasons why we choose goals and objectives help us define the characteristics of good goals and objectives.

1. Good goals and objectives are realistic. They identify outcomes that are feasible for the firm to achieve given its environment and resources. It is not helpful to set unrealistic goals and objectives, even if they impress others. Unrealistic goals and objectives may create unrealistic expectations and lead to frustration later because they cannot be achieved. Good goals and objectives can be achieved with the resources available to the firm and the environment in which it exists.
2. Good goals and objectives answer the question: "What must the firm do to achieve its mission?" Goals and objectives that are consistent with the firm's mission statement provide the criteria for the management of the firm's energy and resources. If the firm goals and objectives fail to provide such criteria, then other goals and objectives should be selected.
3. Good goals and objectives reflect a consensus among those tasked with achieving them. People work hard for money. But they work harder when they feel they are part of a team—that their contributions are valued. Thus, good goals and objectives are the product of serious discussions intended to produce a consensus among those involved in reaching them. Therefore, at the end of the day, good goals and objectives provide a focus for synergistic efforts.
4. Finally, progress toward the achievement of good goals and objectives can be measured. Thomas S. Monson (1970) taught, "when progress is measured, progress improves. And when progress is measured and reported, the rate of improvement increases." Measuring the firm's progress helps guide the firm's future. If the measures signal that the firm is making adequate progress, then the firm is supported in its efforts to keep doing what it has been doing. If the measures signal that the firm is not making adequate progress, then the firm is supported in its efforts to change directions.

So what have we learned? We learned that to realize the firm's vision we must choose goals that are realistic, describe what needs to be done, reflect a consensus, and can be measured. We need goals because they reflect what we believe is possible, they provide criteria for choosing between alternatives, they provide a rallying point for firm members to work together, and they provide measures against which performance can be evaluated.

Identify the Firm's Strengths, Weaknesses, Opportunities, and Threats.

Before developing strategies to accomplish the firm's goals and objectives, a manager needs to identify and evaluate the internal strengths and weaknesses of the firm. This evaluation should include an assessment of the firm's ability to survive financially in both the long run and the short run (solvency and liquidity); its profitability; its efficient management of its resources on which its profitability depends; and the risk inherent in its current financial state. External opportunities and threats that impact the firm's ability to accomplish its objectives also need to be considered. An external opportunity and threat analysis might include evaluating the behavior of close competitors or assessing the condition of the economy and business climate, or the impacts of the business cycle on clients' incomes and the resulting product demand.

So what have we learned? We learned that strengths, weaknesses, opportunities and threats analysis helps the firm understand the current constraints placed on it by both internal and external forces and enables the firm to take corrective action, when possible, to better position itself to accomplish its mission.

Develop and Evaluate the Firm's Strategy.

The firm's strategy is a plan that describes how it intends to achieve its strategic goals and its tactical objectives. Some have claimed that a goal or objective without a plan is only a wish. The firm's strategy is a plan of action that describes who will do what, when, and how. For each goal or objective, the firm must develop the corresponding strategy to accomplish it. The strategy development process includes collecting data and information about possible choices and likelihoods of possible events. Then, the information must be analyzed to determine the impact of a particular strategy on the firm's goals and objectives. Based on these analyses, management must select the proper strategy.

So what have we learned? We learned that unless we create a plan to achieve our goals and objectives consistent with our vision statement we have likely engaged only in wishful thinking. Planning who will do what, when, and how moves us in the direction of realizing our mission, goals, and objectives.

Implement the Firm's Strategy.

Once a strategy is selected, it must be administered throughout the firm. All relevant parts of the business (accounting, purchasing, manufacturing, processing, shipping, sales, administration) must support and take an active role implementing the strategy. There may be changes in the business that are necessary to implement the strategy such as changes in personnel, technology, or financial structure. Implementing the firm's strategy will require a carefully coordinated effort if the firm's strategy leads to the firm achieving its goals and objectives.

So what have we learned? We learned that there is often more said than done and more planned than executed. Implementing one's strategy is the sine qua non—unless the strategy is implemented, nothing else matters.

Evaluate the Firm's Performance.

Firm managers must continually evaluate the strategies they implemented to reach the firm's objectives and goals. They must determine if what the firm has achieved is consistent with its mission statement, goals, and objectives within an environment described by the firm's strengths, weakness, opportunities, and threats. Firm managers must also be prepared to alter strategies in response to changes in technologies, laws, market conditions, and personnel. These changes will make it necessary for the firm to continually reevaluate and make adjustments.

Evaluating the firm's performance must also include a review of its mission statement, goals, objectives, efforts to implement its strategies, and its strengths, weaknesses, opportunities and threats. Since the firm's mission statement and strategic goals are oriented toward the long term, they change infrequently. However, the firm's strategies may change as frequently as its internal strengths and weaknesses and external opportunities and threats change.

So what have we learned? We learned that life is like driving a car. Most of the time we look forward to where we are going. On occasion, however, it is important to look in our rear-view mirror to see where we have been and to learn from our journey.

The Firm Financial Management Process

The firm's financial management process involves the acquisition and use of funds to accomplish its financial goals and objectives consistent with its financial mission statement. The firm's financial management process essentially employs the same six management steps described earlier. The six steps of financial management include: 1) develop the financial mission of the firm; 2) choose the financial goals and objectives of the firm; 3) identify and evaluate the firm's financial strengths, weaknesses, opportunities and threats; 4) develop financial strategies including evaluating and ranking investment opportunities to achieve financial goals and objectives consistent with the firm's mission; 5) implement investment strategies by matching the liquidity of funding sources with cash flow generated from investments, by forecasting future funding needs, and by assessing the risk facing the firm; and, 6) evaluate the firm's financial performance relative to the goals and objectives of the firm. These six steps are described next in more detail.

(1) Develop the Firm's (Financial) Mission Statement.

While financial management usually plays a role in developing the firm's overall mission statement, there are other considerations shaping the firm's mission. As a result, the financial mission of the firm is usually nested within the more general mission of the firm. One financial mission of the firm may be to reach certain financial conditions that allow the firm's owners to pursue other goals and provide firm owners resources in the future. The firm's mission statement may lead naturally to important financial goals such as to maximize profit, reduce costs and increase efficiency, manage or increase the firm's market share, limit the firm's risk, or maximize the owner's equity in the firm. However, there may be a distinction between a firm's financial mission and the firm's overall mission. In other words, the firm's financial mission included in the strategic financial management process may be an objective in the firm's overall strategic management process.

(2) Choose the Firm's (Financial) Strategic Goals and Tactical Objectives.

The part of the firm's mission related to financial management must lead to the firm selecting financial goals consistent with the firm's mission statement and objectives likely to lead to the successful achievement of its strategic goals. The financial objectives may direct how the firm organizes itself and how it manages its tax obligations—subjects discussed in Chapters 2 and 3. However, taxes will also influence the development and implementation of investment strategies—subjects discussed in several chapters of this book.

(3) Identify (Financial) Strengths, Weaknesses, Opportunities and Threats.

The use of coordinated financial statements (CFS) discussed in Chapter 4 can be used to evaluate the firm's internal strengths and weaknesses. We may need to look outside of the firm to identify external opportunities and threats facing the firm. Financial statements are used to formulate ratios that can be compared to other firms to determine how the firm's financial condition compares to normal—or average—firms, the subject of Chapter 5. Chapter 6 uses financial statements and ratios to demonstrate that the firm is a system with interconnected parts. As a result, each financial measure (e.g. solvency, profitability, efficiency, liquidity, and leverage) are connected to each other, and a change in one measure will change all the others.

(4) Develop and Evaluate the Firm's (Financial) Strategy.

Financial managers face an almost limitless set of investment opportunities with a wide variety of characteristics. Some investments will be liquid and easily converted to cash, such as inventories or time deposits. Other investments, such as real estate or production facilities, cannot be easily converted to cash and are considered illiquid. There are investments that provide fairly certain, low risk returns while others will provide uncertain, high risk returns. Some investments are depreciable while others increase in value over time. Firm budget resources for capital or long-term investments and evaluate them using present value (PV) tools. Many of the chapters that follow focus on PV models. Indeed, it may be correct to say that the focus of much of this book is on how to use PV models to evaluate a firm's financial strategy.

- PV models recognize the time value of money; that a dollar today is different than a dollar received in the future (Chapter 7);
- PV models convert future cash flow to their equivalent value in the present (Chapter 8);
- PV models need to be consistent, must be investments of homogeneous size, term, and tax treatments—comparing apples to apples and oranges to oranges (Chapters 9, 10, and 11);
- PV models are similar in construction to accrual income statements (Chapter 12);
- PV models often evaluate incremental changes in a firm's portfolio of investments and project the future values of exogenous

variables (Chapter 13);
- PV models can be used to find the liquidity of investments since investments like firms differ in their degree of liquidity (Chapter 14);
- Risk is ubiquitous and we must account for it in our PV models (Chapter 15).

Equipped with PV models and knowledge of how to conduct proper comparisons of investments using consistent measures, we are prepared to apply our tools to a wide range of investment problems that employ a variety of PV models. Included is a discussion of loan analysis in Chapter 16, land investments in Chapter 17, leasing options in Chapter 18, and investment in financial assets in Chapter 19. Then Chapter 20 introduces yield curves to help us identify outside-of-the-firm external threats and opportunities. Finally, the last chapter in this book, Chapter 21, ends with a cautionary note—there are relational goods that may be more important than money and should not be ignored.

(5) Implement the Firm's (Financial) Strategy.

Financial managers often play an important role in managing the implementation of an investment strategy. The implementation stage of financial management may include interacting with capital markets to raise funds required to support a strategy. Managers decide whether to acquire funds internally or borrow from other investors, commercial banks, the Farm Credit System, life insurance companies, or, depending on how the firm is organized, by issuing stocks or bonds.

In the process of obtaining and allocating funds, financial managers interact directly or indirectly with financial markets. This interaction could be simply obtaining a savings or checking account at your local bank. Or it could involve more sophisticated interactions such as raising funds by issuing ownership (equity) claims in your firm in the form of shares of stock.

Another part of implementing the financial strategy of the firm is to interact with various parts of the business and the household. For example, setting inventory policy is both a financial and business management decision and requires input from the production and sales departments of the firm as well as the firm's financial managers. In addition, financial managers must make trade-offs between risks and expected returns. One tool that can be used to evaluate future returns and risk is the term structure of interest rates, the subject of Chapter 20. There are other kinds of trade-offs as well. One important trade-ff is between commodities and relational goods. We will need both because they each satisfy different needs. This book is mostly about managing commodities, but Chapter 21 reminds us that "money can't buy love" or relational goods. Therefore, one more thing about management is to account for and manage relational goods.

(6) Evaluate the Firm's (Financial) Performance.

Finally, financial concepts and information are often used to evaluate a strategy's performance and to signal investment changes the firm needs to adopt in the future. In this effort, PV models will prove to be particularly helpful.

The relative importance of the six steps included in the management process will differ depending on what is being managed. In the case of financial management, the financial goals, the objectives, the strengths, weaknesses, opportunities, and threats analysis, and the strategies adopted and evaluated will differ from personnel management, for example. Similar to both financial management and other management efforts is their shared responsibility for the firm. What follows is really strategic firm management applied to the financial resources available to the firm.

Our focus on firm financial management. While all six firm financial management processes are important and discussed in this book, we focus on two:

1. Assessing the firm's internal strengths and weakness through the use of coordinated financial statements, ratio analysis, and comparisons with 'average' firms; and
2. Developing strategies described by after-tax cash flow and evaluating them using PV models.

Of course the other parts of the management process are important, especially implementing strategic financial investment plans and evaluating the firm's performance. However, a thorough treatment of these topics which should be pursed in other venues.

Trade-offs between Financial Goals and Objectives

The firm's financial goals and objectives guide the financial manager. There are, of course, a wide range of possible tactical objectives that a firm may adopt to achieve its strategic goals and its mission. However, one maxim should guide the manager's

choice of objectives: “There is no such thing as a free lunch!” Interpreted, this adage reminds us that nearly always, every objective comes at the cost of another objective

For example, consider the objective of maximizing the firm’s profits. Increases in short-term profits may often reduce long-term profits. If the firm desires to reduce its risk, this may require that the firm reduce its profits by investing in less risky–but lower return–investments.

One often-stated firm financial management objective is to maximize the profits of the firm. However, the measure of profits can differ drastically across different accounting practices. For example, cash versus accrual accounting, different depreciation methods, and different inventory accounting methods all lead to different measures of profit. Maximizing profits using one accounting method may not maximize profits using another accounting practice. Furthermore, profits may be difficult to measure when the firm employs unpaid family labor. The real problem is that profits don’t reflect the actual cash flow of the firm.

In addition, the traditional notion of profit ignores the timing of the cash flow received by a firm. Suppose you are given the choice of receiving $1,000 either today or one year from today. Which would you choose? Naturally, you would choose to get the money today because you could invest the money for some positive rate of return and earn more than $1,000 by the end of one year. For instance, suppose you could invest the money in the bank and earn a 5 percent return during the year. At the end of the year you would get back your $1,000 plus $1,000 x (.05) = $50 interest or a total of $1,050. Clearly, the $1,000 today is worth more than the $1,000 one year from today. The notion that dollars at different points in time are not worth the same amounts at a single point in time is known as the time value of money concept. Moreover, it underlies one of the most important financial trade-offs: present profits versus the present value of discounted future after-tax cash flow.

Another trade-off involving the maximize profit objective is that it ignores liquidity. Liquidity can be defined as a firm’s ability to meet unexpected cash demands. These cash demands might be unexpected cash expenses for such things as repairs and overhead expenses, new investment opportunities, or unexpected reductions in revenue. The concept of liquidity is closely related to risk. Liquidity needs are usually met by holding salable assets and/or maintaining the capacity to borrow additional funds. Serious risks may reduce the value of some assets and make liquidation of the assets difficult. Likewise, serious risk may also make it difficult to borrow additional funds.

If you were a firm manager, would your objective be to maximize profits or is some other objective more preferable? In this class, we will argue that traditional profit maximization is not a very desirable objective for a financial manager. We will argue that in most cases, financial decisions should be made so they maximize the value of the firm, which turns out to be the same thing as maximizing the present value of all the future cash generated by the firm. Once again, it is important to note that “cash flow” is much different than “profit.”

This concept of maximizing firm value is easy to defend in large firms where firm ownership is often separate from management. Owners of these firms generally want management to operate the firm in a way that maximizes the value of their investment; however, in smaller firms and households the value maximization principal is often constrained by other considerations such as concerns about quality of life. For example, you would likely be able to increase your personal wealth over time by driving a Chevrolet instead of a Cadillac (you could invest the cost savings), but you may gain enough satisfaction from driving a Cadillac (or a tractor with green paint) that you are willing to accept the lower wealth level. Nevertheless, for most of this course we will assume that financial decisions are made in a manner that is consistent with maximizing value. In cases where a firm does not maximize present value, it is still useful to estimate the present value maximizing decisions as a benchmark in order to understand the cost of alternative decision in terms of wealth loss.

Summary and Conclusions

Like it or not, we are all managers—if not managers of a firm then we are personal managers. We have important management responsibilities for our lives and resources. The management process is a universal process requiring that we first determine our mission and what goals and objectives are consistent with our mission? The goals and objectives we choose must declare what it is that we believe we can and should accomplish and the level of our commitment to reaching our goals and objectives. Finally, we cannot avoid setting goals and objectives because having no goal or objective is a goal or objective—to reach nowhere in particular.

Choosing our goals and objectives is crucial in the management process. We cannot achieve our mission without properly formulated goals and objectives. Goals and objectives lead us to conduct an honest evaluation of our internal strengths and weakness and external threats and opportunities, a process that identifies the resources and constraints likely to contribute to achieving our mission. After formulating our goals and objectives, and after conducting an honest evaluation of our strengths,

weaknesses, opportunities and threats, we next decide on a strategy, a plan to follow that will enable us to accomplish our mission. Strategic management requires that we implement our plan, that we take specific actions to ensure we achieve our goal by implementing our strategies. And finally, we evaluate and, if necessary, modify our goals, objectives, and our understanding of our strengths, weaknesses, threats, and opportunities. Then, when we have completed the management process, we repeat it all over again, continually, and not necessarily following the steps in the management process in the same order. We end this chapter by emphasizing this truism: we are all managers, all the time.

Questions

1. What does the word management mean?
2. Discuss the six steps included in the management process. Should these steps be practiced sequentially? In any order? Or does it depend on the management problem? Defend your answer.
3. What features differentiate management and firm financial management?
4. Consider mission statements and strategic goals and tactical objectives. Is the following statement: "I want to obtain a college degree." a mission statement, a goal, or an objective? If it is a tactical objective, what strategic goals and mission statement are consistent with this tactical objective? If it is a strategic goal, what mission statement and tactical objectives are consistent with this strategic goal? Finally, if it is a mission statement, what strategic goals and tactical objectives are consistent with this mission statement?
5. Mission statements, strategic goals, and tactical objectives reflect motives. Several motives may explain one's desire to achieve a college degree. One motive for obtaining a college degree is to increase one's lifetime earnings. But there are other motives. To help you connect motives to your personal mission statement, goal, or objective to obtain a college degree, identify the relative importance of the five motives described below. To complete the questionnaire below, assume you have 10 weights (e.g. pennies) to allocate among the five motives for attending the university. Distribute the 10 weights (pennies) according to the relative importance of each motive. Write your answer in the blank next to each question. There is no right or wrong allocation of weights among the motives except that the sum must add to 10. Each motive reflects the relative importance of one's need to increase one's own consumption, to earn internal/external validation, and to have a sense of belonging.
    a. I want a college degree so I can increase my lifetime earnings and get a better job.
    b. I want a college degree so important people in my life will be pleased with my achievements.
    c. I want a college degree to live up to the expectations I have for myself.
    d. I want a college degree so I will feel part of groups to which I want to belong.
    e. I want a college degree so that in the future, I will be better able to help others.
6. Develop a management plan for this class, this semester. This should include a brief discussion of each of the six steps in the strategic planning process including a mission statement, strategic goals and objectives, your strengths, weaknesses, opportunities, and threats, your strategies, your plan to implement your strategies, and, lastly, your evaluation process. Discuss what role financial management will play in your strategic planning process.
7. Imagine yourself as the financial manager of a small firm. Write a mission statement focused on profit maximization. What other considerations may be ignored if profit maximization were your primary mission? Suppose your mission statement was to "aid the disadvantaged"? Would profit or value maximization be part of your firm's goals and objectives? Please explain.

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2: Alternative Forms of Business Organizations

Learning Objectives

After completing this chapter, you should be able to: (1) know the different forms of business organizations; (2) compare the advantages and disadvantages of alternative types of business organizations; and (3) identify how alternative forms of business organizations can influence a firm's ability to achieve its financial goals and objectives.

To achieve your learning goals, you should complete the following objectives:

- Learn about the advantages and disadvantages of a sole proprietorship.
- Study the characteristics of firms best suited to be organized as sole proprietorships.
- Learn about the advantages and disadvantages of partnerships.
- Study the characteristics of firms best suited to be organized as partnerships.
- Learn about the advantages and disadvantages of C corporations and S corporations.
- Study the characteristics of firms best suited to be organized as C corporations and S corporations.
- Learn about the advantages and disadvantages of a Limited Liability Company (LLC).
- Study the characteristics of firms best suited to be organized as LLCs.
- Learn about the advantages and disadvantages of a cooperative.
- Study the characteristics of firms best suited to be organized as cooperatives.
- Learn about the advantages and disadvantages of a trust.
- Study the characteristics of firms best suited to be organized as trusts.

Introduction

The way a business is organized influences its ability to reach its goals and objectives. This chapter focuses on legal forms of business organizations that are widely used in the U.S. These include sole proprietorships, general and limited partnerships, limited liability corporations (LLCs), S corporations, and C corporations. Partnerships, LLCs, and C corporations are found across a wide spectrum of business types and sizes. LLCs are becoming increasingly important in the production agricultural sector, particularly with multi-generation family businesses. In family businesses, legal business structures which facilitate intergenerational transfer of assets has become particularly important.

Characteristics of businesses organizations that influence the ability of a firm to reach its goals and achieve its mission include: 1) who makes the management decisions; 2) how much flexibility does it have in its production, marketing, consumption, and financing activities; 3) its liability exposure; 4) its opportunities for acquiring capital; 5) how the life of the business is defined; 6) how the death of its owners affects the firm; 7) methods available for transferring the current owners' interest to others; and 8) Internal Revenue Service definitions of business profits and their taxation. Thus, the way a firm is legally organized provides the framework to make financial management decisions.

Sole Proprietorship

The sole proprietorship is a common organization form especially used by small businesses. A sole proprietorship is a business that is owned and operated by a single individual. Most sole proprietorships are family-owned businesses. The advantages of the sole proprietorship organization include:

1. It is easy and inexpensive to form and operate administratively (simplicity);
2. It offers the maximum managerial control; and
3. Business income is taxed as ordinary (personal) income to the owner.

The disadvantages of the sole proprietorship include:

1. It is difficult to raise large amounts of capital;
2. There is unlimited liability;

3. It is difficult to transfer ownership; and
4. The company has a limited life that is linked to the life of the owner.

Sole proprietorships are typically organized informally and require relatively little paperwork to begin operations. It is the most simple among the alternative business organizations to understand and use. To begin operation, the individual declares himself/herself to be a business. In many cases, a license will be required to operate the business, but often the business begins simply by "opening its door." The day-to-day operations of the firm are also organized informally and may be administered as the owner desires, subject to legal and tax restrictions. For example, certain taxes must be paid by specific dates. The vast majority of regulations small businesses face are independent of the legal form of the business. Metrics such as business size, number of employees, and location determine which regulations business face.

Since sole proprietorships are owned by a single individual, this form of business organization offers the maximum management control. In small firms, the owner of the business is often involved in all aspects of the business: purchasing, inventory control, production, sales, accounting, personnel and customer relations, as well as financial and general management. While the large amount of owner control can be a strength in small firms, it often turns out to be a disadvantage as firms begin to grow and the owner is no longer able to manage all aspects of the business. The owner must then hire competent staff to manage specific aspects of the business.

Additionally, the business is restricted by the financial resources available to the owner. This can restrict the startup of the business as well as its growth over time. In many cases, substantial cash outlays are required to make the capital purchases (land, facilities, equipment, cars, offices) to start the business and to provide initial overhead expenses (salaries, wages, supplies) until the business gets going. Further, it frequently takes two or three years before the business begins to show a profit. The owner of the business has to obtain these funds using his/her own equity (funds owned by the individual) and/or by borrowing funds, and borrowing requires collateral in the form of owner equity The amount that can be borrowed depends on the level of equity as well as the projected cash flow generated by the business. The lack of available financial capital for starting and expanding the business is a major drawback in the sole proprietorship. The profits from the business are taxed as personal income to the owner.

Sole proprietorships are subject to unlimited liability which means that the liability for business debts extends beyond the owner's investment in the firm. For example, if the sole proprietorship is unable to cover its debts and obligations, creditors have the right to collect the personal assets that are not part of the business or other businesses of the owner. The owner may be forced to liquidate assets, such as a personal savings account, a vacation home, or other personal assets just to cover the firm's obligations.

Another disadvantage of a sole proprietorship is that it has a limited life that corresponds to life of the owner. The owner may sell assets from the business to another sole proprietorship or business. However, if the business is not terminated prior to the death of the owner, then after the proprietor's death, the assets remaining in the firm will be distributed according to the owner's will or comparable instrument. When the owner dies, the business is terminated.

Partnerships

A *general partnership* is a business that is owned and operated by two or more individuals. The partners contribute to the business, share in management, and divide any profit. Partnerships are usually created by written contract among the partners, but they can be legally recognized even without a written agreement. If the partnership owns real property, the partnership agreement should be filed in the county where the property is located.

Advantages of partnerships include:

1. They are easy and inexpensive to form and operate administratively;
2. They have the potential for large managerial control;
3. Business income is taxed as ordinary (personal) income to the owner; and
4. A partnership may be able to raise larger amounts of capital than a sole proprietorship.

The disadvantages of a general partnership include:

1. Raising capital can still be a constraint;
2. There is unlimited liability;
3. It is difficult to transfer ownership; and
4. The company has limited life.

The advantages and disadvantages of a general partnership are similar to the sole proprietorship. Partnerships are generally easy and inexpensive to set up and operate administratively. Partnership operating agreements are critical. Like sole proprietorships, profit allocated to the partners is based upon their share in the business.

Managerial control resides with the partners. This feature can be an advantage or disadvantage depending on how well the partners work together and the level of trust in each other. Control by any one partner is naturally diluted as the number of partners increases. Partnerships are separate legal entities that can contract in their own name and hold title to assets

The challenge to partnerships extends beyond possible conflicts with the partners. Divorce and other disputes may threaten the survival of the partnership when a claimant to a portion of the business's assets demands his/her equity.

Unlimited liability remains a strong disadvantage for a general partnership. All partners are liable for the debts of the firm. Due to this unlimited liability, the risks of the business may be spread according to the owners' equity rather than according to their interests in the business. This risk becomes an actual obligation whenever the partners are unable to satisfy their shares of the business's obligations.

Increasing the number of partners can increase the amount of capital that can be accessed by the firm. More partners tends to mean more financial resources and this can be an advantage of a partnership compared to a sole proprietorship. Still, it is generally difficult for partnerships to raise large amounts of capital—particularly when liability is not limited.

Ownership transfer and limited life continues to be a problem in partnerships; however, it may be possible to build provisions into the partnership that will allow it to continue operating if one partner leaves or dies. In some cases, parent-child partnerships can ease the difficulties of ownership transfer.

A *limited partnership* is another way businesses can organize. Limited partnerships have some partners (limited partners) who possess limited liability; limited partners do not participate in management of the firm. There must be at least one general partner (manager) who has unlimited liability. Because of the limited liability feature for limited partners, this type of business organization makes it easier to raise capital by adding limited partners. These limited partners are investors and make no management decisions in the firm.

One difficulty occurs if the limited partners wish to remove their equity from the firm. In this instance, they must find someone who is willing to buy their share of the partnership. In some cases, this may be difficult to do. Another difficulty is that the Internal Revenue Service (IRS) may tax the limited partnership as a corporation if it believes the characteristics of the business organization are more consistent with the corporate form of business organization.

In production agriculture, family limited partnerships serve a number of objectives. For example, parents contemplating retirement may wish to maintain their investment in a farm business but limit their liability and be free of management concerns. To reduce their liability exposure and be free of managerial responsibilities, parents can be limited partners in a business where younger family members are the general partner.

The joint venture is another variation of the partnership, usually more narrow in function and duration than a partnership. The law of partnership applies to joint ventures. The primary purpose of this form or organization is to share the risks and profits of a specific business undertaking.

Corporations

A *corporation* is a legal entity separate from the owners and managers of the firm. Three fundamental characteristics distinguish corporations from proprietorships and partnerships: (1) the way they are owned and managed, (2) their perpetual life, and (3) their legal status separate from their owners and managers.

A corporation can own property, sue and be sued, contract to buy and sell, and be fined—all in its own name. The owners usually cannot be made to pay any debts of the corporation. Their liability is limited to the amount of money they have paid or promised to pay into the corporation.

Ownership in the corporation is represented by small claims (shares) on the equity and profit stream of the firm.

The two most common types of claims on the equity of the firm are common and preferred stock. The claims of preferred stockholders takes preference over equity claims of common stockholders in the event of the corporation's bankruptcy. Preferred stockholders must also receive dividends before other equity claims. The preferred stockholders' dividends are usually fixed

amounts paid at regular intervals that rarely change. In most cases, preferred stock has an accumulated preferred feature. This means that if the firm fails to pay a dividend on preferred stock, at some point in time the corporation must make up the payment to its preferred stockholders holders before it can make payments to other equity claims.

Common stock equity claims are the last ones satisfied in the event of the corporation's bankruptcy. These are residual claims on the firm's earnings and assets after all other creditors and equity holders have been satisfied. Although it appears that common stock holders always get the "leftovers," the good news is that the leftovers can be substantial in some cases because of the nature of the fixed payments to creditors and other equity holders.

Large corporations are usually organized as Subchapter C corporations.

The advantages of a C corporation include:

1. There is limited liability;
2. The corporation has unlimited life;
3. Ownership is easily transferred; and
4. It may be possible to raise large amounts of capital.

The disadvantages of a C corporation include:

1. There is double taxation; and
2. It is expensive and complicated to begin operations and to administer.

Seeing Double

Earnings from the corporation are taxed using a corporate tax rate. When earnings are distributed to the shareholders in the form of dividends, the earnings are taxed again as ordinary income to the shareholder. For example, suppose a corporation, whose ownership is divided among its 3000 shareholders, earns $1,000,000 in taxable profits for the year and is in a flat 40-percent tax bracket. Profits per share equal $1,000,000/3000—or $333.33. The corporation pays 40% of $1,000,000—or $400,000—in taxes to the government. Taxes per share equal $400,000/3000—or $133.33.

Now suppose the corporation distributes its after-tax profits to its 3,000 shares in the form of dividends. Each shareholder would receive a dividend check of $600,000/3,000 = $200. The $200 dividend income received by each shareholder would then be taxed as ordinary personal income. If all the shareholders were in the 30-percent tax bracket, then each would pay 30% of $200 or $60 in taxes, leaving each shareholder with $140 in after-tax dividend income.

So what is the total tax rate paid on corporate earnings? Dividing the taxes paid by the corporation and the shareholder by the profit per share, the total tax rate is ($133.33+$60)/$333.33 = 58%, a higher rate than would be paid on personal income of the same amount.

One of the primary strengths of the corporate form of business organization is that the most the owners of the firm (shareholders) can lose is what they have invested in the firm. This limited liability feature means that as a shareholder, one's personal assets beyond the investment in the corporation can't be taken to satisfy the corporation's debts or obligations.

Ownership can easily be transferred by selling shares in the corporation. Likewise, the corporation has an unlimited life because when an owner dies, the ownership shares are passed to his/her heirs. The common separation of ownership and management in large corporations helps to ease the ownership transfer as the firm management process never ceases.

The easy transfer of ownership, separation of management and ownership, and limited liability features of a corporation combine to create a business structure that is designed to raise large sums of equity capital. Investors in large corporations don't have to become involved in management of the firm. Their risk is limited to the amount of funds invested in the firm, and their ownership interest can be transferred by selling their shares in the firm.

Corporations are more expensive and complicated to set up and administer than sole proprietorships or partnerships. Corporations require a charter, must be governed by a board of directors, pay legal fees, and meet certain accounting requirements. Despite the relatively high setup cost, the primary disadvantage of the corporate form of business is that income generated by the corporation is subject to double taxation.

However, there is a limit on corporate earnings that are double-taxed. The corporation may pay reasonable salaries, and these are deducted from the corporation's profits. Therefore, salaries paid to corporate workers and operators are not taxed at the corporate level. In some cases, the corporation's entire net profit may be offset by salaries to the owners so that no corporate income tax is due. On the other hand, if the corporation pays dividends to the shareholders, those payments are subject to corporate-level income tax. However, the individual does not have to pay self-employment tax on the dividends. And, qualifying dividends (and most United States Corporation dividends can fit into this definition) are taxed at capital gains rates and not the individual's top marginal tax rate. Finally, dividends paid to a shareholder that actively participates in the business are not subject to either the 0.9 percent Medicare surtax on earnings or the 3.8 percent tax on net investment income that are levied on higher-income taxpayers.

Another disadvantage of corporations has to do with the fact that the managers do not own the firm. Managers, who control the resources of the firm, may use them for their own benefit. For example, top management may build extravagantly large headquarters and buy fleets of jets and limousines for transportation. If less were spent on perquisites, then the income of the corporation would be higher. Higher income allows higher dividends to be paid to the owners (shareholder).

The (potential) self-serving behavior by management running contrary to the interests of stockholders is an example of a principal-agent problem. Methods of dealing with the principal-agent exist. One way is to hire auditors to monitor the use of firm resources. Further, a corporation has a board of directors responsible for hiring, evaluating, and removing top management. Boards are often ineffective because they meet infrequently and may not have access to the information necessary to fulfill their responsibilities. Additional problems exist if management personnel also sit on the board of directors.

Another way to deal with the principal-agent problem in corporations is to align the interests of management with those of shareholders. This is accomplished by basing the compensation of management on the value of the firm's stock. A chief executive officer could receive stock options as a part of his/her compensation package. If the stock price rises, the value of the options increase, which benefits the manager financially. The shareholders also benefit when the stock price increases. Such an arrangement may reduce the principal-agent problem. However, very high executive compensation can often trigger criticism from external groups such as consumer or labor activists.

Limitations of linking management's compensation to the value of its stock have been illustrated by Enron and Tyco corporations. These corporations inflated the value of their stocks and eventually bankrupted themselves and lost the investments of their employees. It seems there is still a lot to be learned about aligning the interests of corporate managers and shareholders.

Many small businesses, including farms, use the C corporation structure and operate much like partnership. This is frequently done for reasons of expensing and intergenerational transfer.

The corporation will need to be "capitalized" by some level of equity funds from the shareholders. It is common practice among lenders to require personal guarantees by the owners of small corporations before providing funds to the business. This essentially eliminates the limited liability features for those shareholders. As one might expect, due to these difficulties, many small corporations are not able to generate large amounts of capital by simply selling ownership shares. As a result, many small corporations do not really receive the full benefits of corporate organization but are still subject to the disadvantages, namely double taxation.

C corporations and S corporations. Any corporation is first formed under the laws of a particular state. From the standpoint of state business law, a corporation is a corporation. However, there are two types of for-profit corporations for federal tax law purposes:

- C corporations: What we normally consider "regular" corporations that are subject to the corporate income tax
- S corporations: Corporations that have filed a special election with the IRS. They are not subject to corporate income tax. Instead, they are treated similarly (but not identically) to partnerships for tax purposes.

There is an alternative form of corporate business organization that is often more desirable from a small business perspective. Subchapter S Corporations have limited liability protection, but the income for the business is only taxed once as ordinary income to the individual (Wolters Kluwer. n.d.).

There are restrictions on what type of firms can be organized as Subchapter S corporations. To do so, it must meet several requirements: (1) cannot have more than 100 shareholders; (2) may have only one class of stock; (3) cannot have partnerships or other corporations as stockholders; and (4) may not receive more than 20 percent of its gross receipts from interest, dividends, rents, royalties, annuities, and gains from sales or exchange of securities. In agriculture, these restrictions usually mean that only family or closely-held farm businesses can achieve Subchapter S status.

Federal income tax rules for Subchapter S corporations are similar to regulations governing partnerships and sole proprietors. However, corporations may provide certain employee benefits that are tax deductible. Accident and health insurance, group life insurance, and certain expenditures for recreation facilities all qualify. However, these benefits may be taxable to the employees and subsequently to the shareholders.

There is greater continuity for businesses organized under Subchapter S than for sole proprietorships or partnerships. Upon the death of shareholders, their shares of the corporations are transferred to the heirs and the Subchapter S election is maintained. Surveys suggest that the major reason farms incorporate is for estate planning. The corporate form allows for the transfer of shares of stock either by sale or gift. This is much easier than transferring assets by deed.

Limited Liability Company

The *Limited Liability Company (LLC)* is a relatively new form of business organization. An LLC is a separate entity, like a corporation, that can legally conduct business and own assets. The LLC must have an operating agreement which regulates its business activities and the relationship among its owners (referred to as members). There are no restrictions on the number of members or the members' identities. LLCs are subject to disclosure, record keeping, and reporting requirements that are similar to a corporation.

The attractive feature of the LLC is that all members obtain limited liability, but the entity is taxed as a general partnership. The LLC is similar in most respects to the Subchapter S corporation. The primary differences are: 1) the LLC has less restrictive membership requirements; and 2) the LLC is dissolved in the event of transfer of interest or death unless members vote to continue the LLC. Table 2.1 summarizes the primary characteristics of the business organizations discussed so far.

Table 2.1. Comparison of Business Organizations

	Sole Proprietorship	Partnership	Limited Partnership	S Corporation	C Corporation	Limited Liability Company
ownership	• single • individual	• two or more individuals	• two or more individuals • one or more general partners	• legal person • max 35 shareholders • individuals	• legal person	• legal person • two or more members
management decision	• proprietor	• partners	• general partner	• elected directors • management	• elected directors • management	• members • manager
life	• terminates at death	• terminates at death	• agreed term • terminates at death	• perpetual or fixed • transfer stock	• perpetual or fixed • transfer stock	• agreed term • terminates at death
transfer	• assets	• assets	• assets	• shares	• shares	• assets
income tax	• individual	• individual	• individual	• individual	• corporate • individual	• individual

	Sole Proprietorship	Partnership	Limited Partnership	S Corporation	C Corporation	Limited Liability Company
liability	• unlimited	• unlimited	• general partners unlimited • limited partners limited	• limited	• limited	• limited
capital	• personal • loans	• personal • loans	• personal • loans	• shareholders • bonds • loans	• shareholders • bonds • loans	• members • loans

Cooperative

A *cooperative* is a business that is owned and operated by member patrons. Generally, cooperatives are thought to operate at cost, with all profits going to member patrons. The profits are usually redistributed over time in the form of patronage refunds. Cooperatives often appear to operate as profit making organizations much the same as other forms of business organization. Agricultural cooperatives do not face the same anti-trust restrictions as non-cooperative businesses, and they enjoy a different federal income tax status. In most instances, the concepts and analysis techniques covered in this course will be relevant to financial management in cooperatives.

Trusts

A *trust* transfers legal title of designated assets to a trustee, who is then responsible for managing the assets on the beneficiaries' behalf. The management objectives can be spelled out in the trust agreement. Beneficiaries retain the right to possess and control the assets of the trust and to receive the income generated by the properties owned by the trust. Beneficiaries hold the trust and personal property, rather than title to the assets. The legal status of certain types of land trusts are unclear in some states.

Farm Business Organization Types in US Agriculture

The USDA defines a farm as a place that generates at least $1,000 value of agricultural products per year. In 2007, farms generating between $1,000 and $10,000 of agricultural products made up 60% of the 2.2 million U.S. farms. Farms producing $500,000 or more in 2007 dollars generated 96% of the value of U.S. agricultural production.

Table 2.2 shows the percentage of farms by organizational type and their share of aggregate agriculture product sales according to the 2007 Census of Agriculture. Sole proprietorships are the dominant form of business organization measured by farm count (86.5%) but have only 49.6% of the value of agricultural production. Partnerships and family corporations make up 20.8% of farms but have 43% of the value of agricultural production. Non-family corporations, part of the "other organization" category, accounted for 0.4% of farms and 6.5% of the value of agricultural production.

Table 2.2. Farm Business Organization Types (USDA Census of Agriculture, 2007. Farms in US 1,925,300)

Business Type	% of Farms	% of Cash Receipts
Sole Proprietorships	86.5%	49.6%
Partnerships	7.9%	20.9%
Family Corporations[1]	3.95	22.9%

Business Type	% of Farms	% of Cash Receipts
Other[2]	1.2%	7.3%

More generally, about 80 percent of all businesses (agriculture and non-agriculture) are organized as sole proprietorships while only around 10 percent of businesses are organized as corporations. Conversely, about 80 percent of business sales come from corporations while sole proprietorships account for only about 10 percent of business sales.

So what have we learned? We learned that firms organize differently depending on the size, ability to manage, the need for internal versus external funding, and tax implications. Indeed, the need for different business organizations can be compared to the need for different kinds of transportation—it depends on where you are going.

Summary and Conclusions

Recognizing that we can offer financial management tools that meet a limited set of business organizations, we purposely focus in this text on small to medium-size businesses. As a result, we focus on firms that depend on internal capital and exercise the maximum control of the firm.

Questions

1. Discuss the advantages and disadvantages of organizing a business as a sole proprietorship versus a C corporation.
2. Limited partnerships, limited liability companies, and Subchapter S corporations are also alternative forms of business organization. Discuss the advantages and/or disadvantages these organizations offer relative to sole proprietorships, general partnerships and C corporations.
3. Approximately 85% of all farm businesses in the US are organized as sole proprietorships. Explain why the organization form of farm businesses in the U.S. is dominated by sole proprietorships.
4. Pick an agricultural commodity or product that is produced in the food industry. Describe the different production, processing and marketing steps for the commodity or product and how they are typically coordinated.
5. Can you explain in Table 2.2 why corporations tend to control more land than partnerships and sole proprietorships?
6. What are the advantages or disadvantages of a family corporation compared to a regular corporation?

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1. More than 50% of the stock is owned by persons related by blood or marriage. ↵
2. Nonfamily farms, estates or trusts, grazing associations, American Indian Reservations, etc. ↵

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3: The Federal Tax System

Learning Objectives

At the end of this chapter, you should be able to: (1) describe the major components of the federal tax system; (2) know how the different forms of business organizations are taxed; (3) recognize the difference between marginal and average tax rates; and (4) understand how depreciation, capital gains, and depreciation recapture affect the amount of taxes a firm pays. To achieve your learning goals, you should complete the following objectives:

- Learn to distinguish between the firm's gross income, adjusted gross income, and personal deductions.
- Learn how to calculate the firm's tax liability by finding its taxable income
- Learn how to find the firm's average tax rate and marginal tax rates.
- Learn how to find a firm's tax liabilities by using tables of Federal Income Tax Rates.
- Learn how depreciation can reduce the firm's tax liabilities.
- Learn how to find the amount of taxes paid on interest and dividend income.
- Learn how to distinguish between capital gains and losses and depreciation recapture.
- Learn how taxes are calculated on depreciation recapture and capital gains and losses.
- Learn about the different ways depreciation can be calculated and the advantages and disadvantages of each depreciation method.

Introduction

Chapter 2 proposed that one financial management objective is to organize the firm so that its value is maximized. Of course, "the firm" could mean any business organization ranging from a large corporate firm to a small business or even an individual household. Later, we will see that the firm's value is determined by its after-tax cash flow, which can differ significantly from the firm's profits measured by its accounting income. As a result, we need to understand the differences between a firm's cash flow and its accounting income—this discussion will come later. Fortunately, for the most part, a detailed knowledge of specific accounting differences for the different types of business organizations isn't essential for our purposes. So we begin by discussing some of the major components of the federal tax system that have a bearing on a firm's after-tax cash flow.

Due to the influence of taxes on cash flow, it is important to have an understanding of how the tax system works. This chapter intends to present a few of the basic concepts related to taxes that are important from a financial management perspective. Our focus is on the federal tax code, because of its importance in determining after-tax profit and after-tax cash flow. Nevertheless, there are a number of additional taxes (e.g. state and local taxes) that can have a significant impact on a firm's earnings and cash flow. It is important to consider the impacts of all taxes when making financial decisions. The federal tax laws are written by Congress. The Internal Revenue Service (IRS) is the agency responsible for administering the code and collecting federal income taxes. The IRS issues regulations which are its interpretation of the tax laws. The regulations are effectively the tax laws faced by businesses and individuals. One final word of caution: always remember that tax laws can and do change, and these changes are not always announced in ways that inform small businesses. Nevertheless, ignorance is not an excuse for incorrectly filing one's taxes.

Individual Taxes

Individual (ordinary) tax liabilities are determined by subtracting certain allowable deductions from one's total income to obtain taxable income. Taxable income is then used as the basis from which the tax liability is calculated. The general procedure is:

- *Gross income – Adjustments to income = Adjusted gross income.*
- *Adjusted gross income – Personal exemptions and deductions = Taxable income.*
- *(Taxable income) x (Average tax rate) = Tax liability.*

Gross income consists of all income received during the tax year in the form of money, goods and services, and property. *Adjustments to income*, including some past losses, may include income that is not taxed, such as interest income generated by nontaxable municipal bonds. *Adjusted gross income* may be reduced by subtracting *personal exemptions and deductions*

regardless of whether you itemize or not and includes such things as business expenses and deductions for some types of Individual Retirement Account (IRA) contributions. A personal exemption is the allowable reduction in the income based on the number of persons supported by that income. You may claim deductions for yourself, your spouse, and other dependents who meet certain criteria. As of 2018, the personal exemption amount is zero until 2025. In addition, you can reduce your taxable income by either claiming a standard deduction or itemizing allowable expenses.

The standard deduction is an amount allowed for all taxpayers who do not itemize, and represents the government's estimate of the typical tax-deductible expenses that you are likely to have. As of 2018, the standard deduction is $12,000 for single, $18,000 for head of household, and $24,000 for married filing jointly. These are adjusted annually for inflation. If your tax-deductible expenses are greater than the standard deduction, you can list them separately and deduct the total value of the itemized deductions. Itemized deductions include expenses for such things as medical expenses, certain types of taxes, mortgage interest expense, and charitable contributions. After reducing *Adjusted gross income* by subtracting *Personal exemptions and deductions*, we obtain our *Taxable income*, the amount of income that will be used to calculate your *Tax liability*.

The Federal income tax in the United States is called a *progressive* tax, meaning that the percentage tax rate increases as taxable income increases. In contrast, *regressive* taxes have their tax rate remain constant or decrease as taxable income increases. State sales taxes, property taxes, social security taxes, and in some cases, state income taxes are regressive taxes because as a percentage of one's income paid as taxes they increase with a decline in one's income.

Two different tax rate measures include the Average tax rate and the Marginal tax rate defined below:

$$average\,tax\,rate = \frac{tax\,liability}{taxable\,income}$$

$$marginal\,tax\,rate = tax\,rate\,on\,the\,next\,dollar\,of\,taxable\,income.$$

The average tax rate represents the "average" tax rate that is paid on each dollar of taxable income. The marginal tax rate is the tax rate that is paid on the next dollar of taxable income. In a progressive tax system, the marginal tax rate will always be equal to or greater than the average tax rate.

The federal tax rate schedule for 2018 taxable income is shown in Table 3.1.

Table 3.1. Federal Income Tax Rates in 2018

Tax Bracket	Married Filing Jointly	Single
10% Bracket	$0 – $19,050	$0 – $9,525
12% Bracket	$19,050 – $77,400	$9,525 – $38,700
22% Bracket	$77,400 – $165,000	$38,700 – $82,500
24% Bracket	$165,000 – $315,000	$82,500 – $157,500
32% Bracket	$315,000 – $400,000	$157,500 – $200,000
35% Bracket	$400,000 – $600,000	$200,000 – $500,000
37% Bracket	Over $600,000	Over $500,000

Because of the progressive nature of the tax, the marginal tax rate increases as your income increases. The first $19,050 of taxable income for married couples filing a joint return are taxed at a rate of 10%, the next $58,350 ($77,400 – $19,050) of taxable income are taxed at a rate of 12%, and so on. Suppose a married couple had $120,000 of taxable income in 2018.

Their Federal tax liability would be calculated on each increment of income as follows: The average tax rate paid equals Total tax liability/taxable income = $18,279/$120,000 = 15.2% and the marginal tax rate paid on the last dollar earned would be 22%.

10% tax on first $19,050 ($19,050 – $0) = 10% x $19,050 = $1,905.00
12% tax on next $58,350 ($77,400 – $19,050) = 12% x $58,350 =$7,002.00

22% tax on next $42,600 ($120,000 – $77,400) = 25% x $44,700 = $9,372.00
Total Tax Liability on $120,000 = $18,279.00

It is important to distinguish between the marginal and average tax rate. The average tax rate is useful because it allows us, with a single number, to characterize the proportion of our total income that is taxed. In many cases, however, we are interested in the amount of tax that will be paid on any additional income that is earned, perhaps as a result of profitable investment. In these situations, the marginal tax rate is the appropriate rate to use. For example, suppose your average tax rate is 18.4 percent and you are in the 22 percent tax bracket, and you receive a $1,000 raise. The additional income you earn will be taxed at the marginal tax rate of 22 percent, regardless of the average tax rate, so that your increase in after-tax income is only $1,000(1 – .22) = $780. The marginal tax rate is also significant when considering the impact of tax deductions. Suppose you are in the 22 percent tax bracket and you contribute $2,000 to your favorite charity. This reduces your taxable income by $2,000, saving you $440 in taxes (22% times $2,000) so that your after-tax cost of your contribution is only $2,000(1 – .22) = $1,560.

Total Marginal Tax Rates

As we pointed out earlier, because of the progressive nature of the federal tax code, the effective marginal rate that individuals pay is nearly always greater than the federal marginal tax rate. However, there are more levels of government collecting tax revenues than just the federal government. In addition to the federal tax, most personal income is also subject to state taxes, Social Security taxes, Medicare taxes, and perhaps city taxes. State taxes vary but often run in the 4 – 6 percent range. Social Security (6.2 percent) and Medicare (1.45 percent) taxes are split between employer and employee, and the rate for most employees is 7.65 percent (self-employed pay both the employer and employee half, usually 15.3 percent). In 2018, the social security tax is imposed only on individual income up to $128,400. There is no maximum income limit on the Medicare tax. City taxes can run 3 – 4 percent. Therefore, the effective marginal tax rate for someone in the 24 percent federal tax bracket who pays social security tax will generally be over 37 percent. If the same person were self-employed they would be subject to an additional 7.65 percent, and a marginal tax rate that could exceed 50 percent of taxable income in some cases. It should be noted that one-half of "self-employment tax" is deductible from income subject to federal tax, so the effective marginal tax rate increases by only 7.65(1 – .24) = 5.81 percent for someone in the 24 percent tax bracket. As you can see, it is extremely important to understand what the effective marginal tax rate is when making financial management decisions.

Bracket Creep

Progressive tax systems are subject to an undesirable feature, often termed "bracket creep." *Bracket creep* is an inflation-induced increase in taxes that results in a loss in purchasing power in a progressive tax system. The idea is that inflation increases tend to cause roughly equal increases in both nominal income and the prices of goods and services. Inflation induced increases in income push taxpayers into higher marginal tax brackets, which reduces real after tax income. Consider an example of how inflation reduces real after-tax income and therefore purchasing power.

For the couple in our previous example with a taxable income of $120,000, their real after-tax income, their purchasing power in today's dollars, is equal to their *Gross income less* any *Tax liabilities* which in their case is: $120,000 – $18,279 = $101,721. Another way to calculate their purchasing power is to multiply (1 – average tax rate *T*) times their gross income: (1 – 15.2325%)$120,000 = $101,721.

Inflation is a general increase in price. Suppose that, as a result of inflation, that next year inflation will increase your salary 10%. However, suppose that inflation will also increase the cost of things you buy by 10%. As a result, to purchase the same amount of goods next year that were purchased this year will require a 10% increase in this year's expenditures. For the couple described in our earlier example, this will require expenditures of $101,721 times 110% = $111,893.10.

Now let's see what happens to our couple's after-tax income. A 10% increase in taxable income means they will have $120,000 times 110% = $132,000 in taxable income next year. Recalculating the couple's tax liabilities on their new income of $130,000:

The average tax rate paid equals Total tax liability/taxable income = $20,919/$132,000 = 15.85% while the marginal tax rate paid on the last dollar earned would be 22%. Subtracting couple's tax liabilities from their income leaves them $111,051, less than $111,893.10 which is the amount required to maintain the same purchasing power before inflation. In other words, as a result of the combination of inflation and bracket creep with increases on the average tax rate, the couple's purchasing power is reduced

The IRS recognizes the impacts of bracket creep, and periodically adjusts the tax schedules and/or deductions in an attempt to smooth or eliminate purchasing power losses due to bracket creep.

10% tax on first $19,050 ($19,050 – $0) = 10% x $19,050 = $1,905.00
12% tax on next $58,350 ($77,400 – $19,050) = 12% x $58,350 = $7,002.00
22% tax on next $54,600 ($132,000 – $77,400) = 25% x $54,700 = $12,012.00
Total Tax Liability on $132,000 = $20,919.00

Interest and Dividend Income

Interest and ordinary dividends earned by individuals are generally taxed as ordinary income. One exception is interest income earned on municipal bonds issued by state and local governments (bonds are promissory notes issued by a business or government when it borrows money). Municipal bonds are exempt from federal taxes, which makes them attractive to investors in high marginal tax brackets.

Suppose you are in a 25% marginal tax bracket and are considering investing $1,000 in either a corporate bond that yields 10% per year, (yield means the rate of return the investment provides) or a municipal bond yielding 8% each year. You can calculate your after-tax cash flow from each investment by subtracting the tax liability from the before-tax cash flow from each investment.

The corporate bond provides a $100 cash flow before taxes but only $75 is left after paying taxes. Meanwhile, the municipal bond is exempt from federal taxes and provides an $80 before and after-tax cash flow. This example illustrates the importance of considering after-tax cash flow as opposed to before-tax cash flow when considering investment opportunities.

Asset	Before-tax cash flow return	– Taxes	= After-tax cash flow
Corporate bond	$1,000(.10) = $100	$100(25%)	$75.00
Municipal bond	$1,000(.08) = $80	$0	$80.00

We can also measure an investment's return using percentages. Percentage measures standardize return measures so investments of different sizes can be compared. We calculate the percentage return during a period as the cash flow received during the period divided by the total amount invested. For example, the percentage rate of return on the corporate bond was $75/$1000=7.5%, while the percentage rate of return on the municipal bond was $80/$1000=8.0%.

When the returns on the bonds are expressed as rates of return, it is clear that the municipal bond is the preferred investment.

Suppose you wanted to find the before-tax rate of return on a corporate bond (*r*cb), a pre-tax equivalent rate of return that would provide the same after-tax rate of return as a tax free municipal bond (*r*mb). To find *r*cb , equate the after-tax rates of returns on the corporate and municipal bonds, *r*cb (1 – *T*) = *r*mb , and solve for the following:

$$r^{cb} = \frac{r^{mb}}{(1-T)} \tag{3.1}$$

In our example municipal bond rate is 8% so that for an investor in the 25% marginal tax bracket, the pre-tax equivalent rate of return is:

$$r^{cb} = \frac{8\%}{(1-0.25)} = 10.67\% \tag{3.2}$$

In other words, a corporate bond yielding 10.67% would produce the same after-tax return as the municipal bond yielding 8% for an investor in the 25% tax bracket.

Interest Paid by Individuals

Interest paid by individuals is generally not tax deductible for personal expenditures, such as interest on a loan for a car used solely for personal travel. One primary, and important, exception to this is that interest on a home mortgage is usually deductible from taxable income if filing using itemized deductions. There is also a limited deduction available for student loan interest payments.

Capital Gains and Losses

From an accounting standpoint, we define *book value* (or basis) as acquisition cost less accumulated depreciation, which is determined by tax codes. (Book value may also be altered by improvements to depreciable assets.) For a variety of reasons, an asset's market value is usually different from its book value. When an asset is liquidated at a market value greater than its book value we say it has appreciated in value. If the appreciation is equal to or less than its original acquisition cost, we refer to the appreciation value as *depreciation recapture*. Appreciated value in excess of its acquisition value is referred to as *capital gains*.

Likewise, when an asset's liquidation value is less than its book value, we say that the asset has experienced *capital losses* and define: capital losses = book value – market value (if book value > market value). The tax rate for individuals on "long-term capital gains", which are gains on assets that have been held for over one year before being sold, depends on the ordinary income tax bracket. For taxpayers in the 10% or 15% bracket, the rate is 0%. For taxpayers in the 22%, 24%, 32%, or 35% bracket, it is 15%. For those in the 37% bracket, the rate is 20%. Other rates also exist under certain situations. The tax rate paid on depreciation recapture is the taxpayer's income tax rate since depreciation was used to shield income from income tax payments.

Suppose you are in the 24% tax bracket. You have just sold some land (a non-depreciable asset) for $50,000. The land was purchased 5 years ago for $30,000. Since you sold the land for more than its book value (the purchase price in this case) you will realize a capital gain of $50,000 – $30,000 = $20,000. Your tax liability on the sale will then be the capital gain times the capital gains tax rate or in this case $20,000(15%) = $3,000.

Capital losses must first be used to offset any capital gains realized that year; however, if you have any capital losses remaining after offsetting capital gains, you may offset ordinary income. If you still have capital losses remaining, you may carry the losses forward to offset future capital gains and income. This can be a powerful tool. Suppose the land in the above example brought a selling price of $20,000. Now you have realized a capital loss of $30,000 – $20,000 = $10,000. If you have not realized capital gains for the year, you can use the loss to offset your ordinary income, which in this example is being taxed at 24%. Your tax savings from the capital loss would be $10,000(24%) = $2,400. In other words, you can use the loss to reduce the level of taxable income by $10,000, which saves you $2,400 in taxes you would have been required to pay without the loss. If you had realized at least $10,000 in capital gains on assets held over 12 months during the year, your capital loss would have only saved you $10,000(.15%) = $1,500. It is clear that the current capital laws require some careful planning to capture the full benefits. (Note: some capital losses are limited to $3,000 per year to be taken against regular income. Anything left over can be carried forward to future years.)

Tax Credits

Tax deductions are subtracted from Adjusted Gross income and reduce the level of Taxable income. This results in a tax savings that is equal to the marginal tax rate for each dollar of allowable deduction. Tax credits, on the other hand, are direct deductions from one's Tax liability. Tax credits, therefore, result in a tax savings of one dollar for each dollar of tax credit. For example, consider the tax savings of a $1,000 deduction versus a $1,000 tax credit for an individual facing a 24% marginal percent tax rate:

tax deduction: tax savings = $1,000(24%) = $240; and
tax credit: tax savings = $1,000(100%) = $1,000.

Clearly, you would prefer one dollar of tax credits more than one dollar of tax-deductible expense. Tax credits come and go sporadically in the tax laws. They have been used in the past to stimulate investment in some types of assets. There are still some tax credits available today. For example, certain college tuition and child care expenses may be eligible for tax credits. Also, for each dependent child under age 17, up to $2,000 credit may be available.

Business Taxation

Remember that Subchapter S corporations, partnerships, and sole proprietorships are taxed according to the individual tax rate schedule the owner faces. Many Subchapter S corporations, partnerships, and sole proprietorships may get a 20% reduction of their Qualified Business Income, which might be their net business income. C corporations, on the other hand, are taxed according to corporate income tax rates. The corporate tax ate schedule for 2018 is now 21%.

Not all the income generated by a business's operations is subject to taxes. Most expenses incurred in order to generate the business income are deductible. In addition, there are a number of other adjustments to business income which deserve mention.

Also be aware that many, but not all, states impose an additional tax on corporate income.

Depreciation

When a business purchases assets that can be used in the business for more than one year (often called capital expenditures), the business is generally not allowed to deduct the entire cost of the asset in the year in which it was purchased. *Depreciation* is an accounting expense that allocates the purchase cost of a depreciable asset over its projected economic life. This deduction acts as an expense for both accounting and tax purposes; so increases in depreciation expense result in decreased profit measures for a business. Nevertheless, depreciation expense is a *noncash expense*; that is, you aren't sending a check to anyone for the amount of the depreciation expense. This allows depreciation expense to act as a *tax shield,* which lowers taxable income, resulting in lower tax obligations and consequently higher after-tax cash flow. Accordingly, higher depreciation expenses result in lower profits but higher after-tax cash flow.

For tax purposes, there are rules that determine how you depreciate an asset. You can only depreciate an asset that has been *placed in service,* which means that it is available for use during the accounting period. There are also conventions that determine the point in time during the accounting period you must assume the asset was placed into service. Most depreciable assets are assumed to be placed into service at the mid-point of the year, regardless of the actual date they are placed in service. This is known as the *half-year convention*. There are also rules about the *recovery period* over which an asset can be depreciated. Table 3.2. shows the allowable recovery periods for different classes of farm property.

Table 3.2. Farm Property Recovery Periods[1]

Assets	Recovery Period in Years	
	Gen. Dep. System	Alter. Dep. System
Agricultural structures (single purpose)	10	15
Automobiles	5	5
Calculators and copiers	5	6
Cattle (dairy or breeding)	5	7
Communication equipment	7	10
Computers and peripheral equipment	5	5
Cotton ginning assets		
Drainage facilities	15	20
Farm buildings	20	25
Farm machinery and equipment	7	10
Fences (agricultural)	7	10
Goats and sheep (breeding)	5	5
Grain bin	7	10
Hogs (breeding)	3	3
Horses (age when placed in service)		
Breeding and working (12 yrs or less)	7	10

Breeding and working (more than 12 yrs)	3	10
Racing horses (more than 2 yrs)	3	12
Horticultural structures (single purpose)	10	15
Logging machinery and equipment	5	6
Nonresidential real property	39	40
Office furniture, fixtures and equipment (not calculators, copiers, or typewriters)	7	10
Paved lots	15	20
Residential rental property	27.5	40
Tractor units (over-the-road)	3	4
Trees or vines bearing fruit or nuts	10	20
Truck (heavy duty, unloaded weight 13,000 lbs. or more)	5	6
Truck (weight less than 13,000 lbs.)	5	5
Water wells	15	20

There are two depreciation methods that can be used for tax reporting purposes: *straight line* (SL) and *modified accelerated cost recovery system* (MACRS). Straight line allocates the depreciation expense uniformly each period. The amount of depreciation is found by multiplying the straight line recovery rate for each year times the asset's unadjusted basis (original cost). The SL recovery rate is simply the inverse of the number of years in the asset's recovery period adjusted by the required half-year placed in service convention. The straight recovery rates for 3-year, 5-year, and 7-year assets are shown in Table 3.3.

Table 3.3. Straight Line Recovery Rates for 3, 5, 7 Year Assets

Year	3-Year	5-Year	7-Year
1	16.67%	10%	7.14%
2	33.33%	20%	14.29%
3	33.33%	20%	14.29%
4	16.67%	20%	14.28%
5		20%	14.29%
6		10%	14.28%
7			14.29%
8			7.14%

As you might guess from its name, MACRS produces larger depreciation expenses in early years than the SL method of depreciation and lower depreciation expenses in the later years than the SL method of depreciation. The federal government

specifies the allowable rates of MACRS depreciation. These rates are determined by using a combination of the declining balance (either 150% or 200%) and straight-line depreciation methods. Prior to 2018, farmers were not permitted to use 200% declining balance. The MACRS depreciation expense for each year is calculated by multiplying the unadjusted basis (acquisition cost) of the asset by the appropriate recovery rate for that year. The MACRS (200% method) recovery rates for the 3-year, 5-year, and 7-year asset classes are shown in Table 3.4.

Table 3.4. MACRS Recovery Rates for 3, 5, and 7-Year Assets

Year	3-Year	5-Year	7-Year
1	33.0%	20.00%	14.29%
2	44.45%	32.00%	24.49%
3	14.81%	19.20%	17.49%
4	7.41%	11.52%	12.49%
5		11.52%	8.93%
6		5.76%	8.93%
7			8.93%
8			4.46%

Suppose your business just purchased a $100,000 asset that has a 3-year useful life, and falls into 3-year class of assets. Using the SL method, the depreciation expense each year for the next 3 years would be:

Year	Recovery Rate	Unadjusted Basis	Depreciation Expense	Accumulated Depreciation
1	.1667	$100,000	$16,670	$16,670
2	.3333	$100,000	$33,330	$50,000
3	.3333	$100,000	$33,330	$88,330
4	.1667	$100,000	$16,670	$100,000

Note that the book value or basis of the asset (acquisition cost – accumulated depreciation) would be $0 after it has been fully depreciated at the end of 4 years. Because of the half-year convention, it takes 4 years to depreciate the asset, even though it falls into the 3-year classification.

Depreciation expense for the same asset using the MACRS method would be calculated as:

Year	Recovery Rate	Unadjusted Basis	Depreciation Expense	Accumulated Depreciation
1	.3333	$100,000	$33,333	$33,333
2	.4445	$100,000	$44,450	$77,780
3	.1481	$100,000	$14,810	$92,950
4	.741	$100,000	$7,410	$100,000

Note again that the depreciation expense using MACRS is higher in the early years and lower in later years than with the SL method and that the book value after 4 years is again zero. Businesses often use MACRS for tax purposes and SL for profit reporting. Can you think of any reasons why?

Some businesses that invest small amounts in capital assets are allowed to deduct up to $1,000,000 of the cost of acquired depreciable property as a current expenditure instead of a capital expenditure. This is known as *direct expensing,* and is available only to businesses that don't make large capital purchases each year. The allowable expensing amount is reduced by one dollar for each dollar of capital investment expenditure over $2,500,000 during the year. Other restrictions also apply.

Bonus or Additional Depreciation

Property purchased with a recovery period of 20 years or less may be eligible for 100% additional depreciation the year it is purchased. It is required unless the taxpayer elects out of it by class or recovery period (3, 5, 7, 10, 15, 20 year classes). It can be used on either new or used property but has limitations if purchased from a relative. Passenger automobiles may be limited to an extra $8,000 additional depreciation rather than 100%. The 100% depreciation decreases to 80% in 2024 and reduces by 20% each year after that until 2028 when it disappears.

Interest and Dividends Received

Interest and ordinary dividend income received by sole proprietorships and partnerships is taxed as ordinary income. Interest income received by corporations is taxed as corporate income. However, dividend income received by corporations is allowed eighty percent *tax exclusion* (only 20 percent is taxed) before being taxed as corporate income. The reason for the exclusion is that corporate income is already subject to double taxation, and taxing dividends as corporate income would result in triple (or more) taxation. Can you explain why?

Which is better for a corporation in a 21% marginal tax bracket: $1,000 of interest income or $1,000 of dividend income? Remember what we care about is after-tax cash flow (ATCF) so let's evaluate both investments:

Investment Income	– Taxes	= ATCF
Interest income $1,000	– $1,000(.21)	= $790
Dividend income $1,000	– $1,000(.20)(.21)	= $958

The ATCF from the dividends of $958 would be preferred to the $790 from the interest income. The dividends are effectively taxed at a lower rate than interest income. We can find the effective rate on each dollar of dividend income received by multiplying the two marginal tax rates by the proportion of income subject to tax: that is $1(.20)(*T*) = $1(.20)(.21) = 0.042 or 4.2%. Accordingly, the effective tax rate *T* for dividends in this case is 4.2% as opposed to 21% for interest income at this marginal tax rate. What incentives do federal tax laws give regarding the type of investments that corporate businesses make?

Interest and Dividends Paid

Interest expenses paid by businesses are tax deductible. Dividends or withdrawals paid by businesses are not tax deductible. Consider a firm in a 21 percent tax bracket that pays $1,000 in interest expense and $1,000 in dividends or withdrawals. What is the after-tax cost of each expense?

Expense Type	Expense	– Tax-savings	= After-tax expense
Interest expense	$1,000	– $100(.21)	= $790
Dividend expense	$1,000	– $0	= $1000

Dividends and withdrawals cost the firm $1 of after-tax income for each $1of dividend paid. Interest expense, on the other hand, costs the firm $1(1 – *T*) = $1(1 – .21) = $0.79 of after-tax income for each $1 of interest expense paid. Another way to think about it is to find out how much before-tax income it takes to pay $1 of both interest and dividend expenses. Using the formula discussed earlier:

Before-tax dividend expense = $1.00/(1 – .21) = $1.266
Before-tax interest expense = $0.79/(1 – .21) = $1.00

It takes $1.27 of before-tax income to pay $1 in dividend or withdrawal expense, while it takes only $1 in before-tax income to pay $1 in interest expense. The two ways for a firm to finance its assets are to use debt or equity financing. Which method of financing do the federal tax loans favor?

So what have we learned? It's not what you earn but what you get to keep that matters. And that's where tax management plays an important role. Taxes are an important difference between what you earn and what you get to keep. Thus, an important part of

strategic financial management is managing one's tax obligations.

Summary and Conclusions

The main lesson for managing taxes is to look at investments and cash flow on an after-tax basis, and this requires that all tax obligations are accounted for in the tax management process. Management of the firm's taxes also requires managers to understand how different business organizations can create different tax obligations, the difference between average and marginal tax rates, the different tax obligations associated with interest versus dividend income, capital gains and capital losses, depreciation schedules, book value and market value of assets, tax deductions, and tax credits. Understanding these and other tax-related concepts will help financial managers manage the difference between what you earn and what you get to keep.

Questions

1. Tiptop Farms, a sole proprietorship, had a gross income of $600,000 in 2018. Tiptop Farms expenses were equal to $320,000. Tiptop had interest expenses of $80,000 and depreciation expenses of $75,000 during the year. Tiptop's owner withdrew $25,000 from the business during the year to help pay college expenses for the owners' children. The tax schedule for 2018 is:

Income	Tax Rate
$0 – 40,000	15%
$40,001 – 100,000	28%
$100,000 – 150,000	31%
$150,001 +	35%

   a. What was Tiptop's taxable income during 2018?
   b. What is Tiptop's tax liability for 2018?
   c. What is Tiptop's average and marginal tax rate for the year?
   d. If Tiptop is considering the purchase of a new investment, what tax rate should they use?
2. Consider a couple filing jointly, whose taxable income last year was $130,000. Assume that state taxes on their taxable earnings were 5%. The couple pays social security at the rate of 6.2%, and Medicare taxes at the rate of 1.45%. Half of the couple's social security and Medicare taxes are paid by their employer. Also recognize that social security was only charged on the first $128,400. Find the couples tax liability, its average tax rate, and its marginal tax rate. What is higher, the couple's average tax rate or their marginal tax rate? Explain your answer.
3. Suppose that last year, inflation was 8%, meaning that the price of every- thing you buy has increased by 8%. If your before-tax income increased by 8% would you be just as well off as before? If not, how much more than 8% would you need to increase to buy as much as you did before inflation?
4. A corporate bond is providing a yield of 12% per year, while at the same time, a municipal bond (which is tax exempt) is yielding 9% per year. Each bond only pays interest each year until the bond matures, at which time the principal investment will be returned. Each bond is equally risky, and your marginal tax bracket is 30%. You have $1,000 to invest.
   a. What is the after-tax cash flow from each bond? Which bond would you invest in?
   b. What is the percentage after-tax return on each bond?
   c. What before-tax equivalent on the corporate bond would make you choose it as an investment?
   d. Suppose that you are also subject to a 5% state tax from which neither the corporate or municipal bond is tax exempt. What is the after-tax return on each bond?
5. Suppose you invest in a new tractor during the tax year that costs you $78,000, plus $2,000 for delivery and set up. Your marginal tax rate is 40%. *Note: new farm machinery has a class life of 5 years.*
   a. What is the depreciation expense each year using the straight line method of depreciation?
   b. What is the depreciation expense each year using the MACRS 200% declining balance method of depreciation?
   c. What is the tax shield each year from each method?
   d. Which method would you use for tax purposes? How about profit reporting? Explain.
6. Businesses often use MACRS for tax purposes and SL for profit reporting. Can you think of any reasons why?

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1. Most assets use the General Depreciation System for recovery periods. See IRS guidelines for more information. ↩

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4: Financial Statements

Learning Objectives

After completing this chapter, you should be able to (1) construct consistent and accurate coordinated financial statements (CFS) (2) describe the differences and connections among balance sheets, accrual income statements (AIS), and statements of cash flows; and (3) distinguish between endogenous and exogenous variables and how they influence the construction of CFS.

To achieve your learning goals, you should complete the following objectives:

- Learn how to construct the financial statements included in the CFS.
- Learn how the fundamental accounting equation organizes the firm's balance sheet entries.
- Learn how to organize a firm's cash flow data into a Sources and Uses of Funds (SAUF) statement.
- Learn how to distinguish between a firm's cash income statement and its accrual income statements (AIS).
- Learn how to compute a firm's cash income statement using cash flow and depreciation data.
- Learn how to distinguish between exogenous and endogenous variables.
- Learn how to compute a firm's statement of cash flow using data from its checkbook and other cash flow records.
- Learn the consistency requirements that connect data from the firm's balance sheets, AIS and statement of cash flow.
- Learn the distinction between consistency of financial statements and accuracy of financial statements.
- Learn how to organize firm financial data into a consistent and (as far as possible) accurate set of CFS.

Introduction

Financial statements include balance sheets, the cash income statement, an accrual income statement (AIS), the statement of cash flow (SCF), and the sources and uses of funds (SAUF) statement. Coordinated financial statements (CFS) include beginning and ending period balance sheets, an AIS, and a SCF. We are particularly interested in the financial statements included in CFS because they are interdependent and when constructed consistently can help us identify inaccuracies in our data.

The purpose of financial statements is to provide the firm the information it needs to identify its strengths and weaknesses, to evaluate its performance, to assess its alternative futures, and to guide its choices between alternative futures to the one most consistent with its mission, goals, and objectives.

Financial managers are expected to conduct financial analysis. To be effective in this role, financial managers need to have a basic understanding of financial statements—how to construct them and how to analyze them. The material that follows provides a basic understanding of financial statements and how to construct them. Later chapters will focus on how to analyze them.

There are, of course, other reasons why familiarity with financial statements may be important. We may have an interest in learning about successful firms, and one way to do this is to examine their financial statements. We may be interested in investing in a firm and want to evaluate its financial condition. We may be employed in a job that requires an understanding of financial statements. Or, we may simply want to make informed decisions that influence the financial conditions of firms in which we have an interest.

Ideally, firms maintain properly constructed financial statements which financial managers can use to conduct financial analysis. But the reality is that many firms, especially small firms, do not maintain a properly constructed set of financial statements. Furthermore, many firms do not record all the data necessary to construct an accurate set of financial statements. On the other hand, nearly all firms maintain cash receipts and cash expense records needed to construct income tax liabilities. And these same tax record lists the book value of long-term assets necessary to construct balance sheets. Fortunately, with balance sheet data and a cash income statement computed from tax records, we can construct a set of financial statements including an AIS.

Financial Statements

Financial statements may vary by type of firm. For example, financial statements prepared for corporations must satisfy generally accepted accounting practices (GAAP) in which ownership claims are represented by shares of stock. Financial statements

prepared for sole proprietorships and partnerships are not subjected to GAAP requirements. In addition, some financial statements are based on accrual methods that record transactions when they occur, while others record transactions only when cash is exchanged.

Different financial statements may report the value of assets differently. Some financial statements represent the value of assets at the price at which they could be sold in the current accounting period. These values are referred to as *market* values. Other financial statements represent the value of assets at the price at which they were purchased less depreciation. In these statements, allowable depreciation is usually described in tax codes. These values are referred to as *book* values.

So what have we learned? We learned that financial managers need access to identify the following financial statements: book value balance sheets, accrual and cash income statements, sources and uses of funds (SAUF) statement, and statement of cash flow. Of these statements we make particular mention of the book value balance sheets, the accrual income statement (AIS) and the statement of cash flow. We refer to these as coordinated financial statements (CFS) because they are interdependent and their proper connections to each other—their consistency—can be easily checked. In addition, consistently constructed CFS can help us identify inaccuracies in our data. In what follows we describe firm financial statements.

The Balance Sheet

The balance sheet describes the firm's assets—what the firm owns or controls. It also lists the claims on the firm's assets from outside the firm called liabilities. The difference between assets and liabilities equals net worth and represents the firm's equity. The balance sheet is constructed at a point in time, e.g. the last day of the year, leading some to describe it as a snapshot of the firm's financial condition.

In the balance sheets presented in this class, the value of assets and liabilities is a combination of current and book values. Long-term assets are recorded at their book value—their purchase prices less accumulated depreciation determined by tax codes. Most liabilities and current assets are valued at their current values.

The underlying principal for constructing any balance sheet is the fundamental accounting equation: Assets = Liabilities + Equity. The fundamental accounting equation declares that each of the firm's assets must be financed either by liabilities (funds supplied by those outside the firm) or equity (funds supplied by the firm's owners). Moreover, the fundamental accounting equation separates the firm's assets from its liabilities and equity on the balance sheet. We can check the fundamental accounting equation by noting that the total value of assets equals the total value of liabilities and equity for each year the balance sheet is calculated.

Table 4.1 reports 2016, 2017 and 2018 year-end balance sheets for the hypothetical proprietary firm called HiQuality Nursery. Since we will repeatedly use data associated with this hypothetical firm and refer to the firm frequently, we will refer to it in the future using the acronym HQN. When reporting balance sheets for multiple years, the balance sheets in this text will appear in increasing time periods from least current to most current: 2016, 2017, and 2018 in the case of HQN.

Table 4.1. Year End Balance Sheet for HQN (all numbers in 000s)
Open HQN Coordinated Financial Statement in MS Excel

Balance Sheet	Year		
	12/31/16	12/31/17	12/31/18
Cash and Marketable Securities	$1,200	$930	$600
Accounts Receivable	$1,560	$1,640	$1,200

Inventory	$3,150	$3,750	$5,200
Notes Receivable	$0	$0	$0
CURRENT ASSETS	**$5,910**	**$6,320**	**$7,000**
Depreciable Long-term Assets	$3,270	$2,990	$2,710
Non-depreciable Long-term Assets	$710	$690	$690
LONG-TERM ASSETS	**$3,980**	**$3,680**	**$3,400**
TOTAL ASSETS	**$9,890**	**$10,000**	**$10,400**
Notes Payable	$1,400	$1,500	$1,270
Current Portion LTD	$700	$500	$450
Accounts Payable	$2,400	$3,000	$4,000
Accrued Liabilities	$870	$958	$880
CURRENT LIABILITIES	**$5,370**	**$5,958**	**$6,600**
NONCURRENT LONG-TERM DEBT	**$2,560**	**$2,042**	**$1,985**
TOTAL LIABILITIES	**$7,930**	**$8,000**	**$8,585**
Contributed capital	$1900	$1900	$1,900
Retained earnings	$60	$100	($85)
TOTAL EQUITY	**$1,960**	**$2,000**	**$1,815**

TOTAL LIABILITIES AND EQUITY	**$9,890**	**$10,000**	**$10,400**

Assets

Assets represent everything of value that the firm controls. Assets have value to the firm, mostly because they represent what can be used to generate earnings. The firm's assets are typically listed in order of liquidity, or nearness to cash. In most cases asset liquidity depends on the ease or cost of converting them to cash.

Current Assets. Current assets are cash and near-cash assets that are expected to be liquidated or converted to cash during the next year. Current assets are typically assets whose liquidation will not significantly disrupt the operation of the firm. We describe current assets next in their order of liquidity, or their nearness to cash.

- Cash balances are the firm's most liquid assets.
- Marketable securities are interest-bearing deposits with low risk of losing principal and can easily be converted to cash if needed.
- Accounts receivable include completed sales, for which payment has not been received.
- Notes receivable represent short-term loans the firm has made to others that are expected to be repaid during the coming year. Notes receivables are important for firms for whom lending money and earning interest on their loans is a significant source of income.
- Finally, inventories may be of two kinds. One kind of inventory represents the value of inputs on hand that can be used in future production or manufacturing of goods. A second kind of inventory are products available for sale. Inventories are often the least liquid of the firm's current assets, and their value is often not known until they are sold.

Long-term assets. Long-term assets yield services over several time periods. Liquidation of long-term assets would typically disrupt the operations of the firm and would occur only if the firm were facing a solvency crisis or replacing long-term assets with more productive ones.

Some balance sheets distinguish long-term assets by the length of time they will be held by the firm before being liquidated, referring to them as intermediate versus long-term assets. We prefer to distinguish between long-term assets by whether or not they depreciate. Depreciable long-term assets include machinery, equipment, breeding stock, contracts, long-term notes receivable, building and improvements. Non depreciable long-term assets are mostly land.

Book value versus market value of long-term assets. The book value of long-term assets is equal to their purchase price less accumulated depreciation: book value = acquisition cost – accumulated depreciation.

Accumulated depreciation is intended to reflect the loss in value of long-term assets due to use or the passage of time. As a practical matter, the depreciation rate is usually determined by tax codes.

While long-term assets are valued at their book value in the balance sheet, firms are also interested in the market value of their long-term assets. An asset's market value is the price at which it could be sold in the current market. An asset's book value is almost always different than its market value, the price at which the asset could be sold in the current period. Two reasons why an asset's book value and market value differ are because an asset's book value ignores appreciation and its depreciation is set by predetermined rates (rather than market forces), most often reflected in tax codes.

Tax consequences created by depreciable assets are complicated. Consider an example. Suppose a firm purchased a depreciable asset for $1,000 and then depreciated its value by $100 for four years and then sells the asset for $1,300. The difference between the realized market value of $1,300 and its book value of $600 consists of recaptured depreciation of $400 and capital gains of $300. The $300 of capital gains are taxed at the capital gains tax rate. Because the depreciation shielded the firm's income from income tax, the tax savings from $400 of depreciation is recaptured at the firm's income tax rate.

To better understand why book value is not equal to market value, think about what determines market value. As will be demonstrated later, the market value of an asset generally depends on the discounted value of future cash flow the asset is expected to generate. The cash flow characteristics that are important in establishing the market value of an asset include:

- the size and/or number of expected future cash flow;
- the timing of expected future cash flow; and

- the risk and variability of future cash flow.

In general, the larger the size and/or number of expected future cash flow, the larger will be the market value of the asset. Likewise, the sooner the asset is expected to generate cash flow, the higher will be the asset's market value, because a dollar today is generally preferred to a dollar later. Finally, as the risk or variability of an asset's future cash flow increase, the lower will be the asset's market value. To account for the influence of the size, timing, and risk of an asset's cash flow on an asset's value, later chapters will introduce the concepts of the time value of money and certainty equivalent measures (see Chapters 7 and 14).

So what have we learned? We learned that with so many variables affecting the market value of an asset, we cannot expect that any predetermined depreciation schedule will accurately predict a depreciable asset's market value. Therefore, depreciable assets are valued at their book value in our balance sheets.

Liabilities

Liabilities are obligations to repay debt and accrued interest charges. They are listed according to the date they become due. Current liabilities are debt and interest payments due during the current period and pose the greatest liquidity demands on the firm's resources. Long-term liabilities are debts that will come due after the current year. Equity, which represents residual ownership of the firm's assets, has no fixed due date and is consequently listed after current and fixed liabilities.

Current liabilities. Current liabilities include the following:

- Notes payable are short-term debt (written promises) the firm is obligated to pay during the current year.
- The current portion of long-term debt (LTD) is the portion of LTD that is due during the upcoming year.
- Accounts payable equal the value of purchases made for inputs but not paid.
- Accrued liabilities are expenses that have been incurred through the operation of the firm and the passage of time, but have not been paid. Examples of accrued expenses include taxes payable, interest payable, and salary and wages payable.

Non-current long-term debt is the final category of liabilities, a long-term liability. Non-current long-term debt is that portion of the firm's debt due after the current period. These are usually long-term notes payable, mortgages, or bond obligations.

Equity: Equity, or net worth, is the difference between assets and liabilities, the difference between what one owns and what one owes. The firm's equity is an estimate of what owners of the firm would have left if they sold all their assets at their book value and paid all their liabilities valued at their market value. Therefore, equity is an important indicator of a firm's financial health.

The actual difference between what one owns and what one owes, if required to sell all of one's assets and repay one's liabilities, depends not only on the difference between the market value and book value of assets but also on the liquidity of the firm. Therefore, some caution is called for when interpreting the equity appearing on the firm's balance sheet as an indicator of its financial well-being.

Equity consists of accumulated retained earnings reported in the AIS and contributed capital by the firm's owners. One practical note is that when reconciling assets and liabilities, contributed capital is sometimes treated as a residual endogenous variable.

Checkbooks and Sources and Uses of Funds (SAUF) Statement

Most small firms record cash flow data in checkbooks, credit card statements, or other financial worksheets. These data are central to the construction of financial statements. They also supply the information needed to construct the firm's income tax returns. Consider HQN's cash flow data recorded in its checkbook reported in Table 4.2. To simplify the reporting of cash flow data, individual entries of the same kind have been combined into general categories.

Checkbooks

Beginning cash balance. Beginning cash balance is the cash the firm had on hand at the end of the previous period. It also appears on the first line of the firm's checkbook.

Cash receipts. Cash receipts may include cash received from the sale of tangible products like grain and livestock. Cash receipts may also include cash received from services the firm sells to other firms such as tiling, harvesting, and veterinary services. Cash receipts may include government payments from sponsored activities and insurance payments. Finally, cash receipts include

reductions in accounts receivables and inventories that represent previous sales and production converted to cash in the present period.

Cash costs of goods sold (COGS). Cash COGS reflect the direct cost of materials and labor used to produce the goods that were sold to generate the firm's revenue. Cash COGS vary with the production levels and are usually the largest expense in most businesses. Cash COGS may also include payments on accounts payable. Finally, cash COGS include reductions in accounts payable that represent expenses incurred in the previous periods paid for in the present period.

Cash overhead expenses (OE). Cash OE represent the costs of operating and administrating the business beyond those expenses included in COGS. These expenses typically include such things as administrative expenses, general office expenses, rents, salaries, and utilities. OE are difficult to assign to a particular production activity because they contribute to more than one project. Moreover, they tend not to vary with changes in production levels. Cash OE may also include payments on accrued liabilities. Finally, cash OE include reductions in accrued liabilities incurred in previous periods paid for in the present period.

Taxes. Taxes include all compulsory contributions to and determined by governmental units. Taxes may be imposed on property, profits, and some goods used in production and sales.

Interest. Interest is the cost paid to use money provided by others during the current period. We sometimes distinguish between interest paid on long-term and short-term debt obligations.

Cash purchases or sales of long-term assets. Depreciable and non-depreciable long-term assets provide services for more than one period providing the firm a measure of control over its capital service flow not afforded by rental agreements.

Loan payments and account and note payments. Loan payments and account and note payments reflect payments on the amount of financial resources owed others. Sometimes referred to as principal payments, in the case of loans, note payment reflect reductions in the financial obligations of the firm as opposed to interest payments charged for the use of the financial resources of others.

Owner draw. Owner draw represents funds withdrawn from proprietary firms by its owners. These payments may be in exchange for services rendered by the firm's owner. In other cases, owner draws are funds withdrawn from the firm to meet financial needs of the firm's owner.

Table 4.2. HQN's 2018 Checkbook

Date	Item	Withdrawal	Deposit	Balance
12/31/17	Beginning cash balance			$930
	Cash receipts		$38,990	$39,920
	Cash cost of goods sold (COGS)	$27,000		$12,920
	Cash overhead expenses (OE)	$11,078		$1,842
	Interest paid	$480		$1,362
	Taxes paid	$68		$1,294
	Purchase of long-term assets	$100		$1,194
	Sale of long-term assets		$30	$1,224
	Current portion of long-term debt paid	$157		$1,067
	Long-term borrowings		$50	$1,117
	Notes payable	$230		$887
	Owners' draw	$287		$600
12/31/18	Ending cash balance			$600

The Sources and Uses of Funds (SAUF) statement.

The data recorded in the firm's checkbook and other cash flow records can be organized as an SAUF statement that identifies sources of cash for the firm (cash inflows) and uses of funds (cash outflows). The SAUF statement is consistent with the cash flow data reported in HQN's checkbook in Table 4.2. HQN's SAUF statement is reported in Table 4.3.

At the beginning of the period, firm managers make cash flow projections recorded in the SAUF statement. These projections allow the firm to plan in advance for cash flow shortages or for investment and savings opportunities. Obviously, a negative ending cash balance is not possible; therefore, the firm adjusts its cash expenses or cash receipts so that the firm remains solvent. In the last column of HQN's SAUF, ending period cash balances were projected to equal $51.

Table 4.3. HQN's 2018 SAUF Statement

Date	***Sources of Cash***	Actual	Projected
12/31/17	Beginning cash balance	$930	$930
	Cash receipts	$38,990	$39,000
	Sale of long-term assets	$30	$50
	Long-term borrowing	$50	$25
12/31/18	Total sources of funds	$40,000	$40,005
	Uses of Cash		
	Cash COGS	$27,000	$25,000
	Cash OE	$11,078	$12,000
	Interest paid	$480	$480
	Taxes paid	$68	$70
	Cash purchases of long-term assets	$100	$150
	Pay current portion of long-term debt payment	$157	$1,067
	Notes payable	$230	$887
	Owners' draw	$287	$300
12/31/18	Total uses of funds	$39,400	$39,954
12/31/18	Ending cash balance (Total sources – total uses of funds)	$600	$51

Notice that the entries in the Checkbook reported in Table 4.2 match those in the SAUF statement reported in Table 4.3, except that they are organized as sources of funds coming into the firm and uses of funds representing funds flowing out of the firm. Consistency between the firm's SAUF statement and its checkbook requires cash at the ending periods in the balance sheet and the SAUF are equal.

So what have we learned? We learned that records of cash flow whether recorded in checkbooks or similar records is one of the most important data sources for constructing CFS for the firm. This data can be used to construct SAUF statements and statements of cash flow.

Statement of Cash Flow (SCF)

The firm's balance sheet describes its financial position at a point in time while the firm's statement of cash flow (SCF) describes the firm's cash flow over a period of time between the firm's beginning and ending balance sheets. The main purpose of the SCF is to find the change in the firm's cash position during the accounting year.

Three major cash flow activities. The firm's SCF is similar to its SAUF statement except that it separates cash flow into the three categories: (1) cash flow from operations, (2) cash flow from investments, and (3) and cash flow from its financing activities.

The cash flow from operations reflect the cash flow generated by the firm in producing and delivering its goods and services. Cash flow from operations reflect the firm's management of its production and marketing activities.

The cash flow from investment activities result from the firm's sale and purchase of long-term assets. Sales of long-term assets whose market value exceeds its book value create realized capital gains and depreciation recapture. Cash flow from investment activities reflect the firm's investment management strategies.

The cash flow from financing activities result from borrowing new debt, repaying old debt, raising new equity capital, and returning capital to owners. Cash flow from financing reflect the firm's management of its debt and equity.

Cash flow for the firm during the accounting period are summarized by its change in cash position. By adding cash on hand at the end of the previous period to the change in cash position reported in the statement of cash flow, we obtain cash on hand at the end of the period. As a result, the change in cash position links the beginning and ending cash on hand reported in the balance sheet.

The SAUF statement derived from the checkbook contains all the data required to construct a statement of cash flow (SCF) for the firm. While cash flow can occur in any order in real life, we have arranged them in the SAUF statement by categories: cash flow associated with operations, cash flow associated with investment, and cash flow associated with financing.

Net cash flow from operations. The first entry in HQN's checkbook is cash receipts of $38,990. This represents a source of cash, and is therefore entered in the "credit" or "deposit" column of the checkbook. The next items that appear in the checkbook are cash COGS of $27,000, cash OE of $11,078, interest paid of $480, and taxes paid equal to $68. We find net operating cash flow by subtracting from cash receipts, cash expenditures or COGS, cash OE, interest and taxes.

+	Cash receipts	$38,990
–	Cash COGS	$27,000
–	Cash OE	$11,078
–	Interest paid	$480
–	Taxes paid	$68
=	*Net Cash Flow from Operations*	$364

Net cash flow from investments. Net cash flow from investment activity is calculated from checkbook entries equal to $70 which corresponds to net cash flow from investment. It is calculated as the difference between purchases of long-term assets ($100) less sales of long investments assets equal to $30. These calculations for HQN in 2018 are recorded below.

+	Realized capital gains + depreciation recapture	$0
+	Sales of non-depreciable assets	$0
–	Purchases of non-depreciable assets	$0
+	Sales of long-term assets	$100
–	Purchases of long-term assets	$30
=	*Net Cash Flow from Investments*	($70)

It is important to recognize that some purchases may be paid for with borrowed funds. In this case the borrowed funds would be entered in the SAUF as a source of funds while the purchase reflects own plus borrowed funds expended to acquire the long-term asset.

Net cash flow from financing. Cash flow from financing activities recorded in the checkbook reflect the difference between borrowing and repayment of long-term debt and payments of notes payable. Interest paid on long-term debt and notes payable is included in net cash flow from operations. Finally, dividends paid and owner draw are subtracted and the difference between new equity contributed and purchased is reflected in the net cash flow from financing. HQN's 2018 net cash flow from financing

are recorded below.

–	Long-term debt payments	$157
+	Long-term borrowings	$50
–	Payments on notes payable	$230
–	Owner draw	$287
=	*Net Cash Flow from Financing*	($624)

An alternative to calculating net cash flow from financing is to use the difference between the ending and beginning balance sheet to the find the change in long-term debt and current long-term debt plus the change in notes payable. Finally, we subtract payment of dividends and owner draw. Net cash flow calculated using the balance sheet rather than the checkbook is reported next.

+	Change in non-current LTD	($57)
+	Change in current portion of LTD	($50)
+	Change in notes payable (borrowing less payments)	($230)
–	Payment of dividends and owner draw	$287
=	*Net Cash Flow from Financing*	($624)

We demonstrate that cash flow associated with borrowing LTD, repaying current and noncurrent portions of LTD, and converting noncurrent LTD to current LTD are properly accounted for by adding changes in current and noncurrent LTD. To this end, consider the following. We assume transactions occur at the end of each period. Current and noncurrent LTD at the end of the previous period are designated LTD_0^C and LTD_0^N respectively. Current and noncurrent LTD at the end of the current period are designated LTD_1^C and LTD_1^N respectively. Assume that at the end of the current period: (1) the firm reduces its outstanding LTD by paying amounts LTD^{PayN} and LTD^{PayC} on noncurrent and current LTD balances respectively ; (2) it increases its LTD by borrowing amount LTD^{Borrow}; and (3) some noncurrent LTD becomes current LTD—an amount equal to $LTD^{Converted}$. We can now write the identity:

$$LTD_1^C = LTD_0^C + LTD^{Converted} - LTD^{PayC} \tag{4.1}$$

In words, current LTD at the end of the period equals current LTD at the beginning of the period plus noncurrent LTD converted to current LTD less current LTD payments made at the end of the period. Then we find the difference between current LTD at the end of the previous and current periods as:

$$\Delta LTD^C = LTD_1^C - LTD_0^C = LTD^{Converted} - LTD^{PayC} \tag{4.2}$$

Similarly, we write the identity

$$LTD_1^N = LTD_0^N + LTD^{Borrow} - LTD^{Converted} - LTD^{PayN}. \tag{4.3}$$

In words, noncurrent LTD at the end of period one is equal to noncurrent LTD at the beginning of the period plus additional LTD borrowings, less noncurrent LTD converted to current LTD minus noncurrent LTD payments. Then we find the difference between noncurrent LTD at the end of the previous and current periods as:

$$\Delta LTD^N = LTD_1^N - LTD_0^N = LTD^{Borrow} - LTD^{Converted} - LTD^{PayN} \tag{4.4}$$

Finally we add $\Delta LTD^C + \Delta LTD^N$ to find:

$$\Delta LTD^C + \Delta LTD^N = LTD^{Borrow} - LTD^{PayC} - LTD^{PayN} \tag{4.5}$$

Since $LTD^{Converted}$ cancels when the two equation are added together, we prove that including the difference in current and noncurrent LTD in the financing cash flow section of the statement of cash flow properly accounts for borrowing and payment of LTD and transferring funds from noncurrent to current LTD.

We are now prepared to calculate the change in the net cash position of the firm by combining HQN's cash flow from operations, investing, and financing.

Table 4.4a. HQN's 2018 Statement of Cash Flow

+	Cash receipts	$38,990
–	Cash COGS	$27,000
–	Cash OE	$11,078
–	Interest paid	$480
–	Taxes paid	$68
=	**Net Cash Flow from Operations**	**$364**
+	Realized capital gains / depreciation recapture	$0
–	Purchases of long-term assets	$100
+	Sales of long-term assets	$30
=	**Net Cash Flow from Investments**	**($70)**
+	Long-term borrowing	$50
–	Long-term debt payments	$157
–	Note payments	$230
–	Dividends and owner's draw	$287
=	**Net Cash Flow from Financing**	**($624)**
	Change in Cash Position of the Firm	**($330)**

An alternative statement of cash flow calculates Net Cash Flows from financing, logically equivalent to Table 4.4a, using changes in Current and Non-current LTD and Changes in notes payable, including as before dividends and owner's draw. The alternative to Table 4.4a, especially useful when data on borrowings and payment data is not available, is reported below:

Table 4.4b. HQN's 2018 Statement of Cash Flow
Open HQN Coordinated Financial Statement in MS Excel

+	Cash receipts	$38,990
–	Cash COGS	$27,000
–	Cash OE	$11,078
–	Interest paid	$480
–	Taxes paid	$68
=	**Net Cash Flow from Operations**	**$364**
+	Realized capital gains / depreciation recapture	$0
–	Change in non-depreciable long-term assets	$0

–	Change in depreciable long-term assets + depreciation	$70
=	**Net Cash Flow from Investments**	**($70)**
+	Change in noncurrent LTD	($57)
+	Change in current portion of LTD	($50)
+	Change in notes payable	($230)
–	Payment of dividends and owner's draw	$287
=	**Net Cash Flow from Financing**	**($624)**
	Change in Cash Position of the Firm	**($330)**

Since the ending cash position of the firm in the previous period was $930, a change in the cash position associated with the firm's cash flow implies that the ending cash position of the firm is: $930 – $330 = $600. This amount, $600, corresponds to the cash balance appearing at the end of the current period's balance sheet. Indeed, a check on the consistency of the calculation that uses a checkbook to construct a statement of cash flow is that the change in cash position reconciles the cash balances appearing in the end of period balance sheets. Furthermore, the beginning and ending cash balances in the checkbook must equal the ending cash balances in the end of period balance sheets. In this case, the beginning and ending cash balances in the checkbook are $930 and $600, respectively, which matches the ending cash balances in the previous and current end of period balance sheets.

So what have we learned? We learned that it is essential to understand that the individual financial statements included in CFS are interdependent and all are important for describing the financial condition of the firm. Their interdependence can be verified with the following consistency checks. The fundamental accounting identity requires that total assets equal liabilities plus net worth. The change in cash position calculated in the SCF equals the difference in cash and marketable securities in the beginning and ending period balance sheets. And finally, the change in retained earnings calculated in the AIS must equal the change in retained earnings in the ending and beginning period balance sheets.

Cash Income Statements

Unlike the balance sheet—which is a picture of the firm's assets, liabilities, and net worth at a point in time—the firm's income statement is a record of the firm's income and expenses incurred between two points in time. Profits (losses) reported in the firm's income statement are reflected in the firm's balance sheet as an increase (decrease) in the firm's equity. Therefore, the firm's income statement provides the details that explain changes in the firm's equity. To complicate matters, there are two distinct income statements, cash and accrual. We first discuss the firm's cash income statement.

Cash income statements record the firm's income and expenses when they generate a cash flow. One of the most important uses of a cash income statement is to determine the firm's tax obligations. The cash income statement is also an important tool for determining the liquidity of the firm—to determine if its cash receipts are sufficient to meet its cash expenses.

HQN's 2018 Cash income statement. Using data from the firm's checkbook or its SAUF statement plus the ending balance sheets, we are prepared to complete a cash income statement. Sometimes the cash income statement is referred to as a modified cash income statement because it includes depreciation which is not a cash flow event but which is an expense allowed when computing taxable income. The 2018 HQN cash income statement constructed using data from HQN's checkbook plus book value asset data from ending balance sheets used to calculate depreciation is reported below:

Table 4.5. HQN's 2018 Cash Income Statement (all number in $000s)

	Cash receipts	$38,990
+	Realized Cap. Gains and Depreciation Recapture	0
–	Cash COGS	$27,000
–	Cash Overhead Expenses	$11,078

–	Depreciation	$350
=	**Cash Earnings Before Interest and Taxes (CEBIT)**	**$562**
–	Interest paid	$480
=	**Cash Earnings Before Taxes (CEBT)**	**$82**
–	Taxes paid	$68
=	**Cash Net Earnings After Taxes (CNIAT)**	**$14**
–	Dividends and Owner Draws	$287
=	**Cash Additions to Retained Earnings**	**($273)**

Notice that all of the entries in the cash income statement are entries in the firm's checkbook and SAUF statement except for realized capital gains and depreciation recapture and depreciation. Depreciation is listed as an expense even though it may not reflect a cash flow event because it represents a loss in value to the firm of assets previously purchased. To find the depreciation for HQN we begin with a fundamental relationship that applies to all depreciable long-term assets (DLTA):

Beginning DLTA + purchases of DLTA – sales of DLTA (book value) – depreciation = Ending (book value) DLTA

Notice that the sale of DLTA is listed at their book value. This is necessary to maintain the value of DLTA at their book value in the balance sheets. Solving for depreciation in the identity above and substituting data from the checkbook and balance sheets we find:

Depreciation = Beginning DLTA + purchases of DLTA – sales of DLTA (book value) – Ending DLTA

Depreciation = $2990 + $100 – $30 – $2710 = $350.

Finally, depreciation recapture is treated a separate category of cash receipts to the firm because it represents a value that was previously deducted as an expense included in depreciation and is now received as an unexpected income (or loss). Depreciation recapture plus capital gains is equal to the market value of DLTA sold less the book value of DLTA sold provided the sale price is not greater than the original purchase price.

Realized capital gains (losses) + depreciation recapture = Sale of DLTA (market value) – sale of DLTA (book value)

= $30 – $0 = $30.

In this example, we assume that DLTA were sold at their book value so realized capital gains plus depreciation recapture are zero.

Accrual Income Statement (AIS)

HQN's 2018 accrual income statement (AIS) is reported in Table 4.6. Because the accrued income statement is the more common of the two income statements, we sometimes drop the word accrual and refer to it as the firm's income statement. We create HQN's income statement by adding to cash transactions non cash exchanges affecting the financial condition of the firm.

Table 4.6. HQN's 2018 Accrual Income Statement
(all numbers in $000s)

+	Cash receipts	$38,990
+	Δ Accounts Receivable	($440)
+	Δ Inventories	$1450
+	Realized Capital Gains / Depreciation Recapture	$0
=	**Total Revenue**	**$40,000**

+	Cash COGS	$27,000
+	Δ Accounts Payable	$1,000
+	Cash OE	$11,078
+	Δ Accrued Liabilities	($78)
+	Depreciation	$350
=	**Total Expenses**	**$39,350**
	Earnings Before Interest and Taxes (EBIT)	**$650**
–	Interest	$480
=	**Earnings Before Taxes (EBT)**	**$170**
–	Taxes	$68
=	**Net Income After Taxes (NIAT)**	**$102**
–	Dividends and owner draws	$287
=	**Additions To Retained Earnings**	**($185)**

An important check on the consistency of HQN's balance sheets and its AIS is that additions to retained earning of ($185) should equal the difference between beginning and ending retained earning in the balance sheet.

We now explain in more detail the calculation of the individual entries in the AIS. In addition, by rearranging the numbers used to find AIS entries, we can find cash income statement entries.

Finding accrued income. We first want to calculate accrued income. Our first checkbook entry records cash receipts of $38,990. So, what is the difference between cash receipts and accrued income? Accrued income includes cash receipts from the sale of products, insurance, and government payments plus credit sales—creating accounts receivable. In addition, accrued income includes inventory increases—production essentially sold into inventory. Both increases in accounts receivable and inventory increase accrued income. While cash receipts are recorded in the checkbook, changes in accounts receivable and changes in inventories are found as the differences between accounts receivable and inventory entries in the two ending period balance sheets. Since we can find changes in accounts receivable and changes in inventories, we can now calculate accrued income equals the following:

Accrued Income = *Cash Receipts* + Δ *Accounts Receivables* + Δ *Inventories*

$40,000 = $38990 + ($440) + $1450

Notice that cash receipts understate actual income in this example.

Total revenue. Adding cash receipts, realized capital gains / depreciation recapture, changes in inventory, and changes in accounts receivable provides a measure of the firm's total accrued income.

Finding accrued COGS. The checkbook records cash COGS of $27,000. But accrued COGS must add to cash COGS the COGS that the firm purchased on credit which increased the firm's accounts payable. The amount of goods the firm purchased on credit can be calculated as the difference between accounts payable at the beginning and at the end of the year recorded in the firm's balance sheets. For HQN, changes in accounts payable equal $1,000, allowing us to find accrued COGS as:

Accrued COGS = *Cash COGS* + Δ *Accounts Payable*

$28,000 = $27,000 + $1000

Note that cash COGS understate actual COGS.

Finding accrued OE. The checkbook records cash OE of $11,078. But accrued OE must add to cash OE the overhead expenses that were purchased on credit increasing the firm's accrued liabilities. In general, changes in accrued liabilities equal the difference between accrued and cash overhead expenses, and can be found as the difference between accrued liabilities recorded in the

ending period balance sheets. The difference between ending period accrued liabilities equals ($78), meaning that the firm actually paid off some accrued liabilities incurred in earlier periods in addition to paying for overhead expenses incurred during the current period. We express accrued overhead expenses as:

Accrued OE = Cash OE + Δ Accrued Liabilities

$11,000 = $11,078 + ($78)

Note that cash OE overstate actual overhead expenses.

Calculating depreciation in the accrual income statement. We previously calculated depreciation in the cash income statement and found it to equal to $350. The calculation of depreciation is the same in both the accrual and cash income statement.

Earnings Before Interest and Taxes (EBIT). After subtracting from the firm's total revenue, its cash COGS, change in accounts payable, cash OE, changes in accrued liabilities and depreciation, we obtain a measure of the firm's profits (total revenue minus total expenses). But the profit measure obtained is before subtracting interest costs and taxes. Therefore, we call this profit measure Earnings Before Interest and Taxes are paid (EBIT).

Earnings Before Taxes (EBT). Subtracting interest expenses from EBIT gives the firm's earnings before taxes (EBT), which are the firm's profits after paying all expenses except taxes.

Net Income After Taxes (NIAT). Subtracting the firm's tax liabilities from EBT gives the firm's net income after taxes (NIAT), generally referred to as the firm's profits. NIAT is also what is available to be reinvested in the firm or withdrawn by the owners.

Interest costs, taxes, and withdrawals. The checkbook records interest costs of $480, taxes equal to $68, and withdrawals of $287. These are paid in cash and can be entered directly in the AIS.

Dividends and owner draw. Dividends and owner draw represent payments make to owners of the firms from the firm's profits. In the case of dividends, these reflect payments made to compensate owners of the firm for their investments in the firm. In the case of owner draw, these may include payments to owners for services rendered or to meet the financial needs of the firm's owners.

Finding Cash Receipts, Cash COGS, and Cash OE from accrual statement entries. Usually, firms have access to cash receipts, cash COGS, and cash OE from which it can find accrued income, accrued COGS, and accrued OE. This was our approach in the previous section. However, we could reverse the calculations and beginning with accrued income, accrued COGS, and accrued OE solve for cash receipts, cash COGS, and cash OE. These calculations are described next and are essential in order to complete the firm's statement of Cash Flow when we begin with accrual data rather than cash flow data.

Suppose that we knew that accrued income were equal to $40,000. We could find cash receipts by subtracting from accrued income, change in accounts receivable and change in inventory:

$38,990 = $40,000 – ($440) – $1450

Finding cash COGS. Suppose that we knew that accrued COGS were equal to $28,000. We could find cash COGS by subtracting accrued COGS change in Accounts Payable:

$27,000 = $28,000 – $1,000

Finding accrued OE. Suppose that we knew that accrued OE were equal to $11,078. We could find cash OE by subtracting from accrued OE, change in accrued liabilities.

$11,078 = $11,000 – ($78)

More Complicated Financial Statements

Compared to HQN's balance sheets, income statements, and SCF described in this chapter, balance sheets, income statemetns, and SCF for an actual firm are often much more complicated. In what follows, we highlight the main differences between HQN's financial statements and financial statements of actual firms.

Level of detail. One of the main differences between HQN's financial statements and financial statements of actual firms is the level of detail. For example, the income statement may separate total sales into sales of livestock and livestock products, crops sales, and sales of services. Expenses may be separated into seed, fertilizer, other crop supplies, machinery hire, feed purchased, feeder livestock purchased, veterinary services, livestock supplies, fuel and oil, utilities, machinery repairs, insurance, rents, hired labor to name a few. Additional inventory details may include livestock and crops held for sale and feed, value of growing crops, farm supplies, and prepaid expenses.

Data deficiencies. Another difference between HQN's financial statements and financial statements of actual firms is the quality of the data. HQN's data are assumed to be accurate and consistent. Data supplied by actual firms is sometimes neither accurate nor consistent. Other data deficiencies of actual firms may include the following. 1) The data is rarely complete, especially for large complex firms with several activities. 2) Data from the firm's activities may be reported by different persons using different metrics. 3) Some of the same data are provided from different sources and are not equal. 4) Personal data.

Exogenous and endogenous variables. There are two types of variables in CFS: exogenous versus endogenous. The value of exogenous variables are determined outside of the CFS . It can be observed and supplied by sources other than the firm. Endogenous variables are calculated using exogenous variables so that any change in exogenous variables produces changes in endogenous variables. For example, the cash flow variable is an exogenous variable because it can be observed and recorded. Other data, such as firm's equity or additions to retained earnings are computed using exogenous and sometimes other endogenous variables. The problem occurs where the a value in the CFS can be observed and calculated. For example, end of period cash is calculated from beginning period cash plus change in cash position. But end of period cash may be observed by referring the firm's checkbook. If the exogenous variable is accurate, then the observed and calculated numbers will be equal and consistent. But in some cases they are not equal and the firm will be required to take steps designed to reconcile the differences.

Consistency versus accuracy. Two words describe financial statements: "consistency" and "accuracy". Consistency means that we compute values for variables in the CFS in the same way every time so that changes in primary data produce predictable changes in calculated data. Accuracy means that our measurement conforms to correct value of what is being measured. The financial statements are consistent if the following are true: "additions to retained earnings" reported in the accrued income statement reconciles retained earnings in the ending period balance sheets. The reconciling equation is: beginning retained earnings plus additions to retained earnings equals retained earnings in the ending period balance sheet. A second consistency requirement is that the change in cash position reported in the statement of cash flow reconciles cash balances in the ending period balance sheets. The reconciling equation is: beginning cash plus change in cash position equal ending cash. Finally, the third consistency condition is that the fundamental accounting identity is true; namely, that assets equal liabilities plus equity.

The financial statements may be consistent, but not all the data may be accurately recorded. On the other hand, if the financial statements are not consistent, we can be sure they are not accurate. We summarize our description of financial statements by declaring that consistency is a necessary condition for an accurate set of financial statements but consistency is not sufficient for an accurate set of financial statements.

Hard and soft data. Faced with the data challenges described above, financial managers are charged with the task of preparing the most accurate and consistent set of CFS possible. Guidelines for this task include the following. Determine which data is "soft" and which data is "hard." Soft data is estimated or may lack a supporting data trail. Still, it represents the best guesstimates available. Hard data has a reference or an anchor. For example, the sale of a product is usually recorded although there may be some benefits or costs associated with the sale that are not recorded. Interest costs and taxes are recorded and can be considered hard data. Inventories are more difficult to determine if they are hard or soft because they change over a reporting cycle and their estimated value at the beginning and ending balance sheets may not be available. So, an important task of the financial manager is to assess the reliability of the different data.

Constructing Consistent Coordinated Financial Statements (CFS): A Case Study

We now construct a consistent set of financial statements for an actual firm using the data they supplied. In the process, we will experience some of the data deficiencies described earlier and the challenge of having our CFS be both accurate and consistent. We will call our case study firm, Friendly Fruit Farm (FFF) because producing and selling fruit is the firm's main commercial activity.

To assure that our financial statements are consistent, we will use the template prepared for analyzing HQN. The categories in the HQN template are aggregated compared to actual firms. To fit the data supplied by FFF to the HQN template will require

side-bar calculations that organize the data to correspond with the general categories described in Tables 4.1, 4.4, 4.5, and 4.6. The general rule for deciding when side-bar calculations are required is the following: if for any given entry of the HQN CFS there are two or more instances of that item supplied by FFF, a side bar calculation is required. Therefore, the number of side-bar calculations will depend on the firm being examined and the data which the firm supplies.

FFF balance sheets. FFF supplied the following balance sheet data.

Table 4.7. FFF supplied Ending Period Balance Sheets

	12/31/17	12/31/18
Cash and checking balance	($7,596)	($24,333)
Prepaid expenses & supplies	$7,467	$15,369
Growing crops	$0	$0
Accounts receivable	$33,400	$45,668
Hedging accounts	$0	$0
Crops held for sale or feed	$178,098	$204,530
Crops under government loan	$0	$0
Market livestock held for sale	$7,933	$6,500
Other current assets	$278	$278
Total current farm assets	**$219,581**	**$248,011**
Breeding livestock	$10,583	$18,019
Machinery and equipment	$85,001	$87,387
Titled vehicles	$2,889	$2,667
Other intermediate asses	$9,689	$8,893
Total intermediate farm assets	**$108,162**	**$116,966**
Farm Land	$166,200	$179,348
Buildings and improvements	$76,852	$81,021
Other long-term assets	$6,229	$6,229
Total long-term assets	**$249,281**	**$266,599**
Total Farm Assets	**$577,023**	**$631,576**
Accrued interest	$1,078	$1,487
Accounts payable	$2,080	$1,637
Current notes	$67,935	$74,644
Government crop loans	$0	$0
Principal due on term debt	$28,511	$30,072
Total current farm liabilities	**$99,604**	**$107,840**
Total intermediate farm liabilities	**$43,793**	**$31,782**
Total long-term farm liabilities	**$118,617**	**$124,420**
Total Farm Liabilities	**$262,015**	**$264,042**

Having collected financial data for FFF's balance sheets we proceed to organize it into categories described by the balance

sheets prepared for HQN. The main reason for doing so is to insure consistency. The second reason for doing so is to organize it into categories amenable to ratio analysis important for financial analysis and management. The balance sheet categories used by HQN which we want to duplicate for FFF are described in Table 4.1 .

Cash and marketable securities. We begin by noting that ending period cash balances cannot be negative. Otherwise they are liabilities. But in the balance sheets provided by FFF, they report negative cash balances in both end of period balance sheets. We set them to zero and include them as liabilities included in accounts payable. Our first line in FFF's ending period balance sheets is:

	12/31/17	12/31/18
Cash and marketable securities	$0	$0

Accounts receivable. FFF lists accounts receivables. They also lists prepaid expenses and hedging accounts that have properties similar to accounts receivable. All three measures represent short-term sacrifices by the firm for benefits they have not yet received. Usually, accounts receivables reflect firm sales for which it has not yet been compensated in cash. In the case of prepaid expenses, it represents payments for goods they have not yet received. In the case of a hedging account, they represent funds paid for options not yet exercised. Adding to accounts receivable, prepaid expenses and hedging funds produces a more inclusive measure of accounts receivable equal to:

	12/31/17	12/31/18
Accounts receivable	$33,400	$45,668
Prepaid expenses and supplies	$7,467	$15,369
Hedging accounts	$0	$0
Accounts receivable	$40,867	$61,037

Inventories. FFF lists several inventories in its ending period balance sheets. We list and sum these below.

	12/31/17	12/31/18
Growing crops	$0	$0
Crops for sale or feed	$178,098	$204,530
Crops under government loan	$0	$0
Market livestock for sale	$7,933	$6,500
Other current assets	$278	$278
Inventories	$186,309	$211,308

We now find the sum of FFF's current assets by adding cash and marketable securities, accounts receivable, and inventories and report the results below.

	12/31/17	12/31/18
Cash and marketable securities	$0	$0
Total accounts receivable	$40,867	$61,037
Total inventories	$186,298	$211,308
Current assets	$227,165	$272,345

We now find FFF's depreciable long-term assets. Note that FFF listed intermediate and long-term assets. In the HQN template, we distinguish long-term as either depreciable or non-depreciable. We consider property, plant, and equipment as depreciable long-term assets and land and buildings as non-depreciable long-term assets. Note that in the FFF supplied balance sheets, they list non-farm assets, which we ignore since our analysis is focused on the farm firm. We now list FFF's depreciable and non-depreciable long-term assets and find their sum.

	12/31/17	12/31/18
Breeding livestock	$10,583	$18,019
Machinery and equip.	$85,001	$87,387
Titled vehicles	$2,889	$2,667
Other intermediate assets	$9,689	$8,893
Depreciable long-term assets	**$108,162**	**$116,966**
Land	$166,200	$179,348
Buildings and improvements	$76,852	$81,021
Other long-term assets	$6,229	$6,229
Non-depreciable long-term assets	**$249,281**	**$266,598**
Total long-term assets	**$357,443**	**$383,564**

We find FFF's total assets as the sum of current and long-term assets and report the result below.

	12/31/17	12/31/18
Total current assets	$227,165	$272,345
Total long-term assets	$357,443	$383,564
Total assets	**$584,608**	**$655,909**

Notice that the total firm assets calculated above do not match the total firm assets calculated by FFF. This is because the negative cash balances have been shifted to the liabilities section of the balance sheet.

Next we prepare FFF's liabilities data to match HQN's reduced categories template. We begin by listing notes payable. We include in this category current notes payable and government loans.

	12/31/17	12/31/18
Current notes	$67,935	$74,644
Government crop loans	$0	$0
Notes payable	$67,935	$74,644

Next we include accrued interest and current portion of the long-term debt and current payments on loans in the category current portion of long-term debt.

	12/31/17	12/31/18
Accrued interest	$1,078	$1,487
Principal due on long-term debt	$28,511	$30,072
Current portion of long-term debt	$29,589	$31,559

Next we list FFF's accounts payable including negative balances in cash reported in the asset portion of the balance sheet.

	12/31/17	12/31/18
Accounts payable	$2,080	$1,637
Negative cash balances	$7,596	$24,333
Accounts payable	$9,676	$25,970

Combining our current liabilities entries and summing we find the sum of FFF's current liabilities:

	12/31/17	12/31/18
Notes payable	$67,935	$74,644
Current portion of LTD	$29,589	$31,559
Accounts payable	$9,676	$25,970
Accrued liabilities	$43,793	$31,782
Total current liabilities	**$150,993**	**$163,955**

Now we consider our long-term liabilities described as long-term debt. The first long-term liabilities is noncurrent long-term debt which is listed below.

	12/31/17	12/31/18
Noncurrent long-term debt	$118,617	$124,420

We add current liabilities to noncurrent long-term debt to find FFF's total liabilities.

	12/31/17	12/31/18
Current liabilities	$150,993	$163,955
Noncurrent long-term debt	$118,617	$124,420
Total Liabilities	$269,610	$288,375

Finally, we compute FFF's equity as the difference between its total assets and it total liabilities and find it equal to:

		12/31/17	12/31/18
+	TOTAL ASSETS	$584,608	$655,909
–	TOTAL LIABILITIES	$269,610	$288,375
=	EQUITY	$314,998	$367,534

Table 4.8. FFF's 2018 Ending Period Balance Sheets

	12/31/17	12/31/18
Cash and marketable securities	$0	$0
Accounts receivable	$40,867	$61,037
Inventories	$186,298	$211,308
Total current assets	$227,165	$272,345
Depreciable long-term assets	$108,162	$116,966
Non-depreciable long-term assets	$249,281	$266,598
Total long-term assets	$357,443	$383,564
TOTAL ASSETS	**$584,608**	**$655,909**
Notes payable	$67,935	$74,644
Current portion of LTD	$29,589	$31,559

Accounts payable	$9,676	$25,970
Accrued liabilities	$43,793	$31,782
Total current liabilities	$150,993	$163,955
Noncurrent long-term debt	$118,617	$124,420
TOTAL LIABILITIES	**$269,610**	**$288,375**
EQUITY	**$314,998**	**$367,534**
TOTAL LIABILITIES AND EQUITY	**$584,608**	**$655,909**

Populating FFF's Cash Income Statements to Conform to HQN's Income Templates

To enable FFF to populate its income statements, it supplied the following data.

+	Apples	$274,069
+	Cherries	$52,123
+	Peaches	$23,046
+	Grapes	$1,467
+	Pears	$638
+	Plums	$508
+	Raspberries	$2,580
+	Blueberries	$900
=	**Total cash fruit sales**	**$355,331**
+	Asparagus	$7,872
+	Cordwood	$31
+	Pumpkins	$360
+	Rhubarb	$179
+	Squash	$246
+	Sweet corn	$1,666
+	Tomatoes	$429
+	Other crops	$7,560
=	**Total cash crops and vegetable sales**	**$18,343**

Finally we sum the cash receipts categories:

+	Total cash fruit sales	$355,331
+	Total cash crops and vegetable sales	$18,343
+	Total cash receipts of finish beef calves	$1,898
+	Government payments	$5,376
+	Dividend and Insurance Payments	$1,651
+	Other farm income	$7,144
=	Cash farm income	$389,743
+	Seed	$2,039

+	Fertilizer	$4,652
+	Chemicals	$55,640
+	Crop insurance	$4,523
+	Packaging and supplies	$7,266
+	Marketing	$137
+	Crop miscellaneous	$31,940
+	Feed	$128
+	Livestock supplies	$794
=	**Total Cash COGS**	**$107,119**
+	Supplies	$638
+	Fuel	$13,458
+	Repairs	$19,882
+	Custom hire	$3,317
+	Hired labor	$99,671
+	Land rent	$29,777
+	Machinery lease	$2,827
+	Insurance	$8,593
+	Utilities	$7,177
+	Hauling	$86
+	Dues	$4,288
+	Miscellaneous	$7,092
=	**Total Cash OE**	**$196,806**

FFF also reported that in 2018 it paid $12,712 in interest charges, $4,628 in taxes, and that the owners withdrew $44,402. FFF also reported the purchase and sale of assets as $57,048 and $1,185 respectively. We assume that the sale of assets was at their book value.

We now populate FFF's AIS, using cash receipts and expense data supplied by FFF and changes in FFF's completed balance sheet entries.

Table 4.9. FFF's 2018 Accrual Income Statement
(all numbers in $000s)

+	Cash receipts	$389,743
+	Change in Accounts Receivable	$20,170
+	Change in Inventories	$25,010
+	Realized Capital Gains / Depreciation Recapture	$0
=	**Total Revenue**	**$434,923**
+	Cash Cost of Goods Sold (COGS)	$107,119
+	Change in Accounts Payable	$16,294

+	Cash Overhead Expenses (OE)	$196,806
+	Change in Accrued Liabilities	($12,011)
+	Depreciation	$47,059
=	**Total Expenses**	**$355,267**
	Earnings Before Interest and Taxes (EBIT)	**$79,656**
–	Interest	$12,712
	Earnings Before Taxes (EBT)	**$66,944**
–	Taxes	$4,628
	Net Income After Taxes (NIAT)	**$62,316**
–	Dividends and owner draws	$44,402
	Additions To Retained Earnings	**$17,914**

Detailed explanations of the AIS entries follow.

- FFF reported total cash receipts equal to $389,743.
- Change in accounts receivables was calculated by finding the difference between accounts receivable in FFF's ending period balance sheets equal to ($61,037 – $40,867) = $ 20,170.
- Change in inventory was calculated by finding the difference between inventories in FFF's ending period balance sheets equal to ($211,308 – $186,298) = $25,010.
- FFF reported Cash COGS equal to $107,119.
- Change in Accounts Payable was calculated by finding the difference between accounts payable reported in FFF's ending period balance sheets equal to $25,970 – $9,676 = $16,294.
- FFF reported Cash OE equal $196,806.
- Change in accrued liabilities was calculated by finding the difference between accrued liabilities in FFF's ending period balance sheets equal to ($31,782 – $43,793) = ($12,011).
- The next expense category required by FFF's AIS is depreciation. The data for calculating the change in long-term assets was available in FFF's balance sheets. FFF also reported its sale and purchases of long-term assets as $57,048 and $1,185 respectively. The formula for depreciation is:
- Purchases of depreciable LTAs – sales of depreciable LTAs (book value) – Δ depreciable LTAs (book) = depreciation.
- Making the necessary substitutions, we find FFF's 2018 depreciation:
- $57,048 – $1,185 – ($116,966 – $108,162) = $47,059.
- Summing cash and noncash receipts we find total revenue.
- Summing cash and noncash expenses we find total expenses.
- Subtracting total expenses from total revenue, we find earnings before interest and taxes (EBIT) equal to $79,656.
- FFF reported interest costs equal to $12,712 which were subtracted from EBIT to obtain Earning Before Taxes (EBT) equal to $66,944.
- FFF reported taxes equal to $4,628 which were subtracted from EBT to obtain Net Income after paying interest and taxes (NIAT) of $62,316.
- FFF reported paying dividends and owner withdrawals of $44,402 which were subtracted from NIAT to find changes in retained earnings of $17,914.

Cash income statement. Using the cash receipts and expense data supplied by FFF, we can easily find its cash income statement

Table 4.10. FFF's 2018 Cash Income Statement

	Cash receipts	389,743
+	Realized Capital Gains	$0
–	Cash COGS	$107,119
–	Cash overhead expenses	$196,806

–	Depreciation	$47,059
=	**Cash Earnings Before Interest and Taxes (CEBIT)**	**$38,759**
–	Interest paid	$12,712
=	**Cash Earnings before Taxes (CEBT)**	**$26,047**
–	Taxes paid	$4,628
=	**Cash Net Earnings After Taxes (CNIAT)**	**$21,419**
–	Dividends and owner draws	$44,402
=	**Cash Additions to Retained Earnings**	**($22,983)**

Statement of Cash Flow. We now have all of the data required to find FFF's statement of cash flow. We begin by finding FFF's net cash flow from operations.

+	Cash receipts	$389,743
–	Cash COGS	$107,119
–	Cash OE	$196,806
–	Interest paid	$12,712
–	Taxes paid	$4,628
=	**Net Cash Flow from Operations:**	**$68,478**

Net cash flow from investment activity is calculated from data used to find depreciation and equals

+	Sale of long-term assets	$1,185
–	Purchases of long-term assets	$57,048
=	**Net Cash Flow from Investments**	**($55,863)**

Net cash flow from financing activities reflects the difference between borrowing of long-term debt and notes payable and principal and interest payments on long-term debt and notes payable. Finally, dividends paid are subtracted and the difference between new equity contributed and purchased is computed.

	Change in non-current LTD (borrowing less payments)	$5,803
+	Change in current portion of LTD	$1,970
+	Change in notes payable (borrowing less payments)	$6,709
–	Payment of dividends and owner withdrawal	$44,402
=	**Net Cash Flow from Financing**	**($29,920)**

Table 4.11. Statement of Cash Flow

+	Cash receipts	$389,743
–	Cash COGS	$107,119
–	Cash OE	$196,806
–	Interest paid	$12,712
–	Taxes paid	$4,628

=	**Net Cash Flow from Operations**	**$68,478**
+	Realized capital gains + depreciation recapture	$0
–	Purchases of depreciable long-term assets	$57,048
+	Sales of depreciable long-term assets	$1,185
=	**Net Cash Flow from Investments**	**($55,863)**
+	Change in non-current long term debt	$5,803
+	Change in current portion long term debt	$1,970
+	Change in notes payable	$6,709
–	Dividends and owner's draw	$44,402
=	**Net Cash Flow from Financing**	**($29,920)**
	Change in cash position of the firm	**($17,305)**

What to do? Consider the following example. You are visiting in a new town and are trying to find your way to an important site. Suppose that you stop a person you assume is familiar with the location and ask for directions which the person provides. You thank the person for directions and begin your journey to your destination. But just to make sure you are on the right path, you consult your map and find that the directions you just received are in conflict with your map. You are faced with a choice. Which set of directions do you choose? They both can't be correct.

We face a similar problem when our completed financial statements aren't consistent. Somewhere in our entries there is an error(s). Therefore to populate our template we have to decide what numbers to believe.

In our case the conflict arises when reported primary data and calculated data required to reconcile the various financial statements are inconsistent. The first requirement is to establish consistency beginning with the calculated entries. Consistency is a necessary condition for accuracy and makes it a logical place to begin. Then we determine if the primary data that conflicts with the calculated number is hard or soft. If it is reasonably hard data, and is higher that the calculated data we explore the data for under estimates of cash inflows and over estimates of cash outflows. We follow the reverse process if the primary data is less than the calculated data. If we can find soft data that can be changed to make the calculated data consistent with the primary data—we make the changes.

So what have we learned? We learned from the FFF example that actual firms have more complicated data sets that we illustrated using the HQN example. Yet, the variable categories we used when computing the HQN study apply generally even though some side-bar calculations may be required to reduce actual firm data to the variable categories used to describe HQN.

Summary and Conclusions

In this chapter we have learned how to construct CFS. Coordinated financial statements are tools that we will use in the next chapter to analyze the firm's strengths and weaknesses. Constructing financial statements for actual firms with less than perfect and complete data is somewhat of an art.

Questions

1. Define the differences between consistent financial statements and accurate financial statements.
2. Discuss the statement: "consistent financial statements are necessary but not sufficient for accurate financial statements."
3. What are some conditions required for financial statements to be consistent?
4. In a typical data set provided by a farm firm, what data is most reliable (hard) and which data is least likely to reliable (soft)?
5. Below is a completed 2018 checkbook and 2017 and 2018 ending period balance sheets for the "Grow Green" vegetable farm. Use the numbers in their checkbook and their two balance sheets to create a 2018 cash and AIS and statement of cash flow.

Grow Green 2018 Checkbook				
Date	**Item**	**Check amount**	**Deposit amount**	**Checkbook balance**
12/31/17	Beginning cash balance			$930
	Cash receipts		$40,940	$41,870
	Seed, feed, fertilizer	$20,000		$21,870
	Labor cost of producing products	$8,350		$13,520
	Insurance	$2,000		$11,520
	Utilities	$9,632		$1,888
	Purchase of LTAs	$100		$1,788
	Sale of LTAs (at book value)		$30	$1,818
	Interest paid	$504		$1,314
	Taxes paid	$71		$1,243
	Payment on current-term debt	$50		$1,193
	Long-term debt payments	$57		$1,139
	Payment on notes	$230		$906
	Owner draw	$390		$516
12/31/18	Ending cash balance			$516

Grow Green Balance Sheet		
	12/31/17	**12/31/18**
Cash and Marketable Securities	$930.00	$516.00
Accounts Receivable	$1,640.00	$1,200.00
Inventory	$3,750.00	$5,200.00
Total Current Assets	**$6,320.00**	**$6,916.00**
Depreciable long-term assets	$2,990.00	$2,800.00
Non-depreciable long-term assets	$690.00	$600.00
Total Long-term Assets	**$3,680.00**	**$3,400.00**
Total Assets	**$10,000.00**	**$10,316.00**
Notes Payable	$1,500.00	$1,270.00
Current Portion of LTD	$500.00	$450.00
Accounts Payable	$3,000.00	$4,000.00
Accrued Liabilities	$958.00	$880.00
Total Current Liabilities	**$5,958.00**	**$6,600.00**
Noncurrent Long-term Debt	**$2,042.00**	**$1,985.00**
Total Liabilities	**$8,000.00**	**$8,585.00**
Other Capital	$1,900.00	$1,689.00
Retained Earnings	$100.00	$42.00
Equity	**$2,000.00**	**$1,731.00**

Grow Green Balance Sheet		
	12/31/17	**12/31/18**
Total Liabilities and Equity	**$10,000.00**	**$10,316.00**

6. Compare the differences and the advantages and disadvantages of cash and AIS.
7. In problem 5, you should have found ending cash to equal $630 which is the same as the observed ending cash. Instead assume that the ending cash balance recorded in the balance sheets provided was equal to $650. Describe the possible adjustments you might make to observed variables to make consistent observed ending cash and the calculated ending cash. Provide a revised set of CFS that are consistent with ending cash calculated and observed.
8. Most firm managers who are also the firm's financial manager keep less than the complete data set required to construct a complete set of financial statements. And even the data they collect are not in the format we expect, requiring us to reformat the data we do have. With less than complete data sets, we are forced to do the best we can with what we have. What follows is a typical data set which ABM 435 teams have used in the past to construct a set of consistent and accurate set of financial statements. Using the data provided, construct a consistent and, to the extent possible, accurate set of financial statements.

Farm A	**12/31/18**
Acres owned	488
Acres rented	449
Machinery investment / crop acre @ cost	$95
Machinery investment / crop acre @ market	$520
Average price received	
Corn	$6
Soybeans	$13
Wheat	$7
Hay	$97
Average yield	
Corn	$175
Soys	$54
Wheat	$76
Gross Cash farm income including government payments & patronage dividends	$863,561
Cash Farm Expense including land rent but excluding interest	$637,231
Interest	$23,232

Other balance sheet related data include the following:

	12/31/17	**12/31/18**
Cash and checking	$63,211	$66,696
Crops and Feed	$470,632	$430,532
Market livestock	46,696	$55,463
Accounts receivable	$34,062	$44,550
Prepaid expenses and supplies	$104558	$101,381
Hedging activities	$1916	$2374
Other current assets	$18,901	$15,822

	12/31/17	12/31/18
Other capital assets	$29,140	$36,539
Breeding livestock	$29,140	$40,053
Accounts payable	$26,149	$29,789
Accrued interest	$7,222	$7470
Current notes	$111,819	$127,402
Government crop loans	$1,400	$1,733
Loan principal due	$59,849	$68,559
Intermediate liabilities	$130,409	$124,872
Long-term liabilities	$344,658	$347,834
Machinery and Equipment (book)	$132,656	$179,366
Titled vehicles (book)	$3,096	$3,563
Buildings/improvements (book)	$105,559	$110,232
Land (book)	$508,571	$574,410

Purchases of Breeding Livestock	$901
Sale of Breeding Livestock	$291
Purchases other capital assets	$11,924
Sale of other capital assets	$8,443
Land sales (50% of sale were realized cap. Gains	$21,970
Purchase of titled vehicles	$1,948
Sale of titled vehicles	$636
Investments in buildings/improvements	$21,970

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5: Financial Ratios

Learning Objectives

After completing this chapter, you should be able to: (1) calculate financial ratios using information included in a firm's coordinated financial statements (CFS); and (2) answer the question: "what are the firm's financial strengths and weaknesses?"

To achieve your learning goals, you should complete the following objectives:

- Learn how to interpret ratios.
- Learn how financial ratios allow us to compare the financial condition of different firms.
- Learn how to construct (S)olvency, (P)rofitability, (E) fficiency, (L)iquidity, and (L)everage ratios—what we refer to collectively as SPELL ratios.
- Learn how SPELL ratios help us describe the financial strengths and weaknesses of a firm.
- Learn how the times interest earned (TIE) ratio and the debt-to-service (DS) ratio can provide information about the firm's solvency.
- Learn how the profit margin (m) ratio, the return on assets (ROA) ratio, and the return on equity (ROE) ratio can provide information about the firm's profitability.
- Learn how to find the after-tax ROE where T is the average tax rate paid by the firm on its earnings before taxes (EBT).
- Learn how to relate ROE and ROA to each other.
- Learn how the inventory turnover (ITO) ratio, the inventory turnover time (ITOT) ratio, the asset turnover (ATO) ratio, the asset turnover time (ATOT) ratio, the receivable turnover (RTO) ratio, the receivable turnover time (RTOT) ratio, the payable turnover (PTO) ratio, and the payable turnover time (PTOT) ratio can provide important efficiency information about the firm.
- Learn how the current ratio (CT) and the quick ratio (QK) can provide information about the firm's liquidity.
- Learn how leverage ratios including the debt-to-equity (DE) ratio and the equity multiplier (EM) ratio can be used to monitor and measure the firm's risk.
- Understand how comparing the firm's SPELL ratios to industry standard ratios can help answer the question: what are the financial strengths and weaknesses of the firm?
- Learn how to construct after-tax ROE and after-tax ROA measures.
- Learn how unpaid family labor affects ROE and ROA measures.
- Learn why the firm may consider profit and solvency ratios key to a firm's survival and success.
- Learn how the DuPont ratio demonstrates the interdependencies of some SPELL ratios.

Introduction

Coordinated financial statement (CFS) variables. CFS contain exogenous and endogenous variables. Exogenous variables take on values that can be observed or are determined by activities occurring outside of the firm. Endogenous variables take on values determined by activities within the firm and the values of exogenous variables.

The variables included in CFS become more valuable, especially for analyzing the strengths and weaknesses of the firm, when formed into ratios. We could look at the variables included in the CFS and draw some conclusions about the firm's strengths and weaknesses by comparing them with other firms, but our conclusions would be limited because no two firms are alike. Ratios, however, provide a means for comparing the performance of firms using a standardized measure which is easier to interpret.

A ratio consists of two numbers when one number is divided by the other. Suppose two numbers are represented by the variables X and Y and form a ratio (X/Y). The ratio tells us how many units of X exist for each unit of Y. This standardized number, the number of units of X that exists for each unit of Y , allows us to make comparisons between firms using similarly constructed ratios. One other way to interpret the ratio X/Y is to multiply the ratio by 100 converting the ratio to a percentage. In this case the ratio multiplied by 100 tells us what percentage of Y is X.

To illustrate the importance of ratios, consider the purchase of a breakfast cereal. Suppose you go to the grocery store to purchase your favorite breakfast cereal (Super Sweet Sugar Snacks). You find a 10-ounce box of Super Sweet Sugar Snacks that sells for

$3.20 and a larger 15-ounce box of the same cereal that sells for $4.50. Which box of cereal is the best buy? The price of each box of cereal won't tell you the answer because the more expensive box also contains more cereal. However, if we divide the price of each box by the amount of cereal in the box we can compare "apples to apples," or in this example we can compare the price per ounce of cereal in each box. Finding the ratio of dollars to ounces in the box, we see that cereal in the small box costs $0.32 per ounce ($3.20/10 oz.) while cereal in the large box costs $0.30 per ounce ($4.50/15 oz.). The large box of cereal costs less for each ounce of cereal and is the better buy.

The cereal example illustrates an important fact: one ratio without another ratio to compare it with is not very helpful. Knowing the price of cereal per ounce of the small box makes the information about the price per ounce of cereal in the large box more meaningful. Similarly, having standardized industry ratios against which we can compare our ratios is important for a financial manager's efforts to discover the firm's strengths and weaknesses.

What have we learned? We learned that when answering the question: what are the firm's financial strengths and weaknesses, it is important that we look at the firm from several different points of view represented by the SPELL ratios.

We discuss next the different views required to adequately describe the firm's financial condition. Each of the different views are represented by a set of ratios.

Financial Ratios

Financial ratios constructed using coordinated financial statement variables can be grouped into five categories. The categories can be remembered using the acronym SPELL. The five categories of financial ratios include: (S)olvency ratios, (P)rofitability ratios, (E)fficiency ratios, (L)iquidity ratios, and (L)everage ratios. Ratios in each of these five categories provide a different view of the firm's financial strengths and weaknesses.

Ratios and points in time measures. When constructing financial ratios using data from the CFS, the "point in time" or the "period of time" reflected by the ratio deserves careful attention. Numbers from balance sheets reflect the financial condition of the firm at a point in time. Numbers from income statements and statements of cash flow describe financial activity over a period of time. When forming a ratio using two numbers from the balance sheet, the numbers should reflect the same point in time.

Ratios of points and period of time measures. There are two approaches when forming a ratio with one number from the income statement or statement of cash flow describing activity over a period of time and another number from the balance sheet reflecting financial conditions at a point in time. One approach uses a number from the previous period's end of period balance sheet that corresponds with the point in time in which activities reported in the income statement begin. The second approach uses the average of beginning and ending period balance sheet measures that span the period of time during which activities reported in the income statement occurred. Later we will discuss in more detail when each of the two methods is preferred.

Cash versus accrual ratios. We construct several ratios in this chapter that include an income or a COGS variable. The question is: should these be cash receipts and cash COGS or accrued income and accrued COGS? We use accrual variables that focus on when financial transactions occurred rather than when transactions were converted to cash.

Useful comparisons. The usefulness of ratios depends on having something useful to compare them to. Suppose we wish we compare ratios of different firms. Obviously, we would expect ratios constructed for different firms to have been calculated at comparable points and periods of time. We would also expect that firms being compared are of the same size and engaged in similar activities. Fortunately, we can often find such measures described as industry average ratios.

Sometimes the relevant comparison for the firm is with itself at different points in time. Having the same ratio over a number of time periods for the same firm allows the firm manager to identify trends. One question trend analysis may answer is: in what areas is the firm is improving (not improving) compared to past performances. Of course, trend analysis can be performed using absolute numbers as well as ratios.

What follows. In what follows, we will introduce several ratios from each of the five "SPELL" categories. Then we will discuss how each of them, alone and together with other SPELL ratios, can help answer the question: what are the firm's financial strengths and weaknesses. Since data from HQN's financial statements will be used to form the SPELL ratios, HQN's balance sheets, AIS, and statement of cash flow for 2018 are repeated in Table 5.1.

So what have we learned? We learned that the ratio of variables X and Y (X / Y) tells us how many units of X are associated with each unit of Y. As a result we can compare the ratio X/Y in firms A and B because the two ratios provide the same information

about the same variable in the two firms—the number of units of X that exist for each unit of Y.

Table 5.1. Coordinated financial statement for HiQuality Nursery (HQN) for the year 2018
Open HQN Coordinated Financial Statement in MS Excel

BALANCE SHEET			ACCRUAL INCOME STATEMENT			STATEMENT OF CASH FLOW		
	12/31/17	12/31/18			2018			2018
Cash and Marketable Securities	$930	$600	+	Cash Receipts	$38,990	+	Cash Receipts	$38,990
Accounts Receivable	$1,640	$1,200	+	Δ Accounts Receivable	($440)	–	Cash COGS	$27,000
Inventory	$3,750	$5,200	+	Δ Inventories	$1450	–	Cash OE	$11,078
Notes Receivable	$0	$0	+	Realized Capital Gains / Depreciation Recapture	$0	–	Interest paid	$480
CURRENT ASSETS	$6,320	$7,000	=	**Total Revenue**	**$40,000**	–	Taxes paid	$68
Depreciable Long-term Assets	$2,990	$2,710	+	Cash Cost of Goods Sold (COGS)	$27,000	=	**Net Cash Flow from Operations**	**$364**
Non-depreciable Long-term Assets	$690	$690	+	Δ Accounts Payable	$1,000	+	Realized capital gains + depreciation recapture	$0
LONG-TERM ASSETS	$3,680	$3,400	+	Cash Overhead Expenses (OE)	$11,078	+	Sales non-depreciable assets	$0
TOTAL ASSETS	$10,000	$10,400	+	Δ Accrued Liabilities	($78)	–	Purchases of non-depreciable assets	$0
Notes Payable	$1,500	$1,270	+	Depreciation	$350	+	Sales of depreciable assets	$30
Current Portion LTD	$500	$450	=	**Total Expenses**	**$39,350**	–	Purchases of depreciable assets	$100
Accounts Payable	$3,000	$4,000		**Earnings Before Interest and Taxes (EBIT)**	**$650**	=	**Net Cash Flow from Investments**	**($70)**
Accrued Liabilities	$958	$880	–	Interest	$480	+	Change in noncurrent LTD	($57)

CURRENT LIABILITIES	**$5,958**	**$6,600**		**Earnings Before Taxes (EBT)**	**$170**	+	Change in current portion of LTD	($50)
NONCURRENT LONG-TERM DEBT	**$2,042**	**$1,985**	–	Taxes	$68	+	Change in notes payable	($230)
TOTAL LIABILITIES	**$8,000**	**$8,585**		**Net Income After Taxes (NIAT)**	**$102**	–	Payment of dividends and owner's draw	$287
Contributed Capital	$1,900	$1,900	–	Dividends and owner draws	$287	=	Net Cash Flow from Financing	($624)
Retained Earnings	$100	($85)		**Additions To Retained Earnings**	**($185)**		Change in cash position of the firm	($330)
TOTAL EQUITY	**$2,000**	**$1,815**						
TOTAL LIABILITIES AND EQUITY	**$10,000**	**$10,400**						

Solvency Ratios

Solvency ratios, sometimes called repayment capacity ratios, can be used to answer questions about the firm's ability to meet its long-term debt obligations. Here we will examine two solvency ratios: (1) times interest earned (TIE) and (2) debt-to-service ratio (DS).

Times interest earned (TIE) ratio

The TIE ratio measures the firm's solvency or repayment capacity. The TIE ratio combines two period of time measures obtained from the firm's income statement and is defined as:

$$TIE = \frac{EBIT}{INT} \tag{5.1}$$

In the above formula, INT represents the firm's interest obligations accrued during the period. EBIT measures the firm's earnings during the period before paying interest and taxes. The TIE ratio answers the question: how many times can the firm pay its interest costs using the firm's operating profits (for every dollar of interest costs how many dollars of EBIT exist? Generally, a healthy firm's TIE ratio exceeds one (TIE > 1), otherwise the firm won't be able to pay its interest costs using its current income. HQN's TIE ratio for 2018 is:

$$TIE = \frac{\$650}{\$480} = 1.35 \tag{5.2}$$

HQN's 2018 TIE ratio indicates for every dollar of interest the firm owes, it has $1.35 dollars of EBIT to make its interest payments.

Debt-to-service (DS) ratio

Like the TIE ratio, the DS ratio answers questions about the firm's ability to pay its current long-term debt obligations. In contrast to the TIE ratio, DS ratio recognizes the need to pay the current portion of its long-term debt in addition to interest. Finally, the DS ratio (in contrast to the TIE ratio) adds depreciation to EBIT because depreciation is a non-cash expense. Subtracting

depreciation from revenue to obtain EBIT understates the liquid funds available to the firm to pay its current long-term debt obligations.

To illustrate the logic behind this formula, assume that cash receipts equals $100, depreciation equals $30, other expenses equal $10, and EBIT is equal to $60. But more than $60 is available to pay interest and principal because depreciation of $30 is a non cash expense. Thus we add depreciation to EBIT to improve our measure of income available for interest and debt repayment: $60 + $30 = $90, which is the numerator in the DS ratio equation.

This book recommends that the current portion of the long-term debt at the beginning of the current period be used to calculate the denominator in the firm's DS ratio. After making these adjustments, we obtain the firm's DS ratio equal to:

$$DS = \frac{EBIT + Depreciation}{INT + Current\,portion\,long - term\,debt} \tag{5.3}$$

If the DS < 1, a firm will not be able to make principal and interest payments using EBIT plus depreciation. In this case, the firm will be required to obtain funding from other sources such as restructuring debt, selling assets, delaying investments in assets, and/or increasing EBIT to meet current debt and interest obligations. If the firm were unable to meet its interest and principal payment over the long term, the firm's survival would be threatened.

We can solve for HQN's 2018 DS ratio. According to HQN's income statement EBIT was $650 and depreciation was $350. Interest paid equaled $480 and the current portion of the long-term debt listed on the firm's 2017 end of period balance sheet was $500. Making the substitutions into Equation ??? we find HQN's DS ratio equal to:

$$DS = \frac{\$650 + \$350}{\$480 + \$500} = 1.02 \tag{5.4}$$

According to HQN's DS ratio, its EBIT plus depreciation are sufficient to meet 102 percent of its interest and current principal payments, a more accurate reflections of its solvency than its TIE ratio of 1.35.

Profitability Ratios

Profitability ratios measure the firm's ability to generate profits from its assets or equity. The firm's accrual income statement (AIS) provides three earnings or profit measures useful in finding rates of return: earnings before interest and taxes (EBIT), earnings before taxes (EBT), and net income after interest and taxes (NAIT).

We examine three profitability ratios: (1) profit margin (m), (2) return on assets (ROA), and (3) return on equity (ROE). In some cases, the profitability measures are reported on an after-tax basis requiring that we know the average tax rate for the firm which we calculate next.

Finding the average tax rate. In some cases, particularly with profitability measures, we need to know the average tax rate paid by the firm. We find the average tax rate by solving for *T* in the following formula that equates net income after taxes (NIAT) to EBT adjusted for the after-tax rate *T*:

$$NIAT = EBT(1 - T) \tag{5.5}$$

And solving for T in Equation ???:

$$T = 1 - \frac{NIAT}{EBT} \tag{5.6}$$

Solving T for HQN in 2018 using EBT and NIAT values from Table 5.1 we find:

$$T = 1 - \frac{102}{170} = .4 \tag{5.7}$$

Profit margin (m) ratio

The ratio *m* measures the proportion of each dollar of cash receipts that is retained as profit after interest is paid but before taxes are paid.

The ratio *m*, is defined as:

$$m = \frac{EBT}{Total\ Revenue} \tag{5.8}$$

In 2018, HQN had a before-tax profit margin equal to:

$$m = \frac{\$170}{\$40,000} = .00425 = .425\% \tag{5.9}$$

In other words, for every $1 of revenue earned by the firm, HQN earned $0.00425 in before-tax profits. Meanwhile, the after-tax profit margin m is defined as:

$$m(1-T) = \frac{EBT(1-T)}{Total\ Revenue} = \frac{NIAT}{Total\ Revenue} \tag{5.10}$$

In 2018, HQN had an after-tax profit margin equal to:

$$m(1-.4) = \frac{\$102}{\$40,000} = .00255 = .255\% \tag{5.11}$$

In other words, for every $1 of cash receipts, HQN earned $0.00255 in after-tax profits.

Return on assets (ROA) ratio

The ROA measures the amount of profits generated by each dollar of assets and is equal to:

$$ROA = \frac{EBIT}{A} \tag{5.12}$$

HQN's 2018 before-tax ROA using beginning period assets is equal to:

$$ROA = \frac{\$650}{\$10,000} = 6.5\% \tag{5.13}$$

Interpreted, each dollar of HQN's assets generates $.065 cents in before-tax profits.

Return on equity (ROE) ratio

The ROE ratio measures the amount of profit generated by each dollar of equity after interest payments to debt capital are subtracted but before taxes are paid. Profits after interest is subtracted equals EBT (earnings before taxes). The ROE ratio can be expressed as:

$$ROE = \frac{EBT}{E} \tag{5.14}$$

HQN's 2018 before-tax ROE using beginning period equity is equal to:

$$ROE = \frac{\$170}{\$2,000} = 8.5\% \tag{5.15}$$

After-tax return on equity, ROE(1 – T), can be expressed as EBT adjusted for taxes, or NIAT. Therefore, ROE(1 – T) can be expressed as NAIT divided by equity:

$$ROE(1-T) = \frac{NIAT}{E} \tag{5.16}$$

HQN's 2018 after-tax ROE is equal to:

$$ROE(1-T) = \frac{\$102}{\$2,000} = 5.1\% \tag{5.17}$$

Interpreted, each dollar of equity generated about $0.085 in before-tax profits and $0.051 in after-tax profits during 2018.

The relationship between ROE and ROA. Before leaving profitability ratios, there is one important question: which is greater for a given firm: ROE or ROA? To answer this question, we simply define (ROE) / (E) as equal to the return assets (ROA)(A) less the cost of debt (i)(D):

$$(ROE)(E) = (ROA)(A) - (i)(D) \tag{5.18}$$

After substituting for A, (D + E) and collecting like terms and dividing by equity E, we obtain the result in Equation ???:

$$ROE = (ROA - i)(\frac{D}{E}) + ROA \tag{5.19}$$

Equation (5.19) reveals ROE > ROA if ROA > i; ROE = ROA if ROA = i; and ROE < ROA if ROA < i.

If ROE is not greater than ROA, then the firm is losing money on every dollar of debt. For HQN, ROE is 8.5% and greater than its ROA of 6.5%. Meanwhile HQN's average interest rate on its debt (total interest costs divided by debt at the beginning of the period equal to i) during 2018 was equal to:

$$i = \frac{int}{debt} = \frac{\$480}{\$8,000} = 6\% \tag{5.20}$$

Efficiency Ratios

Efficiency ratios compare outputs and inputs. Efficiency ratios of outputs divided by inputs describe how many units of output each unit of input has produced. More efficient ratios indicate a unit of input is producing greater units of outputs than smaller efficiency ratios.

Consider two types of efficiency ratios: turnover (TO) ratios and turnover time (TOT) ratios. The turnover ratios measure output produced per unit of input during the accounting period, in our case 365 days. For example, suppose our TO ratio is 5. A TO ratio of 5 tells that during 365 days, every unit input produced 5 units of output.

We may want to find the number of days required for a unit of input to produce a unit of output. We can answer the question by dividing 365 days by the number of turnovers that occurred during the year. This tells us the number of days required for a unit of input to produce a unit of output, what we call a turn over time, or TOT, ratio. Continuing with our example, if an input was turned into an output 5 times during the year, then dividing 365 days by 5 tells us that every turnover required (365 days)/5 = 73 days.

We now consider four TO efficiency measures: (1) the inventory turnover (ITO) ratio, (2) the asset turnover (ATO) ratio, (3) the receivable turnover (RTO) ratio, and (4) the payable turnover (PTO) ratio. We also find for each TO ratio their corresponding TOT ratio.

Inventory turnover (ITO) ratio

The ITO ratio measures the output (total revenue) produced by the firm's inputs (inventory). Total revenue is a period of time measure. Inventory is a point-in-time measure. We use the beginning of the period inventory measure because it reflects the inventory on hand when revenue generating activities began. ITO is defined below.

$$ITO = \frac{Total\ Revenue}{Inventory} \tag{5.21}$$

The 2018 ITO ratio for HQN is:

$$ITO = \frac{\$40,000}{\$3,750} = 10.67 \tag{5.22}$$

The ITO ratio indicates that for every $1 of inventory, the firm generates an estimated 10.67 dollars of revenue during the year. A small ITO ratio suggests that the firm is holding excess inventory levels given its level of total revenue. Likewise, a large ITO ratio may signal potential "stock outs" which could result in lost revenue if the firm is unable to meet the demand for its products and services.

We can find the number of days required to sell a unit of the firm's beginning inventory, its inventory turnover time (ITOT) ratio, by dividing one year (365 days) by the firm's ITO:

$$ITOT = \frac{365}{ITO} \tag{5.23}$$

The ITOT ratio for HQN in 2018 is equal to:

$$ITOT = \frac{365}{10.67} = 34.21 \tag{5.24}$$

In other words, a unit of inventory entering HQN's inventory is sold in roughly 35 days.

Asset turnover (ATO) ratio

The ATO ratio measures the amount of total revenue (output) for every dollar's worth of assets (inputs) during the year. The ATO ratio measures the firm's efficiency in using its assets to generate revenue. Like the ITO ratio, the ATO ratio reflects the firm's pricing strategy. Companies with low profit margins tend to have high ATO ratios. Companies with high profit margins tend to have low ATO ratios.

Let A represent the value of the firm's assets. The ATO ratio is calculated by dividing the firm's total revenue by its total assets:

$$ATO = \frac{Total\ Revenue}{A} \tag{5.25}$$

Using beginning of the period assets, HQN's 2018 ATO ratio is equal to:

$$ATO = \frac{\$40,000}{\$10,000} = 4.0 \tag{5.26}$$

We can find the firm's asset turnover time (ATOT) ratio, the number of days required for a dollar of assets to generate a dollar of sales, by dividing 365 by the firm's ATO ratio. Using the ATO previously calculated for HQN in 2018 we find:

$$, ATOT = \frac{365}{ATO} = \frac{365}{4.0} = 91.3 \tag{5.27}$$

In other words, a dollar of HQN's assets generates a dollar of cash receipts in roughly 91 days.

Receivable turnover (RTO) ratio

The RTO ratio measures the firm's efficiency in using its accounts receivables to generate cash receipts. The RTO ratio is calculated by dividing the firm's total revenue by its accounts receivables. Using account receivables measured at the beginning of the year, the firm's RTO ratio measures how many dollars of revenue are generated by one dollar of accounts receivables held at the beginning of the period. The RTO reflects the firm's credit strategy. Companies with high RTO ratios (strict customer credit policies) tend to have lower levels of total revenue than those with low RTO ratios (easy credit policies). We express the RTO ratio as:

$$RTO = \frac{Total\ Revenue}{Account\ Receivables} \tag{5.28}$$

Using data from HQN for 2018, cash receipts from the income statement, and accounts receivables from the ending 2017 balance sheet, we find HQN's RTO to equal:

$$RTO = \frac{\$40,000}{\$1,640} = 24.39 \tag{5.29}$$

In the case of HQN during 2018, every dollar of account receivables generated $24.39 in revenue or an output to input ratio of 24.39.

We can estimate the firm's receivable turnover time (RTOT) ratio or what is sometimes called the firm's average collection period for accounts receivable ratio, the number of days required for a dollar of credit sales to be collected, by dividing 365 by the firm's RTO ratio.

$$RTOT = \frac{365}{RTO} \tag{5.30}$$

In the case of HQN during 2018, we find its RTOT ratio equal to:

$$RTOT = \frac{365}{24.39} = 14.96 \tag{5.31}$$

Interpreted, it takes an average of nearly 15 days from the time of a credit sale until the payment is actually received. The RTOT ratio, like the RTO ratio, reflects the firm's credit policy. If the RTOT is too low, the firm may have too tight of a credit policy and might be losing revenue as a result of not offering customers the opportunity to purchase on credit. On the other hand, remember that accounts receivable must be financed by either debt or equity funds. If the RTOT is too high, the firm is extending a lot of credit to other firms, and the financing cost may become excessive. Another concern is that the longer a firm extends credit, the greater is the risk that the firm's accounts receivable will ever be repaid.

In some cases, it is useful to construct a schedule that decomposes accounts receivable into the length of time each amount has been outstanding. For example, the schedule might break the accounts receivable into: 1) the amount that is less than 30 days outstanding, 2) the amount that is 30–60 days outstanding, and 3) the amount that is more than 60 days outstanding. This breakdown provides additional information on the risk of the firm's accounts receivable and the likelihood of repayment.

Payable turnover (PTO) ratio

The PTO ratio measures the firm's efficiency in using its accounts payable to acquire its accrued COGS. The PTO is calculated by dividing accrued COGS (equal to cash COGS plus change in account payable) by accounts payable measured at the beginning of

the year. The firm's PTO ratio measures how many dollars of accrued COGS are generated by one dollar of accounts payable held at the beginning of the period. The PTO reflects the firm's credit strategy. Does it prefer equity or debt financing. Firms with low PTO ratios tend to favor the use of debt to finance the firm which tends to generate higher variability in its ROE. The PTO ratio is expressed as:

$$PTO = \frac{Accrued\,COGS}{AP} \tag{5.32}$$

Using data from HQN for 2018, we find its PTO to equal:

$$PTO = \frac{\$28,000}{\$3,000} = 9.33 \tag{5.33}$$

In the case of HQN, every dollar of accounts payable produced 9.33 dollars in accrued COGS.

We can estimate the firm's payable turnover time (PTOT) ratio by dividing 365 days by the firm's PTO ratio.

The PTOT ratio measures the number of days before a firm repays its credit purchases. The PTOT formula, like the other average period ratios, is found by dividing 365 by the PTO ratio. The PTOT ratios can be expressed as:

$$PTOT = \frac{365}{PTO} \tag{5.34}$$

HQN's 2018 PTOT ratio is calculated as:

$$PTOT = \frac{365}{9.33} = 39.12 \tag{5.35}$$

Interpreted, HQN's PTOT ratio of nearly 39 days implies that it takes the firm an average of 39 days from the time a credit purchase is transacted until the firm actually pays for its purchase. The PTOT ratio, like the PTO ratio, reflects the firm's credit policy. If the PTOT is too low, the firm may not be using its available credit efficiently and relying too heavily on equity financing. On the other hand, PTOT ratios that are too large may reflect a liquidity problem for the firm or poor management that depends too much on high cost short term credit.

Note of caution. Economists and others frequently warn against confusing causation and correlation between variables. Descriptive data reflected in the ratios derived in this section on efficiency ratios do not generally reflect a causal relationships between variables nor should they be used to make predictions. For example, in the previous section, we are not suggesting that PTOT can be predicted by the PTO or vice versa. The only thing that can be inferred is that PTOT times PTO will always equal 365 days.

Liquidity Ratios

A firm's liquidity is its ability to pay short-term obligations with its current assets. Also implied by liquidity is the firm's ability to quickly convert assets into cash without a loss in their value which would be the case if the exchange of an asset for cash required a large discount. However, before we review important liquidity ratios, we review an important liquidity measure that is not a ratio: a firm's net working capital.

Net working capital (NWC). Even though NWC is not a ratio, it provides some useful liquidity information that should not be ignored. If NWC is positive, then CA which are expected to be converted to cash during the upcoming year will be sufficient to pay for CL, those liabilities expected to come due during the upcoming year. HQN's net working capital, described in Table 5.2, is positive for years 2016, 2017, and 2018, suggesting the firm was capable of meeting its short-term debt obligations by using only the assets expected to be liquidated during the upcoming year.

Table 5.2. Net Working Capital for HQN

Year	rrent CuAssets	–	Current Liabilities	=	Net Working Capital
2016	$5,910,000	–	$5,370,000	=	$540,000
2017	$6,320,000	–	$5,958,000	=	$362,000
2018	$7,000,000	–	$6,600,000	=	$400,000

Another aspect of HQN's NWC is its trend. Is NWC increasing or decreasing over time? We measure the trend in NWC by calculating the change in NWC between calendar years. HQN's NWC decreased by $178,000 during 2017 ($362,000 – $540,000). It increased by $38,000 during 2018 ($400,000 – $362,000).

The decrease in NWC during 2017 and the slight increase in 2018 calls for an explanation. Was the drop in NWC justified? Did it represent a conscious liquidity decision by the firm? Was it due to external forces? It is the duty of financial managers to find answers to these questions.

Current (CT) ratio

Liquidity ratios measure a firm's ability to meet its short-term or current financial obligations with short-term or current assets. The CT ratio is the most common liquidity measure. It combines two point-in-time measures from the balance sheet, current assets (CA), and current liabilities (CL). The point-in-time measures of the two numbers must be the same. We write the CT ratio as:

$$CT = \frac{CA}{CL} \tag{5.36}$$

In principle we would like to see the CT ratio exceed one (CT > 1), because it suggests that for every dollar of CL there is more than one dollar of CA sufficient to cover the liquidation of CL if necessary. If the CT ratio is less than one (CT < 1), then liquidating current assets will not generate enough funds to pay for the firm's maturing current liabilities obligations which may create a significant problem. If the firm's current liabilities exceed its current assets, the firm may have to liquidate long-term assets to meet it current obligations. But liquidating long-term (usually illiquid) assets is often costly to do because they cannot be easily converted to cash and end up being sold for a price less than their value to the firm.

The current ratio is constructed from the firm's balance sheet (see Table 5.1). The CT ratio for HQN at the beginning of 2018 (the end of 2017) was:

$$CT = \frac{\$6,320}{\$5,958} = 1.06 \tag{5.37}$$

HQN's beginning 2018 CT ratio value of 1.06, suggests that its current liquid resources were just sufficient to meet its current obligations.

As with all the ratios we will consider, there is generally no "correct" CT ratio value. Clearly a firm's CT ratio can be too low, in which case the firm might have difficulty paying its maturing short-term liabilities. Nevertheless, a CT < 1 does not mean that a firm will not be able to meet its maturing obligations. The firm may have access to other resources that can be used to help meet maturing obligations, such as earnings from operations, long-term assets that could be liquidated, debt which could be restructured, and/or investments in depreciating assets which can be delayed.

On the other hand, a firm's CT ratio can be too high. CA usually earn a low rate of return and holding large levels of current assets may not be profitable to the firm. It may be more efficient to convert some of the CA to long-term assets that generate larger expected returns. To illustrate, think of the extreme case of a firm that liquidates all of its long-term assets and holds them as cash. The firm might have a large CT ratio and be very liquid, but liquid assets are unlikely to generate a high rate of return or profits.

Quick (QK) ratio

The QK ratio is sometimes called the acid-test ratio. The QK ratio is very similar to the CT ratio, except that inventories (INV),

another point-in-time measure obtained from the firm's balance sheet, are subtracted from CA. The QK ratio is defined as:

$$QK = \frac{CA - INV}{CL} \tag{5.38}$$

In forming the QK ratio, inventories are subtracted because inventories are most often the least liquid of the current assets, and their liquidation value is often the most uncertain. Thus the QK ratio provides a more demanding liquidity measure than the firm's CT ratio.

Using balance sheet data from Table 5.1, we find the beginning 2018 QK ratio for HQN equal to:

$$QK = \frac{(\$6,320 - \$3,750)}{\$5,958} = 0.43 \tag{5.39}$$

In other words, liquidating all current assets except inventory will generate enough cash to pay for only 43 percent of HQN's current liabilities. Once again, there is no right or wrong QK ratio. This partly depends on the form of one's inventories. Product inventories are liquid. Inventories of inputs are less liquid. Clearly, HQN's liquidity is much lower if its inventory is not available to meet currently maturing obligations. Nevertheless, similar to the CT ratio, a QK ratio of less than one does not necessarily mean the firm will be unable to meet the maturing obligations.

Leverage Ratios

A lever is bar used for prying or dislodging something. We can move more weight with a lever than by applying force directly. The concept of leverage has application in finance. In finance, we define leverage ratios as those ratios used to describe how a company obtains debt and assets using its equity, as a lever. There are several different leverage ratios, but the main components of leverage ratios include debt, equity, and assets. A common expression that associates leverage with equity and debt is: How much debt can we raise (leverage) with our equity?

In general, higher leverage ratios imply greater amounts of debt financing relative to equity financing and greater levels of risk. Greater levels of firm risk also imply less ability to survive financial reversals. On the other hand, higher leverage is usually associated with higher expected returns. Here, we consider two key leverage ratios: (1) debt-to-equity ratio (DE) and (2) equity multiplier ratio (EM).

Debt-to-equity (DE) ratio



DE ratios are the most common leverage ratios used by financial managers. They combine two point-in-time measures from the same balance sheet. The DE ratio measures the extent to which the firm uses its equity as a lever to obtain loan funds. As the firm increases its DE ratio, it also increases its control over more assets.

The DE ratio is equal to the firm's total debt (D) divided by its equity (E). If dollar returns on assets exceed the dollar costs of the firm's liabilities, having higher DE ratios (greater leverage) increases profits for the firm. We write the firm's DE ratio as:

$$DE = \frac{D}{E} \tag{5.40}$$

In general, having a lower DE ratio is preferred by creditors, because more equity funds are available to meet the firm's financial obligations. (Why?) HQN's DE ratio at the beginning of 2018 was:

$$DE = \frac{\$8,000}{\$2,000} = 4.0 \tag{5.41}$$

Interpreted, HQN's DE ratio of 4 implies that each dollar of its equity has leveraged $4.00 of debt. As with liquidity ratios, there is no magic value for DE ratios. If too much debt is used per dollar of equity, the risk of being unable to meet the fixed debt obligations can become excessive. On the other hand, if too little debt is used, the firm may sacrifice returns that can be realized through leverage.

Equity multiplier (EM) ratio



The EM ratio is equal to the firm's total assets A divided by its equity E. The EM ratio tells us the number of assets leveraged by each dollar of equity. The EM ratio like the DE ratio combines two point-in-time measures from the balance sheet. The EM ratio is a financial leverage ratio that evaluates a company's use of equity to gain control of assets.

The EM ratio is particularly useful when decomposing the rate of return on equity using the DuPont equation that we will discuss later in this chapter. The formula for EM can be written as:

$$EM = \frac{A}{E} \tag{5.42}$$

HQN's assets and equity are used to calculate its EM at the beginning of 2018, and can be expressed as:

$$EM = \frac{\$10,000}{\$2,000} = 5.00 \tag{5.43}$$

Leverage ratios are often combined with income statement measures to reveal important information about the riskiness of the firm beyond those provided by leverage ratios. We need to include income and cash flow data to answer the question: what is the optimal leverage ratio? We considered these issues when we earlier examined repayment capacity ratios.

Other Sets of Financial Ratios

Other sets of financial ratios besides SPELL ratios have been proposed and used elsewhere. For example, one popular set of ratios is referred to as the Sweet 16 ratios. These are compared to the SPELL ratios in Table 5.3.

Table 5.3. Comparing Sweet 16 Ratios with SPELL ratios

SPELL Ratios	Sweet 16 List	Comments
(S)olvency	Solvency	Same ratios.
(P)rofitability	Profitability	Same ratios.

SPELL Ratios	Sweet 16 List	Comments
(E)fficiency	Efficiency	Same ratios.
(L)iquidity	Liquidity	Same ratios.
(L)everage	Repayment Capacity	Different interpretation. Equates repayment capacity with leverage.

The DuPont Equation

The DuPont equation equals ROE multiplied by two identities assets (A) over A and total revenue over total revenue.

$$ROE = \frac{EBT}{E}\frac{A}{A}\frac{total\ revenue}{total\ revenue} = \frac{EBT}{total\ revenue}\frac{total\ xrevenue}{A}\frac{A}{E} \tag{5.44}$$

The second half of Equation ???, after substituting and rearranging ratios, shows that ROE depends on the asset turnover ratio (ATO), sales margin (m) and the equity multiplier (EM) ratio:

$$ROE = \frac{EBT}{total\ revenue}\frac{total\ revenue}{A}\frac{A}{E} = m(ATO)(EM) \tag{5.45}$$

The DuPont equation is important because it provides a detailed picture of the firm's ability to generate profits efficiently from its equity across several of the SPELL ratios. The first ratio measures operating efficiency using the firm's profit margin ratio m. The second ratio measures asset use efficiency using the firm's asset turnover ratio ATO. And the third ratio measures financial leverage or risk using the firm's equity multiplier ratio EM.

ROE depends on =	(	Efficiency in generating profits from sales	)(	Efficiency in generating sales from assets	)(	Amount of assets leveraged by each dollar of equity	)

The DuPont is only one of a large number of DuPont-like equations. Multiplying by ROE assets/assets and one of the following: accounts receivables/accounts receivables, inventories/inventories, and accounts payables/accounts payables produces many versions of the DuPont equation. We list a few possibilities below:

$$\begin{aligned} ROE &= (m)(ATO)(EM) = (m)(RTO)(EM) \\ &= (m)(ITO)(EM) = (m)(PTO)(EM) \end{aligned} \tag{5.46}$$

The interdependencies described in the DuPont equation help us to perform strengths and weaknesses analysis. HQN's DuPont equation for 2018 is found using previously calculated values for m, ATO, and EM:

$$ROE = (m)(ATO)(EM) = (.00425)(4.0)(5.0) = 8.5\% \tag{5.47}$$

Since our ROE calculation of 8.5% equals the DuPont calculation of ROE, we are confident that our calculations, which mix point and period of time measures, are consistently calculated and reflect the interdependencies of the system. Comparing the components of the DuPont equation with industry standards we find:

ROE depends on =	(	m: lowest quartile of the industry	)(	ATO: highest quartile of the industry	)(	EM: significantly higher than the highest quartile of the industry	)

Based on the above analysis, the ROE is at or near the industry average despite a weak profit margin because its ATO and EM are both high. HQN is efficient in its generation of cash receipts from assets and it is also highly leveraged so that as long as its average cost of debt is less than its ROA, its ROE increases.

To explore the profit margin further, note that the low profit margin is determined by EBT that in turn depends on the level of cash receipts and the cost to generate that level of cash receipts. Our earlier analysis suggested that operating costs and interest costs were relatively high, and these may be having a major impact on the profit margin.

Looking at the ATO ratio, we see that fixed assets impact the ratio, and we were concerned that the firm may not be reinvesting enough in replacing assets. Failing to replace assets as they are used up would artificially inflate the ATO and the firm's ROE. Also, the inventory levels may be too high. Lowering the inventory levels would increase the ATO and improve ROE. Finally, the high level of leverage helped ROE but is putting the firm in a risky position. The large withdrawal of equity in 2018 has further increased this risk.

Comparing Firm Financial Ratios with Industry Standards

Financial ratios calculated for an individual firm can be made more useful by having a set of standards against which they can be compared. One might think of the limited usefulness of one's blood pressure readings without some reference level of what is considered a normal of healthy blood pressure. Consider how one can learn more about a firm by comparing it to similar firms in the industry or by comparing it to the distribution of ratios of similar firms.

Major sources of industry and comparative ratios include: Dun and Brad- street, a publication of Dun and Bradstreet, Inc.; Robert Morris Associates, an association of loan officers; financial and investor services such as the Standard and Poor's survey; government agencies such as the Federal Trade Commission (FTC), Securities and Exchange Commission (SEC), and Department of Commerce; trade associations; business periodicals; corporate reports; and other miscellaneous sources such as books and accounting firms. Table 5.4 shows selected HQN's ratios for 2018, as well as ratios for other firms in the industry. The industry ratios are broken into quartiles. For example, 1/4 of the firms in the industry have current ratios above 2.0.

Table 5.4. HQN ratios for 2018 & Industry Average Ratios in Quartiles
Open HQN Coordinated Financial Statement in MS Excel

Ratios	HQN for 2018	Lower Quartile	Median	Upper Quartile
SOLVENCY RATIOS				
TIE (times interest earned)	1.35	1.6	2.5	5.8
DS (debt-to-service)	1.02	0.9	1.4	3.3
PROFITABILITY RATIOS				
m (margin)	0.43%	0.44%	1.03%	1.79%
ROA (return on assets)	6.50%	0.66%	3.30%	7.00%
ROE (return on equity)	8.50%	2.10%	10.70%	17.20%
EFFICIENCY RATIOS				
ITO (inventory turnover)	10.67	4.8	7.7	14.9
ITOT (inventory turnover time)	34.21	76.04	47.40	24.50
ATO (asset turnover)	4	1.5	3.2	3.9
ATOT (asset turnover time)	91.25	243.33	111.06	93.59
RTO (receivables turnover)	24.40	15.21	11.41	8.90
RTOT (receivables turnover time)	14.96	24	32	41
PTO (payable turnover)	9.33	9.36	12.59	15.21
PTOT (payable turnover time)	39.12	39	29	24
LIQUIDITY RATIOS				
CT (current)	1.06	0.9	1.3	2

Ratios	HQN for 2018	Lower Quartile	Median	Upper Quartile
QK (quick)	0.43	0.5	0.7	1.1
LEVERAGE RATIOS				
DE (debt-to-equity)	4	2.8	1.9	0.9
EM (equity multiplier)	5	3.8	2.2	3.24

Using Financial Ratios to Determine the Firm's Financial Strengths and Weaknesses

Comparing SPELL ratios with industry standards. In what follows we compare the ratios computed for HQN with the ratios calculated for similar firms. Comparing HQN's SPELL ratios with industry standards is the essence of strengths and weaknesses analysis and answers the question: what is the financial condition of the firm?

In Table 5.4, the industry is described by the ratio for the firm, the median firm, and the average of firms in the upper and lower quartile of firms. Consider how comparing HQN to the other firms in its industry might allow us to reach some conclusions about HQN's strengths and weaknesses and to determine its financial condition.

Solvency ratios. HQN's solvency ratio compared to its industry indicates that it may have a difficult time paying its fixed debt obligations out of earnings. The TIE ratio in 2018 is 1.35 which is less than the industry's lowest quartile. HQN's DS ratio in 2018 is 0.94, which implies that only about 94 percent of the firm's interest and principal can be paid out of current earnings which is only slightly higher than the industry's lowest quartile. In effect, compared to industry standards, HQN's significant weakness is its solvency. HQN will need to refinance, raise additional capital, or liquidate some assets in order to make the interest and principal payments and remain in business.

Profitability ratios. Compared to industry averages, HQN is profitable. Its ROE is reasonably close to the industry average and its ROA is close to the upper quartile industry average. Paradoxically, HQN's m margin is close to the industry's lowest quartile average.

Efficiency ratios. Compared to the industry averages, HQN is very efficient. Both HQN's ITO and ATO ratios are near the top in its industry. Its ITO ratio in 2018 was 10.67, indicating that HQN has sold its inventory over 10 times during the year. This ITO ratio is above the median value for firms in this industry of 7.7, and strong. The ATO ratio has a 2018 value of 4.0, indicating the firm had sales of 4 times the value of its assets, compared to an industry median of 3.2 which is in the upper quartile of firms in its industry. The firm appears to be using assets efficiently which undoubtedly contributes to HNQ's profitability even though its margin in low.

The RTOT ratio has a 2018 value of 14.96. This value is well below industry averages, and raises a question about the firm's credit policies. The industry average RTOT was 32 and suggests that HQN might consider a more generous credit policy. On the other hand, HQN's PTOT ratio is 39 and is in the lowest quartile for the industry. This suggests that HQN is depending on dealer supplied credit more than other firms in its industry because of its low solvency. Still, HQN's strength may be its efficiency.

Liquidity ratios. The current ratio is 1.06, which suggests the firm is liquid, but barely. Its ratio is near the lower quartile of firms in the industry. The quick ratio is 0.43, suggesting the firm cannot meet its short-term obligations without relying on inventory. HQN's quick ratio is in the lower quartile of firms in the industry, indicating that the firm is less liquid than most of its competitors and is an HQN weakness.

Leverage ratios. The leverage ratios indicate that the HQN's use of debt is high. Comparison with industry ratios shows that HQN is highly leveraged relative to other firms in the industry. As long as ROA exceeds the average interest costs of debt, high leverage increases the firm's profitability—but increases its risk associated with adverse earnings.

Limitations of Ratios

While ratio analysis can be a powerful and useful tool, it does suffer from a number of weaknesses. We discussed earlier how the use of different ac- counting practices for such items as depreciation can change a firm's financial statements and, therefore, alter its financial ratios. Thus, it is important to be aware of and understand accounting practices over time and/or across firms.

Difficult problems arise when making comparisons across firms in an industry. The comparison must be made over the same time periods. In addition, firms within an “industry” often differ substantially in their structure and type of business, making industry comparisons less meaningful. Another difficulty is that a departure from the “norm” may not indicate a problem. As mentioned before, a firm might have apparent weaknesses in one area that are offset by strengths in other areas.

Furthermore, things like different production practices in a firm may require a different financial structure than other firms in the industry. Additionally, shooting for financial ratios that look like the industry average may not be very desirable. Would you want your business to be average?

Inflation can have a significant impact on a firm’s balance sheet and its corresponding financial ratios. As a results, it is important to keep in mind the difference between a capital item’s book value and it market value. Firms that keep a set of market value financial statements in addition to their book value financial statements should conduct financial analysis with both their book value and market value financial statements.

We should recognize that a single ratio does not provide adequate information to evaluate the strength or weakness of a firm. A weak ratio in one area might be offset by a strong ratio in another area. Likewise, a perfectly healthy firm, from a financial standpoint, may have some special characteristics which result in a ratio which would be out of line for other firms in the industry who do not have these characteristics.

Finally, it must be understood that financial analysis does not in itself provide a management decision. The analysis provides information which will be a valuable input into making management decisions, but there is no “cook book” formula into which you plug the financial analysis number and produce the correct management decisions.

Financial ratios can be an effective strengths and weaknesses analysis tool. Their principal use is to assess the firm’s ability to survive. To survive in the long term, the firm must be profitable and solvent. Profitability is defined as the difference between a firm’s revenues and its expenses. Solvency is the firm’s ability to meet its cash obligations when they become due. Solvency depends on the firm’s holdings of liquid assets—assets that can easily and with little expense be converted into cash in the current period.

If a firm is not both profitable and solvent, it cannot survive in the long term. In the short term, a firm can be solvent but not profitable. For a limited time, an unprofitable firm can convert assets to cash and remain solvent by borrowing, refinancing existing debt, selling inventory, liquidating capital assets, increasing accounts payable, or depleting its capital base. These acts may improve the firm’s solvency in the short run, but are likely to erode the firm’s future profitability.

In contrast, a firm may be profitable and not solvent, in which case it cannot survive even in the short term. Once a firm fails to meet its cash flow obligations, even if it is profitable, in most cases it loses control over its assets. Therefore, short-term survival may require some firms to sacrifice profitability for solvency. Thus, financial managers must monitor both the firm’s solvency and profitability. An appropriately constructed set of financial ratios will allow financial managers to monitor both the firm’s profitability and solvency.

Financial ratios may also provide information about the liquidity of the firm, which is related to the firm’s solvency because the firm’s liquidity position tells us something about the firm’s ability to meet unforeseen outcomes and survive. Also related to the firm’s solvency and liquidity is the probability of achieving different rates of return. Measures of the probability of alternative rates of return are sometimes examined under the general heading of risk, a subject we will return to later in this book (see Chapter 15).

Strengths and Weaknesses Summary

How do we summarize our strengths and weaknesses analysis? One way is to assign a grade to each of the SPELL categories ranging from 5 (superior) to 1 (on life support). Clearly, the grades assigned are somewhat subjective, but perhaps useful, in summarizing a great deal of financial information. Then what? Do we assign equal weight to each of the SPELL categories depending on their relative importance? The answer to this question depends on the vision, goals, and objectives of the firm manager. To complete this discussion of how to assign weights to the SPELL categories, we treat them equally important in this example—although a strong case exists for assigning a greater weight to profitability measures. In Table 5.5 we summarize our strengths and weaknesses ratings.

Table 5.5. Summary of HQN’s 2018 Financial Strengths & Weaknesses

SPELL Category	weights	Grades: 5 (very strong) to 1 (very weak)
Solvency	.2	2.0
Profitability	.2	3.0
Efficiency	.2	4.0
Liquidity	.2	3.0
Leverage	.2	2.0
Weighted Summary		2.8

So what have we learned? We learned that a firm's SPELL ratios can be compared to financial ratios of similar firms to determine the firm's financial strengths and weaknesses. In the case of HQN, we assign to it a strengths and weaknesses score of 2.8 which is less than the median or average financial conditions of similar firms in its industry. Significant in arriving at an overall financial strengths and weaknesses score of 2.8 was HQN's high leverage that places it in a risky position and its weak solvency condition.

Summary and Conclusions

When using financial ratios from one's own firm and comparing them with industry standard ratios, it is often useful to take notes or summarize the major points as you work through the ratio analysis. In our analysis of HQN, the firm is highly leveraged and is in a risky position. We might ask why is the firm relying so heavily on debt and why is its equity being withdrawn at such a relatively high rate? The overhead expenses seem to be too high. Why? How can the situation be improved? Why are the firm's assets being depleted? What is the cause of the increasingly high level of inventory being held?

After gathering information on these questions and others, the firm's financial manager may produce a detailed strengths and weaknesses report. In the report, key financial management issues can be explored, and forecasts of future financial needs and situations can be made. Continued monitoring of the firm's financial statements and ratios will allow the firm's management to gain solid understanding of the relationship between the firm's operations and its financial performance and to recommend changes when required.

So what have we learned? We learned that by using the information contained in its CFS, firms can construct financial ratios that provide five different views of the firm's financial condition: its solvency, its profitability, its efficiency, its liquidity, and its leverage. A logical next step is to a assign a weight to each of the firm's financial conditions reflected by its ratios after comparing them to industry standards. The overall weighted average reflects the firm's financial strengths and weaknesses and answers the question: what is the financial condition of the firm.

Questions

When calculating 2018 ratios, please refer to Tables 4.1, 4.4A or 4.4B, and 4.6 in Chapter 4. When asked for industry standard comparisons, use industry measures provided in Table 5.4.

1. Explain why financial statement data is made more useful by forming SPELL ratios?
2. Describe the kinds of questions related to the firm's financial strengths and weaknesses each of the SPELL ratios can help answer.
3. Calculate the 2018 SPELL ratios for Friendly Fruit Farm (FFF) described in Chapter 4.
4. Do FFF's DS and TIE ratios, both solvency ratios, tell consistent stories? Defend your answer.
5. Explain why a firm might be reluctant to meet its short-term liquidity needs by liquidating long-term assets.
6. Describe the connections between the m ratio and FFF's ROE.
7. What is the essential difference between ROA and ROE profit measures? What do they each measure?
8. What conditions guarantee that ROE > ROA or that ROA > ROE?
9. Explain the connections between efficiency and ROE or ROA measures.
10. Create an efficiency ratio for your class preparation efforts? (Hint: what are the inputs and what are the outputs?) What could you do to improve the efficiency of your class preparation efforts?
11. What might be implied by very high or very low ITO ratios?
12. Calculate ITO ratios using 2018 total revenue measures for FFF. Then compare your results with ITO ratios using accrued

COGS. Explain the differences.

13. The optimum RTOT ratio seeks to balance the need to generate cash receipts by offering easy credit versus the need to meet liquidity need by limiting its accounts receivable. Looking at the financial statements for HQN, what is an ideal RTOT ratio (state a number)? Defend your ideal number RTOT number, and if it is different than HQN's actual number, what actions could you take to align HQN's actual RTOT to its ideal RTOT?
14. Explain why it is difficult to compare net working capital numbers between firms.
15. The DuPont equation allows us to decompose the ROE measure. Replace total revenue with COGS in Equation ??? and recalculate the components of the revised DuPont equation. Interpret the results. Does the resulting equation still equal HQN's ROE?
16. Calculate and compare FFF CT ratios at the end of years 2017 and 2018. What can you learn from the changes in FFF's CT ratios? Compare FFF CT ratios with industry standards. What do you learn from the comparison?
17. Using FFF's QK ratios at the end of years 2017 and 2018, what strengths and weaknesses score would you assign to its liquidity?
18. Suppose FFF's long-term debt was 10% above the book value of their long-term assets and only 50% of the current value of their long-term assets. Calculate DE ratios using current and book values of their long-term assets. If FFF were applying for a loan, which DE ratios would they most likely present?
19. If ROA exceeds the average costs of the firm's liabilities, having higher DE ratios (greater leverage) increases profits for the firm. Why might lenders want lower DE ratios while borrowers may want higher DE ratios?
20. Compare firms with low ITO ratios such as jewelry stores with firms with high ITO ratios like grocery stores or gas stations. How might their profit margin requirements for success differ? Explain.
21. Using FFF's SPELL ratios and the industry standards used to evaluate HQN's strengths and weaknesses, write a brief report of FFF's financial strengths and weaknesses. Organize your report into the five SPELL categories: solvency, profitability, efficiency, liquidity, and leverage. Complete a table similar to Table 5.5 that was prepared for HQN. What is the summary measure of FFF's financial strengths and weaknesses?

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6: System Analysis

Learning Objectives

After completing this chapter, you should be able to (1) define a system; (2) recognize system properties included in coordinated financial statements (CFS); and (3) connect changes in exogenous variables determined outside the CFS system to changes in endogenous variables calculated inside the CFS system to answer"what if" and "how much" kinds of questions.

To achieve your learning goals, you should complete the following objectives:

- Learn what is a system
- Learn how CFS satisfy system requirements.
- Learn how individual financial statements included in the CFS are connected.
- Learn how subsystems can be created from a system.
- Learn how to answer the question: what if the value of an exogenous variable included in the CFS system changes, then how will the values of endogenous variables included in the CFS change?
- Learn how to answer the question: how much does the value of an exogenous variable included in the CFS system need to change for the value of a specified endogenous variable included in the CFS to equal a specified value?
- Learn how asking what if and how much questions about changes in exogenous variables allows firm managers to conduct opportunity and threat analysis.
- Learn how to create a subsystem of the CFS system that describes a firm's rate of return on equity (ROE).
- Learn how to create a subsystem of the CFS system that describes a firm's solvency.
- Learn how to compute common size balance sheets and accrual income statements (AIS) and describe the trade-off perspectives they provide.

Introduction

In what follows we define a system and distinguish between the different kinds of systems. Then we make the point that CFS are a system.

CFS are a system. Indeed we have already used the system properties of CFS to answer the question: "what is" the financial condition of the firm? We answered that question by constructing ratios that described the firm's (S)olvency, (P)rofitability, (E)fficiency, (L)iquidity, and (L)everage (SPELL) ratios. However, the value of information gained from SPELL ratios has its limit. Answering "what is" kinds of questions is a static (timeless) analysis because its focus is on the current financial condition of the firm. We need addition information that is forward looking such as knowing how the financial condition of the firm may change in response to changes in the external environment of the firm. We may also want to know what kinds of changes are required in the firm's external environment for specified changes to occur within the CFS system.

Because CFS is a system, we can use it as our primary strengths, weaknesses, opportunities, and threats analysis tool. We summarize several reasons why the system's properties of CFS are important for financial managers conducting strengths, weaknesses, opportunities, and threats analysis:

- Because it allows us to answer the question: what is the financial condition of the firm reflected by its SPELL ratios. Answering the what is question is the primary means for conducting strengths and weakness analysis.
- Because the relationships between CFS variables and financial statements are consistent (they don't change and cannot produce a contradiction), we can check the accuracy of our numbers by looking for extraordinary numbers in the statements. If we observe unrealistic results, they can only be attributed to the data's inaccuracy.
- Because it allows us to ask what if questions by changing conditions in the external environment of the firm and noting changes in the financial condition of the firm. Answering what if questions is one of two primary means for conducting opportunities and threats analysis.
- Because it allows us to ask how much questions and determine how much of an external change is needed to change a particular variable inside the system. Answering how much questions is one of two primary means for conducting opportunities and threats analysis.
- Because it allows us to define subsystems to focus on parts of the system such as profitability and solvency that we want to

emphasize.

Coordinated Financial Statements as a System

What is a system. A system is an interacting and interdependent group of items forming a unified whole and serving a common purpose (Wikipedia). Every system has boundaries that separate activities that occur within the system from those that occur outside of the system. Each element of the system contributes to a common purpose.

There are, of course, several kinds of systems. An abstract system uses variables to represent tangible or intangible things and may or may not have a real world counterpart. On the other hand, physical systems are generally concrete operational systems made up of people, materials, machines, energy, and other physical things. Physical systems are the actual systems that abstract systems may attempt to represent.

Finally, systems may be closed or open. Open systems allow for exogenous forces outside of the system to influence endogenous activities within the firm. Closed systems are immune to exogenous forces. Finally, systems may be stochastic such that endogenous outcomes within the system can only be described with probabilities. Meanwhile deterministic systems are connected endogenously and exogenously with certain (non probabilistic) relationships.

Coordinated financial statements are a system. The CFS are an abstract system whose variables and statements are described using mathematical equations and numbers. The CFS are designed to represent the financial condition of the firm at the beginning and ending of a period with balance sheets and financial activities between the beginning and ending balance sheets using an accrual income statement (AIS) an a statement of cash flow (SCF). The CFS are an open system. They allow for an external environment represented by exogenous variables to influence activities within the firm represented by endogenous variables. Finally, for our purposes, we assume that the relationships between financial statements and variable values included in the CFS system are deterministic.

To understand the CFS system, we must be able to distinguish between endogenous and exogenous variables. One easy way to distinguish between CFS' endogenous and exogenous variables is to ask: was this variable calculated somewhere in the system? Or was this variable's value determined outside of the system? If the variable was calculated within the system, it is an endogenous variables. If the variable was calculated outside of the system, it is an exogenous variable. A firm's coordinated financial statements for a given time period includes its end-of-period balance sheets, an (accrual) income statement (AIS), and a statement of cash flow (SCF). The firm's CFS constitute a system because their endogenous and exogenous variables are interdependent. The variables are connected with equations that describe how the endogenous variables are calculated and how the financial statements are connected.

To illustrate, cash balances in the previous period's ending balance sheet is an exogenous variable. It's value was determined by activities in previous time periods. In contrast, the current period's ending period cash balances depend on cash balances at the end of the previous period and changes in cash position calculated in the SCF. Therefore the firm's cash balance in the current period's ending balance sheet is an endogenous variable.

Endogenous and exogenous variables create interdependencies between the financial statements in the CFS system. In general, financial activities described in the AIS and statement of cash flow link the beginning and ending balance sheets. To illustrate:

1) Change in cash position calculated in the statement of cash flow equals the difference in cash and marketable securities recorded in the beginning and ending period balance sheets.

2) Cash flow from changes in notes and long-term liabilities in the beginning and ending period balance sheets equal cash flow recorded in the financing section of the statement of cash flow.

3) Addition to retained earning calculated in the AIS equals the difference between the firm's retained earnings recorded in the beginning and ending period balance sheets.

Over identified variables and systems. We discussed over identified variables in Chapter 4. We review and illustrate the concept again here. Suppose that we use the system properties of the CFS to find the ending period cash balances. Then suppose we observe that the ending cash balance calculated as an endogenous variable in our balance sheet is different than the ending cash balance in our check book.

The problem is that the ending cash balance value is over identified. It can be calculated as an endogenous variable within the

financial system or observed externally as an exogenous variable. When the two values differ, we say the system is inaccurate because exogenously and endogenously variable values differ from one another and the financial manager must employ his/her best effort to find the error in the data. Resolving data errors revealed by over identified variables may be the most challenging task facing financial managers whose data is often incomplete and not always accurate. Sometimes the data errors revealed by over identified variables provides financial managers the opportunity to encourage the principals of the firm to reexamine their financial records and look for errors or missing data.

Prioritizing Opportunities and Threats Questions

Financial ratios. Chapter 5 described the financial system of the firm using SPELL ratios. Of course the ratios in each of the SPELL categories provide some information by themselves. Alone, they help us answer what is the financial condition of the firm and what are its strengths and weaknesses. However, ratios are more useful when their interdependence is recognized. In other words a change in the firm's solvency is likely to change the firm's liquidity. A change in the firm's efficiency is likely to change the firm's profitability. And the list of possible interdependencies continues. Since the variables in the financial system are interdependent, the ratios composed of system variables are also interdependent.

How do we proceed to examine the interdependencies of the firm? We limit and organize our examination of the financial interdependencies by focusing on the firm's bottom line—its profitability ratios.

A firm can exist for many reasons. It may satisfy the firm owners' desires to engage in a particular production activity. (For example, I just want to farm!) It may be organized to provide family members and others employment. It may exist to provide some public service. There are undoubtedly other reasons why firms exist. But, the firm financial manager is charged with only one mission—to ensure the firm's survival and its profitability. This requires a proper balance at least between the firm's return and its solvency.

In this view, solvency ratios described by the times interest earned (TIE) and debt-to-service (DSR) ratios and profitability ratios described by margin (*m*), return on equity (ROE), and return on asset (ROA) ratios are the most important of the SPELL ratios. However, efficiency, leverage, and liquidity ratios also matter to the firm because they influence its profitability measured by its ROE and ROA ratios and its solvency measured by its TIE and DSR ratios. One such formula that connects the firm's profitability ratios and other considerations of the firm including leverage and efficiency is the DuPont equation introduced in Chapter 5.

System Metaphors

A metaphor compares two ideas or objects that are dissimilar to each other in some ways and similar to each other in other ways. We use three metaphors to describe how changes in an exogenous variable—a shock—change endogenous variables.

Balloons. One might compare the CFS to a balloon. If you squeeze one part of the balloon (an exogenous force), there will be an (endogenous) bulge somewhere else in the balloon. This action-reaction nature of a system (and balloons) leads to us examine shocks in pairs and answer the question: "what if" a change in an exogenous variable occurs, "then" what happens to the endogenous variables of the system?

Predicting the weather. Predicting the financial future of the firm has characteristics in common with predicting the weather. Meteorologists look at where the weather fronts have been, the direction they have been traveling, and then predict where they will likely be in the future. To hedge their bets, they often predict future weather patterns with probabilities. Predicting the future financial condition of the firm also looks at the condition of the firm now, how it has changed over time, and then predicts with probabilities where it will be in the future.

The detective. Trying to describe how changes in an exogenous variable will affect endogenous variables is like a detective trying to put all the clues together to solve a case that explains who committed the crime. The detective observes a crime—a condition different than what existed before the crime occurred. The financial manager observes changes in the firm's endogenous variables and attempts to link them to changes in one of the exogenous variables.

What does this mean? Since the CFS constitute a system, we ask and answer questions related to changes in the values of exogenous variables and observe how these changes produce changes in the values of endogenous variables within the system. In addition, by creating balance sheet and CFS subsystems, we can examine trade-offs within the firm: if one variable value within the balance sheets or CFS increases, what other variable(s) is likely to decrease, reminiscent of squeezing one part of a balloon and observing a bulge in another part of the balloon. Or we may ask how much an exogenous variable must change to produce a

desired change in an endogenous variable.

CFS systems are not unique because every financial system may define its endogenous and exogenous variables differently. As a result, each system may define its interdependencies differently, which in turn defines how consistency is achieved in the system. Nevertheless, a template that describes the system may be our best and only approach for answering system-wide what if and how much types of questions.

What If Analysis

At the beginning of this chapter, we emphasized how endogenous and exogenous variables created interdependence in the CFS system. Therefore, any change in an exogenous variable in one part of the CFS system will produce changes in endogenous variables in other parts of the system. Tracing the impact of a change in an exogenous variable on endogenous variables in the system is referred to here as what if analysis. The exogenous variable changed by the analyst is often referred to as a control variable—a special kind of exogenous variable.

What if analysis can be helpful in several ways. First, changes in exogenous variables outside of the firm's control may be examined by following their consequences throughout the CFS system. This exercise helps the firm anticipate and plan for different outcomes. A second benefit that may be derived from what if analysis is that changes in exogenous variables may reflect the need for alternative financial strategies that can be evaluated before being adopted and those most beneficial to the firm can be adopted.

The first step in the what if analysis is to introduce the change in an exogenous or control variable. The second step is to recalculate the endogenous variables in the financial statements. The third step is to recalculate the SPELL ratios and compare them to the previous ratio values and to industry averages. And finally, the fourth step is to interpret the results which should include describing connections between changes in the firm's SPELL ratios. Fortunately, steps one, two, and three can be automated using Excel spreadsheets.

To help analyze the changes in exogenous variables described in the scenarios below, we use an Excel integrated set of financial statements described in this chapter. The Excel spreadsheet describes HQN's coordinated financial system and allows those interested in answering what if questions to substitute actual numbers and note the changes in absolute values in the financial statements and ratios.

To illustrate what if analysis, consider the following change in an exogenous variable. Suppose cash receipts for HQN were $39,990 instead of $38,990. The changes that result in HQN's financial statements are highlighted in Table 6.1. As a result of an increase in cash receipts reported in the statement of cash flow (an exogenous variable), EBIT and EBT increased by $1,000. Taxes increased by 40% of the increased earning, from $58 to $468. NIAT increased by $600 of after-tax income to $702 as did retained earnings. Finally assets and equity and liabilities all increased by $600 of after-tax income.

Table 6.1. Coordinated Financial Statements for Hi-Quality Nursery Cash Receipts = $39,990

Open Table 6.1 in Microsoft Excel

BALANCE SHEET			ACCRUAL INCOME STATEMENT		
DATE	**12/31/2017**	**12/31/2018**	**DATE**		**2018**
Cash and Marketable Securities	$930	$1,200	+	Cash Receipts	$39,990
Accounts Receivable	$1,640	$1,200	+	Change in Accounts Receivable	($440)
Inventory	$3,750	$5,200	+	Change in Inventories	$1,450

Notes Receivable	$0	$0	+	Realized Cap. Gains/Depr. Recapture	$0
Total Current Assets	$6,320	$7,600	**Total Revenue**		$41,000
Depreciable Assets	$2,990	$2,710	+	Cash Cost of Goods Sold	$27,000
Non-depreciable Assets	$690	$690	+	Change in Accounts. Payable	$1,000
Total Long-Term Assets	$3,680	$3,400	+	Cash Overhead Expenses	$11,078
TOTAL ASSETS	$10,000	$11,000	+	Change in Accrued Liabilities	($78)
Notes Payable	$1,500	$1,270	+	Depreciation	$350
Current Portion Long-Term Debt	$500	$450	**Total Expenses**		$39,350
Accounts Payable	$3,000	$4,000	**Earnings Before Interest and Taxes (EBIT)**		$1,650
Accrued Liabilities	$958	$880	–	Less Interest Costs	$480
Total Current Liabilities	$5,958	$6,600	**Earnings Before Taxes (EBT)**		$1,170
Non-Current Long-Term Debt	$2,042	$1,985	–	Less Taxes	$468
TOTAL LIABILITIES	$8,000	$8,585	**Net Income After Taxes (NIAT)**		$702
Contributed Capital	$1,900	$1,900	–	Less Dividends and Owner Draw	$287
Retained Earnings	$100	$515	**Addition to Retained Earnings**		$415
Total Equity	$2,000	$2,415			

TOTAL LIABILITIES AND EQUITY	$10,000	$11,000

Table 6.2. HQN Ratio Analysis after a $1,000 increase in Cash Receipts

Open Table 6.2 in Microsoft Excel

Ratios	Industry Average	Activity Ratios	HQN Base	Increase CR by $1000
Solvency				
TIE	2.50	3.44	1.35	3.44
DSR	1.40	2.04	1.02	2.04
Profitability				
ROA	3.30%	16.50%	6.50%	16.50%
ROE	10.70%	58.50%	8.50%	58.50%
m margin	29.00%	2.85%	0.43%	2.85%
Efficiency				
ITO	7.7	10.93	10.67	10.93
ITOT	47.4	33.38	34.22	33.38
ATO	3.2	4.10	4.00	4.10
ATOT	114.1	89.02	91.25	89.02
RTO	11.41	25.00	24.39	25.00
RTOT	32	14.60	14.97	14.60
PTO	12.59	9.33	9.33	9.33
PTOT	29	39.11	39.12	39.11
Liquidity				
Current Ratio	1.30	1.06	1.06	1.06
Quick Ratio	0.70	0.43	0.43	.43
Leverage				
Debt/Assets	0.91	0.80	0.80	.8
Debt/Equity	2.00	4.00	4.00	4.00
Asset/Equity	2.20	5.00	5.00	5.00

In Chapter 5, we focused on ratio analysis. As a result of changes from increased cash receipts, we ask: how many ratios calculated in the previous chapter change? In other words, compare the new and old ratios. The new ratios resulting from an increase in CR of $1,000 are reported in the activity column in Table 6.2. Compare these to the HQN base ratios. The TIE solvency ratio increased from 1.35 to 3.44. The ROA profitability ratio increased from 6.5% to 16.5%. The ITO efficiency ratio increased from 10.67 to 10.93. Liquidity and leverage ratios that depend on beginning balance sheet values remain unchanged. Based on the changes in SPELL ratios resulting from an increase of $1,000 in CR, life is good at HQN!

What If Analysis and Scenarios

In what follows, we describe several scenarios facing HQN that could be analyzed by changing exogenous variables and noting their effect on CFS endogenous variables. Obviously, each of the changes has the capacity to produce changes in HQN's SPELL ratios. Following these consequences throughout the financial system, a what if analysis exercise, is really answering the question: what if something happens, then what?

What complicates our scenario analysis is that more than one exogenous variable change may be required to answer the what if question. Consider several scenarios that we describe next.

Scenario 1. The firm has not been replacing its long-term assets. As a result, its cost of goods sold has been increasing due to increased repairs and maintenance. What are the consequences of this scenario on the financial condition of the firm?

Scenario 2. A financial manager is risk-averse and decides to increase the firm's current assets. What actions can the firm manager take to increase the firm's level of current assets. What are the consequences of increasing the firm's current assets relative to its fixed assets?

Scenario 3. Suppose the firm decides to increase the time it takes to repay its notes payable. What are the advantages/disadvantages of adopting such a strategy? What conditions facing the firm might prompt it to increase the time it takes to repay its notes payable? What ratios would you want to investigate to confirm your assumptions?

Scenario 4. To boost its cash receipts, the firm offers easy credit terms to its customers. What are the implications for the firm? How would you expect the firm's credit policies to be reflected in the firm's financial statements?

Scenario 5. Market conditions have reduced the demand for the firm's products. As a result, cash receipts are falling. Unfortunately, most of the firm's costs are fixed and don't adjust to changing output levels. What changes would you expect to find in future financial statements of the firm?

Scenario 6. The firm's owners face serious medical costs and must extract funds from the business. Describe the impact of these expenses on the firm's financial statements.

Scenario 7. The firm makes a major investment in long-term assets to improve its efficiency. One impact of the change is to reduce its taxes because of the increased depreciation. Describe other consequences on the firm's financial statements.

Scenario 8. Hard economic times have reduced the firm's customers' ability to pay for their purchases in the usual amount of time. Describe the consequences on the firm's financial statements.

Scenario 9. Cash receipts have been inadequate for the firm to meet its notes payable and current long-term liabilities. As a result, it is forced to sell off some of its long-term assets at values less than reported on its balance sheet. What other strategies can the firm adopt to meet its solvency demands?

Scenario 10. Reduced cash receipts without changes in production levels have led to increased inventories. To meet its financial demands, the firm has restructured its debt, decreasing the current portion of the long-term debt. How will these changes be reflected in its financial statements?

Scenario 4 illustration. Consider performing what if analysis on scenario 4. The analysis requires that we assume specific numbers. In our illustration assume that CR increased by 5% from $38,990 to $40,940. Then, because production has increased, assume that cash COGS increase by 8% from $27,000 to $29,160. There may be other changes in exogenous variables, but these are sufficient to illustrate our scenario analysis. After making the changes, we resolve the CFS template and report the consequences in the activity column. In case we want to save our results for later analysis, we save them under scenario 4 column in Table 6.3 below.

Table 6.3. HQN's SPELL Analysis for Scenario 4.
Open Table 6.3 in Microsoft Excel

Ratios	Industry Average	Activity Ratios	HQN Base	Scenario 4

Solvency				
TIE	2.50	0.92	1.35	0.92
DSR	1.40	0.81	1.02	0.81
Profitability				
ROA	3.30%	4.40%	6.50%	4.40%
ROE	10.70%	-2.00%	8.50%	-2.00%
m margin	29.00%	-0.10%	0.43%	-0.10%
Efficiency				
ITO	7.7	11.19	10.67	11.19
ITOT	47.4	32.63	34.22	32.63
ATO	3.2	4.20	4.00	4.20
ATOT	114.1	87.01	91.25	87.01
RTO	11.41	25.58	24.39	25.58
RTOT	32	14.27	14.97	14.27
PTO	12.59	10.05	9.33	10.05
PTOT	29	36.31	39.12	36.31
Liquidity				
Current Ratio	1.30	1.06	1.06	1.06
Quick Ratio	0.70	0.43	0.43	0.43
Leverage				
Debt/Assets	0.91	0.80	0.80	0.80
Debt/Equity	2.00	4.00	4.00	4..00
Asset/Equity	2.20	5.00	5.00	5.00

While HQN successfully increased its CR, its ROA decreased from 6.5% to 5.6% and its ROE decreased from 8.5% to 3.98%. A homework exercise asks you to examine other consequences of increases in CR and COGS. However, one lesson learned from this scenario analysis is to be careful what you wish for—especially if your ultimate goal is to increase the profitability of your firm.

How Much Questions and Goal Seek

CFS system's properties allow us to ask and answer important what if kinds of questions by changing an exogenous variable and observing its effect on the endogenous variables of the system. Goal Seek is an important Excel function that allows us to ask and answer how much kinds of questions. How much questions take the form: how much change is required in an exogenous variable x for variable y to reach a particular value, a goal, equal to a? To illustrate using HQN data, suppose we asked: how much must HQN's CR increase for ROE to equal 9%?

Table 6.4. Coordinated Financial Statements for Hi-Quality Nursery

Open Table 6.4 in Microsoft Excel

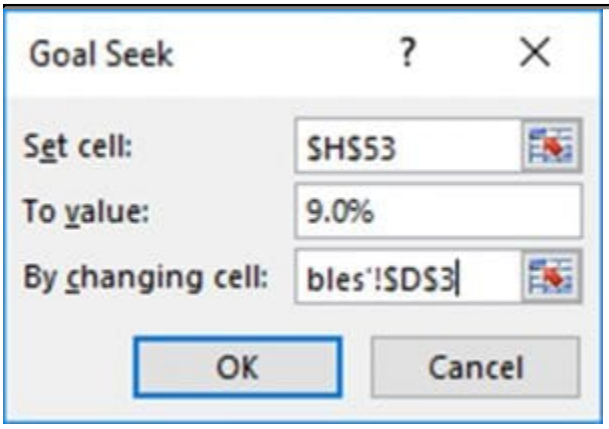
BALANCE SHEET	ACCRUAL INCOME STATEMENT	STATEMENT OF CASH FLOW

DATE	12/31/2017	12/31/2018	DATE		2018	DATE		2018
Cash and Marketable Securities	$930	$606	+	Cash Receipts	$39,000	+	Cash Receipts	$39,000
Accounts Receivable	$1,640	$1,200	+	Change in Accounts Receivable	($440)	–	Cash Cost of Goods Sold	$27,000
Inventory	$3,750	$5,200	+	Change in Inventories	$1,450	–	Cash Overhead Expenses	$11,078
Notes Receivable	$0	$0	+	Realized Cap. Gains/Depr. Recapture	$0	–	Interest paid	$480
Total Current Assets	$6,320	$7,006	**Total Revenue**		$40,010	–	Taxes	$72
Depreciable Assets	$2,990	$2,710	+	Cash Cost of Goods Sold	$27,000	**Net Cash Flow from Operations**		$370
Non-depreciable Assets	$690	$690	+	Change in Accounts. Payable	$1,000	+	Realized Cap. Gains + Depr. Recapture	$0
Total Long-Term Assets	$3,680	$3,400	+	Cash Overhead Expenses	$11,078	+	Sales Non-depreciable Assets	$0
TOTAL ASSETS	$10,000	$10,406	+	Change in Accrued Liabilities	($78)	–	Purchases Non-depreciable Assets	
Notes Payable	$1,500	$1,270	+	Depreciation	$350	+	Sale Depreciable Assets	$30
Current Portion Long-Term Debt	$500	$450	**Total Expenses**		$39,350	–	Purchases Depreciable Assets	$100
Accounts Payable	$3,000	$4,000	**Earnings Before Interest and Taxes (EBIT)**		$660	**Net Cash Flow from Investment**		($70)
Accrued Liabilities	$958	$880	–	Less Interest Costs	$480	+	Change in Non-Current Long Term Debt	($57)
Total Current Liabilities	$5,958	$6,600	**Earnings Before Taxes (EBT)**		$180	+	Change in Current Portion of Long Term Debt	($50)
Non-Current Long Term Debt	$2,042	$1,985	–	Less Taxes	$72	+	Change in Notes Payable	($230)

TOTAL LIABILITIES	$8,000	$8,585	**Net Income After Taxes (NIAT)**		$108	– Less Dividends and Owner Draw	$287
Contributed Capital	$1,900	$1,900	–	Less Dividends and Owner Draw	$287	**Net Cash Flow from Financing**	($624)
Retained Earnings	$100	($79)	**Addition to Retained Earnings**		($179)	**CHANGE IN CASH POSITION**	($324)
Total Equity	$2,000	$1,821					
TOTAL LIABILITIES AND EQUITY	$10,000	$10,406					

To answer this question on an Excel spreadsheet that describes HQN's CFS, we press the [Data] tab and the [What If Analysis] button. Finally, we press "Goal Seek" in the drop down menu. Goal Seeks asks us to supply three pieces of information: the cell where the goal value is located, the numeric value for the variable identified in the goal cell, and the cell location of the variable we wish to change to achieve our goal. The number we wish to change must be an exogenous variable—not a calculated variable. We want to change the ROE located in cell H53 to a value of 9% by changing cash receipts of landscaping services located in cell D3 located on the exogenous variables page. We record this information in the Goal Seek menu below.

Figure 6.1. Goal Seek Pop-up Menu



Then we click [OK] and that find cash receipts from landscaping services must increase to $30,010 to earn an ROE of 9%. Furthermore, increasing cash receipts from landscaping services to $30,010 increases total cash receipts to $39,000. Changes in the endogenous variables included in system are described next in the activity column of the what if page.

Table 6.5. What If Analysis using SPELL Ratios.
Open Table 6.5 in Microsoft Excel

Ratios	Industry Average	Activity Ratios	HQN Base	Goal Seek
Solvency				
TIE	2.50	1.38	1.35	1.38
DSR	1.40	1.03	1.02	1.03

Profitability				
ROA	3.30%	6.60%	6.50%	6.60%
ROE	10.70%	9.00%	8.50%	9.00%
m margin	29.00%	0.45%	0.43%	0.45%
Efficiency				
ITO	7.7	10.67	10.67	10.67
ITOT	47.4	34.21	34.22	34.21
ATO	3.2	4.00	4.00	4.00
ATOT	114.1	91.23	91.25	91.23
RTO	11.41	24.40	24.39	24.40
RTOT	32	14.96	14.97	14.96
PTO	12.59	9.33	9.33	9.33
PTOT	29	39.11	39.12	39.12
Liquidity				
Current Ratio	1.30	1.06	1.06	1.06
Quick Ratio	0.70	0.43	0.43	0.43
Leverage				
Debt/Assets	0.91	0.80	0.80	0.80
Debt/Equity	2.00	4.00	4.00	4..00
Asset/Equity	2.20	5.00	5.00	5.00

Note that increasing CR from landscaping services to $39,000 resulted in an increase of ROE to 9.00%. Of course, there were other consequences. ROA increased to 6.6%. The TIE solvency ratio increased slightly from 1.35 to 1.38. These and other changes we observe by comparing the activity and goal seek columns with the HQN base column.

Creating Subsystems

It is important to be able to answer what if and how much questions, especially about such important endogenous variables as ROE in a smaller system than the CFS. We create subsystems by redefining the boundaries between exogenous and endogenous variables in the system.

To simplify the connections between exogenous and endogenous variables in systems, we define subsystems and ask what if and how much questions about the subsystems. We create a subsystem by redefining some endogenous variables in the system as exogenous variables. This permits us to examine the consequences of particular shocks on a reduced number of particularly interesting endogenous variables.

We can construct a large number of subsystems. However, we focus on the two subsystems that matter most to the firm: those that describe the firm's ROE and its solvency. To illustrate, suppose we wanted to build a subsystem around the firm's ROE. Letting letters represent system endogenous variables, we might begin by assuming that the firm sells each item of what it produces at an exogenously determined price p, that its marginal cost is c, its fixed overhead expenses (OE) cost is b, and that its interest costs are iD where i is the average cost of its debt and D is the sum of the firm's liabilities determined in the previous period. Finally, letting the number of physical units sold equal S, we define our ROE subsystem by assuming all other variables except ROE to be exogenous. We have now created an ROE simplified subsystem. We define Earning Before Taxes (EBT) in the subsystem as:

$$EBT = (p - c)S - b - iD. \tag{6.1}$$

Now we can write the ROE subsystem as:

$$ROE = \frac{EBT}{Equity} = \frac{(p-c)S - b - iD}{Equity} = \frac{EBT}{Total\ Revenue}\frac{Total\ Revenue}{Assets}\frac{Assets}{Equity} = m\ ATO\ EM \tag{6.2}$$

Having defined an ROE subsystem, we are prepared to ask what if questions such as: what would happen to the firm's ROE if we could increase the ATO by increasing cash receipts? Since our subsystem has defined all of the interdependencies, we can find the answer to this what if question by observing the change in the firm's ROE in response to changes in the system's exogenous variables. We illustrate the approach using HQN's data. Initially, HQN's ROE equals:

$$ROE = \frac{EBT}{Equity} = \frac{(p-c)S - b - iD}{Equity} = \frac{\$40,000 - \$39,350 - \$480}{\$2,000} = 8.5\%. \tag{6.3}$$

Suppose the value of the exogenous variable cash receipts increased to $40,100? The results on the firm's ROE can be found to equal:

$$ROE = \frac{EBT}{Equity} = \frac{(p-c)S - b - iD}{Equity} = \frac{\$40,100 - \$39,350 - \$480}{\$2,000} = 13.5\%. \tag{6.4}$$

Another subsystem might involve solvency and the TIE ratio. To analyze this subsystem, we begin with the simplified EBT subsystem defined earlier and remove interest costs to obtain earnings before interest and taxes (EBIT) equal to:

$$EBIT = (p-c)S - b. \tag{6.5}$$

Next we write a DuPont type equation focused on TIE equal to:

(6.6) E}\right). \end{split}\end{equation*}

where DE is the debt-equity leverage ratio. Having now defined a solvency subsystem reflected by the firm's TIE ratio, we can ask the following what if question. What if the firm increased its debt D? Then, what would be the effect on the firm's solvency? To answer this what if question, we substitute the simplified EBIT formula into Equation ??? to obtain:

(6.7) E}\right).\end{equation*}

To illustrate, we substitute HQN's data into Equation ??? to find its initial TIE value. Making the substitution we find:

$$TIE = \frac{ROA}{i}\left(1 + \frac{E}{D}\right) = \frac{6.5\%}{6\%}\left(1 + \frac{\$2,000}{\$8,000}\right) = 1.35. \tag{6.8}$$

Now suppose we ask: what if the firm's equity falls by $1,000? In response to this change in an exogenous variable, HQN's TIE ratio would decline to:

$$TIE = \frac{ROA}{i}\left(1 + \frac{E}{D}\right) = \frac{6.5\%}{6\%}\left(1 + \frac{\$1,000}{\$8,000}\right) = 1.22. \tag{6.9}$$

And what if the firm's interest rate increased by one percent to 7.0%? Then its TIE ratio becomes:

$$TIE = \frac{ROA}{i}\left(1 + \frac{E}{D}\right) = \frac{6.5\%}{7\%}\left(1 + \frac{\$1,000}{\$8,000}\right) = 1.16. \tag{6.10}$$

It is important to recognize that the answers to our what if questions answered in our subsystems are only approximations of what would happen if we considered the entire system. Nevertheless, the subsystem approach provides some useful intuitive explanations that may be disguised in a full system analysis.

So what have we learned? We learned that open systems like CFS require endogenous variables whose values are determined with the system and exogenous variables who values are determined outside the system. However, systems and subsystems are arbitrary constructs, and we can create open subsystems including one that describes the firm's ROE and TIE ratios by arbitrarily defining some exogenous variables.

Common Size Balance Sheets and Common Size Income Statements

When comparing financial statements across time and across firms, it is often useful to standardize the statements. The typical way this is done is by expressing all items in the balance sheet as a percentage of total assets and all items in the income statement as a percentage of total revenue. Common size balance sheets and common size income statements facilitate comparison across time and across firms because absolute size effects are eliminated by expressing numbers in the statements as percentages of the same whole.

One of the significant advantages of common size balance sheets and common size income statements is that they allow all items in either the balance sheet or income statement to be compared to an industry standard. There is one other thing, however, that happens when we convert balance sheets and income statements to ratios reflecting a percentage of the whole: all the variables in the statements become interdependent—an increase in one variable requires decreases in other variables in the statements because their sum must equal 100%. Furthermore, this requirement that they sum to 100% creates a type of closed system in which all the entries except one sum to 100%. This enables us to do trade-off analysis.

Consider what we can learn from common size balance sheet statements. Suppose that a firm's cash position was $100,000 at the end of 2017 and $110,000 at the end of 2018. In addition, suppose the firm's total assets were $2,000,000 at the end of 2017 and $3,000,000 at the end of 2018. The absolute value of the firm's cash position increased by $10,000 over the year which might suggest the firm is now more liquid. However, the firm's cash position must now support a larger amount of total assets. Looking at the cash position as a percentage of total assets, we find the firm's cash position was 5% of total assets at the end of 2017, and only 3.67% at the end of 2018. Thus, the amount of cash available per dollar of assets held by the firm actually decreased during the year. Common size balance sheets for HQN are presented in Table 6.7.

Table 6.6. Common Size Balance Sheets for HQN

	2016	2017	2018	Ind. Ave.
ASSETS				
Cash and Marketable Securities	12.13%	9.30%	5.77%	6.3%
Accounts Receivable	15.77%	16.40%	11.54%	26.4%
Inventory	31.85%	37.50%	50.00%	25.6%
CURRENT ASSETS	59.76%	63.20%	67.31%	58.3%
Depreciable long-term assets	33.06%	29.90%	26.92%	35.7%
Non-depreciable long-term assets	7.18%	6.90%	5.77%	6.0%
LONG-TERM ASSETS	40.24%	36.80%	32.69%	41.7%
TOTAL ASSETS	**100.00%**	**100.00%**	**100.00%**	**100.00%**

LIABILITIES				
Notes Payable	4.16%	15.00%	12.21%	13.9%
Current Portion LTD	7.085%	5.00%	4.33%	3.6%
Accounts Payable	24.27%	30.00%	38.46%	18.7%
Accrued Liabilities	8.80%	9.58%	8.46%	6.8%
CURRENT LIABILITIES	54.30%	59.58%	63.46%	43.0%
NON-CURRENT LONG-TERM DEBT	25.88%	20.42%	19.09%	13.4%
TOTAL LIABILITIES	80.18%	80.00%	82.55%	56.4%
Equity	19.82%	20.00%	17.45%	43.6%
TOTAL EQUITY	**19.82%**	**20.00%**	**17.45%**	**43.6%**
TOTAL DEBT AND EQUITY	**100.00%**	**100.00%**	**100.00%**	**100.00%**

Entries in HQN's common size balance sheet can be examined by comparing them with other firms in the industry described in the last column of Table 6.7. HQN's level of current assets is above the industry average and rising, primarily as a result of relatively high and increasing inventory levels. The accounts receivable levels are low in terms of the industry levels. Long-term asset levels are also low, relative to industry levels, and declining. Current liabilities are well above the average levels in the industry, and rising mostly as a result of increasingly high levels of accounts payable. Although falling, long-term debt is still above industry averages. Owner equity levels are well below the average firm in the industry.

Common size income statements. Entries in HQN's common size income statement can be examined by comparing them with other firms in the industry described in the last column of Table 6.8. Consider what we might learn from HQN's common size income statements. HQN's cost of goods sold (COGS) as a percentage of cash receipts in 2018 was close to the industry average. However, its overhead expenses (OE) were much higher than the industry average in both 2017 and 2018. As a result, its EBIT as a percent of cash receipts was much lower than the industry average. Furthermore, its interest costs as a percentage of cash receipts were almost double the industry average. HQN's high OE and high interest costs relative to the industry were somewhat mitigated by HQN's lower than industry standards depreciation and taxes. Still, HQN's net income after taxes (NIAT) as a percentage of cash receipts in 2018 is low compared to the industry average: 0.25% for HQN versus 2.28% for the industry.

"What if" questions and the common size financial statements. Common size balance financial statements are derived from the CFS. As a result they respond to changes in endogenous and exogenous variables that make up the common size financial statements. Furthermore, we can ask what if and how much kinds of questions of the CFS and observe their changes in the common size financial statements.

For example, suppose that HQN's cash receipts increased from $38,000 in 2017 to $40,000 in 2018. Meanwhile, suppose its COGS increased from $25,600 to $28,000. We may want to know if COGS increased in proportion to its increased cash receipts. From the common size income statement we see that as a proportion of its cash receipts, COGS increased from 67.37% in 2017 to 70% in 2018, suggesting that its COGS increased at a rate greater than its cash receipts—a result that should concern the financial manager.

Table 6.7. Accrued Common Size Income Statements for HQN

	2017	2018	Ind. Ave.
Total Revenue	100.00%	100.00%	100.00%
Cost of Goods Sold (COGS)	67.37%	70.00%	71.40%
Overhead Expenses	29.82%	27.50%	22.50%
Depreciation	1.24%	0.88%	1.75%
EARNING BEFORE INTEREST AND TAXES (EBIT)	1.58%	1.62%	4.35%

Interest	1.22%	1.20%	0.55%
EARNINGS BEFORE TAXES (EBT)	0.36%	0.42%	3.80%
Taxes	0.17%	0.17%	1.52%
NET INCOME AFTER TAXES (NIAT)	0.19%	0.25%	2.28%

Common Size Statements and Trade-offs

That entries in common size statements sum to 100% means that we cannot increase one variable in the statements without decreasing another variable in the statement. Therefore, any change in an exogenous variable that affects the proportion of that variable in the common size statement will have some offsetting impact on at least one other variable in the system. We may refer to these cause-and-effect changes in exogenous variables as trade-offs. There are various ways we can describe these trade-offs.

The Squeeze versus the Bulge. One way to examine trade-offs is to assume the financial system has some characteristics similar to a balloon. If a squeeze happens somewhere on the balloon, a corresponding bulge will occur somewhere else—because balloons require equal pressure on its surface. This balloon-like characteristic is evident in common size financial statements.

For example, suppose the firm wishes to increase its liquidity and so increases the percentage of its assets held as accounts receivable. However, if the percent of assets must add to 100%, increasing the percentage of short-term assets will require that the percentage of long-term assets decrease—and profitability and perhaps efficiency suffers.

CFS and trade-offs. Trade-offs are obvious within common size statements. They exist within the CFS system but may be less obvious. Some usual trade-offs are summarized in the table the follows. Consider the left-hand column as the "squeeze" and the right-hand column is a possible "bulge." However, the "squeeze" and "bulge" comparisons described below are only qualitative possibilities. To find out the quantitative connections, we must look at the firm's ratios and common size statements. Still, the principle is important: when analyzing the firm by examining a particular ratio, look for its companion ratio.

Table 6.8. The Squeeze vs. The Bulge

The Squeeze	**The Bulge**
leverage ratio (D/E): High	rate of return on equity (ROE): High
cash receipts/inventory ratio (ITO): High	cash receipts/accounts receivable (RTO): Low
cash receipts/inventory ratio (ITO): Low	profit margin (m): Low
current assets/current liabilities ratio (CR): High	rate of return on equity (ROE): Low
cash receipts/assets ratio (ATO): High	operating and repair: High
COGS/notes payable (PTO): High	interest costs: High

Companion ratios. We apply the principle of looking for interesting things in pairs to HQN. Is anything else unusual about HQN? Yes! Look at its inventories in the common size balance sheet: 50% of its assets in 2018 versus an industry average of 25.6%. We have found a squeeze. The bulge? Look at accounts receivable: 11.54% versus an industry standard of 26.4%. Does this suggest that the firm has adopted a stringent credit policy that has discouraged customers? Perhaps. It's an area the firm should likely explore. If HQN's stringent credit policy were indeed affecting cash receipts, then its inventory turnover ratio (ITO) would be affected, but this ratio isn't too far out of the ordinary: 10.67% versus the industry median of 7.7%. However, the upper quartile for the industry is 14.9%, suggesting a large variability for the industry. So, the firm's credit policy is not off the hook yet.

Anything else unusual about HQN? Well, yes. Its debt to equity ratio is 4.0 in 2018 versus the industry average of 1.9. Unfortunately for HQN, a high leverage ratio hasn't increased profits or rates of return as much as might be expected because of its low efficiency and possibly an ineffective cash receipts strategy. Continuing, if HQN has unusually high levels of debt relative to its equity, we should expect its interest payments to be above the industry average. They are 1.2% of cash receipts in 2018 versus an industry average of .55%. Already we are alarmed; high leverage is usually accompanied by high risk. One reason that high leverage implies high risk is that the firm's equity relative to its liability is small, and not able to handle and survive a market

reversal. Is HQN's equity low relative to the industry? Very much so: 17.45% in 2018 versus the industry average of 43.6%.

Trend Analysis

Using historical data, we attempt to look ahead to financial conditions likely to be experienced in the future. Consider the common size balance sheets reported earlier in Table 6.7. The first step is to look for any significant changes or trends in the asset or liability accounts. Current assets have increased over the three year period mostly due to a significant increase in inventory levels. Cash levels declined during the three years for the same reason—current assets being tied up in inventory. Long-term assets have fallen primarily as a result of declining values of the firm's property, plant, and equipment. The worrisome result of this trend is that it may project increased maintenance costs associated with aging machinery.

On the debt side of the balance sheet, current liabilities have increased during the three year period mostly as a result of increases in accounts payable. Long-term debt has declined, and owner equity has remained relatively constant. The question associated with this trend is, can increasing dependence on notes payable be sustained? Are there less expensive sources of financing available?

Examining HQN's common size income statement, reported in Table 6.8, we see that both EBIT and NIAT increased in 2018 as a percentage of cash receipts. Comparing HQN's income statement with other firms in the industry, we note that HQN's EBIT and NIAT were low compared to industry averages, primarily as a result of relatively high overhead expenses and interest expenses. High OE, COGS, and interest costs have reduced HQN's taxes.

The statement of cash flow in Table 6.9 confirms that cash levels in the firm have been declining. However, cash flow from operations was positive in both 2017 and 2018. The firm had positive income and additional cash flow was generated by depreciation expense and increases in current liabilities. A large cash inflow was also received in 2017 as a result of a decrease in accounts receivable. One salient point is that the firm sacrificed significant amounts of cash in 2017 and 2018 to increase inventory levels.

Table 6.9. HQN's Coordinated Financial Statements for Year 2018 after a cash receipts change in an exogenous variable

BALANCE SHEET			ACCRUAL INCOME STATEMENT			STATEMENT OF CASH FLOW		
DATE	**12/31/ 2017**	**12/31/ 2018**	**DATE**		**2018**	**DATE**		**2018**
ASSETS			+	Cash Receipts	$39,990	+	Cash Receipts	$39,990
Cash and Marketable Securities	$930	$1,600	+	Change in Accounts Receivable	($440)	–	Cash Cost of Goods Sold	$27,000
Accounts Receivable	$1,640	$1,200	+	Change in Inventories	$1,450	–	Cash Overhead Expenses	$11,078
Inventory	$3,750	$5,200	+	Realized Cap. Gains/Depr. Recapture	$0	–	Interest paid	$480
Current Assets	**$6,320**	**$7,000**	**Total Revenue**		$41,000	–	Taxes	$68
Property, Plant, and Equipment	$2,990	$2,800	+	Cash Cost of Goods Sold	$27,000	**Net Cash Flow from Operations**		$1,364

Other Assets	$690	$600	+	Change in Accounts. Payable	$1,000	+	Realized Cap. Gains + Depr. Recapture	$0
Long-Term Assets	**$3,680**	**$3,400**	+	Cash Overhead Expenses	$11,078	+	Sales Non-depreciable Assets	$0
TOTAL ASSETS	**$10,000**	**$11,400**	+	Change in Accrued Liabilities	($78)	–	Purchases Non-depreciable Assets	
LIABILITIES			+	Depreciation	$350	+	Sale Depreciable Assets	$30
Notes Payable	$1,500	$1,270	**Total Expenses**		$39,350	–	Purchases Depreciable Assets	$100
Current Portion Long-Term Debt	$500	$450	**Earnings Before Interest and Taxes (EBIT)**		$1,650	**Net Cash Flow from Investment**		($70)
Accounts Payable	$3,000	$4,000	–	Less Interest Costs	$480	+	Change in Non-Current Long Term Debt	($57)
Accrued Liabilities	$958	$880	**Earnings Before Taxes (EBT)**		$1,170	+	Change in Current Portion of Long Term Debt	($50)
Total Current Liabilities	$5,958	$6,600	–	Less Taxes	$68	+	Change in Notes Payable	($230)
Non-Current Long Term Debt	$2,042	$1,985	**Net Income After Taxes (NIAT)**		$1,102	–	Less Dividends and Owner Draw	$287
Contributed Capital	$1,900	$1,900	–	Less Dividends and Owner Draw	$287	**Net Cash Flow from Financing**		($624)
Retained Earnings	$100	$915	**Addition to Retained Earnings**		$815	**CHANGE IN CASH POSITION**		$670
Total Equity	**$2,000**	**$2,815**						
TOTAL LIABILITIES AND EQUITY	**$10,000**	**$11,400**						

Cash was used each year to increase investments in long-term assets. However, the amount of assets used each year was significantly greater than the amount reinvested in long-term assets. For example, in 2018 the firm used up $350 in assets (the

amount of depreciation expense) and only had a net investment of $70,000, a depletion of $280,000 in the firm's assets.

Cash was also used each year in the firm's financing activities. The firm made a large payment of long-term debt in 2017. Then in 2018, a large equity withdrawal was made. Both of the payments were larger than the cash flow produced out of the firm's operations.

Common Size Statements and Pro Forma Financial Statements

Consider how financial managers can use financial ratios to forecast the financial condition of the firm by asking what if questions whose answers produce pro forma balance sheet and income statements. Financial forecasting is a method used by firms to help plan for future financial needs. However, using pro forma income statements and pro forma balance sheets both require that each one is defined as a subsystem.

Pro forma income statements and balance sheets are forecasts of what these statements will look like in the future and provide essential planning information. There are several ways to construct pro forma statements. The usual technique is to select a key variable and predict its future value. Then assume constant SPELL ratios which include the key variable and solve for other values using other ratios. We demonstrate this approach in what follows.

Assume that HQN wants to achieve a projected level of cash receipts. Also assume that SPELL ratios will remain constant even in the face of projected total revenue increases. Specifically, assume that next year's projected total revenue for HQN will equal $42,000, an increase of 5%. What does this imply for HQN's level of inventory? From HQN's financial ratios, we see that the inventory turnover ratio was 10.67%. Assuming that this year's ratios will hold next year, we can use the projected level of cash receipts to forecast the pro forma levels of inventories (INV). The first step in these types of problems is to write out the definition of each ratio and their assumed values:

$$ITO = \frac{total\ revenue}{INV} = \frac{\$40,000}{\$3,750} = 10.67\%. \tag{6.11}$$

From the inventory turnover (ITO) equation we find:

$$INV = \frac{total\ revenue}{ITO} \tag{6.12}$$

Next we use the projected cash receipts level of $42,000 and divide by ITO to find projected HQN's inventory in 2019:

$$INV = \frac{total\ revenue}{ITO} = \frac{\$42,000}{10.4} = \$4,038.46. \tag{6.13}$$

But now, we observe an interesting result, inventory increased by 5% to $4,038.46, the same percentage increases as was projected in total revenue. This result, of equal percentage increases, occurs whenever the ratio is held fixed. When one number of the ratio is increased by some percent, the other number in the ratio must increase by the same percent. To illustrate, consider the m ratio:

$$m = \frac{EBIT}{total\ revenue} = \frac{\$170}{\$40,000} = .00425 \tag{6.14}$$

If the *m* ratio is constant and cash receipts increases to $42,000, then EBIT will increase by 5% to $178.50. Again, the technique is to assume that the historical financial relationships will hold in the future and then project the future value of one variable, usually cash receipts. This allows us to calculate the projected values of the remaining financial variables based on the historical financial relationships. Of course, we could create pro forma income statements and balance sheets by increasing all the variables by some common percent, or just the exogenous variables by the common percent. However, increasing all the exogenous variables by a common percent is confusing, and it is highly unrealistic. Instead, we will suggest the change in an exogenous variable analysis approach described next.

Summary and Conclusions

We began this chapter by describing the CFS as a system in which changes in one part of the system affected other parts of the system. We reminded ourselves that while ratios can describe the firm's strengths and weaknesses, they cannot answer what if and how much kinds of questions unless they are embedded in a system.

Then, we faced the fact that the world is a really complicated system and we can't learn much about it without creating subsystems that assume or define some endogenous variables as exogenous variables. In effect, we create subsystems from systems by reducing the number of endogenous variables and increasing the number of exogenous variables—allowing us to describe a subsystem within a system with a reduced number of endogenous variables.

We found our subsystems to be useful because they allow us to understand the connections between some of the most important parts of the financial system, such as its rates of return and its solvency. While there are a large number of subsystems we could create and examine, we emphasized the firm's TIE ratio and its ROE ratios.

Another important concept emphasized in this chapter was that the nature of the system implies that changes in exogenous variables produce consequences on more than one endogenous variable, and these changes in more than one endogenous variable can be described as trade-offs. Trade-offs are one way to answer what if questions. For example, what if the changes in an exogenous variable were increased cash receipts? Then what is the effect on COGS, ROE, or other endogenous system variables? Several of these important trade-offs were described in this chapter. We also answered how much questions with the help of Excel's goal seek. How much questions help us find the requirements for reaching specific financial goals.

Finally, we recommended that we examine changes in exogenous variables in the context of the CFS system. And to practice, we suggested learners consider several scenarios. One way to practice finding the impacts of change in an exogenous variable was an exercise we referred to as what if analysis or how much analysis. This chapter provided several scenarios that could be examined using what if and how much analysis—which is the essence of opportunities and threats analysis.

Questions

1. Describe a system, one that is different than the financial system described in this chapter.
2. In this chapter, we treated taxes as an exogenous variable in the CFS system. Describe how you would convert taxes from being an exogenous variable to an endogenous variable.
3. Perform a what if analysis by describing how an increase in NIAT would influence other endogenous variables in the CFS system.
4. Refer to Table 6.9. Then, list the endogenous and exogenous variables in the end of period Balance Sheets, the Income Statement, and the Statement of Cash Flow.
5. Compare HQN's common size balance sheet at the end of years 2016, 2017, and 2018. Notice that cash and marketable securities were well above the industry average in 2016 and then declined in both 2017 and 2018 to a level below the industry average. By referring to other ratios in the common size balance sheet, explain what changes in the firm likely account for HQN's decline in liquidity.
6. Refer to Table 6.1. Then, observe that HQN's account receivables declined from 2017 to 2018. What effect did this decline have on HQN's liquidity? What effect did the decline have on the firm's cash and marketable securities?
7. Refer to Table 6.1. Then, observe that HQN's current liabilities increased from 59.58% in 2017 to 63.46% in 2018. In all years, current liabilities were well above the industry average of 43%. What changes at HQN's may account for the rise in current liabilities as a percentage of total assets? Is this change a strength or weakness of the firm? Provide a possible explanation for the HQN's change in its current liabilities.
8. Focusing on HQN's common size income statement in Table 6.7, observe that overhead expenses were increasing between 2017 and 2018 while depreciation was low. Are the two connected? Can you connect this change to changes in the firm's long-term assets in the common size balance sheets?
9. The common size income statement's base is cash receipts. Recalculate the common size income statements for 2017 and 2018 using COGS as the new base. What is the effect of changing the base in the calculation of the common size income statement?
10. In Table 6.7, NIAT is especially low compared to the industry average. Can you explain why?
11. Choose three of the ten scenarios described in the text. Then do the following opportunities and threats procedures. First, perform a what if exercise that includes the following steps. Use the Excel worksheet supplied in class and perform what if and how much exercises consistent with the conditions described in the scenario. Identify the responses to the changes in an exogenous variable in financial statements. Recognize that some connections may require more than one exogenous

variable be changed. Then recalculate the affected ratios and the common size balance sheet and income statements. Finally, write a brief opportunities and threats report about how the conditions described in the scenario would change the firm's opportunities and threats.

12. Compare HQN's 2017 and 2018 common size balance sheet in Table 6.6 with the industry average. Based on these comparisons, what are some financial strengths and weaknesses of HQN compared to the industry?
13. Compute a pro forma income statement for HQN for 2019. Assume cash receipts equals $42,000 and the relationships described by the common size income statement for 2018 are maintained.

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7: Homogeneous Measures

Learning Objectives

At the end of this chapter, you should be able to: (1) properly construct a present value (PV) model; (2) understand the need for homogeneous measures when building PV models; and (3) describe PV model dimensions that require homogeneous measures.

To achieve your learning goals, you should complete the following objectives:

- Define PV models and describe their uses.
- Learn how to compare challenging and defending investments.
- Learn how to convert a challenging investment's future earnings and costs to their value in the present.
- Learn how to represent the cost of sacrificing a defending investment by using its internal rate of return (IRR).
- Learn about PV model dimensions that require homogeneous measures to accurately and to consistently compare investments.

Introduction

Firm managers either continue their commitment to an existing investment called a defender or disinvest in the defender and commit to a new investment called a challenger. The financial manager's assignment is to analyze and to rank defenders and challengers. What adds complexity to the ranking process is that defenders and challengers are sometimes measured in different units. These differences in measures between challenging and defending investments may result in unstable and inconsistent rankings when more than one ranking method is used. This chapter intends to describe present value (PV) models that consistently and accurately rank defending and challenging investments using homogeneous (same) measures.

What is a Present Value (PV) Model?

A PV model is a mathematical expression that represents the value of future cash flow in present dollars. The present value of future cash flow earned by a challenging investment exchanged at the discount rate equal to the defender's IRR is called net present value (NPV). The NPV of the future cash flow of an investment discounted by its own IRR is zero.

To rank uniquely a defending and challenging investment requires that we reduce their current and future earning to a single number in the same period. To make this point, that ranking investments uniquely requires a one-dimensional measure, assume that we determine the winner and loser of a sporting event by several different measures. For example, suppose that the winner of the Super Bowl football game depended on the following measures: points earned, yards gained, yards earned on the ground divided by the yards earned passing, yards penalized, injuries sustained, and the number of persons viewing the contest. Most football sports fans would agree that the success measures just described matter—but we will never determine who wins and who loses with such multidimensional measures unless in some rare event one team dominated in all dimensions. So, we must decide which measure matters most and in the case of football—the measure that matters most is points earned.

To avoid the indecisiveness of a multi-dimension metric when ranking investments, we convert future cash flow to the present creating a one-dimensional number that allows us to rank investments uniquely.

The Need for Homogeneous Measures

The idea that future cash flow can be valued by its worth in the present period may be one of the most important and pervasive concepts in financial management. It is the basis for ranking physical investments and for valuing bonds, stocks, insurance, pension funds, housing, land, cars—and almost anything that has more than one period of economic life generating cash flow.

We might have hoped that converting investments' future cash flow to their value in the present would produce one agreed on method for valuing investments. It has not—because PV models have several dimensions and PV models have only resolved

differences in measurement for one dimension—time. Other dimensions such as investment size, term, loan terms, taxes, measures of return, liquidity, and risk also require that we measure them using the same—homogeneous—measures before we can rank investments consistently and accurately.

To make the point that a lack of homogeneous measures may create ranking conflicts, compare ranking investments to a horse race. We organize horse races so that other factors besides the horses' speed do not influence the horse race outcome. For example, we expect that only horses enter the race. We require that the horses all begin the race at the same time and place. We expect that all horses will run the same distance—that implies that not only has the starting point been determined but the finish line as well. Finally, we expect that everyone agree that the criterion for ranking horses is the time interval between when each horse starts and finishes the race.

Ranking investments is, of course, not a horse race. Yet, the process of ranking investments according to their earnings and horses according to their speed have many elements in common. First, we assume that we are comparing investments of the same size (only horses run the race). We also assume that we are comparing investments over the same time-period, term, (the length of the race is the same for all horses). Finally, we assume that the criterion used to rank investments (present value) would provide the same rankings for each investment just as the time required to run the race would be used to rank the speed of the horses.

In the remainder of this chapter and in the following chapters, we focus on several PV model dimensions that must be measured the same, homogeneously, so that our investment rankings will be consistent and accurate.

Homogeneous Time Measures

The first and most important homogeneous measure requires that investment cash flow from defenders and challengers be measured in present dollars. Creating homogeneous time measures by discounting future cash to flow to their value in the present is in essence what PV models achieve. They convert future cash flow to equivalent values in the present. But how do we find the present value of future cash flow? The key is to recognize that prices, including the price of future dollars valued in the present, are ratios. Ratios tell us the per unit value of what is in the denominator. For the ratio A/B, the ratio tells us the number of units of A for one unit of B.

To illustrate, consider the price of a tank of gas. To calculate the price per gallon of gasoline, we divide the money paid for a tank of gas (A) by the gallons of gas purchased (B). The ratio tell us the per unit price of a gallon of gasoline. If the ratio (money paid)/(gallons of gas purchased) were ($20) / (10 gallons) = $2.00, we would say that the price of a one gallon of gasoline is $2.00.

Exchanging future dollars for a present dollar. Now consider a special kind of ratio that describes the rate at which we exchange present dollars (A) and future dollars (B) between time-periods. We exchange present and future dollars every time we lend or borrow money, make or liquidate an investment, or invest in or withdraw money from a retirement account. To illustrate, suppose I borrow (lend) present dollars V_0 (A) now and in one year from now, I repay (receive) future dollars R_1 (B) where the subscript indicates the end of the time-period in which the cash flow occurs. The result of the division (A/B) is equal to (1 + *r*) where r is expressed as a decimal because the dollar units in the ratio cancel. Thus, the ratio of A/B or future dollars to present dollar is equals to 1 plus *r* percent:

$$\frac{R_1}{V_0} = (1 + r) \tag{7.1}$$

Using our convention for describing ratios, we would describe Equation 7.1 as follows: (1 + *r*) future dollars can be exchanged for one present dollar or that a present dollar is compounded to its future value at the rate of r percent.

To illustrate, suppose V_0 equals $100 and R_1 equals $110. In this example the ratio of future to present dollars is: $110/$100 = 110%, and we could say that one present dollar can be exchanged for $1.10 future dollars. Sometimes we simply say that one dollar in the present can be compounded to 1.10 future dollars.

Were we to describe the ratio of future dollars for present dollars, we would say $1.00 future dollar can be exchange for 1/(1 + *r*) present dollars.

$$\frac{V_0}{R_1} = \frac{1}{(1 + r)} \tag{7.2}$$

To illustrate, suppose V_0 equals $100 and R_1 equals $110. In this example the ratio of present to future dollars is $100/$110 =

$0.91 and we could say that one future dollar can be exchanged for $0.91 present dollars. Sometimes we say that one dollar in the future can be discounted to 0.91 present dollars.

At this point, it may be useful to explain the use of the variable r. It appears frequently in PV models that involve comparing money across time. In its most general use, it is a percent or a rate. It could be the market rate of interest. It could be the rate of interest earned on one's assets or equity. Or it could be the rate charged on loans, of which there are several. We will try to make it clear which rate r refers to and in some cases add a superscript to *r* to clarify. For example, the market rate of interest is defined as rm. If we want to refer to r in a particular period, we would subscript it with the appropriate time-period. The rate *r* in the tth period could be written as r^t.

Exchanging present dollars for a future dollar over more than one period. If we know the exchange rate between present and future dollars, then it is a small step to convert present dollars to their equivalent in the future or to convert future dollars to their equivalent in the present. We simply multiply by the appropriate ratio. To convert present dollars to their future value we multiply by (1 + *r*).

$$V_0(1+r) = R_1 \tag{7.3}$$

To convert future dollars to their value in the present we multiply by 1/(1 + *r*).

$$V_0 = \frac{R_1}{(1+r)} \tag{7.4}$$

This last ratio, the present value of future cash flow, is of particular interest. We find this ratio to be of special interest because, as was mentioned earlier, we live and make decisions in the present. Thus, converting future cash flows to their equivalent present value makes it possible to evaluate capital budgets with future consequences in terms of present dollars.

To summarize, using numbers from our previous example where *r* = 10% or .1, the future value of $100 is $100(1.1) = $110. The present value of $110 future dollars is $110[1/1.1)] = $100.

Completing the gasoline purchase analogy, if the price of gasoline were $2 per gallon, we might ask how much will 10 gallons of gasoline cost? Ten gallons times $2 is equal to $20. Or we might ask how many gallons of gasoline can I purchase for $20? Twenty dollars divided by $2 is equal to 10 gallons.

To illustrate the importance of calculating the present value of future cash flow, suppose that a "down-on-her-luck friend" approaches you. She explains that her wealthy aunt has promised her $100 in one year, but she needs the money now. She asks you what you would offer in present dollars in return for her future $100 dollars. You quickly calculate (assuming an exchange rate of 110%), and report that the present value of $100 future dollars (received in one period from now) is $90.91. Or, we might say that the discounted present value of $100 future dollars is $90.91.

Just as we found the present value of one period in future dollars, we can also find the present value of future cash flows received in two or more periods in the future. Returning to our earlier example, suppose this same down-on-her-luck friend offers you R_1 dollars in one period and R_2 dollars two periods in the future and asks how many present dollars would you offer for the exchange if the exchange rate is 1 + *r*.

We can find the present value of R_2 in two steps. First, convert R_2 to its equivalent value in period-one dollars by using the ratio [R_2/(1 + *r*)]. The important point is that the exchange rate is assumed to be (1 + *r*) between any two periods, including between periods one and two. Next, discount period one dollars to their equivalent in the present:

$$V_0 = \frac{\left[R_1 + \frac{R_2}{(1+r)}\right]}{(1+r)} = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} \tag{7.5}$$

We would follow a similar procedure to find the present value of dollars received in three periods from the present. Suppose you were offered $100 for the next three periods. What is the present value of these future cash flows if the exchange rate were 110 percent between any two periods? The answer is:

$$V_0 = \frac{\$100}{1.1} + \frac{\$100}{1.1^2} + \frac{\$100}{1.1^3} = \$90.91 + \$82.64 + \$75.13 = \$248.68 \tag{7.6}$$

In the remainder of this book, models which evaluate financial strategies by converting future cash flows to their equivalent in

the present are referred to as PV models.

So what have we learned? We learned that we can find the value of future dollars in the present by discounting them using one plus the appropriate discount rate *r*. Furthermore, we can find the value of present dollars in the future by compounding then using one plus the appropriate discount rate *r*. All this assumes, of course, that the exchange rate of dollars between time periods is a constant *r*.

Homoegenous Measures and Cash Flow

The second homogeneous measure required when building a PV model is to represent economic activities of the firm by their cash flow. In our horse race analogy, the cash flow principle is equivalent to letting only horses run the race. Some activities of the firm we characterize as noncash flow such as appreciation (depreciation) of assets, increases in inventories of unsold goods, and increases in accounts receivable and payable. But these events are not included in PV models because they do not produce cash flow.

One justification for the cash flow principle is that, at some point, we expect all economic activities to generate cash flow. At some point, we expect inventories to be liquidated and generate cash. At some point, we expect accounts receivable and accounts payable to be settled for cash. At some point we expect long-term assets that have appreciated (depreciated) to be sold and the difference between their acquisition and sale price to capture the noncash flow of depreciation (appreciation). Thus, in effect, we do count all economic activities of the firm by recognizing only cash flow; however, we count them only when they create cash flow.

Defenders and Challengers and Homogeneous Measures

The defender's opportunity cost. An opportunity cost is a benefit, profit, or value of something that must be given up to acquire or achieve something else. A PV model compares the returns from a defender that must be sacrificed to acquire the challenger with the returns from the challenger. Both the opportunity costs of the defender and the returns from the challenger must be measured in homogeneous units.

To represent what we sacrifice by liquidating the defender to purchase the challenger, we find the defender's internal rate of return (IRR) *r* that equates the defender's discounted future earnings to its liquidated value in the present. We represent this trade-off in Equation 7.7. In this equation, $R_t t = 1, \ldots, n$ are the defender's cash flow in period t if it were not liquidated. The variable S_n is the defender's liquidation value assuming it would be held for n periods. And, V_0 is the present value of the defender's future cash flow and liquidation value exchanged for present dollars at rate *r*:

$$V_0 = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} + \frac{S_n}{(1+r)^n} \tag{7.7}$$

The discount rate *r* is the opportunity cost of the sacrificing the defender to acquire the challenger measures as a percent. Another way to describe the defender's opportunity cost is to describe it as the exchange rate between the defender's present and future dollars.

Prices are not opportunity costs. The price of a good is the amount of money, or money equivalent, paid to obtain it. The amount of money or money equivalent paid to obtain the good represents the direct cost of what is given up to obtain something desired or to avoid something disliked. However, there may be other costs to acquire a good—or to avoid a bad—besides the price paid. For example, one might consider the cost of attending a movie to be the price of the ticket. However, transportation costs to and from the theater could add to the actual value of what must be exchanged to attend the movie. And there may be other costs of attending the movie such as lost earnings as a result of missing work.

Suppose the movie was playing at the same time that the moviegoer ate his or her prepaid dinner meal. Now the cost of the movie includes not only the price of the ticket plus transportation costs and lost wages but also the value of the skipped meal. The cost of the ticket, transportation costs, lost wages, and the skipped meal together represent the opportunity cost of attending the movie, which is different from the price of the movie ticket.

So what have we learned about opportunity costs? If we consider only the prices paid for goods and services as costs, we may underestimate true costs, the opportunity costs. When we consider attending a movie, making an investment, or taking out a loan, we should be careful to measure the opportunity costs of these investments. These opportunity costs will be expressed as the

rate at which we exchange present dollars for future dollars.

Opportunity costs and perfect capital markets. The capital (or financial) market is where people trade today's present dollars for future dollars and vice versa. In a perfect capital market, dollars trade between adjacent time-periods at the (same) market rate of interest r^m. For a perfect capital market to exist, the following conditions are required: no barriers to entry; no participant can influence the price; transactions are costless to complete; relevant information about the market is widely and freely available; products and services are homogeneous; no distorting taxes exist; and investment possibilities are continuously divisible. Finally, the firm's opportunity cost of capital is the same regardless of the size or purpose of the amount being borrowed or lent.

Though markets for some financial investments are considered highly efficient, they are not perfect. Rates of return on savings rarely equal the rate paid to borrow funds. Moreover, rates of return on investments typically depend on the size and economic lifetime of the investments. Hence, in the real world, investors face imperfect capital markets. We allow for imperfections in capital markets in our PV models by allowing the rate of return on an investment being considered for adoption, a challenger, to differ from the rate of return on an investment that must be sacrificed to adopt the challenger, a defender

Defenders may include investments that must be liquidated, investments that must be foregone, or credit reserves (unused borrowing capacity) that must be exchanged for debt funds. Thus, an interest rate on a loan equals the opportunity cost of capital on a defender only if the credit reserve used up has no value. In imperfect markets, the opportunity cost of the defender and the market rates of interest on loans are rarely equal. Since PV models focus on retaining the defender or acquiring the challenger, deciding between them requires that they be measured in the same homogeneous measures. Indeed, as we will show in later chapters, one reason why PV models may rank challengers and defenders inconsistently is because they fail to measure them in homogeneous measures. This point is so important that we will discuss it in more detail in later chapters. The focus on homogeneous measures leads us to consider other principles that will guide us when constructing PV models.

Homogeneous Rates of Return Measures

Another homogeneous measure is the rate of return measures used in PV models. The homogeneity of returns measure requires that if the return to the challenger is measured as a return to equity invested, then the discount rate that measures the defender's opportunity cost must also measure the defender's return on equity invested. If the return to the challenger is measured as a return on the investment, then the discount rate that measures the defender's opportunity cost must also measure the defender's return on the investment. The return on the investment (ROI) corresponds to the return on assets (ROA) when describing a firm. To avoid confusion we will describe the rate of return on equity in the investment or in the firm as ROE. And we will describe the return to the firm's assets or investment as ROA.

So how do we build PV models that homogeneously measure returns on assets, ROAs, and returns on equity, ROEs? We begin by finding ROAs. When finding the ROA for the defender, we ignore cash flow that includes borrowing or lending activities. The advantage of this approach is that it measures the rate of return on assets independent of the returns from the loan used to finance the investment. We write the defender's cash flow and its corresponding IRR that measures ROA as:

$$V_0 = \frac{(R_1 + S_1)}{(1 + ROA)} \tag{7.8}$$

Where V_0 is the initial investment, R_1 is net cash flow earned in the first period, and S_1 is the liquidation value of the investment after one period. Then we write the ROA as:

$$ROA = \frac{R_1 + (S_1 - V_0)}{V_0} \tag{7.9}$$

Interpreted, Equation 7.9 equates ROA to cash returns R_1 plus the cash value of capital appreciation (depreciation) equal to the difference between the liquidation value and the beginning value of the asset all divided by the beginning value of the asset. It measures the rate of returns on all of the firm's assets including returns generated by human capital, manufactured capital, social capital, natural capital, and financial capital.

Why would we want to exclude earnings from debt capital while including earnings from other forms of capital? To do so would overstate or understate the earnings generated by the other forms of capital. This is the argument for accounting for cash flow associated with debt capital, an approach consistent with the calculation of the return on equity. To measure the rate of return on the firm's equity, we pay the cost of debt used to finance the firm's assets—netting out the contributions of debt capital. We calculate the firm's rate of return on its equity (ROE).

To include cash flow associated with debt capital, assume that an asset is acquired using debt D_0 plus equity E_0 whose sum is equal to V_0. At the beginning of the project, the firm supplies equity capital to purchase the investment creating a negative cash flow of E0 dollars in the beginning period. Then it receives D_0 from the lender and pays D_0 to the seller to complete the purchase of the investment. The receiving and paying debt capital D_0 at the beginning of the period cancel out making net cash flow from debt capital at the beginning of the investment zero. At the end of the period, the investment repays debt capital D_0 plus interest iD_0 to the lender. Thus at the end of the period, cash flows of D cancel each other out. Thus, we can express these cash flow as:

$$E_0 = \frac{R_1 - iD + S_1 - D_0}{(1 + ROE)} \tag{7.10}$$

And we can write the ROE as follows:

$$ROE = \frac{R_1 - iD + S_1 - D_0 - E_0}{E_0} \tag{7.11}$$

We are interested in the relationship between ROA and ROE. To determine that relationship, we first solve for R_1 in Equation 7.9 and find that it equals:

$$R_1 = (ROA)V_0 - (S_1 - V_0) \tag{7.12}$$

Then we make the substitution for R_1 in Equation 7.11 and obtain the result:

$$ROE = \frac{(ROA - i)D_0}{E_0} + ROA \tag{7.13}$$

What Equation 7.13 reveals is that ROE > ROA as long as the firm earns a positive return on its debt capital (ROA > i) confirming an earlier result.

The cash flow principle requires that careful distinction be made between a cash transaction and a noncash transaction. Sometimes the distinction is not always clear. For example, an asset's book value depreciation does not itself generate a cash flow. Depreciation expenses of an investment do, however, generate a tax shield that creates a cash flow in the form of reduced tax payment. Thus, we include the cash flow associated with tax savings resulting from depreciation of an asset but not the depreciation.

Increased inventories of unsold goods do not create a cash flow. However, when the inventory is liquidated at the end of the period, it is converted to cash and enters in the present value calculations.

Total Costs and Returns Measures

All cash costs and cash returns associated with an investment should be included when determining an investment's present value. Consider how the total costs and returns principle is applied in several practical situations. Whenever low-interest loans or preferential tax treatments are tied to the ownership of a durable, these concessions will influence the present value of the investment. To ignore these benefits (costs) would lead to an under or over evaluation of the worth of the investment.

Sometimes an investment such as land has more than one source of returns. Mineral deposits, potential recreational use, and urbanization pressures may create expected returns over and above those associated with agricultural use. Pollution standards may impose costs in addition to those normally experienced. All these expected cash costs and returns that influence the value of the durable should be included in the PV model.

Other Homogeneous Measures

We require that cash flow associated with both the challenger and defender be measured in homogeneous units. Therefore, we have discussed the requirement that their returns both be measured in the present time period and that only cash flow be included when measuring economic activities of the defender and challenger. There are at least four other homogeneous measure requirement that are so important that we devote a chapter to each one.

- Homogeneous size measures (Chapter 9) require that initial and periodic investment sizes for challengers and defenders must be equal.

- Homogeneous term measures (Chapter 10) require that the defenders and challengers experience economic activity over equal terms.
- Homogeneous tax-rate measures (Chapter 11) require that we account for differences in effective tax rates for defenders and challengers that depend on cash flow patterns and capital gains (loses).
- Homogeneous profit measures (Chapter 12) require that we calculate rates of return in PV models and AIS following the same (homogeneous) methods.
- Homogeneous investment measures (Chapter 13) recognize that PV models may be constructed for an incremental or a stand-alone investment, and homogeneity of investment types requires that both the defender and challenger are either incremental or stand-alone investments.
- Homogeneous liquidity measures (Chapter 14) recognize that firms and investments can be differentiated by their liquidity and requires that the defender and challengers are measured using consistent liquidity measures.
- Homogeneous risk measures (Chapter 15) recognize that risk tolerance is different for individuals. Therefore, to measure risk in homogeneous units, we must account for influence of risk attitudes of certainty equivalent representations of risky investments.

So what have we learned? To compare the value of a potentially new investment, a challenging investment, with an investment to be sacrificed, a defending investment, we need to apply homogeneous measures to both. At least seven cash flow units associated with challengers and defenders must be homogeneous (the same) in order to construct consistent PV models: (1) initial and periodic investment sizes; (2) investment terms; (3) effective tax rates; (4) rates of return; (5) investment types; (6) liquidity; and (7) risk.

If the cash flow for a prospective challenger is measured in after-tax nominal values with no risk over n years with an initial investment of V_0 dollars, then the defender whose ROA or ROE is used as the discount rate when calculating the challenger's net present value (NPV) should be measured in similar units—in after-tax nominal values with no risk over n years with an initial investment of V_0 dollars.

Summary and Conclusions

The discount rate in a PV model represents the rate of return earned by a defender. It is the rate of return sacrificed by disinvesting in the defender to acquire the challenger. The defender's rate of return measures either its ROA or ROE. Choosing the appropriate discount rate for PV models describing a challenger is perhaps the most difficult task of investment analysis. The choice essentially involves the identification of challengers and opportunity costs associated with defenders.

The relevant costs in economic models have always been opportunity costs. These costs reflect what is given up when an alternative is chosen. This chapter has examined how the calculation of opportunity costs is influenced by time.

This chapter also discussed homogeneous measures that should direct the construction of PV models. They should measure cash flow in the present. We should count only cash flow when finding present values for challengers and defenders. And we should measure the defender's opportunity cost of capital and the challenger's cash flow as either ROA or ROE. We will devote entire chapters on special homogeneous measures such as homogeneous sizes, homogeneous terms, homogeneous tax rates, homogeneous investment types, homogeneous rates of return, homogeneous liquidity measures, and homogeneous risk measures.

Our goal is that by using homogeneous measures to construct PV models we can obtain consistent, stable, and accurate investment ranking. Knowing that one's investment rankings are stable, accurate, and consistent is essential for the success of a financial manager whose goal is to maximize the present value of the firm's investments.

Questions

1. Underlying present value (PV) analysis is the fundamental concept that a dollar today is not valued the same as a dollar received in the future. List several reasons that might explain why a dollar today may not be valued the same as a dollar in the future.
2. The idea that the value of a good today is not the same as the value of the good in the future is a universal concept. Suppose you were offered a 2018 Ford F-150 pickup truck for delivery today or the same truck in exactly the same condition delivered in 2023. What would you offer for the truck today versus what you would offer for the identical truck delivered in 2023? If the prices you would offer are different, please explain why?
3. Building on the concept of present and future dollars, suppose that you invest $100 today. Then assume that one year later

your investment returns $110 in cash.
   a. What is the present value of the future earnings?
   b. What is the future value of $100?
   c. What is the ratio that converts dollars received one period in the future to their present value?
   d. What is the ratio that converts present dollars to their value one period in the future?
   e. Interpret the ratios: $100/$110 and $110/$100 in terms of the problem described above. How would you use the two ratios? (Hint: what ratio would convert a present dollar to a future dollar and what ratio would convert a future dollar to its worth in the present?)
4. Suppose you are considering a ten-day spring break vacation. Transportation, lodging, and meals will cost about $2,300. Explain the difference between the cost of $2,300 and your opportunity cost of the vacation?
5. Suppose that you invest $100 today. Then assume that one year later your investment returns $110 in cash. Assume that your opportunity cost of capital is 12%.
   a. What is the present value of the future earnings?
   b. What is the future value of $100?
   c. What is the ratio that converts dollars received one period in the future to their present value?
   d. What is the ratio that converts present dollars to their value one period in the future?
   e. Compare the ratios: $100/$110 and $110/$100 in terms of the problem described in problem 3 and the ratios 1/1.12 and 1.12/1 described in this problem. What is the main difference between the problems and what difference does it make?
6. In a PV model, two investments are being compared, a defender and a challenger. Briefly distinguish between a defending and a challenging investment. Suppose you are considering replacing an aging orchard with new trees. What would be the challenging investment and what would be the defending investment? What consideration would you include when comparing a challenger and a defender? What are the possible decisions you might take in this investment problem?
7. In practice, financial managers face more complicated investment problems than deciding between a single defender and a single challenger. Describe how you might solve an investment problem with more than one challenger? One challenger but more than one defender? When choosing between multiple defenders, what would be the appropriate criterion for the preferred defender? How would you analyze a problem consisting of multiple challengers and defenders? What principle from micro economic theory might guide you in preparing your answer?
8. A financial manager believes that a defending investment can earn $500 for the next six years. He also believe that his defending investment can be sold today for $2,500. What is the opportunity cost of sacrificing the defender to invest in a challenger. In other words find the IRR of the defending investment.
9. Reconsider the defender in question 8 and suppose that the defending investment is financed by a $2,000 that does not require principle payment—only interest payments at the rate of 4% per year. Now find the rate of return on equity invested in the defender. Compare your results with those obtained in question 8 and explain the differences if they exist.

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8: Present Value Models

Learning Objectives

After completing this chapter, you should be able to: (1) identify the different kinds of present value (PV) models; (2) understand the types of questions different PV models can answer; and (3) construct PV models that represent defending and challenging investments.

To achieve your learning goals, you should complete the following objectives:

- Learn about the different kinds of PV models.
- Learn about the unique questions each kind of PV model can answer.
- Learn how to construct the following PV models:
  - net present value (NPV),
  - internal rate of return (IRR),
  - maximum (minimum) bid (sell),
  - annuity equivalent (AE),
  - break-even,
  - optimal life, and
  - payback.
- Learn how to represent the opportunity cost of the defender using its return on equity (ROE) or its return on assets (ROA).

Introduction

Picture yourself traveling across the country driving a luxury car or a tractor. Alternatively, picture yourself pulling a plow with a luxury car or a tractor. While both a luxury car and a tractor can provide transportation and pulling services, they are not equally suited for the two assignments.

One word economists use to describe activities a firm can perform but with unequal efficiency is comparative advantage. Luxury cars are perfectly suited for traveling across country but not for pulling a plow. A tractor is well designed for pulling a plow but not for long-distance travel.

Similarly, there are several different kinds of PV models, but they are designed for answering different kinds of questions. Each model has a comparative advantage for answering a particular kind of question. In what follows we examine some of the most important PV models and the questions they can be used to answer.

Net Present Value Models

In a static (timeless) model, profits describe the difference between costs and returns. In a PV model, an equivalent concept is called the Net Present Value (NPV). The NPV measures the difference in present dollars between cash outflows and cash inflows. NPV models reveal whether the present benefits of an investment outweigh its present costs. The important difference between NPV models and profit calculations is that profits may include non-cash returns and expenses such as increases in inventory and depreciation. The numbers that enter PV models besides the discount rate and the exponents on the discount rate of dollar are cash flow measures where the liquidation value of the investment at the end of its economic life is treated as though it were converted to cash.

Suppose an investment in a challenger requires an outflow of V0 dollars and generates a positive cash flow of R_1 dollars one period later plus the investment's liquidation value S_1. For now, assume that the discount rate, or the rate of exchange between present and future dollars for the defender, is r percent, and represents the opportunity cost of sacrificing the defender to invest in the challenger. To determine if the benefits of this investment outweigh the costs, we find the NPV defined as:

$$NPV = -V_0 + \frac{R_1 + S_1}{(1+r)} \tag{8.1}$$

Suppose a challenging investment costs \$100 and returns R_1 = \$100 and S_1 = \$20 in period one while the defending investment could earn *r* equal to 10%. Then the NPV of this investment would be:

$$NPV = -100 + \frac{\$100 + \$20}{1.1} = \$9.09 \tag{8.2}$$

For this investment, NPV is positive, and the present value of cash benefits outweighs the present value of cash costs. Another way to describe this result is that the challenger earned a higher rate of return than the defender. How much more? The exchange rate for the challenger is (\$100 + \$20)/\$100 = 1.20 compared to \$110/\$100 = 1.10 for the defender.

But what does it mean to say that the challenger earned a higher rate of return than the defender? An investment is a commitment you make to a project. If the project pays you more than the rate of return on the defender, then the investment's NPV is positive. In our example the project pays at the rate of 20%. In NPV models, think of the discount rate as the rate earned by the defending investment. And if NPV is positive, the challenger will earn a rate of return higher than could be earned by continuing to invest in the defender.

Of course, investments can be much more complicated than described above. For example, an investment may generate (positive and negative) cash flows for n periods (R_1, R_2, ⋯, R_n) rather than just one period. Furthermore, most capital assets generate a positive or negative liquidation value (S_n) at the end of its economic life which should be accounted for explicitly.

Using subscripts to indicate the time periods in which the cash flows occur, we write a more complete NPV model as:

$$NPV = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + + \frac{R_n}{(1+r)^n} + \frac{S_n}{(1+r)^n} \tag{8.3}$$

Internal Rate of Return (IRR) Models

Assume that the cash flow represented in Equation 8.3 are largely determined by the market or from other sources of information and are known. But where do we find *r*, the opportunity cost of capital, the rate of return earned on the defender? The answer is that we set the NPV of the cash flow that describes the defender equal to zero and solve for the discount rate which we then refer to as the defender's internal rate of return or its IRR. In such a model, the IRR will be equal to the defender's ROE or its ROA depending on the focus of the PV model. To make clear the distinction between cash flows associated with the challenging and defending investments we adopt the following notation: we superscript cash flow for the challenger with the lowercase letter "c" and cash flow for the defender with the lowercase letter "d".

$$NPV^d = -V_0^d + \frac{R_1^d + S_1^d}{(1+r)} \tag{8.4}$$

Now set NPV^d equal to zero. Such a PV model in which NPV is set equal to zero is called an Internal Rate of Return (IRR) model, and r is the rate at which dollars are exchanged between time periods for the defender. Because NPV is zero in this model, *r* is exactly the rate of return earned on the defending investment, the rate at which future dollars earned on the investment are exchanged for present dollars. Finally, the internal rate of return earned on the defender is what is sacrificed to invest in the challenger, and equals the opportunity cost in the NPV model. (The calculation of the defender's IRR in multi-period models can be much more complex.) To illustrate, we solve for the rate of return earned on the defender or the IRR in the previous model. The IRR equals:

$$r = \frac{R_1^d}{V_0^d} - 1 \tag{8.5}$$

Next substitute the defender's IRR into the NPV model of the challenger:

$$NPV^c = -V_0^c + \frac{R_1^c + S_1^c}{(1+r)} \tag{8.6}$$

If the NPV of the challenger is positive, the challenger is preferred to the defender. Why? Because exchanging future dollars for present dollars at the same rate of exchange as the defender leaves the investor with a positive net present value.

Maximum Bid (Minimum Sell) Models

Different kinds of investment questions created the need for different kinds of PV models. The maximum bid and minimum sell models assume *r* is known and NPV is zero. The models then solve for the purchase price of an investment in a maximum bid price model, or the sale price in a minimum sell price model. In a maximum bid model, the sale price that equates NPV to zero indicates how much the buyer can bid for the investment and still earn the IRR rate *r* on the defender. Or, from the seller's perspective, the minimum sell price is the lowest price a seller can accept in exchange for the cash flow stream generated by the investment and still earn the IRR rate *r* earned on the defender.

To illustrate, begin by assuming that *r*, the IRR of the defender, is known as well as the cash flow that can be earned by the challenger. Now evaluate the challenger by setting NPV equal to zero and finding the maximum bid (minimum sell) price V_0^c:

$$NPV^c = -V_0^c + \frac{R_1^c + S_1^c}{(1+r)} = 0 \tag{8.7}$$

The maximum bid (minimum sell) price is:

$$V_0^c = \frac{R_1^c + S_1^c}{(1+r)} \tag{8.8}$$

The Break-even Model

Assume that the investor knows the investment cost V_0^c, its liquidation value S_1^c, and the IRR of the defender *r* but doesn't know the amount of returns on the challenger R_1^c that would be required if the investment were to earn a rate of return equal to that available on the defender. The firm can find the amount required to return break-even earnings by setting the NPV model to zero and solving for R_1 as follows:

$$NPV^c = -V_0^c + \frac{R_1^c + S_1^c}{(1+r)} = 0 \tag{8.9}$$

and the break-even earnings amount is:

$$R_1^c = (1+r)V_0^c - S_1^c \tag{8.10}$$

If *r* were 10%, the liquidation value were zero, and the initial investment of V_0^c were $100, then the break-even return would be $100(1.1) = $110, the amount the investment would be required to earn in order to break-even. Break-even here has a specific meaning which is to earn the defender's IRR.

The Annuity Equivalent (AE) Model

An annuity is a financial product sold by financial institutions. The essence of an annuity is that an individual pays into a fund that is invested and grows until some point in time when the investment is paid back to the investor as a constant stream of payments for a specified period of time. In this book we define a related concept, an annuity equivalent. An annuity equivalent is a constant stream of payments whose present value is equivalent to some other stream of payments that may not be constant. The annuity equivalent model finds an annuity associated with an investment. An annuity is like a time adjusted average. Suppose we wished to find the annuity equivalent associated with the generalized NPV model below:

$$NPV^A = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} + \frac{S_n}{(1+r)^n} \tag{8.11}$$

Now consider an alternative model in which the NPV^A is equal to the NPV^A in Equation ???:

$$NPV^A = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} \tag{8.12}$$

The value for *R* in Equation ??? is the annuity equivalent cash flow that yields NPV^A in Equation ???.

Because annuity equivalents appear in so many contexts, including constant payment loans, retirement benefit payouts, and others, we adopt a notation to describe them. Factoring *R* in Equation ??? we write:

$$NPV^A = R\left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^n}\right] = RUS_0(r,n) \tag{8.13}$$

The notation $US_0(r,n)$ equals the bracketed expression in Equation ??? and stands for the sum of a uniform series of $1 payments discounted by a rate of *r* for *n* periods. Obviously, there is a direct correlation between NPVs and annuity equivalents, and both are used to rank investments.

To illustrate, suppose that NPV were $10,000, monthly installments were for 5 years, and the discount rate were 3 percent. The AE for this investment is equal to $179.69 and $_{12,60)}$ = 55.65

Optimal Life Models

Optimal life models ask what is the optimal life of this investment? The optimal life model can be written as:

$$NPV = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} + \frac{S_n}{(1+r)^n} \tag{8.14}$$

or in continuous time as:

$$NPV = -V_0 + \int_0^n R(t)e^{-rt}dt + S_n e^{-rn} \tag{8.15}$$

The optimal solution has a specific meaning in the context of the optimal life model. It is that value of *n* that maximizes the NPV. Formally, the solution employs calculus to optimize the NPV. In the discrete models which are most often employed in practice, the optimal value is found through trial and error or through repeated calculations of alternative values for *n*.

A related optimal life model asks: what is the optimal value of n that maximizes NPV if there are replacements for the investment? In this case, NPV^S is the sum of NPVs from individual investments. Such a replacement model is described below:

$$NPV^S = \left[-V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} + \frac{Sn}{(1+r)^n}\right] \left[1 + \frac{1}{(1+r)^n} + \frac{1}{(1+r)^2 n} + \cdots\right] \tag{8.16}$$

The Payback Model

While PV models are generally carefully deduced and the data required to solve them is explicit, sometimes decision makers just want a “ball park” estimate of the desirability of a financial strategy. In such cases, decision makers are willing to sacrifice rigor and precision for approximations. When this is the case, the payback model is often employed. To obtain an approximation, it assumes the discount rate in PV models is zero. In other words, the payback model assumes that present and future dollars are valued the same—a very unrealistic assumption. Then the payback model calculates the number of periods required to earn the investment’s present value. The number of periods required to earn the investment’s value is the payback period, the criterion used to rank investments. All cash flow after the payback period is assumed to have no influence on the criterion. To illustrate, *n* is the payback period in the payback model that follows.

$$V_0 = R_1 + R_2 + \cdots + R_n \tag{8.17}$$

And if periodic cash flows are constant we can express the payback period as:

$$Payback\,period = \frac{(initial\,investment)}{(periodic\,cash\;flows)}$$

To illustrate, consider an initial outflow of $5,000 with $1,000 cash inflows per month. In this case, the payback period would be 5 months. If the cash inflows were paid annually, then the result would be 5 years. More generally, cash flows will not be equal to one another. If $10,000 is the initial outflow investment, and the cash inflows are $1,000 at year one, $6,000 at year two, $3,000 at year three, and $5,000 at year four, then the payback period would be three years, as the first three years are equal to the initial outflow.

Despite its popularity, the payback model is not recommended for several reasons. Mainly, ignoring the time value of money basically treats an inter temporal investment as though it were a static profit problem. Furthermore, it treats cash flows earned after the payback period as though they were of no worth. In sum, in many respects it inadequately accounts for important elements of the investment problem.

Ranking Investments

Questions PV models answer. It may be helpful to categorize PV models into similar types by recognizing the questions they can answer. The first type answers the question: what is the break-even price such that NPV is zero. Included in this category are maximum bid (minimum sell) models, break-even models, and annuity-equivalent models. A second type of model answers the question: what is they optimal life of the investment. These models find the time period that maximizes NPV. The third type of PV model—and the most popular—are PV models that rank investments. They answer the question: which investment is preferred? The two most popular ranking PV models are the NPV and IRR models. In one survey, about 75% of chief financial officers of large corporations used IRR and NPV models nearly equally (Graham and Harvey, 2001). About 50% used payback models.

The advantage of the NPV model is that we can experiment with several different IRRs to find out how sensitive the NPV of the challenger is to alternative discount rates or IRRs. As we will discover in later chapters, consistency between NPV and IRR rankings exists only under homogeneous measures of size, term, taxes, and other measures.

There are alternative, easier ways of ranking investments such as the payback model. They are not recommended here because they fail to recognize a fundamental property of PV models: dollars in different time periods are valued differently and ignore cash flows after the payback period.

More complicated investment ranking problems. When we described the NPV model, the discount rate was equal to the defender's IRR. When the firm is considering the adoption of two mutually exclusive investments, then which investment is the defender and which is the challenger is an arbitrary decision. If the capital budgeting decision is to replace an existing investment with one of two mutually exclusive investments, then typically the defender is the investment in place. Then, the investment problem becomes calculating the NPV of the two challenging investments and selecting the one with the greater NPV.

More complicated ranking problems involve replacing a defender with a newer version of the same investment. Such an investment might be when to replace a cohort of growing chickens or other livestock grown for slaughter with a new cohort. In this case, there are multiple defenders equal to the same investment at several different ages. This class of NPV models and their corresponding criterion is referred to as replacement models and is the focus of Chapter 10.

Consider the investment problem involving the least cost source of financing. We have decided on an investment, but we have alternative ways to fund the investment and we want to find the least cost source of funding. In this problem, we designate one of the funding sources as the challenger and the other as the defender. We then compare the IRRs of the two funding sources. In this case, the funding source with the smallest IRR is preferred and used as the discount rate in finding the NPV of the potential new investment.

One can easily imagine more complicated investment problems where the designation of the defender is not always clear. In these circumstances, we ask the question do the rankings depend on which investment is selected as the defender? The answer depends on whether or not the investments being ranked are compared using homogeneous units. If the investments are adjusted to make the units consistent, then ranking them by their NPVs will result in the same rankings, regardless of which investment is chosen as the defender.

IRR, ROA, and ROE

So far, the opportunity cost of capital has been introduced without specifying whose capital is being invested. Is it equity capital, debt capital, or a combination of debt and equity capital? Or does it matter? The short answer is that it does matter as we will demonstrate in Chapter 12. If the focus is on the return on equity, then the discount rate represents the return on equity (ROE). In this model, interest costs on debt are subtracted from the cash flow included in the model. If the focus is on the return on assets (ROA), then the cost of the investment is subtracted at the beginning of the model and returns reflect a return to the assets and interest costs are irrelevant since the asset is treated as though it were purchased at the beginning of the investment.

The two approaches, ROE versus ROA, represent two schools of thought in investment analysis. The ROA approach focuses on the returns generated by the firm's assets. The ROE approach focuses on the returns generated by the firm's equity invested net of any interest payments on debt.

The difference between the two approaches matter because most firms rely on a combination of debt and equity to fund assets. Reduced to its essence, the issue is whether the opportunity cost of capital reflects the rate of return on the firm's assets or equity. We now describe the two approaches in more detail.

Return on Assets (ROA)

In the ROA approach, the cost of the investment is subtracted in the present period and net cash flow representing return to assets do not subtract debt or principal payments because the investment is paid for "up-front" or at the beginning of the investment. The advantage of this approach is that the analysis considers the rate of return on the entire investment made at the beginning of the period. The NPV for the ROA approach for the single period can be expressed as:

$$NPV = -V_0 + \frac{R_1 + S_1}{1 + ROA} \tag{8.18}$$

Note that in Equation ???, if NPV = 0 as it would in an IRR model, then (1 + ROA)V_0 is equal to R_1 + S_1. If we replace V_0 with E_0 + D, we can write R_1 + S_1 = (1 + ROA)(E_0 + D). This fact will be helpful as we connect ROA and ROE measures.

Return on Equity (ROE)

In the ROE approach, the analysis depends on how the asset is financed. In this approach, the cost of interest and debt payments is subtracted explicitly, and the initial investment is equal to the amount of equity invested, since the debt is paid directly to whoever supplies the investment. However, the debt D plus average interest costs charged at interest rate i (iD) are subtracted at the end of the period. The NPV for the ROE model is expressed as:

$$NPV = -E_0 + \frac{[R_1 + S_1 - (1+i)D]}{(1 + ROE)} \tag{8.19}$$

The fundamental rates of return identity. One of the issues to consider when comparing the ROE versus the ROA approach is that in equations (8.18) and (8.19), E_0 and V_0 are not equal because part of the acquisition of the investment is supported by debt, V_0 = D_0 + E_0. The main point here is that the two approaches are not equivalent. We made this point when we derived Equation ??? which is repeated here as Equation ???.

$$ROE = \frac{(ROA - i)D_0}{E_0} + ROA \tag{8.20}$$

Notice that in Equation ???, if the cost of debt i is equal to the return on assets ROA, then ROA and ROE will be equal. But if i is less than the return on assets, then ROE > ROA. A homework problem at the end of this chapter asks you to explain this result. Equation (8.20) is so important in financial analysis that we give it a special name: the fundamental rates of return identity.

IRRs, ROAs, and ROEs

We earlier defined IRRs as the discount rate that corresponds to NPVs of zero. Now we illustrate how to find IRRs on assets and equity. The fundamental rates of return identity alerts us to the fact that IRRs calculated on equity invested in the project and assets invested in the project will not usually be equal. To find the IRRs on assets invested in the project, we charge the entire investment at the beginning of the period and include its liquidation value as a return at the end of the period. This approach ignores the fact that investments may be financed and paid for over the life of the investment and charging for the investment at the beginning of the project doesn't accurately reflect its cash flow. This is the ROA approach. In this case, the ROA approach, we ignore financing because our interest is in the productive capacity of the long-term asset, independent of the terms under which it can be financed.

We illustrate how to find IRRs for assets (IRR^A) in a simple example. Suppose the firm's defender is a $1,000, non-depreciable investment that will earn $100 for one period and then will be liquidated at its acquisition price. We find the IRR^A associated with $1,000 of assets invested in the defender by setting its NPV equal to zero in Equation ??? and solving for ROA.

$$ROA = \frac{R_1 + S_1 - V_0}{V_0} = \frac{\$1,100 - \$1,000}{\$1,000} = 10\% \tag{8.21}$$

We find that the IRR^A of the defender is equal to 10%.

Now reconsider the same example, except that the $500, or half of the defender, is financed at 9%. The other half of the investment, $500, is financed by the firm's equity. We continue to assume that, after one period, the investment is liquidated for its acquisition value, the loan of $500 is repaid, and the firm recovers its investment of $500. The firm also earns in one period, $100, the same as before. But now it has to pay a rental fee for the use of the loan's funds of 9% times $500, or $45. By setting the NPV model of the defender in Equation ??? equal to zero, we can find its IRR associated with the firm's equity (IRR^E) in the project equal to.

$$ROE = \frac{R_1 + S_1 - (1+i)D - E_0}{E_0} = \frac{\$1100 - \$545 - \$500}{\$500} = 11\% \tag{8.22}$$

Now the defender's IRR on its equity is 11%. In this case, the firm gained access to the use of an asset because of financing. The gains from a lender providing the firm access to $500 of debt capital to acquire a $1000 investment using only $500 of its own money increased its earnings on its equity from 10% to 11%. Meanwhile the investment earned only 10%. The value of the financing increased the rate of return on equity by 1%.

ROE or ROA?

For a variety of reasons, financial managers may prefer to represent the defender's IRR as the defender's ROA or the defender's ROE. However, this same manager must be careful to make sure that the cash flows associated with the challengers are consistent with the method used to find the IRR of the defender. If the IRR of the defender represents the defender's ROE, then debt and interest costs should be accounted for explicitly. If the ROA of the defender is preferred, the cash flows associated with the challenger and the defender do not separate debt and interest costs from the calculations.

In practice, PV models appear to prefer the ROA approach, even though both approaches are valid and provide unique information. Nevertheless, the dominance of the ROA approach has resulted in the identification of ROAs as simply the IRR of the investment, a practice we will also adopt in the remainder of this book.

Summary and Conclusions

This chapter reviewed several different kinds of PV models. They differ because they are designed to answer different kinds of questions. Some PV models, NPV and IRR, help us rank alternative investments. Others like AE models can be used to find the optimal time for replacing an investment—or what our periodic loan payments will be. And still other PV models like the maximum bid (minimum sell) models help us know the maximum (minimum) we can offer to purchase (sell) an investment and still earn our defender's IRR. Finally, still other PV models such as capitalization formulas and payback models offer at most a crude rule of

thumb for evaluating and making investment decisions.

Regardless of the PV model type, they all have one measure in common: it is the opportunity cost of a defending investment represented by the discount rate. The discount rate is the ROA or the ROE of the defender that must be sacrificed to acquire the new or challenging investment. This construction of PV models emphasizes the important role of opportunity costs in the construction of PV models and reminds us that we focus on opportunity rather than direct costs when making applied economic decision.

Questions

1. Explain the connection between investment questions and corresponding PV models.
2. Suppose you found the maximum bid price for a purchase you are considering. What would you conclude from your calculations?
3. Suppose that you found that an investment's AE reached its maximum at year 10 while its NPV reached its maximum at 15 years. How would you interpret your results?
4. When finding the ROE measure in Equation ???, we explicitly accounted for the debt used to finance the investment. However, when finding the ROA measure in Equation ???, we did not account for the debt used to finance the investment. Explain the difference between the two approaches.
5. When comparing the ROAs and ROEs, we concluded that the ROE > ROA as long as the ROA was greater than the average interest rate on the debt used to finance an investment. Please explain these finding and provide an example on these results. Refer to the fundamental rates of return identity when formulating your answer.
6. Provide numerical PV models in which you find an investment's IRR and NPVs using ROA-IRR and ROE-IRR, AE, maximum bid price, and the investment's payback period. Defend your choice of a discount rate.
7. Explain how the "life of the investment" principle guarantees that all economic activity associated with an investment will be captured by its cash flow.
8. Most of the time, we don't identify discount rates in PV models as being either a ROE or a ROA. Instead we seem to prefer to identify the defender's ROA with the letter "r". Why do we tend to prefer ROA to ROE measures? Can you describe a case when it would be important to evaluate the projects using the defender's ROE instead of its ROA?
9. Assume you were considering one investment that could be financed from two different financial institutions. Thus the only difference between the projects were their cash flow associated with their use of debt capital. How would you proceed to rank the two investments?
10. In a previous chapter, we calculated the rates of return on equity and rates of return on assets for HQN for the year 2018 for the entire firm. But this calculation was for only one year and included non-cash items in the calculation. What is different between IRRs calculated as ROA or ROE for the entire firm versus IRRs calculated as ROA or ROE in a PV model?
11. Suppose you ranked two challengers using their IRRs and by finding their NPVs using the IRR of the defender. What would you conclude if the IRR and NPV ranking provided conflicting ranking?

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9: Homogeneous Sizes

Learning Objectives

At the end of this chapter, you should be able to: (1) understand how internal rate of return (IRR) and net present value (NPV) models can produce inconsistent rankings; (2) produce consistent IRR and NPV rankings by adjusting for investment size differences and by adopting common reinvestment rate assumptions; (3) understand why some NPV and IRR rankings are unstable; (4) recognize how different initial size adjustment methods (addition, scaling, or some combination) produce consistent but sometimes different investment rankings; and (5) recognize how investment type differences can be used to identify the proper size adjustment method.

To achieve your learning goals, you should complete the following objectives:

- Recognize that IRR and NPV rankings may be inconsistent and that NPV rankings may be unstable.
- Understand the difference between periodic investment sizes and initial investment sizes.
- Describe how different reinvestment rate assumptions can produce inconsistent IRR and NPV rankings.
- Show how resolving periodic and initial size differences and a common reinvestment rate produce consistent challenging investment rankings.
- Illustrate the different methods available for resolving periodic size investment differences.
- Demonstrate how scaling and addition can be used to resolve initial size differences in investments.
- Demonstrate that while different methods for resolving periodic and initial size differences can each produce consistent rankings, the consistent rankings may not be the same.
- Understand the conditions under which IRR and NPV provide the same rankings as the size adjusted IRR and NPV models.
- Recognize the four basic investment models
- Recommend the appropriate investment model based on challenger characteristics.

Introduction[1]

In the single equation NPV model, we assumed that cash flow was exchanged between time periods at the defender's IRR. In the single equation IRR model, we assumed that cash flow was exchanged between time periods at the challenger's IRR. In the NPV model, the challenger (defender) was preferred to the defender (challenger) if the NPV were positive (negative). Furthermore, the NPV ranking of the defender and challenger was the same ranking obtained by comparing their respective IRRs. As a result, both IRR and NPV criteria produce the same ranking.

This chapter recognizes that when we rank multiple challengers funded by one defender, their IRR and NPV rankings may be different. Furthermore, we observe that changes in the defender's IRR may produce unstable NPV rankings. We demonstrate these results in Table 9.1 that ranks three challengers.

Table 9.1. Inconsistent IRR and NPV rankings and unstable NPV rankings among three mutually exclusive investments of different initial and periodic cash flows.

initial investments V_0^i where i = 1,2,3	period one cash flows R_t^i where t = 1 generated by initial investments V_0^i where i = 1,2,3	period two cash flows R_t^i where t = 2 generated by initial investments V_0^i where i = 1,2,3	NPVi earned by initial investments V_0^i where i = 1,2,3 assuming r = 5% (ranking)	NPVi earned by initial investments V_0^i where i = 1,2,3 assuming r = 10% (ranking)	IRRi earned by initial investments V_0^i where i = 1,2,3 (ranking)
\$1,000	\$800	\$400	\$124.72 (3)	\$57.85 (2)	14.8% (1)
\$2,000	\$1,560	\$800	\$211.34 (2)	\$79.34 (1)	13.3% (2)

initial investments V_0^i where i = 1,2,3	period one cash flows R_t^i where t = 1 generated by initial investments V_0^i where i = 1,2,3	period two cash flows R_t^i where t = 2 generated by initial investments V_0^i where i = 1,2,3	NPVi earned by initial investments V_0^i where i = 1,2,3 assuming r = 5% (ranking)	NPVi earned by initial investments V_0^i where i = 1,2,3 assuming r = 10% (ranking)	IRRi earned by initial investments V_0^i where i = 1,2,3 (ranking)
$3,000	$2,250	$1,200	$231.29 (1)	$37.10 (3)	11.0% (3)

We describe the challenging investments' NPV rankings in columns 4 and 5 for investments of $1,000, $2,000, and $3,000. The investments' IRR rankings appear in column 6. We indicate investment rankings by parenthesized numbers in columns 4, 5, and 6.

If the defender's IRR is 5% for challenging investments 1, 2, and 3, their NPV rankings are 3, 2, and 1 respectively. If the defender's IRR is 10% for challenging investments 1, 2, and 3, their NPV rankings are 2, 1, and 3 respectively. Meanwhile, for challenging investments 1, 2, and 3 their IRR rankings are different still: 1, 2, and 3 respectively. These results demonstrate that IRR and NPV rankings may provide inconsistent rankings and NPV rankings may vary with the defender's IRR.

The inconsistent IRR and NPV rankings require decision makers to choose between conflicting recommendations. They must decide: which ranking method should they believe. Among practitioners, the NPV approach is considered more reliable for wealth maximization. However, the ease in which decision makers can interpret IRRs has made it the more popular of the two ranking methods. Both ranking methods, however, have their drawbacks. Investments may have multiple IRRs and NPV rankings may be unstable—varying with changes in the defender's IRR what is often referred to as the discount rate.

The problem produced by inconsistent IRR and NPV rankings is obvious—it leaves decision makers without a clear recommendation. On the other hand, NPV rankings that vary with the discount rate (see columns four and five) can be just as problematic. The reason is that if we are not sure of the discount rate, then we cannot be sure of the NPV rankings either.

The purpose of this chapter is to demonstrate that by resolving periodic and initial investment size differences between challenging investments and by adopting a common reinvestment rate, we can guarantee consistent IRR and NPV rankings. We will also explain why some NPV rankings vary with changes in the defender's IRR. Finally, we show that depending on how we adjust for size differences and the reinvestment rate we adopt, we an produce four different present value (PV) models. We also discuss the conditions under which each investment model is appropriate for ranking challenging investments—rankings that will be consistent but possibly different across models.

Periodic and Initial Investment Sizes

Sometimes when IRR and NPV methods produce inconsistent rankings, it is because the challenging investments being ranked lack investment size homogeneity. Challenging investments may lack investment size homogeneity in two ways. First, their initial investment sizes may be different. In Table 9.1 note that challenging investments 1, 2, and 3 invested initial amounts of $1,000, $2,000, and $3,000 respectively.

The second way challenging investment sizes may differ is their periodic cash flows. These differences change periodic investment amounts between challenging investments. Notice that in Table 9.1 cash flows in period one are different: $800, $1,560, and $2,250 for investments 1, 2, and 3 respectively. Differences in initial and periodic investment sizes contribute directly to inconsistent and unstable rankings.

Notation. This chapter employs more than the usual amount of mathematical notation. It is required to tell the story about consistent and stable IRR and NPV rankings. The good news is that there are no complicated mathematical operations besides adding, subtracting, canceling like terms, and factoring. The notation that will be used throughout this chapter includes variables with subscripts and superscripts. The superscript on a variable associates it with an investment, i = 1, 2, ..., m. The subscript on a variable associates it with a time period t = 1, ..., n. If a superscripted variable is raised to a power, the variable is placed in parentheses and the power to which the variable is raised is placed outside of the parentheses. To illustrate, the variable $(R_5^2)^3$ describes the value for the variable R associated with investment i = 2 in the t = 5 time period raised to the third power. Finally, when we write an expression like $S(r, R_t^i)$ we are saying that function S depends on values assumed by variables r and R_t^i

without specifying the exact nature of the function or the values of the variables represented by *r* and R_t^i. When we want to represent a vector of values for investment *i* we bold **R** and write $\mathbf{R} = R_1^i, R_2^i, \ldots, R_n^i$.

How differences in periodic cash flow can create periodic investment size differences. Applying the notation described earlier, we now demonstrate how differences in cash flow create differences in investments sizes over time. Assume that initial investments of V_0^i for investments *i* = 1 and *i* = 2 equal V_0. Also assume that cash flows in period one earned by investments one and two are equal to R_1^1 and R_1^2 respectively. These positive (negative) cash flows represent investments (disinvestments) in the underlying investment. If the initial investments earned a rate of return equal to the defender's IRR equal to *r*, then beginning in period two, investment one equals $V_0(1+r) - R_1^1$ and investment two equals $V_0(1+r) - R_1^2$ which are not equal if $R_1^1 \neq R_1^2$. As a result, unequal periodic cash flows create unequal periodic investments, even if their initial investments are equal, a cause of inconsistent rankings.

To illustrate numerically, suppose that V_0 = \$100 and $R_1^1 = 15$ and $R_1^2 = 20$. If r is 10% then after one period the amount invested in investment one is \$100(1.1) – \$15 = \$95 and the amount invested after one period in investment two is \$100(1.1) – \$20 = \$90. After one period the investment amounts are unequal and violate the homogeneity of measures requirement.

The implication of these results is that even though two investments begin with equal initial investments, IRR and NPV rankings may still be inconsistent and unstable if periodic cash flows are unequal.

So what have we learned? We've learned that the two homogeneous investments size conditions required for consistent NPV and IRR rankings are 1) equal initial challenging investments sizes, and 2) equal periodic cash flows except in the last period. We require equal periodic cash flows except in the last period to ensure that the challenging investments will satisfy equal periodic investment sizes. Besides these two size conditions, one of two reinvestment rate assumptions must be adopted for both NPV and IRR rankings. These two reinvestment rate assumption are that cash flow are either reinvested at the defender's IRR or at the challengers' respective IRRs.

Consistent IRR and NPV Rankings When the Reinvestment Rate is the Defender's IRR

To be clear, reinvestment rates indicate where periodic cash flows will be reinvested. If the reinvestment rate is the defender's IRR, then we assume that earnings from the challenging investments will be reinvested in the defender. If the reinvestment rates are the challenging investments' respective IRRs, then we assume that earnings from the investments will be reinvested in the challenging investments. If there is some other reinvestment rate, it must correspond to a separate investment and be evaluated as another challenger. Hence, the only reinvestment rates we consider are the defender's IRR and the challenging investments' IRRs.

NPV models implicitly assume that the reinvestment rate is the defender's IRR and earnings from the investment will be reinvested in the defender. IRR models assume that the reinvestment rate is the investment's own IRR and that the funds will be reinvested in itself. However, to remove ranking inconsistencies, we impose the same reinvestment rate across IRR and NPV models and produce what others have referred to as modified IRR (MIRR) and modified NPV (MNPV) models.

Next we demonstrate that if the two sufficient size conditions are satisfied and challenger earnings are reinvested in the defender, we are guaranteed consistent NPV and IRR rankings. NPV rankings will be consistent and under some conditions stable.

NPV rankings. Consider ranking two mutually exclusive *n* period investments in a similar risk class by their respective NPVs. The NPV model for investment one is described as:

$$NPV^1 = -V_0^1 + \frac{R_1^1}{(1+r)} + \cdots + \frac{R_n^1}{(1+r)^n} \tag{9.1}$$

The NPV model for investment two is described as:

$$NPV^2 = -V_0^2 + \frac{R_1^2}{(1+r)} + \cdots + \frac{R_n^2}{(1+r)^n} \tag{9.2}$$

In the equations above, *r* is the defender's IRR, V_0^1 and V_0^2 are initial investments one and two respectively, and R*t*1 and R*t*2 are periodic cash flows in periods $t = 1, \cdots, n$ generated by mutually exclusive investments one and two. We assume that the terms of the two investments are equal.

To resolve size differences caused by differences in periodic cash flows, we set them equal to zero in every period except the last one by reinvesting them at rate *r* until the last period. Generally, the discount rate and the reinvestment rate in NPV models are equal. To distinguish the original NPV model with the NPV model with funds reinvested until the last period, many finance scholars refer to the later model as the modified NPV model or the MNPV model. We write the MNPV model as:

$$\begin{aligned} MNPV^1 &= -V_0^1 + \frac{(1+r)^{n-1}R_1^1 + (1+r)^{n-2}R_2^1 + \cdots + R_n^1}{(1+r)^n} \\ &= -V_0^1 + \frac{S(r, \mathbf{R}^1)}{(1+r)^n} \end{aligned} \tag{9.2a}$$

where $S(r, \mathbf{R}^1) = (1+r)^{n-1}R_1^1 + (1+r)^{n-1}R_2^1 + \cdots + R_n^1$. Similarly the MNPV model for investment two can be rewritten as:

$$\begin{aligned} MNPV^2 &= -V_0^2 + \frac{(1+r)^{n-1}R_1^2 + (1+r)^{n-2}R_2^2 + \cdots + R_n^2}{(1+r)^n} \\ &= -V_0^2 + \frac{S(r, R^2)}{(1+r)^n} \end{aligned} \tag{9.2b}$$

where $S(r, \mathbf{R}^2) = (1+r)^{n-1}R_1^2 + (1+r)^{n-2}R_2^2 + \cdots + R_n^2$. Finally, we impose the additional condition that $S(r, \mathbf{R}^1), S(r, \mathbf{R}^2) > 0$ 0" title="Rendered by QuickLaTeX.com" height="20" width="169" style="vertical-align: -4px;">, otherwise the investment decision is no longer interesting.

The difference between MNPV^1 and MNPV^2 which ranks the two investments can be expressed as:

$$MNPV^2 - MNPV^1 = [V_0^1 - V_0^2] + \frac{[S(r, \mathbf{R}^2) - S(r, \mathbf{R}^1)]}{(1+r)^n} \tag{9.3}$$

If the initial investment sizes are the same as required by the homogeneity of measures principle, then the difference in the first bracketed expression in Equation ??? is zero and the rankings will depend only on the difference between the sums of reinvested cash flows:

$$MNPV^2 - MNPV^1 = \frac{[S(r, \mathbf{R}^2) - S(r, \mathbf{R}^1)]}{(1+r)^n} \tag{9.4}$$

Equation (9.4) provides an interesting result. We obtained these results by reinvesting and discounting cash flow at the same rate so NPVs of the two investments are unaffected allowing us to write:

$$\frac{[S(r, \mathbf{R}^2) - S(r, \mathbf{R}^1)]}{(1+r)^n} = NPV^2 - NPV^1. \tag{9.5}$$

We now illustrate the results of Equation ???. Let the initial investment equal $100 and cash flows in periods one and two equal $50 and $70 respectively. Finally let the discount rate and the implied reinvestment rate equal 10%. Then we write:

$$NPV = -\$100 + \frac{\$50}{(1+10\%)} + \frac{\$70}{(1+10\%)^2} = \$3.31 \tag{9.6a}$$

Then we write the equivalent MNPV model as:

$$MNPV = -\$100 + \frac{(\$50)(1+10\%) + \$70}{(1+10\%)^2} = \$3.31 \tag{9.6b}$$

The results illustrate NPV and MNPV models are equal and provide the same rankings when the reinvestment rate is the same as the discount rate.

Finally, it should be noted that the difference between the NPVs in Equation ??? may be unstable and change as the defender's IRR changes as we demonstrated in Table 9.1.

IRR rankings. Consider ranking two mutually exclusive n period investments in a similar risk class by their respective IRRs. If the IRR rankings are to be consistent with the NPV rankings, they must satisfy the sufficient conditions described earlier, including assuming the same reinvestment rate r. The IRR model for investment one is described as:

$$V_0^1 = \frac{R_1^1}{(1+IRR^1)} + \cdots + \frac{R_n^1}{(1+IRR^1)^n} \tag{9.7a}$$

The IRR model for investment two is described as:

$$V_0^2 = \frac{R_1^2}{(1+IRR^2)} + \cdots + \frac{R_n^2}{(1+IRR^2)^n} \tag{9.7b}$$

In the equations above, V01 and V02 are initial investments one and two respectively, and R_t^1 and R_t^2 are periodic cash flows in periods *t* = 1, ⋯, *n* generated by mutually exclusive investments one and two.

To resolve differences in periodic cash flows, we set them equal to zero in every period except the last period by reinvesting them at rate *r* until the last period. Generally, the discount rate and the reinvestment rate in IRR models are equal. Here, the reinvestment rate is r rather than their respective IRRs. To distinguish the original IRR model from the IRR model with funds reinvested until the last period, many finance scholars refer to the later model as the modified IRR model, the MIRR model. We write the MIRR model as:

$$V_0^1 = \frac{S\left(r, \mathbf{R}^1\right)}{(1+MIRR^1)^n} \tag{9.3}$$

Solving for MIRR[1] in Equation 9.3 we obtain:

(9.9) n} - 1\end{equation*}

which has one positive root.

Investment two is described as:

$$V_0^2 = \frac{S(r, \mathbf{R}^2)}{(1+MIRR^2)^n} \tag{9.10}$$

Solving for MIRR[2] in Equation ??? we obtain:

(9.11) n} - 1\end{equation*}

which has one positive root.

Finally, we write the difference between MIRR^1 and MIRR^2 assuming initial investments V_0^2 and V_0^1 are equal to V_0 as:

(9.12) n} \end{split}\end{equation*}

If the initial sizes and terms of investments one and two are equal, then their MIRR ranking will depend on the same criterion that ranked the investments' MNPVs—the difference in the sum of their reinvested cash flows $S(r, \mathbf{R}^1)$ and $S(r, \mathbf{R}^2)$ and the MIRR rankings and MNPV rankings will be consistent. However, we cannot be certain that the MIRR rankings will equal the IRR rankings because we changed the reinvestment rate assumption in the IRR models.

Consistent IRR and NPV Rankings Assuming Reinvestment Rates are the Challenging Investments' IRRs

In this section, we demonstrate that we can also guarantee consistent NPV and IRR rankings by imposing the second reinvestment rate assumption—that periodic cash flows are reinvested in the challenging investments' IRRs.

Suppose the reinvestment rates were the IRRs of the challengers being evaluated. This reinvestment rate is appropriate under the condition that the cash flows are reinvested in the challengers as opposed to being reinvested in the defending investment. To demonstrate the consequences of assuming that the reinvestment rates equal the challengers' IRRs, consider the following.

Assume that the initial sizes of the challenging investments are equal and that their cash flows are reinvested in themselves earning their respective IRRs and the discount rate applied to the NPVs is the defender's IRR. Then we could write the difference between MNPVs as:

$$MNPV^2 - MNPV^1 = \frac{[S(MIRR^2, \mathbf{R}^2) - S(MIRR^1, \mathbf{R}^1)]}{(1+r)^n} \tag{9.13a}$$

Meanwhile, we can write the difference between MIRRs as:

(9.13b) n} \end{split}\end{equation*}

Notice that the MNPV and MIRR rankings depend on the same sums making their rankings consistent. However, in contrast to the NPV model, the MNPV rankings are stable and likely different that the original NPV rankings. But there is one thing more. Look at the original IRR equations. They are equivalent to the MIRR equations. Reinvesting and discounting by the challengers' IRRs are offsetting operations and don't change the original IRRs. Therefore, if the initial sizes are equal, and the assumption is that cash flows are reinvested in the challenging investments, then IRR and MIRR rankings are equivalent! This is such an important point, we illustrate it with an example.

Assume net cash flows of $50 and $70 in periods one and two respectively, an initial investment of $100, and a reinvestment rate of the investment's IRR or MIRR. Then we can write the IRR as:

$$\$100 = \frac{\$50}{(1+IRR)} + \frac{\$70}{(1+IRR)^2} \; and \; IRR = 12\% \tag{9.14a}$$

Now we can write the MIRR model as:

$$\$100 = \frac{(\$50)(1+MIRR) + \$70}{(1+MIRR)^2} \; and \; MIRR = 12\% \tag{9.14b}$$

Because the IRR and MIRR values are the same, they will return identical rankings. To emphasize the main point, if the periodic cash flows are reinvested in the challengers, then ranking investments using the investments' IRR is the same as ranking them with their MIRRs.

So what have we learned? We learned that when we satisfy the two sufficient size conditions required for consistent rankings, equal initial and period investment sizes (except in the last period), and satisfy the second reinvestment rate assumption that we reinvest in the challenging investments—then MNPV and MIRR rankings of the challengers will be consistent. And there is one thing more, the MIRR rankings will be the same as the original IRR rankings.

We have also learned that inconsistent challenging investments IRR and NPV rankings are often the results of the two ranking methods adopting different reinvestment rate assumptions. NPV models assume the reinvestment rate is the defender's IRR. IRR models assume the reinvestment rate is the challenger's IRR for the same investment. Consistency requires that the reinvestment rate for each investment be the same regardless of whether the ranking tool is IRR or NPV. Once we have satisfied initial and periodic investment size differences and adopted a consistent reinvestment rate assumption, investments can be ranked consistently using IRR or NPV methods.

Reinvestment rates and inconsistent rankings. To make clear that consistent IRR and NPV rankings require that the same reinvestment rate for each investment regardless of the ranking methods, IRR or NPV, we describe three investments with the same initial sizes and assume the defender's IRR is 5%. We allow NPV models to reinvest at the defender's IRR rate of 5%. We allow the challenger's cash flow to be reinvested at their respective IRRs. The inconsistent rankings produced by allowing investments' reinvestment rate to be different when using IRRs versus NPVs, we describe in Table 9.2. The results demonstrate that even if we adjust for initial and periodic size differences but fail to require consistent reinvestment rates, IRR and NPV rankings could still be inconsistent.

Table 9.2. Rankings Allowing NPV models (IRR models) to reinvest periodic cash flows at the defender's IRR (challengers' IRRs). Investment rankings indicated in parentheses below NPVs and IRRs values.

Period	Variables	Challenger 1 cash flow	Challenger 2 cash flow	Challenger 3 cash flow
0	V_0	– $3,000	– $3,000	– $3,000
1	R_1	– $100	$2,010	$2,250
2	R_2	$3,800	$1,560	$1,200
	NPVs assuming r=5%	$351.47 (1)	$329.25 (2)	$231.29 (3)
	IRRs	10.89% (3)	13.01% (1)	11.03% (2)

Now assume that investment three was the defender and we used its IRR equal to 11.03% to reinvest period one cash flow to period two. Then because we employed the same reinvestment rate for each model regardless of whether we used IRR or NPV ranking models, consistency is restored. We describe these results in Table 9.3.

Table 9.3. Allowing NPV and IRR models to reinvest periodic cash flows at same reinvestment rate equal to challenger 3's IRR. Investment rankings indicated in parentheses below NPVs and IRRs values.

Period	Variables	Challenger 1 cash flow	Challenger 2 cash flow	Challenger 3 cash flow
0	V_0	– $3,000	– $3,000	– $3,000
1	R_1	$0	$0	$2,250

Period	Variables	Challenger 1 cash flow	Challenger 2 cash flow	Challenger 3 cash flow
2	R_2	$3,800 – $100 x 1.1103 = $3688.97	$1,560 + $2010 x 1.1103 = $3791.70	$1,200
	NPVs assuming *r*=11.03%	– $7.57 (3)	$83.82 (1)	$0 (2)
	IRRs assuming *r*=11.03%	10.89% (3)	12.57% (1)	11.03% (2)

So what have we learned? We learned that when we rank challengers using their NPVs and IRRs even when we adjust for initial and periodic size differences—we can produce conflicting rankings because IRR and NPV models assume different reinvestment rates. For IRRs and NPVs to produce consistent rankings, each investment must assume the same reinvestment rates. Different investments may assume different reinvestment rates; however, if IRR and NPV ranking are to be consistent, they must be the same reinvestment rate for the same investment regardless of whether the ranking models are IRRs or NPVs.

Resolving Initial Size Differences by Scaling

Scaling and a reinvestment rate equal to the defender's IRR

Consider the possibility of eliminating initial investment size differences between mutually exclusive investments by scaling the challengers to a common initial size. To scale challengers requires that they are available in continuous sizes. To scale an investment requires that it leaves the relative size of the investment and its cash flows constant over the continuous sizes. For example, scaling an investment by two assumes that there exists an investment twice the size that produces twice the cash flow of the original investment in each period. To illustrate, assume net cash flows of $50 and $70 in periods one and two respectively, an initial investment of $100, and assume the reinvestment rate equals the defender's IRR of r = 10%. Then we can write the MIRR model as:

(9.15a) 2} - 1 = 12\%\end{equation*}

Suppose now that we scale the MIRR model by 2. Then we can rewrite the model as:

(9.15b) 2} - 1 = 12\%\end{equation*}

Notice that the scaling factor "2" cancels so we are left with the original MIRR equation in (9.15a). And even if the reinvestment rate is the IRR, the scaling factor still cancels. Thus, the MIRR and the scaled MIRR are equal. (Can you demonstrate that the scaling factor cancels regardless of the reinvestment rate?)

Scaling and a reinvestment rate equal to the challengers' IRR

What if the reinvestment rate is the challenger's own IRRs, which assumes that the funds are reinvested in the challengers, then MIRRs, scaled MIRRs and the IRR equations are the same.

To make the point absolutely clear, that scaling will not change MIRRs or IRRs when the reinvestment rate is the investment's IRR, we illustrate these results by comparing the original IRR calculations from Table 9.1 with MIRRs, and scaled MIRRs calculated in Table 9.4.

Table 9.4. Consistent MIRR, scaled MIRR, and IRR rankings obtained by scaling initial investment sizes and periodic cash flows assuming the reinvestment rates are equal to the investments' IRR^i where i = 1,2,3.

initial investment V_0^i and scaling factor γ_i where i = 1,2,3	period one cash flows $\gamma_i R_t^i$ where t = 1	period two cash flows $R_2^i + (1 + IRR^i)R_1^i$ where i = 1,2,3	period two cash flows $\gamma_i R_2^i + \gamma_i(1 + IRR^i)R_1^i$ where i = 1,2,3	MIRR^i where i = 1,2,3 assuming r = IRR^i (ranking)	Scaled MIRR^i where i = 1,2,3 assuming r = IRR^i (ranking)	IRR^i where i = 1,2,3 assuming r = IRR^i (ranking)
$V_0^1 = \$1,000$ $\gamma_1 V_0^1 = \$3,000$ $(\gamma_1 = 3)$	0	\$400 + (1.148)\$800 = \$1,318.40	(3)\$400 + (3)(1.148)\$800 = \$3,955.20	14.8% (1)	14.8% (1)	14.8% (1)
$V_0^2 = \$2,000$ $\gamma_2 V_0^2 = \$3,000$ 2)	0	\$800 + (1.133)\$1,560 = \$2,567.48	(3/2)\$800 + (3/2)(1.133)\$1,560 = \$3,851.22	13.3% (2)	13.3% (2)	13.3% (2)
$V_0^3 = \$3,000$	0	\$1,200 + (1.110)\$2,250 = \$3,652.50	\$1,200 + (1.110)\$2,250 = \$3,652.50	11.0% (3)	11.0% (3)	11.0% (3)

We next demonstrate how scaling achieves ranking consistency between MNPVs and MIRRs and scaled MNPVs when the reinvestment rate is the defender's IRR. The MIRR, scaled MIRR and MNPV rankings in Table 9.5 are consistent because the MNPV rankings have been scaled to conform to the MIRR, and scaled MIRR.

Table 9.5. Consistent size adjusted MIRR and MNPV rankings obtained by scaling initial investment sizes and periodic cash flows assuming the reinvestment rate equal to the defender's IRR of r = 5%.

initial investment V_0^i and scaling factor γ_i where i = 1,2,3	period one cash flows $\gamma_i R_t^i$ where t = 1	period two cash flows $\gamma_i R_2^i + \gamma_i(1 + r)R_1^i$ where i = 1,2,3	Scaled MNPV^i where i = 1,2,3 assuming r = 5% (ranking)	MIRRs and Scaled MIRR^i where i = 1,2,3 assuming r = 5% (ranking)
$\gamma_1 V_0^1 = \$3,000$ $(\gamma_1 = 3)$	0	(3)\$400 + (3)(1.05)\$800 = \$3,720	\$374.15 (1)	11.4% (1)
$\gamma_2 V_0^2 = \$3,000$ 2)	0	(3/2)\$800 + (3/2)(1.05)\$1,560 = \$3,657	\$317.01 (2)	10.4% (2)
$V_0^3 = \$3,000$	0	\$1,200 + (1.05)\$2,250 = \$3,562.50	\$231.29 (3)	9.0% (3)

While MIRRs are not affected by scaling, even when the reinvestment rate is the defender's IRR, scaling does have an effect on MNPVs. We observe the effect of scaling on MNPVs by rewriting Equation ??? as:

$$NPV^2 - NPV^1 = [V_0^1 - V_0^2] + \frac{[S(r,\mathbf{R}^2) - S(r,\mathbf{R}^1)]}{(1+r)^n} = \frac{[S(r,\mathbf{R}^2) - S(r,\mathbf{R}^1)]}{(1+r)^n} \tag{9.16}$$

Note that the scaling factor does not cancel out in Equation ??? and may affect the values and rankings of scaled and unscaled MNPVs. To summarize, scaled and unscaled MIRRs are equal in rank and amount. However, scaled and unscaled MNPVs are not necessarily equal in rank or amount.

To illustrate, consider the earlier example reconstructed as an MNPV model. Assume an MNPV model with an initial investment of $100, and cash flows in periods one and two of $50 and $70 respectively and a reinvestment rate *r* = 10% and a discount rate of *r* = 8%

$$MNPV = -\$100 + \frac{(\$50)(1+10\%)+70}{(1+8\%)^2} = \$7.17 \tag{9.17a}$$

Suppose now that we scale the MNPV model by 2. Then, we can rewrite the model as:

$$scaled\, MNPV = -(2)\$100 + \frac{(2)[(\$50)(1+10\%)+70]}{(1+8\%)^2} = \$14.34 \tag{9.17b}$$

Notice that the scaling factor "2" no longer cancels so we are left with a different MNPV equations before and after scaling. And even if the reinvestment rate is the same as the discount rate, the scaling factor still does not cancel. (How can you demonstrate that the scaling factor does changes the MNPV regardless of the reinvestment rate?) Notice something else that is very important: the scaled MNPV is equal to the unscaled MNPV times the scaling factor which in this case is two: $14.34 = (2) $7.17

So what does this mean? If the reinvestment rate is the investment's IRR which assumes that cash flows are reinvested in the challenger, then we should rank investments using IRRs. Even if the investments are of unequal size which can be resolved by scaling, IRR rankings will still be consistent with scaled MNPV rankings which can be obtained by multiplying the unscaled MNPVs by their respective scaling factors.

Resolving Initial Size Differences by Addition

Suppose that the defender is scalable but that the challenging investments are unique and cannot be scaled. Since initial investment size equality is one of the conditions required for consistent MIRR and MNPV rankings, we must adopt an alternative to the scaling method to achieve initial size equality. One approach is to add sufficient amounts of the scalable defender to the smaller challenging investment so that its size equals the size of the larger challenger. For example assume that $(V_2 - V_1) = \delta$ is added to V_1 where it earns the defender's IRR equal to *r* until the nth period. This allows us to rewrite Equation ??? as:

$$MNPV^2 - MNPV^1 = [V_0^1 + \delta - V_0^2] + \frac{[S(r,\mathbf{R}2) - S(r,\mathbf{R}1) - \delta(1+r)^n]}{(1+r)^n} \tag{9.18a}$$

Because compounding and discounting δ are offsetting effects, we can rewrite equation (9.18a) as (9.18b):

$$MNPV^2 - MNPV^1 = [V_0^1 - V_0^2] + \frac{S(r,\mathbf{R}^2) - S(r,\mathbf{R}^1)}{(1+r)^n} \tag{9.18b}$$

But this ranking would have been obtained by comparing the MNPVs before adjusting for size differences by addition. Therefore,

equation (9.18b) can be rewritten as:

$$MNPV^2 - MNPV^1 = NPV^2 - NPV^1$$
$$= [V_0^1 - V_0^2] + \frac{S(r, \mathbf{R}^2) - S(r, \mathbf{R}^1)}{(1+r)^n}$$

(9.18c)

These results demonstrate that when the reinvestment rate is the defender's IRR then NPV, MNPV, and added MNPV rankings will all be equal in amount and rank.

We can write the added MIRRs as:

(9.19) n} \end{split}\end{equation*}

Furthermore, these added MNPV and added MIRR rankings will be consistent because the ranking criterion are the same (see Table 9.6). Note, however, that the consistent rankings obtained through addition are not the same as those obtained when initial size differences were resolved through scaling. Clearly, the final rankings depend on what methods we use to adjust for initial size differences and what reinvestment rate we assume.

Table 9.6. Added MIRRs and added MNPVs assuming the reinvestment rate is the defender's IRR of r = 5%.

initial investment V_0^i and addition amount δ_i where *i* = 1,2,3	period one cash flows R_t^i where *t* = 1	period two cash flows $R_2^i + \delta_i(1+r)^2 + R_1^i(1+r)$ where *i* = 1,2,3	MNPV^i where *i*=1,2,3 assuming *r*=5% (ranking)	MIRR^i where *i*=1,2,3 assuming *r*=5% (ranking)
$(\delta_1 + V_0^1) = \$3,000$ $(\delta_1 = \$2,000)$	0	\$400 + \$2,000(1.05)2 + \$800(1.05) = \$3,445	\$124.72 (3)	7.2% (3)
$(\delta_2 + V_0^2) = \$3,000$ $(\delta_2 = \$1,000)$	0	\$800 + \$1,000(1.05)2 + \$1,560(1.05) = \$3,540.50	\$211.34 (2)	8.6% (2)
$V_0^3 = \$3,000$	0	\$1,200 + \$2,250(1.05) = \$3,562.50	\$231.29 (1)	9.0% (1)

Four Consistent Investment Ranking Models

We have demonstrated that differences in initial and periodic sizes can cause inconsistent ranking. We resolved differences in periodic sizes by reinvesting cash flow until the last period. However, we noted that the reinvestment rate for each investment could be the challenger's IRR or the defender's IRR.

Then we noted that consistency requires that their initial sizes are equal as well as their periodic sizes. Several methods can be employed to resolve initial size differences. We emphasized two: scaling and addition. Each of these methods will generate consistent initial size adjusted MIRR and MNPV rankings as long as cash flows are reinvested until the last period.

The methods employed to adjust for periodic and initial size differences produce four distinct ranking models that depend on which size adjustment method and reinvestment rate we adopt. We describe the four investment ranking models in Table 9.7 below.

Table: 9.7. Alternative methods for adjusting challenging investments to resolve periodic and initial size differences

	Assume the reinvestment rate is the defender's IRR equal to *r*	Assume the reinvestment rate is the challengers' IRR
Resolve initial size differences by scaling the smaller challenger with a factor equal to $\gamma > 1$ 1" title="Rendered by QuickLaTeX.com" height="16" width="42" style="vertical-align: -4px;">	model 1	model 2
Resolve initial size differences by adding amount δ to the smaller challenger	model 3	model 4

We now list below the PV models consistent with models 1 through 4. We first write the unadjusted IRR and NPV models as:

$$V_0 = \frac{R_1}{(1+IRR)} + \frac{R_2}{(1+IRR)^2} + \cdots + \frac{R_n}{(1+IRR)^n} \tag{9.20}$$

and

$$NPV = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} \tag{9.21}$$

Model 1. We write the adjusted IRR* and NPV* model 1 as:

(adjusted IRR model 1)

$$V_0 = \frac{(1+r)^{n-1}R_1 + (1+r)^{n-2}R_2 + \cdots + R_n}{(1+IRR^*)^n} \tag{IRR 1}$$

(adjusted NPV model 1)

$$\begin{aligned} NPV^* &= -\gamma V_0 + \frac{(1+r)^{n-1}\gamma R_1 + (1+r)^{n-2}\gamma R_2 + \cdots + \gamma R_n}{(1+r)^n} \\ &= \gamma NPV \end{aligned} \tag{NPV 1}$$

Model 2. We write the adjusted IRR and NPV as:

(adjusted IRR model 2)

$$\gamma V_0 = \frac{(1+IRR)^{n-1}\gamma R_1 + (1+IRR)^{n-2}\gamma R_2 + \cdots + \gamma R_n}{(1+IRR)^n} \tag{IRR 2}$$

(adjusted NPV model 2)

$$\begin{aligned} NPV^* &= -\gamma V_0 \\ &\quad + \frac{(1+IRR)^{n-1}\gamma R_1 + (1+IRR)^{n-2}\gamma R_2 + \cdots + \gamma R_n}{(1+r)^n} \end{aligned} \tag{NPV 2}$$

Model 3. We write the adjusted IRR* and NPV* model 3 as:

(adjusted IRR model 3)

$$V_0 + \delta = \frac{(1+r)^{n-1}R_1 + (1+r)^{n-2}R_2 + \cdots + R_n + \delta(1+r)^n}{(1+IRR^*)^n}$$

(IRR 3)

(adjusted NPV model 3)

$$\begin{aligned} NPV^* &= -(V_0 + \delta) \\ &\quad + \frac{(1+r)^{n-1}R_1 + (1+r)^{n-2}R_2 + \cdots + R_n + \delta(1+r)^n}{(1+r)^n} \\ &= NPV \end{aligned}$$

(NPV 3)

Model 4. We write the adjusted IRR* and NPV* model 4 as:

(adjusted IRR model 4)

$$\begin{aligned} &V_0 + \delta \\ &= \frac{(1+IRR)^{n-1}R_1 + (1+IRR)^{n-2}R_2 + \cdots + R_n + \delta(1+IRR)^n}{(1+IRR)^n} \end{aligned}$$

(IRR 4)

(adjusted NPV model 4)

$$\begin{aligned} NPV^* &= -(V_0 + \delta) \\ &\quad + \frac{(1+IRR)^{n-1}R_1 + (1+IRR)^{n-2}R_2 + \cdots + R_n + \delta(1+IRR)^n}{(1+r)^n} \\ &= MNPV \end{aligned}$$

(NPV 4)

Comments on the Four Investment Models and Ranking Consistency

A comment about ranking consistency. Before commenting on models 1 through 4, we note the following. In some cases periodic and initial size-adjusted IRRs and NPVs produce the same rankings as the original IRRs and NPVs. In these cases, it simplifies our efforts to be able to use the original NPV or IRR rankings knowing they will provide the consistent rankings produced by the size adjusted IRR and NPV models.

Comments on model 1. Notice that we can factor out the scaling factor in NPV model 1 producing the original NPV multiplied by the scaling factor—γNPV. Moreover, ranking investments by γNPV provides the same ranking as size adjusted IRRs and NPVs. So, if ranking needs to occur under conditions consistent with model 1, our recommendation is to rank challengers using γNPV.

Comments on model 2. Notice that the scaling factor in IRR model 1 cancels so that we are left with our original IRR model except that periodic cash flow are reinvested until the last period. But, compounding and discounting by the same term are offsetting actions. As a result, the IRR rankings provides the same ranking as the adjusted IRR model. So, if ranking needs to occur under conditions consistent with model 2, our recommendation is rank challengers using their original IRRs.

Comments on model 3. Notice that compounding and discounting by the factor (1 + *r*) in model 3 are offsetting actions so that the adjustment produces the original NPV model except for the added factor δ that is added at the beginning and subtracted at the end of the period—eliminating it's influence in the model. As a result, the original IRR rankings are the same as the size adjusted IRR and NPV models. So, if ranking needs to occur under conditions consistent with model three, our recommendation is rank challengers using their original IRRs.

Comments on model 4. Notice that compounding and discounting using the challenger's IRR are offsetting actions so that the adjustment produces the original IRR model except for the added factor δ. However, δ is added at the beginning and subtracted at the end of the period eliminating it from the model. As a result, the IRR rankings are the same rankings provided by the size adjusted IRR and NPV models. So, if ranking needs to occur under conditions consistent with model four, our recommendation is

rank challengers using their original IRRs.

Other size adjustment models. For completeness, it is important to note that there are a number of other size adjustment methods we could employ. For example, we could combine challengers 1 and 2 in Table 9.1 to achieve initial size consistency with challenger 3. Or we could use some combination of scaling and addition to adjust for initial size differences. And the alternative reinvestment rates we could employ to resolve periodic size differences is large indeed. The only consideration when employing these nonstandard size adjustments is that the recommendations included in comments for models one through four no longer apply, requiring decision makers to find size adjusted NPVs or IRRs.

When to Rank Challengers Using Four Models

Adjusting initial investment sizes: scaling versus addition. The appropriate initial size adjustment method depends on the characteristics of the challengers in which investments are made. To compare challengers of different initial sizes, scaling to achieve initial size differences is appropriate when the challengers are available in continuous sizes or approximately so. We describe such investments as producing constant returns to scale. For constant returns to scale, the relationship between the investment and the cash flow it produces is constant. To illustrate, hours of services may often be purchased in divisible units, each unit producing the same output as earlier ones. In sum, models 1 and 2 apply when the investments are models available in continuous sizes.

On the other hand, some investments are available only in a few sizes. Machinery is available in a limited number of sizes. Sometimes, land may be purchased in fixed quantities depending on physical configurations. The same is true for buildings. So how do we compare investments available in a few sizes?

Suppose we are comparing two investments of different sizes. Assume we find the difference in their initial cost and ask: if the smaller challenger is adopted, where is the difference in their initial cost employed and what does it earn? The answer is it remains invested in the defender and these must be included with the earnings of the smaller challenger. In this way we achieve initial size consistency while allowing the underlying investments to be different in their initial sizes.

Adjusting periodic size differences: reinvestment rate assumptions. We resolved periodic size differences by setting cash flow in all of the periods to zero except cash flow in the last period. We set all of the cash flow in all but the last period equal to zero by reinvesting them. The implicit reinvestment rate for NPV and IRR models is the defender's or the challengers' IRRs.

Ranking Investments when the Challengers are not Mutually Exclusive

So far we have treated our challengers as mutually exclusive investments—only one can be chosen. For firms with a fixed investment budget, treating investments as mutually exclusive may be justified. Suppose that we face a set of challenging investments that are not mutually exclusive and their adoption depends on their passing some IRR hurdle. How should the investment problem be analyzed? In other words, suppose that we have a portfolio of different investment challengers and of different sizes. How big or how many challengers should be adopted ?

Economic theory can help us answer this question. Usually we array our challengers according to their IRRs and invest in the most profitable ones first. Of course, the size of the defender must match the size of the challenger, but the criterion for arranging defenders to match the size of our challenger is somewhat different. We give up, sacrifice, the defender with the smallest IRR first. To assume that defenders are infinitely scalable is to believe that the firm is in a perfect market, an assumption we earlier rejected.

Of course, investments may not come in infinitely small amounts; one large investment may be more profitable than a small investment because it is larger. If challengers come in lumpy amounts, that is the size of the challenger we investigate.

So, we are back to the place we started. Begin with the highest rate of return challenger, and sacrifice the lowest rate of return defender. Then analyze the next highest rate of return challenger and sacrifice the next lowest rate of return defender, all the time maintaining size equality. Keep analyzing challengers and defenders sequentially, maintaining size homogeneity, until you fail to adopt the next highest rate of return challenger compared to the next lowest rate of return defender.

This rule is comparable to the marginal cost equals marginal return rule employed in comparative economic static analysis.

The logic behind this arrangement is that we begin by considering the most favorable challenger, and compare it with the least

favorable defender. Then we continue to the next most favorable challenger and second least favorable defender. Once we have found a challenger unable to earn a higher IRR than the IRR of the defender being sacrificed, we end our investment analysis.

Common Practice, Common Problems, and Reinvestment Rates

Common practice is to rank challengers using IRR or NPV methods. We have emphasized that this common practice can frequently produce a common problem of inconsistent rankings. Then we made the point that when challengers were adjusted for initial and periodic size differences and employed a common reinvestment rate—either the defender or the challengers' IRRs—then the ranking were consistent. The question is, can consistent IRR and NPV rankings be achieved under fewer restrictions? For example, could we relax the common reinvestment rate assumption and still find consistent rankings—all the time? The answer is no!

Since we require only one counter-example to disprove the general claim, we provided one in Table 9.2. Consider three challengers of equal initial size and different periodic cash flow. Since reinvesting the cash flow to achieve periodic size equality at the defender's IRR doesn't change NPV rankings and reinvesting at the challenger's IRR doesn't change IRR rankings, we can rank the three challengers using NPV and IRR under initial and periodic size equality. We reported the results of such a comparison earlier in Table 9.2.

Note the conflicting NPV and IRR rankings. NPV ranks the three challengers 1, 2, and 3. IRR ranks the three challengers 3, 1, and 2. These results provide the one counter example needed to disprove the claim that different reinvestment rates are not required for consistent rankings. Furthermore, since we are describing the reinvestment rate for the same investment and are only changing the ranking method, consistency would require the same reinvestment rate assumption regardless of the ranking method employed.

Summary and Conclusions

This chapter has demonstrated that equal initial investments and periodic cash flows reinvested until the last period plus a common reinvestment rate across IRR and NPV models are sufficient conditions for consistent MIRR and MNPV investment rankings. We can make unequal initial investments equal by scaling or adding investments to the small challengers—and by some other methods, if required. Whether or not initial size differences are resolved by scaling or by addition depends on whether the challenger is available in continuous sizes in which case scaling is appropriate. Otherwise, if it is unique and cannot be scaled, size differences are resolved by additions. All other methods for resolving initial size differences ultimately create additional challengers.

Resolving periodic size differences required that we reinvest cash flow until the final period. We considered the defender's IRR and the challengers' IRRs as possible reinvestment rates. Of course, there are a number of other possible reinvestment rate choices. The important point is that if the discount rate and the reinvestment rate are different, then the NPV rankings will be stable and will not vary with the discount rate.

So what have we learned? As long as the homogeneity of size principle is observed, whatever common reinvestment rate is assumed or whatever initial size adjustment method is employed will generate consistent rankings. These possibilities we described as four basic models. In addition, while all size adjusted PV models will produce consistent rankings, the ranking may vary with the methods adopted. The main implication of these results is to shift the debate from which ranking method to use, IRR versus NPV, to what is the appropriate size adjustment model to adopt?

Questions

1. Discuss why some investment analysts prefer NPV methods to rank investments while others prefer IRR methods. Which do you prefer? Defend your answer.
2. Provide three reasons why NPV and IRR rankings may be inconsistent.
3. Refer to the investment problems described in Table Q9.1 below. Notice that initial investment sizes are unequal as are the periodic cash flows in period one which produce the inconsistent NPV and IRR ranking reported in the last two columns and rows of Table Q9.1. We will refer to the results in Table Q9.1 in several questions that follow.

Table Q9.1. NPVs and IRRs for two mutually exclusive investments assuming a discount rate equal to the defender's

IRR of r = 5%.

initial investments V_0^i where *i* = 1,2	period one cash flows R_t^i where *t* = 1 generated by initial investments V_0^i where *i* = 1,2	period two cash flows R_t^i where *t* = 2 generated by initial investments V_0^i where *i* = 1,2	NPV^i earned by initial investments V_0^i where *i* = 1,2 assuming *r* = 5% (ranking)	IRR^i earned by initial investments V_0^i where *i* = 1,2 (ranking)
$V_0^1 = \$3,000$	$2,000	$1,500	$265.31 (1)	11.51% (2)
$V_0^2 = \$1,000$	$800	$400	$124.72 (2)	14.83% (1)

a. Complete Table Q9.2 by finding MNPVs and MIRRs for the two investments assuming a discount rate and reinvestment rate equal to the defender's IRR, of *r*=5%. Include the investment rankings in the parentheses.

Table Q9.2. MNPVs and MIRRs for two mutually exclusive investments assuming a discount and reinvestment rate of r = 5%.

initial investments V_0^i where *i* = 1,2	period one cash flows R_t^i where *t* = 1 generated by initial investments V_0^i where *i* = 1,2	period two cash flows R_t^i where *t* = 2 generated by initial investments V_0^i where *i* = 1,2	MNPV^i earned by initial investments V_0^i where *i* = 1,2 assuming *r* = 5% (ranking)	MIRR^i earned by initial investments V_0^i where *i* = 1,2 assuming *r* = 5% (ranking)
$V_0^1 = \$3,000$		$1,500 + $2,000(1.05) = $3,600	$ ( )	% ( )
$V_0^2 = \$1,000$		$400 + $800(1.05) = $1,240	$ ( )	% ( )

b. Complete Table Q9.3 by finding MNPVs and MIRRs for the two investments assuming a discount rate and reinvestment rate equal to the defender's IRR, of *r* = 5% and a reinvestment rates equal to the investments' IRRs. Include the investment rankings in the parentheses.

Table Q9.3. MNPVs and MIRRs for two mutually exclusive investments assuming a discount and reinvestment rate of *r* = 5%. and reinvestment rates equal to the investments' IRRs

initial investments V_0^i where *i* = 1,2	period one cash flows R_t^i where *t* = 1 generated by initial investments V_0^i where *i* = 1,2	period two cash flows R_t^i where *t* = 2 generated by initial investments V_0^i where *i* = 1,2	MNPV^i earned by initial investments V_0^i where *i* = 1,2 assuming the discount and reinvestment rate equal *r* = 5% (ranking)	MIRR^i earned by initial investments V_0^i where *i* = 1,2 assuming the reinvestment rate equals *r* = 5% (ranking)
$V_0^1 = \$3,000$		$1,500 + $2,000(1.1151) = $3,730.20	$ ( )	% ( )

initial investments V_0^i where i = 1,2	period one cash flows R_t^i where t = 1 generated by initial investments V_0^i where i = 1,2	period two cash flows R_t^i where t = 2 generated by initial investments V_0^i where i = 1,2	MNPV^i earned by initial investments V_0^i where i = 1,2 assuming the discount and reinvestment rate equal r = 5% (ranking)	MIRR^i earned by initial investments V_0^i where i = 1,2 assuming the reinvestment rate equals r = 5% (ranking)
$V_0^2 = \$1,000$		\$400 + \$800(1.1483) = \$1,318.64	\$ ( )	% ( )

c. Complete Table Q9.4 by finding scaled MNPVs and scaled MIRRs for the two investments assuming a discount rate and reinvestment rate equal to the defender's IRR, of r=5%. What is the scaling factor you are using? Include the investment rankings in the parentheses.

Table Q9.4. Scaled MNPVs and MIRRs for two mutually exclusive investments assuming a discount and reinvestment rate of r = 5%.

initial investment $\gamma_i V_0^i$ where i = 1,2	period one cash flows $\gamma_i R_t^i$ where t = 1 generated by initial investments $\gamma_i V_0^i$ where i = 1,2	period two cash flows $\gamma_i R_t^i$ where t = 2 generated by initial investments $\gamma_i V_0^i$ where i = 1,2	Scaled MNPV^i earned by scaled initial investments $\gamma_i V_0^i$ where i = 1,2 assuming the discount and reinvestment rate equal r = 5% (ranking)	Scaled MIRRs earned by scaled initial investments $\gamma_i V_0^i$ where i = 1,2 assuming the discount and reinvestment rate equal r = 5% (ranking)
$V_0^i = \$3,000$		\$1,500 + [\$2,000(1.05)] = \$3,600	\$ ( )	% ( )
$\gamma_2 V_0^2 = \$3,000$ $\gamma_2 = 3$		[(3)\$400] + [(3)\$800(1.05)] = \$3,720	\$ ( )	% ( )

d. Complete Table Q9.5 by finding scaled MNPVs and scaled MIRRs for the two investments assuming a discount rate equal to the defender's IRR, of r = 5% and reinvestment rates equal to the investments' IRRs. Include the investment rankings in the parentheses.

Table Q9.5. Scaled MNPVs and MIRRs for two mutually exclusive investments assuming a discount rate of r = 5% and reinvestment rates equal to the investments' IRRs

initial investment $\gamma_i V_0^i$ where i = 1,2	period one cash flows $\gamma_i R_t^i$ where t = 1 generated by initial investments $\gamma_i V_0^i$ where i = 1,2	period two cash flows $\gamma_i R_t^i$ where t = 2 generated by initial investments $\gamma_i V_0^i$ where i = 1,2	Scaled MNPV^i earned by scaled initial investments $\gamma_i V_0^i$ where i = 1,2 assuming the discount rate r = 5% and reinvestment rates equal to the investments' IRRs (rankings)	Scaled MIRRs earned by scaled initial investments $\gamma_i V_0^i$ where i = 1,2 assuming the reinvestment rate equals r = 5% (ranking)
$V_0^i = \$3,000$		\$1,500 + [\$2,000(1.1151)] = \$3,730.20	\$ ( )	% ( )

initial investment $\gamma_i V_0^i$ where *i* = 1,2	period one cash flows $\gamma_i R_t^i$ where *t* = 1 generated by initial investments $\gamma_i V_0^i$ where *i* = 1,2	period two cash flows $\gamma_i R_t^i$ where *t* = 2 generated by initial investments $\gamma_i V_0^i$ where *i* = 1,2	Scaled MNPV^i earned by scaled initial investments $\gamma_i V_0^i$ where *i* = 1,2 assuming the discount rate *r* = 5% and reinvestment rates equal to the investments' IRRs (rankings)	Scaled MIRRs earned by scaled initial investments $\gamma_i V_0^i$ where *i* = 1,2 assuming the reinvestment rate equals r = 5% (ranking)
$\gamma_2 V_0^2 = \$3,000$ $\gamma_2 = 3$		[(3)$400] + [(3)$800(1.1483)] = $3,955.92	$ ( )	% ( )

e. Complete Table Q9.6 by finding added MNPVs and MIRRs for the two investments assuming a discount and reinvestment rate equal to the defender's IRR, of *r* = 5%. What is the additional investment amount δ_i added to the smaller investment? Include the investment rankings in the parentheses.

Table Q9.6. Added MNPVs and MIRRs for two mutually exclusive investments assuming a discount rate of *r* = 5% and reinvestment rates equal to the defender's IRR of r = 5%.

Initial plus added investments $(V_0^i + \delta_i)$ where i = 1,2	period one cash flows R_t^i where *t* = 1 generated by initial plus added investments $(V_0^i + \delta_i)$ where *i*=1,2	period two cash flows $R_t^i + \delta_i(1+r)^2$ where *t*=2 generated by initial plus added investment $(V_0^i + \delta_i)$ where *i* = 1,2	Added MNPV^i earned by initial plus added investments $(V_0^i + \delta_i)$ where *i* = 1,2 assuming the discount rate and reinvestment rate *r* = 5% (rankings)	Added MIRR^i where *i* = 1,2 earned by initial plus added investments $(V_0^i + \delta_i)$ where *i* = 1,2 assuming the reinvestment rate *r* = 5% (rankings)
$V_0^i = \$3,000$		$1,500 + $2,000(1.05) = $3,600	$ ( )	% ( )
$(V_0^2 + \delta_2) = \$3,000$ where $\delta_2 = \$2,000$		$400 + $800(1.05) + $2,000$(1.05)^2$ = $3,445	$ ( )	% ( )

f. Complete Table Q9.7 by finding added MNPVs and MIRRs for the two investments assuming a discount rate equal to the defender's IRR, of *r* = 5% and reinvestment rates equal to the investments' IRRs. Include the investment rankings in the parentheses.

Table Q9.7. Added MNPVs and MIRRs for two mutually exclusive investments assuming a discount rate of *r* = 5% and reinvestment rates equal to the investments' IRRs

Initial plus added investments $(V_0^i + \delta_i)$ where *i* = 1,2	period one cash flows R_t^i where *t* = 1 generated by initial plus added investments $(V_0^i + \delta_i)$ where *i*=1,2	period two cash flows R_t^i where *t*=2 generated by initial plus added investment $(V_0^i + \delta_i)$ where *i* = 1,2	Added MNPV^i earned by initial plus added investments $(V_0^i + \delta_i)$ where *i* = 1,2 assuming the discount rate *r* = 5% and reinvestment rates equal to the investments' IRRs (rankings)	Added MIRR^i earned by initial plus added investments $(V_0^i + \delta_i)$ where *i* = 1,2 assuming the reinvestment rate *r* = 5% (rankings)
$V_0^i = \$3,000$		$1,500 + [$2,000(1.1151)] = $3,730.20	$ ( )	% ( )
$(V_0^2 + \delta_2) = \$3,000$ where $\delta_2 = \$2,000$		$400 + [$800(1.1483)] + $2,000(1.1483)2 = $3,955.83	$ ( )	% ( )

4. To answer the questions that follow, first complete Table Q9.8 using data from the completed Tables in question 3. Cells indicated with an "n.a." do not require an answer.

Table Q9.8. Summary of rankings described in Tables Q9.1 through Q9.7

Table provide the data	Type of ranking	Assuming reinvestment rate is defender's IRR		Assuming rein
		Investment 1	**Investment 2**	**Investment 1**
Q9.1	NPV			n.a.
	IRR	n.a.	n.a.	
Q9.2	MNPV			n.a.
	MIRR			n.a.
Q9.3	MNPV	n.a.	n.a.	
	MIRR	n.a.	n.a.	
Q9.4	Scaled MNPV			n.a.
	Scaled MIRR			n.a.
Q9.5	Scaled MNPV	n.a.	n.a.	
	Scaled MIRR	n.a.	n.a.	
Q9.6	Added MNPV			n.a.
	Added MIRR			n.a.
Q9.7	Added MNPV	n.a.	n.a.	
	Added MIRR	n.a.	n.a.	

a. Referring to your completed Table Q9.8, which ranking methods produce consistent results? Please explain your answer.
b. Referring to your completed Table Q9.8, which consistent ranking method(s) are consistent with NPV rankings? Please explain your answer.
c. Referring to your completed Table Q9.8, which consistent ranking method(s) are consistent with scaled NPVs? Please explain your answers.
d. Referring to your completed Table Q9.8, which consistent ranking method(s) are consistent with IRRs? Please explain

your answers.

5. Explain why investments of unequal term can be examined using the ranking methods described in this chapter.

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1. Much of the material in this chapter was published in Robison, Myers, and Barry (2015). ↵

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10: Homogeneous Terms

Learning Objectives

Assuming reinvestment rate is investments IRR After completing this chapter, you should be able to: (1) consistently rank one-time investments using internal rate of return (IRR) or net present value (NPV) methods; (2) find time adjusted cash flow averages called annuity equivalents (AE); (3) use AE to find the optimal life of a repeatable investment; and (4) use capitalization rates to find the present value of long-lived investments and repeatable investments.

To achieve your learning goals, you should complete the following objectives:

- Learn appropriate methods for ranking one-time investments with unequal terms using IRR or NPV ranking criteria.
- Learn how to represent the time adjusted average of an investment's cash flow using AE.
- Learn how IRR and NPV methods may provide conflicting optimal replacement ages for repeatable investments.
- Learn how to use an investment's cash flow patterns and its AE to determine the optimal life of a repeatable investment.
- Learn how to find the present value of future earnings from repeatable investments using the capitalization formula.

Introduction

One-time investments. In the previous chapter, we developed methods for ranking one-time mutually exclusive investments with unequal initial and periodic sizes. In that effort, we employed a simplifying assumption: that the terms of the competing challengers and the defender were equal. This assumption is frequently violated. Not all challengers and defenders have equal terms or economic lives. Therefore, we develop methods for ranking one-time mutually exclusive challengers of unequal terms which is the first goal of this chapter. An important characteristic of one-time investments is that owning a one-time investment does not require replacement in order to invest in a similar investment.

Repeatable investments. Some investments provide essential services. As a result, when they wear out, they need to be replaced (think of a light bulb). What replaces the existing investment may be an identical, improved, or remodeled version of the original investment. We call these investments that require replacement, repeatable investments because they are owned in sequence rather than simultaneously. Examples of repeatable investments include orchards, breeding livestock, roofs on houses, and equipment. Since repeatable investments are owned in sequence, we must determine what is the optimal time to replace a repeatable investment.

AE and cash flow patterns. Finding the optimal life of a repeatable investment requires that we review the concept of a time-adjusted average cash flow, an AE. AE, of course, change as new periods of cash flow are included in its calculation. Using AE for different time periods and patterns of cash flow, we can determine the optimal life of the investment.

Capitalization rates. To rank repeatable investments, we must value the present value of the original investment and their future replacement's cash flow. We accomplish this task by using capitalization rates, a common tool used by appraisers to value long-term investments.

Present Values and Capitalization Rates

The economic life and term of an investment. The term of an investment is the number of periods the financial manager expects to manage an investment. The economic life of an investment is the number of periods the investment is expected to generate cash flow. The PV of an investment for an individual depends on its cash flow during the term of the investment plus its liquidation value. The liquidation value of the investment depends on its economic life. Therefore, to estimate the present value for an investment, we must find the discounted present value of all future cash flows. In practice, we often estimate the present value of an investment using the capitalization formula.

The capitalization formula and capitalization rate. Consider a challenging investment that earns R constant cash flow dollars for n period and is liquidated at the end of the nth period for V_n. We assume that the defender funding the investment has an IRR of r. We write the maximum bid (minimum sell) price PV model that equates V_0 to its discounted future earnings over a term of n

periods discounted by its defender's IRR as:

$$V_0 = \frac{R}{(1+r)} + \cdots + \frac{R+V_n}{(1+r)^n} \tag{10.1}$$

Now assume that the second owner of the investment has the same earnings expectations as the first owner of the durable so that we can write:

$$V_n = \frac{R}{(1+r)} + \cdots + \frac{R+V_{2n}}{(1+r)^{2n}} \tag{10.2}$$

Finally, substituting for V_n in Equation ??? the right hand side of Equation ??? we obtain:

$$V_0 = \frac{R}{(1+r)} + \cdots + \frac{R+V_{2n}}{(1+r)^{2n}} \tag{10.3}$$

And if the investment's salvage value were continually exchanged with its expected cash flows we could write:

$$V_0 = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots \tag{10.4}$$

Clearly, the farther away from the present is the constant cash flow *R*, the less it contributes to the present value of the investment. To demonstrate the diminishing contributions of future cash flow, note how the value of discount rate that multiplies *R* decreases with *n*. To illustrate we let *r* = 10%, *R*=100, and alternative values of *n*.

$$\frac{\$100}{(1.1)^{20}} = \$14.86$$

$$\frac{\$100}{1.1)^{40}} = \$2.21$$

$$\frac{\$100}{(1.1)^{60}} = \$0.33$$

$$\frac{\$100}{(1.1)^{80}} = \$0.05$$

And

$$\frac{\$100}{(1.1)^{100}} = \$0.01$$

So what is the sum of an infinite stream of constant payments *R* discounted by *r* percent? To find that sum, multiply both sides of Equation ??? by (1 + *r*) and subtract from the result the original equation:

$$V_0(1+r) - V_0 = R - \frac{R}{(1+r)} + \frac{R}{(1+r)} - \frac{R}{(1+r)^2} + \frac{R}{(1+r)^2} - \cdots - \frac{R}{(1+r)^{mn}} + \frac{V_{mn}(1+r)+V_{mn}}{(1+r)^{mn}} \tag{10.5}$$

After subtracting and simplifying and letting *m* get very large, we find the sum of the infinite series to equal:

$$V_0 = \lim_{m\to\infty} \frac{R}{r}\left[1 - \frac{1}{(1+r)^{mn}}\right] + \frac{V_{mn}}{(1+r)^{mn}} = \frac{R}{r} \tag{10.6}$$

The far right-hand side of Equation ??? we refer to as the capitalization formula where the investment's AE is equal to *R* divided by the defender's IRR equal to *r*, the capitalization rate. The capitalization formula allows us to relate the present value of all future cash flow to the value of the investment.

$$V_0 = \frac{R}{r} \tag{10.7}$$

To illustrate the capitalization formula, suppose that you purchase an annuity for $100 that pays you and your heirs $5 a year forever. The capitalization formula for this investment is equal to:

$$V_0 = \frac{R}{r} = \frac{\$5}{r} \tag{10.8}$$

If we knew the investment's initial value and its infinite stream of AEs, we could always estimate the capitalization rate equal to:

$$r = \frac{R}{V_0} = \frac{\$5}{\$100} = 5\% \tag{10.9}$$

Comments about the capitalization formula and capitalization rate. In textbooks and references to the capitalization rate, it is often expressed as:

$$r = \frac{annual\,cash\,income}{purchase\,price} \tag{10.10}$$

Then when practitioners implement the capitalization formula, Equation ???, they estimate *R* as the first period's net cash flow and an industry capitalization rate for *r*. For example, if the industry standard were 5%, we would write the capitalization formula that estimates the maximum bid price for the investment as:

$$V_0 = \frac{R}{r} = \frac{\$5}{.05} = \$100 \tag{10.11}$$

PV models and capitalization formulas. The capitalization formula is a PV model. The unknown variable in the capitalization formula identifies the kind of PV model represented. If the unknown variable is the discount rate or the capitalization rate r, the capitalization formula is an IRR model. Most often, capitalization formulas solve for V_0 making them maximum bid (minimum sell) type models.

So what have we learned? We learned that we can best understand the capitalization formula and the capitalization rate in the context of a PV model. Despite its various descriptions in applied publications, the capitalization formula is the AE of the investment over its economic life divided by defender's IRR.

More Complicated Capitalization Formulas

In the discussion to this point, we implemented the capitalization formula by assuming that the future cash flow were constants, AEs. This of course, is rarely the case. We now ask: how can we find the capitalization formula if future cash flow are expected to increase (decrease) at *g*% over time as you might expect on rental property? To answer this question we return to our PV model.

Suppose that we wanted to find the capitalization formula for an investment whose cash flow are expected to increase (decrease) at an average rate of *g* percent ($g < 0$ percent). Then the capitalization formula that accounts for *R* is increasing (decreasing) at rate *g* ($g < 0$) as:

$$V_0 = \frac{R(1+g)}{(1+r)} + \frac{R(1+g)^2}{(1+r)^2} + \cdots = \frac{R(1+g)}{(r-g)} \tag{10.12}$$

Now the capitalization rate, the defender's IRR is equal to: $(r - g)/(1 + g)$. We write the capitalization rates depending on alternative values of g and r = 10 in Table 10.1

Table 10.1a. Alternative percentage capitalization rates $(r - g)/(1 + g)$ depending on alternative values of $g > 0$ and r

	g = 0%	g = 1%	g = 2%	g = 3%	g = 4%	g = 5%
r = 10%	0.1	0.089	0.078	0.068	.058	0.048
r = 9%	0.09	0.079	0.069	0.058	0.048	0.038
r = 8%	0.08	0.069	0.059	0.049	0.038	0.029
r = 7%	0.07	0.059	0.049	0.039	0.029	0.019
r = 6%	0.06	0.050	0.039	0.029	0.019	0.01

Table 10.1b. Alternative percentage capitalization rates $(r - g)/(1 + g)$ depending on alternative values of $g < 0$ and r

	g = 0%	g = –1%	g = –2%	g = –3%	g = –4%	g = –5%
r = 10%	0.10	0.111	0.122	0.134	0.146	0.158
r = 9%	0.09	0.101	0.112	0.124	0.135	0.147
r = 8%	0.08	0.091	0.102	0.113	0.125	0.137
r = 7%	0.07	0.081	0.092	0.103	0.115	0.126
r = 6%	0.06	0.071	0.082	0.093	0.104	0.116

So what have we learned? We have learned that expected increases in cash flow decrease capitalization rates and increase the value of capitalized income. For example if R = \$100, r = 10% and g = 0% so that the capitalization rate is 10%, then V_0 = \$1000. If R = \$100, r = 10%, and g = 5% then the capitalization rate is 4.8%, and V_0 = \$2,083.33. We have also learned that expected decreases in cash flow increase capitalization rates and decrease the value of capitalized income. For example, if R = \$100, r = 10%, and g = –5% then capitalization rate is 15.8%, then V_0 = \$632.91.

Ranking One-time Investments with Unequal Terms using NPV Models

Notation. Before proceeding to the first focus of this chapter, we confirm the notation used earlier and which will be used again in this chapter. The mathematical notation will describe two challenging investments and a defender. We assume that the initial investment sizes are equal to V_0. The defender's IRR for the two investments is r. The term of investments one and two are n_1 and n_2 respectively. Periodic cash flows for investments i = 1, 2 in period t = 1, ..., n_i can be expressed as R_t^i. And the vector of cash flows is represented as: $\mathbf{R}^i = R_1^i, R_2^i, \ldots, R_{n_i}^i$ for i = 1, 2.

Finally, we define the sum S of compounded periodic cash flows at rate r as: $S(r, \mathbf{R}^i, n_{in}) = (1+r)^{(n_i-1)}R_1^i + (1+r)^{(n_i-2)}R_2^i + \cdots + R_{n_i}^i$. We also define the sum of periodic cash flows compounded at the investment's IRR as: $S(IRR^i, \mathbf{R}^i, n_i) = (1+IRR^i)^{(n_i-1)}R_1^i + (1+IRR^i)^{(n_i-2)}R_2^i + \cdots + R_{n_i}^i$. Note that the value of the function S depends on three variables defined in the equation: the reinvestment rate r or IRR^i, the vector of periodic cash flows

$\mathbf{R}^i$, and the term of the investment n_i.

Sufficient conditions for consistently ranking mutually exclusive one-time investments using IRRs and NPVs. We discovered in Chapter 9 that there are two sufficient conditions for consistently ranking investments using NPVs and IRRs—assuming their terms were equal. These two sufficient conditions are equal initial investment sizes and equal periodic cash flows except in their common last period.

In this chapter we will assume that initial sizes of investments are equal. However, we will allow for differences in periodic cash flows because investment terms differ. As a result, we can no longer be sure that the NPVs and IRRs rank investments the same. To solve the problem of unequal periodic cash flows, we need to rationalize investment term differences. Our focus in the first part of this chapter is on how to create equal terms and periodic cash flows except for the last period for mutually exclusive challengers.

Compounding and discounting by the same rate are offsetting operations. Essential to rationalizing term differences is the obvious fact that NPVs for investment one and two compounded and discounted at rate r have the same value as their original NPV function. We demonstrate this point using the following equations. For investment one, the result is:

$$\begin{aligned} NPV^1 &= -V_0 + \frac{R_1^1}{(1+r)} + \cdots + \frac{R_{n_1}^1}{(1+r)^{n_1}} \\ &= -V_0 + \frac{(1+r)^{(n_1-1)}R_1^1 + (1+r)^{(n_1-2)}R_2^1 + \cdots + R_{n_1}^1}{(1+r)^{n_1}} \\ &= -V_0 + \frac{S(r, \mathbf{R}^1, n_1)}{(1+r)^{n_1}} \end{aligned}$$

(10.13a)

Similarly, for investment two:

$$\begin{aligned} NPV^2 &= -V_0 + \frac{R_1^2}{(1+r)} + \cdots + \frac{R_{n_2}^2}{(1+r)^{n_2}} \\ &= -V_0 + \frac{(1+r)^{(n_2-1)}R_1^1 + (1+r)^{(n_2-2)}R_2^2 + \cdots + R_{n_2}^2}{(1+r)^{n_2}} \\ &= -V_0 + \frac{S(r, \mathbf{R}^2, n_2)}{(1+r)^{n_2}} \end{aligned}$$

(10.13b)

What equations 10.13a and 10.13b illustrate is the obvious: multiplying by one, the compound rate divided by the identical discount rate, cannot change the value of what is being multiplied.

In Chapter 9, we created equal periodic cash flows by reinvesting the periodic cash flows until the last period, creating MNPV and MIRR models. Then we learned that if the reinvestment rate was equal to the discount rate that NPV and MNPV and IRR and MIRR models produced identical results because reinvesting and discounting by the same rate are offsetting operations. What we learn in this chapter confirms this principle, that reinvesting and discounting by the same rate are offsetting operations. Furthermore, this principle can be used to resolve differences in investment terms.

Resolving term differences. We can convert challengers to the same term by reinvesting and discounting their cash flow to a common term. The compound factor that converts the periodic cash flows and the discount rate from period n_2 to n_1 is $(1+r)^{(n_1-n_2)}$. This is applied to the extreme right-hand side of equation (10.13b) resulting in the expression:

(10.14) $(1 + \mathrm{r})^{n_1}$

Equation (10.14) confirms once again that compounding and discounting by the same rate are offsetting operations even when used to extend the term of investments. Nevertheless, by compounding and discounting by the same rate (multiplying by one), we

convert the term of investment two to the term of investment one without changing the value of the function. It is still equal to the original NPV equation.

The rankings of investments one and two—assuming their reinvestment rates and discount rate are the defender's IRR, *r*—can be expressed as:

$$NPV^1 - NPV^2 = \frac{S(r, \mathbf{R}^1, n_1)}{(1+r)^{n_1}} - \frac{S(r, \mathbf{R}^2, n_1)}{(1+r)^{n_1}} \tag{10.15}$$

Clearly, the only difference between the two NPVs are their vector of cash flow since they have equal initial investments, discount (reinvestment) rates, and terms.

So what have we learned? We learned that it is okay to compute and rank mutually exclusive investments using their original NPV equations even though their terms are not the same as long as the reinvestment rate is the defender's IRR. In other words, the compounding and discounting are offsetting operations so that NPV models that convert future streams of cash flow to the present are unaffected.

Ranking One-time Investments with Unequal Terms using IRR Models.

Now suppose that we replace the reinvestment rate r in equations 10.13a with the IRR of challenger one equal to IRR1. Also assume that we replace the reinvestment rate r in 10.13b with the IRRs of challenger two, IRR2. We continue to assume that the discount rate for both investments is the defender's IRR. The revised MIRR ranking equation can be expressed as:

(10.16) n_1} \end{split}\end{equation*}

So what have we learned about MIRRs and IRRs? Equation (10.16) reveals an interesting result: the MIRRs are equal to the IRRs because compounding and discounting IRR have offsetting effects. Therefore, Equation ??? provides no new information beyond what we already had discovered when we found the investments' IRR.

IRR and NPV Models for Analyzing Repeatable Investments

The two preceding sections resolve term differences by assuming the reinvestment rates were either the defender's or the challengers' IRRs. The problem is that we can no longer be certain that the NPV and IRR methods rank the challengers consistently since we violated the common reinvestment rate assumption—the reinvestment rates were the challengers' own IRRs. If we allow each investment to reinvest in itself, we lose our consistency guarantee.

Another problem using IRR methods besides losing consistency with NPV rankings is that methods for finding the optimal term involve finding the term with the greatest IRR instead of finding the term with the largest AE. This complicates an already complicated subject.

For these and several other reasons, in the remainder of this chapter we will analyze repeatable investments using NPV model assumptions, the most notable of which is that the reinvestment rate is the defender's IRR common to both challengers. To be clear, we could assume a common defender's IRR or each challenger's IRR but having more than one challengers' IRR may produce inconsistent rankings and asymmetry in exchanges. Were we to assume a still different reinvestment rate besides these, the defender's or the challengers' IRR would imply there exists another challenger besides those being considered and, if so, should be considered as a separate challenger.

So in what follows, we assume that earnings from the challengers are reinvested in the defender and asymmetry between exchanges of dollars between time periods. In the next section we begin building the AE tool using NPV assumptions required to analyze repeatable investments.

Present Value (PV) Models and Averages

Ranking investments using their annuity equivalents (AE). Referring to our previous analogy, trying to rank two investments of different terms would be like trying to rank horses in a race in which they each ran different distances. If we did indeed wish to

compare two horses that ran different distances, at least we could compare their average speed—their average speed per mile. Then, even though the comparison might not be perfect, at least the comparisons would be compatible.

This is the essence of ranking investments by their time adjusted averages, referred to earlier as their annuity equivalents (AE). Ranking investments using the AE will be essential when we find the optimal age of repeatable investments.

Arithmetic means, expected values, geometric means and AE. There are several measures of central tendency in a numeric series that include arithmetic means, expected values, and geometric means. An example of an arithmetic mean or average follows. Consider three numbers 3, 5, and 7. The average of these numbers can be calculated by dividing their sum by 3 since there are three numbers: (3 + 5 + 7) / 3 = 5. This is the average of this series. Now suppose we wanted to find the mean of the three numbers weighted by their probability of occurring. If the probability of 3 occurring were 25%, if the probability of 5 occurring were 25%, and the probability of 7 occurring were 50%, then the expected value of the series would be: [(.25)3] + [(.25)5] + [(.5)7] = 5.50. This is the weighted average or expected value of the series.

Next consider an example of a geometric mean. Consider three rates of return: 105%, 110% and 115%. The geometric mean is that number which, when multiplied together three times, equals the product of 105%, 110%, and 115%: [(1.05)(1.10)(1.15)] = $(1.0991)^3$. Alternatively, the geometric mean is $_{n\ =\ 1.0992}$ where n = 3 because there are three numbers in the series. Note that the geometric mean is not equal to the arithmetic mean: [(1.10 + 1.11 + 1.12)] / 3 = 1.11. It is also not necessarily equal to the expected value.

To illustrate AE, the constant R in Equation ??? is an AE whose present value sum equals the present value of the sum of discounted cash flow $R^1, R^2, \cdots, R^n$.

$$\frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} \tag{10.17}$$

For example, consider the AE in the following problem:

$$\frac{15}{(1.1)} + \frac{20}{(1.1)^2} = \frac{17.38}{(1.1)} + \frac{17.38}{(1.1)^2} \tag{10.18}$$

On the left hand side of Equation ??? is a stream of unequal periodic cash flow. On the right hand side of Equation ??? is a stream of equal periodic cash flow each of which is an AE. The important fact, however, is that the discounted AE on the right-hand side of Equation ??? equals the discounted periodic cash flow on the left-hand side of Equation ???. The AE for the series on the right-hand side of Equation ??? is 16.85. We demonstrate how to find an AE using Excel as follows.

Table 10.2a. How to find an AE for an irregular stream of cash flow

Open Table 10.2a. in Microsoft Excel

B6	***Function*:**		=NPV(B2,B3:B4)
	A	**B**	C
1	How to find an AE for an irregular stream of cash flow		
2	rate	0.1	
3	R1	15	
4	R2	20	
5	nper	2	
6	NPV	**$30.17**	NPV(rate, R1:R2)

Calculating AE from an irregular stream of cash flow is a two-step procedure. The first step is to find the net present value (NPV) of the irregular cash flow stream. We illustrate this step using Excel's NPV function. In our example, the NPV of $15 received at

the end of period one and $20 received at the end of period two is $30.17.

The next step is to find the AE, a constant payment, for the NPV equal to $30.17. Using Excel's PMT function we find the AE for an NPV of $30.17 equal to $17.38. We display the Excel solution below.

Table 10.2b. How to find an AE for an irregular stream of cash flow

Open Table 10.2b. in Microsoft Excel

B7		***Function*:**	=PMT(B2,B5,B6,,0)
	A	**B**	C
1	How to find an AE for an irregular stream of cash flow		
2	rate	.1	
3	R1	15	
4	R2	20	
5	nper	2	
6	NPV	$30.17	NPV(rate,R1:R2)
7	AE	**($17.38)**	PMT(rate,nper,NPV,,0)

So what have we learned about AE? An AE is a constant cash flow stream whose discounted value is to equal the present value of another stream of unequal cash flow. Each value in the stream of equal cash flow is an AE. If one calculated the arithmetic mean of the AE in the steam of cash flow, the arithmetic average would also equal the value of any other AE in the series. If the discount rate were an interest rate and V_0 were an amount borrowed, then AE would be the constant loan payment whose present value would equal the original loan amount.

Some observations on NPV and AE rankings. Consider an NPV model and its value expressed as the present value of a series of AE in Equation ???. Since the present value of the AE is equal to the NPV of the investment, then the two sums must provide equal NPV rank. Important to note, however, is that it is the present value of the series of AE payments that is equal to the NPV, while a number of unequal payments could equal the same NPV. Therefore, there is a direct relationship (i.e. they both go up or down together) between NPVs and AEs. Furthermore,

$$\begin{aligned} NPV &= -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} \\ &= \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} \end{aligned} \tag{10.19}$$

any change in NPV must be matched by a corresponding change in the AE and in the same direction. For any two investments of equal size and term, and where investment one has a larger NPV than investment two, then investment two must increase its AE in order for it to equal the higher NPV amount. We illustrate this point in more detail.

Consider again the expression:

$$NPV = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} \tag{10.20}$$

Then, suppose the term of the model in Equation ??? is increased by one period. Then the equality in Equation ??? no longer holds:

$$NPV \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} + \frac{R}{(1+r)^{n+1}} \tag{10.21}$$

To preserve the equality in Equation ???, the AE equal to *R* must be decreased by some amount δ to reestablish the equality allowing us to rewrite the earlier equality:

$$\begin{aligned} NPV &= \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} \\ &= \frac{R-\delta}{(1+r)} + \frac{R-\delta}{(1+r)^2} + \cdots + \frac{R-\delta}{(1+r)^{n+1}} \end{aligned} \tag{10.22}$$

So what have we learned? We learned that NPVs and the present value of their AE provide equal rankings. Furthermore, any sum or vector of cash flow can be converted to its present value equivalent annuity. To maintain the equality between a present value and a stream of annuities requires the following. If the present value sum is increased, then either the term or the amount of the annuity must increase.

Numerical Demonstrations

Term difference and inconsistent rankings. Consider Table 10.3. Note that the two challengers are not periodic size consistent because they withdraw funds at different rates. Challenger one withdraws all of its earnings after one period. Meanwhile, challenger two withdraws some of its earnings in period one and the remainder in period two. As a result, the two investments have unequal terms.

In Table 10.3 panel a, we rank the two challengers using their NPV, IRR, and AE assuming that the discount rate and the reinvestment rate is 10%. In Table 10.3 panel b, we make the two challengers periodic size-consistent by reinvesting period one earnings for one period at the defender's IRR. This operation also resolves term differences between the two investments.

Table 10.3. The Influence of periodic Size and Term Differences Created by Differential Withdrawals.

Panel a. NPV, IRR and AE rankings assuming different periodic cash flows and terms for challengers one and two and a discount rate equal to the defender's IRR of 10%.

Challengers	Initial Outlay	Cash Flows in period one	Cash Flows in period two	NPV (rankings)	IRR (rankings)	AE (rankings)
C1	$1,000	$1,180	$0	$72.73 (2)	18% (1)	$72.73 (1)
C2	$1,000	$160	$1,160	$104.13 (1)	16% (2)	$60.00 (2)

Panel b. NPV, IRR, and AE rankings assuming equal periodic cash flows and terms for challengers one and two where equal periodic cash flows and terms are achieved by reinvesting period one cash flows at the defender's IRR rate of 10% to a common ending period. NPV and IRR rankings after adjusting for term differences assuming a reinvestment rate of r.

Challengers	Initial Outlay	Cash Flows in Period one	Cash Flows in Period two	NPVs and MNPVs (rankings)	IRR (rankings)	AE (rankings)
C1	$1,000	$0	$1,180 (1.10) = $1,298	$72.73 (2)	6.7% (2)	$49.91 (2)
C2	$1,000	$0	$160 (1.10) + $1,160 = $1,336	$104.13 (1)	15.6% (1)	$60.00 (1)

Panel c. NPV, IRR, and AE rankings assuming equal periodic cash flows and terms for challengers one and two where equal periodic cash flows and terms are achieved by reinvesting cash flows at the investments' IRRs. The discount rate is assumed to equal the defender's IRR of 10%

Challengers	Initial Outlay	Cash Flows in Period one	Cash flows in Period two	NPV (MNPV) (rankings)	IRR (rankings)	AE (rankings)
C1	$1,000	$0	$1,180 (1.18) = $1,392.40	$150.74 (1)	18% (1)	$86.66 (1)
C2	$1,000	$0	$160 (1.16) + $1,160 = $1,345.60	$112.07 (2)	16% (2)	$64.57 (2)

Term and periodic cash flow differences in Table 10.3 panel a produced inconsistent rankings using NPV versus IRR and AE methods. However, in Table 10.3 panel b, when term and periodic cash flow differences were eliminated except for the common last period through reinvesting at the defender's IRR of 10%, NPV, IRR, and AE rankings were consistent. In Table 10.3 panel c, when term and periodic cash flow differences were eliminated through reinvesting at the challengers' respective IRRs of 18% and 16%, NPV, IRR, and AE rankings were again consistent but changed from the rankings produced when the reinvestment rate was the defender's IRR.

There are two things to be emphasized about Table 10.3. First, NPVs in panel a and panel b are the same even after adjusting for differences in periodic cash flows and terms. This is because the reinvestment rate was the defender's IRR, and reinvesting and discounting cash flows are offsetting operations. Second, when we adjusted for periodic size inconsistencies using the investments' IRRs as the reinvestment rate, NPVs changed but the investments' IRRs were equal to their MIRRs in panels a and c. This is again the result of reinvesting and discounting by the same rate—the investments' IRRs.

So what have we learned? We learned that if investments have term differences and the reinvestment rate is the defender's IRR then NPV rankings are appropriate. If investments have term differences and the reinvestment rates are the investments' IRRs then IRR rankings are appropriate.

Alternative reinvestment rate assumptions. In Table 10.3, we made the first challenger into a two-period model by reinvesting its earnings at the defender's IRR. Suppose the one-period challenger was available for investment in each period. In other words, suppose that challenger one could be repeated. Since challenger one is an investment of size $1,000, then only $1,000 of period one earnings could be reinvested in the one-period challenger. The difference between the challenger's first-period earnings and

$1,000 we assume will be invested at the defender's IRR. The new investment problem is summarized in Table 10.4.

Table 10.4. Resolving Term Differences Between Two Challengers by Reinvesting $1,000 of Period One Earnings at its One-period IRR of 18% and Reinvesting other Funds at the Defender's IRR of 10%.

Challengers	Initial Outlay	Cash Flow in period one	Cash Flow in period two	NPV assuming defender's IRR is 10% (rankings)	IRR (rankings)	AE (rankings)
C1	$1,000	$0	$1,000 (1.18) + $180 (1.1) = $1,378	$138.84 (1)	17.4% (1)	$80.00 (1)
C2	$1,000	$0	$1,160 + $160 (1.1) = $1,336	$104.13 (2)	15.6% (2)	$60.00 (2)

The interesting result of Table 10.4 provides an example of a blended reinvestment rate not equal to either the defenders IRR of 10% or the investment's own IRR but a weighted average of them both equal to 17.4% for investment one and 15.6% for investment two. However, viewing investment opportunities as combinations of investments in the defender and the challenger must be considered to be a new challenger with a unique reinvestment rate.

Using Annuity Equivalents (AE) to Rank Repeatable Investments

The difficulty of finding finite number of replacements to resolve term differences. In our previous example, we resolved term differences between a one-period investment and a two-period investment by repeating the first investment. Now suppose we have a more complicated term inconsistency problem. For example, assume challenger one's term is 7 periods while challenger two's term is 8 periods. Now repeating an investment one or several times won't resolve term differences. Indeed, to resolve term differences in this problem would require that challenger one be repeated 8 times and challenger two be repeated 7 times. Now we have a 58-period model—which requires a lot of work.

Using Annuity Equivalents (AE) to rank repeatable investments. We can resolve term differences by calculating and comparing the AE of the investments, even though they have different terms. The reason we can use the AE to rank investments of different terms is because the annuity equivalent doesn't change when you increase the term by repeating investments. Thus, the AE from one investment repeated 2, 3, 4, 7, 8, *m*, or an infinite number of times is the same. This is an important fact because the AE calculated over the lives of multiple (infinite) replacements can be compared to the AE of another repeatable investment and the two investments can be ranked by their difference.

We now support the claim that we can rank repeatable investments by their AE. We write the one-period model as:

$$NPV = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} \tag{10.23a}$$

We write the NPV model with one replacement as:

$$NPV + \frac{NPV}{(1+r)^n} = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} + \frac{1}{(1+r)^n}\left[-V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n}\right] \tag{10.23b}$$

And we could write the NPV model with enough replacements to equalize their term as:

$$NPV + \frac{NPV}{(1+r)^n} + \cdots = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} + \frac{1}{(1+r)^n}\left[-V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n}\right] + \cdots \tag{10.23c}$$

Next, factoring, we obtain:

$$NPV\left[1 + \frac{1}{(1+r)^n} + \cdots\right] = \left[-V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n}\right]\left[1 + \frac{1}{(1+r)^n} + \cdots\right] \tag{10.23d}$$

Then, canceling the two bracketed terms at the end of each equation, we regain our original one-investment problem:

$$NPV = -V_0 + \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots + \frac{R_n}{(1+r)^n} = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots + \frac{R}{(1+r)^n} \tag{10.23e}$$

A numerical example that the AE for a single investment is equal to the AE calculated over two investments. Note that the same AE that solved the one-investment problem solves the multiple-replacement problem.

We demonstrate this result in Table 10.5 which calculates AE for one investment and then recalculates the AE for an investment and one replacement.

Table 10.5. Resolving Term Inconsistencies by Calculating AE for an Investment and the Investment and a Replacement Assuming Defender's IRR is 10%

Challenger	V_0	R_1	R_2	R_3	R_4	NPV (rankings)	II (rank
C1	$2,000	$1,200	$1,200			$82.64 (2)	13 (
C1 plus replacement	$2,000	$1,200	$1,200 – $2,000 = –$800	$1,200	$1,200	$150.95 (1)	13 (

Table 10.5 illustrates the importance of AE rankings to resolve term (and size) inconsistencies for repeatable investments. Note

first that term inconsistencies produce different NPVs. The NPVs are positive because the challenger earns a higher rate of return than the defender. Furthermore, collecting these returns for two challengers, the challenger and its replacement, earns more than just one investment. Hence, the NPV for the challenger and its replacement is greater than the NPV for just one challenger. On the other hand, both the IRR and the AE rank the investments the same, because their calculations are adjusted for the term of the investment. These results are also helpful because they confirm that the IRR of a single defender can be used to discount the challenger and still maintain consistency.

So what is our best advice? Resolve term inconsistencies for repeatable investments by calculating AE.

So what have we learned about ranking repeatable investments? When asked to rank two challengers that are repeatable but have unequal term and their reinvestment rate is the defender's IRR, rank them by their respective first investment's AE.

Finding AE that account for technologically improved replacements. Suppose that one of the challengers will be replaced by a technologically improved replacement that perhaps costs more, but also produces higher returns. For γ > 1 the investment problem takes the following form:

$$NPV + \frac{(1+\gamma)NPV}{(1+r)^n} + \cdots \tag{10.24}$$

Including enough replacement to equalize terms, we find the sum of the discounted NPVs. Call this sum S which is equal to:

$$\begin{aligned} S &= NPV\left[1 + \frac{1+\gamma}{(1+r)^n} + \frac{(1+\gamma)^2}{(1+r)^{2n}} + \cdots\right] \\ &= NPV\left[\frac{1}{1 - \frac{1+\gamma}{(1+r)^n}}\right] \end{aligned} \tag{10.25}$$

The details of the derivation are not included here, but require nothing more that the summation of geometric series. The interpretation of Equation ??? is that technological improvements result in NPV increases in the first replacement by γ percent. The second replacement's NPV increase by $(1+\gamma)(1+\gamma)$—over the first one. Thus, the NPVs increased from NPV to $(1+\gamma)NPV(1+\gamma)^2NPV$ etc. We demonstrate the effect of technologically improved replacement on the ranking of investments in Table 10.5. To simplify our calculations, we assume that we have already found the NPV of the two challengers equal to $100 for the first investment and $150 for the second investment. The term differences of the two investments are n_1 = 10 for the first investment and n_2 = 20 for the second investment. The rate of technological improvements are 5% for investment one and 3% for investment two. To demonstrate the importance of accounting for technological improvements, we find the AE unadjusted for technological improvement and the AE accounting for technological improvement.

Table 10.6. Ranking Adjusted for Term Differences and Technologically Improved Replacements. Rankings Assuming Defender's IRR is 10%

Challenger	NPV (rankings)	Terms	Technological change coefficient γ	Adjustment coefficient $\left[\frac{1}{1-\frac{1+\gamma}{(1+r)^n}}\right]$	AE not adjusted for technological change (rankings)	NPVs adjusted for technological change (rankings)
C1	NPV_1 = \$100 (2)	n_1 = 10	γ_1 = 5%	1.68	\$16.27 (2)	\$100(1.68) = \$168 (1)
C2	NPV_2 = \$150 (2)	n_2 = 20	γ_2 = 3%	1.18	\$17.62 (1)	\$150(1.18) = \$177.12 (2)

It is useful to note in Table 10.6 that NPV and AE unadjusted for technological change are consistent as our theory implies. However, once we account for technological change, the investment rankings based on the investments AE are reversed. Investment one is preferred even though its NPV is less than that of challenger's two NPV.

Inconsistent rankings were not caused by failing to adjust for differences in size and terms. As we have already demonstrated, these can be rationalized using AE. What produced the inconsistencies was comparing the rankings without technologically improved replacements (unadjusted AE) versus including the assumption of technologically improved replacements (adjusted for technology rankings).

Capitalizing AE to find the present value of a stream of repeatable investments. If we are comparing repeatable investments with different terms, then the comparisons are not between individual investments but with the present value sum of all the investments in each cash flow stream. In this regard we could compare the AE since the AE ordering of investment is the same as the NPV orderings. Or, we could capitalize the AE to find the present value sum of all of the investments. To understand how to capitalize the AE, that is, to find its value over an infinite number of repeatable investments, we write Equation ???:

$$\begin{aligned} NPV\left[1+\frac{1}{(1+r)^n}+\cdots\right] = \\ \left[\frac{R}{(1+r)}+\frac{R}{(1+r)^2}+\cdots+\frac{R}{(1+r)^n}\right]\left[1+\frac{1}{(1+r)^n}+\cdots\right] \\ = \frac{R}{(1+r)}+\frac{R}{(1+r)^2}+\cdots+\frac{R}{(1+r)^{mn}} \\ = \lim_{m\to\infty}\frac{R}{r}\left[1-\frac{1}{(1+r)^{mn}}\right] = \frac{R}{r} \end{aligned} \tag{10.26}$$

To get some idea of how fast convergent to the capitalization formula R/r occurs in Equation ???, if n = 10 and r = 10%, then for m = 2, then $[1 - 1/(1 + r)mn] = .85$; for m = 3, then $[1 - 1/(1 + r)mn] = .94$, and for m = 4, then $[1 - 1/(1 + r)mn] = .98$. Finally, for m = 5, then $[1 - 1/(1 + r)mn] = .991$.

Finding the Optimal Replacement Age for Different Repeatable Investments

The condition that identifies the optimal replacement age for a repeatable investment. The optimal age for each repeatable investment in a stream of repeatable investments is that age that maximizes the NPV for the entire stream of repeatable investments. Finding the optimal age of a repeatable investment is a ranking problem. Only in this case, each challenger is defined by its replacement age, and each age-differentiated investment is considered to be a different challenger. Our goal is to find optimal replacement age.

The key to understanding when to replace repeatable investments (without employing a lot of calculus) is to think about averages—or in our case, AE. We want to maximize the present value sum of NPVs for the entire stream of repeatable investments—not just the NPV for an individual investment.

If the investments in a series of repeatable investments have identical cash flow patterns, then the rule for maximizing the NPV of the present value sum for all repeatable investment is to find the term that maximizes the AE for a single investment. Thus, for a repeatable investment, if holding the investment for an additional period increases the AE for the challenger, then the investment should be held for at least another period—until holding the investment another period decreases the AE. Of course, the periodic cash flows could still be positive and NPV increasing even though AE are decreasing. Thus holding the investment for the term that maximizes its NPV is definitely not the same rule as holding the investment for the term that maximizes its AE.

The pattern of cash flows is the ultimate determinant of an investment's optimal life. Since the calculus requires a smooth inverted cup-like shape for maximization, we typically assume investment cash flows have corresponding patterns. However, the pattern for an investment's periodic cash flows are not the same as the pattern of AE for an investment at different ages. Consider some different kinds of investments and cash flow patterns and their corresponding optimal lives.

Finding the optimal replacement age for a growth and decay type investment. The growth and decay type of investment, after the initial investment, is identified by increasing periodic cash flows followed by decreasing periodic cash flows. Specifically, suppose that we have a repeatable investment with cash flows reported for 6 periods. Assuming the reinvestment rate and the discount rate are 10%, we find the NPV for each investment assuming it has an economic life of one period, two periods, three periods, and up to six periods. Then we find the AE for the investments at their alternative economic lives. Finally, we capitalized the AE by 10% (divide the AE by .1) to find the lifetime present value of the multiple investments at their alternative ages. The results are reported in Table 10.7.

Table 10.7. An Example of a Growth and Decay Type Investment.

Time period	Cash flow per period	NPV per investment for alternative investment lives discounted at 10%	AE per investment for alternative investment lives discounted at 10%	Present value sum of an infinite number of repeatable investments
0	($300.00)	($300.00)		
1	$150.00	($163.64)	($180.00)	($18,000.00)
2	$275.00	$63.64	$36.67	$3,667.00
3	$130.00	$161.31	$64.87	$6,487.00
4	$70.00	$209.12	$65.97	$6,597.00
5	$30.00	$227.75	$60.08	$6,008.00
6	($10.00)	$222.10	$51.00	$5,100.00

If our goal were to maximize the NPV for one of the repeatable investments, we would hold the investment until it no longer produced positive cash flows—in our example until period 5 with a cash flow of $30 and lifetime present earnings of $6,008.00. But if our goal is to optimize our lifetime earnings from a large number of repeatable investments, then we would hold each investment until they reach age 4 with a cash flow of $70 and lifetime earnings of $6,597.00. Another way to report the results of Table 10.6 is to note that as long as the periodic cash flow exceeds the AE in a period, adding that period to the life of the

investment will increase the AE and the lifetime earnings of the repeatable investments.

Finding the optimal replacement age for a light bulb type investment. Recall that the "light bulb" type of investment describes a category of investments which, after the initial investment, produce a nearly constant level of services that are virtually undiminished over its economic life. Then, at some point, the investment stops providing services and the investment dies a sudden death. An example cash flow pattern for this investment assumes that the investment dies in period 5. The investment is described in Table 10.8.

Table 10.8. An Example of a Light Bulb Type Investment.

Time period	Cash flow per period	NPV per investment for alternative investment lives discounted at 10%	AE per investment for alternative investment lives discounted at 10%	Present value sum of an infinite number of repeatable investments
0	($300.00)	($300.00)		
1	$100.00	($209.09)	($230.00)	($23,000.00)
2	$100.00	($126.45)	($72.86)	($7,286.00)
3	$100.00	($51.31)	($20.63)	($2,063.00)
4	$100.00	$16.99	$5.36	$536.00
5	$0	$16.99	$4.48	$448.00
6	$0	$16.99	$3.90	$390.00

The light bulb type investment has an important pattern that is easily recognized. It is that as long as its constant cash flows are positive, it's NPV and AE are increasing. However, once the investment dies and its positive cash flows end, its NPV is constant but its AE is continually decreasing. Thus, the optimal life of a light bulb is to keep it until it dies. Of course, this recommendation could be modified if there were serious costs associated with an interruption of services and that the exact period in which the investment died was not known with certainty.

Finding the optimal replacement age for a continuous decay type investment. The continuous decay type investment is one in which the investment performs best when new and then, with use and time, its service capacity decreases and its maintenance requirements increase so that its periodic cash flows exhibit a continuous decay. An example of such a periodic cash flow pattern is described in Table 10.9.

Table 10.9. An Example of the Continual Decay Type Investment.

Time period	Cash flow per period	NPV per investment for alternative investment lives discounted at 10%	AE per investment for alternative investment lives discounted at 10%	Present value sum of an infinite number of repeatable investments
0	($4300.00)	($300.00)		
1	$190.00	($127.27)	($139.96)	($13,996.00)
2	$152.00	($1.65)	($.95)	($95.00)
3	$129.00	$95.27	$38.31	$3,831.00
4	$85.00	$153.32	$48.37	$4,837.00
5	$38.00	$175.92	$46.41	$4,641.00
6	$8.00	$181.43	$41.66	$4,166.00

In the continual decay model, after the cost of the initial investment is paid, the periodic cash flows of the investment continually

decrease. Still they increase the NPV of the investment as long as they are positive. At some point, the value of earning high returns during the early life of the investment swamps the cost of acquiring a new investment and the AE decrease indicating the optimal age of the investment. In this example, the optimal life of the investment is 4 periods with a corresponding present value sum of earnings over an infinite number of repeatable investments equal to $4,837.00.

Finding the optimal replacement age for an investment with irregular periodic cash flows. The last category of investments considered are those whose cash flow patterns are unique. That is, the cash flow pattern for an investment owned one year is different than the same investment owned two years, three years, and so on. To illustrate this type of investment, consider a machinery owner who custom hires using his machine to perform services for customers. The cash flow pattern for the machine begins with a capital purchase followed by three years of nearly constant cash flows which then decrease by 25% per year—mostly because of repairs but also because the demand for custom hires using older machines decreases. In the year the machine is replaced, the old machine earns a liquidation value that depends on the age of the machine. In this problem, the machine at each age is considered a unique challenger even though it is the same machine differentiated by age. A description of the cash flows for this problem follows.

Table 10.10 Investments With Irregular Cash Flow Characterized by Constant and then Declining Cash Flow with an Income Spike in the last Period of the Investment's Economic Life. The Discount and Reinvestment Rates are Assumed to Equal 10%.

Period	3 year old challenger	4 year old challenger	5 year old challenger	6 year old challenger	7 year old challenger	8 year old challenger
0	($100)	($100)	($100)	($100)	($100)	($100)
1	$40	$40	$40	$40	$40	$40
2	$40	$40	$40	$40	$40	$40
3	liquidation = $64	$32	$32	$32	$32	$32
4	0	liquidation = $51.20	$25.60	$25.60	$25.60	$25.60
5	0	0	liquidation = $40.96	$20.48	$20.48	$20.48
6	0	0	0	liquidation = $32.77	16.38	$16.38
7	0	0	0	0	liquidation = $26.21	$13.11
8	0	0	0	0	0	liquidation = $20.97
Summary Measures						
NPVs	$17.51	$28.43	$36.38	$42.16	$46.36	$49.42
AE	$7.04	$8.97	$9.60	$9.68	$9.52	$9.26
Capitalized AE	$70.40	$89.70	$96.00	$96.80	$95.20	$92.60

Note that NPVs are increasing with the age of the challengers. The maximum AE is earned in the sixth period and declines for each of the older challengers. Thus the optimal age for the challengers is age 6.

Summary and Conclusions

In Chapter 9, we found two sufficient conditions for consistently ranking mutually exclusive investments. The two conditions are that initial investment sizes and periodic cash flows are equal except for their last common period. In this chapter we extended

the results of Chapter 9 by developing methods to rank investments of unequal terms. Ranking unequal term investments are problematic because unequal terms create unequal periodic cash flows, violating the second of two sufficient conditions for consistently ranking investments using IRRs and NPVs.

In our efforts to find methods for ranking investments of unequal terms, we found that MNPV and MIRR models would rank investments equally. But we also found that under some conditions, IRR and NPV models could produce equal rankings. In this chapter, we emphasized that adjusting for term differences by reinvesting and discounting by the defender's IRR that NPV and MNPV and MIRR models would produce consistent rankings. These results would not hold when some other reinvestment rate applies. These findings led to some important practical results: when the reinvestment rate and the discount rate are the same, rank investments using NPVs. When the reinvestment rates are the investments' IRRs, rank using investments' IRRs.

In the second part of this chapter, we considered repeatable investments. If by repeating investments for a required number of times the investments had a common ending date, then individual investment term differences could be ignored. In effect, the entire stream of repeatable investments could be considered to be a single investment. Of course, if the number of repeatable investments was considered to be infinite then the term problem is resolved.

Assuming equal initial investment sizes, we found that the repeatable investment's optimal age was the age that maximized AE for a single investment. Thus, we can find the optimal replacement age for repeatable investments in a stream of replacements by finding the age that maximizes AE for any one investment. Capitalizing the AE provides us with an estimate of the present value of the earnings from the stream of repeatable investments.

Questions

1. Describe the three sufficient conditions required for consistent IRR and NPV rankings for one-time investments? Explain how investment term differences violate one of the two sufficient conditions.
2. Some investments are one-time investments. Others are repeatable. Describe what conditions produce one-time investments. Then describe what conditions produce repeatable investments. Give examples of one-time and repeatable investments.
3. One way to resolve term differences is to reinvest the periodic cash flows of both investments to a common ending period. Explain the implications of assuming that the reinvestment rates are the defender's IRR, the investments' IRR, or some other rate.
4. Explain the effects on an investment's NPV if term differences are resolved by reinvesting its periodic cash flows to some common period using the defender's IRR, while discounting the reinvested funds over the changed terms by the same rate, the defender's IRR.
5. Explain the difference, if any, between an investment's IRR and its MIRR if the reinvestment rate is the investment's IRR. Depending on your answer, what practical recommendation would you offer to financial managers wanting to rank investments whose earnings would be reinvested in themselves?
6. In the table below, term differences are resolved by reinvesting periodic cash flows to the common ending period assuming the reinvestment rate and the discount rate is the defender's IRR of 10%. Produce a similar table assuming the same initial investment sizes and cash flows, only assume the defender's IRR is 5% not 10%. Then associate your results with the ranking possibilities described in Table 10.1 by declaring which of the four models correspond to your table.

Table Q10.1. The influence of periodic size and term differences created by differential withdrawals assuming a reinvestment rate equal to the defender's IRR of 10%. The discount rate for the NPV and MNPV models equal the defender's IRR. The discount rate for the IRR and MIRR models are the IRRs and MIRRs

Challengers	Initial Outlay	Cash Flow in period one	Cash Flow in period two	NPV (rankings)	IRR (rankings)	MNPV (rankings)	MIRR (rankings)
C1	$900	$1090	0	$90.91 (2)	$21.11% (1)	$90.91 (2)	15.42% (2)

Challengers	Initial Outlay	Cash Flow in period one	Cash Flow in period two	NPV (rankings)	IRR (rankings)	MNPV (rankings)	MIRR (rankings)
C2	$900	$160	$1050	$113.22 (1)	17.27% (2)	$113.22 (1)	16.71% (1)

7. In the table below, term differences are resolved by reinvesting periodic cash flows to the common ending period assuming the reinvestment rates are the investments' IRRs and the discount rate is 10%. Produce a similar table assuming the same initial investment sizes and cash flows, only assume the defender's IRR is 5% not 10%. Then associate your results with the ranking possibilities described in Table 10.1 by declaring which of the four models correspond to your table.

Table Q10.2.The influence of periodic size and term differences created by differential withdrawals assuming a reinvestment rate equal to the challenger's IRRs. The discount rate for the NPV and MNPV models equal the defender's IRR. The discount rate for the IRR and MIRR models are the IRRs and MIRRs.

Challengers	Initial Outlay	Cash Flow in period one	Cash Flow in period two	NPV (rankings)	IRR (rankings)	MNPV (rankings)	MIRR (rankings)
C1	$900	$1090	$0	$90.91 (2)	$21.11% (1)	$190.91 (1)	21.11% (1)
C2	$900	$160	$1050	$113.22 (1)	17.27% (2)	$122.83 (2)	17.27% (2)

8. Referring to the completed tables in Questions 6 and 7, please answer the following. Why are NPV and MNPV ranking consistent and equal in amounts in the Question 6 table but inconsistent and different in amounts in the Question 7 table? And why are IRR and MIRR rankings inconsistent and unequal in the Question 6 table but consistent and equal in amount in the Question 7 table? What are the practical implications of these results?
9. Annuity equivalents are elements in a stream of constant periodic cash flows whose present value equals the present value of some fixed amount or the present value of a non-constant cash flow stream. What is the arithmetic mean of a series of AE? If the discount rate is 8% and the term is 10 periods, find the AE for the periodic cash flows 21, 34, 5, and 13. Then find the AE for a fixed present value amount of $199 assuming the same discount rate and term. Finally, recalculate the AE if the term is decreased from 10 periods to 5 periods.
10. A potential Uber driver can purchase a new car for $18,000. Then the car is expected to earn constant periodic cash flows for the next three years of $6,000. After that, mostly because of decreased demand for rides in older cars and higher repair costs, periodic cash flows decrease by 25% per year. The liquidation value of the new car after three years is $9,000 and then declines each year thereafter by 25%. Find the optimal age at which the Uber driver should replace cars. Then find the capitalized value of an investment in one car assuming each car is owned until its optimal age.

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11: Homogeneous Tax Rates

Learning Objectives

After completing this chapter, you should be able to: (1) construct effective tax rates on defending and challenging investments; (2) describe how effective tax rates depend on investment types; and (3) find effective tax rates on defending and challenging investments by calculating tax adjust coefficients.

To achieve your learning goals, you should complete the following objectives:

- Learn how to compute a defender's after-tax IRR by finding its tax adjustment coefficient.
- Learn how capital gains alter a defending investment's tax adjustment coefficient.
- Learn how to compute the tax adjustment coefficient for a single period.
- Learn how to compare the average effective tax rate for an investment.
- Learn how to find the tax adjustment coefficient for loans and investments with increasing (decreasing) cash flow.
- Learn how taxes and tax adjustment coefficients influence investment rankings.

Introduction

According to Benjamin Franklin, taxes are one of the two constants in life (the other one is death). Taxes need to be included in present value (PV) models because only what the firm earns and keeps after paying taxes really matters. Therefore, proper construction of PV models requires that taxes be consistently applied to defending and challenging investments.

A single-equation PV model compares two investments: a challenger, described by time-dated cash flow, and a defender, usually described by its internal rate of return (IRR) of its time-dated cash flow. Homogeneous tax rates require that the same units are used when describing the cash flow of the challenger as the cash flow used when finding the IRR of the defender. And in particular, homogeneous tax rates require that when the cash flow of the challenger are adjusted for taxes, the IRR of the defender must also be adjusted for taxes. This chapter shows how to find homogeneous tax rates in PV models.

It is popular to adjust a defender's before-tax IRR to an after-tax IRR by multiplying it by (1 – T), where T is the average tax rate applied to the investment. This chapter points out that this method for adjusting the defender's IRR for taxes is only appropriate in a few special cases. Finally, this chapter shows how to find appropriate tax rates for adjusting the defender's IRR for taxes in a variety of tax environments.

After-tax PV Models for Defenders and Challengers

The goal is to find a defender's after-tax IRR. The first step is to find the defender's before-tax IRR. Recall that the defender's before-tax IRR is that rate of return such that the NPV of the defender's before-tax net cash flow is zero. The defender's after-tax IRR discounts the defender's after-tax cash flow so that its NPV remains equal to zero.

There is an easy test to determine if taxes have been properly introduced into a PV model. First consider the defender's cash flow stream and its IRR. Then introduce taxes into the defender's cash flow stream and its IRR. If taxes have been properly introduced, then the NPV of the defender's after-tax cash flow stream discounted by its after-tax IRR is still zero.

To illustrate, consider a defending investment that earns constant net cash flow of R dollars per period in perpetuity. The maximum bid (minimum sell) price V_0 for this defending investment whose before-tax IRR is r. We describe this investment in Equation ???.

$$V_0 = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots = \frac{R}{r} \tag{11.1}$$

where $r = R/V_0$ is the before-tax IRR. Now suppose that we describe the defender's cash flow and the IRR for this same investment

on an after-tax basis. We do so by introducing the constant marginal income tax rate T into the model:

$$V_0 = \frac{R(1-T)}{[1+r(1-T)]} + \frac{R(1-T)}{[1+r(1-T)]^2} + \cdots = \frac{R}{r} \tag{11.2}$$

Income R and the before-tax IRR of the defender are both adjusted for taxes by multiplying by (1 – T). We can be sure that r(1 – T) is the after-tax IRR, since V_0 is the same in the before and after-tax models. However, we need to be aware that only in the special case of constant infinite income can we multiply r by (1 – T) to obtain the after-tax IRR.

A General Approach for Finding the Defender's After-tax IRR

A general approach for finding the defender's after-tax IRR follows. Consider a defender that has a market-determined value of V_0 and earns a before-tax cash flow stream of R_1, R_2, ..., and whose defender's before-tax IRR is r. A before-tax PV model for this defender can be written as:

$$\begin{aligned} V_0 &= \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots \\ &= \frac{R_1}{(1+r)} + \frac{1}{(1+r)}\left[\frac{R_2}{(1+r)} + \frac{R_3}{(1+r)^2} + \cdots\right] \end{aligned} \tag{11.3}$$

Now we express the PV model for the defender one period later as:

$$V_1 = \frac{R_2}{(1+r)} + \frac{R_3}{(1+r)^2} + \cdots \tag{11.4}$$

Note that the bracketed expression in Equation ??? equals the right-hand side of Equation ???. This equality allows us to substitute V1 for the bracketed expression in Equation ???. After making the substitution in Equation ??? we solve for capital gains equal to:

$$V_1 - V_0 = rV_0 - R_1 \tag{11.5}$$

Using the expression in Equation ??? we can solve for the defender's before-tax IRR equal to:

$$r = \frac{(V_1 - V_0) + R_1}{V_0} \tag{11.6}$$

We now want to find the after-tax IRR for the defender described in Equation ???. To do so, the IRR and the cash flow for the defender must be adjusted for taxes in such a way that V_0 in Equation ??? is not changed (and the defender's NPV is still zero). The defender's cash flow is adjusted for taxes by multiplying them by (1 – T). The defender's before-tax IRR is adjusted for taxes by multiplying it by $(1-\theta T)$ where θ, a tax adjustment coefficient, adjusts r to its after-tax equivalent such that V_0 is the same whether calculated on a before-tax or after-tax basis. Besides these tax adjustments, let T_j equal other taxes applied to the defender's cash flow that may include property taxes in the case of land, depreciation tax credits in the case of depreciable investments, and capital gains taxes when appropriate for assets earning capital gains. What all these taxes have in common is their functional dependence on the previous period's asset value.

The after-tax PV model corresponding to Equation ??? that leaves V_0 unchanged can be written as:

$$\begin{aligned} V_0 &= \frac{R_1(1-T) + T_j f(V_0)}{[1+r(1-\theta T)]} \\ &+ \frac{1}{[1+r(1-\theta T)]}\left[\frac{R_2(1-T) + T_j f(V_1)}{[1+r(1-\theta T)]} + \frac{R_3 + T_j f(V_2)}{[1+r(1-\theta T)]^2} + \cdots\right] \end{aligned} \tag{11.7}$$

Similarly, V_1 can be expressed as:

$$V_1 = \frac{R_2(1-T) + T_j f(V_1)}{[1 + r(1-\theta T)]} + \frac{R_3 + T_j f(V_2)}{[1 + r(1-\theta T)]^2} + \cdots \tag{11.8}$$

Replacing the bracketed expression in Equation ??? with V_1, we find the after-tax IRR for the defender as:

$$r(1-\theta T) = \frac{(V_1 - V_0) + R_1(1-T) + T_j f(V_0)}{V_0} \tag{11.9}$$

Finally, substituting for *r* in Equation ??? from the right-hand side of Equation ??? and solving for θ, we obtain:

$$\theta = \frac{R_1 - \left(\frac{T_j}{T}\right) f(V_0)}{R1 + (V_1 - V_0)} \tag{11.10}$$

The value for θ in Equation ??? adjusts the defender's before-tax IRR to obtain homogeneity of measures between the defender's after-tax IRR and the defender's after-tax cash flow in Equation ???. When homogeneity of measures is maintained, the defender's NPV is still zero whether calculated on a before-tax or after-tax basis. We now find some specific after-tax IRRs for various types of defenders.

Case 11.1. $(V_1 - V_0) = 0$ and $T_j f(V_0) = 0$. In this case, the defender earns neither capital gains nor suffers capital losses, in which case T_j = 0 . This case is illustrated by an infinite constant cash flow series, described in equations 11.1 and 11.2. Because the cash flow is constant, capital gains are zero. Furthermore, the defender's return in each period is equal to its cash receipts, which is fully taxed at income tax rate *T*. Therefore, the entire before-tax rate of return must be adjusted by (1 – *T*). In Equation ???, substituting zero for capital gains and depreciation or capital gains tax results in:

$$\theta = \frac{R_1 - \left(\frac{T_j}{T}\right) f(V_0)}{R_1 + (V_1 - V_0)} = \frac{R_1}{R_1} = 1 \tag{11.11}$$

Case 11.2. $(V_1 - V_0) > 0$ 0" title="Rendered by QuickLaTeX.com" height="18" width="103" style="vertical-align: -4px;"> and $T_j f(V_0) = 0$. In this case the investment earns capital gains that are not taxed, which lowers the defender's effective tax rate. Thus, $\theta < 1$ and $(1 - \theta T) < (1 - T)$. The greater the capital gains, the lower the effective tax rate in period t. We illustrate this type of model next.

Consider a defending investment whose before-tax cash flow grows geometrically at rate *g*. Then before-tax cash flow in period *t*, R_t, equals: $R_0 (1 + g)^t$, and we write the investment's IRR model as:

$$\begin{aligned} V_0 &= \frac{R_0(1+g)}{(1+r)} + \frac{1}{(1+r)}\left[\frac{R_0(1+g)^2}{(1+r)} + \frac{R_0(1+g)^3}{(1+r)^2} + \cdots\right] \\ &= \frac{R_0(1+g)}{(r-g)} \end{aligned} \tag{11.12}$$

One period later, we can write:

$$V_1 = \frac{R_0(1+g)^2}{(1+r)} + \frac{R_0(1+g)^3}{(1+r)^2} + \cdots = V_0(1+g) \tag{11.13}$$

and capital gains equal:

$$V_1 - V_0 = V_0(1+g) - V_0 = gV_0 \tag{11.14}$$

Then, substituting the right-hand side of Equation ??? for V_0 in Equation ???, we obtain:

$$gV_0 = \frac{gR_0(1+g)}{r-g} \tag{11.15}$$

Now we are ready to find the tax-adjustment coefficient in this model. Substituting into Equation ??? for capital gains and first period cash flow, we obtain:

$$\begin{aligned}\theta &= \frac{R_1 - \left(\frac{T_j}{T}\right) f(V_0)}{R_1 + (V_1 - V_0)} = \frac{R_0(1+g)}{R_0(1+g) + gV_0} \\ &= \frac{R_0(1+g)}{R_0(1+g) + \frac{gR_0(1+g)}{r-g}} = \frac{r-g}{r}\end{aligned} \tag{11.16}$$

Suppose that we naively introduced taxes into the growth model described in Equation ??? and obtained:

$$\begin{aligned}V_0 &= \frac{R_0(1+g)(1-T)}{[1+r(1-T)]} + \frac{R_0(1+g)^2(1-T)}{[1+r(1-T)]^2} + \cdots \\ &= \frac{R_0(1+g)(1-T)}{r(1-T)-g}\end{aligned} \tag{11.17}$$

In this case, an increase in T increases V_0:

$$\frac{dV_0}{dT} = \frac{R_0(1+g)g}{[r(1-T)-g]^2} > 0. \tag{11.18}$$

0. \end{equation*} " title="Rendered by QuickLaTeX.com">

Since increasing T increases V_0, we know that $r(1 - T)$ cannot equal the after-tax IRR for the defending investment.

So what have we learned? We learned how to interpret θ in the geometric growth model. Earlier we demonstrated that in the geometric growth model, capital gains are earned at rate g. But in this model, capital gains are not converted to cash, and so they are not taxed. Therefore, not all earnings on the investment are taxed, only the portion earned as cash. As a result, the effective tax rate is not T but θT, where θ is the percentage of returns earned as cash.

If, for example, r = 8%, g = 3%, then

$$\theta = \frac{.08 - .03}{.08} = .63. \tag{11.19}$$

Furthermore, for an investor in the 35% tax bracket whose cash flow is increasing at 3%, the effective tax rate would be $\theta T = (.63)(.35) = 22$.

Case 11.3. $(V_1 - V_0) < 0$ and $T_j f(V_0) = 0$. In this case the investment suffers capital losses for which no tax savings are allowed, which increases the defender's effective tax rate. Thus, $\theta > 1$ 1" title="Rendered by QuickLaTeX.com" height="13" width="40" style="vertical-align: -1px;"> and $(1 - \theta T) > (1 - T)$ (1 - T)" title="Rendered by QuickLaTeX.com" height="18"

width="145" style="vertical-align: -4px;">. The greater the capital losses, the higher is the effective tax rate in period *t*. We illustrate this type of model next.

Consider a defending investment whose before-tax cash flow declines geometrically at rate $d > 0$. Then before-tax cash flow in period *t*, R_t, equals: $R_0(1-d)^t$, and we write the investment's maximum bid price as:

$$V_0 = \frac{R_0(1-d)}{(1+r)} + \frac{1}{(1+r)}\left[\frac{R_0(1-d)^2}{(1+r)} + \frac{R_0(1+d)^3}{(1+r)^2} + \cdots\right] = \frac{R_0(1-d)}{(r+d)} \tag{11.20}$$

One period later, we can write:

$$V_1 = \frac{R_0(1-d)^2}{(1+r)} + \frac{R_0(1+d)^3}{(1+r)^2} + \cdots = V_0(1-d) \tag{11.21}$$

and capital losses equal:

$$V_1 - V_0 = V_0(1-d) - V_0 = -dV_0 \tag{11.22}$$

Then, substituting the right-hand side of Equation ??? for V_0 in Equation ??? we obtain:

$$dV_0 = \frac{dR_0(1-d)}{r+d} \tag{11.23}$$

Now we are ready to find the tax-adjustment coefficient in this model. Substituting into Equation ??? for capital losses and first period cash flow, we obtain:

$$\begin{aligned}\theta &= \frac{R_1 - \left(\frac{T_j}{T}\right) f(V_0)}{R_1 + (V_1 - V_0)} = \frac{R_0(1-d)}{R_0(1-d) - dV_0} \\ &= \frac{R_0(1-d)}{R_0(1-d) + \frac{dR_0(1-d)}{r+d}} = \frac{r+d}{r}\end{aligned} \tag{11.24}$$

So what have we learned? We learned that in the geometric decay model, capital losses are suffered at rate d. But since capital losses are not converted to cash, they do not create a tax shield. As a result, the effective tax rate is not *T* but θT, where θ is (*r* + *d*)/*r* which is greater than one and depends on the decay rate d. It should be obvious that the two previous cases are equivalent since *d* could be replaced by $g < 0$. On the other hand, it may be helpful later on to distinguish between growth in cash receipts *g*% and book value depreciation *d*%.

If for example, *r* = 8%, *d* = 3%, then,

$$\theta = \frac{.08 + .03}{.08} = 1.375. \tag{11.25}$$

Furthermore, for an investor in the 35% tax bracket, the effective tax rate on cash flow described by a geometric growth pattern would be $\theta T = (1.375)(.35) = 48$ The conclusion of Case 11.3 is that suffering losses on investments which are not used to shield income from taxes increases the investor's effective tax rate.

Case 11.4. $(V_1 - V_0) > 0$ " title="Rendered by QuickLaTeX.com" height="18" width="103" style="vertical-align: -4px;"> and $T_j f(V_0) = -T_p(1-T)V_0.$ In this model, the defending investment earns capital gains but pays property tax on its

previous period's value times the property tax rate T_p. However, property taxes are tax deductible, so we can write the after-tax IRR model for the defending land investment as:

$$\begin{aligned} V_0 &= \frac{R_0(1+g)(1-T) - T_p(1-T)V_0}{[1+r(1-\theta T)]} \\ &\quad + \frac{R_0(1+g)^2(1-T) - T_p(1-T)V_0(1+g)}{[1+r(1-\theta T)]^2} + \cdots \\ &= \frac{R_0(1+g)(1-T)}{r(1-\theta T)-g} - \frac{V_0(1+g)T_p(1-T)}{r(1-\theta T)-g} \end{aligned} \tag{11.26}$$

We can sum the IRR model above by recognizing that it consists of two geometric sums. However, after summing, V_0 appears on both sides of the equation. Therefore, we solve for V_0 using Equation ??? and obtain:

$$V_0 = \frac{R_0(1+g)(1-T)}{r(1-\theta T) - g + T_p(1-T)} \tag{11.27}$$

Capital gains and V_0 in this model are the same as in Case 11.2, which allows us to write:

$$\begin{aligned} \theta &= \frac{R_1 - \left(\frac{T_f}{T}\right) f(V_0)}{R_1 + (V_1 - V_0)} = \frac{R_0(1+g) + \left(\frac{T_p(1-T)}{T}\right) V_0}{R_0(1+g) + gV_0} \\ &= \frac{R_0(1+g) + \left(\frac{T_p(1-T)}{T}\right) \frac{R_0(1+g)}{r-g}}{R_0(1+g) + \frac{gR_0(1+g)}{r-g}} = \frac{r - g + \frac{T_p(1-T)}{T}}{r} \end{aligned} \tag{11.28}$$

Consider the effect of property taxes compared to the geometric model without property taxes. In Case 11.2, we assumed that r = 8% and g = 3%, so that $.08 = .63$. We continue with the assumption that the decision maker is in the 35% tax bracket and add the assumption that the property tax rate is 2%. We now substitute into Equation ??? and find:

$$\begin{aligned} \theta &= \frac{r - g + \frac{T_p(1-T)}{T}}{r} \\ &= \frac{.08 - .03 + \frac{.02(1-.35)}{.35}}{.08} = 1.09 \end{aligned} \tag{11.29}$$

So what have we learned? We learned that property taxes increase the effective tax rate. To illustrate, for an investor in the 35% tax bracket, the effective tax rate on cash flow described by a geometric growth pattern with property taxes would be $\theta T = (1.09)(.35) = 38\%$. Obviously, property taxes increase an investor's effective tax rate.

Case 11.5. $(V_1 - V_0) > 0$ 0" title="Rendered by QuickLaTeX.com" height="18" width="103" style="vertical-align: -4px;"> and $T_j f(V_0) = TdV_0$. A most fortuitous tax environment is an investment that appreciates but is considered to depreciate at rate d for tax purposes. In this case the investor is not required to pay taxes on the capital gains and is allowed a tax credit for depreciation that occurs only on the books. An example of such a model is the following:

$$V_0 = \frac{R_0(1+g)(1-T) - TdV_0}{[1+r(1-\theta T)]} + \frac{R_0(1+g)^2(1-T) + TdV_0(1-d)}{[1+r(1-\theta T)]^2} + \cdots$$
$$= \frac{R_0(1+g)(1-T)}{r(1-\theta T) - g} + \frac{V_0 Td}{r(1-\theta T) + d} \tag{11.30}$$

Because the before-tax model is the geometric growth model, we can write:

$$\theta = \frac{R_1 - \left(\frac{T_f}{T}\right) f(V_0)}{R_1 + (V_1 - V_0)} = \frac{R_0(1+g) - dV_0}{R_0(1+g) + gV_0}$$
$$= \frac{R_0(1+g) - d + \frac{R_0(1+g)}{r-g}}{R_0(1+g) + \frac{gR_0(1+g)}{r-g}} = \frac{r-g-d}{r} \tag{11.31}$$

So what have we learned? We learned that the effective tax rate for depreciable assets whose value is actually increasing while the investor is able to claim a tax deduction for book value depreciation results in a significant reduction in the investor's effective tax rate.

To illustrate, suppose that *d* = 5%. Then, using numbers from our previous example, we substitute into Equation ??? and find:

$$\theta = \frac{r-g-d}{r} = \frac{.08 - .03 - .05}{.08} = 0\% \tag{11.32}$$

In other words, the tax rate for the investment is effectively zero.

Tax Adjustment Coefficients in Finite Models

So far we have obtained tax adjustment coefficient models for infinite time examples. As a result, the tax adjustment coefficients have been the same for each period. This approach has been convenient for exposition purposes. However, this result is not generally applicable. In practice, the tax adjustment coefficients vary by time period. We demonstrate using a finite time horizon model, a loan model in which the interest paid is tax deductible.

In many applications, the defender is a loan. When a loan is the defender, we immediately want to know if the interest paid is tax deductible. To keep our analysis simple, assume a constant-payment loan made for n periods at an interest rate of *i* percent. In which case, loan amount L_0 at interest rate i with annuity payments A can be written as:

$$L_0 = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \cdots + \frac{A}{1+i)^n} \tag{11.33}$$

And the firm's before-tax IRR is *i*. The loan balance after one period can be written as:

$$L_1 = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \cdots + \frac{A}{(1+i)^{n-1}} \tag{11.34}$$

Furthermore, capital loss for loans can be expressed as:

$$L_1 - L_0 = -\frac{A}{(1+i)^n} \tag{11.35}$$

The after-tax IRR model can be expressed as:

$$\begin{aligned} L_0 &= \frac{A - AT\left[1 - \frac{1}{(1+i)^n}\right]}{[1+i(1-\theta T)]} + \frac{A - AT\left[1 - \frac{1}{(1+i)^{n-1}}\right]}{[1+i(1-\theta T)]^2} \\ &\quad + \cdots + \frac{A - AT\left[1 - \frac{1}{(1+i)}\right]}{[1+i(1-\theta T)]^n} \\ &= \frac{A(1-T) + \left[\frac{AT}{(1+i)^n}\right]}{[1+i(1-\theta T)]} + \frac{A(1-T) + \left[\frac{AT}{(1+i)^{n-1}}\right]}{[1+i(1-\theta T)]^2} \\ &\quad + \cdots + \frac{A(1-T) + \left[\frac{AT}{(1+i)}\right]}{[1+i(1-\theta T)]^n} \end{aligned} \tag{11.36}$$

With Equation ??? in a familiar form, we can write θ as:

$$\theta = \frac{R_1 - \left(\frac{T_f}{T}\right) f(V_0)}{R_1 + (V_1 - V_0)} = \frac{A - \frac{A}{(1+i)^n}}{A - \frac{A}{(1+i)^n}} = 1 \tag{11.37}$$

In words, the effective after-tax IRR for a loan whose interest is tax-deductible is (1 – *T*).

Ranking Investments and Taxes

In this section, we acknowledge that the effective tax rates vary by the different types of investments being considered. Furthermore, if the effective tax rate for defenders and challengers differ, then before and after-tax investment rankings may be changed. To make this point, consider the following example. Assume the defender's before-tax IRR is 8%, the income tax rate is 35%, the rate of growth in net cash flow is 3%, and the last year's before-tax net cash flow was $100. The maximum bid price for this investment with geometric growing income is:

$$V_0^b = \frac{R_0(1+g)}{r-g} = \frac{\$100(1.03)}{(.08-.03)} = \$2,060. \tag{11.38}$$

If the minimum sell price is $3,000, then the NPV for the investment is a negative $940 and the challenger is rejected.

Now suppose we introduce taxes into the model and recalculate the investment's NPV. To simplify we assume an infinite life for the investment and assume $\theta = 1$ for the defender and find:

$$
\begin{aligned}
NPV &= -V_0 + \frac{R_0(1+g)(1-T)}{[1+r(1-T)]} + \frac{R_0(1+g)^2(1-T)}{[1+r(1-T)]^2} \\
&\quad + \frac{R_0(1+g)^3(1-T)}{[1+r(1-T)]^3} + \cdots \\
&= -V_0 + \frac{R_0(1+g)(1-T)}{r(1-T)-g} \\
&= -\$3,000 + \frac{\$100(1.03)(1-.35)}{.08(1-.35)-.03} = \$43.18
\end{aligned}
\tag{11.39}
$$

Now the challenger's NPV is positive. Introducing taxes into the NPV model has reversed the ranking: the challenger is now preferred to the defender. These results should alert financial managers to the importance of after-tax rankings.

Summary and Conclusions

Taxes are important when valuing and ranking investments because what really matters is the amount of your earnings you get to keep after paying taxes. It's not the before-tax IRR that matters but the after-tax IRR that counts because that's the rate you really earned on your defending investment.

In this chapter, we have introduced a method for finding a defender's after-tax IRR. The method required that when a defender's cash flow adjusted for taxes and was discounted by its after-tax IRR its change in NPV was zero. Then we used the equality to find the after-tax discount rate IRR. The implied tax rate in the after-tax IRR was compared to the firm's average marginal income tax-rate.

While the usual practice is to multiply the defender's before-tax IRR by (1 – *T*) to obtain the defender's after-tax IRR, this chapter demonstrated that this cannot be generally relied on to obtain the defender's properly adjusted after-tax IRR. Indeed, we demonstrated that capital gains that are not taxed can lower the effective tax rate. Capital losses that do not create tax shields can increase the effective tax rate. Property tax paid on land and other real property increases the effective tax rate paid. Allowing investors to write off an investment's depreciation lowers the effective tax rate.

So what have we learned? We learned that when comparing defenders and challengers, it is important to find the correct after-tax IRR of the defender to discount the challenger's after-tax cash flow. Other things being equal, we prefer challengers whose effective tax rate is less than the effective tax rate of the defender.

Finally, this chapter has employed simplifying assumptions, such as large values for *n* and average depreciation and growth rates, to find effective after-tax discount rates that have nice, closed-form solutions. Usually, this is not the case. It is often more difficult to find effective after-tax discount rates, and often these are not closed-form solutions.

Questions

1. Describe the appropriate test for determining whether or not taxes have been properly introduced into the defender's IRR.
2. When finding after-tax IRRs for defenders, we solve for θ and claim that θT is the defender's effective tax rate. Interpret the meaning of θ.
3. Under what condition is the defender's effective tax rate equal to its income tax rate *T* or that $\theta = 1$?
4. Assume that a defending investment's price is V_0, that *d* is the defender's book value depreciation rate, that *R* is the constant stream of income earned by the defender, that *T* is the income tax rate, and that *r* is the defender's before-tax IRR. Also assume that the defender is allowed to write off book value depreciation even though its income stream is constant. We write such a model as:

$$
V_0 = \frac{R(1-T)+TdV_0}{[1+r(1-\theta T)]} + \frac{R_0(1-T)+TdV_0}{[1+r(1-\theta T)]^2} + \cdots
\tag{Q11.1}
$$

In this model, the depreciation is constant for 100/d periods beyond which the discounted cash flow is small enough to be ignored. The before-tax IRR model can be written as:

$$V_0 = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \cdots = \frac{R}{r} \tag{Q11.2}$$

Solve for θ in the after-tax IRR model by substituting for V_0 the right hand side of the second model R/r. Next, solve for a numerical value for θ assuming that d = 3% and r = 8%.

5. Suppose that you have found the NPV for a challenger. What impact will an increase in the defender's effective tax rate have on the challenger's IRR? Please explain your answers.
6. Calculate the effective tax rate θT for a depreciating challenger where d is the depreciation rate, T is the income tax rate, r is the defender's before-tax IRR, and V_0 is the price of defender. The IRR model is written below. (Hint: find θ by setting the right hand side of the equation below to the equivalent model without taxes (T = 0) and solve for θ.

$$\begin{aligned} V_0 = & \frac{R_0(1-d)(1-T)+TdV_0}{[1+r(1-\theta T)]} \\ & + \frac{R_0(1-d)^2(1-T)+TdV_0(1-d)}{[1+r(1-\theta T)]^2} + \cdots \\ & = \frac{R_0(1-d)(1-T)+TdV_0}{r(1-\theta T)+d} + \end{aligned} \tag{Q11.3}$$

7. Discuss the following. Suppose that cash flow was constant ($R_t = R_0$), yet tax laws allowed the owner of the durable to claim depreciation at rate d. Without solving for θ, can you deduce whether it would be equal to, less than, or greater than one? Defend your answer.
8. Assume you are investing in land and that you borrow money to finance its purchase. Also assume that you pay property taxes on the land. In other words, land is the challenger and the loan is the defender. Which investment has the higher effective tax rate under three scenarios: g = 0, g > 0, and g < 0? What do you need to know about the property tax rate T_p to answer this question?
9. In this chapter, five different investment tax scenarios were described. Please provide an example of each type of investment and how the tax scenario might change its rankings compared to before-tax rankings.

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12: Homogeneous Rates of Return

Learning Objectives

After completing this chapter, you should be able to: (1) construct present value (PV) models using multi-period equivalents of an accrual income statement (AIS); (2) find multi-period before and after-tax rates of return by solving internal rate of return (IRR) models; and (3) solve practical investment problems using present value model templates that are consistent with generalized AIS construction principles.

To achieve your learning goals, you should complete the following objectives:

- Describe the similarities and differences between an AIS and IRR PV models.
- Construct multi-period IRR models by generalizing AIS earning measures.
- Demonstrate that before and after-tax return on assets (ROA) and return on equity (ROE) measures derived from an AIS are equivalent (equal) to rate of return measures derived from multi-period (one-period) IRR models.
- Learn how to divide the PV model construction process into three phases: the acquisition phase, the operating phase, and the liquidation phase.
- Construct net present value (NPV) models by discounting a challenger's cash flow using a defender's IRR.
- Show how NPV models that discount a challenger's cash flow and changes in operating and capital accounts with a defender's IRR can be used to rank defenders and challengers.
- Develop PV construction skills by using Excel templates.

Introduction

In this chapter, we find before and after-tax one-period rates of return on assets and equity by calculating cash flow and changes in operating and capital accounts. Operating accounts include accounts receivable (AR), inventories (Inv), accounts payable (AP), and accrued liabilities (AL). Depreciation (Dep) plus changes in realized capital gains measure changes in the firm's capital account.

Next, we point out the similarities and differences between AIS and IRR PV models. Then using HQN data introduced in Chapter 4, we derive ROA and ROE measures in an AIS and a one-period IRR model demonstrating that the two models produce equal results. Next, we show how AIS construction principles guide the construction of multi-period IRR PV models.

We acknowledge that AIS and IRR models are descriptive tools. They describe the rate of return on assets or equity of the firm (industry or sector) or investment. Prescriptive PV models that produce investment recommendations require that we combine before and after-tax ROA and ROE measures for a defending investment, the investment owned by the firm, and cash flow and changes in operating and capital accounts for a challenging investment, a potential replacement for the defending investment. Continuing, we divide the IRR and NPV PV model construction into three stages: the acquisition phase, the operating phase, and the liquidation phase. Finally, we operationalize the construction of PV models by introducing a generalized PV model.

Accrual Income Statement (AIS) versus Internal Rate of Return (IRR) Present Value Models

Consider the similarities and differences between an AIS and IRR models.

Descriptive versus prescriptive focus. An AIS generates one-period before and after-tax ROA and ROE. IRR models generate one or multi-period before and after-tax IRR*A* and IRR*E* that estimate average rate of return measures on assets and equity respectively. One-period IRR*A* and IRR*E* are equal to ROA and ROE AIS measures respectively. However, using multi-period IRR models we can find IRR*A* and IRR*E* that estimate average before and after-tax rates of return over several periods for which there are no AIS equivalent measures. However, one-period AIS construction processes can guide the construction of multi-period IRR models.

IRR models like AIS are descriptive in nature—they do not enable us to rank firms or investments unless we compare them to other firms or investments' rates of return. In contrast, NPV PV models can rank investments because they contain information about two investments, a defending and a challenging investment. A defending investment is one in use. A challenging investment is a potential replacement for a defending investment. NPV models represent a defending investment by its IRR. NPV models represent a challenging investment by its cash flow and changes in its operating and capital accounts over one or more periods. Maximum bid (minimum sell) PV models are variations of NPV models that find break-even investment prices that make the firm indifferent between a defender and a challenger. A positive (negative) NPV infers that the challenger earns a higher (lower) rate of return than does the defender that it replaces.

Past versus future time focus. Most of the time, an AIS measure cash flow and changes in operating and capital accounts resulting from financial and production activities already completed or in the process of being completed. Of course, it is possible to forecast future coordinated financial statements (CFS) by predicting future values of exogenous variables or by changing control variables in the CFS. Multi-period IRR models, on the other hand, are forward looking, attempting to measure anticipated cash flow and changes in operating and capital accounts resulting from projected future financial and production activities.

The firm versus an investment. An AIS focuses on finding returns on the firm's assets and equity. Meanwhile, IRR models focus on finding the rate of return on an investment. We may say that the focus of an AIS is the firm while the focus of PV models is an investment.

Measuring returns to assets and equity. We can divide returns to assets and equity into three categories:

- cash flow during the period(s) of analysis,
- changes in the firm's noncash operating accounts; and
- changes in the firm's capital accounts.

Liquidation of noncash operating and capital accounts. Both AIS and IRR models include the liquidated value of noncash operating and capital accounts. An AIS includes the liquidated value of accounts at the end of one-period. IRR models include the liquidated value of operating and capital accounts at the end of the analysis that may be one or several periods. Liquidated values depend on what the next owners of the operating and capital accounts are willing to pay to acquire them.

So what have we learned? We learned that before and after-tax ROE and ROA derived from AIS and one-period IRR models are equal. In addition, we learned that AIS construction principles can guide the construction of multi-period IRR models. Finally, we learned that rate of return measures derived from AIS and IRR models are descriptive and that we can build prescriptive PV models by combining IRR measures from a defending investment and cash flow and changes in operating and capital accounts for a challenging investment.

Finding Changes in Assets and Equity

We now describe how to find cash flow and changes in operating and capital accounts using AIS and IRR models. Consider the AIS for HQN in Table 4.6 in Chapter 4 and repeated here as Table 12.1. Notice that we organize AIS into total revenue and total expenses. Then, after adjusting for interest, taxes, and dividend/owner draw, we find Addition to Retained Earnings.

Table 12.1. 2018 Accrual Income Statement for HQN.
Open HQN Coordinated Financial Statement in MS Excel

ACCRUAL INCOME STATEMENT		
DATE		**2018**
+	Cash Receipts	$38,990
+	Change in Accounts Receivable	($400)
+	Change in Inventories	$1,450
+	Realized Cap. Gains/Depr. Recapture	$0
Total Revenue		$40,000

ACCRUAL INCOME STATEMENT		
DATE		**2018**
+	Cash Receipts	$38,990
+	Cash Cost of Goods Sold	$27,000
+	Change in Accounts. Payable	$1,000
+	Cash Overhead Expenses	$11,078
+	Change in Accrued Liabilities	($78)
+	Depreciation	$350
Total Expenses		$39,350
Earnings Before Interest and Taxes (EBIT)		$650
–	Less Interests Costs	$480
Earnings Before Taxes (EBT)		$170
–	Less Taxes	$68
Net Income After Taxes (NIAT)		$102
–	Less Dividends and Owner Draw	$287
Addition to Retained Earnings		($185)

Equation (12.1) summarizes the AIS statement in Table 12.1. It equates total revenue to cash receipts (CR) plus changes in accounts receivable (ΔAR) plus change in inventory (ΔInv) plus realized capital gains/depreciation recapture (RCG). It equates total expenses to cash cost of goods sold (COGS) plus changes in accounts payable (ΔAP) plus cash overhead expenses (OE) plus changes in accrued liabilities (ΔAL) plus the change in the book value of capital assets or depreciation (Dep).

The difference between total revenue and total expenses equals HQN's asset earnings. We refer to a firm's asset earnings as earnings before interest and taxes (EBIT).

$$\begin{aligned} EBIT &= (CR + \Delta AR + \Delta INV) - (CE + \Delta AP + \Delta AL + Dep) \\ &= total\ revenue\ -\ total\ expenses\ -\$40,000 - \$39,350 = \$650 \end{aligned} \tag{12.1}$$

We find HQN's rate of return on its assets, ROA, by dividing its EBIT by its beginning assets (A_0) of $10,000 reported in Table 4.1 and repeated here in Table 12.2.

Table 12.2. Year End Balance Sheets for HQN (all numbers in 000s)
Open HQN Coordinated Financial Statement in MS Excel

BALANCE SHEET		
DATE	**12/31/2017**	**12/31/2018**
Cash and Marketable Securities	$930	$600
Accounts Receivable	$1,640	$1,200
Inventory	$3,750	$5,200
Notes Receivable	$0	$0
Total Current Assets	$6,320	$7,000
Depreciable Assets	$2,990	$2,710

Non-depreciable Assets	$690	$690
Total Long-Term Assets	$3,680	$2,400
TOTAL ASSETS	$10,000	$10,400
Notes Payable	$1,500	$1,270
Current Portion Long-Term Debt	$500	$450
Accounts Payable	$3,000	$4,000
Accrued Liabilities	$958	$880
Total current Liabilities	$5,958	$6,600
Non-Current Long Term Debt	$2,042	$1,985
TOTAL LIABILITIES	$8,000	$8,585
Contributed Capital	$1,900	$1,900
Retained Earnings	$100	($85)
Total Equity	$2,000	$1,815
TOTAL LIABILITIES AND EQUITY	$10,000	$10,400

$$ROA = \frac{EBIT}{A_0} = \frac{\$650}{\$10,000} = 6.5\% \tag{12.2}$$

We find HQN's ROE the rate of return on its equity by subtracting from EBIT interest costs that represent return on debt. We refer to the result as earnings before taxes (EBT). Dividing EBT by the firm's beginning equity E_0 reported in Table 12.2 as $2,000 we find:

$$ROA = \frac{EBIT}{E_0} = \frac{\$170}{\$10,000} = 8.5\% \tag{12.3}$$

AIS estimates of EBIT and EBT represent changes in the firm's assets and equity. However, these changes in assets and equity may not be reflected by the difference between assets and equity in the beginning and ending balance sheets unless we make some adjustments. In Table 12.2, the change in equity equals $1815 – $2,000 = ($185) compared to an EBT of $170 estimated from its AIS. If we subtract from EBT taxes of $68 and owner draw of $287 we find the same difference:

$$EBT - taxes - owner\,draw = \$170 - \$68 - \$237 = (\$185) \tag{12.4}$$

Table 12.2 reports a change in HQN's assets of $400 ($10,400 minus $10,000). Meanwhile HQN's AIS reports a change in HQN's assets, EBIT, of $650. Again part of this difference can be explained by owner draw of $287 that when added to $400 equals $687 still leaving a discrepancy of $37. In this case, the discrepancy occurs because reducing cash balances to retire some liabilities also reduces assets but not equity.

Reorganizing AIS data entries. We can reorganize AIS data into cash flow and changes in operating and capital accounts to build an IRR model that is more appropriate for analyzing multi-period investment problems. This reorganization is required because cash flow and changes in operating and capital account liquidations in IRR models may occur in several different periods while in an AIS they occur in the same period. Separating AIS entries into cash flow and changes in operating and capital accounts produces Tables 12.3 and 12.4:

Table 12.3. HQN 2018 Cash Receipts minus Cash Expenses

Cash Receipts (CR)		
+	Cash receipts from operations	$38,990
+	Realized capital gains	$0
=	**Cash Receipts (CR)**	**$38,990**
Cash Expenses (CE)		
+	Cash COGS	$27,000
+	Cash OE	$11,078
=	**Cash Expenses (CE)**	**$38,078**
Cash Receipts – Cash Expenses = CR – CE		**$912**

Table 12.3 divides cash flow into cash receipts (CR) and cash expenses (CE). The sources of cash receipts include cash sales from operations, realized capital gains, reductions in accounts receivable ($\Delta AR < 0$), and reductions in inventories held for sale ($\Delta Inv < 0$). The sources of cash expenses include cash payments for costs of goods sold (COGS) and cash overhead expenses (OE), reductions in accounts payable ($\Delta AP < 0$), and reductions in accrued liabilities ($\Delta AL < 0$). We measure reductions in the book value of capital assets in the *tth* period as depreciation (Dep*t*). We record changes in operating and capital accounts in Table 12.4. Asset account changes include ΔAR and ΔInv. Changes in liability accounts include ΔAP and ΔOE

Table 12.4. HQN 2018 Changes in operating accounts and depreciation

+	Change in accounts receivable (ΔAR)	($440)
+	Change in inventories (ΔInv)	$1450
–	Change in accounts payable (ΔAP)	$1,000
–	Change in accrued liabilities (ΔAL)	($78)
=	**Changes in operating accounts**	**$88**
–	Depreciation (Dep)	$350
=	**Changes in capital accounts**	**$350**
=	**Changes in operating and capital accounts**	**($262)**

To summarize the calculations included in Tables 12.3 and 12.4 we express EBIT as:

$$\begin{aligned} EBIT &= (CR - CE) + (\Delta AR + \Delta INV - \Delta AP - \Delta AL - Dep) \\ &= \$912 + (\$262) = \$650 \end{aligned} \tag{12.5}$$

Notice that the sum of cash flow recorded in Table 12.3 of $912 plus changes in operating accounts (ΔAR + ΔINV – ΔAP – ΔAL – Dep) recorded in Table 12.4 of ($262) equal EBIT of $650, an amount equal to the difference in total revenue and total expenses recorded in Equation ???. Of course, the IRR model estimates of ROA and ROE must equal the AIS estimates of ROA and ROE since the changes in assets and equity are the same—merely rearranged entries in the two estimation methods.

Accrual Income Statements (AIS) and Internal Rate of Return (IRR) Models

EBIT and ROA

We previously calculated ROA as EBIT divided by beginning assets A_0. Multiplying ROA by A_0 we equate asset earnings to EBIT:

$$\begin{aligned}(ROA)(A_0) &= EBIT \\ &= (CR - CE) + (\Delta AR + \Delta INV - \Delta AP - \Delta AL - Dep)\end{aligned} \tag{12.6}$$

Now suppose that we add A_0 to both sides of Equation ??? and factor A0 on the left hand side of the equation to obtain (1 + ROA)A_0. Finally, assume that we divide both sides of Equation ??? by (1 + ROA) to obtain an IRR PV model:

$$A_0 = \frac{A_0 + (CR - CE) + (\Delta AR + \Delta INV - \Delta AP - \Delta AL - Dep)}{(1 + ROA)} \tag{12.7}$$

Generally, a PV model is one that converts future values of changes in assets and equity to their present value equivalent. Consistent with that PV model definition, IRR is a PV model that discounts future assets earnings to an amount equal to its beginning assets. To avoid double counting returns and expenses in PV models, we only account for changes in operating and capital account when we convert them into cash.

To write the multi-period equivalent of Equation ???, we first subscript our variables for time *t*. To convert cash flow and liquidated values of noncash operating and capital accounts to their present value we discount them at the discount rate (1 + ROA). However, the discount rate in the multi-period equation is not the ROA derived from the one-period AIS but a multi-period average internal rate of return on assets (IRR^A)that we substitute for ROA. We summarize our results in Equation ???:

$$\begin{aligned}A_0 &= \frac{CR_1 - CE_1}{(1 + IRR^A)} + \cdots \\ &+ \frac{CR_n - CE_n}{(1 + IRR^A)^n} + \frac{\sum_{t=1}^{n}(\Delta AR_t \Delta Inv_t - \Delta AP_t - \Delta AL_t - Dep_t)}{(1 + IRR^A)^n}\end{aligned} \tag{12.8}$$

We simplify Equation ??? by substituting $\Delta Accounts_t$ for $\Delta AR_t + \Delta Inv_t - \Delta AP_t - \Delta AL_t$ and by recognizing the equality between beginning and ending account balances and the sum of periodic changes in accounts:

$$\begin{aligned}\sum_{t=1}^{n} Accounts_t &= \sum_{t=1}^{n}(Accounts_t - Accounts_{t-1}) \\ &= Accounts_n - Accounts_0.\end{aligned} \tag{12.9}$$

Finally, since $Accounts_0$ and V_0 equal acquisition investment cash flow and $Accounts_n$ and V_n equal liquidation investment cash flow, we enter them in the period in which they occur and rewrite Equation ??? as Equation ???:

$$\begin{aligned}Accounts_0 + V_0 &= \frac{CR_1 - CE_1}{(1 + IRR^A)} + \cdots + \frac{CR_n - CE_n}{(1 + IRR^A)^n} \\ &+ \frac{Accounts_n + V_n^{liquidation}}{(1 + IRR^A)^n}\end{aligned} \tag{12.10}$$

EBT and ROE

The difference between ROA and ROE is that ROA computes earnings generated by all of the firm's assets. ROE computes only earnings produced by the firm's equity by subtracting from total asset earnings interest earned by the firm's liabilities. To find the multi-period IRR for equity, IRR*E*, we subtract in each period *t* interest cost iD_{t-1} where D_{t-1} equals the firm's debt at the end of the previous period and *i* equals the average cost of debt. Cash flow changes in debt are ignored since they do not represent earnings from equity. To find the amount of equity invested, we subtract from initial assets initial debt D_0. Outstanding debt during the period of analysis collects interest. No changes in debt occur in the last period and debt at the end of the period

n–1, D_{n-1}, is retired in the last period. Revising Equation ??? to account for interest costs and debt—and replacing IRRA with the multi-period equivalent of ROE, IRR^E, we can write:

$$V_0 + Accounts_0 - D_0 = \frac{CR_1 - CE_1 - iD_0}{(1+IRR^E)} + \cdots + \frac{CR_n - CE_n - iD_{n-1}}{(1+IRR^E)^n} + \frac{V_n + Accounts_n + V_n^{liquidation} - D_{n-1}}{(1+IRR^E)^n} \tag{12.11}$$

After-tax ROA and ROE Measures

PV models mostly use after-tax cash flow (ATCF). They focus on ATCF because it represents what the firm/investors keep after paying all their expenses including taxes. In what follows we present tax obligations in a simplified form to illustrate their impact on after tax rate of return measures.

Let the average tax rate applied to equity earnings be T. Let the average tax rate applied to asset earnings be T^* that depends on T. Our goal is to find the average tax rate T that adjusts ROE to ROE(1–T) and T^* that adjusts ROA to ROA(1–T^*). We do not try to duplicate the complicated processes followed by taxing authorities to find T and T^*.

AIS report taxes paid by the firm and subtracts them from EBT to obtain net income after taxes (NIAT). We assume interest costs found by multiplying the average interest rate i times beginning period debt D_{t-1} (iD_{t-1}), have already been subtracted from earnings and used to reduce tax obligations. As a result, NIAT represents changes in equity after both interest and taxes have been paid. In 2018, HQN paid $68 in taxes. To find the average tax rate HQN paid on its changes in equity we set taxes equal to the average tax rate T times EBT:

$$Taxes = (T)(EBT) = \$68 \tag{12.12}$$

Solving for the average tax rate T HQN paid on its earnings we find:

$$T = \frac{Taxes}{EBT} = \frac{\$68}{\$170} = 40\% \tag{12.13}$$

Finally, we adjust ROE for taxes and find HQN's after-tax rate of return to be:

$$ROE(1-T) = \frac{EBT(1-T)}{E_0} = \frac{NIAT}{E_0} = \frac{\$102}{\$2,000} = 5.1\% \tag{12.14}$$

AIS Statements and After-tax ROA. AIS compute taxes paid by the firm on its returns to equity but not on its returns to assets. AIS record only one value for taxes paid and these estimates account for interest payment tax savings. As a result, the average tax rate T calculated for taxes on equity cannot be used to adjust ROA for taxes. To find the average tax rate T^* that adjusts ROA to ROA(1–T^*), we calculate taxes "as if" there were no interest costs to reduce the average tax rate T^*. We find ROA(1–T^*) in Equation ??? as:

$$ROA(1-T^*) = \frac{EBIT - Taxes}{A_0} = \frac{\$650 - \$68}{\$10,000} = 5.82\% \tag{12.15}$$

Solving for T^* we find:

$$T^* = 1 - \frac{(EBIT - Taxes)}{(A_0)(ROA)} = 1 - \frac{\$650 - \$68}{\$650} = 10.46\% \tag{12.16}$$

Equation (12.16) emphasizes an important point, that adjusting ROE and ROA for taxes nearly always requires different average tax rates. The only time that $T = T^*$ is when interest costs are zero. In that particular case, we can easily demonstrate that $T^* = T$ since EBIT = EBT:

$$T = T^* = \frac{Taxes}{EBIT} = \frac{Taxes}{EBT} = \frac{\$68}{\$6500} = 10.46\% \tag{12.17}$$

A note on discount rates and the weighted cost of capital (WCC). It is common in many finance texts to assume that the discount rate is an exogenous variable observed in the financial markets. In small firms, the focus of this book, the discount rate depends on the rate of return earned by the defender and sacrificed to acquire the challenging investment.

The weighted average cost of capital (WACC) is the rate that a company is expected to pay on average to finance its assets. The WACC is commonly referred to as the firm's cost of capital. Importantly, it is dictated by the external market and not by management. For small firms, we find a concept similar to the WCC that depends on the IRR of the defender and its average cost of debt. We can derive such a measure by solving for ROA in Equation ??? that is repeated below:

$$ROE = (ROA - i)\frac{D_0}{E_0} + ROA \tag{12.18}$$

from which we find that ROA is an equivalent measure to the WCC:

$$ROA = ROE\left(\frac{E}{A}\right) + i\left(\frac{D}{A}\right) \tag{12.19}$$

After-tax Multi-period IRR Models

After-tax Multi-period IRR^E Model

Having found the after-tax discount rates ROE(1 – *T*) we are prepared to introduce taxes into multi-period IRR^E PV model described in Equation ???. We begin by replacing IRR^E with IRR^E(1 – *T*). Then we adjust cash flow and changes in operating and capital accounts for taxes. Consider additional changes required in our IRR^E PV model described in Equation ??? to account for taxes.

Interest costs. Interest costs are returns paid for the use of borrowed money, similar to rent. Therefore, interest costs represent an expense to the firm or investment and should reduce taxable income.

Depreciation. Depreciation is a noncash event and by itself does not create a cash flow. However, it has cash flow consequences because it provides a tax shield for some income because it can be treated as an expense that reduces taxable income. Therefore ATCF must account for tax savings from depreciation equal to the average tax rate *T* times depreciation for the period: (*T*)(*Dep*).

Capital gains (losses) and taxes. At the end of the analysis, PV models liquidate capital accounts for an amount equal to $V_n^{liquidation}$. For tax purposes, capital assets are valued at their depreciated book value of V_n^{book}. If the difference between the liquidation and book value of capital assets is positive, $(V_n^{liquidation} - V_n^{book}) > 0,$ 0," title="Rendered by QuickLaTeX.com" height="22" width="195" style="vertical-align: -4px;"> the firm or the investment has earned capital gains or depreciation recapture whose after-tax value is $(1-T)(V_n^{liquidation} - V_n^{book}) > 0.$ 0." title="Rendered by QuickLaTeX.com" height="22" width="252" style="vertical-align: -4px;"> On the other hand, if the difference is negative $(V_n^{liquidation} - V_n^{book}) < 0,$ then the firm has suffered a capital loss and earned tax credits. In the case of capital losses, the difference that represents a loss in value to the firm is reduced to $(1-T)(V_n^{liquidation} - V_n^{book}) < 0.$ Finally, to adjust capital accounts for taxes, we add the original book value V_n^{book}. We summarize these results below:

$$(1-T)(V_n^{liquidation}-V_n^{book})+V_n^{book}$$
$$=(1-T)V_n^{liquidation}+TV_n^{book} \quad (12.20)$$

We make similar adjustments to the firm's operating account balances. We adjust changes in account balances for tax consequences. Letting the liquidation value of accounts equal $Accounts_n$ and the acquisition value of accounts equal to $Accounts_0$, we write the after-tax liquidation value of accounts as:

$$(1-T)(Accounts_n-Accounts_0)+Accounts_0$$
$$=(1-T)Accounts_n+TAccounts_0 \quad (12.21)$$

Now we can write the after-tax IRR^E model for changes in equity consistent with principles followed when we constructed one-period AIS as:

$$V_0+Accounts_0-D_0=\frac{(CR_1-CE_1-iD_0)(1-T)+TDep_1}{[1+IRR^E(1-T)]}$$
$$+\cdots+\frac{(CR_n-CE_n-iD_{n-1})(1-T)+TDep_n}{[1+IRR^E(1-T)]^n}$$
$$+\frac{(1-T)V_n^{liquidation}+TV_n^{book}+(1-T)Accounts_n+TAccounts_0-D_{n-1}}{[1+IRR^E(1-T)]^n} \quad (12.22)$$

After-tax Multi-period IRR^A Model

There is a paradox in applied PV models. The paradox is that there is no explicit measure for T^* that can be used to find ROA(1 – T^*). This peculiar result occurs because taxes must account for interest costs that are not considered in EBIT measures. Yet, most applied IRR models solve for after-tax return on assets that should be guided by ROA(1 – T^*) calculations.

To properly account for taxes when finding in an IRR^A model we should apply the tax rate T^* to earnings on assets. Adjusting (12.20) by eliminating interest costs while allowing them to influence taxes paid and replacing IRR^E with IRR^A, we can express the after-tax multi-period IRR^A model as:

$$V_0+Accounts_0=\frac{(CR_1-CE_1)(1-T^*)+T^*Dep_1}{[1+IRR^A(1-T^*)]}$$
$$+\cdots+\frac{(CR_n-CE_n)(1-T^*)+T^*Dep_n}{[1+IRR^A(1-T^*)]^n}$$
$$+\frac{(1-T^*)V_n^{liquidation}+T^*V_n^{book}+(1-T^*)Accounts_n+T^*Accounts_0}{[1+IRR^A(1-T^*)]^n} \quad (12.23)$$

Consistency and $IRR^A(1-T^*)$ and $IRR^E(1-T)$ PV model rankings

We can construct PV models that find either before or after-tax returns on equity or assets and use them to rank investments. Furthermore, the $IRR^A(1-T^*)$ and $IRR^E(1-T)$ rankings need not be consistent with IRR^A and IRR^E rankings. One reason they need not be consistent is that their initial and periodic sizes are different (see Chapter 9). Furthermore, interest costs included in the IRR^E model and not in the IRR^A model allow for ranking variations

So, how do we decide on our ranking criterion? Do we focus on equity or asset earnings? Equity or asset earnings before or after-

taxes are paid? Although there may not be any one satisfactory answer to this question, it surely must depend on, among other things, on what questions the financial manager wants to answer.

Net Present Value Models

The homogeneous measures principle requires that cash flow and changes in accounts for defenders and challengers be measured in consistent units. Both must be measured in either before or after-taxes. Both must be measured in return to assets or equity. Both must be measured in real or current dollars. Both must be similar in investment sizes. Both must reflect similar terms. And both should be adjusted to similar risk classes.

The homogeneous measure requirements guarantee that if two investments are compared using their IRRs or NPVs—they will be ranked the same. Furthermore, satisfying homogeneity requirements also guarantee the usual number system properties apply. In particular, if investment A is preferred to B and B is preferred to C, then A is preferred to C.

While any particular NPV model that satisfies homogeneity requirements will rank investments A and B consistently, it is not true that NPV and IRR models will produce the same rankings when homogeneity conditions change. Earlier we showed that rankings depended on which size adjustment method we adopted for rationalizing investment size differences. Now we show that rankings depend not only on what size adjustment method we adopt but also by what homogeneity conditions we adopted. For example, we can easily demonstrate that equity derived IRRs and NPVs may produce different rankings than asset derived IRRs and NPVs.

To make clear that equity versus asset derived IRR and NPV measures may produce different rankings, consider HQN's one-period IRR^A of 6.5% ($650/$10,000) and its one-period IRR^E of 8.5% ($170/$2,000) respectively. Let the original HQN asset and earning measures describe investment A. Now without changing beginning assets or equity, suppose a different investment B incurred no interest costs. Then investment B's ROA would be unchanged ($650/$10,000); however, its ROE would increase to 32.5% ($650/$2,000). Meanwhile the investor would continue to be indifferent between investments A and B based on their IRR^As (and NPVs) since both equal 6.5%. However, clearly investment B is preferred to A based on their IRR^Es (and NPVs).

So what have we learned? By changing the homogeneity of measures requirement from measuring returns on assets to returns on equity, we showed that we can change investment rankings. Other changes in homogeneity conditions can produce similar conflicts. So what does it all mean? It means that decision makers must decide which homogeneity measures are most relevant because their investment rankings may depend on their choice.

PV Model Templates

PV models answer practical questions about investments whose costs and benefits extend over time. They can be descriptive such as IRR models that provide information about an investment's rate of return. Or, they can be prescriptive such as NPV models that compare defender's rate of return with a challenger's cash flow.

To facilitate the calculation of IRRs and NPVs and to emphasize the decision maker's important role in providing accurate and consistent data, we provide two PV model templates that correspond to equations (12.22) for equity and (12.23) for assets. In each template we calculate before and after-tax NPVs, their respective IRRs, and annuity equivalents (AE). Finally, we note that we can use the PV templates to solve one-period problems producing results equivalent to those produced by AIS. More typically, we can use them to solve multi-period problems. Finally, while we can use PV models to describe and rank the firm's financial performance, it is more common to use PV models to describe and rank investments.

We illustrate the two templates by using HQN 2018 data. In effect, we calculate one-period NPV, AE, and IRR estimates that correspond to AIS estimates. In Chapter 13 we apply the model to describe multi-period investments.

Before-tax PV Asset Models and EBIT

We now introduce a template for assets described in Table 12.5 using HQN data for 2018 described in Tables 12.1 and 12.2. By setting T^* equal to zero, the template corresponds to EBIT calculations in Table 12.1. We describe the template in Table 12.5 that follows.

Table 12.5. PV Template for Rolling NPVs, AEs, and IRRs Earned by Assets Before Taxes are Paid.

Open Table 12.5 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	
3	Yr	Capital accounts values	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR-INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OEs)	The savings from depr. (TxDepr)	A c (A op
4	0	$8,558		$8,558	$1,442						
5	1	$0	$8,218	$8,208	$1,520	$350	$38,990	$27,000	$11,078	$0	

The column headings in Table 12.5 describe exogenous and endogenous variables used to find rolling (every year) NPV, AE, and IRR estimates.

- Column A lists the periods at the end of which financial activity occurs.
- Column B lists capital investment amounts.
- Column C lists the liquidation value of capital investments.
- Column D lists the book value of capital investments.
- Column E lists the value of operating accounts at the end of each period (AR + Inv – AP – AL).
- Column F calculates depreciation equal to the change in the investment's book value determined by tax regulations.
- Column G reports CR equal to the sum of cash sales plus negative changes in AR and Inv.
- Column H reports cash COGS equal to cash operating expenses plus reductions in AP.
- Column I reports cash OE equal to cash overhead expenses plus reductions in AL.
- Column J calculates tax saving created by depreciation, relevant only in after-tax models.
- Column K calculates cash flow from operations equal to CR less cash COGS and cash OE.
- Column L calculates cash flow from liquidation of capital assets and operating accounts.
- Column M sums cash flow from operations and liquidations.
- Column N calculates rolling NPVs as though the investment ended in each year.
- Column O calculates rolling AEs associated with the NPV for each year.
- Finally, column P finds rolling IRRs for each investment life.

Note that operating cash flow equal to $912 (Column K) correspond to the sum of operating cash flow in Table 12.3 and is found by adding cash receipts (Column G) less cash COGS (Column H) and cash OE (Column I). We report the total liquidation value in column L as an amount equal to the investment's liquidation value reported in column C plus the change in the sum of accounts in successive years reported in Column E. The sum of operating cash flow reported in column K plus the liquidation value of operating and capital accounts reported in Column L equals the value of the firm reported in Column M. Finally, the change in assets equals the value reported in Column M less the beginning value of assets reported in cell B4 equals $650 corresponding the HQN's EBIT value reported in Table 12.1. It follows that EBIT divided by assets of $10,000 is the one-period ROA calculated from the AIS and the one-period IRR*A* calculated in the PV template reported in Table 12.5, both equal to 6.5%. Furthermore, if we discount the ending value of the firm by its IRR, its NPV is zero as is its AE since the discount rate used in cell N4 is the IRR.

*Change in operating account*s. Table 12.5 is intended to model Equation ???. And for the most part, the similarities are transparent. The initial investment that includes operating accounts and capital assets equals total assets of $10,000. The change in the capital accounts equals depreciation of $350. Cash flow during the period is cash receipts less cash expenses equal to $912 is easy to associate with Equation ???. However, the change in operating accounts need some explanation.

Beginning (ending) operating accounts are calculated from HQN's beginning (ending) balance sheet. Beginning assets less beginning operating account balances equal beginning capital account value. Beginning capital account balances less depreciation equals ending book value for capital account balances.

Before-tax PV Equity Models and EBT

We now introduce a PV template for equity described in Table 12.6 that corresponds to equation 12.20. We populate the template using HQN data for 2018 described in Tables 12.1 and 12.2. By setting the tax rate T equal to zero, the results corresponds to EBT

calculations in Table 12.1.

Table 12.6. PV Template for Rolling Estimates of NPVs, AEs, and IRRs for Equity Earnings Before Taxes
Open Table 12.6 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K
3	Yr	Capital accounts values	Debt Capital	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR-INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OEs)	Intere Cos
4	0	$8,558	$8,000		$8,558	$1,442					
5	1	$0	$8,080	$8,208	$8,208	$1,530	$350	$38,990	$27,000	$11,078	$4
6	2	$0	$8,000	$20,000	$20,000	$1,200	-$6,792	$29,800	$13,500	$11,189	$4

The column headings in Table 12.6 are only slightly modified from those reported in Table 12.5.

- Column A lists the periods at the end of which financial activity occurs.
- Column B lists investment (equity) amounts for which NPVs, AEs, and IRRs are calculated.
- Column C lists the debt and other liabilities supporting the investment.
- Column D lists the liquidation value of the investment (equity).
- Column E lists the book value of investments.
- Column F lists the net value of operating accounts at the end of each period (AR + Inv – AP – AL).
- Column G calculates depreciation equal to the change in the investment's book value determined by tax regulations.
- Column H reports cash receipts equal to the sum of cash sales plus negative changes in AR and Inv.
- Column I reports cash COGS equal to cash operating expenses plus reductions in AL.
- Column J reports cash OE equal to cash overhead expenses plus reductions in the AL account.
- Column K reports interest paid on debt and other liabilities.
- Column L calculates the tax saving created by depreciation, relevant only in after-tax models.
- Column M calculates cash flow from operations.
- Column N calculates cash flow from liquidation.
- Column O sums cash flow from operations and liquidation.
- Column P calculates NPV as though the investment ended in each year.
- Column Q calculates the AE associated with the NPV for each year.
- Finally, column R finds the IRR for each investment life.

Note that operating cash flow of $432 (Column M) corresponds to the sum of operating cash flow in Table 12.3 and is found by adding cash receipts (Column H) less cash COGS (Column I), cash OE (Column J) and interest costs (Column K). The equity liquidation value of the investment is reported in column D. The book value of the investment is listed in column E. The total liquidation value less debt and liabilities outstanding at the end of the previous period are reported in Column N. The sum of operating cash flow plus the liquidation value of operating and capital accounts less outstanding debt are reported in Column O . Finally, the change in equity equals the value reported in cell O5 less the value reported in cell O4, an amount equal to $170 corresponding to HQN's EBT in Table 1. It follows that EBT divided by equity of $2000 is the one-period ROE and the one-period IRR*E*, both equal to 8.5%. Furthermore, if we discount the ending value of the firm by its IRR reported in cell P4, its NPV is zero as is its AE since the discount rate is its IRR.

Change in operating accounts. Table 12.6 is intended to model Equation ??? and for the most part, the similarities are transparent. The initial investment that includes accounts and capital assets less initial debt is the same and equal to $2,000. The change in the capital accounts is equal to depreciation and the difference between beginning and ending capital accounts of $350. The cash flow during the period of cash receipts less cash expenses equal to $912 less interest costs of $480 are also easy to connect. However, the change in operating accounts need some explanation.

Beginning operating accounts are included in the beginning investment (assets). They are also included in the book value of the investment unchanged from the beginning investment amount. To include changes in operating account to adjust beginning

investment to liquidated investment value, we add the difference between the operating accounts at the beginning and ending period.

After-tax PV Equity Models and NIAT

AIS introduce taxes into its return on equity calculations by subtracting taxes paid from EBT (see Table 12.1). The result is net income (earned on equity) after taxes, NIAT. We introduce taxes into Table 12.6 by adjusting cash to an after-tax basis by multiplying operating cash flow less interest costs by (1 – T) where T was found earlier to equal to T = .4. The after-tax cash flow from operations, operating ATCF, we report in column M. In addition, we must account for tax savings generated by depreciation reported in column L. Finally, tax consequences from any capital gains (losses) adjust liquidation cash flow reported in column N. The template corresponding to NIAT and after-tax return to equity we report in Table 12.7.

Table 12.7. PV Template for Rolling Estimates of NPVs, AEs, and IRRs for Equity Earnings After Taxes
Open Table 12.7 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K
3	Yr	Capital accounts values	Debt Capital	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR- INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OEs)	Interests Costs
4	0	$8,558	$8,000		$8,558	$1,442					
5	1	$0	$8,080	$8,208	$8,208	$1,520	$350	$38,990	$27,000	$11,078	$480

Note that in column O, the difference between the beginning and ending equity is $102, the same value reported for NIAT in Table 12.1. Furthermore, dividing $102 by the beginning equity of the firm produce an after-tax ROA and an after-tax $IRR^E(1 - T)$ of 5.1%. Finally, if we change the discount rate in cell P4 we find NPV and AE are equal to zero since the discount rate is also the after-tax IRR.

Summary and Conclusions

We now make explicit the main point of this chapter. PV models are multi-period expressions of information found in single-period AIS. IRR^A and IRR^E derived in IRR models correspond to ROA and ROE measures derived from single-period AIS. Furthermore, EBIT measures calculated in AIS models correspond to earnings from assets in PV models. EBT corresponds to earnings from equity in PV models, and NIAT corresponds to after-tax earnings from equity in PV models. While there is no explicit AIS measure of after-tax earnings from assets, it can be implied and easily measured.

Rates of return measures derived in AIS and IRR models are descriptive in nature—they describe a single investment. However, we can use also use them prescriptively by comparing them with IRRs of other firms and investments. However, we can construct a prescriptive PV model, an NPV model, by discounting cash flow from a challenging investment using the IRR of a defending investment. (The challenging investment is one considered for a replacing to a defending investment already in use by the firm.). If the NPV is positive (negative), we recommend replacing (leaving in place) the defending investment.

There are other PV models besides IRR and NPV models. We described these in Chapter 8. Most other PV models are variants of the NPV or IRR model. Maximum bid (minimum sell) models find the maximum bid (minimum sell) that can be paid (received) for a challenger and still earn the IRR of the defender. Loan formulae are special PV models that describe the amount that must be paid to receive a challenging loan while the defending loan earns its interest rate. AE simply find a multi-period time adjusted average of a challenger's cash flow. Replacement flows are special NPV models that find the optimal life of a repeatable investment that maximizes its NPV and the NPV of future replacements.

The next step in our study of PV models is to generalize them to multi-periods and use them to solve practical investment problems. Both of these tasks, we undertake in chapter, Chapter 13.

Questions

1. List similarities and differences between an AIS statement and a PV model.
2. Both AIS and PV models find rates of return on assets and equity using cash flow and changes in operating and capital asset accounts. However, they organize them differently. Explain how AIS and PV models organize cash flow and changes in operating and capital asset accounts differently.
3. We can describe ROA and ROE as measures of changes in assets and equity respectively divided by original asset and equity values. Please explain.
4. EBIT measures the change in the value of the firm's assets before adjusting for taxes during a single period. Do you agree or disagree? Defend your answer.
5. EBT measures the change in the value of the firm's equity before adjust for taxes during a single period. Do you agree or disagree? Defend your answer.
6. Measures the change in the value of the firm's equity after adjusting for taxes during a single period. Do you agree or disagree? Defend your answer.
7. There is no measure in an AIS statement that finds the change in assets after adjusting for taxes. Can you explain why?
8. Use the template reported in Table 12.5 to find the change in assets after adjust for taxes. To find the after-tax change in assets, use an average tax rate of T^{*}=10.46% and a discount rate of ROA$(1 - T^{*})$ = (6.5%)(1 – 10.46%) = 5.8%.
9. Describe the conditions under which NPV or IRR asset and equity models would rank challenging and defending investment consistently.
10. Give an example in which NPV or IRR asset and equity models rank challenging and defending investments inconsistently.
11. As a financial manager tasked with ranking a defending and a challenging investment, which ranking criteria would you employ: NPV earned by equity or NPV earned by assets? Defend your answer.

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13: Homogeneous Investment Types

Learning Objectives

After completing this chapter, you should be able to: (1) distinguish between incremental and stand-alone investments; (2) understand how time and use costs determine the optimal service extraction rate from investments; (3) distinguish between net present values (NPV), annuity equivalent (AE), and internal rates of return (IRR) used to measure earnings by an investment or by equity committed to funding an investment; and (4) use Excel spreadsheets to find NPV, AE, and IRR for investments and equity funds committed to some practical multi-period investments.

To achieve your learning goals, you should complete the following objectives:

- Learn how to distinguish between incremental and stand-alone investments.
- Learn how an investment's time and use costs determine its optimal service extraction rates.
- Learn how an investment's liquidation and acquisition values determine its fixity.
- Learn how to find contributions from incremental investments by finding changes in the firm's cash receipts (CR), cash cost of goods sold (COGS), cash overhead expenses (OE), and change in operating and capital asset accounts.
- Learn how the order of changes influences the returns on incremental investments.
- Learn how to project cash flow and changes in operating and capital accounts.
- Learn how to use Excel templates to find NPV, AE, and IRR measures earned by investments or by equity committed to some practical multi-period investments.

Introduction

The nature of investments. An investment is a commitment of resources for one or more periods. We invest instead of consume because we expect future investment earnings will more than compensate for the present investment sacrifice. If the commitment is to a firm, we talk about return on the firm's assets (ROA). If the commitment is to an investment, we talk about the returns on an investment (ROI). However, since it is more common to use PV models to examine returns on an investment rather than a firm, rather than differentiate between commitments to a firm or an investment, we will refer to both as investment analysis. To make things a little more confusing, rather than introduce a new acronym, ROI, that would undoubted annoy the amazing editor of this book, we will refer to both returns on an investment and returns on the assets of a firm as ROA measures.

Returns on an investment versus returns on equity invested in an investment. We are interested in the rate of return on funds committed to an investment. We are also interested in the rate of return on equity committed to an investment. The main distinction between measures that reflect earnings on an investments and equity funds committed to an investment is the following. Returns on equity subtract from total returns interest costs earned by debt capital. Returns earned by the investment do not subtract interest costs earned by debt capital.

Return measures. In previous chapters and especially in the previous chapter we described several measures designed to measure the returns earned by an investment versus returns earned by equity committed to an investment. These measures include NPV that find the present value of profit earned by the investment or equity committed to the investment over its economic life. AE derived from NPV convert the present value of earnings to their equivalent annuity—the constant amount that could be paid each period over the economic life of the investment. Finally, IRR measure the average rate of return earned by the investment or by equity committed to the investment earned over its economic life.

Two kinds of investments. There are two kinds of investments: stand-alone and incremental. Stand-alone investments are independent of other investments. They are self-contained. Furthermore, we do not expect that increases or decreased in stand-alone investments will influence the returns or affect investment decisions of other investments. Stand-alone investment may include more than one capital investment but when they do—they are interdependent so that one investment cannot be studied independent of the other investments.

Incremental investments add to, replace, or modify existing investments. Therefore, when considering the PV of incremental investments, we must account for the contributions of other inputs and services from other investments. In most studies, we ignore the problem of incremental investments by assuming that we can measure their contributions independent of the

contributions of other investments and nondurable inputs—treating them as stand-alone investments. The reality is quite different. Most of our investments are incremental ones and measuring their unique contributions is difficult because when we add an investment to another investment(s), the use of other investments and nondurable inputs often change as well—making it difficult to identify what part of the change in the investment's returns and expenses we can attribute to the new investment.

Investment identities. Capital investments have a distinct property. They can supply services for more than one period without losing their identity and without exhausting their service potential. Think of a battery as an example. It can provide services for more than one period without losing its identity. Another property of capital investments is that their service extraction most often requires services from other durables and nondurable inputs–whose service capacity and identity change with a single use.

Fixed investments. A fixed investment is one already owned by the firm and unlikely to be removed from service. Glenn Johnson describe the conditions required for an investment to be fixed; namely, that the investment's acquisition value (V_0) and liquidation value ($V_0^{liquidation}$) bound its value in use (V_0^{use}). In other words an investment is fixed or unlikely to be removed from service if: $V_0^{liquidation} < V_0^{use} < V_0$. Because the investment's value in use is less than what it would cost to acquire another one and because its liquidation value is less than what it is earning—the firm has no incentive to invest or disinvest in them. An investment with a zero or negative liquidation value is one likely fixed as long as it contributes something positive to the firm.

Accrual income statements (AIS) and investment analysis. An AIS makes no effort to find the contributions of a single or subset of a firm's investments. However, isolating earnings from a single or subset of the firm's investments is exactly what we do when constructing PV models for incremental capital investments—we attempt to isolate returns from capital investments that add to, replace, or alter capital investments used by the firm. Otherwise, we have no basis for incremental investment decisions.

ROA, ROE, and IRR. AIS summarize the collective contributions of a firm's assets and equity during a single period by accounting for cash flow and changes in operating and capital accounts. Furthermore, we can calculate AIS earning measures as a percentage of the firm's beginning assets and equity and report them as one period ROA and ROE respectively. Incremental investment analysis finds IRR for investments and equity committed to investments for one or several periods and report the results as the internal rate of return earned by an investment (IRR^A) and internal rate or return earned by equity committed to the investment (IRR^E). To find IRR^A and IRR^E for incremental investments require that we find the incremental investment's unique contributions over their economic life. As a result, we cannot use the firm's AIS to evaluate an incremental investment, although we organize our analysis around changes in operating cash flow and changes in operating and capital accounts—just the same as we did when analyzing a firm or stand-alone investment.

What follows. To find NPV, AE, IRR that reflect investment earnings and equity committed to stand-alone and incremental investments, we organize the remainder of this chapter as follows. First, we describe the connection between incremental and stand-alone investments. Second, we describe how to project cash flow and changes in operating and capital account used to describe stand-alone and incremental investments. We are careful when calculating NPV, AE, and IRR to indicate returns on stand-alone versus incremental investments whether they measure returns on the investment or returns from equity committed to the investment. Finally, we demonstrate how to find NPV, AE, and IRR for several practical multi-period stand-alone and incremental investments problems using Excel spreadsheets.

Incremental Investments

Incremental investment analysis intends to separate the contributions of an incremental investment from those of existing investments and other nondurable inputs and services. To be clear, incremental investment analysis does not hold all other inputs constant when considering its contributions because doing so would underestimate its contributions and lead to less than optimal investment choices.

The contributions of a stand-alone versus an incremental investment. To emphasize the connection between a stand-alone and an incremental investment, consider investing in scissor-type investments whose blades we purchase separately. With only one blade, the scissors' output is small—equivalent to a knife. When we purchase a second blade, the scissor output increases toward normal. However, the increase in the scissors' production when we added the second blade changed because we changed the use of both blades. If we analyzed the contribution of the second blade keeping the first blade idle, output would have remained low. Is it fair to attribute to the second blade the contributions of the first blade? Yes, and that is the approach we follow in incremental investment analysis because we want to know how adding the second blade changes the productivity of the scissors. Since the firm already owns the first blade, the investment decision focus is on the second blade, not the first one.

Optimal Service Extraction Rates

We want to determine the rate of return on multi-period capital investments. In this analysis, we assume that our investment provides services at its optimal rate. Finding the optimal service extraction rate for capital investments is complicated and we often employ complex computer programs to find these optimal rates. To do so, we must determine the optimal rate at which to extract services from our capital investments. At our level of investment analysis, we assume that optimal service extraction rates have already been determined. Yet it is important to understand what determines optimal service extraction rates even though the focus of our analysis is on the investment's rate of return.

To be clear, when we discuss the cost of extracting services from our capital investment, we include its change in salvage value, the cost of using other nondurable inputs, and the change in the value of other capital investments whose services support our investment. As it turns out, the costs of using a capital investment within a period conform to the categories already identified in our PV templates. The cost of using nondurable inputs are measures as cash costs of goods sold (COGS). The cost of extracting capital investment services that depend mostly on the passage of time, we measure as cash overhead expenses (OE). The cost equal to the change in the liquidation value of the capital investment we measure as depreciation (Dep) and most of the time we assume that this rate of depreciation is determined by the passage of time.

Service extraction rates when the cost of using the investment depends mostly on the passage of time. Consider, for example, an investment whose use cost depends mostly on the passage of time such as a storage facility. The roof, for example, has a finite time during which it can provide services. It simply wears out over time. Much the same is true for other parts of the building including its painted surfaces, floors, and other building features. While there may be some important costs associated with heating, ventilation, and air conditioning services, these costs depend mostly on the passage of time and weather conditions than on the amount of items stored or the activity that occurs in the building. Finally, investments in buildings are usually fixed because their liquidation value is often low. To use a building requires that the new owner relocate to the building site. Thus, location fixity reduces the liquidity of the investment and contributes to its fixity in the firm's investment portfolio.

So what is the optimal use of an investment whose costs depend mostly on the passage of time and whose low salvage value contributes to its fixity within the firm? Since its marginal use cost is usually low, their optimal use is their maximum capacity as long as the investment is making some positive contributions to the firm. For investments whose use costs depend mostly on the passage of time, we are not likely to change their use when adding an incremental investment to the firm unless we operate them at less than full capacity before the incremental investment.

Service extraction rates when the cost of using an investment depends mostly on its use. Consider another kind of investment whose use costs depends mostly on services provided by other capital investments, including maintenance costs, and nondurable inputs. Electric motors or gasoline powered equipment have their cost of service extraction dependent mostly on the cost of using nondurable inputs and services from other durables.

So what determines the optimal service extraction rates from investments whose cost depends mostly on use and other inputs? The optimal service extraction rate depends mostly on the marginal use costs. Furthermore, we expect that service extraction rates change when incremental investments are added to the firm that alter these marginal use costs. In sum, investments whose costs depends mostly on their use will find that their optimal use is not fixed nor at their maximum capacity.

Opportunity costs. All investments that provide services for one period or more incur opportunity costs equal to the defender's IRR times the value of the investment at the beginning of the period. We incur the cost when we sacrifice a defending investment to employ a challenging one. We account for these costs when discounting future costs and returns to their present value.

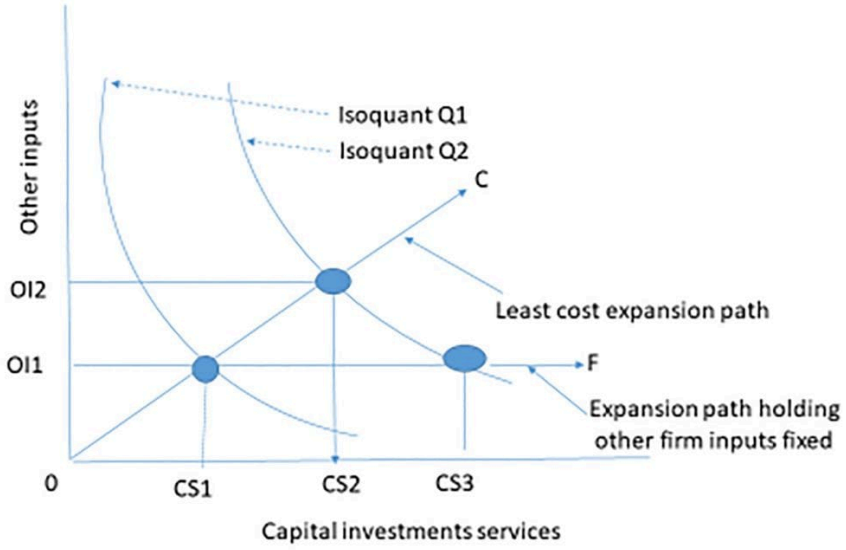
Therefore, when we analyze incremental investments that provide services for more than one period, we must account for their opportunity cost and treat them as a mostly time related cost and unlikely to influence optimal service extraction rates within the period.

Least cost expansion paths. We have described how the optimal service extraction rates for stand-alone and incremental investments depend on whether the cost of extracting services depend mostly on their use or the passage of time. If the cost of extracting services depends mostly on the passage of time, their use will not likely change even with the addition of an incremental investment.

On the other hand, if the cost of extracting services from an investment depends mostly on its use, then their optimal service extraction rates will depend on the variable cost of using other inputs and the marginal use cost of extracting services from other investments.

To find the optimal service extraction rate, we employ a tool from economic analysis we refer to as the least-cost expansion path that we describe in Figure 13.1.

Figure 13.1. A least cost expansion path that employ other inputs (OI) and capital investment services (CS) to produce outputs Q1 and Q2 using various combinations of OI and CS.



Assume that a firm produces output using a combination of capital services and other inputs. We graph the combinations of capital services and other inputs that produce the same output and call the result an isoquant (i.e. same quantity). In Figure 13.1 we represent two levels of output by isoquants Q1, and Q2 drawn convex to the origin to reflect diminishing marginal productivity—a subject covered in microeconomics.

Now suppose that the firm considers a capital investment that will allow the firm to increase its use of capital services from CS1 to CS3 and increase output from Q1 to Q2. If the cost of using other inputs depend mostly on the passage of time, we hold their contributions fixed at OI1. On the other hand, if the cost of using other inputs is variable, then economic production theory teaches that increasing production from Q1 to Q2 holding other inputs constant would be inefficient. Instead, the theory teaches that the firm should increase capital services and other inputs along some least cost line OC determined by the relative costs of increasing both the capital services from the new investment and from the other inputs.

Increasing production along the least cost line OC, the firm would increase capital service from CS1 to CS2 an amount less that was required to reach Q2 when we held other inputs constant. In other words, increasing the use of other inputs allowed the firm to use less capital services from the new investment compared to the capital services required if other inputs were held constant (CS2 – CS1) < (CS3 – CS1) while still producing the same level of output. Furthermore, expanding other inputs and services from the new durable along the least cost expansion path assures us that the cost of increasing other inputs from OI1 to OI2 is more than offset by reducing the cost of capital services from (CS3 – CS1) to (CS2 – CS1).

Which expansion path? We have demonstrated that whether to measure the cost of increasing production along the least cost line OC or the expansion path OF that holds other inputs constant at OI1 depends on the nature of the costs of increasing the use of other inputs depends mostly on the passage of time or use. If increases in other inputs makes the investment more productive, this productivity increase is credited to the investment. Finally, if we measure changes along the least cost path, we must recognize that we are answering a different kind of question than “what are the unique returns to a challenging investment?” When performing PV analysis on a stand-alone investment, we ask: does the investment earn a positive net present value (NPV)—greater than the PV sacrificed by liquidating the defender. In contrast, the incremental investment analysis asks: is this change in the capital structure of the firm and use of other inputs more efficient and profitable than before the change?

Incremental investments and comparing two challengers. Incremental investment analysis is a legitimate approach for attempting to isolate the change in a firm’s revenue and cost in response to an incremental capital investment and changes in the use of other inputs. We justify the incremental approach because we are adding an incremental investment that may change the optimal use of other nondurable inputs and use of existing durables.

But what about a firm choosing between two or more challenging investments? In this case, three scenarios are being considered: the defending firm with its existing capital structure and the firm with incremental investments one and two. In some instances, the analysis finds the present value difference between the two incremental investment's cash flow discounted by the defender's IRR. Although this approach is convenient, it not generally an acceptable approach.

The problem of discounting the cash flow difference between two possible incremental investments is that we design NPV models to rank only two investments, not three. We illustrate the problem of ranking two challengers using their differences and the defender's IRR as the discount rate. In our example, we assume the defender's IRR is 12% and incremental challenging investments C1 and C2 have similar two-period terms with the following cash flow:

Table 13.1. Ranking investments C1, C2, and the difference (C1–C2) with rankings in parentheses

Period	Cash flow for C1	Cash flow for C2	Cash flow for (C1–C2)
0	–$10	$15	–$25
1	$1	1	0
2	$15	-$10	$25
NPV at 12%	$2.85 (2)	$7.92 (1)	–$5.07 C2 is preferred to C1
IRR	27.58% (1)	–21.62% (2)	0% Indifferent between C1 and C2

The first thing to note about the example is that NPV(C1) minus NPV(C2) equals NPV(C1–C2) Therefore, individual NPV rankings in this example are consistent with NPV difference rankings as long as their cash flows are independent. However, IRR and NPV rankings for C1 and C2 are inconsistent because they violate size homogeneity conditions. Furthermore, IRR rankings depend on multiplicative operations. As a result, IRR rankings for C1 versus C2 are inconsistent with the IRR of C1 versus C2 based on the IRR of the difference in cash flows (C1–C2).

So what have we learned? We have learned that investments may be of two kinds: stand-alone and incremental. Regardless of the type of investment being analyzed, the optimal rate of service extraction will depend on whether changes in the liquidation of the investment depends mostly on the passage of time or on the level of the investment's use. Because we can likely never separate the productive contributions of the new versus the existing investments, we simply acknowledge that the best we can do is to measure the changes in cash flow and operating and capital asset accounts and that the order of incremental investments will matter.

Forecasting Future Values for PV Templates

Future values for PV template variables. Chapter 12 introduced two PV templates, one for assets and one for equity. We illustrated the templates in Tables 12.5 through 12.7 using period one data reported in the coordinated financial statements (CFS) for HQN. The two templates apply regardless of whether we are analyzing the PV of incremental or stand-alone investments. We now ask, where can we find future values for template variables that will permit us to solve multi-period PV incremental or stand-alone problems? The answer is that we estimate (guess) their values!

Excel PV templates often create a tab for each exogenous variable as well as a tab for discount rate and tax rate assumptions. These tabs feed exogenous variable values to the PV templates that calculate NPV, AE, and IRR. The columns/tabs may include:

- Investment. Typically, there will be several investment components for a project aggregated to a single value for the PV tab.

In addition, some investments may take place over the course of the project. For example, for long-term projects, earnings may be reinvestment in shorter-lived investments. In the case of dairy expansions populated with purchased cows, there will be an initial investment followed by reinvestments of cows until the herd can generate its replacements.
- Annual investment market liquidation values.
- Annual investment accounting book value. This tab must also include depreciation schedules.
- Annual account balances.
- Annual CR that may reflect a wide range of patterns over the investment's economic life including no change, constant rate of change, lower receipts in start-up years, growth to a plateau followed by a decline, and several years with no receipts followed by growth to a plateau followed by a decline (e.g., orchards).
- Annual cash COGS.
- Annual cash OE.

Our confidence in the estimates of future values for exogenous variables decreases as the distance in time of the projection from the present increases—simply because we know more about the present than the future. Still, our estimates of near present exogenous variable values are often seriously wrong. For example, present investment amount is usually a well-documented number. Nevertheless, we frequently hear reports of investment expenditure overruns. Yet current and recent past data may still represent our best starting point for projecting future exogenous variable values.

We often project future values as stable trends. However, the future is hardly ever so predictable. Consider, for example, nominal corn prices in the mid-1960s. Corn rice fell by two and one-half times in the 1970s, leading to a fall in related prices and the financial crisis of the 1980s. Land values fell as much as 50 percent. Later, in the 2006–2013 period corn prices nearly doubled over the previous twenty-year period only to fall again in 2013 with input prices substantially higher than in the previous period. In real terms, current profitability is lower than in was in the post 1980's period.

Historically, it has taken grain and oilseed markets six to twelve years to find their "new normal" following large, sustained shocks. History again provides a guide to the future, but the internal confidence around these future projections should be wide.

There are research groups such as the Food and Agricultural Policy Research Institute (FAPRI) at the University of Missouri (fapri.missouri.edu/publication/2019-u-s-baseline-outlook/ FAPRI) who annually make 10-year baseline projections for grain, oilseed, livestock markets, and a variety of other indicators. Even here, it is important to moderate forecasts using microeconomic principles and experience in making projections including a range of scenarios. Understanding intermediate and longer-term industry supply curves including regional and international dimensions is very important for most projects.

Figure 13.2. Past, Current, and Forecasted values for Corn Prices in the U.S. from 1965 to 2020 (National Agricultural Statistics Service).



Testing for robustness. That we populate our PV templates with estimates suggests that our templates will not be completely accurate. They will be consistent because the templates relate variables to each other consistently, but they are not likely to

be completely accurate. We simply are not likely to estimate all of the future values of our variables correctly. As a result, we should calculate NPV, AE, and IRR under a variety of assumptions about future values so that we have some impression about the robustness of our estimates.

Exogenous and Endogenous variables in the PV templates. Similar to CFS, we populate PV templates with exogenous and endogenous variables. The distinction we made between exogenous and endogenous variables when constructing CFS is modified only slightly when constructing PV templates. The distinction is required because CFS modeled financial activity for one period. PV templates can model financial activity for several periods, requiring that we project variable values for future periods.

We define exogenous variables in PV templates as those whose values are determined outside of the template. We define endogenous variables in PV templates as those whose values are determine by current and previous values of template variables.

We can identify exogenous variables within the template because their cell values do not reference variable values in other cells. We can identify endogenous variables within the template because they include a formula signaled by an "=" sign that references other cells.

We must project all variables values in a PV template except some of those described at the beginning and end of period one. What distinguishes between projected values is whether we project their values using current and previous periods or whether they are projected outside of the PV template. Projections made using current or previous period template values are endogenous. Projections made outside of the PV template are exogenous.

Expert opinion and forecasts. Often, we rely on experts to project revenue related variables such as yields, prices, sales, and margins between revenue and cost. For example, services such as the Food and Agricultural Policy Research Institute (FAPRI) at the University of Missouri (fapri.missouri.edu/publication/2019-u-s-baseline-outlook/ FAPRI) presents a summary of 10-year baseline projections for U.S. agricultural markets, farm program spending, farm income and a variety of other indicators. Extension specialists, such as Dr. Jim Hilker, who have distinguished themselves with accurate forecasts can also be an invaluable resource.

Populating the PV Templates

We now describe in more detail the variables and data needed to populate our PV templates that enable us to find rolling NPV, AE, and IRR estimates for investments and invested equity. We begin with the only information we know for sure, future dates: 2019, 2020, 2021, etc. Therefore, we begin populating our PV models templates with the one thing we know about the future: future dates recorded in column 1.

The investment acquisition amount. In both the investment and equity PV templates, we record the investment acquisition amount in cell B4. Its value is determined outside of the template making it an exogenous variable. We treat the investment's acquisition value as though it were a cash expenditure unless funded by debt, an amount recorded in cell C4. When we employ debt to acquire the investment, the focus is on the equity invested—equal to the investment amount less the debt used to acquire the investment. The investment amount may be the most accurate data in our template because investments occur mostly in the present and we know much more about "now" than "later".

The investment amount is usually a well-documented number. Durable sellers post their offer prices. Markets establish and record durable exchange prices. References for used farm equipment similar to Kelly's Blue Book for cars are generally available.

Complicating our investment data are projects that incur investments costs over several time-periods. Indeed, one might consider repair and maintenance expenditures designed to improve the life and durable performance to represent additional investments. In other cases, the investment includes several durable that we replace at different intervals. For example, feeder calf operations routinely replaces one cohort of calves with another but the feeding equipment and physical facilities we replace less often than the feed calves.

Liquidation values. To find rolling NPV, AE, and IRR estimates, we require not only an investment acquisition amount, but investment liquidation and market values over time. An investment's acquisition value, V_0, is the amount of money exchanged to acquire the investment. An investment's book value, V^{book}, equals it acquisition value minus its accumulated depreciation. An investment's liquidation value, $V^{liquidation}$, is the amount of money a buyer is willing to exchange for the investment when it the investment is sold. If an investment's liquidation value is greater than its acquisition value, we refer to the difference as capital gains. If an investment's liquidation value is less that its acquisition value but greater than its book value, we refer to the difference as depreciation recovery. Finally, if the liquidation value is less than its book value, we refer to the difference as capital losses. We describe capital gains, depreciation recovery, and capital losses in the following equations:

for

(13.1) $V^{liquidation} > V_0 > V^{book}, capital\ gains = V^{liquidation} - V^{book} > 0,$ V_0 > V^{book}, \,capital\, gains\, = V^{liquidation} - V^{book} > 0, \end{equation*} " title="Rendered by QuickLaTeX.com">

for

(13.2) $$\begin{aligned} V_0 > V^{liquidation} > V^{book}, depreciation\, recapture \\ = V^{liquidation} - V^{book} > 0, \end{aligned}$$ V^{liquidation} > V^{book}, \,depreciation\, recapture\, \ & = V^{liquidation}- V^{book} > 0, \end {split} \end{equation*} " title="Rendered by QuickLaTeX.com">

and for

(13.3) $V_0 > V^{book} > V^{liquidation}, capital\ losses = V^{book} - V^{liquidation} > 0.$ V^{book}> V^{liquidation}, \,capital\, losses\, = V^{book}- V^{liquidation} > 0. \end{equation*} " title="Rendered by QuickLaTeX.com">

We determine accounting book values by applying tax regulations that specify depreciation amounts depending on the type and age of the investment. Since depreciation forecasts depend on the age and amount of the investment, we generally consider them endogenous variables. Furthermore, we recognize that liquidation values of investments are also generally determined by projecting market values and cost data and may be endogenously or exogenously variables. When we do not expect to liquidate the investments, but want to estimate their NPV, AE and IRR, we may equate market liquidation value with book values.

Endogenous variables in PV templates. When projecting COGS data we generally assume they are related to CR projections. COGS are likely to depend on CR because they mostly vary with production levels making them endogenous variables. However, COGS may also vary with time as relative supplies of inputs and inflation add a trend line to our COGS estimates. We might express this relationship between future value of COGS, an endogenous projection, and their connection to CR and time *t* as:

(13.4) $COGS_t = \alpha_0 + \alpha_1 t + \alpha_2 CR_t + \epsilon_t$

where $\alpha_0, \alpha_1,$ and α_2 are estimated in such a way to reduce the average error in period *t*, ε_t, involved in the estimation of COGS while treating time *t* and CR as exogenous variables. When we have past values of the variables in Equation ??? we can use statistical techniques such as linear regression analysis to find the coefficients.

Similarly, we can treat future OE as endogenous and estimate then from values of other variables. However, OE in contrast to COGS we expect to be more related to time and their previous values than CR values. This connection between OE, time and their previous values we can express as:

(13.5) $OE_t = \beta_0 + \beta_1 t + \beta_2 OE_{t-1} + \epsilon_t$

where $\beta_0, \beta_1, and\, \beta_2$ are estimated in such a way to reduce the average error in period *t*, εt.

Endogenous variable projections. We often begin building a PV template data set by observing previous values of exogenous and endogenous variable reported in the current and past CFS. For example, suppose that we observe that prices and yields in the past have predictable trends and changes. We might use these observations of past template variable values to project their future value. To illustrate, suppose that over the past 10 years, corn yields g_1, g_2, ⋯, g_{10} have increased by 13%. Then letting [/latex]\bar{g}_y[/latex] equal the average percentage increase in corn yields we find its value in the equation equal to:

(13.6) $(1 + g_1)(1 + g_2) \cdots (1 + g_10) = (1 + \bar{g}_y)^10 = 113\%$

And solving for the annual average increase in yield we find:

(13.7) $\bar{g}_y = (1.13)^{0}.1 - 1 = 1.23\%$

Assuming the past trends in yield continue and if the yield last year were 140 bushels per acre, in 5 years we would expect yields to have increased to:

$$\begin{aligned} &current\,yield\,x(1+\ average\,annual\,increase\,in\,yields\,)^5 \\ &= 140x(1.0123)^5 = 148.8\,bushels\,of\,corn\,produced\,per\,acre\,in\,five\,years \end{aligned} \tag{13.8}$$

Finally, assume we employ the professional forecasting services to find future values corn prices which when multiplied by our projected yields produces an estimate of CR.

Projecting margins. One of the main ways we can improve our variable value forecasts is to observe their values and relationships to one another in the past—and the more observations, usually the better. One observation that is important using past data is the margin between revenue and costs. We can be confident that this difference is bounded, otherwise, competitors would flock to join our industry and begin replicating our operations. If the difference is negative for a significant period of time, we will become insolvent and leave the industry.

Past margin observations may guide our projection in the future. Taxes will be some function of the difference between revenue and sales. Changes in comparatives advantages and preferences may also be included in the description of how the variables are related. Future preferences will influence demand for what we produce. Changes in comparative advantage—who can produce least expensively and where—will also influence revenue and expenses. Favorable increases in comparative advantage will reduce our costs and put us in a position to expand our operation. And as we increase in sophistication, we might worry about local, national, and international politics that may influence tariffs and the challenges of competing in a global economy.

Other forecasts. Sometimes, failing to find forecasts of direct interest to the calculation of our investment's NPV, AE, and IRR—we can often times find forecasts highly correlated with our forecasts of interest. For example, suppose we are interested in the price of hard red winter wheat but can only observe past prices of corn and soybeans. Using linear regression, a statistical technique that allows us to relate an endogenous variable to one or more exogenous variable—we can use the future estimates of corn and soybean prices to predict hard red winter prices.

Real versus nominal data values. In our PV templates, we project CR, cash COGS and cash OE. We can employ several assumptions about future cash flow associated with these values. The first one is that there is no change from data entered in year one. This assumption is consistent with capitalization formula that assume zero change in future cash flow. It may also be relevant if the model assumes that data are in real numbers so that numbers do not change with inflation. Furthermore, we may assume that the future is represented by the present and these relationships between variables in the present should be preserved.

Solving Practical Investment Problems: Green and Clean Services

Lon is considering investing in a lawn care and snow removal business that will require that he purchase lawnmowers, snow blowers, and other equipment. Lon intends to name his business Green and Clean Services (GCS). Assume that Lon has hired you to advise him on whether or not he should invest in the business. To solve Lon's investment problem you intend to solve Equation ??? (see Equation ??? in Chapter 12).

$$\begin{aligned} &V_0 + Accounts_0 \\ &= \frac{(CR_1 - CE_1)(1-T^*) + T^* Dep_1}{[1 + IRR^A(1-T^*)]} + \cdots + \frac{(CR_n - CE_n)(1-T^*) + T^* Dep_n}{[1 + IRR^A(1-T*)]^n} \\ &+ \frac{(1-T^*)V_n^{liquidation} - T^* V_n^{book} + (1-T^*)Accounts_n + T^* Accounts_0}{[1 + IRR^A(1-T*)]^n} \end{aligned} \tag{13.9}$$

To complete the PV template corresponding to Equation ??? and described in Table 12.5, he provides the information that follows.

Acquisition cash flow. Lon estimates the initial investment will cost $40,000. The equipment falls into the MACRS 3-year depreciation class (25%, 37.5%, 25%, and 12.50%) and in four years Lon estimates he can sell his equipment for $10,000. With this information, Lon completes the acquisition stage of his investment analysis.

Operating cash flow. Lon intends to charge $40 per service for both lawn care and snow removal services. He estimates that his expenses (labor, fuel, maintenance) will average $18 per service. Lon projects the number of services for the next four years to

be 500 in year one, 750 in year two, 900 in year three, and 1,000 in year four. Lon recognizes that not all his customers will pay for his services when he provides them. He expects his accounts receivable (AR) to equal $1,000 at the end of his first year of operation, to grow to $1200 at the end of year two, to decline to $800 at the end of year three, and to decline to $400 at the end of year four. The outstanding $400 balance at the end of year four, he expects to liquidate when he sells his business.

To simplify, we assume that Lon's marginal tax rate and capital gains tax rate both equal 20%. Lon also assumes that the before-tax IRR of the defending investment equals 10% and that the after-tax IRR of the defending investment is 8%.

To determine Lon's operating ATCF, we need to find CR, cash COGS, and cash OE. We begin with CR. CR equals sales less increases (plus decreases) in AR. Increases in AR represent sales for which Lon did not receive payment. Decreases in AR represent previous sales paid for in the current period in cash. We could also adjust CR for increases (decreases) in inventories. But in this example, GCS has no inventories.

To find sales we multiply the number of services per year by the price paid per service. We find that cash sales equal $40 x 500 = $20,000 in year one; $40 x 750 = $30,000 in year two; $40 x 900 = $36,000 in year three; and $40 x 1000 = $40,000 in year four. In year one AR increased by $1,000. As a result, CR equal sales of $20,000 less $1,000 increase in AR or $19,000. In year two AR increased by $200 so that CR equal sales of $30,000 less $200 increase in AR or $29,800. In year three, AR declined by $400 so the CR was greater than sales of $36,000 by $400: $36,000 + $400 equals $36,400. Finally, at the end of year four, the last year Lon intends to operate GCS, AR declines to $400 increasing CR by $400 to $4,400. Finally, the remaining $400 balance of CR Lon liquidates when he sells the business.

Similarly, we find COGS by multiplying the number of services provided times the cost to deliver a service unit. We find they equal $18 x 500 = $9,000 in year one; $18 x 750 = $13,500 in year two; $18 x 900 = $16,200 in year three, and $40 x 1000 = $18,000 in year four.

We calculate depreciation by multiplying MACR 3-year class rates (25%, 37.5%, 25% and 12.5%) times the value of the capital accounts equal to $40,000. Depreciation is ($40,000 x 25%) = $10,000 in year one; $40,000 x 37.5% = $15,000 in year two, $40,000 x 25% = $10,000 in year three, and $40,000 x 12.5% = $5,000 in year four. Finally, we summarize our data and estimations in columns A through M in Table 13.2 below.

Table 13.2. PV Template for Rolling Estimates of NPV, AE, and IRR for Green and Clean Services (GCS)
Open Table 13.2 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K	L
3	Yr	Capital accounts values	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR- INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OE)	Tax savings from depr. (TxDepr)	After-tax cash flow (ATCF) from operations	ATCF fr liquidati Capita and operati accoun
4	0	$40,000		$40,000	$0							$40,0
5	1	$0	$30,000	$30,000	$1,000	$10,000	$19,000	$9,000	$0	$2,000	$10,000	$30,8
6	2	$0	$20,000	$15,000	$1,200	$15,000	$29,800	$13,500	$0	$3,000	$16,040	$19,9
7	3	$0	$15,000	$5,000	$800	$10,000	$36,400	$16,200	$0	$2,000	$18,160	$13,6
8	4	$0	$10,000	$0	$400	$5,000	$40,400	$18,000	$0	$1,000	$18,920	$8,3

To find after-tax cash flow (ATCF) from operations during each year, we subtract from CR in column G, cash COGS in column H and cash OE in column I and multiply the result by $(1 - T^*)$ where T^* equals the average tax rate paid on investment earnings that we set equal to 20%. Finally, we add tax savings from depreciation T^*Dep reported in column J. These operations on exogenous variables estimate ATCF from operations reported in column K.

Liquidation. At the end of four years of operating GCS, Lon's equipment has a salvage value of $10,000 and an accounting book value of $0. Therefore, the after-tax liquidated equipment salvage value equals $V_4 (1 - T^*) = \$10{,}00(1 - .2) = \$8{,}000$. (See equation

13.6). The AR account has a value of $400 and when liquidated is treated as after-tax cash income of $AR_4(1 - T^*) = \$400(1 - .2) = \$320$. The sum of ATCF from liquidation is equal to $8000 + $320 plus $T^* V_n^{book} = \$0$ and T*Accounts0 = $0 (see equation 13.6) or an after-tax liquidation value at the end of year four equal to $8,320.

Rolling NPV, AE, and IRR calculations. Lon assumes he will operate GCS for four years. He might have asked: what is the optimal life of GCS? To answer the optimal life question, we need to find annual or rolling NPV, AE, and IRR values. In this case, Lon is correct, four years is preferred to one, two, or three years of operation. Rolling NPV reported in column N increased from a negative NPV of ($2,222.22) if he operated the business for one year to an NPV of $17,449.18 if he operated his business for four years as planned. That the optimal life of operation is confirmed by increasing AE from a negative ($2,222.22) to a positive $4,362.30 in year 4. Meanwhile, IRR increases from 2.00% to 23.46% in year four.

To find rolling NPV, AE, and IRR estimates we divide the numbers in Table 13.2 into acquisition, operating, and liquidation cash flow. We record the acquisition value in all for years in cell L4. We record the ATCF from operations in column K and the ATCF from liquidation in column L. The sum of ATCF from operations and liquidations for the last year of operations we record in column M. We record the ATCF for each year of possible economic life in the IRR tab. In our case the ATCF for years 1,2,3, and 4 are recorded in columns B, C, D, and E in the Excel spreadsheet corresponding to the IRR tab.

Table 13.3. ATCF, NPV, AE, and IRR for GCS for economic lives of 1, 2, 3, and 4 years for an investment of $40,000.
Open Table 13.3 in Microsoft Excel

	A	B	C	D	E
1	Economic Life	1 year	2 years	3 years	4 years
2	ATFC year 0 (investment)	-$40,000.00	-$40,000.00	-$40,000.00	-$40,000
3	ATFC year 1	$40,800.00	$10,000.00	$10,000.00	$10,000
4	ATFC year 2		$36,000.00	$16,040.00	$16,040
5	ATFC year 3			$31,800.00	$18,160
6	ATFC year 4				$27,240
7					
8	NPVs	-$2,222.22	$123.46	$8,254.84	$17,449.18
9	AEs	-$2,222.22	$61.73	$2,751.61	$4,362.30
10	IRRs	2.00%	8.19%	17.15%	23.46%

Equity invested in Green and Clean Services. Lon is not only interested in knowing his earnings from the GCS investment, he is also interested in knowing his earnings from his equity invested in GCS. To that end, he accounts for the investment less the debt supporting his investment in Table 13.3. Lon intends to borrow $10,000 and pay interest costs at a rate of 6% on principal that he intends to reduce each year by $1,000. As a result, at the end of the first year Lon expects his outstanding loan balance will equal $9,000, $8,000 at the end of year two, $7,000 at the end of year three, and $6,000 at the end of year four. Lon plans to liquidate his loan balance of $6,000 when he sells his business at the end of four years of operations.

To find his equity earnings, we solve Equation ??? (see equation 12.20 in Chapter 12). The template corresponding to Equation ??? and is only slightly changed from the template described in Table 13.2. We describe it next in Table 13.4. It records outstanding debt in column C, interest payments that reduce operating income and taxes, we record in column K.

$$
\begin{aligned}
&V_0 + Accounts_0 - D_0 \\
&= \frac{(CR_1 - CE_1 - iD_0)(1-T) + TDep_1}{[1 + IRR^E(1-T)]} + \cdots \\
&+ \frac{(CR_n - CE_n - iD_{n-1})(1-T) + TDep_n}{[1 + IRR^E(1-T)]^n} \\
&+ \frac{(1-T)V_n^{liquidation} - TV_n^{book} + (1-T)Accounts_n + TAccounts_0 - D_{n-1}}{[1 + IRR^E(1-T)]^n}
\end{aligned}
\tag{13.10}
$$

To find rolling NPV, AE, and IRR estimates we divide the numbers in Table 13.4 into acquisition, operating, and liquidation cash flow. We record the acquisition value in all for years in cell N4. We record the ATCF from operations in column M and the ATCF from liquidation in column N. The sum of ATCF from operations and liquidations for the last year of operations is recorded in column O.

Table 13.4. PV Template for Rolling Estimates of NPV, AE, and IRR for Equity Invested in Green and Clean Services (GCS)

Open Table 13.4 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K	L
3	Yr	Capital accounts values	Debt Capital	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR- INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OE)	Interest Costs	Tax savings from depr. (TxDepr)
4	0	$40,000	$10,000		$40,000	$0						
5	1	$0	$9,000	$30,000	$30,000	$1,000	$10,000	$19,000	$9,000	$0	$600	$4,000
6	2	$0	$8,000	$20,000	$15,000	$1,200	$15,000	$29,800	$13,500	$0	$540	$6,000
7	3	$0	$7,000	$15,000	$5,000	$800	$10,000	$36,400	$16,200	$0	$480	$4,000
8	4	$0	$6,000	$10,000	$0	$400	$5,000	$40,400	$18,000	$0	$420	$2,000

In the IRR tab we record the ATCF for each year of possible economic life. In our case the ATCF for years 1,2,3, and 4 are recorded in columns B, C, D, and E. In the IRR tab in rows 8, 9, and 10 we report NPV, AE, and IRR for each year and repeat them for convenience in columns P, Q, and R in Table 13.5.

Table 13.5. ATCF, NPV, AE, and IRR for GCS for economic lives of 1, 2, 3, and 4 years for equity of $30,000 invested in GCS.

Open Table 13.5 in Microsoft Excel

	A	B	C	D	E
1	Economic Life	1 year	2 years	3 years	4 years
2	ATFC year 0 (investment)	-$30,000	-$30,000	-$30,000	-$30,000
3	ATFC year 1	$30,240	$9,640	$9,640	$9,640

4	ATFC year 2		$25,176	$15,456	$15,456
5	ATFC year 3			$19,312	$15,832
6	ATFC year 4				$14,428
7					
8	NPVs	-$1,471.70	$1,500.89	$9,064.85	$17,571.30
9	AEs	-$1,471.70	$750.44	$3,021.62	$4,392.83
10	IRRs	0.80%	9.07%	19.89%	27.80%

Excel Spreadsheets for Describing Exogenous Variable Forecasts

As we increase the sophistication of our forecasts, we recognize the need to use additional Excel spreadsheets identified by tabs. These describe the procedures we followed to estimate the future values of exogenous variables and the values of exogenous variables we estimated. The tabs appear at the bottom of Tables 13.2 that we describe next. Similar tabs describe the exogenous variable projection procedures and values for Table 13.4.

Constants tab

After the template tab that contains Table 13.2 and Table 13.4 is the “constants” tab that contains key assumptions employed in the calculation of the Tables 13.2 and 13.4. Although they vary with projection method, they may include such constants as the average federal, state, and local tax rates paid on earnings from the investment or equity earnings, the defenders IRR, growth rate assumptions for debt, liquidation values, CR, COGS, OE and other variables. We represent arbitrary constant value assumption below that would be input to the PV template

Table 13.6a. Excel Template Constants Worksheet Tab
Open Template in Microsoft Excel

	A	B
1	**IRRs, tax rates, and growth rates**	
2	defender’s IRR	10.00%
3	average tax rate	20.00%
4	After-tax rate	8.00%
5	average interest rate	6.00%
6	debt growth rate	1.00%
7	liquid. growth rate	-2.00%
8	Book value growth rate	-2.00%
9	acct.bal.growth rate	1.00%
10	CR growth rate	1.00%
11	COGS growth rate	1.00%
12	OEs growth rate	1.00%

IRR tab

To calculate IRR required that we assemble ATCF for each PV model with a different term. These along with NPV, AE, and IRR computations we report in the IRR tab. Data for the IRR tab for GCS were reported in Tables 13.3 and 13.5.

Table 13.6b. Excel Template IRRs Worksheet Tab
Open Template in Microsoft Excel

	A	B	C	D	E
1	Economic Life	1 year	2 years	3 years	4 years
2	ATFC year 0 (investment)	-$40,000	-$40,000	-$40,000	-$40,000
3	ATFC year 1	$40,800	$10,000	$10,000	$10,000
4	ATFC year 2		$36,000	$16,040	$16,040
5	ATFC year 3			$31,800	$18,160
6	ATFC year 4				$27,240
7					
8	NPVs	-$2,222.22	$123.46	$8,254.84	$17,449.18
9	AEs	-$2,222.22	$61.73	$2,751.61	$4,362.30
10	IRRs	2.00%	8.19%	17.15%	23,46%

Capital Account tab

The capital account describes the capital investment that is the focus of the investment. Usually, the investment occurs at the beginning of the model, period zero. However, it may describe other investments during the economic life of the analysis.

Table 13.6c. Excel Template Capital Account Worksheet Tab
Open Template in Microsoft Excel

	A	B
1	Capital account value	
2	$40,000	-> investment in lawn and snow removal equip
3	$0	
4	40	

Liquidation Value of Capital Investments tab

To calculate rolling NPV, AE, and IRR estimates, we must estimate the liquidation value for capital investments. Generally, we treat capital account liquidation values as exogenous. We represent arbitrary projected capital account liquidation values below that would be input to the PV template.

Table 13.6d. Excel Template Liquidation Value Worksheet Tab
Open Template in Microsoft Excel

3	Yr	Capital accounts values	Capital accounts liquidation value
4	0	$40,000	
5	1	$0	$30,000
6	2	$0	$20,000
7	3	$0	$15,000
8	4	$0	$10,000
9	5	$0	$10,000
10	6	$0	
11	7	$0	

Book Value of Capital Assets tab

The purchase price of capital assets established a book or accounting value of capital assets. In the CFS, we record the book value of capital assets in firm's balance sheets. The book value of the firm's capital assets are adjusted by depreciation rates determined by taxing authorities. There are more than one methods for determining depreciation allowing capital investment owners some flexibility in establishing depreciation amounts. Generally, the time value of money encourages investment owners to depreciate their investments as rapidly as possible. In this tab, we generally describe the depreciation method that is used to establish the capital investments' book values. We represent arbitrary projected book values of capital accounts below that would be input to the PV template.

Table 13.6e. Excel Template Book Value Worksheet Tab

Open Template in Microsoft Excel

	A
1	Capital Accounts Book Value
2	$40,000
3	$30,000
4	$15,500
5	$5,000
6	$0
7	$0

Depreciation Rate tab

The Excel spreadsheet that corresponds to the depreciation rate tab (Dep) describes the depreciation method applied to the capital investments to determine their book value. This tab most often includes the depreciation base for capital investments so that multiplying the depreciation rates times the base calculates the depreciation amount in each year. We represent arbitrary projected depreciation values below that would be input to the PV template

Table 13.6f. Excel Template Depreciation Worksheet Tab

Open Template in Microsoft Excel

	A	B	C	D
1	Year	Depr. Base	Depr. Rate	Depr. Amount
2	0	$40,000		
3	1		25%	$10,000
4	2		37.50%	$15,000
5	3		25%	$10,000
6	4		12.50%	$5,000
7	5		0%	0

Sum of Operating Accounts tab

Increases in the sum of operating accounts (AR+Inv) represent income not received as cash. Decreases in the sum of operating accounts (AR+Inv) represent income earned previously received as cash in the current period. Increases in the sum of operating accounts (AP+AL) represent expenses not paid for in cash. Decreases in the sum of operating accounts (AP+AL) represent expenses incurred previously paid in the current period. The sum of operating accounts are reported in this tab. We represent arbitrary projected sums of operating accounts below that would be input to the PV template.

Table 13.6g. Excel Template Operating Accounts Worksheet Tab
Open Template in Microsoft Excel

	B	C	D	E	F
1	AR	INV	AP	AL	Op. Acc. Sum
2	1640	3750	3000	958	1432
3	1200	5200	4000	880	1520
4					1550
5					1580
6					1610
7					1500

Cash Receipts tab

Cash receipts are the oxygen of the firm and investment. We might say that cash receipts are the sine qua non tab because without it nothing else exists for long. In addition, we might also claim that it is the leading exogenous variable because other cash flow variables such as COGS and OE are related to it. Finally, it is the variable most often the focus on professional forecasts or at least the focus of variables highly correlated with CR such as prices. We will have more to say about the Excel sheet that correspond to the CR tab later on. We represent arbitrary projected cash CR estimates below that would be input to the PV template.

Table 13.6h. Excel Template Cash Receipts Worksheet Tab
Open Template in Microsoft Excel

	A	B
1	Cash Receipts	

2	Year	
3	0	$19000.00
4	1	$19190.00
5	2	$19381.90
6	3	$19575.72
7	4	$19771.48
8	5	$19969.19

Cash COGS tab

Cash COGS follow closely CR values since by definition they are the costs that vary with production. Therefore, most forecasts of Cash COGS use CR as a variable in the COGS forecast. COGS forecasts may also benefit from projections of input prices. We represent arbitrary projected cash COGS estimates below that would be input to the PV template.

Table 13.6i. Excel Template COGS Worksheet Tab
Open Template in Microsoft Excel

	A	B
1	Cost of Goods Sold	
2	Year	
3	0	
4	1	$9,000.00
5	2	$9,090.00
6	3	$9,180.90
7	4	$9,272.709
8	5	$9,365.436

Cash OE tab

Cash OE may be related to CR but less so than cash COGS. The initial value for OE may be determined by the size of the business and production activity but in later periods overhead expenses may be less tied to production that input prices that vary over time. Therefore, we sometimes estimate cash OE based on their previous period's value plus some function of time designed to capture changes in utility, labor, and general maintenance costs. We represent arbitrary projected OE estimates below that would be input to the PV template.

Table 13.6j. Excel Template Cash OE Worksheet Tab
Open Template in Microsoft Excel

	A	B
1	Cash Overhead Expenses	
2	Year	
3	0	

4	1	$300.00
5	2	$305.00
6	3	$310.00
7	4	$315.00
8	5	$320.00

More Sophisticated Forecasts for GCS

Multi-period PV models depend on forecasts of future values of exogenous variables. Our forecasts of exogenous variables differ in their sophistication and resources committed to their estimation. Obviously, we can never validate our forecasts until the time of the forecast has passed, leading us to infrequently report the accuracy of our predictions—which are usually "wide of the mark".

So as the quip goes, if you cannot guess correctly, guess often! That strategy is exactly what we recommend here: make frequent forecast updates as new information becomes available. We begin with a base forecast, the one we expect most likely to occur. In our case, we reported the base forecasts for GCS in Tables 13.2 and 13.4. Then we make additional forecasts employing a variety of methods and resources.

Status quo

We begin with the least sophisticated forecast—that the future will be the same as the present. The rational for this approach is that our best forecast is likely the closest to the present. In our case, we assume future CR and COGS in period one continue for the next four years and that AR remain at $1,000. We continue with our other assumptions and resolve the PV template for investments. We describe the results in Table 13.7.

Table 13.7. PV Template for Rolling Estimates of NPV, AE, and IRR for Green and Clean Services (GCS) Assuming Constant CR and COGS

Open Template in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K	L
3	Yr	Capital accounts values	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR + INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OE)	Tax savings from depr. (TxDepr)	After-tax cash flow (ATCF) from operations	ATCF fror liquidatin capital and operating accounts
4	0	$40,000		$40,000	$0							$40,00
5	1	$0	$30,000	$30,000	$1,000	$10,000	$19,000	$9,000	$0	$2,000	$10,000	$30,80
6	2	$0	$20,000	$15,000	$1,000	$15,000	$19,000	$9,000	$0	$3,000	$11,000	$19,80
7	3	$0	$15,000	$5,000	$1,000	$10,000	$19,000	$9,000	$0	$2,000	$10,000	$13,80
8	4	$0	$10,000	$0	$1,000	$5,000	$19,000	$9,000	$0	$1,000	$9,000	$8,80
9	5	$0	$10,000	$0	$1,000	$0	$19,000	$9,000	$0	$0	$8,000	$8,80

We can observe these consequences of our more conservative growth assumptions. NPV and AE remain negative throughout the four years. Only in the fifth year does the IRR exceed the after-tax discount rate of 8% turning NPV and AE positive. The IRR for GCS is increasing and we expect that by year five will exceed the after-tax IRR of the defender. Contributing to the positive NPV in period five is the liquidation of the capital accounts that Lon assumes will be the same value as in year four.

Constant percentage change assumptions

Next in forecast sophistication, we assume a constant growth (decay) rate assumption about CR and COGS. In this forecast we project $CR_t = (1 + g_{CR})CR_{(t-1)}$ and similarly we project COGS as: $COGS_t = (1 + g_{COGS})COGS_{(t-1)}$ .Our assumptions for g_{CR} and g_{COGS} we describe in the Excel sheet accessed through the constant tab—each assumed to equal 1%. We project the future values for CR and COGS and report them in the Excel sheets reference by the CR and COGS tabs. Then we copy them into the GCS investment template and report the results in Table 13.8.

Table 13.8. PV Template for Rolling Estimates of NPV, AE, and IRR for Green and Clean Services (GCS) Assuming Constant % Δ in CR and COGS

Open Table 13.8 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K	L
3	Yr	Capital accounts values	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR + INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OE)	Tax savings from depr. (TxDepr)	After-tax cash flow (ATCF) from operations	ATCF fr liquidati capita and operati accoun
4	0	$40,000		$40,000	$0							$40,0
5	1	$0	$30,000	$30,000	$1,000	$10,000	$19,000	$9,000	$0	$2,000	$10,000.00	$30,8
6	2	$0	$20,000	$15,000	$1,000	$15,000	$19,190	$9,090	$0	$3,000	$11,080.00	$19,8
7	3	$0	$15,000	$5,000	$1,000	$10,000	$19,382	$9,181	$0	$2,000	$10,160.80	$13,8
8	4	$0	$10,000	$0	$1,000	$5,000	$19,576	$9,273	$0	$1,000	$9,242.41	$8,8
9	5	$0	$10,000	$0	$1,000	$0	$19,771	$9,365	$0	$0	$8,324.83	$8,8

Note that increasing CR and COGS by the same percent each period increased the margin between them over time. To illustrate, in year one, (CR – COGS) = ($19,000 – $9,000) = $10,000. In year five, we found the margin to be (CR – COGS) = ($19,771 – $9,365) = $10,406. Because of compounding CR and COGS, NPV turned positive in year four (one year earlier than under the constant value assumption) and the IRR in the constant percentage change model exceeded the IRR in the constant model in each year after the first one.

Linear growth rate assumption

To project linear grow rates is generally a most conservative assumption that projecting constant percentage growth rates—other things being equal. There are several different linear assumptions we can adopt. For example, we might assume that $CR_t = CR_1 + \alpha_t$ and $COGS_t = COGS_1 + \beta_t$ where t = 1, 2, 3, 4, 5 and constants $\alpha = 20$ and $\beta = 25$. We project the future values for CR and COGS using our estimating and report them in the Excel sheets reference by the CR and COGS tabs. Then we copy them into the GCS investment template and report the results in Table 13.9.

Table 13.9. PV Template for Rolling Estimates of NPV, AE, and IRR for Green and Clean Services (GCS) Assuming Linear Δ in CR and COGS

Open Table 13.9 in Microsoft Excel

	A	B	C	D	E	F	G	H	I	J	K	L

3	Yr	Capital accounts values	Capital accounts liquidation value	Capital accounts book value	Operating accounts values (AR + INV – AP – AL)	Capital Accounts Depr.	Cash Receipts (CR)	Cash Cost of Goods Sold (COGS)	Cash Overhead Expenses (OE)	Tax savings from depr. (TxDepr)	After-tax cash flow (ATCF) from operations	ATCF fro liquidatir capital and operatin account
4	0	$40,000		$40,000	$0							$40,00
5	1	$0	$30,000	$30,000	$1,000	$10,000	$19,000	$9,000	$0	$2,000	$10,000	$30,80
6	2	$0	$20,000	$15,000	$1,000	$15,000	$19,100	$9,110	$0	$3,000	$10,992	$19,80
7	3	$0	$15,000	$5,000	$1,000	$10,000	$19,200	$9,330	$0	$2,000	$9,896	$13,80
8	4	$0	$10,000	$0	$1,000	$5,000	$19,300	$9,660	$0	$1,000	$8,712	$8,80
9	5	$0	$10,000	$0	$1,000	$0	$19,400	$10,100	$0	$0	$7,440	$8,80

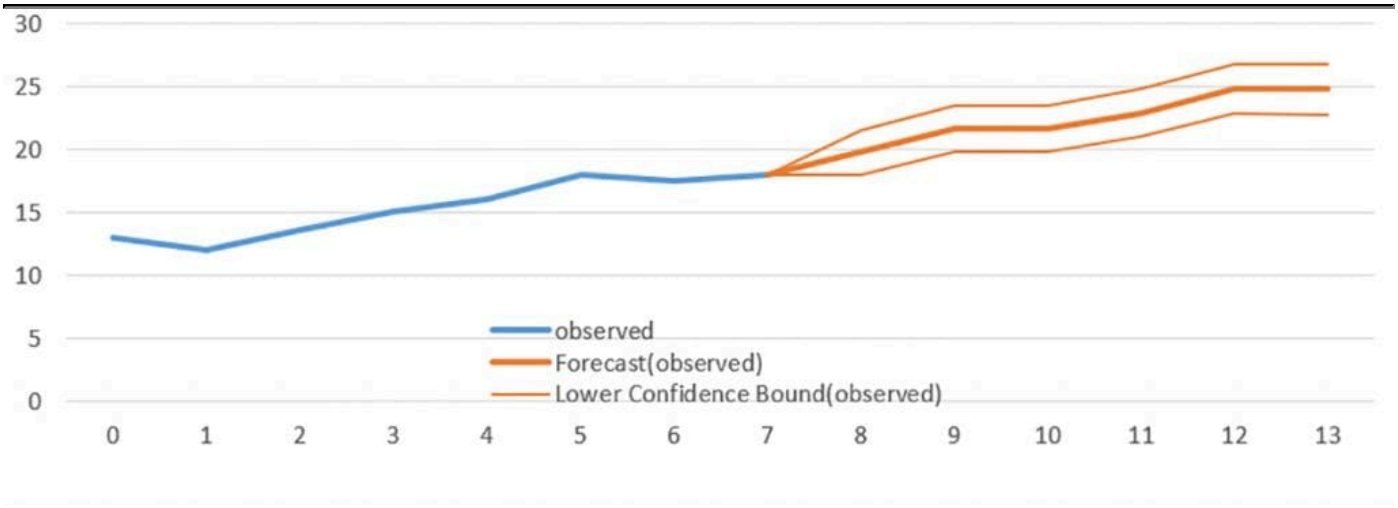
Projections based on related forecasts

To this point, we have projected CR and COGS based on their past values and time. Most sophisticated projections take advantage of expert predictions based on observed value of related variables in the past. Suppose that for the last seven years, prices of lawn care and snow removal services were $13, $12, $13.5, $15, $16, $18, $17.5, and $17.9 for t = 0, 1, 2, 3, 4, 5, 6, 7. We project future prices p for t = 8, 9, 10, 11, 12, 13 for lawn care and snow removal services using Excel's forecast option.

Table 13.10. Excel Forecast Estimate
Open Template in Microsoft Excel

	A	B	C	D	E
1	time	observed	Forecast(observed)	Lower Confidence Bound(observed)	Upper Confidence Bound(observed)
2	0	13			
3	1	12			
4	2	13.5			
5	3	15			
6	4	16			
7	5	18			
8	6	17.5			
9	7	17.9	17.90	17.90	17.90
10	8		19.73214458	17.99	21.47
11	9		21.59962998	19.81	23.39
12	10		21.62645857	19.78	23.47
13	11		22.85993116	20.96	24.76
14	12		24.72741656	22.78	26.68
15	13		24.75424515	22.75	26.75

Figure 13.3. Graph of Projection Estimates
Open Template in Microsoft Excel



Having obtained price forecasts, our next step would be to re-estimate CR for GCS based on the forecasted prices. In addition, we may use the confidence interval forecasts to find a most optimistic forecast using the upper confidence interval forecasts and a pessimistic forecast using the lower bound forecasts.

The forecasts above have been presented as coming from a "black box". The black box, in this case contains well-known statistical methods that can easily be learned but beyond an already over ambitious collection of topics covered in this book.

So what have we learned? We learned that PV models analysis requires projections of future values for exogenous variables. Furthermore, we learned that there is no way to validate our future estimates except by comparing them to their past values. Finally, project future values of exogenous variables from the past values is an art and unlikely to be accurate. Therefore, our approach should be to estimate several alternative forecasts to determine the robustness of our results under a variety of assumptions. Still, we recommend that when any new information that might affect our forecast becomes available, we should incorporate it into our models. Another approach to forecasting and making investment decisions based on our forecasts is to require a "cushion". In other words, to commit to an investment if it not only earn the defender's IRR—but that it earns much more—a cushion—so that if our forecasts are wrong, too optimistic, we are still likely to earn a positive NPV.

Incremental Investments

Incremental investments may replace, modify, or expand an existing investment. Regardless, the incremental investment analysis approach is the same: it compares the defending investment with changes that result from an incremental investment. What is common to incremental investments is that something of the original investment remains.

In what follows, we illustrate an incremental investment with a bakery considering replacing its doughnut-making machine. Other investments supporting the doughnut making machine are assumed to be unaffected by the incremental investment.

Brown and Round Doughnuts

We next consider two alternative machines for making doughnuts. One machine is the one already in operation. In this case the challengers are an older version of the original machine and a new machine. Should you continue with the used machine, buy a new machine, or get out of the doughnut business?

Continuing to make doughnuts with the old machine.

In this example the challenger is an older version of the original investment. To be specific, suppose you bought a doughnut machine 3 years ago for $90,000. You are depreciating the machine over 5 years using the MACRS method, and the expected market value 5 years from today is $10,000. The machine generates $30,000 in cash revenues and produces $15,000 in cash expenses each year. If you sold the machine today, you could get $30,000 from a local competitor.

Your after-tax ROA-IRR, $r(1 - T)$, is 10%, your marginal tax rate is 40%, and the capital gains tax is 20%. Assume book value

depreciation can be used to offset ordinary income.

Let's begin with the old machine. Keeping the old machine is equivalent to reinvesting its $30,000 current value ($V_0$) in the business and forfeiting the tax refund from capital losses. To determine the tax refund forfeited, recall that the used machine was purchased 3 years earlier for $90,000, and has been depreciated using 5-year MACR, so the machine's book value is $90,000(100% – 15% – 25.5% –17.85%) = $37,485.

Since book value exceeds liquidation value, a tax credit is owed to the seller equal to $T(V_0 - V_0^{book}) = .4(30,000 - 37,485) = \$2,994.$ Now we can write the acquisition ATCF as $ATCF_0$ = –$30,000 – $2,994 = –$32,994. Finally, 5 year MACR depreciation rates on the old machine equal: 16.66%, 16.66%, and 8.33%. We write the ATCF for the tth period as:

$$\begin{aligned} ATCF_t &= (CR_t - CE_t - Dep_t)(1-t) + Dep_t - \Delta V_n \\ &= (CR_t - CE_t)(1-t) + TDep_t - \Delta V_n \end{aligned} \tag{13.11}$$

However, because ATCF per period differs, we describe it using Table 13.11. below.

Table 13.11. Finding ATCF for Continuing to Operate with the Old Doughnut Machine

Old Doughnut Machine Operating ATCF						
Year	CR_t	CE_t	Dep_t (MACR)	$(CR_t - CE_t)(1- T) + T\ Dep_t$	ΔV_{n_t}	$ATCF_1$
1	30,000	15,000	14,994 (16.66%)	(15,000 x .6) + (14,994 x .4) = 14,998	0	$14,998
2	30,000	15,000	14,994 (16.66%)	(15,000 x .6) + (14,994 x .4) = 14,998	0	$14,998
3	30,000	15,000	7,470 (8.33%)	(15,000 x .6) + (7470 x .4) = 11,988	0	$11,988
4	30,000	15,000	0	(15,000 x .6) = 9,000	0	$9,000
5	30,000	15,000	0	(15,000 x .6) = 9,000	0	$9,000

Finally, we write the salvage value for the old machine. Recalling that the salvage value of the old machine, V_5, is $10,000, that the old machine is completely depreciated, and the capital gains tax rate T_g is 0.2, we can write the salvage value as:

$$\begin{aligned} ATCF_5 &= V_5(old) - T[V_5(old) - V_5(book\,of\,old)] \\ &= \$10,000 - .2(10,000 - 0) = \$8,000 \end{aligned} \tag{13.12}$$

Finally, we are prepared to combine the ATCF from the acquisition, operation, and liquidation of the old doughnut machine. We express these in Table 13.12.

Table 13.12. Old Doughnut Machine Operating ATCF

Year	CR_t	CE_t	Dep_t (MACR)	$(CR_t - CE_t)(1- T) + T\ Dep_t$	ΔV_{n_t}
0	$-V_0 - T(V_0 - V_0^{book}) = -\$30{,}000 - (.4)(\$30{,}000 - \$37{,}485) = -\ \$30{,}000 - \$2{,}994 = -\$32{,}994$				
1	30,000	15,000	14,994 (16.66%)	(15,000 x .6) + (14,994 x .4) = 14,998	
2	30,000	15,000	14,994 (16.66%)	(15,000 x .6) + (14,994 x .4) = 14,998	
3	30,000	15,000	7,470 (8.33%)	(15,000 x .6) + (7470 x .4) = 11,988	
4	30,000	15,000	0	(15,000 x .6) = 9,000	
5	30,000	15,000	0	(15,000 x .6) = 9,000	
5	$V_5 - T(V_5 - V_5^{book}) = \$10{,}000 - (.2)(\$10{,}000 - 0) = \$10{,}000 - \$2{,}000 = \$8{,}000$				

The only other calculation left is to compute the NPV of the old doughnut machine which we complete using our Excel spreadsheet.

Table 13.13. Old Doughnut Machine
Open Table 13.13 in Microsoft Excel

	A	B	C
1	*Old Doughnut Machine*		
2	variables	data	formulas
3	rate	0.1	
4	Acquisition Cost	($32,994)	
5	ATCF1	$14,998	
6	ATCF2	$14,998	
7	ATCF3	$11,998	
8	ATCF4	$9,000	

9	ATCF5	$9,000	
10	Salvage Value	$8,000	
11	NPV	$18,745	"= -V_0 + NPV(rate, ATCF1:ATCF5) + V_n / (1 + rate)^5
12	IRR	30%	"=IRR(Acquisition Cost, ATCF1:ATCF5, Salvage Value)
13	AE	($4,944.92)	"=PMT(rate, nper, NPV)
14	nper	5	

Purchasing a new doughnut machine.

Assume that, after completing the PV analysis of the old doughnut machine, a salesman for a "new and improved" doughnut machine stops by and wants to sell you a new machine. The new machine will increase your revenues to $45,000 each year, and decrease your expenses to only $10,000 each year. The new machine costs $90,000 and will require $10,000 for delivery and installation costs. Thus, the acquisition ATCF0 = $100,000. The machine will fall into the MACRS five-year depreciation class (15%, 25.5%, 17.85%, 16.66%, 16.66%, and 8.33%). The machine is expected to have a $40,000 salvage value after 5 years.

Next, we need to determine the operating ATCF for the new machine. Tax rates, the defender's after-tax IRR, and MACR depreciation rates are the same as before—equal to those used to evaluate the continued use of the old doughnut machine. Because of the level of sales, accounts receivable and accounts payable increase by $5,000 and $2,000 in the first year respectively. They are reduced by the same amount in the fifth year. The formula for operating ATCF is the same as before except that sales and expenses are no longer just cash items and must be adjusted by the term ΔV_n = ΔCA – ΔCL = $5,000 – $2,000 = $3,000

$$\begin{aligned} ATCF_t &= (Sales_t - Expenses_t - Dep_t)(1-T) + Dep_t - \Delta V_n \\ &= (Sales_t - Expenses_t)(1-T) + TDep_t - \Delta V_n \end{aligned} \tag{13.13}$$

The new machine will be depreciated using the 5-year MACRS method. The depreciation for each machine, and the change in depreciation each year for the next 5 years is:

Table 13.14a. New Doughnut Machine Depreciation

Year	$100,000 x MACR rate	=	Depreciation
1	$100,000 x 15%	=	$15,000
2	$100,000 x 25.5%	=	$25,500
3	$100,000 x 17.85%	=	$17,850
4	$100,000 x 16.66%	=	$16,660
5	$100,000 x 16.66%	=	$16,660

Remember that depreciation expense is a noncash expense, which by itself doesn't generate a cash flow. However, you can use the depreciation expense to reduce taxable income. As a result, the firm realizes an additional tax savings from depreciation. The amount of the tax savings are described below.

Table 13.14b. New Doughnut Machine Tax Savings

Year	Tax Savings	=	(Depreciation)(T = .4)
1	$15,000 x (.4)	=	$6,000
2	$25,500 x (.4)	=	$10,200
3	$17,850 x (.4)	=	$7,140
4	$16,660 x (.4)	=	$6,664
5	$16,660 x (.4)	=	$6,664

Finally, we find the salvage value as follows. We first recognize that the sale of the doughnut machine in 5 years will increase cash flow by $40,000. Next, we find the asset's book value in year five as the difference between its acquisition value less its accumulated depreciation: $100,000 – ($15,000 + $25,500 + $17,850 + $16,600 + $16,660) = $8,330. Since the asset's book value is less than its salvage value, there are capital gains taxes to be paid. Therefore, the salvage value ATCF can be written as the salvage value less the capital gains tax:

$$\begin{aligned} ATCF &= V_5 - T[V_5 - V_5(book)] \\ &= \$40,000 - .2(\$40,000 - \$8,330) = \$33,666 \end{aligned} \tag{13.14}$$

Finally, we combine acquisition, operation, and liquidation ATCF associated with the new doughnut machine in Table 13.15.

Table 13.15. New Doughnut Machine Operating ATCF

Year	Sales	Expense	Dep. (MACR %)	(Sales – Expenses) x (1 – T) + T Dep.
0 $ATCF_0$(Acquisition) = $-V_0$ =				
1 $ATCF_1$	45,000	10,000	15,000 (15%)	(35,000 x .6) + (15,000 x .4) = 27,000
2 $ATCF_2$	45,000	10,000	25,500 (25.5%)	(35,000 x .6) + (25,500 x .4) = 31,200
3 $ATCF_3$	45,000	10,000	17,850 (17.85%)	(35,000 x .6) + (17,850 x .4) = 28,140
4 $ATCF_4$	45,000	10,000	16,660 (16.66%)	(35,000 x .6) + (16,660 x .4) = 27,664
5 $ATCF_5$	45,000	10,000	16,660 (16.66%)	(35,000 x .6) + (16,660 x .4) = 27,664
5 $ATCF_5$ (Liquidation) = $V_5 - T[V_5 - V_5$(book)] = $40,000 – .2($40,000 – $8,330) =				

We find the NPV for the new doughnut machine in Table 13.16:

Table 13.16. New Doughnut Machine NPV

Year	$\frac{1}{(1+r(1=T))^2} = \frac{1}{(1.10)^t}$	ATCF	$\frac{ATCI}{(1+r(1-}$
0	1	–$100,000	–$100,000
1	.91	$24,000	$21,840

Year	$\frac{1}{(1+r(1=T))^2} = \frac{1}{(1.10)^t}$	ATCF	$\frac{ATCI}{(1+r(1-$
2	.83	$31,200	$25,896
3	.75	$28,140	$21,105
4	.68	$27,664	$18,812
5	.62	$30,664 + $33,666 = $64,330	$39,885
$NPV = \sum_{t=0}^{5} \frac{ATCF_t}{[1+r(1-T)]^t} = \$27,538$ (27,584 using CF worksheet)			

Finally, we find the NPV for the new doughnut machine using our Excel Spreadsheet.

Table 13.17. New Doughnuts Machine
Open Table 13.13 in Microsoft Excel

	A	B	C
1	*New Doughnut Machine*		
2	variables	data	formulas
3	rate	0.1	
4	Acquisition Cost	($100,000)	
5	ATCF1	$24,000	
6	ATCF2	$31,200	
7	ATCF3	$28,140	
8	ATCF4	$27,664	
9	ATCF5	$30,664	
10	Salvage Value	$33,666	
11	NPV	$27,584	"= -V_0 + NPV(rate, ATCF1:ATCF5) + V_n / (1 + rate)^5
12	IRR	18%	"=IRR(Acquisition Cost, ATCF1:ATCF5, Salvage Value)
13	AE	($7,276.60)	"=PMT(rate, nper, NPV)
14	nper	5	

The result of the analysis is that NPV will be positive for both options by continuing to use the old machine or by investing in the new one. However, the firm's NPV will increase the most if it adopts the new machine—an increase of $8,839 ($27,584 – $18,745) over continuing to use the old machine. However, the IRR of the old machine is greater, and we have a conflict between IRR and NPV rankings. The ranking is resolved by deciding on the reinvestment rate for the differences in funding required by the two investments. If the difference in funding levels is invested at the discount rate, the NPV rankings are appropriate. If the investments can be scaled, then the IRR rankings are appropriate. It is assumed here that the reinvestment rate is the after-tax IRR of the defender, 10%, so that any differences in size would not change their respective NPVs, and NPV rankings are appropriate.

Summary and Conclusions

This chapter operationalized PV analysis by introducing two PV multi-period templates, one for analysis of investments and the second one for analysis of equity committed to investments. In this effort we adjusted the average tax rate to correspond to the average tax rate paid on investments versus the average tax rate paid on equity committed to investments.

We reached an important conclusion when we analyzed returns on investments (assets) and equity. They need not rank investments and equity committed to investment consistently. These results add you our earlier finding that NPV and IRR models need not rank investments consistently homogeneous size and term conditions are satisfied.

That returns on investments and equity committed to investments may produce inconsistent ranking leads to an important question: when should we rank investment using returns on investments (assets) versus returns on equity committed to investments? The general answer is: it depends on who is asking. An individual investor whose financial conditions including loan amounts and costs are likely to be most interested in knowing returns to his/her specific conditions. Those more interested in the general condition of an industry or class of investments will more likely be most interested in returns on the investment or class of firms.

Not only have we focused on analyzing firms and investments versus returns on equity invested in firm's and investments, we have also focused on the analysis of stand-alone versus incremental investments. In Chapter 12 we used accrual income statement (AIS) to find rates on return on assets and equity for a firm. The analysis was for a single period and focused primarily on describing the condition of the firm. In contrast, this chapter has focused on investments that add to, alter, or replace firm investments. We referred to investments that add to, replace, or change the firm's capital structure as incremental investments. Sometimes the language used to describe incremental investments is confusing and describes them as partial budgeting analysis. We may use partial budgets to describe an incremental investment—but should not be confused with the actual PV analysis of incremental investments.

An irony is that most often practical applications of PV analysis are focused on stand-alone investments while most practical investments are incremental ones. Included in this irony is that tax rates most often refer to equity commitments rather than investment commitments.

To illustrate PV analysis, we focused on two practical investments. One a stand-alone investment we referred to as Green and Clean Services (GCS) and the second, an incremental investment we referred to as Round and Brown Doughnuts. In the first case we focused on how to distinguish between investment versus equity committed to investment analysis. The important point is this example was how the financial terms of the loan can change the attractiveness of the investment and, of course, change the rankings between investments.

When conducting PV analysis and making prescriptive statements based on the analysis, we have to estimate the future. Of course, some may argue that this involves sophisticated guessing—or estimating as some would refer to the efforts. Estimates of the future range in their sophistication—but they have one thing is common, they are never completely accurate. To remedy this problem, we recommend analyzing one's investment and equity committed to the investment using several different forecasts as illustrated in this chapter and only adopting an investment is the returns are sufficiently positive that we can still be too optimistic in our forecasts and yet earn a comfortable rate of return.

PV analysis and forecasting the future is an important and unavoidable task. Hopefully, the tools described in this chapter will aid in that effort.

Questions

1. Please describe the difference in investment focus between an AIS and a PV model?
2. What is the main difference between a stand-alone versus an incremental investment? Explain why some sources describe incremental investment analysis as partial budgeting analysis.
3. This chapter presented two main PV model templates. One focused on analyzing investments. The second one focused on analyzing equity committed to the investment. Describe the differences between the two templates. Then explain how these differences permit us to calculate returns associated with investments versus returns on equity committed to investments.
4. Describe the nature of a fixed investment.
5. A capital investment provides services over several periods. A nondurable provides services once. Explain how this difference in capital goods and nondurable inputs are recognized in PV models.
6. A capital investment provides services for more than one period without losing its identity. The cost of providing these

services is the change in the liquidation of the capital asset and the use of nondurable goods. The change in the liquidation value of a capital asset may mostly depend on the passage of time and the intensity of its use. Please explain how these two costs that result from the passage of time an use influence the optimal service extraction rate from the capital asset.
7. Operating accounts include AR, INV, AP, and AL. Describe how decreases (increases) in capital accounts influence operating and liquidation ATCF.
8. What is the difference between exogenous and endogenous variables used to solve for NPV, AE, and IRR in PV models?
9. Solving PV models requires that we forecast the values of exogenous and endogenous variables for the economic life of the investment. Describe how these forecasts might vary in their sophistication.
10. Assume you are attending school, working part time, and need transportation. To satisfy your transportation needs, you consider three options: You could continue driving your old clunker. You could buy a new car. Alternatively, you could buy a newer used car.

Your parents, who will need to loan you the money if you buy a new or newer used car, prefer that you continue to drive your clunker. You have done your homework to determine which of the three transportation options is the least cost one—although there may be other considerations. The financial data for the three investments follows.

Continue driving the clunker. You list the following financial details associated with continuing to drive the clunker. The clunker's current market value is V = $3,000 which is the amount that you would sacrifice if you keep the clunker. As a result, you consider it to be your acquisition price. You estimate that you will travel 1,000 miles per month for 24 months in the clunker. In addition you estimate repair costs will equal 14 cents per mile, maintenance costs will equal 2.5 cents per mile, fuel costs/mile ($3.00/ gallon divided by 17 miles/gallon) will equal 17.6 cents per mile, insurance costs will equal 10 cents per mile, and the clunker's salvage value in 2 years (24 months) you estimate will equal V24 = $375. The total operating cost per mile for the clunker you estimate to equal 44.1 cents per mile and monthly operating cost you estimate will equal $441.

A new car. It would take a lot of coaxing for your parents to loan you enough money to buy a new car, but you would prefer this option. You can imagine how impressive you would be offering rides to your friends in your new car. You found one that you think would work. The financial characteristics of the new car include an acquisition price of V_0 = $24,000, and you estimate that you will travel 1,000 miles per month for 10 years or 120 months in the new car. In addition, you estimate repair costs will equal 3.5 cents per mile, maintenance costs will equal 2.5 cents per mile, fuel costs ($3.00/ gallon divided by 27 miles/gallon) will equal 11.1 cents per mile, insurance costs will equal 12 cents per mile, and the salvage value in 10 years (120 months) you estimate will equal V_{120} = $8,000 The total operating cost per mile for the new car you estimated will equal 29.1 cents per mile and monthly operating costs you estimate will equal $291.

A used car. In case you cannot get the loan to buy a new car, you consider a newer used car to be an acceptable alternative to continuing to drive your clunker. You found one that you think would work. The financial characteristics include an acquisition price of V_0 = $19,800, and you estimate that you will travel 1,000 miles per month for 8 years or 96 months in the newer used car. You estimate repair costs will equal 5 cents per mile, maintenance costs will equal 2.5 cents per mil, fuel costs ($3.00/gallon divided by 21.4 miles/gallon) will equal 14 cents per mile, insurance costs will equal 11 cents per mile, and the salvage value in 8 years (96 months) you estimate will equal V_{96} = $7,000 The total operating cost per mile for the used car you estimate will equal 32.5 cents per mile and you estimate monthly operating costs will equal $325.

Summary of data. We summarize the financial characteristics of the three car options in the Table Q13.1 below. Included in the table are the three phases of the investment: the acquisition costs, the operating costs per month, and the salvage values. Note that the operating costs are lowest for the new car. The acquisition cost is lowest for the clunker. Not included in the Table Q13.1 data summary is the average value of the investment lost per year—the average depreciation cost. We find the average depreciation cost by subtracting from the acquisition value the liquidation value and this difference we divide by the number of service months. This cost is ($3,000 –$ 375) / 24 = $109.38 for the clunker, ($24,000 – $8,000) / 120 = $133.3 3 for the new car, and ($19,800 – $7,000) / 96 = $133.33 for the used car. Since we account for these costs as the acquisition and liquidation values, they are not included in our monthly costs. We note, however, that one significant advantage of the clunker is its lower monthly depreciation.

Table Q13.1. A summary of per mile and monthly costs of driving a clunker, a new car, and a used car.

	Clunker	New Car	Used Car
– V_0	$3,000	$24,000	$19,800

	Clunker	New Car	Used Car
Miles driven per month	1,000	1,000	1,000
Repair costs per mile	$0.14	$0.035	$0.05
Maintenance costs per mile	$0.025	$0.025	$0.025
Miles per gallon	17	27	25
Fuel cost per mile ($3 / gal)	$3 / 17 = $0.176	$3 / 27 = $0.111	$3 / 25 = $0.12
Insurance cost per mile	$0.10	$0.12	$0.11
Total operating cost per mile	$0.441	$0.291	$0.325
Operating cost per month (1000 miles/month * cost/mile)	$441	$291	$325
Economic life (months)	24	120	96
Salvage Value (V*n*)	V_{24} = $375	V_{120} = $8,000	V_{96} = $7000
Interest rate per year / month	12% / 1%	12% / 1%	12% / 1%

Net present cost (NPC). Your goal is to select the least cost transportation option. What makes this problem especially interesting is that the term is different for each car option requiring that we compare car transportation costs using their annuity equivalent (AE) cost per month. Your assignment is to find the NPC, AE, and IRR for the useful life of each car using the template that corresponds to equation 13.21 with *T* set equal to zero. Then describe your rankings.

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14: Homogeneous Liquidity

Learning Objectives

After completing this chapter, you should be able to: 1. Learn how to measure an investment's liquidity. 2. Learn how to compare liquidity for a firm versus liquidity for an investment. 3. Learn how to rank investments according to their liquidity using cash flow measures.

To achieve your learning goals, you should complete the following objectives:

- Learn to distinguish between two types of returns earned by an investment: time dated cash flow and capital gains (losses).
- Learn how to distinguish between a firm's liquidity and an investment's liquidity.
- Learn how to describe an investment's liquidity at a point in time by using its current-to-total returns (CTR) ratio.
- Learn how to describe an investment's liquidity over time by using its inter temporal CTR ratio.
- Learn how to connect CTR ratios to price-to-earnings (PE) ratios.
- Learn how PE ratios can be used to infer the liquidity of an investment.
- Learn how coverage (C) ratios can be used to infer the liquidity of an investment.

Introduction

An investment may earn two types of returns (losses) for investors: (1) time-dated cash flow called current returns, and (2) capital gains (losses). This chapter demonstrates that capital gains earned on an investment depend on the investment's pattern of future cash flow.

This chapter also demonstrates that the combination of current returns versus capital gains (losses) has important liquidity implications for investors, especially when an investment is financed with debt capital. Debt-financed investments whose earnings are expected to grow over time may experience a cash shortfall called a financing gap, in which the cash returns are less than the scheduled payments of principal plus interest. This gap is most likely to occur early in the investment's life and is exacerbated by inflation. As time passes and the cash returns grow in size, they will eventually exceed the repayment obligation, and the liquidity problem is solved.

The term liquidity is used to describe "near-cash investments" because of the similarity between liquids and liquid investments. A liquid such as water can fill the shape of its container and is easily transferred from one container to another. Similarly, liquid funds easily meet the immediate financial needs of their owners. Illiquid investments such as land and buildings, like solids, are not easily accessed to meet financial needs because their ownership and control are not easily transferred.

A firm's liquidity reflects its capacity to generate sufficient cash to meet its financial commitments as they come due. A firm's failure to meet its financial commitments results in bankruptcy, even though the firm might be profitable and have positive equity. Consequently, firms must account for an investment's liquidity in addition to earning positive net present value (NPV).

There are several measures that reflect a firm's or an investment's liquidity. One measure used to reflect the liquidity of the firm is the current ratio, the ratio of current assets divided by current liabilities. Because current assets can be quickly converted to cash, they are an important source of liquidity for the firm.

While the current ratio measures the liquidity of the firm, we want a liquidity measure specific to an investment, and one that describes the liquidity of an investment over time—what we will refer to as a periodic liquidity measure. The liquidity measure that provides investment-specific liquidity information over time is the current-to-total returns (CTR) ratio. The CTR ratio is derived from market equilibrium conditions where NPV is zero and represents the ratio of current (cash) returns to total returns. The price-to-earnings (PE) ratio is a commonly reported ratio reflecting the price of an investment reflected by its current earnings. A liquidity measure related to the CTR ratio is the coverage (C) ratio. A C ratio is the ratio of cash returns to loan payments or other debt obligations in any particular period.

Current-to-Total-Returns (CTR) Ratio

The current-to-total returns (CTR) ratio is a periodic liquidity measure derived from the relationship in a given period between an investment's current (cash) returns, its capital gains (losses), and its total return, or the sum of current returns and capital gains (losses). To derive the CTR measure, we begin with an expression of an investment's total returns:

Total investment returns = current returns + capital gains (losses)

The above relationship is expressed symbolically as follows. Let the investment's total return equal the investment's internal rate of return r times the value of the investment at the beginning of the period V_0 or rV_0. Meanwhile, let R_1 be defined as the asset's net cash, or current returns earned at the end of the first period. In addition, let V_0 and V_1 equal the investment's beginning and end-of-period values respectively, while their difference (V_1 – V_0) is equal to the investment's capital gains (losses) in the first period. Summarizing, we write the investment's total returns as:

$$rV_0 = R_1 + (V_1 - V_0) \tag{14.1}$$

Next we solve for the investment's internal rate of return earned in each period, r. We solve for this measure by dividing the investment's total returns by the investment's value at the beginning of the period V_0:

$$r = \frac{R_1}{V_0} + \frac{(V_1 - V_0)}{V_0} \tag{14.2}$$

A useful measure obtained from Equation ??? is the ratio of current returns R_1 divided by total returns rV_0. This ratio indicates the percentage of total returns the firm receives as cash. The higher this ratio, the greater the likelihood that the firm has the liquidity required to meet its debt obligations in the current period. Rearranging the previous equation yields the ratio of current to total returns in the first period, equal to CTR_1:

$$CTR_1 = \frac{R_1}{r_1 V_0} = 1 - \frac{\left(\frac{V_1 - V_0}{V_0}\right)}{r_1} = 1 - \frac{\eta_1}{r_1} \tag{14.3}$$

where $_{V\_0}$ is the capital gains (loss) rate in period one.

To illustrate the CTR ratio, we now derive the CTR ratio for the geometric series of cash flow. Consider an investment that is expected to generate a perpetual series of cash flow that grow at average geometric rate of g percent per period. If R_0 is the initial net cash flow, and r is the IRR of the investment, then the PV of the growth model can be expressed as:

$$V_0 = \frac{R_0(1+g)}{(1+r)} + \frac{R_0(1+g)^2}{(1+r)^2} + \frac{R_0(1+g)^3}{(1+r)^3} + \cdots \tag{14.4a}$$

One year later:

$$V_1 = \frac{R_0(1+g)^2}{(1+r)} + \frac{R_0(1+g)^3}{(1+r)^2} + \frac{R_0(1+g)^4}{(1+r)^3} + \cdots = V_0(1+g) \tag{14.4b}$$

and in general

$$V_t = V_0(1+g)^t \tag{14.4c}$$

and the capital gains rate for the geometric growth model is found to equal:

$$\eta = \frac{V_t - V_{t-1}}{V_{t-1}} = \frac{V_{t-1}(1+g) - V_{t-1}}{V_0} = g \tag{14.4d}$$

Using our results from Equation ??? we can find our CTR ratio as:

$$CTR_1 = \frac{R_1}{r_1 V_0} = 1 - \frac{\eta_1}{r_1} = 1 - \frac{g}{r} = \frac{r-g}{r} \tag{14.5}$$

To illustrate, if an investment earns cash flow in perpetuity described by R_0, r = 10% and g = 5%, the investment's CTR ratio can be found equal to:

$$CTR_1 = \frac{.1 - .05}{.1} = 50\% \tag{14.6}$$

It is left as an exercise to demonstrate that if the CTR ratio is constant over time, the CTR ratio is 1. Namely, that:

$$CTR_1 = \frac{r-g}{r} \; for \; t = 1, 2, \cdots, n. \tag{14.7a}$$

To illustrate equation (14.7a), if in period one, r = 10% and g = 5%, then 50% of the investment's returns will be earned in the form of capital gains. On the other hand, if the geometric mean of changes in future cash flow was negative, (g < 0), the current returns to total returns would exceed total returns. If g = –5%, the CTR1 ratio would equal:

$$CTR = \frac{r-g}{r} = \frac{.15}{.1} = 150\% \tag{14.7b}$$

In words, equation (14.7b) informs us that 150% of the investment's return in period one will be received as current returns in order to compensate for the 50% capital losses experienced by the investment.

So what does this mean? It means that the investments with increasing cash flow are less liquid than those with decreasing cash flow. A less (more) liquid investment means that a part of its return in each period is earned in capital gains (losses) that are not converted to cash until the investment is liquidated.

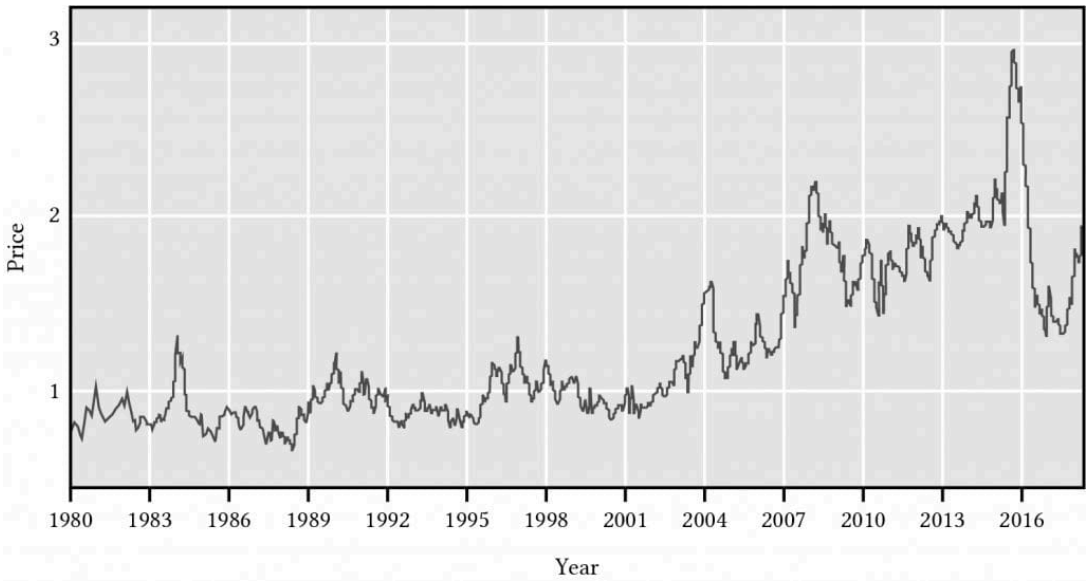
Liquidity, Inflation, and Real Interest Rates

Prices reflect the ratio of money exchanged for a unit of a good. For example, assume that the price for gasoline is $3.00 per gallon. If you purchased 10 gallons of gasoline, you would pay $30.00 ($3.00 x 10). Conversely, the ratio of $30.00 expended for gas divided by the gallons of gas purchased is the price of gas: $30.00/10 equal to $3.00.

Now suppose that last year you purchased roughly $50,000 worth of goods. For convenience, let p_j be the price of good j where j = 1, ..., n. Furthermore, let q_j represent the quantity of good j purchased. Multiplying the price times the quantity of each good purchased equals your total expenditures of $50,000. To summarize, $\$50,000 = \sum_{j=1}^{n} p_j q_j$ equals the sum of price times quantity of all of the goods you purchased.

In case you had not noticed, prices for goods and services change over time. Consider the price of a dozen large eggs. The price of a dozen of large eggs over the past 38 years is graphed in Figure 14.1 ranging from less than $1 per dozen in 1980 to almost $3 per dozen in 2016.

Figure 14.1. The price of a dozen large eggs over the period 1980 to 2016. (Bureau of Labor Statistics, 2016)



Prices over time for an individual good can change for several reasons. The demand for eggs can rise—more people wanting to consume more eggs will likely cause egg prices to rise. Or the cost of supplying eggs may change—chicken have become more efficient at producing eggs making them cheaper or chicken feed may become more expensive making eggs more expensive to produce.

The quantity theory of money

There is another reason why prices may change over time—there is more money in circulation. This explanation for why prices rise we call the quantity theory of money (QTM). The quantity theory of money states that there is a direct relationship between the quantity of money in an economy and the level of prices of goods and services sold. What constitutes the money supply is a complicated subject that we can avoid here and still make our point.

Returning to our earlier discussion, let the total of goods and services sold times their prices equal the amount of money *M* in the economy times its velocity (*V*) or the number of times the same unit of money is used during a time period. We write this relationship as

$$\sum_{j=1}^{n} p_j q_j = MV \tag{14.8}$$

According to the QTM, if people purchased the same goods in the same quantity and if *V* were held constant and *M* increased by *i* percent, then the price of each individual good must also increase by *i* percent.

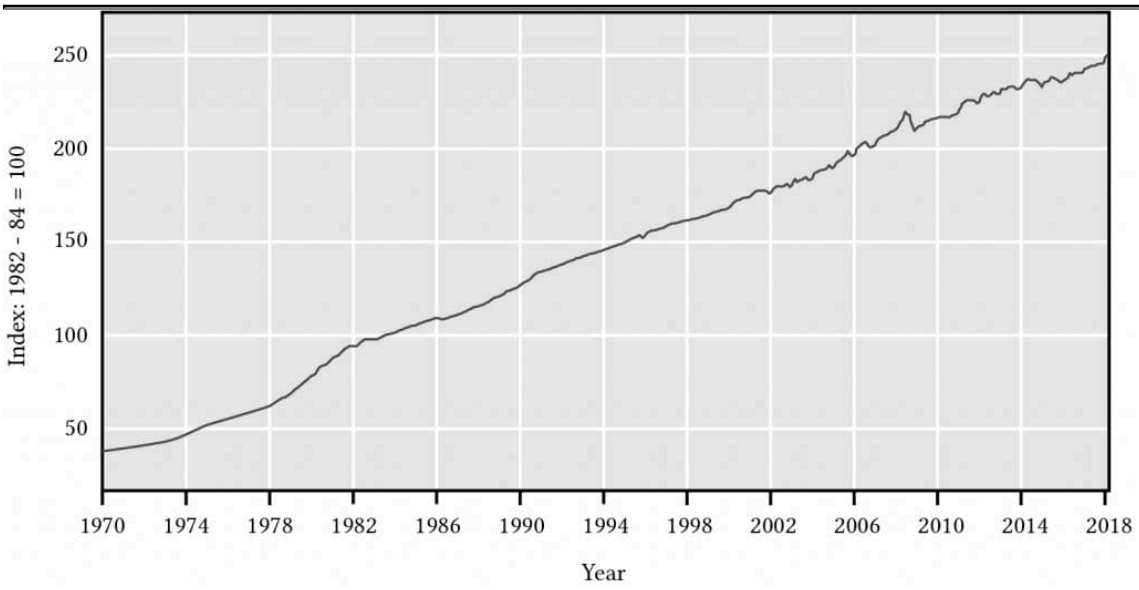
$$\sum_{j=1}^{n} (1+i) p_j q_j = (1+i) MV \tag{14.9}$$

Finally, if we measured the prices times the same goods in each year and calculated the ratio of the current year by the previous year we could obtain an estimate of the how prices in general have changed or the rate of inflation. Adding a superscript *t* to the price variable, we find the inflation rate plus one in year *t* is equal to:

$$\frac{\sum_{j=1}^{n} p_j^t q_j}{\sum_{j=1}^{n} p_j^{t-1} q_j} = (1+i) \tag{14.10}$$

An index (1 + *i*) can be computed for all different bundles of purchased goods. One popular inflation index is called the Consumer Price Index (CPI) that measures the rate of increase in prices holding the bundle of goods constant or at least as far as it is possible. Consider how inflation has affected consumer purchases over time. The vertical axis measures the CPI. The horizontal axis measures time. All indices select one year at their base in which the index is one. In Figure 14.2, the base year is somewhere between 1982-1984.

Figure 14.2. The consumer price index for all the U.S. letting average prices between 1982-84 equal 100.



According to Figure 14.2, in 2018 you paid on average two and one-half times as much money to purchase the same good as you did in 1982-1984.

So why is inflation important to our discussion of PV models and profit measures? If your income increases by *i* percent and the cost of everything you purchase increases by the same percent *i*, then in real terms what you can actually purchase is the same as before since your income and costs increased by the same rate.

Real interest rates. Let say that your after-tax cash flows increased by rate *g*% but that *g* is influenced by the rate of inflation. Then after accounting for inflation, what is left is attributed to other factors besides an increase in the money supply. Let this real growth in after-tax cash flow (ATCF) be g^*. Then we can write:

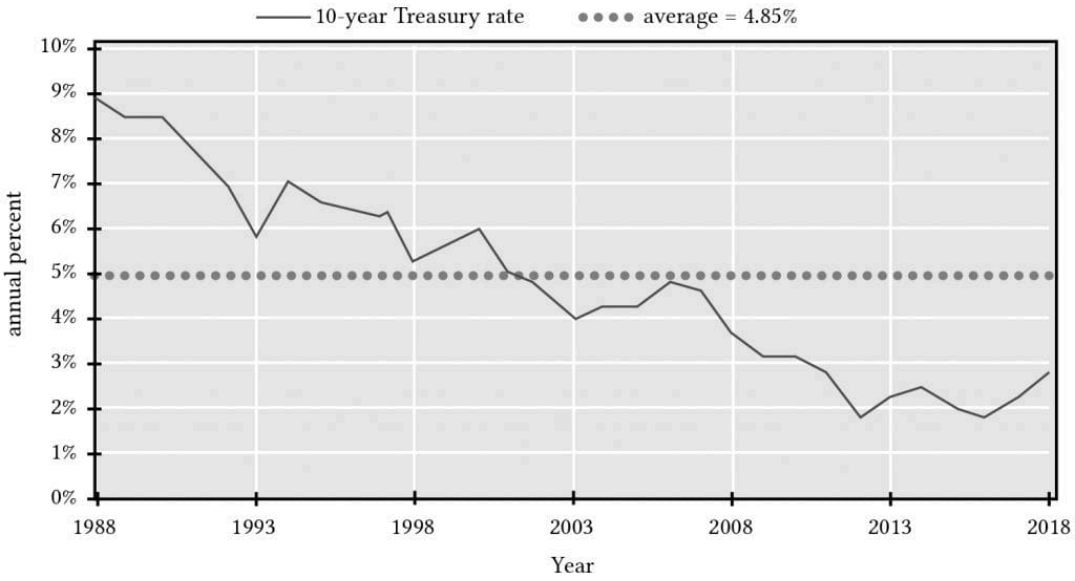
(14.11) $$(1+g) = (1+g^*)(1+i)$$

Interest rates, discount rates, and inflation. Inflation also influences interest rates paid to borrow money and the discount rate that reflects the opportunity cost for one's resources. To motivate this discussion intuitively, assume you lent a friend $1000 for one year. Also assume that inflation is 5% so that when your friends repays the loan, to make the same purchases at the time the loan was made, they would have to pay back $1,000 * (1.05) = $1,050. But they would have to pay more than 5% for being able to rent your money for a year—let's call this rate the real interest rate and denote it as r^*. To account for both the reduced purchase price of your money because an increase in prices by inflation *i* and the cost of postponing the use of one's money, the interest rate (and opportunity cost of resources invested in a defender) is *r* equal to:

(14.12) $$(1+r) = (1+r^*)(1+i)$$

We graph real interest rates r^* over time in Figure 14.3.

Figure 14.3. 10-year Treasury constant maturity interest rate, U.S., 1988 – 2018



So what does all this mean? It mean that real interest rates have been falling since 1988 and have recently started to increase. Increasing real interest rates will make long-term investments less liquid because it increases interest rate changes on loans and also increases the rate of growth g that we already demonstrated increases capital gains relative to current earnings.

Liquidity Implications

The mix of capital gains (losses) versus current returns depends directly on the average rate of growth ($g > 0$) or decline ($g < 0$) in future cash flow. As inflation increases the average rate of growth g, then inflation also reduces an investment's liquidity, especially during the investment's initial years.

For geometric decay in earnings ($g < 0$), current returns are more than 100% of total earnings—and current returns have increased in importance relative to capital loss or depreciation. These results clearly show the paradoxical conditions that growth in projected earnings can weaken the liquidity of the investment due to the increasing relative importance of capital gains. In contrast, a decline in projected earnings can strengthen the liquidity due to the increasing relative importance of current returns (See Table 14.1).

Also note in Table 14.1 that increases in the capital gains rate g holding r constant always leads to a reduction in CTR ratio. This is easily explained. Since the total rate of return r is fixed and composed of current rate of returns and capital gains rate of returns, increases in the capital gains rate of return must necessarily decrease the current rate of returns.

Table 14.1. Values of CTR ratios where $CTR_t = (r - g) / r$

	r%				
g%	10	8	6	4	2
-5	150%	163%	183%	225%	350%
-3	130%	138%	150%	175%	250%
0	100%	100%	100%	100%	100%
3	70%	63%	50%	25%	
5	50%	38%	17%		

Price-to-earnings (PE) Ratios

Related to the CTR ratio is the price-to-earnings (PE) ratio equal to:

$$PE = \frac{V_0}{R_1} \tag{14.13}$$

If we describe the investment using the PV expression in equation (14.4a), then we can write:

$$V_0 = \frac{R_0(1+g)}{(1+r)} + \frac{R_0(1+g)^2}{(1+r)^2} + \frac{R_0(1+g)^3}{(1+r)^3} + \cdots + \frac{R_0(1+g)}{r-g} \tag{14.14}$$

Next we substitute $R_0(1 + g)$ for R_1, and the right-hand side of Equation ??? for V_0 into the PE expression in Equation ???, and obtain:

$$PE = \frac{V_0}{R_1} = \frac{1}{r-g} \tag{14.15}$$

But from equation (14.7a) we know that $rCTR = (r - g)$, which allows us to connect CTR and PE ratios, namely:

$$PE = \frac{1}{r-g} = \frac{1}{rCTR} \tag{14.16}$$

In words, what we learn from Equation ??? is that, in general, higher CTR ratios imply lower PE ratios and lower CTR ratios imply higher PE ratios. These results may at first appear to be somewhat counter intuitive—that, for liquidity purposes, we prefer investments with lower PE. Upon reflection, it makes sense—assets whose returns are mostly capital gains will be earning a lower percentage of current returns compared to their values than assets that earn little or no capital gains.

To make the point that CTR and PE ratios are inversely related, we construct Table 14.2 for PE ratios. Compare the results with those found in Table 14.1 to confirm in your minds the implications of Equation ???. Note that PE ratios uniformly increase with increases in g and the rate of return earned in the form of capital gains.

Table 14.2. Values of PE ratios where $PE_t = 1/(r - g) = 1 / (rCTR_t)$

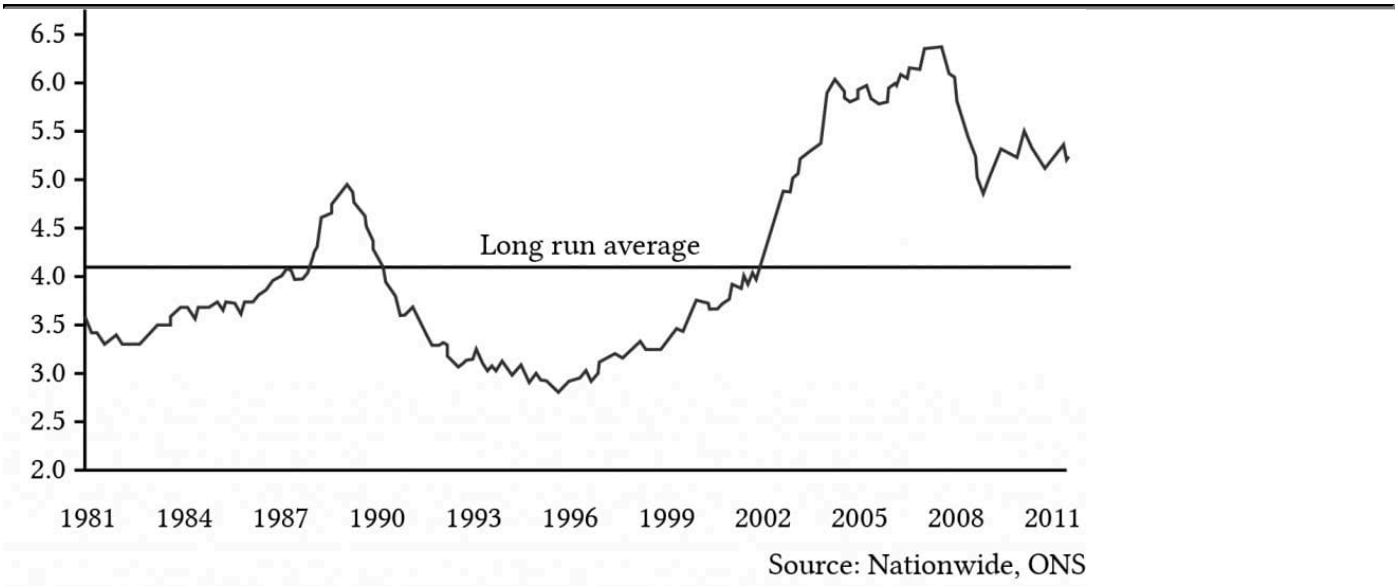
	r%				
g%	10	8	6	4	2
-5	6.67%	7.67%	9.11%	11.11%	14.29%
-3	7.69%	9.06%	11.11%	14.29%	20.00%
0	10%	13.50%	16.67%	25.00%	50.00%
3	14.29%	20.00%	33.33%	100.00%	
5	20.00%	33.33%	100.00%		

PE ratios are frequently used to describe the financial condition of an asset. Generally speaking, high PE ratios reflect an expectation of income growth for an asset. Unfortunately, high levels of PE ratios may reflect bubbles or unrealistic expectations for income growth for a particular investment—and high PE ratios are often followed with a market adjustment in which the earnings expectation of an investment are adjusted downward, which reduces the investment's PE ratio. For example, in 1929 for stocks (see Figure 14.4), and 2010 for housing (see Figure 14.5), expected income growth was unrealistic, and in both cases, PE ratios fell when earnings expectations were adjusted downward.

Figure 14.4. Historic PE ratios for stocks described on the Standard and Poor's Stock Exchange. (Earnings are estimated based on a lagged 10-year average.)



Figure 14.5. Historic PE ratios for UK housing stock.



Compare the value of the PE ratios in Figures 14.4 and 14.5. PE values for stocks that reflect depreciable assets ($g < 0$) have average values around 20. PE values for housing stock that reflect non-depreciable assets ($g > 0$) have recently averaged around 5. (See Table 14.2.)

Coverage (C) Ratios

Investments can be evaluated according to both profitability and periodic liquidity criteria. Profitability analysis focuses on whether the acceptance of an investment will increase the investor's present wealth. Periodic liquidity measure analysis considers whether an investment will generate cash flow consistent with the terms of financial capital, especially debt capital, that are used to finance the investment. Is an investment capable of generating sufficient net cash flow in each period to satisfy the requirements for repayment of the loan's principal plus interest? If not, then the investment is illiquid because liquid funds drawn

from other sources are required to meet cash flow requirements associated with the investment. Thus, a simple test of investment liquidity is to compare the net cash flow generated by the investment in a given period with the debt-servicing requirement.

The ratio of net cash flow in a period t, R_t, divided by the loan plus principal payment in the same period, A_t, is called the coverage ratio, C_t:

$$C_t = \frac{R_t}{A_t} \tag{14.17}$$

If the coverage ratio is equal to or exceeds 1.0, the investment is liquid in that period. If coverage is less than one, the investment is illiquid in that period. In most cases, the loan payment that includes principal plus interest is a constant "A," that depends on the price of the purchased asset, V_0; the financed proportion of the purchase price γ; the term of the loan n; and the interest rate charged on the loan r^f. Using our previous notation, we can express the relationship between the amount financed, the fixed loan payment, the interest rate on the loan, and the term of the loan as:

$$\gamma V_0 = AUS_0(r^f, n) \tag{14.18}$$

We solve Equation ??? for A and substitute the result for A_t in Equation ???. In the same equation,we also substitute $R_0(1 + g)^t$ for R_t, and obtain:

$$C_t = \frac{R_0(1+g)^t}{\gamma V_0} US_0(r^f, n) \tag{14.19}$$

Replacing V_0 with $[R_0 (1 + g)]/(r - g)$, we can rewrite Equation ??? as:

$$C_t = \frac{(r-g)(1+g)^{t-1}}{\gamma} US_0(r^f, n) \tag{14.20}$$

We illustrate Equation ??? as follows. Assume r = 8%, g = 2%, n = 20, and r^f = 7%. Also assume the investment is 80% financed. Using an Excel spreadsheet, we find that US0(7%, 20) = 10.59.

Table 14.3. Calculating $US_0(r^f$=7%, n=20)

Open Table 14.3 in Microsoft Excel

B6	***Function*:**		=PV(B3,B4,B5,,0)
	A	**B**	C
1	Calculating $US_0(r^f$ = 7%, n=20)		
2			
3	rate	7%	
4	nper	20	
5	payment	-1	
6	NPV	**$10.59**	"=PV(rate,nper,payment,,0)

Then we find the coverage value for the first year of the investment to equal:

$$C_1 = \frac{(.08 - .02)(1.02)^0 10.59}{.8} = .79 \tag{14.21a}$$

Interpreted, in year one, the investment described above will only pay for 79 percent of the loan payment due. However, 10 years later and half way into the term of the loan, the coverage ratio has increased to:

$$C_{10} = \frac{(.08 - .02)(1.02)^9 10.59)}{.8} = .95 \tag{14.21b}$$

Finally, in the last year of the loan, the coverage ratio is:

$$C_{20} = \frac{(.08 - .02)(1.02)^{19} 10.59)}{.8} = 1.16 \tag{14.21c}$$

In other words, in year 20, the last year of the loan, the investment cash flow not only repays the loan but 16 percent more than the required loan payment. Of course, after the loan is repaid, the periodic cash flow is available to the firm to meet its cash flow requirements.

One word about the term of the investment, it is assumed to be infinite. However, for any one investor, the term may be finite, but since the terminal value for any one owner is simply the present value of the continued cash flow, we can express the investment as having an infinite life. Therefore, we are justified in ignoring the term of the infinite-life investment.

When is the coverage equal to 100 percent? There is another way to approach the issue of coverage. It is to ask in what year is the coverage equal to 1? To answer this question for the geometric growth model with an infinite life and using the already established notation, we set:

$$A = R_0(1+g)^t \tag{14.22}$$

and solving for *t* we find:

$$t = \frac{\ln A}{\ln R_0 + \ln(1+g)} \tag{14.23a}$$

And after making substitutions for *A* and R_0 and simplifying, we can write:

$$t = \frac{\ln \gamma - \ln(r-g) - \ln US_0(r^f, n)}{\ln(1+g)} \tag{14.23b}$$

We illustrate equation using numbers from the previous example where: γ = .8, *r* = .08, *g* = .02, r^f = .07, and *n* = 20. We find:

$$t = \frac{-.22 + 2.81 - 2.36}{.02} = 11.50 \tag{14.24}$$

In words, what we have found that the 12th payment will be the first period that more than pays for its financing.

So what does this mean? As the previous example demonstrates, it means that for investments with increasing cash flow, its liquidity increases over time.

Summary and Conclusions

An investment may earn two types of returns: time dated cash flow and capital gains (losses). Moreover, these returns can be earned over the lifetime of the asset. The relative importance of the two forms of returns, cash flow versus capital gains (losses), will determine the inter temporal liquidity of the asset. The greater the portion of the return is earned as cash, the more liquid will be the investment.

This chapter demonstrated whether an investment earns capital gains (losses) is determined in PV models by its pattern of expected future cash flow. Expected increases (decreases) in the number, frequency, or amount of the expected future cash flow

will produce capital gains (losses). Inflation causes long-term investments to become less liquid because they increase the rate of growth in the investment's cash flow.

Investments that earn capital gains are called appreciating investments. Investments that experience capital losses or depreciation are referred to as depreciating investments.

Perhaps paradoxically, investments that earn capital gains are less liquid than investments that experience capital losses. Assuming investments earn similar total rates of return, the greater the cash returns are compared to total returns, the greater are the liquid funds available to meet liquidity needs including repayment of funds and interest required to maintain control of the asset.

In this chapter we developed current-to-total returns (CTR) measures to indicate an investment's liquidity over time. Furthermore, CTR measures were shown to be related to price-to-earnings (PE) ratios which also depended on an investment's liquidity. Finally, one feature of an investment, is its coverage, its ability in any one period to repay principal and interest payments due in any one period. If funds borrowed to purchase an appreciating investment are repaid with a constant loan payment,then initially, it is likely that cash flow generated by the investment will not cover the interest and principal payments. This pattern is very likely during periods of inflation. Coverage ratios will also indicate the liquidity of investments over their productive lives.

In summary, the periodic liquidity of an investment is an important characteristic that should influence the capital decisions of financial managers along with the impact of an investment on the firm's solvency, profitability, efficiency, and leverage.

Questions

1. Please describe three possible appreciating and three depreciating investments that are available for most farm financial managers.
2. Describe the connections between capital gains (losses) with an investment's periodic liquidity.
3. Describe the connection between the inflation rate *i* and an investment's CTR ratio.
4. How is an investment's periodic liquidity related to its CTR (current-to-total returns) ratio and its C (coverage) ratio? How are the two periodic liquidly measures related to each other?
5. Compare the firms current (CT) ratio and its debt-to-service (DS) ratio with an investment's CTR ratio and C ratio. In what ways are they similar? How do they differ?
6. Consider the CTR ratio described in Equation ???. Find the CTR ratio for the investment whose anticipated cash flow is a constant *R* in perpetuity described below:

$$V_0 = \frac{R}{(1+r)} + \frac{R}{(1+r)^2} + \frac{R}{(1+r)^3} + \cdots \tag{Q14.1}$$

   (Hint: Observe the CTR ratio described in Equation ???. Then derive capital gains (losses) measures for the PV model described in this question.)

7. Find the CTR ratio for an investment whose defender's IRR is 8% and its anticipated growth rate is 2%. Then find the PE ratio for the same investment.
8. Assume that PE ratios can be approximated using Equation ???. What would account for the large swings in PE ratios described in housing stock described in Figure 14.2—changes in expected *r* or *g*? Or due to some other influence on the value of housing stock?
9. Find the coverage (C) ratio for year 5 for the investment described in the text where: γ = .7, *r* = .08, *g* = .02, r^f = .07, and *n* = 20. Call the result, the result for the base model.
   a. Recalculate the base model C ratio where *n* = 20 is changed to *n* = 15. Describe how reducing *n* changed the coverage compared to the base model. Then provide an explanation for the change in coverage.
   b. Recalculate the base model C ratio where *g* = .02 is changed to *g* = –.02. Describe how reducing *g* changed the coverage compared to the base model. Then provide an explanation for the change in coverage.
   c. Recalculate the base model C ratio when γ = .8 is changed to γ = .6. Describe how increasing γ changed the coverage compared to the base model. Then provide an explanation for the change in coverage.
10. In this chapter we derived Equation ??? which solved for the time period in which the coverage was 100%. Find the time period in which coverage is 100% when γ = .7, *r* = .08, *g* = .02, r^f = .06, and *n* = 20. Call the result, the base model result. Then use Equation ??? to find how *t* associated with 100 percent coverage changes for the following cases:
    a. Recalculate the base model and find a new *t* value associated with 100% coverage where *n* = 20 is changed to *n* = 15.

Describe how reducing n changed t and then provide an explanation for the change in *t*.

b. Recalculate the base model and find a new *t* value associated with 100% coverage where r^f = .06 is changed to r^f = .08. Describe how increasing the interest rate r^f changed *t* associated with 100% coverage. Then provide an explanation for the change in *t*.
c. Recalculate the base model and find a new *t* value associated with 100% coverage where *r* = .08 is changed to *r* = .06. Describe how decreasing the discount rate *r* changed *t* associated with 100% coverage. Then provide an explanation for the change in *t*.

11. Suppose the firm you managed was facing financial stress due to falling prices. Moreover, the stress is reflected in low liquidity and solvency measures for the firm. It appears that there is no way the firm can survive its current crises without liquidating not only short-term assets, but also long-term investments. Based on what you have learned about investment liquidity, propose a plan for the firm to meet it liquidity crisis.

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15: Homogeneous Risk Measures

Learning Objectives

After completing this chapter, you should be able to: (1) define and measure risk; (2) understand how a person's risk aversion affects his/her resource allocation; (3) distinguish between direct and indirect outcome variables; and (4) evaluate alternative risk response strategies available to financial managers including sharing risky outcomes, purchasing insurance, diversifying investments, purchasing risk reducing investments, and choosing an optimal capital structure.

To achieve your learning goals, you should complete the following objectives:

- Learn how to describe risky events by assigning a random variable to their outcomes and by assigning a probability density function (pdf) to the random variable.
- Learn how measure the variability and central tendencies of random variables using the expected values and variances of their probability density functions.
- Learn how risk premiums can be used to measure the cost of risk.
- Learn how to describe the risky choice set facing firm managers using expected value-variance (EV) efficient sets.
- Learn about the normal probability density function and understand why it is so important when describing risky events.
- Learn how sharing risk with others can be a useful response to risk.
- Learn how purchasing insurance can be a useful response to risk.
- Learn how diversification can be a useful response to risk.
- Learn how purchasing risk-reducing inputs can be a useful response to risk.
- Learn how leverage affects the level of risk facing the firm.

Introduction

The adjective "uncertain" describes an event (such as a hurricane or a football game) whose outcomes are not known with certainty. An uncertain event must have at least two possible outcomes, and it usually has more. In an earlier time, a distinction was made between risky events and uncertain events based on the information available for identifying an event's possible outcomes and predicting the probabilities of those outcomes. For example, the flip of a coin is a familiar event with two outcomes: heads (H) and tails (T). Based on past trials or logic, we can infer that the probability of a heads (tails) appearing when a fair coin is tossed is about 50 percent. Some may call this event risky because we have good information about the possible outcomes and the probability of their occurrence.

Now consider a different event: the toss of a thumbtack.[1] What is the probability that the thumbtack will land on its side versus landing on its head? Because we are not familiar with this event and cannot assign probabilities to its outcomes based on our past experience or logic, this event might be called uncertain.

For many, the distinction between risky events and uncertain events is no longer important, and most researchers assign the adjectives uncertain and risky to events interchangeably. One reason many do not distinguish between risky and uncertain events is because the assignment of probabilities to event outcomes is subjective (we never have enough information to be absolutely certain about either possible outcomes or their probabilities) and may be based on other factors besides logic and past observations including hunches, omens, experiences of others in irrelevant circumstances, and advice from unqualified persons to name a few.

A distinction between risky and uncertain events may still be useful. People appear to refer to events as uncertain when the event's outcomes are not known with certainty. It also appears popular to refer to uncertain events as risky when they are uncertain and their occurrence alters the decision maker's well-being. Thus, only risky events matter, regardless of one's confidence in the probabilities of various outcomes (e.g. the toss of a coin versus the toss of a thumbtack).

The study of risky events, of course, has application to capital budgeting problems. When we estimate future cash flow used in analyzing capital budgeting decisions, we are estimating risky outcomes associated with a risky event. Thus, we are faced with the challenge of satisfying the homogeneous measures principle applied to risk by adjusting future cash flow projections of a challenger and the cash flow predictions used to find the IRR of the defender to their certainty equivalent value that we will define

in this chapter.

Statistical Concepts Useful in Describing Risky Events and Risky Outcomes

Probability density function. A probability density function (pdf) is a function that assigns probabilities to outcomes of risky events. For example, the toss of a fair coin is an event with outcomes showing a heads (H) or a tails (T). The pdf for this event may assign to H the probability of 50% occurrence and 50% to the occurrence of T. A pdf may be discrete or continuous. If the outcomes of an event are finite, then their likelihood of occurring is described by a discrete pdf. If the outcomes of an event are infinite, then their likelihood of occurring is described by a continuous pdf.

Random variables. A random variable is a numerical value assigned by a function or rule to outcomes of risky events. The probability of a particular value described by a random variable is the same as the probabilities of its underlying event outcome. For example, suppose that an event were the toss of a coin. We might assign the outcome of heads the number one, and the outcome of tails the value of zero. Then the probability of a random variable taking on the value one is the same probability as H occurring when tossing a coin.

Expected values. An expected value is one measure used to describe the properties of a pdf. It is sometimes called the first moment of the pdf because it measures the center of a pdf's mass (like the fulcrum of a teeter totter).

The expected value of a pdf is determined by calculating a weighted average of the possible values of the random variable values times their likelihood of occurring. To illustrate, consider two possible investments A and B. The event is the operation of the economy with three possible outcomes: a recession with 20% likelihood, a stable economy with 60% likelihood, and a growth economy with 20% likelihood. The value of the random variables describing the three outcomes for investments A and B are described in Table 15.1 and represent alternative rates of return on the investments.

Table 15.1. pdfs and Random Variables Associated with Investments A and B

Economic outcomes	pdf associated with economic outcomes	Returns on investment A (a random variable)	Returns on investment B (a random variable)
Recession	20%	-20%	-40%
Stable	60%	20%	20%
Growth	20%	40%	60%
Expected values of investments A and B		16%	16%
Variances (standard deviations) of investments A and B		.039 (19.6%)	.103 (32.0%)

The expected value operator is expressed as *E*( ). The expected value of investment A is the sum of A's random variables weighted by their respective probabilities and is written as:

$$E(\text{Investment } A) = (.2)(-.20) + (.6)(.20) + (.2)(.4) = 16\% \tag{15.1}$$

The expected value of investment B is written as:

$$E(\text{Investment } B) = (.2)(-.40) + (.6)(.20) + (.2)(.60) = 16\% \tag{15.2}$$

In general, the expected value of random variable y_j which occurs with discrete probability p_j with $j = 1, ..., n$ outcomes is expressed as:

$$E(y) = \sum_{j=1}^{n} p_j y_j \tag{15.3}$$

and where the sum of all probabilities of yj occurring equal 1:

$$\sum_{j=1}^{n} p_j = 1 \tag{15.4}$$

A special kind of expected value is the mean. A mean is calculated for *n* observations from an unknown pdf where every observation is equally likely. Suppose we observed five draws from an unknown distribution equal to 8, 4, 0, 2, and 6. Since we assume each observation was equally likely, 1/*n* , or 1/5, in this case the expected value is equal to the mean calculated as

$$\bar{x} = \frac{\sum x_i}{n} \tag{15.5}$$

$$= \frac{8+4+0+2+6}{5} = \frac{20}{5} = 4 \tag{15.6}$$

Variance and standard deviation. Even though the expected values of investments A and B described above are equal, most investors would not consider them equally attractive because of the wide differences in the variability of the values assumed by their random variables. One approach to measuring the variability of a random variable is to calculate its variance or the square root of its variance equal to its standard deviation. We can find the variance of a random variable y_j with *n* possible outcomes by summing y_j minus $E(y)$ quantity squared weighted by the probability of random variable occurring. We write the variance formula for random variable y_j as:

$$\text{Variance}(y) = \sigma_y^2 = \sum_{j=1}^{n} p_j \left[y_j - E(y)\right]^2 \tag{15.7}$$

We illustrate the variance formula by calculating the variances and standard deviations for investments A and B. The variance for investment A is calculated as:

$$\sigma_A^2 = .2(-.2 - .16)^2 + .6(.2 - .16)^2 + .2(.4 - .16)^2 = .039 \tag{15.8}$$

Meanwhile the standard deviation for investment A can be found by calculating the square root of the variance of investment A and is equal to:

$$\sigma_A = \sqrt{\sigma_A^2} = \sqrt{.039} = .196 \text{ or } 19.6\% \tag{15.9}$$

The variance for investment B is calculated as:

(15.9) $$\sigma_B^2 = .2(-.4 - .16)^2 + .6(.2 - .16)^2 + .2(.6 - .16)^2 = .103$$

Meanwhile the standard deviation for investment B is found to equal:

$$\sigma_B = \sqrt{\sigma_B^2} = \sqrt{.320} = .320 \text{ or } 32\% \tag{15.10}$$

One reason for measuring the variability of the random variable by squaring its deviations from its expected value is because were we to find the average of the deviation of the random variable from their expected values, they would always sum to zero. They would sum to zero because probability weighted deviations above the expected value exactly equal probability weighted deviations below the expected value. Taking the square root of the variance of returns converts the deviation measure to units comparable with those of the original random variable. Thus, from the standard deviations calculated above, we can infer that, on average, the random variable representing investment A will deviate 19.6% from its expected value while the random variable representing investment B will deviate 32.0% from its expected value. Clearly, risky outcomes for investment B are more variable—and some would say more risky—than outcomes associated with investment A. We add to the descriptions in Table 15.1 of investments A and B their respective variances (standard deviations).

The variance of the sample is found as before by summing and squaring the deviations from the mean weighted by their probability of occurrence. However, for reasons not discussed here, the variance of the sample of observations is divided by *n*–1 instead of *n* where *n* is the number of observations. Thus the standard deviation from a sample distribution is denoted S_x for observations on the random variable *x*. Otherwise the variance of the random variable *x* drawn from the true population is divided by *n*. Therefore in our example, the sample standard deviation, the square root of the variance is:

$$S = \sqrt{\frac{\left(\sum x_i - \bar{x}\right)^2}{n-1}} \tag{15.11}$$

$$S = \sqrt{\frac{(8-4)^2 + (4-4)^2 + (0-4)^2 + (2-4)^2 + (6-4)^2}{4}}$$
$$= \sqrt{\frac{40}{4}} = \sqrt{10} \tag{15.11b}$$

Risk premiums and certainty equivalent incomes. While not properties of pdfs, important concepts connected to pdfs describing investments are risk premiums and certainty equivalent incomes. While these concepts are related to pdf properties, they also depend on risk preferences of individual decision makers for the variance and expected return inherent in the investment. Risk premiums, certainty equivalent incomes, and risk preferences can be easily explained.

Suppose an investor faced investment A described in Table 15.1 and had the opportunity to receive its expected value with certainty in exchange for paying a risk premium. What is the largest risk premium the investor would pay to receive the investment's expected value with certainty? The answer would depend on the investor's risk preferences. If the investor would pay a positive risk premium to receive investment A's expected value with certainty, then the investor is risk averse. If the investor would pay nothing to receive investment A's expected value with certainty, the investor is risk neutral. If the investor would pay to keep the variability (the investor enjoys gambling), we would say the investor is risk preferring.

Once we know the investor's risk premium for a particular investment's pdf, we can find his/her certainty equivalent income by subtracting it from the investment's expected value. We can describe the relationship between the ith investor's risk premium, certainty equivalent income, risk attitudes, and the expected value and variance of the random variable in the following expression:

$$y_{CE}^{i} = E(y) - \frac{\lambda^{i}}{2}\sigma_{y}^{2} \tag{15.12}$$

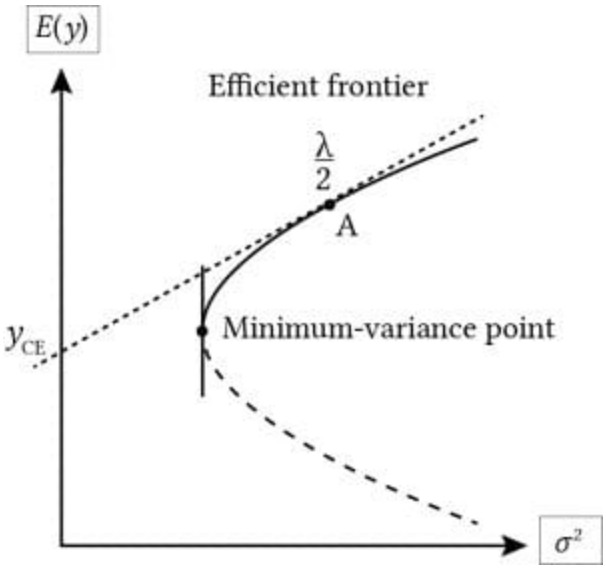
In Equation 15.12, the ith investor's certainty equivalent income y_{CE}^{i} for an investment whose possible values are represented by the random variable *y* is equal to the expected value of *y* less a risk premium. In Equation ???, we can solve for the largest insurance premium the investor would be willing to pay to receive the investment's expected value with certainty.

(15.13)

The ith's investor's risk premium is equal to the decision maker's risk preference measure λ^{i} divided by 2 times the variance of the random variable *y*. The investor's risk aversion coefficient $\lambda^{i}/2$ is best understood as a slope coefficient that indicates the response on the investor's certainty equivalent income by an increase in a unit of variance of the random variable *y*.

Expected value-variance criterion. Suppose we had a set of pdfs each representing the likelihood of m alternative random variables. Furthermore, suppose each of the *m* investments were described using their expected values and variances. Then, assuming that all investors were risk averse and without knowing each investor's risk aversion coefficient, we know that for every two investments with equal expected values (variances) and unequal variances (expected values) the investor would prefer the investment with the smaller variance (largest expected value). On the other hand, we could not rank two investments in which one had a larger expected value and variance than the other. The set of investments and ranked preferred for risk averse decision makers is called the expected value-variance (EV) efficient set. The graph of a particular EV set follows.

Figure 15.1. An Efficient Expected Value-Variance Frontier of Investments Represented by their Expected Values E(y) and Variances σ^2.

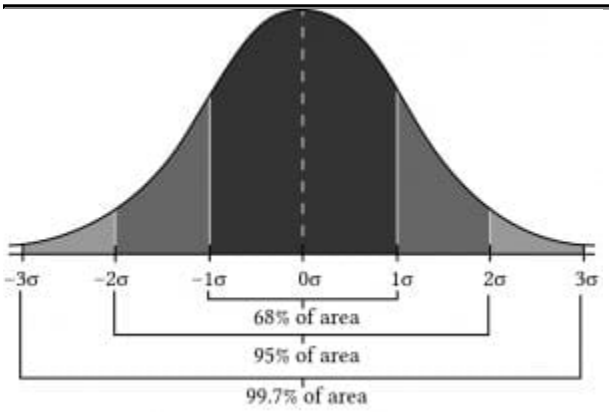


Suppose that we were to draw a line tangent to the relevant section of the EV at point A, representing an investment A. Then the slope of the line at point A would equal the decision maker's risk coefficient at the point. Furthermore, were we to extend the tangent line to intersect with the vertical axis, the point of intersection would equal the certainty equivalent of the investment.

The Normal Probability Density Function (pdf)

If you have a large enough sample of outcomes from a normal distribution, the distribution will look like a bell-shaped curve.

Figure 15.2. A normal probability distribution that describes probability in areas divided to standard deviations from the mean.



The normal distribution is symmetric about its expected value. The numbers in the graph correspond to standard deviations measured from the mean of the normal distribution. All of the characteristics of the normal distribution are completely described by the mean and variance (or equivalently, the standard deviation). The mean specifies the average value of the rate of return. The probability of getting a return above or below the mean by a certain amount is determined by the size of the standard deviation.

If the returns are normally distributed, there is a 68% chance that the return for any given period will be within one standard deviation of the expected value; a 95% probability that any particular observed return will be within 2 standard deviations of the expected value; and a 99.74% probability that any return will be within 3 standard deviations of the mean. Clearly, the larger the standard deviation, the more spread out the actual returns that occur will be. Suppose the common stock returns we looked at earlier were generated by a normal distribution. We estimated the expected value of the distribution to be r = 18.92% and the standard deviation to be $\sigma(r)$ = 16.18%. If we were interested in predicting what next year's return on common stocks will be, we could say that the expected value will be about 18.92%, but, in addition, there is a 68.26% probability that returns will be between

2.74% and 35.1%; a 95.44% probability that returns will be between –13.44% and 51.28%; and a 99.74% chance that the returns will be between –29.62% and 67.46%. The smaller the standard deviation, the less spread out the returns will be and the more accurate the mean will be as a predictor of future returns.

Sampling error. The normal distribution is an important distribution from both a theoretical and a practical standpoint. It is important to note that if you are looking at a small sample of data, its empirically based pdf is likely not to have the shape of a normal distribution even if it is actually generated from a normal distribution. In a small sample, the distribution will have gaps and holes in it and is unlikely have a shape that looks like the true distribution that generated the data. If you keep adding observations from the distribution, eventually the gaps will fill in, and it will start to look like the true distribution that generated the data. The point here is, even if the sample pdf generated by a normal distribution does look like a normal distribution, this may be because the sample size is too small. Therefore, because of its usefulness and tractability, we often assume that our sample pdfs are normally distributed.

Direct and Indirect Outcome Variables

Typically, we connect a risky event ϵ to a random variable $y(\epsilon)$. For example, ϵ may represent uncertain prices and $y(\epsilon)$ may represent income which depends on uncertain prices ϵ. We will refer to ϵ as the direct outcome variable and $y(\epsilon)$ as the indirect outcome variable over which the firm's utility is defined.

In most risk models, the relationship between ϵ and $y(\epsilon)$ is monotonic, if the direct outcome variable goes up (down) so does the indirect outcome variable. If prices increase, so does income; if the variance of ϵ increases, so does the variance of $y(\epsilon)$. However, one can easily construct examples in which the linkage is not so direct. For example, suppose the firm faces financial stress and that only very favorable outcomes will permit the firm to meet its cash flow obligations and survive. Under such circumstances, the firm may increase its expected income and its probability of surviving by choosing a strategy that increases its variance of direct outcomes. Consider such a problem by defining an indirect outcome variable $w = 0$ if the firm fails to survive and $w = 1$ if the firm survives. In this model the direct outcome variable is y. Then we connect the indirect outcome variable w to the direct outcome variable y by defining a survival income yd and defining w in terms of y as follows:

\[w=\left\{\begin{array}{ll}{0} & {\text { for } \quad y

The ad hoc decision rule can be expressed as:

\[V(y)=\left\{\begin{array}{lll}{U(0)} & {\text { for }} & {y

Since U(0) and U(1) are arbitrarily assigned values, let U(0) = 0 and U(1) = 1. Then, the ad hoc decision rule to be maximized is:

(15.14) $$Max\ 1 - Pr(y < y_d)$$

which calls for minimizing $Pr(y < y_d)$. This rule is known as Roy's safety-first rule. When direct outcomes are defined in terms of winning or losing, Roy's safety-first rule is consistent with an expected utility model.

Of course, one can think of other direct and indirect outcome variable relationships. For example, y could represent uninsured income and w could represent insured income, or y could represent unhedged income and w could represent hedged income, or y could represent income produced without risk reducing inputs and w could represent income produced with risk reducing inputs.

So what have we learned? We learned that risk responses are defined over direct outcome variables. Failure to distinguish between indirect and direct outcome variables may lead us to view responses to indirect outcome variables as risk preferring when they are indeed risk averting.

Firm Responses to Risk

Individuals and businesses all face risky events, including the possibility of losses that leave them less well off than before the outcome associated with the risky event occurred. These outcomes may include you or someone you care about becoming ill or unemployed. Business also face the possibility of losing customers, production failures, declining prices, and loss of financial support.

The next section describes alternative responses to risky events available to firms and individuals. If firms and individuals limit

their risk responses described by alternatives on an EV frontier and select risky alternatives consistent with their underlying risk preferences, then they will maximize their certainty equivalent income. Higher risk-averse decision makers will select investments with lower expected values and variances. Less risk-averse decision makers will select investments with higher expected values and variances. If the firm's current risk position is an expected value-variance combination off the frontier, then the firm's risk responses may be designed to move the firm closer to a position on the EV frontier.

More to the point, moving up or down its EV frontier involves paying another unit to absorb part or all of its risk (e.g. buying insurance) or changing the relative amount of safe and risky investments in its portfolio.

Someone once claimed that economists can predict the past correctly only 50% of the time. Imagine how successful economists are at predicting future outcomes for risky events? This should suggest that even though we present risk response solutions in closed forms, we should be cautious and explore alternative assumptions and predictions to explore the robustness of our estimates. With that caution in mind, we proceed to explore external risk responses (pay others to absorb our risk) and internal risk responses (change the firm's holdings of risky and safe investments).

Our discussion of risky and uncertain events and how to measure the probabilities of their outcomes has prepared us to evaluate alternative risk responses including: (1) sharing risk with others through various arrangements including forming partnerships and cooperatives, (2) buying insurance, (3) diversifying one's holdings of risky investments, (4) purchasing risk reducing inputs, and (5) choosing optimal capital structure. In addition, we will introduce the expected value-variance criterion for ranking alternative risky investments. Finally we will explore how converting risky cash flow to their certainty equivalents can provide the means for introducing risk into PV models so that the homogeneous measures principle applied to risk is satisfied.

Sharing Risky Outcomes

One way to mitigate the impact of adverse outcomes associated with risky events is by forming risk sharing arrangements with others even when they are facing similar risks. The key to successful risk reduction combinations with others is to combine operations that are statistically independent of each other. Statistical independence has a precise meaning, but essentially it means that the expected value of the product of two random variables equals the product of their expected values. In a later section we will discuss combining firms' operations within other firms whose returns tend to move together.

To make the point that combining units experiencing independent risky outcomes can mitigate risk, consider the following. Let *x* be a random variable with expected value μ_x and variance σ_x^2 respectively. Let "a" and "b" equal numerical constants. Then, define a new random variable:

(15.15) $$y = a + bx$$

The variance of *y*, σ_y^2, is equal to:

(15.16) $$\sigma_y^2 = b^2\sigma_x^2$$

The explanation for Equation ??? is that the variance of the constant "a" or any constant is always zero while the variance of a constant times a random variable is the constant squared times the variance of the random variable.

In Equation ??? we created a new random variable *y* by linearly transforming the random variable *x*. Now consider another way to create a new random variable—by taking the average of two (or more random variables). For example, suppose two businesses were facing independently distributed risky earnings. One business owner faced the random variable x_1 and the second business owner faced the random variable x_2. Also suppose that the two business owners decided to combine their businesses and agreed that each would receive half of the average earnings. In this process we define a new random variable.

(15.17) $$y = \frac{x_1 + x_2}{2}$$

First, recognize that each random variable is multiplied by the constant (1/2). Then each partner would earn on average the expected value

$$E\left(\frac{x_1+x_2}{2}\right)=\left(\frac{\mu_{x_i}+\mu_{x_2}}{2}\right) \tag{15.18}$$

and the variance each owner would face would be the variance of the expected return multiplied by the constant (1/2) squared. The variance of the average earnings each firm would receive, σy^2, is equal to:

$$\sigma_y^2=\left(\frac{1}{4}\right)\left(\sigma_{x_1}^2+\sigma_{x_2}^2\right) \tag{15.19}$$

n the case that the distributions were identically distributed with expected value and variance of μ_x and σ_x^2, each partner would face the same expected value as before, μ_x. But, the variance of their individual earnings would be $_2$, half of what it was before without combining their businesses. Furthermore, the standard deviation of the earnings each partner would face would be:

$$\sqrt{\frac{\sigma_x^2}{2}}=\frac{\sigma_x}{\sqrt{2}} \tag{15.20}$$

And if *n* partners joined together, then they would each face the same expected value as before, but the variance each partner would receive is \sqrt{n}. We now illustrate these important results.

Assume that business one's earnings are determined by outcomes associated with the toss of a fair coin. If the outcome of the coin toss is tails, the firm pays (loses) $5,000. If the toss is a heads, the firm wins $8,000. Thus, the firm wins either $8,000 or loses $5,000 and earns on average (.5) (–5,000) + (.5) (8,000) = $1500.

The standard deviation of this risky outcomes is:

$$\sqrt{(.5)(-\$5,000-\$1,500)^2+(.5)(\$8,000-\$1,500)^2}=\$6,500 \tag{15.21}$$

Furthermore, assuming a normal distribution, 68% of the time, the average outcome will be between the mean and plus or minus one standard deviation: ($1,500 + $6,500) = $8,000 and ($1,500 – $6,500) = –$5,000.

Now suppose that two persons decide to combine their operations and share the average of the outcomes. Then the possible outcomes of two coin tosses are two heads (H, H) which earns on average $16,000 / 2 = $8,000 and occurs with a probability of .25; two tails (T, T) which earns on average –$10,000 / 2 = –$5,000 and occurs with a probability of .25, and one head and one tail (H, T) or one tail and one head (T, H) which both earn on average $3,000 / 2 = $1,500 and each occurs with a probability of .25. The expected value for each of the two players can now can be expressed as:

$$(.25)(\$8,000)+(.25)(-\$5,000)+(.25)(\$1,500)+(.25)(\$1,500)=\$1,500 \tag{15.22}$$

The two players now receive on average the same as before, $1,500, but consider the standard deviation of the average outcome:

(15.23) 2} \\ & =\$ 4,596\end{split} \end{equation*}

Furthermore, assuming a normal distribution, 68% of the time, the average outcome will be between the mean and plus or minus one standard deviation: ($1,500 + $4,596) = $6,096 and ($1,500 – $4,596) = –$3,096. Note that even though the expected value did not change when outcomes were averaged over two persons, the standard deviation was reduced almost 30%. Also note that the results are in accord with Equation ???:

$$\frac{\sigma_x}{\sqrt{n}}=\frac{\$6,500}{\sqrt{2}}=\$4,596 \tag{15.24}$$

Now imagine ten persons each facing independent and identical distributed earnings outcomes, then the standard deviation each would face would equal:

$$\frac{\sigma_x}{\sqrt{n}} = \frac{\$6,500}{\sqrt{10}} = \$2,055 \tag{15.25}$$

Furthermore, assuming a normal distribution, 68% of the time, the average outcome will range between ($1,500 – $2,055) = –$555.48 and ($1,500 + $2,055) = $3,555. Note that even though the expected value did not change when outcomes were averaged over ten persons, the standard deviation was reduced almost over 70%. Reducing the variability for each person by over 70% by combining ten independent risky events (facing each of 10 independent firms) illustrates the power of reducing risk by sharing independent risky events.

In the previous example we assumed that each partner contributed an equal share. Now assume two persons decided to form a partnership and share the risk and expected returns weighted by their shares contributed to the business. Assume partner 1 contributed w_1 = 30% of the assets and partner 2 contributed w_2 = 70% of the business. Each partner's business rate of return can be described by random variable y_1 and y_2 with expected values and variances for partner one of $\mu_1 = 10\%$ and $\sigma_1^2 = (5\%)^2$ for the first partner and $\mu_2 = 12\%$ and $\sigma_2^2 = (6\%)^2$ for the second partner. We want to find expected value of the partnership and each partner's share of the expected value. Then, we want to find the variance of the partnership and the variance of returns each partner would face.

First, to find the expected value of the partnership, we weight each partner's contribution by the percentage of their contributions. Call the expected return of the partnership $E(y_p)$:

$$E(y_p) = w_1\mu_1 + w_2\mu_2 = (.3)(10\%) + (.7)(12\%) = 11.4\% \tag{15.26}$$

Meanwhile, the variance of expected return from the partnership is the sum of the individual variances multiplied by the partners' shares squared:

$$\sigma_y^2 = w_1^2\sigma_{x_1}^2 + w_2^2\sigma_{x_2}^2 = (.3)^2(.05)^2 + (.7)^2(.06)^2 = .002 = (4.5\%)^2 \tag{15.27}$$

Furthermore, the standard deviation of the portfolio is 4.5%. Thus, the partnership earns 11.4% on its investments and faces a standard deviation of returns equal to 4.5% which is less than their standard deviation of returns faced alone equal to 5.0% or 6.0%. Of course, they would have to agree on how to distribute profits, but we assume it would be based on the shares they contributed.

Reducing Risk by Purchasing Insurance

It may be difficult for an individual firm to agree with other independent firms on how to share profits and risk. However, an insurance company that absorbs individual firms' risk in exchange for an insurance premium can reduce the overall risk by combing the risk facing large numbers of individual firms. Furthermore, by carefully selecting businesses from different geographic areas and of different types, the risk absorbed by individual firms can be close to negligible.

For most individuals and businesses, insurance offers a way to reduce their risk when other measures are not available. Insurance is a practical arrangement by which a company or government agency provides compensation for a wide variety of losses and adverse outcomes. For example, we can purchase health insurance in case we become ill, fire insurance in case of a fire, term and whole life insurance for our heirs if we die, revenue insurance in case our expenses exceed income, trip insurance in case our flight gets canceled, and almost any other adverse outcome as long as we are willing to pay someone to assume the possibility of loss. This kind of insurance, discrete disaster event insurance, is described next.

Discrete disaster event insurance. Consider a firm with wealth comprised of a risky asset, whose value is W in the best state of nature and zero in the worst state of nature, and risk-free assets valued at W_0 regardless of the state of nature. An insurance company is willing to absorb the risk of possible losses of wealth in exchange for an insurance premium π_s. The firm must determine the maximum insurance premium π_b it can pay to avoid a disaster without reducing the level of its certainty equivalent wealth below the level attained with no insurance coverage. The firm can increase its certainty equivalent income by purchasing insurance if $\pi_s < \pi_b$.

Suppose the firm is considering a comprehensive fire insurance policy and W represents the value of the firm's flammable property being insured. To the find the maximum insurance premium π_b that the firm can pay without reducing its certainty

equivalent income, we form the decision matrix displayed in Table 15.2.

The matrix has two possible states of nature, two choices, and four different possible outcomes. The states of nature are (1) a fire state s_1, and (2) a no-fire state s_2 . The choices are buy insurance (choice A_1) and remain uninsured (choice A_2). If choice A_1 is selected, the outcome in both states s_1 and s_2 is initial wealthy $W + W_0$ less an insurance premium π_s. This result is obtained because if a fire does occur, the insurance company reimburses the firm for its losses and receives a risk premium. If there is no fire, the insurance company pays for no losses while still earning the insurance premium. In both states the firm purchasing insurance pays a premium. If, on the other hand, the firm decides to remain uninsured (choice A_2) and no fire occurs, the firm will be left with both its safe wealth W_0 and its risky wealth W and will have saved the insurance premium because it didn't purchase insurance. However, if a fire does occur, the firm will lose its risky wealth W. These results are summarized below.

Table 15.2. Decision Matrix for Insurance Versus No Insurance with a Discrete Disaster Outcome.

		Choices	
States of nature	Probability of outcomes	A_1 (buy insurance) outcomes	A_2 (Don't buy insurance) outcomes
(s_1) fire	$0 < p < 1$	$W + W_0 - \pi_s$	W_0
(s_2) no fire	$1 - p$	$W + W_0 - \pi_s$	$W + W_0$

If the probability of fire is p and the probability of no fire is $1 - p$, the expected values of the two choices $E(A_1)$ and $E(A_2)$ are:

(15.28) $$E(A_1) = W + W_0 - \pi_s$$

And

(15.29) $$E(A_2) = pW_0 + (1 - p)(W + W_0) = W + W_0 - pW$$

The difference between $E(A_1)$ and $E(A_2)$ can be expressed as:

(15.30) $$E(A_1) - E(A_2) = pW - \pi_s$$

If the decision maker was risk neutral and decided between options based on their expected value, the maximum the client would pay for the fire insurance would be $pW = \pi_b$, and as long as $\pi_b < \pi_s$ the client would be better off purchasing insurance than not purchasing insurance.

To illustrate, suppose that the flammable property was W = \$100,000 and p = 1%. Then, the most a client could pay and break-even based on his or her expected values would be pW = (.01)(\$100,000) = \$1,000. If an insurance policy was available for less than \$1,000, the client would be advised to purchase the insurance. Suppose the client is risk averse and lays awake at night worrying about the possibility of a fire, and perhaps the client is willing to pay an additional insurance premium based on some function of p and W equal to $U(p,W)$ \$150 to know that, no matter what, the outcome will be $W + W_0 - \pi_b - U(p, W) = \$100,00 + W_0 - \$1,150$.

Revenue insurance. One for the most important forms of insurance available to individual and firms is revenue insurance. The general principles of revenue insurance can be complicated. We describe a simplified version of revenue insurance with discrete outcomes.

Suppose an outcome from a crop operation is a risky event with three possible outcomes: normal income y, reduced income γy where γ is a percentage between one and zero, or a failed crop resulting in $y = 0$. Let the probability of y be p_1. Let the probability of γy be p_2. And let the probabilities of a failed crop and zero income be $(1 - p_1 - p_2)$.

The insurance provided does not fully compensate farmers for their losses for moral hazard reasons; they want the farmers to experience some losses for not producing a normal crop.

As a result, lost revenues are only compensated by δ percent. Furthermore, there is a complicated process to determine what is a normal yield that produces y. If a failed crop outcome occurs, the firm receives δy or δ percent of what it normally earns. If a partial crop outcome occurs, then the firm also earns δy because the insurance company pays δ of the firm's lost earnings equal to $\delta y(1-\gamma)$.

To describe the revenue insurance program described above we construct Table 15.3.

Table 15.3. Decision Matrix for Revenue with Insurance Versus no Insurance with Discrete Outcomes.

		Choices	
States of nature	Probability of outcomes	A_1 (buy insurance) outcomes	A_2 (Don't buy insurance) outcomes
(s_1) full crop	p_1	$y-\pi_s$	y
(s2) partial crop	p_2	$\delta y-\pi_s$	γy
(s_3) crop failure	(1 – p_1 – p_2)	$\delta y-\pi_s$	0

To solve for π_s we equate the expected value for the two choice options:

(15.31) $$p_1(y-\pi_s)+(1-p_1)(\delta y-\pi_s)=p_1 y+p_2(\gamma y)$$

And solving for the insurance premium π_s we find:

(15.32) $$\pi_s=[\delta(1-p_1)-p_2\gamma]y$$

To illustrate, suppose $y=100$ and $\gamma y=50$ so that $\gamma=50\%$. Assume that the probability of a normal income is 60%, the probability of a partial crop and a reduced income is 30%, and the probability of a complete crop failure and no income is 10%. Finally, assume that your revenue insurance policy covers δ = 80% of lost revenue. We restate these conditions in Table 15.4 and then solve for the break-even insurance premium π_s.

Table 15.4. Decision Matrix for Revenue Insurance Versus No Insurance with Discrete Outcomes.

		Choices	
States of nature	Probability of outcomes	A_1 (buy insurance) outcomes	A_2 (Don't buy insurance) outcomes
(s_1) full crop	p_1 = 60%	$y-\pi_s=100-\pi_s$	$y=100$
(s_2) partial crop	p_2 = 30%	$\delta y-\pi_s=80-\pi_s$	$\gamma y=50$
(s_3) crop failure	(1 – p_1 – p_2) = 10%	$\delta y-\pi_s=80-pi_s$	0

Finally, we solve for the break-even insurance premium π_s :

(15.33) $$\pi_s=[\delta(1-p_1)-p_2\gamma]y=[(0.8)(0.40)-(0.30)(0.5)]100=17$$

In other words, a manager could afford to pay up to 17% of its normal income as revenue insurance under the conditions described in Table 15.3.

Diversification of Firm Investments

Investors rarely hold investments in isolation. Indeed, holding a single investment by itself may be very risky. Most investors attempt to reduce risk by holding a portfolio of (two or more) investments. Adding a risky investment to a portfolio of investments may actually decrease the risk of the portfolio without adversely affecting the expected return on the portfolio. We illustrate the point that adding risky investments to one's portfolio may decrease risk with an example.

Umbrellas and sunglasses. Suppose that on any given day there are three possible weather outcomes: there may be rain, there may be a mix of clouds and sun, and there may be bright sunny skies. For simplicity, assume that the probability of each outcome is 1/3. A firm whose outcomes depend on the weather state can invest in umbrellas or sunglasses or a mix of the two. Both investments in umbrellas and sunglasses earn an expected rate of return equal to 10%. These results are summarized in Table 15.5 below:

Table 15.5. Expected Returns and Variances on Investments in Sunglasses and Umbrella

Weather states i = 1,2,3:	Probability of weather states	Random Returns on Sunglasses in the ith weather state: r_i^S	Random Returns on Umbrellas in the ith weather state: r_i^W	Return on portfolio
Rain	1/3	0%	20%	.5(0) +.5(20%) = 10%
Mix clouds and sunshine	1/3	10%	10%	.5(10) +.5(10%) = 10%
Sunny	1/3	20%	0%	.5(20) +.5(0%) = 10%
Expected return on investments:		$E(r_i^S)$ = (1/3)(0% + 10% + 20%) = 10%	$E(r_i^W)$ = (1/3)(20% + 10% + 0%) = 10%	$.5E(r_i^S) + .5E(r_i^S)$ = (1/3)(10% + 10% + 10%) = 10%
Standard deviation of returns:		8.16%	8.16%	0%

Notice that when it rains, return on umbrellas is favorable (20%) but the return on sunglasses is low, 0%. The reverse is true when there are bright sunny skies; the return on umbrellas is low (0%) while the return on sunglasses is favorable, 20%. The standard deviation for both investments equals:

(15.34a) $$\sqrt{.33(0-.10)^2+.33(.10-.10)^2+.33(.20-.10)^2}=8.16\%$$

Now assume that the firm diversified and created a portfolio in which 50% of its investments were in sunglasses and the other 50% were in umbrellas. The results are described in the last column of Table 15.5. Notice that the return in each state is 10% because when returns on low on sunglasses, return on umbrellas are favorable and vice versa. Note also that while each individual investment has a standard deviation of returns equal to 8.16%, the return on the portfolio is constant and the standard deviation of portfolio returns is zero.

(15.34b) $$\sqrt{.33(.10-.10)^2+.33(.10-.10)^2+.33(.10-.10)^2}=0\%$$

This is an extreme example of how adding a risky investment may actually decrease the firm's risk. However, this favorable result occurred because the returns on umbrellas and sunglasses were perfectly and negatively correlated.

Covariance measures. To be perfectly and negatively correlated means that returns on one investment is above its mean by exactly same amount as the other investment is below its mean in the same state. One measure of correlation between two random variables is the covariance measure. Using the notation from the umbrellas and sunglasses example, we define the covariance as:

$$
\begin{aligned}
Cov(r_i^S, r_i^U) &= E[r_i^S - E(r_i^S)][r_i^U - E(r_i^U)] \\
&= \left(\frac{1}{3}\right)(0\% - 10\%)(20\% - 10\%) \\
&+ \left(\frac{1}{3}\right)(10\% - 10\%)(10\% - 10\%) \\
&+ \left(\frac{1}{3}\right)(20\% - 10\%)(10\% - 20\%) \\
&= \left(\frac{2}{3}\right)(-10\%)(10\%) = -0.00666
\end{aligned}
\tag{15.35}
$$

Notice that the covariance is similar to a variance measure except that instead of a deviation from the expected value being squared, the deviation for both variables in the same state are multiplied, so the covariance measures whether the two variables are moving in opposite directions from their means (negative covariance) or whether the two variables are moving in the same direction from their means (positive covariance). To emphasize the difference between variance and covariance measures, when calculating variance, we squared deviations from the mean, and as a result all variances are positive. In contrast, deviation in covariance measures are not squared which allows them to be positive or negative. Note that the first and third term in the covariance calculation in Equation ??? were negative while the second term was zero. Thus, the covariance of investments in sunglasses and umbrellas is negative.

Obviously the sunglasses and umbrellas example is simplified to illustrate a point, that risk is completely eliminated because the returns from the two investments have perfect negative correlation. The level of correlation between returns is measured by the correlation coefficient ρ defined as:

$$
\rho = \frac{Cov(r_i^S, r_i^U)}{\sigma_{r_i^S}, \sigma_{r_i^U}} \tag{15.36}
$$

Thus, in our example, the correlation coefficient is negative one:

$$
\rho = \frac{Cov(r_i^S, r_i^U)}{\sigma_{r_i^S}, \sigma_{r_i^U}} = \frac{-.00666}{(.0816)(.0816)} = -1.0 \tag{15.37}
$$

Obviously, other things being equal, we would prefer to add investments to our portfolio that are negatively correlated with our overall portfolio returns.

Let's return to our investigation of the partnership, only this time allow for a single firm to consider its rate of return on its portfolio of investments. Assume that it has two investments and the percent of the total portfolio invested in each investment is indicated by a weight w_1 and w_2 that sum to one. The expected value for the firm's portfolio can be expressed as before:

$$
E(y_p) = w_1\mu_1 + w_2\mu_2 \tag{15.38}
$$

Now consider the variance of the investor's portfolio. If the investments are represented by independent random variables, the portfolio variance is as before—the weighted sum of the individual variances. When the investments are not independent, the portfolio variance also includes a covariance term. We can write the variance of the portfolio allowing for dependence between the two investments as:

$$
\sigma_y^2 = w_1^2\sigma_{x_1}^2 + w_2^2\sigma_{x_2}^2 + 2w_1w_2Cov(r_i^1, r_i^2) \tag{15.39}
$$

We now apply our portfolio approach to the umbrellas and sunglasses example. Recall that both investments earned an expected rate of return of 10% and their standard deviations were both equal to 8.16%. If the firm divided their portfolio between the two

investments, then we would write the expected value and variance of the portfolio as:

(15.40) $$E(y_p) = w_1\mu_1 + w_2\mu_2 = (.5)(10\%) + (.5)(10\%) = 10\%$$

And we write the portfolio variance as:

(15.41) $$\begin{aligned}\sigma_y^2 &= w_1^2\sigma_{x_1}^2 + w_2^2\sigma_{x_2}^2 + 2w_1w_2Cov(r_i^1, r_i^2) \\ &= (.5)^2(.0816)^2 + (.5)^2(.0816)^2 + 2(.5)(.5)(-.00666) = 0\end{aligned}$$

In this special example, that the returns on sunglasses and umbrellas moved in perfectly opposite direction means that combining investments in both eliminated the variability of returns on the firm's portfolio.

Beta coefficients and risk diversification. An important risk concept applied to securities markets but which also has application to the firm is the beta coefficient (β). The beta coefficient is a measure associated with an individual investment which reflects the tendency of an investment's returns to move with the average return in the market. Applied to the individual firm, the beta coefficient measures the tendency of an individual investment's returns to move with the average return on the firm's portfolio of investments.

To explain beta coefficient, suppose we have past rate of return observations r_t^j on a potential new investment and on the firm's portfolio of returns r_t^p in time period *t*. Table 15.6 summarizes our observations:

Table 15.6. Observations of Returns on the Firm's Portfolio of Investments r_t^p and on a Potential New Investment (a Challenger).

Time *t*	Observed returns on the firm's portfolio over time r_t^p	Observed returns on a potential new investment for the firm's r_t^j
2012	10%	7%
2013	6%	8%
2014	7%	5%
2015	3%	2%
2016	5%	3%

Another way to represent the two rates of return measures and their relationship to each other is to represent them in a two dimensional scatter graph.

We may visually observe how the two sets of rates of return move together by drawing a line through the points on the graph in such a way as to minimize the squared distance from the point to the line. Our scatter graph is identified as Figure 15.3.

Figure 15.3. Scatter Graph of Returns on the Firm's Portfolio of Investments and Returns on the Potential New Investment



The relationship between the returns on the new investment and the firm's portfolio can be expressed as:

$$r_t^j = a + \beta r_t^j + \epsilon_t \tag{15.42}$$

Notice that the equation above describes the straight line drawn through the point plus the vertical distance from the line to each point. The slope of the line is the beta coefficient β and tells us how the returns on the portfolio and potential new investment have moved together in the past. We can find the equation for this line using a statistical method called "least squares" regression analysis. The method essentially finds a line so that the average squared deviations from the line, ε_t^2, are minimized. The formula for beta is equal to the covariance between the returns on the new investment r_t^j and the returns on the firm's portfolio r_t^p over some time period divided by the variance of portfolio returns:

$$\beta_j = \frac{Cov(r_t^j, r_t^p)}{\sigma_p^2} \tag{15.43}$$

Fortunately, the calculations for the beta coefficient as well as the coefficients in Equation ??? can be found using Excel.[2] We find the beta coefficient and the coefficients for Equation ??? using the data in Table 15.6.

The estimated equation for the line is:

$$E(r_t^j) = a + \beta r_t^p + E(\epsilon_t) = a + \beta r_t^p = .02 + .48 r_t^p \tag{15.44}$$

So what have we learned? We learned that, in particular, a beta coefficient of .48 means that a 10% change in the return on the firm's portfolio will likely be accompanied by an increase of 4.8% in the expected returns on the potential new investment. Furthermore, a decrease of 10% in the returns on the firm's portfolio of investments will likely be accompanied by a 4.8% decrease in the expected returns on the potential new investment. It also means that the variability of the returns on the potential investment are less than the variability of returns on the firm's portfolio of returns. It also says that adding the potentially new investment will not diversify all of the firm's risk. There will still be 48% that is not diversified; it varies with the returns on the firm's overall rate of return. It reduces some of the risk faced by the firm, but not all of it—only about 52%.

Diversifiable and non-diversifiable risk. Assume a beta coefficient of minus one (–1). This would mean that for a 10% increase (decrease) in the firm's overall rate of return, that the expected rate of return on the potentially new investment decreases (increases) by 10%. Like investments in sunglasses and umbrellas, a sufficient investment in the new investment would eliminate the firm's risk. Thus, a beta of –1 means that all of its risk can be diversified. In contrast, assume a beta coefficient of one. This would mean that for a 10% increase (decrease) in the firm's overall rate of return, that the expected rate of return on the potentially new investment increases (decreases) by 10%. Unlike investing in sunglasses and umbrellas, adding the potentially new investment to the firm's portfolio of investment only accentuates its risk, and the new investment has no potential to diversify the firm's overall risk.

Purchase Risk Reducing Investments

It is useful to distinguish between two primary reasons for investing: first, to increase expected earnings for the firm and second, to reduce the variability of earnings. If one can increase expected returns without increasing variance of return, certainty equivalent income has increased. If one can reduce variance of returns without also reducing expected returns, certainty equivalent income has increased. See Equation 15.12. Of course if from one's current expected value-variance position, one could increase one's expected value of returns without increasing the variance of returns—or if one could reduce one's variance without also reducing one's expected value—then the firm would be in an inefficient expected value-variance position off the EV frontier. However, we can identify investments whose primary purpose is to reduce variability even though they alter expected incomes. We call these risk reducing investments. We analyze risk reducing inputs using the certainty equivalent income model described in Equation 15.12 that accounts for both variance and expected value in the ranking criterion.

We will illustrate risk reducing investments with a case study involving an irrigation investment. Consider a firm facing five moisture states with an equally likely chance of occurring: normal, low stress, moderate stress, high stress, and drought. The returns per acre with and without the irrigation system are reported in Table 15.7. The annualized cost of the irrigation system, which is expected to have a 20 year life and no liquidation value, is π per acre. We assume the defender's IRR associated with the certainty equivalent cash flow stream to be 8%.

Table 15.7. Returns per Acre under Alternative Moisture States with and without an Irrigation System.

Moisture states	Probability of moisture states	Return per acre without the irrigation investment $r_i^{w/0}$	Return per acre with the irrigation system r_i^w minus irrigation costs per acre π
Normal	20%	$128.00	$113– π
Low stress	20%	$105.00	$100 – π
Moderate stress	20%	$90.00	$80 – π
High stress	20%	$75.00	$75 – π
Drought	20%	$50.00	$70– π
Expected returns		$89.60	$87.60
Standard deviation of returns		$26.43	$16.28

Assume that π is $15 per acre. Then the expected value of the crop production per acre without irrigation is still $89.60 and greater than $87.60. If the decision maker were risk neutral, he or she would not invest in the irrigation system. Allow that the decision maker is risk averse and chooses between investments based on their certainty equivalent incomes rather than the difference in their expected values. If this were the case, then the certainty equivalent income without irrigation is:

(15.45) 0} = E(y) - \dfrac{\lambda}{2} \sigma_y^2 = \$89.60 - \dfrac {\lambda}{2}(26.43)^2 \end{equation*}

In contrast, the certainty equivalent income with the irrigation system is:

$$y_{CE}^{w} = E(y) - \frac{\lambda}{2}\sigma_y^2 = \$87.60 - \frac{\lambda}{2}(16.28)^2 \tag{15.46}$$

We cannot decide between the two systems because one has a higher expected value and the other has a lower variance. It all depends on how risk averse the decision maker is. Recall that λ reflects the decision maker's preferred trade-off between expected return for variance. The break-even λ in this case is found by equating the two certainty equivalent incomes:

$$\$89.60 - \frac{\lambda}{2}(26.43)^2 = \$87.60\frac{\lambda}{2}(16.28)^2 \tag{15.47}$$

and solving for λ:

$$\lambda = \frac{2(89.60 - 87.60)}{(26.43)^2 - (16.28)^2} = .009 \tag{15.48}$$

Thus, all decision makers more risk averse than is reflected by the risk aversion coefficient of λ = .009 will be earning a lower expected value on average than would be earned without the irrigation system.

Choosing an Optimal Capital Structure

Leverage and risk. In an earlier chapter, we used leverage as a measure of the firm's risk without explicitly stating the connection. We now make the connection explicit by reconsidering equation 8.5 and making one adjustment. The adjustment is that since we only consider realized capital gains when finding ROA–IRR, we ignore unrealized capital gains (V_1 – V_0) which allows us to rewrite equation 8.5 as:

$$ROE = (ROA - i)\frac{D}{E} + ROA \tag{15.49}$$

In Equation ???, note the leverage ratio D/E and that it multiplies the difference between the ROA and the average interest rate *i*. And now we return to a risk principle introduced earlier—that multiplying a random variable by a constant, in this case the leverage ratio, increases the variance of the random variable by the constant squared. Consider the application of this principle.

Suppose the random variable ROA is described by pdf $f(r^{ROA})$ with expected value r^{ROA} and variance σ^2. Now suppose that we were to multiply the random variable ROA by some scalar, say 2. Then the expected value of the random variable would be $2r^{ROA}$ and the variance would be $2^2\sigma^2$, or $4\sigma^2$.

In Equation ???, the debt-to-equity ratio is the scaler that multiplies and exaggerates the difference between r^{ROA} and *i*. To illustrate, suppose that ROA can take on values of –8%, –3%, 3%, 5%, and 12% and *i* = 3%. Then in Table 15.8 we find ROE for leverage ratios of 0, 2, 5, and 10.

Table 15.8. ROEs whose Expected Values and Standard Deviations depend on Leverage Ratios.

Values of the random variable ROA (*i* = 3%)	ROE values for alternative Leverage Ratios (*L* = *D*/*E*) and values of the random variable ROA			
	L = 0	*L* = 2	*L* = 5	*L* = 10
–8%	–8%	–30%	–63%	–118%
–3%	–3%	–15%	–33%	–63%
3%	3%	3%	3%	3%
5%	5%	9%	15%	25%
12%	12%	30%	57%	102%
Standard deviations for ROEs associated with each leverage ratio *L*				
Expected values	1.8 %	–.6%	–4.2%	–10.2%
Standard deviation σ	.077	.23	.460	.843

The first thing to note about the outcomes in Table 15.8 is that whenever the ROA exceeds the average interest rate *i*, that ROE > ROA. For example, for a leverage ratio of 2 and an ROA of 5%, ROE is 9%. The second point to observe is that even though the *E*(ROA) > 0, as the leverage ratio increased, the *E*(ROE) was mostly less than zero. In other words, the effect of leverage was more pronounced when the ROA < *i* than when ROA > *i*. Another thing to note is that if the ROA outcome is –8% and the firm has a leverage ratio of 10, it loses 118% of its equity. In other words, one unfavorable outcome with a high leverage ratio can destroy the firm. Finally, due to the cost of debt that must be paid regardless of ROA outcomes, ROA's less-than-average cost of debt with high leverage ratios have significant adverse effects on the firm's equity. Thus, we conclude that a high leverage ratio, even though it can exaggerate unusually high ROA outcomes, is still a risky state for the firm. For that reason, many firms view leverage reduction as an important strategy for reducing the riskiness of the outcomes they face.

Capital structure. A firm's capital structure is its combination of debt and equity used to finance its overall operations and growth. The small to medium-size firm may finance its overall operations and growth by using long-term debt, equity, and notes payable. We will discuss the small to medium-size firm's optimal capital structure using a simplified expected value-variance (EV) profit model.

In our simplified model, we let the firm's assets A be funded by a combination of debt *D* and equity *E*. We let $\tilde{r}^a$ be the stochastic rate of return on the firm's assets whose variance is σ_a^2 and whose expected rate of return is r^a. We let the average non-stochastic cost of debt per dollar be represented by the variable i^D. The firm decision maker's risk-return trade-off is measured by $_2$.

We represent the firm's stochastic profits $\tilde{\pi}$ to equal the stochastic rate of return on assets times assets less the firm's average cost of debt times the firm's debt. Then we substitute for assets *A* the sum of debt *D* plus equity *E* and collect like terms and express the stochastic results in Equation ???.

$$\tilde{\pi} = \tilde{r}^a A - i^D D = \tilde{r}^a (D + E) - i^D D = (\tilde{r}^a - i^D) D + \tilde{r}^a E \tag{15.50}$$

We write the expected value of stochastic profits as:

$$E(\tilde{\pi}) = r^a A - i^D D = (r^a - i^D) D + r^a E \tag{15.51}$$

We write the variance of profits as:

$$\sigma^2(\tilde{\pi}) = (D + E)^2 \sigma_a^2 \tag{15.52}$$

Finally, we substitute the expected value and variance of profits into Equation ??? to find the firm's certainty equivalent of profits π_{CE} that we refer to as the EV model.

$$\pi_{CE} = E(\tilde{\pi}) - \left(\frac{\lambda}{2}\right) \sigma^2(\tilde{\pi}) = (r^a - i^D) D + r^a E - \left(\frac{\lambda}{2}\right) (D + E)^2 \sigma_a^2 \tag{15.53}$$

Finding the firm's optimal capital structure.[3] Having our EV model defined over the expected value and variance of profits and accounting for the decision maker's risk attitudes, we use calculus to find the optimal debt level by differentiating the certainty equivalent function π_{CE} with respect to *D*.

$$\frac{d\pi_{CE}}{dD} = (r^a - i^D) - \lambda (D + E) \sigma_a^2 = 0 \tag{15.54}$$

The second order conditions are satisfied allowing us to solve for the optimal debt D^* (assuming fixed equity). We find the optimal debt D^* to equal:

$$D^* = \left(\frac{r^a - i^D}{\lambda \sigma_a^2}\right) - E \tag{15.55}$$

Equation (15.55) reveals an interesting detail. If the cost of debt equals the expected return on assets, the firm holds negative debt—preferring to lend out its equity at a safe rate i^D rather than earning a stochastic return on firm assets.

Dividing Equation ??? by the firm's equity *E*, we can find its optimal leverage ratio l^* equal to:

$$l^* = \frac{D^*}{E} = \left(\frac{r^a - i^D}{\lambda \sigma_a^2 E}\right) - 1 \tag{15.56}$$

We illustrate Equation ??? using HQN's data. We substitute for r^a the ROA value equal to 6.5% (equation 5.13), the average cost of debt i^D equal to 6% (equation 5.21), equity *E* equal to $8,000 (Table 4.1), the risk aversion coefficient calculated in Equation ??? equal to .009, and finally, we let the standard deviation—the amount that returns on assets vary on average—equal 1.25% or .0125 that we square to find the variance of profits $\sigma^2(\tilde{\pi})$ equal to .000156. Making the substitutions for the variables in Equation ??? we find the firm's optimal leverage ratio equal to:

$$l^* = \frac{D^*}{E} = \left(\frac{r^a - i^D}{\lambda \sigma_a^2 E}\right) - 1 = \frac{(.065 - .06)}{(.009)(.000156)(\$8,000)} - 1 = 3.45 \tag{15.57}$$

compared to the firm's actual leverage ratio of 4.0.

Changing variables affecting the optimal capital structure. We can imagine how the optimal leverage would change in response to changes in the value of the variables included in Equation ???. In other words, we ask: how would the optimal leverage change if the value of one of the variables in Equation ??? changed? We can infer the answer to this question by looking at changes in the optimal leverage ratio in response to a change in one of the variables holding the other variables constant.

Increasing the expected value of asset returns r^a makes it more profitable to use borrowed funds and increases the optimal leverage ratio. Increasing the average cost of debt i^D makes using debt less profitable and reduces the optimal leverage ratio. As a decision maker becomes more risk averse, represented by an increase in the risk aversion coefficient λ, the decision maker is less willing to risk losing equity with an unfavorable outcome and reduces leverage. As the firm's equity increases, it can achieve the same risk return combination with less debt and the optimal leverage ratio decreases. Finally, as the variance of return on firm assets σ_a^2 increases, the firm reduces its leverage ratio to return to its preferred trade-off between equity and debt.

In addition to describing how the firm's finds its optimal capital structure in response to changes in the value of the variables that determine the optimal capital structure, we create Table 15.9. Columns in Table 15.9 include the list of variables, their original values for HQN, a column showing increased values of the variables, the revised optimal leverage ratio, and the change in the optimal leverage ratio compared to the original optimal leverage ratio of 3.45.

Table 15.9 Optimal leverage ratios and changes in the optimal leverage ratio in response to increases in one of the variables in Equation ??? holding the other variables constant.

Variable	Original value	Increased value	Revised optimal leverage ratio	Change in the optimal leverage ratio
r^a	6.5%	7.0%	7.903134	4.45
i^D	6%	6.5%	(1)	(4.45)
λ	.009	.001	3.01	(.44)
σ_a^2	0.000156	.000175	2.97	(.48)
E	$8,000	$9,000	2.96	(.49)

So what have we learned? We learned that firms generally dislike risk (however it is defined) and prefer expected earnings. Generally, we are willing to assume some risk if the increase in expected returns is sufficient. The firm selects its optimal combination of debt and equity to achieve its preferred expected profits and variance of profits. Increases in the value of variable that increase the firm's expected profits increase the firm's optimal leverage ratio. Increases in the value of variables that increase the firm's variance of profits reduce the firm's optimal leverage ratio.

Summary and Conclusions

Some sage is reported to have said that only death and taxes are certain. If that statement is anywhere close to being true, then risk and uncertainty fill the world we live in and try to manage. One important step toward managing the outcomes of risky events is to understand the tools that have been developed to report and measure it. In this effort, precision is not expected. It is best to explore the influence of risk in a variety of settings and assumptions.

The second thing to note about risk, emphasized in the irrigation example, is that individual risk preferences may have significant effects. As a result, two individuals facing the same investment opportunities may make different choices because of the different risk preferences. As a result, it is important for managers to explore their own risk preferences and apply them when making risky decisions.

Questions

1. This question has several parts.
   a. What is the difference between a sample of observations and the population of possible values?
   b. Explain the difference between an expected value and variance (standard deviation) calculated from a sample and the

expected value and variance (standard deviation) of a population.

c. Find the expected value and variance (standard deviation) for the numbers 5, 8, –3, 9, and 0. Assume each number has an equally likely chance of being observed.
d. Find the expected value and population variance (standard deviation) for the numbers 5, 8, –3, 9, and 0 if their probability of occurring were .1, .2, .4, .2, and .1 respectively.
e. Compare the results obtained in parts c and d.

2. Return to Table 15.1 in the text. Suppose that the investor decided to invest half of her assets in investment A and half in investment B. Describe the random variable for the combined investment. Then describe its pdf, expected value, and variance (standard deviation). Based on the respective expected values and variances for investment A, investment B, and the combined investment—which would you prefer, assuming you are risk averse?
3. Assume two people decide to form a partnership and share the risk and expected returns based on their shares contributed to the business. Assume partner 1 contributed 40% of the assets and partner 2 contributed 60% of the assets. Each partner's business can be described by random variables y_1 and y_2 with expected values and variances of $\mu_1 = 8\%$ and $\sigma_1^2 = 0.006$ for the first partner and $\mu_2 = 12\%$ and $\sigma_2^2 = .007$ for the second partner. Find the expected value standard deviation of the partnership.
4. Assume that Kelly wants to provide for her heirs in case she dies during the coming year. Therefore, she purchases a term life insurance policy that pays $1,000,000 in case she dies in return for an insurance premium of $800. Assuming Kelly is risk neutral, what must Kelly assume is the probability of her death in order for her to purchase the insurance policy?
5. Assume the conditions described in Table 15.3 except allow for the insurance coverage δ to increase from 80% to 85%. Find the increase in the break-even insurance premium.
6. Assume the conditions described in Table 15.4. Also assume that instead of purchasing revenue insurance the investor could purchase an irrigation system that would increase the probability of a normal revenue income year from 60% to 75%, reduce the probability of a reduced income year from 30% to 20%, and reduce the probability of zero income from 10% to 5%. What would be the most that the manager could pay to reduce its risk through the purchase of an irrigation system and still be as well as he was before? (Hint: compare the value provided by the irrigation system less the cost of the irrigation system compared to the outcomes without an irrigation system.)
7. One of the differences between the purchase of an irrigation system and revenue insurance is that one has to purchase revenue insurance each year while the irrigation system continues to provide risk reduction services during its useful life. If the irrigation system described in the previous question were available for 10 years and the discount rate were 8%, what is the NPV of the irrigation system?
8. Use the data in Table 15.5 to find the beta coefficient for the investment in umbrellas and sunglasses.
9. A firm has two investments in its portfolio. The historical rates of return on the two investments are reported below. Find the expected rate of return for the firm's portfolio, the covariance between the two investments, and the variance of the portfolio returns. Rank investment 1, investment 2, and the combined investment using the EV criterion.

Table Q15.1. Observations of Returns on the Firm's Two Investments

Time *t*	Observed returns on investment one.	Observed returns on investment two.
2012	10%	7%
2013	6%	8%
2014	7%	5%
2015	3%	6%
2016	5%	4%

1. The authors thank Jack Meyer for the thumbtack example of an uncertain event. ↵
2. The linear regression equation that includes the Beta coefficient is found in Excel by first plotting a scatter diagram and then hovering over a data point in the graph. A complete discussion of linear regression models is outside the scope of this class. ↵
3. This section uses calculus to find the firm's optimal debt level and its optimal capital structure or leverage ratio (D/E). Those not interested in the derivation may skip to the next section without loss of continuity in the discussion. ↵

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16: Loan Analysis

Learning Objectives

After completing this chapter, you should be able to: (1) understand the different ways interest can be calculated on a loan; (2) recognize the different kinds of interest rates that are used to calculate interest costs on loans; (3) computer comparable interest rates by finding a loan's effective interest rate; (4) use Excel worksheets to calculate the loan payment, interest rate, term, elasticity of term, and amortization schedule for a particular loan; and (5) evaluate alternative loans using present value (PV) models developed earlier.

To achieve your learning goals, you should complete the following objectives:

- Learn the differences between the following interest rates: the annual percentage rate (APR), the actuarial rate, and the effective rate.
- Learn how to compute the APR, the actuarial rate, and the effective rate on alternative loans.
- Learn how to use the loan equality equation to find a loan's effective interest rate, its constant payment, its term, and its original loan amount.
- Learn how to find the elasticity relationship between interest rates, loan payments, and the term of the constant payment loan.
- Learn how to create an amortization table for constant payment loans.
- Learn how to find break-even points for refinancing loans.
- Learn how to find the effective rate for several disguised interest rate loans including discount loans and points added loans.

Introduction

Loan formulas and PV models have many similarities. Interest rates on loans are like opportunity costs, the loan amount is like an investment, and loan payments are like an investment's cash flow. These and other similarities between PV models and loan formulas allow us to use PV tools to analyze different types of loans. The next section focuses on alternative interest rate definitions. This chapter will also identify the relationship between the term of a loan and the size of the loan payment. Finally, this chapter finds break-even points when refinancing loans and effective interest rates for a variety of disguised interest rate loans.

Comparing the Actuarial Rate, Annual Percentage Rate (APR), and Effective Interest Rate

Loans charge interest rates of which there are at least three kinds that are closely related to each other. These rates and their commonly used synonyms are listed below. The interest rate name used in this chapter is italicized. They are: (1) actuarial rate, compound rate, true rate, or periodic rate; (2) Annual Percentage Rate (APR), annual rate, or nominal rate; and (3) effective interest rate or effective annual rate.

Actuarial Rate. In financial transactions, interest may be computed and charged more than once a year. For example, interest on savings deposits is usually calculated on a daily basis while many corporate bonds pay interest on a semiannual basis. The interest rate used in computations for periods of less than one year is called an actuarial interest rate. The actuarial rate is defined as the interest rate per compounding period or the interest rate per period of conversions. It is the actuarial rate used to charge interest on the principal sum during each successive conversion period. For example, consider a 1% actuarial rate charged monthly on $1,000. In this case, in the first month of the loan, 1% of $1,000 or $10 of interest is charged. In the second month, interest is charged on $1,010 equal to $10.10, etc.

Annual Percentage Rate (APR). Let r^f represent the APR. Let m stand for the number of times during the year the interest is calculated or charged. Thus, m equals the number of compounding periods per year. The ratio of r^f / m is the actuarial rate, the compound rate, the true rate, or the periodic rate.

We find the APR from actuarial rates by expressing the actuarial rate on an annual basis. To convert the actuarial rate to an APR, we multiply the actuarial rate by *m*. In the previous example, we multiply the actuarial rate of 1% per month by 12 to yield an APR of 12%. When the compound period or conversion period is one year in length, then the actuarial rate and the APR are equal.

Consider two savings institutions, both offering the same APR. The only difference is that institution *A* offers monthly compounding of interest, while institution *B* offers annual compounding. Which one should the saver prefer? Obviously, monthly compounding is preferred because the saver earns interest on the interest earned during the same year. With institution *B*, interest is earned during the year only on the principal saved and on interest earned in previous years.

Effective interest rates. Effective interest rates are the actual interest charged measured on an annual basis. When APRs have different numbers of compound periods per year, the different actuarial rates should be converted to their effective interest rates for comparison. The effective rate is obtained by compounding the actuarial rate for a period of one year. As the number of compounding periods per year increases, the difference between the APR and the effective rate increases.

Relationship between interest rates. The relationships between an actuarial rate, an APR, and an effective rate can be easily summarized. Let *m* be the number of compounding periods per year, let r^f be the APR, let r^f/m be the actuarial rate, and let *re* be the effective rate. The relationship between the effective rate r^e, the APR rate r^f, and the actuarial rate r^f/m can be expressed as:

$$r^e = \left[\left(1 + \frac{r^f}{m}\right)^m - 1\right] \tag{16.1}$$

Note that when $m = 1$, the effective interest rate, the APR rate, and the actuarial rate are equal. However, when *m* is not 1, the rates are no longer equal. For example, suppose we wish to find the *re* assuming r^f were compounded quarterly. To solve this problem let $m = 4$, and $r^f = 12$. Substituting .12 for r^f and 4 for *m* in Equation 16.1, we obtain the results described in Table 16.1 using Microsoft Excel. In cell B3 we enter the function f_x = ((1 + (B1 / B2))^B2) – 1 which returns 12.55%.

Table 16.1. Finding an effective interest rate compounded quarterly. Open Table 16.1 in Microsoft Excel.

B4		***Function*:**	**=(1+B2/B3)^B3 -1**
	A	**B**	C
1	Finding an effective interest rate		
2	APR	0.12	
3	m	4	
4	effective rate	12.55%	"=(1 + APR/m)^m – 1

If *m* is increased to 12, or monthly compounding periods, the effective rate is found as before. Let $m = 12$, and $r^f = 12$ in Equation 16.1. Then

$$\left[\left(1 + \frac{.12}{12}\right)^{12} - 1\right] \tag{16.2}$$

To solve this equation in Excel, we change cell B3 to 12 and find the effective interest rate to equal 12.68% as shown in Table 16.2.

Table 16.2. Finding an effective interest rate compounded monthly. Open Table 16.2 in Microsoft Excel.

B4		***Function*:**	**=(1+B2/B3)^B3 -1**
	A	**B**	C
1	Finding an effective interest rate		
2	APR	0.12	
3	m	12	
4	effective rate	12.68%	"=(1 + APR/m)^m – 1

A special compounding formula is obtained by allowing the number of periods compounded to be very large. This idea is expressed as:

$$\lim_{m\to\infty}\left(1+\frac{r^f}{m}\right)^m \tag{16.3}$$

which means that as *m* approaches infinity the effective rate *re* equals:

$$r^e=\lim_{m\to\infty}\left(1+\frac{r^f}{m}\right)^m-1=e^{r^f}-1 \tag{16.4}$$

To solve this problem using Excel, we use the EXP function. In Excel, Euler's number "e" is found as the function EXP(1) and 12% compounded continuously can be calculated as f_x = EXP(.12) – 1 = 12.75. Using the previous cell designation, to find the value of continuously compounding of 12% we would enter the formula in B4 as: f_x = EXP(B2) – 1 = 12.75.

Table 16.3. Finding an effective interest rate compounded continuously. Open Table 16.3 in Microsoft Excel

B4		***Function*:**	=EXP(B2) – 1
	A	**B**	C
1	Finding an effective interest rate		
2	APR	0.12	
3	m	Infinite	
4	effective rate	12.75%	"=EXP(APR) – 1

Constant Payment Loans

Having defined interest rates in financial models, we now use PV models to analyze the most common type of loan, the constant payment loan. Constant payment loans are repaid with a series of equal payments *A* at equal time intervals. These payments may occur *m* times during a year over *n* years, yielding a total of *mn* payments.

The fundamental equality is that the sum of the loan payments discounted at the actuarial interest rate must equal the amount loaned. The relationship between loan amount, L_0, received in time-period zero, with payment *A*, made for *mn* periods, at actuarial interest rate (r^f/m) beginning in period one, is:

$$L_0=\frac{A}{1+\frac{r^f}{m}}+\frac{A}{\left(1+\frac{r^f}{m}\right)^2}+\cdots+\frac{A}{\left(1+\frac{r^f}{m}\right)^{mn}} \tag{16.3}$$

In this formula, the actuarial rate (r^f/m) is the IRR for the PV model, and since loan payments are constant, it is unique.

A sum that will prove useful in several calculations is the following:

$$US_0\left(\frac{r^f}{m},mn\right)=\frac{A}{1+\frac{r^f}{m}}+\frac{A}{\left(1+\frac{r^f}{m}\right)^2}+\cdots+\frac{A}{\left(1+\frac{r^f}{m}\right)^{mn}} \tag{16.6}$$

The notation $US_0(r^f/m, mn)$ stands for the present value of a uniform series of $1 payments discounted at the actuarial rate (r^f/m) for (mn) periods. This shorthand notation allows us to rewrite Equation 16.2 as:

$$L_0 = AUS_0\left(\frac{r^f}{m}, mn\right) \tag{16.7}$$

Recall that with one equation we can solve for at most one unknown variable. From Equation ??? if we know *A* and $US_0(r^f/m, mn)$ we can find loan amount L_0 that *mn* payments of amount A discounted at actuarial rate (r^f/m) will repay.

Suppose we solve for *A* in Equation ???. The result is:

$$A = \frac{L_0}{US_0\left(\frac{r^f}{m}, mn\right)} \tag{16.8}$$

From Equation ??? if we know L_0 and $US_0(r^f/m, mn)$, we can find constant loan payment *A*, that, if discounted at actuarial rate (r^f/m) for *mn* periods, will repay loan amount L_0.

Suppose we solve for $US_0(r^f/m, mn)$ in Equation ???. The result is:

$$US_0\left(\frac{r^f}{m}, mn\right) = \frac{L_0}{A} \tag{16.9}$$

If we know *A* and loan amount L_0 in Equation ??? we can find r^f, *m*, or *n*, that constant loan payment *A* discounted at actuarial rate (r^f/m) for *mn* periods will repay.

Fortunately, the calculations described in equations (16.7), (16.8), and (16.9) can be easily performed using Excel.

Example 16.1: Loan amount supported by a constant payment loan

Suppose a borrower can make constant payments of $150 per month for 48 months (four years). The borrower wants to know what size loan can be repaid if the APR interest rate is 5% and the actuarial rate is .05/12 = 0.42%. Using Equation ??? we can solve for the loan amount supported by the constant loan payment of $150 using the formula below:

$$L_0 = AUS_0\left(\frac{r^f}{m}, mn\right) = \$150\, US_0\left(\frac{.05}{12}, 48\right) \tag{16.10}$$

We solve the equation above using Excel's PV function in Table 16.4.

Table 16.4. Finding the Loan Amount in a Constant Payment Loan. Open Table 16.4 in Microsoft Excel

B7		***Function:***	=PV(B3/B4,B4*B5,B6,,0)
	A	**B**	C
1	Finding the loan amount in a constant payment loan		
2			
3	rate	.05	
4	m	12	
5	n	4	
6	pmt	-150	
7	PV	$6,513.44	"=PV(rate/m,mn,pmt,,0)

The answer displayed is: $6,513.44. In other words, 48 monthly payments of $150 on a loan charging a 5% APR interest rate and a monthly actuarial interest rate of .42% will repay a loan in the amount of $6,513.44.

Example 16.2: Constant Payment Loan Annuities

Suppose $5,000 is borrowed from a lending institution for five years at an APR of 12% or a monthly actuarial rate of 1%. The loan is to be repaid with 60 equal monthly installments. What is the payment or annuity necessary to retire the loan?

Using Equation ??? we can solve for the payment that repays the $5,000 loan:

$$A = \frac{L_0}{US_0\left(\frac{r^f}{m}, mn\right)} = \frac{\$5,00}{US_0(.01, 60)} \tag{16.11}$$

We solve for the loan payment using Excel's PMT function in Table 16.5. The answer displayed is: –$111.22 which means that 60 payments of $111.22 on a loan charging 12% APR interest and a monthly actuarial interest rate of 1% will repay a loan in the amount of $5,000.

Table 16.5. Finding the Constant Loan Payment. Open Table 16.5 in Microsoft Excel

B6		***Function*:**	=PV(B3/B4,B4*B5,B7,,0)
	A	**B**	C
1	Finding the loan amount in a constant payment loan		
2			
3	rate	0.12	
4	m	12	
5	n	5	
6	pmt	($111.22)	"=pmt(rate/m,mn,PV,,0)
7	PV	$5,000.00	

Example 16.3. *Loan term required to retire a constant payment loan.* Suppose a borrower can make constant payments of $150 per month. The borrower wants to know how many monthly payments will be required to retire a loan of $8,000 if the APR interest rate is 6% and the monthly actuarial rate is .5%. Using Equation ??? we can solve for the term of the loan required to retire the loan.

$$US_0(.005, mn) = \frac{L_0}{A} = \frac{\$8,000}{\$150.00} \tag{16.12}$$

We solve the equation using Excel's NPER function shown in Table 16.6. The answer displayed is 62.19 which means that 62 regular payments and one partial payment will be required to retire a loan of $8,000 if the APR interest rate charged on the loan is 6%.

Table 16.6. Finding the Number of Payments required to Retire a Loan
Open Table 16.6 in Microsoft Excel

B8		***Function*:**	=NPER(B3/B4,B6,B7,,0)
	A	**B**	C

1	Finding the number of payments required to retire a loan		
2			
3	rate	0.06	
4	m	12	
5	n		
6	pmt	-150	
7	PV	$8,000.00	
8	nper	62.18593	"=NPER(rate/m,pmt,PV,,0)

Comparing Interest Paid, Loan Term, and Payment Amounts for Constant Payment Loans

An important relationship exists between the loan's term and total interest paid. To illustrate, consider a $30,000 loan at 15% APR to be repaid in monthly payments over 30 years. The monthly payment for this loan is $379.33. Total interest TI paid on a constant payment loan is found by multiplying the constant loan payment A times the term of the loan *mn*, minus the amount of the loan L_0:

(16.13) $$TI = (A)(mn) - L_0 = (\$379.33)(360) - \$30,000 = \$106,560$$

In this case, the total interest paid is $106,560.

Increasing the payment amount by 10% to $417.27 reduces the term of the loan by 48% to just over 15.36 years (verify the results above using Excel). Meanwhile, total interest paid is reduced by 56% to $46,961.

The term reduction in response to an increased loan payment is not always so significant. For example, if the above loan had an 8% APR, the monthly payment would equal $220.13 instead of $379.22. Increasing the payment by 10% would decrease the term of the loan by only 27% from 30 years to 21.93 years, and the total interest paid would be decreased by 32% from $49,247 to $33,722.

It would be useful to know how changing the term of the loan affects payment size and total interest paid. It can be shown that as *mn* becomes large, the payment *A* approaches the interest cost per period, i.e., the smallest payment possible equals the interest charged on the outstanding loan balance. If the borrower wished to minimize his or her payment, the appropriate term is the one that permits the borrower to repay only interest. The shortest repayment period, on the other hand, is one. Obviously, there is a trade-off between the size of the loan payment and the length of the loan.

The point elasticity of term, measured in years n with respect to the payment *A*, measures the percentage change in the term n in response to a 1% change in the loan payment. The term elasticity, *E(n,A)*, has been calculated as:

(16.14) $$E_{n,A} = \frac{1 - e^{rm}}{rn} < 0$$

For example, a 30-year loan and an APR of 15% would have an elasticity of term equal to:

(16.15) $$E_{n,A} = \frac{1 - e^{(.15)(30)}}{(.15)(30)} = -19.78\% < 0$$

In other words, increasing the payment by 1% would decrease the term by approximately 19.78%. In contrast, the arc elasticity, rather than the point elasticity, compares the percentage change in the loan term to a 10% increase in the loan payment and finds the percentage change in the term to equal 48%, or an arc elasticity of term equal to 4.8%. Note that a point elasticity of 19.78% versus an arc elasticity of 48% is the result of comparing large changes in loan payments of 10% versus comparing tiny changes in loan payments (e.g. .00001%). See Table 16.7 of point elasticities below.

Table 16.7. Point elasticity measures for loans of alternative terms and interest rates.

n / *r* %	1%	5%	7.5%	10%	15%	20%
1	1.01	1.03	1.04	1.05	1.08	1.11
5	1.08	1.14	1.21	1.30	1.49	1.72
10	1.17	1.30	1.49	1.72	2.32	3.19
15	1.26	1.49	1.85	2.32	3.77	6.36
20	1.37	1.72	2.32	3.19	6.36	13.40
25	1.49	1.99	2.94	4.47	11.07	29.48
30	1.49	2.32	3.77	6.36	19.78	67.07
60	2.99	6.36	19.78	67.07	900.23	13,563.00

Example 16.4. *Term and Loan Payment Trade-Offs*. Lucy Landlord is financing the renovation of a property. She needs a loan for $28,000. Her lender offers her a loan for 20 years at the current interest rate of 15%. She calculates her annual payment to be $4,473.32. If she increases her payment by 1% to $4,518.05, her term is reduced to 19 years, or a reduction of 5%. This percentage reduction is nearly equal to the tabled value of 6.36 in the Table 16.1, found at the intersection of the row labeled 20 and the column labeled 0.15. Large percentage increases in *A*, such as 10%, may not be accurately reflected in the table of point elasticities. This is because the percentage changes in n with respect to *A* are large compared to the very small changes in n with respect to *A* used to calculate the table.

Creating an Amortization Table for Constant Payment Loans

The word "amortize" originally meant "to kill." Thus when we amortize a loan, we kill or extinguish it by making regular payments—killing the loan if you will. One feature of the constant payment loan is that, while the loan payment is constant, the amount of the payment devoted to paying off the loan—the principal portion of the payment—and the amount of the payment devoted to paying the interest on the loan are constantly changing. As the loan principal is reduced or killed off, the amount of the payment devoted to interest charges is reduced and the amount of the payment devoted to reducing the loan is increased. Lending institutions, when asked, will provide amortization tables that detail the amount of interest and principal paid on each payment during the life of the loan. Fortunately, Excel provides us the tools needed to create our own amortization tables.

Finding principal portion of the tth loan payment using Excel. The Excel PPMT function can be used to find the principal portion of the tth loan payment. The function is expressed as:

(16.16) $$PPMT(rate, per, nper, PV, , 0)$$

To illustrate the PPMT function, consider at $25,000 loan to be repaid in monthly installments for four years. The APR for the loan is 5%. We want to know what portion of the 5th payment (period) will be applied to the loan's principal. The Excel solution is represented in Table 16.8.

Table 16.8. Finding the Loan Principal Paid on the tth Payment
Open Table 16.8 in Microsoft Excel

B8		***Function*:**	=PPMT(B3/B4,B6,B4*B5,B7,,0)
	A	**B**	C
1	Finding the loan principal paid on the tth payment		

2			
3	rate	0.05	
4	m	12	
5	n	4	
6	period	5	
7	PV	$25,000	
8	Principal paid	($479.47)	“=PPMT(rate/m,period,m*n,PV,,0)

It turns out that $479.47 of the 5th payment is applied to the outstanding loan principal. In addition, students may verify that the principal portion of the 25th payment is $521.05. This is because the outstanding principal on which interest is charged decreases over the life of the loan. As interest decreases, more of the loan payment can be applied to the outstanding loan principal.

Finding interest payment IP(t) on the tth loan payment using Excel. The Excel IPMT function can be used to find the interest portion of a loan payment in the tth period and is expressed as:

(16.17) $$IPMT(rate, per, nper, PV, , 0)$$

To illustrate the IPMT function, we return to the earlier example: a loan amount of $25,000, a term of 48 monthly payments, at 5% APR, and a desire to find the interest paid on the 5th loan payment. We illustrate the Excel solution for finding interest paid on the tth period. The solution is entered in cell B8 as $54.68. Table 16.9 describes the solution in more detail.

Table 16.9. Finding the Loan Interest Paid on the tth Payment
Open Table 16.9 in Microsoft Excel.

B8		***Function*:**	=IPMT(B3/B4,B6,B4*B5,B7,,0)
	A	**B**	C
1	Finding the loan interest paid on the tth payment		
2			
3	rate	0.05	
4	m	12	
5	n	4	
6	period	5	
7	PV	$25,000	
8	Interest paid	($96.26)	“=IPMT(rate/m,period,m*n,PV,,0)

Together, the interest payment of $96.26 and the principal payment of $479.47 equals the constant loan payment of $575.73.

PVs of Special Loans

Loans and credit (one’s borrowing capacity) make possible a modern economy and successful firms. Sometimes sellers offer special loan arrangements to encourage the potential buyers to purchase their products. These may include concessionary interest rate loans, skip payment loans, skip principal payment loans, variable interest rate loans, and balloon payment loans. Other times firms may want to expand their investment base and need to refinance their loans, or decreases in interest rates may provide them an incentive to refinance. Fortunately, all of these special loans can be analyzed using PV models developed earlier. We consider the benefits and costs of several special loans in what follows.

Concessionary interest rate loans. Assume that Jane Doe's IRR = *r* is 10%. Recall that an IRR is the opportunity cost of the defender and is the appropriate rate to use when discounting investment cash flow. Jane is considering an investment loan for $50,000 to be repaid in monthly installments over 15 years (*mn* = 180). The APR for the loan is 7%, while the actuarial rate is 7% / 12 = .58%.

To find Jane's NPV for this loan, we first determine her loan payment *A*. This we can do using the Excel function PMT(rate,nper,PV,,0) = PMT(.58,180,$50,000,,0). The payment *A* for this loan is: $449.41.

The next step is to treat the loan payment as a cash flow in an investment problem and discount the payments using the borrower's IRR = *r* of 10%, or the borrower's actuarial opportunity cost rate of 10%/12=.83%. Using Equation ???, we find:

$$\begin{aligned} PV &= \frac{A}{\left(1+\frac{r^f}{m}\right)} + \frac{A}{\left(1+\frac{r^f}{m}\right)^2} + \cdots + \frac{A}{\left(1+\frac{r^f}{m}\right)^{mn}} \\ &= AUS_0\left(\frac{r^f}{m}, mn\right) = \$449.41\, US_0(.83, 180) = \$41,916.34 \end{aligned} \tag{16.18}$$

Some interpretation of the above results may be helpful. The first important fact is that the borrower's IRR(*r* = 10%) was more than the interest rate on the loan (r^f = 7%). Therefore the borrower can borrow at a rate less than what she will earn by investing the loan. The borrower's actual cost of the loan, what is paid back after adjusting for what the loan earns as an investment, is $41,916.34, not $50,000. Another way to express this result is that present value of the loan was $41,916.34. The Net Present Value (NPV) of the $50,000 loan is:

$$\begin{aligned} NPV(\$50,000\, loan) &= \$50,000 - \$41,916.34 \\ &= \$8,083.66 \end{aligned} \tag{16.19}$$

In other words, the borrower received in the form of a loan $50,000 in present value dollars. What the borrower paid back in present value dollars was $41,916.34. The difference—the NPV—was $8,083.66.

Refinancing a Constant Payment Loan. A common financial transaction is the refinancing of a constant payment loan. What complicates this transaction is loan closing costs or points charged as a percentage of the loan required to close. If the current interest rate is less than one's interest rate on the existing loan, a reasonable borrower would prefer to refinance. What if refinancing requires a fee, percentage points charged as a percentage of the new loan, to be paid at the loan closing? We want to know what the borrower can afford to pay as a refinancing cost, or points of the loan to break even. Finding break-even points is what we do next. We will use as our starting point the numbers introduced into the previous example.

The refinance problem is clearly an NPV problem. Its solution requires that we identify which is the defending and which is the challenging investment. The defender in this case is the loan the borrower now holds. The challenger is the new loan with its reduced interest rate and points charged to close the loan. What the borrower will earn on the loan is not relevant here because we assume that those earnings will be the same regardless of whether they are financed with the new loan or continue to be financed with the old loan.

To solve this problem, we recognize that if the PV of loan payments on the new loan plus points charged to close the loan is the same as the PV of loan payments on the old loan (and ignoring taxes), points charged are break-even. So using Equation ???, we find break-even points charged to refinance as:

$$A = \frac{L_0 + pL_0}{US_0\left(\frac{r^N}{m}, mn\right)} \tag{16.20a}$$

where pL_0 is the cost the borrower pays to refinance the loan, *p* percent of the new loan must be paid as a refinance cost, and r^N is the interest rate on the new loan. The loan payment on the old loan is:

$$A = \frac{L_0}{US_0\left(\frac{r^O}{m}, mn\right)} \tag{16.20b}$$

where r^O is the interest rate on the old or original loan. Finally, equating equation (16.20a) to (16.20b) and solving for *p,* we find the break-even points to refinance the loan; that is, we find the percentage of the loan *p* that could be paid as a refinancing charge by the borrower to obtain a loan with a lower interest rate and still break-even. The formula for *p* is:

$$p = \frac{US_0\left(\frac{r^N}{mv}, mn\right)}{US_0\left(\frac{r^O}{m}, mn\right)} - 1 \tag{16.21}$$

To illustrate, let r^O = 8%, r^N = 6%, and let *mn* = 180. We first solve for *p* using Excel:

$$US_0\left(\frac{r^O}{m}, mn\right) = US_0\left(\frac{8\%}{12}, 180\right) \tag{16.22}$$

by keying in PV(rate,nper,PMT,,0) = PV(.66,180,1,,0) and find:

$$US_0\left(\frac{8\%}{12}, 180\right) = \$105.15 \tag{16.23}$$

We follow the same procedures to find:

$$US_0\left(\frac{r^N}{m}, mn\right) = US_0\left(\frac{6\%}{12}, 180\right) = \$118.50 \tag{16.24}$$

Finally, we find *p* by making the appropriate substitutions into Equation ???:

$$p = \frac{US_0\left(\frac{r^N}{m}, mn\right)}{US_0\left(\frac{r^O}{m}, mn\right)} - 1 = \frac{118.50}{105.15} - 1 = 13\% \tag{16.25}$$

So what have we learned? We learned that we could afford to pay 13% of the loan as a closing fee to acquire the same loan at a lower interest rate and still be as well off as we would have been keeping the old loan.

A variation of the problem would be the following: Suppose that we knew what points would be charged to refinance the loan, and we wanted to know what APR would be required to be indifferent between the new and old loan. In other words, we want to find the break-even r^N given that we know *p*, the points charged to refinance the loan. The solution is found by rearranging Equation ??? so that:

$$US_0\left(\frac{r^N}{m}, mn\right) = (1+p)US_0\left(\frac{r^O}{m}, mn\right) \tag{16.26}$$

To illustrate, suppose that *p* is known to be 10%, and the conditions attached to the old loan are as before, so that:

$$(1+p)US_0\left(\frac{r^O}{m}, mn\right) = (1.1)(105.15) = \$115.67 \tag{16.27}$$

To find the break-even APR on the new loan we equate:

$$US_0\left(\frac{r^N}{m}, mn\right) = (1+p)US_0\left(\frac{r^O}{m}, mn\right) = \$115.67 \tag{16.28}$$

We now solve this problem using Excel RATE function:

$$RATE(nper, pmt, PV, , 0) = RATE(180, 1, -115.67, , 0) = 53\% \tag{16.29}$$

Displayed is the break-even actuarial rate, .53, which when multiplied by 12 equals the break-even loan rate of 6.38%. Compared with our earlier results, if points paid to refinance the loan were reduced from 13% to 10%, the borrower could afford to pay 6.38% on the new loan and still break even.

Consider another example with p = 15% in the previous example. First equate:

$$US_0\left(\frac{r^N}{m}, mn\right) = (1+p)US_0\left(\frac{r^O}{m}, mn\right) = (1.15)(105.15) = \$120.92 \tag{16.30}$$

To find the corresponding interest rate for $US_0(r^N/12, 180)$ = \$120.92, we key into Excel's RATE function:

$$RATE(nper, pmt.PV, , 0) = RATE(180, -1, 120.92, , 0) = 47\% \tag{16.31}$$

The answer displayed is: .47 which, when multiplied by 12, equals the break-even loan rate of 5.69%. Before, a new interest rate of r^N = 6% corresponded to break-even points of 13%. If points charged were 15% instead of 13%, the new interest rate would need to equal 5.69% to be indifferent between continuing with the old loan versus paying closing costs and taking out a new loan.

Practical refinance problems facing agricultural firms. Agricultural firms in particular face cash receipts variability. This means that firm's liquidity measures such as the current ratio (CT) or the times-interest-earned (TIE) ratio may also face significant variability over time making mortgage payments problematic. And sometimes, the original mortgage for an investment was written for a much shorter period than the investment's productive life. Whatever the case, liquidity concerns may require the firm to renegotiate the terms of their loans with their lenders. To illustrate, consider the payment calculated in Example 16.2. In this case the required payment was found to equal \$111.22. Suppose the lender were willing to increase the term of the loan by 12 monthly payments. What would be the new loan payment? Resolving the Excel PMT function returns:

$$PMT(rate, nper, PV, , 0) = PMT(.01, 72, 5000, , 0) = \$97.75 \tag{16.32}$$

The solution is the loan payment that will retire a loan of \$5,000 with 72 payments rather than 60 payments at an actuarial rate of 1%. The loan payment is reduced from \$111.22 to \$97.75.

Disguised Interest Rates and Effective Interest Rates

One of the challenges financial managers face when considering borrowing decisions is knowing the actual cost of borrowing—or, stated another way, knowing the effective APR interest rate. Sometimes lenders offer loans designed to disguise the real cost of their loans. We call their loans disguised interest rate loans. Disguised interest rate loans have effective interest rates increased by methods other than increasing the interest rate on the loan. For example, interest costs can be subtracted in the initial period, reducing the actual loan amount received by the borrower (a discount loan). Interest can be charged as though the original loan balance was outstanding throughout the life of the loan (an add-on loan). Alternatively, the lender can charge a loan closing fee, reducing the actual loan balance received by the borrower. Additionally, the interest can compound more frequently than loan payments occur. Each of these methods will increase the effective interest rate above the stated interest rate. Consider several types of disguised interest rate loans.

The Discount Loan: A borrower approaches his lender for a loan of L_0 for *mn* periods. The borrower learns that the stated interest rate or disguised interest rate is r*d*. When the borrower picks up the check for his loan, the amount he receives equals only:

$$L_0^d = L_0(1 - r^d n) \tag{16.33}$$

the amount of the loan requested less the stated interest rate times the term of the loan. Meanwhile, the constant loan payment is calculated as:

$$A = \frac{L_0}{mn} \tag{16.34}$$

To calculate the APR associated with this loan, treat payments of amount A as if they were associated with a constant payment loan that retires a principal amount of L_d. The relationship is expressed as:

$$L_0^d = L_0(1 - r^d n) = AUS_0\left(\frac{r^d}{m}, mn\right) \tag{16.35}$$

Next, substituting for A, L_0/mn, we find:

$$US_0\left(\frac{r^f}{mv}, mn\right) = mn(1 - r^d n) \tag{16.36}$$

To illustrate the discount loan, assume a consumer obtains an installment loan for $10,000, from which $2,500 is deducted for interest costs. The loan is to be repaid over 2 years, with monthly payments equal to $416.67 = ($10,000/24). To solve this example, we first must find the stated rate r_d:

$$r^d = \frac{L_0 - L_0^d}{nL_0} = \frac{\$2,500}{(2)(\$10,000)} = 12.5\% \tag{16.37}$$

Normally, the stated rate is given, but it is advisable to confirm the rate as we have done above. Now we enter our numbers into Equation ??? and obtain:

$$US_0\left(\frac{r^f}{m}, mn\right) = mn(1 - r^d n) = 24[1 - (.125)(2)] = 18.000 \tag{16.38}$$

To find the actuarial rate, we key into our Excel RATE function:

$$RATE(nper, pmt, pv, , 0) = RATE(24, -1, 18, , 0) \tag{16.39}$$

Displayed is the actuarial monthly rate of 2.44% or, after multiplying by 12, we find the corresponding APR rate of 29.3%. The effective rate r^e is: r^e = (1.0244)12 – 1 = 33.55%.

This is quite a difference compared to the stated interest rate of 12.5%. Hence, the discount loan effectively disguises its true APR.

Points-added loans. Sometimes lenders charge points p to close a loan. The fee has the effect of increasing the interest rate on the loan since the lender earns more than the stated rate suggests. The APR rate for such a loan can be calculated by first computing the payment which retires the loan, plus the points added. The payment equals:

$$A = \frac{L_0 + pL_0}{US_0\left(\frac{r^d}{m}, mn\right)} \tag{16.40}$$

Next, express the relationship between the payment A in Equation ???, APR rate r^f, and the actual amount of the loan received as:

$$A = \frac{L_0 + pL_0}{US_0\left(\frac{r^f}{m}, mn\right)} \tag{16.41}$$

Equating equations (16.40) and (16.41), we find r^f from the equality:

$$US_0\left(\frac{r^f}{m}, mn\right) = \frac{US_0\left(\frac{r^d}{m}, mn\right)}{(1+p)} \tag{16.42}$$

To illustrate how to find the effective interest rate for a points-added loan, consider the following problem. A bank offers a loan rate of 12% with monthly payments for three years, with a 3% loan-closing fee. What is the APR interest rate r^f? Using Equation ??? we first find:

$$US_0\left(\frac{r^f}{m}, mn\right) = \frac{US_0\left(\frac{r^d}{m}, mn\right)}{(1+p)} = \frac{US_0(1\%, 36)}{1.03} = \frac{30.1075}{1.03} = 29.23 \tag{16.43}$$

To find the true actuarial rate, we key into our Excel RATE function:

$$RATE(nper, pmt, pv, , 0) = RATE(36, -1, 29.33, , 0). \tag{16.44}$$

Displayed is the actuarial monthly rate of 1.15%, or after multiplying by 12, we find the corresponding APR rate of 13.83%. The effective rate r^e is: $r^e = (1.0115)^{12} - 1 = 14.71\%$. This is quite different compared to the stated interest rate of 12%.

Summary and Conclusions

In this chapter we demonstrated the versatility of PV models by using them to analyze loans. For constant payment loans, we used PV models to solve for constant loan payments, terms, loan amounts, and interest rates—remembering that one PV equation can solve for at most one unknown.

Using PV models to analyze loans required that we identify the various kinds of interest rates. This was an important exercise because we discovered the difference between stated interest rates and effective interest rates—the interest rate actually paid on the loan funds made available.

Another important exercise was discovering the sensitivity of the relationship between the size of the loan payment and the term of the loan. In most cases, the relationship is not one-to-one. In other words, a 1% increase in the size of the loan payment rarely leads to a 1% drop in the term. The corresponding percent decline in the term of the loan is usually much, much more. Hence, we discovered that, when applying for loans, it pays to explore various terms and sizes of loan payments and find the best match—the one with the optimal trade-off between term and liquidity.

A common problem is that existing loans often need to be refinanced. Such may be the case when interest rates drop or a project currently financed is expanded and additional funds are required. In the text we considered refinancing existing loans and found break-even loan closing points. In the supplemental materials at the end of this chapter, we will find the more general formula for refinancing when the term, interest rate, and size of the new loan may be different than on the existing loan.

Lastly, we demonstrated how some loans may disguise the true interest rate. While we only illustrated the solution for the discount loan and the points-added loan, there are several other kinds of loans that disguise the true interest rate.

Questions

1. Which would you prefer to earn on your savings? An APR rate of 12.5% or a 1% actuarial rate compounded monthly? Given an APR of r percent, what is the most that the effective rate can earn above the APR rate if it is compounded continuously?
2. Consider a loan of $80,000 at an APR of 13%. What is the loan payment that would retire the loan if repaid in monthly payments for 10 years? If repaid in monthly installments for 9 years? Compare the percentage change in the term versus the percentage change in the loan payment (the arc elasticity). Finally, find the point elasticity E*(n,A)* on the original loan.
3. Assume a loan of $54,000 with a remaining term of 21 years. The existing loan requires monthly payments at an APR of 11.25%. For a 3% closing fee, the borrowers could refinance their loan at an APR rate of 10% for the same term. What is the effective interest rate on the new loan? What are the break-even points for refinancing the loan? What is the total interest paid on the two loans?
4. A consumer obtains an installment loan of $12,000 from which $2,700 is deducted for interest costs. The loan is to be repaid over two years with monthly payments equal to $500 ($12,000/24). Please determine the effective interest rate *re* on this loan.
5. A farm supply store offers its customers 30 days same-as-cash arrangements. That is, for bills paid within 30 days after purchases are made, no interest is charged. On the other hand, to encourage early payments, the supply store offers a 2% discount on bills paid with 10 days. Please calculate the effective interest rate the store offers its buyers for giving up 20 days of free credit.
6. Suppose you borrowed $5,000 for 3 years at an APR rate of 8%. Create an amortization table for this loan.
7. Home Depot mailed to some of its customers a coupon entitling them to either a 10% discount on their next purchase or two years of free credit. Under what conditions would you be indifferent between the two options? (Hint: the answer does not depend on the amount purchased.)
8. A farm firm has a mortgage loan for $150,000 at an APR rate of 5%. The term of the loan is 15 years and the payments equal $14,451. Cash flow problems from reduced farm income leaves the firm only able to pay $10,000 on this loan. What would the new term equal if the lender allowed the borrower to repay over a longer term?

Supplemental Materials

The refinancing problem described in this chapter can be more complicated. What if not only the new and old APR loan rate were different but also the term and the amount refinanced? Let r^O and r^N equal the APR on the old and new loans respectively. Let's assume that the new loan included not only the refinanced loan L_0 but also an additional amount equal to L_0. Also assume that the term on the old loan is mn^O compared to the term on the new loan, mn^N. We want to know if the borrower should refinance. The IRR used for calculating the NPVs for the new and old loans is *r*.

Recall that if the IRR is also the reinvestment rate, then an investment's NPV is not affected by changing its size and term. Therefore, we can determine if the borrower should refinance by comparing the NPVs on the old and new (refinanced) loan.

Assume for the moment that we know the points charged to refinance a loan and we want to know our loan payment for the refinanced loan. Using the notation already defined, we solve the problem by revising equation (16.21a) and write the revised equation as:

$$A^N = \frac{(L_0 + \Delta L_0)(1+p)}{US_0\left(\frac{r^N}{m}, mn^O\right)} \tag{16.i}$$

The loan payment for the original (old) loan, for which there are no points charged, can be written as:

$$A^O = \frac{L}{US_0\left(\frac{r^O}{m}, mn^O\right)} \tag{16.ii}$$

At this point,we make a critical assumption; namely, that funds are reinvested at the defender's IRR, which allows us to write the NPVs for the new and old loans as:

$$NPV^N = L_0 + \Delta L_0 - A^N US_0\left(\frac{r}{m}, mn^N\right) \tag{16.iii}$$

and

$$NPV^0 = L_0 - A^O US_0\left(\frac{r}{m}, mn^O\right) \tag{16.iv}$$

If the points charged make the two loans equal in NPV, then we can equate equations (16.iii) and (16.iv) and solve for the break-even points:

$$p = \left[\frac{\gamma}{1+\gamma} + \frac{1}{1+\gamma}\frac{US_0\left(\frac{r}{m}, mn^O\right)}{US_0\left(\frac{r^O}{m}, mn^O\right)}\right]\left[\frac{US_0\left(\frac{r^N}{m}, mn^O\right)}{US_0\left(\frac{r}{m}, mn^O\right)}\right] - 1 \tag{16.v}$$

Where $\gamma = \delta L_0$. Note that if $\gamma < em >< /em >= 0$ = 0" title="Rendered by QuickLaTeX.com" height="18" width="174" style="vertical-align: -5px;"> and $n^O = n^N$, equation (16.v) reduces to Equation ???. Therefore, equation (16.v) allows us to find break-even points in general, even if the interest rates, term, and size of the refinanced loans are different compared to the loan being refinanced.

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17: Land Investments

Learning Objectives

After completing this chapter, you should be able to: (1) describe land's unique investment characteristics; (2) understand how land's endurable nature affects its price variability; (3) recognize how transaction costs reduce the liquidity of land investments; and (4) evaluate land investments using present value (PV) models developed earlier.

To achieve your learning goals, you should complete the following objectives:

- Learn what makes land distinct from other types of investments.
- Learn why land prices are so volatile compared to other investments.
- Learn how to distinguish between real and nominal discount rates.
- Learn how to distinguish between inflationary and real growth in earning rates.
- Learn how expected growth rates in earnings from land are capitalized into land values.
- Learn how to find the real growth rate for land.
- Learn how to calculate the maximum bid (minimum sell) price for land.
- Learn how transaction costs associated with buying and selling land influence land's liquidity.
- Learn how taxes influence the maximum bid (minimum sell) price for land.
- Learn how to find the tax adjustment coefficient for investments in land.
- Learn how to use land price-to-earnings ratios to predict adjustments in the price of land.

Introduction

Land's immobility and durability make it unique among investments and deserving of special attention in PV analysis. Land's immobility means that it cannot be moved and its services must be extracted by those physically on site. Durability means that land has the capacity to provide services over time without significant change in its service provision capacity.

Earlier, an asset's liquidity was defined as it nearness to cash. One dimension of an asset's liquidity depends on the form of its earnings—cash versus capital gains (see Chapter 13). In this chapter we discuss a different dimension of liquidity—the cost of converting an asset to cash through its sale. Land's immobility makes land less liquid than assets that can be moved because land cannot be moved to meet the convenience of the buyer. Land's immobility also limits the potential buyers to those near enough to the land to extract its services. Another reason that land is illiquid is because buyers and sellers pay fees to complete its purchase and sale, including Realtor fees, legal costs of changing and recording its title, and other related fees—but not to each other. Evidence of farmland's low liquidity is its infrequent transfer. On average, only 2% to 3% of the privately owned farmland in the United States is sold each year.

On the other hand, lenders prefer land as collateral for loans for the same reason that makes land illiquid—its immobility. Land's immobility reduces the riskiness of it being stolen, hidden, or moved. Lenders also prefer land as collateral for loans because of its durability, which reduces the riskiness of it losing its value as security for loans. To offset some of land's illiquidity, special institutions and programs have developed for financing residential and farm real estate.

The immobility and durability of land also make it a popular object on which to assess taxes. Taxing agencies have easy and indisputable records of the amount of land subject to tax and who owns the land. Thus, most land-owners pay property taxes on land and buildings but pay no similar tax on more mobile and less durable investments. Consequently, land is one of the few investments for which taxes are based on the market value of the investment at the beginning of the period as well as on the earnings of the investment during the period.

This chapter develops PV models to estimate maximum bid (minimum sell) prices for land investments, to describe how transaction costs contribute to land's illiquidity, to understand how land's durability increases its price volatility, and to demonstrate the influence of taxes on maximum bid (minimum sell) prices. We begin by describing the connection between land's durability and its price volatility and why land and other durables are subject to inflationary bubbles and crashes.

Why are land prices so volatile?

Durable price volatility. One interesting feature of farmland prices, and other durables prices such as housing stock, is their historically high swings in values. Figure 17.1 describes year-to-year changes in Michigan farmland values over the 2000 to 2010 period. Figure 17.2 describes changes year-to-year changes in housing values over the same period. Part of these changes can be attributed to capital gains (losses). But there is another explanation for changes in the value of land and other durables, and it has to do with changes in real interest rates and real growth rates.

Figure 17.1. Year-to-year changes in farmland values.

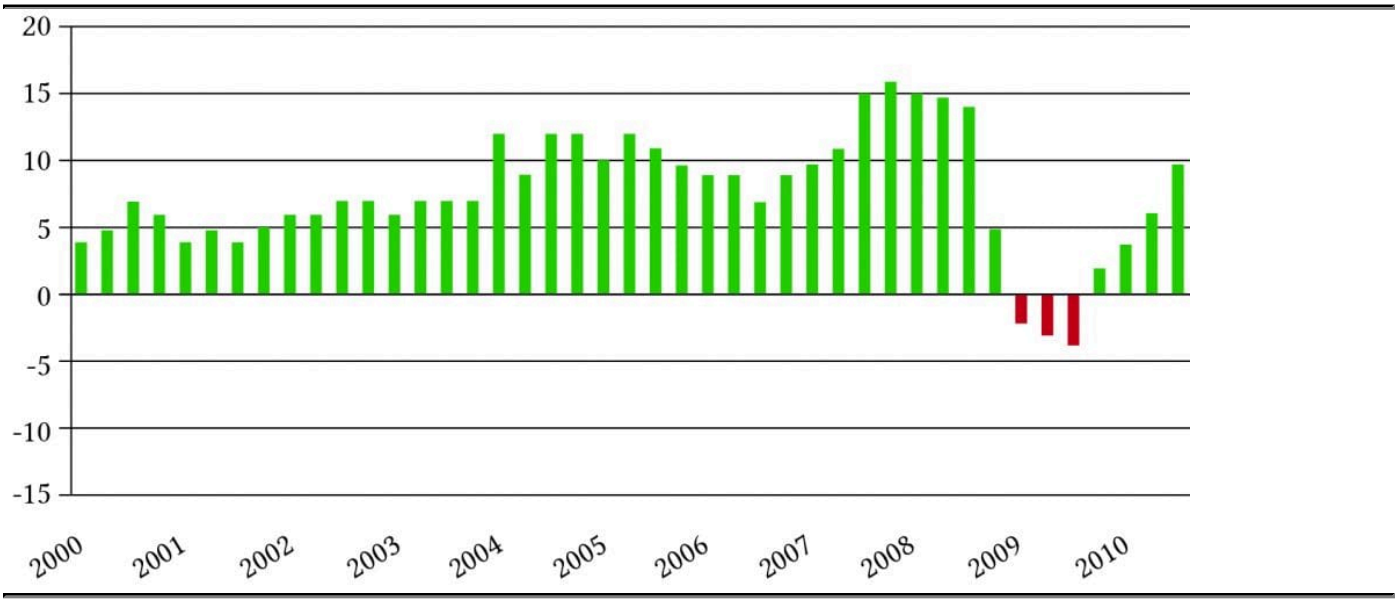
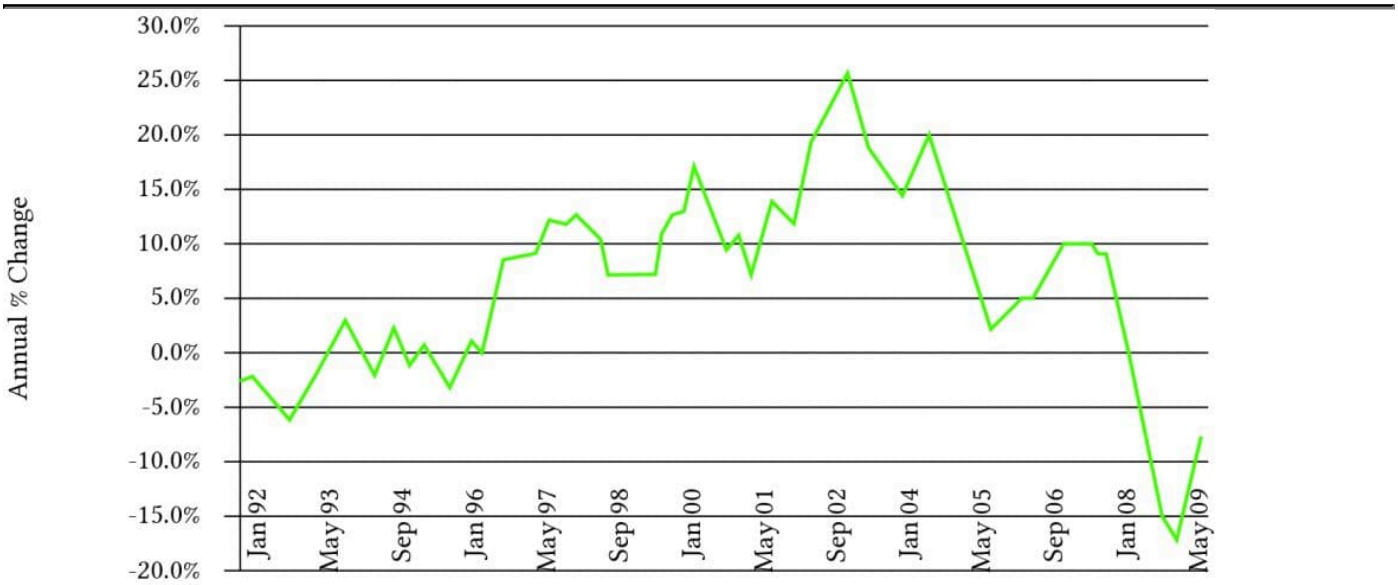


Figure 17.2. Year-to-year changes in housing prices.



Inflationary, nominal, and real interest rates. To understand price volatility of durables, it is necessary to describe inflationary, nominal, and real interest rates. Recall from your earlier training that the inflation rate i is equal to the rate of change in average prices, changes often linked to monetary or fiscal policies of governments. The nominal interest rate r depends on the rate of inflation and a real component that is dependent on factors other than the rate of inflation such as changing market conditions or

changes in productivity. To describe the effects of inflation on the nominal interest, let one plus the nominal interest rate r equal one plus the real rate r^* times one plus the inflation rate i so that:

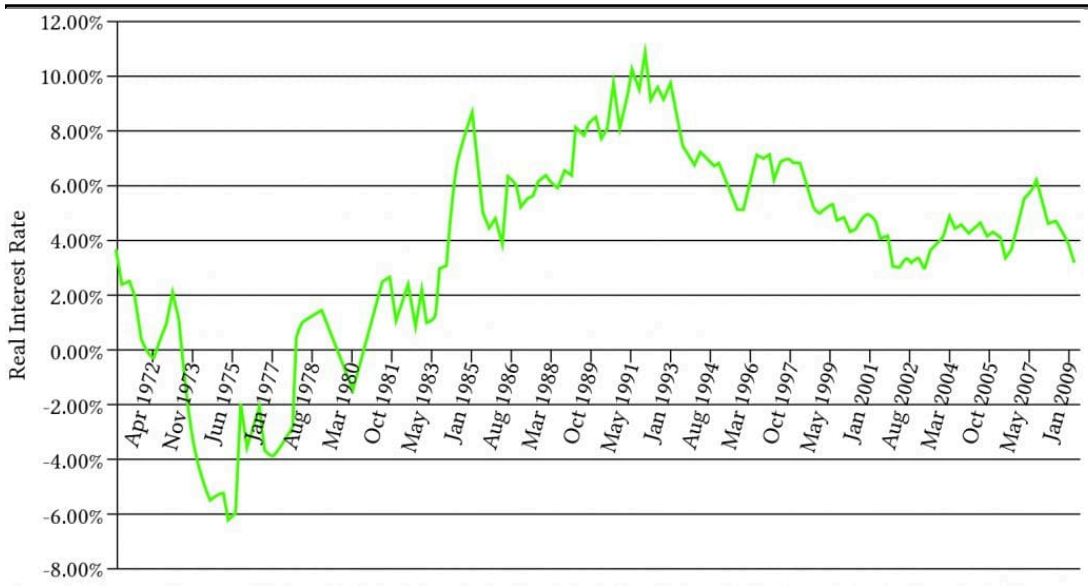
$$(1+r) = (1+r^*)(1+i) \tag{17.1}$$

We solve for the real interest rate in Equation ??? as:

$$r^* = \frac{r-i}{(1+i)} \tag{17.2}$$

Historically, real interest rates vary by sectors but are highly correlated. Real interest rates on home loans are described graphically in Figure 17.3:

Figure 17.3. Real home loan interest rates.



We can describe the effects of changes in the real growth rate on durable asset prices by letting the nominal growth rate g equal one plus the real growth rate g^* times one plus the inflation rate i, so that:

$$(1+g) = (1+g^*)(1+i) \tag{17.3}$$

We solve for the real growth rate in Equation ??? as:

$$g^* = \frac{g-i}{(1+i)} \tag{17.4}$$

To understand the volatility of farmland prices, we build a general PV model where V_0 is land's present value, g is the average nominal growth (decay) rate of net cash flow, r is the nominal interest rate, and R_0 represents initial cash flow. For the moment we ignore taxes and write the relationship between the variables just described in a growth model as:

$$V_0 = \frac{R_0(1+g)}{(1+r)} + \frac{R_0(1+g)^2}{(1+r)^2} + \frac{R_0(1+g)^3}{(1+r)^3} + \cdots = \frac{R_0(1+g)}{(r-g)} \tag{17.5}$$

We call the result the value in use model because there are no sales or purchases implied. Suppose we replace (1 + r) with the right-hand side of Equation ??? and (1 + g) with the right-hand side of Equation ???. After the substitution, we obtain the result:

$$V_0 = \frac{R_0(1+g^*)(1+i)}{(1+r^*)(1+i)} + \frac{R_0[(1+g^*)(1+i)]^2}{[(1+r^*)(1+i)]^2} + \frac{R_0[(1+g^*)(1+i)]^3}{[(1+r^*)(1+i)]^3} + \cdots \tag{17.6}$$

Notice that the inflationary effects on the discount rate and the growth (decay) rate cancel, so that we can write Equation ??? as a function of only real rates and initial cash flow:

$$V_0 = \frac{R_0(1+g^*)}{(1+r^*)} + \frac{R_0(1+g^*)^2}{(1+r^*)^2} + \frac{R_0(1+g^*)^3}{(1+r^*)^3} + \cdots = \frac{R_0(1+g^*)}{(r^*-g^*)} \tag{17.7}$$

Suppose that in Equation ???, r^* changes. To measure the impact of a change in r^*, we differentiate V_0 with respect to r^* and obtain:

$$\frac{dV_0}{dr^*} = \frac{-R_0(1+g^*)}{(r^*-g^*)^2} \tag{17.8}$$

The percentage change in V_0, with respect to a change in r^*, equals the change in the asset's value in response to a change in the real interest rate, Equation ???, divided by the asset's initial value V_0, the right-hand side of Equation ???:

(17.9) dr^*}{V_0} = \dfrac {-1}{(r^*-g^*)} \end{equation*}

Now suppose that g^* changes. To measure the impact of a change in g^* we differentiate V_0 with respect to g^* in Equation ??? and obtain:

$$\frac{dV_0}{dg^*} = \frac{R_0(1+g^*)}{(r^*-g^*)^2} \tag{17.10}$$

The percentage change in V_0 with respect to a change in g^* equals the change in the asset's value in response to a change in the real interest rate, Equation ??? divided by the asset's initial value V_0 described in Equation ???.

The percentage change in V_0 with respect to a change in g^* equals:

(17.11) dg^*}{V_0} = \dfrac {1}{(r^*-g^*)} \end{equation*}

We now describe the percentage changes in asset values in response to changes in the real discount rate and the real growth rate for alternative real interest rates and real capital gains (capital loss) rates. In the previous chapter we described changes in asset values over time. This analysis is different (and static or timeless). We consider what happens to an asset's value at a point in time if one of the underlying variables that determines its value is changed.

In Table 17.1 calculations are based on Equation ???. In the calculations we set the percentage change in asset values for alternative values of r^* and g^*. We set r^* equal to 2%, 4%, 6%, 8%, and 10%. We set g^* equal to –20%, –10%, –5%, 0%, 2% and 4%. Negative values for g^* correspond to depreciable durable assets such as buildings and machinery. Positive values of g^* are harder to justify and may reflect speculative bubbles in which asset earnings are expected to grow in real terms in perpetuity—an assumption which is hard to justify.

Table 17.1. Percentage changes in asset values corresponding to small changes in real interest rates for alternative values of r and for assets with various degrees of durability characterized by the variable g*.*

	g^* = –20%	g^* = –10%	g^* = –5%	g^* = 0%	g^* = 2%	g^* = 4%
r^* = 2%	–4.5%	–8.3%	–14.3%	–50%	n.a.	n.a.
r^* = 4%	–4.1%	–7.1%	–11.1%	–25%	–50%	n.a.
r^* = 6%	–3.8%	–6.3%	–9.1%	–16.7%	–25%	–50%

	g^* = –20%	g^* = –10%	g^* = –5%	g^* = 0%	g^* = 2%	g^* = 4%
r^* = 8%	–3.6%	–5.6%	–7.7%	–12.5%	–16.7%	–25%
r^* = 10%	–3.3%	–5%	–6.7	–10%	–12.5%	–16.7%

Consider Table 17.1. Note that an asset's durability is reflected by its real growth (decay) rate—an asset's durability decreases with its g^* value. Also note that the percentage change in asset values in response to increases in real interest rates decreases with an asset's durability. For example, if the real interest rate were 4%, a small increase in the real interest rate would cause a 4.1% decrease in the value of a depreciable asset with a negative real growth rate of –20%. Meanwhile, a small increase in the real interest rate for a durable asset with a 4% real growth rate would result in a 50% decrease in the value of an asset.

Now consider Table 17.2, which examines the effect of small increases in the real growth rate for the same scenarios described in Table 17.1 and also based on Equation ???. The difference between Table 17.1 and Table 17.2 is that the signs within the body of the tables are reversed. An increase in the real interest rate reduces asset values because future dollars are worth less with a higher discount rate. In contrast, an increase in the real growth rates means there are more future cash flows to discount—making the change in the value of the assets positive in response to a small increase in the real growth rate.

*Table 17.2. Percentage changes in asset values corresponding to 1% changes in real growth rate for alternative values of r*and for assets with various degrees of durability characterized by the variable g*.*

	g^* = –20%	g^* = –10%	g^* = –5%	g^* = 0%	g^* = 2%	g^* = 4%
r^* = 2%	4.5%	8.3%	14.3%	50%	n.a.	n.a.
r^* = 4%	4.1%	7.1%	11.1%	25%	50%	n.a.
r^* = 6%	3.8%	6.3%	9.1%	16.7%	25%	50%
r^* = 8%	3.6%	5.6%	7.7%	12.5%	16.7%	25%
r^* = 10%	3.3%	5%	6.7	10%	12.5%	16.7%

One interesting feature of Table 17.2 is that the percentage change in asset values in response to small increases in the real growth rate are dampened by higher real interest rates. For that reason, when the economy is "overheated" and prices are rising, one response is to reduce the money supply, increasing the real interest rate and slowing price increases. For example, suppose the real growth rate were zero% and the real interest rate was 2%. If the real growth rate increased slightly, the asset value would increase by 50%. If, instead, the real interest rate were 10% and the real growth rate increased slightly, the asset value would increase by only 10%.

So what have we learned? We learned that durable assets such as land and houses are subject to significant percentage changes in value in response to changes in real interest rates and real growth rates—much more so than depreciable assets. In addition, changes in the asset's real growth rate in earnings have less effect when real interest rates are high.

To test your understanding of the material just covered, demonstrate that a 1% increase in the real growth rate produces the same percentage changes in asset values as those described in Table 17.1, except opposite in sign.

Maximum Bid (Minimum Sell) Price Models and Transaction Costs

Maximum bid (minimum sell) price models. We now deduce maximum bid price models for buyers and minimum sell price models for sellers. At least two costs differentiate maximum bid price models for buyers and minimum sell price models for sellers: taxes and transactions costs. Let c equal the percentage of land's sale price paid by the buyer to purchase the land—but not paid to the seller. These may include loan closing fees, title examination and registration fees, attorney's costs, and closing points paid by the buyer. Let s equal the percentage of the land's sale price paid by the seller to sell the land—but not paid to the buyer. This may include loan closing fees, Realtor fees, advertising, and other fees.

Denote V_0^b as the buyer's maximum bid price and V_0^s as the seller's minimum sell price. Ignoring taxes, and assuming that the

land will never be sold again, the expression for V_0^b can be written as:

$$(1+c)V_0^b = \frac{R_0(1+g)}{(1+r)} + \frac{R_0(1+g)^2}{(1+r)^2} + \frac{R_0(1+g)^3}{(1+r)^3} + \cdots = \frac{R_0(1+g)}{(r-g)} \tag{17.12}$$

Expression $(1+c)V_0^b$ represents the maximum bid price, including the effect of closing costs. To isolate the maximum bid price, we solve for V_0^b.

$$V_0^b = \frac{R_0(1+g)}{(r-g)(1+c)} \tag{17.13}$$

In the maximum bid price model, the discounted stream of future cash flow represents the challenging investment—the present value of earnings the buyer will receive if the land is purchased.

Example 17.1: *Finding the Maximum Bid Price for Land.* What is the maximum bid price of a parcel of land if the buyer pays 6% of the acquisition price as a closing fee? To solve the problem, assume last year's net cash flow was $22,000, which is expected to grow in perpetuity at 4.5%. Assume that the buyer's opportunity cost of capital is 12%. The maximum bid price is calculated using Equation ???:

$$V_0^b = \frac{R_0(1+g)}{(r-g)(1+c)} = \frac{\$22,000(1.045)}{(.12-.045)(1.06)} = \$289,182 \tag{17.14}$$

If closing costs, as a percentage of the price, increase from 6% to 9%, V_0^b decreases to $281,223.

Minimum sell price models. Realtor fees and other closing costs paid by the seller are now introduced into the minimum sell price model. To build the minimum sell price model, let sV_0^s be subtracted from the seller's proceeds. Ignoring taxes and assuming that the land will only be sold one time, we can write the minimum sell price model as:

$$(1-s)V_0^s = \frac{R_0(1+g)}{(1+r)} + \frac{R_0(1+g)^2}{(1+r)^2} + \frac{R_0(1+g)^3}{(1+r)^3} + \cdots = \frac{R_0(1+g)}{(r-g)} \tag{17.15}$$

Solving for V_0^s, we find the minimum sell price equal to:

$$V_0^s = \frac{R_0(1+g)}{(r-g)(1-s)} \tag{17.16}$$

In the minimum sell price model, the discounted stream of future cash flow represents the defending investment—it represents the earnings the seller would not earn if the land were sold.

Example 17.2: *Finding the Minimum Sell Price.* Letting s = 2% and using the data from Example 17.1, we find the minimum sell price using Equation ???:

$$V_0^s = \frac{R_0(1+g)}{(r-g)(1-s)} = \frac{\$22,000(1.045)}{(.12-.045)(.98)} = \$312,789 \tag{17.17}$$

Now, suppose s increases from 2% to 3%. In turn, V_0^s increases to $316,014. In contrast to the effects of an increase in *c* that reduced the maximum bid price, an increase in s increases the minimum sell price:

Land Values, Transaction Costs, and Liquidity

Suppose a buyer and a seller had the same earnings expectations and opportunity costs for a durable such as farmland. It must follow that their present-value for the durable in use would be the same. But if the buyer had to pay closing fees to acquire the durable, his maximum bid price would be reduced. And if the seller received only a portion of the present value after the sale, her minimum sell price would increase. Therefore, transaction costs that increase the seller's minimum sell price and reduce the buyer's maximum bid price reduce the liquidity of the durable, even when the buyer and seller have the same earnings expectations and opportunity costs.

In other words, an asset is illiquid if its value in use (V_0) is bounded below by a buyer's maximum bid price (V_0^b) and bounded above by the seller's minimum sell price (V_0^s). We can easily show that assets for individuals with identical expectations of land's value in use are illiquid when transaction costs are imposed on its sale and purchase.

Substitute the value in use from Equation ??? into the maximum bid price model in Equation ???, and the result is:

$$V_0^b = \frac{R_0(1+g)}{(r-g)(1+c)} = \frac{V_0}{(1+c)} < V_0 \tag{17.18}$$

It should be clear that for $c > 0$, $V_0^b < V_0$.

Next, substitute the value in use from Equation ??? into the minimum sell price model in Equation ??? and the result is:

$$V_0^s = \frac{R_0(1+g)}{(r-g)(1-s)} = \frac{V_0}{(1-s)} > V_0 \tag{17.19}$$

V_0 \end{equation*}" title="Rendered by QuickLaTeX.com">

It should be clear that for $s > 0$, $V_0 < V_0^s$. Finally combining these results allows us to express the effects of transaction costs on the maximum bid, minimum sell, and value-in-use models:

$$V_0^b < V_0 < V_0^s \tag{17.20}$$

Despite equal earning expectations, as a result of transaction costs, the maximum bid price is less than the minimum sell price making land illiquid. To make this point another way, suppose that we formed the ratio of the maximum bid price and the minimum sell price for land and assumed that c = 5% and s = 6%. Then the buyer in this example would only be willing to offer 89.2% of the seller's minimum sell price, and a sale would not occur.

$$\frac{V_0^b}{V_0^s} = \frac{(1-s)}{(1+c)} = \frac{1-.06}{1+.05} = 89.2\% \tag{17.21}$$

Other Factors Contributing to Asset Liquidity

Differences in financing costs for buyers and sellers may also contribute to an asset's illiquidity. For example, a loan tied to the ownership of durables at below-market interest rates may create a wedge between the maximum bid price and the minimum sell price. If the loan has a due-on-sale clause, the seller gives up something of value that is not received by the buyer when the investment is sold. Thus, concessionary interest rate loans with due-on-sale clauses may create illiquidity.

Differences in information between the buyer and seller may also contribute to an asset's illiquidity. When two trusted friends exchange a durable, the information costs are insignificant—they freely exchange information. However, when two strangers exchange a durable, the buyer must commit resources to discover the quality of the information provided by the seller. If an investment's service capacity is easily observed, as with certified seed or fertilizer, then the information cost is small and the investment will be more liquid. Durables like used cars or tractors, whose remaining service capacity is not easily observed, will be less liquid. The issuance of a warranty is a signal of higher quality, and thus a warranty is an attempt to reduce information costs for the prospective buyer and to increase the liquidity of the asset.

The cost of gathering information about the asset's service capacity may also contribute to its liquidity. To illustrate, a new asset's service capacity may be well known. Once the durable is used, however, the uncertainty associated with the intensity of use and care of the durable creates an immediate decline in the value of the durable. On the other hand, a used durable whose service extraction is nearly depleted is more liquid, because there is nearly perfect information about the durable's remaining capacity.

The liquidity of durables is also affected by disposal costs. Disposal cost is the cost of removing durables from service paid by its current owner. Most durables are mobile. Buildings, however, must be disassembled at considerable costs. In some cases, the investment is so valuable that assembly and disassembly are warranted. An example is the London Bridge. Most durables with high disposal costs become completely illiquid and remain in service to the first owner until destroyed. Often it is easier to transfer the owner rather than the investment itself. Examples of durables with high disposal costs are land and houses.

Durables whose value increases with time and use are called appreciating durables. Durables whose values decrease with time and use are called depreciating durables. A durable whose value increases with time and use will be more liquid than one whose value declines with time and use, with all other liquidity attributes held constant. Appreciating durables provide greater security for lenders than depreciating durables because their value and the lender's security are increasing. Finally, liquidity is affected by whether or not time or use determines the durables' service capacity. Durables whose service capacity is determined by time have a fixed pattern of changes in service capacity and cannot adjust to changes in the market. Durables whose change in service capacity is determined by use can adjust to changes in the marketplace for the value of its services. Thus, durables whose service capacity is tied to use are more liquid than those whose service capacity is linked to time.

The liquidity of a durable is also influenced by the number of services the durable can provide. Durables become more liquid if the buyer has opportunities for the durables' services that are not available to the seller. A durable is more likely to have multiple uses when it is not fixed geographically. Multiple use capability is also tied to the acquisition characteristics of the durable. If the durable is lumpy in acquisition, potential buyers must take it or leave it. Durables divisible in acquisition, on the other hand, provide options for prospective buyers. They can buy a gallon or a tankful of gas, a sack of seeds or a truckload.

Finally, durables whose service extraction rate is fixed, and hence irreversible, are less liquid than durables whose service extraction rate is variable. Variations in service capacity provide greater adjustment potentials, and thus reduce the differences in perceptions between buyers and sellers. Moreover, durables whose service extraction rates vary tend to have multiple uses, while those with fixed service extraction rates tend to be single-use durables.

Maximum Bid (Minimum Sell) Land Price Models with Taxes

Taxes are now introduced into our maximum bid (minimum sell) price for land models. As mentioned earlier, land is subject to a wide variety of taxes, not all of which are imposed on other assets. Four kinds of taxes must be considered in our maximum bid (minimum sell) price for land models: First, the cash flow stream is taxed at the average income tax rate *T*. Second, the opportunity cost of capital *r* is expressed on an after-tax basis. Since the discount rate may reflect the opportunity cost of capital associated with investments that earn both capital gains and cash, the tax rate is θT. Third, property taxes (T_p) are another tax charged against the market price of land in each period. Thus, land's maximum bid price in future periods must be included in each period. The study of capital gains indicated that land prices should increase at the same rate as land earnings. Finally, one tax not included in the land price models is tax-savings rate T_g, resulting from durable depreciation. The reason is simple—land is not considered a depreciable investment.

The Maximum Bid Price Model with Taxes. The expanded maximum bid price model with taxes is written as:

$$(1+c)V_0^b = \frac{R_0(1+g)(1-T) - V_0^b T_p(1-T)}{[1+r(1-\theta T)]} + \frac{R_0(1+g)^2(1-T) - V_0^b(1+g)T_p(1-T)}{[1+r(1-\theta T)]^2} + \cdots = S_1 + S_2 \quad (17.22)$$

The cash flow stream contains two geometric series. In both series, the geometric factor is $[1 + r(1 - \theta T)]$ and the sums of the two series are equal to:

$$S_1 = \frac{R_0(1+g)(1-T)}{[1+r(1-\theta T)]} + \frac{R_0(1+g)^2(1-T)}{[1+r(1-\theta T)]^2} + \cdots$$
$$= \frac{R_0(1+g)(1-T)}{r(1-\theta T)-g} \quad (17.23)$$

while

$$S_2 = \frac{-V_0^b T_p(1-T)}{[1+r(1-\theta T)]} + \frac{-V_0^b(1+g)T_p(1-T)}{[1+r(1-\theta T)]^2} + \cdots$$
$$= \frac{-V_0^b T_p(1-T)}{r(1-\theta T)-g} \quad (17.24)$$

Finally, setting $(1+c)V_0^b$ equal to the sum of S_1 and S_2 and solving for V_0^b, the maximum bid price with taxes of land can be written as:

$$V_0{}^b = \frac{R_0(1+g)(1-T)}{[r(1-\theta T)-g](1+c)+T_p(1-T)} \quad (17.25)$$

It is still the case that the challenging investment in Equation ??? is represented by the discounted after-tax cash flow stream while the defender is the maximum bid price plus closing fees, which earns the discount rate *r* given up to acquire the income earning durable.

Example 17.3: *Maximum Bid Price with Taxes.* An investor is interested in buying a crop farm. According to the financial statements, last year the farm generated a net cash receipts of $48,000. Past data indicated that net cash returns have been growing at 6% per year. The investor's opportunity cost of capital is 11%, and closing fees for the transaction are 3%. The investor's income tax bracket is 30%, the property tax is 2%, and we assume that $\theta < em >< /em >= .75$ = .75" title="Rendered by QuickLaTeX.com" height="18" width="186" style="vertical-align: -5px;">. Using Equation ???, the maximum bid price for the farm is

$$V_0^b = \frac{\$48,000(1.06)(1-.3)}{\{.11[1-(.75)(.3)]-.06\}(1+.03)+.02(1-.3)} = \frac{\$35,616}{.026+.014}$$
$$= \$890,400 \quad (17.26)$$

The Value in Use model. It will be convenient later on to find the value of what the seller will sacrifice instead of selling his or her land and continuing to extract services from it. In this case there are no transaction costs, and property taxes are associated with its use value as opposed to its maximum bid price. We can derive the value in use with taxes model, V_0^u, by setting *c* = 0 in Equation ???. The result is:

$$V_0^u = \frac{R_0(1+g)(1-T)}{[r(1-\theta T)-g]+T_p(1-T)} \quad (17.27)$$

To illustrate, the value in use model for the previous example is:

$$V_0^u = \frac{\$48,000(1.06)(1-.3)}{\{.11[1-(.75)(.3)]-.06\}+.02(1-.3)} = \$913,230 \quad (17.28)$$

One way to interpret the value in use with taxes model is that it represents the defending asset for the seller of land—the present value of future earnings it would receive, provided the land isn't sold.

The Minimum Sell Price Model with Taxes. The minimum sell price model with taxes is similar to the maximum bid price model with taxes, with a few exceptions: First, assume the seller originally bought his or her land *t* periods earlier at a price of V^b_{-t}-t}^b" title="Rendered by QuickLaTeX.com" height="24" width="113" style="vertical-align: -9px;">. Then, if V^b_{-t}-t}^b" title="Rendered by QuickLaTeX.com" height="24" width="113" style="vertical-align: -9px;"> is less than (more than) the current selling price of V^s_0, the seller pays (earns) capital gains taxes (credits) at a rate of α times the average income tax rate of $\alpha T\%$ or αT.

Another difference is the transaction costs. These costs affect taxable capital gains because the capital gains tax is only paid on the net gain from the sale of land. Like the maximum bid price model, however, the property tax rate is assessed against the inflating value in use since there is no benchmark sell price to calibrate its tax value.

The value in use with taxes is the challenging investment for the seller. We equate the after-tax sale price to the value in use with taxes in Equation ???. We summarize these results in Equation ???:

$$(1-s)V^s_0 - [(1-s)V^s_0 - (1+c)V^b_{-t}\alpha T = V^u_0 = \frac{R_0(1+g)(1-T)}{[r(1-\theta T)-g]+T_p(1-T)} \tag{17.29}$$

Solving for V^s_0 the minimum sell price with taxes we find:

$$V^s_0 = \frac{V^u_0 - (1+c)V^b_{-t}\alpha T}{(1-s)(1-\alpha T)} \tag{17.30}$$

Example 17.4:*Minimum Sell Price with Taxes.* ABC Corporation wants to sell a large farm. They want to know their minimum sell price to accept—or the price a buyer would need to offer in order for them to be as well off selling as they would be continuing to farm the land. ABC has already calculated the farmland's value in use is equal to $913,230 (see equation 17.27). This is the present value of their defender.

Recognizing its minimum sell price adjusted for transaction costs and capital gains taxes (credits) as its challenger, ABC equates the two, and solves for its minimum sell price with taxes. The result is expressed in Equation ???. Summarizing the information needed to find ABC's minimum sell price: $V^u_0 = \$913,230$, $c = 3\%$, $V^b_{-t} = \$600,000$, $\alpha = .4$, $T = 30\%$, and s = 5%.

The result is:

$$V^s_0 = \frac{V^u_0 - (1+c)V^b_{-t}\alpha T}{(1-s)(1-\alpha T)} = \frac{\$913,230 - (1.03)(\$600,000)(.4)(.3)}{(.95)[1-(.4)(.3)]} = \$1,003,672 \tag{17.31}$$

Clearly, the buyer and seller in Examples 17.3 and 17.4 would never exchange land—the minimum sell price of $1,003,672 exceeds the buyer's maximum bid price of $890,400. What would have to change for land to become liquid—for the buyer to offer the seller at least his or her minimum sell price? Well, for one thing, the buyer would have to expect initial earnings greater than $48,000. Or the buyer could find means of reducing its property tax rate. Or the buyer could reduce its transaction costs. Or the buyer may have a lower opportunity cost than the seller. Other changes, such as changes in *g* and *T*, have ambiguous effects on the maximum bid price V^b_0.

Finding the Tax Adjustment Coefficient θ

The homogeneity of measures principle requires that after-tax cash flow be discounted by an after-tax discount rate—in most cases the IRR of the defender. The question is, how do we find the after-tax IRR that corresponds to the after-tax cash flow of the

defender? First, we specify the defender's cash flow stream and PV and solve for the discount rate or the IRR of the investment. We are guaranteed that we have found the IRR of the defender in a maximum bid (minimum sell) price model and value in use model because the NPV in these models is always zero. We demonstrate how to find the defender's IRR first for the simple geometric growth model and then for more complicated models that include taxes.

Now we introduce taxes into Equation ???, holding V_0 constant, and express the after-tax geometric growth model as:

$$V_0 = \frac{R_0(1+g)(1-T)}{[1+r(1-\theta T)]} + \frac{R_0(1+g)^2(1-T)}{[1+r(1-\theta T)]^2} + \frac{R_0(1+g)^3(1-T)}{[1+r(1-\theta T)]^3} + \cdots$$
$$= \frac{R_0(1+g)(1-T)}{r(1-\theta T)-g} \tag{17.32}$$

To summarize, because Equation ??? describes a defender's after-tax cash flow stream, then $r(1-\theta T)$ is the defender's after-tax IRR—because NPV in Equation ??? is zero. Another way to summarize the results above is to note that taxes in Equation ??? are neutral—meaning that they are introduced so that NPV remains zero. Since V_0 is the same in both equations (17.5) and (17.32), we can equate their right-hand sides and write:

$$V_0 = \frac{R_0(1+g)(1-T)}{r(1-\theta T)-g} = \frac{R_0(1+g)}{(r-g)} \tag{17.33}$$

Up to this point, we have not specified the value of θ. However, we do require that its value be chosen in such a way that $r(1-\theta T)$ is the after-tax IRR of the defender's after tax cash described using a geometric growth model. We find such a θ for the geometric cash flow model using Equation ???, which equals:

$$\theta = \frac{r-g}{r} \tag{17.34}$$

The above results are intuitive. For $g \neq 0$ in Equation ???, capital gains (losses) are earned (lost) without creating any tax consequences. If the before-tax IRR is *r*, an after-tax rate of *r*(1 – *T*) implies cash and capital gains (losses) are taxed at the average income tax rate T, which is not true because only cash receipts are taxed. Since capital gains (losses) are earned (lost) at rate *g*, subtracting from *r*%, *g*%, makes taxes neutral in the geometric growth model described above. To confirm that θ coefficient makes taxes neutral, the right-hand side of Equation ??? is substituted into Equation ??? to obtain the result:

$$V_0 = \frac{R_0(1+g)(1-T)}{\left\{r\left[1-\left(\frac{r-g}{r}\right)T\right]-g\right\}} = \frac{R_0(1+g)}{(r-g)} \tag{17.35}$$

Following a similar procedure, we can find the value for θ in the value in use model for land. In this case, we equate the right-hand side of the value in use land model to the right hand side of the geometric growth model. This new equality can be written as:

$$V_0^u = \frac{R_0(1+g)(1-T)}{[r(1-\theta T)-g]+T_p(1-T)} = \frac{R_0(1+g)}{r-g} \tag{17.36}$$

As before, θ in Equation ??? is the coefficient that makes taxes neutral in the land value in use model because it ensures that the effects of taxes on the defender's cash flow are equally offset by changes in the discount rate. Therefore, the discount rate continues to equal the defender's IRR. We can find such a θ for a value in use land model by solving for θ in Equation ???:

(17.37) T)(1-T)}{r} \end{equation*}

Clearly, property taxes increase the tax adjustment coefficient and the effective tax rate.

Non-neutral taxes. When we calculated the tax-adjustment coefficient, we assumed tax-neutrality. This was an appropriate

assumption for finding the after-tax IRR of the defender. In other PV models, there is no reason to assume that the after-tax IRR of the defender used to calculate the maximum bid (minimum sell) price or NPV will be tax-neutral. For example, assume the challenger's cash flow is represented by a geometric growth equation and assume that the tax adjustment coefficient of the defender is one—an assumption consistent with a defender's whose returns are only cash. In this case, we find the value in use as:

$$\begin{aligned} V_0 &= \frac{R_0(1+g)(1-T)}{[1+r(1-T)]} + \frac{R_0(1+g)^2(1-T)}{[1+r(1-T)]^2} + \cdots \\ &= \frac{R_0(1+g)(1-T)}{r(1-T)-g} \end{aligned} \tag{17.38}$$

To test for tax-neutrality, we differentiate V_0 with respect to T and find:

$$\frac{dV_0}{dT} = \frac{gR_0(1+g)}{[r(1-T)-g]^2} > 0 \tag{17.39}$$

0 \end{equation*}" title="Rendered by QuickLaTeX.com">

In this particular case, taxes are not neutral, and increasing the tax rate makes the defender less attractive relative to the challenger because the challenger's capital gains are shielded from income taxes.

Price-to-earnings Ratios for Land

An interesting ratio can be derived from the value in use with taxes model, Equation ???. It is the ratio of land's value in use, divided by the previous period's cash flow, approximated by cash rents. We write the ratio as:

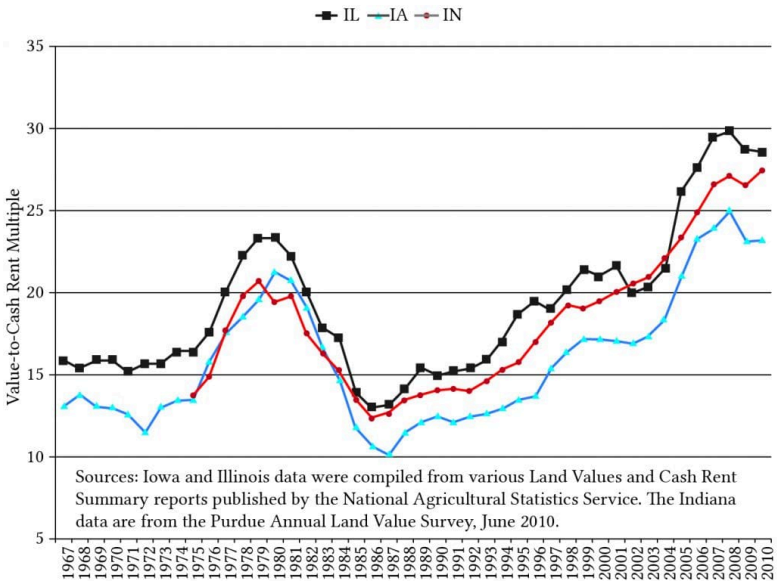
$$\frac{V_0}{R_0} = \frac{(1+g)(1-T)}{[r(1-T)-g]+T_p(1-T)} \tag{17.40}$$

Note that the value-to-earnings ratio for land depends on specific tax rates, opportunity costs, and expected growth rates, and these vary from location to location. Figure 17.4 below describes land values to rent ratios for land in Iowa, Illinois, and Indiana since 1967. Suppose that in Equation ??? we were to ignore taxes and assume a real growth rate of zero. Then Equation ??? reduces to the traditional capitalization ratio of:

$$\frac{V_0}{R_0} = \frac{1}{r^*} \tag{17.41}$$

This is how the traditional capitalization ratio is derived. As an earlier graph demonstrated, however, the ratio has not been constant over time.

Figure 17.4. Land values to rent ratios for land in Iowa, Illinois, and Indiana since 1967.



An interesting exercise would be to approximate the discount rate in Equation ??? with the real interest rate described in an earlier figure and then compare the results with the ratios described above.

Summary and Conclusions

Land's immobility and durability make it an important asset for securing loans. Its immobility means that lenders know where to find it, and its durability means that it will have the capacity to supply services and earn a return almost undiminished into the future. Both of these characteristics provide assurances to lenders that if they loan money to buy land, they will be able to recover their loans even if the borrowers fail to meet their obligations. On the other hand, land's immobility and durability also mean that its owners' can be easily identified and taxed. Land sales are carefully recorded to establish its value. Thus, land is a favorite source of tax revenue not only for its market value but also at the time of sale if capital gains (losses) are incurred.

Land has another feature. Land is infrequently traded, and transfers of ownership are costly for both buyers and sellers—often requiring the help of outside agents who must be paid for their services. As a result, both buyers and sellers pay funds they do not receive when buying and selling land. These transaction costs, including tax payments, mean that land exchange will likely not occur unless the buyer expects to earn more from land's services than currently earned by sellers. As a result, land is illiquid and its purchase and sale is a very infrequent occurrence.

In this chapter, PV models were used to model three types of prices: maximum bid prices, the most a buyer could offer to purchase the land; minimum sell prices, the least a seller would accept to sell the land; and value in use prices, the value of the land assuming continued use. These models accounted for three different types of taxes (property tax, income tax, and capital gains tax) as well as cash flow. These models provide helpful tools for assisting buyers and sellers in determining investment strategies.

Questions

1. Describe the features that make land an illiquid investment.
2. Why do lenders seem to prefer land to secure their loans? Compare the loan security provided by land compared to that offered by used equipment or breeding livestock?
3. Figures 17.1 and 17.2 compare year-to-year changes in farmland and housing prices. What is similar between those forces contributing to the variability in the prices of land and housing stock?
4. Commodities are nondurable goods and are likely used up in a single period. Meanwhile, land and houses are durables and last for many years. Can you describe how these differences between durable and nondurable goods may contribute to differences in the variability of their prices? Does the fact that the supply of land is relatively fixed versus the supply of commodities that can vary from year to year influence their variability?
5. Equation (17.9) describes the percentage change in the price of a long term asset (in this case land) in response to a change in the real interest rate r^*. Assume that the real growth rate is 3% and the real interest rate is 4%. Find the percentage

increase in V_0 if r^* increases by 1%. Find the percentage increase in V_0 if g^* decreases by 1%.

6. Consider the maximum bid price and minimum sell price models. Consider the following data: the closing fee for the buyer is 3%, the closing fee for the seller is 4%, and in the last period land rented for $150 per period. Land is liquid if the maximum bid price is greater than the minimum sell price. Using this data, please answer the following questions. Calculate the ratio of the maximum bid price to the minimum sell price. Is land liquid or illiquid? What must occur for the land to become liquid?
7. An investor is interested in buying a crop farm. According to the financial statements last year, the farm generated a net cash flow of $52,000. Past data indicates that net cash returns have been growing at 3% per year. The investor's opportunity cost of capital, the IRR on its defender, is 8%, and closing fees for the transaction are 4%. The property is taxed at 1.5%, and the investor is in the 35% tax bracket. The tax adjustment coefficient θ is assumed to equal 75%. Find the maximum bid price for the parcel of land using maximum bid price with taxes model.
8. Farmland owners Rob and Ruthie Bell want to retire and sell their farm. They purchased their farm over 30 years ago for $325,000. They recognize that their minimum sell price must leave them as well off as they would be if they continued to operate their farm. They estimate that closing fees will equal 3%, they are in the 30% tax bracket, and their capital gain tax rate αT is 12%. Finally, they realize that their closing fees will be 5%. Find the minimum sell price for Rob and Ruthie Bell. (Hint: first find the value in use with taxes for the farmland. Then using the value in use with taxes and data described in this question, calculate the minimum sell price using the minimum sell price with taxes model.)
9. In previous problems, the tax adjustment coefficient was assumed to be 75%. Using the data supplied for calculating the maximum bid price for land with taxes, find the tax adjustment coefficient using Equation ???.
10. Use Equation ??? and land value to cash rent data described in Figure 17.4 to approximate real interest rates over time for Iowa, Indiana, or Illinois. If you believe that real interest rates are constant, what other explanations can you offer for time varying land value to rent ratios?

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18: Leases

Learning Objectives

After completing this chapter, you should be able to: (1) understand how leasing can provide an alternative method for gaining control of and access to capital resources; and (2) evaluate leases using present value (PV) models developed earlier.

To achieve your learning goals, you should complete the following objectives:

- Learn how leasing arrangements offer a different method for gaining control of and access to capital resources.
- Distinguish between the different forms of leases including the sale and leaseback lease, the operating lease, and the financial or capital lease.
- Learn the differences between leases and rental agreements.
- Learn how lease agreements are taxed.
- Learn the advantages and disadvantages of entering a lease agreement versus owning a capital resource.
- Learn how to find the break-even purchase price when compared to a leasing option.

Introduction

Control of a durable can be transferred between parties in several ways. A lease agreement is one important way. A lease is a contract through which control over the right to use a durable is transferred from one party, the lessor, to another party, the lessee who acquires control of the durable. In exchange for the right to use or control a durable for a specified time period, the lessee pays the lessor a rental payment or share in the output produced by the leased durable. The lease payment or shared output must cover the opportunity cost of funds invested in the leased item, depreciation of the durable, and other incidental ownership costs incurred by the lessor. Thus, leasing is a method of financing the control of a durable that separates its use from its ownership.

There are several kinds of lease arrangements and several kinds of durables that are leased. Farmland is frequently leased as are machines, houses, cars, computers, coping equipment, buildings, breeding livestock, and many kinds of management services. In this chapter, we describe various types of leases, evaluate their advantages and disadvantages, and apply present value (PV) tools to analyze leasing benefits for both the lessor and the lessee.

Types of Lease Agreements

Sale and leaseback. Under the sale-and-leaseback agreement, a firm owning a durable to be leased sells the durable to a financial institution and simultaneously executes an agreement to lease the durable back. Thus, the lease becomes the alternative to a purchase with a specific advantage to the lessee: namely, he or she is allowed to write off the entire lease payment as an expense instead of just interest costs and depreciation. The lessor will not enter into this type of lease agreement unless the lease payments are sufficient to return the full purchase price plus a return on the investment to the lessor.

Operating leases. Operating leases, sometimes called service leases, provide for both financing and maintenance. These leases ordinarily call for the lessor to maintain the leased equipment. The cost of the maintenance is built into the lease payment. Computers and office copiers, together with cars and trucks, are the primary types of equipment involved in operating leases.

In the case of operating leases, lease payments are often insufficient to recover the full cost of the equipment. To offset this feature, the lease is written for a time period considerably less than the expected life of the leased equipment, leaving the lessor to recover his or her investment in renewal payments of the lease or through disposal of the leased equipment.

One of the main advantages of the operating lease for lessees is the cancellation of the short lease period which allows them to adopt and bring into use more advanced equipment. Thus, for durables subject to rapid changes in technology, an operating lease is often a preferred method for gaining control of a durable.

Financial or capital leases. A financial lease is a fully amortized lease whose PV of the lease payments equals the full price of the leased equipment. It does not provide for maintenance service nor may it be canceled. The financial lease begins with a

lessee selecting the specific items it requires and agreeing with the lessor about the price and the delivery of the item. The lessor arranges with a bank or another financial institution to purchase the equipment and simultaneously executes a financial lease with the lessee intending to use the equipment. The purchase of the equipment builds into the lease payments a rate of return equivalent to what would be charged on a loan, and the lease is canceled when the purchase price of the durable plus a return for the lessor is paid.

Under financial leases, lessees generally pay property taxes and insurance and, in many cases, can acquire ownership of the durable at the end of the lease. The significant difference between the sale-and-leaseback lease and the financial lease is that the lessee purchases the durable directly from the manufacturer rather than from the lessor under the terms of the financial lease.

Lease Agreements and Taxes

One of the major effects of the lease is to alter the tax obligations of the lessee and lessor. Therefore, special attention is required to make sure that lease agreements are acceptable under current tax codes as interpreted in the United States by the Internal Revenue Service. There is an important distinction between a lease agreement and a loan for tax purposes. If the lease agreement cannot be distinguished from an ordinary loan agreement, then any special tax provisions associated with the lease agreement are lost.

To distinguish a lease from a sale agreement, the term over which the investment durable is leased must be less than 75 percent of the economic life of the durable. Nor should the lessee be granted any special repurchase option not available to others not involved in the lease. There are other conditions as well; tax codes are evolving documents that are constantly being updated.

Lease Liquidity and Risk

Normally, the decision to acquire the durable is not at issue in the typical lease analysis. The issue at hand is whether to acquire control of the durable through a lease or by purchasing it—often by borrowing funds to finance the purchase. The decision requires a careful examination of the advantages of leasing. Leasing offers several possible advantages relative to owning a durable that we describe next.

Release of cash and credit. When a firm leases or sells a durable on a leaseback arrangement, the lessee avoids the cash down payment required to purchase the investment. More generally, the liquidity of a lease versus the liquidity of a purchase depends on which option requires the most accelerated payments.

Some texts discuss in detail the effect on the firm's credit reserve as a result of leasing. If leasing uses up credit at a slower rate than borrowing, there may be credit incentives for leasing rather than borrowing. However, lenders are likely to recognize that long-term lease agreements place the same requirements on the firm's future cash flow as loans do.

Obsolescence risk. Investments may experience significant obsolescence risks. In addition, there is some risk that the need for durables' services will change before the durables' service capacity is exhausted. Part of this obsolescence and use risk may be reduced through a lease arrangement for a short time period, especially if the lessor is less subject to obsolescence risk than the lessee. This is likely the case where the leased equipment has alternative uses in other firms or industries and where the risk can be spread over many lessees.

Idle capacity risk. Another risk that can often be reduced through lease arrangement is the risk of holding idle equipment. If the demand for services from a durable is not sufficient to employ the durable full time, the lessee can reduce idle capacity risk by leasing rather than owning the durable. From the lessor's point of view, the durable can be completely employed because many lessees will use the equipment.

Foreclosure risk. In many respects, leasing is similar to borrowing because it represents an obligation of the firm to a series of future cash payments. There is one significant difference between agreeing to lease and borrowing to purchase the durable. In the case of financial difficulties, the lessor simply takes back the equipment because he or she holds the legal title. In the case of a loan, inability to meet loan payments may result in more complex foreclosure proceedings.

Tax advantages. A tax advantage may be gained when the term of the lease is shorter than the allowable tax depreciation period for ownership. However, tax incentives for the lessee must be a result of a lower total tax burden for the lessee and lessor. This situation implies that the lease arrangement has legitimately allowed for a reduction in the total amount of taxes paid. Needless to say, the Internal Revenue Service imposes conditions on what does and does not constitute a legal lease.

Net Present Cost of Leasing versus Purchasing

The decision to lease or to buy a durable depends on the net present cost (NPC) of leasing versus NPC of purchasing the durable. Since the size and timing of the cash flow, influence of taxes, and risks associated with buying and leasing are different, they represent two different investment opportunities which are amenable to analysis using PV models. In the following discussion, the after-tax cash flow is considered risk-adjusted, and the problem is treated as riskless.

To compare the lease with the purchase option, the firm considers itself in both roles—that of a lessee and lessor—and finds the maximum lease payment it could pay and be indifferent between purchasing or leasing the durable. We begin by describing the NPC of the lease (NPC*L*). Because we are describing the NPC of the lease, cash costs are treated as positive flow, and income that reduces the cost of leasing is treated as negative cash flow. For simplicity, we will assume a maximum constant lease payment equal to C. Then we assume the lease payment at age t is C*t*, the lessee's constant marginal tax rate is T, the lessee's before-tax opportunity cost of capital is r, and the lessee's tax adjustment coefficient is θ as described in Chapter 11. With these definitions in place, the net present cost of a lease for n periods, NPC*L*, can be expressed as:

$$-NPC^L = C(1-T) + \frac{C(1-T)}{[1-r(1-\theta T)]} + \cdots + \frac{C(1-T)}{[1-r(1-\theta T)]^{n-1}} \tag{18.1}$$

Notice in Equation 18.1 that the first lease payment is made at the beginning of the lease payment rather than at the end of the lease period, which would be the case of a loan. Therefore, leases can be viewed as an annuity due type of financial arrangement. Multiplying both sides of Equation 18.1 by $[1 + r(1 - \theta T)]$ and summing the resulting geometric series, we write the result as:

$$NPC^L = C(1-T)\left\{1 + US_0[r(1-\theta T), n-1]\right\} \tag{18.2}$$

where:

$$US_0[r(1-\theta T), n-1] = \frac{C(1-T)}{[1-r(1-\theta T)]} + \cdots + \frac{C(1-T)}{[1-r(1-\theta T)]^{n-1}} \tag{18.3}$$

We now consider the net present cost of a cash purchase, NPC^P, for the same financial manager who previously considered the lease option. We begin by assuming that the market value of the durable leased or purchased is equal to V_0. We also assume that V_t^B is the book value of the depreciable durable at age t. We continue to assume that the purchaser's opportunity cost of capital is *r*, the average tax rate is *T*, and γ_t is the percentage of the investment's value allowed to be deducted in year *t* where *t* = 1, ..., *n*, and the tax adjustment coefficient is θ. Of course, the sum of the amount deducted cannot exceed the value of the investment so that $\gamma_1 + \cdots + \gamma_n \leq 1$. All these assumptions allow us to write the NPC^P as:

$$NPC^P = V_0 - \frac{T\gamma_1 V_0}{[1+r(1-\theta T)]} - \cdots \tag{18.4}$$

$$-\frac{T\gamma_n V_0}{[1+r(1-\theta T)]^n} - \frac{V_n - T\gamma_n V_n}{[1+r(1-\theta T)]^n} \tag{18.5}$$

The purchaser's cost on the right-hand side of Equation 18.5 is the purchase price V_0, less *n* periods of tax savings from depreciation equal to $T\gamma_t V_0$ in the period discounted to the present period. The liquidation value V_n minus a tax adjustment term $T(V_n - V_n{}^B)$ reduces the purchaser's cost. The adjustment term accounts for the tax consequences of a difference between the liquidation value and the book value of the investment. If the investment is completely depreciated by the nth period, and the liquidation value is zero, then the NPC^P can be written as:

$$NPC^P = V_0 - \frac{T\gamma_1 V_0}{[1+r(1-\theta T)]} - \cdots - \frac{T\gamma_n V_0}{[1+r(1-\theta T)]^n} \tag{18.6}$$

Finally, if the NPC^L, expressed as the right-hand side of Equation 18.2, is equated to NPC^P, expressed as the left-hand side of Equation 18.6, we can solve for the lease payment C, a maximum bid lease, which equates the two alternatives for controlling the services from the durable.

$$C = \frac{NPC^p}{(1-T)\{1+US_0[r(1-\theta T), n-1]\}} \tag{18.7}$$

Consider an example. Go Green is a lawn service that requires a new truck to service its customers. It wants to know the largest lease payment it could afford and still be as well off as it would be if it purchased the truck. The truck in question has a new sticker price of $30,000 and can be depreciated using a straight-line method over five years. At the end of the lease, the truck has a liquidation value of $5,000. To find the maximum bid payment, the following assumptions are used: T = 32%, $\gamma t = 10\%$ for t = 1, ..., 5, r = 14%, θ = 1, n = 5, V_n = $5,000, and $V_n^B = 0$. Finally, the after-tax discount rate is $r(1 - T)$ = .14(1 – .32) = .095

To find the maximum bid payment, we first find NPC^P by substituting numerical values into Equation 18.7:

$$NPC^P = \$30,000 - (.32)(\$6,000)US_0(0.095, 5) - \frac{\$5,000(1-.32)}{(1.0952)^5} \tag{18.8}$$

We record the solution to this problem using Excel in Table 18.1 below.

Table 18.1. ***Finding the NPC of the Purchase.*** *Open Table 18.1 in Microsoft Excel*

C10	***Function:***	=NPV(C3,C4:C9)	
	A	B	**C**	D
1	Finding the NPC of the Purchase			
2	Purchase Price		$30,000.00	
3	rate		.095	
4	Depreciation tax savings period 1		($1,920)	
5	Depreciation tax savings period 2		($1,920)	
6	Depreciation tax savings period 3		($1,920)	
7	Depreciation tax savings period 4		($1,920)	
8	Depreciation tax savings period 5		($1,920)	
9	After-tax salvage value period 6		($3,100)	
10	NPV of depreciation tax savings + after-tax salvage		($9,170.60)	“=NPV(rate, value1:value6)
11	NPC: purchase price – (PV of tax-savings + salvage value)		$20.829.40	“=NPV of depreciation tax savings + after-tax salvage plus the purchase price (C10+C2)

In cell C10 we enter the function: = NPV(rate,values) = NPV(C3,C4:C9) which returns the value ($9,170.60). In cell C11 we add the purchase price of $30,000 to the NPC of after-tax depreciation savings plus the liquidation value which returns the NPC of the purchase equal to $20,829.36.

Having found NPC^P equal to $20,829.40, we can find C using Equation 18.7.

We solve this problem using Excel in Table 18.2 by finding the present value of Annuity Due with 5 payments of –1. We enter the arguments of the PV function and label the terms in adjacent cells. Excel’s PV function has the following arguments listed in column A: rate (rate), number of payments (nper), annuity (pmt), future value (FV), annuity type (type). For this problem, the discount rate is 9.5% = .095, the number of payments equals 5, the payments are –1, the future value FV is zero, and the type of annuity is an annuity due which requires we enter 1.

Table 18.2. Finding the PV of the Annuity Due
Open Table 18.2 in Microsoft Excel

B6		***Function*:**	=PV(B1,B2,B3,B4,B5)
	A	**B**	C
1	rate	.095	
2	nper	5	
3	pmt	-1	
4	FV	0	
5	type	1	
6	PV	$4.20	"=PV(rate,nper,pmt,FV,type)

In cell B6 we type in the PV function: = PV(B1,B2,B3,B4,B5). The result returned is PV = $4.20. Then we multiply by (1 – *T*) as required in Equation 18.7. Finally we find C:

$$C = \frac{NPC^P}{(1-T)\left\{1+US_0[r(1-\theta T), n-1]\right\}} = \frac{\$20,829.40}{.68(4.2)} = \$7,283.01 \tag{18.9}$$

Now suppose that we wanted to know the number of periods required to reach a future value of $2,000. With the data already entered, we turn to the "goal seek" option in Excel. To access goal seek from the main menu, press the [Data] tab, then on the ribbon menu press the [What-if Analysis] button, and on the drop down menu press [Goal Seek...]. The goal seek menu allow us to identify a target value for a particular cell and a variable to change. In our case the target cell is B6 and the target amount is $2,000. The cell we allow to vary is the number of periods cell, B2. Then entering these into Excel's solver, the solution returned is 18.57. Thus, by continuing to invest for 8.57 periods, we obtain a future value of $2,000 compared to the future value of $1,005.18 reached after 10 years of investing.

Alternatively, we could have asked what is the payment amount required to reach a future value of $2,000 during the original term. In this case we vary the annuity in cell is B3, and the solver returns the value of $182.65. Thus, by increasing our annuity due from $90 to $182.65, we reach a future value amount of $2,000 in the same term required to reach a future value of $1,005.18 with $90 annuity due payments.

What we have found in this example is that we would be indifferent between purchasing the truck for $30,000 and making five lease payments of $7,283.01. Obviously, the high discount rate of 14% and making the payment for the truck up front contributes to the high lease payment. In the questions at the end of this chapter, students will be asked to repeat the problem assuming the same values except that the opportunity cost of capital is 5%.

Comparative Advantages for Leasee and Leasor

So far, we have described the technical properties of leases and how a financial manager may decide between gaining control of a durable through a lease or purchase arrangement. In this section, we allow the lessee and the lessor to be two different persons whose comparative advantages lead them into an exchange arrangement in which one party purchases an asset and then leases it to another party. The solution in this case considers the NPC from the lessee's (lessor's) position, assuming that the lessor (lessee) pays their maximum bid purchase price (lease payment). What makes this an interesting problem is that differences in the lessee's and lessor's tax rates, as well as opportunity costs of capital, can create incentives to exchange.

Comparative advantage. A fundamental principle of economics is that firms should specialize in tasks for which nature, institutions, or luck has granted them an advantage. To illustrate, suppose two farmers, A and B, can both produce dry edible beans and carrots. Also assume A can grow dry edible beans and carrots more profitably than B, and B can grow carrots more profitably than it can grow dry beans. If both beans and carrots must be produced, then A should produce beans, and B should grow carrots. They could then trade to obtain what they did not produce, and both would be better off.

Even though A produced both beans and carrots more profitably than B, it would still be to A's and B's advantage to specialize. They should decide in which product A had the greatest advantage in production, or B had the least comparative disadvantage, and then specialize and trade as before.

An application of the law of comparative advantage explains why two firms may lease. Suppose firm A is able to purchase a durable at a lower price than firm B. But firm B, not firm A, has need of the services of the durable. In this case, firm A's advantage

is in purchasing while B's advantage is in using the durable. Thus, A might purchase the durable and either sell or lease to B.

Another reason why A and B might both agree to lease has to do with relative tax rates. Suppose A, the purchaser of a durable, pays taxes at an average rate T^P, which is higher than T^L, the average rate at which B, the lessee, pays taxes. As the durable is depreciated, the depreciation creates a tax shield of greater value to the purchaser than to the lessee. Thus, we might say that A, the purchaser, has a comparative tax advantage over B, the lessee, in claiming tax depreciation. The lease allows the purchaser, A, and the lessee, B, to benefit from their comparative advantages associated with the tax shields created by depreciation.

Still further reasons why A and B might purchase and lease a durable are that they face different opportunity costs of capital, have different opportunities for using the durable's services, or face different marginal costs of credit.

Comparative advantages for lessee and lessor created by different tax rates and opportunity costs of capital. To demonstrate the idea of comparative advantage with a lease agreement, consider that person B, the lessee, will not lease for more than it would cost to control the durable through a purchase agreement. We want to find B's NPC if the durable were purchased. To simplify, assume the purchase price of the durable is V_0, the book value depreciation rate in year t is $\gamma t = \gamma(1-\gamma)t$ where t = 0, ..., n – 1, r^B is B's opportunity cost of capital, T^B is B's average tax rate, and assume the lessee tax adjustment coefficient is 1. The NPC^P of the purchase for B is:

$$NPC^L(\text{purchase}) = V_0 - \frac{T^B \gamma V_0}{[1 + r^B(1 - T^B)]} - \frac{T^B \gamma V_0 (1-\gamma)}{[1 + r^B(1 - T^B)]^2} - \frac{T^B \gamma V_0 (1-\gamma)^2}{[1 + r^B(1 - T^B)]^3} - \cdots \tag{18.10}$$

Equation 18.10 is equivalent to Equation 18.5 except that it is expressed in a form that makes it analytically tractable. Obviously, more detailed expressions with finite time horizons could easily be calculated. What makes Equation 18.10 tractable is that it is expressed as an infinite series whose solution can be written as:

$$NPC^L(\text{purchase}) = V_0 - \frac{T^B \gamma V_0}{r^B(1 - T^B) + \gamma} \tag{18.11}$$

Now consider what happens to B's NPC resulting from the purchase if B's tax rate increases. We find the change in B's NPC associated with the purchase of the durable by differentiating NPC^P with respect to T^B. (This operation in essence examines the change in NPC^P with respect to a small change in T^A. The results is that NPC^P decreases with an increase in T^B. Thus, a firm in a higher tax bracket can buy a durable at a lower after-tax cost than someone in a lower tax bracket.

Next, we find B's NPC^L associated with leasing the durable which, after summing geometrically, can be expressed as:

$$NPC^L(\text{lease}) = \frac{C(1 - T^B)}{[1 + r^L(1 - T^B)]} + \frac{C(1 - T^B)}{[1 + r^L(1 - T^B)]^2} + \frac{C(1 - T^B)}{[1 + r^L(1 - T^B)]^3} + \cdots \tag{18.12}$$

$$= \frac{C[1 + r^L(1 - T^B)]}{r^L} \tag{18.13}$$

To illustrate Equation 18.11 consider the following example: Suppose Affordable Assets (AA) buys durables and leases them to other firms. As a well-to-do established firm, AA's tax rate, T^A is high—45 percent. On the other hand, the **Unendowed User (UU)** lacks capital and prefers to lease rather than own in order to protect its limited credit. Because of its lower earnings, UU's average tax rate T^U is 21 percent. In addition, assume that the leased durable wears out at the rate of γ = 10 percent, and assume the market rate of return for both AA and UU is 14 percent. Finally, assume that the purchase price of the durable is $4,000.

Making the appropriate substitutions into Equation 18.11, we find the NPC of purchasing the durable for UU is:

$$NPC^{UU} = \$4,000 - \frac{[(.21)(.1)(\$4,000)]}{[(0.14)(1 - 0.21) + 0.1]} = \$3,600 \tag{18.14}$$

In other words, the after-tax purchase price of the durable for UU is $3,600.

Now consider the after-tax NPC of the purchase for firm AA. Making the appropriate substitutions into Equation 18.11, we find the NPC of purchasing the durable for AA is:

$$NPC^{AA} = \$4,000 - \frac{[(0.45)(0.1)(\$4,000)]}{[(0.14)(1-0.45)+0.1]} = \$3,000 \tag{18.15}$$

In other words, the after-tax cost of the durable purchase for AA was only $3,000 compared to $3,600 for UU.

Having calculated NPC for purchasing the durable for firms UU and AA, we can find their respective maximum bid lease prices. Making the appropriate substitutions into Equation 18.13, we find

$$NPC^{UU}(leasing) = \frac{C^{UU}[1+(0.14)(1–0.21)]}{0.14} = \$3,600. \tag{18.16}$$

Solving for C^{UU} we find $C^{UU} = \$453.81$.

Similarly we find

$$NP^{AA}(leasing) = \frac{C^{AA}[1+(0.14)(1–0.45)]}{0.14} = \$3,000. \tag{18.17}$$

Solving for C^{AA} we find $C^{AA} = \$389.97$.

Reflections on comparative advantages for firms UU and AA. Because of their differences in tax rates, firms UU and AA have different effective after-tax cost of the durable even though the market price of the durable is the same for each firm. In this case, the effective cost for AA is less than for firm UU—$3,000 versus $3,600. On the other hand and as a result of the differences in the effective after tax cost of the durable, the maximum bid lease price for firm UU is greater than for firm AA: $453.81 versus $389.97.

These results provide opportunities for firms AA and UU to take advantage of their comparative advantages: firm AA purchases the durable at a lower effective cost than is available to UU. UU, on the other hand, is quite willing to pay a higher lease price than AA's maximum bid lease price. Under these arrangements, both firms AA and UU are made better off.

Leases and NPV. Assume that firm AA buys the durable and UU leases the durable from AA. Now the lease price in an expense for firm UU but an income for AA. We may want to ask: what is the minimum lease payment AA could accept from UU and still break even? To find this amount we set AA's NPV as:

$$NPV^{AA} = \frac{C\left[1+r\left(1-T^{AA}\right)\right]}{r - NPC^{AA}(\text{purchase})} \tag{18.18}$$

After making the appropriate substitutions we find

$$C = \frac{NPC^{AA}(\text{purchase})\left(r^{AA}\right)}{\left[1+r^{AA}\left(1-T^{AA}\right)\right]} = \frac{\$3,000(.14)}{[1+(.14)(1-.45)]} = \$389.97 \tag{18.19}$$

Thus, any arrangement with UU which generates a lease payment more than $389.97 will result in a positive NPV for AA.

Summary and Conclusions

Trades occur when each party to the exchange gives up something of value in return for something of greater value. In most trades, a physical object or service is exchanged for an agreed-on amount of cash. Moreover, in most cases, ownership is transferred along with the good or service.

This chapter has considered leasing, a different kind of trade, in which the control over the use of a durable is transferred, but ownership of the durable is not. Leasing exists because it provides benefits to the lessee and lessor that might not be realized if control of a durable also required ownership.

Different types of leases described in this chapter include sale-and-leaseback agreements, operating leases, and financial or capital leases. Leases offer particular advantages for lessees including the avoidance of capital requirements associated with a purchase. As a result, lessees may use their limited credit for other purchases or as a credit reserve. Obsolescence risk of ownership is also reduced for lessees. Additionally, leasing offers the chance to better match service requirements to the delivery of services. Ownership of durables may require holding idle capacity—less likely when services of a durable are leased.

Tax considerations are critical in the decision to lease or to purchase. Ownership allows for tax depreciation shields; leasing allows the entire lease payment to be claimed as an expense.

An important principle of comparative advantage is involved in the lease decision. If the tax depreciation from ownership is greater for one firm than for another, leasing permits the firm which can benefit the most from the depreciation shield to claim it by purchasing the asset and leasing it to another firm. Not only can comparative tax advantages be optimally used through leasing, there can also be comparative advantages in acquiring financing. Comparative advantage and the incentive to lease may also result from differences in opportunity costs, access to credit, and use for the durables' services. In essence, leasing is a critical tool that allows firms to benefit from their comparative advantages.

Questions

1. Describe three different lease types and their essential differences.
2. For every lease agreement, there is a lessor and a lessee who both believe they will be better off by executing the lease. Describe how a sale and leaseback agreement might make both the lessor and lessee better off.
3. Assume a lease agreement that requires $100 payments at the beginning of each period for 15 years. The lessee's marginal tax bracket is 25% and opportunity cost of capital is 8%. The tax adjustment coefficient is assumed to equal 80%. Find the NPC of this lease. Now assume that the lessor agreed to accept payments at the end of each period instead of the beginning. What would be the NPC of the lease agreement under the new arrangement?
4. Resolve the Go Green lease problem assuming that the firm's opportunity cost of capital is 5% rather than 14%. Compare your solution with the previous one. Did the lease payment go up or down? Can you explain why?
5. Resolve the Go Green lease problem by assuming that the lessee will lease the durable for an extra period. The lessee will make 6 lease payments instead of 5, but the depreciation schedule will not change. In other words, find the maximum lease payment and explain your results.
6. Explain in your own words the concept of comparative advantage. Provide one example of how the concept of comparative advantage has influenced your choice or the choice of others.
7. Explain how the principle of comparative advantage might explain why two firms may agree to a lease arrangement.
8. In the example of a lessee UU and a lessor AA, the lessee and lessor are in different average tax brackets (45 versus 21 percent) but were assumed to face the same opportunity cost of capital. Resolve the example by assuming UU's opportunity cost of capital is 7% instead of 14%. Find the effective after-tax purchase price of the durable for the two firms. Then find the maximum bid lease price for the two firms. Do the results allow for the two firms to take advantage of the principle of comparative advantage? Please explain.
9. Consider the case of firms UU and AA described in the text. Since AA has the lowest NPC for purchasing the durable, it should do so. Meanwhile the maximum bid lease price for UU is higher than for AA, so UU should lease rather than purchase the durable. Suppose that UU agrees to pay AA a lease price of $410.00. What is AA's NPV from buying the durable and leasing it back to UU? (Hint: In this arrangement, the lease cost for UU is an income for AA.)

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19: Financial Investments

Learning Objectives

After completing this chapter, you should be able to: (1) distinguish between financial investments and capital investments; (2) recognize the different kinds of financial investments; and (3) evaluate the different kinds of financial investments using present value (PV) models developed in earlier chapters.

To achieve your learning goals, you should complete the following objectives:

- Distinguish between financial investments and nonfinancial investments.
- Learn how PV tools can be used to analyze financial investments using methods similar to those used when comparing nonfinancial investments.
- Learn to distinguish between financial and nonfinancial objectives.
- Learn the role of brokers, dealers, and financial intermediaries in the securities markets.
- Learn how to value riskless securities such as time deposits.
- Learn why Albert Einstein called compounding interest “the greatest mathematical discovery of all time”.
- Learn the nature of bonds and the variables that determine their value.
- Learn how tax rates affect the value of financial investments.

Introduction

In its broadest meaning, to invest means to give up something in the present for something of greater value in the future. Investments, what we invest in, can differ greatly. They may range from lottery tickets and burial plots to municipal bonds and corporate stocks. It is helpful to organize these investments into two general categories: real and financial. Capital, or real, investments involve the exchange of money for nonfinancial investments that produce services such as storage services from buildings, pulling services from tractors, and growing services from land. Financial investments involve the exchange of money in the present for a future money payment. Financial investments include time deposits, bonds, and stocks.

This chapter applies PV models developed earlier to financial investments. Large corporations and other business organizations require financial investments from a large number of small investors to provide funds for operation and growth. The collection of these funds would be impossible if each investor were required to exercise a managerial role over them. Moreover, the collection of investment funds from a large number of small investors would be impossible unless the investors could be assured of investment safety and limited liability. A number of financial investments have been designed to overcome these and other investment obstacles.

The market in which financial investments are traded is called the securities market or financial market. Activities in the financial markets are facilitated by brokers, dealers, and financial intermediaries. A broker acts as an agent for investors in the securities markets. The securities broker brings two parties together to obtain the best possible terms for his or her client and is compensated by a commission. A dealer, in contrast to a broker, buys and sells securities for his or her own account. Thus, a dealer also becomes an investor. Similarly, financial intermediaries (e.g. banks, investment firms, and insurance companies) play important roles in the flow of funds from savers to ultimate investors. The intermediary acquires ownership of funds loaned or invested by savers, modifies the risk and liquidity of these funds, and then either loans the funds to individual borrowers or invests in various types of financial assets.

Numerous books discuss the institutional arrangements associated with securities trading. In this chapter, the focus in not on the details of trading in the securities market but on how to value financial investments or securities.

Valuation of Riskless Securities

The prices of financial investments, like other prices, are determined in markets—financial markets. The characteristics of financial markets, however, may differ significantly depending on the type of financial investment traded. Common to all financial markets, though, is that prices are established by matching the desires of buyers and sellers. Moreover, in equilibrium there is

neither excess demand nor excess supply. However, equilibrium does not mean that all prospective buyers and sellers agree that an investment's price is equal to its value. It only means that a price has been found that, in a sense, balances the different goals of buyers and sellers.

Time deposits. Time deposits are money deposited at a banking institution for a specified period of time. The main difference between different kinds of time deposits is their liquidity. Generally speaking, the less liquid the time deposits are, the greater their yield is.

The most liquid form of time deposits are sometimes known as "on call" deposits. On call deposits can be withdrawn at any time and without any notice or penalty. Examples of on call deposits include money deposited in a savings or checking account in a bank. Another kind of time deposit is a certificate of deposit (CDs). When CDs mature, they can be withdrawn or they can be held for another term. CDs are generally considered to be liquid because they are negotiable and can be re-discounted when the holder needs some liquidity.

Some time deposits must be held until maturity or suffer penalties for early withdrawal. The rate of return for these time deposits is higher than on call deposits because the requirement that the deposits be held for a designated term gives the bank the ability to invest in financial products that earn greater returns. In addition, the return on time deposits is generally lower than the average return on long-term investments in riskier products like stocks and some bonds. Some banks offer market-linked time deposit accounts which offer potentially higher returns while guaranteeing principal.

Investments in time deposits may consist of a one-time investment or a series of deposits. What all time deposits have in common is that the payment(s) is compounded until some ending date at which time they are partially or totally withdrawn. Thus, our interest is not in the present value of the payments but in their future value. The market determined price in the case of financial investments is the compound rate at which time deposits earn interest. Typically, the compound rate of interest earned on time deposits is low—partly because they provide nearly perfect liquidity—especially for time deposits for which there is no penalty nor transaction cost associated with withdrawal of funds.

Ordinary annuities and annuities due. Previously we described an annuity as a time series of constant payments made or received at the end of each period. Annuities paid and received at the end of a period are referred to as ordinary annuities. Annuities paid (or received) at the beginning of each period are referred to as annuity due payments.

To illustrate an annuity due, consider a series of deposits R made every period, the first one occurring in the present. We write the future value V*n* of the time series of deposits R compounded at rate $\hat{r}$ for withdrawal in period n as:

$$R(1+\hat{r})^n + R(1+\hat{r})^{n-1} + \cdots + R(1+\hat{r})^2 + R(1+\hat{r}) = V_n \tag{19.1}$$

Note that the first time deposit R is compounded for n periods—and is equal in value to its compounded value at the beginning of period n. Similarly, the second payment is compounded for n – 1 period and is equal to its compounded value at the beginning of period n. Finally, the last payment is made at the beginning of period n – 1 and is compounded once to find its compounded value at the beginning of period n.

To illustrate, suppose a saver deposits a $90 payment (PMT) at the beginning of each period for n = 10 periods. If the deposits are compounded at a rate of r = 2%, what amount is available for withdrawal at the beginning of period 10? We solve this problem using Excel in Table 19.1. Note that to indicate annuity due payments at the beginning of the period, "type" in the Excel future value FV function is changed from its default of zero to 1. In place of PV in the equation, we enter a blank closed with a comma.

Table 19.1. The Future Value of 10 Annuity Payments of $90 Compounded at 2%

Open Table 19.1 in Microsoft Excel

B6		***Function*:**	=FV(B4,B5,B3,,B2)
	A	**B**	C
1	The Future Value of 10 Annuity Due Payments of $90 Compounded at 2%		
2	type	1	
3	PMT	($90.00)	

4	rate	0.02	
5	n	10	
6	FV	**$1,005.18**	“=FV(rate,nper,pmt,[pv],type)
7	PV	0	

What our calculations determine is that at the beginning of the 10th period, we have $1,005.18 available. However, there are several other interesting questions that we might answer using our Excel formulas. For example, we might want to know how many periods would be required to have $2,000 available at some future date. Continuing with the entries already made in Excel, we add the following:

Table 19.2. Finding the Number of Annuity Due Payments Required to Earn a Future Value of $2,000
Open Table 19.2 in Microsoft Excel

B5		***Function*:**	=NPER(B4,B3,B6,B2)
	A	**B**	C
1	Number of Annual $90 Annuity Due Payments Compounded at 2% Required to Earn a FV of $2,000		
2	type	1	
3	PMT	($90.00)	
4	rate	0.02	
5	n	29.69347722	“=NPER(rate,pmt,FV,type)
6	FV	$2,000	
7	PV	0	

We might be interested in knowing the periodic investment required to have $2,000 available in 10 years if the investments were compounded at 2%. The required Excel formula is represented in Table 19.3.

Table 19.3. Finding the Annuity Due Payment Required to Earn a Future Value of $2,000 in 10 Years
Open Table 19.3 in Microsoft Excel

B3		***Function*:**	=PMT(B4,B5,B7,B6,B2)
	A	**B**	C
1	Annual Annuity Due Payment Compounded at 2% Required to Earn a FV of $2,000 in 10 years.		
2	type	1	
3	PMT	($179.07)	“=PMT(rate,nper,PV,FV,type)
4	rate	0.02	
5	n	10	
6	FV	$2,000	
7	PV	0	

To reach one’s savings goal of $2,000 in 10 years at a compound rate of 2% would require a periodic payment into a time deposit at the beginning of each period equal to $179.07.

Compound Interest

The magic of compounding. It is claimed that Albert Einstein called compound interest "the greatest mathematical discovery of all time". It is probable that no other concept in finance has more importance for investors than what is sometimes called the "wonder of compound interest." Compounding interest creates earnings not only on the original amount saved or invested but also creates earning on the interest on earnings, and it does so period after period. Another way to describe this process is to say that compounding interest generates earnings on the investment and reinvested earnings from the investment. To realize the power of compound interest requires two things: the re-investment of earnings and time.

Our previous calculations confirm the importance of compound interest. In 10 payments of $90 we accumulated a future value of $1,005.18. By making an additional 8.57 payments, we doubled our future value. We found a similar effect when we increased loan payments and observed significant decreases in term. When finding the effect of increased loan payments on loan terms, we calculated the elasticity of the term of the loan with respect to the size of the loan payment. Similarly, we can find the impact of an increase in the term of periodic savings on future values (FV). We refer to this elasticity as the FV elasticity with respect to term. The derivation of the FV elasticity with respect to term is found in Derivation 19.1 at the end of this chapter. The discrete approximation to the FV elasticity with respect to *n* is:

$$E_{FV,n} = \frac{dFV}{dn}\frac{n}{FV} \approx \frac{n}{US_0(r,n)} \tag{19.2}$$

Table 19.4 provides elasticity estimates for alternative reinvestment rates and terms.

Table 19.4. Tabled Values of elasticity $E_{FV,n} = n/US_0(r,n)$

n	*r* = .01	*r* = .03	*r* = .05	*r* = .08	*r* = .10	*r* = .15
1	1.00%	1.03%	1.05%	1.08%	1.10%	1.15%
3	1.02%	1.06%	1.10%	1.16%	1.21%	1.31%
5	1.03%	1.09%	1.15%	1.25%	1.32%	1.49%
10	1.05%	1.17%	1.30%	1.49%	1.63%	1.99%
20	1.08%	1.34%	1.60%	2.04%	2.35%	3.20%
30	1.16%	1.53%	1.95%	2.66%	3.18%	4.57%
60	1.34%	2.17%	3.17%	4.85%	6.02%	9.00%

A related question with a more intuitive answer is how long does it take to double, triple, or quadruple one's investment? The derivation for the equation that answers this question can be found in Derivation 19.2 at the end of this chapter.

Let *m* be the number of times one wants to multiply his or her FV obtained at period *n*. Let *nm* equal the period in which the last deposit is made. Then we can find the number of periods (*nm* – *n*) required to increase the original FV by *m* times. The formula found in Derivation 19.2 is provided below:

$$n_m - n = \frac{\ln\left[m + (1-m)e^{-rn}\right]}{r} \tag{19.3}$$

To illustrate, let *r* = .1, *n* = 5, and *m* = 2. The number of periods required to double the FV of one's original investment equals:

$$n_2 - n = \frac{\ln\left[2 - e^{-(.1)(5)}\right]}{.1} = \frac{\ln(1.39)}{.1} = 3.3 \tag{19.4}$$

In this example, an investment consisting of 5 equal payment compounded at 10% could be doubled in additional 3.3 payments. And the investment could be tripled in:

$$n_3 - n = \frac{\ln\left[3 - e^{-(.1)(5)}\right]}{.1} = \frac{\ln(1.79)}{.1} = 5.8 \tag{19.5}$$

The second example illustrates the power of compounding interest. Starting after 5 periods of investing, in 3.3 more periods the FV of the investment doubled and in 5.8 additional periods the FV of the original investment tripled.

Finally, we conclude this section by constructing Table 19.5 demonstrating the power of compounding interest. The cells of the table indicate the time required to double, triple, and quadruple an initial investment for reinvestment rates of 1%, 3%, 5%, and 10%.

Table 19.5. Periods required to double, triple, or quadruple the FV of an investment consisting of 5 equal payments assuming alternative reinvestment rates.

Alternative re-investment rates	Number of time periods required to increase the FV of the original investment *m* times		
	m = 2 (double the original FV)	*m* = 3 (triple the original FV)	*m* = 4 (quadruple the original FV)
1%	4.8	9.3	13.7
3%	4.4	8.2	11.6
5%	4.0	7.3	10.2
10%	3.3	5.8	7.8

The rule of 72 approximates the amount of time it takes to double your investment at a given rate of return. To apply the rule, you divide the rate of return by 72. For example, assume you invest $1000 at an interest rate of 5%. It would take 14.4 years (72/5) to double your investment to $2000.

Bonds

A bond is a financial asset frequently traded in financial markets. Bonds represent debt claims on the assets and income of the entity issuing the bond. A bond's value is equal to the present value of its future cash flow (interest and principal) discounted at an appropriate interest rate. Bonds usually have a known maturity date at which time the bond holder receives the bond's face or par value. Bonds are unique because their redemption or liquidation value is fixed. Typical redemption amounts are $1,000 or $10,000.

Consider the following example. Suppose a bond can be purchased at a price of V_0 (an initial cash outflow) and redeemed n periods later at a cash value of V_n, a cash inflow to the bond holder. Moreover, suppose that the bond generates no cash return except when it is sold. Further, assume that the before-tax discount rate is r percent. Ignoring any tax consequences, the NPV of this bond is the sum of the cash outflow plus the present value of the cash inflow.

$$NPV = -V_0 + \frac{V_n}{(1+r)^n} \tag{19.6}$$

Those who purchase bonds may want to calculate the "yield" on a bond, which is the discount rate that equates the present value of the bond's cash flow to its present market value—the bond's internal rate of return (IRR). If, for example, the bond's market value is $321.97 and its cash flow of $1,000 occurs at the end of year 10, then the bond's yield, or its IRR, is 12 percent.

$$V_0 = \frac{\$1,000}{(1.12)^{10}} = \$321.97 \tag{19.7}$$

Now consider the effect of taxes. First, assume capital gains taxes are paid by the bond purchaser at rate T_g and income taxes are paid at rate T. The after-tax NPV of the bond is calculated by adjusting the discount rate to its after-tax equivalent and by

subtracting from the liquidation value of the bond, the capital gains tax:

$$NPV = -V_0 + \frac{V_n - T_g(V_n - V_0)}{[1 + r(1 - \theta T)]^n} \tag{19.8}$$

where θ is the tax adjustment coefficient defined in Chapter 11. If the NPV in Equation ??? is zero then $r(1 - \theta T)$ is the bond's after-tax IRR. To find the effective tax rate we set NPV equal to zero in equations (19.8) and (19.6) so that before-tax cash flow discounted by the before-tax IRR in Equation ??? is equal to the after-tax cash flow discounted by the after-tax IRR in Equation ???. Then we solve for θ that makes the two equations equal. The result is:

(19.9) n}\right\} \end{equation*}

To illustrate using the numbers from our previous example, let the before-tax IRR equal 12%, let V_0 equal $321.97, let V_n equal $1,000, and let the income tax and the capital gains tax T and T_g both equal 15 percent. Next making the appropriate substitutions into Equation ??? we find θ equal to:

(19.10) 10}\right\} = .66. \end{equation*}

For $\theta = .66$ the effective tax rate is reduced from 0.15 to (.66)(0.15) = 0.10.

In the example, just completed, we were able to deduce a closed form solution for θ. In most cases, closed form solutions are very difficult to obtain. Nevertheless, in most practical cases involving numerical estimates, we can still find estimates for the effective tax rate and after-tax IRRs. We will demonstrate the empirical approach to finding effective tax rates in the next section.

Coupons and Bonds

Most bonds, in addition to capital gains (or losses), provide "coupon" (interest) payments. The number and amount of the coupon payments will alter the NPV as well as the price of the bond. Usually, the coupon rate r^c is a percentage of the redemption value of the bond.

The before-tax NPV of a bond with n coupon payments is:

$$\begin{aligned} NPV &= -V_0 + \frac{r^c V_n}{(1+r)} + \frac{r^c V_n}{(1+r)^2} + \cdots + \frac{r^c V_n + V_n}{(1+r)^n} \\ &= -V_0 + (r^c V_n) US_0(r, n-1) + \frac{r^c V_n + V_n}{(1+r)^n} \end{aligned} \tag{19.11}$$

To illustrate, suppose an investor wants to find the maximum bid price V_0^b for a three-year $1,000 bond if it offers coupon payments of $r^c V_n$ = (.05)(1,000) = $50, and r = 10%. Then using Equation ??? and setting NPV equal to zero, find the maximum bid price V_0^b equal to:

$$\begin{aligned} {V_0}^b &= (r^c V_n) US_0(r, n) + \frac{V_n}{(1+r)^n} = \$50 US_0(.10, 2) + \frac{\$1,050}{(1+.10)^3} \\ &= \$875.65 \end{aligned} \tag{19.12}$$

Taxes, of course, affect the NPV of bonds with coupon payments. Only now, taxes may or may not be paid on the coupon payments and capital gains. For example, coupon payments of many municipal bonds are not taxed but their capital gains are taxed. Tax exemptions, of course, raise the maximum bid price and NPVs of the bonds for all investors but especially for higher tax bracket investors. The after-tax present value of the bond with tax-exempt coupon payments is written as:

$$NPV = -V_0 + (r^c V_n) US_0[r(1-\theta T), n-1] + \frac{V_n - (V_n - V_0)T_g}{[1+r(1-\theta T)]^n} \tag{19.13}$$

To illustrate, suppose an investor wants to find the maximum bid price V_0^b for a three-year $1,000 bond if it offers coupon payments of $r^c V_n$ = (.05)(1,000) = $50 and r = 10% and pays capital gains and income taxes at the rate of 10%. Then using Equation ??? and setting NPV equal to zero, we find the maximum bid price V_0^b for the bond whose earlier maximum bid price was calculated equal to $875.65:

$$\begin{aligned} {V_0}^b &= (r^c V_n) US_0[r(1-\theta T), n] + \frac{V_n - (V_n - V_0)T_g}{[1+r(1-\theta T)]^n} \\ &= \$50 US_0[.10(1-\theta .20), 3] + \frac{\$1,050 - (\$1,000 - \$875.65)(.10)}{[1+.1(1-\theta .2)]^3} \end{aligned} \tag{19.14}$$

In the application of Equation ???, the discount rate was the IRR—because it was the discount rate that resulted in NPV equal to zero. We introduced taxes into the cash flow, but we don't know the effective after-tax discount rate that would set NPV equal to zero. We have to solve for θ that will adjust the discount rate for taxes in the same magnitude as were the cash flow adjusted for taxes. For Equation ???, we find the after-tax IRR using Excel:

Table 19.6. Finding the After-tax IRR
Open Table 19.6 in Microsoft Excel

B6		***Function:***	=IRR(B2:B5)
	A	**B**	C
1	Finding the After-tax IRR		
2	Bond's max bid price	-875.65	
3	first coupon payment	50	
4	second coupon payment	50	
5	Salvage value (1000) + coupon payment	1037.6	
6	IRR	9.59%	"=IRR(B2:B5)

The IRR calculation of 9.59% equals the after-tax IRR of 9.59% compared to the before tax IRR of 10%. We can find θ and the effective tax rate by setting

$$r(1-\theta T) = .1[1-\theta(.1)] = .0959. \tag{19.15}$$

From the above equation we find θ = .41. Thus the effective tax rate is (.2) (.41) = 8%—far less than the actual income tax rate of 20%.

To review the process, we began by solving for the maximum bid price of the bond. This required that the stated discount rate was the before-tax IRR for the bond. Then we reasoned as follows. If the maximum bid price is the same whether calculated on a before-tax or after-tax basis, then the effect of taxes on the cash flow—in this case the capital gains tax—must have the same effect on the discount rate. Thus, we required that the maximum bid price in the after-tax model be the same maximum bid determined in the before-tax model and solved for the after-tax IRR and the tax adjustment coefficient θ .

Common Stocks

In contrast to bonds, common stocks have neither a fixed return nor a fixed cost. The terminal value of bonds is usually fixed,

but the terminal value of stocks depends on the market value of the stock at the time of sale. The equity capital generated by the sale of stock is an alternative to debt capital generated by the sale of bonds. It also is a means of sharing risk among numerous investors.

Stocks offer significant benefits for stockholders as well as the companies issuing the stocks. Stockholders have the opportunity for ownership in the major businesses of the world with the consequent share in profits while their liability is limited to their investments. Moreover, stock ownership frees them from decision-making responsibilities in the management of the company, although common stock allows its owners to vote for directors and sometimes other matters of significance facing the company.

Stock owners receive dividend payments on their stocks, usually on a quarterly basis. The amount of dividends paid on stocks is determined by a corporation's board of directors. The board of directors' dividend policies may influence the kinds of stock they issue and the kinds of investors they attract.

The relevant question for a potential stock purchaser is what is the maximum bid price for a particular stock? If r is the nominal discount rate, and R_1, R_2, $\cdots$ are projected dividends paid on the stock in periods 1, 2, $\cdots$, the maximum bid price for the stock is:

$$V_0^b = \frac{R_1}{(1+r)} + \frac{R_2}{(1+r)^2} + \cdots \tag{19.16}$$

The model above assumes an infinite life. This assumption is consistent with the "life of the investment" principle discussed in Chapter 8 because to know the terminal value of the stock, V_n, we must know the value of the dividends in periods $(n + 1)$, $(n + 2)$, $\cdots$. Knowing all future dividends converts the problem to one in which the number of periods equals the life of the firm.

A simplified form of Equation ??? is possible if expected dividends R_1, R_2, $\cdots$ are replaced by their expected annuity equivalent R. Then we can write:

$$V_0^b = RUS_0(r, \infty) = \frac{R}{r} \tag{19.17}$$

And r, the stock's IRR, is R/V_0. Thus, with long-term constant dividends of \$100 and the stock valued at \$1000, the rate of return is 10%. Another important ratio derived from Equation ??? is what is commonly referred to as the price-to-earnings ratio, $V_0 / R = 1 / r$ which is often viewed as the bellwether of financial irregularities in the financial market. Higher than usual price-to-earnings ratios may signal what has been called "irrational exuberance" for the investment. Lower than usual price-to-earnings ratios may signal that the investment is undervalued.

Finally, debt capital must be repaid regardless of the financial fortunes of the business. However, the return to stockholders depends critically on the performance of the company. This makes NPV analysis of stock investments subject to considerable uncertainty.

Figure 19.1. S&P 500 PE ratio peaks at record highs.



So what have we learned? We learned to distinguish between real and financial investments. Earlier chapters in this book focused on real investments. Our focus in this chapter was on financial investments including time deposits, certificates of deposits, bonds, and stocks. The important point is that we can examine both real and financial investments using PV tools.

Summary and Conclusions

This chapter has introduced concepts related to financial investments which involve exchanges of money over time. In relation to financial investments, the focus is on the amount of one's investment available at some future period of time. We find the FV of a financial investment by compounding interest. As we demonstrated in Table 19.1, the power of compounding is truly amazing. Therefore, the advice for most investors is to invest early and continuously, and then let one's investment grow.

Time deposits represent an important financial investment opportunity. These differ mostly in their liquidity—those that are least liquid are also the ones with the higher yields. Bonds are an important class of financial investment. In contrast to most other financial investments, their liquidation value is fixed. What is not fixed is their purchase price which is established in the market and depends on their yield—their expected IRR.

One of the interesting aspects of bonds is the way they are taxed. Often when bonds are issued by municipalities and some other institutions, they receive special tax considerations. In the extreme neither their coupon payments nor their capital gains are taxed. In other circumstances, capital gains are taxed and coupon payments are not. Regardless, an important question is how do special tax provisions provided with some bonds affect their after-tax IRRs. While the effective after-tax IRR can sometimes be found in a formulaic expression, often they are too complicated to be expressed in a closed form solution but can be found numerically by calculating the before and after-tax IRRs.

Finally, we only introduced the concept of common stocks. Investments in common stocks are most often discussed in the context or risk analysis and include discussions of many alternative of risk response strategies. A more complete discussion of investments in stocks is beyond the scope of this financial management text.

Questions

1. To invest means to give up something in the present for something of value in the future. Can you describe three of your most important personal investments? Describe what you sacrificed including money and time, and describe your expected future returns.
2. Describe the difference between real and financial investments. Also describe how the investment settings for real and financial investments may differ.
3. Describe the main characteristics of alternative types of time deposits? What are the rates of return currently offered by major banks or credit unions on alternative types of time deposits?
4. Suppose a saver invests $65 in a time deposit at the beginning of each period for 10 periods. If the deposits are compounded at a rate of 4%, what amount is available for withdrawal at the beginning of period 11?
5. Suppose a young couple now renting an apartment wants to invest in their own home. The average price range of their desired home is approximately $250,000. If they make monthly deposits in a time deposit account at the beginning of the

period for 5 years and their deposits earn an APR compound rate of 3%, what would be the required size of their monthly deposit to pay for the 20% required down payment of the price of their home ($50,000)? If they desired to save the required down payment in three years, what would be the required size of their monthly deposits?

6. It is claimed that Albert Einstein called compound interest "the greatest mathematical discovery of all time." What is it about compounding that is so important that he would make such a claim? Do you agree? If so, why? If you disagree, what would be an alternative mathematical discovery of greater importance?
7. Describe in words the meaning of the elasticity of the future value of an investment with respect to its term. Then calculate the elasticity of the future value of an investment compounded at an APR rate of 4% for 12 periods. How would the elasticity measure change if the compound rate were increased to 7%? Can you explain the direction of the change?
8. Find the number of periods required to double the future value of an investment compounded for 5 periods at alternative reinvestment rates of 2%, 4%, and 8%.
9. An investor desires to save $10,000. If the compound rate is 3% per month and the investor plans on saving $200 at the beginning of each month, how many months will be required for the investor to reach her savings goal of $10,000? Once the saving goal is reached, how many addition months will be required for the saver to double the amount saved if she saves at the same rate?
10. Consider an 8-year bond with a par value of $10,000. If the purchase price of the bond is $4,250, what is the yield on the bond? (Ignore the influence of taxes.)
11. What is the relationship between the bond's yield (its IRR) and the bond's purchase price? What would you expect to happen to the bond's purchase price if the expected yield on the bond increased? Please explain.
12. In the example illustrating Equation ???, the effective tax rate for a bond purchaser was found to equal θT = .66(.15) = 10%. Please recalculate the effective tax rate in the example assuming the capital gains tax rate is only 50% of the income tax rate. In other words, if the capital gains tax rate is 7.5%, what is the bond purchaser's effective tax rate? How does lowering the effective tax rate change the firm's effective after-tax IRR?
13. Consider the equation:

$$V_0 = (r^c V_n)(1-T)US_0[r(1-\theta T), n] + \frac{V_n - (V_n - V_0)T_g}{[1+r(1-\theta T)]^n} \tag{Q19.1}$$

where $r(1-\theta T)$ is the after-tax IRR of a bond.

Next consider the equation:

$$V_0 = (r^c V_n)US_0(r, n) + \frac{V_n}{(1+r)^n} \tag{Q19.2}$$

where *r* is the before-tax IRR of the bond.

Then assume that V_0 = $500, V_n = $1000, $r^c V_n$ = .04($1,000) = $40, T = 20%, and T_g = 10%. Find the before-tax and after-tax IRRs using the two equations described in this question and the values listed. Then use the before-tax and after-tax IRRs to find the value of the tax-adjustment coefficient θ.

14. Compare the rule of 72 with the actual time required to double your investment.

Derivation 19.1.

The derivation of the elasticity of future value (FV) with respect to term:

The elasticity of FV with respect to n is defined as:

$$E_{FV,n} = \frac{dFV}{dn}\frac{n}{FV} \tag{19.i}$$

The future value of *n* payments compounded at rate *r* is equal to:

$$FV = \int_0^n Re^{rt}dt = \frac{R}{r}(e^{rn} - 1) \tag{19.ii}$$

And the derivative of FV with respect to *n* is:

$$\frac{dFV}{dn} = Re^{rn} \tag{19.iii}$$

Substituting into the elasticity formula we find:

$$E_{FV,n} = \frac{dFV}{dn}\frac{n}{FV} = \frac{nRe^{rn}}{\frac{R}{r}(e^{rn} - 1)} = \frac{rn}{1 - e^{-rn}} \tag{19.iv}$$

The discrete approximation of the elasticity of FV with respect to *n* is:

$$E_{FV,n} = \frac{dFV}{dn}\frac{n}{FV} = \frac{nRe^{rn}}{\frac{R}{r}(e^{rn} - 1)} = \frac{rn}{1 - e^{-rn}} \approx \frac{n}{US_0(r, n)} \tag{19.v}$$

Derivation 19.2.

Deriving the formula for calculating the number of periods to double the FV of an *n* period investment reinvested at an interest rate of *r*%.

To begin, the future value (FV) of an *n* period investment *R* compounded at rate *r* is equal to:

$$FV = \int_0^n Re^{rt}dt = \frac{R}{r}(e^{rn} - 1) \tag{19.vi}$$

Now *m* times the FV of the original investment is set equal to the same investment compounded for *nm* periods:

$$m\int_0^n Re^{rt}dt = \frac{mR}{r}(e^{rn} - 1) = \int_0^{n_m} Re^{rt}dt = \frac{R}{r}(e^{rn_m} - 1) \tag{19.vii}$$

and simplifying by equating the two integrated equations above we find:

$$m(1 - e^{-rn}) = (e^{rn_m - rn} - e^{-rn}) \tag{19.viii}$$

Then solving for the difference between (*nm*– *n*), we find the time it takes to increase the FV of the original investment at period *n* by *m* times.

$$n_m - n = \frac{\ln[m + (1 - m)e^{-rn}]}{r} \tag{19.ix}$$

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20: Yield Curves

Learning Objectives

After completing this chapter, you should be able to: (1) understand how interest and earning rates on a variety of debt and investments instruments are related to yield curves and periodic interest rates; (2) describe how yield curves and periodic interest rates defined over future time periods can be used to predict market participants' expectation of future opportunities and threats; and (3) use yield curves and periodic interest rates to better understand and predict future financial opportunities and threats.

To achieve your learning goals, you should complete the following objectives.

- Learn how to calculate single period discount rates from investments of varying terms.
- Understand the relationship between yield curves and single period discount rates.
- Learn how yield curves can be used to predict future financial opportunities and threats.

Introduction

In most present value (PV) models, the discount rate is a constant even when the term of the model has changed. In reality, each period's discount rate may be different because factors that influence the discount rate are not constant. These factors include the level of economic growth in the economy, the inflation rate, national and international events likely to influence our economy, activities in the stock and bond market, housing and land markets, the unemployment rate, and election outcomes.

One way to observe the difference in period discount rates is to observe the interest rates on loans with different terms. They are not constant. To demonstrate, note the changes in variable interest rate loans. They aren't constant either. Or, relevant to this discussion, notice the difference in average yields on bonds with different maturities even in the same risk class. Alternatively, notice the change in the internal rate of return (IRR) of a bond as its time to maturity changes over times.

In what follows, we pursue two objectives. The first objective is to demonstrate how to calculate the periodic discount rate for no-coupon bonds. The approach we describe can also be used to find periodic discount rates implied by bonds that are more complicated as well as other financial instruments. Our second objective is to connect the shapes and patterns of periodic interest rate curves and their corresponding yield curves patterns to predict future economic activity and opportunities and threats. Yield curves, as we will explain in more detail later, are the geometric means of periodic interest rates at a point in time with various maturities.

Geometric Means and Periodic Discount Rates

Geometric means and long-term discount rates. Suppose we have information that allows us to predict n future periodic discount rates $r_1, r_2, \cdots, r_n$. The geometric mean for the n periodic rates satisfies the following equation:

$$(1+r)^n = (1+r_1)(1+r_2)\cdots(1+r_n) \tag{20.1}$$

In words, one plus the geometric mean multiplied by itself n times would equal the product of 1 plus n periodic discount rates. We can solve for the geometric mean of the n periodic discount rates in Equation ??? by finding the n*th* root of the products of the n periodic discount rates and subtracting one:

(20.2) n)} -1 \end{equation*}

To demonstrate, suppose that periodic discount rates for period one through three were r_1 = .07, r_2 = .03, and r_3 = .075. We can solve for the geometric mean of the three periodic interest rates as:

(20.3) 3)} -1 = .058\, or\, 5.8\% \end{equation*}

How to calculate periodic discount rates. Consider a zero coupon bond that can be purchased for V_0^1 at the beginning of the period and redeemed at the end of the period for its par value of $1,000. The periodic rate of return r_1 for the one-period bond satisfies the PV model:

$$V_0^1 = \frac{\$1,000}{(1+r_1)} \tag{20.4}$$

Solving for the one-period discount rate r_1 we find:

$$r_1 = \frac{\$1,000}{V_0^1} - 1 \tag{20.5}$$

For example if $V_0^1 = 960$ then

$$r_1 = \frac{\$1,000}{\$960} - 1 = .042 \, or \, 4.2\% \tag{20.6}$$

In the case of a one-period bond, the one-period discount rate is also the geometric mean.

Now consider a zero-coupon bond that matures in two periods with similar risk and tax provisions as the one-period bond described in Equation ???. Assume the zero-coupon two-period bond can be purchased for V_0^2 at the beginning of the period and redeemed at the end of the period for its par value of $1,000. The periodic rate of return r_2 for the two-period bond satisfies the PV model:

$$V_0^2 = \frac{\$1,000}{(1+r_1)(1+r_2)} \tag{20.7}$$

Solving for the second period discount rate r_2 we find:

$$r_2 = \frac{\$1,000}{V_0^2(1+r_1)} - 1 \tag{20.8}$$

For example if $V_0^2 = 910$ then

$$r_2 = \frac{\$1,000}{\$910(1.042)} - 1 = 0.55 \, or \, 5.5\% \tag{20.9}$$

When we purchase a two period bond, we acquire a financial instrument with a single yield for two periods. The yield is the geometric mean of the product of the single period discount rates. In our example, the yield or geometric mean is equal to:

(20.10) 2} -1 = .048 \,or\, 4.8\% \end{equation*}

Continuing our example, if $V_0^3 = 855$ then

$$r_3 = \frac{\$1,000}{\$855(1.042)(1.055)} - 1 = .064 \, or \, 6.4\% \tag{20.11}$$

Finally, consider a zero-coupon n period bond that can be purchased for V_0^n at the beginning of the period and redeemed at the end of the period for its par value of $1,000. The periodic rate of return r_n for the n period bond satisfies the PV model equal to:

$$V_0^n = \frac{\$1,000}{(1+r_1)(1+r_2)\cdots(1+r_n)} \tag{20.12}$$

Solving for the one-period discount rate r_n we find:

$$r_n = \frac{\$1,000}{V_0(1+r_1)(1+r_2)\cdots(1+r_{n-1})} - 1 \tag{20.13}$$

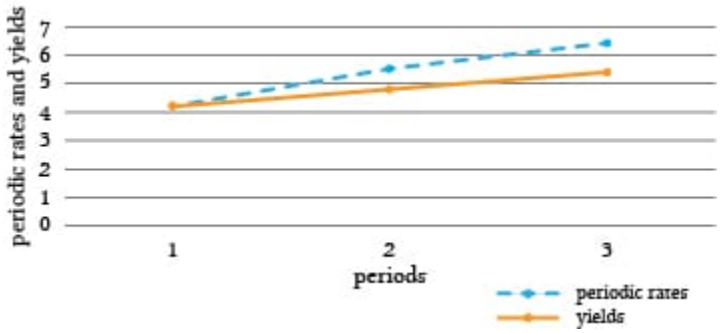
Periodic rates of return and geometric means. When we purchase a three-period bond, we acquire a single yield for three periods. Consistent with the PV model, the yield is the geometric mean of the product of the single-period discount rates. In our example, the yield or geometric mean for the three-period bond is equal to:

(20.14) 3} -1 = .054 \, or \, 5.4\% \end{equation*}

While one could expect to earn the bond's yield to maturity if the investor held the bond to maturity, a three-year bond becomes a two-year bond after one year, and if sold, the bond's price would reflect the yield on a two-year bond.

Graphing the geometric means of bonds against their varying time to maturity produces the bond's yield curve. In Figure 20.1, we graph the periodic discount rates and the corresponding yield curves for one, two, and three period bonds.

Figure 20.1. A comparison of periodic interest rates and the corresponding yield curves of bonds of varying maturities

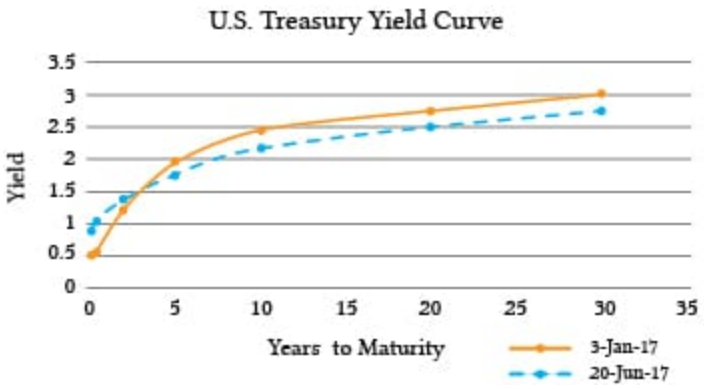


Predicting Future Economic Activities and Opportunities and Threats

Marginal periodic rates and average geometric mean yield curves. The periodic rate is a marginal rate while the yield rate is an average or mean—in this case a geometric mean. In our example, the periodic rate is increasing. As a result, the yield curve is also increasing but at a slower rate because previous and lower values of the periodic rate influence the yield. Therefore, if the yield curve is increasing, it is because the marginal periodic rates added to the series are greater than the geometric mean of the previous values included in the calculation. Moreover, if the yield curve were decreasing, it would require that periodic rates added were less than the geometric mean. To be complete, if the yield curve were constant, it would suggest that the marginal periodic rate were equal to the geometric mean.

Of course, there are many bond yield curves depending on the type of bond considered. When producing a yield curve, it is essential that bonds used to produce the yield curve belong to a similar risk class—even though this may be difficult because term differences produce different risks. To do the best we can to hold risk constant when producing yield curves, government backed debt is often considered. The most frequently reported yield curve compares three-month, two-year, five-year and thirty-year U.S. Treasury debt. We present graphs of U.S. Treasury yield curves at two points in time in Figure 20.2 below

Figure 20.2. Yield curves using U.S. Treasury debt calculated on January 3, 2017 and June 20, 2017



Interpreting the shapes of yield curves. In the previous section, we described the relationship between the period or marginal interest rates and the yield curve that, according to our PV model, is the geometric mean of the periodic rates. We now suggest some interpretations of the yield curves. Figures of periodic rate calculated from yield curves are not generally available.

- *Positively sloped yield curves.* To invest or lend one's financial capital to another person or entity requires a sacrifice on the part of the lender. Furthermore, the longer the funds are committed to another person or entity, the greater is the sacrifice. As a result, many economists claim that upward sloping yield curves predict a healthy economy in the future where borrowers have a bright view of future earnings and are willing to pay increased interest rates for the privilege of investing in the future. Another explanation is that borrowers expect periodic interest rates in the future to rise that in turn will produce an increasing yield curve. One reason for periodic rates to rise is expected increases in inflation and subsequent response to inflation by the Federal Reserve to increase interest rates on government debt instruments to offset inflationary pressures.
- *Flat or humped yield curves.* A flat yield curve is consistent with constant periodic interest rates so that all bond maturities have similar yields. A humped yield curve implies that periodic interest rates for a period lie above then fall below the yield curve and are constant before and after the hump. Economists generally view constant or humped yield curves as uncertain indicators of the future well-being of the economy.
- *Negatively sloped or inverted yield curves.* During periods when financial market participants expect periodic rates of return to decrease, yield curves have been downward sloping or inverted. Some financial economist connect inverted yield curves with pending downturns in the economy or recessions. In support of this connection, an inverted yield curve has indicated a worsening economic situation in the future 7 times since 1970 (Adrian, 2010). See Table 20.1.

Summary and Conclusions

Previous chapters have treated the multi-period discount rate in PV models (including IRR) as constants. This chapter has emphasized that these constant discount rates are composed of time varying periodic discount rates. Many time-varying factors would prevent periodic discount rates from being constant. Such forces acting on periodic interest rates include monetary and fiscal policies, inflation rates, unemployment rates, and national and international trade and treaties to name a few.

So what have we learned? We learned that rate of return expectations built into periodic discount rates and reflected in yield curves of bonds (and interest rates) of varying maturities reflect expected future economic conditions.

While there is not universal agreement on how to interpret yield curves, and indeed different yield curves may be subject to varying interpretations, there is support for interpreting downward sloping or inverted yield curves as foreshadowing a slow-down in future economic activities.

The goal of this chapter has been to acquaint students with another resource for predicting future opportunities and threats.

Table 20.1. The connection between inverted yield curves and future recessions (1970-2009).

Event	Date of Inversion Start	Date of the Recession Start	Time from Inversion to Recession Start	Duration of Inversion	Duration of Recession
			Months	Months	Months

1970 Recession	Dec-68	Jan-70	13	15	11
1974 Recession	Jun-73	Dec-73	6	18	16
1980 Recession	Nov-78	Feb-80	15	18	6
1981–1982 Recession	Oct-80	Aug-81	10	12	16
1990 Recession	Jun-89	Aug-90	14	7	8
2001 Recession	Jul-00	1-Apr	9	7	8

Questions

1. Compare geometric means and periodic discount rates.
2. Find the geometric mean of three periodic discount rates equal to $r_1 = 8\%, r_2 = 6.5\%, and\, r_3 = 2\%$.
3. Suppose the geometric mean of two periodic discount rates was 6%, and the periodic discount rate in the second period was 7%. Find the periodic discount rate in the first period.
4. Suppose a one-year zero-coupon bond with a par value of $1,000 was selling for $962. Find the bond's yield.
5. Suppose the yield curve was decreasing. Some economics would view this as a sign the economy is slowing down. Do you agree or disagree? Defend your answer.
6. Find data on the yield curve in the economy and discuss what it portends.

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21: One Thing More...

Learning Objectives

After completing this chapter, you should be able to: (1) understand that, as humans, our needs include socio-emotional goods as well as those goods that can be purchased with money; and (2) that our needs motivate us to allocate resources to achieve both economic as well as socio-emotional goals.

To achieve your learning goals, you should complete the following objectives:

- Learn the difference between “econs” and “humans”.
- Be reminded of the five distinct motives for pursuing a goal.
- Learn why humans exchange on different terms and levels than econs.
- Learn why humans exchange goods on terms and levels that econs view as irrational.
- Learn the difference between commodities and relational goods.
- Learn how including relational goods in our models explains exchanges that many econs would call irrational.

Introduction

The famous Beatles singing group titled one of their hit songs, “Can’t Buy Me Love.” While coming from an unlikely source of philosophical insight, their song title did proclaim an important truth: money can’t buy love nor most any other goods whose value depends on relationships. So, the purpose of this chapter is to remind us that relationships and the intangible goods produced in relationships may be our most important resources that need careful managing.

Econs versus humans. The behavioral economist Richard Thaler (2008) described two types of decision makers: “econs” and “humans.” Econs, according to Thaler, make decisions like persons described in graduate school economic textbooks. They are perfectly selfish, possess perfect information about the outcome of their choices, and their will power is absolute. Humans, on the other hand, fail to satisfy any of those assumptions. Their choices are influenced by relationships with other humans that lead them to unselfish choices. They lack complete information about the outcomes of their choices and about alternative choices. Finally, they sometimes overeat, drink too much, and also often don’t follow through on their choices.

Motives. A homework question at the end of Chapter 1 asked you what are your motives for wanting a college degree? The list of possible motives are reported below.

- I want a college degree so I can increase my lifetime earnings and get a better job.
- I want a college degree so important people in my life will be pleased with my achievements.
- I want a college degree to live up to the expectations I have for myself.
- I want a college degree so I will feel part of groups to which I want to belong.
- I want a college degree so that, in the future, I will be better able to help others.

An econ would have said that 100% of his or her motive was to increase lifetime earnings. Assuming your responses were not those of an econ, then you undoubtedly make decisions like a human. Your selection of motives can be used to infer that you care about and are motivated by other considerations than your desire to obtain a college degree to increase your potential income. The evidence from past surveys supports the conclusion that we are all human.

Anomalies

One outcome of managers behaving like humans instead of econs is that we observe behavior that is inconsistent with the prediction of profit and net present value (NPV) maximizing behavior associated with econs. One such example occurs regularly in the sale and purchase of farmland. Some of the anomalies we observe in the farmland market include terms and level of trade and selection of trading partners that depend on relationships, unselfish exchanges where sellers sacrifice higher prices for lower ones, and prefer to sell to friends and family rather than strangers and unfriendly neighbors.

To illustrate anomalies in the U.S. farmland markets, consider the minimum sell price of land. Compared to the arm's length market price, fifteen hundred farmland owner-operators in Illinois, Michigan, and Nebraska were surveyed. They reported discounting farmland prices to friends and family members by 5.57% and 6.78% respectively. These same owner-operators reported that they would require a minimum sell price premium of 18.4% to sell their land to their unfriendly neighbors (Siles, et. al., 2000).

Another farmland market study found that strangers entering the Linn County, Oregon, farmland market were at a decided disadvantage because they were forced to rely on public advertisements and Realtors to gain access to farmland sales information. Friendly neighbors and family of sellers accessed farmland purchase opportunities directly from the sellers. One consequence of this differential information access was that a stranger buying an 80-acre parcel of Class II non-irrigated farmland though a Realtor was projected to pay $2,535 per acre while a neighbor of the seller buying the same land was projected to pay 20% less (Perry and Robison, 2001).

Finally, as a result of premiums and discounts and preferential access to farmland markets that depend on relationships, farmland sellers reported that less than 2% of their sales were to unfriendly neighbors while up to 70% of land sales were to friendly neighbors and family. Others reported similar observations in which relationships altered the terms and level of farmland trades. Indeed, terms and level of trade and selection of trading partners in farmland exchanges that are not influenced by relationships have a special name, "arm's length sales."

How do we explain the regular anomalies we observe in farmland markets? One explanation follows. Econs trade and consume only commodities. Humans trade and consume both commodities and relational goods. When we ignore relational goods in human exchanges and consumption, the results appear as anomalies. Consider next the nature of relational goods.

Commodities and Relational Goods

Expanding the decision maker's choice set. For the most part, economic theory focuses on physical goods and services that decision makers obtain for themselves and whose values do not depend on their connection to a particular person(s). We call these goods commodities. Describing this focus on commodities, Becchetti, Pelloni and Rossetti (2008) wrote: "in mainstream economics agents are mostly considered in isolation as they impersonally interact through markets, and consumption goods and leisure are assumed to be 'sufficient statistics' of their utility."

Nothing in economic theory, however, prevents us from expanding the set of properties used to describe goods in decision makers' choice sets. For example, we could add the goods' relational properties to the description of those goods. Relational properties of goods include the identity of persons who produce, exchange, consume, and store goods in the choice set. Furthermore, this added description of goods could be justified if it were shown that decision makers' preference ordering depended on the relational properties of goods.

Social scientists in the past have connected a good's relational properties to its preference orderings (Bruni and Stanca, 2008). Indeed, Adam Smith may have foreshadowed the concept of relational goods when he described fellow-feeling or sympathy as essential for human happiness.

Emphasizing that the identity of exchange partners matters when defining relational goods, Uhlaner (1989) wrote, "goods which arise in exchanges where anyone could anonymously supply one or both sides of the [exchange] are not relational". Luigino and Stanca (2008) concluded in their review of relational goods that "genuineness" is foundational, and the identity of the other person is essential for the value, and in some cases even for the existence, of the relational good. Gui and Sugden (2005) defined relational goods as "the affective components of interpersonal relations [that] are usually perceived as having value through their sincerity or genuineness".

Defining relational goods. Relational goods are those goods whose value depends at least in part on their connections to people who produce, exchange, consume, and store them. Three concepts describe relational goods. The flow of relational goods is called socio-emotional goods (SEG). The stock or inventory of relational goods is called attachment value goods (AVG). SEG embedded in persons are said to create investments in social capital (Robison and Flora, 2003). Finally, social capital is required to produce SEG. We describe these, SEG, AVG, and social capital, in more detail in what follows.

SEG are intangible goods that satisfy socio-emotional needs. While there is no universally accepted list of socio-emotional needs that relational goods are expected to satisfy, generally accepted needs include the need for internal validation or self-actualization, the need for external validation, the need for connectedness (belonging, love and friendship), and the need for knowing (Robison, Schmid, and Siles, 2002). SEG differ from other intangible goods and services because they are produced by

social capital—sympathy (empathy), regard, or trust that one person has for another person or group.

SEG like other intangible goods can become attached to, associated with, or embedded in durable goods and change the meaning and value of the durable goods they act on. Durable goods embedded with SEG are called attachment value goods (AVG) and represent a stock of SEG. Since SEG and AVG "spring out of interpersonal relationships, and comprise the often intangible, interpersonal side of economic interactions", they qualify as relational goods (Robison and Ritchie, 2010).

In mainstream economics, the production of commodities employs manufactured capital (tools and implements), natural capital, human capital, and financial capital. All of these contribute to the creation of a good or service valued for its mostly observable physical properties. In contrast, relational goods are produced in sympathetic (empathetic), trusting, and high regard relationships referred to here and by others as social capital. While there are other definitions of social capital, many of these do not satisfy the requirement of being capital or social. Instead they focus on where social capital lives (networks), what it can produce (cooperation), the rules that organize its use (institutions) and how to produce it (Robison, Schmid and Siles, 2002).

Human needs are satisfied by commodities and relational goods. The distinguishing properties of commodities and relational goods are described next.

Distinguishing properties of commodities. The properties that describe commodities have little or no connection to people or relationships among people and are described next. (1) Commodities are exchanged in impersonalized markets. (2) The terms and level at which commodities are exchanged are determined by the aggregate of market participants and apply generally. (3) Commodities are standardized goods of uniform quality which makes them perfect substitutes for each other so that little or no connection exists between their value and those who produce, exchange, consume, and store them. (4) The value of commodities can be inferred from their (mostly) observable properties. (5) Manufactured, natural, human, and financial capital may all play an important role in the production of commodities. (6) The value of commodities can be altered by changing their form, function, location, or other physical properties. (7) Commodities satisfy mostly physical needs and wants. (8) Commodities are mostly nondurable goods not likely to become embedded with SEG because of their short useful lives. (9) Commodities are most likely to have their quantity and quality certified by arm's length agencies established for that purpose.

Distinguishing properties of relational goods. The properties that describe relational goods are wholly or partially dependent on the good's connection to people who produce, exchange, consume, and store them and are described next. (1) Relational goods are exchanged in personalized settings in which either the buyer or the seller or both are known to each other. (2) The terms and level of relational goods exchanged are influenced by the social capital inherent in the relationships of those producing, exchanging, consuming, and storing them. (3) Relational goods are poor substitutes for each other because they are produced in unique relational settings. (4) The value of relational goods depend on their mostly unobservable intangible properties. (5) While other forms of capital may be used in their production, relational goods cannot be produced without social capital. (6) The value of relational goods can be altered by changing their connections to people who produce, exchange, consume, or store them. (7) Relational goods satisfy mostly socio-emotional needs and wants. (8) Relational goods are mostly durable goods likely to become embedded with SEG because of their extended useful lives. (9) Relational goods are not likely to have their quantity or quality value certified by arm's length agencies established for that purpose.

We summarize the differences between commodities and relational goods in Table 21.1.

Table 21.1. Properties of commodities and relational goods.

Property	Commodities	Relational goods
1. Exchange setting.	Impersonal setting in which buyer and seller are not known to each other.	Personalized setting in which buyer and seller are connected through a social relationship.
2. How terms and level of exchange are determined.	Terms and level of goods exchanged are determined by the aggregate influence of market participants.	Terms and level of goods exchanged are uniquely determined by the social capital inherent in the persons engaged in the exchange.
3. Substitutability of goods.	Standardized goods with uniform quality which allows one commodity to substitute for another.	Unique good with few substitutes because its value is uniquely determined by the social capital involved in its exchange.
4. Value determining properties.	Mostly observable, physical properties.	Mostly unobservable intangible SEG exchanged directly or embedded in an AVG.

Property	Commodities	Relational goods
5. Capital used in their production.	Manufactured, natural, human, and financial capital may all be important in the production of commodities.	While other forms of capital may be used, social capital is required in the production of relational goods.
6. How the value of the good is changed.	Value is altered by changing the physical properties of the good including its form, function, location, taste, color, and other physical properties.	Value is altered by humanistic acts that produce SEG that may lead to increased investments in social capital or that may become embedded objects creating AVG.
7. Needs satisfied.	Mostly physical	Mostly socio-emotional including the need for validation, belonging, and knowing.
8. Durability.	Mostly nondurable or used infrequently.	Mostly durable or if nondurable, used frequently.
9. Certification.	Arm's length agencies empowered with regulatory and inspection duties.	Within the relationships associated with the good's production, consumption, or exchange.

Two setting for exchanging relational goods

There are two types of relational good exchanges. In the first type of exchange, the focus is on the relationships, and goods exchanged are mostly SEG. In the second type of exchange, the focus is on the good exchanged that is almost always an AVG, a tangible thing embedded with SEG.

Exchanges focused on relationships. In relationship focused exchanges, SEG are exchanged directly between persons in social capital rich relationships and require no object besides the persons involved in the exchange to complete the transaction. For example, two persons with strong feelings of affection for each other may express those feelings, SEG, in any number of settings including meals, cultural events, conferences, religious gatherings, or work settings. And if there is an object exchanged, it is incidental to the exchange of SEG.

Exchanges focused on objects. In object focused exchanges, AVG are exchanged, things or objects embedded with SEG. AVG result from prior or anticipated connections between social capital rich persons in which SEG are produced and embedded in objects. AVG are most likely a durable. However, AVG may sometimes be non durables that are often exchanged repeatedly such as a meal prepared to celebrate special occasion or a song or dance performed to mark milestones.

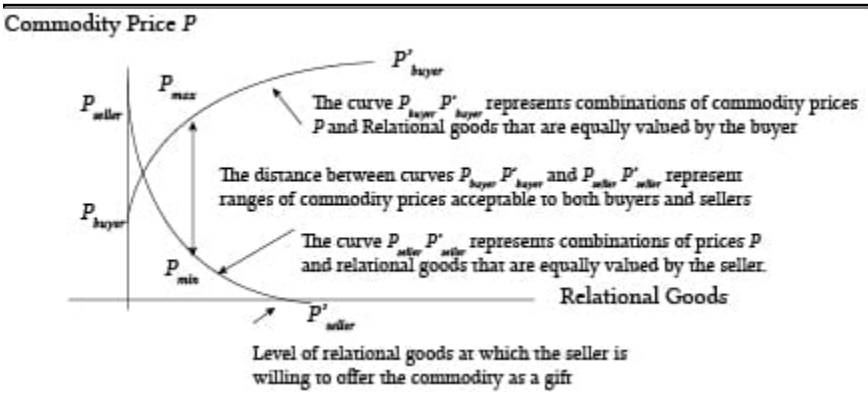
Anomalies and Isoutilities

At the beginning of this chapter, we claimed that econs and humans made decisions differently. Sometimes these differences are called anomalies. Furthermore, because many human decisions differ from econ choices, human choices are sometimes called irrational. The claim here is that human choices can appear irrational because relational goods are excluded from the analysis.

To illustrate how including relational goods in exchanges can resolve important economic anomalies, consider the following example. Suppose a seller has the option of exchanging his farmland with a stranger for a commodity (the market price) or exchanging his farmland with a friend or family member for a combination of commodities and relational goods. If the seller prefers the combination of relational goods and commodities offered by friends and family members to the commodities only offered by a stranger even though the commodities offered by the stranger exceed those offered by friends and family members, we might consider that an economic exchange anomaly has occurred. The seller accepted a lower commodity price from a friend or family member when a higher commodity price was available from a stranger. This is only an anomaly if the relational goods included in the exchange are ignored.

To explain further how including relational goods in exchanges can resolve anomalies, we consider the concept of an isoutility line. Suppose that a decision maker is offered alternative combinations of two goods, a commodity and a relational good. Furthermore, allow that the amounts of the commodity and relational good can be exchanged at some rate that leaves the well-being of the decision maker unaffected. The combinations of relational goods and commodities that leave the decision maker's well-being unaltered are referred to isoutility combinations and are represented in Figure 21.1 as PbuyerPbuyer. Curve PsellerPseller represents the seller's isoutility combinations of relational goods and commodity prices.

Figure 21.1. Combinations of Commodity Prices and Relational Goods that Leave Buyers' and Sellers' Well-Being Unchanged.



The implication of the graph in Figure 21.1 is that as more of a relational good is received, the seller (buyer) would be willing to accept (offer) a lower (higher) commodity price without suffering a loss in well-being. Furthermore, as relational goods are included in the transaction, the range of commodity prices acceptable to both buyers and sellers increases which also increases the likelihood that persons rich in social capital will exchange. For example, in Figure 21.1 persons without social capital would not trade since, with no relational goods exchanged, the minimum sell price is above the maximum bid price.

In one of the first studies designed to test the influence of relationships on terms and level of exchange, Robison and Schmid (1989) asked faculty and graduate students what would be their minimum sell price of a used car to people who offered them various levels of social capital. Since the Robison and Schmid article was published, the essence of the study has been repeated multiple times with similar results. A recent unpublished survey by Richard Winder found the results reported in Table 21.2. The mode of the distributions of responses by relationship are in bold.

Table 21.2. Average minimum sell price for a used car with a market value of $3,000 reported by 600 survey respondents.

	Nasty neighbor	Stranger	Friend	Family
> $3,500	65			
$3,500	263	39	2	
$3,250	21	33	5	1
$3,000	236	476	122	29
$2,750	11	30	135	24
$2,500	4	22	298	199
< $2,500			38	348

Notice that in the absence of social capital (exchanges with a stranger), the distribution of minimum sell prices centers around the commodity exchange price of $3,000. However, when the exchange is conducted with a social capital rich partner such as a friend or family member, the minimum sell price is significantly below the market price with a mode of $2,500 for a friend and a mode price below $2,500 for a family member.

Summary and Conclusions

The material covered in this book is intended to provide instructions about how to behave like an econ. We hope you don't always follow our advice. There are times and places and circumstances when commodity considerations should be softened by social capital and the importance of relational goods. Learning how to make the appropriate trade-offs between commodities and investments in social capital and relational goods, when to behave like an econ versus a human, may be our most important management task. Good luck.

Questions

By 2013 Bill and Melinda Gates had donated 38 billion dollars to various charities and especially to fight hunger in Africa. They have donated billions more to these causes since. If Bill and Melinda Gates were to have taken the same survey you took at the beginning of the class where "I want a college degree" is replaced with "We donated billions of dollars…", how do you think they would have answered? To better speak for them, read a brief interview of Bill Gates by Neil Tweedie at: http://www.telegraph.co.uk/technolog...Gods-work.html. Now answer the survey that follows as though you were channeling either Bill or Melinda Gates by writing the relative importance of each motive using percentages in the blank besides each motive. The sum of the motives must equal 100%.

- We have donated billions of dollars so that we could increase our lifetime earnings and get a better job.
- We have donated billions of dollars so that important people in our lives would be pleased with our achievements.
- We have donated billions of dollars to live up to the expectations we have for ourselves.
- We have donated billions of dollars so we will feel part of different groups to which we want to belong.
- We have donated billions of dollars to help others.

Consider a hypothetical gas purchase.

You gas tank holds 15 gallons and is nearly empty. You normally fill up your car with gas at a station on your way home. How many additional miles would you drive to fill up your car with gas if you could save 10 cents per gallon? I would drive an additional ____ miles to save 10 cents per gallon.

You gas tank holds 15 gallons and is nearly empty. You normally fill up your car with gas at a station on your way home. How many additional miles would you drive to fill up your car with gas at a gas station owned and operated by your favorite uncle? I would drive an additional ____ miles to purchase gas at a gas station owned and operated by my favorite uncle.

If your answer to parts a) and b) were different from zero, can you explain why?

Summarize the difference between relational goods and commodities as discussed in this chapter. Please list two commodities and two relational goods that you own.

An isoutility line describes different combinations of two different goods that provide equal satisfaction. Use the concept of isoutility to explain why you might sell your used car at different prices to a friend, a stranger, a family, member, and someone you disrespect.

Commodities are sold in the market place, and their prices are determined by anonymous market forces. The terms of trade of relational goods depend on the relationship between persons exchanging them. Give an example in which you have exchanged relational goods in which relationships altered the terms and level of trade. Then give another example in which you have exchanged commodities in which relationships had no influence on the terms and level of trade.

One could donate one's blood or blood plasma at a local Red Cross and receive in return a small amount of juice served in a paper cup and possibly a cookie. One could also sell one's blood at a number of places (the current price is $25 to $60 per bag). Since some people sell their blood for money and others donate it for free, explain the difference in the way these two groups of people dispose of their blood.

Suppose you needed a medical procedure that required a skilled physician. Assume that a number of equally skilled physicians were available to perform the procedure. Would your choice of a physician to perform the procedure depend on your relationship to the physician? If he were a family friend? If he were a stranger? If you knew the physician only by reputation—that he performed volunteer work in developing countries? If he was rude to his/her patients, inconsiderate to his/her assistants, and lived a lavish life style? For each of the physicians, answer the questions that follow using the scale included in each question:

If the physician were skilled and a family friend, the likelihood I would select this physician to perform my procedure is:

Not likely 1 3 5 7 Very likely (circle one)

If the physician were skilled and a stranger, the likelihood I would select this physician to perform my procedure is:

Not likely 1 3 5 7 Very likely (circle one)

If the physician were skilled and someone I knew only by reputation—that he performed volunteer work in developing countries, the likelihood I would select this physician to perform my procedure is:

Not likely 1 3 5 7 Very likely (circle one)

If the physician were skilled and someone I knew only by reputation—that he was rude to his/her patients, impatient with his assistants, and lived a lavish lifestyle, the likelihood I would select this physician to perform my procedure is:

Not likely 1 3 5 7 Very likely (circle one)

If you were equally likely to select one of the physicians described in question 7, explain why. If you were not equally likely to select one of the physicians described above, explain why.

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